

# PHILCO

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# PHILCO TECHREP DIVISION BULLETIN

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## CONTENTS

Editorial .....	2
<i>By John E. Remich</i>	
Antenna Pattern Plots .....	3
<i>By Gail W. Woodward</i>	
What's Your Answer? .....	6
Introduction to Transistor Electronics— Installment IV .....	7
<i>By John Buchanan</i>	
Rocket and Rocket Circuit Testing .....	37
<i>By Donald Love</i>	
Solution to Last Month's "What's Your Answer?"	41
Eliminating Switchboard-Ringing Interference in Carrier-VHF Systems .....	42
<i>By F. C. Farrar</i>	

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If any information contained herein conflicts with a technical order, manual, or other official publication of the U.S. Armed Forces, the information in the official publication should be used.

# *Editorial*

## **SAFETY**

by John E. Remich

Manager, Technical Department

In spite of the fact that many millions of dollars are spent each year on safety campaigns throughout America, and that much of the effort is directed toward reducing the incidence of injury or death from electric shock, yet many thousands of persons die needlessly in this manner every year. Furthermore, many of these deaths are the result of pure carelessness on the part of individuals who should and do know most about the danger involved.

Typical of such cases is one written up recently in several Philadelphia and Camden newspapers. The man involved had been an electronics technician for many years, and an employee of one of the country's largest electronics manufacturers for 16 years. At the time of the accident, he was testing a power amplifier in a production-model television transmitter from which primary power had just been disconnected. Although he had tested many similar units before, and was well aware that the unit contained large 3000-volt filter capacitors, he failed to take the simple precaution of using a shorting stick to discharge them. The result, when he accidentally touched one of the still-charged capacitors, was instant death.

This is not an isolated type of incident—in fact, there are many exactly like it each year in American industry and the Armed Forces. Thus, each of us in electronics should be particularly careful to maintain the proper respect for potentially dangerous electronic equipment. Always use a shorting stick (the bleeder might be open), keep one hand in your pocket if you *must* work on energized equipment, and don't short out interlocks.

In brief, always observe common-sense safety precautions when working with electrical or electronic equipment, and never become complacent about, or contemptuous of, the possibility of serious injury or death from electric shock. It can happen to *any* of us, don't let it happen to you.

# ANTENNA PATTERN PLOTS

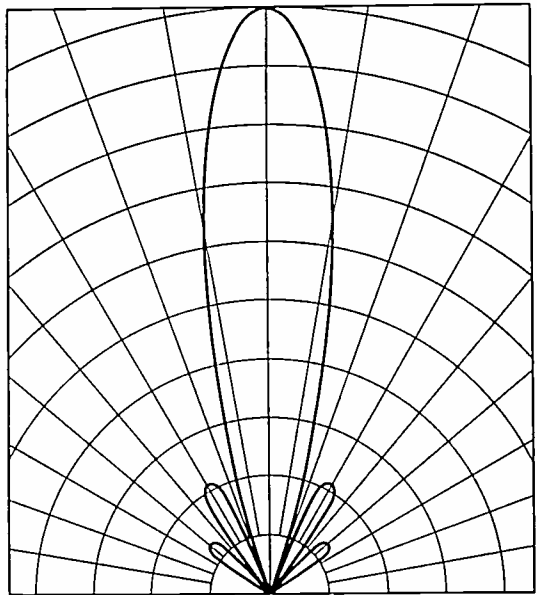
by Gail W. Woodward  
Headquarters Technical Staff

**A comparison of the various methods used to display antenna radiation patterns.**

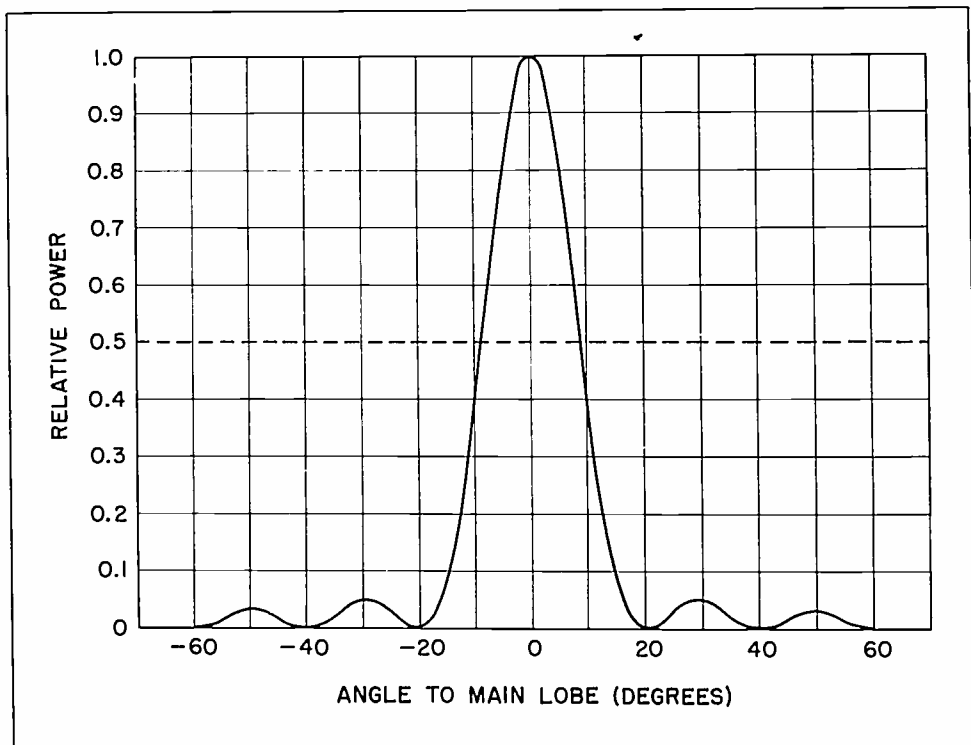
**T**HE EVALUATION of published antenna patterns sometimes becomes confusing, because the appearance of a plot taken in the field often looks quite different from the manufacturer's claim or from the textbook pattern. A great deal of difficulty arises from failure to show coordinates or failure on the part of the technician to take note of the type of presentation. Another confusion factor arises when a manufacturer shows his antenna patterns in one set of coordinates while he displays a comparison pattern using another set of coordinates.

## DISPLAY METHODS

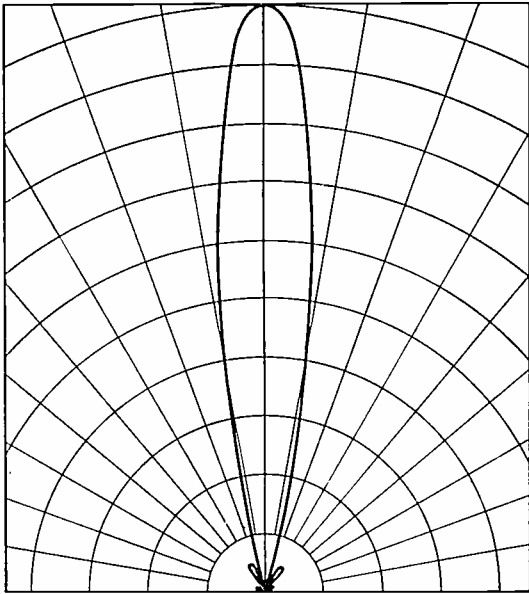
There are two general methods of displaying patterns. One uses polar-coordinate graph paper, and the other



**Figure 1. Power Plot Using Polar Coordinates**



**Figure 2. Power Plot Using Rectangular Coordinates**



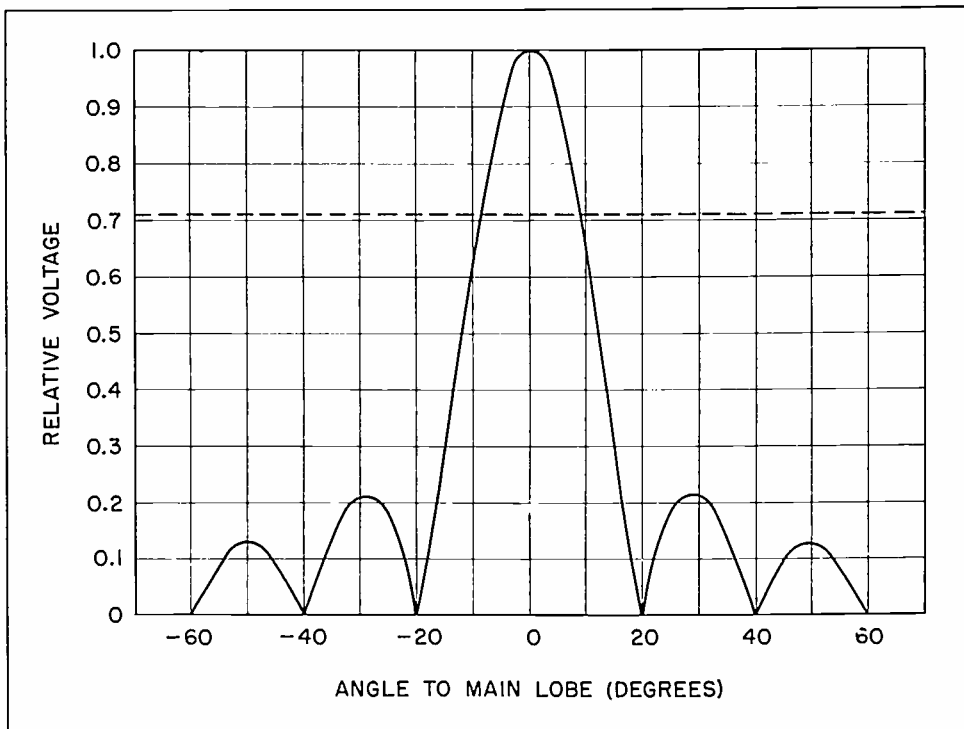
**Figure 3. Voltage Plot Using Polar Coordinates**

uses rectangular-coordinate graph paper. Figure 1 shows a plot of an antenna pattern using polar coordinates, and figure 2 shows the same plot with rectangular coordinates. At first glance the two patterns do not appear at all related, but a careful check will show the equality. Figure 2 could be con-

verted to figure 1 by reducing the horizontal base line to zero length, while leaving the upper portions proportionately spread out.

To simplify the following discussion, a theoretically perfect antenna is considered. All of the patterns shown in this article are based upon a perfect parabolic reflector, uniformly excited by a point source located at the focal point of the reflector. Wavelength and reflector size are such that the beam-width at the half-power points is somewhat less than 20 degrees. Only four minor lobes are shown but a large number is actually present. Since the relative amplitudes of the minor lobes decrease as the angle to the main lobe increases, it is necessary to consider only the larger ones.

The pattern shown in figure 3 appears much inferior to the one shown in figure 1; however, the two plots are taken on the same antenna—figure 3 is a plot of voltage (or current) while figure 1 is a plot of power. Figure 4 is a



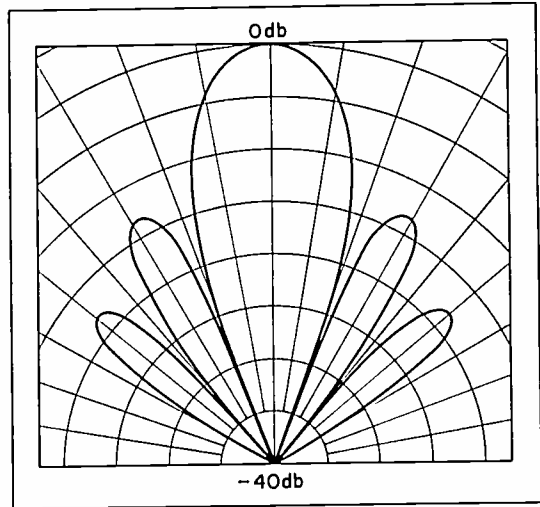
**Figure 4. Voltage Plot Using Rectangular Coordinates**

voltage plot using rectangular coordinates. A good point of comparison is the beamwidth—notice that the beamwidth is the same for the half-power points as it is for the 70% voltage points ( $70\% E = 50\% W$ ).

It is a safe bet that if one were selling antennas to relatively uninformed consumers, he would publish only power plots with polar coordinates.

Figure 5 is a rather horrible looking antenna pattern of the same antenna used to derive the first four illustrations. Here, a polar-coordinate plot in terms of decibels is used. The level at the point of maximum radiation is taken as the zero-db reference. Figure 6 shows the same plot using rectangular coordinates—again note that the half-power (3-db) points correspond to the other beamwidth points. The same general shape of pattern would have been obtained if the power scales in figures 1 and 2 were made logarithmic.

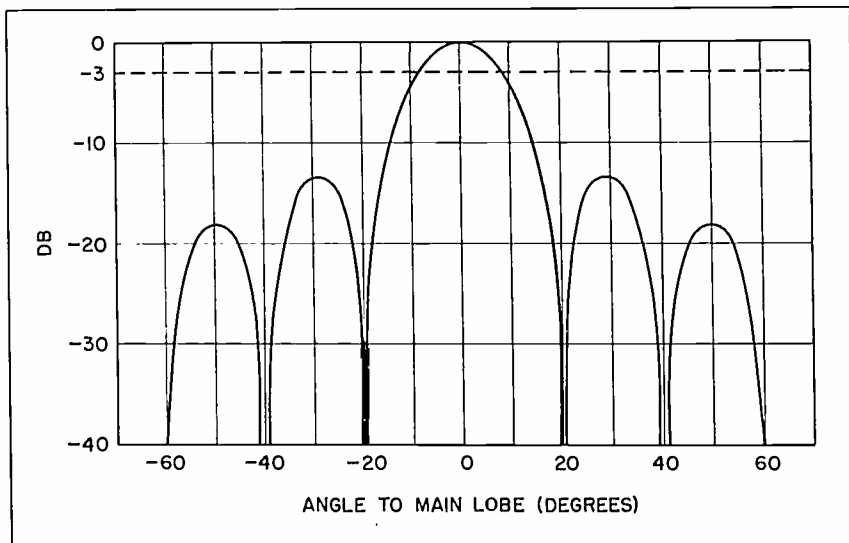
The plots in figures 5 and 6 have been made to look especially bad by choosing a large range of db values (40 in this case). Figure 6 shows that the base line could have easily been drawn at 20 db. This would make the minor lobes appear much smaller. A very interesting point



**Figure 5. Decibel Plot Using Polar Coordinates**

regarding the db plot is the significance of the minima values. If the voltage (or power) were actually zero, the db value would be equal to minus infinity; therefore, the minima shown do not represent actual values. This type of plot is by far the most informative and versatile, but, since the antenna characteristics look rather poor at first glance, plots of this type are seldom found in published data.

A fairly accurate plot of a radar antenna pattern can be obtained by scanning the antenna past an echo box located a few radar-dish diameters away



**Figure 6. Decibel Plot Using Rectangular Coordinates**

from the antenna. The horizontal antenna pattern will be traced out on the indicator. If a B-scan presentation is used, the plot will have rectangular coordinates, while if a PPI-type scan is used, the plot will have polar coordinates. A fairly accurate plot of the free-space pattern will be obtained if the echo box is backed up by a power absorber whose dimensions subtend the

angle included in the antenna's larger minor lobes.

### CONCLUSION

It should be pointed out that merely comparing the published shapes of antenna patterns is not enough. The technician must take careful note of the units used as well as the type of plot. If a published plot does not indicate units, it is very likely to be in terms of power.

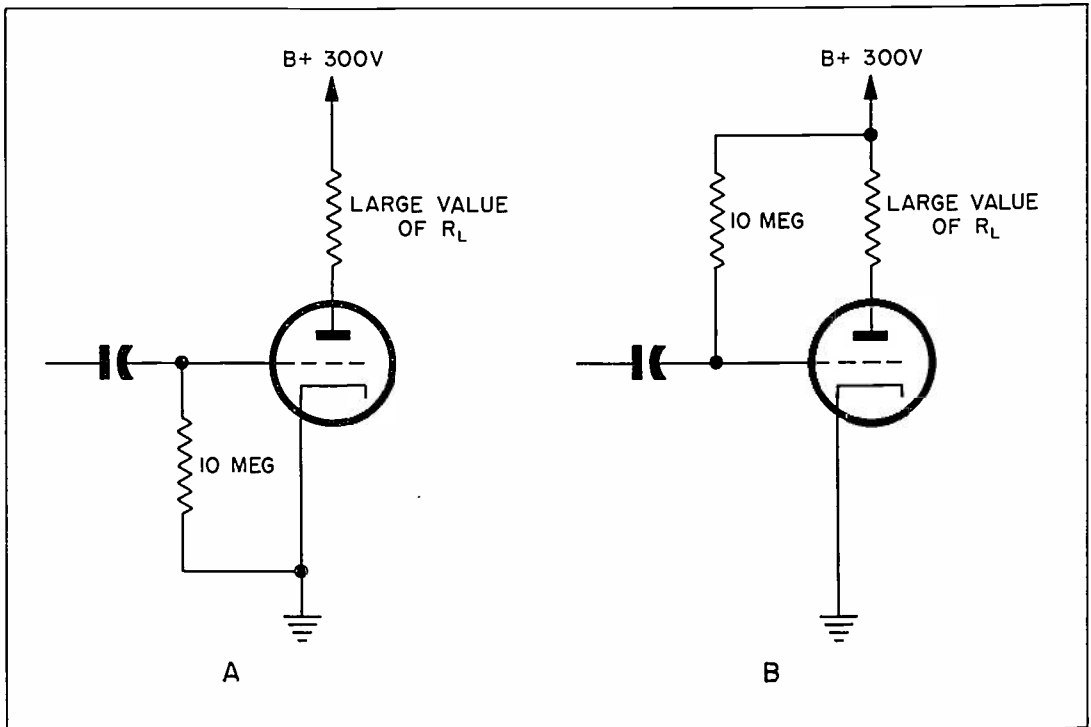
## What's Your Answer?

Consider the contact-biased a-f voltage amplifier as it is often found in home radios (see drawing A). Assume that the zero-signal bias measures one volt.

It is apparent that if a low-resistance 0.1 ma. meter were connected across the grid-leak resistor, the bias voltage would be essentially zero, and the meter would read maximum contact current. Assume in the case shown that this current value is 0.5 ma.

Now, suppose that instead of returning the grid leak to the cathode, it is returned to a positive voltage such as the B+ point as shown in figure B. Roughly, how far positive will the grid be driven in respect to the cathode?

(Solution next month)





# INTRODUCTION TO TRANSISTOR ELECTRONICS

## Installment IV

### Classical Background for the Free-Electron View (Continued)

by John Buchanan  
Technical Publications Dept.

A further continuation of the discussion of the methods and theories of classical physics, in which some of the concepts of statistical mechanics are introduced, and the derivation of the Maxwell-Boltzmann distribution law is presented.

#### SYNOPSIS

*This is the fourth in a series of articles covering the fundamentals of transistor electronics. The first article of the series, which appeared in the January, 1952, issue of the BULLETIN, covered the atomic view of solids. This view maintains the particle nature of the electron, but introduces the quantum to restrict the electron's motion within the atom to definite shells, or energy levels. Each shell, in turn, permits a finite number of planetary orbits, or electron states, at the shell's respective energy level. Outside an atom, an electron is assumed to be completely free to move about with random thermal motions. Electronic conduction is visualized as a drift of free electrons, or as a drift of empty electron energy states (electron holes), or as a combination of the two.*

*The second article, published in the April issue, laid the groundwork for the more involved free-electron view of solids. The first elements of the kinetic*

*theory of gases were presented as a point for beginning the transition from the classical laws of ideal gases to the wave-mechanical laws of electron gases.*

*The third article, published in the September issue, continued the discussion of classical theories as applied to systems of ideal particles. Here the "degree of freedom" was introduced as a mathematical concept useful in interpreting particle behavior and in analyzing the distribution of energy within a system. The discussion led to a theory of metallic heat capacity based upon the atomic view that the free electrons behaved approximately as ideal gas molecules. Actual measurements, however, revealed that the "free" electrons have a negligible capacity for absorbing thermal energy at room temperatures—a complete contradiction of the atomic view of classical electrons in thermal equilibrium with the atoms.*

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#### COMING ATTRACTIONS

The unprovoked betrayal of the classical ideals by the metallic electrons was, in truth, the signal which opened the quantum's campaign against free bodies. It will be recalled that the first conquest found its victim in the electro-

magnetic wave—which was reduced from its ethereal role to the status of a mere photon, and forced to share the laws of particles. Now it is that the particles are under attack, and the wily quantum, with an implacable thirst to dictate a classless nature for the whole universe for all time, is maneuvering to

force the particles to share the laws of waves. The fortifications of the classical mechanics, however, are too deeply entrenched to risk a frontal attack; and hence the campaign is to be waged primarily from within.

The intrigue begins inside the metals, where the free electrons, like their planetary brethren, are attracted by the quantum's promises of order and security. The electron seems to have the ruling passion of one who has tasted freedom, but knows not its flavor beyond the primal urge of a blind body in soundless space to feel unbound. Its passion is not for degrees of freedom it cannot distinguish, but only for an absence of obstacles in the path ahead. Wherever the quantum can guarantee the privileges of a two-track orbit—be it about an atom, molecule, or a crystal—a crowded electron pair will trade their degrees of freedom for the exclusive property rights. It was through the quantum's success in cornering the phase-space market inside of copper that led to the catastrophic results of the heat-capacity experiment reported in the previous discussion. However, the means by which this feat was engineered, and the ultimate implications, involve the degeneracy of the grandest of all the ideal gas laws—the Maxwell-Boltzmann law for the distribution of velocities.

This law will be the last of the preliminary classical theories of particles that we discuss; but it will prove first in importance as a standard of reference for gauging the strange social order of the quantum and the electron in solids.

### TODAY'S FEATURE

The laws of ideal molecules which have been discussed thus far, although fundamental as points of departure in passing from classical to quantum statistics, have involved only the behavior of molecules as averaged over the system as a whole, and thus have been of

relatively elementary derivation. The conduction properties of a solid, however, depend primarily upon the actual distribution of energy among the electrons, rather than merely the energy per average electron. The distribution in metals is described by Fermi-Dirac statistics, and in semiconductors and insulators by wave functions that must take into account the crystal lattice of positive ions. The new quantum concepts are certain to prove a challenge to our normal mental habits, but compounding the difficulty will be the ways of thinking that are peculiar simply to statistical mechanics—classical, as well as quantum. It is our purpose, in this article, to become acquainted with some of the concepts and methods of statistical mechanics before they become involved with quantum considerations. Other than this broad goal, however, there is the more immediate problem of understanding the equations that express the actual microscopic behavior of a system of particles. Each of these goals should be fulfilled if, by starting with familiar principles, we discover for ourselves the Maxwell-Boltzmann distribution law—the law which defines the distribution of energy among the molecules of an ideal gas at thermal equilibrium. The concepts gained will then form a principal bridge in crossing from ideal to electron gases.

We begin with a brief examination of the mechanics of impacts between ideal molecules. Next, assuming such impacts take place, we make the assumption of equal probabilities for all directions of motion, and “extract” from that assumption as much knowledge of the nature of a gas at equilibrium as possible. Afterwards, another equal probability is to be assumed—the equal probability of each position in phase space that the given gas system can conceivably occupy. This postulate will be pursued only to indicate the path which will link the energy states of ideal *dis-*

*tinguishable* particles with the energy states of *indistinguishable* electrons. The discussion up to this point is to be largely suggestive, rather than concrete, except that new terms and statistical methods are to be introduced. Following this, however, we trace the logic of the steps that Maxwell employed, or implied, in his original derivation of the velocity distribution law. Although our derivation will not be mathematically rigorous, the opportunity will be afforded to see how the constants and variables find their places; and the similar quantum distribution laws, such as the Bose-Einstein and the Fermi-Dirac laws, may then appear later without seeming unduly mysterious. Next, a breather is taken while a symposium is held pitting the "deterministic" approach in physics against the "statistical" approach. And last, there is a brief introduction to the term *degeneracy* as applied to systems of particles. At the end of this discussion, we may consider ourselves to have arrived at the turn of the century, and thus to be standing at the same point from which we originally departed in developing the atomic theory of solids. This time, however, we shall follow those events that lead to the undoing of the atomic point of view, and to the development of the free-electron theory.

### FREE TRADE AMONG IDEAL MOLECULES

Throughout the following discussion

we assume a system containing an ideal gas of monatomic molecules which are perfectly spherical and frictionless, and are all of the same size and mass. No forces, except those caused by impacts, act upon the molecules. It is also assumed that the volume is maintained constant within fixed boundaries, that the laws of conservation of energy and momentum hold, and that the system is in a state of thermal equilibrium. The number of molecules,  $N$ , is very large, say something on the order of  $10^{24}$ , so that the chance of any measureable fractional variation from the equilibrium state is negligible.

Figure 1 shows the result of a head-on impact between an ideal molecule in motion and an ideal molecule at rest. Assuming the mass is the same for each molecule, the laws of conservation of momentum and energy require that the striking molecule lose all its momentum to the one formerly at rest. In other words, such an impact results in a complete exchange of velocity, momentum, and energy between the two molecules.

Figure 2 shows the result of a head-on impact between molecules, both of which are in motion relative to the observer. The vectors indicate the direction and magnitude of the velocities before and after the impact. Here, as in figure 1, the laws of conservation of energy and momentum require that the impact result in an exact exchange of velocity, momentum, and energy be-

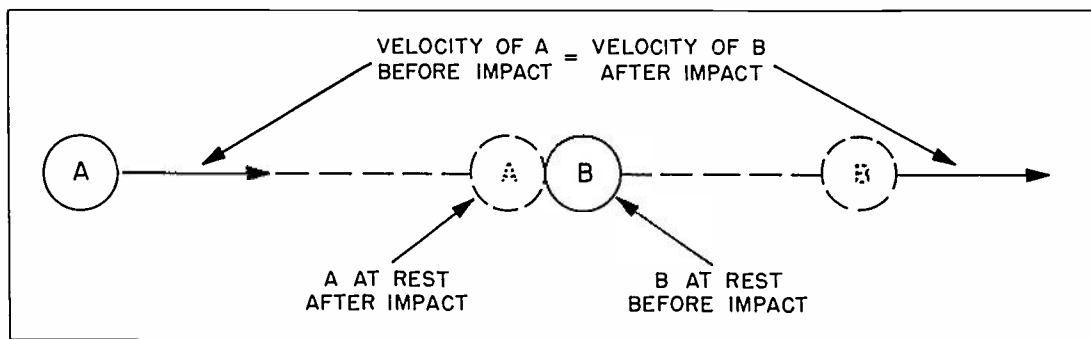
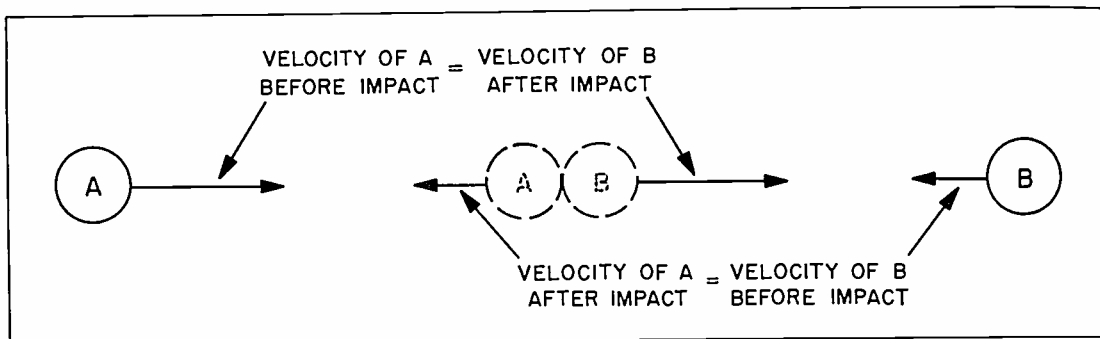


Figure 1. Result of Head-on Impact of Ideal Molecule in Motion, with Ideal Molecule at Rest

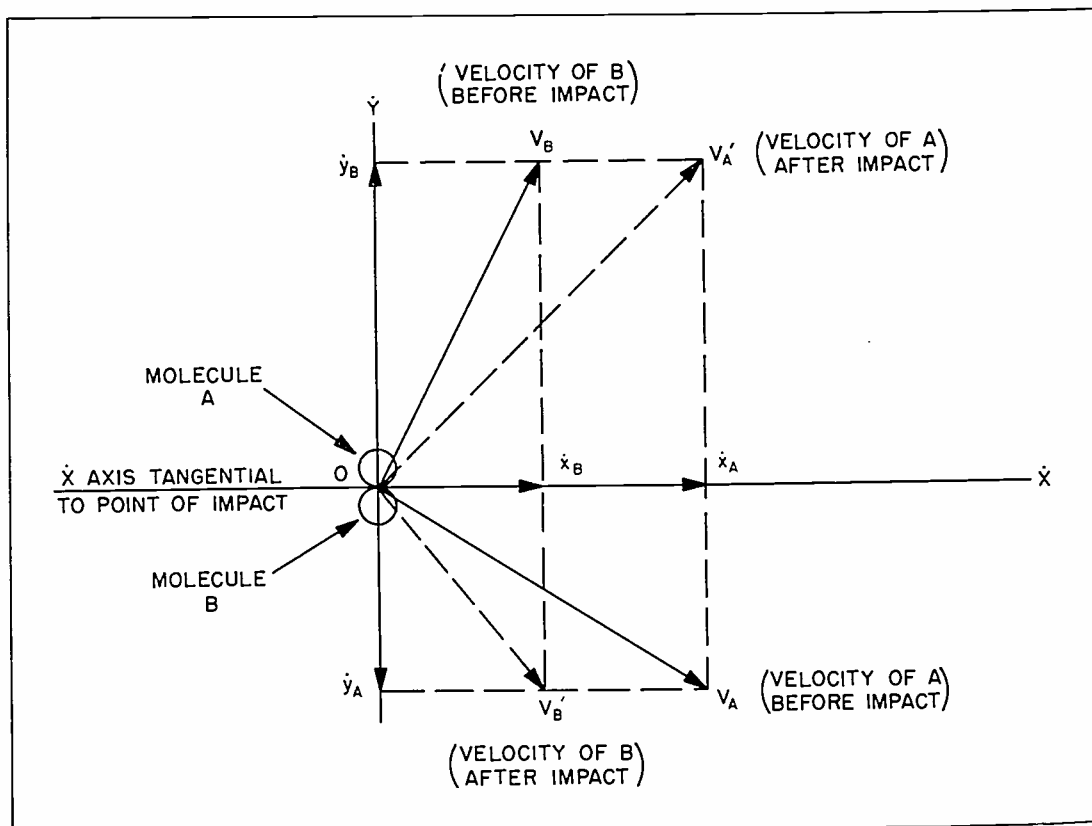


**Figure 2. Result of Head-On Impact Between Two Ideal Molecules, Both of Which are in Motion Relative to the Observer**

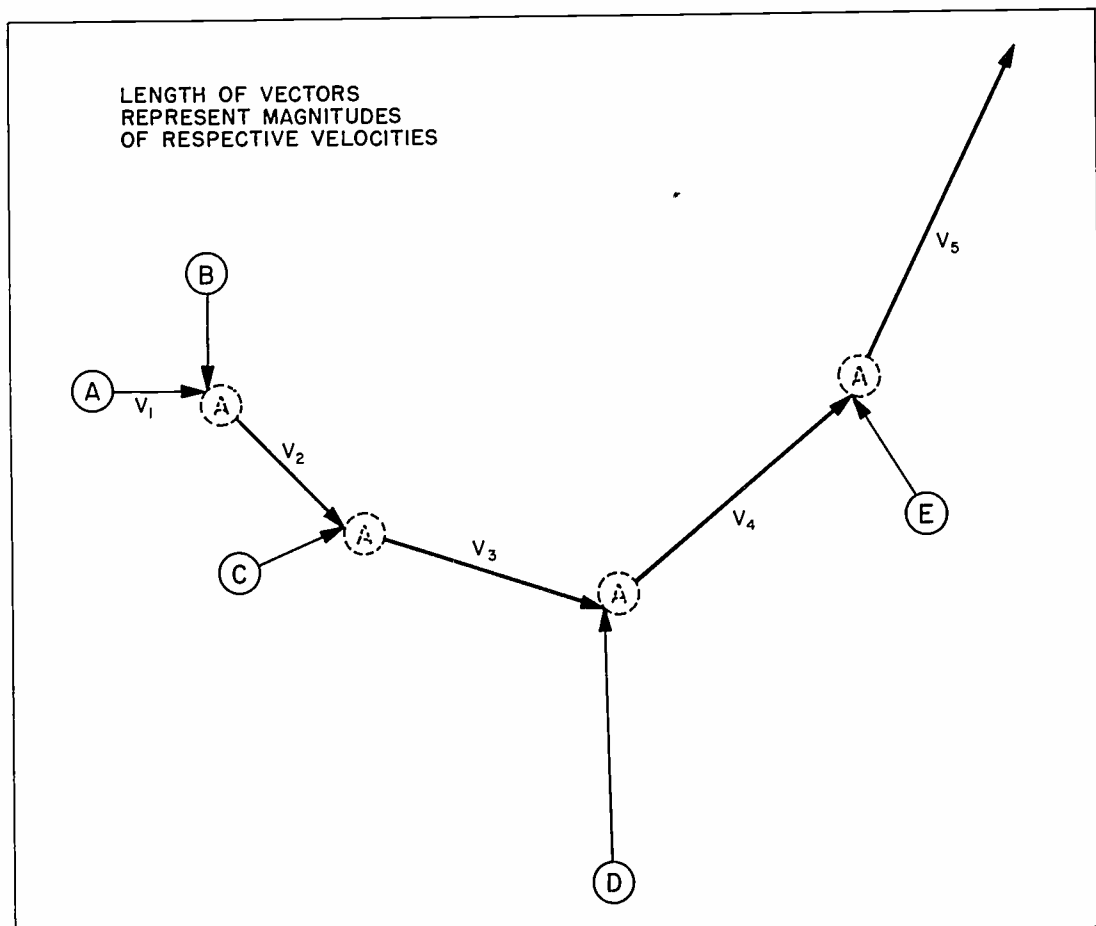
tween the two molecules. After the impact, molecule A has the same velocity that molecule B had before the impact, and vice versa.

Figure 3 shows the result of an impact between two molecules which are not traveling along the same straight line. We let point 0 represent the point of impact, and draw a velocity axis ( $\dot{X}$ ) through 0 tangential to the two molecular spheres. We then draw the  $\dot{Y}$  axis so

that the  $\dot{X}\dot{Y}$  plane includes the direction lines of the molecular velocities. With this reference system, the velocities of the two molecules before the collision can be divided into two rectangular components. The  $\dot{x}$  component of each molecule is in the tangential direction, and is *not affected* by the collision. The  $\dot{y}$  component of each molecule is in the direction of a head-on collision, and hence is exchanged for the  $\dot{y}$  component of the other molecule. Before im-



**Figure 3. Velocity Exchange Resulting from Random Impact Between Two Ideal Molecules**



*Figure 4. Imaginary Sequence of Collisions of Molecule A, in Which Molecule A Absorbs Energy from Molecules B, C, D, and E, in That Order, Thereby Increasing the Magnitude of its Velocity from  $V_1$  to  $V_5$ . (In each collision, the initial velocity of A is assumed to be tangential to the surfaces at the point of impact, and the initial velocity of the colliding molecule to be normal to the surfaces at the point of impact.)*

At the time of impact the energies of the molecules are:

$$E_A = \frac{1}{2} m \dot{x}_A^2 + \frac{1}{2} m \dot{y}_A^2$$

(molecule A)

$$E_B = \frac{1}{2} m \dot{x}_B^2 + \frac{1}{2} m \dot{y}_B^2$$

(molecule B)

After impact the energies of the molecules are:

$$E_A = \frac{1}{2} m \dot{x}_A^2 + \frac{1}{2} m \dot{y}_A^2$$

(molecule A)

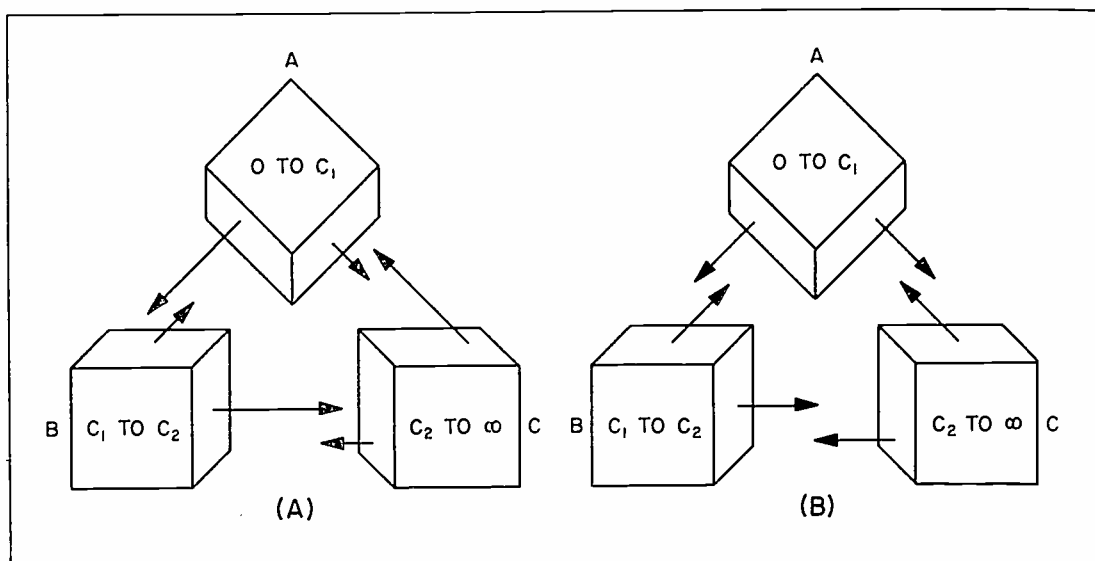
$$E_B = \frac{1}{2} m \dot{x}_B^2 + \frac{1}{2} m \dot{y}_B^2$$

(molecule B)

Likewise, the velocity and momentum of each molecule after impact are the vector sums of the molecule's original  $\dot{x}$  component, and its new  $\dot{y}$  component. It should be noted, however, that if the masses of the two molecules differed, momentum and energy exchanges would

require much more complicated equations than those above.

Figure 4 indicates how the energy of a molecule can be increased by a succession of collisions. Each increase of energy requires that the  $\dot{y}$  component gained by impact be greater than that lost in exchange. For a given angle of impact, however, the higher the initial velocity of a molecule, the less probable will be its chance to have a smaller  $\dot{y}$  component than that of the molecule with which it collides. In a system containing as many as  $10^{24}$  molecules, where each molecule is colliding millions of times a second, there should be at all instants large numbers with ex-



**Figure 5. Equilibrium Conditions for Ideal Gas.** (Each box is imagined to contain all the molecules with speeds between the limits indicated. At equilibrium, the number in each box remains constant. *A*—Imaginary state of equilibrium, with a net flow of molecules “circulating” counterclockwise—the number in each speed box, however, remaining constant. *B*—Actual thermodynamic state of equilibrium, with the traffic between any two boxes the same in both directions.)

tremely high energies above the average. Theoretically, assuming an infinite time, every conceivable combination of impacts, with their resulting distribution of energy, would occur and reoccur an infinite number of times. However, because of the phenomena of thermal equilibrium, there is every reason to believe that at least one distribution of energy is vastly more probable than others. Such a distribution of energy, of course, will also correspond to the most probable distribution of speeds, and of velocities. Indeed, if either an energy, speed, or velocity distribution is known, it is a simple matter to determine the other two. We shall begin with a preliminary investigation of the equilibrium state as represented by the speed distribution.

### TRAFFIC PROBLEMS OF THE EQUILIBRIUM STATE

A function which defines a distribution of speeds will indicate the number of molecules with speeds within any chosen range—provided the range is sufficiently wide to justify a statistical

average. The distribution, remember, is not primarily concerned with *which* molecules, but simply the *number* of molecules, that have certain speeds. Nevertheless, to determine the distribution, we must assume that one molecule can be distinguished from another. The reason for this will become apparent later.

An equilibrium state, is by definition, a state with a static distribution—that is, the number of molecules within each speed range remains constant.

In figure 5, a system at equilibrium is arbitrarily divided into three “speed boxes.” Since, at equilibrium, the number of molecules in each box remains constant, this state is reached only when the rates at which molecules enter all three boxes through collisions equal the rates at which they are lost through collisions. Figure 5A shows an equilibrium with a net flow from box A to box B, balanced by an equal net flow from B to C back to A. Although this would define an equilibrium state, Boltzmann proved that the conditions indicated in figure 5B, where no net flow

exists between any of the boxes, is the one and only equilibrium consistent with thermodynamical postulates. In other words, the equilibrium rate of impacts converting A molecules to B molecules equals the reverse B-to-A rate, regardless of which particular speed limits we assign respectively to A and B. Even if a system is imagined to be divided among a billion speed boxes, this same equilibrium condition would hold between any two of the boxes. This, then, is the type of distribution we are now setting out to discover, purely by hypothetical reasoning.

Figuratively speaking, we are, in effect, to analyze the behavior of a self-contained society of completely independent and equal bodies. All the forces that hold the society together are represented by the impacts against the walls of the container, and all the forces that control the "social order" arise from the impacts of the members against each other. Experience encourages one to suppose that such a society does not remain completely lawless, but gradually drifts toward a state of equilibrium. This state represents the division of activity most favorable to ideal nature. The problem is to determine what the *a priori* probabilities are, "for the sake of" which the society cannot remain a complete anarchy, but must approach an ideal limit of evolution.\*

### SPEED LIMITS IN VELOCITY SPACE

Consider now a system consisting of a large number of molecules,  $N$ , at thermal equilibrium. We wish to investigate the relationships between the three com-

ponents of velocity of all the molecules which have a speed between  $c$  and  $c + \Delta c$ . We cannot consider simply the molecules with a speed  $c$ , for theoretically no two ideal molecules can have *exactly* the same speed, unless we assumed an infinite number of molecules. However, if we imagine  $\Delta c$  to be very, very small, for example, on the order of one micron per second (one millionth of a meter per second), then the behavior of the molecules within that range, for all practical purposes, will be the same as their behavior at  $c$ . To specify that  $\Delta c$  is very small, we shall use the calculus notation  $dc$ , in place of  $\Delta c$ . However,  $dc$  is not to be considered so small that it approaches zero as a limit; for then, since we are assuming a finite number of molecules, there would not be a sufficient number,  $N_c$ , with speeds between  $c$  and  $c + dc$  to warrant a statistical average.

Remember that the system is in a state of equilibrium, and so the number of molecules within any given region of the system remains constant. This is true whether the particular region represents a volume in geometrical, velocity, or phase space.  $N_c$ , as defined above, is the equilibrium "population" of a small volume in velocity space.

Since  $c$  is speed, not velocity, it is not represented by a vector in velocity space. Instead, it is the surface of a sphere as shown in figure 6. Any point on the spherical surface, however, represents a velocity with a magnitude equal to  $c$ , and a direction according to the position of the point. The outer surface shown in figure 6 is that of  $c + dc$ , and hence the distance (greatly magnified) between it and the inner surface is equal to  $dc$ . Any vector that ends in the thin shell formed by the two surfaces is thus a velocity with a speed between  $c$  and  $c + dc$ . In a similar manner, we imagine the whole of velocity space to be divided into a series of concentric spheres, each one separated by "speed

\* A thermodynamicist would never agree that an isolated thermodynamical system progresses from a less orderly to more orderly state. To him, all the molecules circulating in a uniform current would represent ordered energy, whereas a state of equilibrium would represent maximum chaos. But this is because he is judging the system by the amount of work it can be made to do, rather than the stability it has been able to achieve.

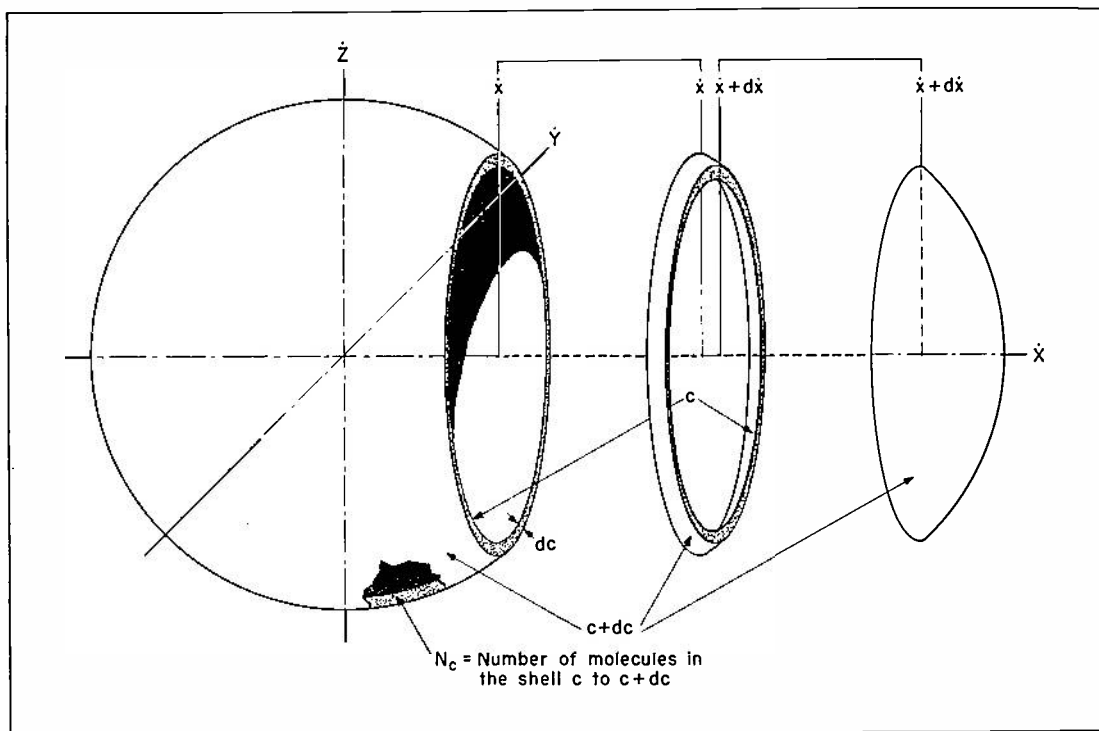


Figure 6. Spherical Shell in Velocity Space, Representing All Speeds Between  $c$  (Inner Surface) and  $c + dc$  (Outer Surface). (The volume of the ring represents all velocities contained in the speed shell which have an  $\dot{X}$  component between  $\dot{x}$  and  $\dot{x} + d\dot{x}$ .)

distances" of  $dc$  from its inner and outer neighbors. Each dot, or vector, in velocity space represents the velocity of one molecule, and by assuming that a molecule in any shell has an equal chance to move in any direction, then the velocity dots will on the average be evenly distributed throughout any one shell.

This last assumption is the first of the *a priori equal probabilities* to be made—an equal probability for each direction of motion, regardless of the particular speed a molecule may have. The first inference from this postulate is that of the equal distribution of velocities within any small shell of thickness  $dc$ .

### MOLECULAR DENSITIES, PROBABILITIES, AND NUMBERS\*

The number of molecules in any shell per thickness of the shell is called the *speed density* for all speeds represented by the shell. Thus, the density at speed  $c$  is  $D_c = N_c/dc$ . The concept of speed

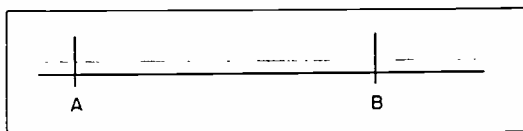


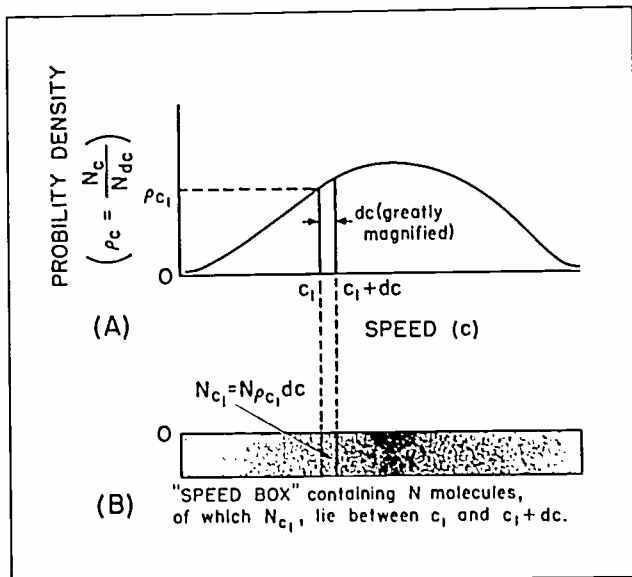
Figure 7. Random Distribution of Dots Over an Interval, Showing Method of Determining Dot Density. (The average density of dots per unit length at any point between A and B may be assumed to be:  $D_{AB} = N_{AB}/L_{AB}$ , where  $D_{AB}$  = average dot density,  $L_{AB}$  = length of interval from A to B, and  $N_{AB}$  = number of dots between A and B.)

density is similar to that of density of mass when it is defined as the mass per unit length. For example, in figure 7, the average density at any point be-

\* We should not fret too much over the symbols defined here.  $N$ ,  $N_c$ ,  $N_v$ ,  $p_c$ , and  $p_v$  are easy to remember;  $\rho_c$  and  $\rho_v$  are the ones to watch.  $D_c$  and  $D_v$  are given primarily for the record, and to block a possible misinterpretation of  $\rho_c$  or  $\rho_v$  as speed or velocity density.



Figure 8. A—Representative Distribution Curve, Plotted as Probability Density Vs. Speed  
 B—Same Distribution, With Speed Density Vs. Speed Shown Pictorially



tween A and B may be considered the number of dots between A and B divided by the distance AB. These dots may represent a quantity of molecules, mass, or anything. Likewise, the AB distance may represent an interval of speed, of length, or of any other denomination.

The number of molecules per shell divided by the *volume* of the shell is the *velocity density* for all points (molecular velocities) within the shell. Thus the velocity density for all points in the shell between  $c$  and  $c + dc$  is:

$$D_v = \frac{N_c}{4\pi c^2 dc}$$

(assuming that the area of the outer surface is approximately the same as that of the inner surface,  $4\pi c^2$ ).

The *fraction* of the molecules that lie within the shell  $c$  to  $c + dc$  is the *probability* of a molecule having a speed within that range at a given instant of time. The fraction, and hence the probability, is  $p_c = N_c/N$ .

The *probability density* at any speed lying within a shell is  $\rho_c = p_c/dc$ . Speed distribution curves are normally plotted as *probability density vs. speed*. See figure 8. To find the fraction of molecules within any range,  $c$  to  $c + dc$ , the average probability density for that range is

multiplied by  $dc$ . For instance,

$$p_{c_1} = \frac{p_{c_1}}{dc} dc = p_{c_1} dc.$$

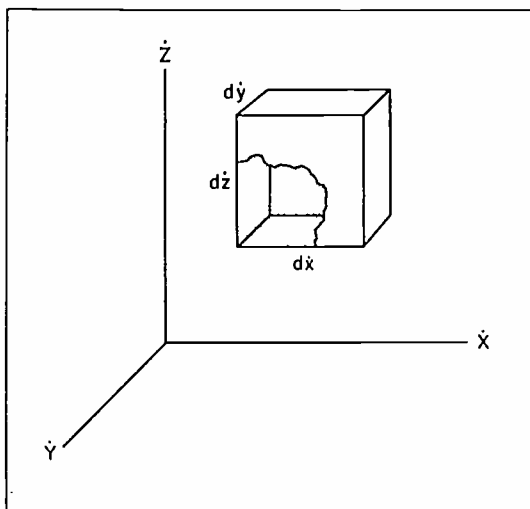
This probability is simply the area under the probability density curve between  $c_1$  and  $c_1 + dc$ . The area under the complete curve is equal to one, which means that the probability of a given molecule having a speed somewhere between zero and infinity is certain. The "speed box" below the chart indicates the distribution of the actual molecules corresponding to the curve above. The number of molecules in the compartment with sides  $c_1$  and  $c_1 + dc$  is  $N_{c_1} = N p_{c_1} = N \rho_{c_1} dc$ , where  $N$  is the total number of molecules in the system.

The *velocity probability density* is the probability per unit volume in velocity space. Thus  $\rho_v = p_v/dv$ , where  $p_v$  is the fraction of molecules in the velocity volume  $dv$ .<sup>\*</sup> Figure 9 illustrates a small velocity cell with a volume  $dv = dx dy dz$ .

If  $\rho_v =$  velocity probability density in cell  $dv$

$$\text{then } p_v = \rho_v dv = \rho_v dx dy dz \\ = \text{fraction of molecules in cell,}$$

<sup>\*</sup> The symbol  $dv$  represents a small range of velocities, and, as such,  $v$  symbolizes *velocity* rather than *volume*.



**Figure 9. Small Cell in Velocity Space, with a Volume  $dv = dx dy dz$ , Containing  $N_v$  Molecules**

and  $N_v = N p_v = N \rho_v dv$   
 = number of molecules in cell.

Similarly

$\rho_c$  = speed probability density in shell  $dc$

and  $p_c = \rho_c dc$   
 = fraction of molecules in shell

and  $N_c = N p_c = N \rho_c dc$   
 = number of molecules in shell.

### $\dot{X}$ , $\dot{Y}$ , AND $\dot{Z}$ PROBABILITIES

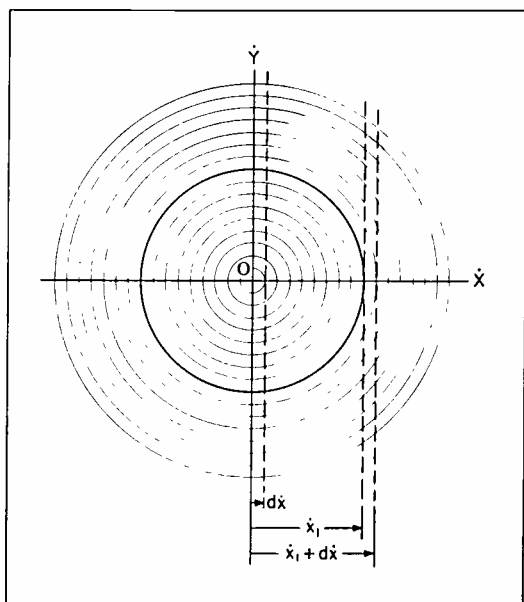
Referring again to figure 6, what fraction of the molecules in the shell,  $c$  to  $c + dc$ , will have an  $\dot{x}$  component from  $\dot{x}$  to  $\dot{x} + d\dot{x}$ , where both  $c$  and  $\dot{x}$  are taken at random (except that  $\dot{x}$  cannot be greater than  $c$ ), and with  $d\dot{x}$  representing a small change,  $\Delta\dot{x}$ , in velocity of approximately the same magnitude as  $dc$ ? Since the  $N_c$  molecules are assumed to be evenly distributed about the shell, this fraction will be represented by the ratio of the volume occupied by all the velocities with  $\dot{x}$  components between  $\dot{x}$  and  $\dot{x} + d\dot{x}$ , to the total volume of the shell. Hence, the fraction  $p_{cx} = \text{volume of ring} / \text{volume of shell}$ . On calculating this fraction, we find that  $p_{cx} = d\dot{x} / 2c$ , approximately the ratio of the  $\dot{x}$ -range width to the diameter of the shell.

Now, if a molecule has a given speed

between  $c$  and  $c + dc$ ,  $p_{cx}$  is the probability that it will have an  $\dot{x}$  component between  $\dot{x}$  and  $\dot{x} + d\dot{x}$ . But note that  $d\dot{x} / 2c$  is completely independent of the particular value of  $\dot{x}$ .  $\dot{x}$  could be zero or it could be approximately equal to  $c$  in magnitude. This tells us that within any small range of speeds,  $c$  to  $c + dc$ , the  $\dot{x}$  components, and in the same manner, the  $\dot{y}$  and  $\dot{z}$  components of velocity, will be equally distributed from zero up to a magnitude equal to  $c$ . In other words, the fact that  $c$  is large or small does not favor one possible velocity component over another.

For example, in the shell of molecules with speeds between 300 and  $300 + 10^{-6}$  meters per second, there would be just as many molecules with  $\dot{X}$ -axis velocities between 0 and  $10^{-6}$  meters per second, as between 150 and  $150 + 10^{-6}$  meters per second. Thus, when we assume that the velocities are equally distributed in all directions, we also assume that the  $\dot{x}$ ,  $\dot{y}$ , and  $\dot{z}$  coordinates are equally distributed over their respective range within a given shell.

Figure 10 shows a cross section of



**Figure 10. Cross-Section of Speed Shells in  $XY$  Plane. (Shells which intersect the dotted lines contain  $\dot{x}$  components within the respective  $d\dot{x}$  ranges.)**

velocity space in the  $\dot{X}\dot{Y}$  plane. We shall assume, however, that each of the concentric rings represents a complete speed shell. In each shell there are the same number of molecules with  $\dot{x}$  components between 0 and  $d\dot{x}$  as there are molecules with  $\dot{x}$  components between any other two velocities that differ by  $d\dot{x}$ . However, a shell cannot contain an  $\dot{x}$  component of greater magnitude than its outer speed limit. Thus, the larger the  $\dot{x}$  component, the fewer the rings it can occupy. From this we infer that there will be more molecules in the system with  $\dot{x}$  components between 0 and  $d\dot{x}$  than within any other  $d\dot{x}$  range, since 0 to  $d\dot{x}$  is the only range of  $\dot{x}$  components available to all of the rings.

In figure 10, for example, the range  $\dot{x}_1$  to  $\dot{x}_1 + d\dot{x}$  does not appear in any of the rings smaller than  $c_1$ . If we consider just that part of the system from  $c_1$  on out, however, there will be the same number of molecules with  $x$  components between  $\dot{x}_1$  and  $\dot{x}_1 + d\dot{x}$  as between 0 and  $d\dot{x}$ .

The arguments above apply equally to the distribution of the  $\dot{y}$  and  $\dot{z}$  coordinates. We now see that the initial assumption of equal probabilities for all directions of motion requires a distribution function in which the probability of a molecule's having a particular  $\dot{x}$ ,  $\dot{y}$ , or  $\dot{z}$  component increases as the component becomes smaller. Note that this is true regardless of how high the temperature may be. It is also true whether or not we postulate a state of equilibrium, as long as we assume the state to be a reasonably random distribution, or even whether or not we postulate the existence of collisions. A curve such as the one shown in figure 11 is a good first guess as to the probable distribution of velocity components in the  $\dot{x}$ ,  $\dot{y}$ , and  $\dot{z}$  directions.

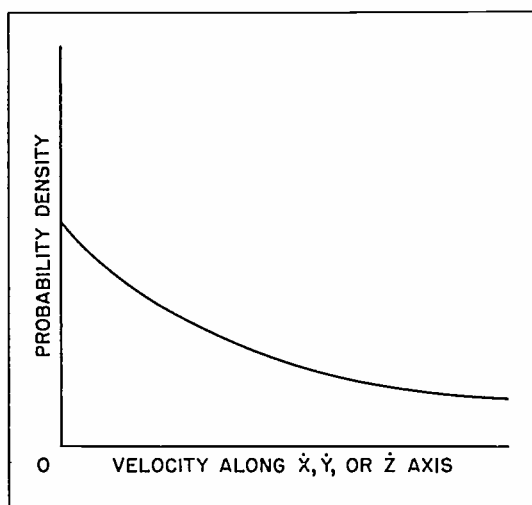
### FIRST THOUGHTS ON THE DISTRIBUTION OF ENERGY

Consider a velocity expressed in polar

coordinates,  $V(c, \phi, \Theta)$ . As long as equal probabilities of direction are assumed, the directional components  $\phi$  and  $\Theta$  are clearly independent of each other, and the probabilities that  $\phi$  and  $\Theta$  of a given molecule will lie within certain limits are fixed, regardless of the magnitude of the individual speed  $c$ . Thus, it may be said that all three variables of velocity are independent of each other. Assuming that the total energy of the system remains constant, the only factor that can cause the probability density to vary from one point in velocity space to another is the speed  $c$ . In other words, the probability density has but one degree of freedom—the variable  $c$ .

Clearly, the most probable distribution of velocities throughout the entire system is the one that corresponds to the most probable distribution of the total energy among the molecules. If over a long period of time a system approaches and hovers about an equilibrium state, the corresponding distribution of energy, since it is the one most often approached, is, by definition, the most probable distribution. Otherwise, we would have to assume that a less probable distribution at a random instant of time is more probable than the most probable.

The question arises: What can cause one distribution of energy to be more probable than others? This question can lead to problems concerning the respective roles of necessity and chance—a bottomless controversy which we shall fall into shortly—but for the present, the possibility of both competing points of view will be implied. Briefly, on the one hand, is the everyday point of view that an effect (in this case, the equilibrium distribution) is “pushed” into being by necessary physical causes, but on the other hand, there is the statistician's tendency to interpret an effect as being “pulled” into being by its own *a priori probabilities*. We shall imply an accept-



**Figure 11.** *Approximate Shape of Distribution Curve for Velocity Along a Single Axis, That Can Be Expected from Assumption of Equal Probabilities of Direction*

ance of the former viewpoint by assuming that the state of an isolated system at a future instant is completely predestined by its state at a prior instant, and an acceptance of the latter viewpoint by assuming that the most probable distribution of energy is the one that has the most number of ways of existing, and by not being concerned with the impacts that generate this distribution. Actually, the latter viewpoint is not a departure from our approach up to now, for a reconsideration of the curve in figure 11 will recall to mind that its general shape was determined by an estimate of the number of ways in which the  $\hat{x}$ ,  $\hat{y}$ , or  $\hat{z}$  velocities can occur. That such a distribution might contradict the probabilities of the impacts seems unlikely, but this is a question that was not considered.

Assume that the velocity space is divided into a series of concentric shells as in figures 6 and 10, but that instead of the thickness of each shell representing equal changes in speed,  $dc$ , assume that they represent equal changes in energy,  $dE$ . As before, the changes are very, very small, but are large enough to allow vast numbers of molecules within each of the first few billion

shells. The problem is to determine the maximum number of ways in which  $N$  molecules can be divided among the shells, but with the restriction that the total energy of the system must lie between  $E$  and  $E + dE$ . Obviously, the outermost shell that need be considered is the one corresponding to this total energy, for the probability of finding a molecule with more energy than that contained in the system is, of course, zero.

### FROM VELOCITY SPACE TO PHASE SPACE

It will be recalled that within a single shell the probability density is everywhere the same, and that the chance of an  $\hat{x}$ ,  $\hat{y}$ , or  $\hat{z}$  component of velocity falling within a given range  $d\hat{x}$ ,  $d\hat{y}$ , or  $d\hat{z}$ , respectively, is everywhere equal as long as we remain within the shell. If the entire system were confined to a single shell of *uniform density*, the probability of a particular molecule's being found in a given velocity cell of volume  $dv = d\hat{x}d\hat{y}d\hat{z}$  would merely be the ratio  $dv/\text{total volume of the shell}$ . (The total volume of the shell should not be confused with the volume of the sphere which is enclosed by the shell.) Now the entire system *can* be confined to a single shell of uniform density if we imagine it to exist as a point in phase space. The phase-space shell will thus be that between the two surfaces corresponding to  $E$  and  $E + dE$ , the limits of the total energy. Outside this shell, the probability of finding the system point, by definition, will be zero. Inside the shell, the probability density is assumed to be everywhere the same—an assumption, however, which is by no means obvious, and indeed, which has no direct proof, although it can be made to appear reasonable. With the assumption of uniform density throughout, then the chance of the system's being in one state will be assumed equal to its chance of being in any other state. The most prob-

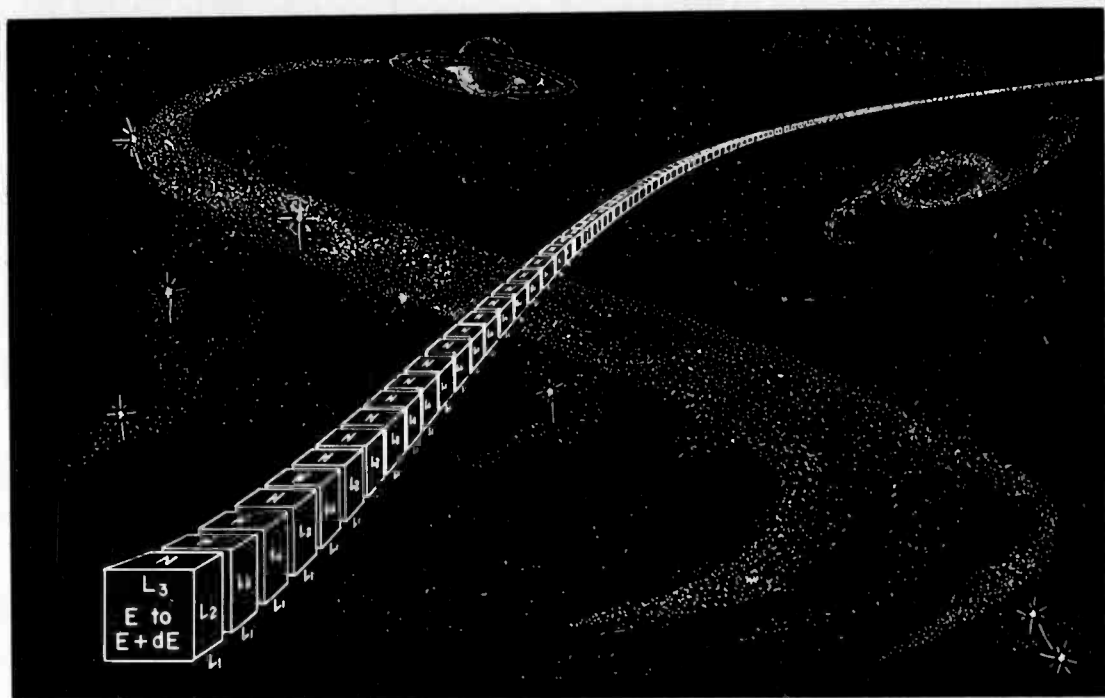


Figure 12. Ensemble of Identical Systems

able distribution, then, will be that which embraces the greatest number of states, or in other words, occupies the largest volume in phase space.

The phase space of the system will have  $6N$  dimensions, or degrees of freedom—allowing three coordinates of position,  $q$ , and three coordinates of momentum,  $p$ , for each molecule. We imagine each of these dimensions perpendicular to each of the others. The state, or phase, of the system at any instant is defined by the phase position  $P(q_1, q_2 \dots, q_{3N}, p_1, p_2 \dots, p_{3N})$ . Since the three-dimensional geometrical space in which the system is actually contained is fixed, the  $q$  coordinates will have upper limits. Furthermore, since the total energy is fixed between  $E$  and  $E + dE$ , the  $p$  coordinates will also have upper limits—those corresponding to the momenta of a single molecule which has absorbed all the energy,  $E + dE$ . As the molecules within the system are continually changing positions and momenta, the phase point of the system will likewise be continually on the move.

### AN IDEAL PHASE-SPACE GAS

We now imagine that this same system is duplicated a vast number of times, so that instead of one system of  $N$  molecules, we have a vast number of systems (see figure 12), each with  $N$  molecules identical with the molecules in the original system, and each system contained in a separate box of exactly the same dimensions as the first, and each system with a total energy, between  $E$  and  $E + dE$ . The systems are isolated from each other, so that each is permitted to vary its phase independently of the other systems. Such an assemblage of identical systems is called an ensemble of systems.

Each system in the ensemble can thus be represented by a point in the shell  $E$  to  $E + dE$  in the same  $6N$ -dimensional phase space. The entire ensemble appears as a streaming "gas" of phase points that circulates through the shell. Now within the limits of our knowledge, there is no reason to assume that the ensemble gas, when it reaches equilibrium, would not have the same density throughout the shell. Note that the equi-

librium here refers to that of the gas of system points, not to the actual gas in each system.

That the ensemble density at equilibrium is everywhere uniform is a conclusion reached indirectly. It is beyond the scope of our discussion to follow the mathematical theorems which make this assumption acceptable; however, at the risk of shadowy reasoning, we can at least make an intuitive grasp at its plausibility.

For this, we begin with the deterministic assumption that if the phase, or state, of an isolated system is completely determined at a given instant of time, then theoretically, an infinitely exact and comprehensive mathematical mind could compute the system's phase for any other instant of time. For instance, if a system consisted of a single molecule whose momentum, energy, size, shape, and position were exactly known at some instant, and if the dimensions of the system boundaries were also known, then it would be possible to compute the molecule's position and momentum at any other instant. In other words, the fact that an isolated ideal molecule has a particular state at one given instant means that the molecule can have one, and only one, particular state at another given instant. The same reasoning applies to a self-contained system of  $N$  molecules. If its state at a given instant is plotted as a point in phase space, the trajectory of that point is predetermined for every interval of time theretofore and thereafter. Furthermore, the trajectory can never cross itself, for such an occurrence would be possible only if ideal molecules could have the same positions and momenta, while heading in different directions—which, of course, is self-contradictory. Thus, if a system is ever to have the same state twice, its phase space trajectory must eventually return to its starting position, and begin the entire cycle over again.

Suppose that we started with a system in which all the molecules but one were at absolute zero, and followed the path of the phase point until the system reached thermal equilibrium. Since each point position on the trajectory is the necessary consequence of every other point position, it cannot be said that a particular position representing an equilibrium state is more probable than the position from which we started, where all the energy was contained by one molecule. Let us now see if this equal probability density *along* the trajectory cannot be extended to the immediate vicinity *around* it.

Instead of starting with a single system, imagine a huge number of systems, such as  $N^N$ , where  $N$ , as before, is the number of molecules per system. Assume that all of the systems are so nearly "in phase"—say, that only the position of a single molecule varies infinitesimally—that their phase points almost, but not quite, occupy the same position at the start. In effect, they form what we might call a "phase molecule," which occupies an infinitesimal volume in phase space. The surface of this "p-molecule" we imagine to be completely defined by the tightly clustered outer points, and the interior to be of uniform density. After starting a p-molecule at some phase position, we might expect its volume to gradually become greater, for it seems that the  $N^N$  systems might gradually become more and more out of step with the passage of time. However, none of the phase points can penetrate the outer surface formed by the original surface points, for that would be equivalent to one trajectory crossing another. Also, there must be some limit to the expansion, since the phase-space shell is finite. On the other hand, there intuitively seems to be little possibility that a p-molecule will contract, for this would require the assumption that the corresponding molecules in the  $N^N$  isolated systems, if started at

slightly different positions, will automatically tend to become more nearly in phase as time progresses. Since this seems contrary to experience, let us assume that a contraction cannot take place, and thus that a p-molecule will either expand or maintain a constant volume.

We now pack the entire phase-space shell between  $E$  and  $E + dE$  with these p-molecules and deliberately arrange their shapes so that the surface of each coincides exactly with the adjacent surfaces of its surrounding neighbors. In this way we "force" a uniform probability density for an ensemble of systems in the  $E$  to  $E + dE$  phase-space shell. The question is, will this uniform density, where every possible phase of a system of  $N$  molecules of given energy is equally represented, be maintained if the ensemble is permitted to run through its paces without further adjustment?

First, we know that the surface of one p-molecule cannot penetrate the surface of another, else it would follow that one set of trajectories can cross another set. Thus, if we assume that one p-molecule expands, it must also be assumed that others will simultaneously contract, for we have purposely packed them one against the other. However, if we can hold to the intuitive implausibility of the contraction of a p-molecule, in the same measure we may assume that it is equally improbable that a p-molecule will expand. Its shape may vary, but not its volume, nor its density. These arguments, of course, do not constitute a proof of the ensemble properties. Nevertheless, they may aid in making it easier to assume that the equilibrium density of an ensemble in the phase space is one that is uniform throughout, and that the various trajectories, each of which is equally probable, do not tend to "bunch" together in any region.

The fact that the density in the vicin-

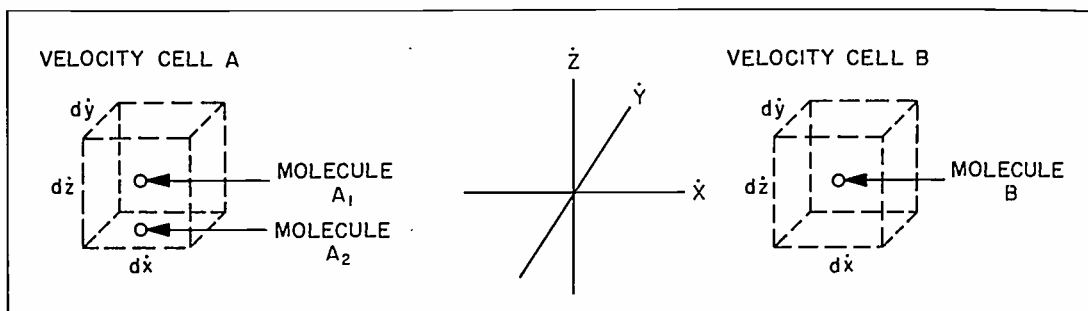
ity of a moving phase point will not vary, and hence that the volume containing a fixed number of points remains constant, can be proven by statistical mechanics. The former property is called the *conservation of density in phase*, and the latter, *the conservation of extension in phase*.

Of major importance to us is to gain a visual image of the ensemble "gas" of ideal systems whose individual energies lie between  $E$  and  $E + dE$ . This gas is incompressible, and, at equilibrium, is of equal density throughout. As long as the energy,  $E$ , is not zero, the phase points are continuously on the move. However, the points never collide with one another, but follow predetermined tracks with no crossings. A single phase point has no volume, and hence its trajectory may be of infinite length before it circles back upon itself. However, a phase molecule does have volume, and thus its trajectory sooner or later will occupy all the phase space available to it, and within a finite time must begin at least an approximate repetition of its previous cycle. Whether the cycle will include every possible phase position consistent with the energy, we do not know, but it seems probable that this would be the case.

The key assumption, however, is that the equilibrium probability density for the ensemble is everywhere uniform, for this permits us to postulate an equal probability for each phase position in the shell.

### PHASE-SPACE INTERPRETATION OF THERMAL EQUILIBRIUM

We should be careful not to confuse the equilibrium of the ensemble of systems with the equilibrium of a single system. The equilibrium conditions of the former tell us that every state of an ideal gas is equally probable. The equilibrium conditions of the latter are then derived from this assumption, by supposing that the most probable distribution in *velocity space* corresponds to



**Figure 13. Effects of Switching Molecules from One Velocity Cell to Another.** (Switching either of the A molecules with the B molecule does not change the velocity distribution, but does change the system's phase-space cell. Switching both A molecules to the B cell and the B molecule to the A cell changes the velocity distribution as well as the phase-space cell. Switching the positions of the  $A_1$  and  $A_2$  molecules changes neither the distribution nor the phase-space cell, but does shift the phase point to a new position in the phase-space cell.)

the one having the largest volume (i.e., largest number of ways of existing) in phase space.

A small cell in phase space,  $dP = dq_1, dq_2, \dots, dq_{3N}; dp_1, dp_2, \dots, dp_{3N}$ , divided by the total volume of the phase-space shell will equal the probability that a system with energy between  $E$  and  $E + dE$  will be found in that cell at any given instant of time. However, remember that there are six dimensions to the cell for each molecule in the system, and that each molecule is distinguishable by its own particular set of coordinates. Thus, molecule A may be assumed to control the coordinates  $(q_1, q_2, q_3, p_1, p_2, p_3)$ , molecule B the coordinates  $(q_4, q_5, q_6, p_4, p_5, p_6)$ , and so on. If only a single coordinate of molecule A is exchanged for a corresponding coordinate of molecule B, the entire system moves to a different phase cell—provided the exchanged coordinates differ by more than  $dq$  or  $dp$ , otherwise the phase point moves to a new position in the same cell,  $dP$ .

Figure 13 shows two small cells in velocity space, each with a volume equal to  $dx \cdot dy \cdot dz$ . If a molecule in the A cell changes position with a molecule in the B cell, the velocity distribution remains the same, for this is determined

solely by the *number* of molecules in the respective cells—not which molecules. On the other hand, switching the molecules does change the phase position of the entire system to a different cell in phase space. Thus it is that more than one phase cell corresponds to the same distribution of velocity (also of speed or energy) in a system. Although the distribution, itself, is not affected by *which* molecules are in *which* cells, the *probability* of the distribution is affected thereby, and the most probable distribution is the one that offers the greatest number of ways for switching molecules without changing the number in any velocity cell. This, in turn, is the distribution having the greatest number of cells, or the greatest volume, in phase space.

If the number of molecules,  $N$ , is large, it can be shown that the most probable distribution is so very much more probable than all the other distributions combined that the latter may be ignored in determining the average distribution probabilities. In other words, the phase-space volume corresponding to the distribution at thermal equilibrium occupies almost the entire  $E$  to  $E + dE$  shell, so that once equilibrium is reached, the resulting distribution may be considered the certain distribution thereafter, without intro-



ducing an appreciable error in the functions defining the probabilities of a molecule having the various ranges of velocity.

### **SUMMARY OF PHASE-SPACE METHOD FOR DERIVING DISTRIBUTION LAW**

We now summarize the key postulates from which the Maxwell-Boltzmann distribution law may be derived:

1. The system is composed of a large fixed number,  $N$ , of distinguishable ideal molecules contained in a volume of fixed boundaries.
2. The total energy  $E_t$  is constant, and lies within a small range  $E$  to  $E + dE$ .
3. The equilibrium distribution is the most probable distribution, and corresponds to the distribution having the maximum number of equal ways of dividing the energy.
4. There is an equal probability of finding the system in any cell of equal volume in the system's phase-space shell.
5. Any probability, whether that of a particular distribution, or that of finding a particular molecule in a particular velocity cell, will equal the fraction of the phase-space shell occupied by the phase cells corresponding to the respective distribution. In other words, the phase-space dimensions are the only factors influencing the the probabilities.

We shall not follow the actual derivation of the Maxwell-Boltzmann law employing the foregoing set of postulates, but shall follow, instead, Maxwell's original derivation, which is less rigorous, and more useful in understanding the introduction of the constants. The postulates above, however, are important as a bridge to link classical mechanics

with wave mechanics, because their wave-mechanical interpretations are the basic assumptions behind the quantum distribution laws. These latter laws are the more accurate for real gases, and describe the equilibrium state of a thermodynamic system under any conditions of temperature, pressure, and volume; whereas the classical laws break down at low temperatures and high pressures. The quantum distribution laws differ from the classical laws because of changes in the above postulates, which have the net effect of reducing the number of phase-space cells available to a system. A discussion of these differences, however, is best postponed until the wave properties of electrons have been established. Of importance now is to remember that the Maxwell-Boltzmann distribution law can be derived by the same general method that is used to derive the quantum distribution laws—i.e., the Bose-Einstein and Fermi-Dirac statistics.

This method begins by determining the appropriate general equation for the number of ways  $N$  objects may be divided among  $M$  boxes under the specified conditions. Next, the function identifying the most probable distribution is determined. Finally, the actual values of the constants that enter the function are derived.

### **MAXWELL'S LAW OF VELOCITIES\***

With this we leave the dust clouds in phase space, but with plans to return to see the changes wrought after the quantum takes over. The descent to three-dimensional space, via Maxwellian logic, is tricky, however, and the occa-

\* The discussion here is rather tedious and offers little in the way of spectator interest. All the fun to be had lies in joining the game. So if the reader feels out of practice, or more hurried to learn "what goes" than curious about "how come," this section can be lightly scanned or saved for a rainy day. However, the dogged sports who cling to the problem step by step will find their investment sound.

sional traveler is well advised to hold to his thinking cap when taking one or two of the turns. Maxwell's method has been accused of being imperfect, although it leads to the correct results. However, the derivation admirably illustrates the mysterious ways by which the mathematical theorist, in quantum, as well as classical mechanics, seemingly picks solutions out of the air while leaving a proof to be supported by its own boot straps. If the reader delights in those things which are quite unusual, he will enjoy seeking out the false turn in Maxwell's logic, for if he does, it will be quite unusual. Now we are thinking in the Maxwellian vein.

Maxwell begins with assumptions that are essentially the same as postulates 1, 2, and 3 listed above, but in place of postulates 4 and 5, he makes two other assumptions, which however, are implicit within the postulates of phase-space probabilities. One is the assumption of equal probabilities for all directions of motion, and the other is the assumption that the  $\dot{x}$ ,  $\dot{y}$ , and  $\dot{z}$  components of velocity are independent of each other. The first of these we have already fully exploited; the second, however, is by no means as obvious.\*

Maxwell assumed the  $\dot{x}$ ,  $\dot{y}$ , and  $\dot{z}$  components of velocity to be independent because they are at right angles to each other, and consequently any component of force that would change the one would not affect the others. When a velocity is expressed in polar coordinates ( $c$ ,  $\phi$ ,  $\Theta$ ), it is clear that there are three independent degrees of freedom for the

\* Actually, it seems that Maxwell's assumption that the  $\dot{x}$ ,  $\dot{y}$ , and  $\dot{z}$  components are independent need not be presented as a basic premise; for it will be recalled that the three polar coordinates ( $c$ ,  $\phi$ , and  $\Theta$ ) must be independent if equal probabilities for direction are assumed. In the presence of a rigorous proof that the number of degrees of freedom cannot be changed by a change in coordinate systems, the hypothesis of equal probabilities for direction would be sufficient.

velocity, but when the polar coordinates are converted to rectangular coordinates each of the latter coordinates ( $\dot{x}$ ,  $\dot{y}$ ,  $\dot{z}$ ,) is revealed as a function of  $c$ . The first intuition might be to suppose that the probability of finding a random molecule with an  $\dot{x}$  component between  $\dot{x}$  and  $\dot{x} + d\dot{x}$  depends somewhat upon whether the  $\dot{y}$  and  $\dot{z}$  components are large or small. If the constant-energy system consisted of only two or three molecules, this dependence would certainly exist, but for equilibrium among a large number of molecules, the assumption of independence among the rectangular coordinates leads to the same distribution formula as that derived from the phase-space method. In other words, if we imagine that each molecule in a *system at equilibrium* is composed of three little baskets of energy corresponding to three degrees of freedom, and that all the baskets are disassociated and then rejoined at random into  $N$  groups of three, the same average distribution results as that at which we started; for each of the two distributions may be assumed to be the most probable distribution of the same three independent quantities.

Suppose that the probabilities that a molecule picked at random from anywhere in the system—not just from a particular speed shell—are such that

$p_x = 1/10 =$  chance that  $\dot{x}$  lies between 0 and 50 meters/sec.

$p_y = 1/100 =$  chance that  $\dot{y}$  lies between 400 and 450 meters/sec.

$p_z = 1/20 =$  chance that  $\dot{z}$  lies between 200 and 250 meters/sec.

If the  $x$ ,  $y$ , and  $z$  velocities are independent of each other, the chance that a molecule picked at random will have a velocity,  $v$ , that lies within the velocity box that combines all these ranges is:

$$\begin{aligned} p_v &= p_x p_y p_z \\ &= 1/10 \cdot 1/100 \cdot 1/20 \\ &= 1/20,000 \end{aligned}$$

In other words, 1/10 of the molecules

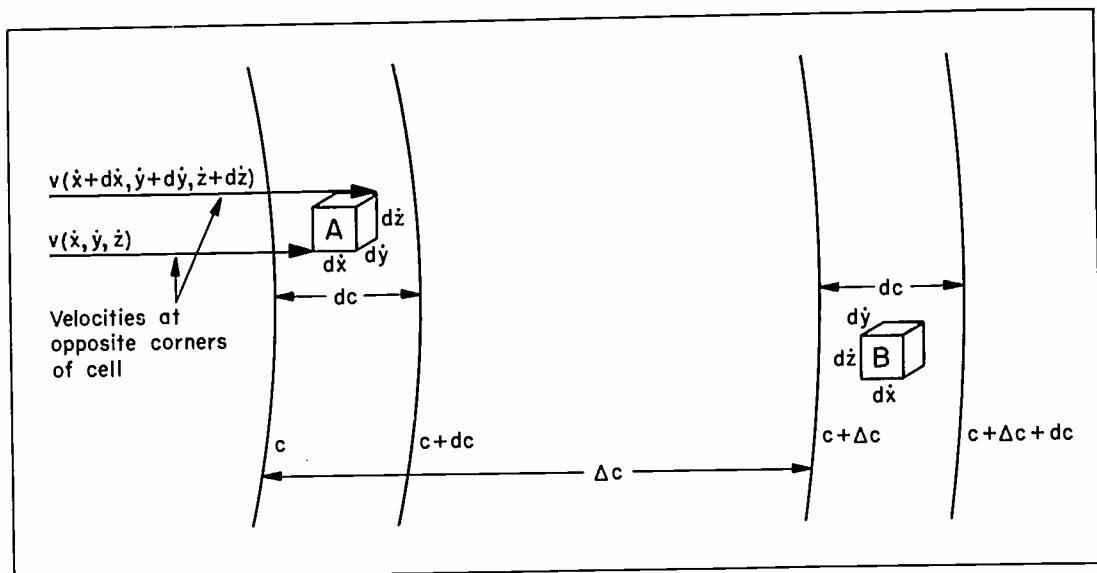


Figure 14. Two Velocity Cells of Equal Volume, Lying in Different Speed Shells

are to be found in the given  $\dot{x}$  range; of these, 1/100 will be in the  $\dot{y}$  range; and of these, 1/20 will be in the  $\dot{z}$  range.

Figure 14 shows two small velocity cells, A and B, both of the same volume  $dv = d\dot{x}d\dot{y}d\dot{z}$ , which are located at random in separate speed shells. Cell A is in the  $c$  to  $c + dc$  shell, and cell B is in the  $c + \Delta c$  to  $c + \Delta c + dc$  shell. The probability of finding a particular molecule in the A cell is  $p_v = p_x p_y p_z$ , where  $p_x$  is the fraction of  $x$  velocities in the entire system between  $x$  and  $x + dx$ , etc. Dividing both sides by the volume of the cell:

$$\frac{p_v}{dv} = \frac{p_x}{d\dot{x}} \frac{p_y}{d\dot{y}} \frac{p_z}{d\dot{z}}$$

or

$$\rho_v = \rho_x \rho_y \rho_z \quad (1)$$

where the  $\rho$ 's are the probability densities. It should be noted that the density,  $\rho_v$ , is the same at each point in the  $c$  to  $c + dc$  shell. Remembering that the logarithm of a product is equal to the sum of the logs of its factors, we can write

$$\log \rho_v = \log \rho_x + \log \rho_y + \log \rho_z$$

Since the speed is approximately the same throughout the  $c$  to  $c + dc$  shell, we can assume that cell A represents a

squared speed of

$$c^2 = v^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2$$

where  $\dot{x}$ ,  $\dot{y}$ , and  $\dot{z}$  are the average velocity coordinates within the cell.

Now, in moving from cell A to cell B, the change in  $c^2$  is

$$*\Delta c^2 = \Delta \dot{x}^2 + \Delta \dot{y}^2 + \Delta \dot{z}^2 \quad (2)$$

and the change in  $\log \rho_v$  is

$$\Delta \log \rho_v = \Delta \log \rho_x + \Delta \log \rho_y + \Delta \log \rho_z \quad (3)$$

If we divide each side of equation 3 by the corresponding sides of equation 2, and then remove the denominator on the right side, the result is

$$\frac{\Delta \log \rho_v}{\Delta c^2} (\Delta \dot{x}^2 + \Delta \dot{y}^2 + \Delta \dot{z}^2) = \Delta \log \rho_x + \Delta \log \rho_y + \Delta \log \rho_z \quad (4)$$

It may seem strange that we can combine two equations which seem, at the most, only vaguely related. However, it should be remembered that in terms of polar coordinates we found that the only variable effecting the distribution, and hence the probability densities, is the speed  $c$ . Regardless of this connection, however, even if  $c$  represented, say, the speed of a meteor, and  $\rho_v$  was a proba-

\*  $\Delta c^2$ ,  $\Delta \dot{x}^2$ , etc. are to be interpreted as  $\Delta(c^2)$ ,  $\Delta(\dot{x}^2)$ , etc. respectively, and not as  $(\Delta c)^2$ ,  $(\Delta \dot{x})^2$ , etc.

bility density giving the fraction of votes cast per minute in the last election, equation 4 would still be mathematically valid.

The next step involves a trick or two in mental juggling. Remember that it is assumed that the x, y, and z velocities are independent of each other, and consequently  $\Delta \log \rho_x$  is not affected by the changes in  $\log \rho_y$  or  $\log \rho_z$ . The only variable that causes  $\log \rho_x$  to change is the velocity  $\dot{x}$ , or, for the sake of convenience,  $\dot{x}^2$ . If  $\Delta \dot{y}^2$  and  $\Delta \dot{z}^2$  are both zero, then so are  $\Delta \log \rho_y$  and  $\Delta \log \rho_z$ . (In other words, if there is no change in  $\dot{y}$  or  $\dot{z}$ , there can be no change in the probability density for either  $\dot{y}$  or  $\dot{z}$ , and hence no change in the logs of the probability densities.) Thus, with  $\Delta \dot{y}^2$ ,  $\Delta \dot{z}^2$ , and  $\Delta \rho_y$ , and  $\Delta \rho_z$  all equal to zero, equation 4 becomes:

$$\Delta \log \rho_x = \frac{\Delta \log \rho_v}{\Delta c^2} \Delta \dot{x}^2 \quad (5)$$

But suppose that simultaneously with the change in  $\dot{x}^2$ , there is also a change in  $\dot{y}^2$ . Since  $\Delta \dot{x}^2$  remains the same, so also must  $\Delta \log \rho_x$ . Furthermore, equation 5 must still hold, for  $(\Delta \log \rho_v / \Delta c^2) \Delta \dot{x}^2$  is the only term on the left-hand side of equation 4 that can account for a change in  $\log \rho_x$ . By equation 5:  $\Delta \log \rho_v = \Delta \rho_x$  if  $\dot{y}$  and  $\dot{z}$  remain constant, for then  $\Delta c^2 = \Delta \dot{x}^2 + 0 + 0$ . Now, however,  $\Delta \log \rho_v$  and  $\Delta c^2$  have new values, for

$$\Delta \log \rho_v = \Delta \log \rho_x + \Delta \log \rho_y$$

and

$$\Delta c^2 = \Delta \dot{x}^2 + \Delta \dot{y}^2$$

But, since equation 5 still holds, and  $\Delta \log \rho_x$  and  $\Delta \dot{x}^2$  remain the same, the ratio  $\Delta \log \rho_v / \Delta c^2$  must be a constant. In other words, the log of the speed probability density varies directly with the square of the speed, or equivalently, with the energy.

We recall that the log of a number is the exponent of another number that is called the *base* of the logarithm. Thus, in the equation  $100 = 10^2$ , 2 is the log of 100 to the base 10. But an exponent,

and hence a log, is a pure number, for it is meaningless to raise a base to the power of a quantity of energy, velocity, etc. Thus,  $\log \rho_v$ ,  $\Delta \log \rho_v$ , or  $\Delta \log \rho_x$  can have no physical dimensions. The ratio,  $\Delta \log \rho_v / \Delta c^2$ , then, has the dimensions of  $1/\text{speed}^2$ , and we shall give it the symbol,  $-1/\alpha^2$ . Here,  $\alpha$  represents some speed; though just what speed, we do not know at this time. The minus sign is inserted because of the fact (see figure 11) that the probability density along each velocity axis must decrease as the velocity increases. If  $\dot{x}^2$  increases,  $\Delta \dot{x}^2$  is positive, and  $\Delta \log \rho_x$  must be negative to represent a decrease in the log of the density. Equation 5 thus becomes

$$\left. \begin{aligned} \Delta \log \rho_x &= -\frac{\Delta \dot{x}^2}{\alpha^2} \\ \log \rho_y &= -\frac{\Delta \dot{y}^2}{\alpha^2} \\ \log \rho_z &= -\frac{\Delta \dot{z}^2}{\alpha^2} \end{aligned} \right\} \text{Similarly} \quad (6)$$

and

Equation 6 reveals that if  $\log \rho_x$  were plotted against  $\dot{x}^2$ , the resulting curve would be a straight line with a negative slope such as those in figure 15, where

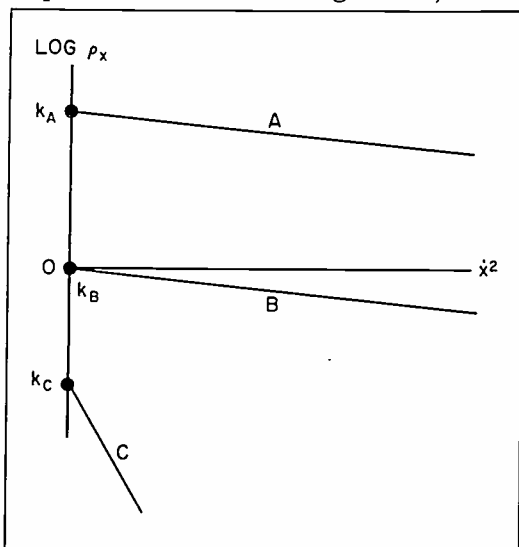


Figure 15. Three Possible Curves of the Equation:

$$\log \rho_x = -\dot{x}^2/\alpha^2 + k$$

log  $\rho_x$  is shown decreasing at a steady rate of  $-1/\alpha^2$  as  $\dot{x}^2$  increases. Each of the three lines expresses the following equation, which is equivalent to equation 6:

$$\log \rho_x = -\frac{\dot{x}^2}{\alpha^2} + k \quad (7)$$

For line A,  $\alpha$  is large,  $k$  is positive.

For line B,  $\alpha$  is large,  $k$  is zero.

For line C,  $\alpha$  is small,  $k$  is negative.

Equation 7 does not specify from what base the log of  $\rho_x$  is to be taken, since the parameter  $k$  can always be adjusted to agree with any particular selection. However, for convenience in calculus operations, the base,  $e = 2.71828 \dots$ , of the natural logarithms is always the choice. Thus, equation 7 may be expressed as

$$\rho_x = e^{-\frac{\dot{x}^2}{\alpha^2} + k}$$

or

$$\rho_x = e^k e^{-\frac{\dot{x}^2}{\alpha^2}}$$

Since  $k$  is simply a constant of proportionality,  $e^k$  is also a constant of proportionality, and may be represented by  $A = e^k$ .

Hence

$$\rho_x = A e^{-\frac{\dot{x}^2}{\alpha^2}} \quad (8)$$

It will be recalled that the probability of finding an  $\dot{x}$  velocity between  $\dot{x}$  and  $\dot{x} + d\dot{x}$  is

$$p_x = \rho_x d\dot{x}$$

On substituting from equation 8, we get

$$p_x = d\dot{x} A e^{-\frac{\dot{x}^2}{\alpha^2}} \quad (9)$$

The number of molecules with  $x$  velocities between  $\dot{x}$  and  $\dot{x} + d\dot{x}$  is

$$N_x = N p_x$$

On substituting from equation 9, we get

$$N_x = dx N A e^{-\frac{\dot{x}^2}{\alpha^2}} \quad (10)$$

Since every molecule has some value of  $\dot{x}$ , adding together the molecules,  $N_x$ , in every interval,  $d\dot{x}$ , from  $\dot{x} = -\infty$  to  $\dot{x} = +\infty$ , must give a total of  $N$  mole-

cules. The only way that the right side of equation 10 can be summed over all values of  $\dot{x}$  to  $\dot{x} + d\dot{x}$  to equal  $N$ , is

$$\text{when } A = \frac{1}{\alpha \sqrt{\pi}}$$

Thus, equation 8 becomes

$$\rho_x = \frac{e^{-\frac{\dot{x}^2}{\alpha^2}}}{\alpha \sqrt{\pi}}$$

Similarly

$$\rho_y = \frac{e^{-\frac{\dot{y}^2}{\alpha^2}}}{\alpha \sqrt{\pi}} \quad (11)$$

and

$$\rho_z = \frac{e^{-\frac{\dot{z}^2}{\alpha^2}}}{\alpha \sqrt{\pi}}$$

By equation 1

$$\rho_v = \rho_x \rho_y \rho_z$$

And on substitution of  $\rho_x$ ,  $\rho_y$ , and  $\rho_z$  from equations 11, we get

$$\rho_v = \frac{e^{-\frac{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}{\alpha^2}}}{\alpha^3 \pi^{3/2}}$$

Since  $\dot{x}^2 + \dot{y}^2 + \dot{z}^2 = c^2$  approximately,

$$\rho_v = \frac{e^{-\frac{c^2}{\alpha^2}}}{\alpha^3 \pi^{3/2}} \quad (12)$$

The probability of finding a molecule in a velocity cell of volume  $d\dot{x}d\dot{y}d\dot{z}$ , at a speed between  $c$  and  $c + dc$ , is

$$p_{xyz} = \frac{e^{-\frac{c^2}{\alpha^2}}}{\alpha^3 \pi^{3/2}} d\dot{x}d\dot{y}d\dot{z} \quad (13)$$

Note that the smaller the speed  $c$ , the greater will be the probability.

The probability of finding a molecule in a speed shell  $c$  to  $c + dc$ , where both spherical surfaces may be assumed to have an area equal to  $4\pi c^2$ , and hence where the shell volume is approximately equal to  $4\pi c^2 dc$ , is

$$p_c = \frac{e^{-\frac{c^2}{\alpha^2}}}{\alpha^3 \pi^{3/2}} 4\pi c^2 dc$$

$$p_c = \frac{4c^2 e^{-\frac{c^2}{\alpha^2}}}{\alpha^3 \sqrt{\pi}} dc \quad (14)$$

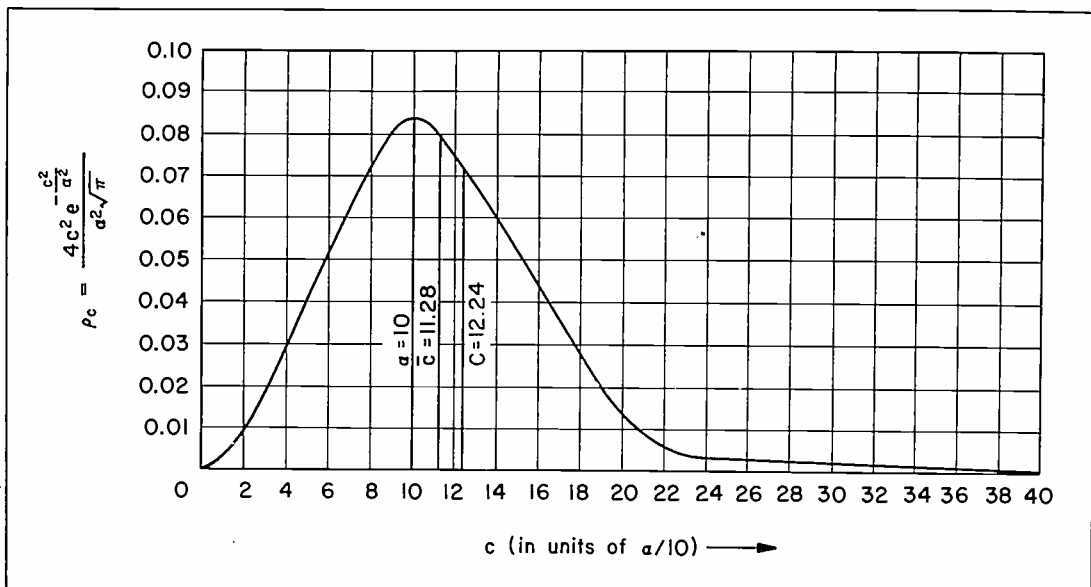


Figure 16. The Maxwell-Boltzmann Distribution Curve

Note that as  $c$  approaches zero, the probability also approaches zero. This may seem contradictory to our expectations, for equation 13 reveals that the probability of a molecule's being found in a given-sized velocity cell increases as the cell approaches the zero region of velocity space. Although this is true, the total number of equal-sized cells that can be contained in a speed shell progressively becomes less the smaller the speed  $c$ . Thus, even though the *velocity* probability density is greatest at zero, the number of molecules in the 0 to  $dc$  shell is not.

The number of molecules with speeds between  $c$  and  $c + dc$  is, by equation 14,

$$N_c = \frac{4 N c^2}{\alpha^3 \sqrt{\pi}} e^{-c^2/2} dc \quad (15)$$

The first factor approaches zero as  $c$  approaches zero, and the second factor approaches zero as  $c$  approaches infinity. Thus, as  $c$  is varied from zero to infinity,  $N_c$  must increase from zero to a maximum, and then gradually decrease toward zero again. The maximum represents the speed of greatest probability. At that point the probability neither increases nor decreases with

a small change in speed. It can be shown that the only speed at which this occurs is when  $c = \alpha$ . In other words,  $\alpha$  is the most probable speed.

To find the average speed,  $\bar{c}$ , we add the speeds of all the molecules together, and divide the total by  $N$ . This gives a value of

$$\bar{c} = \frac{2\alpha}{\sqrt{\pi}}$$

To find the average squared speed,  $C^2$ , we add the squares of the speeds of all the molecules together, and divide the total by  $N$ . This gives a value of

$$C^2 = \frac{3}{2} \alpha^2$$

or

$$C = \alpha \sqrt{3/2} \quad (16)$$

Figure 16 shows the speed probability density according to the Maxwellian law of velocities with  $\alpha$ ,  $\bar{c}$ , and  $C$  indicated, and with  $\alpha/10$  taken as the unit speed.

Rearranging equation 16, we see that

$$\alpha^2 = 2C^2/3$$

and

$$m\alpha^2/2 = 2/3 mC^2/2$$

where  $m$  is the mass of one molecule,

and  $mC^2/2$  is the average energy per molecule.

Thus,

$$m\alpha^2/2 = 2/3 E_{ave}$$

Certainly, this is an open invitation to recall our old friend,  $kT$ ; for one of our earlier discoveries was that  $kT$  equals two-thirds of the average energy per molecule. So it must be that

$$m\alpha^2/2 = kT.$$

By making the proper substitutions in equation 15, we obtain

$$N_c = 4\pi N \left( \frac{m}{2\pi kT} \right)^{3/2} C^2 e^{-\frac{mc^2}{2kT}} dc \quad (17)$$

It can be shown that

$$c^2 dc = dE \sqrt{2E/m^3}$$

where  $E$  is  $\frac{1}{2} mc^2$ . So if the molecular energy  $E$  is substituted for  $c$  as the variable in equation 17,

$$N_E = \frac{2\pi N}{(\pi kT)^{3/2}} \sqrt{E} e^{-\frac{E}{kT}} dE \quad (18)$$

This is the Maxwell-Boltzmann law for the distribution of energy among ideal molecules at thermal equilibrium at an absolute temperature,  $T$ . If the number of molecules is known for a given volume, and the temperature is known, then the number of molecules having a particular energy between  $E$  and  $E + dE$  can be readily computed from equation 18. The use of this equation, of course, depends upon whether the molecules can be assumed to behave approximately as an ideal gas. If not, then the more exact quantum statistics must be employed.

### SIMILARITIES BETWEEN CLASSICAL AND QUANTUM STATISTICS

It should be remembered that the probabilities expressed by the above equations are all directly proportional to the size of the cells or shells in velocity space that are chosen. Thus, the

number of molecules in a cell  $dx dy dz$  will depend on the magnitude of each of the three dimensions. Likewise, the number of molecules in a shell of volume  $4\pi c^2 dc$  will depend upon the magnitude of  $dc$ . It will be discovered later that the quantum effectively partitions a system's phase space into a fixed sequence of unit cells all of the same size. The number of these unit cells that permit a molecule to have an energy between  $E$  and  $E + dE$  will depend upon the magnitude of  $dE$ , and may be represented by the general function  $g(dE)$ . This quantity will be equivalent to the number of ways a molecule can have an energy between  $E$  and  $E + dE$ . If  $g(dE)$  is substituted for the volume of the velocity shell between  $E$  and  $E + dE$ , equation 18 can be transformed into the equivalent equation:

$$N_E = \frac{g(dE)}{Ae^{E/kT}} \quad (19)$$

This is the Maxwell-Boltzmann law expressed in a general form.

Only for the purpose of a preliminary comparison, the Bose-Einstein and Fermi-Dirac distribution laws are shown below. The Bose-Einstein statistics apply generally to all atoms or molecules containing an even number of elementary particles (electrons, protons, and neutrons), and also to photons. The Fermi-Dirac statistics apply to the elementary particles themselves, except photons, and to atomic nuclei containing odd numbers of protons and neutrons.

The Bose-Einstein distribution law is

$$N_E = \frac{g(dE)}{Ae^{E/kT} - 1} \quad (20)$$

And the Fermi-Dirac distribution law is

$$N_E = \frac{g(dE)}{Ae^{E/kT} + 1} \quad (21)$$

The parameter,  $A$ , and the variable  $g(dE)$  have the same significance in each equation, but the values may vary from one equation to the other. However, whenever the term  $Ae^{E/kT}$  become large relative to 1, equations 20 and 21 become equivalent to the classical distribution equation.

### APPLICATION OF THE DISTRIBUTION LAWS

Whenever an analytical interpretation of a thermodynamic property is required, one or the other of the statistics discussed above is likely to enter the problem. For example, equations that have been derived with their aid are those for thermionic emission of atoms and electrons from heated surfaces, electrical and thermal conductivities, the thermocouple effect, the Hall effect, heat capacities, electrical discharges through gases, noise levels in vacuum-tube circuits, the photoelectric effect, heat radiation, the rates of gas diffusion, of sound propagation, and of chemical reactions, and a host of other physical phenomena. Indeed, a complete analysis of any physical property of any system ultimately leads to the statistics that govern the actions of the particles that make up that system. The importance of the distribution laws in explaining the behavior of semiconductors is the reason we have concentrated so much of our attention upon the classical derivation.

The factor,  $e^{B/kT}$ , is a variable in many of the equations which appear in treatises on semiconductors. It is well to remember that  $kT$  represents the most probable kinetic energy of a mass point, inasmuch as that point may be assumed to behave as an ideal molecule. Since  $kT$  has the dimensions of energy, so also must the quantity represented by  $B$ , for the over-all exponent must be a pure number. It should also be noted that  $e^A$  is always a pure number, as also is any probability,  $p$ . However, a probability

density,  $\rho$ , or any other non-trivial density for that matter, is never a pure number. For example, the formula giving the density of electron holes in a semiconductor under equilibrium conditions at a temperature  $T$  is

$$\rho = Ae^{- (qV/kT)}$$

Since  $\rho$  has the dimensions of charge per unit volume, so also must  $A$ . The factor  $q$  is the charge of the hole, and  $V$  the electrostatic potential of the region. The variable  $e^{- (qV/kT)}$  indicates that the density is least in regions of high potential energy, but increases with the temperature.

Another point to remember is that the distribution laws are formulas which give the "population" of a particular range at a given instant of time, not the "rates of birth and death." At equilibrium, however, the "birth and death rates" are equal to each other, so that the "population" equals the equilibrium "birth rate" times the average "lifetime." For an ideal gas it can be readily imagined that the average "lifetime" of a high-velocity particle will be shorter than that of a low-velocity particle, since the high-velocity particle will collide with another particle in a shorter time. Thus, for a given  $N_c$  of a high-speed shell, the rate at which the speeds are produced must be greater than the rate for a low-speed shell of the same  $N_c$ . This is a factor that would need to be considered when determining the rate at which particles escape through a surface, such as the rate of escape of the electrons during thermionic emission.

### A BRIEF PREVIEW OF WAVE-MECHANICAL CONCEPTS

Figure 17 illustrates the Maxwell-Boltzmann distribution of velocities plotted in the  $xy$  plane of velocity space. It will be noted that the area of greatest density is at the zero point. A shell with



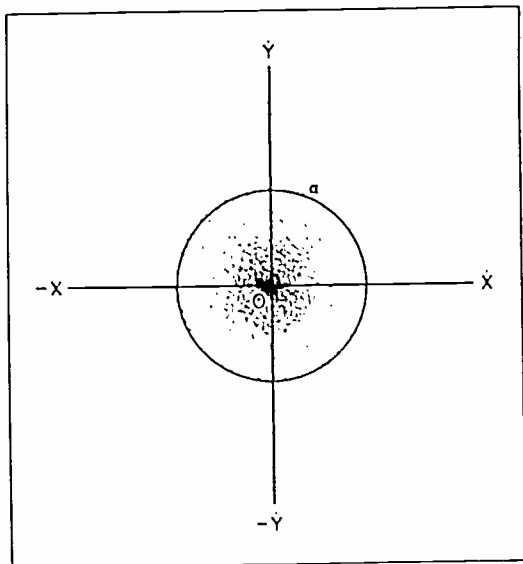


Figure 17. *Maxwell-Boltzmann Distribution of Molecules in Velocity Space*

the maximum number of molecules, however, is the one with the  $\alpha$  ring at its center, corresponding to the most probable level of energy,  $m\alpha^2/2$ , or  $kT$ .

It is of interest to note that if the velocity axes in figure 17 were changed to represent an  $xy$  plane in geometrical space, and the zero point were designated as the position of a proton, the illustration could roughly pass as the wave-mechanical picture of a hydrogen atom with its electron in the lowest energy state. The  $\alpha$  ring, representing the most probable distance from the nucleus, would correspond to the first Bohr orbit, even though the probability density is greatest in the immediate vicinity of the nucleus. The actual distribution curve, plotted as a function of the distance from the nucleus, is very similar in appearance to a Maxwellian curve, but its significance and the method by which it is derived are totally different. One or two of these differences can be mentioned now without prematurely becoming over-involved in wave mechanics.

First, if figure 17 were changed to illustrate a hydrogen atom, the dots would represent the average positions of one electron over a period of time. Furthermore, the representation would

be only that of a single energy shell. Thus, the total energy is the same for every dot—presumably the electron bounces around so that the point of maximum kinetic energy and lowest potential energy is at the nucleus, and vice versa at points infinitely removed from the nucleus. This, then, is not to be confused with the statistical laws governing the distribution of energy in a system, as exemplified by equations 19, 20, and 21. Rather, there is a much more revolutionary significance. Briefly, such a picture of a hydrogen atom represents the most accurate description, even theoretically, that can be given of the atom's lowest energy state. In classical physics, and even in the older quantum mechanics of the Bohr atom, it was assumed that the state of a system at a given instant can, at least, be theoretically known, so that all the positions, directions of motion, and divisions of energy might be predicted as the state at some other instant. However, the new wave mechanics reveals that, in the first place, there are insufficient grounds for assuming a completely defined position and momentum for a particle at a given instant of time; and in the second place, even if we did make such an assumption, there is even less reason to believe that a future state at another instant can be definitely predicted, even if we assumed that it could be observed—an event which is, itself, an impossibility. In short, the Newtonian laws of motion, which were formerly assumed to completely govern the state of a body at every instant, have been replaced by wave equations which also define the state of a body, but a state that cannot be pin-pointed to a definite position, velocity, momentum, energy, or instant of time—it is a state of possibilities, instead of actualities.

It is also a sad state of affairs for the loyal physicist who struggles to uphold the old creed of strict determinism; for at the same time he is honor bound to

admit that neither observation nor equation can directly support the belief that the exact state of a system at one instant is a necessary consequence of its exact state at a previous instant. The determinism that once was a positive doctrine of materialism has now become an uncertain faith leading towards subjectivism.

The new concepts, however, did not arrive overnight in a single packet. The fact is that for some time the old philosophy was gradually being undermined by various points of view that were taking root in the different branches of physics. It was in the development of wave mechanics that these different approaches suddenly converged. First, there was the revolutionary change in the physicist's thinking being wrought by the older quantum mechanics; then there was the equally revolutionary approach of the relativity theory; even before this the Newtonian equations of motion had been duplicated in various forms in which the concept of force was no longer necessary; and finally, there was the approach via statistical mechanics, where, through mental habit, chance often appears as a more concrete reality than physical force. All of these avenues of thought have contributed to the downfall of classical determinism, and to the rise of wave-mechanical indeterminism. However, it is only the conflict between the viewpoint of the classical determinist and that of the statistician that we shall touch upon now.

### **THE ETERNAL COMBATANTS— NECESSITY AND CHANCE**

*"Certainly the early physicists found no place for chance among the causes which they recognized—love, strife, mind, fire, or the like."*

—Aristotle (384-323 B.C.)

The early successes of Newtonian mechanics in explaining and predicting the behavior of nature inevitably nour-

ished the faith that all physical events are brought into being by physical forces. Paralleling this thought is the earlier belief that all things exist by necessity, and nothing by chance—for if it were supposed that something could exist by chance, the possibility that something, which does exist, need not exist, must also be assumed. Since this latter assumption is incapable of proof, and since everyday experience continuously encourages the belief that every effect can be assigned a cause, there seems little to gain by asserting the actuality of chance. An event which is normally ascribed to chance, such as a particular throw of dice, thus is recognized as a necessary event which differs from a deterministic event only in that the mind cannot assimilate the causes into a parade leading to the final effect. Even if a law of chance were believed to exist, it would be a variable function of the believer's mind. For example, the probability that the second card drawn from a deck is an ace would depend upon whether or not the first card drawn were known. It was this meeting of the concepts of necessity and physical cause that led to a strongly fortified positivistic and deterministic approach among the classical theorists.

Nor have the empiricists and engineers been of lesser faith, for certainly their achievements, in themselves, pay respect to a belief in the search and application of physical cause. "Human knowledge and human power meet in one; for where the cause is not known the effect cannot be produced"—this statement by Francis Bacon, an early prophet of the deterministic faith, might well be called the first dictum of the empirical and the applied sciences.

The physical sciences—empirical and applied, as well as theoretical—are thus actually the products of determinism. Classical physics expressed this determinism by assuming that all changes in the position of a body were caused by

its motion, and that all changes in its motion were caused by physical forces in obedience to Newton's laws. Thus, all physical events were seen as the necessary consequence of prior motions and forces.

Chance, however, in spite of this atmosphere, would not be denied a lawful hearing. Its influence was particularly felt by the statistical theorists, even to the extent that some of them came to embrace the cause of indeterminism.

One of these theorists was Clerk Maxwell, a Scotsman of great intellectual wealth, whom we remember chiefly for the development of the electromagnetic field and wave theories, but to whom we are also indebted for contributions in astronomy—such as his theoretical proof that Saturn's rings must consist of small, separate satellite bodies—and for valuable experiments in the study of color vision, as well as for the discovery of the velocity-distribution law of ideal gases.

In contrast to Maxwell was his contemporary, Ludwig Boltzmann, a Viennese physicist and strict determinist, who, however, is chiefly famous for his application of the theory of probability in the kinetic theory of gases.

It is known that according to the second law of thermodynamics, every irreversible transfer of energy from one system to another is always accompanied by an increase in the amount of "disordered" energy in the universe—energy which is no longer available for conversion into work. Of such a nature are the frictional losses in any machine, or the radiation losses in a resistor. The function which measures this "disordered" energy is called the *entropy* of a system. For the universe as a whole, the entropy is seen as constantly increasing. Boltzmann was the first to show that the macroscopic function, entropy, can be identified with the probability of a microscopic distribution of energy. Boltzmann was also

the first to propose the method of calculating the equilibrium distribution of thermal energy in a system from its number of corresponding phase states. It should be recalled, however, that this method is essentially deterministic; for the equal probabilities of the different states require a uniform ensemble density, and this uniformity is in turn based upon the assumption that the state of an isolated system at any given instant is completely determined by its state at any other given instant.

Maxwell, however, derived his velocity law with a minimum of physical postulates; in fact, not even collisions between molecules were involved. On this account, Boltzmann vigorously argued that Maxwell's method was not valid, for it is only by means of impacts between molecules that an equilibrium distribution can be achieved. Maxwell's proof would hold even if the absence of collisions were postulated, thus proving an impossibility. Boltzmann did not rest upon these objections, but followed through with a magnificent contribution to the deductive arts, rigorously proving through "the probabilities of collisions that there can be but one law of velocities at thermal equilibrium. Boltzmann's law, however, was exactly the same as Maxwell's. Although Maxwell was the originator, Boltzmann was the first to actually derive the distribution law from the laws of classical mechanics, and for this reason the honors are usually divided by calling it the *Maxwell-Boltzmann distribution law*.

We say that Boltzmann held the classical point of view by his insistence that the equilibrium state of a gas can only be explained as the effect of forces inherent within the mechanical impacts. This places Maxwell closer to the pure statistician's point of view, where the tendency is to see an equilibrium state simply as the effect of a priori probabilities.

The pure statistician, however, has

more than mental habit to support his point of view. For instance, if an effect has a certain probability of occurring, then its necessary physical cause must also have that same probability, and likewise the cause of the cause, and so on ad infinitum. Thus, can it not be argued that the probability is prior to and has a certain independence of the physical cause?

Again, the heads-or-tails probability of a coin toss is readily determined by assuming equal a priori probabilities for the two ways in which the coin may land. The "Boltzmann," or Newtonian method of determining the probabilities of impact, angle, spin, height, etc. of the flip would be quite an achievement. However, could it be correctly argued that a "Maxwellian" method of determining the probabilities solely from the effect would be invalid because it does not necessarily assume that the coin be tossed?

If, with all known factors considered, a function *accurately* defines the probability of occurrence of an event, that is, by definition, as far as one can go in predicting the event. Thus, since even the most probable of physical causes are not known with absolute certainty, is it not true that all our equations predicting physical states, if rigorously defined, would assume the nature of probability functions?

Finally, there are the elementary events, such as the actions of electrons, which seem to just happen, not from known physical causes, but in ways which obey probability equations. Since all physical events are aggregates of these elementary events, should we not expect the elementary probability functions to eventually replace the laws of force, conservation of energy, etc. as the basic physical laws of the universe—from which the classical laws are to be derived as statistical averages?

Thus, the statistician tends to evoke

the laws of probability to wield a non-physical authority over physical events, in which Newtonian cause is merely an interval in a sequence of chance effects; and the statistician's mind is tempted by the vision of a universe evolving, not through flights before the states of greater physical force, but through attractions towards the states of greater probability.

Our discussion here is not intended to be a plea for one mode of thinking as opposed to the other, nor even to imply that there is a fixed line of cleavage between the two. The problem is an open controversy, and philosophical overtones outside the realm of the physical sciences could not fairly be ignored if one were seeking to resolve the issue in his own mind. Within the physical sciences there are good teachers and good theorists in both schools of thought. Our subject will necessarily emphasize the statistical point of view, for the Newtonian laws break down completely in interpreting, even qualitatively, the new free-electron mechanics. The reader will find that different interpretations in different treatises of the same wave function may simply be a reflection of this difference in basic approach. Thus, a statistical bias might view the electron as a small particle relative to the area at its command; whereas a deterministic viewpoint might interpret the electron as simply a smear of charge filling all the space in which it is free to move. These interpretations and viewpoints have frequently become caught in their own cross fire, for it is the particle that traditionally has been closely associated with Newtonian mechanics, whereas the smear of charge suggests the nature of a statistical average. Our present purpose is not to draw a definite conclusion, but merely to practice an acquaintanceship with the statistical mind. It is important to remember, however, that in the deterministic point of view, probability is a func-

tion of an observer's knowledge, and not that of an actual physical state. Probability, in the indeterministic point of view, can be a law that actually guides the course of the universe.

### DYNAMIC DEGENERACY

Insofar as the qualitative fundamentals of ideal molecular systems are understood, we now stand approximately with the physicist of the early 1900's. At least we are in a position to keep pace with the quantum's main campaign as it lays waste to the classical ideals. The word *degeneracy* will appear with increasing frequency. Since the meaning of the term has, itself, somewhat "degenerated" from the original connotation, the word is often a source of confusion when used in different contexts. For this reason the general usage of the term will be given now, and the later modifications when they arise.

Originally, the word *degenerate* was an expression used in the kinetic theory of gases to describe a gas that did not obey the ideal gas laws. In other words, a gas was either *ideal* or *degenerate*. All real gases are degenerate in some degree, and the greater their deviation from the ideal, the greater, it is said, is their *degeneracy*. This degeneracy may be caused by many independent factors. For example, the molecules may have small attractions for each other, or they may be so crowded together that the actual space in which they are free to move is considerably less than the total volume of the container. The first factor would cause the pressure to be less than that predicted by the ideal gas equation of state,  $PV = nRT$ , whereas the second factor would cause the pressure to be greater.

Degeneracy of a gas also results if there are any restrictions on the distribution of velocities or energy. Insofar as the quantum restricts the movements of electrons, it will add to the degeneracy of an electron gas. It is at this point

that confusion can arise concerning the meaning of *degeneracy*. The reason is that the original application as a broad descriptive term for any variation from the ideal laws has been gradually abandoned through lack of use. *Degeneracy* now is more commonly identified with particular quantum conditions, and the quantum texts rarely refer to its older meaning. However, it will be seen that electrons that are not degenerate according to the quantum conditions, are also those that approximately obey the ideal gas laws.

Now the quantum always makes its appearance in connection with periodic actions of one sort or another. For example, in the Bohr atom, it determines the orbits for the "bi-cycling" of electron pairs about the nucleus. In searching for terms to describe the restricted state of an electron in cyclic motion, scientists borrowed the words *degenerate* and *degeneracy*. These words, which were originally used in connection with gases, thus assumed, via the quantum and the electron, special meanings with reference to periodic functions.

Imagine a particle of constant energy with a fixed periodic motion for each of its three translational degrees of freedom. Our astronomers call such a state of motion a "conditioned periodicity." Normally, the resultant motion of a body with a conditioned periodicity is not, itself, periodic; that is, the body does not repeat the same orbit over and over again. However, if the frequencies ( $f_x, f_y, f_z$ ) of the three component motions are such that the ratios  $f_x/f_y$ ,  $f_y/f_z$ , and  $f_z/f_x$  are rational numbers—i.e. integers, or fractions that can be expressed as ratios of integers, such as  $7/3/29$ , etc., as opposed to numbers such as  $\sqrt{2}$  or  $\pi$ , which are endless *non-repeating* decimals—the resultant motion of the body will be periodic, and the path it describes will be a closed curve. A particle thus confined to motion along a single line is said to have

but one *independent* periodicity, and is classed as *totally degenerate*. Or, from the point of view of the number of degrees of freedom that has been surrendered, the particle is said to have a *degeneracy of two*.

If only one of the ratios of the component frequencies is a rational number, the resultant motion will not be periodic, but the path will be confined to a fixed surface, and over an infinite time will touch every included point. The particle is thus said to have two independent periodicities; and, since one of its degrees of freedom is not in use, it has a *degeneracy of one*.

If none of the frequency ratios is a rational number, the resultant motion will not be periodic, but the path will be confined to a fixed volume, and over an infinite time will touch every included point. The particle thus has three independent periodicities, and is said to be *non-degenerate*.

When a system of many particles with periodicities is considered—a solid, for example—the normal modes of molecular vibration, or the periodic functions of the electrons may be described in a similar manner. If there are two normal vibrations having a rational ratio, the resultant mode has a degeneracy of one; if there are a hundred normal vibrations whose frequency ratios are rational numbers, the resultant mode has a single periodicity, and hence a degeneracy of ninety-nine. Normal vibrations of the same frequency are of special interest, and the reader may recall that these are called *degenerate vibrations*; although if rigorously defined,

degenerate vibrations would be any whose frequency ratios are rational.

It should be noted that the interpretation of periodic degeneracy does not in itself describe a condition that restricts the exchange of energy between particles, or that decreases the number of degrees of freedom for partitioning the energy. A change of energy in any of the component motions of an "ideal" degenerate particle will simply mean a change of orbits. However, when the quantum enters the picture, it limits the number of possible orbits, and the periodic degeneracy finds a new interpretation—a degeneracy of energy states. When applied to an electron gas, the term implies the existence of many electrons that obey the same periodic function. It will be recalled, however, that the classical conception of a degenerate gas does not require such an assumption. Because of the differences between the old and new interpretations of degeneracy, a newcomer is often confused by being confronted with definitions from different sources that appear, at best, only remotely related. However, any quantum state that is classed as degenerate will in one way or another prevent a system of particles from obeying the Maxwell-Boltzmann distribution law. Since *all* the laws of ideal gases, including the equation of state,  $PV = nRT$ , can be derived from the Maxwell-Boltzmann distribution law, it can be said that a quantum degeneracy, which describes a microscopic state, reflects a classical degeneracy when its effects are observed at the macroscopic level.

The stage is now set for the reappearance of the quantum.



# ROCKET AND ROCKET CIRCUIT TESTING

by Donald Love  
Philco Field Engineer

**Construction details and theory of operation of a simple device for testing HVAR rocket igniters and the firing circuits used to launch these rockets from aircraft.**

**T**HROUGH THE COLD CLEAR SKIES over North Korea, Captain Dale D. Ryder hurled his B-26 light bomber at a communist flak position. Earlier in the conflict, the communists had learned to take cover whenever a B-26 aimed its fourteen forward-firing machine guns at them. However, this aircraft had been modified to accommodate a bombardier, and its original hard nose which mounted eight .50-caliber machine guns had been replaced with a plexiglass nose. The aircraft was still formidable, however, because in addition to the three guns in each wing, it carried eight HVAR rockets beneath its wings. Four men were risking their lives to destroy an enemy gun position. Captain Ryder took careful aim and turned loose his entire fire power. Of the eight rockets at his command not one would fire! On the second trip to the target these four men again risked their lives to no avail. Still not one rocket was fired, and all eight remained on the aircraft as it taxied into service position on its home base.

This regrettable incident sparked the imagination of Captain Ryder, who was not only a pilot, but also assistant Wing Communication Officer. He sketched his plans for a rocket circuit tester and turned the development of a working model over to Philco Field Engineers George Hukle and Donald Love. Their experiments ranged from a set capable of detecting the reduction in resistance of a circuit caused by a person's touching the wiring, to a set which would not only test the rocket firing squib, but also the entire firing circuit.

Several months of spare-time work resulted in the final complete tester shown in figure 1. Its use eliminates the need for Circuit Continuity Tester Type #680 and type B-2 Circuit Tester. It permits ohmic resistance tests of the rocket squib, voltage tests of the rocket-firing circuits in the aircraft, and continuity tests of those firing circuits using type A-2 Projector Release intervalometers.

In portable form, the tester facilitates the testing of rocket motors by Armament personnel, and when installed in an aircraft it may be used to preflight-test the rocket-firing system, even to determining that rocket motors are properly armed. Before takeoff, the pilot can be assured that each of his rockets is ready for action. After takeoff, the tester can reveal to the pilot

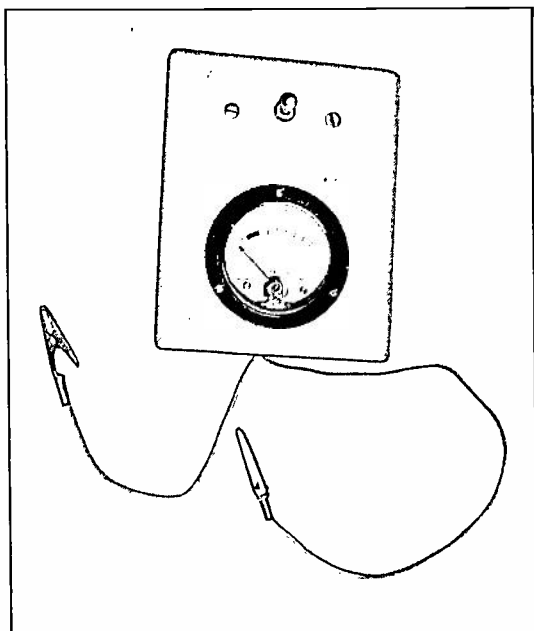


Figure 1. Completed Rocket-Firing-Circuit Tester

exactly how many useable rockets he is carrying, and in which positions they are mounted beneath his wings.

The tester consists of a D'Arsonval-type milliammeter and associated circuitry, mounted in a small, sheet-metal box, as shown in figure 1. It serves three functions — high-voltage test (0-30v.), low-voltage test (0-5v.) and continuity test (10 ohms, center-scale). The high-voltage test is used before the plane is loaded. It reveals to the armorer that no crossed wiring has placed a voltage across any of the firing cables. A second test, using the 0-5 volt range, shows the presence of low voltages which may have shorted to the firing circuits and are capable of firing a rocket. The third test before loading brings to light any firing cable which has become grounded to the aircraft frame. Next, the armorer may energize the Projector Release intervalometer and measure the firing pulse as it is applied to the rotor of the intervalometer. Should this release mechanism be defective, the condition is readily apparent. If any of the circuits are defective, a rocket need not be transported from the supply dump to be needlessly loaded on the aircraft. However, every operating rocket position may be utilized.

After loading, rockets are usually not armed until immediately prior to take-off at the end of the runway. As each rocket is plugged in, the pilot may check the circuit continuity from the Projector Release intervalometer rotor, out through the firing cable to the rocket, and through the rocket motor to the frame of the aircraft. If dust, rust, or corrosion makes arming difficult, the continuity test will reveal the defective circuit, and the armorer can easily rectify the situation and provide proper arming before takeoff.

(Note: Any continuity test of a rocket motor must not cause firing of the rocket. Since this check involves the

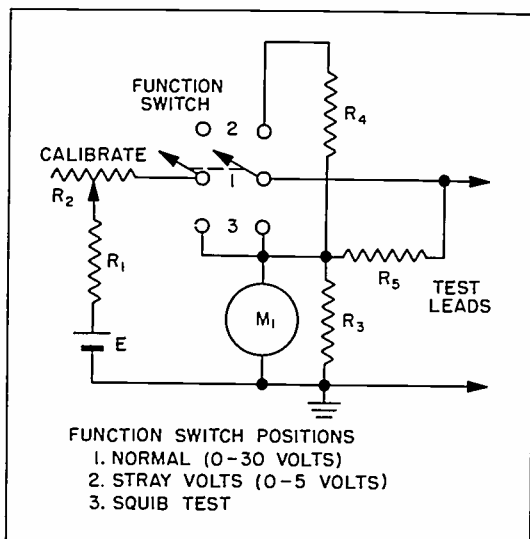


Figure 2. Schematic Diagram of Complete Tester

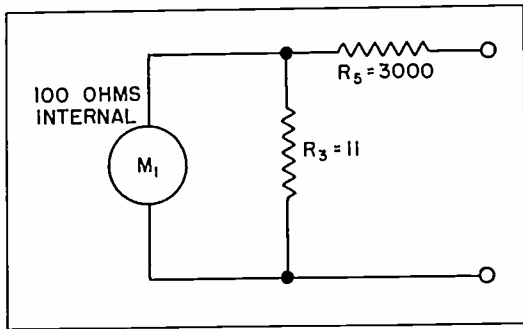
passage of a measurement current through the squib, provision must be made to limit the current to a safe value. The HVAR rocket squib requires several amperes for reliable instantaneous firing, and a current well in excess of 60 ma. must be applied over a long period of time to cause firing. This tester permits a maximum value of only 10 ma. to flow during any measurement — *this value should be considered safe only for the HVAR rocket. Before using this tester upon any other rocket type, make sure that the maximum safe-current rating is well over 10 ma.*)

In flight, before reaching the target, or after releasing some of the rockets, the pilot may flip the intervalometer selector switch through its various positions to determine in what positions armed rockets are mounted. Rarely will a pilot return to his home base with a rocket beneath his wing. Of course, a break in the wiring may occur, or the rocket arming plug may become disconnected; however, either condition is revealed to the pilot before he dives upon his target.

### THEORY OF OPERATION

The full schematic diagram of the rocket tester is shown in figure 2. The





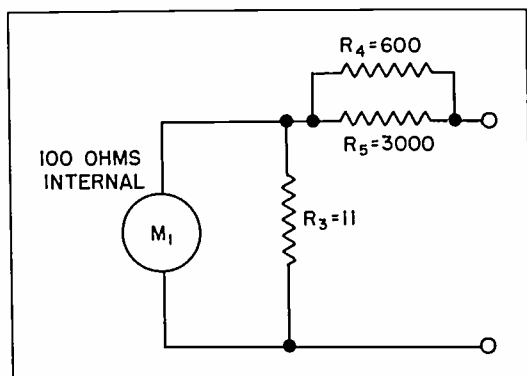
**Figure 3. Effective Circuit of Tester with Function Switch in NORMAL Position (Voltmeter, 0-30 Volts)**

d-p-d-t, spring-return, center-off, function switch is normally in the NORMAL position (figure 3), and the tester serves as a voltmeter which has a full-scale deflection of thirty volts. As the intervalometer is rotated through its various positions, any voltage above five volts is readily indicated. When the function switch is held in the STRAY VOLTS position (figure 4), a shunting resistor is introduced into the multiplier circuit. In this position, the tester is also a voltmeter, with a full-scale deflection of only five volts. Again, flipping through the positions of the intervalometer, any stray voltage capable of firing a rocket motor will be indicated. In the remaining, or SQUIB TEST, position, the tester is a low-range ohmmeter (figure 5). Ohmic resistance of the firing circuits may vary from one aircraft type to another, but

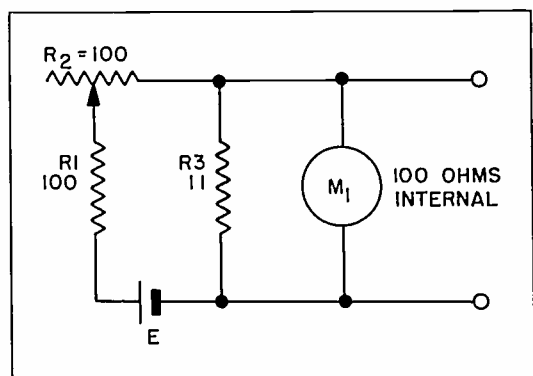
in general, the resistance is about 0.5 ohm. Rocket-motor squibs also vary in resistance, but exhaustive tests have revealed that they range from 0.5 to 1.5 ohms. Squibs which differ from these values may not fire at all or may fire with improper timing, thus causing the rocket to miss the aiming point. Hence, a green scale should be superimposed on the face of the milliammeter from the one-ohm mark to the two-ohm mark, to facilitate obtaining a "GOOD-BAD" indication. Of course, should a particular tester be prepared for use by Armament Supply personnel as a portable rocket-testing instrument, the green scale should be positioned between the 0.5- and the 1.5-ohm marks.

This tester is designed so that it can be readily constructed from parts normally available in any electronic-maintenance activity, or in any radio-parts supply house. Parts required are listed in Table I.

Resistor  $R_2$  is for calibration purposes, and is desirable because of voltage changes during the life of the battery. Calibration is easily achieved in the following manner: With the tester installed and all rocket stations cleared (disconnected), operate the function switch to the SQUIB TEST position, and adjust  $R_2$  for full-scale deflection on the meter.



**Figure 4. Effective Circuit of Tester with Function Switch in STRAY VOLTS Position (Voltmeter, 0-5 Volts)**



**Figure 5. Effective Circuit of Tester with Function Switch in SQUIB TEST Position (Ohmmeter, 10-Ohms at Center Scale)**

**Table I. Parts List for Rocket-Firing Circuit Tester**

Schematic Symbol	Quantity	Description
M <sub>1</sub>	1 ea.	Milliammeter*, Weston, 0-1 ma. (approx. 100 ohms).
R <sub>1</sub>	1 ea.	Resistor, carbon, 100-ohm, 1/2-watt.
R <sub>2</sub>	1 ea.	Potentiometer, wire-wound, 100-ohm, 1/2-watt.
R <sub>3</sub>	1 ea.	Resistor, carbon, 11-ohm, 1/2-watt.
R <sub>4</sub>	1 ea.	Resistor, carbon, 600-ohm, 1/2-watt.
SW	1 ea.	Switch, toggle (or rotary), two-pole-double-throw, center-position off, spring-return, (low amperage).
E	1 ea.	Battery, 1.5-volt dry cell (BA-30 or equiv.).
	3 ft.	Cable, aircraft, AN-20 (or equiv.).
	4 ft.	Wire, flex-lead (for use as test leads on portable model only).
	2 ea.	Clip, battery or alligator type, (for test leads on portable model only).

To calibrate the green portion of the scale, secure or construct two precision resistors. One should measure exactly 0.5 ohm, the other 1.5 ohms. A resistance bridge is most useful in this operation. Connect the 0.5-ohm resistor in place of the rocket motor, and mark the meter indication on the face plate of the milliammeter. Next, substitute the 1.5-ohm resistor and again mark the deflection on the face plate of the milliammeter. Color the area green between these two marks. If extremely accurate resistances are not available, it is advisable to use several different resistors of values as near to these values as possible. An average of the readings should be marked upon the face plate of the milliammeter.

For installation in aircraft, only one external lead is connected to the aircraft wiring (the other connection being the aircraft frame). This connection is shown in figure 2 as the ungrounded lead. This lead is soldered to the rotor arm of the type A-2 Projector Release intervalometer. This rotor arm receives

the firing voltage and in turn passes it to the firing cables leading to the rocket positions under the aircraft wing. Testing is complete because each firing pulse will indicate upon the meter.

### OPERATION

The following procedure is recommended for preflight testing of the above aircraft rocket-firing system:

1. Set the intervalometer for automatic firing of all rocket stations *before* aircraft is loaded.
2. Trigger the intervalometer through all positions. A reading should be noted on the tester in the NORMAL position of the function switch. (Failure to obtain a reading for any position indicates either a poor contact within the intervalometer or a grounded cable.)
3. Run through all intervalometer positions without triggering, while holding the function switch in the STRAY VOLTS

\* The circuit shown in figure 2 is designed for a 0-1 ma. meter with about 100 ohms internal resistance (a typical commercial value). However, almost any basic movement with a sensitivity equal to or greater than 10 ma. can be used. For example, if a 0-10 ma. meter is to be used, merely omit R<sub>3</sub>. If any other movement is used, choose a value for R<sub>3</sub> that will shunt the movement to 10 ma. (for a 0-1 ma. meter, the shunting

resistor should have a value of 1/9 of the internal meter resistance). Of course, different resistance values will alter the mid-scale-ohms value, but, since the units are calibrated individually, no impairment of function or accuracy will result. (The Weston, Model-301, 0-10 ma. meter has a two-ohm internal resistance. This means that the center-scale reading on SQUIB TEST would be two ohms—a desirable condition.)

position. Any voltage capable of firing a rocket motor will be indicated. (At this point, if all checks are ok, it is safe to load and arm the rockets.)

4. Run through each of the intervalometer positions while the function switch is held in the SQUIB TEST position. Any grounded cable will be revealed by this test as a low-resistance indication.
5. *After loading and arming*

*rockets*, hold function switch in SQUIB TEST position, and without triggering run intervalometer through all positions. Each position should produce a reading in the green range of the meter.

The total time required for one man to perform these simple steps is considerably less than the time formerly required for two men to make comparable tests using much more complex test equipment.

## *Solution To . . .*

### **Last Month's "What's Your Answer?"**

Since the circuit shown in the problem was not overheating, it is evident that no short circuits are present. The only other possible cause of zero voltage between points A and B would be an in-phase condition between the two halves of the high-voltage-secondary winding of the transformer.

In manufacture, the two halves of the secondary are wound as two separate windings and the two are joined when the leads are attached. It is conceivable that one winding might be connected in reverse, thus converting the circuit to a half-wave rectifier with the two diodes conducting simultaneously. Since the two secondary voltages are in the same phase they will buck each other with respect to points A and B.

# ELIMINATING SWITCHBOARD-RINGING INTERFERENCE IN CARRIER-VHF SYSTEMS

by F. C. Farrar  
Philco Field Engineer

A simple modification to eliminate the transmission interruptions caused by blocking of the phase-modulator stage of a T-14/TRC transmitter by high-amplitude, 20-cycle ringing voltage leaking through an EE-101A ringer before relay closure.

A RINGING-INTERFERENCE PROBLEM which was causing considerable trouble on practically all carrier-VHF systems in Korea was recently presented to this writer for analysis and solution. After a study of the equipment used in a carrier-VHF system, the source of interference was determined and a simple solution was evolved. Since this particu-

lar kind of interference probably causes trouble wherever carrier-VHF equipment is used, the findings are presented for others who might be confronted with the same problem.

In the carrier-VHF system shown in figure 1, a ringer unit is required between the switchboard and the carrier equipment, because the low-frequency

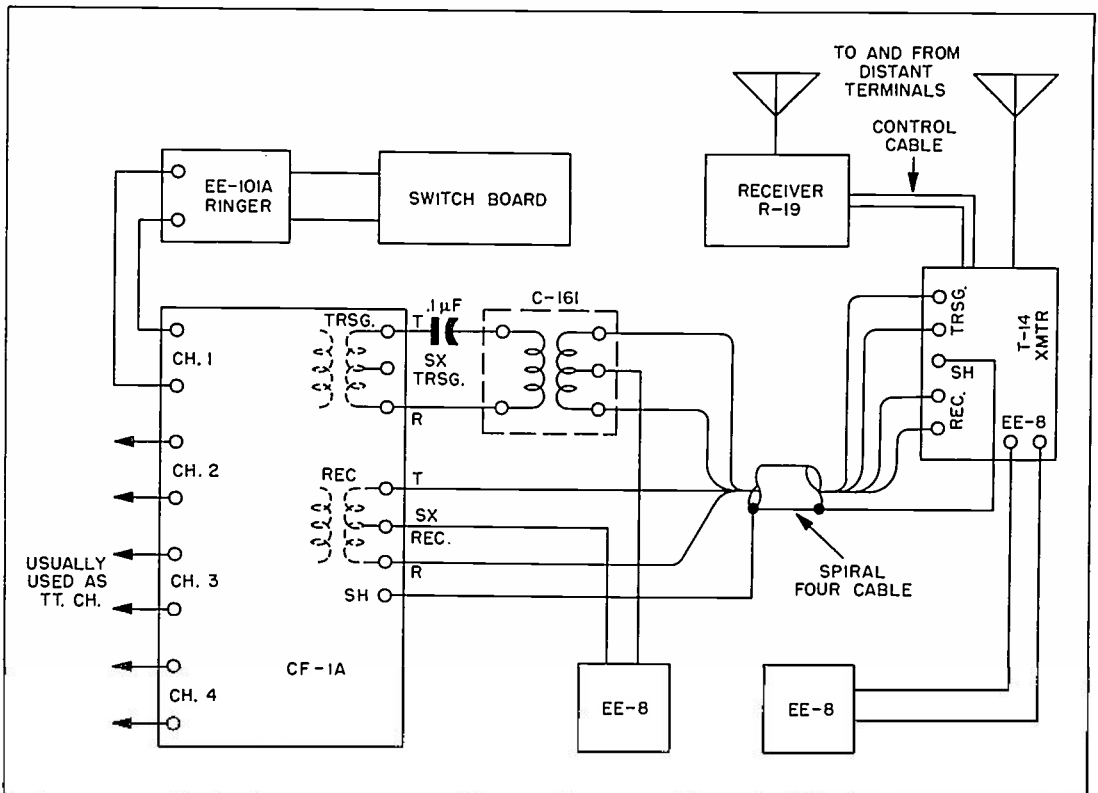


Figure 1. Simplified Schematic Diagram of Carrier-VHF System, Showing Modification

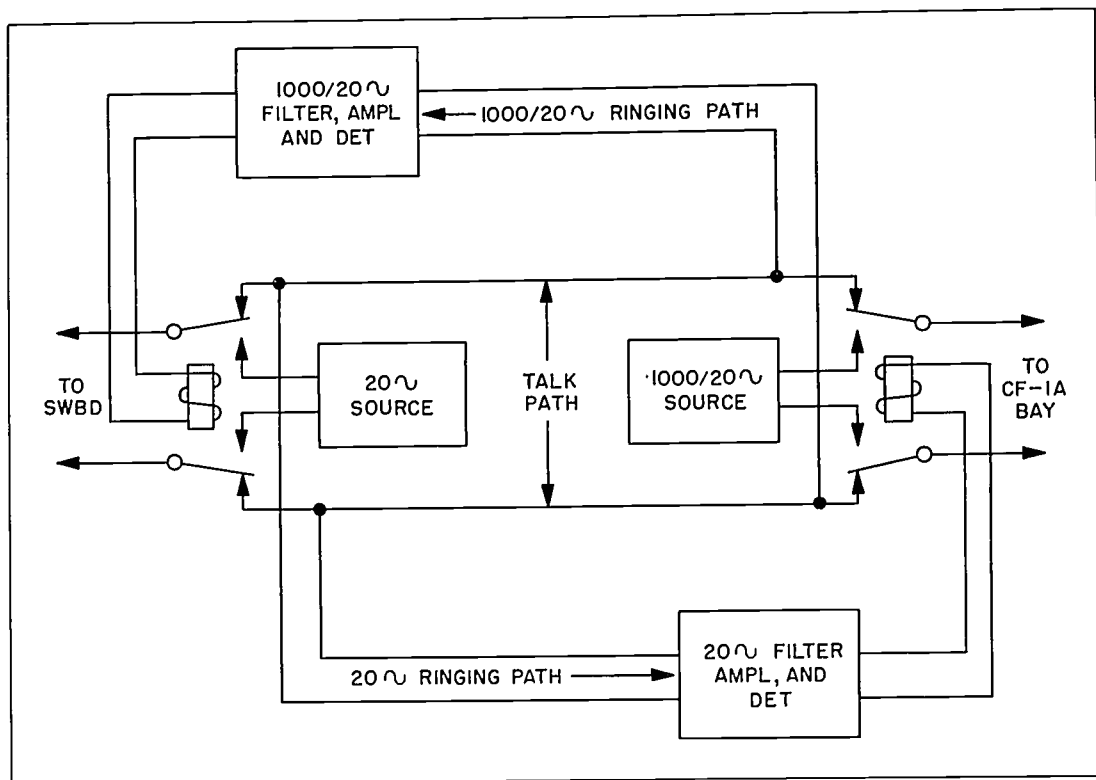


Figure 2. Simplified Block Diagram of EE-101A Voice-Frequency Ringer

(20 cycles) signal which is used to ring a switchboard or magneto telephone is not transmitted satisfactorily through the carrier equipment. The ringer unit generates a ringing signal of 1000 cycles (easily transmitted) interrupted at a 20-cycle rate. At the distant terminal, a similar ringer unit is also required between the carrier bay and the switchboard. In this ringer unit, the 1000/20-cycle current received through the carrier equipment is used to actuate a relay which connects a source of low-frequency ringing current to the switchboard for operating the switchboard signals or ringing the telephone bell.

Whenever the switchboard operator throws the ring key, an 85-volt, 20-cycle ringing signal is applied through the two-wire talk path to the EE-101A ringer unit. (See figure 2.) The talk path through the ringer unit has a varistor-arrangement bridge connected across it to rectify the switchboard ringing current. The rectified current operates a relay at B, which applies the 1000/20-

cycle current at a level of approximately 0 dbm to the line side of the ringer, and then to the channel of the carrier bay. Also, the operation of the relay breaks the talk path through the ringer between the switchboard and the carrier bay, and keeps the path open as long as the ring key is held in the ring position. (Each ringer operates in the reverse direction, of course, for signaling from the remote terminal.)

The undesirable part of the operation described in the preceding paragraph is that, when the ring key is first thrown, a short period of time is required for the bridge circuit to rectify sufficient current to operate the relay which breaks the talk path through the ringer and applies the 1000/20-cycle ringing current to the carrier bay. As a result of this delay, the 85-volt ringing-current signal is applied momentarily to the channel binding post of the carrier bay. The level of the signal is much greater than the 0-dbm signal level that the equipment is designed to accept from the ringer unit.

Each channel of the carrier bay has a 2- $\mu$ f. capacitor in series with the two-wire side of the hybrid coil to attenuate by about one-half the frequencies around 20 cycles. In addition, channels 2, 3, and 4 have bandpass filters which block audio frequencies below 200 cycles. Channel 1, which is a voice-frequency channel, contains a filter that permits frequencies from 0 to 2800 cycles to pass. Although channel 1 has a voltage-limiting device which tends to suppress high-amplitude signals, the device is effective only against signals such as those caused by loud speaking.

These circuit features seem to do a good job in preventing undesirable effects when the CF-1 bays are operated in conjunction with land lines; however, serious interference results when the equipment is operated with VHF (AN/TRC-1) radio relay equipment.

The T-14 transmitter, which is used in relaying the multichannel frequencies to the distant carrier terminal, is designed to accept signals of not over +8 dbm in signal strength. When signals above this level are supplied to the transmitter, its phase-modulator stage is biased below cutoff on the negative half-cycles. During the brief time interval that is required for operation of the relay in the ringer unit, a voltage considerably greater than a +8-dbm level is applied to the transmitter, thereby blocking it for this time duration. This condition may also be caused by jiggling of the hook on a common-battery telephone connected to channel 1. Since one channel of most CF-1 bays is used for telegraph operation, the teletype and cryptographic equipment either runs open or garbles at the distant terminal whenever the T-14 transmitter is thus blocked, because during the blocking interval no transmission can occur over any of the four channels.

Thus, a solution had to be found which would prevent the interference from occurring and yet would not appreciably affect the quality of the transmissions. It was found that if a .1- $\mu$ f. capacitor was connected in series with either the tip or ring conductor of the transmitting pair from the CF-1 bay to the transmitter, the undesirable effects could be overcome. This capacitor offers sufficient capacitive reactance to keep the interfering low-frequency currents from reaching the transmitter, and has negligible attenuation on the frequencies used in channels 2, 3, and 4. In channel 1, at 1000 cycles the signal level is reduced by about 4 or 5 db, which is sufficient to prevent blocking of the transmitter. This attenuation produces no undesirable effect, because the loss can be easily compensated for by means of the equalization and circuit-net-loss adjustments at the distant CF-1 bay.

A test was made with the capacitor in the circuit. After the circuit was realigned, there was no discernible difference in a person's voice. Since telephone handsets have a limited frequency response, the very low frequencies in the range in which the capacitor provides attenuation would not be reproduced even if no attenuation were introduced.

In cases where it is desirable to use a phantom circuit as an order-wire circuit between the CF-1 and the VHF equipment, the phantom circuit can be easily obtained by placing a C-161 repeating coil in the circuit, as shown in figure 1.

In most of the installations where ringing interference existed, a CF-1A bay was used. Since the CF-1B bay has improved circuit features, it is not so susceptible to this trouble; however, the modification suggested above may also provide an improvement for some systems that employ the CF-1B bay.



