DUP

AUSTRALIAN BROADCASTING CONTROL BOARD

373 Elizabeth Street, Melbourne, C.1

TECHNICAL SERVICES DIVISION

REPORT NO.27

TITLE: The Absorption of Medium Frequency Sky-Waves by Close Coupling to the Extraordinary Mode.

Issued by -

The Chairman,
Australian Broadcasting Control Board,
373 Elizabeth Street,
MELBOURNE.

TELEPHONE: 30 4321 — CODE ADDRESS: CONBOARD. MELBOURNE

TECHNICAL REPORT NO.27

Title: The Absorption of Medium Frequency Sky-Waves by Close Coupling to the Extraordinary Mode.

Prepared by J.M. Dixon, September, 1965.

a J McKenzie

(A. J. McKenzie)
Acting Director.
Technical Services Division.

Abstract

A method of transmission is described whereby medium frequency sky-wave signals are subjected to considerably greater absorption than that imposed on a vertically polarised transmission. Details are given of propagation tests conducted to confirm the predicted reduction in sky-wave field strength for propagation to the north in the southern hemisphere. The observed reduction varied from 12db when the field strength was low, to 20db when the field strength was high, with a median reduction of 16db.

Under favourable conditions, the proposed method of transmission should be applicable to the reduction of common channel interference and to the extension of primary service areas restricted by fading zones.

The Absorption of Medium Frequency Sky-Waves by Close Coupling to the Extraordinary Mode.

By: J. M. Dixon

A radio wave incident upon the ionosphere splits into two waves referred to as the ordinary wave and the extraordinary wave. Both waves are elliptically polarised, the resultant vector rotations are in opposite directions, and the major axes are at right angles*. The whole phenomenon is a direct result of the earth's magnetic field.

1, 2, 3.

The ordinary wave critical frequency of the E layer is the same as that calculated for no magnetic field, but that for the extra-ordinary wave is lower when the gyro-frequency exceeds the transmission frequency, and consequently this wave penetrates deeper into the ionosphere than the ordinary wave. Under certain conditions it is subjected to considerably greater absorption than the ordinary wave.

As a corollary to this description of radio wave behaviour in the ionosphere, it may be argued that a wave can not propagate through the medium unless it possesses one or other of the required elliptical polarisations, and that if an incident wave does not possess the required characteristics, then these will be generated within the ionosphere.

---

* Major axes are not at right angles for small values of $\left(\mu H e/m\right)\left(\frac{1}{2}\sin^2\theta/\cos\theta\right)\frac{1}{\nu}$ i.e. when the collision frequency of the electrons is not negligible.

..2/..

A vertically polarised wave incident upon the ionosphere is closely coupled to the ordinary mode except for east-west or west-east propagation at or near the gyro-frequency in the vicinity of the magnetic dip equator. The division of power between ordinary and extraordinary modes depends upon the incident wave polarisation and the particular polarisation form of the ordinary and extraordinary modes. This latter condition varies with magnetic dip, magnetic bearing of the path, angle of incidence, transmission frequency and gyro-frequency.[4,5.]

The purpose of this paper is to suggest a more efficient propagation system than that at present in use, whereby the polarisation of a medium frequency transmission is changed from vertical to the particular elliptical form required for propagation by the extraordinary mode in the ionosphere. This will ensure virtually no propagation via the ordinary mode, and consequently the sky-wave signal received at the surface of the earth would be severely attenuated due to high absorption of the extraordinary wave. In the remaining sections of the paper this system will be referred to as orthogonal transmission.

Successful application of this system requires a knowledge of the conditions under which the extraordinary wave is more severely attenuated than the ordinary wave. It is also essential to know the extraordinary wave polarisation at the lower edge of the noctural E layer.

Power Division Between Ordinary and Extraordinary Modes.

The degree of coupling between an incident elliptically polarised wave and a propagation mode in the ionosphere has been determined by Phillips[5] and is given by the power coupling factor F.

..3/..

$$F = \frac{(1+M_i M_m)^2 \cos^2(\psi_i - \psi_m) + (M_i + M_m)^2 \sin^2(\psi_i - \psi_m)}{(1+M_i^2)(1+M_m^2)}$$

Where F is the ratio of the induced power in the mode of propagation to the incident power,

M_i is the ratio of minor axis to major axis of the polarisation ellipse for the incident wave,

M_m is the ratio of minor axis to major axis of the polarisation ellipse for the mode of propagation,

ψ_i is the angle from vertical of the major axis of the polarisation ellipse for the incident wave,

ψ_m is the angle from vertical of the major axis of the polarisation ellipse for the mode of propagation.

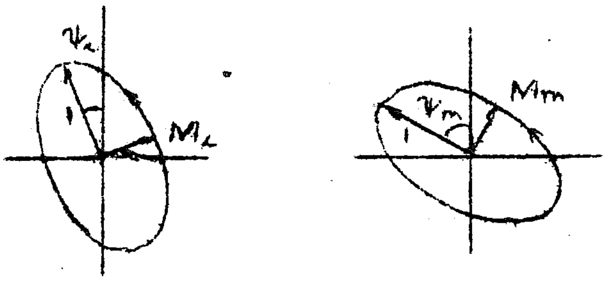


Figure 1

Case I

Consider the propagation through the ionosphere by way of the ordinary mode.

Let $\psi_i = \psi_m \pm 90°$

$$F = \frac{(M_i + M_m)^2}{(1+M_i^2)(1+M_m^2)}$$

Let $M_i = -M_m$

$$F = 0$$

..4/..

All the incident power is therefore propagated by the extraordinary mode. The proportion of total power in the vertical component increases with increase in M which increases the system efficiency for ground wave reception.

At the gyro-frequency M is given by the cosine of the angle between the direction of propagation and the direction of the earth's magnetic field. If the electron collision frequency is negligible, the major axis of the ordinary wave polarisation ellipse is in the same plane as that containing the direction of the earth's magnetic field and the direction of propagation. With a magnetic dip of -67°, M_m for the ordinary wave varies from 1.0 to 0.54 when the direction of propagation is towards the north. For propagation to the south M_m varies from 0.92 to -0.22, and for propagation towards the east or west M_m varies from 0.92 to 0.16.

Case II

Consider propagation through the ionosphere by way of the ordinary mode.

Let $\psi_{\ell} = \psi_m = \psi$

$$F = \frac{(1 + M_{\ell} M_m)^2}{(1+M_{\ell}^2)(1+M_m^2)}$$

If $M_{\ell} = -M_m = 1$ (circular polarisation)

$$F = 0$$

Case III

Consider propagation through the ionosphere by way of the ordinary mode.

$$\text{Let } \psi_t = \psi_m \pm 90^\circ$$
$$F = \frac{(M_t + M_m)^2}{(1+M_t^2)(1+M_m^2)}$$
$$\text{Let } M_t = M_m = 0$$
$$F = 0$$

For east-west or west-east propagation at the magnetic dip equator, $\psi_m = 90^\circ$, and all the energy of a vertically polarised wave is propagated through the ionosphere via the extraordinary mode.[6] There is experimental evidence from Africa[7] which indicates that the attenuation on paths with a magnetic bearing of $90^\circ \pm 12^\circ$ or $270^\circ \pm 12^\circ$ is approximately 17db greater than that for north-south paths. Attenuation along the magnetic dip equator is expected to exceed this figure.

Application Within the Medium Frequency Band

By comparing absorption indices of the ordinary and extraordinary waves[6] appropriate to the lower region of the noctural E layer, it is possible to determine the range of transmission frequencies which would provide sufficient difference in absorption for the method to be successfully applied. The conclusions reached are therefore based entirely on the assumption that most medium frequency sky-wave absorption is nondeviative.

The absorption index is given by the Appleton-Hartree formula.

$$n^2 = (\mu - j\chi)^2 = 1 - \frac{X}{1 - jZ - \frac{\frac{1}{2}Y_T^2}{1-X-jZ} \pm \left\{ \frac{\frac{1}{4}Y_T^4}{(1-X-jZ)^2} + Y_L^2 \right\}^{\frac{1}{2}}}$$
$$= \mu^2 - \chi^2 - j2\chi\mu$$

Where $X = \frac{4\pi N e^2}{\epsilon_0 m \omega^2}$ = 1 at the critical frequency for no applied magnetic field.

$$Y = \frac{\omega_H}{\omega}$$

$$Y_T = Y \sin\theta$$

$$Y_L = Y \cos\theta$$

$$Z = \frac{\nu}{\omega}$$

ν = collision frequency of electrons with heavy particles.

$$\omega = 2\pi f$$

$$\omega_H = \mu_0 H_0 \frac{e}{m} = 2\pi f_H$$

f_H = gyro-frequency

N = number density of electrons

θ = angle between the earth's magnetic field and the direction of propagation.

Quasi Longitudinal Propagation

For propagation along the magnetic field lines

$$\theta = 0 \qquad Y_T = 0 \qquad Y_L = Y$$

$$n^2 = 1 - \frac{X}{1 - jZ \pm Y}$$

As X is less than unity in the lower E region, the + sign corresponds to the ordinary wave and the - sign corresponds to the extraordinary wave.

Y	X_-/X_+
0.5	$(2.25+Z^2)/(0.25+Z^2) \simeq 9$ for Z small
1	$\frac{4}{Z^2}$ for Z small
2	9 for Z small
3	4 for Z small

With an E region gyro-frequency of 1600 Kc/s (Victoria and Southern New South Wales), the method appears to have general application throughout the medium frequency band for quasi longitudinal propagation.

Quasi Transverse Propagation

For propagation at right angles to the magnetic field lines.

$$\theta = 90^\circ \qquad Y_T = Y, \qquad Y_L = 0$$

$$n^2 = 1 - \frac{X}{1 - jZ - \frac{1}{2}Y^2/(1-X-jZ) \pm \frac{1}{2}Y^2/(1-X-jZ)}$$

$$n_+^2 = 1 - \frac{X}{1-jZ}$$

$$n_-^2 \simeq 1 - \frac{X}{1-jZ-Y^2(1+jZ)/(1+Z^2)} \quad \text{for X small}$$

$$\simeq 1 - \frac{X}{1-jZ-Y^2(1+jZ)} \quad \text{for Z small}$$

Y	$\frac{X_-}{X_+}$
0.5	2.2
0.67	5
1	$\frac{1}{2Z^2}$
1.25	8
1.4	3.2
2	0.5

These calculations indicate that the lower frequency limit for quasi-transverse propagation is about 1150 Kc/s when the gyro-frequency is 1600 Kc/s. Application of the method for quasi-transverse propagation is therefore more restrictive than for quasi-longitudinal propagation.

Limits of Improvement Under Error Conditions

1. Quasi Longitudinal Propagation

Let $M_L = -M_m + \Delta M$ where $\Delta M << 1$

$\psi_L = \psi_m + 90^\circ - \Delta\psi$ where $\Delta\psi$ is small

$$F_1 = \frac{(1 - M_m^2 + M_m \Delta M)^2 \sin^2 \Delta\psi + \Delta M^2 \cos^2 \Delta\psi}{(1+M_m^2)^2}$$

$$\simeq \frac{(1-M_m^2)^2 \Delta\psi^2 + \Delta M^2}{(1+M_m^2)^2}$$

Let F_2 be the power coupling factor between a vertically polarised wave and the ordinary mode.

$$F_2' = \frac{\cos^2\psi_m + M_m \sin^2\psi_m}{1+M_m^2}$$

$$\psi_m = 0$$

$$F_2 = \frac{1}{1+M_m^2}$$

Let L be the ratio of received sky-wave power for an elliptically polarised transmission, to received sky-wave power for a vertically polarised transmission with the same total power radiated in each case. It is convenient to restrict this ratio to power propagated by way of the ordinary wave. The conclusions reached will therefore apply only to those cases in which it may be assumed that power propagated by way of the extra ordinary wave is completely absorbed in the ionosphere.

$$L = \frac{F_1}{F_2}$$

Let L_p be the ratio of received sky-wave power for an elliptically polarised transmission, to received sky-wave power for a vertically polarised transmission with same power radiated from the vertical aerial in each case.

$$L_p = \frac{F_1}{F_2}\left(1+\frac{1}{M_1^2}\right)$$

$$= \frac{\left[(1-M_m^2)^2\Delta\psi^2 + \Delta M^2\right]}{(1+M_m^2)^2}(1+M_m^2)\frac{(1+M_1^2)}{M_1^2}$$

$$\simeq \left(\frac{\Delta M}{M}\right)^2 + \left(\frac{\Delta\psi}{M}\right)^2(1-M^2)^2$$

$$L_p \simeq \left(\frac{\Delta M}{M}\right)^2 \qquad \text{when} \quad M \simeq 1$$

$$L_p \simeq \left(\frac{\Delta M}{M}\right)^2 + \left(\frac{\Delta\psi}{M}\right)^2 - 2\,\Delta\psi^2 \quad \text{when} \quad M \ll 1$$

These equations indicate that the available improvement is independent of ΔM and M when ΔM is a fixed proportion of M, but that if either ΔM or M is varied independently, the improvement decreases with increase in ΔM and decrease in M.

When the mode of propagation is elliptically polarised, ($M \ll 1$) the improvement available is, to a first order approximation, independent of $\Delta\psi$ when $\Delta\psi$ is a fixed proportion of M, but if either $\Delta\psi$ or M is varied independently, the improvement decreases with increase in $\Delta\psi$ and decrease in M.

If practical limits to the error quantities ΔM and $\Delta\psi$ are found to be related to M, then the application of this method would be limited only by economic considerations of the total transmitter power required, as the improvement attained would be independent of M. Table 1 shows the improvement to be expected for various degrees of error in M_α for values of M_m between 0.3 and 1.0 with $\Delta\psi = 0$.

Table 1

	$M_m=1$			$M_m=0.7$			$M_m=0.5$			$M_m=0.3$		
Error in M_α db	-3	-2	-1	-3	-2	-1	-3	-2	-1	-3	-2	-1
L (increase in transmission loss) db	13.7	16.8	22.2	15.6	18.6	24.1	17.7	20.7	26.2	21.5	24.6	30
Ratio of total effective transmitted power (northwards) to effective transmitted power from the vertical aerial (northwards). db	4.8	4.2	3.6	7.1	6.3	5.5	9.5	8.7	7.8	13.7	12.5	11.8
L_p (improvement in db for constant power to the vertical aerial)	8.9	12.6	18.6	8.5	12.3	18.6	8.2	12.0	18.4	7.8	12.1	18.2

..11/..

2. Quasi Transverse Propagation

$$\text{Let } \psi_{\ell} = \psi_m + 90^\circ - \Delta\psi \qquad \text{where } \Delta\psi \text{ is small}$$

$$M_m = 0 = M_{\ell} - \Delta M \qquad \text{where } \Delta M \ll 1$$

$$F_1 = \frac{\sin^2\Delta\psi + \Delta M^2\cos^2\Delta\psi}{1+\Delta M^2}$$

$$\simeq \Delta\psi^2 + \Delta M^2$$

$$F_2 = \frac{\cos^2\psi_m + M_m^2\sin^2\psi_m}{1+M_m^2} \simeq \cos^2\psi_m$$

$$L_p = \left(\frac{\Delta\psi^2 + \Delta M^2}{\cos^2\psi_m}\right)(1+\cot^2\psi_m)$$

$$= (\Delta M^2 + \Delta\psi^2)(2 + k + \frac{1}{k^2}) \qquad \text{where } k = \frac{E_H}{E_\psi}$$

The improvement therefore decreases with increase in the error terms ΔM and $\Delta\psi$ and with increase in magnetic dip beyond -45°. For southern Australia we have –

$L_p = -9$ db $\qquad \begin{cases} \Delta\psi = 0.1 \\ \Delta M = 0.1 \end{cases}$

$L_p = -15$ db $\qquad \begin{cases} \Delta\psi = 0.05 \\ \Delta M = 0.05 \end{cases}$

Approximately 7.5db more power would be required in the east-west direction.

Effective Power Gain

Where a secondary service is provided by medium-frequency sky-wave propagation, a maximum signal will be produced by strong coupling of the radiated power to the ordinary mode.

..12/..

In the case of south-north propagation we have -

$$M_{\perp} = M_{m} \quad , \qquad \psi_{\perp} = \psi_{m}$$

$$F_1 = 1$$

Let F_2 be the power coupling factor between a vertically polarised wave and the ordinary mode.

$$F_2 = \frac{1}{1+M^2}$$

$$L = \frac{F_1}{F_2} = 1+M^2$$

Therefore, if the effective radiated power is divided between a horizontally polarised component and a vertically polarised component to give strong coupling to the ordinary mode, instead of being radiated only from a vertical aerial, an effective power increase of $1+M^2$ will be evident in the received sky-wave signal. In this case it is not necessary to obtain precise matching to the ordinary mode.

Propagation Tests

Propagation tests were conducted with orthogonal transmission in August, 1965, on a north-south path between Melbourne and Hillston, a distance of 295 miles. The purpose of the test was not to provide a specific improvement in service, but merely to demonstrate in the simplest possible manner that a reduction in sky-wave field strength could be produced by a phenomenon peculiar to the propagation medium, as described in the first section of this paper. It was also considered desirable to conduct the test under conditions which were favourable and which would provide engineering data appropriate to the most likely application in Australia.

..13/..

As a general guide in the selection of conditions for the test, it was decided that -

1. the transmission frequency should be close to the gyro-frequency,

2. the value of M should be large,

3. both E and F modesshould be considered.

Details of the conditions selected for the test are as follows -

<u>Transmission frequency</u> 1230 Kc/s, 0.77f_H
<u>Transmitter power</u> 2 Kw.
<u>Transmitter location</u> Sydenham near Melbourne (50 Kw. National broadcasting transmitters 3AR and 3LO are operated at this site).
<u>Transmitting aerials</u>

<u>Vertical</u> - The 708 feet guyed mast used for 3AR and 3LO.

<u>Horizontal</u> - A half wave dipole slung from the vertical mast (at a height of 445 feet and supported by nylon cords tethered 1000 feet on either side of the mast at ground level. Magnetic bearing of anchor points 88° and 268°.

<u>Receiving Sites</u>

1. Hillston at a distance of 295 miles and magnetic bea^r^ing of 359° from the transmitter.

..14/..

2. Hay at a distance of 222 miles and magnetic bearing of 351.5° from the transmitter.

Duration of test Four nights with transmission from 00 30 hours to 04 45 hours E.S.T.

The circuit shown in figure 2 was designed to provide a variable power ratio between the two aerials and continuous phase change through 360°. The latter was obtained by a $0-90^\circ$ phase change circuit, a 180° phase change switch and provision to shift the phase change circuit from one aerial feed to the other. Figure 3 shows the calculated vertical polar diagrams of each aerial and the accompanying M plot which was produced by assuming the aerials to have phase centres located at the same point and similar phase polar diagrams for $70^\circ > \alpha > 20^\circ$. The M plot indicates a fairly satisfactory match to ideal M_m values for angles of elevation up to 60°, and consequently both E and F modes may be expected to satisfy the required test conditions in the lower region of the E layer simultaneously, and within reasonable limits, over an appreciable range in distance.

During the test, power was switched periodically from the horizontal dipole to dummy load in order to measure the improvement factor. In this manner a search was made for the greatest reduction in field strength with variation of phase for a constant power ratio. This was repeated for several power ratios and the optimum phase setting determined in each case. By plotting the optimum phase against power ratio and sky-wave reduction against power ratio (figures 4 and 5) a pattern was established which indicates the region of greatest improvement. A detailed search was made of this region to determine the optimum conditions of power ratio and phase. After establishing these values the circuit was left unaltered for the remaining period of the test and recordings made (figure 6 (a) with power switched alternatively to the horizontal dipole (3 minutes) and to dummy load (2 minutes). Figure 7 (a) shows the variation observed in the improvement factor with change in field strength. This factor has a median value of -15db to -16db but varies with field strength in such a way that a maximum improvement of -20db is obtained with maximum field strength and a mimimum improvement of -12db is obtained with minimum field strength. The distribution of field strength short term values changes from skew, for a vertically polarised transmission, to normal for orthogonal transmission.

After completing the test at Hillston, the field party moved 81 miles to Hay, where much the same improvement factors were obtained (figures 6(b), 7(b)) as at Hillston using the same optimum settings of power ratio and phase as those established at Hillston.

Figure 8 shows the variation in improvement factor with change in phase for different power ratios. The optimum improvement obtained for each power ratio error is in good agreement with that predicted in Table 1, but the variation with phase change is greater than that anticipated.

The decrease in field strength improvement factor, $^{E}(V+H)/^{E}(V+D)$, with increase in field strength, $^{E}(V+D)$, is a fortunate characteristic of orthogonal transmission. It is due to the relatively steady median values of $^{E}(V+H)$ compared with the changes which occur in $^{E}(V+D)$. Results for Hay show a better correlation between $^{E}(V+H)$ and $^{E}(V+D)$ than those for Hillston and there is less variation of the improvement factor with change in field strength. If sporadic E transmission is ignored, it may be argued that reception at Hillston could have been influenced to some extent by the ordinary ray critical frequency for one hop E, which on this path would be close to the transmission frequency. As the gyro-frequency exceeds the transmission frequency, the extraordinary ray appropriate to one hop E would pass through the E layer and play no further part in reception at Hillston. Therefore, one hop E transmission would be severely attenuated by close coupling to the extraordinary mode, irrespective of the absorption characteristics. This leaves transmission by way of the F layer for both ordinary and extraordinary modes. According to the calculated transmitting aerial vertical polar diagrams, the effective radiated power for one hop F transmission to Hillston is expected to be approximately the same as that for one hop E. Consequently the extraordinary wave must have been subjected to considerably greater absorption than the ordinary wave, whether Hillston was within the skip distance for one hop E (ordinary), or whether it was not.

The vertical incidence critical frequency would have to exceed 630 kc/s for one hop E transmission to have occurred between Melbourne and Hay, which seems unlikely, particularly for the time of night when these tests were conducted. It must therefore, be assumed that reception at Hay was by way of one hop F and two hop F transmission. In this respect it is significant that much the same improvement was measured at Hay as at Hillston. The influence of one hop E transmission on the improvement measured at Hillston, therefore seems to have been small.

The situation will be somewhat different in tropical regions where the gyro-frequency is about 1,000 kc/s. In this case, sky waves propagated on frequencies above the gyro frequency and received beyond the E layer skip distance, will always be accompanied by an extraordinary wave as this has less penetration into the ionosphere than the ordinary wave.

In order to test the predicted effective power gain, a phase change of 180° was made from the optimum setting for sky-wave reduction, thereby closely coupling the transmission to the ordinary mode. The same test procedure was adopted as that described previously. Power was switched alternatively from the horizontal aerial to dummy load while radiating continuously from the vertical aerial. An increase in field strength of 5.5db median was observed compared with an expected increase of 6db. This represents an effective power gain of 0.5db compared with an exepcted effective power gain of 1db.

After considering the significance of the results obtained at Hillston and Hay, it was decided to verify the calculated radiation appropriate to one hop E reception at Hillston. Field strength measurements were made 4.1 miles north of the transmitter, at ground level, and from an aircraft flying at an altitude of 8500 feet. The same optimum power ratio and phase settings were used for these measurements as those established for minimum sky-wave field strength at Hillston.

Measurements made in the aircraft were in accordance with the polar diagrams shown in figure 3 (a); the ratio of vertically polarised to horizontally polarised field, with power fed to both aerials, being 0.68. When power was diverted from the horizontal aerial to dummy load, the change in vertically polarised field strength was 2db at the aircraft and 1db at ground level. In this condition, virtually no horizontally polarised field was evident at the aircraft. Taken together, the ground wave and aircraft measurements indicate the presence of an unattenuated ground based lobe extending to 0.17 units in figure 3 (a) for the vertical aerial.

Conclusion

A method of transmission has been described which will produce a substantial reduction in medium frequency sky-wave field strength without altering the shape of ground wave service areas. At mid latitudes the method may be employed for propagation to the north in the southern hemisphere and to the south in the northern hemisphere. At or near the magnetic dip equator the method is expected to produce a substantial reduction in sky-wave field strength for propagation to the north or to the south.

Propagation tests have been conducted to confirm the predicted reduction in sky-wave field strength for propagation to the north in the southern hemisphere. The observed improvement varied from 12db when the field strength was low to 20db when the field strength was high with a median improvement of 16db. In terms of practical engineering, this involves an exchange of increased transmitter power for sky-wave field strength reduction.

It is expected that fading zones of clear channel stations may be placed well beyond their normal range without a large increase in the required transmitter power, but sky-wave reduction between common channel stations will require additional power comparable with that at present employed for the ground-wave service. In the latter case, high transmitting aerials would be required to obtain sufficient horizontally polarised power at low angles of departure.

Application of the method, in temperate regions, to paths which are not predominately northward in the southern hemisphere or southward in the northern hemisphere, will depend upon the results of further propagation tests.

Acknowledgement

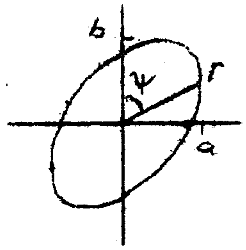
The author wishes to acknowledge the helpful suggestions and assistance given by officers of the Postmaster-General's Department for the successful conduct of the propagation tests.

APPENDIX 1.

Whereas the polarisation of an ordinary mode in the lower ionosphere is stated in terms of the ratio of minor to major areas (M) of the polarisation ellipse and the inclination (ψ) of the major axis, orthogonal transmission will be specified and varied in terms of the phase between horizontally and vertically polarised components and the power ratio of these components.

In order to transpose equations which indicate the effect of small errors in M_L and ψ_L on the degree of coupling between an incident wave and the ordinary mode, it is necessary to determine the relationship between these different sets of paramenters. A rigorous determination of this relationship is quite complicated and unnecessary at this stage of investigation. A simple examination at various degrees of elliptical polarisation will be a sufficiently accurate indication for practical purposes.

Let β be the phase difference between the horizontally and vertically polarised components.



$$r = a^2 \sin^2\theta + b^2 \sin^2(\theta+\beta)$$

$$\frac{dr}{d\theta} = a^2 \sin 2\theta + b^2 \sin 2(\theta+\beta)$$

$= 0$ for maximum and minimum values of r.

Let θ_1 be the value of θ at r max and $r$min.

Therefore at $r$max and $r$min we have

$$\cot 2\theta_1 = -\frac{a^2}{b^2 \sin 2\beta} - \cot 2\beta$$

Let a = b

$$\theta_1 = \frac{\pi}{2} - \frac{\beta}{2} \quad or \quad \pi - \frac{\beta}{2}$$

$$M = \tan\frac{\beta}{2}$$

..1/..

Relationship between $\Delta\beta$ and ΔM, $\Delta\psi$ with 'a' and 'b' held constant

$$M+\Delta M = \tan\left(\frac{\beta+\Delta\beta}{2}\right) \simeq \tan\frac{\beta}{2} + \frac{\Delta\beta}{2} \quad , \text{ for } \Delta\beta \text{ small}, \; 0<\beta<\frac{\pi}{2}$$

$$\Delta M \simeq \frac{\Delta\beta}{2}$$

$$\frac{\Delta M}{M} \simeq \frac{\Delta\beta}{2} \tan\frac{\beta}{2}$$

As θ_1 is a function of β, there may also be an accompanying change in ψ for any $\Delta\beta$.

β	$\frac{\Delta M}{M}$
90°	$\Delta\beta/2$
45°	$1.2\,\Delta\beta$
0°	∞

$$\tan\psi = \frac{a \sin\theta_1}{b\sin(\theta_1+\beta)}$$

Let $a=b$

$$\tan\psi = 1$$

$$\psi = 45^\circ$$

Therefore when $a = b$, ψ is a constant and independent of β.

$$\Delta\psi = 0$$

In those cases where $a \neq b$ we have

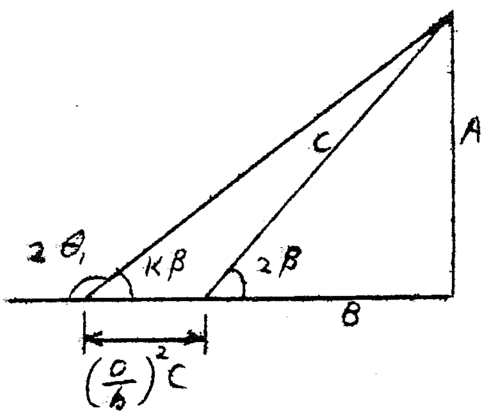
$$\tan\psi = \frac{a\sin\left(\frac{\pi}{2}-K\frac{\beta}{2}\right)}{b\sin\left(\frac{\pi}{2}-K\frac{\beta}{2}+\beta\right)}$$

This is a constant only when $\frac{a}{b}$ is constant and

$$-\frac{K\beta}{2} = -\left(\beta - \frac{K\beta}{2}\right)$$

ie $K=1$

$a=b$



$$\frac{1}{\tan 2\theta} = \frac{-\left(\frac{a}{b}\right)^2 C - B}{A}$$

where $\frac{C}{A} = \frac{1}{\sin 2\beta}$

$$\frac{B}{A} = \cot 2\beta$$

$$2\theta_1 + K\beta = \pi$$

$$\theta_1 = \frac{\pi}{2} - \frac{K\beta}{2}$$

..2/..

$$\Delta\theta_1 = -\frac{K\Delta\beta}{2}$$

$$\tan(\psi+\Delta\psi) = \frac{a\sin(\theta_1 - K\frac{\Delta\beta}{2})}{b\sin(\theta_1+\beta+\Delta\beta(1-\frac{K}{2}))}$$

$$\simeq \frac{a}{b}\left[\frac{\sin\theta_1 - \cos\theta_1 . \frac{K\Delta\beta}{2}}{\sin(\theta_1+\beta) + \cos(\theta_1+\beta) . \Delta\beta(1-\frac{K}{2})}\right]$$

$$\simeq \frac{a}{b} . \frac{\sin\theta_1\sin(\theta_1+\beta) - \sin\theta_1\cos(\theta_1+\beta).\Delta\beta(1-\frac{K}{2}) - \cos\theta_1\sin(\theta_1+\beta).\frac{K\Delta\beta}{2}}{\sin^2(\theta_1+\beta)}$$

$$\therefore \Delta\psi \simeq -\frac{a}{b}\frac{\sin\theta_1\cos(\theta_1+\beta).\Delta\beta(1-\frac{K}{2})}{\sin^2(\theta_1+\beta)} - \frac{a}{b}\frac{\cos\theta_1}{\sin(\theta_1+\beta)} . \frac{K\Delta\beta}{2}$$

$$\simeq -\frac{\Delta\beta}{2\sin(\theta_1+\beta)}\left(\tan\psi\cos(\theta_1+\beta) + \frac{a}{b}\cos\theta_1\right)$$

$$(r+\Delta r)^2 = a^2\sin^2(\theta_1 - \frac{K\Delta\beta}{2}) + b^2\sin^2(\theta_1+\beta+\Delta\beta - \frac{\Delta\beta . K}{2})$$

$$\simeq a^2\left(\sin\theta_1 - \cos\theta_1 . \frac{K\Delta\beta}{2}\right)^2 + b^2\left(\sin(\theta_1+\beta) + \cos(\theta_1+\beta).\Delta\beta(1-\frac{K}{2})\right)^2$$

$$\simeq a^2\left(\sin^2\theta_1 - 2\sin\theta_1\cos\theta_1\frac{K\Delta\beta}{2}\right) + b^2\left(\sin^2(\theta_1+\beta) + 2\sin(\theta_1+\beta)\cos(\theta_1+\beta).\Delta\beta(1-\frac{K}{2})\right)$$

$$2r\Delta r \simeq -a^2\sin 2\theta_1 . \frac{K\Delta\beta}{2} + b^2\sin 2(\theta_1+\beta) . \Delta\beta(1-\frac{K}{2})$$

$$\Delta r \simeq \frac{\left[-\frac{a^2K}{2}\sin 2\theta_1 + b^2(1-\frac{K}{2})\sin 2(\theta_1+\beta)\right]}{\left[a^2\sin^2\theta_1 + b^2\sin^2(\theta_1+\beta)\right]^{\frac{1}{2}}} . \frac{\Delta\beta}{2}$$

Let $a = \frac{b}{2}$, $\beta = 45^\circ$

$b = 1$, $\theta_1 = 52^\circ$ or 142°

$K = 1.7$, $\psi = 21.8^\circ$

$r_{max} = 1.07$

$\Gamma min = 0.33$

$\Delta\Gamma max = -0.104\Delta\beta$

$\Delta\Gamma min = 0.38\Delta\beta$

$$M+\Delta M = \frac{\Gamma min + \Delta\Gamma min}{\Gamma max + \Delta\Gamma max}$$

$$\Delta M = \frac{\Delta\Gamma min}{\Gamma max} - \frac{\Gamma min}{\Gamma^2_{max}} \cdot \Delta\Gamma_{max}$$

$$\Delta M = 0.38\Delta\beta$$

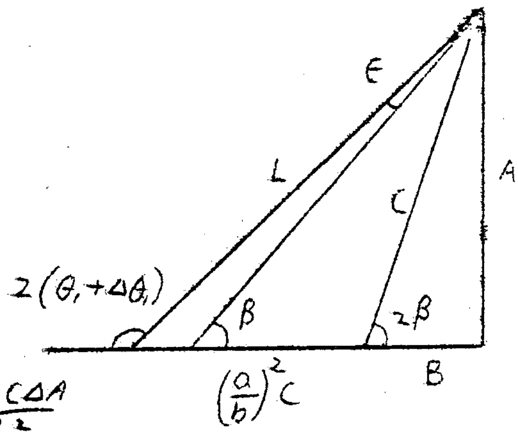
$$\frac{\Delta M}{M} = 1.23\,\Delta\beta$$

$$\Delta\psi = -0.13\Delta\beta$$

A change in the ratio a/b from unity has therefore made no substantial change in the relationship between the two sets of error parameters.

<u>Relationship between Δa and $\Delta M, \Delta\psi$ when β and "b" are held constant</u>

Let a=b=1



$$\left(\frac{a+\Delta a}{b}\right)^2 C \simeq \left(\frac{a}{b}\right)^2 C + \frac{2aC\Delta A}{b^2}$$

$$\Delta\left(\frac{a}{b}\right)^2 C \simeq \frac{2aC\Delta a}{b^2}$$

$$2\frac{c\,a\,\Delta a}{b^2}\sin\beta = L\epsilon = 2c\cos\beta\,.\,\epsilon$$

$$\therefore \epsilon = \frac{a\Delta a}{b^2}\tan\beta$$

$$2(\theta_1+\Delta\theta_1)+\beta-\epsilon=\pi = 2\theta_1+2\Delta\theta_1+\beta-\frac{a\Delta a}{b^2}\tan\beta$$

$$2\Delta\theta_1 - \frac{a\Delta a}{b^2}\tan\beta = 0$$

$$\Delta\theta_1 = \frac{a\Delta a}{2b^2}\tan\beta$$

$$(r+\Delta r)^2 = (a+\Delta a)^2\sin^2(\theta_1+\Delta\theta_1) + b^2\sin^2(\theta_1+\beta+\Delta\theta_1)$$

$$r^2+2r\Delta r \simeq (a^2+2a\Delta a)(\sin\theta_1+\cos\theta_1.\Delta\theta_1)^2+b^2(\sin(\theta_1+\beta)+\cos(\theta_1+\beta).\Delta\theta_1)^2$$

$$\simeq a^2\sin^2\theta_1+a^2\sin 2\theta_1.\Delta\theta_1+2a.\Delta a.\sin^2\theta_1$$

$$+b^2(\sin^2(\theta_1+\beta)+\sin 2(\theta_1+\beta).\Delta\theta_1)$$

$$\therefore \Delta r \simeq \frac{a^2\sin 2\theta_1.\Delta\theta_1+2a\Delta a\sin^2\theta_1+b^2\sin 2(\theta_1+\beta).\Delta\theta_1}{2r}$$

$$\simeq \frac{2\Delta a\sin^2\theta_1+\frac{\Delta a}{2}\tan\beta(\sin 2\theta_1+\sin 2(\theta_1+\beta))}{2(\sin^2\theta_1+\sin^2(\theta_1+\beta))^{\frac{1}{2}}}$$

$$\simeq \frac{\Delta a\sin^2\theta_1}{(\sin^2\theta_1+\sin^2(\theta_1+\beta))^{\frac{1}{2}}}$$

$$r_{max} = \sqrt{2}\cos\frac{\beta}{2}, \quad r_{min} = \sqrt{2}\sin\frac{\beta}{2}$$

$$\Delta r_{max} = \frac{\Delta a}{\sqrt{2}}.\cos\frac{\beta}{2}, \quad \Delta r_{min} = \frac{\Delta a}{\sqrt{2}}\sin\frac{\beta}{2}$$

$$\Delta M = \frac{\Delta a\sin\frac{\beta}{2}}{2\cos\frac{\beta}{2}} - \frac{\sin\frac{\beta}{2}}{2\cos^2\frac{\beta}{2}}.\Delta a\cos\frac{\beta}{2}$$

$$= 0$$

This also follows from the equation from M

$$M = \tan\frac{\beta}{2}$$

As β is held constant

$\Delta M = 0$ for Δa small

The variation in ψ is given by -

$$\tan(\psi+\Delta\psi) = \frac{(a+\Delta a)\sin(\theta_1+\Delta\theta_1)}{b\sin(\theta_1+\Delta\theta_1+\beta)}$$

$$= \frac{(a+\Delta a)(\sin\theta_1+\cos\theta_1 . \Delta\theta_1)}{b(\sin(\theta_1+\beta)+\cos(\theta_1+\beta).\Delta\theta_1)}$$

$$\therefore \tan\psi + \Delta\psi \simeq \left(\frac{a+\Delta a}{b}\right)\left[\frac{\cos\frac{\beta}{2}+\sin\frac{\beta}{2}.\Delta\theta_1}{\cos\frac{\beta}{2}-\sin\frac{\beta}{2}.\Delta\theta_1}\right]$$

when $a=b=1$

$\tan\psi=1$

$\theta_1=\frac{\pi}{2}-\frac{\beta}{2}$

$$= \left(\frac{a+\Delta a}{b}\right)\left(\frac{1+\tan\frac{\beta}{2}.\Delta\theta_1}{1-\tan\frac{\beta}{2}.\Delta\theta_1}\right)$$

Provided $\beta < \frac{\pi}{2}$ we have

$$1+\Delta\psi \simeq (1+\Delta a)\left(1+\tan\frac{\beta}{2}.\Delta\theta_1\right)^2$$

$$\simeq (1+\Delta a)\left(1+2\tan\frac{\beta}{2}.\Delta\theta_1\right)$$

$$\therefore \Delta\psi \simeq 2\tan\frac{\beta}{2}.\Delta\theta_1 + \Delta a$$

$$= 2\tan\beta . \frac{\Delta a}{2}\tan\beta + \Delta a$$

$$= \Delta a\left(1+\frac{2}{\cot^2\frac{\beta}{2}-1}\right)$$

$$\frac{\Delta\psi}{M} = \Delta a\left(\cot\frac{\beta}{2}+\frac{2}{\cot\frac{\beta}{2}-\tan\frac{\beta}{2}}\right)$$

β	$\Delta\psi$	$\frac{\Delta\psi}{M}$
90°	∞	∞
45	$1.4\,\Delta a$	$3.4\,\Delta a$
0°	Δa	∞

..6/..

In the special case of $\beta = 0$ we have

$$\Delta M = 0$$

$$\tan\psi = \frac{a}{b}$$

$$\tan(\psi + \Delta\psi) = \frac{a+\Delta a}{b}$$

$$\therefore \Delta\psi = \frac{\Delta a}{b}$$ provided ψ is small.

When ψ is large,

$$\Delta\psi = \Delta a \frac{\cos\psi}{(a^2+b^2)^{\frac{1}{2}}}$$

<u>For propagation from south to north</u>

$$\psi_m = 0$$

$$\beta = \frac{\pi}{2}$$

Let the error be in "a".

$$\Delta M = \frac{\Delta a}{b}$$

$$\frac{\Delta M}{M} = \frac{\Delta a}{a}$$

$$\Delta\psi = 0$$

$$L_p = \left(\frac{\Delta a}{a}\right)^2$$

Let $\frac{\Delta a}{a} < 0.1$

$$L_p < -20\text{db}$$

Let the error be in β

$$a = \frac{b}{2}$$

$$\theta_1 = \Delta\beta$$

..7/..

$$\tan \Delta\psi \simeq \frac{a \sin \Delta\beta}{b \sin (\Delta\beta + \frac{\pi}{2})}$$

$$\therefore \Delta\psi \simeq \frac{a}{b}\Delta\beta$$

$$\frac{\Delta\psi}{M} \simeq \Delta\beta$$

$$\Delta r \text{ max} = \frac{-(\Delta\beta)^2}{4}$$

$$\Delta r \text{ min} = \frac{(\Delta\beta)^2}{4}$$

$$\Delta M = \frac{(\Delta\beta)^2}{2}$$

$$\frac{\Delta M}{M} = (\Delta\beta)^2$$

$$L_p \simeq (\Delta\beta)^2$$

Let $\Delta\beta < 0.1$ radians

$L_p < -20$db

For Propagation from East to West or West to East

$\tan \psi = \frac{a}{b}$, $M = 0$, $\beta = 0$

Let the error be in "a"

$\Delta\psi \simeq \Delta a/b$ for ψ small

$\Delta M = 0$

$$L_p = \left(\frac{\Delta a}{b}\right)^2 \left(2 + k^2 + \frac{1}{k^2}\right), \quad k = \frac{a}{b}$$

Let $\frac{\Delta a}{a} = 0.1$, $k = \frac{1}{2}$, $a = 1$

$L_p = -18$db

Let the error be in β

$\Delta\psi = 0$

$$\tan 2\theta_1 \simeq -2\Delta\beta$$

$$\therefore \quad \theta_1 \simeq \frac{\pi}{2} - \Delta\beta$$

$$\Delta r_{max} = -\frac{(\Delta\beta)^2}{4}$$

$$\Delta r_{min} = \frac{\Delta\beta}{2}$$

$$\Delta M \simeq \frac{\Delta\beta}{2}$$

$$L_p = \left(\frac{\Delta\beta}{2}\right)^2 \left(2 + k^2 + \frac{1}{k^2}\right)$$

Let $\Delta\beta$ = 0.1 radians

$$L_p = -18\text{db}$$

Legend to Figures

Figure 2 Circuit diagram of the network provided at the transmitter to obtain variable phase change and power division.

Figure 3 (a) Calculated vertical radiation patterns $f(\alpha_H)$ and $f(\alpha_V)$ of the horizontal and vertical aerials towards the north, where α is the angle of departure in degrees.

(b) Calculated M_m values for the **extraordinary** wave, and M_α values obtained from the vertical polar diagrams.
$M_\alpha = f(\alpha_V)/f(\alpha_H)$

	α°	
	1 hop E	1 hop F
Hillston	22.5	51.5
Hay	29	58

Figure 4 Optimum phase setting ϕ and corresponding power ratio P_H/P_V of the power division network shown in figure 2.

Figure 5 Minimum improvement factor $\frac{E_{(V+H)}}{E_{(V+D)}}$ and corresponding power ration P_H/P_V of the power division network shown in figure 2.
$E_{(V+H)}$ is the field strength, either instantaneous or median, when power is transmitted from both vertical and horizontal aerials. $E_{(V+D)}$ is the field strength, either instantaneous or median, when the same power is transmitted from the vertical aerial but that which was previously fed to the horizontal aerial is diverted to dummy lead.

Figure 6 Field strength recordings showing the reduction which occurs when power is fed to both horizontal and vertical aerials. Transmission intervals are $P_{(V+H)}$, 3 minutes, $P_{(V+D)}$ 2 minutes.

(a) Recordings made at Hillston with optimum settings of power ratio and phase.

(b) Recordings made at Hay with the same optimum settings as those established at Hillston. The distance between Hillston and Hay is 81 miles.

Figure 7 Improvement factor variation with change in field strength. Median field strength values have been used in figure 7 (a), Hillston, and figure 7 (b), Hay.

Figure 8 The variation in field strength improvement factor with change in phase for different power division ratios of the circuit shown in figure 2.

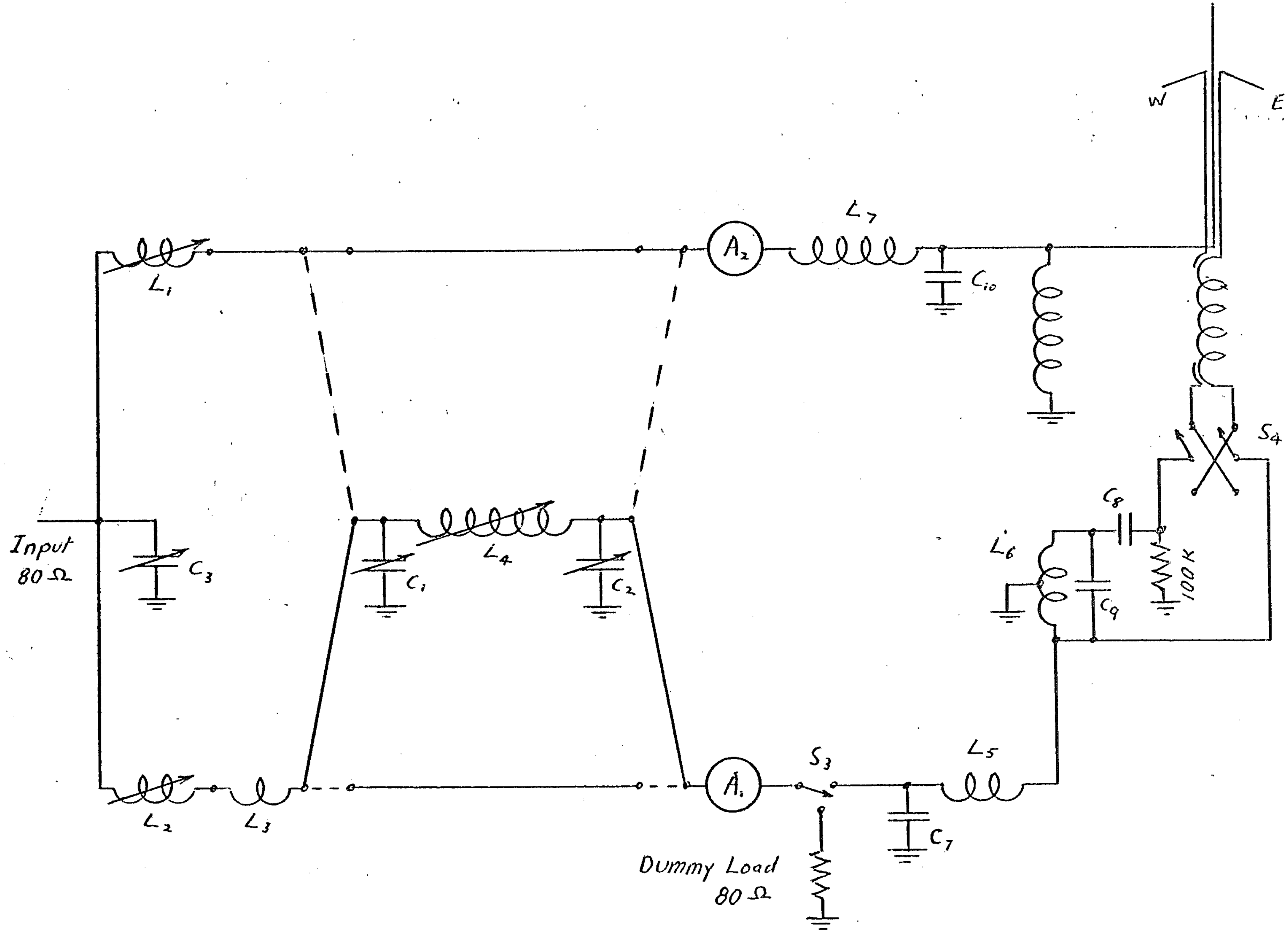

W
E
L_7
A_2
C_{10}
L_1
S_4
C_8
Input
80 Ω
C_3
L_4
C_1
C_2
L_6
C_9
100 K
S_3
L_5
A_1
L_2
L_3
C_7
Dummy Load
80 Ω

FIGURE 2

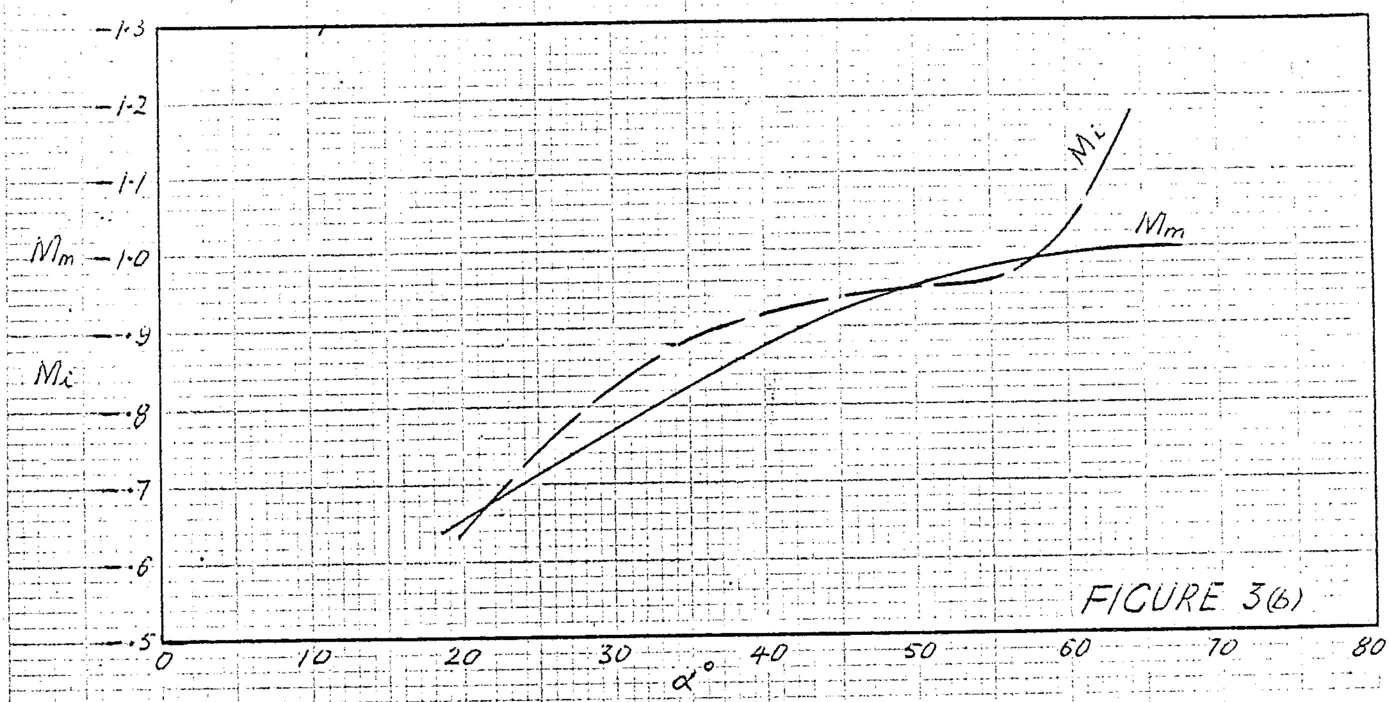


FIGURE 3(b)

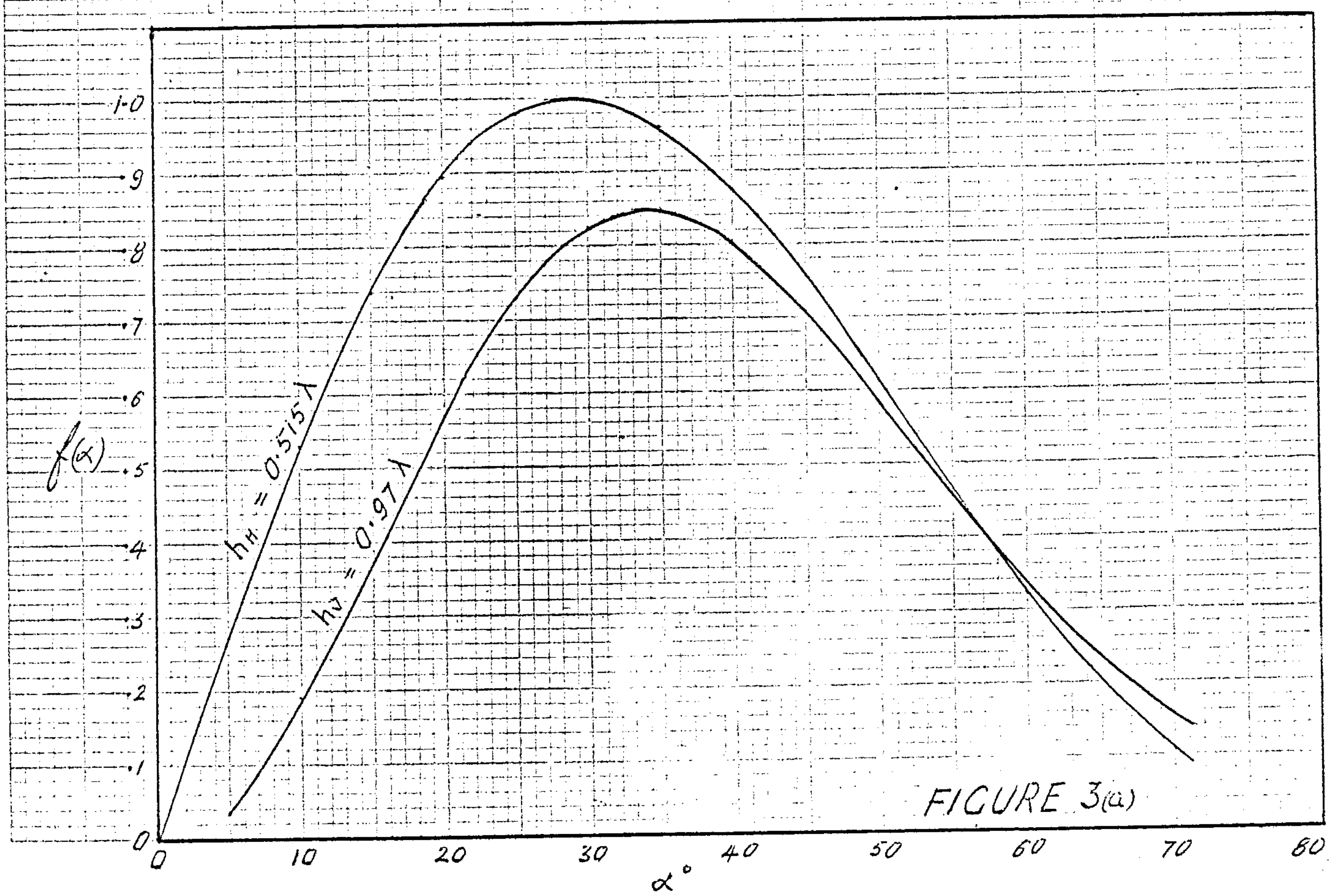
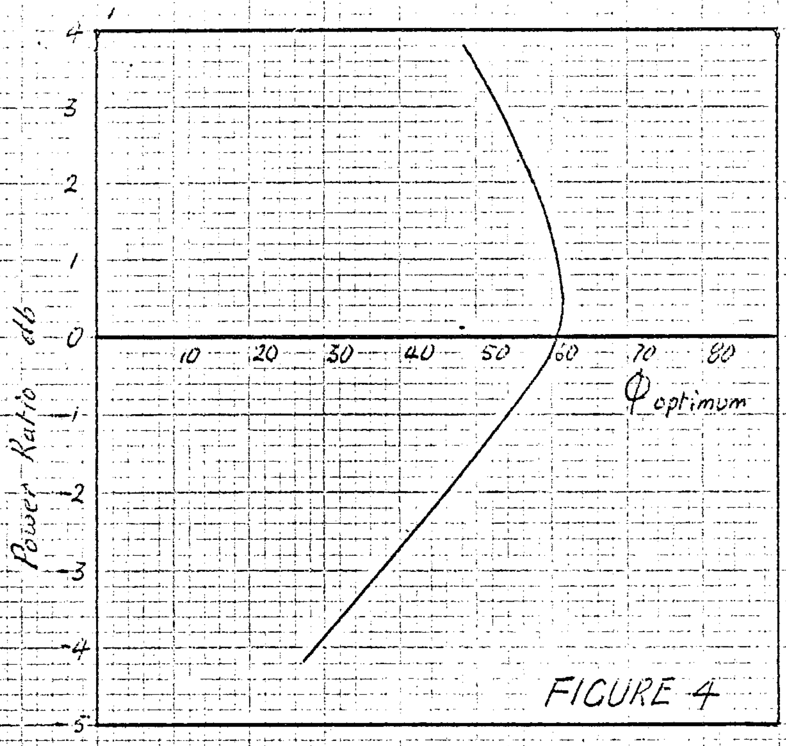
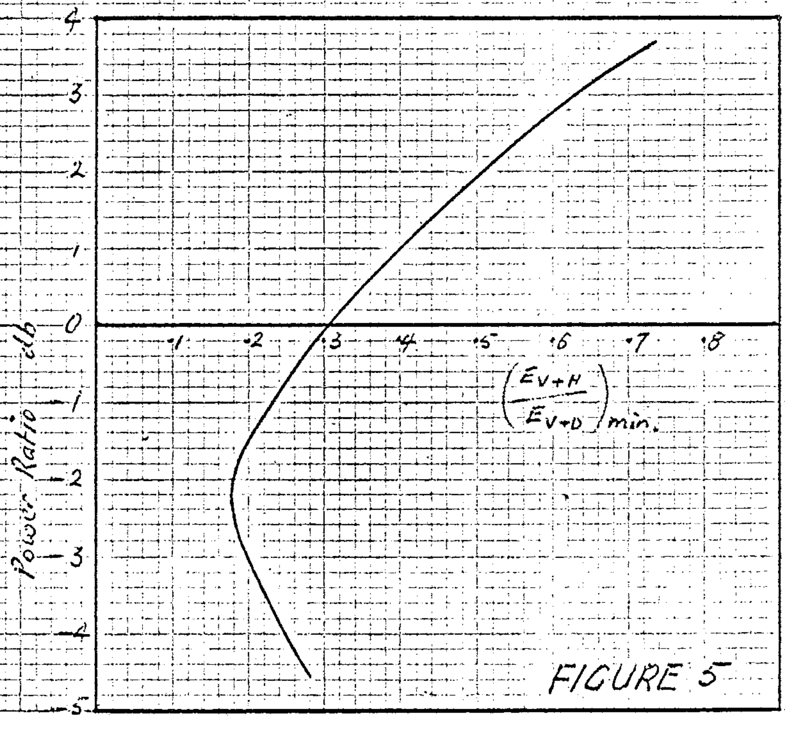


FIGURE 3(a)


4
3
2
1
0
-1
-2
-3
-4
-5
10 20 30 40 50 60 70 80
Φ optimum
Power Ratio db
FIGURE 4


4
3
2
1
0
-1
-2
-3
-4
-5
·1 ·2 ·3 ·4 ·5 ·6 ·7 ·8
$\left(\frac{E_{V+H}}{E_{V+D}}\right)_{min.}$
Power Ratio db
FIGURE 5

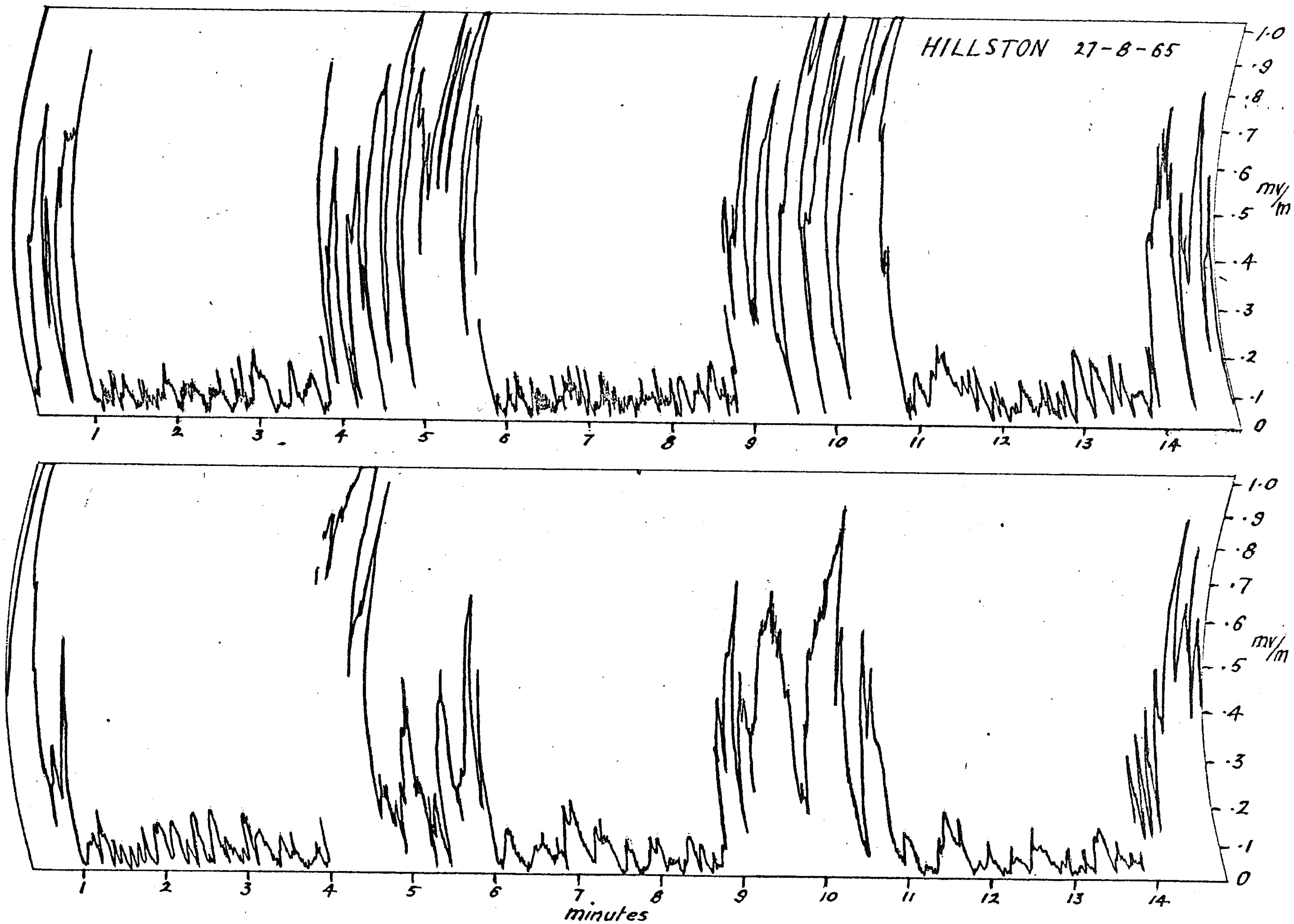

HILLSTON 27-8-65
1·0
·9
·8
·7
·6
·5
·4
·3
·2
·1
0
mv/m
1
2
3
4
5
6
7
8
9
10
11
12
13
14
minutes

FIGURE 6(a)

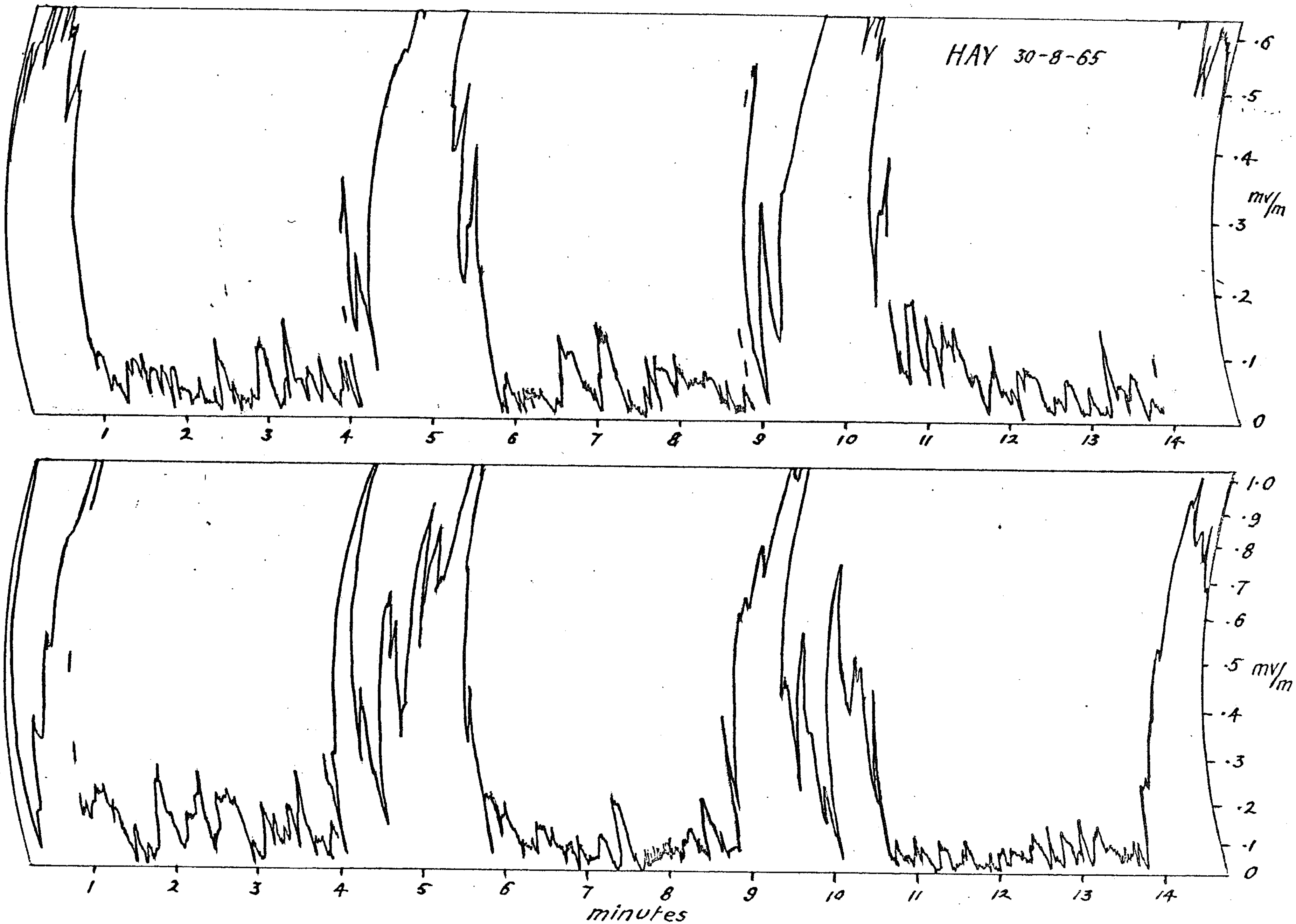
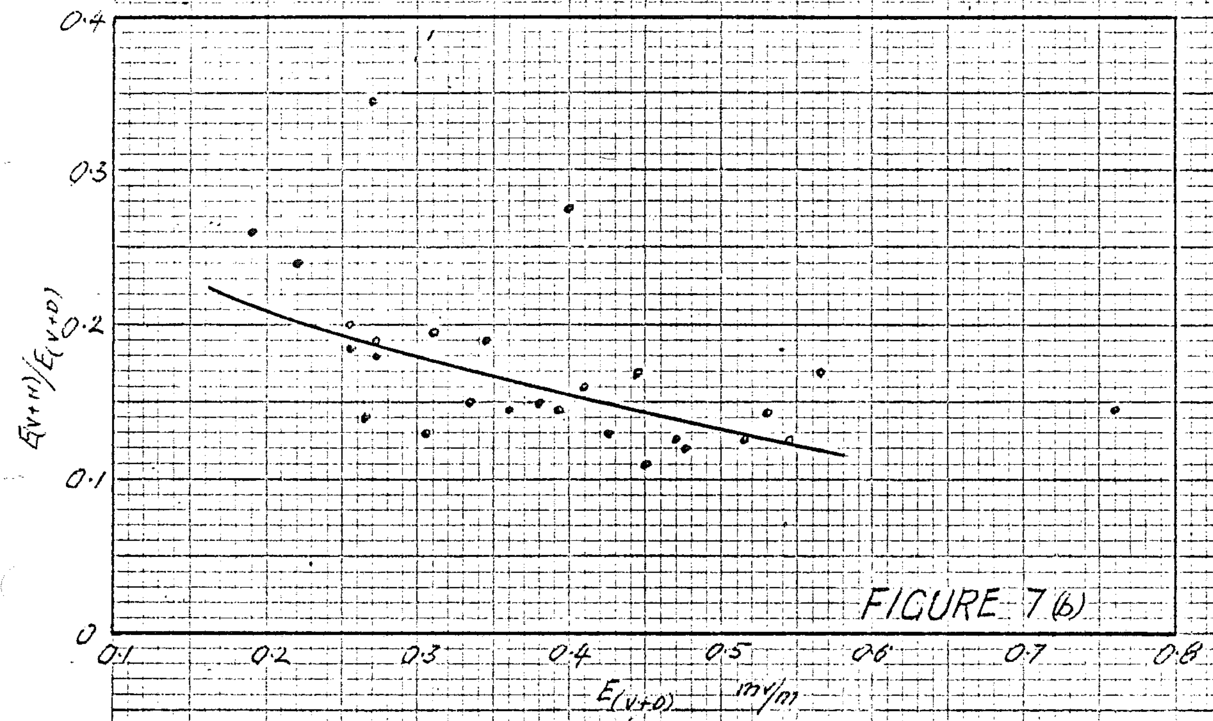
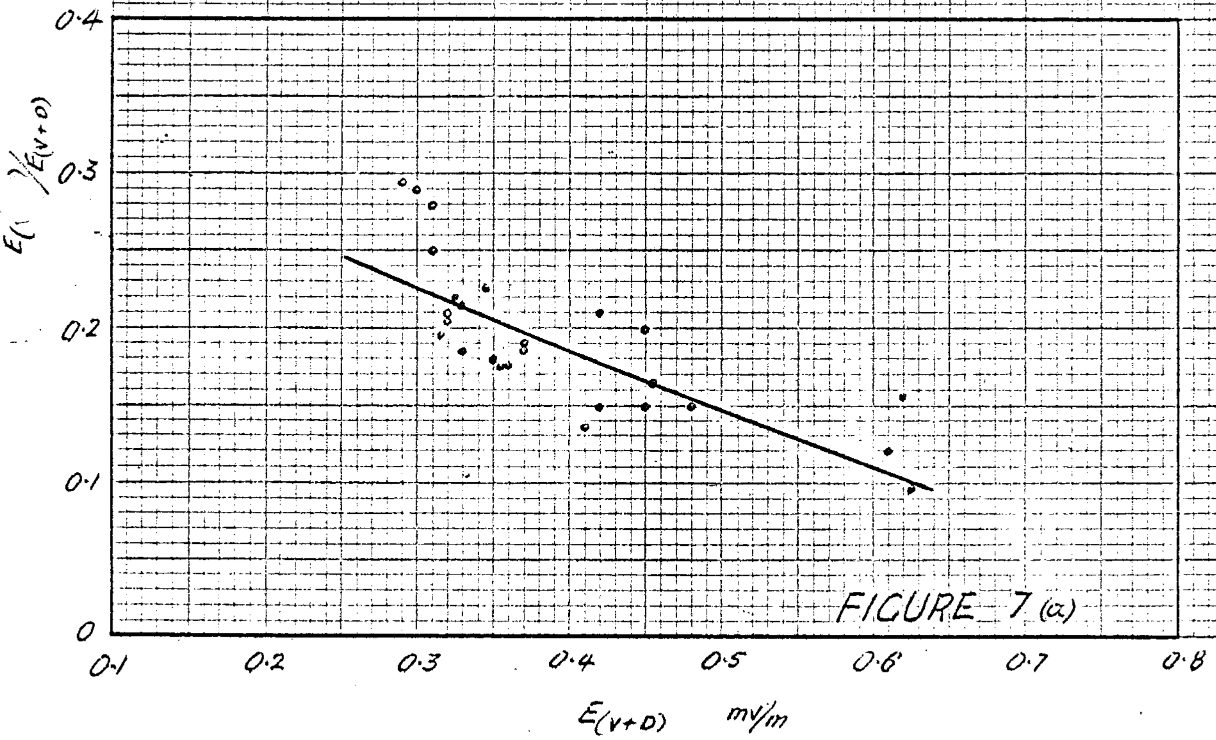

HAY 30-8-65
mV/m
·6
·5
·4
·3
·2
·1
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
1·0
·9
·8
·7
·6
·5 mV/m
·4
·3
·2
·1
0
minutes

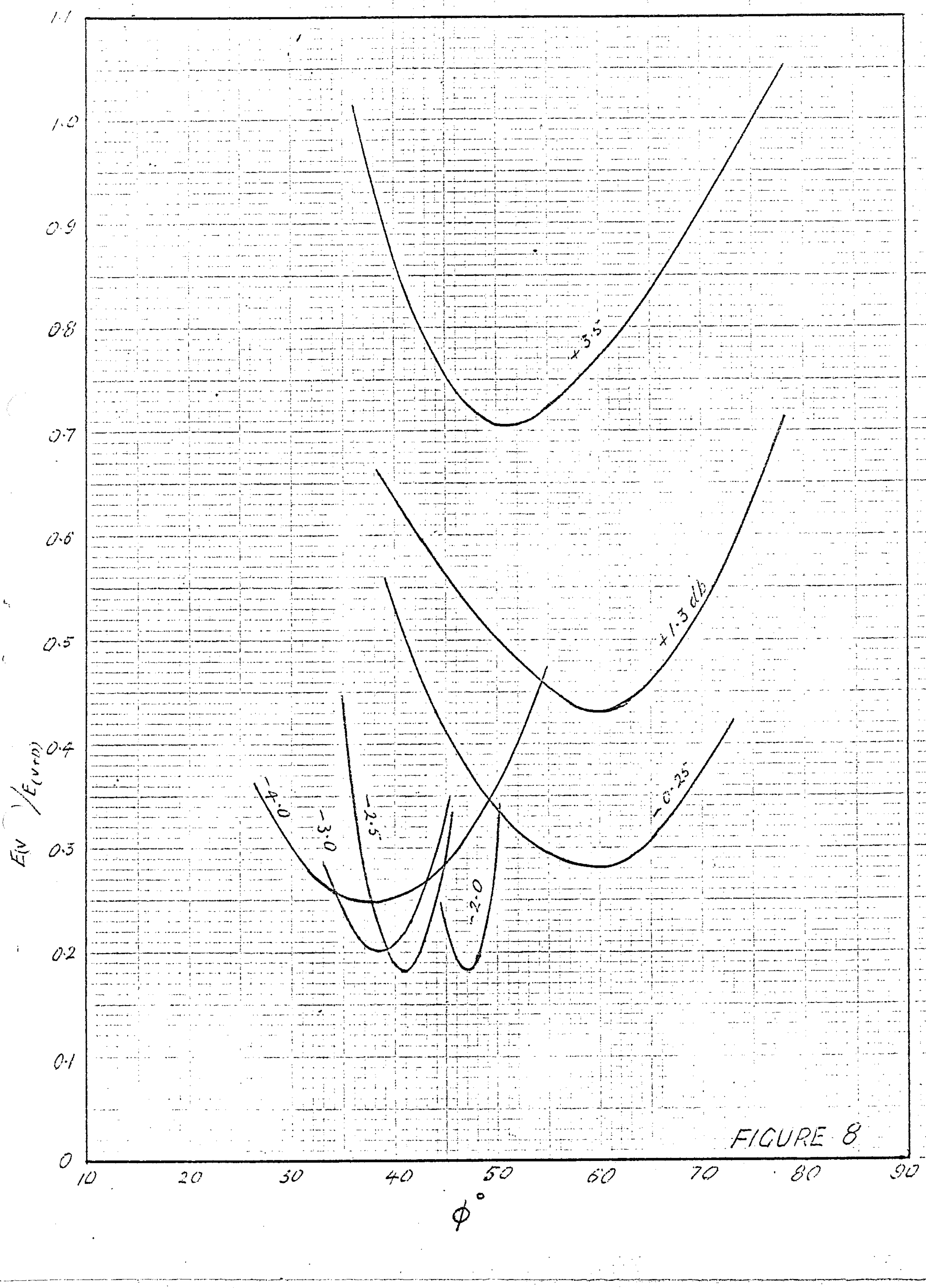
FIGURE 6(b)



0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$E_{(V+H)}/E_{(V+D)}$
$E_{(V+D)}$ mV/m
FIGURE 7(b)



0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$E_{(V+D)}$ mV/m
FIGURE 7(a)



1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$E(V) / E(V+D)$
10
20
30
40
50
60
70
80
90
ϕ°
+3.5
+1.3 db
-0.25
-4.0
-3.0
-2.5
-2.0

FIGURE 8