AEROVOX

RESEARCH WORKER

The Aerovox Research Worker is published by the Aerovox Corporation to bring authoritative, first hand information on state-of-the-art developments in electronics. Copyright material herein contained can be reprinted in whole or part only with the written consent of the Aerovox Corporation.

VOL. 37 NOS. 1-6 | JANUARY - JUNE 1967 | Subscription By Application Only

Schedule Method of Harmonic Analysis

By the Engineering Department of Aerovox Corporation

When no wave analyzer is available, a harmonic analysis may be made of any wave from the latter's oscillogram obtained either from an oscilloscope screen or recorder chart. The basic technique is a solution of the Fourier series:

(1-1)

$$y = A_0 + A_1 \cos wt + A_2 \cos 2wt + A_3 \cos wt\ 3wt + A_4 \cos 4wt + A_5 \cos 5wt + A_6 \cos 6wt + B_1 \sin wt + B_2 \sin wt + B_3 \sin 3wt + B_4 \sin 4wt + B_5 \sin 5wt$$

Here, A_0 is the d-c component, and the A and B terms are a-c components:

$$A_0 = 1/\pi \int_0^{2\pi} y\,dx,\quad A = 1/\pi \int_0^{2\pi} y \cos nx\,dx,$$

and $B_n = 1/\pi \int_0^{2\pi} y \sin nx\,dx$.

If the technician knows no calculus, he quickly abandons the desire to find the harmonic content from the wave pattern he sees on the oscilloscope screen. But he need not do this, for the *Schedule Method* permits evaluation of the Fourier series, using simple arithmetic. This method has been known for some time (it was described in the *Bureau of Standards Bulletin* in 1913 and is discussed in several current radio and electronic handbooks); nevertheless, no great number of technicians seem to have tried to apply it. While the method does involve a number of calculations, they all are simple and the method provides an extremely useful tool. This article describes the Schedule Method and gives a step-by-step illustrative example. The version detailed here has been simplified by consolidating the first two series of calculations of the original method into the third series, and presenting this combined operation as our first series of calculations. The technician may use this method to analyze any periodic wave (sine, distorted audio, square, pulse, sawtooth, etc.) that he can display on the oscilloscope screen or recorder chart.

PROCEDURE FOR ANALYSIS

The version of the Schedule Method described here involves 6 steps, in the last three of which there are several simple calculations.

STEP 1. Obtain a workable-sized pattern of the waveform of interest. Do this by (a) directly viewing an oscilloscope screen, (b) photographing the pattern from the screen, or (c) taking the traced chart from a recorder.

STEP 2. See Figure 1. Along the x-axis, divide 1 complete period of the waveform into 12 equal parts (as between O and X in Figure 1): Start shortly to the right of the beginning of the period. From each of the 11 points which lie within the half-cycles of the pattern, carefully measure the vertical distance from the point to the pattern (vertical lines — ordinates — may be drawn up to the positive half-cycle and down to the negative half-cycle, as shown in Figure 1), and label these heights y_0 to y_{11} as shown. The lengths of these ordinates may be measured in any consistent units (inches, centimeters, millimeters, oscilloscope screen divisions, recorder chart divisions), provided that tenths can be measured. Upgoing ordinates (e.g., y_0 to y_6) are positive values; downgoing ordinates (e.g., y_7 to y_{11}) are negative.

STEP 3. Record these y-values in a table (see Figure 2).

STEP 4. Using these y-values, calculate the sums (S-values) and differences (D-values), according to Equations (1-2) to (1-17), below.

(1-2) $S_0 = y_0 + y_6$

(1-3) $S_1 = y_0 + y_5 + y_7 + y_{11}$

(1-4) $S_2 = y_2 + y_4 + y_8 + y_{10}$

(1-5) $S_3 = y_3 + y_9$

(1-6) $S_4 = y_1 + y_5 - y_7 - y_{11}$

(1-7) $S_5 = y_2 + y_4 - y_8 - y_{10}$

(1-8) $S_6 = y_3 - y_9$

(1-9) $S_7 = y_0 + y_2 + y_4 + y_6 + y_8 + y_{10}$

(1-10) $S_8 = y_1 + y_3 + y_9 + y_{11}$

(1-11) $D_0 = y_0 - y_6$

(1-12) $D_1 = y_1 - y_5 - y_7 + y_{11}$

(1-13) $D_2 = y \;- y_4 - y_8 + y_{10}$

(1-14) $D_3 = y_1 - y_5 + y_7 - y_{11}$

(1-15) $D_4 = y_2 - y_4 + y_8 - y_{10}$

(1-16) $D_5 = y_1 - y_3 + y_5 - y_7 + y_9 - y_{11}$

(1-17) $D_6 = y_0 - y_2 + y_4 - y_6 + y_8 - y_1{}^0$

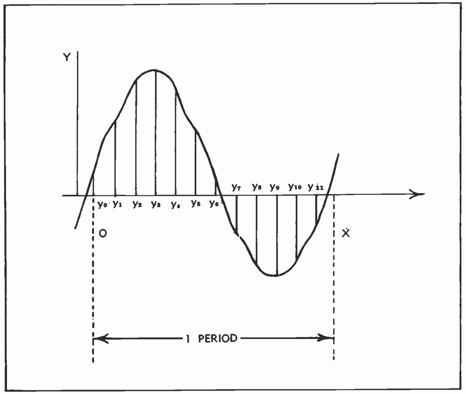

FIGURE 1. DISTORTED WAVE FOR ANALYSIS

STEP 5. Using the S- and D-values obtained in Step 4, calculate the A and B Fourier coefficients, according to Equations (1-18) to (1-29), below.

(1-18) $A_0 = (S_7 + S_8)/12$

(1-19) $A_1 = (D_0 + 0.866D_1 + 0.5D_2)/6$

(1-20) $A_2 = (S_0 + 0.5S_1 - 0.5S_2 - S_3)/6$

(1-21) $A_3 = D_6/6$

(1-22) $A_4 = (S_0 - 0.5S_1 - 0.5S_2 + S_3)/6$

(1-23) $A_5 = (D_0 - 0.866D_1 + 0.5D_2)/6$

(1-24) $A_6 = (S_7 - S_8)/12$

(1-25) $B_1 = (0.5S_4 + 0.866S_5 + S_6)/6$

(1-26) $B_2 = (0.866(D_3 + D_4))/6$

(1-27) $B_3 = D_5/6$

(1-28) $B_4 = (0.866(D_3 - D_4))/6$

(1-29) $B_5 = (0.5S_4 - 0.866S_5 + S_6)/6$

STEP 6. Using the A- and B-values obtained in Step 5, calculate the various components of the wave, according to Equations (1-30) to (1-36), below. The wave contains a d-c component, A_0, only when the half-cycles are sufficiently asymmetrical to result in a positive or negative dominance. (For a symmetrical sine wave, A_0=O.)

(1-30) D-C Component. $DC = A_0$

(1-31) Fundamental. $h_1 = \sqrt{A_1^2 + B_1^2}$

(1-32) 2nd Harmonic. $h_2 = \sqrt{A_2^2 + B_2^2}$

(1-33) 3rd Harmonic. $h_3 = \sqrt{A_3^2 + B_3^2}$

(1-34) 4th Harmonic. $h_4 = \sqrt{A_4^2 + B_4^2}$

(1-35) 5th Harmonic. $h_5 = \sqrt{A_5^2 + B_5^2}$

(1-36) 6th Harmonic. $h_6 = A$

ILLUSTRATIVE EXAMPLE

The following example illustrates analysis of the distorted wave shown in Figure 1. The ordinates on the original pattern were measured in centimeters. STEP 1. The obtained wave pattern is shown in Figure 1.

STEP 2. One period of the wave is divided into 12 x-axis intervals (O to X in Figure 1). Here, the ordinates are measured in centimeters.

STEP 3. The y-values are recorded in the Table in Figure 2.

RESEARCH WORKER AEROVOX

ORDINATE	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}
HEIGHT	0.63	1.95	3.10	3.30	2.90	1.78	0.45	-0.92	-1.89	-2.14	-1.70	-0.82

FIGURE 2. TABLE OF Y-VALUES

STEP 4. The S- and D-values are calculated as shown in Equations (2-2) to (2-17), below. These calculations apply the basic equations (1-2) to (1-17), respectively.

(2-2) $S_0 = 0.63 + 0.45 = 1.08$

(2-3) $S_1 = 0.63 + 1.78 + (-0.92 + (-0.82) = 0.67$

(2-4) $S_2 = 3.10 + 2.90 + (-1.89) + (-1.70 = 2.41$

(2-5) $S_3 = 3.30 + (-2.15) = 1.16$

(2-6) $S_4 = 1.95 + 1.78 - (-0.92) - (-0.82) = 5.47$

(2-7) $S_5 = 3.10 + 2.90 - (-1.89) - (-1.70) = 9.59$

(2-8) $S_6 = 3.30 - (-2.14) = 5.44$

(2-9) $S_7 = 0.63 + 3.10 + 2.90 + 0.45 + (-1.89) + (-1.70) = 3.49$

(2-10) $S_8 = 1.95 + 3.30 + (-2.14) + (-0.82) = 2.29$

(2-11) $D_0 = 0.63 - 0.45 = 0.18$

(2-12) $D_1 = 1.95 - 1.78 - (-0.92) + (-0.82) = 0.27$

(2-13) $D_2 = 3.10 - 2.90 - (-1.89) + (-1.70) = 0.39$

(2-14) $D_3 = 1.95 - 1.78 + (-0.92) - (-0.82) = 0.07$

(2-15) $D_4 = 3.10 - 2.90 + (-1.89) - (-1.70) = 0.01$

(2-16) $D_5 = 1.95 - 3.30 + 1.78 - (-0.92) + (-2.14) - (-0.82) = 0.03$

(2-17) $D_6 = 0.63 - 3.10 + 2.90 - 0.45 + (-1.89) - (-1.70) = -0.21$

STEP 5. The A- and B-values are calculated from the S- and D-values obtained in Step 4), according to Equations (2-18) to (2-29), below. These calculations apply the basic equations (1-18) to (1-29), respectively.

(2-18) $A_0 = (3.49 + 2.29)/12 = 0.482$

(2-19) $A_1 = (0.18 + 0.866(0.27) + 0.5(0.39))/6 = (0.18 + 0.234 + 0.195)/6 = 0.609/6 = 0.1015$

(2-20) $A_2 = (1.08 + 0.5(0.67) - 0.5(2.41) - 1.16)/6 = (1.08 + 0.335 - 1.205 - 1.16)/6 = -0.95/6 = 0.158$

(2-21) $A_3 = 0.21/6 = 0.035$

(2.22) $A_4 = (1.08 - 0.5(0.67) + 1.16)/6 = (1.08 - 0.335 + 1.16)/6 = 1.905/6 = 0.3175$

(2-23) $A_5 = (0.18 - 0.866(0.27) + 0.5(0.39)/6 = (0.18 - 0.234 + .195)/6 = 0.141/6 = 0.0235$

(2-24) $A_6 = (3.49 - 2.29)/12 = 1.20/12 = 0.10$

(2-25) $B_1 = (0.5(5.47) + 0.866(9.59) + 5.44)/6 = (2.735 + 8.305 + 5.44)/6 = 16.48/6 = 2.75$

(2-26) $B_2 = (0.866(0.07 + 0.01))/6 = (0.866(0.08))/6 = 0.0693/6 = 0.0115$

(2-27) $B_3 = 0.03/6 = 0.005$

(2-28) $B_4 = 0.866(0.07 - 0.01))/6 = (0.866(0.06))/6 = 0.05196/6 = 0.00866$

(2-29) $B_5 = (0.5(5.47) - 0.866(9.59) + 5.44)/6 = (2.735 - 8.305 + 5.44)/6 = -0.13/6 = 0.0217$

STEP 6. Finally, using the A- and B-values obtained in Step 5, calculate the value of each component in the wave, according to Equations (2-30) to (2-36), below. These calculations apply the basic equations (1-30) to (1-36), respectively.

(2-30) D.-C Component.

$DC = A_0 = 0.482$

(2-31) Fundamental.

$$h_1 = \sqrt{0.1015^2 + 2.75^2} = \sqrt{0.0103 + 7.56} = \sqrt{7.57} = 2.75$$

(2-32) 2nd Harmonic.

$$h_2 = \sqrt{0.158^2 + 0.0115^2} = \sqrt{0.02964 + 0.00013} = \sqrt{0.0298} = 0.173$$

(2-33) 3rd Harmonic.

$$h_3 = \sqrt{0.035^2 + 0.005^2} = \sqrt{0.00122 + 0.000025} = \sqrt{0.00124} = 0.0352$$

(2-34) 4th Harmonic.

$$h_4 = \sqrt{0.3175^2 + 0.00866^2} = \sqrt{0.10 + 0.000075} = \sqrt{0.10} = 0.317$$

(2-35) 5th Harmonic

$$h_5 = \sqrt{0.0235^2 + 0.0217^2} = \sqrt{0.0552 + 0.00047} = \sqrt{0.0557} = 0.236$$

(2-36) 6th Harmonic. $h_6 = .1$

If desired, an individual percentage may be determined in terms of the fundamental amplitude: $100(h/h_1)$, where h is the amplitude of the harmonic component of interest (determined by means of the appropriate one of Equations 2-30 to 2-36) and h_1 is the fundamental amplitude. Example: In the wave analyzed in the foregoing sections, what is the percentage of 2nd harmonic? Here, from Equation (2-31) the fundamental amplitude is 2.75, and the 2nd harmonic amplitude is 0.173 (from Equaiton 2-32). 2nd harmonic=100 (0.173/2.75) = 100 (0.0629) = 6.29%.

Similarly, the total harmonic distortion (D%) of the wave may be determined:

(2-37)

$$D\% = \frac{\sqrt{h_2^2 + h_3^2 + h_4^2 + h_5^2 + h_6^2}}{h_1} \times 100$$

From this equation and the component values obtained in Equations (2-30) to (2-36), the total harmonic distortion of the wave analyzed in the preceding sections is

$$D\% = \frac{\sqrt{0.173^2 + 0.0352^2 + 0.317 + 0.236^2 + 0.1^2}}{2.75} \times 100 = \frac{\sqrt{0.0298 + 0.000124 + 0.10 + 0.0557 + 0.01}}{2.75} \times 100 = \frac{\sqrt{0.1967}}{2.75} \times 100 = \frac{0.4435}{2.75} \times 100 = (0.1613)\,100 = 16.13\%$$