T12-1

EASY DX CALCULATIONS -------------------------------- de Ghoti, Ph.Dx

A number of articles have appeared in past bulletins explaining how to compute the great circle distance between two points on the Earth's surface; for example, the distance between a BCB station and a DXer's location.

Presented here is a simple equation whose accuracy is basically "good" for much of the work of a BCB DXer -- especially the Domestic DXer. It is simple in that it requires little effort to compute. Without explaining in detail the error analysis of the equation with respect to those for calculating the great circle path, a brief description of its background derivation and some examples will be given.

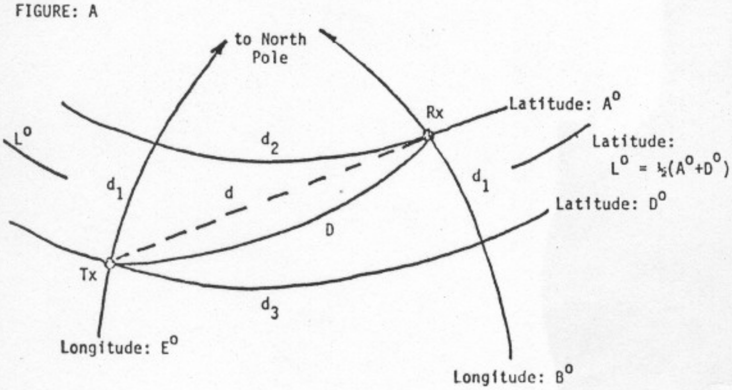
First, the mean value of the radius of the Earth is about 3957 statute miles. This means the circumference of an Earth great circle is equal to 2π (3957) or 24862.56 miles. Every great circle can be divided into 360 one-degree arcs, so that each degree on a great circle of the Earth corresponds to 24862.56/360 miles or 69.06 miles/degree. If we reduce this (for computational convenience) to an "even" 69.0 miles/degree, the Earth's radius would be reduced to 3953.41 miles or a reducation of 0.09% which will be taken as acceptable for this development. To recap: We assume each degree of an arc on a great circle of the Earth corresponds to a distance of 69.0 statute miles on the Earth's surface.

Second, a circle of longitude is a great circle of the Earth passing through both the North and South Geographic Poles. Thus, there are 69 miles for every degree on longitude circles.

Third, a circle of latitude on the Earth is not a great circle, save for the 0^{o} Latitude circle which corresponds to the Equator. The radius of a circle of latitude whose latitude is L^{o} is equal to: $(R)COS(L^{o})$ where R is the Earth's radius. The circumference of such a circle of latitude L^{o} is then $(2\pi R)(COS(L^{o}))$ miles and there are $(2\pi R)(COS(L^{o}))/360$ miles per degree on such a latitude circle. But, $(2\pi R)(COS(L^{o}))/360$ is equal to $(2\pi R/360)(COS(L^{o}))$ and since $(2\pi R/360)$ equals 69 (see above), we have $(69)(COS(L^{o}))$ miles per degree of latitude on such a circle. Since latitude satisfies the inequality: $-90^{o} \leq L^{o} \leq +90^{o}$, we have the inequality: $0 \leq COS(L^{o}) \leq 1.0$ so that the number of miles per degree of a latitude circle will be less than 69, except for the Equator, $L^{o} = 0^{o}$, where there are 69 miles/degree.

Fourth, let Rx(receiver) and Tx(transmitter) be two points on the Earth's surface whose coordinates (Latitude^{o}, Longitude^{o}) are (A^{o}, B^{o}) and (D^{o}, E^{o}), respectively. Thus, diagramatically (in the Northern Hemisphere) we have Figure A below, where D is the great circle distance between Rx and Tx; d_1 is the distance measured along longitude circles B^{o} and E^{o} between latitude circles A^{o} and D^{o}; d_2 is the distance measured along latitude circle A^{o} between longitudes B^{o} and E^{o}. d is the approximation (computed below) we shall take for the actual distance D. Define: $L^{o} = 0.5(A^{o} + D^{o})$. This corresponds to the latitude circle midway between latitudes A^{o} and D^{o} (which is indicated in the above diagram). Note that d_2, in the diagram, being higher in latitude, closer to the North Pole, is less than d_3.

FIGURE: A



Define d* and d** as follows:

Since these are taken to be right triangles, from the Pythagorean Theorem we have:

$$d^{*} = SQRT(d_1^2 + d_3^2) \quad \text{and} \quad d^{**} = SQRT(d_1^2 + d_2^2)$$

where SQRT denotes the square root function. Also note that, as shown, the approximation d** is less than the approximation d* since d_2 is less than d_3 (in the Northern Hemisphere). From above we can write:

$$d_1 = 69(A^{o} - D^{o}); \quad d_2 = 69(B^{o} - E^{o})COS(A^{o}); \quad d_3 = 69(B^{o} - E^{o})COS(D^{o})$$

miles. We define a better approximation to D than d* or d**, namely:

$$d = (69)SQRT\left[(A^{o} - D^{o})^2 + \left[(B^{o} - E^{o})COS(L^{o})\right]^2\right] \quad \ldots \text{ miles}$$

where $L^{o} = \frac{1}{2}(A^{o} + D^{o})$.

Note that from the right triangles above and the Pythagorean Theorem:

$$d^{*} = (69)SQRT\left((A^{o} - D^{o})^2 + ((B^{o} - E^{o})COS(D^{o}))^2 \right) \quad \ldots \text{ miles}$$

$$d^{**} = (69)SQRT\left((A^{o} - D^{o})^2 + ((B^{o} - E^{o})COS(A^{o}))^2 \right) \quad \ldots \text{ miles.}$$

An examples of calculations for d and d**: Rx @ South Bend, IN with $(A^{o},B^{o}) = (41.62^{o},86.22^{o})$ and Tx @ Jackson, MI with $(D^{o},E^{o}) = (42.18^{o}, 84.43^{o})$. Then:

$$\begin{aligned}
d &= (69)SQRT((41.62 - 42.18)^2 + ((86.22 - 84.43)COS(\tfrac{1}{2}(41.62^{o}+42.18^{o})))^2) \\
&= (69)SQRT((-0.56)^2 + ((1.79COS(41.90^{o}))^2) \\
&= (69)SQRT((0.31360) + ((1.79)(0.74431)^2) \\
&= (69)SQRT((0.31360) + (1.33232)^2) = (69)SQRT((0.31360)+(1.77508)) \\
&= (69)SQRT(2.08868) = (69)(1.44523) = 99.72 \text{ miles.} \\
d^{**} &= (69)SQRT((-0.56)^2 + ((1.79)COS(41.62^{o}))^2) \\
&= (69)SQRT((0.31360) + ((1.79)(0.74757))^2) = (69)SQRT(0.3136+1.79063)) \\
&= (69)(1.45060) = 100.09 \text{ miles.}
\end{aligned}$$

The actual great circle distance: 99.80 miles.

We now show by examples the kind of accuracy that d, d*, d** give with respect to the actual distance D. It will be noted that if $D \leq 750$ miles or so, that d** can be used without much error which allows $COS(A^{o})$ to be computed once and for all and is thus a constant in all calculations.

EXAMPLES

Rx location is taken to be Louisville, KY: Latitude: 38.19^{o}N, Longitude: 85.52^{o}W so that $A^{o} = 38.19$, $B^{o} = 85.52$, and for d**, $COS(A^{o}) = COS(38.19^{o}) = 0.78595$ or 0.786.

Tx: Station	Latitude: D^{o}	Longitude: E^{o}	D	d	d*	d**
C64KFI	33.88	118.01	1892.12	1837.1	1884.8	1786.9
C65KORL	21.30	157.86	4382.93	4487.8	4794.3	4092.6
C65WSM	36.00	86.79	166.63	166.5	166.9	166.1
C66KFAR	64.87	147.82	3063.82	3246.6	2592.6	3847.6
C70WLW	39.35	84.33	102.58	102.5	102.2	102.8
C71KIRO	47.40	122.44	1954.80	1974.4	1837.7	2100.7
C72WGN	42.01	88.04	295.46	295.2	293.5	296.9
C76WJR	42.17	83.22	300.42	300.2	298.7	301.6
C82WBAP	32.94	96.99	738.54	738.7	756.6	719.8
C88WCBS	40.86	73.99	640.71	640.7	629.3	651.9
C88KRVN	40.52	99.39	757.06	757.3	745.1	769.2
C102KDKA	40.56	79.95	339.32	339.1	334.7	343.5
C112KMOX	38.72	90.05	247.68	247.5	246.6	248.4
C118KOFI	48.20	114.25	1590.59	1605.9	1490.9	1704.3
R137WDEF	35.04	85.34	217.77	217.6	217.6	217.6
R160WKWF	24.58	81.73	965.84	965.3	968.7	961.3

so for D on the order of 1000 miles, the approximation d is "pretty good".

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