



SECTION 2

**ADVANCED
PRACTICAL
RADIO ENGINEERING**

TECHNICAL ASSIGNMENT

SERIES-PARALLEL CIRCUITS

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SERIES-PARALLEL CIRCUITS

FOREWORD

This assignment gives you an opportunity to test your overall knowledge of applied mathematics and a.c. circuit theory in applications to radio frequency networks. Almost all r.f. circuits can be broken down into series or parallel circuits or combinations of the two. In the preliminary design state almost any group of specifications may be given with the requirement that the remainder be calculated.

For example, the tuned r.f. network between a transmitter power amplifier and an antenna may include a tuned tank circuit, a harmonic suppression circuit, a tuned antenna circuit and provision for coupling the first to the last. In addition, in a television transmitter there also will be a vestigial side-band filter to remove a portion of one of the side-bands. This is usually—but not always—done between the power amplifier and the antenna. Normally most of the L, C and R values must be calculated on the basis of given frequency, required power output, known amplifier tube characteristics, and measured antenna resistance. If a directional antenna array is to be used, the factor of phase angle becomes important. Practically all television and FM broadcasting radiating systems employ multi-element units which require the use of phasing networks.

This assignment will give you an overall picture of such r.f. circuits and will prepare you to handle specific applications of such circuits when they are taken up in later assignments.

Note particularly in this assignment how easily the most complex circuit is broken down for analysis into simple component circuits. *This is extremely important.* Just as the expert mathematician looks at a complex algebraic equation as a series of simple algebraic processes

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and attacks the problem in a systematic manner, so should the solution of a complex electrical circuit be viewed and attacked. Problems such as those studied in this assignment will help you to acquire the habit of orderly thinking. Nothing else will simplify the analysis of complex circuit operation so much as the orderly approach.

The fact that you have reached this assignment indicates that you have the necessary background to handle it without difficulty. You should enjoy it immensely. When you have completed the exam, check your work carefully and then go on to the next assignment which takes up practical radio applications of r.f. tuned circuits.

E. H. Rietzke,
President.

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SIMPLE PARALLEL CIRCUIT: THE CHARACTERISTICS OF PARALLEL CIRCUITS HAVE BEEN COMPARED WITH THE CHARACTERISTICS OF SERIES CIRCUITS UNDER SIMILAR CONDITIONS. MOST OF THE CHARACTERISTICS ARE EXACTLY OPPOSITE IN THE TWO TYPES OF CIRCUITS. CONDITIONS HAVE BEEN ASSUMED AND BOTH SERIES AND PARALLEL CIRCUITS HAVE BEEN DISCUSSED BOTH THEORETICALLY AND MATHEMATICALLY. IT IS NECESSARY TO THOROUGHLY UNDERSTAND THE PARALLEL CIRCUIT MATHEMATICALLY IN ORDER THAT THE OPERATION OF SUCH A CIRCUIT MAY BE PREDICTED FROM ITS KNOWN VALUES. A MATHEMATICAL UNDERSTANDING OF A CIRCUIT WILL GREATLY SIMPLIFY THE STUDY OF THE THEORY AND OPERATION OF THAT CIRCUIT. BEFORE PROCEEDING TO A MORE COMPLEX CIRCUIT ANOTHER SIMPLE PARALLEL CIRCUIT PROBLEM WILL BE SOLVED. PARALLEL LCR CIRCUITS AND COMBINATIONS INVOLVING SUCH CIRCUITS ARE VERY EXTENSIVELY USED IN RADIO, BOTH TRANSMITTERS AND RECEIVERS.

CONSIDER THE CIRCUIT IN FIGURE 1. THIS REPRESENTS A PARALLEL TUNED CIRCUIT IN WHICH THE RESISTANCE SHOWN IN SERIES WITH THE INDUCTANCE IS THE RESISTANCE DISTRIBUTED THROUGHOUT THE INDUCTANCE COIL, AND THE RESISTANCE IN SERIES WITH THE CAPACITY REPRESENTS THE RESISTANCE LOSSES IN THE CONDENSER, PARTLY IN THE DI-ELECTRIC AND PARTLY IN THE

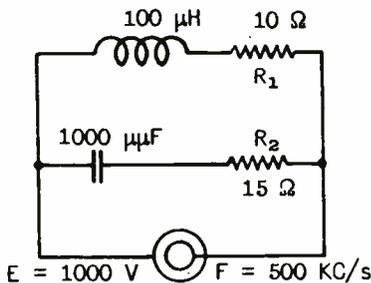


FIG. 1.

PLATES AND CONNECTING LEADS. AS WAS SHOWN IN THE STUDY OF SERIES CIRCUITS, THE EFFECTS OF THESE RESISTANCES ARE AS IF THE RESISTANCES WERE LUMPED IN SERIES WITH PURE INDUCTANCE AND PURE CAPACITY. IT IS EVIDENT THAT EACH BRANCH OF THE PARALLEL CIRCUIT MUST BE TREATED FIRST AS A SEPARATE SERIES CIRCUIT.

IN ORDER TO FIND THE TOTAL IMPEDANCE OF A PARALLEL CIRCUIT IT IS FIRST

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NECESSARY TO DETERMINE THE TOTAL CURRENT IN THE EXTERNAL CIRCUIT, THAT TOTAL CURRENT DIVIDED INTO THE APPLIED VOLTAGE GIVING THE TOTAL IMPEDANCE OF THE CIRCUIT AT THAT FREQUENCY. THE EXTERNAL CIRCUIT CURRENT EQUALS THE VECTOR SUM OF THE CURRENTS THROUGH THE INDIVIDUAL BRANCHES. THE CURRENT THROUGH EACH BRANCH CIRCUIT EQUALS THE APPLIED VOLTAGE DIVIDED BY THE IMPEDANCE OF THAT BRANCH. THE IMPEDANCE OF EACH BRANCH CIRCUIT IS A FUNCTION OF THE RESISTANCE AND REACTANCE OF THAT INDIVIDUAL BRANCH. IN THE SOLUTION OF THIS PROBLEM IT IS CLEAR THAT EACH BRANCH CIRCUIT MUST FIRST BE SOLVED INDIVIDUALLY. FIRST, SOLVE FOR ALL THE VALUES OF THE UPPER CIRCUIT CONSISTING OF AN INDUCTANCE OF 100 MICROHENRIES AND RESISTANCE OF 10 OHMS. STATED IN EQUATION FORM:

$$R_1 = 10 \text{ OHMS}$$

$$X_L = 2\pi fL = 628 \times 5 \times 10^{-1} = 314 \text{ OHMS}$$

$$Z_1 = \sqrt{R^2 + X_L^2} = \sqrt{10^2 + 314^2} = 314.1 \text{ OHMS}$$

$$I_L = E/Z = 1000/314.1 = 3.18 \text{ AMPERES}$$

$$\tan \theta_1 = X/R = 314/10 = 31.4$$

$$\theta_1 = 88^\circ 10' \text{ LAG}$$

IN THE INDUCTIVE BRANCH THE IMPEDANCE IS 314.1 OHMS, THE CURRENT IS 3.18 AMPERES, AND THE CURRENT LAGS 88 DEGREES 10 MINUTES BEHIND THE APPLIED VOLTAGE. THIS CURRENT IS SHOWN IN FIGURE 2 AS I_L . IN THE CAPACITY BRANCH:

$$R_2 = 15 \text{ OHMS}$$

$$X_C = 1/2\pi fC = \frac{10^6}{628 \times 5} = 318.4 \text{ OHMS}$$

$$Z_2 = \sqrt{R^2 + X_C^2} = \sqrt{15^2 + 318.4^2} = 318.7 \text{ OHMS}$$

$$I_C = E/Z = 1000/318.7 = 3.13 \text{ AMPERES}$$

$$\tan \theta_2 = X/R = 318.4/15 = 21.2$$

$$\theta_2 = 87^\circ 18' \text{ LEAD}$$

IN THE CAPACITY BRANCH THE IMPEDANCE IS 318.7 OHMS, THE CURRENT 3.13 AMPERES, AND THE CURRENT LEADS THE APPLIED VOLTAGE BY 87 DEGREES AND 18 MINUTES.

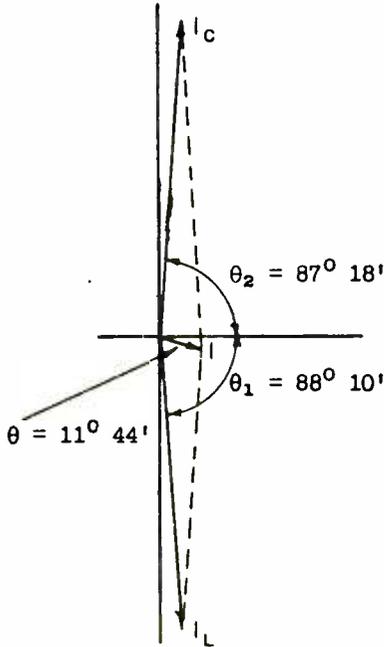


FIG. 2.

THIS CURRENT IS SHOWN ON FIGURE 2 AS I_C .

THERE ARE TWO CURRENTS, I_L AND I_C , BOTH CAUSED TO FLOW THROUGH THEIR RESPECTIVE CIRCUITS BY THE SAME APPLIED VOLTAGE, E . THEY MAY THEREFORE BE CONSIDERED AS TWO FORCES ACTING UPON A COMMON POINT, AND THE TOTAL RESULTING CURRENT IN THE EXTERNAL CIRCUIT IS EQUAL TO THE VECTOR SUM OF THE INDIVIDUAL CURRENTS.

A REFERENCE TO THE ASSIGNMENTS ON VECTOR ANALYSIS AND PARALLEL CIRCUITS WILL SHOW THE METHOD OF HANDLING THIS PROBLEM. SINCE THE ANGLE BETWEEN THE TWO CURRENTS IS NEITHER ZERO DEGREES NOR 180 DEGREES, THAT IS,

THEY ARE NEITHER EXACTLY IN PHASE NOR EXACTLY OPPOSITE, IT IS NECESSARY TO FIND THE TWO COMPONENTS OF EACH CURRENT IN ORDER TO ADD THEM **vectorially**. Each current has two forces acting on it, reactance and resistance. Each current therefore has a resistance component and a reactive component. THE RESISTANCE COMPONENT OF CURRENT TENDS TO FLOW IN PHASE WITH E AND IS EQUAL TO $I \cos \theta$. THE REACTIVE COMPONENT TENDS TO LEAD OR LAG BY 90° AND IS EQUAL TO $I \sin \theta$. A STUDY OF FIGURE 2 WILL SHOW THAT THE RESISTANCE COMPONENTS OF BOTH ARE TO THE RIGHT OF THE VERTICAL BISECTOR THEREFORE ADDING. THE TOTAL RESISTANCE COMPONENT IS EQUAL TO $I_C \cos 87^\circ 18' + I_L \cos 88^\circ 10'$.

THE REACTIVE COMPONENTS, $I \sin \theta$, ARE IN EXACT OPPOSITION, THE SMALLER THEREFORE SUBTRACTING FROM THE LARGER AND THE TOTAL REACTIVE COMPONENT EQUALING $I_L \sin 88^\circ 10' - I_C \sin 87^\circ 18'$.

IN THIS CASE WHERE THE DIFFERENCE BETWEEN THE REACTIVE COMPONENTS IS VERY SMALL AND A CASUAL INSPECTION DOES NOT INDICATE WHICH IS THE LARGER, THE SINE

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VALUE OF I_L OR THE SINE VALUE OF I_C , THE TOTAL MAY BE WRITTEN AS ABOVE AND IF THE RESULTS SHOW $I_C \sin \theta$ AS GREATER THAN $I_L \sin \theta$ IT MAY BE REVERSED AND WRITTEN $I_C \sin \theta - I_L \sin \theta$, OR THE RESULT MAY SIMPLY BE CONSIDERED AS A NEGATIVE VALUE.

AGAIN REFERRING TO VECTOR ANALYSIS, THE RESULTANT IS EQUAL TO THE HYPOTENUSE OF A RIGHT TRIANGLE FORMED BY THE TOTAL RESISTANCE COMPONENT AND THE TOTAL REACTIVE COMPONENT. THE TOTAL CURRENT IS THEN,

$$I = \sqrt{I_R^2 + I_X^2}$$

BUT, $I_R = I_C \cos 87^\circ 18' + I_L \cos 88^\circ 10'$

AND $I_X = I_L \sin 88^\circ 10' - I_C \sin 87^\circ 18'$

THEREFORE, $I = \sqrt{(I_C \cos 87^\circ 18' + I_L \cos 88^\circ 10')^2 + (I_L \sin 88^\circ 10' - I_C \sin 87^\circ 18')^2}$

$$I_L = 3.18 \text{ AMPERES}$$

$$I_C = 3.13 \text{ AMPERES}$$

$$\cos 87^\circ 18' = .04711$$

$$\sin 87^\circ 18' = .99889$$

$$\cos 88^\circ 10' = .03199$$

$$\sin 88^\circ 10' = .99949$$

THEREFORE, $I_C \cos 87^\circ 18' = 3.13 \times .04711 = .1475$

$$I_L \cos 88^\circ 10' = 3.18 \times .03199 = .1017$$

AND $I_L \sin 88^\circ 10' = 3.18 \times .99949 = 3.1783$

$$I_C \sin 87^\circ 18' = 3.13 \times .99889 = 3.1265$$

SUBSTITUTING THESE VALUES FOR THE SYMBOLS IN THE ABOVE EQUATION, IT BECOMES

$$I = \sqrt{(.1475 + .1017)^2 + (3.1783 - 3.1265)^2}$$

ADDING, $I = \sqrt{.2492^2 + .0518^2}$

SQUARING THE VALUES AND ADDING,

$$I = \sqrt{.0621 + .00268} = \sqrt{.06478}$$

EXTRACTING THE SQUARE ROOT, $I = .254 \text{ AMPERE.}$

IT WILL BE SEEN THAT THERE IS A CURRENT OF 3.18 AMPERES IN THE INDUCTIVE BRANCH AND 3.13 AMPERES IN THE CAPACITY BRANCH BUT ONLY .254 AMPERE IN THE EXTERNAL CIRCUIT OR LINE THROUGH THE ALTERNATOR. THE TOTAL IMPEDANCE IS EQUAL TO

$$E/I = 1000/.254 = 3936 \text{ OHMS.}$$

THIS CIRCUIT, EVEN THOUGH NOT OPERATED EXACTLY AT RESONANCE, HAS AN IMPED-

ANCE OF 3936 OHMS WHILE THE INDIVIDUAL IMPEDANCES OF THE TWO BRANCHES IN PARALLEL ARE ONLY 314 AND 318 OHMS RESPECTIVELY. THE SMALL DIFFERENCE BETWEEN THE TWO IMPEDANCES SHOWS THAT THE FREQUENCY OF 500 KC/S IS NOT FAR FROM THE RESONANT FREQUENCY OF THE CIRCUIT AT WHICH FREQUENCY THE RESULTING IMPEDANCE WOULD HAVE BEEN SOMEWHAT HIGHER.

SINCE THE REACTIVE VALUE OF I_L IS SLIGHTLY GREATER THAN THE REACTIVE VALUE OF I_C , THE RESULTING CURRENT WILL TEND TO LAG BEHIND THE APPLIED VOLTAGE. THE TANGENT OF THE ANGLE OF LAG OF THE LINE CURRENT BEHIND THE APPLIED VOLTAGE IS EQUAL TO THE TOTAL REACTIVE COMPONENT OF CURRENT DIVIDED BY THE TOTAL RESISTIVE COMPONENT. THE EQUATION THEN BECOMES

$$\tan \theta = I_X / I_R = \frac{I_L \sin \theta - I_C \sin \theta}{I_C \cos \theta + I_L \cos \theta}$$

$$\tan \theta = \frac{.0518}{.2492} = .20786$$

$$\theta = 11^{\circ}44' \text{ Lag.}$$

TABULATING THE RESULTS OF THIS PROBLEM IT IS FOUND THAT AT A FREQUENCY OF 500 KC/S WITH AN APPLIED VOLTAGE OF 1000 VOLTS,

$$\text{TOTAL IMPEDANCE} = 3936 \text{ OHMS}$$

$$\text{TOTAL CURRENT} = .254 \text{ AMPERE}$$

$$\text{TOTAL CURRENT LAGS THE VOLTAGE BY } 11^{\circ}44'$$

AT THE RESONANT FREQUENCY OF THE CIRCUIT THE RESULTING CURRENT WOULD HAVE BEEN STILL LESS AND THE TOTAL IMPEDANCE HIGHER. THE RESONANT FREQUENCY IS FOUND BY THE USE OF THE EQUATION

$$F = \frac{1}{2\pi \sqrt{LC}}$$

L IN HENRIES, C IN FARADS, F IN CYCLES.

$$2\pi = 628 \times 10^{-2}$$

$$C = 1000 \times 10^{-12} = 10^{-9} \text{ F}$$

$$L = 100 \times 10^{-6} = 10^{-4} \text{ H}$$

$$\text{THEREFORE, } F = \frac{1}{628 \cdot 10^{-2} \cdot \sqrt{10^{-9} \times 10^{-4}}}$$

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$$F = \frac{1}{628 \cdot 10^{-2} \sqrt{10} \times 10^{-14}} = \frac{1}{628 \cdot 10^{-2} \sqrt{10} \cdot 10^{-7}}$$

$$F = \frac{1}{628 \cdot \sqrt{10} \cdot 10^{-9}} = \frac{10^9}{628 \sqrt{10}} = \frac{10^9}{628 \times 3.16}$$

$$F = \frac{10^{11}}{198448} = 503,910 \text{ CYCLES OR } 503.91 \text{ KC/s.}$$

IT HAS BEEN SHOWN IN A PRECEDING LESSON THAT AT A FREQUENCY LOWER THAN ITS RESONANT FREQUENCY, THE CIRCUIT WILL ACT AS AN INDUCTANCE. AT RESONANCE THE CIRCUIT ACTS AS A RESISTANCE. IN THIS PROBLEM THE CIRCUIT HAS A RESONANT FREQUENCY OF 503.91 KC/s. IT IS OPERATED AT A LOWER FREQUENCY OF 500 KC/s AND A CURRENT LAG OF SLIGHTLY MORE THAN 11 DEGREES RESULTS. A LAGGING CURRENT IS THE RESULT OF AN INDUCTIVE CIRCUIT, THIS CIRCUIT THEREFORE HAS THE EFFECT OF A HIGH RESISTANCE AND A COMPARATIVELY SMALL INDUCTANCE IN SERIES.

IN CIRCUIT CALCULATIONS IT IS OFTEN HELPFUL TO REPLACE A COMPLEX CIRCUIT WITH AN EQUIVALENT MORE SIMPLE CIRCUIT. IT IS INTERESTING AT THIS POINT TO SEE JUST WHAT COMPONENTS AN EQUIVALENT SERIES CIRCUIT MUST HAVE TO REPLACE THE PARALLEL COMBINATION OF FIGURE 1 AT 500 KC/s WITHOUT CHANGING IN ANY WAY THE LOAD ON THE GENERATOR. THE EQUIVALENT SERIES CIRCUIT MUST BE SUCH THAT THE SAME GENERATOR CURRENT, .254 AMPERE, WILL FLOW UNDER THE PRESSURE OF 1000 VOLTS WITH A CURRENT LAG OF $11^{\circ}44'$. THE EQUIVALENT CIRCUIT WILL OBVIOUSLY CONSIST OF INDUCTANCE AND RESISTANCE IN SERIES. SEE FIGURE 3(A). $Z = 3936$ OHMS, MADE UP OF

$$Z = \sqrt{R^2 + X^2} = 3936 \text{ OHMS}$$

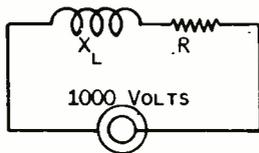


FIG. 3(A).

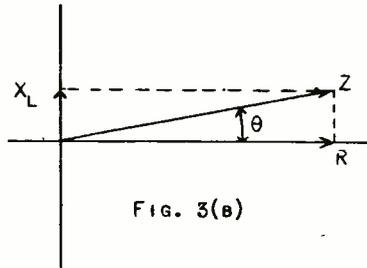


FIG. 3(B)

A LARGE RESISTANCE WHICH WILL BE THE "IN PHASE" OR COSINE COMPONENT, AND A COMPARATIVELY SMALL

INDUCTIVE REACTANCE WHICH WILL BE THE "QUADRATURE" OR SINE COMPONENT. THIS IS SHOWN IN FIGURE 3(B).

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FROM FIGURE 3(B) IT IS EVIDENT THAT $R = Z \cos \theta$ AND $X_L = Z \sin \theta$.

$$R = Z \cos \theta = 3936 \cos 11^{\circ}44' = 3936 \times .97910 = 3854 \text{ OHMS}$$

$$X_L = Z \sin \theta = 3936 \sin 11^{\circ}44' = 3936 \times .20336 = 800 \text{ OHMS}$$

$$L = \frac{X_L}{2\pi f} = \frac{800}{6.28 \times 5 \times 10^5} = 254.7 \mu\text{H.}$$

THUS AT THIS FREQUENCY, 500 KC/S, SO FAR AS THE LOAD EFFECT ON THE GENERATOR IS CONCERNED, THE PARALLEL CIRCUIT OF FIGURE 1 COULD BE REPLACED WITH THE EQUIVALENT SERIES CIRCUIT OF FIGURE 3(A) HAVING THE FOLLOWING CONSTANTS: $R = 3854 \text{ OHMS}$, $L = 254.7 \mu\text{H}$.

A MENTAL CONCEPT OF THE OPERATION OF A COMPLEX CIRCUIT MAY OFTEN BE SIMPLIFIED BY RESOLVING IT INTO A SIMPLE EQUIVALENT CIRCUIT.

IN THE PARALLEL CIRCUIT PROBLEM DISCUSSED ABOVE, WHEN SOLVING FOR THE IMPEDANCE OF THE ENTIRE PARALLEL CIRCUIT IT WAS NECESSARY FIRST TO FIND THE CURRENT THROUGH EACH OF THE BRANCHES, AND AFTER ADDING THOSE CURRENTS VECTORALLY TO DIVIDE THE RESULTING CURRENT INTO THE APPLIED VOLTAGE TO OBTAIN THE IMPEDANCE; $Z = E/I$. TUNED CIRCUITS OF THIS TYPE WILL ORDINARILY BE USED WITH VARYING VOLTAGES, AND WHEN IT IS REQUIRED TO FIND THE IMPEDANCE NO VOLTAGE WILL BE SPECIFIED. IN THAT CASE IT IS MERELY NECESSARY TO ASSUME A VOLTAGE. THIS IS SEEN TO BE CORRECT WHEN IT IS REMEMBERED THAT THE CURRENT THROUGH ANY CIRCUIT VARIES DIRECTLY AS THE VOLTAGE. IF THE VOLTAGE IS DOUBLED THE CURRENT THROUGH THE CIRCUIT IS ALSO DOUBLED. IF THE VOLTAGE IS DECREASED TO ONE-HALF ITS NORMAL VALUE THE CURRENT IS ALSO DECREASED TO ONE-HALF.

SINCE $Z = E/I$, IF THE APPLIED VOLTAGE IS 100 VOLTS AND THE CURRENT IS ONE AMPERE, THE IMPEDANCE IS EQUAL TO $100/1$ OR 100 OHMS. IF THE VOLTAGE IS INCREASED BY TEN TIMES, THE CURRENT WILL ALSO BE INCREASED TEN TIMES, AND THE EQUATION $Z = E/I$ BECOMES $Z = 1000/10 = 100 \text{ OHMS}$, THE SAME AS BEFORE. IT IS EVIDENT THAT THE IMPEDANCE OF THE CIRCUIT DOES NOT CHANGE WITH A VARIATION OF THE APPLIED VOLTAGE AND THEREFORE ANY VOLTAGE MAY BE ASSUMED WHEN SOLVING FOR THE IMPEDANCE.

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IT MUST BE REMEMBERED HOWEVER THAT THE SAME ASSUMED VOLTAGE MUST BE USED THROUGHOUT THE ENTIRE PROBLEM. WHEN WORKING WITH FAIRLY HIGH VALUES OF IMPEDANCE IT IS USUALLY SOMEWHAT MORE SIMPLE IF A COMPARATIVELY HIGH ASSUMED VOLTAGE IS USED BECAUSE UNDER THAT CONDITION COMPARATIVELY LARGE CURRENTS WILL BE OBTAINED IN THE PARALLEL BRANCH CIRCUITS. THE HIGHER VALUES OF CURRENT ARE USUALLY CONSIDERED EASIER TO HANDLE THAN VERY SMALL DECIMAL AMOUNTS. OF COURSE, IF A CERTAIN VOLTAGE IS SPECIFIED IN THE PROBLEM, IT IS UNNECESSARY TO USE AN ASSUMED VOLTAGE.

PARALLEL COMBINATION OF SERIES LCR CIRCUITS: IT MAY BE NECESSARY TO DETERMINE THE ACTUAL VOLTAGE ACROSS A PARALLEL CIRCUIT AND THE TOTAL CURRENT IN THE EXTERNAL OR GENERATOR CIRCUIT WHEN ONLY THE CIRCUIT CONSTANTS AND THE CURRENT THROUGH ONE BRANCH OF THE PARALLEL CIRCUIT ARE GIVEN. THE FREQUENCY MUST BE SPECIFIED IN ALL A. C. WORK INVOLVING INDUCTANCE AND CAPACITY. SEE FIGURE 4.

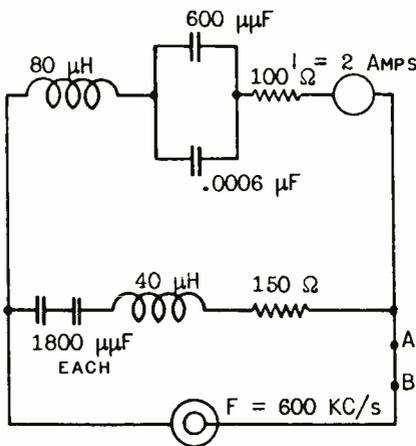


FIG. 4.

IN THIS DIAGRAM THERE ARE TWO SERIES CIRCUITS, EACH COMPOSED OF L, C AND R, IN PARALLEL.

IT IS FIRST NECESSARY TO SOLVE FOR THE IMPEDANCE, ANGLE OF LEAD OR LAG, AND THE CURRENT IN EACH BRANCH.

NO APPLIED VOLTAGE IS SPECIFIED BUT THE CURRENT THROUGH THE UPPER BRANCH IS GIVEN AS 2 AMPERES. SINCE $E = IZ$, IF THE IMPEDANCE OF THE UPPER BRANCH IS FIRST DETERMINED, THE

VOLTAGE ACROSS THE CIRCUIT MAY BE FOUND BY MULTIPLYING THAT IMPEDANCE BY THE CURRENT OF 2 AMPERES. THIS BEING A PARALLEL CIRCUIT, THE SAME VOLTAGE IS ALSO APPLIED ACROSS THE LOWER BRANCH, AND BY DIVIDING THIS COMMON VOLTAGE BY THE IMPEDANCE OF THE LOWER BRANCH, THE CURRENT THROUGH THE LOWER BRANCH MAY ALSO BE CALCULATED. THIS PROBLEM IS STARTED IN THE SAME WAY AS WAS THE PRECEDING

PROBLEM, I. E., THE INDIVIDUAL BRANCHES ARE TREATED AS SIMPLE SERIES CIRCUITS AND SOLVED FOR IMPEDANCES AND CURRENTS.

UPPER BRANCH:

$$R = 100 \text{ OHMS.}$$

$$X_L = 2\pi FL = 628 \times 6 \times 8 \times 10^{-2} = 301 \text{ OHMS.}$$

$$C = 600 \mu\mu F + .0006 \mu F = 600 + 600 = 1200 \mu\mu F. \quad (\text{THE TWO CAPACITIES BEING IN PARALLEL, ADD.})$$

$$X_C = \frac{1}{628 \times 6 \times 12 \times 10^{-7}} = \frac{10^7}{45216} = 221 \text{ OHMS.}$$

$$Z_1 = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{100^2 + (301 - 221)^2} = \sqrt{100^2 + 80^2} = 128 \text{ OHMS.}$$

$$\text{TAN } \theta_1 = X/R = 80/100 = .8$$

$$\theta_1 = 39^\circ \text{ LAG.} \quad (X_L \text{ IS GREATER THAN } X_C)$$

$$E = IZ = 2 \times 128 = 256 \text{ VOLTS.}$$

AS WAS PREVIOUSLY EXPLAINED, THIS IS ALSO THE VOLTAGE ACROSS THE LOWER BRANCH; IN OTHER WORDS, THE TOTAL APPLIED VOLTAGE. SOLVING FOR THE LOWER BRANCH IN A SIMILAR MANNER:

$$R = 150 \text{ OHMS.}$$

$$X_L = 628 \times 6 \times 4 \times 10^{-2} = 151 \text{ OHMS.}$$

$$C = \frac{1}{\frac{1}{1800} + \frac{1}{1800}} = \frac{1}{\frac{2}{1800}} = \frac{1800}{2} = 900 \mu\mu F \quad (\text{THE TWO CAPACITIES ARE IN SERIES.})$$

$$X_C = \frac{1}{628 \times 6 \times 9 \times 10^{-7}} = \frac{10^7}{33912} = 295 \text{ OHMS.}$$

$$Z_2 = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{150^2 + (295 - 151)^2} = \sqrt{150^2 + 144^2} = 208 \text{ OHMS}$$

$$\text{TAN } \theta_2 = X/R = 144/150 = .960$$

$$\theta_2 = 44^\circ \text{ LEAD.} \quad (X_C \text{ IS GREATER THAN } X_L.)$$

$$I_2 = E/Z = 256/208 = 1.23 \text{ AMPERES.}$$

IT HAS BEEN DETERMINED THAT THE APPLIED VOLTAGE OF 256 VOLTS CAUSES A CURRENT FLOW OF 2 AMPERES, I_1 , IN THE UPPER BRANCH, THAT CURRENT LAGGING 39° BEHIND THE APPLIED VOLTAGE; ALSO THAT THE SAME VOLTAGE CAUSES A CURRENT FLOW OF 1.23 AMPERES, I_2 , THROUGH THE LOWER BRANCH, THIS CURRENT LEADING THE SAME VOLT-

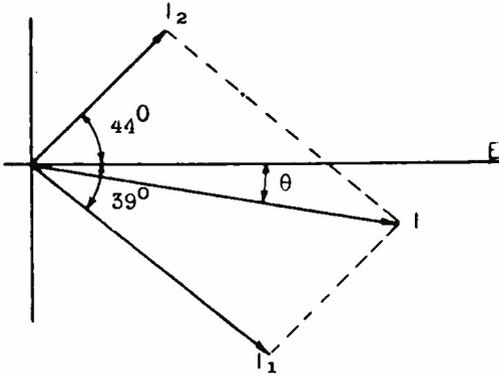


FIG. 5.

BY 44° . THIS IS SHOWN VECTORIALLY IN FIGURE 5.

THE RESULTANT CURRENT, THAT IS, THE LINE CURRENT IN THE EXTERNAL CIRCUIT, IS ALSO SHOWN AS THE VECTOR SUM OF THE CURRENTS THROUGH THE INDIVIDUAL BRANCHES. THE VECTOR SUM OF THE CURRENTS MAY BE EXPRESSED BY THE FOLLOWING EQUATION:

$$I = \sqrt{(I_1 \cos 39^{\circ} + I_2 \cos 44^{\circ})^2 + (-I_1 \sin 39^{\circ} + I_2 \sin 44^{\circ})^2}$$

$$I_1 \cos 39^{\circ} = 2 \times .777 = 1.554 \text{ AMPERES}$$

$$I_2 \cos 44^{\circ} = 1.23 \times .719 = .884 \text{ AMPERES}$$

$$I_1 \sin 39^{\circ} = 2 \times .629 = 1.258 \text{ AMPERES}$$

$$I_2 \sin 44^{\circ} = 1.23 \times .695 = .855 \text{ AMPERES}$$

REPLACING THESE FIGURES IN THE ABOVE EQUATION, IT BECOMES

$$I = \sqrt{(1.554 + .884)^2 + (-1.258 + .855)^2}$$

$$I = \sqrt{2.438^2 + .403^2} = \sqrt{6.1062} = 2.47 \text{ AMPERES.}$$

$$\tan \theta = I_X / I_R = \frac{-.403}{2.438} = -.165$$

$$\theta = 9.5^{\circ} \text{ LAG.}$$

SINCE THE TOTAL IMPEDANCE OF A PARALLEL CIRCUIT IS EQUAL TO THE APPLIED VOLTAGE DIVIDED BY THE TOTAL CURRENT, THE IMPEDANCE OF THIS PARALLEL COMBINATION IS EQUAL TO:

$$Z = E/I = 256/2.47 = 103 \text{ OHMS.}$$

TABULATING THE RESULTS:

$$I \text{ (UPPER BRANCH)} = 2 \text{ AMPERES}$$

$$\theta \text{ " " } = 39^{\circ} \text{ LAG}$$

$$I \text{ (LOWER BRANCH)} = 1.23 \text{ AMPERES}$$

$$\theta \text{ " " } = 44^{\circ} \text{ LEAD}$$

TOTAL CURRENT = 2.47 AMPERES

θ (TOTAL CURRENT) = 9.5° LAG

APPLIED VOLTAGE = 256 VOLTS.

Z(TOTAL CIRCUIT) = 103 OHMS.

ADDITION OF SERIES CIRCUIT TO PARALLEL COMBINATION: SUPPOSE IT IS DESIRED TO ADD ANOTHER IMPEDANCE, COMPOSED OF L, C AND R IN SERIES, TO THE PARALLEL COMBINATION AND IN SERIES WITH IT. IN FIGURE 6 IS SHOWN THE SERIES CIRCUIT,

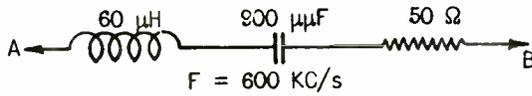


FIG. 6.

AND ITS COMPONENT VALUES. THIS CIRCUIT IS TO BE INSERTED BETWEEN POINTS A AND B IN FIGURE 4 AND THOSE POINTS ARE ALSO SHOWN IN FIGURE 6.

THE FREQUENCY IS THE SAME AS IN FIGURE 4, I. E., 600 KC/s.

$R = 50 \text{ OHMS}$

$X_L = 2\pi fL = 628 \times 6 \times 6 \times 10^{-2} = 226 \text{ OHMS.}$

$X_C = \frac{1}{628 \times 9 \times 6 \times 10^{-7}} = \frac{10^7}{628 \times 9 \times 6} = 295 \text{ OHMS.}$

$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{50^2 + (295 - 226)^2} = \sqrt{50^2 + 69^2} = 85 \text{ OHMS}$

$\text{TAN } \theta = X/R = 69/50 = 1.38$

$\theta = 54^\circ \text{ LEAD } (X_C \text{ IS GREATER THAN } X_L.)$

THE IMPEDANCE OF THE PARALLEL CIRCUIT Z_p WAS FOUND TO BE 103 OHMS AND TO BE SUCH AS TO CAUSE A CURRENT LAG OF 9.5 DEGREES. THE IMPEDANCE OF THE SERIES CIRCUIT Z_s IS 85 OHMS AND ITS EFFECT IS TO CAUSE A CURRENT LEAD OF 54 DEGREES. THE TOTAL IMPEDANCE OF THE TWO CIRCUITS, IN SERIES, WILL BE THE VECTOR SUM OF Z_p AND Z_s AND IS SHOWN IN FIGURE 7. BOTH WILL HAVE AN EFFECT ON THE CURRENT FLOW THROUGH THE CIRCUIT ACCORDING TO THEIR INDIVIDUAL MAGNITUDES AND ANGLES.

THE TOTAL IMPEDANCE FOR THE ENTIRE CIRCUIT CONSISTING OF THE PARALLEL COMBINATION SHOWN IN FIGURE 4 IN SERIES WITH THE SERIES COMBINATION SHOWN IN FIGURE 6 IS EXPRESSED IN THE FOLLOWING EQUATION:

12.

SERIES-PARALLEL CIRCUITS

$$Z = \sqrt{(Z_P \cos 9.5^\circ + Z_S \cos 54^\circ)^2 + (Z_S \sin 54^\circ - Z_P \sin 9.5^\circ)^2}$$

$$Z_P \cos 9.5^\circ = 103 \times .987 = 101.6$$

$$Z_S \cos 54^\circ = 85 \times .588 = 49.9$$

$$Z_S \sin 54^\circ = 85 \times .809 = 68.7$$

$$Z_P \sin 9.5^\circ = 103 \times .165 = 16.9$$

REPLACING SYMBOLS WITH NUMERICAL VALUES, THE EQUATION BECOMES:

$$Z = \sqrt{(101.6 + 49.9)^2 + (68.7 - 16.9)^2} = \sqrt{151.5^2 + 51.8^2} = 160 \text{ OHMS.}$$

$$\tan \theta = X/R = 51.8/151.5 = .341$$

$$\theta = 19^\circ \text{ LEAD}$$

THE TOTAL IMPEDANCE OF THE TWO CIRCUITS CONNECTED IN SERIES IS 160 OHMS AT

A FREQUENCY OF 600 KC/S. THE REACTIVE COMPONENT OF Z_S IS LARGER THAN THE REACTIVE COMPONENT OF Z_P , THEREFORE SINCE THE EFFECT OF Z_S IS TO CAUSE A LEADING CURRENT THE EFFECT OF THE TOTAL IMPEDANCE IS TO CAUSE A CURRENT LEAD OF 19° .

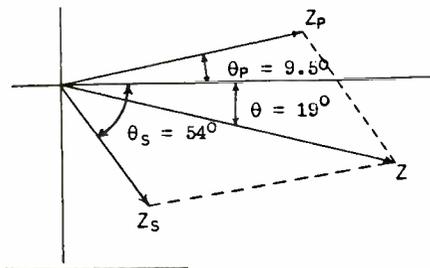


FIG. 7.

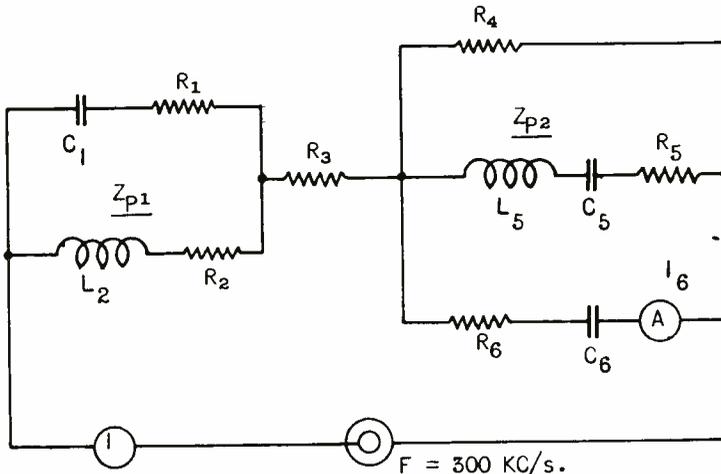
COMPLEX NETWORK: IN ORDER TO

DEMONSTRATE THE EASE WITH WHICH ALL THE VOLTAGE, CURRENT, AND IMPEDANCE RELATIONS IN A SOMEWHAT COMPLEX NETWORK MAY BE DETERMINED FROM WHAT MAY AT FIRST APPEAR TO BE RELATIVELY LITTLE INFORMATION, CONSIDER THE SOLUTION OF THE CIRCUIT OF FIGURE 8.

FUNDAMENTALLY THIS IS A SERIES CIRCUIT CONSISTING OF Z_{p1} , R_3 AND Z_{p2} IN SERIES. IT IS FIRST NECESSARY TO SOLVE FOR Z_{p1} AND Z_{p2} IN TERMS OF OHMS AND θ . IN A PROBLEM OF THIS TYPE THE MOST ESSENTIAL FACTOR IS AN ORDERLY AND METHODICAL ATTACK.

TO FIND THE LINE CURRENT I_T , THE IMPEDANCE OF THE TOTAL LOAD ACROSS THE GENERATOR, THE GENERATOR VOLTAGE, AND THE PHASE ANGLE OF THE LOAD.

FIRST, SOLVE Z_{p1} .



- $C_1 = .004 \mu F$
- $C_5 = .0015 \mu F$
- $C_6 = 1000 \mu F$
- $L_2 = 200 \mu H$
- $L_5 = 400 \mu H$
- $R_1 = 100 \text{ OHMS}$
- $R_2 = 200 \text{ OHMS}$
- $R_3 = 300 \text{ OHMS}$
- $R_4 = 400 \text{ OHMS}$
- $R_5 = 200 \text{ OHMS}$
- $R_6 = 300 \text{ OHMS}$
- $I_6 = 3 \text{ AMPERES}$

FIG. 8.

$F = 300 \text{ KC/s.}$

UPPER BRANCH:

Z_{P1}

$$R_1 = 100 \text{ OHMS}$$

$$X_{C1} = \frac{1}{6.28 \times .3 \times 10^6 \times 4 \times 10^{-9}} = \frac{10^3}{7.536} = 133 \text{ OHMS}$$

$$Z_1 = \sqrt{R^2 + X_{C1}^2} = \sqrt{100^2 + 133^2} = 166 \text{ OHMS}$$

$$\tan \theta_1 = \frac{X}{R} = \frac{133}{100} = 1.33 \quad \theta_1 = 53^\circ \text{ LEAD}$$

ASSUME VOLTAGE DROP IS 166 VOLTS, THEN $I_1 = \frac{166}{166} = 1 \text{ AMP.}$

LOWER BRANCH:

$$R_2 = 200 \text{ OHMS}$$

$$X_{L2} = 6.28 \times 3 \times 10^5 \times 2 \times 10^{-4} = 376.8 \text{ OHMS}$$

$$Z_2 = \sqrt{200^2 + 376.8^2} = 427 \text{ OHMS}$$

$$\tan \theta_2 = \frac{X}{R} = \frac{376.8}{200} = 1.88 \quad \theta_2 = 62^\circ \text{ LAG}$$

$$I_2(\text{ASSUMED}) = \frac{166}{427} = .39 \text{ AMPERE}$$

THE ASSUMED CURRENTS ARE SHOWN IN THE VECTOR OF FIG. 9.
TO FIND THE TOTAL LINE CURRENT WE MUST COMBINE THE TWO BRANCH CURRENTS VECTORIALLY AS FOLLOWS:

$$I_T = \sqrt{(I_1 \cos 53^\circ + I_2 \cos 62^\circ)^2 + (I_1 \sin 53^\circ - I_2 \sin 62^\circ)^2}$$

$$= \sqrt{(.602 + .183)^2 + (.799 - .344)^2} = \sqrt{.785^2 + .455^2}$$

$$= .907 \text{ AMPERE}$$

SERIES-PARALLEL CIRCUITS

14.

$$Z_{P1} = \frac{E \text{ ASSUMED}}{I \text{ ASSUMED}} = \frac{166}{.907} = \underline{\underline{183 \text{ OHMS}}}$$

$$\text{TAN } \theta_{P1} = \frac{.455}{.785} = .58 \quad \theta_{P1} = \underline{\underline{30^\circ \text{ LEAD}}}$$

THIS IS THE TRUE IMPEDANCE OF Z_{P1} EVEN THOUGH AN ASSUMED VOLTAGE WAS USED IN ITS DETERMINATION. THE ASSUMED VOLTAGE AND THE BRANCH CURRENTS WILL NOT BE USEFUL FOR ANY OTHER WORK IN THIS PROBLEM.

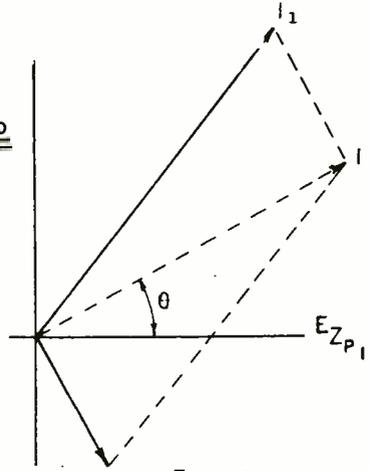


FIG. 9.

NEXT CONSIDER THE THREE BRANCH PARALLEL COMBINATION Z_{P2} . THIS WILL BE SOLVED IN A SIMILAR MANNER TO THE FIRST CIRCUIT BUT IT WILL NOT BE NECESSARY TO ASSUME A VOLTAGE IN THIS CASE SINCE ONE BRANCH CURRENT IS GIVEN.

LOWER BRANCH: Z_{P2}

$$R_6 = 300 \text{ OHMS} \quad C_6 = 1000 \mu\text{F} = 10^{-9} \text{ FARAD} \quad I_6 = 3 \text{ AMP.}$$

$$X_{C6} = \frac{1}{6.28 \times 3 \times 10^5 \times 10^{-9}} = \frac{10^4}{18.84} = 531 \text{ OHMS}$$

$$Z_6 = \sqrt{300^2 + 531^2} = 609 \text{ OHMS} \quad \text{TAN } \theta_6 = \frac{531}{300} = 1.77$$

$$= 60.5^\circ \text{ LEAD}$$

$$E_{Z_{P2}} = I_6 Z_6 = 3 \times 609 = 1827 \text{ VOLTS (THIS IS THE ACTUAL VOLTAGE ACROSS THE CIRCUIT, NO ASSUMED VOLTAGE NECESSARY).}$$

MIDDLE BRANCH:

$$R_5 = 200 \text{ OHMS}$$

$$X_{L5} = 6.28 \times 3 \times 10^5 \times 4 \times 10^{-4} = 75.36 \times 10 = 753.6 \text{ OHMS}$$

$$X_{C5} = \frac{1}{6.28 \times 3 \times 10^5 \times 15 \times 10^{-8}} = \frac{10^3}{282.6} = 354 \text{ OHMS}$$

$$Z_5 = \sqrt{200^2 + (753.6 - 354)^2} = \sqrt{200^2 + 399.4^2} = 447 \text{ OHMS}$$

$$\text{TAN } \theta_5 = \frac{399.4}{200} = 1.997 \quad \theta_5 = 63.5^\circ \text{ LAG}$$

$$I_5 = \frac{1827}{447} = 4.09 \text{ AMPERES}$$

UPPER BRANCH:

$$Z = R_4 = 400 \text{ OHMS} \quad \theta_4 = 0^\circ$$

$$I_4 = \frac{1827}{400} = 4.57 \text{ AMPERES}$$

THE CONDITIONS IN EACH PART OF THE CIRCUIT ARE CLEARLY SHOWN IN FIG. 10.

I_6 LEADS THE APPLIED VOLTAGE OR VOLTAGE DROP ACROSS Z_{P2} , I_5 LAGS AND

I_4 IS IN PHASE WITH E . FIG. 11 MORE CLEARLY SHOWS THE OPERATION OF THE CIRCUIT AS A WHOLE. IT IS SEEN THAT THE TOTAL CURRENT THROUGH Z_{P2} IS QUITE LARGE AND THE CURRENT LAGS THE CIRCUIT VOLTAGE BY A FEW DEGREES. THE TOTAL CURRENT I WILL NEXT BE FOUND.

FIRST NOTE THAT IN FIG. 11 THE CURRENTS ARE THOSE FOUND FROM ACTUAL CALCULATION WITHOUT ASSUMING ANY VOLTAGE AND THE REFERENCE VECTOR IS THE VOLTAGE DROP ACROSS THE PARALLEL COMBINATION. THE TOTAL CURRENT LAGS THIS VOLTAGE DROP

AS SHOWN IN THE FIGURE. THE TOTAL APPLIED VOLTAGE TO THE ENTIRE CIRCUIT IS THE VECTOR SUM OF THE VOLTAGE DROPS ACROSS THE SEPARATE CIRCUITS Z_{P1} , R_3 , AND Z_{P2} . THIS CAN BE USED AS A CHECK ON THE VOLTAGE AS FOUND LATER AND ON THE SOLUTION AS A WHOLE.

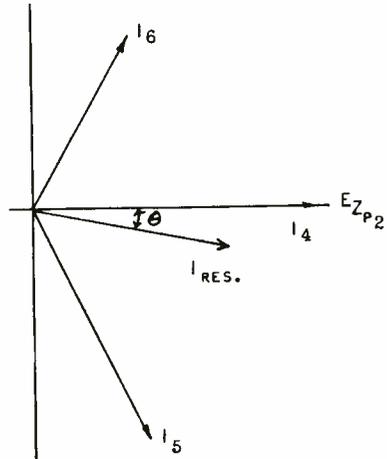


FIG. 10.

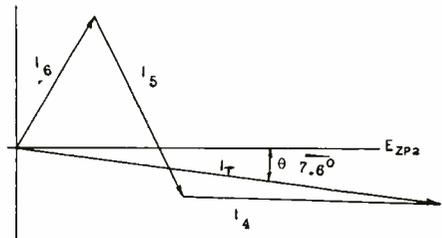


FIG. 11.

$$\begin{aligned} I_T &= \sqrt{(I_6 \cos 60.5^\circ + I_5 \cos 63.5^\circ + I_4)^2 + (I_5 \sin 63.5^\circ - I_6 \sin 60.5^\circ + I_4 \sin 0^\circ)^2} \\ &= \sqrt{(3 \cos 60.5 + 4.09 \cos 63.5 + 4.57)^2 + (4.09 \sin 63.5 - 3 \sin 60.5 + 0)^2} \\ &= \sqrt{(1.48 + 1.82 + 4.57)^2 + (3.66 - 2.61)^2} \\ &= \sqrt{7.87^2 + 1.05^2} = 7.94 \text{ AMPERE} \end{aligned}$$

16.

SERIES-PARALLEL CIRCUITS

$$\tan \theta = \frac{1.05}{7.87} = .133 \quad \theta_{P_2} = \underline{\underline{7.6^\circ \text{ LAG}}}$$

$$Z_{P_2} = \frac{1827}{7.94} = \underline{\underline{230 \text{ OHMS}}}$$

NOTE THAT THIS CURRENT I_T IS THE TRUE CURRENT OF THE ENTIRE CIRCUIT WHICH IS SUPPLIED BY THE GENERATOR.

NOW THAT THE THREE PARTS OF THE ORIGINAL CIRCUIT OF FIG. 8 HAVE BEEN SOLVED THEY CAN BE CONSIDERED AS AN EQUIVALENT SERIES CIRCUIT AS ILLUSTRATED BY THE VECTORS IN FIG. 12 WHICH CAN THEN BE COMBINED INTO A RESULTANT TOTAL IMPEDANCE AS SHOWN IN FIG. 13.

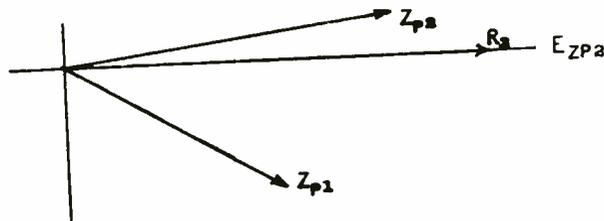


FIG. 12.

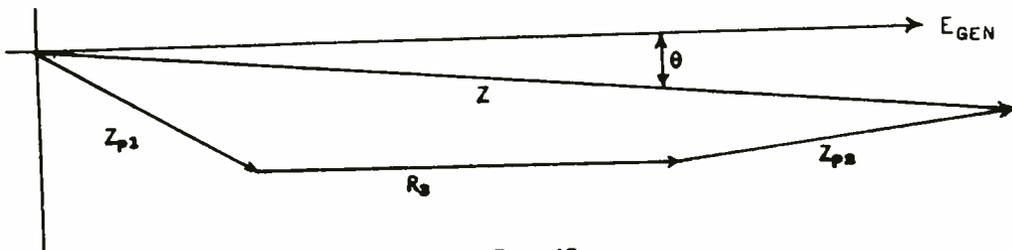


FIG. 13.

THE SOLUTION OF TOTAL Z CAN BE DONE GRAPHICALLY BY PLOTTING THE VECTORS TO SCALE OR BY TRIGONOMETRY AS SHOWN BELOW:

$$\begin{aligned} Z &= \sqrt{(Z_{P_1} \cos 30^\circ + Z_{P_2} \cos 7.6^\circ + R_3)^2 + (Z_{P_1} \sin 30^\circ - Z_{P_2} \sin 7.6^\circ)^2} \\ &= \sqrt{(183 \cos 30^\circ + 230 \cos 7.6^\circ + 300)^2 + (183 \sin 30^\circ - 230 \sin 7.6^\circ)^2} \\ &= \sqrt{(158 + 228 + 300)^2 + (91.5 - 30.4)^2} = \sqrt{686^2 + 61.1^2} = 689 \text{ OHMS} \end{aligned}$$

$$I_T = 7.94 \text{ AMP.} \quad E_{\text{GEN}} = IZ = 7.94 \times 689 = 5471 \text{ VOLTS}$$

AS A CHECK WE CAN SOLVE FOR THE VECTORIAL SUM OF THE VOLTAGES.

$$E_{Z_{P_1}} = 183 \times 7.94 = 1453 \text{ VOLTS } 30^\circ \text{ LEAD} \quad E_3 = 300 \times 7.94 = 2382 \text{ V.}$$

$$\begin{aligned} E_G &= \sqrt{(1827 \cos 7.6^\circ + 1453 \cos 30^\circ + 2382)^2 + (1827 \sin 7.6^\circ - 1453 \sin 30^\circ)^2} \\ &= \sqrt{(1827 \times .9912 + 1453 \times .866 + 2382)^2 + (1827 \times .13226 - 1453 \times .5)^2} \\ &= \sqrt{(5451)^2 + (513.5)^2} = 5452 \text{ VOLTS (A CLOSE CHECK)} \end{aligned}$$

$$\tan \theta = 61.1/686 = .08906$$

$$\theta = 5.1^{\circ} \text{ LEAD.}$$

THIS IS THE ENTIRE SOLUTION. THIS COMPLEX CIRCUIT HAS A TOTAL IMPEDANCE AT 300 KC/S OF 689 OHMS. TO CAUSE 3 AMPERES TO FLOW IN THE BRANCH CIRCUIT C_6R_6 REQUIRES A GENERATOR VOLTAGE OF 5471 VOLTS, AND THIS VOLTAGE CAUSES A TOTAL LINE CURRENT OF 7.94 AMPERES WHICH LEADS THE GENERATOR VOLTAGE BY 5.1⁰. SO FAR AS THE GENERATOR LOAD IS CONCERNED, THIS COMPLEX CIRCUIT COULD BE REPLACED BY A SIMPLE SERIES CIRCUIT CONSISTING OF RESISTANCE AND CAPACITY, THE TOTAL IMPEDANCE AND PHASE ANGLE AT THIS FREQUENCY BEING EQUAL TO THE CALCULATED VALUE, THAT IS, $Z = 689 \text{ OHMS}$, $\theta = 5.1^{\circ} \text{ LEAD}$. THIS WOULD REQUIRE A RESISTANCE OF 686 OHMS AND CAPACITY SUCH THAT $X_C = 61.1 \text{ OHMS}$, AS IS APPARENT FROM THE FINAL EQUATION FOR Z ABOVE.

IT SHOULD BE NOTED THAT, EVEN THOUGH AT FIRST GLANCE THE SOLUTION OF THIS CIRCUIT SEEMS QUITE COMPLEX, ACTUALLY WHEN HANDLED IN A SYSTEMATIC MANNER IT IS NOT AT ALL DIFFICULT. ANY SERIES-PARALLEL COMBINATION CAN BE SOLVED BY THE GENERAL METHODS USED IN THIS PROBLEM. IT SHOULD BE EMPHASIZED THAT THE SOLUTION IS CORRECT ONLY FOR THE SINGLE FREQUENCY AT WHICH THE CALCULATIONS ARE MADE.

VARIOUS PRACTICAL APPLICATIONS OF SERIES AND PARALLEL CIRCUITS WILL BE DISCUSSED IN CONSIDERABLE DETAIL ALONG WITH THEIR CONSTRUCTION, IN FOLLOWING LESSONS.

SUMMARIZING THE VOLTAGE, CURRENT AND IMPEDANCE RELATIONS IN SERIES AND IN PARALLEL CIRCUITS:

1. SERIES.

VOLTAGE ACROSS CIRCUIT EQUALS VECTOR SUM OF INDIVIDUAL VOLTAGES.

CURRENT IS THE SAME THROUGH ALL PARTS OF A SERIES CIRCUIT.

IMPEDANCE IS EQUAL TO THE VECTOR SUM OF THE INDIVIDUAL IMPEDANCES.

2. PARALLEL.

SERIES-PARALLEL CIRCUITS

VOLTAGE IS THE SAME ACROSS EACH BRANCH OF THE CIRCUIT.

CURRENT EQUALS THE VECTOR SUM OF THE INDIVIDUAL CURRENTS.

IMPEDANCE EQUALS THE TRUE OR ASSUMED VOLTAGE DIVIDED BY THE TRUE OR ASSUMED CURRENT WHICH THE TRUE OR ASSUMED VOLTAGE FORCES THROUGH THE CIRCUIT.

THE LC TABLE.

THE LC TABLE, A COPY OF WHICH IS INCLUDED IN THE BACK OF THIS LESSON, IS ONE OF THE MOST CONVENIENT TABLES USED IN RADIO WORK. IT HAS BEEN SHOWN THAT FOR ANY ONE FREQUENCY OR ANY ONE WAVELENGTH, THERE IS ONLY ONE VALUE OF THE LC PRODUCT. THAT IS, THE INDIVIDUAL L AND C VALUES FOR TWO CIRCUITS MAY BE ENTIRELY DISSIMILAR BUT SO LONG AS THE PRODUCTS OF L TIMES C IN THE TWO CIRCUITS ARE THE SAME, BOTH CIRCUITS WILL HAVE THE SAME WAVELENGTH OR RESONANT FREQUENCY.

WHY IS THIS LC TABLE SO USEFUL? BECAUSE IN ALMOST ANY TYPE OF RADIO PROBLEM DEALING WITH WAVELENGTH OR FREQUENCY THE LC PRODUCT IS ENCOUNTERED. FOR EXAMPLE,

$$\text{FREQUENCY} = \frac{1}{2\pi\sqrt{LC}} \quad F \text{ IN CYCLES, } L \text{ AND } C \text{ IN UNITS.}$$

$$\text{FREQUENCY} = \frac{159}{\sqrt{LC}} \quad F \text{ IN KILOCYCLES, } L \text{ AND } C \text{ IN MICRO-UNITS.}$$

$$\text{WAVELENGTH} = 1884 \sqrt{LC}. \quad \text{WAVELENGTH IN METERS, } L \text{ AND } C \text{ IN MICRO-UNITS.}$$

GIVEN A CIRCUIT CONSISTING OF KNOWN VALUES OF INDUCTANCE AND CAPACITY, IT IS POSSIBLE OF COURSE TO FIND EITHER THE WAVELENGTH OR THE RESONANT FREQUENCY FROM ONE OF THE EQUATIONS SHOWN ABOVE. THIS INVOLVES SEVERAL STEPS IN EITHER CASE. BY THE USE OF THE TABLES, HOWEVER, IT IS ONLY NECESSARY TO MULTIPLY THE VALUES OF L AND C, (BOTH IN MICRO-UNITS), AND REFER TO THE TABLE FOR THE CORRESPONDING VALUE OF EITHER WAVELENGTH OR FREQUENCY. IT SHOULD BE OBSERVED THAT IN THE ENCLOSED TABLES, FREQUENCY IS DESIGNATED BY "N" AND IS EXPRESSED IN CYCLES. TO CONVERT TO KILOCYCLES IT IS ONLY NECESSARY TO DIVIDE THE FREQUENCY IN CYCLES BY 1000.

ANOTHER EXAMPLE WHERE THE LC TABLE CAN BE USED TO ADVANTAGE. SUPPOSE THERE IS GIVEN A CIRCUIT OF KNOWN WAVELENGTH OR RESONANT FREQUENCY, AND THE VALUE OF C; TO FIND L. IT IS ONLY NECESSARY TO REFER TO THE LC PRODUCT FOR THE KNOWN FREQUENCY OR WAVELENGTH AND DIVIDE THE LC PRODUCT BY THE KNOWN VALUE OF C IN MICROFARADS. THE ANSWER WILL BE THE INDUCTANCE IN MICROHENRIES. IF L IS KNOWN AND C MUST BE DETERMINED, THE LC PRODUCT IS DIVIDED BY THE VALUE OF L IN MICROHENRIES, THE ANSWER BEING THE CAPACITY IN MICROFARADS.

THE ENCLOSED TABLES ARE COMPUTED FOR A MINIMUM WAVELENGTH OF 100 METERS OR 3000 KILOCYCLES. IN MODERN RADIO WORK IT IS OFTEN NECESSARY TO CALCULATE VALUES IN WAVELENGTHS MUCH SHORTER THAN 100 METERS. IT IS A VERY SIMPLE MATTER TO USE THE LC TABLE FOR SUCH VALUES.

FOR EXAMPLE, THE LC PRODUCT FOR A WAVELENGTH OF 1000 METERS IS .282; FOR 100 METERS THE LC PRODUCT IS .00282; FOR 10 METERS THE LC PRODUCT WILL BE .0000282.

THE LC PRODUCT FOR 3000 METERS IS 2.53; FOR 300 METERS, THE LC PRODUCT IS .0253; FOR 30 METERS THE LC PRODUCT WILL BE .000253; FOR 3 METERS, .00000253.

IN OTHER WORDS, FOR EACH PLACE IN THE WAVELENGTH COLUMN THE DECIMAL POINT IS MOVED TO THE LEFT, THE DECIMAL POINT MUST BE MOVED TWO PLACES TO THE LEFT IN THE COLUMN OF LC PRODUCTS. THIS IS DUE TO THE FACT THAT THE WAVELENGTH VARIES AS THE SQUARE ROOT OF THE LC PRODUCT; $WAVELENGTH = 1884 \sqrt{LC}$. IN EXTRACTING THE SQUARE ROOT OF THE PRODUCT OF LC, THE PRODUCT IS POINTED OFF IN GROUPS OF TWO PLACES FROM THE DECIMAL POINT.

FREQUENCY VALUES MAY BE HANDLED IN A SIMILAR MANNER EXCEPT THAT AS THE DECIMAL POINT IN THE NUMERICAL VALUE OF FREQUENCY IS MOVED ONE PLACE TO THE LEFT, THE DECIMAL POINT IN THE LC PRODUCT IS MOVED TWO PLACES TO THE RIGHT. THIS IS DUE TO THE FACT THAT THE FREQUENCY VARIES INVERSELY AS THE SQUARE ROOT

OF THE LC PRODUCT: $F = \frac{1}{2\pi \sqrt{LC}}$.

SERIES-PARALLEL CIRCUITS

REGULAR USE OF THE LC TABLES WILL GREATLY DECREASE THE TIME REQUIRED IN THE SOLUTION OF PROBLEMS INVOLVING L, C, WAVELENGTH AND FREQUENCY, AND WILL ALSO DECREASE THE CHANCES OF ERROR.

A PROBLEM INVOLVING THESE VALUES WILL SHOW CLEARLY THE USE OF THIS TABLE. GIVEN A VARIABLE CONDENSER OF 500 MICRO-MICROFARADS MAXIMUM CAPACITY AND 40 MICRO-MICROFARADS MINIMUM. IT IS DESIRED TO USE THE CONDENSER WITH AN INDUCTANCE THAT WILL ENABLE THE CIRCUIT TO BE TUNED TO 550 METERS WITH ALL THE CAPACITY IN THE CIRCUIT. WHAT MUST BE THE EFFECTIVE VALUE OF INDUCTANCE?

REFERRING TO THE LC TABLE, IT IS FOUND THAT THE LC PRODUCT AT 550 METERS IS .0852; $500 \mu\mu\text{F} = .0005 \mu\text{F}$. $L = LC/C = .0852/.0005 = 170.4 \mu\text{H}$.

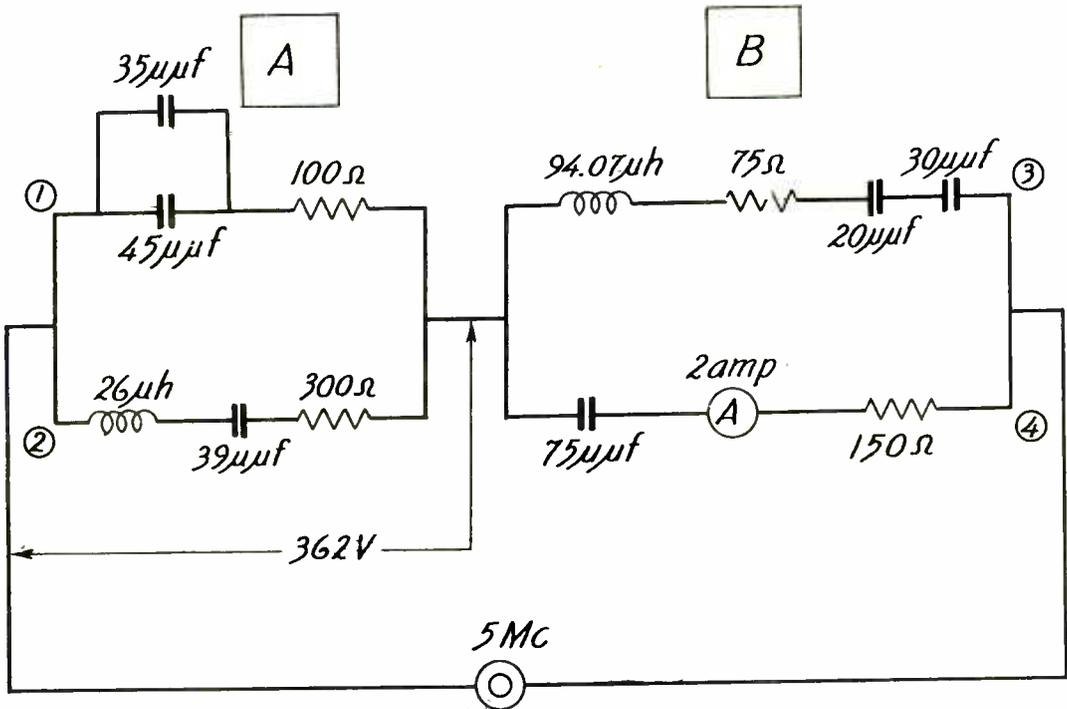
WITH THIS VALUE OF INDUCTANCE WHAT WILL BE THE SHORTEST WAVELENGTH TO WHICH THIS CIRCUIT CAN BE TUNED? MINIMUM CAPACITY IS $40 \mu\mu\text{F}$ OR $.00004 \mu\text{F}$. L IS $170.4 \mu\text{H}$. $LC = 170.4 \times .00004 = .006816$.

THE LC TABLE STATES THAT .00633 REPRESENTS A WAVELENGTH OF 150 METERS AND THAT .00721 REPRESENTS 160 METERS. .006816 IS ABOUT HALF-WAY BETWEEN THESE TWO VALUES SO THAT THIS CIRCUIT WOULD TUNE DOWN TO APPROXIMATELY 155 METERS. AN ACCURATE FIGURE COULD BE DETERMINED BY INTERPOLATION, BUT GREATER ACCURACY IS USUALLY NOT NECESSARY IN A PROBLEM OF THIS TYPE.

WHEN WORKING WITH QUITE HIGH FREQUENCIES, THAT IS, FREQUENCIES IN EXCESS OF 1000 KC/s, THE FREQUENCY IS COMMONLY EXPRESSED IN MEGACYCLES. THUS 15,000 KC/s = 15 MC, 8700 KC/s = 8.7 MC, 2300 KC/s = 2.3 MC, 120,000 KC/s = 120 MC, ETC. ONE MEGACYCLE EQUALS ONE THOUSAND KILOCYCLES. TO CONVERT MC TO KC, MULTIPLY BY 10^3 . TO CONVERT KC TO MC, MULTIPLY BY 10^{-3} .

Meters	n	L x O	Meters	n	L x O	Meters	n	L x O	Meters	n	L x O	Meters	n	L x O	Meters	n	L x O
100	8,000,000	0.00282	800	875,000	1.1901	1800	187,500	0.721	3200	93,800	2.88	5500	54,400	8.52	11000	27,300	84.1
110	7,277,000	0.00341	806	873,000	1.1824	1810	186,500	0.730	3220	92,200	2.92	5550	54,100	8.67	11100	27,000	84.7
120	2,500,000	0.00408	810	870,000	1.1747	1820	185,500	0.739	3240	92,600	2.96	5600	53,600	8.83	11200	26,800	85.3
130	2,308,000	0.00476	815	868,000	1.1870	1830	184,000	0.748	3260	92,000	2.99	5650	53,100	8.99	11300	26,500	85.9
140	2,143,000	0.00552	820	866,000	1.1833	1840	182,900	0.757	3280	91,500	3.03	5700	52,700	9.15	11400	26,300	86.5
150	2,000,000	0.00638	825	864,000	1.1916	1850	181,800	0.766	3300	90,900	3.06	5750	52,200	9.31	11500	26,100	87.1
160	1,875,000	0.00721	830	861,000	1.1939	1860	180,700	0.776	3320	90,400	3.10	5800	51,700	9.47	11700	25,800	88.5
170	1,764,000	0.00813	835	859,000	1.1982	1870	179,600	0.785	3340	89,800	3.14	5850	51,300	9.63	11800	25,600	89.2
180	1,667,000	0.00912	840	857,000	1.1966	1880	178,500	0.794	3360	89,300	3.18	5900	50,900	9.80	11900	25,400	89.9
190	1,579,000	0.01016	845	855,000	1.2021	1890	177,400	0.804	3380	88,800	3.22	5950	50,400	9.98	12000	25,200	90.5
200	1,500,000	0.01128	850	853,000	1.203	1900	176,500	0.813	3400	88,300	3.25	6000	50,000	10.13	12000	25,000	91.2
210	1,429,000	0.01241	855	851,000	1.206	1910	175,400	0.823	3420	87,700	3.29	6050	49,600	10.30	12100	24,800	91.8
220	1,364,000	0.01382	860	849,000	1.2098	1920	174,400	0.833	3440	87,200	3.33	6100	49,200	10.47	12200	24,600	92.5
230	1,304,000	0.01489	865	847,000	1.211	1930	173,400	0.842	3460	86,700	3.37	6150	48,800	10.65	12300	24,400	93.2
240	1,250,000	0.01619	870	845,000	1.2121	1940	172,400	0.852	3480	86,200	3.41	6200	48,400	10.82	12400	24,200	93.8
250	1,200,000	0.01769	875	843,000	1.216	1950	171,400	0.862	3500	85,700	3.45	6250	48,000	11.00	12500	24,000	94.4
260	1,154,000	0.01903	880	841,000	1.218	1960	170,500	0.872	3520	85,300	3.49	6300	47,600	11.17	12600	23,800	94.7
270	1,111,000	0.0205	885	839,000	1.220	1970	169,500	0.882	3540	84,800	3.53	6350	47,200	11.35	12700	23,600	95.4
280	1,071,000	0.0221	890	837,000	1.2223	1980	168,500	0.892	3560	84,300	3.57	6400	46,800	11.53	12800	23,400	96.1
290	1,034,000	0.0237	895	835,000	1.225	1990	167,600	0.902	3580	83,800	3.61	6450	46,400	11.71	12900	23,300	96.8
300	1,000,000	0.0253	900	833,000	1.228	1800	166,700	0.912	3600	83,400	3.65	6500	46,200	11.89	13000	23,100	97.6
310	968,000	0.0270	905	831,000	1.231	1810	165,700	0.922	3620	82,900	3.69	6550	45,800	12.08	13100	22,900	98.0
320	938,000	0.0288	910	830,000	1.233	1820	164,800	0.932	3640	82,400	3.73	6600	45,500	12.28	13200	22,700	98.0
330	909,000	0.0306	915	828,000	1.236	1830	163,900	0.943	3660	82,000	3.77	6650	45,100	12.48	13300	22,600	98.8
340	883,000	0.0325	920	826,000	1.238	1840	163,000	0.953	3680	81,500	3.81	6700	44,800	12.64	13400	22,400	99.5
350	857,000	0.0345	925	824,000	1.241	1850	162,200	0.963	3700	81,100	3.85	6750	44,400	12.83	13500	22,200	100.3
360	834,000	0.0365	930	823,000	1.243	1860	161,300	0.974	3720	80,700	3.90	6800	44,100	13.02	13600	22,100	101.1
370	811,000	0.0385	935	821,000	1.246	1870	160,400	0.984	3740	80,200	3.94	6850	43,800	13.21	13700	21,900	102.8
380	790,000	0.0406	940	819,000	1.249	1880	159,600	0.995	3760	79,800	3.98	6900	43,500	13.40	13800	21,700	103.6
390	769,000	0.0428	945	817,000	1.251	1890	158,700	1.006	3780	79,400	4.02	6950	43,200	13.60	13900	21,600	104.4
400	750,000	0.0450	950	815,000	1.254	1900	157,900	1.015	3800	79,000	4.06	7000	42,900	13.79	14000	21,400	105.2
410	732,000	0.0473	955	814,000	1.257	1910	157,100	1.026	3820	78,600	4.11	7050	42,600	13.99	14100	21,300	106.0
420	715,000	0.0496	960	813,000	1.259	1920	156,300	1.037	3840	78,200	4.15	7100	42,300	14.19	14200	21,100	106.8
430	698,000	0.0520	965	811,000	1.262	1930	155,400	1.048	3860	77,700	4.19	7150	42,000	14.39	14300	21,000	107.6
440	682,000	0.0545	970	809,000	1.265	1940	154,600	1.059	3880	77,300	4.24	7200	41,700	14.59	14400	20,800	108.4
450	667,000	0.0570	975	808,000	1.268	1950	153,800	1.070	3900	76,900	4.28	7250	41,400	14.79	14500	20,700	109.2
460	652,000	0.0595	980	806,000	1.271	1960	153,000	1.081	3920	76,500	4.32	7300	41,100	15.00	14600	20,600	110.0
470	639,000	0.0622	985	804,000	1.273	1970	152,200	1.092	3940	76,200	4.37	7350	40,800	15.21	14700	20,400	110.8
480	625,000	0.0649	990	803,000	1.276	1980	151,500	1.103	3960	75,800	4.41	7400	40,500	15.41	14800	20,300	111.6
490	612,000	0.0676	995	802,000	1.279	1990	150,800	1.114	3980	75,400	4.46	7450	40,300	15.62	14900	20,100	112.5
500	600,000	0.0704	1000	800,000	1.282	2000	150,000	1.126	4000	75,000	4.50	7500	40,000	15.83	15000	20,000	113.3
505	594,000	0.0718	1010	797,100	1.287	2020	148,500	1.148	4020	74,700	4.55	7550	39,700	16.04	15100	19,870	114.2
510	588,000	0.0732	1020	794,200	1.293	2040	147,100	1.171	4040	74,300	4.59	7600	39,600	16.26	15200	19,740	115.0
515	583,000	0.0747	1030	791,300	1.299	2060	145,600	1.194	4060	73,900	4.64	7650	39,200	16.47	15300	19,610	115.9
520	577,000	0.0761	1040	788,500	1.304	2080	144,200	1.218	4080	73,500	4.69	7700	39,000	16.69	15400	19,480	116.7
525	572,000	0.0776	1050	785,700	1.310	2100	142,900	1.241	4100	73,200	4.73	7750	38,700	16.90	15500	19,350	117.6
530	566,000	0.0791	1060	783,000	1.316	2120	141,500	1.265	4120	72,800	4.78	7800	38,500	17.12	15600	19,230	118.5
535	561,000	0.0806	1070	780,400	1.322	2140	140,100	1.291	4140	72,500	4.82	7850	38,200	17.34	15700	19,110	119.4
540	556,000	0.0821	1080	777,800	1.328	2160	138,900	1.313	4160	72,100	4.87	7900	38,000	17.56	15800	18,990	120.3
545	551,000	0.0836	1090	775,200	1.334	2180	137,600	1.335	4180	71,800	4.92	7950	37,700	17.79	15900	18,870	121.2
550	546,000	0.0852	1100	772,700	1.341	2200	136,400	1.358	4200	71,500	4.96	8000	37,500	18.01	16000	18,750	122.1
555	541,000	0.0867	1110	770,300	1.347	2220	135,100	1.382	4220	71,200	5.01	8050	37,300	18.24	16100	18,630	123.0
560	536,000	0.0883	1120	767,900	1.353	2240	133,900	1.417	4240	70,900	5.06	8100	37,100	18.47	16200	18,510	123.9
565	531,000	0.0899	1130	765,500	1.359	2260	132,700	1.443	4260	70,600	5.11	8150	36,900	18.70	16300	18,400	124.8
570	527,000	0.0915	1140	763,200	1.366	2280	131,600	1.468	4280	70,300	5.16	8200	36,800	18.93	16400	18,290	125.7
575	522,000	0.0931	1150	760,900	1.372	2300	130,400	1.494	4300	69,900	5.20	8250	36,600	19.16	16500	18,180	126.6
580	517,000	0.0947	1160	758,600	1.379	2320	129,300	1.519	4320	69,500	5.25	8300	36,400	19.39	16600	18,070	127.5
585	513,000	0.0963	1170	756,400	1.385	2340	128,200	1.541	4340	69,100	5.30	8350	36,200	19.62	16700	17,960	128.4
590	509,000	0.0979	1180	754,200	1.392	2360	127,100	1.568	4360	68,800	5.35	8400	36,000	19.86	16800	17,850	129.3
595	504,000	0.0996	1190	752,100	1.399	2380	126,000	1.594	4380	68,500	5.40	8450	35,800	20.11	16900	17,740	130.4
600	500,000	0.1013	1200	750,000	1.405	2400	125,000	1.621	4400	68,200	5.45	8500	35,600	20.3	17000	17,640	131.3
605	496,000	0.1030	1210	747,900	1.412	2420	124,000	1.648	4420	67,900	5.50	8550	35,400	20.5	17100	17,540	132.3
610	492,000	0.1047	1220	745,900	1.419	2440	122,900	1.676	4440	67,600	5.55	8600	35,200	20.8	17200	17,440	133.3
615	488,000	0.1065	1230	743,900	1.426	2460	121,900	1.703	4460	67,300	5.60	8650	35,000	21.1	17300	17,340	13

EXERCISE PROBLEMS
SERIES AND PARALLEL CIRCUITS



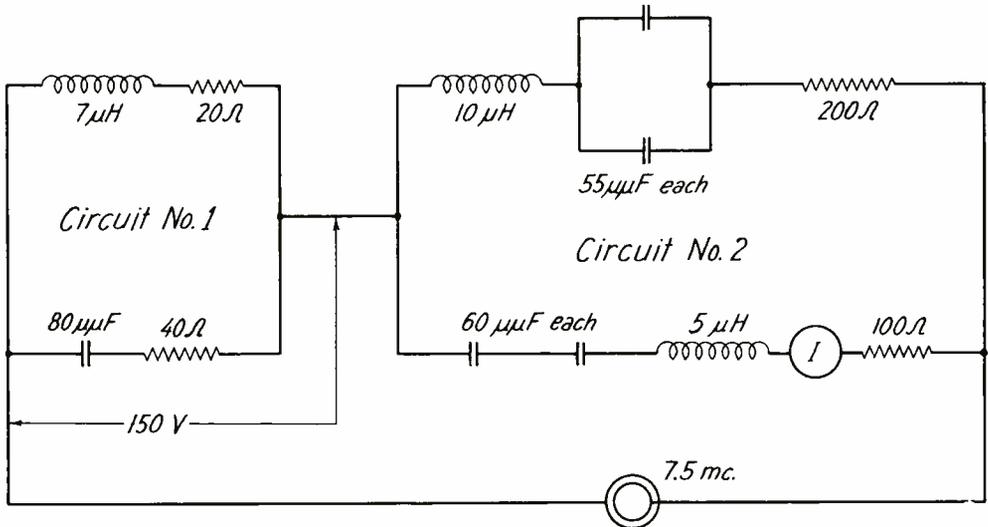
- 1). Impedance, and current of branch 1
 - 2). Impedance, and current of branch 2
 - 3). Impedance, and current of circuit **A** By two methods
 - 4). Impedance, and voltage of branch 4
 - 5). Impedance, and current of branch 3
 - 6). Impedance, and current of circuit **B** by two methods
 - 7). Total Impedance of the circuit by two methods
 - 8). Total line current
 - 9). Total voltage drop
 - 10). Total Power dissipated in the circuit by two methods.
- (Note: Vector diagrams for each circuit will be helpful).

ANSWERS

1. $Z_1 = 100 - j398.0 = 410.4/\underline{-75^\circ 54'}$
 $I_1 = .882 \text{ amps.}$
2. $Z_2 = 300 + j0 = 300/\underline{0^\circ}$ ohms
 $I_2 = 1.206 \text{ amps.}$
3. $Z_A = 186.93 - j112.54 = 218.2/\underline{-31^\circ 4'}$ ohms
 $I_A = 1.66 \text{ amps.}$
4. $Z_2 = 150 - j424.6 = 450.2/\underline{-70^\circ 32'}$ ohms
 $E_4 = 900.4 \text{ volts}$
5. $Z_3 = 75 + j300 = 309.8/\underline{75^\circ 58'}$ ohms
 $I_3 = 2.906 \text{ amps}$
6. $Z_B = 449.23 + j303.67 = 542.3/\underline{34^\circ 4'}$ ohms
 $I_B = 1.66 \text{ amps.}$
7. $Z_t = 636.14 + j191.13 = 664.45/\underline{16^\circ 43'}$ ohms
8. $I_t = 1.66 \text{ amps.}$
9. $E_t = 1102 \text{ volts.}$
10. $P_t = 1752 \text{ watts.}$

SERIES-PARALLEL CIRCUITS

EXAMINATION



FIND:

1. Impedance of circuit No. 1.
2. Impedance of circuit No. 2.
3. Total impedance of both circuits.
4. Voltage drop across circuit No. 2.
5. Current through lower branch circuit No. 2.
6. Power supplied by generator (power loss each branch equals I^2R).

All steps should be shown just as in the solution of the circuit in Fig. 8 of text. Tabulate the answers 1 to 6 for easy inspection of your work as a whole.

7. What LC product corresponds to a frequency of 40.3 kilocycles? 4095 kilocycles?

SERIES-PARALLEL CIRCUITS

EXAMINATION, Page 2.

8. You are designing a 50 mc circuit and decide to use a 10 micromicrofarad condenser:
- (A) What value of L will you use?
 - (B) What will be the L/C ratio?
9. What frequencies correspond to the following LC values?
- (A) .1801
 - (B) .01903
 - (C) 18.01
10. (A) What will be the LC product for a wavelength of 1,550 meters?
- (B) If a circuit tuned to this wavelength employs a capacity of 860 micromicrofarads, what will be the inductance of the circuit?

