The Lenkurt® Demodulator

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# INFORMATION THEORY and CODING

## Part 2

*There is a never-ending need to overcome the various kinds of interference inherent in today's communications systems, so that information can be transmitted faster and with fewer errors. One way of providing improvements is to design special coding schemes which better organize the messages sent over communications systems. This article discusses some of the considerations in coding and decoding messages and the methods of detecting and correcting transmission errors.*

COMMUNICATION requires that information be transported from one place to another and, for this purpose, must be converted into a form suitable for handling. Electrical communication requires additional conversions to prepare the words or other symbols for transmission. Sound waves are converted to a variable voltage; electrical pulses, like drum beats or smoke signals, provide the means for transmitting letters and numerical data over today's modern communications systems.

Regardless of the exact means of transmission, some form of symbolic language or code must always be used to carry information from its source to its destination, and most of these codes and languages are inherently wasteful. In language, some words are used more than others and letters occur in predictable patterns. This predictability and pattern in sounds, letters, and words make it possible to receive the meaning of a spoken or written message, even when some part of it is altered or deleted in transmission. A reader's familiarity with the words and syntax of a language allow him to supply missing or incorrect letters and words in the text. The prolonged sounds of speech, and their inflection and pattern preserve the intelligibility of speech except in the presence of extreme interference.

A simple experiment will confirm how predictable language actually is. A short passage of written prose is selected, and someone is asked to guess the characters (including spaces and punctuation) one at a time. The subject continues guessing until he names each character correctly. As each character is guessed, it is written down as an aid in predicting the next character. The results of such an experiment are shown in Figure 1. The numerals show the number of guesses required for each character. Of the 109 symbols in the text, the subject guessed correctly on his first try 79 times, and was able to identify all 109 characters in 235 attempts. This is an average of only about two guesses (or information "bits") per character. Further experiments have indicated that long passages of English text have an information content of only about one bit per letter. This means that, theoretically, it should be possible to transmit text by pulses no more numerous than the letters themselves, thus enabling 24 of the 26 letters to be discarded without loss of communication. Although this ideal cannot be achieved, it provides a goal to be approached in the design of coding techniques.

Transmission codes can be made more efficient by designing them to fit the statistics of the language. Thus, letters which occur most frequently—E, T, and A, for instance—are represented by the shortest code symbols, while the least probable characters have longer symbols. Figure 2 shows such a code which has an average information content of about 4 bits per character. By contrast, the standard teletypewriter code employs 5 bits per character, not counting synchronizing pulses.

Although the additional redundant symbols and pattern in language may help overcome errors, *unsystematic* redundancy is wasteful, and merely lowers the rate of communication. It follows logically that the more redundancy removed, the more efficient the communications channel, but the greater the likelihood of error due to interference. Since interference is always present to some degree, a very efficient communications system would use a code in

```
11 3  1  1  1  1  1  19 3  2  3  1  1  1  1  1  1  1  1  1  1
I  N  –  T  H  E  –  M  I  D  D  L  E  –  O  F  –  T  H  E  –

10 1  1  3  1  8  5  6  3  1  1  1  6  1  1  1  1  1  1  2
D  A  Y  ,  –  I  –  W  E  N  T  –  D  O  W  N  –  T  O  –

1  1  1  1  2  1  1  1  1  1  1  1  1  5  1  9  1  1  1
T  H  E  –  S  H  O  R  E  –  T  O  –  W  A  T  C  H  –

1  1  1  1  3  2  1  2  1  5  1  13 1  4  1  1  1  1
T  H  E  –  C  R  A  B  S  ,  –  L  I  T  T  L  E  –

2  1  1  1  1  1  1  1  1  1  1  1  2  1  1  2  2  1  1  1  1
R  E  A  L  I  Z  I  N  G  –  T  H  A  T  –  I  –  W  A  S  –

16 2  1  1  6  1  1  1  1  1
N  O  T  –  A  L  O  N  E  .
```

*Figure 1. Predictability of language is indicated by large number of characters correctly guessed on first try. Numerals indicate number of guesses required to identify each character.*

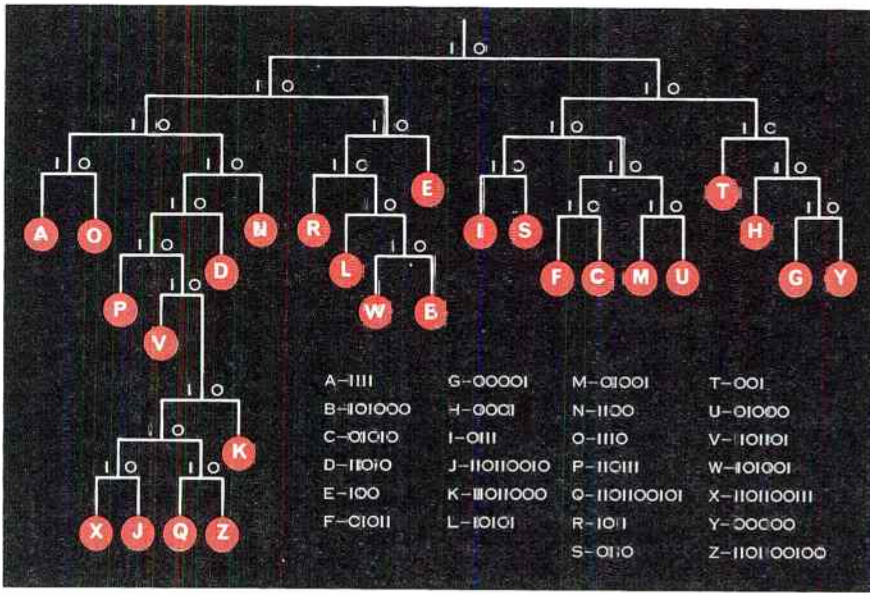


*Figure 2. Efficient binary code for English requires average of only 4 bits per character by taking advantage of language statistics.*

which all message redundancy was eliminated to obtain maximum information rate; then, just enough redundancy would be re-inserted to overcome the interference present in the transmission path.

Unlike the redundancy in spoken and written languages, data-type messages have no inherent redundancy. Machine-generated characters occur without pattern, and errors cannot be detected by inspection, as in the case of text. To complicate matters, data errors cannot be tolerated to the same extent as errors in text, because control operations or machine calculations may be completely ruined by a single error. Yet the high speed with which data is generated and transmitted makes the occurrence of errors more likely.

One way of overcoming errors in handling and transmitting high-speed data is to design codes which, by their very construction and organization, are able to detect or even correct errors automatically. Unfortunately, most such codes cannot be created without adding redundancy. The problem then becomes one of finding a coding method that provides maximum error-free transmission with the least possible redundancy.

## Error Probability

The information capacity of a communications system with a finite bandwidth depends primarily upon the effective signal-to-noise ratio at the detector or receiver. Noise power in the system is generally considered to be completely random and adds to or subtracts from the signal power. The addition of this noise to digital signals makes it difficult for a detector to always make a correct decision, thus causing errors.

In a typical binary system, for example, the digital codes I and O are represented by different amplitudes as shown in Figure 4. The detector must determine whether a signal pulse is a I or O by its amplitude at the time of sampling. (Signals are usually sampled at the center of the pulse.) If the signal amplitude at the time of sampling exceeds a set level, called the *decision threshold* or *slicing level,* a binary I will be indicated. If the signal amplitude is less than the slicing level at the time of sampling, a binary O will be indicated.

The amplitude of the pulse at the sampling time is proportional to the vector sum of the signal power and the noise power. If the signal at the detector is a binary O, then the noise power would have to be of such amplitude and phase that it would *raise* the amplitude of the pulse above the slicing level to produce an error. Conversely, if the signal is a binary I, the noise power would have to be of such amplitude and phase that it would *lower* the amplitude of the pulse below the slicing level to produce an error.

In Figure 4, the slicing level is set at half the peak signal amplitude. This means that whenever the amplitude of a signal pulse at the sampling time is distorted by an amount equal to half the peak amplitude, an error will occur because the detector will indicate the wrong binary symbol. If random noise is considered to be the only cause of signal distortion, then the chance of error is related to the probability of the noise power becoming greater than half the peak amplitude set for the signal pulse. This implies, of course, that the greater the signal-to-noise ratio, the less chance there is for error. Therefore, given the signal-to-noise ratio, the probability of random noise peaks causing errors can be estimated by using the mathematics of statistics and the so-called *normal* or *gausian distribution* values which have been well tabulated. The error rates established by this sta-

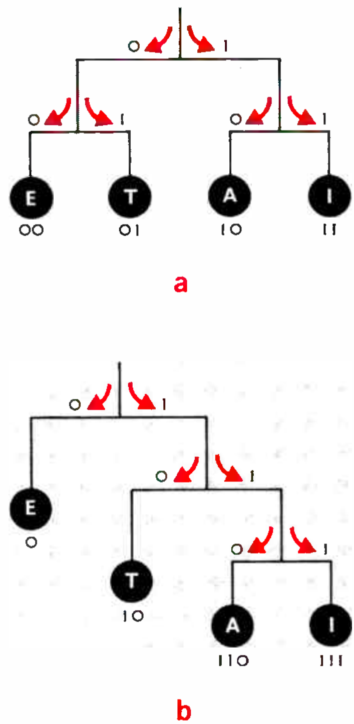


*Figure 3. Two methods of coding information. In (a), characters are equally probable and two bits are required for each. In (b), characters known to appear more frequently are assigned a shorter code thus reducing the total number of bits required.*

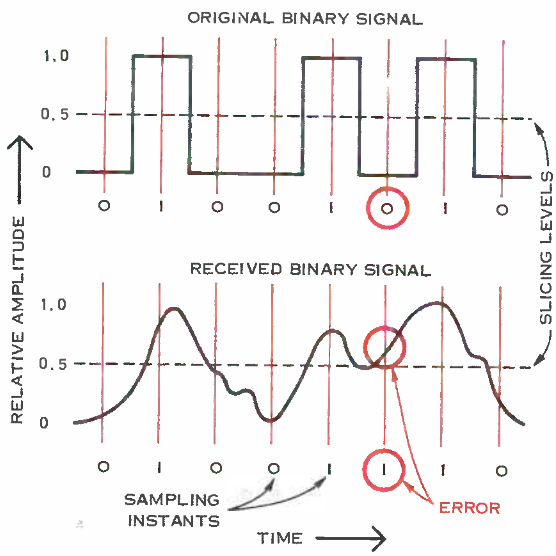


*Figure 4. In a binary system, digital codes 1 and 0 are represented by different amplitude levels. When a pulse is sampled at the detector, its amplitude is proportional to the vector sum of the signal power and the noise power. The addition of this noise may distort the signal to such a degree that the detector will indicate the wrong code at the time of sampling.*

tistical method provide an excellent measure of performance that is especially useful in rating digital communications systems. The error rate performance of today's communications systems ranges from one bit in 100,000 to over one bit in 1,000,000. Figure 5 shows curves of error probability versus signal-to-noise ratio (in decibels) for three types of digital systems.

It is important to note that in comparing multilevel codes, such as ternary and quaternary, with a binary code, the probability of error increases when the peak-to-peak signal range is the same. As shown in Figure 6, two slicing levels are required for a ternary signal and three slicing levels are required for a quaternary signal. Although these additional levels increase the information capacity of the signal, when compared to a binary signal, the margin against noise is reduced by a factor of $1/(n-1)$ where n equals the number of levels. Thus, for a quaternary signal, where n equals 4, the margin against noise is reduced by one-third.

## Error Control

Error control has become an essential part of pulse or data transmission systems since it is not practical to make circuits perfectly error free. The method adopted depends on whether or not the circuit provides one-way or two-way transmission and its error performance—that is, the type and distribution of errors. The use of error detecting and error correcting techniques can increase the overall accuracy of the transmission within the capacity of the channel to any accuracy required but at the expense of equipment complexity.

Shannon's formula described in Part 1, arrives at the remarkable conclusion that even a noisy channel has a definite errorless capacity. No matter how low the error rate must be, it can be achieved while still transmitting a signal over the channel at the desired rate provided that the rate of information does not exceed the channel capacity. However, as the error requirements become more stringent, it becomes more difficult to transmit the signal but only because the

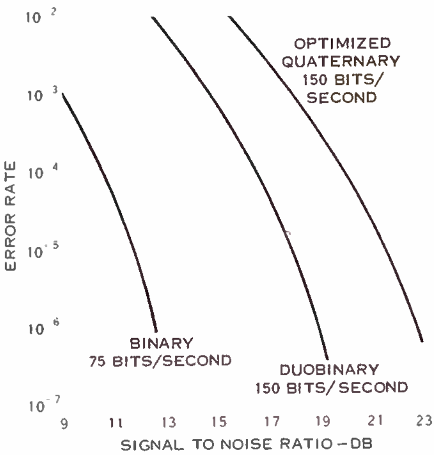


*Figure 5. Error rates versus signal-to-noise ratio for three different types of digital systems.*

encoding becomes very complex and imposes a long transmission delay in coding and decoding. Practical considerations necessitate using a simple encoding alphabet, thus wasting a good fraction of a channel's capacity. Shannon showed that the information capacity of a channel expressed in bits per second is:

$$C = W \log_2 (1 + \frac{S}{N})$$

where

$W$ = *Bandwidth in cycles per second*

$S$ = *Average signal power*

$N$ = *rms noise power in a one cycle band*

Take for example a typical telephone-type voice channel:

$W$ = *3000 cps*

$\frac{S}{N}$ = *20 db (signal S at —15 dbm0 and noise N at —35 dbm0)*

$C$ = *20,000 bits per second*

In existing systems only a small fraction of this maximum possible channel capacity is obtained. Indeed the equation gives no indication of how such ideal encoding of the message may be realized. In any practical system yet proposed there will be a finite probability of error for a finite transmission rate. The sophistication needed to reach the channel capacity of Shannon's formula would result in an extremely complex system.

Error control systems today are either error detecting or error correcting. One of the simplest methods of reducing errors is to repeat the message several times. A more elaborate approach is to use some form of coding which enables a block of characters to be tested for errors. If no errors are found, a feedback signal is sent which acknowledges receipt of the block and asks for the next block to be transmitted. If no acknowledgment is received, the original block is retransmitted. These systems achieve more accuracy at the price of a slower rate. The transmission efficiency of this type of system can be expressed as the number of information bits per block divided by the sum of the information bits plus the redundancy bits, plus the number of bits that could have been sent in the waiting time for answer-back signals together with the average additional time for repeated transmission.

Thus:

$$E = \frac{Bi}{Bi + Bh + Bw}$$

where:

$E$ = *Transmission efficiency*

$Bi$ = *Information bits per block*

$Bh$ = *Redundant bits per block*

$Bw$ = *Waiting bits per block*

Figure 7 shows how waiting time effects the transmission efficiency of the system. It also indicates that adding redundant bits seriously limits the overall rate of transmission.

It is interesting to note that the coding system based on the Lenkurt developed duobinary technique (described in the February 1963 DEMODULATOR) gives a degree of error detection without adding redundant digits. This capability is achieved by increasing the amount of information per digit. The duobinary code is a more powerful error detection system than the simple *parity check* and has the additional advantage that the data does not have to be processed before the error becomes apparent.

## Parity Checks

A widely used error detection scheme is the so-called parity check. An extra digit is added to the regular binary code group so that there will always be an even (or odd) number of 1's in each group. A single error will cause an odd number of 1's to appear at the receiver, indicating an error. A single parity check will detect all odd numbers of errors, but will not detect double errors or other even-count errors, since the count of 1's will still provide the required even number. By adding an additional parity check for every other digit, all odd numbers of errors and about half the even number of errors can be detected. A third parity check added for the remaining digits will further reduce the undetectable errors.

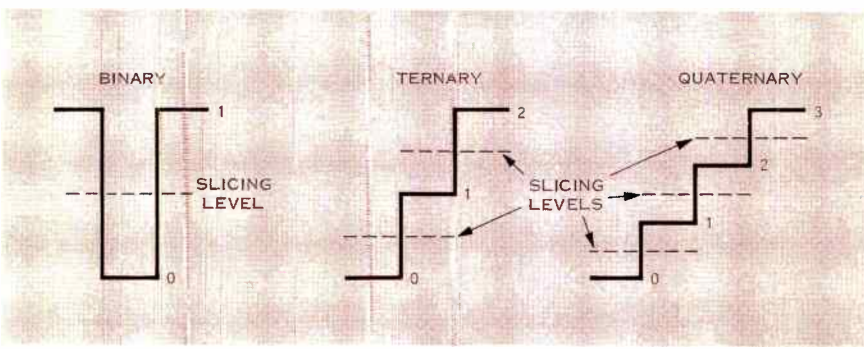


*Figure 6. Multilevel codes such as ternary and quaternary increase the information capacity of the signal. However, when the peak-to-peak amplitude is the same as for a binary signal the margin against noise is reduced.*

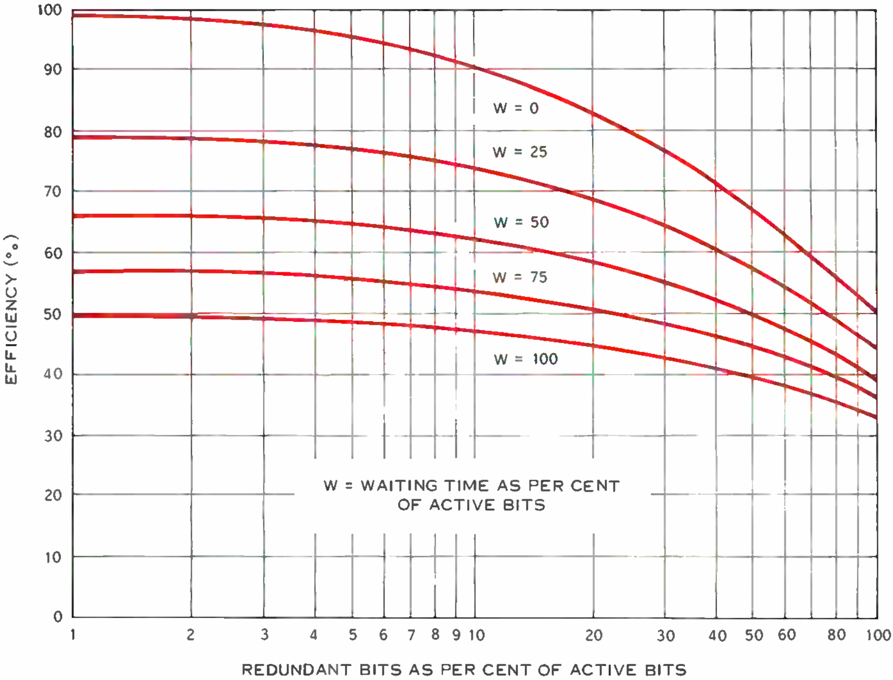


*Figure 7. Effect of waiting time on transmission efficiency for one type of error-control system where a* block *of characters are tested for errors. Curves show that adding redundant bits limits the overall rate of transmission.*

Parity checks provide some protection against errors, but like all redundancy, they slow down the transmission of the message. If a single parity check is used with each five-digit code group, as shown in Figure 8, the message will contain about 16% redundancy. This can be reduced by increasing the number of information digits for each check digit, but this increases the probability of undetectable errors occurring.

Many types of parity check systems exist. Where parallel transmission is used (tape-to-tape computer data, for instance), parity checks may be used in both the horizontal and vertical directions, in order to reduce the chance of data errors going undetected.

A related approach to error detection uses a fixed ratio of marks and spaces for all code characters. When designed to reduce the likelihood of compensating errors, this code can be very effective in detecting most errors.

Essentially, error detection coding and retransmission make an excellent system for reliable communication if the transmission channel introduces only a few scattered errors, and if a high quality return channel is available. The system deteriorates rapidly as the error rate increases.

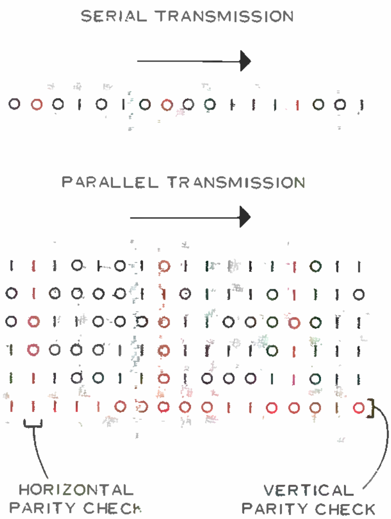


*Figure 8. Typical parity checks for serial and parallel transmission. Check digit is chosen so that sum of marks in each block is always even (or odd) — wrong count reveals error.*

The fraction of errors actually detected in transmission will depend on the characteristics of channel noise. Adding more parity bits will increase the protection afforded by the code. If a block of bits of arbitrary length includes parity bits, the number of undetected errors will amount to the fraction 1/2p of all possible errors if the error detecting code is efficient.

Thus:

$$d = (1 - \frac{1}{2p}) \times 100$$

where

$p$ = *Number of parity bits*

$d$ = *Percent of errors detected.*

Figure 9 illustrates the dependency of detection efficiency on the number of parity bits.

When properly arranged, parity checking by block is a very powerful

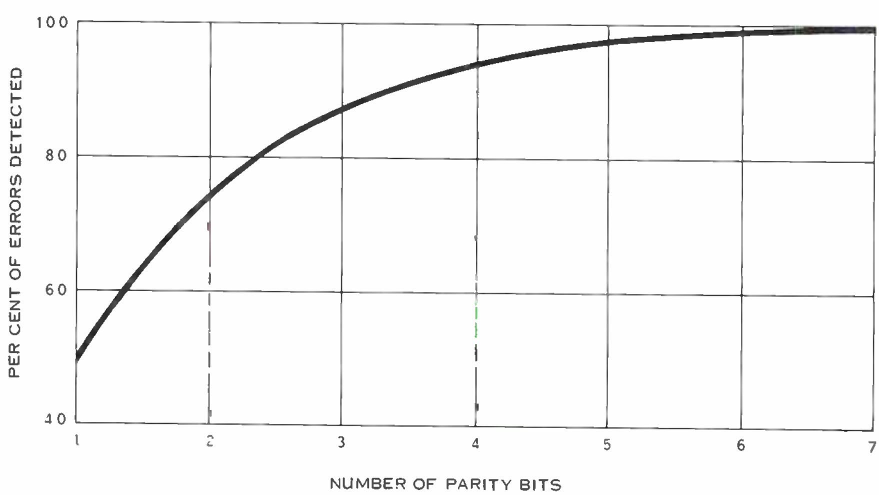


*Figure 9. Efficiency of parity check error correction system depends on the number of parity bits.*

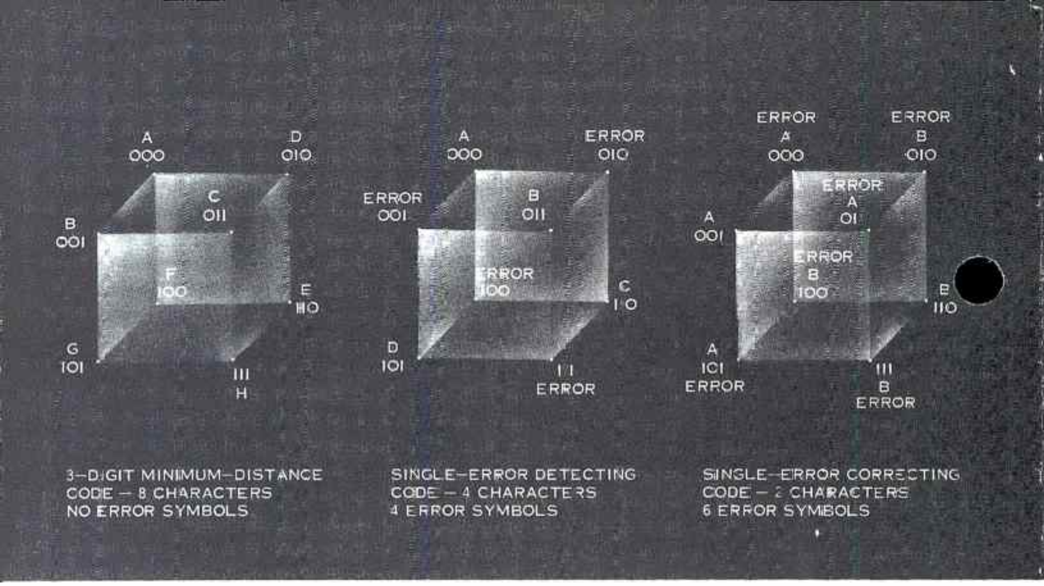


*Figure 10. The efficiency and certain other characteristics of complicated codes are analyzed easily by mathematicians through the use of solid geometry. Simple 3-digit code shown in the example requires three-dimensional figure. Each vertex represents a code combination assignable to characters or errors.*

device. Consider, for example, a block of 80 6-bit characters used to transmit the information contained on a standard 80-column punched card. A single parity check on each character is only 50 per cent efficient and requires 80 parity bits to a card. By contrast in a block detection system, 9 parity bits could be 99.8 per cent efficient and detect any burst of errors up to 8 bits long. The redundancy will be less than 2 per cent.

## Error Correction

It is not enough merely to identify the existence of errors. Some means of correcting the message is required in order to complete the transmission or control function. One basic form of error correction is to transmit the message several times in the hope that errors will not destroy identical portions of each mesage. A similar approach would be to transmit each digit several times and count the bits received. A majority count would presumably reveal the correct digit. Obviously, this method fails if more than half the digits are in errors.

Error-correcting codes which do not require retransmission have been devised, using principles similar to those used in the code of Figure 1. Error correction is obtained by adding additional redundant digits so that an erroneous code group still most nearly resembles the intended group, despite changes occurring in one (or more) binary digits. Obviously, the redundancy is greatly increased.

Mathematicians specializing in information theory and advanced coding techniques find it useful to describe codes in terms of geometry, so that each character in the code is located at a "corner" or vertex of a geometrical figure. Thus, a code having 2 digits could be described by a square with all four combinations located at the four corners. A code with three digits would require a three-dimensional figure for the eight possible combinations, and a four-digit code requires a solid having four dimensions to adequately describe its properties. Although it is difficult or impossible to diagram multi-dimensional figures accurately on paper, they are relatively easy to handle mathematically.

Since each code combination, whether an error or a correct symbol, lies at a vertex of the solid figure, a change in one digit represents the difference between one vertex and an adjacent one. Two changes move it two places, and so forth. The ideal code, then, will use the least possible number of code combinations, but separates all *valid* (non-error) code groups by as many locations as possible. The less the "distance" between correct symbols, the lower the redundancy. If additional "distance" is placed between valid characters, the code can either detect multiple errors or correct single errors, depending on how the code is set up. Figure 10 diagrams how a geometric figure can be used to express "distance" between symbols, and shows how efficiency or information capacity can be traded for error correction or detection capability.

## Conclusion

Information theory has provided the designers of communications systems with new insight into the intangible commodity with which they work. By providing engineers with a specific measuring stick, information theory enables them to measure the efficiency of their communications apparatus and make improvements. The theory is important not only to conventional communications media, but it has important implications in computers, control systems, and data systems where machines communicate directly with machines.

Information theory studies have revealed two basic approaches to improving communications. One is based on improved coding of the signal to be transmitted, the other stems from new knowledge of the relationship between signal power, noise, and bandwidth. There are indications that both approaches lead to a common goal; the most efficient coding method will possibly be the most efficient way of compressing bandwidth and overcoming noise in a transmission channel.

### Bibliography

1. Shannon, C. E., and W. Weaver, *The Mathematical Theory of Communications*, University of Illinois Press, Urbana, Illinois, 1949.
2. "Information Theory and Modern Communications," *The Lenkurt Demodulator*, June, 1959.
3. "Methods for Transmitting Data Faster," *The Lenkurt Demodulator*, March, 1960.
4. Reza, Fazlollak M., *An Introduction to Information Theory*, McGraw-Hill Book Co., Inc., New York, 1961.
5. "Special Code Techniques for Improved Data Transmission," *The Lenkurt Demodulator*, June, 1961.
6. Abramson, Norman, *Information Theory and Coding*, McGraw-Hill Book Co., Inc., New York, 1963.
7. Bennett, W. R., and J. R. Davey, *Data Transmission*, McGraw-Hill Book Co., Inc., New York, 1965.

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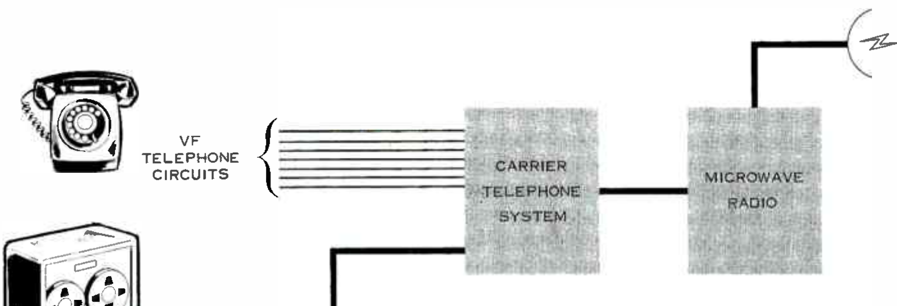


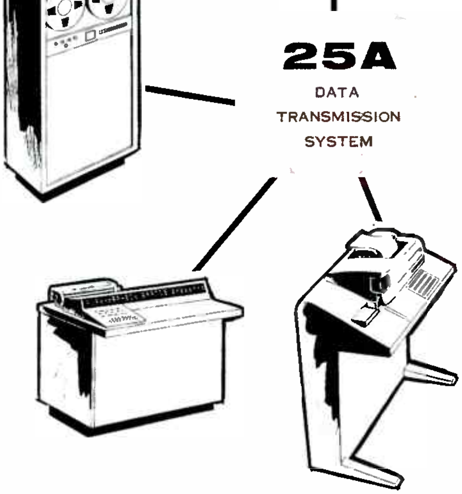


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