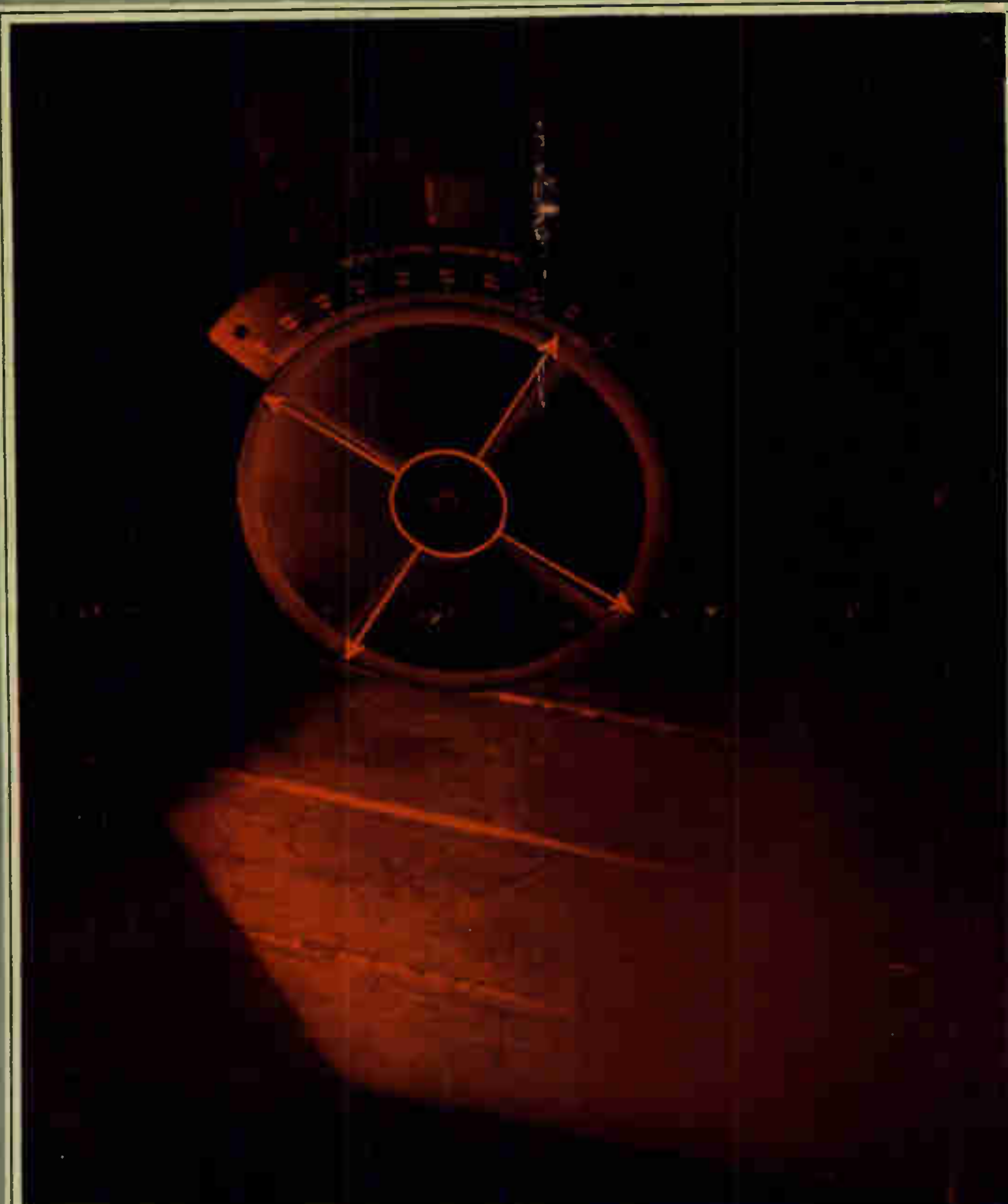
THREE DOLLARS A YEAR

THIRTY CENTS A COPY

GENERAL ELECTRIC REVIEW

VOL. 31, No. 3

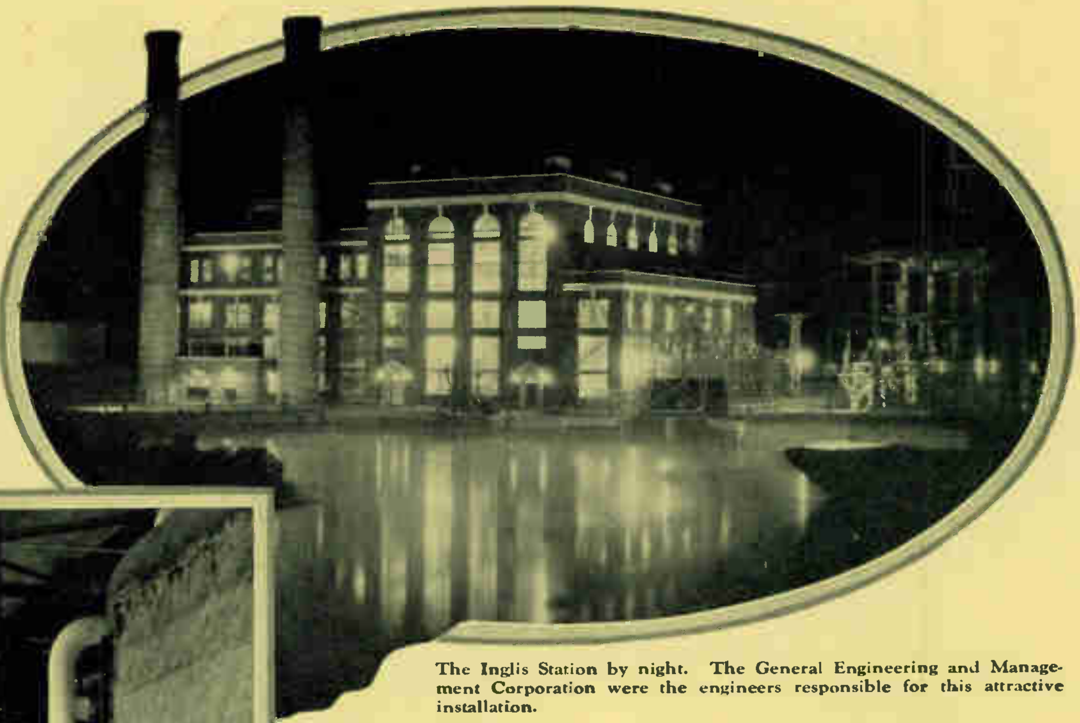
MARCH, 1928



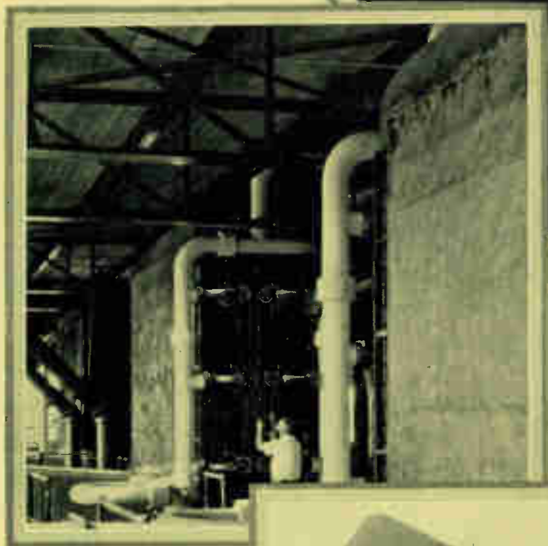
TIME AND PLACE

Continuously rotating arrows as they appear—sharply defined and stationary—in the light of the Neon-electric Stroboscope under the following conditions: disk driven by synchronous motor under constant load and stroboscope operated from same source as the motor. A variation in load causes a displacement in the position of the arrows

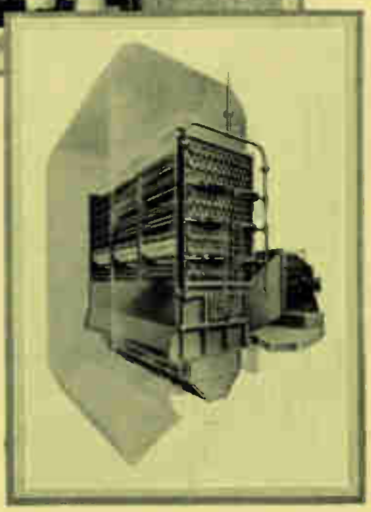
(*See p. 136*)



The Inglis Station by night. The General Engineering and Management Corporation were the engineers responsible for this attractive installation.



General view of the Foster Economizers installed at the Inglis plant. Characteristic features are—low maintenance, high efficiency, compactness with accessibility.



Phantom view of the Foster Economizer. Catalog upon request.

Inglis, Florida, Uses Foster Equipment

Six 936 hp. Edge Moor longitudinal drum boilers have been installed at the Inglis steam station of the Florida Power Corporation. The boilers are equipped with Foster Superheaters and Foster Economizers, all designed for operating loads up to 300% of rating. The superheaters add 200 deg. of superheat to the steam generated. Economizers of 4368 sq. ft. heating surface raise the temperature of the boiler feed water to 350 deg. F., while handling 85,200 lbs. of water per hour.

Foster Wheeler power plant equipment is being used at leading power stations throughout the world in both large and small units. It is rendering continuous high-grade service and can always be depended upon for unfailing reliability.

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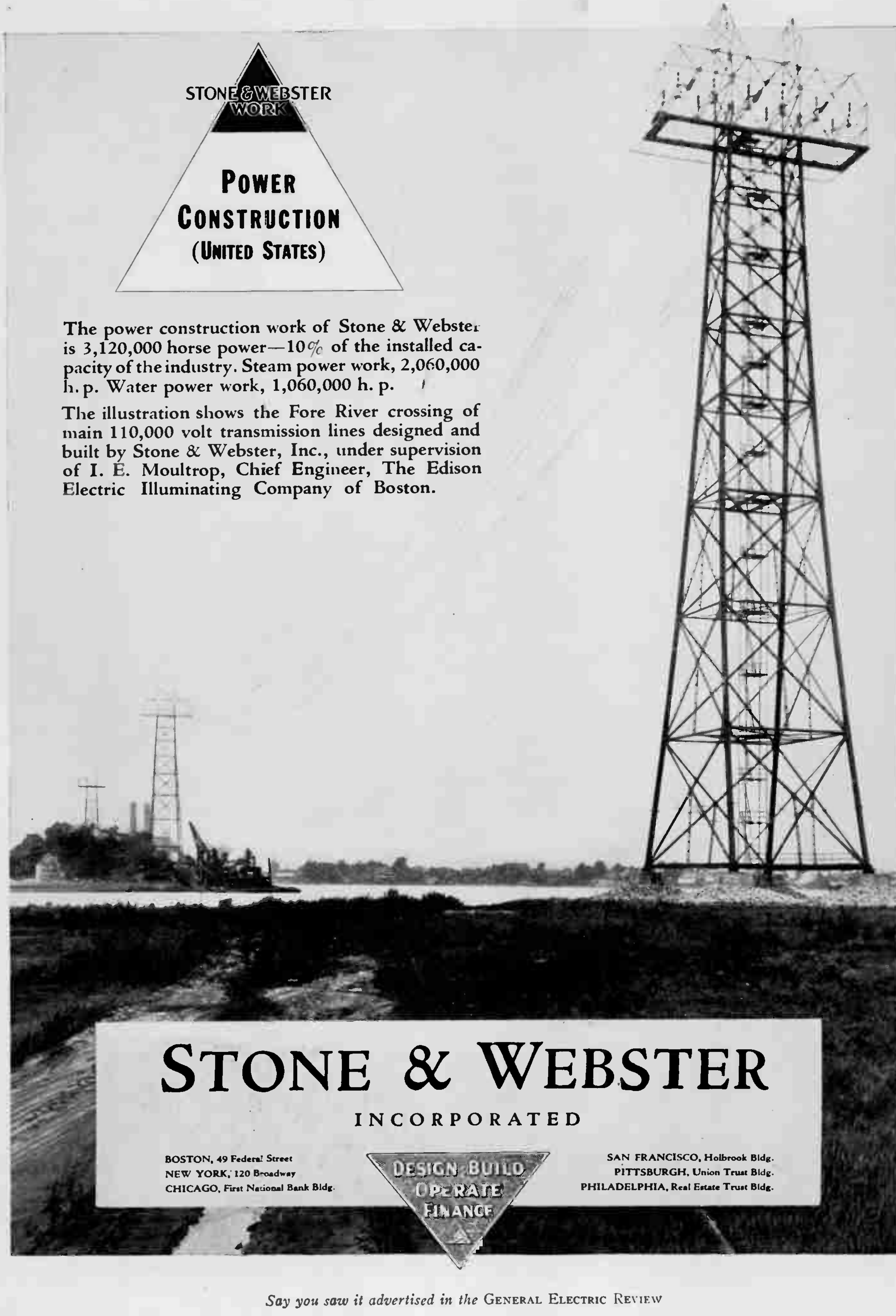
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The power construction work of Stone & Webster is 3,120,000 horse power—10% of the installed capacity of the industry. Steam power work, 2,060,000 h. p. Water power work, 1,060,000 h. p.
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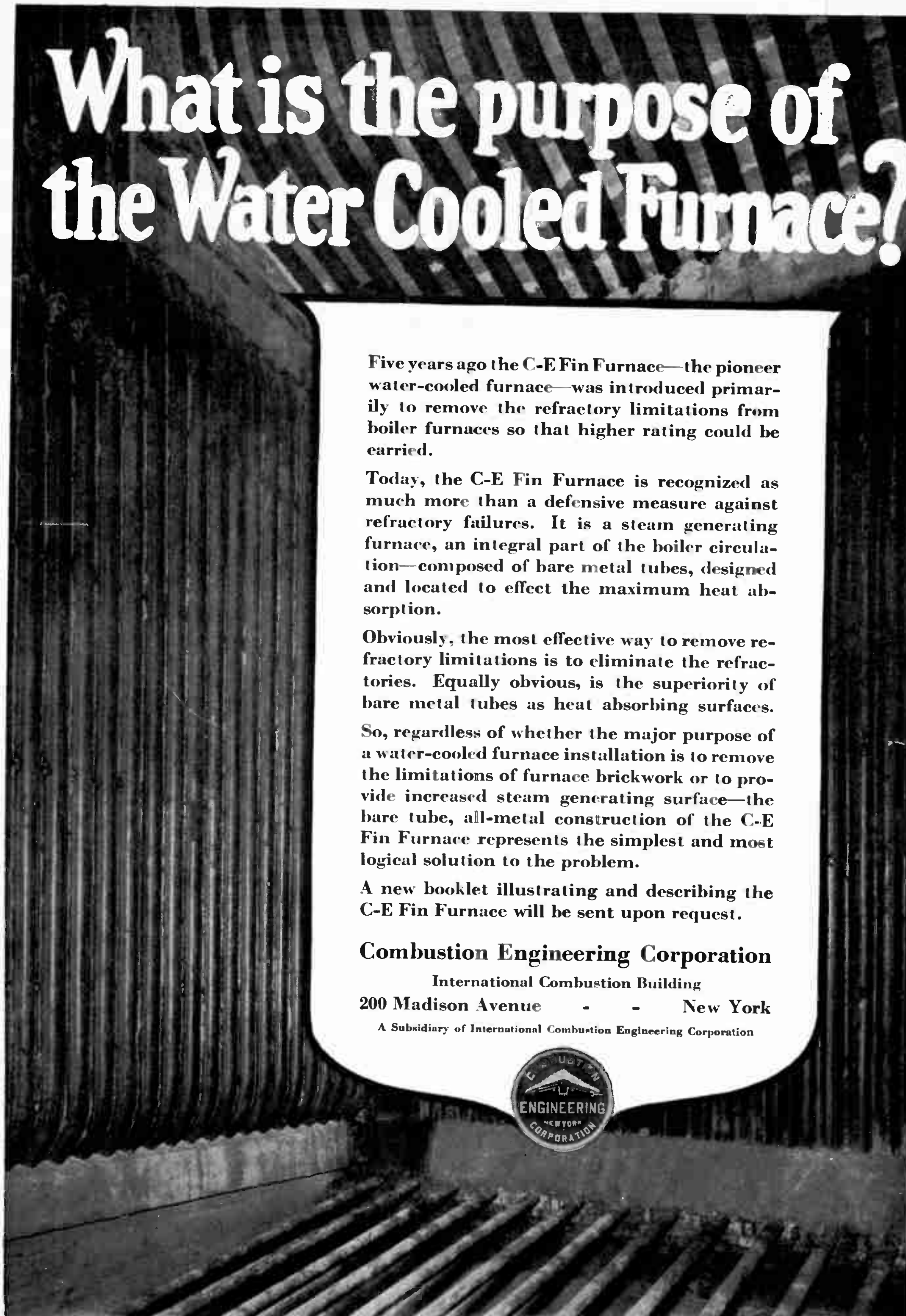
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Five years ago the C-E Fin Furnace—the pioneer water-cooled furnace—was introduced primarily to remove the refractory limitations from boiler furnaces so that higher rating could be carried.
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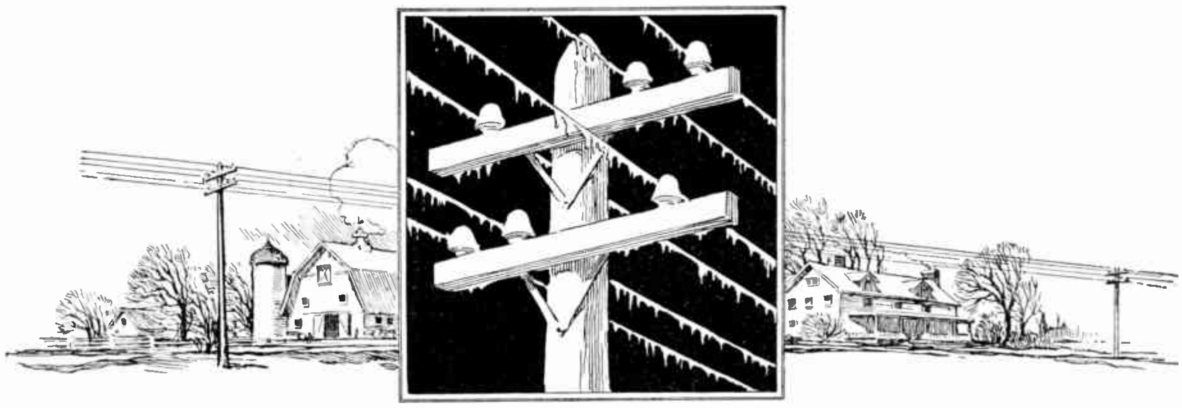

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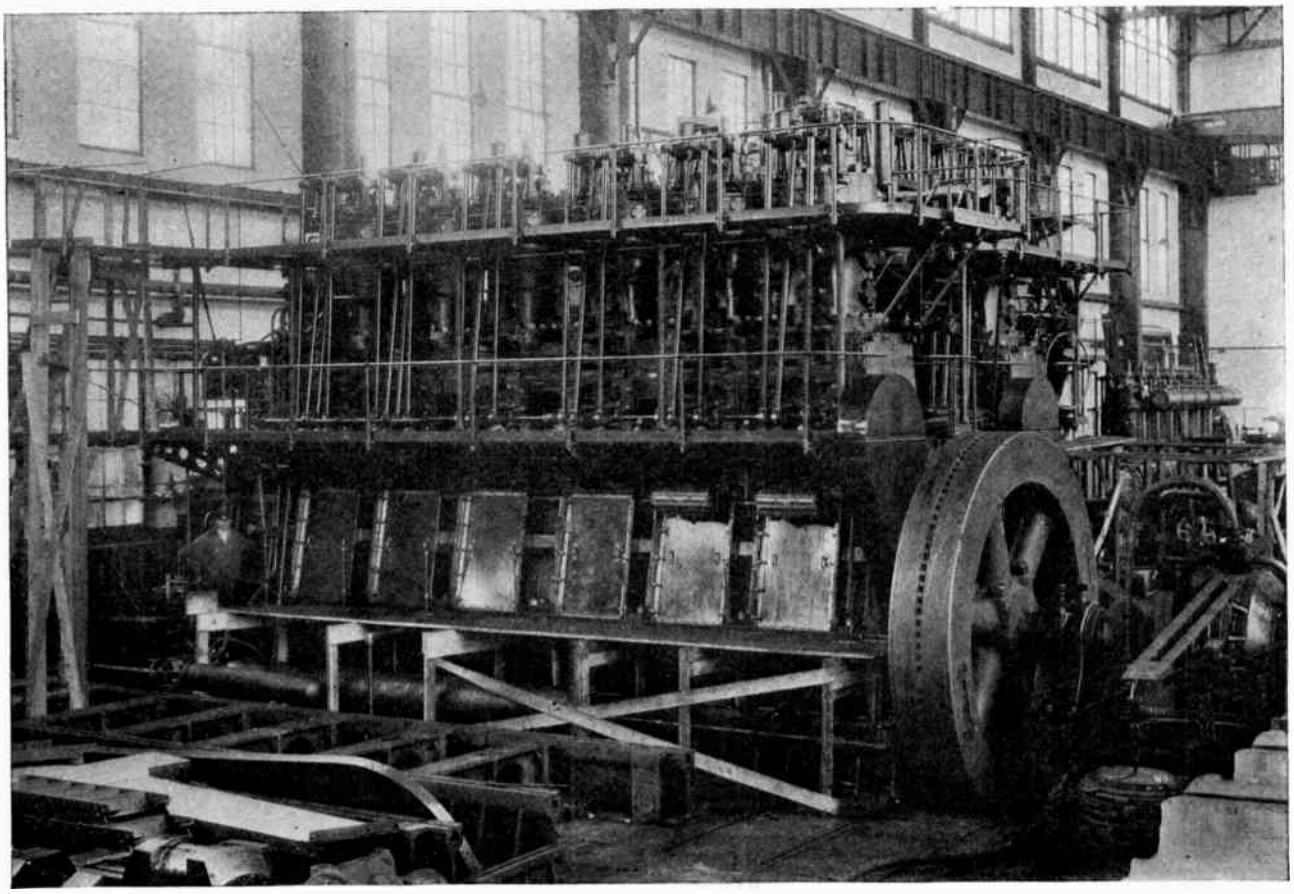

Rockville Center will have the finest Municipal Lighting and Power Plant on Long Island

WITH a view to freeing this village from the necessity of buying supplementary current outside, and to assure reasonable and future residence and power rates, the Commissioner of Public Utility for several years made a careful investigation of the power problem. As a result, the present partially obsolete and inadequate steam-electric plant is being replaced by complete Diesel-electric equipment, which when installed in its new building, will comprise one of the largest and best plants of its kind in the Eastern United States.

All the current needed will then be supplied from one 560-kw. and two 850-kw. generators driven respectively by one 825-b.hp. and two 1250-b.hp. McIntosh & Seymour Diesel Engines. One of the latter is shown as it appeared completed on our testing floor.

This replacement of steam with Diesel power is in line with the modern practice, and the selection of McIntosh & Seymour units is consistent for dependable uninterrupted service with very low year-in-year-out operating and maintenance expense. Division of the load between the three entirely independent sets, each of which can be operated singly or with either or both of the others, provides excellent flexibility to meet all demands and at the same time permits units in service to work at the high loads at which fuel rate is most favorable.

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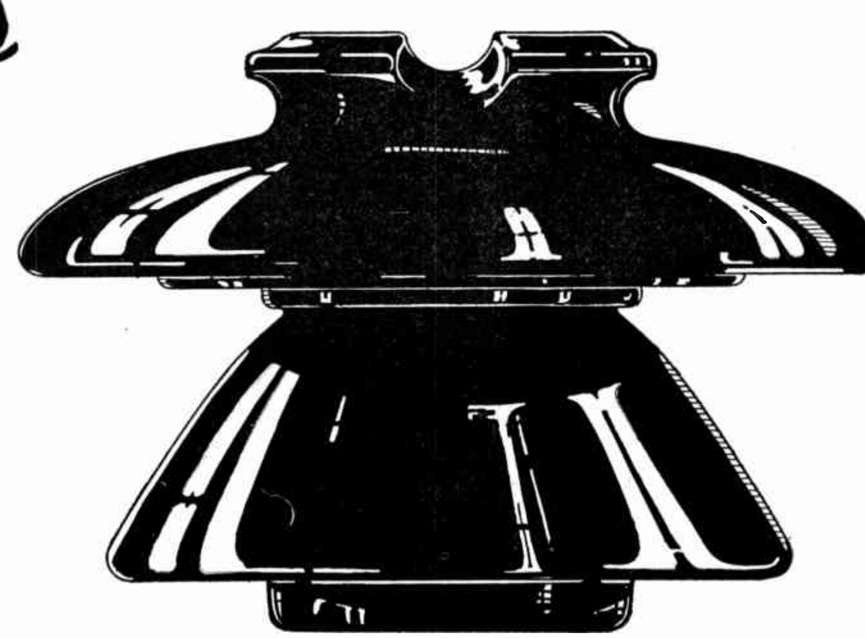

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Locke Pin Type Insulators are the outcome of thirty-four years of research and experience in the design and manufacture of high voltage insulators. They are designed to give maximum flashover values per pound of weight without sacrificing ruggedness or strength. Equal voltage distribution between parts is scientifically assured. Under lighting conditions these insulators show a remarkable freedom from cascading. Due also to this equal voltage distribution corona values are high, largely eliminating the possibility of radio interference. When service counts specify LOCKE.

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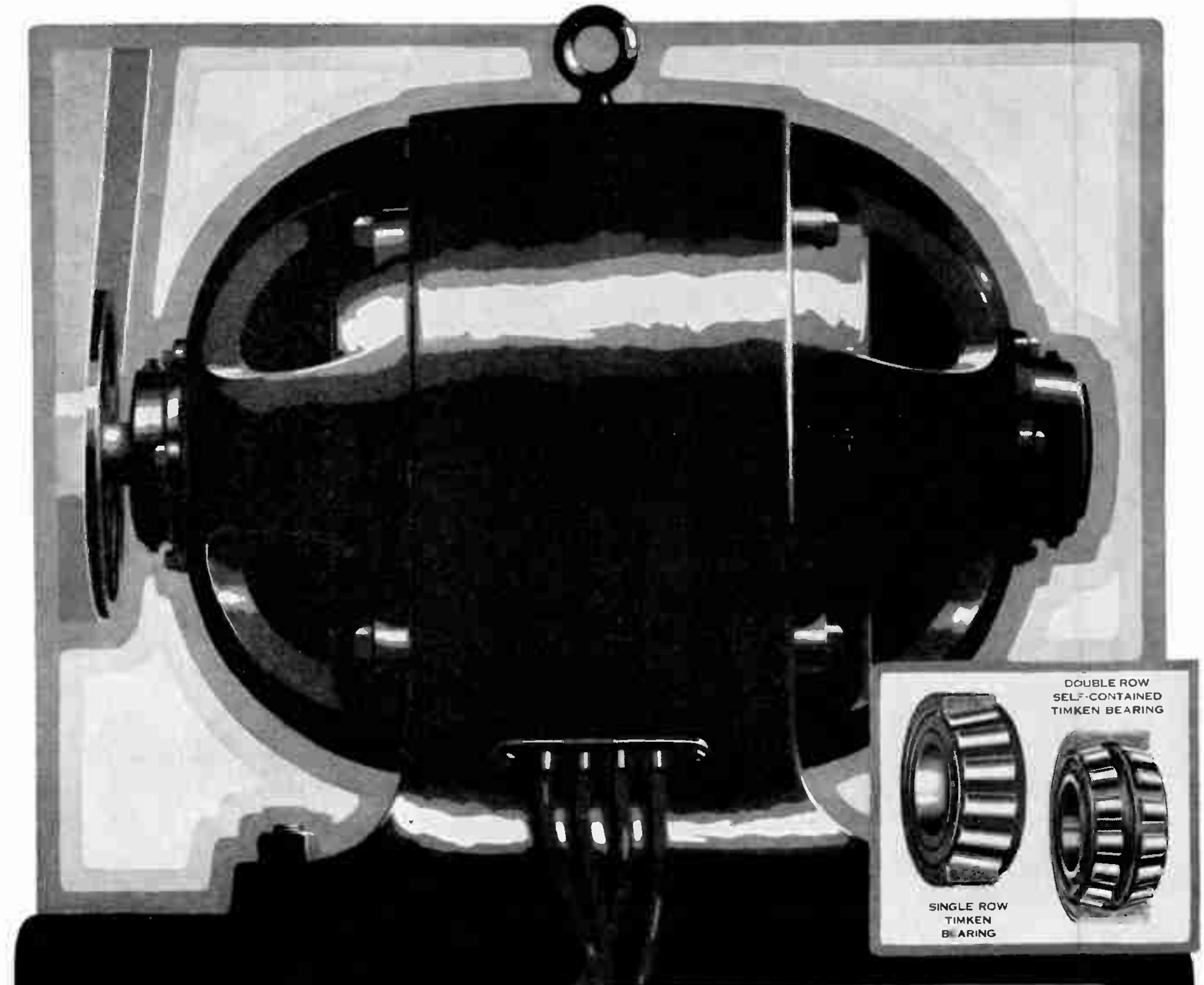

Electric Motors for Any Hook-Up

No longer must you buy motors that fit only one job! No longer need the combination of thrust and radial load concern you. No longer do type of drive or mounting position make any difference. For the *thrust-radial* electric motor is here!

Tell the motor manufacturer merely the horse-power, the general nature of the service—and specify Timken Tapered Roller Bearings. Then the motors you get are sure to work perfectly in floor, wall or ceiling positions—on direct drive—or with flat belt, V-belt, chain, or rope—and spur, helical, or herringbone gear. It is all made possible by Timken Bearings—*the* modern electric motor bearings, with far broader functions than friction-elimination alone!

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To motors, as to all types of equipment throughout all Industry, Timken Bearings bring the greatest endurance and economy ever known. The records of thousands of Timken-equipped motors, during years of service, are the proof. Capitalize these tremendous advantages for yourself, by specifying Timken Bearings in your orders for electric motors.

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—about motor performance by installing Stackpole—the Carbon Brush that's made for the job—by a company that has made carbon brush manufacture an exact science—a company that has specialized on carbon brushes for over 20 years.

The individual commutation requirements of every type and make of motor have been carefully studied—no expense has been spared to make Stackpole the most efficient and economical brush scientific skill can devise.

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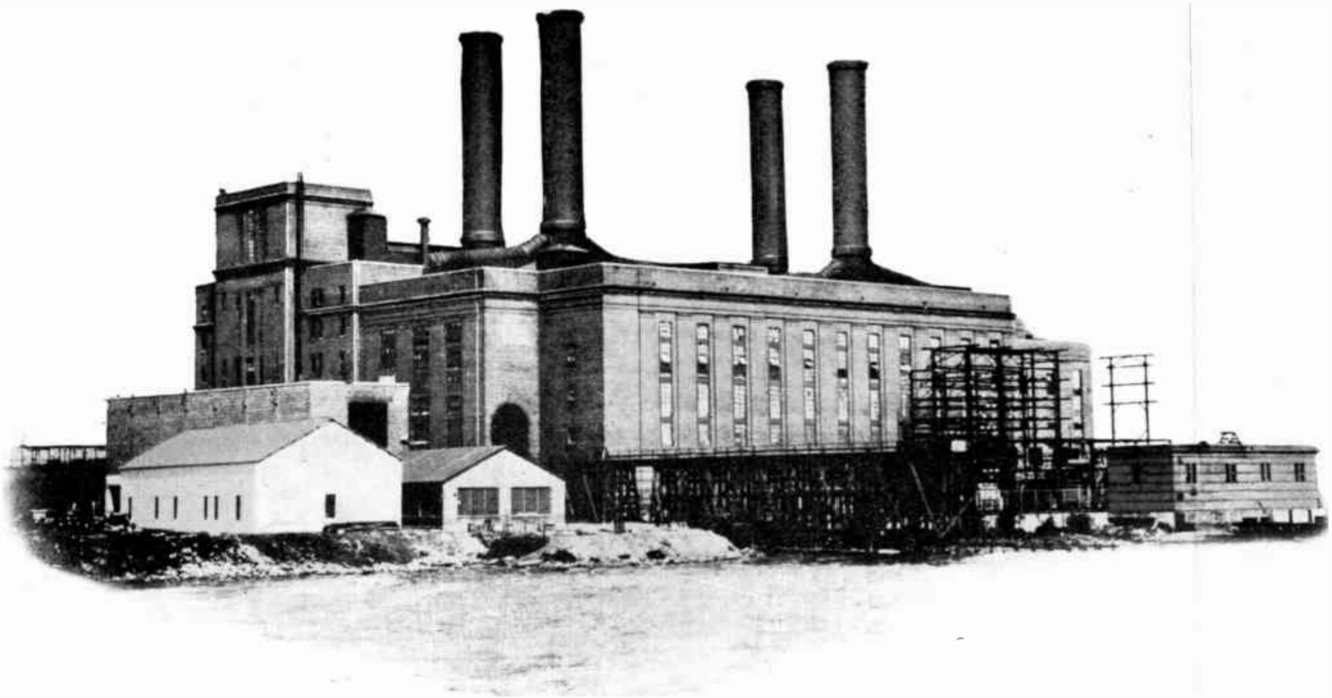

How hundreds of thousands of people are insured light and power

Six days a week thousands of men and women work in more than 21,000 stores and offices lighted by the Buffalo General Electric Company. Through power supplied by this company, wheels turn in more than 4500 industrial plants. 137,000 families can see to cook dinner and read the evening newspaper because this service station sees to it that its turbines operate without fail.

5 turbine units carry the load; one 35,000-kw., three 20,000-kw. and one 60,000-kw. Their continuous and economical operation is assured by correct lubrication with Gargoyle D.T.E. Oils.

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Today these high-grade lubricants are assuring reliable operation of the majority of capital turbine units of 5,000-kw. capacity and up in the United States.

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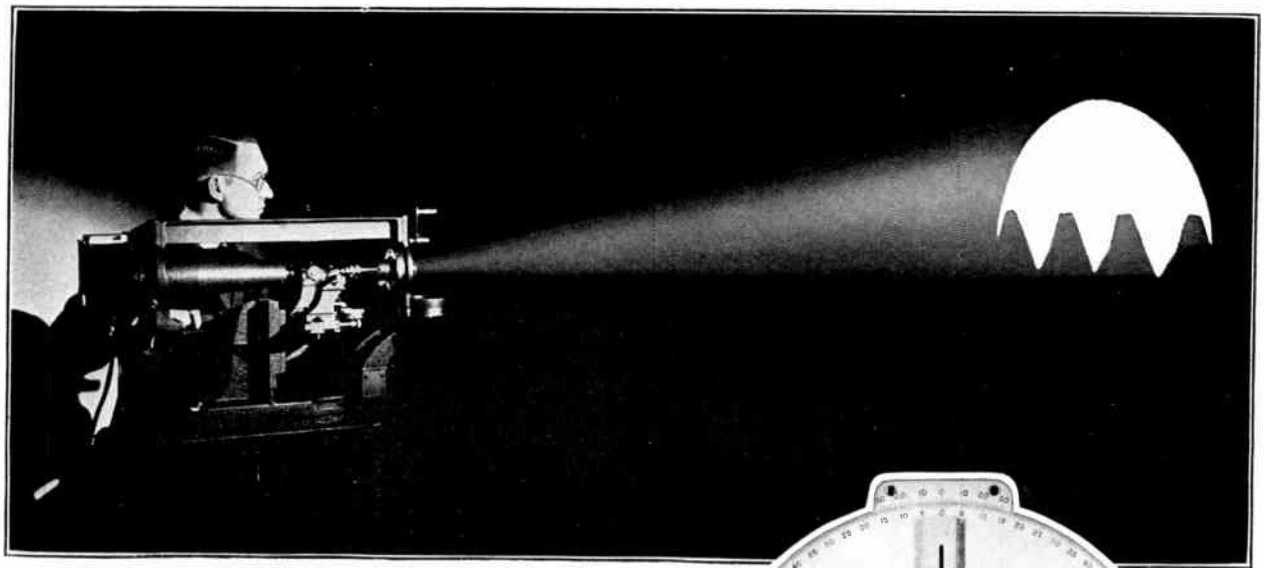

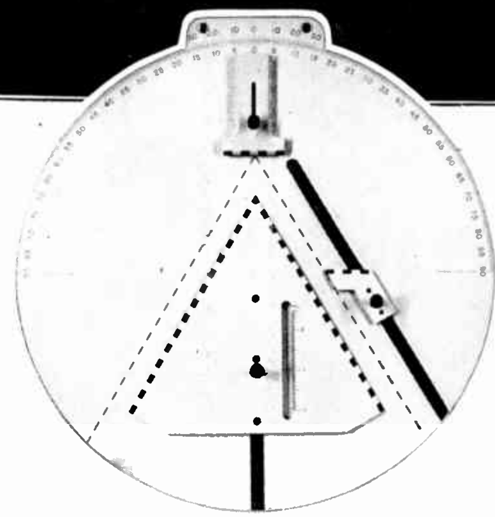

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The Bausch & Lomb Contour Measuring Projector is a remarkable time saver in inspecting screw threads. The image can be projected on to a screen, magnified many times, and errors detected at once.

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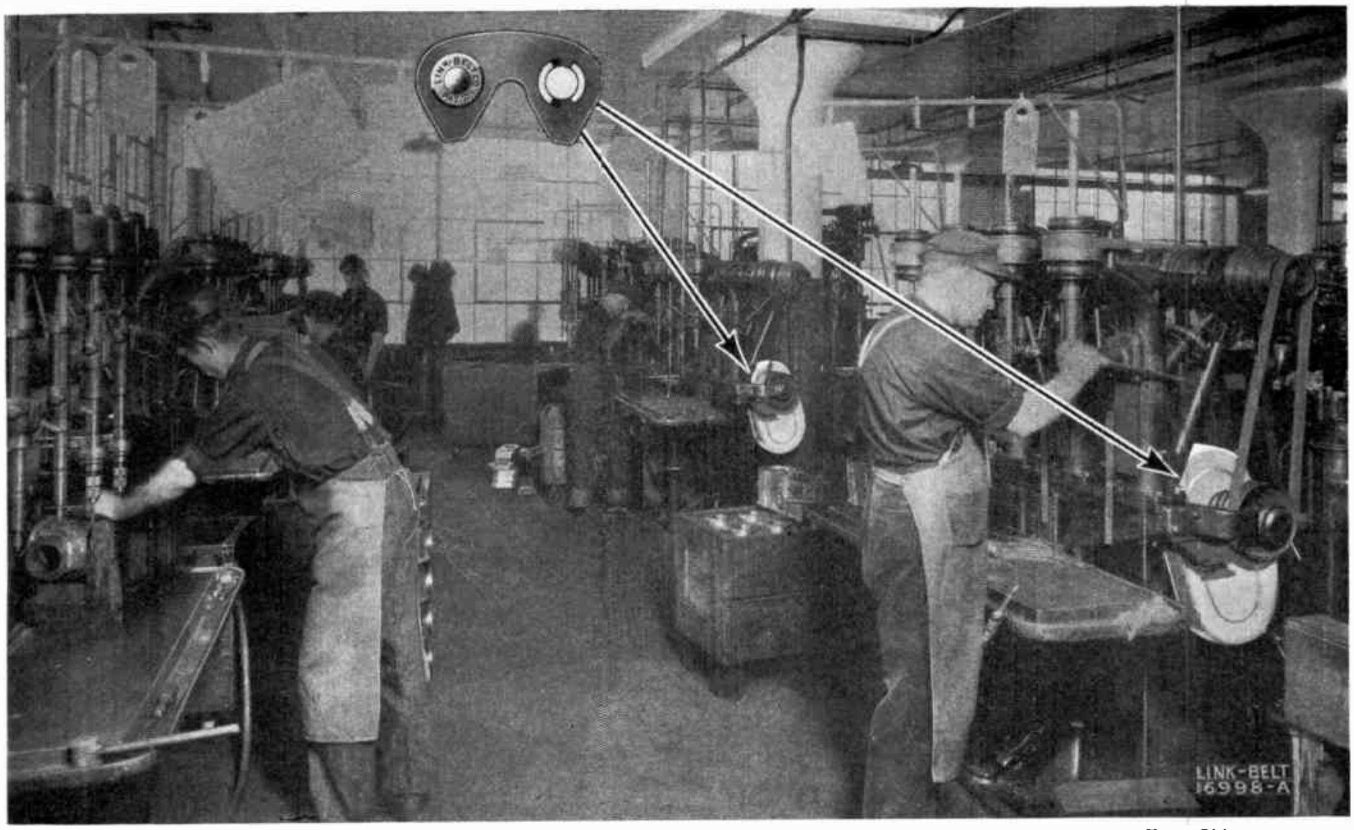

7 Avey drills and 2 Cincinnati Bickford drills using Link-Belt Silent Chain Drives at the Hobart Manufacturing Company, Troy, Ohio.

Link-Belt Silent Chain Drives Increase Production 11% to 15%

AT the new Hobart Manufacturing Company's plant at Troy, Ohio, an increase in drilling machine production of 11% to 15% has been effected by replacing belt drives with Link-Belt Silent Chain Drives.

All belt troubles are eliminated.

Transmission of power at an efficiency of 98.2% is maintained.

Link-Belt Silent Chain Drives are made in sizes from ¼ to 1000 H.P. and over.

Drives ½ to 15 H.P. are carried in stock in many cities. Sendfor a copy of our Machine Tool Book No. 812. Also copy of Stock List No. 725.

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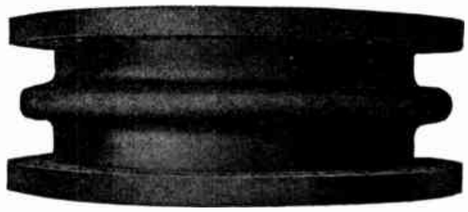

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[1] Will compress one quarter inch axially. when subject to a compressive force of 125 pounds per inch nominal diameter of pipe.

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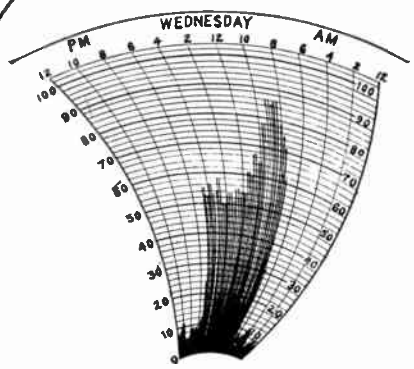
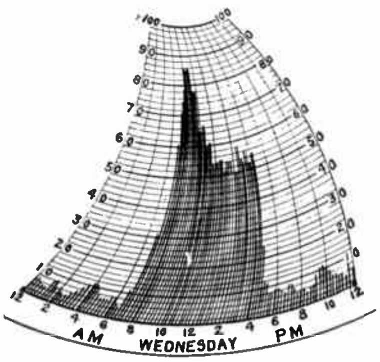


Section of standard 7-day chart, as used to show accurately the time of maximum demand as well as the value.



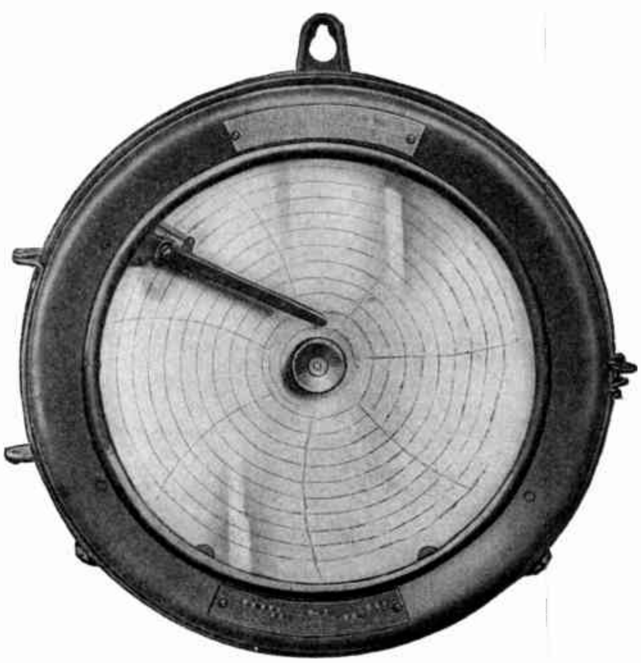
Record of same all-day load as above but on inverted chart, which is advantageous for reading the time and the value of the smaller demands.

The G type demand meters can be used with any type or make of watthour meter—they are flexible. They give a complete record for a week or longer on one circular 8-in. wax-coated chart that is easily filed. They produce a well-defined record that can be accurately read.

Send for Bulletin GEA-903

While these new meters adhere to the same fundamental principles of operation as the G-8 and GS-8, which they supersede, they embody several changes in design. These minor modifications, which are directed toward a general improvement of the line, are as follows:

1. The clocks are equipped with temperature-compensated balance wheels together with improved winding key and arbor.
2. The thermostat and its mounting also are improved.
3. Bronze bearing bushings have been added to important bearings in the mechanism and clock, and the stylus arm has been stiffened.
4. The general appearance has been made much more attractive; the meters are now finished in nickel and black throughout.



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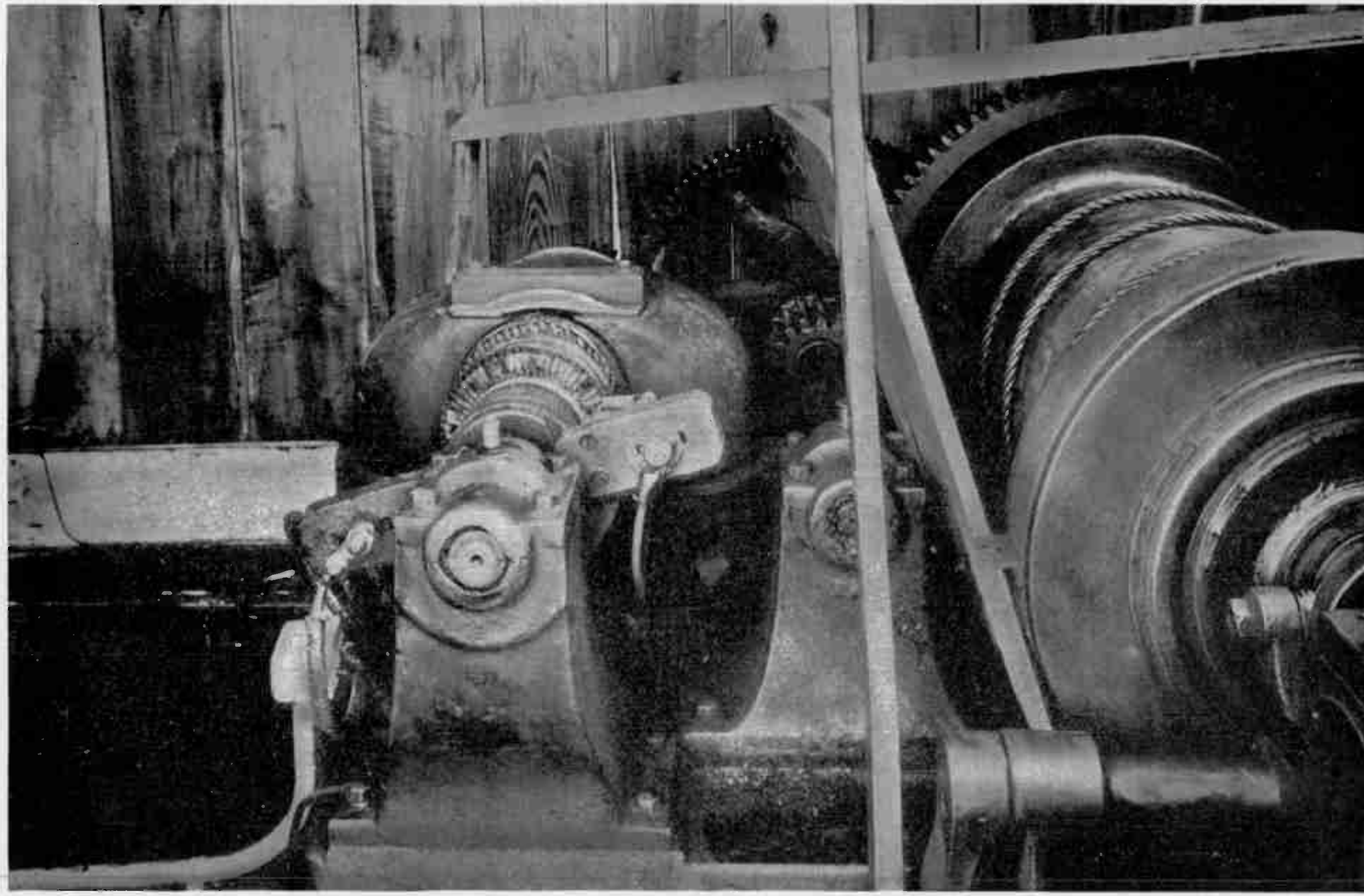

STILL IN ACTIVE SERVICE AFTER THIRTY-EIGHT YEARS

This Thomson-Houston motor was built in 1889 and was exhibited at the Maritime Exposition in Boston in the fall of that year, following which it entered upon its long career in hoisting operations in various localities. For many years it has been at work in its present location on a dock at Lynn (Mass.)

(An article on the development of d-c. motor design appears on p. 116 of this issue)

GENERAL ELECTRIC REVIEW

Vol. 31 March, 1928 No. 3

THE WHY, WHERE, AND HOW OF DIRECT-CURRENT UNITS

Mark Twain once said: "Everybody talks about the weather but nobody does anything about it." The reverse is very nearly true of direct-current applications for they are seldom discussed but are very much used.

Prof. Elihu Thomson reveals in this issue how great has been the advance in direct-current design; and we are prompted to look into present-day applications to learn why, where, and how direct-current units are being used.

Disregarding their almost exclusive use in traction work, we find that direct-current motors deliver 31 per cent of the electric power in industry and by number constitute 36 per cent of all motors in service. The limited areas where direct-current distribution prevails account for only a small fraction of these applications. Most of them are found in the presence of alternating-current power supply; and the extra fixed charges and operating costs involved in making the transition, or in first-hand generation, indicate that there must be some very good reasons for the practice. There are; and "adjustable speed" is by no means the principal one, although it is the one most often mentioned.

Direct current is required for battery charging, for electrolytic work, and for the excitation of the fields of many a-c. and d-c. machines. These uses account for great numbers of direct-current generators in the smaller sizes. That wonderfully flexible system of control known as "generator-voltage control" or "Ward-Leonard control" inherently requires a generator for each motor, or for each group of motors that operates as a fleet. One big motor, that comes to mind, is waited upon by a retinue of two major generators, a lesser generator, three exciters, and two litt e pages or messenger generators. These generator-voltage control applications employ many generators some of which run into large sizes.

Process steam requirements, at one or more reduced pressures, have caused the introduction of the steam turbine from which reduced-pressure steam is taken from the stages or from the exhaust, or both. The turbine drives a generator which furnishes power to the motors driving the process machinery. In many cases this is a d-c. system within an industry which is mainly operated on an a-c. power supply.

Direct-current motors are preferred for some applications and are practically indispensable for others, because of certain characteristics peculiar to them. Base speeds are not limited to synchronous zones and they may be made constant, adjustable, varying, or adjustable-varying by choice. The control devices handle only small currents for speed adjustments or reversals. Stable speeds from zero to maximum in either rotation are easily obtained and controlled by the generator-voltage control system.

Dynamic braking is effective even at very low speeds, thus reducing friction braking to practically a holding function only. Starting and accelerating torques of high values are available with comparatively small energy demands because speeds may be inherently tempered to compensate for the high torques.

Commutation, the one-time bugbear of direct-current design, has been so thoroughly mastered that it is no longer an important factor in the equations leading to a choice between a-c. and d-c. power.

Direct current is used extensively in the iron and steel industry for both major and minor operations. It is indispensable in the making of paper. Machine tools which enter into hundreds of industries are largely equipped with direct-current motors. High-speed passenger elevators as well as the slowest heavy-duty freight elevators are usually operated by direct current though the primary supply is usually alternating current. Power shovels of the large sizes rely on the "iron hand in the velvet glove" qualities of generator-voltage control to perform operations that would otherwise tear them to pieces. Mining requires direct current for cutting, hauling, and hoisting. Material handling operations throughout the whole industrial field consistently employ this flexible power medium. Feed mechanisms for all kinds of materials take advantage of the adjustable-speed direct-current motor.

How d-c. motors are used would require pages to tell and only a few examples can be cited here. A car dumper that hoists and upsets 120-ton carloads of coal every 40 sec. uses generator-voltage-controlled motors on the cradle hoist, on the "mule" that delivers the cars, and on the apron that guides the coal into the ship; yet no excessive load peaks can break through to the a-c. supply lines. A 1000-hp. mine-hoist motor, subject to 1800-hp. peaks, works on a fast cycle yet the a-c. motor driving the generator takes 600 kw. from the line as steadily as though it were driving a fan.

The paper machine, a line of ten disconnected units, makes "news" 25 ft. wide at the rate of 1200 ft. per min. Each section is driven by a d-c. motor. The ten motors are speed-adjustable as a fleet by a single rheostat. Each can be dropped to a stable low speed for the servicing of its section. Each section runs at an independently micro-adjustable speed slightly faster than the preceding section. The paper must not break.

The giant reversing blooming-mill motor with its retinue previously mentioned is regularly fed with white-hot steel ingots the size of office tables. Its speed from zero to 80 turns per minute, its reversals as rapid as one every three seconds, and its torque up to 2,400,000 lb. are controlled by the manipulation of comparatively microscopic currents. The bumps incident to reducing ingots of steel to long slender blooms hardly cause a ripple in the a-c. supply system.

The one-time general impression that a-c. supply required a-c. motors has faded away and most of the fading has taken place in very recent years.

R. H. Rogers

A Review of the Development of Direct-current Motor Design

By PROF. ELIHU THOMSON
Director, Thomson Research Laboratory, General Electric Company

WHEN reviewing the developments that have brought direct-current motors into existence as we know them, all those crude early efforts to obtain power from the currents yielded by voltaic batteries may be neglected. We may also exclude motors whose power depended on the intermittent pulls of iron armatures toward magnet poles, the magnetic drawing of cores into solenoids, and the successive exertions of force on iron plates or armatures mounted on wheels for rotation by electromagnets whose circuits were closed and opened by a breakpiece or cam on the shaft.


ELIHU THOMSON

Not indeed until the development of machines of the Gramme dynamo type, and its modification, the Siemens type—both inherently foreshadowed in the middle sixties of the past century by Paccinotti—did we have the basis for a real electric motive power development. In the early seventies it was not unusual to find references to the reversal of dynamos, such as the Gramme, and their use as motors. It began to be recognized that a well-organized dynamo-electric machine would make a good motor if supplied with such a current as it would produce when driven as a generator. This dual characteristic was exemplified at the 1876 Philadelphia Centennial Exhibition where a Gramme dynamo driven by power was connected to a similar machine receiving current and running as a motor. This dynamo motor was first connected to a centrifugal pump which raised water a few feet. The water then flowing over a dam gave the appearance of a small waterfall. No other exhibit in Machinery Hall was more impressive. This application took place long before the appearance of the first incandescent lamp, and was contemporaneous with the first announcement and exhibition of the then new, speaking telephone of Alexander Graham Bell.

In those early days there was much to learn regarding the proper proportioning and relationship of windings in the construction of dynamos, and necessarily much more as to the factors which must enter into the construction of a satisfactory direct-current motor; *e.g.*, the endeavor to reduce "idle wire." This attempt led to elongated cores and small armature diameters in both dynamos and motors. The bugbear of idle wire as found in the Gramme and similar machines was corrected through improvements in design made by the writer and others. Another superstition of that time influenced existent design; *viz.*, that long rather than short magnet cores should be used. Long magnet cores were supposed to project the lines of force through the armature, which the shorter magnet cores would fail to do. We early recognized that after all it was a question of ampere-turns, and that if these turns were allowed to exert their full effect on the armature core there would be no need for any special length in a field magnet core.

In the early years, or those around 1878-80, there was very little demand for electric motors. There were no lines to which they could be connected. Consequently, the electrical engineer of the time devoted his efforts to dynamo-generators, feeding arc lamps as the sole system of distribution. Not until 1880 was there any promise of extension in incandescent lighting. This broadening of the field came through the invention by Edison of his high-resistance carbon-filament lamp in October, 1879. With the gradual development of electric stations, generating direct current in large centers, a field was opened for direct-current motors to replace small steam or gas engines economically.

When the factory of the American Electric Company was established in New Britain, Conn., in 1880, this plant furnished one of the earliest examples of transmission to and operation of an electric motor of several horsepower. The available space for a pattern shop was too far away from the main engine to allow power transmission by shafting and belting. In this case, a T-H dynamo was placed near the main shaft and belted to it; two wires from the dynamo were led to the pattern room where they were connected to a second dynamo which, however, took current as a motor and was belted to the woodworking lathes, saws, etc., of the pattern room. Novel appliances were added for changing power and speed. This drive proved to be very satisfactory for the woodworkers and was continued in use while the plant was operated, or until the Works were removed late in 1883 to Factory A of the West Lynn Plant of the Thomson-Houston Electric Co.

Outside of a few small motors for use on series lines operating arc lamps, there was still little demand for electric motors until about 1885. A few appeared as exhibits in the Franklin Institute Electrical Exhibition in Philadelphia in the Fall of 1884. The great convenience and effectiveness of direct-current motors, especially for isolated printing-plant machines, woodworking, pumping, and similar drives, resulted in a considerable and increasing demand for central generating stations to furnish the driving current.

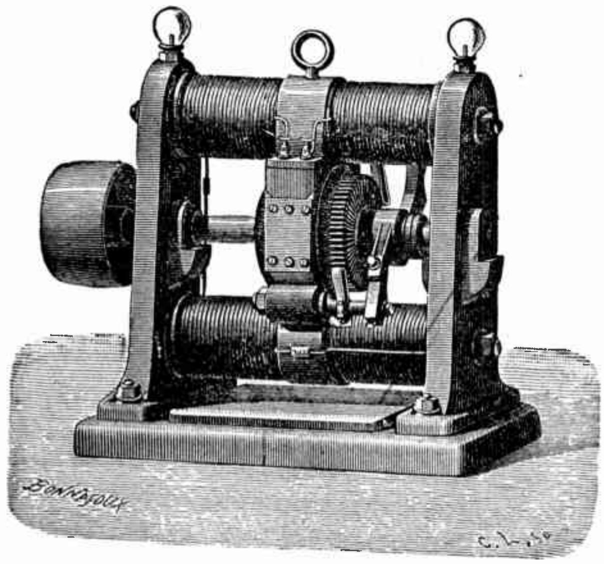

Fig. 1. Gramme Dynamo, "A" Pattern

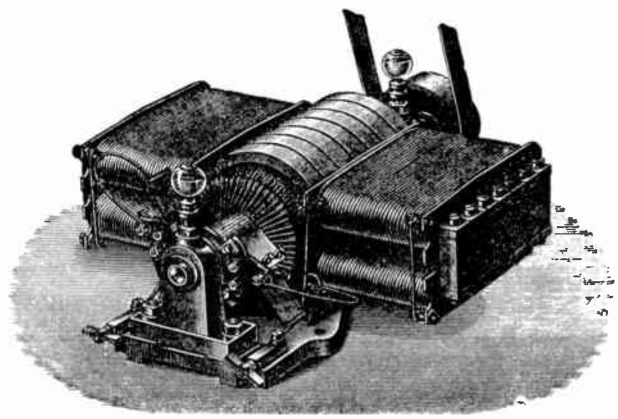

Fig. 2. Siemens Dynamo, Horizontal Pattern

Fig. 3. Typical Two-pole Riveted-frame General-purpose Direct-current Motor of Size Ranging from ½ to 3 hp.

Fig. 4. Typical Four-pole Cast-magnet-frame General-purpose Direct-current Motor of Size Ranging from 3 to 200 hp.

Improvements in design took place rapidly; composite carbon brushes replaced copper wire or gauze, thereby reducing commutator and brush wear to a minimum. Fortunately, the problem of securing good commutation was more easily solved for motors than for generators, because the reactions which in generators are additive, are differential or compensatory in motors. As a later development, the so-called commutating field pole was made available in dynamo construction, including, of course, in such designation, direct-current motors.

Beginning about 1887, sturdily constructed direct-current motors were first applied to electric traction, diverting the skill of the best designing engineers to machinery demanding mechanical perfection, robustness, reliability in electrical qualities, good insulation, and other properties to meet the harsh conditions of use under street cars.

The wonderful flexibility of a well-built direct-current motor is probably not equalled by any other device producing power. As its running speed depends on the development of a counter e.m.f., speed variations within fairly wide limits can be obtained by simple modification of the effective magnetizing ampere-turns of the field magnets. The load can be widely varied under any given speed, for the speed is self-regulating within a small per cent even though the load vary from nil up to full or the reverse. Also, starting under normal or heavy load is accomplished easily and promptly by using resistance in the armature circuit, this resistance being cut out when the motor reaches full speed. The efficiency factor in a well-designed direct-current motor, even of relatively small output, leaves practically nothing to be desired; while reliability in operation depends on the excellencies of mechanical and electrical design coupled with good workmanship. Change of direction of rotation can, in like manner, be attained by the simple reversal of current in either the armature or the field circuit, practically no other adjustment being required.

This ease of reversal and very close regulation of speed control assures an indefinite continuance of demand for direct-current motors and, in fact, compels their adoption and use for driving many of the automatic and special machines now so largely required in mass and repetition manufacture.

As an indication of the satisfaction that has attended the use of direct-current motors, there are today great numbers of such machines in operation which were placed in service 20 or 30 years ago, and which may reasonably be expected to continue in satisfactory operation for years to come.

Briefly traced, we have seen the direct-current motor, progenitor of all the commercially practicable types, first appear as a crude conversion of the Gramme or Siemens dynamos.

Slowly improved to secure the proper proportioning and relationship of windings, these "trial and error" initial methods employed were the necessary precursors of a technique which, step by step, has made dynamo-electric design and construction an exact science. We now reach culmination in the modern direct-current industrial motor with its detailed wealth of mechanical and electrical features.

Carefully proportioned main and commutating poles and stabilizing windings permit close speed regulation. Refinements of commutator, brush rigging, and other essentials insure practically complete freedom from commutation troubles.

Bearings, for years a source of recurrent trouble, have as a result of refined accuracy secured through modern design, manufacture, and assembly, become available in sleeve, ball, and roller types with a degree of trustworthiness beyond cavil.

In summary, the arrangement and distribution of all active and inactive materials embodied in the latest direct-current motors provide a combination of strength, compactness, attractive appearance, stability, and efficiency guaranteeing permanent satisfaction under stress of hard and continuous service.

Professor Thomson Honored

Prof. Elihu Thomson, director of the Thomson Research Laboratory of the General Electric Company, has been named a member of the American Committee of the World Congress of Engineers to be held in Tokyo (Japan) in November, 1929. The appointment was made by Herbert Hoover, Secretary of Commence.

This is the first congress of its kind ever held and according to Baron K. Furuichi, President of the Engineering Society of Japan, is for the purpose of promoting international coöperation in the study of engineering in all its branches and in stimulating a sense of brotherhood among the engineers of the world.

Among the other well-known members of the American Committee appointed by Secretary Hoover are Thomas A. Edison, John Hays Hammond, Charles M. Schwab, and Orville Wright.

Electric Heaters Prevent Freezing Troubles in Compressed-air Application in Cascade Tunnel

In connection with the construction of the new Cascade Tunnel, seven and three-fourths miles long, and two shorter tunnels for the Great Northern Railway Company, A. Guthrie & Company, of St. Paul (Minn.), has demonstrated the versatility of electric heating equipment in a unique manner. This consists in applying water immersion heaters to the preheating of air employed to operate an air-driven shovel.

The heating units, which were of General Electric manufacture, are used to preheat the compressed air that operates a No. 40 Marion shovel employed in tunnel excavation work. This shovel is of the ordinary steam engine-driven type, from which the boilers have been removed and replaced by a large air receiver.

Much engine trouble was experienced as a result of the freezing of the moisture in the air on expansion, to avoid which it was found necessary to heat the air.

The receiver on the shovel is four feet in diameter by seven and one-half feet high. Water immersion heaters of helical type were chosen because such units are easily installed. To prevent the units from burning out in the air receiver it was necessary to use two 220-volt units in series, on a 220-volt circuit. Nine of these groups were placed in multiple, 18 units thus being employed.

When the shovel is operating, sufficient air passes through the receiver to keep the heating units from burning out. To prevent overheating the air, and to guard against damaging the units, an indicating thermometer is placed where the shovel operator can readily note the temperature of the air. When this temperature rises above 300 deg. F., the operator shuts off the heaters. Although a higher temperature might not burn them out, it might occasion other difficulties, as, for example, those arising from a breakdown of the lubricating oil.

A large share of the heat energy placed in the air is regained through the expansion cycle of the engine on the shovel.

Load Ratio Control

PART I

Method Employed in Voltage Control—Principle of Operation of Various Types of Load Ratio Control—Possible Reduction in Reactor Capacity and Losses—Symmetrical, Unsymmetrical, and Bridging Connections—Special Considerations

By L. F. BLUME

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Pittsfield Works, General Electric Company

At a joint meeting of the Electrical Section of the Western Society of Engineers and the Chicago Section of the A.I.E.E. the author presented the material contained in this article, which will be published in the Review *in two parts. The present installment deals with the general principles of load ratio control; a later one will treat of its fields of application.—*Editor

IT frequently happens in the progress of applied science or engineering that an invention does not find a positive field of application until many years after its original development. This long delay may not be due so much to the undeveloped state of the invention, as to the fact that the field in which it eventually will be used successfully has not yet been established. Of this the present subject is a very good illustration. Many years ago, the inherent weakness of constant-potential electric lines in general, and transformers in particular, to the effect that voltage could not be maintained constant at all points if the load varied through a wide range, was clearly recognized, and various steps were taken to overcome or neutralize this inherent defect.

The problem was approached by three different methods. First, serious attempts were made to overcome the regulation drop inherent not only in the transformer, but also in the line by designing the transformer so that the secondary voltage would actually increase in value with increasing loads.

Some very ingenious schemes were suggested, such as connecting two transformers having different ratios in series in both primary and secondary sides. By properly choosing the magnetizing characteristics of the two transformers, the combination can be made to give inherently rising voltage characteristics with increasing loads.

Ingenious as this scheme was, it had many practical limitations which caused it to be abandoned. As it had rising voltage characteristics only at high power-factors, it could not be made to compensate correctly for variations in applied voltages; and it had other serious limitations.

The second method attempted was to change the ratio of transformation by mechanical means, by providing the transformer with a number of taps and a suitable switching device so that, at the will of the operator, the voltage ratio could be varied. Many years ago, a considerable number of these outfits were built and put into successful operation, and the principle upon which they were based is quite similar to that embodied in what are now being built and known as transformer ratio control or tap-changing equipments (Figs. 1 and 2).

However, the induction regulator was being developed at the same time, as the third method of obtaining voltage control; and for many years, the induction regulator has very successfully dominated the field. It is, therefore, a pertinent question to ask for the reasons that have caused the present rapid development of transformer ratio control equipments. The following two facts, if taken together, seem quite sufficient to account for the growing demand for them.

In the first place, the kv-a. capacity concentrated in central stations, and the kv-a. of individual transformer banks has been steadily increasing from year to year, and these stations are being more and more connected into networks. With this naturally has come about an increasing demand for voltage control devices in larger units than heretofore, and at higher voltages. The second fact is found in the comparison of the cost curves of induction regulators and ratio control equipments, plotted as a function of the kv-a. ratings, which shows that the induction regulator is the less expensive for the smaller ratings and for moderate voltages; whereas for the larger transformer ratings, and especially for higher voltages, ratio control equipments are proving to be considerably less expensive. Thus, the present return of the tap-changing method has been largely brought about by the development of a field of application in which it is proving to be the most economical, and at the same time the most efficient, solution for voltage control.

The principle of operation underlying transformer ratio control should first be examined, so that the various types can be compared as to electrical performance. It is quite possible, by employing two transformers operating in parallel on a given line, to shift from tap to tap without dropping the load, if the operating handles of the ratio adjusters are brought out of the tanks, and so located that they may be operated with excitation on the transformers. The diagram in Fig. 3 represents two transformers connected in parallel on both high- and low-voltage sides; and each transformer is provided with ratio adjusters. A tap change can be made without dropping the load by first opening circuit breaker A, which shifts the load from transformer A but does not remove the

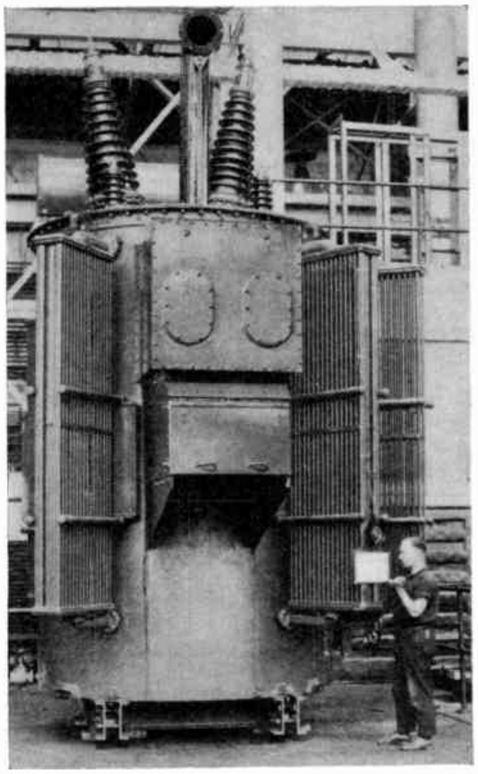


Fig. 1. External View of a Load Ratio Control Transformer

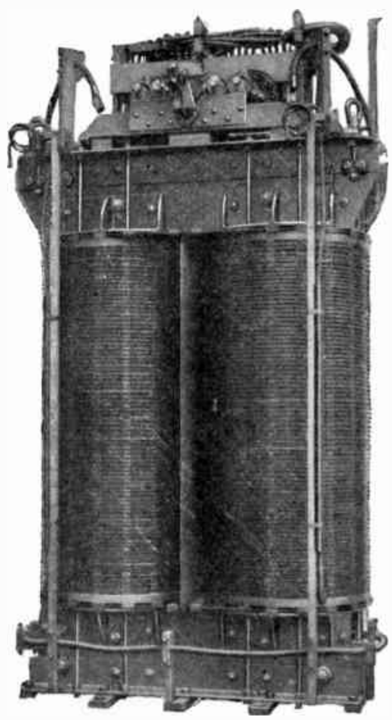


Fig. 2. Load Ratio Control Transformer Removed from Tank

excitation since the high-voltage side is still connected to the line. The ratio adjuster in transformer A can now be moved to the next position, after which circuit breaker A is again closed. The two transformers are now operating, with unequal ratios, in parallel; and a circulating current as indicated by the arrows flows between them. Next, circuit breaker B is opened so as to permit the ratio adjuster in transformer B to be shifted to the adjacent position. Circuit breaker B is then closed and the two transformers are again operating under normal conditions, but with a changed ratio.

Examining more closely what happens during this process, it is important to note that when one of the

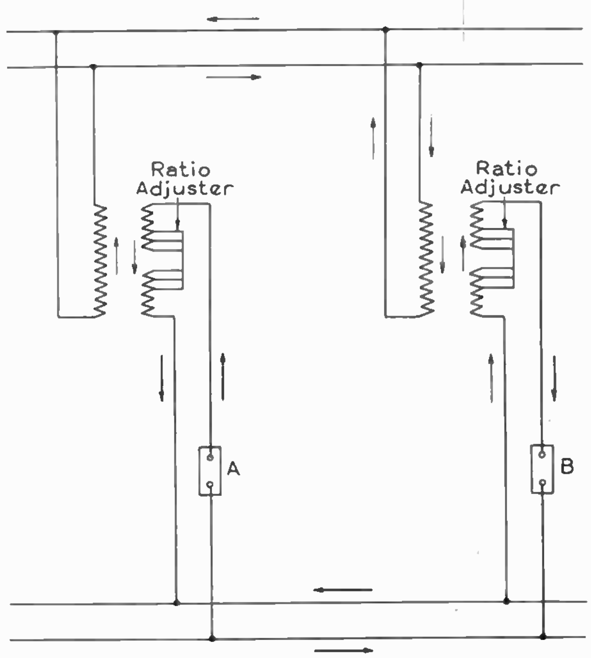


Fig. 3. Elementary Scheme of Connections for Voltage Adjustment by the Use of Two Parallel Transformers Having Suitable Means for Changing Taps

circuit breakers is open, the voltage which exists across the open breaker is not the line voltage, but a very small fraction of it, owing to the fact that the disconnected transformer receives excitation from the high-voltage side. Thus the voltage which the breakers are obliged to open is equal to that arising from the difference in the ratio of transformation of the two transformers, plus the impedance drop which exists in the loaded one.

Aside from the inconvenience of operation in this manner, and the liability of the operator to make serious mistakes, this method of tap changing is undesirable, because the doubling of the load on one winding doubles the impedance drop, so that, during the switching period, the increase in impedance results in an undesirable fluctuation in secondary voltage. Thus, with 10 per cent transformer reactance, at full load the reactance drop is increased to 20 per cent which is almost five times the voltage between adjacent taps.

By designing one transformer with a multiple circuit, and connecting it as shown in Fig. 7, these various difficulties are readily overcome. The reactance between the multiple paths can now be controlled independently of the main transformer reactance, and it can be made small enough to prevent objectionable fluctuations in voltage during the process of switching, and large enough to prevent excessive circulating current.

When transformer ratings are such that a multiple circuit can be provided without additional expense, this arrangement is very desirable, since it does not involve additional kv-a. capacity of apparatus. This scheme, however, possesses two inherent objections:

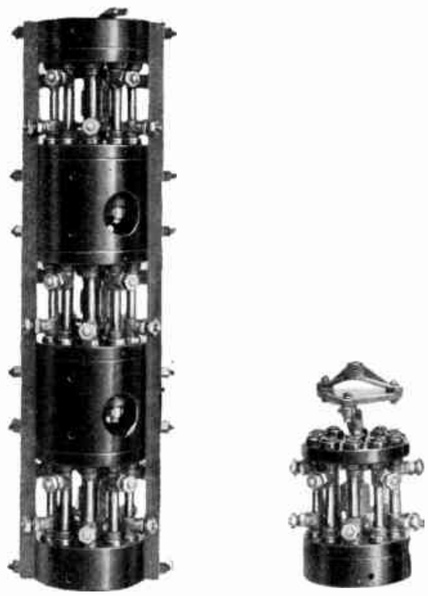


Fig. 4. Ratio Adjusters Used on Load Ratio Control Transformers. Three-phase at left; single-phase at right

First, during the process of switching, momentary full load is thrown on one-half of the transformer winding and, in the very remote possibility of the operating motor failing to complete its cycle, this load may be present for a sufficient time to seriously overheat the windings. To provide against this contingency, means either to design the windings with an appreciably greater copper cross-section (and this cannot be done without increase in size and cost) or, resorting to a less expensive method, to provide circuits which warn the operator of the stopping of the motor in a non-operating position. The second limitation of the circuit is that in some ratings, the transformer design does not lend itself so readily to a multiple circuit without appreciable increase in cost.

These practical considerations make it desirable to depart in many cases from the simple arrangement shown in Fig. 7, by placing only a small portion of the circuit in multiple, as shown in Fig. 8. Here, the multiple circuit consists of only the ratio adjusters and circuit breakers, and an auxiliary reactance is employed to limit the circulating current. Here, also, nine taps are used to derive nine operating voltages. Whether this or the circuit shown in the Fig. 7 should be used depends largely upon design considerations; and the choice should, for the most part, be determined by whether the multiple circuit with extra taps and leads costs more than the current-limiting reactor.

It will be well at this point to examine in greater detail the condition that the current-limiting reactor must meet during the tap change. In the operating

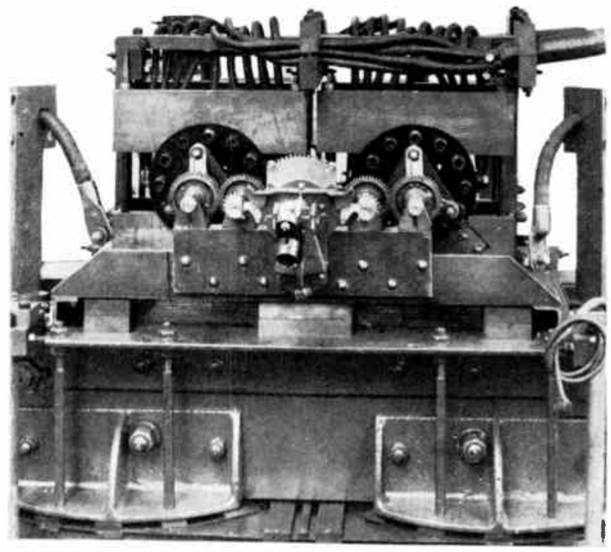


Fig. 5. Close-up View of Intermittent Gear Used to Drive Load Ratio Control Equipment. Single-phase application

position, both sides of the reactor being connected symmetrically to the same tap, the only voltage which appears across it is due to the leakage reactance between the reactor halves, and this may be made negligible by interlacing. The load current divides equally between the two halves.

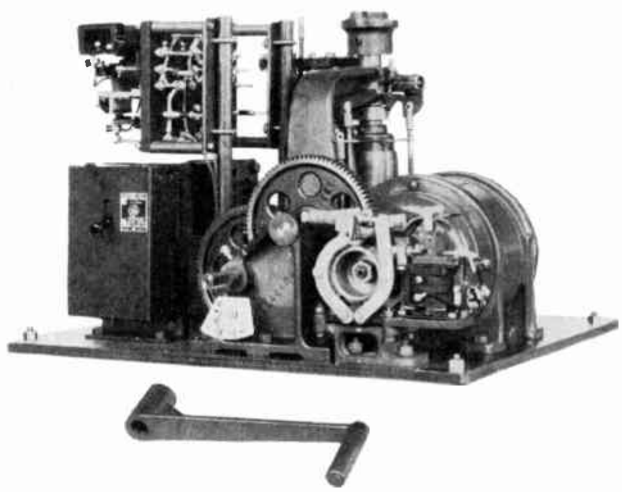


Fig. 6. Driving Mechanism for a Transformer Load Ratio Control. Direct-current Drive

In the process of switching from one tap to the next, there are two unsymmetrical positions: the first, in which one ratio adjuster only is connected; the second, in which both ratio adjusters are connected, but on adjacent taps. In Fig. 9, a close-up of all three positions are shown; diagram (a) being the operating position with the load current dividing equally in the two reactor halves; diagram (b) with connection through reactor to one tap only, in which case the full load current flows through one half of the reactor; diagram (c) where the reactor is bridging adjacent taps, and therefore tap voltage is impressed on the reactor. To aid in considering these diagrams, let us call (a) the symmetrical, (b) the unsymmetrical, and (c), the bridging positions. In the bridging position, two currents are flowing in the reactor, *viz.*, the load current, which divides in two equal halves as in (a), and a magnetizing current or circulating current, which flows as a result of the reactor bridging across adjacent taps. The conditions imposed by the bridging and unsymmetrical positions are conflicting; since to design with relatively high reactance, so as to have

Occasionally, the arrangement of windings is such as to make it desirable to reduce the number of taps to a minimum, and there are other cases where a particularly large number of taps are desired. For example, in one instance when eighteen operating voltages were wanted, and where, on account of the design of the windings, it was not desirable to have a multiplicity of taps, the design selected had only five tap sections; and by using all positions for permanent operation, the eighteen steps were obtained.

In the majority of cases, however, it has been found desirable to make the unsymmetrical and bridging connections only transitional or switching positions and not operating positions, in order to make the size of the reactor smaller and, at the same time, to save in losses. Permanent operation on only the symmetrical position incurs no losses due to circulating current, or to core loss in the reactor. Furthermore, since twice the number of taps must be used for permanent operation on only the symmetrical position, the voltage between taps is only one-half, and therefore the voltage impressed on the reactor during

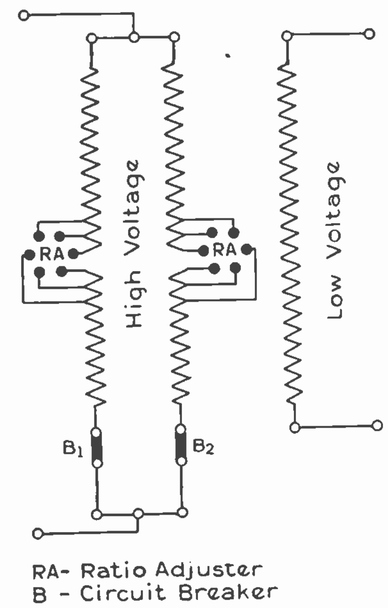


Fig. 7. Diagram of a Transformer Designed with a Multiple Circuit, by Means of which Many Difficulties which Accompany Tap-changing Under Load Are Overcome

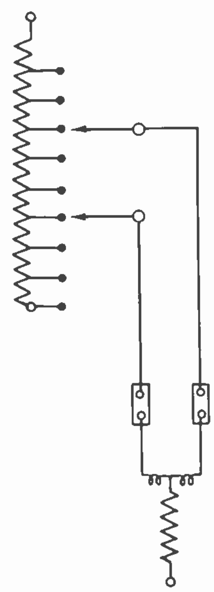


Fig. 8. A Connection Diagram for Load Ratio Control in which Nine Taps Are Provided for Deriving Nine Operating Voltages

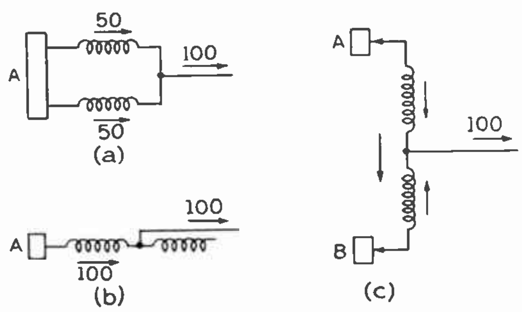


Fig. 9. Current-limiting Reactor Connections in Load Ratio Control
(a) Symmetrical (b) Unsymmetrical
(c) Bridging

a small circulating current, will result in a relatively large reactance drop in the unsymmetrical position and, consequently, an undesirably large fluctuation in line voltage. If the reactor is of the iron type, gaps are necessary in the magnetic circuit. A magnetizing current of not less than 50 per cent full-load current is considered necessary to give good results.

It would be logical to ask, at this point, why better use were not made of all of these positions by making them permanent operating positions. This is possible; and it has been done. For example, the connection shown in Fig. 8, in which nine taps are used to derive nine operating voltages, could be replaced by the connection shown in Fig. 10, if all positions are made operating positions. Here, nine voltages are derived by using five taps instead of nine, and three-point instead of nine-point ratio adjusters. In this, the symmetrical position is not used, the sequence of operation being alternately the unsymmetrical and bridging positions.

switching is only one-half of what it would be if all possible positions were made operating positions. The net result is that the kilovolt-ampere rating of the reactor is about one-third, and the energy losses in the reactor only one tenth. In the larger ratings, this is by no means negligible. For example, in a 60,000-kv-a. bank having ratio control in 2 per cent steps, *the reactor rating can be reduced from 3000 to 1000 kv-a.* by doubling the number of taps and operating only on the symmetrical position; and at the same time, *the reactor losses can be reduced from 10 to a single kilowatt.* These considerations alone have made it very desirable to use a larger number of taps in equipments involving the larger ratings.

Another advantage of operating permanently on only the symmetrical positions is that the voltage steps will always be uniform. For the purpose of comparing the voltages derived, the diagrams in Fig. 11 are useful, (a) giving the voltages when operating only on symmetrical positions and (b) the voltages when operating alternately on unsymmetrical and bridging positions. The diagrams are drawn to the same scale, and both show the voltages for two steps, the desired voltage of each step being 100. In Fig. 11 (a) the three operating tap positions are *A*, *B*, and *C*, the voltages derived on the switching positions being *1*, *2*, *3* in going from *A* to *B*, and *3*, *4*, *5* in going from step *B* to *C*. Both diagrams are plotted for full load at 80 per cent power-factor, and it is assumed that the reactor has been designed for a circulating current of 60 per cent. In (b), Fig. 11, the voltage between adjacent taps *A* and *B* is 200, but the tap voltage derived on the operating positions *1*, *2* and 3 are alternately 50 and 150. The reactance drops introduced by the reactor during operation on the unsymmetrical position are *A-1* and *B-3*, whereas in the bridging position, no drop, or relatively little drop, is introduced by the reactor. The resulting non-uniformity in tap voltage may be improved by designing the reactor for greater circulating current, and of course, the conditions are better for lighter loads and higher power-factor; but on the other hand, with over-loads, it is quite possible for one of the steps to become reversed so that when the equipment is operated to gain in the voltage, actually a lower voltage is obtained.

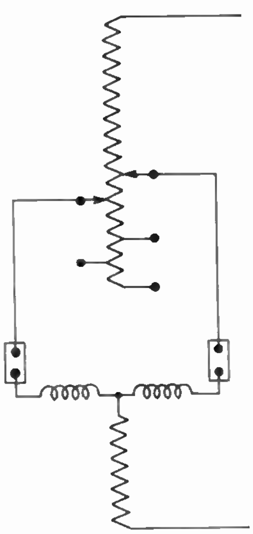


Fig. 10. A Connection Diagram for Load Ratio Control in which All Positions Are Made Operating Positions, Thus Making it Possible to Obtain Nine Voltages by Using Only Five Taps

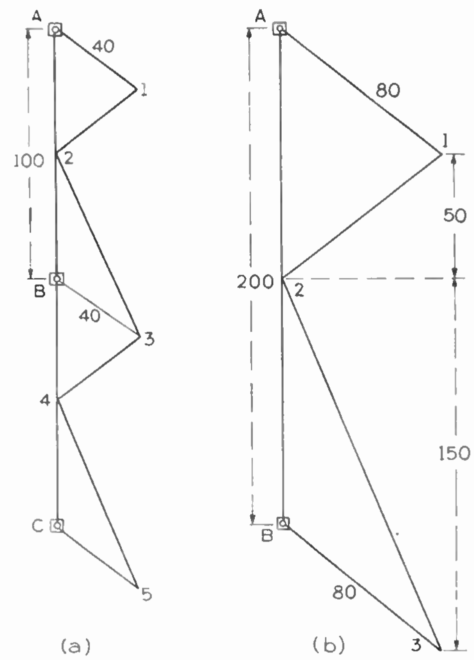


Fig. 11. Diagram Showing the Voltages Derived in Tap-changing with a Circulating Current of 60 Per Cent Full Load at 80 Per Cent Power-factor

(a) Voltages derived from symmetrical position shown in (a), Fig. 9.
(b) Voltages derived from unsymmetrical and bridging positions (b) and (c), Fig. 9. Note the inequality of voltage steps produced.

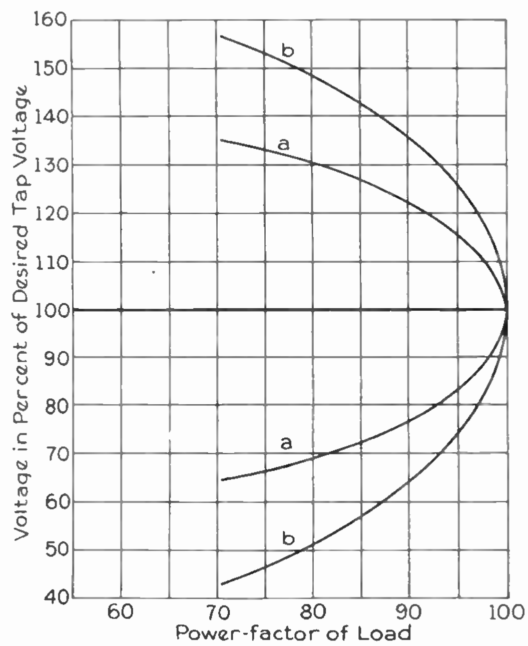


Fig. 12. Unequal Voltage Steps, Caused by Operating Alternately with One and Two Taps Closed, Plotted as Functions of Power-factor

Curve *aa*—Reactor designed for 100 per cent circulating current.
Curve *bb*—Reactor designed for 60 per cent circulating current.

In Fig. 12, the relations between unequal voltage steps are plotted as a function of power-factor. Curve *aa* is obtained when the reactor is designed for 100 per cent circulating current and curve *bb* when the reactor is designed for 60 per cent circulating current. The curves show that, even at the higher power-factors, a considerable inequality in adjacent voltage steps is obtained; for example, at 90 per cent power-factor and full load, we obtain from curve *bb* adjacent voltage steps equal to 65 per cent and 135 per cent, where two 100-per cent steps are desired.

Another objection to operating with connections on only one tap is that short-circuit currents are concentrated on one circuit; consequently, unless the current-carrying parts are designed much more liberally, the safety-factor, under short-circuit conditions, will be very greatly reduced.

Furthermore, during short circuits, the voltage drop across the reactor will rise, so that a relatively high voltage appears across the open breaker. For example, a short-circuit current of ten times normal, flowing through one side of the reactor, may increase the voltage across the open circuit breaker in the neighborhood of 50 per cent of line voltage. With symmetrical operation, both breakers being closed, this voltage rise cannot occur.

The circuits which have just been described show what can be done with the combination of ratio adjusters and circuit breakers, to obtain voltage control. When a group of circuit breakers or contactors are used alone, they are connected in a manner shown in Fig. 13. This method[1] is one of the oldest forms of ratio control arrangements; and it was used in connection with relatively small units many years ago.

[1] It was fully described in the paper, "Tap Changing Transformer Under Load," by L. H. Hill, at the A.I.E.E. Convention, Pittsfield, Mass., May, 1927.

The arrangement just described involves two kinds of operating positions, *viz.*, one in which only one contactor is closed, and another in which two adjacent contactors are closed and bridged by the current-limiting reactor. The voltage diagram at the right-hand side of Fig. 13, showing the voltages obtained in three successive positions at full load, marked **1**, **2**, and **3**, indicates that the steps are not uniform as was previously described. However, by adding (Fig. 14) another contactor, 6, which is connected so as to short-circuit the reactor in alternate steps, uniform voltages may be obtained. This arrangement involves the operating voltages 1, 2, and 3, shown in the voltage diagram at the right-hand side. Thus, a considerable improvement in operating characteristics is obtained

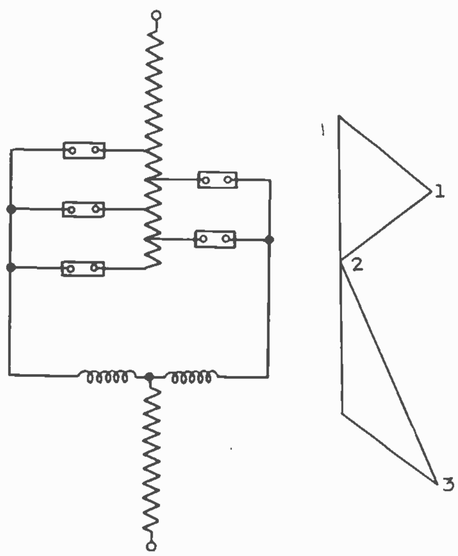


Fig. 13. Connection Diagram Showing How Load Ratio Control May be Accomplished by the Use of Circuit Interrupting Equipment Alone. The voltage diagram appears on the right

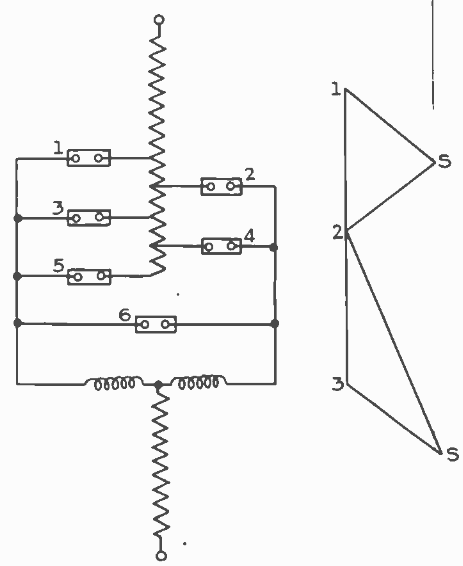


Fig. 14. The Switching Equipment of Fig. 13, Supplemented by the Use of an Additional Unit, 6, Permits Tap-changing in Uniform Voltage Steps. In the voltage diagram, on the right, *1, 2, 3* are operating positions; *S, S* are transition positions

in that all steps are now uniform, and two contactors are always closed in an operating position. In alternate positions the reactor bridges adjacent taps, and therefore, the reactor is under excitation and circulating current is flowing. As both the core losses and the copper losses are independent of load because of the circulating or magnetizing current in the bridging positions, they should be included in the no-load losses of the equipment, although allowance should be made for the fact that they are present only on alternate operating positions. Of course, by avoiding the use of the bridging positions as operating positions these losses can be eliminated and, at the same time, the reactor size can be reduced to about one-third, as previously explained. However, to the writer's knowledge, this has not been attempted, since it involves doubling the number of contactors.

The foregoing characteristics have been considered in rather minute detail, in order to show that although various ratio control equipments differ very greatly from each other in mechanical and electrical design, they are all nevertheless reducible to equivalent electrical circuits which are similar to each other; and the major difference in electrical characteristics depends mostly upon the positions in a cycle of operation which are selected by the designer to be the permanent operating positions. By using all possible positions, the number of mechanical parts is reduced; but when this is done it is necessary to design the electrical circuits more liberally and even then, in some cases, the electrical performance is not particularly good.

While having the two types of equipments in mind, it is worth noting that the segregation of the switching duty from the tap-changing duty, which is accomplished by the combination of circuit breakers and ratio adjusters, materially reduces the voltage between parts in the switch tank. This is because, in the one case, the full tap range must be brought out into the switch tank; whereas when ratio adjusters are used, only the voltage of one tap appears across the circuit breaker. Take for example the case of ratio control inserted in the grounded neutral end of a 132,000-volt line. Without ratio adjusters, it is necessary to bring out of the transformer tank the full tap range, *i.e.*, from 7000 to 20,000 volts; whereas with ratio adjusters the voltage of only one tap is brought out which is one-tenth as much. These values are also the voltages to ground. Considering that the oil in a switching compartment is very quickly carbonized, and also contains particles of copper, it is certainly advantageous to keep the voltage in these tanks as low as possible.

(*To be continued*)

Industrial Electric Heating

PART IV

MELTING NON-FERROUS METALS (Cont'd)

By N. R. STANSEL
Industrial Engineering Department, General Electric Company

THE second division of the non-ferrous metals in the melting point scale, Group II, Table VIII, begins with silver and is extended to include all alloys containing a metal from that group as one of the major constituents of the alloy. Copper and nickel are the two metals in Group II which are of greatest utility in the industrial arts. The copper alloys, brass, bronze, nickel-silver, and some of the bearing metals which contain copper are first in importance in this group of alloys. Of these, brass is most widely used; the term "brass" including all the copper-zine alloys which contain 55 per cent or more of copper.

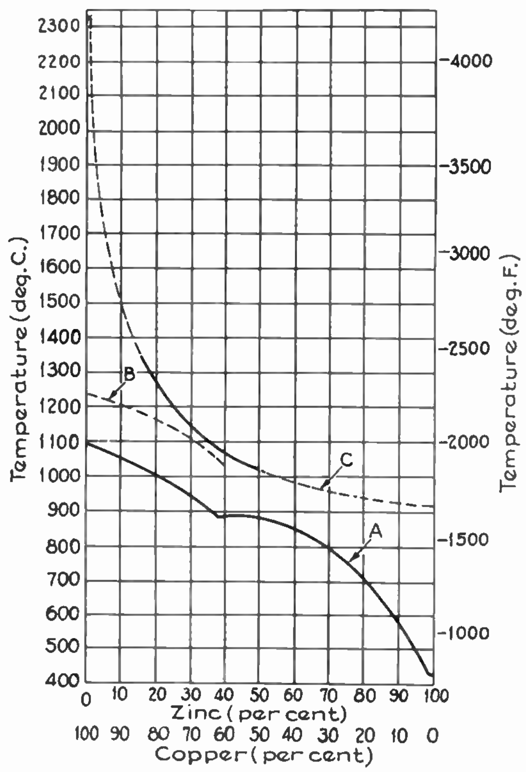


Fig. 58. Temperature Chart for Molten Copper-zinc Alloys (*Gillett*)

(*A*) Melting temperatures.
(*B*) Pouring temperatures, assumed 150 deg. C. above melting point.
(*C*) Approximate boiling points.

The extent of the present use of electric heat for melting in this branch of the non-ferrous metal industry is shown in Table X.[16] While the resistor-type electric furnace has been used to some extent for this class of melting, by far the larger part of electric heat for this service is now obtained from induction furnaces and arc furnaces.

TABLE X

Electric Brass and Bronze Furnaces in Active Use in United States and Canada, Feb., 1927. Excludes idle or spare furnaces. Includes those melting copper, nickel brass, and aluminum bronze. Includes only arc and induction furnaces

Number of Furnaces	Connected Load Kw.	Tons of Melt for 1926 (Estimated)	Kilowatt-hours Used (Estimated)
Wrought Brass Industry			
321	27,100	*450,000	113,000,000
Brass Foundry Industry			
303	29,200	†225,000	67,500,000
Total 624	66,300	‡675,000	180,500,000

* Approximately 90 per cent of the total output of mills.
† Approximately 30 per cent of the total output of foundries.
‡ Approximately 50 per cent of the grand total.

The figures of Table X show that while the numbers of electric furnaces in active use for melting in the two divisions of industry noted are about the same, the electric energy used for melting alloys for the production of bars, sheets, tubes, etc., is about twice that used in foundry practice. This wrought-metal production is concentrated into a few large plants which use electric heat almost exclusively for melting. The total cast-metal output comes from a large number of comparatively small, widely-scattered foundry plants. In these plants the growth of the adoption of electric heat for melting has been much slower.

Pouring Temperatures

The pouring temperatures of non-ferrous metals of high melting temperatures and their alloys vary with the character of the metal or alloy, the purpose for which the melt is to be used, and with the type of mold. In general, the pouring temperatures range from 150 to 350 deg. F. above the melting point. The chart in Fig. 58, by Gillett and Mack[17], gives

(16) Gillett: "Twenty-five Years of Non-ferrous Electrothermics," *Trans. Amer. Electrochem. Soc.* Apr., 1927.

(17) Gillett and Mack: "Electric Brass Furnace Practice." *Bul. No. 202*, U.S. Dept. of the Interior.

References made to material hitherto published in this series of articles may be readily located from the following table:

	Issue	*Figs.*	*Equations*	*Examples*	*Tables*	*Footnotes*
Part I	Oct. 1927, p. 488	1 to 23	(1) to (8b)	1 to 3	I to III	(1) and (2)
Part II	Nov. 1927, p. 551	24 to 43	(9) and (10)	4 to 6	IV to VII	(3) to (13)
Part III	Dec. 1927, p. 600	44 to 57	(11) and (12)	8 to 11	VIII to IX	(14) and (15)

approximate boiling points, melting points and assumed pouring temperatures for the entire range of commercial copper-zinc alloys.

Heat Content

Only a comparatively small amount of work in the determination of the heat content of molten non-ferrous alloys has been done. The values in Table XI are given by Gillett and Mack [17] who state: "According to the probable specific heat of molten copper alloys it will probably take from 10 to 15 kw-hr. per ton of brass or bronze extra for each 100 deg. C.

TABLE XI

Alloy	Temperature Deg. F.	Temperature Deg. C.	Heat Content Kw-hr. per Ton
61 Cu 36 Zn 3 Pb	1832	1000	146 (Gillett and Mack)
65 Cu 35 Zn	1832	1000	136 (Richards)
66⅔ Cu 33⅓ Zn	1832	1000	138 (Clamer and Hering)
80 Cu 20 Zn	2012	1100	161 (Hansen)
85 Cu 15 Sn	1922	1050	136 (Richards)

above the melting point to which the metal is raised;" also, "It seems likely that 150 kw-hr. per ton for average yellow brass poured at about 1050 deg. C. (1922 deg. F.) and 170 kw-hr. per ton for average red brass poured at 1200 deg. C. (2192 deg. F.) are better figures for the theoretical power requirements." The heat content curves of Fig. 59 for two copper-zinc alloys and of Fig. 60 for two nickel silver alloys are given by Tama. [18]

The heat content curves of non-ferrous metals and their alloys, for example, Figs. 46, 59 and 60, consist of a line broken into three sections:

(*a*) A section showing the heat absorption up to the melting point.

(*b*) An intermediate section representing the transition from the solid to the liquid state.

(*c*) A section showing the heat absorption above the melting point.

TABLE XII

Temperature, Deg. C.	Microhms Per Centimeter Cube
20	1.7347
1000	9.42
1082.6	10.30 solid state
1082.6	21.30 liquid state
1100	21.43
1340	23.39
1450	24.22

The change from the solid to the liquid state may be sudden or gradual, depending upon the character of the metal or alloy. If this change is gradual, we have the pasty stage noted with reference to the equilibrium diagrams, Figs. 44 and 45. The change from the solid to the liquid state is accompanied by a marked increase in the specific resistance of the metal.

[18] Tama: *Zeitschrift fur Metalkunde*, Vol. 18, p. 7.

For example, Northrup [19] obtained the values given in Table XII for copper (99.39 per cent conductivity).

In each case the value given for the heat content represents only the heat absorbed by the metal or metals. If the charge of metals to be melted contains impurities, such as oxides, dirt, moisture, etc., these also absorb heat. Usually the most important impurity, from the standpoint of heat absorption, is water. Moisture in a charge may account for an appreciable quantity of heat in the melting operation. The evaporation of one pound of water requires approximately 1200 B.t.u. (0.35 kw-hr.) From the figures just given the heat content of one pound of a molten non-ferrous alloy runs from 250 to 300 B.t.u. If the charge contains, say, three per cent moisture the heat absorption of the total charge will be from 10 to 15 per cent greater than the theoretical value for the metal alone.

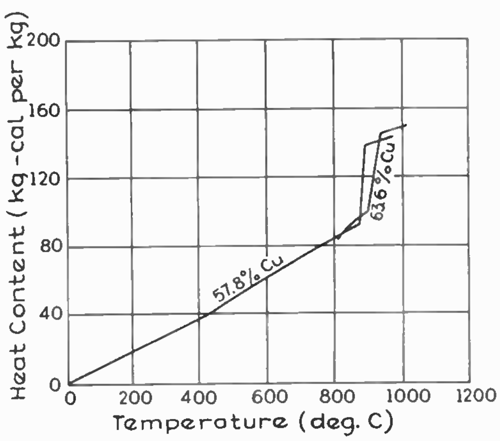


Fig. 59. Heat Content Curves of Two Copper-zinc Alloys (*Tama*)

Factors in Melting

In non-ferrous melting practice the more important factors which guide the melter are:

(*a*) The low vapor pressures of some metals, notably zinc, which cause volatilization at comparatively low temperatures;

(*b*) The readiness with which oxidation of these metals occurs at high temperatures;

(*c*) The tendency in case of some alloys towards segregation while in the molten state;

(*d*) The effects of underheating, overheating, and of unequal distribution of temperature within the mass of the charge;

(*e*) The effects that impurities have upon the metal or alloy.;

(*f*) The analysis of the charge and the nature of the different parts of the charge; and

(*g*) The kind and size of the castings to be poured.

In brass melting it is easy to lose zinc by vaporization. Values of the vapor pressure of zinc at different temperatures, together with the corresponding values for some other metals for comparison, are given in

[19] Northrup: *Jour. Franklin Institute*, Vol. 177, p. 21.

Table XIII. It will be noted that zinc has an appreciable vapor pressure not far above its melting point. Also by reference to Fig. 58 it is seen that the pouring temperatures of the copper-zinc alloys near the 60-40 proportion (which constitute a large part of the brass used) are close to the boiling points of these alloys.

If brass is melted in an open crucible the rate of loss of zinc from the charge depends upon:

(*a*) The percentage of zinc in the charge;
(*b*) The temperature of the metal;
(*c*) The rate of diffusion of zinc through the mass to the surface; and
(*d*) The velocity of gases over the surface of the metal.

The total loss of zinc from a given weight of charge, as determined by these factors, will depend upon the area of the exposed surface and upon the length of time that the mass of metal is held at the stated temperature. With a sealed melting chamber the loss by vaporization during melting would of course be zero.

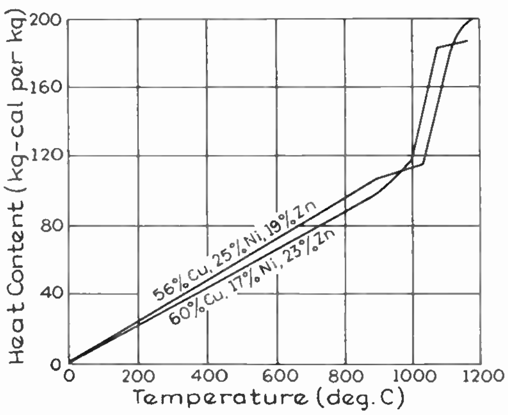


Fig. 60. Heat Content Curves of Two Nickel Silver Alloys (*Tama*)

A loss of a part of one or more of the constituents of a charge of metals during melting is undesirable in itself as a metal loss. A more important consideration, in the case of alloys, is the consequent loss of the control of the analysis of the alloy. In the process of melting steel a check analysis of each charge is usually made before pouring. This checking of the melt before pouring is not practicable, under average conditions, in melting non-ferrous alloys.

The poisonous character of zinc fumes is another factor in melting alloys which contain that metal. Before the advent of the electric melting furnace in the brass industry the malady known as "brass shakes" was common among workmen employed around open crucible melting furnaces. While zinc has been singled out as the metal requiring much care in melting to prevent loss by vaporization, it should be kept in mind that other metals (see Table XIII) have vapor pressures at elevated temperatures that must be considered in melting operations.

As the constituents of non-ferrous alloys are readily oxidizable at the pouring temperatures of these alloys, the atmosphere of the melting chamber should be neutral.

Stirring of an alloy while it is in the molten state is often essential to prevent segregation of the metals. This is particularly the case with alloys containing lead, a metal quite common in brass mixtures. The degree of the stirring necessary in any particular case depends upon the character of the alloy, *i.e.*, upon its tendency towards segregation and also upon the nature and the quantity of the impurities which may be present. The stirring of the molten mass should be a uniform procedure. This requires that the action be automatic. Stirring also brings about equalization of temperatures within the mass and prevents overheating of any portion of the metal.

The Ideal Melting Furnace

A convenient method of considering equipment for a given service is to first consider a specification that represents ideal conditions. This method, however, will lead to wrong conclusions unless the degree of importance attached to each item of the specification is based upon the conditions under which the equipment will operate. The ideal specification for a furnace with which to melt non-ferrous metals and their alloys would include many items. However, what is thought to be a reasonably complete catalogue of requirements follows:

(*a*) A closed melting chamber to prevent loss of metal by volatilization and for the control of the atmosphere of the chamber.
(*b*) A continuous and uniform stirring of the molten metal the degree of which can be controlled to suit the particular alloy being melted.
(*c*) A close control over the flow of energy into the metal. This is necessary in order to obtain accuracy in pouring temperature of the individual melt and uniformity in the pouring temperatures of successive melts.

TABLE XIII

Metal	Melting Point Deg. C.	Vapor Pressure in mm. of Mercury							
		10^{-3}	10^{-2}	10^{-1}	1	10	50	100	760
		Temperature (Deg. C.)							
Zinc.....	419	290	350	420	500	610	700	750	920
Copper...	1083	1080	1200	1340	1520	1740	1930	2030	2350
Lead.....	327	620	710	820	960	1130	1290	1360	1640
Tin......	232	1010	1130	1270	1440	1660	1850	1940	2260

(*d*) A temperature gradient from the metal to the outer boundary of the furnace walls. This prevents a flow of heat from the furnace structure into the metal when the supply of heat to the furnace is cut off, as in holding a charge before pouring.

(*e*) There should be no contamination of the metal or alloy by impurities or gases during the melting process.

(*f*) The furnace should be suitable for melting all classes of charges, *i.e.*, virgin metal, scrap of all kinds, and in fact any mixture of metals.

(*g*) The design of the furnace as regards size should be such that the type can be fitted to operations both large and small.

(*h*) The furnace should be adaptable to both intermittent and continuous operations and to the melting of alloys of different compositions in intermittent service.

(*i*) The furnace should be suitable for location at any point in the line of manufacturing operations.

(*j*) The conversion efficiency must be high and the design such that a high operating efficiency can be obtained easily under average operating conditions.

(*k*) The nature of the ambient atmosphere due to the operation of the furnace should affect neither the efficiency nor the health of the operators.

(*l*) The refractory lining of the melting chamber should have a long life.

(*m*) Simplicity and ruggedness of design and construction are essential.

(*n*) The furnace should be simple in operation.

(*o*) Safety in operation is essential.

(*p*) The labor cost of operation should be a small item.

(*q*) The maintenance charge should be low.

(*r*) Adaptability of design to any desired method of charging and pouring should be possible.

In addition, for the electric melting furnace, the following features relating to the power system which supplies the furnace load should be considered:

(*s*) The utilization of standard voltages and frequencies;

(*t*) A unity power-factor load;

(*u*) A balanced polyphase load;

(*v*) An even demand upon the power supply; and, with all these requirements, the furnace must meet the economic phase of the service.

Obviously no one type of furnace complies wholly with this specification. Hence in each case the class of service required and the local conditions should be considered to determine the features which should be given priority. There is a wide variety in melting service. In some cases two or more types of furnaces will serve the purpose. In others a furnace of a particular type is essential for satisfactory results.

Electric Melting Furnaces

Both the induction furnace and the arc furnace meet in a large measure the ideal specification. The wide use of both of these types shows that electric heat for melting meets the economic demands of melting service besides being of particular value in the production of high-grade metals and alloys of uniform quality; the latter often being the only reason assigned to the selection of electric heat for melting service.

The induction furnace and the arc furnace differ fundamentally in the methods of conversion of electric energy into heat and in the methods of applying the heat to the metal to be melted. In induction heating,

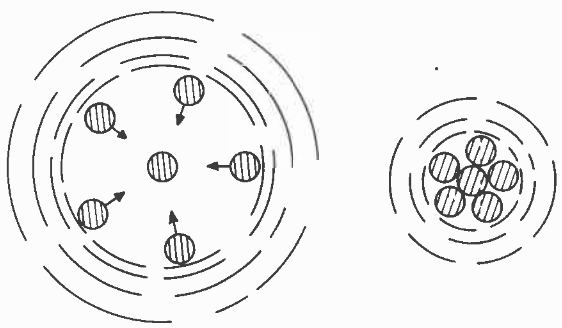


Fig. 61. The Constricting Effect of Electromagnetic Forces Between Parallel Conductors Carrying Currents in the Same Direction

the energy is transferred to the metal by electromagnetic induction and the electric currents thus induced in the metal develop heat in the mass as expressed by Joule's law, given as Equation (1). In the arc furnace, the electric energy is converted into heat in the arc and the heat is transferred to the metal by heat radiation, in accordance with Equation (8). These fundamental differences necessitate separate considerations of the two types of furnaces, which will be taken up in turn.

The Induction Furnace

The induction furnace is a combination of an air-cooled transformer and a direct-resistance furnace; a transformer in that the energy supplied to a primary coil is transferred by induction to a secondary coil, and a furnace in that this energy is converted into heat within this secondary coil. The secondary coil is a closed loop formed by the charge of metal to be melted. The character of this secondary circuit is the distinguishing feature of the types of induction melting furnaces.

There are certain principles which are peculiar to the use of induction heating for melting metals, *viz.:*

(*a*) The heat is developed within the mass of the metal and the time required to melt the charge does not depend upon the thermal resistance of the surface.

The Theory and Characteristics of Radiotrons

PART I

Outcome of the Edison Effect—Electron Current—Space Charge Effect of Positive Ions—Three-electrode Tubes—Detectors

By DR. LEWIS R. KOLLER and HENRY SCHROEDER

Research Laboratory Edison Lamp Works

General Electric Company

Certainly there is no lack in the abundance of magazine literature on radio tubes, but unfortunately it exists as isolated and partial descriptions widely scattered with respect to time and place of appearance. To fill the need for a logical and up-to-date assembly of the related facts, the authors have prepared the following two-part article.—EDITOR

IN 1881 Thomas A. Edison noticed, in his work of further developing the incandescent lamp which he had invented, that a glow frequently appeared in the bulb when current was passed through the filament while the lamp was being exhausted. It seemed strange that this should occur, as it appeared that it was probably due to current flowing through the rarefied gases in the bulb, even though there was apparently a much lower resistance path for the current through the filament itself.

In order to determine if current did pass through the rarefied gases, he inserted a wire in the bulb between the ends of the filament, as shown in Fig. 1, and connected this wire through a galvanometer to one terminal of the lamp outside the bulb. He found that a small current did flow through this wire but only when it was connected to the positive terminal of the lamp. This phenomenon was called the "Edison Effect."

Nothing practical was done with this discovery until 1905, when Prof. J. A. Fleming worked out his idea of using it for its rectifying effect on alternating currents as a detector of radio signals in place of the "coherer" used by Marconi.

The underlying principles of this two-element tube, or Fleming valve, are applied in rectifiers for converting alternating into direct current, such as the Tungar rectifier, battery eliminators for radio, etc. In order to understand the principles which govern the action of the three-electrode tube, the present day Radiotron, it is necessary to outline very briefly certain parts of the Electron Theory.

The Electron Theory

In 1897 Prof. J. J. Thomson showed experimentally that the atom, heretofore considered the smallest particle of matter, contained still smaller particles called electrons which are negative charges of electricity. Prof. Ernest Rutherford and Prof. Neils Bohr have carried Thomson's work still further. The atom is now considered to consist of a nucleus about which rotate one or more of these electrons, just as the planets rotate about the sun in our solar system.

In the solar system the gravitational force of attraction between the sun and the planets is balanced by the centrifugal force due to the rotation. In the case of the atom (since unlike charges attract and like charges repel) the rotating electron is attracted by the positively charged nucleus, but is kept from falling in toward the nucleus by the centrifugal force due to its rotation. The mass of the nucleus is large compared with that of the electron.

An electric current is caused by the motion of positive or negative electricity through matter or through space. Most currents, however, are due to the motion of negative electricity; that is, the electrons alone. It is customary to speak of the direction of a current as being just opposite to the direction in which the electrons are flowing. In substances like metals, which are good conductors, there are a large number of free electrons, which are not attached to individual atoms.

If a metal is heated in a vacuum, some of these free electrons will "evaporate" from the metal and enter the space within the bulb. Since these are all of the same charge (negative) they repel each other and also tend to repel the electrons which are being emitted from the surface. Thus after a short time, the space becomes practically saturated with electrons and when any leave the filament others are forced back in.

Plate Current

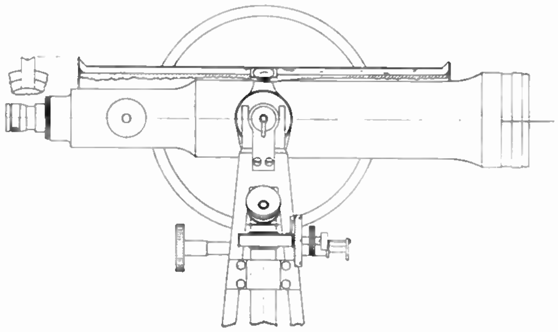
If a positive charge is placed on a wire or plate in the bulb, some of the electrons will be attracted to the plate, reducing the repulsion on the electrons tending to leave the filament, permitting more of these to evaporate. If the plate is connected to the positive terminal of a battery and the negative end is connected to the filament, as in Fig. 2, there will be a continual flow of electrons from the filament through the space to the plate, then through the battery circuit and back to the filament. This constitutes the plate current, and as it is customary to speak of the direction of the current as opposite to the motion of electrons, the current flows from plate to filament within the tube. This plate current can be measured by an ammeter placed in the circuit. In

Pointing Device for Precision Theodolite

By DOUGLAS L. PARKHURST
Chief of Instrument Division, U. S. Coast and Geodetic Survey

THE U. S. Coast and Geodetic Survey, in its work of establishing accurately located control points for mapping and other purposes, has been confronted with the problem of providing a suitable device for easily directing the telescope of the instrument approximately upon signal lights, the pointing device being only sufficiently accurate so that the signal will appear in the field of view. The pointing device is desired because observations must be made at night in order to minimize the effects of atmospheric refraction.

The instrument used, a theodolite, is set up upon some commanding elevation, such as a mountain peak or high tower, and is directed upon a signal lamp of special design, which need not be described in this article other than to state that it is so powerful that the rays may be seen for many miles by the naked eye, such lamps having been observed upon with instruments over a distance of 152 miles.



Clear Fused Quartz Pointing Device Shown Mounted on the Telescope of a Precision Theodolite

No lights can be used in the observation tent as the comparative brilliance would make it impossible for the observer to pick up the faint signal light. In consequence, the pointing of the approximately 45-power telescope of the instrument upon the signal is a somewhat difficult feat; for while the dim light from the recorder's lantern outside the tent is sufficient for the observer to see the outline of the instrument, yet upon attempting to sight along the telescope without a suitable pointing device its outlines resolve themselves into a blur so that sufficiently close direction to insure the signal coming into the field of view is impossible.

Various attempts have been made to devise a pointing apparatus, but until now none has achieved any marked success. A device developed by the Survey, with the co-operation of the General Electric Company, is now being tried out, however. In this the property of fused quartz of transmitting light longitudinally and around corners is made use of, the basic idea being that if two tiny points of light could be placed upon the barrel of the telescope in line with the axis of collimation, and could be seen anywhere in the tent to guide the observer these points would serve as a sight in directing the instrument.

The device shown in the illustration was accordingly designed. It consists of a shielded flashlight bulb placed upon the barrel of the telescope. Two rods of fused quartz have one end of each placed against this bulb. The othe: ends are bent at 90 deg. and are chamfered off to form points, the chamfered surfaces being lightly frosted. A V-shaped baffle is placed directly before the eyepiece point in order to reduce the area of illumination to a minimum, this being necessary because of the apparent magnification due to its being so close to the observer's eye. The light from the bulb travels along the quartz and causes the exposed tips of the rods to glow, the intensity being regulated by a small rheostat. The points are readily discernible anywhere in the tent and when the eye is placed in line with them the theodolite may be readily trained upon the signal light.

This device is still in the trial stage but has given promising results in laboratory tests. A wooden holder was made and strapped to the telescope of the theodolite and placed approximately in collimation by sighting upon a target about 50 ft. distant. The instrument was then trained by means of the pointing device upon a small lamp some distance away in broad daylight, and upon looking through the telescope the lamp was found to be practically in line although no special efforts had been made to insure accuracy.

With this device it is expected that the observer will be readily able to train his telescope in both altitude and azimuth upon the signal light so closely that upon looking through the telescope the light will be in the field of view and it will be necessary only to make use of the slow motion adjustments to bring the cross wires directly to bear. The device will also be useful when making star observations, and it is expected that a considerable saving in observing time will be effected to say nothing of the greater satisfaction to the observer than the use of the cut-and-try method now in vogue.

the ordinary radio receiving tube this current is small, of the order of a few thousandths of an ampere, so a milliammeter is ordinarily used.

When the electrons strike the plate they generate heat, their impact heating the plate just as repeated blows of a hammer on a piece of metal will heat it. The heat generated at the plate of the ordinary radio receiving tube is insufficient to materially raise its temperature, but in the larger power tubes used in broadcasting, the heat is sufficient to make the plates red hot. In a very large tube, the plate is made also a part of the container and is cooled by circulating water around it. The temperature reached depends on the relative size of the plate, the plate current and the plate voltage.

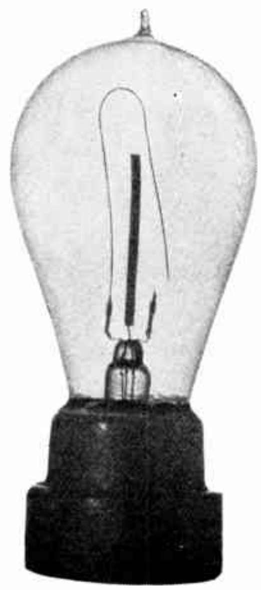


Fig. 1. Edison's Experimental Lamp

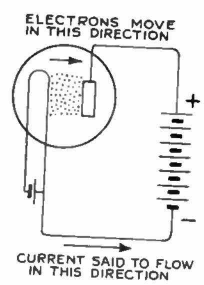


Fig. 2. Diagram Showing Flow of Electrons in Vacuum

Effect of Gas in the Bulb

It is impossible to produce a perfect vacuum, since there are always some molecules of gas remaining in the bulb. If an electron, in traveling to the plate, strikes one of these gas molecules with sufficient speed, it will dislodge one of the electrons of the gas molecule. The original electron will then continue to the plate at reduced speed together with the electron dislodged from the gas molecule. The gas molecule is now positively charged, as one of the electrons has been taken away from it. In such a condition it has become ionized, and the positive ions will be attracted toward the filament. The positive ion strikes the filament with considerable energy as its mass is large compared with a single electron. If a large number of positive ions strike the filament, they will not only heat it to a still higher temperature, just as the plate is heated by electrons striking it, but they will also disintegrate the filament by knocking off small particles.

If the ionized molecule collides with a slow moving electron before reaching the plate, it may combine with it to form a neutral molecule again. When this happens light is given out and, if a sufficient number of positive ions and electrons are recombining, a faint bluish glow is visible within the bulb. This is the effect Edison noticed in 1881. The glow can be made to appear in a "gassy" or "soft" tube, if the plate voltage be high enough. The pull on the electrons then is so great, making their speed of travel to the plate so high, that they can readily ionize a large number of molecules which, in recombining with electrons, give out light.

The Two-element Tube or Fleming Valve

Wireless telegraph signals at the time that Prof. Fleming developed the two-element tube, in 1905,

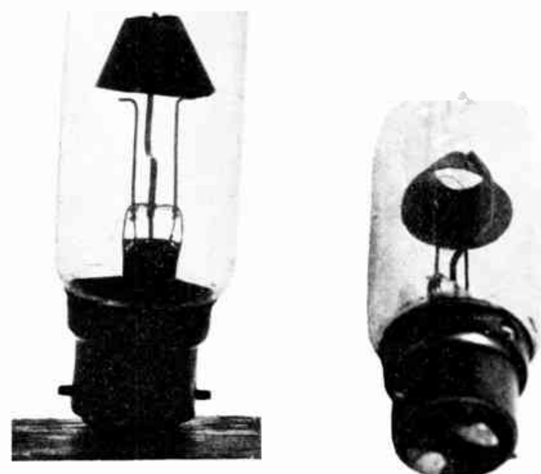


Fig. 3. Two Views of a Two-element Tube or Fleming Valve

were sent out by a spark transmitter. The radio waves sent out were of very high frequency, in the neighborhood of 100,000 cycles per second, which is far beyond the audible range whose upper limit is about 10,000 cycles. The alternating current, generated in an antenna by these waves, would produce no audible sound, even if it were possible to send it through a telephone, as the diaphragm cannot respond to such high frequencies. Very little of this current passes through the telephone, however, for at these high frequencies the impedance of the coils is very large.

While each spark sent out a group of high-frequency waves, the series of sparks or groups of waves sent out were of a frequency within the audible range. A device was therefore necessary to detect these groups of waves, the frequency of these groups fixing the pitch of the sound and their duration denoting whether the signal was a dot or a dash. Until the advent of the two-element tube, this was accomplished by means of a coherer.

The two-element tube, Fig. 3, consists of two metal pieces, called electrodes, one maintained at a higher temperature than the other, both mounted within a vacuum tube. The most convenient hot electrode is a tungsten filament heated by current. The other electrode is a metal plate. If an alternating potential be impressed between the filament and plate, through a load of some sort, a pulsating direct current will flow through this load. Edison had found that current would flow only when the plate is positive, so the tube acts as a rectifier or valve for the alternating current, hence the name "valve."

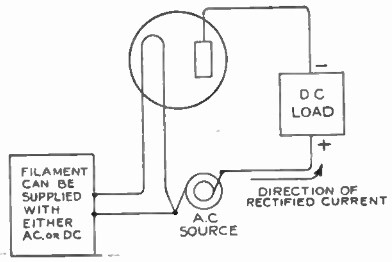


Fig. 4. Diagram of Connections Illustrating Rectifying Action of Two-element Tube

During the half cycle in which the plate is positive it will attract electrons from the filament, thus causing a current to flow through the external circuit from the filament to plate. During the succeeding half cycle, the plate will be negative and so will repel electrons and hence no current will flow. The diagram of connections illustrating this action is shown in Fig. 4.

This principle is now used in such devices as the Tungar rectifier to produce a direct current for charging storage batteries and in so-called "B" Battery Eliminators to produce direct current for radio sets in place of batteries. In the latter device, condensers and impedances or equivalent electrical devices must be used in the direct-current circuit to smooth out the pulsations, otherwise the pulsations will produce a hum.

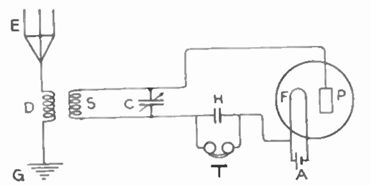


Fig. 5. Diagram of Connections of Fleming Valve as Detector of Radio Signals

Fig. 5 shows a diagram of connections illustrating the method by which Prof. Fleming utilized the two-element tube as a means of detecting radio telegraph signals. The radio signal produces an alternating current in the antenna circuit *EG*. This current passes through the primary *D* of an air core transformer which induces an alternating potential across the terminals of the secondary *S*. A variable condenser *C* is connected across the terminals of the secondary *S*, this condenser being adjusted or "tuned" to resonance with the induced alternating potential in order to get the highest possible voltage. One end of the secondary *S* is connected through a telephone *T*, shunted by a fixed condenser *H*, to the filament *F* of the two-element tube. The filament *F* is heated by passing current through it from the battery *A*. The other terminal of the secondary *S* is connected to the plate *P*. The radio signals reaching the antenna thus make the plate alternately positive and negative with respect to the filament so that a pulsating direct current flows from filament to plate through the fixed condenser *H* and telephone *T*.

Suppose the radio signal produced an alternating current in the antenna circuit of the form shown by the oscillograph curve (*A*), indicated in Fig. 6. The electron current between filament and plate, known as the plate current, is a pulsating direct current as indicated by (*B*). In passing through the telephone, the fluctuations are smoothed out by the condenser and by the inductance of the telephone coils, so the current passing through the telephone is as indicated by (*C*).

It will be noted that the frequency of this pulsating telephone current is the same as the number of groups of radio waves per second and, being of audio frequency, produces a sound in the telephone. This is the detection of the radio signal. Each group of radio waves consists of several thousand cycles and, if the groups be sent out at a rate of about 1000 per second, there will be 1000 impulses given each second to the diaphragm of the telephone which will produce a high-pitched audible note. The length of time between intervals at which these groups are sent out will indicate whether the signal be a dot or a dash.

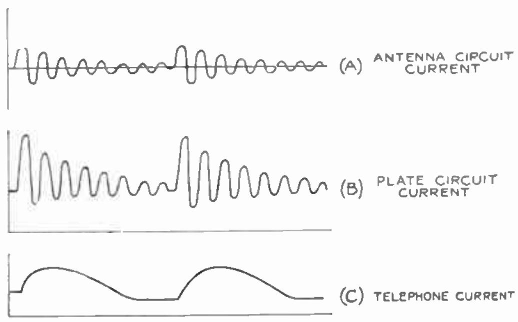


Fig. 6. Oscillographs of Current

Plate or B Battery

Later, the use of an additional battery to increase the current flowing from plate to filament was found to improve greatly the operation of the tube. This improvement depends upon a characteristic of the tube, that is, the relation between the voltage impressed on the plate and the resultant current flowing from plate to filament. This relation is known as the plate voltage—plate current characteristic.

Suppose a given voltage be steadily impressed on the plate. This voltage will determine the value of the plate current. By measuring the plate current at different plate voltages and plotting the relation between the two, a curve will be obtained having a characteristic appearance similar to that shown in Fig. 7. Note that when the plate is 10 volts positive, the current is about nine-tenths of a milliampere. With 20 volts plus on the plate the current is more than doubled, and with 30 volts is more than tripled. With any negative voltage on the plate the current is always zero, showing the rectifying action of the tube.

Now suppose a battery, usually called the "plate" or "B" battery, were inserted in the circuit between the filament and telephone (see Fig. 5) with the negative end connected to the filament. If the voltage of this battery were 10 volts, there would be, according to the characteristic curve in Fig. 7, a steady direct current of about 0.9 milliamperes in the plate circuit. This steady direct current flows through the telephone and will produce no sound.

The alternating voltage, induced in the secondary S, Fig. 5, of the transformer, due to the radio signals, alternately adds to and subtracts from the voltage between the plate and filament. Suppose this variation is of the order of one volt plus and minus. It will be seen from the shape of the characteristic curve, Fig. 7, that the increase in plate current HK caused by the increase in plate voltage AD from plus 10 to plus 11 volts is greater than the decrease in plate current HG caused by the decrease in plate voltage AC from plus 10 to plus 9 volts.

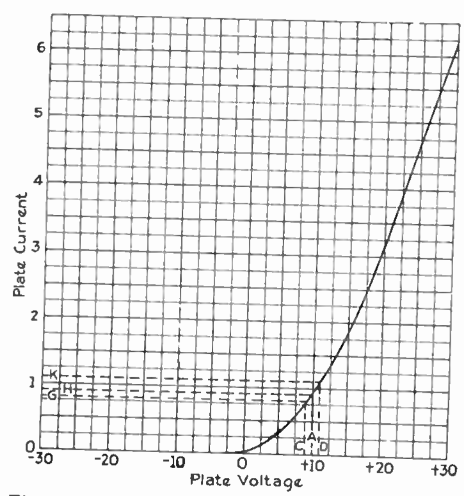


Fig. 7. Plate Voltage—Plate Current Characteristics

Thus for each group of radio waves, the average value of the current in the plate circuit is changed by this difference HK less HG. It will be noted that this difference is greater than the change in plate current when the plate voltage varies from minus one to plus one (as would have been the case without any "B" battery). As the amount of change in plate current affects the amount of change in the current flowing through the telephone, which is a measure of the loudness of the received signal, the use of a plate battery of proper voltage will increase the strength of the received signal.

A study of the characteristic curve, Fig. 7, will indicate that the best plate battery voltage to use will be at that part of the curve where there is the greatest curvature. It should be noted that the object is to obtain the greatest change in telephone current,

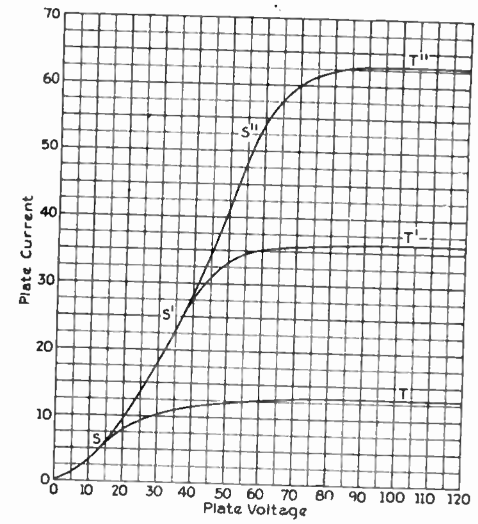


Fig. 8. Plate Voltage—Plate Current Relation at Different Filament Temperatures

which may be either an increase or a decrease. The foregoing explanation has been based on an increase, but similar reasoning will indicate that a decrease will produce the same results. The plate battery voltage to use can therefore be either at the lower bend of the curve or a higher one at the upper bend in the curve. The upper bend is not shown as the size of the abscissas and ordinates taken, to demonstrate this properly, would require an inconveniently large illustration. The upper bend of such a curve would appear as in Fig. 8. The lower bend, however, gives more satisfactory results.

The Three-element Tube

Dr. Lee DeForest made a notable invention, about 1907, in his discovery that if a third element, called the grid, be put in a two-element tube between the filament and plate, slight fluctuations in voltage impressed between the filament and grid would produce very great variations in the current flowing in the filament-plate circuit. (Three early tubes of the three-element type are shown in Fig. 9.) The variations in the plate current of three-element tubes are much greater than would be obtained if the voltage fluctuations were, instead, impressed between the filament and plate as in the two-element tube. For example, in the average UX-201A Radiotron with the filament operating at its rated voltage, if the plate voltage be increased from 90 to 100 volts, the plate current will only increase from 6.00 to 7.2 milliamperes, an increase of 0.12 milliamperes plate current per volt increase in plate voltage. If, however, with 90 volts on the plate, the grid voltage be increased from zero to plus one volt, the plate current will increase from 6.00 to 7.00 milliamperes, an increase of one milliampere per volt increase in grid voltage. That is, a given change in voltage is over eight times as effective when impressed on the grid as when impressed on the plate.

The three-element tube can be used in three ways. First, as a "detector" to eliminate the "carrier" wave of radio frequency so that its variations of audio frequency will pass through the telephone to reproduce the signal as sound. Second, it may be used as an amplifier to increase the strength of the received waves either by amplifying the voltage produced by the carrier wave before it goes through the detector (radio-frequency amplification), or by amplifying the voltage produced by the audio wave after it has passed through the detector (audio-frequency amplification). Third, it may be used as a generator of high-frequency currents.

Space Charge Effect

Before explaining the action of these three uses of the three-element tube, it is desirable to explain the theory of the space charge effect.

If an electrode such as the plate is placed near a heated filament and no voltage is applied to that plate, the electrons given off from the filament exert a repulsion on each other with the result that the space around the filament soon becomes saturated with electrons. No emission current is observed, as the electrons filling the space repulse the electrons tending to come off the filament. This repulsion of the electrons in the space on each other, and on the electrons tending to leave the filament, is called the "space charge." If a slight positive voltage is put on the plate, the electrons are attracted to the plate. This reduces the repulsion of the electrons in the space on the electrons tending to leave the filament, allowing electrons to leave the filament at a definite rate. Thus there is a definite flow of current for a given voltage on the plate. As the voltage on the plate is increased, the current is increased in proportion to the 3/2 power of the voltage. A point is reached, however, depending upon the temperature of the filament, at which all the electrons emitted from the filament are drawn to the plate, so that any further increase in plate voltage will make no further increase in plate current.

This space charge discovery furnished a simple explanation of the facts which led previous investigators to believe that in a very good vacuum with the low plate voltages they used, there was no electron emission. With a small amount of gas present, the collision of electrons with gas molecules produces a small number of positive ions which tend to neutralize the space charge; consequently more electrons flow to the plate than would flow if there were no gas present, as one positive ion will neutralize the field produced by about 300 to 500 electrons. Thus the better the vacuum the smaller the plate current will become, leading the investigators to believe that the plate current would cease in high vacuum.

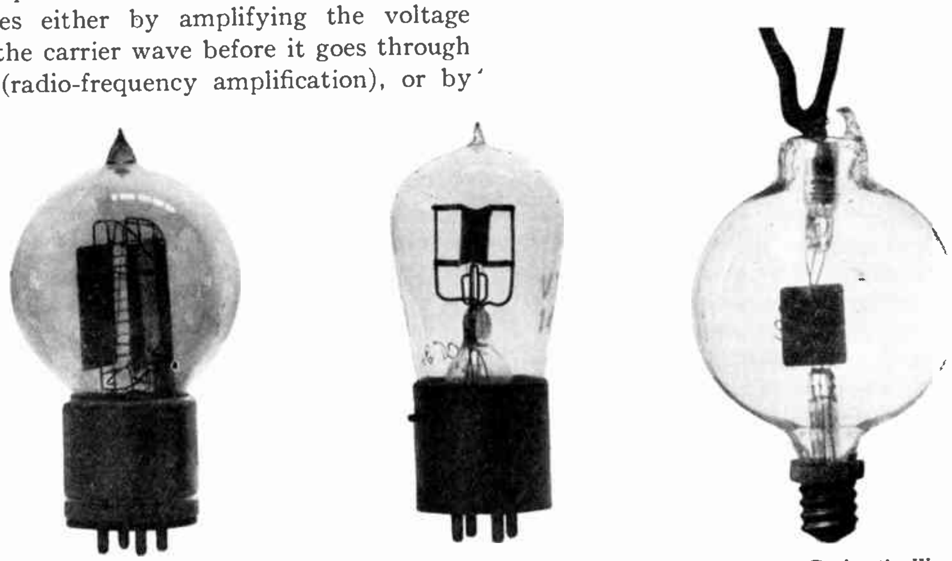


Fig. 9. Three of the Three-element Tubes Used by the Signal Corps of the U. S. Army During the War

The Three-electrode Tube as a Detector

Dr. Irving Langmuir showed that the current between two electrodes in a *good vacuum* varies, up to a certain point, as the 3/2 power of the voltage between the electrodes. This certain point is called the "saturation" point as at this point the plate current has reached its maximum and any further increase in plate voltage produces no further increase in plate current. This is caused by the fact that all the electrons emitted from the filament are being pulled over to the plate and any further increases in voltage on the plate above the saturation voltage cannot attract any more electrons than are being emitted.

The saturation current, however, will depend upon the temperature at which the filament operates; the higher the temperature, the more electrons emitted. Hence, with higher filament temperatures a higher saturation voltage is necessary to reach the higher saturation current thus made possible. With a given filament, the plate current below saturation is independent of filament temperature. This is indicated in the curves in Fig. 8, which show a typical characteristic relation between plate current and plate voltage at different filament temperatures.

It will be seen that as the voltage on the plate is increased, for a given filament temperature *T*, the plate current increases up to the saturation point *S*, the plate current increasing from *O* to *S* as the 3/2 power of the plate voltage. If the filament temperature be increased to *T′*, the saturation point is increased to *S′* and the 3/2 power law then applies from *O* to *S′* along the same line *OS* and extends to *S′*. Similarly, if the temperature be further increased to *T″*, the 3/2 power law will apply along the line *O S S′* and extend further to *S″*, a still higher saturation point.

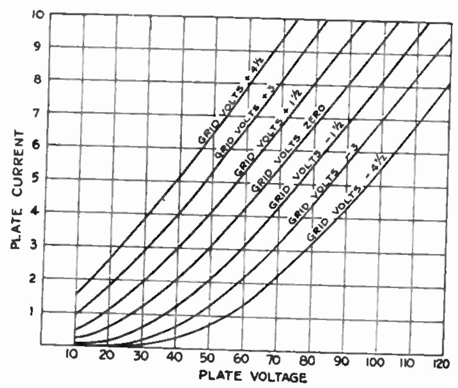


Fig. 10. Plate Voltage—Plate Current Curves for Different Grid Voltages

The curve does not change suddenly at *S* from the 3/2 power relation to a horizontal line but passes through a transition stage. One reason for this is that, due to the voltage drop along the filament, the plate has a different potential with regard to different parts of the filament. Thus, if the plate be connected to the negative end of the filament, the plate voltage with respect to the negative end is several volts more (depending upon the voltage on the filament) than with respect to the positive end. The result is that the saturation voltage is different for each portion of the filament. Another reason for this gradual transition is that the electrons do not all have the same velocities when they leave the filament and the voltage which is just sufficient to pull them over to the plate varies according to the distribution of their initial velocities.

Suppose a grid were placed between the plate and filament and kept at a low positive voltage, the filament being maintained at a certain temperature and a certain voltage being maintained on the plate: the grid will attract electrons from the filament. The plate having a higher voltage than the grid, relatively few electrons will actually be caught by the grid and the remainder will pass on, through the meshes of the grid, to the plate.

The effect of slight positive voltage on the grid is to partially neutralize the space charge. The space charge is due to the repelling forces of those electrons in the neighborhood of the plate on other electrons near the filament. This nearly balances the attractive force due to the positively charged plate. As the forces exerted by electric charges on each other are inversely proportional to the square of the distance between the charges, a very small charge on the grid has a much larger effect in neutralizing space charge than is obtained from the charge on the more distant plate. Thus a slight positive grid voltage "boosts" the electrons along on their way to the plate. The higher the grid voltage the greater the number of electrons going to the plate for a given plate voltage.

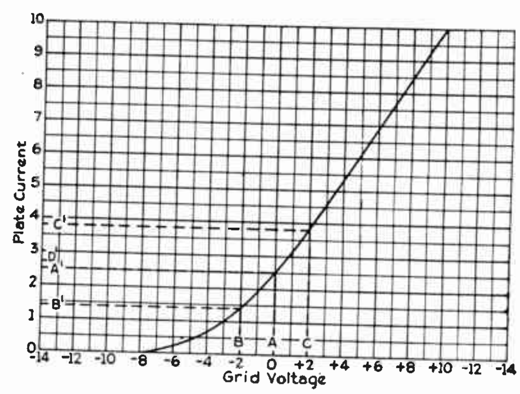


Fig. 11. Grid Voltage—Plate Current Characteristic to Illustrate Action of Three-element Tube Detector without Grid Leak and Condenser

Similarly a negative voltage on the grid would repel electrons and tend to prevent them passing through the grid and so diminish the plate current.

Suppose a series of plate voltage–plate current curves be plotted for different grid voltages. This family of curves will appear as in Fig. 10. It will be noted that at any plate voltage below saturation, the more positive the grid the greater the plate current, and the more negative the grid the smaller the plate current. A vertical line drawn through a given plate voltage will give values of plate current at the different values of grid voltage for this plate voltage.

If the values of the plate current are plotted against the grid voltage, the curve thus obtained is called the grid voltage–plate current characteristic, illustrated in Fig. 11. Comparing the curve in Fig. 11 with the zero grid voltage curve in Fig. 10, it will be seen that a one-volt change in grid voltage will produce 0.90 milliamperes change in plate current, whereas a one-volt change in plate voltage will produce but 0.15 milliamperes change in plate current.

This suggests the way in which the three-element tube is used as a detector of radio signals. Instead of impressing the voltage fluctuations due to the incoming signals on the plate, they are impressed on the grid where they exert a much greater effect on the plate current. The explanation of the detector action of the three-element tube is similar to that of the Fleming valve. Fig. 12 shows the connections for a three-element detector tube as used before the advent of the grid leak and condenser.

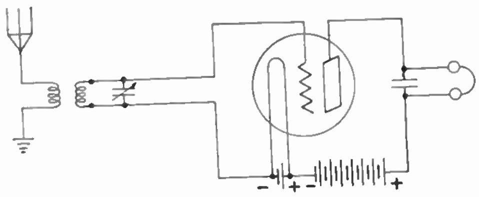


Fig. 12. Diagram of Connections of Three-element Tube as Detector of Radio Signals Without Grid Leak and Condenser

The grid is connected to the negative end of the filament, and the normal voltage on the grid with no signals coming in is therefore zero, as also indicated in the grid voltage–plate current characteristic, Fig. 11. The point of zero potential is always considered as the negative terminal of the filament.

When signals come in, the alternating voltage induced in the secondary coil, Fig. 12, will make the grid voltage fluctuate between B and C, Fig. 11, which causes the plate current to vary between B' and C'. When the grid voltage increases from A to C, the plate current is increased by the amount $A'C'$, and when the grid voltage decreases from A to B, the plate current is decreased by the amount $A'\ B'$. From the shape of the characteristic curve it will be seen that the increase in plate current $A'\ C'$ is greater than the decrease $A'\ B'$, the net increase being $A'\ D'$.

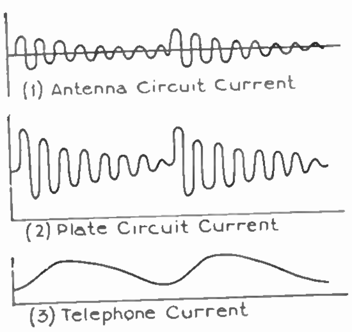


Fig. 13. Oscillograph Curves—Three-element Tube Used as Detector without Grid Leak and Condenser

While this explanation and those following make use of static characteristics to explain action under dynamic conditions, this is not strictly correct. It makes possible, however, a simpler explanation of the fundamental principles.

Suppose the radio signal produced an alternating voltage on the grid as shown by the oscillograph curve (*1*), Fig. 13, which produces the varying plate current (*2*). In passing through the telephone, the fluctuations in the plate current are smoothed out by the condenser and the inductance of the telephone. The telephone current is shown by (*3*) in Fig. 13. The variations in the telephone current reproduce the signal, as they are of audio frequency.

The operation of the detector tube without grid leak and condenser depends upon the curvature of the characteristic curve. The greatest curvature produces the greatest change in average plate current (the telephone current) so that by connecting the grid to the negative end of the filament (zero grid bias) and using a relatively low plate voltage, the part of the curve of greatest curvature will be utilized.

If the operation were on the straight portion of the curve there would be no difference between the increase and decrease in plate current and hence no net change in telephone current and therefore no detector action.

Grid Leak and Condenser

A small condenser shunted by a high resistance provides another method of using the three-electrode tube. The condenser is usually of about 0.00025 microfarad capacity and the resistance may be from two to five million ohms. They are placed between the grid and the tuning circuit as shown in Fig. 14 and are known as the grid condenser and grid leak.

The grid is connected to the positive terminal of the filament and when no signals are coming in it has a positive potential. The grid being normally positive, a small electron current flows within the tube from filament to grid and returns, through the grid leak in the outside circuit, to the filament. Thus there is voltage drop across the high-resistance grid leak, which has the effect of making the grid potential less positive with respect to the negative terminal of the filament.

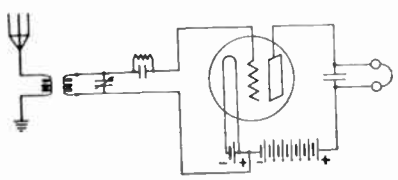


Fig. 14. Diagram of Connections of Three-element Tube Detector Used with Grid Leak and Condenser

Suppose when a signal is impressed on the grid, and the voltage wave readily passes through the grid condenser to the grid, the grid becomes positively charged during the first half of the radio frequency cycle. The grid then being momentarily more positive than before, attracts more electrons to it from the filament. The high resistance of the grid leak materially retards the flow of these extra electrons back to the filament through the circuit outside the tube. Thus each cycle draws more electrons to the grid, these electrons tending to stay there and give the grid a negative charge, that is, make the grid less positive with respect to the negative end of the filament.

The first few cycles lower the grid potential, the succeeding cycles holding it at this potential which is dependent on the strength of the signal less the loss through the grid leak. When the signal has lowered the grid potential so that the positive part of the radio signal cycle equals the amount the grid voltage has been lowered, the grid, not becoming positive at the peak of the radio wave, cannot attract any more electrons and so cannot be further reduced in voltage. Hence there is a limiting value beyond which the grid voltage cannot be lowered. With weak signals the rate of leakage of the voltage on the grid becomes an appreciable part of the signal voltage, so that a high-resistance grid leak is desirable in order to keep this loss small.

The oscillographs of the signal and grid voltage would then appear as in (*1*) and (*2*) in Fig. 15. For the sake of simplicity these graphs are those of telegraph signals.

Each group of incoming telegraph signals reduces the grid voltage from a certain positive figure to a lower figure. This reduction in grid voltage let us say is AB, in Fig. 16, which reduces the plate current by the amount A' B'. The grid voltage is pulled down from A in the first few radio frequency cycles to B and kept at B during the remaining cycles. The plate current is then quickly pulled down from A' to B' and kept at B' where it stays during the remaining and longer portion of the signal. The telephone current is then reduced by the amount A' B', the oscillograph of the telephone current being shown as (*3*) in Fig. 15.

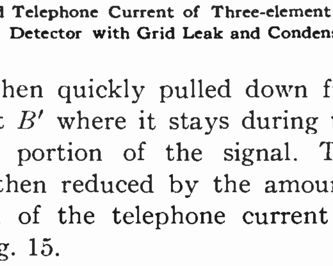


Fig. 15. Oscillographs of Signal Voltage, Grid Voltage and Telephone Current of Three-element Tube Detector with Grid Leak and Condenser

It will be seen that the amount of the change in telephone current A' B' (Fig. 16) is much greater than the amount of the change obtained without the use of the grid leak and condenser A' D' (Fig. 11). Thus with proper values of grid leak and condenser, louder signals will be obtained than without their use.

A grid current is essential for detector operation with grid leak and condenser. There are two other requirements for the best detector action: first, a maximum change in plate current and, second, a maximum change in grid current, both for a given grid voltage change. The maximum change in grid current, however, does not necessarily occur at the steepest part of the plate current curve.

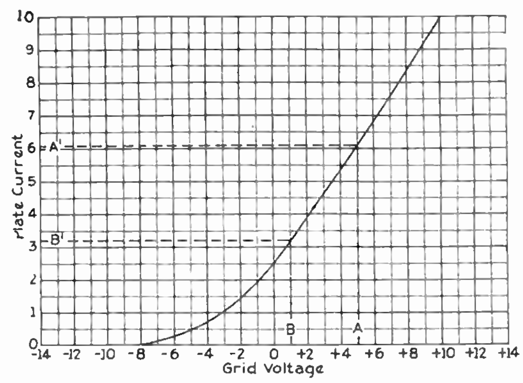


Fig. 16. Grid Voltage—Plate Current Characteristic to Illustrate Action of Three-element Tube Detector with Grid Leak and Condenser

The grid leak must be of a value which will keep the grid at a potential corresponding to the most favorable part of the characteristic curve. It must be small enough to allow the charge to leak off across the grid condenser between wave trains, though not so small as to prevent a charge from building up across the condenser and yet high enough to prevent most of the electrons from leaving the grid during the negative part of the radio frequency cycles.

The grid condenser must be small enough so that it becomes nearly fully charged during the period of a group of incoming waves, one audio-frequency cycle, and yet not so small as to take up too large a portion of the voltage drop due to the incoming signal and so prevent the potential from reaching the grid.

In the case of radio telephony, the smallest group of waves may be considered as corresponding to the highest overtones ordinarily heard in sound waves which are probably under 10,000 cycles per second. The lower of the broadcasting radio frequencies are about 500,000 cycles (600 meters) so that the smallest audio group is of a period of at least fifty radio-frequency cycles. The grid leak and condenser should be of values that operate well within this minimum of fifty radio-frequency cycles.

(*To be continued*)

The Characteristics of Tungsten Filaments as Functions of Temperature

PART III

By Dr. HOWARD A. JONES and Dr. IRVING LANGMUIR
Research Laboratory, General Electric Company

TRANSIENT PHENOMENA IN HEATED FILAMENTS INVOLVING CHANGES IN TEMPERATURE

Large Displacements of Temperature

(1) *Time of Cooling.* Let us assume that we have an ideal tungsten filament 0.01 cm. in diameter operating at 2000 deg. K in good vacuum. By the function A' we see that

$$A = 1022 \times (0.01)^{3/2} = 1.022 \text{ amperes}$$

is the heating current through the filament. Suppose we wish to know how long it takes the filament to cool from 2000 deg. to 500 deg. K on turning off the current.

Since the time of cooling for an ideal filament in good vacuum is a function of the energy content, U, and the energy radiated, W, we may use equation (23) or column 15, Table I, to calculate this time in seconds. Hence from column 15 we see that

$$(6914.6 - 18.906)\ 0.01 = 68.96 \text{ seconds}$$

is time of cooling for an ideal filament of this diameter.

(2) *Sudden Heating of Filament.* Let us assume that we have an ideal tungsten filament 0.01 cm. in diameter and 41.7 cm. in length operating in good vacuum with 30.7 volts drop over the filament.

Since $V' = \dfrac{V\sqrt{d}}{l}$ we calculate

$$V' = \frac{30.7\sqrt{0.01}}{41.7} = 0.07375 \text{ volts cm.}^{-1/2}$$

From the function V' in column 5, Table I, we note, therefore, that the filament temperature is 2000 deg. K.

Suppose, now, we wish to heat the filament to 2800 deg. K by a sudden surge of energy, such as may be obtained by the discharge of a condenser or an inductance. We first inquire how much energy is needed. By equation (21) the total energy U_T of the filament is equal to $U'_T ld^2$. Thus the energy increment between 2000 and 2800 deg. K by the data of Table I is

$$(6610 - 4320)\ 41.7 \times (0.01)^2 = 9.53 \text{ watt sec.}$$

The energy of a charged condenser in watt sec. is $1/2\ CV^2$, C being the capacitance in farads and V the potential in volts. Thus, if we have a condenser of 10 microfarads charged to 1382 volts, the stored energy is 9.53 watt sec. and this should be able to raise the temperature of the filament from 2000 to 2800 deg. K. This may be accomplished by placing a large choke coil in the filament heating circuit and discharging the condenser directly across the terminal of the filament.

The energy of an inductance carrying a current I is $1/2\ LI^2$, L being the inductance in henries. It is convenient to connect the secondary of a transformer to the filament. Thus by interrupting a d-c. current through the primary, a current surge is generated by the secondary.

Small Displacements of Temperature

Consider a filament which at any instant is heated by an electric power input W_a. The filament radiates the energy W_b per sec. The wattage loss by conduction is W_c and W_d is the heat generated on the surface of the filament by positive ion bombardment or by a chemical action of the surrounding gas. Let H be the heat capacity of the filament in watt seconds per degree. A wire of length l cm. and diameter d cm. of metal of atomic weight M, atomic heat c_p in cal. per gm. atom and density ρ has a heat capacity

$$H = \frac{4.185\, c_p\, \pi d^2 l \rho}{4M} \text{ watt sec. per degree.} \qquad (43)$$

For tungsten $H = 0.3456\, c_p\, d^2 l$ watt sec. per degree. For a filament in the steady state, we have

$$W_a + W_d - W_b - W_c = 0 \qquad (44)$$

If, however, a small or infinitesimal variation in temperature ΔT is made to occur, then we have

$$\left.\begin{aligned} &\left(W_a + \frac{dW_a}{dT}\Delta T\right) + \left(W_d + \frac{dW_d}{dT}\Delta T\right) \\ &-\left(W_b + \frac{dW_b}{dT}\Delta T\right) - \left(W_c + \frac{dW_c}{dT}\Delta T\right) \\ &= \frac{H\, d\, \Delta T}{dt} \text{ where } t = \text{time in sec.} \end{aligned}\right\} \qquad (45)$$

But by (44) this becomes

$$\frac{dW_a}{dT} + \frac{dW_d}{dT} - \frac{dW_b}{dT} - \frac{dW_c}{dT} = \frac{H\, d\, ln\, \Delta T}{dt} \qquad (46)$$

In general, W_a, W_b, W_c and W_d are functions of the temperature and over even a fairly wide range of temperature each may be assumed to vary with a certain power of the temperature. These exponents we may designate by n_a, n_b, n_c and n_d.

Thus we may write

$$W_x = KT^{n_x} \text{ where } K \text{ is a constant.} \qquad (47)$$

Expressing logarithmically and differentiating, we obtain

$$\frac{dW_x}{dT}=\frac{n_x W_x}{T} \quad (48)$$

Letting x equal successively a, d, b and c and substituting in equation (46) gives

$$n_a W_a+n_d W_d-n_b W_b-n_c W_c=HT\frac{d\ ln\ \Delta t}{dt} \quad (49)$$

For small changes in temperature, Δt, each of the derivatives $\frac{dW_x}{dT}$ and the quantity HT may be considered constant and thus from equation (49) we see that $\frac{d\ ln\ \Delta T}{dt}$ is a time constant α which shows how the temperature of the filament varies after any change in conditions.

By dividing equation (49) by HT we obtain

$$\alpha=\frac{n_a W_a+n_d W_d-n_b W_b-n_c W_c}{HT} \quad (50)$$

where

$$\alpha=\frac{d\ ln\Delta T}{dt} \quad (51)$$

Integrating equation (51) gives

$$\Delta T=\Delta_0\epsilon^{\alpha t} \quad (52)$$

where Δ_0, the constant of integration, is the initial temperature displacement.

It is clear from equation (52) that if α is negative in sign the temperature returns to or approaches its stationary value according to this equation.

If, however, a change is brought about in the conditions governing the temperature of the filament such that α as given by equation (50) is positive in sign, we see from equation (52) that the equilibrium is unstable and the temperature will either rise or fall indefinitely or until the temperature displacement is so great that equation (46) no longer applies.

EXAMPLES INVOLVING SMALL TEMPERATURE DISPLACEMENTS

Change of Input Power

Consider an ideal filament 0.002 cm. in diameter and 35.1 cm. in length operating in good vacuum with a constant drop of 100 volts over the filament so that it is, according to the data of Table I, at a temperature of 2400 deg. K. The current will be 0.1272 amperes.

Let the applied voltage now be suddenly raised to 105 volts and maintained constant at this new value. The question arises: How much will the filament temperature increase and how rapidly will it approach its new value?

The temperature increase can be obtained from the function V' in column 5, Table I, or from the semi-logarithmic plot of this function in Fig. 1. By this method we find that the temperature increases to 2440.1 deg. K.

We may also calculate the temperature rise due to the increase of five per cent in voltage from the data in Table III. We note that when the diameter and length are held constant

$$\frac{d\ ln\ T}{d\ ln\ V}=0.339 \text{ at 2400 deg. K}$$

Hence we have in our example

$$\frac{dT}{T}=0.339\times0.050=0.01695$$

or $dT=40.7$ degrees, which checks well with the above value.

The current change corresponding to this change in temperature is given by

$$\frac{\Delta A}{A}=n_A\frac{\Delta T}{T}$$

where $n_A=\frac{2}{n_w-n_R}$ and may be calculated from the data in Table II. Thus we find

$$\Delta A=0.5725\times0.0167\times0.1272=0.00122 \text{ amperes.}$$

The new current value is, therefore, 0.1284 amperes.

According to equation (52) the increase in temperature from 2400 deg. to 2440 deg. occurs gradually. To calculate the rate, we find the value of α by equation (50). In this equation, $W_d=0$, $W_c=0$, and $n_b=n_w$. Thus by equation (44) we see that W_a and W_b are equal and are the same as the power input which is $105\times0.1284=13.50$ watts. Equation (50) then becomes

$$\alpha=\frac{W}{HT}(n_a-n_w)$$

Since $W=\frac{V^2}{R}$ and V is maintained constant at 105 volts during the time interval that the filament temperature is increasing toward 2440 deg. K, W varies inversely as R or $n_a=-n_R=(-1.196)$ for this case.

Similarly, from Table II, we see that $n_W=4.69$ at 2440 deg. K and by equation (11), $H=H'ld^2=4.06\times10^{-4}$ watt sec. per degree.

Therefore

$$\alpha=\frac{13.50\,(-1.196-4.69)}{4.06\times10^{-4}\times2440}=-80.2$$

Substituting in equation (52), we obtain

$$\Delta T=40\ \epsilon^{-80.2t}$$

which may be written

$$t=\frac{2.303}{80.2}\log_{10}\frac{40}{\Delta T}=0.02770\log_{10}\frac{40}{\Delta T}$$

Hence we may calculate the time necessary for the filament temperature to reach the values 2420, 2439 and 2439.9, *i.e.*, $\triangle T = 20$, 1.0 and 0.1 successively. Thus we find that

$$t_{2420} = 0.00833 \text{ seconds}$$
$$t_{2439} = 0.0443 \text{ seconds}$$
$$\text{and } t_{2439.9} = 0.0720 \text{ seconds.}$$

We have seen that since $W = \frac{V^2}{R}$ $n_a = -n_R$ when a filament is maintained at constant voltage. Similarly, since $W = RI^2$ it follows that $n_a = +n_R$ when a filament is maintained at constant current.

In general, if we have a filament of resistance R, heated by a current from a constant voltage source which passes through a resistance R_2, the exponent n_a for the filament input power is

$$n_a = n_R \left[\frac{R_2 - R_1}{R_2 + R_1}\right] \qquad (53)$$

From equation (53) it follows that if R_2/R_1 is large, the circuit acts as a constant current circuit and n_a is approximately equal to $+1.2$, while if R_2/R_1 is small the circuit is a constant voltage circuit and $n_a = -1.2$ approximately.

TRANSIENT EFFECTS IN PRESENCE OF GAS

Temperature Displacements in a Hot Wire Pressure Gauge

The changes in resistance of a heated filament caused by the cooling effect of gas is used in the Pirani-Hale [23] gauge to measure pressure. The time required to take a reading with such a gauge depends on the rate at which the temperature approaches its final value after a displacement of temperature produced, for example, by a change of pressure.

Let us assume that we have a tungsten filament 0.005 cm. in diameter and 10 cm. in length operating at a temperature of 1050 deg. K in nitrogen at 400 baryes pressure. The bulb temperature T_1 we will assume to be 300 deg. K.

Suppose now that while the filament current is maintained constant there is an increase in pressure sufficient to cool the filament from 1050 deg. to 1000 deg. K. The question arises: How long a time will it take the filament to approach sufficiently near to the final temperature to permit an accurate pressure reading?

We have from equation (50), since $W_d = 0$

$$n_a W_a - n_b W_b - n_c W_c = \alpha H T$$

From equation (44), since $W_d = 0$, we have

$$W_a = W_b + W_c$$

By the condition of constant current, we have

$$n_a = +1.20 \text{ at } 1000 \text{ deg. K.}$$

By Table II,

$n_b = n_W = 5.65$ and by equation (2), $W_b = W''ld = 0.0946$ watts.

[23] C. F. Hale, *Trans. Amer. Electrochem. Soc.* Vol. *20*, 243 (1911). See also N. Campbell, *Proc. Phys. Soc.* London, *33*, 287 (1921).

The watts conducted in nitrogen at low pressure is given by equation (32a), *i.e.*,

$$W'_c = \frac{W_c}{ld} = 18.8 \times 10^{-6}\, p\, (T - T_1)$$

whence we calculate

$$W_c = 18.8 \times 10^{-6} \times 400\, (1000 - 300) \times 10 \times 0.005$$
$$= 0.264 \text{ watts}$$

Hence $W_a = 0.0946 + 0.264 = 0.359$ watts

Expressing equation (32a) logarithmically and differentiating we have

$$\frac{d \ln W_c}{d \ln T} = n_c = \frac{T}{T - T_1} = \frac{1000}{1000 - 300} = 1.428$$

Substituting in equation (50), we obtain

$$\alpha = \frac{(1.20 \times 0.359) - (5.65 \times 0.0946) - (1.428 \times 0.264)}{0.00055 \times 1000}$$

$$= \frac{0.431 - 0.534 - 0.377}{0.550} = 0.873$$

Substituting in equation (52), we have

$$\triangle T = 50\, \epsilon^{-0.873t}$$

which may be simplified to

$$t = \frac{2.303}{0.873} \log_{10} \frac{50}{\triangle T} = 2.64 \log_{10} \frac{50}{\triangle T}$$

Solving we find

$t_{(\text{sec.})}$	$\triangle T$	T_f
1.052	20	1020
1.84	10	1010
4.48	1	1001

Hence we note that under these conditions an accurate pressure reading could be obtained in approximately five seconds.

In a similar manner we have calculated α', t'_g and S' for several other filament temperatures and nitrogen pressures. Table V gives some of the results which may be useful in designing Pirani-Hale gauges.

TABLE V
TRANSIENT EFFECTS IN RESISTANCE MANOMETERS

1	2	3	4	5	6	7
T_f deg. K	$P_{(N)_2}$	$\alpha' = \alpha d$	$\triangle T$	$\triangle_0$	t'_g sec. $= t/d$	$\triangle V'/V' = \triangle V/V = S'$
500	0	−0.000157	0.01	1.0	29,300	0.123
500	10	−0.000196	0.01	1.0	23,500	0.0553
500	100	−0.000792	0.01	1.0	5,820	0.00925
500	500	−0.00145	0.01	1.0	3,175	0.00197
800	0	−0.00142	0.01	1.0	3,240	0.0178
800	10	−0.00144	0.01	1.0	3,200	0.0151
800	100	−0.00177	0.01	1.0	2,600	0.00640
800	500	−0.00318	0.01	1.0	1,450	0.00180
1000	400	−0.00437	0.01	1.0	1,055	0.00184
1200	.0	−0.00830	0.01	1.0	555	0.00325
1200	10	−0.00832	0.01	1.0	553	0.00315
1200	100	−0.00842	0.01	1.0	547	0.00245
1200	500	−0.00887	0.01	1.0	520	0.00124

Column 1 gives the final temperature of the filament in degrees Kelvin; column 2, the pressure of nitrogen in baryes, while column 3 gives the function α' for a filament 1 cm. in diameter. Columns 4 and 5 have been so chosen that the data on the time of approach (t') in column 6 is expressed as the number of seconds required for the temperature to approach within 1 per cent of the final value after a small displacement of temperature has occurred.

Column 7 gives the sensitivity (S') of a constant current filament per barye of nitrogen.

It follows that for any filament

$$t = t'_g \times d, \tag{54}$$

and $$\alpha = \frac{\alpha'}{d} \text{ where } d \text{ is expressed in cm.} \tag{55}$$

Instability of a Filament Heated by Ion Bombardment

Let us calculate α for the case of a filament operating in ionized mercury vapor where the positive ion bombardment causes considerable heating.

Consider an ideal filament 0.04 cm. in diameter and 20 cm. in length operating at 2200 deg. K in mercury vapor at a low pressure. The filament will radiate (W_b) 95.75 watts and the current will be 9.730 amperes.

Let us assume that there is an anode in the tube at 100 volts positive with respect to the filament and that for every 10 electrons leaving the filament there is one mercury positive ion striking the cathode with a kinetic energy corresponding to 100 volts.

The energy delivered to the filament by one positive ion striking it is the product of the charge on an electron and the sum of the cathode drop (100 volts), the ionizing potential of mercury (10.4 volts) minus the work function of the tungsten surface for electron emission (4.5 volts). Therefore, the total energy received by the filament from positive ions is

$W_d = 0.1\ (I) \times 105.9 = 10.59\ (I)$ (where I is the electron emission in amperes)

Since $I = I'ld$ we find that

$$I_{2200} = 0.0417 \times 20 \times 0.04 = 0.0335 \text{ amperes}$$

Hence

$$W_d = 10.59 \times 0.0335 = 0.3545 \text{ watts.}$$

If we imagine that a temperature change has just occurred while the filament current is maintained constant, which brings the filament to 2200 deg. K as a steady value, we may determine whether the filament temperature will be stable by calculating α from equation (50).

By the condition that we are operating in mercury vapor at a low pressure, we see by equation (31b) that we are justified in assuming $W'_d = 0$ so that equation (50) becomes

$$n_a W'_a + n_d W'_d - n_b W'_b = \alpha HT$$

Similarly, since $W_d = 0$, we have from equation (46)

$$W_a + W_d = W_b$$

By maintaining the filament current constant, we have $n_a = n_R = 1.19$ at 2200 deg. K.

By equation (44) we have $W'_a = W'_b - W'_d = 95.75 - 0.3545 = 95.39$ watts.

Also, we have $n_d = n_i$ which is given in column 10, Table II. Thus $n_d = n_i = 25.8$ at 2200 deg. K.

$n_b = n_W = 4.81$ at 2200 deg. and $H = H'ld^2 = 2.76 \times 20 \times 1.6 \times 10^{-3} = 0.0884$ watt sec. per degree.

Substituting in equation (50), we have

$$\alpha = \frac{(1.19 \times 95.39) + (25.8 \times 0.3545) - 4.81 \times 95.75}{0.0884 \times 2200}$$
$$= \frac{113.5 + 9.15 - 460.5}{194.5}$$
$$= -1.730$$

whence it follows from equation (52) that the filament temperature is stable under these conditions and the rate of approach to the new temperature is fairly rapid.

We have calculated α for different filament temperatures under the conditions previously imposed.

T_f	α
2200	−1.730
2300	−1.91
2400	−1.975
2500	−1.82
2600	−1.193
2700	+0.314
2800	+3.425

From a plot of α against T we find that $\alpha = 0$ at 2680 deg. K. Hence at temperatures higher than this the filament is unstable and the temperature will rise indefinitely or until, because of a large temperature displacement, some factor not considered in our present theory limits the current.

TRANSIENT TEMPERATURE EFFECTS ON ALTERNATING CURRENT

Small and Rapid Temperature Displacement

When a filament is heated by alternating current [24] of frequency f, the temperature of the filament fluctuates with a frequency $2f$. The temperature of the filament varies between the limits $T - \Delta$ and $T + \Delta$ where T is the average temperature and Δ, the amplitude of the temperature fluctuation, is given by

$$\Delta = \frac{W}{4\pi f H} = \frac{W'}{4\pi f d H'} \tag{56}$$

W being the average power input in watts.

APPENDIX

It may be of interest to analyze in detail the method suggested by Mr. H. M. Mott-Smith, Jr., to obtain the partial derivatives of lamp variables by means of matrices and determinants.

[24] "The Flicker of Incandescent Lamps on Alternating Currents," Langmuir, General Electric Review 17, 294 (1914).

Let W, R, L, T, l and d be lamp variables where

W = watts
R = ohms
L = lumens
T = temperature
l = length
d = diameter

The symbols $\mathbf{W}$, $\mathbf{R}$, $\mathbf{L}$, $\mathbf{T}$, $\mathbf{l}$, and $\mathbf{d}$ in bold-faced Roman type will now be used to represent the logarithms of these quantities.

We have already shown that when we have added two conditions, we have five equations containing the above variables. These equations are

$$\mathbf{W} = \mathbf{A_W} + n_W\mathbf{T} + \mathbf{l} + \mathbf{d} \tag{38}$$

$$\mathbf{R} = \mathbf{A_R} + n_R\mathbf{T} + \mathbf{l} - 2\mathbf{d} \tag{39}$$

$$\mathbf{L} = \mathbf{A_L} + n_L\mathbf{T} + \mathbf{l} + \mathbf{d} \tag{40}$$

$$\lambda_W\mathbf{W} + \lambda_R\mathbf{R} + \lambda_L\mathbf{L} + \lambda_l\mathbf{l} + \lambda_d\mathbf{d} = \mathbf{C_1} \tag{41}$$

$$\epsilon_W\mathbf{W} + \epsilon_R\mathbf{R} + \epsilon_L\mathbf{L} + \epsilon_l\mathbf{l} + \epsilon_d\mathbf{d} = \mathbf{C_2} \tag{42}$$

Since we are to deal only with the total derivative of one variable with respect to another variable when two other variables are held constant, the constant terms in the equations become zero.

A matrix may be formed from the coefficients of the variables in these different equations. Thus

$$\begin{array}{cc} & \begin{matrix} \mathbf{W} & \mathbf{R} & \mathbf{L} & \mathbf{T} & \mathbf{l} & \mathbf{d} \end{matrix} \\ M = & \left|\begin{matrix} 1 & 0 & 0 & -n_W & -1 & -1 \\ 0 & 1 & 0 & -n_R & -1 & +2 \\ 0 & 0 & 1 & -n_L & -1 & -1 \\ \lambda_W & \lambda_R & \lambda_L & 0 & \lambda_l & \lambda_d \\ \epsilon_W & \epsilon_R & \epsilon_L & 0 & \epsilon_l & \epsilon_d \end{matrix}\right| \end{array}$$

There remains only one degree of freedom and we can determine the derivative of any variable with respect to any other variable.

We may simplify the above matrix by the ordinary rules of determinants, eliminating columns and rows until there remain only the two variables of which we want the derivatives. An example will illustrate the method.

Let us fulfil one of the conditions by postulating that the length of the filament shall remain constant. Then $\lambda_l = 1$ and $\lambda_W = \lambda_R = \lambda_L = \lambda_d = 0$. Substituting in the above matrix, we have

$$\begin{array}{ccc} & \begin{matrix} \mathbf{W} & \mathbf{R} & \mathbf{L} & \mathbf{T} & \mathbf{l} & \mathbf{d} \end{matrix} & \\ M = & \left|\begin{matrix} 1 & 0 & 0 & -n_W & -1 & -1 \\ 0 & 1 & 0 & -n_R & -1 & +2 \\ 0 & 0 & 1 & -n_L & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \epsilon_W & \epsilon_R & \epsilon_L & 0 & \epsilon_l & \epsilon_d \end{matrix}\right| & l = \text{const.} \end{array}$$

Since all the coefficients except one in the fourth row are zero, the matrix may be reduced by dropping the column and row in which this coefficient occurs, the new matrix being multiplied by the value of this coefficient, *i.e.*, unity

$$\begin{array}{ccc} & \begin{matrix} \mathbf{W} & \mathbf{R} & \mathbf{L} & \mathbf{T} & \mathbf{d} \end{matrix} & \\ M = & \left|\begin{matrix} 1 & 0 & 0 & -n_W & -1 \\ 0 & 1 & 0 & -n_R & +2 \\ 0 & 0 & 1 & -n_L & -1 \\ \epsilon_W & \epsilon_R & \epsilon_L & 0 & \epsilon_d \end{matrix}\right| & l = \text{const.} \end{array}$$

Let us now fulfil the second condition by postulating that the volts (V) shall remain constant. Since $\frac{\mathbf{W}+\mathbf{R}}{2} = \mathbf{V}$, we have the condition that $\epsilon_W = \epsilon_R = 1$ and $\epsilon_L = \epsilon_d = 0$. Substituting these values in the above matrix and subtracting column 2 from column 1, we obtain the matrix:

$$\begin{array}{ccc} & \begin{matrix} \mathbf{W} & \mathbf{R} & \mathbf{L} & \mathbf{T} & \mathbf{d} \end{matrix} & \\ M = & \left|\begin{matrix} 1 & 0 & 0 & -n_W & -1 \\ -1 & 1 & 0 & -n_R & +2 \\ 0 & 0 & 1 & -n_L & -1 \\ 0 & 1 & 0 & 0 & 0 \end{matrix}\right| & \begin{matrix} l = \text{const.} \\ V = \text{const.} \end{matrix} \end{array}$$

The second column and fourth row may be eliminated from this matrix by the rule given above to obtain the matrix.

$$\begin{array}{ccc} & \begin{matrix} \mathbf{W} & \mathbf{L} & \mathbf{T} & \mathbf{d} \end{matrix} & \\ M = & \left|\begin{matrix} 1 & 0 & -n_W & -1 \\ -1 & 0 & -n_R & +2 \\ 0 & 1 & -n_L & -1 \end{matrix}\right| & \begin{matrix} l = \text{const.} \\ V = \text{const.} \end{matrix} \end{array}$$

The second column and third row may now be dropped to obtain the matrix

$$\begin{array}{ccc} & \begin{matrix} \mathbf{W} & \mathbf{T} & \mathbf{d} \end{matrix} & \\ M = & \left|\begin{matrix} +1 & -n_W & -1 \\ -1 & -n_R & +2 \end{matrix}\right| & \begin{matrix} l = \text{const.} \\ V = \text{const.} \end{matrix} \end{array}$$

If we now add row one to row two we obtain

$$\begin{array}{ccc} & \begin{matrix} \mathbf{W} & \mathbf{T} & \mathbf{d} \end{matrix} & \\ M = & \left|\begin{matrix} 1 & -n_W & -1 \\ 0 & -(n_R+n_R) & +1 \end{matrix}\right| & \begin{matrix} l = \text{const.} \\ V = \text{const.} \end{matrix} \end{array}$$

Column one and row one may be dropped from this matrix to give

$$\begin{array}{ccc} & \begin{matrix} \mathbf{T} & \mathbf{d} \end{matrix} & \\ M = & \left|\begin{matrix} -(n_W+n_R) & +1 \end{matrix}\right| & \begin{matrix} l = \text{const.} \\ V = \text{const.} \end{matrix} \end{array}$$

Whence we observe that when length (l) and volts (V) remain constant $\frac{d\mathbf{T}}{d\mathbf{d}} = \frac{1}{n_W+n_R} = 0.169$ at 2400 deg. K.

Similarly, we may reduce the original matrix until any two terms remain. In this case, the total derivative of the first term with respect to the second is always the quotient obtained by dividing the second term by the first term with its sign reversed.

(*Concluded*)

Equation (1) applied to melting by induction heating can be further rearranged thus:

$$t' = \frac{h_a}{I_s^2 \times R_s \times 1000} \tag{1d}$$

$$= \frac{h_a}{I_s \times E_s \times 1000} \tag{1e}$$

in which

t' = theoretical time in hours for the metal to reach a given temperature.
h_a = kilowatt hours absorbed by the metal from Equation (2).
I_s = current induced in the secondary circuit.
R_s = resistance of the secondary circuit.
E_s = voltage induced in the secondary circuit.

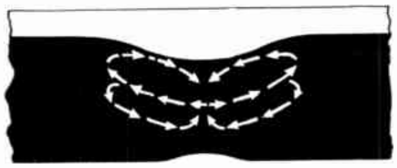


Fig. 62. A Diagram of a Portion of a Molten Metal Conductor, Showing the Constriction of Its Sectional Dimension Occasioned by Electromagnetic Forces

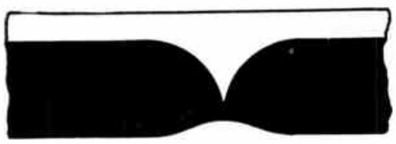


Fig. 63. The So-called "Pinch Effect," which Sometimes Occurs When the Constrictive Effect of Electromagnetic Forces Becomes Pronounced

Equations (1d) and (1e), together with Equation (2), state *first* that the theoretical time required to melt a given charge of metal depends upon the specific heat, the latent heat, and the final temperature; and, *second*, that this time is inversely proportional to the input of energy into the metal. The value h_a in a given case is a definite quantity. Hence the minimum value of t' is only a matter of the rate at which energy can be supplied. The actual time is greater than the theoretical time by an amount depending upon the limitations of the power input imposed by the design of the furnace and upon the rate at which heat can be accumulated in the metal, the last mentioned being a matter of the degree to which the mass of metal is insulated against the flow of heat to the surrounding atmosphere.

(*b*) The secondary coil of an induction furnace becomes, when melted, a liquid conductor at high temperature, *i.e.*, molten metal. A liquid conductor can be considered as an infinite number of current-carrying elements or conductors in parallel. The law of electro-magnetic forces between parallel conductors is:

$$F = \frac{I_1 \times I_2 \times c}{s} \tag{13}$$

in which

F = force acting per unit length; attractive if the currents flow in the same direction, repulsive if in opposite directions.
I_1, I_2 = currents in the conductors.
c = a constant.
s = the distance between the conductors.

In the case of parallel conductors free to move and in which the currents flow in the same direction this force will tend to bring the conductors together to form a closed bundle of circular cross section as indicated in Fig. 61. If the conductor is a liquid contained in an open channel the radial pressure created tends to force the liquid towards the center and out at the ends thus decreasing the cross section of the conductor. If there is an irregularity in the channel of the liquid conductor, as shown in Fig. 62, the liquid will be constricted and consequently the current density and the pressure will be increased at this point. The result is that the liquid is forced away from the point of higher pressure, a movement being set up as indicated. If the current in the liquid conductor is great enough the liquid will be forced away faster than it can return and the circuit will be ruptured, as in Fig. 63. This phenomenon is known as the "pinch effect."[20] It is a factor which has much bearing upon the design of induction furnaces.

Equation (14) by Hering[21] applies to liquid conductors in open channels whose width is one-half the initial height of the liquid. The value I_c in this equation is the current which will cause rupture of the circuit and is termed the critical current.

$$I_c = 449\sqrt{G \times H^3} \tag{14}$$

in which

I_c = critical current.
G = specific gravity of the liquid.
H = initial height of the liquid.

From Table XIV it will be noted that the current densities corresponding to the critical currents for various sizes of the channel varies with the size of the channel. In the use of induction furnaces with open channels the current in the liquid conductor (secondary circuit) must be limited to a safe operating value below the critical value for the size and type of channel used. As a general rule, three-fourths of the critical value provides sufficient margin.

(*c*) The high temperature of the liquid conductor makes it necessary to use a refractory container for the secondary circuit of the furnace. This construction requires a comparatively large amount of space in the transformer construction, *i.e.*, between the primary and secondary coils. Thus the reactance is high; and unless the power supply is of low frequency the power-factor of the furnace load will be low.

[20] Northrup: *Physical Review*, Vol. 24, p. 474.
[21] Hering: *Trans. Amer. Electrochem. Soc.*, Vol. 11, p. 330, and Vol. 15, p. 255.

(*d*) The heating effect of the currents in the secondary circuit being proportional to the electric resistance of the circuit, the specific resistance of the metal to be melted is a determining factor in the use of induction heating for melting. Data on the specific resistances of metals and alloys in the molten state are limited. The values of Table XV were compiled from various sources. In any case the value of specific resistance depends, of course, upon the temperature of the metal. Tama[18] gives the chart of Fig. 64, showing the relative values of specific resistance of copper-zinc alloys in the solid state at 0 deg. C. and in the liquid state at the melting temperature. The chart[22] in Fig. 65 shows the resistivity of a brass of composition 63 Cu, 34.58 Zn, 2.42 Pb. It will be noted that the molten metal has a negative resistance temperature coefficient. According to Northrup this is due to the zinc in the alloy, he having determined previously[23] that the resistance temperature coefficient of zinc in the molten state (up to 750 deg. C.) is negative.

(*e*) The forces acting upon the liquid conductor which forms the secondary circuit of the furnace tend to cause motion of the liquid, *i.e.*, the molten metal. In general, the more viscous the molten metal the more sensitive it is to the forces tending to cause motion of the metal. The viscosities of molten metals vary. However there is but little, if any, data on this property.

From the foregoing considerations we can classify the properties of metals with reference to melting by induction heating, as in Table XVI.

The lower the values of specific heat, latent heat and melting temperature, the lower the heat content for any given pouring temperature. None of these values affect the permissible operating current in the secondary circuit. The higher the specific gravity, the higher may be the operating current, as in Equation (14). The higher the specific resistance, the higher will be the rate of heat development by the permissible current. The heat developed by the current in the secondary circuit must supply the heat loss of the furnace as well as raise the temperature of the charge of metal. The limit of this temperature is reached when the rate of heat loss equals the rate at which heat is developed. If in a given furnace the specific gravity and the specific resistance of the metal are low, it may not be possible with the limited current to develop heat at a sufficient rate to raise the temperature of the metal to the desired value.

TABLE XIV

CHANNELS		2 IN. BY 1 IN.		4 IN. BY 2 IN.		6 IN. BY 3 IN.		8 IN. BY 4 IN.	
Metal	G	I_c	CD_c	I_c	CD_c	I_c	CD_c	I_c	CD_c
Tin	6.99	3300	1650	9800	1225	17000	945	26000	810
Lead	10.60	4100	2050	11500	1440	21000	1165	33000	1030
Zinc	6.48	3200	1600	9100	1140	16500	920	25500	800
Aluminum	2.67	2000	1000	5800	725	10500	585	16500	515
Silver	9.51	3900	1950	11000	1375	20000	1110	31000	970
Copper	8.20	3600	1800	10000	1250	18500	1030	29000	900
Nickel	8.80	3700	1850	10500	1310	19500	1080	30000	940
Steel	6.88	3300	1650	9500	1185	17000	945	26500	830

G = specific gravity.
I_c = critical current for liquid conductors in open channels whose width is one-half the initial height of liquid.
CD_c = current density corresponding to I_c.

TABLE XV

	MICROHMS PER CENTIMETER CUBE	
	Solid	Liquid
Metal		
Tin	11.3	54.6
Lead	20.8	102.8
Zinc	6.1	36.6
Aluminum	2.94	24.0
Silver	1.65	17.0
Copper	1.78	24.8
Nickel	11.8	...
Steel	10.8	1.66
Alloys		
Cu 66⅔ Zn 33⅓	4.0	41
Monel Metal	42.5	80
Ni 66 Cr 22 Fe 10 Mn 2	109	...

TABLE XVI

Property	Favorable	Unfavorable
Specific heat	low	high
Latent heat	low	high
Melting point	low	high
Specific gravity	high	low
Specific resistance	high	low
Viscosity	low	high

Types of Induction Furnaces

There are two types of induction furnaces in use today, *viz.*, the core type and the coreless type. The transformer of the former type has a magnetic core and is designed for use on circuits of a standard frequency—60, 50, 40, 30 or 25 cycles—and in some cases for a special low frequency, below 25 cycles.

The coreless, or high-frequency type, furnace has no magnetic core and requires a frequency considerably higher than those adopted as standard for power service. Here the term high frequency means, say,

(22) Northrup: *Metallurgical and Chemical Engineering*, Vol. 12, p. 161.
(23) Northrup and Suydam: *Jour. Fran. Inst.*, February, 1913.

500 cycles or higher; although no particular value of frequency is assigned to the term.

For reasons of simplicity of design and construction, induction furnaces are as a rule single-phase. Some three-phase induction furnaces have been built; but since the three-phase design has only the added merit of a balanced load as compared with the single-phase furnace, the three-phase design has made but little progress. With the growth of the size of power supply systems the magnitude of the permissible single-phase load has increased; and it is now not uncommon to find single-phase furnace loads up to 300 kv-a. In non-ferrous melting service large melting furnace units are not usually required. As a rule, two or more

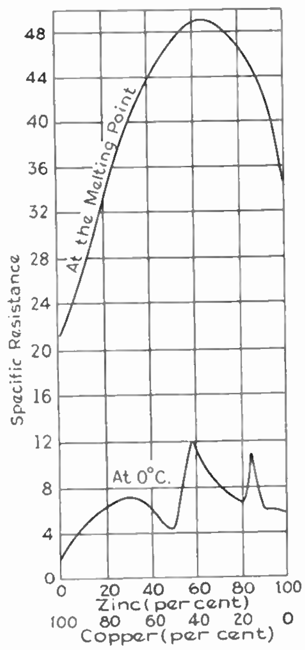


Fig. 64. Relative Values of Specific Resistances for Copper-zinc Alloys in Solid and Molten States (*Tama*)

small units are better adapted to the operating conditions; and this arrangement lends itself to obtain a balanced load on the power system. For special frequencies a motor-generator set is required which, of course, constitutes a balanced load. The load of an induction furnace is inherently steady, the load chart being practically a straight line. For holding the temperature of a charge of molten metal a low voltage, obtained by a transformer tap, is used. The power-factor depends upon the design of the furnace and upon the metal being melted. This will be referred to again in the later considerations of furnace types.

Two forms of the core-type induction furnace have been developed, *viz.*, the Horizontal Ring furnace and the Vertical Ring furnace. While the Horizontal Ring furnace is designed primarily for melting and refining ferrous alloys and non-ferrous alloys of high specific resistance and is not intended for general application to melting non-ferrous alloys, it is logical to include this form at this point in the treatment of induction furnaces.

The Horizontal Ring Induction Furnace. The name of this furnace itself indicates the arrangement of the secondary circuit of the transformer, *i.e.*, a ring or loop of metal in a horizontal plane. The general design of the furnace is shown in Fig. 66. The entire mass of the charge of metal is contained in a ring crucible, which is an *open* channel in a refractory material. The charge of metal is thus formed into a single-turn loop for the secondary

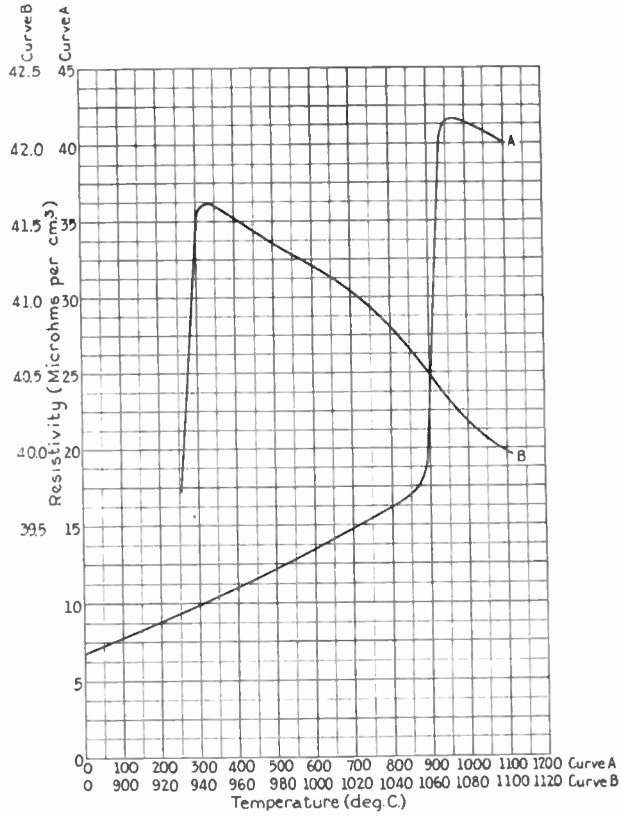


Fig. 65. Resistivity of a Copper-zinc-lead Alloy (*Northrup*)

circuit. The magnetic core is of the three-leg type made up of laminated steel in accordance with standard transformer practice. The primary coil, of the circular disk type insulated with mica and asbestos, is located above the secondary coil (the molten metal loop) so that it will not be damaged in the event of a leak in the crucible. A small motor-driven blower supplies air under a low pressure, two to four ounces, for cooling the primary coil. A description of a 6-ton furnace[24] of this type has been previously published. A typical installation is shown in Fig. 67.

In this furnace there is an open channel for the liquid conductor, and the forces acting upon the secondary circuit so formed produce the following results:

(*a*) Keeping within the permissible limits of operating current value for the size and proportions of the open channel used and for the metal to be melted (see Table XIV) there is an unbalanced pressure set up within the molten metal, as indicated in

[24] "A Six-ton Induction Furnace Installation," by M. Unger, General Electric Review, August, 1924, p. 498.

Fig. 62, which causes a more or less gentle motion of the metal along the horizontal axis of the channel. If there are slag or gases—comparatively poor conductors—in the metal these impurities act in the same way as irregularities in the channel, *i.e.*, they reduce the cross-section of the liquid conductor and thereby aid in producing motion of the molten metal.

(*b*) Owing to the annular shape of the secondary circuit, the current density in the loop of metal is greater at the inner face of the ring, *i.e.*, where the path of the current is the shortest and resistance lowest. Hence, the attractive force within the liquid conductor is not uniform, being greatest at the inner face. This inequality of forces tends to set up a circulating motion in the molten metal as indicated in Fig. 68.

(*c*) With the arrangement of primary and secondary coils as shown in Fig. 66, the repulsive force between these two circuits, given by Equation (13), tends to repel the current-carrying elements of the liquid conductor towards the bottom of the channel. This action forces the impurities in the molten metal to the surface.

The combined effect of these forces in the operation of the Horizontal Ring furnace may be summarized thus: The molten metal is forced towards the center of the channel so that any impurities in the metal are squeezed out. The molten metal is furthermore subjected to circulation, one motion in a vertical

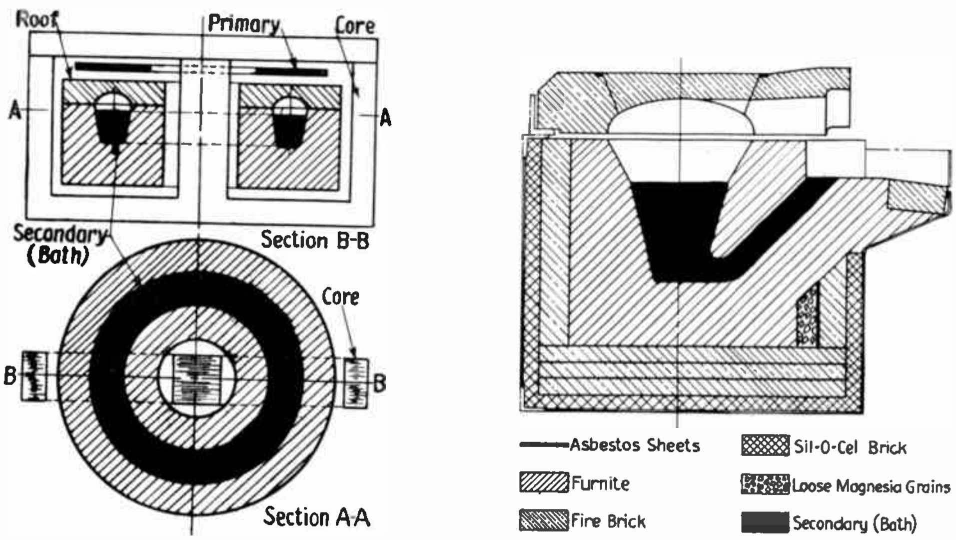


Fig. 66. A Diagram of the Horizontal Ring Induction Furnace, Showing a Section Along the Transverse Horizontal Axis and Another Through the Pouring Spout

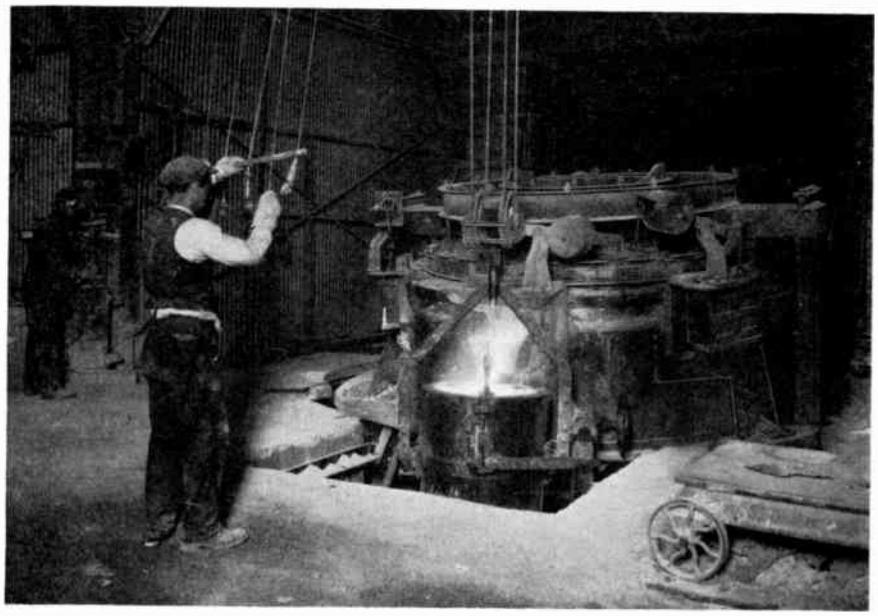


Fig. 67. An Installation of a Horizontal Ring Induction Furnace of 4000-lb. Capacity. The electrical rating is 250 kw., 2200 volts, 15 cycles

plane, perpendicular to the axis of the channel, and another in horizontal planes in the direction of the axis of the channel. This combination of forces provides a stirring action the degree of which is governed by the extent of the unevenness of the channel, the amount of impurities present, and upon the value of the current in the conductor. The current is regulated to give the desired degree of stirring as dictated by the metallurgical conditions of the melting or refining operation.

The starting of the Horizontal Ring furnace is accomplished either by pouring molten metal into channel for the secondary circuit or by the use of solid starting rings made to fit the contour of the channel, one of which is shown in Fig. 69. These rings must of course be of the same metal as the charge. In the production of alloys, instead of using a single starting ring which may cause some trouble due to rupture

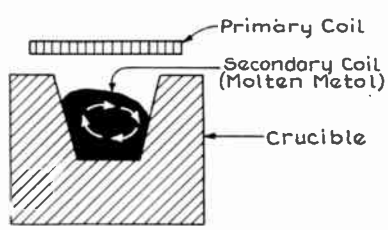


Fig. 68. A Sectional Diagram Showing the Forces Acting Upon the Secondary Circuit of a Horizontal Ring Induction Furnace

of the circuit, two rings of different melting points are used. After the starting rings are melted, cold metal is added to complete the charge. When pouring, about one-third of the charge is left in the furnace for starting the subsequent charge.

The power-factor of this type of furnace is comparatively low owing to the loose coupling between the primary and secondary coils. The higher the specific resistance of the metal of the charge the higher will be the power-factor; moreover, the furnace will operate at a higher power-factor with a portion of a charge than with a full charge. With the larger sizes of furnaces the power-factor at a standard frequency is lower than is desirable; and a motor-generator set is used which both supplies a low frequency for the furnace circuit and balances the load on the power system. Also, if desirable, a synchronous motor can be used for the motor-generator set and capacity added to this machine for power-factor correction. A furnace having a capacity of 500 lb. of steel has a power-factor of approximately 50 per cent on a 60-cycle circuit. This value of power-factor is also obtained with a 1500-lb. furnace on a 25-cycle circuit, and with a 4000-lb. furnace on a 15-cycle circuit, in each case with a full charge of molten metal.

The Vertical Ring Induction Furnace. The distinguishing feature of this form of core type induction furnace is the use of a *closed* channel of a construction and location which eliminate the current density limitation of open channels and adapt induction heating to the melting of metals and alloys which are outside of the range of furnaces with open channels. The Vertical Ring induction furnace is particularly well suited to the melting of copper-zinc alloys and is widely used for that purpose by the brass industry. The construction is shown in Fig. 70.

The secondary circuit of the transformer is a single-turn loop of molten metal in a channel which forms a V in a vertical plane below, and connecting with, the main body of the charge of metal. The mass of metal in the V loop is comparatively

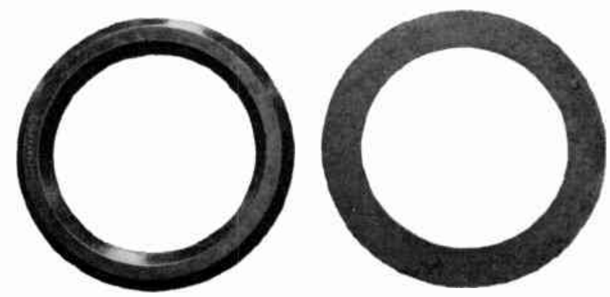


Fig. 69. Starting Rings for a Horizontal Ring Induction Furnace

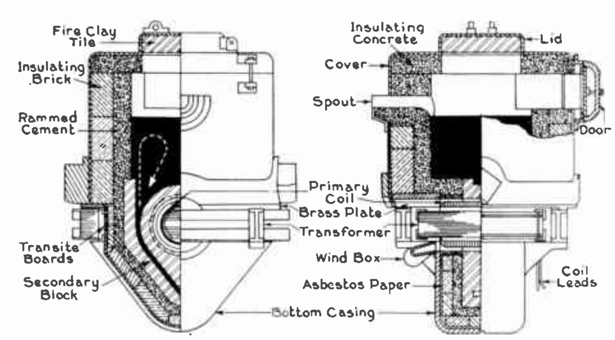


Fig. 70. Sectional Views of a Vertical Ring Induction Furnace

small, from 3 to 10 per cent of the total mass of the charge, the amount in each case depending upon the size of the furnace and the properties of the metal or alloy for which the furnace is designed. The heat developed by the current induced in the secondary circuit is transferred to the main body of the charge by liquid convection. The V loop of molten metal is thus an indirect heating unit. The convection movement of molten metal in the channel is effected by the combined actions of the force of gravity and the electrodynamic forces acting on the liquid conductor.

The force of gravity effect is that which follows from the difference in densities of the particles of molten metal in the channel and of the cooler (and hence denser) particles in the main body of the charge. The electrodynamic forces acting are due in part to the geometrical arrangement of the liquid conductor, this element of the forces being a maximum at the apex of the triangle formed by the channel, and in part to the "pinch effect" phenomenon which action is a maximum at the junctions of the channel with the main body of the charge. A complete analysis of the actions of these forces is given elsewhere.[25] The channel in the refractory material for the secondary circuit is designed to utilize these forces to produce a movement, or circulation, of the molten metal, *i.e.*, the liquid convection noted in a preceding paragraph and indicated by the arrow in Fig. 70. The channel is designed as a secondary circuit of a transformer with a cross-section and length corresponding to rate of melting desired; and in addition there must be taken into account the viscosity of the molten metal and the frictional resistance of the channel. The primary coil and magnetic core follow in design standard transformer practice. The primary coil is cooled by low-pressure air supplied by a small motor-driven blower.

The rate of melting a given metal or alloy in the Vertical Ring furnace is a matter of the permissible power input into the secondary circuit. There is, of course, a limit to the temperature to which the liquid conductor of the secondary circuit can be carried both on account of the volatilization of the metal and the temperature limit of the refractory material which contains the channel. As in all transformers, the primary coil is limited also to a definite kilowatt input at the voltage for which it is designed. Furnaces of this type operating with pouring temperatures up to 2400 deg. F. are in commercial service.

The power-factor of this type of furnace is comparatively high, being 75 to 80 per cent on 60-cycle circuits and somewhat higher on 25-cycle circuits. In operation, the loop forming the secondary circuit is constantly kept full by the hydrostatic head of the charge above; and the resistance of the circuit is practically constant. A chart showing the uniformity of the load is reproduced in Fig. 71. A regulating transformer with a number of taps is provided for holding the temperature of a charge. A diagram of the furnace connections is shown in Fig. 72.

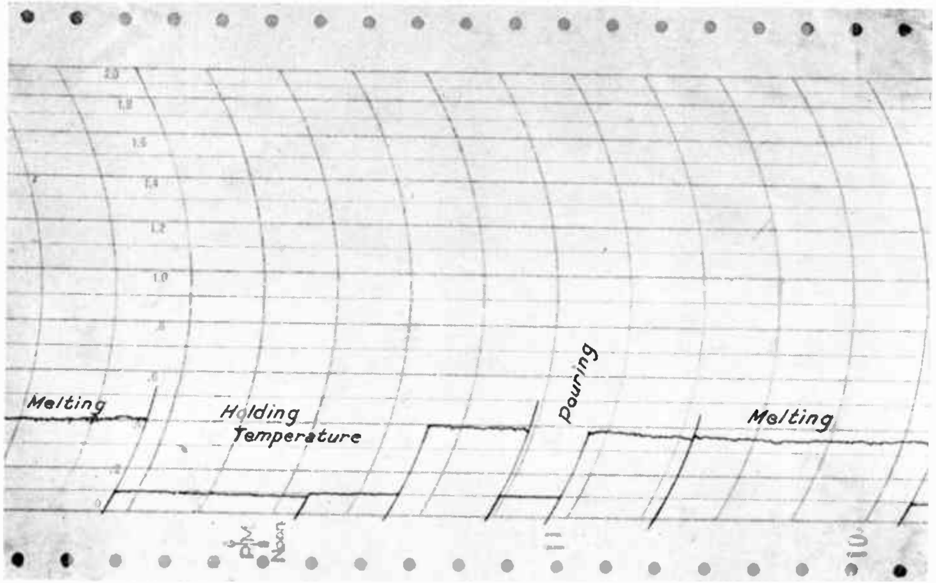


Fig. 71. A Typical Load Chart of a Single-phase 60-kw., 220-volt Vertical Ring Induction Furnace

In starting the Vertical Ring furnace it is first necessary to fill the V channel with molten metal: and this loop must be kept in a molten state as long as the furnace is in service. This requirement makes the furnace inflexible as regards frequent changes of alloy compositions. In general, this type of furnace

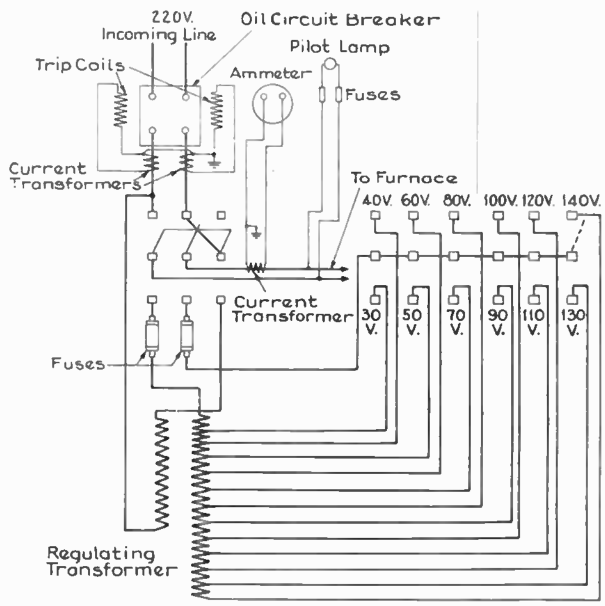


Fig. 72. The Connection Diagram of a Single-phase 60-kw. 220-volt Vertical Ring Induction Furnace

[25] Northrup: "Nature and Explanation of the 'Motor Effect' in the Ajax-Wyatt Furnace," *Jour. of the Franklin Institute.* Vol. 190, p. 817.

is intended for a continuous melting service although there may be idle periods, as nights, holidays, and Sundays in the schedule of operations. During such periods the metal in the furnace is held molten by a low-voltage tap on the regulating transformer.

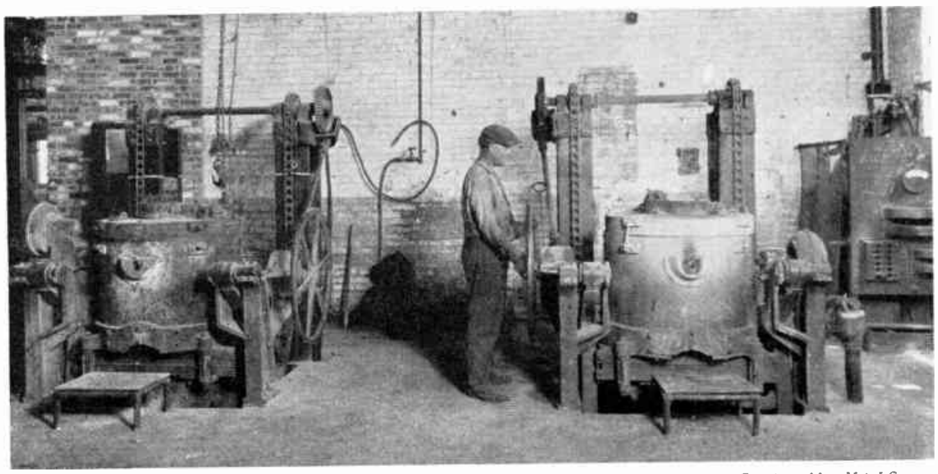
TABLE XVII

	Output, Lb.	Per Cent
Gross Charges	3,352,581	100
Net Castings	3,285,062	97.99
Net recovery on skimmings and splashings	53,552	1.59
Total metal shrinkage including oil and non-metallics in original charge	13,967	0.42

The data of a test[26] of this type of furnace melting brass for metal loss, analysis of output, and energy consumption are given in Tables XVII, XVIII, and XIX respectively.

The operating efficiency, *i.e.*, the kilowatt-hours per ton of castings over an extended period of operation of a furnace, depends of course upon the many factors that enter into a schedule of manufacturing operations. Thus the operating efficiency of a furnace on the same metal will vary as the operating conditions are changed. The highest operating efficiency is obtained when the furnace runs continuously, when the metal is poured as soon as it reaches the pouring temperature, and when the charging and pouring times are a minimum percentage of the total time.

The manufacturers of the Ajax-Wyatt Vertical Ring induction furnace give as representative results with this furnace the data of Table XX.



Courtesy Ajax Metal Co.

Fig. 73. A Typical Installation of Two 60-kw. 220-volt Single-phase Furnaces of the Vertical-ring Induction Type in Operation in a Foundry

TABLE XVIII

ANALYSES OF POURINGS FOR LEAD CONTENT

Heat No.	First Pouring Cu	First Pouring Pb	Last Pouring Cu	Last Pouring Pb
1	64.44	1.03	64.38	1.05
2	64.38	1.01	64.38	1.08
3	64.27	1.09	64.24	1.17
4	64.49	1.10	64.52	1.02

TABLE XIX

Alloy	Estimated Pouring Temperature	Kw-hr. Per Ton
60 Cu 40 Zn	1850 deg. F.	172
65 Cu 35 Zn	1960 deg. F.	186
66.7 Cu 33.3 Zn	2000 deg. F.	192
70 Cu 30 Zn	2070 deg. F.	200

(26) Heuer: "The Ajax-Wyatt Furnace in the Brass Mill Casting Shop," *Jour. Ind. and Eng. Chem.*, Vol. 14, p. 1021.

TABLE XX

Foundry Operation: 9 hr. per day.
Product: brass, 75 Cu, 23 Zn 2 Pb medium weight castings, 7 lb. per mold.
Charge: Clean medium-weight scrap and ingots.
Pour: 500 to 800 lb.
Average melting time per heat (600 lb.): 60 min.
Production (8 heats per day): 2½ to 3 tons per day.

	Melting Cost Per Ton
Energy consumption, 275 kw-hr. per ton (includes stand-by nights and week ends) at 2 cents per kw-hr	$5.50
Lining (life, 500 tons)	0.30
Maintenance	0.18
Metal loss (0.5 per cent) at 11 cents per lb.	1.10
Labor at 65 cents per hr.	1.17
Ladle heating	0.40
Total Cost per ton of brass melted	$8.65

Refractories

The vital part of any melting furnace is the refractory lining. This is a general subject common to all types of furnaces and is reserved as the subject matter of a later article of this series.

(To be continued)

The Neon-electric Stroboscope

By E. E. STEINERT
Engineering General Department, General Electric Company

ABOUT three years ago there arose an urgent need for a stroboscope that could be used in the study of power system stability. The system involved in the study consisted of an alternator, a synchronous condenser, a synchronous motor, and two "π" sections of line. Among other things it was necessary to measure various displacement angles caused by different load conditions. This need brought about the development of the neon-electric stroboscope. It is the purpose of this article to review some of the previous types of stroboscopes, and to describe the new form developed by C. A. Nickle.

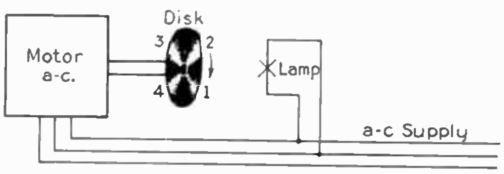


Fig. 1. Diagram of a Simple Form of Stroboscope

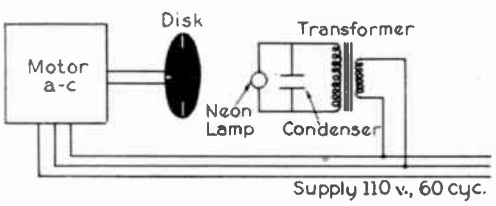


Fig. 2. Diagram of the Neon-electric Stroboscope

Fig. 3. The Neon-electric Stroboscope

General

The word "stroboscope," derived from Greek, means "whirling mark." A stroboscope usually consists of a disk rotating with the shaft of the machine under consideration and is provided with some device which allows it to be seen by the eye only at certain regular intervals. The operation depends on the property of the eye to retain an image. For example, if a moving object attains a certain position twenty times or more a second, and is allowed to be seen by the eye only when in the stated position, it appears to be stationary. There are a variety of stroboscopes designed to meet specific problems. A simple form is shown diagrammatically in Fig. 1.

If the motor in Fig. 1 is a four-pole synchronous machine, there is utilized in the procedure a disk having four black and four white sectors equally and alternately spaced. The disk is attached to the motor shaft and rotates with it. Light is furnished by an electric lamp (usually a carbon arc) connected to the a-c. supply and giving to the disk an illumination which pulsates at a frequency twice that of the a-c. supply.

Action

If in the case under consideration the motor is connected to a 60-cycle supply, it will run at 1800 r.p.m., or 30 r.p.s. The current through the arc will be positive for one-half cycle and negative for the next half cycle, but the light intensity will be the same for each half cycle. Thus the pulsations of light from the arc lamp will have a frequency equal to twice that of the supply, or 120 cycles per second. While the disk is turning through 90 degrees, the intensity of illumination passes through one cycle. During the next light cycle, the disk moves through another 90 degrees and allows a second image to be registered. Suppose the first image shows the white sectors in positions as indicated in Fig. 1; then in the second image, sector two occupies the position previously occupied by sector one, sector three occupies that of sector two, etc. If the intensity of illumination is great for a very small portion of each cycle, the shift in sector position is not detected. Thus the impression transmitted to the observer is that of a stationary disk. Usually, however, the image is blurred because the light furnished by the a-c. arc, instead of giving an instantaneous flash each cycle, has a more or less continuous variation.

In any study which requires precise measurement by the use of a stroboscope, it is necessary that the image recorded by the eye be very sharp. This is particularly the case when it is desired to measure the displacement angle of a synchronous motor.

Types of Stroboscopes

One form of stroboscope[1] makes use of a slotted disk, back of which is located a small neon lamp.

[1] "A Simple Method of Determining Slip of Induction Motors and Torque Angle of Synchronous Motors by Means of the Neon Lamp," by W. E. Maserve and D. Ramadanoff, *Sibley Journal of Engineering*, June, 1927.

the neon lamp being chosen because of its nominally instantaneous responsiveness to applied voltage and because of its low thermal capacity which prevents its continuing as a light source while the voltage wave is crossing the zero axis. When the disk revolves, it interrupts the passage of light between the neon lamp and the observer. In this connection there is described also an ingenious method of reading high values of induction motor slip by means of two disks and a system of pulleys.

Another form of the stroboscope is that of Dr. D. Robertson[2] in which the light interruption is obtained by means of an electrically-driven tuning fork placed between a segmented disk and the observer's eye. The prongs of the fork have slotted aluminum wings. With the fork vibrating, light reaches the observer's eye when the slots are opposite and is cut off when they are not opposite. Thus the number of interruptions per second depends on the fork frequency.

Still another form is that of Dr. C. V. Drysdale[2] which makes use of a tuning fork, an induction coil, and a neon tube. The tuning fork, which is electrically driven, makes and breaks the primary current of the induction coil. The secondary of the coil is connected to the neon tube. Thus the vibrating fork causes the neon tube to flash, producing the usual stroboscopic results.

Of particular significance is the fact that all previous stroboscopes which give a fair degree of accuracy involve moving parts in addition to the rotating disk.

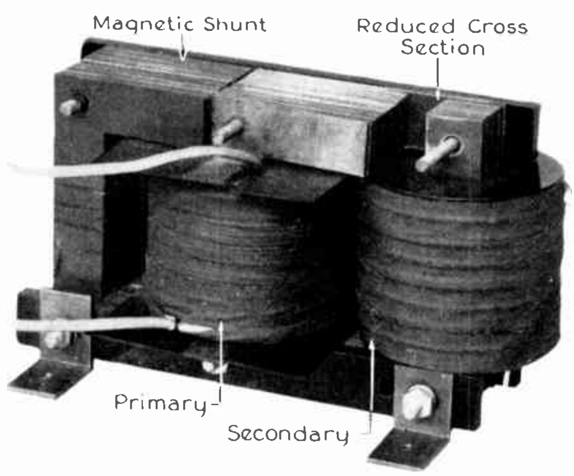


Fig. 4. The Special Transformer

Neon-electric Stroboscope

The new or neon-electric form of stroboscope is unique in that it has no moving parts except the disk. This stroboscope consists of four parts: a special transformer, a condenser, a neon lamp, and a disk. The description of these parts will follow. Fig. 2 shows the general arrangement of the parts. The transformer, condenser, and neon lamp are enclosed in a case (see Fig. 3) for protecting the lamp against breakage and the operator against high voltage. The only electrical connections to the case are those leading to the transformer primary, which operates from a 110-volt 60-cycle supply; and the only technique required in its operation is to place the box where the light can fall upon the disk. No tuning or adjusting is necessary. Moreover, one looks directly at the disk, and not through slots or past vibrating parts. Hence it has the merit of great convenience.

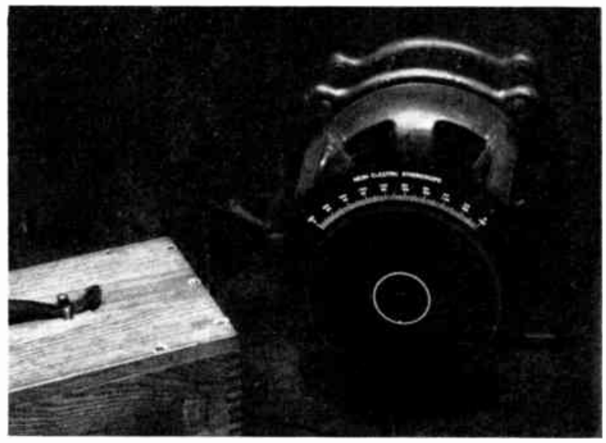


Fig. 5. A Laboratory Set-up with the Neon-electric Stroboscope in Position but Without the Neon Lamp in Operation, for which Reason the Arrows on the Rotating Dssk Are Invisible
(*See cover illustration*)

The element which makes this stroboscope effective is the special transformer shown in Fig. 4. To describe its action it will be necessary to treat the functioning of the various parts separately. It is a well-known fact that the voltage induced in a coil is proportional to the rate of change of flux with respect to time. Thus, if a transformer is supplied with a sine-wave voltage, and saturation does not occur, the secondary voltage will also be a sine wave. With an approximately rectangular flux wave, as shown in Fig. 6, the voltage wave will consist of lines perpendicular to the time axis.

For the purpose in hand, the flux wave of Fig. 6 would be the one to strive for, because it would produce instantaneous flashes in the neon lamp. Even ordinary grades of transformer iron will give a slightly peaked voltage because of saturation; and special grades which saturate readily, and thus permit only a very slight increase of flux after the knee of the curve is passed, might further increase the peaked effect. However, to radically accentuate the peak of the induced voltage wave, a magnetic circuit was constructed which had one part of much smaller cross-section than the remainder (see Fig. 4).

The effect of the reduced section is to increase the rapidity with which the core becomes saturated and thus give a flux wave which approaches rectangular shape. Hence the voltage generated in the secondary is characterized by a sharp, well-defined peak. The nature of this wave shape (Fig. 7) was determined in a test by means of an exploring coil and the oscillograph.

(2) Miles Walker: "The Diagnosis of Troubles in Electrical Machines."

Still better results were obtained by tuning the secondary circuit with a condenser. With this arrangement, an oscillogram of voltage in the secondary was obtained, the nature of which was determined by test and is shown in Fig. 8.

This voltage wave, when impressed on a suitable neon lamp, produces the sharp flashes of light which are essential for satisfactory operation of the stroboscope. There remained one difficulty to be overcome in the use of the oscillatory voltage wave, *viz.*, that the

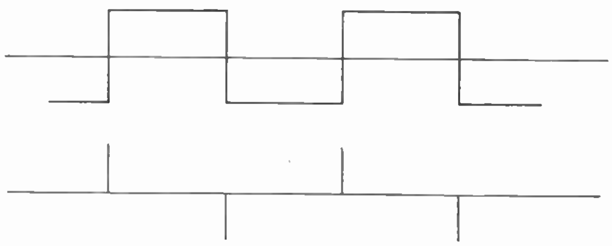


Fig. 6. Rectangular Flux Wave and Induced Voltage

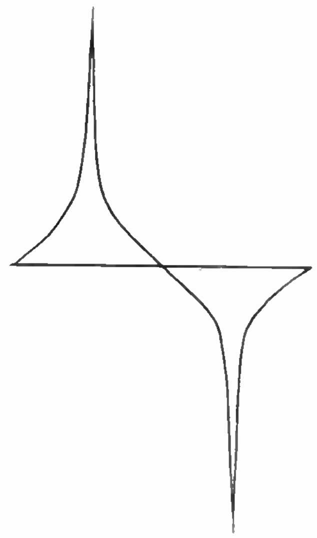


Fig. 7. Secondary Voltage Generated by the Special Transformer Used with the Neon-electric Stroboscope

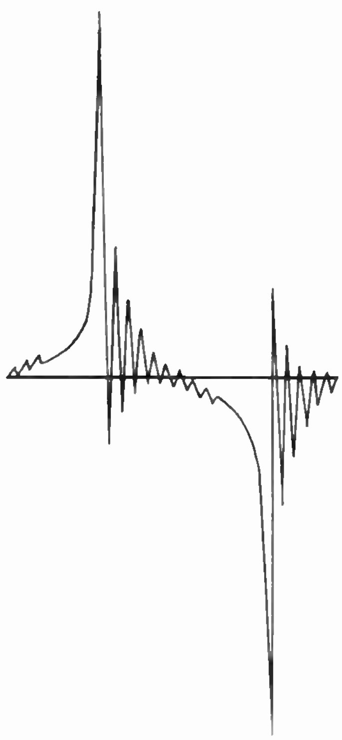


Fig. 8. Tuned Secondary Voltage Obtained Through the Use of a Condenser

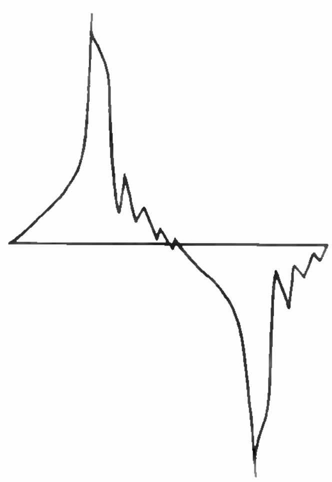


Fig. 9. Tuned[Secondary Voltage with Load

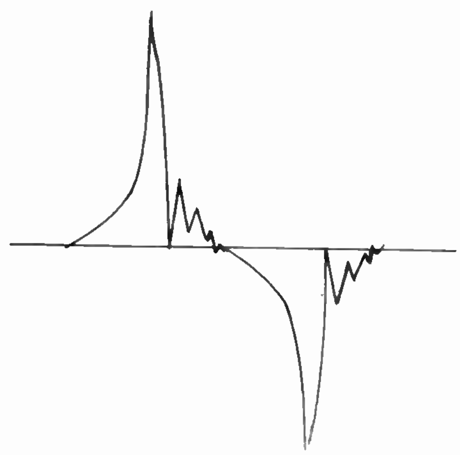


Fig. 10. The Wave Shape for which the Neon Lamps Are Designed to Operate in the New Stroboscope

neon lamp responds not only to the first peak of the cycle, but also to the second and third peaks. However, there was no difficulty in distinguishing the fundamental image from the others, for it is much brighter and is separated from them by a considerable distance.

There is another factor entering to aid the lamp design, and that is the effect of transformer load on the sharpness of tuning. Increasing the secondary load produces not only the detrimental effect of making the peaks flatter, but it produces also the favorable result of decreasing the secondary peak with respect to fundamental peak (see Fig. 9).

The neon lamp was designed to operate on a compromise wave shape (Fig. 10) between that of Fig. 8 and Fig. 9. The result is that a single sharp flash of high intensity is obtained each half cycle.

Another feature of the transformer is the magnetic shunt shown in Fig. 4. By varying the air gap between the shunt and the primary core, the primary current can be adjusted with respect to the resultant variation of light intensity from the neon lamp. The shunt also makes the transformer more compact, since without it an external reactance would be needed. During some tests, a small exploring coil was wound on the shunt and oscillograms taken. The voltage induced in this coil by flux passing through the magnetic shunt is shown in Fig. 11, while the voltage applied to the transformer primary is shown in Fig. 12. The sudden dip in voltage wave shown in Fig. 11 occurs when the restricted magnetic section between the primary and secondary windings becomes non-saturated.

Disk

It is not necessary to have a very large disk to secure quite precise measurements. In the case mentioned earlier in this article, black disks one foot in diameter were used. The marking employed consisted of a $\frac{1}{16}$-in. radial white line. (Enough equally spaced lines must be employed so that the frequency of appearance of the image shall be at least twenty per second. Furthermore, the number of lines must be an integral divisor of the number of poles on the machine to which the disk is attached.) Lines as narrow as 0.02 in. have been used on a one-foot disk rotating at 1800 r.p.m., giving readings which were accurate to within 0.02 in. at the periphery of the disk. Thus the effective duration of illumination of the disk is about 1/50,000 of a second for each flash of the neon lamp.

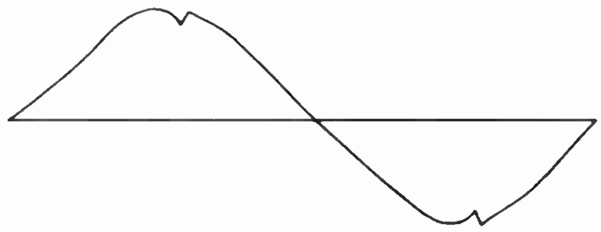


Fig. 11. Voltage Induced in Exploring Coil Used in Making Oscillographic Investigations of Stroboscope Performance

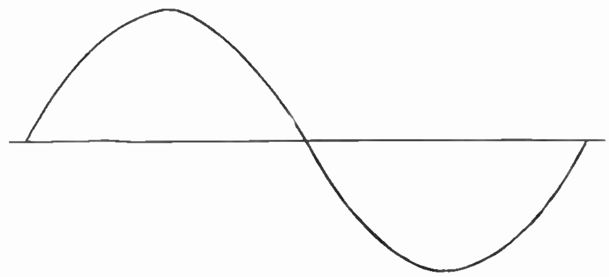
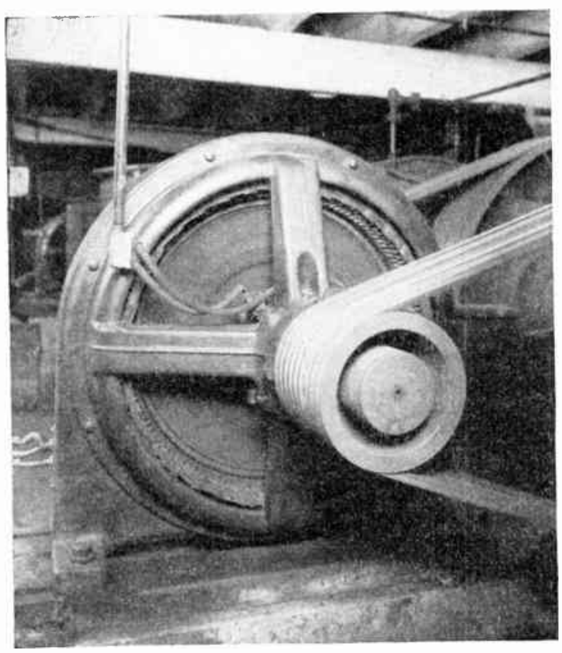


Fig. 12. Primary Voltage Wave Applied to Stroboscope Transformer

Summary

The success with which the neon-electric stroboscope has been used in recent tests is a direct result of its high degree of accuracy, which is possible only because of the extreme clearness of the image. Other features of this type of stroboscope are that it is compact and quite rugged. Also, it is as portable as an ordinary voltmeter.

Synchronous Motors Replace Steam Engine in Paper Plant



One of the Synchronous Motors Installed

A 250-hp. Corliss engine, belted to four beaters in the Camden (N. J.) plant of the West Jersey Paper Mfg. Co., has been displaced by two 140-hp. 600-r.p.m. 440-volt 60-cycle 0.8-p-f. synchronous motors each driving two beaters.

These motors not only provide the power-factor correction that was needed but successfully accelerate and carry the load. With the beaters filled with pulp under certain conditions, and particularly when working manila rope stock, the partly ground stock has a tendency to pack between the beater rolls and the housing. To obviate this difficulty it was found that by modifying the control so that the motors could be given a jogging start, in the reverse direction, the mass in the beater is loosened. On again reversing to standard rotation, the motors pick up the load without difficulty.

Low-voltage A-c. Networks

PART II

TYPES OF COMBINED LIGHT AND POWER NETWORK SYSTEMS

By D. K. BLAKE

Central Station Engineering Department, General Electric Company

IN residential areas, where the lighting and household appliance loads predominate, the single-phase transformer is used to supply three-wire secondary mains to which the consumers are connected as shown in Fig. 5. The voltage supplied at the socket may be 110/220, 115/230 or 120/240. The 115/230 predominates and is the standard preferred by the N.E.L.A., while the other voltages are recognized departures from the standard. The National Electric Safety Code requires that the circuits supplying lamps and appliances must be grounded so that the nominal voltage to ground does not exceed 150 volts. In order to meet this requirement, the neutral of the three-wire circuits is grounded. Wherever motor

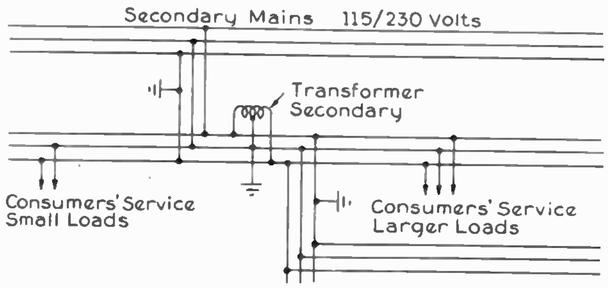


Fig. 5. A Three-wire Secondary Distribution System for Supplying Power for Lighting and Household Appliances

loads are encountered the electric service company usually specifies that single-phase motors must be used in applications of 7½ hp. or less. The distribution cost to supply three-phase service is excessive for such small ratings. Three-wire services with three-wire meters are used for the larger loads occasionally encountered.

The motor load predominates in the industrial areas. Where the load consists of 220-volt motors, the delta-connected transformer secondary is used almost universally. The consumer's lighting load, being a small percentage of the total load, is supplied from a separate single-phase transformer in some cases; in others, it is supplied from one phase of the power transformers as illustrated in Fig. 6. The latter method is more economical than the former, but is limited to applications where the motor load characteristic is such that the motor starting currents will not cause objectionable flicker. The transformer on the phase supplying the lighting load is usually greater in capacity than the other two. The unbalance on the primary circuit is as a rule not objectionable because the lighting load is small and different phases are used for the lighting load of different consumers. The combined light and power supply requires three-coil protection on the control equipment because of the ground.

The power consumption in the commercial areas of large American cities is divided approximately into 50 per cent lighting and 50 per cent motor load. Some of the smaller cities may show various ratios approaching 90 per cent lighting and 10 per cent motors. An appreciable percentage of the motor load in the commercial area is made up of numerous small consumers. The d-c. Edison system now

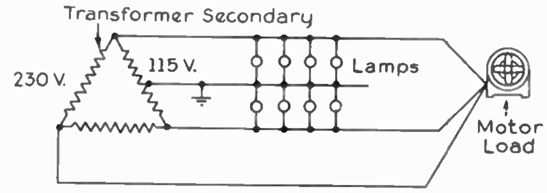


Fig. 6. A Method of Supplying a Lighting Load from One Phase of a Power Transformer that Furnishes Motor Service

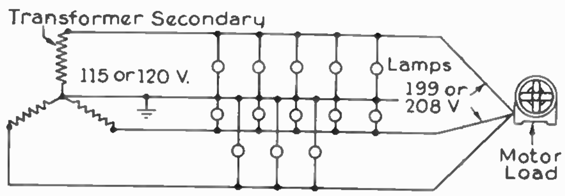


Fig. 7. The Y-secondary System of Distribution for Supplying Combined Lighting and Motor Loads

supplies both lighting and power loads from a single set of mains. It is very desirable to likewise supply light and power from a common set of mains when an a-c. network is used. The reasons for this are obvious, such as the diversity between light and power loads, the saving in duct and transformer vault space where there is already so much congestion, the use of one set of meters for certain loads where the rate structure permits, and the simplicity. The combined light and power a-c. network, however, presents several problems of a practical nature which are very important and very complex owing to the prevalent voltage standards of the devices constituting the load and existing practices of distribution and metering. There is also the additional technical problem of motor starting currents causing lamp flicker due to the voltage drop in the reactance of the transformers and secondary mains.

A combined light and power network should permit the load to be easily balanced over the phases, should permit grounding to limit the voltage on lighting circuits below 150 volts to ground, and should have the same nominal voltage for lamps as the circuits that are not in the network area.

About fifteen of the networks now installed use the Y-connected transformer secondary illustrated in Fig. 7. The lamps are supplied from phase to neutral and the motor load from the phase wires. Most of the networks supply 120 volts for light and 208 volts for motors measured at the customer's meter. A much smaller number of networks supply 115/199 volts. There are two important things to consider when contemplating this system. The first is that the voltage supplied to polyphase motors is appreciably below the standard rating of 220 volts. The second is that the three-wire service for lighting precludes the usual three-wire meter, and requires either two single-phase meters or the two-element polyphase meter because the voltage across the phases is 30 deg. out of phase with the current in the line.

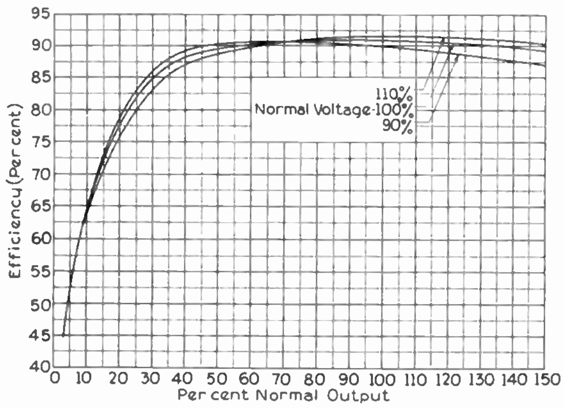


Fig. 8. Approximate Efficiency Curves for a Standard Induction Motor

The various operating companies who have adopted the Y-secondary system have advanced the following reasons for their action:

(*a*) The standard 220-volt induction motor is designed to operate successfully on potentials as low as 198 volts.

(*b*) The great majority of motor applications have larger motors than necessary for successful performance. (Various estimates are given, such as that 75 per cent of the motors are 50 to 60 per cent loaded).

(*c*) The network system will be regulated to give a plus or minus variation of three to four per cent, which is far better than now given on the usual power circuit. (This accounts for the large number of 120/208-volt systems, since a variation from 200 volts to 216 volts is above the lower limit of 198 volts.)

(*d*) The power-factor is slightly improved above 70 per cent output, and the efficiency slightly decreased but it is increased below 70 per cent output which covers the great majority of motor applications. The speed is not appreciably affected. (The approximate characteristics of standard general-purpose 220-volt motors at 90, 100, and 110 per cent voltage are given in Figs. 8, 9, and 10.)

(*e*) In the very few cases where the load is such as to demand full voltage applied to the motor, auto-transformers may be used to step up the voltage. (These cases are estimated to be less than 5 per cent of the total number of installations.)

(*f*) An increase in the lighting voltage to 120 volts makes 208 volts available for motor service, and this is adequate as mentioned in Item (*a*). Standard lamps of 120 volts rating are available; and large operating companies report that, in their experience, no difficulties are encountered

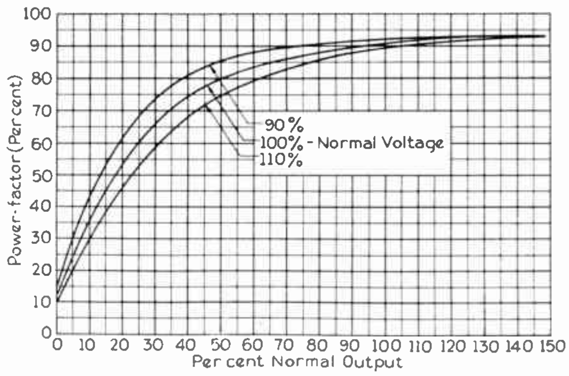


Fig. 9. Typical Power-factor Curves for a Standard Induction Motor

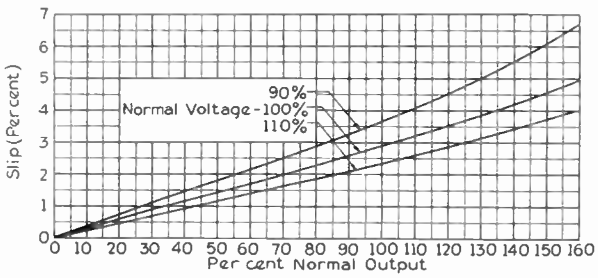


Fig. 10. Approximate Slip Characteristics of a Standard Induction Motor

with conservatively rated 110-volt appliances even though the voltage variations exceed the manufacturer's limit of 120 volts maximum. Also, that distribution transformers may be excited above 120 volts to maintain 120 volts at the service. (Some companies find it too difficult to raise their system voltage to supply 120 volts.)

Two companies already using two-phase for distribution purposes considered the three-phase Y-connected system when planning their network; but in view of the subnormal voltage supplied to motors, the three-wire metering difficulties, and the cost of changeover from two-phase to three-phase, decided to continue two-phase operation with the five-wire connection shown in Fig. 11. These two companies consider their demand for two-phase motors large enough to assure their consumers of no difficulty in readily obtaining two-phase motors. Aside from this consideration this system fits present voltage standards and meter practice perfectly.

Three other companies of moderate size consider it preferable to accept a more expensive system than to adopt a system which supplies subnormal voltages. One of these companies installs two sets of mains with auto transformers stepping up the voltage from the lighting mains to the power mains as illustrated in Fig. 12. Another company uses a similar system except that the transformer secondary winding is extended to 133 volts and a tap is provided at 115 volts as shown in Fig. 13. The 18-volt portion of the winding presents difficulties to the transformer de-

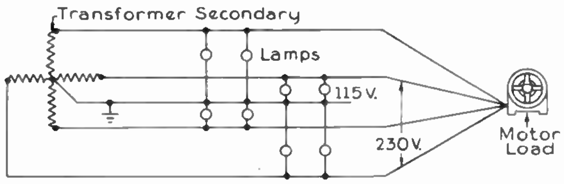


Fig. 11. A Two-phase Five-wire System for Supplying Lighting and Motor Loads

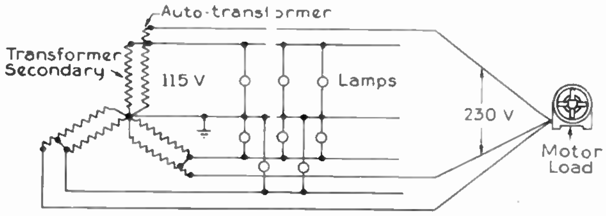


Fig. 12. Auto-transformers Used in Conjunction with a Lighting Supply Circuit to Provide for a Motor Load

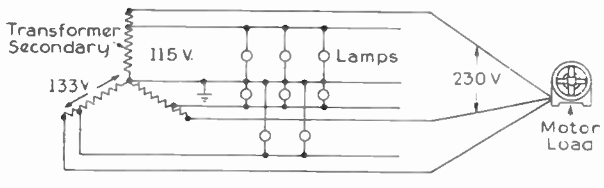


Fig. 13. A System Providing Proper Voltages for Lamps and Motors Through the Employment of Transformer Taps

signer when parallel operation of different sizes is required because of the small number of turns and the high voltage per turn. Neither of these systems eliminates the three-wire meter difficulty.

The third company uses the well-known delta secondary illustrated in Fig. 6. The network is segregated into three separate areas in order to balance the load on the three phases. The installed transformer capacity on the lighting phase of a particular area exceeds that on the other phases. This system, of course, fits standard voltages and permits the usual three-wire meter practice. The necessity of three separate phase areas is the reason why this system is not more generally used. The delta-connected secondary also presents difficulties in the relay characteristic for the automatic network protector now extensively used. This company proposes to try out the so-called translator system, employing devices which are essentially auto-transformers for tying together the secondary mains into one large network with the lighting load on a different phase in different areas.

The translator network may be understood by referring to Fig. 14, 15 and 16. The connection of the translators between two transformers having different phases grounded is shown in Fig. 14. The vector relation of the two transformers is shown in Fig. 15. The translator is simply an iron core with four 115-volt coils as illustrated in Fig. 16. The vector diagram in Fig. 15 shows that there is a difference of potential of 115 volts between A_1 and A_2, B_1 and B_2,

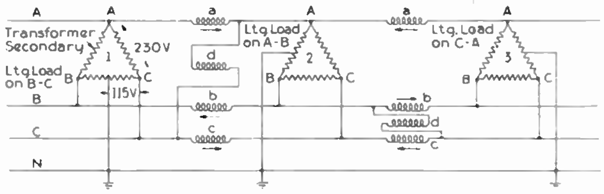


Fig. 14. The Translator Network, Consisting of a Combination of Mid-point Transformer Taps in Separate Phases and Special Devices for Supplying Lighting Loads

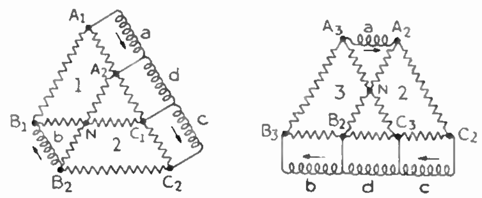


Fig. 15. Diagrams Showing the Relationship of Voltages in the Translator System

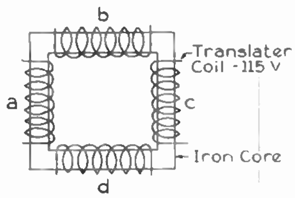


Fig. 16. Elementary Construction of an A-c. Network Translator

C_1 and C_2 due to the grounds on the different phases being at zero potential. Coil a connects A_1 to A_2, coil b connects B_1 and B_2 and coil c connects C_1 to C_2. Therefore, each coil is energized at 115 volts and excites the iron core. Suppose the lighting load on Bank 2 is very heavy and some of this load is supplied from Bank 1. A current will flow from A_1 to A_2 through coil a and from B_2 to B_1 through coil b. If the lighting load is on Bank 1 then Bank 2 will likewise supply current which will flow from B_2 to B_1 through coil b and from C_1 to C_2 through coil c. The arrows show clearly that the load currents flow through the coils in opposite directions and therefore pass readily through the translator as they would through a one to one ratio transformer. It is also evident from the arrows that a heavy three-phase load may be readily passed through the translater because the current in one coil is equal and opposite to the vector sum of the currents in the two other coils. Fig. 15 also shows the diagram for Banks 2 and 3. The load is always carried by coils a, b, and c unless the lighting load is unbalanced; then coil d carries some of the current. The chief function of coil d is to maintain the three-phase relation when the transformers of a particular section are removed. Suppose transformer No. 1 is removed. It is then obvious from Fig. 15, that coils d and c are in series across A_2 and C_2 and that conductors A_1 and B_1 are maintained at the same position as when the transformer is energized. Therefore, coil d need only be designed for a small percentage of the current rating of the other coils.

Each of the coils a, b and c should be designed with the same current-carrying capacity as the cables connected to it. The translator is preferably located between transformer vaults; but usually subway conditions demand that they be located in the transformer vault. It is obvious that this system requires an additional piece of apparatus at an increase in cost, losses, space requirement and maintenance. All of these are borne by the electric service company and none by the consumer. The consumer is not called upon to operate his single-phase devices at excessive voltage or his polyphase motors at subnormal voltage. It seems, therefore, that the selection of the translater system must be upon the grounds that the electric service company does not want to run any risk of inconvenience to, or complaint from, the customer but prefers to accept the extra cost and complications of the translater. The electric service company also gains the advantage of being thus enabled to use standard 3-wire meters on three-wire circuits.

(To be continued)

Heaviside's Operational Calculus as Applied to Engineering and Physics

Part IV: The Expansion Theorem Applied to Some Definite Problems
Part V: Operations on Unit Function Squared Have No Physical Significance
Part VI: Additional Operators Employed when a Network is Suddenly Connected to an Alternator Instead of to a Battery of Constant Voltage

By DR. ERNST JULIUS BERG
Consulting Engineer, General Electric Company
Professor of Electrical Engineering, Union College

To illustrate the application of the Expansion Theorem to the problem cited in Part II when a leaky condenser in series with a resistance is suddenly connected to a storage battery of potential difference E, we have $Z_{(p)} = r + r_2 + rr_2pC$. (See Equation (8) and Fig. 7 in Part II.)

Therefore $\quad Z_{(p)} = 0$ gives $p = -\dfrac{r+r_2}{rr_2C}$

$$\text{and } p_1 = -\frac{r+r_2}{rr_2C}$$

$$\frac{dz}{dp} = rr_2C$$

Therefore $\quad p\dfrac{dz}{dp} = prr_2C$

$$p\frac{dz}{dp} \text{ for } p = p_1 = -\frac{r+r_2}{rr_2C} \times rr_2C = -(r+r_2)$$

$$Y_{(p_1)} = 1 - \frac{r+r_2}{rr_2C} \times rC = 1 - \frac{r}{r_2} - 1 = -\frac{r}{r_2}$$

$$\frac{Y_{(0)}}{Z_{(0)}} = \frac{1}{r+r_2}$$

Therefore $\quad i = E\left[\dfrac{1}{r+r_2} + \dfrac{r}{r_2(r+r_2)}\epsilon^{-\frac{r+r_2}{rr_2C}t}\right]$

For $t=0$, $i = \dfrac{E}{r_2}$ and for $t=\infty$, $i = \dfrac{E}{r+r_2}$.

It is of interest to note that the initial and final currents can always be obtained directly from the operational solution. In this case

$$i = E\frac{(1+pCr)}{r+r_2+pCrr_2} 1 \tag{19}$$

When we remember the shape of the unit function, we realize that for $t=0$, $p = \dfrac{d}{dt} = \infty$; for $t=\infty$, $p=0$.

Therefore, for $t=0$ when $p=\infty$ or is very large compared with any finite quantity, we get

$$i = \frac{EpCr}{pCrr_2} 1 = \frac{E}{r_2}$$

For $t=\infty$, $p=0$, and therefore $i = E\dfrac{1}{r+r_2}$.[2]

(2) The operational solution is the solution of the permanent alternating-current condition when vector representation is used if for p is substituted $j\omega$, thus $p^2 = -\omega^2$, $p^3 = -j\omega^3$, etc.

Heaviside brought out this fact in 1892, or perhaps earlier; it had also been shown by Kennelly in 1892.

The proof is simple when it is recollected that a convenient equation of a revolving vector is:

$$\dot{I} = I\epsilon^{j\omega t}$$

$$\text{Thus } \frac{d}{dt}\dot{I} = p\dot{I} = j\omega I\epsilon^{j\omega t} = j\omega\dot{I}$$

Referring then to Equation (19)

$$\dot{I} = \dot{E}\frac{(1+j\omega rC)}{r+r_2+j\omega Crr_2}$$

$$= \dot{E}\frac{r+r_2+\frac{r^2}{x_c^2}r_2 + j\frac{r^2}{x_c}}{(r+r_2)^2+\frac{r^2}{x_c^2}r_2^2}$$

where $x_c = \dfrac{1}{\omega C}$.

Other examples involving complex roots will be given in Parts VII and VIII.

The operational equation brings out some other interesting features. Since for $t=0, p=\infty$, and for $t=\infty$, $p=0$, it is evident that if the denominator is of a higher power of p than the numerator, the initial current must be zero.

If numerator and denominator are of the same power in p, the initial current is of a definite value, not zero.

If the numerator is of higher power of p than the denominator, then the initial rush of current is infinite; but after an infinitely short time it decreases to a definite value. It might be well to say here that this condition can never be met in any actual problem, whether mechanical or electrical. It would exist in the case, for instance, where a condenser is suddenly connected to a battery with wires of no resistance, which obviously is impossible.

However, even then the method throws some light on the subject. We would obviously expect to find an instantaneous infinite rush of current during an exceedingly short time such that $\int idt$ would be the charge which corresponds to the voltage impressed.

The operational solution obviously is:

$$i=E\frac{1}{\frac{1}{pC}}1=pCE\,1$$

which means an instantaneous rush of current of infinite magnitude and the charge $=\frac{i}{p}=CE$, which is the proper charge.

Problem

A condenser made up with two different kinds of dielectric is suddenly connected to a storage battery of voltage E as in Fig. 8. Find the charge on the middle plate.

Let the leakage conductance and the capacity of the upper portion be g_1 and C_1 and the corresponding values for the lower part be g_2 and C_2. Let e_1 be the instantaneous voltage consumed by the upper part and e_2 that of the lower part, then the current taken by the upper part is obviously $i_1=e_1\,(g_1+pC_1)$ (the resistance operator of the condenser being $\frac{1}{pC_1}$; the corresponding admittance is $\frac{1}{\frac{1}{pC_1}}$). Similarly, the current taken by the lower part is $e_2\,(g_2+p\,C_2)$. These two currents are obviously the same. Thus

$$e_1\,(g_1+p\,C_1)=e_2\,(g_2+p\,C_2)$$

But $e_1+e_2=E\,1$. Therefore

$$E\,1=\frac{i}{g_1+pC_1}+\frac{i}{g_2+pC_2}$$

$$\text{or } i=E\frac{(g_1+pC_1)\;(g_2+pC_2)}{g_1+g_2+p\;(C_1+C_2)}1$$

Therefore

$$e_1=\frac{i}{g_1+pC_1}=E\frac{g_2+pC_2}{g_1+g_2+p\;(C_1+C_2)}1$$

and the charge on the top plate is

$$e_1\,C_1=E\,C_1\frac{g_2+p\,C_2}{g_1+g_2+p\;(C_1+C_2)}1.$$

The corresponding charge on the top side of the middle plate is numerically the same but of course with minus sign.

Consider next the lower part of the condenser. Its upper plate has a charge equal to C_2e_2 which is easily shown to be

$$C_2e_2=E\,C_2\frac{g_1+p\,C_1}{g_1+g_2+p\;(C_1+C_2)}1.$$

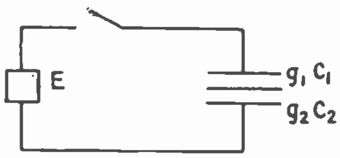


Fig. 8

Thus the total charge on the middle plate is

$$q=E\frac{C_2\,(g_1+p\,C_1)-C_1\,(g_2+p\,C_2)}{g_1+g_2+p\;(C_.+C_2)}1$$

$$=E\frac{g_1\,C_2-g_2\,C_1}{g_1+g_2+p\;(C_1+C_2)}1$$

$$=E\frac{A}{G_0+p\,C_0}1$$

where

$$A=g_1\,C_2-g_2\,C_1$$
$$G_0=g_1+g_2$$
$$C_0=C_1+C_2$$

This can be written

$$q=\frac{A_1}{p+a}1$$

$$\text{where } A_1=\frac{A}{C_0}$$

$$\text{and } a=\frac{G_0}{C_0}$$

The solution can be written down at once by comparing this equation with Equation (4) in Part I. It is

$$q=E\frac{A}{C_0}\frac{C_0}{G_0}\left[1-\epsilon^{-\frac{G_0}{C_0}t}\right]$$

$$=E\frac{g_1C_2-g_2C_1}{g_1+g_2}\left[1-\epsilon^{-\frac{g_1+g_2}{C_1+C_2}t}\right]$$

The student should verify this by the use of the Expansion Theorem.

PART V: OPERATIONS ON UNIT FUNCTION SQUARED HAVE NO PHYSICAL SIGNIFICANCE

The student may be tempted to write the operational expression for electric power and expect from this to get by the Expansion Theorem the instantaneous values. He will fail because the operational solution will contain 1^2 and neither Heaviside nor anybody else has shown how to operate on 1^2.

To illustrate, assume that it is desired to find the power supplied to the inductance of an inductive circuit. The power is the instantaneous product of the voltage consumed by the inductance and the current flowing in the inductance.

The operational solution for the current is

$$i=\frac{E}{r+pL}1$$

and since the voltage consumed by the inductance is

$$e=pLi=E\frac{pL}{r+pL}1$$

he might write

$$P=E^2\frac{pL}{(r+pL)^2}1^2$$

and erroneously proceed to use the Expansion Theorem. It must be remembered that we know only how to operate on 1; we know nothing about rules of operations on 1^2.

The solution is of course the product of the voltage and current when these are given as functions of t, not of p. Thus the power in this case is

$$P=E^2\frac{\epsilon^{-\frac{r}{L}t}}{r}\left[1-\epsilon^{-\frac{r}{L}t}\right]$$

Incidentally this difficulty is similar to that encountered when trying to get the power in an alternating-current circuit from the vector expressions of the current and voltage. The student will remember that the answer is obtained by "telescoping" the vectors; ordinary multiplication does not "work."

PART VI: ADDITIONAL OPERATORS EMPLOYED WHEN A NETWORK IS SUDDENLY CONNECTED TO AN ALTERNATOR, INSTEAD OF TO A BATTERY OF CONSTANT VOLTAGE

Referring to Equation (3) in Part I

$$\frac{p}{p-a}1=\epsilon^{at}.$$

Thus $$\frac{p}{p\pm j\omega}1=\epsilon^{\mp j\omega t}\qquad \text{when } a=j\omega$$

$$\sin\omega t=\frac{\epsilon^{j\omega t}-\epsilon^{-j\omega t}}{2j}=\frac{1}{2j}\left[\frac{p}{p-j\omega}1-\frac{p}{p+j\omega}1\right]$$

$$=\frac{p}{2j}\left[\frac{p+j\omega-p+j\omega}{p^2+\omega^2}\right]1=\frac{p\omega}{p^2+\omega^2}1.$$

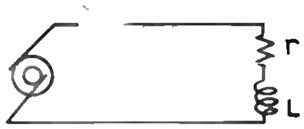


Fig. 9

This shows how a sine wave can be converted to a "Unit function."

Similarly $$\cos\omega t=p\frac{\sin\omega t}{\omega}=\frac{p^2}{p_2+\omega^2}1\ ^{(3)}$$

Therefore $$\sin(\omega t\pm\varphi)=\sin\omega t\cos\varphi\pm\cos\omega t\sin\varphi$$

$$=\frac{p\omega\cos\varphi\pm p^2\sin\varphi}{p^2+\omega^2}1$$

and $$\cos(\omega t\pm\varphi)=\frac{p^2\cos\varphi\mp p\omega\sin\varphi}{p^2+\omega^2}1$$

This relation the writer believes was first shown by Pleijel.

Similarly

$$\epsilon^{-\beta t}\sin(\omega t\pm\varphi)=\frac{p\omega\cos\varphi\pm p(p+\beta)\sin\varphi}{(p+\beta)^2+\omega^2}1$$

$$\epsilon^{-\beta t}\cos(\omega t\pm\varphi)=\frac{p(p+\beta)\cos\varphi\mp p\omega\sin\varphi}{(p+\beta)^2+\omega^2}1.$$

(3) For the development of $p^n \sin \omega t$ where n is a fraction, see Part XXVI.

In connection with the last two equations it should be noted that, while $\epsilon^{-\beta t}=\frac{p}{p+\beta}1$ and $\sin\omega t=\frac{p\omega}{p^2+\omega^2}1$, $\epsilon^{-\beta t}\sin\omega t$ cannot be obtained directly by multiplying the two operators because the resultant operator would involve the unit function squared.

An application of these operators will be given in the case of an alternator of e.m.f. $E\sin(\omega t+\varphi)$ being suddenly (at $t=0$) connected to an inductive circuit as shown in Fig. 9.

The procedure is to write first the operational solution as if unit e.m.f. were impressed. It is $i=\frac{E}{r+pL}1$. Then introduce the additional operator which converts the sine wave to a wave of unit function. Thus the operational solution becomes

$$i=\frac{E}{r+pL}\cdot\frac{\omega p\cos\varphi+p^2\sin\varphi}{\omega^2+p^2}1\qquad(20)$$

Therefore $$Y_{(p)}=\omega p\cos\varphi+p^2\sin\varphi$$

and $$Z_{(p)}=(r+pL)(\omega^2+p^2)$$

The solution of Equation (20) gives the instantaneous values of the current. The denominator is of a higher power of p than the numerator, thus the initial current value is zero. But in this case we cannot say that for $t=\infty$, $p=0$. It is not zero because the final current is obviously not steady. If we desire to know the final value without solving the equation by the Expansion Theorem, we may resort to the vector representation as discussed previously and write

$$I=\frac{E}{r+pL}$$

where $p=j\omega$.

Therefore

$$I=\frac{E}{r+j\omega L}$$

which is the well-known solution.

It is now perfectly simple to solve Equation (20) by the Expansion Theorem. Since $Z_{(p)}=(r+pL)(\omega^2+p^2)$, we get three roots, $p_1=-\frac{r}{L}$, $p_2=+j\omega$, $p_3=-j\omega$. We proceed then to find $d\frac{Z_{(p)}}{dp}$ and then $p\frac{dZ_{(p)}}{dp}$, for $p=p_1$, p_2 and p_3, etc.

The result will be three terms ($\frac{Y_{(o)}}{z_{(o)}}$ is easily seen to be zero in this case), *viz.*,

$$A\epsilon^{-\frac{r}{L}t}+B\epsilon^{j\omega t}+C\epsilon^{-j\omega t}$$

The last two terms will combine to a pure sine wave without decrement and are, therefore, the permanent condition, which usually is well known to engineers.

In this case the permanent alternating-current is, of course,

$$i_p=\frac{E}{Z}\sin(\omega t+\varphi-\alpha)$$

so that it is only necessary to calculate the value of i from the Expansion Theorem for the root $p=p_1=-\frac{r}{L}$, and then to add i_p to the solution.

However, the solution is not always so easily obtained. We will therefore consider a general simplification of work when the denominator $Z_{(p)}$ is in the form of a product.

Let

$$\frac{Y_{(p)}}{Z_{(p)}}=\frac{Y_{(p)}}{h_{1(p)}\cdot h_{2(p)}}$$

Therefore

$$Z'_{(p)}=\frac{dZ_{(p)}}{dp}=h_{1(p)}\,h_2'{}_{(p)}+h_{2(p)}\,h_1'{}_{(p)}$$

and

$$pZ'_{(p)}=p\,[h_{1(p)}\,h_2'{}_{(p)}+h_{2(p)}\,h_1'{}_{(p)}]$$

When in this expression we substitute the root or roots obtained when $h_{1(p)}=0$, we get only the second term left since $h_{1(p)}$ obviously becomes zero. A similar argument applies in regard to the second set of roots, so that the net result is, for instance, in case $h_{1(p)}$ has only one root, p_1, and $h_{2(p)}$ two roots, p_2 and p_3,

$$i=E\Bigg[\frac{Y_{(o)}}{Z_{(o)}}+\frac{Y_{(p)}\,\epsilon^{pt}}{ph_{2(p)}\,h_1'{}_{(p)}}\text{ for }p=p_1$$
$$+\frac{Y_{(p)}\,\epsilon^{pt}}{ph_{1(p)}\,h_2'{}_{(p)}}\text{ for }p=p_2$$
$$+\frac{Y_{(p)}\,\epsilon^{pt}}{ph_{1(p)}\,h_2'{}_{(p)}}\text{ for }p=p_3\Bigg]$$

This relation the writer believes was first worked out by Pleijel or Herlitz.

In the particular problem under consideration

$h_{1(p)}=r+pL$; therefore $h_1'{}_{(p)}=L$ and $ph_1'{}_{(p)}=pL$

$h_{2(p)}=\omega^2+p^2$; therefore $h_2'{}_{(p)}=2p$ and $ph_2'{}_{(p)}=2p^2$

Thus $\qquad ph_{2(p)}\,h_1'{}_{(p)}=pL\,(\omega^2+p^2)$

and $\qquad ph_{1(p)}\,h'_{2(p)}=2p^2\,(r+pL)$

$$ph_{2(p)}\,h_1'{}_{(p)}\text{ for }p=p_1\text{ becomes }-\frac{r}{L^2}z^2$$

where $\qquad z^2=r^2+\omega^2L^2=r^2+x^2$

$$Y_{(p)}\text{ for }p=p_1\text{ becomes }z\frac{r}{L^2}\sin(\varphi-a)$$

where $\qquad \tan\alpha=\frac{x}{r}$

Thus the first term becomes

$$-\frac{E}{z}\epsilon^{-\frac{r}{L}t}\sin(\varphi-a)$$

$ph_{1(p)}\,h_2'{}_{(p)}$ for $p=p_2=j\omega$ becomes $-2\omega^2\,(r+j\omega L)$

$Y_{(p)}$ for $p=p_2=\omega^2\,(j\cos\varphi-\sin\varphi)$

Therefore

$$\frac{Y_{(p)}}{ph_{1(p)}\,h_2'{}_{(p)}}\text{ for }p=p_2=\frac{j\cos\varphi-\sin\varphi}{-2\,(r+j\omega L)}$$
$$=\frac{\cos\varphi+j\sin\varphi}{2j\,(r+j\omega L)}=\frac{\cos(\varphi-a)+j\sin(\varphi-a)}{2jz}.$$

$$\frac{Y_{(p)}}{ph_{1(p)}\,h_2'{}_{(p)}}\text{ for }p=p_3=\frac{j\cos\varphi+\sin\varphi}{2\,(r-j\omega L)}$$
$$=\frac{-\cos\varphi\,(\varphi-a)+j\sin(\varphi-\alpha)}{2jz}.$$

Thus the last two terms become

$$\frac{E}{z}\left[\cos(\varphi-\alpha)\frac{\epsilon^{j\omega t}-\epsilon^{j\omega t}}{2j}+\sin(\varphi-\alpha)\frac{\epsilon^{j\omega t}+\epsilon^{jwt}}{2}\right]$$
$$=\frac{E}{Z}\sin(\omega t+\varphi-\alpha)$$

which is the permanent value and the solution is

$$i=\frac{E}{z}\left[\sin(\omega t+\varphi-\alpha)-\epsilon^{-\frac{r}{L}t}\sin(\varphi-\alpha)\right]$$

The process is somewhat lengthy, but as has been stated, when it is once done, it is known that the roots $\pm j\omega$ give the permanent condition, so that it is really only necessary to calculate the condition for the other root or roots, if the permanent condition is known. If not, it is simplest to obtain it first in vector form by substituting $j\omega$ in the operational solution, $i=\frac{E\,1}{r+pL}$ in this particular case.

This problem will be treated in an entirely different way in Part XXVI. The object then will be to bring out certain peculiarities of power series developments and to show the application of Heaviside's "shifting."

(To be continued)

Vacuum Tubes as Oscillation Generators

PART IV

SPECIAL CONSIDERATIONS BEARING ON THE DESIGN AND OPERATION OF OSCILLATING CIRCUITS

By D. C. PRINCE and F. B. VOGDES
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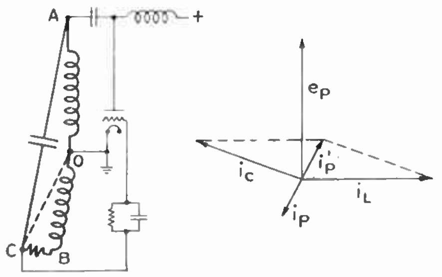
Grid Phase-angle Corrections

IN dealing with the design of oscillating circuits the effects of the load resistance, the plate-choke, and the blocking condenser were described as though only their primary functions were fulfilled and, this accomplished, they exerted no other influence on the circuit. All three parts of the circuit can, however, affect its operation; and, though the changes produced are usually small, it is satisfying to know just what may be expected. In general, the effects consist of the introduction of small voltages or currents into the simple scheme of things first described. This results in a small change in magnitude of practically all of the various quantities, combined with a slight change in the phase relations.



(A) (B)

Fig. 24. Effect of Load Resistance in a Hartley Circuit

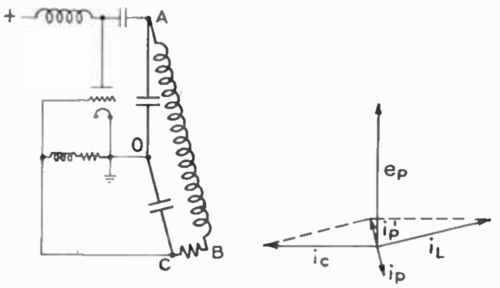
In order to simplify the discussion, only the fundamental frequency component of the current supplied to the oscillating circuit through the blocking condenser will be considered. The harmonics are relatively much less important, and their omission will entail no serious error. The value of the fundamental component can be obtained by dividing the watts input to the oscillating circuit by the voltage between the leads to the plate and filament of the tube.

Fig. 24 illustrates the effect of the load resistance in a Hartley circuit. In (A) the oscillating circuit itself, *AOBC*, is drawn in such a way that the angles between various parts of the circuit correspond to the electrical phase differences of the voltages across them. Thus the grid coil *OB* and the condenser *AC* are in geometrically parallel sections of circuit, for they carry the same current, and both are pure reactances. The load resistance is drawn at right angles to them for similar reasons. Part of the load resistance might be located in the plate coil section *AO*, but this would not affect the diagram, as far as the object in hand is concerned, which is to show the relation between plate and grid voltages. It will be observed that the grid current has been neglected. This is not the case with the plate current, which may be represented by a sinusoidal current flowing between the leads to the circuit at *A* and *O*. For this reason the plate coil in Fig. 24 is not drawn parallel with the grid coil but instead there is a phase difference between their voltages. For the present it will be assumed that there is no drop across the blocking condenser and no current passed by the plate choke. Hence, *OA* will be the alternating component of the plate voltage.

The grid voltage will be represented by the dotted line *OC* in Fig. 24(A), and the plate current will be in phase with it. Thus, in Fig. 24(B), the plate voltage e_p is drawn parallel with *OA* in Fig. 24 (A) and i_p, the alternating component of the plate current, is drawn parallel with *OC*. The currents through the condenser and plate coil are 90 degrees out of phase with the corresponding voltages and are represented by i_c and i_L perpendicular to *AC* and *OA* respectively. Adding the two vectorially results in i_p', which must be equal and opposite to the alternating component of the plate current i_p. It will be seen that i_p' must always lag behind e_p or, in other words, the oscillating circuit absorbs power not as a perfect resistance, but as a resistance in connection with an inductance. The circuit itself produces this effect by running slightly below the resonant frequency, thus drawing the lagging component of current by increasing the current through the inductance and decreasing the current through the capacity. The deviation of the frequency from that corresponding to the natural frequency of oscillation can be calculated by applying numerical values to the vectors.

The corresponding phenomena in a Colpitts circuit are shown in Fig. 25, (A) and (B). In this case i_p' leads e_p by some angle which requires that the oscillating circuit operate slightly above its natural frequency in order to produce this power-factor. This self adjustment of the simpler types of circuit is a very valuable property under many conditions, as it automatically insures against serious losses due to the grid excitation being out of phase.

In Fig. 26, (A) and (B) indicate the effect of the plate choke and the blocking condenser upon the operation of the circuit. This is the same Hartley circuit as shown in Fig. 24(A) but the blocking condenser and plate choke are no longer considered to be perfect in operation. The alternating component of the plate voltage is, therefore, no longer represented by OA but by a vector OD displaced from OA by the addition of AD, the drop in the blocking condenser. The plate choke may be considered to be grounded on the side next to the high voltage generator as far as the high frequency is concerned, so this choke appears as an inductance connected between O and D.



(A) (B)

Fig. 25. Effect of Load Resistance in a Colpitts Circuit

By choosing a blocking condenser of the correct impedance, it is possible to bring the plate voltage OD exactly 180 degrees out of phase with the grid voltage, the conditions to be desired for efficient operation. This will result in the oscillating circuit plus the blocking condenser drawing a load with a leading current component. This component may then be neutralized by arranging the choke so that it will draw a lagging current of equal magnitude. In the vector diagram 26(B), the current through AC is represented by i_c and that through AO by i_L. These combine to form a short vertical current vector to which is added the choke current, i_s, to form a total current equal and opposite to i_p supplied by the tube. The voltage across OA has been taken to be vertical, and, if the sum of i_c and i_L is vertical, the drop which this current causes in the blocking condenser must be represented by a horizontal line. This, when added to that representing the voltage across OA, results in the alternating component of the plate voltage, e_p. This voltage can, by this means, be made opposite to i_p and e_g; *i.e.*, by control of the drop in the blocking condenser.

To make a complete calculation of a circuit, including the points just discussed, the circuit $OBCA$ is treated as though the choke and blocking condenser were not present. This circuit may be equivalent either to a pure resistance between A and O (as shown in the vector diagram) or the equivalent load may contain some reactance. Let it be so proportioned that it will be equivalent to a resistance. The angle COA, and its supplement AOD, can then be calculated. The equivalent resistance between A and O is known since it represents a given load at a given voltage. The capacity reactance AD can, therefore, be selected so that $AD/OA = \tan(180° - AOC)$.

The alternating voltage is impressed at D, and the circuit DAO will draw some leading current, the amount being easily ascertainable. It is then only necessary to choose the choke DO of such a value that it will draw the same amount of lagging current.

The procedure in arriving at the proper values of blocking condenser and line choke will be clearer if the solution of a numerical example is carried through.

The first item to be determined is the ratio of the inductance on which the alternating components of the plate and grid voltage depend. In Fig. 26, the plate voltage is represented by OD and the drop across the inductance by OA; and it will be noticed that the two may be considered equal in magnitude unless the circuit is far from normal in design. The effect of the resistance in the grid inductance on the magnitude of the grid voltage can also be neglected. The two inductances will then be in the same ratio as the alternating components of plate and grid potentials, E_p and E_g respectively, as determined from the data on optimum operating conditions. Assume that this gives

$$\frac{E_p}{E_g} = \frac{OA}{OC} = 4.$$

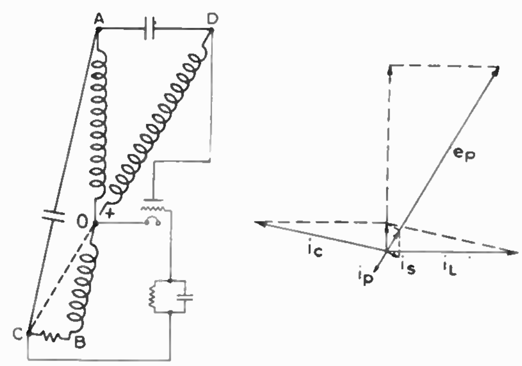
Let $OA = 100$ ohms inductance with 5 ohms resistance.

$OB = 25$ ohms inductive reactance.

$BC = 2.5$ ohms resistance.

$CA = 125$ ohms capacity reactance.

Then angle $OAC = \sin^{-1}\frac{2.5}{100} = 1$ deg., 27 min.



(A) (B)

Fig. 26. Effect of Plate Choke and Blocking Condenser

If 100 volts be impressed across OA

$$i_L = 1, \text{ watts in } OA = (i_L)^2 \times 5 = 5.0$$
$$i_c = 1, \text{ watts in } BC = i_c^2 \times 2.5 = 2.5$$
$$\text{Total watts} = 7.5$$

Equivalent resistance $= \frac{E^2}{W} = \frac{100^2}{7.5} = 1333$ ohms. It will be arranged to have this circuit operate at the resonant point so that its reactance between O and A is zero.

In the triangle AOC

$$\frac{OA}{\text{Sin } ACO}=\frac{OC}{\text{Sin } OAC}=\frac{AC}{\text{Sin } AOC}$$

$$\frac{100}{\text{Sin } ACO}=\frac{25}{0.025}, \text{ Sin } ACO=0.1$$

$$\text{Angle } ACO=5 \text{ deg., } 45 \text{ min.}$$

$$\text{Angle } AOD=\text{Angle } OAC+\text{Angle } ACO=7 \text{ deg. } 12 \text{ min.}$$

$$AD=\text{impedance between } A \text{ and } O\times\tan AOD$$

$$=1333\times0.126=168 \text{ ohms capacitance.}$$

The total impedance between O and D through A is 1343 ohms, so that the ratio between the plate and grid voltages suffers no appreciable change due to the presence of the plate-blocking condenser. If 100 volts are impressed between D and O, the current is

$$\frac{100}{1343}=0.0745 \text{ amp.}$$

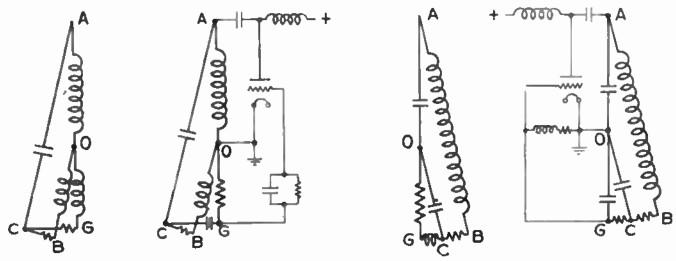


Fig. 27. Special Circuits to Compensate for Errors in Phase of Grid Voltage

and the wattless volt-ampere component is

$$0.0745^2\times168=0.933 \text{ volt-amperes.}$$

In order to correct for these leading volt-amperes, the choke should draw the same amount lagging, thus giving

$$DO=\frac{100^2}{0.933}=10{,}700 \text{ ohms}$$

If the impedance of the plate-blocking condenser had been high, the desired ratio between grid and plate voltages would not have been obtained. It would then have been necessary to assume an initial value somewhat larger than desired for the final result, proceeding by a series of approximations. However, for other reasons, it is not likely that a high-impedance blocking condenser would be desirable. A high-impedance blocking condenser corresponds to a low-impedance choke which would allow radio frequency currents to flow in circuits with high effective resistance and thus, possibly, damage power generating apparatus.

Proportioning the plate-blocking condenser and line choke is not the only method of bringing plate voltage and current into the 180-degree phase relation. The angle OAC may be compensated for by de-phasing the grid in such a way as to cause the oscillations to occur at the natural resonant frequency and the plate current and voltage to be properly related.

In Fig. 27, diagrams (A) and (B) show two methods by which a Hartley circuit may be restored to operation at the natural resonant frequency of the circuit

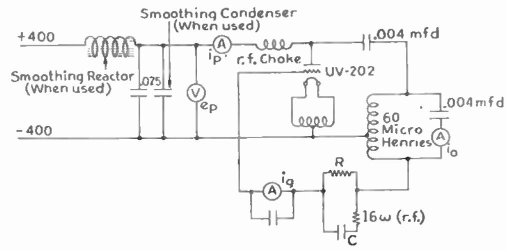


Fig. 28. Circuit Used in Taking Oscillograms Shown in Figs. 30, 31 and 32

with proper phase relations, while (C) and (D) show the two equivalent methods for the Colpitts circuit. In (A) the phasing is accomplished by connecting resistance CG and inductance GO in series. The grid is attached at G. The same result is accomplished in (B) by a resistance from O to G and a condenser from G to C. The elements are reversed in (C) and (D) to produce lag instead of lead.

Although grid phasing may correct the various angles, it is probable that adjustments of the choke and the blocking condenser are to be preferred since these devices are normally present and so do not constitute added complication.

Grid Bias Condenser

It will be noted that no criteria have yet been developed governing the choice of a grid-blocking condenser. Since the function of this condenser is to

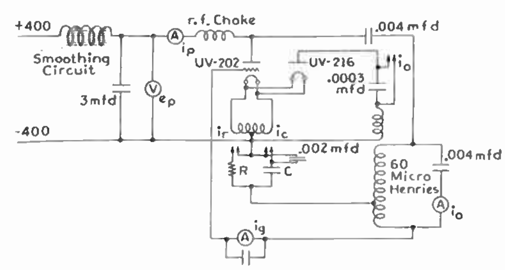


Fig. 29. Circuit Used in Taking Oscillogram Shown in Fig. 33

pass the alternating-current component of grid excitation without the occasion of serious voltage drop while forcing the direct component to flow through the grid leak or biasing resistance, its value is not critical. Its value should be large enough to make the grid-plate capacity small by comparison. The values of tube capacity are normally so small that this requirement causes no concern. The individual pulses of direct current should cause no considerable change in the bias, but this requirement also causes small concern.

Intermittent Oscillation

Troubles may be encountered because of excessive capacity of the grid leak condenser. Since the reason for this is not apparent at first glance, it will be developed in more detail. The evidence of trouble appears as a falling off in input and output, and the radiation of an interrupted wave.

Fig. 30. That the phenomenon is not due to resonance is indicated by the irregularities shown in Fig. 32. In the foregoing oscillograms, plate current is used as a measure of oscillations. Fig. 33 shows that the actual oscillation amplitude follows the same general form. This was taken with the circuit shown in Fig. 29.

The explanation of the phenomenon just described is arrived at as follows: Referring to Fig. 34, let the ratio of alternating plate and grid volts, and other adjustments be fixed. If a constant bias in volts

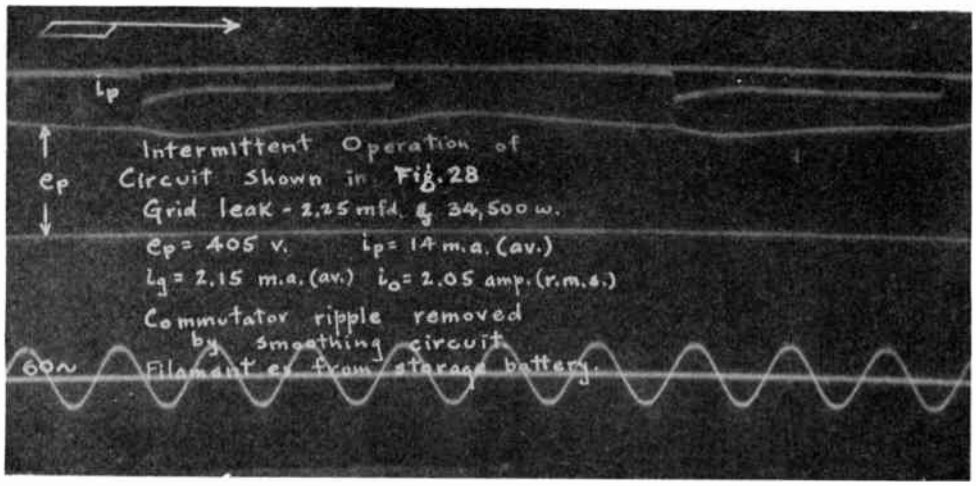


Fig. 30. Intermittent Operation with a Very Large Grid Condenser

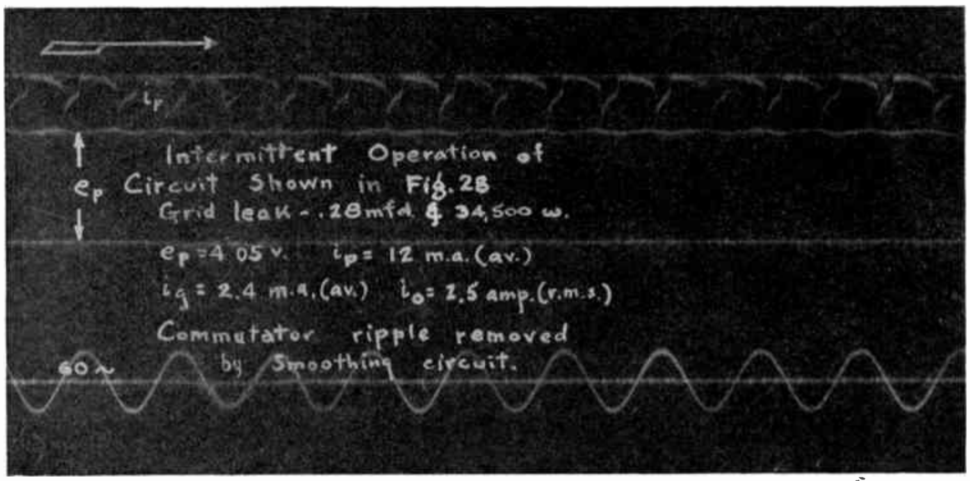


Fig. 31. Intermittent Operation with a Grid Condenser of Smaller Capacity Than is Used for Fig. 30

To show the effects of different grid capacities, the circuits illustrated in Figs. 28 and 29 were subjected to oscillographic study. The result of using a very large grid condenser is shown in Fig. 30. This oscillogram was taken in connection with the circuit shown in Fig. 28. The oscillations alternately build up and die out, the period being quite long. By reducing the capacity of the condenser, the period is changed as shown in Fig. 31. The phenomenon persists even when considerable care is used in smoothing out the impressed voltage by means of a filter, as shown in

is then assumed, the tube output for any amplitude of oscillation can be calculated directly by the method developed in the latter half of Part II of this serial. Several such curves are plotted. The resistance loss in the circuit is proportional to the square of the voltage and is also plotted. An intersection between tube output and circuit loss represents a point of equilibrium. If a decrease in oscillation amplitude causes the tube output to become higher than the losses, the point is stable; if the reverse is the case, it is unstable. By determining the grid current for various points on the curves of output at constant bias, the corresponding leak resistances can be determined. These points may then be joined by curves which represent output variation with change in oscillation amplitude at constant leak resistance. It will be observed that there is a value of bias such that the bias curve and the loss curve are tangent, as at C. A greater bias than this cannot be used as there are no intersections with the loss curve and hence no points of equilibrium.

For a small grid condenser, the constant resistance line is more nearly the criterion of stability. It is apparent that in the region of C, in the direction of point A, stable operation is obtained with either constant resistance or constant bias. In the direction B, constant bias operation is unstable, while constant resistance operation is still stable.

Best operation, especially for a highly efficient tube, is very likely to fall in the C—B zone; thus stability requires a grid condenser small enough to

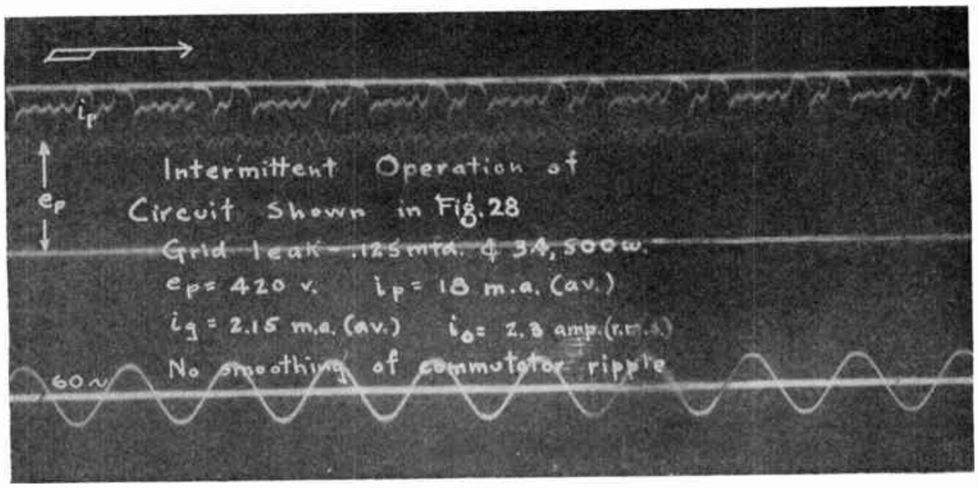


Fig. 32. Irregularities in Intermittent Operation

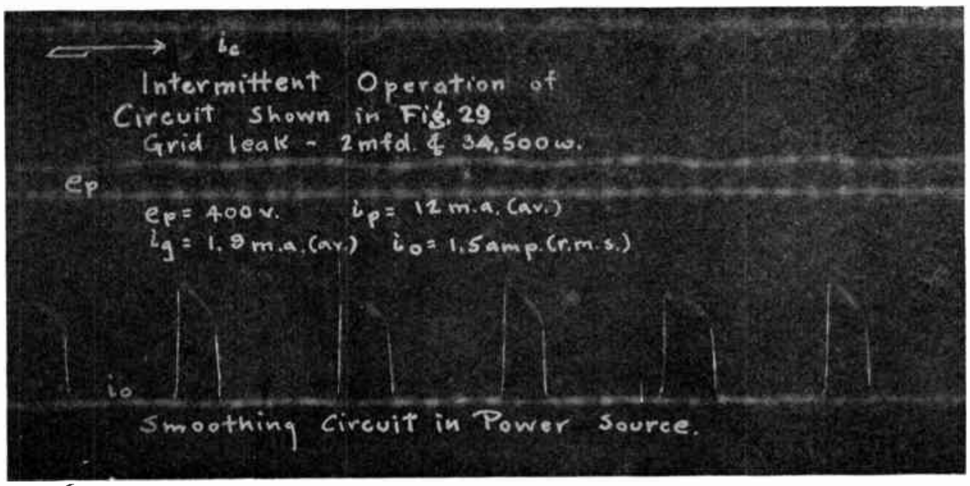


Fig. 33. Intermittent Nature of Oscillating Current

Through any point, as A or B, on the loss curve two lines can be drawn, one representing the change of output variation at constant bias and the other the change of output variation at constant leak resistance. If a fixed bias is being used, the equilibrium test must be applied to the bias line. If resistance bias is being used, the test is applied with reference to the constant-resistance line. The grid-blocking condenser tends to hold constant bias; therefore, with a large condenser, the diagram of operation, as far as equilibrium is concerned, utilizes a constant bias line.

cause the slope of the resultant output curve for the probable maximum rate of amplitude variation to be less than the slope of the loss curve. This condition is also frequently observed in regenerative receiver sets.

Having established a condition of either instability or a very narrow margin of stability, equilibrium may be disturbed by a large number of causes, including generator ripple, line regulation, keying, and the like. The cycle followed is somewhat as follows: Assuming the 20,000-ohm value of grid leak, the tube, starting with zero bias, will build up both oscillations and grid bias until equilibrium is reached at 7.3 watts output and approximately 50 volts bias. If the grid condenser is small, the operation will be stable. If the condenser is large, any momentary disturbance will shift the operating point up or down on the 50-volt bias line, the condenser tending to hold the bias constant over a brief interval. If it shifts down, the tube output becomes less than the circuit losses, and the oscillations die out. The grid condenser now discharges through the leak until the bias is low enough for oscillations to start again and the cycle is then repeated.

Operating Difficulties

If a circuit does not operate as expected after making the calculations, the trouble will usually be found to be due to one of two causes. The tube may not correspond to the characteristics, or the circuit constants may not be as they seem. The most frequent variation in tube characteristics is in the emission. If this is low, the tube is very apt to refuse to operate at all. If it should be too high, the operation will be good, but the tube life can be extended by decreasing the filament current. In cases where it is desirable, output and efficiency can both be increased at the expense of tube life by overburning the filaments.

Occasionally the grid current will differ from that given by the characteristic curves. In this case a satisfactory adjustment can usually be obtained by varying the grid leak resistance.

Circuit difficulties are of two sorts: the measured values may be in error, or the circuit unexpectedly may operate at a much higher frequency than desired, utilizing the inductance of condenser leads and the internal capacities of coils rather than the normal reactances of these elements.

The resistance of the oscillating circuit is perhaps the most difficult constant to determine, besides being especially apt to make trouble if it actually has a greater value than supposed. Hence the greatest care should always be used to see that it is correctly determined. Certain objects, such as concrete floors or walls within a few feet of radio-frequency apparatus, are likely to have considerable losses produced in them which means that they increase the effective resistance of the circuits. In general, any object near a high-frequency circuit should be built of either very good insulating materials or good conductors. Very little loss can occur in good insulators; and good conductors shield themselves by the currents induced in them which flow with only a relatively small loss

It is often difficult to foresee the causes of parasitic oscillations in which an inductance coil will behave as a condenser due to its internal capacity, or a lead

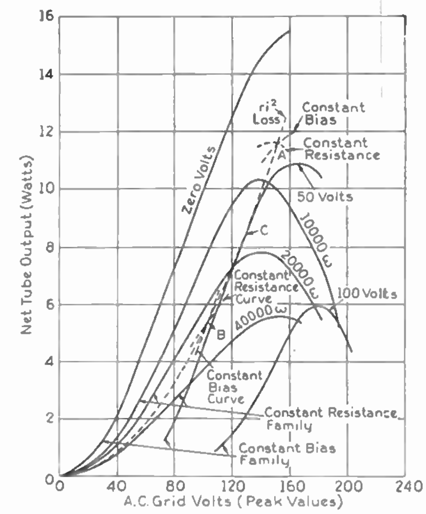


Fig. 34. Curves Showing Grid Circuit Stability of a Small Pliotron Assuming the Alternating Plate Volts as 1.7 Times the Alternating Grid Volts and a Direct Plate Potential of 350 Volts

whose reactance is negligible at the desired frequency develops a high reactance due to the greatly increased frequency. Such annoyances are always to be expected; sometimes they will require the replacement of some part of the circuit, but often they can be stopped by placing a resistance in some part of the circuit where it will cause very little loss under the normal operating conditions, but form a heavy burden for the parasitic oscillations. A very good place to put such a resistance is in series with the grid leak condenser. In many cases, a few hundred ohms at this point will be all that is required.

(To be continued)

Graphical Determination of Magnetic Fields

PART II

RULES FOR CALCULATING AND PLOTTING FIELDS

By A. R. STEVENSON, JR., and R. H. PARK
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ALTHOUGH various authors have given rules for plotting flux in air, it seems well in the interest of completeness to briefly sum up the more important principles, in the group of brief rules which follows.

The method used by Doherty and Shirley can best be described by reference to a particular problem illustrated in Fig. 23 which represents a pair of alternator field poles.

In attacking a problem of this sort, Doherty and Shirley made the following assumptions:

(1) That the magnetomotive force is due to an infinitely thin coil distributed along the pole[17].

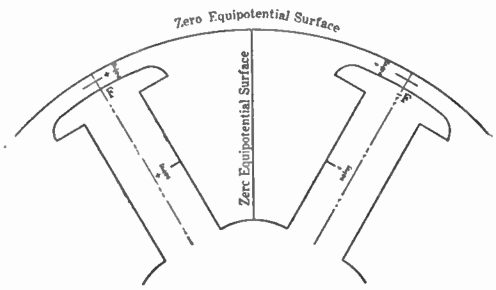


Fig. 23

The armature reaction is neglected while plotting the flux due to the field m.m.f.; and later, after another sketch is made of the field due to the m.m.f. of armature reaction, the two are superposed.

(2) The magnetic potential of one pole due to the m.m.f. of its own winding is $+F$. The magnetic potential of the other pole due to its own m.m.f. is $-F$.

(3) Assuming infinite permeability, the armature and field pole faces are equipotential surfaces.

(4) By symmetry, a zero equipotential surface is drawn midway between the two poles.

(5) The armature surface is also assumed to be a zero equipotential surface.

(6) At the surface of the left-hand pole, the potential is $+F$. As the air-gap is crossed, this drops to 0 at the surface of the armature. In re-crossing the gap from the armature surface to the right-hand pole, the magnetic potential drops to $-F$. Therefore, the total drop from one pole face to the other is $2F$.

(7) In any place where the flux lines are known to be straight and parallel, as is approximately true in the air-gap, the potential drop is a linear function of distance. Thus, a line half-way across the gap represents one-half potential.

(8) The equipotential planes $+F/2$ and $-F/2$ will curve from the center of the air-gap to the mid-points of their respective poles. The beginning and ends of the one-quarter potential planes can be similarly located. In sketching the traces of these planes through points known to be at the same potential, there must be discontinuity in the gradual change in shape of adjacent potential lines, *i. e.*, when near the pole they follow its configuration, but those near the zero equipotential line must approach its rectangular shape.

(9) After the equipotential lines have been sketched, the lines of force are drawn perpendicular to them. The whole surface is thus divided into chequers. If enough lines are drawn, these chequers become very small rectangles. The sketch is not correct until all the angles are right angles.

(10) In space through which no current is flowing, all these rectangles must be similar, *i.e.*, the spacings of lines of force at two different places in the field must be proportional to the spacing of equipotential lines at the same respective points. It is recommended as a matter of convenience that the flux density be represented by such a number of lines as will make these rectangles curvilinear squares.

This method of plotting magnetic fields is a cut-and-try method. The first few sketches will obviously be wrong, but there are sufficient conditions to provide that the final picture obtained after several readjustments will be approximately correct.

The actual flux density can be calculated at any point from this picture as follows:

a. The total m.m.f. per pole,

$$F = 4\,\pi\,n\,I \tag{22}$$

where n is the number of turns per pole and I is the field current in abamperes.

b. If there are m equipotential surfaces including one of the boundary surfaces drawn between the pole and the zero equipotential surface, the potential gradient between any two of these surfaces is

$$\frac{d\,P}{d\,s} = \frac{4\,\pi\,n\,I}{m\,\delta} \tag{23}$$

where δ is the perpendicular distance measured in centimeters between adjacent equipotential surfaces at the point in question; *i. e.*, $m\,\delta$ = air-gap.

[17] This approximation is more accurate than neglecting the distribution of the coil entirely, as has been done by some others, and apparently does not very much affect the accuracy of flux distribution in the air-gap.

c. Since the permeability of air is unity, the density in lines of force per sq. cm. at every point is exactly equal to the potential gradient.

$$B=\frac{d\,P}{d\,s}=\frac{4\,\pi\,n\,I}{m\,\delta} \qquad (24)$$

d. But, if the figure has been drawn so that the small chequers are square, the lines of force will have the same spacing, δ, as the equipotential surfaces. Therefore, the flux included in the tube between consecutive lines of force is

$$\Delta\,\phi=B\,\delta=\frac{4\,\pi\,n\,I}{m} \qquad (25)$$

e. Attention is called to the fact that if δ is the spacing of lines of force, the area of a tube δ wide and 1 cm. thick is δ, and the density at any point is

$$B=\frac{\Delta\,\phi}{\delta}=\frac{4\,\pi\,n\,I}{m\,\delta} \qquad (26)$$

Equation (26) is identical, of course, with Equation (24), and merely represents another viewpoint.

In order to check the accuracy of the free-hand method, 23 different men determined the effective air-gap of a motor by making free-hand sketches of the flux. By Carter's equation, the correct value of the effective gap was 0.575. The results of the free-hand sketches were as follows:

Results Between	Number
0.570–0.605	8
0.605–0.640	6
0.640–0.675	4
Rejected for gross inaccuracy	5
Total	23

The above table indicates that the more accurate sketches gave a minimum reluctance. This agrees with Arnold's[18] statement: "The most nearly correct distribution of the flux will be shown by the sketch which makes the permeance of the flux tubes a maximum."

It is probably obvious that the flux will arrange itself in such a manner as to follow the path of least resistance; but it is interesting to note that a comparison of the reluctances determined from several independent flux plots with the mathematically correct permeance confirms this theory so well.

Method of Plotting Two-dimensional Magnetic Fields in Space Occupied by Current-carrying Conductors

Consider a section of an infinitely long cylindrical conductor, shown in Fig. 24. The work done in making a complete circuit about such a conductor with a unit pole is

$$W=4\,\pi\,I \qquad (28)$$

[18] E. Arnold: *Die Wechselstromtechnik* (Julius Springer, Berlin), p. 78.

The difference of potential between two points is usually defined as the work done against the field in transporting a unit pole from one point to the other by any path whatever.

Let the potential at A be zero. Then the potential at D is

$$P_D=2\,\pi\,I \qquad (29)$$

The potential, however, continues to increase as the circle is completed. On returning to A after a complete circuit, the potential is no longer zero, but

$$P_A=4\,\pi\,I \qquad (30)$$

Every time the unit pole is taken around the conductor, the potential increases by this amount.

It might be asked, "How does the law of conservation of energy apply?" The answer is that, in taking the unit pole around the conductor, a voltage has been

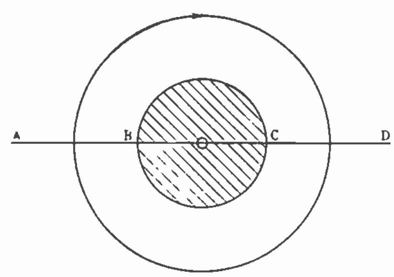


Fig. 24

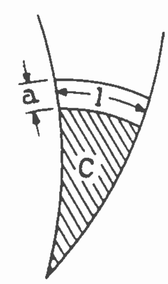


Fig. 25

induced. The product of this induced voltage multiplied by the current in the conductor represents electrical power which, integrated for the elapsed time, gives electrical energy. Thus the mechanical work done in carrying the unit pole about the conductor is converted into electrical energy.

Again referring to Fig. 24, it should be noticed that no work would be done in going from A to D by the path $A\ B\ O\ C\ D$, because this path at all points is perpendicular to the lines of force. Such lines, therefore, can be called lines of no work.

The following are the most important rules to be followed in plotting flux in regions containing current-carrying conductors:

(1) The equipotential lines in the air space, when projected as lines of no work into the copper, must divide the copper into equal areas. It is thus seen that each particular part of the ampere conductors may be regarded as responsible for a particular part of the field.

(2) The work done in carrying a unit pole along any line of force from one point to another is proportional to the current flowing in the area enclosed by the line of force and the lines of no work passing through the two points.

(3) The spacing of lines of force must be inversely proportional to the copper enclosed and proportional

to the length of the part of the tube enclosing the copper.[19] Thus

$$H\,l = 4\,\pi\,C$$

If k is the number of lines in a tube, the width is

$$a = \frac{k}{H} = \frac{k\,l}{4\,\pi\,C}$$

The spacing between lines of force must be proportional, therefore, to the ratio l/C, where l is the length of the tube under consideration between the lines of no work and C is the total current included by this length of tube and the boundary lines of no work in Fig. 25.

(4) The line of half potential outside the copper, when extended as a line of no work into the copper, does not necessarily intersect every line of force at a

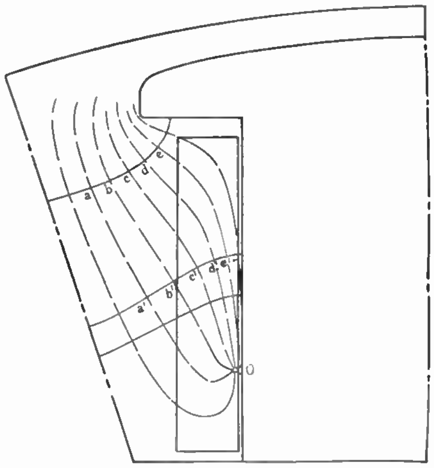


Fig. 26

point which would divide into two equal halves the work done in carrying a unit pole around the line. A similar statement can be made, of course, for the other potential lines.

Brief Rules for Construction of Field Flux Plots Inside the Copper

The following directions as given by L. P. Shildneck have proved very helpful in making flux plots of the fields of salient-pole machines.

Consider the case of a copper and iron distribution as given by a field pole (Fig. 26), with lines of no work $o\,a$, $o\,b$, $o\,c$, $o\,d$, $o\,e$, etc., drawn so as to divide the current into equal areas. Then the work done in transporting a unit pole from a to b is equal to the work done in transporting a unit pole around the path $a\,b\,o\,a$.

$$\begin{aligned} \text{Work } (a\,b) &= \text{work } (a\,b\,o\,a) \\ &= \text{work } (b\,c) = \text{work } (b\,c\,o\,b) \\ &= \text{work } (c\,d) = \text{work } (c\,d\,o\,c), \text{ etc.} \end{aligned}$$

$$\text{for work } (a\,o) = \text{work } (b\,o) = \text{work } (c\,o) = 0.$$

That is, the work done in going from one equipotential line such as a to another such as b, is equal to the m.m.f. between these two lines, or $4\,\pi$ times the current enclosed by the lines $a\,b$, $b\,o$, and $o\,a$. Consequently, work $a\,b$ equals work $a'\,b'$; but work $c\,d$ is greater than work $c'\,d'$, for more current is enclosed by $c\,d$. It is well to remember that the m. m. f. between any two points in the copper, such as c' and d', is proportional to the amount of current enclosed by the line of flux $c'\,d'$ and the lines of no work $c'\,o$ and d' o. Therefore, if a tube of flux is desired along $a'\,b'\,c'\,d'$, so that it may enclose the same flux at all points of the tube, the reluctance must vary in direct proportion to the m.m.f. And since the m.m.f. varies in direct proportion to the amount of current enclosed, the ratio l/a[20] for the curvilinear rectangles must vary in direct proportion to the amount of current enclosed. This relation makes it possible to extend the plot into the copper.

The subsequent rules will aid materially in shortening the time necessary to obtain an accurate field plot. The reasons for following the directions in the order given will be obvious.

(1) Draw bisectors of the angles at the points a, b, c, d, e, f, as shown in Fig. 27. These are the directions of the lines of flux at these points. Continue them wherever it is possible to do so with any degree of accuracy, as at a, b, c, d, entering the opposite side at right angles.

(2) Divide the current-carrying conductor section into eight equal regions, as shown in Fig. 27. The

(20) The reluctance is proportional to the length l of the path and inversely proportional to the area a of the path. Since unit thickness is chosen for the plot, the width of a tube represents its area.

(19) L. F. Richardson has given a rule for the plotting of fields in current-carrying copper to the effect that in a region occupied by current "the difference of successive chequer ratios in a direction perpendicular to the lines of force, divided by the mean chequer area is equal to a constant times the current density in the region." A proof of this relation is briefly outlined below.

Let n and t represent distance measured along lines of no work and along lines of force, respectively. Let small quantities of the order x be represented by the symbol $o\,(x)$. Then, (referring to Fig. A), if n and t are small enough increments so that the field intensity and current density change only slightly for points included in their variation, the relations exist:

$$H = o\,(0)$$

$$n,\ t = o\,(1)$$

$$H_1\,t_1 - H_0\,t_0 = 4\,\pi\,i\,n_1\,t_1 + o\,(3) \qquad (30a)$$

$$H_2\,t_2 - H_1\,t_1 = 4\,\pi\,i\,n_2\,t_2 + o\,(3) \qquad (30b)$$

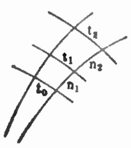


Fig. A

where

H_n = field intensity along t_n,

i = current density at any point near n_1, n_2, t_1, t_2.

If n_1 and n_2 are chosen so that the flux between t_2 and t_1 is the same as the flux between t_1 and t_0, then

$$(H_1 + H_0)\,n_1 = (H_2 + H_1)\,n_2 + o\,(3) = 2\phi + o\,(3) \qquad (30c)$$

where

ϕ = the flux between t_1 and t_0, and between t_2 and t_1.
$= o\,(1)$.

Also,
$$n_2 - n_1 = o\,(2)$$
$$n_2\,t_2 = n_1\,t_1 + o\,(3) = n\,t + o\,(3)$$

Adding Equations (30a) and (30b), there results:

$$(H_2 + H_1)\,t_2 - H_1\,(t_2 - t_1) - (H_1 + H_0)\,t_1 + H_0\,(t_1 - t_0) = 8\,\pi\,i\,n\,t + o\,(3) \qquad (30d)$$

But
$$H_0\,(t_1 - t_0) - H_1\,(t_2 - t_1) = (H_0 - H_1)\,(t_1 - t_0) - H_1\,(t_2 + t_0 - 2t_1) = o\,(3)$$

Therefore, Equation (30d) becomes

$$(H_2 + H_1)\,t_2 - (H_1 + H_0)\,t_1 = 8\,\pi\,i\,n\,t + o\,(3)$$

Substituting in this equation from Equation 30(c), there results

$$\phi\left(\frac{t_2}{n_2} - \frac{t_1}{n_1}\right) = 4\,\pi\,i\,n\,t + o\,(3)$$

$$\frac{t_2}{n_2} - \frac{t_1}{n_1} = \frac{4\,\pi\,i\,n\,t}{\phi} + o\,(2)$$

Thus, omitting second order terms, we obtain a relation involving quantities of the first order on either side:

$$\frac{\dfrac{t_2}{n_2} - \dfrac{t_1}{n_1}}{n\,t} = \frac{4\,\pi}{\phi}\,i$$

which is the rule stated by Richardson.

reason for the peculiar division is that, later, the lines of no work will roughly coincide with the construction lines, thereby providing an easy method of sketching the lines of no work as that they will divide the current into equal parts. If it is found later that some of the construction lines are not placed to best advantage, others may be drawn.

(3) Draw a trial set of seven lines of no work as in Fig. 28, crossing the lines of flux a, b, c, d at right angles, hugging the sharp projections closely, as at b and c, keeping away from the inverted corners as at a and d. This trial set must divide the current region into eight equal portions. Arbitrarily choose some point P in the corner closest to the iron, for the kernel. If no iron were present, it would lie in the center of the copper; if iron were touching the copper along one surface, the kernel would be at the copper-iron boundary; if iron were touching the copper along two surfaces, the kernel would be on both copper-iron boundaries, at the corner. Obviously, the kernel, in all practical cases, would lie somewhere between the center of the copper and the lower corner near the iron. Line No. 4 (Fig. 28) must divide the current into two equal regions. The advantage of the straight construction line between Regions 4 and 5 of Fig. 27, dividing the current into two equal portions, is evident. The other construction lines also are located so as to be most useful in enabling an accurate division of the current to be made by judging only small differences in area with the eye.

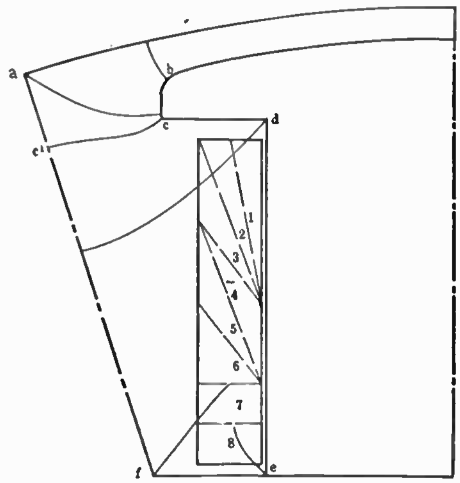


Fig. 27

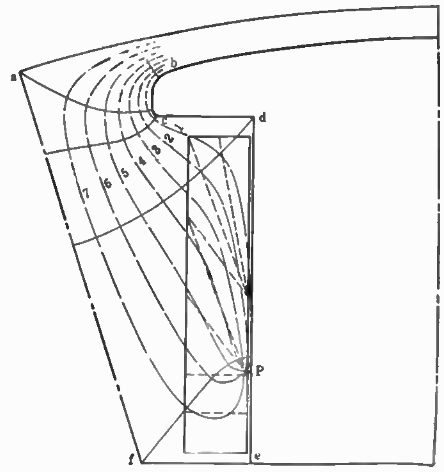


Fig. 28

(4) Starting from some line of flux c c' (Fig. 29), draw lines of flux, making curvilinear squares along the line a c' f in the region where there is no current. Then extend the lines of force into the current region orthogonal to the lines of no work.

(5) The correctness of the plot may now be tested as follows:

a. Lines of flux must cross lines of no work at right angles.

b. Lines of flux must enter the iron at right angles (assuming infinite permeability).

c. All rectangles outside the current must be curvilinear squares (ratio l/a = unity).

d. Within the current region, the rectangles must have a ratio l/a less than unity and equal to the ratio between the current enclosed and the one-eighth portion of the total current. Thus at a point g (Fig. 29), if the flux line and the two lines of no work intersected by it enclose one-half of the eighth portion of current, then the length of the rectangle at this point should be one-half of the width.

(6) If the various ratios of l/a are *not* proportional to the current enclosed, then the plot must be redrawn, either changing the position of the kernel or else shifting the position of the lines of no work, and making the correspondingly necessary changes in the lines of flux. It is well to use tracing paper, for then each previous trial may be used for a guide.

The final plot is shown in Fig. 30. Any required accuracy may be attained by continuing the process indefinitely. By this method, however, a large portion of the cut-and-try is eliminated because as many of the required conditions as possible are fulfilled in the first part of the construction, and after three or four trials a surprisingly accurate sketch can be obtained.

Vector Potential in Coplanar Magnetic Fields

Although neither the problems nor the general methods given in this connection are new it is believed that the application of the little-used conception of vector potential to a group of familiar problems will be of interest.

In problems where the flux distribution can be represented by a two-dimensional sketch,[21] the vector potentials are all perpendicular to the plane of the paper and consequently can be added and

[21] Such fields are said to be coplanar in the plane of the paper.

subtracted exactly like scalar quantities. This makes the construction of field plots by lines of equi-vector potential very much easier than by the more usual method which employs the two components of the vector field intensity.

Let X and Y be the x and y components of the field. Then, integration around any small area in the x y plane will show that [22]

$$\frac{\partial Y}{\partial x} - \frac{\partial X}{\partial y} = 4\pi i \tag{27}$$

where i is positive when directed upward out of the plane of the paper.

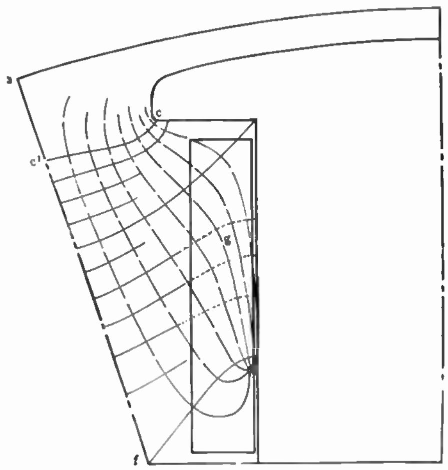


Fig. 29

Let R be a function [23] such that

$$X = -\frac{\partial R}{\partial y} \tag{28}$$

$$Y = +\frac{\partial R}{\partial x} \tag{29}$$

Then

$$\frac{\partial^2 R}{\partial x^2} + \frac{\partial^2 R}{\partial y^2} = +4\pi i \tag{30}$$

The function R is known as the vector potential of the field. In a coplanar magnetic field, a line of force is characterized by the equation $R=\theta=$a contant, because

$$dR = \frac{\partial R}{\partial x}dx + \frac{\partial R}{\partial y}dy = 0 \tag{31}$$

and solving for the slope of the line $R=$a constant:

$$\frac{dy}{dx} = -\frac{\partial R/\partial x}{\partial R/\partial y} = \frac{Y}{X} \tag{32}$$

which shows that this line has the same direction as the magnetic field.

(22) "Fundamental Theory of Flux Plotting," by A. R. Stevenson, Jr., GENERAL ELECTRIC REVIEW, Vol. 29, No. 11, Nov., 1926, p. 797-804.

(23) As in Part I, in order to make R positive in sign, it is defined by the relations.

$$Y = \frac{\partial R}{\partial} \qquad X = -\frac{\partial R}{\partial y}$$

instead

$$Y = -\frac{\partial R}{\partial x} \qquad X = \frac{\partial R}{\partial y}$$

as in Rogowski's work

Also, $\Delta\phi=\phi_1=\phi_2$ is equal to the flux included between the lines of force $R=\phi_1$ and $R=\phi_2$, for

$$\Delta\phi = \int_1^2 dR = \int_1^2 Y\,dx - X\,dy \tag{33}$$

may be seen to give the flux included between the points 1 and 2 whatever the path of integration employed.

In any problem in which the only sources of m.m.f., are currents within the regions under consideration, the boundary conditions imposed consist in variations of permeability frum the region under consideration to the adjoining regions and, in general, from

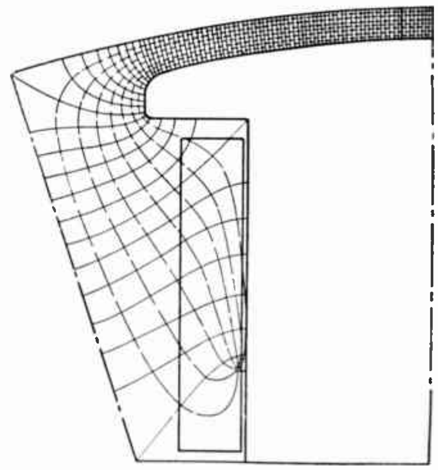


Fig. 30

the adjoining regions to other regions. This type of boundary condition may be recognized as consisting of a sum of terms containing constants and partial derivatives operating on R.

It is then clear that if two solutions R_1 and R_2 satisfy these boundary conditions and the point Equation (30) for current densities i_1 and i_2 separately, their sum must also satisfy these boundary conditions and the point Equation (30) for a current density i_1+i_2.

It is possible, therefore, to superpose solutions for vector potential due to two distributions of current density and the result may be extended to any number of superpositions; and thus from a knowledge of the vector potential of an element we may by integration determine the vector potential of a complicated distribution of current density.

The Vector Potential of an Isolated Straight Wire of Circular Section. Let the radius of the wire be a and let the total current through the wire be I.

Then inside the wire the field intensity [24] is

$$H = \frac{2Ir}{a^2} \tag{34}$$

and outside the wire

$$H = \frac{2I}{r} \tag{35}$$

(24) $H = \sqrt{X^2 + Y^2}$.

If we choose $R=0$ at $r=0$, then inside the wire

$$R=\int_{r=0}^{r=r} H\,d\,r=\frac{I\,r^2}{a^2} \tag{36}$$

and outside the wire

$$R=\int_{r=a}^{r=r} H\,d\,r+\frac{I\,a^2}{a^2}=2\,I\left[\log_\epsilon\left(\frac{r}{a}\right)+\frac{1}{2}\right] \tag{37}$$

The Vector Potential of Two Isolated Straight Wires of Circular Section. Let r_1 and r_2 be the radius vectors from wires 1 and 2, respectively, and let a_1 and a_2 be the respective radii of the conductors. Then inside conductor 1

$$R=2\,I_2\left[\log_\epsilon\left(\frac{r_2}{a_1}\right)+\frac{1}{2}\right]+I_1\frac{r_1^2}{a_1^2} \tag{38}$$

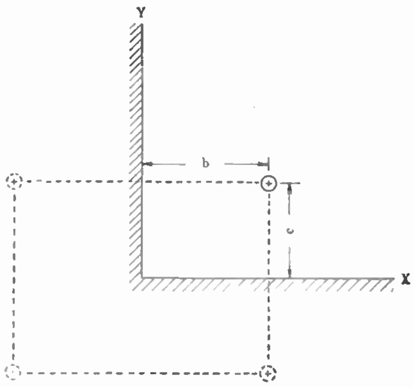


Fig. 31

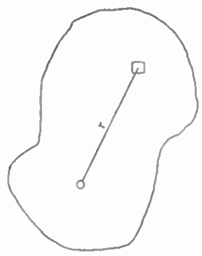


Fig. 32

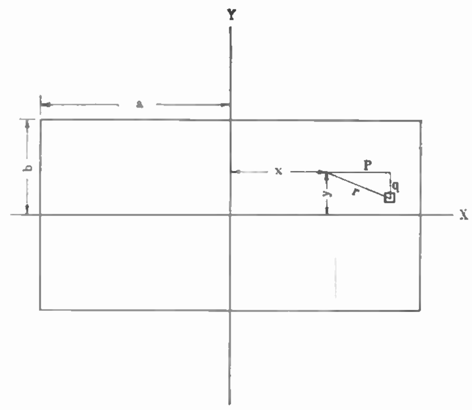


Fig. 33

Inside conductor 2

$$R=2\,I_1\left[\log_\epsilon\left(\frac{r_1}{a_1}\right)+\frac{1}{2}\right]+I_2\frac{r_2^2}{a_2^2} \tag{39}$$

Outside both conductors

$$R=2\,I_1\left[\log_\epsilon\left(\frac{r_1}{a_1}\right)+\frac{1}{2}\right]+2\,I_2\left[\log_\epsilon\left(\frac{r_2}{a_2}\right)+\frac{1}{2}\right] \tag{40}$$

(a) In the *special case* where the currents are in opposite directions, $I_2=-I_1=I$, and $a_1=a_2$, the vector potential outside the conductors is

$$R=2\,I\,\log_\epsilon\frac{r_1}{r_2} \tag{41}$$

and the equation of a line of force, $R=$a constant, is

$$\frac{r_1^2}{r_2^2}=\epsilon^{R/I} \tag{42}$$

The well-known fact follows that the lines of force outside the conductors are circles.

In particular, if the centers of the wires are located at $x=-b$, and $x=b$, respectively, Equation (42) becomes

$$[x+b\coth(R/2\,I)]^2+y^2=\frac{b^2}{\sinh^2(R/2\,I)} \tag{43}$$

(b) In the *special case* where the currents are in the same direction, $I_1=I_2=I$, and $a_1=a_2=a$,

$$R=2\,I\left[\log_\epsilon\left(\frac{r_1\,r_2}{a^2}\right)+1\right] \tag{44}$$

The equation of the line of force in this case is of the fourth degree.

The Vector Potential of a Straight Wire Near the Corner of two Infinitely Permeable Planes. The wire and images for this case are shown in Fig. 31. Outside the wire

$$\frac{R}{I}=\log_\epsilon\frac{(x-b)^2+(y-c)^2}{a^2}+\log_\epsilon\frac{(x-b)^2+(y+c)^2}{a^2}+\log_\epsilon\frac{(x+b)^2+(y-c)^2}{a^2}+\log_\epsilon\frac{(x+b)^2+(y+c)^2}{a^2}+4 \tag{45}$$

Equations of the type of (45) may be solved by plotting since only the real roots are desired. In the case under consideration, R/I should be plotted as a function of x with y as a parameter. Points on a line of force are then determined by the intersections of the curves along lines of $R/I=$a constant.

The Vector Potential of an Isolated Wire of Any Cross-Section in Air. Referring to Fig. 32, the vector potential due to an element of the wire is,

$$d\,R=i\,(\log_\epsilon r^2)\,da \tag{46}$$

where r is the distance from the point at which R is determined to the element under consideration, i is the current density, and da the area of the element. The value of R for the whole wire is, then,

$$R=\int_S i\,(\log_\epsilon r^2)\,da+\text{a constant} \tag{47}$$

the subscript S indicating that the integral is taken over the whole surface of the wire. If the current density over the cross-section is constant, we have

$$\frac{R}{i}=\int_S(\log_\epsilon r^2)\,da+\text{a constant}^{(25)} \tag{48}$$

(25) This result may be expressed in the form $\frac{R}{i}=\log D^2+$ a constant, where D equals the geometric mean distance of the point at which R is to be computed from the area of the section of the conductor.

(*a*) For the *special case* of an isolated wire of rectangular section we have, referring to Fig. 33.

$$\frac{R}{i} = \int_{q=y-b}^{q=y+b} \int_{p=x-a}^{p=x+a} \log_\epsilon (p^2+q^2)\, dp\, dq \qquad (49)$$

Integrating and adding the constant terms.

$$-4\, a\, b \log_\epsilon (a^2+b^2) - 6\, a\, b - 4\, a^2 \tan^{-1}\frac{b}{a} - 4\, b^2 \tan^{-1}\frac{a}{b}$$

In order to make $\frac{R}{i}=0$ at the origin, Equation (49)

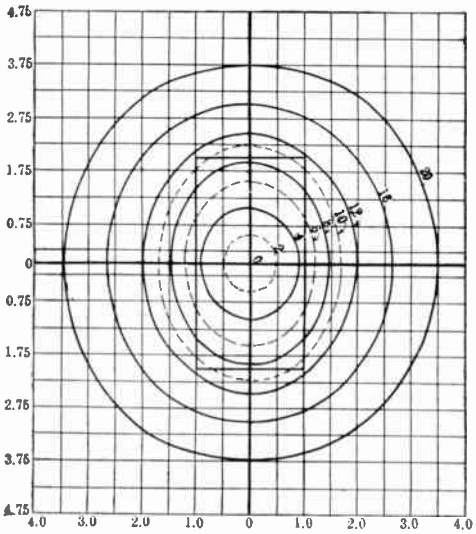


Fig. 34

may be reduced to the symmetrical form:[26]

$$\begin{aligned}
\frac{R}{i} = {} & (x+a)\,(y+b) \log_\epsilon \left[\frac{(x+a)^2+(y+b)^2}{a^2+b^2}\right] \\
& -(x-a)\,(y+b) \log_\epsilon \left[\frac{(x-a)^2+(y+b)^2}{a^2+b^2}\right] \\
& -(x+a)\,(y-b) \log_\epsilon \left[\frac{(x+a)^2+(y-b)^2}{a^2+b^2}\right] \\
& +(x-a)\,(y-b) \log_\epsilon \left[\frac{(x-a)^2+(y-b)^2}{a^2+b^2}\right] \\
& +(x+a)^2 \left[\tan^{-1}\frac{y+b}{x+a} - \tan^{-1}\frac{y-b}{x+a}\right] \\
& -(x-a)^2 \left[\tan^{-1}\frac{y+b}{x-a} - \tan^{-1}\frac{y-b}{x-a}\right] \\
& +(y+b)^2 \left[\tan^{-1}\frac{x+a}{y+b} - \tan^{-1}\frac{x-a}{y+b}\right] \\
& -(y-b)^2 \left[\tan^{-1}\frac{x+a}{y-b} - \tan^{-1}\frac{x-a}{y-b}\right] \\
& -4\, a^2 \tan^{-1}\frac{b}{a} - 4\, b^2 \tan^{-1}\frac{a}{b} \qquad (50)
\end{aligned}$$

[26] The variable terms in Equations (50) and (51) may be checked against those given by Maxwell "Electricity and Magnetism," (paragraph 692) for the mean geometric distance of a point from a rectangle and a straight line, respectively.

The field in and around a rectangular conductor of dimensions two units by four units, carrying a current of 80 amp., *i. e.*, $a=2$, $b=1$, $i=1$, has been plotted from Equation (50) by one of the writers' associates, R. S. Arthur, and is shown in Figs. 34 and 35. Figs. 36 and 37, showing R as a function of x and y, were used in plotting Figs. 34 and 35.

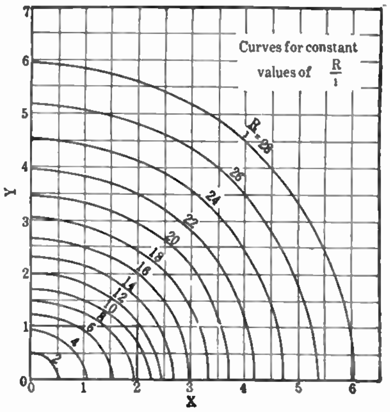


Fig. 35

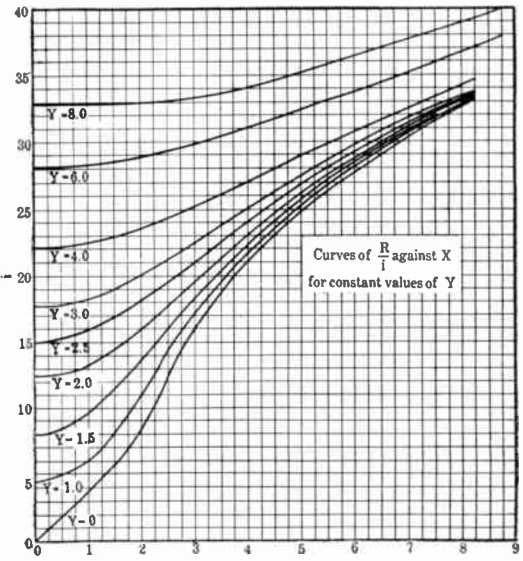


Fig. 36

(*b*) *Special Case.* Infinitely thin ribbon.

For sufficiently small values of b, that is, if the rectangle degenerates into a ribbon, putting $2\,b\,i=i'$ = current per unit length o the ribbon and adding terms such that $\frac{R}{i'}=0$ at $x=y=0$, Equation (50) may be written[26]

$$\begin{aligned}
\frac{R}{i'} = {} & (a+x) \log_\epsilon \frac{(a+x)^2+y^2}{a^2} \\
& +(a-x)\ \text{og}_\epsilon \frac{(a-x)^2+y^2}{a^2} \\
& +2\,y \left[\tan^{-1}\frac{a+x}{y} + \tan^{-1}\frac{a-x}{y}\right] \qquad (51)
\end{aligned}$$

Equation (51) corresponds precisely to the case of a vanishingly thin isolated conductor, or to a narrow strip of conductor on an infinite iron surface.

In the case of a ribbon conductor two units long, carrying a current of 20 amp., the expression for vector potential is

$$R=(1+x)\log_\epsilon[(1+x)^2+y^2]+(1-x)\log_\epsilon[(1-x)^2+y^2]$$
$$+2y\left[\tan^{-1}\left(\frac{1+x}{y}\right)+\tan^{-1}\left(\frac{1-x}{y}\right)\right] \quad (52)$$

The field for this case as plotted by one of the writers is shown in Fig. 38. The curves of R as a function of x and y which were used in plotting Fig. 38 are shown in Figs. 39 and 40.

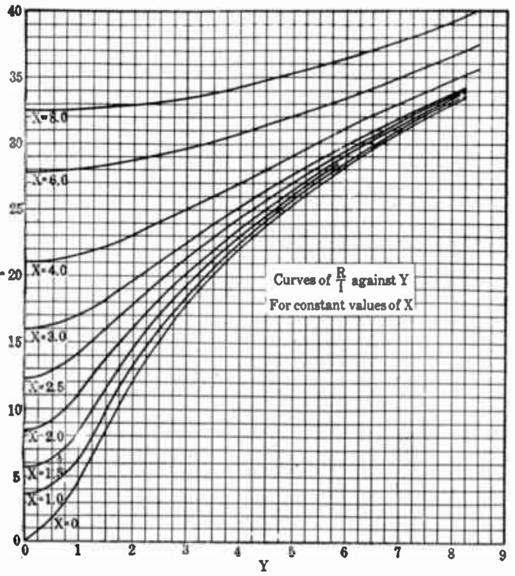


Fig. 37

The Relation of the Scalar Potential Function to the Vector Potential Function in Coplanar Magnetic Fields

For points outside of regions in which current density exists, it is legitimate to construct a potential function V having the property:

$$H_n = \frac{\partial}{\partial n} V \quad (53)$$

where H_n is the field intensity in a direction n and where n is distance measured in any direction from the point at which H_n exists.

Then, in particular,

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy = H_x\,dx + H_y\,dy$$
$$V_2 - V_1 = \int_1^2 H_x\,dx + H_y\,dy \quad (54)$$

where x and y form any convenient set of coördinate axes. The path of integration must be arranged so that it does not traverse regions in which current density exists. At the same time, it is necessary to construct arbitrary boundary surfaces, one beginning at some point of each current-carrying region and dividing the field into arbitrary sections such that within any section it is impossible to enclose current by any path of integration outside the zone of current density. Equipotential lines may be carried through these regions by joining lines which differ by an amount 4π times the current enclosed by any circuit which does not cut through any boundaries, and starting at one side of the boundary and ending at the other side at the same point.

If the vector potential of the field is known, we may use Equation (54) in the form:

$$V_2 - V_1 = \int_1^2 \frac{\partial R}{\partial x}dy - \frac{\partial R}{\partial y}dx \quad (55)$$

Solutions of scalar potential due to particular distributions of current density will be superposable in

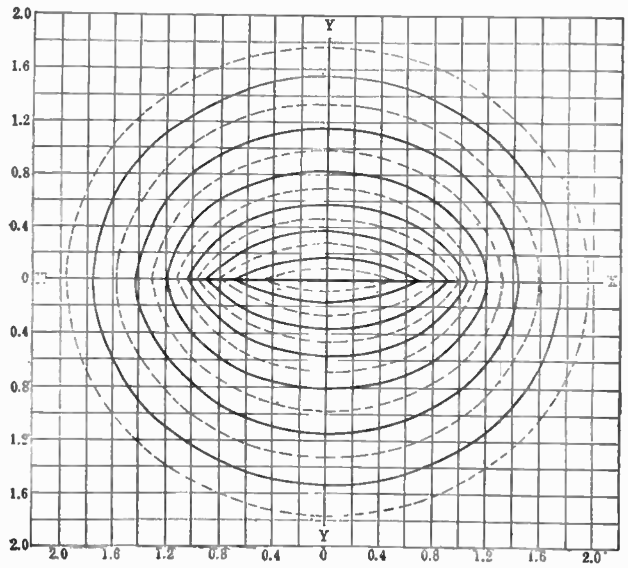


Fig. 38

the same way and subject to the same limitations as apply to solutions of vector potential.

In the following, the scalar potential functions in the various examples of magnetic fields given in other parts of the article will be worked out briefly.

Field Pole, or Slot. Referring to Fig. 41, it will be convenient to divide Region II into a and b parts, the a part being between the current-carrying zones and the b part between the current-carrying zones and the iron, and to choose the line $x=a$ from $\theta=\pi-\theta_2$ to $\theta=\pi$, and from $\theta=\theta_2-\pi$ to $\theta=-\pi$ as the arbitrary boundary surface previously referred to.

Then, choosing $V=0$ at $x=y=0$, and remembering $y=\frac{\theta}{k}$,

$$V=\int_0^{\theta,x} -k\frac{\partial R}{\partial \theta}dx + \frac{1}{k}\frac{\partial R}{\partial x}d\theta = \int_0^{\theta}\frac{1}{k}\frac{\partial R}{\partial x}d\theta \quad (56)$$

Thus, for Region I:

$$\frac{\partial R}{\partial x} = \sum_1^\infty kn\left[a_n\epsilon^{-kn(a-x)} - a_n'\epsilon^{-knx}\right]\cos n\theta$$

$$V_1 = \sum_1^\infty \left[a_n\epsilon^{-kn(a-x)} - a_n'\epsilon^{-knx}\right]\sin n\theta \quad (57)$$

For Region II:

$$\frac{\partial R}{\partial x} = 4\pi\alpha_0(x-a) + \sum_1^\infty n\,[b_n\,\epsilon^{-kn(b-x)} - b_n'\,\epsilon^{-kn(x-a)}]\cos n\theta$$

$$V_{IIa} = 4\pi\alpha_0(x-a)\,y + \sum_1^\infty [b_n\,\epsilon^{-kn(b-x)} - b_n'\,\epsilon^{-kn(x-a)}]\sin n\theta \qquad (58)$$

In order to obtain the scalar potential for Region IIb, it is convenient to subtract from the known potential at the iron surface the potential difference from the point in question referred to that surface. Thus:

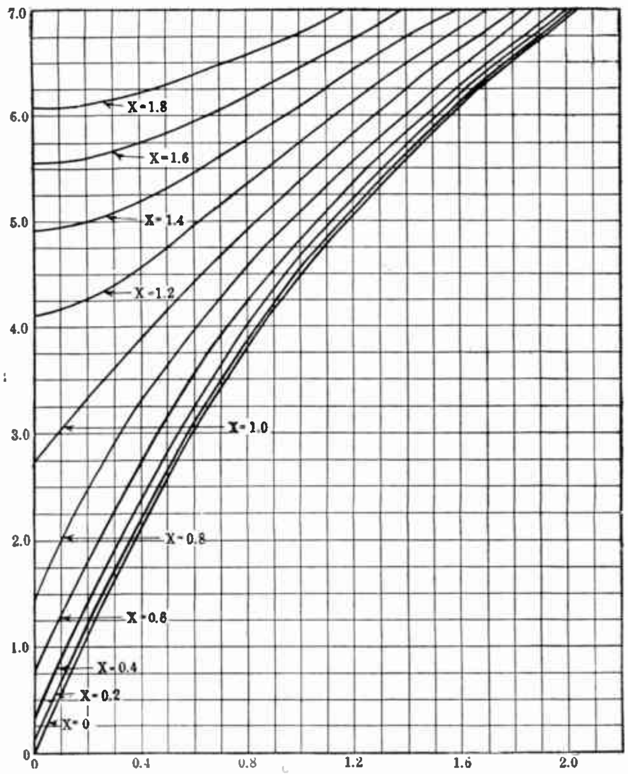


Fig. 39

$$V_{IIb} = 4\pi\alpha_0(b-a)\frac{l}{2} - \int_\theta^\pi \frac{1}{k}\frac{\partial R}{\partial x}\,d\theta$$

$$= 4\pi\alpha_0\left[(b-a)\frac{l}{2} - (x-a)\left(\frac{l}{2}-y\right)\right]$$

$$+ \sum_1^\infty [b_n\,\epsilon^{-kn(b-x)} - b_n'\,\epsilon^{-kn(x-a)}]\sin n\theta$$

$$= V_{IIa} + 2\pi\alpha_0\,l\,(b-x) \qquad (59)$$

For Region III:

$$\frac{\partial R}{\partial x} = 4\pi\alpha_0(b-a) + \sum_1^\infty [c_n\,\epsilon^{-kn(c-x)} - c_n'\,\epsilon^{-kn(x-b)}]\cos n\theta$$

which gives

$$V_{III} = 4\pi\alpha_0(b-a)\,y + \sum_1^\infty [c_n\,\epsilon^{-kn(c-x)} - c_n'\,\epsilon^{-kn(x-b)}]\sin n\theta \qquad (60)$$

The solution for an infinitely deep slot may be obtained by inserting $c=\infty$.

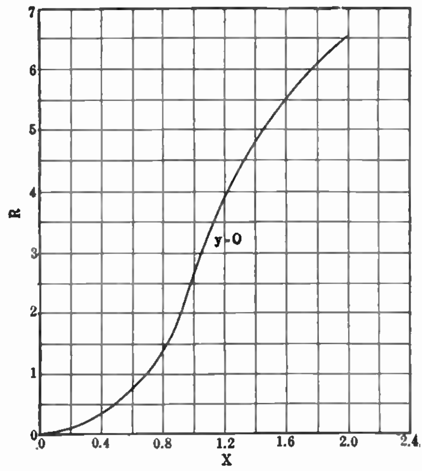


Fig. 40

Circular Slot. In polar coördinates, Equations (54) and (55) become

$$V_2 - V_1 = \int_1^2 H_r\,dr + H_\theta\,r\,d\theta \qquad (61)$$

$$= \int_1^2 -\frac{1}{r}\frac{\partial R}{\partial\theta}dr + \frac{\partial R}{\partial r}r\,d\theta \qquad (62)$$

Taking $V=0$ at $\theta=0$, there results:

$$V = \int_0^\theta r\frac{\partial R}{\partial r}\,d\theta \qquad (63)$$

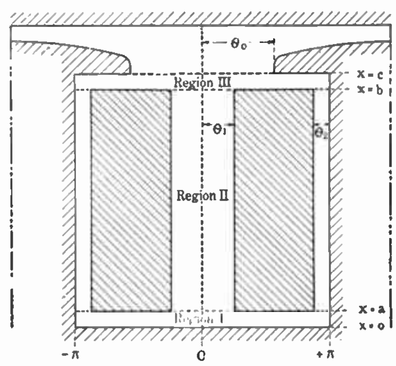


Fig. 41
(Fig. 10 from Part I, repeated)

For Region II:

$$\frac{\partial R}{\partial r} = \sum_1^\infty \frac{n}{r_0}\left(\frac{r}{r_0}\right)^{n-1}[b_n\cos n\theta] + \frac{2I}{r}$$

which gives:

$$V_{II} = \sum_1^\infty \left(\frac{r}{r_0}\right)^n b_n\sin n\theta + 2I\theta \qquad (64)$$

Scalar Potential of a Wire of Any Section. Evidently, for an isolated circular wire,

$$V = 2\, I\, \theta$$

The solution for several circular wires is obtained by superposition. The solution for a wire of any section is

$$V = \int_S 2\, i\, \theta\, da \tag{65}$$

the integral to be extended over the whole section in which i exists. It may be found more convenient, however, to obtain V indirectly through Equations (47) and (55).

Calculation of Inductance from a Knowledge of Vector Potential

In general, in order that it shall be permissible to employ the conception of inductance to a conductor of large section, it is necessary that the conductor consist of a bundle of smaller conductors, all of these conductors supposedly connected in series, or in such a way that the same current flows in each.[27] If there is a sufficient number of small conductors, it is permissible as an approximation to calculate the inductance of the system as the average inductance of a continuous distribution of small current filaments, the density of the filaments being in proportion to the density of small conductors. It will further be assumed that the current distribution may be regarded as continuous within the section of the large conductor.[28]

Let ϕ_0 be the total flux outside some particular line of force $R = R_0$ and between that line and the line with respect to which induced voltage is to be computed, where R is the vector potential function of the field.

Then $(\phi_0 + R_0 - R)$ will be the flux outside any line R.

Let n = the density of small conductors.

The average flux linkages per conductor are

$$\int_S \frac{(\phi_0 + R_0 - R)\, n\, da}{N} = (\phi_0 + R_0) - \int_S \frac{R\, n\, da}{N}$$

where

$$N = \int_S n\, da$$

= the total number of small conductors in the section.

If I is the total current through the section, the component of average inductance due to flux up to the point that ϕ_0 is computed, is

$$L = \left(\frac{\phi_0 + R_0}{I}\right) - \int_S \frac{R\, n\, da}{N\, I}$$

[27] From the restricted point of view that stored magnetic energy equals one-half the product of inductance and current squared, it is permissible to apply the conception of inductance to conductors which are not subdivided.

[28] If desired, this approximation may be corrected. A correction of this type is given by Maxwell, "Electricity and Magnetism," par. 693.

But

$$i = \frac{n}{N} I$$

$$I = \int_S i\, da$$

or

$$L = \left(\frac{\phi_0 + R_0}{I}\right) - \int_S \frac{R\, i\, da}{I^2} \tag{66}$$

Equation (66) may be put in the form,

$$L = 4\, \pi\, (P_0 - P) \tag{67}$$

where P_0 and P are permeance factors:

$$P_0 = \frac{\phi_0 + R_0}{4\, \pi\, I} \tag{68}$$

$$P = \frac{\int_S R\, i\, da}{4\, \pi\, [\int_S i\, da]^2} \tag{69}$$

In Equations (68) and (69), the quantity P_0 is the permeance factor that would apply if the flux $\phi_0 + R_0$ linked all of the conductors, while the factor P takes into account the effect of partial linkages.

It is interesting that in the foregoing equation, the vector potential R need not be computed so that the minimum value of $R = 0$. Thus it is not necessary to calculate the value of R at the kernel. If the kernel is known to exist on some line such that one coördinate x or y, or r or θ, for example, is fixed, the calculation of the value of R at the kernel is comparatively easy. If, as in the field-pole problem, however, the value of neither coördinate is known, then on account of the unsatisfactory convergence which usually occurs at the kernel, the calculation of the position and value of R at the kernel is a task of considerable difficulty. It is fortunate, therefore, that this calculation is not required.

Inductance of Two Parallel Cylindrical Conductors. As a simple example of Equation (66), consider the case of two straight conductors of circular section carrying currents I and $-I$, respectively. From Equation (38), the vector potential at a point inside one wire is

$$R = I \frac{r_1^2}{a^2} - 2\, I \left[\log\left(\frac{r_2}{a}\right) + 1/2\right]$$

where a = the radius of the section.

Let $r_1 = r$

l = distance between centers of the conductors.

The component of inductance due to half the flux will be calculated. Thus ϕ_0 will be chosen equal to 0 for $r_1 = r_2$. But also, from Equation (41), $R = R_0 = 0$ at $r_1 = r_2$. Thus, Equation (66) gives

$$L = -\int_S \frac{R\, i\, da}{I^2}$$

which may be put in the form:

$$L=\frac{1}{\pi a^2}\int_0^a\int_0^{2\pi}\left[1-\frac{r^2}{a^2}\right.$$

$$\left.+\log\left(\frac{l^2+r^2-2\,l\,r\cos\theta}{a^2}\right)\right]r\,d\theta\,dr \qquad (70)$$

$$=\frac{2}{a^2}\int_0^a\left[1-\frac{r^2}{a^2}+\log\frac{l^2}{a^2}\right]r\,dr$$

$$=2\log\frac{l}{a}+\frac{1}{2} \qquad (71)$$

which is the usual expression for one-half the inductance of a circuit formed by two parallel wires.

Application to Field-Pole Problem. The use of Equation (69) will now be illustrated by applying it to the field-pole problem. In this case, and considering all the copper as comprising a single coil side, there is

$$P=\frac{2\,i\int\limits_{\theta_1}^{\pi-\theta_2}\int\limits_{x=a}^{x=b}R_{\text{II}}\,dx\,dy}{4\,\pi\,[\alpha_0\,(b-a)\,l]^2} \qquad (72)$$

This integral reduces to the expression

$$P=\frac{b-a}{6\,l}-P'$$

where

$$P'=\sum_1^\infty\frac{\alpha_n}{16\,\pi^2\,\alpha_0^2\,(b-a)^2\,n}\,[(b_n+b_n')\left\{1-\epsilon^{-kn(b-a)}\right\}$$

$$+2\,K_n\,k\,n\,(b-a)] \qquad (73)$$

Thus, for the field-pole shown in Fig. 11

$$P'=0.0045$$

For the case of an infinite slot, $c=\infty$, Equation (73) becomes explicitly:

$$P'=\sum_1^\infty\frac{1}{8\,\pi^2}\left(\frac{\alpha_n}{n\,\alpha_0}\right)^2\left(\frac{l}{b-a}\right)\left[1\right.$$

$$\left.-\frac{(2-\epsilon^{-2kna}+\epsilon^{-kn(b+a)})\,(1-\epsilon^{-kn(b-a)})}{k\,n\,(b-a)}\right] \qquad (74)$$

Thus, for the "infinite" slot shown in Fig. 6,

$$P'=0.0037$$

In the case of an infinitely deep slot, it may be verified that the factor P' provides a correction which gives the increase of inductance due to concentration of current above the inductance which would be calculated on the assumption that the lines of force were everywhere perpendicular to the slot sides.

Circular Conductors in a Circular Slot. The expression for P' in the case of a circular conductor is very simple because for b_0 chosen equal to 0, the integral over the area of the copper of the term which involves the effect of the slot is zero; and there remains only the term which would exist were the conductor isolated in space.

Thus, with $b_0=0$:

$$P'=\frac{i}{4\,\pi}\int\limits_S\frac{R\,i\,d\,a}{(\int_S i\,d\,a)^2}=\frac{1}{4\,\pi^2\,r_2^2}\int_0^{2\pi}\int_0^{r_2}\frac{r^2}{r_2^2}\,r\,dr\,da \qquad (75)$$

$$=\frac{1}{8\,\pi} \qquad (76)$$

Conductors in Air. The calculation of the inductance of conductors in air permits the development of special formulas involving the conception of geometric mean distance. Thus, for a system of positive currents with return currents within a finite distance, the vector potential calculated from Equation (47) will be zero at any point which is an infinite distance from the system in question, if the constant term is taken equal to zero. Thus, if the voltage due to flux between the system of positive currents and infinity is to be calculated, and if the system of currents fulfills the requirements permitting the calculation of inductance, then

$$L_1=-\int\limits_{S_1}\frac{R\,i\,da}{I^2} \qquad (77)$$

where S_1 refers to the area of the sections of conductors carrying positive currents and

$$R=\int\limits_{(S_1+S_2)}i\log r^2\,da$$

Thus

$$L_1=\frac{-1}{I^2}\int\limits_{S_1}\left[\int\limits_{S_1+S_2}i\log r^2\,da\right]i\,da \qquad (78)$$

The voltage due to flux from infinity to the return is

$$L_2=\frac{-1}{I^2}\int\limits_{S_2}\left[\int\limits_{S_1+S_2}i\log r^2\,da\right]i\,da \qquad (79)$$

The total inductance of the circuit is

$$L=L_1+L_2=\frac{-1}{I^2}\int\limits_{S_1+S_2}\int\limits_{S_1+S_2}i\log r^2\,da\,i\,da \qquad (80)$$

which may be put in the symmetrical form:[29]

$$L=+2\frac{\int\limits_{S_1+S_2}\int\limits_{S_1+S_2}i\log r\,da\,i\,da}{[\int\limits_{S_1}i\,da]\,[\int\limits_{S_2}i\,da]} \qquad (81)$$

[29] It is understood that, subject to the conditions imposed $\int\limits_{S_1}i\,da=-\int\limits_{S_2}i\,da$

For uniform current density, remembering that the current density is negative over the section of the return conductors, Equation (81) becomes:

$$L = -2 \frac{\left\{ \int_{S_1} - \int_{S_2} \right\} \left\{ \left[\int_{S_1} - \int_{S_2} \right] \log r \, da \right\} da}{A_1 A_2} \qquad (82)$$

$$= -2 \frac{\left[\int_{S_1}\int_{S_1} \log r \, da \, da - 2 \int_{S_1}\int_{S_2} \log r \, da \, da + \int_{S_2}\int_{S_2} \log r \, da \, da \right]}{A_1 A_2}$$

$$= 2 \log \frac{R_{12}^2}{R_{11} R_{22}} \qquad (83)$$

where

R_{11} = geometrical mean distance of section 1 from itself.

R_{22} = geometrical mean distance of section 2 from itself.

R_{12} = geometrical mean distance of section 1 from section 2. [30]

A_1 = area of section 1.

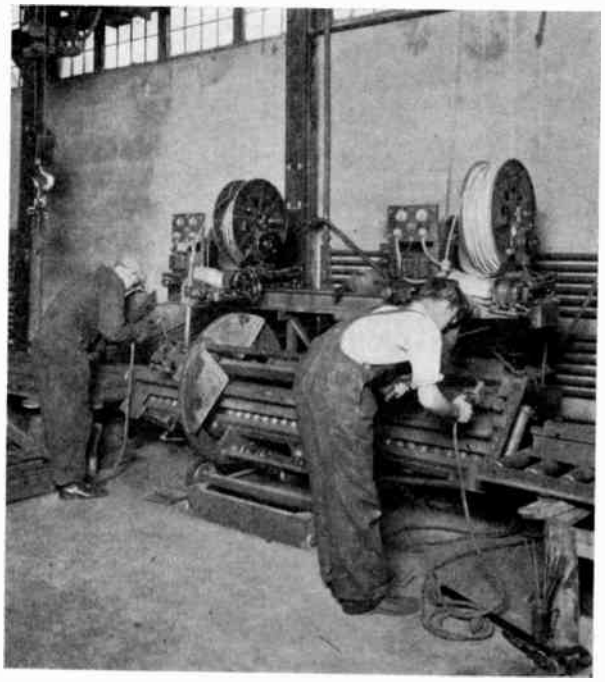
A_2 = area of section 2.

[30] The geometrical mean distance for sections of several types are given in Maxwell, loc. cit. p. 63.

(Concluded)

A New Machine for Producing Welded Railroad Ties

Following the development of a new type of railroad tie constructed from scrap rails, as described in the June, 1927, issue of this magazine, there has come the application of automatic welding machinery to the production of such ties.



The Apparatus by Means of which Ties Are Quickly Produced from Scrap Rails by the Simultaneous Application of Automatic and Hand Welding Operations

The component parts of the tie are assembled and clamped rigidly together on a jig which rotates with the work into positions most favorable to the application of the process. While two automatic welding heads are fastening the tie plates to which the rails are keyed when installed, angle bars which further strengthen the construction and provide for tie spacing are welded to the ends of the tie by the operators, both of whom are provided with manual welding equipment.

The current is supplied by a 1500-amp-hr. motor-generator welding set. Separate control panels provide two circuits for the hand welding electrodes and two circuits for the automatic welding heads.

By means of this equipment it is possible for two operators to produce about twelve ties per hour. The attendants work simultaneously upon the same tie, but each independently of the other, since the machine embodies two separate sets of welding appliances.

IN MEMORIAM

ELMER ELLSWORTH FARMER CREIGHTON

A CAREER of much technical achievement was terminated by the untimely death of Elmer Ellsworth Farmer Creighton on the twelfth of last January, at Schenectady, N. Y. He was in his fifty-fifth year, having been born on April 11, 1873, at Vallejo, California.

For nearly twenty-five years, beginning in 1904, Prof. Creighton was a special research engineer with the General Electric Company, devoting most of his time to the subject of protective equipment for electrical circuits and transmission systems. During the greater part of that period he was a technical associate and a personal friend of the late Dr. Charles Proteus Steinmetz. Together they studied lightning in its relation to electric power transmission; but Prof. Creighton also performed much independent engineering work in this field.

He was educated at Leland Stanford University, graduating in 1895, and receiving the degree of Electrical Engineer in 1897. These early years were divided between service as special testing engineer for the Pacific Postal Telegraph Company, teaching at his alma mater, and assisting Dr. David Starr Jordan on the Fur Seal Commission among the Pribilof Islands, Alaska.

From 1898 to 1900, he was assistant to Andre Blondel, in the Sorbonne, Paris, and at the Ecole Superieure d'Electricite. Upon returning to this country he again taught, for a time, at Leland Stanford.

In 1901, Prof. Creighton became the head of the Experimental Department at the Stanley Electric Manufacturing Company's plant at Pittsfield, Massachusetts, removing to Schenectady in 1904 to become assistant professor to Dr. Charles P. Steinmetz, who was at that time the head of the Electrical Engineering Department at Union College.

In 1913, a research laboratory for investigating protective methods, which Prof. Creighton had assisted in conducting at Union College in behalf also of the General Electric Company, was moved to the General Electric Works, and was made part of Dr. Steinmetz's consulting engineering department. A few years ago this laboratory was combined with the Standardizing Laboratory to form the present General Engineering Laboratory, under which Prof. Creighton had since served.

Prof. Creighton was a contributor to the electrical engineering profession both by inventions and by technical papers. He held sixty-six United States patents, almost all in the field of protective devices. These were principally the aluminum arrester, the compression-chamber arrester, the dry-film arrester, concrete reactors, and several types of direct-current and alternating-current arresters.

He served on many technical committees, particularly those on meetings and papers and on protection for the A.I.E.E., as well as on several American engineering standards committees. Since 1906, he had contributed more than seventy-five technical papers and discussions to the transactions of the A.I.E.E.

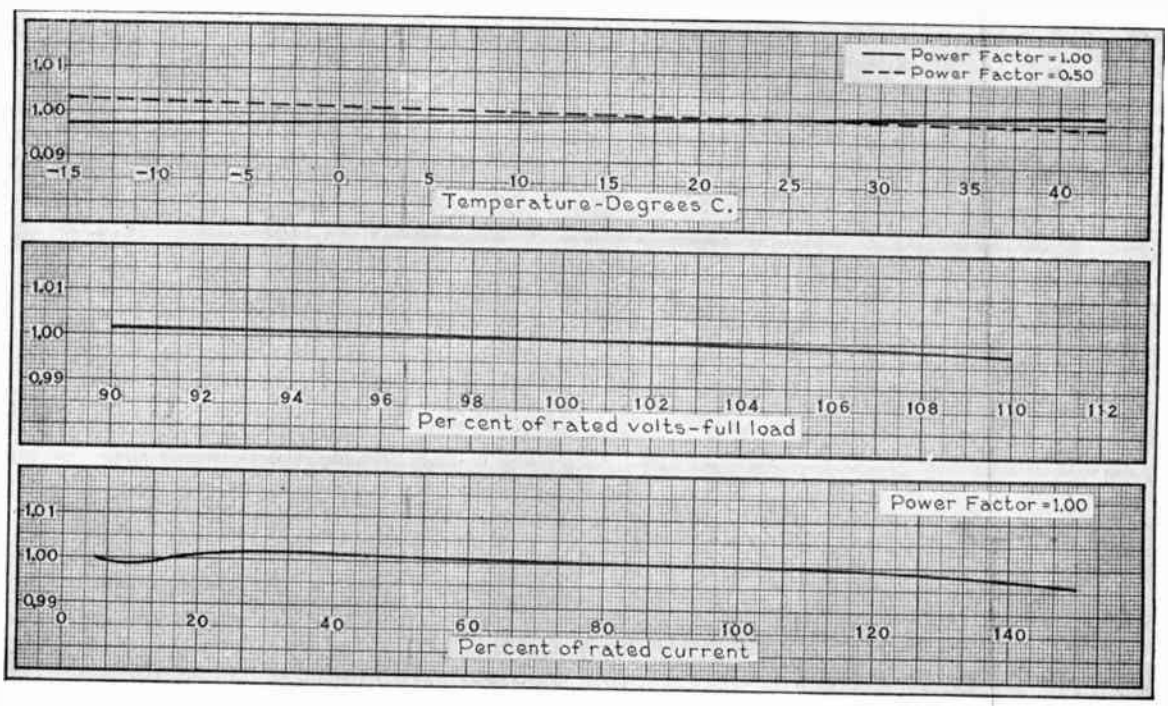
He was a Fellow of the American Institute of Electrical Engineers, a member of the Society for the Promotion of Engineering Education, the American Association for the Advancement of Science, the National Electric Light Association, the American Society of Mechanical Engineers, the American Physical Society, the American Electro-Chemical Society, the American Ceramic Society, and the Society Francaise des Electriciens. He was also a member of the Sigma Xi fraternity.

Superior Rotating

These meters possess exceptional performance characteristics and improved mechanical features.

The effects of variation in voltage, frequency, and power-factor are reduced to negligible values. There is almost entire freedom from temperature errors. As in the case of the other G-E watthour meters, high torque is an outstanding characteristic. These facts unquestionably establish the superiority of these meters for precision work.

These are some of the new mechanical features: A metal case which is attractive in appearance and durable. A moulded bakelite top, with a gasket which renders the meter moisture-proof and dust-proof. An improved zero set-back device, furnished with both meters. The commutating switch in the type IB-8 meter for changing current capacity is also of improved design.



Typical performance curve of both meters

GENERAL

GENERAL ELECTRIC COMPANY, SCHENECTADY, N. Y.

Standards

Type IB-7 Portable Test Meter
110-200 volts 1/5/20 amp.

A small, light-weight standard, considerably smaller than the type IB-8, with ratings limited to three values. It is intended for testing service meters of moderate ratings or those used with instrument transformers.

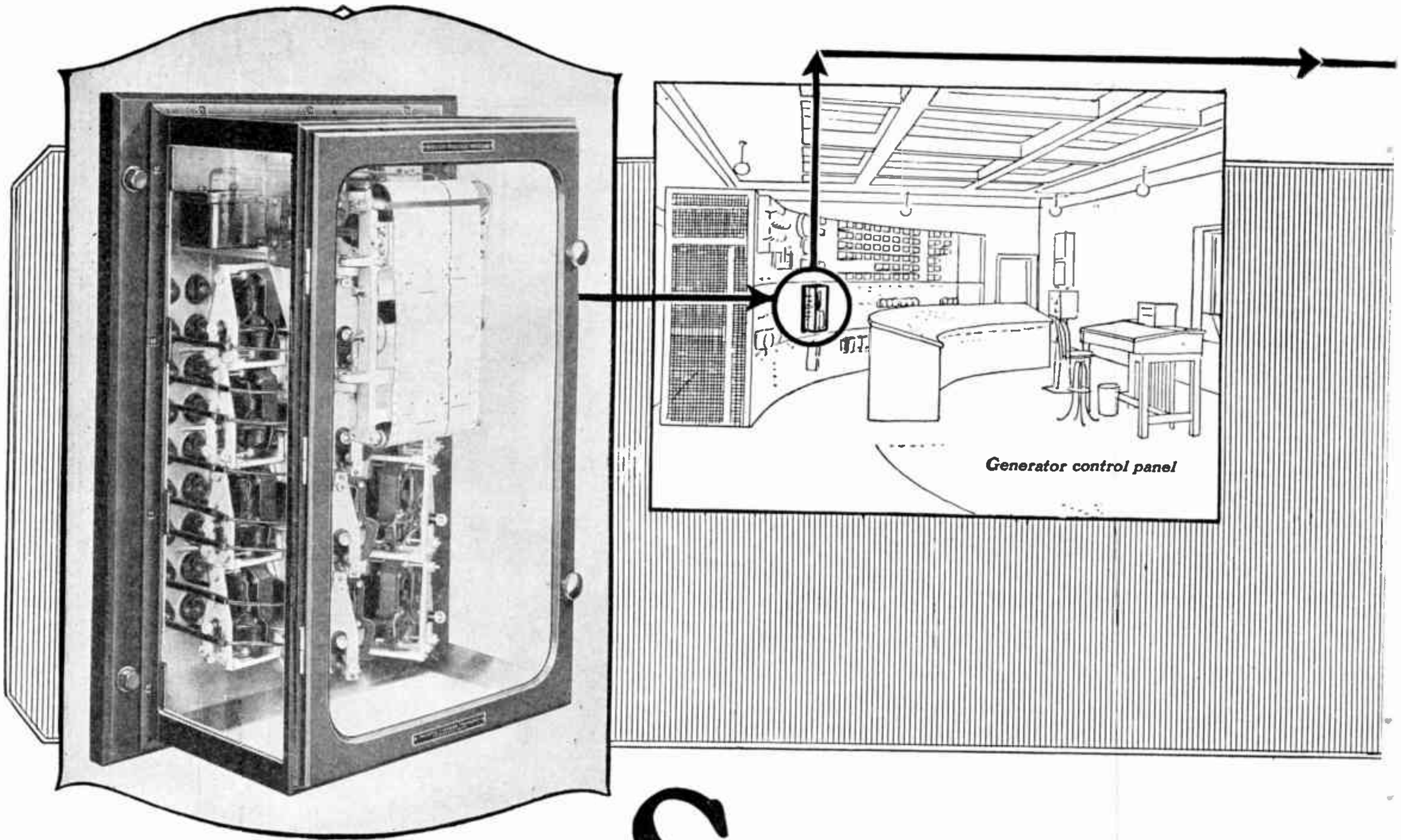
Type IB-8 Portable Test Meter
110-220 volts, 1/5/10/50/100 amp.

This meter embraces a wide range of ratings and is suitable, therefore, for testing service meters of practically any size. The commutating switch facilitates changing the connections to get the desired rating.

600-18

ELECTRIC

SALES OFFICES IN PRINCIPAL CITIES



Station totalized, recorded,

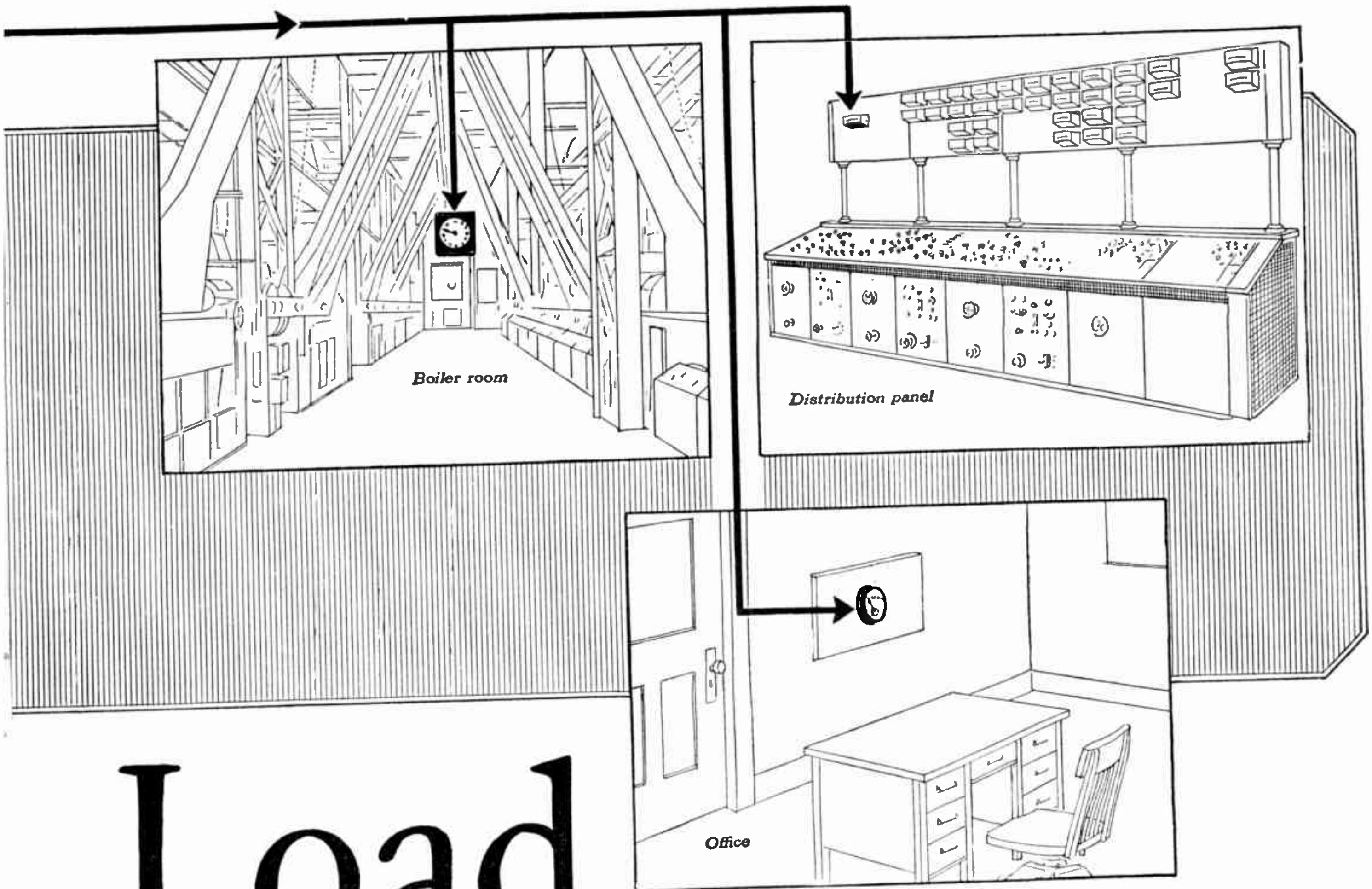
Totalized and recorded, the station load is constantly visible on the chart of this instrument at the generator control panel.

The G-E Totalizing Recording Wattmeter is designed to facilitate the analysis of station economies and plant operation.

Features that contribute to the utility and accuracy of this instrument are the direct connection between moving element and pointer, provision for adjustment of the individual metering elements as required by the transformer ratio, and a generous amount of torque to transmit the indication.

GENERAL

GENERAL ELECTRIC COMPANY, SCHENECTADY, NEW YORK



Load
and transmitted

In coincidence with the indication of the Totalizing Instrument, the station load is accurately indicated at points remote from the generator control panel.

The Remote Indicators employ the G-E Selsyn Transmitting System and are available in many types to conform with the requirements of plant and office.

G-E Load-demand Indicators for the boiler room enable the attendants to read at a glance the station output, the anticipated load, and the time at which it is to occur. These are controlled by the Totalizing Instrument, supplemented by the Manual Transmitter in the operating room or office.

The summation and remote indication of station load can be easily accomplished in any plant. G-E Meter Specialists have a descriptive publication compiled for those interested in the details of this type of installation. Ask for GEA-584.

610-8

ELECTRIC

SALES OFFICES IN PRINCIPAL CITIES

The Improved Conducell!

The high tension underground cable joint insulation...

FOR more than ten years Conducell has been eliminating joint troubles... the outstanding cause of cable breakdowns.

Now, the improved Conducell does away with porcelain separators in the conductor crotch and employs interlocking mica rings. The advantage of this change is evident in that it requires no sharp bend in the conductors, and leaves sufficient free space to permit the compound to flow freely around the whole length of each conductor between belt insulation.

By assuring a predetermined and uniform amount of insulation, by eliminating air pockets and voids, by insuring an equal and permanently fixed mechanical relation between conductors... in short by removing the uncertainty of manhole workmanship... this factory-formed insulator has proved its effectiveness in making the joint as dependable as the cable itself.

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World's Largest Manufacturers of Mica Insulation

New York: 200 Varick Street Chicago: 542 So. Dearborn St.
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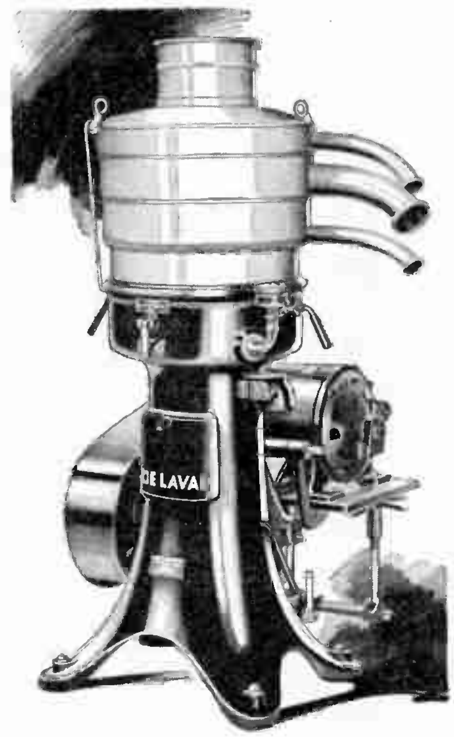
The Improved Conducell is recommended for and will be shipped for use on circuits of over 9000 volts.

CONDUCELL

Leadership

~ among the leaders



SINCE 1878, when De Laval invented the first continuously operated centrifugal separator, no one has thought of such a machine without thinking "De Laval." This year, as we observe our Golden Anniversary, De Laval leadership in centrifugally separating, purifying or clarifying various liquids is just as pronounced as it was 50 years ago.

De Laval's leadership is proved by the number and character of the organizations which are today adding more De Laval Centrifugals to their plant equipment. It is the result of rigid adherence to the De Laval policy of building the finest centrifugal machines that can be designed and selling them at the lowest prices made possible by its modern mass production methods; making no concessions in design, quality or price to offset the claims made for other machines.

THE list below shows a few of the nationally known organizations from which *repeat* orders for De Laval Centrifugals have been received during the past few months:

American Gas & Electric Co.
American Steel & Wire Co.
Brooklyn Edison Co.
Carnegie Steel Co.
Devoe & Raynolds Co.
The Ford Motor Co.
General Electric Co.
The Glen Alden Coal Co.
Humble Pipe Line Co.
Jones & Laughlin Steel Corp.
The Lehigh Portland Cement Co.
Nash Motors Co.
Public Service Production Co.
Reo Motor Car Co.
Southern California Edison Co.
Stone & Webster, Inc.
U. G. I. Contracting Co.
Union Electric Light & Power Co.

Three De Laval Oil Purifiers in the Hudson Avenue Station of Brooklyn Edison Company.

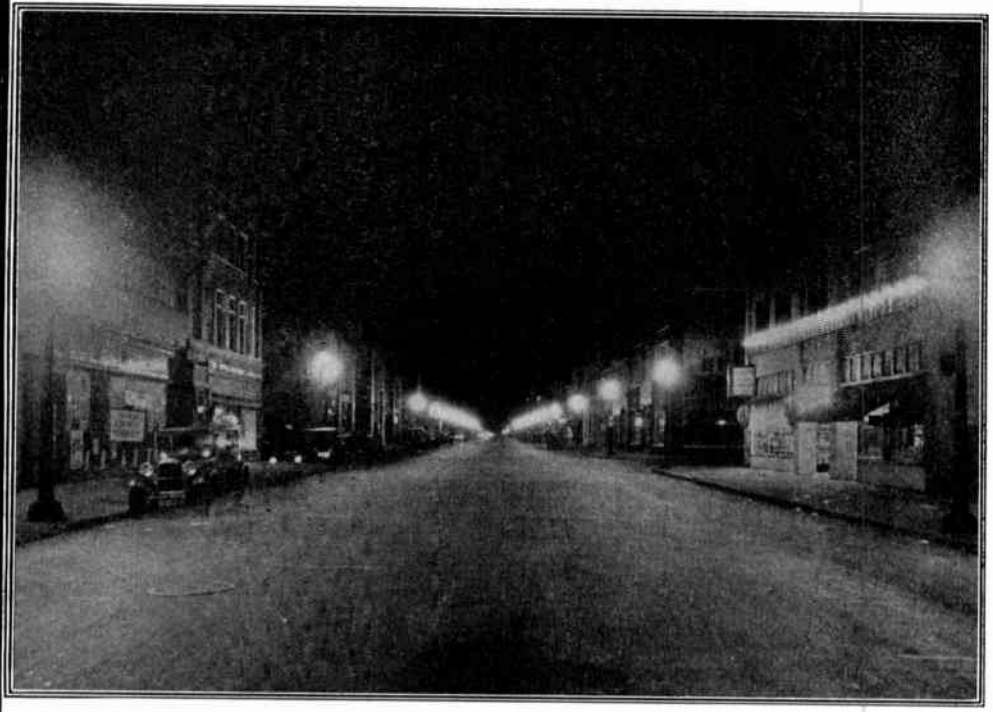
In writing for information as to how De Lavals can best work for you, please ask for Bulletin 106-G on lubricating oil, or Bulletin 107-G on insulating oil.

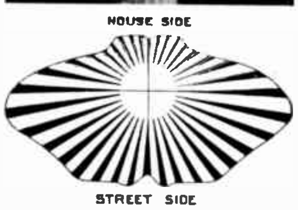
THE DE LAVAL SEPARATOR COMPANY
165 Broadway, New York 600 Jackson Blvd., Chicago
DE LAVAL PACIFIC COMPANY, 61 Beale St., San Francisco
ALFA-LAVAL CO., Ltd., 34 Grosvenor Road, London, S. W. 1.

DE LAVAL
OIL PURIFIERS

Types for Lubricating Oil and Insulating Oil

REFRACTORS





Light distribution of the B-SYM-ETRIC Dome Refractor, looking down from above

Holophane-General Electric A-SYM-METRIC Dome Refractor with G-E Ornamental Rippled Globe on Union Metal Post.

Planned Street Lighting With A-SYM-METRIC Refractors In Canton, Ohio.

Canton, Ohio, better known as the headquarters of the Ohio Power Company and home of the Union Metal Mfg. Company, manufacturers of street lighting standards has lighted its thorofares and residence streets with Planned Street Lighting using Holophane Refractors. The proper amount of illumination, the proper location of units, the correct mounting height, the correct spacing, and the proper size of lamp was all planned and then A-SYM-METRIC REFRACTORS were used to give the right kind of illumination and the right kind of light distribution. A-SYM-METRIC REFRACTORS direct the light toward the street and spread it up and down the street to give efficient illumination, even distribution and better visibility with economical spacing of light units.

HOLOPHANE COMPANY

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NORMA BALL BEARINGS
NORMA
THE increasing use of "Norma" Precision Ball Bearings and "Hoffmann" Precision Roller Bearings, in electric motors and motored appliances, is simply an answer to the demand for longer life, higher efficiencies, and lower upkeep. The slightly higher first cost is more than repaid in the continuing lower after-costs.
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You Can Eliminate Shut-downs
for Coupling Repairs by
Replacing old type Flexible Couplings with Fast's Self-Aligning Couplings

ONCE or twice in every generation a new mechanical principle sweeps in, revolutionizing current practice. That is exactly what has happened to flexible couplings.

Heretofore it was accepted as axiomatic that a flexible coupling had to be built with flexible materials. But eight years ago Fast's design cast aside all flexible materials and eliminated replacements forever.

No More Replacements

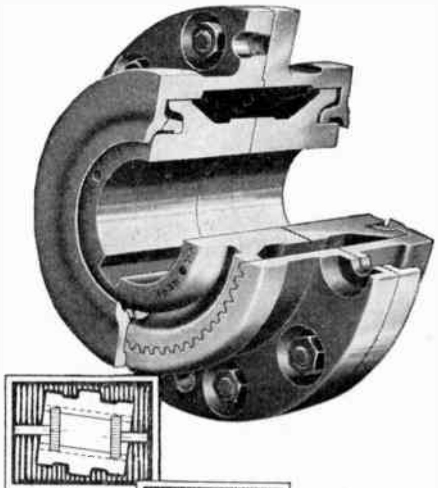
There are today thousands of plants that carry replacement parts to repair broken pins, bushings, springs, discs or grids in old type flexible couplings. But there are thousands of other plants that have written couplings off the replacement list forever, by standardizing on Fast's Couplings without flexible materials.

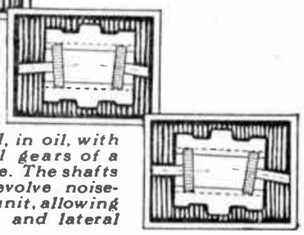
Note the Principle

See diagram above. Read the explanation. See how this simple principle of a floating sleeve compensates for lateral and angular misalignment, by distributing the error between the lubricated tooth faces of generated gears. Simple, isn't it? And because of its very simplicity it will solve your coupling problems.

What is Your Problem?

It makes no difference whether your coupling failures occur on small pumps or fans; or on main units in power plants, or on huge steel-mill reversing drives—in any case, hanging over your head is "shut-down for coupling repairs" unless you do away with flexible materials, and thus do away with shut-downs. Fast's Coupling is the answer.





Two spur gears, one on each shaft end, are completely and continually meshed, in oil, with the internal gears of a floating sleeve. The shafts and sleeve revolve noiselessly as one unit, allowing free angular and lateral movement.

FAST'S
Self-Aligning
COUPLING

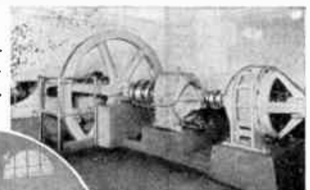
Without flexible bushings, pins, springs, discs, grids or any other flexible materials.

Patented and Manufactured by
The BARTLETT HAYWARD Co.
BALTIMORE, MARYLAND

Send for the Book

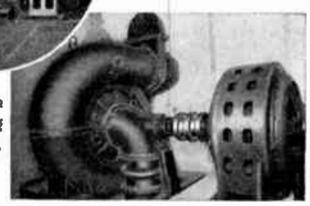
Here is a book that tells the whole story—from both a practical and an engineering standpoint. It clearly illustrates Fast's principle of mechanical flexibility and centrifugal lubrication. It also gives the mathematics of design for the engineer who is interested in how misalignment is "divided by 10" and practically eliminated before it gets to the gears.

Also the book shows typical installations, coupling ratings, and utility factors for all varieties of drives. Write today and see how this coupling can eliminate from your plant, forever, the now costly shut-downs for coupling repairs.



General Motors Building, Detroit.

Frequency Changer Appalachian Power Company.



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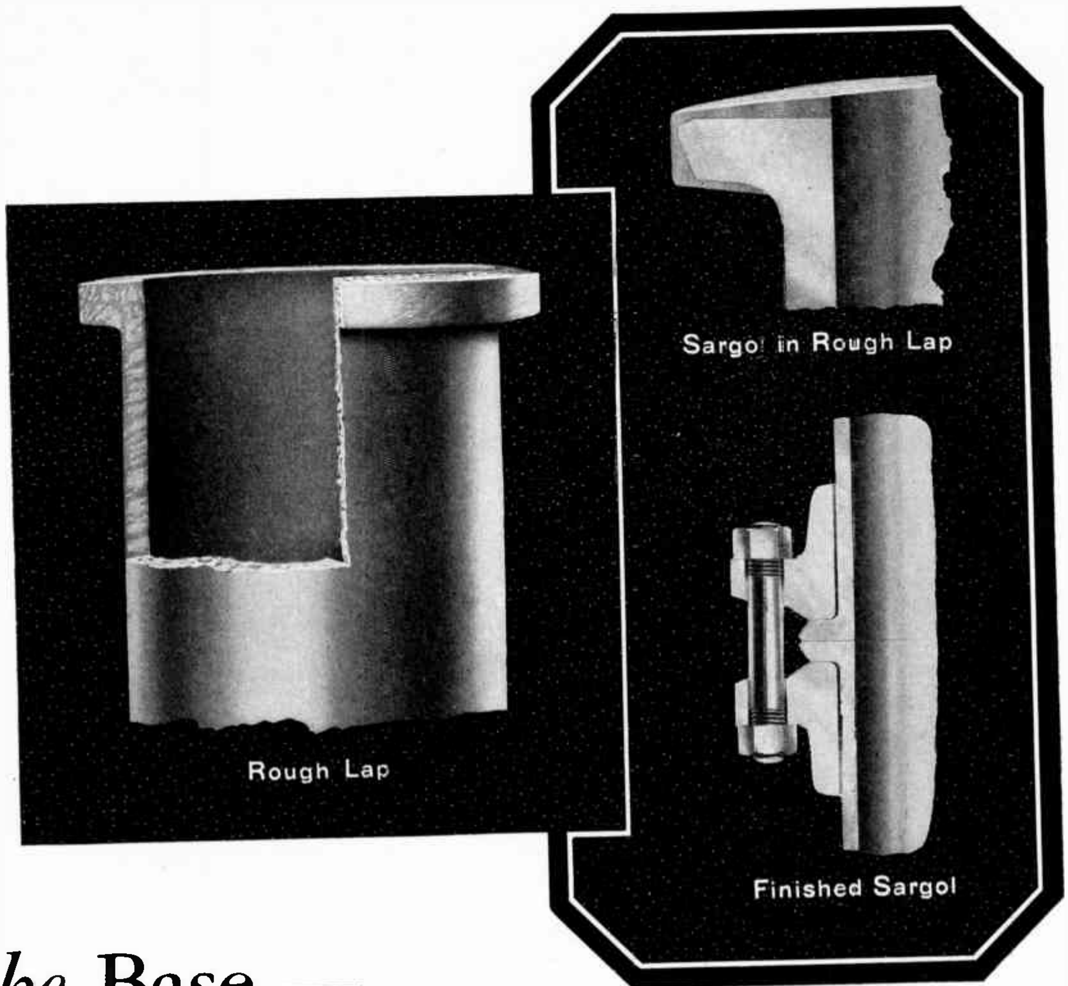

The Base—

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ABOVE is shown the rough lap which is the base-product from which all joints in the Grinnell TRIPLE XXX Line are machined. This lap is forged to twice the thickness of the pipe or seamless tubing by the specially built Grinnell Lap Joint Machine—the greatest in the world—without changing the metal structure.

Likewise shown, is a finished Grinnell TRIPLE XXX Sargol Joint from which it can be seen that the double thickness of the rough lap allows for machining front and back and gives a finished joint of more than 100% strength. This applies not alone to Sargol but to Tongue and Groove, and Square types as well. "Freedom of choice as to type—certainty of strength in the finished product" is the fundamental principle of Grinnell TRIPLE XXX Pipe Joints. For illustrated folder giving complete details address Grinnell Company, Inc., 268 W. Exchange Street, Providence, R. I.

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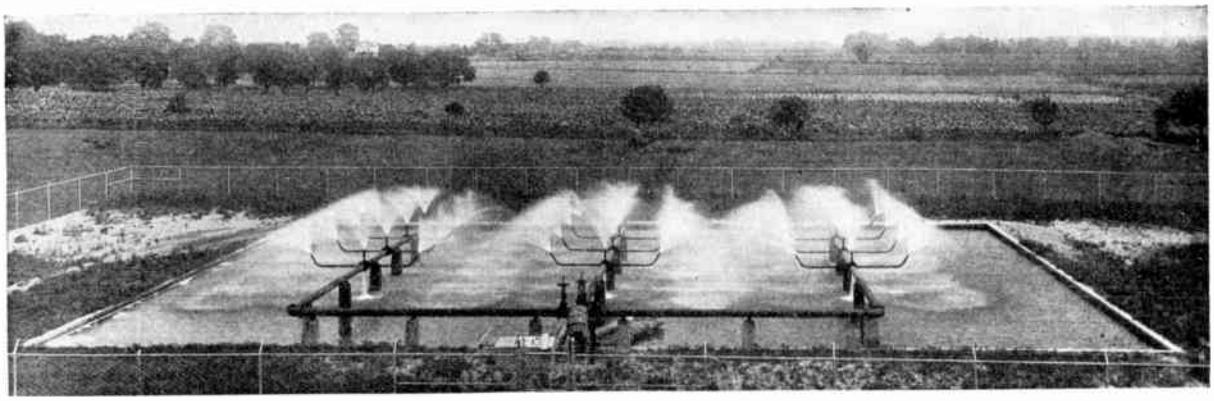

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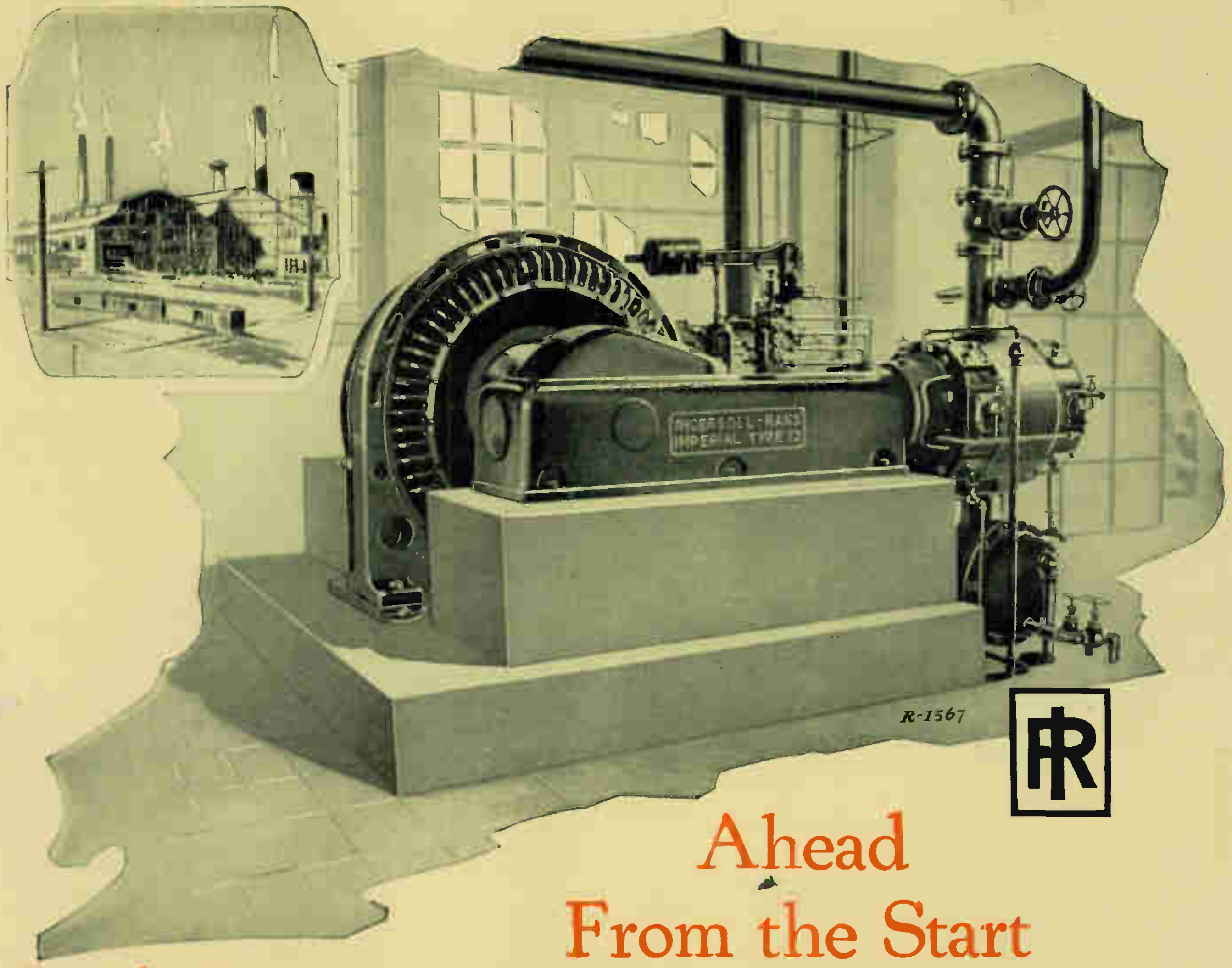

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You can get 'em from the leading motor manufacturers

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NOW more than FORTY electrical manufacturers supply ball bearings in their general purpose motors as standard or optional equipment.

Many dollars and cents saved in maintenance have created the demand to which this progressive step is the answer.

Further reasons why electric motors should be equipped with ball bearings are fully outlined in two booklets published by our Engineering Department. "Booklet K" is of interest to designers. "Cutting Your Costs" is of interest to owners, operators and maintenance men. Either or both sent upon request.

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