ACHIEVEMENT THROUGH ELECTRONICS

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DC CIRCUIT CALCULATIONS

X105

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REFERENCE TEXT X105

STUDY SCHEDULE

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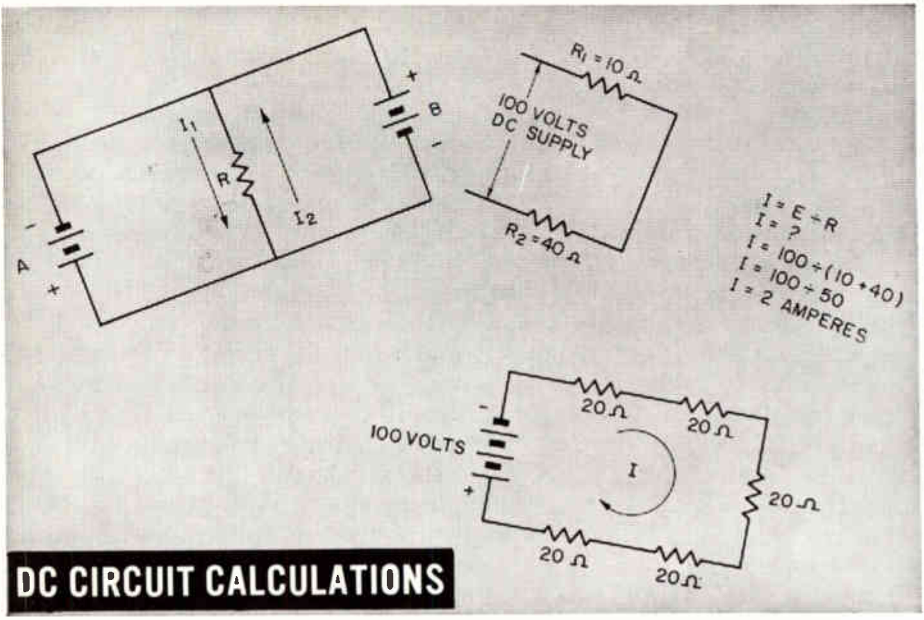


DC CIRCUIT CALCULATIONS

One of the very first things that is explained to anyone learning electronics is Ohm's Law. You will remember that this is the basic law covering the relations of voltage, current, and resistance in electrical circuits. In addition to being one of the most basic of the fundamentals of electronics, it is also one of the most important rules or laws. You will find that no matter how far you go in the study of industrial electronics, radio communications, or electronics engineering, this law which states that "the current flowing in a circuit is equal to the applied voltage divided by the resistance" will be used and applied over and over again.

At the same time that you learned about Ohm's Law, you also learned that it could be expressed much more simply by using the symbols for current, voltage, and resistance in the formula: $I = E \div R$. This is the mathematical expression of the law using symbols instead of words and is called a formula. Most of the rules or laws of electronics, or of any other science for that matter, are expressed with formulas for two reasons. One reason is that these simple mathematical expressions of the laws are much easier to memorize. The other reason is that they automatically provide a workable relationship of the laws for use in calculations. Thus, once you have learned the formulas, you not only have learned the rules, but you also have them expressed properly for use in circuit calculations.

One of the best things about formulas is that they can be arranged to find a particular quantity that we want to know. For example, the formula for Ohm's Law, $I = E \div R$, is used when we know the voltage and resistance and want to find the current in a circuit. We also learned that another way of saying the same thing is to state that $E = I \times R$. We usually simplify this expression still further by dropping the × sign and writing the formula simply $E = IR$. This way of expressing the basic formula is used when we know the current and resistance and want to find the voltage.

By still another arrangement, we can state the formula so we can easily find the resistance by saying $R = E \div I$. We use the formula in this form when we know the voltage and current in the circuit and want to find the resistance. All three of these statements of Ohm's Law say exactly the same thing. They are simply arranged in different ways for convenience in making circuit calculations.

If you do not already know the three forms of Ohm's Law, stop right now and memorize them. You will save yourself a great deal of time in the long run, and in addition, knowing these formulas and understanding the way in which voltage, current, and resistance in a circuit are related will help you to understand electronic circuits. The three forms of Ohm's Law are:

$$I = E \div R$$
$$E = IR$$
$$R = E \div I$$

Actually the three forms of Ohm's Law are really only one formula with the letters manipulated around the equals sign. We are able to change these formulas around to suit our purposes by applying some very basic rules of mathematics. In electronics, we must learn quite a few formulas to perform certain calculations so that we can understand how the circuits work and what may be wrong with them. Of course, the formulas can be arranged depending on what we want to find out and what we already know. Obviously, if we have to learn not only all the formulas but all the different forms of the formulas as well, we would have to do a lot of memorizing.

This would be impractical because we can easily learn a few of the basic laws of mathematics and then change the formulas to suit our own purposes. By doing this, we need to learn only one form of each formula, plus the rules for changes. In this way, mathematics becomes a useful tool, both in studying, and working in the electronics field. You will learn how to rearrange formulas later, so that then all you will have to do is remember one form of Ohm's Law and you will be able to get the other two. Right now, however, to save time, be sure that you know all three forms.

These lessons in mathematics may be just review for you. However, don't skip over them lightly. They are just as important as any other lessons in your course. You must study them carefully and send in answers to the questions in the back of the book. The only difference between these mathematic lessons and your regular study lessons is the order in which you will do them. The math lessons should be studied immediately after the regular lesson text with the same number. For example, the first lesson should be studied after you finish lesson five.

We will break up our study of mathematics in this way for two reasons. The first and most important reason is that these math lessons are different from any you have ever seen. Most math textbooks teach general mathematics so that the learning can be applied to any subject. Here, we are primarily interested in mathematics from the standpoint of usefulness in electronics. In other words, we are interested in its application to a specific purpose. Therefore, we will take up the subjects in the order that you will need them and will use practical examples from the text that you have already studied. In this way you will be sure to learn the mathematics that you need, and you will also learn how to use it in practical examples.

The other reason that we break it up into several books spaced among the technical lessons is that we don't want to take you away from your technical progress long enough to study the math lessons all at once. By spacing the math lessons, you will be able to keep up with your technical lessons too. In this first lesson, we will do a number of problems in electronics which involve only addition, subtraction, multiplication and division. We will then start a detailed review of arithmetic starting with fractions and decimals. Examples and problems include circuit applications to help you learn how to make calculations in dc circuits. It may seem rather simple, however, you should read it all to be sure you remember it. Also, we have put in a few short cuts which you probably did learn in school and presented some of the material in a special way to help you later in your more advanced studies.

If mathematics has always bothered you before, don't be discouraged by the fact that you have to study it now. We present it very simply, and having a practical use for it makes it easier to understand.

Simple Arithmetic Review

Before going ahead with fractions and decimals, we want to point out here that the basic arithmetic operations of addition, subtraction, multiplication and division are important in electronics as they are in every other science. We assume that you are able to perform these basic operations; if you cannot, stop and get a book on basic arithmetic from your library. If you know how to add, subtract, multiply and divide, but have become rusty because you have not had occasion to perform these basic arithmetic operations, take some time now to do a little practicing. You will be surprised how quickly you will be able to pick up speed again after a little practice.

ADDITION

You might think that you will not have much occasion to use such a basic arithmetic operation as addition. However, this is not the case. As an example, in a series circuit, where two resistances are connected in series, the total resistance is equal to the sum of the two resistors. This means that if you have two 100-ohm resistors in series, to get the total resistance you add the resistance of the two resistors, 100 + 100 = 200. If you have a 100-ohm resistor in series with a 25-ohm resistor, to find the total resistance you add 100 + 25 = 125. If you have a number of resistors connected in

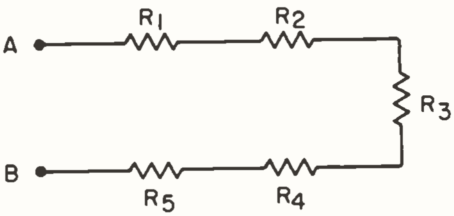


Fig. 1. A simple series circuit.

series, such as shown in the series circuit in Fig. 1, to get the total resistance between terminals A and B you add the resistance of the individual resistors. Suppose in this circuit that R1 = 100 ohms, R2 = 250 ohms, R3 = 150 ohms, R4 = 175 ohms, and R5 = 250 ohms. To find the total resistance between terminals A and B you write down the value of the individual resistors as shown and add.

$$\begin{array}{r} 100 \\ 250 \\ 150 \\ 175 \\ \underline{250} \\ 925 \end{array}$$

The preceding example is a simple example of addition in electronics to find the total resistance in a series circuit. Sometimes you have to be somewhat careful because the value of the resistors may vary quite widely and then it is important to get the digits arranged in the proper columns. In other words, you simply have to make sure that you arrange your work neatly so that the addition can be performed easily. As a second example, suppose the resistors have the following values: R = 5 ohms, R2 = 75 ohms, R3 = 6 ohms, R4 = 125, R5 = 32 ohms. To add the resistance of these resistors to get the total resistance between terminals A and B you write the resistors down as shown below and add.

$$\begin{array}{r} 5 \\ 75 \\ 6 \\ 125 \\ \underline{32} \\ 243 \end{array}$$

SUBTRACTION

There are occasions when you will have to subtract. In Fig. 2 we have shown a series circuit where the total resistance of the circuit is 197 ohms. The value of four of the resistors is known, but the value of the fifth resistance is unknown.

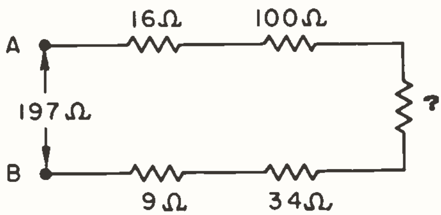


Fig. 2. A series circuit where one resistance is unknown.

You will remember that in a series circuit the total resistance is equal to the sum of the individual resistances. Therefore, the sum of the resistances must be 197 ohms. Since we know the value of four of the resistances we can find what their resistance is and then subtract this one value from 197 ohms to get the value of the unknown resistance.

To solve this problem we first write down the value of the known

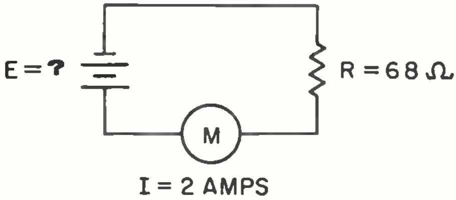


Fig. 3. An example where the formula E=IR is used to find the voltage.

resistors and add as shown:

$$\begin{array}{r} 16 \\ 100 \\ 34 \\ 9 \\ \hline 159 \end{array}$$

Thus the total resistance of the four known resistors is 159 ohms. Since the total resistance is 197 ohms we can find the resistance of the unknown resistor by subtracting 159 from 197. We set the problem down as shown below and subtract:

$$\begin{array}{r} 197 \\ -159 \\ \hline 38 \end{array}$$

MULTIPLICATION AND DIVISION

Simple multiplication and division are also important in electronics. The various forms of Ohm's Law that you memorized demonstrate the importance of being able to multiply and divide.

An example of the use of the formula E = IR is shown in Fig. 3. In this circuit we know that the resistance is 68 ohms and the current flowing in the circuit is 2 amps and we want to find the value of the applied voltage. Using the formula E = IR in substituting 2 amps for I and 68 ohms for R we get:

$$E = IR$$
$$E = 2 \times 68$$
$$E = 136 \text{ volts}$$

In case you wonder why we used this particular form of Ohm's Law, the answer is that this is the form which is used to find the voltage, which is the unknown, when we know the value of the current and resistance, which are the known values. We have the unknown on one side of the equals sign and the two known values on the other side of the equals sign.

In the circuit shown in Fig. 4 we know the value of the voltage and resistance and want to find the current. Therefore, we will use a form of Ohm's Law which places the unknown on one side of the equals sign and the known values on the other side. This means that I must be on one side of the equals sign, and E and R on the other side. Thus, we use the formula I = E ÷ R. Substituting 32 volts for E and 16 ohms for R, we can get the value of the current:

$$I = E \div R$$
$$I = 32 \div 16$$
$$I = 2 \text{ amps}$$

Fig. 5 is an example in which we use the remaining form of Ohm's Law, R = E ÷ I. Again, in this case we have R, which is the unknown

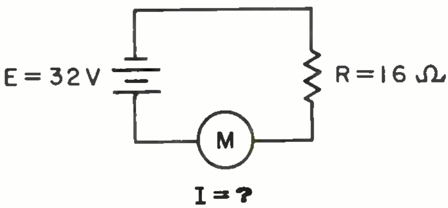


Fig. 4. An example of the formula I=E÷R used to find the current.

value, on one side of the equals sign, and the two known values E and I on the other side. Substituting 26 volts for E, and 2 amps for I we get:

$$R = E \div I$$
$$R = 26 \div 2$$
$$R = 13 \text{ ohms}$$

Notice that in each of the three preceding examples, not only did we carry out the mathematical operations to get a numerical answer, but we also gave the answer in its correct units. For example, in the first case, in the circuit shown in Fig. 3, where we wanted to find the value of the applied voltage, we gave the answer in volts. In the second case, where we had to find the current we gave the answer in amps. In the third case, where we had to find the resistance, we gave the answer in ohms. Remember this, it is important. You should always indicate the units of your answer. If you simply give a numerical answer, it has no meaning. If you are looking for the voltage in the circuit, the answer should be given in volts or some fraction or multiple of a volt. If you are looking for the current in a circuit then the answer should be given in amps or again in some fraction or a multiple of an ampere. Similarly, if you are looking for the resistance in a circuit, then the answer should be given in ohms or some fraction or multiple of an ohm.

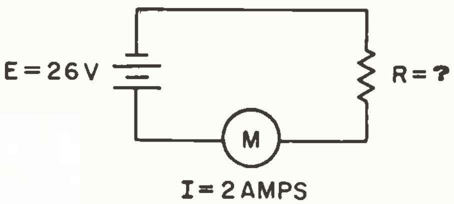


Fig. 5. An example of where the formula R = E ÷ I is used to find the resistance.

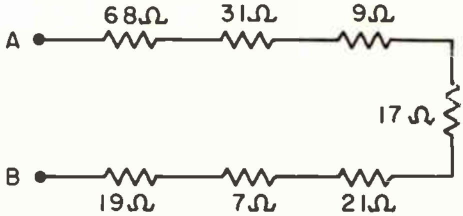


Fig. 6. Circuit for problem no. 1.

So far we have shown simple applications of addition, subtraction, multiplication and division in electronics. The examples were very simple, and in all probability you can handle problems of this type without any trouble. However, just to be sure, we have included ten practice problems which we have called, "Self-Test Questions." Even though you may think the problems are extremely simple, we urge you to do all ten to get practice finding the resistance in a circuit and in using Ohm's Law. A little practice now will help you become so familiar with these basic operations that when you have more complicated calculations to perform later, you will see that in many cases you're simply performing these simple operations several times.

You will find the answers to these questions at the back of the lesson along with a brief solution of each problem. Try to do each problem before you look at the answer so you will be working on your own. Be sure to check your answer and if you have made a mistake be sure to find out where the mistake is before going on.

SELF-TEST QUESTIONS

1. Find the total resistances between terminals A and B in the circuit shown in Fig. 6.

2. In the circuit shown in Fig. 7 the total resistance is 81 ohms. Find the resistance of the unknown resistor R.

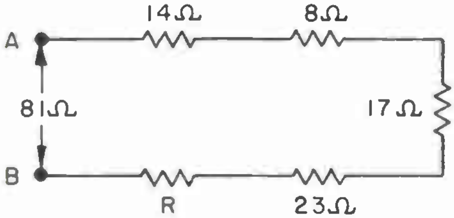


Fig. 7. Circuit for problem no. 2.

3. Find the total resistance between terminals A and B in the circuit shown in Fig. 8.

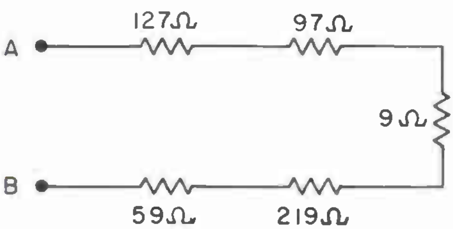


Fig. 8. Circuit for problem no. 3.

4. In the circuit shown in Fig. 9, the total resistance between terminals A and B is 823 ohms. Find the value of the unknown resistor R.

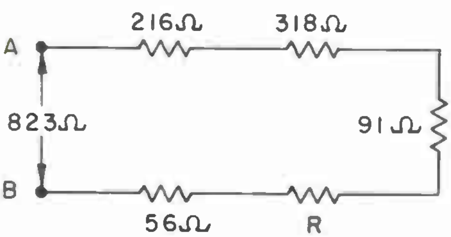


Fig. 9. Circuit for problem no. 4.

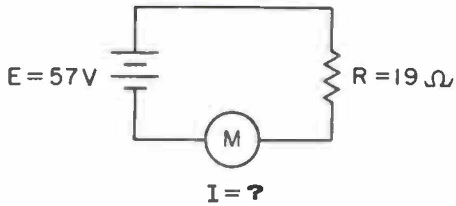


Fig. 10. Circuit for problem no. 5.

5. In the circuit shown in Fig. 10, the voltage is 57 volts, and the resistance in the circuit is 19 ohms. Find the current flowing in the circuit.

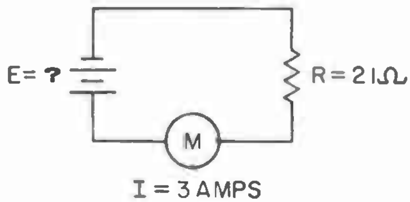


Fig. 11. Circuit for problem no. 6.

6. In the circuit shown in Fig. 11, the current flowing is 3 amps and the resistance in the circuit is 21 ohms. Find the value of the applied voltage.

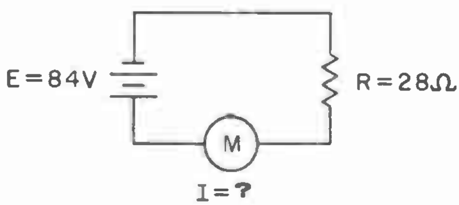


Fig. 12. Circuit fo· problem no. 7.

7. In the circuit shown in Fig. 12, find the current, if the applied voltage is 84 volts, and the resistance if the circuit is 28 ohms.

8. In the circuit shown in Fig. 13, the applied voltage is 96 volts, and the current in the circuit is 4 amps. Find the resistance in the circuit.

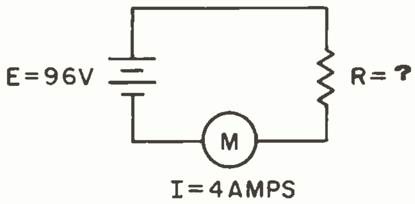


Fig. 13. Circuit for problem no. 8.

9. Find the voltage in the circuit shown in Fig. 14, if the current flowing in the circuit is 5 amps, and the resistance in the circuit is 17 ohms.

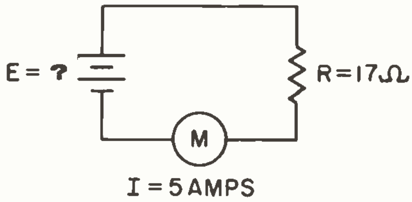


Fig. 14. Circuit for problem no. 9.

10. In the circuit shown in Fig. 15, the applied voltage is 32 volts, and the current flowing in the circuit is 2 amps. If R1 is equal to R2, find the value of these two resistors. (We have not covered an example exactly like this, but here is a chance for you to try a problem that is a little new on your own. Be sure to give it a good try, before looking at the solution at the back of the book.)

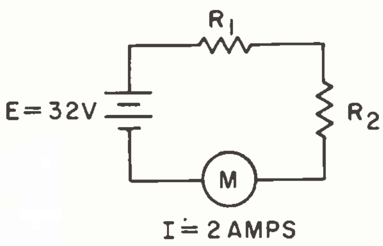


Fig. 15. Circuit for problem no. 10.

RULES OF ORDER

So far we have done problems involving addition, subtraction, multiplication and division. These are all very basic mathematical operations which we will use over and over again in our study of electronics. In the preceding problems, we have been concerned with just one operation at a time. In actual practice we will find a need to do several, or perhaps all these operations in order to find an answer to these problems.

While it may not seem at first glance that there is anything special about this, most of the time there will be a definite order in which we should do them. For example, take a simple problem like "find the value of 10 × 5 + 2." Let us look at this problem closely.

If we do the multiplication first and then the addition, we get 10 × 5 = 50, then we add the 2 and find the answer 52. However, if we look at the problem and say 5 + 2 = 7 and then 7 × 10 we come up with an answer of 70. As you can see, there is quite a difference between our first answer of 52 and our second answer of 70. Thus, the order in which we do a problem is important.

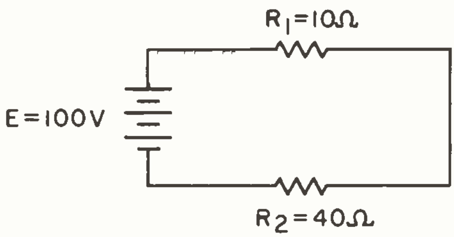


Fig. 16. Practical circuit where rules of order are important.

Let us take another more practical operation that we might find in our work in electronics. In the circuit shown in Fig. 16, we have a voltage supply of 100 volts and two resistors. One resistor is 10 ohms and the other is 40 ohms. Now the current in the circuit will be equal to this voltage divided by the resistance ($I = E \div R$). Let us take the problem mathematically and then substitute the values in the formula:

$$I = E \div R$$
$$I = 100 \div 10 + 40$$

Which operation do we do first? We know from our lessons on Ohm's Law that we are dealing with a total voltage of 100 volts. Therefore, we will be finding the total current, and to do this we will want to divide the total voltage by the total resistance. For this reason, we must add the two resistances first to get 10 + 40 = 50 ohms and then divide 100 by 50 to get a current of 2 amperes.

We did the problem this way because we know something about electronics; Ohm's Law states that total current equals total voltage divided by total resistance. We also know that the total resistance is equal to the sum of the resistances in the circuit. Therefore, in our problem, though we might not be aware of it, we actually thought this way:

$$I_T = E_T \div R_T$$
$$R_T = R_1 + R_2$$

so we kept the two resistances together.

If a person who did not know anything about electronics tried to work this problem he might not know the importance of keeping the two resistances together. Yet, one of the big advantages of using formulas in electronics is to express things simply so that problems can be easily worked out. Therefore, so that there will be no misunderstanding about the fact that R_1 and R_2 should be kept together, we enclose them in parentheses and write them as $(R_1 + R_2)$. Now the formula becomes:

$$I_T = E_T \div (R_1 + R_2)$$

Then we substituted to get:

$$I_T = 100 \div (10 + 40)$$

thus, $I_T = 100 \div 50$

$= 2$ amps

If someone saw this problem without the parentheses, and he knew nothing about electronics, or math, other than addition and subtraction, multiplication and division, he might come up with a different answer. For example:

$$I = E \div R$$
$$I = 100 \div 10 + 40$$

Then, they might first divide 10 into 100 and get 10 so that for the current they would get

$$I = 100 \div 10 + 40 = 10 + 40$$
$$= 50 \text{ amperes}$$

From the preceding you can see the value of using parentheses, and also the need for establishing some rules in which the various arithmetic operations should be performed. These rules are called the rules of order and they insure that everyone everywhere will always know in what order to tackle a problem. The rules of order are very easy to learn and must always be followed. Always start at the left of a problem and work towards the right and do all the operations inside the parentheses first. Next, start at the left again and work towards the right and do all the multiplication and division in the order in which they occur. Then go back to the left of the problem again and once again work to the right doing the addition and subtraction.

By following these rules there is no possibility of coming up with the wrong answer or doing the wrong operations at the wrong time. A problem such as:

$$(6 + 3) \times 3 - 81 \div 9 + 4$$

can have only one answer. We start at the left and do the operations enclosed in the parentheses first. Since there is only one parenthesis, the first time, working from left to right, there is only one operation to perform: we add 6 + 3. Thus our problem becomes:

$$(6 + 3) \times 3 - 81 \div 9 + 4 =$$
$$9 \times 3 - 81 \div 9 + 4$$

Now we start at the left and go through from the left to the right again doing the multiplication and division in the order in which they occur. The first multiplication we hit is 9×3 and this is 27. The next operation we must perform is the division of 81 by 9; 9 goes into 81, 9 times. Since there are no other multiplications or divisions indicated these are the only operations we perform through this time. Thus we have:

$$9 \times 3 - 81 \div 9 + 4 = 27 - 9 + 4$$

Now we go through the problem again, working from the left to the right, doing the addition and subtraction.

$$27 - 9 = 18 \text{ and adding } 4$$

to this would give us 22. Thus we have

$$27 - 9 + 4 = 22$$

Sometimes all of the operations covered in the preceding example are not found in a problem. For example in the problem

$$8 \times 7 + 2 \times 4 - 16 \times 3$$

there are no parentheses and therefore we start right in going from left to right to do any multiplication or division that might be indicated. You will notice that in this problem there is no division so you simply go through doing the indicated multiplication. $8 \times 7 = 56$, $2 \times 4 = 8$ and $16 \times 3 = 48$; therefore, our problem becomes:

$$8 \times 7 + 2 \times 4 - 16 \times 3 =$$
$$56 + 8 - 48 = 16$$

The problem might have parentheses in it, but no multiplication or division. As long as the parentheses are there, you must perform the operations inside the parentheses. As an example, in the problem,

$$29 - (17 + 4) + 11$$

we must perform the operation 17 + 4 which is inside the parentheses first. Since 17 + 4 = 21 our problem becomes:

$$29 - (17 + 4) + 11 = 29 - 21 + 11 = 19$$

Sometimes we need to do two or

more things in a special order. For this reason we also use brackets [ ] which are really a different kind of parentheses to indicate which comes first. Thus, we might have a problem:

$$5 \times 300 \div [2 \times (15+35)] + 20 - 5$$

Here we do the operations within the parenthesis, $(15 + 35) = 50$ first, and rewrite the bracket operation, replacing the $(15 + 35)$ with 50 to get

$$5 \times 300 \div [2 \times 50] + 20 - 5$$

then we do the operation inside the bracket to get:

$$2 \times 50 = 100$$

Now we rewrite the whole problem, placing everything inside the brackets with 100 and leaving the brackets out, thus, our problem becomes:

$$5 \times 300 \div 100 + 20 - 5$$

By following our rules of order, we start at the left and multiply 5×300 to get 1500, and then we divide this by 100 to get 15. Now we do our addition and subtraction: $15 + 20 - 5 = 35 - 5 = 30$ to find our final answer.

Since these rules of order are so important, let us state them again. First we do all the operations within the parentheses. Second, if we have one parenthesis within another, we do the operations inside the inner parenthesis first, and then do the operations within the outer parenthesis. When all the parenthetical operations are out of the way, we remove the parentheses, replacing the data within them with the answer we got. Then we rewrite our problem, starting at the left and working towards the right, doing all the multiplication and division in the order in which they occur. Then we return to the left and work to the right, doing the addition and subtraction in the order in which they occurred to get our answer.

You are going to run into problems in which you will have to perform the various operations in the correct order to get the correct answer. You will run into problems of this type in this lesson. Therefore, to get some practice doing the various operations in the right order, do the following five Self-Test Questions. You will find the answers in the back of the book.

SELF-TEST QUESTIONS

11. $25 + 16 \times 3 - 28 \div 7$
12. $5 \times (11 - 8) + 3 \times (7 - 5) \div 2$
13. $4 + (5+2) \times 20 - (10 - 6) \div (7 - 5)$
14. $3 \times 500 \div [2 \times (28+22)] + 25 - 6$
15. $95 \div (22 - 17) - 6 \times 2 - 3 + 8$

Fractions

In the simple circuit shown in Fig. 17, we have 100 volts applied to a circuit containing a total resistance of 100 ohms. This means that we will have a current flowing through the circuit of 1 ampere and a voltage drop across the resistances equal to the applied voltage of 100 volts. However, in this circuit we do not have a single 100-ohm resistor. Instead, we have two 50-ohm resistors in series, which makes up the total resistance of 100 ohms. Therefore, our voltage drop of 100 volts does not occur as one voltage, it occurs as two voltage drops of 50 volts across each resistor. In a case like this, where we have two equal voltage drops of 50 volts that add up to a total voltage of 100 volts, we often say that each drop of 50 volts is equal to one half of the total voltage. Just what do we mean when we say that we have one half of something? First, we mean that the "something," in this case 100 volts, is split up into parts. Further, since we have only two parts and they are equal we mean that the whole 100 volts is split into two equal parts. Thus, when we say one half of one hundred, it is like saying one of two equal parts of one hundred, or, more simply we mean 100 ÷ 2

Likewise, in a circuit such as the one shown in Fig. 18, the total voltage drop of 90 volts is split up into three equal parts of 30 volts each. This is just the same as saying that the voltage is split into thirds and one drop of 30 volts is equal to one-third of the total or 90 ÷ 3.

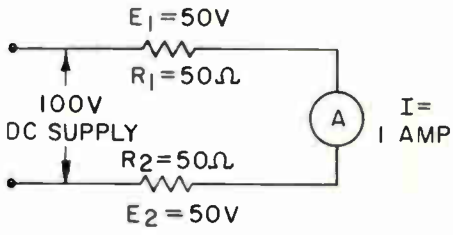


Fig. 17. One hundred volts divided into two equal parts or halves.

This can be written as $\frac{90}{3}$

100 ÷ 2 can be written as $\frac{100}{2}$

Whenever we split anything into parts we call the parts fractions.

Thus, $\frac{100}{2}$ is a fraction and $\frac{90}{3}$ is also a fraction. Now these particular fractions can easily be worked out by performing the actual division so that $\frac{100}{2}$ is 100 ÷ 2, or 50; and $\frac{90}{3}$ is 90 ÷ 3, or 30. When they are worked out like this, 50 and 30 actually become whole numbers in themselves because they each represent an individual voltage drop. However, when we consider them as part of the total

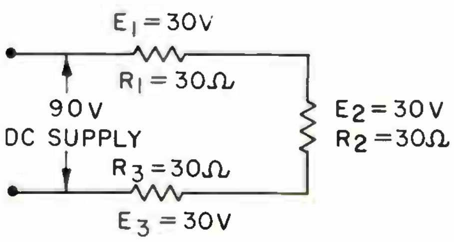


Fig. 18. Ninety volts divided equally into three parts or thirds.

voltage drop, they are both fractional parts of something and therefore they are also fractions.

When we want to represent 50 volts as some fraction of 100 volts, or 30 volts as a fraction of 90 volts, we would write them as $\frac{50}{100}$ or $\frac{30}{90}$, respectively. The fraction $\frac{50}{100}$ can be written more simply by dividing both the top of the fraction 50 and the bottom of the fraction 100 by the same number. For example, if we divide 100 by 50 we get 2 for an answer, and if we divide 50 by 50 we get an answer of 1. But placing the 1 in place of the 50 and the 2 in the place of the 100, our fraction becomes $\frac{1}{2}$, or one-half. Thus $\frac{50}{100}$ can be changed to $\frac{1}{2}$, or as we say, one-half, by dividing both the top and the bottom of the original fraction by 50. In the same way we can write $\frac{30}{90}$ or can simply divide both 30 and 90 by 30. This gives us $30 \div 30 = 1$ and $90 \div 30 = 3$. Then by replacing 30 with 1 and 90 with 3 we have the fraction $\frac{1}{3}$, which we pronounce "one-third." This fraction $\frac{1}{3}$ and the fraction $\frac{1}{2}$ which we found previously, are the simplest forms of the original fraction. We call this process of changing a fraction to its simplest form reducing the fraction.

Thus, in the circuit in Fig. 17 either one of the voltage drops can be considered several ways. They can be considered as one-half of a hundred which we write mathematically as $\frac{1}{2}$ of 100. They can be considered as $\frac{50}{100}$ which we can reduce to one-half or $\frac{1}{2}$ or they can be considered $\frac{100}{2}$ which is equal to 50. Now, these are all just different ways of saying the same thing. Likewise, any one of the voltage drops in the circuit in Fig. 18 can be expressed $\frac{1}{3}$ of 90, $\frac{30}{90}$ which equals $\frac{1}{3}$, or $\frac{90}{3}$ which equals 30.

We also have many other fractions. In fact, just as there is no limit to the largest number we can write by using combinations of digits from 0 to 9, there is no limit to the smallest part of something we can write by using the same digits. Just as $\frac{1}{2}$ means a whole something divided into two parts and $\frac{1}{3}$ means something divided into three parts, we can write $\frac{1}{6}$, which means the whole something is divided into six equal parts. We can continue in this way indefinitely. For example, $\frac{1}{64}$ means one of sixty four parts; $\frac{1}{128}$ means one of one hundred and twenty-eight equal parts and $\frac{1}{2465}$ means one of two thousand four hundred and sixty-five equal parts of something.

Notice, however, that a fraction by itself does not mean anything specific. For example, one half, one third, $\frac{1}{128}$, or $\frac{1}{2465}$ are fractions, but until we say what they are fractions of, we do not have any idea to what they are equal. One half of 50 volts is 25 volts, but as we have already seen one half of 100 volts is 50 volts. So a fraction to indicate anything definite must be accompanied by the whole something that we are talking about. Thus, we always say one-half of a gallon, or one-third of a quart, $\frac{1}{128}$ (pronounced one, one hundred twenty-eighth) of an ounce, one-fifth of the voltage, etc.

There are two parts to every fraction: there is the top part written above the line which is called the numerator, and there is the bottom part below the line called the denominator. The number in the numerator always tells us how many parts we have, and the number in the denominator tells us the size of the parts. Just as we can have one third of a gallon, it is also possible to have two $\frac{1}{3}$'s of a gallon which we would write $\frac{2}{3}$ of a gallon. The two indicates that we have two parts of a gallon and the three indicates that each part equals $\frac{1}{3}$ of a gallon.

There are also two kinds of fractions. One kind is called a proper fraction and always has a numerator that is smaller than the denominator, thus, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{4}{7}$, and $\frac{5}{128}$ are all proper fractions because their numerator is smaller than the denominator. The other kind of fraction is called an improper fraction. An improper fraction is one in which the numerator is larger than the denominator, such as $\frac{100}{50}$, $\frac{100}{2}$, $\frac{755}{4}$, etc.

Improper fractions can always be converted into whole numbers or whole numbers and proper fractions. For example, the improper fraction $\frac{100}{50}$ becomes 2 if we perform the division and divide 50 into 100. The improper fraction $\frac{100}{2}$ becomes 50 if we divide 2 into 100. In the improper fraction $\frac{755}{4}$, when we divide 4 into 755, it will not go an even number of times. It will go one hundred and eighty-eight times with a remainder of 3. This means that in the improper fraction $\frac{755}{4}$ there are one hundred and eighty-eight whole parts and three $\frac{1}{4}$ parts. Thus we can write the improper fraction $\frac{755}{4}$

as $188\frac{3}{4}$

Both proper and improper fractions can be added, subtracted, multiplied and divided just like any whole numbers. After all, it is possible to have several halves of something, or several fifths of something which we might want to add or subtract from each other. The basic operations with fractions are much the same as with whole numbers, but there are certain rules that we must follow. In this section of the lesson we will learn the rules and see how to apply them.

ADDITION OF FRACTIONS

In adding fractions, we must remember one of the basic rules of any addition problem. Only like things can be added together. For example, we can add six oranges and four oranges and say we had ten oranges. Similarly, we can add twenty apples and nine apples and say we had twenty-nine apples. However, we could not add six apples and eight oranges and say we had fourteen oranges or fourteen apples. If we want to call the oranges and apples pieces of fruit then we could add the six and eight and say we had fourteen pieces of fruit because here we equated them to a common name. Similarly, we can add any number of volts to any other number of volts and get a total number of volts. We can add ohms to ohms and amperes to amperes, but we cannot add ohms to amperes or volts. The same rule applies to fractions except that we have an additional item of similarity to consider.

The denominators of fractions must be alike if we are going to add them. For example, one half a gallon can be added to another one half of a gallon to get a whole gallon. One third of a gallon can be added to another one third of a gallon to get two thirds of a gallon. Thus, fractions of like things with like denominators can be added together very simply by adding their numerators. Thus:

$$\frac{1}{2} + \frac{1}{2} = \frac{1 + 1}{2} = \frac{2}{2} = 1$$

$$\frac{2}{3} + \frac{1}{3} = \frac{2 + 1}{3} = \frac{3}{3} = 1$$

$$\frac{1}{3} + \frac{1}{3} = \frac{1 + 1}{3} = \frac{2}{3}$$

$$\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$$

$$\frac{12}{64} + \frac{13}{64} = \frac{12 + 13}{64} = \frac{25}{64}$$

$$\frac{55}{137} + \frac{67}{137} = \frac{55 + 67}{137} = \frac{122}{137}$$

Now we have just seen that all fractions with like denominators can be added together simply by adding their numerators. However, fractions with unlike denominators cannot be added so simply. Before we can add fractions with unlike denominators, we must arrange them in a way that their denominators are all alike. This is called finding the lowest common denominator.

The Common Denominator.

When we first started our discussion of fractions, we discovered that we could "reduce" a fraction such as $\frac{50}{100}$ to a fraction $\frac{1}{2}$ by dividing both the numerator and the denominator by the same number. In this case, the number was 50 because with both the numerator and denominator of $\frac{50}{100}$ divided by 50, we get $\frac{1}{2}$. When we did this, we realized that either $\frac{50}{100}$ or $\frac{1}{2}$ meant exactly the same thing. Since either of these two ways of writing the fraction is correct, the two fractions must be equal. Thus, by dividing the numerator and the denominator by the same number, we have not changed the value of the number.

If this is true, we must also be able to multiply the numerator and denominator of a fraction by the same number without changing its value. 50 times both the numerator and denominator of $\frac{1}{2}$ is $50 \times 1 = 50$ and $50 \times 2 = 100$, or $\frac{50}{100}$. From this we can see that we can either multiply or divide the numerator and the denominator of a fraction by the same number without changing the value of the fraction. Accordingly, a fraction such as $\frac{1}{2}$ might be written in any one of several ways as follows:

$$\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}, \frac{2}{4} \times \frac{2}{2} = \frac{4}{8}$$

$$\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}, \frac{4}{8} \times \frac{4}{4} = \frac{16}{32}$$

$$\frac{16}{32} \times \frac{100}{100} = \frac{1600}{3200}$$

All these fractions are exactly equal to $\frac{1}{2}$ because they can all be reduced to $\frac{1}{2}$.

Likewise, we can change a fraction such as $\frac{1}{3}$ to any of the following fractions:

$$\frac{1}{3} \times \frac{3}{3} = \frac{3}{9}, \frac{1}{3} \times \frac{2}{2} = \frac{2}{6},$$

$$\frac{2}{6} \times \frac{5}{5} = \frac{10}{30}, \text{ etc.}$$

Let's see how this will help us in adding fractions. Suppose we want to add $\frac{1}{2} + \frac{1}{3}$. Since their denominators are not alike, we know that we can't add them as they are. However, suppose we change $\frac{1}{2}$ to $\frac{3}{6}$ which we can do by multiplying both the numerator and denominator by 3. Then, if we also change $\frac{1}{3}$ to $\frac{2}{6}$ which we can do by multiplying both numerator and denominator by 2, we now have two fractions which have the same denominator. They are $\frac{3}{6}$ and $\frac{2}{6}$ and they can be added together in the usual way to get $\frac{3+2}{6}$ or $\frac{5}{6}$. We have changed $\frac{1}{2}$ and $\frac{1}{3}$ to fractions with the same denominator so that they can be added without changing the values of the fractions themselves. $\frac{3}{6}$ is exactly the same as $\frac{1}{2}$ and $\frac{2}{6}$ is exactly the same as $\frac{1}{3}$. Added together they make $\frac{5}{6}$ or five-sixths.

When two fractions have the same denominator, we say they have a common denominator. When we change two or more fractions to equal fractions having the same common denominator so that we can add them, we call it finding the common denominator. Let's try a few more examples. For example:

$$\frac{1}{5} + \frac{1}{3} + \frac{4}{5} =$$

This is a very simple problem and we can readily see that both 5 and 3 will go into 15. As a matter of fact, 15 is the lowest common denominator of 5 and 3. 5 goes into 15 three times and therefore we change $\frac{1}{5}$ to a fraction and with 15 as the denominator we must multiply both the top and bottom by 3. Thus $\frac{1}{5}$ becomes $\frac{3}{15}$.

Similarly, 3 goes into 15 five times and therefore to change $\frac{1}{3}$ into a fraction with 15 as the denominator, we must multiply both the top of the fraction by 5 and the problem becomes $\frac{5}{15}$. To change $\frac{4}{5}$ into a fraction with 15 as the denominator, we again multiply the numerator and the denominator by 3 and therefore $\frac{4}{5}$ will become $\frac{12}{15}$. Thus our problem becomes:

$$\frac{1}{5} + \frac{1}{3} + \frac{4}{5} =$$

$$\frac{3 + 5 + 12}{15} =$$

$$\frac{20}{15}$$

While $\frac{20}{15}$ is the correct sum of the three fractions, it is not the usual custom to leave a fraction in the form of an improper fraction. We normally simplify the fraction. 15 goes into 20 once with a remainder of 5. Therefore,

$$\frac{20}{15} = 1\frac{5}{15}$$

Thus, we can say our answer is $1\frac{5}{15}$ However, $\frac{5}{15}$ can be simplified by dividing both the numerator and denominator by 5 and this would give us $\frac{1}{3}$. Therefore our sum is $1\frac{1}{3}$

If we look back at the original problem we can see that this is the answer we should expect. Notice that the first fraction we have is $\frac{1}{5}$ and the third one is $\frac{4}{5}$

$\frac{4}{5} + \frac{1}{5} = \frac{5}{5}$ which is equal to 1. Now we add $\frac{1}{3}$ to 1 and the answer is $1\frac{1}{3}$

In this problem it is quite obvious that the lowest common denominator of 5 and 3 is 15. However, suppose that instead of using 15 as the common denominator we had used 30. Both 5 and 3 will go into 30. This will not cause any difficulty; we will get exactly the same answer, but we will be working with bigger numbers because we did not use the lowest possible common denominator. Using 30 as the common denominator the problem becomes:

$$\frac{1}{5} + \frac{1}{3} + \frac{4}{5} =$$

$$\frac{6 + 10 + 24}{30} =$$

$$\frac{40}{30} =$$

$$1\frac{10}{30} =$$

$$1\frac{1}{3}$$

Similarly, if we had used 45 as a common denominator, once again we would get the same answer, but we would have to work with larger numbers. In this case the problem is,

$$\frac{1}{5} + \frac{1}{3} + \frac{4}{5} =$$

$$\frac{9 + 15 + 36}{45} =$$

$$\frac{60}{45} = 1\frac{15}{45} = 1\frac{1}{3}$$

In the preceding example it was easy to find the lowest common denominator. We were dealing with fifths and thirds and we got the lowest common denominator simply by multiplying 5 and 3 together. However, we cannot always do this and get the lowest common denominator. Suppose, for example, that you had the problem:

$$\frac{1}{6} + \frac{2}{9}$$

If we simply multiply the two denominators together we will get $6 \times 9 = 54$. Therefore, 54 is a common denominator. Now our problem is:

$$\frac{1}{6} + \frac{2}{9} =$$

$$\frac{9 + 12}{54} =$$

$$\frac{21}{54} = \frac{7}{18}$$

When we get the answer $\frac{21}{54}$ we immediately could see that 3 could be divided into both the numerator and the denominator and therefore we could reduce the fraction to $\frac{7}{18}$. Often when you get a fraction that can be reduced it is a sign that you did not use the lowest common denominator. However, as long as you reduce the fraction and perform your addition correctly, you will come out with the same answer as you would have if you had used the lowest common denominator. When you can find the lowest common denominator, it is best to use it, because you will be working with smaller numbers and there will be less chance of making a mistake.

There is an easy way to find the lowest common denominator. To do this take the denominators of the various fractions and factor them. Now you might wonder what a factor is. A factor is a number that when multiplied by another number gives you the original number. For example, $1 \times 6 = 6$. Therefore, 1 and 6 are factors of 6. Also $3 \times 2 = 6$ and therefore 3 and 2 are factors of 6. We call 3 and 2 prime factors because they themselves cannot be broken down into factors other than the number and one. In other words, you could say that 2 was equal to 2×1, but there is no other way you could factor it. The factor 2 still appears in 2×1; therefore, 3 and 2 are prime factors. When we factored 6 into 6×1 we did not factor it into prime factors because the 6 could be broken down into 3 and 2. Therefore, we break our denominators down into prime factors. The prime factors of 6 are 3×2 and the prime factors of 9 are 3×3. Now to find the lowest common denominator we look for common factors in the two denominators. We see immediately that we have a 3 in each denominator and therefore we write down 3 as one of the factors in our lowest common denominator and then place a stroke through the 3 in 3×2 and one 3 in 3×3. This leaves us two unused prime factors: the 2 from the factoring of 6 and one of the 3's from factoring 9. Since these are not common, but are different numbers, we must write both of these down beside the first three we set up in determining our lowest common denominator. Therefore, our lowest common denominator will be $3 \times 2 \times 3 = 18$. Now if we add $\frac{1}{6} + \frac{2}{9}$ using 18 as the common denominator we have:

$$\frac{1}{6} + \frac{2}{9} =$$

$$\frac{3 + 4}{18} =$$

$$\frac{7}{18}$$

Now let's try another example. Let us do the problem,

$$\frac{3}{7} + \frac{5}{14} + \frac{8}{21}$$

We can find a common denominator in this problem by multiplying $7 \times 14 \times 21$. However, you can see immediately if we do this we are going to have a rather large number as our common denominator. There is no point in getting involved in such big numbers because we see at a glance that 7, 14 and 21 are all divisible by 7. Therefore, it is worthwhile to factor the three denominators into prime factors to see if we can find the lowest common denominator. When we factor them we get:

$$\begin{aligned} 7 &= 7 \times 1 \\ 14 &= 7 \times 2 \\ 21 &= 7 \times 3 \end{aligned}$$

Now to get our lowest common denominator we first look for prime factors that are common to all three denominators. We notice first that 7 is a prime factor of all three so the first digit in our common denominator will be 7. Now we mark out the three 7's to be sure that we see they have been used. This leaves us with 1 in the first number, 2 in the next and 3 in the third. Therefore our complete lowest common denominator will be $7 \times 1 \times 2 \times 3$. This is equal to 42 and therefore 42 is the lowest common denominator. Now the problem becomes:

$$\frac{3}{7} + \frac{5}{14} + \frac{8}{21} =$$

$$\frac{18 + 15 + 16}{42} =$$

$$\frac{49}{42} = 1\frac{7}{42} = 1\frac{1}{6}$$

Let us do one more example. Add the following:

$$\frac{5}{18} + \frac{7}{27} + \frac{8}{45}$$

Again, we see immediately that if we find a common denominator by multiplying the three denominators together we will have a very large denominator. Therefore it is worthwhile to factor the denominator to see if we can find a smaller common denominator. 18 is equal to 9×2, but 9 is not a prime factor because it in turn can be factored into 3×3. Therefore, the prime factors of 18 are $3 \times 3 \times 2$. Similarly, 27 and 45 can be factored into prime factors so that we will get:

$$\begin{aligned} 18 &= 3 \times 3 \times 2 \\ 27 &= 3 \times 3 \times 3 \\ 45 &= 3 \times 3 \times 5 \end{aligned}$$

Now to get our lowest common denominator we first look for factors that are common to the three numbers. We see that the first digit in each factor, which is 3, is common to all three so we put down the 3 as the first factor in our common denominator and then mark out the first 3 in each of the factors to indicate that this 3 has been used. The second digit is also a 3 so we use a second 3 in our lowest common denominator to give us 3×3 and we mark out the second 3 in each of the three numbers. Now this leaves us 2, 3, 5 and since these are not common in any of the three numbers we must include them in our lowest common denominator. Thus the lowest common denominator becomes $3 \times 3 \times 2 \times 3 \times 5$. Multiplying these together we get 270 as the lowest common denominator. 18 goes into 270, 15 times. 27 goes into 270, 10 times and 45 goes into it 6 times. Therefore, our problem becomes:

$$\frac{5}{18} + \frac{7}{27} + \frac{8}{45} =$$

$$\frac{75 + 70 + 48}{270} =$$

$$\frac{193}{270}$$

So far, in finding common factors in the denominators in order to find the lowest common denominator, we have had a common factor in each of the denominators. However, this will not always be the case. Sometimes the common factor may appear in only two of the denominators. For example, in the problem,

$$\frac{1}{3} + \frac{1}{14} + \frac{1}{21}$$

we will have two different common factors which appear in only two of the denominators. Factoring the denominators we get:

$$\begin{aligned} 3 &= 3 \times 1 \\ 14 &= 2 \times 7 \\ 21 &= 3 \times 7 \end{aligned}$$

Now we start looking for common factors. Looking at the first number factored, we see that 3 is one of the factors so we place a stroke through it to indicate that we have used it. Looking at the second number we see that there is no 3 in its factors, but we see a 3 in the factors of the third number so we place a stroke through it indicating that it has been used and then put down 3 as the first number in the product which will eventually give us our lowest common denominator. Now looking at the first number again we see that the only factor left is 1, so we skip it and go on to the second number. Here we see that the factors are 2 and 7 and we take the first number which is a 2 and place a stroke through it. Now we write a × sign next to the 3 we have in the common denominator and place the 2 to the right of the × sign. This gives us 3 × 2 as the first two numbers in the common denominator. Since there is no 2 in the third group of factors we start over again. Looking at the second number we see that we still have a 7 left so we place a ×7 as the next factor in our common denominator and place a stroke through the 7 to indicate it has been used. Moving on to the third group of factors we see that we also have a 7 so we place a stroke through it to indicate that it has been used. Now the factors we have for our lowest common denominator are 3 × 2 × 7 and if we multiply these out we get 42 which is the lowest common denominator.

From the preceding we can see that some of the factors that go to make up our lowest common denominator might not appear in all of the numbers factored. However, when a common factor appears in two or three of the denominators that have been factored there is no point in placing it in the product that is going to make up our lowest common denominator more than once unless it appears in one of the factors more than once.

Another situation that you should be on the lookout for is a problem in which one of the denominators is a factor of the denominator in one of the other factors. When you run into this situation you can completely forget about the smaller denominator. For example, in the problem,

$$\frac{1}{3}+\frac{1}{7}+\frac{1}{14}$$

we see that 7 is a factor of 14. Therefore any number that 14 will divide into, 7 will divide into also. Therefore, we can simply forget about the 7 and find the lowest common denominator for 3 and 14.

You should also be on the lookout for a fraction which may not be reduced to its simplest form. For example, in the addition,

$$\frac{1}{5}+\frac{1}{4}+\frac{6}{8}$$

we could go ahead and find a common denominator. The lowest common denominator in this case is 40. However, $\frac{6}{8}$ can be reduced to $\frac{3}{4}$ by dividing the top and bottom of the fraction by 2. If we do this our problem becomes:

$$\frac{1}{5}+\frac{1}{4}+\frac{3}{4}$$

Now we see that our lowest common denominator is 20. In this problem it would not make a great deal of difference if we used 20 or 40 as a common denominator in performing the addition so long as we reduced the fraction to the simplest form after the addition; in some problems the lower figure could make the addition a great deal simpler.

Now to get practice adding fractions do the following problems. Be sure to work each problem carefully before looking at the answers at the back of the book. If by any chance you find that you have made a mistake in one of the additions, be sure to check your work over carefully comparing it with our solution to see where you made your mistake.

SELF-TEST QUESTIONS

16. $\frac{3}{7}+\frac{5}{7}+\frac{6}{7}$

17. $\frac{1}{2}+\frac{1}{3}+\frac{1}{12}$

18. $\frac{1}{2}+\frac{1}{3}+\frac{1}{5}$

19. $\frac{3}{8}+\frac{1}{2}+\frac{3}{4}$

20. $\frac{3}{4}+\frac{1}{16}+\frac{1}{8}$

21. $\frac{1}{3}+\frac{1}{7}+\frac{3}{14}$

22. $\frac{6}{23}+\frac{8}{46}+\frac{19}{69}$

23. $\frac{1}{5}+\frac{2}{9}+\frac{3}{11}$

24. $\frac{1}{9}+\frac{3}{4}+\frac{1}{8}$

25. $\frac{7}{25}+\frac{9}{35}+\frac{2}{5}$

SUBTRACTION OF FRACTIONS

Subtracting fractions is just the reverse of adding fractions. All the rules that apply to the addition of fractions apply to subtraction. First, the fractions must be parts of like things and they must have the same denominator in order to subtract them. If they do not have a common denominator, we must find the lowest common denominator for them. We do this in exactly the same way as we did for addition.

When we are subtracting one fraction from another, we subtract only the numerators and when we have finished our subtraction, we always reduce the answer as much as possible. For example:

$$\frac{5}{6} - \frac{1}{6} = \frac{5 - 1}{6} = \frac{4}{6} = \frac{2}{3}$$

To subtract $\frac{1}{3}$ from $\frac{1}{2}$, we must find the lowest common denominator which is 6. Thus the subtraction becomes:

$$\frac{1}{2} - \frac{1}{3} = \frac{3 - 2}{6} = \frac{1}{6}$$

As you can see, the procedure is essentially the same as adding fractions, however, in this case we subtract the numerators instead of adding them.

Just as in problems involving addition of fractions where we had more than two fractions to add, sometimes we have several subtractions to perform. You proceed in essentially the same way. For example, in the problem,

$$\frac{11}{12} - \frac{1}{4} - \frac{1}{3}$$

we first find the lowest common denominator, which in this case is 12. Then we perform first one subtraction and then the other. If we wanted to do so we could add the numerators of the two fractions to be subtracted and then perform a single subtraction. The problem will be worked out as follows:

$$\frac{11}{12} - \frac{1}{4} - \frac{1}{3} =$$

$$\frac{11 - 3 - 4}{12}$$

Now we can subtract 3 from 11 which would give us 8 and 4 from 8 which gives a remainder of 4 and an answer of $\frac{4}{12}$ which we reduce to $\frac{1}{3}$. The other method would be to first add the 3 and 4 together to get 7 and then subtract the 7 from 11 which again gives us $\frac{4}{12}$. We get the same answer in either case so you can do the problem whichever way you want. The usual procedure is to start at the left and go from left to right and perform one subtraction after the other.

There is no way to really learn how to subtract fractions other than by doing problems involving subtraction of fractions. Therefore, you should do the following problems. Do each problem carefully before looking at the answers in the back of the book. Once again, if you should fail to get the same answers we got, be sure to check your work to find out where the mistake is.

SELF-TEST QUESTIONS

26. $\frac{5}{7} - \frac{3}{7}$

27. $\frac{2}{3} - \frac{1}{2}$

28. $\frac{7}{10} - \frac{3}{5}$

29. $\frac{8}{9} - \frac{2}{5}$

30. $\frac{3}{4} - \frac{3}{8}$

31. $\frac{25}{27} - \frac{1}{3}$

32. $\frac{3}{4} - \frac{1}{6} - \frac{1}{3}$

33. $\frac{7}{8} - \frac{1}{9} - \frac{1}{5}$

34. $\frac{4}{5} - \frac{3}{25} - \frac{3}{50}$

35. $\frac{35}{36} - \frac{3}{9} - \frac{1}{4}$

MIXED NUMBERS

Often you will have to add and subtract mixed numbers. You will remember that a mixed number is a number made up of a whole number and a fraction. For example, add,

$$1\frac{2}{3} + 2\frac{1}{6}$$

There are two ways you can do this problem. One method is to add the whole numbers first and then add the fractions and then add the sum of the fractions and the sum of the whole numbers together. The other method is to convert each mixed number to an improper fraction and then go ahead and perform the addition and then convert the answer back to a mixed number. Usually the easiest way to do this type of problem is to add the whole numbers and the fractions separately. However, we will go through both methods here. Using the first method first, that is of adding the whole numbers and fractions separately we get:

$$1\frac{2}{3} + 2\frac{1}{6} =$$

$$1 + 2 + \frac{2}{3} + \frac{1}{6} =$$

$$3 + \frac{4 + 1}{6} =$$

$$3 + \frac{5}{6} = 3\frac{5}{6}$$

As you can see, in using this method we simply add the whole numbers and then add the fractions, following the same procedure as we used before, that of finding the lowest common denominator and then adding.

When we convert the numbers to improper fractions we sometimes have to deal with slightly larger numbers, but this method works out equally as well. For example,

$1\frac{2}{3}$ can be converted to $\frac{5}{3}$

1 is $\frac{3}{3}$ and $\frac{3}{3} + \frac{2}{3} = \frac{5}{3}$

$2\frac{1}{6}$ can be converted to $\frac{13}{6}$

Now our problem becomes

$$\frac{5}{3} + \frac{13}{6}$$

And using the lowest common denominator of 6 we get:

$$\frac{10 + 13}{6} = \frac{23}{6}$$

$$\frac{23}{6} =$$

$$3\frac{5}{6}$$

As you see, we got $3\frac{5}{6}$ as the answer both ways. One method is as good as the other; use whichever method you prefer.

Subtraction of mixed numbers can be performed in essentially the same way as addition. However, sometimes we come up with a small complication in subtraction. Let us look at the problem:

$$3\frac{3}{4} - 2\frac{1}{8}$$

We can perform this subtraction by subtracting the whole numbers first and then by subtracting the fractions, using the same method as we used previously. Following this procedure the problem becomes:

$$3\frac{3}{4} - 2\frac{1}{8} =$$

$$3 - 2 + \frac{3}{4} - \frac{1}{8} =$$

$$1 + \frac{6 - 1}{8} =$$

$$1 + \frac{5}{8} =$$

$$1\frac{5}{8}$$

We can also perform this subtraction by converting both numbers to improper fractions and then subtracting. To convert $3\frac{3}{4}$ to a mixed number we multiply 3×4 which gives us 12; there are twelve quarters in 3 plus 3 which gives us $\frac{15}{4}$

$2\frac{1}{8}$ becomes $2 \times 8 + 1 = \frac{17}{8}$

Now our problem is

$$\frac{15}{4} - \frac{17}{8}$$

As in previous subtraction problems we must convert to the lowest common denominator which in this case is 8 so we get:

$$\frac{30}{8} - \frac{17}{8} =$$

$$\frac{13}{8} =$$

$$1\frac{5}{8}$$

Once again, you see that we get the

same answer so it does not matter which method you use. The first method is simple enough in most cases, but sometimes when using this method you have to borrow in order to perform one of the subtractions. We can best see what this involves by looking at the example:

$$4\frac{1}{8} - 2\frac{3}{4}$$

If we proceed with the subtraction by subtracting the whole numbers first, we get 4 - 2 which is 2. Now when we try to subtract the fractions we have

$$\frac{1}{8} - \frac{3}{4}$$

and when we convert this to the lowest common denominator which is 8 we have

$$\frac{1 - 6}{8}$$

If we subtract 6 from 1 we end up with a minus answer. Therefore, to perform this subtraction we must borrow from the whole number. In the original problem the whole number was 4 - 2 which gave us 2. We can change 2 to $1 + \frac{8}{8}$ and then add the $\frac{8}{8}$ to $\frac{1}{8}$ and get $\frac{9}{8}$

Now we can subtract $\frac{6}{8}$ from $\frac{9}{8}$ and get $\frac{3}{8}$

and our answer is $1\frac{3}{8}$

If you do this problem by converting to improper fractions, you do not run into this problem.

$4\frac{1}{8}$ becomes $4 \times 8 + 1 = \frac{33}{8}$

$2\frac{3}{4}$ becomes $2 \times 4 + 3 = \frac{11}{4}$

Now our problem is

$$\frac{33}{8} - \frac{11}{4} =$$

$$\frac{33 - 22}{8} = \frac{11}{8} = 1\frac{3}{8}$$

Some problems will involve addition and subtraction of mixed numbers. When you encounter this type of problem you can do the addition and subtraction of the whole numbers first and then the addition and subtraction of the fractions or you can convert all the numbers to improper fractions and work the problem this way. As an example, consider the problem:

$$7\frac{1}{4} - 2\frac{3}{8} + 4\frac{7}{8} - 3\frac{9}{16}$$

Adding and subtracting the whole numbers we get 7 - 2 + 4 - 3 = 6. Now we must handle the fractions and to do this we must convert to the lowest common denominator which in this problem is 16. Thus, we have:

$$\frac{4 - 6 + 14 - 9}{16}$$

Now adding together the numbers to be added and at the same time adding the numbers that are to be subtracted we get:

$$\frac{4 + 14 - 6 - 9}{16} =$$

$$\frac{18 - 15}{16} = \frac{3}{16}$$

Therefore, our complete answer is

$$6\frac{3}{16}$$

Now if we decide to do the problem by converting the fractions to improper fractions

$7\frac{1}{4}$ becomes $7 \times 4 + 1 = \frac{29}{4}$

$2\frac{3}{8}$ becomes $2 \times 8 + 3 = \frac{19}{8}$

$4\frac{7}{8}$ becomes $4 \times 8 + 7 = \frac{39}{8}$

and $3\frac{9}{16}$ becomes $3 \times 16 + 9 = \frac{57}{16}$

Thus our problem becomes:

$$\frac{29}{4} - \frac{19}{8} + \frac{39}{8} - \frac{57}{16}$$

and changing these fractions to a common denominator of 16 we get:

$$\frac{116 - 38 + 78 - 57}{16} =$$

$$\frac{99}{16} = 6\frac{3}{16}$$

Once again we have the same answer so the choice of which way you do the problem is yours. Decide on which way you think is the easier and then do all of them the same way. Usually it is best to stick to one method rather than jump back and forth between the two because this often results in confusion. Now to get practice handling mixed numbers do the following problems. Again, be careful of your work to be sure that you get the right answer. Be sure you check each problem carefully before comparing your answers with ours which are at the back of the book.

SELF-TEST QUESTIONS

36. $1\frac{1}{4} + 2\frac{1}{5}$

37. $2\frac{1}{2} + 3\frac{1}{3} + 1\frac{1}{6}$

38. $4\frac{7}{8} - 3\frac{1}{4}$

39. $5\frac{1}{8} - 2\frac{3}{7}$

40. $8\frac{1}{9} - 3\frac{2}{7} - 2\frac{2}{3}$

41. $\frac{1}{7} + \frac{8}{9} - \frac{1}{3}$

42. $\frac{3}{4} - \frac{1}{9} + \frac{3}{18} - \frac{1}{2}$

43. $4\frac{3}{8} + 1\frac{1}{4} - 2\frac{1}{7}$

44. $1\frac{7}{8} - 3\frac{1}{7} - 1\frac{1}{6} + 8\frac{2}{9}$

45. $6\frac{1}{3} - 4\frac{3}{4} + \frac{1}{9} - 1\frac{1}{3}$

MULTIPLYING FRACTIONS

There is a very simple rule that we follow in multiplying fractions. We simply multiply the numerators of the fractions together to get the numerator of the product and multiply the denominators together to get the denominator of the product. For example, $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. We have multiplied the two numerators, 1×1 and got 1 as the product and then multiplied the two denominators, 2×2 and got 4 as the new denominator. Actually, we can take the problem $\frac{1}{2} \times \frac{1}{2}$ and write it as $\frac{1 \times 1}{2 \times 2}$ and then it becomes quite obvious that our product will become $\frac{1}{4}$. It might at first disturb you that when you multiply two fractions together the product is smaller than either fraction. However, if you consider the multiplication as $\frac{1}{2}$ of $\frac{1}{2}$, you will see that the resultant should be $\frac{1}{4}$ In other words, you have $\frac{1}{2}$ of something and you are taking $\frac{1}{2}$ of that so that the resultant will be $\frac{1}{4}$

Often after you have multiplied two fractions together you can reduce the fraction to its simplest form. For example:

$$\frac{2}{3} \times \frac{1}{2} =$$

$$\frac{2 \times 1}{3 \times 2} =$$

$$\frac{2}{6} = \frac{1}{3}$$

We can also multiply several fractions together at once. For example, $\frac{2}{9} \times \frac{3}{4} \times \frac{5}{7}$ would be set up for the multiplication like this:

$$\frac{2 \times 3 \times 5}{9 \times 4 \times 7} = \frac{6 \times 5}{36 \times 7} =$$

$$\frac{30}{252} = \frac{15}{126} = \frac{5}{42}$$

In this example the resultant fraction is reducible: this usually means that it is possible to reduce some of the fractions before we perform the multiplication by what we call division or cancellation, before we multiply. Thus, in the problem,

$$\frac{2 \times 3 \times 5}{9 \times 4 \times 7}$$

the first number in the numerator is 2 which can be divided into the second number in the denominator which is 4. This would leave us:

$$\frac{\overset{1}{\cancel{2}} \times 3 \times 5}{9 \times \underset{2}{\cancel{4}} \times 7}$$

We can also divide the 3 in the numerator into 9 in the denominator which leaves us:

$$\frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{3}} \times 5}{\underset{3}{\cancel{9}} \times \underset{2}{\cancel{4}} \times 7}$$

Now when we multiply we have $1 \times 1 \times 5$ or 5 in the numerator, and $3 \times 2 \times 7$ or 42 in the denominator and we get our answer $\frac{5}{42}$ directly. Thus, by cancellation before we multiply we simplify the problem.

You can perform your multiplication of fractions either way: you can multiply them out and then cancel later or you can do the cancellation first. Usually the best method is to do the cancellation first because you will then be multiplying smaller numbers together and there will be less chance of your making an error.

For practice do the following multiplication problems. As before, be sure to do each problem carefully before checking your answer with those given in the back of the book and if you do make a mistake be sure to find out where your mistake lies before leaving the problem.

SELF-TEST QUESTIONS

46. $\frac{1}{3} \times \frac{1}{4}$

47. $\frac{1}{7} \times \frac{2}{9}$

48. $\frac{3}{4} \times \frac{3}{8}$

49. $\frac{7}{8} \times \frac{4}{7}$

50. $\frac{5}{13} \times \frac{26}{30}$

51. $\frac{3}{7} \times \frac{7}{8} \times \frac{2}{3}$

52. $\frac{3}{8} \times \frac{16}{19} \times \frac{19}{21}$

53. $\frac{1}{8} \times \frac{8}{9} \times \frac{9}{23}$

54. $\frac{4}{7} \times \frac{21}{28} \times \frac{7}{9}$

55. $\frac{18}{20} \times \frac{30}{36} \times \frac{2}{3}$

DIVIDING FRACTIONS

Division of whole numbers is just the opposite of multiplication of whole numbers. For example, if we are multiplying 3×4 we get 12. If we divide 4 into 12 we get 3. Likewise, division of fractions is just the reverse of multiplication of fractions. In division we have a fraction for our dividend and a fraction for our divisor and we are asked what number or fraction, when multiplied by the divisor, will give us the product that equals the dividend.

If $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$\frac{1}{4} \div \frac{1}{2}$ must equal $\frac{1}{2}$

Likewise,

if $\frac{2}{3} \times \frac{1}{2} = \frac{2}{6}$ or $\frac{1}{3}$

then $\frac{1}{3} \div \frac{1}{2}$ must equal $\frac{2}{3}$

To divide one fraction into another, all we do is invert the divisor, in other words, we turn it upside down, and then multiply the two fractions together. Thus, in the problem,

$$\frac{1}{6} \div \frac{2}{3}$$

$\frac{2}{3}$ is the divisor so to perform the division we invert it and multiply so that our problem becomes:

$$\frac{1}{6} \div \frac{2}{3} =$$

$$\frac{1}{6} \times \frac{3}{2} = \frac{1}{4}$$

Likewise,

$\frac{3}{7} \div \frac{5}{6}$ becomes $\frac{3}{7} \times \frac{6}{5} = \frac{18}{35}$

When performing the multiplication it is usually worthwhile to see if it is possible to do any cancelling. This will simplify the problem and usually you will be working with smaller numbers so there is less chance of your making a mistake. For example, in the problem,

$$\frac{3}{4} \div \frac{9}{16}$$

we invert the $\frac{9}{16}$ and multiply so we have

$$\frac{3}{4} \times \frac{16}{9}$$

We immediately see that the 4 in the denominator of $\frac{3}{4}$ will go into 16 in the numerator of $\frac{16}{9}$ four times. At the same time we see that the 3 in the numerator of the fraction $\frac{3}{4}$ will go into the 9 in the denominator of $\frac{16}{9}$ three times. Thus, dividing by 4 and 3 we get:

$$\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{4}{\cancel{16}}}{\underset{3}{\cancel{9}}} =$$

$$\frac{4}{3} = 1\frac{1}{3}$$

In some problems we will have more than one division to perform. In this case you can set the problem up as one problem by inverting all the divisors. For example, in the problem,

$$\frac{3}{8} \div \frac{7}{16} \div \frac{3}{7}$$

we have two divisors. The first divisor is $\frac{7}{16}$ which when inverted becomes $\frac{16}{7}$ and the second divisor which is $\frac{3}{7}$ becomes $\frac{7}{3}$ when it is inverted. Thus, if we invert both divisors our problem becomes:

$$\frac{3}{8} \times \frac{16}{7} \times \frac{7}{3}$$

Again, we can multiply all the numbers in the numerator to get the numerator product and then multiply all the numbers in the denominator to get the denominator product, but it is easier if we cancel first. Notice that in the first fraction, which is $\frac{3}{8}$, the 3 in the numerator will cancel the 3 in the denominator of the last fraction, $\frac{7}{3}$. Similarly, the 8 in the denominator of the first fraction will go into the 16 of the numerator of the second fraction twice. The 7 in the denominator of the second fraction will go into the 7 in the numerator of the third fraction. Thus when we perform the divisions our problem becomes:

$$\frac{\cancel{3}}{\underset{1}{\cancel{8}}} \times \frac{\overset{2}{\cancel{16}}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{3}} =$$

$$\frac{2}{1} = 2$$

Now you need to get practice dividing fractions so do the following problems carefully and compare your answers with those at the back of the book to be sure that you have the correct answer for each problem.

SELF-TEST QUESTIONS

56. $\frac{1}{3} \div \frac{2}{9}$

57. $\frac{2}{7} \div \frac{3}{7}$

58. $\frac{3}{4} \div \frac{9}{16}$

59. $\frac{47}{49} \div \frac{1}{7}$

60. $\frac{19}{23} \div \frac{38}{41}$

61. $\frac{2}{7} \div \frac{1}{14} \div \frac{1}{2}$

62. $\frac{1}{9} \div \frac{2}{3} \div \frac{3}{7}$

63. $\frac{2}{11} \div \frac{4}{7} \div \frac{3}{14}$

64. $\frac{2}{9} \div \frac{4}{5} \div \frac{5}{8}$

65. $\frac{13}{15} \div \frac{2}{5} \div \frac{4}{9}$

MIXED NUMBERS AND IMPROPER FRACTIONS

To multiply and divide mixed numbers, you should convert the mixed number to an improper fraction and then proceed as you do with simple fractions. For example, in the problem,

$$1\frac{3}{4} \times 2\frac{2}{7}$$

we convert the $1\frac{3}{4}$ to fourths; to do this we multiply 1×4 which gives us 4 plus 3 or the total of $\frac{7}{4}$. We convert the $2\frac{2}{7}$ to sevenths by multiplying 2×7 to get 14 plus 2 equals $\frac{16}{7}$. Our problem then becomes:

$$\frac{7}{4} \times \frac{16}{7}$$

and rather than perform the indicated multiplications we cancel the 7 in the numerator of $\frac{7}{4}$ and the 7 in the denominator of $\frac{16}{7}$. Similarly, we divide 4 into the denominator of $\frac{7}{4}$ and 4 into the numerator in the fraction $\frac{16}{4}$. Thus, our problem becomes:

$$\frac{\cancel{7}}{\underset{1}{\cancel{4}}} \times \frac{\overset{4}{\cancel{16}}}{\cancel{7}} =$$

$$\frac{4}{1} = 4$$

We do our division the same way: we convert both mixed numbers to improper fractions. For example:

$$2\frac{5}{7} \div 1\frac{3}{7} =$$

$$\frac{19}{7} \div \frac{10}{7} =$$

$$\frac{19}{\cancel{7}} \times \frac{\cancel{7}}{10} = 1\frac{9}{10}$$

Sometimes you will run into problems with both multiplication and division involving mixed numbers. To do these problems you convert the mixed numbers to improper fractions and then go ahead and proceed as you would in a multiplication and division of simple fractions. Remember that to divide you simply invert the divisor and multiply. For example:

$$3\frac{1}{4} \div 1\frac{1}{8} \times 3\frac{6}{13} =$$

$$\frac{13}{4} \div \frac{9}{8} \times \frac{45}{13} =$$

$$\frac{\cancel{13}}{\cancel{4}} \times \frac{\overset{2}{\cancel{8}}}{\cancel{9}} \times \frac{\overset{5}{\cancel{45}}}{\cancel{13}} =$$

$$\frac{10}{1} = 10$$

If you worked all these preceding multiplication and division problems you should have no difficulty doing the following problems involving mixed numbers. Be sure to check your work carefully as you go along and if you should get the wrong answer be sure to find out where you made your mistake before leaving the question.

SELF-TEST QUESTIONS

66. $1\frac{1}{4} \times 2\frac{1}{2}$

67. $2\frac{3}{8} \times 4\frac{1}{7}$

68. $4\frac{3}{7} \times 5\frac{4}{9}$

69. $6\frac{7}{8} \div 2\frac{1}{4}$

70. $5\frac{1}{8} \div 3\frac{1}{7}$

71. $8\frac{2}{9} \div 5\frac{1}{6}$

72. $8\frac{1}{9} \times 3\frac{1}{5} \div 9\frac{1}{7}$

73. $6\frac{2}{3} \div 2\frac{2}{9} \times 4\frac{3}{4}$

74. $3\frac{1}{4} \div 8\frac{1}{2} \times 2\frac{1}{8}$

75. $1\frac{7}{8} \div 6\frac{6}{9} \times 2\frac{1}{2}$

RULES OF ORDER

In some problems you will have addition, subtraction, multiplication and division all in the same problem. In problems of this type you follow the rules of order that we established for addition, subtraction, multiplication and division of whole numbers. You will remember that you do any operations enclosed inside of brackets or parentheses first. Then going through the problem, working from the left to the right, you do the multiplication and division in the order in which they occur and then you go back to the left and work through to the right doing the addition and subtraction this time all the way through.

Where there is a multiplication and division side by side, rather than perform either operation separately, you can set the problem up so you can cancel as you did in earlier examples and, in this way, sometimes save yourself some work.

We are not going to give you any detailed examples on how to do problems of this type because you have done all the operations involved many times and you should be able to work out this type of problem. However, we have included five problems at the end of this section for you to do. Try working these problems and then after you have worked them out check with the answers at the back of the book to see how you made out. If you have made any errors be sure to check the sample solution carefully to be sure where your error lies.

SELF-TEST QUESTIONS

76. $\frac{1}{2} + \frac{1}{4} \times \frac{3}{5} - \frac{1}{10} + \frac{1}{10} \div \frac{3}{5}$

77. $\frac{1}{2} \div \frac{1}{8} \times \frac{5}{7} - \frac{1}{8} \times \frac{8}{9} \div \frac{1}{3}$

78. $\frac{1}{2} - \frac{1}{4} \times \frac{1}{3} + \frac{1}{2} \div \frac{3}{4} - \frac{1}{6}$

79. $1\frac{1}{4} + 1\frac{3}{8} \times \frac{3}{8} \div 2\frac{3}{4} - 3 \times \frac{1}{9} + \frac{3}{4}$

80. $\left(\frac{3}{4} + \frac{1}{8}\right) \div \frac{7}{8} - \left(\frac{3}{4} - \frac{1}{8}\right) \times \frac{8}{9}$

SUMMARY

We have now gone through all the rules for operating with fractions and we can summarize our results for easy reference as follows:

(1) The product of two or more fractions is a new fraction whose numerator is a product of the numerators of all these fractions, and whose denominator is the product of the denominators of all the factors. Any whole number may be considered as a fraction with a denominator of 1. For instance, 127 means $\frac{127}{1}$

(2) The quotient of two fractions is the product of the dividend times the divisor inverted (turned upside down).
Notice that neither multiplication nor division of fractions require the use of a common denominator.

(3) To add fractions, they must be reduced to equivalent fractions with a common denominator; then their sum is the sum of the numerators divided by the common denominator.

(4) Similarly, to subtract one fraction from another, we must use a common denominator and the difference is the difference of the numerators, divided by the common denominator.

(5) All results should be reduced to their simplest form by dividing both numerator and denominator by the largest common divisor.

Decimals

A decimal is simply a fraction whose denominator is 10 or some multiple of 10, for example, 100, 1000 etc. For example, the fraction $\frac{6}{10}$ can be written as .6 and .6 is called a decimal or a decimal fraction. We omit the denominator because it is understood. The decimal fraction $\frac{65}{100}$ can be written as .65 and the fraction $\frac{655}{1000}$ can be written as .655

At first you might think decimals are something new or something difficult to deal with, but this is not the case. You deal with decimals everyday when you handle money. You are acquainted with these coins: a cent, a nickel, a dime, a quarter dollar, and a half dollar.

$\frac{1}{100}, \frac{1}{20}, \frac{1}{10}, \frac{1}{4}, \frac{1}{2}$ of a dollar

or $\frac{1}{100}, \frac{5}{100}, \frac{10}{100}, \frac{25}{100}$ and $\frac{50}{100}$ of a dollar.

It is just second nature to write these as \$.01, \$.05, \$.10, \$.25 and \$.50. That is, \$1.25 means one dollar and twenty-five cents or $1\frac{25}{100}$ dollars. On this basis you can easily add up such amounts as:

$$\begin{array}{r} \$\ 3.25 \\ 7.12 \\ 2.84 \\ 6.33 \\ \hline \$19.54 \end{array}$$

Just as we separate the dollars and cents (the cents are hundreths of a dollar), we can separate our whole numbers and fractions with a point. This is called a decimal point, and we can write all our fractions with denominators of 10, 100, 1000, etc. as whole numbers by learning to use this decimal point. To do this, we use the first place to the right of the point for tenths, the next for hundredths, the third for thousandths etc.

In this fashion $1\frac{1}{10}$ becomes 1.1

$1\frac{2}{10}$ becomes 1.2

$1\frac{1}{100}$ becomes 1.01

and $1\frac{1}{1000}$ can be written as 1.001

CONVERTING FRACTIONS TO DECIMALS

With a little practice many fractions can be converted to decimals simply by inspection. This is true of fractions whose denominators are multiples of 10. However, there is a standard rule that we can follow for converting other fractions to decimals so that the procedure becomes almost automatic. Let us take as an example the fraction $\frac{1}{8}$.

To convert $\frac{1}{8}$ into a decimal we simply divide 8 into 1. We do this by setting up the problem for division and then placing a decimal point to the right of the 1 and then adding as many zeros as we may need to the right of the 1. Now we start by dividing 8 into 1 and since it cannot go, we place a decimal point above

```
      .125
  8/1.00000
     8
     20
     16
      40
      40
```

Fig. 19. Converting 1/8 to a decimal.

the line immediately above the decimal point to the right of the 1 and now divide 8 into 10 as shown in Fig. 19. 8 will go into 10 once so we place a 1 to the right of the decimal point and place an 8 beneath the 10. Subtracting 8 from 10 we get 2, so we bring down the next 0 and now divide 8 into 20. It will go twice so we place the 2 in our answer to the right of the 1 and multiplying 8 by 2 we get 16 which we place beneath the 20. Now subtracting 16 from 20 gives us 4 and we bring down the next 0. 8 will go into 40 five times so we place a 5 to the right of the 2 in our answer and since 5 times 8 is 40 we place this 40 beneath the other 40 and subtracting, our remainder is 0 so the decimal equivalent of $\frac{1}{8}$ is .125.

To convert $\frac{5}{8}$ to a decimal we proceed in exactly the same manner. We set down the division as shown in Fig. 20 and divide 8 into 5 and get as our answer .625.

```
      .625
  8/5.000
    4 8
      20
      16
       40
       40
```

Fig. 20. Converting 5/8 to a decimal.

Any fraction can be converted to a decimal by following this simple procedure. However, not all fractions will work out to an even value. Sometimes you will get a number in the answer or a combination of numbers that will repeat indefinitely. In this case you simply carry out the division as far as necessary. How far you will actually have to carry it out depends upon what accuracy you want in your answer. If you are dealing with money, there would not be much point in carrying the division past two decimal places because the third decimal is less than a cent and a cent is the smallest denomination of money we have.

```
      .3333
  3/1.00000
     9
     10
      9
      10
       9
       10
        9
        1
```

Fig. 21. Converting 1/3 to a decimal produces a repeating 3. No matter how far we carry the division we will always have the remainder of 1 in this problem.

An example of a common fraction that cannot be converted to an exact decimal value is $\frac{1}{3}$. Fig. 21 shows the conversion of $\frac{1}{3}$ to a decimal. We have shown four decimal places and you will notice that no matter how far we carry the division we will always have a remainder of 1 and the next figure to the right will always be a 3. If we wanted to express the fraction $\frac{1}{3}$ as a decimal to 3

```
   .6666
 3/2.0000
   1 8
     20
     18
      20
      18
       20
       18
        2
```

Fig. 22. Converting 2/3 to a decimal; to three decimal places, 2/3=.667.

places we would write it as .333

When we try to convert $\frac{2}{3}$ to a decimal we run up against a repeating 6 as shown in Fig. 22. Here again we have to decide how many figures we want and then round off our answer. If we wanted to express $\frac{2}{3}$ as a decimal to three decimal places we would write it as .667. Notice that we changed the digit in the third decimal place from a 6 to a 7. The rule for rounding off decimals in this way is that if the next number to the right is more than 5 we add 1. If it is less than 5 we leave the decimal figure as it is. If the next digit to the right should happen to be a 5 then we add 1 if it will make the last number an even number. If the last digit is already an even number we do not add 1. For example, to express the decimal .7635 to three decimal places, since the 3 is an odd number, adding 1 will make it an even number so we would round that off as .764. On the other hand, to express the decimal .8665 to three decimal places, since the third digit is a 6 which is already an even number, we simply drop the 5 and write the decimal as .866

```
     .0625
 16/1.0000
     96
      40
      32
       80
       80
```

Fig. 23. Converting 1/16 to a decimal.

Another example of converting a fraction to a decimal is shown in Fig. 23. Here we have converted $\frac{1}{16}$ to a decimal. Notice that in the first division when we try to divide 16 into 10 it will not go so we place a 0 to the right of the decimal point. Then we try to divide 16 into 100 and it will go 6 times. 6 sixes are 96 which we subtract from 100 to give us a 4. Now we bring down another 0 and 16 into 40 goes twice. 2 × 16 are 32 and subtracting we get 8. Bringing down another 0 we get 80, and 16 goes into 80, 5 times. Thus the decimal equivalent of $\frac{1}{16}$ is .0625. You will run into this situation quite frequently in converting small fractions to decimals, however, do not forget, if the denominator of the fraction will not go into 10, we must place a 0 to the right of the decimal point. If the denominator of the fraction is so large it will not go into 100 then you have to put two zeros to the right of the decimal point and try to divide it into a thousand etc.

Now to get practice converting fractions to decimals, convert the following fractions to decimals. If a fraction does not convert evenly to a decimal, round your answer off correctly to four decimal places. You will find the answers in the back of the book so you can check your results.

SELF-TEST QUESTIONS

81. $\frac{5}{16}$

82. $\frac{7}{8}$

83. $\frac{1}{6}$

84. $\frac{5}{7}$

85. $\frac{13}{16}$

ADDITION AND SUBTRACTION OF DECIMALS

The operations of addition and subtraction of numbers involving decimals are precisely the same as those involving whole numbers. In setting up the problems, the decimal points must be all placed in a vertical line and the decimal point in the sum or difference will be in that same line. Here are several examples:

ADD	123.45
	23.41
	1745.00
	1.12
	.03
	1893.01

From	985.00
Subtract	27.43
	957.57

For practice, try the following:

(1) ADD	2543.67
	100.24
	78.29
	2.27
	.09

(2) From	768.08
Subtract	129.29

Answers: (1) 2724.56 (2) 638.79.

MULTIPLYING DECIMALS

Multiplication of decimal numbers is exactly the same as multiplication of whole numbers except that we need a rule for determining the position of the decimal point in the product. The rule is that we count the number of decimal places in each factor. Then, starting at the right of the product we count off the same number of decimal places to the left as the sum of the number of places in the two factors. For example, in the multiplication

$$232.7 \times 4.89$$

there is one decimal place in the number 232.7 and two decimal places in the number 4.89 and therefore in our answer we will count off three decimal places to the left starting at the right of the product. The multiplication is shown in Fig. 24 and notice that we have a total of 3 decimal places in our answer.

```
   232.7
    4.89
   20943
  18616
  9308
1137.903
```

Fig. 24. There are three decimal places in the factors and therefore there must be three places in the product.

We follow this rule for placing the decimal point at all times even if it means adding several zeros in our answer. For example, if we find the product of

$$.1273 \times .0032$$

we will get as our product 40736 as shown in Fig. 25. However, there are four decimal places in the number .1273 and four decimal places in the number .0032. Therefore, there must be eight decimal places in our answer. Starting at the right of the product and counting to the left, we find that there are only five digits in our answer and therefore we must add three zeros to the left of the 4 before placing the decimal point so that our answer becomes .00040736.

.1273	4 decimal places
.0032	4 decimal places
2546	8 decimal places
3819	
40736	5 digits
.00040736	8 decimal places

Fig. 25. **We must add three zeros to get the required eight decimal places.**

DIVIDING DECIMALS

It is no more difficult to divide numbers involving decimals than it is to divide whole numbers, if we remember a few simple facts. You will remember that a division problem is really a fraction. In other words, 1000 ÷ 18 can be written as $\frac{1000}{18}$

You will also remember that in a fraction you can multiply the numerator and the denominator by the same number without changing the value of the fraction.

In division involving decimals, if the divisor has a decimal in it we get rid of the decimal by moving the decimal point to the right. If we move the decimal point one place to the right. This is the equivalent of multiplying the divisor by 10 so we must multiply the dividend by 10 also. We do this by moving the decimal point one place to the right also. For example, in the problem 42.97 ÷ 4.8, we can get rid of the decimal in the divisor by moving it one place to the right and at the same time moving the decimal point in the dividend one place to the right so that our problem then becomes 429.7 ÷ 48.

Sometimes in order to move the decimal point to the right in the dividend we have to add a 0. For example, in the problem 634 ÷ 82.7, to get rid of the decimal point in the divisor we move the decimal point one place to the right and the divisor becomes 827. We must move the decimal point one place to the right in the dividend also and to do this we add a 0 so that the dividend becomes 6340.

When performing a division involving decimals we must keep track of the decimal point. This is done by placing the decimal point in the quotient immediately above the decimal point in the dividend. For example, in the problem 207.09 ÷ 3.9, the first thing we do is move the decimal point one place to the right to get rid of the decimal in the divisor. Then we proceed with the divisor as follows:

53.1
39/2070.9
195
120
117
39
39

Notice that the decimal point in the quotient is placed immediately above the decimal point in the dividend and the first two digits in the quotient were obtained before we used the decimal in the quotient. This means that the 5 and the 3 to the left of the decimal point and the 1 which was obtained using the .9 from the dividend obtained from the right of the decimal point.

Now to get practice adding, multiplying, dividing and subtracting with decimals do the following problems. Be sure that you do your arithmetic carefully and watch the decimal point to be sure you have it in the right place. Check your answers with the answers given in the back of the book.

SELF-TEST QUESTIONS

86. Add
$$\begin{array}{r} 1.34 \\ 26.2 \\ 8.41 \\ \underline{91.74} \end{array}$$

87. Add 8.33 + 92.1 + 17.41 + 6.3

88. Subtract
$$\begin{array}{r} 91.31 \\ \underline{-80.94} \end{array}$$

89. Subtract 137.42 - 43.8

90. 137.6 × 4.88

91 .43 × .0061

92. 108.33 ÷ 2.3

93. 45.227 ÷ .049

94. .01887 ÷ .051

95. .00173 × 21

MULTIPLYING AND DIVIDING BY TEN

One of the greatest advantages of the decimal system is that multiplication or division by 10,100, or 1000, or any power of ten can be accomplished by simply moving the decimal point as many places to the right (in multiplication) or left (in division) as there are zeros in the particular power of ten. Thus, to multiply a number like 2.35 by 1000, all we have to do is move the decimal point three places to the right, filling in the vacant spaces with zeros to get 2350.

Similarly, to divide by any power of ten requires only that we move the point to the left. Thus, 3500 ÷ 1000 = 3.5.

You will remember that the basic electric units - ampere, volt, farad, cycle, henry, watt, etc. -- are sometimes too large for convenient handling in electronics. In other cases they are much too small. So a set of five prefixes for measurement are used to remedy these situations. These are:

		Units
M	MEGA	1,000,000
k	KILO	1000
m	MILLI	.001
μ	MICRO	.000,001
p	PICO	.000,000,000,001

Pico = $\mu\mu$ or micro-micro.

These units help us by making it possible to express electrical terms in more convenient figures. For example, a radio station in the standard broadcast band might operate on a frequency of 1,470,000 cycles. By moving the decimal point three places to the left, we can convert this frequency in cycles to a frequency in kilocycles. The frequency in kilocycles would be 1,470 kilocycles. If we wanted to go a step further, we could move the decimal point six places to the left instead of 3 places and convert from cycles to megacycles. In this case the frequency would be 1.470 megacycles. Since a kilocycle equals a thousand cycles and a megacycle equals a million cycles it follows that a megacycle equals a thousand kilocycles. Therefore, we can convert from kilocycles to megacycles by moving the decimal point three places to the left. The terms milli, micro and pico are used to express values smaller than one. Thus, to change a current of 1 amp to milliamps, we would move the decimal point three places to the right so that 1 amp = 1000 milliamps. If we had a current .047 amperes, we would convert this to milliamperes by moving the decimal point three places to the right and the current would then be 47 milliamperes. If the current was .000047 amperes, we could convert this to milliamperes again by moving the decimal point three places to the right and the current would be .047 milliamperes. If instead of converting to milliamperes we converted to micro-amperes, we would move the decimal point six places to the right, in which case the current would be 47 micro-amperes. To convert from units such as farads to picofarads, we move the decimal point twelve places to the right. This term has recently come into use and previously was referred to as micro-micro. You will probably see both terms used; you should remember that they mean the same thing. To summarize, to convert from a unit to a larger value we move the decimal place to the left. To convert from units to kilo you move three places to the left and to convert from units to megaunits move it six places to the left. To convert from kilo units to mega units move it three places to the left. It follows that if you are given a value in megohms and you want to convert to ohms, you would move the decimal point six places to the right and if you were given the value in kilohms and you wanted to convert to ohms, you would move the decimal point three places to the right. Thus, in a resistor that had a value of 4.7 megohms, to convert to ohms, you move the decimal point six places to the right and get 4,700,000 ohms. If you had a resistance of 4700 ohms and wanted to convert this to kilohms, you would move the decimal point three places to the left and get 4.7K (kilohms).

From	⟶	To
MEGA	3 places	KILO
KILO	3 places	UNIT
UNIT	3 places	MILLI
MILLI	3 places	MICRO
MICRO	3 places	NANO
NANO	3 places	MICRO-MICRO
MICRO	6 places	MICRO-MICRO
To	⟵	From
PICO	equals	MICRO-MICRO

Fig. 26. Electrical unit conversion table.

A chart which shows which way to move the decimal point and how far to move it to convert if from one unit to another is shown in Fig. 26. After you have used the chart a few times it will become second nature and you will find it comparatively easy to convert from one unit to another.

Remember that when you move the decimal point and there are spaces not filled by numbers, they must be filled by zeros. Now to get practice converting from one unit to another do the following problems:

a. 2.3 kilohms to ohms
b. 437,000 ohms to kilohms
c. .023 megohms to ohms
d. 1.5 amperes to milliamperes
e. 13,000 microamperes to amperes
f. 3 kilovolts to microvolts

g. 1.28 megacycles to kilocycles
h. 4,000 cycles to megacycles
i. 1690 kilocycles to megacycles
j. 3000 microamperes to amperes

Answers

a. 2,300 ohms
b. 437 kilohms
c. 23,000 ohms
d. 1500 ma
e. .013 amperes
f. 3,000,000,000 μv
g. 1280 kc
h. .004 mc
i. 1.69 mc
j. .003 amps

PERCENTAGE

It is very expensive to make any electronic part to an exact value. Fortunately, considerable tolerance is permissible in most electronic circuits so that the components used in it do not have to be made to an exact value. Parts usually have a certain tolerance and this tolerance is expressed as a percentage of the rated or required value.

Percentage is a fractional part expressed in hundredths. In other words, 1% is $\frac{1}{100}$, 2% is $\frac{2}{100}$ and 10% is $\frac{10}{100}$. We normally use the symbol % to stand for the word percent. Thus 5 percent is written 5%. This means $\frac{5}{100}$.

If we say that a resistor has a tolerance of 10%, what we mean is that its actual measured resistance will be within 10% of the value it is supposed to be. In other words, if a resistor is supposed to be a 1,000 ohm resistor and it has a tolerance of 10%, 10% of 1000 is

$$\frac{10}{100} \times 1000 = 100 \text{ ohms}$$

This means that the resistor is within 100 ohms of 1000 ohms. The resistor might have a value as low as 900 ohms or as high as 1100 ohms. In other words, the resistance of the resistor will fall somewhere between 900 ohms and 1100 ohms - it is rare that the value would fall on the exact value of 1000 ohms.

Most resistors used in communications and electronics equipment have tolerances of 5% or 10%. If you want to find an exact range of resistance that a resistor might have, you find how much it might vary from its value by determining the percentage variation from its rated value. If the resistor is a 5% resistor multiply the value of the resistor by 5 over 100 to get the amount it could vary from its rated value and if it is a 10% resistor, multiply the value by 10 over 100. For example, a 470 ohm 5% resistor may have a tolerance of $\frac{5}{100} \times 470 = 23.5$ ohms. This means its value will lie somewhere between 470 - 23.5 ohms and 470 + 23.5 ohms.

The same value resistor that has a 10% tolerance could vary by as much as

$$\frac{10}{100} \times 470 = 47 \text{ ohms}$$

Therefore, it might have a value anywhere between 470 - 47 and 470 + 47 ohms.

In accurate measuring equipment, resistors having a tolerance of 1% or $\frac{1}{2}$ of 1 % are frequently encountered. If you want to find how much a 1% resistor can vary from its rated value, you multiply 1 over 100 times the value of the resistor. If you want to find how much a one half percent resistor varies, then multiply $\frac{1}{2}$ over 100 times its rated value or $\frac{1}{200}$ times its rated value to find how much the one half percent resistor could vary. Then to get the limits of the resistor, you subtract the variation to find how low the resistance can actually be and then you add the variation to find out how high the resistance can actually be.

Sometimes you know the value by which a part varies from its rated value and you want to find what percentage this is of the rated value. To find the percent that one number is to the second number, divide the first number by the second and multiply the quotient by 100. In other words, if you had a 600 ohm resistor and found that it actually measured 650 ohms and you wanted to find what percentage the resistor was of its rated value you would subtract 600 from 650. In other words, the resistor was 50 ohms over its rated value. To convert this to percentage you set the problem up as

$$\frac{50}{600} \times 100 = 8.33\%$$

Since percentage is a fraction of 100 we can easily convert percentage to its decimal number. For example, 40% means $\frac{40}{100}$

To divide 40 by 100 we move the decimal point two places to the left.

Thus $40\% = \frac{40}{100} = .40$

Similarly, $12\% = \frac{12}{100} = .12$

$6\% = \frac{6}{100} = .06$

You will find it useful to be able to find the given percent of a number. For example, what number is 40% of 350? To find this number we convert 40% to a decimal and then multiply 350 by the decimal.

$$40\% \text{ of } 350 = 350 \times \frac{40}{100} = 350 \times .40 = 140$$

To get practice doing percent problems you should do the following ten problems. If you find that you are unable to do one particular type, be sure to refer to the model solution in the back of the book to see how the problem is worked and then go back and work the other problems of the same type. Percentage is useful not only in electronics, but in many other activities of every day life.

SELF-TEST QUESTIONS

96. What percent of 105 is 35?
97. What percent of 40 is 8?
98. What is 15% of 60?
99. What is 36% of 4286?
100. A 680 ohm resistance has a tolerance of 10%. What is the lowest value the resistance may have and still be in tolerance?
101. A 2200 ohm, 5% resistor measures 2300 ohms. Is this resistance (a) above, (b) below, (c) within its rated tolerance?
102. A 4.7K-ohm, 10% resistor measures 5200 ohms. Is this resistance (a) above, (b) below, (c) within its rated tolerance?
103. If $\frac{1}{4}$ of the voltage applied to a circuit is dropped across a certain resistor, what percentage of the total voltage does this represent?
104. If $\frac{1}{8}$ of the voltage applied to a circuit is dropped across a certain resistor, what percent-

age of the total voltage does this represent?

105. If the total voltage applied to a circuit is 200 volts and 75% of this voltage is dropped across one resistor, what is the voltage across the resistor?

Solving Circuit Problems

Now that we have reviewed the operations of basic arithmetic let us use these operations to solve some simple dc circuit problems. This will help you in two ways. First, you will get a chance to review some of the facts you learned in this lesson and secondly, you will get a chance to practice applying your arithmetic to actual electronic circuits. This will help you to better understand how these circuits work.

Earlier in this lesson you did a number of problems involving Ohm's Law. We won't do any more examples involving simple applications of Ohm's Law at this time. If you have forgotten the three forms of Ohm's Law, be sure to memorize them again. Remember the three forms are:

$$I = E \div R$$
$$R = E \div I$$
$$E = I \times R$$

You can use Ohm's Law to solve many circuit problems. If you know the voltage and the resistance in the circuit, you can use the formula $I = E \div R$ to find the current. If you know the voltage and the current in a circuit, you can use the formula $R = E \div I$ to find the resistance and if you know the current and the resistance in the circuit, you can use the formula $E = I \times R$ to find the voltage. Remember, if you know any two of the quantities - resistance, current or voltage - you can use the appropriate form of Ohm's Law to find the other quantity.

Remember to give your answer in the proper units. If you are finding the voltage in the circuit, give your answer in volts; if you are finding the current in the circuit, give the answer in amps, and if you are finding the resistance in the circuit, give the answer in ohms. Simply giving a numerical answer without the correct units after it is unsatisfactory.

Remember also that to use Ohm's Law you must have the voltage in volts, the current in amps, and the resistance in ohms. If you are given the current in milliamperes, you must change it to amperes to use Ohm's Law. Similarly, if the voltage is given in millivolts or kilovolts, change it to volts and if the resistance is given in megohms or kilohms, change it to ohms.

In your regular lesson text you learned the power formulas that let you find the power consumed in watts. These formulas are:

$$P = E \times I$$
$$I = P \div E$$
$$E = P \div I$$

Using these formulas, if you know any two of the quantities, power, voltage or current, you can find the other.

For example, find the power in a

circuit if the voltage is 150 volts and the current is 3 amps. Using the formula $P = E \times I$ and substituting for E and I we get:

$$\begin{aligned} P &= E \times I \\ &= 150 \times 3 \\ &= 450 \text{ watts} \end{aligned}$$

If in another problem you are given that the power in a circuit is 660 watts and the voltage is 110 volts, you can find the current in the circuit using the formula $I = P \div E$. For example:

$$\begin{aligned} I &= P \div E \\ &= 660 \div 110 = 6 \text{ amps} \end{aligned}$$

If you are given that the power in the circuit is 800 watts and the current is 2.5 amps, you can find the voltage using the formula $E = P \div I$. You proceed as follows:

$$\begin{aligned} E &= P \div I \\ &= 800 \div 2.5 \\ &= 320 \text{ volts} \end{aligned}$$

There is another formula for power when we know the value of current and resistance, $P = I^2R$. I^2 is equal to $I \times I$. We would use this formula in examples where current and resistance are given and we want to find power. For example, if the current in the circuit is 3 amperes and the resistance in the circuit is 15 ohms, we can find the power dissipated in the circuit.

$$\begin{aligned} P &= I^2R \\ &= 3 \times 3 \times 15 \\ &= 135 \text{ watts} \end{aligned}$$

We can also rearrange this formula to get $I^2 = P \div R$ or $R = P \div I^2$ These two forms are also useful in solving certain problems. For example, if the power in a circuit is 400 watts and the resistance in the circuit is 100 ohms, we can find the current flowing in the circuit using the formula:

$$\begin{aligned} I^2 &= P \div R \\ &= 400 \div 100 \\ &= 4 \end{aligned}$$

This gives us the value of I^2 or $I \times I$. To get the value of I we need a number that when multiplied by itself will give us four. Obviously the answer is 2 and therefore the current is 2 amps. 2 is called the square root of 4. In a later lesson you will learn how to find the square root of a number, but for the present the only problems we will have you do involving square roots will be simple numbers which you will be able to recognize readily. For example, the square root of 4 is 2, the square root of 9 is 3, the square root of 16 is 4, the square root of 25 is 5 etc.

Another version of the power formula that is useful is $P = E^2 \div R$. We use this formula when we know the voltage and the resistance in the circuit. To find the power in the circuit, we divide the voltage squared, which is equal to $E \times E$, by the resistance and this will give us the power in watts. We can also rearrange this formula into the form $E^2 = P \times R$ and $R = E^2 \div P$ and use it to find the voltage in the first case when the power and resistance are known and the resistance in the second case when the voltage and power are known. We will not give any examples in the use of these formulas because they are exactly the same as the other power formulas.

You already learned how to find the resistance of resistors in series, but sometimes you have to find the value of resistors in parallel. The total resistance of two resistors in parallel can be found using the formula:

$$R_t = \frac{R_1 \times R_2}{R_1 + R_2}$$

We would use this formula to find the resistance of two resistors in parallel if we have been given the resistance of the individual resistors. For example, if a 20 ohm resistor and a 30 ohm resistor are connected in parallel, find the resistance of the two resistors in parallel. Substituting these values in the formula:

$$R_t = \frac{R_1 \times R_2}{R_1 + R_2}$$

$$= \frac{20 \times 30}{20 + 30}$$

$$= \frac{600}{50}$$

$$= 12 \text{ ohms}$$

Sometimes you will have a circuit where there are more than two resistors connected in parallel. You can use the same formula for finding the resistance of the parallel combination by finding the resistance in groups. For example, if there are three resistors, find the resistance of two of the resistors in parallel while ignoring the third resistor. When you have found the resistance of two of the resistors in parallel, treat this parallel resistance as a single resistance and find the resistance of it in parallel with the third resistor. If there should happen to be four resistors in parallel, group them into two groups of two each. Find the parallel resistance of each group and then treat each group as a single resistor and then find the parallel resistance of the two groups of resistors.

Now to get practice applying the power formulas, Ohm's Law and the various resistance formulas, do the following problems. You will find many of the problems are very similar to many of the problems you have worked in this lesson, but the time you will spend on these additional problems will be spent in a very worthwhile way. It will not only help you with the mathematics, but it will also help you remember the various formulas and how to use them - and give you a better understanding of electronics. Also, if you will work these problems, you should have no difficulty doing the lesson questions because they are very similar to the problems in this group.

SELF-TEST QUESTIONS

106. What is the total resistance of a circuit that has a 35-ohm resistor in parallel with a 75-ohm resistor? Round off your answer to the nearest ohm.
107. If we have a voltage of 120 volts applied to a lamp with a resistance of 100Ω, how many watts of power will be consumed?
108. In the circuit shown in Fig. 27, the total resistance of the circuit is 335 ohms. What is the resistance of R_2?
109. If the current flowing through the circuit in Fig. 27 is 6 amps, what is the power in watts?
110. What is the maximum rated current-carrying capacity of a 500-ohm resistor marked: 500 ohms, 2000 watts?
111. If a vacuum tube that has a filament rating of .25 amps at 5 volts is to be operated from a 6-volt battery, what value of series-dropping resistor would we need?

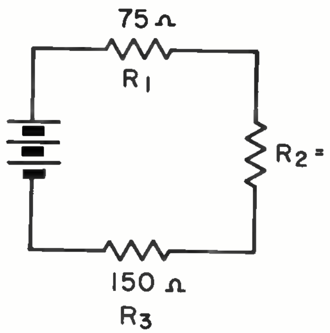


Fig. 27. Circuit for problems 108 and 109.

112. A tube with a filament resistance of 500Ω is designed to operate when 200 milliamperes flow through the filament. What value of resistance must be connected in series with the filament to limit the current to this value if we operate it from 110 volts dc?
113. If resistors of 5, and 15 ohms are connected in parallel, what is the total resistance?
114. If 120 volts dc is applied to a 50Ω resistor connected in series with two resistors of 50Ω in parallel with each other, how much current will flow in the circuit?
115. What is the total resistance of the circuit shown in Fig. 28?
116. A 450Ω resistor has a rated tolerance of ± 10%. When we measure it with an ohmmeter, we find that it actually has a resistance of 410Ω. Is it within its tolerance?

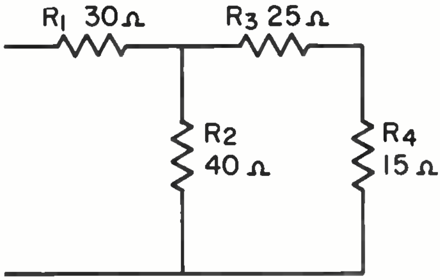


Fig. 29. Circuit for problems 118 and 119.

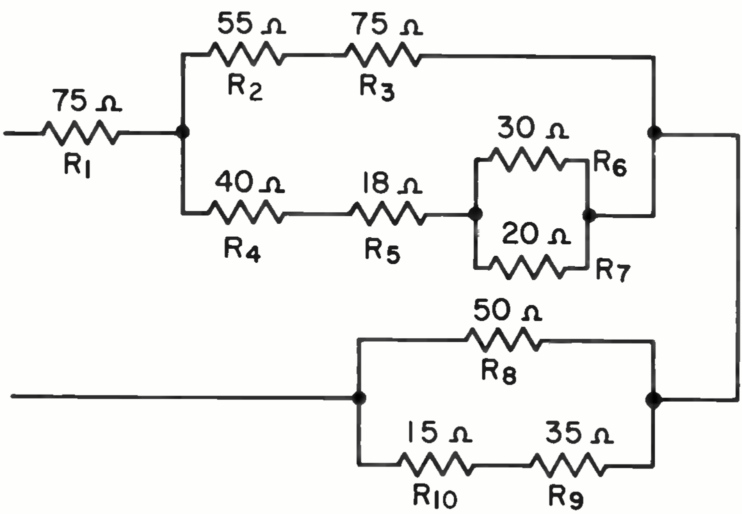


Fig. 28. Circuit for problem 115.

117. Three resistors in series have voltage drops of 36 volts, 24 volts, and 40 volts. What percentage of the total voltage is the 24-volt drop?

118. In the circuit shown in Fig.29, 450 watts of power are consumed. What is the voltage drop across R_1?

119. How many amperes of current flow through R_4 in the circuit described in Problem 118?

120. A current of 400 milliamps flows through a resistance of 2.2K. What is the voltage drop across the resistor?

Now, as soon as you feel sure that you are ready, work out the answers to the ten questions at the end of the lesson and submit your answers for grading.

Answers To Self-Test Questions

The following solutions to the various problems are to be used after you have tried to work the problems out yourself. Don't look at the answer or the solution until you have made an attempt to do the problem yourself. However, if you do not get the same answer as we do, be sure to go through the solution very carefully to be sure that you find out where you made your mistake. If you work each problem and then check your answer with ours, and if you make any mistakes find out where the mistake is, you should have no difficulty doing the questions at the end of the lesson.

1. The problem here is simply to find the resistance of a number of resistors in series. To do this you simply add the value of the individual resistors. Setting down the values as below we add and get 172 ohms.

$$\begin{array}{r} 68 \\ 31 \\ 9 \\ 17 \\ 21 \\ 7 \\ \underline{19} \\ 172 \end{array}$$

2. In this problem you know the total resistance in the circuit, and you know the resistance of four of the resistors. To find the resistance of the unknown resistor you add the resistance of the four known resistors. We know that this value plus the resistance of the unknown resistor must be equal to 81 ohms. Therefore, to get the correct resistance of the unknown resistor you subtract the resistance of the four resistors in series from the 81 ohms. Putting down the values of the four known resistors to get their total resistance we have:

$$\begin{array}{r} 14 \\ 8 \\ 17 \\ \underline{23} \\ 62 \end{array}$$

Since the total resistance of the four known resistors is 62 ohms, we now subtract 62 from 81 and get 19 ohms; this is the value of the unknown resistor.

$$\begin{array}{r} 81 \\ \underline{-62} \\ 19 \end{array}$$

3. This is simply a series circuit with five resistors connected in series. To find the total resistance we simply add the resistance of the resistors as shown below and get as our answer 511 ohms.

$$\begin{array}{r} 127 \\ 97 \\ 9 \\ 219 \\ 59 \\ \hline 511 \end{array}$$

4. The first step in solving this problem is to determine the resistance of the four known resistors. To do this we put down the values and add as below:

$$\begin{array}{r} 216 \\ 318 \\ 91 \\ 56 \\ \hline 681 \end{array}$$

Thus, the total resistance of the four known resistors is 681 ohms. Since the total resistance in the circuit is 823 ohms, the value of the unknown resistor must be equal to 823 - 681. Thus, we set down the problem subtracting as shown below and get as our answer 142 ohms.

$$\begin{array}{r} 823 \\ -681 \\ \hline 142 \end{array}$$

5. Here we have a simple application of Ohm's Law. The unknown value is I so we want this on one side of the equation. The known values are E = 57 volts and R = 19 ohms, so we want these values on the other side of the equation. Thus we use the formula:

$$\begin{aligned} I &= E \div R \\ &= 57 \div 19 \\ &= 3 \text{ amps} \end{aligned}$$

6. To solve this problem we want to use Ohm's Law in the form E = IR. Here we have the unknown E on one side of the equals sign and the known values I = 3 amps and R = 21 ohms on the other side of the equals sign. Thus we have:

$$\begin{aligned} E &= IR \\ &= 3 \times 21 \\ &= 63 \text{ volts} \end{aligned}$$

7. To do this simple Ohm's Law problem we use, I = E ÷ R
E = 84 volts and R = 28 ohms so we have:

$$\begin{aligned} I &= E \div R \\ &= 84 \div 28 \\ &= 3 \text{ amps} \end{aligned}$$

8. In this problem the known values are E and I, E = 96 volts and I = 4 amps. We have to find R so we use the formula:

$$\begin{aligned} R &= E \div I \\ &= 96 \div 4 \\ &= 24 \text{ ohms} \end{aligned}$$

9. We are given that I = 5 amps and R = 17 ohms. To find the voltage we use Ohm's Law in the form:

$$\begin{aligned} E &= IR \\ &= 5 \times 17 \\ &= 85 \text{ volts} \end{aligned}$$

10. In a problem such as this, where we have the total voltage equal to 32 volts and the total current equal to 2 amps, we can find the total resistance using the formula:

$$\begin{aligned} R &= E \div I \\ &= 32 \div 2 \\ &= 16 \text{ ohms} \end{aligned}$$

Since $R_1 = R_2$ and since the two

are in series, $R_1 + R_2 = 16$ ohms and each resistor must be equal to 8 ohms.

11. Using the rules of order, we go through the problem doing the multiplication and division first as we go from left to right. Thus we have:

$$25 + 16 \times 3 - 28 \div 7 =$$
$$25 + 48 - 4 =$$
$$69$$

12. In this problem we have to do the operations inside the parentheses first and then we go from left to right doing the multiplications and divisions in the order in which they occur. Thus we have:

$$5 \times (11 - 8) + 3 \times (7 - 5) \div 2 =$$
$$5 \times 3 + 3 \times 2 \div 2 =$$
$$15 + 3 =$$
$$18$$

13. $4 + (5+2) \times 20 - (10-6) \div (7-5) =$
$$4 + 7 \times 20 - 4 \div 2 =$$
$$4 + 140 - 2 = 142$$

14. In this problem you must perform the operation inside the parentheses first, and then we do the operation inside the bracket. Then we go through the problem from left to right doing the multiplication and division in the order in which they occur and then finally go through the problem again from left to right doing the addition and subtraction.

$$3 \times 500 \div [\,2 \times (28 + 22)] + 25 - 6 =$$
$$3 \times 500 \div [\,2 \times 50] + 25 - 6 =$$
$$3 \times 500 \div 100 + 25 - 6 =$$
$$15 + 25 - 6 = 34$$

15. $95 \div (22 - 17) - 6 \times 2 - 3 + 8 =$
$$95 \div 5 - 6 \times 2 - 3 + 8 =$$
$$19 - 12 - 3 + 8 = 12$$

16. $\frac{3}{7} + \frac{5}{7} + \frac{6}{7} =$

$$\frac{3 + 5 + 6}{7} =$$

$$\frac{14}{7} = 2$$

17. $\frac{1}{2} + \frac{1}{3} + \frac{1}{12} =$

$$\frac{6 + 4 + 1}{12} =$$

$$\frac{11}{12}$$

18. $\frac{1}{2} + \frac{1}{3} + \frac{1}{5} =$

$$\frac{15 + 10 + 6}{30} =$$

$$\frac{31}{30} = 1\frac{1}{30}$$

19. $\frac{3}{8} + \frac{1}{2} + \frac{3}{4} =$

$$\frac{3 + 4 + 6}{8} =$$

$$\frac{13}{8} = 1\frac{5}{8}$$

20. $\frac{3}{4} + \frac{1}{16} + \frac{1}{8} =$

$$\frac{12 + 1 + 2}{16}$$

$$= \frac{15}{16}$$

21. $\frac{1}{3} + \frac{1}{7} + \frac{3}{14} =$

$$\frac{14 + 6 + 9}{42} =$$

$$\frac{29}{42}$$

22. $\frac{6}{23} + \frac{8}{46} + \frac{19}{69} =$

$$\frac{6}{23} + \frac{4}{23} + \frac{19}{69} =$$

$$\frac{18 + 12 + 19}{69} =$$

$$\frac{49}{69}$$

23. $\frac{1}{5} + \frac{2}{9} + \frac{3}{11} =$

$$\frac{99 + 110 + 135}{495} =$$

$$\frac{344}{495}$$

24. $\frac{1}{9} + \frac{3}{4} + \frac{1}{8} =$

$$\frac{8 + 54 + 9}{72} =$$

$$\frac{71}{72}$$

25. $\frac{7}{25} + \frac{9}{35} + \frac{2}{5} =$

$$\frac{49 + 45 + 70}{175} =$$

$$\frac{164}{175}$$

26. $\frac{5}{7} - \frac{3}{7} =$

$$\frac{5 - 3}{7} =$$

$$\frac{2}{7}$$

27. $\frac{2}{3} - \frac{1}{2} =$

$$\frac{4 - 3}{6} =$$

$$\frac{1}{6}$$

28. $\frac{7}{10} - \frac{3}{5} =$

$\frac{7 - 6}{10} =$

$\frac{1}{10}$

29. $\frac{8}{9} - \frac{2}{5} =$

$\frac{40 - 18}{45} =$

$\frac{22}{45}$

30. $\frac{3}{4} - \frac{3}{8} =$

$\frac{6 - 3}{8} =$

$\frac{3}{8}$

31. $\frac{25}{27} - \frac{1}{3} =$

$\frac{25 - 9}{27} =$

$\frac{16}{27}$

32. $\frac{3}{4} - \frac{1}{6} - \frac{1}{3} =$

$\frac{9 - 2 - 4}{12} =$

$\frac{3}{12} = \frac{1}{4}$

33. $\frac{7}{8} - \frac{1}{9} - \frac{1}{5} =$

$\frac{315 - 40 - 72}{360} =$

$\frac{203}{360}$

34. $\frac{4}{5} - \frac{3}{25} - \frac{3}{50} =$

$\frac{40 - 6 - 3}{50} =$

$\frac{31}{50}$

35. $\frac{35}{36} - \frac{3}{9} - \frac{1}{4} =$

$\frac{35 - 12 - 9}{36} =$

$\frac{14}{36} = \frac{7}{18}$

36. $1\frac{1}{4} + 2\frac{1}{5} =$

$1 + 2 + \frac{1}{4} + \frac{1}{5} =$

$3 + \frac{5 + 4}{20} =$

$3 + \frac{9}{20} = 3\frac{9}{20}$

37. $2\frac{1}{2} + 3\frac{1}{3} + 1\frac{1}{6} =$

$2 + 3 + 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{6} =$

$6 + \frac{3 + 2 + 1}{6} =$

$6 + \frac{6}{6} = 6 + 1 = 7$

38. $4\frac{7}{8} - 3\frac{1}{4} =$

$4 - 3 + \frac{7}{8} - \frac{1}{4} =$

$1 + \frac{7 - 2}{8} =$

$1 + \frac{5}{8} = 1\frac{5}{8}$

39. $5\frac{1}{8} - 2\frac{3}{7} =$

$5 - 2 + \frac{1}{8} - \frac{3}{7} =$

$3 + \frac{7 - 24}{56} =$

$2 + \frac{63 - 24}{56} =$

$2 + \frac{39}{56} = 2\frac{39}{56}$

40. $8\frac{1}{9} - 3\frac{2}{7} - 2\frac{2}{3} =$

$8 - 3 - 2 + \frac{1}{9} - \frac{2}{7} - \frac{2}{3} =$

$3 + \frac{1}{9} - \frac{2}{7} - \frac{2}{3} =$

$2 + \frac{10}{9} - \frac{2}{7} - \frac{2}{3} =$

$2 + \frac{70 - 18 - 42}{63} =$

$2 + \frac{10}{63} = 2\frac{10}{63}$

41. $\frac{1}{7} + \frac{8}{9} - \frac{1}{3} =$

$\frac{9 + 56 - 21}{63} =$

$\frac{44}{63}$

42. $\frac{3}{4} - \frac{1}{9} + \frac{3}{18} - \frac{1}{2} =$

$\frac{27 - 4 + 6 - 18}{36} =$

$\frac{33 - 22}{36} = \frac{11}{36}$

43. $4\frac{3}{8} + 1\frac{1}{4} - 2\frac{1}{7} =$

$4 + 1 - 2 + \frac{3}{8} + \frac{1}{4} - \frac{1}{7} =$

$3 + \frac{21 + 14 - 8}{56} =$

$3 + \frac{27}{56} = 3\frac{27}{56}$

44. $1\frac{7}{8} - 3\frac{1}{7} - 1\frac{1}{6} + 8\frac{2}{9} =$

$1 - 3 - 1 + 8 + \frac{7}{8} - \frac{1}{7} - \frac{1}{6} + \frac{2}{9} =$

$5 + \frac{441 - 72 - 84 + 112}{504} =$

$5 + \frac{553 - 156}{504} = 5 + \frac{397}{504} = 5\frac{397}{504}$

45. $6\frac{1}{3} - 4\frac{3}{4} + \frac{1}{9} - 1\frac{1}{3} =$

$6 - 4 - 1 + \frac{1}{3} - \frac{3}{4} + \frac{1}{9} - \frac{1}{3} =$

$1 + \frac{12 - 27 + 4 - 12}{36} =$

$1 + \frac{16 - 39}{36} =$

$\frac{52 - 39}{36} = \frac{13}{36}$

46. $\frac{1}{3} \times \frac{1}{4} =$

$\frac{1 \times 1}{3 \times 4} = \frac{1}{12}$

47. $\frac{1}{7} \times \frac{2}{9} =$

$\frac{1 \times 2}{7 \times 9} = \frac{2}{63}$

48. $\frac{3}{4} \times \frac{3}{8} =$

$\frac{3 \times 3}{4 \times 8} = \frac{9}{32}$

49. $\frac{7}{8} \times \frac{4}{7} =$

$\frac{\overset{1}{\cancel{7}} \times 4}{8 \times \underset{1}{\cancel{7}}} = \frac{4}{8} = \frac{1}{2}$

50. $\frac{5}{13} \times \frac{26}{30} =$

$\frac{\overset{1}{\cancel{5}} \times \overset{2}{\cancel{26}}}{\underset{1}{\cancel{13}} \times \cancel{30}} = \frac{2}{6} = \frac{1}{3}$

51. $\frac{3}{7} \times \frac{7}{8} \times \frac{2}{3} =$

$\frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{7}} \times 2}{\underset{1}{\cancel{7}} \times 8 \times \underset{1}{\cancel{3}}} = \frac{2}{8} = \frac{1}{4}$

52. $\frac{3}{8} \times \frac{16}{19} \times \frac{19}{21} =$

$\frac{\overset{1}{\cancel{3}} \times \overset{2}{\cancel{16}} \times \overset{1}{\cancel{19}}}{\underset{1}{\cancel{8}} \times \underset{1}{\cancel{19}} \times \underset{7}{\cancel{21}}} = \frac{2}{7}$

53. $\frac{1}{8} \times \frac{8}{9} \times \frac{9}{23} =$

$\frac{1 \times \overset{1}{\cancel{8}} \times \overset{1}{\cancel{9}}}{\underset{1}{\cancel{8}} \times \underset{1}{\cancel{9}} \times 23} = \frac{1}{23}$

54. $\frac{4}{7} \times \frac{21}{28} \times \frac{7}{9} =$

$\frac{\overset{1}{\cancel{4}} \times \overset{1}{\cancel{21}} \times \overset{1}{\cancel{7}}}{\underset{1}{\cancel{7}} \times \underset{1}{\cancel{28}} \times \underset{3}{\cancel{9}}} = \frac{1}{3}$

55. $\frac{18}{20} \times \frac{30}{36} \times \frac{2}{3} =$

$\frac{\overset{1}{\cancel{18}} \times \overset{1}{\cancel{30}} \times \overset{1}{\cancel{2}}}{\underset{1}{\cancel{20}} \times \underset{2}{\cancel{36}} \times \underset{1}{\cancel{3}}} = \frac{1}{2}$

56. $\frac{1}{3} \div \frac{2}{9} =$

$\frac{1}{\underset{1}{\cancel{3}}} \times \frac{\overset{3}{\cancel{9}}}{2} = \frac{3}{2} = 1\frac{1}{2}$

57. $\frac{2}{7} \div \frac{3}{7} = \frac{2}{\underset{1}{\cancel{7}}} \times \frac{\overset{1}{\cancel{7}}}{3} = \frac{2}{3}$

58. $\frac{3}{4} \div \frac{9}{16} =$

$\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{4}{\cancel{16}}}{\underset{3}{\cancel{9}}} = \frac{4}{3} = 1\frac{1}{3}$

59. $\frac{47}{49} + \frac{1}{7} =$

$\frac{47}{\underset{7}{\cancel{49}}} \times \frac{\overset{1}{\cancel{7}}}{1} = \frac{47}{7} = 6\frac{5}{7}$

60. $\frac{19}{23} \times \frac{41}{38} =$

$\frac{\overset{1}{\cancel{19}}}{23} \times \frac{41}{\underset{2}{\cancel{38}}} =$

$\frac{41}{46}$

61. $\frac{2}{7} + \frac{1}{14} + \frac{1}{2} =$

$\frac{2}{\underset{1}{\cancel{7}}} \times \frac{\overset{2}{\cancel{14}}}{1} \times \frac{2}{1} = \frac{8}{1} = 8$

62. $\frac{1}{9} + \frac{2}{3} + \frac{3}{7} =$

$\frac{1}{9} \times \frac{\overset{1}{\cancel{3}}}{2} \times \frac{7}{\underset{1}{\cancel{3}}} = \frac{7}{18}$

63. $\frac{2}{11} + \frac{4}{7} + \frac{3}{14} =$

$\frac{\overset{1}{\cancel{2}}}{11} \times \frac{7}{\underset{1}{\cancel{4}}} \times \frac{\overset{7}{\cancel{14}}}{3} =$

$\frac{49}{33} = 1\frac{16}{33}$

64. $\frac{2}{9} + \frac{4}{5} + \frac{5}{8} =$

$\frac{2}{9} \times \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{1}{\cancel{5}}} = \frac{4}{9}$

65. $\frac{13}{15} + \frac{2}{5} \div \frac{4}{9} =$

$\frac{13}{\underset{1}{\cancel{15}}} \times \frac{\overset{1}{\cancel{5}}}{2} \times \frac{\overset{3}{\cancel{9}}}{4} =$

$\frac{39}{8} = 4\frac{7}{8}$

66. $1\frac{1}{4} \times 2\frac{1}{2} =$

$\frac{5}{4} \times \frac{5}{2} = \frac{25}{8} = 3\frac{1}{8}$

67. $2\frac{3}{8} \times 4\frac{1}{7} =$

$\frac{19}{8} \times \frac{29}{7} = \frac{551}{56} = 9\frac{47}{56}$

68. $4\frac{3}{7} \times 5\frac{4}{9} =$

$\frac{31}{\underset{1}{\cancel{7}}} \times \frac{\overset{7}{\cancel{49}}}{9} = \frac{217}{9} = 24\frac{1}{9}$

69. $6\frac{7}{8} + 2\frac{1}{4} =$

$\frac{55}{8} + \frac{9}{4} =$

$\frac{55}{\underset{2}{\cancel{8}}} \times \frac{\overset{1}{\cancel{4}}}{9} =$

$\frac{55}{18} = 3\frac{1}{18}$

70. $5\frac{1}{8} \div 3\frac{1}{7} =$

$\frac{41}{8} \div \frac{22}{7} =$

$\frac{41}{8} \times \frac{7}{22} = \frac{287}{176} = 1\frac{111}{176}$

71. $8\frac{2}{9} \div 5\frac{1}{6} =$

$\frac{74}{9} \div \frac{31}{6}$

$\frac{74}{\underset{3}{\cancel{9}}} \times \frac{\overset{2}{\cancel{6}}}{31} = \frac{148}{93} = 1\frac{55}{93}$

72. $8\frac{1}{9} \times 3\frac{1}{5} \div 9\frac{1}{7} =$

$\frac{73}{9} \times \frac{16}{5} \div \frac{64}{7} =$

$\frac{73}{9} \times \frac{\overset{1}{\cancel{16}}}{5} \times \frac{7}{\underset{4}{\cancel{64}}} =$

$\frac{511}{180} = 2\frac{151}{180}$

73. $6\frac{2}{3} \div 2\frac{2}{9} \times 4\frac{3}{4} =$

$\frac{20}{3} \div \frac{20}{9} \times \frac{19}{4} =$

$\frac{\overset{1}{\cancel{20}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{20}}} \times \frac{19}{4} = \frac{57}{4} = 14\frac{1}{4}$

74. $3\frac{1}{4} \div 8\frac{1}{2} \times 2\frac{1}{8} =$

$\frac{13}{4} \div \frac{17}{2} \times \frac{17}{8} =$

$\frac{13}{4} \times \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{17}}} \times \frac{\overset{1}{\cancel{17}}}{\underset{4}{\cancel{8}}} = \frac{13}{16}$

75. $1\frac{7}{8} \div 6\frac{6}{9} \times 2\frac{1}{2} =$

$\frac{15}{8} \div \frac{60}{9} \times \frac{5}{2} =$

$\frac{\overset{1}{\cancel{15}}}{8} \times \frac{9}{\underset{4}{\cancel{60}}} \times \frac{5}{2} =$

$\frac{45}{64}$

76. $\frac{1}{2} + \frac{1}{4} \times \frac{3}{5} - \frac{1}{10} + \frac{1}{10} \div \frac{3}{5} =$

$\frac{1}{2} + \frac{3}{20} - \frac{1}{10} + \frac{1}{\underset{2}{\cancel{10}}} \times \frac{\overset{1}{\cancel{5}}}{3} =$

$\frac{1}{2} + \frac{3}{20} - \frac{1}{10} + \frac{1}{6} =$

$\frac{30 + 9 - 6 + 10}{60} = \frac{43}{60}$

77. $\frac{1}{2} \div \frac{1}{8} \times \frac{5}{7} - \frac{1}{8} \times \frac{8}{9} \div \frac{1}{3} =$

$\frac{1}{\underset{1}{\cancel{2}}} \times \frac{\overset{4}{\cancel{8}}}{1} \times \frac{5}{7} - \frac{1}{\cancel{8}} \times \frac{\cancel{8}}{\underset{3}{\cancel{9}}} \times \frac{\overset{1}{\cancel{3}}}{1} =$

$\frac{20}{7} - \frac{1}{3} = \frac{60 - 7}{21} = \frac{53}{21} = 2\frac{11}{21}$

78. $\frac{1}{2} - \frac{1}{4} \times \frac{1}{3} + \frac{1}{2} \div \frac{3}{4} - \frac{1}{6} =$

$\frac{1}{2} - \frac{1}{12} + \frac{1}{\underset{1}{\cancel{2}}} \times \frac{\overset{2}{\cancel{4}}}{3} - \frac{1}{6} =$

$\frac{1}{2} - \frac{1}{12} + \frac{2}{3} - \frac{1}{6} =$

$\frac{6 - 1 + 8 - 2}{12} = \frac{11}{12}$

79. $1\frac{1}{4} + 1\frac{3}{8} \times \frac{3}{8} \div 2\frac{3}{4} - 3 \times \frac{1}{9} \div \frac{3}{4} =$

$\frac{5}{4} + \frac{\cancel{11}}{8} \times \frac{3}{\cancel{8}_{2}} \times \frac{\cancel{4}}{\cancel{11}} - \frac{\cancel{3}}{1} \times \frac{1}{9} \times \frac{4}{\cancel{3}} =$

$\frac{5}{4} + \frac{3}{16} - \frac{4}{9} =$

$\frac{180 + 27 - 64}{144} = \frac{143}{144}$

80. $\left(\frac{3}{4} + \frac{1}{8}\right) \div \frac{7}{8} - \left(\frac{3}{4} - \frac{1}{8}\right) \times \frac{8}{9} =$

$\left(\frac{6 + 1}{8}\right) \div \frac{7}{8} - \left(\frac{6 - 1}{8}\right) \times \frac{8}{9} =$

$\frac{\cancel{7}}{\cancel{8}} \times \frac{\cancel{8}}{\cancel{7}} - \frac{5}{\cancel{8}} \times \frac{\cancel{8}}{9} =$

$1 - \frac{5}{9} = \frac{9 - 5}{9} = \frac{4}{9}$

81.
.3125
16/5.00
4 8
20
16
40
32
80
80

82.
.875
8/7.000
6 4
60
56
40
40

83.
.16666 = .1667
6/1.00000
6
40
36
40
36
40
36
40
36
4

84.
.71428 = .7143
7/5.0000
4 9
10
7
30
28
20
14
60

85.
.8125
16/13.000
12 8
20
16
40
32
80
80

86.
1.34
26.2
8.41
91.74
127.69

87.
8.33
92.1
17.41
6.3
124.14

88.
91.31
-80.94
10.37

89.
$$\begin{array}{r} 137.42 \\ -\ 43.8 \\ \hline 93.62 \end{array}$$

90.
$$\begin{array}{rl} 137.6 & \text{1 decimal place} \\ 4.88 & \text{2 decimal places} \\ \hline 110\ 08 & \text{3 decimal places} \\ 1100\ 8 & \\ 5504 & \\ \hline 671.488 & \text{3 decimal places} \end{array}$$

91.
$$\begin{array}{rl} .43 & \text{2 decimal places} \\ .0061 & \text{4 decimal places} \\ \hline 43 & \text{6 decimal places} \\ 258 & \\ \hline .002623 & \text{6 decimal places} \end{array}$$

92. $2.3\overline{)108.33}$

$$\begin{array}{r} 47.1 \\ 23\overline{)1083.3} \\ \underline{92} \\ 163 \\ \underline{161} \\ 23 \\ \underline{23} \end{array}$$

93. $.049\overline{)45.227}$

$$\begin{array}{r} 923 \\ 49\overline{)45227} \\ \underline{441} \\ 112 \\ \underline{98} \\ 147 \\ \underline{147} \end{array}$$

94. $.051\overline{).01887}$

$$\begin{array}{r} .37 \\ 51\overline{)18.87} \\ \underline{15\ 3} \\ 3\ 57 \\ \underline{3\ 57} \end{array}$$

95.
$$\begin{array}{rl} .00173 & \text{5 decimal places} \\ 21 & \text{0 decimal places} \\ \hline 173 & \text{5 decimal places} \\ 346 & \\ \hline .03633 & \text{5 decimal places} \end{array}$$

96. $\dfrac{\overset{1}{\cancel{35}}}{\underset{3}{\cancel{105}}} \times 100 = 33\frac{1}{3}\%$

97. $\dfrac{\overset{1}{\cancel{8}}}{\underset{5}{\cancel{40}}} \times \overset{20}{\cancel{100}} = 20\%$

98. $\dfrac{\overset{3}{\cancel{15}}}{\underset{1}{\underset{\cancel{20}}{\cancel{100}}}} \times \overset{3}{\cancel{60}} = 9$

99. $\dfrac{36}{100} \times 4286 = 1542.96$

100. 10% of 680 =

$\dfrac{10}{100} \times 680 = 68$ ohms

$\therefore$ Lowest value = 680 - 68 = 612 ohms.

101. 5% of 2200 =

$\dfrac{5}{100} \times 2200 = 110$ ohms.

2200 + 110 = 2310 ohms.

$\therefore$ 2300 ohms is within its rated tolerance.

102. 10% of 4.7 K =

$\dfrac{10}{100} \times 4700 = 470$ ohms

4700 + 470 = 5170 ohms.

5200 is greater than 5170

$\therefore$ resistor is above its rated tolerance

103. $\dfrac{1}{4} \times 100 = 25\%$

104. $\frac{1}{8} \times 100 = 12.5\%$

105. 75% of 200 volts =

$$\frac{75}{100} \times 200 = 150 \text{ volts}$$

106. $R_t = \dfrac{R_1 \times R_2}{R_1 + R_2}$

$= \dfrac{35 \times 75}{35 + 75}$

$= \dfrac{2625}{110}$

$= 23.8$ ohms

$= 24$ ohms

107. $P = E^2 \div R = \dfrac{12\cancel{0} \times 120}{10\cancel{0}}$

$= \dfrac{12 \times 12\cancel{0}}{1\cancel{0}} = \dfrac{12 \times 12}{1}$

$= \dfrac{144}{1} = 144$ watts

108. $R_1 + R_2 = 75 + 150 = 225\,\Omega$
$R_t = 335\,\Omega$
$\therefore R_2 = 335 - 225 = 110\,\Omega$

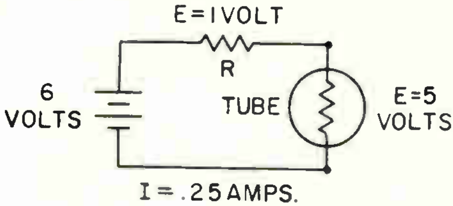
109. $P = 1^2 \times R$
$= 6 \times 6 \times 335$
$= 12{,}060$ watts

110. $P = 2000$ watts $R = 500\,\Omega$
$I^2 = P \div R$

$= \dfrac{2000}{500} = 4$

If $I \times I = 4$, then $I = 2$ amps.

111. In this problem you have 2 resistors in series. One resistor is the tube filament which has a voltage drop of 5 volts across it. The other resistor must use up or drop the extra voltage which is the difference between the battery voltage and the tube filament voltage in this case. 6-5 = 1 volt. Therefore we have the circuit:



We have E across R = 1 volt, I = .25 amps

$\therefore R = \dfrac{E}{I} = \dfrac{1}{.25} = 4$ ohms

112. The current through the tube filament = 200 milliamps

200 milliamps = .2 amps.

resistance of filament = 500 ohms.

$\therefore$ voltage across filament

$E = I \times R$
$= .2 \times 500$
$= 100$ volts

$\therefore$ series resistor must drop 110-100 = 10 volts

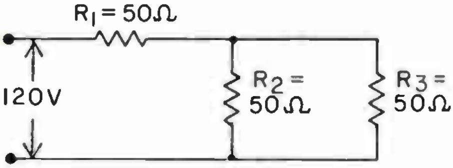
$\therefore R = \dfrac{10}{.2} = 50$ ohms.

113. $R_t = \dfrac{R_1 \times R_2}{R_1 + R_2}$

$= \dfrac{5 \times 15}{5 + 15}$

$= \dfrac{75}{20} = 3.75$ ohms

114. The circuit we have in this problem is:



First we find the value of R^2 and R^3 in parallel:

$$R_t = \frac{R_2 \times R_3}{R_2 + R_3}$$

$$= \frac{50 \times 50}{50 + 50} = \frac{2500}{100} = 25$$

∴ total series resistance is R_1 in series with 25 Ω

$50 + 25 = 75\Omega$

Now we know E = 120V and R = 75Ω so we can find I:

$$I = \frac{E}{R}$$

$$= \frac{120}{75} = 1.6 \text{ amps}$$

115. At first glance, this problem might look difficult, but it is just a matter of solving it a step at a time.

First we find the value of $R_6 \times R_7$ in parallel:

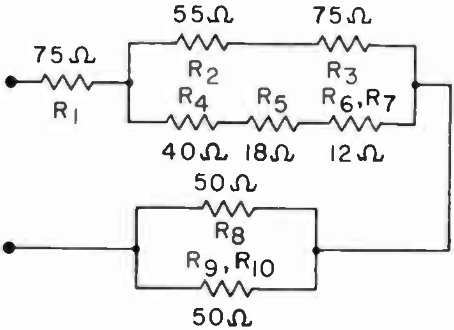
$$R_t = \frac{R_6 \times R_7}{R_6 + R_7}$$

$$= \frac{30 \times 20}{30 + 20} = \frac{600}{50} = 12\Omega$$

Next we find the value of R_9 and R_{10} in series:

$R_9 + R_{10} = 35 + 15 = 50$

Now we can redraw the circuit and substitute for R_6 and R_7 and for R_9 and R_{10}:



Now we find R_2 and R_3 in series:

$R_2 + R_3 = 55 + 75 = 130\Omega$

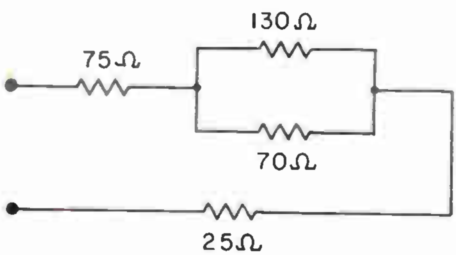
and R_4, R_5 and $R_6 - R_7$ in series:

$40 + 18 + 12 = 70\Omega$

Also we find R_8 in parallel with $R_9 - R_{10}$:

$$R_t = \frac{50 \times 50}{50 + 50} = \frac{2500}{100} = 25\Omega$$

Now we can redraw our circuit and substitute this value:



Now we find the value of 130Ω in parallel with 70Ω:

$$R_t = \frac{130 \times 70}{130 + 70} = \frac{9100}{200} = 45.5\Omega$$

Now we have 75 + 45.5 + 25 = 145.5

116. 10% of 450Ω =

$$\frac{10}{100} \times 450 = 45\Omega$$

∴ 450-45 = 405Ω is the lowest value in tolerance

∴ 410Ω is in tolerance

117. Total voltage = 36 + 24 + 40 = 100 volts.

∴ 24 volts $= \frac{24}{100} \times 100 = 24\%$

118. The first step in solving this problem is to find the total R.

$R_3 + R_4 = 25 + 15 = 40\Omega$

∴ $R_3 + R_4$ in parallel with R_2 =

$$\frac{40 \times 40}{40 + 40} = \frac{1600}{80} = 20\Omega$$

∴ R_1 + parallel combination = 30 + 20 = 50Ω

power = 450 watts

∴ $I^2 = \frac{450}{50} = 9$

∴ I × I = 9 and I = 3 amps.

∴ Voltage drop across R_1

E = I × R = 3 × 30 = 90 volts

119. Total current = 3 amps.
Part flows through R_2 and part through R_3 and R_4.

Since $R_2 = 40\Omega$ and $R_3 + R_4 = 40\Omega$

current flowing in each branch will be equal

∴ $I_{R4} = \frac{3}{2} = 1.5$ amps

120. I = 400 milliamps = .4 amps
R = 2.2K = 2200 ohms.
∴ E = I × R = .4 × 2200 = 880 volts.

Lesson Questions

Be sure to number your Answer Sheet X105.

Place your Student Number on *every* Answer Sheet.

Most students want to know their grades as soon as possible, so they mail their answers immediately. Others, knowing they will finish the next lesson within a few days, send in two sets of answers at a time. Either practice is acceptable. However, don't hold your answers too long; you may lose them. Don't hold answers to more than two sets of lessons at any time, or you may run out of lessons before new ones can reach you.

1. A supply voltage of 120 volts is applied to three resistors in series. One resistor has a drop of 55 volts, another a drop of 30 volts. What is the voltage drop across the third resistor?

2. A 750-ohm resistor has a rated tolerance of ±5%. When it is measured with an accurate ohmmeter, we find that the meter reads 697 ohms. Is the resistor (a) above, (b) below, (c) within its rated tolerance?

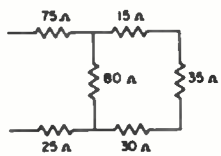
3. If 640 watts are consumed in a circuit with a total resistance of 160 ohms, how much current will flow in the circuit?

4. A current of 200 milliamps flows through a resistance of 1.6K. What is the voltage drop across the resistor?

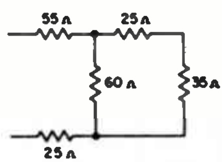
5. Find the answer to the problem 5 + 60 X 3 ÷ 4 – 6.

6. If 1/16th of the voltage applied to a circuit is dropped across a certain resistor, exactly what percentage of the total voltage does this represent?

7. What is the total resistance of the circuit shown at the right?



8. If 440 volts is applied to the circuit at the right, how much voltage will be dropped across the 35Ω resistor?



9. If a vacuum tube filament has a resistance of 20 ohms, and is rated at 250 milliamps, how much voltage should we apply so that it will draw its rated current?

10. The power consumed in a circuit is 160 watts. The voltage applied is 80 volts. What is the resistance of the circuit?

LEARNING NEVER ENDS

More and more it becomes evident that learning is a continuous process--that it is impossible to break the habit of studying without slipping backward. Look around you at all the marvelous developments of the last twenty years. You have the advantage of having "grown up" with them--yet there are probably many things you wish you knew more about. Then, consider what can happen in the years ahead if you do not keep abreast of the stream of new things that are bound to come!

Your NRI Course is preparing you for the problems of today and tomorrow, but what about the day after tomorrow? In five or ten years, will you still be up-to-date? Yes, if you plan your future. Resolve now--that you WILL keep up. You have the fundamentals; keep them fresh in your mind by constantly reviewing. Read and study technical literature and textbooks; join in discussion groups and listen to lectures; take advantage of every possible educational opportunity. Then, and only then, can you face the future unafraid, no matter what technical developments the future may hold.

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ACHIEVEMENT THROUGH ELECTRONICS

nri

AC CIRCUIT CALCULATIONS

REFERENCE TEXT X109

NATIONAL RADIO INSTITUTE • WASHINGTON, D. C.

AC CIRCUIT CALCULATIONS

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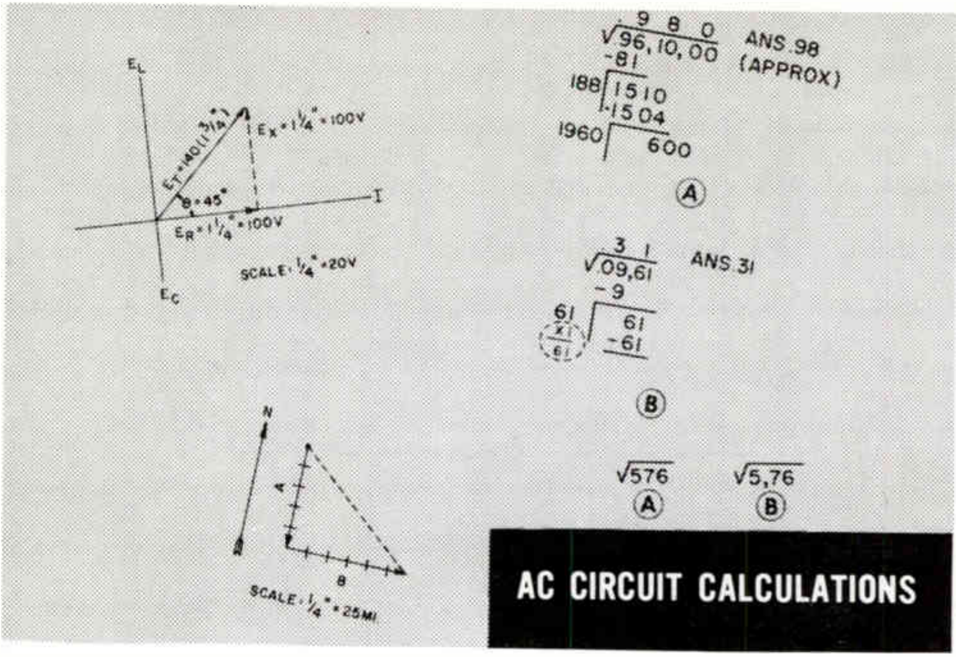
STUDY SCHEDULE

KL5M373 **1975 EDITION** **Litho in U.S.A.**



AC CIRCUIT CALCULATIONS

In your reference lesson on dc circuit calculations, you reviewed and studied many of the operations of basic arithmetic. You learned how simple addition, subtraction, multiplication, and division can be used in your work with electronics and electronic circuits. In this reference lesson you will take another step in your study of electronics by learning how to apply these same fundamental operations when solving ac circuit problems. However, as you learned when you studied coils and capacitors, alternating current reacts much differently from direct current in certain circuits. For this reason, you will have to expand your knowledge of basic mathematics in order to handle some of these conditions.

One of the first things you learned in dealing with alternating current is that the reactance of coils and capacitors causes the voltage and current to be out-of-phase with each other, while any resistance tries to keep them in phase. Consequently, you cannot add or subtract these ac circuit quantities without taking these phase differences into consideration. Although this might have seemed pretty difficult at first, you quickly learned that it could be done quite easily by using vectors to represent the phase relationships as well as the values. Then, by adding and subtracting the vectors, you were able to account for both the phase and the size of the circuit quantities.

While there is really no limit to the circuit solutions you can obtain by using simple vector measurement methods, they are very awkward and clumsy for the more complex circuits. For this reason, a number of simpler methods have been worked out to use in practical circuit solutions. Some of them involve square roots, others trigonometry, and one very common system uses a principle known as the j-operator. Like many other new subjects or methods, you will have to learn a few basic rules in order to use them with confidence and accuracy.

In addition to knowing how to do square roots, you should be very familiar with positive and negative numbers. In this lesson we will discuss finding the square roots of numbers, which is really a special type of division. You will also study positive and negative numbers, ratio and proportion, and take a closer look at vectors. Remember, you are not going to try to learn all there is to know about these subjects. You are simply going to look at them from the standpoint of their practical application in electronics.

Even though this is a reference lesson, it is still required for your course. Submit the answers to the questions at the back of the book just as you do for the technical lessons.

Square Roots

In our discussion of coils and capacitors we mentioned that if you knew how to do square roots, you could use a mathematical solution for finding the impedance in ac circuits. We even discussed the formula for this method, which is

$$Z = \sqrt{R^2 + X^2}$$

where Z is the impedance, R is the resistance, and X is the total reactance. The reactance may, of course, be either the inductive reactance, X_L, capacitive reactance, X_C, or a combination of both X_L and X_C.

This is a very handy formula for finding impedance and you will probably use it many times in your work with ac circuits. However, like many good methods of working problems, it is of no use unless you know how to do square roots. So let's take a look at this process, which is known as finding the square root of a number. Even if you have already studied square roots it will still be a good idea to read this section to refresh your memory.

When you studied multiplication you learned that the product of a number multiplied by itself was called a "square." You also learned a special way of indicating the process of finding a square by placing a small "2" above and to the right of the number. Thus, 6 X 6 can be indicated as 6^2, or 136 X 136 as 136^2. Any other number times itself can be indicated the same way.

Many times in your work with electronics you will have the "square" of a number and will want to know the number. There is a special process of division that can be used to find the "squared" number. Since the number that makes a square when it is multiplied by itself is sometimes called the "root" of a square, this special division process is called finding the "square root."

Let's make sure you understand this. A number multiplied by itself, or "squared," makes a product called a "square." The number that is squared is called the "root" of the square. The special division to find the root of a square is called finding the "square root."

SIMPLE SQUARE ROOTS

Now that you have seen what we mean by square root, let's look at a typical example and learn how to solve it. Suppose you are asked to find the square root of the number 576. What you want to know is: "What number multiplied by itself will give me a product of 576?" The main difference between this and any other division problem is that here you are given only the product to work with. Instead of being given a product and one number and being asked what number when multiplied by the given number will equal the product, you are simply given a product and are asked to find the one number that can be multiplied by itself to give this product.

This process is not as difficult as it may sound. It is just a matter of learning a few rules and how to apply them. First, you must set up the number as you would any other division problem, as shown in Fig. 1A. However, you will notice that there is one major difference besides the fact that

$\sqrt{576}$ $\sqrt{5,76}$

A B

Fig. 1. Setting up a square root problem.

there is no divisor. It's the symbol that looks like a lopsided letter V that is in front of the dividend (√). It replaces the straight vertical line in a standard division symbol and is called a "radical" sign. This radical sign is the symbol for finding the root of a number and is always used in a square root problem.

In a square root, just as in any other division, you do not tackle the whole number at once. You split it up into smaller numbers and work with them a few at a time. However, in square roots you do it a little bit differently. You will notice that in Fig. 1B we have placed a comma between the 5 and the 7. This breaks our number up into two numbers, 5 and 76. The method for breaking these numbers up in this way is quite simple. You merely start at the extreme right of the whole number and work toward the left, placing a comma after each group of two numbers. Thus, starting with the 6 and working toward the left we have our first group, 76, and then the 5. Since the 5 is the last number on the left in this particular problem, you have only two groups, one of which is a single number.

Grouping the numbers in this way under the radical sign completes the setup. You are now ready to go to work. You will remember that in division you used a basic multiplication table containing all the products for various combinations of the numbers from 1 to 9. After you had a division problem set up, you tried various products to see which one would go into the dividend. In a square root, you do much the same thing except that you need to know the squares of the numbers from 1 to 9. Fig. 2 is a basic multiplication table showing all the squares of the numbers from 1 to 9 marked with stars. This table and these squares are all you need to work any square root problem.

Now, let's look at the problem again, as shown in Fig. 3A, where it is set up and ready to work on. Since we have broken the number up into two groups, and since the number 5 is alone in the first group, we consider the 5 first. We don't have a divisor to divide into the 5, so we must make one. We do this by determining the largest square that will go into 5. Looking at the table, we see that 2 squared is equal to 4, which is smaller than 5, and 3 squared is equal to 9, which is larger than 5. Since the square of 3 is larger than 5, the square of 2 is the largest perfect square that will go into 5.

Therefore, 4 becomes the first trial product and the 2, which is the square root of 4, is the first trial divisor. Now

*1	2	3	4	5	6	7	8	9
2	*4	6	8	10	12	14	16	18
3	6	*9	12	15	18	21	24	27
4	8	12	*16	20	24	28	32	36
5	10	15	20	*25	30	35	40	45
6	12	18	24	30	*36	42	48	54
7	14	21	28	35	42	*49	56	63
8	16	24	32	40	48	56	*64	72
9	18	27	36	45	54	63	72	*81

Fig. 2. The basic multiplication table showing squares of numbers from 1 to 9.

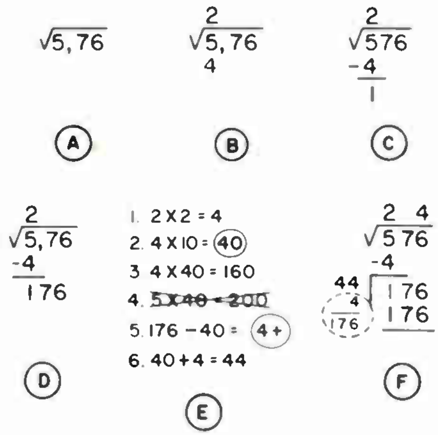


Fig. 3. Steps for working out a square root problem.

that we have found that 4 is the largest square that will go into 5, we place it under the 5 as shown in Fig. 3B. Then, we take the 2, which we squared to get the 4, and place it above the line over the 5 to indicate that it is the first digit of the quotient. This is also shown in Fig. 3B. Next, we subtract the 4 from the 5 to get the first remainder, which is 1, as shown in Fig. 3C. By finding the largest square that will go into the first group in our dividend, we have obtained the first trial product, the first remainder, and the first digit in the quotient.

This first step is the only time we have to use the square of a number when doing square roots. As you can see we never have more than two numbers in any group. The largest number we can possibly have in the first group is 99, and 9 squared, or 81, is the largest square that can go into 99 because 10 squared is 100. This is the reason why we never need more than the basic squares of the numbers from 1 to 9 in order to find the square root of any number.

From now on the problem becomes more like regular division except for the way of obtaining the trial divisors and trial products.

Looking at Fig. 3D, you see that the next step is to bring down the next group of numbers, which is 76. Notice that we bring down the whole group, not just the 7. When we have placed the 76 beside the 1 as shown, we have a trial dividend of 176. Now we must learn the rule by which we establish the next trial divisor. To do this, we take the partial quotient of 2, double it, and then multiply the result by 10.

Following this rule, the partial quotient is 2. If we double it, we get 4. Multiplying the 4 by 10 gives us 40. We use the 40 as the trial divisor. Now we ask ourselves: "How many times will 40 go into 176?" We know that 4 times 40 is 160 which will go into 176 easily. Since 5 times 40 is 200, which is too large to go into 176, 4 is the number we want.

However, we are still not quite finished with the trial divisor. Before we can use it, we have one more step to do. After we have determined that 4 is the largest number of times that 40 will go into 176, we must then *add* the 4 to 40 as shown in Step 6 of Fig. 3E. This gives us 44 which we use as the final trial divisor, as shown in Fig. 3F.

Now, we must multiply 44 by 4 to see if it will still go into 176. As you can see, 4 times 44 is exactly 176 and we will have no remainder when we subtract. Since the trial divisor did go into 176 four times, the second number of the quotient is 4 and can be placed above the line over the second group of numbers. There is no remainder; therefore, the complete quotient is 24, which is the square root of 576. We can prove this by squaring 24, which will give the product of 576.

Although the process of finding the

square root of a number is not difficult, it requires a firm knowledge of some important rules. For this reason we will work out another problem dealing with a larger square. Then you will have a good review of the process as well as some practical experience. Remember, no matter how large the number may be you proceed in the exact same way.

Working Square Root Problems. Let's find the square root of 186,624.

First, set up the problem under the radical sign. Next, separate the number into groups of two numbers, starting at the right and working toward the left, as shown in Fig. 4A. In this problem the radicand (the special name given to the dividend in a square root) is divided evenly into three groups of two numbers each. Since the first group is the number

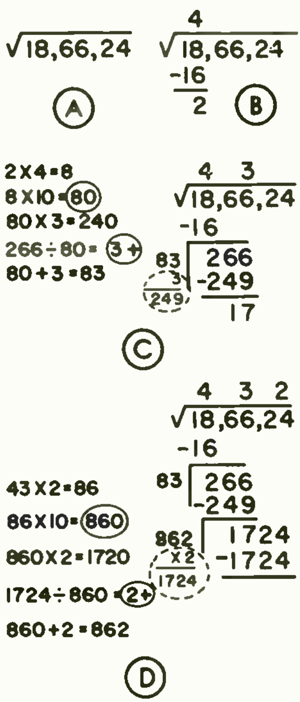


Fig. 4. Finding the square root of a six digit number.

18, we will find the largest square that will go into 18.

Looking at the multiplication table in Fig. 2 we see that the square of 4 is 16 and that the square of 5 is 25. Twenty-five is larger than 18, so 16 must be the largest square smaller than 18. Therefore, we place the 16 under the 18 in our radicand and subtract, as shown in Fig. 4B. Since 4 is the number that we squared to obtain this trial product, it is the first trial divisor and we place it in the quotient over the number 8 in the first group.

Next, subtract 16 from 18 to get a remainder of 2. Then bring down the next group of two numbers, 66, and place them beside the 2. This gives us a new number, 266, to use as the dividend for the next step, as shown in Fig. 4C. Now you must determine the second trial divisor. Remember, double the existing quotient (4) and then multiply by 10, giving us 80 as shown. Then see how many times 80 will go into 266. For this particular problem, 80 will go into 266 three times. Therefore, we *add* 3 to 80, giving 83 as the second trial divisor. Now, multiply 83 by 3 to get the trial product of 249 which can be subtracted from 266. This subtraction gives a remainder of 17, which is shown in Fig. 4C. Since the trial divisor of 83 went into 266 three times, enter the 3 in the quotient above the line over the second group of numbers, 66.

In this problem we still have another group of two numbers left in our radicand, so we are not finished yet. Therefore, we must bring the 24 down beside the 17 to get the dividend for the next step. As you can see in Fig. 4D, this gives 1724 and you must find a trial divisor for it so that you can find the next number for the quotient.

Proceeding as before, you take the partial quotient, 43, double it to get 86, and then multiply the product by 10, giving you 860. You can see that 860 will go into 1724 only two times. Therefore, you *add* 2 to 860, making 862 the trial divisor. Now, multiplying 862 by 2 gives exactly 1724 and you can subtract without having any remainder. Finally, you place the 2 in the quotient above the 24, giving you the complete square root of 186,624, which is 432. You can check this, of course, by multiplying 432 by itself to see if the product is equal to the radicand you started with.

Although the rules for doing square roots are a little different from those involved in other types of division, they are not any more difficult. Like anything else, it takes practice to become proficient at finding square roots. Most of us don't get this practice after we leave school, so we are likely to forget how to do it. However, in electrical work, square roots can be quite important. Let's try another problem.

In Fig. 5 we have set up the number 7,306,209 to find its square root. Notice that the complete radicand contains seven numbers so it can't be divided evenly into groups of two. Since we always start at the right end of the radicand, we form the groups as shown In this way, a single number will be the first number to the right of the radical sign.

$2^2 = 4$

$2 \times 2 = 4 \times 10 = 40$

$40 \times 8 = 320$

$40 + 8 = 48 \times 8 = 384$

$40 + 7 = 47 \times 7 = 329$

$54 \times 10 = 540$

$540 \times 10 = 5400$

$5400 + 3 = 5403 \times 3 = 16209$

```
              2  7  0  3
            √7,30,62,09
             -4
        47  | 330
        x7  |-329
       329
      5403  |    1 62 09
            |   -1 62 09
```

Fig. 5. Another square root problem.

Now, we examine the number in the first group to find the largest square that will go into it. In this problem, the first group is only one number, 7. The largest square that will go into it is 2^2, or 4. Thus, we place a 2 in the quotient or root, directly over the 7. Then we place the square of 2, which is 4, under the 7 and subtract, giving a remainder of 3.

Bring the next group, 30, down beside the 3. Double the root number, 2, making it 4 and multiply it by 10 for the next trial divisor. Now, we determine how many times 40 will go into 330. It looks as if 8 will work because $4 \times 8 = 32$. However, when we add 8 to 40 to get a trial divisor of 48, we find that $48 \times 8 = 384$, which is larger than 330. Therefore, we will have to try the next smallest number which is 7.

Notice that we don't multiply 48×7. We change the whole trial divisor of 48 to 47 and then multiply 47×7 which is equal to 329. In a square root, we often have to reject trial divisors and use new ones that are smaller. Since 329 is smaller than 330, we place the 7 in the root and subtract 329 from 330. This gives a remainder of 1 and we bring down the next group, 62, from the radicand, giving us our new remainder, 162.

We double the two numbers in the root, 27, to get a new trial divisor, which is 54 times 10 or 540. It is obvious at a glance that 540 will not go into 162. Therefore, we place a zero in the root above 62 and then bring down the next two numbers, which are 09. The remainder now becomes 16209.

We double the three numbers in the root, 270, to get the next trial divisor, which is 540 times 10 or 5400. This number (5400) looks as if it will go into

16209 about 3 times. So we add 3 to 5400 to get 5403 and then multiply this number by 3 and get exactly 16209. Since we have no remainder, the problem is completed and we place the 3 in the quotient, or root, above the last group of numbers. Thus, we have found that the square root of 7,306,209 is 2703, which we can prove to be correct by squaring 2703 to get the original radicand.

IMPERFECT SQUARES

So far all the answers have been the roots of perfect squares. Although this is convenient, it does not often happen in practice. There will usually be a remainder, which must be accounted for by making the root end with a decimal number or a fraction. For example, suppose the radicand for the last problem that you worked had been 7,306,976 instead of 7,306,209. If this had been the case, the problem would have worked the same way until you had reached the last part of it, as shown in Fig. 6.

As you can see, the trial product of 5403 X 3 is equal to 16209, but because the original radicand was changed you have a remainder of 767 when you subtract. This is not large enough to

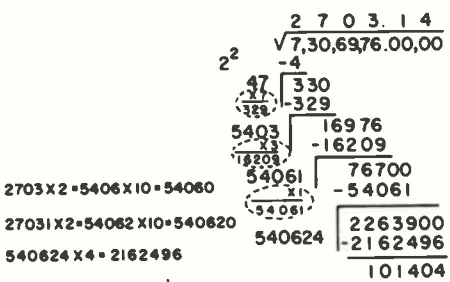


Fig. 6. An imperfect square root carried two places.

increase the root by another whole number to make it 2704, but it does leave a fraction. If the need for accuracy is such that you want to carry the root of 2703 into its fractional part, you would continue the problem as shown in Fig. 6.

To do this, you place a decimal after the last number in the radicand and then add a group of two zeros for each decimal place you want in the root. Notice that there are two zeros added for each decimal place in a square root instead of only one zero, as there is in regular division. In this particular problem, we have carried the answer to two decimal places and have added four zeros as shown.

After you have added the zeros in the radicand and the decimal point in both the radicand and in the root number, you continue in the same manner as before to find the new root numbers. Bring down the first group of two zeros beside the remainder of 767 to give 76700 for the next dividend. Then you double the existing root number, 2703, and multiply the product by 10 to get the trial divisor, 54060. You can quickly see that this will go into 76700 only once, so you add the 1 to it as shown. Then, since any number multiplied by 1 is that same number, you have 54061 for the trial product. Place the 1 in the root.

Subtracting 54061, you get a remainder of 22639. Bring down the next two zeros as shown. Now, double the quotient and multiply by 10, ignoring the decimal in the quotient. In other words, multiply 27031 by 2 to get 54062 and then multiply by 10, making it 540620. Continue as before to find the next root number, which is 4. As you can see, this gives a root of 2703.14 to two decimal places. Since we still have a remainder we do not have a perfect root. Except where extreme accuracy is needed, two decimal places in the answer are enough. Of course, if you do want more accuracy, you would simply continue in the same way by adding two more zeros for each additional decimal place.

You can check your answer in the usual way by squaring the root to see if you get your original radicand. In this problem, if you square 2703.14, you will get an answer of 7,306,965.8596. It is not exactly the same as the original radicand because we did not have a perfect square to start with. However, you can see that the root is close enough and is correct to two decimal places, which is all we are interested in.

FRACTIONS AND DECIMALS

Many times in electronics you will need to find the square root of a fraction or decimal. While the process for finding the roots of fractions or decimals is nearly the same as it is for whole numbers, there is one important thing to remember. The square root of a fraction or decimal will always be larger than its square. You will recall that the product of any two fractions or decimals is always smaller than either of the two numbers. Therefore, the square of any fraction or decimal must also be smaller than the numbers squared.

Finding the square root of fractions is usually very easy, because most fractions have small numbers in both the numerator and the denominator. Also, finding the square roots of small numbers does not involve much work. The first thing to do to find the square root of a fraction is to reduce the fraction as much as possible. Thus, if you wanted to find the root of 6/32, you would first reduce it to 3/16, as shown in Fig. 7A.

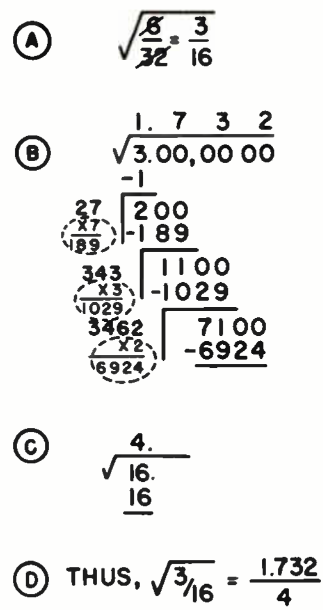


Fig. 7. Finding the square root of a fraction.

Next, separate the numerator and the denominator and find the square root of each one separately, using the same method that you learned for whole numbers. Thus, you would find the square root of 3, which is 1.732 to three places, as shown in Fig. 7B. Then find the square root of 16, which is 4. Now use the root of 3 as the new numerator and the root of 16 as the new denominator. Thus, the square root of 3/16 is 1.732/4 as shown in Fig. 7C. By using this simple method, you can find the square root of any fraction quickly and accurately.

In finding the square root of a decimal, you proceed exactly the same as for a whole number except the grouping will be different when you set up the problem. You still separate the decimal number in groups of two numbers, but with a decimal you start at the decimal point and work to the right. You must always have an even number of digits to the right

$\sqrt{.96,10}$ (A) $\quad$ $\sqrt{.96,12,35}$ (B)

Fig. 8. Setting up a decimal for a square root.

of the decimal place. That is, each group must have two digits. Thus, to set up the decimal .961 for finding its square root, you proceed as shown in Fig. 8A.

After you have placed the number under the radical sign, start at the decimal point and count two numbers to the right and place a comma between the 6 and the 1. Then add a zero after the 1, so that the second group has two digits. If the number were .961235, you would place commas after the 6 and after the 2, as shown in Fig. 8B. The fact that you work from the decimal point to the right in the grouping of the digits is the only difference in the method of finding the square root of a decimal from that of a whole number. In Fig. 9 we have worked out the square root of two decimals. You should not have any trouble in following these examples.

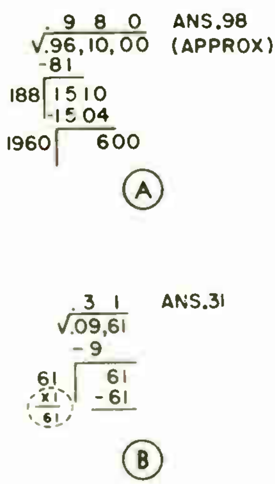


Fig. 9. Finding decimal square roots.

ESTIMATING SQUARE ROOTS

Now that you have seen how to work out the square roots of numbers in detail, let's see how we can apply this process in ac circuits. Suppose, for example, that we want to find the impedance of the circuit shown in Fig. 10. By using the formula

$$Z = \sqrt{R^2 + X^2}$$

and substituting for the values of R and X, we have:

$$Z = \sqrt{50^2 + 60^2}$$

as shown in Step 2 of Fig. 10.

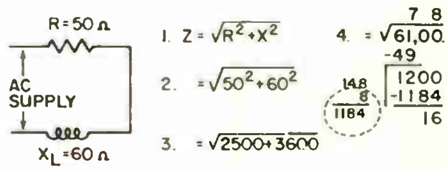


Fig. 10. Using a square root to find impedance.

In working square roots you must follow the rules of order just as in doing any other math. Therefore, the next thing you must do is to perform all the multiplication under the radical sign. This is shown in Step 3. Next add the two squares, which gives 6100. Now go ahead as shown in Step 4. This gives the circuit impedance of 78 ohms and automatically takes care of the phase relationships without having to work with vectors.

Many times, instead of working out the square root of problems accurately, it will be much easier to estimate the answers. For example, take the number 6436. You can see that 80 X 80 is equal to 6400, which gives a remainder of 36. For most purposes this would be close enough, so we would use 80 as the answer.

Likewise, with a number such as 7548, you can see that it is more than 80^2 (6400), but less than 90^2 (8100). Therefore, it would be easy to guess that 84^2 might be close, since 7500 is a little less than halfway between 6400 and 8100. If we square 84, we will find that it is equal to 7056, which is somewhat less than 7548.

When we are this far off in our estimate, the next number we would try would probably be 86 or 87. Let's square 87 and see what we get.

$$\begin{array}{r} 87 \\ \underline{87} \\ 609 \\ \underline{696} \\ 7569 \end{array}$$

As you can see, this is very close. The difference between 7569 and 7548 is only 21, and unless you require extreme accuracy, you would use 87 as the square root of 7548. By estimating in this way, you can come very close to the square root of any number with a little practice. For large numbers it is generally much easier than working out the root in detail. Try the following problems and see if you can get the answers we have shown:

Find the square root of 625.

Solution:

```
         2   5
       √6, 25
       -4
  45 |  2 25
     |-2 25
```

Find the square root of 11025.

Solution:

```
          1   0   5
        √1, 10, 25
        -1
  205 |   10 25
      | - 10 25
```

What number when multiplied by itself gives 8,094,025?

Solution:

```
           2   8   4   5
         √8, 09, 40, 25
         -4
  48   |  4 09
       | -3 84
  564  |   25 40
       | - 22 56
  5685 |    2 84 25
       | -  2 84 25
```

SELF-TEST QUESTIONS

(1) What is the name given to the symbol which is used to indicate a square root ($\sqrt{}$)?

(2) What is the impedance of an ac circuit containing, in series, a resistance of 15 ohms and a total reactance of 20 ohms?

(3) Find the square root of 2 to three decimal places.

(4) Find the square root of 25/625.

(5) What number when multiplied by itself gives 12,321?

(6) Find $\sqrt{449.44}$.

(7) An inductor with an inductive reactance of 30 ohms is connected in series with a 40 ohm resistor. What is the total impedance?

(8) Find $\sqrt{439569}$.

(9) Find $\sqrt{.000441}$.

(10) Find $\sqrt{25.1001}$.

Ratio and Proportion

There are many times in electronics work when we want to compare quantities. Sometimes we can simply say that something is either larger or smaller than something else, but usually this does not give us enough information. In electronics, as in any other scientific work, we need exact information as to sizes or quantities. For example, saying that one resistor is larger than another is not enough. We must know how much larger it is if we are going to use the resistors for anything practical.

Although we can subtract one quantity from another to find out how much larger it is, there are many times when this type of specific information is not too useful. For example, suppose we want to compare the efficiencies of two electrical circuits. Let's say that one circuit has an input of 400 watts and an output of 300 watts, while the other has an input of 568 watts and an output of 426 watts. If we subtract the input from the output in the first circuit, we find that it has a loss of 100 watts (400 watts – 300 watts). The second circuit, on the other hand, has a loss of 142 watts (568 watts – 426 watts).

By subtracting we find that the loss in the second circuit is 142 watts as compared to a loss of only 100 watts in the first circuit. However, we still don't know which circuit is the more efficient. We know that 42 watts more power is consumed in the second circuit, but since the input and output of this circuit are also larger, this does not tell us anything about its efficiency.

However, there is a method by which we can quickly and accurately compare the losses in the two circuits and determine their relative efficiencies. This is actually a form of division and is known as establishing ratios. For example, if we divide the output of the first circuit by its input and reduce the resulting fraction, we have;

$$\frac{300}{400}=\frac{3}{4}$$

This fractional value of 3/4 tells us that three-quarters of the input power appears as useful output. The remaining one-quarter is the loss in the circuit.

If we do the same thing to the second circuit, we have,

$$\frac{426}{568}=\frac{3}{4}$$

because 142 will go into the numerator 3 times and into the denominator 4 times. Thus, the ratio of the output to the input of the second circuit is exactly the same as the ratio of the output to the input of the first circuit. Their ratios are both 3/4 and, therefore, their efficiencies are the same. In other words, for every 4 watts of input we will get 3 watts of output.

By establishing ratios in this way, we can make many accurate comparisons between various quantities. In addition, through a process known as proportion, we can use an established ratio to compute circuit values much more simply and quickly than we could in any other way. In this section of the lesson, you will learn the rules for establishing ratios and how to apply the ratios in circuit computations.

ESTABLISHING RATIOS

When two quantities are compared by division, we call the ratio of the two quantities the "quotient." Thus, when we divided 426 by 568 and then reduced the resultant fraction to its lowest possible form, we had the ratio of 3/4. We can write such a ratio as the fraction 3/4, or we can use two dots as a ratio sign and write it 3:4. In either case, we say that the ratio is "3 to 4."

In establishing ratios for use in comparing quantities, we must always be sure to express the two quantities in the same units. For example, we can't compare 10 volts with 10 millivolts. We must either change the 10 volts to 10,000 millivolts or change the 10 millivolts to .010 volts before we can establish a ratio between them. Also, the quantities themselves must be of the same kind. We can't, for example, compare a volt with an ampere, or an ohm with a watt. However, there are many times when we can compare seemingly unlike quantities by changing both of them to a third quantity.

For example, suppose a motor's output is 5 horsepower and its input is 4 kilowatts. If we want to establish an output-to-input efficiency ratio for this motor, we can do it quite simply by converting our values. One horsepower is equivalent to 746 electrical watts. To simplify our calculations we will use 750 watts as an approximation. Thus, we can convert 5 horsepower to 5 X 750 or 3750 watts. We also know that 4 kilowatts is equal to 4000 watts. So, by converting both the output and the input to a common unit such as watts, we are able to establish the following ratio:

Given: Output = 5hp
Input = 4kW
Find: Efficiency

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}}$$

$$= \frac{5\text{hp}}{4\text{kW}} = \frac{5 \times 750}{4 \times 1000}$$

$$= \frac{3750}{4000} = \frac{15}{16} = 15{:}16$$

Thus, the efficiency of the motor can be expressed as the ratio of 15 to 16. If we prefer, we can change the ratio to a percentage value by dividing as follows:

```
     .9375 or 93.75%
16)15.0000
   144
     60
     48
     120
     112
       80
       80
```

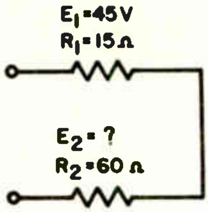
In this way, we can express a ratio as a fraction, a decimal value, or a percentage.

The rules for establishing ratios are quite simple. To find the ratio of two similar quantities:

1. Convert the quantities to the same units of measurement.
2. Form a fraction using one quantity as a numerator and the other quantity as a denominator.
3. Reduce the fraction to its lowest possible form.
4. If you wish, you can divide the denominator into the numerator to express the ratio as a decimal or as a percentage.

PROPORTION

Even though ratios are extremely useful for comparing similar quantities, they



1. $\frac{E_1}{E_2} = \frac{R_1}{R_2}$
2. $\frac{45}{E_2} = \frac{15}{60}$
3. $\frac{45}{E_2} = \frac{1}{4}$
4. $45 \times 4 = 180$
5. $\frac{45}{180} = \frac{1}{4}$
6. $\frac{1}{4} = \frac{1}{4}$

THUS, E_2 MUST EQUAL 180 VOLTS

Fig. 11. Using ratio and proportion to solve for circuit values.

are probably even more useful as a computing tool. By setting certain ratios equal to each other in what is known as a proportion, we are able to make many shortcuts in circuit calculations. For example, suppose we had a circuit such as the one shown in Fig. 11 and wanted to know the voltage across R_2. Ordinarily, in a series circuit such as this, we would first find the current through R_1 by using the formula $I = E_1 \div R_1$. Then, since the current is the same in all parts of a series circuit, we would find E_2 by multiplying I by R_2. However, by using ratio and proportion we can find either the voltage across R_2 or the total voltage in one simple operation, without ever knowing the current at all. Thus, ratio and proportion save time and work by eliminating the step of finding the current.

In order to use proportion, you have to remember one simple rule. That is, a proportion is a mathematical statement that two ratios are equal. For example, refer back to the efficiency problem in the two circuits you studied earlier in this lesson. You recall that in the first circuit we had an efficiency of

$$\frac{300}{400} = \frac{3}{4}$$

Likewise, in the second circuit the efficiency was

$$\frac{426}{568} = \frac{3}{4}$$

Thus, both of the ratios are equal and we can actually indicate this mathematically as

$$\frac{300}{400} = \frac{426}{568}$$

because, when we reduce both fractions, we have 3/4 on both sides of the equal sign. Therefore, it is a proportion because it contains two equal ratios.

Now, let's see how we can apply this type of thinking to the circuit in Fig. 11. First, let's establish a resistance ratio for the circuit. We do this by dividing R_1 by R_2 to get the ratio 15/60 or 1/4. Now, let's find the current in the circuit by dividing E_1 by R_1 and then find the voltage across E_2 by using the method we are familiar with. If we do this, we find that:

$$\begin{aligned} I &= E_1 \div R_1 \\ &= 45 \div 15 = 3 \text{ amps} \\ \text{Then, } E_2 &= R_2 \times I \\ &= 60 \times 3 \\ &= 180 \text{ volts} \end{aligned}$$

If we form another ratio from the voltages across E_1 and E_2, we will have

45/180 = 1/4. Notice that this voltage ratio is exactly the same as the resistance ratio and, therefore, the two ratios must be equal. Accordingly, we can establish a proportion with the two equal ratios by stating them mathematically as:

$$\frac{R_1}{R_2} = \frac{E_1}{E_2}$$

or

$$\frac{15}{60} = \frac{45}{180}$$

or

$$\frac{1}{4} = \frac{1}{4}$$

All three expressions are proportions and say exactly the same thing.

Suppose we knew that the resistance and voltage ratios were equal to begin with. Actually, we can easily see that they would be, because the voltage drops around a circuit must distribute themselves in accordance with the size of the resistances. If we had realized this, we could have set up our proportion as shown in Fig. 11 to begin with, and substituted all our known values as follows:

$$\frac{E_1}{E_2} = \frac{R_1}{R_2}$$

Substituting, we have:

$$\frac{45}{E_2} = \frac{15}{60}$$

This, of course, gives us what we call an "equation" that has one unknown value.

Now that we know that the two ratios are equal, we can find the value of E_2 quite simply. If we reduce the 15/60 to its lowest form of 1/4, we have

$$\frac{45}{E_2} = \frac{1}{4}$$

Since one side of our proportion will reduce to 1/4, the other side must also reduce to 1/4 so that the two sides can be equal. Thus, all we have to do is replace the denominator, E_2, in our first ratio, with a number that is equal to 4 times 45. Since 4 X 45 = 180, E_2 must be equal to 180 because 45/180 is the only fraction with 45 as the numerator that will reduce to 1/4. We must be able to reduce both sides to 1/4 in order to have a proportion.

Although this may seem a little complex at first, let's consider the following conditions which should clear it up. If 3 resistors cost 75 cents, we know that 6 resistors must cost $1.50. At the given rate, the cost of the resistors must depend only on the number bought. The more resistors we buy, the larger the cost will be. The fewer we buy, the less the cost will be. When two quantities depend on each other in this way, they are said to be in proportion.

We have already said that a proportion is a mathematical statement that says two ratios are equal. In this problem, the two quantities that make up the ratios are the number of resistors, N, and the cost, C. Since the cost depends on the number of resistors bought, we can write a proportion.

First Purchase	Second Purchase
N_1 = 3 resistors	N_2 = 6 resistors
C_1 = \$.75	C_2 = \$1.50

The ratio of the number of resistors is:

$$\frac{N_1}{N_2} = \frac{3}{6} = \frac{1}{2}$$

The ratio of the cost is:

$$\frac{C_1}{C_2} = \frac{75}{150} = \frac{1}{2}$$

Since the two ratios are equal, the proportion is written mathematically as:

$$\frac{N_1}{N_2} = \frac{C_1}{C_2}$$

or

$$\frac{3}{6} = \frac{75}{150}$$

or

$$\frac{1}{2} = \frac{1}{2}$$

This is an example of a direct proportion. When two quantities depend on each other so that one increases as the other increases, or one decreases as the other decreases, they are said to be directly proportional. This is what we mean when we say that the current through a fixed resistance is directly proportional to the voltage applied. As the voltage increases, the current increases, etc.

Notice that when we set up two equal ratios as a direct proportion, we must compare the two ratios in the same order. In other words, we used the quantities in the first situation, N_1 and C_1, as numerators, and the quantities in the second situation, N_2 and C_2, as denominators. Thus, they are in the same order because the second ratio, C_1/C_2, is patterned after the first, N_1/N_2. However, we could have written the proportion in the opposite form. For example,

$$\frac{C_1}{C_2} = \frac{N_1}{N_2}$$

or

$$\frac{C_2}{C_1} = \frac{N_2}{N_1}$$

or

$$\frac{N_2}{N_1} = \frac{C_2}{C_1}$$

As you can see, it is not important which ratio is written first or how it is written. However, in a direct proportion, the second ratio must always be written in the same order as the first.

Thus, the rules for setting up a direct proportion for two variables that depend on each other are:

1. Make a ratio of either one of the variables.
2. Make a ratio of the other variable in the same order.
3. Make the two ratios equal to each other.

SOLVING PROPORTIONS

In the first example of a proportion that we solved, we reduced the completed ratio to its smallest possible form. Then we found a number for the unknown in the incomplete ratio that would allow it to be reduced to the same form. While this is actually what we must do in order to find the solution for any proportion, there is a shortcut which we can use that makes it much easier. This shortcut is called "cross-multiplication" and we will learn later how it can be used. It always works and we should learn how to use it.

For example, suppose we have a length of cable that is 78 ft. long and another length of the same kind that is 4 ft. long. We want to know how much the longer piece of cable weighs, but because it is bulky and hard to handle it will be difficult to weigh it. In this case, we could weigh the smaller length of cable and then set up a proportion and find the weight of the longer piece.

Let's say that the shorter piece weighs ten pounds. We would set up the proportion as follows:

$$\frac{L_L}{L_S} = \frac{W_L}{W_S}$$

L_L = length of longer cable
L_S = length of shorter cable
W_L = weight of longer cable
W_S = weight of shorter cable

Now, by substituting values, we have:

$$\frac{L_L}{L_S} = \frac{W_L}{W_S}$$

$$\frac{78}{4} = \frac{W_L}{10}$$

To apply our shortcut using cross-multiplication, we multiply the numerator of one fraction by the denominator of the other as follows:

$$\begin{aligned} 4 \times W_L &= 78 \times 10 \\ 4W_L &= 780 \end{aligned}$$

Now we have a familiar equation form to work with and we know that if

$$\begin{aligned} 4W_L &= 780\text{, then} \\ W_L &= 780 \div 4 = 195 \end{aligned}$$

Therefore, the weight of the large, bulky piece of cable is 195 lbs.

Another example of the same problem in a slightly different situation might be quite common in your work in electronics. Suppose the longer piece of cable were wound on a reel and you wanted to know how long it was without unwinding it. Since it was wound on a reel, it would be easy to handle and we could weigh it quite easily. Then, by weighing the shorter piece, and setting up the proportion using the two weights and the length of the short piece, we could find the length of the long piece. Suppose the cable on the drum weighed 425 pounds, while the short piece was 4 ft. long and weighed ten pounds. First, we would have to subtract the weight of the reel itself, say 25 pounds, and then set up the proportion:

$$\frac{L_L}{L_S} = \frac{W_L}{W_S}$$

$$\frac{L_L}{4} = \frac{(425 - 25)}{10}$$

$$\frac{L_L}{4} = \frac{400}{10}$$

By cross-multiplying, we have:

$$\begin{aligned} 10 \times L_L &= 4 \times 400 \\ 10\,L_L &= 1600 \\ L_L &= 1600 \div 10 \\ L_L &= 160\text{ ft.} \end{aligned}$$

The steps for solving any direct proportion are always the same.

1. Set up the direct proportion:

$$\frac{X_1}{X_2} = \frac{Y_1}{Y_2}$$

2. Substitute numbers where possible:

$$\frac{10}{100} = \frac{50}{X}$$

3. Cross-multiply:

$$10 \times X = 50 \times 100.$$

4. Simplify: $10X = 5000$.

5. Solve for the unknown by dividing:

$$X = 5000 \div 10$$

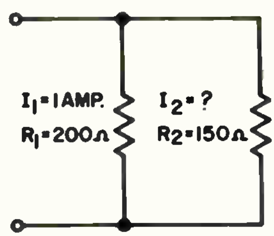
6. Answer: $X = 500$.

Inverse Proportion. So far in our discussion of proportion, we have considered only what happens when two quantities vary directly. Many times two quantities depend on each other, but instead of varying directly they do just the opposite. When one increases, the other must decrease an appropriate amount. When this occurs, we say the two quantities vary indirectly or "inversely" and that they are inversely proportional. The current and resistance in an electrical circuit, with a fixed voltage, is a good example of an inverse proportion. As the resistance increases, the current decreases.

We can set up inverse proportions mathematically just as we do direct proportions except for one major difference. In an inverse proportion, we always set up the second ratio in the opposite or inverse order. For example, consider the parallel circuit shown in Fig. 12.

In this circuit we are given the values of the two resistances and the current through one of them. We could find the current, I_2, by finding the voltage across R_1, and since the voltage across R_2 would be the same we could find I_2 by using this voltage. However, we can solve this problem in a much simpler manner. There is just one thing to remember. The current and the resistance vary inversely and, therefore, we must use an inverse proportion. To do this, we set up the first ratio. It doesn't make any difference which one we use first or how we set it up, as long as we set up the next one in the opposite manner.

For example, in the circuit of Fig. 12, let's use the current ratio first as I_1/I_2. Then, the resistance ratio in the reverse order R_2/R_1, and then set them equal to each other as follows.



1. $\frac{I_1}{I_2} = \frac{R_2}{R_1}$

2. $\frac{1}{I_2} = \frac{150}{200}$

3. $150 \times I_2 = 200 \times 1$

4. $150\, I_2 = 200$

5. $I_2 = 200 \div 150$

6. $I_2 = 1.33$ AMPS

Fig. 12. Using an inverse proportion to solve for current.

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}$$

Substituting, we get:

$$\frac{1}{I_2} = \frac{150}{200}$$

$$\begin{aligned} 150 \times I_2 &= 200 \times 1 \\ 150\, I_2 &= 200 \\ I_2 &= 200 \div 150 \\ I_2 &= 1.33 \text{ amps.} \end{aligned}$$

As you can see, the solution is obtained in exactly the same manner as in a direct proportion. The only difference is that we reversed the order of the second ratio from that of the first when we set up the ratios.

You will recall that when we first discussed establishing ratios we mentioned that a ratio could be written as 5:2 as well as 5/2. We can also write proportions in two ways, depending upon which way we indicate our ratios. For example, the proportion

$$\frac{10}{20} = \frac{50}{100}$$

would be written as 10:20 :: 50:100, using the two dots as ratio signs and the four dots to indicate proportion. When a ratio is written in this way, there is no cross-multiplication indicated, as there is in the fractional form, and we use a different way of indicating the solution.

In the form 10:20 :: 50:100, we give a name to both parts of the proportion. We call the two outside numbers, 10 and 100, the "extremes" of the proportion. The two inside numbers, 20 and 50, are called the "means" of the proportion. Now, we say that the product of the means is equal to the product of the extremes. In other words, the product of the two outside numbers is equal to the product of the two inside numbers. As you can see, remembering this and using it with this form of the proportion gives us exactly the same thing as cross-multiplying a proportion that is written in the fractional form.

$$\frac{10}{20} = \frac{50}{100}$$

$$20 \times 50 = 10 \times 100$$

or

$$10:20 :: 50:100$$

$$20 \times 50 = 10 \times 100$$

Using ratios in proportion to solve for unknown quantities is one of the handiest tools in mathematics. It is not only easy to use and work with, but it is easy to remember the rules. No matter where you go or what you do, you can almost always find some use for ratio and proportion in solving problems. You will be very pleased with the amount of work it can save you.

SELF-TEST QUESTIONS

(11) Explain the difference between a ratio and a proportion.
(12) A 2-horsepower motor requires 1.75 kilowatts of input power. What is the efficiency of the motor?
(13) Find the ratio of 6V to 18V.
(14) What is the ratio of 250 millivolts to 1 volt?
(15) Find the efficiency of a 3-horsepower motor which requires 2.5 kilowatts of input power.

(16) A is directly proportional to B. $A_1 = 15$, $A_2 = 6$, $B_1 = 90$; find B_2.

(17) A is inversely proportional to B. $A_2 = 4$, $B_1 = 100$, $B_2 = 150$; find A_1.

(18) R_1 and R_2 are connected in series across a battery. R_1 is a 1200-ohm resistor and drops 10 volts. R_2 is a 240-ohm resistor. What is the applied battery voltage?

(19) R_1 and R_2 are connected in parallel. R_1 is a 66-ohm resistor passing a current of 2 amps. R_2 passes a current of 2.2 amps. What is the value of R_2?

Positive and Negative Numbers

Many of the calculations, graphs, and tables that are used to solve problems in both ac and dc circuits require an understanding of positive and negative numbers. These numbers are commonly called "signed" numbers and are used to indicate opposite amounts. These opposite amounts might be some of the following: gain or loss in voltage, increase or decrease in volume, currents that flow in opposite directions, capacitive or inductive reactance, etc.

In our everyday lives we commonly indicate opposite quantities by various pairs of words, such as north and south, up and down, and gain or loss. In electronics, we use opposite quantities so often that it is much easier to indicate them by using plus (+) and minus (−) signs. For example, 5° above zero is written as +5°, and 5° below zero is written as −5°. A current in one direction would be +10 amps, but a current in the opposite direction would be −10 amps.

Numbers preceded by a minus sign are called negative numbers. A positive number is indicated by a plus sign. However, many times a positive number will not have any sign at all. Thus, a number with no sign is always considered a positive number. A number is never considered to be negative unless it has a minus sign. Generally, increases and gains, and directions to the right and upward are considered to be positive (+). Losses and decreases, directions to the left and downward are considered negative (−).

Since your work in electronics involves signed numbers so often, you must be very familiar with them. You will have to be able to add and subtract them from each other as well as be familiar with multiplying and dividing them. In this section of the lesson you will learn how to perform these basic operations with signed numbers.

ADDING SIGNED NUMBERS

Probably the best way to understand positive and negative numbers is to represent them on a graph as shown in Fig. 13.

Fig. 13. Graph showing arrangement of positive and negative numbers.

As you can see, there is a reference point or zero mark at the center of the scale with the positive numbers extending to the right and the negative numbers to the left. A scale of numbers like this is very handy for showing both addition and subtraction of signed numbers. For example, if we want to add +3 and +5, we start at zero and count three numbers to the right which will bring us to +3. Then, we start at +3 and count five more numbers to the right, which brings us to +8, as shown by the arrows in Fig. 14.

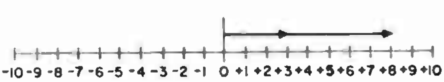


Fig. 14. Adding +3 and +5 graphically.

Thus, adding +3 and +5 gives us +8 which is the same as the addition we studied in basic arithmetic. Likewise, suppose we want to add −4 and −3. We can also do this graphically as shown by the arrows in Fig. 15. Notice that we first count four units to the left of the zero because all negative numbers increase in size as we move toward the left. This brings us to −4. Then we start from −4 and count three more units to the left which brings us to −7. Thus, the sum of −7 is obtained by adding −4 and −3.

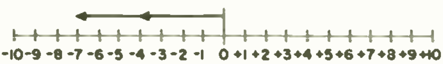


Fig. 15. Adding −4 and −3 graphically.

This brings us to the first rule in dealing with signed numbers. When two or more numbers have the same sign (all positive or all negative), they are said to have "like signs."

The first rule for addition of signed numbers is: ***To add two or more numbers with like signs, find the sum of the numbers as you would in ordinary arithmetic and place the sign of the numbers added in front of this sum.*** Thus, the sum of −3, −5, and −7 would be −15; and the sum of +6, +8, +9, and +2 would be +25.

However, we will also be dealing with both positive and negative numbers at the same time. For example, suppose we have to find the sum of +2 and −5. We know how to add two and five when both numbers have the same sign, but here each has a different sign. How will we handle it?

To begin with, let's start with a graph as we did for numbers with like signs. First, we start at the zero reference point and count two units to the right which brings us to +2, as shown by the short arrow in Fig. 16. This takes care of the +2 in our addition and now we can consider the −5. We know that in order to arrive at −5 we would normally start at zero and count five units to the left. However, we are at +2 and must start at +2 instead of zero when we begin to add our −5. Therefore, instead of starting at the zero reference and counting five units to the left, we start at +2 and count five units to the left, as shown by the long arrow in Fig. 16.

This brings us to −3 on the graph. Accordingly, since we have added two arrows, one which is +2 units long and the other −5 units long, and arrived at −3, it follows that the sum of +2 and −5 must be −3. If we look at this addition of two numbers with "unlike" signs closely, we can see that we have actually found the difference between the two numbers (5 − 2 = 3) and then used the sign of the largest number (−5) in front of our answer (−3).

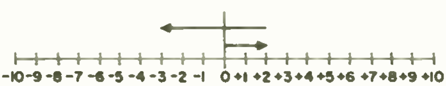


Fig. 16. Adding numbers with unlike signs graphically.

The second rule for addition of signed numbers is: ***The sum of two signed numbers with unlike signs is equal to the difference between the two numbers, preceded by the sign of the largest number.*** Remember that although we find the difference of the two numbers, it is not subtraction. It is the addition of numbers with unlike signs.

Here are a few examples of the addition of signed numbers following the two rules which we have learned. Try them and see if you get the same answers that we do.

-7	-9	$+10$	-63	$+1/2$
-8	$+7$	-6	$+46$	$-1/4$
-15	-2	$+4$	-17	$+1/4$

$-.25$	$+5$	$+6$	-20
$+.05$	$+6$	-5	$+18$
$-.20$	$+11$	$+1$	-2

SUBTRACTING SIGNED NUMBERS

When you studied basic subtraction in earlier lessons, you asked, "What number must be added to a given number to give another given number?" In other words, in subtracting 5 from 9 you asked, "What number added to 5 will give 9?" The answer is 4, because 4 + 5 is equal to 9. In subtracting signed numbers you do exactly the same, but you have to be very careful to watch the signs.

For example, consider the problem of subtracting -4 from -9. In order to do this, we ask what number added to -4 will give us -9. Of course, there is only one number and that is -5, because $-4 + (-5)$ is equal to -9. Therefore, -4 from -9 is equal to -5.

We can also illustrate this graphically as shown in Fig. 17. First, we draw an arrow from the zero point to -9 as shown by the long arrow. Then, we draw an arrow from zero to -4 to represent the value (-4) that we are subtracting, as shown by the short arrow. Now, if we count the units between -4 and -9 as shown by the dotted arrow, we can see that we have five units to the left which means that -5 is our answer.

Now that we know what happens when we subtract a positive number from a positive number, $(+9) - (+5) = +4$, and also what happens when a negative number is subtracted from a negative number, $(-9) - (-4) = -5$, let's consider the subtraction of numbers with unlike signs. Suppose that we want to subtract -3 from $+6$. First, we ask ourselves, "What number added to -3 will give us $+6$?" If we look at this closely, we will see that $+9$ is the only number that can be added to -3 to give us $+6$ because,

$$(+9) + (-3) = +6$$

Therefore, $(+6) - (-3)$ must be equal to $+9$.

Fig. 18 shows this graphically. First, we draw an arrow from zero to $+6$ to represent $+6$. Then, we draw an arrow from zero to -3 to represent -3. Now, we start at the -3 and count towards the right to see how many units would have to be added to -3 to give us $+6$. If we count off these units, we will find that there are nine of them. Since we move from left to right, the 9 must be $+9$. Thus, we can prove that

$$(+6) - (-3) = +9$$

Suppose, however, that we have the same numbers with the signs reversed. What is $(-6) - (+3)$ equal to? Once again,

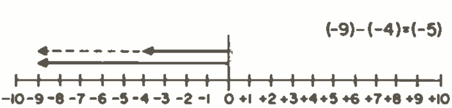


Fig. 17. Subtracting signed numbers graphically.

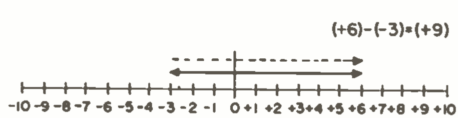


Fig. 18. Subtracting numbers with unlike signs.

we ask ourselves, "What number added to +3 will give −6? Of course +3 plus −9 is the only answer. Therefore, (−6) − (+3) must be equal to −9. Graphically, we can show this in Fig. 19. We draw an arrow from zero to −6. Then we draw an arrow from zero to +3. Now, we can see that −6 is nine units to the left of +3, as shown by the dotted arrow. Since we go to the left, this 9 must be −9.

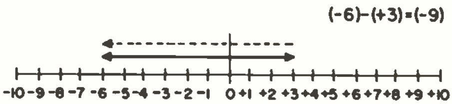


Fig. 19. Subtracting +3 from −6 graphically.

Now look carefully at the examples that we have just discussed:

$$\begin{array}{r} +9 \\ \underline{-\,(+5)} \\ +4 \end{array} \quad \begin{array}{r} -9 \\ \underline{-\,(-4)} \\ -5 \end{array} \quad \begin{array}{r} +6 \\ \underline{-\,(-3)} \\ +9 \end{array} \quad \begin{array}{r} -6 \\ \underline{-\,(+3)} \\ -9 \end{array}$$

Notice that they all have one thing in common. That is if we change the sign in the subtrahend (the number subtracted), and then add the two numbers, we will get the proper answer. For example, consider the problem of:

$$\begin{array}{r} -9 \\ \underline{-\,(-4)} \\ -5 \end{array}$$

If we set this up and change the −4 to a +4 as

$$\begin{array}{r} -9 \\ \underline{+4} \\ \end{array}$$

and then add, we will get −5 for the answer.

The rule for subtracting signed numbers is: ***To subtract signed numbers, change the sign of the number you wish to subtract (subtrahend) and then add the two numbers.*** This rule will work for subtracting any two signed numbers no matter how small or large they may be. We have given some examples for you to try. Notice that the numbers and their signs are enclosed in parentheses so they will not be confused with the subtraction (−) sign.

1. (−25) − (−15)
   = (−25) + (+15) = −10

2. (−18) − (+6)
   = (−18) + (−6) = −24

3. (+29) − (+7)
   = (+29) + (−7) = +22

4. $\left(-\dfrac{1}{8}\right) - \left(+\dfrac{1}{16}\right)$
   $= \left(-\dfrac{2}{16}\right) + \left(-\dfrac{1}{16}\right) = -\dfrac{3}{16}$

5. (+.36) − (−.05)
   = (+.36) + (+.05) = +.41

MULTIPLYING SIGNED NUMBERS

You learned that multiplication is the addition of a number to itself an indicated number of times. Thus, 5 X 6 tells us to either add 6 to itself five times or to add 5 to itself six times. The multiplication of positive and negative numbers is just the same, except that we must consider what to do about the signs. In order to do this, let's consider all the possible combinations of signs that we might have in multiplying two numbers.

There are only four possible combinations and they are as follows:

(1) $(+2) \times (+3) = ?$
(2) $(-2) \times (+3) = ?$
(3) $(+2) \times (-3) = ?$
(4) $(-2) \times (-3) = ?$

Now, you already know that the first situation is the same as saying that +2 is to be added three times, or:

$$(+2) + (+2) + (+2) = +6$$

Therefore,

$$(+2) \times (+3) = +6.$$

In the same manner, the second situation simply states that -2 is to be added together three times, or:

$$(-2) + (-2) + (-2) = -6$$

Therefore,

$$(-2) \times (+3) = -6.$$

From what we have seen so far, multiplying two positive numbers together gives us the product of the two numbers preceded by a plus sign. Also, in a similar manner, the product of a negative number multiplied by a positive number is the product of the two numbers preceded by a minus sign. Thus, we have taken care of the first two situations.

The third situation says that we must add +2 to itself -3 times. If we stop to consider this for a minute, we can see that if we add a number to itself a minus number of times, it will be the same as subtracting the +2 from zero three separate times. Therefore, $(+2) \times (-3)$ is the same as $-(+2) - (+2) - (+2)$. If we change the signs and add as we do in any problem of subtracting signed numbers, we have:

$$(-2) + (-2) + (-2) = -6.$$

Thus, $(+2) \times (-3)$ must be equal to -6. This is the same answer we got for the second situation which was

$$(-2) \times (+3) = -6.$$

This is right because you learned that the order in which the numbers are arranged does not make any difference in multiplication. Thus, we can now say that the product of any two numbers with unlike signs is always negative.

Now, let's look at the fourth situation. Here we have $(-2) \times (-3)$, which is the same as saying -2 added to itself -3 times. Once again, adding a number to itself a minus number of times must be the same as subtraction. Therefore, we can write it as

$$-(-2) - (-2) - (-2) = ?$$

However, once again we are subtracting signed numbers and we must change the signs and add. Consequently, we would rewrite the problem

$$(+2) + (+2) + (+2) = ?$$

which of course equals +6. From this, we can say that

$$(-2) \times (-3) = +6$$

and, accordingly, the product of any two negative numbers is always positive.

Reviewing all that we have just discussed, we find that there are two simple rules for the multiplication of two signed numbers:

1. The product of any two numbers with like signs is always positive.

2. The product of any two numbers with unlike signs is always negative.

With these two rules you can handle the multiplication of any two signed numbers.

Sometimes you may have more than two signed numbers to multiply together. In such a case, you simply take the numbers two at a time and then multiply the product by the next number. For example:

$$(-2) \times (+3) \times (-4) = ?$$

First take $(-2) \times (+3)$, which equals -6, then the product $(-6) \times (-4) = +24$. Therefore, $(-2) \times (+3) \times (-4) = +24$.

Another example is:

$$(+2) \times (-5) \times (-7) \times (+11) = ?$$

First, $(+2) \times (-5) = (-10)$
Then, $(-10) \times (-7) \times (+11) = ?$
Now, $(-10) \times (-7) = +70$
Then, $(+70) \times (+11) = +770$

DIVIDING SIGNED NUMBERS

Because division is just the reverse of multiplication, you should not have any trouble learning to divide signed numbers. Remember, when you divide one number by another you ask, "What number, when multiplied by the divisor (the number you divide by), will equal the dividend?" In other words, if you wish to divide 30 by 5 you say, "Since 5×6 is equal to 30, then $30 \div 5$ must equal 6."

In dividing signed numbers you do the same thing, except that you must be careful to obtain the proper sign for the quotient. Once again, let's consider all the possible combinations of signs that we might have in dividing one number by another. There can be only four combinations as follows:

(1) $(+30) \div (+6) = ?$
(2) $(-30) \div (+6) = ?$
(3) $(+30) \div (-6) = ?$
(4) $(-30) \div (-6) = ?$

Because division is the opposite of multiplication, we must have the following:

(1) $(+30) \div (+6) = +5$ because $(+5) \times (+6) = +30$
(2) $(-30) \div (+6) = -5$ because $(-5) \times (+6) = -30$
(3) $(+30) \div (-6) = -5$ because $(-5) \times (-6) = +30$
(4) $(-30) \div (-6) = +5$ because $(+5) \times (-6) = -30$

Therefore, our two rules for division of signed numbers are as follows:

1. If both numbers have like signs, the quotient is always positive.

2. If the numbers have unlike signs, the quotient is always negative.

This is all you need to know in order to handle the division of any two signed numbers.

EXPONENTS AND ROOTS

Let's consider the effect of signs on numbers with exponents or roots. You have already learned that the square of a number is the product of a number that is multiplied by itself. You also learned that you can use a small number "2" written above and to the right of the number to indicate that it is to be squared. Thus, 13^2 means 13×13, which is 169. We call the small "2" above the 13 an exponent. When we use 2 as an exponent to indicate the operation of squaring a number, we sometimes say that it means raising the number to its second power.

Just as we can raise a number to its second power by multiplying it by itself, we can also raise numbers to other powers. For example, 13^3 means that the number is to be raised to its third power, or 13 X 13 X 13, which equals 169 X 13 or 2197. In this case, the exponent is a 3 and indicates that the number must be raised to its third power. In the case of the third power of a number, we have a special name for the operation just as we do for the second power (square). For the third power, we call it "cubing" a number, or finding the cube of a number.

Of course, we can raise a number to any power that we desire, simply by multiplying it by itself the proper number of times. In each case, the exponent indicates the power to which the number must be raised. Thus, 23^4 means 23 X 23 X 23 X 23, and 17^6 means 17 X 17 X 17 X 17 X 17 X 17. However, beyond the third power (cube) we have no special names because the operation is not common enough. We simply say the "fourth power of the number" or the "sixth power" or whatever power the exponent may indicate.

If a number has a sign in front of it, we proceed just as we would in multiplying any series of signed numbers. For example, $(-3)^2$ would be

$$(-3) \times (-3) = +9$$

and $(-3)^3$ would be

$$\begin{aligned} & (-3) \times (-3) \times (-3) \\ = & (+9) \times (-3) \\ = & -27 \end{aligned}$$

Likewise, $(-2)^4$ would be

$$\begin{aligned} & (-2) \times (-2) \times (-2) \times (-2) \\ = & (+4) \times (-2) \times (-2) \\ = & (-8) \times (-2) \\ = & +16 \end{aligned}$$

It is interesting to notice that a negative number squared always gives a positive product, while a negative number cubed always gives a negative product. This is true because any two negative numbers multiplied always give a positive product and any three negative numbers multiplied always give a negative product. With a negative number, any even-numbered exponent, such as 4, 8, 28, or 32, always gives a positive product and any odd-numbered exponent gives a negative product.

Just as every number can be raised to any power, every number has an infinite number of roots. You learned earlier in this lesson what is meant by the square root of a number and that this operation is indicated by the use of the radical sign $\sqrt{}$. Therefore, if 13^2 = 13 X 13 or 169, then $\sqrt{169}$ is equal to 13. You also learned that $-13^2 = (-13) \times (-13)$ which is also 169. Therefore, we have two possible square roots of 169, either +13 or −13. In fact, any positive square has two possible square roots: a positive root or a negative root. If there is any question as to the sign of a square root, we write the root and use both signs. Thus, we would indicate the square root of 169 as ±13. We read this as "plus or minus 13."

The symbol for a square root is the radical sign $\sqrt{}$. The same basic sign is used to indicate the root of any number, except that we use an "index" number in the notch of the sign to indicate that particular root. Thus, $\sqrt[3]{27}$ means the cube root of 27. We find that it is 3, because 3 X 3 X 3 equals 27. Likewise, $\sqrt[4]{16}$ means that we are to find the fourth root of 16 which is 2, because 2 X 2 X 2 X 2 equals 16. Of course the square or second root of any number should be indicated with an index of 2 in the radical sign as $\sqrt[2]{4}$. However, in square roots it is common practice not to write 2 as the index so we just use the radical sign by itself. Thus, a radical sign with no index always indicates a square or second root and any other desired root must be indicated by using the proper index.

Just as the root of any positive square may be either positive or negative, the even-numbered roots of any number raised to a power may also be either positive or negative. In other words, $\sqrt[4]{16}$ may be either ± because either -2^4 or $+2^4$ is equal to +16. Cube roots, however, or any other odd-numbered roots will be positive or negative depending on the sign of the number. Thus, $\sqrt[3]{27}$ must be +3 because $(+3) \times (+3) \times (+3)$ equals 27. Therefore, $(-3) \times (-3) \times (-3)$ can never be equal to +27; it will always equal −27. So $\sqrt[3]{-27}$ equals −3. The cube root of any negative number must be equal to a negative number. Other odd-numbered roots follow the same rule. For example, $\sqrt[5]{32}$ always equals +2 because;

$$(+2) \times (+2) \times (+2) \times (+2) \times (+2) = +32$$

while

$$(-2) \times (-2) \times (-2) \times (-2) \times (-2) = -32$$

You will notice that we have not mentioned how to find the square root of a negative number. This is because it is impossible. As far as we know, there is no number multiplied by itself which can give a negative square. A negative number squared is always positive, and a positive number squared is always positive. The product $+2 \times (-2)$ is not a square because, if you notice the signs, we are not multiplying the same number by itself. We will learn more about this later in the course.

MULTIPLYING AND DIVIDING BY POWERS OF TEN

One of the greatest advantages of the decimal system is that in multiplying or dividing by 10 we simply move the decimal point. For example, if we multiply 237 by 10 the answer is 2370 because we move the imaginary decimal point, which is after the seven, one place to the right. To divide by 10, we move the decimal point one place to the left. Thus, 237 ÷ 10 is 23.7; in other words 10 goes into 237 twenty-three and 7/10 times.

You know that 2×2 is often written 2^2. Also, $2 \times 2 \times 2$ is written 2^3. Similarly 10×10 is written 10^2. Also, $10 \times 10 \times 10 = 10^3$, and $10 \times 10 \times 10 \times 10 = 10^4$ and so on. Thus, summarizing:

$10^0 = 1$
$10^1 = 10$
$10^2 = 10 \times 10 = 100$
$10^3 = 10 \times 10 \times 10 = 1000$
$10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000$
$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100{,}000$
$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1{,}000{,}000$

Why $10^0 = 1$ will be explained later in this section.

To multiply 237 by 10 we move the decimal point one place to the right. To multiply by 100, or 10^2, we move the decimal point two places to the right. To multiply by 1000, or 10^3, we move the decimal point three places to the right, etc.

To divide, we do the opposite. To divide by 100, or 10^2 we move the decimal point two places to the left, etc. Thus:

$237 \times 10 = 2370$
$237 \times 100 = 237 \times 10^2 = 23700$
$237 \times 1000 = 237 \times 10^3 = 237000$
$237 \div 10 = 23.7$
$237 \div 100 = 237 \div 10^2 = 2.37$
$237 \div 1000 = 237 \div 10^3 = .237$

Any number can be expressed in terms of a number between 1 and 10 times a power of 10. For example, 237 can be written as $2.37 \times 10 \times 10$. Rather than write the number out this way we usually write it 2.37×10^2. Similarly, 7648 can be written 7.648×10^3. Also, you can write 93486 as 9.3486×10^4.

Now consider the problem of multiplying 237×393. We rewrite 237 as 2.37×10^2, and 393 as 3.93×10^2. This breaks the problem down to:

$$2.37 \times 10^2 \times 3.93 \times 10^2$$

which we can regroup as:

$$2.37 \times 3.93 \times 10^2 \times 10^2$$

When two powers of 10 are multiplied together, you perform the multiplication simply by adding the exponents. In the expression 10^2, the exponent is 2. In 10^3 the exponent is 3. To multiply $10^2 \times 10^3$, you simply add the exponents and the problem becomes:

$$10^2 \times 10^3 = 10^{2+3} = 10^5$$

To multiply $10^3 \times 10^5$ proceed the same way:

$$10^3 \times 10^5 = 10^{3+5} = 10^8$$

If this seems somewhat confusing to you, or if you doubt that $10^3 \times 10^5 = 10^8$, you can write out 10^3 as $10 \times 10 \times 10$, and write out 10^5 as $10 \times 10 \times 10 \times 10 \times 10$, and then multiply them together, and you'll find that the answer is 10^8.

Now to get the rest of our answer we have to determine the value of $10^2 \times 10^2$. The term 10^2 is equal to 10×10, and therefore:

$$10^2 \times 10^2 = 10 \times 10 \times 10 \times 10$$

Since there are four tens, then $10^2 \times 10^2 = 10^4$ and our complete answer is:

$$9.31 \times 10^4$$

Another example is 767×839. Again, we write our factors in powers of 10, and so the problem becomes:

$$7.67 \times 8.39 \times 10^2 \times 10^2$$

We know from our previous problem that $10^2 \times 10^2$ is 10^4, so immediately it becomes:

$$7.67 \times 8.39 \times 10^4$$

Thus, our answer becomes:

$$64.4 \times 10^4$$

We can leave the answer in this form, but the usual procedure is to write the answer in terms of a number between 1 and 10. We can do this by writing our answer as:

$$6.44 \times 10^1 \times 10^4$$

There is no point in leaving the answer like this, because it is more complex than it need be, so we simply multiply $10^1 \times 10^4$ by adding the exponents to get:

$$6.44 \times 10^5$$

Multiplying Negative Numbers by Powers of Ten. When 347 is a positive number it can be written as:

$$347 = 3.47 \times 10^2$$

But what about −347? How would you write it? You might think you would write it as -3.47×-10^2. But if you go back to our rules of multiplication you'll see that $-10^2 = -10 \times -10 = 100$. Thus, there's no point in writing the minus sign in front of the ten unless we write it as $-(10)^2$. This means $-(10 \times 10) = -100$. Is this what we want? Let's look and see:

$$-3.47 \times -100 = 347$$

Obviously, we do not want to write our power of ten as $-(10)^2$. However,

$$-3.47 \times 100 = -347$$

so we can write −347 as -3.47×10^2. Similarly −47 is -4.7×10, and −5762 is -5.762×10^3. A negative number in powers of ten is written as a negative number between 1 and 10 times a power of ten.

The problem 347×-162 is written in powers of ten as:

$$3.47 \times 10^2 \times (-1.62) \times 10^2$$

The problem -114×-262 can be handled in the same way:

$$\begin{aligned} &-114 \times -262 \\ &= -1.14 \times 10^2 \times (-2.62) \times 10^2 \\ &= 2.99 \times 10^4 \end{aligned}$$

The 2.99 is positive, because a negative number times a negative number gives a positive product.

Division by Powers of Ten. You might wonder how we handle decimal numbers using this system. The number .147 is equal to $1.47 \div 10$. However, rather than leaving it in that form we write the number 1.47×10^{-1}. Notice the minus sign in front of the exponent 1. This indicates the number is divided by 10, or that the decimal point has been moved to the right one place. We write the number .0756 as 7.56×10^{-2}. This indicates that 7.56 is divided by 10^2 or 100.

Consider the problem of multiplying $.342 \times .266$. We rewrite the problem as:

$$3.42 \times 2.66 \times 10^{-1} \times 10^{-1}$$

The product 3.42×2.66 is approximately 9.10. Now to handle $10^{-1} \times 10^{-1}$, we simply add the exponents, and get 10^{-2}. Thus, our answer is 9.10×10^{-2}. If we want to express the number as a decimal number we simply divide it by 100, and we do this by moving the decimal point two places to the left. Notice that the exponent tells you how many places to move the decimal point. Thus,

$$9.10 \times 10^{-2} = .0910$$

Now consider the problem $.0187 \times -475$. We can write these numbers as powers of ten as follows:

$$1.87 \times 10^{-2} \times -4.75 \times 10^2$$

Notice in one case we have a negative exponent, in the other a negative number. There is no connection between the minus signs in front of the exponent and the number. They mean different things. The minus sign in front of the exponent means the number is divided by 10^2, but the entire number 1.87×10^{-2} is positive. The minus sign in front of 4.75

means the entire number, -4.75×10^2, is negative. Thus $1.87 \times 10^{-2} \times -4.75 \times 10^2$ is handled as two separate problems.

$$1.87 \times -4.75 = -8.88$$

and

$$10^{-2} \times 10^2 = 10^0 = 1$$

Therefore, our answer is -8.88.

You may wonder about 10^0 being equal to 1. You may think it should be zero. Let's see why it isn't.

We said earlier that when a number is multiplied by ten with a negative exponent, it indicates the number is to be divided by that power of ten. Therefore, $10^{-2} \times 10^2$ could be written as $10^2/10^2$, which equals 1. Also, $10^{-2} \times 10^2$ can be written as 10^0, because to multiply by powers of ten we add the exponents, and adding a +2 and a −2 gives us zero. Therefore, $10^0 = 1$. In fact, in the same way you can prove that any number to the zero power is equal to 1. Thus, $2^0 = 1$, $3^0 = 1$, $500^0 = 1$, $5{,}000{,}000^0 = 1$, and so on.

For now, we have all we need to know about signed number and their exponents and roots.

SELF-TEST QUESTIONS

(20) What is the rule for the addition of numbers with like signs?

(21) What is the rule for the addition of numbers with unlike signs?

(22) What is the rule for subtracting signed numbers?

(23) Add the following numbers:

−72, +13, −12, +57, +17, −6.

(24) Add the following:

(a) $\begin{array}{r} -16 \\ -\ 7 \\ \hline \end{array}$ (b) $\begin{array}{r} +13 \\ -\ 4 \\ \hline \end{array}$

(c) $\begin{array}{r} -22 \\ +48 \\ \hline \end{array}$ (d) $\begin{array}{r} -34 \\ +12 \\ \hline \end{array}$

(25) Subtract the following:

(a) $\begin{array}{r} -16 \\ -\ 7 \\ \hline \end{array}$ (b) $\begin{array}{r} +13 \\ -\ 4 \\ \hline \end{array}$

(c) $\begin{array}{r} -22 \\ +48 \\ \hline \end{array}$ (d) $\begin{array}{r} -34 \\ +12 \\ \hline \end{array}$

(26) State two rules which are used to determine the sign of the product when multiplying two signed numbers.

(27) State two rules used for determining the sign of the quotient when dividing signed numbers.

(28) Multiply:

(a) −6 by −7
(b) +9 by −2
(c) −11 by +17
(d) +12 by +3

(29) Divide:

(a) +60 by −15
(b) −144 by −6
(c) −153 by +51
(d) +516 by +12

(30) If −131 were raised to the sixth power, would the result be a positive or a negative number?

(31) Add the following:

(a) $\begin{array}{r} -\ 4 \\ -11 \\ +\ 7 \\ \hline \end{array}$ (b) $\begin{array}{r} +17 \\ -\ 6 \\ +13 \\ \hline \end{array}$

(c) $\begin{array}{r} -5 \\ -9 \\ \hline -13 \end{array}$

(d) $\begin{array}{r} +21 \\ -41 \\ \hline +20 \end{array}$

(32) Subtract the following:

(a) $\begin{array}{r} +81 \\ -57 \\ \hline \end{array}$

(b) $\begin{array}{r} -36 \\ -14 \\ \hline \end{array}$

(c) $\begin{array}{r} -42 \\ +17 \\ \hline \end{array}$

(d) $\begin{array}{r} +19 \\ +31 \\ \hline \end{array}$

(33) Multiply:

(a) $(+6) \times (-3) \times (-7) =$
(b) $(-4) \times (+10) \times (+3) =$
(c) $(-1) \times (+7) \times (+4) \times (+2) =$
(d) $(-2) \times (-3) \times (-4) \times (-5) =$

(34) Divide:

(a) -258 by -6
(b) $+363$ by -33
(c) -87 by $+348$
(d) $+112$ by -7

(35) Find the following:

(a) $(-2)^4 =$
(b) $(+2)^4 =$
(c) $(-3)^3 =$
(d) $(+3)^3 =$

(36) Do the following divisions and multiplications:

(a) $3247 \div 10^3$
(b) 7625×10^2
(c) $23 \div 10^4$
(d) $967 \div 10{,}000$
(e) $23 \times 100{,}000$
(f) $9327 \div 10^4$
(g) 82×10^2
(h) $.032 \div 10^2$
(i) $.0756 \times 10^3$

(37) Do the following divisions using powers of 10:

(a) $875 \div 326$
(b) $-526 \div 234$
(c) $.671 \div .0341$
(d) $-470 \div 621$
(e) $.234 \div 875$
(f) $1.46 \div 26.2$
(g) $735 \div .0234$
(h) $426 \div 621$
(i) $36.2 \div .0465$
(j) $9.21 \div -11.3$

Vectors

You learned in your technical lessons that an ac voltage actually consists of a series of different instantaneous values of voltage and that these different voltages all occur at specified times in the ac cycle. You also learned that the current forced through a complete electrical circuit by an ac voltage was likewise made up of a series of instantaneous values of current that occurred at specified times in a cycle. However, one of the most important factors that we must consider in dealing with ac circuits is that these peak ac voltages and peak ac currents do not necessarily occur at the same instant of time.

In your study of coils you found that the current actually lags the voltage by 90° in a purely inductive circuit. Conversely, you discovered that in a purely capacitive circuit the current leads the voltage by 90°. In fact, the only time that the instantaneous values of the current and voltage can be in phase with each other throughout the entire circuit is in a purely resistive circuit. Actually, most practical circuits contain some combination of resistance, inductance, and capacitance, and the phase relationship between the ac current and voltage is a result of the combined action of these effects.

Because of this difference in phase between the voltage and current in ac circuits, we cannot use ordinary arithmetic in our circuit calculations. In ordinary arithmetic we have only simple numbers or "scalars", as they are sometimes called, which we can use. The numbers or scalars can only indicate the size or the magnitude of the voltage or current quantities. They cannot, in any way that we know of, indicate the time difference or phase angle which we must also consider. However, through the use of what we call "vectors" and vector arithmetic, we can indicate both the size of the quantities and the times at which they occur. Actually, the vectors used in ac circuit calculations are not vectors at all, but are phasors. However, they are similar to vectors and are usually called vectors, so we will use that name instead. By using these vectors, we are able to perform any ac circuit calculation quite simply and accurately.

In this section of the lesson, you will learn about vectors and how they can be used in ac circuit calculations. You will apply many of the rules of ordinary arithmetic, square roots, signed numbers, and even some ratio and proportion in working with vectors. Vectors are extremely important in work with ac circuits, because without them any of the methods used for solving ac circuit calculations would be useless. They are all based on vector principles and you will discover later that many of the explanations for circuit characteristics are easier to understand if you use vectors.

DEFINITION OF A VECTOR

A vector is a straight line having a definite length and direction. Fig. 20 shows several different vectors, each with a certain length and direction. Although these vectors do not represent any particular values or functions, they are all true vectors and could be used for any

Fig. 20. Vectors showing definite length and direction.

number of purposes. Notice that each vector starts at a certain point and ends a certain distance away in a specific direction. The starting point of any vector is usually represented by a dot and is called the "tail" of the vector. The ending point of the vector is represented by an arrowhead and is called the "head" of the vector.

The distance between the head and tail of the vector is used to indicate or designate the magnitude of a quantity. The direction of the vector, from a common reference point or line, represents the second factor which we must consider. This second factor may be either "direction" or "time." For example, the vectors in Fig. 21 represent the flight of an airplane. They are drawn so that vector A represents a 125-mile flight to the south, while vector B represents a 150-mile flight to the east. The tail of vector A shows where the airplane started, and the head of vector B shows where the flight ended. The fact that the

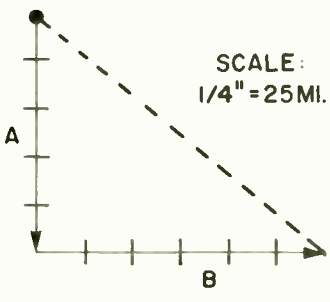


Fig. 21. Use of vectors to show results of motion.

tail of vector B starts from the head of vector A, tells us that the airplane flew south first and then east. The arrow in the diagram shows us where north is and becomes the reference line from which the direction of the vectors may be determined. The notation "Scale: 1/4" = 25 mi." tells how far the airplane traveled in each direction, because each quarter-inch of vector length is equal to 25 miles of travel. Thus, the vector diagram gives an accurate and descriptive picture of where the airplane went. As you can see, this is much more valuable than saying the airplane flew 275 miles.

Another thing about this vector diagram is that it can show how far the airplane actually went from the starting point. Notice in Fig. 21 that even though the airplane actually flew 275 miles, it did not end its flight 275 miles from the starting point. The head of vector B is not 275 miles away from the tail of vector A. Since the diagram is drawn to scale, you can measure this distance with a ruler, as shown by the dotted line in Fig. 21. If you do this carefully, you will find that it is just under 2 inches from the tail of A to the head of B. Since each quarter-inch on the diagram equals 25 miles, you know that the airplane ended its flight less than 200 miles from the starting point even though it flew 275 miles to get there.

Further, by comparing the direction of this dotted line with the reference arrow, you can determine the direction of the end of its flight from the starting point. If you do this, you can see that the flight ended approximately southeast of the starting point. Therefore, the vector diagram gives a complete picture of the airplane's flight as well as giving its progress away from the starting point.

Vectors in Electronics. In electronics work, we do not use vectors very often to indicate motion. Instead, we use the direction of vectors to indicate the time of an occurrence. For example, suppose that we have a series circuit containing a coil and a resistor as shown in Fig. 22. The ammeter in the circuit shows that there is an ac current of 5 amps flowing through the circuit and we know that the current in a series circuit is the same through any part of the circuit. Therefore, there is a current of 5 amps through the coil.

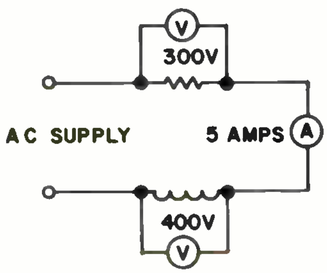


Fig. 22. AC circuit containing resistance and inductance.

The voltmeter across the resistor indicates 300 volts and the one across the coil indicates 400 volts. You know from your studies of ac circuits that these voltages across the coil and resistor do not occur at exactly the same instant of time. For example, if we were to draw a sine wave diagram of the ac current through the circuit, we might have a wave shape like the solid line in Fig. 23A. This sine wave rises from zero to a maximum of such a value that the effective value of the alternating current is 5 amperes. Using this ac sine wave as a reference point for time, and comparing the rise and fall of the voltage sine waves with it, we can

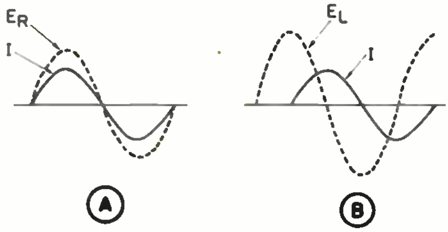


Fig. 23. Sine waves of voltage and current in a resistor (A), and a coil (B).

actually see that the two voltages do not occur at the same instant.

We know that in a purely resistive circuit the current and voltage actually rise and fall together. We say that they are in phase. Therefore, the voltage across the resistor, E_R, must rise and fall so that it will be maximum when the current is maximum. Further, the maximum value of this ac voltage must be large enough so that its effective value will be 300 volts, as measured by the voltmeter. Accordingly, if we were to draw a voltage sine wave for the voltage E_R, using the current sine wave as a time reference, we would have a wave shape like that shown by the dotted line in Fig. 23A.

On the other hand, the current and the voltage do not rise and fall together as far as the coil is concerned. The coil current actually lags the coil voltage by 90°. Therefore, we would draw their respective sine waves 90° out-of-phase, as shown in Fig. 23B, where the solid line, I, represents the coil current and the dotted line, E_L, represents the coil voltage. The current sine wave is the same sine wave that we used in Fig. 23A and occurs at exactly the same instant. The current is common throughout the circuit. The voltage wave, E_L, has a maximum value necessary to produce the effective value of 400 volts, as indicated by the voltmeter in Fig. 22.

Since the current sine waves in Figs. 23A and 23B represent the same current at the same instant of time, we can combine the two drawings as shown in Fig. 24. Here the one current sine wave, I, represents the common circuit current through both the coil and the resistor. The voltage wave, E_R, represents the sine wave voltage that gives the effective value of 300 volts across the resistor. The voltage wave, E_L, represents the sine wave that produces the effective value of 400 volts across the coil. It is easy to see from this drawing that the two voltages do not rise and fall together. They are 90° out-of-phase.

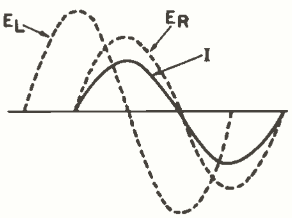


Fig. 24. Sine waves showing E_R lagging E_L.

Through the use of vectors, we are able to represent this difference in "time" between the two voltages and between the current and the coil voltage much more simply than by using sine waves. We do this by means of a rotating vector. As the vector rotates through 360°, the projection from the end of the vector traces out a sine curve. Looking at the current waveform in Fig. 23A, we see that the current is zero at the start of the cycle. This is represented by the vector shown in Fig. 25A. One-quarter of a cycle later, the current has reached its peak positive value. This is represented by rotating the vector 90° (one-quarter of a turn) to the position shown in Fig. 25B. At the end of another quarter-cycle, the current waveform will have gone through one-half of a cycle, and will be back to zero. This is represented by the vector shown in Fig. 25C. Here the vector has rotated through 180° (one-half turn). The vector in Fig. 25D represents the current waveform one quarter-cycle later when the current is at its peak negative value. The vector in Fig. 25E represents the current at the end of one complete cycle. Here the vector has rotated through 360° (one complete turn) and is back at the starting point.

Notice that we rotated the vector in Fig. 25 in a counterclockwise direction. Be sure to remember the direction of rotation; you'll need to know this to understand the vector diagrams in the rest of this lesson and in later lessons.

Now that you've seen how we use a rotating vector to show the current phase throughout a cycle, let's see how vectors can be used to show the phase relationship between the voltage and current across the resistor in Fig. 22, throughout a cycle.

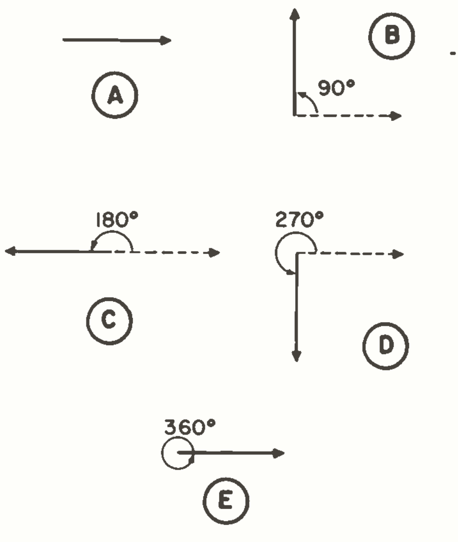


Fig. 25. A rotating vector showing one complete ac cycle.

At the start of the cycle at 0°, E_R and I are in phase. We show the phase relationship by drawing the two vectors, E_R and I, as shown in Fig. 26A. Notice that the vectors are drawn superimposed on each other because at the start of the cycle both the voltage and current are zero. One quarter-cycle later, the voltage and current have reached their maximum position values. This is shown by the vectors in Fig. 26B. Again the vectors are superimposed because the voltage and current are in phase - - they are both at their maximum values at the same instant, one quarter-cycle or 90° after they were both zero. In Fig. 26C we have shown the vectors at the end of one half-cycle, in Fig. 26D at the end of three quarter-cycles and in Fig. 26E at the end of a complete cycle.

Now, let's see how vectors can be used to show the phase relation between the

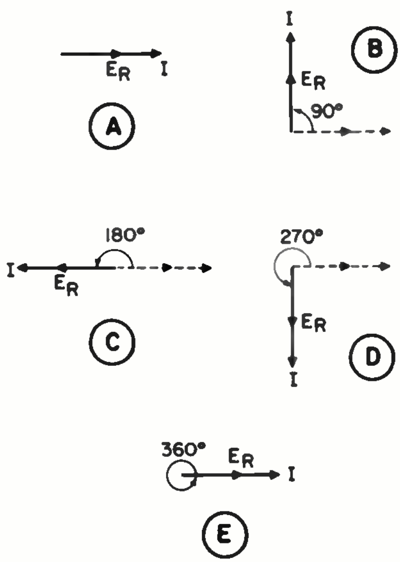


Fig. 26. Vector diagrams showing phase relationship between the voltage across and the current through a resistor, throughout a complete cycle.

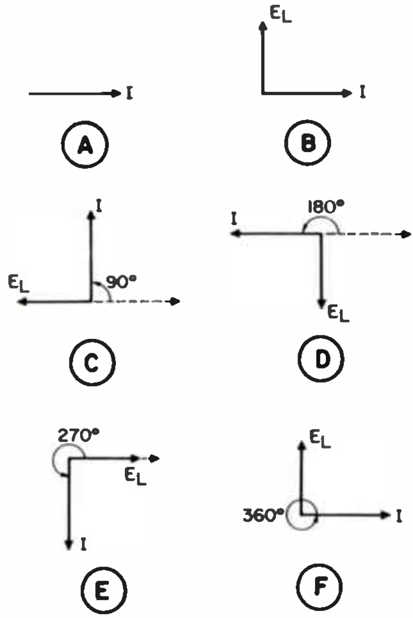


Fig. 27. Vector diagrams showing phase relationship between the voltage across and the current through a coil, throughout a complete cycle.

voltage across the coil and the current through it. Let's start with the current vector at zero representing the start of the current cycle, as shown in Fig. 27A. This vector is the same as the current vector in Fig. 25A. Now, to draw the vector representing the voltage across the coil we must consider the phase relationship. You will remember that the voltage across a coil leads the current by 90°. This is shown in Fig. 23B. Thus, if the current vector is drawn at zero, we must advance the voltage vector 90° (rotate it counterclockwise) as shown in Fig. 27B. Thus, the vectors in Fig. 27B show the phase of the voltage and current at the start of the current cycle.

One quarter-cycle later, both vectors will have rotated through 90°, as shown in Fig. 27C. At that point, the current is at its peak position value and the voltage has dropped to zero, as shown in Fig. 23. The vector diagram indicates this condition. A quarter-cycle later, when the current has gone through one half-cycle, the current will be zero again and the voltage will be at its peak negative value. This is shown in Fig. 23 and also in Fig. 27D by means of vectors. Fig. 27E shows the phases one quarter-cycle later when the voltage is back to zero and the current is at its peak negative value. Fig. 27F shows the voltage and current vectors at the end of a complete cycle.

Notice that throughout the illustrations in both Figs. 26 and 27, the phase relations remain constant. Fig. 26 shows the resistor voltage and current in phase through the entire cycle. Fig. 27 shows the coil voltage leading the current by 90° through the entire cycle.

In Fig. 24 we have shown the current, coil voltage, and resistor voltage on the same diagram. It does not matter whether we start working with the current, coil voltage, or resistor voltage in constructing our diagram, as long as we show the correct phase relationship. However, in circuits of this type the usual practice is to start by drawing the current vector I, as shown in Fig. 28A. Let's draw the vector to represent zero current at the start of the current cycle. We can draw the current vector any convenient length because we are not going to use it for anything other than a reference vector.

Now, let's draw a vector to represent the voltage across the resistor. In Fig. 22 we see that the voltage is 300 volts. Let's draw the vector to scale so the length of the vector represents 300 volts. If we use a scale of 1/2″ = 100 volts then the vector should be 1-1/2″ long. Since the voltage across the resistor is in phase with the current, the voltage vector is drawn superimposed on the current vector, as shown in Fig. 28B.

To draw the vector representing the coil voltage we first note, from Fig. 22, that the voltage is 400 volts. Therefore, using the scale of 1/2″ = 100 volts, the vector should be 2″ long. Since the coil voltage leads the current by 90°, we draw this vector as shown in Fig. 28C. Now we have a vector diagram showing the phase relationship between the current and the two voltages at the start of the current cycle. As a matter of fact, this diagram shows the phase difference between the coil voltage, the resistor voltage, and the current throughout the entire cycle because, as we saw from Figs. 26 and 27, these relationships did not change. The coil voltage always leads the resistor voltage and current by 90°.

We can use the vector diagram of Fig. 28C to determine the total voltage across the coil and the resistor. This is equal to the source voltage in Fig. 22. We do this by adding the vector representing the coil voltage to the vector representing the resistor voltage. Since we have drawn these vectors to scale, we should be able to scale the resulting vector to get the total voltage.

There are two ways of making this addition. The first way is to place the coil voltage vector at the end of the resistor voltage vector, like we did in Fig. 21, to determine how far the plane traveled. This addition is shown in Fig. 28D. Notice the resultant vector, E_T, which represents the total voltage that leads the current by a value somewhat less than 90°. This is what we might expect. In a circuit with pure resistance, the voltage and current will be in phase -- in other words the phase difference will be 0°. In a circuit with pure inductance, the volt-

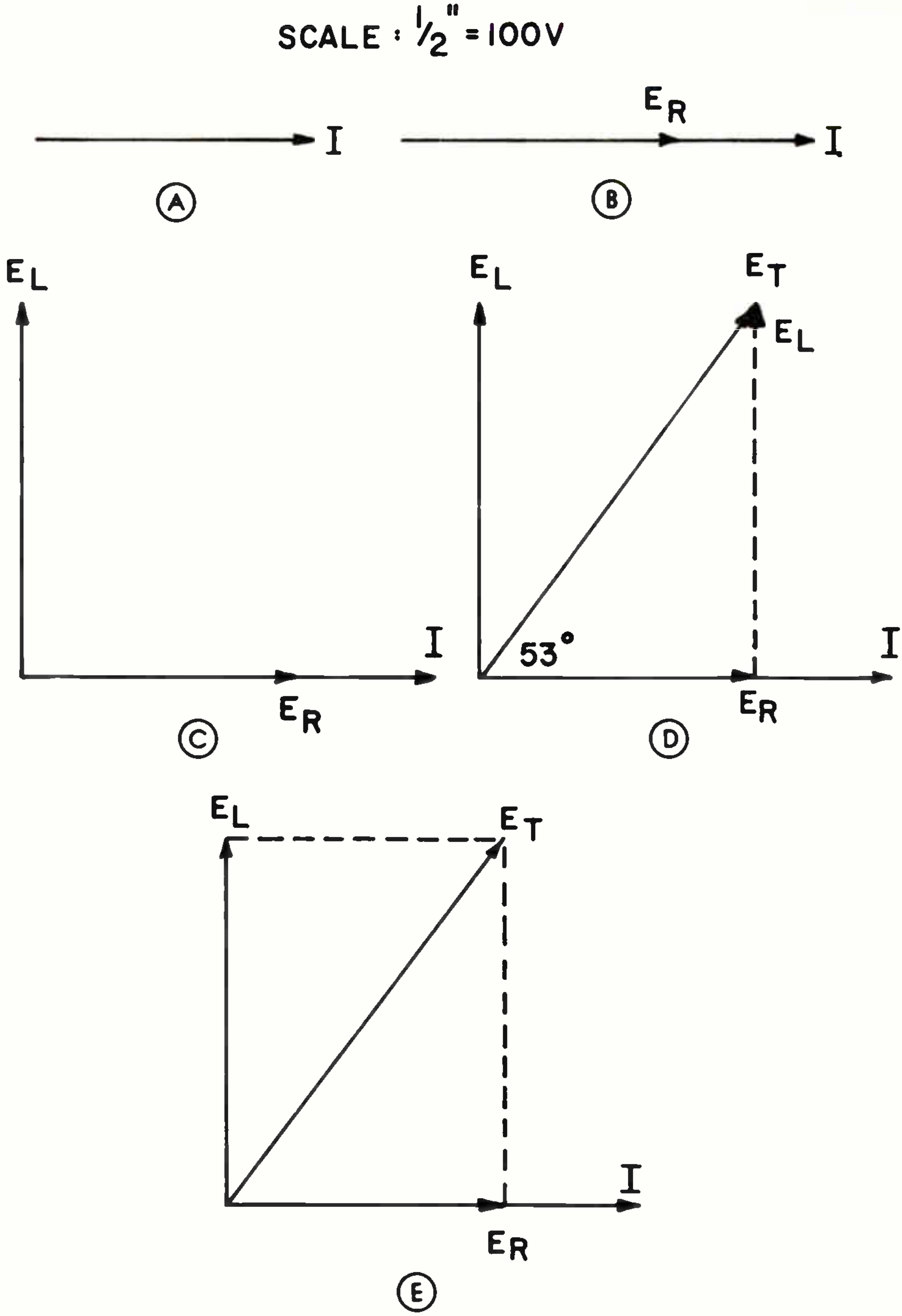


Fig. 28. Vector addition of resistor and coil voltages.

age leads the current by 90°. Thus, in a circuit with both resistance and inductance, we would expect both to influence the phase relationship between the voltage and current so that the phase difference will be somewhere between 0° and 90°. If you construct the diagram carefully and measure the angle between E_T and I, you will find that it is about 53°.

To determine the amplitude of E_T, measure the length of the vector. You will find it is 2-1/2″ long. Since the scale used in constructing the diagram is 1/2″ = 100 volts, the amplitude of E_T must be 500 volts.

The other method of adding the two vectors is shown in Fig. 28E. Here a dotted line is drawn from the end of vector E_R parallel to vector E_L. A second dotted line is drawn from the end of vector E_L parallel to E_R. The point where the two dotted lines meet locates the end of vector E_T. The angle between it and vector I will be the same when obtained by this method as it would be using the previously discussed method.

To really see the advantage of the vector method of representing these voltages, look at Fig. 29. Fig. 29A shows the sine waves of the two voltages and the current, and Fig. 29B shows how these sine waves can be added to get the total voltage. To do this, many instantaneous

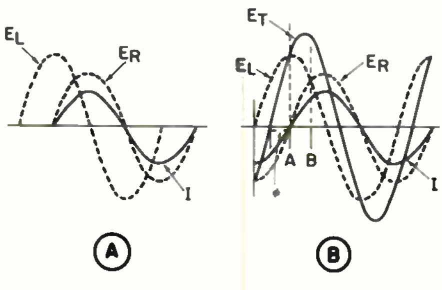


Fig. 29. Adding sine waves to obtain voltage total, E_T.

values must be added to enable you to plot the curve of E_T. This is tedious and, in addition, is not nearly as easy to evaluate as was Fig. 28D or Fig. 28E.

You might wonder why we started our construction of Fig. 28 by putting I in the 0° position. We did this simply because it was convenient. We could actually put I in any position. As long as we keep the correct position between I, E_R, and E_L, the value of E_T and the phase angle between it and I will be the same.

VECTOR ARITHMETIC

Now that we have seen what vectors are and how they can be used in electronics to represent size and phase or time for ac circuit calculations, let's learn more about handling them. Actually, the rules for working with vectors are quite simple and are similar to any arithmetic that involves signed numbers. The most important differences are learning to deal with the angles and establishing the qualifications for lead and lag.

To begin with, we have seen that any vector diagram must have a reference line so that the directions of the individual vectors can be established in accordance with a common reference for comparison. The reference we used in the preceding example was the current. The best type of reference to use when learning about vectors is a scale similar to the one we used when learning to deal with positive and negative numbers. In Fig. 30, we have drawn such a scale with a center reference point. All positive values extend to the right of the center and all negative values extend to the left.

Now suppose that we have two quantities representing the same direction or instant of time: one is +5 units long and

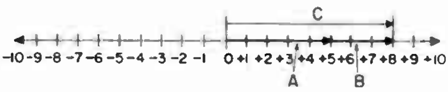


Fig. 30. Adding in-phase vectors.

the other +3 units long. In order to add them vectorially, we simply lay them out along the reference line (to scale) in the same direction, with the tail of one starting at the head of the other, as shown in Fig. 30. Vector A starts at the reference point, 0, and continues for five units. Vector B starts at the head of vector A and continues along the reference for three units. To add the two vectors, we simply draw a new vector, C from the tail of A to the head of B. Since A and **B** both point in the same direction, this new vector lays along the same line as A and B and is +8 units long. Therefore, we have a rule which states: ***The sum of two vectors extending in the same direction is a new vector equal to the combined length of the two vectors and pointing in the same direction.***

In Fig. 31, we have added two vectors [(–6) + (–3)] that point in the same direction, but both of them are negative so their direction is just opposite to those in Fig. 30. Therefore, their sum is a new vector that is –6 units long plus –3 units long which makes it –9 units long. Thus, as long as two or more vectors point in the same direction, regardless of what that direction is, their sum is a new vector, pointing in that direction, that is equal to the combined length of the individual vectors.

Another problem in working with vec-

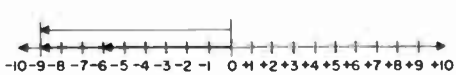


Fig. 31. Adding in-phase vectors.

tors is when they point in opposite directions. This will often come up when two voltages or other circuit quantities are exactly 180° out-of-phase. For example, suppose we have two opposing voltages working against each other in an ac circuit. One of these has a peak value of 90 volts and the other a peak of 40 volts. The two are exactly out-of-phase at all times, so we can lay them out vectorially, as shown in Fig. 32, by using a scale of 1 unit equals 10 volts. Notice that since the two are 180° out-of-phase, one vector points from the reference point 0, along the reference line in one direction, while the other starts at the reference point and extends in the opposite direction.

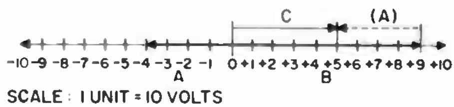


Fig. 32. Adding vectors 180° out-of-phase.

It does not make any difference which vector we use for which direction in this particular case. We know that each one changes sign during every cycle and we are just stopping the action at a particular time. No matter what we get for an answer, its sign will automatically change during the next alternation because we are working with ac. The thing that really does matter is that both vectors represent the same instant of time so that the 180° phase difference is represented by the vector directions.

To add these two vectors we must do the same thing that we did in other vector additions. We place the tail of one vector against the head of the other, being careful not to change the direction or length of the vector that we relocate. In this problem, we have drawn a dotted line from the head of the 90-volt vector B that is exactly the same length and points in the same direction as the 40-volt vector A. Notice that we have drawn the dotted line slightly above the reference line so that we can see it better. Now, we complete our addition by drawing a new vector, C, from the tail of the 90-volt vector. This new vector is 5 units, or 50 volts long, and points in the direction of the longest vector. Thus, we can see that the vector sum of the two out-of-phase voltages is a new vector, extending from the tail of one vector to the head of the other, after they have been properly joined (head to tail) for addition.

Representing Lead or Lag. As you can see, all the vector addition that we have considered so far follows the same basic pattern. We lay the vectors out to their proper scale length and orient them in their proper direction. Then, we lay them head to tail, being careful not to alter either their length or direction in the process, and draw a resultant vector, between the tail of the first vector and the head of the last, to represent their sum. The length of this resultant represents the magnitude of the vector sum, and its direction in relation to the reference line indicates the phase or time of the resultant quantity.

All vector computations follow these same basic rules. However, the problems that we have considered so far have dealt with vectors that are exactly in phase or exactly 180° out-of-phase. As you know, many of our problems in electronics deal with reactance calculations where the phase shift will be only 90°. Also, this reactance phase shift may be either 90° leading, in the case of the capacitance, or it may be 90° lagging in problems dealing with inductance. Accordingly, we must now consider what to do about laying out and computing vectors that are affected by reactance.

To do this let's go back to our basic idea of a phasor. A phasor is a rotating vector. We have shown that this vector rotates counterclockwise. Thus, if we start a vector at 0°, as in Fig. 33A, and rotate it 90° to represent one-quarter of a cycle, it will move to the position shown in Fig. 33B. The vector at B has passed through one-quarter of a cycle more than the one at A. Thus, vector B is leading vector A by 90°. The vector diagram in Fig. 33C shows how the voltage leads the current by 90° in a coil.

In Fig. 34 we have shown an example of a lagging voltage. We started with our current vector at A and then drew a second vector 90° behind it at B. Vector B is following vector A by 90°. A complete vector diagram showing how the current leads the voltage in a capacitor is shown in Fig. 34C.

You might think that since the two

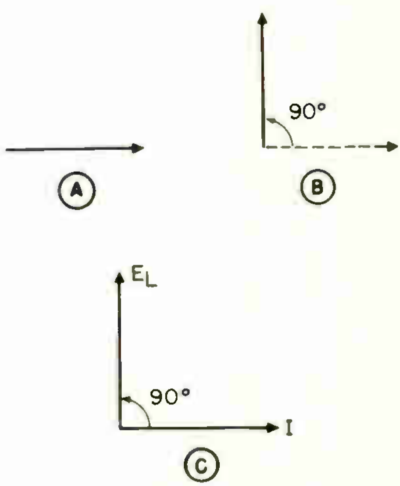


Fig. 33. Vector diagrams showing 90° phase shift where the voltage leads the current.

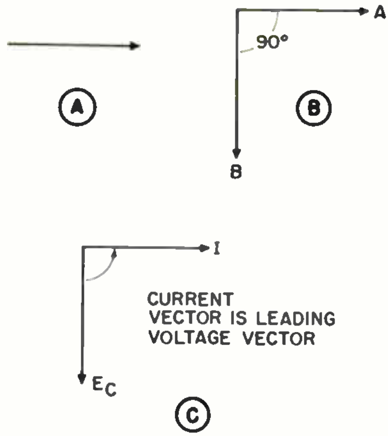


Fig. 34. Vector diagrams showing a 90° phase shift where the voltage lags the current.

vectors are rotating counterclockwise, the voltage vector is leading the current by 270°. Since the voltage is lagging one current cycle by 90°, it will indeed be leading the following current cycle by 270°. However, it is the 90° phase difference we are interested in.

Now it is fairly easy to see that, if we consider our discussions of vectors up to this point, we can call the right-hand end of the scale the "in-phase reference line." Likewise, the end of the horizontal line extending to the left of the center towards 180° can be called the "180° out-of-phase reference line." If we do this, all vectors parallel to the horizontal reference and pointing to the right will be in-phase vectors, and those pointing to the left will be 180° out-of-phase. Thus, our horizontal line, divided in the center in this way, can represent a phase shift or time lag of 180°, depending on whether we point our vectors from the center to the right of 0° or from the center to the left of 180°.

The vertical line represents a phase shift of 90°. Since our vector rotates counterclockwise, the vector representing the voltage in Fig. 33C is leading the current vector by 90°. It's not always necessary or even advisable to place the current vector at 0°. However, regardless of how we start the diagram and what we place on the 0° axis, all voltages or current leading the reference value are shown rotated in a ***counterclockwise*** direction. Voltages or currents lagging the reference value are shown in a ***clockwise*** direction. Any voltage or current can be taken as our reference value, but in most series circuits it is easiest to use the current as a reference.

Sometimes we may want to construct an impedance diagram of resistance, and inductive or capacitive reactance. You might wonder if one of these reactive elements should be drawn above or below the reference line. The rule is to draw inductive reactance above and capacitive reactance below. Inductive reactance is considered positive and capacitive reactance negative. You'll see later, when you study the j-operator, that this is necessary in order to get the currents to have the correct phase. Thus, if we consider our reference line extending from the center right towards 0°, as our current reference line in ac circuit problems, our inductive reactance (voltage leading current) values will go above the reference and our capacitive reactance (voltage lagging current) values will go below the reference. Another thing that will help you to remember this rule is to notice that vectors are always considered to rotate counterclockwise from zero.

Also notice that the 90° head point from the center can also be considered to lag the 0° reference line by 270°. It doesn't make any particular difference which way you think of it. The important thing is to make sure that when you construct a scale like this and use the current as the reference vector, you should remember:

1. Right from center: in phase I, R, or E_R.

2. Left from center: 180° out-of-phase.

3. Straight up from center: 90° lead, X_L or E_L.

4. Straight down from center: 90° lag, X_C or E_C.

VECTOR CALCULATIONS

Now that we have discussed some of the basic rules and principles for laying out vectors and computing with them, we need to gain a little practice to become thoroughly familiar with this type of computation. We considered a problem dealing with a coil and a resistor a little earlier. Now, let's consider a circuit containing a resistor and capacitor in series like the one shown in Fig. 35A. In this circuit we are told the capacitor voltage and the resistor voltage and are asked to find the supply voltage, E_T, and the phase angle, θ.

In order to do this, we first lay out the current vector as the reference vector because the current in the circuit, I, is the same in all parts of the circuit. Using the horizontal line extending to the right toward 0° to represent the current, as we have learned to do, completes our preliminary setup for the problem, as shown in Fig. 35B. Notice that we do not have the value of this current. It is not needed to work the problem. We are only using I as a reference line from which to indicate our voltage-phase relationships. Its value is not important. Its position as the common reference is what concerns us here.

Using the scale of 1/2″ = 100V, we can lay out the voltage vector, E_R, to scale along the reference line to show that the voltage across the resistor is in phase with the current through it. We call this voltage the in-phase component of the total voltage. The vector diagram of this stage is shown in Fig. 35C. Then we can lay out the voltage vector, E_C, to scale 90° behind the reference current because I will be leading it by 90°. The diagram at this point is shown in Fig. 35D. To add these two vectors, we can move one of them so that the two are head to tail, as shown by the dotted vector, E_C, in Fig. 35E and then draw in E_T, the total voltage. Notice that we are careful to construct the dotted vector, E_C, so that its length and direction are not changed. We can also get E_T by drawing a line parallel to E_R from the end of E_C, and a second line parallel to E_C from the end of E_R. Where the two lines meet locates the end of E_T, as shown in Fig. 35F.

Now, we can draw the resultant vector, E_T, that represents the vector sum of the two quantities. If we measure E_T carefully and our other vectors are to scale, we will find that E_T is 2-1/8″ long. Since 1/2″ = 100V, 1″ must equal 200 volts, and 2″ must equal 400 volts. Now, since 1/8″ is equal to 1/4 of 1/2″, 1/8″ is equal to 25 volts. Accordingly, our E_T vector, which is 2-1/8″ long, must equal 400 + 25 or 425 volts.

We are also asked to find the phase angle, θ, for our supply voltage. We can do this by using a protractor which measures angles. We will find that the angle between E_T and the reference I is 45°. As a matter of fact, we can estimate that E_T lies about halfway between E_C and I, which are 90° apart, so E_T must be approximately 45° out-of-phase with the current. Thus, through the use of vectors, we have determined that E_T is 425 volts and is 45° out-of-phase with the current. Since E_T is below the reference I, the

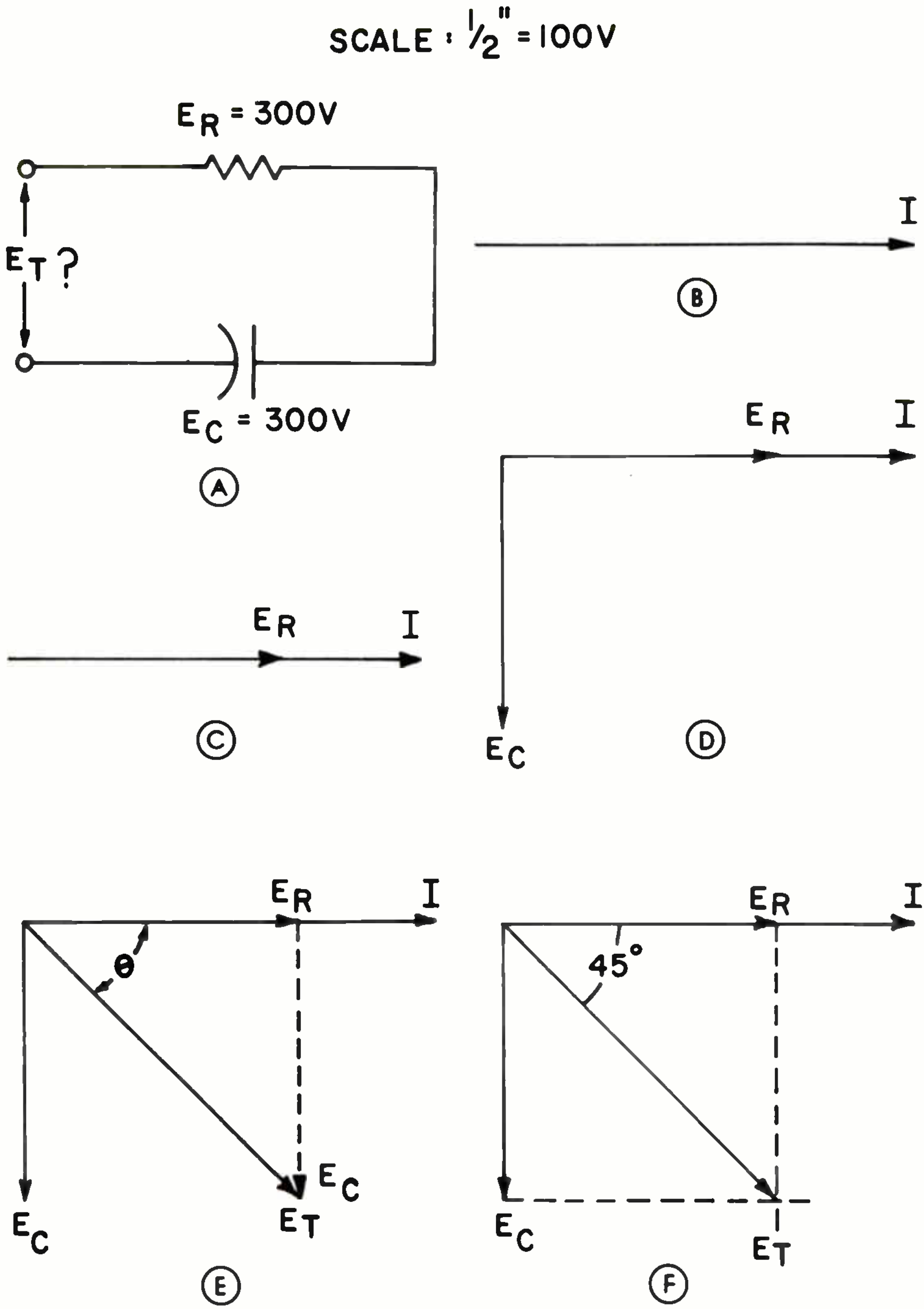


Fig. 35. Vector addition of the voltage E_R and E_C in the circuit at (A) shown in detail.

current must *lead* the voltage (or the voltage lags the current, whichever you prefer) by 45°.

Before we leave this circuit, let's consider one other thing. In converting our length of E_T of 2-1/8" to 425 volts, we went through a series of steps. Ratio and proportion would have saved us a lot of time. For example, we could have set up two ratios as follows:

$$\frac{\frac{1}{2}''}{2\frac{1}{8}''} \quad \text{and} \quad \frac{100V}{E_T}$$

Now our proportion:

$$\frac{\frac{1}{2}''}{2\frac{1}{8}''} = \frac{100V}{E_T} \quad \text{or}$$

$$\frac{\frac{1}{2}''}{\frac{17''}{8}} = \frac{100V}{E_T}$$

Now cross-multiplying, we have:

$$\frac{17}{8} \times 100 = \frac{1}{2} \times E_T$$

$$\frac{1700}{8} = \frac{E_T}{2}$$

and then:

$$8 \times E_T = 1700 \times 2$$

$$8E_T = 3400$$

$$E_T = \frac{3400}{8} = 425 \text{ volts}$$

This is just another example of a good use for ratio and proportion.

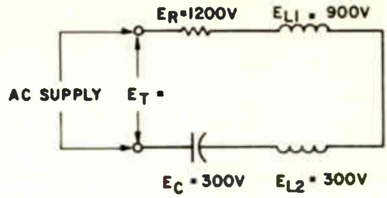


Fig. 36. Finding E_T in a simple series ac circuit containing R, L, and C.

In Fig. 36, we have shown a more difficult problem. Here we have a resistor, two coils, and a capacitor in series with each other. Let's see how we would handle the problem of finding the total voltage for this circuit vectorially.

First, we lay out our vector reference scale and call the horizontal line, which is from center to 0°, the current reference. This is shown in Fig. 37. Then we can draw in the voltage vector E_R superimposed on I to show the two in phase. Since E_R is 1200 volts, we used a scale of 1/4" = 100 volts, instead of 1/2" = 100 volts as before, in order to keep the diagram a reasonable size in the book. If you try drawing this diagram you can use a scale of 1/2" = 100 volts if you wish. Then E_R would be twice as long. Next, we draw vector E_{L1} leading I by 90°. This vector represents 900 volts. Using the scale of 1/4" = 100 volts, we make it 9 X 1/4" = 9/4 = 2-1/4" long.

The next step is to add the vector E_{L2} to the diagram. Since this voltage also leads I by 90°, we draw E_{L2} starting at the head of E_{L1} to add these two in-phase voltages. E_{L2} is made 3/4" long to represent 300 volts.

The next vector we draw is E_C. Since this represents 300 volts we know it should be 3/4" long. Also, since it represents a voltage across a capacitor, we know it will lag I by 90°. The position of this vector is shown on the diagram. Notice that it is 180° out-of-phase with

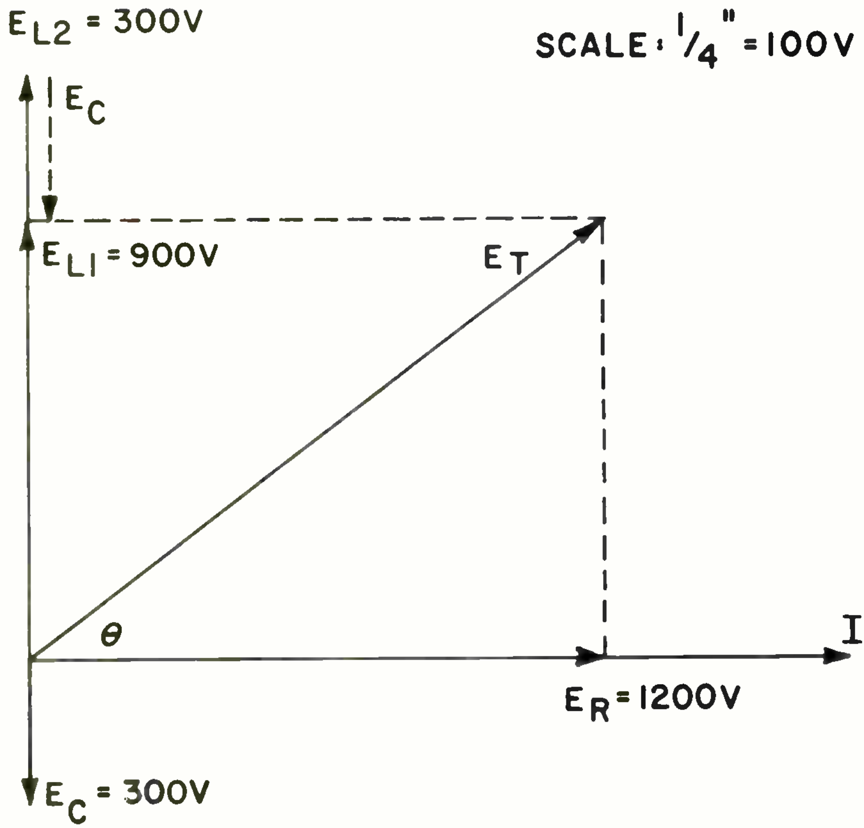


Fig. 37. Vector solution of multiple reactance circuit.

E_{L1} and E_{L2}. We now add this vector to the sum of E_{L1} and E_{L2} following the same procedure as shown in Fig. 32. We move vector E_C to the head of E_{L2} as shown by the dotted line. Then the head of E_C represents the total reactive voltage. We have shown E_C dotted and slightly to one side of E_{L2} so you can see it; actually it should be superimposed on E_{L2}.

To complete the vector diagram and get E_T, we draw a line parallel to vector E_R from the end of the vector representing the sum of E_{L1} + E_{L2} + E_C. Then we draw a second line parallel to the E_{L1} vector from the end of E_R. The point where these two lines intersect locates the end of E_T. Now we can draw vector E_T to represent the sum of our reactance and resistance vectors and carefully measure it. We find that it is exactly 3-3/4" long. Now, by setting the ratios and proportion between our scale values and our measurements, we can convert our vector measurements to volts as follows:

$$\frac{\frac{1}{4}''}{3\frac{3}{4}''} = \frac{100}{E_T}$$

$$\frac{1}{4}'' \times E_T = 3\frac{3}{4}'' \times 100$$

$$\frac{E_T}{4} = \frac{15}{4} \times 100$$

$$\frac{E_T}{4} = \frac{1500}{4}$$

$$4E_T = 4 \times 1500$$

$$E_T = 1500 \text{ volts}$$

We can then determine the value of θ by measuring it: $\theta = 36\text{-}1/2^\circ$.

So far, in working with vectors, we have considered only the addition of vectors to find a total. We can also break down a given resultant vector to find some of its components. In other words, if we have E_T, we can find E_R and the total circuit reactance voltage. This process is important when working with parallel ac circuits, so let's see how it is done.

Suppose we are given a statement regarding the voltage of a circuit as follows: "The voltage applied to a series circuit is equal to 140 volts and it leads the current by a phase angle of 45°. What is the value of the resistance in the circuit if the current is 5 amperes?" Although this might seem difficult at first glance, it is really quite easy to solve. First, let's see what we know about the voltage.

We can see that it leads the current (or that the current lags the voltage) and, therefore, the total reactance in the circuit must be inductive. We know the phase angle is 45° and that the voltage value is 140 volts. With this information we can lay out our standard reference diagram for vector solutions and construct a vector that represents the total voltage applied. Using a scale of 1/4″ = 20 volts, our total voltage vector, E_T, can be laid out to scale as shown in Fig. 38. We

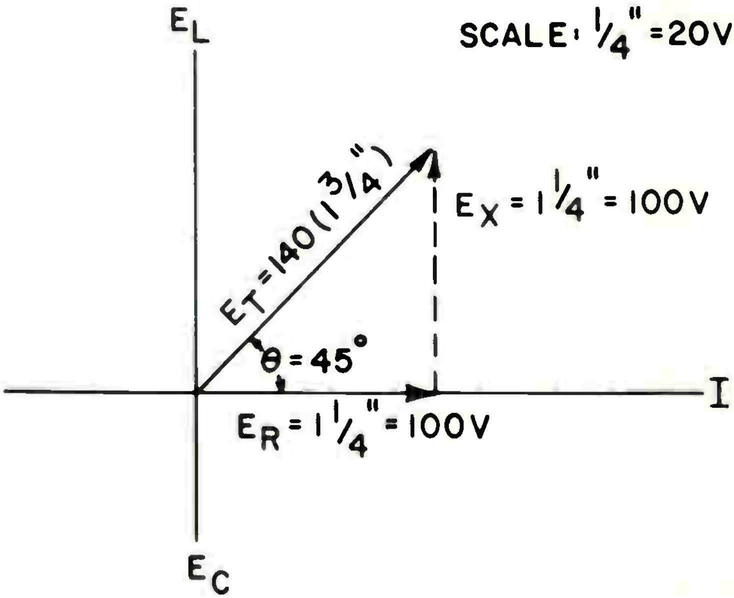


Fig. 38. Breaking a vector down into two components.

have drawn this vector above the reference line I, at the phase angle of 45°, to show that it is inductive and leads the current. Now this vector, E_T, is a resultant vector and must be made up of at least two component vectors; one resistive, and one reactive. Any vector that is not out-of-phase with the reference by some multiple of 90° can always be broken down into at least two components that are at right angles to each other. Because of this rule, we can separate the vector E_T into two component vectors.

Doing this, we find that in the circuit the reactive component, E_X, is equal to 100 volts and the resistive component, E_R, is also equal to 100 volts. Now that we have the resistive voltage drop of 100 volts, it is easy to find the resistance. We know that it is a series circuit and, therefore, the current is common. Accordingly,

$$R = \frac{E_R}{I} = \frac{100}{5} = 20 \text{ ohms}$$

Thus, by breaking down the vector value of applied voltage into its two components, we are able to determine quite a lot about the circuit. You will find that

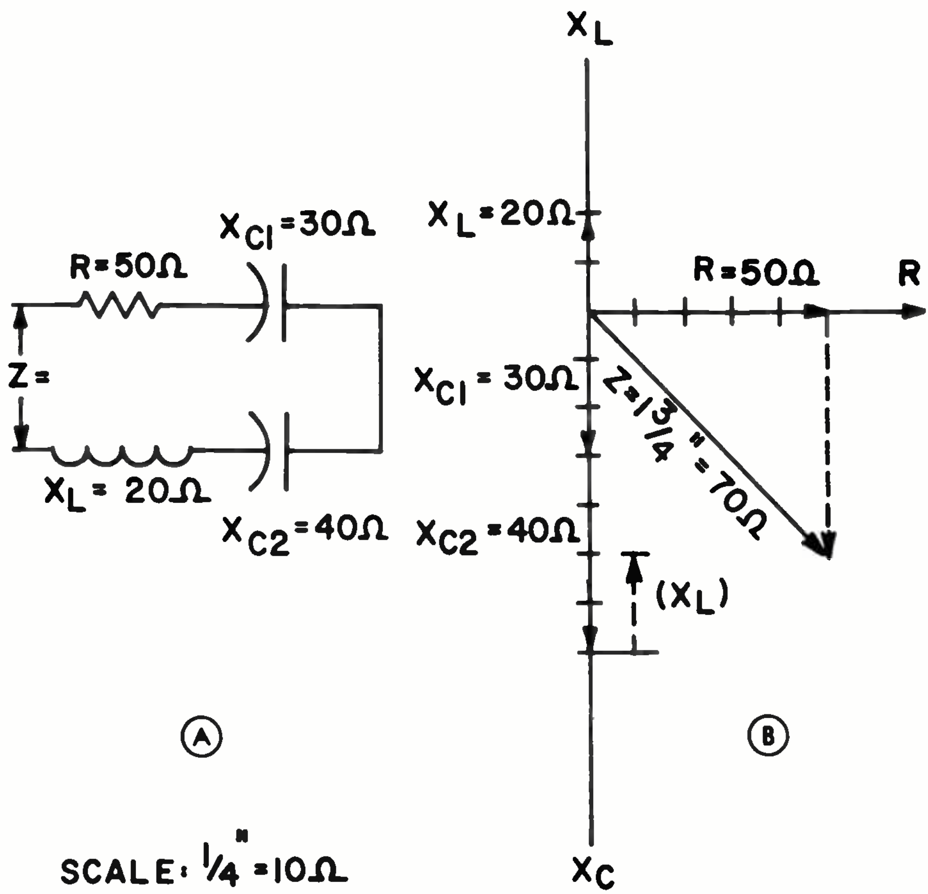


Fig. 39. Solving for impedance with vectors.

this is a very valuable process in your electronics work.

Up until now, we have been working primarily with the voltages in a circuit. We can use our vector diagrams and computation methods to find the impedance of a circuit. For example, suppose we wanted to find the total impedance of the circuit shown in Fig. 39A. We would simply lay out our vector diagram and add the component vectors just as we did when we were working with voltage, as shown in Fig. 39B. Notice that we use R as the reference line because it represents the in-phase component of the impedance Z. You should have no trouble following the solution of this simple vectorial computation.

PYTHAGOREAN THEOREM FOR VECTOR SOLUTIONS

You will remember that earlier in this lesson and in your technical lessons, we mentioned that we could use formulas instead of vectors for ac circuit solutions. One of the most familiar is the formula for impedance which is,

$$Z = \sqrt{R^2 + X^2}$$

This formula is really just a mathematical solution of a vector diagram. By using it, we involve ourselves in the problem of finding a square root, but it is often much more desirable and usually more accurate than solving vectors by measurement. In measuring vectors for the solution of vector diagrams, we need to take care in laying out the diagrams, and measuring the angles and lengths of the vectors.

In working with the formulas, we are given a method of solving for the lengths of the vectors, but not the angles. Since the formula allows us to use a mathematical solution for the length of the vectors in vector diagrams, it eliminates the need for the accuracy of layout and measurement. Let's look at a typical vector diagram and see how it is possible to consider the formula $Z = \sqrt{R^2 + X^2}$ as the mathematical solution.

In Fig. 40A, we have laid out the solution of a typical vector diagram for finding the impedance of an ac circuit. The vector representing R, the dotted vector representing X_L, and the resultant vector Z, representing the impedance of the circuit, forms a three-sided, completely enclosed figure. Such a three-sided figure, as you probably know, is called a triangle. However, this is a special type of triangle, called a right triangle. Any triangle that contains one angle that is equal to 90°, such as the angle between vectors X_L and R, is a right triangle, no matter what the other two angles or the lengths of the sides may be. Any triangles involved in vector solutions for ac circuits will also be right triangles, because two of the sides must be 90° displaced from each other.

It is important that you realize this and understand it, since many of the laws for ac circuit solutions depend on this fact. In Fig. 40B, we have drawn the triangle of Fig. 40A, leaving out the arrowheads and values so that we can show you a very important fact about all right triangles. First of all, notice that we have given names to the sides of our right triangle in Fig. 40B. The longest side, or the side opposite the right angle, is always called the hypotenuse. The other two sides are simply known as legs or sides.

Many years ago, a mathematician named Pythagoras discovered a very interesting fact about right triangles. He discovered that if you squared each side

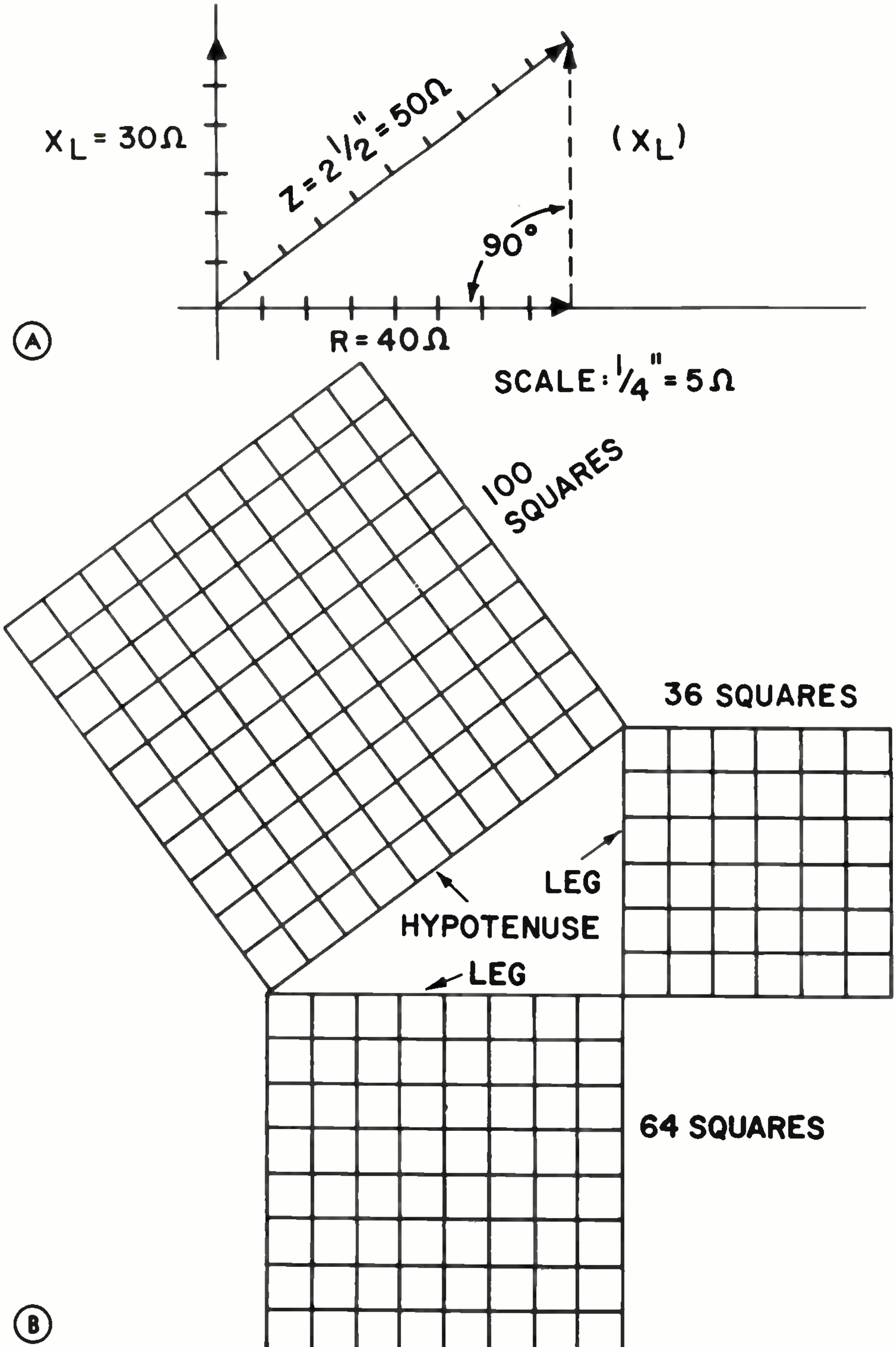


Fig. 40. The derivation of the Pythagorean Theorem.

of the right triangle and filled in all the squares in between as we have shown, the sum of all the squares (created by squaring the two legs) is exactly equal to the number of squares created by squaring the hypotenuse. You can actually prove this to yourself by counting all the squares in the two legs. You will find that there are 64 squares in the bottom leg and 36 squares in the vertical leg. The sum of 64 and 36 is, of course, 100. Now, if you count all the squares made by squaring the hypotenuse, you will find that there are also exactly 100. Thus, we have the theorem which Pythagoras discovered:

The square of the hypotenuse of any right triangle is equal to the sum of the squares of the other two sides.

If we apply this theorem to our vector diagram in Fig. 40A, we have:

$$Z^2 = R^2 + (X_L)^2$$

Then, by substituting our values, we have:

$$\begin{aligned} Z^2 &= 40^2 + 30^2 \\ &= 1600 + 900 = 2500 \end{aligned}$$

Then, if $Z^2 = 2500$, $Z = \sqrt{2500} = 50$. You will notice that this is the same answer we obtained by measurement. You will also notice that our statement $Z^2 = R^2 + X_L{}^2$ is the same as one of our formulas; $Z = \sqrt{R^2 + X_L{}^2}$. We usually use the general formula;

$$Z = \sqrt{R^2 + [X_L + (-X_C)]^2}$$

because we always find our total reactive component, X, before we find our squares. In this way, we can solve for any right angle vector diagram without using a measurement solution.

SELF-TEST QUESTIONS

(38) What is a vector?

(39) Draw a simple vector diagram which shows the phase relationship between the voltage across and the current through a coil.

(40) A coil and resistor are connected in series across an ac generator. The voltage drop across the coil is 10 volts and the drop across the resistor is 20 volts. Draw a vector diagram which shows the amplitude and phase relationship of the two voltages.

(41) Find the total voltage from the generator using the vector diagram in Question 40.

(42) Draw a vector diagram which shows the phase relationship between the voltage across and the current through a capacitor.

(43) A coil, resistor, and capacitor are connected in series across an ac generator. The coil drops 12.5 volts, the resistor 15 volts, and the capacitor 7.5 volts. Draw a vector diagram showing amplitude and phase relationship of the 3 voltages.

(44) Find the total voltage supplied by the generator described in Question 43.

(45) State the Pythagorean Theorem.

(46) What is a right triangle?

(47) What quantity is represented by the hypotenuse of a right triangle which has one side representing resistance and the other side representing reactance?

Circuit Calculations

In this lesson and in your previous lessons you have learned many of the important facts concerning ac circuits. You have also studied some of the mathematical procedures that can be used in considering these facts in ac circuit calculations. Although you may feel that at the present time you have mastered all this information, it is quite possible that you will forget many of the details. One of the best ways to prevent forgetting this information and to insure that you really do understand it thoroughly is to get some practice using it.

Therefore, at the end of this section we will present several typical circuit problems and ask you to solve them. Remember that many times it may be necessary to solve for intermediate values in order to obtain the necessary factors for use in finding the answers. If you have learned and understood the mathematics and electronics that you have studied so far, you should be able to obtain all the answers without too much difficulty.

Most of the information in this lesson parallels the technical lessons from 6 through 9. Therefore, the problems will deal mainly with the formulas and data covered in these same technical lessons. If you have any real difficulty with the solutions of these problems, it will be a good idea to review the appropriate subjects in other lessons that you have had. Remember, only by working out the solutions of these problems will you be sure that you have thoroughly understood the material presented. It is very important for you to do this before you continue with your remaining lessons.

To help refresh your memory and prevent your having to look up too much information regarding these problems, we have listed some of the more important formulas that you have studied. If you find that you do not thoroughly remember and understand these useful and common formulas, it will be a good idea to review them before you start working the problems.

Inductance of coils in series:

$$L_T = L_1 + L_2 \pm 2M$$

Inductance of coils in parallel:

$$L_T = \frac{L_1 \times L_2}{L_1 + L_2}$$

Inductive reactance:

$$X_L = 2\pi fL$$

(f = Hertz, L = henrys)

Q of a coil:

$$Q = \frac{X_L}{R}$$

Capacitance of capacitors in series:

$$C = \frac{C_1 \times C_2}{C_1 + C_2}$$

Capacitance of capacitors in parallel:

$$C = C_1 + C_2 + C_3$$

Capacitive reactance:

$$X_C = \frac{1}{2\pi fC} \text{ or}$$

$$X_C = \frac{159000}{fC}$$

where f is in Hertz and C is in mfd.

Ohm's law for ac circuits:

$$E = I \times Z$$

Impedance in ac circuits:

$$Z = \sqrt{R^2 + X^2}$$

Total voltage in ac circuits:

$$E_T = \sqrt{E_R{}^2 + E_X{}^2}$$

Resonant frequency:

$$f = \frac{1}{2\pi\sqrt{L \times C}}$$

or

$$f = \frac{.159}{\sqrt{L \times C}}$$

Turns ratio of transformers:

$$\frac{N_1}{N_2} = \frac{E_1}{E_2}$$

$$\frac{N_1}{N_2} = \frac{I_2}{I_1}$$

$$\frac{N_1}{N_2} = \sqrt{\frac{Z_1}{Z_2}}$$

EXAMPLES

Study the following examples carefully. They are purposely difficult to give you practice and show you how to get the correct answers in an orderly manner. Break down a problem into smaller parts and solve for each part step by step. You will note that it is necessary to find a quantity not asked for but required before you can get the final answer. Review of these examples, although quite advanced for you at this time, will give you the confidence to tackle the exercise at the end of this section. Do not be discouraged should you not get the right answer on the first try.

Be sure you have all quantities in the correct units. If you are asked to find power in a circuit and are given the voltage in volts and the current in milliamps, you must convert the current to amperes by moving the decimal point three places to the left. When you need to find either inductive or capacitive reactance, be sure that you have frequency in Hertz and inductance in henrys or capacitance in farads. To convert microhenrys to henrys or microfarads to farads, move the decimal point six places to the left.

Example 1: Find the current in a series circuit consisting of a 150-ohm resistor, a 292 millihenry coil, and a 7 microfarad capacitor if the line voltage is 120 volts and the line frequency is 150 Hertz.

Solution: To find the current in an ac circuit, you divide the voltage by the impedance of the circuit. This is Ohm's Law for ac circuits.

$$I = \frac{E}{Z}$$

We know E, but do not know Z so we must find Z.

$$Z = \sqrt{R^2 + [X_L + (-X_C)]^2}$$

We know R, but we do not know X_C or X_L so we must find these values.

$$X_L = 2\pi fL$$

and

$$X_C = \frac{1}{2\pi fC}$$

Since we know f, L, and C we can find X_L and X_C. Then we can get Z, and Z is used to find I. Therefore, our first steps in the problem are to find X_L and X_C. Let's do that now:

$$X_L = 2\pi fL$$

$2\pi = 6.28$, f = 150 Hertz and L = 292 millihenrys.

To use our formula, f must be in Hertz, which it is, and L must be in henrys. So we must convert 292 millihenrys to henrys by moving the decimal three places to the left.

Thus, 292 millihenrys = .292 henrys. Now $X_L = 6.28 \times 150 \times .292$.

$$\begin{array}{r} 6.28 \\ \underline{1\,50} \\ 314\,00 \\ \underline{628} \\ 942.\,00 \end{array}$$

$$\begin{array}{r} 942 \\ \underline{.292} \\ 1884 \\ 8478 \\ \underline{1884} \\ 275.064 \end{array}$$

Thus, X_L = 275.064 ohms, which we can round off to 275 ohms.

Now let's find X_C using the formula

$$X_C = \frac{1}{2\pi fC}$$

Again $2\pi = 6.28$, f = 150 Hertz and C = 7 microfarads. C must be in farads so we move the decimal six places to the left.

Thus, 7 microfarads = .000007 farad.

$$X_C = \frac{1}{6.28 \times 150 \times .000007}$$

$6.28 \times 150 = 942$ (we did this when we were finding X_L).

$$\begin{array}{r} 942 \\ \underline{.000007} \\ .006594 \end{array}$$

Now dividing $.006594 \overline{)\,1}$

$$\begin{array}{r} 151.6 \\ 6594\overline{)1000000.} \\ \underline{6594} \\ 34060 \\ \underline{32970} \\ 10900 \\ \underline{6594} \\ 43060 \end{array}$$

Thus, X_C = 151.6 ohms, which we round off to 152 ohms. Now that we know R, X_L, and X_C, we can get Z using

$$Z = \sqrt{R^2 + [X_L + (-X_C)]^2}$$

To add X_L and X_C we must remember they are opposite reactances. X_L is positive and X_C is negative. Thus, when we add we have 275 + (−152) so, because of the negative number, we subtract

$$\begin{array}{r} 275 \\ -152 \\ \hline 123 \end{array}$$

Therefore, $X_L + X_C = 123$ ohms.

Now, squaring 123 we get

$$\begin{array}{r} 123 \\ 123 \\ \hline 369 \\ 246 \\ 123 \\ \hline 15129 \end{array}$$

And squaring R we get

$$\begin{array}{r} 150 \\ 150 \\ \hline 7500 \\ 15 \\ \hline 22500 \end{array}$$

Now, adding $R^2 + [X_L + (-X_C)]^2$

$$\begin{array}{r} 22500 \\ 15129 \\ \hline 37629 \end{array}$$

To find Z we must take the square root of 37629.

```
        1  9  3. 9
      √3,76,29.00
        1
  29  | 276
        261
  383 | 1529
        1149
  3869 | 38000
         34821
```

Thus, Z is 193.9 ohms, which we can round off to 194 ohms.

Now, to find I we use

$$I = \frac{E}{Z}$$

$$I = \frac{120}{194}$$

```
        .618
  194 ) 120.0
        1164
         360
         194
         1660
         1552
          108
```

The line current is .618 amps. Notice how we solved this problem. We worked backwards to find what we had to determine in order to get the required answer and then began by evaluating the terms needed to get the final solution.

Example 2: Find the voltage across the inductance in a series circuit having an inductive reactance of 4 ohms, a capacitive reactance of 12 ohms and a resistance of 6 ohms, if the voltage applied to the circuit is 50 volts at 60 Hertz.

Solution: To find the voltage across the coil we need to know the current flowing in the circuit. Then we can use the formula

$$E_L = I \times X_L$$

To get I, we need the impedance. Then we can use

$$I = \frac{E}{Z}$$

We start by getting Z.

$$Z = \sqrt{R^2 + [X_L + (-X_C)]^2}$$

$X_L = 4$ and $X_C = -12$ so we have

$$\begin{array}{r} 4 \\ +(-12) \\ \hline -8 \end{array}$$

Thus, $Z = \sqrt{6^2 + (-8)^2}$.

Remember the $(-8)^2$ is $(-8) \times (-8)$ and your rules for multiplying signed numbers tell you that $(-8) \times (-8) = 64$

Therefore,

$$\begin{aligned} Z &= \sqrt{36 + 64} \\ &= \sqrt{100} \\ &= 10 \text{ ohms} \end{aligned}$$

Thus, $I = 50/10 = 5$ amps and the voltage across the coil is $E_L = I \times X_L = 5 \times 4 = 20$ volts.

You might wonder about the 60 Hertz. We didn't use this, because we didn't have to. We had the reactance of the coil and capacitor so we could get the impedance directly. Often, in problems and in actual practice, you'll find you have more data than you need. You have to learn to take what information is needed and use it and ignore unneeded information.

SELF-TEST QUESTIONS

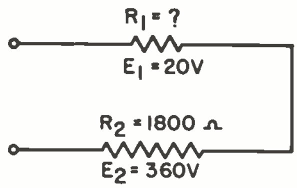
(48) If a 500-ft. roll of copper hookup wire has a resistance of 30 ohms, how much resistance will an 850-ft. roll of the same wire have?

(49) What ac voltage will be needed to force a current of .02 amps through an 8K-ohm resistor?

(50) What value of ac current will flow through a 10-henry coil with negligible resistance if the voltage supplied is 120-volt, 60 Hertz ac?

(51) Find the current sent through a .03-henry choke coil by an ac voltage of 188.4 volts at 1 kHz.

(52) In the circuit shown below, find the resistance of R_1.



(53) A 10 mfd capacitor draws 300 ma of current at a frequency of .4 kHz. What is the voltage drop across the capacitor?

(54) What turns ratio should we have for a transformer that we wish to use to match a source impedance of 490 ohms to a load of 10 ohms?

(55) What is the total inductance of two coils connected in series if the inductance of one is .2 henrys and the other is .8 henrys and their mutual inductance is zero?

(56) What is the impedance of a series ac circuit having an inductive reactance of 14 ohms, a resistance of 6 ohms, and a capacitive reactance of 6 ohms?

(57) What is the resonant frequency of a series circuit containing a 500 picofarad capacitor, a 150 microhenry choke and a 10-ohm resistor?

(58) What is the impedance of an ac circuit containing a 3-ohm resistor in series with an inductive reactance of 7 ohms?

(59) If three capacitors of 1, 3, and 5 microfarads are connected in parallel, what will the total capacitance be?

(60) If we assume that a coil has a negligible resistance and that 215 ma of current is forced through it

by a supply voltage of 110 volts at 25 Hertz what is the inductance of the coil?

(61) If we supply 240 volts at 60 Hertz to a capacitor and obtain a current of 452 ma, what is the capacitance?

(62) If a circuit containing a 175-ohm resistor is connected in series with a 5-microfarad capacitor across a source of 150 volts at 120 Hertz, what current will flow in the circuit?

(63) If a series circuit contains a capacitive reactance of 10 ohms, an inductive reactance of 25 ohms, and a resistance of 15 ohms, what will the phase angle be?

(64) What is the capacitive reactance of a capacitor at a frequency of 1200 kHz if its reactance is 300 ohms at 680 kHz?

(65) If a series circuit has a resistance of 4 ohms, an inductive reactance of 4 ohms, and a capacitive reactance of 1 ohm, and it is supplied with an ac voltage of 50 volts, what will the voltage drop across the inductance be?

Answers to Self-Test Questions

(1) The radical sign.

(2) 25 ohms.

$$Z = \sqrt{R^2 + X^2}$$

$$= \sqrt{(15)^2 + (20)^2}$$

$$= \sqrt{225 + 400}$$

$$= \sqrt{625}$$

$$= \begin{array}{rl} & \ \ 2\ \ \ 5 \\ & \sqrt{\ 6,25} \\ & -4 \\ 45 & \overline{)2\ 25} \\ & -2\ 25 \end{array}$$

$Z = 25$ ohms

(3) Approximately 1.414.

$$\begin{array}{rl} & 1.\ \ 4\ \ 1\ \ 4 \\ & \sqrt{2.00,00,00} \\ & -1 \\ 24 & \overline{)1\ 00} \\ & -96 \\ 281 & \overline{)4\ 00} \\ & -2\ 81 \\ 2824 & \overline{)1\ 19\ 00} \\ & -1\ 12\ 96 \end{array}$$

(4) $\dfrac{1}{5}\sqrt{\dfrac{25}{625}} = \dfrac{1}{25} = \dfrac{\sqrt{1}}{\sqrt{25}} = \dfrac{1}{5}$

(5) 111. This is the same as saying, "What is the square root of 12,321."?

$$\begin{array}{rl} & \text{“}\ 1\ \ 1\ \ 1 \\ & \sqrt{1,23,21} \\ & -1 \\ 21 & \overline{)23} \\ & -21 \\ 221 & \overline{)2\ 21} \\ & -2\ 21 \end{array}$$

(6) 21.2.

$$\begin{array}{rl} & 2\ \ 1.\ 2 \\ & \sqrt{4,49.44} \\ & -4 \\ 41 & \overline{)49} \\ & -41 \\ 422 & \overline{)8\ 44} \\ & -8\ 44 \end{array}$$

(7) 50 ohms.

$$Z = \sqrt{R^2 + X^2}$$

$$= \sqrt{(40)^2 + (30)^2}$$

$$= \sqrt{1600 + 900}$$

$$= \begin{array}{l} \ \ 5\ \ 0 \\ \sqrt{25,00} \\ -25 \end{array}$$

$Z = 50$ ohms

(8) 663.

$$\begin{array}{rl} & 6\ \ 6\ \ 3 \\ & \sqrt{43,95,69} \\ & -36 \\ 126 & \overline{)7\ 95} \\ & -7\ 56 \\ 1323 & \overline{)39\ 69} \\ & -39\ 69 \end{array}$$

(9) .021.

$$\begin{array}{r|l} & \ .0\ \ 2\ \ 1 \\ & \sqrt{.00,04,41} \\ & -00 \\ 2 & \quad 04 \\ & \ -04 \\ 41 & \quad\ \ 41 \\ & \quad -41 \end{array}$$

(10) 5.01.

$$\begin{array}{r|l} & \quad 5.\ 0\ \ 1 \\ & \sqrt{25.10,01} \\ & -25 \\ 1001 & \quad 10,01 \\ & \ -10\ 01 \end{array}$$

(11) A ratio is a comparison of two numbers or similar quantities. For example, the efficiency of a motor is expressed as the ratio of output power to input power. A proportion is a comparison of two ratios. To be more precise, a proportion is a mathematical statement that two ratios are equal.

(12) $\text{Efficiency} = \dfrac{\text{Output power}}{\text{Input power}}$

$= \dfrac{2 \text{ horsepower}}{1.75 \text{ kW}}$

(1 horsepower equals approximately 750 watts.)

$= \dfrac{2 \times 750}{1.75 \times 1000}$

$= \dfrac{1500}{1750}$

$= \dfrac{6}{7}$

$= .857$ or 85.7%

(13) $\dfrac{6V}{18V} = \dfrac{1}{3} = 1{:}3$

(14) $\dfrac{250 \text{ mV}}{1V} = \dfrac{250 \text{ mV}}{1000 \text{ mV}} = \dfrac{1}{4} = 1{:}4$

(15) $\text{Efficiency} = \dfrac{\text{Output power}}{\text{Input power}}$

$= \dfrac{3\text{hp}}{2.5\text{kW}}$

(1 horsepower equals approximately 750 watts.)

$= \dfrac{3 \times 750}{2.5 \times 1000}$

$= \dfrac{2250}{2500}$

$= \dfrac{9}{10}$

$= .9 = 90\%$ efficiency

(16) $\dfrac{A_1}{A_2} = \dfrac{B_1}{B_2}$

$\dfrac{15}{6} = \dfrac{90}{B_2}$

$15B_2 = 540$
$B_2 = 36$

(17) $\dfrac{A_1}{A_2} = \dfrac{B_2}{B_1}$

$\dfrac{A_1}{4} = \dfrac{150}{100}$

$$100A_1 = 600$$
$$A_1 = 6$$

(18) The battery voltage is equal to the sum of the voltage drops across the two resistors. The voltage across R_1 is 10 volts. Therefore, we can find the voltage across R_2 using direct proportion:

$$\frac{E_1}{E_2} = \frac{R_1}{R_2}$$

$$\frac{10}{E_2} = \frac{1200}{240}$$

$$1200E_2 = 2400$$
$$E_2 = 2 \text{ volts}$$

The applied battery voltage is the sum of $E_2 + E_1$ or $2V + 10V = 12V$.

(19) Since current is inversely proportional to resistance, our equation is:

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}$$

$$\frac{2}{2.2} = \frac{R_2}{66}$$

$$2.2R_2 = 132$$
$$R_2 = 60 \text{ ohms}$$

(20) To add two or more numbers with like signs, find the sum of the numbers as you would in ordinary arithmetic and place the sign of the numbers added in front of this sum.

(21) The sum of two signed numbers with unlike signs is equal to the difference between the two numbers, preceded by the sign of the larger number.

(22) To subtract signed numbers change the sign of the subtrahend and then add the two numbers according to the rules for adding signed numbers.

(23) -3

(24) (a) -23 (b) $+9$ (c) $+26$ (d) -22

(25) (a) -9 (b) $+17$ (c) -70 (d) -46

(26) The product of two numbers with like signs is always positive. The product of two numbers with unlike signs is always negative.

(27) If the numbers have like signs, the quotient is always positive. If the numbers have unlike signs, the quotient is always negative.

(28) (a) $+42$ (b) -18 (c) -187 (d) $+36$

(29) (a) -4 (b) $+24$ (c) -3 (d) $+43$

(30) A positive number.

(31) (a) -8 (b) $+24$ (c) -27 (d) 0

(32) (a) $+138$ (b) -22 (c) -59 (d) -12

(33) (a) $+126$ (b) -120 (c) -56 (d) $+120$

(34) (a) $+43$ (b) -11 (c) $-.25$ (d) -16

(35) (a) $(-2)^4 = (-2) \times (-2) \times (-2) \times (-2) = +16$

(b) $(+2)^4 = (+2) \times (+2) \times (+2) \times (+2) = +16$

(c) $(-3)^3 = (-3) \times (-3) \times (-3) = -27$

(d) $(+3)^3 = (+3) \times (+3) \times (+3) = +27$

(36) (a) 3.247

(b) 762,500

(c) .0023

(d) .0967

(e) 2,300,000

(f) .9327

(g) 8200

(h) .00032

(i) 75.6

(37) (a) 2.68

(b) -2.25

(c) 1.97×10

(d) -7.57×10^{-1}

(e) 2.67×10^{-4}

(f) 5.57×10^{-2}

(g) 3.14×10^{4}

(h) 6.85×10^{-1}

(i) 7.78×10^{2}

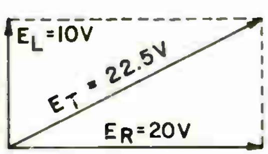
(j) -8.15×10^{-1}

(38) A vector is a straight line of definite length and direction which shows the magnitude and direction or time of a quantity.

(39) See Fig. 27B.

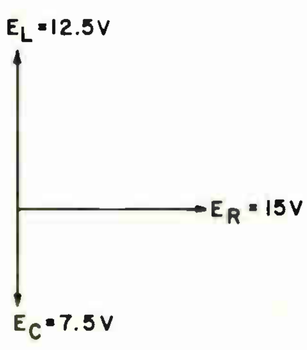
(40)

(41) Approximately 22.5 volts.

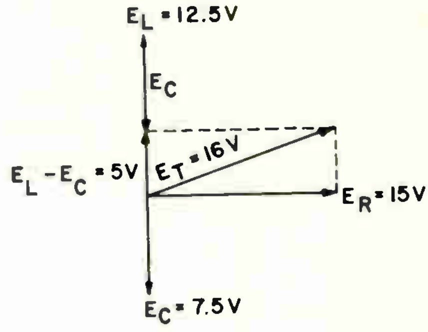


(42) See Fig. 34C.

(43)



(44) Approximately 16 volts.



(45) The Pythagorean Theorem states that the square of the hypotenuse of any right triangle is equal to the sum of the squares of the other two sides.

(46) Any triangle which contains one angle that is equal to 90°.

(47) Impedance.

(48) 51 ohms.

L_L = Length of long wire

L_S = Length of short wire

R_L = Resistance of long wire

R_S = Resistance of short wire.

Now, since the resistance of a wire is directly proportional to its length, we can establish a proportion.

$$\frac{L_L}{L_S} = \frac{R_L}{R_S}$$

$$\frac{\overset{17}{\cancel{850}}}{\underset{10}{\cancel{500}}} = \frac{R_L}{30}$$

$$10R_L = 510$$

$$R_L = 51 \text{ ohms}$$

(49) 160 volts. First convert 8K-ohms to ohms.

8K-ohms = 8000 ohms

Then use Ohm's Law to find the voltage.

$E = IR$
$E = .02 \times 8000$
$E = 160$ volts

(50) .0318 amps. Use Ohms's Law for ac circuits.

$$I = \frac{E}{Z}$$

The only impedance in the circuit is the X_L of the coil. Therefore, $I = E/X_L$, so we must first find X_L.

$X_L = 2\pi fL$
$f = 60$ Hertz
$L = 10$ henrys
$X_L = 6.28 \times 60 \times 10$
$X_L = 3768$ ohms

Now,

$$I = \frac{E}{X_L}$$

$$I = \frac{120}{3768}$$

$I = .0318$ amps

(51) 1 amp.

$X_L = 2\pi fL$
$f = 1$ kHz or 1000 Hertz
$L = .03$ henry
$X_L = 6.28 \times 1000 \times .03$
$X_L = 188.4$ ohms

$$I = \frac{E}{X_L}$$

$$I = \frac{188.4}{188.4}$$

$I = 1$ amp.

(52) $R_1 = 100$ ohms.

$$\frac{R_1}{R_2} = \frac{E_1}{E_2}$$

$$\frac{R_1}{1800} = \frac{\overset{1}{\cancel{20}}}{\underset{18}{\cancel{360}}}$$

$18R_1 = 1800$
$R_1 = 100$ ohms

(53) 12 volts. First find X_C.

$$X_C = \frac{.159}{fC}$$

$f = .4$ kHz or 400 Hertz
$C = 10$ mfd or .00001 farad

$$X_C = \frac{.159}{400(.00001)}$$

$$X_C = \frac{.159}{.004}$$

$X_C = 39.75$ ohms or approximately 40 ohms

$E = IX_C$
$I = 300$ ma or .3 amps
$E = .3 \times 40$
$E = 12$ volts

(54) 7 to 1.

$$\frac{N_1}{N_2} = \sqrt{\frac{Z_1}{Z_2}}$$

$$\frac{N_1}{N_2} = \sqrt{\frac{490}{10}}$$

$$\frac{N_1}{N_2} = \sqrt{49}$$

$$\frac{N_1}{N_2} = 7$$

Therefore, the turns ratio is 7 to 1.

(55) 1 henry.

$$L_T = L_1 + L_2 \pm 2M$$
$$L_T = .2 + .8$$
$$L_T = 1 \text{ henry}$$

(56) 10 ohms.

$$Z = \sqrt{R^2 + [X_L + (-X_C)]^2}$$

$$Z = \sqrt{6^2 + [14 + (-6)]^2}$$

$$Z = \sqrt{6^2 + 8^2}$$

$$Z = \sqrt{36 + 64}$$

$$Z = \sqrt{100}$$

$$Z = 10 \text{ ohms}$$

(57) Approximately 580 kHz.

$$f = \frac{.159}{\sqrt{LC}}$$

$$f = \frac{.159}{\sqrt{1.5 \times 10^{-4} \times 5 \times 10^{-10}}}$$

$$f = \frac{.159}{\sqrt{7.5 \times 10^{-14}}}$$

$$f = \frac{.159}{2.74 \times 10^{-7}}$$

$$f = 5.8 \times 10^5$$

$$f = 580 \text{ kHz}$$

(58) 7.6 ohms.

$$Z = \sqrt{R^2 + X_L{}^2}$$

$$Z = \sqrt{3^2 + 7^2}$$

$$Z = \sqrt{9 + 49}$$

$$Z = \sqrt{58}$$

$$Z = 7.6 \text{ ohms}$$

(59) 9 microfarads.

$$C_T = C_1 + C_2 + C_3$$
$$C_T = 1 + 3 + 5$$
$$C_T = 9 \text{ mfd.}$$

(60) Approximately 3.3 henrys. First find the X_L of the coil.

$$X_L = \frac{E}{I} = \frac{110}{.215} = 512 \text{ ohms}$$

Now, since $X_L = 2\pi fL$
Then,

$$L = \frac{X_L}{2\pi f}$$

$$L = \frac{512}{6.28(25)}$$

$$L = \frac{512}{157}$$

$$L = 3.3 \text{ henrys.}$$

(61) 5 microfarads. First find the X_C of the capacitor.

$$X_C = \frac{E}{I} = \frac{240}{.452} = 531 \text{ ohms}$$

And since $X_C = \dfrac{.159}{fC}$

Then

$$C = \frac{.159}{fX_C}$$

$$C = \frac{.159}{60(531)}$$

$$C = \frac{.159}{31860}$$

$$C = .000005 = 5 \text{ mfd}$$

(62) .472 amps. I = E/Z, so we must first find Z. But to find Z we must find X_C.

$$Z = \sqrt{R^2 + X_C^2}$$

$$X_C = \frac{.159}{fC}$$

$$X_C = \frac{.159}{120(.000005)}$$

$$X_C = \frac{.159}{.0006}$$

X_C = 265Ω. Now find Z.

$$Z = \sqrt{R^2 + X_C^2}$$

$$Z = \sqrt{175^2 + 265^2}$$

$$Z = \sqrt{30625 + 70225}$$

$$Z = \sqrt{100850}$$

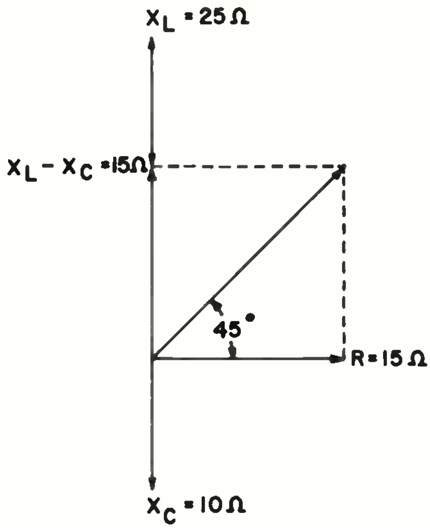
Z = 318 ohms. Now find I.

$$I = \frac{E}{Z}$$

$$I = \frac{150}{318}$$

I = .472 amps.

(63) 45° inductive.



(64) 170 ohms. There are two ways to work this problem. We will consider both ways. The easier method is to set up and solve a proportion. We know that X_C is inversely proportional to frequency. Therefore,

$$\frac{X_{C_1}}{X_{C_2}} = \frac{f_2}{f_1}$$

$$\frac{X_{C_1}}{300} = \frac{680}{1200}$$

$$1200\, X_{C_1} = 204000$$

$$X_{C_1} = \frac{204000}{1200} = 170 \text{ ohms.}$$

The second method is to find the capacitance of X_C and frequency given. Then find X_C of that capacitance at the new frequency.
That is,

$$C = \frac{.159}{fX_C} = \frac{.159}{680{,}000(300)}$$

$$= \frac{.159}{204{,}000{,}000}$$

C = 780 picofarads. Now find the reactance of this capacitance at the new frequency.

$$X_C = \frac{.159}{1{,}200{,}000(.000000000780)}$$

$$X_C = \frac{.159}{.000936}$$

X_C equals approximately 170 ohms. The answer does not work out exactly because we rounded off 2π and the value of C. By working both methods, it becomes obvious that the first method is not only easier, it is also more accurate.

(65) 40 volts. First find the impedance of the circuit.

$$Z = \sqrt{R^2 + [X_L + (-X_C)]^2}$$

$$Z = \sqrt{4^2 + [4 + (-1)]^2}$$

$$Z = \sqrt{16 + 9}$$

$$Z = \sqrt{25}$$

$$Z = 5 \text{ ohms.}$$

Now, find the current in the circuit.

$$I = \frac{E}{Z}$$

$$I = \frac{50}{5}$$

$$I = 10 \text{ amperes.}$$

Now, find the voltage across the coil (E_L).

$$E_L = I(X_L)$$
$$E_L = 10(4)$$
$$E_L = 40 \text{ volts.}$$

Lesson Questions

Be sure to number your Answer Sheet X109.

Place your Student Number on every Answer Sheet.

Most students want to know their grades as soon as possible, so they mail their set of answers immediately. Others, knowing they will finish the next lesson within a few days, send in two sets of answers at a time. Either practice is acceptable to us. However, don't hold your answers too long; you may lose them. Don't hold answers to send in more than two sets at a time or you may run out of lessons before new ones can reach you.

1. Find the square root of:
   (a) 576 (b) 4489 (c) 16384

2. Find the square root of:
   (a) 41616 (b) 73652 (c) .0625

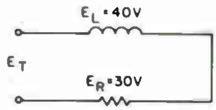
3. Find the value of X in each of the following:
   (a) 4 : 5 :: X : 20 (b) 47 : 329 :: 7 : X

4. Find the value of X in each of the following:
   (a) $\frac{62}{124} = \frac{3}{X}$ (b) $\frac{X}{72} = \frac{1}{12}$

5. Solve the following:
   (a) $(-47) \times (51)$
   (b) $(-23) \times (-17)$
   (c) $(37) \times (-62)$
   (d) $(-11) \times (-7) \times (-6)$

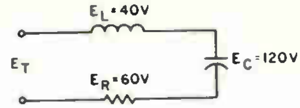
6. Solve the following:
   (a) $72 \div (-8)$
   (b) $(-144) \div (-12)$
   (c) $\frac{(-64)(-7)}{(-8)}$
   (d) $\frac{(-48)(-4)}{(12)}$

7. Find the value of E_T in the circuit at the right by means of a vector diagram. (Show your diagram.)



(OVER)

8. Find the value of E_T in the circuit at the right by means of a vector diagram. (Show your work.)



9. What is the total impedance of a circuit that has an inductive reactance of 124 ohms in series with a 95 ohm resistance?

10. Find the current in a series circuit made up of a coil with an inductance of 325 millihenrys, a capacitor of 4 microfarads, and a resistor of 120 ohms if the source voltage is 120 volts at 100 Hertz.

1" = 40v

E_L = 1"

THE VALUE OF KNOWLEDGE

Knowledge comes in mighty handy in the practical affairs of everyday life. For instance, it increases the value of your daily work and thereby increases your earning power. It brings you the respect of others. It enables you to understand the complex events of modern life, so you can get along better with other people. Thus, by bringing skill and power and understanding, knowledge gives you one essential requirement for true happiness.

But what knowledge should *you* look for? The first choice naturally goes to knowledge in the field of your greatest interest – electronics. Become just a little better informed than those you will work with, and your success will be assured.

It pays to know – but it pays even more to know how to use what you know. You must be able to make *your* knowledge of value to others, and to the rest of the world, in order to get cash for knowledge.

The NRI Course gives you knowledge, and in addition shows you how to use what you learn. Master thoroughly each part of your course, and you'll soon be getting cash for *your* knowledge.

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STUDY SCHEDULE

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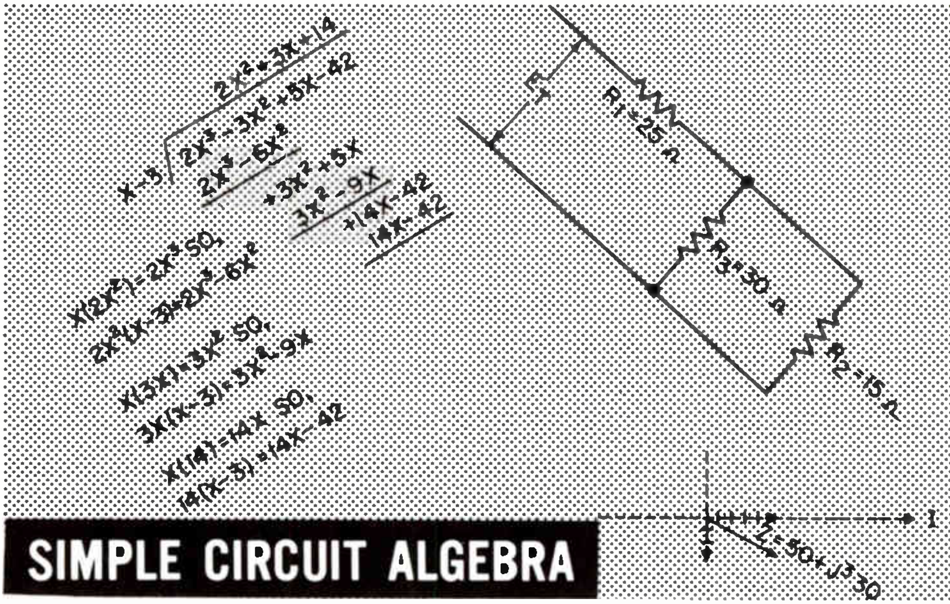


SIMPLE CIRCUIT ALGEBRA

In many ways the study of tube and transistor circuits is just an extension of your study of ac and dc circuits. The tuning circuits, coupling circuits, filter circuits, and voltage dividers that are used with tubes and transistors are simply special designs of circuits that you are already familiar with to a certain extent. In these circuits, coils and capacitors and resistors of various values are assembled in different series and parallel combinations to give special effects. Either ac, dc, or both may be applied to these circuits depending on their function and use.

For the most part, the basic arithmetic and operations with signed numbers and vectors that you studied for use with ac and dc circuit calculations can be applied in the calculations for tube and transistor circuits. However, as the equipment and circuits that you study become more involved, it will become increasingly difficult to keep up with their operation and maintenance if you rely only on the mathematical processes that you have already learned. You will need many new shortcuts and some new mathematical tools to keep your studies and work in electronics simple and straightforward.

In your previous lessons, you saw how important vectors are in analyses and calculations dealing with ac circuits. As you continue with your studies, these simple vectors will become even more important. However, the simple measurement solutions that you have been using to solve vector problems will become very awkward to use as the circuits become more complex. In addition to requiring careful construction and measurement, they require a lot of space and can become very involved, especially in parallel circuits. Although we can overcome these problems to some extent by using the Pythagorean Theorem, it also has its limits.

However, there is a handy method for working with ac circuit vectors so they can be solved mathematically. It involves using a special tool known as the "j" operator. This simple operator allows us to easily add, subtract, multiply, and divide vectors, regardless of their complexity. No course in electronics can really be considered complete without at least a working knowledge of this method of determining the solutions to circuit problems.

In order to use the j operator successfully, you should have at least a basic understanding of the essentials of another mathematical process known as algebra. As many of you already know, algebra is simply a form of mathematics that simplifies complex operations in arithmetic by using letters. Through these letter solutions of practical problems, we are able to speed up and simplify operations that would take a long time and involve a lot of tedious work if we used numbers alone. Thus, time spent learning the fundamentals of algebra will be worthwhile.

Therefore, in this lesson on circuit calculations, you will learn to use and apply the fundamentals of algebra and the j operator in electronic circuit calculations. If you have never studied these subjects, you may be a little uneasy about tackling them. However, you have already seen that math makes a lot more sense and becomes much easier when you have a practical use for it, such as your work in electronics.

As you study this lesson, remember these subjects are like all the others. All you have to do is learn a few rules and get some practice using them. Once you have done this, you have accomplished your goal of learning new processes that will help make your work much easier and more efficient. If you have already studied these subjects, you will find that this lesson will give you a good review and some valuable pointers on circuit applications.

Basic Algebra

Anyone who can add and subtract, multiply and divide, and perform the other operations of basic arithmetic should find algebra easy to understand. The only difference between algebra and arithmetic is that in algebra you work with letters as well as with numbers. Therefore, we can consider that algebra is simply arithmetic with letters. Consequently, all we have to do is get used to applying the rules of arithmetic to combinations of letters.

You are probably wondering just how we can use letters to compute with, because letters have no indicated values such as numbers have. We merely select letters and let them represent the values that we wish to work with. For example, you are already accustomed to working with formulas such as Ohm's Law which states that $E = I \times R$. Here we have simply used certain letters to indicate various quantities in our circuit and have used these letters in an equation that represents their relationship in an electrical circuit. This is algebra.

In ordinary arithmetic, the next step would be to substitute the actual number values in place of the letters and solve the problem. In algebra, we may also do this, but many times we will find that it is easier to work with the letters awhile before we substitute the numbers. In this section of the lesson, we will start at the very beginning and learn just what algebra is all about.

THE LANGUAGE OF LETTERS

In your earlier lessons you learned how to substitute numbers for letters in certain formulas. As you continue in your work in electronics, it will be handy to know more about the process of computing with both numbers and letters. You are already familiar with this sort of reasoning: "If I have one resistor and you give me another resistor, I will have two resistors." Or, "If you give me four capacitors and then someone else gives me five more capacitors, I will have nine capacitors." This is simply addition as applied to physical things.

Addition, as you know, can only be performed with like things. We can add resistors to resistors, or capacitors to capacitors, but we can never add a number of resistors to a number of capacitors and get a sensible answer. Thus, the result of any addition is the sum of the number of things added followed by their name. We know that it is possible to represent various quantities by letters. For example, suppose that we receive three orders of parts as follows:

(1) 5 choke coils, 7 resistors, 4 capacitors.

(2) 3 choke coils, 2 resistors, 3 capacitors.

(3) 7 choke coils, 4 resistors, 5 capacitors.

If we want to know the total number of parts received in these orders we can simply add the number of like parts in each order together to give us a total of 15 choke coils, 13 resistors, and 12 capacitors.

However, in doing this we must be very careful to keep the numbers associated with the coils separated from the numbers associated with the resistors, and resistors separated from the capacitors, etc. Otherwise, there is a possibility that we might get the numbers confused and start adding the coils to one of the other parts. To keep them separated, we can list them carefully in separate columns and write down the names of the parts as we just did. While this will work very well, you can easily see that it would require a lot of tedious writing if we needed to add a number of different things.

A much simpler way would be to choose abbreviations for the parts. For example, we could decide to let the letter "c" stand for the coils, the letter "r" stand for the resistors, and then let another letter such as "a" stand for the capacitors to be sure that the coils and capacitors are kept separate, since they both begin with "c." Now our additions and notations would look something like this (where c = choke coils, r = resistors, a = capacitors):

(1) 5c, 7r, 4a

(2) 3c, 2r, 3a

(3) 7c, 4r, 5a

15c, 13r, 12a

By doing this we could save ourselves a lot of work and still perform the addition in such a way that it would make sense and eliminate the chance of getting the different parts confused with each other.

In this way, it is possible to represent any number of different things or quantities with letters. Once we have chosen letters to indicate quantities, we can add or subtract them, or multiply and divide them by simply performing the operations with the letters instead of using the quantities themselves.

For example, let's say that we choose the letter "a" to represent a quantity. This means that the letter "a" written by itself will always mean "1 a," the term "2a" will mean "2 a's," "3a" will mean "3 a's," etc. Thus, a term such as "5a" means that the quantity represented by the letter "a" is to be multiplied by 5. When we get ready to substitute and actually find the value or the meaning of the term "5a," we will need to know what the letter "a" stands for. In the meantime, we can go along and work with the term itself without worrying about what it means.

Another example of a term that we are likely to meet is one such as "6ab." This simply means that a quantity represented by "a" is to be multiplied by a quantity represented by "b." Then the product of quantity "a" times quantity "b" is to be multiplied by 6. Thus, 6ab means "6" times "a" times "b" or 6 X a X b. In working with algebra we usually do not use the symbol X as a "times" sign because it can be confused with the letter "x" which can be used to represent a quantity. Sometimes dots are used between letters to indicate multiplication, as "6 · a · b," but generally the letters are simply written close together without any sign between them.

Before we go on to learn the rules of using letters, there are some special names given letter combinations that we should be familiar with. A letter by itself is called a "term." Thus "a" is a term, "b" is a term, "x" is a term, etc. The indicated product of a group of letters such as "6ab" is also called a term. In this term, the 6, the a, and the b, are all factors of the indicated product, just as 6 and 8 are factors of 48.

We may have "like" terms or "unlike" terms in algebra. For example, 6ab, 3ab, and 5ab are all like terms because their letter factors are all the same. Terms such as 6xy, 10ax, 7ab, are unlike terms because their letter factors are not all the same.

The number tells you how many times the letter term is to be multiplied. It is called the "numerical coefficient" of the term or, more simply, just the "coefficient." Thus, a term may be a single letter, or it may consist of an indicated product between two or more letters, or between one or more letters and a numerical coefficient. Remember, a term may consist of one or more letters and numbers, but if more than one letter appears, it is a term only if multiplication is indicated. The expression, "a + b", is two separate terms because addition is indicated between the two letters. Likewise, an expression such as 6ab − cd is also made up of two terms.

An algebraic expression made up of one term such as "7xyz" is called a "monomial" term. An expression made up of two or more terms, such as 6ir + 7abc, or 8yx + 7cd − ab is called a "polynomial." If a polynomial has only two terms, it is usually called a "binomial" and one with three terms is often called a "trinomial." Thus, ab + ac is a binomial, and ab + ac + ad would be called a trinomial. We have no special names for polynomials that consist of more than three terms. For example, an expression such as xy + ab − ac − yb would simply be called a four-term polynomial.

Now, with the names given to these various expressions firmly in mind, let's see how to perform simple arithmetic with letters.

Addition of Letters. The rules for performing arithmetic with letters are the same as those with numbers. We can add like terms to each other, but we cannot add unlike terms. The rules for working with signed numbers also apply to working with letters. Likewise, the rules of order for performing a series of operations apply to letter arithmetic, or algebra, just as they do to numbers.

A term consisting of a letter with a coefficient, such as 5a, means that "a" is to be taken five times, or a+a+a+a+a. A term such as 6a means a+a+a+a+a+a. Therefore, to add 5a and 6a together really means (a+a+a+a+a) + (a+a+a+a+a+a), or a total of 11 a's which we would write as 11a. Accordingly, we can say that 5a + 6a = 11a. We can perform the indicated addition because the letter factors are the same and we can add like things together. Notice that in adding these like terms, we simply added their numerical coefficients (6 and 5) and used this sum as the new coefficient for the common letter. Thus, *the sum of like terms is the sum of the coefficients of the terms followed by the common letters.* For example,

$$6ab + 3ab = 9ab$$

$$4abc + 3abc + 5abc = 12abc$$

$$xy + 3xy + 8xy = 12xy$$

When working with unlike terms, however, we can only indicate the addition to be performed. Thus, a + b can only be written as a + b. Likewise, 6a + 5b must remain as 6a + 5b as far as the addition is concerned. Thus, addition of unlike terms always results in a polynomial term. Consequently, when we have addition indicated in a problem such as

$$6a + 7b + 9ab + 4a + 3b + b,$$

we would proceed as follows.

First, we would arrange the terms so that all the like terms were grouped together. Thus, we would have 6a + 4a + 7b + 3b + b + 9ab. Now, adding the like terms gives us

$$6a + 4a = 10a,$$

and

$$7b + 3b + b = 11b.$$

Since we now have 10a + 11b + 9ab, which are all unlike terms and cannot be added any further, our answer to our problem of

$$6a + 7b + 9ab + 4a + 3b + b$$

is simply

$$10a + 11b + 9ab.$$

Thus, to add a group of terms in algebra arrange the like terms so that they are together, then add all the like terms by adding their coefficients and then use these sums in an indicated addition of the unlike terms. The process of rearranging and adding the like terms is often called "collecting" terms.

In working with letters, we will often run into terms with negative signs just as in working with numbers. We handle these signed algebraic terms like signed numbers. For example, to add two terms with like signs, we add the coefficients and use the common sign in front of the sum. Thus, (−6ab) + (−4ab) would be equal to −10ab, just as +3ab plus +4ab would equal +7ab.

If we have to add terms with unlike signs, we simply find the difference between the coefficients and use the sign of the largest coefficient. Thus,

$$+5c + (-3c) = +2c$$

and

$$-7cd + 4cd = -3cd.$$

In a more complicated problem that consists of like and unlike terms as well as like and unlike signs, we would simplify the problem by collecting all like terms with like signs and then perform the addition. Thus, for a problem such as

$$4c + (-9d) + 6e + (-3e) + 12d + (-4x) + 2c + (-c) + 3d + 4e + (-3c) + (-3d),$$

we would first collect all our like terms and like signs as follows:

$$4c + 2c + (-c) + (-3c)$$

$$+ 12d + 3d + (-3d) + (-9d)$$

$$+ 6e + 4e + (-3e) + (-4x).$$

$$+ (-4x).$$

Then, combine like terms:

$$6c + (-4c)$$

$$+ 15d + (-12d)$$

$$+ 10e + (-3e) + (-4x).$$

$$+ (-4x).$$

Now,

$$6c + (-4c) = 2c$$

$$15d + (-12d) = 3d$$

$$10e + (-3e) = 7e$$

$$+ (-4x) = -4x$$

and our answer would be

$$2c + 3d + 7e + (-4x)$$

or simply $2c + 3d + 7e - 4x$.

We will also find terms like this to add:

$$(6ab-7xy) + (5ab-xy) + (-3ab+4xy).$$

Here we have to add three binomials but the terms in each binomial are alike, so we can add them quite easily like this:

$$\begin{array}{rr} & 6ab - 7xy \\ + & 5ab - xy \\ & -3ab + 4xy \\ \hline & 8ab - 4xy \end{array}$$

Or, collecting terms like this:

$$6ab + 5ab + 4xy - 3ab - 7xy - xy$$

then:

$$11ab + 4xy - 3ab - 8xy$$

and:

$$11ab - 3ab + 4xy - 8xy = 8ab - 4xy.$$

Now suppose we had a problem like this:

$$(4x - 3y + 6c) + (-3x + 2y - 3d) + (2x - 7c + 2d).$$

Here we have three trinomials with terms that are not all alike. In a case like this we can set up our problem as follows:

$$\begin{array}{rrrrr} & 4x & - 3y & + 6c & \\ + & -3x & + 2y & & - 3d \\ & 2x & & - 7c & + 2d \\ \hline & 3x & - y & - c & - d \end{array}$$

Since we cannot add unlike terms, this is as far as we can go with our answer. Notice that we could also have proceeded like this:

$$(4x - 3y + 6c) + (-3x + 2y - 3d) + (2x - 7c + 2d)$$

Collecting like terms and like signs:

$$4x + 2x - 3x + 2y - 3y + 6c - 7c + 2d - 3d$$

Then:

$$3x - y - c - d$$

which is the same answer we got before.

Now you should be able to add the following terms without any trouble:

(1) $3x - 2y + 4z + 2x + 8y - 2z + 12x + y + z =$

(2) $5ab - 6xy - 3ab + 12xy + 3ax - 5xb + ab - 3xy - 2xb =$

(3) $(12a + 6c - 3d) + (-20a + 8c - 5d) + (10a - 2c + d) =$

Answers:

(1) $17x + 7y + 3z$

(2) $3ab + 3xy + 3ax - 7xb$

(3) $2a + 12c - 7d$

Subtraction of Letters. When you

learned to subtract signed numbers, you found by experimenting with numbers of various signs that a simple rule would apply to all subtraction with signed numbers. This rule stated that: *To subtract signed numbers, change the sign of the number in the subtrahend (the number you are subtracting) and then proceed as in adding signed numbers.* Following this rule, + 7 minus − 6 would be handled this way:

$$\begin{array}{r} +7 \\ \underline{-\ (-6)} \end{array} = \begin{array}{r} +7 \\ \underline{+6} \\ +13 \end{array}$$

Thus, $+7 - (-6) = 13$.
To prove this, add the subtrahend, − 6, and the answer, + 13, which gives $-6 + 13 = +7$, which is the minuend. Likewise, $-7 - (-6) =$

$$\begin{array}{r} -7 \\ \underline{-\ (-6)} \end{array} = \begin{array}{r} -7 \\ \underline{+6} \\ -1 \end{array}$$

The proof is that $-1 + (-6) = -7$.

Subtracting terms in algebra is just like subtracting signed numbers in arithmetic. Thus, if we want to subtract 6a from 8a we would have: + 8a minus + 6a, which is written (+ 8a) − (+ 6a). Now, changing both signs in front of 6a, we get

$$(+8a) + (-6a).$$

Adding:

$$\begin{array}{r} +8a \\ \underline{-6a} \\ +2a \end{array}$$

To prove this, we add + 6a (the subtrahend) and + 2a which gives us the + 8a that we started with. As with addition in algebra, we can subtract only like terms. Thus, we can state a rule for subtraction in algebra which is: *To subtract in algebra, change the sign of the terms in the subtrahend and then add the coefficients of the like terms.*

This rule for subtraction in algebra holds true for either single terms or for polynomials. For example, to subtract (2a − 2b − 3c) from (3a − 4b + 5c) we would first change the sign of the subtrahend. 2a − 2b − 3c then becomes − 2a + 2b + 3c. Now, we proceed to add:

$$(3a - 4b + 5c) + (-2a + 2b + 3c)$$

collecting terms

$$= 3a - 2a - 4b + 2b + 5c + 3c$$

$$= a - 2b + 8c.$$

Or, we could set it up like this:

$$\begin{array}{lr} & 3a - 4b + 5c \\ \underline{+} & \underline{-2a + 2b + 3c} \\ & a - 2b + 8c \end{array}$$

In either case, the difference is equal to a − 2b + 8c, which is the correct answer. To prove it, add the difference of a − 2b + 8c to the subtrahend 2a − 2b − 3c which, by collecting terms, gives the sum:

$$a + 2a - 2b - 2b + 8c - 3c$$
$$= 3a - 4b + 5c$$

Thus, the important thing to remember in subtracting letters is to change the sign of the subtrahend and then add.

Now that we have seen how to add and subtract with letters, let's prove that what

LETTERS	NUMBER SUBSTITUTES
(5a+3b)+(-2a-6b)	[5(115)+3(95)]+[-2(115)-6(95)]
=5a-2a+3b-6b	=(575+285)+(-230-570)
=3a-3b	= 860-800=60

Fig. 1. Substituting numbers in place of letters in addition.

we are doing with the letters is correct by substituting numbers in place of the letters. Suppose that we want to add 5a + 3b to − 2a − 6b. Collecting terms, this gives us:

$$5a - 2a + 3b - 6b = 3a - 3b.$$

Now, let's substitute some numbers in this same problem. For example, suppose that we have chosen the letter "a" to represent "115" and the letter "b" to represent "95." We would set up the problems side by side, one using the coefficients and letters, and the other using the number values as shown in Fig. 1.

In this way, we find that the term answer is 3a − 3b, while the answer we got by substituting numbers is 60. Therefore 3a − 3b must be equal to 60. To prove this substitute the letter values in our answer 3a − 3b. Doing this, we have:

$$\begin{aligned} &3a - 3b \\ &= 3(115) - 3(95) \\ &= 345 - 285 \\ &= 60 \end{aligned}$$

Thus, if we substitute the numbers in place of the letters in the beginning, we get an answer of 60. If we wish to work with the letters as long as we can, we get an answer 3a − 3b. However, we find that this is also equal to 60 when we substitute at the end of the problem. Since we get an answer of 60 either way, our process of adding letters must be correct.

Now, let's check a problem in subtraction the same way. Again, let a = 115 and b = 95. This time the problem is to subtract 3a − 2b from 5a − 7b as shown in Fig. 2. As you can see, the answer with the letters is 2a − 5b and the answer with the number substitutes is −245. Now let's substitute in our letter answer to see if we also get −245 for our final solution. Doing this, we have:

LETTERS:	NUMBER SUBSTITUTES:
FROM 5a − 7b TAKE 3a − 2b	5(115) − 7(95) TAKE 3(115) − 2(95)
5a − 7b − (3a − 2b)	5(115) − 7(95) − [3(115) − 2(95)]
CHANGING SIGNS:	
= 5a − 7b + (−3a + 2b)	= 575 − 665 + [−3(115) + 2(95)]
= 5a − 7b − 3a + 2b	= 575 − 665 − 345 + 190
= 5a − 3a − 7b + 2b	= 575 − 345 − 665 + 190
= 2a − 5b	= 230 − 475 = −245

Fig. 2. Substituting numbers in place of letters in subtraction.

$$2a - 5b$$

$$= 2(115) - 5(95)$$

$$= 230 - 475$$

$$= -245$$

Consequently, our method of handling subtraction with letters must also be correct, since in either case we get -245 for the final solution.

Next, let's take a look at multiplying and dividing with letters.

Multiplication of Letters. Multiplication with letters such as 3 times "a" may be indicated simply as 3a, which means "a" taken three times, or a + a + a. Likewise, $2 \times b$ is written 2b, which means "b" taken two times, or b + b. In multiplying two letters together such as "a" × "b" we write "ab", which means "a" taken "b" times, or "b" taken "a" times. For multiplication of two terms with coefficients, such as 3a times 2b, we actually perform the multiplication of the coefficients and then indicate the letter multiplication. We can do this because $3a \times 2b$ really means

$$3 \times a \times 2 \times b$$

$$= 6 \times a \times b$$

$$= 6 \times ab$$

$$= 6ab$$

Following this method,

$$3a \times 4b \times 2c$$

would equal

$$3 \times 4 \times 2 \times a \times b \times c = 24abc.$$

Now suppose that we want to multiply two like terms such as $2a \times 3a$. This can be rewritten as

$$2 \times 3 \times a \times a$$

or

$$6 \times a \times a = 6aa.$$

However, in arithmetic when we wanted to multiply a number by itself such as 5×5, we found that we would simply say, 5^2. The small "2" indicated the 5 was to be raised to the second power (multiplied by itself) or squared, and we called the "2" an exponent. We can also use exponents in algebra to indicate that a letter is to be multiplied by itself. Thus, $2a \times 3a = 6aa$ or $6a^2$. A multiplication such as $a \times a \times a$ is aaa or a^3 (read "a cubed"), and $a \times a \times a \times a \times a$ is aaaaa or a^5 (read "a to the fifth").

Using exponents in this way saves time. For example, 4a times 3ab becomes $4 \times 3 \times a \times a \times b$ or $12a^2b$. Now, there is an interesting thing about exponents that we should know. When we write the letter "a" alone, we really mean "a" taken once or a^1. However, just as we never indicate a coefficient of one, we never indicate an exponent of one. We say that the one is understood. For right now though, let's use the exponent "1" for a moment in order to examine the exponents as we multiply.

When we multiply $a \times a$, we can say that we have $a^1 \times a^1$. We know that this is equal to aa or a^1a^1 or a^2. Now, notice that the exponent "2" in a^2 is the sum of the exponents in the indicated multiplication.

$$a^1a^1 = a^{1+1} = a^2$$

Likewise,

$$a \times a \times a$$

$$= a^1 \times a^1 \times a^1$$

$$= a^{1+1+1}$$

$$= a^3$$

Thus, we have a rule for exponents in multiplication which states that: *To find the product of two or more powers of the same base, add the exponents.* According to this rule, $a^2 \times a^3 = a^{2+3}$ or a^5. If we do it the long way, we find that $a^2 = a \times a$ and

$$a^3 = a \times a \times a.$$

Therefore, $(a \times a) \times (a \times a \times a)$ = a taken five times, which is:

$$a \times a \times a \times a \times a = a^5.$$

Therefore, the rule for adding exponents must be correct.

Accordingly, a multiplication such as $5ab \times 3a^2b$ must equal:

$$5 \times 3 \times a^{1+2} \times b^{1+1}$$

$$= 15 \times a^3 \times b^2$$

$$= 15a^3b^2.$$

Likewise,

$$(3a^2b^3c^4) \times (25ab^2c^3)$$

$$= 3 \times 25 \times a^{2+1} \times b^{3+2} \times c^{4+3}$$

$$= 75a^3b^5c^7$$

Can you multiply $6a^3b^4$ by $5a^2b^3c$? The answer is $30a^5b^7c$.

These few simple steps cover the process of multiplying one single term (monomial) by another. However, in algebra, we must not only consider multiplying one monomial by another monomial, but we must also consider multiplying one polynomial by a monomial, and a polynomial by another polynomial. In considering the multiplication of a polynomial by a monomial, let's go back to our work with numbers for a moment.

For example, consider a number problem such as: $7 \times (3 + 2 + 5)$. We can write this down and solve it in a number of different ways. We can do the addition separately first, which gives us $7 \times (3 + 2 + 5) = 7 \times 10 = 70$, or we can multiply each number by seven and then add. This would give:

$$(7 \times 3) + (7 \times 2) + (7 \times 5) = 21 + 14 + 35$$

which also equals 70. Since this is true with numbers, it must also be true with letters.

Let's multiply the polynomial

$$b + c + d$$

by the monomial a. We would set it up like this:

$$a \times (b + c + d)$$

$$= a(b + c + d)$$

$$= ab + ac + ad$$

Thus, we can say that the product of a monomial and a polynomial is the sum of the products of the monomial and each term of the polynomial. Accordingly,

$$a(b - c + d - e) = ab - ac + ad - ae.$$

Notice that, as with signed numbers,

multiplication of unlike signs in algebra always gives a negative product, while the product of two terms with like signs is always positive. Or,

$$-b(a - c + d - e) = -ab + cb - db + eb.$$

With this in mind, we are ready to do a multiplication problem such as $3a^2b(2a + 3b - 7c)$. This equals

$$(3a^2b \times 2a) + (3a^2b \times 3b)$$

$$+ [(3a^2b) \times (-7c)]$$

$$= 6a^3b + 9a^2b^2 - 21a^2bc$$

Or, we can set up the problem a different way. Let's try it with this problem: $4I^2R(3IR + 5I + 6R)$. Now, multiplying each term of the polynomial by the monomial gives us:

$$4I^2R \times 3IR = 12I^3R^2$$

$$4I^2R \times 5I = 20I^3R$$

$$4I^2R \times 6R = 24I^2R^2$$

The sum of the products is $12I^3R^2 + 20I^3R + 24I^2R^2$.

Multiplying one polynomial by a binomial or by another polynomial is much the same. For example, multiplying the polynomial $(a + b - c)$ by the binomial $(a - b)$, we can think of $(a - b)$ as being a single multiplier. Thus, multiplying $(a - b)$ by each of the terms in the polynomial, we would have $(a - b)a$, then $(a - b)b$, and then $(a - b)(-c)$. Putting them together, we have

$$(a - b)a + (a - b)b + (a - b)(-c).$$

Now, the partial products would be:

$$(a - b)a = a^2 - ab$$

$$(a - b)b = ab - b^2$$

$$(a - b)(-c) = -ac + bc$$

Then, adding these products gives us $a^2 - ab + ab - b^2 - ac + bc$. Collecting terms, we get $a^2 - b^2 - ac + bc$ as the answer. Notice that the $-ab$ and the $+ab$ cancel each other out just as $+2 + (-2)$ would do.

In this way, we can say that the product of

$$(a - b)(a + b - c) = a^2 - b^2 - ac + bc.$$

We can prove that the solution to this problem is correct by substituting any numbers we want in place of the letters. For example, let's have $a = 5$, $b = 3$, and $c = 2$. If we do this and substitute these values for the letters we have:

$$(a - b)(a + b - c) = a^2 - b^2 - ac + bc.$$

Then,

$$(5 - 3)(5 + 3 - 2) = 5^2 - 3^2 - (5 \times 2) + (3 \times 2)$$

and

$$2 \times 6 = 25 - 9 - 10 + 6$$

$$12 = 25 - 19 + 6 = 6 + 6 = 12$$

Since the problem works out so that $12 = 12$, our multiplication of the letters must be correct.

We can also perform the multiplication by setting it down as in Fig. 3.

As you can see, we simply multiply all the terms in the polynomial by each term of the binomial. We do this by multi-

	$a + b - c$
	$\times\ a - b$
MULTIPLYING BY a	$a^2 + ab - ac$
MULTIPLYING BY $-b$	$\quad\ - ab \qquad - b^2 + bc$
ANSWER	$a^2 \qquad - ac - b^2 + bc$

Fig. 3. Multiplying a polynomial by a binomial.

plying $a + b - c$ first by a, then by $-b$, and then adding the two partial products.

There are three polynomial products that we will find quite often in our work with algebra. They are:

(1) $(a + b)(a - b) = a^2 - b^2$

(2) $(a + b)(a + b)$ or $(a + b)^2$
$= a^2 + 2ab + b^2$

(3) $(a - b)(a - b)$ or $(a - b)^2$
$= a^2 - 2ab + b^2$

If we work each one of these out, we will find that the listed products are correct. Thus:

(1)
$$\begin{array}{lll} a & +b & \\ a & -b & \\ \hline a^2 & +ab & \\ & -ab & -b^2 \\ \hline a^2 & & -b^2 \end{array}$$

(2)
$$\begin{array}{lll} a & +b & \\ a & +b & \\ \hline a^2 & +ab & \\ & +ab & +b^2 \\ \hline a^2 & +2ab & +b^2 \end{array}$$

(3)
$$\begin{array}{lll} a & -b & \\ a & -b & \\ \hline a^2 & -ab & \\ & -ab & +b^2 \\ \hline a^2 & -2ab & +b^2 \end{array}$$

Sometimes these products are stated in words and used as rules:

1. The product of the sum of two terms $(a + b)$ and the difference of the same two terms $(a - b)$ is equal to the square of the first term minus the square of the second term $(a^2 - b^2)$.

This is a handy rule, because any time that we have to multiply the sum of two terms by the difference of the same two terms, we can just set down the answer without working it out. For example,

$$(4abc + 6xyz)(4abc - 6xyz)$$

must equal

$$(4abc)^2 - (6xyz)^2.$$

Likewise,

$$(4a^2b^3c^5 + 7x^2yz^3)(4a^2b^3c^5 - 7x^2yz^3)$$

must equal

$$(4a^2b^3c^5)^2 - (7x^2yz^3)^2.$$

The second example stated in words is:

2. The square of the sum of two terms $(a + b)^2$ is equal to the square of the first term plus the square of the second term plus twice the product of the terms $(a^2 + 2ab + b^2)$.

By using this rule, we automatically know that a binomial such as

$$(5xy + 3ab)^2$$

is equal to

$$(5xy)^2 + 2(15xyab) + (3ab)^2.$$

The third example covers the square of the difference of two terms:

3. The square of the difference of two terms $(a - b)^2$ is equal to the square of the first term plus the square of the second term minus twice the product of the terms.

Thus,

$$(16cd - 5x^2y)^2$$

is equal to

$$(16cd)^2 - 2(80cdx^2y) + (5x^2y)^2.$$

Since we often have to work with either the square of the sum or the square of the difference of two terms, we will use these rules quite a lot.

Can you find the products for the following problems?

(1) $(2x - 3)(3x + 7)$

(2) $(2x + 3)^2$

(3) $(5x^2 - 4y^2)(5x^2 + 4y^2)$

(4) $(6ab - c^2)^2$

Answers:

(1) $6x^2 + 5x - 21$

(2) $4x^2 + 12x + 9$

(3) $(5x^2)^2 - (4y^2)^2$ or $25x^4 - 16y^4$

(4) $(6ab)^2 - 2(6abc^2) + (c^2)^2$ or

$$36a^2b^2 - 12abc^2 + c^4.$$

Division with Letters. Division in algebra is just the reverse of multiplication. In division, we are given a product and one of the factors of the product and are asked to find the other factor. Remember, there are special names for the quantities in a division problem. The dividend is the product that is to be divided. The divisor is the factor by which the dividend is to be divided. The quotient is the result of the division, or the factor which we are to find. Thus: dividend ÷ divisor = quotient.

Also, multiplication is the proof of a division problem. Thus, $24 \div 6 = 4$ because $4 \times 6 = 24$; likewise,

$$24 \div 4 = 6$$

because $4 \times 6 = 24$. Accordingly, we can say that $ab \div a = b$ because $a \times b = ab$. The rules for the division of signed numbers also apply to the division of signed terms in algebra. Thus:

$+24 \div +6 = +4$ because $+4 \times (+6) = +24$

$-24 \div +6 = -4$ because $-4 \times (+6) = -24$

$+24 \div -6 = -4$ because $-4 \times (-6) = +24$

$-24 \div -6 = +4$ because $+4 \times (-6) = -24$

Accordingly, our rules for division of signed numbers and signed terms are:

If the dividend and the divisor have like signs, the quotient is positive.

If the dividend and the divisor have unlike signs, the quotient is negative.

Thus:

$ab \div b = a$ because $a \times b = ab$

$-ab \div b = -a$ because $-a \times b = -ab$

$ab \div -b = -a$ because $-a \times -b = ab$

$-ab \div -b = a$ because $a \times -b = -ab$

With this review and application of the general rules for division to letter problems, we are ready to look at the rules for handling exponents in division. You are already familiar with the fact that a^4 means

$$a \times a \times a \times a$$

and a^2 means $a \times a$. With this in mind, let's divide a^4 by a^2 and see what we get for an answer.

$$\begin{aligned} a^4 \div a^2 &= \frac{\cancel{a} \times \cancel{a} \times a \times a}{\cancel{a} \times \cancel{a}} \\ &= \frac{a \times a}{1} \\ &= a \times a \\ &= a^2 \end{aligned}$$

Likewise:

$$\begin{aligned} b^6 \div b^4 &= \frac{b^6}{b^4} \\ &= \frac{\cancel{b} \times \cancel{b} \times \cancel{b} \times \cancel{b} \times b \times b}{\cancel{b} \times \cancel{b} \times \cancel{b} \times \cancel{b}} \\ &= \frac{b \times b}{1} = b^2 \end{aligned}$$

And:

$$c^3 \div c^2 = \frac{c^3}{c^2} =$$

$$\frac{\cancel{c} \times \cancel{c} \times c}{\cancel{c} \times \cancel{c}} = \frac{c}{1} = c.$$

If you look at these examples closely, you will see that we could have obtained the same results by subtracting exponents.

$$a^4 \div a^2 = a^{4-(+2)} = a^2$$

$$b^6 \div b^4 = b^{6-(+4)} = b^2$$

$$c^3 \div c^2 = c^{3-(+2)} = c$$

Thus, just as we can multiply powers with the same base by adding exponents, we can divide two powers with the same base by subtracting exponents. Consequently,

$$a^6 b^3 \div a^5 b^2 = ab$$

and

$$x^3 y^2 \div xy = x^2 y$$

We can prove these answers by multiplying the quotients by the divisors to see if we get the original dividends. Doing this, we would have

$$ab \times a^5 b^2 = a^{1+5} b^{1+2} = a^6 b^3$$

and

$$(xy)(x^2y) = x^{1+2}y^{1+1} = x^3y^2$$

As we start working with division, we will find a few new situations regarding exponents. We know that any number divided by itself is equal to one. Thus,

$$\frac{6}{6} = 1, \frac{a}{a} = 1, \frac{3}{3} = 1$$

Now, if we follow our rules for dividing by subtracting exponents, we can see that if

$$\frac{a^3}{a^3} = 1,$$

that

$$a^3 \div a^3 = a^{3-(+3)} = a^0$$

which must also equal one. Likewise,

$$\frac{a}{a} = 1$$

$$a \div a = a^{1-(+1)}$$

$$= a^0 = 1$$

and

$$\frac{a^6}{a^6} = 1$$

$$a^6 \div a^6 = a^{6-(+6)}$$

$$= a^0 = 1$$

Thus, we have a new situation brought on by division which gives us an exponent of zero, and any factor with a zero exponent must equal 1. Remember, a factor by itself, such as "x", is considered to have an exponent of one, or x^1, and is equal to itself; but, a factor with an exponent of zero, such as b^0, can only be equal to the number 1.

If we look further into this problem of dividing by subtracting exponents, we will find that we can not only have positive exponents, such as 1, 2, 5, etc., and zero exponents, but we can also have negative exponents. This would occur if we had a division problem, such as $a^2 \div a^5$. This would be written either as

$$\frac{a^2}{a^5} = \frac{\cancel{a} \times \cancel{a}}{\cancel{a} \times \cancel{a} \times a \times a \times a}$$

$$= \frac{1}{a \times a \times a}$$

$$= \frac{1}{a^3}$$

or it could be written as

$$a^2 \div a^5 = a^{2-(+5)} = a^{-3}$$

If our answer can be either $1/a^3$ or a^{-3}, then a^{-3} must equal $1/a^3$. Thus, we can say that any factor with a negative exponent is equal to one divided by the factor with the exponent positive. Accordingly, $x^{-5} = 1/x^5$ and $c^{-3} = 1/c^3$.

Once again, we can prove that the reasoning behind negative exponents is correct by multiplying. For example, $x^4 \div x^7 = x^{4-(+7)} = x^{-3}$ because $x^{-3} \times x^7 = x^{-3+7} = x^4$ or $(x^4/x^7) = (1/x^3)$ because

$$\frac{1}{x^3} \times x^7$$

$$= \frac{x^7}{x^3} = x^4$$

The problems of division in algebra can be broken down into three general considerations the same as multiplication. First, we have the division of one monomial by another. Second, we have the division of a polynomial by a monomial. Third, we have the division of a polynomial by another polynomial.

In our review of division in general and our studies of handling exponents in division, we have covered the problem of dividing one monomial by another monomial. There is only one more thing that we must learn and that is what to do with the coefficients of terms. For example, suppose we want to divide $-12a^3x^4y$ by $4a^2x^2y$. We can set this up as

$$\frac{-12a^3x^4y}{4a^2x^2y},$$

and then break it up into

$$\left(\frac{-12}{4}\right)\left(\frac{a^3}{a^2}\right)\left(\frac{x^4}{x^2}\right)\left(\frac{y}{y}\right)$$

We can see that this will reduce to:

$$(-3)(a)(x^2)(1).$$

Now, putting the quotients together we have $-3ax^2 1$ or just $-3ax^2$, since any quantity times one equals itself.

By breaking up our division problems in this way and following the rules for division of signed numbers and exponents, we can see how division is accomplished. As a general rule, we will not need to do this, because the division of most monomials by another monomial can be worked out mentally. For example, see if you can follow these monomial divisions:

(1) $$\frac{-14a^2b^4c}{-7ab^2c^3} = \frac{2ab^2}{c^2}$$

(2) $$\frac{4x^3y^5}{8x^5y^2} = \frac{y^3}{2x^2}$$

(3) $$\frac{28a^2b^4c^3}{-7b^3c^3} = -4a^2b$$

(4) $$\frac{-16e^3i^2r^5}{-4e^2i^2r^3} = 4er^2$$

In order to divide a polynomial by a monomial, let's consider numbers for a moment. $16 \div 2 = 8$ because $2 \times 8 = 16$. Thus, if $3(a + 4) = 3a + 12$, then $(3a + 12) \div 3$ must equal

$$\frac{3a + 12}{3} = a + 4.$$

Similarly, if

$$3x(2x + 3y) = 6x^2 + 9xy,$$

then

$$\frac{6x^2 + 9xy}{3x}$$

must equal

$$2x + 3y.$$

Thus, we have a very simple rule for dividing a polynomial by a monomial. It is: *Divide each term in the polynomial dividend by the divisor, and then collect the terms in the quotient with the proper signs.*

For example,

$$8a^2b^3c - 12a^3b^2c^2 + 4a^2b^2c$$

divided by $4a^2b^2c$ can be set up as follows:

$$\frac{8a^2b^3c - 12a^3b^2c^2 + 4a^2b^2c}{4a^2b^2c}$$

equals

$$\frac{8a^2b^3c}{4a^2b^2c} = 2b$$

and

$$\frac{-12a^3b^2c^2}{4a^2b^2c} = -3ac$$

and

$$\frac{4a^2b^2c}{4a^2b^2c} = 1$$

Now, collecting terms we have

$$2b - 3ac + 1$$

for our answer.

Another example:

$$-27x^3y^2z^5 + 3x^4y^2z^4 - 9x^4y^3z^5$$

divided by $-3x^3y^2z^4$ is equal to:

$$\frac{-27x^3y^2z^5}{-3x^3y^2z^4} = 9z$$

and

$$\frac{3x^4y^2z^4}{-3x^3y^2z^4} = -x$$

and

$$\frac{-9x^4y^3z^5}{-3x^3y^2z^4} = 3xyz$$

$$3X - 2 \overline{\big)\, 3X^2 - 8X + 4}$$

Fig. 4. Setting up a polynomial for division by another polynomial.

Now, collecting our quotient terms, we have our answer: $9z - x + 3xyz$. Any polynomial can be divided by any monomial in this way.

In order to divide one polynomial by another polynomial, we must arrange the terms in a certain order before we actually divide. To do this, we simply make sure that all the terms in the dividend are arranged in the same order as those of the divisor. In doing this, we always place the term with the largest exponent first. Thus, in the problem $3x^2 + 4 - 8x$ divided by $3x - 2$, the divisor is correctly arranged, but the dividend isn't. Therefore, we must arrange it properly before we can proceed. Properly

$$\begin{array}{rl} & X \\ 3X-2 \,\big) & 3X^2 - 8X + 4 \\ & \underline{3X^2 - 2X} \\ & \quad -6X \end{array}$$

$X(3X) = 3X^2$ SO,

$X(3X-2) = 3X^2 - 2X$

(A)

$$\begin{array}{rl} & X - 2 \\ 3X-2 \,\big) & 3X^2 - 8X + 4 \\ & \underline{3X^2 - 2X} \\ & \quad -6X + 4 \\ & \quad \underline{-6X + 4} \end{array}$$

$-2(3X) = -6X$ SO,

$-2(3X-2) = -6X + 4$

(B)

Fig. 5. (A) First steps in polynomial division. (B) Next step in polynomial division.

arranged, it should be written

$$3x^2 - 8x + 4.$$

Now that we have our terms arranged properly, we can set up our problem exactly as we did with long division of numbers shown in Fig. 4. Notice that we have the dividend set up under the division sign and the divisor at the left. Our process now is really just plain long division, as shown in Fig. 5A.

First, we see how many times the first term of our divisor will go into the first term of our polynomial dividend. For example, $3x$ will go into $3x^2$ x times, because $3x$ times x is equal to $3x^2$. Thus, x becomes our first term in our quotient as shown. Now we multiply our entire divisor, $3x - 2$, by x to give us our first trial product of $3x^2 - 2x$. We place this trial product under the proper terms of the divident and subtract.

Our remainder from this subtraction, plus the other term which we bring down from the dividend, can be considered to be a new dividend, as shown in Fig. 5B. Notice the sign of the first term. Signs are very important in algebra. Now, we see how many times the first term in our divisor will go into the first term in this new dividend. Since -2 times $3x$ equals $-6x$, we will try the number 2 as the second term in our quotient. To do this,

$$\begin{array}{r} 3X-2 \\ X-2 \\ \hline 3X^2-2X \quad\;\; \\ -6X+4 \\ \hline 3X^2-8X+4 \end{array}$$

Fig. 6. Checking the answer in polynomial division.

$$\begin{array}{lr} & 2X^2+3X+14 \\ & X-3\,\overline{)\,2X^3-3X^2+5X-42} \\ & \underline{2X^3-6X^2} \qquad\qquad \\ X(2X^2)=2X^3 \text{ SO,} & +3X^2+5X \qquad \\ 2X^2(X-3)=2X^3-6X^2 & \underline{3X^2-9X} \qquad \\ & +14X-42 \\ X(3X)=3X^2 \text{ SO,} & \underline{14X-42} \\ 3X(X-3)=3X^2-9X & \\ X(14)=14X \text{ SO,} & \\ 14(X-3)=14X-42 & \end{array}$$

Fig. 7. Another problem in polynomial division.

we place the -2 beside the x in our quotient, as shown, and then multiply our entire divisor by -2. As you can see, this gives us $-6x + 4$ as a trial product to subtract from the dividend. Since $-6x + 4$ from $-6x + 4$ leaves no remainder, our division is complete.

In this way, we find that $3x^2 - 8x + 4$ divided by $3x - 2$ is equal to $x - 2$. To check our answer, we simply multiply the divisor by the quotient to see if we can get our dividend, as shown in Fig. 6. Since our answer checks, our problem is correct.

To make sure that we understand this, let's do another problem following the rules. Divide

$$5x - 42 + 2x^3 - 3x^2$$

by $x - 3$. Our first step is to rearrange the dividend in the proper order, which would give us

$$2x^3 - 3x^2 + 5x - 42$$

Now, we set up the problem for division, as shown in Fig. 7. Then, we see how

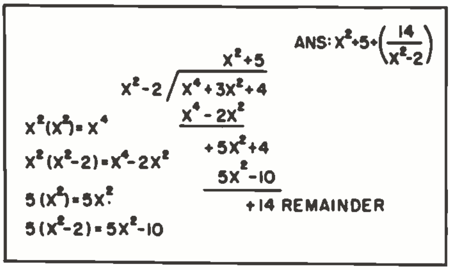


Fig. 8. Polynomial division with a remainder.

many times x will go into $2x^3$. Since $2x^2$ times x is equal to $2x^3$, we place $2x^2$ in our quotient and multiply the entire divisor by it. Since $2x^2(x - 3) = 2x^3 - 6x^2$, we use this as our first trial product and subtract it from the proper terms in the dividend.

Our remainder from this subtraction, plus the next term of our dividend, gives us a new dividend of $3x^2 + 5x$ to work with. x will go into $3x^2$, 3x times, so 3x becomes our next quotient term.

$$3x(x - 3) = 3x^2 - 9x,$$

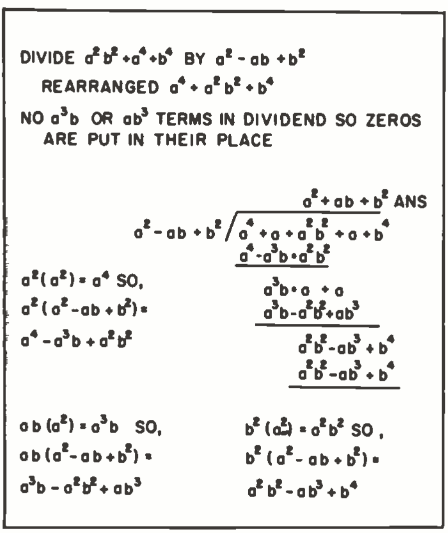


Fig. 9. A polynomial divided by a trinomial.

which is the term we subtract from our new dividend. This makes our next dividend $14x - 42$, as shown, and $x - 3$ will go into it exactly 14 times. Thus, our answer is $2x^2 + 3x + 14$. We can check this in the usual way, by multiplying the quotient and the divisor.

Some problems in division may not come out exactly even. It is possible to have a remainder in algebraic division, just as we do when working with numbers. An example of such a problem is shown in Fig. 8. Notice that we proceed to work it out just as we would any other problem until we get to a point where the first term of the divisor will not go into the dividend. When we come to this point, we simply stop and carry the remainder as a fraction in our answer, just as we do in ordinary arithmetic.

In Fig. 9, we have worked a problem where the divisor is a trinomial. As you can see, this is really no different from the problems we have been working, where the divisor is a binomial. You shouldn't have any trouble following this example.

SELF-TEST QUESTIONS

1. What is a monomial?
2. What is a polynomial?
3. Define binomial and trinomial.
4. What is an exponent?
5. What is the numerical coefficient of the term $6a^2b^3c$?
6. Add the following binomials: $3a^2b + 2b$; $a^2b - b$; and $-2a^2b + 4b$.
7. Add the following:

(a)
$$\begin{array}{r} 8x^2y + 9xy + 4y - 3 \\ -\,3x^2y + 2xy - 3y + 7 \\ \underline{2x^2y + xy + 2y - 2} \end{array}$$

(b) $$\begin{array}{r} 2a^4b^2 - a^2b \\ -2a^4b^2 - 3a^2b + 3 \\ -6a^4b^2 + 4a^2b - 5 \\ \hline \end{array}$$

8. Add the following: $4a^2b^2 - 2b$; $3ab^2 + 2a$; $a^2b^2 - 3a^2b + 3b + 3$.
9. Add the following:

(a) $$\begin{array}{r} 4ab + 2a \\ -2ab - 4a - 3 \\ ab - a \\ \hline \end{array}$$

(b) $$\begin{array}{r} 3x^2y + 2xy - 2y \\ 5x^2y + 3xy + 4y \\ -4x^2y - xy + y \\ \hline \end{array}$$

10. Add the following: $ab^2 + ab - 3a - b$; $a + b$; $3ab^2 - b$; $ab + 7a$; $ab + 3$.
11. Subtract $6a - 4b + 2c$ from $11a + b - 2c$.
12. Subtract $6a^2b + 3ab^2 - b^3$ from $a^3 - a^2b + 4ab^2$.
13. Subtract $a + b + c + d$ from $3a - 4b + c - 6d$.
14. Subtract $4a + 7b$ from $2a + 6b$.
15. Subtract $6a^3 - a^2b + ab^2 - b^3$ from $8a^3 + 3a^2b - ab^2 + b^3$.
16. Multiply $(a + 2b)$ times $(a - b)$.
17. Multiply $(a^2 + 2ab + b^2)(a + b)$.
18. Multiply $(2a + 3b)$ times $(2a - 3b)$.
19. Multiply $(a^2 - 2ab + b^2)$ by $(a + b)$.
20. Multiply $(a - b)(a + 2b^2)$.
21. Divide $(a^3 - 3a^2b + 3ab^2 - b^3)$ by $(a - b)$.
22. Divide $(64a^4 - 81b^6)$ by $(8a^2 + 9b^3)$
23. Divide $(a^5 - 3a^3 + a)(a)$.
24. Divide $(x^3 + 2x^2 + x)$ by $(x^2 + x)$.
25. $(a^4 + 2a^2b^2 + b^4) \div (a^2 + b^2)$.
26. $(6x^3 + 12 - 7x - x^2) \div (2x + 3)$.
27. Divide $26x^2 + 15x^3 + 10 - 39x$ by $3x - 2$.
28. What is the sum of the following polynomials?
$(-9a^3b + 6a^2b^2 - 5ab^3) + (14a^3b + 6a^2b^2 - 5ab^3) + (a^3b - 3a^2b^2 - ab^3)$

Equations

You have now learned how to do arithmetic with letters. Since many of these fundamental operations of algebra were new to you, you had a lot to learn so we did not take the time to see how they could be put to practical use in your work in electronics. Now, however, we have covered most of the elementary processes in algebra and it is time to see how to put these new mathematical tools to work in the solution of circuit problems. This can be done through the use of equations.

An equation is simply a mathematical statement that two quantities are equal to each other. The two equal quantities in an equation are called the "members" of the equation and they are always separated by an equal sign (=). Thus, the mathematical statements that $12 = 12$, $6 \times 2 = 6 \times 2$, $6 \times 2 = 12$, or $6 \times 2 = 3 \times 4$ are all equations, because the quantities on each side of the equal sign are equal to each other. Sometimes, when we want to be specific, we call the quantities on the left of the equal sign the "left member" of the equation and the ones on the right, the "right member."

Although you may already be somewhat familiar with equations and their use, it will be helpful to review some of the more common facts that you will use in working with them. Of course, the most important thing to remember is that an equation is always a statement of equality between the two members, and that in order to use it, we must never upset this equality or balance between the members. Thus, in an equation such as $24 = 24$, if we make any changes in one member, we must be very careful not to upset the balance of the whole equation. For example, we can change $24 = 24$ to $24 \times 1 = 24$, $12 \times 2 = 24$, $6 \times 4 = 24$, $6 \times 4 = 12 \times 2$, $3 \times 2 \times 4 = 6 \times 2 \times 2$, etc., because our changes do not upset the equality of the two members. Likewise, an equation such as $4Ir + 4IR = 4Ir + 4IR$ may be written in any of the following ways:

$$4(Ir + IR) = 4Ir + 4IR$$

$$4I(r + R) = 4Ir + 4IR$$

$$4I(r + R) = 4(Ir + IR)$$

because in any of these cases the equations remain balanced.

We can also do other things to equations without disturbing their equality. For example, we can add or subtract a quantity from one member of an equation as long as we perform the same operation to the other member with the same quantity. Thus, if we have an equation such as $x = x$, we can add the same number to each side without destroying the equation. For example, let's add 2 to each side of the equation $x = x$. This would give:

$$x + 2 = x + 2$$

We can see that this is still an equation, because if we let $x = 4$, and substitute for x, we have:

$$4 + 2 = 4 + 2, \text{ or } 6 = 6,$$

which is still an equation because both members are equal. Likewise, if $x = x$, we can subtract a number from either side, as:

$$x - 3 = x - 3$$

and if x = 4, then $x - 3 = x - 3$ becomes

$$4 - 3 = 4 - 3 \text{ or } 1 = 1$$

We can also multiply or divide both members by the same quantity. For example, if ab = ab and we multiply both members by 2, we have: 2ab = 2ab. Or, dividing by 2, we have (ab/2) = (ab/2). In either case, our equality can be proved by substitution. Thus if a = 3 and b = 4, substituting in the equation ab = ab, $3 \times 4 = 3 \times 4$ or 12 = 12. And, 2ab = 2ab, or $2 \times 3 \times 4 = 2 \times 3 \times 4$, which is 24 = 24. Likewise,

$$\frac{ab}{2} = \frac{ab}{2},$$

or

$$\frac{3 \times 4}{2} = \frac{3 \times 4}{2},$$

or

$$\frac{12}{2} = \frac{12}{2},$$

or

$$6 = 6.$$

In all of these cases our equations remain balanced, because one member always equals the other.

From this, we can make the general statement that we can do anything to one side of an equation as long as we do exactly the same thing to the other side. There is only one exception to this rule, and that is that we can never multiply or divide either member by zero. We will show you why we cannot divide by zero a little later after you have become familiar with working with equations.

These rules for working with equations are very valuable in working with formulas. Formulas are, of course, equations, but they are a special kind of equation. A formula is a rule or a law that is stated as an equation. Thus, both the equations ab = ab, and $E = I \times R$ are equations, but only $E = I \times R$ is a formula, because there is a law that makes it a true equation. In other words, ab = ab, or $I \times R = I \times R$ are equations because they meet the requirements of any equation automatically. Both members are exactly the same, and therefore equal. However, the fact that $E = I \times R$ is an equation is not apparent, and it wouldn't be recognized as an equation unless we knew that it was a statement of Ohm's Law. Here, both members are equal only by definition.

Since formulas are equations, we can use the rules for equations when working with formulas. Let's see how this can help us with a simple formula such as $P = E \times I$. Suppose we want to use this formula to find the power in a circuit, but we don't know the voltage, E. Instead of knowing the values of E and I, we have the values of I and R. Since, according to Ohm's Law, $E = I \times R$, we can substitute $I \times R$ in place of E in the power formula. Then, instead of $P = E \times I$, we would have $P = I \times R \times I$, or $P = I^2R$. By doing this, we have arranged our formula so that it contains the quantities that we know the values of, but we have not destroyed its equality. We have simply replaced one value, E, with an equal quantity, $I \times R$.

The rules for equations also help us to rearrange formulas so that they indicate directly the quantities we want to find. For example, the formula

$$Z = \sqrt{R^2 + (X_C)^2}$$

tells us how to find the impedance, Z, of a circuit. Suppose, however, we want to find X_C, but do not know the value of C or the frequency of the circuit. However, we have been given the impedance and can measure the resistance. In this case, we can apply the rules for working with equations to rearrange

$$Z = \sqrt{R^2 + (X_C)^2}$$

so that it indicates X_C from Z and R.

We do it like this:

$$Z = \sqrt{R^2 + (X_C)^2}.$$

Then, squaring both members, we have

$$(Z)^2 = \left(\sqrt{R^2 + (X_C)^2}\right)^2$$

which equals

$$Z \times Z = \sqrt{R^2 + (X_C)^2} \times \sqrt{R^2 + (X_C)^2}$$

or

$$Z^2 = R^2 + (X_C)^2$$

Now, subtracting R^2 from both members, we have

$$Z^2 - R^2 = R^2 - R^2 + (X_C{}^2)$$

or

$$Z^2 - R^2 = (X_C)^2$$

This indicates the value of $(X_C)^2$. But we want only X_C itself, so we take the square root of both members:

$$\sqrt{Z^2 - R^2} = \sqrt{(X_C)^2}$$

or

$$\sqrt{Z^2 - R^2} = X_C$$

or

$$X_C = \sqrt{Z^2 - R^2}$$

Now our one basic formula is rearranged to give us X_C directly when Z and R are known.

Likewise, C may be found with the formula $X_C = (1/2\pi fC)$ by rearrangement as follows:

If

$$X_C = \frac{1}{2\pi fC},$$

then

$$X_C \times C = \frac{1}{2\pi fC} \times C$$

or

$$X_C \times C = \frac{1}{2\pi f}.$$

Then,

$$X_C \times C \div X_C = \frac{1}{2\pi f} \div X_C$$

or

$$\frac{X_C \times C}{X_C} = \frac{1}{2\pi f X_C}$$

or

$$C = \frac{1}{2\pi f X_C}$$

Many of our formulas themselves are the result of the use of algebra and the rules for equations. They are found or derived from the knowledge of other facts.

For example, we often have the inductance of a circuit in microhenrys and the capacity in microfarads and want to find the resonant frequency of the circuit. We can do this using the formula

$$f = \frac{159}{\sqrt{LC}}$$

where L is in microhenrys, C is in microfarads, and f is in kilohertz. This formula is developed through the knowledge of other facts. For instance, at resonance we know that:

$$X_L = X_C$$

also $$X_L = 2\pi fL$$

and $$X_C = \frac{1}{2\pi fC}$$

In the formula $X_L = 2\pi fL$, f is in Hertz and L is in henrys; and in $X_C = (1 \div 2\pi fC)$, f is in Hertz and C is in farads.

Since

$$X_L = X_C,$$

We can substitute for X_L and X_C and get

$$2\pi fL = \frac{1}{2\pi fC}$$

Now, multiplying both sides by $2\pi fC$ we get

$$2\pi fL \times 2\pi fC = \frac{2\pi fC}{2\pi fC}$$

or

$$4\pi^2 f^2 LC = 1$$

Now, dividing both sides by $4\pi^2 LC$ we get

$$\frac{4\pi^2 f^2 LC}{4\pi^2 LC} = \frac{1}{4\pi^2 LC}$$

or $$f^2 = \frac{1}{4\pi^2 LC}$$

and taking the square root of both sides

$$\sqrt{f^2} = \frac{1}{4\pi^2 LC}$$

or

$$f = \frac{\sqrt{1}}{\sqrt{4} \times \sqrt{\pi^2} \times \sqrt{LC}} = \frac{1}{2\pi\sqrt{LC}}$$

$2\pi = 6.28$ and dividing 1 by 6.28 gives .159 so we can rewrite the equation as

$$f = \frac{.159}{\sqrt{LC}}$$

where f is in Hertz, L is in henrys and C is in farads.

If we substitute L in microhenrys and C in microfarads in this equation, we must divide each value by 1,000,000 to convert them to henrys and farads. Let's do this in the equation:

$$f = \frac{.159}{\sqrt{\frac{L}{1{,}000{,}000} \times \frac{C}{1{,}000{,}000}}}$$

which can be written

$$f = \frac{.159}{\frac{\sqrt{LC}}{\sqrt{1{,}000{,}000^2}}}$$

which is

$$f = \frac{.159}{\frac{\sqrt{LC}}{1{,}000{,}000}}$$

This is the same as

$$f = \frac{.159}{1} \div \frac{\sqrt{LC}}{1{,}000{,}000}$$

Now recall that to divide one fraction by another we invert the divisor and multiply. For example,

$$\frac{1}{3} \div \frac{1}{4} = \frac{1}{3} \times \frac{4}{1} = \frac{4}{3}$$

Similarly,

$$\frac{6}{14} \div \frac{1}{2} = \frac{6}{14} \times \frac{2}{1} = \frac{12}{14}$$

and

$$\frac{.159}{1} \div \frac{\sqrt{LC}}{1{,}000{,}000} = \frac{.159}{1} \times \frac{1{,}000{,}000}{\sqrt{LC}}$$

Therefore,

$$f = \frac{159{,}000}{\sqrt{LC}}$$

where f is in Hertz, L is in microhenrys and C is in microfarads.

To convert Hertz to kilohertz we divide by 1000. Therefore,

$$f = \frac{159{,}000}{\sqrt{LC}} \div 1000$$

$$f = \frac{159{,}000}{\sqrt{LC}} \times \frac{1}{1000}$$

$$f = \frac{159}{\sqrt{LC}}$$

where f is in kilohertz, L in microhenrys, and C in microfarads. Thus, through algebraic manipulation of letters and using the rules for equations, we derive a simple, easy-to-remember formula for finding the resonant frequency.

SHORTCUTS FOR EQUATIONS

Although we can work with any equations with the rules and information that we have already studied, there are some shortcuts which will let us work much faster and more efficiently. They are all derived from the basic rules, so we won't have to learn anything new. We will simply study the rules closely so we can see what the end results of the operations are and learn to apply them directly.

Moving a term from one member of an equation to the other member is an operation that is quite common and is called "transposing." The rule for transposing is:

A term may be transposed from one member of an equation to the other member by changing the sign of the term.

Thus, in an equation such as

$$Z^2 = R^2 + X^2$$

we can transpose the X^2 by simply changing the sign to give

$$Z^2 - X^2 = R^2$$

or transpose the R^2 to give

$$Z^2 - R^2 = X^2$$

or both, to get

$$Z^2 - R^2 - X^2 = 0$$

Using an equation with numbers shows that doing this does not destroy the equality. For example:

If

$$4 + 2 = 6,$$

then

$$4 = 6 - 2,$$

or

$$2 = 6 - 4$$

or

$$0 = 6 - 4 - 2.$$

Likewise, if

$$10 - 4 - 2 = 4,$$

then

$$10 - 4 = 4 + 2$$

or

$$10 - 2 = 4 + 4$$

or

$$10 = 4 + 2 + 4.$$

The basic rule of equations that states that we can add or subtract the same quantity from both members of the equation allows us to transpose. For example, in the equation $Z^2 = R^2 + X^2$, if we subtract X^2 from both members, we have:

$$Z^2 - X^2 = R^2 + X^2 - X^2$$
$$Z^2 - X^2 = R^2$$

Or, in the equation $10 - 4 - 2 = 4$, adding 4 to both members, we have $10 - 4 - 2 + 4 = 4 + 4$ which is equal to $10 - 2 = 4 + 4$. *Thus, transposing terms by changing the sign is simply a shortcut for adding or subtracting quantities to both members.*

Using the same basic rule, we can also make the statement that:

We can cancel out like terms from the members of an equation, if the same term appears in each member, and is preceded by the same sign.

Thus, if we have an equation like $x + y = z + y$, we can cancel the y's out to give $x = z$. Or, an equation with numbers like $4 \times 3 + 2 = 12 + 2$ can be reduced to $4 \times 3 = 12$ by canceling the 2's. As you can see, all we are doing when we cancel is to subtract the same term from both members. Thus, $x + y = z + y$ becomes $x + y - y = z + y - y$ or $x = z$. Likewise, $4 \times 3 + 2 = 12 + 2$ becomes $4 \times 3 + 2 - 2 = 12 + 2 - 2$ or simply

$$4 \times 3 = 12.$$

Another common rule is one that involves the signs of the terms in the equations. Stated simply, it is:

The signs of all the terms of an equation may be changed without changing the equality.

Thus, an equation such as $-x + y = -4 + 3$ may be rewritten as $x - y = 4 - 3$. In doing this, we are simply multiplying both sides of the equation by the same number, -1. In our example,

$$(-x + y)(-1) = (-4 + 3)(-1)$$

or

$$x - y = 4 - 3.$$

When you studied ratio and proportion, you learned to cross-multiply. Thus, $(x/y) = (a/b)$ could be rewritten as $xb = ya$. Cross-multiplication is also made possible through the rules for working with equations. When we do this, we are really multiplying both members by one term and then multiplying both members again by another term. Thus, $(x/y) = (a/b)$ becomes $xb = ya$, because: If $(x/y) = (a/b)$, then

$$b\frac{x}{y} = \frac{a}{b}b$$

or

$$\frac{xb}{y} = a.$$

Again,

$$\frac{xb}{y}y = (a)y$$

and xb = ya. Thus, cross-multiplication is just a quick way of following the basic rules.

Then, of course, we have the many operations with multiplication and division which help us so much with rearranging our formulas. For example:

$$I = \frac{E}{R} \text{ because } E = I \times R$$

and

$$\frac{E}{R} = \frac{I \times R}{R} \text{ or } \frac{E}{R} = I \text{ or } I = \frac{E}{R}.$$

Likewise,

$$R = \frac{E}{I} \text{ because } E = I \times R$$

and

$$\frac{E}{I} = \frac{IR}{I} = \frac{E}{I} = R.$$

With these rules and shortcuts in mind, and our knowledge of basic algebra, we are ready to practice solving equations.

SOLVING EQUATIONS

The purpose of learning to work with letters and equations is to make it easier to solve the problems in working in electronics. While many of the problems will be straightforward and can be solved by applying basic formulas, others will require more thinking and reasoning before the answer is found. The use of algebra and a good working knowledge of equations will be very helpful in these more difficult solutions. As you have seen, working with letters is not difficult and the rules for operating with equations are both simple and logical. However, to become really proficient with algebra and equations requires a lot of practice.

One of the biggest difficulties in arriving at circuit solutions is not in solving the equations themselves, but in setting up the equations in the first place. This also takes a lot of practice. Although it is difficult, if not impossible, to operate by a strict set of rules for solving problems, there are a few general procedures that are worth following.

First, you should read the problem so carefully that you thoroughly understand everything about it. Then, you should determine exactly what you want to know and represent it with a letter. If there are two or more unknown quantities, you should represent them in terms of the first one. Next, you should try to apply the formulas that will allow you to find the unknown quantity from the known facts. If this is not possible, you should try to set up letter equations that will allow you to state the problem in terms of the unknown quantity. Finally, you should solve the equations for the unknown value by substituting letter and number equivalents that are available. Remember, you will often save yourself a lot of time and effort by working with letters as long as possible before substituting numbers.

Now let's solve some simple equations, and later some problems, to see how we can apply these rules. In the problem

$$3i + 14 + 2i = i + 26$$

solve for i. The first thing to do is to get all like terms together. We can do this by transposing the "i" terms to one side and the numbers to the other side. Thus,

$$3i + 14 + 2i = i + 26$$

becomes

$$3i + 2i - i = 26 - 14$$

Then, collecting terms, we have: $4i = 12$ and then dividing both members by 4 to solve for i gives us

$$\frac{4i}{4} = \frac{12}{4}$$

or $i = 3$.

We can always check this answer by substituting this value of $i = 3$ back into our original equation. Doing this:

given $\quad 3i + 14 + 2i = i + 26$

then $\quad 3 \times 3 + 14 + 2 \times 3 = 3 + 26$

and $\quad 9 + 14 + 6 = 29$ or $29 = 29$

Thus, our answer of $i = 3$ must be correct because our equation is balanced if this value is used to check it.

Solve for y in the equation:

$$3(y - 2) - 10(y - 6) = 5.$$

Here, we follow the rules of order and get rid of the values within the parentheses first. This gives us:

$$3y - 6 - 10y + 60 = 5$$

Transposing:

$$3y - 10y = 5 + 6 - 60$$

Then:

$$-7y = -49$$

Changing signs:

$$7y = 49$$

Solving for y:

$$\frac{7y}{7} = \frac{49}{7}$$

or

$$y = 49 \div 7 = 7$$

Now let's try solving for E in the equation:

$$19 - 5E(4E + 1) = 40 - 10E(2E - 1)$$

Removing parentheses:

$$19 - 20E^2 - 5E = 40 - 20E^2 + 10E$$

Transposing:

$$-5E - 10E = 40 - 19$$

Then:

$$-15E = 21$$

Solving for E:

$$\frac{-15E}{-15} = \frac{21}{-15} \text{ or } E = \frac{21}{-15} = -1.4$$

Notice the cancellation of equal terms in the second step.

Earlier in our discussion of equations, we mentioned that we could never multiply or divide an equation by zero. This is easy enough to remember, but it is not always so easy to realize that we are in danger of doing it. Now that you are more familiar with working with equations, let's examine this important rule more thoroughly by working the following equation. First, let:

$$a = b$$

Multiply by a:

$$a^2 = ab$$

Subtract b^2:

$$a^2 - b^2 = ab - b^2$$

Now,

$$a^2 - b^2 = (a + b)(a - b)$$

and

$$ab - b^2 = b(a - b)$$

Therefore:

$$(a + b)(a - b) = b(a - b)$$

Divide by (a – b):

$$\frac{(a + b)\cancel{(a - b)}}{\cancel{(a - b)}} = \frac{b\cancel{(a - b)}}{\cancel{(a - b)}}$$

Then:

$$a + b = b$$

But,

$$a = b$$

Therefore:

$$2b = b$$

Divide by b:

$$\frac{2b}{b} = \frac{b}{b}$$

and $$2 = 1.$$

Obviously, 2 cannot equal 1, and somewhere in our manipulation of the equation, we have made a mistake that has destroyed its equality. Although all of our steps seem justified, because we never did anything to one member that we didn't do to the other, we actually have divided by zero at one point. Can you find it? If a = b, then (a – b) must equal zero. Therefore, when we divided both sides of our equation by (a – b), we were dividing by zero, which we can never do.

Setting up Equations. Now let's see what sort of reasoning we have to do to set up an equation for solving a simple problem. For example, consider the following problem: "What value of inductance will produce resonance at 50 Hertz if it is placed in series with a 20 μf capacitor?" Looking at the problem carefully, we see that it deals with resonance and that a resonant frequency and a value of capacitance are given. We are asked for the inductance. Thus, we have:

Given

$$C = 20\,\mu f \quad f = 50 \text{ Hertz}$$

Find L

Since our problem deals with resonance,

we naturally think of our formula for resonance:

$$f = \frac{.159}{\sqrt{LC}}$$

Comparing this with what is given and with what we want to find, we can see that we have the necessary information to use this formula and that L can be found with it, if it is rearranged. Accordingly, we would first rearrange our formula to indicate the value of L.

Doing this:

$$f = \frac{.159}{\sqrt{LC}} \quad \text{or} \quad f^2 = \frac{.159^2}{LC}$$

then

$$Lf^2 = \frac{.159^2}{C}$$

$$L = \frac{.159^2}{f^2 C}$$

Now, we can substitute our values in the formula and solve for L. However, before we do this, we must check our units of measurement to see if the given values can be substituted directly. In this particular problem, we cannot substitute them directly because the formula $f = (.159/\sqrt{LC})$ is in kilohertz when L is in microhenrys and C is in microfarads. Therefore, we must convert 50 Hertz to kilohertz by moving the decimal three places to the left. Thus,

$$50 \text{ Hertz} = .05 \text{ kHz}$$

Now, using the formula

$$L = \frac{.159^2}{f^2 C}$$

$$= \frac{25{,}281}{.05 \times .05 \times 20}$$

$$= \frac{25{,}281}{.0025 \times 20}$$

$$= \frac{25{,}281}{.05}$$

$$= 505{,}620 \text{ microhenrys}$$

$$= .51 \text{ henrys (approx.)}$$

While this is a simple problem, it does show the basic reasoning behind the handling of any problem. First, examine the problem. Find a formula, if possible. Arrange the formula to indicate the unknown. Check for proper units of measurement. Substitute and solve for the unknown. Now, let's try the procedure again on a more complex situation.

In the circuit shown in Fig. 10, suppose we are asked to find the resistance of R_4 from the values given. First of all, examination of the problem shows that we are given all the resistances except R_4 and we are also given the supply voltage and the current. Listing these values, we have:

Given:

$E_T = 100V$

$I_T = .2A$

$R_1 = 100\Omega$

$R_2 = 200\Omega$

$R_3 = 800\Omega$

Find: R_4

If we had the total resistance of the circuit, we could set up an equation because we know the total resistance must be equal to R_1 plus the resistance of the parallel branch made up of R_3 in parallel with R_2 and R_4. The resistance of this branch can be expressed using the formula for parallel resistors and treating R_2 and R_4 in series like a single resistance. The resistance of the parallel branch R_p is

$$R_p = \frac{R_3(R_2 + R_4)}{R_3 + (R_2 + R_4)}$$

Thus, the total resistance of the circuit R_T is

$$R_T = R_1 + \frac{R_3(R_2 + R_4)}{R_3 + R_2 + R_4}$$

Now in this equation we do not know the value of R_T or R_4. But we do know the total voltage E_T and the total current I_T so we can find R_T.

$$R_T = \frac{E_T}{I_T}$$

Thus, it looks like we can use the equation expressing R_T in terms of R_1, R_2, R_3, and R_4 to solve for R_4. Indeed we can do this, but look at the term for the resistance of the parallel branch. Notice we have R_4 in both the top and bottom of this expression. We will have to do a great deal of manipulation before we can solve for R_4. Before we start on this task, let's look at the circuit again to see if any easier solution is available.

First, notice that the total current is .2 amp. This means that the current through R_1 is .2 amp so we can easily find the voltage drop across the resistor using:

$$E = I_T R_1$$

$$= .2 \times 100$$

$$= 20 \text{ volts}$$

If we have a source voltage of 100 volts and a voltage drop of 20 volts across R_1, we must have $100 - 20 = 80$ volts across the parallel branch. Now let's find the current through R_3 which we can do using

$$I = \frac{E}{R}$$

$$= \frac{80}{800}$$

$$= .1 \text{ amp}$$

If the total current is .2 amp and .1 amp flows through one branch of the parallel circuit, the current in the other branch must also be .1 amp. Therefore we have .1 amp flowing through R_2 and R_4.

We know the voltage across R_2 and R_4 in series is 80 volts. Let's find the voltage across R_2 using:

$$E = IR_2$$

$$= .1 \times 200$$

$$= 20 \text{ volts}$$

This means the voltage across R_4 must be $80 - 20 = 60$ volts. Now we know the voltage across R_4, 60 volts, and the current through it, .1 amp, so we can find R_4 using:

$$R = \frac{E}{I}$$

$$= \frac{60}{.1}$$

$$= 600 \text{ ohms}$$

Thus we have solved the problem, using a series of simple steps and avoided some complicated work by taking a second look at the problem.

In a similar fashion, we could solve for E_T if we had the following values given for the circuit in Fig. 10

Given: $R_1 = 350\Omega$

$R_2 = 300\Omega$

$R_3 = 500\Omega$

$R_4 = 600\Omega$

Find E_T if the voltage drop across R_4 is 60V.

First, since you know the voltage across R_4 and the resistance of R_4, find the current through R_4. Once you have this current you can find the voltage across R_2 because the same current flows through R_2 and R_4. When you get the voltage across R_2, you can find the current through R_3 because the voltage across R_3 will be equal to the voltage across R_2 plus the voltage across R_4.

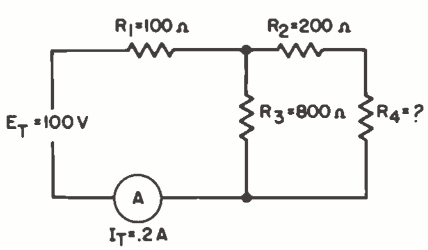


Fig. 10. Circuit used for solving for the resistance of R_4.

Now you can determine the total current flow in the circuit and then find the voltage across R_1. Once you have this voltage you should be able to find the total voltage. Work out this problem using the values given. The answer is 188 volts.

Thus, by applying the simplest formula or equation that we can, and working through the problem a step at a time, we can find the solutions to many different types of problems. As you can see, one of the greatest difficulties is in choosing a basic equation that can be made to use our known quantities. We want to be sure to choose the equation that will lead to the simplest solution. This takes sound reasoning and a lot of practice. Once you learn to do this, your knowledge of algebra and equations will let you solve the problems readily. You will get some more practice in this type of work as you study the "j" operator in the next section.

SELF-TEST QUESTIONS

29. What is an equation?
30. Which of the following can we *not* do to an equation?
    (a) Add the same number to each side.
    (b) Multiply each side by the same number.
    (c) Divide each side by 0.
    (d) Subtract the same number from each side.
    (e) Square each side.
31. What must be done to a term before it can be transposed from one side of an equation to the other?
32. What is a formula?
33. Using the power formula $P = I^2R$, solve for R.

34. Using the formula $P = I^2R$, solve for I.

35. Solve for L in the formula $X_L = 2\pi fL$.

36. Solve for f in the formula:

$$X_C = \frac{1}{2\pi fC}$$

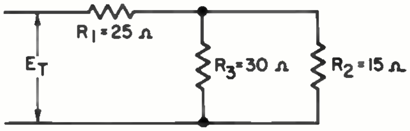
37. Solve for E using the power formula:

$$P = \frac{E^2}{R}$$

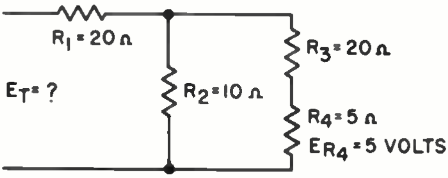
38. Solve for X_C in the formula:

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

39. In this diagram, if the current through R_2 is 2 amps, what is the value of E_T?



40. Find the source voltage in the circuit shown.



The J Operator

The "j" operator, or j multiplier as it is sometimes called, is simply a device that allows us to represent a vector mathematically. Through the use of this j operator, we are able to simplify a great deal of work in ac circuits. Instead of having to lay out a vector accurately for each separate value of resistance or reactance, we can simply state them all mathematically and then compute their final value algebraically. This is a great advantage in dealing with the complex arrangements found in tube and transistor circuits as well as any other complex ac circuit.

Being able to compute vectors mathematically means that we can multiply and divide vectors as easily as we can add or subtract them. This in itself is something that we have never been able to do before. In this section of the lesson, we will see exactly what we mean by the term "j" and how it can be used as an operator in ac circuits. We will learn how to do j arithmetic, and then we will apply these new principles to ac circuit calculations.

NUMERICAL REPRESENTATION OF A VECTOR

When you studied vectors you learned that they could be used in electronics to represent the time or phase as well as the magnitude of ac circuit quantities. In constructing vector diagrams, we used two scales at right angles to each other, like those shown in Fig. 11. Our reference vectors were laid out from the center of the scale to the right towards 0° and were used to represent zero time or in-phase components. Those that represented quantities that led the reference vector by 90° were laid out vertically from the center towards 90°. Those that were exactly 180° out of phase were laid out on the horizontal scale, pointing from the center towards the left, or 180°. Those that represented quantities that lagged the reference vector by 90° were drawn down the vertical scale from the center towards 270°.

Thus, any vector that was laid out so that it pointed towards 90° was considered to lead a vector at 0° and lag a vector at 180°. Similarly, a vector pointing down towards 270° was considered to lag a vector at 0° and lead a vector at 180°. Because of this, we arrived at the statement that vectors could be rotated counterclockwise about a common point to indicate phase relationships or the time of an occurrence.

In this way, vector A in Fig. 11 leads 0° but lags 90°. In the same way, vector B leads 90° but lags 180°, vector C leads 180° but lags 270°, and vector D leads 270° and lags 0°.

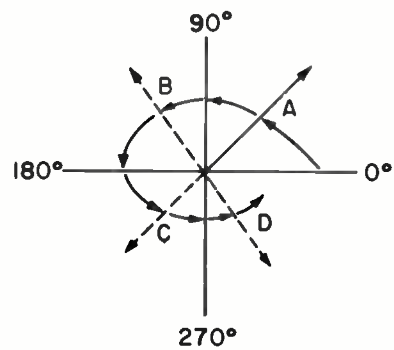


Fig. 11. Rotation of vectors.

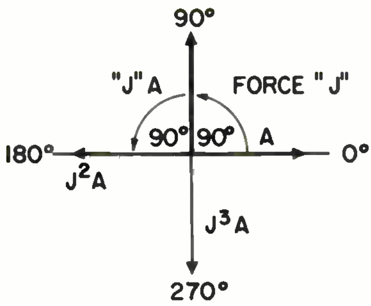


Fig. 12. Using j to rotate a vector.

In your study of algebra, you learned that you could represent any quantity or value by a letter. Therefore, let's consider that a force acts upon vectors to cause them to rotate in this way, and that this force can be represented by a letter value. Further, let's assume that the amount of this force necessary to rotate a vector 90° is represented by the letter j.

Now, let's draw a vector, A, ten units in length, along the reference line from the center toward 0° as shown in Fig. 12. In this position the vector is in phase with the reference and occurs at time zero. If we now multiply the vector by j, which represents a rotating force of 90°, we must consider that the vector will rotate 90° counterclockwise and point towards 90° as shown by vector jA in Fig. 12. Thus, multiplying the base vector A by j has resulted in its being rotated through 90° until it becomes the new vector jA.

Likewise, if we multiply our new vector jA by j, it will rotate another 90° and become vector j·jA or j^2A and will point toward 180° as shown. Multiplying by j again will make our vector rotate another 90° to become $j \cdot j^2A$ or j^3A pointing towards 270°. One more multiplication by j or $j \cdot j^3A$ gives us j^4A and brings the vector back to its starting point.

When we studied signed numbers, we used a horizontal scale similar to the one we use in our reference diagram for vectors. We represented positive numbers as starting from the center at 0 and working toward the right, as shown in Fig. 13. Our negative numbers started at the center and progressed toward the left. In Fig. 13, we have shown the same basic vector reference diagram as we used in Fig. 11 and 12, but we have also included the positive and negative scales along the horizontal line as shown.

Along our reference line, we have drawn vector A to represent an in-phase vector + 5 units long. If we multiply this vector A by j^2, it will rotate 180° and point towards 180° as shown. Now, according to our scale of positive and negative numbers, this new vector j^2A will equal −5. This is as it should be because anything 180° out of phase with +5 must be equal to −5 because it is exactly opposite. Just what is minus 5? One explanation is that minus five is plus five times minus one, because $+5 \times (-1) = -5$. If this is the case, then j^2 must be equal to −1, because $j^2 \times (+5) = -5$, just as $-1 \times (+5) = -5$.

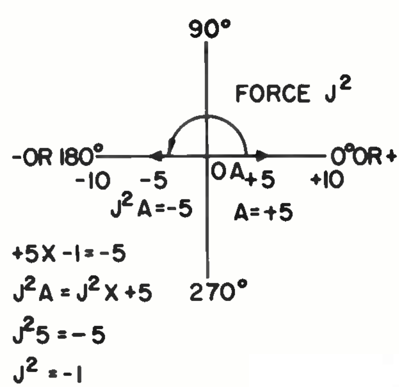


Fig. 13. Diagram of $j^2 = -1$.

Thus, any time that j^2 is used to represent a force for rotating vectors, j^2 will always be equal to -1, and -1 is substituted immediately for j^2. If $j^2 = -1$, then j must be equal to $\sqrt{-1}$. Thus, the value of j is often referred to as being imaginary because there is no number equal to $\sqrt{-1}$ because $1 \times 1 = 1$ and $-1 \times (-1)$ also $= 1$. There is no number you can multiply by itself to get -1! Whenever a j^2 term appears in a problem solution, we eliminate it by substituting -1, but where a j term appears we simply leave the j in the term because there is nothing we can substitute for it. Thus, in the term $6 + j8$, the 6 is called the real or in-phase component and the j8, the imaginary or quadrature component.

Now let's go a step further. If $j^2 = -1$, then $j \cdot j^2$ or j^3 must be equal to $-1 \cdot j$ or $-j$. We have already represented vectors drawn down the vertical line toward 270° as being a reference vector times j^3, so either j^3 or $-j$ times a vector must rotate it so that it points downward toward 270°. If $j \cdot j = j^2$ or -1 and $j \cdot j^2$ or $j^3 = j \cdot -1$ or $-j$, then j^4 representing a full 360° rotation of a vector is equal to $j^3 \cdot j$, or $j^2 \cdot j^2$ or $-1 \times (-1) = +1$. Once again this is as it should be, because any vector rotated completely around the diagram will be back where it started and represents a positive or in-phase value.

Once we understand this use of the letter "j" as an operator for determining the final position of a vector, we can use it in our work in electronics. Any time that we have a quantity multiplied by j, we will immediately know that it is a vector quantity pointing toward 90°. Similarly, if we have a $-j$ or j^3 quantity, we will know that it represents a vector drawn down towards 270°. Any positive quantity without a j or one with a j^4 multiplier can be treated as an in-phase component drawn towards 0°, and any minus quantity or one with a j^2 multiplier will represent a vector drawn out of phase towards 180°.

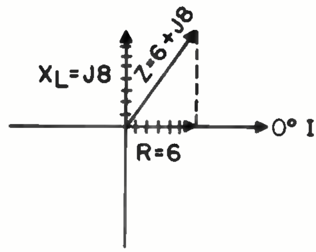


Fig. 14. Diagram of $Z = 6 + j8$.

Thus, if we have a series circuit consisting of a resistor and a coil, we can represent the impedance vector with a binomial term. For example, suppose the resistor has a resistance of 6Ω and the coil has an inductive reactance of 8Ω. We can say that the impedance of the circuit is equal to (6Ω + j8Ω). As soon as we see the j in the impedance notation, we can visualize a vector diagram like the one shown in Fig. 14. Here the 6Ω has no multiplier so it is drawn along the reference line I and represents the in-phase component. The j in the +j8Ω tells us that this quantity is drawn upward at right angles to the in-phase component, as shown.

Similarly, if we see a notation such as: $E = (-100 - j60)$, we can visualize a

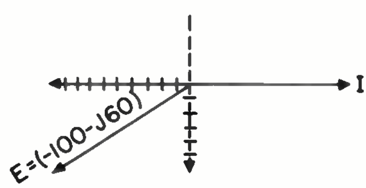


Fig. 15. Diagram of $E = (-100 - j60)$.

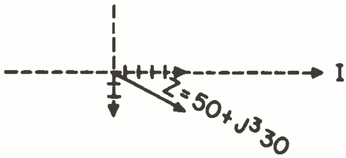


Fig. 16. Diagram of $Z = 50 + j^2 30$.

resultant voltage vector that has an E_R of -100 volts for one component and an E_{X_c} of 60 volts as another component, as shown in Fig. 15. Another vector such as

$$Z = 50 + j^3 30$$

would be immediately recognized, as shown in Fig. 16. In this way, we can represent any vector as a simple binomial term. All we have to do is remember the position values of our various j multipliers.

J ARITHMETIC

Since we are able to represent any vector mathematically as a binomial term through the use of j as a multiplier, we can solve any vector problem through the use of algebra. For example, we learned that the sum of two binomials such as (5a + 66) and (3a – 46) would be

$$5a + 66 + 3a - 46 = 8a + 20.$$

Likewise, the sum of a vector such as (10 + j5) and another equal to

$$(5 - j10)$$

would be

$$(10 + j5) + (5 - j10) = 10 + 5 + j5 - j10,$$

or a new vector equal to 15 – j5. To prove that this mathematical solution is correct we can check it against a measurement solution.

First, let's draw our two vectors, (10 + j5) and (5 – j10) as shown in Fig. 17A. Now, there are two methods which we can use to add vectors. We can break them both down into their components and add the components as we learned to do in our lesson on vectors, and as shown in Fig. 17B. Or, we can add the two vectors head to tail on the same diagram by being careful to place them in their proper position regarding the reference line, and then draw a resultant vector, as shown in Fig. 17C. In either case, the components of the resultant vector are the same and equal 15 – j5, which is exactly what we got mathematically so our mathematical solution must be correct.

To subtract one vector from another, we can also work mathematically with our binomial terms, or we can solve them with diagrams. For example, let's subtract vector B from vector A, as shown in Fig. 18. As you can see from the diagram in Fig. 18A, vector A can be written as 8 +

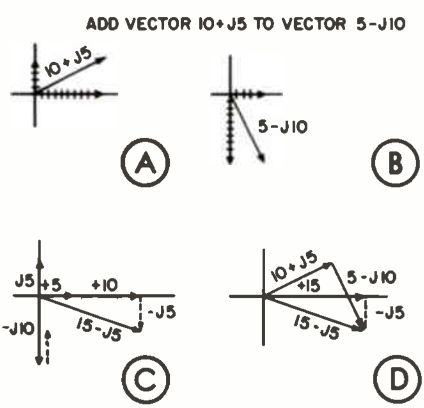


Fig. 17. Adding vectors with diagrams to prove mathematical solution.

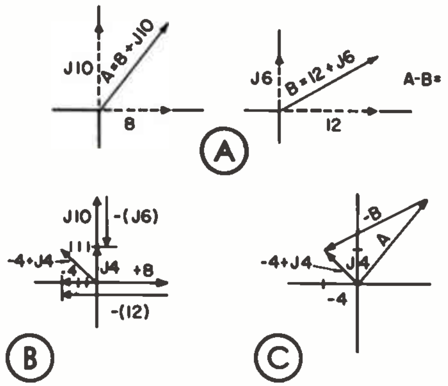


Fig. 18. Subtracting vectors with diagrams.

j10 and vector B can be written as 12 + j6. There are also two ways that we can subtract vectors with a diagram. Let's examine them.

First, there is the resolution method where we break each vector into its two components. Then we take the components of the vector that we are subtracting, reverse their directions and then add them head to tail, as we do in adding vectors. Notice how similar this is to the subtraction of signed numbers. We reverse the direction of the subtrahend (change the signs) and then proceed as in addition.

We have done this in Fig. 18B where the j6 and +12 components of vector B have been reversed in direction and then added vectorially to the j10 and +8 components of vector A. As you can see, this gives us a new vector with components of −4 and j4, or simply −4 + j4. We can also subtract vectors by subtracting them directly, as shown in Fig. 18C. Here we simply reverse the direction of vector B and add it head to tail to vector A, being careful not to change its position in regard to the reference. Then, the result drawn from the tail of A to the head of B is equal to −4 + j4, as it was with the other method.

Subtracting vectors mathematically is much simpler. We simply subtract the binomial notations of the vectors just as we would subtract any binomial terms. For example, vector B from vector A will equal:

$$(8 + j10) - (12 + j6)$$

$$= 8 + j10 - 12 - j6$$

$$= -4 + j4$$

which is the same as we got with our diagrams. This mathematical method of subtracting vectors is especially valuable in complex problems dealing with many vectors at many different angles. For example, consider the vectors A, B, C, and D in Fig. 19. Suppose we want to add vector A to vector B and then subtract vectors C and D from this sum. Mathematically this becomes:

$$(15 + j3) + (6 - j9) - (-8 + j4) - (+9 - j12)$$

$$= 15 + j3 + 6 - j9 + 8 - j4 - 9 + j12$$

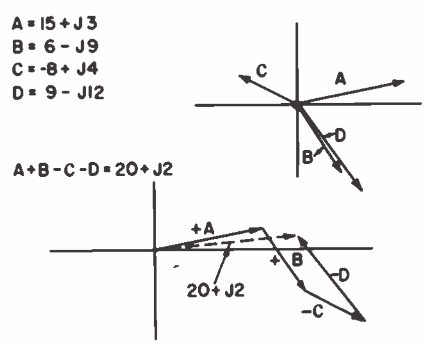


Fig. 19. Addition and subtraction of vectors by diagram.

$$= 15 + 6 + 8 - 9 + j3 + j12 - j9 - j4$$

$$= 29 - 9 + j15 - j13$$

$$= 20 + j2$$

The diagram gives us the same thing, but what a lot of work and confusion it is!

Multiplication and Division. In our work in electronics we may want to multiply or divide two or more vectors. The vectors may represent voltages, currents, or impedances of various values at different phase angles. The j operator will be very handy in this case because there is no purely graphical means of multiplying or dividing vectors with different phase angles. However, as we learned in algebra, it is quite simple to multiply or divide binomials.

Since we studied the multiplication and division of binomials earlier in this lesson, we should not have any trouble with the mathematics. Our only job now is to make sure we understand how we represent our vector resultant. Suppose that we want to multiply vector A by vector B. As shown in Fig. 20A, vector A is equal to $(2 + j3)$ and vector B is equal to $(4 + j2)$. To multiply these two vectors we simply multiply the binomials which gives us:

$$\begin{array}{r} 2 + j3 \\ \times\ 4 + j2 \\ \hline 8 + j12 \qquad \\ + j4 + j^2 6 \\ \hline 8 + j16 + j^2 6 \end{array}$$

But, remember j^2 is equal to -1, so $8 + j16 + j^2 6$ becomes

$$8 + j16 + 6(-1)$$

$$= 8 + j16 - 6$$

$$= + 2 + j16.$$

Thus, our resultant vector from this multiplication is equal to a vector of $2 + j16$, as shown in Fig. 20B.

Notice that we have not only increased the length of the resultant vector by multiplying, but we have also increased the angle of this vector from the reference line. If we stop and think a moment, we will have to agree that this should happen because we are multiplying a rotating force by a rotating force when we multiply j by j. Further, remember that j alone is enough to rotate a vector 90°, while j^2 rotates it 180°. Looking at this,

$$90^\circ + 90^\circ = 180^\circ \text{ and } j \times j = 180^\circ$$

Thus, multiplying j by j is the same as adding the two 90° angles. Now, if you measure the angle that vector A makes with the reference line and add it to the angle that vector B makes with the reference line, the sum of these two angles will equal the angle of the resultant vector.

In our problem: Since $\phi A = 56^\circ$ and $\phi B = 27^\circ$, then

$$\phi A + \phi B = 56^\circ + 27^\circ = 83^\circ.$$

If we measure the angle of the resultant vector AB, we will find that its angle is exactly 83°. In addition to this relationship between the angles, there is a relationship between the lengths of vector A and vector B in the resultant vector AB. If we determine the length of vectors A and B through measurement or by using the Pythagorean theorem, we will find that A is equal to 3.61 and B is equal to

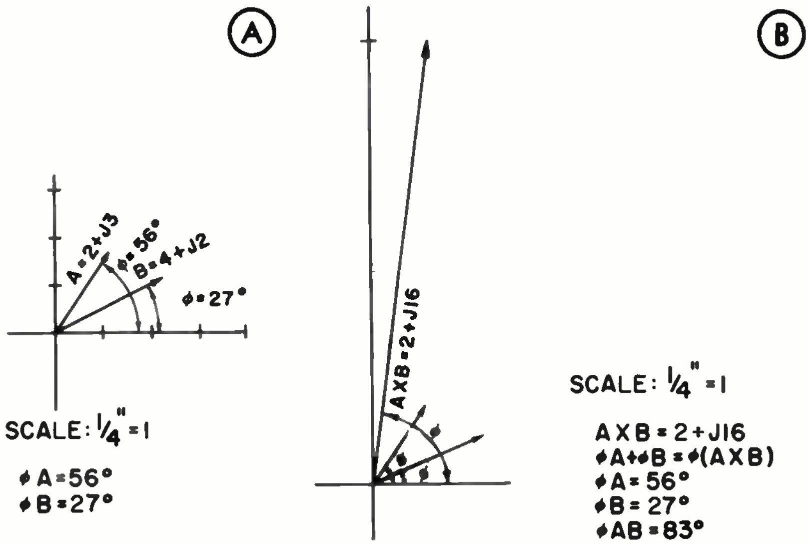


Fig. 20. Multiplying vectors.

4.47. Then if we multiply these actual lengths of A and B we find that A X B = 3.61 X 4.47 = 16.13. Now, either through measurement or by using the Pythagorean theorem, we can also determine the length of our resultant vector AB. It is equal to:

$$\sqrt{2^2 + 16^2}$$

$$= \sqrt{4 + 256}$$

$$= \sqrt{260}$$

$$= 16.13$$

to two decimal places. Thus, our resultant in vector multiplication is a new vector that is equal to the product of the length of all the vectors multiplied and that forms an angle with the reference that is equal to the sum of the angles of all the vectors multiplied. If we stop and think a moment and have understood our previous operations with vectors, we will see that this is what should happen.

In the problem that we just discussed, both of the vectors that we multiplied were made up of positive values. Let's see what happens if we multiply a vector such as 12 + j9 by another vector equal to 7 − j6. Multiplying our binomial, we have:

$$(12 + j9) \times (7 - j6)$$

$$= 84 + j63 - j72 - j^2 54$$

$$= 84 - j9 - 54(-1)$$

$$= 84 + 54 - j9$$

$$= 138 - j9$$

Here, as you can see, we had one vector to the right and above the reference, and another to the right and below the reference. The resultant is a vector that is to the right and below the reference.

Now, suppose we wanted to multiply the following vectors together:

$$(8 + j16) \times (5 + j2) \times (2 + j3)$$

This would give us:

$$\begin{array}{r} 8 + j16 \\ 5 + \ \ j2 \\ \hline 40 + j80 \\ + j16 + j^2 32 \\ \hline 40 + j96 + j^2 32 \end{array}$$

Then:

$$\begin{array}{l} 40 + j96 + j^2 32 \\ 2 + j3 \\ \hline 80 + j192 + j^2 64 \\ + \quad j120 + j^2 288 + j^3 96 \\ \hline 80 + j312 + j^2 352 + j^3 96 \end{array}$$

Then:

$$\begin{aligned} & 80 + j312 + j^2 352 + j^3 96 \\ &= 80 + j312 + (352 \times -1) \\ &\qquad + (96 \times -j) \\ &= 80 - 352 + j312 - j96 \\ &= -272 + j216 \end{aligned}$$

Even though all our multipliers were to the right and above the line, our resultant is to the left and above. Notice that the j^2 term was resolved to its value of -1 and that the j^3 term resolved to its equal value of $-j$.

In order to divide vectors, we simply divide our binomial representations of the vectors involved. The easiest way to do this is to set up the division as a fraction and then clear the j term from the denominator. For example, if we wish to divide a vector such as $2 + j16$ by a vector equal to $2 + j3$, we would set our division up as a fraction:

$$\frac{2 + j16}{2 + j3}$$

Then, if we multiply both the numerator and the denominator by $2 - j3$, we will not change the value of our fraction, but we will get rid of the j term in our denominator. For example, we will have:

$$\frac{(2 + j16)(2 - j3)}{(2 + j3)(2 - j3)}$$

$$= \frac{4 + j26 - j^2 48}{4 - j^2 9}$$

$$= \frac{4 + j26 + 48}{4 + 9}$$

$$= \frac{52 + j26}{13}$$

$$= \frac{13(4 + j2)}{13}$$

$$= 4 + j2$$

Thus, $2 + j16 \div 2 + j3 = 4 + j2$. For proof of this, check Fig. 20 again. As you can see $(4 + j2) \times (2 + j3)$ are the vectors we previously used in this multiplication problem and our product was $2 + j16$. Similarly:

$$138 - j9 \div 12 + j9$$

$$= \frac{138 - j9}{12 + j9}$$

$$= \frac{(138 - j9)(12 - j9)}{(12 + j9)(12 - j9)}$$

$$= \frac{1656 - j1350 + j^2 81}{144 - j^2 81}$$

$$= \frac{1656 - j1350 - 81}{144 + 81}$$

$$= \frac{1575 - j1350}{225}$$

$$= (7 - j6)$$

To prove our answer we simply multiply our quotient $(7 - j6)$ by our divisor $(12 + j9)$ to get our dividend of $138 - j9$. Notice that each time we clear our j term from the denominator by multiplying our numerator and denominator by the same number. This number is always a binomial that is exactly the same as the denominator except that the sign of the j term is reversed. Such a term is called a "conjugate" term. You'll notice that each time we get a j^2 term or any even power of j, the j term disappears because $j^2 = -1$. Remember, in algebra we pointed out that $(a - b)(a + b) = a^2 - b^2$. Thus, if we have $(a - jb)$, we can multiply it by $(a + jb)$ to get $a^2 - j^2b^2$ and eliminate the j. Similarly, if we have $a + jb$, we can multiply it by $a - jb$ to eliminate the j. Thus, we can say that we multiply both the numerator and the denominator by the conjugate of the denominator to clear the j term from the denominator.

When we multiplied two vectors together, we discovered that the product was a new vector equal in length to the product of the vector values at an angle equal to the sum of the angles of the vectors multiplied. In dividing vectors, the opposite relationship exists. If we divide one vector by another and then lay out the dividend vector, the divisor vector, and the quotient vector in a diagram, we will find that:

1. The quotient vector is equal in length to the quotient of the length of the dividend vector divided by the length of the divisor vector.

2. The quotient vector will be at an angle to the reference line that is equal to the difference between the angles of the vectors divided.

Thus, we have two ways that we can multiply or divide vectors. We can multiply or divide the binomial representation of the vectors as we have learned to do in this section, or if we know the vector length we can use it. To multiply, we find the product of the lengths and the sum of the angles. To divide, we find the quotient of the lengths and the difference of the angles.

We mentioned earlier that there was no purely graphical way to multiply and divide vectors. While we can do some of the work graphically, we must always perform some mathematics on the side. Even then, the process of finding the product or quotient in this way is very tedious and involved. Since we already have two methods for finding the products or the quotients mathematically, and since either of these methods is much simpler and faster than the simplest graphical method, it will not be worthwhile for us to study the graphical (plus some math) methods.

In this section of the lesson, you have learned to perform arithmetic operations with vectors. You have learned how to add and subtract vectors, how to multiply and divide by vectors. You will perform all four operations in solving even fairly simple ac circuit problems.

In the next section we will complete the study of the j operator and the binomial representation of vectors, by applying what you have learned to some circuit problems. To test your understanding of this chapter perform the indicated operations in the self-test questions which follow.

SELF-TEST QUESTIONS

Make the following computations:

41. $(3 + j6) + (7 + j2)$
42. $(7 + j2) + (9 - j17)$
43. $(9 - j4) + (-3 + j5)$
44. $(8 + j3) - (4 + j7)$
45. $(17 - j6) - (11 - j8)$
46. $(3 + j7) + (8 - j13) + (7 + j8)$
47. $(16 - j13) + (-11 + j4) + (5 - j2)$
48. $(23 + j14) - (17 + j26) + (1 - j3)$
49. $(2 - j11) - (19 - j17) - (4 + j6)$
50. $(-6 - j18) - (-12 - j14) - (-2 - j21)$
51. $(7 + j6)(3 + j2)$
52. $(11 + j2)(2 - j7)$
53. $(-2 - j9)(-3 + j4)$
54. $(8 - j7)(-3 + j5)$
55. $(3 + j2)(4 + j6)(5 - j8)$
56. $(30 + j30) \div (4 + j2)$
57. $(60 - j11) \div (5 - j6)$
58. $(69 + j17) \div (4 - j3)$
59. $(44 - j168) \div (7 - j9)$
60. $255 \div (6 + j7)$

Using The J Operator In Circuit Operations

The best way to make sure that you understand representing vectors with binomials by using the j operator is to work with them in circuit calculations. In this way, you will get some practice with j arithmetic as well as some more experience in solving circuit problems. We will start with some simple series ac circuits, and then examine some parallel and series-parallel combinations. If, after we have done this, you feel that you still need more practice, try applying these methods to some of the ac circuits you have worked with in your other lessons.

In analyzing and working the circuit problems in this section, we will use the mathematical solutions and the j operator. However, we will still use vector diagrams to help visualize the circuit quantities and their relationships. But, since we are not going to use the diagrams for our actual calculations, we will not need to draw them to scale. Thus, for every problem, we will have a simple diagram to use in our analysis and a mathematical solution for the diagram. This is by far the best way to work with any ac circuit problem.

Series Circuits. In the circuits shown in Fig. 21 we are to find the current. Let's consider the circuit at A first. Since we have an inductance in the circuit along with a resistance, we know that the voltage will lead the current, or another way of saying the same thing is that the current will lag the voltage. Thus, since we are given the value E = 234V, and we draw it at 0°, as in Fig. 21C, then the current must lag it as shown. To position the current vector in this position, we must have a –j term in the current.

Now let's look at the circuit in Fig. 21B. Here we have a resistance and capacitance in series. We know the current must lead the voltage so we must have a phase relationship like the one shown in Fig. 21D. This means we must have a +j term in the current.

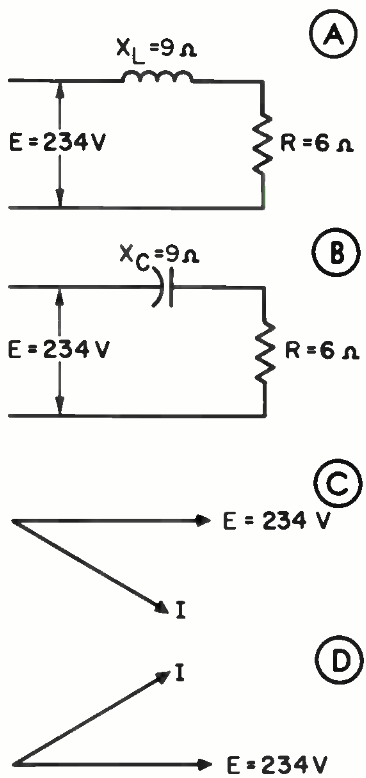


Fig. 21. The vector diagrams at C and D show the phase relationships between the voltage and current in the circuits shown at A and B.

We know that capacitive reactance is the opposite of inductive reactance so one must have a +j sign and the other a –j sign. But which should be + and which should be –? The answer is we must use the signs that make the current come out with the correct sign. Let's see what this means. We know that in an ac circuit

$$I = \frac{E}{Z}$$

In the circuits in Fig. 21 the voltage is 234 volts, the resistance 6 ohms, and the reactance 9 ohms. Thus, in one circuit Z = 6 + j9, and in the other circuit Z = 6 – j9. Now, let's solve the current in both circuits and then we can see whether a +j term represents inductive or capacitive reactance.

$$I = \frac{E}{Z}$$

$$= \frac{234}{6 + j9}$$

$$= \frac{234(6 - j9)}{(6 + j9)(6 - j9)}$$

$$= \frac{1404 - j2106}{36 - j^2 81}$$

$$= \frac{1404 - j2106}{117}$$

$$= 12 - j18$$

Now, since we already know that in the circuit with the inductive reactance we need a –j term in the current, this represents the current in Fig. 21A, and 6 + j9 must represent the impedance of the circuit in Fig. 21A. Therefore, it appears that inductive reactance should be represented by a +j term which means that capacitive reactance will be represented by a –j term. Now let's solve Fig. 21B, using 6 – j9 as the impedance, and see if we get a +j in the current term.

$$I = \frac{E}{Z}$$

$$= \frac{234}{6 - j9}$$

$$= \frac{234(6 + j9)}{(6 - j9)(6 + j9)}$$

$$= \frac{1404 + j2106}{36 - j^2 81}$$

$$= \frac{1404 + j2106}{117}$$

$$= 12 + j18$$

Thus we have a +j term in the current. In fact, notice that the only difference in the two current values is in the sign of the j term which is what we might expect since the reactances are equal.

Remember: *Inductive reactance gets a + sign and capacitive reactance a – sign.* Now, let's do another example.

In the circuit shown in Fig. 22A, we are asked to find the impedance. An examination of the circuit shows that it is a series circuit consisting of resistances, coils, and capacitors. Accordingly, we know that the impedance must be equal to the vector sum of the resistances and reactances. Therefore, in the diagram in Fig. 22B, we have made a simple sketch of the vector relationship of all the

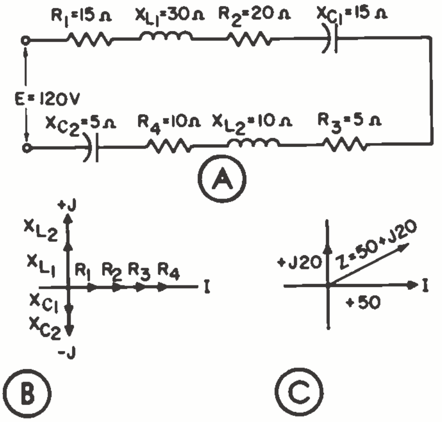


Fig. 22. Series ac circuit and vector representation.

components. Since it is a series circuit and the current is common, we have used a reference line, I, as a base for the diagram. All the resistance vectors are indicated along this reference line to show the total effect of the "in phase" components. Voltages across the resistances will all be in phase with I and hence fall along this reference vector.

The voltage across any coil in the circuit will lead the current by 90° if we neglect its resistance, so X_L vectors are drawn so that they lead the resistance vectors by 90°. This conforms with what we just discovered, that inductive reactance terms are +j terms. The voltage across the capacitors, on the other hand, will lag the current, so the X_C vectors are drawn so that they lag the resistance vectors by 90°. Now, notice that the X_C vectors are –j vectors. The resistance vectors, of course, are in phase and are simply represented as the positive number terms.

Now, from our knowledge of circuit laws, vectors, algebra, and the j operator, we can write the following equation for the circuit impedance:

$$Z = R_1 + R_2 + R_3 + R_4 + jX_{L1} + jX_{L2} - jX_{C1} - jX_{C2} \text{ and,}$$

$$Z = 15 + 20 + 5 + 10 + j30 + j10 - j15 - j5$$

$$Z = 50 + j40 - j20 = 50 + j20$$

Thus, we can draw a resultant vector diagram as shown in Fig. 22C where Z = 50 + j20. Since the j term in our resultant vector is only used to indicate the direction of the final reactive component, or the sign of the resultant phase angle, we can drop it while we compute the impedance with our formula $Z = \sqrt{R^2 + X^2}$. Thus, the impedance is:.

$$Z = \sqrt{50^2 + 20^2}$$

$$= \sqrt{2500 + 400}$$

$$= \sqrt{2900}$$

$$= 54\Omega \text{ (approximately)}$$

Therefore, we can write the impedance of our circuit in two ways: As a vector, Z = 50 + j20 or from the result of our computation as: Z = 54Ω (approx).

To show that either of these answers is perfectly correct and acceptable, we can examine the circuit a little further. Suppose that we are told that the current in the circuit is equal to 4 amps and asked to find the voltage. We know that E = IZ, so let's substitute both of our answers for Z in this formula and see what we get. First, if E = I X Z, then E = 4 X 54 (approximately) or about 216 volts. Next, if E = I X Z, then

$$E = 4(50 + j20) = (200 + j80) \text{ volts.}$$

Now, since $E_T = \sqrt{E_R{}^2 + E_X{}^2}$ and 200 =

E_R and j80 = E_X, we have, by dropping the j,

$$E_T = \sqrt{200^2 + 80^2}$$

$$= \sqrt{40{,}000 + 6400}$$

$$= \sqrt{46{,}400}$$

$$= 216 \text{ volts (approx.)}$$

Although either the vector representation of the answer or the numerical representation is correct and acceptable, the vector answer is often preferred as it indicates our phase angle and leading voltage. Thus, we would say that our impedance was (50 + j20)Ω and our voltage was (200 + j80)V.

Now, let's look at the circuit in Fig. 23. Here we also have a simple series circuit, and are asked to find the impedance. But, instead of being given all the resistances and the reactances, we are given an assortment of values. However, we still know that Z is equal to the sum of the resistances and the reactances, and we can draw our vector diagram as shown in Fig. 23B. Also, Z will be equal to $E_T \div I$ and E_T will be equal to the sum of the individual voltage vectors, as shown in Fig. 23C. Since we are given the total current, the frequency, some of the individual resistances or reactances, some of the voltage drops, and a value of capacitance, we can find the impedance of the circuit either way. The information given is adequate to give us anything we need to know. For example, using Z = R±jX, we have: $Z = R \pm jX = R_1 + R_2 + jX_L - j\lambda_{C1} - jX_{C2}$ and Z =

$$R_1 + \frac{E_{R2}}{I} + j\,\frac{E_L}{I} - jX_{C1} - j\left(\frac{159000}{fC_2}\right)$$

Therefore,

$$Z = 50 + 15 + j50 - j40 - j\left(\frac{159000}{60 \times 20}\right)$$

and

$$\frac{159000}{60 \times 20} = \frac{1590}{12} = 132.5\Omega$$

and

$$Z = 65 + j50 - j40 - j132.5 = 65 - j122.5$$

as shown in Fig. 23D.

Using the other method, we would have:

$$Z = E_T \div I = (E_{R1} + E_{R2} + jE_{XL} - jE_{XC1} - jE_{XC2}) \div I,$$

then

$$Z = [(IR_1) + E_{R2} + jE_{XL} - j(IX_{C1}) - j(IX_{C2})] \div I$$

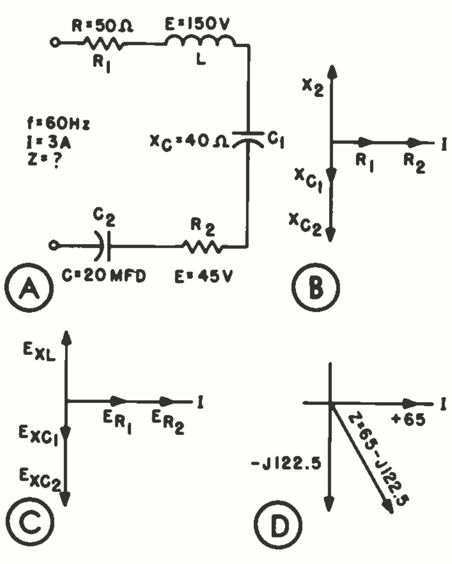


Fig. 23. Series ac circuit with vector diagrams.

Now:

$$j(IX_{C2}) = [jI(159000 \div fC_2)]$$

$$= [jI(159000 \div 1200)]$$

$$= j(132.5I)$$

Thus, Z =

$$\frac{IR_1 + E_{R2} + j(E_{XL}) - j(IX_{C1}) - j(132.5I)}{I}$$

Now,

$$Z = R_1 + \frac{E_{R2}}{I} + \frac{j(E_{XL})}{I} - j(X_{C1}) - j132.5$$

$$= 50 + \frac{45}{3} + \frac{j(150)}{3} - j40 - j132.5$$

$$= 50 + 15 + j50 - j172.5$$

$$= 65 - j122.5$$

which is the same answer we got the other way. By using the Pythagorean theorem, we can further find that:

$$Z = \sqrt{65^2 + 122.5^2}$$

$$= \sqrt{4225 + 15006}$$

$$= \sqrt{19231}$$

$$= 139\Omega \text{ (approximately)}$$

Notice that we always drop the j when we use the Pythagorean theorem, because the j only indicates the position of the impedance vector and there is no way to indicate this in a monomial such as 139. However, we can use the sign of the j operator to indicate the direction by saying "139Ω capacitive."

Parallel Circuits. In our earlier ac circuit calculations, we have worked almost exclusively with series circuits. Although parallel circuits and series-parallel circuits can be solved by using vector measurement solutions alone, the vector diagrams generally become quite complex and difficult to work with. However, now that we have a method of solving ac circuits mathematically, we shall be able to handle these more complex circuits.

The major difference in working with parallel circuits is in the choice of a reference. In your study of dc circuits, you learned that the current divides in the branches of a parallel circuit while the voltage across all the branches is common. This is just the opposite from a series circuit where the current is common and the voltage divides. The same is true for ac circuits, so the general rules for dc circuits will apply to ac circuit solutions. Therefore, in our work with ac parallel circuits, we will use the circuit voltage as a reference instead of the current as we did in most of the series circuits. This difference in the choice of the reference value is very important.

Now, let's look at the simple parallel circuit in Fig. 24. Here, our circuit contains a coil in one leg and a resistor in the other leg. The voltage applied to the circuit is applied equally to each branch. However, the current as shown by an ammeter in each leg is different in each branch. The total current in the circuit is equal to the sum of the current in the branches. What is this total current?

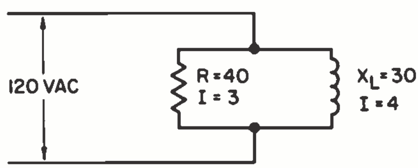


Fig. 24. Simple ac parallel circuit.

Since one branch has an inductive current and the other branch has a resistive current, we cannot add the two currents together numerically. We must add them together vectorially just as we would the voltages in a series circuit. If we draw a vector diagram for this addition, we have to use the voltage as our reference line since the voltage across each branch is the same. Thus, we would lay out our reference line and label it E as shown in Fig. 25A. Next, we want to represent our current vectors for each leg. First, we take a vector representing the current in the resistance branch and draw it along the reference line, E, and label it I_R, as shown in Fig. 25B. We draw this vector along the reference to show that the current through the resistive branch is in phase with the common voltage.

Next, we want to represent the current through our inductive branch as a vector. Now, we know that neglecting the resistance of the coil, this current will lag the voltage by exactly 90°. Since our E reference is along the horizontal and points to the right, we must draw this current vector downward, as shown in Fig. 25C, in order to show this lagging effect. Thus, our I_L vector is a $-j$ value. Therefore, our total current vector for the circuit would be indicated mathematically as

$$I_T = I_R - jI_L,$$

as shown in Fig. 25C.

Now, if we substitute the given values for the two currents shown in Fig. 24, our total current would equal:

$$I_T = I_R - jI_L$$

$$= 3 - j4$$

dropping the j

$$I_T = \sqrt{3^2+4^2}$$

$$= \sqrt{9+16}$$

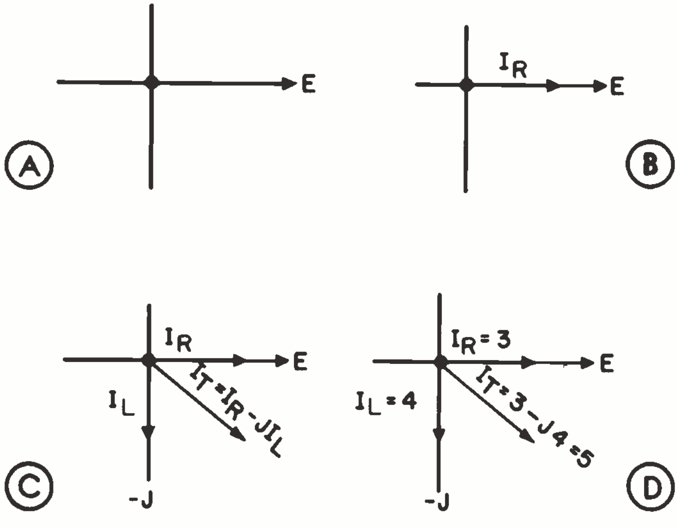


Fig. 25. Parallel circuit solution.

$$= \sqrt{25}$$

$$= 5 \text{ amps}$$

Our j operator was minus, so our total circuit current is 5 amps (lagging) as shown by the vector $I_T = 3 - j4 = 5$, in Fig. 25D.

If we want to find the impedance of this parallel circuit, we can proceed in two ways. The simplest way is to use the total current and the applied voltage in our formula E = IZ and therefore Z = (E/I) or, in our circuit, Z = (120/5) = 24Ω. If we were relying on only vector measurement solutions for parallel circuits, this would be the only way we could find the impedance. The reason for this is that the formula for resistances or impedances in parallel is

$$Z_T = \frac{Z_1 \times Z_2}{Z_1 + Z_2}$$

and we have no purely graphical way of multiplying or dividing the vectors.

However, since we know how to use the j operator, we can multiply or divide these vectors mathematically. Therefore, we can use this formula to find the total impedance. In the circuit shown in Fig. 24, we would have:

$$Z_T = \frac{R(jX_L)}{R + jX_L}$$

$$= \frac{40(j30)}{40 + j30}$$

$$= \frac{j1200(40 - j30)}{(40 + j30)(40 - j30)}$$

$$= \frac{j48000 - j^2 36000}{1600 - j^2 900}$$

$$= \frac{j48000 + 36000}{1600 + 900}$$

$$= \frac{j480 + 360}{25}$$

$$= 14.4 + j19.2$$

and our impedance total written as a vector would be (14.4 + j19.2). Then, applying the Pythagorean theorem to this vector, we would find:

$$Z = \sqrt{(14.4)^2 + (19.2)^2}$$

$$= \sqrt{207.36 + 368.64}$$

$$= \sqrt{576} = 24\Omega$$

This, of course, is the same answer that we got for the impedance by dividing the total voltage by the total current.

While the impedance of any parallel circuit can be found using either method, you can see that it is much simpler and quicker to use the first method. The current can be found by addition of vectors and then a simple division allows us to find the impedance if we know the voltage. The other way requires both the multiplication and division of vectors which can become quite complex. In fact, in complex circuits, it becomes so involved mathematically that it is almost never used.

Because of this complexity, a method of finding impedance has been worked out that involves the addition of current vectors, even though the voltage is not known. For example, consider the circuit in Fig. 26. Here we have a resistor, a coil, and a capacitor in parallel. We are asked to find the impedance and we have the values of X_C, X_L, and R given. Since we have no values of either current or voltage

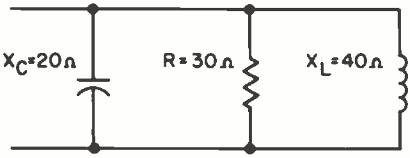


Fig. 26. Parallel circuit with X_C, R_1, and X_L. Find Z.

given, it would seem that we will be forced to use our impedance formulas.

Suppose, however, that we *assume* a circuit voltage of 120 volts. If we do this, then we can find the current that would flow in each branch with this assumed voltage applied to the circuit. It would be:

$$I_C = \frac{E}{X_C} = \frac{120}{20} = 6 \text{ amps}$$

$$I_R = \frac{E}{R} = \frac{120}{30} = 4 \text{ amps, and}$$

$$I_L = \frac{E}{X_L} = \frac{120}{40} = 3 \text{ amps.}$$

Now, we can add these currents using the j operator to find the total current that would flow for the value of assumed voltage we have chosen.

Laying out a vector diagram for reference as shown in Fig. 27, we would use our common reference E as a base. Then our I_L vector would be drawn down toward $-j$ to indicate the current lag through the coil. The vector for I_C would be drawn up toward $+j$, indicating the leading current through the capacitor. Finally, the I_R vector would be drawn along the reference to indicate the in phase current through the resistance. Now, our problem becomes mathematically:

$$I_T = I_R + jI_C - jI_L$$

$$= 4 + j6 - j3$$

$$= 4 + j3$$

Then,

$$I_T = \sqrt{4^2 + 3^2}$$

and I_T = 5 amps leading (notice the sign of j) with an assumed voltage of 120 volts.

Now, applying our formula $Z = (E/I)$, we have $Z = (120/5) = 24\Omega$. Thus, an assumed voltage forces a total current through the circuit that gives us an impedance of 24Ω. The interesting thing is, that no matter what voltage we assume, the computed current will always be a value such that our impedance for this circuit will work out to 24Ω. Thus, we can assume any voltage for a parallel circuit, compute the total current forced

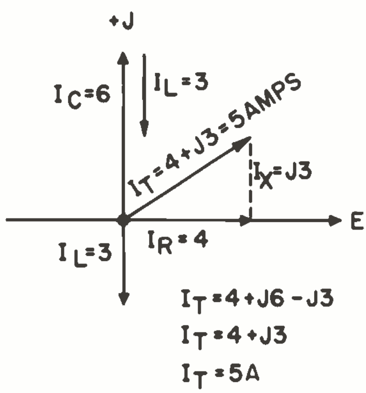


Fig. 27. Vector solution of I for circuit in Fig. 26.

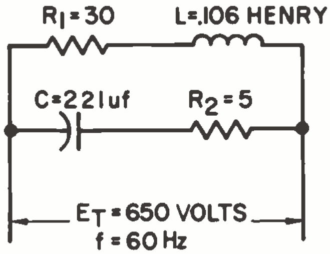


Fig. 28. Series-parallel ac circuit.

through the circuit by this voltage, and then divide to find the impedance. Naturally, in doing this, we always assume a value of voltage that will make our problem as simple as possible. Try assuming a couple of different voltages for the circuit in Fig. 26, and then compute the current and impedance. You will see that 24Ω is always your answer for this circuit.

Series-Parallel Circuits. The next problem is to learn to combine our knowledge of series circuits with our knowledge of parallel circuits for series-parallel combinations. Generally, we do this just as we would for dc circuits. We break our circuit down into simple circuits which we solve one at a time, and then combine our answers. For example, let's consider the circuit shown in Fig. 28.

Here we have a coil in series with a resistor in one branch which is in parallel with another branch containing a resistor and a capacitor. We are given R_1, R_2, L and C, the total voltage, and the frequency. We are asked to find the total current and the total impedance of the circuit. First solve each of the two branches separately, then combine them to find the total current, and then find the total impedance.

The best way is to proceed as follows: Let's call the branch with the coil, branch A, and the one with the capacitor, branch B. Since we want the total current, we would want to find the current in each leg and then combine them. Starting with branch A, we must first find X_L and then find I_A as follows:

$$
\begin{aligned}
I_A &= E_A \div Z_A \\
&= E_A \div (R_1 + jX_L) \\
&= E_A \div (R_1 + j2\pi fL) \\
&= 650 \div (30 + j6.28 \times 60 \times .106) \\
&= 650 \div (30 + j40) \\
&= \frac{650\,(30 - j40)}{(30 + j40)\,(30 - j40)} \\
&= \frac{19500 - j26000}{900 - j^2\,1600} \\
&= \frac{195 - j260}{9 + 16} \\
&= \frac{195 - j260}{25} \\
&= 7.8 - j10.4
\end{aligned}
$$

Now, notice that I_A is the current through the series circuit of branch A, yet we have it broken up into a j binomial. This probably seems strange since you know that the current is common in a series circuit and the current through the coil is the same as the current through the resistance. Although it is true that we have only one current through the series branch, this current is made up of the combined effects of the resistor and the coil. Therefore, this current is a vector that can be considered to consist of two components just the same as any other vector.

The vector diagrams in Fig. 29 may help you to understand this. In Fig. 29A we have shown the impedance vector 30 + j40 which we found in the first few steps of our equation. Since this is a series circuit, we have used the current as a

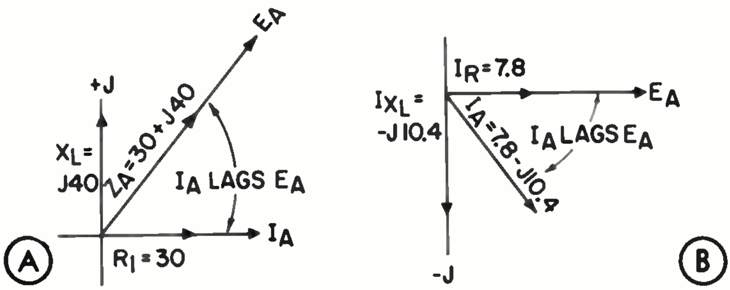


Fig. 29. Vector relationships for branch A. (Not to scale.)

reference for this diagram. Now, if we were to represent the voltage for this series circuit vectorially, it would extend along the same line as the impedance vector, as shown. Thus, the vector diagram in Fig. 29A shows the relationships of the impedance, the current, and the voltage. Notice that the current lags the voltage.

Now, when we get ready to combine branch A and branch B, we will want to use the voltage as a reference because it is common to both branches. When we do this, we would have to show the current for branch A as a vector lagging the voltage, as shown in Fig. 29B. Thus, either Fig. 29A or 29B shows the proper relationship between the current and the voltage. In order to compute with this current vector using the j operator, we would want to break it up into its components. We can do this in the diagram, shown in Fig. 29B, because E is the reference. That is why we simply divided the voltage E_A by the binomial of the impedance vector rather than solving for the monomial impedance. In this way, our current is already broken into its binomial term, ready for use in combining with branch B as soon as we divide the voltage by the impedance.

Now, we follow the same general procedure and solve for the current in branch B as follows:

$$I_B = E_B \div Z_B$$

$$= E_B \div (R_2 - jX_C)$$

$$= E_B \div \left(R_2 - j\frac{159000}{fC}\right)$$

$$= 650 \div \left(5 - j\frac{159{,}000}{60 \times 221}\right)$$

$$= 650 \div (5 - j12)$$

$$= \frac{650\,(5 + j12)}{(5 - j12)\,(5 + j12)}$$

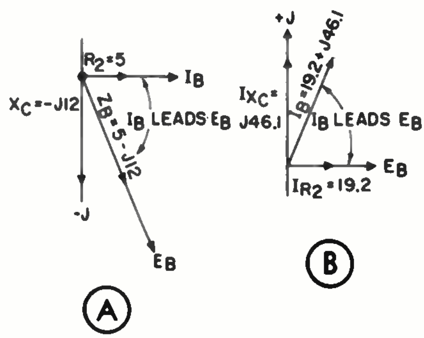


Fig. 30. Vector relationships for branch B. (Not to scale.)

$$= \frac{3250 + j7800}{25 - j^2 144}$$

$$= \frac{3250 + j7800}{25 + 144}$$

$$= \frac{3250 + j7800}{169} = 19.2 + j46.1$$

The vector diagrams for this are shown in Fig. 30 in the same manner as those in Fig. 29. Notice that in the diagram in Fig. 29A, the inductive terms are positive or +j to show that the voltage leads the current reference. However, when we use the voltage as a reference, as in Fig. 29B, the current term must be negative, or −j, in order to show this same lag. Likewise, the sign changes in Fig. 30A and 30B show the same thing except that they are opposite because we are dealing with capacitance or leading current.

Now that we have found the current in the two branches, we simply add them as shown by the vector diagram in Fig. 31 and the following mathematical solution:

$$I_T = I_A + I_B$$

$$= (7.8 - j10.4) + (19.2 + j46.1)$$

$$= 7.8 + 19.2 + j46.1 - j10.4$$

$$= 27 + j35.7$$

$$= \sqrt{27^2 + 35.7^2}$$

$$= \sqrt{729 + 1274.5}$$

$$= \sqrt{2003.5}$$

$$= 44.7 \text{ amps}$$

Now, the impedance is:

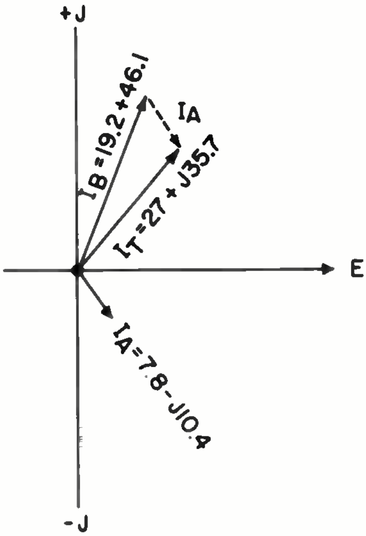


Fig. 31. Vector addition of current in Figs. 25 and 26 (not to scale).

$$Z = \frac{E}{I} = \frac{650}{44.7} = 14.5 \text{ ohms}$$

In solving circuit problems such as this it is wise to set up a complete equation in the beginning. In this way you get straight to the heart of the problem and save yourself from doing a lot of work finding things that you do not need. As an example, we found only the binomial expressions for current and impedance in the circuit we just completed. We did not bother to find the numerical values of the quantities until the last moment. Also, we did not have to find the voltage drops across the individual components. The proper circuit equations will keep you from spending unnecessary time solving for quantities you do not need. Stated as a complete equation, this last circuit would have been:

Given: $E_T = 650V$

$$f = 60 \text{ Hertz}$$

$$R_1 = 30\Omega$$

$$R_2 = 5\Omega$$

$$L = .106\text{h}$$

$$C = 221\ \mu f$$

find I_T Z_T
Then,

$$Z_T = \frac{E_T}{I_T}$$

and

$$I_T = I_A + I_B$$

Therefore:

$$Z_T = E_T \div (I_A + I_B)$$

But: $I_A = E_A \div Z_A$

$I_B = E_B \div Z_B$

But: $E_B = E_A = E_T$

Then

$$I_A = E_T \div Z_A$$

$$I_B = E_T \div Z_B$$

Therefore:

$$Z_T = \frac{E_T}{\frac{E_T}{Z_A} + \frac{E_T}{Z_A}}$$

Now: $Z_A = R_1 + jX_L$
$Z_B = R_2 - jX_C$

Therefore:

$$Z_T = \frac{E_T}{\frac{E_T}{R_1 + jX_L} + \frac{E_T}{R_2 - jX_C}}$$

And: $jX_L = j(2\pi fL)$
$jX_C = j(159000 \div fC)$

Therefore:

$$Z_T = \frac{E_T}{\frac{E_T}{R_1 + j2\pi fL} + \frac{E_T}{R_2 - j\frac{15900}{f_C}}}$$

This is the complete circuit equation and gives us Z_T in terms of our known values. We can now solve for Z_T and apply the Pythagorean theorem to get Z_T as a monomial answer. Ohm's Law will then give us I_T.

In Fig. 32, we have a more complex series-parallel circuit. In this problem, the resistances and reactances are given; you are to find the total circuit impedance.

Examination of the circuit will show that the total impedance "Z_T" is equal to the vectorial sum of R_1 X_{L1} and X_{C1} in series with the combined impedance of the three parallel branches. For con-

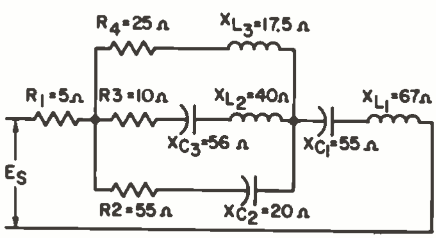


Fig. 32. Series-parallel circuit problem.

venience, we will refer to the combined impedance of the three parallel branches as "Z_b". We can now write the equation for the total circuit impedance:

$$Z_T = (R_1 + jX_{L1} - jX_{C1}) + Z_b$$

We will first find the value of Z_b. Later, we can find the total circuit impedance.

Neither the source voltage E_S nor the voltage drop across any part of the circuit is given. However, we can simplify the work by assuming that a voltage "E_b" exists across the three parallel branches and use this voltage as a reference. Now, we can write the equation:

$$Z_b = \frac{E_b}{I_b}$$

where I_b is the total current flowing in the three parallel branches. Regardless of the assumed voltage E_b, the impedance of the parallel branches will remain the same since the current is proportional to E_b divided by Z_b.

The current I_b is the vectorial sum of the currents flowing through the three individual parallel branches. We will use I_1 to represent the current through R_2 and X_{C2}, I_2 for the current through R_3, X_{C3} and X_{L2}, and I_3 to represent the current through R_4 and X_{L3}. Then, we can write:

$$I_b = I_1 + I_2 + I_3$$

Substituting this in the equation for the impedance of the parallel branches gives us:

$$Z_b = \frac{E_b}{I_1 + I_2 + I_3}$$

The current flowing through each of the parallel branches is equal to the assumed voltage divided by the impedance of the individual branches. The branch currents then are:

$$I_1 = \frac{E_b}{Z_1} \quad I_2 = \frac{E_b}{Z_2} \quad I_3 = \frac{E_b}{Z_3}$$

We can now write the equation for the combined impedance of the parallel branches:

$$Z_b = \frac{E_b}{\dfrac{E_b}{Z_1} + \dfrac{E_b}{Z_2} + \dfrac{E_b}{Z_3}}$$

Let us assume that E_b is equal to 100V (we can assume any value for E_b and still get the same final answer for Z_b) and substitute 100 for E_b in the equation:

$$Z_b = \frac{100}{\dfrac{100}{55 - j20} + \dfrac{100}{10 + j40 - j56} + \dfrac{100}{25 + j17.5}}$$

$$Z_b = \frac{100}{\left(\frac{100}{55 - j20} \cdot \frac{55 + j20}{55 + j20}\right) + \left(\frac{100}{10 - j16} \cdot \frac{10 + j16}{10 + j16}\right) + \left(\frac{100}{25 + j17.5} \cdot \frac{25 - j17.5}{25 - j17.5}\right)}$$

$$Z_b = \frac{100}{\frac{100(55 + j20)}{3025 - j^2 400} + \frac{100(10 + j16)}{100 - j^2 256} + \frac{100(25 - j17.5)}{625 - j^2 306}}$$

Now, since $j^2 = -1$

$$Z_b = \frac{100}{\frac{100(55 + j20)}{3025 + 400} + \frac{100(10 + j16)}{100 + 256} + \frac{100(25 - j17.5)}{625 + 306}}$$

$$Z_b = \frac{100}{\frac{5500 + j2000}{3425} + \frac{1000 + j1600}{356} + \frac{2500 - j1750}{931}}$$

$$Z_b = \frac{100}{(1.6 + j.58) + (2.8 + j4.5) + (2.7 - j1.88)}$$

$$Z_b = \frac{100}{7.1 + j3.2}$$

$$Z_b = \frac{100}{7.1 + j3.2} \cdot \frac{7.1 - j3.2}{7.1 - j3.2}$$

$$Z_b = \frac{710 - j320}{60.2}$$

$$Z_b = 11.8 - j5.3\Omega$$

Now solve for the total circuit impedance:

$$Z_T = (5 + j67 - j55) + (11.8 - j5.3)\Omega$$

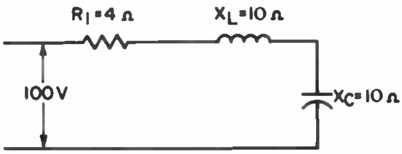
$$Z_T = 16.8 + j6.7\Omega$$

Then from the Pythagorean theorem, the total circuit impedance is equal to 18.1 ohms.

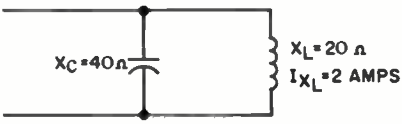
In this lesson, we have studied two math subjects that will be especially important in your work in electronics – algebra and the j operator. With a knowledge of these two subjects, you will be able to resolve complex ac circuit vectors mathematically. As you can see, this is a far more accurate and less tedious method than solving these problems by laying out and measuring vectors. However, we are still missing one very important factor of ac circuits. We are able to determine whether the current leads or lags the voltage, but we cannot tell what the actual phase angle is by mathematics alone. The only way we can determine the phase angle is to draw our final vector resultant to scale and then measure the angle of lead or lag with a protractor. In our next reference lesson on mathematics, we will learn how to compute the phase angle mathematically and then go on to consider power and resonance in ac circuits in more detail.

SELF-TEST QUESTIONS

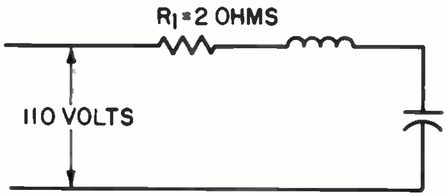
61. One leg of a parallel circuit contains an impedance equal to $(3 + j4)$, the other leg impedance equals $(8 - j6)$. What is the total impedance of the circuit?
62. If the impedance vector of a series circuit equals $40 - j30$ and the applied voltage is 100 volts, what is the current in amps?
63. A series circuit consists of R_1, R_2, R_3, C_1, C_2, L_1, and L_2 connected in series. What is the impedance of the circuit if $R_1 = 12$ ohms, $R_2 = 17$ ohms, $R_3 = 11$ ohms, $X_{C1} = 75$ ohms, $X_{C2} = 50$ ohms, $X_{L1} = 40$ ohms, and $X_{L2} = 60$ ohms?
64. What is the total impedance of a series circuit that contains the following impedances: $1 + j6$, $3 - j2$, $4 - j7$, $3 + j14$, $7 - j1$?
65. In the circuit shown find the total current, the voltage across the coil, and the voltage across the capacitor. What is the name given to this type of circuit?



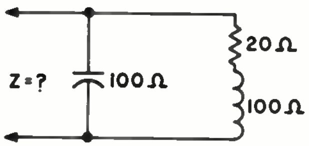
66. If an alternating voltage of 117 volts is connected across a parallel circuit made up of three legs, with a 30Ω resistance in one leg, an inductive reactance of 117Ω in one leg, and a capacitive reactance of 39Ω in one leg, what is the total current drawn from the source?
67. A parallel circuit is made up of four branches, three of the four branches being pure resistances of 16, 16, and 8 ohms, respectively. The fourth branch has an inductive reactance of 6Ω. What is the total impedance of the circuit?
68. A series circuit consisting of a 12-ohm resistor, a 20-mfd capacitor and a .1 henry coil is connected across a 150-volt, 120-Hertz ac source. What is the current in the circuit?
69. Find the current through the capacitor in the circuit shown.



70. What will the current be in the circuit shown when it is operated at its resonant frequency?



71. Find the impedance of the circuit shown below.



Answers To Self-Test Questions

1. A monomial is a mathematical expression containing only one term.
2. A polynomial is a mathematical expression containing two or more terms.
3. A binomial contains two terms while a trinomial contains three terms.
4. An exponent is a number which is normally placed to the right and above a term. It means the number of times the term is to be multiplied by itself.
5. 6.
6. $$\begin{array}{r} 3a^2b + 2b \\ a^2b - b \\ -2a^2b + 4b \\ \hline 2a^2b + 5b \end{array}$$

7. (a) $7x^2y + 12xy + 3y + 2$

   (b) $-6a^4b^2 - 2$

8. $$\begin{array}{rrrrrr} 4a^2b^2 & & & & -2b & \\ & & +3ab^2 & +2a & & \\ a^2b^2 & -3a^2b & & & +3b & +3 \\ \hline 5a^2b^2 & -3a^2b & +3ab^2 & +2a & +b & +3 \end{array}$$

9. (a) $3ab - 3a - 3$

   (b) $4x^2y + 4xy + 3y$

10. $$\begin{array}{rrrrr} ab^2 & +ab & -3a & -b & \\ & & +a & +b & \\ 3ab^2 & & & -b & \\ & +ab & +7a & & \\ & +ab & & & +3 \\ \hline 4ab^2 & +3ab & +5a & -b & +3 \end{array}$$

11. $$\begin{array}{r} 11a + b - 2c \\ -\;\; 6a + 4b - 2c \\ \hline 5a + 5b - 4c \end{array}$$

12. $$\begin{array}{rrrr} a^3 & -a^2b & +4ab^2 & \\ & -6a^2b & -3ab^2 & +b^3 \\ \hline a^3 & -7a^2b & +ab^2 & +b^3 \end{array}$$

13. $$\begin{array}{rrrr} 3a & -4b & +c & -6d \\ -a & -b & -c & -d \\ \hline 2a & -5b & & -7d \end{array}$$

14. $$\begin{array}{rr} 2a & +6b \\ -4a & -7b \\ \hline -2a & -b \end{array}$$

15. $$\begin{array}{rrrr} 8a^3 & +3a^2b & -ab^2 & +b^3 \\ -6a^3 & +a^2b & -ab^2 & +b^3 \\ \hline +2a^3 & +4a^2b & -2ab^2 & +2b^3 \end{array}$$

16. $$\begin{array}{lrr} a & +2b & \\ a & -b & \\ \hline a^2 & +2ab & \\ & -ab & -2b^2 \\ \hline a^2 & +ab & -2b^2 \end{array}$$

17. $$\begin{array}{rrrr} a^2 & +2ab & +b^2 & \\ & a & +b & \\ \hline a^3 & +2a^2b & +ab^2 & \\ & a^2b & +2ab^2 & +b^3 \\ \hline a^3 & +3a^2b & +3ab^2 & +b^3 \end{array}$$

18. $(2a + 3b)(2a - 3b) = 4a^2 - 9b^2$

19. $$\begin{array}{rrrr} a^2 & -2ab & +b^2 & \\ & a & +b & \\ \hline a^3 & -2a^2b & +ab^2 & \\ & a^2b & -2ab^2 & +b^3 \\ \hline a^3 & -a^2b & -ab^2 & +b^3 \end{array}$$

20. $$\begin{array}{l} a - b \\ \underline{a + 2b^2} \\ a^2 - ab \\ \underline{\qquad + 2ab^2 - 2b^3} \\ a^2 - ab + 2ab^2 - 2b^3 \end{array}$$

21. $$\begin{array}{r} a^2 - 2ab + b^2 \\ a - b \,\big)\overline{\,a^3 - 3a^2b + 3ab^2 - b^3} \\ \underline{a^3 - a^2b} \qquad\qquad \\ -2a^2b + 3ab^2 \qquad \\ \underline{-2a^2b + 2ab^2} \qquad \\ ab^2 - b^3 \\ \underline{ab^2 - b^3} \end{array}$$

or $(a - b)^2$

22. $$\frac{64a^4 - 81b^6}{8a^2 + 9b^3}$$

$$= \frac{(8a^2 + 9b^3)(8a^2 - 9b^3)}{8a^2 + 9b^2}$$

$= 8a^2 - 9b^3$

23. $$\frac{a^5 - 3a^3 + a}{a}$$

$$= \frac{a(a^4 - 3a^2 + 1)}{a}$$

$= a^4 - 3a^2 + 1$

24. $$\frac{x^3 + 2x^2 + x}{x^2 + x}$$

$$= \frac{x(x^2 + 2x + 1)}{x(x + 1)}$$

$$= \frac{(x + 1)(x + 1)}{x + 1}$$

$= x + 1$

25. $$\frac{a^4 + 2a^2b^2 + b^4}{a^2 + b^2} = \frac{(a^2 + b^2)(a^2 + b^2)}{a^2 + b^2}$$

$= a^2 + b^2$

26. $$\begin{array}{r} 3x^2 - 5x + 4 \\ 2x + 3 \,\big)\overline{\,6x^3 - x^2 - 7x + 12} \\ \underline{6x^3 + 9x^2} \qquad\qquad \\ -10x^2 - 7x \qquad \\ \underline{-10x^2 - 15x} \qquad \\ + 8x + 12 \\ \underline{+ 8x + 12} \end{array}$$

27. $$\begin{array}{r} 5x^2 + 12x - 5 \\ 3x - 2 \,\big)\overline{\,15x^3 + 26x^2 - 39x + 10} \\ \underline{15x^3 - 10x^2} \qquad\qquad \\ + 36x^2 - 39x \qquad \\ \underline{+ 36x^2 - 24x} \qquad \\ - 15x + 10 \\ \underline{- 15x + 10} \end{array}$$

28. $$\begin{array}{r} - 9a^3b + 6a^2b^2 - 5ab^3 \\ + 14a^3b + 6a^2b^2 - 5ab^3 \\ \underline{+ a^3b - 3a^2b^2 - ab^3} \\ + 6a^3b + 9a^2b^2 - 11ab^3 \end{array}$$

29. An equation is a mathematical statement that two quantities are equal to each other.

30. (c) Division by 0 can not be done in mathematics.

31. The sign of the term must be changed.

32. A formula is a rule or law which is stated as an equation.

33. $R = \dfrac{P}{I^2}$

34. $I = \sqrt{\dfrac{P}{R}}$

35. $L = \dfrac{X_L}{2\pi f}$

36. $f = \dfrac{I}{2\pi C X_C}$

37. $E = \sqrt{PR}$

38. $X_C = X_L + \sqrt{Z^2 - R^2}$

$$Z^2 = R^2 + (X_C - X_L)^2$$

$$(X_C - X_L)^2 = Z^2 - R^2$$

$$X_C - X_L = \sqrt{Z^2 - R^2}$$

$$X_C = X_L + \sqrt{Z^2 - R^2}$$

39. 105 volts. First solve for the voltage across R_2.

$E = IR$

$E_{R2} = 2\,(15)$

$E_{R2} = 30$ volts.

Since R_3 is in parallel with R_2 it must also drop 30 volts. Thus, we can find the current through R_3:

$I = \dfrac{E}{R}$

$I = \dfrac{30}{30}$

$I = 1$ amp.

Now, the total current in the circuit is equal to the sum of the currents in the two parallel branches or 3 amps. This means that 3 amps of current is flowing through R_1. Therefore, R_1 drops (25 ohms X 3 amps) = 75 volts. Thus, the total applied voltage is 75 volts plus 30 volts equals 105 volts.

40. 95 volts. First find the current through R_4. $I = \frac{E}{R} = \frac{5}{5} = 1$ amp. Then find the voltage dropped by R_3. $E = IR = 1(20) = 20$ volts. Thus, the voltage across R_2 must be 5 volts + 20 volts = 25 volts. Consequently, the current through R_2 is: $I = E/R$; $I = 25/10 = 2.5$ amps. This means that the total current through R_1 is 1 amp + 2.5 amps = 3.5 amps. Now, find the voltage dropped across R_1: $E = IR$; $E = 3.5(20)$; $E = 70$ volts. Thus E_T = 70 volts + 25 volts = 95 volts.

41. $\begin{array}{r} 3 + j6 \\ 7 + j2 \\ \hline 10 + j8 \end{array}$

42. $\begin{array}{r} 7 + j\,2 \\ 9 - j17 \\ \hline 16 - j15 \end{array}$

43. $\begin{array}{r} 9 - j4 \\ -3 + j5 \\ \hline 6 + j1 \end{array}$

44. $\begin{array}{r} 8 + j3 \\ -4 - j7 \\ \hline 4 - j4 \end{array}$

45. $\begin{array}{r} 17 - j6 \\ -11 + j8 \\ \hline 6 + j2 \end{array}$

46.
$$\begin{array}{r} 3+j\ 7 \\ 8-j13 \\ 7+j\ 8 \\ \hline 18+j\ 2 \end{array}$$

47.
$$\begin{array}{r} 16-j13 \\ -11+j\ 4 \\ 5-j\ 2 \\ \hline 10-j11 \end{array}$$

48.
$$\begin{array}{r} 23+j14 \\ -17-j26 \\ 1-j\ 3 \\ \hline 7-j15 \end{array}$$

49.
$$\begin{array}{r} 2-j11 \\ -19+j17 \\ -\ 4-j\ 6 \\ \hline -21 \end{array}$$

50.
$$\begin{array}{r} -\ 6-j18 \\ +12+j14 \\ +\ 2+j21 \\ \hline 8+j17 \end{array}$$

51.
$$\begin{array}{l} 7+j\ 6 \\ 3+j\ 2 \\ \hline 21+j18 \\ \quad +j14+j^2\,12 \\ \hline 21+j32+j^2\,12 \end{array}$$

$$= 21+j32-12 = 9+j32$$

52.
$$\begin{array}{l} 11+j\ 2 \\ \ \ 2-j\ 7 \\ \hline 22+j\ 4 \\ \quad -j77-j^2\,14 \\ \hline 22-j73-j^2\,14 \end{array}$$

$$= 22-j73+14 = 36-j73$$

53.
$$\begin{array}{l} -2-j9 \\ -3+j4 \\ \hline +6+j27 \\ \quad -j\ 8-j^2\,36 \\ \hline +6+j19-j^2\,36 \end{array}$$

$$= +6+j19+36 = 42+j19$$

54.
$$\begin{array}{l} 8-j\ 7 \\ -\ 3+j\ 5 \\ \hline -24+j21 \\ \quad +j40-j^2\,35 \\ \hline -24+j61-j^2\,35 \end{array}$$

$$= -24+j61+35 = 11+j61$$

55.
$$\begin{array}{l} 3+j2 \\ 4+j6 \\ \hline 12+j8 \\ \quad +j18+j^2\,12 \\ \hline 12+j26+j^2\,12 \end{array}$$

$$= 12+j26-12 = j26$$

Now multiply j26 times $5-j8$

$$\begin{array}{l} \quad 5-j8 \\ j\ 26 \\ \hline j130-j^2\,208 \end{array}$$

$$= j130+208 = 208+j130$$

56.
$$\frac{30+j30}{4+j2} = \frac{30+j30\,(4-j2)}{(4+j2)\,(4-j2)}$$

$$= \frac{120+j120-j60-j^2\,60}{16-j^2\,4}$$

$$= \frac{180+j60}{20} = 9+j3$$

57. $\frac{60 - j11}{5 - j6} = \frac{60 - j11\,(5 + j6)}{(5 - j6)\,(5 + j6)}$

$= \frac{300 + j305 - j^2 66}{25 - j^2 36} = \frac{366 + j305}{61}$

$= 6 + j5$

58. $\frac{69 + j17}{4 - j3} = \frac{69 + j17\,(4 + j3)}{(4 - j3)\,(4 + j3)}$

$= \frac{276 + j275 + j^2 51}{16 - j^2 9}$

$= \frac{225 + j275}{25} = \frac{25(9 + j11)}{25}$

$= 9 + j11$

59. $\frac{44 - j168}{7 - j9} = \frac{44 - j168\,(7 + j9)}{(7 - j9)\,(7 + j9)}$

$= \frac{308 - j780 - j^2 1512}{49 - j^2 81}$

$= \frac{1820 - j780}{130} = \frac{130\,(14 - j6)}{130}$

$= 14 - j6$

60. $\frac{255}{6 + j7} = \frac{255\,(6 - j7)}{(6 + j7)\,(6 - j7)}$

$= \frac{1530 - j1785}{36 + 49}$

$= \frac{1530 - j1785}{85} = \frac{85\,(18 - j21)}{85}$

$= 18 - j21$

61. $(4 + j2)$ ohms. There are two ways to work this problem. One method is to substitute the given impedance values into the formula for two impedances in parallel, $Z_T = \frac{Z_1\,Z_2}{Z_1 + Z_2}$. Another method is to assume an applied voltage; solve for the current in each leg; solve for the total current; and finally solve for the total impedance. Although the latter method sounds involved, it is usually the easier of the two. The solution using the second method is given below. An applied voltage of 100 volts is assumed. You could have assumed any other voltage and arrived at the same answer.

$I_1 = \frac{E}{Z}$

$I_1 = \frac{100}{3 + j4} = \frac{100\,(3 - j4)}{3 + j4\,(3 - j4)}$

$= \frac{100\,(3 - j4)}{9 - j^2 16} = \frac{100\,(3 - j4)}{25}$

$= 12 - j16$

$I_2 = \frac{E}{Z}$

$I_2 = \frac{100}{8 - j6} = \frac{100\,(8 + j6)}{(8 - j6)\,(8 + j6)}$

$= \frac{100\,(8 + j6)}{64 - j^2 36} = \frac{100\,(8 + j6)}{100}$

$= 8 + j6$

$I_T = I_1 + I_2$

$$= (12 - j16) + (8 + j6)$$

$$= 20 - j10$$

$$Z = \frac{E}{I_T} = \frac{100}{20 - j10} = \frac{100}{10(2 - j1)}$$

$$= \frac{10}{2 - j1} = \frac{10(2 + j1)}{(2 - j1)(2 + j1)}$$

$$= \frac{10(2 + j1)}{5} = 4 + j2$$

62. 2 amps. First convert the impedance from j operator form to ohms.

$$Z = \sqrt{R^2 + X^2}$$

$$Z = \sqrt{40^2 + 30^2}$$

$$Z = \sqrt{1600 + 900}$$

$$Z = \sqrt{2500}$$

$$Z = 50 \text{ ohms}$$

Now find the current.

$$I = \frac{E}{Z} = \frac{100}{50} = 2 \text{ amps}$$

63. $(40 - j25)$ ohms.

$$Z = R_1 + R_2 + R_3 - jX_{C1} - jX_{C2} + jX_{L1} + jX_{L2}$$

$$Z = 12 + 17 + 11 - j75 - j50 + j40 + j60$$

$$Z = 40 - j125 + j100$$

$$Z = 40 - j25$$

64. $(18 + j10)$ ohms.

$$Z_T = Z_1 + Z_2 + Z_3 + Z_4 + Z_5$$

$$Z_T = (1 + j6) + (3 - j2) + (4 - j7) + (3 + j14) + (7 - j1)$$

$$Z_T = (18 + j10)$$

65. $I_T = 25$ amps.

$EX_L = 250$ volts.

$EX_C = 250$ volts.

Series-resonant circuit. This is a series resonant circuit since $X_C = X_L$. This means that the 10 ohms of capacitive reactance is exactly canceled by the 10 ohms of inductive reactance. Therefore, the only opposition to current flow is the 4-ohm resistor. Thus, $I_T = \frac{E}{Z} = \frac{100}{4} =$ 25 amps. And $EX_L = I(X_L) = 25 \times 10 = 250$ volts. Also, $EX_C = I(X_C) = 25(10) = 250$ volts.

66. $(3.9 + j2)$ amps or approximately 4.4 amps.

$$I_T = I_R + jI_C - jI_L$$

$$I_R = \frac{E}{R} = \frac{117}{30} = 3.9 \text{ amps}$$

$$I_C = \frac{E}{X_C} - \frac{117}{39} = 3 \text{ amps}$$

$$I_L = \frac{E}{X_L} = \frac{117}{117} = 1 \text{ amp}$$

$$I_T = 3.9 + j3 - j1$$

$$I_T = (3.9 + j2) \text{ amps}$$

$$I_T = \sqrt{(3.9)^2 + (3-1)^2}$$

$$I_T = \sqrt{(3.9)^2 + (2)^2}$$

$$I_T = \sqrt{15.21 + 4}$$

$$I_T = \sqrt{19.21}$$

I_T = 4.4 amps

67. 3.33 ohms.

First assume an applied voltage. Any value will work but 48 volts is particularly convenient.

$$I_T = I_{R1} + I_{R2} + I_{R3} - jI_L$$

I_{R1} = 48/16 = 3 amps

I_{R2} = 48/16 = 3 amps

I_{R3} = 48/8 = 6 amps

I_L = 48/6 = 8 amps

$$I_T = (12 - j8)$$

$$Z = \frac{E}{I_T}$$

$$Z = \frac{48}{12 - j8} = \frac{48}{4\,(3 - j2)}$$

$$= \frac{12\,(3 + j2)}{(3 - j2)\,(3 + j2)}$$

$$= \frac{36 + J24}{13} = 2.77 + j1.84$$

$$= \sqrt{(2.77)^2 + (1.84)^2} = 3.33 \text{ ohms}$$

68. (8 – j6) amps or 10 amps.

$$I = \frac{E}{Z}$$

$$I = \frac{E}{R + jX_L - jX_C}$$

$$I = \frac{E}{R + j\,2\pi fL - j\,\dfrac{1}{2\pi fC}}$$

$$I = \frac{E}{12 + j\,(6.28)\,(120)\,(.1) - j\,\dfrac{.159}{120\,(.000\,02)}}$$

$$I = \frac{150}{12 + j75 - j66}$$

$$I = \frac{150}{12 + j9}$$

$$I = \frac{150}{3\,(4 + j3)} = \frac{50}{4 + j3}$$

$$I = \frac{50\,(4 - j3)}{(4 + j3)\,(4 - j3)}$$

$$= \frac{50\,(4 - j3)}{16 - j^2 9} = \frac{50\,(4 - j3)}{25}$$

$$I = 8 - j6$$

$$I = \sqrt{(8)^2 - (j6)^2}$$

I = 10 amps

69. 1 amp.

$$I_C = \frac{E_C}{X_C};\ E_C = E_L;\ E_L = I_L\,(X_L);$$

E_C = 40 volts

$$I_C = \frac{40}{40} = 1 \text{ amp}$$

70. 55 amps.

At resonance $X_L = X_C$ and the two cancel each other. Thus, the only opposition to current flow is the 2-ohm resistor. I = E/Z; I = 110/2; I = 55 amps.

71. 510 ohms.

$$Z_T = \frac{Z_1 \times Z_2}{Z_1 + Z_2}$$

$$Z1 = 0 - j100$$

$$Z2 = 20 + j100$$

$$Z_T = \frac{-j100\,(20 + j100)}{-j100 + (20 + j100)}$$

$$Z_T = \frac{-j2000 - j^2\,10{,}000}{20}$$

$$= \frac{10{,}000 - j2000}{20}$$

$$Z_T = 500 - j100$$

$$Z_T = \sqrt{500^2 + 100^2}$$

$$Z_T = \sqrt{260{,}000}$$

$$Z_T = 510\Omega$$

Lesson Questions

Be sure to number your Answer Sheet X202.

Place your Student Number on *every* Answer Sheet.

Most students want to know their grade as soon as possible, so they mail their set of answers immediately. Others, knowing they will finish the next lesson within a few days, send in two sets of answers at a time. Either practice is acceptable to us. However, don't hold your answers too long; you may lose them. Don't hold answers to send in more than two sets at a time or you may run out of lessons before new ones can reach you.

1. Add the following:
   (1) $(3a^2 + 2b^2 + 6c + 3d^2)$ plus $(2a^2 - b^2 + c^2 - 4d^2)$
   (2) $(ax^3 + bx^2y + cxy^2 + y^3)$ plus $2ax^3 + ax^2y + 6cxy^2 - 4y^3$.

2. Perform the following subtractions:
   (1) From $6a - 3b + 2c + d$ take $2a - 4b - c + 3d$
   (2) From $12ax^2 - 6by^2 - 4cz^2$ take $6ax^2 + 2by^2 - 5cz^2$

3. Multiply:
   (1) $(a^2 - b^2 + 3c)$ times $(a + b)$ (2) $(3x^2 + y)$ times $(3x^2 - y)$

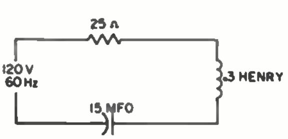
4. Divide:
   (1) $(4a^2 - 8ab + 4b^2) \div (2a-2b)$
   (2) $(18a^5 + 33a^4b + 6a^3b^2 - 11a^2b^3 + 20ab^4 + 32b^5) \div (3a + 4b)$

5. (1) Add $(4 + j17) + (3 - j2)$ (2) Subtract $(16 - j2) - (4 + j6)$

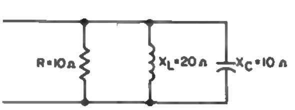
6. (1) Multiply $(6 + j9)(7 + j3)$ (2) Divide $(10 + j62) \div (8 + j2)$

7. If a resistance of 62 ohms is connected in series with a coil with a reactance of 42 ohms and a capacitor with a reactance of 100 ohms, what is the impedance of the circuit expressed in j-operator form?

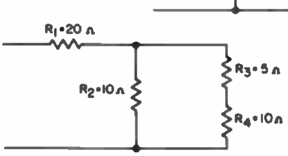
8. Find the current in the series circuit shown at the right. Give your answer in j-operator form.



9. Find the impedance of the circuit shown at the right. Give your answer in j-operator form.



10. Find the source voltage in the circuit shown at the right if the voltage across R_4 is 10 volts.



SUCCESS

The word "SUCCESS" means different things to different people. But the definition of "Success" which appeals to me most is this one, written by Mrs. A. J. Stanley.

"He has achieved success who has lived well, laughed often and loved much; who has gained the respect of intelligent men and the love of little children; who has filled his niche and accomplished his task; who has left the world better than he found it, whether by an improved poppy, a perfect poem, or a rescued soul; who has never lacked appreciation of earth's beauty or failed to express it; who has looked for the best in others and given the best he had; whose life was an inspiration; whose memory is a benediction."

Those of us who can even come close to achieving success of this kind will truly be contented, happy men.

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ACHIEVEMENT THROUGH ELECTRONICS

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MATHEMATICS IN PRACTICAL ELECTRONICS

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MATHEMATICS IN PRACTICAL ELECTRONICS

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STUDY SCHEDULE

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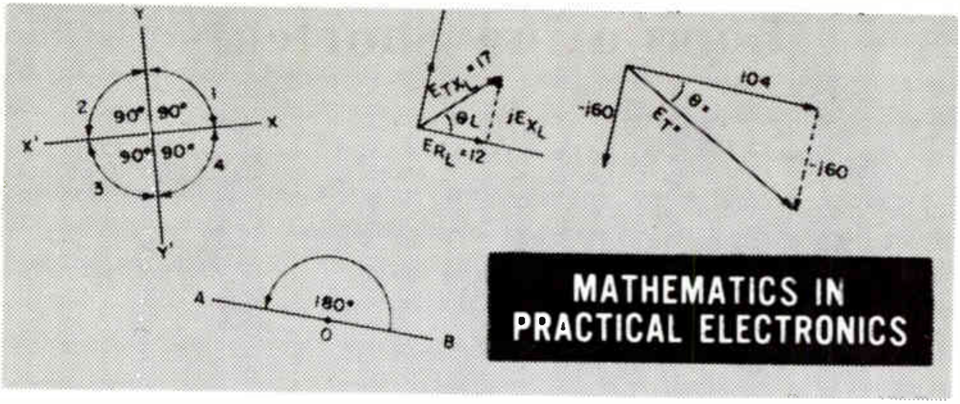


MATHEMATICS IN PRACTICAL ELECTRONICS

This text begins with a summary of shortcuts, some of which you have been using in your earlier lessons. Related to this summary is a discussion of significant figures. You will learn when it does not pay to be too exact and yet have reliable practical results.

You have learned many ways of dealing with the problems involved in making ac circuit computations. You know how to lay out the resultant vectors and measure the angles of lead or lag to determine their values. Although this is satisfactory for many problems and circuits, you will find it awkward when working with precision timing circuits, filter networks, and frequency control circuits that are so important in electronics work.

Many of these difficulties can be overcome by applying the principles of trigonometry to vector solutions. In this way we are able to account for the angles formed by vectors, as well as the length of the vectors, without having to rely on any construction or measurement. Although you may never have used trigonometry, or "trig" as it is usually called, it is a very valuable mathematical tool. It is not difficult to learn or use; in fact, it is simpler than many of the processes you have already learned.

By studying trigonometry, you will not only learn another method of computing with vectors, but you will also learn more about vector principles. You will see why a sine wave of alternating current is called a "sine wave" and learn why ac vectors are often called "phasors." In addition, you will learn to work with power in ac circuits and study some of the factors concerning the importance of "power factor." This knowledge of trigonometry will be of value to you throughout your career in electronics. For example, this knowledge of trigonometry is essential when working with computer equipment.

In this lesson you will also learn how to construct and use different types of graphs that will enable you to present complex information clearly and precisely. The importance of graphs cannot be overemphasized, because the technical texts and references which you will constantly need in your work use this form of presentation. In many ways, this is the most important lesson on mathematics since you will be using and reviewing all previous material and "polishing up" the rough edges.

Computing Shortcuts

Many simple calculations in electronics can become quite tedious because of the size of the numbers involved. Take the case of finding the plate current of a tube by measuring the voltage drop across the plate load resistor. Assuming there is a 7.0-volt drop across a 22,000-ohm resistor, what is the current? Ohm's Law tells us that we divide the voltage by the resistance to get the current. This is simple enough, but look at the arithmetic involved:

```
          .00031818
22000)7.00000000
       6 6000
         40000
         22000
         180000
         176000
           40000
           22000
           180000
           176000
              4000
```

There are five digits in the divisor, nine in the dividend, and eight in the quotient. No matter how many places there are in the quotient, there will still be a remainder with either four or five digits in it.

Practically speaking, a lot of needless work was done in performing this division. This operation could have been simpler. For one thing, the quotient was carried three decimal places too many. The second simplification is to get rid of those three zeros in the divisor. When these things are done, your division looks like this:

```
     .31
22)7.00
   6 6
     40
     22
     18
```

Admittedly, the quotients obtained by these two divisions have their decimal points in different places. However, the method that was used to get rid of the three zeros in the second divisor also shows us how to shift the decimal point in the second quotient.

How to take the unnecessary work out of practical calculations will be the subject of this section of the lesson. There are two parts of this section: One is concerned with the number of digits that should be used in any arithmetic operation; the other is how to get rid of zeros whose only purpose is to locate the decimal point.

The rules that you will learn in this section are not only labor-saving tricks, they greatly reduce the possibility of mistakes. Everyone makes mistakes in arithmetic. The more marks you have to make on a piece of paper, the more likely you are to make a mistake. By using no more figures than are absolutely necessary, you can reduce the likelihood of an incorrect answer.

EXPONENTIAL NUMBERS

Many of the numbers used in science represent either very large or very small quantities. The field of electronics is no exception. In many cases, the majority of the digits in the numbers are zeros. These zeros serve only to locate the decimal point. They are necessary but very inconvenient to work with.

Mathematicians working in the different fields of science have developed another way of writing these numbers. The method is simply to express the number as the product of two factors. One factor, called the *digit factor*, contains the significant digits (this term is explained later in this section). The other factor, called the ***exponential factor***, is a whole numbered power of 10 which properly locates the decimal point. This method of writing large and small numbers is sometimes called the scientific method of expressing numbers.

As examples, let's use the resistance and current values in the sample division earlier in this section. The resistance was 22,000 ohms. Using the two-factor method, this number is expressed as 2.2×10^4. The current is .000318. This is written using exponential numbers: 3.18×10^{-4}. To show that these new figures are correct, we multiply them. 10^4 is 10,000; multiply this by 2.2 and we get 22,000. 10^{-4} is .0001; multiply this by 3.18 and we get .000318.

Conversion from one system to the other is very simple and involves only determining the correct power of 10. One rule tells the whole story: The power of 10 is given by the number of places the decimal point must be moved to obtain the digit factor. Moving the decimal point to the left gives a positive exponent; moving the decimal point to the right gives a negative exponent. For convenience, the digit factor is usually written with only one digit to the left of the decimal point.

Here are some examples: Convert 473,000 to exponential form. The decimal point in the digit factor will come between the 4 and the 7; this is five places to the left, giving an exponent of +5. The digit factor is then 4.73 and the exponential factor is 10^5. The complete expression is 4.73×10^5. Convert 6,720,000. The decimal point moves six places to the left, giving 6.72×10^6. Convert .000706. The decimal point moves to the right four places making the exponent −4. The complete expression is 7.06×10^{-4}. Convert .0000000123. The decimal point moves eight places to the right giving 1.23×10^{-8}.

Converting the number back is just as easy: Move the decimal point as many places as the power of 10 shown by the exponent. If the exponent is positive, move the decimal point to the right; if the exponent is negative, move the decimal point to the left. After this conversion, every place must be shown. If the decimal point is moved more places than are occupied by digits in the digit factor, fill in the blank places with zeros.

To convert 3.14×10^{-2}, we must move the decimal point to the left two places. There is only one place in the digit factor to the left of the decimal point so that we must add a zero to the left of the 3. This results in .0314. To convert 3.14×10^2, the decimal point must move two places to the right. This time there are digits in the digit factor for each place moved over and no zeros are added. 3.14×10^2 is equal to 314.

As simple as this system is for general use, it is even easier to make conversions in electronics. Many times it is not even necessary to count the number of decimal places. The method of giving the values of voltage and current, and of components has the digits all counted. You seldom see 1,500,000 ohms written out in full; instead, it is written 1.5M or 1.5 meg. Either way it means 1.5 million. One million is equal to 10^6, so as soon as you see the expression megohm, you know that the exponential factor is 10^6.

There are a number of other common methods of indicating size in the name of a unit. For instance, 1 kilohertz is 1000 or 10^3 Hertz; 1K ohm means 1 kilohm or 10^3 ohms. The prefix "kil" or "kilo" immediately tells you that the exponential factor is 10^3. Similarly, 1 milliampere is 1/1000 of an ampere; 1 millihenry is 1/1000 of a henry; 1 millivolt is 1/1000 of a volt, etc. 1/1000 is $1/10^3$ or 10^{-3}. The prefix "milli" is just another way of writing 10^{-3}. In the same way, "micro" means 1/1,000,000 or 10^{-6}. "Micro-micro" means one millionth of a millionth or 10^{-12}. Basic units such as volts, amperes, henrys, etc. have an exponential factor of 10^0 or 1.

Multiplication and Division. Exponential numbers are at their best for multiplication and division. It is in these operations that they save the most work. These operations are actually performed in two parts. The indicated operation is performed on the digit factors and then on the exponential factors. As an example, suppose a calculation called for multiplying .0022 X 670 X 3.14. Converting and grouping digits gives $(2.2 \times 6.70 \times 3.14) \times (10^{-3} \times 10^2 \times 10^0)$. First multiply the digit factors together and you get 46.2836. Next, multiply the exponential factors by adding the exponents algebraically; $-3 + 2 + 0 = -1$. Combining the two factors you get 46.2836×10^{-1}. This can be simplified by moving the decimal point in the digit factor one place to the left giving $4.62836 \times 10^1 \times 10^{-1}$. Now the two exponents cancel, leaving 4.62836 as the final answer.

Division is just as easy. For example, divide .0572 by .0026. Converting and grouping gives

$$(5.72 \div 2.6) \times (10^{-2} \div 10^{-3})$$
$$5.72 \div 2.6 = 2.2$$

The division of the exponential factor is performed by subtracting the exponent of the divisor (−3) from the exponent in the dividend (−2).

$$-2 - (-3) = -2 + 3 = 1$$

The complete quotient of this division is 2.2×10^1 or 22. The same basic procedures are followed for operations with combined multiplications and divisions. As an example, take

$$(22{,}000 \div 80) \times (.032 \div 308) \times 7$$

Fig. 1 shows how this is set up and solved.

Power and Roots. Raising a number to a power is a special form of multiplication; taking a root is a special form of division. As in multiplication and division, we must perform the indicated

$$22000 = 2.2 \times 10^4$$
$$80 = 8.0 \times 10^1$$
$$032 = 3.2 \times 10^{-2}$$
$$308 = 3.08 \times 10^2$$
$$7 = 7 \times 10^0$$

$$\left(\frac{2.2}{8.0} \times \frac{3.2}{3.08} \times 7\right) \left(\frac{10^4}{10^1} \times \frac{10^{-2}}{10^2} \times 10^0\right)$$

$$= \frac{2.2 \times 3.2 \times 7}{8.0 \times 3.08} \times \frac{10^4 \times 10^{-2} \times 10^0}{10^1 \times 10^2}$$

$$= \frac{49.28}{24.64} \times \frac{10^2}{10^3} = 2 \times 10^{-1} = .2$$

Fig. 1. Solving combined multiplication-division problems using exponential numbers.

operation on the digit factor and treat the exponential factor separately. An exponential number is raised to a power by multiplying the exponents. You can see that this will give the correct answer by considering the following:

$$10 \times 10 = 10^2$$
$$10 \times 10 \times 10 = 10^3$$

and therefore

$$10^2 \times 10^2 \times 10^2 = (10^2)^3$$

But we know that

$$10^2 \times 10^2 \times 10^2 = 10^{2+2+2} = 10^6$$
$$(10^2)^3 = 10^{(2)(3)} = 10^6$$

Extracting the root is the reverse of raising to a power. A root of an exponential number is taken by dividing the exponent by the digit indicating the root, 2 for square root, 3 for cube root, etc. Suppose you want the square root of 10^6. Dividing the exponent 6 by 2 gives 10^3. We know that $10^3 \times 10^3 = 10^6$ which shows that dividing the exponent by the root gives the correct result.

We know from arithmetic that $6 \div 2$ is the same as $6 \times ½$. Thus, $\sqrt{10^6}$ may be written as $(10^6)^{½}$. Fractional exponents indicate that roots must be taken. When this method of indicating the roots is used, square root is handled as the 1/2 power, cube root as the 1/3 power, and so on. The square root of 10^6 could be written as $10^{6/2}$. The square root of 10^3 would be written as $10^{3/2}$.

At the beginning of this section, you learned that the exponent of 10 in an exponential number must be a whole number. This can lead to a slight complication when taking roots. Consider taking the square of 8.1×10^3. We would write this as $\sqrt{8.1 \times 10^{3/2}}$. But our exponential number system does not allow fractional or decimal exponents. In order to extract the square root of 8.1×10^3, we must have an exponent divisible by 2. We get this by increasing or decreasing the exponent and moving the decimal point in the digit factor accordingly.

$$8.1 \times 10^3$$
$$= 81 \times 10^2$$
$$= .81 \times 10^4$$

The square root of 81×10^2 is 9×10; the square root of $.81 \times 10^4$ is $.9 \times 10^2$. Both of these roots convert to 90 in the decimal system. 8.1×10^3 converts to 8100, the square root of which is 90 and you can see that we have obtained the correct result. To extract the cube root, the exponent of 10 must be divisible by 3; the exponent for a fourth root must be divisible by 4, etc.

Addition and Subtraction. There is nothing to be gained by converting decimal numbers into exponential numbers for addition and subtraction. However, addition and subtraction may occur as part of a calculation when exponential numbers are used to simplify multiplication and division. When addition and subtraction are indicated, you must remember that you can only add and subtract exponential numbers having the same power of 10. The reason for this is simple. For example, $160 + 16 = 176$. But we can write 160 as 1.6×10^2 and 16 as 1.6×10. If we simply add $1.6 + 1.6$, we get 3.2 which is not the correct answer for $160 + 16$ regardless of whether we multiply it by 10 or 10^2. If we first change the numbers so that we have the same power of 10, then we will get the correct answer. For example, $160 = 16.0 \times 10$, and $16 = 1.6 \times 10$. Then,

$$16 \times 10 + 1.6 \times 10 = 17.6 \times 10 = 176$$

As you can see, the use of exponential numbers greatly simplifies multiplication and division when working with large numbers having many zeros adjacent to the decimal point and on either side of it. Still greater simplification can be obtained when some of the digits in a number like 4.62836 can be dropped. This is possible when we are working with measured values. In most practical work, it is seldom necessary to use more than four digits in any factor. The rules for dropping digits from a number which has been obtained by a measurement are the next subject that we take up.

SIGNIFICANT FIGURES

When you took arithmetic in grade school, the teacher probably listed numbers like 67,530, 4156, 873, and told you to "round them off" to the nearest thousand or hundred or ten. You did this sort of thing for homework two or three nights and that was the end of it. The next and last time you did this was on a test. Well, that time wasn't wasted; studying significant figures is just learning when to round off and how much.

The term "significant" is used here to mean: "having a meaning." In working with significant figures, you retain only those figures which have meaning and drop all others by "rounding off." The use of significant figures applies only to numbers which are connected in some way with measurements.

The figures that you used when studying mathematics were considered exact. 1 was 1, and 2 was 2. Each digit meant exactly what is said; not almost or approximately, or a little more or less. This is not true of numbers that are obtained by measurement. There is always a certain amount of estimating or guesswork in taking any measurement. The first step in using significant figures is to properly record the results of the measurement. This means putting down meaningful figures in a manner that shows how exact the measurement is.

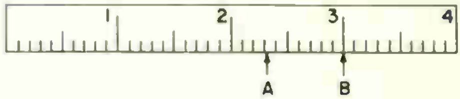


Fig. 2. Scale for showing use of significant figures in recording a measurement.

Fig. 2 shows a scale with two arrows at the bottom edge. The scale is used to show the position of pointers represented by the arrows. The scale is divided into four major units and each of these units is subdivided into ten parts. Suppose we want to read the position of arrow **A**. Arrow **A** lies between two and three units so the first figure is 2. Since the arrow is slightly past the third mark following the 2, the second figure is .3. Two figures are all that can be read directly from the scale marks. We estimate the third digit by mentally dividing the space between the scale marks into ten parts. Then we must decide which of these ten parts the arrow is nearest. 3 is the third digit for arrow **A**.

Perhaps you disagree with the reading of 2.33. Maybe you think the reading should be 2.32 or 2.34. Perhaps it should. The last digit is an estimate, not an exact figure. Because it is an estimate, different observers will record different values for this digit. However, if each observer reads the scale carefully, the readings should span only three digits; in this case, .02, .03, and .04. This is what you are saying when you record any measurement. Only the right-hand digit is an estimate and the error in reading is not more than ±1 in the right-hand figure.

Perhaps you think the arrow is exactly one-quarter of the way between 2.3 and 2.4; in other words, the reading should be 2.3 $\frac{1}{4}$. However, a mixed decimal and fraction is awkward to use. Since $\frac{1}{4}$ = .25, why not record the reading as 2.325? This, however, gives misleading information. To anyone using this figure, it means that you could tell the difference between 2.324, 2.325, and 2.326. Obviously you cannot, so 2.325 should not be recorded. You could, however, record 2.3 $\frac{1}{4}$. This indicates that you could tell the difference between 2.3, 2.3 $\frac{1}{4}$, and 2.3 $\frac{1}{2}$. It also says that no attempt was made to read the scale any closer than one-quarter of the smallest division. But instead of using a mixed decimal and fraction, it is better to estimate so that you will have only 3 digits.

The arrow B appears to be exactly on the "3" line. If it is recorded as 3, it indicates only that the reading is nearer to 3 than to 2 or 4. If it is recorded as 3.0, it means nearer to 3 than to 2.9 or 3.1. The correct way to record this is as 3.00. The two zeros to the right of the decimal point say that the scale can be read to 1/100 of a unit. 3.000 would be wrong because you cannot read the scale to thousandths. Even when a pointer falls directly on a scale mark, you cannot assume a more precise reading than at any other point on the scale.

Each of the readings, 2.33 and 3.00, has three digits. Therefore, we say that they have three significant figures. If we had read the scale only to the nearest scale mark, the readings would have been 2.3 and 3.0 with two significant figures each.

Zeros in a decimal number may or may not be significant, depending on where they are in the number. The decimal number .00678 has only three significant figures. The two zeros between the decimal point and the 6 are not considered significant.

.0067800 has five significant figures. The two zeros between the decimal point and the 6 are not significant, but the two zeros following the 8 are. If these last two digits had not been significant, they should not have been written; since they were written, you must assume that the measurement could be made to five figures.

Numbers like 22,000 create a problem. The "2's" are significant, but what about the zeros? As the number is written you cannot tell how many significant figures it has. Unless there is some note with the data, you must assume only two significant figures. However, if this were written as an exponential number, there would be no uncertainty. 2.2×10^4 has two significant figures. 2.20×10^4 has three significant figures. This is another advantage of exponential numbers; only significant figures appear in the digit factor.

Rules for Significant Figures. For convenience the rules for using significant figures will be listed by number. Then, examples of the application of the rules will be shown. In the examples, the rules that apply will be shown by number.

1. Only one uncertain figure should be recorded in giving the numerical value of any measured quantity. The uncertainty of the last figure will be ±1 unless otherwise stated.
2. When adding and subtracting with significant figures, keep only as many

decimals as are given in the number having the fewest decimals.

3. When multiplying and dividing, retain enough figures in each factor so that no factor has a greater uncertainty than the factor with the least number of significant digits.

4. When dropping nonsignificant digits by rounding off, increase the most right-hand retained figure by 1, if the figure to its right is 5 or greater.

5. Products and quotients should be rounded off so that the uncertainty is the same as that of the factor with the least number of significant figures.

Now some examples: Add 14.16 + .0078 + 1.234. The least number of decimal places is 2, so the last two numbers must be rounded off to two decimal places (Rule 2). .0078 becomes .01, and 1.234 becomes 1.23 (Rule 4). The sum is 15.40 (not 15.4). Since there are two decimal places in the numbers added, there must be two decimal places in their sum. (Common sense.)

Multiply 14.16 × .0078 × 1.234. The decimal number .0078 has the least number of significant figures (two), and an uncertainty of one part in 78. The other two factors must be rounded off. 14.16 rounded off to three figures becomes 14.2 (Rule 4) with an uncertainty of one part in 142. Rounded off to two figures it becomes 14 with an uncertainty of one part in 14. This uncertainty is much greater than one part in 78, so three significant figures must be used (Rule 3). 1.234 rounds off to 1.23 with an uncertainty of one part in 123. The product of 14.2 × .0078 is .11076; we round this off to .111 (Rule 5, Rule 4). .111 × 1.23 = .13653 which when rounded off becomes .137 (Rule 5, Rule 4).

What effect does this rounding off have on the accuracy of the calculation? None. If we had not rounded off any of the figures, the final product would have been .136292832. If we had not rounded off .11076, the final answer would have been .1362348. Both of these numbers round off to .136. Our answer was .137, just one unit greater. Since the last digit has an uncertainty of ±1, we can say the three answers were practically identical. By rounding off we have saved a lot of needless work without any loss of accuracy.

Multiply 19.7 × 9.81. Both numbers have the same number of significant figures. However, the larger number can be rounded without loss of accuracy. 19.7 has an uncertainty of one part in 197 or about .5%. 9.81 has an uncertainty of one part in 981 or about 1/10%. Rounding off 9.81 gives 9.8 with an uncertainty of one part in 98 or about 1%. This is much closer to the uncertainty of 19.7. The product without rounding off is 193.257; if 9.81 is rounded off to 9.8, the product is 193.06. Rounded to three figures, the answers are the same. It is apparent that Rule 4 can be modified at times to say that the percentage uncertainty of two factors should be nearly the same. It is hard to state this as a fixed rule and give exact values of how much the uncertainties can differ, but it does show how the use of common sense fits in with significant figures.

It is seldom necessary to use more than three significant figures in electronics. Except for precision parts, tolerances range from ±1% to ±20%; a good service meter has an accuracy of ±2% for voltage and current and ±5% for resistance. Keep the accuracy of your instruments and the tolerances of your parts in mind when performing calculations. Do not carry a lot of meaningless digits. Three significant figures are accurate enough for all but the most exacting laboratory work. Two significant digits are enough when using ±20% parts.

SELF-TEST QUESTIONS

1. Express the following in exponential form:
   (a) 1,100,000
   (b) 7200
   (c) 0.00015
   (d) 0.64
2. Convert the following exponential numbers to standard form:
   (a) 326×10^{-12}
   (b) 1.22×10^{6}
   (c) 7.7×10^{-4}
   (d) 9×10^{0}
3. Perform the following division and give the answer to three significant figures.

$$\frac{1}{6.28 \times 1550 \times 10^{3} \times 452 \times 10^{-7}}$$

4. Divide:
   $.0058 \times .000983$ by $.0000071$

5. Applying the rules for significant figures, add the following numbers:

$$\begin{array}{r} 7.92 \\ 3.0094 \\ 6.101 \\ \underline{0.0076} \end{array}$$

Trigonometry

Trigonometry is the study of the mathematical relationships that exist between the sides and angles of triangles. The word trigonometry itself is derived from two Greek words which mean the measurement of angles. The origin and earliest uses of trigonometry were for measuring distances and objects by using triangles. Today surveyors and construction engineers use trigonometry for this same purpose. However, the science of trigonometry has been developed to such a point that it is now commonly used for many other purposes.

As an electronics technician your most important use for trigonometry will be in the solution of the triangles formed by vector diagrams to determine phase relationships. Before you can actually learn to use trigonometry in this way, you must first learn some of the basic fundamentals and principles of angles, triangles, and coordinate systems.

ANGLES

When two straight lines meet at a point, an angle is formed. Thus, when the two lines OX and OY meet at the point 0 as shown in Fig. 3A, an angle is formed between the two lines. Similarly, the two lines OA and OB meeting at the point 0 in Fig. 3B also form an angle. The point 0, where the lines meet, is called the *vertex* of the angle, and the lines themselves are known as the sides of the angle. The angle in Fig. 3A is called "angle XOY" to show that it is the angle formed by lines OX and OY. Likewise, the angle in Fig. 3B is "angle AOB."

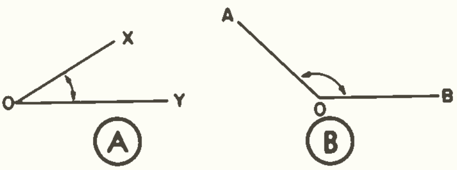


Fig. 3. Angles are formed when two straight lines meet.

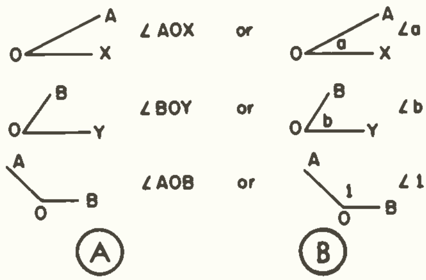


Fig. 4. Naming angles.

To save time and space, the symbol "∠" is used in mathematics to represent the word "angle." Thus, the angles in Fig. 4A could be designated as ∠ AOX, ∠ BOY, and ∠ AOB. In working with a large number of angles, it is often awkward to describe angles in this way. Instead, we insert a letter or a number in the vertex of the angles, as shown in Fig. 4B and simply call them "∠ a", "∠ b", or "∠ 1" as shown. Many times special designations are used to describe angles. For example, we often use the Greek letter theta, θ, in electronics to designate the phase angle. This would be written $\angle \theta$, and if we are working with several different phase angles we would indicate them with appropriate subscripts such as $\angle \theta_2$, $\angle \theta_3$, or perhaps $\angle \theta_a$ and $\angle \theta_b$.

The size or the magnitude of an angle is a measure of the space or distance between the sides and is determined by the difference in ***direction*** of the sides. Notice that it is only the difference in direction of the two sides that determines the size of an angle. The lengths of the sides do not in any way affect the size of the angle itself. You can easily see that either shortening or lengthening the sides of the angles shown in the figures will not change the size of the angles.

In working with the size or measurement of angles, as in measuring anything else, some standard unit of measurement must be chosen. Although it is easy to see that an angle is either smaller or larger or nearly the same size as another angle, this is not enough definition for the precision required in mathematics or electronics. Although there are three generally accepted units of measurements for angles, we will be concerned with only two of them: the degree (°), and the radian. To understand these two units of measurement, let's examine angles more closely, especially their relationship to circular measure.

An angle should be thought of as being generated by a line that starts at a certain initial position and rotates about the vertex of the angle until it stops at its final position. This is shown in Fig. 5. Consider the two straight lines: the short, heavy line OX and the lighter, longer line OY.

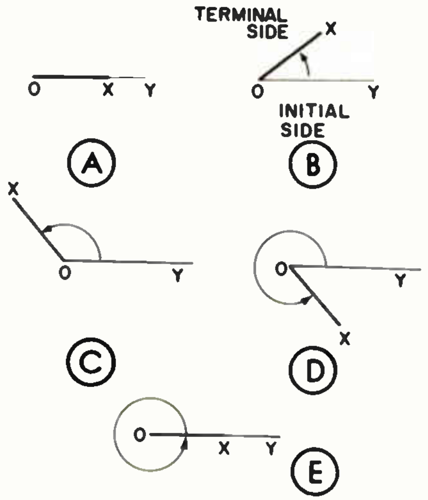


Fig. 5. Generation of angles.

In Fig. 5A, line OX is drawn on top of line OY and no angle is formed. However, in Fig. 5B, an angle is formed because the line OX is rotated counterclockwise from its initial position on line OY. Thus, the various angles XOY in Figs. 5B, 5C, and 5D are formed by the rotation of line OX from its initial position. The side of the angle that represents the original or initial position of the rotating side is known as the ***initial side***. The final position of the rotating side determines the size of the angle and is known as the ***terminal side***.

If the terminal side of the angle is rotated one complete revolution before it is stopped, the two lines are back at their original position, as shown in Fig. 5E. Thus, an angle is said to be formed by a line rotating about a point from one position to another. The unit of measure called the degree is based upon this formation by rotation. By definition, there are 360° in one complete revolution or 1° equals 1/360 of a complete revolution.

As we progress with our study of angles, triangles, and trigonometry, we will find that the degree is often a large unit of measurement. For this reason, the degree can be divided into smaller units called ***minutes***, written (′), and the minute can be further divided into units called ***seconds*** written (″). There are 60 minutes in one degree, and 60 seconds in each minute. Thus, the size of a certain angle might be written as 35° 46′ 57″ to tell us that the angle is 46 minutes and 57 seconds more than 35/360 of a revolution of the terminal side. Although these minutes and seconds may seem to be ridicuously small units, we will soon see that they can be very important. Remember, there are 360° in one revolution; 60 minutes in each degree (360 X 60 or 21,600 minutes in a revolution); and 60 seconds in each minute (60 X 60 or 3600 seconds in a degree and 60 X 60 X 360 or 1,296,000 seconds in a revolution).

In trigonometry, we often use the decimal system instead of minutes and seconds. For example, instead of saying that an angle is 36° 30′, we can write it as 36.5° because 30′ is half of 60′ and 1/2 equals .5. When using decimals, however, we must remember that there are 60 parts to a degree. For example, 36.25° is 36-1/4° which is 36 and 1/4 of 60, or 36° and 15′. Likewise, in converting 36° 12′ to decimals, we have 36-12/60° = 36-1/5° = 36.2°. It is very easy to make errors in converting from decimals to minutes or seconds.

Let's again use Fig. 5 to examine the other unit of angular measure: the radian. As the line OX rotates about the vertex to form the various angles, it must pass through every possible position from an angle of 0° (or no angle at all, as shown in Fig. 5A) to an angle of 360° (or one complete revolution, as shown in Fig. 5E). If we assume that the line OX never changes in length as it is rotating through this one revolution and place a pencil at the point X, we would find that a complete circle is drawn, as shown in Fig. 6.

Thus, we can say that since there are 360° in one revolution of side OX there are 360° in the circle. Also, since the ***length*** of OX has no bearing on the number of degrees in the revolution or in the size of the angles that could be formed by any one partial revolution, we can say that there are 360° in every circle, no matter how small or large it

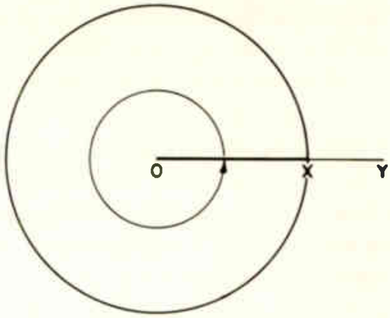


Fig. 6. Point X describes a complete circle in one complete revolution of terminal side OX.

may be. Changing the length of OX would change the radius of the circle and its area, but not the number of degrees in it.

If we were to mark off each degree on the circumference of the circle shown in Fig. 6, the degree marks would be very close together. In fact, they would be so close together that it might be difficult to show any space between them at all. In such a small circle, the difference between a degree or two would be insignificant as far as the linear distance between the marks is concerned.

However, suppose that we were considering a circle as large as the earth. At the equator, where it is about 25,000 miles around the earth, there would be nearly 70 miles between the degree marks and each degree would be extremely important. In fact, even a difference of a minute (1/60°) would be nearly 1.2 miles. Thus, a degree can be a very small unit or a very large unit, depending on where and how it is used. Consequently, the minute and second subdivisions can be quite important.

Although we don't often stop to realize it, a straight line is really a 180° angle and is therefore the most common angle. We can show that there are 180° in any straight line by rotating the terminal

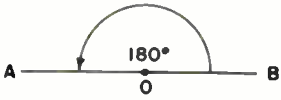


Fig. 7. Straight line is 180° angle.

side OA of any angle such as ∠ AOB in Fig. 7 until the line AB is straight. When the line AB is a straight line, the ∠ AOB equals one half a revolution of the rotating side, which is 180°.

Even though the straight line is the most common angle, the right angle is the most important. We are already familiar with the fact that there are 90° in a right angle and that there is a system of angular measurement based on right angles. The Pythagorean relationships of angles in a triangle having a right angle make it easy to solve electronics problems using trigonometry.

When two straight lines intersect each other so that four right angles are formed, the lines are said to be perpendicular to each other or mutually perpendicular. In Fig. 8, the two lines X′X and Y′Y are mutually perpendicular because angles 1, 2, 3, and 4 are all equal to 90° and are right angles. This is, of course, the basis of our coordinate systems which are used in making graphs, surveying, navigating, etc. We will learn more about this a little later.

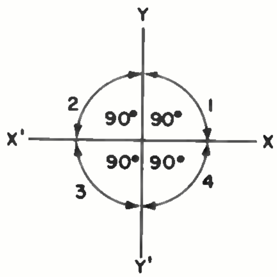


Fig. 8. Right angles formed when two lines are mutually perpendicular.

Any angle that is less than 90° is called an acute angle and any angle larger than 90° is called an obtuse angle. Two acute angles whose sum is equal to a right angle or 90° are called complementary angles. Either one of such acute angles may be called the complement of the other. Two angles whose sum is two right angles (180° or a straight line) are called supplementary angles.

Angles, of course, may be added, subtracted, multiplied, or divided, using the rules of arithmetic or algebra. We even have positive and negative angles to consider sometimes. A positive angle is generated when the terminal side is rotated counterclockwise to form the angle. If the angle is formed by the terminal side rotating clockwise it is called a negative angle.

RADIANS

The radian is a unit of measure that is based upon the length of an "arc" of a circle as compared with the radius of the circle. An "arc" is simply a part or section of the curved line that forms the circumference of a circle. An arc can be any length, but it must be a section of a true circle in order to be called an arc. Otherwise it would simply be called a curved line, or curve. An arc that is exactly equal in length to the radius of the circle of which the arc is a part is said to be a radian. A more formal way of saying it is that a radian is an angle that, when placed with its vertex at the center of a circle, intercepts an arc equal in length to the radius of the circle. Thus, if the ∠ XOY in the circle shown in Fig. 9A is to be equal to one radian, the length of the arc XY measured along the circumference of the circle must be equal to the

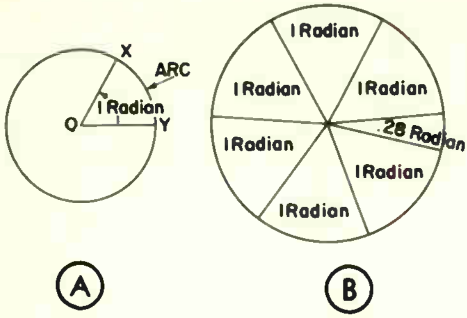


Fig. 9. Radian measure.

radius of the circle or the sides OX or OY of the angle.

If we lay out and mark off a circle using angles that are each equal to 1 radian, as shown in Fig. 9B, we will find that there are 6.28 radians in a circle. This must hold true for any circle regardless of its size since the length of arc intercepted by an angle of 1 radian must change directly as the radius of the circle changes. We are already familiar with the Greek letter π which we use in working with the area of circles. We know that it is a constant equal to 3.14. Therefore, we usually say that there are 2π radians in every circle since $6.28 \div 3.14 = 2$.

Many times you will want to change from radian measure to degrees, etc. Therefore, you should know how many degrees there are in a radian and how to convert from one to the other. Since there are 360° in every circle and 2π radians in every circle, 2π radians = 360°. From this:

$$\begin{aligned} 2\pi \text{ radians} &= 360^\circ \\ \pi \text{ radians} &= 180^\circ \\ 1 \text{ radian} &= 180/\pi \\ &= 57.32/57.3^\circ \end{aligned}$$

or approximately 57.3°. Accordingly, to change radians to degrees we would multiply the number of radians indicated by 57.3. Since 57.3 or $180/\pi$ is the multiplier when changing radians to degrees we would multiply the number of degrees by $\pi/180$ or .01745 in order to change them to radians.

Now let's consider triangles.

TRIANGLES

A triangle is a three-sided, closed plane figure. It is probably quite obvious what we mean by a closed, three-sided figure. However, if you have not studied geometry you may wonder what we mean by a

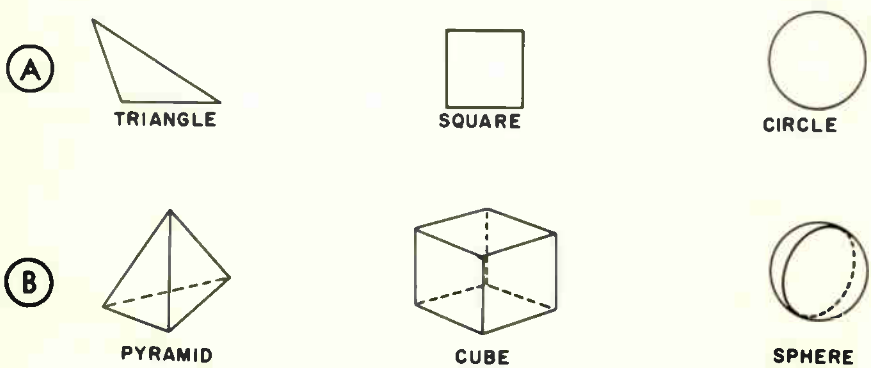


Fig. 10. (A) Plane figures. (B) Solid figures.

"plane" figure. A plane figure is simply a figure that has height and width, but no depth. Thus, a triangle, a square, a circle, or any other figure that is drawn flat on a piece of paper is a plane figure. A pyramid, a cube, or a sphere are all what we call "solid" figures, whether they actually exist or whether they are drawn with their depth indicated. Thus, the figures in Fig. 10A are plane figures and those in Fig. 10B are solid figures. The study of trigonometry includes both plane and solid figures, but in your work you will only need to be familiar with trigonometry for plane figures unless you enter some very specialized field work.

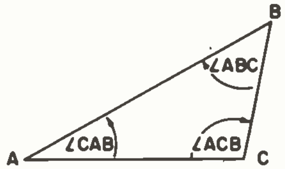


Fig. 11. A triangle has three angles and three sides.

Since a triangle has three sides, it must also contain three angles, as shown in Fig. 11. A triangle is named for reference purposes by naming the three vertexes of the three angles in order around the triangle. Thus, the triangle in Fig. 11 would be called triangle ABC. It might also be called triangle CBA, triangle BCA, triangle CAB, triangle BAC, or triangle ACB, depending on which vertex we start with and in which direction we go around. The mathematical symbol for a triangle is "Δ." Thus, the triangle in Fig. 11 could be written ΔABC.

The sum of the three angles in a triangle is always 180°. It can never be any more or any less no matter how large or small the triangle may be. This is very important in your work in trigonometry

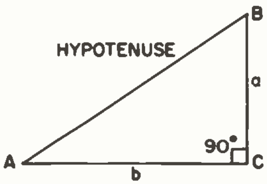


Fig. 12. A right triangle contains one right angle.

since you can always find the value of the third angle of any triangle if you know the other two.

If one of the angles of a triangle is a right angle, the triangle is called a ***right triangle***. Accordingly, since a right triangle always has one 90° angle, the other two angles must be acute angles whose sum is also 90°. This relationship allows us to find one acute angle of a right triangle if we know the other acute angle. A right triangle is shown in Fig. 12. The fact that it is a right triangle is shown by drawing a small square at the vertex. The side of a right triangle that is opposite the right angle has been given the special name ***hypotenuse***. When a right triangle is in standard position as shown in Fig. 12, side a is called the ***altitude*** and side b the ***base***.

Two triangles are said to be ***similar*** when their corresponding angles are equal. In other words, similar triangles are triangles that are identical in shape, but not necessarily in size. Thus, although the corresponding angles of similar triangles are equal, the sides are not equal. However, there is a special relationship between the sides of similar triangles that forms the basis of trigonometry.

The corresponding sides of similar triangles are always proportional.

For example, the triangle in Fig. 13A is similar to the triangle in Fig. 13B,

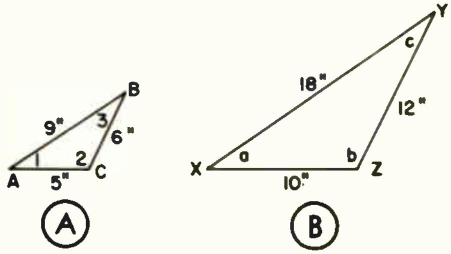


Fig. 13. Similar triangles have corresponding angles equal.

because ∠ 1 = ∠ a, ∠ 2 = ∠ b, and ∠ 3 = ∠ c. If we establish a ratio between any two sides of one of the similar triangles, we will find that it is equal to the ratio established between the corresponding sides of the other similar triangle. Thus, the ratio of side AC to side CB of ΔABC is:

$$\frac{AC}{CB} = \frac{5''}{6''}$$

The ratio of the corresponding sides of ΔXYZ would be:

$$\frac{XZ}{ZY} = \frac{10''}{12''} = \frac{5''}{6''}$$

Accordingly, we can establish a proportion from the two ratios as:

$$\frac{AC}{CB} = \frac{XZ}{ZY}$$

Also,

$$\frac{AB}{CB} = \frac{XY}{ZY} \text{ since } \frac{9''}{6''} = \frac{18''}{12''}$$

and

$$\frac{AC}{AB} = \frac{XZ}{XY} \text{ since } \frac{5''}{9''} = \frac{10''}{18''}$$

Remember that this proportionality between corresponding sides always exists when two or more triangles are similar.

Since we know that there are 180° in all triangles, we can determine if triangles are similar by knowing only two of the angles. If two angles of one triangle equal two angles of another triangle, the third angle must also be equal to each other and the triangles will be similar.

In the case of right triangles, the right angle of one is always equal to the right angle of another. Consequently, if one of the acute angles of one right triangle equals an acute angle of another right triangle, the two right triangles must be similar.

Suppose we have two similar right triangles such as the ones shown in Fig. 14. In these triangles, angle 1 and angle a are both equal to 30°. Any other right triangle that has one of its acute angles equal to 30° will be similar to these right triangles. If we examine the ratios of any two sides of these similar right triangles, we will discover a very important fact.

The ratio of the side opposite the 30° angle to the hypotenuse of any right triangle containing a 30° angle is always equal to .5. For example, the ratio of the side opposite the 30° angle, BC, to the

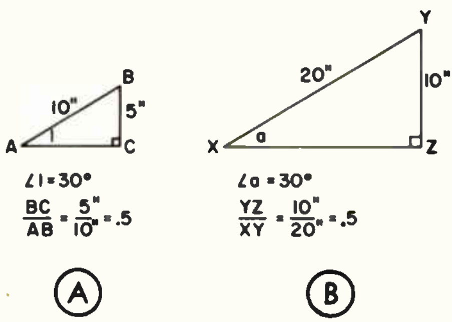


Fig. 14. Right triangles are similar if one acute angle equals the corresponding acute angle.

hypotenuse, AB, of ΔABC in Fig. 14 is equal to:

$$\frac{BC}{AB} = \frac{5''}{10''} = .5$$

Likewise, the same ratio exists for the corresponding sides of ΔXYZ because:

$$\frac{YZ}{XY} = \frac{10''}{20''} = .5$$

Since any other right triangle containing an acute angle of 30° must be similar to these, the ratio of the side opposite the 30° angle to the hypotenuse must always be equal to .5 for any right triangle that contains a 30° angle. Mathematically, this can be written as:

$$\frac{\text{side opp } 30^\circ}{\text{hypotenuse}} = .5$$

By applying this equation, we can determine any one of the factors if one of the others is known.

If we have a right triangle where one side equals 12.5 and the hypotenuse equals 25, we can tell that the angle opposite the 12.5 side must be equal to 30° whether it is given to us or not. If one of the acute angles is 30°, the other must be 60° because 30° + 60° + 90° = 180°. Or, if we know that the hypotenuse of a right triangle equals 50 and one of the angles equals 60°, we can find the value of one of the sides because:

$$180^\circ - 90^\circ - 60^\circ = 30^\circ$$

and

$$\frac{\text{side opp } 30^\circ}{\text{hypotenuse}} = .5$$

and

$$\frac{\text{side opp } 30^\circ}{50} = .5$$

and

$$\text{side opp } 30^\circ = 25$$

Now, if we were to construct a right triangle containing a 29° angle and compute the same ratio for it, we would find that it would equal .4848. Thus, any time a ratio of the side opposite the angle to the hypotenuse works out to .4848, we know that the angle would be 29° because the ratios of corresponding sides of similar triangles are always equal. By continuing in this way we could work out the ratios for all possible angles that can exist in a right triangle and use these ratios to compute other unknown facts about their triangles.

This is trigonometry in its most basic form. Mathematicians have worked out ratios for all the angles it is possible to have in a right triangle and listed them in tables. These ratios are called the ***trigonometric functions*** of angles and can be used for computing unknown facts about similar right triangles. Since the functions are computed for all the angles that can exist, any right triangle you may work with will be similar to one for which the functions are listed.

TRIGONOMETRIC FUNCTIONS

We have seen that certain ratios can be established between two of the sides of a right triangle and that these same ratios will exist between the corresponding sides of any similar right triangle, no matter how large or how small it may be. These ratios are called the trigonometric functions of the angles to which they are related. If you examine any right triangle carefully, you will see that there are six of these ratios or "functions" that can be established for each of the acute angles of the triangle. These six functions have been given special names that you must learn and thoroughly understand in order to make any practical use of trigonometry.

Let's look at the typical right triangle shown in Fig. 15 and consider the angle θ at the lower left. Notice that the three sides of this right triangle have been given special names.

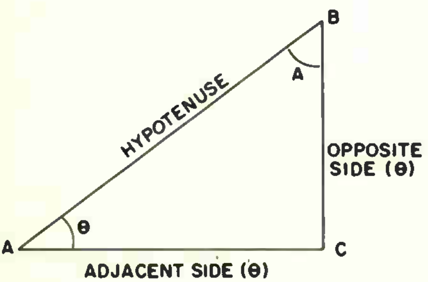


Fig. 15. Right triangle showing names of sides with respect to $\angle\theta$.

The side, BC, opposite the angle θ is called the ***opposite side***. The side, AC, of angle θ is called the ***adjacent side***. The side, AB, opposite the right angle is called the ***hypotenuse***. The hypotenuse and the adjacent side form the angle θ. Using these special names, the six separate ratios or functions that can be established for the acute angle θ are:

1. $\dfrac{BC}{AB}$ or $\dfrac{\text{opposite side}}{\text{hypotenuse}}$ is the function called the ***sine*** of the angle θ.

2. $\dfrac{AC}{AB}$ or $\dfrac{\text{adjacent side}}{\text{hypotenuse}}$ is the function called the ***cosine*** of the angle θ.

3. $\dfrac{BC}{AC}$ or $\dfrac{\text{opposite side}}{\text{adjacent side}}$ is the function called the ***tangent*** of the angle θ.

4. $\dfrac{AC}{BC}$ or $\dfrac{\text{adjacent side}}{\text{opposite side}}$ is the function called the ***cotangent*** of the angle θ.

5. $\dfrac{AB}{AC}$ or $\dfrac{\text{hypotenuse}}{\text{adjacent side}}$ is the function called the ***secant*** of the angle θ.

6. $\dfrac{AB}{BC}$ or $\dfrac{\text{hypotenuse}}{\text{opposite side}}$ is the function called the ***cosecant*** of the angle θ.

The first three of these functions are the most commonly used in electronics. If you examine them carefully, you will see that the cotangent is simply the reciprocal of the tangent, the secant is the reciprocal of the cosine, and the cosecant is the reciprocal of the sine. Since all the sides of the triangle are taken into consideration in the first three functions (sine, cosine, and tangent), the reciprocal relationships expressed by the cotangent, secant, and cosecant do not tell us anything really new. They just express it in a different way to provide certain conveniences for persons who work extensively with trigonometry. Since you will probably not be using trigonometry enough to make it worthwhile, you do not have to memorize the last three functions. However, it is important that the first three functions be memorized.

These trigonometric functions are usually abbreviated as follows:

$$\sin \theta = \frac{opp}{hyp}$$

$$\cos \theta = \frac{adj}{hyp}$$

$$\tan \theta = \frac{opp}{adj}$$

$$\csc \theta = \frac{hyp}{opp}$$

$$\sec \theta = \frac{hyp}{adj}$$

$$\cot \theta = \frac{adj}{opp}$$

In addition to the angle θ in the triangle shown in Fig. 15, we also have another acute angle. This is $\angle$ A at the top of the triangle and it is the "complement" of the angle θ because $\angle \theta + \angle A$ must equal 90° in a right triangle. Angle A also has six separate ratios or functions which can be established between its sides. These six functions of angle A are stated just the same as those for $\angle \theta$. In other words:

$$\sin A = \frac{opp}{hyp}$$

$$\cos A = \frac{adj}{hyp}$$

$$\tan A = \frac{opp}{adj}, \text{etc.}$$

However, notice that the side ***opposite*** angle A is the side that was adjacent to angle θ. Likewise, the side adjacent to angle A is the side that was opposite angle θ. Thus, although the trigonometric functions of sin, cos, tan, etc., are stated the same for either of the acute angles, the sides actually referred to in these functions as "opposite" and "adjacent" are different.

For this reason, even though we can simply say:

$$\sin = \frac{opp}{hyp}$$

$$\cos = \frac{adj}{hyp}$$

$$\tan = \frac{opp}{adj}$$

as a general statement of the trigonometric functions, we must express the specific angle as:

$$\sin \theta = \frac{opp}{hyp}$$

or

$$\cos A = \frac{adj}{hyp}$$

in order for our expression to have any specific meaning for a particular triangle. In fact, as you can see from studying the sides and angles of Fig. 15

$$\sin \theta = \frac{opp}{hyp} = \frac{BC}{AB}$$

but BC is adjacent to angle A, so

$$\frac{BC}{AB} = \frac{adj}{hyp}$$

or the cosine of angle A. Thus,

$$\sin \theta = \frac{BC}{AB} = \cos A$$

and sin θ equals cos A.

The following relationships between the functions of one acute angle and its complement can be worked out by referring to Fig. 15.

Angle (θ)	Sides Used	Complement (A)

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{BC}{AB} = \frac{\text{adj}}{\text{hyp}} = \cos A$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{AC}{AB} = \frac{\text{opp}}{\text{hyp}} = \sin A$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{BC}{AC} = \frac{\text{adj}}{\text{opp}} = \cot A$$

$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{AC}{BC} = \frac{\text{opp}}{\text{adj}} = \tan A$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{AB}{AC} = \frac{\text{hyp}}{\text{opp}} = \csc A$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{AB}{BC} = \frac{\text{hyp}}{\text{adj}} = \sec A$$

Tables of Functions. We mentioned earlier that mathematicians have worked out trigonometric functions or ratios of the sides of triangles for all possible angles and listed them in tables. They are called Tables of Functions. Some of them are very detailed, such as those used in navigation where the functions are listed in minutes and seconds for all the angles. This is necessary for this type of work because, as we have seen, even a few minutes can mean several miles when we are considering the whole earth.

However, such detailed tables require a thick book which would not be practical for most of your work in electronics. Usually, accuracy to one degree, or possibly a few tenths of a degree, will be close enough. A typical Table of Functions that is simple and efficient is shown in Fig. 16. This table lists the sine, tangent, cotangent and cosine for angles from 0° through 90° in a convenient form.

The angles from 0° to 45° are listed in steps of 1° in the column marked "Degrees" at the left of the table. In the next column, the sine of all the angles from 0° to 45° are listed, then the tangent, the cotangent, and the cosine of the angles in the indicated columns. You should be familiar enough with tables of this sort to find the indicated functions of the angles from 0° through 45° without any trouble. For example, to find any of the functions for an angle, say 36°, we read down the degree column until we come to 36. Then, reading to the right, the sine of 36° is .5878, the tangent is .7265, the cotangent is 1.3764, and the cosine is .8090.

We know that the sine of an angle is equal to the cosine of its complement. That is, in Fig. 15, the sine θ equals:

$$\frac{BC}{AB} = \cos A$$

Therefore, if the sine of 36° is equal to .5878, as shown in the table, this same value of .5878 must be equal to the cosine of 54° which is the complement of 36° (36° + 54° = 90°). Likewise, the cosine of 36°, which according to the table is .8090, must be equal to the sine of 54°. Thus, if we know the functions of the angles from 0° to 45°, we automatically know the functions of all the angles from 0° to 90° if we remember the relationships of the functions of complementary angles.

Most tables are made so that they can be read up as well as down, like the one in Fig. 16. Notice that the functions are

Degrees	Sine	Tangent	Cotangent	Cosine	
0	.0000	.0000	......	1.0000	90
1	.0175	.0175	57.290	.9998	89
2	.0349	.0349	28.636	.9994	88
3	.0523	.0524	19.081	.9986	87
4	.0698	.0699	14.301	.9976	86
5	.0872	.0875	11.430	.9962	85
6	.1045	.1051	9.5144	.9945	84
7	.1219	.1228	8.1443	.9925	83
8	.1392	.1405	7.1154	.9903	82
9	.1564	.1584	6.3138	.9877	81
10	.1736	.1763	5.6713	.9848	80
11	.1908	.1944	5.1446	.9816	79
12	.2079	.2126	4.7046	.9781	78
13	.2250	.2309	4.3315	.9744	77
14	.2419	.2493	4.0108	.9703	76
15	.2588	.2679	3.7321	.9659	75
16	.2756	.2867	3.4974	.9613	74
17	.2924	.3057	3.2709	.9563	73
18	.3090	.3249	3.0777	.9511	72
19	.3256	.3443	2.9042	.9455	71
20	.3420	.3640	2.7475	.9397	70
21	.3584	.3839	2.6051	.9336	69
22	.3746	.4040	2.4751	.9272	68
23	.3907	.4245	2.3559	.9205	67
24	.4067	.4452	2.2460	.9135	66
25	.4226	.4663	2.1445	.9063	65
26	.4384	.4877	2.0503	.8988	64
27	.4540	.5095	1.9626	.8910	63
28	.4695	.5317	1.8807	.8829	62
29	.4848	.5543	1.8040	.8746	61
30	.5000	.5774	1.7321	.8660	60
31	.5150	.6009	1.6643	.8572	59
32	.5299	.6249	1.6003	.8480	58
33	.5446	.6494	1.5399	.8387	57
34	.5592	.6745	1.4826	.8290	56
35	.5736	.7002	1.4281	.8192	55
36	.5878	.7265	1.3764	.8090	54
37	.6018	.7536	1.3270	.7986	53
38	.6157	.7831	1.2799	.7880	52
39	.6293	.8098	1.2349	.7771	51
40	.6428	.8391	1.1918	.7660	50
41	.6561	.8693	1.1504	.7547	49
42	.6691	.9004	1.1106	.7431	48
43	.6820	.9325	1.0724	.7314	47
44	.6947	.9657	1.0355	.7193	46
45	.7071	1.0000	1.0000	.7071	45
	Cosine	Cotangent	Tangent	Sine	Degrees

Fig. 16. Table of functions.

listed again at the bottom of the table, but in the reverse order. In addition, at the extreme right we have another heading marked "Degrees." This we read from the bottom, 45°, to the top, 90°, in conjunction with the function headings at the bottom. Thus, to find the cosine of 63°, we read up the right-hand degree column to 63, then across to the extreme left to the column which is marked "cosine" at the bottom. This shows us that cosine 63° is .4540; cotangent 63° is .5095; tangent 63° is 1.9626; and sine 63° is .8910.

Thus, sine 63° is .8910 which is the cosine of its complement 27°. By studying the table, you will notice some important relationships between the various functions. The sine of 0° is .0000, but the cosine is 1.0000, while the sine of 90° is 1.0000 and the cosine is .0000. Thus, the sines of angles start from 0° and .0000 and work up to a maximum of 1.0000 at 90°. The cosine works out exactly the opposite; it has a value of 1 at 0° and decreases until it is .0000 at 90°. The value of the sine or the cosine of an angle can never be more than 1. The tangent also starts at 0° and .0000, but the tangent functions have no upper limit. At 89° it is 57.290, but as you can see, it is increasing in value rapidly as it approaches this upper limit. From 89° it increases to some infinite (unmeasurable) value at 90°.

Interpolation. Although accuracy to one degree is usually satisfactory, you may want to be accurate to a fraction of a degree. You can get this additional accuracy from the Table of Functions even though it shows only whole degree steps. We do this by a process known as interpolation. Suppose we want to find the sine of an angle of 36.5°. Since it is between 36° and 37°, we know that its sine must be more than the sine of 36° and less than the sine of 37°. Therefore, we look up the sine of both 36° and 37° and proceed as follows:

First, we subtract to find the difference between the values of the sines of the two angles. Thus:

$$\begin{array}{rr} \text{sine } 37^\circ = & .6018 \\ -\text{ sine } 36^\circ = & .5878 \\ \hline \text{difference} & .0140 \end{array}$$

The angle that we are trying to find the sine for is 36.5°, which is an increase of .5° over 36°. Therefore the sine of 36.5° must be the sine of 36° plus .5 of the difference between 36° and 37°.

$$.5 \times .0140 = .00700$$

Then, .5878 (sine 36°) plus .007 equals .5948, which is the sine of 36.5°.

To make sure that we understand this, let's try another example. What is the sine of 28° 15′? First,

$$\begin{array}{rr} \text{sine } 29^\circ = & .4848 \\ -\text{ sine } 28^\circ = & .4695 \\ \hline \text{difference} & .0153 \end{array}$$

Now, 28° 15′ is 15/60 or .25 more than sine 28°. Therefore:

$$.25 \times .0153 = .003825$$

and

$$.4695 + .003825 = .473325$$

which is the sine of 28° 15′. To interpolate sine functions, find the difference between the functions of the next smaller and the next larger angle. Then, multiply this difference by the amount of increase and add the product to the function of the smaller angle. The same procedure is used to interpolate tangent functions.

Interpolation of cosine and cotangent functions starts the same: Find the difference between the next smaller and next larger functions and multiply this difference by the amount of increase. Now, since the values of cosine and cotangent become smaller as the angle becomes larger, you must ***subtract*** the product from the value of the function of the smaller angle.

You have learned to find the value of a function of an angle by interpolation when it lies between two angles. A similar process is used to find an angle when the value of a function does not appear in the table.

For example, suppose we want to find the angle θ whose tangent is .5978. The Table of Functions does not show a tangent for .5978. But the tangent of 31° is .6009 and the tangent of 30° is .5774. Therefore, the angle θ lies somewhere between 30° and 31° since its tangent lies between the tangents of 30° and 31°.

First find the difference of the tangent values:

$$\begin{array}{r} \tan 31^\circ = .6009 \\ -\tan 30^\circ = .5774 \\ \hline \text{difference} = .0235 \end{array}$$

Therefore, an increase of 1° between 30° and 31° makes an increase of .0235 in tangent value.

Next, subtract the tangent of the smaller angle (30°) from the tangent of angle θ:

$$\begin{array}{r} \tan \theta = .5978 \\ -\tan 30^\circ = .5774 \\ \hline \text{difference} = .0204 \end{array}$$

The tangent shows an increase of .0204 from the tangent of 30°, therefore the angle which lies between 30° and 31° is determined by the fraction:

$$\frac{.0204}{.0235}$$

or .868 when expressed as a decimal. The angle θ whose tangent is .5978 must be 30.868°. Working to the nearest one tenth of a degree will usually be accurate enough. Therefore angle θ is equal to 30.9°.

Interpolation applies to all functions and all angles not shown in a table of functions.

SELF-TEST QUESTIONS

6. Define trigonometry.
7. How many minutes are there in 13 degrees? How many seconds?
8. How many radians are there in 360°?
9. What is the difference between an acute angle and an obtuse angle?
10. One acute angle of a right triangle is 32°, what is the other acute angle?
11. List and define the six trigonometric functions.
12. Using the table in Fig. 16, find the following:

    (a) sin 13°
    (b) cos 46°
    (c) tan 19°
    (d) sin 57°

13. Find the following:

    (a) sin 3.2°
    (b) tan 51.6°
    (c) cos 19.5°
    (d) sin 16.8°

14. Find the angle:

(a) whose sin is .3584
(b) whose tan is 1.9626
(c) whose sin is .7771
(d) whose cos is .5592

15. Find the angle:

(a) whose sin is .8141
(b) whose tan is .6081
(c) whose sin is .1581
(d) whose cos is .6665

Coordinate Systems

We have become use to expressing a vector as a binonimal term by using the j operator. Now that we have learned a little about trigonometry, we can also express a vector as a numerical value and an angle. For example, suppose we have the simple circuit shown in Fig. 17. The coil has an inductive reactance of 15 ohms and is in series with a 15-ohm resistance. Neglecting the resistance of the coil itself, what is the impedance and the phase angle of the circuit?

We already know two ways to solve such a problem. The first method is to construct a very accurate vector diagram using the resistive and reactive components and then measure the resultant impedance vector and the phase angle. Although this can be done for simple circuits, the drawings can become complex and difficult to work with if the circuits are the least bit complicated. However, the greatest disadvantage in this method is that the accuracy depends on drawing neatly and precisely. Since most of us are not draftsmen or artists, it may be difficult to make a completely accurate drawing to scale and this gives a large margin for error when using this method.

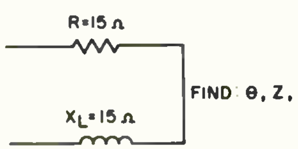
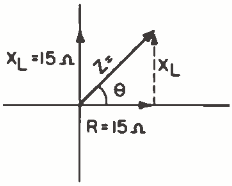


Fig. 17. Simple ac circuits.

The other method is to use the j operator and express the vector mathematically using a binomial number. This eliminates the need for many of the accurate diagrams because we can work with vectors mathematically to combine them into a final binomial representation of the resultant. Then, by using the Pythagorean Theorem, we can find the numerical value of the binomial representation which gives the length of the final vector. However, we still must construct a diagram and actually measure the angle of lead or lag to find the phase angle.

Using trigonometry to solve vector diagrams will eliminate the need for any construction or measurement in finding the phase angle as well as the impedance. It is also easier than finding the square roots of numbers as we do when using the Pythagorean Theorem. Now, let's use trigonometry to solve the circuit shown in Fig. 17.

The best way to begin is to lay out a simple sketch of the vectors involved. Since we are not going to make any



FIND θ, Z

$$\text{TAN}\,\theta = \frac{\text{Opp}}{\text{Adj}} = \frac{X_L}{R} = \frac{15}{15} = 1.0000$$

$$\theta = 45°$$

$$\text{SIN}\,\theta = \frac{\text{Opp}}{\text{Hyp}} = \frac{X_L}{Z}$$

$$Z = \frac{X_L}{\text{SIN } 45°} = \frac{15}{.7071} = 21.2\ \Omega$$

Fig. 18. Trigonometric solution to circuit in Fig. 17.

measurements, this can be a rough diagram as shown in Fig. 18. Here the resistance component becomes a vector that forms the base of a triangle. The reactance component becomes a vector that represents the altitude of a triangle. Since the phase angle between a pure resistance and a pure inductance is exactly 90°, these two vectors form a 90° angle. Therefore, the resultant impedance vector which we draw from the tail of the resistance vector to the head of the reactance vector becomes the hypotenuse of a right triangle. We want to know the value of the length of this impedance vector and the size of the phase angle θ that is formed by it.

Looking at this triangle in terms of what we know about it as compared to what we want to know, we see that we know the value of the side opposite the angle θ and the value of the side adjacent to the angle θ. Now, we consider the trigonometric functions that we have just learned to see which one of them fits the unknown angle θ in terms of the known values. If we go down the list of functions, the first one we come to that uses the opposite and adjacent sides is the tangent. This states that

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

If we use this function as an equation, and substitute the known values, we have:

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{X_L}{R} = \frac{15}{15} = 1.0000$$

This tells us that the tangent of the angle θ is equal to 1.0000 when the opposite side and the adjacent side are both equal to 15.

Now, we turn to the Table of Functions in Fig. 16 and look down the column headed "Tangent" until we come to 1.0000. Then, looking to the left, we find that this number is the tangent of an angle of 45°. Thus, the phase angle θ in our diagram must equal 45°, because our equation states that $\tan\theta = 1.0000$ and our Table of Functions shows us that only 45° has a tangent equal to 1.0000.

Now that we have found our phase angle, we will want to find the value of our impedance vector. We can do this quite easily now that we know the value of θ because:

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{X_L}{Z}$$

Stated in terms of Z, this becomes:

$$Z = \frac{X_L}{\sin\theta} = \frac{15}{\sin 45°}$$

Then, from the Table of Functions,

$$\sin 45° = .7071 \text{ and } Z = \frac{15}{.7071}$$

Performing the mathematics:

$$Z = 15 \div .7071 = 21.2$$

Thus, using two steps of simple algebra and arithmetic and the Table of Functions, we can find both the phase angle (45°) and the impedance (21.2Ω) for the circuit shown in Fig. 17. By using the trigonometric functions of angles in this way, we can find the value of either of the acute angles of a right triangle if two of the sides are known, or the value of the two unknown sides if one of the sides and an acute angle are known.

Thus, we can say that the impedance is $21.2\ \underline{/+45^\circ}\Omega$. As you recall from an earlier lesson, the impedance can also be expressed as a binomial: $(15 + j15)\Omega$. That is, the impedance can be expressed as a number and an angle or as a binomial. Either expression gives us a number picture of the vector and allows us to construct an accurate diagram of both the resultant vector and its components. They also allow us to compute with the vectors mathematically.

These two methods of noting vectors mathematically are given special names. The first, using a binomial such as 15 + j15 ohms is termed a ***rectangular*** or ***Cartesian*** coordinate. The second, using a number and an angle such as $21.2\ \underline{/+45^\circ}$ ohms is called a ***polar*** coordinate. Notice that in the polar form we give the angle a positive value to show that we are measuring it in a counterclockwise direction from the reference to vector Z.

We give the angle a negative sign when measuring clockwise from the reference to the vector Z. The angle is usually given the same sign as the corresponding j term when we use rectangular coordinates. Thus vector $Z = 20 + j20$ ohms would be written in polar form as $Z = 28.3\ \underline{/+45^\circ}$ ohms.

Both of these methods of describing a vector or locating a point are commonly used. Measurements and computations are made either way, depending on the information desired. Often, when mechanical or electronic computers are used, conversions from rectangular coordinates to polar coordinates and back again are made constantly, depending on the nature of the information supplied, the type of information needed, and the equipment in the computer. Although we have actually learned nearly everything about these two systems of describing a vector, our work in electronics is such that we have used them constantly without knowing some of the basic concepts of these systems of coordinates. Now is a good time to catch up on some of these basic considerations.

RECTANGULAR COORDINATES

A coordinate system is simply a standard frame of reference for describing some particular value, condition, or place. Unless we have these standard references, even everyday occurrences would be difficult to explain or describe. For example, the common directions of north, south, east and west have no meaning unless we know what they are north, south, east or west from. Describing the voltage-current relationship of a circuit as 120 volts lagging a current of 1 ampere by 30° has no meaning unless we are familiar with a standard condition to compare it with. For this reason, standard reference frames have been established for universal use so that everyone will have the same means for describing a situation so that everyone else can understand it.

Originally, the rectangular coordinate system was devised for giving directions in a standard manner. Since this system is so simple and so widely understood, it

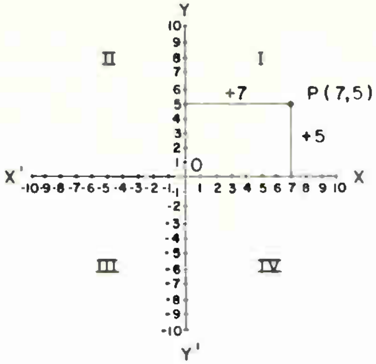


Fig. 19. Coordinate reference frame.

has been adopted for locating values and describing conditions throughout the fields of science and mathematics.

To begin with, let's call a rectangular coordinate system a device for associating points with pairs of numbers. The standard reference for this system consists of two mutually perpendicular lines, as shown in Fig. 19. One line is always horizontal and is labeled X′X as shown. Therefore, the other line labeled Y′Y is always vertical as shown, and the lines intersect at the point 0. We have used this device constantly in our work with vectors and it is nothing new except that we have never used these particular letters for the axes.

Now the two lines are called the coordinate axes and are said to be made up of the X axis (line X′X) and the Y axis (line Y′Y). The point 0 is called the origin. We can lay off scales along these axes to suit our particular purpose, but the scale from the origin, 0, to the right along the X axis is always positive, while the scale is always negative from the origin, 0, to the left along the X axis. Likewise, the scale from the origin, 0, up toward Y is always positive and the scale down from the origin, 0, to Y′ is always negative.

This reference frame consisting of the two coordinate axes serves as a means of locating any point in the plane of the axes by referring to two numbers. The two numbers completely express the position or distance of the point from the origin of the coordinate axes. For example, consider the point P in Fig. 19.

We can completely describe its position so that anyone familiar with this system can immediately locate it by saying it lies +5 from the X axis and +7 from the Y axis. The distance of the point from the Y axis, measured along the X axis is called the *abscissa* of the point. The distance of the point from the X axis measured by the scale on the Y axis is called the *ordinate* of the point. These two numbers, each with their proper algebraic sign, are called the *coordinates* of the point. In writing the coordinates of a point, the abscissa is written first and the ordinate second. Thus, we would write the coordinates of the point P in Fig. 19 as "the coordinates of P are (7, 5)."

The coordinate axes divide the area into four sections or quadrants as they are called. These quadrants are numbered in Fig. 19 by the Roman numerals I, II, III, IV. Notice that a point must have two positive coordinates to lie in the first quadrant and two negative coordinates to lie in the third quadrant. A point in the second quadrant must have a negative abscissa and a positive ordinate, while a point in the fourth quadrant has a positive abscissa and a negative ordinate.

This system of referring to a point by its coordinate is sometimes called the "Cartesian" coordinate system in honor of the French mathematician Descartes. It is also called the rectangular coordinate system because a rectangular figure is drawn when the points are completely located.

In electronics work, we use this standard coordinate reference frame constantly, but we usually use some other method of labeling the coordinate axes. However, regardless of the symbols we may use, the quadrant designations always remain the same, the signs of the scales are always the same, and we always consider the positive section of the X axis as the starting point or reference line. When the frame is assigned a degree system of reference, 0° is always at X, 90° at Y, 180° at X′, and 270° at Y′. Although most of our work will lie in the first and fourth quadrants, which is all we need to represent vectors consisting of coordinates using positive resistance, +jX and −jX, we will have some occasion to get into the second and third quadrants when we consider polyphase systems and "negative" resistances later in the course.

POLAR COORDINATES

As we have seen in studying ac circuits and trigonometry, degrees can be used to designate the points in the standard coordinate reference frame as shown in Fig. 20. Thus, we can locate a point P accurately and clearly by saying it is 10 units from the "0" of the graph and a line connecting the point "P" with "0" forms an angle of 45° with the positive X axis, as shown in Fig. 20. This would be written $10\underline{/+45^\circ}$ to show the length of a line from the origin, 0, to the point as 10 units; and the displacement of the line from 0° (measured counterclockwise) as $\underline{/+45^\circ}$. Thus, saying point P is $10\underline{/+45^\circ}$ would describe the location of the point in terms of polar coordinates. We call this the polar coordinate system because of its use in reference to the poles of the earth for navigation and surveying. Remember, a negative sign is used for angles generated or measured from 0° in a clockwise direction.

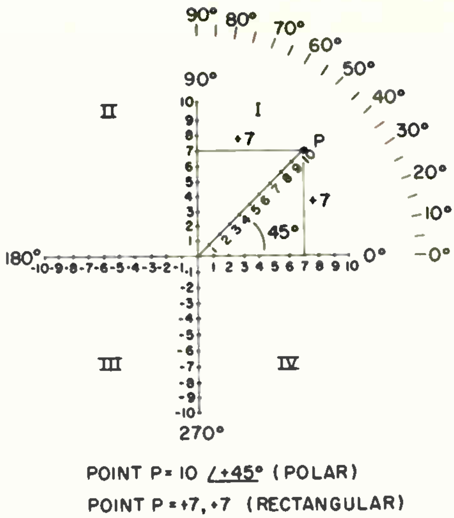


Fig. 20. Reference frame for polar coordinates.

CONVERTING POLAR AND RECTANGULAR COORDINATES

Point P in Fig. 20 can be completely described or located by using either polar coordinates or rectangular coordinates. Therefore, we may want to convert polar coordinates to rectangular coordinates and vice versa. This can easily be done by using the fundamentals of trigonometry.

For example, suppose we wish to refer to the point P in Fig. 21 in rectangular

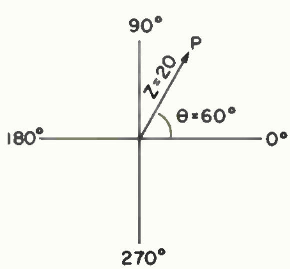


Fig. 21. Converting polar coordinates to rectangular coordinates.

coordinates so that it can be combined with other points that are also expressed as rectangular coordinates. When point P is expressed in polar form as an impedance vector and a phase angle it is written as $Z = 20\underline{/+60^\circ}$. To convert this impedance vector to its rectangular coordinates, we would express it as a resistance ± reactance.

The resistance component is always the adjacent side of $\angle\theta$ and the reactive component is always the opposite side of $\angle\theta$, representing the ±j term. Accordingly:

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

or

$$\text{adj} = \text{hyp} \times \cos\theta$$

Substituting:

$$R = Z\cos\theta$$

Likewise:

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

or

$$\text{opp} = \text{hyp} \times \sin\theta$$

Substituting:

$$X = Z\sin\theta$$

Then, since

$$Z = R \pm jX$$

we can substitute:

$$Z = Z\cos\theta \pm j(Z\sin\theta)$$

or

$$Z = Z(\cos\theta \pm j\sin\theta)$$

By applying this equation, we can convert polar coordinates to rectangular coordinates.

In Fig. 21, we would have:

$$Z = Z(\cos\theta \pm j\sin\theta)$$

Substituting:

$$Z = 20(\cos 60 + j\sin 60)$$

From our table:

$$Z = 20(.5 + j.866)$$

and:

$$Z = 10 + j17.32$$

would express

$$Z = 20\underline{/+60^\circ}$$

in rectangular coordinates.

For example, to convert the rectangular coordinates of an impedance, $Z = 250 - j100$ ohms to polar coordinates we have in three steps:

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

Substituting:

$$\tan\theta = \frac{-jX}{R}$$

$$= \frac{-100}{250} = -.4$$

and from our tables:

$$\theta = -21.8^\circ$$

(θ is negative because jX is negative.)

Then:

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

or:

$$\text{hyp} = \frac{\text{opp}}{\sin\theta}$$

Substituting:

$$Z = \frac{-jX}{\sin\theta} = \frac{100}{\sin 21.8^\circ}$$

$$= \frac{100}{.3714} = 269\Omega$$

Accordingly, in polar coordinates:

$$Z = 269\underline{/-21.8^\circ}\Omega$$

Notice that once we found θ, and since all the necessary values are available, we could have used:

$$\cos\theta = \frac{adj}{hyp}$$

instead of:

$$\sin\theta = \frac{opp}{hyp}$$

However, our answer would still have worked out to the same value of $269/-21.8^\circ\Omega$.

MULTIPLICATION AND DIVISION USING POLAR COORDINATES

You will recall from your study of operator j that complex quantitites can be multiplied and divided to form other complex quantities. For example, if both the current and impedance were known in an ac circuit, we can find the voltage by multiplying the current by the impedance:

$$E = I \times Z$$

To do this, we would use the following procedure (keeping in mind significant figures). Assume:

$$I = 1.72 + j.38 \text{ amperes}$$
$$Z = 246 - j12 \text{ ohms}$$
$$E = I \times Z$$

$$\begin{array}{rrrr} & (1.72 & + j.38) & \\ \times & (24.6 & - j\,12) & \\ \hline & 42.3 & + j9.35 & \\ & & -j20.6 & + \; 4.56 \\ \hline & 42.3 & -j11.3 & + \; 4.56 \end{array}$$

$$= 46.9 - j11.3$$

$$E = 46.9 - j11.3 \text{ volts}$$

We can perform the same calculations using the polar form for the current and impedance. To do this we first convert the current and impedance expressions to polar form:

$$I = 1.72 + j.38 = 1.76\underline{/+12.5}$$
$$Z = 24.6 - j12 = 27.4\underline{/-26^\circ}$$

The rules for multiplication in polar form are:

1. *Multiply* the magnitudes.
2. *Add* the angles.

Using the figures for I and Z already determined, we obtain:

$$E = 1.76 \times 27.4\underline{/(+12.5) + (-26)}$$
$$E = 48.3\underline{/-13.5^\circ}$$

To check our calculation, we can convert $48.3\underline{/-13.5^\circ}$ to get rectangular form and we get $E = 46.9 - j11.3$ volts.

We can also divide using the polar form for complex numbers. You will recall that to divide complex numbers in rectangular form we multiply numerator and denominator by the conjugate of the denominator to remove the j term from the denominator. This is a long and complicated operation which can be simplified by using the polar form. The rules for division are:

1. *Divide* the magnitude of the numerator by the magnitude of the denominator.
2. *Subtract* the angle of the denominator from the angle of the numerator.

As an example, suppose we have $E = 123\underline{/20^\circ}$ and $I = 1.4\underline{/-46^\circ}$. To find the impedance we divide E by I:

$$Z = \frac{E}{I} = \frac{123\underline{/20^\circ}}{1.4\underline{/-46^\circ}} = 87.9\underline{/(20^\circ) - (-46^\circ)}$$

$$Z = 87.9\underline{/66^\circ} \text{ ohms}$$

As an exercise, convert E and I to rectangular form and perform the calculation using conjugates. Then see if your result is equal to 35.8 + j80.3 ohms.

PHASORS AND SINE WAVES

In the lessons on alternating currents, you learned that an alternating current is sinusoidal in nature. In other words, the various values of current and voltage generated by an alternating current source describe what is called a sine wave which repeats itself periodically. However, we have not yet discussed what a sine wave is nor why ac current values produce it.

Additionally, in your first introduction to vectors, we mentioned that the vectors used in ac circuit calculations were not considered vectors by electronic engineers. In a most accurate technical sense, they are *phasors*. This is because vectors usually express force as magnitude and direction, while phasors represent sine wave values with respect to time or phase.

Since many of you learned to work with vectors in high school or college physics, and since vectors and phasors are treated the same in circuit computations, we will continue to call them vectors.

However, many modern engineering texts refer to electrical vectors as phasors and we should know why they are phasors. This is a good time to examine both the sine wave and electrical vectors in greater detail.

Angles Greater Than 90°. In studying trigonometry, we have considered the functions of angles only up to 90°. We can, however, have angles of any magnitude, depending on where the terminal side of an angle stops and on how many complete revolutions it makes in generating the angles. In our study of sine waves, we will need to consider the functions of all possible angles up to 360°.

To begin with, let's consider the sine of the acute angle θ shown in Fig. 22A. Since $\angle\theta$ is equal to 30°, the sine of θ is equal to the ratio of the length of the opposite side over the hypotenuse which always works out to .5.

Now let's consider the angle θ shown in Fig. 22B. Angle θ is equal to 150° which is larger than 90° and so cannot be one of the angles in a right triangle. We know that the trigonometric functions are based on the sides and angles of right triangles. Yet, $\angle\theta = 150^\circ$ is considered to have a sine function, even though it cannot be part of a right triangle. In fact, the sine of 150° is .5, just the same as the sine of 30°.

To see how this is determined, we must consider what is known as the "associated acute angle" of the angle θ. First of all, an angle is said to be in *standard position*

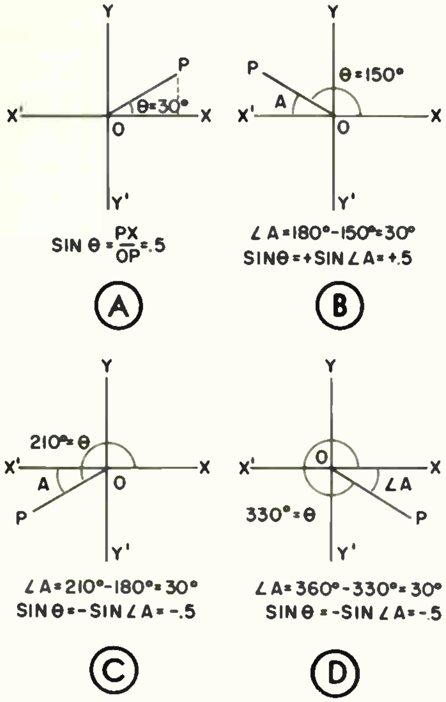


Fig. 22. Sines of angles greater than 90°.

when its ***initial*** side coincides with the ***positive*** side of the ***X axis*** of the standard coordinate reference frame, regardless of where its terminal side may be. Thus, the $\angle\theta = 150°$ in Fig. 22B is in the standard position. Now, the ***associated acute angle*** of any angle θ larger than 90° is the acute angle which the terminal side of the angle makes with the X axis when the angle θ is in the standard position. Since $\angle\theta$ (Fig. 22B) is by definition in standard position, $\angle A$ is its associated acute angle. $\angle A$ is the supplement of $\angle\theta$, and therefore $\angle A = 180° - \angle\theta(150°)$ or 30°.

In Fig. 22C we have another angle θ which is equal to 210°. Its associated acute angle is $\angle A$ because, by definition, the associated acute angle is the acute angle formed between the terminal side and the X axis. Since θ equals 210°, and since the $\angle X'OX$ is equal to 180°, the $\angle A$ in Fig. 22C must be equal to $210° - 180°$ or $\angle A = 30°$ again.

In Fig. 22D, we show still another angle θ. This time $\angle\theta = 330°$ and its associated acute angle, $\angle A$, must equal $360° - 330°$ or 30° once again.

Remember, the associated acute angle always lies between the terminal side of $\angle\theta$ and the X axis as shown in Fig. 22. Notice that from this definition, $\angle\theta$ in Fig. 22A is really its own associated acute angle.

There is a trigonometric statement that says:

The function of any angle greater than 90° is equal to plus or minus the function of its associated acute angle.

This simply means that a 150° angle has the same sine as its associated acute angle, which is $\angle A$. Since $\angle A$ is 30°, the sine of the 150° angle is the same as the sine of the 30° angle. Likewise, the sine of 210° is equal to the sine of 30° except that it is negative, and the sine of 330° is also equal to the sine of 30° except that it is negative.

To determine the sine of any angle greater than 90°, find the value of the function of its associated acute angle. Then affix a plus or minus sign, depending on the quadrant in which the terminal side of the angle lies.

As you have probably guessed, all the other functions can also exist for any angle greater than 90°. The rule for the sine function applies to the other functions as well.

In Fig. 23A, the cosine of $\angle\theta = 252°$ is equal to minus the cosine of 72° or $-.3090$ because 72° is the associated acute angle of 252° and the terminal side lies in the third quadrant. Likewise, the tangent of $\angle\theta = 305°$ is equal to minus

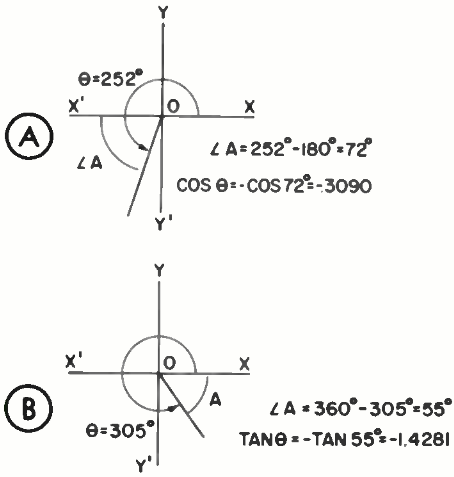


Fig. 23. All functions exist for all angles.

the tangent of 55° or −1.4281, as shown in Fig. 23B.

Now let's consider the sign of the functions in the different quadrants.

SIGNS OF FUNCTIONS IN QUADRANTS

In the first quadrant, all six functions are positive. In the second quadrant, the sine and cosecant are positive, all other functions are negative. In the third quadrant the tangent and its reciprocal (the cotangent) are positive and the other functions are negative. In the fourth quadrant, the cosine and secant are positive and the other four functions are negative.

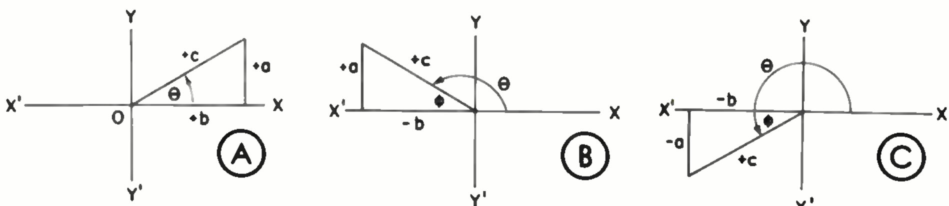
To see why the functions have these signs, look at Fig. 24. First consider that the X–X′ axis and the Y–Y′ axis divide the figure into four quadrants. A line drawn from the Y–Y′ axis along or parallel to the X axis is considered positive if it is drawn to the right, and negative if it is drawn to the left. Similarly, a line drawn from the X–X′ axis along or parallel to the Y axis is positive if it is drawn upward and negative if it is drawn below the X–X′ axis.

Now look at angle θ in Fig. 24A. Side a is positive because it is drawn above the X–X′ axis, and side b is also positive because it is drawn to the right of the Y–Y′ axis. *The hypotenuse of the triangle is always considered positive* and therefore the functions are as follows:

$$\sin\theta = \frac{+a}{+c}$$

$$\cos\theta = \frac{+b}{+c}$$

$$\tan\theta = \frac{+a}{+b}$$



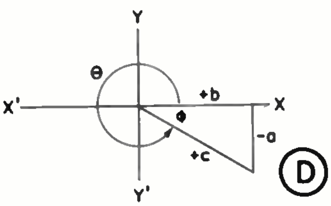


Fig. 24. Determining the signs of the functions in the four quadrants.

$$\cot \theta = \frac{+b}{+a}$$

$$\sec \theta = \frac{+c}{+b}$$

$$\csc \theta = \frac{+c}{+a}$$

In each case we have a plus term divided by another plus term, so the result in each case is positive.

Now look at Fig. 24B and consider the second quadrant. Again we are interested in angle θ. The functions are equal to the functions of the associated acute angle which we have labeled Φ (Greek capital letter Phi). The hypotenuse c is always positive. Side a which is parallel to the Y axis and drawn above the X–X′ axis is positive. Side b which is parallel to the X axis is drawn to the left of the Y–Y′ axis and is therefore negative. So now the functions are as follows:

$$\sin \theta = \sin \Phi = \frac{+a}{+c}$$

$$\cos \theta = -\cos \Phi = \frac{-b}{+c}$$

$$\tan \theta = -\tan \Phi = \frac{+a}{-b}$$

$$\cot \theta = -\cot \Phi = \frac{-b}{+a}$$

$$\sec \theta = -\sec \Phi = \frac{+c}{-b}$$

$$\csc \theta = \csc \Phi = \frac{+c}{+a}$$

From this you can see that the sin and csc will be positive because you have a plus term divided by another plus term and the other four functions will be negative because in each of these expressions there is one plus term and one negative term.

Now look at θ in the third quadrant shown in Fig. 24C. The hypotenuse c is positive, but now both a and b are negative. Therefore:

$$\sin \theta = -\sin \Phi = \frac{-a}{+c}$$

$$\cos \theta = -\cos \Phi = \frac{-b}{+c}$$

$$\tan \theta = \tan \Phi = \frac{-a}{-b}$$

$$\cot \theta = \cot \Phi = \frac{-b}{-a}$$

$$\sec \theta = -\sec \Phi = \frac{+c}{-b}$$

$$\csc \theta = -\csc \Phi = \frac{+c}{-a}$$

In the case of the tan and cot functions you have a minus term divided by a minus term which gives you a plus result so the tangent and cotangent are positive in the third quadrant and all other terms are negative.

Now look at θ in the fourth quadrant as shown in Fig. 24D. Again c is positive. Side b will also be positive, but side a is negative. Therefore:

$$\sin \theta = -\sin \Phi = \frac{-a}{+c}$$

$$\cos\theta = \cos\Phi = \frac{+b}{+c}$$

$$\tan\theta = -\tan\Phi = \frac{-a}{+b}$$

$$\cot\theta = -\cot\Phi = \frac{+b}{-a}$$

$$\sec\theta = \sec\Phi = \frac{+c}{+b}$$

$$\csc\theta = -\csc\Phi = \frac{+c}{-a}$$

Thus cos and sec are positive and the other four functions are negative in the fourth quadrant.

The Sine Wave. Now that we can determine the sine of any angle up to 360°, we are ready to continue with the definition of a sine curve. A sine curve is merely a graph of all the sines of all the angles up to 360° that repeats itself periodically. Since there are an infinite number of angles that can be formed between 0° and 360° (if all the possible stopping points of the terminal side are considered), there are also an infinite number of sine functions to plot a curve. Therefore, we simply choose angles in 15° steps from 0° to 360° and plot their functions. We then round off the sine functions of these angles to two decimal places because this is as accurate as we can be if we are to keep the graph to a reasonable size.

Now let's examine and list the sine functions of the angles, starting with $\angle\theta = 0°$. When θ equals zero, there is no angle or opposite side, so when $\angle\theta$ equals 0°, the opposite side must also be zero. Therefore, since $\sin\theta$ equals the ratio of opp/hyp, and $\sin\theta$ equals the ratio of 0/hyp, when $\angle\theta$ equals 0°, $\sin\theta$ must also equal 0. Therefore, our list of functions begins with:

$$\theta = 0°; \quad \sin\theta = 0$$

We have seen that the ratio of opp/hyp is always equal to the sine of $\angle\theta$ as listed in the tables, no matter what the relative lengths of the sides of the right triangle may be. Therefore, you can use the tables directly to make a list of the sine functions of all the angles you wish to use.

The next step is to lay out a graph as shown in Fig. 25. Here the base line is laid out on the X axis and is marked in 15° steps from 0° to 360°. The numbers 0 to 1.0 are laid out vertically in steps of

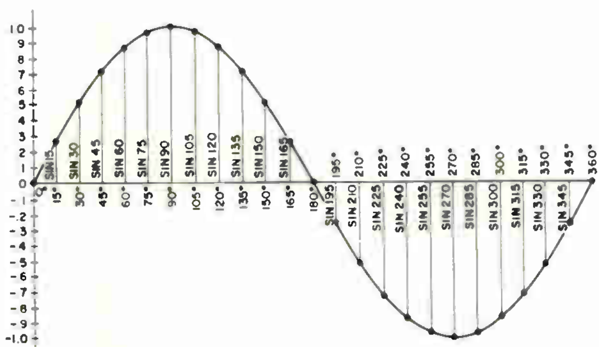


Fig. 25. Graphical plot of the sines of angles in 15° steps from 0° to 360°.

.1 in the positive direction along the Y axis. The numbers from 0 to –1.0 are laid out in steps of –.1 in the negative direction along the Y axis. At each 15° step along the X axis, the appropriate value of the sine of the angle is indicated in terms of the numbers located on the Y axis. Thus, we have a graph of ***various values*** of the Y values of the sines of all the angles indicated along the X axis. This is generally called a graph of Y = sin X.

When these points are joined with a smooth curve, a curve is formed which represents the sines of all the angles between 0° and 360°. This is called a sine curve or sine wave. We can consider the terminal side of angles generated to form a sine curve as a rotating vector, as shown in Fig. 26. This rotating vector generates the angles and consequently the functions of the angles indicated by the sine curve. If sine θ equals

$$\frac{\text{opp}}{\text{hyp}}$$

and hyp (rotating vector) equals 1 unit, then sin θ equals

$$\frac{\text{opp}}{1} = \text{opp}$$

Thus, if this vector equals 1″, 1′, 1 volt, or any other value, the sine functions generated will be equal to the length of the perpendiculars, which will also equal the actual value of the sine functions as listed in the tables.

Notice that as the vector starts from 0°, the function is 0; as it passes through 90° the maximum function is generated. At 180° the function is again 0; at 270° in the negative direction it is maximum; and at 360° it is back again to no function. Also notice that the value of

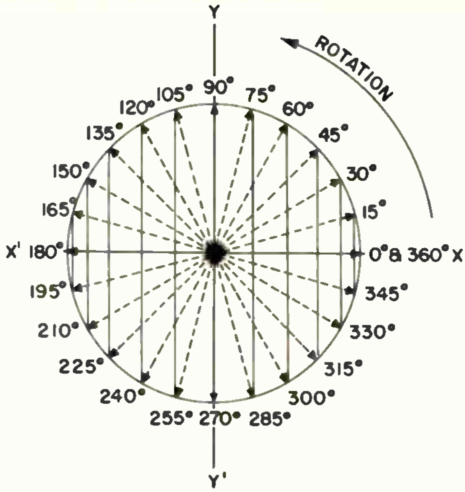


Fig. 26. Diagram of rotating vector generating sine functions shown in the sine curve of Fig. 25.

the function changes most rapidly when the vector is rotating through 0°, 180°, and 360°. This rotating vector can be compared to the rotation of the armature of a basic ac generator as shown in Fig. 27.

As the armature moves parallel to the lines of force as shown in Fig. 27A, no voltage will be generated. As it moves at right angles to the lines of force as in Fig. 27B, the maximum number of lines of force will be cut and the maximum voltage will be generated. While it is

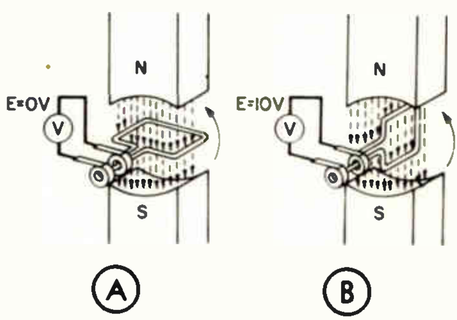


Fig. 27. Generator armature can be compared to rotating vector in Fig. 26.

moving from its position shown in Fig. 27A to that shown in Fig. 27B, the voltage generated will increase from zero to maximum in proportion to the sine of its instantaneous angular position. Thus, if we consider the maximum voltage as 10 volts at 90° of rotation as shown in Fig. 27B, and zero voltage as 0 volts at 0° of rotation as shown in Fig. 27A, the instantaneous value of voltage at a rotation of 15° would be 2.7 volts. It would be 5 volts at 30°, 7.1 volts at 45°, etc. If we were to measure all these instantaneous voltages at each 15° step of angular rotation of the basic armature in a standard magnetic field from 0° through 360°, we would obtain a sine wave graph similar to the one in Fig. 25. This is why an ac voltage is said to vary sinusoidally, and is called a sine wave.

If the armature continues to rotate again and again at the same speed, the sine wave voltage will repeat itself indefinitely. We will get a complete 0° – 360° sine wave during each complete revolution or period, so the sine wave is a periodic repetition of the same functions.

Phase Relationships. We have seen how the voltage generated by the rotating armature describes a sine wave just as the sine functions of the rotating vector describe the sine curve of the angles it passes through. Now suppose we have two generators represented by rotating vectors, as shown in Fig. 28. They are exactly the same except that when one armature is passing through 90° the other is passing through 0°. The sine wave outputs of the two generators are superimposed on each other as shown by the solid and dotted sine waves in Fig. 29.

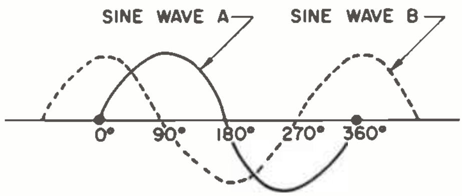


Fig. 29. Sine waves generated by two identical phasors operating 90° apart.

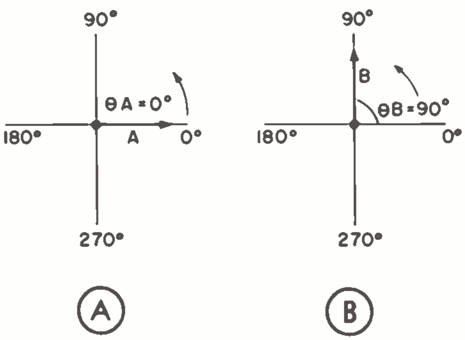


Fig. 28. Two generators represented by rotating vectors. (A) shows instantaneous value of sin θ to be zero. (B) shows instantaneous value of sin θ to be maximum at the same instant.

As you can see, these voltage sine waves are generated 90° out-of-phase with each other just as the vectors are 90° out-of-phase. Thus, the rotating vectors represent the relative phase of the two sine waves and are called phasors. By applying the principles of trigonometry to the angle generated by one of the phasors, we can determine the instantaneous value of voltage generated for any position of the phasor if we know its value at $\sin\theta = 1$.

Thus, if at $\sin\theta = 1$, or 90°, the phasor produces a voltage of 120 volts, then at $\angle\theta = 30$, the instantaneous value of voltage generated will equal

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

And, hyp equals phasor length at $\sin \theta = 1$, therefore hyp equals 120. Then,

$$\sin \theta = \frac{\text{opp}}{120}$$

and θ equals 30°, therefore

$$\sin 30° \text{ or } .5 = \frac{\text{opp}}{120}$$

and opp equals $120 \times .5 = 60$ volts of instantaneous voltage. In this way phasors can be used to represent the instantaneous values of all the sine wave voltages occurring across circuit components at any instant.

Phasors can be expressed in rectangular coordinates as well as polar coordinates All the principles of the parallelogram measurement method for solving vector diagrams can be applied to phasors. Similarly, the j operator and trigonometry can be used in the solution of phasor problems. Consequently, we shall continue to call our phasors "vectors" in this course. Remember, however, that although a phasor can perhaps be considered a special type of vector as is done in most older texts, a vector cannot be considered a phasor.

SELF-TEST QUESTIONS

16. A 15-ohm resistor is connected in series with a 15-ohm capacitive reactance. What is the impedance of the circuit? Express your answer in both polar and binomial form.
17. R_1, R_2, C_1, C_2, L_1 and L_2 are connected in series. $R_1 = 10$ ohms, $R_2 = 30$ ohms, $X_{C1} = 9$ ohms, $X_{C2} = 16$ ohms, $X_{L1} = 20$ ohms, $X_{L2} = 35$ ohms. What is the impedance? Express your answer in both polar and binomial form.
18. In the Cartesian coordinate system,
    (a) in which quadrants is X positive?
    (b) in which quadrants is Y positive?
    (c) in which quadrants are both X and Y negative?
19. What is the value of
    (a) sin 150°?
    (b) cos 150°?
20. What is the value of
    (a) tan 135°?
    (b) cot 225°?
21. What is the value of
    (a) sin −60°?
    (b) cos −60°?
22. Express the following polar coordinates as rectangular coordinates.
    (a) $6\underline{/36°}$
    (b) $5\underline{/-50°}$
23. Convert the following rectangular coordinates into polar coordinates.
    (a) (5, −11)
    (b) (−6, −7)
24. Convert 11 + j15 ohms to polar form.
25. Convert 7 − j4 ohms to polar form.
26. Convert $9\underline{/-50°}$ ohms to j-operator form.
27. Convert $12\underline{/32°}$ ohms to j-operator form.

Trigonometry in AC Circuits

In an earlier part of this lesson we saw how trigonometry is used in simple series circuit calculations. In this section we will apply what we have learned about vector diagrams, the j operator, algebra, and trigonometry to more complicated ac circuits. Before discussing series parallel circuits, let's take a look at a series circuit and get familiar with voltage computations.

Resistance in Coils. Until now we have neglected the resistance that exists in the windings of coils in our ac circuit calculations. Although we can do this without being too far off, occasionally we will want to be more accurate. This does not present any particular problem where both the resistance and either the inductance or inductive reactance of the coil are given. However, in many circuits, this information is not easy to obtain. Although, it is easy to measure the resistance of a coil with an ohmmeter, the inductance cannot be found as easily unless we have a special meter.

However, through the use of trigonometry and standard measuring instruments, it is quite easy to determine the inductance of a coil and separate the inductive reactance from the resistance. For example, consider the circuit shown in Fig. 30. Here we have a coil, a capacitor, and a resistor in series with each other. If we measure the voltage drops across each of these units as shown, and then compute the total voltage of the circuit, we will find that the total voltage computed does not agree with the total voltage indicated by the measurement. Let's see why this difficulty occurs and how to overcome it.

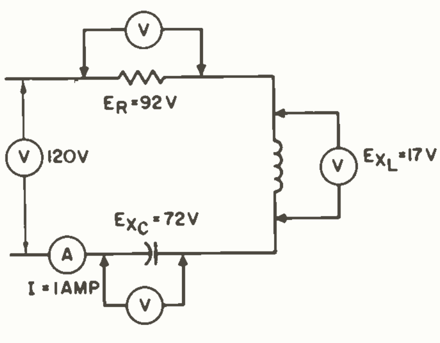
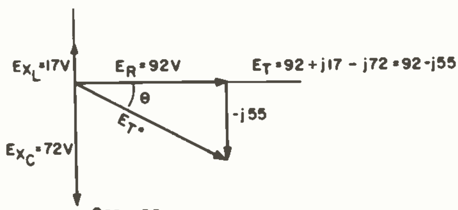


Fig. 30. Typical series circuit voltage measurements.



$$\text{TAN } \theta = \frac{\text{Opp}}{\text{Adj}} = \frac{55}{92} = .5978 = \text{TAN } 30.868° \text{ or } 30.9°$$

$$\theta = 30.9°$$

$$\text{SIN } \theta = \frac{\text{Opp}}{\text{Hyp}} = \frac{55}{E_T}$$

$$E_T = \frac{55}{\text{SIN } 30.9°} = \frac{55}{.5135} = 107.1 \text{ VOLTS}$$

Fig. 31. Computation of total voltage without considering coil resistance.

First, computing the total voltage, E_T, without considering any possible resistance that the coil might have, we get the results shown in Fig. 31. Drawing our rough vector diagram as shown, we have E_{X_L} of 17V as the inductive vector component, E_{X_C} of 72V as the capacitive vector component, and E_R of 92V as the resistive vector component. Mathematically, this gives us: 92 + j17 – j72, or 92 – j55 as our final vector stated as a binomial. Thus, E_T is the hypotenuse of a right triangle with a base of 92V and an altitude of 55V. Then, using trigonometry, we find that the phase angle θ equals:

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{55}{92} = .5978$$

Therefore:

$$\theta = \text{angle whose tangent is } .5978$$

By using our Table of Functions and interpolating we find that the angle θ equals 30.9°.

Now that we have the angle θ, we can find the value of E_T through the function:

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \text{ or hyp} = \frac{\text{opp}}{\sin\theta}$$

or $$E_T = \frac{55}{\sin 30.9^\circ}$$

Since our table lists the functions in one-degree steps we must interpolate to find the sine of 30.9°. Doing this, we find:

$$\sin 30.9^\circ = .5135$$

Substituting:

$$E_T = \frac{55}{\sin 30.9^\circ}$$

$$= \frac{55}{.5135} = 107.1 \text{ volts}$$

Thus, by computing, we find that E_T = 107.1 volts, even though the measured value for E_T in the circuit of Fig. 30 is 120 volts. The reason for this difference is that the Q of the coil is low; therefore, the ratio of the resistance of its winding to its inductance is large. This means that the voltage E_{X_L}, which we assumed to be purely inductive, does not lead the current in the circuit by a full 90°. Consequently, our E_{X_L} of 17 volts is really voltage made up of $E_{R_L} + jE_{X_L}$ and cannot be written +j17 and added directly to our −j72. It must be broken down into its two components, $E_{R_L} + jE_{X_L}$. Thus, the vector diagram of our circuit must be drawn as shown in Fig. 32, and our voltage vector E_T must be found from:

$$E_T = E_R + E_{TX_L} - jE_{X_C}$$

and

$$E_T = E_R + (E_{R_L} + jE_{X_L}) - jE_{X_C}$$

$$= 92 + E_{R_L} + jE_{X_L} - j72$$

In order to work out a solution to this problem, we must know the values of both E_{R_L} and E_{X_L}. Neither the value of the resistance of the coil nor the value of its inductance is given. However, the chances are that if we have a meter or meters capable of reading the voltages of the circuit and the current through the circuit, as shown in Fig. 30, we will also

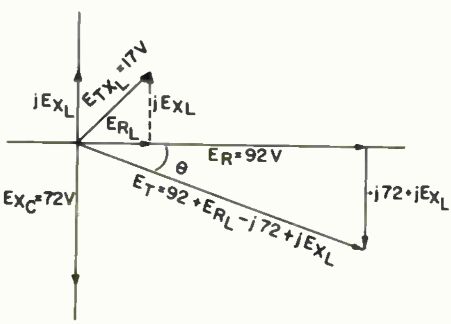


Fig. 32. Vector diagram of circuit in Fig. 30 considering coil resistance.

have an ohmmeter. With the ohmmeter we can determine the resistance of the coil, and then by computing we can find E_{R_L} and E_{X_L}.

For example, suppose we measure the resistance of the coil and find it to be 12 ohms. Our circuit ammeter shows us that the current through the circuit is 1 amp. Thus,

$$\begin{aligned} E_{R_L} &= I \times R_L \\ &= 1 \times 12 \\ &= 12 \text{ volts} \end{aligned}$$

This gives us one of our components of E_{X_L}, so our equation for E_T becomes:

$$E_T = 92 + E_{R_L} + jE_{X_L} - j72$$

$$= 92 + 12 + jE_{X_L} - j72$$

Now, all we have left to determine is the value of jE_{X_L}.

This is where trigonometry really helps. We have the total coil voltage drop and have been able to compute the resistance component of this total drop. Therefore, we have the value of both the hypotenuse and the base of a right triangle as shown in Fig. 33A, and can

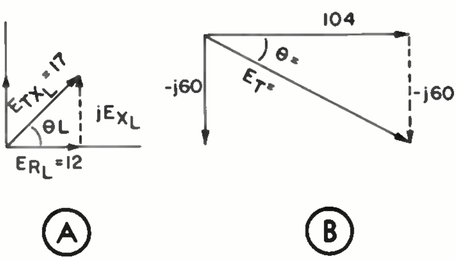


Fig. 33. Trigonometric solution of vector diagram of Fig. 32.

compute the value of jE_{X_L}. First we list our values as:

Given: $E_{TX_L} = 17V$

$I_L = 1A$

$R_L = 12$ ohms

$$E_{R_L} = I_L \times R_L$$

$$= 1 \times 12$$

$$= 12V$$

Find: E_{X_L}

Now, sketching the vector diagram with 17 as the hypotenuse and 12 as the base of a right triangle, as shown in Fig. 33A, allows us to use our trigonometry. First we want to find the phase angle θ_L. To do this, we use the cosine function which is:

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{E_{R_L}}{E_{TX_L}} = \frac{12}{17} = .7057$$

In checking our Table of Functions we find that there is no cosine given as exactly .7057. Although we could interpolate to find the exact value for θ, our cosine of .7057 is so close to .7071, which is the cosine of 45°, that we can use the approximate value of 45° as the angle θ_L.

Now that we know that θ_L is equal to 45°, we can proceed to find the value of jE_{X_L} (the opposite side) by using one of the other functions. For example,

$$\tan\theta_L = \frac{\text{opp}}{\text{adj}}$$

and:

$$opp = adj \times \tan \theta_L$$

Substituting:

$$jE_{X_L} = E_{R_L} \times \tan 45^\circ$$

$$jE_{X_L} = 12 \times 1.0000 = 12$$

Thus, jE_{X_L}, which is the reactive component of the total coil voltage, is also equal to 12 volts. Notice that we could also have used the sine since

$$\sin \theta_L = \frac{opp}{hyp}$$

and therefore

$$opp = \sin \theta_L \times hyp$$

or

$$jE_{X_L} = \sin 45^\circ \times E_{TX_L}$$

$$= .7071 \times 17 = 12 \text{ (approx.)}$$

The function used is a matter of personal choice or convenience as long as the necessary values are known.

Notice what we have done. We took a voltage drop across a coil and broke it up into its resistive and reactive components. We could not measure these components because they do not exist separately; they exist together as a total. The fact that this vector sum is not exactly in phase with the current tells us that it must have both of the components. In order to ***compute*** the circuit values accurately, we must have the values of the components, not their vector sum. Since the resistance of a coil, the current through the coil, and the total voltage across the coil are all easy to measure with instruments, we have used these values, together with trigonometry, to obtain the two components of total coil voltage.

If we have the inductance or the inductive reactance of the coil, we can also use them to compute the value of the components. However, in practical circuits the value of the inductance or inductive reactance is often not known. Since these values can only be measured with instruments, which are usually not available in the average shop, we have shown you a method to use to solve the problem.

Since we now have all the values required to find E_T as shown in Fig. 32, we can continue with our computation as shown in Fig. 33B. We have already established the fact that:

$$E_T = E_R + E_{TX_L} - jE_{X_C}$$

By substitution:

$$E_T = E_R + (E_{R_L} + jE_{X_L}) - jE_{X_C}$$

and:

$$E_T = 92 + 12 + j12 - j72$$

Then:

$$E_T = 104 - j60$$

and we can draw the vector diagram as shown in Fig. 33B and compute the value of E_T and θ by using trigonometry. First:

$$\tan \theta = \frac{opp}{adj} = \frac{60}{104} = .5769$$

and

$$\theta = 30^\circ \text{ (approx.)}$$

Then:

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

and

$$\text{hyp} = \frac{\text{opp}}{\sin\theta} = \frac{60}{.5} = 120\text{V}$$

This is the value of E_T that we obtained by measurement in Fig. 30 and is therefore correct.

We have used a very low Q coil to demonstrate the importance of considering the resistive component of voltage in a coil.

$$Q = \frac{12}{12} = 1$$

Although such a low Q coil will not be found often in practical circuits, it is still important to know how to handle these effects when they do occur. You are probably wondering about the resistance of capacitors and if it too has to be considered. It does not. Although all capacitors do have some resistance, it is never large enough to be noticed in a practical capacitor.

SERIES-PARALLEL CIRCUITS

Now let's look at a more complicated circuit, such as the one shown in Fig. 34. This is a series-parallel circuit with the various values given. We are asked to find the total impedance Z_T, the total current I_T, and the final phase angle θ_T.

First, we want to find the impedance of the three parallel branches, a, b, and c, which we can do as follows:

$$Z_a = R + jX_L = 25 + j17.5 \text{ ohms}$$

$$\tan\theta_a = \frac{\text{opp}}{\text{adj}} = \frac{X_{La}}{R_a} = \frac{17.5}{25} = .700$$

$$\theta_a = 35^\circ$$

Then:

$$\sin\theta_a = \frac{\text{opp}}{\text{hyp}} = \frac{X_{La}}{Z_a}$$

or

$$Z_a = \frac{X_{La}}{\sin\theta_a} = \frac{17.5}{\sin 35^\circ}$$

$$= \frac{17.5}{.574} = 30.5 \text{ ohms}$$

Thus:

$$Z_a = 30.5\underline{/35^\circ} \text{ ohms}$$

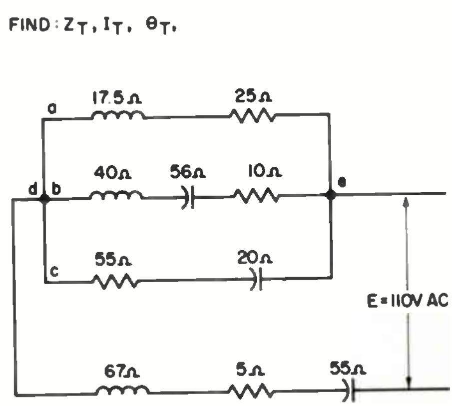


Fig. 34. Series-parallel ac circuit.

Next:

$$Z_b = R + jX_L - jX_c$$

$$= 10 + j40 - j56$$

$$= 10 - j16$$

$$\tan \theta_b = \frac{\text{opp}}{\text{adj}} = \frac{X_b}{R_b}$$

$$= \frac{-16}{10} = -1.6$$

$$\theta_b = -58^\circ$$

Then:

$$\sin \theta_b = \frac{\text{opp}}{\text{hyp}} = \frac{X_b}{Z_b}$$

or

$$Z_b = \frac{X_b}{\sin \theta_b} = \frac{-16}{\sin -58^\circ}$$

$$= \frac{-16}{-.848} = 18.9$$

Thus:

$$Z_b = 18.9\underline{/-58^\circ} \text{ ohms}$$

Next:

$$Z_c = R - jX_c = 55 - j20$$

$$\tan \theta_c = \frac{\text{opp}}{\text{adj}} = \frac{-20}{55} = -.364$$

$$\theta_c = -20^\circ$$

Then:

$$\sin \theta_c = \frac{\text{opp}}{\text{hyp}} = \frac{X_c}{Z_c}$$

or

$$Z_c = \frac{X_c}{\sin \theta_c} = \frac{-20}{\sin -20}$$

$$= \frac{-20}{-.342} = 58.5$$

Thus:

$$Z_c = 58.5\underline{/-20^\circ} \text{ ohms}$$

Now assume a voltage between d and e and find I for each branch as follows:

$$I_a = \frac{E_{de}}{Z_a} = \frac{100\underline{/0^\circ}}{30.5\underline{/35^\circ}} = 3.28\underline{/-35^\circ} \text{ amps}$$

$$I_b = \frac{E_{de}}{Z_b} = \frac{100\underline{/0^\circ}}{18.9\underline{/-58^\circ}} = 5.3\underline{/58^\circ} \text{ amps}$$

$$I_c = \frac{E_{de}}{Z_c} = \frac{100\underline{/0^\circ}}{58.5\underline{/-20^\circ}} = 1.71\underline{/20^\circ} \text{ amps}$$

Now convert branch currents I_a, I_b, and I_c from polar to rectangular coordinates for easy addition as follows:

$$\begin{aligned} I_a &= I_a (\cos \theta_a + j \sin \theta_a) \\ &= 3.28 (\cos -35^\circ + j \sin - 35^\circ) \\ &= 3.28 (.8192 - j.5736) \\ &= 2.69 - j1.88 \text{ amps} \end{aligned}$$

Since -35° lies in the fourth quadrant, cos -35° is positive and sin -35° is negative. This explains why the resistive term is positive and the reactive term is negative.)

$$
\begin{aligned}
I_b &= I_b(\cos\theta_b + j\sin\theta_b) \\
&= 5.3(\cos 58^\circ + j\sin 58^\circ) \\
&= 5.3(.5299 + j.8480) \\
&= 2.81 + j4.5 \text{ amps}
\end{aligned}
$$

$$
\begin{aligned}
I_c &= I_c(\cos\theta_c + j\sin\theta_c) \\
&= 1.71(\cos 20^\circ + j\sin 20^\circ) \\
&= 1.71(.9397 + j.3420) \\
&= 1.61 + j.585 \text{ amps}
\end{aligned}
$$

Now,

$$I_{de} = I_a + I_b + I_c$$

and:

$$
\begin{aligned}
I_a &= 2.69 - j1.88 \\
I_b &= 2.81 + j4.5 \\
I_c &= 1.61 + j.585 \\
\hline
I_{de} &= 7.11 + j3.205
\end{aligned}
$$

thus,

Now

$$\tan\theta_{de} = \frac{\text{opp}}{\text{adj}} = \frac{I_{de_x}}{I_{de_r}}$$

$$= \frac{3.2}{7.11} = .45$$

$$\theta_{de} = 24.2^\circ$$

And I_{de} in polar form:

$$\sin\theta_{de} = \frac{\text{opp}}{\text{hyp}} = \frac{I_{de_x}}{I_{de}} \text{ or } I_{de} = \frac{I_{de_x}}{\sin\theta_{de}}$$

Then:

$$I_{de} = \frac{I_{de_x}}{\sin\theta_{de}} = \frac{3.2}{\sin 24.2^\circ}$$

$$= \frac{3.2}{.4099} = 7.8$$

Thus:

$$I_{de} = 7.8\underline{/24.2^\circ} \text{ amps}$$

And:

$$Z_{de} = \frac{E_{de}}{I_{de}} = \frac{100\underline{/0^\circ}}{7.8\underline{/24.2^\circ}}$$

$$= 12.8\underline{/-24.2^\circ} \text{ ohms}$$

Converting Z_{de} to rectangular coordinates:

$$
\begin{aligned}
Z_{de} &= Z_{de}(\cos\theta_{de} + j\sin\theta_{de}) \\
&= 12.8(\cos -24.2 + j\sin -24.2^\circ) \\
&= 12.8(.9121 - j.4099) \\
&= 11.7 - j5.25 \text{ ohms}
\end{aligned}
$$

Now combining:

$$
\begin{aligned}
Z_T &= Z_{de} + j67 - j55 + 5 \\
Z_T &= 11.7 + 5 - j5.25 + j67 - j55 \\
Z_T &= 16.7 + j6.75 \text{ ohms}
\end{aligned}
$$

Then:

$$\tan\theta_T = \frac{\text{opp}}{\text{adj}} = \frac{6.75}{16.7} = .404$$

$$\theta_T = 22^\circ$$

Converting Z_T to polar form:

$$\sin\theta_T = \frac{\text{opp}}{\text{hyp}} = \frac{X_T}{Z_T}$$

or

$$Z_T = \frac{X_T}{\sin\theta_T} = \frac{6.75}{\sin 22^\circ} = \frac{6.75}{.3746} = 18$$

$$Z_T = 18\underline{/22^\circ} \text{ ohms}$$

Then:

$$I_T = \frac{E_T}{Z_T} = \frac{110\underline{/0^\circ}}{18\underline{/22^\circ}}$$

$$= 6.11\underline{/-22^\circ} \text{ amps}$$

With this as a typical example of using trigonometry in ac circuits, try some of the problems you solved using the j-operator and the Pythagorean Theorem in your previous lessons.

Power in AC Circuits. Up until now we have not considered power in ac circuits because we have not had a simple method of finding it. In a purely resistive circuit, where the current and voltage are in phase, the power is equal to the voltage times the current as it is in a dc circuit. In a purely reactive circuit where there is no resistance whatsoever, power is alternately stored up by the reactive elements and then returned to the line. Such a circuit can exist only in theory, of course, because practical circuits always have some resistance. However, oscillatory tank circuits, which you have studied, do come fairly close to being resistance-free and, consequently, small properly timed surges of current can keep them going indefinitely.

Thus, we can say that the resistive elements of a circuit consume the only power expended. Let's see what this means in terms of the circuit we have just studied. If we construct the resultant vector diagram of the circuit from θ_T = 22°, $Z_T = 18\Omega$, we have a vector $Z_T = 18\underline{/22^\circ}$ as shown in Fig. 35. The reactive component of this impedance is jX_T as shown, and the resistive component is equal to R_T as shown. The resistive component is equal to 16.7 ohms, while the reactive component is 6.75 ohms, as we discovered while solving for Z_T.

Now, if the resistive component is all that consumes power, the power must be equal to $E_R \times I$ since only the resistive component of voltage forces current through the resistance to consume power. However, in the circuit we have just solved, we are not given the value of E_R, nor did we find it in any of our computations. However, if we lay out a vector representing the conditions of our circuit, we have a vector of the total voltage E_T of 110V leading the current vector, which we have taken as our reference vector, by 22°. We can find E_R now by using:

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{E_R}{E_T}$$

and

$$E_R = E_T \cos\theta$$

Now, if $P = E_R \times I$ equals the power expended and $E_R = E_T \cos\theta$, by substitution:

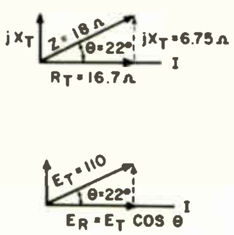
$$P = (E_T \cos\theta)I$$

or

$$P = EI \cos\theta$$

Then, substituting values:

$$\begin{aligned} P &= 110 \times 6.11 \times \cos 22^\circ \\ &= 110 \times 6.11 \times .9272 \\ &= 623 \text{ watts} \end{aligned}$$



$P = EI \cos\theta$

$P_a = EI$

$P.F. = \cos\theta$

Fig. 35. Vector representation of power factor.

This is the power actually consumed by the circuit.

Now, in ac circuits we have another value of power which is called the apparent power. This is simply the product of E_T and I_T without considering their relative instantaneous values at any particular moment. Thus, in the circuit we have just solved, the apparent power is simply 110 × 6.11 or 672.1W. The apparent power in a circuit is designated as P_a to separate it from the true power P which does take into consideration the relative instantaneous values of E and I through multiplication by cos θ.

Now, we have another situation in ac power that must be considered. This is the power factor PF which is the ratio of the true power P to the apparent power P_a. Mathematically, this is stated:

$$PF = \frac{P}{P_a}$$

then

$$PF = \frac{EI \cos\theta}{EI} = \cos\theta$$

and the power factor can be found from PF = cos θ. In the circuit we have just computed, PF = .927 lagging because the current lags the voltage by 22°. Power factor is also expressed as a percentage, 92.7%.

There are other formulas for power factor in ac circuits, but these are the most common. Since the formula $P = I^2 R$ uses only resistance and current, it will also give us the true power. Here we are not multiplying the effective values of current by an effective value of voltage without considering their relative instantaneous values. We are simply squaring the effective value of current and multiplying it by the total resistive component of Z. Thus, $P = I^2R$ gives us: $6.11^2 \times 16.7 = 623$ watts which is true power.

We can also write another formula for power factor. If

$$PF = \frac{P}{P_a}$$

and

$$P = I^2 R$$

and

$$P_a = EI$$

Then:

$$PF = \frac{I^2 R}{EI} = \frac{IR}{E}$$

But

$$E = IZ$$

so

$$PF = \frac{IR}{IZ} = \frac{R}{Z}$$

Therefore

$$PF = \frac{R}{Z}$$

However, this is just another way of saying cos θ, because

$$\cos\theta = \frac{adj}{hyp} = \frac{R}{Z}$$

It also should be noted that true power P equals PF × P_a because

$$PF = \frac{R}{Z} \text{ or } \cos\theta$$

and

$$P_a = EI$$

and our first formula for power was $P = EI \cos \theta$. Any of the relationships may be used, and if you remember that:

$$P_a = EI$$

and

$$P = EI \times PF$$

$$PF = \cos \theta$$

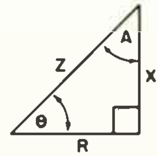
you can use algebra and trigonometry to work out the other formulas.

Although you will not become an expert in the science of trigonometry from what you have learned in this lesson, you have covered most of the basic fundamentals and their application in electronics. You will be able to handle nearly every problem when you understand the elementary principles of trigonometry. Like any other math, trig requires a lot of practice to become familiar with it. You should practice solving the right triangles of circuits in other lessons to obtain this practice.

Now answer the following Self-Test Questions which are an overall review of trigonometry. They will give you further practice in solving ac circuit problems.

SELF-TEST QUESTIONS

28. An angle is equal to 3 radians. What does it measure in degrees?
29. How many degrees are there in 1.7 radians?
30. Convert 15.6° to radians.
31. In the right triangle shown below side X is equal to 25 ohms and side R is equal to 76.0 ohms. What does side Z equal?



32. In the triangle shown above, if side Z equals 45 and $\angle \theta$ equals 45°, what does side X equal?
33. What is the sine of 345°?
34. In the triangle shown above, find $\angle \theta$ if $\sin \angle A = .4415$.
35. If the voltage of an ac circuit is leading the current by 35.8°, what is the power factor of the circuit?
36. If the total voltage applied to the circuit discussed in Question 35 is 220 volts and the current through the circuit is 1.79 amps, what is the true power consumed by the circuit?
37. When the impedance of a circuit is described as: $Z = 5\underline{/36.9^\circ}\Omega$ in polar coordinates, how would you express it in rectangular coordinates?
38. A choke coil draws 2 amps of current when it is connected across 110V dc. When connected to 110V, 60 cycles ac, the current drawn is .25 amp. What is the resistance and the inductive reactance of the coil?
39. The following 60-Hertz impedances are connected in series:
    $Z_1 = 3 - j6\Omega$
    $Z_2 = 10 + j19\Omega$
    $Z_3 = 2 - j7\Omega$
    $Z_4 = 5 + j14\Omega$
    What is the impedance of the circuit in polar coordinates?
40. In the circuit described in Question 39, what value of capacitance would we have to add to the circuit to make

the power factor 70.7% leading?

41. A supply of 220 volts ac is applied to an impedance of $Z_a = 55\angle 40^\circ \Omega$ in parallel with an impedance of $Z_b = 71\angle -36^\circ \Omega$. What is the power consumed in Z_b?

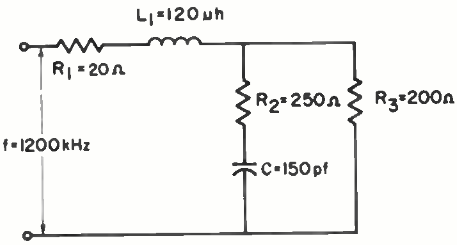
42. In the circuit shown, what is the total impedance?

43. In the circuit shown below, find the total impedance of the circuit.

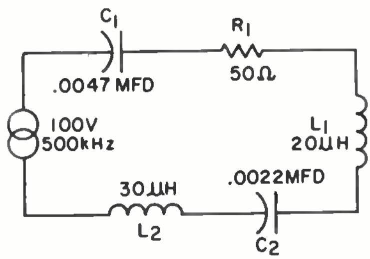
44. Find the total current in the circuit shown below.

45. Find the power dissipated in the circuit shown below.

46. Find the power dissipated by R_1 in the circuit shown below.



Circuit for Question 42.



Circuit for Questions 43, 44, 45, and 46.

Graphs

A graph is a picture that shows the effect of changes in one variable on a second variable. Graphs are very common in electronics literature since they provide a simple means of describing circuit operations, illustrating equations and formulas, showing relationships when no formulas exist, and displaying results of experiments.

Graphs are not new to you; you studied them in grade school. They are commonly used in newspapers to show economic trends. Although the same things could be shown with columns of figures, the line on a graph puts the idea over much better. It is hard to visualize trends or patterns from a column of figures, but a line on a graph lays the pattern out in front of you in a way that is easy to grasp.

USING GRAPHS

The most common type of graph is drawn on paper ruled with uniformly spaced horizontal and vertical lines. This type of paper is known as ***rectilinear*** or ***cross section*** or just plain graph paper. Cross-sectional paper is available with many different line spacings, but 4, 5, 10, and 20 lines per inch are the most common.

The data for plotting graphs may be obtained by measurements of both quantities. Or the data may be obtained by repeated solutions of a formula. A graph with two plots obtained in the latter manner is shown in Fig. 36. This graph shows the voltage across a 75-ohm resistor and the power dissipated in it for different values of current.

Fig. 36 was plotted by assuming different values for the current and calculating the corresponding voltage drops and power dissipations using the formulas shown. Since the calculated values of voltage and power depend on the assumed values of the current, voltage and power are called the ***dependent variables***. The current could be given any desired value and changed at will, so current is called the ***independent variable***. Following custom, the dependent variable is scaled along the vertical axis, and the independent variable is scaled along the horizontal axis.

Several important things about graphs are illustrated here. Both scales are labeled to show the quantity they represent, and the units in which that quantity is measured. The values assigned to each division of the scales are marked along them. Note that the two scales are not equally divided; one division of the horizontal scale is equal to .05 units and one division on the vertical scale equals 5 units. Each plot is labeled with the

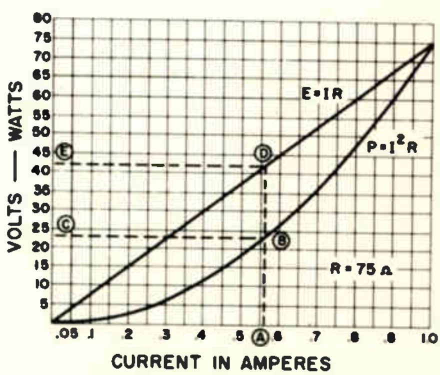


Fig. 36. Change in voltage across, and power in a 75-ohm resistor as current changes.

formula used to obtain the data. The formulas also serve as titles, telling what the graph shows. The value of the resistance used in the calculation completes the labeling. All the information needed to use or identify this graph appears upon it. Nothing is left to memory or imagination. Without this information no graph is complete.

Graphs like Fig. 36 are frequently made up to avoid repeated computations. For example, suppose you are checking the effect of changing tube voltages on the output of an amplifier. You are measuring the output with an ammeter in series with the 75-ohm load of the amplifier. In order to avoid making a very large number of calculations of voltage and power, you have constructed this graph. Now you can obtain the voltage and power for each value of current without having to work out each solution with pencil and paper.

The dashed lines on Fig. 36 show how the voltage and power for a current of .56 ampere would be read from the graph. Starting at .56 (point A) on the horizontal scale, trace upward to the intersection with the power curve (point B). Then trace over to the vertical scale and read 23 watts at point C. The .56 line intersects the voltage line at D. Tracing over to the vertical scale from D gives 42 volts at point E. The same graph could be used to determine the current for a specific power. You would simply reverse the procedure and start at the required power on the vertical scale. Then trace over to the power curve and down to the horizontal scale to read current.

It actually takes longer to tell how to read values from a graph than it takes to do it. The dashed lines which were drawn on Fig. 36 are not necessary in practice. They were used here only to demonstrate the procedure. With a little practice you will find that you can do the reading right at the curve without tracing to the scales. Try to determine the voltage drop and the power dissipation for the following currents: .72, .23, .07, and .48. What current is necessary for 60 watts, 36 watts, 7 watts, and 53 watts? Remember when reading the scale of a graph that, since an estimate is involved, all the rules for significant figures apply. You can check your readings by using the formulas to calculate the values.

Slope. One look at a graph can tell you a great deal about the way the dependent variable changes with changes in the independent variable. You have only to glance at Fig. 36 to know that voltage drop and power dissipation do not change in the same way with changes in current. The graph of voltage against current is a straight line. The increase in voltage for a given increase in current is the same at every point on the line. An increase of .1 ampere always produces an increase of 7.5 volts; an increase of .2 ampere always produces an increase of 15 volts. These relationships hold true no matter what value the current has at the start.

A special name is given to the rate at which the dependent variable changes with changes in the independent variable. This rate of change is called the slope. The slope is determined by dividing the span of the dependent variable over a section of the line by the span of the independent variable over the same section. Two examples of slope calculation are shown in Fig. 37. The two lines on this graph are plots of voltage against current for two different values of resistance. The slope of the line, R = 75 ohms, was calculated for the section between point A (.53, 40) and point B (.80, 60). The span of current is equal to the scale

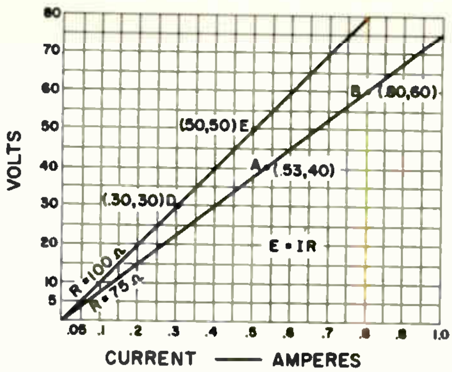


Fig. 37. Determining the slope of a line.

reading of B − A. The value of B (.80) minus the value of A (.53) is equal to .27 ampere. On the vertical scale, the reading of B is 60. We subtract the reading of A on the vertical scale (40) from B (60) to obtain the voltage span, which is 20. Dividing the voltage span by the current span gives 20 ÷ .27 = 75. The slope of the R = 75 ohms line is 75. The slope of the R = 100 ohms line is calculated between point D (.30, 30) and point E (.50, 50). The slope is (50 − 30) ÷ (.50 − .30) which works out to be 100.

You have undoubtedly already noticed that in both the examples of slope calculation the slope was numerically equal to the value of the resistance used in the formula which was plotted. Whenever the dependent variable is equal to the independent variable multiplied by a constant, the slope will always be equal to the constant. It is also true that the graph of the relationship will always be a straight line on rectilinear graph paper. Because the graph is a straight line, the relationship is said to be linear and the formula which expresses the relationship is called a linear equation.

The slope of a straight line is constant; that is, the slope is the same for all parts of the line. This is not true for all graphs. Unless the graph is a straight line on rectilinear paper, the slope will be different at different parts of the curve. In other words, the value of the slope depends on the value of the independent variable. Because the slope is continually changing, we must use a slightly different method to determine it. Fig. 38 shows how the slope of a curved line on a graph is obtained. First, it is necessary to draw a line tangent to the curve at the point at which the value of the slope is desired. (A tangent line is a line which touches the curve at only one point.) Three such tangent lines are shown on the curve. One is tangent at the point (.20, 3.0), another at (.40, 12), and the third at (.80, 48). The slope of the tangent line is determined by the same way as the slope of a straight line graph. The slope of the curve at the point where the tangent line touches it is the same as the slope of the line.

It is obvious that each of the three tangent lines has a different slope. It is also apparent that the slope is least when the current is least. As the current increases, the slope of the curve becomes greater. From a practical viewpoint this

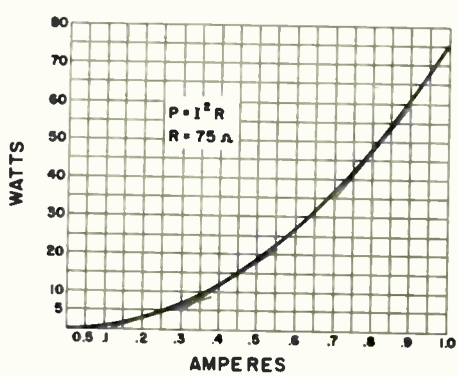


Fig. 38. Determining the slope of a nonlinear graph.

means that when the current is low, a small change in current results in only a small change in power, but when the current is large, a small change in current results in a large change in power. This type of curve will always result when the dependent variable is directly proportional to the square of the independent variable. Because the graph of the formula is not a straight line on rectilinear paper, the formula is said to be a nonlinear equation.

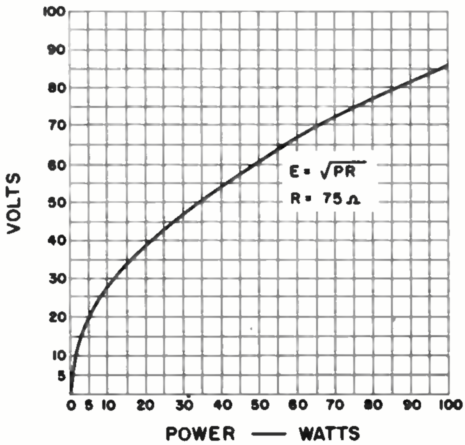


Fig. 39. Curve resulting when dependent variable is proportional to square root of independent variable.

Another common nonlinear curve results when the dependent variable is proportional to the square root of the independent variable. The curve of the formula $E = \sqrt{PR}$ for determining the voltage drop across a 75-ohm resistor for a given power dissipation is shown in Fig. 39. This curve has a high initial slope which decreases as the value of the independent variable becomes larger.

A third common nonlinear curve is shown in Fig. 40. This is the plot of current against resistance for a constant voltage drop.

The tangent to the curve at one point is also drawn in. Two points, A (2, 30) and B (8, 8) are marked on the tangent line and are used to determine the slope. In determining the span of the dependent variable, the value of the dependent variable at the point nearer the Y-Y′ axis is always subtracted from the value of the dependent variable at the farther point. Here we have

$$(8 - 30) \div (8 - 2) = \frac{-22}{6} = -3.66$$

This curve differs from the others you have studied in that it has a negative slope. The practical meaning of a negative slope is simply that the dependent variable becomes smaller as the independent variable becomes larger. The slope of this curve is greatest for small values of the independent variable, and least for high values. This curve shape and negative slope are characteristic of the graph of any equation in which the dependent variable is inversely proportional to the independent variable.

Each of the four types of formulas you are likely to use has its own characteristic graph curve. These curves show the way the dependent variable changes when the independent variable changes. The slope

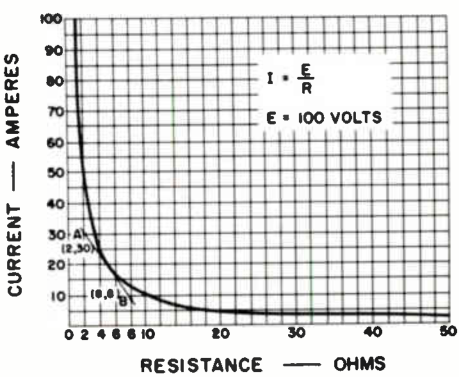


Fig. 40. Curve of a reciprocal relationship.

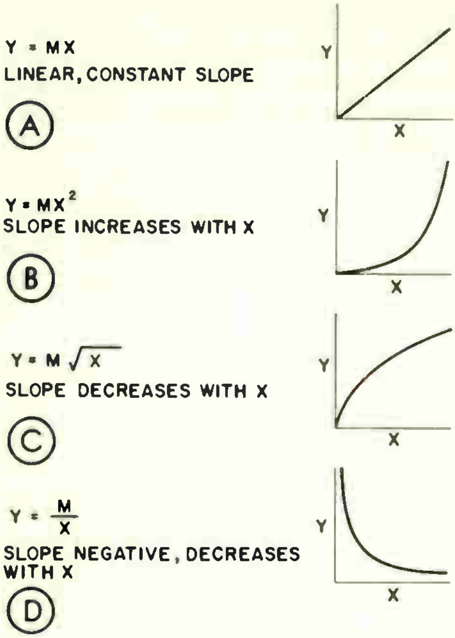


Fig. 41. Four common equations and their curves.

of the curves gives a numerical value to the rate of change. Fig. 41 summarizes the four types of equations and their curves. In the equations, X stands for the independent variable, Y for the dependent variable, and M for the factor of proportionality. Remember the general characteristics of the curve of each equation; they are a big help in visualizing the relationship expressed in a formula.

TYPES OF GRAPH PAPER

Common rectilinear graph paper is best for showing the relationship between variables. However, it is not always the easiest type to use. An accurate graph of some formulas can be obtained only by plotting a large number of points. It is difficult to read values from a curve when the curve is nearly parallel to either the horizontal or vertical grid lines. The uncertainty of readings near the low end of either scale is much greater than the uncertainty near the high end.

The plotting and reading of graphs can be made much easier by using graph paper which has special scales. There are many special types of graph paper for use in science, engineering and business. Three of these types are common in electronics.

Logarithmic. Most of the disadvantages of rectilinear paper can be overcome by plotting the logarithms of the variables instead of the variables themselves. To avoid having to look up the logarithms of every number, a special type of graph paper is used. The scales on this paper are laid out so that the distance of each number from the lower left corner is proportional to the logarithm of the number. Fig. 42 shows rectilinear and logarithmic scales side by side for comparison. The numbers on the rectilinear scale are ten times the logarithm of the numbers opposite them on the logarithmic scale. Notice in particular that there is no "0" on the logarithmic scale. There is no logarithm for zero.

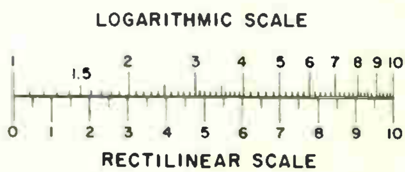


Fig. 42. Comparison of linear and logarithmic scales.

In order to extend the scale from 10 to 100, it is necessary only to repeat the 1 to 10 scale to the right of the 10. For numbers less than 1, it would be necessary to add additional 1 to 10 scales to the left of 1. Each complete 1 to 10 scale along an axis is called a cycle. Logarithmic graph paper is described by the

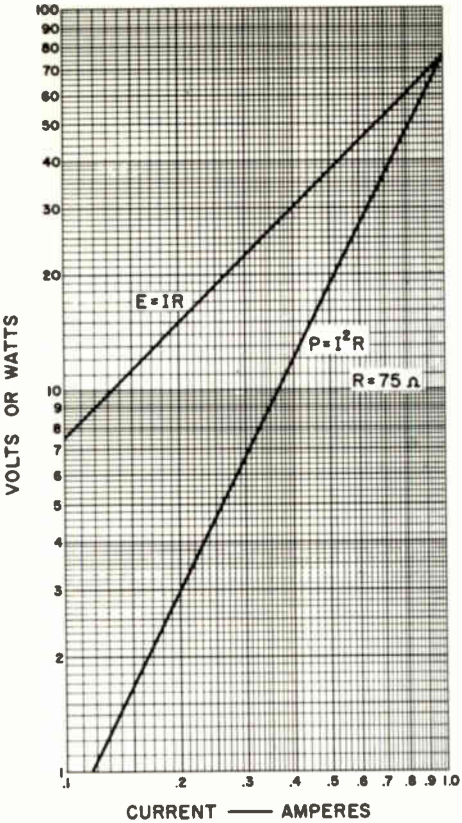


Fig. 43. Power against current and voltage against current plotted on logarithmic paper.

number of cycles along the horizontal and vertical scales. The chart on which Fig. 43 is drawn is called a "1 X 2 cycle."

One big advantage of logarithmic graph paper is that the uncertainty in reading numbers from the scale is the same at both ends of a cycle. The low end of the cycle can be read to three significant figures; the upper end of the cycle can be read to two significant figures. In both cases the uncertainty is about 1 part in 100. This is very important when a graph is used as an aid to computation.

Another advantage of logarithmic scales is shown in Fig. 43. The two lines on this graph are plots of the same relationships that gave one straight line and one curved line in Fig. 36. Both the voltage and the power plots are straight lines on this type of paper. In fact, all four of the typical equations in Fig. 41 are straight lines when plotted on logarithmic paper. This greatly simplifies the work of plotting. No more than three points need be calculated. (Actually two points are enough; the third is just a check.)

Although it is much easier to plot and read values from graphs on logarithmic paper, these plots have two big disadvantages. They do not show the exact manner in which changes in one variable affect the value of the other variable. Furthermore, you cannot determine the slope except in the case of the linear equation ($y = mx$).

A second disadvantage is the fact that there is no zero on the scales. On the 1 X 2 cycle paper used here, currents below .1 ampere, voltage drops below 7.5 volts, and powers below 1 watt do not show. If these lines had been plotted on 2 X 3 cycle paper, the lowest values would have been .01 ampere, .75 volt, and .1 watt. However, no matter how many cycles were used, zero would not appear.

Semilogarithmic Paper. Another type of special graph paper has a logarithmic scale on one axis and a linear scale on the other. The logarithmic scale may have from 1 to 5 cycles; the linear scale may have any convenient number of lines. The most common are 10 or 20 lines to the inch. One use for this type of paper is to obtain a straight line plot of an equation in which one variable is proportional to the logarithm of the other. Another use is where a large range of numbers must be covered on one scale. A linear scale would not show details on the low end, whereas a logarithmic scale would open up the

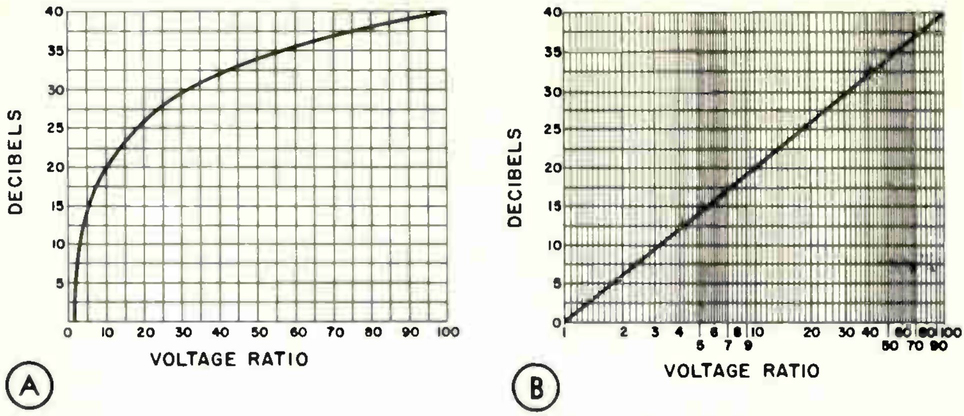


Fig. 44. Plots of voltage against decibels: (A) rectilinear; (B) semilogarithmic.

low end and make all parts equally readable.

Two plots of a logarithmic relationship between variables are shown in Fig. 44. Fig. 44A shows the plot of voltage ratio against decibels on rectilinear paper. At least 20 points must be calculated and marked to get a smooth plot. Even then the graph is very difficult to read below 15 on the voltage ratio scale. Fig. 44B shows the same relationship plotted on semilogarithmic paper. It can be drawn with only three calculations, and can be read with equal ease on all parts of the line.

Polar Graphs. A third type of special graph paper is laid out in polar coordinates. Points are located by means of radial lines marked in degrees and a series of circles with common centers which show the distance along the radials. This paper is used when you want to show the radiation patterns of antennas, loudspeakers, light sources, and other forms of energy transmitters. A polar plot is used for this type of graph since it gives a pattern of direction in space that is immediately apparent. Fig. 45 is an example of a graph plotted on polar coordinate paper. The graph shows the radiation pattern of an ideal quarter-wavelength antenna in free space. The distance of the points along the radials is proportional to the field strength in percent of the field strength in the direction of maximum radiation.

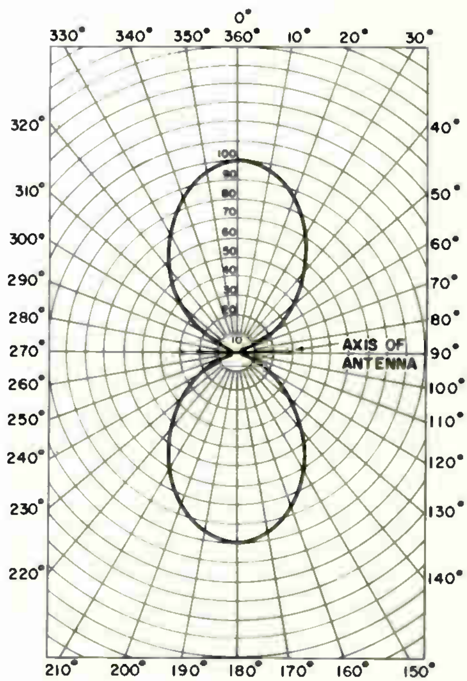


Fig. 45. Polar plot of antenna field strength.

SELF-TEST QUESTIONS

47. What is meant by the "slope" of a line?
48. How can the slope of a curved line be determined?
49. What is the difference between logarithmic graph paper and semilogarithmic graph paper?
50. What are some uses for polar graph paper in electronics?

Answers to Self-Test Questions

1. (a) 1.1×10^6
   (b) 7.2×10^3
   (c) 1.5×10^{-4}
   (d) 6.4×10^{-1}
2. (a) .000 000 000 326
   (b) 1,220,000
   (c) .00077
   (d) 9
3. 2.27×10^{-3}
4. 8.03×10^{-1}
5. 17.04

$$\begin{array}{r} 7.92 \\ 3.01 \\ 6.10 \\ 0.01 \\ \hline 17.04 \end{array}$$

6. Trigonometry is the study of the mathematical relationships that exist between the sides and angles of triangles.
7. 780 minutes; 46,800 seconds.
8. 360° contains 6.28 or 2π radians.
9. An acute angle is any angle that is less than 90°. An obtuse angle is an angle which is larger than 90°.
10. 58°.
11. The ***sine*** θ is the ratio of the side opposite the angle to the hypotenuse:

$$\frac{\text{opp}}{\text{hyp}}$$

The ***cosine*** θ is the ratio of the side adjacent to the angle to the hypotenuse:

$$\frac{\text{adj}}{\text{hyp}}$$

The ***tangent*** θ is the ratio of the opposide side to the adjacent side:

$$\frac{\text{opp}}{\text{adj}}$$

The ***cotangent*** θ is the ratio of the adjacent side to the opposite side:

$$\frac{\text{adj}}{\text{opp}}$$

The ***secant*** θ is the ratio of the hypotenuse to the adjacent side:

$$\frac{\text{hyp}}{\text{adj}}$$

The ***cosecant*** θ is the ratio of the hypotenuse to the opposite side:

$$\frac{\text{hyp}}{\text{opp}}$$

12. (a) .2250
    (b) .6947
    (c) .3443
    (d) .8387

13. (a) .0558
(b) 1.2619
(c) .9426
(d) .2890
14. (a) 21°
(b) 63°
(c) 51°
(d) 56°
15. (a) 54.5°
(b) 31.3°
(c) 9.1°
(d) 48.2°
16. (15 – j15) ohms;
21.2 $\underline{/-45^\circ}$ ohms

17. (40 + j30) ohms;
50 $\underline{/36.9^\circ}$ ohms

$$Z = R_1 + R_2 + jXL_1 + jXL_2 - jXC_1 - jXC_2$$

$$Z = 40 + j55 - j25$$

$$Z = 40 + j30$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{X}{R}$$

$$\tan\theta = \frac{30}{40} = .75$$

$$\theta = 36.9^\circ$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\text{hyp} = \frac{\text{opp}}{\sin\theta} = \frac{30}{.5904} = 50 \text{ ohms}$$

18. (a) X is positive in the first and fourth quadrants. These are the quadrants to the right of the X axis.
(b) Y is positive in the first and second quadrants. These are the quadrants above the X axis.
(c) X and Y are both negative in the third quadrant. This is the quadrant beneath the X axis and to the left of the Y axis.

19. (a) The value of sin 150° is .5

$$\sin 150^\circ = \sin(180^\circ - 150^\circ) = \sin 30^\circ = .5$$

(b) The value of cos 150° is –.866

$$\cos 150^\circ = -\cos(180^\circ - 150^\circ) = -\cos 30^\circ = -.866$$

20. (a) The value of tan 135° is –1

$$\tan 135^\circ = -\tan(180^\circ - 135^\circ) = \tan 45^\circ = -1$$

(b) The value of cot 225° is 1

$$\cot 225^\circ = \cot(225^\circ - 180^\circ) = \cot 45^\circ = 1$$

The tangent is negative because it falls in the second quadrant, the cotangent is positive because it falls in the third quadrant.

21. (a) The value of sin –60° is –.866

$$\sin -60^\circ = -\sin 60^\circ$$
$$\sin 60^\circ = .866$$

Therefore

$$\sin -60^\circ = -.866$$

(b) The value of cos –60° is .5

$$\cos -60^\circ = \cos 60^\circ$$
$$\cos 60^\circ = .5$$

The cosine is positive in the fourth quadrant.

22. (a) (4.85; 3.53)

$X = 6 \cos 36^\circ = 4.85$
$Y = 6 \sin 36^\circ = 3.53$

(b) (3.21; –3.83)

$X = 5 \cos - 50^\circ = 3.21$
$Y = 5 \sin - 50^\circ = -3.83$

23. (a) $12.1 \underline{/-65.5^\circ}$ or $12.1 \underline{/294.5^\circ}$

$$\tan \theta = \frac{-11}{5}$$

Therefore

$$\theta = -65.5^\circ$$

$$H = \frac{11}{\sin 65.5^\circ} = 12.1$$

(b) $9.23 \underline{/229.4^\circ}$

$$\tan \theta = \frac{-7}{-6}$$

Therefore

$$\theta = (49.4^\circ + 180^\circ) = 229.4^\circ$$

$$H = \frac{7}{\sin 49.4^\circ} = 9.23$$

24. $18.6 \underline{/+53.7}$ ohms

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{+jX}{R} = \frac{15}{11} = 1.3636$$

$$\theta = +53.7^\circ$$

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\text{hyp} = \frac{\text{opp}}{\sin \theta}$$

$$Z = \frac{+jX}{\sin \theta}$$

$$Z = \frac{15}{.8059}$$

$$= 18.6 \text{ ohms}$$

25. $8.1 \underline{/-29.7^\circ}$ ohms

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{-jX}{R} = \frac{-4}{7} = -.5714$$

$$\theta = -29.7^\circ$$

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\text{hyp} = \frac{\text{opp}}{\sin \theta}$$

$$Z = \frac{-jX}{\sin \theta}$$

$$Z = \frac{4}{.4954}$$

$$= 8.1 \text{ ohms}$$

26. (5.8 – j6.9) ohms

$$\begin{aligned} Z &= Z(\cos \theta \pm j \sin \theta) \\ &= 9(\cos 50^\circ - j \sin 50^\circ) \\ &= 9(.6428 - j.7660) \\ &= 5.8 - j6.9 \end{aligned}$$

27. (10.2 + j6.4) ohms

$$Z = Z(\cos\theta \pm j\sin\theta)$$
$$= 12(\cos 32^\circ + j\sin 32^\circ)$$
$$= 12(.8480 + j.5299)$$
$$= 10.2 + j6.4$$

28. 171.9°
29. 97.4°
30. .272 radians
31. 80 ohms

$$\tan\theta = \frac{opp}{adj} = \frac{X}{R} = \frac{25}{76} = .3289$$

$$\theta = 18.2^\circ$$

$$\sin\theta = \frac{opp}{hyp}$$

$$hyp = \frac{opp}{\sin\theta}$$

$$Z = \frac{X}{\sin\theta}$$

$$Z = \frac{25}{\sin 18.2^\circ}$$

$$= \frac{25}{.3123} = 80 \text{ ohms}$$

32. 31.8 ohms

$$\sin\theta = \frac{opp}{hyp} = \frac{X}{Z}$$

$$X = \sin\theta Z$$

$$= \sin 45^\circ Z$$

$$= .7071(45)$$

$$= 31.8$$

33. –.2588
34. 63.8°
35. .811 or 81.1%

$$PF = \cos\theta$$
$$\theta = 35.8^\circ$$
$$PF = \cos 35.8^\circ$$
$$= .8110$$

36. 319 watts

$$P_a = E_T I = 393.8 \text{ watts}$$

$$PF = \frac{P}{P_a}$$

$$P = PF(P_a)$$

$$P = .811(393.8)$$

$$P = 319 \text{ watts}$$

37. (4 + j3) ohms

$$Z = Z(\cos\theta \pm j\sin\theta)$$
$$= 5(\cos 36.9^\circ + j\sin 36.9^\circ)$$
$$= 5(.7996 + j.6004)$$
$$= 4 + j3$$

38. R = 55 ohms; X_L = 437 ohms. First find the dc resistance of the coil:

$$R = \frac{E}{I} = \frac{110 \text{ volts}}{2 \text{ amps}} = 55 \text{ ohms}$$

This means that a 55-ohm resistor is in series with a perfect coil. We can find the impedance of the coil when connected to ac by:

$$Z = \frac{E}{I} = \frac{110 \text{ volts}}{.25 \text{ amps}} = 440 \text{ ohms}$$

Now we know the impedance and the resistance in the circuit. This is equivalent to a right triangle in which the hypotenuse and the side adjacent to the angle θ are known. Now find angle θ:

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{R}{Z} = \frac{55}{440} = .1250$$

$$\theta = 82.8^\circ$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{X_L}{Z}$$

$$X_L = \sin\theta\,(Z)$$

$$= \sin 82.8^\circ\,(440)$$

$$= .9921\,(440)$$

$$= 437 \text{ ohms}$$

39. 28.2 $\underline{/45^\circ}$ ohms

$$Z_T = Z_1 + Z_2 + Z_3 + Z_4$$

$$= 3 - j6 + 10 + j19 + 2 - j7 + 5 + j14$$

$$= 20 - j13 + j33$$

$$= 20 + j20$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{X}{R} = \frac{20}{20} = 1$$

$$\theta = 45^\circ$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{X}{Z}$$

$$Z = \frac{X}{\sin\theta}$$

$$Z = \frac{20}{.7071} = 28.2$$

40. 66.3 mfd

$$PF = \cos\theta$$
$$.707 = \cos\theta$$
$$\theta = 45^\circ$$

And, since a leading power factor is required, the angle must be -45°. Therefore, at 60 Hertz the X_C of the capacitor must be large enough to cancel the 20-ohm X_L and still have enough reactance left over to shift the current 45° ahead of the voltage. We have seen that a 45° shift occurs when $R = X$. Therefore, the X_C of the capacitor must be $R + X_L$ or 40 ohms. Thus,

$$C = \frac{.159}{f X_C} = \frac{.159}{60(40)} = \frac{.159}{2400} = .0000663$$

$$C = 66.3 \text{ mfd}$$

41. 553 watts. First find the current through Z_b.

$$I_b = \frac{E}{Z_b} = \frac{220}{71\underline{/-36^\circ}} = 3.1\underline{/36^\circ} \text{ amps}$$

Now find the apparent power of Z_b:

$$P_a = E_T I_b = 682 \text{ watts}$$

Since $P = P_a(PF)$ and $PF = \cos\theta$, then:

$P = P_a(\cos\theta)$
$P = 682(\cos 36^\circ)$
$P = 682(.809)$
$P = 553$ watts

42. $891\underline{/77.2^\circ}$ ohms. First find X_L and X_C.

$X_L = 904$ ohms
$X_C = 883$ ohms

Now find the overall impedance of the two parallel branches. To do this you must first find the impedance (Z_{RC}) of R_2 and C in series.

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{XC}{R_2} = \frac{883}{250} = 3.5320$$

$$\theta = -74.2^\circ$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\text{hyp} = \frac{\text{opp}}{\sin\theta}$$

$$Z_{RC} = \frac{X_C}{\sin 74.2^\circ} = \frac{883}{.9618} = 920$$

$$= 920\underline{/-74.2^\circ}$$

Next, assume a voltage across the two parallel branches to establish a current.

$$I_{RC} = \frac{100\underline{/0^\circ}\text{ volts (assumed)}}{920\underline{/-74.2^\circ}\text{ ohms}}$$

$$= .11\underline{/74.2^\circ}\text{ amps}$$

$$I_{R3} = \frac{100\text{ volts}}{200\text{ ohms}} = .5\text{ amp}$$

Then convert the two currents to j-operator form so that they can be added.

$I_{RC} = I_{RC}(\cos\theta + j\sin\theta)$
$I_{RC} = .11\,(.2720 + j.9618)$
$I_{RC} = .03 + j.106$

And since the other branch contains only R_3:

$I_{R3} = .50 + j0$

Now add the two currents to find the total current:

$I_T = I_{RC} + I_{R3}$
$= (.03 + j.106) + (.50 + j0)$
$I_T = .53 + j.106$

Next convert I_T to polar form:

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{.106}{.53} = .2000$$

$$\theta = 11.3^\circ$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\text{hyp} = \frac{\text{opp}}{\sin\theta} = \frac{.106}{.1929} = .55$$

$$I_T = .55\underline{/11.3^\circ}$$

Thus the overall impedances of the parallel branches can now be found:

$$Z_p = \frac{E\text{ (assumed)}}{I_T} = \frac{100\underline{/0^\circ}}{.55\underline{/11.3^\circ}}$$

$$Z_p = 182\underline{/-11.3^\circ}\text{ ohms}$$

Then convert Z_p to j-operator form so that it can be added to the series impedance:

$$Z_p = Z_p\,(\cos\theta - j\sin\theta)$$
$$Z_p = 182\,(.9805 - j.1929)$$
$$Z_p = 178 - j35$$

Now add Z_p to the series impedance of R_1 and L_1.

$$Z_T = 20 + j904 + 178 - j35$$
$$Z_T = 198 + j869$$

Finally, convert to polar form:

$$\tan\theta = \frac{opp}{adj} = \frac{X}{R} = \frac{869}{198} = 4.3889$$

$$\theta = 77.2^\circ$$

$$\sin\theta = \frac{opp}{hyp}$$

$$hyp = \frac{opp}{\sin\theta}$$

$$Z = \frac{X}{\sin 77.2^\circ}$$

$$= \frac{869}{.9751}$$

$$= 891\underline{/77.2^\circ}$$

43. $75\underline{/-48^\circ}$ ohms. First find the reactances of the capacitors and coils.

$X_{C_1} = 67.7$ ohms

$X_{C_2} = 145$ ohms

$X_{L_1} = 62.8$ ohms

$X_{L_2} = 94.2$ ohms

$$Z = 50 - j67.7 - j145 + j62.8 + j94.2$$

$$Z = 50 - j55.7 \text{ ohms}$$

$$\tan = \frac{-55.7}{50} = -1.11$$

$$\theta = -48.0^\circ$$

$$\sin\theta = \frac{X}{Z}$$

$$Z = \frac{X}{\sin\theta}$$

$$Z = \frac{-55.7}{\sin -48^\circ}$$

$$Z = \frac{-55.7}{-.7431} = 75 \text{ ohms}$$

44. $1.33\underline{/48^\circ}$ amps

$$I = \frac{E}{Z}$$

$$I = \frac{100/0^\circ}{75\underline{/-48^\circ}}$$

$$I = 1.33\underline{/48^\circ} \text{ amps}$$

45. 89 watts.

$$P = P_a\,(PF)$$
$$P_a = E_T I = 100 \times 1.33 = 133 \text{ watts}$$
$$PF = \cos\theta = \cos 48^\circ = .6691$$
$$P = 133\,(.6691)$$
$$P = 89 \text{ watts}$$

46. 89 watts. R_1 is the only component which can dissipate power. Therefore, it dissipates the entire 89 watts.

You can arrive at this conclusion by:

$$P = I^2 R = (1.33)^2 (50) = 89 \text{ watts}$$

47. The slope of a line is the rate at which the dependent variable changes with changes in the independent variable.
48. The slope of a curved line can be determined by drawing a line tangent to the curve at the point of interest and calculating the slope of the tangent line.
49. Both the horizontal and vertical scales are logarithmic on logarithmic graph paper. On the other hand, semilogarithmic paper has a logarithmic scale on one axis and a linear scale on the other.
50. Polar graph paper is useful for plotting the radiation patterns of antennas, loudspeakers, and light sources.

Lesson Questions

Be sure to number your Answer Sheet X206.

Place your Student Number on every Answer Sheet.

Most students want to know their grades as soon as possible, so they mail their set of answers immediately. Others, knowing they will finish the next lesson within a few days, send in two sets of answers at a time. Either practice is acceptable to us. However, don't hold your answers too long; you may lose them. Don't hold answers to send in more than two sets at a time or you may run out of lessons before new ones can reach you.

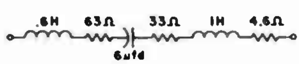
1. Give the sines of the following angles: 23°, -47.5°, 290°, 163°, 215°.

2. Convert $12 + j10.8$ to polar coordinates.

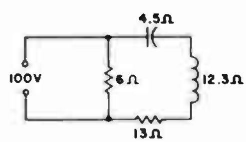
3. Convert $30\underline{/-50.5^\circ}$ to rectangular coordinates.

4. What are the power factors of circuits having the following impedances: $67.1\underline{/68^\circ}$; $123\underline{/-84^\circ}$; $.015\underline{/72^\circ}$; $1.49\underline{/60^\circ}$; $15.1 - j7.7$?

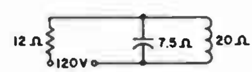
5. What is the impedance in polar coordinates at 60 Hertz of the circuit at the right?



6. What is the current through the circuit at the right? Give your answer in polar form.



7. What is the power dissipated in the circuit at the right?



8. Express as exponential numbers: 57 pf, 10 megohms, .16 microsecond, 26.7K ohms, 4503 kHz.

9. Solve the following problem and express your answer as an exponential number with the correct number of significant figures:

$$\frac{.0073(14.689 - 3.2)}{569} \times \frac{117 \times 9.64}{.00857}$$

10. Sketch graphs, as in Fig. 44, for capacitive reactance with capacity as the independent variable, and inductive reactance with inductance as the independent variable. Label the axes.

CONFIDENCE

Whatever project you undertake, *confidence* in yourself, and in your ability, will make the job easier.

By *confidence*, I do not mean "cockiness," and I most certainly do not refer to *fake confidence* which is simply "bluffing."

I'm speaking of the *confidence* which comes only from a thorough understanding of your work – and a genuine desire to do a good job.

This kind of *confidence* is felt by the people with whom you associate. It *causes them to have confidence in you* – to rely upon your judgment – and to entrust important work to your care.

Successful businesses – important jobs – are managed by men with *confidence* in their ability and desire to do a good job.

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