COMMONWEALTH OF AUSTRALIA

AUSTRALIAN BROADCASTING CONTROL BOARD
TECHNICAL SERVICES DIVISION

REPORT No. 10

TITLE: CALCULATION OF THE EFFECT OF AN EARTH SYSTEM ON THE UNATTENUATED FIELD STRENGTH AT ONE MILE.

Issued by :—
The Chairman,
Australian Broadcasting Control Board,
Rialto Building,
497 Collins Street,
Melbourne, C.1.

C.4170/50

CALCULATION OF THE EFFECT OF AN EARTH SYSTEM ON THE UNATTENUATED FIELD STRENGTH AT ONE MILE.

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Date of Issue: 14th October, 1953.

CALCULATION OF THE EFFECT OF AN EARTH SYSTEM ON THE UNATTENUATED FIELD STRENGTH AT ONE MILE

Abstract:

This report shows a method of calculating the unattenuated field strength at 1 mile for a medium frequency vertical aerial, in terms of aerial height, soil conductivity, frequency, and details of the earth system using data given in a paper by F.R. Abbot. Four examples are given covering considerable ranges of frequency and soil conductivity. It is concluded that with a good earth system the soil conductivity has little effect on the unattenuated field strength at 1 mile.

1. Introduction:

The field strength at any appreciable distance from a grounded medium frequency vertical aerial is dependent on a number of factors, particularly the power input to the aerial, the nature of aerial and earth system, the soil conductivity of the medium, and the frequency.

It has been customary for the calculation of field strength to employ families of curves of field strength against distance with parameters of frequency and soil conductivity, for a fixed soil dielectric constant.[1] These curves are customarily drawn for an unattenuated field strength of 100 mV/m. at one mile, and their use requires a knowledge of the unattenuated field strength at one mile for a given power, aerial, and earth system.

In order to determine the unattenuated field strength at one mile in any prediction of field strengths, the experimental data of G.H. Brown[2] may be employed. This data gives unattenuated field strength at 1 mile for a range of aerial and earth systems for a fixed frequency and conductivity, it being assumed without complete proof that the unattenuated field strength at 1 mile is substantially independent of soil conductivity and frequency, provided that the aerial and ground system bear a fixed relationship to the wavelength.

A recent article by F.R. Abbot[3] dealing with the design of ground systems enables a calculation to be made of the unattenuated field strength at a mile in terms of the height of the vertical aerial, configuration of the radial earth system, frequency and ground conductivity. Calculations have been made, and four examples worked out to check a few points in the data provided by G.H. Brown, and in particular some indication is obtained of the effect of soil conductivity on the losses in the earth system and on the unattenuated field strength at a mile.

2. Method:

A study of the fundamentals of aerials commencing with the simple dipole will make it clear that in the case of a grounded vertical aerial, the ground in the vicinity of the aerial must be considered as part of that aerial.

The question then arises as to what radius may be taken as part of the aerial. Losses within this radius may be considered as effecting aerial efficiency, while losses outside this radius may be considered as propagation losses. It has been suggested by Williams [4] that the boundary be set at the limit of the induction field for which $r = \frac{\lambda}{2}$. This appears to be a logical choice as the total radial earth current becomes constant for all values of aerial height h at $\frac{r}{\lambda} > \frac{1}{2}$, indicating that a variation in h will not cause a variation in the watts loss outside the radius $r = \frac{\lambda}{2}$ for constant input power.

To obtain the aerial efficiency the watts loss is calculated within the radius $r = \frac{\lambda}{2}$. Let this be p and the aerial input power be P.

$$\text{Aerial efficiency} = \frac{P - p}{P} \times 100\% \qquad (1)$$

$$\text{Unattenuated field strength for P kW input at one mile} = Eo\sqrt{P-p} \qquad (2)$$

Where Eo is the no loss field strength at one mile for 1 kw input.

Eo varies because of the change in the vertical polar diagram with change in aerial height and is given in figure 1.

3. Calculation of Loss:

It is assumed that the earth is flot and homogeneous except for radial earth wires and that the aerial has a sinusoidal distribution of current. To enable an expression for the total radial earth current to be obtained in a comparatively simple form it is further assumed that the earth is a perfect conductor.

We then have from the paper quoted by F.R. Abbot.[3]

$$J = Io\,\frac{\left(e^{-j2\pi\frac{\sigma}{\lambda}} - e^{-j2\pi\frac{r}{\lambda}}\cos 2\pi\frac{h}{\lambda}\right)}{\sin 2\pi\frac{h}{\lambda}} \qquad (3)$$

Where J = total radial current at any radius r given in figure 2 amps.

Io = total current at the base of the aerial amps.

σ = distance from the top of the aerial metres.

r = distance from the base of the aerial metres.

h = height of the aerial metres.

λ = wave length metres.

j = $\sqrt{-1}$

It is then assumed that with earth of finite conductivity the earth currents remain the same. This assumption has been used by Brown [2] and others successfully.

The power dissipated per square metre of ground surface is given by $\left(\frac{J}{2\pi r}\right)^2 \frac{1}{Y}$ (real part).

Where Y = admittance of a square metre of soil.

Hence for an earth system we have

$$Y = Y_s \div Y_g \text{ for } d \ll \lambda \quad \text{(Ref. 4)}$$

Where Y_s = admittance of the soil per square metre.

Y_g = admittance of the wires per square metre.

d = spacing between wires in metres.

$$Y_s = \sqrt{\frac{\epsilon - j\frac{\sigma}{\omega}}{\mu}} \quad \text{mhos (Ref. 4)} \quad \ldots\ldots \quad (4)$$

$$Y_g = \frac{-j}{f\mu d \log_e \frac{d}{2\pi a}} \quad \text{mhos (Ref. 4)} \quad \ldots\ldots \quad (5)$$

Where ϵ = the dielectric constant of the soil M.K.S.

σ = the conductivity of the soil mho/metre.

μ = the permeability of the soil M.K.S.

a = the diameter of the wire metres.

f = the frequency cycles/sec.

In cases where $\epsilon \ll \frac{\sigma}{\omega}$

$$Y_s \simeq (1-j)\sqrt{\frac{\sigma}{4\pi f\mu}} \quad \ldots\ldots \quad (6)$$

and $$Y \simeq (1-j)\sqrt{\frac{\sigma}{4\pi f\mu}} - \frac{j}{f\mu d \log_e \frac{d}{2\pi a}} \quad \ldots\ldots \quad (7)$$

w = watts loss per square metre = real part of $\left(\frac{J}{2\pi r}\right)^2 \frac{1}{Y}$

$$= \frac{\left(\frac{J}{2\pi r}\right)^2 \sqrt{\frac{\sigma}{4\pi f\mu}}}{\left[\frac{\sigma}{4\pi f\mu} + \left(\sqrt{\frac{\sigma}{4\pi f\mu}} + \frac{1}{f\mu d \log_e \frac{d}{2\pi a}}\right)^2\right]} \quad \text{watts/ metre} \quad \ldots\ldots \quad (8)$$

In cases where the dielectric constant of the soil cannot be neglected we have -

$$Y_s = U + jV$$

$$W = \frac{\left(\frac{J}{2\pi r}\right)^2 U}{U^2 + \left(V - \frac{1}{f\mu d \log_e \frac{d}{2\pi a}}\right)^2} \qquad \ldots\ldots (9)$$

Outside the earth system and neglecting the effect of the dielectric constant of the soil

$$w = \frac{\left(\frac{J}{2\pi r}\right)^2}{1-j} \sqrt{\frac{4\pi f\mu}{\sigma}} \qquad \text{real part}$$

$$= \left(\frac{J}{2\pi r}\right)^2 \sqrt{\frac{\pi f\mu}{\sigma}} \qquad \text{watts/square metre} \quad \ldots (10)$$

$$\Delta W = 2\pi r \Delta r\, w$$

$$W = \sum_{r=r_1}^{r=r_2} 2\pi r\, w\, \Delta r \qquad \text{watts.}$$

Neglecting the dielectric constant of the soil we have within the earth mat -

$$W_1 = \sum_{r=r_1}^{r=r_2} \frac{J^2 \sqrt{\frac{\sigma}{4\pi f\mu}}\ \Delta r}{2\pi r\left[\frac{\sigma}{4\pi f\mu} + \left(\sqrt{\frac{\sigma}{4\pi f\mu}} + \frac{1}{f\mu d \log_e \frac{d}{2\pi a}}\right)^2\right]} \qquad \ldots (11)$$

Outside the earth mat -

$$W_2 = \sum_{r=r_2}^{r=r_3} \frac{J^2 \sqrt{\frac{\pi f\mu}{\sigma}}}{2\pi r}\ \Delta r \qquad \ldots\ldots (12)$$

Because of the assumption that the earth currents remain the same for soil of finite conductivity these integrals cannot be taken to the limit $r_3 \longrightarrow \infty$ as then $W \longrightarrow \infty$ which is impossible.

4. Determination of the attenuated field strength at any given distance:

The method of using the FCC curves must be modified as the earth losses within $\frac{\lambda}{2}$ of the aerial have been taken as aerial losses and not propagation losses. The actual field has been taken as the unattenuated field at $r = \frac{\lambda}{2}$ so that any conductivity curve σ must be moved up to coincide with the inverse distance curve at $r = \frac{\lambda}{4}$.

Alternatively the field at any distance given by the F.C.C. curves may be multiplied by the ratio of the inverse and attenuated fields read off at $r = \frac{\lambda}{2}$.

5. Comments on Method.

A method has been presented which enables the field strength from a vertical aerial to be calculated taking the aerial earth system into account.

It has been assumed that the current distribution along the aerial is sinusoidal. This may not be precisely the case particularly with aerials for which $h > \frac{\lambda}{2}$. In these cases the effect is to make the actual aerial efficiency about 5% less than that calculated.

In the examples given it is shown that the dissipation within the earth system area is greater for high conductivity soil than for low conductivity soil. However this is of no real importance as the dissipation within the earth system area is shown to be small.

6. Examples:

The unattenuated field strength at one mile has been calculated in four cases summarised in the following table and in three cases a comparison is possible with the figure given in G.H. Brown's article.

Table.

Example	1	2	3	4
Aerial height (fraction of wavelength)	0.125	0.125	0.25	0.5
Earth system radius (fraction of wavelength)	0.274	0.274	0.274	0.02
Number of radials	113	113	113	113
Frequency kc/s.	1600	1600	600	600
Ground conductivity ($x\ 10^{-14}$) e.m.u.	3	40	10	10
Unattenuated field strength at 1 mile calculated.	172	183	191	229
From G.H. Brown	172	172	180	-
Case of earth system without losses	187	187	195	237

The calculated unattenuated field strengths at one mile are in comparatively close agreement with the figures obtained from G.H. Brown's paper, particularly in the case of poor conductivity. It is evident that where a good earth system is used, the ground conductivity has a small effect on the unattenuated field strength at one mile. In example 1 the worst medium wave case which can be contemplated (1600 kc/s. and poor conductivity) the unattenuated field strength at a mile is only 8% less than the no loss case.

An inference from example 4 is that a poor earth system does not seriously effect the unattenuated field strength at 1 mile when a half wave aerial is used. However this does not mean that a good earth system is unnecessary, since it no doubt ensures a good approximation to the no loss vertical radiation characteristic at high angles where radiation is low.

Example 1.

Find the attenuated field strength at 1 mile for a $\frac{\lambda}{8}$ aerial with an earth system consisting of 113 radials No. 8 copper wire extending out to 0.274λ.

Frequency 1600 K.C. Earth conductivity 3×10^{-14} e.m.u.

Power into aerial 1 K.W.

$$\sigma = 3 \times 10^{-3} \text{ mho/metre.}$$

$$\epsilon = 1.33 \times 10^{-10} \text{ M.K.S.}$$

$$\mu = 4\pi \times 10^{-7} \text{ M.K.S.}$$

$$\lambda = 187 \text{ metres}$$

$$.274\lambda = 51 \text{ metres}$$

$$0.5\,\lambda = 94 \text{ metres}$$

In this case the dielectric constant of the soil cannot be neglected.

$$Y_s = \left(\frac{1.33 \times 10^{-10}}{4\pi \times 10^{-7}} - j\,\frac{3 \times 10^{-3}}{2\pi \times 1.6 \times 10^{6} \times 4\pi \times 10^{-7}}\right)^{\frac{1}{2}}$$

$$= 10^{-2}\,(1.06 - 2.37j)^{\frac{1}{2}}$$

$$= 10^{-2}\,(1.35 - 0.87j)$$

$$\frac{1}{fud \log_e \frac{d}{2\pi a}} = \frac{1}{1.6 \times 10^{6} \times 4\pi \times 10^{-7} \times d \times 2.303 \log_{10} \frac{d}{2\pi \times .01}}$$

$$= \frac{0.216}{d \log_{10} 15.9d}$$

$$\Delta w = \frac{2\pi r \Delta r \left(\frac{J}{2\pi r}\right)^2 1.35 \times 10^{-2}}{1.82 \times 10^{-4} + \left(-0.87 \times 10^{-2} - \frac{0.216}{d \log_{10} 15.9d}\right)^2}$$

metres	$\frac{r}{\lambda}$	Δr	d = 0.0556	15.9d	$\log_{10}$ 15.9d	J	J^2	$\frac{.216}{d\log_{10} 15.9d}$	Δ w watts
10	.0535	10	.556	8.85	0.947	9.3	86	.41	1.1
20	.107	10	1.11	17.7	1.248	7.5	56	.156	2.2
30	.160	10	1.67	26.6	1.425	6.6	43.5	.091	3.1
40	.214	10	2.22	33.7	1.528	6.1	37	.064	3.2
48	.257	6	2.67	40.9	1.612	5.8	33.5	.05	2.5

$$W_1 = \sum_{r=5}^{r=51} \Delta w_1 = 12.1 \text{ watts}$$

$$\Delta W_2 = 2\pi r \Delta r w$$

$$w = \left(\frac{J}{2\pi r}\right)^2 \frac{1}{Y} \quad \text{real part}$$

$$= \left(\frac{J}{2\pi r}\right)^2 \frac{10^2}{1.35 - 0.87j} \quad \text{real part}$$

$$= \left(\frac{J}{2\pi r}\right)^2 \frac{10^2 \times 1.35}{1.82 + 0.75}$$

$$W_2 = \sum_{r=51}^{r=94} 2\pi r \Delta r \left(\frac{J}{2\pi r}\right)^2 \frac{1.35 \times 10^2}{2.57}$$

$$= \sum_{r=51}^{r=94} 8.36\, J^2 \frac{\Delta r}{r}$$

As J does not vary greatly over this range of r assume it constant at 5.4 amps.

$$W_2 = 559 \log_{10} r \Big]_{51}^{94}$$

$$= 559\ (1.973 - 1.708)$$

$$= 148 \text{ watts}$$

$$W_1 + W_2 = 160 \text{ watts}$$

Unattenuated field strength at 1 mile = 188 $\sqrt{1 - .16}$ = 172 mV/m.

From the F.C.C. curves the curve $\sigma = 3 \times 10^{-14}$ e.m.u. is 0.87 of the unattenuated value at $r = \frac{\lambda}{4}$ and 0.57 of the unattenuated value at 1 mile.

Attenuated field strength at 1 mile = 172 x $\frac{.57}{.87}$ = 112 mV/m.

Example 2.

Find the attenuated field strength at 1 mile for a $\frac{\lambda}{4}$ aerial with an earth system consisting of 113 radials No. 8 # copper wire extending out to 0.274λ.

Frequency 1600 K.C.

Earth conductivity 40×10^{-14} e.m.u.

Power to aerial 1 K.W.

$\sigma = 4 \times 10^{-2}$ mho/metre.

$\epsilon = 1.33 \times 10^{-10}$ M.K.S.

$\mu = 4\pi \times 10^{-7}$ M.K.S.

$\lambda = 187$ metres

$.274\lambda = 51$ metres

$.5\lambda = 94$ metres

The dielectric constant of the soil can be neglected.

$$\sqrt{\frac{\sigma}{4\pi f \mu}} = \left(\frac{4 \times 10^{-2}}{4\pi \times 1.6 \times 10^{6} \times 4\pi \times 10^{-7}}\right)^{\frac{1}{2}} = 0.04$$

$$W_1 = \sum_{r=5}^{r=51} J^2 \frac{.04\,\Delta r}{2\pi r\left[16 \times 10^{-4} + \left(4 \times 10^{-2} + \frac{.216}{d \log_{10} 15.9d}\right)^2\right]}$$

$$= \sum_{r=5}^{r=51} \frac{J^2\, 0.636 \times 10^{-2}}{16 \times 10^{-4} + \left(4 \times 10^{-2} + \frac{.216}{d \log_{10} 15.9d}\right)^2} \frac{\Delta r}{r}$$

r	Δr	J^2	$\frac{.216}{d \log_{10} 15.9d}$	ΔW_1
10	10	86	.41	2.7
20	10	56	.156	4.7
30	10	43.5	.091	4.9
40	10	37	.064	4.7
48	6	33.5	.05	2.7

$$W_1 = \sum_{r=5}^{r=51} \Delta W_1 = 19.7 \text{ watts}$$

$$\sqrt{\frac{\pi f \mu}{\sigma}} = \sqrt{\frac{\pi\, 1.6 \times 10^{6} \times 4\pi \times 10^{-7}}{4 \times 10^{-2}}} = 4\pi$$

$$W_2 = \int_{r=51}^{r=94} \frac{J^2 4\pi \Delta r}{2\pi r}$$

As J does not vary greatly over this range of r assume it constant at 5.4 amps.

$$W_2 = 5.4^2 \times 2 \int_{r=51}^{r=94} \frac{\Delta r}{r}$$

$$= 133 \log_{10} r \Big]_{51}^{94}$$

$$= 133\ (1.973 - 1.708)$$

$$= 35 \text{ watts}$$

$$W_1 + W_2 = 55 \text{ watts}$$

Unattenuated field strength at 1 mile = 188 $\sqrt{1 - .945}$ = 183 mV/m.

Attenuated field strength at 1 mile = 183 x .96 = 175 mV/m.

Example 3.

Find the unattenuated field strength at 1 mile for a $\frac{\lambda}{4}$ aerial with an earth system consisting of 113 radials No. 8# copper wire extending out to .274λ.

Aerial input power 1 K.W.

Frequency 600 K.C.

Earth conductivity 10×10^{-14} e.m.u.

Dielectric constant of earth 15.

Expressing the constants in M.K.S. units we have

$$\sigma = 10 \times 10^{-14} \times 10^{11} = 10^{-2} \text{ mho/metre.}$$

$$\epsilon = 15 \times \frac{10^{11} \times c^{-2}}{4\pi} = 15 \times 8.85 \times 10^{-12} = 1.33 \times 10^{-10}$$

$$\mu = 4\pi \times 10^{-7} \text{ M.K.S.}$$

$$a = .01 \text{ metres}$$

$$\lambda = 500 \text{ metres}$$

$$.274\lambda = 137 \text{ metres}$$

$$0.5\lambda = 250 \text{ metres}$$

$$\frac{\sigma}{\omega} = \frac{10^{-2}}{2\pi \cdot .6 \times 10^6} = 26.5 \times 10^{-10}$$

The dielectric constant of the soil can therefore be neglected.

$$\sqrt{\frac{\sigma}{4\pi f \mu}} = \left(\frac{10^{-2}}{4\pi \ .6 \times 10^6 \times 4\pi \times 10^{-7}} \right)^{\frac{1}{2}} = 3.24 \times 10^{-2}$$

$$\frac{1}{f \mu d \log_e \frac{d}{2\pi a}} = \frac{1}{.6 \times 10^6 \times 4\pi \times 10^{-7} \ d \times 2.303 \log_{10} \frac{d}{2\pi \times .01}}$$

$$= \frac{0.576}{d \log_{10} 15.9d}$$

$$w_1 = \sum_{r=10}^{r=137} 2\pi r \Delta r \left(\frac{J}{2\pi r}\right)^2 \frac{3.24 \times 10^{-2}}{10.5 \times 10^{-4} + \left(3.24 \times 10^{-2} + \frac{.576}{d \log_{10} 15.9d}\right)^2}$$

$$J = \text{constant} = 5.25 \text{ amps.}$$

$$w_1 = \sum_{r=10}^{r=137} \frac{5.25^2}{2\pi} \times \frac{3.24 \times 10^{-2}}{10.5 \times 10^{-4} + \left(3.24 + 10^{-2} + \frac{.576}{d \log_{10} 15.9d}\right)^2} \frac{\Delta r}{r}$$

$$= \sum_{r=10}^{r=137} \frac{14.2 \times 10^{-2}}{10.5 \times 10^{-4} + \left(3.24 \times 10^{-2} + \frac{.576}{d \log_{10} 15.9d}\right)^2} \frac{\Delta r}{r}$$

r	Δr	d=.0556	15.dd	$\log_{10} 15.9d$	$\frac{.576}{d \log_{10} 15.9d}$	Δw
30	20	1.67	26.6	1.425	24.2×10^{-2}	1.2
50	20	2.78	44.2	1.645	12.6×10^{-2}	2.2
70	20	3.9	62	1.792	8.25×10^{-2}	2.8
90	20	5.1	81	1.908	5.93×10^{-2}	3.4
110	20	6.11	97.2	1.989	4.74×10^{-2}	3.5
128.5	17	7.1	113	2.053	3.97×10^{-2}	3.0

$$w_1 = 16.1 \text{ watts}$$

$$\sqrt{\frac{\pi f \mu}{\sigma}} = \left(\frac{\pi .6 \times 10^{6} \times 4\pi \times 10^{-7}}{10^{-2}}\right)^{\frac{1}{2}} = 4.9\pi$$

$$W_2 = \sum_{r=137}^{r=250} \frac{J^2}{2\pi r} 4.9\pi \Delta r$$

$$= \sum_{r=137}^{r=250} 67.5 \frac{\Delta r}{r} = 155 \log_{10} r \Big]_{137}^{250}$$

$$= 155\ (2.3118 - 2.1367)$$

$$= 27 \text{ watts}$$

$$W_1 + W_2 = 43 \text{ watts}$$

Unattenuated field strength at 1 mile = $195 \sqrt{1-.043}$ = 191 mV/m.

Example 4.

Find the unattenuated field at 1 mile for a $\frac{\lambda}{4}$ aerial with an earth mat extending out to 10 metres with 113 radials.

Frequency 600 K.C.

Soil conductivity 10×10^{-14} e.m.u.

Power input to aerial 1 K.W.

$\sigma = 10^{-2}$ mho/metre

$$W = \sum_{r=10}^{r=250} \frac{\sqrt{\frac{\pi \times .6 \times 10^{6} \times 4\pi \times 10^{-7}}{10^{-2}}}}{2\pi} \frac{\Delta r}{r}$$

$$= \sum_{r=10}^{r=250} J^2\, 2.45 \frac{\Delta r}{r}$$

r	$\frac{r}{\lambda}$	Δr	J	J^2	ΔW
20	.04	20	.8	.64	1.6
40	.08	20	1.4	1.96	2.4
60	.12	20	2.1	4.4	3.6
80	.16	20	2.6	6.75	4.1
100	.2	20	3.1	9.6	4.7
130	.26	40	3.7	13.7	10.3
170	.34	40	4.9	24	13.8
220	.44	60	5.2	27	18.7

W = 59.2 watts

Unattenuated field strength at 1 mile = $236 \sqrt{.941} = 229$ mV/m.

The number of radials in this case has little effect on the unattenuated field strength at 1 mile, since the loss within the extent of the earth system is in any case small.

References.

1. F.C.C. Standards of Good Engineering Practice.

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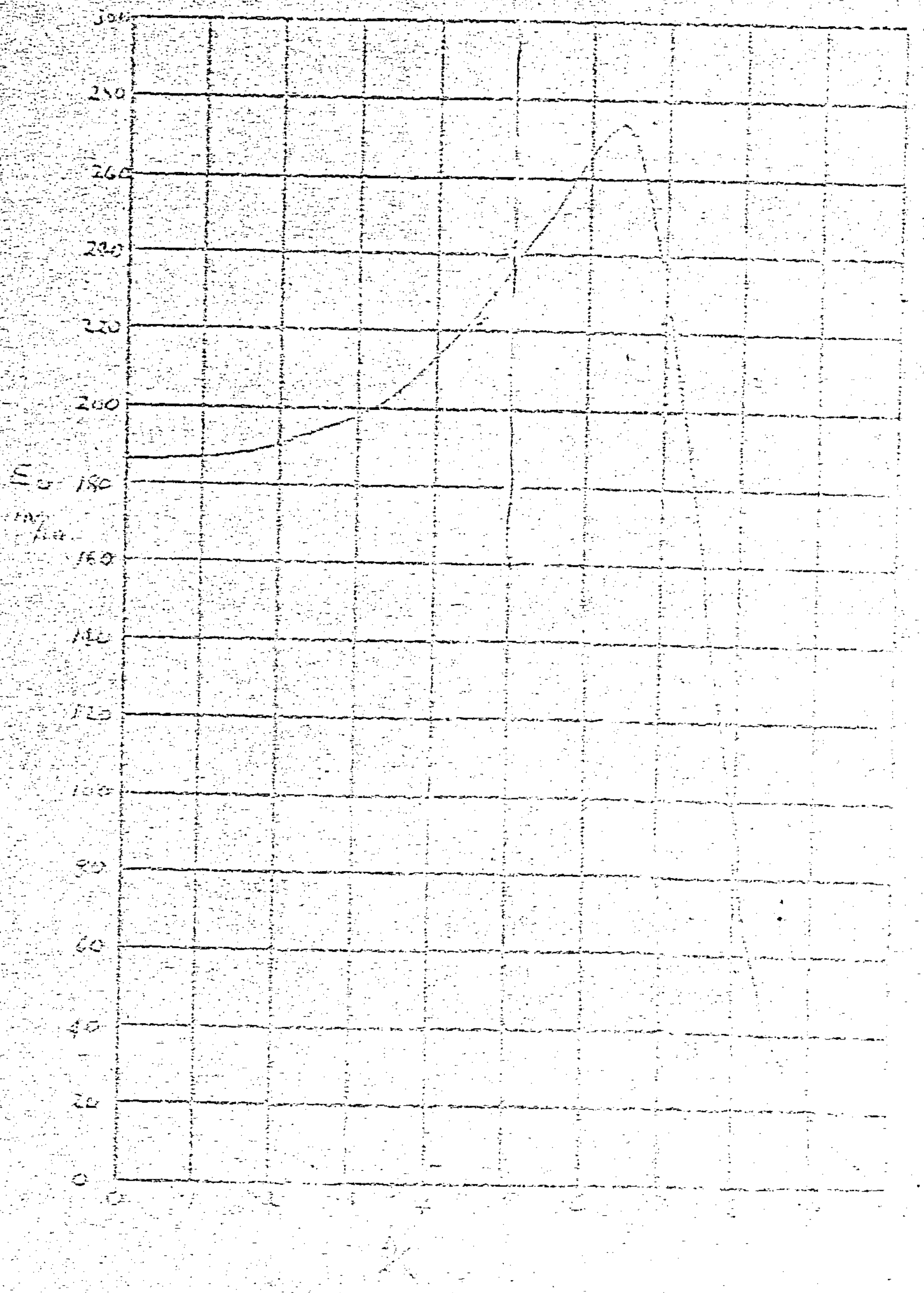
4. Antenna Theory and Design - H.P. Williams. (A book).



Figure

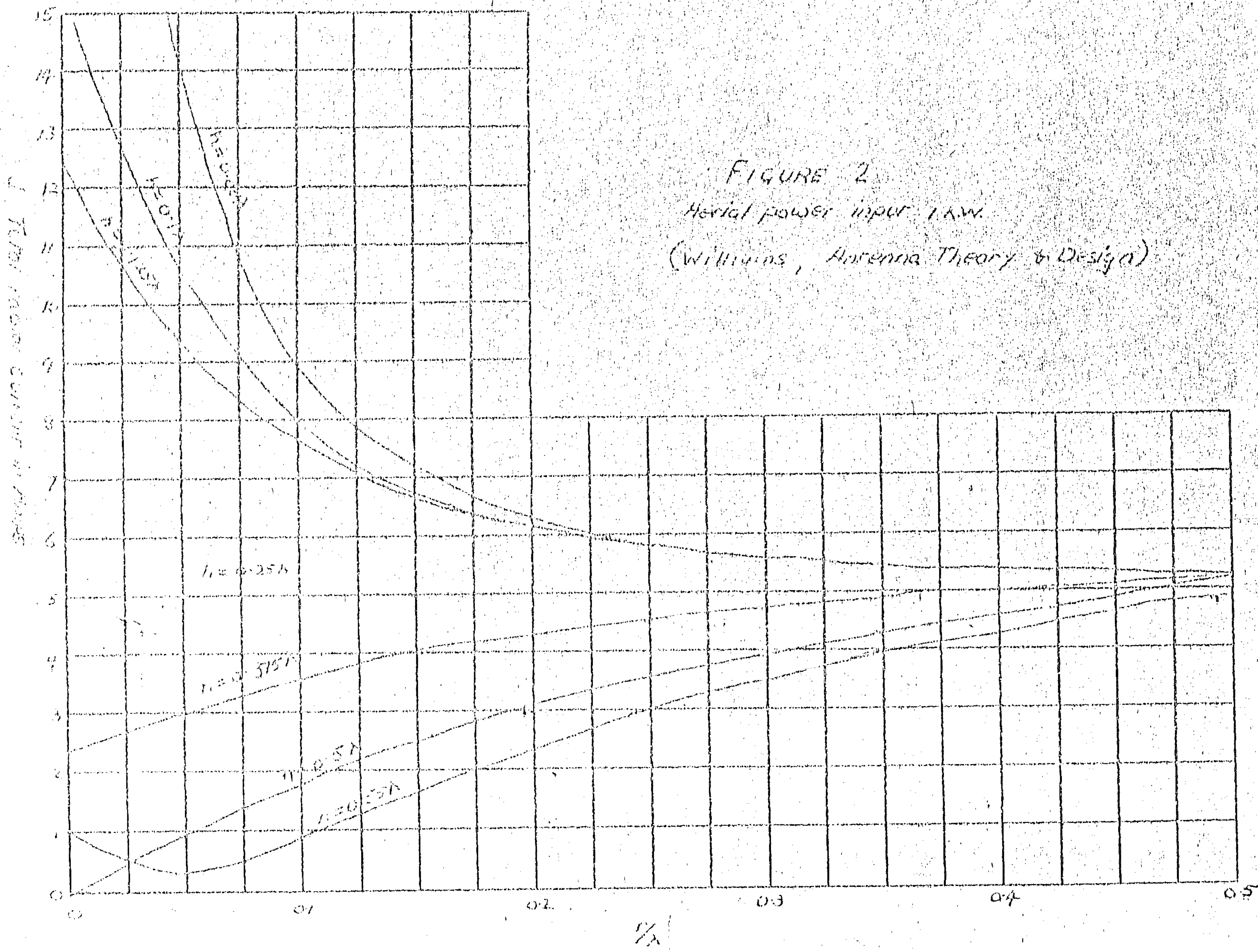


FIGURE 2
Aerial power input 1 kW
(Williams, Antenna Theory & Design)