REPORT 17
NOT FOR DISTRIBUTION

AUSTRALIAN BROADCASTING CONTROL BOARD

TECHNICAL SERVICES DIVISION

REPORT No. 17

TITLE: Tropospheric Propagation at 64.25 Mc/s,
182.25 Mc/s and 196.25 Mc/s.

Issued by — AUSTRALIAN BROADCASTING CONTROL BOARD
Marland House
562-574 Bourke Street, Melbourne Vic, 3000

~~The Chairman,~~
~~Australian Broadcasting Control Board,~~
~~Rialto Building,~~
~~497 Collins Street,~~
~~MELBOURNE C.1~~

Report No. 17

Title Tropospheric Propagation at 64.25 Mc/s, 182.25 Mc/s and 196.25 Mc/s.

Prepared by J. M. Dixon, November, 1958.

(D. McDonald)
Director
Technical Services Division

Date of issue

23/2/1959

Abstract

An analysis of v.h.f. field strength recordings for propagation beyond the horizon is presented. The study results from measurements made to establish the interference field strength between common channel television transmitters when the propagation path is over undulating country without mountain ranges.

The received field strength is shown to be greater in band I than in band III for points well beyond the horizon. There is a marked dependency of field strength and fading rate on meteorological conditions up to an altitude of about 6000 feet. When ropospheric layers ($\frac{dn}{dh}$ greater than normal) occur, there is a simultaneous decrease in the fading rate and an increase in field strength. The effect of layer thickness, layer height and rate of change of dielectric constant through the layer is shown.

Under standard atmospheric conditions the received field strength is found to agree well with that calculated for diffraction around the earth and reflection from the troposphere.

The average observed field strength exceeded for 10% of the time during winter at points well beyond the horizon is found to agree well with values extracted from F.C.C. and C.C.I.R. curves, but large discrepancies occur in the case of field strength values exceeded for 50% of the time.

Contents

Tropospheric Propagation at 64.25 Mc/s, 182.25 Mc/s and 196.25 Mc/s

1. Introduction

In expanding the television service throughout Australia, it has been proposed to limit the minimum geographical separation of common channel television transmitters to not closer than 170 miles for stations with an e.r.p. of 100 kw, though in most cases it is likely that the separation will be in excess of 200 miles.

Under such spacing conditions mutual interference has been observed and in recent years numerous theoretical studies and experimental observations have been made of beyond the horizon transmissions particularly by those interested in fixed point to point communication.

In this report an analysis is presented of television field strength recordings made at two locations beyond the geometrical horizon in southern Australia, to determine co-channel interference field strength characteristics.

2. The Recording Programme

Recordings were made at Camperdown, 123 miles from the transmitters at Mt. Dandenong from March 20th to May 20th, 1958 and at Warrnambool, (162 miles) from May 22nd to July 20th, 1958.

The aerial height at both sites was 30 feet above ground level with an almost unobscured view towards the transmitters over fairly flat country. The aerial employed was a commercial multi channel television receiving aerial of the modified Yagi type.

The picture carrier of each channel (ABV.2 64.25 Mc/s, HSV.7 182.25 Mc/s and GTV.9 196.25 Mc/s) was recorded for approximately one hour three times per week. Tables 1 and 2 show the signals exceeded for 10% and 50% of the time.

3. Signal Characteristics

In some respects it is misleading to state average values of field strength without giving the distribution which may be expected about the mean. The results obtained in these tests show a large variation from the mean with most of the observations lying below the mean (figure 2).

On three occasions strong steady signals were observed for periods of one or more hours at Camperdown. Very weak signals were frequently observed at Warrnambool but these rarely disappeared completely. There is some evidence to suggest that when high signal conditions prevail, the increase is more pronounced for channels 7 and 9 than for channel 2. The ratio of the 10% to 50% field strength has a mean value of 1.8 with a standard deviation of 0.3.

..2/.

In general a greater voltage is delivered to receivers by transmissions on channel 2 than on channels 7 and 9 due to the larger capture area of channel 2 aerials and the higher field strength at great distances.

Table 3 shows the mean field strength exceeded for 10% of the time compared with that given in C.C.I.R. Recommendation No. 111 London 1953, and values obtained from F.C.C. curves (Rules Governing Radio Broadcast Services) on the basis of the following deviations of effective radiated power from 100 kw in the direction of Camperdown and Warrnambool.

Channel 2	- 1 db
Channel 7	+ 2 db
Channel 9	- 2 db

Table 3

Location	Channel	Mean measured 10% field strength 100 kw e.r.p.	10% field strength FCC 100 kw e.r.p.	10% field strength CCIR 100 kw e.r.p.
Camperdown	2	43 db above 1µV/m	38 db above 1µV/m	37 db above 1µV/m
	7	42	36	33
	9	45	36	33
Warrnambool	2	31	27	30
	7	23	22	25
	9	22	22	25

The average figure for channel 7 measured at Camperdown is probably not characteristic value for even sampling as several channel 7 recordings were discarded because of unreliable calibrations. Since these discarded recordings were taken during periods of high field strength the figure of 42 db above 1 µV/m is probably too low.

Plots of one channel against another for results obtained on the same night show better correlations for the Warrnambool recordings (figure 4).

4. Consideration of Meteorological Data

Regular radiosonde data is available for Laverton which is along the propagation path near Melbourne. There is however a time difference of about twelve hours between the vhf recordings and the meteorological observations which were commenced at 0915 hrs. EST on the day following the vhf recordings. Since changes in the troposphere may be expected to occur in less than twelve hours and as the meteorological conditions may be expected to differ along the radio path, a considerable spread of results is likely in any test for correlation.

A preliminary study of the meteorological data showed that the signal strength increased when a sharp drop occurred in the mixing ratio with increase in height. At Laverton this comparatively rapid change in the amount of water vapour carried by the air occurs over a layer thickness range of 300 feet to 2,300 feet. These layers occur most frequently between 3,000 feet and 6,000 feet above sea level and are usually associated with subsidence.

In order to determine the reason for the improvement in vhf propagation which occurs when such layers appear, the electrical characteristics of the atmosphere must be expressed in terms of the meteorological measurements.

$$\tfrac{1}{2}(k - 1) \simeq n - 1 = \frac{79.5 \times 10^{-6}}{T} \left(P_1 + \frac{4800}{T} P_2\right) \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_(1)}$$

where k = dielectric constant
n = refractive index
T = absolute temperature
P_1 = partial pressure of air in millibars
P_2 = partial pressure of water vapor in millibars

The partial pressure of water vapour is given by

$$P_2 = 0.00161\, P\, S \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_(2)}$$

where P = the barometric pressure in millibars
S = the mixing ratio in gm/kg.

Substituting in (1) we have

$$n - 1 = 79.5 \times 10^{-6}\, \frac{P}{T} \left(1 + 7.73\, \frac{S}{T}\right) \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_(3)}$$

From appendix 1 we have

$$\frac{1}{\rho} \doteq -\frac{1}{n}\frac{dn}{dh} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_(4)}$$

where ρ is the radius of curvature of the refracted ray while in the layer

$$= -\frac{dn}{dh} \quad \text{since } n \simeq 1$$

$$= - 79.5 \times 10^{-6} \frac{d}{dh} \left\{ \frac{P}{T} \left(1 + 7.73 \frac{S}{T}\right) \right\} \qquad (5)$$

$$\frac{dn}{dh} \simeq \left(0.33 \frac{dP}{dh} + 8.2 \frac{ds}{dh} - 1.36 \frac{dT}{dh}\right) 10^{-6} \qquad (6)$$

Figures 5 and 6 show the 10% field strength plotted against the maximum radius of curvature calculated from equation (5). Where no refraction layer exists the average radius of curvature from ground level to about 6,000 feet is taken. All plots display a significant correlation between the calculated radius of curvature of the refracted ray and the observed field strength. When layers occur with a thickness less than 700 feet the resulting field strength values show a tendency to be less than those obtained from thicker layers which produce the same radius f curvature.

To assess the effect of layer height the results have been plotted for thick layers which produce approximately the same radius of curvature (figure 7). The field strength at Warrnambool appears to be almost independent of layer height although no results were obtained for low layers giving a small radius of curvature of the refracted ray.

The Camperdown recordings show that provided a layer exists which will give the refracted ray a radius of curvature less than approximately 11,000 miles, the field strength depends upon the layer height and to a first approximation is independent of $\frac{dn}{dh}$.

The field strength at points beyond the geometrical horizon is therefore determined by the existence of a layer in the troposphere in which the dielectric constant falls rapidly with increase in height. In the absence of such a layer the 50% field strength of all three channels, at a distance of 123 miles, is close to that calculated for diffraction around a smooth earth. If the assumptions discussed in Appendix I are made concerning reflections in the troposphere, the received field strength is also close to that calculated for reflection from the lower atmosphere. At a distance of 162 miles, the results show considerably more dispersion but have a mean which is close to the field calculated for reflection from the lower atmosphere. Table 5 shows the observed and calculated field strengths.

....5/.

Table 5

Channel	Distance miles	Calculated field for reflection from the lower atmosphere. No layer present. $\frac{dn}{dh}=\frac{1}{16000}\text{miles}^{-1}$ db above 1 uv/m	Calculated diffraction field with refraction from a standard atmosphere. 100kw e.r.p. db above 1 uv/m	Field strength from FCC curves F (50,50) 100 kw e.r.p. db above 1 uv/m	Observed 50% field strength for 100kw e.r.p.	
					Mean of observations when no layers were present $\frac{dn}{dh}=\frac{1}{16000}$ miles^{-1} db above 1 uv/m	Mean value of all observations db above 1 uv/m
2	123	26	28	26	27	38
7	123	20	21	19	25	—
9	123	20	21	19	22	40
2	162	14	+ 3	14	20	26
7	162	10	- 18	5	9	18
9	162	10	- 18	5	10	17

Ray traces (figures 8 and 9) show ray bending to be small when layers with a linear refractive index gradient occur at a greater height than that of the transmitting aerial. The observed increase in field strength is therefore assumed to be due to re-radiation from the layer in the direction of the receiver.

The resultant field strength due to reflection in the troposphere is shown in Appendix 1 to be in fair agreement with the observed results, although the calculated increase in field strength due to an elevated layer is about 4 db less than that observed.

In developing an expression for the field due to reflection, it was assumed that no change in refractive index occurred with time. Measurements have shown that this assumption is in error and that variations in refractive index occur due to turbulence. It is on the basis of this change in refractive index, and the volume effected, that the scatter theory $_{5,\ 6,\ 7}$ has developed.

Although the fading characteristics of the signal received when a layer exists do not suggest scatter propagation, the observed dependency on layer thickness seems better explained by the scatter theory than by the reflection theory. Other signal characteristics are in approximate agreement with those predicted from a model in which the main signal is due to the ray which is first reflected from the ground and then reflected from an elevated layer to the receiver. Using this model, the calculated field strength for reception at 123 miles is in agreement with the observed dependency on layer height (figure 7) while the calculated field at 162 miles is in agreement with the observed dependency on $\frac{dn}{dh}$.

..6/.

Nevertheless a scatter propagation mode cannot be dismissed. The equation developed by Booker and Gordon for the received power from a scattering medium is -

$$\frac{P}{P_{f.s.}} = K d^2 \int \frac{\sigma(\theta_1, \psi)}{R_o^{\,2} \quad R^2} \, dV$$

$$\text{where } \sigma(\theta_1, \psi) = \frac{\left(\frac{\overline{\Delta k}}{k}\right)^2 \sin^2\psi}{16\, S \sin^4\frac{\theta}{2}}$$

$$\text{for } 2\, S \sin\frac{\theta}{2} \gg \lambda$$

d = distance between transmitter and receiver
R_o = distance from the transmitter to the scattering element
R = distance from the receiver to the scattering element
dV = volume of macroscopic element
P = power scattered to a receiver
P_{fs} = free space power
S = effective size of the scatterer
$\left(\frac{\overline{\Delta k}}{k}\right)^2$ = mean square fluctuation of the dielectric constant
ψ = scattering angle from the incident electric vector
θ = scattering angle from the direction of incidence

Villars and Weisskopf[7] have shown that the scatter field in the absence of a layer is proportional to -

$$E_{f.s.} \left(\frac{A}{R}\right)^{\frac{1}{2}} \frac{\lambda^{\frac{1}{2}}}{\psi^{2.5}} \frac{dn}{dh}$$

where A is the effective aperture of the receiving aerial

on the assumption that fluctuations in the refractive index are proportional to $\left(\frac{dn}{dh}\right) S$. The dependency on frequency and angle of incidence is therefore similar to that shown in Appendix I for reflection without turbulence. Refractometer measurements have also shown the intensity of dielectric constant fluctuations[6, 8, 9] within a layer to be several times that observed above and below the layer.

When the bottom of the layer lies below the height of the transmitting aerial, the reflection theory predicts a maximum signal. Comparatively few low layers occurred during the recording period and therefore the effect on field strength is not known with any precision, however the tendency is towards high signals in the case of the Camperdown recordings, some of which are presumed to be due to the formation of ducts.

The layers referred to are characterised by a rapid decrease in the amount of water vapour with increase in height, which is usually accompanied by an increase in temperature. As these conditions exist in anticyclone regions, a correlation may be expected between the barometric pressure at ground level and field strength (figure 10). Values of atmospheric pressure relate to the mid point of the path at 1800 hrs. E.S.T. on the night of the recording.

5. Fading Characteristics

The fading rate is defined as the number of fades below the value exceeded for 50% of the time over a period of one hour.

These results show that fading rates increase with increase in distance and frequency, and for any one distance the fading rate is greatest when no layer exists. When layers are present, short period fading is frequently absent; a trend which is more pronounced for lower frequencies and shorter distances.

Average fading rates are shown in the following table -

Table 6

Distance	Fading rate			
	Layer present		No layer present	
Miles	Channel 2	Channels 7 and 9	Channel 2	Channels 7 and 9
123	11	23	48	70
162	20	50	70	140

The response of the Elliott type 230 recorder used is - 30 db at 10 c/s, - 25 db at 5 c/s, - 13 db at 1 c/s and - 7db at 0.5 c/s.

6. Loss of Aerial Gain

Several receiving aerials were available at the Warrnambool site. These included a sixteen element end fed half wave aerial array for channel 2 at a height of 40 feet, a similar aerial for channels 7 and 9 at a height of 75 feet and a vertical stack of four rhombic aerials with twelve wave-length sides for channels 7 and 9.

..8/.

The signals obtained on channels 7 and 9 maintained approximately the same ratio between the aerials at all times with the rhombic aerial delivering the greatest voltage at the receiver terminals. However, powers obtained from the channel 2 array and rhombic aerial were frequently found to be only a little greater than that obtained from the low gain yagi aerial used for the field intensity measurements. This was particularly noticeable when strong signals were received.

Figure 11 shows average curves of picture rating as a function of the voltage obtained from the measuring aerial. An improvement was later obtained when the main lobe of the channel 2 array was tilted up by increasing the length of feeder to the two top sections. The arrival of radiation at high angles of elevation has been reported elsewhere (10).

7. Polarisation Discrimination

During the last week of the recording period at Warrnambool, recordings were made of channel 2 from a vertically polarised aerial. The average ratio $\frac{E_H}{E_v}$ was found to be 16 db. A noticeable increase in the noise level was observed when switching from the horizontally polarised aerial to the vertically polarised aerial.

8. Comparison with F.C.C. values

Table 7 summarises a comparison of measured field strengths with those calculated from the F.C.C. curves.

Table 7

Comparison with F.C.C. values

Location	Channel	Field strengths for 100 kw e.r.p. db above 1 microvolt per metre							
		F.C.C.			Measured			Ratio measured/F.C.C.	
		50%	10%	ratio 10/50%	50%	10%	ratio 10/50%	50%	10%
Camperdown	2	26	38	12	38	43	5	12	5
	7	19	36	17					
	9	19	36	17	40	45	5	21	9
Warrnambool	2	14	27	13	26	31	5	12	4
	7	5	22	17	18	23	5	13	1
	9	5	22	17	17	22	5	12	0

This table shows the outstanding differences between measured field strength values and those predicted from F.C.C. curves, particularly in the case of the median values where the discrepancy is about 13 db, and also in the case of the 10% values which exhibit values between 0 and 9 db higher than the F.C.C. figures.

..9/.

The F.C.C. ratio of 10% to median value is commensurately higher, being about 16 db whereas the measured ratio is about 5 db. The reason for these differences is open to speculation. The F.C.C. values are statistical for average terrain (50% of locations) whereas the measured values were obtained at good receiving sites with open country looking towards the transmitters, but surrounded by houses in the Camperdown case.

9. Conclusion

The field strength for channels 2, 7 and 9 at points beyond the geometrical horizon depends upon the existence of a layer in the troposphere in which the dielectric constant falls rapidly with increase in height. Under these conditions the fading rate is usually less than that observed when no layer exists.

In the absence of a layer the field strength received at 123 miles is close to that calculated for diffraction around the earth and that calculated for reflection from the lower atmosphere, while at a distance of 162 miles the received signal is close to that calculated for reflection from the lower atmosphere.

The observed dependency on meteorological conditions means in its turn a dependency on season, time of day and geographical location. Although no attempt is made here to investigate the effect of geographical location it may be assumed that average values for interference field strength levels will change with location in accordance with the occurrence and intensity of tropospheric layers.

Acknowledgement

The author wishes to acknowledge the assistance given by Mr. H. S. Fuller who made the recordings at Warrnambool, the officers of the Postmaster-General's Department who made the recordings at mperdown and officers of the Bureau of Meteorology who supplied the radio sonde data.

Report 17

References

1. K. Bullington "Radio Propagation at Frequencies above 30 Mc/s" Proc. I.R.E. October, 1947.

2. C. Millington "The Reflection Coefficient of a Linearly Graded Layer" Marconi Review 1949 p. 140.

3. C.R. Burrows and S.S. Attwood "Radio Wave Propagation" Consolidated Summary Technical Report of the Committee of Propagation of the National Defence Research Committee 1949.

4. K. Toman, W.C. Albright and E.C. Jordan "Meteorological Effects on VHF Propagation" I.R.E. Transactiors, Antennas and Propagation December 1952.

5. H.G. Booker and W.E. Gordon "A Theory of Radio Scattering in the Troposphere" Roc. I.R.E. April, 1950.

6. W.E. Gordan "Radio Scattering in the Troposphere" Proc. I.R.E. January 1955.

7. F. Villars and V.F. Weisskopf "On the Scattering of Radio Waves by Turbulent Fluctuations of the Atmosphere" Proc. I.R.E. October, 1955.

8. C.M. Crain "Survey of Airborne Microwave Refractometer Measurements" Proc. I.R.E. October, 1955.

9. C.M. Crain, A.W. Straiton and C.E. Von Rosenburg "A Statistical Survey of Atmospheric Index of Refraction Variation" I.R.E. Transactions, Antennas and Propagation October, 1953.

10. A.W. Straiton, D.F. Metcalf and C.W. Tolbert "A Study of Tropospheric Scattering of Radio Waves" Proc. I.R.E. June, 1951.

11. Documents of the Plenary Assembly C.C.I.R. Vol. 1 London 1953 or Warsaw 1956.

12. Rules Governing Radio Broadcast Services issued by the F.C.C. Federal Register July 4, 1953.

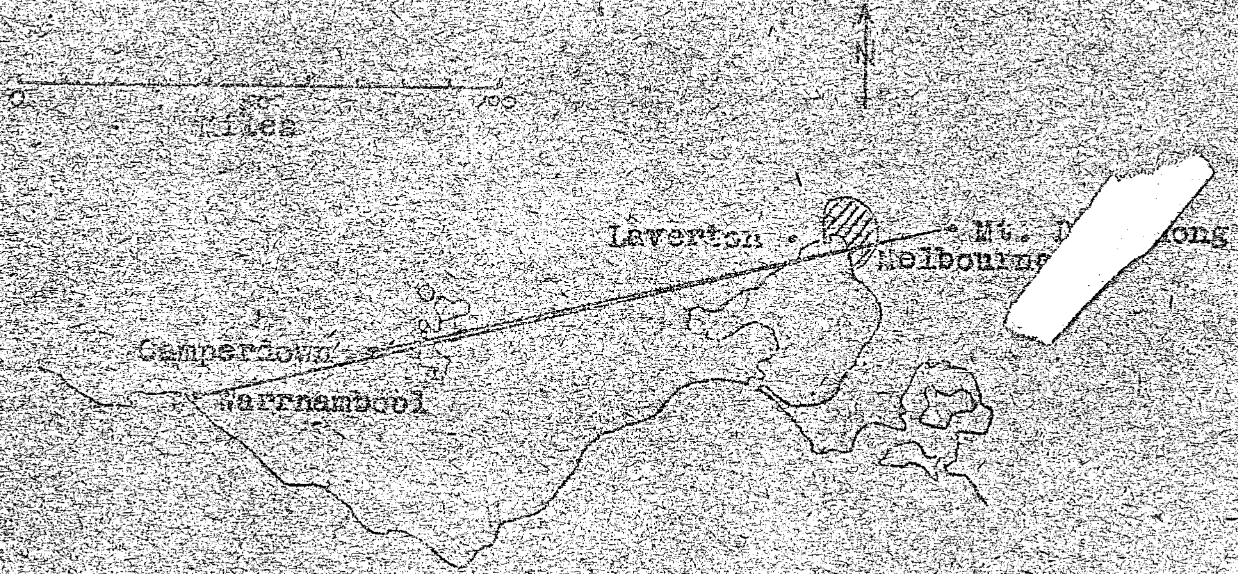

0
50
100
Miles
N
Laverton
Mt. D ong
Melbourne
Camperdown
Warrnambool

FIGURE 1

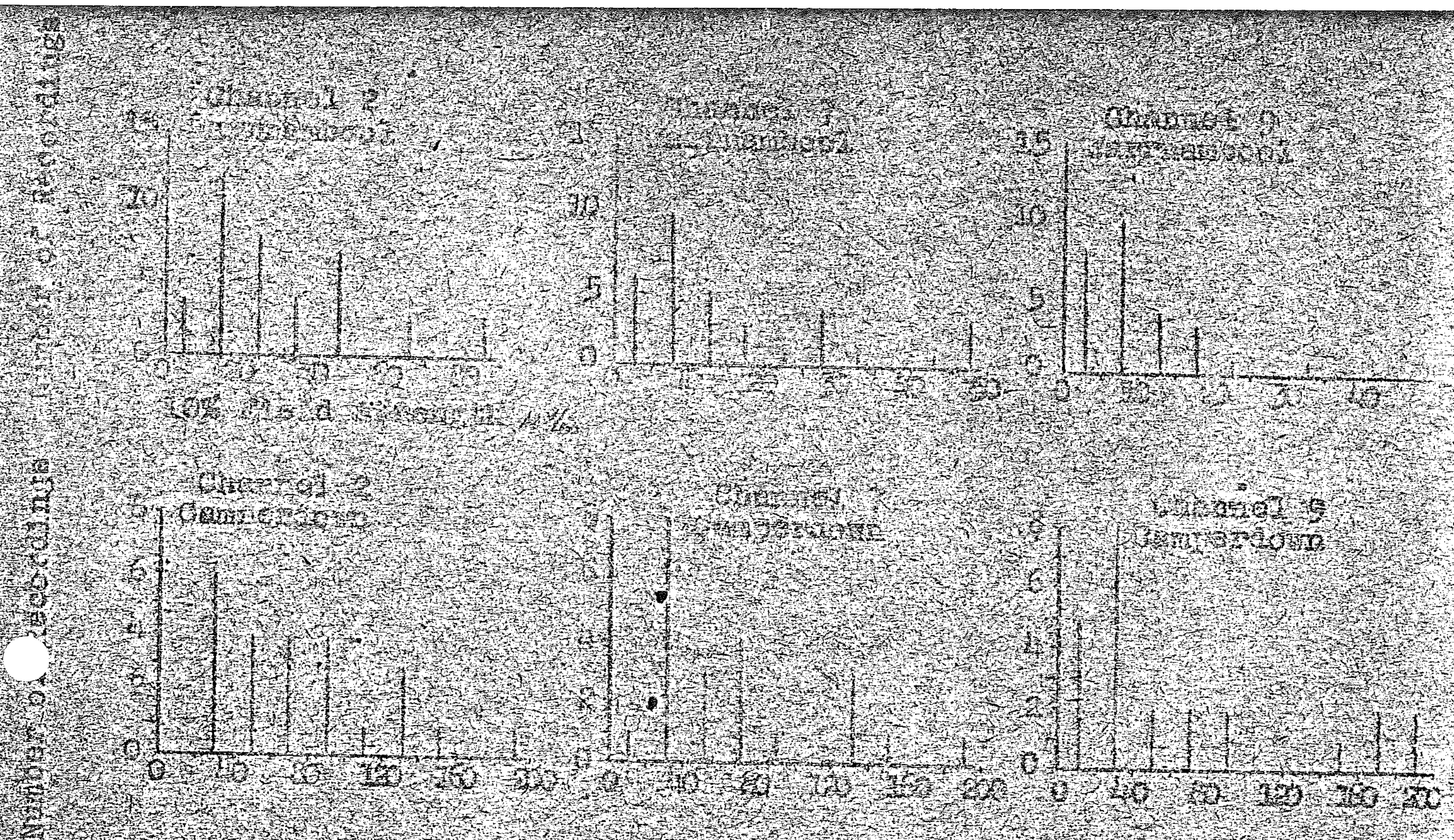
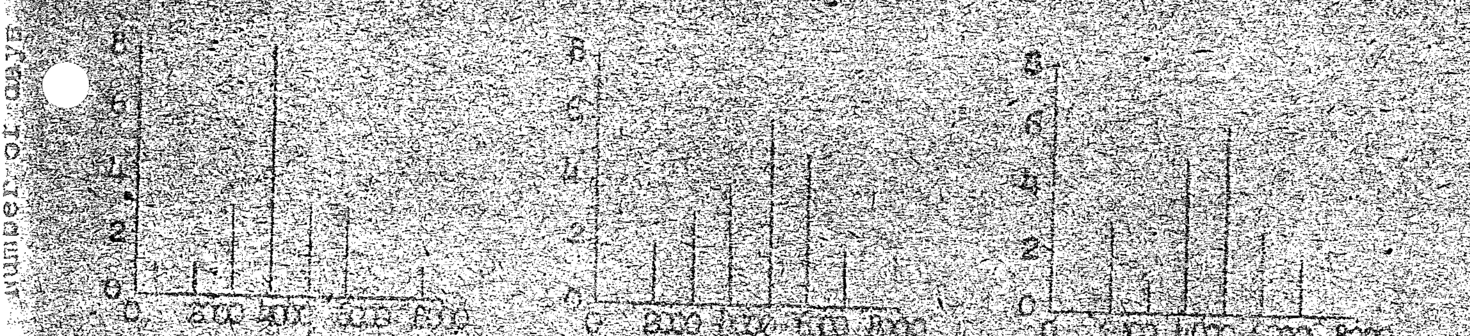


FIGURE 2



Layer height - feet
Laverton 20.3.58 to 21.5.58
Total Number of days 25

Layer height - feet
Laverton 28.5.58 to 20.7.58
Total Number of days 40

Layer height - feet
Laverton [illegible]
Total Number of days [illegible]

FIGURE 3

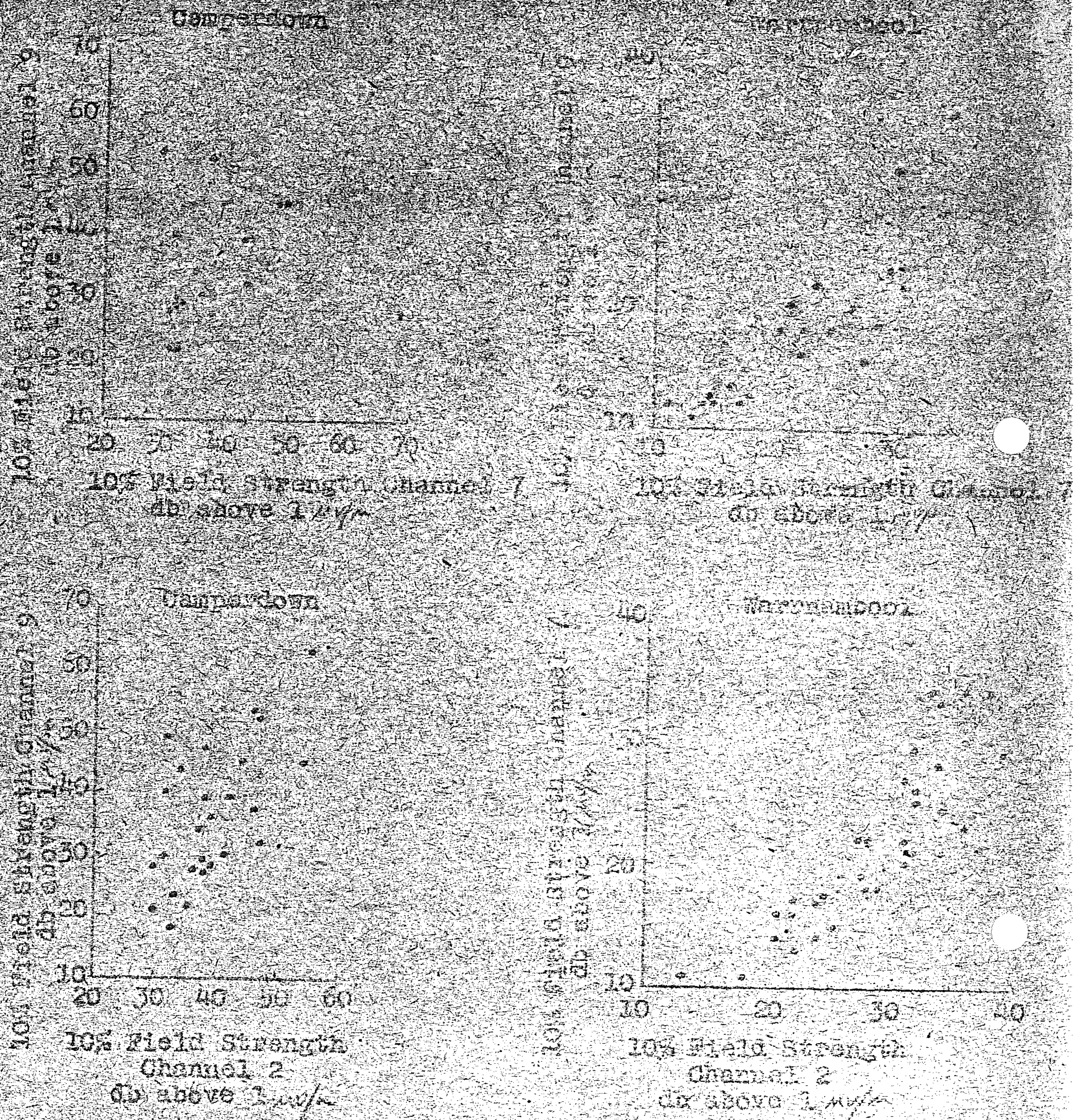

Camperdown
10% Field Strength Channel 9
db above 1 μv/m
10
20
30
40
50
60
70
20 30 40 50 60 70
10% Field Strength Channel 7
db above 1 μv/m
Warrnambool
10
20
30
40
10 20 30
10% Field Strength Channel 7
db above 1 μv/m
Camperdown
10% Field Strength Channel 9
db above 1 μv/m
20 30 40 50 60
10% Field Strength
Channel 2
db above 1 μv/m
Warrnambool
10% Field Strength Channel 7
db above 1 μv/m
10 20 30 40
10% Field Strength
Channel 2
db above 1 μv/m

FIGURE 4

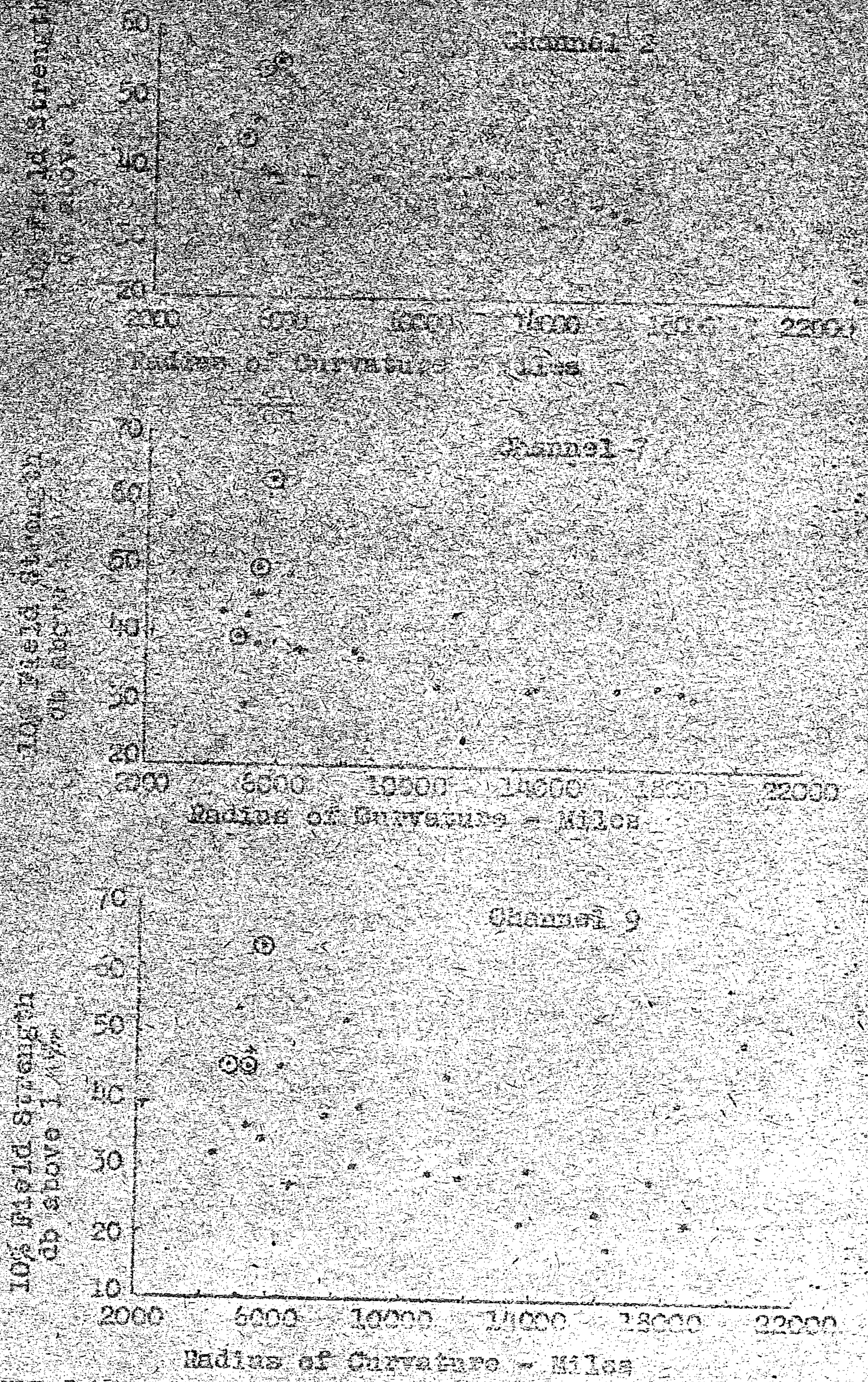


FIGURE 5 Camperdown Recordings

⊙ Transmitting aerial and refracting layer at same height
Thickness of layer less than 700 feet.

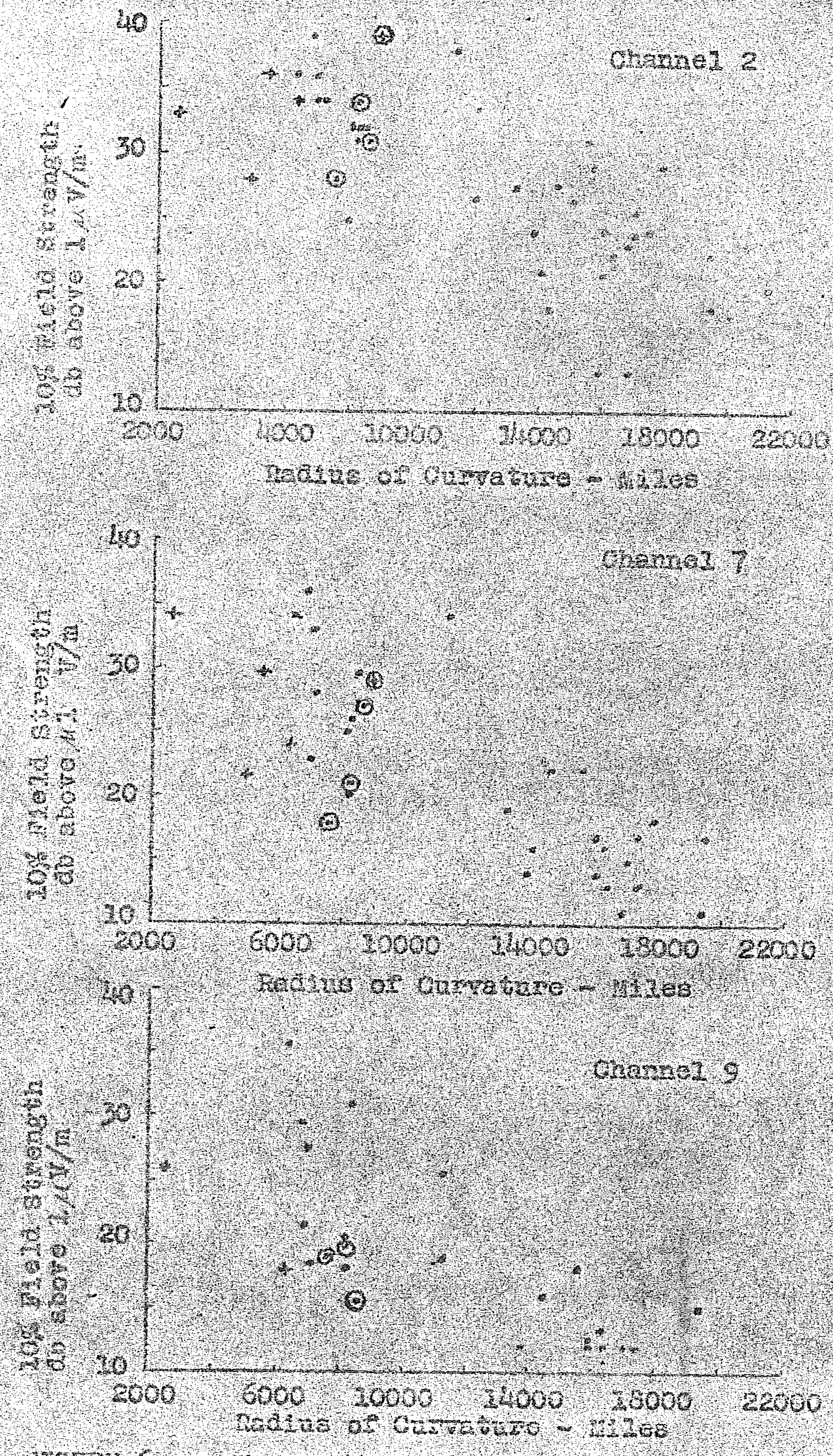


FIGURE 6 Warrnambool Recordings

⊙ Transmitting aerial and refracting layer at same height

\+ Thickness of layer less than 700 feet

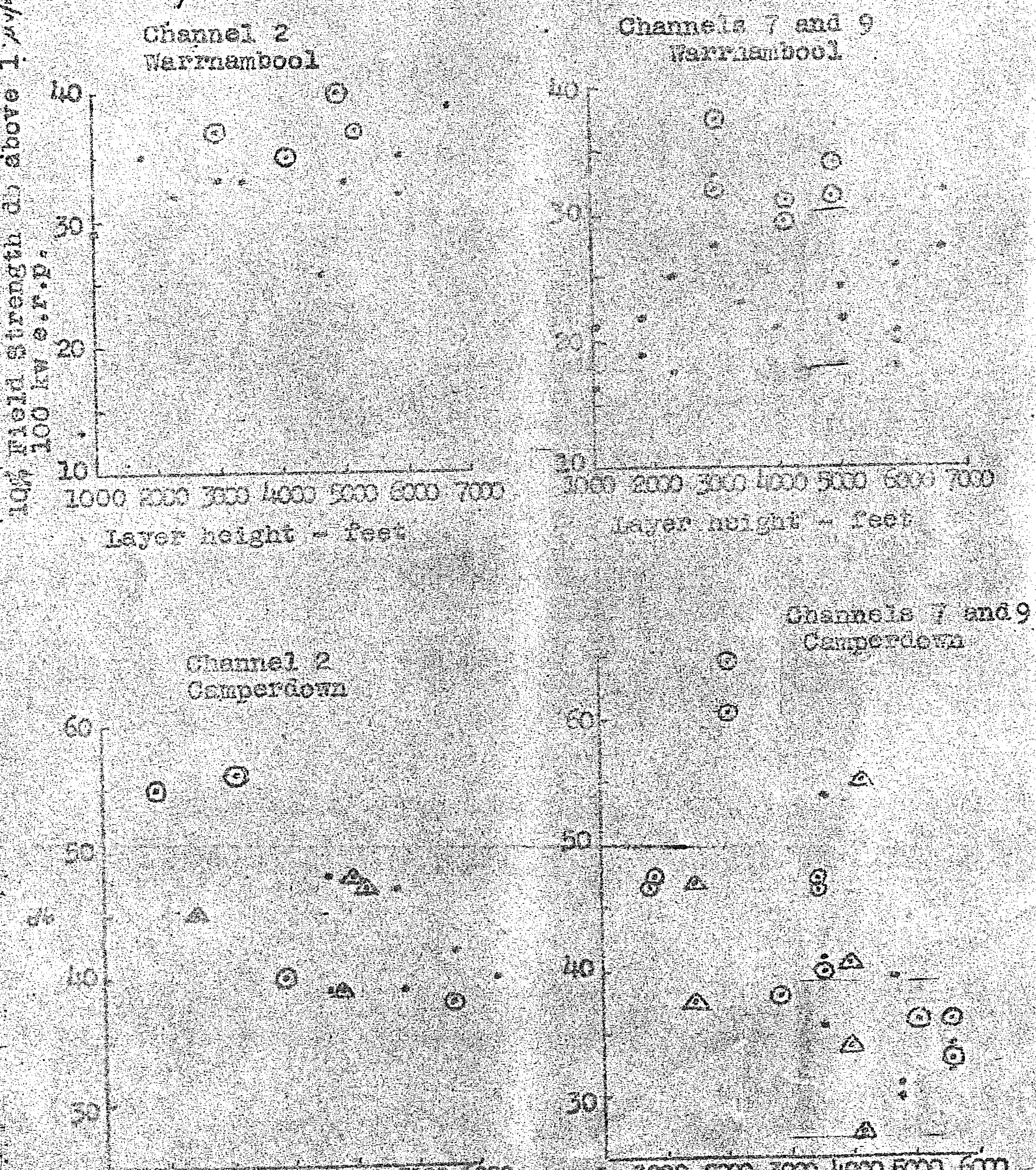


FIGURE 7

Effect of Layer Height

- • Radius of Curvature between 11000 and 7000 miles
- ⊙ " " " " 7000 " 5000 "
- △ " " " less than 5000 miles

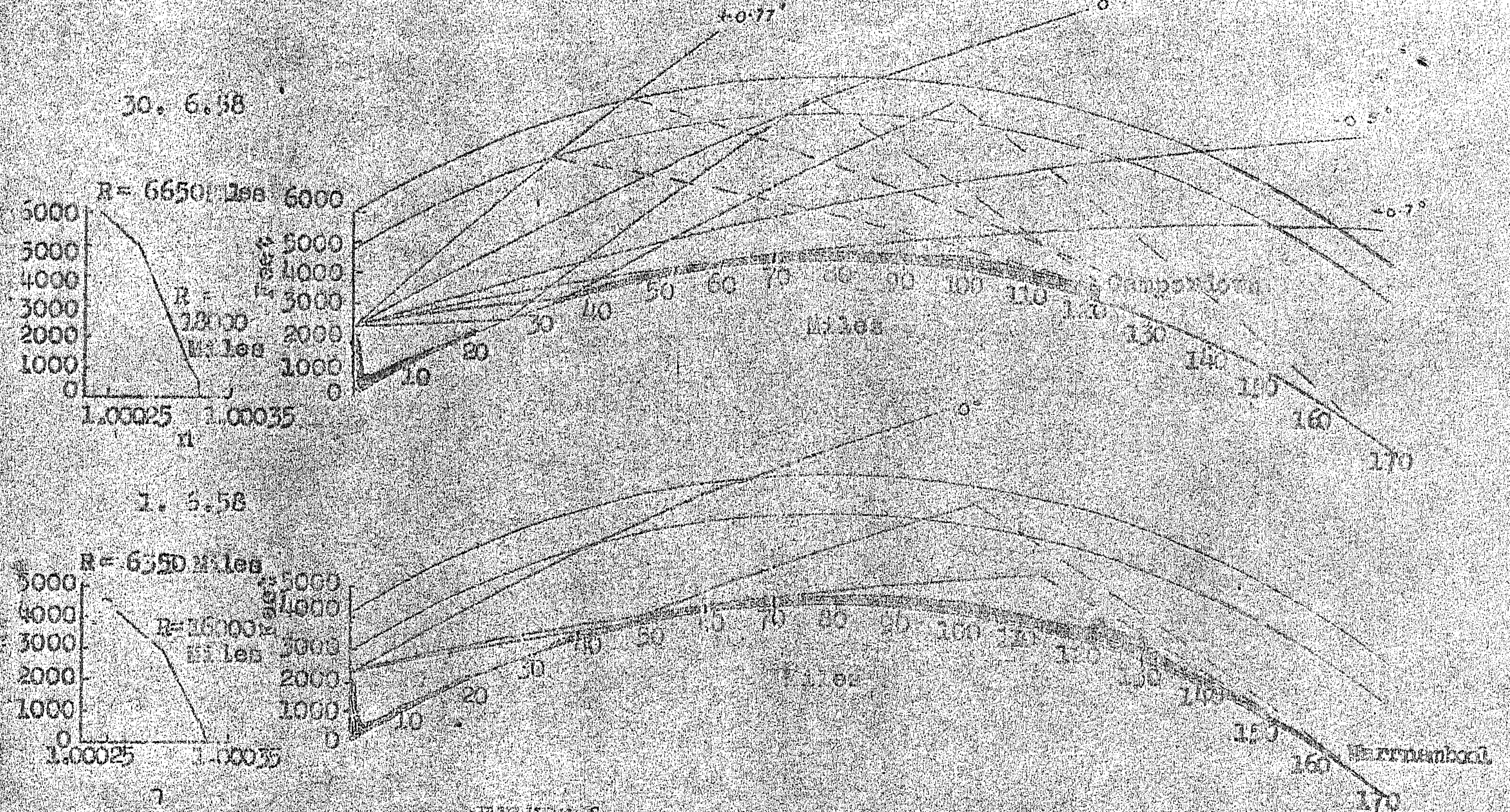

30. 6.58
R = 6650 Miles
R = 12000 Miles
1.00025
1.00035
n
0
1000
2000
3000
4000
5000
6000
Feet
Miles
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
Camperdown
+0.77
0°
0.5°
0.7°
1. 5.58
R = 6550 Miles
R=16000 Miles
Warrnambool

FIGURE 7

25. 4.58

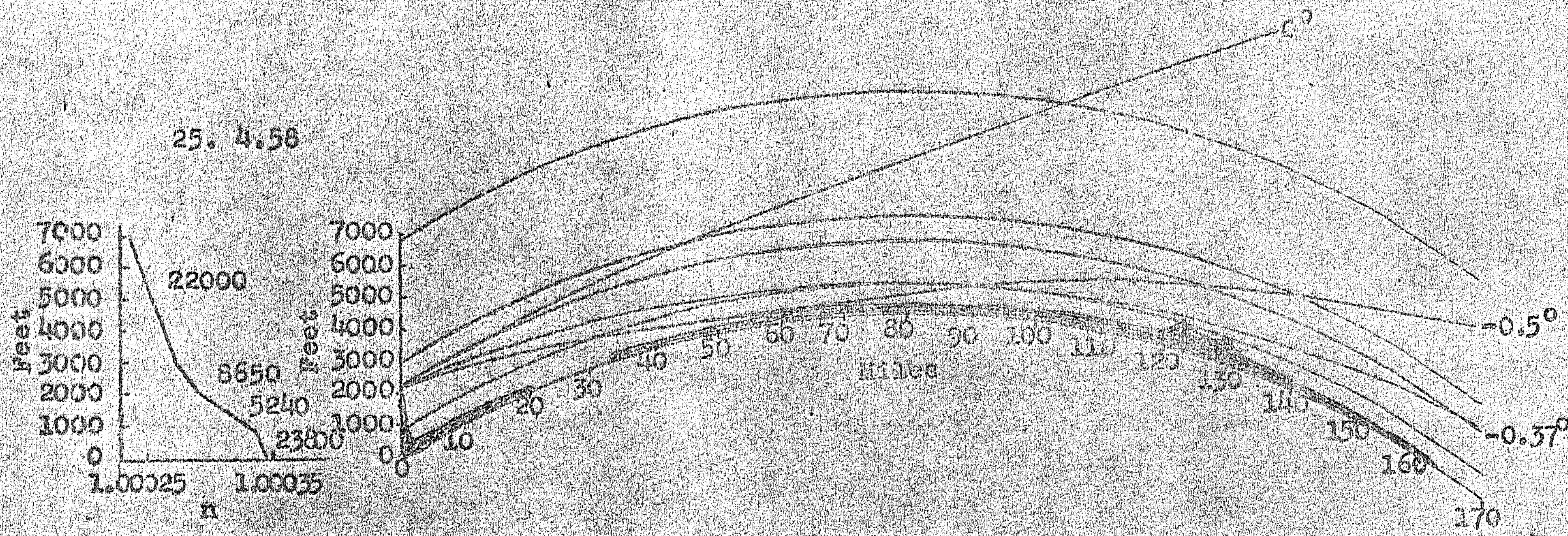


FIGURE 9

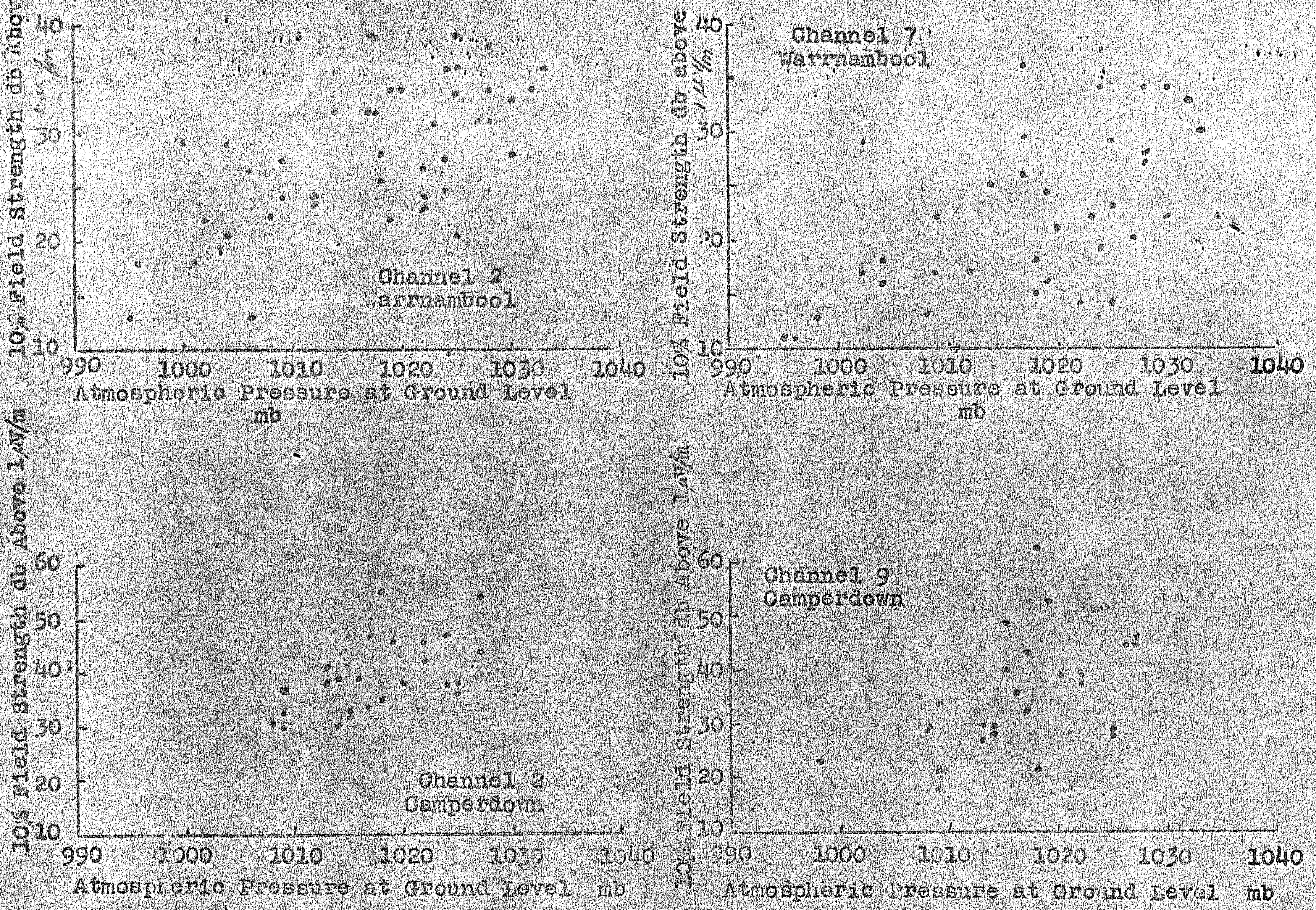

10% Field Strength db Above 1 µV/m
40
30
20
10
990
1000
1010
1020
1030
1040
Atmospheric Pressure at Ground Level mb
Channel 2
Warrnambool
Channel 7
Warrnambool
60
50
Channel 2
Camperdown
Channel 9
Camperdown

FIGURE 10

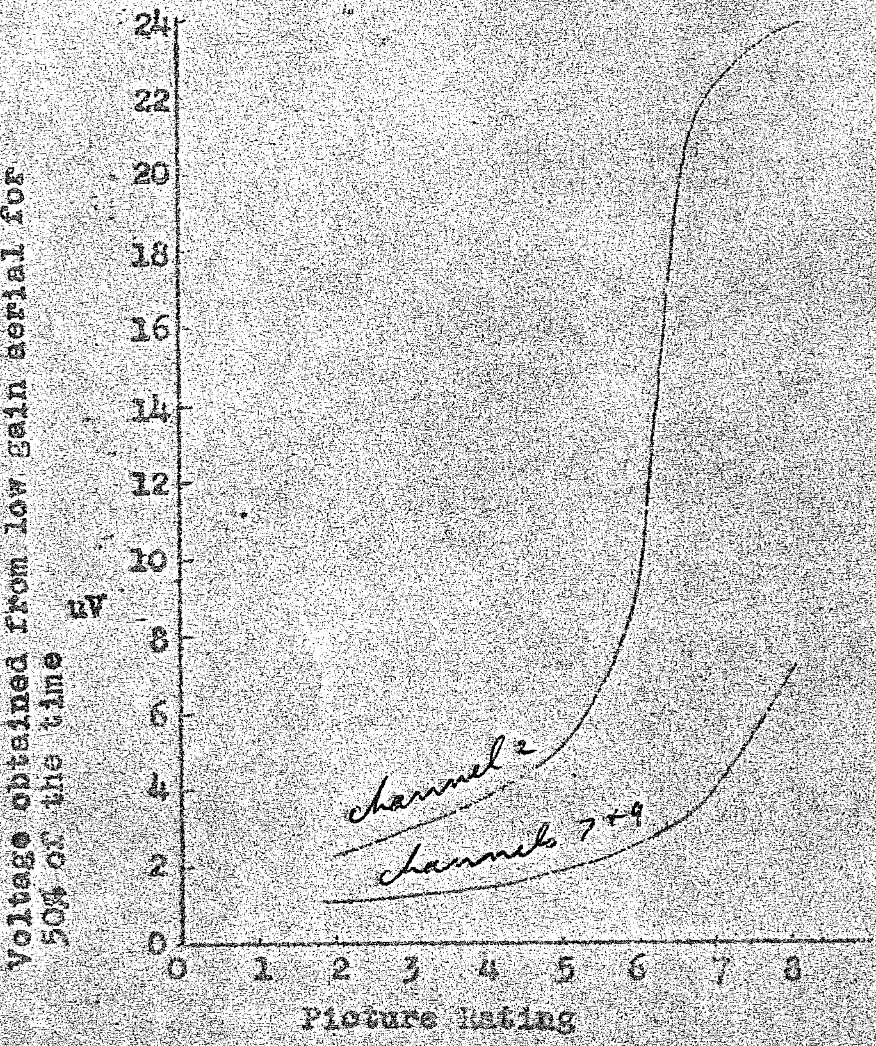


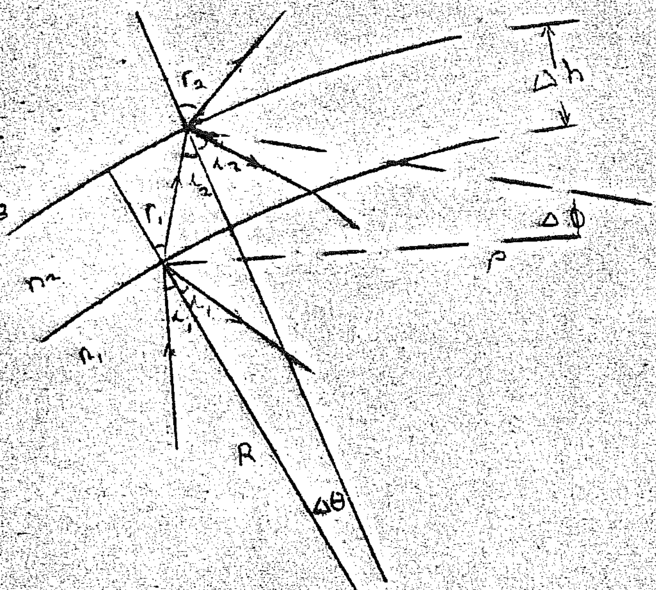
FIGURE 11

Picture rating

1 no picture
2 occasional faint picture
3 severe fading, loss of synch.
4 programme followed with difficulty 25% picture
5 50% picture
6 some fading 60% picture
7 good usable picture, some noise
8 very little noise, excellent picture, occasional fade.

Appendix I

Let the layer formed by air in which the refractive index decreases linearly with height, be represented by a large number of layers in which the refractive index is constant but decreases slightly from one layer to the next. At the boundary of two such layers a portion of the incident energy will continue in a slightly different direction as a refracted ray and, neglecting absorption, the remaining energy will be propagated as a reflected ray. In developing an expression for the field due to reflection it is assumed that there is no change in refractive index with time.



$\Delta n = n_1 - n_2 = n_2 - n_3$

$n_1 > n_2 > n_3$

For the refracted ray we have –

$n_1 \sin i_1 = (n_1 + \Delta n) \sin r_1$ where n = refractive index

$= (n_1 + \Delta n) \sin (i_2 + \Delta\theta)$

Let $i_2 = i_1 + \Delta i$

$\therefore n_1 \sin i_1 = (n_1 + \Delta n) \sin (i_1 + \Delta i + \Delta\theta)$

Neglecting small terms we have

$n_1 \cos i_1 . (\Delta i + \Delta\theta) + \Delta n \sin i_1 = 0$

$\therefore n_1 (\Delta i + \Delta\theta) = - \Delta n \tan i_1 \simeq - \frac{\Delta n}{\Delta h} R \Delta\theta$ ______________(1)

$\Delta\phi + i_1 = \Delta\theta + i_2$

$\therefore i_2 - i_1 = \Delta i = \Delta\phi - \Delta\theta$ ______________(2)

From (1) and (2) we have

$$\frac{1}{R}\frac{\Delta\phi}{\Delta\theta} = -\frac{1}{n}\frac{\Delta n}{\Delta h}$$

but $R\,\Delta\hat{\theta} \simeq \rho\Delta\phi$

$$\therefore \frac{1}{\rho} = -\frac{1}{n}\frac{\Delta n}{\Delta h} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(3)}$$

which gives the radius of curvature of the refracted ray.

The reflection coefficient for propagation in the troposphere is given by -

$$r = \frac{\sqrt{\frac{k_1}{k_2} - \cos^2\psi} - \sin\psi}{\sqrt{\frac{k_1}{k_2} - \cos^2\psi} + \sin\psi} \qquad \text{where } \psi = \frac{\pi}{2} - i_1$$

$$\simeq \frac{\sqrt{k_1 - k_2 + \psi^2} - \psi}{\sqrt{k_1 - k_2 + \psi^2} + \psi} \qquad \text{where } k_1 \simeq k_2 \simeq 1 \text{ and } \psi \text{ is small}$$

$$\simeq \frac{\sqrt{\Delta k + \psi^2} - \psi}{2\psi} \qquad \text{where } \Delta k = k_1 - k_2 = 2\Delta n \ll \psi^2$$

$$\simeq \frac{\Delta k}{4\psi^2} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(4)}$$

d the phase change θ_1 from one elementary layer to the next is $\frac{2\psi\Delta h}{\lambda}$ 360 degrees.

The resultant signal received from the region is

$$E = E_{f.s.} \sum_{r_1}^{r_n} \{ \; r_1 \, e^{j.\theta_1} + \sum_{r_1}^{r_n} r_2 \, e^{j.\theta_2} + \qquad \}$$

where $E_{f.s.}$ = free space field. ____________(5)

The locus of the resultant for any one ray is a spiral formed by the contributing vector elements from each elementary layer. The resultant therefore oscillates in amplitude as the total layer thickness increases, assuming $\frac{dn}{dh}$ to be constant.

The point of intersection of the reflected rays is at a distance $\rho_1 \simeq R\psi$ from the reflection point and for a single ray the resultant field is

$$E \simeq E_{fs} \frac{2r}{\theta_1} \cos \left(\frac{\pi - m\theta_1}{2}\right) \quad \text{\_\_\_\_\_\_\_\_(6)}$$

where $\theta_1 \leqslant 35^\circ$

m = number of elementary layers

The partial contribution from other rays forms a second spiral of resultant vectors, but, since the point of focus depends upon the angle of incidence and the position of the reflection point it may be assumed that the median value will not exceed

$$E = E_{fs} \frac{2r}{\theta_1}$$

$$= E_{fs} \frac{\lambda}{4\pi\psi^3} \frac{dn}{dh} \quad \text{\_\_\_\_\_\_\_\_(7)}$$

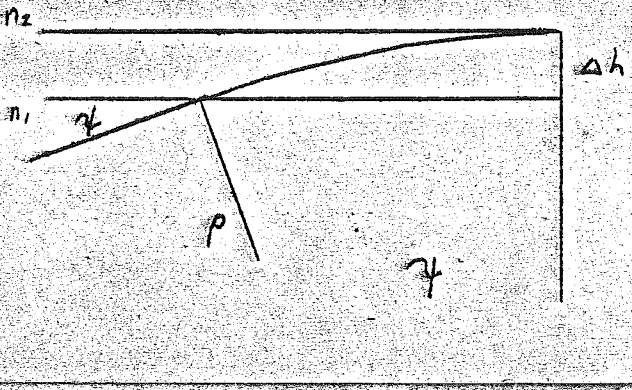
It is sometimes convenient to transform ray traces for a circular earth to an equivalent flat earth. The geometry of the refracted ray is preserved by assuming an equivalent refractive index such that $\Delta M = \frac{\Delta h}{R} - \Delta n$, but the reflection coefficient may be calculated only from Δn. Reflection coefficients have been published in a paper by Millington [2], but the values given are for flat layers and should be used with caution when the compliment of the angle of incidence (ψ) becomes very small.

The refracted ray will leave a flat layer on the side from which it entered when

$$\psi^2 < \frac{2\Delta h}{\rho} = 2\Delta n$$

and the ray will return to the earth when

$$\frac{2h}{R} < \psi^2 < 2\left(\Delta n - \frac{\Delta h}{R}\right)$$



A tabulated calculation of the field strength due to reflection from an elevated layer such as that shown in figure 8 and for reflection from the lower atmosphere is given below.

	Layer reflection		
Channel 2	First reflection at ground	First reflection at layer	Lower troposphere reflection
Distance from transmitter to receiver	123 miles	123 miles	123 miles
Distance from receiver to tropospheric reflection region	49 miles	85 miles	30 miles
Elevation of lower section of reflecting region above receiver height	4200 feet	4200 feet	340 feet
ψ effective	0.019 rad.	0.0175 rad.	0.0065 rad.
$\frac{dn}{dh}$ in reflection region	$\frac{1}{6650}$ miles^{-1}	$\frac{1}{6650}$ miles^{-1}	$\frac{1}{18000}$ miles^{-1}
Attenuation due to reflection in troposphere	46 db	44 db	26 db
Attenuation due to angle of anival for a receiving aerial height of 30 feet.	8 db	19 db	29 db
Relative phase of received signal	240°	22°	240°
E 100 kw	$103 + 20 - 42 - 46 - 8 = 27$ db above 1 V/m	$103 + 20 - 42 - 44 - 19 =$ 18 db above 1 V/m	$103 + 20 - 42 - 26 - 29 =$ 26 db above 1 V/m
Resultant field due to tropospheric reflection	31 db above 1 V/m (median)		

	Layer reflection		Lower troposphere reflection
Channels 7 and 9	First reflection at ground	First reflection at layer	
Distance from transmitter to receiver	123 miles	123 miles	123 miles
Distance from receiver to tropospheric reflection region	49 miles	85 miles	30 miles
Elevation of lower section of reflecting region above receiver height	4200 feet	4200 feet	340 feet
ψ effective	0.019 rad.	0.0175 rad.	0.0065 rad.
$\frac{dn}{dh}$ in reflection region	$\frac{1}{6650}$ miles^{-1}	$\frac{1}{6650}$ miles^{-1}	$\frac{1}{18000}$ miles^{-1}
Attenuation due to reflection in troposphere	56 db	54 db	36 db
Attenuation due to angle of arrival for a receiving aerial height of 30 feet	3 db	15 db	25 db
E 100 kw	$103 + 20 - 42 - 56 - 3 =$ 22 db above 1 V/m	$103 + 20 - 42 - 54 - 15 =$ 12 db above 1 V/m	$103 + 20 - 42 - 36 - 25 =$ 20 db above 1 V/m

	Layer reflection		
Channel 2	First reflection at ground	First reflection at layer	Lower troposphere reflection
Distance from transmitter to receiver	162 miles	162 miles	162 miles
Distance from receiver to tropospheric reflection region	64 miles	no case	50 miles
Elevation of lower section of reflecting region above receiver height	4800 feet		1800 feet
ψ effective	0.023 rad.		0.0133 rad.
$\frac{dn}{dh}$ in reflection region	$\frac{1}{6650}$ miles^{-1}		$\frac{1}{18000}$ miles^{-1}
Attenuation due to reflection in troposphere	50 db		45 db
Attenuation due to angle of arrival for a receiving aerial height of 30 feet	11 db		20 db
Relative phase of recived signal	76°		50°
E 100 kw	103 + 20 − 44 − 50 − 11 = 18 db above 1 V/m		103 + 20 − 44 − 45 − 20 = 14 db above 1 V/m
Resultant field due to tropospheric reflection	22 db above 1 V/m (Median)		

Channels 7 and 9	Layer reflection		Lower troposphere reflection
	First reflection at ground	First reflection at layer	
Distance from transmitter to receiver	162 miles	162 miles	162 miles
Distance from receiver to tropospheric reflection region	64 miles	no case	50 miles
Elevation of lower section of reflecting region above receiver height	4800 feet		1800 feet
ψ effective	0.023 rad.		0.0133 rad.
$\frac{dn}{dh}$ in reflection region	$\frac{1}{6650}$ miles^{-1}		$\frac{1}{18000}$ miles^{-1}
Attenuation due to reflection in troposphere	60 db		55 db
Attenuation due to angle of arrival for a receiving aerial height of 30 feet	7 db		14 db
E 100 kw	$103 + 20 - 44 - 60 - 7 =$ 12 db above 1 V/m (Median value)		$103 + 20 - 44 - 55 - 4 =$ 10 db above 1 V/m (median value)

Table 1

Recordings made at Camperdown (distance 123 miles)

Note These voltages should be multiplied by the following factors to obtain the field strength in μ V/m

Channel 2	1.34
Channel 7	3.31
Channel 9	3.96

Date	Channel	Time EST	Voltage in 50 ohm line uv		Ratio 10%/50%	Atmospheric pressure at ground level (path mid point)
			10%	50%		mb 1800 hrs.EST
20. 3.1958	7	1630-1730	12	7.2	1.7	1008
	9	1845-1945	7.6	5.6	1.4	
	2	1950-2050	28	18.4	1.5	
21. 3.1958	9	1900-1950	10.4	6.0	1.7	1018
	2	2000-2050	164	90	1.8	
	7	2100-2150	42	24	1.75	
24. 3.1958	2	1550-1650	57	38	1.5	1020
	9	1930-2030	22	14	1.6	
26. 3.1958	2	1530-1645	19	13	1.5	1013
	9	1815-1915	6.4	3.2	2.0	
	7	1920-2020	12	8	1.5	
	2	2030-2115	24	14	1.7	
28. 3.1958	2	2000-2050	32	21	1.5	1018
	9	2100-2150	2.0	1.3	1.5	
	7	2200-2245	12	6	2.0	
31. 3.1958	2	2020-2115	64	35	1.8	1017
	9	2120-2215	16	8	2.0	

Date	Channel	Time EST	Voltage in 50 ohm line uv		Ratio 10%/50%	Atmospheric pressure at ground level (path mid point) mb 1800 hrs.EST
			10%	50%		
2. 4.1958	2	1900-2000	96	42	2.3	1022
	9	2000-2100	24	12	2.0	
	7	2115-2210	42	24	1.75	
3. 4.1958	9	1530-1630	58	32	1.8	1018
	7	1845-1945	410	310	1.3	
	2	2000-2100	920	230	1.8	
	9	2105-2200	346	154	2.25	
8. 4.1958	7	1900-2000	22	12	1.8	1025
	2	2000-2100	166	97	1.7	
	9	2100-2200	102	64	1.6	
9. 4.1958	2	1000-1100	96	50	1.9	1024
	9	1320-1415	3	1.6	1.9	
	7	1430-1515	4.8	2.7	1.8	
	9	1830-1930	7	3.4	2.1	
	7	1930-2030	12	6.8	1.8	
	2	2045-2145	56	32	1.75	
10. 4.1958	2	1830-1930	117	67	1.75	1027
	7	1936-2035	28	15.5	1.8	
	9	2040-2145	46	24	1.9	
14. 4.1958	7	1910-2010	8.5	3.8	2.2	1019
	9	2015-2115	110	53	2.1	
	2	2120-2205	150	112	1.3	

Date	Channel	Time EST	Voltage in 50 ohm line uv		Ratio 10%/50%	Atmospheric pressure at ground level (path mid point)
			10%	50%		mb 1800 hrs.EST
16. 4.1958	7	1340-1420	14	7.5	1.9	1025
	2	1440-1600	96	57	1.7	
	7	1820-1925	11	2.5	4.4	
	9	1930-2030	6	3.5	1.7	
	2	2040-2135	48	22	2.2	
17. 4.1958	7	1045-1155	80	48	1.7	1026
	9	1200-1300	45	24	1.9	
18. 4.1958	2	2000-2100	23	17	1.35	1010
	7	2105-2200	10.3	5.0	2.1	
	9	2205-2305	3.0	1.6	1.9	
21. 4.1958	7	1830-1930	40	30	1.3	1027
22. 4.1958	9	1945-2040	19	11	1.7	1023
	2	2045-2145	148	84	1.8	
23. 4.1958	7	1400-1500	24	10	2.4	1027
25. 4.1958	9	1900-2000	45	36	1.25	1027
	7	2010-2100	90	51	1.8	
	2	2110-2200	384	320	1.2	
28. 4.1958	2	1850-1920	60	42	1.4	1024
	9	1920-2020	57	32	1.4	
	7	2020-2120	56	46	1.2	
30. 4.1958	9	1910-2010	121	35	3.5	1015
	2	2135-2200	27.5	17	1.6	

Date	Channel	Time EST	Voltage in 50 ohm line uv		Ratio 10%/50%	Atmospheric pressure at ground level (path mid point) mb 1800 hrs.EST
			10%	50%		
2. 5.1958	2	1945-2045	64	30	2.1	1014
	9	2055-2145	6.3	3.7	1.7	
5. 5.1958	9	1830-1930	5.8	3.3	1.8	1012
	2	1940-2030	60	39	1.5	
	7	2040-2135	23	11	2.1	
7. 5.1958	2	1820-1920	86	48	1.8	1013
	7	1920-2025	19	9.5	2.0	
	9	2050-2130	7.9	3.5	2.3	
9. 5.1958	7	1830-1930	11.4	7.4	1.5	1015
	2	1940-2040	29	16	1.8	-
	9	2045-2145	25	16.5	1.5	
12. 5.1958	9	1815-1915	39	22	1.8	1017
	2	1920-2020	39	31	1.3	
	7	2025-2125	43	28	1.5	
14. 5.1958	2	1820-1920	34	28	1.2	999
	9	1925-2025	3.7	2.3	1.6	
16. 5.1958	7	1815-1915	16	10	1.6	1009
	9	1915-2015	12	6.3	1.9	
	2	2015-2030	55	43	1.3	
21. 5.1958	7	1900-2000	11.5	6	1.9	1018
	2	2030-2125	40	23	1.7	
	9	2200-2300	3	1.8	1.7	

Table 2

Recordings made at Warrnambool (distance 162 miles)

Date	Channel	Time EST	Voltage in 50 ohm line uv 10%	Voltage in 50 ohm line uv 50%	Ratio 10%/50%	Atmospheric pressure at ground level (path mid point) mb 1800 hrs.EST
22. 5.1958	2	1925-2025	16	6.8	2.35	1006
23. 5.1958	2	1930-2030	3.3	2.5	1.3	1006
25. 5.1958	2	1810-1855	8.0	4.0	2.0	1004
	7	2145-2240	2.0	0.8	2.5	
26. 5.1958	7	1930-2025	1.3	0.9	1.4	1008
	2	2120-2220	9.5	4.7	2.0	
	9	2230-2320	0.9	0.7	1.3	
28. 5.1958	2	1230-1325	30	15.5	1.9	1017
	7	1940-2040	8.5	4.4	1.9	
	9	2050-2150	2.4	1.45	1.7	
	7	2245-2330	5.9	2.9	2.0	
30. 5.1958	2	1910-2015	25	12	2.1	1027
	7	2030-2130	3.0	2.0	1.5	
	9	2145-2245	2.0	1.5	1.3	
1. 6.1958	7	1930-2020	15	7	2.15	1024
	2	2040-2135	48	42	1.1	
	9	2215-2300	4.7	2.3	2.0	
2. 6.1958	2	1900-2000	28	15	1.9	1017
	7	2030-2130	9.0	4.0	2.25	
	9	2145-2245	3.2	1.8	1.8	
3. 6.1958	7	1930-2000	5.5	3.0	1.8	1014
	2	2010-2100	30	17	1.8	

Date	Channel	Time EST	Voltage in 50 ohm line uv		Ratio 10%/50%	Atmospheric pressure at ground level (path mid point) mb 1800 hrs. EST
			10%	50%		
4. 6.1958	2	2020-2145	3.5	2.7	1.3	995
	7	2125-2230	1	less than 1	-	
	9	2235-2315	less than 1	"	-	
6. 6.1958	7	1910-2005	4.5	2.5	1.8	1025
	9	2015-2145	3.0	1.5	2.0	
	2	2200-2230	50	26	1.9	
8. 6.1958	7	1800-1855	15	8.5	1.8	1030
	2	1900-2005	32	22	1.5	
	9	2015-2110	5.0	3.0	1.7	
9. 6.1958	7	1800-1900	7.0	3.0	2.3	1028
	2	2055-2200	27	17	1.6	
	9	2200-2300	1.5	less than 1	-	
11. 6.1958	7	1845-2000	8.0	3.3	2.4	1028
	2	2008-2105	37	26	1.4	
	9	2110-2215	2.1	1.2	1.8	
12. 6.1958	2	1900-1955	65	45	1.45	1025
	7	2140-2235	8.5	4.5	1.9	
	9	2245-2335	3.5	1.7	2.1	
13. 6.1958	2	1920-2045	14	7.0	2.0	1018
	7	2055-2155	1.7	1.0	1.7	
	9	2200-2235	less than 1	Less than 1	-	
15. 6.1958	2	1850-1955	36	18	2.0	1032
	7	2130-2225	13	8.0	1.6	
	9	2230-2250	6.0	4.6	1.3	

Date	Channel	Time EST	Voltage in 50 ohm line uv		Ratio 10%/50%	Atmospheric pressure at ground level (path mid point) mb 1800 hrs.EST
			10%	50%		
16. 6.1958	7	1900-2000	15	9.0	1.7	1028
	9	2010-2105	4.7	2.2	2.1	
	2	2112-2217	60	30	2.0	
18. 6.1958	2	1900-1957	38	22	1.7	1019
	7	2008-2100	5.0	2.5	2.0	
	9	2108-2215	2.0	1.0	2.0	
20. 6.1958	7	1910-2035	3.4	1.8	1.9	1020
	2	2045-2208	36	18	2.0	
	9	2215-2325	2.6	1.7	1.5	
23. 6.1958	2	1940-2035	9.0	5.2	1.7	1019
	7	2045-2145	1.8	1.0	1.8	
	9	2155-2255	1.2	1.0	1.2	
24. 6.1958	2	1940-2035	20	10	2.0	1030
	7	2200-2255	4.0	2.3	1.7	
25. 6.1958	7	1735-1830	9.8	5.0	2.0	1033
	2	1845-2000	47	23.5	2.0	
26. 6.1958	9	2135-2230	2.2	1.3	1.7	1027
27. 6. 1958	2	1905-2008	36	25	1.4	1025
28. 6.1958	7	1900-1955	3.7	1.7	2.2	1023
	9	1955-2125	2.0	1.2	1.7	
	2	2130-2221	25	18	1.4	
30. 6.1958	7	1840-2000	19	10	1.9	1017
	2	2005-2100	65	33	2.0	
	9	2110-2210	7.7	3.7	2.1	

Date	Channel	Time EST	Voltage in 50 ohm line uv		Ratio 10%/50%	Atmospheric pressure at ground level (path mid point) mb 1800 hrs.EST
			10%	50%		
2. 7.1958	7	1900-2000	3.6	2.0	1.8	1009
	2	2010-2100	18	12	1.5	
	9	2120-2210	1.6	1.0	1.6	
3. 7.1958	7	1800-1905	2.4	1.5	1.6	1004
	2	1920-2025	21	15	1.4	
4. 7.1958	2	1925-2035	12	7.0	1.7	1009
	7	2130-2245	2.0	1.3	1.5	
	9	2245-2320	1.0	less than 1	-	
6. 7.1958	2	1850-1950	16	8.0	2.0	1022
7. 7.1958	7	1845-1950	1.5	1.0	1.5	1025
	2	2000-2115	8.0	4.7	1.7	
	9	2150-2245	1.0	less than 1	-	
8. 7.1958	7	1850-1945	2.8	1.3	2.2	1024
	2	1950-2050	18	8.0	2.25	
9. 7.1958	2	1815-1910	11.8	7.0	1.7	1022
	7	1915-2035	1.5	less than 1	-	
	9	2045-2155	1.0	"	-	
10. 7.1958	2	2010-2055	13	8.0	1.6	1024
11. 7.1958	2	1905-2010	10.5	4.7	2.2	1022
13. 7.1958	2	1930-2030	19	12	1.6	1018
	7	2050-2150	2.3	1.0	2.3	
	9	2200-2250	2.3	1.1	2.1	

Date	Channel	Time EST	Voltage in 50 ohm line uv		Ratio 10%/50%	Atmospheric pressure at ground level (path mid point)
			10%	50%		mb 1800 hrs.EST
14. 7.1958	2	1850-1955	12	6.7	1.8	1012
	7	2020-2125	2.2	1.0	2.2	
	9	2145-2255	less than 1	less than 1	-	
15. 7.1958	2	1810-1955	6.0	4.3	1.4	1001
16. 7.1958	7	2030-2125	1.3	less than 1	-	998
17. 7.1958	7	1905-1950	less than 1	less than 1	-	996
	2	2000-2040	5.8	3.0	1.9	
18. 7.1958	2	1905-2005	9.5	5.6	1.7	1002
	7	2030-2125	2.2	1.5	1.5	
	9	2130-2230	1.4	1.0	1.4	
19. 7.1958	2	1030-2130	21	13	1.6	1000
20. 7.1958	2	2000-2040	11.5	5.8	2.0	1012