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PGA NEWS

CONTRIBUTIONS

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PGA News

BENJAMIN B. BAUER APPOINTED TO HEAD AUDIO AND ACOUSTICAL RESEARCH FOR CBS LABORATORIES

The appointment of Benjamin B. Bauer to head the Section for Audio and Acoustical Research in the CBS Laboratories, effective immediately, was announced August 15 by Dr. Peter C. Goldmark, President of CBS Laboratories Division.

In his new position, Mr. Bauer has assumed responsibility for the laboratory section dealing with research and development in the fields of audio and acoustics. It is this part of CBS Laboratories which is concerned with advanced development and research for Columbia Records, also a division of the Columbia Broadcasting System, Inc.

Prior to joining CBS, Mr. Bauer was with Shure Brothers, Inc., Chicago, Ill., manufacturers of electroacoustical devices, where he was Vice-President in charge of Engineering and Research. He joined Shure Brothers in 1936 prior to his graduation from the University of Cincinnati in 1937.

He was responsible for the development of a great many electroacoustical devices such as microphones, phonograph pickups, and tape recording heads. He has also made numerous inventions in the fields of audio and acoustics and has published over thirty papers covering a wide range of the entire audio and acoustic field.

He is a Fellow of IRE, the Acoustical Society of America, and the Audio Engineering Society. He is also associate editor of the *Journal of the Acoustical Society of America* and in 1955, as Cofounder and past National Chairman of the IRE Professional Group on Audio, was recipient of the group's "Achievement Award." Mr. Bauer is also a past editor of the IRE TRANSACTIONS ON AUDIO and is at present the Secretary-Treasurer of PGA. His column, "With Other Acoustical and Audio Societies," is a regular contribution to these TRANSACTIONS. Mr. Bauer is an honorary member and recipient of the "Recognition Award" of Eta Kappa Nu and an honorary member of Tau Beta Pi and Sigma Xi. As an active member of the American Management Association, he has conducted seminars on various phases of research administration.

Mr. Bauer is married and has two children.

PGA CHAPTER ACTIVITIES

Dayton, Ohio

At a meeting held on October 3 at the Engineers Club, Edgar Villchur, of Acoustic Research, Inc., presented a paper on "Problems of Bass Reproduction in Loud Speakers." Villchur's paper covered some of the special problems associated with harmonic distortion, power capability, frequency, and transient response for operation in the bass range. He dealt with the three basic approaches to the solution of these problems through horn-loaded, resonant, and direct-radiator systems. Particular emphasis was given to the acoustic suspension system.

The new officers for this Chapter are: A. J. Falkowski, *Chairman;* J. S. Stanton, *Vice-Chairman;* E. O. Valentine, *Secretary.*

San Francisco, Calif.

A paper on the "History, Tonal Design, and Acoustical Environment of the Modern Church Pipe Organ" was presented at the St. Mark's Episcopal Church, 600 Colorado Avenue, Palo Alto, on September 17. The speakers were Roy Long, Dick Stenger, and Lambert Dolphin, of the Stanford Research Institute. A demonstration and recital on the new Casavant organ was provided by C. Thomas Rhoads, church organist at St. Mark's.

New officers for the San Francisco Chapter are: Don L. Broderick, *Chairman;* John Leslie, *Vice-Chairman;* Lambert Dolphin, *Secretary-Treasurer.*

Mr. Broderick was born in Chico, Calif., in 1928. He enlisted in the U. S. Navy in 1945 and served as an electronic technician's mate. After his discharge he attended the University of California and received the B.S. degree in electrical engineering in 1950. Formerly employed with Boeing Airplane Company in Seattle, Mr. Broderick is now a development engineer in the laboratory of Hewlett-Packard Company.

Mr. Leslie has been associated with Ampex Corporation since 1950. He participated in the development of broadcast studio-quality audio recorders, multichannel, fm-carrier data recorders, and most recently, video recorders. He was born in Springfield, Mo., in 1921. In 1940 he entered the University of California, but his studies were interrupted by service in the U. S. Navy as an instructor of radio and radar at Treasure Island. He returned to the University of California after the war and received the B.S. degree in electrical engineering in 1949. He was formerly employed with Bendix Radio and Television in Towson, Md. Mr. Leslie is a member of the Society of Motion Picture and Television Engineers.

Mr. Dolphin was born in Shoshone, Idaho. He received the A.B. degree in physics from San Diego State College in 1954. His college studies included considerable work in electronics, in addition to a minor in mathematics. He has since completed two years of graduate work in physics at Stanford University with a minor in electrical engineering. In June, 1956, he joined Stanford Research Institute as a research physicist in the special techniques group of the radio systems laboratory where he has been engaged in radio propagation studies. He is a member of Sigma Phi Sigma.

TAPESCRIPT COMMITTEE ACTIVITIES

The best way to utilize tapescripts for local presentation is to have someone review the material and prepare a short discussion of the subject. This person should be prepared and qualified to answer questions that may come up. It is of the utmost importance that those presenting a tapescript run through the material from a purely mechanical standpoint to make sure they have the copies expected and that they have the technical equipment to reproduce the sound and picture.

Tapescripts are loaned by the IRE-PGA Tapescript Committee. The only cost is the return postage on the material. It is very important for the program chairman to request a tapescript in advance so he may review the material and prepare his program.

Technical standards for tapescripts are as follows: Sound on $7\frac{1}{2}$ inches per second, $\frac{1}{4}$-inch tape, full track on 7-inch reels. The slides are 2×2-inch cardboard mounts, double 35-mm slides. In a few cases where a limited number of copies are available $3\frac{1}{4}\times 4$-inch slides are used.

The following tapescripts are available.

"Phonograph Reproduction," B. B. Bauer, Shure Brothers, Inc. Grooves and needles, fidelity and efficiency, pickup arms, and recording-reproducing characteristics are some of the subjects discussed in this one-hour recorded paper.

"An Experimental Cochannel Television Booster Station Using Crossed Polarization," J. H. DeWitt, Jr., WSM, Nashville, Tenn. The special receiving and transmitting antennas and equipment developed to improve fringe and reception by cochannel booster methods are discussed. A report on the operation of an experimental installation at Lawrenceburg, Tenn., is given.

"Method for Time or Frequency Compression-Expansion of Speech," G. Fairbanks, W. L. Everitt, and R. P. Jaeger, University of Illinois.

"Magnetic Recording," M. Camras, Armour Research Foundation of Illinois Institute of Technology. Discusses fundamentals of wire and tapes; heads; bias; circuits; equalization; present problems; and future developments. Thirty minutes.

"Push-Pull Single-Ended Audio Amplifier," A. Peterson and D. B. Sinclair, General Radio Company. A convention paper presented by Dr. Peterson. Twenty-five minutes. $3\frac{1}{4}\times 4$-inch slides.

"The Electrostatic Loudspeaker—An Objective Evaluation," R. J. Larson, Jensen Manufacturing Co., Chicago, Ill. The condenser loudspeaker, following development of new materials and methods, is now practical for high-frequency use in multichannel loudspeaker systems. Theoretical and mechanical design considerations illustrate the limitations at this stage of the art, including inherently high distortion at high output levels, and inability to withstand overloads. Principal advantages are low cost and efficient reproduction at extremely high frequencies. Demonstration will illustrate the operation of a typical electrostatic "tweeter" in combination with lower frequency dynamic loudspeaker channels.

"Efficiency and Power Rating of Loudspeakers," R. W. Benson, Armour Research Foundation, Chicago, Ill. The specification of the performance of loudspeakers is a subject of international controversy at the present time. Several groups in this country are concerned with standardized methods of evaluating the performance of loudspeakers in order to have a basis for comparison. Several European countries are also concerned with similar problems. The measurement of the response-frequency characteristic and directional characteristics of loudspeakers are routinely performed by various laboratories. Sufficient agreement can be attained by the various laboratories to standardize this measure of performance. It is important to have a measure of the efficiency, that is, sound power output vs electrical power input, and also a method of specifying the power-handling capacity of a loudspeaker. Various methods are in use at the present time to indicate the characteristics of a loudspeaker concerning these two measures. The method of using a reverberation chamber in integrated acoustic power output and thus determine the efficiency will be discussed in comparison to the more tedious method of analytical integration of measurements performed in free space. Power-handling capabilities of loudspeakers as determined by distortion measurements and mechanical or electrical failure will also be discussed. A summary of the various methods of specifying both efficiency and power-handling capacity as used in various laboratories in both the United States and Europe will be included.

"Sound Survey Meter," A. Peterson, General Radio Co. A convention paper. Twenty minutes. $3\frac{1}{4}\times 4$-inch slides.

"Microphones for High Intensity and High Frequencies," J. K. Hilliard, Altec Lansing Co. A convention paper. Twenty minutes. $3\frac{1}{4}\times 4$-inch slides.

"The Ideo-Synchronizer," J. M. Henry and E. R. Moore, Boston Bell. A humorous satire on technical writing, specifications, and engineering. Good for mixed audience. Twelve minutes.

"An Improved Optical Method for Calibrating Test Records," B. B. Bauer, Shure Brothers Inc., Chicago, Ill.

"Electronically-Controlled Audio Filters," L. O. Dolansky, Northeastern University, Boston, Mass.

"Energy Distribution in Music," J. P. Overley, Radio Manufacturing Engineers, Inc., Peoria, Ill. A knowledge of the manner in which the acoustic power encountered in music varies with respect to frequency can be a useful tool in the decision of components to be used in audio reinforcement or reproduction systems. The author believes that energy distribution information available up to this time is not fully applicable for such a use because it concerns primarily the average energy distribution. This paper deals with the amplitude of fractional-second energy peaks, without reference to the rate of their occurrence. It is these peaks which must be considered when distortion is of primary consideration; average power is useful only in predicting temperature rise (where applicable) of signal-handling components. Throughout the discussion, emphasis is placed upon the difference between average and peak energy considerations. The source material from which the distribution analysis is drawn consisted of recent commercial vinyl recordings played on a carefully equalized reproducing system. Ten various types of music are classified and a distribution curve for each is drawn. Methods used in arriving at a typical curve are shown by breaking the spectrum into octaves with a band-pass filter. This distribution information mentioned above is applied to design of a three-channel loudspeaker system as an example of use. Other possible applications are mentioned.

"Bells, Electronic Carillons and Chimes," F. H. Slaymaker, Stromberg-Carlson Co., Rochester, N. Y. Bells and chimes, unlike the more familiar string and wind instruments, produce tones in which the overtone structure cannot be expressed as a series of harmonics. The accuracy of tuning of the various overtones varies widely but the better cast bell carillons, electronic carillons, and tubular chimes do have very accurately tuned overtones. This paper describes the results of measurements on cast bells, electronic carillons, and tubular chimes. The individual overtones of the three types of instruments will be demonstrated on a loudspeaker and data will be given on relative amplitude and decay rates of the various overtones. The reaction of "out-of-tuneness" will be discussed and explained. A new type of tone source for electronic carillons will be described and demonstrated. With this new tone source, a rod of carefully controlled rectangular cross section is used which can give in a single unified structure essentially the same overtones array as that of an accurately tuned cast carillon bell. The new rectangular bar tone source will be demonstrated.

ANDREW B. JACOBSEN

WITH OTHER ACOUSTICAL AND AUDIO SOCIETIES

The April, 1957, issue of *The Journal of the Acoustical Society of America* contains four papers from the Second International Commission on Acoustics Congress, which was held at Massachusetts Institute of Technology and Harvard University in June, 1956. Two papers from this issue will be of interest to those working with transducers and ultrasonics.

Dieter Goetze of Raytheon Manufacturing Company describes "Effect of Pressure between Tool Tip and Workpiece on the Rate of Ultrasonic Machining in Ketos Tool Steel." In this paper data are presented for the ultrasonic machining rate in Ketos tool steel obtained by using rectangular tool tips having different perimeters and circular tool tips having different radii. It is shown that the pressure rather than the force maintained between the tool tip and the workpiece is pertinent in the ultrasonic machining process, and that the ratio C/A of the tool tip perimeter C to the tool tip area A is of importance. It is also shown that for rectangular and circular tool tips a critical machining rate V_0 and a critical pressure P_0 exist, the values of which are related to the ratio C/A by the two simple expressions $V=2\xi\nu dM(C/A)$ and $P_0=L(C/A)$, where 2ξ and ν are the peak-to-peak amplitude and the frequency of vibration, respectively, d is the abrasive particle diameter, and M and L are parameters independent of 2ξ, ν, d, C, or A. On the basis of the above two equations, a phenomenological expression is derived which for circular and rectangular tool tips relates the machining rate in general to the pressure in general.

"Dynamic Magnetostrictive Properties of Ni-Fe Alloys" was contributed by C. M. Davis, Jr., H. H. Helms, and S. F. Ferebee of the U. S. Naval Ordnance Laboratory, Silver Spring, Md. The dynamic magnetostrictive properties of Ni-Fe alloys containing from 35 per cent to 67.5 per cent nickel were investigated to determine their suitability for use in electromechanical transducers. The material, in the form of toroids made from ring laminations, was evaluated by the motional impedance method. Annealing temperatures were varied from 600°C to 1220°C, and the effect of various annealing techniques was investigated. Measured values of the electromechanical coupling coefficient, the reversible permeability, the dynamic magnetostrictive constant, and other parameters are given.

This issue contains, in addition, fifteen regular contributions on ultrasonics, underwater sound, music, speech, and general theory.

The May, 1957, issue of *The Journal of the Acoustical Society of America* contains nine papers from the abovementioned ICA Congress, several of which will be of interest to PGA members.

Richard V. Waterhouse of the National Bureau of Standards contributed an interesting paper entitled "On Standard Methods of Measurement in Architectural Acoustics." Suggestions are made pertaining to standard methods of measuring the sound absorption of acoustical materials and the sound transmission loss (stl) of partitions. 1) Concerning the reverberation chamber method of measuring the sound absorption of materials, it is shown that the absorption varies with the position of the sample in the chamber, owing to interference patterns. An estimate is made of the difference in absorption of a sample on the floor both centrally and touching one side wall. It is suggested that sample position should be specified as distant at least half a wavelength ($\lambda/2$) from walls and reflecting surfaces other than the one backing the sample. 2) Regarding the measurement of stl it is suggested that microphone position be specified as at least $\lambda/2$ from walls and the corresponding distances from edges and corners of reverberation chambers. The technique in which the microphone is placed near a wall is not recommended. 3) As a single figure for the over-all sound insulation value of a partition, the arithmetic mean of the decibel stl figures at various frequencies is often used. It is suggested that a better average figure is obtained if the linear ratios corresponding to these decibel figures are averaged instead of the decibel figures themselves. This removes the logarithmic weighting of the latter average, for which there is no physical justification.

E. Zwicker, B. Flottorp, and S. S. Stevens of the Psycho-Acoustic Laboratory at Harvard University have contributed an interesting paper on "Critical Band Width in Loudness Summation." The concept of the critical band, or *Frequenzgruppe*, is shown to apply to loudness summation. When the spacing between a group of pure tones is increased, the loudness remains constant until a critical point is reached, after which the loudness increases. The same effect occurs when the width of a band of noise of constant spl is made larger. The critical bandwidth at which loudness summation begins to depend on the spread of energy is approximately the same as the critical bandwidth determined previously by methods involving thresholds, masking, and phase. The critical band as measured directly by these three methods (plus the method of loudness summation) is about two and a half times as wide as the critical band derived from the assumptions made by Fletcher, but its dependence on frequency is approximately the same. The relation of the critical band to other functions is noted.

Yoshimitsu Kikuchi of Tohoku University, Sendai, Japan, writes on "Performance of Magnetostrictive Transducers Made of Aluminum-Iron Alloy or Nickle-Copper Ferrite." A brief history of the finding and development of aluminum-iron alloy introduces the description of our industrial specification as finally decided upon by the organized group. The description covers the specification for the melting and rolling processes, both of which are main factors affecting the magnetostriction characteristics. Improvement of transduction efficiency is then discussed in general, and it is concluded that no other material which has larger coupling factor than that of nickel, or "Alfer," (*i.e.*, the 13.5 per cent aluminum-iron alloy) is necessary, in so far as the transducers to be used at their resonance are concerned. Improvement in the efficiency can be attained only by the considerable elimination of the eddy current, and this is accomplished by our several magnetostrictive transducers made of Ni-Cu ferrites which are shown to be excellent for generating underwater ultrasound. Electroacoustic efficiency is as high as 90 per cent even at such higher frequencies as 70 and 100 kc. Intense generation of ultrasonic waves is successfully carried on up to the rate of 3 w/cm^2 of the radiation surface.

"Research on the Acoustic Air-Jet Generator: A New Development" was contributed by E. Brun, Université de Paris, Paris, France, and R. M. G. Boucher, Collaborateur Scientifique de Ministere de l'Air, Paris, France. The structure and operation of the Hartmann air-jet ultrasonic generator has been reviewed critically, and modifications are described which substantially increase the efficiency and available power output of the original device. Experimental results are presented in confirmation of the theoretical analysis. An important part of the development has been the employment of a secondary resonator and projecting exponential horn. A novel design using a large number of whistles in a single horn (Multiwhistle R.B.) is described. This unit has a wide frequency and power range and has been employed in France for agglomeration of aerosols, ultrasonic drying, fog dispersal, and other interesting applications.

In addition, this issue contains ten regular contributions.

In an interesting letter to the editor, "On the Low-Frequency Radiation Load of a Bass-Reflex Speaker," R. H. Lyon of the Department of Electrical Engineering, University of Minnesota, presents a novel approach to the analysis of this device.

Both issues contain "Reviews of Acoustical Patents" by Robert W. Young, and the May issue contains "References to Contemporary Papers on Acoustics" by Robert N. Thurston, which appears in the odd-numbered issues of the *Journal*.

BENJAMIN B. BAUER

Principles of Loudspeaker Design and Operation*

JOSEPH CHERNOF†

Summary—The electrical and physical parameters which are of interest in loudspeaker design are discussed. The analysis of loudspeaker action on the basis of its analogy to a vibrating rigid disk is presented. The limitations of such an analysis are indicated. Electromechanical analogies are used to formulate an equivalent electromechanical circuit from which loudspeaker performance factors can be derived. Design criteria for commercial and "hi-fi" units are developed. Means of improving loudspeaker performance, particularly at low audio frequencies, are discussed.

List of Symbols

D = differential operator d/dt.
F = force.
f = frequency.
w = angular frequency.
R = radius.
ρ = density.
σ = resistivity.
c, a = velocity of sound in free air.
J_1 = Bessel's function of the first order.
H_1 = Struve's function of the first order.
μ = restoring force.
r = radial distance.
ξ, x = displacement.
k = wave number $= w/c$.
A = area.
M, m = mass.
m_e = effective mass.
s = stiffness.
$R = R_{vc}$, dc resistance of voice coil.
$L = L_{vc}$, self-inductance of voice coil.
$\left.\begin{matrix} R_m \\ L_m \\ C_m \end{matrix}\right\}$ = motional quantities referred to equivalent series circuit.
$\left.\begin{matrix} R_{m'} \\ L_{m'} \\ C_{m'} \end{matrix}\right\}$ = motional quantities referred to equivalent parallel circuit.
C = electromagnetic conversion factor.
r_e = equivalent mechanical resistance.
m_i = accession to inertia.
R_A = amplifier output impedance.
η = output transformer turns ratio.
w_r = mechanical resonant frequency.
z_e = mechanical impedance.
P = power.
r_r = mechanical radiation resistance.
η_m = mechanical efficiency.
η_a = acoustic efficiency.
m_c = voice coil mass.
p = sound pressure.
B = flux density.
l = length of voice coil conductor.

* Manuscript received by the PGA, July 15, 1957; revised manuscript received, October 18, 1957.
† Bell Aircraft Corp., Buffalo, N. Y.

Introduction

THE loudspeaker is one of a class of electroacoustical transducers which includes the microphone and telephone transmitter and receiver, as well as magnetostrictive elements used in the propagation of sound waves under water. The loudspeaker is actuated by electrical signals to produce acoustical energy through the mechanical vibrations of a radiating element. The two general classifications for loudspeakers are the horn and direct radiator types. In the horn type, a small vibrating element is coupled to the air through a horn structure which is usually approximately exponentially flared. The direct radiator type has a comparatively large radiating element which is directly coupled to the air. This paper discusses the principles of operation of direct radiator loudspeakers with particular emphasis on obtaining adequate response in the low-frequency portion of the audio spectrum. To eliminate baffle effects, the speaker is considered to be mounted in an infinite baffle throughout the discussion.

The first patent on a direct radiator loudspeaker was issued to Siemens in 1877.[1] It was intended to be used in telephone equipment, and as outlined in the original patent application, bears a striking resemblance to present-day units. The modern inertia controlled speaker was first described by Rice and Kellogg in 1925.[2]

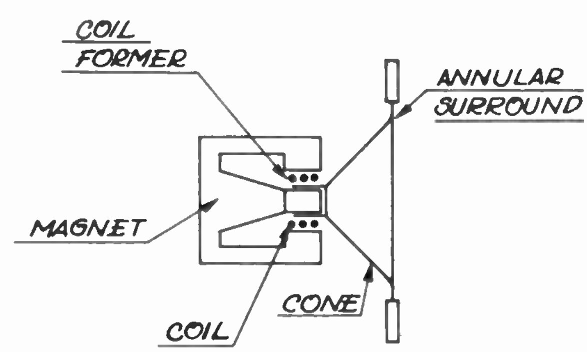


Fig. 1—The direct radiator loudspeaker.

In its simplest form, the direct radiator loudspeaker may be represented as in Fig. 1. A conical shell or diaphragm formed from cloth or paper is driven axially by a moving coil, or voice coil. The voice coil is supported on a coil former which is fastened to the inner radius of the conical shell. A centering device is provided to insure axial motion of the coil, thus preventing wobble.

[1] M. S. Corrington, "75th anniversary of the first dynamic loudspeaker," *Audio Eng.*, pp. 12–13; February, 1953.
[2] C. W. Rice and E. W. Kellogg; "Notes on the development of a new type of hornless loud speaker, *J. AIEE*, vol. 44, pp. 982–991, 1017–1020; September, 1925.

The conical shell is supported at its outer radius by a flexible annular surround, which in turn is fastened to a metal supporting frame. The voice coil is situated in the circular air gap of a permanent magnet or electromagnet which creates a radial flux in the air gap. Thus, when a current passes through the coil, the coil, coil former, and diaphragm are driven as a single unit by the reaction of the magnetic field. The suspension, consisting of the annular surround and centering device, represents a mechanical constraint on the vibrating system and must possess linear force-displacement characteristics over the expected amplitude range of the diaphragm. The mechanical elements represented by diaphragm and coil, the mechanical constraint, and the dissipation elements of friction and radiation of sound energy form a simple one degree of freedom vibrating system in the low-frequency operating range of the loudspeaker. Mechanical resonance of this vibrating system is a major factor in speaker design and performance.

The Vibrating Rigid Disk

The operation of the mechanical portion of the direct radiator loudspeaker is usually analyzed by analogy with the behavior of the rigid circular plate vibrating harmonically in an equal circular aperture cut out of an infinite plane plate. This provides a good approximation to loudspeaker performance up to frequencies of the order of 1 kc. Above this frequency, the diaphragm ceases to behave as a rigid disk, as it proceeds to break up into several modes of oscillation. This treatment by analogy is necessary since the solution of the vibrational problem for the conical shell has not been accomplished as yet.

The behavior of the vibrating rigid disk has been presented very completely by Rayleigh.[3] He has found the reaction due to air presented to one side of the vibrating plate in an infinite plane to consist of a frictional force:

$$F_f = a\rho\pi R^2\left(1 - \frac{J_1(2kR)}{kR}\right) D\xi, \tag{1}$$

and an inertia force of

$$F_i = \frac{\rho\pi}{2k} K_1(2kR)\, D^2\xi. \tag{2}$$

K_1 is not a Bessel function, but is rather related to Struve's functions by

$$K_1(z) = zH_1(z), \tag{3}$$

where H_1 is Struve's function of the first order.

If the actual mass of the vibrating plate is M and the restoring force due to the mechanical constraints is $\mu\xi$, the equation of motion for the plate when acted upon by an impressed force F, proportional to $e^{i\omega t}$, will be as given by Rayleigh:

$$\left(M + \frac{\rho\pi}{2k^3} K_1(2kR)\right) D^2\xi + \rho a\pi R^2\left(1 - \frac{J_1(2kR)}{kR}\right) D\xi + \mu\xi = F. \tag{4}$$

In the region of interest, below 1 kc, kR is usually sufficiently small that (4) may be rewritten as

$$\left(M + \frac{8\rho R^3}{3}\right) D^2\xi + \tfrac{1}{2}\pi a\rho k^2 R^4 D\xi + \mu\xi = F. \tag{5}$$

Rayleigh[4] gives the power radiated from one side of the disk as

$$P = \frac{A^2 w^4 \xi^2}{2a\pi}, \tag{6}$$

for a diaphragm small enough in comparison to the shortest sound wavelengths to be reproduced that it may be considered to behave as a point source. The radiated power can also be expressed as the product of the resistive (functional) component of the air load and the square of the velocity of the system, and is thus given by

$$P = r_A D\xi = \frac{(a\rho k^2 R^4)}{2} D\xi, \tag{6a}$$

in the frequency region of interest, $(kR < 0.5)$.

It should be noted that m_i, the accession to inertia, is relatively constant at low frequencies, starting to drop off rapidly for $kR > 1.5$, and becoming negligible with respect to M for larger values of kR. However, r_A, the resistive component of radiation resistance, is proportional to the square of the frequency in the low-frequency region. For large values of $kR(kR > 1.9)$, $J_1(kR)$ tends to vanish so the radiation resistance becomes

$$r_A = a\rho\pi R^2, \tag{7}$$

a constant for a given cone configuration.

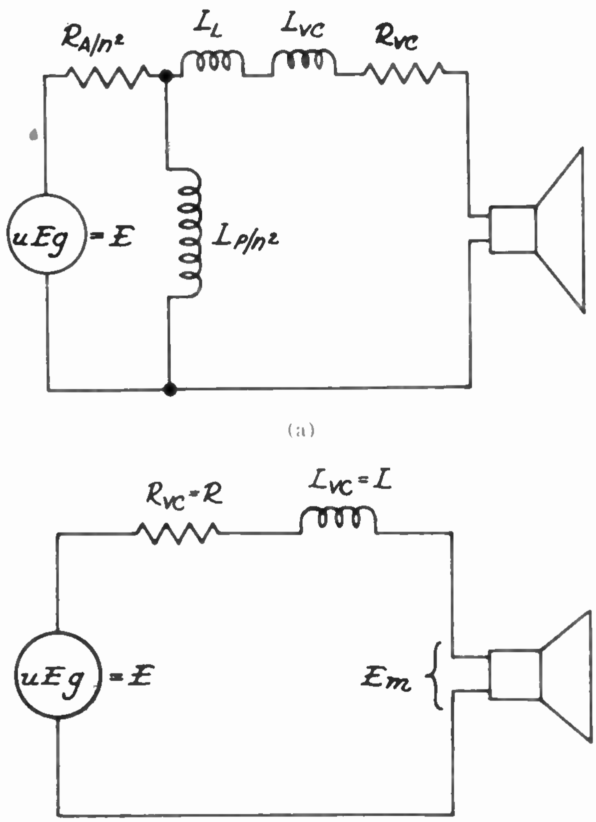
The Electromechanical Circuit

It is useful to analyze the operation of the loudspeaker in terms of electromechanical analogies; that is, in terms of equivalent electrical and mechanical circuits. To facilitate this procedure, frequently used electrical quantities and a set of mechanical equivalents are tabulated below. A very excellent discussion of this technique is found in a paper by Firestone.[5]

[3] Rayleigh, "Theory of Sound," Dover Publications, Inc., New York, N. Y., vol. II, pp. 162–169; 1945.

[4] *Ibid.*, p. 113.

[5] F. A. Firestone, "Mobility method of computing the vibration of linear mechanical and acoustical systems: mechanical-electric analogies," *J. Appl. Phys.*, vol. 9, pp. 373–387; June, 1938.



(a)

(b)

Fig. 2—(a) Equivalent electrical circuit for loudspeaker, (b) simplified equivalent electrical circuit for loudspeaker.

Electrical	*Mechanical*
Voltage $=E$,	Force $=F$,
Current $=I$,	Velocity $=v$,
Charge $=Q$,	Displacement $=\xi$, x,
Resistance $=R$,	Resistance $=r$,
Reactance $=X$,	Reactance $=x$,
Impedance $=Z$,	Impedance $=z$,
Inductance $=L$,	Mass $=M$, m,
Capacity $=C$.	Compliance $=1/$Stiffness $=1/s$.

Electromechanical conversion factor $C=Bl=$ the product of the length of wire and the mean magnetic flux density in the air gap of the magnet in absolute electromagnetic units. C is the mechanical force on the coil per unit current (absolute) or the voltage induced in the coil per axial velocity. It should be noted that, in magnitude, one electrical ohm $=10^9$ mechanical ohms, as a unit of resistance.

On the basis of the following assumptions (after McLachlan),[6] the equivalent electrical and mechanical circuits for the loudspeaker and driving amplifier are as shown in Fig. 2(a) and Fig. 3.

1) The diaphragm and driving coil behave dynamically as a rigid structure.
2) The linear axial constraint of stiffness represents the net effects of the suspension, consisting of annular surround and centering device.
3) The resistive force due to sound radiation and mechanical losses is proportional to the axial velocity.
4) Mutual inductance and capacity between the voice coil and magnet are negligible over the frequency range of interest.
5) The radial magnetic field is uniform throughout the travel of the coil and is undistorted by the current.

[6] N. W. McLachlan, "Loud Speakers," Oxford University Press, New York, N. Y., ch. 7; 1934.

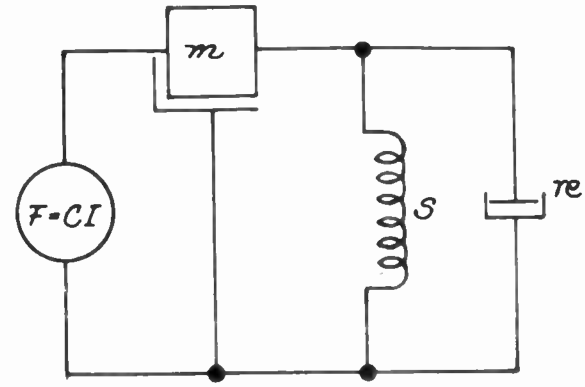


Fig. 3—Simplified equivalent mechanical circuit for loudspeaker.

If we further assume that the driving power is being supplied from a tube and transformer of 1:1 turns ratio without losses or leakage, the electrical circuit shown in Fig. 2(b) is obtained. Considering the electrical voltage drops in the circuit:

$$LDi + Ri + E_m = E \tag{8}$$

but,

$$E_m = CDx \tag{9}$$

from the definition of the electromechanical conversion factor. $D=d/dt$ and E_m represent an induced voltage in the voice coil due to its motion through the magnetic field. Eq. (8) can thus be rewritten as

$$LDi + Ri + CDx = E. \tag{10}$$

A differential equation similar to (5) can be written for the mechanical circuit shown in Fig. 3:

$$mD^2x + r_eDx + sx = Ci, \tag{11}$$

where $m=$ the natural mass of the coil, coil former, and diaphragm + accession to inertia, $r_e=$ the effective mechanical resistance of the vibrating system = mechanical losses + radiation resistance, and s is the stiffness or spring constant of the suspension.

On the steady-state basis, we can substitute $D=i\omega$, so that (10) and (11) become

$$i\omega LI + RI + i\omega Cx = E, \tag{12}$$

and

$$-m\omega^2x + ir_e\omega x + sx = CI. \tag{13}$$

If we solve (13) for x and substitute in (12), we obtain

$$E = IR + iwL + \frac{C^2}{r_e + i(wm - s/w)}. \tag{14}$$

Since $E = IZ$, the effective electrical impedance, Z, is immediately obtainable from (14). This impedance may be written symbolically as

$$Z = (R + R_m) + iw(L + L_m) \tag{15}$$

where R_m and L_m are the electrical motional resistance and inductance, respectively. R_m and L_m have no physical meaning. Their existence is due solely to the motion of the voice coil in the magnetic field and the emf thereby induced in the coil. R_m represents the electrical equivalent of the mechanical resistance of the vibrating system. These quantities may be determined experimentally as the difference between the measured effective resistance and inductance with the driving mechanism in motion, and the effective resistance and inductance with the driving mechanism blocked. In terms of the mechanical speaker parameters:

$$R_m = \frac{r_e C^2}{r_e^2 + (wm - s/w)^2} = \frac{r_e C^2}{z_e^2} \tag{16}$$

$$L_m = \frac{(s/w^2 - m)C^2}{z_e^2}. \tag{17}$$

The motional inductance, L_m, can be positive, negative, or zero, depending on the relative values of angular frequency, w; mass, m; and stiffness, s. The negative condition can be treated as a capacitance effect, and the resulting motional capacity is expressed as

$$C_m = \frac{1}{w^2 L_m}. \tag{18}$$

The equivalent series electrical circuit can now be represented as in Fig. 4.

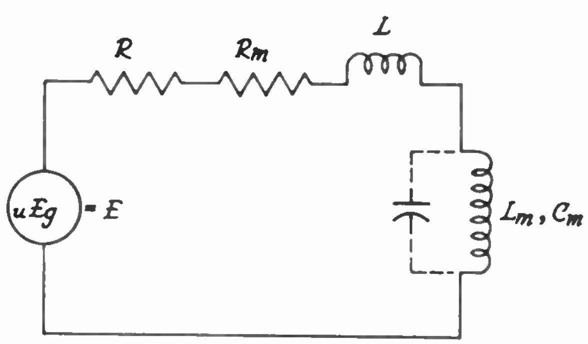


Fig. 4—Equivalent series electromechanical circuit for loudspeaker.

It can be seen from (11) that the natural resonant frequency of the mechanical system occurs when $w_r = \sqrt{s/m}$, or $m = s/w^2$, as in any simple linear mechanical system consisting of mass, spring, and damping. The reactive portion of the mechanical impedance may be considered as an effective mass, where

$$m_e = m - s/w^2. \tag{19}$$

At resonance, $m_e = 0$. Above resonance, the mass reactance predominates, while below resonance $m_e \to$ infinity as $w \to$ zero, and the stiffness reactance predominates. It should be noted that the static inductance L and the motional capacity C_m form a resonant circuit. In practice, this resonant frequency is inconspicuous.[6]

The mechanical impedance having been defined as $[r_e^2 + (wm - s/w)^2]^{1/2}$, it is possible to also define a mechanical power factor,

$$\cos\theta = \frac{r_e}{z_e}, \tag{20}$$

this being the phase angle between the driving force and the resulting axial cone velocity. Cos θ is also the motional electrical power factor since θ also represents the phase angle betseen the driving current and the induced voltage in the voice coil, or

$$\cos\theta = \frac{R_m}{Z_m}. \tag{21}$$

From (21) and (20),

$$\cos^2\theta = \frac{r_e R_m}{z_e Z_m} \tag{22}$$

$$R_m = \frac{r_e C^2}{z_e^2} \tag{23}$$

$$= \frac{C^2 \cos^2\theta}{r_e}. \tag{24}$$

From (22) and (24), we finally obtain

$$C^2 = z_e Z_m, \tag{25}$$

further indicating the significance of the electromechanical coupling factor C.

Inertia Control

Present-day direct radiator loudspeakers are almost exclusively of the inertia controlled type. That is, the effective mass reactance of the conical diaphragm is large compared to the stiffness reactance and radiation resistance at frequencies above w_r. It can be seen from (6) that for a constant acoustic power output over the piston range of the diaphragm, w^4x^2 must remain constant. If (6) is rewritten for the case where $wm \gg r$ and s/w:

$$w^2mx = CI = \text{the driving force } F, \tag{26}$$

or

$$w^2x = F/m. \tag{27}$$

This latter expression indicates that with a constant driving force over the operating frequency range, w^2x, or w^2x^4, also remains constant. Thus inertia control results in an acoustic output which is independent of frequency over the range of interest.

The use of inertia control, however, implies diaphragm motion nearly 90° out of phase with the driving force, low-power factor, and resulting low efficiency. Also, large cone amplitudes are required at low frequencies since x is inversely proportional to w^2 for constant driving force. At higher frequencies ($kR>1.9$), the radiated power is given by (6a). If $x=F/w^2m$ is substituted into (6a), we obtain

$$P = \frac{a\rho R^2F^2}{w^2m'}, \tag{28}$$

where $m'=m-m_i$ since m_i is negligibly small in this range of frequencies. The power output at higher frequencies thus decreases as the square of the frequency for the case of the rigid disk. An important secondary effect which causes a further decrease in high-frequency output is a reduction in driving force, due to the increased inductive reactance of the coil and reduced current.

Efficiency

The acoustic efficiency of the loudspeaker is defined by McLachlan[7] as

$$\eta = \frac{\text{power radiated as sound}}{\text{power input to driving agent}}$$

$$\eta = \frac{R_r}{R+R_m}, \tag{29}$$

in terms of the equivalent electrical circuit where R_r is the portion of R_m due to sound radiation. $R_r=R_m-R_1$ where R_1 is the component of R_m due to mechanical losses, measured in air. For a very good speaker, it can be assumed that $R_r \gg R_m$, and the acoustic efficiency would be given by

$$\eta = \frac{R_m}{R+R_m}. \tag{30}$$

Since $C^2=z_eZ_m$, $R_m=C^2r_e/z_e^2$, so it can be seen that R_m and, therefore, efficiency fall off as the inverse square of the frequency. For typical 8 to 12-inch speakers, the maximum low-frequency efficiency is under 10 per cent. Speaker efficiency at any frequency may be increased by making C larger. This is usually accomplished by increasing the magnetic field strength. It should be mentioned that the use of an exponential horn as a coupling device between the sound radiator and the air will increase acoustic efficiency from 25 per cent to 50 per cent.

The acoustic efficiency is not easily determined, except by accurate measurement of the acoustic output. The mechanical efficiency, however, which is defined as the ratio of the mechanical power transmitted to the diaphragm to the electrical power input to the driving agent, can be computed from quantities which may be expressed in terms of the national parameters. Seabert[8] has found the following approximate expression for η_m:

$$\eta_m = \frac{1}{1+\dfrac{\sigma\rho}{B^2m_er_e}r_e^2+w^2(m_c+m_e)^2 10^9}. \tag{31}$$

σ is the resistivity of the voice coil conductor, r_e and m_e may be expressed in terms of C, R_m, and Z_m; and m_c; is the actual mass of the voice coil.

It has been shown by Olson[9] and McLachlan[10] that the maximum theoretical speaker efficiency occurs when the mass of the cone = the mass of the voice coil, although this may not be an optimum operating condition from the standpoint of fidelity. This result is obtained from the condition for maximum undistorted amplifier output of $R+R_m=\mu R_p$, where R_p is the plate resistance of the output tube. μ is a constant, depending on the curvature of the tube characteristics, usually lying in the range of 2 to 3. The value of voice coil impedance which maximizes the power output for this condition is thereby found to be

$$wm_c = \sqrt{r_e^2+w^2m_e'^2}, \tag{32}$$

where m_e' includes the diaphragm, coil former and insulation, and accession to inertia. This analysis is valid from above the mechanical resonant frequency to about 1200 cycles for typical direct radiators. Since both r_e and m_e' vary over the above frequency range, a particular value for m_c will be optimum only for a restricted subrange of frequencies.

Radiation Pattern

At low frequencies, when the radius of the vibrating disk is small in comparison to the wavelength of the sound being emitted, the sound distribution at a considerable distance from the disk is uniform and spherical. However, at higher frequencies where the radius of the disk is of the same order as or larger than the wavelength, radiation at any point in space due to separately vibrating elements of the disk will arrive displaced in phase, and annulment may occur. Thus, interference of the radiation from the vibrating disk takes place, and the sound output is propagated in the form of a beam whose angular width decreases with the frequency of vibration.

Assuming harmonic vibrations, the sound pressure at a point P at a distance R (R being at least 10 times the radius of the disk) from an elemental area dA is given by Rayleigh[11] as

$$dp = \frac{i\rho wDxe^{-ikr}}{2\pi r}dS. \tag{33}$$

Integrating this expression over the area of the disk, McLachlan finally obtains

[7] *Ibid.*, ch. 15.

[8] J. D. Seabert, "Electrodynamic speaker design considerations," Proc. IRE, vol. 22, pp. 738–750; June, 1934.

[9] H. F. Olson, "A new cone loudspeaker for high fidelity sound reproduction," Proc. IRE, vol. 22, pp. 33–46; January, 1934.

[10] McLachlan, *op. cit.*, ch. 8.

[11] Rayleigh, *op. cit.*, pp. 244, 302.

$$p = \frac{wDxR^2}{r} \frac{J_1(kR \sin \phi)}{kR \sin \phi}. \qquad (34)$$

e^{-ikr} having been omitted since it represents a constant phase factor at any given value of r. On the axis, sin $\phi = 0$ and the bracketed expression $= \frac{1}{2}$ so that the axial pressure at a great distance is

$$\text{axial } p = \rho w \frac{DxR^2}{2}. \qquad (35)$$

The sound pressure at any angle ϕ is thus equal to this axial pressure times d, where d is given by

$$d = \frac{J_1(kR \sin \phi)}{kR \sin \phi}. \qquad (36)$$

A plot of D is given in Fig. 5. For a disk 10 cm in radius, McLachlan[12] shows that the angle ϕ is approximately 30° at 4096 cycles, while the effective beamwidth is reduced to 14.5° at 8192 cycles. It can be seen that an increase in either the radius of the disk, or in the frequency, accentuates this focusing effect.

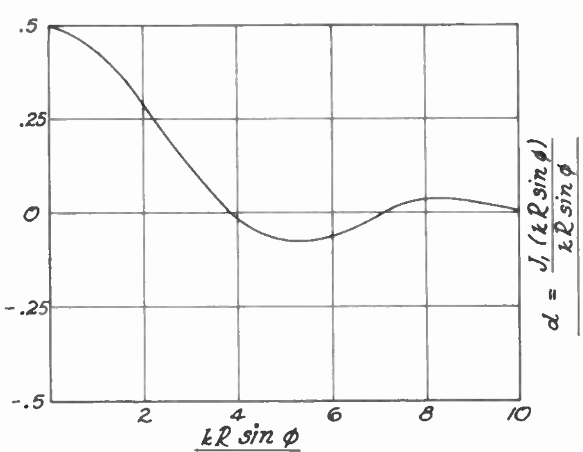


Fig. 5—Directivity characteristics of direct radiator loudspeaker.

One benefit derived from the directivity of the vibrating rigid disk at high frequencies is that the axial sound intensity for a given acoustical output is greater at high frequencies than would be the case if the radiation were spherically distributed. This is accomplished, of course, at the expense of the sound output of locations not on the axis.

Speaker Operation at High Frequencies

It is apparent from the preceding discussion that the rigid disk is a very poor radiator of sound in the high-frequency portion of the audio spectrum. Fortunately, actual loudspeaker performance indicates that at all but the very lowest frequencies of operation, there is superimposed on the motion of the cone as a piston or rigid disk an elastic vibration of the cone material itself. The net effect of this cone break-up into various modes of oscillation is a decrease in effective mass at each of those resonant frequencies with a resulting increase in acoustical output.

A direct treatment of the vibrational modes of the conical shell is not available in the literature. However, it is instructive to examine the vibrational modes of the circular membrane since empirical data on the conical shell have indicated similarities between these two structures. The solution for the problem of the vibrating circular membrane is given in Rayleigh,[13] as well as in a number of other references.[14,15] For purposes of illustration, the case of the vibrating circular membrane, clamped at its outer circumference, will be considered.

The Helmholtz equation in two dimensions, for cylindrical coordinates, is

$$r^2 \frac{\partial^2 \xi}{\partial r^2} + r \frac{\partial \xi}{\partial r} + \frac{\partial^2 \xi}{\partial \theta^2} + k^2 r^2 \xi = 0 \qquad (37)$$

where ξ is assumed to be a harmonic function of time. Separation of variables leads to the general solution:

$$\xi(r, \theta) = J_n(kr)[C_n \cos n\theta + D_n \sin n\theta)]. \qquad (38)$$

Note that the term in $Y_n(kr)$ has been eliminated to avoid a discontinuity at $r = 0$. To satisfy the boundary condition that $\xi = 0$ for $r = R$, $J_n(kR) = 0$. The multiple roots of this equation yield the vibrational frequencies for the circular membrane. The roots are of the form $k_{m,n}R$, this being the mth root for $J_n(kR)$. Two modes of vibration are present, symmetrical and radial, there being n radial modes and $m-1$ symmetrical modes. The surface of the membrane is divided into segments by the resulting nodal lines in such a way that the sign of the vibration changes each time a nodal line is crossed. This is illustrated very clearly in Rayleigh,[13] from which the nodal patterns shown in Fig. 6 have been obtained. The numbers shown in parentheses in Fig. 6 are the frequency ratios to the fundamental for each pattern, the fundamental being given by $kR = 2.405$.

In practice, the radial modes of vibration are substantially suppressed by the use of a suitable annular surround, so that the symmetrical modes are of the greatest interest.[16] McLachlan[17] has analyzed the case of a centrally driven aluminum disk, clamped at the outer edge, and has found that the effective mass is given by

$$m_e = \frac{2\pi mb}{k} \left[\frac{J_0(kR)Y_0(kb) - J_1(kb)Y_0(kR)}{J_0(kR)Y_0(kb) - J_0(kb)Y_0(kR)} \right]. \qquad (39)$$

[12] McLachlan, *op. cit.*, ch. 5.

[13] Rayleigh, *op. cit.*, pp. 244, 302.

[14] N. W. McLachlan, "Theory of Vibrations," Dover Publications, New York, N. Y., ch. 8; 1951.

[15] A. G. Webster, "Partial Differential Equations of Mathematical Physics," Dover Publications, Inc., New York, N. Y., pp. 353-355; 1955.

[16] M. J. O. Strutt, "On the amplitude of driven loudspeaker cones," Proc. IRE, vol. 19, p. 839–850; May, 1951.

[17] McLachlan, "Loud Speakers," ch. 4.

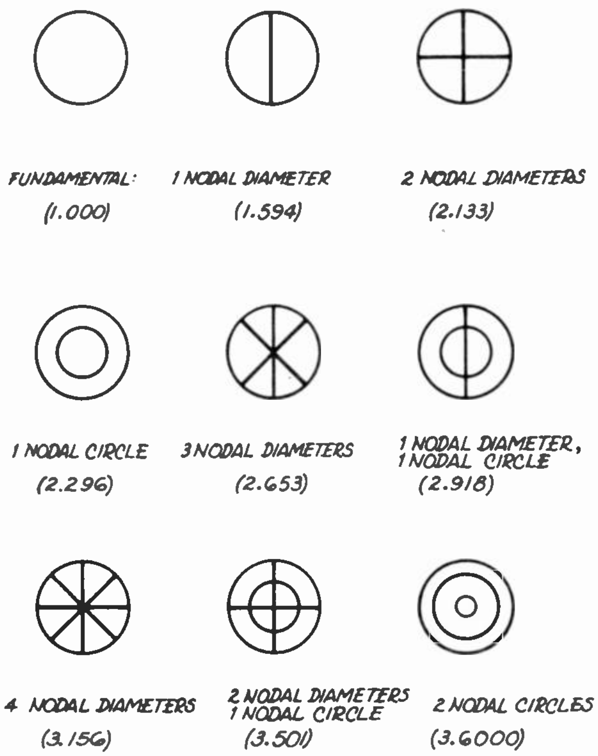


Fig. 6—Vibrational modes for circular membrane.

The inner radius of the disk is b, and the mass per unit area is given by m. Fig. 7, given by McLachlan, shows curves of m_e vs frequency for a centrally driven aluminum disk.

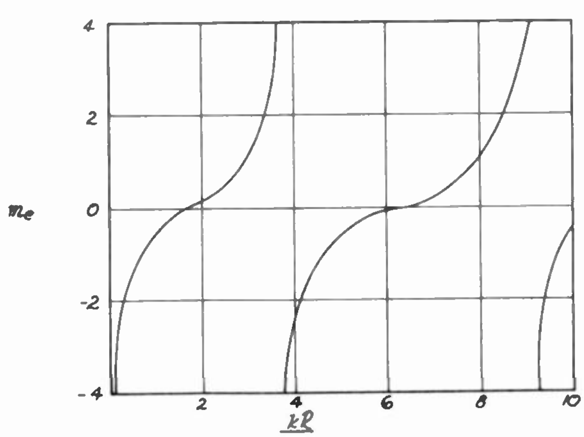


Fig. 7—Variation of m_e for centrally driven aluminum disk.

The waves propagated radially out along the disk are elastic in nature and are reflected at the edge. As the driving frequency is increased, points are reached where the reflected wave and the direct wave cancel each other along nodal circles. At this point, $m_e = 0$. The first frequency for which the effective mass = 0 corresponds to a single nodal circle, the second corresponds to two; this cycle of events thus continues to repeat itself. The frequencies for which m_e is infinite correspond to a nodal circle at the center (driving point). The first of these is a center stationary mode only, while the second is a center stationary plus one nodal circle mode, and so on.

Actually, mechanical losses decrease the amplitude of the reflected wave, the damping effect increasing with frequency, so that the variation of effective mass with frequency consists of a series of hills and valleys with the amplitude excursions falling off with increasing frequency.

The symmetrical modes for the circular disk were seen to occur at relatively widely separated frequencies. Experiments with conical shells of the type used in typical direct radiator loudspeakers have indicated that the symmetrical vibrational modes tend to cluster together over a particular frequency range, the 2000 to 4000-cycle range in one typical case.[16–18] Thus, by suitably modifying the cone structure to damp these resonant frequencies, one method being the insertion of concentric corrugations at predetermined radii, the acoustic response may be extended far beyond the limits indicated by the vibrational behavior of the rigid disk.

Olson[19] describes a similar technique for extending the response of a loudspeaker to 10,000 cycles. The cone is divided into sections of mass: $m_1, m_2, m_3, \cdots$, separated by compliances introduced by corrugations in the cone. To maintain constant sound output over the entire frequency range, the sum of the outputs of the elements $m_1, m_2, m_3, \cdots$, each of which is determined by the product of the radiation resistance of each element and the square of the velocity of that element, is thus made substantially independent of frequency. This technique is analogous to the design of wide-band amplifiers using multiple resonant circuits. A compliance is added to the voice coil also in the form of a capacitor shunting part of the coil so that this portion is effectively short-circuited at high frequencies. Thus, m_e is also reduced to a value suitable for high-frequency performance.

The Magnetic Field

Perhaps the only area of speaker performance about which there is not an aura of mystery is the magnetic structure. Given the air gap dimensions and the desired flux density, usually 5000 to 10,000 Gauss, the design of the field structure follows rather standard procedures. Both electromagnets and permanent magnets are used to produce the magnetic field. Up to a few years ago, it was impossible to produce a powerful magnetic field with a permanent magnet without resorting to the use of a very large and heavy field structure. Electromagnets requiring external field excitation were then widely used. Development of new magnetic materials, such as Alnice V, has made it possible to achieve high intensity fields with permanent magnet structures of moderate size. Electromagnetic units are seldom used today.

[18] C. R. Cosens, "Moving coil loud speakers," *Exper. Wireless and Wireless Eng.*, pp. 353–368; July, 1929.
[19] Olson, *loc. cit.*

The electromagnetic conversion factor $C = Bl$, so that the greater the flux density B, the more efficient the conversion of energy. As will be discussed later, a high flux density also improves electromagnetic damping which serves to suppress the mechanical resonant frequency, w_r.

Nonlinearities

The chief causes of nonlinearity and resulting distortion of the acoustic output in the loudspeaker are nonlinear suspension characteristics and nonuniform distribution of air gap flux. Both these effects are most important with large cone excursions. The suspensions, consisting of annular surround and centering device, were considered to have the force-displacement characteristics of a linear spring. As in the case of most actual springs, the suspension has a straight characteristic for small deflections only, becoming stiffer for larger deflections. For the direct radiator loudspeaker, the amplitude of the cone excursions is inversely proportional to w^2 for constant power output. The greatest distortion due to this nonlinearity will therefore occur at the low-frequency end of the audio spectrum, where the largest excursions are produced.

A typical cone suspension force-displacement characteristic is given by

$$F_m = f(x) = \alpha x + \beta x^3, \tag{40}$$

F_m being the mechanical force which produces the displacement.[20] The stiffness of the suspension is then $x/F_m = \alpha + \beta x^2$. The resulting differential equation for the mechanical vibrating system is then

$$m\ddot{x} + r_e\dot{x} + \alpha x + \beta x^3 + F \cos wt, \tag{41}$$

assuming a sinusoidally varying signal is applied to the voice coil. Since r_e is $\ll wm$, except at mechanical resonance, (41) may be rewritten as

$$mx + \alpha x + \beta x^3 = F \cos wt. \tag{42}$$

An approximate solution for (42) for small β is given by

$$x = A \cos wt + \frac{\beta A^3}{(\alpha + 3/4\beta A^2 - F/Am)32} \cos 3wt. \tag{43}$$

The arbitrary amplitude A maybe obtained from the following relationship between A and w

$$w^2 = \frac{\alpha}{m} + \frac{3/4\beta A^2}{m} - \frac{F}{Am}. \tag{44}$$

The importance of the cos $3wt$ component for large amplitudes is clearly apparent.

It is also found that there are particular solutions for (41) which yield subharmonic components with respect to the frequency of the applied force.[21] However, these subharmonics are difficult to detect experimentally since the frequencies involved are quite low.

Large cone excursion amplitudes may cause the voice coil to move out of the air gap an appreciable distance, so that the mean flux density cut by the coil decreases and the driving force is no longer proportional to the voice coil current. This may be avoided by maintaining an axial gap length considerably exceeding the length of the voice coil. McLachlan[22] has shown that an electromechanical rectification effect occurs with large cone excursions outside the main field. As a result, the coil fails to return to its original equilibrium position.

Transient Response

Many sounds, both musical and nonmusical, are associated with sharply rising or falling wavefronts, indicative of high harmonic content. The transient response of a loudspeaker is a measure of the fidelity with which these sounds will be reproduced. A rigorous analysis of transient response for a step input is fairly complicated since m, r_e, and coil current all are functions of frequency. In addition the directivity of the speaker radiation pattern for higher frequency components will have some effect.

From a qualitative standpoint, when a square wave of voltage is applied, the coil and cone overshoot their final position until brought to rest by the mechanical constraints. The kinetic energy stored in the moving mass thus becomes potential energy stored in the springiness of the suspension. The coil then reverses direction, continuing to oscillate about its mean position until the energy stored in the moving mass is dissipated in frictional loss, finally coming to rest at a new equilibrium position. The energy expended in this oscillation is a portion of the signal energy, thus the loudspeaker acts as a signal converter, serving to redistribute the frequency spectrum of the applied pulse to emphasize the natural mechanical resonant frequency, w_r.

After each pulse the coil is finally brought to rest due to the energy which had been stored in the moving mass being dissipated by the losses in the cone and constraints, air friction losses due to the air trapped in the gap between the voice coil and pole pieces, the energy radiated as sound, and the losses due to the back emf induced in the moving coil. When the coil overshoots its final position, the voltage generated in the coil exceeds the driving voltage, thus causing a current to flow back through the driving amplifier output circuit with a resultant I^2R loss.

[20] H. F. Olson, "Action of a direct radiator loudspeaker with nonlinear cone suspension," *J. Amer. Stand. Assoc.*, vol. 16, p. 1–4; July, 1944.

[21] R. V. L. Hartley, "The production of inharmonic sub-frequencies by a loud speaker," *J. Amer. Stand. Assoc.*, vol. 16, pp. 206–210; January, 1945.

[22] McLachlan, *op. cit.*, ch. 14.

Design Considerations

The diaphragm structure is usually developed through an empirical or trial-and-error technique. The high-frequency resonant modes can be controlled through the choice of cone material, the use of circular corrugations, or by varying the interior angle of the conical shell. Cones are made by rolling and glueing a paper conical section, or may be moulded in one piece. For high-frequency reproduction, small cones or specially constructed larger units are used to reduce m_c. The smaller cone surface also broadens the high-frequency radiation pattern. Hard springy cones for high-frequency speakers create the impression of greater output through the production of more accentuated resonances.

The use of a large cone is a necessity for low-frequency reproduction. For the same acoustic output, the excursion of a 5-inch speaker cone is 6.8 times that of a 12-inch unit. Thus, the low-frequency distortion of the smaller speaker would be much greater.

Voice coil impedances are usually maintained in the range from 3.2 to 16 ohms. This requires the use of an output transformer, or other impedance matching device, in the output circuit of a driving amplifier which uses vacuum tubes. It is possible, however, to drive directly a conventional speaker voice coil without impedance matching devices through the use of transistors. High-impedance voice coils have been used in the past; however, their relatively large mass and inductance proved unsatisfactory for wide-range frequency response. For low-frequency operation, a fairly large voice coil diameter is required for efficient coupling to the diaphragm. Copper wire is used to obtain low resistance in the interests of efficiency and damping. At high frequencies the coil mass must be minimized, and aluminum wire is frequently used because of its lighter weight. The voice coil—air gap structure must be carefully designed to avoid distortion due to the coil being driven outside the main magnetic field. The voice coil mass is usually matched to the cone mass at the mid-range to balance the over-all frequency response.[23]

A commonly used means of cone suspension employs a corrugation along the outer rim of a one-piece cone. The outer radius of the corrugation is then glued to the speaker frame. Less common is the use of a flexible surround of cloth, soft leather, or rubber. Care must be taken when using a stiff suspension to avoid the generation of a prominent resonant frequency resulting from the surround vibrating as an annular membrane. When properly controlled, however, this resonance may be used to fill in gaps in the speaker's frequency characteristic.

Centering devices are required to maintain axial coil motion. A flexible ring, attached to the voice coil at its inner rim and connected to the speaker frame by extension arms, has been used. Another commonly used centering device is a flexible corrugated surround fastened to the coil former at the point where it is glued to the inner radius of the diaphragm.[17]

Reasonably flat frequency response over a wide range is a necessity for a high quality speaker. However, inexpensive units, designed with a powerful bass resonance from the mechanical vibrating system, and sharp resonances in the high-frequency range may give the illusion of better fidelity of reproduction than a much better unit with relatively flat frequency response.[23] The bass resonance gives a "juke box" or one note bass response that is popularly characterized as mellow or boomy.

Specifications for an ideal direct radiator loudspeaker would probably encompass the following. The voice coil would be wound from wire of zero resistance and fed from an amplifier with zero output impedance. The self-inductance of the voice coil, when held stationary, would be zero. The coil velocity would have to be such as to generate a motional counter emf exactly equal to the amplifier output voltage. The coil velocity waveform would then be exactly the same as the amplifier output voltage. Mechanically, the coil would be attached to a light, but perfectly rigid, diaphragm, working into an infinite baffle.

Improvement of Low-Frequency Performance

As mentioned in the Introduction, the secondary objective of this paper was a discussion of the problem of obtaining satisfactory speaker performance in the low-frequency portion of the audio spectrum. Speaker cost per undistorted watt rises sharply at this end of the audio spectrum. The principles of design for low-frequency radiators have already been discussed. Each commercially available unit represents a compromise between these principles and such economic factors as cost and size. Fortunately, means exist for improving the response of existing units through modification of the driving amplifier, or the use of specially designed enclosures. The latter is a very complete subject in itself and will not be discussed here.

The most prominent characteristic in the low-frequency operating range of the direct radiator loudspeaker is the mechanical resonant frequency w_r. For frequencies below w_r, the acoustic output of the speaker falls off as the fourth power of the signal frequency since the mode of operation shifts from inertia control to stiffness control, the amplitude of vibration thus becoming independent of frequency. The chief objectives in improving low-frequency performance are therefore to lower the frequency of mechanical resonance and to damp the peak in acoustic output at that frequency.[24,25]

Since $w_r = (s/m)^{1/2}$, the resonant frequency can be

[23] H. S. Knowles, "Speaker cost vs quality," *Electronics*, pp. 240–244; September, 1933.

[24] H. T. Souther, "Design elements for improved bass response in loudspeaker systems," *Audio Eng.*, pp. 16–19; May, 1951.

[25] E. W. Kellogg, "Means for radiating large amounts of low frequency sound," *J. Amer. Stand. Assoc.*, vol. 3, pp. 94–110; July, 1931.

lowered by either decreasing the suspension stiffness s, or increasing the mass of the diaphragm. Decreasing suspension stiffness has the undesirable result of allowing increased cone amplitudes for the same driving force. Thus, the driving power to an equivalent speaker with reduced stiffness must be decreased to avoid cone excursion into nonlinear regions of operation, unless a proportional increase in air loading is supplied to the cone. Increasing the mass of the cone will result in a reduction in w_r proportional to $m^{1/2}$, but there will also be a corresponding loss of mechanical efficiency proportional to m^2. Very recently, a number of speaker-enclosure combinations have been made commercially available which consist of conventional low-frequency radiators in which the outer suspension has been completely removed and replaced with strips of soft cloth. This, very large reduction in stiffness, is then compensated for by the design of the enclosure which produces a large increase in the air load at low frequencies.

As previously mentioned, the moving coil speaker provides some inherent damping through its motional resistance, this being a fictitious quantity representing the circuit losses due to the induced back emf during voice coil motion. The effect of the motional resistance on the low-frequency resonance can best be seen from an equivalent electrical circuit for the speaker where the motional impedance is represented as a parallel combination. This is shown in Fig. 8. The defining expressions for this equivalent are

$$R_m' = C^2/r_c, \tag{45}$$

$$C_m' = m/C^2, \tag{46}$$

$$L_m' = C^2/s. \tag{47}$$

The mechanical resonant frequency can then be expressed as

$$w_r = 1/L_m'C_m'. \tag{48}$$

A well-known property of parallel resonant circuits such as that shown for the motional impedance in Fig. 8 is that oscillations will not occur for values of R_m' less than $R_m''=(L_m'/C_m')^{1/2}$, R_m'' being defined as the parallel resistance necessary for critical damping. Thus it can be seen that it is very desirable to minimize R_m'. This can be achieved by increasing the magnetic field strength, although this is limited by the size of the magnetic structure and its cost.

Fig. 8 shows that the voice coil resistance and output impedance of the generator (driving amplifier) add up up to a parallel resistance across $R_m' \cdot R_{vc}$ is a fixed quantity for a given voice coil; however, the output impedance of the amplifier may be reduced. Triode output tubes have inherently low plate resistance; however, the more popular beam power output tubes do not. In either case, it is possible to reduce the output impedance (as reflected to the secondary of the output transformer) to a very low value through the use of negative voltage

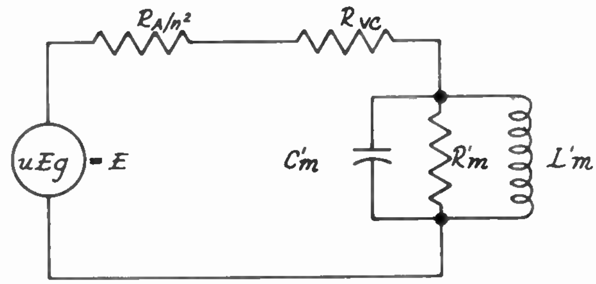


Fig. 8—Equivalent parallel electromechanical circuit for loudspeaker.

feedback[26,27] over a loop from the output terminals to the input terminals of the amplifier. This also results in improved linearity and frequency response for the amplifier at the expense of the over-all gain of the unit. Even with a very large amount of negative voltage feedback, so that the effective output impedance is nearly zero, R_{vc} remains in series. Thus there would be little gained from reducing the output impedance below, say; 20 per cent of R_{vc}.

A combination of negative voltage feedback and positive current feedback in the driving amplifier has proved to be an attractive means of compensating for R_{vc}.[28–30] Positive current feedback also reduces the amplifier impedance; in fact, it can be made effectively negative to a degree that R_{vc} can be cancelled out. In addition, over-all amplifier gain is increased over the case of negative feedback alone. One disadvantage is that great care must be taken in designing the feedback loops to avoid system oscillation at certain frequencies.

Another means of suppressing the effects of bass resonance is through motional feedback.[31,32] It can be seen from Fig. 8 that if it were physically possible to measure the voltage across the motional impedance, this, being a linear function of cone velocity, would be a direct indication of speaker fidelity. If this motional voltage could be fed back degeneratively to the input, or some intermediate point in the amplifier circuit, a means would be provided of including the nonlinearities of cone suspension and magnetic field in the feedback loop. Thus, at least theoretically, all important speaker deficiencies may be compensated for. This may be practically accomplished in several ways. The most feasible of these is to place an additional winding over the existing voice coil. The voltage induced in this auxiliary winding is nearly a pure motional voltage at most frequencies. This method is most suitable for

[26] J. Moir, "Transients and loud speaker damping," *Wireless World*, pp. 166–170; May, 1950.

[27] J. P. Wentworth, "Loudspeaker damping by the use of inverse feedback," *Audio Eng.*, p. 21; December, 1951.

[28] W. Clements, "A new approach to loudspeaker damping," *Audio Eng.*, pp. 20–23; August, 1951.

[29] A. Preisman, "Loudspeaker damping," *Audio Eng.*, pp. 22–24; March, 1951, and p. 24, April, 1951.

[30] T. Roddam, "Output impedance control," *Wireless World*, pp. 48–49; February, 1950.

[31] R. L. Tanner, "Improving loudspeaker response with motional feedback," *Electronics*, pp. 141–144; March, 1951.

[32] J. De Boer and G. Schenkel, "Electromechanical feedback," *J. Amer. Stand. Assoc.*, vol. 20, pp. 641–647; September, 1948.

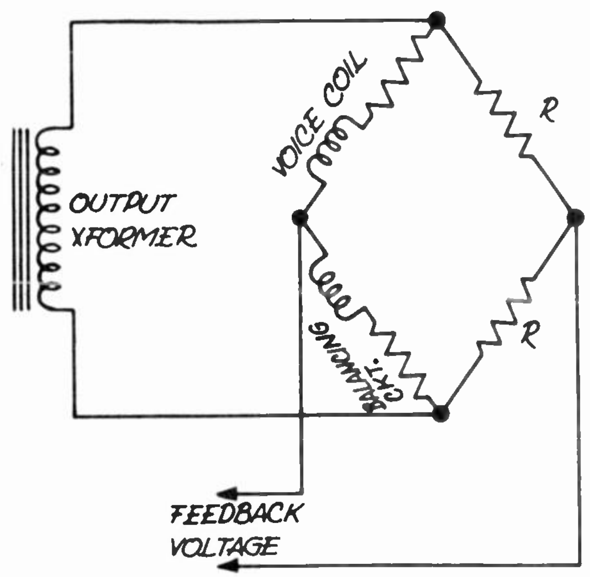


Fig. 9—Circuit for obtaining motional feedback voltage from existing voice coil.

speakers intended solely for low-frequency reproduction since voltages will be induced in the feedback winding at higher frequencies due to mutual inductance which do not depend on the motion. These mutual voltages may be balanced out by adding external mutual inductance to the electrical circuit. A means of obtaining a motional feedback voltage from the existing voice coil is shown in Fig. 9. The voice coil is used as one arm of a bridge circuit and balanced with an equal inductance and resistance, thus enabling the back emf to be separated from the driving voltage.

Conclusion

It is apparent that the direct radiator loudspeaker, in spite of its wide usage over a considerable number of years, is still an imperfectly understood device. However, by employing analogies with similar electrical and mechanical systems, it is possible to gain an understanding of the principles of operation of the speaker that compares fairly well with actual results.

A Loudspeaker Installation for High-Fidelity Reproduction in the Home*

G. J. BLEEKSMA† AND J. J. SCHURINK†

Summary—For more than a quarter of a century the normal broadcast receiver has been equipped with a single loudspeaker fitted inside the cabinet. The introduction of sets with two separate speakers, each reproducing part of the frequency spectrum on the Philips "Bi-Ampli" principle, dates only from the last few years.

The installation described goes a big step further, the loudspeakers being entirely separated from the amplifying part; two low-note speakers are housed together in a special cabinet and two high-note speakers separately in their own boxes. The quality of reproduction has been remarkably improved in this way, which appears to full advantage with fm reception and the playback of gramophone records or tape recordings.

A complete system for sound reproduction, from performance to playback, can be considered as a chain whose first link is formed by one or more microphones and whose final link is made up of one or more loudspeakers. In between are one or more amplifiers, and further, either a radio link or some "memory" with playback facilities. (The "memory" is normally a tape recording or a gramophone record.)

The links in this chain are not equally strong. "Strong" links are those that are hardly likely to be improved upon by present engineering practice. As such we may consider the condenser microphone and the amplifiers. The memory is among the weaker links. However high the quality of magnetic recordings[1] and modern gramophone records[2] may be, certain improvements in the step function response and in the dynamic characteristic are still desirable.

Loudspeakers too should be considered as among the weaker links. A recent development in this field forms the subject of this article. The installation described is, in our opinion, about the very best attainable for home entertainment that can be realized with the technical means now at our disposal.

I. Brief Description of the Installation

The installation comprises four loudspeakers, *viz.*, two for the low notes (with a range below 420 cps) and two for the frequency range above 420 cps (referred to here briefly as the high range). The two low-note speak-

* Reprinted from *Philips Tech. Rev.*, vol. 18, pp. 304–315; 1956/57.
† Philips Res. Labs., N. V. Philips' Gloeilampenfabrieken, Kastanjelaan, Eindhoven, Netherlands.

[1] See, *e.g.*, D. A. Snel, *Philips Tech. Rev.*, vol. 14, pp. 181–190; 1952/53, and W. K. Westmijze, *Philips Tech. Rev.*, vol. 15, pp. 84–96; 1953/54.
[2] See L. Alons, *Philips Tech. Rev.*, vol. 13, pp. 134–144; 1951/52, and J. L. Ooms, *Philips Tech. Rev.*, vol. 17, pp. 101–109; 1955/56.

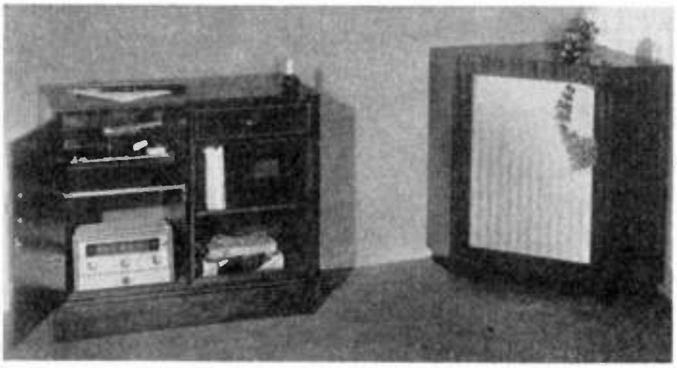


Fig. 1—Left: gramophone AG 1005 (with magnetodynamic pickup AG 3020/21)[3] and amplifier AG 9006. Right: the low-note cabinet. The installation is completed by two separate loudspeakers for the high notes, above 420 cps. (See Figs. 2 and 3.)

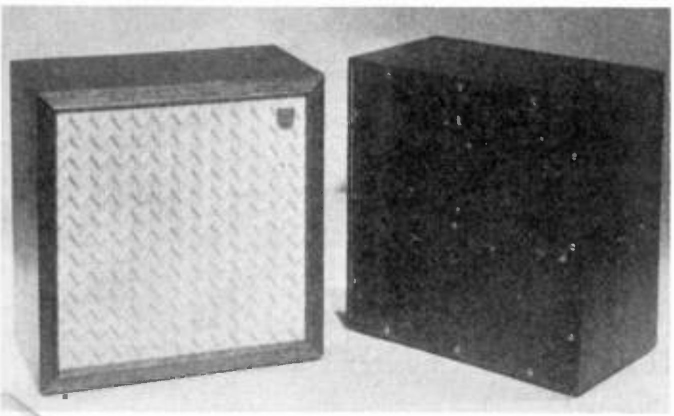


Fig. 2—The two high-note loudspeakers. Each box contains one double-cone loudspeaker and has a width and height of 26 cm.

ers are housed together in a corner cabinet (Fig. 1), while the high-note speakers are contained in separate boxes (Fig. 2). All these loudspeakers are fed by one amplifier (type AG 9000 or AG 9006), via a low-pass and a high-pass filter, respectively.

The loudspeakers for the frequency range above 420 cps radiate nearly all their energy in a forward direction, and for this reason they are called projectors. Separate projectors can be so placed that their sound reaches the listeners only indirectly, *i.e.*, via one or more reflections from the walls or the ceiling of the room. The importance of this method is that it diffuses the sound, so that the result approaches far more closely the actual conditions in the concert hall. Measurements in various concert halls have revealed that at most places in the hall the greater part of the sound intensity is attributable to indirect sound and only a small part to direct radiation.[4] Similar conditions can be created in the living room by directing the projectors at different walls of the room in such a way that the listeners are outside the direct beams (Fig. 3). The position of the low-note cabinet is less critical, since the directions from which the low notes issue are more difficult to distinguish.

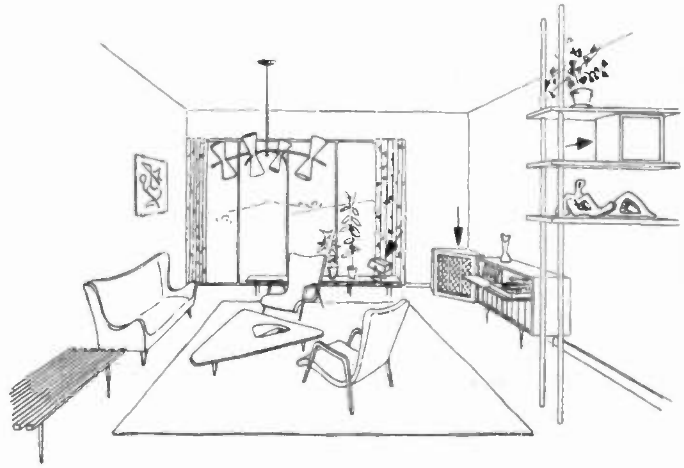


Fig. 3—A typical arrangement of the units. To the right is the cabinet containing the gramophone and the amplifier, and in the corner is the low-note cabinet. The two high-note projectors are so arranged that a diffused sound is obtained.

The use of two projectors, placed some distance apart from each other and from the listeners, effectively avoids the "hole-in-the-wall" effect.[5] This can be explained as follows. Let us assume that we are listening to an orchestra via one loudspeaker only. Even if the whole reproduction channel reproduced uniformly the entire frequency spectrum and were completely devoid of distortion and dynamic limitations, the reproduction would still remain unsatisfactory in one respect: the entire orchestra, set up on a wide podium, would be heard as if it were compressed into the small aperture of the loudspeaker, *i.e.*, as if the orchestra were listened to through a hole in the wall of the concert hall.

An effective means of remedying this hole-in-the-wall effect is stereophony,[6] but this is only seldom used in radio broadcasts or with gramophone records. However, by placing the two projectors in such a way that the sound reaches the listeners only indirectly, the hole-in-the-wall effect can be eliminated.

The high-note speakers (type 9710 M) are distinguished from the low-note speakers (type 9710) in that they are double-cone loudspeakers.[7] As can be seen in Fig. 4, the moving coil drives not only a normal cone (C_1) but also a small auxiliary cone (C_2). The latter extends the frequency range, which would normally reach no further than 8 kc, to about 18 kc, and at the same time blurs the sharp outlines of the beam.

The reproduction of the high notes is further improved by the use of loudspeakers with nearly constant impedance: at 20 kc the impedance of a type 9710 M loudspeaker is only 1.5 times as high as at 400 cps, whereas for conventional loudspeakers this ratio is at least 5. When fed with a constant voltage, the current flowing through such "constant impedance" loudspeakers will therefore decrease considerably less with rising

[3] N. Wittenberg, *Philips Tech. Rev.*, vol. 18, pp. 101–109, 173–178; 1956/57.

[4] Cf. *Philips Tech. Rev.*, p. 258; 1955/56.

[5] *Philips Tech. Rev.*, vol. 17, p. 171; 1955/56.

[6] *Ibid.*, p. 173 *et seq.*

[7] J. J. Schurink, *Philips Tech. Rev.*, vol. 16, pp. 241–249; 1954/55.

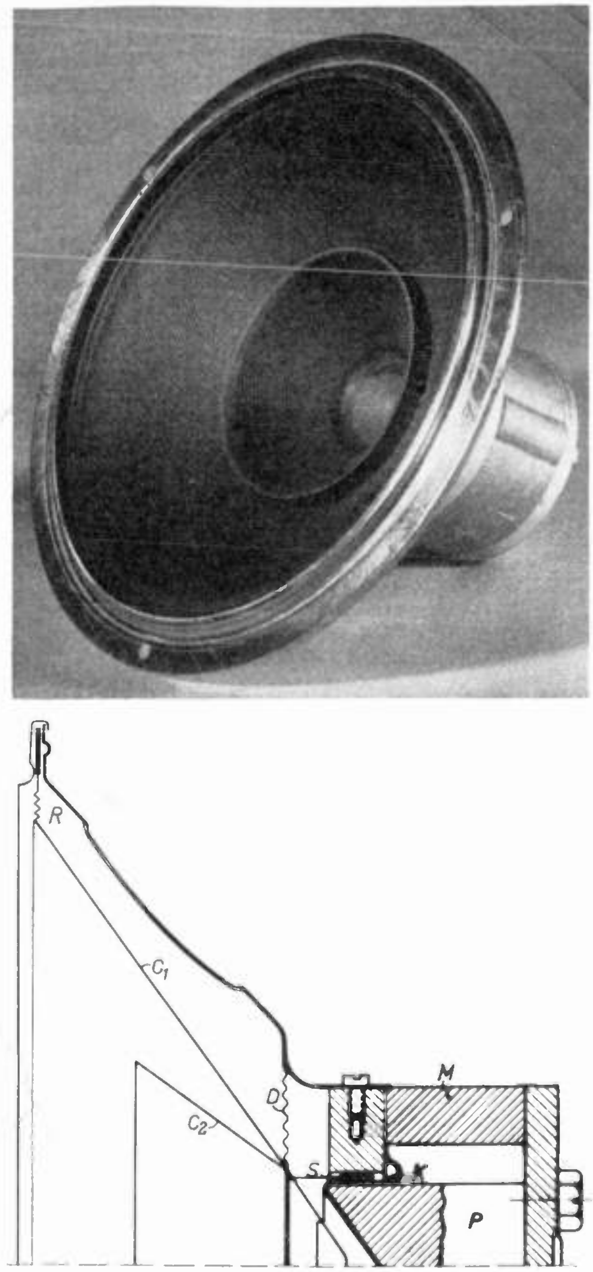


Fig. 4—The double-cone loudspeaker 9710 M of the high-note projector. In the cross section, C_1 is the main cone, C_2 the auxiliary inner cone, S the coil, D the centering ring, R the corrugated cone edge, M the permanent magnet, and K the copper ring ensuring a constant impedance (see further in this article). The core P of the magnetic circuit has a conical hole for accommodating the top of cone C_1.

frequency than the current flowing through conventional loudspeakers. The reproduction of the high-note range will consequently be superior since it is the current that determines the force with which the cone is driven.

The cabinets in which the loudspeakers are housed are completely closed behind the cone. In the high-note projectors this prevents any back radiation (possibly directed at the listeners) which would mar the diffusion effect aimed at. In the low-note loudspeakers the closed box prevents the air vibrations from traveling around the cone; it thus represents an infinitely large baffle, which greatly enhances the reproduction of the very low notes. The enclosed volume of air, as we shall demonstrate later, also helps in reducing nonlinear distortion. To attain the best possible enclosure, "complete cones" have been used instead of the conventional truncated cone (Fig. 4).

After this brief description, we shall now consider some of the above-mentioned features more closely, starting with the forms of distortion caused by nonlinear phenomena.

II. Nonlinear Distortion in Loudspeakers

The speech coil of a moving-coil loudspeaker, through which a current I flows, is situated in a magnetic field with an induction B. A force $F=BlI$ then operates on the coil (l denoting the length of wire in the speech coil). Since a force is numerically equal to the reaction it produces, this force is equal to the product of the mechanical impedance Z_m and the velocity v:

$$BlI = Z_m v = \left(j\omega M + \frac{1}{j\omega C} + R_m\right)v. \tag{1}$$

In this expression ω is the angular frequency of the current, M the mass of the coil and cone and that of the air that moves with the cone, C the total compliance (*i.e.*, the reciprocal of the stiffness), and R_m the mechanical resistance (both the resistance of the effective sound radiation and the mechanical loss resistance, the latter mainly occurring in the edge of the cone and in the centering ring).

The displacement x of the coil at the moment t is given by $x=\int v\,dt$, and hence

$$x = \frac{v}{j\omega} = \frac{BlI}{\dfrac{1}{C} - \omega^2 M + j\omega R_m}. \tag{2}$$

The resonance frequency f_0 of the system is therefore:

$$f_0 = \frac{1}{2\pi\sqrt{MC}}. \tag{3}$$

Because of the resistance term in (2), the displacement will be greatest at a frequency below f_0. It also appears from (2) that at frequencies that are sufficiently *above* the resonance frequency the displacement varies about proportionally with ω^{-2}, and hence by 12 db per octave (the current remaining constant). In this frequency range the amplitude is so small that no distortion is noticeable. At frequencies sufficiently *below* the resonant frequency, however, the displacement is given by

$$x \approx CBlI. \tag{4}$$

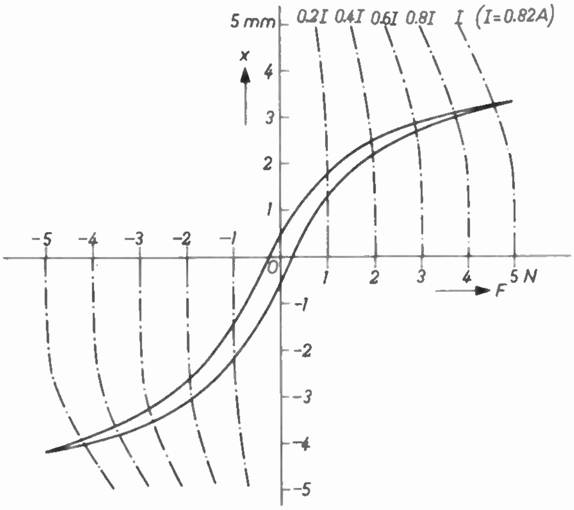
Here the amplitude can become so large that the relation between x and I is no longer a linear one as the result of two causes: 1) the induction B in the air gap is not quite homogeneous, so that B depends upon x (at very large amplitudes the coil comes even partly out of the air gap), and 2) the compliance C is dependent on x

to an even higher degree. With a sinusoidal current this nonlinear distortion will produce harmonics, so that the timbre of the sound will be different. Moreover, if the current consists of two (or more) sinusoidal components, nonharmonic overtones are likely to occur which are displeasing to the ear. We shall now consider separately these two forms of distortion.

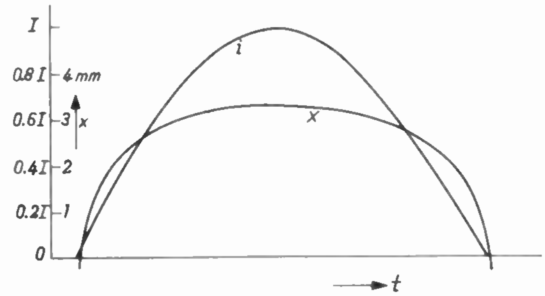
Harmonic Distortion

It can be seen from Fig. 5(a) that the factors B and C in (4) are not constants. The dot-dash lines of this diagram represent the force F (plotted horizontally) operating on the coil in the magnetic field at different values of a direct current I, passing through the coil. The force F is strictly proportional to I, but varies with x, since B varies with the position in the air gap. If the field were homogeneous, all these lines would be straight and vertical. If the displacement x of the currentless coil is measured statically as a function of an external force F, we arrive at a hysteresis loop as represented by the fully drawn curve in Fig. 5(a). This shows that the total compliance C_e of the cone edge and the centering ring (the only compliances operative here) depends upon x and thus likewise has a nonlinear character.

The variation of x as a function of time, for a sinusoidal current, can be derived from the curves in Fig. 5(a). As expected, the original sinusoidal waveform is considerably distorted and a number of higher harmonics occur in the displacement. As regards the sound pressure p, this effect will be even more pronounced since the pressure variations are proportional to $\omega^2 x$, so that the subsequent harmonics in p are, respectively, 1, 4, 9, . . . times greater than those in x. For instance, 1 per cent of third harmonic in x results in 9 per cent of third harmonic in p.



(a)



(b)

Fig. 5—(a) Dot-dashed curves: force F on the coil of a moving-coil loudspeaker with a direct current of 1/5 I, 2/5 I, . . . , I passing through the coil (I=0.82 A). Full curve: static displacement x of the (currentless) coil as a function of an externally applied force F. (b) Displacement x as a function of time t, showing distortion as compared with the sinusoidal current i passed through the speech coil.

Nonharmonic Distortion

Let us now consider the case that the current is made up of two sinusoidal components, so that its instantaneous value i can be written as:

$$i = \hat{I}_1 \sin 2\pi f_1 t + \hat{I}_2 \sin 2\pi f_2 t. \tag{5}$$

The nonlinear relation between displacement x and current i can be expressed by a series:

$$x = x_0 + x_1 i + x_2 i^2 + x_3 i^3 + \cdots. \tag{6}$$

Substituting (5) in (6) shows that the displacement will contain components with the frequencies $\pm mf_1 \pm nf_2$ (m and n being integers 0, 1, 2, $\cdots$). This effect is known as intermodulation.

The above is only valid for frequencies *below* the resonance frequency, in the so-called stiffness range, in which the amplitude is independent of the frequency. Above the stiffness range the amplitude does depend upon the frequency, which greatly complicates a quantitative evaluation. Qualitatively speaking, however, intermodulation appears here as well.

Nonharmonic distortion can be attributed to still another cause, *viz.*, the Doppler effect. If a loudspeaker cone vibrates simultaneously with a low frequency f_1 and a higher frequency f_2, then it may be considered as a source of sound with frequency f_2 whose distance to the ear varies with the frequency f_1. Owing to the Doppler effect, the pitch will then fluctuate with the rhythm of the low frequency f_1 around the higher frequency f_2, so that frequency modulation occurs. This can be interpreted as the presence of a sound with central frequency f_2 and two sidebands in which the frequencies $f_2 \pm mf_1$ occur. Let the amplitudes of the vibrations with the frequencies f_1 and f_2 be $\hat{x}_1$ and $\hat{x}_2$, respectively; then, the amplitudes of the first and the second sideband components will amount to

$$10^{-2} \times \hat{x}_1 f_2\% \quad \text{and} \quad 5 \times 10^{-7} \times (\hat{x}_1 f_2)^2\% \text{ of } \hat{x}_2,$$

$\hat{x}_1$ being expressed in cm and f_2 in cps. For example, if $\hat{x}_1 = 0.1$ cm and $f_2 = 10{,}000$ cps we find: 10 per cent for the components with the frequencies $f_2 + f_1$ and $f_2 - f_1$, and 0.5 per cent for the components with the frequencies $f_2 + 2f_1$ and $f_2 - 2f_1$.

III. Means of Reducing Distortion

Doubling the Number of Loudspeakers

For reducing the nonlinear distortion there exists a solution so simple that its value is not always fully appreciated. We refer to the use of more than one loudspeaker (quite apart from the separation of low and high notes). To attain an equivalent total sound volume each individual cone can then vibrate with a smaller amplitude, so that a shorter and less curved part of the hysteresis loop [Fig. 5(a)] is used, and the coil operates within a more uniform magnetic field. For this reason we are using two low-note loudspeakers in our installation. (The two high-note speakers are used mainly to diffuse the sound, distortion at higher frequencies being very slight, anyway.)

As for the low-note reproduction, this particular aim—less distortion through smaller amplitude, at equal total sound volume—might also be attained in principle by using instead of two adjacent loudspeakers with radius a [Fig. 6(c)], a single loudspeaker with radius $a\sqrt{2}$ [Fig. 6(b)].

Let us first compare one "small" loudspeaker [radius a, Fig. 6(a)] with one "large" loudspeaker [radius $a\sqrt{2}$, Fig. 6(b)]. Their radiation resistances are proportional to the fourth power of their radii. The large speaker, therefore, has a radiation resistance $(\sqrt{2})^4=4$ times greater than that of the small loudspeaker. At the same amplitude $\hat{x}$ the large loudspeaker will radiate four times more power.

When the large loudspeaker is to be compared with a combination of two small ones then one would expect, if each small speaker operating separately radiates a power P, that they would produce together a power $2P$, and that therefore the large loudspeaker, with $4P$, would be twice as powerful. It should be taken into account, however, that the effect of two small loudspeakers close together is different from that of the same two speakers some distance apart. Two loudspeakers (vibrating in phase) in close proximity produce the favorable effect that they stimulate each other, as it were, to radiate a greater power.

This effect can be readily explained as follows: if the loudspeakers are far apart [Fig. 6(d)], then each yields energy to the air since the cone is moving against self-produced pressure variations. If, however, the loudspeakers are close together [Fig. 6(c)], then in addition to the self-produced pressure variations there will be those from the adjacent loudspeaker. Owing to the small distance, these pressure variations are virtually equal in amplitude and phase. Each of the cones, then, operates against a sound pressure that is twice as high and thus produces double the amount of power. The two loudspeakers together now produce a power that is twice as large as when they are placed far apart (for a constant amplitude $\hat{x}$ of the cone displacement).

Klapman has evaluated this effect for intermediate cases, *viz.*, for two loudspeakers, considered as flat pistons (radius a, distance between centers d).[8] Fig. 7(a) shows the curve plotted for $R_r/\rho c A$ (R_r = radiation resistance per piston, ρ = air density, c = velocity of sound, A = area of each piston) as a function of $2\pi fa/c$, with d as parameter. For the type 9710 loudspeaker $2a=17.5$ cm; the frequency scales added to Fig. 7 apply to this value. In the cabinet the loudspeakers are placed at a distance $d=2.8a$. In the frequency range in question (below 420 cps), R_r is substantially greater than would be the case with a single loudspeaker (curve for $d=0$).

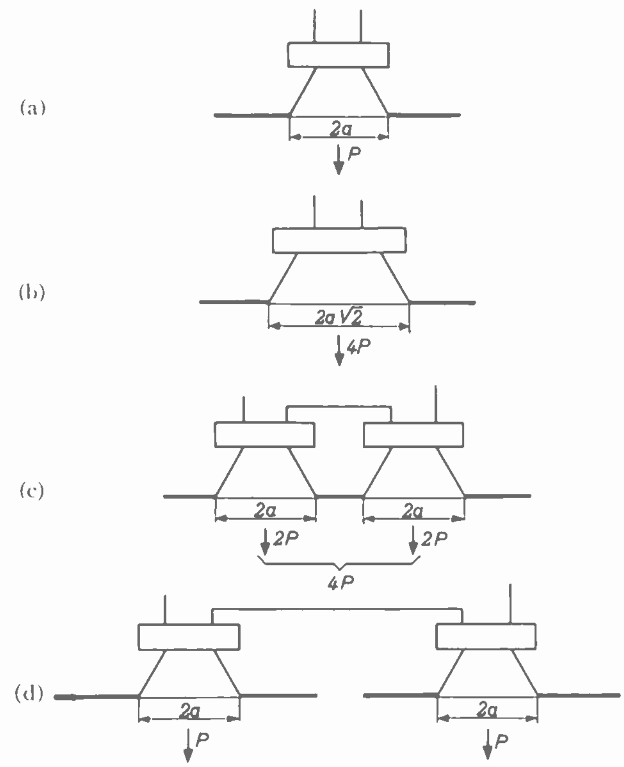
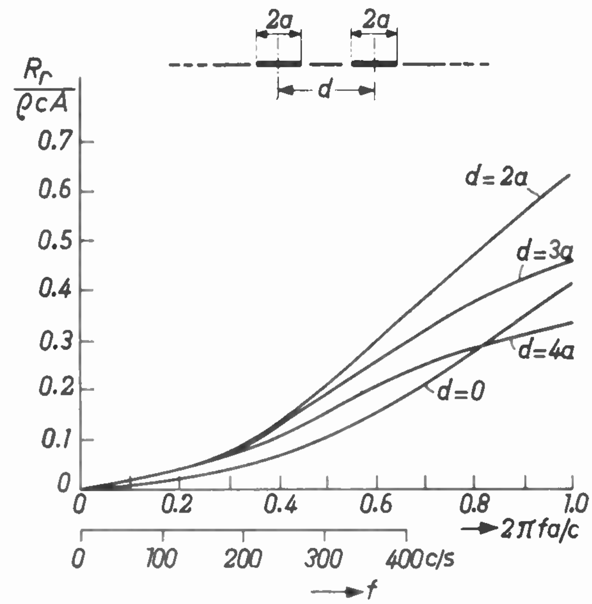


Fig. 6—(a) Loudspeaker with diameter $2a$. The radiated sound power is P. (b) Loudspeaker with diameter $2a\sqrt{2}$. Sound power $4P$. (c) Two loudspeakers as in (a), placed closely together. Now either radiating a sound power $2P$, they produce together $4P$. (d) Two loudspeakers as in (a), placed a distance apart. As in (a), either produces a power P; total power $2P$. The loudspeakers, of course, are supposed to be of the same type, mounted in large baffles, and vibrating with the same amplitude $\hat{x}$.

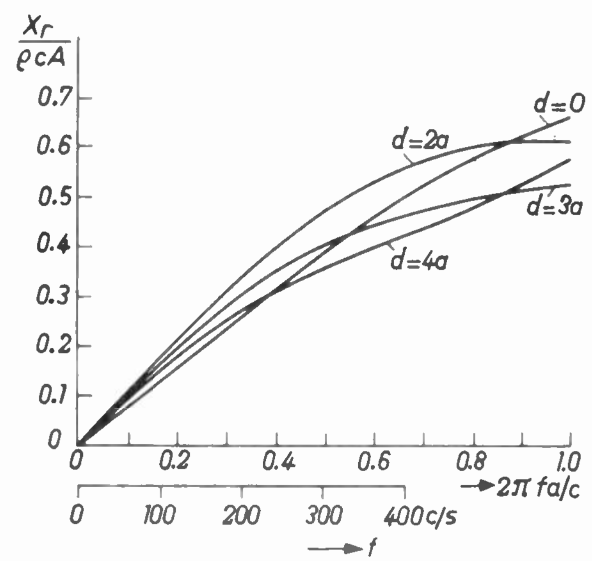
In the above imaginary experiment, in which two loudspeakers are brought closer together, the amplitude $\hat{x}$ was assumed to remain constant. If, however, the supply current is kept constant when the loudspeakers are approaching each other, $\hat{x}$ will decrease a little (thus reducing the advantage), because an increasing amount of air will move with the cones. This moving mass of air gives rise to a reactive component (X_r) of the mechanical impedance. The curve of this component has likewise been evaluated by Klapman [Fig. 7(b)].

A large cone, however, has the drawback that it must be relatively heavy to be sufficiently rigid. The loudspeakers used here (type 9710), with $2a=17.5$ cm, are

[8] S. J. Klapman, "Interaction impedance of a system of circular pistons," *J. Acoust. Soc. Amer.*, vol. 2, pp. 289–295; 1939/40.



(a)



(b)

Fig. 7—(a) The quantity $R_r/\rho cA$; (b) the quantity $X_r/\rho cA$, plotted against $2kfa/c$, for two circular pistons in an infinite baffle, vibrating in phase (according to Klapman[8]). The frequency scale is that for a diameter $2a = 17.5$ cm (loudspeaker 9710).

preferable in this respect to a loudspeaker with $2a = 17.5\sqrt{2} = 25$ cm, the cone of which is more than twice as heavy.

Loudspeakers in Closed Cabinets

A second means of reducing distortion is to enclose a given volume of air behind the cone. The enclosed air represents a stiffness S_a which, unlike the rigidity S_c of cone edge and centering ring, remains satisfactorily constant and thus tends to improve the linearity of the entire system.

In order to find to what extent S_a still deviates from true linearity, we shall consider an enclosed quantity of air under a piston. If the latter is given a displacement

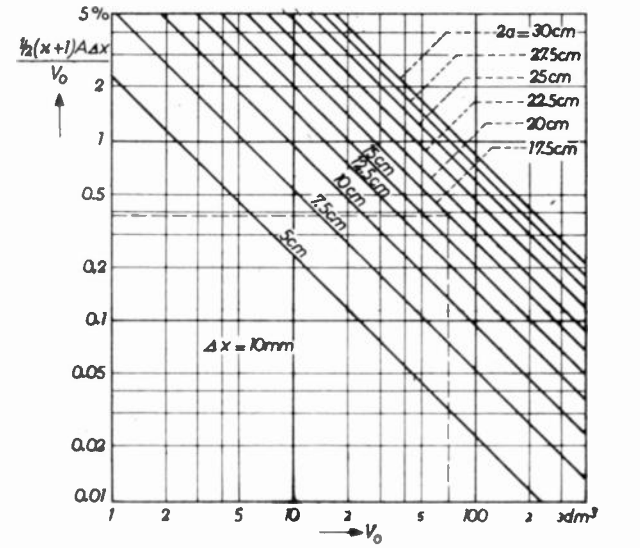


Fig. 8—The first nonlinear term in the rigidity of an enclosed volume of air, $\frac{1}{2}(\varkappa+1)A\Delta x/V_0$, as a percentage of the main term, plotted against V_0, for various values of the cone diameter $2a$, for $\Delta x = 10$ mm [see (7)].

Δx, then the air rigidity S_a exerted on the piston can be written as:

$$S_a \approx \frac{\rho_0 c^2 A^2}{V_0}\left[1 + \tfrac{1}{2}(\kappa + 1)\frac{A}{V_0}\Delta x\right], \tag{7}$$

(the process being assumed as adiabatic). Here ρ_0 represents the density; V_0 the volume of the air in the state of equilibrium; A, the area of the piston; and $\varkappa$, the ratio c_p/c_v of the specific heat of air at constant pressure to that of air at constant volume ($\varkappa \approx 1.4$).

Eq. (7) can be derived as follows. An adiabatic process occurs according to

$$pV^{\kappa} = \text{constant}$$

(p = air pressure, V = air volume). For a displacement of the piston causing the pressure to increase from p_0 to $p_0 + \Delta p$ and the volume to decrease from V_0 to $V_0 - \Delta V$, we arrive at

$$(p_0 + \Delta p)(V_0 - \Delta V)^{\kappa} = p_0 V_0^{\kappa}.$$

From this we obtain

$$\frac{\Delta p}{\Delta V} = \kappa p_0\left[V_0^{-1} + \frac{\kappa + 1}{2!}V_0^{-2}\Delta V + \frac{(\kappa + 1)(\kappa + 2)}{3!}V_0^{-3}(\Delta V)^2 + \dots\right].$$

The stiffness S_a can be defined as $\Delta F/\Delta x$, ΔF being the force necessary to bring about a displacement Δx of the piston. Here $\Delta F = A\Delta p$ and $\Delta x = \Delta V/A$, so that:

$$S_a = \frac{\Delta F}{\Delta x} = A^2\frac{\Delta p}{\Delta V} = \frac{\kappa p_0 A^2}{V_0}\left[1 + \frac{\kappa + 1}{2}\frac{A}{V_0}\Delta x + \frac{(\kappa + 1)(\kappa + 2)}{6}\frac{A^2}{V_0^2}(\Delta x^2) + (\dots)(\Delta x)^3 + \dots\right]. \tag{8}$$

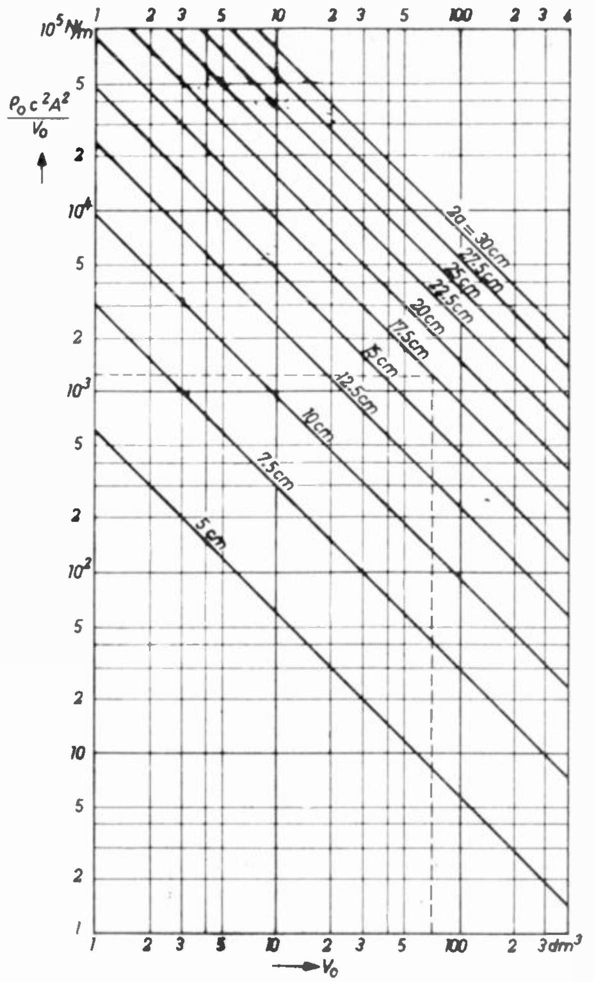


Fig. 9—Main term $\rho_0 c^2 A^2/V_0$ of the rigidity of an enclosed volume of air [see (7)], vs V_0, for various values of the cone diameter $2a$.

With the help of the relation $c=\sqrt{x p_0/\rho_0}$, the product $x p_0$ in the right-hand term of (8) can be written as $\rho_0 c^2$. By substituting this value and disregarding the terms containing $(\Delta x)^2$, $(\Delta x)^3$, etc., in (8), we arrive at (7).

The term $\frac{1}{2}(x+1)A\Delta x/V_0$ represents the nonlinearity of S_a. In Fig. 8 this term is expressed as a percentage and plotted as a function of the enclosed air volume V_0, with the diameter $2a$ of the cone as parameter; this diagram applies to the extremely large amplitude $\hat{x}(=\Delta x)=10$ mm. It then appears that the nonlinearity in s_a can be kept below 0.5 per cent by a proper selection of $2a$ and V_0 (*e.g.*, $2a=17.5$ cm, such as that of the loudspeaker 9710, and $V_0=70$ 1, which is half the volume of the low-note cabinet; this gives 70 1 per loudspeaker).

Owing to this enclosure of a quantity of air behind the cone, the total stiffness S becomes S_c+S_a; the nonlinearity of the cone stiffness S_c appears from the hysteresis loop in Fig. 5(a). It is evident that because S_a is virtually linear, S will show relatively less distortion than S_c, and the less so as S_a is greater and therefore as the air volume V_0 is smaller. [See (7) and also Fig. 9, which represents the linear part of S_a as a function of V_0 with $2a$ as parameter.]

The air rigidity cannot be indiscriminately enlarged since it is associated with an increase in the resonance frequency of the cone. This is revealed by (3), which now assumes the following form:

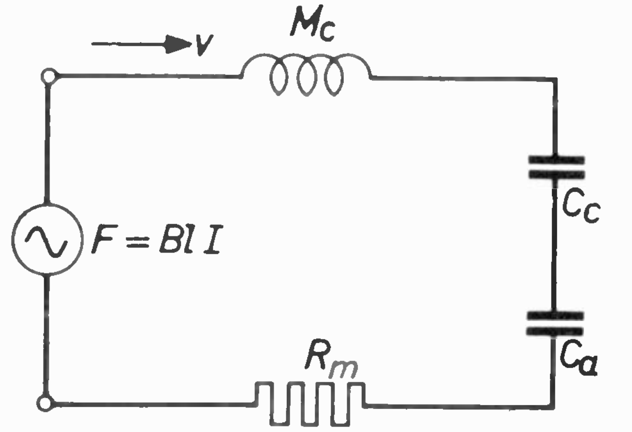


Fig. 10—Equivalent diagram of a loudspeaker with a cone compliance C_c, for a compliance C_a of the enclosed air volume behind the cone. The self-inductance M_c represents the effective mass of the cone and of the air moving with it; the resistance R_m the sum of the radiation resistance and the loss resistance.

$$f_0=\frac{1}{2\pi\sqrt{M\dfrac{C_cC_a}{C_c+C_a}}}=\frac{1}{2\pi\sqrt{\dfrac{M}{S_c+S_a}}}.$$

($C_c=1/S_c$ and $C_a=1/S_a$ are the compliance values; cf. the equivalent diagram Fig. 10.) The resonance frequency lies only slightly below f_0, and the former must be confined to the lowest regions of the frequency range to be reproduced. The resonance frequency of the type 9710 loudspeakers without cabinet is about 40 cps. The cabinet for the low-note speakers (Fig. 1) is made so large (70 l per loudspeaker) that S_a considerably exceeds S_c($S_a=1200$ N/m, $S_c=750$ N/m). This has raised the resonance frequency to about 60 cps. By using electric damping (cf. Section VI) this resonance peak has been sufficiently flattened out.

The high-note speakers are likewise accommodated in closed boxes, but here the object is to prevent back radiation. The boxes can be small (Fig. 2)—and therefore the air rigidity great—since the resonance frequency of these speakers can quite permissibly be raised to about 300 cps. In fact, a high resonance frequency is an advantage since it helps to suppress low notes, and so reinforces the effect of the electric filter, to be discussed presently.

Division of the Frequency Range

In the foregoing, intermodulation and Doppler effect were mentioned as causes of nonharmonic distortion. Both can be effectively combated by splitting up the audio range and reproducing it by separate loudspeakers.

It was found that in practice a division into *two* parts, as applied in the installation discussed here, is sufficient. As to the question at what frequency this division can best be made, it was decided to make the crossover frequency 420 cps. Combination tones of the type $\pm mf_1 \pm nf_2$, where f_1 is low (*e.g.*, 50 cps), are especially objectionable if f_2 is higher than about 400 cps, so that the crossover frequency should preferably not be fixed much higher than this value.

The filter effecting the separation of the high and low notes consists of two coils with self-inductance L and two capacitors of capacitance C, connected as shown in Fig. 11. The impedance of each of the speakers is a nearly frequency-independent resistance R_1, amounting to 7 Ω for the low-impedance speakers, and to 400 or 800 Ω for the high-impedance ones. (How this constant impedance has been achieved will be discussed later in this article.) Let the internal resistance of the amplifier be negligible, and E be its output voltage; we then find for the current I_l through the low-note speakers and I_h through the high-note projectors:

$$I_l = \frac{E}{2(1-\omega^2 LC)R_1 + j\omega L}$$

and

$$I_h = \frac{E}{2\left(1 - \dfrac{1}{\omega^2 LC}\right)R_1 - \dfrac{j}{\omega C}}.$$

The formulas may be written as:

$$I_l = \frac{E/2R_1}{\sqrt{\left\{1-\left(\dfrac{\omega}{\omega_0}\right)^2\right\}^2 + \left(\dfrac{\omega}{\omega_0}\delta\right)^2}} \tag{9}$$

and

$$I_h = \frac{E/2R_1}{\sqrt{\left\{1-\left(\dfrac{\omega_0}{\omega}\right)^2\right\}^2 + \left(\dfrac{\omega_0}{\omega}\delta\right)^2}}, \tag{10}$$

where

$$\omega_0^2 = \frac{1}{LC}$$

and

$$\delta = \frac{1}{2R_1}\sqrt{\frac{L}{C}}, \tag{11}$$

δ being the damping factor.

In order to give each network the desired pass band (Fig. 12), L and C must be so chosen that

$$\frac{1}{2\pi\sqrt{LC}} = 420 \text{ cps}, \tag{12}$$

and there must also be adequate damping δ. For critical damping (aperiodic system without free vibrations), δ must be at least 2. In that case, however, the currents I_l and I_h are insufficiently constant in their pass bands; at the crossover frequency they will drop by as much as

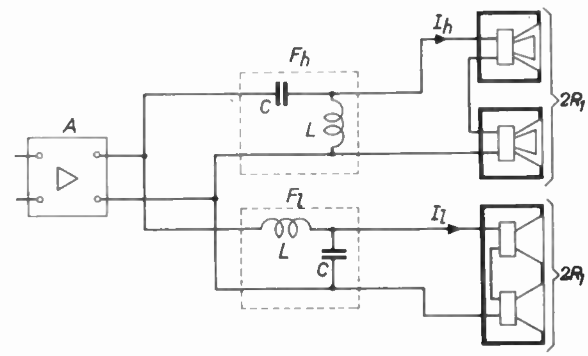


Fig. 11—F_l low-pass filter ($f=420$ cps), F_h high-pass filter ($f>420$ cps), A amplifier. For loudspeakers with a resistance $R_1=7\ \Omega$, $L=7.4$ mh and $C=\mu$f.

6 db below the value $E/2R_1$ which they respectively assume at very low and very high frequencies (see the dotted curves in Fig. 12). For obtaining a flatter characteristic we have preferred a somewhat smaller damping, *viz*, $\delta=\sqrt{2}$. The fully drawn curves in Fig. 12, which drop only 3 db below $E/2R_1$ at the crossover frequency, represent this damping value. Outside the pass bands the drop in the curves approaches 12 db per octave, *i.e.*, I_l decreases at high frequencies proportionally to f^{-2} and I_h increases at low frequencies proportionally to f^2, which is also apparent from (9) and (10).

If R_1 is known, L and C are completely determined by (11), (12), and the value selected for δ. For example, when $R_1=7\ \Omega$ we find: $L=7.4$ mh and $C=20\ \mu$f, which are the values used for low-impedance loudspeakers.

IV. Some Further Details Regarding the Cabinets

One advantage of operating the loudspeakers in completely enclosed cabinets, as we demonstrated earlier, is a reduction of distortion, as a result of the additional air rigidity. Another advantage, at least as regards the low notes, is the fact that the sound waves cannot travel around the cone (the same as if the baffle were infinitely large). This latter point may be elucidated as follows.

The sound radiation of a vibrating diaphragm is impaired if the air vibrations are allowed to travel along a short path around the diaphragm. This is why this path is usually lengthened by placing the diaphragm in a baffle,[9] *e.g.*, inside a cabinet. Let l_0 be the shortest path from the front to the rear of the diaphragm around the baffle; then, we may apply the rule of thumb that the sound emission decreases by 6 db per octave if the frequency drops below the value relating to a wavelength $\lambda = 2l_0$. For a frequency of 50 cps, for instance, the wavelength is more than 6 m. An enormous baffle would thus be required for a satisfactory reproduction of very low notes.

In the installation described here, this has been circumvented by fitting the bass-note speakers in a completely enclosed cabinet which, just as an infinite baffle,

[9] See, *e.g.*, Th. van Urk and R. Vermeulen, *Philips Tech. Rev.*, vol. 4, pp. 213–222; 1939.

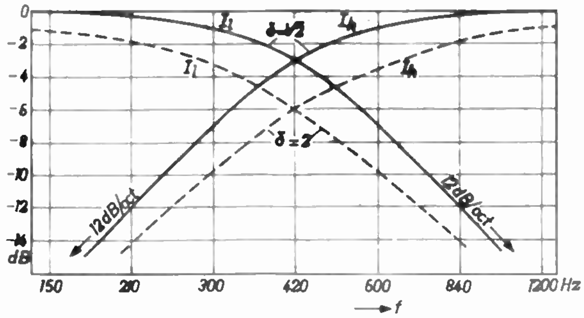


Fig. 12—Currents I_l and I_h through the low-note and high-note loudspeakers respectively plotted against the frequency f, for a constant signal voltage and a negligible internal resistance of the amplifier, both scales being logarithmic. The dotted curves apply for critical damping ($\delta_{cr}=2$), the full curves for $\delta=\sqrt{2}$.

prevents even the longest waves from traveling around.

With any cabinet, however, whether closed or not, there is a risk that it will act as a resonant cavity and give rise to standing waves due to reflection against the walls of the cabinet. This has been avoided by covering the interior of both the high and the low-note cabinets with a sound absorbent material.

No special problems were involved in this procedure for the high-note cabinet since there are several materials commercially available that adequately absorb the higher audio frequencies. For the lower frequencies, however, the solution was not so simple. In this case the principle of "panel resonance" was adopted. The inside of the cabinet was fitted with panels mounted on a framework of laths, leaving an air cushion between these panels and the walls of the cabinet (Fig. 13). Owing to the resonator interaction of the mass of the panel with the stiffness of the air cushion, an adequate absorption through the low-note range is achieved, mainly as a result of the dissipation in the clamped-in edges of the vibrating panels. The panels, consisting of an absorbent material, furthermore bring about ordinary absorption in the upper part of the low-note range.

There must be sufficient absorption over a certain frequency range. This range is determined by the following. The most important of the natural vibrations of the cabinet are those for which $\lambda/4$ or $\lambda/2$ is equal to one of the interior dimensions of the cabinet. The largest interior dimension is 70 cm, so that the lowest frequency of the natural vibrations is $c/4\times 0.70)=120$ cps. By giving the panels a natural frequency $f_p=240$ cps and making this resonance not very selective, adequate absorption is obtained in the entire range from 120 to 420 cps (in the upper part of this range, assisted by the absorption of the material itself). In connection with dimensioning the resonator such that $f_p=240$ cps, the following should be pointed out. By analogy with (3), we may write:

$$f_p = \frac{1}{2\pi}\sqrt{\frac{S}{M}}. \tag{13}$$

S represents mainly the stiffness S_a of the air cushion

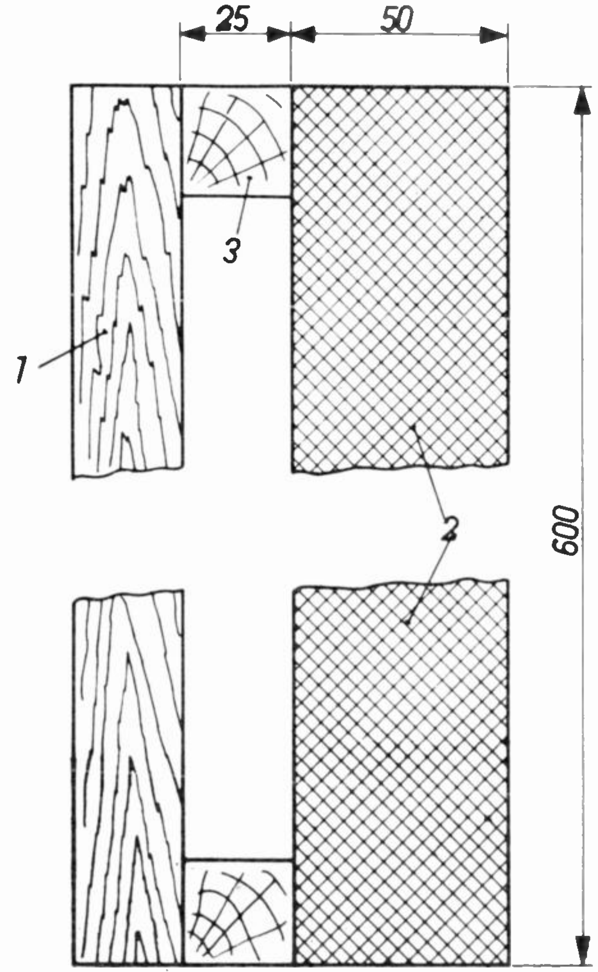


Fig. 13—Standing waves have been avoided in the low-note cabinet by fitting the inside of the walls 1 with panels 2 of absorbent material. The spacing laths 3 provide for an air cushion between wall and panel. Dimensions in mm.

behind the panel, M mainly the mass M_p of the panel; the stiffness of the panel at the edges and the mass of the air moving with the panel may be neglected to a first approximation. According to (7), S_a can be written as

$$S_a = \frac{\rho_0 c^2 A_p^2}{A_p d_a}, \tag{14}$$

ρ_0 being the density of the air; A_p, the area of the panel; and d_a, the thickness of the air cushion (panel-wall distance). M_p can be written as

$$M_p = A_p d_p \rho_p, \tag{15}$$

d_p being the thickness of the panel and ρ_p, the density of the absorbent material. By substituting (14) and (15) in (13) we arrive at:

$$f_p = \frac{c}{2\pi}\sqrt{\frac{\rho_0}{\rho_p d_p d_a}}.$$

Now $c=340$ m and $\rho_0=1.3$ kg/m^3. The density ρ_p of the absorbent material used is 55 kg/m^3. For $d_p=50$ mm and $d_a=25$ mm (Fig. 13), we find $f_p=$ approximately 240 cps.

V. Loudspeakers with Constant Impedance

The self-induction of the speech coil causes its impedance to increase with the frequency (see the dotted curve in Fig. 14). If the coil is fed with a frequency-independent voltage, the current, and consequently the force driving the cone, will decrease with increasing frequency. As mentioned in the description of the installation, the loudspeakers used have a virtually constant impedance. This is owing to the introduction of a copper ring (K, Fig. 4) within the coil. This ring operates as a short-circuited winding and thus mainly eliminates the self-induction of the coil.[10] The result is the impedance curve shown as the fully drawn line in Fig. 14. Between the frequencies 400 cps and 18 kc the impedance changes only in the ratio 1:1.5; without the short-circuiting ring this ratio would be between 1:5 and 1:6.

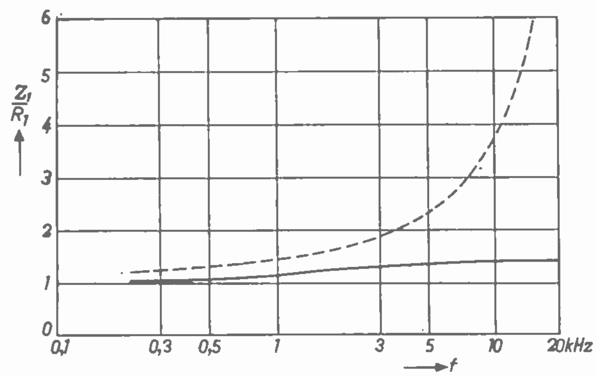


Fig. 14—Impedance Z_1 of a loudspeaker divided by the dc resistance R_1, as a function of the frequency f. Ordinary loudspeaker—dotted curve; loudspeaker with short-circuiting ring—full curve.

The optimum shape and dimensions of the short-circuiting ring can be determined from the following analysis.

Let E_1 be the terminal voltage and I_1 the current through the coil (of resistance R_1 and self-inductance L_1), and I_2 the current induced in the short-circuiting ring (of resistance R_2 and self-inductance L_2); the impedance $Z_1 = E_1/I_1$ of the coil is then given by:

$$Z_1 = R_1 + \frac{k^2 L_1 R_1 / L_2}{1 + \left(\frac{R_2}{\omega L_2}\right)^2} + j\omega L_1 \left[1 - \frac{k^2}{1 + \left(\frac{R_2}{\omega L_2}\right)^2}\right] + Z'. \qquad (16)$$

In this equation k represents the coupling factor $= \sqrt{M_{k2}/L_1 L_2}$ (M_k being the mutual induction between coil and ring), and Z' represents a term accounting for the emf induced in the moving coil. The motion of the coil, however, does not alter the alternating flux linked by the ring. In order to determine the influence of the ring on the impedance of the coil it is therefore permissible to consider the coil as being stationary, *i.e.*, to put $Z' = 0$; to distinguish Z_1 in this case we shall call it Z_{10}. As regards "low" frequencies ($\omega \ll R_2/L_2$), (16) can be written as

$$Z_{10} \approx R_1 + j\omega L_1, \qquad (16a)$$

and as regards "high" frequencies ($\omega \gg R_2/L_2$) as

$$Z_{10} \approx (R_1 + k^2 L_1 R_2 / L_2) + j(1 - k^2)\omega L_1. \qquad (16b)$$

The dimensions of the ring should be such that the following conditions are satisfied.

1) The frequency at which $\omega L_2 = R_2$ must be so low that ωL_1 is still small with respect to R_1 (*e.g.*, below 2 kc), so that at "low" frequencies [see (16a)] $Z_{10} \approx R_1$.
2) The coupling between ring and coil must be very tight, so that the factor $(1-k^2)$ in (16b) is sufficiently small to keep the term $(1-k^2)\omega L_1$ small with respect to the resistance term up to the highest audio frequencies.

Both requirements can be satisfactorily met. It should be noted that compliance with 1) means that also the quantity $k^2 L_1 R_2 / L_2$, by which the resistance term in (16b) differs from R_1, is small with respect to R_1. With an appropriately dimensioned ring, therefore, the efficiency of the loudspeaker is not appreciably reduced.

The question arises whether, with a loudspeaker without copper ring, the two soft-iron boundaries of the air gap would not act as short-circuiting rings. This effect, however, is negligible, owing to the skin effect, which is far more pronounced in iron with its fairly high permeability and rather poor conductivity than in copper. With the copper ring no skin effect is noticeable since the penetration depth of the current is greater than the thickness of the ring, even at 20 kc. This means that the current is virtually uniformly distributed over the cross-section of the ring, so that R_2 is practically equal to the dc resistance of the ring.

VI. The Amplifier

As mentioned above, the bass-note loudspeakers in their cabinet show a mild resonance peak around 60 cps. To provide a further damping of this resonance, the amplifier should have a low internal resistance.

The influence of the internal resistance upon the damping can be explained as follows. Owing to the motion of the coil (wire length l, velocity v) an emf Blv is induced in it. This emf operates on the resistance R_1 of the coil in series with the internal resistance R_i of the amplifier (R_i measured at the output terminals). These two resistances thus have to dissipate a power $(Bvl)^2/(R_1+R_i)$. Instead of this, let us imagine an equal dissipation in an imaginary *mechanical* resistance R_m':

[10] A ring of this type was mentioned in *Philips Tech. Rev.*, vol. 4, pp. 301; 1939. See also Snel and Westmijze, *loc. cit.*

$$\frac{(Blv)^2}{R_1 + R_i} = R_m'v^2,$$

and hence

$$R_m' = \frac{B^2l^2}{R_1 + R_i}.$$

The mechanical damping resistance already present is consequently raised by this amount, R_m' being larger the lower the internal resistance R_i of the amplifier.

An effective means of reducing the internal resistance is the use of voltage feedback, *i.e.*, part of the output voltage is applied in antiphase to the input voltage of the output stage or of a previous stage.[11]

The splitting-up into two frequency ranges *after* the amplifier requires an amplifier that is adequately designed to prevent any appreciable intermodulation. The amplifiers AG 9000 and AG 9006 (Fig. 1) are very suitable in this respect, respectively supplying 10 and 20 w output at 2 per cent intermodulation. The former amplifier has been designed for loudspeakers with the conventional low resistance (two 7-Ω speakers in series) and is therefore equipped with a step-down output transformer. The amplifier AG 9006, on the other hand, contains a novel output circuit, capable of directly feeding high-impedance loudspeakers;[12] the output transformer with its inevitable distortion is thus obviated here.

VII. Conclusion

The high-fidelity loudspeaker installation for the home described here comprises a corner cabinet with two bass-note loudspeakers, two separate boxes for the higher frequencies, each containing a double-cone loudspeaker, and filters for splitting up the audio spectrum into two ranges, one below and one above 420 cps. By appropriately positioning the high-note loudspeakers, good diffusion of the sound can be achieved. The listener then hears mainly indirect sound, just as in the concert hall. The so-called hole-in-the-wall effect, which is a drawback of reproduction by a single loudspeaker, is thus eliminated.

One cause of nonlinearity in moving-coil loudspeakers is the amplitude dependence of the stiffness of the cone suspension. The influence of this has been greatly reduced, 1) by doubling the number of loudspeakers, so that the cones can vibrate with a smaller amplitude, and 2) by adding a considerable, practically linear air stiffness. The latter was achieved by using completely closed cabinets. For low-note reproduction this has the additional advantage that the air waves cannot travel around the cone, while the high-note speakers will not emit any sound to the rear. Standing waves inside the cabinet are prevented by lining the cabinets with an absorbent material (in the low-note cabinet in the form of panels separated from the walls by an air cushion). Nonharmonic distortions (intermodulation and Doppler effect) have been greatly reduced by dividing the total frequency range into two parts. The over-all reproduction extends from about 20 cps to about 18 kc, the high upper limit resulting from the use of double-cone loudspeakers. Inside the coil of the loudspeakers is a fixed short-circuited ring, which virtually eliminates the self-inductance of the coil, so that the impedance rises only very little with the frequency. The amplifier feeding the installation should have a low internal resistance, which can be achieved by voltage feedback. Amplifier types AG 9000 and AG 9006 are very suitable in this respect.

[11] B. D. H. Tellegen, *Philips Tech. Rev.*, vol. 2, pp. 289–294; 1937.

[12] An article on output circuits for high-impedance loudspeakers will be published in *Philips Tech. Rev.*

A Transistorized Decade Amplifier for Low-Level Audio-Frequency Applications*

ALEXANDER B. BERESKIN†

Summary—The amplifier descrbied in this paper has an input resistance of approximately 400,000 ohms in the audio-frequency range. The output noise level is equivalent to 5 μv at the input terminals with a response that is down 3 db at 5 cycles and at 100 kc. The amplifier has been designed in a self-contained package, the size of a frozen orange juice can, suitable for plugging into the standard banana plug terminals of a sensitive vacuum tube voltmeter. The design may be modified, with considerable reduction in volume, to incorporate the amplifier in a small package with a low signal source.

In audio-frequency work it is often necessary to make measurements at voltage levels below the lowest indication on logarithmic scale vacuum tube voltmeters or in very inaccurate regions of units with semilinear scales. If a supplementary decade amplifier can be made with a sufficiently low noise and microphonic level, it would be very useful in extending the range of these voltmeters.

In the case of vacuum tube amplifiers operating with microvolts of input signal, the features necessary to supply the filament power and to control the microphonics do not usually lead to a convenient package. The use of transistors in these amplifiers eliminates both the filament and the microphonic problem and greatly simplifies the problem of obtaining a suitably filtered and shielded supply voltage.

The problems introduced by the use of transistors are those of getting a sufficiently high input impedance and a sufficiently low noise level. Both of these quantities depend to a large extent on the transistors selected and the circuits in which they are used.

The low input impedance of the common base connection rules it out for this application. The common collector circuit has a sufficiently high input impedance but no voltage amplification. It could be used as an input stage in conjunction with other stages that would contribute voltage amplification.

A common emitter circuit with a forward current transfer ratio[1] ($h_{fe}=\beta$) of about 50 could be expected to have an input impedance of about 5000 ohms and a voltage amplification of about 50 for a load resistance of 10,000 ohms. The addition of an unbypassed emitter resistor will substantially increase the input resistance at the expense of the voltage amplification. The use of a transistor with a higher forward current transfer ratio would lead to both higher input resistance and a higher voltage amplification. The increase in voltage amplification could in turn be traded for an additional increase in input resistance by using a larger unbypassed emitter resistor.

This approach is limited largely by the fact that transistors with forward current transfer ratios appreciably in excess of 50 are the exception rather than the rule. There is available, however, a circuit frequently referred to as the super-alpha[2,3] connection, shown in Fig. 1, in which two transistors are suitably interconnected and appear to the external circuit as though they were one transistor with an unusually high value of forward current transfer ratio.

When the composite super-alpha transistor of Fig. 1 is used in the common emitter connection, the input terminal is the composite base B which is the same as base B_1 of TR_1. An amplified current flows from E_1 into B_2. The highly amplified output current in the C_2 terminal of TR_2 combines with the current in the C_1 terminal of TR_1 to form the total output current flowing in the composite collector terminal C.

The following equations may be used to compute the common emitter composite hybrid parameters from the common emitter hybrid parameters of the individual transistors:

$$h_{ie} = h_{ie_1} + \frac{(1 - h_{re_1})(1 + h_{fe_1})h_{ie_2}}{(h_{oe_1}h_{ie_2} + 1)}$$

$$\doteq h_{ie_1} + (1 + h_{fe_1})h_{ie_2}$$

$$h_{fe} = h_{fe_1} + \frac{(h_{fe_2} - h_{oe_1}h_{ie_2})(1 + h_{fe_1})}{(h_{oe_1}h_{ie_2} + 1)}$$

$$\doteq h_{fe_1} + h_{fe_2} + h_{fe_1}h_{fe_2}$$

$$h_{oe} = h_{oe_2} + \frac{(1 + h_{fe_2})(1 - h_{re_2})h_{oe_1}}{(h_{ie_2}h_{oe_1} + 1)}$$

$$\doteq h_{oe_2} + h_{oe_1}(1 + h_{fe_2})$$

$$h_{re} = h_{re_2} + \frac{(h_{ie_2}h_{oe_1} + h_{re_1})(1 - h_{re_2})}{(h_{ie_2}h_{oe_1} + 1)}$$

$$\doteq h_{re_2} + h_{re_1} + h_{ie_2}h_{oe_1}.$$

In a typical application it might be desired to operate TR_2 with a collector current of 0.5 ma. If this transistor has a dc current gain of 35 it will require a B_2 input current of about 14 μa. This current is obtained from E_1 of

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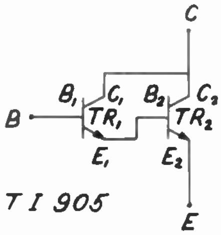
[1] The notation used in this paper follows the "IRE standards on letter symbols for semiconductor devices, 1956," PROC. IRE, vol. 44, pp. 934–937; July, 1956.

[2] A. R. Pearlman, "Some properties and circuit applications of super-alpha composite transistors," IRE TRANS., vol. ED-2, pp. 25–43; January, 1955.

[3] Patent No. 2,663,806—S. Darlington assigned to Bell Labs.



$r_e = 85\ \Omega$
$r_b = 29{,}000\ \Omega$
$r_c = 15.45\ M\Omega$
$r_m = 15.42\ M\Omega$

Quantity	TR_2	TR_1	Comp.
V_{cc}-Volts	4.5	4.5	4.5
I_c-μA	500	14	500
I_B-μA	14	2	2.8
h_{ie}-ohms	7,740	23,600	69,300
h_{fe}	80	5.3	472
h_{oe}-μmhos	21.2	.21	30.6
h_{re}-$\times 10^{-4}$	13.1	3.1	26.0

$$h_{fe} \doteq h_{fe_1} + h_{fe_2} + h_{fe_1} h_{fe_2}$$

Fig. 1—Transistor relations in super α connection.

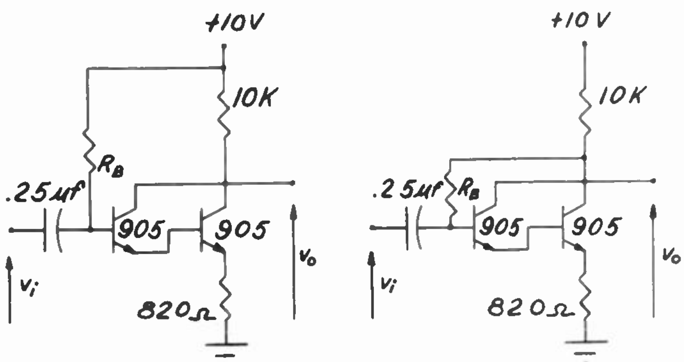


Fig. 2—Input stage.

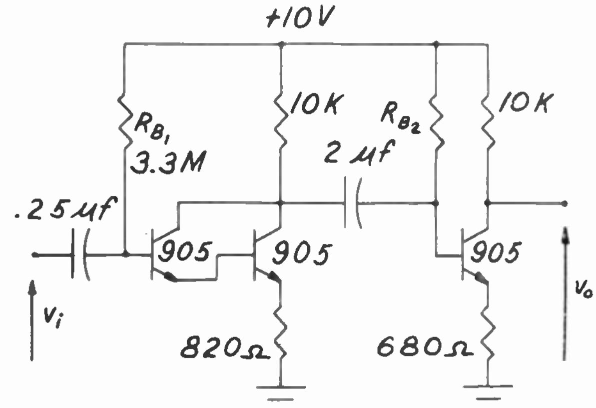


Fig. 3—Complete X100 amplifier.

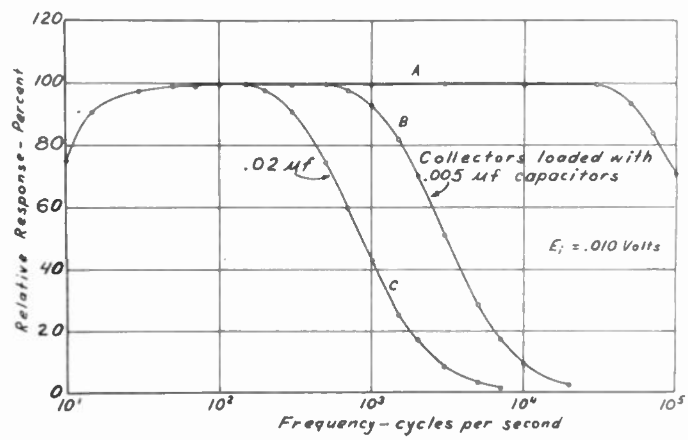


Fig. 4—Frequency response characteristics.

TR_1 so that it is essential that the current I_{co} for transistor TR_1 be substantially lower than 14 μa. Germanium transistors have to be very carefully selected to satisfy this requirement while silicon transistors do so quite readily.

If TR_2 operates at a typical value of current it is obvious that TR_1 will be operating at a value considerably below that which would be considered typical. Because of this the values of h_{fe_1} and h_{oe_1} are very much lower than the values of h_{fe_2} and h_{oe_2} while the value of h_{ie_1} is very much larger than that of h_{ie_2}. Measured values of hybrid parameters are listed in Fig. 1 for two individual TI 905 transistors, TR_2 and TR_1, and for these same transistors in the composite super-alpha connection. The resistance parameters listed below the circuit diagram were computed from the measured hybrid parameters of the composite super-alpha transistor.

If both transistors had the same forward current transfer ratio as TR_2 it would be expected that the composite forward current transfer ratio would be close to 6500, which would correspond to a common base forward current transfer ratio of -0.99985. In actual practice the reduced effectiveness of TR_1 only permits a composite forward current transfer ratio of about 500 corresponding to a common base forward current transfer ratio of -0.9980. This is still a very considerable and useful improvement over a single transistor with a forward current transfer ratio of only about 50.

Fig. 2(a) shows the basic input stage used to provide a voltage amplification of 10 with a bandwidth of approximately 100 kc. The resistance R_B is chosen to establish a dc collector current of approximately 0.5 ma. For the transistors specified previously this resistor is 3.3 megohms. The dynamic input resistance of this stage is 380 K in the audio-frequency range.

A variation of this circuit is shown in Fig. 2(b). In this case, since R_B is returned to the composite collector, the value of resistance required to establish a dc collector current of 0.5 ma is only 1.8 megohms. The dynamic input resistance of this stage is 170 K in the audio frequency range. The values of R_B stated above are nominal values while the values of input resistance are measured values at 1 kc.

If the transistors used in the circuits of Fig. 2(a) and 2(b) have been carefully selected for low noise characteristics the output noise level will be low enough to permit the use of an additional stage, shown in Fig. 3, which would make the over-all gain 100. The noise generated by the additional stage could be 3 times greater than that due to the input stage with only about 5 per cent increase in the total equivalent input noise voltage. The resistor R_{B_2} is selected to produce a collector current of approximately 0.5 ma in the second stage.

Curve A in Fig. 4 is the experimental frequency response characteristic for the amplifier of Fig. 3 and for

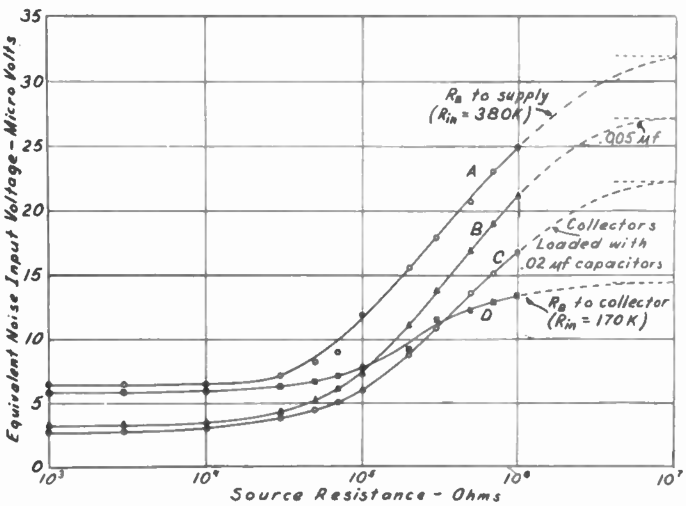


Fig. 5—Noise vs source resistance.

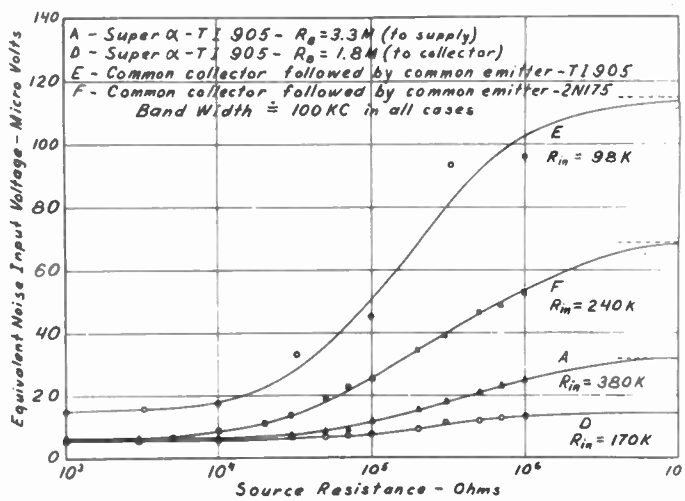


Fig. 6—Noise vs source resistance for alternate circuits.

the modification of this amplifier discussed in connection with Fig. 2(b). In many cases the wide bandwidth of Curve A in Fig. 4 may not be necessary and it may be desirable to reduce noise voltage by reducing bandwidth. In the case of Curve B the bandwidth has been reduced by connecting a 5000-$\mu\mu$f capacitor between the collector of each of the two stages and ground. For Curve C the bandwidth has been further reduced by replacing the 5000-$\mu\mu$f capacitors with 20,000-$\mu\mu$f capacitors.

If a uniform (white) noise spectrum were involved we would expect the amplifier corresponding to Curve B to have $\frac{1}{7}$ as much noise as that corresponding to Curve A, while the amplifier of Curve C should have $\frac{1}{14}$ as much noise as that corresponding to Curve A.

Curves A, B, and C in Fig. 5 show that the restricted bandwidths provide considerably less reduction in noise voltage than that which would be expected with white noise. For source resistances below 10,000 ohms Curve B is approximately one half as high as Curve A while Curve C has an almost negligible improvement over Curve B. At high values of source resistance the improvement of Curves B and C over A is relatively small. The horizontal dotted lines are the measured "open circuit input" noise values for the various conditions. Curve D is essentially the same as Curve A at low values of source resistance but less than half as great as A at high values of source resistance. While the reduction in noise of Curve D over A is appreciable, it is obtained at the price of a substantially reduced value of dynamic input resistance.

It was mentioned previously that an alternative method would have been to use a straight common collector circuit for its high input impedance properties, followed by a number of amplifier stages to supply the required gain. It must be remembered that the total input resistance is a function of the biasing circuit also and in this case the input stage biasing resistors are considerably lower than the one in Fig. 2(a). This will lead, in general, to values of total input resistance which are lower than those obtained with the circuit of Fig. 3.

Curves A and D in Fig. 6 are the same as those in Fig. 5. Curve E is for an amplifier with the same transistors, in the same order as Curve A but connected with a common collector input stage. At low source resistances Curve E is approximately $2\frac{1}{2}$ times higher than Curve A while at high source resistance values it is almost four times higher. Notice also that in addition to the increase in noise there has been a 4-to-1 reduction in the input resistance. Curve F is for special low noise 2N175 transistors in the common collector input stage circuit. These transistors had too high a value of I_{co} to be used in the super-alpha connection. It is seen that Curve F is essentially the same as Curves A and D at low values of source resistance. At high values of source resistance the noise voltage is more than twice as great as that of Curve A while the input resistance is lower than that of Curve A. Typical measured hybrid parameters for the 2N175 transistors at $V_{CE}=4.5$ volts and $I_C=0.5$ ma were:

$$h_{ie} = 3340 \text{ ohms} \qquad h_{fe} = 60$$
$$h_{oe} = 10.7\ \mu\text{mho} \qquad h_{re} = 3.8 \times 10^{-4}.$$

From the curves of Figs. 4 and 5 it is obvious that the transistor noise is not white. The curves of Fig. 7 show the spectral distribution of this noise and indicate that there is a very strong 1/f noise source in some of the cases. These curves were obtained by amplifying the output of the amplifier under test with two other similar amplifiers with unrestricted bandwidth. A 20-db resistive attenuating network was inserted between the second and third amplifier so that the system had a total gain of 10^5. The output of this system was measured with a Hewlett Packard wave analyzer, set at its minimum bandwidth position, over a frequency range of 100 to 10,000 cps. The letters A, C, and D on the curves of Fig. 7 correspond to the same letters used in previous figures.

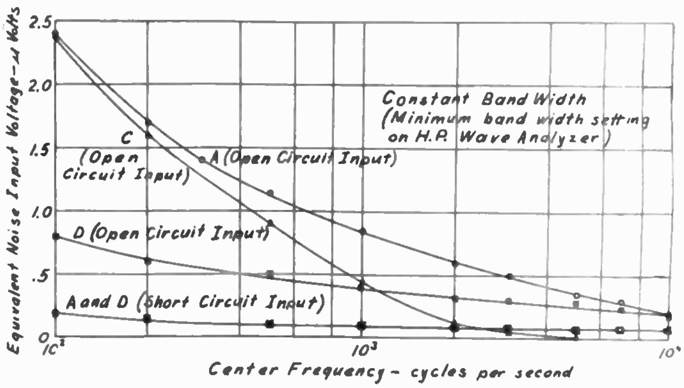


Fig. 7—Noise spectrum characteristics.

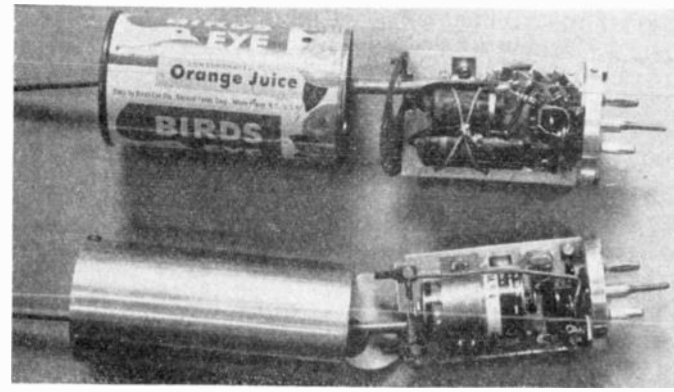


Fig. 9.

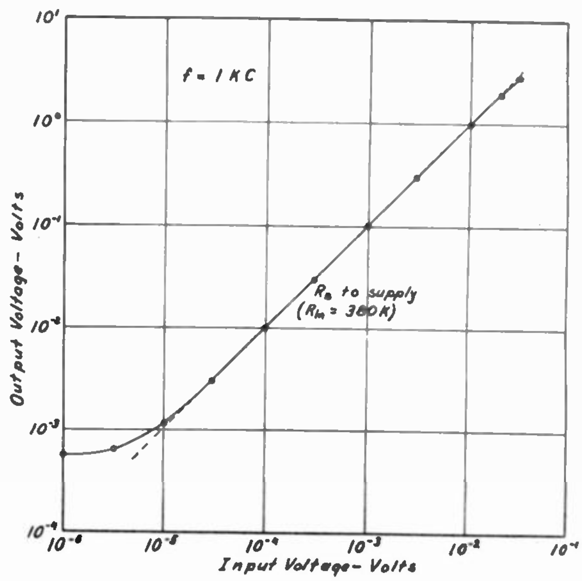


Fig. 8—Linearity characteristic.

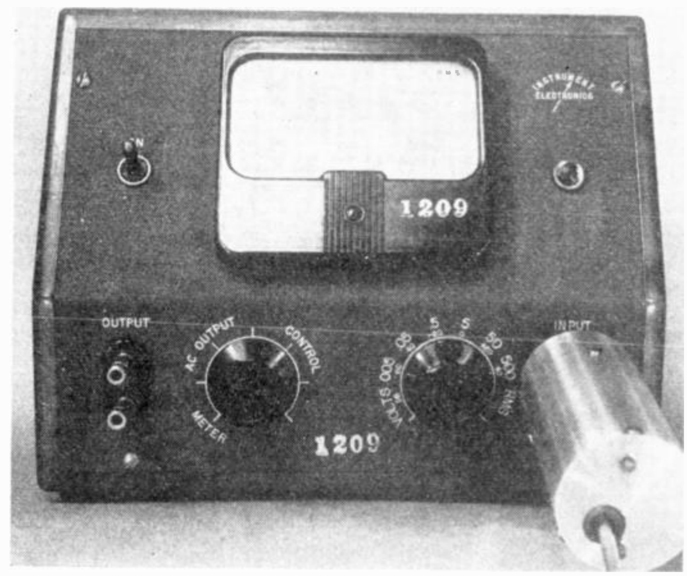


Fig. 10.

At the higher values of frequency the noise per unit bandwidth appears to be independent of whether the biasing resistor is returned to the supply terminal (Curves A) or the collector terminal (Curves D). At the lower values of frequency this noise voltage is also independent of the connection if the amplifier is operating with zero source resistance. With open circuit input the low-frequency noise is considerably higher when the bias resistor is returned to the supply terminal than when it is returned to the collector terminal. The obvious conclusion is that the 1/f noise is generated in a manner which permits a feedback path from the collector, through R_B, to the base to control the level of this noise.

Loading the collectors with capacitance to ground, on the other hand, while greatly reducing the high-frequency noise still leaves an appreciable amount of 1/f noise contribution at the low frequencies. An amplifier of this type operating with a restricted bandwidth above 10 kc would have a substantially reduced value of noise voltage.

The linearity characteristic of Fig. 8 shows that the amplifier is linear over a dynamic range of 60 db. The low end deviation is due to residual noise while the high end deviation is due to excessive drive. The high end linearity could be extended by the use of a higher value of supply voltage.

It should be emphasized that transistor selection was required to obtain the results presented in this paper. In the case of the TI 905 transistors, the two highest snr units, in a group of 15 units, were used for the input stage. In the case of the RCA 2N175 the two highest snr units in a group of 4 units were selected. The 2N175 units tested were found to have extremely uniform hybrid parameters although there was a 1.5:1 spread in the snr between the best and the poorest unit. The two best units chosen had snr which differed by only 2 per cent.

Fig. 9 is a photograph of two X100 amplifier units. The top unit, showing one side of the chassis, was the original one made while the bottom one, showing the other side of the chassis, represents a later model. The orange juice can was about the right size to house and properly shield the amplifier and gives the viewer an idea of the physical size of the device. This amplifier was designed to be plugged into the input terminals of most laboratory model vacuum tube voltmeters (vtvm). In order to eliminate a manually operated switch, a plunger which operates a small snap switch was placed between the two banana plugs. When the X100 amplifier is plugged into a vtvm it is automatically energized by having the center plunger actuate the snap switch. In order to turn the X100 amplifier off it must be pulled far enough out of the input terminals of the vtvm to permit the snap switch to open.

Fig. 10 is a photograph of a X100 amplifier plugged into a logarithmic scale vtvm and shows the effective shielding provided for the input terminals.

While this amplifier was originally designed to extend the range of a vtvm it has proven quite useful as a general amplifier for low-level signals. In special applications where space is at a premium and there is no need to handle low frequencies, the switch can be replaced with some smaller and ligher device and the coupling capacitors and the battery can be greatly miniaturized. It is assumed that the higher level device to which this amplifier is connected will have a suitable input capacitor. The complete amplifier therefore requires only one battery, three transistors, six resistors, and two coupling capacitors.

Contributors

Alexander B. Bereskin (A'41–M'44–SM'46—F'58) was born in San Francisco, Calif., in1912. He received the electrical engineering degree in 1935 and the M.Sc. in engineering degree in 1941, both from the University of Cincinnati.

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