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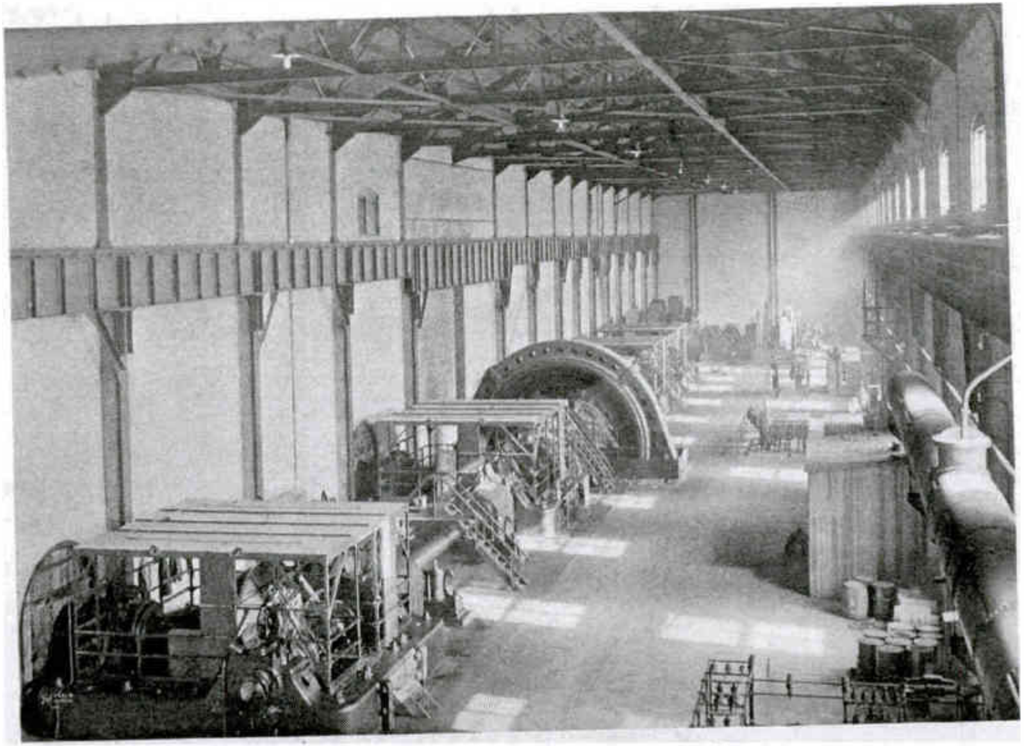

Section of Motor Room in Billet Mill, Indiana Steel Company, Gary, Ind.

(See page 210)

GENERAL ELECTRIC
REVIEW

REVIEW SUPPLEMENT AND TRANSMISSION LINE CALCULATIONS

The REVIEW this month publishes a supplement in which the method of calculating transmission line problems by the use of hyperbolic functions is explained; two numerical examples being given to illustrate the method of working; *i.e.*, a 300 mile line operating at 60 cycles, using three No. 000 wires triangularly spaced 10 ft. apart; and a line 100 miles long operating at 25 cycles, using three No. 0 wires equally spaced in a plane with 8 ft. between centers. In the latter case, approximate formulæ are used which have been derived from the hyperbolic equations. The formulæ are given in the supplement with explanatory notes as to their use. The constants and hyperbolic functions necessary for the evaluation of numerical results are also tabulated. No references need therefore be made to other publications or tables, and the supplement is complete and self-contained.

As noted in the first part of Mr. W. E. Miller's article, the method followed is Kennelly's, as given in McMahon's Hyperbolic Functions. The work was undertaken in consequence of the discussion on Mr. Thomas's paper, at Frontenac, reported in the A.I.E.E. Proceedings for November, 1909, where more than one speaker referred to the hyperbolic method as that best adapted to transmission calculations, though there was considerable divergence of opinion on this question. Reference to the supplement ought to remove any doubt as to the simplicity of the hyperbolic method, and to convince engineers of its ready application to the solution of transmission problems when the constants and hyperbolic functions are properly tabulated.

The second part of the article discusses at some length the physical aspect of corona viewed in accordance with one of the modern theories of electricity the contrast between corona and capacity current being emphasized. The law connecting the no load loss with the length of short transmission lines is also given.

It must be understood that the formulæ and constants given can be directly applied to transmission line problems only if the generator current and voltage follow a simple harmonic law; that is to say, only if harmonics of considerable magnitude are absent. The latter is generally the case, but occasionally the capacity or even the load current introduces harmonics into the generator waves. From oscillograph records these waves can be analyzed into their harmonics, and the formulæ can then be applied to the fundamental wave and each harmonic separately. The constants, of course, apply only to the fundamental wave, and new constants must be calculated for each harmonic.

In the present state of knowledge, it is impossible to include corona effect in the equations. Where the corona current is considerable, the no load loss cannot be obtained from the equations, but they are sufficiently reliable in such cases for calculating the electrical conditions along lines under load.

The equations and discussions refer to the electrical characteristics of transmission lines after the normal state has been reached, and the transient phenomena which occur when the electrical conditions are suddenly changed are ignored. The method for calculating these is given in Steinmetz's "Transient Phenomena." These phenomena are under certain circumstances extremely important and it would be well worth while if numerical results were computed for, say, two cases as examples; *i.e.*, a long line on open circuit operating at 60 cycles when the generator is connected to the line at maximum voltage, and when it is connected at zero voltage. The volts and current should be plotted for each case for different points along the line at the moment of closing the switch and after successive time intervals, until the normal state is reached. The advance of the voltage and current waves and their reflection when they reach the end of the line would then be graphically shown.

If the calculations were made at many points along the line and at sufficiently close intervals of time, and each curve were photographed, the series so obtained could be run through a cinematograph machine and show a continuous record of the phenomena by projection on a screen. This would be extremely valuable from an educational point of view, since such visual presentations help towards a physical understanding of the chief phenomena underlying the problem. Were more of the abstruse, and for that matter the simpler, problems which enter into electrical engineering treated in this manner, much clearer ideas would be formed than can be obtained from discussions of or calculations from formulæ. The labor and expense involved in the preparation of these curves and their photographic reproduction are far from prohibitive, so that there is no reason why such methods should not be used occasionally as an auxiliary for college training or lecture work.

THE SINGLE-PHASE INDUCTION MOTOR

The REVIEW is fortunate in being able to present with the present issue the first part of an article on the single-phase motor, by Professors Morecroft and Arendt of Columbia University. This article, which the authors have kindly given us permission to print, was written to form part of a treatise on the subject of electrical motors. The book, which will appear later, will include the articles on the synchronous a.c. motor and the d.c. series motor that were published respectively in the May and June issues and the August and September issues of last year.

Coming from this source, the editors have not presumed to pass upon the accuracy of the statements in the article, which, considering the high authority of the authors, has been left entirely with them.

As with the articles on the synchronous a.c. and the d.c. series motor, our readers will find this discussion of the single-phase motor of much interest and value. While the mathematics employed is not difficult, the authors have also presented their conclusions and much of the reasoning in simple non-mathematical language.

The article begins with a description of the interaction between the impressed and induced magnetic fluxes, which is followed by a lucid explanation of how the revolving field is developed and the torque produced.

The first part of the article closes with the torque equations and a clear statement of the conclusions to be deduced from their analysis.

The second part, which will be published in the next issue, opens with the subject of the characteristic curves. A circle diagram for plotting the curves is described and the results in a specific case are tabulated and discussed. The various methods of starting are then taken up and concisely but amply treated. This second portion of the article is wholly free from mathematics.

UNDERGROUND ELECTRICAL SYSTEMS

Under the title Underground Electrical Systems, Mr. W. E. Hazeltine has contributed a remarkably succinct article covering the choice of conduits and cables for various classes of work; the subject being treated in an entirely practical way, without theoretical discussion.

The conditions to be met in underground systems are described; the material used for conduits are then given, and the advantages and disadvantages briefly stated.

The relative utility of single and double ducts, of single and double manhole covers, the construction of manholes and the methods of supporting the cables within them, are given briefly as are also the size of cables and the several kinds of insulation employed in different cases. The essentials to be considered in drawing in the cables and otherwise installing the systems are also treated.

In short, the article forms a very complete and practical summary of the subject of underground electric systems.

THE SINGLE-PHASE INDUCTION MOTOR*

Part I

By Profs. J. H. Morecroft and M. Arendt

Columbia University

In small single-phase alternating current plants, the constant speed motor that is most extensively used is of the induction type. Structurally it is very similar to the corresponding polyphase machine;† in fact any polyphase induction motor will operate as a single-phase machine of somewhat smaller capacity and lower power factor, if it is at first caused to rotate at nearly synchronous speed by some starting arrangement. The necessity of providing some such auxiliary device arises from the fact that the single-phase motor, *per se*, has no starting torque. That such is the case may be readily seen without the introduction of mathematical proof.

Absence of Starting Torque

Consider a bi-polar single-phase motor, provided with a squirrel-cage rotor. The distribution of current in the secondary at standstill is as indicated in Fig. 1. The current in bars aa' is zero, because these are equivalent to a closed loop the plane of which is located parallel to the flux. The maximum current is set up in bars bb'. However, this equivalent loop, if it moves at all, must move parallel to the direction of the lines of force; hence it exerts no turning effort.

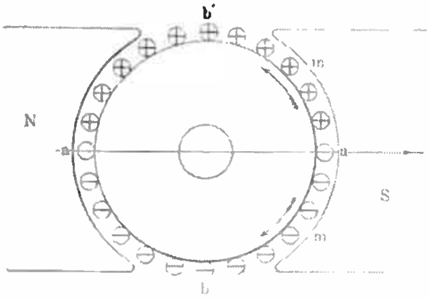

Fig. 1. Distribution of Current in Stationary Rotor of Single-Phase Induction Motor

The bar m, carrying current as indicated, will exert a torque upon the rotor, as shown by the arrow alongside it. However, owing to the symmetry of the secondary winding, for every bar m there is another m' having a current of equal amplitude but of opposite sign. This latter bar being in a field of the same strength and direction as that in which m is located, will exert a torque equal to that developed by m, but in the reverse direction, as indicated by the corresponding arrow. In the same way the effort exerted due to the current in any bar of the winding will be neutralized by that of another bar symmetrically located with respect to the axis of the primary field; consequently at standstill no turning effort is developed and the motor fails to accelerate.

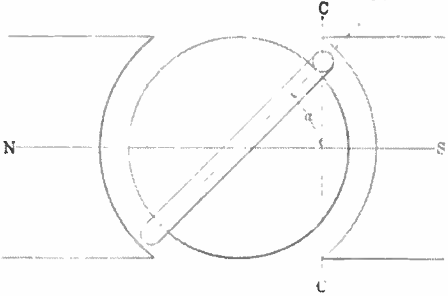

Fig. 2. Short Circuited Coil Inclined to Axis of Oscillating Field

The above fact may be proved as follows: Assume the rotor winding as composed of symmetrically placed short-circuited coils, and consider one having its plane at any angle α to the axis of the field NS, as illustrated in Fig. 2. Further suppose the flux distribution to be a cosine function of α; this is approximately the case with actual motors provided with distributed stator windings: then let

B represent the maximum flux density at $\alpha, = 0^\circ$,

$B \cos pt$ is the instantaneous flux density at $\alpha, = 0^\circ$,

$B \cos pt \cos \alpha$ is the corresponding value at the inductors selected, and with A as the area

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*To appear later as part of a book.

†The first successful motor of this type was built by C. E. L. Brown. See London *Electrician*, Vol. XXX, pages 358, 1893.

of the coil the flux passing through it becomes

$$\Phi = \int_0^{\alpha} AB \cos pt \cos \alpha d\alpha = BA \cos pt \sin \alpha. \tag{1}$$

The e.m.f. induced in the selected coil is

$$e = -\frac{d\Phi}{dt} = BA\ p \sin pt \sin \alpha. \tag{2}$$

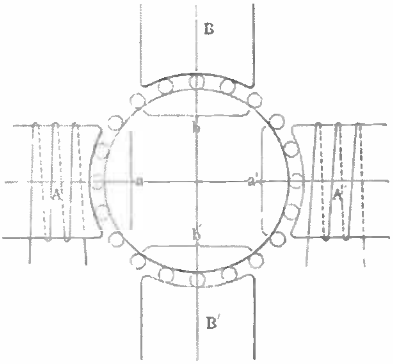

Fig. 3. Main and Quadrature Fields, Single-Phase Induction Motor

The instantaneous value of the corresponding current is

$$i = BA\ p \sin(pt-\theta) \sin \alpha \div Z'. \tag{3}$$

Naturally in the case of a single coil this current will react upon the stator field and produce flux distortion; but as we are going to sum up the effects of all the rotor coils the individual reactions balance and the field distortion becomes negligible. It is to be noted that the impedance of a coil will be modified by the action of the neighboring coils, consequently Z' in equation 46 represents the effective impedance. The angle $\theta = \cos^{-1} (r' \div Z')$, wherein r' is the effective resistance of the coil and Z' the impedance as above defined.

If there are n coils on the rotor equally spaced from one another, the effort of the Kth coil will be

$$t_k = lB^2 A p[\sin(2\ pt-\theta) + \sin \theta] \times \sin \frac{2K}{n} \pi \div 2\ Z', \tag{4}$$

wherein l is the length of one coil.

The instantaneous torque exerted by the whole rotor is

$$T = \Sigma t = lB^2 A p[\sin(2\ pt-\theta) + \sin \theta] \times \Sigma_1^n \sin \frac{2\ K}{n} \pi \div 2\ Z' = 0.^* \tag{5}$$

Development of Revolving Field

We have just shown that when we have an oscillating magnetic field the rotor placed therein fails to exert any starting torque. Therefore, if a single-phase induction motor does develop a turning effort after it is caused to revolve, it must be because it has, by some reactions of the rotor currents upon the stator flux, provided for itself a rotating magnetic field. That such is the case may be shown non-mathematically. Assume a two-pole motor (Fig. 3) the stator winding of which is supplied with a single-phase alternating current, producing an oscillating field between the poles AA'. The rotor currents produce a field at right angles to the main field, and for convenience we will assume this to be represented by the poles BB'. In commercial machines no such empty pole spaces exist, as practically all of the stator is covered with coils.

The inductors of the revolving rotor have e.m.f's., induced in them due to two actions; namely, by motion through the field and by the time rate of change of the flux threading the coils. The first we shall designate as a *rotational* e.m.f. and the second as a *transformer* e.m.f.

The inductors aa' will always have a rotational e.m.f. set up in them except when the stator field passes through zero value. The amplitude of this e.m.f. for any given speed will be proportional to the instantaneous value of the stator flux. Conductors aa' may be considered equivalent to closed coils, and the current flowing in them will produce a field in direction BB'. Neglecting temporarily the IR drop in the rotor, the e.m.f. induced in aa' may be placed equal to $\frac{d\Phi_r}{dt}$, where Φ_r denotes the cross field developed by the currents due to the motion of the rotor in the main field. The rotational e.m.f. is in time phase with the main field, hence the cross field Φ_r will be in time quadrature with it. The direction of the main field and the motion of the rotor inductors are such that the e.m.f. generated in aa' is positive.† The rotor currents are in such direction that when pole A is of north polarity and decreasing, pole B will be of like sign but increasing,

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*This same result is obtained from analysis of equa. 16.

†Currents flowing away from the reader into the plane of the paper are called positive.

reaching its maximum strength one quarter of a period later. The strength of pole B decreases after a similar lapse of time, the main field reverses and a north pole begins to build up at A'. *That is, the main field and quadrature field so combine that a north pole travels around the stator in the direction $ABA'B'$ at synchronous speed.* Hence, there exists a rotating field produced by the combined action of stator and rotor currents. This simple explanation gives an idea of the production of the rotating field in the single-phase induction motor, but it does not consider all the reactions which occur.

The inductors bb' moving in the quadrature field have a rotational e.m.f. induced in them, in the same manner as those passing through the main field, and this is of maximum positive value when the north pole at B attains its highest value. In addition to these *two rotational* e.m.fs., the varying fields AA' and BB' set up *transformer* e.m.fs., in coil groups bb' and aa' respectively. Consequently, there are *four* e.m.fs., to be considered before the actual rotor currents which produce the quadrature field can be determined.

The rotational e.m.f. induced in inductors aa' is of maximum positive value when the pole A is at its greatest north polarity, but the transformer e.m.f. set up in these bars by the quadrature field is at the same moment of maximum negative value. Hence the actual e.m.f. (Ea) existing in AA' is the algebraic sum of these two voltages. The rotational e.m.f. due to the main field must be greater than the transformer e.m.f. of the quadrature field; in fact the latter is of such strength that the actual e.m.f., Ea, will be just enough to establish the current which produces the field BB'. Since this quadrature field is at right angles to the main field, its m.m.f. cannot be furnished directly by the stator magnetizing current, so we must investigate further to see how it is taken, as it must be, from the line. It must be remembered that the impedance of the rotor coils is here assumed to be such that the IZ drop is negligible; if this is not the case, the rotational and transformer e.m.fs. will not be in time opposition and their *vector sum* instead of algebraic sum, must be considered.

The main field, by transformer action, induces an e.m.f. in bars bb', and this is opposed to the e.m.f. developed in the same inductors by their motion through the quadrature field. The resultant e.m.f. Eb in these conductors sets up a current affecting the main field and, consequently, the current drawn from the line. The current flowing in inductors bb' due to Eb is equal to that existing in bars aa', which is that producing the cross m.m.f. Moreover, the current bb' is in such direction that it increases the magnetizing current taken from the line, the increment being that which would be necessary to directly magnetize the quadrature field. The reluctance of the cross field's magnetic circuit is substantially the same as that of the main field, consequently the m.m.f. required for both will be the same, and obviously, therefore, a two-phase motor run on one phase will draw twice its normal magnetizing current. This conclusion is borne out by actual practice, tests showing that *the magnetizing current of a single-phase motor is double that taken per phase by a two-phase and three times that required by a three-phase machine, the potential difference, frequency and turns per phase winding being the same.*

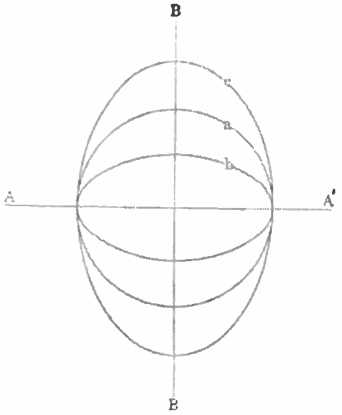

Fig. 4. Forms of Rotating Field at Various Rotor Speeds

At synchronous speeds the two component fields are of equal strength; accordingly they combine to give a circularly rotating field. Below synchronous speed the rotating e.m.f. in the bars aa' is reduced in inverse proportion to the slip, and thus the quadrature field diminishes, while the main field remains constant. Consequently the rotating field developed below synchronous speed is of an elliptical form, the shorter axis being in the direction of the quadrature field BB'. When driven above synchronous speed the field is also of elliptical form, the major axis, however, being in the direction of the cross field. The field forms for different speeds are as illustrated in Fig. 4, a, b, c, respectively, corresponding to synchronous, sub-synchronous and super-synchronous speeds.

The maximum torque which a motor is capable of exerting, other things being equal, depends upon the average value of the magnetic field in which the rotor moves. This mean value, neglecting IR drop and leakage, is in the polyphase induction motor independent of the slip, while for the corresponding single-phase machine the average value of the field decreases as the slip increases; thus the pull-out torque of a polyphase machine connected single-phase will be less than when normally operated.

Many interesting facts concerning the rotor currents as well as the development of the rotating field may be derived through a simple mathematical analysis. Let us consider the elementary bipolar single-phase induction motor represented in Fig. 5 with a coil at an angle α to the main polar axis. Assume as before that the flux distribution is a cosine function of time, and adopt the following notation:

A = area of coil.

ω = angular velocity of the coil, or $\alpha = \omega t$.

$A \sin \alpha = \sin \omega t$ = projected area of coil on plane CC' perpendicular to the flux NS.

B = maximum flux density, its instantaneous value being $B \cos pt$.

Instantaneous flux interlinking coil α is

$$\Phi = AB \cos pt \sin \omega t$$

$$= \tfrac{1}{2} AB\Big(\sin (p+\omega) t - \sin (p-\omega) t\Big); \quad (6)$$

the e.m.f. induced in coil α is

$$e = -\frac{d\Phi}{dt} = \tfrac{1}{2} AB \Big((p-\omega)\cos(p-\omega)t - (p+\omega)\cos(p+\omega) t.\Big) \quad (7)$$

Let r_1 and L_1 represent respectively the effective resistance and inductance of the coils; the values of these constants being based not only upon the character of an individual coil but also to some extent upon the action of neighboring coils. With this notation the current in any secondary coil can be considered as resulting from the e.m.f. of equation 57, or

$$I = 0.5\, AB\Bigg(\frac{p-\omega}{(r_1^2+(p-\omega)^2L_1^2)^{\frac{1}{2}}} \times \cos[(p-\omega)t-\theta_1] - \frac{p+\omega}{(r_1^2+(p+\omega)^2L_1^2)^{\frac{1}{2}}} \times \cos[(p+\omega)t-\theta_2]\Bigg); \quad \text{(S)}$$

wherein

$$\theta_1 = \cos^{-1} \frac{r_1}{(r_1^2+(p-\omega)^2L_1^2)^{\frac{1}{2}}}$$

and

$$\theta_2 = \cos^{-1} \frac{r_1}{(r_1^2+(p+\omega)^2L_1^2)^{\frac{1}{2}}}.$$

The flux produced by one rotor coil and the main field will so react upon each other that the value of the secondary current, if but a single coil be considered, can only be expressed by an infinite series. It has been experimentally shown, however, that the flux-distorting reactions between primary and secondary do not exist with a rotor winding composed of a number of coils which are divisible into pairs, the members of which are placed at 90 degrees (electrical) to each other. The rotor winding of a commercial machine substantially satisfies this condition; consequently the higher harmonics of the rotor current disappear and the current is correctly represented by equa. given above. This equation indicates that the rotor current consists of two parts having different frequencies and amplitudes.

At standstill, any coil spaced an angle y from the axis of the magnetic field will have a current of the following form:

$$I \text{ (standstill)} = \frac{ABp \cos(pt+y-\theta_{ss})}{(r_1^2+pL_1^2)^{\frac{1}{2}}}, \quad (9)$$

which shows that the secondary current at standstill is of line frequency. The current component with frequency $(p-\omega)$ decreases in value as the rotor speed rises toward synchronism, being zero at that limit, and the secondary current then becomes

$$I \text{ (syn)} = \frac{ABp \cos(2\,pt+y-\theta \text{ syn})}{(r^2+2\,pL_1^2)^{\frac{1}{2}}}, \quad (10)$$

which is of double-line frequency.

These variations of rotor current frequencies as well as the presence of the differential $(p-\omega)$ and additive $(p+\omega)$ components may be conveniently observed by the application of a reed frequency meter. Connect such an instrument across the slip rings of the wound rotor of a polyphase motor, excite the stator with single-phase current and then start the machine. As the speed of the rotor increases the frequency meter will indicate the presence of two currents, one increasing and the other diminishing from the line frequency.

Let us now select a coil on the rotor displaced any angle β from the loop α we have just considered, Fig. 5. The flux through this new coil at *synchronous speed* ($\alpha = \omega t = pt$) will be, from equa. 6,

$$\Phi = AB \cos pt \sin (pt+\beta),$$

$$= \frac{AB}{2}\left\{\sin(2\,pt+\beta)+\sin\beta\right\}, \qquad (11)$$

$$\text{e.m.f. coil } \beta = e = -\frac{d\Phi}{dt} = -ABp(\cos 2\,pt+\beta), \qquad (12)$$

$$\text{current coil } \beta = i = -\frac{ABp}{\sqrt{r^2+(2pL)^2}}\cos(2\,pt+\beta-\theta), \qquad (13)$$

$$= K_1 \cos(2\,pt+\beta-\theta). \qquad (14)$$

The total magneto-motive force of all the coils on the rotor may be expressed as $K_1 \Sigma i$. The maximum m.m.f. exists in the plane of the coil in which the current is equal to zero, and hence the poles of the rotor will be in the same plane. Let β' be the angle of that particular coil; then

$$i = K_1 \cos(2\,pt+\beta'-\theta).$$

But since i is equal to zero,

$$K_1 \cos(2\,pt+\beta'-\theta) = 0,$$

whence

$$2\,pt+\beta'-\theta = \frac{\pi}{2}$$

and

$$\beta' = \left(\frac{\pi}{2}+\theta\right) - 2\,pt.$$

This means that the angle between the reference coil and the magnetic pole of the rotor changes at the rate of $-2\,pt$. It also indicates that the pole rotates backwards on the rotor. The latter, however, is turning forward at a rate pt, consequently the rotor poles revolve backward in space at a rate pt, and the equation of this pole in space is

$$\beta' = \left(\frac{\pi}{2}+\theta\right) - pt.$$

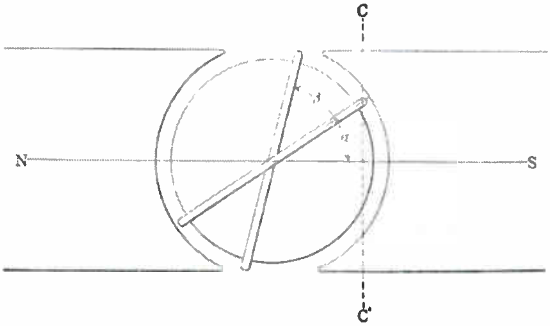

Fig. 5. Coils Inclined to Axis of Oscillating Field

If the equation for the current in the general coil is referred to the magnetic axis instead of to the reference coil, we have

$$i = K_1 \cos\left\{(2\,pt+\beta-\theta)+\left(\frac{\pi}{2}+\theta-2\,pt\right)\right\}$$

$$= K \cos\left(\beta+\frac{\pi}{2}\right).$$

That is, referred to the magnetic axis of the rotor the current distribution is constant; hence the m.m.f. of these currents is constant and rotates backward at synchronous speed, as above proved.

The relative value of the stator and rotor m.m.fs. may be derived as follows: Assume the rotor stationary; this corresponds to considering it as the short-circuited secondary of a transformer. Thus the relations existing between primary and secondary m.m.f's. of a transformer apply or, neglecting resistance and leakage, the secondary m.m.f. is equal and opposite to that of the primary. The current distribution in the bars on the rotor on the basis of the above assumption is expressed by equation (3) as

$$i_s = \frac{ABp}{\sqrt{r^2+pL^2}}\sin(pt-\theta)\sin\beta,$$

which upon neglecting r makes $\theta = \frac{\pi}{2}$ and reduces to

$$i_s = -\frac{ABp}{pL}\cos pt \sin\beta;$$

This, if $t = 0$, becomes

$$i_s = -\frac{AB}{L} \sin \beta. \qquad (15)$$

It is to be noticed that when $t = 0$, the equation of the rotor currents at synchronous speed equation (13) reduces to

$$i_r = -\frac{ABp}{\sqrt{r^2 + (2pL)^2}} \cos (\beta - \theta)$$

which can be still further simplified, if r is negligibly small with respect to pL, to the following form

$$i_r = -\frac{AB}{2L} \sin \beta. \qquad (16)$$

Comparing these values of i_s and i_r we see that these currents have the same distribution in the rotor, but that amplitude of the latter is only one half that of the former. Consequently, since the m.m.f's. of the stationary rotor and of the stator are equal, the m.m.f. of the synchronously revolving rotor is one-half that of the stator winding.

The magneto-motive force effective in developing the flux $B \cos pt$. *when the two fields coincide*, may be expressed as $Y - X$, wherein Y represents the maximum m.m.f. developed by the stator and X that due to the rotor. But, as above shown, $X = Y \div 2$, hence the excitation necessary to produce the flux $B \cos pt$ throughout the magnetic circuit of the machine is $\frac{Y}{2}$, or X.

The two magneto-motive forces *acting at any instant* in this type of machine are:

$Y \cos pt$, stationary in space,

X, constant in value, but rotating backward at synchronous speed. Since X rotates backwards it may be written $X = X \cos pt - X \sin pt$, and consequently $Y - X$, the total magneto-motive force acting at any instant, becomes

$$Y \cos pt - X \cos pt + X \sin pt = X \cos pt + X \sin pt.$$

This means that *the total m.m.f. acting at any instant is of constant value and rotates forward at synchronous speed*.

The magnetic reluctance of commercial single-phase motors, due to the use of uniformly distributed windings, is practically the same, whatever the axis of the field; consequently the reactions existing between stator and rotor currents produce at or near synchronous speed a circular rotating field, and the formulæ which apply to polyphase motors may be utilized. The effect of leakage and rotor resistance will modify this rotating field somewhat, changing it from circular to elliptical form.

Torque Equations

It has been shown in the derivation of equa. 8 that, when the secondary of a single-phase induction motor is caused to rotate at any rate ω, its current may be expressed as

$$I = \frac{AB}{2}\left(\frac{(p-\omega)}{\sqrt{r_1^2 + (p-\omega)^2 L_1^2}} \times \cos[(p-\omega)t - \theta_1] - \frac{(p+\omega)}{\sqrt{r_1^2 + (p+\omega)^2 L_1^2}} \times \cos[(p+\omega)t - \theta_2]\right).$$

Inspection of this equation shows that the rotor current is composed of two parts, one of a lower and the other of a higher frequency than the rotating field. We may consequently consider that this current is set up through the action of two synchronously rotating fields, one revolving in the same direction as the rotor and the other oppositely.* The frequency of the rotor current component, due to the suppositional field revolving in the same direction as the rotor, is naturally less (by the velocity of the rotor) than synchronous value or it is $(p - \omega)$. The component due to the oppositely rotating field has a frequency higher than that of the line, its value being $(p + \omega)$.

The per cent. slip of the rotor with respect to the field first is $\left(\frac{p-\omega}{p}\right)$ 100, and referred to the second field it is $\left(\frac{p+\omega}{p}\right)$ 100.

The effective turning effort of the motor is the resultant of the interaction between the rotor current and two oppositely rotating fields. But, since the rotor and one field turn in the same direction, the torque due to this latter field must be greater than that set up by the other. The torque developed by a polyphase induction motor may be expressed by the following equation:

$$T = \frac{N_2^2 e^2 s r_2}{\omega_1 (r_2^2 + s^2 x_2^2)},$$

wherein s is the per cent. slip between rotating field and rotor core; N, inductors in series per phase of the rotor; e, volts per turn; r_2, resistance; x_2, reactance at standstill per motor phase, and $\omega_1 = p$, the angular velocity of the revolving field. We may accordingly

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*G. Ferraris, Mem. Reale Accad. di Scienze Torino, Series II, Vol. XLIV, December, 1893. *Electrician*, Vol. 33, pages 110, 129, 132, 184. London, 1894.

write the two component torques existing in the single-phase motor as

$$T_1=\frac{N_2^2e^2s_1r_2}{\omega_1(r_2^2+s_1^2x_2^2)},$$

$$T_2=-\frac{N_2^2e^2s_2r_2}{\omega_1(r_2^2+s_2^2x_2^2)},\text{ wherein}$$

$$s_1=\frac{p-\omega}{p}=\frac{\omega_1-\omega}{\omega_1}\text{ and}$$

$$s_2=\frac{p+\omega}{p}=\frac{\omega_1+\omega}{\omega_1}.$$

The total effective torque is

$$T=T_1+T_2=\frac{N_2^2e^2r_2(s_2-s_1)(s_1s_2x_2^2-r_2^2)}{\omega_1(r_2^2+s_1^2x_2^2)(r_2^2+s_2^2x_2^2)},\tag{17}$$

wherein s_2-s_1 is positive for speeds below synchronism, while s_1s_2 is variable but never greater than unity.

Analysis of this equation brings out the following facts:

1. That the torque of the single-phase machine varies as the square of the impressed voltage, this being the same relation as obtains in polyphase induction motors.

2. That the motor exerts no torque at standstill because s_2-s_1 then equals zero, which fact makes the numerator of the same value.*

3. The motor cannot operate at synchronous speed, because this makes s_1 zero, in which case the torque developed is of negative value ($s_1s_2\,x_2^2-p_2^2$ reducing to $-r_2^2$), and the machine tends to act as a generator. Consequently the single-phase induction motor must rotate at less than synchronous speed.

4. The fact that the maximum value of s_1s_2 is unity indicates that the single-phase induction motor cannot operate unless the reactance of its rotor winding at standstill is greater than its resistance. Unless such is the case $s_1s_2x_2^2-r_2^2$ will have a negative value, which means that the machine would tend to develop a negative torque or act as a generator. Fig. 6 indicates how the speed-torque curves of a single-phase induction motor are affected by change in the value of rotor resistances. Curves *A* and *B* may be considered as representative of standard machines. Curves *C* and *D* indicate the effects produced by inserting relatively large resistances into the rotor winding. It is apparent from these curves that the introduction of resistance into the rotor circuit for purposes of speed regulation is attended by a marked reduction of the overload capacity of the motor, and cannot be used as conveniently or advantageously as with polyphase motors. It is, however, employed to limit the starting current.

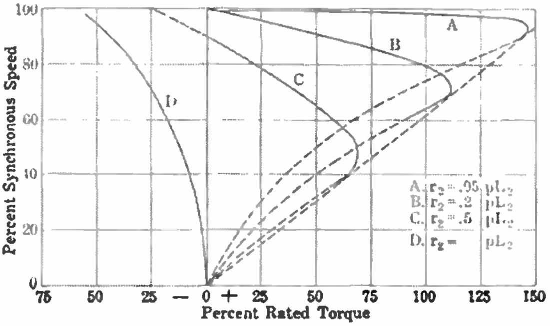

Fig. 6. Speed-Torque Curves of Single-Phase Induction Motor, with Different Values of Rotor Resistance

5. The torque developed by a polyphase motor operated as a single-phase machine is less than that produced when normally connected, because of the presence of the counter torque T_2.

6. If we take the first differential coefficient of equation (17) with respect to r_2 and place it equal to zero, we find that the maximum torque developed for any rotor speed ω exists when

$$r_2=(x_2s_1s_2)^{\frac{1}{2}}\div\left((s_1s_2)^{\frac{1}{2}}+2\right),$$

and that the maximum torque

$$T_{max}=N_2^2e^2s_1s_2(s_2^{\frac{1}{2}}-s_1^{\frac{1}{2}})\div\omega_1r_2.\tag{18}$$

This equation shows that the torque at any selected speed is greater the less the value of r_2.

(*To be continued.*)

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[*] See equations (5) and (6).

COMMERCIAL ELECTRICAL TESTING

PART VII

BY E. F. COLLINS

ROTARY CONVERTER—(Cont'd)

D.C. Circulating Current

Fig. 32 shows the connections for two three-phase converters wired for a pump back heat run without a potential regulator to control the load. The core losses and C^2R losses are supplied from the direct current end. The diagram shows, also, the standard starting panel, which should always be used when two converters are tested together.

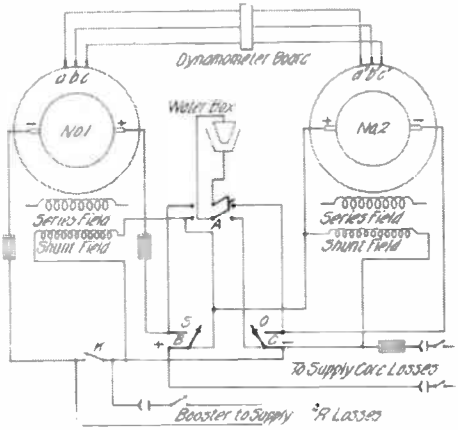


Fig. 32. Connections for Pumping Back Rotary Converters Without the Use of a Regulator

To start the rotaries, for instance No. 1, close the shunt field switch and the switch K, the latter short circuiting the armature of the loss supply. Note that the shunt fields are wired across the core loss supply, which in turn is wired to buses B and C of the starting panel, and that the series fields are left open. Throw switch A to the left and slowly reduce the resistance of the water rheostat until it is practically short circuited, when the switch S may be closed. The blade of the water rheostat is now drawn out of the water and the switch A thrown to the right. Machine No. 2 is then started in a similar manner.

The field strength of each machine is then reduced until both machines run at normal speed. Next connect a number of incandescent lamps in series, the rated voltage of which is equal to the sum of the machine voltages across rings $A A$; *i.e.*, across switches located on the dynamometer board. Two sets of lamps should be provided, one being connected across one of the switches while the other is stepped across each of the other switches in turn. Should one set show a rise and fall in voltage displaced in time with relation to that of the other, the two phases are reversed and must be corrected. When all phases show a simultaneous rise and fall, the machines may be phased together and their speeds brought to the same value by changing the field on one of them. When the time between rise and fall of voltage, as shown by the lamps, decreases to a period of five seconds or longer, all switches are closed simultaneously and the lamps become dark.

During the period of starting and phasing the machines together, the fields of the booster should be opened and the armature short circuited. When the rotaries are synchronized, the switches across the armatures of the boosters are opened and a weak field applied, the line meter on machine No. 1 being watched. The reading of this meter should reverse from that given on motor load, if machine No. 1 is taking load as a rotary. By reversing the booster field either machine can be made to run as a rotary.

After balancing the current in each phase, full load phase characteristics may be taken by holding the speed constant by means of the field of the inverted machine, and the load constant by means of the booster, the shunt field of the rotary being varied throughout its range and the current input read. Full load voltage ratio should next be taken, after which the heat runs may be made.

A line shunt must be used in each side of the direct current circuit, otherwise one line will have more resistance than the other and the currents flowing through them will have unequal values, the unbalanced current returning through the alternating current ends of the machines. The currents in these lines can be balanced by decreasing the resistance in the low reading line. The direct currents should be balanced before attempting to balance the alternating current.

In running a pump back test there will be a slight difference in the direct current voltages of the two machines, equal to the CR drop of the set. The field of the inverted machine will be less than that required for minimum input and will carry the additional current necessary for supplying the core losses.

This method of supplying the C^2R losses from a booster requires such a large low voltage booster that it is not often used, except for small rotaries.

With a Booster in the A.C. Side

A second method of pumping back rotaries on full load heat runs is to use an induction voltage regulator in the alternating current side of the machines, as shown in Figs. 33 and 34. The regulator is connected with its secondaries in series with the alternating current lines and its primaries across the alternating current terminals of the inverted machine. It is always preferable to connect the regulator between the inverted rotary and the dynamometer board. The regulator takes the place of the booster used in the previous method, and is very satisfactory for supplying the C^2R losses.

Starting the machines, checking the phase rotation, phasing in, and other operations already described, are repeated with this method. Always see that the regulator is set at the no boost point before phasing in, otherwise load will be thrown on when the switches are closed.

Load is increased by turning the core of the regulator in the direction of boost, the ammeter of machine No. 1 being watched at the same time. If the reading reverses from motor load, then No. 1 is running as a rotary; if, however, No. 1 does not reverse, the regulator should be turned in the opposite direction. This shows that the regulator is wrongly connected in reference to its markings; there is no necessity, however, to change connections.

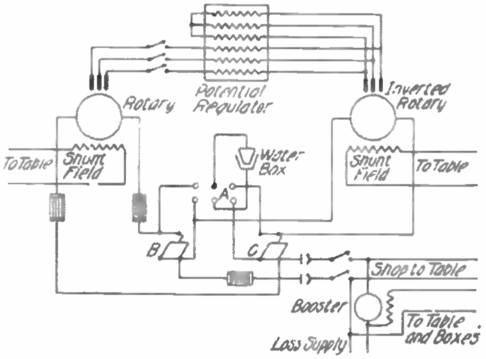


Fig. 33. Connections for Pumping Back Rotary Converters with Regulator

Using A.C. Loss Suppy

If, instead of supplying the losses from a direct current source of power, an alternator is connected across the alternating current lines, between the inverted rotary and the regulator as in the preceding method, the losses can be supplied at the alternating current end. When the alternator is large enough to start the rotaries, the wiring on the direct current end is greatly simplified. The starting panel is omitted and the shunt fields are connected according to the print of connections for the machine. Load is obtained by means of the regulator as before and the test carried out as already described.

If the alternator is too small to start the machines, the latter may be started singly from the direct current side as before, and the two phased together. The alternator is then synchronized with the pair. If only one machine can be started by the alternator, bring it up to speed, open all its circuits, and let it run by its own momentum while the second machine is quickly started. The excitation is then removed from the alternator field and the switches on the first machine are closed. Excite the alternator field and bring both machines up to speed together. After the machines are once started, they can be brought up to speed without an excessive current from the alternator.

Alternating Current Generators

Complete Tests consisting of special tests and temperature tests.

Special tests include saturation and synchronous impedance, and from these the regulation of the machine is calculated as follows:

Let V = normal voltage line, C = amperes line, R = hot resistance between lines.

For three-phase machines $C = \frac{\text{Kw.}}{\text{Voltage}\ \sqrt{3}}$

For two-phase machines $C = \frac{\text{Kw.}}{2\ \text{Voltage}}$

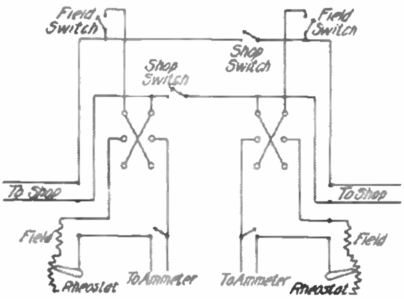


Fig. 34. Table Connections for Rotary Converter Pump Back

For three-phase machines, voltage drop in armature, $C_1R_1 = \frac{\sqrt{3}\,CR}{2}$

For two-phase machines $C_1R_1 = CR$.

Let a_1 = amperes field on saturation curve corresponding to $V + C_1R_1$ and a_2 = amperes field on the synchronous impedance curve corresponding to C.

The amperes field required to produce normal rated voltage with full load on the generator will be $a_3 = \sqrt{a^2_1 + a^2_2}$.

Let the voltage on the saturation curve corresponding to $a_3 = V_1$.

Then the per cent. regulation $= \frac{V_1 - V}{V}$

If it is desired to calculate the regulation of the machine at any power factor, then C becomes $\frac{C}{\%\ P.F.}$ and $a_3 = \sqrt{a^2_1 + a^2_2 + 2a_1a_2 \sin\theta}$ when θ is the angle of which the per cent. power factor is the cosine.

Input-output efficiency test is made by the input-output method.

Standard efficiency test is made by the method of losses.

The calculation of a standard efficiency test is made as follows:

Let V_L = volts line
W_b = output = $\sqrt{3}V_L C_L$ for three-phase and $2\ V_L C_L$ for two-phase
C_L = amperes line R_1 = hot res. of armature between lines
C_1 = amperes field
R_2 = hot res. of field
W_1 = open circuit core loss corresponding to $V_L + CR$ on the core loss curve
W_2 = short circuit core loss corresponding to C_L on the short circuit loss curve
W_3 = friction and windage obtained from core loss test

C_1 is calculated for each load, as in the test for regulation.

CR = the drop in the armature = $\frac{\sqrt{3}}{2}\, C_L\, R_1$ for three-phase machines and $C_L\ R_1$ for two-phase.

$\Sigma W = W_1 + \frac{1}{2}\ W_2 + W_3 + \frac{3}{2}\ C_L^2\ R_1 + C_1^2\ R_2$
for three-phase machines
$= W_1 + \frac{1}{2}\ W_2 + W_3 + 2\ \ C_L^2R_1 + C_1^2R_2$
for two-phase machines
Watts input $= W_a = W_b + \Sigma W$

Efficiency $= \frac{W_b}{W_a}$

W_3 need not be considered if the machine is furnished without base, shaft or bearings.

The above method of calculation is used when the machine is to operate at unity power factor.

If it is desired to calculate the efficiency at any power factor, the following calculations must be made.

$C_L = \frac{\text{Kw.}}{V_L \times \sqrt{3} \times \%\ \text{P.F.}}$ and
$W_b = \sqrt{3} \times V_L \times C_L \times \%$ P.F. for three-phase machines.

$C_L = \frac{\text{Kw.}}{V_L \times 2 \times \%\ \text{P.F.}}$ and
$W_b = 2V_L \times C_L \times \%$ P.F. for two-phase machines.

C_1 should be calculated for various power factors as given under regulation.

The change in the line current will affect C_1, W_1, W_2, and the C^2R of the armature. See Fig. 33 and Table XIII.

Non-inductive normal load heat runs consist in running the machine under normal load at unity power factor until constant temperatures are reached. These final temperatures are then recorded and readings taken of regulation with unity power factor.

Non-inductive overload heat runs consist in bringing the machine to normal load temperatures, applying the overload at unity power factor for the specified time, and recording the overload temperatures. Readings for regulation at unity power factor should be taken.

Normal load and overload power factor heat runs are made in the same way as normal and overload non-inductive runs, except that the machine is operated at a specified power factor. Wattmeters should be used with the voltmeter and ammeters to determine the power factor.

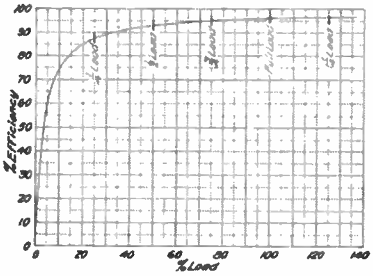
SYNCHRONOUS MOTORS

The preliminary tests taken on synchronous motors consist of drop on spools, air gap, resistance measurement, balancing of phase voltages, phase rotation and running free minimum output.

Complete tests consist of special tests and normal and overload heat runs.

Special tests consist of starting tests, open and short circuited core loss, saturation, synchronous impedance, no load and full load phase characteristics, and wave form. The method of taking phase characteristics has previously been described.

Starting tests should be made both with and without a compensator, if the motor is of a new type and rating and is to be started with a compensator when installed. If the motor does not form part of a motor-generator set, it should be belted to a generator so that it will have some load at starting.



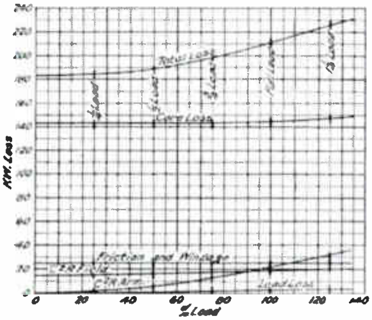


Fig. 35. Efficiency and Losses on a 5000 Kw., 11000 Volt, 3-Phase A.C. Generator

TABLE XIII—Eff. and Losses of a 5000 Kw., 11000 V., 28-Pole, 60-Cycle, 3-Phase Generator

% Load	0	25	50	75	100	125
Volts Line	11000	11000	11000	11000	11000	11000
Amps. Line	0	65.5	131	196.5	262	317
Amps. Fld.	220	224	228	235	245	257
CR		12	24	36	48	60
V+CR	11000	11012	11024	11036	11048	11060
Core Loss	143000	143000	143100	143600	144100	147000
⅓ Short Cir. Core Loss	—	—	200	580	1300	2500
C²R Arm.	0	1330	5320	12000	21300	31100
C²R Fld.	14500	15000	15600	16600	18000	19800
Friction	25000	25000	25000	25000	25000	25000
Total Losses	182500	184330	189220	197700	209700	225400
Kw. Output	0	1250	2500	3750	5000	6250
Kw. Input	182.5	1434	2689	3948	5210	6475
% Efficiency	0	87.3	93.0	95.0	96.0	96.5

Res. Arm. (Line) .1927 Ohms 25° C. .207 Ohms Hot.
Res. Fld. .2795 Ohms 25° C. .3003 Ohms Hot.

The motor should first be tested for starting without the compensator. The center line of one pole is placed in line with the center line of the frame and 180 electrical degrees marked off in a clockwise direction from this line on the head end of the motor. The total length of this scale should be two-thirds of the distance between the center lines of adjacent poles for three-phase machines, one-half for two-phase machines, and one-third for six-phase machines. The scale should be divided into four equal parts, each division line being numbered. On each one of these scale divisions, the center line of the marked pole should be placed and the motor started. Thus five tests are made to insure that the motor will not stick in any position. See Fig. 36.

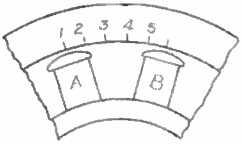


Fig. 36

With the pole A moved to position No. 1 and the machine at rest, sufficient current should be sent through the armature to give a reasonable reading of amperes and volts on the various phases, and induced volts on the field. The induced volts field should be read by a potential transformer and alternating current voltmeter. These readings are taken to determine which phase gives maximum readings of current and voltage.

The voltmeter and ammeter should be placed in this phase and the armature current increased until the motor starts. Volts armature, amperes armature and induced volts field should be simultaneously read. The starting voltage is now held constant until the motor comes to synchronism, and the time required to reach this point recorded. The machine attains synchronism when the induced volts on the field fall to zero. The machine is then shut down and the tests are repeated for each of the other positions.

If a motor shows a tendency to remain at half speed, the alternating current voltage should be increased until the motor breaks from half speed and comes up to synchronism, the voltage required to accomplish this being held until full speed is reached and then recorded.

If the test is required to be made with a compensator, the motor should be set with its field in the position where greatest starting current is taken and allowed to rest in that position for at least six hours until the oil is well pressed out of the bearings. This is done in order to obtain the worst starting conditions likely to occur in normal operation. Connections are then made to the lowest tap of the compensator, and with normal voltage held on the line the starting switch of the compensator is closed. If the motor fails to start, the voltage must at once be switched off and connections made with the next higher taps on the compensator, and so on until the motor starts. Readings should be taken on each of the taps of the compensator in the starting position, with the machine

TABLE XIV—Starting Test on a 425 Kw., 11000 V., 8-Pole, 25-Cycle, 3-Phase Syn. Motor

	VOLTS LINE			AMP. LINE			Ind. Volts per Spool	Pos. at Start	Time to Syn.
	1-2	2-3	1-3	1	2	3			
Rest	1340	1430	1480	15	17.5	15.2	52	1	
Start			2650		35		90.7		
Syn.	2650	2650	2650	9.2	9	8.9			66 Sec.
Rest	1255	1340	1340	15	16	13.6	47	2	
Start		2560			30		88.3		
Syn.	2560	2560	2560	9.5	9.3	9.2			70 Sec.
Rest	1153	1300	1320	15	14	12.7	45	3	
Start			2380	29.5			84.7		
Syn.	2380	2380	2380	10	10.2	10			70 Sec.
Rest	1248	1260	1165	15	12.8	13.8	44	4	
Start		2590		33			80.8		68 Sec.
Syn.	2590	2590	2590	9	9.3	9.5			
Rest	1400	1308	1302	15	13.9	16.2	49	5	
Start	2620					32	87		
Syn.	2620	2620	2620	8.9	9.1	9.3			64 Sec.

Is there any tendency to stick at half speed? No.

at rest, to determine the voltage ratio of the taps of the compensator. All these tests should be made with the field circuit of the motor open, and enough time allowed between trials to permit the compensator to cool, since it is designed for intermittent service only. Table XIV.

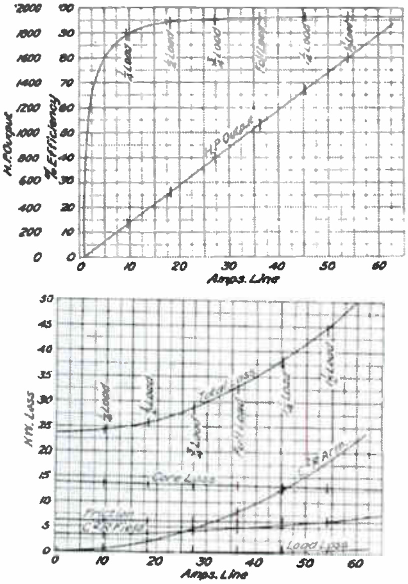


Fig. 37. Efficiency and Losses on a 1070 H.P., 13200 Volt, 3-Phase Synchronous Motor

Input-output efficiency test is made by the input-output method.

Standard efficiency tests are made by the method of losses. In calculating efficiency, the same nomenclature is used as that employed for alternating current generators. C_1 is either taken from the phase characteristics or is calculated in the same manner as for alternating current generators.

Watts input $W'_a = V'_L C_L + C_1^2 R_2$.

Watts output $= W'_b = W'_a - \Sigma W'$.

$$\text{Efficiency} = \frac{W'_b}{W'_a}$$

W' = open circuit core loss corresponding to $V'_L - CR$ on the core loss curve.

$$\text{Horse-power output} = \frac{W'_b}{746}.$$

See Table XV and Fig. 37.

The non-inductive load heat run is made as follows: Run the machine under load at unity power factor until it has reached constant temperature and record temperatures. Take readings of regulation at normal and no load and full load phase characteristics.

The non-inductive overload heat run consists in bringing the machine to normal load temperature, applying the overload for the specified time, recording temperatures and taking readings of regulation at unity power factor.

Normal load power factor heat run is similar to the normal load non-inductive run, except that the machine is operated at a specified power factor. Wattmeters should be used as described for alternating current generators.

Overload power factor heat run is similar to the overload non-inductive run, except that the power factor is less than unity.

(To be Continued)

TABLE XV—Eff. and Losses of a 1070 H.P., 13200 V., 6-Pole, 25-Cycle, 3-Phase Syn. Motor

% Load	0	25	50	75	100	125	150
Volts Line	13200	13200	13200	13200	13200	13200	13200
Amps. Line		9.5	19.0	28.5	38.0	47.5	57
Amps. Fld.	50.1	50.6	51.0	55.1	59.7	63.8	68.9
CR	—	31.5	69.0	103	168	172	205
(V – CR)	13200	13165	13131	13097	13032	13028	12995
Core Loss	13900	13800	13700	13600	13500	13400	13300
½ Short Cir. Core Loss	—	47	107	190	310	473	700
C²R Arm.		565	2265	5100	9040	14100	20400
C²R Fld	3550	3620	3680	4300	5050	5770	6710
Friction	6272	6272	6272	6272	6272	6272	6272
Total Losses	23722	24314	26024	29462	34172	40015	47442
Kw. Input	23.72	220.63	436.68	654.3	870.05	1089.8	1306.7
Kw. Output	0	196.32	410.66	624.8	835.9	1049.8	1259.3
H.P. Output	0	263.2	550	837	1121	1408	1689
% Efficiency	0	89.0	94.1	95.5	96.1	96.3	96.4

Res. Arm. (Line) 3.86 Ohms 25° C. 4.18 Ohms Hot 47. Res. Fld. 1.34 Ohms 25° C. 1.42 Ohms Hot 40.

A MOTOR OPERATED BILLET MILL.

By B. E. Semple

The Indiana Steel Company, Gary, Indiana, started its billet mill in August, 1909. This was the second of the several large motor-operated mills installed by this company for the manufacture of steel, that was put into regular operation.

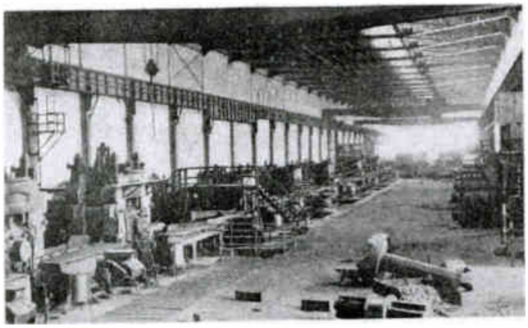


Fig. 1. Last Two Stands of 40 In. Blooming Mill Shown in Foreground: Five Stands of 32 In. Blooming Mill in Background

The principal work in this mill is accomplished by five 25 cycle, 3 phase, slip ring type induction motors, the ratings of which are as follows:

2 motors, 14 poles, 2000 h.p., 214 r.p.m., 6600 volts.

3 motors, 36 poles, 6000 h.p., 83 r.p.m., 6600 volts.

These motors are designed to carry full rated load continuously, with a temperature rise not in excess of 40 degrees C.; 25 per cent. overload continuously, with a temperature rise of not more than 50 degrees C.; and 50 per cent. overload for one hour, with a temperature rise not in excess of 60 degrees C.

Like the rail mill motors described in the Review for Feb., 1910, they were purposely designed for heavy rolling mill duty, their proportions being extremely liberal. The entire five motors represent a total weight of 1518 tons, and will carry 3½ times their rated load before dropping out of step.

The method of connecting the motors to the rolls in this mill differs considerably from that employed in the rail mill, except in the case of the two 2000 h.p. motors, which differ only in that the gearing is located in the motor room instead of in the mill proper.

The illustration on page 194 shows the east half of the south motor room, the two motors in the distance being the 2000 h.p. machines which drive the 40 in. blooming rolls. The motor in the foreground is a 6000 h.p. machine, and drives the five stands of 32 in. blooming rolls, each of which is connected to the motor driven shaft through bevel gears.

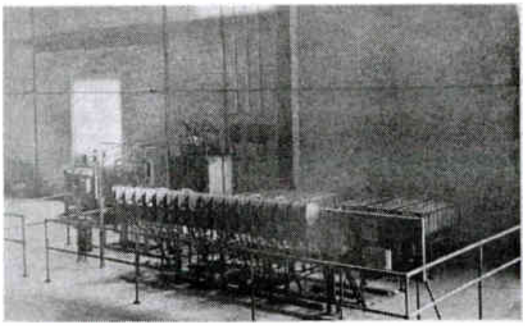


Fig. 2. Primary and Secondary Control for 6000 H.P. Motor

Fig. 1 is a view on the other side of the wall, showing in the left foreground the last two stands of the 40 in. mill, and in the background all five stands of the 32 in. mill; the former driven by the 2000 h.p. motors and the latter by the 6000 h.p. motor.

The west half of the motor room contains another of the three 6000 h.p. motors. This motor operates the 24 in. mill, consisting of six stands of rolls, each connected to the motor driven shaft through bevel gears, and drives from one end only, instead of from both ends, as in the case of the first mentioned 6000 h.p. motor.

Fig. 2 shows the primary and secondary control for the last named motor, the 6600 volt motor-operated primary switch being located in the rear and to the left, and the secondary contactor panel in the front, with the secondary resistance directly behind it. The master controller is located to the left and in front, directly beneath the panel containing the instruments.

In this mill the motors are started and stopped by the motor attendant rather than by the mill operators in the mill proper.

Fig. 3 is a view of the 18 in. mill, comprising five stands of rolls and driven by the third 6000 h.p. motor, shown in Fig. 4. The motor is located in the north motor room, and the connections to the rolls are made through bevel gears, as in the cases of the other 6000 h.p. motors.

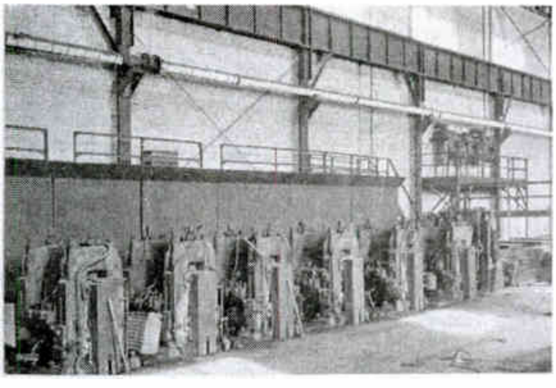


Fig. 3. 18 in. Blooming Mill Operated by 6000 H.P. Induction Motor

Each of the five motors is equipped with a heavy fly wheel to assist in smoothing out the peaks that would be demanded by the motors from the generating station if the fly wheels were not used. The wheels on the two 2000 h.p. motors are external, as seen in Fig. 6, while in the case of the three 6000 h.p. motors, the additional weight necessary to obtain the desired fly wheel effect is added directly onto the rotor. The fly wheel effect of the 6000 h.p. rotor is equal to 10,330,000 pounds, and that of the 2000 h.p. rotors, 4,720,000 pounds, at a one foot radius.

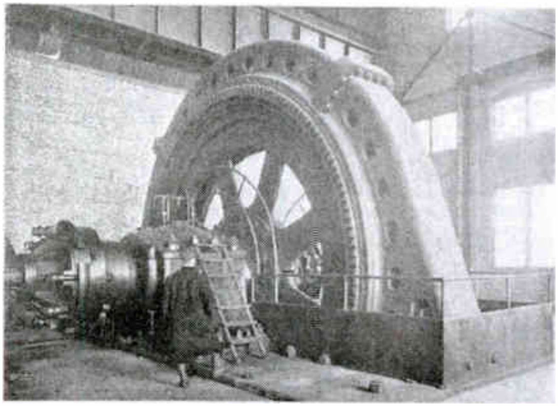


Fig. 4. 6000 H.P. Three-Phase Induction Motor

The 2000 h.p. motors were assembled and tested at the works of the General Electric Company before shipment, and the fly wheels, which are laminated, were assembled at the point of installation. The 6000 h.p. motors were entirely too large to ship, even partially assembled, and consequently were completely assembled at the Gary plant by experts sent from the Company's works.

All of the motors have water-jacketed bearings, those for the 6000 h.p. being 30 in. in diameter and 70 in. long and those for the 2000 h.p., 24 in. in diameter and 60 in. long. However, water is not used on the bearings excepting in instances of heating to such an extent as to demand it; owing, for instance, to the failure of the oiling system.

This mill was designed to roll 4000 tons in twenty-four hours and is the largest straightaway billet mill in existence; it is strictly modern in every detail and its operation throughout has been entirely successful.

The rolling cycle begins with the receipt upon the approach table to the first pass, of an 8000 pound ingot from the reheating furnaces, measuring 20 ft. by 24 ins. sq. section. Twenty-one passes are made in reducing this ingot to either a 2 in. by 2 in., or a 1¾ in. by 1¾ in. billet.

The first four passes are made in the 40 in. mill, driven by the two 2000 h.p. motors; the next five in the 32 in. mill, driven by a 6000 h.p. motor; the next six in the 24 in. mill, driven by a 6000 h.p. motor; and the final six in the 18 in. mill, also driven by a 6000 h.p. motor.

The apparatus for controlling the five large motors in this mill is almost identical to that employed in the rail mill. Reversing switches are provided in order that the motors may be reversed if necessary, and provision has been made to introduce a predetermined amount of resistance into the secondary circuit to increase the slip and thus allow the fly wheel to share the load with the motor.

The feeder circuits entering this mill are protected against lightning and surges by aluminum cell arresters.

THE ELEMENTS OF TRANSFORMER CONSTRUCTION

Part II

By W. A. Hall

As already mentioned, the prime consideration in the lighting transformer is insulation, particularly between primary and secondary. The material used at this point consists principally of a heavy layer of built-up mica, augmented by high grade material carrying varnished film, which is one of the best insulators. There is thus afforded a certain protection, not only under normal conditions, but also under those of severe overload, short circuit, or external fire, which may completely disintegrate the internal transformer before the windings come together and thus allow the high potential of the primary to pass to the secondary line, with possibly serious consequences. While the transformers are designed for operation at 2400 volts or less, they are regularly tested by the manufacturer, between primary and secondary, at not less than ten thousand volts, the average breakdown strength being probably more than double that amount. The other insulation between turns, layers, sections and coils are generally fibrous, untreated materials, particularly adapted to receive and retain the oil-proof insulating compound which is applied by high pressure to the coils after they have been subjected to vacuum and which permeates the innermost fibres and interstices, forming a compact structure. This serves to preserve, protect, insulate and conduct away the heat generated within during operation. The otherwise spongy mass of wire and insulation is thus also made capable of resisting the mechanical stresses. Fig. 11 shows a group of transformers finished and ready for test. An interesting comparison with the design of a quarter of a century may be had by referring to Fig. 10, which shows one of the first commercial transformers made in this country.


Fig. 10. Transformer of Quarter Century Ago

Next in importance are durability, reliability and longevity, to insure continuity of service and low rate of depreciation. These features demand superior insulation and mechanical construction and moderate temperatures, the latter with particular reference to an even distribution thereof. It is now common practice to make the case tight and fill it with a specially prepared oil, completely submerging the transformer. In operation, the heat starts a natural circulation upward

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Errata: April Review, page 182, 2d column, line 14, should read Figs. 3, 4 and 5; page 184, 2d column, 5th line from bottom, should read Fig 4 in the center, from the warm transformer outward to the sides of the case at the top, and thence by contact with the cooler sides of the case downward to the bottom, when it is cooled ready for return through the transformer. In addition to its cooling properties, the oil possesses a very high insulating quality, and in consideration is practically indispensable.

Small masses of coil must be opened by ventilating channels which will direct the oil to their innermost parts in the course of its circulation; while the larger masses must be subdivided into several coils interspersed by generous channels which will give the cooling medium access to parts alike, thus maintaining uniform low temperatures.

The primary wires of small transformers, which are of very small cross-section, are round, all other conductors being of rectangular section. The latter improve the space factor and afford ample bearing surfaces to resist crushing of insulation through mechanical stresses, and operate in conjunction with the coil filler to produce a solidity of coil of even temperature to prevent unequal expansion and contraction. These precautions, well executed, insure reliability and long service.

Closely following these qualities in importance comes efficiency. A consideration of this factor must take account of a somewhat unusual condition, which may be summed up as follows: These transformers are built in sixteen standard sizes ranging from 6/10 kw. to 50 kw., the average of all transformers built being about 7½, although as is evident, sizes smaller than this predominate in number. These small units are installed on lighting circuits in vast numbers, and their use is at present increasing at the rate of more than 50,000 per annum. These circuits are excited continuously night and day at normal voltage, and since the core loss of the transformer is dependent only upon the voltage of the system, it is constant for all loads on the transformer, including no load. The copper loss varies as the square of the load, and for ordinary service is considered to be about equivalent to that corresponding to full load for three hours, for each day. From this fact alone, the ratio of these two losses in a well-designed transformer might be expected to be one to eight. In operation, however, the cost of supplying the energy for the two purposes is not equal. The core loss is maintained largely during hours of light load, when plant efficiency is at a minimum. On the other hand, the copper loss is largely carried at a time when the station is operating at highest efficiency, but when it is frequently taxed to supply the demand for power, and the loss then becomes a limitation on the output. It would be difficult to fix the exact cost of supplying this waste energy, but the

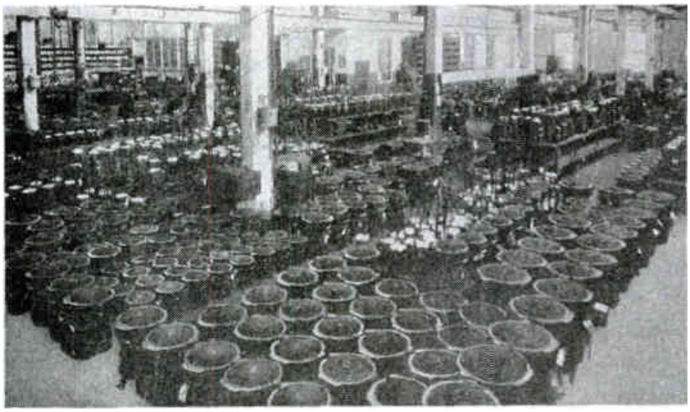

Fig. 11. Group of Small Transformers Completed and Ready for Test

mean of a number of values obtained from many of the large stations places it at 1 cent per kilowatt hour for core loss and 4 cents per kilowatt hour for copper loss; whence, in consideration of the time during which each is maintained, the relative costs are as 2:1.

In designing the transformer, other factors enter to distort this relation. With existing materials, cost of labor and present practice, it costs more, generally speaking, to produce a transformer with low core loss than one with low copper loss. Coincident with high core loss is likely to be high magnetizing current, which is detrimental to satisfactory operation. Opposed to this, however, is the fact that the regulation—another measure of merit—varies substantially in direct proportion to the copper loss and should be kept low. Likewise, the temperature during operation depends primarily upon the copper loss and demands a small value. Core loss also affects temperatures somewhat, although to a less degree than the copper loss, owing to more ready means of dissipation.

A compromise between these many dependent variables of manufacture and operation, as well as cost of depreciation, interest on investment, and other fixed charges, has resulted in a design which gives substantially equal losses in the smallest sizes, and in the largest sizes a copper loss twice that of the core loss; the losses of the intermediate sizes having a relation varying along a smooth curve between these extremes. The full load efficiencies vary from 95 per cent. on the smallest to 98.5 per cent. on the largest sizes, disclosing the fact that these small pieces of apparatus, without moving parts, transform energy at a very high efficiency.

A multiple transformer must not only operate continuously with good efficiency, but at the same time must maintain a constant potential on the secondary lines at all loads within rating. In other words, its regulation (defined as the per cent. of secondary voltage variation from no load to full load) must be very low. The principal factor affecting regulation is the voltage loss caused by the ohmic resistance of the copper, commonly termed the IR drop. In fact, the other principal component, reactive drop, need not be considered in approximations, except under special conditions of load involving low power factors. Since the IR drop bears the same relation to the normal voltage that the copper loss does to the transformer capacity, the regulation of a transformer may be estimated with a fair degree of accuracy by dividing the copper loss by the capacity of the transformer, both expressed in the same units. The actual regulation can never be better than this; in fact, it will be usually about 5 per cent. higher.

Fig. 12. General View of Winding and Clamping Pancake Coils for Large Shell Type Transformers

Now the evolution from this lighting transformer to others in this group consists merely in magnitude of figures and the accentuation of certain characteristics, due to either the physical proportions or a change in the demands of service for which it is intended. The same fundamental factors enter into the design and construction of each, although their relative importance is modified. For example, the output of transformers is approximately proportional to their weights, or masses; the losses, therefore, are also proportional to this factor. The only means of dissipating these losses, however, is in form of heat, through the surfaces of the transformers. That is, the losses or heat units to be dissipated increase as the cube of the linear dimension, whereas the surfaces increase only as the square. Obviously, then, if the same temperature of operation is to be maintained, a different means to the accomplishment of that end must be introduced as the unit grows larger. Here, then, is one characteristic which, while negligible in a 6/10 kw. transformer, requires a few oil channels, then more, and finally artificial cooling in the form of air-blast, forced oil circulation, or water-cooling coils immersed in oil, as the size of the unit increases.

In like manner the mechanical strain, which is always exerted in a transformer as the resultant of the magnetic forces, and which tends to tear asunder turns, coils and cores, is quite insignificant in the small transformer protected by its relatively large surface and compact form; but in the large power transformers, this item demands most careful consideration from the designer. Think for a moment of the possibilities for damage when 10,000 kw.—approximately 13,500 h.p.—is suddenly short circuited on a transformer, and it will not be surprising to know that solid coils well constructed and carefully supported on extensive bearing surfaces crush under the enormous pressure developed.

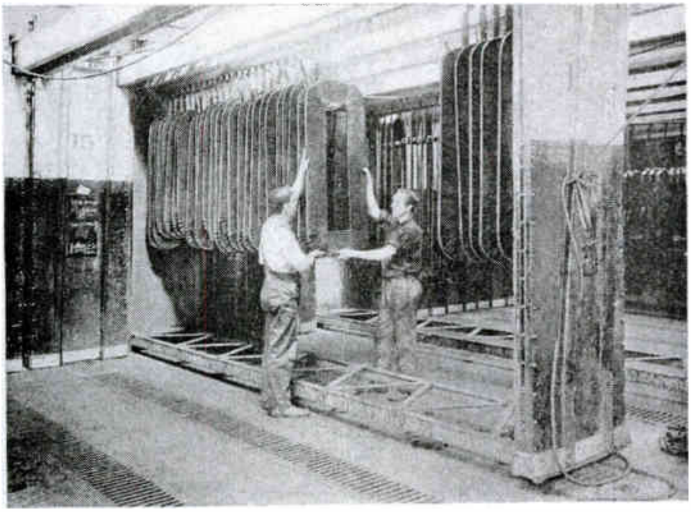

Fig. 13. Removing Shell Type Coils from the Baking Oven in Insulation Department

Efficiency and regulation, so important in the small lighting transformer, become of relatively minor importance. This class of apparatus operates continuously upon transmission lines, the load upon which varies far less than that upon the small lighting transformer, or one operating a small motor load. The regulation drop extends over such a comparatively narrow range that it can be largely corrected by taps in the winding, by means of which the ratio of transformation can be changed.

Again, the position of these large units on long transmission lines, which are subject to sudden excessive rises of potential from either lightning or line disturbances, makes it necessary to strengthen the ends of the windings by supplementary insulation of a very high value; while rushes of current from short circuits must be minimized by an amount of reactance in the design consistent with the best interests of all considerations.

All these requirements, for reasons already cited, have led to the almost universal adoption of the shell type for such service. The air-blast and water-cooled types are constructed much the same fundamentally, although they differ materially in details and external appearance. The coils, both primary and secondary, are wound in the so-called pancake type (Fig. 12); *i.e.*, they employ flat rectangular wire and are wound one turn per layer, in many layers, forming a spiral, the insulation between turns consisting of paper, mica or varnished cambric, or all three together, as conditions demand.

These thin coils are treated with coil filler and wound with a number of layers of tape, depending upon the voltage for which they are designed, each layer being given several coats of insulating varnish, baked on (Fig. 13). These coils are then assembled into groups (Fig. 14) and the groups into complete windings, the primary and secondary being intermixed and the whole interspersed by suitable barriers of insulating collars. The various groups are effectively encased in a box-like structure which, while serving as an electrical and mechanical protection, is so arranged that it will not obstruct the oil channels which are found adjacent to every coil.

These windings are then set up vertically in the bottom frame and the magnetic circuit built around them in the form of rectangular sheets of steel (Fig. 15). The top frame is next added and securely clamped to the bottom, compressing and securing the core. After connection board, leads, etc., are added, the transformer is ready for its casing. If of an air-blast design, the casing is arranged to form a blower, receiving air at its base through a conduit in the floor. The circulation is through channels about the coils and iron, the air gaining access to all parts and conveying heat out through the discharge at top of casing.

If the transformer is designed for water-cooling, it is hung to a heavy cast-iron cover or cap fitting on a tank of boiler steel, and a coil of water pipe placed around it near the top. The whole is then lowered into position in the tank. In operation, these water coils, which are located in the upper or warmer strata of oil, cool the oil so that it falls along

Fig. 14. Assembling Coils for Large Shell Type Transformers

the outside, near the walls of the tank, aiding the natural circulation to the extent that all parts are kept at a substantially uniform temperature.

The air-blast type is used for moderate voltages where cooling water is expensive or unavailable, and has been built for voltages up to 35,000, in sizes up to 5000 kw. The water-cooled type, which is better adapted for high potentials by reason of its superior facilities for insulation afforded by oil immersion, has been constructed for voltages up to 140,000 and capacities up to 10,000 kw. The general construction of this interesting class of apparatus is shown in Fig. 16.

Between these two divisions of the multiple transformers, falls a class of moderate capacity and wide range in voltage. This class demands many turns of small wire and generally follows the core type in design. The units of this class are nearly always installed in buildings or sub-stations where power is generated or received and transformed for testing, for mill work, or for supplying rotary converters for railway work. They differ from the lighting transformer only in that, because of capacity and voltage, they require a greater subdivision of coils, the proper supporting of which demands a more complicated mechanical structure. Fig. 17 shows a representative type of this division.

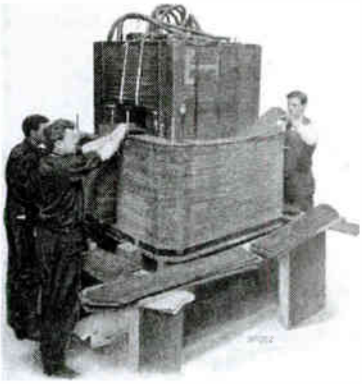

Fig. 15. Building Up the Core of a Large Shell Type Transformer

This class of multiple transformers is completed by a multiplicity of miscellaneous styles to which this paper can only briefly refer, such as sign-lighting, individual incandescent lamp, telephone line insulating, bell-ringing, wireless telegraph, signal, instrument or switchboard, and railway transformers. The wireless and signal transformers are examples of designs employing active elements other than steel and copper; the former, intended for very high frequencies, substituting air for steel in the core, while the latter, for certain reasons, is wound with high resistance wire.

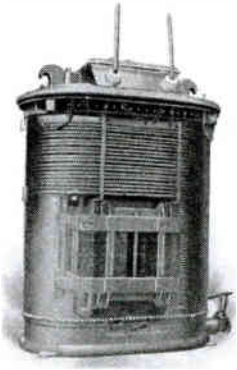

Fig. 16. Transparent View of 2000 Kw. Water Cooled Transformer

The prime function of the multiple transformers so far described, is to transform electrical energy at a fixed ratio of voltages; and that of the series transformer now to be considered is to make the transformation at a fixed ratio of currents. These two classes, as a matter of fact, perform both of these functions simultaneously, the essential difference being that the relative importance of certain characteristics differs in the two cases. For example, in the multiple transformer, any loss of current in magnetizing the core is important only so far as it affects the voltage regulation or the power factor of the circuit upon which it operates. On the other hand, a considerable loss of this nature in a series transformer is prohibitive. Conversely, the voltage loss in a series transformer is of importance only so far as it affects the current regulation, while in the multiple transformer, it is of the greatest moment.

Fig. 17. High Voltage Core Type Transformer for Power and Testing Purposes.

By far the most common form of the series class of apparatus is that generally styled "current transformers." These transformers are mounted on the framework of switchboards and introduced into a main bus or feeder as a multiplier and insulator for the instruments that measure the current or power in the feeder, or to operate the protective devices which open the switches in emergencies. Their use with meters measuring large amounts of power delivered to a distributing company or large consumer immediately suggests the necessity of a refinement in accuracy, not required or attainable in the ordinary transformer.

They are called upon to deliver but a fraction of a horse-power; yet, controlling the protective devices, they are by far the most important elements in the structure, assuming a prominence out of all proportion to size or cost. The ratio of test to operating voltage is, on this account, generally three instead of two as in other transformers. Considering the remarks made in the early part of this paper concerning space factor and its effect upon efficiency, it may be rightly inferred that the additional insulation necessary imposes a serious obstacle to accuracy, especially in those transformers that have been built for circuits of 110,000 volts. Fig. 18, which shows a transformer constructed for this voltage, is impressive when the 40 watt output is contrasted with the total height of about 9 feet.

Up to the present time, the best transformers of this class have been made on either a ring core without joints, or of a shell-type design, with joints in the magnetic circuit. The former economizes material at the expense of labor, particularly in winding, where the wire is threaded through the center of the ring—a laborious process for skilled labor. The latter design, necessarily demanding lower magnetic densities and consequently more material to compensate for the detrimental effect of the joint in the circuit upon accuracy, is less expensive in labor as the coils may be machine wound and subsequently assembled.

Fig. 18. Current Transformer for 110,000 Volt Circuit

The only other examples of this class of transformers requiring mention here are those used in compounding self-excited generators, and those inserted in series lighting circuits carrying arc or incandescent lamps of a certain current rating, for the purpose of operating a local circuit carrying series lamps of the same or different type but of different current capacity. The first of these is inserted in the line from the generator and transforms a portion of the current, which is then rectified to direct current and sent through the field coils of the machine. Neither requires the extreme accuracy of the switchboard or current transformer, although the capacities are much larger and operating voltages moderately high. In construction and appearance, they resemble small multiple transformers.

Thus far we have considered transformers with stationary parts and fixed characteristics. We now come to the third general class of apparatus, in which, by means of moving parts, a combination of the properties of the two previous groups is acquired. The constant current transformer, by which name this third class is usually known, is made upon a long slender shell-type core with pancake coils. In the simplest form, there is one primary and one secondary coil, occupying not more than a quarter of the length of the core window. The primary is fixed at the lower end of this space by means of a suitable clamping device attached to the core clamps. The secondary is hung by flexible cables to rocker arms and counterbalanced by weights, so that it is free to move throughout the length of the window; although it naturally rests upon the secondary because heavier than the counter weight. The primary is connected in multiple with the line supplying energy, and the secondary in series with the lighting line of series arc or incandescent lamps.

If the coils are separated as far as possible and the circuit closed, the magnetic field established by the primary is opposed by that of the secondary, and the coils are forced apart by this electromagnetic force. Most of the lines of force are driven back and cross the windows, or "leak" to the outer legs, while a small portion threads the secondary, producing voltage and current in the line. Now, if the counter weight is lessened, the weight of the coil causes it to overcome the repulsion and to settle nearer the primary, embracing more flux and therefore developing more current and voltage and reducing the reactive or leakage flux. By a suitable adjustment of the weights, any current value within the limits of the design may be obtained. The number of lamps on the line may vary at will, demanding more or less voltage, but the coil will always float upon the leakage flux, threading enough to give the voltage necessary to maintain constant current on the secondary lines. This action is not unlike that of a floating body, which always displaces its own weight, regardless of the specific gravity or density of the supporting medium.

The fourth or final group, as we have classified them, broadly designated regulators, performs a function quite similar to that of the apparatus just considered, but in a different manner. The secondary voltage rather than the current is the factor regulated, although the same apparatus may be adjusted to control current under certain special conditions of load.

The principal purpose of these devices is to receive a voltage which, although nominally of constant value, nevertheless varies excessively, due to poor regulation of generating and distributing apparatus under heavy or changing loads, and convert this into one of constant value. They are generally installed in a generating or sub-station, on feeders supplying energy to the centers of heaviest load upon the system, that the proper voltage may be maintained at these important points.

These "feeder regulators," as they are commonly called, are designed in two general types, both of which are made for either hand or automatic control. The simpler form of this device consists of either a core-type or shell-type transformer, the secondary of which is subdivided into several equal coils successively cut in or out of circuit by means of a switch. The other, or induction type, is in reality a generator of special design, in which the rotor is connected in multiple with the primary line, and the stator in series with the feeder.*

In conclusion, it should be observed that this paper presents only a very general view of transforming apparatus, dwelling relatively upon the salient points of the prevalent type, the multiple transformer. Again let it be pointed out that all of the other classes, types and forms possess inherently the same characteristics of design and operation, which differ only in relative importance, depending upon the requirements of the service for which they are intended.

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*Note. Descriptions of these regulators will be found in the issues of the Review for July, 1908, and June, 1909.—*Editor.*

HYPERBOLIC FUNCTIONS AND THEIR APPLICATION TO TRANSMISSION LINE PROBLEMS

PART II

BY W. E. MILLER

Sign Convention

The positive direction of rotation has been taken as contra-clockwise, as this is the convention usually employed in mathematics including trigonometry. Steinmetz uses the opposite notation which has advantages also. In the clockwise rotation impedance is written $r-jx$; in the contra-clockwise rotation it is written $r+jx$.

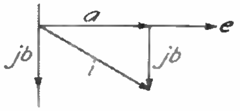


Fig. 5

A leading current is represented in the contra-clockwise notation, $i=a+jb$; $a-jb$ representing a lagging current, jb being drawn downwards as shown in Fig. 5.

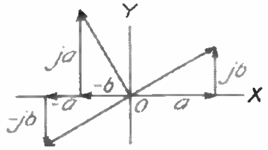


Fig. 6

If a vector is multiplied by j, it rotates it contra-clockwise through one right angle. For instance, if the vector $a \pm jb$ is multiplied by j, the result is $ja \mp b$, which means that a has twisted forward through one right angle and b also, which now lies in the opposite direction to the original direction of a. If a further multiplication by j is performed, the result is $-a \mp jb$ and the vector has been rotated into the third quadrant, and so on. Similarly, multiplying by $-j$ rotates the vector clockwise or in the negative direction. See Fig. 6.

Forms Used for Complex

The complex $a+jb$ is often written $\sqrt{a^2+b^2}\ \underline{|\theta}$ where $\theta = \tan^{-1}\left(\frac{b}{a}\right)$. Another form in which the complex is written is $\sqrt{a^2+b^2}\,(\cos\theta + j\sin\theta)$ this form being immediately obtained from inspection of Fig. 5. This method of writing the complex is very useful in many cases, for instance, when it is required to write down the complex of a current which lags or leads a voltage taken as the standard phase, and the power factor is given. In this case, $i=\sqrt{a^2+b^2}\,(PF \mp j\sqrt{1-PF^2})$. For example, if the power factor is .90 lagging and the R.M.S. current is 120 amperes, then $i = 120\,(.90-j\sqrt{.19}) = 108-52.3j$.

Meaning of $\sqrt{a+jb}$

The value of this quantity is required later, hence, the following method is given to show how to extract its square root:

Let $\sqrt{a+jb}=c+jd$ then $a+jb=c^2-d^2+2jcd$

Then, since the real parts must be equal to one another, and also the unreal,

$$a=c^2-d^2 \text{ and } b=2cd, \text{ hence } c^2+d^2=\sqrt{a^2+b^2}$$

Now $c+jd=\sqrt{c^2+d^2}\,(\cos\theta^1+j\sin\theta^1)$ where $\theta^1 = \tan^{1-}\left(\frac{d}{c}\right)$

And $a+jb=\sqrt{a^2+b^2}\,(\cos\theta+j\sin\theta)$ where $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ or $\tan\theta=\frac{b}{a}$

$$\text{But } \tan\theta=\frac{b}{a}=\frac{2cd}{c^2-d^2}=\frac{2\frac{d}{c}}{1-\frac{d^2}{c^2}}=\frac{2\tan\theta^1}{1-\tan^2\theta^1}$$

$$=\tan 2\theta^1$$

Therefore $\theta^1=\frac{\theta}{2}$ hence, as $c+jd=\sqrt{c^2+d^2}\ \underline{|\theta^1}$

and $c^2+d^2=\sqrt{a^2+b^2}$ and $\theta^1=\frac{\theta}{2}$ or $\dfrac{\tan^{-1}\left(\frac{b}{a}\right)}{2}$

it follows that $\sqrt{a+jb}=(a^2+b^2)^{\frac{1}{4}}\ \underline{\left|\dfrac{\tan^{-1}\left(\frac{b}{a}\right)}{2}\right.}$

or $\sqrt{a+jb}$

$$=(a^2+b^2)^{\frac{1}{4}}\left(\cos\frac{\tan^{-1}\left(\frac{b}{a}\right)}{2}+j\sin\frac{\tan^{-1}\left(\frac{b}{a}\right)}{2}\right) \quad (16)$$

Hence, the rule is as follows: Find the fourth root of the sum of the squares of a and b. Find the value of the angle whose tangent is $\frac{b}{a}$, then halve it and find the cosine and sine of half the angle. If this angle is ϕ the resulting complex can be written

$$(a^2+b^2)^{\frac{1}{4}}\,[\cos\phi+j\sin\phi]$$

If the original complex was $\sqrt{jb-a}$, the angle whose tangent is $\left(\frac{b}{-a}\right)$ lies in the second quadrant, halving it, however, brings it back into the first quadrant, where the sine and cosine are both positive. Hence, $\sqrt{jb-a}$ must have the positive sign placed between its components. Similar rules apply for any root, and in general

$$\sqrt[n]{a+jb}$$

$$= (a^2+b^2)^{\frac{1}{2n}}\left(\cos\frac{\tan^{-1}\left(\frac{b}{a}\right)}{n}+j\sin\frac{\tan^{-1}\left(\frac{b}{a}\right)}{n}\right)$$

Division by Complex

If the value of $\frac{a+jb}{c+jd}$ is required, multiply both numerator and denominator by $c-jd$; then

$$\frac{a+jb}{c+jd}=\frac{(a+jb)(c-jd)}{(c+jd)(c-jd)}=\frac{ac+bd+j(bc-ad)}{c^2+d^2}$$

which eliminattes the j term from the denominator and brings the result into the form $p+jq$. If the denominator had been $c-jd$, the multiplier should, of course, be $c+jd$ in order to clear the denominator of terms involving j.

The following example is given here showing the application of this rule. Take the ordinary equation connecting volts and amperes in an inductive circuit

$v=ri+L\frac{di}{dt}$, i being equal to $I \sin pt$ where $p=2\pi f$.

Then this equation can be written $v=ri+jLpi$ or

$$i=\frac{v}{r+jLp}=v\frac{r-jLp}{r^2+L^2p^2}$$

which immediately solves the problem and shows that the current lags behind the e.m.f. by an angle $\tan^{-1}\frac{Lp}{r}$.

Hyperbolic Complex

These functions are involved in the solution of transmission line problems (with distributed capacity, self induction and leakage). They appear in the form $\cosh(x+jy)$ and $\sinh(x+jy)$.

By the addition and subtraction formulæ (14 and 15) which must apply generally, the following formulæ are at once obtained:

$\cosh(x\pm jy)=\cosh x \cos jy \pm \sinh x \sin jy$ and
$\sinh(x\pm jy)=\sinh x \cos jy \pm \cosh x \sin jy$

$$\text{Now } \cos jy = 1+\frac{(jy)^2}{2}+\frac{(jy)^4}{\underline{|4}}+\text{etc.}=1-\frac{y^2}{2}+\frac{y^4}{\underline{|4}}$$
$$-\text{ etc.}=\cos y$$

Proceeding in a similar manner $\sin jy = j \sin y$ (see formulæ 12 and 13).

Therefore $\cosh(x\pm jy)=\cosh x \cos y \pm j \sinh x \sin y$ (17)

and $\sinh(x\pm jy)=\sinh x \cos y \pm j \cosh x \sin y$ (18)

from which formulæ the hyperbolic complexes have been calculated.

Thus if $\cosh(x+jy)=a+jb$ and $\sinh(x+jy)=c+jd$

Then $a=\cosh x \cos y$ and $b=\sinh x \sin y$
$c=\sinh x \cos y$ and $d=\cosh x \sin y$

Equations (17 and 18) show that $\cosh u$ and $\sinh u$ are periodic with an imaginary period of $2\pi j$, since $\cosh(u+2\pi j)=\cosh u \cos 2\pi + j \sinh u \sin 2\pi = \cosh u$

Similarly $\sinh(u+2\pi j)=\sinh u$. These functions change sign when u is increased by $j\pi$ whence $\cosh(u+j\pi)=-\cosh u$, and similarly for $\sinh u$.

Again by substitution in the addition formulæ, the following holds

$$\cosh\left(u+\frac{j\pi}{2}\right)=j\sinh u \text{ and } \sinh\left(u+\frac{j\pi}{2}\right)=j\cosh u$$

$$\text{also } \cosh\left(u+\frac{3j\pi}{2}\right)=-j\sinh u \text{ and } \sinh\left(u+\frac{3j\pi}{2}\right)$$
$$=-j\cosh u.$$

If it is necessary, as in long telephone lines, to calculate the hyperbolic functions in which the j term is greater than $\frac{\pi}{2}$, a great saving in labor can be effected by using the above results. For example,

$$\cosh(u+2.57\,j)=j\sinh\left[u+\left(2.57-\frac{\pi}{2}\right)j\right]$$
$$=j\sinh(u+1.0\,j)$$

$$\cosh(u+3.42\,j)=-\cosh\left[u+(3.42-\pi)\,j\right]$$
$$=-\cosh(u+.28\,j)$$

$$\cosh(u+6.00\,j)=-j\sinh\left[u+\left(6.00-\frac{3\pi}{2}\right)j\right]$$
$$=-j\sinh(u+1.29\,j)$$

$$\cosh(u+7.00\,j)=\cosh\left[u+(7.00-2\pi)\,j\right]$$
$$=\cosh(u+.72\,j)$$

Similar formulæ hold for $\sinh(u+j\pi)$

The real part of the complex cannot, however, be reduced, only the unreal or j term. Hence, if large values of the real term x or x are required, they must be calculated from a series, if no tables are available, or from the exponential values of the hyperbolics.

Transmission Line Equations

Let r = resistance per mile, L = self induction per mile measured between line and neutral, C = capacity per mile measured between line and neutral, and g = coefficient of dielectric conductance per mile. Let x be the distance to any point of the line measured from the receiving end in miles, e the voltage at any point, and i the current. Then the following equations give the relations between "e" and "i":

$$\frac{di}{dx} = ge + C\frac{de}{dt} \qquad (19)$$

provided that the value of g is independent of the voltage e, which is not true of corona effect; or provided that the voltage is practically constant along the line.

And

$$\frac{de}{dx} = ri + L\frac{di}{dt} \qquad (20)$$

That is to say, the increment of current at any point per infinitely small length equals the vector sum of the leakage current (or the leakage conductance multiplied by the volts at that point) and the capacity current, which is at right angles to the leakage current. In the same way, the increment of voltage at any point per infinitely small length, equals the volts consumed in resistance added vectorially to the inductive volts at right angles to the resistance volts.

Now $C\frac{de}{dt} = jpCe$, since $\frac{de}{dt}$ is at right angles to e

And $L\frac{di}{dt} = jpLi$, since $\frac{di}{dt}$ is at right angles to i

Where $p = 2\pi f$

Therefore, $\frac{di}{dx} = (g+jpC)e$ and $\frac{de}{dx} = (r+jpL)i$

$$\text{Whence, } \frac{d^2e}{dx^2} = (r+jpL)\,(g+jpC)e \qquad (21)$$

$$\text{And } \frac{d^2i}{dx^2} = (r+jpL)(g+jpC)i \qquad (22)$$

Hence, the solution is hyperbolic, because in both equations, the second differential is proportional to the quantity itself, a law which sinhu and coshu both follow.

Therefore, the solution is:

$e = A \cosh mx + B \sinh mx$

And: $i = F \cosh mx + D \sinh mx$, of which only two of the constants are arbitrary.

And $m^2 = (r+jpL)\,(g+jpC)$

If the receiving end terminal values of e and i be E_r and I_r respectively, the general solution is

$$e = E_r \cosh mx + m_1 I_r \sinh mx \qquad (23)$$

$$i = I_r \cosh mx + \frac{E_r}{m_1} \sinh mx \qquad (24)$$

Where x = distance in miles, measured from the receiving end, and

$$m_1 = \frac{m(g-jpC)}{g^2+p^2C^2} \quad \text{and} \quad \frac{1}{m_1} = \frac{g+jpC}{m}$$

If E_s and I_s are given at the sending end, and the line is measured from that point towards the receiving end, equations 23 and 24 become

$$e = E_s \cosh mx - m_1 I_s \sinh mx \qquad (25)$$

$$i = I_s \cosh mx - \frac{E_s}{m_1} \sinh mx \qquad (26)$$

Where x is the distance in miles measured from the sending end.

Calculation of Constants

As already stated, $m^2 = (r+jpL)\,(g+jpC)$

In the majority of lines, except those using wires of small diameter at very high voltage, where corona effect is noticeable, g, the leakage conductance, can be neglected.

Then, $m^2 = (r+jpL)\,jpC = pC\,(jr-pL)$

$$\text{Therefore, } m = \sqrt{pC}\,(r^2+p^2L^2)^{\frac{1}{4}}\,\underline{\Big|\frac{\tan^{-1}\left(\frac{r}{-pL}\right)}{2}}$$

$$= \sqrt{pC}\,(r^2+p^2L^2)^{\frac{1}{4}}\left(\cos\frac{\tan^{-1}\left(\frac{r}{-pL}\right)}{2} + j\sin\frac{\tan^{-1}\left(\frac{r}{-pL}\right)}{2}\right) \qquad (27)$$

The above is, of course, of the form $a+jb = m$

$$\text{Since } m_1 = \frac{m}{jpC} = \frac{(a+jb)\,(-j)}{pC} = \frac{b-ja}{pC} \qquad (28)$$

$$\text{and } \frac{1}{m_1} = \frac{pC}{b-ja} = \frac{pC(b+ja)}{b^2+a^2} \qquad (29)$$

The tables for m, m₁ and $\frac{1}{m_1}$, in the Supplement have been calculated from these formulæ, C being given in farads per mile and L in henrys per mile.

Volt and Current Phase Shift and Power Propagation Velocity

Equations 23 and 24 prove that when there is no load current, there is a complete reversal of phase in volts and amperes along a transmission line in a distance $x = \frac{\pi}{b}$ where $(a+jb) = m$ because both $\sinh mx$ and $\cosh mx$ change sign every half period πj. In a distance

$\frac{2\pi}{b}$ the amperes and volts are in the same phase respectively as they are at the receiving end. Hence, if the frequency be f, then the velocity of propagation of the voltage or current wave along the line will be $v=\frac{2\pi f}{b}$.

This must not be taken as the velocity of the power wave along the line, as the apparent velocity of the current and volt wave vary at different points from the receiving end and the shift of phase does not vary uniformly along the line, owing to capacity current, the current leading the volt wave 90 degrees at the receiving end. If a lagging load current, however, is taken at the receiving end, the power factor can be approximately unity along the line, in which case the volts and current are nearly in phase at every point and the velocity of either gives the velocity of the power wave along the line. In the majority of lines used for long distance work, the resistance is not large enough to affect the velocity and in such cases the velocity of power propagation is practically equal to the velocity of light. For $r=o$, the velocity is

$$\frac{2\pi f}{b\sqrt{LC}}=\frac{1}{\sqrt{LC}}$$

or independent of the frequency. From this formula, the formula for the natural period of a transmission line can be derived equal to $\frac{1}{4l\sqrt{LC}}$ where l is the length of the line; $\frac{1}{\sqrt{LC}}$ being the velocity of light nearly, the expression only representing the velocity of light when the self induction inside the wires is negligible, which is true of very high frequencies, practically perfect conductors, etc. The natural frequency of the fundamental wave, for a transmission line 400 miles long is about 115, the velocity of power propagation being approximately, 1.8 per cent. less than the velocity of light. The closer the wires are together or the larger they are, the slower does the power travel, and the velocity can be taken as lying between 1.5 and 2.5 per cent. less than the velocity of light. For transmission lines, 184,000 miles per second can be taken, as an average velocity.

It must be remembered that power is a double frequency quantity and cannot, therefore, be represented vectorially in the same plane as a vector of different frequency; hence, if the power wave is obtained by multiplying the complexes of current and volts together, difficulties are encountered. The best way to obtain the electric power is to plot the instantaneous values of the current and volts along the line, the values of which are immediately given by equations 23 and 24. Then multiply the instantaneous values together and plot the power curve from the result. The distance between the maxima, minima, or corresponding points on this curve multiplied by double the line frequency gives the velocity of power propagation along the line; the distance between the maxima on this curve will, of course, be half that between the maxima on the current or volt curve, provided that practically unity power factor obtains through the distance taken.

The shift of phase of volts or amperes along commercial transmission lines is not large, since the maximum frequency used is only 60 cycles and with this frequency the half period length is about 1500 miles. In long telephone lines, on the contrary, shift of phase is very large and will amount to a number of complete reversals along the line owing to the necessarily high frequency used in speech, 800 per second being a representative frequency.

Approximate Formulæ for Short Lines

Since $\cosh u=1+\frac{u^2}{\underline{|2}}+\frac{u^4}{\underline{|4}}+$

for small values of u

$$\cosh u=1+\frac{u^2}{\underline{|2}} \text{ nearly}$$

Similarly $\sinh u=u$ nearly.

Hence for short lines, if $mx=x(p+jq)$

$$e=E_r\left(1+\frac{x^2(p^2-q^2)}{2}+jx^2pq\right)+m_1 I_r(p+jq) \quad (23)$$

$$i=I_r\left(1+\frac{x^2(p^2-q^2)}{2}+ix^2pq\right)+\frac{E_r}{m_1}(p+iq) \quad (24)$$

and

$$e=E_s\left(1+\frac{x^2(p^2-q^2)}{2}+ix^2pq\right)-m_1 I_s(p+iq) \quad (25)$$

$$i=I_s\left(1+\frac{x^2(p^2-q^2)}{2}+ix^2pq\right)-\frac{E_s}{m_1}(p+jq) \quad (26)$$

These formulæ are accurate to 1 per cent. for lines, 120 miles long at 60 cycles and 150 miles long at 25 cycles, greater accuracy being obtained for shorter lines.

Corona Effect

The escape of electricity through the atmosphere from one wire of a transmission line to another is an example of the increase of conductivity of a gas due to high dielectric stress. The conductivity of gases is enormously augmented under special conditions; such as, when subjected to radio activity, when the temperature is raised above a certain value, when drawn from the neighborhood of flames or electric arcs, or after being in contact with incandescent metals or carbon, etc. A gas through which an electric discharge is passing is also affected in a similar manner, this being the cause of the increase in conductivity of the air between transmission lines when the voltage rises above a certain critical value. The physical aspect of these phenomena has been studied by many scientists, notably by Kelvin, J. J. Thomson, Rutherford, Hittorf, etc., and a very full discussion of the whole matter has been given in Thomson's work "Conduction of Electricity Through Gases."

According to one of the modern theories of matter, each atom or molecule is composed of or associated with negative and positive ions or minute electrified particles, the negative ion possessing a mass small compared to that of the hydrogen atom, and the positive ion a larger mass than that of the negative ion. The electric charge of these ions is a constant. On this assumption, the following discussion may help to picture what happens when the voltage stress is increased in the dielectric between two conductors, beyond the dielectric strength and the gas becomes ionized. It can, however, only be regarded as a rough approximation to the phenomena.

Capacity Current

Suppose that a potential difference is applied to two electrodes separated by an air space, the potential being gradually increased. At first, the current passing across the air, which completes the electric circuit, is exceedingly small and consists of a displacement or charging current in the surrounding dielectric. The greater part of this current is due to an ether displacement, but part is caused by a displacement of the ions, which are elastically attached to the gas molecules. The strain or displacement of the ions in each molecule is greatest when the voltage stress is at a maximum, the ions then being at rest and the current zero. If the voltage is alternating, at the moment the voltage passes through zero, the ions in the molecule are in midswing and move at their highest velocity; and the displacement current is then maximum. Thus the elastically controlled displacement current in the air constitutes a small part of the capacity current between the electrodes and is in quadrature with the voltage. This is, of course, also true of the ether displacement current. As practically no friction enters into the motion which beats rhythmically with the voltage, no energy loss occurs in the dielectric, the energy being alternately potentially stored in the dielectric and kinetically released in the moving ions.

Corona Current

If now the voltage is further increased, the electric stress at a certain critical point becomes sufficiently great to tear off some of the ions attached to the gas molecules. This disruption occurs first, in a layer of air a short distance from the surface of the electrode or conductor, since although the electric stress is greatest at the surface of the conductor, it has been found that the dielectric strength of the air immediately surrounding the conductor is considerably higher than that further off. For small conductors, the breakdown point is approximately .07 inches distant from the surface. Here the ions are first released and are then free to move under the force of the electric field in the same way as an electrically charged pith ball moves in an electrostatic field. At the moment of release, the inertia of these ions is small and, therefore, their speed is rapidly accelerated until they are stopped by collision with other gas molecules or ions. If, at the moment of collision, the kinetic energy of the ion is above a certain value, it may shake off other ions, and in this manner the whole space between the electrodes becomes filled with electrified particles or ions, the positive, on the average, all moving in one direction, and the negative in the opposite direction. Of these collisions, some may cause ionic recombinations and a neutralization of the electric charge, the number of recombinations increasing if the gas pressure is raised. Hence, a transfer of electricity occurs from one conductor to another, the carriers being the ions torn off the molecules, either by the electric stress or by collision, the current at any point being proportional to the number of ions passing per second. At every collision molecular vibration is started and part of the electric energy is transformed into heat. If the voltage is still further increased, a larger number of ions are released which attain a greater velocity between molecules and cause more heat waste and current. It follows, that this current is independent of frequency, and has, therefore, the same value at a given voltage, whether the voltage alternates or is held constant.

Directly the voltage falls below the critical value for ionization, all action ceases, only to begin again when the voltage rises to its proper value in the opposite direction. This is practically true, except that a minute time lag exists, which is short compared to the period of commercial frequencies. It will be readily seen from the above that the current is a true convection current and is in phase with the voltage. It is, therefore, at right angles to the capacity current.

If the pressure of the gas be diminished, the number of gas molecules between the conductors is proportionally decreased, and therefore the distance between the gas molecules is correspondingly increased. Under such circumstances, the ions have, on the average, a longer path to travel before collisions occur and, therefore, their speed and kinetic energy are greater at collision. Hence, each collision is more likely to tear off other ions from the molecules, and as the number of recombinations producing neutral ions is diminished, the current is increased. If the gas pressure be further decreased, the ionization current, for a given voltage stress, increases until at a certain critical pressure where the number of molecules per unit volume has been very much reduced, the current becomes maximum, that is, the space is saturated with ions and on a further decrease of pressure, the current falls.

Under the conditions which exist in high voltage transmission lines, it is found that a decrease of pressure near the atmospheric pressure causes a distinct increase of corona loss, and when pressures as low as 20 in. are encountered, the loss is considerable. As the height of many transmission lines exceeds 8,000 ft., the highest point reached by the Central Colorado Power Line being 13,700 ft., it is abundantly evident that the relation of pressure to corona loss is of the first importance.

Since the critical voltage for No. 2 wire is in the neighborhood of 90,000 volts effective or 126,000 volts maximum, only a small part of the cycle is effective in producing corona current at ordinary transmission voltages. If very much higher voltages were employed, not only would the corona current be enormously increased (so long as the saturation point is not reached between the conductors), but also the loss would last during the greater part of the period. If the no load current oscillograph record is taken, a kick in the curve at maximum voltage is very apparent at high voltages when corona is present and the shape of the current wave is considerably altered.

When ionization takes place, a brush discharge can be observed near the surface of the wire, where the greatest number of ions per unit volume occur and the current density is at a maximum. The resistance, therefore, of the air layers surrounding the wire is considerably decreased, and the effective conductor diameter can, therefore, be regarded as greater than that bounded by its metallic surface. This increase in conductor diameter increases the capacity between the conductors and, therefore, the capacity current. Thus, if the loss at no load be measured on a transmission line subject to a large corona loss, the capacity loss cannot well be calculated from formulæ and oscillograph records should be taken if the corona and capacity loss require separating. The high inductive capacity of insulator material increases the voltage stress near the wires where they are fastened to the insulators, and hence the corona loss is increased at these points as well as the capacity loss.

Mr. H. J. Ryan considers these matters in his paper before the American Institute of Electrical Engineers, February 26, 1904, and a considerable amount of work has been done by Messrs. C. F. Scott, R. D. Mershon and others, especially in connection with the effect of the barometric pressure on corona loss. Many more experiments are, however, needed, before the phenomena can be considered as subject to calculation. The variation of corona loss with voltage, size of wire, spacing between conductors, atmospheric pressure, state of atmopshere and wire surface, and many other conditions must be determined before equations can be formed to give reliable results.

Capacity Loss

The extra current carried by the conductor due to corona is exceedingly small and can be neglected; in consequence, the i^2r loss in the conductor is negligible. The loss all occurs in the dielectric between the conductors. Thus, for constant voltage along the line, which will obtain at no load in lines up to 200 miles long at 25 cycles, the corona loss per mile is constant along the line; it being, independent of frequency. The capacity current loss, on the other hand, all occurs in the conductor, the current varying directly as the voltage and frequency. In this case, practically no loss takes place in the dielectric between the wires.

In lines, up to about 200 miles long at 25 cycles, and in slightly shorter lines at 60 cycles, the capacity current along the line follows practically a straight line law, being a maximum at the generator end and zero at the receiving end. Hence, if r is the resistance per mile of wire, I the capacity current per wire at the generator end, and l is the length of line, the current per mile $i = \frac{I}{l} =$ constant, and the loss for a line l miles in length is $3\int_0^l r i^2 l^2 dl = i^2 r l^3$. The total loss is therefore Al^3 where $A = i^2 r$, which is a constant at a given frequency, for a definite size of wire and given line capacity. Hence, the capacity loss for a given transmission line varies as the cube of its length up to the limits of length just given. Above this length, the voltage rises from the generator end toward the receiving end, and the capacity current does not fall off uniformly from the generator end but more gradually; that is, the curve of capacity current along the line is concave towards the abscissa representing line length, provided the line resistance is not too high. The reason for this is that the increase of voltage holds up the current at the generating end, and prevents it decreasing as fast as it does near the receiving end. The total loss, however, with a given voltage at the receiving end, does not vary as rapidly as the cube of the distance of the line length, since the drop in voltage at the generator end reduces the capacity current correspondingly, which more than compensates for the slower drop of current along the line. See Fig. 7.

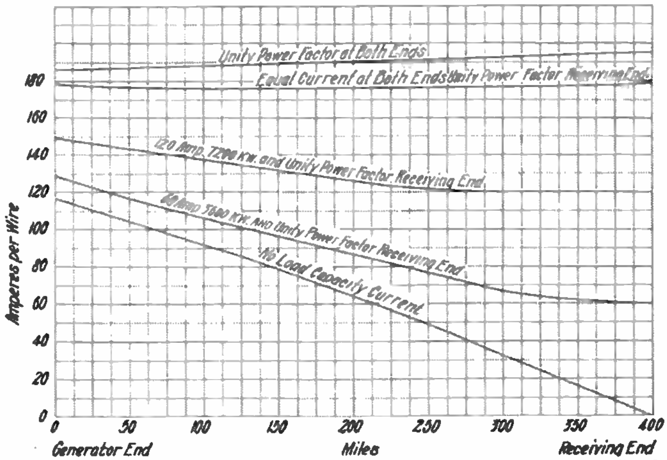


Fig. 7. Capacity Current at 60 Cycles along Three-Phase Line 400 Miles in Length, Using Three No. 0000 Wires Spaced 10 ft. Apart, 104,000 Volts between Wires at Receiving End.

Calculation of Dielectric Conductance Constant

The curves, Fig. 8, showing corona and capacity loss, have been obtained by substituting a value for g in the transmission line equations. This value was derived from the observed loss along a 50 mile line operating under similar conditions and using the same size of wire and spacing. As, therefore, the voltage is nearly constant, the corona loss may be regarded as approximating the real value provided the capacity current is not seriously altered by the corona loss and that no appreciable insulator loss exists. Note the change in phase at generator and receiving ends due to corona indicated on Fig. 9.

The dielectric conductance coefficient was calculated as follows: The total loss of a 50 mile line at no load was experimentally found to be 25 kw., of which 3 kw. was calculated as capacity loss, and the remainder assumed as corona loss. The voltage was

110,000 volts between wires. Then the loss per mile equals $\frac{22000}{50} = 438$ watts per mile.

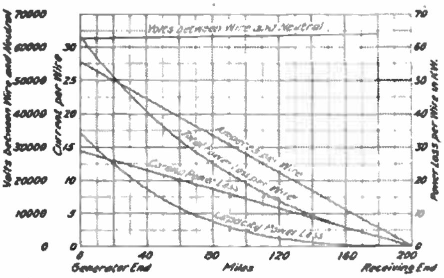


Fig. 8. Amperes, Volts and Power Losses (Capacity and Corona) at no Load along 200 Mile Three-Phase Line using Three No. 1 Wires 8 Ft. Apart, Operating at 25 Cycles, with 110,000 Volts between Wires at Receiving End

Then since the dielectric conductance is $\frac{1}{r}$ where r is the dielectric resistance per mile between wires, and since $\frac{\mathrm{I}''^2}{r} =$ the loss per mile, therefore, $\frac{1}{r} = \frac{438}{110{,}000^2} = .036 \times 10^{-6} = g.$

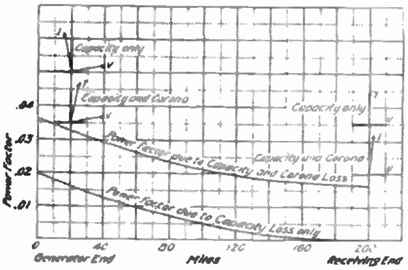


Fig. 9. Power Factors Due to Corona and Capacity Losses along 200 Mile Line at 25 Cycles

Corona effect is less masked by capacity loss at 25 cycles than at 60 cycles, and the example was, therefore, chosen at the lower frequency. The constant m in the transmission line equations, where g is included is

$$\sqrt{(r^2+p^2L^2)(g^2+p^2C^2)}\left[\cos\frac{\tan^{-1}\theta}{2}+j\sin\frac{\tan^{-1}\theta}{2}\right]$$

where $\tan^{-1}\theta = \frac{p(Lg+Cr)}{rg-p^2LC}$

From these values m_1 and $\frac{1}{m_1}$ can be readily calculated. *(To be Continued)*

UNDERGROUND ELECTRICAL DISTRIBUTION

By W. E. Hazeltine

Supply Dept., General Electric Company

The ungainly appearance and danger of overhead electrical wires have in a large measure been accountable for the adoption of the underground system of distribution.

While the first cost of an underground system exceeds that of an overhead, security of operation tends to counter-balance this difference, for underground conductors are entirely free from the effects of storms and weather conditions.

One of the earliest attempts at placing wires underground was over fifty years ago when Professor Morse of Boston undertook to install a telegraph line between Washington and Baltimore. His method of laying was by means of a large plow drawn by sixteen yoke of oxen, the cable being placed on a reel secured to the plow and played out in the furrow as the plow advanced. It is well known that this attempt proved unsuccessful.

Several years after, the so-called "pump log" was brought into use. This consisted of eight-foot lengths with a 3-inch bore, the ends butting together with socket joints and laid directly in the trench. Logs were sometimes of plain wood and, again, treated with tar or creosote as a preservative.

The cement-lined iron pipe was also used to a considerable extent, but this type of conduit proved unsatisfactory, one disadvantage being that the pipe was affected by electrolysis and that the inside coating of cement sometimes caused corrosion. Both types proved inadequate to the requirements of an underground conduit.

The theoretical conduit should be one that in itself possesses high insulating properties; one upon which the action of water, gas and chemical elements have no effect, and one which is permanent and practically indestructible. These requirements are found in the vitrified clay conduit, which is best adapted to the purpose and in addition is comparatively low in first cost and in expense of installing.

Another type of conduit, which is at present used to some extent, is the fibre conduit. It is made from wood fibre treated with an asphalt compound which the makers claim renders it water, acid and alkaline proof. The standard length is about five feet, with a 3 in. bore and is smooth inside; therefore, cables are safe from injury when being drawn in. This type of conduit is usually laid in concrete and lengths are joined by a butt joint which keeps them in alignment.

To return to the vitrified clay conduit, this is without doubt the most popular type in use. Duct sections are supplied with one, two, three, four, six, nine and twelve holes, the standard single duct being 18 in. in length and 3 in. internal diameter. Sections of more than six ducts are difficult to handle and are not made to a great extent on account of the liability to warp during manufacture.

The flexibility of this system allows obstructions such as gas, water and sewer pipes, to be overcome by laying the duct line over or under them, and in some cases to split the duct line, placing part above and part underneath. In any case, ducts should be laid with such a gradual grade as to permit cables to be pulled in without injury to the lead sheath. Also, the use of short lengths permits the laying of curves of long radius, oftentimes doing away with additional manholes. The duct is generally laid on a bed of concrete, usually 3 in. thick, surrounded by walls and covered with concrete of the same thickness.

In laying single duct, a mandrel about 30 in. long, slightly smaller than the internal diameter of the hole and having a rubber gasket on the end slightly larger than the diameter of the hole, is drawn through the duct as it is being laid. This removes all loose particles of cement and stones and makes sure that there are no obstructions to injure the cables; further, care should be taken to insure that ducts are perfectly aligned.

In laying a section of conduit the engineer in charge should lay out his grades so that all the ducts in each section drain into one manhole, or else break the grade so as to drain into two adjacent manholes, thus preventing injury to the cables after they are installed, from the freezing of water which may find its way into the duct line and settle in any pocket there may be.

For long straight runs, when there are no obstructions, the multiple forms of duct are usually used. These are laid in practically the same manner as the single ducts, except that joints are aligned by the use of dowel pins.

One objection to multiple duct is, that between any two duct sections there is only one wall and it is impossible to break joints as with single duct; therefore, there is a possibility of a burnout in one duct finding its way to a neighboring one.

With single conduit, where there are two thicknesses of wall between any two ducts and where joints can be staggered, this danger is practically eliminated. Single duct, however, requires experienced labor in installing, while multiple duct may be laid by ordinary laborers.

Manholes are necessary in order to facilitate the drawing in or out of cables in the system and are generally located at street intersections or at sharp bends in the duct line. The maximum distance between holes should not exceed 500 feet, for at greater distances the cable is liable to break or stretch from the excessive strain during the process of pulling.

The general manhole construction is of brick, although the present tendency is towards concrete holes whenever possible, as the average cost of concrete manholes is approximately two-thirds that of first class brick. Concrete holes are usually made from wooden forms of take-down design which may be used indefinitely. Bottoms of manholes are usually of concrete, with a hollow in the center which allows water to gather. When possible, connection with the sewer through a trap should be provided to remove any surface water which may work in around the cover.

Referring to manhole covers, authorities do not agree as to whether a single or double cover should be used. The single cover simply fits in a cast iron frame at street level. Inside of this there is sometimes another cover resting on a rubber gasket, bolted and secured so as to prevent water entering. The main disadvantage of this inner cover is that, in case sewer gas or illuminating gas escaping from leaky mains finds its way into the hole, there is no way of escape and this accounts for the majority of manhole explosions which occur.

It is believed by many authorities that the single cover is preferable and this should be supplied with several air holes to allow gas to escape. Theoretically these vent holes should be conical in shape with the small opening on top to prevent them from becoming clogged. It is true that surface water finds its way through these vents into the hole, but this is taken care of by the sewer connection. It is well known that, with a large number of cables carrying heavy loads, considerable heat is generated and a large percentage of this is dissipated through the surrounding earth, but by using perforated manhole covers, it may be got rid of more easily.

In order to support the cables which must necessarily pass through the manhole it is customary to provide some sort of a device on which they may rest. In brick holes, brick shelves are built into the sides at suitable distances apart upon which cables are placed. Also with concrete manholes the wooden forms may be so designed as to provide for concrete shelves. The use of shelves for cable-supports is especially desirable, for in case of trouble occurring on one cable, the neighboring cables are protected to a certain extent from injury. Oftentimes with brick or concrete manholes, iron cable racks, provided with arms adjustable at will, are built in.

A conduit line composed of a large number of ducts is undesirable, one reason for this being that it is almost impossible to support a large number of cables in one manhole. It is advisable, therefore, to divide the underground lines from the generating station, installing a portion through two or more streets, if possible; but if the station is so situated that the entire output must pass through one street, a single conduit line with twin manholes may be used.

With a single duct line entering the station it is rather difficult to dispose of the cables satisfactorily.

Coming to the question of cable, the size of duct determines in a way the size of cable to be used. For the standard 3 in. duct for working pressures of 1500 volts or less, the largest single conductor that should be installed is a 2,500,000 cir. mil., or a concentric 1,000,000 cir. mil. cable, while the largest three-conductor cable is one of 400,000 cir. mils. From 1500 to 3000 volts, the largest single conductor should be a 2,000,000 cir. mil. cable or a concentric cable of 750,000 cir. mils.; and the largest three-conductor cable, one of 400,000 cir. mils. For 6000 volts (usually three-phase delta connected), the largest three-conductor cable is one of 250,000 cir. mils.; for 13,000 volts, 3-conductor 4/0; and for 20,000 volts, 3-conductor 1/0.

As the cost of the duct line is independent of the cable cost, it is advisable to choose such cable as will reasonably fill the duct area, thereby cutting down the conduit investment to a minimum for the amount of energy transmitted. In laying out an underground system, it is advisable to provide extra ducts to take care of future requirements.

For underground work, three types of insulation are used; *viz.*, paper, varnished cambric and rubber; paper being the cheapest, varnished cambric intermediate and rubber the most expensive. For dry ducts, where there is no danger from corrosion of the lead sheath or where electrolysis is absent or may be guarded against, paper cables may be used. Paper is also used to a great extent for trunk lines. Paper cable must not be used without a lead sheath, for the life of a paper cable is dependent upon the sheath, the presence of moisture causing the insulation to break down almost immediately. Electrolysis, therefore, proves disastrous to paper cables.

Varnished cambric cables have all the good qualities of paper cables and may be used in almost any place where rubber cables could be used. These cables are built up of successive layers of lapped, varnished cambric tape, with plastic compound between layers, this compound permitting the layers to slide on themselves when the cable is bent, without reducing the thickness of insulation between conductor and lead. This type of insulation is waterproof, and the ends of the cable do not necessarily have to be sealed to prevent moisture entering, as with paper cable. This is also true of rubber insulated cable. Since varnished cambric tape is used in insulating, the copper core must be in the center of the cable, while with rubber insulation for heavy copper cores, used for horizontal runs at high temperature, there is a tendency for the rubber to soften, thus allowing the core to drop and reduce the thickness of insulation between copper core and lead sheath.

Varnished cambric cables with a braided finish may be used for inside work, as the insulation does not absorb moisture. These cables, unlike paper insulated ones, are not seriously affected by electrolysis.

Rubber insulated cables are used where there is constant moisture and almost invariably for submarine use.

Paper cables may be bent to a radius equal to eight times the outside diameter of the cable, while rubber and varnished cambric may be bent to a radius of six times this value.

For direct current low tension and railway feeders, single conductor cables are generally used. In some cases where two or three small feeders run parallel for any considerable distance, it is frequently desirable to combine them into one large cable running to the station.

For the grounded side of street railway feeders, or the neutral of three-wire Edison systems, a bare wire may be used, but this should not be run in the same duct with leaded cables. An ordinary weatherproof finished wire is often substituted for this bare wire.

Low tension feeder cables are frequently of the two-conductor concentric type with pressure wires in the outer conductor. The carrying capacity is slightly less than that of a two-conductor cable leaded flat as there is less chance of radiation; but the concentric type is easier to install and is much more economic of duct space.

For alternating current single-phase two-wire systems, the duplex type is preferable unless many taps are called for, and if so, single conductors are sometimes used on account of the greater ease in making joints.

The largest solid conductor recommended is 4; larger conductors are too stiff to handle and should be made stranded.

Duplex or figure 8 cables larger than 250,000 cir. mils. are liable to kink in handling and are not used to any great extent. For larger cables the two conductors may be stranded up with fillers to make them round, and the lead applied. Any size of duplex (Fig. 8) cable must have special care in installing to prevent kinking.

For three-phase work, it is advisable to use three-conductor cables; for, with this construction, there is no loss theoretically in the lead sheath, and, if necessary, telephone cable may be run in the same duct system without disturbance.

On low tension systems, single conductors are frequently used on account of the ease in making service taps.

The chief advantages in using three-conductor cable are: cost of installing is less and installation is easier, while the first cost of a three-conductor is approximately the same as that of three single conductor cables.

Three-phase cables are more economical of duct space than either single or two-conductor cables. Three-phase Y connected cables are generally run with grounded neutral and the thickness of insulation between conductor and ground need only be seven-tenths the insulation between conductors, thereby allowing a slightly larger cable to be installed in conduit than a three-phase delta connected, where insulation between conductors and between conductor and ground is the same.

The general practice in three-phase cable work is to use the so-called split type of insulation, placing half of the total thickness required on each conductor, stranding the three conductors up with jute fillers to make round, wrapping the three conductors with the second half of insulation, and applying the lead finish. This makes a more compact cable than when applying all the insulation on each conductor and is somewhat cheaper.

For arc circuits, single-conductor and also duplex cables are in general use. Where several circuits run parallel for any distance, they are sometimes combined into a multiple-conductor cable, for if several single conductors are run in one duct the lead sheath is liable to be injured in installing, and if one cable burns out, it is likely to injure one or more conductors.

One danger in underground cables which should be guarded against is electrolysis, for no manufacturer will guarantee his product against electrolytic action. The amount of electrolysis depends primarily on how near the cables are to electric railway lines, the distance they run parallel, and the condition of the return circuit of the railway, also the proximity of water pipes and gas mains. Electrolytic action occurs at the point where current leaves the lead sheath; therefore, with leaded cables, it is customary where this danger exists to provide suitable grounds at intervals along the system.

Sometimes this is accomplished by driving an iron pipe into the earth at each manhole. This can be tested out with an electric current of about 110 volts, connecting one side of circuit to pipe and other side to an adjacent hydrant, and then driving in pipe until sufficient current passes to make sure that a good ground is obtained. It may be necessary to drive as much as thirty feet of pipe or more in dry soil before a good ground is reached. It is sometimes customary to provide grounds by burying large copper plates in the earth, embedded in coke. In any case all lead sheaths in the manhole should then be connected to ground.

With railway systems, the negative side of the generator is usually grounded and the lead sheath of the return circuit should be connected to the negative side of generator by suitable copper cable. If precautions are taken, the danger from electrolysis may be reduced to a minimum.

After the duct line is installed it is good practice to pass a mandrel through each duct, thus removing all obstacles and making sure that ducts align. This is sometimes accomplished with rods about three feet long, provided with a coupling device, and as many rods as are required to reach from one manhole to another are successively joined.

At the same time that the mandrel is pulled through, an iron "fish" wire is also drawn in and left until it is desired to install cables in that particular duct, when the fish wire serves to pull through the heavy rope which is fastened to cable by a cable grip.

There are several methods of pulling cable. Short runs of light cable are sometimes pulled in by hand, but for heavy cable it is necessary to use some form of winch or manhole capstan. Both electric and gasoline-driven winches are being used with success.

It is advantageous in drawing in cable to have a man in the manhole where he can watch cable and make sure that it is not being pulled in faster than it is unreeled, thus preventing sharp bends which might prove injurious. He can also smear the sheath with a cheap grade of vaseline, which in the case of heavy cables makes them slide easier.

All cables to be installed in one duct should be drawn in at the same time, for if a cable is pulled in afterwards, it is almost sure to injure the lead of cables already installed. It is also poor practice to draw out one cable from among one or more others.

Enough slack should be allowed to permit cables being passed around the sides of manhole, and also to permit jointing, for occasionally cable ends are injured during the process of drawing in, necessitating cutting back far enough to remove injured portion.

With single conductor cables, where a butt joint is used, the ends in the manhole should overlap slightly, and for multiple conductor cables, where joints are staggered, the overlap should be enough to take care of this.

Ends of duct should be provided with lead collars on which the cable rests, thus preventing sharp corners of duct from injuring the lead sheaths. It is also good practice to use rubber bushings made of old hose between iron hangers and cable, which prevent leakage of stray current from one cable to another.

Cables in manholes are often protected by asbestos listing or by enclosing them in split duct, thus preventing the danger of a burnout on one cable from affecting another. This also protects cables against injury from careless workmen and prevents their being used for steps in descending into a manhole. All sharp bends should be avoided.

With high voltage leaded cables of 2500 volts and over, there is a tendency to puncture the insulation at the ends of lead sheath; therefore end bells are required at the station end of system, and also at the farther end where cables change from the leaded underground type to the braided overhead. These are generally of spun brass wiped to the lead sheath, their object being to flare out the lead, thus preventing a breakdown at the ends.

With paper insulated cables, the bells are made long so as to allow of the joint being made inside the bell. A cap is provided, through which the overhead cable end passes, and after the joint is completed, the bell is filled with a compound which prevents moisture entering and also acts as an insulator.

Underground systems when connected to overhead should be protected from lightning discharges by suitable arresters placed on second or third pole from the end of the cable.

It is desirable, after cables are installed and connected, to test them for five minutes with about twice the working pressure to make sure that there are no weak points due to imperfect jointing, or injury during installation.

Junction boxes are a necessity on low voltage systems and are installed in manholes at feeding points or street intersections so that in case of local trouble the feeders and mains may be disconnected.

Services are usually run from manholes or from service boxes located between manholes at street surface. Iron pipe is frequently used, so laid as to drain into manhole. For long services, cable with band iron armor finish is laid directly in the earth. The band iron protects the cable from injury, but it is customary to place a heavy plank over the cable so that in future excavations workmen will not injure the cable with a pick.

Service cables are sometimes connected to mains through service boxes placed in manholes. This arrangement is inconvenient in case of trouble; therefore it is customary to place service boxes on customers' premises.

There are several installations of so-called Edison tube systems still in use, but at present the popular drawing-in system which this article deals with is used almost exclusively throughout the country.

MOTIVE EQUIPMENT FOR ELECTRIC AUTOMOBILES

By H. S. Baldwin

Since the introduction of the electric automobile manufacturers of electrical apparatus have been active in the development of automobile equipments, consisting of motors, controllers and resistances designed to give the most efficient and economical service and to conserve the battery.

In the earlier automobiles, designers generally employed two low-speed motors, each of which operated the driving wheels by means of single reduction spur gears. Of late years, however, this practice has been largely abandoned in favor of the single motor and countershaft with double reduction chain transmission.

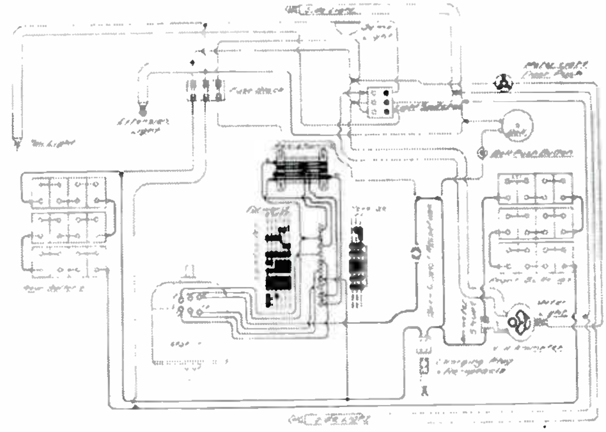


Fig. 1. Diagram of Connections of Electric Automobile Equipment

The advantages claimed for the single motor drive may be briefly summed up as follows:

1. It affords a material saving in weight, cost and space occupied, as against two motors of practically the same capacity.

2. By the use of double reduction gearing with countershaft, it is possible to design the motor for higher speed, which tends to lessen weight and insure electrical efficiency.

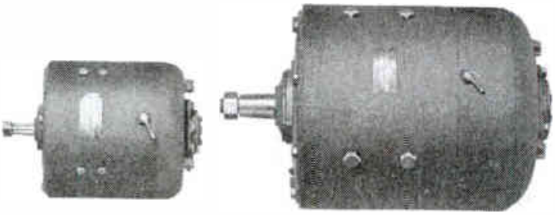
3. It further permits the motor to be mounted on the chassis, well up from the ground, which arrangement was not possible with the single reduction gear drive of the past.

It was realized by the General Electric Company that the high-speed single motor drive represented the simplest, most practical and efficient form of equipment that could be offered, and as a result they proceeded with the development of the so-called GE-1020 line of motors, of uniform mechanical construction throughout.

In addition to motive equipment, this Company supplies specially designed charging equipments, comprising rectifiers, motor generator sets, switch boards, rheostats complete with controlling panels, and many other useful and desirable appliances.

In order to meet the requirements of automobile manufacturers, the motors have been made in six different sizes, each of which can be supplied in several electrical ratings. It will readily be seen that this line is the most complete ever offered, and that a motor can be selected for any vehicle, from the smallest runabout to the heavy 5-ton truck.

The motors are of unique mechanical design (Fig. 2), the frame and one head being made from a single piece cylindrical steel casting, machined from end to end. By this construction the minimum of weight and maximum of strength are combined. This feature is original with these motors and is not to be found in those of other make.



Runabout Type; Weight 150 Lb. **5-Ton Truck Type: Weight 660 Lb.**

Fig. 2. General Electric Automobile Motors

It is of paramount importance to protect the storage battery, and all of the motors under consideration are designed with this object in view. They have a steep torque curve and give about five times the torque for two-and-a-half times the current, throughout the limits of their capacity. It might be well to add a word of explanation as to this statement. To obtain long life and efficient operation, the storage battery should not be discharged more rapidly than at the one hour rate. To start and accelerate an electric automobile requires approximately five times the running torque. Again, the average maximum grade encountered in cities is about 7.6 per cent., to climb which also requires about five times normal torque. It will, therefore, readily be seen that a properly designed electric automobile motor, having the above characteristics, will accelerate the vehicle and climb any ordinary grade without exceeding the one hour discharge rate of the battery.

The standard motors have cast iron heads fitted with the most improved annular ball bearings, which somewhat increase the efficiency, reduce the overall length of the frame and require only occasional lubrication. All electrical factors are liberal, permitting these motors to be run at high overloads for considerable periods of time without injury. The commutators are composed of a large number of bars, and observations covering several years' use of motors in service indicate that the commutation is practically perfect, great care having been taken to secure this result. Special graphite brushes of large area are used and the current per square inch of brush contact is lower than usually obtains in electric motor practice.

Two motors, namely the GE-1022 and the GE-1027, have only recently been added to the line, the former being suitable for 3-ton and the latter for 5-ton single motor trucks. All motors are of the 4-pole type, and with the exception of the smaller sizes for runabouts are designed to operate at 85 volts, experience indicating this voltage to be most advantageous when the lead battery is employed. The runabout motors are built to operate at 48 or 60 volts, as desired.

Special attention is called to the fact that all motors are constructed so that the shaft can be removed without disturbing the commutator or winding. This feature affords great flexibility and permits a change of shaft at small expense, to accommodate special conditions or when worn.

In the manufacture of the field coils, railway practice is closely followed, especially as regards insulation and treatment; thus the coils are adapted to withstand severe service conditions. Copper is used liberally throughout in the complete line of motors. This is especially true as regards the field coils, a feature which, together with high grade brushes, ball bearings, and commutators of

STANDARD AUTOMOBILE MOTORS

Type	Volts	Amps	R.P.M.	Vehicle	[illegible]	
					Forward	Backward
GE-1028	48	26	2000	Runabouts	6	3
GE-1020	85	20	2000	Light delivery	6	3
GE-1025	85	22	1200	1000 lb. del.	4	2
GE-1026	85	28	1200	2000 lb. del.	[illegible]	[illegible]
GE 1022	85	40	1200	3-ton truck	4	2
GE-1027	85	60	900	5-ton truck	4	2

small diameter, insures the highest possible electrical efficiency, so important when the storage battery is the source of power.

It will readily be seen that the General Electric automobile motors are practically universal in form, and can be adapted to many different methods of suspension and mounting. By use of the accurately machined motor frame, it is possible to meet all practical requirements of automobile manufacturers as to mounting, since supporting brackets or cradle can be attached directly to the steel casting by means of screws. This results in quicker deliveries and lower costs, for the reason that a large amount of engineering and developmental expense is eliminated.

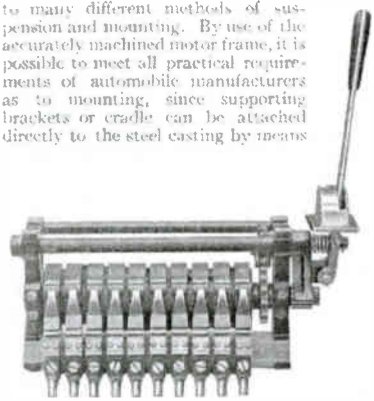


Fig. 3. New Controller for Automobile Motor

A good controller is scarcely less important than a good motor, and it is essential that each be selected with reference to the other and the nature of the service to which the automobile is to be put. All General Electric automobile controllers embody the continuous torque principle, which insures freedom from jolts due to opening of circuit when passing from series to multiple connection of field coils. This is an important feature, which adds much to the pleasure of operating small cars and to the life of large trucks. With the single motor equipment, series parallel arrangement of fields is the standard form of control, resistance being used on intermediate steps. In the case of controllers for pleasure vehicles, it is customary to have a comparatively large number of points, to permit of slow operation in cities where the traffic is congested, and higher speed over park and country roads. Commercial trucks as a rule do not require this fine gradation of speed and, therefore, have only sufficient notches to safe-guard the chain or gearing of the transmission.

A new controller has recently been designed (Fig. 3), which contains all the good points of the several types heretofore offered, with the additional advantage that many of the same parts can be used for different systems of connection, thereby insuring uniformity of construction. The new controller is of the cylindrical drum type and is operated by a pinion and sector at one end. The sector is mounted on a countershaft which carries the operating hand lever. Drum contacts are made from drawn copper tubing, screwed in place on a treated wood drum, and horn fibre spacers are inserted to insure smoothness of operation and to prevent sparking. Contact fingers are of rolled copper stock, secured to phosphor bronze springs. The controller is designed throughout with a view to withstanding rough usage.

An operating handle of new design has been provided, made from drop-forged steel and having the advantage that it can be formed to suit the automobile manufacturer. This is very desirable, since there is a great diversity of opinion as to shape of seat and body outline.

To make the equipment complete, a light cast iron grid resistance is employed (Fig. 4). This again is of sturdy construction and heavily insulated with mica. All terminals are drop-forged.

An important point in connection with these motive equipments is that the terminals, leads and contacts of each component part are marked with letters in accordance with the wiring diagram, so that the necessary connections in an electric automobile can be made by those not having special electrical knowledge.

Fig. 4. Cast Iron Grid Rheostat

APPARENT CHANGE OF RATIO OF TRANSFORMATION IN THREE-PHASE TRANSFORMERS

By G. Faccioli

Sometime ago three single-phase transformers were installed to operate a rotary converter. The primaries of the three transformers were connected "Y", and the secondaries "Y" diametrical. The difference of potential between primary lines was 11,000 volts, and the normal voltage of each secondary winding 210 volts.

The three transformers were connected to the high tension feeders (11,000 volts) and the voltage across each secondary winding was measured at no load. This voltage resulted to be 235 volts instead of 210.

The leg voltage corresponding to 11,000 volts "Y" is 6360 volts and the ratio of the transformer windings was exactly 6360:210; therefore there was no apparent reason for the higher secondary voltage. An investigation of the trouble immediately disclosed the fact that the secondary voltage was increased at no load from 210 to 235 volts by a triple frequency component of the voltage.

A brief review of the phenomena involved in the case will probably prove of some interest.

It is known that if a single-phase transformer is excited by a sinusoidal electromotive force, the magnetizing current is considerably distorted, owing to the characteristics of the iron in the core.

Fig. 1 shows the curve of exciting current of a 25 kw. transformer at normal excitation. The electromotive force applied across the exciting winding was a perfect sine wave, and its effective value 460 volts. The analysis of the complex wave of current gives the following results: If the maximum value of the complex wave is assumed to be 100, the fundamental component will have a maximum value of 57.3, the third harmonic a maximum value of 30.2, the 5th harmonic a maximum value of 10.6, and the 7th harmonic a maximum value of 2.96. The predominant overtune of this wave is, therefore, the third harmonic, and this is generally the case with every transformer.

Now let us take three of these 25 kw. transformers and connect their exciting windings in "Y", leaving the secondary windings disconnected, then apply across the lines 795 volts, which corresponds to a leg voltage of 460 volts. Each of the three transformers requires for its magnetization a triple frequency current, and since the electromotive forces across each transformer are 120 degrees apart, it is evident that the magnetizing currents and their high frequency components will have the same phase displacement. The triple harmonics of the magnetizing currents will then be displaced 120 degrees; but 120 degrees constitutes exactly one wave length of the triple harmonic, and therefore the three triple frequency components of the magnetizing current in the three transformers will be in phase with each other. The arrows in Fig. 2 represent the directions of the three triple frequency currents in the legs of the "Y" at any instant. It is obvious that under these conditions such currents cannot flow, and

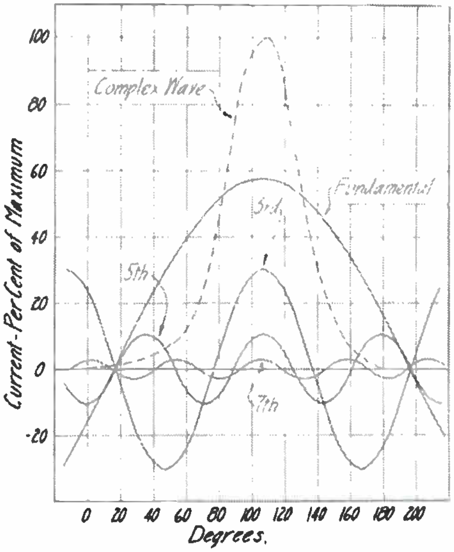


Fig. 1

therefore the flux in each core can no longer be a sinusoidal flux and the electromotive force across each individual transformer cannot be a sine wave. In other words, although a sinusoidal e.m.f. is applied between AB, BC, and CA, the electromotive forces across AN, BN and CN must contain some high frequency components which are necessary to restore the equilibrium.

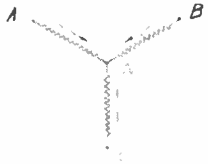


Fig. 2

795 volts were applied across the lines of the "Y" system, giving a leg voltage of 460 volts, and the electromotive force across AN, BN and CN was measured and resulted to be 525 volts instead of 460. The secondary windings of the transformers were not connected together. Fig. 3 gives the curve of this electromotive force and its analysis. If the maximum value of the complex wave is taken as 100, the maximum values of the fundamental and the 3rd and 5th harmonics are respectively 64.5, 33.3 and 2.2. The wave of the line current was taken at the same time and is given in Fig. 4. The analysis of this current wave gives 100 maximum complex, 84.2 maximum fundamental, 21.8 maximum 5th, and 3.3 maximum 7th. This current is then free from third harmonics, as we had anticipated; but the triple frequency distortion, which could not appear in the wave of current, appears in the wave of electromotive force. If we neglect the 5th harmonic, which is comparatively small, and assume that the electromotive force across each transformer is composed of a fundamental and third harmonic, we can immediately deduce the value of this third harmonic.

The fundamental is equal to 460 volts (the normal leg voltage corresponding to 795 volts across lines) and the third harmonic is equal to

$$\sqrt{525^2-465^2}=250.$$

In fact, it is well known that the effective value of the sum of two effective vectors of different frequency is equal to the square root of the sum of their squares. We see then that the "Y" connection on the exciting side does not allow the flow of any triple frequency currents, and that, in consequence, the voltage across each transformer is composed of a fundamental wave of 460 volts plus a triple frequency component of 250 volts. This latter component is equal in all three transformers and affects equally the three voltages AN, BN and CN. Furthermore, the difference of potential between the point N and the neutral of the generating system is evidently equal to 250 volts, and has a frequency three times the fundamental.

To remedy this distortion of the voltages, two methods can be followed: First, the neutral N of the "Y" can be connected to the neutral of the generating system; and second, the secondary windings of the three transformers can be delta connected.

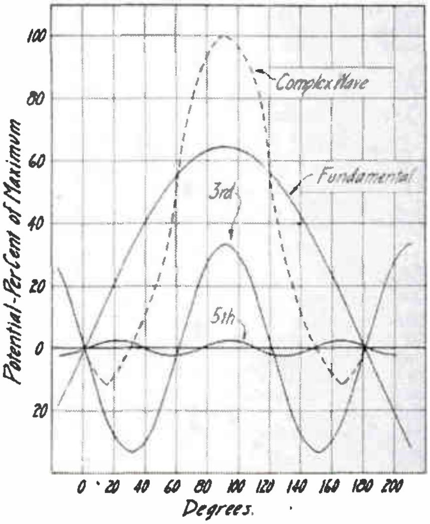


Fig. 3

In the first case, the three triple frequency currents of each leg will flow in the neutral wire and the magnetizing current of each transformer will have the same value and shape of wave as in the case of single-phase connection. If follows that the electromotive force across each transformer will be a sine wave and equal to 460 volts. In the second case, the triple frequency currents which cannot flow in the primary winding circulate in the closed secondary delta because the direction of these currents is the same in the three sides of the triangle.

This can easily be seen by remembering that the secondary electromotive force of each transformer must be an exact reproduction of its primary electromotive force. Then, if the primary electromotive force has a triple frequency component, the electromotive force induced across each side of the secondary delta must also have a triple frequency component. These triple frequency electromotive forces induced across each side of the delta assist each other, as shown in Fig. 5, and produce in the closed delta a triple frequency current which is magnetizing in character and excites the triple frequency flux necessary to give a sine wave of flux in the transformer. The final result is that across each side of the delta there will be only a small triple frequency electromotive force, active in sending the triple frequency magnetizing current through the resistance and leakage reactance of the windings, and that the voltage across each transformer will be practically a sine wave.

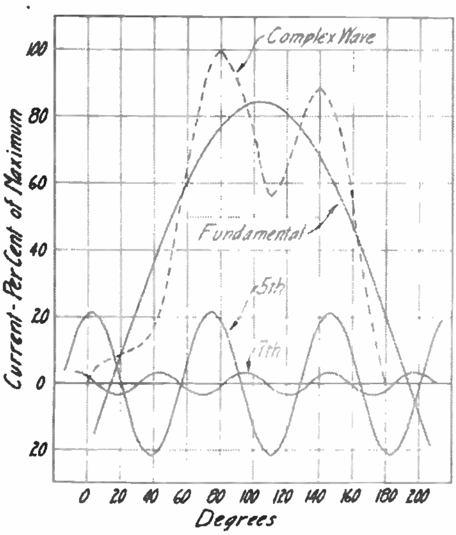


Fig. 4

But if the secondary delta is open and the primary neutral not connected to the neutral of the generating system, each leg of the "Y" has a triple frequency component of the electromotive force across its terminals, which is also present at the terminals of each secondary winding.

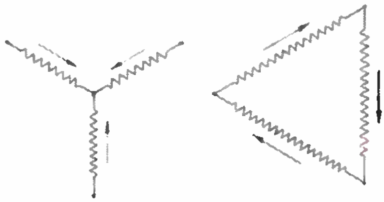


Fig. 5

The common method of deducing the voltage across the secondary windings of a three-phase system, the primary of which is "Y" connected, consists in dividing the voltage between primary lines by $\sqrt{3}$. This gives the voltage across each leg of the primary, and this voltage multiplied by the ratio of turns gives the secondary voltage. In the case just mentioned, this method of calculation is incorrect, because, as we have seen, the leg voltage of the primary and the voltage across the secondary windings are considerably increased by the presence of a triple harmonic.

This is the reason for the apparent discrepancy in the ratio voltages referred to at the beginning of this article. In that case 11,000 volts were impressed across the primary lines, giving 6360 as the corresponding leg voltage. Since, however, the neutral was not connected to the neutral of the generating system, and the secondary windings were open (diametrical. "Y"), a triple frequency component of the voltage was active across each leg of the "Y". This component was reproduced across the terminals of the secondary and increased the secondary voltage from 210 to 235 volts. The value of this component is

$\sqrt{235^2-210^2}=105$ volts on the secondary side. This means that the triple frequency component of the e.m.f. on the primary side must be

$$105 \times \frac{6360}{210} = 3190 \text{ volts.}$$

This is the difference of potential between the neutral of the "Y" and the neutral of the generating system, and the potential across each primary leg is

$$\sqrt{6360^2 + 3190^2} = 7100$$

Now $\frac{6360}{210} = \frac{7100}{235}$, that is to say, the ratio of voltage measured across primary and secondary windings of each transformer is equal to the ratio of the turns; but it is impossible to deduce the leg voltage from the voltage across lines by dividing it by the coefficient 1.73. Conditions of this nature are very frequent in three-phase systems.

If the transformers are loaded, the apparent change in the ratio of voltages disappears at once. Therefore, the presence of the triple frequency e.m.f. in this case has practically no effect on the operation of the transformers.

FURNACE ECONOMY

By F. W. Caldwell

Many power stations are operated uneconomically, due to indifference or ignorance in regard to the operation of the boiler plant.

Although there has been a great deal of discussion concerning the value of determining the quantity of CO_2 (and neglecting other gases) in boiler furnaces, the opinion seems to be very definite that some form of indicating apparatus is of great advantage to the firemen as well as to the plant operator, but that there is no apparatus which can entirely replace the trained eye in determining the best kind of fire.

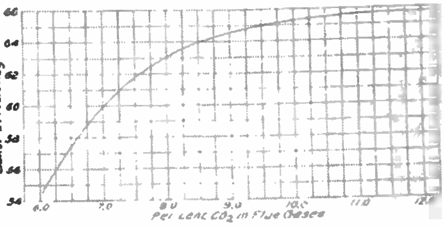
Perfect combustion, high furnace temperature, high velocity of gases over heating surfaces, and low stack temperature are all advantageous, and the best efficiency of evaporation is attained when all of these are a simultaneous maximum. Gas analysis is influenced by the first two conditions, slightly by the third, not at all by the fourth, and is itself a perfect measure of none.

If there are holes in the fire bed, the oxygen content will rise and the amount of carbonic acid will fall in proportion. Gas analysis will reveal the presence of such holes, and so will the eye coupled with an examination of the fire bed with the usual fire tools.

If a fire is thin and is passing too much air, gas analysis will give the same indication that is given when a fire bed contains holes, and it will not determine which of these troubles exists. The usual draft gauge and fire tool are the final instruments and might have been used in the first place. Again, leaks in the setting give this same oxygen indication and must be determined separately, independently of gas analysis.

The most common errors are the admission of too much air to the furnace, uneven fires and poor methods of firing. An analysis of the flue gases is naturally the best evidence of what is taking place in the furnaces.

The flue gases consist principally of nitrogen (N), carbon dioxide (CO_2), oxygen (O_2), and carbon monoxide (CO), the proportions depending upon the amount of air admitted to the furnace, the completeness of combustion and the quality of the coal used.



Curve Showing Percentage of CO_2 in Flue Gases for Different Boiler Efficiencies

Generally speaking, a low percentage of CO_2 indicates the admission of too much air to the furnaces and a low boiler efficiency. A high percentage of CO always indicates incomplete combustion and a low boiler efficiency. The correct percentage of CO_2 has always been subject to more or less discussion, the estimate varying from 9 to 14 per cent. Certain boilers which are of the water-tube, internally-fired type, have given the best results where CO_2 was 15 per cent., although in some types of water-tube boilers, when attempting to run at high values of CO_2, the arches and side walls have been burned out. The correct percentage undoubtedly varies for different boiler settings, the quality of coal burned, etc., but a percentage of 10 to 12 per cent. usually indicates the most economical operation. The attached curve was taken from the Government Boiler Testing Plant at St. Louis, Mo., Bulletin No. 325 of the U. S. Geological Survey. This curve indicates that CO_2 should have in general a value of about 10 per cent. The upward slope of the curve indicates that a higher efficiency is obtained by raising the percentage of CO_2.

The improvement in boiler efficiency effected by increasing the percentage of CO_2 in flue gases from 6 to 11 per cent., as shown in this curve, is 11.8 per cent., and corresponds to a saving of 20 per cent. in the coal burned. This curve probably shows the improvement that could be expected in the average boiler plant. The percentage of CO_2 in the flue gases can be almost entirely regulated by proper damper control, careful firing, etc.

In order to act intelligently, the boiler plant operators must have an analysis of the flue gases as often as possible. Due to the small percentage of CO compared with the total volume of flue gas, the usual gas analysis does not give reliable figures on this content. The usual practice, therefore, is to obtain the percentage of CO_2 only. The oldest method of obtaining this analysis is with the Orsat apparatus, which is very reliable but not automatic. With this apparatus, one man who does nothing else analyzes a sample of the flue gases about every twenty to thirty minutes. Today there are several fairly reliable devices on the market which record the percentage of CO_2 automatically, giving the operator a continuous record to work by.

A very satisfactory method is to have an automatic device before each fireman. This device need not have great accuracy, as anything that makes a mark varying with the firing will constantly urge the man to his best endeavours. If, in addition to this, supervision is exercised by one well trained in the fireroom and the results for the day are accurately summated, a good degree of economy should be secured.

It would appear that the curve can be taken to represent general values, although there is no doubt that a few boiler tests on each type of boiler would indicate that the best results could be obtained by very slightly raising or lowering this curve. There can, however, be no question but that a percentage as low as 6 makes a tremendous difference in the economy of the boiler.

BOOK REVIEWS

THEORY AND CALCULATION OF TRANSIENT ELECTRICAL PHENOMENA AND OSCILLATION

By Charles Proteus Steinmetz

McGraw Publishing Co. 556 Pages Price Net $5.00

The increasing use of the alternating current within the past few years has rendered the subject of transient phenomena of vital importance; there has been, however, no work available which treated the subject in a thorough and consistent manner. With the publication of Dr. Steinmetz's book, a treatise has been placed in the hands of engineers which, for the first time, adequately discusses these complex phenomena. The book is therefore a pioneer work; it is, in fact, epoch making.

An exact physical definition of the expression "transient phenomena," one which shall be sufficiently inclusive and at the same time non-mathematical and easily understood, is rather difficult to frame. In his preface, the author defines the term by giving the common characteristic of the phenomena. He says: "the characteristic of all transient phenomena is that they are transient functions of the independent variable time or distance." Transient phenomena may be described as all those phenomena that are episodial in character; *i.e.*, that begin at a certain moment or place and vary continuously either gradually or in an oscillatory manner with the time or the distance, finally becoming constant at a maximum or zero value. The building up of a dynamo is a transient phenomena, the rise of current in a circuit upon closing the switch, the discharge of a condenser, the surge in a transmission line, etc., etc.

While the inherent nature of the subject absolutely necessitates the employment of higher mathematics, the author wherever possible has used the simpler algebraic forms; furthermore, after developing the various theories, he has applied them practically to working conditions, and has given concrete numerical examples. This is an especially valuable feature of the work, as it renders the conclusion of the theoretical discussions available to all classes of readers, and thus does not limit the book's usefulness merely to those whose mathematical training enables them to follow the discussions in their entirety.

From such a mass of uniformly valuable material, it is difficult to make selections for special comment. The book covers the subject thoroughly, and considers transient phenomena as involved in generation, transformation, rectification and transmission under both normal and abnormal conditions. A beautifully lucid presentation of the subject of artificial leakage and loading is also included. Skin effect and other properties of wires and cables are fully discussed, as is also the theory of lighting and lightning protection.

THE ELECTRIC SOLICITORS' HANDBOOK

This book is issued by the National Electric Light Association under an editorial committee with Mr. Arthur Williams as chairman, and as might therefore be expected is a thoroughly practical and useful production. It is written for the use of central station solicitors and all others directly interested in the applications of electricity. The book is divided into three chief sections, entitled, "Illuminating Engineering, Heating Engineering and Power Engineering." The three sections are prefaced by some valuable information on business getting and talking points. A complete index is provided which shows a very wide range of subjects dealt with. The use of very concise and simple methods and round numbers, etc., is of course, necessary in a book of this type, but it forms the most valuable addition to the library of the central station man that we have had for a long time.

The range of the book can possibly be best illustrated by a few examples. The following, for instance, are extracts taken at random:

Cost of Central Station Service Compared with Isolated Plants; Horse-Power Required to Drive Various Machines; Load Factors for Different Classes of Service; Data on High Pressure Exhaust Fans; Application of Motors to Machine Tools; Electric Heating Calculations; Power Taken by Different Heating Devices; Estimation of Illumination; The Lighting of Factories; Power Consumption of Various Forms of Lamps; Table of Reflection Coefficients; The Electric Motor in the Household; The Electric Motor Compared with the Gas and Gasolene Engine; The Electric Motor Compared with an Isolated Steam Plant; Electric Light Advertising; Specific Advantages of Electric Light; The Relation of the Company to the Consumer; Methods of Keeping Records, Etc.; How to Meet Opposition and Competition.

This book would have been still more valuable if it could have been published at the time the manuscripts were completed by the different competitors—now more than two years ago—since it now contains a large amount of matter either not available elsewhere or difficult to find. At the time referred to, it must have been a still more exceptional production. We note with pleasure that a large share of the credit of this production belongs to one old employee and one present employee of the General Electric Company.

OBITUARY

James J. Mahony, who had been connected with the General Electric Company since its organization, died on March 19th at Holyoke, Mass., at the home of his sister, Mrs. A. J. McDonald.

Mr. Mahony was born at Worcester, Mass., June 16, 1863. His parents were Maurice and Mary White Mahony, both of Ireland.

He received his education in the Public Schools of Worcester, entering the High School in 1876, where he stood well in all his classes, and in particular showed marked ability in mathematics.

At the end of his third high school year he left school to accept employment at Forehand & Wadsworth's pistol factory, where his father had been employed for some years. He remained with this concern a year or more and then served an apprenticeship as machinist with the McMahone & Carver Tool Company. During this time he also took up the study of engineering and mechanical drawing.

In the spring of 1888 Mr. Mahony entered the employment of the Thomson-Houston Company at Lynn, where for the first six months he worked as a machinist under Mr. John Riddell, and was then transferred to the expert corps. A few months later he was sent to take charge of the installation of car equipments and to supervise the operation of street railway apparatus during their trial period. While in this position he had charge of a number of important railway installations, among others the street railways of Albany and the original West End power station of Boston.

In 1891 the Foreign Department of the Company sent him to Australia to take charge of a number of installations, one of importance being the Sydney Tramways.

Returning about two years later, he was placed in charge of similar work in and about New York City. He erected the electrical machinery in the Kent Avenue power station in Brooklyn and built the first large direct connected generators in both Brooklyn and Boston. He also accomplished a great deal of important inspection work throughout the United States and Canada.

About twelve years ago Mr. Mahony became connected with the Commercial Department of the New York Office; and here, by reason of his tactfulness, diplomacy and unfailing courtesy, he made one of his greatest successes.

In 1905, at the Company's request, he made a trip to South America, from which, after a few months, he returned to his duties at the New York Office, where he remained until the time of his death.

Through his inherent ability and by his own unaided efforts Mr. Mahony rose to a high position in the electrical profession, his sterling character and perseverance commanding the respect and admiration of his associates. He possessed a personal charm that was exceptional.

From his boyhood, he was very fond of outdoor sports, being particularly interested in baseball and golf; he was also an exceptionally expert sailor and was the winner of a number of cups. He was a member of the Engineer's Marine and Field, Dyker Meadow, and Scarsdale Golf and Country Clubs.

The funeral services, which were very largely attended, were held in St. Paul's church, Worcester, and the interment was made at St. John's cemetery.

FORMULAE, CONSTANTS AND HYPERBOLIC FUNCTIONS
FOR
TRANSMISSION LINE PROBLEMS

With Explanation and Examples of their Use

By W. E. Miller

"General Electric Review" Supplement
May, 1910

FORMULÆ, CONSTANTS AND HYPERBOLIC FUNCTIONS

FOR

TRANSMISSION LINE PROBLEMS

NOTE. In using the formulæ given below, the following points should be noted:

All voltages given in the formulæ must be those between wire and neutral. That is, the line voltage for three-phase lines must be divided by $\sqrt{3}$ before substituting in the equations, and must be divided by 2 for single-phase lines. The currents are given in amperes per wire. Hence, the power lost, delivered or generated, calculated from the equations, only refers to one phase and must be multiplied by 3 to obtain the total power in three-phase lines, and by 2 in the case of single-phase lines.

Phase Convention

When the voltage is given at some point, as at the receiving or generating end, it is usually taken as the standard phase, and all other voltages or currents, calculated or given, refer to this phase. Thus, according to the sign convention adopted throughout *viz*, contra clockwise rotation, the given voltage v (say at the receiving end) is written v without any j term. If the voltage or current at any point is $a+jb$, the $+$ sign indicates that the voltage or current leads the voltage v by the angle $\tan^{-1}\frac{b}{a}$; a negative sign would show that they lag behind the voltage v by the same angle. The form $-a+jb$ indicates that the current or voltage at the point considered leads the voltage v, at the receiving end, by the angle $\tan^{-1}\left(\frac{b}{-a}\right)$, that is, by an angle greater than 90°.

A current or voltage $a+jb$ means that a amperes or volts are in phase with the standard voltage phase and b amperes or volts are in quadrature with the standard phase. The prefix j is equal to $\sqrt{-1}$, therefore, $j\times j=-1$. The resultant value of voltage or current $a\pm jb=\sqrt{a^2+b^2}$. The product $(a+jb)(c+jd)=ac-bd+j(bc+ad)$ and so on. For division $\frac{1}{a+jb}=\frac{a-jb}{a^2+b^2}$ and so on.

The cosine of the angle that the current leads or lags the voltage at any point is the power factor at that point. Hence, if the power factor is given (at the receiving end say) the current $i=a\pm jb$ at this end can be obtained from the formula $\sqrt{a^2+b^2}\left(PF\pm j\sqrt{1.0-PF^2}\right)$. Thus if the resultant effective current is 100 amperes at .80 P.F. lagging, $i=100(.80-j\sqrt{1.0-.64})$ $=80-60j$ and must be substituted in the equations, in this form.

Constants m, m_1 and $\frac{1}{m_1}$. Tables II and III, Pages 6 and 7

These are calculated to include the capacity and self-induction between wire and neutral, and the resistance per mile of single wire, whether for three-phase or single-phase lines.

When calculating single-phase lines, the constants determined for the wire spacings lying in a plane must not be used, but only those for triangular spacings. The constants for spacings in a plane must only be used for three-phase lines, when a sufficient number of transpositions has been made to produce balanced electrical conditions along the line.

The constants for any wire spacings between those given can be readily determined by interpolation. As noted in the tables, the values for m and $\frac{1}{m_1}$ must be divided by 1000. The values of m_1 are correct as they stand.

Resistance

The resistances included in the constants refer to hard drawn stranded copper wires, the value of the resistance used being given in the tables. If the resistance of a given line is slightly greater or less than that for which the tables have been calculated, the proper percentage increase or decrease in the constants can be obtained from Table I, page 5.

Hyperbolic Tables. Tables IV to XIII, Pages 8 to 17 inclusive

The values of *cosh* $(x+jy)$, where x is given in the top row and y in the extreme left or right hand column will be found in the columns headed a and b, and the values of *sinh* $(x+jy)$ in the columns headed c and d. Hence, the value of *cosh* $(x+jy)$ will be $a+jb$, and *sinh* $(x+jy)=c+jd$. The values of these functions lying between those tabulated can be readily obtained by interpolation, see example A.

Equations

The accurate equations for transmission lines are given below, provided the generator voltages and currents are simple harmonic functions of the time, and there is no corona effect.

When the electrical conditions are given at the receiving end and E_r = volts at receiving end between wire and neutral, I_r the current per wire, e_l the voltage at any point l, and i_l the current at the same point. The distance l is given in miles and is measured from the receiving end, then

$$e_l = E_r \cosh ml + I_r m_1 \sinh ml \tag{1}$$

$$i_l = I_r \cosh ml + \frac{E_r}{m_1} \sinh ml \tag{2}$$

If the conditions are determined at the generator end and E_g and I_g are the voltage and current respectively at this end, then the voltage and current at any point l along the line can be obtained from equations (3) and (4) following, l being measured from the generator end.

$$e_l = E_g \cosh ml - I_g m_1 \sinh ml \tag{3}$$

$$i_l = I_g \cosh ml - \frac{E_g}{m_1} \sinh ml \tag{4}$$

Approximate Formulæ for Short Lines

These formulæ can be used with an accuracy of 1 per cent. for lines using No. 2 wire up to 120 miles long at 60 cycles and 150 miles at 25 cycles. Greater accuracy will be obtained if larger wires than No. 2 are used, though the difference is immaterial. See example B.

If $m = p+jq$ and the conditions are given at the receiving end, then

$$e_l = E_r\left(1+\frac{l^2(p^2-q^2)}{2} + jpql^2\right) + I_r m_1 l(p+jq) \tag{5}$$

$$i_l = I_r\left(1+\frac{l^2(p^2-q^2)}{2} + jpql^2\right) + \frac{E_r}{m_1} l(p+jq) \tag{6}$$

If the conditions are given at the generator end

$$e_l = E_g\left(1+\frac{l^2(p^2-q^2)}{2} + jpql^2\right) - I_g m_1 l(p+jq) \tag{7}$$

$$i_l = I_g\left(1+\frac{l^2(p^2-q^2)}{2} + jpql^2\right) - \frac{E_g}{m_1} l(p+jq) \tag{8}$$

A. Example of Accurate Solution

Three-phase line, 300 miles long using hard drawn stranded copper wire No. 000 B.&S. triangularly spaced, with wires 10 ft. apart. Frequency 60 cycles.

From the tables $m = \dfrac{.421+2.11j}{1000}$ $\quad m_1 = 302 - 78.0j \quad \dfrac{1}{m_1} = \dfrac{2.44+.483j}{1000}$.

Suppose the following conditions are determined at the receiving end. Line voltage 104,000 volts, or 60,000 volts between wire and neutral. Load current 100 amperes at receiving end at .90 power factor lagging. Then, $E_r = 60{,}000$ and $I_r = 100(.90 - j\sqrt{1.0^2 - .9^2}) = 90 - 43.5j$. If the power factor had been unity $I_r = 100$, or if .9 leading $I_r = 90+43.5j$.

At the sending or generating end, $ml = \dfrac{(.421+2.11j)\ 300}{1000} = .126+.633j$.

Then by interpolation from the tables of hyperbolics, the following values are obtained:

$\cosh ml = \cosh(.126+.633j) = .812+.075j,$

$\sinh ml = \sinh(.126+.633j) = .102+.597j$

The interpolation can be obtained as follows:

From the tables

$\cosh(.12+.62j)=.820+.070j$

$\cosh(.12+.64j)=.808+.072j$

Therefore, $\cosh(.12+.633j)=.812+.072j$

From tables

$\cosh(.14+.62j)=.822+.081j$

$\cosh(.14+.64j)=.810+.083j$

Therefore, $\cosh(.14+.633j)=.814+.082j$

But $\cosh(.12+.633j)=.812+.072j$

Therefore, $\cosh(.126+.633j)=.812+.075j$

By the same method $\sinh(.126+.633j)$ can be determined. The above steps, for obtaining the interpolations, were given more for the purpose of showing how to use the tables than for determining the values of the functions; since with a little practice, it will be found that practically all values can be immediately obtained from the tables by inspection.

Substituting in equation (1) the voltage at the generator end is given as follows:

$$e_g=60{,}000(.812+.075j)+(90-43.5j)(392-78j)(.102+.597j)$$
$$=48{,}700+4500j+17{,}500+16{,}500j=66{,}200+21{,}000j$$

Hence $e_g=\sqrt{66200^2+21000^2}=60{,}500$ volts at generator end, between wire and neutral.

The generator voltage leads the receiving voltage by the angle $\tan^{-1}\frac{210}{662}=\tan^{-1}.318=17°39'$.

To find the current of the generator end, substitute in (2), then

$$i_g=(90-43.5j)(.812+.075j)+\frac{60{,}000(2.44+.485j)(.102+.597j)}{1000}$$
$$=76.3-28.3j-1.7+90.2j=74.6+61.9j$$

Therefore, generator current $=\sqrt{74.6^2+61.9^2}=96.8$ amperes per wire.

The generator current, therefore, leads the voltage at the receiving end by the angle $\tan^{-1}\frac{61.9}{74.6}$ $=\tan^{-1}.83=39°\ 42'$.

Therefore, the current at the generator end leads the voltage at the generator end by the angle $(39°\ 42')-\text{angle}(17°\ 39')=22°\ 03'$.

The power factor at the generator end is, therefore, $\cos(22°\ 03')=.927$ leading.

Transmission efficiency is thus $\frac{60{,}000\times100\times.90}{60{,}500\times96.8\times.927}=.87$

The total power delivered by the transmission line is $3\times60{,}000\times90=16{,}200$ kw., the total power lost in transmission being 2,400 kw.

To obtain the regulation, find the voltage at the generator end with no load current, that is, since $I_r=0$.

$e_g=60{,}000(.812+.075j)=48{,}700+4{,}500j=48{,}900$ volts between wire and neutral.

Hence, a voltage rise occurs between wires of $20{,}600\times\sqrt{3}$ volts $=35{,}500$ volts at the generator end when the load is increased from nothing to 100 amperes at .90 power factor lagging at the receiving end, with constant voltage at the receiving end.

Since $I_r=0$ at no load, the capacity current is

$$i_g=\frac{60{,}000(2.44+.485j)(.102+.597j)}{1000}=90.2j-1.7=90.2 \text{ amperes per wire.}$$

At no load, the voltage at the generator end leads the voltage at the receiving end by the angle $\tan^{-1}\left(\frac{4.5}{48.7}\right)=\tan^{-1}.093=5°19'$; and the current at the generator end leads the voltage at the receiving end by the angle $\tan^{-1}\left(\frac{90.2}{-1.7}\right) = \tan(-53.1) = 91° \, 04'$.

Hence, the current at the generator end leads the voltage at the generator end by the angle $(91° \, 04') - (5° \, 19') = 85° \, 45'$; hence, the power factor at no load is $\cos(85° \, 45') = .074$ leading; and the total no load transmission loss due to capacity current is $.074 \times 3 \times 48{,}900 \times 90.2 = 980$ kw.

B. Example of Solution by Approximate Formulæ

Three-phase line 100 miles long, using hard drawn stranded copper wires No. 0 B.&S. wires equally spaced in a plane and 8 ft. between wires. Frequency 25 cycles.

In this case, suppose that the generator conditions are determined, being 50,000 volts between wire and neutral, and 100 amperes per wire at unity power factor.

From the tables $m = \frac{.555 + 1.025j}{1000}$ $\quad m_1 = 470 - 254j \quad$ $\frac{1}{m_1} = \frac{1.65 + .88j}{1000}$

Therefore, ml at the receiving end is $.0555 + .1025j$ and since $ml = pl + jql$ therefore, $pl = .0555$ and $ql = .1025$.

By substituting these values in formula (7), the received voltage is obtained

$$e_r = 50{,}000\left(1 - \frac{.0074}{2} + .0057j\right) - 100(470 - 254j)(.0555 + .1025j)$$
$$= 49{,}800 + 280j - 5{,}200 - 3{,}420j = 44{,}600 - 3{,}140j.$$

Therefore, the received voltage is $\sqrt{44{,}600^2 + 3{,}140^2} = 44{,}800$ volts between wire and neutral, and this voltage lags behind the voltage at the generator end by the angle $\tan^{-1}\left(\frac{-314}{4460}\right)$
$= \tan^{-1}(-.0704) = -4° \, 02'$.

The current at the receiving end is given by substituting in equation (8)

$$i_r = 100(.996 + .0057j) - \frac{50{,}000\,(1.65 + .88j)\,(.0555 + .1025j)}{1000}$$
$$= 99.6 + .57j - .10 - 10.9j = 99.5 - 10.3j = 100 \text{ amperes per wire.}$$

Therefore, the current received is 100 amperes which lags behind the voltage at the generator end by the angle $\tan^{-1}\left(\frac{-10.3}{99.5}\right) = \tan^{-1}(-.1035) = -5° \, 55'$.

Hence, this current lags behind the voltage at the receiving end by the angle $(5° \, 55') - (4° \, 02') = 1° \, 53'$ and the power factor at the receiving end is $\cos(1° \, 53') = .9995$ lagging.

Since the received current at no load is 0, the capacity current is given by the equation

$$I_c\left(1 + \frac{l^2(p^2 - q^2)}{2} + jpql^2\right) = \frac{E_c}{m_1} l(p + jq)$$

therefore, capacity current $I_c = \frac{.10 + 10.9j}{.996 + .0057j} = \frac{(.10 + 10.9j)(.996 - .0057j)}{.996^2 + .0057^2}$

that is, capacity current or I_c at no load $= .164 + 10.9j$:
hence, the capacity current is 10.9 amperes per wire and leads the voltage at the sending end by the angle $\tan^{-1}\frac{10.9}{.164} = \tan^{-1} 66.5 = 89° \, 08'$. Thus the power factor at the sending end at no load is $\cos(89° \, 08') = .0151$.

By substituting the capacity current or I_c at no load in equation (7), the voltage at the receiving end at no load can be obtained, and therefore the regulation at the receiving end between no load and 100 amperes. Substituting the values

$$e_r = 49{,}800 + 280j - (.164 + 10.9j)(470 - 254j)(.0555 + .1025j)$$
$$= 49{,}800 + 280j + 363 - 572j = 50{,}200 - 292j.$$

Thus the received voltage is 50,200 volts, lagging by a small angle behind the generator voltage.

The regulation at the receiving end is, therefore, 50,200 − 44,800 = 5,400 volts between wire and neutral, or betwen wires = $\sqrt{3} \times 5{,}400 = 9{,}300$ volts drop between no load and 100 amperes load at the receiving end, when the generator voltage is kept constant.

Example of how to use Table No. I

Assume that the No. 00 wire used in a transmission line operating at 25 cycles has a resistance of .423 ohms per mile, instead of .417 ohms as given in the tables for m, etc. Then, in this case, the increase of resistance is 1.4% nearly. Hence, the following changes must be made in m, m_1 and $\frac{1}{m_1}$ in accordance with Table No. I.

The real term of m must be increased 1.4×.9% = 1.3% nearly

The j term of m must be increased 1.4×.4% = 0.6% nearly

The real term of m_1 must be increased 0.6% and the j term 1.3% nearly

The real term of $\frac{1}{m_1}$ must be decreased 1.4×0.2% = 0.3% nearly

The j term of $\frac{1}{m_1}$ must be increased 1.4×0.3% = 0.4% nearly.

If the resistance were 1.4% less instead of greater, the values must be decreased where they were increased in the above example and *vice versa*.

TABLE I

Percentage change of constants m, m_1 and $\frac{1}{m_1}$ for change in resistance

For every 1% variation in resistance, change the real and j terms in the constants by the percentage amounts given in the table. If the resistance is increased, the + sign means, increase the term and − sign decrease the term. The opposite rule holds when the resistance is decreased. This table covers both methods of spacing and any distance between wires.

Wire B.&S.	R in Ohms per Mile	60 CYCLES						25 CYCLES					
		m		m_1		$\frac{1}{m_1}$		m		m_1		$\frac{1}{m_1}$	
		Real +%	j Term +%	Real +%	j Term +%	Real −%	j Term +%	Real +%	j Term +%	Real +%	j Term +%	Real −%	j Term +%
250,000	.223	+.75	None	None	+.75	None	+.75	+.85	+.05	+.05	+.85	−.10	+.70
0000	.263	+.70	None	None	+.70	None	+.70	+.80	+.20	+.20	+.80	−.10	+.60
000	.330	+.70	None	None	+.70	None	+.70	+.80	+.30	+.30	+.80	−.15	+.35
00	.417	+.70	None	None	+.70	−.05	+.65	+.90	+.40	+.40	+.90	−.20	+.30
0	.523	+.75	+.05	+.05	+.75	−.05	+.65	+.90	+.50	+.50	+.90	−.25	+.15
No. 1	.663	+.85	+.10	+.10	+.85	−.10	+.60	+.70	+.30	+.30	+.70	−.25	+.15
No. 2	.833	+.90	+.20	+.20	+.90	−.15	+.55	+.80	+.20	+.30	+.80	−.20	+.10

★ TABLE II

Values of m, m_1 and $\frac{1}{m_1}$ per mile for triangular spacing at 25 and 60 cycles

Spacing between Wires Inches	60 CYCLES			25 CYCLES			Spacing between Wires Inches
	m Divide by 1000	m_1	$\frac{1}{m_1}$ Divide by 1000	m Divide by 1000	m_1	$\frac{1}{m_1}$ Divide by 1000	
	250,000 B.&S. R = .223 ohms per mile			230,000 B.&S. R = .222 ohms per mile			
72	.323 +2 11j	346 −52 7j	2 82 +.439j	.306 +.918j	361 −120j	2 61 +.870j	72
96	.306 +2 10j	362 −52 7j	2 70 +.394j	.294 +.912j	377 −121j	2 41 +.772j	96
120	.294 +2 09j	376 −52 8j	2 60 +.365j	.282 +.907j	390 −122j	2.33 +.730j	120
144	.287 +2 09j	387 −53.1j	2 53 +.347j	.275 +.906j	401 −122j	2.27 +.691j	144
	No. 0000 B.&S. R = .263 ohms per mile			No. 0000 B.&S. R = .263 ohms per mile			
72	.374 +2.11j	352 −62 2j	2.78 +.488j	.352 +.932j	372 −141j	2 35 +.892j	72
96	.357 +2 10j	366 −62 4j	2.63 +.448j	.337 +.927j	388 −141j	2 28 +.822j	96
120	.344 +2 10j	382 −62 7j	2.55 +.418j	.326 +.922j	402 −142j	2 21 +.780j	120
144	.334 +2 09j	391 −62 7j	2 49 +.399j	.316 +.916j	411 −142j	2.17 +.749j	144
	No. 000 B.&S. R = .33 ohms per mile			No. 000 B.&S. R = .33 ohms per mile			
72	.467 +2 12j	362 −78 0j	2 64 +.568j	.420 +.960j	391 −171j	2 14 +.937j	72
96	.437 +2 12j	378 −78 0j	2 53 +.521j	.403 +.953j	406 −172j	2 09 +.885j	96
120	.421 +2 11j	392 −78 0j	2 44 +.485j	.390 +.946j	420 −173j	2 04 +.840j	120
144	.408 +2 11j	402 −78 0j	2.36 +.460j	.381 +.941j	430 −174j	2 00 +.809j	144
	No. 00 B &S. R = .417 ohms per mile			No. 00 B &S. R = .417 ohms per mile			
72	.556 +2 15j	372 −97 0j	2 52 +.652j	.497 +.996j	414 −207j	1 93 +.962j	72
96	.526 +2.14j	391 −97.8j	2 40 +.600j	.482 +.989j	434 −211j	1 86 +.907j	96
120	.517 +2 13j	405 −98 1j	2 32 +.565j	.468 +.980j	446 −213j	1 83 +.875j	120
144	.504 +2.13j	414 −98.2j	2 29 +.545j	.456 +.973j	453 −213j	1 81 +.890j	144
	No. 0 B &S. R = .525 ohms per mile			No. 0 B &S. R = .523 ohms per mile			
72	.678 +2 18j	385 −120j	2 34 +.727j	.591 +1 05j	445 −250j	1 71 +.960j	72
96	.653 +2 17j	402 −121j	2 28 +.685j	.569 +1 03j	458 −253j	1 67 +.924j	96
120	.632 +2 16j	415 −121j	2 22 +.649j	.554 +1 02j	470 −255j	1 64 +.892j	120
144	.615 +2 15j	424 −121j	2 17 +.622j	.541 +1 01j	479 −256j	1 62 +.868j	144
	No. 1 B &S. R = .665 ohms per mile			No. 1 B.&S. R = .665 ohms per mile			
72	.828 +2 22j	402 −149j	2 17 +.802j	.691 +1 10j	476 −300j	1 51 +.946j	72
96	.791 +2 21j	418 −149j	2.11 +.765j	.670 +1 09j	498 −304j	1 47 +.901j	96
120	.766 +2.20j	430 −150j	2 07 +.728j	.655 +1.08j	507 −307j	1 44 +.873j	120
144	.749 +2 19j	441 −151j	2 02 +.696j	.640 +1 07j	516 −309j	1 43 +.855j	144
	No. 2 B &S. R = .835 ohms per mile			No. 2 B &S. R = .833 ohms per mile			
72	.989 +2 29j	422 −182j	2 00 +.850j	.800 +1 17j	518 −354j	1 31 +.896j	72
96	.948 +2 27j	438 −182j	1 94 +.810j	.779 +1 16j	536 −360j	1 28 +.861j	96
120	.920 +2 26j	450 −183j	1 90 +.776j	.756 +1 14j	548 −362j	1.27 +.840j	120
144	.905 +2 25j	460 −184j	1 88 +.751j	.744 +1.14j	559 −364j	1 26 +.821j	144

★ This table can also be used for single-phase lines.

* TABLE III

Values of m, m_1 and $\frac{1}{m_1}$ per mile for equally spaced wires lying in a plane at 25 and 60 cycles

Spacing between Wires Inches	60 CYCLES • • •			25 CYCLES • • •			Spacing between Wires Inches
	m Divide by 1000	m_1	$\frac{1}{m_1}$ Divide by 1000	m Divide by 1000	m_1	$\frac{1}{m_1}$ Divide by 1000	
	250,000 B.&S. R = .222 ohms per mile			250,000 B.&S. R = .222 ohms per mile			
72	.310+2.11j	360−82.8j	2.71+.398j	.296+.917j	378−121j	2.41+.730j	72
96	.297+2.11j	375−82.6j	2.60+.366j	.284+.913j	390−121.8j	2.34+.729j	96
120	.288+2.10j	386−82.8j	2.53+.345j	.275+.910j	404−122j	2.27+.687j	120
144	.278+2.09j	397−82.9j	2.47+.329j	.265+.909j	415−122j	2.23+.684j	144
	No. 0000 B.&S. R = .263 ohms per mile			No. 0000 B.&S. R = .263 ohms per mile			
72	.351+2.11j	365−82.3j	2.63+.463j	.340+.929j	386−141j	2.39+.840j	72
96	.345+2.10j	380−82.6j	2.57+.422j	.326+.924j	401−142j	2.31+.785j	96
120	.332+2.10j	395−82.8j	2.47+.390j	.315+.920j	415−142.5j	2.16+.742j	120
144	.324+2.09j	405−82.6j	2.41+.373j	.308+.917j	426−143j	2.10+.706j	144
	No. 000 B.&S. R = .33 ohms per mile			No. 000 B.&S. R = .33 ohms per mile			
72	.439+2.12j	374−77.4j	2.55+.530j	.406+.955j	408−172j	2.10+.894j	72
96	.421+2.12j	390−77.6j	2.47+.491j	.391+.949j	420−173j	2.05+.848j	96
120	.406+2.11j	405−77.9j	2.38+.457j	.379+.943j	434−174j	1.96+.797j	120
144	.396+2.11j	416−78.0j	2.32+.424j	.370+.939j	446−176j	1.95+.766j	144
	No. 00 B.&S. R = .417 ohms per mile			No. 00 B.&S. R = .417 ohms per mile			
72	.540+2.16j	385−97.0j	2.42+.603j	.484+.990j	426−206j	1.89+.924j	72
96	.519+2.15j	403−97.1j	2.30+.565j	.469+.984j	443−211j	1.84+.877j	96
120	.500+2.13j	418−97.5j	2.27+.532j	.456+.977j	453−214j	1.79+.836j	120
144	.487+2.12j	435−97.5j	2.23+.512j	.447+.973j	467−218j	1.74+.812j	144
	No. 0 B.&S. R = .525 ohms per mile			No. 0 B.&S. R = .525 ohms per mile			
72	.657+2.18j	398−120.0j	2.30+.695j	.573+1.038j	455−251j	1.68+.930j	72
96	.630+2.17j	415−120.5j	2.22+.648j	.556+1.028j	470−254j	1.63+.880j	96
120	.609+2.16j	428−121.0j	2.16+.611j	.542+1.020j	485−257j	1.61+.854j	120
144	.595+2.16j	439−121.0j	2.11+.585j	.530+1.011j	496−260j	1.56+.830j	144
	No. 1 B.&S. R = .665 ohms per mile			No. 1 B.&S. R = .665 ohms per mile			
72	.802+2.23j	418−150.0j	2.12+.760j	.673+1.095j	493−303j	1.468+.902j	72
96	.769+2.21j	432−150.0j	2.06+.716j	.655+1.084j	509−307j	1.440+.868j	96
120	.748+2.20j	445−150.5j	2.02+.682j	.638+1.070j	520−309j	1.420+.846j	120
144	.726+2.19j	456−151.0j	1.95+.654j	.625+1.064j	531−312j	1.398+.810j	144
	No. 2 B.&S. R = .835 ohms per mile			No. 2 B.&S. R = .835 ohms per mile			
72	.959+2.28j	436−182j	1.96+.820j	.780+1.160j	532−358j	1.290+.878j	72
96	.924+2.27j	452−184j	1.88+.770j	.759+1.150j	550−363j	1.268+.835j	96
120	.898+2.26j	465−185j	1.85+.739j	.741+1.135j	562−366j	1.250+.814j	120
144	.881+2.26j	476−186j	1.82+.712j	.729+1.127j	570−369j	1.235+.800j	144

* This table may be used for three-phase lines, only when a sufficient number of transpositions has been made to obtain balanced electrical conditions along the line. In no case must the table be used for single-phase lines. The table can be used, whether the plane in which the wires lie is vertical or horizontal.

TABLE IV

Values of cosh (x + jy) = a + jb and sinh (x + jy) = c + jd

y	x = 0.00				x = 0.02				x = 0.04				x = 0.06				x = 0.08				y
	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d	
0.00	1.000	0.00	0.00	.000	1.000	.000	.020	.000	1.001	.000	.040	.000	1.002	.000	.060	.000	1.003	.000	.080	.000	0.00
.02	1.000	0.00	0.00	.020	1.000	.000	.020	.020	1.000	.001	.040	.020	1.002	.001	.060	.020	1.003	.002	.080	.020	.02
.04	1.000	0.00	0.00	.040	1.000	.000	.020	.040	1.000	.002	.040	.040	1.001	.002	.060	.040	1.002	.003	.080	.040	.04
.06	.998	0.00	0.00	.060	.998	.001	.020	.060	.999	.002	.040	.060	1.000	.004	.060	.060	1.001	.005	.080	.060	.06
.08	.997	0.00	0.00	.080	.997	.002	.020	.080	.998	.003	.040	.080	.999	.005	.060	.080	1.000	.006	.080	.080	.08
.10	.995	0.00	0.00	.100	.995	.002	.020	.100	.996	.004	.040	.100	.997	.006	.060	.100	.998	.008	.080	.100	.10
.12	.993	0.00	0.00	.120	.993	.002	.020	.120	.994	.005	.040	.120	.995	.007	.060	.120	.996	.010	.080	.120	.12
.14	.990	0.00	0.00	.140	.990	.003	.020	.140	.991	.006	.040	.140	.992	.008	.059	.140	.993	.011	.079	.140	.14
.16	.987	0.00	0.00	.159	.987	.003	.020	.159	.988	.006	.040	.159	.989	.010	.059	.159	.990	.013	.079	.159	.16
.18	.984	0.00	0.00	.179	.984	.004	.020	.179	.985	.007	.039	.179	.986	.011	.059	.179	.987	.014	.079	.179	.18
.20	.980	0.00	0.00	.199	.980	.004	.020	.199	.981	.008	.039	.199	.982	.012	.059	.199	.983	.016	.078	.199	.20
.22	.976	0.00	0.00	.218	.976	.004	.020	.218	.977	.009	.039	.218	.978	.013	.059	.218	.979	.017	.078	.218	.22
.24	.971	0.00	0.00	.238	.971	.005	.019	.238	.972	.010	.039	.238	.973	.014	.058	.238	.974	.019	.078	.239	.24
.26	.966	0.00	0.00	.257	.966	.005	.019	.257	.967	.010	.039	.257	.968	.015	.058	.257	.969	.021	.077	.258	.26
.28	.961	0.00	0.00	.277	.961	.006	.019	.277	.962	.011	.038	.277	.963	.017	.058	.277	.964	.022	.077	.278	.28
.30	.955	0.00	0.00	.296	.955	.006	.019	.296	.956	.012	.038	.296	.957	.018	.057	.296	.958	.024	.076	.297	.30
.32	.949	0.00	0.00	.315	.949	.006	.019	.315	.950	.013	.038	.315	.951	.019	.057	.315	.952	.025	.076	.316	.32
.34	.943	0.00	0.00	.334	.943	.007	.019	.334	.944	.013	.038	.334	.945	.020	.057	.334	.946	.027	.076	.335	.34
.36	.936	0.00	0.00	.353	.936	.007	.019	.353	.937	.014	.037	.353	.938	.021	.056	.353	.939	.028	.075	.354	.36
.38	.929	0.00	0.00	.371	.929	.007	.019	.371	.930	.015	.037	.371	.931	.022	.056	.372	.932	.030	.074	.372	.38
.40	.921	0.00	0.00	.389	.921	.008	.018	.389	.922	.016	.037	.389	.923	.023	.055	.390	.924	.031	.074	.390	.40
.42	.913	0.00	0.00	.408	.913	.008	.018	.408	.914	.016	.037	.408	.915	.024	.055	.409	.916	.033	.073	.409	.42
.44	.905	0.00	0.00	.426	.905	.009	.018	.426	.906	.017	.036	.426	.907	.026	.054	.427	.908	.034	.073	.427	.44
.46	.896	0.00	0.00	.444	.896	.009	.018	.444	.897	.018	.036	.444	.898	.027	.054	.445	.899	.036	.072	.445	.46
.48	.887	0.00	0.00	.462	.887	.009	.018	.462	.888	.018	.035	.462	.889	.028	.053	.463	.890	.037	.071	.463	.48
.50	.878	0.00	0.00	.479	.878	.010	.018	.479	.879	.019	.035	.479	.880	.029	.053	.480	.881	.038	.070	.480	.50
.52	.868	0.00	0.00	.497	.868	.010	.017	.497	.869	.020	.035	.497	.870	.030	.052	.498	.871	.040	.070	.499	.52
.54	.858	0.00	0.00	.514	.858	.010	.017	.514	.858	.021	.034	.514	.859	.031	.052	.515	.860	.041	.069	.516	.54
.56	.847	0.00	0.00	.531	.847	.011	.017	.531	.848	.021	.034	.531	.849	.032	.051	.532	.850	.043	.068	.532	.56
.58	.836	0.00	0.00	.548	.836	.011	.017	.548	.837	.022	.033	.548	.838	.033	.050	.549	.839	.044	.067	.550	.58
.60	.825	0.00	0.00	.564	.825	.011	.016	.564	.826	.023	.033	.564	.827	.034	.049	.565	.828	.045	.066	.566	.60
.62	.814	0.00	0.00	.581	.814	.012	.016	.581	.815	.023	.033	.581	.816	.035	.049	.582	.817	.047	.065	.583	.62
.64	.802	0.00	0.00	.597	.802	.012	.016	.597	.803	.024	.032	.597	.803	.036	.048	.598	.805	.048	.064	.599	.64
.66	.790	0.00	0.00	.613	.790	.012	.016	.613	.791	.025	.032	.613	.792	.037	.047	.614	.793	.049	.063	.615	.66
.68	.777	0.00	0.00	.629	.777	.013	.016	.629	.778	.025	.031	.629	.779	.038	.047	.630	.780	.050	.062	.631	.68
.70	.765	0.00	0.00	.644	.765	.013	.015	.644	.766	.026	.031	.645	.767	.039	.046	.645	.767	.052	.061	.646	.70
.72	.752	0.00	0.00	.659	.752	.013	.015	.659	.753	.026	.030	.660	.754	.040	.045	.660	.754	.053	.060	.661	.72
.74	.738	0.00	0.00	.674	.738	.013	.015	.674	.739	.027	.030	.675	.740	.041	.044	.675	.740	.054	.059	.676	.74
.76	.725	0.00	0.00	.689	.725	.014	.014	.689	.725	.027	.029	.690	.727	.041	.044	.690	.727	.055	.058	.691	.76
.78	.711	0.00	0.00	.703	.711	.014	.014	.703	.712	.028	.028	.704	.712	.042	.043	.704	.713	.056	.057	.705	.78
.80	.697	0.00	0.00	.717	.697	.014	.014	.717	.698	.029	.028	.718	.698	.043	.042	.718	.699	.057	.056	.719	.80
.82	.682	0.00	0.00	.731	.682	.015	.014	.731	.683	.029	.027	.732	.684	.044	.041	.732	.684	.059	.055	.733	.82
.84	.667	0.00	0.00	.744	.667	.015	.013	.744	.667	.030	.027	.745	.668	.045	.040	.746	.669	.060	.053	.746	.84
.86	.652	0.00	0.00	.758	.652	.015	.013	.758	.653	.030	.026	.759	.653	.046	.039	.759	.654	.061	.052	.760	.86
.88	.637	0.00	0.00	.771	.637	.015	.013	.771	.637	.031	.025	.771	.638	.046	.038	.772	.639	.062	.051	.773	.88
.90	.622	0.00	0.00	.783	.622	.016	.012	.783	.622	.031	.025	.784	.623	.047	.037	.784	.624	.063	.050	.786	.90
.92	.606	0.00	0.00	.796	.606	.016	.012	.796	.606	.032	.024	.797	.607	.048	.036	.797	.608	.064	.049	.799	.92
.94	.590	0.00	0.00	.808	.590	.016	.012	.808	.590	.032	.024	.809	.591	.048	.035	.809	.592	.065	.047	.811	.94
.96	.574	0.00	0.00	.819	.574	.016	.011	.819	.574	.033	.023	.820	.575	.049	.034	.820	.576	.066	.046	.822	.96
.98	.557	0.00	0.00	.830	.557	.017	.011	.830	.557	.033	.022	.831	.558	.050	.033	.831	.559	.066	.044	.833	.98
1.00	.540	0.00	0.00	.841	.540	.017	.011	.841	.540	.034	.022	.842	.541	.050	.032	.843	.542	.067	.043	.844	1.00

TABLE V

Values of cosh (x + jy) = a + jb and sinh (x + jy) = c + jd

y	x = .10				x = .12				x = .14				x = .16				x = .18				y
	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d	
0.00	1.005	.000	.100	.000	1.007	.000	.120	.000	1.010	.000	.140	.000	1.013	.000	.161	.000	1.016	.000	.181	.000	0.00
.02	1.005	.002	.100	.020	1.007	.002	.120	.020	1.010	.003	.140	.020	1.013	.003	.161	.020	1.016	.003	.181	.020	.02
.04	1.004	.004	.100	.040	1.006	.005	.120	.040	1.009	.006	.140	.040	1.012	.006	.160	.040	1.015	.007	.180	.041	.04
.06	1.003	.006	.100	.060	1.005	.007	.120	.060	1.008	.008	.140	.061	1.011	.010	.160	.061	1.014	.011	.180	.061	.06
.08	1.002	.008	.100	.080	1.004	.010	.120	.080	1.007	.011	.140	.081	1.010	.013	.160	.081	1.013	.014	.180	.081	.08
.10	1.000	.010	.100	.100	1.002	.012	.120	.101	1.005	.014	.140	.101	1.008	.016	.160	.101	1.011	.018	.180	.102	.10
.12	.998	.012	.099	.121	1.000	.014	.120	.121	1.003	.017	.139	.121	1.006	.019	.160	.122	1.009	.022	.180	.122	.12
.14	.995	.014	.099	.141	.997	.017	.119	.141	1.000	.020	.139	.141	1.003	.022	.159	.142	1.006	.025	.179	.142	.14
.16	.992	.016	.099	.160	.994	.019	.119	.160	.997	.022	.138	.161	1.000	.026	.159	.161	1.003	.029	.178	.162	.16
.18	.989	.018	.098	.180	.991	.022	.119	.180	.994	.025	.138	.181	.997	.029	.158	.181	1.000	.032	.178	.182	.18
.20	.985	.020	.098	.200	.987	.024	.118	.200	.990	.028	.138	.201	.993	.032	.158	.202	.996	.036	.177	.202	.20
.22	.981	.022	.098	.219	.983	.026	.118	.220	.986	.031	.137	.220	.988	.035	.157	.221	.992	.039	.177	.221	.22
.24	.976	.024	.097	.239	.978	.029	.117	.240	.981	.033	.136	.240	.983	.038	.156	.241	.987	.043	.176	.242	.24
.26	.971	.026	.097	.258	.973	.031	.116	.259	.976	.036	.136	.259	.978	.041	.156	.260	.981	.046	.175	.261	.26
.28	.966	.028	.096	.278	.968	.033	.116	.279	.970	.039	.135	.280	.973	.044	.155	.281	.976	.050	.174	.281	.28
.30	.960	.030	.095	.297	.962	.036	.115	.298	.964	.041	.134	.299	.967	.048	.154	.299	.971	.053	.173	.300	.30
.32	.954	.031	.095	.316	.956	.038	.115	.317	.958	.044	.133	.318	.961	.051	.152	.319	.964	.057	.172	.320	.32
.34	.948	.033	.094	.335	.950	.040	.114	.336	.952	.047	.132	.337	.955	.054	.151	.338	.958	.060	.171	.339	.34
.36	.941	.035	.094	.354	.943	.042	.113	.355	.945	.049	.131	.356	.948	.057	.150	.357	.951	.064	.169	.358	.36
.38	.934	.037	.093	.373	.936	.045	.112	.374	.938	.052	.130	.375	.941	.059	.149	.376	.944	.067	.168	.377	.38
.40	.926	.039	.092	.391	.928	.047	.111	.392	.930	.055	.129	.393	.933	.062	.148	.394	.936	.070	.166	.395	.40
.42	.918	.041	.091	.410	.920	.049	.110	.411	.922	.057	.128	.412	.925	.065	.147	.413	.928	.073	.165	.414	.42
.44	.910	.043	.091	.428	.912	.051	.109	.429	.914	.060	.127	.430	.916	.068	.146	.431	.919	.077	.164	.432	.44
.46	.901	.044	.090	.446	.902	.053	.108	.447	.904	.062	.126	.448	.907	.071	.144	.449	.910	.080	.162	.451	.46
.48	.891	.046	.089	.464	.893	.055	.107	.465	.896	.065	.125	.467	.898	.074	.142	.468	.901	.083	.160	.469	.48
.50	.882	.048	.088	.482	.884	.057	.106	.483	.887	.067	.123	.484	.889	.077	.141	.486	.892	.087	.159	.487	.50
.52	.872	.050	.087	.500	.874	.060	.104	.501	.877	.069	.122	.502	.879	.080	.140	.503	.882	.090	.157	.505	.52
.54	.861	.051	.086	.517	.863	.062	.103	.518	.865	.072	.121	.519	.868	.082	.138	.520	.871	.093	.155	.522	.54
.56	.851	.053	.085	.534	.853	.064	.102	.535	.855	.074	.119	.536	.858	.085	.136	.537	.861	.096	.153	.540	.56
.58	.840	.055	.084	.551	.842	.066	.100	.552	.844	.077	.118	.553	.847	.087	.134	.555	.850	.099	.151	.557	.58
.60	.829	.056	.083	.567	.831	.068	.099	.569	.833	.079	.116	.570	.836	.091	.132	.571	.838	.102	.149	.573	.60
.62	.818	.058	.081	.584	.820	.070	.098	.585	.822	.081	.114	.587	.824	.093	.130	.588	.827	.105	.147	.590	.62
.64	.806	.060	.080	.600	.808	.072	.097	.602	.810	.084	.112	.603	.812	.096	.128	.604	.815	.108	.145	.607	.64
.66	.794	.061	.079	.616	.796	.074	.096	.618	.798	.086	.110	.619	.800	.098	.127	.620	.803	.111	.143	.623	.66
.68	.781	.063	.078	.632	.783	.076	.094	.634	.785	.088	.109	.635	.787	.101	.125	.636	.790	.114	.140	.639	.68
.70	.769	.064	.076	.647	.771	.078	.092	.649	.773	.090	.108	.650	.775	.103	.123	.652	.777	.116	.138	.654	.70
.72	.756	.066	.075	.662	.758	.079	.091	.664	.759	.092	.106	.665	.762	.105	.121	.667	.764	.118	.136	.670	.72
.74	.742	.067	.074	.677	.743	.081	.089	.679	.745	.094	.104	.681	.748	.108	.119	.683	.750	.122	.134	.685	.74
.76	.729	.069	.072	.692	.730	.083	.087	.694	.732	.096	.102	.696	.734	.111	.117	.698	.737	.124	.131	.700	.76
.78	.714	.070	.071	.706	.716	.085	.086	.708	.718	.099	.100	.710	.720	.113	.114	.712	.722	.127	.128	.714	.78
.80	.700	.072	.070	.720	.702	.086	.084	.722	.704	.101	.098	.724	.706	.115	.112	.726	.708	.130	.126	.729	.80
.82	.685	.073	.068	.734	.687	.088	.082	.736	.689	.102	.096	.738	.691	.118	.110	.740	.693	.132	.123	.743	.82
.84	.670	.074	.067	.747	.672	.090	.080	.749	.674	.104	.094	.751	.676	.120	.107	.753	.679	.134	.120	.756	.84
.86	.656	.076	.065	.761	.657	.091	.079	.763	.658	.106	.092	.765	.661	.122	.105	.768	.663	.137	.118	.770	.86
.88	.640	.077	.064	.775	.642	.093	.077	.777	.643	.108	.090	.779	.645	.124	.102	.781	.647	.139	.115	.783	.88
.90	.625	.078	.062	.787	.626	.094	.075	.789	.628	.110	.087	.791	.630	.126	.100	.793	.632	.141	.112	.796	.90
.92	.609	.080	.061	.800	.610	.096	.073	.802	.612	.112	.086	.804	.614	.128	.098	.806	.616	.143	.109	.809	.92
.94	.593	.081	.059	.812	.594	.097	.071	.814	.596	.113	.083	.816	.598	.130	.095	.818	.600	.146	.108	.821	.94
.96	.577	.082	.057	.823	.578	.098	.069	.825	.580	.115	.081	.827	.581	.132	.092	.829	.583	.148	.104	.832	.96
.98	.560	.083	.056	.834	.561	.100	.067	.836	.563	.116	.076	.838	.564	.134	.090	.841	.566	.150	.101	.843	.98
1.00	.543	.084	.054	.846	.544	.101	.065	.847	.546	.118	.076	.849	.547	.135	.087	.852	.549	.152	.098	.855	1.00

TABLE VI

Values of cosh (x + jy) = a + jb and sinh (x + jy) = c + jd

y	x = .20				x = .22				x = .24				x = .26				x = .28				y
	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d	
0.00	1.020	.000	.201	.000	1.024	.000	.221	.000	1.029	.000	.242	.000	1.034	.000	.263	.000	1.039	.000	.284	.000	0.00
.02	1.020	.004	.201	.020	1.024	.004	.221	.020	1.029	.005	.242	.021	1.034	.005	.263	.021	1.039	.006	.284	.021	.02
.04	1.019	.008	.201	.041	1.023	.009	.221	.041	1.028	.010	.242	.041	1.033	.010	.263	.041	1.038	.011	.284	.042	.04
.06	1.018	.012	.201	.061	1.022	.013	.220	.061	1.027	.014	.241	.062	1.032	.016	.262	.062	1.037	.017	.283	.062	.06
.08	1.017	.016	.200	.082	1.021	.018	.220	.082	1.026	.019	.241	.082	1.031	.021	.262	.083	1.036	.023	.283	.083	.08
.10	1.015	.020	.200	.102	1.019	.022	.220	.102	1.024	.024	.240	.103	1.029	.026	.261	.103	1.034	.028	.282	.104	.10
.12	1.013	.024	.200	.122	1.017	.027	.220	.123	1.022	.029	.240	.123	1.027	.031	.261	.124	1.032	.034	.281	.124	.12
.14	1.010	.028	.199	.142	1.014	.031	.220	.143	1.019	.034	.239	.144	1.024	.037	.260	.144	1.029	.040	.280	.145	.14
.16	1.007	.032	.199	.162	1.011	.036	.219	.163	1.015	.038	.239	.164	1.021	.043	.259	.164	1.026	.045	.279	.165	.16
.18	1.004	.036	.198	.183	1.008	.040	.218	.183	1.012	.043	.238	.184	1.017	.047	.258	.185	1.023	.051	.278	.186	.18
.20	1.000	.040	.197	.203	1.004	.044	.217	.204	1.008	.048	.237	.204	1.013	.052	.257	.205	1.019	.057	.277	.206	.20
.22	.996	.044	.197	.223	1.000	.048	.216	.223	1.004	.053	.236	.224	1.009	.056	.256	.225	1.014	.062	.276	.226	.22
.24	.990	.048	.196	.243	.995	.052	.215	.244	.999	.058	.235	.245	1.004	.064	.255	.246	1.009	.067	.275	.247	.24
.26	.985	.052	.195	.262	.989	.057	.214	.263	.994	.062	.234	.264	.999	.067	.254	.266	1.004	.073	.274	.267	.26
.28	.980	.056	.194	.282	.984	.061	.213	.284	.989	.066	.232	.284	.994	.072	.252	.286	.999	.078	.272	.286	.28
.30	.974	.059	.192	.301	.978	.066	.212	.303	.983	.071	.231	.305	.987	.077	.251	.306	.993	.084	.270	.307	.30
.32	.968	.063	.191	.321	.972	.070	.211	.322	.976	.076	.230	.324	.981	.083	.250	.325	.986	.089	.269	.327	.32
.34	.962	.067	.190	.341	.966	.074	.209	.342	.970	.081	.228	.344	.975	.088	.248	.345	.980	.094	.267	.347	.34
.36	.955	.071	.188	.359	.959	.078	.207	.361	.963	.085	.226	.363	.968	.093	.246	.364	.973	.100	.266	.367	.36
.38	.948	.075	.187	.378	.951	.082	.206	.380	.955	.090	.225	.382	.960	.097	.244	.384	.965	.105	.263	.386	.38
.40	.939	.078	.185	.397	.943	.086	.205	.398	.948	.094	.223	.400	.952	.102	.242	.403	.957	.110	.261	.404	.40
.42	.931	.082	.184	.416	.935	.090	.203	.418	.939	.099	.221	.420	.944	.107	.240	.422	.949	.116	.259	.424	.42
.44	.923	.086	.182	.435	.927	.094	.201	.436	.931	.103	.219	.438	.935	.112	.238	.440	.941	.120	.256	.443	.44
.46	.914	.089	.180	.453	.918	.098	.199	.455	.922	.107	.216	.457	.926	.116	.235	.459	.931	.125	.254	.461	.46
.48	.905	.093	.178	.471	.909	.102	.197	.473	.913	.112	.214	.476	.917	.121	.233	.477	.922	.131	.251	.480	.48
.50	.895	.096	.177	.489	.899	.106	.195	.492	.903	.116	.212	.493	.907	.126	.230	.496	.912	.136	.249	.498	.50
.52	.885	.100	.176	.507	.889	.110	.193	.509	.893	.120	.210	.511	.897	.130	.228	.513	.902	.141	.246	.517	.52
.54	.874	.103	.172	.524	.878	.114	.190	.526	.882	.125	.207	.528	.886	.135	.225	.530	.891	.146	.243	.534	.54
.56	.864	.106	.170	.542	.868	.118	.188	.544	.872	.128	.205	.546	.876	.140	.223	.549	.880	.150	.239	.552	.56
.58	.853	.110	.168	.559	.856	.122	.185	.561	.860	.132	.202	.563	.864	.144	.219	.566	.870	.155	.236	.570	.58
.60	.842	.114	.166	.576	.846	.126	.183	.578	.849	.137	.199	.580	.851	.148	.216	.583	.858	.160	.233	.586	.60
.62	.830	.117	.164	.593	.834	.129	.181	.595	.838	.141	.197	.598	.842	.152	.213	.601	.846	.164	.231	.604	.62
.64	.818	.120	.162	.609	.821	.132	.178	.611	.826	.144	.194	.614	.829	.157	.210	.617	.833	.169	.227	.620	.64
.66	.806	.123	.159	.626	.809	.136	.175	.628	.813	.148	.191	.631	.817	.161	.207	.634	.821	.174	.224	.637	.66
.68	.793	.126	.156	.642	.796	.140	.173	.644	.799	.152	.188	.647	.803	.166	.204	.650	.808	.178	.220	.654	.68
.70	.780	.130	.154	.657	.784	.143	.170	.660	.787	.156	.185	.663	.791	.170	.201	.666	.795	.183	.217	.669	.70
.72	.767	.133	.152	.672	.770	.146	.167	.676	.773	.160	.182	.678	.778	.173	.198	.681	.782	.186	.213	.686	.72
.74	.753	.136	.149	.687	.756	.150	.164	.690	.760	.163	.179	.693	.763	.177	.194	.696	.768	.190	.209	.701	.74
.76	.740	.139	.146	.703	.743	.153	.161	.706	.746	.166	.175	.709	.750	.181	.191	.712	.754	.195	.205	.716	.76
.78	.726	.141	.143	.717	.728	.156	.158	.720	.731	.170	.172	.723	.735	.185	.187	.726	.739	.200	.201	.731	.78
.80	.711	.144	.140	.732	.714	.159	.155	.734	.717	.172	.169	.738	.721	.188	.183	.741	.724	.203	.197	.746	.80
.82	.696	.147	.137	.746	.699	.162	.152	.749	.702	.176	.165	.752	.705	.192	.179	.755	.708	.207	.193	.760	.82
.84	.680	.150	.134	.760	.683	.165	.148	.762	.686	.180	.161	.766	.690	.196	.176	.770	.693	.210	.189	.773	.84
.86	.666	.153	.131	.773	.668	.168	.144	.776	.671	.183	.158	.780	.674	.199	.171	.783	.677	.214	.186	.788	.86
.88	.650	.155	.128	.786	.653	.171	.141	.790	.655	.186	.154	.794	.659	.203	.167	.798	.663	.218	.180	.802	.88
.90	.634	.158	.125	.799	.637	.174	.138	.802	.640	.190	.151	.806	.642	.206	.162	.810	.647	.222	.176	.814	.90
.92	.616	.160	.122	.812	.621	.177	.134	.816	.623	.192	.147	.819	.627	.209	.159	.822	.630	.225	.171	.827	.92
.94	.602	.162	.119	.824	.604	.179	.131	.828	.607	.196	.143	.832	.610	.212	.155	.835	.613	.228	.167	.840	.94
.96	.586	.164	.116	.836	.588	.182	.127	.839	.591	.197	.139	.843	.594	.215	.151	.847	.597	.231	.163	.851	.96
.98	.568	.167	.112	.847	.571	.184	.123	.850	.573	.200	.135	.854	.576	.218	.146	.858	.579	.236	.158	.863	.98
1.00	.551	.169	.109	.858	.553	.186	.120	.861	.556	.204	.131	.866	.559	.221	.142	.870	.561	.238	.153	.874	1.00

TABLE VII

Values of cosh (x + jy) = a + jb and sinh (x + jy) = c + jd

y	x = .30				x = .32				x = .34				x = .36				x = .38				y
	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d	
0.00	1.046	.000	.304	.000	1.052	.000	.326	.000	1.058	.000	.347	.000	1.065	.000	.368	.000	1.073	.000	.389	.000	0.00
.02	1.046	.006	.304	.021	1.051	.006	.326	.021	1.058	.007	.346	.021	1.065	.007	.367	.021	1.073	.008	.389	.021	.02
.04	1.044	.012	.304	.042	1.051	.013	.325	.042	1.057	.014	.346	.042	1.064	.015	.367	.043	1.072	.016	.389	.043	.04
.06	1.043	.018	.303	.063	1.050	.019	.324	.063	1.056	.021	.345	.064	1.063	.022	.366	.064	1.071	.023	.388	.064	.06
.08	1.042	.024	.303	.084	1.048	.026	.324	.084	1.055	.028	.345	.085	1.062	.029	.366	.085	1.070	.031	.387	.086	.08
.10	1.040	.030	.303	.104	1.046	.032	.324	.105	1.053	.035	.344	.106	1.060	.037	.366	.107	1.068	.039	.387	.107	.10
.12	1.038	.036	.302	.125	1.044	.039	.323	.126	1.051	.041	.344	.127	1.058	.044	.365	.128	1.066	.047	.386	.129	.12
.14	1.035	.042	.301	.146	1.041	.045	.322	.147	1.048	.048	.343	.148	1.055	.051	.364	.149	1.062	.054	.385	.150	.14
.16	1.032	.048	.300	.166	1.038	.052	.321	.167	1.045	.055	.342	.168	1.052	.059	.363	.170	1.059	.062	.384	.171	.16
.18	1.028	.055	.299	.187	1.035	.058	.320	.188	1.042	.062	.341	.189	1.048	.066	.362	.191	1.056	.070	.382	.192	.18
.20	1.024	.061	.298	.208	1.031	.065	.319	.209	1.037	.069	.340	.211	1.044	.074	.360	.212	1.052	.078	.380	.214	.20
.22	1.019	.066	.297	.228	1.026	.071	.317	.229	1.033	.076	.338	.231	1.039	.080	.358	.233	1.047	.085	.378	.234	.22
.24	1.014	.072	.296	.249	1.021	.077	.316	.251	1.028	.082	.336	.253	1.034	.087	.356	.254	1.042	.092	.377	.255	.24
.26	1.009	.078	.294	.269	1.016	.083	.314	.271	1.022	.089	.334	.273	1.029	.095	.354	.274	1.037	.100	.376	.276	.26
.28	1.004	.084	.293	.289	1.011	.090	.313	.291	1.017	.096	.332	.294	1.024	.102	.353	.295	1.031	.108	.373	.297	.28
.30	.999	.090	.291	.309	1.004	.096	.310	.311	1.011	.102	.331	.313	1.017	.108	.351	.314	1.025	.115	.371	.318	.30
.32	.993	.096	.289	.329	.998	.102	.308	.331	1.004	.109	.329	.333	1.011	.116	.349	.334	1.018	.122	.369	.338	.32
.34	.987	.102	.287	.349	.992	.108	.306	.351	.998	.116	.327	.353	1.004	.123	.347	.355	1.012	.130	.366	.358	.34
.36	.980	.107	.285	.369	.984	.115	.304	.371	.991	.122	.324	.373	.997	.130	.343	.375	1.004	.138	.363	.379	.36
.38	.972	.113	.283	.388	.977	.121	.302	.390	.983	.128	.321	.393	.990	.134	.341	.395	.997	.146	.361	.398	.38
.40	.963	.118	.280	.407	.968	.126	.300	.409	.975	.134	.319	.412	.981	.143	.339	.414	.988	.151	.358	.417	.40
.42	.954	.124	.277	.426	.960	.133	.297	.428	.966	.141	.317	.431	.973	.150	.336	.435	.980	.158	.355	.438	.42
.44	.945	.130	.276	.445	.952	.138	.294	.447	.958	.148	.313	.450	.964	.156	.333	.453	.971	.166	.352	.457	.44
.46	.936	.135	.273	.464	.942	.144	.291	.466	.948	.154	.310	.469	.955	.163	.329	.473	.961	.172	.348	.476	.46
.48	.928	.140	.270	.483	.933	.150	.288	.485	.939	.160	.307	.488	.945	.170	.325	.492	.952	.179	.344	.495	.48
.50	.917	.146	.267	.501	.923	.156	.285	.504	.929	.166	.304	.507	.936	.176	.322	.510	.942	.186	.341	.514	.50
.52	.908	.151	.263	.519	.913	.162	.282	.522	.919	.172	.300	.526	.925	.182	.319	.530	.932	.193	.338	.533	.52
.54	.898	.156	.260	.537	.901	.167	.278	.540	.907	.178	.296	.543	.913	.189	.315	.548	.920	.200	.334	.552	.54
.56	.887	.161	.257	.555	.891	.172	.276	.558	.896	.184	.293	.561	.902	.195	.311	.566	.909	.206	.329	.570	.56
.58	.876	.166	.254	.573	.879	.178	.272	.576	.885	.190	.290	.580	.891	.201	.307	.584	.897	.212	.326	.588	.58
.60	.863	.172	.251	.590	.867	.183	.268	.593	.873	.195	.286	.597	.879	.207	.303	.601	.886	.219	.321	.606	.60
.62	.851	.177	.248	.607	.856	.189	.264	.611	.862	.201	.282	.615	.867	.214	.299	.619	.874	.226	.317	.623	.62
.64	.839	.181	.244	.624	.843	.194	.260	.628	.849	.207	.279	.633	.854	.219	.295	.636	.861	.233	.312	.641	.64
.66	.826	.186	.240	.641	.831	.199	.256	.645	.836	.212	.274	.649	.842	.225	.290	.653	.848	.239	.307	.658	.66
.68	.813	.191	.236	.657	.817	.204	.252	.661	.822	.218	.269	.666	.828	.231	.285	.670	.834	.246	.302	.675	.68
.70	.800	.196	.233	.673	.806	.209	.249	.677	.810	.223	.265	.682	.815	.237	.283	.686	.821	.251	.297	.691	.70
.72	.787	.200	.229	.689	.791	.214	.244	.693	.796	.228	.261	.698	.801	.242	.276	.702	.807	.256	.293	.707	.72
.74	.773	.206	.224	.705	.776	.219	.240	.709	.781	.234	.255	.714	.786	.248	.271	.718	.791	.262	.288	.723	.74
.76	.758	.210	.221	.720	.762	.224	.236	.724	.767	.239	.251	.729	.772	.254	.267	.734	.777	.268	.282	.739	.76
.78	.744	.214	.216	.736	.746	.229	.231	.739	.753	.243	.246	.744	.758	.259	.261	.749	.763	.274	.276	.754	.78
.80	.728	.218	.212	.750	.733	.233	.226	.754	.738	.248	.242	.759	.743	.264	.256	.764	.748	.279	.271	.768	.80
.82	.714	.222	.207	.764	.717	.237	.222	.769	.722	.254	.236	.774	.727	.269	.251	.779	.732	.284	.265	.784	.82
.84	.697	.227	.203	.778	.701	.242	.217	.783	.706	.257	.231	.787	.711	.274	.245	.793	.716	.289	.259	.798	.84
.86	.682	.230	.198	.792	.686	.246	.212	.797	.690	.262	.226	.802	.695	.279	.240	.808	.700	.296	.254	.814	.86
.88	.666	.235	.194	.806	.670	.250	.207	.811	.674	.266	.220	.817	.679	.284	.234	.821	.684	.300	.248	.827	.88
.90	.650	.238	.189	.819	.654	.255	.202	.823	.658	.271	.215	.829	.663	.288	.229	.834	.668	.306	.242	.840	.90
.92	.634	.242	.184	.832	.637	.259	.197	.837	.641	.275	.210	.843	.643	.293	.223	.848	.651	.309	.236	.854	.92
.94	.617	.246	.179	.846	.620	.262	.192	.849	.626	.280	.204	.854	.628	.297	.216	.861	.634	.314	.229	.867	.94
.96	.600	.249	.175	.858	.604	.266	.186	.861	.608	.284	.199	.867	.612	.300	.211	.873	.617	.319	.223	.879	.96
.98	.582	.252	.169	.868	.586	.270	.181	.873	.589	.287	.193	.879	.593	.305	.205	.884	.598	.323	.216	.891	.98
1.00	.565	.256	.164	.880	.568	.274	.176	.884	.572	.291	.187	.890	.576	.310	.199	.896	.580	.327	.210	.903	1.00

TABLE VIII

Values of cosh (x + jy) = a + jb and sinh (x + jy) = c + jd

y	x = .40				x = .42				x = .44				x = .46				x = .48				y
	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d	
0.00	1.081	.000	.411	.000	1.089	.000	.433	.000	1.098	.000	.454	.000	1.108	.000	.476	.000	1.117	.000	.499	.000	0.00
.02	1.081	.008	.411	.022	1.089	.009	.432	.022	1.098	.009	.454	.022	1.108	.009	.476	.022	1.117	.010	.498	.022	.02
.04	1.080	.016	.410	.043	1.088	.017	.431	.044	1.097	.018	.453	.044	1.107	.019	.475	.044	1.116	.020	.498	.045	.04
.06	1.079	.025	.410	.065	1.087	.026	.431	.065	1.096	.027	.452	.066	1.106	.029	.474	.066	1.115	.030	.498	.067	.06
.08	1.078	.033	.409	.086	1.086	.035	.430	.087	1.095	.036	.452	.088	1.105	.038	.474	.089	1.113	.040	.497	.089	.08
.10	1.076	.041	.409	.108	1.084	.043	.430	.109	1.093	.045	.451	.110	1.102	.048	.473	.112	1.110	.050	.496	.112	.10
.12	1.073	.049	.408	.129	1.082	.052	.429	.130	1.091	.054	.450	.132	1.101	.057	.472	.133	1.108	.060	.495	.134	.12
.14	1.070	.057	.407	.151	1.079	.061	.428	.152	1.088	.063	.449	.154	1.098	.067	.471	.156	1.105	.070	.494	.156	.14
.16	1.067	.065	.406	.172	1.076	.069	.427	.174	1.085	.072	.448	.176	1.095	.076	.470	.177	1.102	.079	.493	.178	.16
.18	1.064	.074	.404	.194	1.072	.078	.426	.196	1.081	.081	.446	.197	1.091	.085	.468	.198	1.098	.089	.491	.200	.18
.20	1.059	.082	.403	.215	1.068	.086	.424	.217	1.076	.090	.444	.219	1.086	.096	.466	.220	1.093	.099	.489	.222	.20
.22	1.055	.090	.401	.236	1.063	.094	.422	.238	1.072	.099	.442	.241	1.081	.104	.464	.242	1.089	.109	.487	.244	.22
.24	1.050	.098	.399	.257	1.058	.103	.420	.259	1.067	.108	.440	.262	1.076	.113	.462	.264	1.083	.118	.484	.266	.24
.26	1.044	.106	.397	.278	1.053	.111	.418	.281	1.062	.117	.438	.283	1.071	.123	.460	.284	1.077	.129	.482	.287	.26
.28	1.039	.114	.395	.299	1.047	.120	.415	.302	1.055	.126	.435	.304	1.066	.132	.457	.306	1.071	.138	.479	.309	.28
.30	1.033	.121	.392	.319	1.040	.128	.413	.322	1.049	.134	.432	.325	1.058	.141	.454	.327	1.063	.147	.476	.330	.30
.32	1.026	.129	.389	.340	1.034	.136	.409	.343	1.042	.143	.429	.346	1.051	.150	.451	.348	1.059	.157	.473	.351	.32
.34	1.019	.138	.386	.360	1.027	.144	.406	.364	1.036	.152	.426	.367	1.044	.159	.448	.370	1.052	.166	.470	.372	.34
.36	1.012	.145	.383	.381	1.020	.152	.403	.384	1.029	.160	.423	.387	1.037	.168	.445	.390	1.044	.176	.467	.393	.36
.38	1.004	.152	.381	.401	1.012	.160	.401	.404	1.021	.169	.420	.407	1.029	.177	.442	.410	1.037	.186	.464	.414	.38
.40	.996	.160	.378	.421	1.003	.168	.398	.424	1.012	.176	.417	.427	1.020	.185	.439	.430	1.029	.194	.460	.435	.40
.42	.987	.168	.375	.441	.995	.176	.395	.444	1.003	.185	.414	.447	1.011	.194	.435	.450	1.019	.203	.456	.455	.42
.44	.978	.176	.372	.460	.986	.184	.392	.464	.994	.193	.411	.467	1.002	.203	.431	.470	1.010	.212	.452	.475	.44
.46	.968	.182	.368	.479	.976	.192	.388	.484	.986	.201	.407	.487	.992	.211	.427	.490	1.000	.221	.448	.496	.46
.48	.959	.190	.364	.498	.966	.200	.384	.503	.978	.209	.403	.507	.983	.220	.423	.510	.991	.230	.443	.516	.48
.50	.949	.197	.360	.518	.957	.207	.379	.522	.966	.217	.399	.526	.973	.228	.418	.530	.981	.239	.436	.535	.50
.52	.938	.204	.356	.537	.946	.215	.376	.541	.954	.225	.395	.545	.962	.236	.413	.548	.970	.249	.433	.554	.52
.54	.927	.213	.352	.555	.934	.222	.371	.560	.942	.233	.390	.564	.950	.244	.408	.568	.958	.257	.428	.573	.54
.56	.916	.219	.348	.574	.923	.230	.367	.578	.930	.241	.385	.582	.938	.252	.403	.587	.946	.265	.423	.592	.56
.58	.904	.225	.344	.592	.911	.236	.362	.595	.918	.249	.380	.601	.926	.260	.398	.606	.934	.274	.416	.611	.58
.60	.892	.232	.339	.610	.899	.244	.357	.614	.906	.256	.374	.619	.914	.268	.392	.624	.922	.282	.412	.630	.60
.62	.880	.239	.334	.628	.887	.250	.352	.633	.892	.264	.369	.637	.901	.276	.387	.642	.909	.290	.406	.648	.62
.64	.867	.245	.329	.645	.874	.256	.347	.650	.880	.271	.364	.655	.888	.284	.382	.660	.896	.298	.400	.666	.64
.66	.854	.251	.324	.662	.861	.263	.342	.667	.867	.278	.359	.672	.874	.292	.376	.678	.882	.306	.394	.684	.66
.68	.840	.258	.319	.679	.847	.272	.337	.684	.854	.285	.353	.690	.861	.300	.370	.696	.869	.314	.388	.702	.68
.70	.827	.265	.314	.696	.833	.278	.331	.701	.840	.292	.347	.707	.847	.307	.364	.713	.856	.321	.382	.719	.70
.72	.813	.270	.309	.712	.819	.285	.325	.717	.826	.299	.341	.724	.833	.314	.358	.730	.841	.329	.376	.736	.72
.74	.798	.276	.304	.728	.804	.292	.319	.733	.811	.306	.335	.740	.818	.321	.352	.746	.825	.337	.369	.753	.74
.76	.784	.283	.298	.744	.790	.298	.313	.749	.795	.313	.329	.756	.803	.328	.346	.762	.809	.344	.362	.769	.76
.78	.769	.289	.292	.760	.775	.304	.307	.766	.781	.319	.322	.772	.787	.336	.339	.778	.794	.351	.355	.785	.78
.80	.753	.295	.286	.775	.759	.310	.301	.781	.766	.325	.316	.788	.772	.341	.332	.794	.779	.356	.348	.801	.80
.82	.737	.300	.280	.790	.743	.315	.295	.796	.750	.331	.309	.803	.756	.348	.325	.809	.763	.365	.341	.816	.82
.84	.721	.306	.274	.805	.727	.322	.289	.811	.734	.337	.302	.818	.740	.354	.316	.824	.747	.372	.334	.831	.84
.86	.705	.311	.268	.819	.710	.328	.283	.825	.717	.342	.295	.832	.723	.360	.311	.838	.729	.378	.326	.846	.86
.88	.689	.316	.262	.833	.694	.334	.275	.839	.700	.349	.288	.846	.706	.366	.304	.853	.712	.384	.316	.861	.88
.90	.672	.322	.256	.847	.676	.339	.269	.853	.683	.356	.281	.860	.689	.372	.296	.867	.695	.390	.310	.876	.90
.92	.655	.326	.249	.860	.660	.343	.262	.866	.666	.361	.274	.873	.672	.378	.289	.880	.678	.396	.302	.889	.92
.94	.638	.332	.243	.873	.643	.349	.255	.879	.648	.366	.267	.886	.653	.384	.281	.893	.659	.402	.294	.902	.94
.96	.621	.334	.236	.885	.625	.354	.248	.892	.630	.371	.260	.899	.635	.390	.273	.906	.641	.408	.286	.915	.96
.98	.602	.341	.229	.897	.607	.359	.241	.904	.612	.376	.253	.912	.617	.396	.266	.919	.623	.414	.278	.928	.98
1.00	.584	.346	.222	.910	.588	.364	.234	.916	.593	.381	.245	.924	.598	.400	.257	.931	.604	.420	.270	.940	1.00

TABLE IX

Values of cosh (x + jy) = a + jb and sinh (x + jy) = c + jd

y	x = .50				x = .52				x = .54				x = .56				x = .58				y
	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d	
0.00	1.128	.000	.521	.000	1.138	.000	.544	.000	1.149	.000	.567	.000	1.161	.000	.590	.000	1.173	.000	.613	.000	0.00
.02	1.127	.010	.521	.022	1.137	.011	.544	.023	1.149	.011	.567	.023	1.161	.012	.590	.023	1.173	.012	.613	.023	.02
.04	1.126	.021	.521	.045	1.136	.022	.543	.045	1.148	.023	.566	.046	1.160	.024	.589	.046	1.172	.024	.612	.047	.04
.06	1.126	.031	.520	.068	1.135	.033	.542	.068	1.146	.034	.566	.069	1.159	.035	.588	.070	1.171	.037	.611	.070	.06
.08	1.124	.043	.519	.090	1.133	.043	.542	.091	1.144	.045	.564	.092	1.158	.047	.587	.093	1.170	.049	.611	.094	.08
.10	1.122	.052	.518	.112	1.131	.054	.541	.114	1.142	.057	.563	.115	1.157	.059	.586	.116	1.169	.061	.610	.117	.10
.12	1.119	.062	.517	.136	1.129	.065	.540	.136	1.139	.068	.562	.138	1.155	.071	.585	.139	1.167	.074	.608	.141	.12
.14	1.116	.073	.516	.158	1.126	.076	.539	.159	1.136	.079	.560	.161	1.152	.083	.584	.163	1.164	.086	.606	.164	.14
.16	1.113	.082	.515	.180	1.123	.086	.537	.181	1.133	.090	.558	.183	1.149	.094	.582	.185	1.160	.098	.604	.187	.16
.18	1.109	.093	.513	.203	1.118	.097	.535	.204	1.129	.102	.556	.206	1.145	.106	.580	.208	1.156	.110	.602	.211	.18
.20	1.106	.103	.511	.226	1.114	.106	.533	.227	1.125	.113	.555	.229	1.140	.118	.578	.231	1.150	.122	.600	.234	.20
.22	1.100	.113	.509	.246	1.109	.119	.531	.249	1.120	.124	.552	.250	1.135	.129	.576	.253	1.148	.136	.598	.256	.22
.24	1.095	.123	.507	.268	1.104	.130	.529	.271	1.115	.135	.549	.273	1.129	.141	.573	.276	1.140	.148	.595	.279	.24
.26	1.089	.134	.504	.290	1.099	.141	.526	.293	1.110	.146	.546	.295	1.123	.153	.570	.299	1.134	.158	.592	.302	.26
.28	1.083	.144	.501	.312	1.093	.151	.523	.315	1.104	.157	.543	.318	1.117	.163	.567	.323	1.127	.170	.589	.326	.28
.30	1.077	.154	.498	.333	1.087	.161	.520	.337	1.098	.168	.540	.340	1.110	.174	.564	.344	1.120	.182	.585	.348	.30
.32	1.070	.164	.495	.355	1.080	.172	.516	.359	1.092	.179	.537	.362	1.102	.186	.560	.366	1.113	.194	.581	.370	.32
.34	1.063	.174	.492	.376	1.072	.182	.512	.380	1.083	.190	.534	.384	1.095	.196	.556	.388	1.106	.206	.577	.392	.34
.36	1.055	.184	.488	.397	1.064	.192	.508	.401	1.075	.200	.530	.405	1.087	.208	.552	.410	1.098	.217	.573	.414	.36
.38	1.047	.194	.484	.418	1.056	.202	.504	.422	1.067	.210	.526	.426	1.079	.219	.548	.431	1.089	.228	.569	.436	.38
.40	1.039	.203	.480	.439	1.048	.212	.500	.443	1.058	.220	.521	.446	1.070	.230	.543	.452	1.081	.239	.565	.457	.40
.42	1.030	.213	.476	.460	1.039	.222	.496	.464	1.048	.231	.516	.468	1.061	.241	.538	.474	1.072	.250	.560	.479	.42
.44	1.021	.222	.472	.481	1.029	.232	.492	.485	1.040	.241	.511	.490	1.051	.252	.533	.495	1.063	.261	.555	.500	.44
.46	1.011	.232	.467	.501	1.019	.242	.488	.505	1.030	.251	.506	.510	1.041	.263	.528	.516	1.052	.272	.550	.521	.46
.48	1.001	.241	.462	.521	1.009	.251	.483	.526	1.019	.261	.501	.531	1.031	.273	.523	.537	1.042	.283	.545	.542	.48
.50	.990	.250	.457	.541	.997	.260	.478	.546	1.008	.271	.496	.550	1.020	.283	.518	.557	1.031	.294	.539	.563	.50
.52	.979	.259	.452	.561	.986	.270	.473	.565	.997	.281	.491	.571	1.009	.293	.512	.577	1.019	.305	.532	.583	.52
.54	.967	.268	.447	.580	.975	.279	.467	.585	.985	.291	.485	.590	.997	.303	.506	.597	1.007	.316	.526	.603	.54
.56	.955	.277	.442	.599	.963	.289	.461	.604	.973	.301	.479	.610	.983	.313	.500	.617	.995	.327	.519	.623	.56
.58	.943	.286	.436	.618	.951	.298	.456	.623	.961	.310	.473	.630	.973	.323	.494	.637	.983	.337	.512	.643	.58
.60	.931	.294	.430	.637	.939	.307	.448	.642	.948	.319	.467	.648	.960	.333	.487	.656	.970	.347	.505	.663	.60
.62	.919	.303	.424	.656	.926	.316	.443	.661	.935	.329	.461	.667	.947	.343	.480	.676	.957	.357	.498	.683	.62
.64	.906	.311	.418	.674	.912	.325	.436	.679	.922	.338	.455	.686	.933	.353	.473	.694	.943	.367	.491	.701	.64
.66	.892	.320	.412	.692	.898	.334	.430	.697	.908	.347	.447	.704	.919	.362	.466	.713	.929	.377	.484	.720	.66
.68	.877	.328	.406	.709	.884	.343	.424	.715	.894	.356	.440	.722	.905	.371	.459	.731	.914	.386	.477	.738	.68
.70	.862	.336	.399	.726	.870	.351	.417	.733	.879	.365	.433	.739	.890	.380	.451	.749	.899	.395	.469	.756	.70
.72	.847	.344	.392	.743	.856	.359	.410	.750	.864	.374	.426	.757	.875	.389	.443	.766	.884	.404	.461	.774	.72
.74	.833	.352	.385	.760	.840	.367	.403	.767	.849	.382	.418	.774	.860	.398	.436	.784	.868	.413	.453	.792	.74
.76	.817	.360	.378	.777	.825	.376	.394	.784	.833	.390	.410	.791	.844	.406	.427	.801	.852	.422	.445	.809	.76
.78	.802	.367	.371	.793	.809	.383	.386	.801	.817	.396	.402	.808	.828	.414	.419	.818	.836	.431	.436	.826	.78
.80	.786	.374	.363	.809	.793	.390	.378	.817	.800	.405	.394	.824	.811	.423	.411	.834	.819	.440	.427	.843	.80
.82	.770	.381	.356	.824	.776	.398	.371	.833	.783	.413	.386	.840	.794	.431	.403	.850	.802	.449	.418	.859	.82
.84	.753	.388	.348	.839	.759	.406	.363	.848	.766	.420	.378	.855	.777	.439	.394	.865	.784	.457	.409	.875	.84
.86	.736	.395	.340	.854	.742	.412	.355	.863	.749	.428	.370	.871	.760	.447	.385	.880	.766	.466	.400	.890	.86
.88	.719	.402	.332	.869	.726	.419	.347	.877	.732	.436	.361	.886	.742	.454	.376	.895	.748	.473	.391	.906	.88
.90	.701	.408	.324	.883	.707	.426	.339	.891	.714	.443	.352	.900	.723	.461	.367	.910	.730	.481	.381	.920	.90
.92	.683	.414	.316	.897	.689	.433	.330	.906	.696	.449	.343	.916	.704	.468	.357	.924	.713	.488	.371	.934	.92
.94	.666	.420	.308	.910	.671	.439	.321	.918	.678	.456	.334	.928	.685	.475	.348	.938	.693	.496	.361	.948	.94
.96	.647	.426	.299	.925	.653	.445	.312	.931	.659	.462	.326	.940	.666	.482	.339	.952	.674	.502	.351	.962	.96
.98	.628	.432	.290	.936	.634	.451	.303	.944	.640	.469	.318	.954	.647	.489	.329	.965	.655	.509	.341	.975	.98
1.00	.609	.438	.281	.949	.616	.457	.293	.957	.620	.476	.306	.966	.627	.496	.319	.977	.636	.516	.331	.988	1.00

TABLE X

Values of cosh (x + jy) = a + jb and sinh (x + jy) = c + jd

y	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d	y
0.00	1.185	.000	.637	.000	1.198	.000	.660	.000	1.212	.000	.686	.000	1.226	.000	.709	.000	1.240	.000	.734	.000	0.00
.02	1.185	.013	.637	.024	1.198	.013	.660	.024	1.212	.014	.686	.024	1.226	.014	.709	.025	1.240	.015	.734	.025	.02
.04	1.184	.026	.637	.047	1.197	.026	.659	.048	1.211	.027	.684	.049	1.225	.028	.708	.049	1.239	.029	.733	.050	.04
.06	1.183	.038	.636	.071	1.196	.040	.659	.072	1.210	.041	.683	.073	1.224	.042	.707	.074	1.238	.044	.732	.074	.06
.08	1.181	.051	.636	.095	1.193	.053	.658	.096	1.209	.055	.682	.097	1.222	.057	.706	.098	1.237	.059	.731	.099	.08
.10	1.179	.064	.633	.118	1.191	.066	.657	.120	1.207	.068	.681	.121	1.220	.071	.705	.123	1.235	.073	.729	.124	.10
.12	1.176	.076	.632	.142	1.188	.079	.656	.144	1.204	.082	.679	.146	1.217	.085	.703	.147	1.232	.088	.727	.149	.12
.14	1.173	.089	.630	.166	1.185	.092	.654	.166	1.201	.095	.677	.170	1.213	.099	.701	.172	1.228	.103	.726	.174	.14
.16	1.169	.101	.628	.189	1.181	.106	.652	.190	1.198	.109	.675	.193	1.209	.113	.699	.196	1.224	.116	.723	.197	.16
.18	1.166	.114	.626	.212	1.177	.118	.650	.214	1.194	.122	.673	.217	1.205	.127	.697	.219	1.220	.131	.721	.222	.18
.20	1.162	.126	.624	.236	1.173	.131	.648	.238	1.190	.136	.670	.241	1.200	.141	.694	.244	1.215	.146	.719	.247	.20
.22	1.158	.139	.622	.258	1.169	.144	.645	.261	1.185	.149	.667	.264	1.195	.154	.691	.267	1.210	.161	.716	.270	.22
.24	1.153	.151	.619	.281	1.164	.157	.642	.285	1.180	.163	.664	.289	1.190	.169	.688	.292	1.204	.175	.713	.295	.24
.26	1.147	.164	.616	.304	1.158	.170	.639	.308	1.174	.176	.661	.313	1.184	.182	.684	.316	1.198	.189	.709	.319	.26
.28	1.140	.176	.612	.327	1.151	.183	.636	.332	1.167	.190	.658	.336	1.177	.196	.680	.340	1.192	.203	.705	.343	.28
.30	1.133	.188	.608	.350	1.143	.196	.631	.354	1.159	.202	.654	.360	1.170	.210	.676	.363	1.184	.217	.700	.367	.30
.32	1.125	.200	.604	.373	1.135	.208	.627	.377	1.151	.216	.650	.383	1.163	.223	.672	.386	1.177	.231	.695	.391	.32
.34	1.117	.212	.600	.396	1.127	.220	.623	.400	1.143	.228	.645	.405	1.155	.236	.667	.409	1.169	.245	.690	.414	.34
.36	1.109	.224	.596	.418	1.119	.232	.619	.422	1.135	.241	.640	.428	1.147	.250	.662	.432	1.160	.259	.685	.437	.36
.38	1.101	.236	.591	.440	1.111	.245	.614	.444	1.126	.254	.635	.450	1.136	.263	.657	.455	1.151	.272	.680	.460	.38
.40	1.092	.248	.586	.462	1.102	.257	.609	.466	1.117	.266	.630	.472	1.129	.276	.652	.477	1.141	.285	.675	.483	.40
.42	1.082	.260	.581	.484	1.093	.270	.604	.488	1.107	.279	.625	.496	1.119	.289	.645	.499	1.131	.299	.669	.506	.42
.44	1.072	.272	.576	.505	1.083	.283	.598	.510	1.097	.292	.619	.518	1.109	.302	.640	.521	1.121	.312	.663	.528	.44
.46	1.062	.283	.571	.526	1.072	.295	.592	.532	1.087	.305	.613	.539	1.098	.316	.636	.543	1.110	.325	.657	.550	.46
.48	1.051	.294	.565	.547	1.061	.306	.586	.553	1.076	.317	.607	.561	1.087	.327	.628	.566	1.099	.338	.650	.572	.48
.50	1.040	.305	.559	.568	1.050	.316	.580	.574	1.066	.329	.601	.583	1.076	.339	.622	.587	1.088	.351	.643	.594	.50
.52	1.028	.316	.553	.589	1.038	.328	.573	.595	1.053	.341	.595	.603	1.064	.351	.615	.608	1.076	.364	.636	.616	.52
.54	1.016	.327	.546	.610	1.026	.340	.566	.615	1.040	.353	.588	.624	1.051	.363	.608	.629	1.063	.377	.629	.636	.54
.56	1.004	.338	.539	.630	1.014	.351	.559	.636	1.027	.364	.581	.646	1.038	.375	.601	.650	1.050	.389	.621	.657	.56
.58	.991	.349	.532	.650	1.001	.362	.552	.656	1.014	.375	.573	.666	1.024	.387	.593	.671	1.036	.401	.613	.678	.58
.60	.978	.359	.525	.669	.988	.373	.545	.675	1.000	.385	.565	.686	1.010	.399	.585	.691	1.022	.413	.605	.699	.60
.62	.964	.370	.518	.689	.974	.384	.537	.695	.986	.397	.557	.706	.996	.411	.577	.711	1.008	.425	.597	.719	.62
.64	.950	.380	.511	.707	.960	.394	.529	.715	.972	.408	.549	.725	.982	.423	.569	.731	.994	.437	.588	.739	.64
.66	.936	.390	.504	.726	.946	.405	.521	.734	.959	.419	.541	.745	.968	.434	.560	.751	.980	.449	.579	.759	.66
.68	.922	.400	.496	.745	.931	.415	.513	.753	.944	.430	.531	.764	.954	.445	.551	.771	.965	.461	.570	.779	.68
.70	.907	.410	.487	.764	.916	.426	.505	.772	.929	.440	.524	.782	.939	.456	.542	.790	.950	.472	.560	.799	.70
.72	.891	.420	.479	.782	.901	.436	.497	.789	.912	.451	.515	.800	.923	.467	.533	.809	.934	.483	.551	.818	.72
.74	.875	.430	.471	.799	.886	.445	.488	.808	.896	.461	.506	.819	.907	.478	.524	.827	.917	.494	.541	.836	.74
.76	.859	.439	.462	.816	.869	.456	.479	.825	.879	.471	.497	.837	.890	.488	.514	.845	.900	.505	.531	.854	.76
.78	.842	.448	.453	.833	.853	.464	.470	.842	.863	.481	.487	.854	.873	.498	.504	.863	.883	.516	.521	.872	.78
.80	.826	.457	.444	.850	.836	.474	.460	.858	.846	.491	.477	.871	.855	.508	.494	.880	.865	.526	.511	.890	.80
.82	.809	.466	.435	.866	.818	.482	.451	.874	.829	.500	.467	.888	.837	.518	.484	.897	.847	.536	.500	.907	.82
.84	.791	.475	.426	.882	.800	.491	.441	.890	.812	.509	.457	.904	.819	.527	.474	.913	.829	.546	.489	.924	.84
.86	.773	.483	.416	.898	.782	.501	.431	.906	.794	.518	.447	.921	.801	.536	.463	.929	.810	.556	.478	.940	.86
.88	.755	.491	.406	.914	.764	.510	.421	.922	.778	.527	.437	.936	.782	.545	.452	.946	.791	.565	.467	.956	.88
.90	.737	.499	.396	.929	.745	.518	.411	.938	.755	.536	.426	.951	.763	.554	.441	.960	.772	.574	.456	.972	.90
.92	.718	.507	.386	.944	.726	.526	.401	.953	.735	.545	.416	.966	.743	.563	.430	.976	.752	.583	.444	.987	.92
.94	.699	.515	.376	.958	.707	.533	.390	.967	.715	.552	.404	.981	.723	.572	.418	.990	.732	.592	.432	1.001	.94
.96	.680	.522	.366	.972	.687	.541	.370	.981	.695	.561	.393	.995	.703	.580	.406	1.003	.712	.600	.420	1.015	.96
.98	.661	.529	.355	.986	.667	.549	.358	.994	.675	.568	.382	1.008	.683	.588	.394	1.017	.691	.608	.408	1.029	.98
1.00	.641	.536	.344	.998	.646	.556	.357	1.008	.655	.575	.370	1.020	.662	.596	.382	1.031	.670	.616	.395	1.043	1.00

TABLE XI

Values of cosh (x + jy) = a + jb and sinh (x + jy) = c + jd

y	x = .70				x = .72				x = .74				x = .76				x = .78				y
	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d	
0.00	1.255	.000	.759	.000	1.271	.000	.784	.000	1.287	.000	.809	.000	1.303	.000	.836	.000	1.320	.000	.861	.000	0.00
.02	1.255	.015	.759	.025	1.271	.016	.784	.025	1.286	.016	.809	.026	1.303	.017	.836	.026	1.319	.017	.861	.026	.02
.04	1.254	.030	.758	.050	1.270	.031	.783	.051	1.285	.032	.808	.051	1.302	.033	.834	.052	1.318	.034	.860	.053	.04
.06	1.253	.045	.757	.075	1.269	.047	.782	.076	1.284	.049	.806	.077	1.301	.050	.833	.078	1.317	.052	.859	.079	.06
.08	1.251	.061	.756	.100	1.267	.063	.781	.102	1.282	.065	.806	.103	1.300	.067	.832	.104	1.316	.069	.858	.106	.08
.10	1.249	.076	.755	.125	1.265	.078	.780	.127	1.280	.081	.806	.129	1.298	.083	.831	.130	1.314	.086	.857	.132	.10
.12	1.246	.091	.753	.150	1.262	.094	.778	.153	1.277	.097	.803	.154	1.295	.100	.829	.156	1.311	.103	.855	.158	.12
.14	1.242	.106	.751	.175	1.259	.110	.775	.178	1.274	.113	.801	.180	1.292	.117	.827	.183	1.307	.121	.853	.185	.14
.16	1.239	.121	.749	.200	1.255	.125	.773	.202	1.270	.129	.798	.204	1.288	.133	.826	.207	1.303	.137	.851	.210	.16
.18	1.235	.136	.746	.224	1.251	.140	.770	.228	1.266	.145	.795	.230	1.284	.149	.822	.234	1.299	.154	.848	.236	.18
.20	1.230	.151	.742	.249	1.247	.156	.767	.253	1.260	.161	.792	.256	1.279	.166	.819	.260	1.294	.171	.845	.263	.20
.22	1.225	.165	.740	.274	1.242	.172	.764	.278	1.255	.176	.788	.280	1.273	.182	.816	.284	1.288	.188	.841	.288	.22
.24	1.219	.180	.737	.298	1.236	.187	.760	.303	1.249	.192	.784	.306	1.267	.199	.811	.310	1.282	.205	.837	.314	.24
.26	1.212	.195	.733	.323	1.229	.202	.756	.328	1.243	.208	.780	.331	1.260	.215	.807	.335	1.276	.221	.833	.339	.26
.28	1.206	.210	.729	.348	1.222	.217	.752	.353	1.236	.224	.776	.356	1.253	.231	.802	.361	1.268	.239	.828	.366	.28
.30	1.199	.224	.725	.371	1.214	.232	.748	.377	1.229	.239	.772	.381	1.245	.247	.797	.386	1.261	.256	.823	.391	.30
.32	1.191	.239	.720	.395	1.206	.247	.743	.401	1.221	.255	.767	.405	1.237	.263	.792	.411	1.252	.271	.818	.416	.32
.34	1.183	.254	.715	.418	1.198	.262	.738	.425	1.212	.270	.762	.430	1.228	.279	.787	.435	1.243	.287	.812	.440	.34
.36	1.174	.268	.710	.442	1.189	.277	.733	.449	1.203	.285	.757	.454	1.219	.295	.782	.460	1.234	.303	.806	.466	.36
.38	1.166	.281	.704	.465	1.180	.291	.727	.473	1.193	.300	.752	.477	1.210	.310	.776	.484	1.225	.319	.800	.490	.38
.40	1.156	.295	.699	.489	1.171	.306	.721	.496	1.183	.315	.746	.500	1.200	.325	.770	.508	1.216	.335	.794	.514	.40
.42	1.146	.309	.693	.512	1.162	.319	.715	.518	1.173	.330	.739	.524	1.190	.340	.763	.533	1.204	.351	.787	.538	.42
.44	1.136	.323	.687	.535	1.152	.333	.709	.541	1.162	.345	.732	.547	1.180	.355	.756	.556	1.193	.367	.780	.562	.44
.46	1.125	.336	.680	.558	1.141	.347	.702	.564	1.151	.359	.725	.570	1.169	.370	.749	.580	1.182	.383	.772	.586	.46
.48	1.114	.350	.672	.580	1.129	.361	.695	.587	1.140	.373	.718	.593	1.158	.385	.741	.603	1.171	.398	.764	.609	.48
.50	1.102	.364	.666	.602	1.117	.376	.688	.609	1.129	.387	.710	.616	1.146	.399	.733	.626	1.159	.412	.756	.632	.50
.52	1.090	.377	.658	.624	1.104	.389	.680	.631	1.116	.401	.702	.638	1.133	.414	.725	.649	1.146	.428	.747	.656	.52
.54	1.077	.389	.650	.646	1.091	.402	.672	.653	1.103	.415	.694	.660	1.120	.428	.716	.670	1.132	.443	.738	.678	.54
.56	1.064	.402	.642	.667	1.078	.415	.664	.675	1.090	.429	.685	.682	1.106	.443	.707	.692	1.118	.457	.729	.700	.56
.58	1.050	.416	.634	.688	1.064	.428	.655	.697	1.076	.443	.676	.704	1.092	.457	.698	.714	1.104	.471	.720	.722	.58
.60	1.036	.427	.626	.709	1.050	.441	.646	.718	1.061	.456	.667	.725	1.077	.471	.689	.735	1.090	.485	.711	.744	.60
.62	1.019	.440	.617	.729	1.035	.454	.637	.739	1.046	.469	.658	.746	1.062	.485	.680	.756	1.076	.499	.701	.766	.62
.64	1.003	.453	.608	.749	1.020	.467	.628	.759	1.031	.482	.649	.767	1.047	.498	.670	.778	1.059	.513	.691	.787	.64
.66	.988	.466	.599	.769	1.005	.480	.619	.779	1.016	.495	.639	.788	1.031	.511	.660	.800	1.043	.527	.681	.808	.66
.68	.976	.477	.590	.789	.990	.492	.609	.799	1.001	.509	.629	.808	1.015	.524	.650	.820	1.027	.541	.670	.829	.68
.70	.960	.489	.580	.809	.974	.504	.599	.819	.985	.521	.619	.828	.999	.537	.639	.840	1.011	.555	.659	.850	.70
.72	.943	.501	.570	.828	.957	.516	.589	.838	.968	.533	.609	.848	.982	.550	.626	.860	.994	.568	.648	.870	.72
.74	.926	.512	.560	.846	.940	.528	.579	.857	.951	.545	.598	.867	.965	.562	.617	.880	.976	.581	.637	.890	.74
.76	.909	.523	.550	.864	.923	.540	.568	.875	.933	.557	.587	.886	.947	.574	.606	.899	.958	.594	.625	.909	.76
.78	.892	.534	.539	.882	.905	.551	.557	.894	.916	.569	.576	.906	.929	.586	.594	.918	.940	.606	.613	.928	.78
.80	.874	.544	.529	.900	.887	.562	.546	.912	.897	.580	.564	.923	.910	.598	.582	.936	.921	.618	.601	.947	.80
.82	.856	.554	.517	.917	.869	.573	.535	.930	.878	.591	.552	.941	.891	.610	.570	.954	.902	.630	.589	.965	.82
.84	.837	.564	.506	.934	.850	.583	.523	.947	.859	.602	.540	.958	.872	.621	.558	.971	.882	.641	.576	.983	.84
.86	.818	.574	.494	.951	.831	.593	.511	.964	.840	.613	.528	.973	.852	.632	.546	.988	.862	.652	.563	1.000	.86
.88	.799	.584	.483	.967	.812	.603	.499	.980	.820	.623	.516	.991	.832	.643	.533	1.004	.842	.663	.550	1.017	.88
.90	.780	.594	.472	.983	.792	.613	.487	.996	.800	.633	.503	1.007	.812	.654	.520	1.020	.821	.674	.536	1.034	.90
.92	.760	.603	.460	.998	.771	.623	.476	1.013	.780	.643	.490	1.023	.791	.664	.507	1.036	.800	.685	.522	1.050	.92
.94	.740	.612	.447	1.013	.750	.632	.462	1.028	.759	.653	.477	1.038	.770	.674	.493	1.051	.779	.695	.508	1.066	.94
.96	.720	.621	.435	1.028	.729	.641	.449	1.042	.738	.663	.464	1.053	.749	.684	.479	1.066	.758	.705	.494	1.081	.96
.98	.699	.630	.423	1.043	.708	.650	.436	1.057	.717	.672	.450	1.068	.727	.693	.465	1.081	.736	.715	.480	1.096	.98
1.00	.678	.638	.410	1.056	.687	.659	.423	1.070	.696	.681	.438	1.083	.706	.702	.451	1.096	.714	.725	.466	1.110	1.00

TABLE XII

Values of cosh (x + jy) = a + jb and sinh (x + jy) = c + jd

y	x = .80				x = .82				x = .84				x = .86				x = .88				y
	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d	
0.00	1.337	.000	.888	.000	1.355	.000	.915	.000	1.374	.000	.942	.000	1.393	.000	.970	.000	1.413	.000	.998	.000	0.00
.02	1.337	.018	.888	.027	1.355	.018	.915	.027	1.374	.019	.942	.028	1.393	.019	.970	.028	1.413	.020	.998	.028	.02
.04	1.336	.036	.887	.053	1.354	.037	.914	.054	1.373	.038	.941	.055	1.392	.039	.969	.056	1.412	.040	.997	.057	.04
.06	1.334	.053	.886	.080	1.353	.055	.913	.081	1.372	.056	.940	.083	1.391	.058	.968	.084	1.411	.060	.996	.085	.06
.08	1.333	.071	.885	.107	1.351	.073	.912	.108	1.371	.075	.939	.110	1.390	.078	.967	.112	1.410	.080	.995	.113	.08
.10	1.331	.089	.884	.134	1.349	.091	.910	.135	1.369	.094	.937	.137	1.388	.097	.966	.139	1.408	.100	.993	.141	.10
.12	1.328	.106	.882	.160	1.346	.110	.908	.162	1.366	.113	.935	.165	1.385	.116	.963	.167	1.406	.120	.991	.170	.12
.14	1.324	.124	.880	.187	1.342	.128	.906	.190	1.363	.132	.933	.192	1.381	.135	.961	.195	1.401	.140	.988	.198	.14
.16	1.320	.141	.877	.213	1.338	.146	.903	.216	1.358	.150	.930	.219	1.377	.154	.958	.222	1.397	.159	.985	.226	.16
.18	1.316	.159	.874	.239	1.334	.164	.900	.243	1.353	.169	.927	.246	1.373	.173	.954	.250	1.393	.179	.982	.253	.18
.20	1.311	.176	.870	.266	1.329	.182	.897	.270	1.348	.187	.923	.274	1.368	.193	.950	.278	1.387	.199	.978	.283	.20
.22	1.305	.193	.867	.292	1.323	.199	.893	.296	1.342	.206	.920	.300	1.362	.212	.946	.304	1.380	.218	.974	.309	.22
.24	1.300	.211	.863	.318	1.317	.218	.889	.322	1.336	.224	.915	.328	1.355	.231	.942	.332	1.373	.238	.969	.337	.24
.26	1.293	.228	.859	.344	1.310	.235	.884	.349	1.329	.242	.911	.354	1.348	.249	.937	.359	1.366	.257	.964	.364	.26
.28	1.284	.246	.854	.370	1.302	.253	.879	.376	1.321	.261	.906	.381	1.340	.269	.932	.386	1.358	.276	.959	.392	.28
.30	1.278	.262	.848	.395	1.294	.271	.874	.401	1.312	.279	.900	.407	1.331	.287	.926	.413	1.350	.296	.953	.419	.30
.32	1.270	.279	.843	.421	1.286	.288	.869	.427	1.303	.297	.894	.434	1.322	.306	.920	.440	1.341	.314	.947	.446	.32
.34	1.261	.296	.837	.446	1.277	.306	.863	.453	1.295	.314	.888	.460	1.313	.324	.914	.466	1.331	.333	.941	.473	.34
.36	1.252	.313	.831	.472	1.268	.322	.857	.479	1.286	.332	.881	.486	1.303	.342	.908	.492	1.321	.352	.934	.500	.36
.38	1.242	.330	.825	.496	1.258	.339	.850	.504	1.276	.349	.874	.510	1.293	.360	.901	.518	1.311	.370	.927	.526	.38
.40	1.232	.346	.818	.521	1.248	.356	.843	.528	1.266	.366	.867	.536	1.282	.377	.894	.543	1.301	.389	.919	.550	.40
.42	1.221	.362	.811	.546	1.237	.373	.836	.554	1.254	.384	.860	.562	1.272	.396	.886	.570	1.290	.407	.911	.577	.42
.44	1.210	.378	.803	.570	1.226	.390	.828	.578	1.243	.401	.852	.586	1.261	.413	.878	.595	1.279	.425	.903	.603	.44
.46	1.198	.394	.795	.594	1.214	.406	.820	.602	1.232	.418	.844	.610	1.249	.430	.869	.619	1.267	.443	.894	.628	.46
.48	1.184	.410	.787	.618	1.203	.422	.813	.627	1.220	.435	.836	.636	1.237	.447	.860	.645	1.253	.461	.885	.653	.48
.50	1.174	.426	.779	.641	1.189	.439	.803	.650	1.207	.451	.827	.659	1.224	.464	.851	.668	1.242	.478	.876	.678	.50
.52	1.161	.441	.770	.664	1.176	.455	.794	.674	1.193	.467	.818	.684	1.210	.481	.841	.693	1.228	.496	.866	.702	.52
.54	1.147	.456	.761	.687	1.161	.471	.785	.697	1.179	.483	.808	.707	1.196	.498	.831	.718	1.214	.513	.856	.727	.54
.56	1.133	.471	.752	.710	1.147	.486	.776	.721	1.164	.499	.798	.731	1.181	.514	.821	.740	1.199	.530	.846	.751	.56
.58	1.119	.486	.743	.733	1.133	.501	.765	.744	1.149	.515	.788	.754	1.166	.530	.811	.763	1.184	.547	.834	.775	.58
.60	1.104	.501	.733	.755	1.118	.516	.755	.765	1.134	.531	.777	.776	1.150	.546	.800	.786	1.168	.563	.823	.799	.60
.62	1.088	.516	.723	.777	1.103	.531	.745	.789	1.118	.546	.766	.797	1.134	.562	.789	.809	1.151	.580	.812	.822	.62
.64	1.073	.531	.712	.799	1.088	.546	.735	.810	1.102	.561	.755	.819	1.118	.578	.778	.833	1.134	.596	.800	.846	.64
.66	1.057	.545	.701	.820	1.072	.561	.724	.832	1.086	.576	.744	.841	1.102	.594	.766	.854	1.117	.613	.788	.867	.66
.68	1.040	.559	.690	.841	1.055	.576	.712	.853	1.068	.591	.732	.862	1.085	.609	.754	.876	1.099	.630	.775	.889	.68
.70	1.023	.573	.679	.862	1.037	.589	.700	.874	1.051	.606	.720	.883	1.067	.624	.742	.898	1.081	.643	.763	.911	.70
.72	1.006	.586	.667	.882	1.020	.603	.688	.894	1.033	.621	.708	.905	1.049	.639	.730	.919	1.063	.658	.750	.932	.72
.74	.988	.598	.656	.902	1.003	.617	.676	.914	1.016	.635	.695	.927	1.031	.654	.717	.940	1.045	.673	.736	.955	.74
.76	.970	.611	.643	.921	.985	.630	.664	.934	.997	.649	.683	.947	1.012	.668	.704	.961	1.026	.688	.724	.973	.76
.78	.951	.624	.631	.940	.966	.643	.651	.954	.978	.663	.670	.967	.993	.682	.690	.981	1.007	.702	.712	.993	.78
.80	.932	.637	.619	.959	.946	.656	.638	.973	.959	.676	.657	.987	.973	.696	.676	1.001	.987	.716	.696	1.013	.80
.82	.912	.649	.606	.979	.926	.669	.626	.991	.938	.689	.644	1.006	.953	.709	.662	1.020	.966	.730	.681	1.032	.82
.84	.893	.661	.593	.996	.906	.681	.611	1.009	.918	.702	.630	1.024	.933	.722	.648	1.039	.945	.743	.666	1.051	.84
.86	.873	.673	.580	1.014	.883	.693	.597	1.027	.897	.714	.616	1.042	.912	.735	.633	1.057	.924	.757	.651	1.070	.86
.88	.852	.685	.566	1.031	.864	.705	.583	1.044	.876	.726	.601	1.060	.890	.747	.618	1.076	.903	.769	.636	1.088	.88
.90	.831	.696	.552	1.048	.843	.717	.569	1.061	.855	.738	.586	1.077	.868	.759	.603	1.092	.881	.781	.621	1.106	.90
.92	.810	.707	.538	1.064	.821	.728	.554	1.078	.833	.749	.571	1.094	.846	.771	.588	1.109	.859	.794	.605	1.124	.92
.94	.789	.717	.524	1.080	.799	.739	.539	1.094	.811	.760	.556	1.110	.824	.783	.572	1.125	.836	.806	.589	1.141	.94
.96	.767	.727	.510	1.095	.777	.750	.524	1.110	.789	.771	.541	1.125	.801	.794	.556	1.141	.812	.817	.573	1.158	.96
.98	.745	.737	.496	1.110	.755	.760	.509	1.126	.766	.782	.525	1.141	.778	.805	.540	1.157	.789	.828	.556	1.174	.98
1.00	.723	.747	.480	1.125	.732	.770	.496	1.140	.743	.792	.509	1.156	.754	.816	.524	1.173	.766	.839	.539	1.190	1.00

TABLE XIII

Values of cosh (x + jy) = a + jb and sinh (x + jy) = c + jd

y	x = .90				x = .92				x = .94				x = .96				x = .98				
	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	d	
0.00	1.433	.000	1.026	.000	1.454	.000	1.055	.000	1.475	.000	1.085	.000	1.497	.000	1.114	.000	1.520	.000	1.145	.000	0.00
.02	1.433	.021	1.026	.029	1.454	.021	1.055	.029	1.475	.022	1.085	.030	1.497	.022	1.114	.030	1.519	.023	1.144	.030	.02
.04	1.432	.041	1.025	.057	1.453	.042	1.054	.058	1.474	.043	1.084	.059	1.496	.045	1.113	.060	1.518	.046	1.144	.061	.04
.06	1.431	.062	1.024	.086	1.452	.063	1.053	.087	1.473	.065	1.083	.089	1.495	.067	1.112	.090	1.517	.069	1.143	.091	.06
.08	1.429	.082	1.023	.115	1.450	.084	1.052	.116	1.471	.087	1.081	.118	1.493	.089	1.111	.120	1.515	.092	1.141	.122	.08
.10	1.426	.102	1.021	.143	1.447	.105	1.050	.145	1.469	.108	1.079	.147	1.490	.111	1.109	.150	1.512	.114	1.139	.152	.10
.12	1.423	.123	1.019	.172	1.443	.127	1.048	.175	1.465	.130	1.077	.177	1.486	.134	1.107	.180	1.508	.137	1.137	.182	.12
.14	1.419	.144	1.017	.201	1.439	.148	1.045	.204	1.462	.152	1.074	.206	1.482	.156	1.105	.210	1.504	.160	1.134	.213	.14
.16	1.414	.163	1.013	.228	1.435	.168	1.042	.232	1.457	.172	1.072	.234	1.477	.177	1.102	.238	1.500	.182	1.131	.242	.16
.18	1.409	.184	1.010	.256	1.430	.189	1.038	.261	1.452	.194	1.068	.264	1.472	.200	1.096	.268	1.495	.206	1.127	.272	.18
.20	1.404	.204	1.006	.285	1.425	.210	1.034	.290	1.446	.216	1.063	.294	1.466	.222	1.092	.296	1.489	.228	1.122	.303	.20
.22	1.399	.224	1.002	.313	1.419	.230	1.030	.317	1.439	.236	1.058	.323	1.459	.243	1.087	.326	1.482	.250	1.117	.332	.22
.24	1.392	.244	.997	.341	1.413	.251	1.025	.346	1.432	.258	1.053	.351	1.452	.266	1.082	.356	1.475	.273	1.112	.362	.24
.26	1.385	.264	.992	.368	1.405	.271	1.020	.374	1.425	.279	1.048	.380	1.445	.287	1.076	.386	1.467	.294	1.106	.391	.26
.28	1.377	.284	.987	.396	1.398	.292	1.014	.402	1.418	.300	1.042	.409	1.437	.309	1.070	.415	1.459	.317	1.100	.421	.28
.30	1.369	.303	.981	.424	1.390	.312	1.008	.431	1.410	.321	1.036	.436	1.429	.330	1.064	.443	1.451	.339	1.093	.450	.30
.32	1.360	.323	.974	.451	1.381	.332	1.002	.458	1.402	.341	1.029	.466	1.420	.351	1.057	.472	1.442	.361	1.086	.479	.32
.34	1.351	.342	.967	.478	1.372	.352	.995	.485	1.393	.362	1.023	.494	1.411	.373	1.050	.500	1.432	.383	1.079	.508	.34
.36	1.341	.362	.960	.505	1.362	.372	.988	.514	1.382	.383	1.015	.521	1.401	.393	1.042	.529	1.422	.404	1.071	.536	.36
.38	1.331	.381	.953	.532	1.352	.391	.980	.540	1.371	.402	1.007	.548	1.390	.414	1.034	.556	1.411	.425	1.063	.564	.38
.40	1.320	.400	.945	.558	1.341	.410	.972	.566	1.359	.422	.999	.574	1.378	.434	1.026	.583	1.399	.446	1.054	.591	.40
.42	1.309	.419	.937	.584	1.329	.431	.964	.592	1.347	.443	.990	.601	1.366	.455	1.017	.610	1.386	.467	1.045	.620	.42
.44	1.297	.437	.929	.610	1.317	.450	.955	.618	1.335	.462	.981	.628	1.353	.475	1.008	.637	1.373	.488	1.036	.646	.44
.46	1.284	.456	.920	.636	1.304	.469	.945	.644	1.323	.481	.973	.654	1.340	.495	.999	.664	1.360	.509	1.026	.675	.46
.48	1.273	.474	.911	.662	1.290	.487	.936	.670	1.308	.500	.962	.680	1.327	.515	.989	.691	1.347	.529	1.016	.703	.48
.50	1.258	.492	.901	.687	1.276	.506	.927	.696	1.294	.519	.952	.706	1.313	.534	.979	.717	1.333	.549	1.005	.728	.50
.52	1.244	.510	.891	.712	1.263	.523	.916	.721	1.279	.538	.941	.732	1.298	.553	.968	.743	1.318	.569	.994	.756	.52
.54	1.229	.528	.881	.737	1.247	.541	.905	.746	1.265	.557	.930	.758	1.282	.572	.956	.769	1.303	.589	.982	.781	.54
.56	1.214	.545	.870	.761	1.232	.559	.894	.771	1.249	.576	.919	.783	1.266	.591	.944	.795	1.287	.608	.970	.807	.56
.58	1.199	.563	.859	.785	1.216	.577	.883	.796	1.233	.594	.907	.808	1.250	.610	.932	.820	1.270	.627	.958	.833	.58
.60	1.183	.580	.847	.809	1.200	.596	.871	.821	1.217	.612	.895	.833	1.233	.629	.920	.845	1.253	.646	.945	.858	.60
.62	1.166	.597	.836	.833	1.184	.613	.859	.845	1.200	.630	.883	.867	1.216	.647	.908	.869	1.235	.665	.932	.883	.62
.64	1.149	.614	.823	.856	1.167	.630	.847	.868	1.182	.648	.870	.891	1.199	.665	.895	.893	1.217	.684	.919	.908	.64
.66	1.132	.630	.811	.879	1.149	.647	.834	.891	1.164	.665	.857	.914	1.181	.683	.882	.917	1.199	.702	.906	.932	.66
.68	1.114	.646	.798	.901	1.131	.664	.821	.914	1.146	.682	.844	.937	1.162	.701	.868	.941	1.180	.720	.891	.956	.68
.70	1.096	.661	.785	.923	1.112	.680	.808	.937	1.128	.698	.830	.960	1.143	.718	.854	.964	1.161	.738	.876	.979	.70
.72	1.078	.676	.772	.945	1.093	.696	.794	.959	1.109	.714	.816	.972	1.124	.735	.839	.987	1.142	.755	.861	1.002	.72
.74	1.058	.692	.758	.966	1.072	.712	.780	.981	1.090	.730	.802	.994	1.105	.751	.824	1.009	1.122	.772	.846	1.026	.74
.76	1.039	.706	.744	.987	1.052	.727	.765	1.002	1.070	.746	.787	1.016	1.085	.768	.809	1.031	1.102	.789	.830	1.047	.76
.78	1.019	.721	.730	1.008	1.033	.742	.751	1.023	1.050	.762	.772	1.037	1.065	.784	.794	1.052	1.081	.805	.814	1.069	.78
.80	.998	.736	.715	1.028	1.013	.757	.735	1.044	1.029	.778	.756	1.058	1.044	.800	.777	1.073	1.060	.821	.798	1.090	.80
.82	.977	.750	.700	1.048	.992	.772	.721	1.064	1.006	.793	.740	1.078	1.022	.815	.761	1.094	1.038	.837	.781	1.111	.82
.84	.956	.764	.685	1.067	.971	.786	.705	1.083	.985	.808	.724	1.098	1.000	.830	.745	1.114	1.016	.852	.764	1.131	.84
.86	.935	.778	.670	1.086	.949	.800	.689	1.102	.964	.822	.708	1.118	.978	.846	.728	1.134	.993	.867	.747	1.151	.86
.88	.913	.791	.654	1.105	.926	.813	.673	1.121	.941	.836	.692	1.137	.955	.859	.711	1.153	.970	.882	.730	1.171	.88
.90	.891	.804	.638	1.123	.906	.826	.656	1.139	.916	.849	.675	1.156	.932	.873	.694	1.172	.946	.897	.712	1.190	.90
.92	.868	.817	.621	1.140	.883	.839	.639	1.157	.895	.862	.658	1.174	.908	.887	.676	1.190	.922	.911	.694	1.209	.92
.94	.845	.829	.605	1.157	.860	.852	.622	1.175	.871	.875	.641	1.191	.884	.900	.658	1.208	.897	.924	.675	1.227	.94
.96	.822	.841	.589	1.174	.836	.864	.605	1.192	.847	.888	.622	1.208	.859	.913	.640	1.225	.872	.937	.656	1.245	.96
.98	.798	.853	.572	1.190	.811	.876	.588	1.208	.822	.900	.605	1.225	.836	.926	.621	1.242	.847	.950	.637	1.262	.98
1.00	.774	.864	.555	1.206	.786	.888	.570	1.224	.797	.913	.586	1.241	.809	.937	.602	1.258	.821	.963	.618	1.279	1.00