

# PHILCO

## TECHREP DIVISION BULLETIN



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part III*

JANUARY  
1953



# PHILCO

## TECHREP DIVISION

### BULLETIN

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If any information contained herein conflicts with a technical order, manual, or other official publication of the U.S. Armed Forces, the information in the official publication should be used.

## *Editorial*

### **Navy Emphasizes Need for Accelerated OJT Program**

*by John E. Remich, Manager, Technical Dept.*

During the BuShips Electronics Conference held in Washington last May, the subject of on-the-job training in electronics maintenance received considerable attention. Summaries of certain of the discussions have now been cleared by the Navy, and were published in a recent RTMA Industry Report.

Of particular interest were several comments by Lt. Commdr. L. A. Adair, who discussed the equipment-maintenance situation on destroyers of the Atlantic Fleet. He stated that the new equipment planned for fleet use will be more complex in design as well as more versatile in application, and that this calls for a program of intense practical training of both operators and technicians. He further stated that as a result of the electronic equipments which were contemplated, it is expected that the need for shipboard technicians will increase by 60 percent in the next few years, and that the need for shipyard support will increase by 100 percent during the same period. As the most logical approach to the problem of building and maintaining an adequate technical force, he advocated an accelerated on-the-job training program.

Captain J. B. Berkley of the Bureau of Ships, who discussed the problems of shipborne electronics installations, also emphasized the importance of properly trained maintenance personnel. He pointed out that skilled personnel are necessary to maintain present-day electronic equipments, but that, as a group, those men now holding electronics-technician ratings are relatively inexperienced. In fact, he gave an estimate that about 70 percent of equipment "off-the-air" time is directly attributable to personnel factors. However, he too emphasized that training can do much to alleviate this problem.

As a result of its close association with the electronics-maintenance activities of all branches of the Armed Forces, the Philco TechRep Division has been continuously aware of these problems, and recognizes them as not confined to any one branch of the Services, or even the Services alone.

On-the-job training has proven the best approach to meeting the need for skilled electronics personnel, but maximum results from the on-the-job training potential cannot be achieved without careful organization and full standardization of the program.

# COMPUTER ATTACHMENT FOR CP-7/APQ-13

by H. G. Burkey  
Philco Field Engineer

**A device used to facilitate setting the sighting angles on Computer CP-7/APQ-13 so that greater accuracy can be achieved in aerial bombing.**

*(Editor's Note: This article was reviewed and approved for BULLETIN publication by the Director of Armament, Office, Deputy for Development, Air Research and Development Command.*

*The article was originally submitted, more than a year ago, to TechRep Division Headquarters as an invention disclosure by Mr. Burkey, and has been modified only to the extent necessary to put it into article form.)*

**P**RIOR TO THE DEVELOPMENT of the attachment described in this article, the radar observer was compelled to divide his time during the bombing run between watching the target on the indicator scope and coordinating the scope data with the sighting-angle computations needed by the bombardier. This compelled the observer to make continual reference to the range-angle conversion chart, during which time his eyes were removed from the scope. The time of a bomb run, at 30,000 feet and 250 knots, from the 70-degree to the 46-degree sighting angle is about 1 minute, 50 seconds. An average of 50 seconds was used to set the computer. During this time, the bombardier had no time to detect drift; therefore, the target was often lost. The modification makes it possible for the radar observer to view the scope 100% of the time, so that loss of the target for this reason is eliminated.

Pre-bomb-run sighting-angle data is correlated with the bombardier's data. This data is electrically stored for use by the radar observer during the bombing run. When the initial point is reached, the observer can devote all of his time to watching the target, and simply by turning a knob he can transmit to his scope the predetermined slant-range circles corresponding to the eight different

sighting angles to be utilized during the run. Viewing the target in relation to these range circles will enable him to make all necessary computations and corrections for transmittal to the bombardier up to the bomb-release point.

During the original exploration of the problem of facilitating the setting of sighting angles, certain fundamentals were established:

1. Modifications of existing radar equipment should be held to a minimum.
2. The attachment must be complete, accurate, and simple to operate.
3. The cost of the unit must be held to a minimum.
4. The design must permit the use of readily available parts.
5. In no case should reliability be sacrificed (such as changing characteristics of a tube).
6. The cathode circuit of the range-delay diode, V1602, is the logical control point.
7. The value of cathode voltage and resistance to ground is of extreme importance.
8. A meter bridge must cause no loading or paralleling effects on the range-delay network, particularly at the point of circuit balance.
9. Any substitute circuit must be equivalent to the range network, and



required to calibrate the attachment after the radar is calibrated. Figure 3 shows the operating controls.

### FIELD TESTS

Tests with an experimental unit have proved the modification to be satisfactory. Approximately 65 hours of operation were accumulated on supersonic trainers, during which time, numerous department and section heads, maintenance and operations personnel, various Wing and Air Division commanders and other personnel, and an estimated 95% of all locally assigned radar observers operated or viewed a demonstration of the computer attachment. The reaction to the modification was so gratifying that a prototype experimental unit was constructed for installation in an aircraft for flight test purposes. As of May 28, 1951, the computer attachment had been operated on three RBS missions. As a result, construction of one unit per B-29 aircraft and one unit per supersonic trainer was directed. Additional interest was evidenced by requests from several other organizations for available data on the modification, for possible application to their own equipment.

Trainer-plane bomb runs with the computer attachment have been made at ground speeds of nearly 600 m.p.h. at 30,000 feet with no difficulty; all sighting angles were utilized during the runs. The same results were obtained at faster-than-normal ground speeds at lower altitudes (5000-1500 feet).

### CONCLUSION

Application of the proposed computer attachment permits the radar observer to:

1. Calibrate his radar accurately, and use the accuracy on the bomb run.



Figure 3. Operating Controls of Computer Attachment

2. View the scope 100% of the time which further enables the observer to:
  - a. Adapt his eyes for "night vision" by continuous viewing of the scope.
  - b. Detect target shift, drift, and breakup at all times by continuous scope-viewing, thereby reducing the possibility of target loss and bombing error.
  - c. Operate with lower gain and intensity settings, because the observer's eyes can adapt themselves to the viewing.
3. Immediately select the preset slant-range release point in the event of bomb-sight or auto-pilot failure prior to or during the bomb run (ref. SAC emergency SOP for AN/APQ-13).
4. Calibrate the preset sighting angles in the event of failure of the meter circuit, merely by superimposing the adjustable slant range to be coincident with the selected slant range on the computer by visual reference to the scope.
5. Bomb accurately at all speeds greater than normal, at all altitudes, and by use of numerous sighting angles.
6. Operate the radar in a completely normal manner by selecting the NORMAL position of the computer attachment FUNCTION switch.

# ELECTRONIC COMPUTERS—PART III

## Basic Computer Circuits and Principles of Binary Addition

by Warren Kitter  
Technical Publications Department

The third of a series of articles discussing electronic computers. This article deals with the basic concepts of logical AND, OR, and NOT circuits, and the principles of binary addition.

A DISTINCT ADVANTAGE of modern electronic computers is the ability of the electronic circuits to instantly switch the results of computation from one section of the computer to another, without any intermediate process of transcribing the data. To provide the switching functions, three different types of switching circuits are used: logical AND, OR, and NOT circuits.

The logical AND circuit, which is shown in block-diagram form in figure 1, is designed to perform the computational function implied by the word *and* (logical concept) as used in number relationships; for example, X *and* Y. As the name of the circuit indicates, an output pulse occurs at C only when input pulses appear at both A *and* B simultaneously.

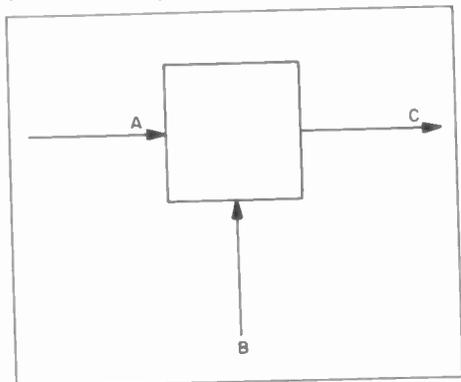


Figure 1. Block Symbol for Logical AND Circuit

Figure 2 shows the schematic diagram of a typical logical AND circuit, which employs a tube of the dual-control-grid type. The circuit constants, as well as the positive voltage applied

to the cathode, are of the proper values to bias the tube well below cutoff. The bias is great enough that a positive pulse arriving only on line A or on line B does not have sufficient amplitude to

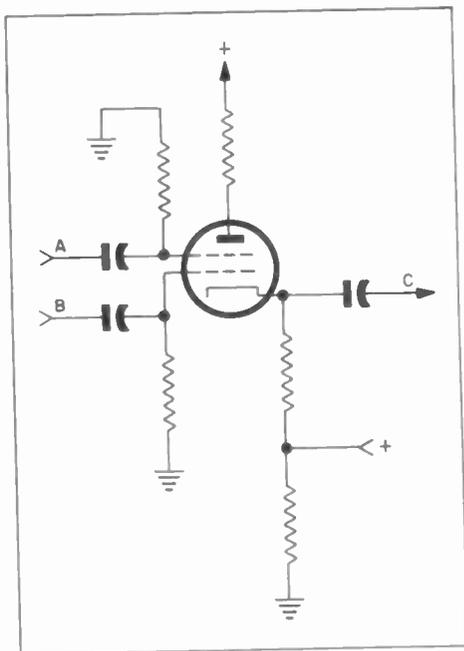


Figure 2. Schematic Diagram of Logical AND Circuit

cause the tube to conduct. Therefore, for this condition there is no output pulse on line C. However, if pulses arrive simultaneously on both lines A and B, the tube is driven into conduction and produces a positive pulse as an output on line C.

If the presence of a pulse is denoted by binary number 1 and the absence of a pulse by 0, the status of a logical

AND circuit for each set of input conditions may be summarized as in Table I.

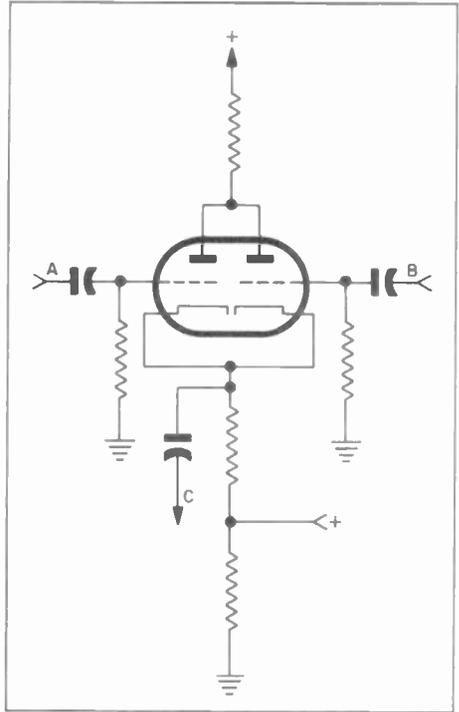
**Table I. Status of Logical AND Circuit for All Possible Input Conditions**

A	B	C
0	0	0
1	0	0
0	1	0
1	1	1

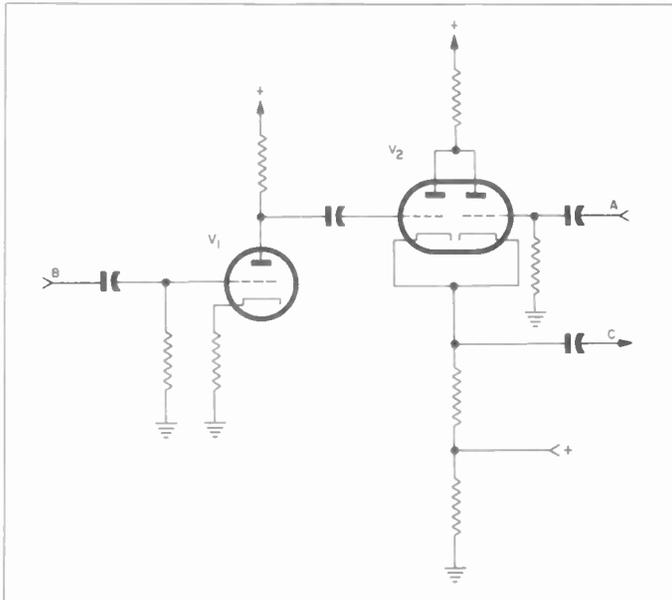
The logical OR circuit is designed to perform the computational function implied by the word *or* (logical concept); for example, X *or* Y. In this circuit, an output is produced if either line A *or* line B receives a pulse.

A simple form of a logical OR circuit is shown in figure 3. In this circuit, the positive voltage applied to the cathode is just sufficient to bias both sections of the tube at a point slightly below cutoff. A positive pulse applied to either line A *or* line B will cause that section of the tube to conduct and produce a pulse on line C. The status of a logical OR

circuit for each set of input conditions may be summarized as in Table II.



**Figure 3. Schematic Diagram of Logical OR Circuit**



**Figure 4. Schematic Diagram of Logical NOT Circuit**

**Table II. Status of Logical OR Circuit for All Possible Input Conditions**

A	B	C
0	0	0
1	0	1
0	1	1
1	1	1

The logical NOT circuit is designed to perform the computational function implied by the word *not* (logical concept). In this circuit, a pulse appearing on line A does *not* produce a pulse on line C if a pulse appears simultaneously on line B.

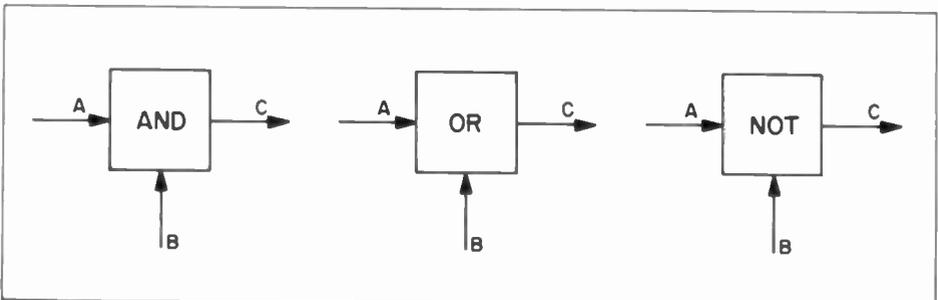
A simplified logical NOT circuit is shown in figure 4. This circuit is the same as the logical OR circuit shown in figure 3, with the exception that a phase inverter,  $V_1$ , precedes it. The positive voltage applied to the cathode of  $V_2$  is just sufficient to keep both sections of the tube cut off. When a positive pulse is applied to line B, it is inverted by  $V_1$  (which should have an amplification factor close to unity), and is applied to the grid of the first half of  $V_2$  as a negative-going pulse. The circuit design is such that the negative pulse is equal in amplitude to the positive pulse applied to the other half of  $V_2$  from line A. With these conditions, the net change in the plate current of  $V_2$  is zero, and consequently no output pulse is produced on line C. The status of a logical NOT circuit for each set of input conditions may be summarized as in Table III.

**Table III. Status of Logical NOT Circuit for All Possible Input Conditions**

A	B	C
0	0	0
1	0	1
0	1	0
1	1	0

In actual computer practice, logical AND, OR, and NOT circuits may be more complex than the ones just discussed. However, the principle of operation is the same. Throughout the remainder of this series of articles, these circuits will be represented by symbols like those illustrated in figure 5.

In the computing process, as a pulse is passed through several computer stages, loss of amplitude and distortion of the pulse shape may occur. Amplitude distortion is not very serious, and can be overcome by simply inserting amplifiers at intervals along the pulse lines. Phase distortion presents a much greater problem, however, in that cumulative effects can destroy the proper timing and sequencing of computer operations. To overcome the latter type of distortion, a switching circuit similar to the logical AND circuit in figure 2 may be used to reshape the pulse and thus restore it to its original form. Pulse-reshaping is accomplished by applying a standard master pulse to line A of the circuit, and the distorted pulse to line B. If the distortion is within tolerable limits, an output pulse which is a



**Figure 5. Block Symbols for Logical AND, Logical OR, and Logical NOT Circuits**

duplicate of the master pulse will appear on line C.

Another circuit that must be considered before the block diagram of an adding system can be discussed is the delay line.

By definition, a delay line is simply an artificial transmission line, in which lumped values of inductance and capacitance replace the distributed inductance-capacitance values present in a normal transmission line.

If AB in figure 6 represents a conventional transmission line, it can be seen that some finite time is required for a pulse applied to point A to travel to the opposite end of the line at point B. It will take some fraction of that time, of course, for the pulse to travel to any intermediate point between A and B.

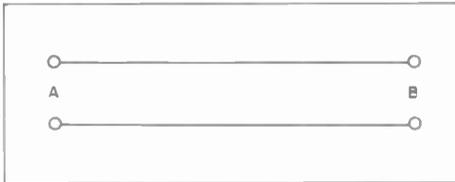


Figure 6. Schematic Representation of Conventional Transmission Line

An artificial transmission (or delay) line can be formed from inductors and capacitors arranged as shown in figure 7. If the proper values are selected, the delay time between each section of the line can be made equal to a pulse interval. Thus, if a pulse is started along the line at A, it will arrive at B after a delay of one interval; at C after a delay

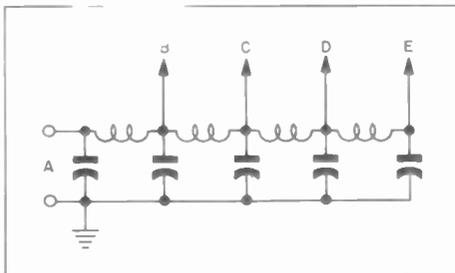


Figure 7. Schematic Representation of Artificial Transmission (Delay) Line

of two intervals; at D after a delay of three intervals; and at E after a delay of four intervals. The line can be extended, by adding sections, to introduce any desired amount of delay between the time a pulse is applied to the line, and the time it reaches a given delay-line tap. In this discussion, a delay line will be represented by the symbol shown in figure 8. The number appearing in the block indicates the number of delay intervals introduced into the circuit.

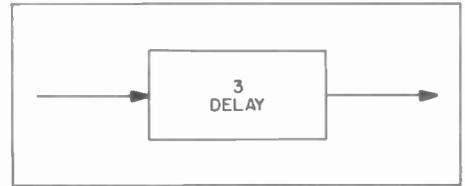


Figure 8. Block Symbol for Computer Delay Line

## BINARY ADDITION

Before a discussion of binary addition is begun, three important terms that are encountered in any process of addition should be defined. These terms are:

- Addend—the number to be added.
- Augend—the number to which the addend is added.
- Carry—the number of tens, hundreds, or thousands, etc., resulting from the addition of the digits of one order and added to the digits of the next higher order, on the left.

Binary\* addition is relatively simple in that there are only four possible combinations of an augend and addend consisting of one-place numbers, as compared to 100 combinations in the decimal system. These combinations are:  $0 + 0$ ,  $1 + 0$ ,  $0 + 1$ , and  $1 + 1$ . Of these, the first three combinations may be added directly:  $0 + 0 = 0$ ,  $1 + 0 = 1$ ,  $0 + 1 = 1$ . In adding binary  $1 + 1$ , we must increase beyond the limits of the highest digit (1) in the

\* Binary notation was discussed in the article, Electronic Computers—Part I.

binary system. To do this, we use the rule given in the first article (Part I) of this series: *substitute the lowest digit in the system for the highest digit, and increase the digit in the next higher place (order) by one.* This means that the binary sum of  $1 + 1$  is 0 with a carry of 1. The carry must be added to the digits in the next order to the left. In order that the reader may become familiar with the binary addition combinations, a list of the combinations is given below in Table IV:

**Table IV. Binary Addition Combinations**

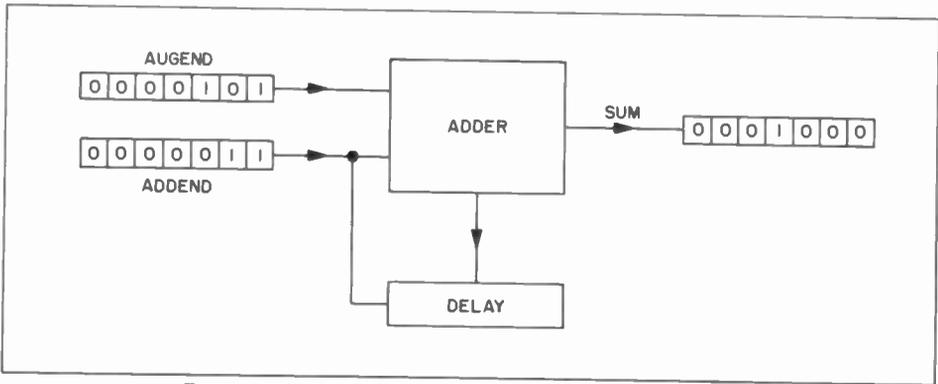
AUGEND	ADDEND	SUM	CARRY
0	0	0	0
1	0	1	0
0	1	1	0
1	1	0	1

To further illustrate binary addition, consider  $0000101 + 0000011$ , ( $5 + 3$ ). The digits in each order of the binary numbers must be added separately start-

3rd-order digits:  $1 + 0 + 1$  (carry)  
 $= 0$  carry 1  
 4th-order digits:  $0 + 0 + 1$  (carry)  
 $= 1$   
 Result = 0001000 (8)

Now let us determine, briefly, using the block diagram of the serial addition system shown in figure 9, how the same addition is performed in one type of computer. Both the augend (figure 9) and addend, which may be stored as pulses in a flip-flop counting circuit, are applied to the inputs of the adder. The addition of the first-order digits,  $1 + 1$ , produces a sum of 0, which may be sent to a storage register, and a carry of 1, which is sent as a carry pulse to a delay line. The delay line holds the carry pulse and applies it to the adder input circuit along with the second-order digits. In this way, the carry pulse is added to the two second-order digits. The process continues until the addition is completed.

We have now seen how logical AND, OR, and NOT circuits produce an out-



**Figure 9. Block Diagram of Serial Addition System**

ing with the first-order digits and proceeding to the left, as follows:

1st-order digits:  $1 + 1 = 0$  carry 1  
 2nd-order digits:  $0 + 1 + 1$  (carry)  
 $= 0$  carry 1

put pulse, and have learned the purpose of a delay line. In the next article it will be shown how these circuits can be combined into a little black box called an adder.



# IMPROVEMENTS IN THE NBS PRIMARY STANDARD OF FREQUENCY

Some details of modifications made recently in the primary standard of frequency maintained by the National Bureau of Standards.

*(Editor's Note: The information contained in this article was supplied by the National Bureau of Standards, U. S. Dept. of Commerce, and appears here through the courtesy of that agency.)*

ACCOMPANYING THE MAINTENANCE of the Nation's primary standard of frequency is a continuing investigation by the National Bureau of Standards of methods for improving the constancy and reliability of the standard. Some modifications incorporated within the last few years include the use of resonator crystals to sustain the accuracy of the standard, more sensitive and reliable temperature controls, and precise clock mechanisms to monitor time signals. The use of new and improved components has resulted in a reduction in the number of replacement parts, and represents a considerable saving of time normally required for preventive-maintenance procedures.

The NBS primary standard of frequency is the foundation upon which are based all time and frequency transmissions from the Bureau's radio broadcasting stations WWV, in Beltsville, Maryland, and WWVH, Maui, Territory of Hawaii.\* From these stations, standard radio frequencies of 2.5, 5, 10, 15, 20, and 25 mc. are transmitted continuously, night and day, with accuracies of two parts in 100 million. Two standard audio frequencies, 600 and 440 cycles (the standard musical pitch, A above middle C) are broadcast on all of the radio frequencies, and every 5 minutes are interrupted for intervals of one min-

ute. A pulse of 0.005-second duration occurs on each carrier frequency at intervals of one second. The time intervals, as transmitted, are accurate within  $\pm$  (two parts in  $10^8$  + one microsecond). An announcement of radio-propagation conditions, pertinent only to transmission paths in the North Atlantic area, is broadcast in code on each of the standard radio frequencies.

The National Bureau of Standards' primary standard of frequency consists of nine crystal-controlled oscillators and eight quartz-crystal resonators. Three of the oscillators are located at the Beltsville installation of WWV—one acting as the main oscillator for all of the transmitters, the second as the standby, and the third as a spare. The remaining six oscillators and the eight quartz-crystal resonators are maintained in the Bureau's Washington laboratories. All of the crystal-controlled oscillators are kept in continuous operation and the best ones—those having the least amount of deviation from 100 kc. for the immediately preceding six-month period or longer—are the units from which the standard frequency is determined.

The oscillators are controlled by specially made GT-cut quartz crystals; the resonant frequency of each crystal is 100 kc. In examining the crystals, it has been observed that generally their performance curves (frequency vs. amplitude) have a flat region within which

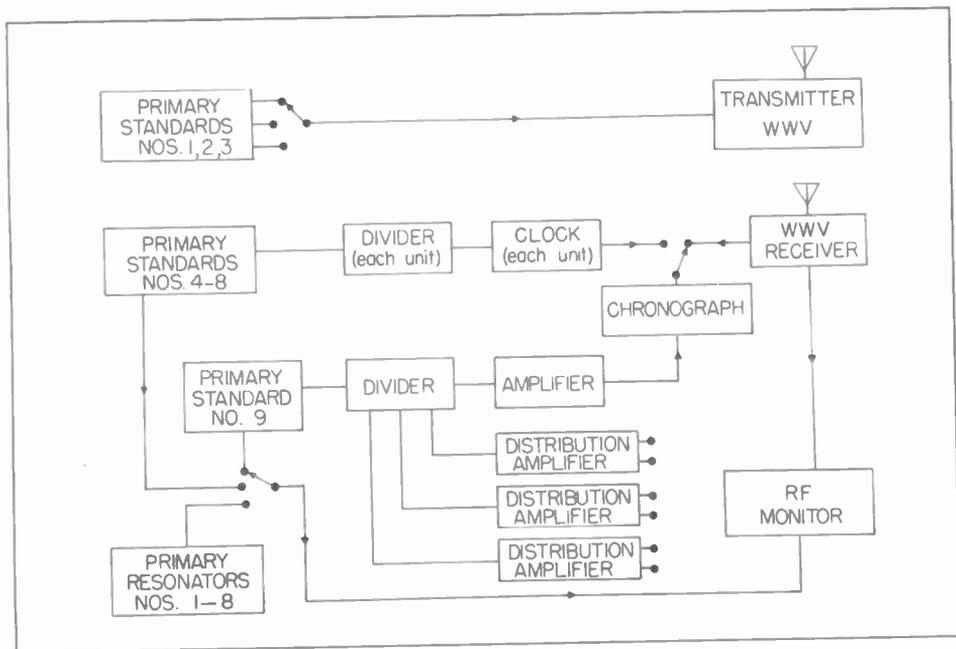
\* For additional information, see "Standard Frequencies and Time Signals from WWV and WWVH," NBS Letter Circular LC 1009.

the crystal frequency is relatively constant. When the driving current reaches a value of about 150 microamperes, the frequency decreases sharply. In view of this fact, the driving current applied to the crystal units of the newer NBS oscillators is less than 100 microamperes. A decided improvement in performance occurs and is especially evident when compared to the older oscillators with driving currents of over 500 microamperes. Increased short-time stability and over-all reliability have also been achieved.

The eight resonator crystals have been part of the frequency standards for about one and a half years. Each resonator's frequency is used in the analysis of the accuracy and constancy of the other nine oscillators. All eight crystals, each also with a resonant frequency of 100 kc., are installed in a

single temperature-controlled oven. They do not incorporate additional components such as tubes, resistors, or capacitors. They are not driven continuously, but are used only once a day as part of a balanced-bridge network for comparison with one of the standard oscillators. Furthermore, the current driving the crystals is only 10 microamperes.

Once a day, the value of each resonator crystal and each standard oscillator is determined, using the system shown in figure 1. First, a precision variable oscillator is adjusted to the frequency of one of the resonators. The variable oscillator is then compared to one of the standard oscillators, and the beat, or difference, frequency is counted on an electronic frequency counter with a precision of the order of parts in  $10^{10}$ . The variable oscillator is readjusted to the second resonator crystal and again

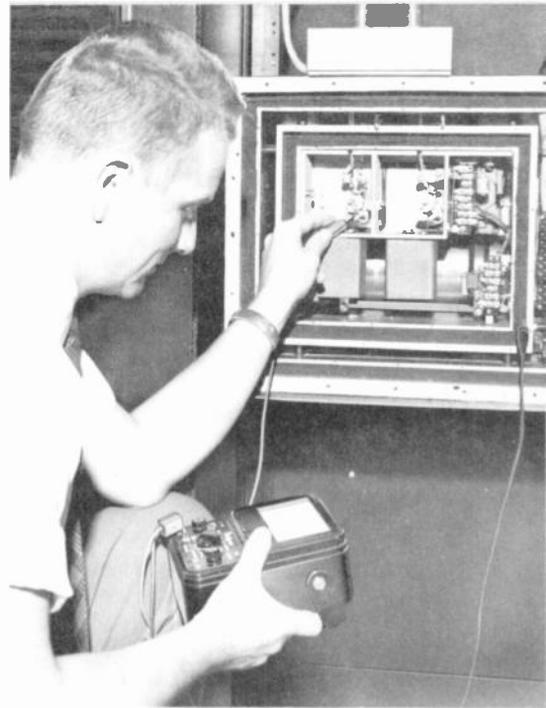


**Figure 1. Block Diagram of the NBS Primary Standard of Frequency**

The primary standard is composed of nine crystal-controlled oscillators and eight resonator crystals. Three of the oscillators, located at the NBS radio station WWV in Beltsville, Maryland, control the transmission of the standard frequencies and time signals. The remaining six oscillators are installed in the Bureau's Washington laboratories. The time signals derived from these oscillators are continuously monitored by a spark chronograph and chronoscope that measures time differences as small as 20 millionths of a second. The time and frequency signals are also fed into distribution amplifiers and over wire lines for use by the scientific staff within the Washington laboratory.

compared to the same standard oscillator. The difference frequency between these two oscillators is again recorded. This procedure is continued until data are available indicating the amount of frequency deviation present between the standard oscillator and each of the resonator crystals. One of the remaining eight oscillators is used as a reference against which all of the other oscillators are compared. Thus, data are available for precisely determining any changes, relative to the system, which may occur in any oscillator or resonator in the system.

The reference oscillator is also instrumental in obtaining a continuous record of the frequencies of the three standard oscillators at the WWV installation. Automatically, the main, stand-



**Figure 2. Temperature-Control Compartment Containing the Crystal and Temperature-Sensitive Components of One of the Nine Crystal-Controlled Oscillators of the NBS Primary Standard of Frequency**

*The compartment is made up of four concentric cubical chambers. The quartz-crystal unit of the oscillator is contained in an electronically controlled cylindrical oven (at back of center chamber). A special resistance bridge and feedback amplifier arrangement maintains the temperature constant to 0.001° C. The unit on top of the oven contains the vacuum-tube-amplifier portion of the oscillator.*

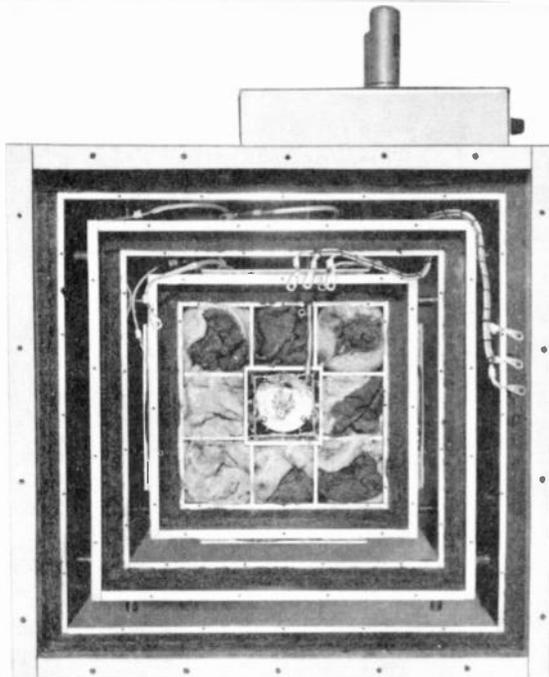
by, and spare oscillators are successively switched at preset intervals to a low-power, v-h-f transmitter. The signals are beamed to the NBS laboratory in Washington, and the received signals are compared to signals derived from the reference oscillator.

A more precise and reliable temperature control of the ovens enclosing the oscillators has been developed. The oven (shown in figure 2) is essentially four concentric cubical chambers; the center chamber holds the oscillator unit, and the spaces of the next and outer chambers are filled with felt insulation. An air-chamber containing mat heaters separates the insulated chambers. The outer heater, designed for coarse temperature control, is controlled by a simple mercury thermostat. Control of the inner heater, designed to respond to very small changes in temperature, is achieved by using a network in which the heater element is part of the sensing circuit. In

effect, one pair of arms of a resistance bridge is made up of wire with a high temperature coefficient, and the other pair, of wire of negligible temperature coefficient. Current through both pairs of wires supplies the necessary heat. An oscillatory circuit, composed of the bridge connected between the input and output of a high-gain amplifier—essentially a feedback loop—controls the temperature. When the temperature is near the desired value, the bridge is only slightly unbalanced and the amplifier is in a stable condition. As the outer temperature of the oven decreases, the bridge becomes further unbalanced and the amplitude of the output oscillations increases so as to supply more current to the bridge wires and, consequently, more heat to the oven. This condition will continue until the temperature regains the assigned operating value. Under normal room conditions, the temperature is controlled to better than 0.001° C.

The temperature-controlled oven for the eight resonator crystal units is constructed with six concentric cubical chambers (see figure 3). All eight crystals are enclosed in the one oven and the temperature control is achieved with mercury thermostats connected to both the inner and outer heaters. Inner temperature variation is less than  $0.005^{\circ}$  C. for average variations in room temperature. The standard oscillators at the NBS transmitting station in Beltsville are installed in a room approximately six feet on a side, built about 25 feet below the ground. The complete room is temperature- and humidity-controlled.

Each of the frequency standards is equipped with individual power supplies, and includes improved filters in



**Figure 3. Temperature-Control Compartment Containing the Resonator Crystals of the NBS Primary Standard of Frequency**

*The oven is made up of six concentric cubical chambers. The partitioned center chamber contains the eight quartz-crystal resonators and a switch for connecting each crystal to an external measuring circuit. The box on top of the oven contains heater controls and power-supply line filters.*

order to achieve better regulation and control. The supplies all have plate and filament batteries that are continuously float-charged, and which, in the event of an a-c power failure, can carry the full load for many hours.

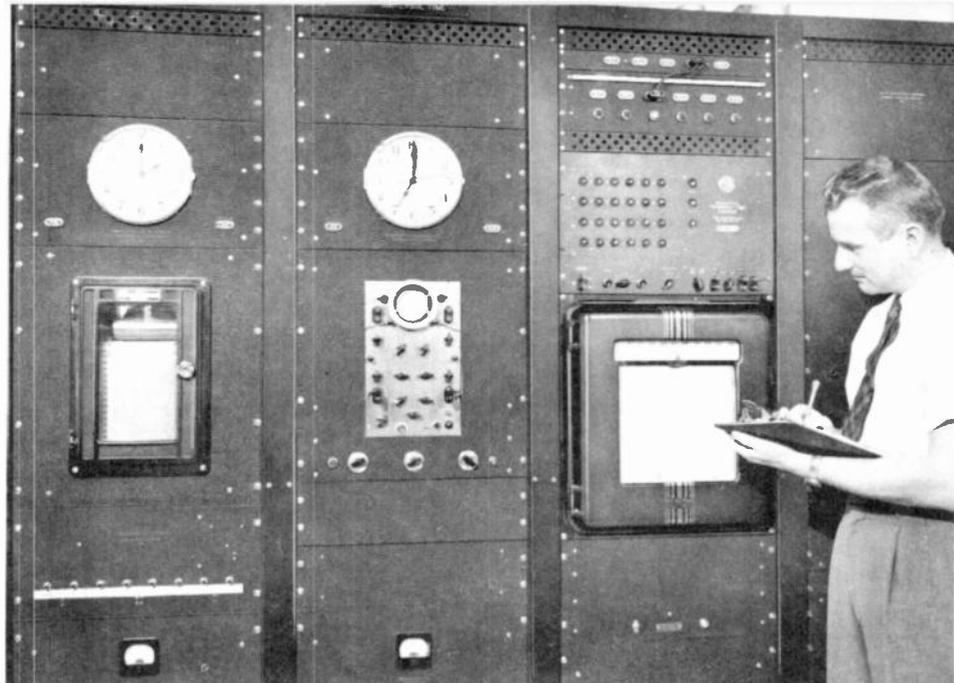
In order to accurately monitor the time signal generated by the frequency standard, one of the NBS standard oscillators is used to drive a synchronous clock (see figure 4). The 100-kc. output of each oscillator is electronically divided to a frequency suitable for driving a spark chronograph and chronoscope.\* The instruments are designed so that the driving oscillator may be compared to the time signals of the other five oscillators in the Washington laboratory, and those at the WWV installation to differences as small as 20 millionths of a second.

The time differences or variations in each clock are reported each day to the

U. S. Naval Observatory and are used in evaluating the mean solar time. The Observatory issues weekly corrections to the WWV signals with reference to mean solar time. Quarterly corrections for the slight deviations in absolute time and frequency as broadcast by WWV are also available from the National Bureau of Standards.

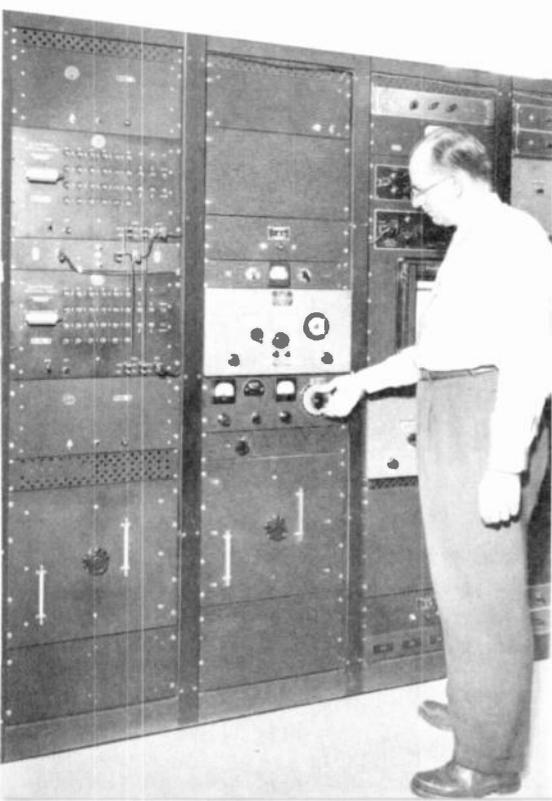
One of the principal uses of the WWV transmission of standard frequencies is that of calibrating precision oscillators. The system commonly employed is demonstrated by the manner in which the WWVH (Hawaii) transmissions are synchronized with those from WWV. Three oscillators are used to obtain accurate results. One oscillator unit is adjusted to one of the standard frequency transmissions from WWV. Each of the other two oscillators is compared to the first and the difference in frequency between them is measured. A better average of the deviation from the WWV transmissions is achieved by this method than would be possible if only one oscillator were involved.

\* For additional technical details, see New Spark Chronograph and Chronoscope NBS Technical News Bulletin, Vol. 35, No. 1, p. 14 (Jan. 1951).



**Figure 4. Frequency-Monitoring Equipment of the NBS Primary Standard of Frequency and Time**

The two racks at the left contain the spark chronograph and chronoscope which are used to measure time differences between any two standard-controlled clocks to a precision of  $\pm 0.02$  millisecond. The third rack from the left contains the automatic beat-frequency recorder for intercomparing the primary frequency standards. An automatic switching unit (top panel) connects each of the six oscillators sequentially to a dual, electronic frequency counter (next panel below). A recording frequency meter with a sensitivity of  $\pm 1$  part in  $10^{10}$  provides a permanent record of the frequency differences between the standards. The remainder of the rack contains power supplies, a proportional voltage generator that converts the count summation to a recorded voltage, and a dual frequency multiplier and demodulator that provides difference frequencies between the oscillators.



One of the major continuing projects of the Bureau's frequency-standard laboratory is the development of more precise quartz oscillators. Another is the investigation of the use of atomic and molecular-resonance standards as a means for obtaining higher accuracies. Reference to such invariant standards would greatly simplify the maintenance of precise frequency and time standards.

**Figure 5. NBS Primary Resonator Frequency Standard**

The rack at the right contains a temperature-controlled oven (constant to  $0.001^\circ \text{C.}$ ) in which the resonator crystals are located (panel with two thermometers). The rack also includes a special bridge for comparing an oscillator with the quartz resonator crystals, a precision adjustable oscillator (operator is shown making an adjustment), power supplies, and a high-gain, narrow-band receiver. The rack at the left contains dual electronic counters for counting the frequency differences between oscillators. Also included is an additional temperature-controlled compartment for use in measuring experimental resonator crystals.

# GUIDED MISSILES

by Merle E. McDougall  
Philco Field Engineer

A summary of available declassified information on the characteristics and capabilities of modern guided missiles.

*(Editor's Note: All material contained in this article has previously appeared in unclassified or declassified publications, extract copies of which are on file in the BULLETIN editorial office.)*

PROGRESS IN THE GUIDED-MISSILE FIELD has been phenomenal since World War II. At that time, the only guided missiles in existence were the German V-2 rocket and the V-1 buzz bomb which was used against London during the latter days of the war. These missiles had relatively short range (150 to 250 miles), and their accuracy was poor (approximately 8 miles error in 150 miles). About 15,000 V-1 and 2500 V-2 missiles were fired during World War II by Germany. Since that time the countries of the world have been spending large amounts in both manpower and money to develop the guided missile into a tactical weapon. As a result, the modern guided missile can fly at a speed several times that of the fastest airplane, and hunt out and destroy an aircraft or deliver a warhead on a distant target with quarter-mile accuracy. The mind can visualize missiles of the future that will do the work of artillery and long-range aircraft, and do it better, carrying either regular explosives or atomic bombs. Guided missiles are of necessity very complex; scientists and engineers who are engaged in propulsion and guidance system development must draw on many fields of knowledge, including aerodynamics, electronics, propulsion, optics, and many others.

Guided missiles can be divided into a number of categories according to their tactical purpose; such as air-to-air (launched from an airplane toward an airborne target), air-to-surface, surface-to-air, and surface-to-surface.

Surface-to-air missiles are designed to be used against enemy aircraft. This type of missile, of which the Lark in figure 1 is representative, is usually 10 to 15 feet long and less than a foot in diameter. Its range is usually on the order of 10 to 30 miles, and it travels at supersonic speed.

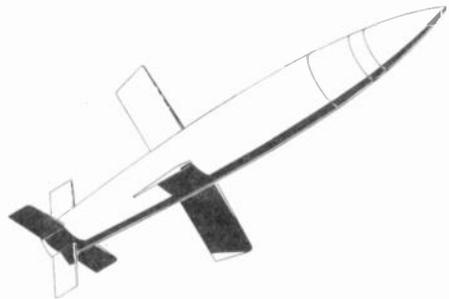


Figure 1. Lark

Air-to-surface missiles are usually larger than the surface-to-air type. A typical length is 30 feet, with a diameter of 1.5 feet. These missiles usually travel at subsonic speed.

Air-to-air missiles are the smallest of the guided missiles. Designed as armament for fighter aircraft, these missiles have so great a speed that they cannot be shot down or dodged. The Sparrow, shown in figure 2, is an example of an air-to-air missile.

Surface-to-surface missiles have a wide range of application, from their use as short-range missiles to replace artillery, to their use as intercontinental bombers which can deliver a payload

halfway around the world with the accuracy of a conventional bomber and without the risk to men. A typical long-range surface-to-surface missile is about the length of a V-2 (46 feet long), but is slimmer and presumably has much greater range.

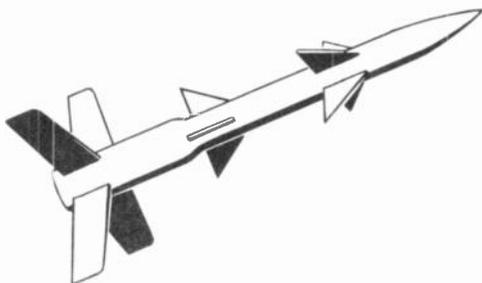


Figure 2. Sparrow

The United States has several test sites for guided missiles, of which the three major sites are White Sands, New Mexico; Point Mugu, California; and Banana River, Florida. The largest of these firing ranges is Banana River, which spreads over an area of 100,000 square miles from Florida to Puerto Rico. At present, this range is limited to a distance of 1000 miles, but, if necessary, it can be extended into the South Atlantic to a maximum distance of 5000 miles. Upon final completion, the range will have a launching site at Cape Canaveral, Florida, and eight instrumentation stations along the 1000-mile course to Puerto Rico.

### GUIDANCE SYSTEMS

For a missile to be a tactical weapon, its movements must be controllable, either before it gets into the air or while it is on its way to the target; control of the missile's movements is termed "guidance." Many types of guidance systems are possible, each with its advantages and disadvantages. All of these types will be discussed in the following paragraphs.

**PRESET SYSTEM:** As the name of this guidance system implies, all of the functions that are to be performed

within the missile during flight are set upon a programmer before launching. For instance, the V-2 was launched vertically, and after it ascended straight up for a certain period of time a programmer caused it to tip toward the horizontal and commence its trajectory toward the target. After another predetermined time period, other functions were performed, such as cutting the fuel off and causing the missile to dive into the target area. If unforeseen elements arose, such as changes in wind direction and variations in atmospheric pressure, they could not be compensated for by the missile guidance system; therefore, the probable error of the preset missile was large.

**COMMAND SYSTEM:** Command guidance can be used in surface-to-air missiles designed for anti-aircraft use. When an enemy aircraft is located in the vicinity, the missile is launched, and both the missile and the target are viewed on a radar screen. The missile operator has control of the missile from the ground by way of radio. The operator's duty is to guide the missile close to the aircraft while watching both on the radar scope. The missile contains a proximity fuze which will explode the missile when it comes in the close vicinity of the aircraft. This method of guidance is simple, but brings into play the human element, which with high-speed aircraft is not sure. Since the missile is radio-controlled, counter measures, such as the radiation of radio-frequency noise by enemy jamming equipment, can be used to prevent the commands from reaching the missile. Proper missile antenna design and high signal strength will make jamming more difficult.

**BEAM-RIDER SYSTEM:** This type of guidance system is primarily used in anti-aircraft missiles. The target is tracked by radar, which produces a narrow beam of radar-frequency energy be-

tween the radar transmitter and the target. The missile is fired into the beam and follows it for a certain distance; then computers in the missile cause the missile to lead the target, like a hunter leads a duck, so as to produce a collision course which will destroy the missile and the target.

**SEEKER SYSTEM:** Missiles using the seeker guidance system can be divided into two classifications: active, and passive. Active seeking missiles contain a radar transmitter and receiver. The missile is steered from the ground to the target vicinity, after which the missile radar takes over, sends out radar pulses, and steers the missile toward anything solid enough to reflect the radar echoes back to the missile radar receiver. Passive seeking missiles have sensitive elements which pick up such radiations from the target as light or heat, and steer the missile to the target. The seeker becomes more accurate as it approaches the target, and if the target changes course, the missile also changes course and produces a collision. Seeker missiles can be made to travel at supersonic speed and cannot be evaded or dodged.

**TERRESTRIAL SYSTEM:** The terrestrial system of guidance, generally useful for surface-to-surface missiles, makes use of the earth's phenomena to keep it on a predetermined course. Such phenomena as the earth's gravitational, electric, and magnetic fields, and the atmospheric density can be used. An example of a terrestrial reference missile is the German V-1, which used a magnetic compass to keep it on course, an altimeter to keep it at the proper altitude, and an airlog to determine distance traveled, so as to keep the missile on the predetermined course and cause it to go into its terminal dive at the proper time and place.

**CELESTIAL SYSTEM:** This guidance system is useful for long-range-

bombardment missiles, and is based on the method used by navigators in "shooting" two stars to determine present position. Automatic star-tracking telescopes and a recording of the predetermined course are contained in the missile. The present position of the missile as determined by the telescopes is compared periodically with the position at which the missile should be. Any deviation from the correct position results in the production of error data, which cause the steering mechanism to bring the missile back to its proper course. The accuracy of this type of guidance system is as good at the target site as at the launching site. No countermeasures can be effective against a missile incorporating a celestial guidance system unless faster missiles are designed to intercept it. The system is limited primarily to night-time operation, but this is not too much of a disadvantage when it is used on supersonic missiles; the course can be flown during a time when the background intensity of the sky is low.

**RADIO-NAVIGATION SYSTEM:** The radio-navigation guidance system makes use of a type of navigation known as LORAN. The missile contains a radio receiver which receives signals sent simultaneously from two fixed ground transmitters, and the missile measures the time delay between the times of reception of the two signals. Before the missile is launched, its electronic brain is "instructed" as to the correct time delays for the predetermined course. If the missile deviates from the correct path, the time delay becomes greater or less than the predetermined value, and the missile is guided back to the correct course. This type of guidance system is accurate, but is subject to interference and enemy electronic countermeasures.

**INERTIAL SYSTEM:** The inertial guidance system employs devices called accelerometers, which are sensitive to

accelerations. These accelerometers are installed in the missile, and a flight plan containing all known accelerations along the correct flight course is fed into the guidance system before the missile is launched. If accelerations not contained in the flight plan (such as those due to wind) take place, the accelerometers record and doubly integrate them, to obtain the distance that the missile has traveled off course. Corrections are then fed to the steering mechanism to put the missile back on its predetermined course. This guidance system is entirely self-contained, and therefore cannot be jammed. Its only drawback is the complexity of the missile equipment.

**TELEVISION SYSTEM:** For television guidance of a missile, a small television transmitter is installed in the nose of the missile. This equipment transmits a television picture of the area ahead of the missile to the missile operator, who controls the missile from a "mother" airplane or ground station. Examples of this type of missile are the Navy's F6F Grumman Hellcats which were used in Korea as television-controlled missiles. The control airplanes were Douglas AD-2's, which stayed well out of range while the missile airplanes flew into the target. Television guidance is good up to approximately 500 miles from the control site.

### PROPULSION SYSTEMS

Two basic types of propulsion systems are available for missile use—rockets and jet engines. The rocket is a self-contained unit which carries both the fuel and the oxidizer, and therefore can fly out of the earth's atmosphere. Another advantage of the rocket is that its top speed is unlimited. Rockets can be divided into two classes: the solid-propellant rocket and the liquid-propellant rocket. The solid-propellant rocket is very simple in construction and may be stored easily in the field until needed (one of the popular solid propellants at present is potassium perchlorate in asphalt). Its disadvantages are that the

combustion chamber must be heavy and large, and its burnout time is short. The liquid-propellant rocket can be turned on and off during flight and has a much longer range than the solid-propellant rocket. The disadvantage of the liquid-propellant rocket is the problem of handling and storing the fuel. The liquids are sometimes acids, such as nitric acid, or liquid gases such as liquid oxygen, which must be kept at very low temperatures ( $-200$  degrees F.).

The jet engine, the other propulsion system, generally exhibits less thrust than the rocket, although it has a lower rate of fuel consumption, which makes it applicable for long-range use. The jet engine is more costly than the rocket, but a successful mission would justify the cost. Jet engines usually use kerosene or gasoline as a fuel. The ceiling of the jet engine is limited, as it must have oxygen for operation.

### SPECIFIC MISSILES

*Note: Because of security classifications, the types of propulsion and guidance systems used in some of the missiles discussed below cannot be given. Only as much information as security regulations permit is included.*

**GERMAN V-1:** The V-1 was the German buzz bomb used in World War II against London. Its propulsion system consisted of a pulse-jet engine (40 pulses per second), which produced the buzzing sound. The V-1 had a speed of 360 m.p.h. and a range of 150 miles. It had a gross weight of 4750 lbs. and a length of about 25 ft. A drawing of the V-1 is shown in figure 3.

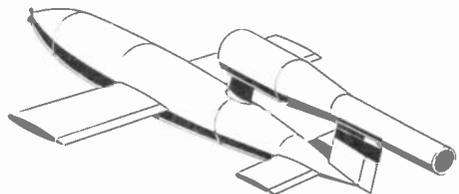
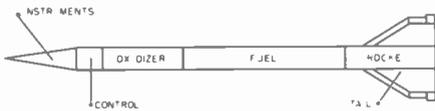


Figure 3. German V-1 Buzz Bomb

**GERMAN V-2:** The guidance system of the V-2 was preset before launching and could not be corrected after launching. The V-2 had a top speed of 3750 m.p.h. Its power plant was a liquid-propellant rocket, using liquid oxygen and ethyl alcohol, which generated 50,000 pounds thrust (equivalent to 500,000 horsepower). Its range was approximately 250 miles. The V-2 had a gross weight of 28,400 pounds and carried a 2000-pound warhead, which contained a low-force explosive. The aerodynamic friction at 3750 m.p.h. produced so much heat that a high explosive would have been detonated before flight termination. In its 65 seconds of powered flight, the V-2 consumed 20,000 pounds of fuel.

**VIKING:** The Viking, shown in figure 4, is primarily a test vehicle and a sounding rocket. It is 45 ft. long and has a diameter of about 2½ ft. The Viking has been launched at White Sands, New Mexico, to investigate conditions in the upper atmosphere. It is powered by a liquid-propellant rocket, which has driven it at a top speed of 5000 m.p.h. and to an altitude of 200 miles. The Viking weighs 10,000 lbs., and can carry a payload of 1400 lbs. It is launched vertically. Heading errors are detected by gyroscopes, and the resultant signals are fed to servos which operate vanes to correct the error heading. Roll-control error signals are produced in the same way, and are used to control exhaust ducts and tabs to correct errors. Aluminum alloys are used for all surface areas of the missile except the nose and leading edges of the fins, where high temperatures are pro-

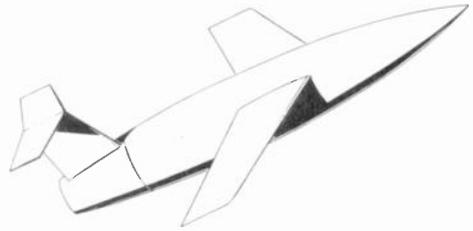


**Figure 4. Viking, Showing Internal Compartmentation**

duced by aerodynamic heating. Stainless steel is used for these high-temperature areas.

**TERRIER:** The Terrier is a surface-to-air missile developed for the Navy Bureau of Ordnance and soon to go into large-scale production at Convair. It is slated for fleet use during the next few years.

**MATADOR:** The Matador is a surface-to-surface missile designed for medium-range bombardment. The Matador, figure 5, is about 30 ft. long, is jet and rocket propelled, and has a top speed of over 1000 m.p.h.



**Figure 5. B-61 Matador**

According to reports, it is radar-controlled and has a 500-mile range. The Matador is currently being tested at Banana River, Florida.

**SPARROW:** The Sparrow, shown in figure 2, is a tiny air-to-air Navy missile on which production is to be handled by Sperry. It is intended for fighter use against bombers.

**NIKE:** The Nike, which is designed for antiaircraft defense, has a solid-propellant booster and a liquid-fuel rocket. It is being jointly developed by Bell and Douglas.

**CORPORAL E:** The Corporal is a 40-ft. Army missile, designed for use against ground installations. It weighs 12,000 lbs. and has a range of 100 miles. It is supersonic, with a speed of four to five times the speed of sound, and is reported to have an accuracy of 500 ft. at a range of 100 miles.

**SNARK:** The Snark is a surface-to-surface missile, designed for long-range bombardment; it is being developed by Northrup.

**OERLIKON:** The Oerlikon is a Swiss anti-aircraft missile. The missile is about 16.5 ft. long and 1.5 ft. in diameter, and weighs about 550 lbs. with a warhead of approximately 45 lbs. The propulsion system is a liquid-fuel rocket using nitric acid, with a burning time of 6 seconds and a top altitude of 66,000 ft. The guidance system is of the beam-rider type with a maximum control range of about 12 miles.

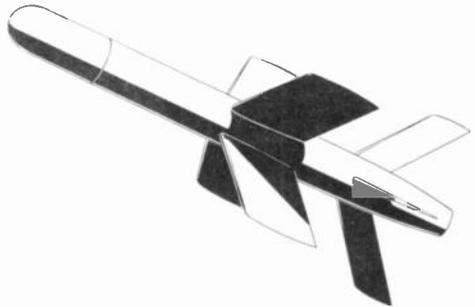
**AEROBEE:** The Aerobee is a free-flight sounding rocket, used for investigation of the upper atmosphere. It is 20 ft. long and 15 in. in diameter, and uses a liquid rocket with a JATO booster to propel its 150-lb. payload to a height of 80 miles.

**LARK:** The Lark, shown in figure 1, is an anti-aircraft missile which is propelled by a liquid-fuel rocket using red fuming nitric acid and aniline. In this missile, the fuel and oxidizer are forced into the rocket engine by bottles of compressed air at a pressure of 500 pounds per square inch. The Lark is 173 in. long. Its guidance system uses radar homing, which becomes more accurate as the missile approaches its target. In tests, the Lark has been fired at the Naval Air Missile Test Center, Point Mugu, California, and from the Norton Sound, a seaplane tender used for missile firings. The Lark is launched from a zero-length launcher (the launcher is no longer than the missile).

**WAC CORPORAL:** The WAC Corporal, which has a weight of 1000 lbs., has been used as a high-altitude sounding rocket. For high-altitude use, the missile is attached to the nose of a V-2. The two missiles are then launched as a two-stage rocket, with the V-2 dropping away at an altitude of 20 miles and a speed approaching 5000 m.p.h. The

highest altitude reached so far with a two-stage rocket of this type has been 250 miles.

**FIREBIRD:** The Firebird, shown in figure 6, is produced by Ryan. It is an air-to-air weapon, 10 ft. long and 6 in. in diameter, with a rocket motor. A radar homing device is used to steer the missile to the target.



*Figure 6. Firebird*

**RUSSIAN DEVELOPMENTS:** The Russians have been using the former German rocket base at Peenemunde as a test site. At present they are known to be working on four different missiles. One is the former German Wasserfall anti-aircraft rocket, which was, and probably still is, a rocket-propelled missile using nitric acid and aniline as propellants. The German missile was 25½ ft. long and 3 ft. in diameter, with a gross weight of 8500 lbs. Maximum speed was around Mach 3.

Another missile that the Russians are working on is the A-9, a winged version of the V-2. The A-9 was developed by the Germans in an attempt to get more range out of the V-2. It was simply a V-2 to which a set of swept-back wings had been welded. Only three of the A-9 missiles were fired by the Germans; it was hoped that their range would be 500 miles. The dream of Germany was the use of the A-9 as the second stage of a two-stage rocket for the bombing of New York.

The A-5 rocket is a half-size V-2 which is used by the Russians for ex-

perimentation. As developed by the Germans, it was rocket-propelled by a so-called "cold" rocket, which used the decomposition of hydrogen peroxide to furnish superheated steam as the exhaust gas.

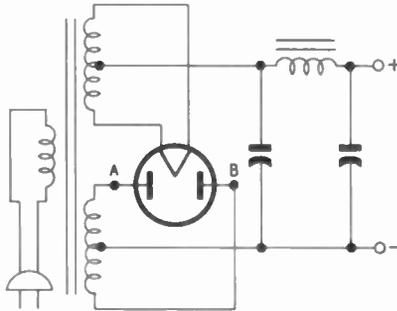
A missile of the V-1 or Matador type is also being used at Peenemunde; six new zero-length launching ramps have been built for a missile of this type.

### CONCLUSION

From the preceding discussion some idea can be gained of the development, use, and future of guided missiles. In addition to their use by world powers as deadly, precision-controlled weapons of war, there is the possibility that one day guided missiles will provide the means for peacetime interplanetary travel, which has been the dream of mankind for many centuries.



## What's Your Answer?



This problem, submitted by Philco Field Engineer G. P. Rumble, was accepted by your editor in a fit of anger when it was learned that the recent "Jewel Robbery" case (see last month's "What's Your Answer?") was broken by a shipping clerk.

A field engineer running a routine check of a power supply (see drawing for schematic) found that even though the d-c output voltage was essentially normal, there was no a-c voltage between the two rectifier plates (point A to point B). Since no overheating was in evidence, the field engineer concluded that one of the two halves of the high-voltage secondary winding was open. After removing the input power and discharging the filters, a resistance check showed that the two halves of the secondary were intact (OK).

What is wrong with the circuit?

(Solution next month)

# A REDISCOVERY OF LONG-STANDING ARITHMETICAL TECHNIQUES

by Mark Flomenhoff  
Philco Research Division

A discussion of the writer's rediscovery of an old but useful method of checking the accuracy of arithmetical operations based upon the principle of "casting out nines," supplemented by a discussion of the divisibility of numbers and of methods of multiplying and squaring numbers rapidly by the use of well-known algebraic laws.

WHILE BROWSING THROUGH AN ALGEBRA BOOK recently, the writer encountered a long-standing theorem of numbers that struck home with thought-provoking impact. This theorem is the basis of the principle of "casting out the nines" as a means of checking arithmetical operations. Since the rule is reputed to be well-known, many readers probably are already familiar with it. Indeed, it would seem that since the first edition of the aforementioned text appeared in 1888, the principle should now be a commonplace among people who perform the innumerable calculations required in any technical field. Nevertheless, the writer was completely ignorant of the principle.

This article, therefore, is offered to those who are in the same plight as the writer had been. Because numbers and the arithmetical processes are the primary tools of scientific activity, it is unthinkable for people who continually perform calculations to remain unaware of the time-worn rules for checking accuracy. In the discussion immediately following, these rules and various other properties of numbers will be stated without proof. For those readers who would like to justify the validity of this information, an appendix of mathematical proofs based on standard texts has also been included.

## CHECKING MULTIPLICATION

Since multiplication is one of the more arduous arithmetical operations,

let us consider it first. The best way of explaining the process of casting out the nines is by illustration, so let us undertake the problem of multiplying 361,496 by 3,127. Could 1,130,397,992 be the correct answer? The simplest way of finding out is as follows:

1. Add the digits of 361,496 and "cast out" the nines. Thus, three and six are discarded, since they total nine, and the nine is also eliminated. A one, four, and a six remain, and their sum is 11. From 11 subtract nine, leaving two. (Note also that the sum of the two ones in the 11 also equals two. This is not a coincidence.)

2. Add the digits of 3,127 and again cast out the nines. It is evident that the two and the seven can be eliminated, leaving a sum of four. (Note that if all the digits were added, the number 13 would be obtained, and that the sum of these digits again is four.)

3. Multiply the two of step 1 and the four of step 2 to obtain eight.

4. Now add the digits of the answer, which was given as 1,130,397,992. Casting out the nines, we have one, one, three, and three. Their sum is eight, which agrees with step 3. According to the rule of casting out the nines, therefore, it is possible that the number 1,130,397,992 is the product of 361,496 and 3,127.

It is essential to recognize that the foregoing procedure is not an absolute check of the multiplication process. For

example, suppose that the digits of the product were rearranged as 9,792,311,093. Since the same digits are involved, eight will still remain after the nines are cast out. Obviously, both numbers cannot be the correct answer. On the other hand, suppose that the multiplication process had resulted in a product of 1,120,397,992. Summing the digits and eliminating the nines in this event gives seven. This not only demonstrates that 1,120,397,992 is absolutely incorrect, but also *indicates* that one digit of the answer is off by one.

### CHECKING ADDITION

Checking an addition is even more straightforward. We shall consider the following example:

$$\begin{array}{r} 57493 \\ 47 \\ 6620 \\ 100488 \\ 4783 \\ \hline 169431 \end{array}$$

To check the answer, perform the steps given below:

1. Cast out the nines of the first number. 57,493. This results in the elimination of the five, the four, and the nine. Seven plus three are 10, which subtracted by nine leaves one. (Note that the one and the zero of the 10 when added give one also.)

2. The number 47, when stripped of its nines, gives the number two. (Nine from 11 equals two, or one plus one equals two.)

3. The number 6,620 yields the number five.

4. The number 100,488 yields the number three.

5. The number 4,783 yields the number four.

6. Add the numbers obtained in each of the preceding steps. Their sum is 15. Cast out the nine. The final number is six.

7. Cast out the nines of the answer, 169,431. Thus, the six, three, and nine

are discarded, leaving a sum of six. This tallies with step 6. Therefore, we conclude that 169,431 can be the correct answer.

### CHECKING SUBTRACTION

Virtually the same process is used to check a subtraction. This is true because the difference added to the subtrahend is equal to the minuend. Consider the following example:

$$\begin{array}{r} 64325 \\ 59091 \\ \hline 5234 \end{array}$$

1. Cast out the six, three, four, and five of the minuend. 64,325. This leaves the number two.

2. Cast out the two nines of the subtrahend, 59,091. This leaves six.

3. Cast out the five and the four of the difference, 5,234. This leaves five.

4. At this point, it probably is advisable to add the five obtained from the difference and the six of the subtrahend. The result is 11, which yields two when stripped of a nine. This two tallies with the result obtained in step 1, indicating that the subtraction is correct.

5. Another method of checking the subtraction is to subtract the number yielded by the subtrahend from the number yielded by the minuend. If the number corresponding to the minuend is the smaller, add *nine*. Hence, two plus nine equals 11. Six from 11 gives five. This checks with step 3.

### CHECKING DIVISION

Now we must devise a check for division. Usually, when a number  $N$  is divided by a number  $a$ , there is both a quotient  $q$  and a remainder  $r$ . This can be expressed as the equation:

$$N/a = q + r/a,$$

where  $r$  is less than  $a$ . Multiplying both sides of the equation by  $a$ , we obtain

$$N = qa + r, \text{ or } N - r = qa.$$

From the last form it is seen that the problem can be regarded as a multiplication, where the number  $N - r$  is the

product of  $q$  and  $a$ . The check procedure is described below, based on the following division:

$$\begin{array}{r}
 5396)37794611.0(7004.1 \\
 \underline{37772} \\
 22611 \\
 \underline{21584} \\
 10270 \\
 \underline{5396} \\
 4874
 \end{array}$$

1. Cast out the nines of the divisor, 5,396. This leaves the number five.

2. Sum the digits in the quotient, 7004.1. This gives a total of 12. Casting out the nine, there is left three.

3. Cast out the three, six, seven, one, one, and nine of the dividend, 37,794,611. This leaves seven and four, the sum of which is 11. Subtracting nine (or adding the two ones), there is left two.

4. The best way to strip the remainder, 4,874, of its nines is to add the digits. This gives a sum of 23. Now add the two and the three. This gives five.

5. Multiply the number obtained from the divisor (step 1) by the number obtained from the quotient (step 2). This gives the number 15, which reduces to six.

6. Add the number obtained from the remainder (step 4). This gives the number 11, which reduces to two. This result tallies with the number obtained from the dividend (step 3).

7. As an alternate check, it is permissible to subtract the number yielded by the remainder from the number yielded by the dividend. Add nine to the number two obtained in step 3 to produce 11, and subtract the number five obtained in step 4. This gives the number six, which agrees with the number obtained in step 5.

The procedure for checking division is valid, irrespective of the number of decimal places in the quotient. Thus, if

the division had been halted after a quotient of 7,004 had been obtained, there would have been a remainder of 10,270. The two of the quotient, in this instance, multiplied by the five of the divisor would yield 10, which reduces to one. This one added to the one of the remainder would give two, again conforming with the number obtained from the dividend. Therefore, we have successfully adapted the rule of casting out nines for checking all four arithmetical operations.

### DIVISIBILITY OF NUMBERS

As a supplement to the preceding information, the remainder of this article shall discuss various elementary properties of numbers in the hope that the reader may find them useful.

Let us begin with the subject of divisibility of numbers. Practically everyone knows that a number is divisible by two when the last digit of the number can be divided by two. In fact, such a number is defined as an *even* number. Not everyone, however, knows the criterion for determining when a number is divisible by four, eight, 16, and so on. Such knowledge is especially helpful with regard to the cancellation process in fractional operations. Fortunately, the criterion is very simple. A number is divisible by four if the last two digits of the number are divisible by four; a number is divisible by eight if the last three digits are divisible by eight; by 16 if the last four digits are divisible by 16; and so on. Similarly, a number is divisible by five, 25, 125, . . . if the last digit, the last two digits, the last three digits, . . . of the number are divisible by five, 25, 125, . . .

The rule for casting out nines has the additional application of disclosing when a number can be divided by three and nine. Simply cast out the nines and examine the remainder. If there is no remainder, the number is divisible by nine. If there is a remainder and it is divisible by three, then the entire num-

ber is divisible by three. For example, 9,264 leaves a remainder of three, showing that the entire number is divisible by three. The number 863,901, on the other hand, is divisible by nine.

Occasionally, there is need to transform a fraction into a decimal. It may be helpful to know when to expect a nonterminating decimal, and when to expect one that terminates. To answer this question, reduce the denominator of the fraction to its prime factors. If there is any prime factor other than two and five, the decimal will be nonterminating.

Now we shall consider two points that probably are not particularly useful, but which may prove rather intriguing. The first is illustrated by a number like 3,856,215, in which the sum of the even-numbered digits is equal to the sum of the odd-numbered digits. A number for which this condition holds is always divisible by 11. Try it.

The second point concerns a number like 295,295, in which the first three digits are followed by a repetition of the same digits in the same order. Such a number is always divisible by seven, 11, and 13. The number also is divisible by the lowest common multiple of any two or three of these factors, which are 77, 91, 143, and 1001. Test these statements, too.

### MULTIPLICATION TRICKS

Finally, let us quickly review a few well-known artifices for performing certain multiplication processes rapidly. One example concerns the multiplication of two numbers ending in five. The procedure is as follows: multiply the two numbers formed by temporarily eliminating the fives; now add the two numbers; if the sum is even, write down 25 as the last two digits of the answer; if the sum is odd, write down 75 as the last two digits of the answer; divide the sum by two; if there is a remainder, ignore it; add the result to the product

obtained previously; and write this number down ahead of the 25 or 75 previously recorded.

Example:  $65 \times 85 = 5525$

1. Multiply 6 x 8 and obtain 48.
2. Add 6 and 8, producing 14.
3. Divide 14 by 2, giving 7.
4. Add 7 to 48, giving 55. The answer therefore is 5525.

Example:  $35 \times 45 = 1575$

1. Multiply 3 x 4 and obtain 12.
2. Add 3 and 4, producing 7.
3. Divide 7 by 2, giving 3. Ignore the remainder.
4. Add 3 to 12, giving 15. The correct answer therefore is 1575.

Example: Find the square of 165.

1. Square 16 and obtain 256.
2. Add 16 to 256, giving 272.
3. The correct answer therefore is 27225.

### SUM-AND-DIFFERENCE RULE

Numbers that can be converted into the sum and differences of two numbers can be multiplied with particular ease. It will be recalled that  $(a + b)(a - b) = a^2 - b^2$ . A few examples showing the facility gained through this law now follow.

Example:  $35 \times 45 = 1575$

1. Convert 35 to  $(40 - 5)$ . Convert 45 to  $(40 + 5)$ .
2. The square of 40 is 1600. The square of 5 is 25. Hence, the answer is their difference, 1575.

Example:  $19 \times 17 = 323$

1. Convert 19 to  $(18 + 1)$ . Convert 17 to  $(18 - 1)$ .
2. The square of 18 is 324. The correct answer therefore is 323.

Example:  $28 \times 24 = 672$

1. Convert 28 to  $(26 + 2)$ . Convert 24 to  $(26 - 2)$ .
2. The square of 26 is 676. The square of 2 is four.
3. The correct answer therefore is 672.

## SQUARE-OF-A-BINOMIAL RULE

The algebraic law governing the square of a binomial,  $(a \pm b)^2 = a^2 \pm 2ab + b^2$ , is often useful in computing the squares of various numbers. Suppose, for example, it were not remembered that 324 is the square of 18. The correct answer is easily obtained by performing the steps below.

1. Convert 18 to  $(20 - 2)$ .
2. Square 20, arriving at 400.
3. Multiply  $2 \times 2 \times 20$ , arriving at 80.
4. Square 2, arriving at 4.
5. The answer therefore is  $400 - 80 + 4$ , or 324.

Example: Find the square of 34.

1. The square of 30 is 900.
2. Multiply  $30 \times 4 \times 2$ , arriving at 240.
3. The square of 4 is 16.
4. The answer is  $900 + 240 + 16$ , or 1156.

In addition to the preceding artifices, there are various tricks which are suggested by common sense, according to the circumstance. Suppose a multiplication of 24 by 13 is required. Express 13 as  $10 + 3$ . multiply 24 by 3 to obtain 72, and add to 240. The answer is 312. Of course, this is exactly what is done in the conventional multiplication operation, but if visualization of rows of figures is difficult, it may be helpful to resort to the suggested procedure.

## APPENDIX

*It shall first be proved that the remainder of any number divided by nine is equal to the remainder obtained by dividing the sum of the digits of the number by nine.*

*Any number in the conventional system can be expressed in the following manner:*

$$N = a_0 + 10a_1 + 10^2a_2 + 10^3a_3 + \dots + 10^na_n + \dots$$

where

$N =$  the number,

$a_0 =$  the digit in the unit column,

$a_1 =$  the digit in the tenths column, and so on.

$$\text{Let } S = a_0 + a_1 + a_2 + \dots + a_n + \dots$$

$$\text{Then } N - S = (10 - 1)a_1 + (100 - 1)a_2 + (1000 - 1)a_3 + \dots + (10^n - 1)a_n + \dots$$

*Evidently, all the coefficients on the right-hand side are a series of nines; that is,  $N - S = 9a_1 + 99a_2 + 999a_3 + \dots$*

*Since the right-hand member is divisible by nine,  $(N - S)/9$  is equal to an integer.*

*This can be expressed as*

$$(N - S)/9 = 1$$

*where 1 is the integer.*

*Therefore,*

$$N/9 = 1 + S/9.$$

*This relation shows that the same remainder obtains when a number is divided by nine and when the sum of its digits is divided by nine. The proposition has therefore been proved.*

*Multiply both sides of the preceding equation by nine, resulting in  $N = 9I + S$ .*

*It follows that if S is divisible by three, nine, the number N is divisible by three, nine.*

*To find the remainder of a number divided by nine, it is permissible to continue the process of summing the digits indefinitely. For if the number 37,597 yields the same remainder when divided by nine as does the sum of its digits, 31, it follows that the sum of three and one when divided by nine will leave the same remainder as 31. Thus, the remainder is four. Now let us approach the same question by recognizing that division is the process of finding how many units of the divisor are contained in the dividend. Therefore, when the digits are added, we may exclude all units of nine. For example, in summing 3, 7, 5, 9, 7, we exclude (cast out) the nine immediately, leaving 22, which contains two units of nine (18) plus the remainder four. This procedure*

gives us a second method of summing digits as a means of finding the remainder of a number divided by nine.

Let us devise a check for addition. Consider the number  $N_1, N_2, N_3, \dots$ , which can be expressed:

$$N_1 = 9q_1 + r_1$$

$$N_2 = 9q_2 + r_2$$

$$N_3 = 9q_3 + r_3$$

and so on, where the  $q$ 's represent the number of times nine is contained in a number and the  $r$ 's represent the remainders. Therefore,

$$N_1 + N_2 + N_3 \dots = 9(q_1 + q_2 + q_3 \dots) + r_1 + r_2 + r_3 \dots$$

It follows that if the individual remainders are added and the nines of this sum are cast out, the resulting number should be equal to the remainder obtained by casting out nines of the answer to the addition.

Now let us devise a check for multiplication. Since  $N_1 = 9q_1 + r_1$  and  $N_2 = 9q_2 + r_2$ , their product is  $81q_1q_2 + 9q_1r_2 + 9q_2r_1 + r_1r_2$ . Thus, the remainder of the product is equal to the remainder obtained by dividing  $r_1r_2$  by nine; in other words, by casting out the nines of the product of the two remainders.

The rule for divisibility by two, four, eight, . . . is proved by examining a number expressed in terms of ten.

$$N = a_0 + 10a_1 + 10^2a_2 + \dots + 10^na_n \dots$$

where the  $a$ 's have the meaning described previously. Since the second, third, fourth, etc., columns from the right represent increasing powers of 10, as shown in the preceding equation, each such column is clearly divisible by two, regardless of the digit. For the entire number to be divisible by two, therefore, it is necessary only that the digit in the unit column,  $a_0$ , be divisible by two. If the number is to be divisible by four, the number composed of the digits in the units and tens columns must be divisible by four, since four will divide into  $10^2, 10^3$ , etc. Similarly, a number

is divisible by eight if the number composed of the digits in the units, tens, and hundreds columns is divisible by eight, since eight will divide into  $10^3, 10^4$ , etc. The same reasoning holds for divisibility by 16.

The criterion for nonterminating decimals is easily deduced. Any decimal represents a fraction for which the denominator is  $10^n$ , where  $n$  equals the number of places in the decimal. If the original fraction is first reduced to lowest terms, and then the denominator is resolved into prime factors, it follows that the number  $10^n$ , which is actually the denominator of the decimal, can be a multiple of the original denominator only if five and two are the sole prime numbers of that denominator.

Now it shall be proved that if the sum of the digits in the even places is equal to the sum of the digits in the odd places, the number is divisible by 11.

Let

$$N = a_0 + 10a_1 + 10^2a_2 + \dots + 10^na_n + \dots$$

$$D = a_0 - a_1 + a_2 - a_3 + \dots$$

$$N - D = a_1(10 + 1) + a_2(10^2 - 1) + a_3(10^3 + 1) + \dots$$

The last term on the right is  $a_n(10^n + 1)$  or  $a_n(10^n - 1)$  as  $n$  is odd or even. Hence, every term on the right is divisible by  $(10 + 1)$ , or 11. If the sums of the digits in the odd and even places are equal,  $D = 0$ , and the proposition is proved.

Suppose the number  $N$  observes the sequence  $a_1, a_2, a_3, a_1, a_2, a_3$ . Then

$$N = a_3 + 10a_2 + 10^2a_1 + 10^3a_3 + 10^4a_2 + 10^5a_1$$

Immediately, it is evident that  $D = a_3 - a_2 + a_1 - a_3 + a_2 - a_1 = 0$ , showing that the number is divisible by 11. Subtract  $D$  from  $N$ .

$$\begin{aligned} N - D &= a_2(10 + 1) + a_1(10^2 - 1) + a_3(10^3 + 1) + a_2(10^4 - 1) + a_1(10^5 + 1) \\ &= a_1(100,100) + a_2(10,010) + a_3(1001) \\ &= (7)(11)(13)(100a_1 + 10a_2 + a_3) \end{aligned}$$

Since  $D = 0$ ,  $N$  is divisible by 7 and 13 as well as by 11.

The rule for multiplying numbers ending in five will first be proved for two-digit numbers. Write such numbers as  $(10a_1 + 5)$  and  $(10a_2 + 5)$ . For example, 65 is equivalent to  $(6 \times 10 + 5)$ .

$$\begin{aligned} & (10a_1 + 5)(10a_2 + 5) \\ &= 100a_1a_2 + 50(a_1 + a_2) + 25 \\ &= 100a_1a_2 + 100(a_1 + a_2) \\ & \quad (5/10) + 25 \\ &= 100a_1a_2 + 100(a_1 + a_2) \\ & \quad /2 + 25 \end{aligned}$$

A study of the foregoing relationship will show that the rule for multiplying numbers ending in five has been proved. This will be made clear by considering the example  $35 \times 45$ . Here  $a_1 = 3$  and  $a_2 = 4$ . The first term of the answer,

$100a_1a_2$ , is equal to 1200. The second term,  $100(a_1 + a_2)/2$ , is equal to  $700/2 = 350$ . The last term is 25. Consequently, their sum is 1,575.

To extend the rule for larger numbers ending in five, such as 135, 5,675, and so on, let  $a_1 = b_1c_1 \dots$  and let  $a_2 = b_2c_2 \dots$ , where the letters designate the respective digits of the number. Since  $a_1a_2$  now represents  $(b_1c_1 \dots)(b_2c_2 \dots)$ , and  $(a_1 + a_2)$  represents the sum of  $b_1c_1 \dots$  and  $b_2c_2 \dots$ , the original proof is still valid.

### REFERENCES

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 Wentworth, G. A., *College Algebra*, Ginn & Company, 1888.



## Last Month's "What's Your Answer?"

### The Great Joule Robbery

The disappearance of 200 joules is explained by the fact that the little demon carried them off in his box. A reconstruction of the crime requires an application of the following fundamentals:

1. The voltage across the coils must at all instants equal the applied voltage.
2. Inductance is effectively the "mass" of a current, and as such opposes only a *change* in an electron flow.
3. Inductance in henries equals the induced voltage per change in amperes per second. Or, rearranging, the change in amperes per second equals the induced voltage per henry.
4. Energy is absorbed by a circuit element when the electrons enter at the negative terminal and leave at the positive terminal, and energy is lost if the electrons flow in the reverse direction relative to the voltage across the circuit element.
5. One watt equals one joule per second, or the number of joules equals the average power in watts times the time in seconds.

When  $S_1$  is closed, the voltage across  $L_1$  is  $(12 \text{ volts} - 4 \text{ volts}) = 8 \text{ volts}$ . This voltage can only occur if the current is rising at a rate  $1/\text{second} = 8 \text{ volts}/8 \text{ henries} = 1 \text{ ampere per second}$ .

Ten seconds later, when  $S_2$  is closed, the current through  $L_1$  will have risen to 10 amperes. Closing  $S_2$  shorts out the 12-volt battery, and the fuse action removes it completely from the circuit.

The two coils are now connected in parallel, so the total inductance is now  $8 \text{ henries}/2 = 4 \text{ henries}$ , and the applied voltage is now 4 volts, but in the opposite direction. This means that the current must now change in the op-

posite direction at a rate  $1/\text{second} = 4 \text{ volts}/4 \text{ henries} = 1 \text{ ampere per second}$ .

In the absence of frictional forces (resistance) or an up-hill potential gradient (the 4-volt battery), the inductance  $L_1$  would maintain the 10-ampere current, just as the inertia of a moving ball would maintain the ball's velocity after an initial force were removed. Nor would  $L_1$  share the current with  $L_2$ , since both would oppose any change in the electron flow. The change that actually occurs, then, is due entirely to the presence of the 4-volt battery.

The reverse voltage across  $L_1$  can only occur if the current falls at a rate  $1/\text{second} = 4 \text{ volts}/8 \text{ henries} = 1/2 \text{ ampere per second}$ .

Similarly, 4 volts across  $L_2$  can only occur if the current rises (in a direction opposite to the flow through  $L_1$ ) at a rate of  $1/2 \text{ ampere per second}$ .

The fall of current in  $L_1$  and the rise of current in  $L_2$  provide an effective decrease for the circuit as a whole of 1 ampere per second.

At the end of 10 seconds, the current through the battery circuit will have fallen to zero, the current through  $L_1$  will have fallen to 5 amperes, and the current through  $L_2$  will have risen to 5 amperes. At this point, the little demon disconnects the box from the external circuit, and thus runs away with a circulating current of 5 amperes.

With an average current of 5 amperes for 10 seconds, the 12-volt battery donated  $J = VI_{\text{ave}}t = 12 \times 5 \times 10 = 600 \text{ joules}$ .

With an average current of 5 amperes for 20 seconds, the 4-volt battery received  $J = 4 \times 5 \times 20 = 400 \text{ joules}$ .

The stolen energy in the magnetic field is the difference or  $J = 1/2 LI^2 = 1/2 (L_1 + L_2)I^2 = 16/2 \times 5^2 = 200 \text{ joules}$ .

*Material Needed for*

**"What's Your Answer?"**

One of the most difficult problems faced by the BULLETIN staff is that of obtaining suitable problems for the "What's Your Answer?" feature. It is an interesting fact that this feature, by far the most popular item in each issue, has produced a greater number of written comments than any other portion of the magazine, but, strangely enough, fewer submitted manuscripts.

Certainly, among the nearly 8000 BULLETIN readers, there must be a great number who know of suitable problems as yet unpublished. Therefore, any reader who wishes to do so may consider this as a hint.

To be acceptable for publication, your problem must pertain to some phase of electronics, it must be briefly stated, and it must have a valid solution (supplied with the original manuscript).

Do you have one that fits these requirements?

## **In Coming Issues**

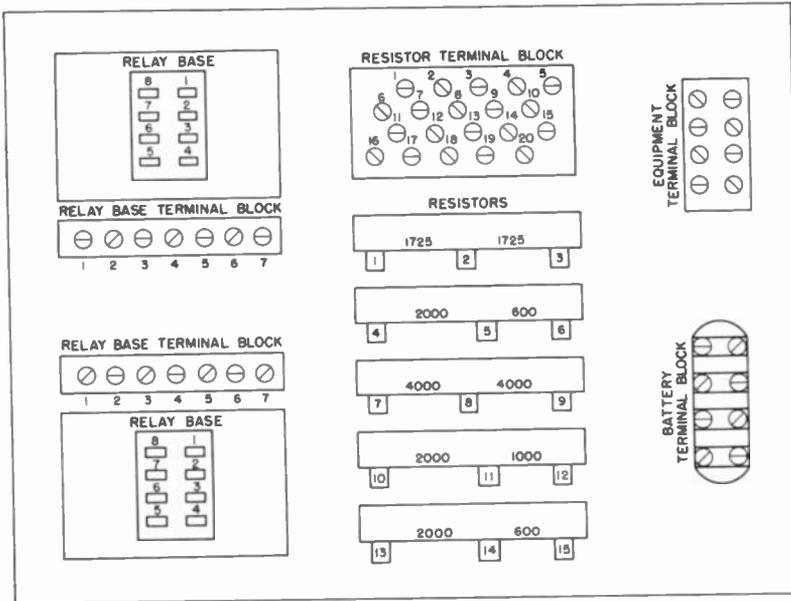
Did you ever wonder why two manufacturers of a particular antenna type often publish radiation patterns which appear widely different in shape and in relative size of corresponding lobes, and yet obviously apply to the same basic design? If you did, a new article by Gail W. Woodward, titled "Antenna Pattern Plots," will explain the situation for you in next month's issue.

Also to appear in the February issue, is Installment IV of John Buchanan's very popular "Introduction to Transistor Electronics," as well as a very practical writeup on "Switchboard Ringing Interference Elimination," by Philco Field Engineer F. C. Farrar.

## A Teletype Training Aid

The accompanying sketch illustrates a breadboard-type training aid which I recently manufactured for training teletype and radio-maintenance personnel. The board's versatility is limited only by the imagination of the instructor using it, and at this station it has proved invaluable as a training aid for demonstrating teletype-loop circuits and Ohm's Law problems.

The resistors used are of the 10-watt variety, and are standard teletype parts, of values commonly employed in teletype-loop circuits. The equipment and battery terminal blocks have nothing connected to them, and are used for connecting rectifiers and external equipment to the circuits. The relay bases are standard 255-A bases, and are connected to the associated terminal blocks, num-



When the device is used in conjunction with standard teletype equipment and rectifier power supplies, it is possible to simulate nearly all polar or neutral circuits. Open circuits and reversals can easily be demonstrated, as well as conversions from neutral to polar operation and from polar to neutral operation. Calculations of necessary resistances for various circuits can be made and then proved under actual operating conditions with this board.

ber-to-number. (Since pin 8 is not used in the relay, no provisions were made for it.) The resistors and resistor terminal blocks are connected in a similar manner. All connecting leads are passed through holes drilled in the board and soldered in place beneath it.

In use, various lengths of wire with spade clips attached are used to wire up the desired circuits. Relays must, of course, be inserted into the relay bases.

Regis E. Gates  
Philco Field Engineer



