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COMMENT ON FJP EQUATION FOR ARRIVAL-TAKEOFF ANGLES ------- de Ghoti, Ph.Dx

FJP published an article on the topic of skyline-horizon blockage in both DXN and DXN and gave there the derivation of the following formula for the angle of arrival or angle of takeoff for a signal travelling via an N-mode (an ionosphere-Earth hop):

$$A = ASIN\ \frac{(R+H)COS(X) - R}{SQRT(2R(R+H)(1-COS(X)))}\quad \text{degrees}$$

where R is the Earth's radius and H is the height of the point of "reflection" in the ionosphere and are both of the same linear measure; e.g., miles or kilometers. Typical values (miles) are R = 3957, H(F-layer) = 186 and H(E-layer) = 65. X is half the angle subtended at the Earth's center by the great circle path of the signal of distance D. X is measured in degrees and for D in miles; X = D/(138.12k) where k is the number of hops the signal takes to travel from the transmitter to the receiver. The maximum distance (miles) a signal can travel by one hop in an N-mode is given by: D_{max} = *138.12 ACOS(R/(R+H)) where R and H are defined as above and of the same linear measure.*

FJP's original derivation contains an error: viz., in Equation 4 of his work in simplifying the Law of Cosines applied to his illustration. Specifically:

$$\text{Eq. 4:}\quad L^2 = R^2 + (R+H)^2 - 2R(R+H)COS(X) \neq 2R(R+H)(1-COS(X))$$

where X denotes the angle above (FJP's symbol here is theta). The error is denoted by the absence of the unequal sign in the above equation. Thus, properly stated, FJP's equation becomes:

$$A = ASIN\ \frac{(R+H)COS(X) - R}{SQRT(R^2 + (R+H)^2 - 2R(R+H)COS(X))}\quad \text{degrees.}$$

This equation can be replaced by a perhaps simpler one which has been derived in PATTERNS IV (an NRC Monograph) which also considers such genetic modes of propagation. There are a multitude of eqations which follow from applying the trigonometric equations to such a problem and from PATTERNS IV, the following is offered:

$$A = ATAN\left(\frac{H + R(1-COS(X))}{(R)SIN(X)}\right) - X \quad \text{degrees.}$$

The error introduced by the published equation given by FJP increases as the angle X decreases for a fixed height H. Thus values given by that formula tend to be larger than the true value of arrival/takeoff, A.

Finally, it should be noted that all trigonometric arguments (dependent variables) and all trigonometric inverse values (independent variables) are measured in degree measure for the equations given above.

The article on TRANS-POLAR DX published in DXN uses the original FJP equation and thus some errors are introduced. For those interested, the following table from that article is reproduced with correct values entered below:

STATION	D(miles)	2F2	3F2	4F2	5F2	6F2	7F2	5E	6E	7E
Urumchi	6472	n	1.7	6.8	11.0	14.7	18.1	0.7	2.6	4.2
Khabarovsk etc.	6500	n	3.0	8.2	12.5	16.4	13.9	1.5	3.4	5.1
Petropavlovsk	5163	n	5.7	11.0	15.6	19.8	23.6	3.0	5.0	6.8
Bulun	4850	n	6.8	12.3	17.0	21.3	25.2	3.7	5.7	7.6
Sredne-Kolumsk	4550	0.8	8.0	13.6	18.5	22.9	26.9	4.4	6.5	8.4
Helsinki	4155	2.4	9.7	15.5	20.5	25.2	29.4	5.4	7.6	3.8
Murmansk	4000	3.0	10.4	16.4	21.6	26.2	30.4	5.8	8.0	10.1
Reykjavik	2683	10.3	18.9	26.1	32.2	37.5	42.1	11.0	13.8	16.4
Thule	2534	11.4	20.2	27.6	33.3	33.2	43.9	11.8	14.7	17.5
Calcutta	7921	n	n	3.2	7.2	10.6	13.7	n	0.5	2.1
Bangkok	8452	n	n	2.1	6.0	9.4	12.4	n	n	1.4

with R = 3957 miles; $H(F_2)$ *= 186 miles and H(E) = 62 miles. It is also proper to round off to the nearest whole degree due to the uncertainty of the path geometry in real ionospherics—in fact variants of several degrees are possible.*