DX MATHEMATICS!

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PART I: TRIGONOMETRY AND GREAT CIRCLE DISTANCE CALCULATION

In past issues of the club bulletins DX NEWS and DX Monitor technical articles have appeared in which, in order to be applied, the use of "math" was required. As a result many DXers mused over them and then placed them aside because of their "rusty" or non-existent ability to perform the required mathematics. Trigonometry certainly has to be the most "avoided" simply because it is almost always involved! It is therefore very important to note that in almost every case the "mathematics" to be performed requires nothing more than the ability to use a set of mathematical tables-which are in abundant supply! This series of articles is therefore presented to the DX Clan with the sincere hope that a large number of this great group will make the necessary <u>small</u> effort to apply these technical feats because the intent here is to completely remove this mask of mathematical perplexity thus allowing, at least, the computation of great circle distances and bearings, the analysis of ionospheric geometry giving path lengths, arrive/departure angles, number of hops, etc., and the ability to derive the latitude and longitude of a distant station by radio direction finding techniques. All of these applications will be covered in this series and others may also be included.

TRIGONOMETRY, both plane and spherical, is a simple but elegant subject! Six trigonometric functions exist but four are eliminable by definition in terms of the "primary" functions called sine and cosine. We shall employ three of these functions, namely: sine, cosine, and tangent whose algebraic names are SIN, COS, TAN respectively. The use of trigonometry in most of the previous articles has required nothing more difficult than the ability to look up numbers ("angles") in tables of natural trig functions and then to subtract(-), multiply (x or · or ( )), divide (÷ or /) or add (+) among the values obtained from such tables. Furthermore, with the modern low cost accurate digital "pocket" calculators available today, the arithmetic tasks of addition, multiplication, subtraction, and division have been reduced to mere button pushing! For our consumption, trigonometry involves (a) "converting" angles into other numbers and (b) "converting" certain numbers back into angles--both processes taking place by use of trig tables. The use of these tables is quite simple and straight forward and is hopefully explained below!

First, ANGLES. There are two basic angular measurements--degree measure and radian measure. For our purposes, ALL ANGLES ARE TO BE OF DEGREE MEASURE. Degree measure gives angles in the following units: degrees (°), minutes ('), and seconds ("). The degree is divided into 60 equal parts, each part called a minute. Similarly, each minute is divided into 60 parts, each part called a second. Thus we may write: 1° = 60' = 3600".

Second, TABLES. Obtain from your local library, bookstore, or elsewhere a set of "tables of the natural trigonometric functions for each minute of angles 0°-90° with values accurate to five significant places." We assume such a table (at least for SIN, COS, and TAN) is in the hand of the reader for all that follows°°!!°°!! An excellent reference is:"Handbook of Mathematical Tables and Formulas" by R.S. Burington, McGraw-Hill, 4th ed., 1965 (pp. 262-284, Table Number 7). Also: "C.R.C. Standard Mathematical Tables" C.D. Hodgman, Editor-in-Chief, Chemical Rubber Publishing Company, Cleveland, Ohio 11th ed. 1957. (Note: the "CRC" tables are referenced as "Natural Trigonometric Functions: Natural Sines, Tangents, Cotangents and Cosines, for the tables desired here.) These sets of tables give the values of the trig functions for every degree (0°-90°) broken into minute-divisions (0'-60'). This subdivision of angle is necessary to achieve the acceptable accuracy for the calculations that follow. Division of each minute of angle into 60 seconds is not necessary for, as we shall demonstrate, seconds of angular measurement represent a refinement in accuracy not required for our needs. Furthermore, tables of trig functions for this decrement are very difficult to procure. Also note that tables of the trig functions for the angular range 0°-90° are sufficient to evaluate these functions at ANY angle whatsoever! Use of these tables is simple. Hence, let A be an angle in degree measure, say A = B°D', and assume A lies in the range 0°-90°. The <u>value</u> of the function, SIN, COS, TAN, at the angle A is given the <u>name</u> SIN(A), COS(A), TAN(A) or SIN(B°D'), COS(B°D'), TAN(B°D') and is the number appearing in the SIN, COS, TAN column of the table specified from angle B° in the row corresponding to D'. Note that these tables (probably) show angles at the top as well as the bottom of the function columns with the minute decrement columns left and right. The left (') column corresponds to angles at the top of the table and the right (') column corresponds to angles at the bottom of the table--e.g., where ****** denotes the actual values are not given in this "sample" table------

8°

(')	SIN	COS	TAN	***	(')
0	******	******	******	******	60
1	******	0.99023	******	7.10038	59
2	0.13975	*******	******	*******	58
3	*******	*******	0.14143	*******	57
58	*******	*******	*******	*******	2
59	0.15615	0.98773	0.15809	6.32566	
(')	COS	SIN	***	TAN	(')

81°

Now note the following values:

SIN (8°2') = 0.13975
COS (8°1') = 0.99023
TAN (8°3') = 0.14143
SIN (8°59') = 0.15615
COS (8°59') = 0.99773
TAN (8°59') = 0.15809
SIN (81°1') = 0.98773
COS (81°1') = 0.15615
TAN (81°1') = 6.32566
TAN (81°59') = 7.10038
SIN (81°59') = 0.99023
COS (81°58') = 0.13975

For reference below (this is important terminology here!) the minute columns of any table are called the ANGLE column and the numbers in the function column are said to be INSIDE the table.

Now, use your set of tables to directly verify ALL the following: COS(36°41') = 0.80195; SIN (36°42') = 0.59763; TAN(17°37') = 0.31754; SIN(47°3') = 0.73195; COS(45°) = 0.70711 = SIN(45°); SIN(44°57') = 0.70649 = COS (45°3'); TAN(45°) = 1.00000; TAN(3°37') = 0.06321; COS 83°21') = 0.11580; TAN(56°7') = 1.48909; and TAN(89°37') = 149.465.

For reference, the TAN is defined in terms of SIN and COS, namely: let A be an angle, then TAN(A) = SIN(A) / COS(A), i.e.. the value of TAN@A equals the value of SIN@A divided by the value of COS@A. Now that you are an expert at using the table tables (that is all there is to it!!!!!) note these facts: for any angle A the values SIN(A) and COS(A) are always numbers between -1.0 and +1.0 while TAN(A) could be any number--i.e., TAN has no limits or restrictions. One more point in terminology: in COS(A), SIN(A), and TAN(A), the angle A is called the ARGUMENT of the functions and, as before, COS(A), SIN(A) and TAN(A) are the VALUES of COS, SIN, and TAN (at the argument a) respectively.

The set of "facts and figures" for trigonometry is essentially endless! Thus, we shall consider only those relevent to the applications to be discussed and these shall be of the type that allows evaluation of the functions SIN, COS and TAN for ANY argument A by converting to an angle and a function that can be evaluated directly from the tables on hand for the range 0°-90°. All these equations will be presented as they are needed in the applications and each will be numbered for reference throughout this series of writings.

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There are three functions related to SIN, COS, and TAN that will also be required in the applications, viz: inverse sine, inverse cosine and inverse tangent whose algebraic names (here) will be ASIN, ACOS, and ATAN respectively. Evaluation of these requires using the tables "backwards" -i.e., locating an INSIDE value and then reading the correspoiding ANGLE. For example: let X be a number twixt -1.0 and +1.0. Then W = ACOS(X) means "find the angle W whose COS is X". That is, W = ACOS(X) is equivalent to X = COS(W). Analogous remarks apply to ASIN and ATAN. Note: the arguments of ACOS and ASIN are numbers from -1.0 to +1.0 and the argument of ATAN can be any number. The values ("angles") of ACOS range from 0° to 180° while the values of ASIN and ATAN range from -90° to +90°. Now, use your tables to verify all the following: ACOS(0.82462)= 34°27' ; ASIN(0.57667)=35°13' ; ATAN(0.49278)=26°14' ; ASIN(0.80576)=53°41' ; ACOS(0.18681)=79°14' ; ATAN(1.84561)=61°33' ; ASIN(0.12345)=7°5' ; ACOS(0.24687) =75°42' and ATAN(0.54321)=28°31'. Note that the last three have arguments that do not appear exactly INSIDE the tables —merely pick the inside table value nearest the argument and then read out the corresponding angle!

We now state those equations necessary for some of the applications below but will show their use in the illustrative examples that follow.

(E1) --- Let M and P be any numbers: (M+P) means add M and P; (M-P) means subtract P from M; (M)(P) means multiply M and P; (M/P) means divide M by P; -M = -(M) = (-1)(M); (-1)(-1) = +1; and -(-M) = +M. Examples of the last several items: -3.42 = -(3.42) = (-1)(3.42) and -(-5.2) = +5.2 and -(-(-2.6)) = -2.6.

Let A be any angle:

(E2) ---------------- COS(-A) = COS(A) USE: when angle is negative
(E3) ---------------- SIN(-A) = -SIN(A) USE: when angle is negative
(E4) ---------------- COS(A) = -SIN(A-90°) USE: A between 90° and 180°
(E5) ---------------- SIN(A) = COS(A-90°) USE: A between 90° and 180°
(E6) ---------------- COS(A) = -COS(A-180°) USE: A between 180° and 270°
(E7) ---------------- SIN(A) = -SIN(A-180°) USE: A between 180° and 270°
(E8) ---------------- COS(A) = SIN(A-270°) USE: A between 270° and 360°
(E9) ---------------- SIN(A) = -COS(A-270°) USE: A between 270° and 360°

Let B be a number between -1.0 and +1.0 (inclusive):

(E10) --------------- ACOS(B) = 90° + ASIN(-B) USE: B negative
(E11) --------------- ASIN(B) = -ASIN(-B) USE: B negative

Note: (E4-E9), USE: .... between means inclusive of the limits listed
Note: (E10-E11): If B is negative (-) then -B is positive (+); see (E1).
Now use tables to verify: COS(-36°)=0.80902; SIN(-42°)= -0.66913; COS(94°)= -0.06976; SIN(117°)=0.89101; COS(187°)= -.99255; SIN(211°)= -0.51504; COS(314°)=0.69466; SIN(296°)= -0.89879; ASIN(-0.78776)= -(51°59') and ACOS(-0.12345)=97°5'. Now to the first application!

GREAT CIRCLE CALCULATIONS of distance and azimuth bearings have proven very valuable to many DXers. But, an even larger number of DXhounds are presently without the simple knowledge as to how to generate these data.

GREAT CIRCLE DISTANCE: GCD. The GCD is the shortest distance on the surface between two points on a sphere and is necessarily an arc of the great circle passing thru these points. On the BCB the signals propagate between the Earth's surface and the ionosphere above the great circle path. Only under adverse auroral conditions does the signal deviate from the direction taken by this great circle path and even then to a very minor extent! Thus, GCD calculations between transmitters (TX) and the receiver (RX) are of value and interest!

Each point on the Earth's surface has assigned to it geographical coordinates of latitude and longitude. The coordinates are given in degree measure —degrees, minutes, seconds (°,',"). The Earth's Equator is the dividing line between North and South Latitude and thus any point on the Equator has 0° Latitude. The True Geographic North Pole has latitude 90°N, and the True Geographic South Pole has latitude 90°S. The Prime Meridian of Longitude passes thru Greenwich Observatory in England and is the dividing line between East and West Longitude. Thus, any point on the Prime Meridian has 0° Longitude. Algebraically, latitude varies from -90° to +90° and longitude varies from -180° to +180°. To obtain Geographical Coordinates of TX and (your) RX, again check your local library, bookstore, etc., for reference materials. Numerous atlases have this information and for North American BCB, the NARBA listing show precise coordinates of every transmitter. To take coordinates from a map is to be avoided unlesss low accuracy of GCD (or GCB, Part II) is acceptable!

Seconds (") of coordinates will be neglected here. The reasoning follows: On the Earth, whose mean radius R* is about 3957 statude (land) miles or 6368 kilometers (km.), the total distance around (circumference of) any great circle is 2π R* or 24862.56 statute miles or 40011.32 kilometers. Since there are 360° in any circle, on Earth, 1° = (24862.56/360)=69.06 miles or (40011.32/360)=111.14 km. Since also, 1°=60', we find 1' = 69.06/60 = 1.151 miles or 111.14/60 = 1.85 km. Furthermore, 1' = 60" so that 1" = 1.151/60 = 0.0192 miles = 101.3 feet or 1.85/60 = 0.0308 km = 30.8 meters. Also, for interest, 1' = 1.0 nautical miles. Hence, if we neglect (") of arc then our GCD can be in error at most 2.3 miles or 3.7 km -- provided the calculations themselves carry the necessary accuracy! Using five-place trig tables affects the accuracy to only a very minor degree and the values so obtained are still highly accurate for DXing purposes. For examples: North Central Greenland (40°N,80°W) to Tasmania (42°S,145°E), GCD = 10108.8 miles (5-place accuracy) and GCD = 10108.46 miles (10-place accuracy), and Oregon to Florida (43°N, 122°W to 30°N,80°W), GCD=2467.74 miles (5-places) and GCD=2468.14 miles (10-places). Thus, we shall simply omit (") whenever specified in coordinates --for example: 43°37'47" can either be rounded to 43°38' or truncated to 43°37' by simply dropping (47") --the resulted GCD error no greater than 2.3 miles.

Latitudes are given as N-Latitude or S-Latitude indicating that the location is North or South of the Equator respectively. Similarly, longitudes are given as E-Longitude or W-Longitude indication the location is East or West of the Prime Meridian respectively. We shall adhere to the following STANDARD conventions on the assignmnt of algebraic sign (+ or -) to coordinates: viz,

North-Latitude and W-Longitude are positive (+)
South-Latitude and E-Longitude are negative (-).

Note the table for TX and RX below assumes the algebraic signs are properly assigned!

With the above understood, we now present the trig formulas that compute GCD:

	LATITUDE	LONGITUDE
TX:	T1° T2'	F1° F2'
RX:	R1° R2'	H1° H2'

Δ = (H1°H2') - (F1°F2')
A = SIN(T1°T2')SIN(R1°R2') + COS(T1°T2')COS(R1°R2')COS(Δ)
B = B1° B2' = ACOS(A) = _____°_____'

GCD = (69.06)B1 + (1.15)B2 = ____________ statute miles
= (111.14)B1 + (1.85)B2 = ____________ kilometers
= (60.00)B1 + B2 = ____________ nautical miles

The trig equations required in this application are (E1)-(E4),(E6),(E8),(E10):

(E2): COS(-A) = COS(A) USE: when angle negative
(E3): SIN(-A) = -SIN(A) USE: when angle negative
(E4): COS(Δ) = -SIN(Δ -90°) USE: 90° ≤ Δ ≤ 180°
(E6): COS(Δ) = -COS(Δ -180°) USE: 180°≤ Δ ≤ 270°
(E8): COS(Δ) = SIN(Δ -270°) USE: 270°≤ Δ ≤ 360°
(E10):ACOS(A) = 90° + ASIN(-A) USE: when A is negative

Note: for GCD calculations, A is a value between -1.0 and +1.0; B is an angle between 0° and 180° and in (E10), observe if A is negative, then -A is positive.

EXAMPLE (I):

TX: (160R-WCWC, Ripon, WI): 43°49'N;88°51'W: T1°T2'=43°49'; F1°F2'=88°51'

RX: (Asheville, NC) @ 35°38'N, 82°35'W : R1°R2'=35°38' ; H1°H2' =82°35'

Δ = 82°35' - 88°51' = -(88°51'-82°35') = -(6°16')

A = SIN(43°49')SIN(35°38') + COS(43°49')COS(35°38')COS(6°16')

= (0.69235)(0.58260) + (0.72156)(0.81276)(0.99402) =

= (0.40336) + (0.58295) = 0.98631 so that

B = B1° B2' = ACOS(0.98631) = 9°29' and finally,

GCD = (69.06)(9) + (1.15)(29) = 654.92 statute miles.

NOTE: COS(-(6°16')) = COS(6°16') by use of (E2)

EXAMPLE (II):

TX: (Dakar, Senegal)@ 14°39'N; 17°28'W; T1°T2'=14°39'; F1°F2'=17°28'

RX: (San Diego, CA) @ 32°45'N; 117°10'W; R1°R2'=32°45'; H1°H2'=117°10'

Δ = 117°10' - 17°28' = 116° 70' - 17°28' = 99°42'

A =SIN(14°39')SIN(32°45') + COS(14°39')COS(32°45')COS(99°42')

= (0.25291)(0.54097) + (0.96749)(0.84104)(-0.16849) =

= (0.13682) + (-0.13710) = -0.00028 so that

B = B1°B2' = ACOS(-0.00028) = 90° + ASIN(0.00028)=90°+0°1'=90°1'

GCD = (69.06)(90) + (1.15)(1) = 6216.55 miles

NOTE: COS(99°42')= -SIN(99°42'-90°) = -SIN(9°42') by (E4) and by (E10)
ACOS(-0.00028) = 90° + ASIN(0.00028).

EXAMPLE (III):

TX: (Lima, Peru)@ 12°6'S; 76°55'W : T1°T2'= -(12°6'); F1°F2'=76°55'

RX: (Boston, MA)@ 42°15'N; 71°7'W : R1°R2'= 42°15' ; H1°H2'=71° 7'

Δ = 71°7'-76°55' = -(76°55'-71°7') = -(5°48')

A = SIN(-(12°6'))SIN(42°15') + COS(12°6')COS(42°15')COS(5°48')

= (-0.20962)(0.67237) + (0.97778)(0.74022)(0.99488) =

= (-0.14094) + (0.72007) = 0.57913 so that

B = B1°B2' = ACOS(0.57913) = 54°37' and finally,

GCD = (69.06)(54) + (1.15)(37) = 3771.83 miles

NOTE: by (E2) COS(-(5°48'))=COS(5°48'); COS(-(12°6'))=COS(12°6') and by
(E3) SIN(-(12°6')) = -SIN(12°6')

EXAMPLE (IV):

TX: (Munich, W. Germ)@48°8'N; 11°35'E : T1°T2'=48°8' ; F1°F2'= -(11°35')

RX: (Ft. Worth, TX)@32°45'N; 97°20'W: R1°R2'=32°45'; H1°H2'= 97°20'

Δ = 97°20' - (-(11°35')) =97°20' + 11°35' = 108°55'

A = SIN(48°8')SIN(32°45') + COS(48°8')COS(32°45')COS(108°55')

= (0.74470)(0.54097) + (0.66740)(0.84104)(-0.32419) =

= (0.40286) - (0.18197) = 0.22089

B = B1°B2' = ACOS(0.22089) = 77°14' so that

GCD = (69.06)(77) + (1.15)(14) = 5333.73 miles

NOTE: COS(108°55') = -SIN(108°55'-90°) = -SIN(18°55') by use of (E4)

EXAMPLE (V):

TX: (Cape Town, S.Af.)@33°48'S; 18°28'E : T1°T2'= -(33°48'); F1°F2'=-(18°28')

RX: (Youngstown, Ohio)@41° 5'N; 80°40'W : R1°R2'=41°5' ; H1°H2'=80°40'

Δ = 80°40' -(-(18°28')) = 80°40' +18°28' = 99°8'

A = SIN(-(33°48'))SIN(41°5') + COS(33°48')COS(41°5')COS(99°8')

= (-0.55630)(0.65716) + (0.83098)(0.75375)(-0.15873) =

= (-0.36558) + (-0.09942) = -0.46500 so that

B = B1°B2'= ACOS(-0.46500) =90°+ASIN(0.46500)=90°+27°43'=117°43'

GCD = (69.06)(117) + (1.15)(43) = 8129.51 miles

NOTE: SIN(-(33°48'))= -SIN(33°48') by (E4); COS(-(33°48'))=COS(33°48') by (E2)
and ACOS(-0.46500) = 90° + ASIN(0.46500) by (E10)

EXAMPLE (VI):

TX:(Tokyo, Japan)@35°41'N; 139°44'E : T1°T2'=35°41' ; F1°F2'=-(139°44')

RX:(Houston, TX)@29°46'N; 95°21'W : R1°R2'=29°46' ; H1°H2'= 95°21'

Δ = 95°21' - (-(139°44')) =95°21'+139°44'= 234°65'=235°5'

A = SIN(35°41')SIN(29°46') + COS(35°41')COS(29°46')COS(235°5')

= (0.58330)(0.49647) + (0.81225)(0.86805)(-0.57238) =

= 0.28959 + (-0.40357) = -0.11390 so that

B = B1°B2' = ACOS(-0.11390) = 90° + ASIN(0.11390) =90°+6°32'=96°32'

GCD = (69.06)(96) + (1.15)(32) = 6666.60 miles

NOTE: COS(235°5')= -COS(235°5'-180°) = -COS(55°5') by (E6)

EXAMPLE (VII):

TX: (Melbourne, Au)@ 37°52'S; 145°8'E : T1°T2'= -(37°52'); F1°F2'= -(145°8')

RX: (Wahoo, Nebr.)@ 41°14'N; 96°39'W : R1°R2'=41°14' ; H1°H2'=96°39'

Δ = 96°39' -(-(145°8')) = 96°39' + 145°8' = 241°47'

A = SIN(-(37°52'))SIN(41°14') + COS(37°52')COS(41°14')COS(241°47')

= (-0.61383)(0.65913) + (0.78944)(0.75203)(-0.47281) =

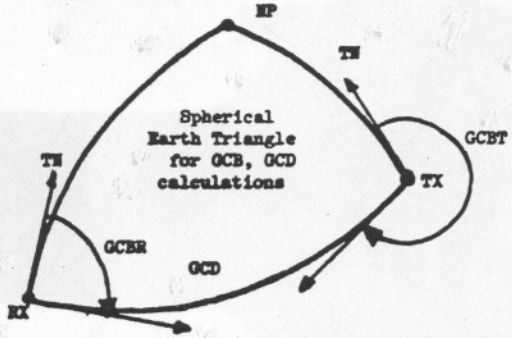
= (-0.40459) + (-0.28070) = -0.68529 so that

B = B1!B2' = ACOS(-0.68529) = 90°+ASIN(0.68529) =90°+43°16'=133°16'

GCD = (69.06)(133) + (1.15)(16) = 9203.40 miles

PART II: Great Circle Bearings

In Part I of this Series, the essentials of trigonometry relevant to the application of computing GCD, great circle distance, was discussed and developed. Below are the trigonometric equations that allow evaluation of the great circle azimuth bearing of the TX as measured at the RX which is termed GCBR and the great circle ("back" or "reciprocal") azimuth bearing of RX as measured at TX and termed GCBT.



NP: True Geographical North Pole
TN: True Geographical North

LATITUDE
TX: T1°T2'
RX: R1°R2'

LONGITUDE
TX: F1°F2'
RX: H1°H2'

Δ = (H1°H2') - (F1°F2') = ____°

A = SIN(T1°T2')SIN(R1°R2') + COS(T1°T2')COS(R1°R2')COS(Δ)

B = B1°B2' = ACOS(A) = ____° ____'

GCD = (69.06)B1 + (1.15)B2 = ____________ statute miles

= (111.14)B1 + (1.85)B2 = ____________ kilometers

J = SIN(B1°B2') = SQRT(1.0 - A^2) = ____________

V1 = SIN(T1°T2') - (A)SIN(R1°R2')

V2 = (J)COS(R1°R2')

V = (V1/V2)

W1 = SIN(R1°R2') - (A)SIN(T1°T2')

W2 = (J)COS(T1°T2')

W = (W1/W2)

RAZ = ACOS(V) = ____° ____'

TAZ = ACOS(W) = ____° ____'

If (0° ≤ Δ ≤ 180°), then GCBR = RAZ and GCBT = 360° - TAZ

If (Δ< 0°, or, Δ>180°), then GCBR = 360 - RAZ and GCBT = TAZ

The trig function equations required in this application are (E1-E6), (E8), (E10):

(E2)	COS(-X) = COS(X)	USE: when angle is negative
(E3)	SIN(-X) = -SIN(X)	USE: when angle is negative
(E4)	COS(Δ) = -SIN(Δ -90°)	USE: 90° ≤ Δ ≤ 180°
(E5)	SIN(X) = COS(X-90°)	USE: 90° ≤ X ≤ 180°
(E6)	COS(Δ) = -COS(Δ -180°)	USE: 180° ≤ Δ ≤ 270°
(E8)	COS(Δ) = SIN(Δ -270°)	USE: 270° ≤ Δ ≤ 360°
(E10)	ACOS(X) = 90° + ASIN(-X)	USE: when angle is negative

Note: this algorithm is valid for GCB and GCD between any two points on the Earth's surface save for the Poles where the analysis is straightforward algebraic computation. Also: -1 ≤ A ≤ +1; 0° ≤ B ≤ 180°; -1 ≤ J ≤ +1;

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-1 ≤ V ≤ +1; -1 ≤ W ≤ +1; 0° ≤ RAZ ≤ 180°; 0° ≤ TAZ ≤ 180°; 0° ≤ GCBR ≤ 360° and 0° ≤ GCBT ≤ 360°. Also: in the examples which follow: T = T1°T2', R = R1°R2', H = H1°H2' and F = F1°F2' .

Verify each detail of the following examples!

EXAMPLE (I):
TX: (16R WCWC, Ripon, WI) @ 43°49'N, 88°51'W so that T = 43°49' and F =88°51'
RX: (Asheville, NC) @ 35°38'N , 82°35' W so that R = 35°38' and H = 82°35'
Δ = 82°35' - 88°51' = -(88°51' - 82°35') = -(6°16') NOTE: Δ < 0°
A = (0.69235)(0.58260) + (0.72156)(0.81276)(0.99403) = 0.98631
B = ACOS(0.98631) = 9°29' so that GCD = 654.9 miles
J = SIN(9°29') = 0.16476 and V2 = (0.16476)COS(35°38') = 0.13391
V1 = SIN(43°49') - (0.98631)SIN(35°28') = 0.11773 and V = V1/V2 = 0.87917
W1 = SIN(35°38') - (0.98631)SIN(43°49') = -(0.10028) , W2=(0.16476)COS(43°49')
W = W1/W2 = -(0.84354) RAZ = ACOS(0.87917) = 28°27' = 0.11888
TAZ = ACOS(-0.84354) = 90° + ASIN(0.84354) = 90° + 57°31' = 147°31'
and (Δ < 0°) so that GCBR = 360°-28°27' = 359°60'-28°27'= 331°33'; and
GCBT = 147°31'

EXAMPLE (II):
TX: (Dakar) @ 14°39'N, 17°28'W so that T = 14°39' and F = 17°28'
RX: (Kansas City) @ 39°5'N , 94°35'W so that R = 39°5' and H = 94°35'
Δ = 94°35' - 17°28' = 77°7' NOTE: (0° ≤ Δ ≤ 180°)
A = (0.63045)(0.25291) + (0.77623)(0.96749)(0.22297) = 0.32690
B = ACOS(0.32690) = 70°55' so that GCD = 4897.5 miles
J = SIN(70°55') = 0.94504, V1 = SIN(14°49') - (0.32690)(SIN(39°5'))=0.04682
V2 = (0.94504)COS(39°5') = 0.73357 and V = V1/V2 = 0.06382
W1 = SIN(39°5') -(0.32690)SIN(14°39') = 0.54777, W2=(0.94504)COS(14°39') =
W = W1/W2 = 0.59910; RAZ = ACOS(0.06382) = 86°20' =0.91432
TAZ = ACOS(0.59910) = 53°12' and (0° ≤ Δ ≤ 180°) so that GCBR = 86°20'
and GCBT = 360° - 53°12' = 359°60' - 53°12' = 306°48'

EXAMPLE (III):
TX: (South Magnetic Pole) @ 68°12'S ; 145°24' E so that T=-(68°12') & F=-(145°24')
RX: (North Magnetic Pole) @ 76° N ; 102° W so that R = 76° and H = 102°
Δ = 102° - (-(145°24')) = 247°24' NOTE: Δ > 180°
A = (-0.92849)(0.97030) + (0.37137)(0.24192)(-0.38430) = -0.93544
B = ACOS(-0.93544) = 90° + 69°18' = 159°18' so that GCD= 11001,3 miles
J = SIN(159°18') = COS(159°18' - 90°) = COS(69°48') = 0.35347
V1 = -SIN(68°12') -(-0.93544)SIN(76°) = -0.02083 and
V2 = (0.35347)COS(76°) = 0.08551 and V = V1/V2 = -0.24360
W1 = SIN(76°) -(-0.93544)SIN(-68°12') =0.10175, W2=(0.35347)COS(68°12')=0.13127
W = W1/W2 = 0.77512; RAZ = ACOS(-0.24360) = 90° + 14°6' = 104°6' and
TAZ = ACOS(0.77512) = 39°11' and (Δ > 180°) gives GCBT = 39°11' and GCBR=255°54'

EXAMPLE (IV):
TX: (Montevideo, Uru) @ 34°50'S , 56°10'W so T = -(34°50') and F = 56°10'
RX: (Orlando, FL) @ 28°32'N , 81°22'W so that R = 28°32' and H = 81°22'
Δ = 81°22' - 56°10' = 25°10' NOTE: (0° ≤ Δ ≤ 180°)
A = (-0.57119)(0.47767) + (0.82082)(0.87854)(0.90508) = 0.37983
B = ACOS(0.37983) = 67°41' so that GCD = 4674.2 miles, J = SIN(67°41')=0.92510
V1 = -SIN(34°50') - (0.37983)SIN(28°32') = -0.75262, V2=(0.9251)COS(28°32')=
W1 = SIN(28°32') - (0.37983)(-SIN(34°50'))=0.69462 and =0.81274
W2 = (0.92510)COS(34°50') = 0.75934 so V = V1/V2 = -0.92603 , W =W1/W2=0.91477
RAZ = ACOS(-0.92603) = 157°49' and TAZ = ACOS(0.91477) = 23°50'
so (0° ≤ Δ ≤ 180°) gives GCBT = 336°10' and GCBR = 157°49'

EXAMPLE (V):
TX: (Lvov, USSR) @ 49°51'N, 28°31'E so T = 49°51' and F = -(28°31')
RX: (McCook, NE) @ 40°13'N, 100°37'W so that R=40°13' and H = 100°37'
Δ = 100°37' - (-28°31')) = 128°68' = 129° 8' NOTE: 0° ≤ Δ ≤ 180°
A = (0.76436)(0.64568)+ (0.64479)(0.76361)(-0.63113) = 0.18278
B = ACOS(0.18278) = 79°28' so that GCD = 5488.0 miles, J=SIN(79°28')=0.98315
V1 = SIN(49°51') -(0.18278)SIN(40°13') = 0.64634, V2=(0.98315)COS(40°13')=
W1 = SIN(40°13') -(0.18278)SIN(49°51') = 0.50597 =0.75074
W2 = (0.98315)COS(49°51')=0.63393; V=V1/V2=0.86093 & W=W1/W2=0.79815 so
RAZ = ACOS(0.86093)=30°35' and TAZ = ACOS(0.79815) = 37°3' and
(0° ≤ Δ ≤ 180°) gives GCBT = 322°57' and GCBR = 30°55'

EXAMPLE (VI):
TX: (Blantyre, Mala) @ 15°48'S, 35°7'E so that T = -(15°48') and F= -(35°7')
RX: (Riverside, CA) @ 33°59'N, 117°21'W so R = 33°59' and H = 117°21'
Δ = 117°21' - (-(35°7')) = 152°28' NOTE: 0° ≤ Δ ≤ 180°
A = (-0.27228)(0.55895) + (0.96222)(0.82902)(-0.88674) = -0.85970
B = ACOS(-0.85970) = 149°17' so that GCD = 10309.5 miles
J = SIN(149°17') = 0.51079, V1=SIN(-15°48') -(-0.85970)SIN(33°59')=0.20825
V2 = (0.51079)COS(33°59') = 0.42355 and v = V1/V2 = 0.49168
W1 = (0.51079)COS(15°48') = 0.32487, W2 = (0.51079)COS(15°48') = 0.49149
W = W1/W2 = 0.66099 and RAZ = ACOS(0.49168) = 60°33' while
TAZ = ACOS(0.66099) = 48°37' and (0° ≤ Δ ≤ 180°) gives the bearings
GCBT = 311°23' and GCBR = 60°33'

EXAMPLE (VII):
TX: (Dunedin, NZ) @ 45°48'S, 141°6'E so T = -(45°48') and F = -(141°6')
RX: (Watertown, MA) @ 42°23'N, 71°7'W so R = 42°23' and H = 71°7'
A = (-0.71691)(0.67409) + (0.69716)(0.73865)(-0.84604) = -0.91894
Δ = 71°7' - (-(141°6')) = 212°13' and NOTE: (Δ > 180°)
B = ACOS(-0.91894) = 156°46' so that GCD = 10826.3 statute miles
J = SIN(156°46') = 0.39448, V1 = SIN(-45°48')-(-0.91894)SIN(42°23') =
V2 = (0.39448)COS(42°43')=0.29138, V=V1/V2= -0.33448 =-0.09746
W1 = SIN(42°23')-(-0.91894)SIN(-45°48') = 0.01529, W = W1/W2 = 0.05560
W2 = (0.39448)COS(45°48') = 0.27502 and RAZ = ACOS(-0.33448) = 109°32'
and TAZ = ACOS(0.05560) = 86°49' and since we have that (Δ > 180°) we
find: GcBT= 86°49' and GCBR = 360° - 109°32' = 250°28'

For those interested in working other examples, the following are based upon a RX location of 18°30'N and 69°55'W (near Santo Domingo, Dominican Republic)

LOCATION	LATITUDE	LONGITUDE	GCBR	MILES	GCBT
Bogota, Columbia	4°38'N	74°6'W	196°56'	998.3	16°6'
Vladivostok, ASSR	43° 6'N	131°47'E	342°38'	7958.7	22°49'
Kingston, Jamaica	18°21'N	77°31'W	270° 6'	498.0	87°37'
Blantyre, Mala	15°48'S	35°7'E	100°54'	7516.8	284°35'
Lima, Peru	12° 6'S	76°55'W	193°14'	2166.3	12°50'
Honololu, HI	23° 8'N	157°28'W	287°26'	5485.4	109°23'
Wellington, NZ	41°25'S	174°45'E	232°25'	8352.4	91°56'

PART III: Radio Direction Finding

Some time ago an article titled "Radio Direction Finding: A practical approach on the BCB with an algorithm for manual or computer evaluation" appeared in both DXN and DXM. In that writing, two receiver locations RA and RB took loop bearings (by nulls) on a distant transmitter (TX) and the calculations that followed allowed computation of the great circle distances RA to TX and RB to TX. The method is rather limited since there are restrictions on the input parameters to that algorithm not mentioned in the article.

Below a complete algorithm is presented which not only allows the computation of the distances involved but also the geographical coordinates of the distant station.

An introduction to the techniques of measuring such bearings (azimuth angles) of a distant transmitter by use of a loop antenna at the receiving site are discussed in the above named article and thus will be omitted here.

Figure I is fundament to this development. NP denotes the geographic North Pole, RA is the location of Receiver "A", RB is the location of Receiver "B", TN denotes True Geographical North, GCAB is the great circle passing thru RA and RB, DAB is the great circle distance between RA and RB, AB is the azimuth bearing of RB as measured at RA, BA is the azimuth bearing of RA as measured at RB, DAF is the great circle distance between RA and Transmitter "F" (TXF), DBF is the great circle distance between RB and TXF, DAG is the great circle distance between RA and Transmitter "G" (TXG), and

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DBG is the great circle distance between RB and TXG. DAF+DAG=DBF+DBG=πR. (πR = 12430.8 miles for Earth radius R=3956 to 3957 miles and is thus one-half the circumference of the Earth) TXF and TXG are thus "endpoints" of a diameter of the sphere of radius R (i.e., they lie at opposite "ends" of the Earth).

Since measurement of a loop bearing of a distant transmitter is inherently ambigious by 180°, one cannot be certain which transmitter (TXF or TXG) is being heard. For this development, we introduce the terms "Half A" and "Half B" which are determined by the great circle GCAB passing thru RA and RB. This great circle divides the Earth into two halves and the two DXers at RA and RB simply agree which half will be designated Half A, the other half then called Half B. The important fact is that loop bearings taken on a TX at both RA and RB must "point" into the same Half. With this understood, A denotes the bearing of the distant TX as measured at RA and B denotes the bearing of the distant TX as measured at RB. (Figure I merely shows A and B in the Half B) The most probably Half in which the station is to be found is thus decided by the DXers at RA and RB by their own experience with propagation conditions, time of day, program language, etc. Furthermore, due to the nature of the BCB, DAB is not likely to exceed several thousand miles. Also, for practical reasons, DAB should not be less than several hundred miles for RDFing stations more distant than a thousand miles or so.

Also, in almost every instance, one of the possible transmitter locations will be nearer to RA or RB than the other. For this discussion, we assume that TXF is the nearer--that is, DAF ≤ DAG or DBF ≤ DBG. There is a simple determination of which TX is being heard (the "short haul" path, TXF or the "long haul" path to TXG), namely: if J3 = J1 + J2 < 180°, then TXF is the distant transmitter and if J3 > 180°, then TXG is the DX. If J3 = 180°, the DX could be either TXF or TXG.

For the following: All trigonometric functions have arguments in degree measure, values in radian measure and all inverse trigometric functions have arguments in radian measure and values in degree measure. The trigonometric functions used are sine (SIN) and cosine (COS) and the inverse trigonometric functions used are arcsine (ASIN) and arccos (ACOS). Also, ABS(X) denotes the absolute value of the number X, i.e., if X is negative (-), ignore the "minus" sign. For example: ABS(+3)=ABS(-3)=3, ABS(-7.98)=7.98, etc.

Let the coordinates of RA be Latitude (A1°A2'A3"), Longitude (A4°A5'A6") and coordinates of RB be Latitude (B1°B2'B3"), and Longitude (B4°B5'B6"). The standard convention of algebraic sign for coordinates is used, viz: N-Latitude and W-Longitude are positive (+) and S-Latitude and E-Longitude are negative (-).

We now present the necessary trigonometry: For notational convenience, let A* denote A1°A2'A3", A** denote A4°A5'A6", B* denote B1°B2'B3" and B** denote B4°B5'B6".

```
T° = A** - B**
D° = ACOS[(SIN(A*)SIN(B*)) + (COS(A*)COS(B*)COS(T°))]
M  = COS(D°)
N  = SIN(D°)
E° = ACOS[(SIN(B*) - (M)SIN(A*))/(N)COS(A*)]
V° = ACOS[(SIN(A*) - (M)SIN(B*))/(N)COS(B*)]
DAB = (69.06)D°   ... statute miles
IF(0° ≤ T° ≤ 180°), THEN: AB = E° and BA = 360° - V°
IF(T° < 0° or T° > 180°), THEN: AB = 360° - E° and BA = V°

J1 = ABS(AB-A)
J2 = ABS(BA-B)
IF(J1 > 180°), REPLACE J1 by 360° - J1
IF(J2 > 180°), REPLACE J2 by 360° - J2
J3 = J1 + J2
IF(J3 > 180°), REPLACE J1 by 180° - J1
IF(J3 > 180°), REPLACE J2 by 180° - J2
P° = ACOS[(COS(D°)SIN(J1)SIN(J2)) - (COS(J1)COS(J2))]
J4 = SIN(D°)/SIN(P°)
J5° = ASIN[(J4)SIN(J2)]
J6° = ASIN[(J4)SIN(J1)]
J7 = SIN(A*)COS(J5)
J8 = COS(A*)SIN(J5)COS(A)
J9 = J7 + J8
IF(J3 > 180°), REPLACE J9 by J7 - J8
J10° = ACOS[J9]
J11 = 90° - J10
J12° = ACOS[(COS(J5) - (SIN(A*)SIN(J11)))/(COS(A*)COS(J11))]
IF(J3 ≤ 180°), then
        (a) IF(0° ≤ A ≤ 180°), THEN J13 = A** - J12
        (b) IF(180° < A < 360°), THEN J13 = A** + J12
IF(J3 > 180°), then
        (a) IF(0° ≤ A ≤ 180°), THEN J13 = A** + J12
        (b) IF(180° < A < 360°), THEN J13 = A** - J12
IF(J3 ≤ 180°), then TXF is the transmitter logged, and
        Latitude TXF:    J11
        Longitude TXF:   J13
        Distance DAF = (69.06)J5°  ... statute miles
        Distance DBF = (69.06)J6°  ... statute miles

IF(J3 > 180°), then TXG is the transmitter logged, and
        Latitude TXG:  -J11
        Longitude TXG: (a) IF(J13 ≥ 0°),Long(TXG)=J13-180°
                       (b) IF(J13 < 0°),Long(TXG)=J13+180°
        Distance DAG = (69.06)(180° - J5°) .. statute miles
        Distance DBG = (69.06)(180° - J6°) .. statute miles
```

Several Examples follow: In each case RA and RB have coordinates A* = 41°0'0", A** = 97°0'0", B* = 38°0'0" and B** = 92°0'0". Thus, for all examples:

T° = 97° - 92° = 5°	D° = 4.886° = 4°53'09"
M = 0.99637	N = 0.08517
E° = 126.256° = 126°15'21"	V° = 50.562° = 50°33'43"
DAB = 337.43 miles	and since T°=5°, we have
AB = E° = 126.256°	BA = 360°-V° = 309.438° =309°26'18"

	EXAMPLE (I)	EXAMPLE (II)
Loop Bearing: A°	102°	282°
Loop Bearing: B°	63°	243°
J1 =	24.256°	155.744°
J2 =	113.562°	66.438°
J3 =	137.818° < 180°	222.182° > 180°
P° =	42.299°	42.299°
J4 =	0.12655	0.12655
J5 =	6.662°	6.662°
J6 =	2.980°	2.980°
J7 =	0.65163	0.65163
J8 =	-0.01820	0.01820
J9 =	0.63343 (=J7+J8)	0.63343 (=J7-J8)
J10 =	50.696°	50.696°
J11 =	39.304°=39°18'14"	39.304°=39°18'14"
J12 =	8.433°	8.433°
J13 =	88.567° = 88°34'01" =(A** - J12)	88.567° = 88°34'01" =(A** - J12)

[J3 < 180°] so TXF is the DX TX and	[J3 > 180°] so TXG is the DX TX and
Lat(TXF): 39°18'14" (N)	Lat(TXG): 39°18'14" (S)
Long(TXF): 88°34'01" (W)	Long(TXG): 91°25'59" (E)
DAF = 460.08 miles	DAG = 11970.72 miles
DBF = 205.80 miles	DBG = 12225.00 miles

A study of Examples (I) and (II) will show that, in either case, J3 ≤ 180° or J3 > 180°, that one always computes the coordinates of TXF, namely Lat(J11) and Long(J13). If J3 > 180°, then TXG is the transmitter of interest, but since TXF and TXG are "endpoints" of a spherical diameter, the coordinates of one follows quickly from knowledge of the coordinates of the other. To wit: if Lat(TXF)=J11 and Long(TXF)=J13, then Lat(TXG)= -J11 and Long(TXG) = J13+180° or J13 - 180°.

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The large agreement in computed values for Examples (I) and (II) follows from the fact that the loop bearings in one example are reciprocal loop bearings of the other example, to wit: 282° = 102° + 180° and 243° = 63° + 180°. In Example (I): J2 = 246.438°, but J2 > 180° requires J2 replacement by 360° - J2 = 113.562°. In Example (II), J1 = ABS(-155.744°) = 155.744° and J2 = 66.438° so that J3 = 222.182° > 180°. Now, in this case J1 is replaced by 180° - J1 = 24.256° and J2 is replaced by 180° - J2 = 113.562° which are identical with the J1, J2 values in Example (I). Thus, the values used for J1 and J2 are the same for both Example (I) and Example (II).

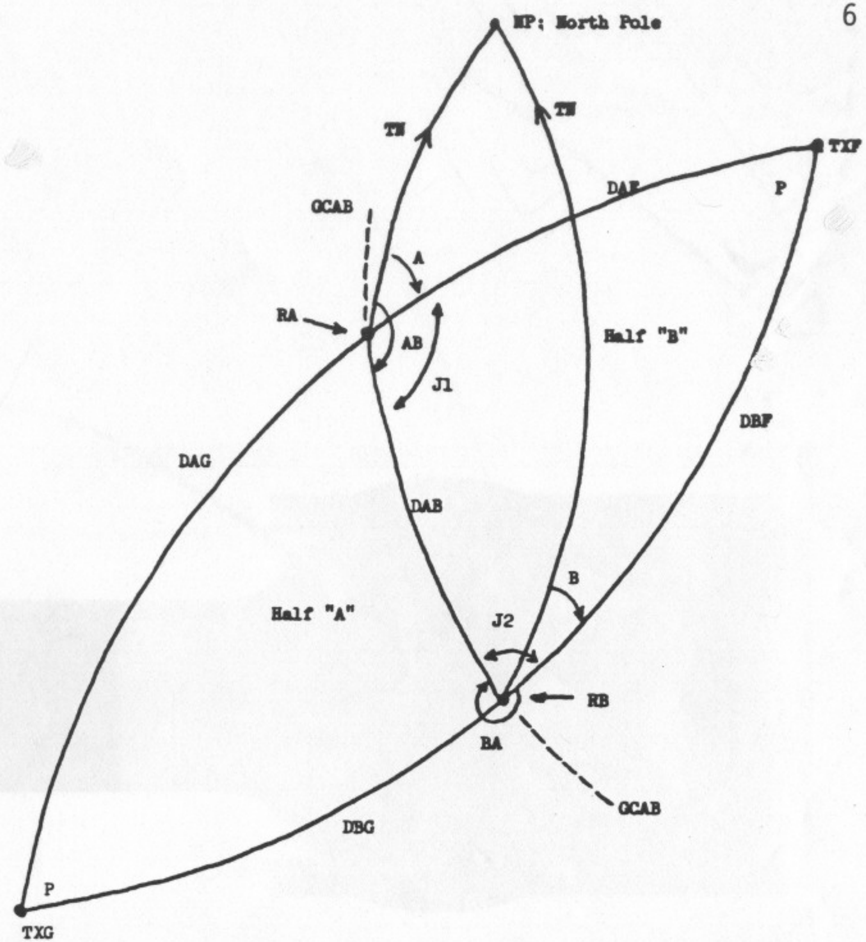
In the remaining examples, J1 and J2 will be listed as their "final" values as computed by lines 10-16 of the algorithm.

		Example(III)	Example(IV)	Example(V)	Example(VI)
Loop Bearing:A°		200	300	235	235
Loop Bearing:B°		300	300	234	235
J1°	=	73.744	6.256	71.256	71.256
J2°	=	9.438	170.562	104.562	105.562
J3°	=	83.182 < 180	183.182 > 180	184.182 > 180	183.182>180
P°	=	96.851	3.248	6.276	5.649
J4	=	0.08579	1.50329	0.77913	0.86528
J5°	=	0.806	14.271	48.947	56.466
J6°	=	4.724	9.428	47.545	55.024
J7	=	0.65599	0.63581	0.43087	0.36243
J8	=	-0.00997	0.09302	-0.32644	-0.36083
J9	=	0.64602	0.54279	0.75731	0.72326
J10°	=	49.758	57.126	40.772	43.676
J11°	=	40.242	32.874	49.228	46.324
J12°	=	0.361	14.725	71.067	81.400
J13°	=	97.361	82.275	25.933	15.600
TX logged	:	TXF	TXG	TXG	TXG
Latitude	:	40°14'31"(N)	32°52'26"(S)	49°13'41"(S)	46°19'26"(S)
Longitude	:	97°21'40"(W)	97°43'30"(E)	154°04'01"(E)	164°24'00"(E)
Distance DA-	:	55.67	11445.24	9050.52	8531.26
Distance DB-	:	326.24	11779.70	9147.34	8630.84

Note Examples (V) & (VI) indicate the extreme sensitivity of the algorithm to very small changes in A° and B° [1° in these Examples] with the resulting transmitters being different by some 1000 miles. Thus, RDF work must be done with as great an effort as is possible to achieve the "best" loop bearing of a station—especially on the long haul paths!!!

The author has great interest in this matter of RDF work on the BCB and would welcome any comments from all parties so interested. Should there warrant a need for analysis of considerable data with this algorithm, the author would consider the job of computing the resulting RDF data and send the results to all concerned. Certainly on the BCB there are numerous instances where RDF work can be used: unId testers, clandestine stations, split frequency stations and the like. If DXers would want, the author would be willing to act as a "clearing house" for all such activities, and would be willing to correspond with those requesting RDF analysis on transmitters they have logged. The accuracy is best achieved with numerous DXers reading bearings on the same stations and the analysis carried out between all pairs of such DXers to ascertain the transmitter location!

RDF de Ghoti, Ph. Dx



************************** FIGURE I **************************

RADIO DIRECTION FINDING MODEL

Note: All bearings A, AB, BA, B are measured clockwise positive (0° - 360°) from True Geographic North (TN) at the respective receiver locations RA and RB.