Australian Broadcasting Control Board

TECHNICAL SERVICES DIVISION

REPORT No. 22

TITLE: Envelope Modulation

Prepared by J. M. Dixon - June, 1960

Issued by:-

The Chairman,
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Abstract

A study of envelope modulation by signal synthesis reveals three solutions to the generation of wave modulation with no appreciable distortion. In each case three signals are required for distortion free sine wave modulation. One solution corresponds to double sideband transmission while the other two correspond to single sideband transmission.

Signals produced in the Kahn compatible single sideband system are calculated and the system performance evaluated according to the developed criterion.

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(D. McDonald)
Director
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Date of issue
26/10/1960

Envelope Modulation

1. Introduction

Amplitude modulation of a sinusoidal signal may be accomplished by several simple circuits. The resulting signal is represented by equation (1) which, when expanded shows the familiar upper and lower sideband components.

$$E = Em\ (1 + f(t))\ \sin \Omega t \qquad \ldots\ldots\ldots\ldots(1)$$

$$= Em\ \Big(\sin \Omega t + \frac{m_1}{2} \sin (\Omega + \omega_1)t + \frac{m_1}{2} \sin (\Omega - \omega_1)t$$

$$+ \frac{m_2}{2} \sin (\Omega + \omega_2)t + \frac{m_2}{2} \sin (\Omega - \omega_2)t$$

$$+ \text{--------}\Big) \qquad \ldots\ldots\ldots\ldots(2)$$

The envelope of this signal is immune to harmonic generation by symmetrical amplitude/frequency and symmetrical phase/frequency networks tuned to the carrier frequency, phase changes by such networks being transferred to the envelope. While these characteristics have been found to be advantageous in sound broadcasting, they are obtained in exchange for bandwidth in sections of the radio frequency spectrum, fast becoming overcrowded. Service demands in the medium frequency and high frequency bands have increased rapidly, and if met would inevitably result in reductions of some imposed limits at the expense of desirable transmission standards. There is however one degree of freedom left since typical receivers employ diode envelope detectors which are insensitive to phase modulation. The first section of this report is confined to a consideration of envelope modulation. Within limits imposed only by multiple sinusoidal signal transmission, the combination given in equation (2) is shown to be one of three possible solutions to the synthesis of an envelope signal. Some features of the additional solutions show that more efficient use may be made of the spectrum.

A theoretical analysis of a modulator developed by Kahn[1, 2] is presented in the second section of this report.

..2/.

1.2 Signal Synthesis

Since modulation is the result of signal additions, possible systems may be investigated by a study of equivalent vector additions. It must be assumed from the outset that no consideration could be given to a system which impaired reception on existing domestic receivers. The signal envelope must therefore remain undistorted while incidental phase modulation is permitted.

To simplify the initial analysis a sinusoidal envelope will be considered. The addition of two signals results in a complex waveform especially for high degrees of modulation. More components are therefore necessary if the envelope is to be free from inherent distortion. Figure 1 shows the vector diagram for three components where the angular frequency of each vector is chosen so that

$$\omega_a - \omega_e = \omega_b - \omega_a \qquad \ldots\ldots\ldots\ldots(3)$$

which corresponds to equal frequency spacing. The relative phase is chosen so that at some stage in the modulation cycle all three vectors are in phase.

$$|R| = (a^2 + b^2 + c^2 + 2a\,(b + c)\cos\theta + 2bc\cos 2\theta)^{\frac{1}{2}}$$

$$= d + e\cos\theta \qquad \ldots\ldots\ldots\ldots(4)$$

A solution in this form is obtained if

$$b = c \text{ or } \frac{a^2}{4c} \qquad \ldots\ldots\ldots\ldots(5)$$

in which case

$$d = \frac{a(b + c)}{2(bc)^{\frac{1}{2}}} = a \text{ or } b + c$$

and $e = 2\,(bc)^{\frac{1}{2}} = 2\,b \text{ or } a$

If A represents a carrier, i.e., constant amplitude and constant frequency, the solution $b = \frac{a^2}{4c}$ does not permit a small depth of modulation and is therefore discarded. The only solution in this case, b = c is independent of the carrier and corresponds to the double sideband system shown in equation (2).

..3/.

If C is chosen as a carrier, the solution b = c = constant does not permit a change in envelope signal level and is therefore discarded. The only solution in this case is $b = \frac{a^2}{4c}$ which depicts a system having two sideband signals on one side of the carrier as shown in equation (6).

$$E = c \sin \Omega t + a \sin (\Omega + \omega)t + \frac{a^2}{4c} \sin (\Omega + 2\omega)t \qquad \text{..........}(6)$$

Envelope detection of this signal gives

$$c + \frac{a^2}{4c} + a \cos \omega t \qquad \text{..........}(7)$$

which is free from harmonic distortion.

The conception of three signals with appropriate relative phase and equal frequency spacing is therefore fundamental in the synthesis of a distortion free envelope. No restriction is necessarily placed on which of the three signals represents a fixed frequency carrier although the choice determines the required amplitude ratio of the remaining components.

A system in which sideband signals are placed on one side of the carrier does not at first appear to offer any advantages in spectrum economy over double sideband transmission since ideally both require the same bandwidth. However when the receiver audio bandwidth is limited to that for which the system is designed, only a reduction in amplitude will result when modulating frequencies exceed half the design bandwidth. Harmonics generated in the detector as a result of sideband component elimination in the intermediate frequency section of the receiver, are not passed by the audio section and therefore the output is free from inherent harmonic distortion at any frequency. Such a system eliminates the redundant character of double sideband transmission requiring a transmission bandwidth double that of the maximum modulating frequency.

In the general case of two or more modulating signals, double sideband transmission requires the addition of sideband pairs as in equation (2). Apparently no rigorous solution is possible for single sideband transmission and a choice must be made which will produce a minimum error. This conclusion also follows from the fact that the amplitude of one component is a function of the other two. A simple case to study is that where a second pair of sideband signals is added which obey all the conditions for zero distortion shown in equation (6). This requires the phase of the added sideband signals to be relative to R_1 which is phase modulated.

$$|R_2| = |R_1|' + \frac{g^2}{4|R_1|} + g \cos \omega_2 t$$

$$= c + \frac{a^2}{4c} + \frac{g^2}{4(c + \frac{a^2}{4c} + a \cos \omega_1 t)} + a \cos \omega_1 t + g \cos \omega_2 t$$

........(8)

The error factor in equation (8) reaches a maximum of $\pm$ 4% and although this example is not precisely what may be expected in a practical system, it serves to show the order of error involved. A reduction in this error may be expected from envelope feedback applied at the transmitter.

2. Compatible Single Sideband Signal Generation

Kahn has produced an adapter which when combined with a conventional medium frequency sound broadcasting transmitter develops a low distortion single sideband envelope signal. This is accomplished by suitable phase modulation of the radio frequency signal fed to the modulated amplifier. A simplified block diagram of the system is shown in figure (2).

Amplitude modulation in the single sideband generator output is eliminated by limiters leaving a phase modulated signal. Audio output from the product demodulator is free from harmonic distortion but apparently contains phase changes of the single sideband component. The phase of the audio signal appearing at the modulator may therefore be assumed to follow any additional change in phase of the limiter output due to the single sideband generator. Phase changes of this nature will therefore not be considered in developing the output signal equation.

2.1 Phase Modulation

Let the single sideband generator output be 100% modulated. The resulting signal phase increases linearly with time relative to the carrier and contains discontinuities at times $\frac{\pi}{\omega}, \frac{3\pi}{\omega}$ etc. which produce a saw-tooth phase characteristic of the form

$$\theta = \frac{2\theta m}{\pi}\left(\sin \omega t - \tfrac{1}{2}\sin 2\omega t + \tfrac{1}{3}\sin 3\omega t - \tfrac{1}{4}\sin 4\omega t + - - - - -\right) \qquad ....(9)$$

The limiter output is therefore

$$E = E_m \sin (\Omega t + \sin \omega t - \tfrac{1}{2} \sin 2\omega t + \tfrac{1}{3} \sin 3\omega t - \tfrac{1}{4} \sin 4\omega t + \text{-------}) \qquad (10)$$

Frequency multiplication of this signal increases the phase deviation by a factor of 1.4, the operation being multiplication by 7 and division by 5. This manipulation will be shown later to be necessary if the required wanted sideband signals are to be produced with minimum energy in the unwanted sideband. Equation (10) now becomes -

$$\begin{aligned} E = E_m \sin (\Omega t &+ 1.40 \sin \omega t - .70 \sin 2\omega t \\ &+ .466 \sin 3\omega t - .350 \sin 4\omega t \\ &+ .280 \sin 5\omega t - \text{--------}) \end{aligned} \qquad (11)$$

A solution to this equation is found by studying the solution to the general equation for a phase modulated signal [3,4,5] which is

$$\begin{aligned} E &= E_m \sin (\Omega t + \beta \sin \omega t) \\ &= E_m \Big[J_0(\beta) \sin \Omega t + J_1(\beta) [\sin (\Omega + \omega)t - \sin (\Omega - \omega)t] \\ &\qquad + J_2(\beta) [\sin (\Omega + 2\omega)t + \sin (\Omega - 2\omega)t] \\ &\qquad + J_3(\beta) [\sin (\Omega + 3\omega)t - \sin (\Omega - 3\omega)t] \\ &\qquad + J_4(\beta) [\sin (\Omega + 4\omega)t + \sin (\Omega - 4\omega)t] \\ &\qquad + \text{-------} \Big] \end{aligned} \qquad (12)$$

A second phase modulation term such as $\beta_2 \sin \omega_2 t$ (Appendix 1), produces phase modulation of each component signal in this general equation. Therefore each term in equation (12) acts as a carrier for the second phase modulating signal and each term produces its own sidebands in accordance with equation (12). It may therefore be assumed that each succeeding phase modulating term, as in equation (11) acts in the same manner. A difficulty arises over the sign of the phase modulation terms which alternate positive and negative in equation (11). This may be overcome by studying the influence of each sideband pair in equation (12). Whereas both even and odd sideband pairs contribute to the maintenance of a constant amplitude signal, only odd sideband pairs determine the direction of phase swing and in a sense, only odd sideband pairs contribute to phase modulation since their withdrawal would leave the signal unmodulated in phase.

Therefore as a negative sign in the phase modulation term indicates a phase swing in the opposite direction, the effect on the solution to equation (11) must be a change in the sign of each odd sideband pair which changes the direction of phase swing. The sideband distribution of a phase modulated signal with more than one modulating signal may therefore be determined by adapting equation (4) Appendix 1 to the general solution.

Equation (11) converges rather slowly and consequently a large number of terms should be taken. It is assumed that modulation factors up to β_5 will be sufficient to show typical signal forms produced in the system although such a limitation on bandwidth will produce phase ringing and amplitude modulation of the phase modulated signal. Equation (13) shows the unsymmetrical sideband components produced after limiting and frequency multiplication.

$\beta_1 = 1.4$	$\beta_2 = .70$	$\beta_3 = .466$	$\beta_4 = .35$	$\beta_5 = .28$
$J_0(\beta_1) = .567$	$J_0(\beta_2) = .881$	$J_0(\beta_3) = .946$	$J_0(\beta_4) = .969$	$J_0(\beta_5) = .980$
$J_1(\beta_1) = .542$	$J_1(\beta_2) = .329$	$J_1(\beta_3) = .226$	$J_1(\beta_4) = .172$	$J_1(\beta_5) = .138$
$J_2(\beta_1) = .207$	$J_2(\beta_2) = .059$	$J_2(\beta_3) = .027$	$J_2(\beta_4) = .015$	$J_2(\beta_5) = .010$
$J_3(\beta_1) = .050$	$J_3(\beta_2) = .007$	$J_3(\beta_3) = .002$		
$J_4(\beta_1) = .009$				

$$
\begin{aligned}
E = Em \Big[.439 \sin \Omega t &+ .772 \sin (\Omega + \omega)t - .212 \sin (\Omega - \omega)t \\
&- .110 \sin (\Omega + 2\omega)t + .155 \sin (\Omega - 2\omega)t \\
&+ .020 \sin (\Omega + 3\omega)t - .131 \sin (\Omega - 3\omega)t \\
&+ .020 \sin (\Omega + 4\omega)t + .126 \sin (\Omega - 4\omega)t \\
&- .053 \sin (\Omega + 5\omega)t - .221 \sin (\Omega - 5\omega)t \\
&+ .101 \sin (\Omega + 6\omega)t + .155 \sin (\Omega - 6\omega)t \\
&+ .002 \sin (\Omega + 7\omega)t - .100 \sin (\Omega - 7\omega)t \\
&+ \text{--------} \Big] \qquad \ldots\ldots\ldots\ldots (13)
\end{aligned}
$$

Amplitude modulation by the audio signal from the product demodulator gives

$$E_1 = E(1 + m \cos \omega t) \qquad \ldots\ldots\ldots\ldots (14)$$

The modulation index m may be chosen to produce -

(1) sideband signals which satisfy the conditions determined in the first section of this report, having no regard for signal levels in the unwanted sideband,

(2) a minimum energy level in the unwanted sideband,

(3) a compromise of (1) and (2) above.

Solution (1) would require a sideband filter at the transmitter, solution (2) results in a distorted envelope while solution (3) produces an undistorted envelope provided the first component in the unwanted sideband is received. Frequency multiplication after limiting enables a solution to be obtained with low energy levels in the unwanted sideband.

Let m = 0.86

$$E_1 = E_m \Big[.68 \sin \Omega t + .91 \sin (\Omega + \omega)t + .04 \sin (\Omega - \omega)t + .23 \sin (\Omega + 2\omega)t + .01 \sin (\Omega - 2\omega)t - .02 \sin (\Omega + 3\omega)t - .01 \sin (\Omega - 3\omega)t + .01 \sin (\Omega + 4\omega)t - .02 \sin (\Omega - 4\omega)t + \text{-------} \Big] \quad \ldots\ldots\ldots\ldots (15)$$

The detected output of this signal is $.95\,E_m(1 + \cos \omega t)$ with about 5% distortion.

Equation (15) should be compared with test results obtained by Harmon[6] on a Kahn adapter at KDKA which are repeated below.

carrier	87%
1st upper sideband	95%
2nd upper sideband	25%
1st lower sideband	3%

These signal amplitudes are in correct proportion to produce a low distortion envelope according to the solution to equation (4) and agree reasonably well with the calculated values except for the carrier amplitude.

..8/.

2.2 Detected Output at Medium Depths of Modulation

Let the single sideband generator output be 50% modulated. A graphical analysis gives the phase deviation produced before frequency multiplication as

$$\theta \simeq .51 \sin \omega t - .127 \sin 2\omega t + .053 \sin 3\omega t \qquad \text{...........(16)}$$

$\beta_1 = .714$	$\beta_2 = .178$	$\beta_3 = .074$
$J_0(\beta_1) = .876$	$J_0(\beta_2) = .995$	$J_0(\beta_3) = .998$
$J_1(\beta_1) = .335$	$J_1(\beta_2) = .088$	$J_1(\beta_3) = .037$
$J_2(\beta_1) = .061$	$J_2(\beta_2) = .004$	
$J_3(B_1) = .007$		

$$E = Em \sin(\Omega t + .714 \sin \omega t - .178 \sin 2\omega t + .074 \sin 3\omega t)$$

$$\begin{aligned} = Em\Big[.97 \sin \Omega t &+ .37 \sin (\Omega + \omega)t - .30 \sin (\Omega - \omega)t \\ &- .02 \sin (\Omega + 2\omega)t + .12 \sin (\Omega - 2\omega)t \\ &+ .01 \sin (\Omega + 3\omega)t - .07 \sin (\Omega - 3\omega)t \\ &+ .01 \sin (\Omega + 4\omega)t + .02 \sin (\Omega - 4\omega)t \\ &+ - - - - - \Big] \qquad \text{...........(17)} \end{aligned}$$

Equation (17) shows the sideband distribution of the signal applied to the modulated amplifier.

$$E_1 = (1 + m \cos \omega t)E$$

Let m = 0.58 for minimum unwanted sideband level.

$$\begin{aligned} E_1 = E_m \Big[.99 \sin \Omega t &+ .65 \sin (\Omega + \omega)t + .016 \sin (\Omega - \omega)t \\ &+ .094 \sin (\Omega + 2\omega)t + .017 \sin (\Omega - 2\omega)t \\ &- .005 \sin (\Omega + 3\omega)t - .028 \sin (\Omega - 3\omega)t \\ &+ .015 \sin (\Omega + 4\omega)t + - - - - \Big] \qquad \text{..........(18)} \end{aligned}$$

The detected output of this signal is -

$$Em\ (1.09 + .66 \cos \omega t) \text{ with 4\% distortion.}$$

Table 1 shows the calculated detector output for high, medium and low depths of modulation when m is chosen for minimum energy level in the unwanted sideband and minimum harmonic distortion in the detector output. These conditions are maintained through all depths of modulation provided the adapter audio section gain varies with signal level. Neither the required amplitude modulation index nor the detector output are linearly related to the input. Optimum conditions for minimum distortion are also influenced by the first lower sideband signal and to this extent reception is not completely single sideband.

Input index	Maximum phase deviation of the phase modulated signal	Modulated amplifier amplitude modulation index	Envelope detector output index
1.0	2.2 radians	.86	.95
0.5	.73	.58	.66
0.1	.14	.14	.14

Table 1 Calculated index values required for low envelope distortion and low unwanted sideband energy when a phase modulated signal is amplitude modulated as in the Kahn system.

2.3 Two Modulating Signals

Let the input signals $.9 \cos \omega_1 t$ and $.05 \cos \omega_2 t$ be applied simultaneously so that the single sideband generator output is

$$E = \sin \Omega t + .9 \sin (\Omega + \omega_1)t + .05 \sin (\Omega + \omega_2)t \qquad \text{..........(19)}$$

where $\omega_2 \gg \omega_1$

After limiting and frequency multiplication we have -

$$\begin{aligned} E = Em \sin (\Omega t &+ 1.4 \sin \omega_1 t - .70 \sin 2\omega_1 t \\ &+ .47 \sin 3\omega_1 t - .35 \sin 4\omega_1 t \\ &+ .28 \sin 5\omega_1 t + - - - - - \\ &+ \beta_1^1 \sin \omega_2 t - \beta_2^1 \sin 2\omega_2 t + - - - -) \text{..........(20)} \end{aligned}$$

This equation is difficult to manipulate as the factors β_1^1, β_2^1 etc. are functions of the signal amplitude before passing through the limiters. Dominant phase modulation will alternate between the two signals according to the amplitude of the single sideband generator output. The radiated signal will therefore be examined at extreme stages of the first modulating signal cycle.

..10/.

Let $-\frac{\pi}{8\omega_1} < t < \frac{\pi}{8\omega_1}$

$$\beta_1^1 = .037$$

$$J_o\ (\beta_1^1) = 1.00$$

$$J_1\ (\beta_2^1) = .018$$

Apart from the sideband components giving phase modulation at frequencies f_1 and f_2 there will be sum and difference components of the form

$$\sin\ (\Omega + \omega_1 + \omega_2)t, \qquad \sin\ (\Omega + \omega_1 - \omega_2)t$$

$$\sin\ (\Omega - \omega_1 + \omega_2)t, \qquad \sin\ (\Omega - \omega_1 - \omega_2)t, \text{ etc.}$$

which for the limits imposed reduce to

$$E \simeq Em \left[\sin \Omega t + .018 \sin\ (\Omega + \omega_2)t - .018 \sin\ (\Omega - \omega_2)t\right] \qquad \text{..........}(21)$$

The audio section gain is assumed to be determined by the largest signal component which results in a modulated amplifier output of

$$E_1 = (1 + .82 \cos \omega_1 t + .043 \cos \omega_2 t)E \qquad \text{..........}(22)$$

$$\simeq Em\ (1.82 \sin \Omega t + .055 \sin\ (\Omega + \omega_2)t - .011 \sin\ (\Omega - \omega_2)t$$

The detected output is therefore

$$Em\ (1.82 + .044 \cos \omega_2 t)$$

Let $t \simeq \frac{\pi}{\omega_1}$

At this stage of the modulation cycle the single sideband generator output is reduced in level due to partial cancellation of the carrier by the sideband component of the first modulating signal, while the instantaneous frequency is reduced by flyback phase sweep.

..11/.

$$E \simeq Em \sin (\Omega t + 1.4 \sin \omega_1 t - .70 \sin 2 \omega_1 t$$
$$+ .47 \sin 3\omega_1 t + - - - - -$$
$$+ .71 \sin \omega_2 t - .18 \sin 2\omega_2 t + .07 \sin 3\omega_2 t) \quad \ldots\ldots\ldots\ldots (23)$$

$$E1 \simeq E_m \left[.18 + .043 \cos \omega_2 t \right] \left[.97 \sin \Omega t + \right.$$
$$.369 \sin (\Omega + \omega_2)t - .303 \sin (\Omega - \omega_2)t$$
$$- .017 \sin (\Omega + 2\omega_2)t + .125 \sin (\Omega - 2\omega_2)t$$
$$+ .013 \sin (\Omega + 3\omega_2)t - .070 \sin (\Omega - 3\omega_2)t$$
$$\left. + - - - - - \right] \quad \ldots\ldots\ldots\ldots (24)$$

$$= Em \left[.18 \sin \Omega t + .087 \sin (\Omega + \omega_2)t - .031 \sin (\Omega - \omega_2)t \right.$$
$$+ .005 \sin (\Omega + 2\omega_2)t + .015 \sin (\Omega - 2\omega_2)t$$
$$+ .002 \sin (\Omega + 3\omega_2)t - .010 \sin (\Omega - 3\omega_2)t$$
$$\left. + - - - - \right] \quad \ldots\ldots\ldots\ldots (25)$$

The detected output of this signal is

$$Em (.18 + .056 \cos \omega_2 t)$$

with harmonic distortion depending on the bandwidth received. There is therefore at least 12% cross modulation in the case considered. The output signal will in fact contain sum and difference components which have been removed by the simplifications involved between equations (24) and (25).

3. Greater Exploitation of the Medium Frequency Band

Medium frequency broadcast channel spacings of 10 kc/s and modulation up to 10 kc/s result in a certain degree of band sharing between adjacent channels. Adequate geographical spacing of adjacent channel stations ensures that mutual sideband signal interference is reduced to a low level within the service area. This spacing is usually dictated, not by the interfering modulation sideband signal but by the adjacent carrier which is detected as a

..12/.

10 kc/s tone. Many receivers rely upon restricted bandwidth to ensure sufficient attenuation of adjacent carrier signals despite the fact that the transmission standards permit wideband reception in high signal areas. This practice results in a radio frequency reception bandwidth of 6 kc/s or 8 kc/s with a guard band of 6 kc/s or 7 kc/s on each side for adjacent channel interference protection.

Against this background one must determine the possible opportunities offered by a low distortion single sideband system.

With the present channel structure and existing receivers it would appear possible to -

(1) Reduce interference from one adjacent channel station by tuning receivers away from the unwanted signal. Since the response to non linear phase/frequency and non uniform amplitude/frequency networks is harmonic distortion, detuning would probably be limited to about 2 kc/s from the carrier resulting in an improvement of 4 db to 8 db. Alternatively the permitted maximum adjacent channel signal strength could be increased by a similar amount when only one adjacent channel is involved.

(2) Decrease the geographical spacing between co-channel stations by operating one on the upper sideband and one on the lower sideband of the channel; or improve reception in those cases where co-channel interference has proved to be more severe than expected.

These improvements are of relatively minor magnitude and must be assessed against the increase in sideband chatter level resulting from a greater energy concentration in one sideband. Consideration of the conditions required for low distortion single sideband reception shows that where a standard receiver response may be taken, greater detuning becomes possible by pre-distortion of the radiated signal components in phase and amplitude with a corresponding reduction in adjacent channel and co-channel interference. When sufficient discrimination is obtained it seems quite possible for the tête-bêche channel arrangement to provide more channels than are at present available.

The existing 10 kc/s adjacent channel band sharing method could be employed with a guard band of 5 kc/s on the unwanted sideband. This technique would release one additional channel for each three used at present, but would require a standard receiver and probably more critical receiver tuning than for double sideband reception.

..13/.

As already stated in the first section of this report, whatever form the amplitude/frequency and phase/frequency characteristics of receivers assume, no inherent harmonic distortion is generated for double sideband reception provided these responses are symmetrical and tuned centrally on the carrier. As a consequence it follows that no restriction is placed on the bandwidth of a receiver in so far as the modulation system is concerned. This is not the case for most efficient use of the single sideband system described here. The detuning required for efficient channel use and the consequent compensation necessary in amplitude and phase for low harmonic content can be expected to limit the choice of receiver bandwidth for satisfactory reception, and it is in this sense that the term standard receiver has been used.

Results of field tests conducted in the United States of America[6, 7] have shown that except for the reduction in adjacent channel interference possible with compatible single sideband, no clear difference in reception was consistently reported between compatible single sideband and double sideband signals. Encouraging as these results may be, the tête-bêche channel arrangement should not be applied to broadcast allocation planning without first conducting field tests in this type of operation, together with a thorough study of existing receiver characteristics, since the success of such a venture would depend largely on receiver performance in the system.

Other problems not directly related to engineering technique are involved, and must be solved before any general application of the system. The only incentive at present likely to persuade a broadcaster to install single sideband equipment is an improvement of reception in the presence of co-channel or adjacent channel interference.

Symbols

θ Instantaneous phase of one signal relative to another.

θ_m Maximum phase deviation.

F A carrier frequency

$\Omega = 2\pi F$

f An audio frequency

$\omega = 2\pi f$

Em A constant

E A phase modulated signal

E_1 A low distortion single sideband signal

t Time

f(t) A function of time

A, B, C, R Vectars

a, b, c, g Vectar magnitudes

m Amplitude modulation index

β Phase modulation index

$J_0(\beta)$, $J_1(\beta)$ etc. Bessel factors.

Appendix 1

When two phase modulating tones are applied simultaneously, the phase deviation will be

$$\beta_1 \sin \omega_1 t + \beta_2 \sin \omega_2 t$$

$$E = Em \sin (\Omega t + \beta_1 \sin \omega_1 t + \beta_2 \sin \omega_2 t)$$

$$= Em \Big\{ \left[\sin \Omega t . \cos (\beta_1 \sin \omega_1 t) + \cos \Omega t . \sin (\beta_1 \sin \omega_1 t)\right] \cos (\beta_2 \sin \omega_2 t)$$

$$+ \left[\cos \Omega t . \cos (\beta_1 \sin \omega_1 t) - \sin \Omega t . \sin (\beta_1 \sin \omega_1 t)\right] \sin (\beta_2 \sin \omega_2 t) \Big\} \qquad \ldots\ldots\ldots\ldots (1)$$

Since

$$\cos (\beta \sin\theta) = J_0(\beta) + 2 \left[J_2(\beta) \cos 2\theta + J_4(\beta) \cos 4\theta + \text{-----}\right] \qquad \ldots\ldots\ldots\ldots (2)$$

and

$$\sin (\beta \sin \theta) = 2 \left[J_1(\beta) \sin \theta + J_3 (\beta) \sin 3\theta + \text{-------}\right] \qquad \ldots\ldots\ldots\ldots (3)$$

We have

$$E = Em \Big\{ J_0(\beta_1) \Big[J_0(\beta_2) \sin \Omega t + J_1(\beta_2) \left[\sin(\Omega + \omega_2)t - \sin (\Omega - \omega_2)t\right]$$

$$+ J_2(\beta_2) \left[\sin(\Omega + 2\omega_2)t + \sin (\Omega - 2\omega_2)t\right]$$

$$+ J_3(\beta_2) \left[\sin(\Omega + 3\omega_2)t - \sin (\Omega - 3\omega_2)t\right]$$

$$+ \text{- - - - -} \Big]$$

$$+ J_1(\beta_1) \Big[J_0(\beta_2) \left[\sin (\Omega + \omega_1)t - \sin (\Omega - \omega_1)t\right]$$

$$+ J_1(\beta_2) \big[\sin (\Omega + \omega_1 + \omega_2)t - \sin (\Omega + \omega_1 - \omega_2)t$$

$$- \sin (\Omega - \omega_1 + \omega_2)t + \sin (\Omega - \omega_1 - \omega_2)t\big]$$

$$+ J_2(\beta_2) \big[\sin (\Omega + \omega_1 + 2\omega_2)t + \sin (\Omega + \omega_1 - 2\omega_2)t$$

$$- \sin (\Omega - \omega_1 + 2\omega_2)t - \sin (\Omega - \omega_1 - 2\omega_2)t\big]$$

$$+ \text{- - - - - -} \Big]$$

../.

$$
\begin{aligned}
&+ J_2(\beta_1)\Big[J_0(\beta_2)\big[\sin(\Omega + 2\omega_1)t + \sin(\Omega - 2\omega_1)t\big] \\
&\qquad + J_1(\beta_2)\big[\sin(\Omega + 2\omega_1 + \omega_2)t - \sin(\Omega + 2\omega_1 - \omega_2)t \\
&\qquad + \sin(\Omega - 2\omega_1 + \omega_2)t - \sin(\Omega - 2\omega_1 - \omega_2)t\big] \\
&\qquad + J_2(\beta_2)\big[\sin(\Omega + 2\omega_1 + 2\omega_2)t + \sin(\Omega + 2\omega_1 - 2\omega_2)t \\
&\qquad + \sin(\Omega - 2\omega_1 + 2\omega_2)t + \sin(\Omega - 2\omega_1 - 2\omega_2)t\big] \\
&\qquad + - - - - - \Big] \\
&+ J_3(\beta_1)\Big[J_0(\beta_2)\big[\sin(\Omega + 3\omega_1)t - \sin(\Omega - 3\omega_1)t\big] \\
&\qquad + J_1(\beta_2)\big[\sin(\Omega + 3\omega_1 + \omega_2)t - \sin(\Omega + 3\omega_1 - \omega_2)t \\
&\qquad\qquad - \sin(\Omega - 3\omega_1 + \omega_2)t + \sin(\Omega - 3\omega_1 - \omega_2)t\big] \\
&\qquad + J_2(\beta_2)\big[\sin(\Omega + 3\omega_1 + 2\omega_2)t + \sin(\Omega + 3\omega_1 - 2\omega_2)t \\
&\qquad\qquad - \sin(\Omega - 3\omega_1 + 2\omega_2)t - \sin(\Omega - 3\omega_1 - 2\omega_2)t\big] \\
&\qquad + - - - - - - \Big] \\
&+ - - - - - - \Big\} \qquad \ldots\ldots\ldots\ldots(4)
\end{aligned}
$$

This solution shows that the carrier and each sideband produced by the first modulating signal alone act as carriers for the second modulating signal producing a series of terms for each which are typical of a phase modulated signal.