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IRE PROFESSIONAL GROUP ON AUDIO

The Professional Group on Audio is an organization, within the framework of the IRE, of members with principal professional interest in Audio Technology. All members of the IRE are eligible for membership in the Group and will receive all Group publications upon payment of an annual fee of \$2.00.

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Professor Alexander B. Bereskin is presented a gavel in honor of his selection to National Chairman of the PGA Administrative Committee. Left to right: B. B. Bauer, A. B. Bereskin, W. C. Wayne (Chairman Cincinnati PGA), and D. W. Martin. (Cincinnati PGA Meeting, April 14, 1959; LaRue Photo.)

A Message from the New Chairman

It is indeed an honor and a pleasure to be elected chairman of the Professional Group on Audio and to have the opportunity to help the PGA serve you during the coming year. The PGA provides two major services to its members. The first of these consists in the dissemination of technical information and chapter news through the publication of the *TRANSACTIONS ON AUDIO*. The other service consists in the organization of technical sessions at the IRE NATIONAL CONVENTION, WESCON, and the National Electronics Conference.

During the past year, under the able chairmanship of Frank Slaymaker, excellent audio sessions were organized at all of the technical conferences. We hope to be able to do as well during the coming year. If we are to achieve this success, however, it will require the cooperation of our membership in preparing material for presentation at these sessions. The large attendance that we have experienced in the past definitely makes the effort of preparation worthwhile.

Papers published in the *TRANSACTIONS* reach an audience in excess of 5,000. This is a highly concentrated audience of people with a major interest in audio techniques.

During the coming year, Ben Bauer will continue as Secretary-Treasurer and Marvin Camras will continue as Editor of the *TRANSACTIONS*. Both of these offices involve heavy effort and the Editor, especially, should receive utmost cooperation in the form of suitable technical papers submitted for publication in the *TRANSACTIONS*.

Chapter news is particularly important since it provides a means by which the chapters can share program ideas and techniques. Monthly reports on chapter activities should be sent to J. Ross Macdonald.

Additional committees are being formed at the present time and their membership will be announced at an early date.

ALEXANDER B. BERESKIN

PGA News

NEWLY ELECTED NATIONAL OFFICERS, 1959-1960

Alexander B. Bereskin (A'41-M'44-SM'46-F'58) was born in San Francisco, Calif., on November 15, 1912. He received the B.S. degree in electrical engineering in 1935 and the M.S. degree in engineering in 1941, both from the University of Cincinnati, Ohio. He is a member of the faculty of the University of Cincinnati with the rank of Professor of Electrical Engineering. Before joining the faculty, he was affiliated with the Commonwealth Manufacturing Corp. and the Cincinnati Gas & Electric Co. He is also active in consulting engineering and is a registered professional engineer in the State of Ohio.



A. B. BERESKIN
Chairman, 1959-1960

He is National Chairman of the IRE Professional Group on Audio and past Editor of the IRE TRANSACTIONS ON AUDIO. He is a member of the Education Committee, and Institute Representative at the University of Cincinnati. He is Past Chairman, Vice-Chairman, and Treasurer of the Cincinnati Section. He is also a member of the AIEE, Sigma Xi, Eta Kappa Nu, and Tau Beta Pi.

Mr. Bereskin has published work on vacuum tube and transistor audio power amplifiers, low-level transistor audio amplifiers, video amplifiers, regulated power supplies, and power factor meters. He has also done work in the fields of special RC oscillators, frequency selective amplifiers, low-jitter multivibrators, special stabilized power supplies, and transistor pulse amplifiers.

John K. Hilliard (A'25-M'29-SM'43-F'52) was born in Windemere, N. D., on October 22, 1901. He graduated from Hamline University, St. Paul, Minn., in 1925 and was awarded the honorary Doctor of Science degree in 1951.

Until 1942 he was engaged in motion picture sound engineering with Metro-Goldwyn-Mayer Studios. He joined the staff of M.I.T. and worked on radar design during 1942-1943. From 1943 to the present time, he has been with the Altec Lansing Corporation, Anaheim, Calif., where he is now Staff Vice-President in charge of advanced engineering. He pioneered in the development of high intensity environmental test chambers including the measurement and generation of high sonic energy. In addition to his work in this field, he is also engaged in the design of numerous other electro-acoustical products, including telephone products.



J. K. HILLIARD
Vice-Chairman, 1959-1960

Mr. Hilliard is a member of the Acoustical Society of America, Society of Motion Picture and Television Engineers, and the Audio Society. He is also a member of the Armed Forces National Research Committee on Hearing and Bio-Acoustics and the Society of Environmental Engineering. He has served the IRE and the PGA in numerous capacities; he was a member of the Editorial Committee from 1953-1955; Chairman of the Awards Committee, 1954; West Coast Program Committee, 1955-1956; and National Vice-Chairman, 1951-1953.

Peter C. Goldmark (A'36-M'38-F'43) was born in Budapest in 1906 and studied at the Universities of Berlin and Vienna, receiving the Ph.D. degree in physics from the University of Berlin, Ger., in 1931.

From 1931-1933 he was engineer in charge of television development for Pye Radio Ltd., Eng. In 1933 he came to the United States and in 1936 joined Columbia Broadcasting System as chief television engineer, later becoming director of the Research and Development Division. The first practical color television system was developed under his direction in the CBS Laboratories, and on August 27, 1940, the first color

broadcast in history was made from the CBS Television transmitter in New York. Under his direction during World War II, CBS Laboratories were responsible for many military developments in the field of electronic countermeasures and reconnaissance. After the war, he and his associates developed the long-playing record. The development of the first high-fidelity compact phonograph, the Columbia 360, followed. In 1954 he became President of CBS Laboratories, and Vice-President of CBS, Inc.



P. C. GOLDMARK
Administrative Committee, 1959-1960

Dr. Goldmark is a Fellow of the AIEE, the Society of Motion Picture and Television Engineers, and the British Television Society. In 1945, he was awarded a medal by the Television Broadcasters Association for his color television pioneering work, and in 1946 the IRE awarded him the Morris Liebman Memorial Prize for electronic research. He is also a visiting Professor for Medical Electronics at the University of Pennsylvania Medical School.

J. Ross Macdonald (S'44-A'48-SM'54-F'59) was born on February 27, 1923, in Savannah, Ga. He received the B.A. degree in physics from Williams College, Williamstown, Mass., and the B.S. degree in electrical engineering from M.I.T., Cambridge, both in 1944. After teaching at the M.I.T. Technical Radar School

and serving as a radar officer in the U.S.N.R., he returned to M.I.T. in 1946 and received the S.M. degree in electrical engineering in 1947. At the completion of further graduate study in physics at M.I.T., he spent two years at Oxford University as a Rhodes Scholar, receiving the D.Phil. degree in physics in 1950.



J. R. MACDONALD
Administrative Committee, 1959-1960

After two years of carrying out and directing research in experimental and theoretical physics at Armour Research Foundation, he spent a year's leave of absence at Argonne National Laboratory working on solid-state physics problems. He joined Texas Instruments, Inc. in 1953 and is presently Director of the Solid-State Physics Research Department. In addition, as Clinical Associate Professor of Medical Electronics at the Southwestern Medical School of the University of Texas, he consults in the fields of physics and electronics.

Dr. Macdonald is a Fellow of the American Physical Society, and a member of Phi Beta Kappa and Sigma Xi. He has published more than fifty technical papers in the fields of physics and electronics. In 1957, he was selected for the IRE-PGA Senior Paper Award. He has served as Program Chairman of the Dallas Section of the IRE, member of the IRE Electron Devices Committee, and member of the IRE Semiconductor Device Research Conference Technical Committee. He is presently an editorial reviewer for the PROCEEDINGS OF THE IRE, a member of the Editorial Committee of the PGA, and Chairman of the PGA Committee on Chapters.

AWARDS FOR 1958

The Awards Committee of the Professional Group on Audio is proud to announce the following awards for the year 1958.

IRE-PGA Achievement Award

Daniel W. Martin—To honor a member of the PGA who, over a period of years, has made outstanding contributions to audio technology documented by papers in IRE publications. The certificate and \$200 award will be presented at a forthcoming meeting of the Cincinnati IRE Section.

IRE-PGA Senior Award

Murlan S. Corrington and T. Murakami—For their paper entitled, "Tracing Distortion in Stereophonic Disc Recording," 1958 IRE NATIONAL CONVENTION RECORD, pt. 7, p. 73. The certificate and \$100 award will be presented at a forthcoming meeting of their local IRE Section.

Murlan S. Corrington (SM'49-F'54) was born on May 26, 1913 in Bristol, S. D. He received the B.S. degree in electrical engineering in 1934 from the South Dakota School of Mines and Technology, Rapid City, and the

M.S. degree in 1936 from Ohio State University, Columbus. From 1935 to 1937 he was a graduate assistant in the Physics Department of Ohio State University, where he specialized in mathematical physics. In 1937 he joined the Rochester Institute of Technology where he taught mathematics, mechanics, and related subjects. Since 1942 he has been engaged in mathematical engineering and is a leader of the development engineering section of the RCA Defense Electronics Products Division. He is now working in the field of communication theory and coding.



M. S. CORRINGTON
Senior Award, 1958

He is a member of Sigma Pi Sigma, the Society of Industrial and Applied Mathematics, and a Fellow of the Acoustical Society of America. In the IRE he is active on many committees, a member of the National Administrative Committee of the Professional Group on Audio and Past Chairman of the Philadelphia Section. He is a member of Commission 1 of the U.S.A. National Committee of the International Scientific Radio Union, the international study group in the field of basic measurements.

He has written many technical papers on frequency modulation, circuit theory, transients and cone motion in loudspeakers, distortion in phonograph records, etc., and is the author of textbooks on mathematics and machine shop practice.

Daniel W. Martin (M'50-SM'54-F'57) was born on November 18, 1918, at Georgetown, Ky. He graduated from Georgetown College, Ky., in 1937, and received the M.S. degree in physics in 1939 and the Ph.D. degree in physics in 1941 from the University of Illinois, Urbana.

From 1941 to 1946 he was engaged in acoustical development of sound-powered telephones, throat microphones, and military acoustic systems at RCA-Victor Division, Indianapolis, Ind. During this same period he served as instructor in mathematics for Purdue University extension. From 1946 to 1949, at RCA-Victor in Camden, N. J., he worked on theater sound, elec-

tronic piano, and served as technical coordinator for development of the AN/AIC-10 intercommunication system used by the United States Air Force since that time. In 1949 he joined the research staff of The Baldwin Piano Company, Cincinnati, Ohio, as an acoustical consultant, becoming supervising engineer of the Acoustical Laboratory in 1950. Since then he has carried on and directed research on pianos, electronic tone radiation systems, reverberation loudspeakers, psychoacoustics of musical tone, airborne intercommunication systems and components, and a high-power airborne announcing system. Since 1957 he has been assistant chief engineer, in charge of research. He is the author or co-author of numerous papers on acoustical and audio subjects published in the *Journal of the Acoustical Society of America*, and these TRANSACTIONS. He has numerous patents on acoustical devices and musical systems.



D. W. MARTIN
Achievement Award, 1958

Dr. Martin is a Fellow of the Acoustical Society of America, and a member of its Executive Council. He is also Chairman of its Technical Committee on Musical Acoustics. He is the IRE representative on the American Standards Association Sectional Committee S1 on physical acoustics, and has served as chairman and member of various standards committees. He has served as a patent reviewer for the *Journal of the Acoustical Society of America*, since 1950. In the PGA, he was Chairman in 1956, and a member of the Administrative Committee from 1954-1957. He has served on the Editorial Committee since 1951 and was Editor of these TRANSACTIONS from 1953-1955. He was a co-recipient of the PGA Senior Award in 1958.

Tomomi Murakami (A'45-M'51-SM'55) was born on May 6, 1922, in Los Angeles, Calif. After attending Compton Jr. College, Compton, Calif., for two years, he continued his education at Swarthmore College, Pa., where he received the B.S. degree in electrical engineering in 1944. In 1947 he received the M.S. degree in electrical engineering from the Moore School of Electrical Engineering of the University of Pennsylvania, Philadelphia.



T. MURAKAMI
Senior Award, 1958

From 1944 to 1946 he was an assistant and research associate in the Department of Electrical Engineering at

Swarthmore College. Since 1946 he has been associated with the Advanced Development section of the RCA-Victor Radio and Television Division, Camden, N. J. From 1946 to 1953 he was engaged in design and development of radio, FM and television circuits, and in these areas he has six issued patents with several others pending. For the past six years he has been doing theoretical and consulting work in circuit theory and system analysis.

Mr. Murakami has published numerous technical papers on circuits and circuit theory, and at present is engaged as co-author of a book to be published by John Wiley and Sons, Inc. He is a member of Sigma Xi and Sigma Tau.

CHAPTER NEWS

Chicago, Ill.

"Some aspects on the future of high fidelity and stereophonic sound" was presented by Robert Oakes Jordan, on Friday, April 10, 1959, at the Western Society of Engineers Building in Chicago. According to the April issue of *Scanfax*:

Jordan, who has been working in high fidelity and stereophonic sound, discussed the present and future technical trends of these communications arts.

He spoke both as a nationally known columnist on these subjects and as an R and D physicist involved in the design and scientific aspects. New and interesting facts about stereo were brought out.

Robert Oakes Jordan, Director of Robert Oakes Jordan and Associates, and executive vice-president of Sonic Arts, has worked on infrared projects for the Navy and teletype work at Kleinschmidt Laboratories.

He has pioneered in stereo research, is the inventor of a telephone for the totally deaf, and holds some 20 patents. The author of several columns, he also has written 11 books on audio subjects.

San Francisco, Calif.

Members of the San Jose Symphony participated in woodwind demonstrations at a talk on "Live vs Recorded A-B Tests of Loudspeaker Systems" by M. Ward Widener of Ampex Audio on Tuesday, January 20, 1959. In the February issue of *The Grid*, Lawrence Johnson reported that:

The first portion of the dual presentation was made up of live music by a quintet of wind instrumentalists from the San Jose Symphony. It was designed to form a frame of reference for the second portion, which included a series of comparisons of live and specially recorded music. The meeting, held in Stanford University's Dinkelspiel Auditorium, was a joint session with the Audio Engineering Society.

The live quintet was made up of flute, oboe, horn, and bassoon, the last-named being played by the speaker. Eight selections were played, from the works of Bach, Poulenc, Leclair, and Debussy. Audience response indicated thorough approval of the performances of Widener and his guests.

The program then turned to the live/recorded comparison. A stereo tape recording of the Debussy selection was played, followed by a section-by-section intermixture of live and stereo recorded segments. The results were interesting and stimulating to both audience and performers.

Purpose of the special comparison was to determine how "live" the sound from an electronic reproducing system can be made. Toward this end, Widener had made the stereo recordings some time earlier in the Ampex Audio anechoic chamber, so as effectively to eliminate reverberation in the recording process. When such a recording is played back in a listening room, only that room's own characteristic reverberation is present, and the over-all result is, as the demonstrations showed, a remarkably close facsimile of the sound produced by the musicians performing live in the same listening room.

Success in such an effort depends on a number of critical points, involving microphone and speaker placement, as well as equipment quality. Widener pointed out that his objective of making the best possible facsimile of a live performance is separate and distinct from the alternative objective of making the most pleasing recording. The latter involves listener preference—a psychological no-man's land inhabited principally by marketing specialists.

The dual three-way speaker systems employed sealed enclosures for the low-, middle-, and high-frequency speakers. Ampex recorders, McIntosh power amplifiers, and an Ampex stereo control center were used.

The demonstration then turned to the intricacies of the tone quality of musical instruments—the bassoon in particular. Slides of recorded waveforms were presented, showing the startling variation of the waveform of the bassoon tone over its frequency range. While the higher notes begin to approach a pure sinusoid, the lower notes turn out to be almost completely lacking in the fundamental frequency. Nevertheless the psychological conditioning of our responses is such that our ears unhesitatingly tell us that a bassoon is playing, no matter what isolated note we may hear.

To get around this natural conditioning, Widener presented what he called "Concerto for Tape Recorder and Speaker Basket," which was in truth far simpler than its name might lead the reader to believe. It consisted of live and recorded comparison of a sound about which the listeners could have no preconceived notions. The sound was made by striking a bare loudspeaker frame with a plastic hammer; the result was a somewhat chime-like sound.

The prepared program concluded with the playing of more bassoon and quintet music, this time recorded in the relatively reverberant Ampex Audio listening room rather than in the anechoic chamber. In this, as in the other tape work, Widener was assisted by Roy Long of SRI, whose contributions added considerably to the success of the presentation.

A question and discussion period was followed by a-b demonstrations of the effects of small variations in speaker system frequency response. Widener pointed out that although such changes are often—though not always—easily noticed, it is difficult to say which of two conditions is more pleasing. Part of the audience may have come away convinced of the reliability of a careful listener's judgment of speaker system response; others, including the writer, felt instead that even most carefully thought-out opinions must have been wrong about as often as they were right.—J. ROSS MACDONALD

The Editor's Corner

MUSIC SPECTRA, EQUALIZATION, AND NOISE

Background noise remains the earmark of artificially reproduced sound, especially when an audio system is almost perfect in other respects. A significant improvement in signal-to-noise ratio can often be made in a recording system by a technique of pre-equalization followed by complementary post-equalization. The same considerations also apply to a radio system, or in general to any chain having an intermediate noisy link.

For example, suppose that we want to minimize high-frequency noise of a recording system where hiss or needle scratch are annoying. In this case, we boost the high frequencies of our music during recording by sending the program material through an equalizer of rising frequency response. Upon playback, we listen to the output through a second equalizer with a falling frequency response which just compensates for the equalizer used in recording. The result is an over-all flat response for the signal, but a falling response for the noise which goes through the second equalizer only.

Many factors are considered in choosing frequency response of the equalizers. A ceiling on the amount of pre-equalization at each frequency is set by the overload point of the recording system at that frequency. But whether we reach the overload point depends not only on the maximum capability of the system, but also on the frequency spectrum of the program material which we are recording. If the program is deficient in high frequencies, we can increase the high-frequency equalization without fear of overload.

It is thus apparent that before we can design a pre-equalization system, we must know the frequency spectrum of the program to be recorded. The most comprehensive study of spectra of orchestral and instrumental music was carried out nearly thirty years ago by Sivian, Dunn, and White at the Bell Laboratories. Their findings have served as a fundamental basis for equalizer design over the years; although there have been minor arguments, re-evaluations, etc., their measurements have been confirmed by later tests. Most of the tentative disagreements turned out to be matters of interpreting the data, rather than disagreeing with the measure-

ments. A major cause of misinterpretation was the form in which the results were presented, which was reasonable at the time, but turned out to be somewhat outside the vocabulary of later-day acousticians and audio engineers.

Recently Dr. Dunn undertook the formidable task of revising his original data in accordance with present-day usage. We believe that in its revised form the information will be more useful than ever. We consider it a privilege to publish this classic material in the present issue of these TRANSACTIONS.

—MARVIN CAMRAS, *Editor*

NEGATIVE FEEDBACK IN CANADA

Referring to your recent editorial,¹ we did avail ourselves of this "gimmick" in our 3-way portables built in 1949 and 1950.

A 4.7 megohm resistor was connected from plate of 3V4 output tube to plate of 1U5 detector-audio. The purpose was "to increase undistorted power output." These portables were notably successful in the field.

The 3V4 output is very limited and the increase realized was noticeable. With the usual ac, ac/dc sets, and TV. sets the output is ordinarily adequate so the extra is hardly appreciable.

E. OLSON,
Canadian Westinghouse Co., Ltd.
Brantford, Ont., Can.

COMPLETE CYCLE

Of course, you have heard of the school boy—all excited—because his teacher had a new kind of portable record player with no tubes, no plug-in, just turn a crank? It happened to a friend of mine who was flabbergasted when the youngster rushed in with this announcement.

EDWARD W. LOGAN, JR.
953 N. Highland
Memphis 12, Tenn.

¹ M. Camras, IRE TRANS. ON AUDIO, vol. AU-7, p. 1; January-February, 1959.

Absolute Amplitudes and Spectra of Certain Musical Instruments and Orchestras*

L. J. SIVIAN†, H. K. DUNN‡, AND S. D. WHITE§

Summary—Measurements made on instruments and orchestras, during the playing of selections, include average amplitudes in long intervals (15 seconds) and distribution of peaks in very short intervals (one-eighth second). Octave and half-octave bands are measured, as well as unfiltered music. The instruments tested were selected as possibly contributing extreme frequencies and amplitudes. Calculations of peak acoustic powers range as high as 27 watts.

Although different techniques would be applied today, such as the use of a recorded tape to insure exact duplication of a selection as different frequency bands are explored, and the measurement of rms rather than average amplitudes, it is nevertheless felt that the measurements were reasonably accurate. The type of presentation, however, left something to be desired. Both kinds of curves will be found here in new forms. For the peaks, the plan used later in presenting similar data on speech¹ has been adopted. The average amplitudes are reduced to a per-cycle basis in a different manner. Both types are plotted in absolute rather than relative units, and are reduced to a common distance. The table of peak powers is recalculated, using the same assumptions as before, but making the estimate for the 1 per cent level of intervals, in all cases. All new reductions and calculations are made from the original measured data.

Some changes in the text are necessary to describe the revisions of the figures, and a few comments have been added. The term "bar" which was in current use in 1931 for "dyne per square centimeter," is changed here to "microbar." The "Historical Note," which appeared at the end of the original paper, has been shortened somewhat and made a part of the introduction.

INTRODUCTION

THE analysis of instrumental musical sounds is a problem of rather old standing. It has been approached at various times by both physicists and psychologists, since the days of Helmholtz's work with his resonators. A summary of the earlier work and a description of his own notable contributions to the subject may be found in Miller's book.² The results of these and other investigations were largely confined to the determination of the frequencies of those components which the investigators held to be responsible for the musical quality of the instrument.

In the earlier measurements, neither the relative nor absolute physical magnitudes of the frequency components were determined. This was not possible until

microphones admitting of absolute calibration over the frequency range became available. The condenser microphone, calibrated by the thermophone method, or by some one of several other methods more recently applied,³ has proved useful for this purpose. A summary of its application, in conjunction with electric filters, to the statistical analysis of various sounds has been given by one of the authors.⁴ Several other investigators have recently made similar application. In particular, H. Lueder⁵ has made a statistical analysis of the amplitudes and spectra of a number of musical instruments and orchestras.

In the previously mentioned paper,⁴ measurements have been described on average and peak amplitudes in speech, using apparatus in which the speech spectrum was divided into thirteen bands of frequencies. The same apparatus has been used in a series of measurements on musical instruments, which are reported in this paper.

As with the speech measurements, the data are statistical in nature, and are taken with a view to their engineering applications. These applications are concerned, chiefly, with the transmission and reproduction of music, and the data should show the power and frequency requirements for systems which are called upon to perform these functions without distortion. In carrying out this purpose, it has been thought well to measure both individual instruments and instruments playing together in orchestras; to make measurements on actual musical selections, rather than on single notes; and to take the measurements in such a way as to obtain an average or integrated picture of the selection, as well as the distribution of amplitudes in magnitude and frequency, the extreme values being particularly important.

APPARATUS

The apparatus was developed for the purpose of making just this sort of statistical measurement on sounds. Since it was not described in the previous paper, a brief description will be given here.

A block diagram is shown in Fig. 1. The fourteen band-pass filters were designed so that when connected in parallel on the input side, and with the output of each

* Manuscript received by the PGA, February 27, 1959; manuscript revised by H. K. Dunn. This paper was presented before the Acoustical Society of America, December 14, 1929, and was first published in *J. Acoust. Soc. Amer.*, vol. 2, pp. 330-371; January, 1931.

† Deceased.

‡ Bell Telephone Labs., Inc., Murray Hill, N.J.

§ Bell Telephone Labs., Inc., New York, N. Y.

¹ H. K. Dunn and S. D. White, "Statistical measurements on conversational speech," *J. Acoust. Soc. Amer.* vol. 11, pp. 278-288; January, 1940.

² D. C. Miller, "The Science of Musical Sounds," The Macmillan Co., New York, N. Y.; 1916.

³ See for example L. J. Sivian, "Absolute calibration of condenser transmitters," *Bell Sys. Tech. J.*, vol. 10, pp. 96-115; January, 1931.

⁴ L. J. Sivian, "Speech power and its measurement," *Bell Sys. Tech. J.*, vol. 8, pp. 646-661; October, 1929.

⁵ H. Lueder, "Zur statistik der intensitätsverteilung in spektrum natürlicher klangbilder," *Veröff. Siemens-Konzern*, vol. 9, pt. 2, pp. 167-225; 1930.

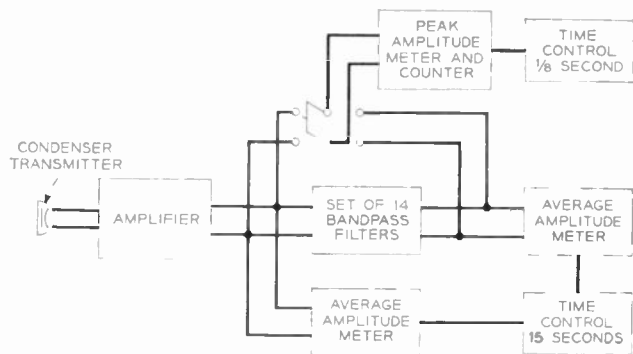


Fig. 1—Diagram of circuit for measuring peak and average pressures in music.

filter terminated in a 600-ohm resistance, they present a uniform impedance of 600 ohms at all frequencies. Only one filter at a time is used, this one being connected to the 600-ohm measuring circuit, while all the others are connected to 600-ohm resistances. Fig. 2 shows a typical filter attenuation characteristic. The ranges of frequencies covered by the filters are as follows: 62.5 cps low-pass, three one-octave bands from 62.5 to 500 cps, nine half-octave bands from 500 to 11,300 cps, and in addition, an 8000 cps high-pass.

Two distinct types of measurement are made. The "average-amplitude meter" consists of an amplifier, a rectifier, and a shunted fluxmeter. The rectifier is a two-element vacuum tube with a resistance in series, of such value that within the range of amplitudes expected, it is very nearly linear in response. Fig. 3 shows a curve of the output against a sine-wave input, the lower limit of actual use being marked. The dotted line shows the true linearity which the curve approaches. The possible error at the lowest output is 1.5 db, but this rapidly diminishes as the output increases.

The indicating instrument in the average-amplitude meter is a Grassot fluxmeter.⁶ This is a very convenient instrument for measuring quantity of electricity over periods much greater than those in which a ballistic galvanometer may be used. It is essentially a galvanometer in which the restoring torque is made very small, yet which has a high electromagnetic damping when placed in a low resistance circuit. Under these conditions the needle moves at a rate proportional to the current and remains almost stationary when the current stops. In a vacuum-tube circuit, the low resistance required for damping must be provided by means of a shunt. The sensitivity is proportional to the resistance of the shunt, but a limit is set by the fact that the torque cannot be reduced entirely to zero, and results in a drift of the needle with a rate proportional to the sum of the resistances of shunt and meter. The limiting effect of this drift is not directly upon the quantity sen-

⁶ H. K. Dunn, "The Grassot fluxmeter as a quantity meter," *Rev. Sci. Instr.*, vol. 10, pp. 368-370; December, 1939. A companion paper describes the use of thermocouple and fluxmeter for measuring rms amplitudes, but this combination had not been devised at the time of the music measurements.

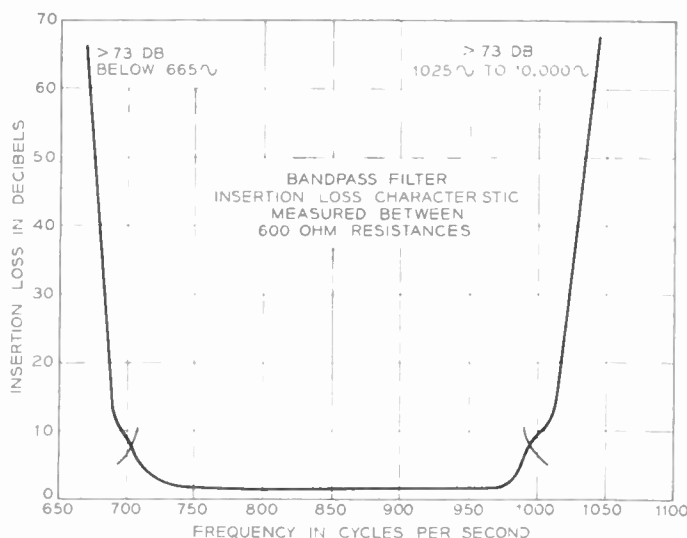


Fig. 2—Attenuation of 700-1000-cps band-pass filter. Lines at left and right show where curves for adjacent filters cross. Attenuations shown are typical of all filters used.

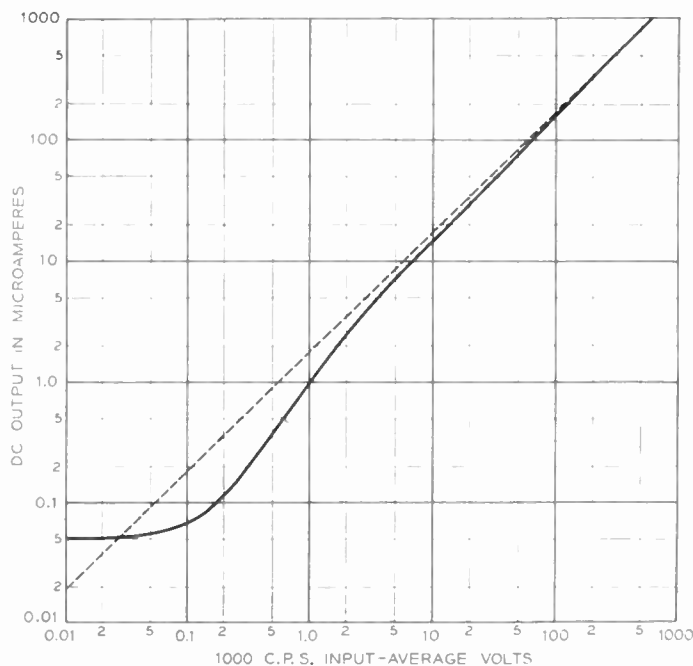


Fig. 3—Response of two-electrode, vacuum-tube rectifier, with 300,000-ohm resistance in series. Cross mark shows lowest average output used in measurements, while dashed curve shows the characteristics of a perfect linear rectifier.

sitivity, but rather upon the relation of current and time for a given sensitivity, the maximum allowable error having been established. Thus, with the meters used, the resistance being 23.5 ohms, it was found that with a shunt of 24.5 ohms the drift in 15 seconds would not be greater than 5 per cent of the deflection from the natural point of rest. If 5 per cent is set as the limit of error, greater intervals than 15 seconds could not be used without decreasing the sensitivity by lowering the shunt; and greater sensitivity could be used only by shortening the interval. A final limit to the length of interval that may be used, for a given percentage error, is set by the

resistance of the meter itself. With the meters used, this limit is about half a minute for a 5 per cent error. In all of our work, the 15-second interval has been used. In order to make use of the entire scale with equal percentage error for all deflections, the natural point of rest is adjusted to one end of the scale. Provision is made for returning the needle to this point electrically after reading a deflection. Calibration of the meters is made with constant currents over 15-second intervals, so that errors in reading due to drift arise only when the distribution of current over the interval is very irregular.

It is seen that the fluxmeter, used with the linear rectifier over a definite time interval, shows the average amplitude during the interval. Two of the average-amplitude meters are used, the "total" responding to all frequencies, the "band" to those passing the filter in use. Measurements with both meters are made regularly in alternate 15-second intervals, the intervening intervals being used for reading and resetting the fluxmeters. The range covered in the band average-amplitude meter is 35 db, and in the total 33 db. This is accomplished in the first case by using the mirror attached to the fluxmeter coil, for very small deflections; and in the other case by using two fluxmeters in series, one with a smaller shunt than the other, and read only when the other meter is off scale.

The other type of measurement is that of peak amplitudes. The most important part of the "peak-amplitude meter" is a set of gas-filled vacuum tubes.⁷ In a tube of this kind, with a given plate voltage and grid bias, a peak voltage of a certain definite magnitude applied to the grid causes an arc to strike between plate and cathode, and this current then continues to flow. It is large enough to operate a sensitive relay, being of the order of 10 to 20 ma. The arrangement of these tubes is shown in Fig. 4. Ten tubes are used, divided into two groups of five each. These groups are fed by amplifying tubes, one of which is set at a level 30 db above the other. Within a group of five tubes, the grid biasing voltages are adjusted so that the peak voltages required to strike the different tubes are progressively higher. The difference between successive tubes is set at 6 db, so that the entire range covered between first and tenth tube is 54 db. A relay is shown in the plate circuit of each tube. These relays operate electric counters, which count each flash of the corresponding tubes. In order to insure a count even on a very brief flash, intermediate relays are used which hold themselves closed until the counter armature has completed its stroke, at which point it makes a contact. Also, to prevent repeated counting, the intermediate relay does not release until the first relay has fallen back, the back contact of the first relay and the contact of the counter being connected in series to release the intermediate relay. The tubes are

extinguished by means of a relay opening the common plate circuit. In reconnecting, however, in order not to flash the tubes at once, the precaution is taken of introducing 100,000 ohms into each plate circuit, and shorting it out again when the connection has been made. Other means of extinguishing may be used, as for instance the brief connection of a condenser around the plate circuit resistor of each tube; but the method described was found to give the most reliable results in a very short interval.

In taking measurements with the peak-amplitude meter, the tubes are connected to the circuit for one-eighth second, and the highest tube striking indicates within 6 db the highest peak during that interval. Each tube striking records the fact by operating its relay and counter. The tubes are then extinguished during the next one-eighth second, then reconnected to the circuit. Peak measurements are made in this way during alternate one-eighth-second intervals, for as long a time as desired. The counter readings at the end show how many times each tube has flashed, and these divided by the total number of active intervals (by a separate counter) show the percentages of intervals during which the peak voltages are above the corresponding tube thresholds. As indicated in Fig. 1, the peak-amplitude meter may be connected either to the input or to the output side of the filters.

Time controls for the 15-second and one-eighth-second intervals are provided by means of synchronous motors. These drive switches operate relays to perform the desired functions. The various amplifiers in the circuit are provided with variable gain, for adjustment to different sound levels. Frequency characteristics of the circuit are shown in Fig. 5. These include all electric parts of the circuit, but not the pressure-voltage characteristics of the condenser transmitters. The latter, obtained by means of a thermophone calibration,⁸ are shown for two transmitters in Fig. 6. The transmitter No. 2 was used only for the bands above 8000 cps, where it has greater sensitivity than No. 1. Reduction of diaphragm pressures to free-field pressures will be described later.

METHODS OF OBTAINING DATA

The individual instruments chosen for measurements were chiefly those believed to be possible extreme cases for reproduction, either in frequency range or in absolute amplitude. The list is as follows: bass drums of three sizes (29×12, 36×15, and 34×19 inches), snare drum, cymbals, triangle, bass viol, bass saxophone, bass tuba, trombone, trumpet, French horn, clarinet, flute, piccolo, piano, and pipe organ. In addition, measurements have been made on orchestras of fifteen, eighteen, and seventy-five pieces.

With the exception of the pipe organ and seventy-five-piece orchestra, the instruments were located in a large sound-treated laboratory room, 29×29×13 feet in size, and having a reverberation time of approxi-

⁷ These 3-electrode, hot-cathode, argon filled tubes, made in Bell Telephone Labs., were of the type later named "thyratrons." A relay tube of this general type was first devised by A. W. Hull, "Hot-cathode thyratrons," *G. E. Rev.*, vol. 32, pp. 212-223, 390-399; April and July, 1929.

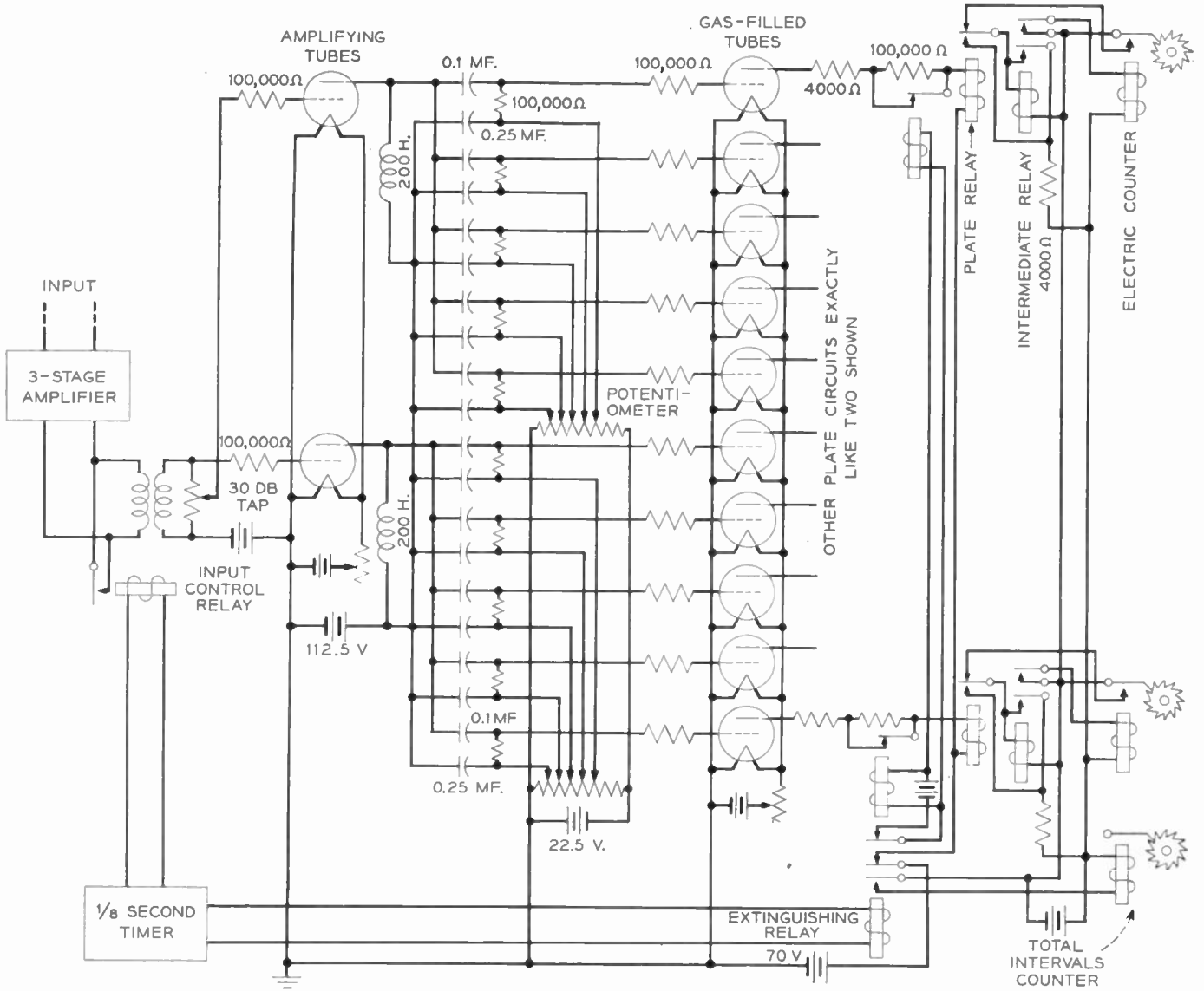


Fig. 4—Circuit of peak-amplitude meter. Peak amplitudes in alternate one-eighth-second intervals are measured and counted over a period of any desired length.

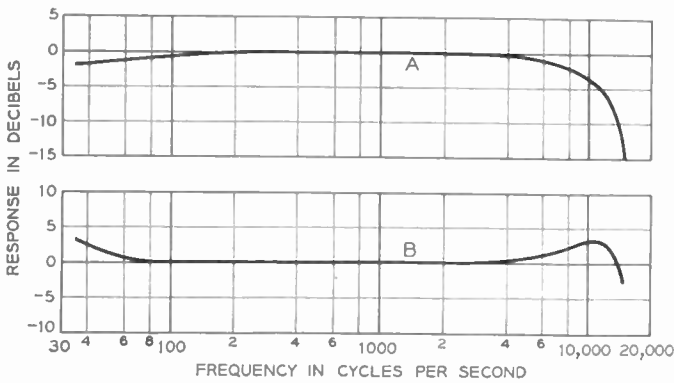


Fig. 5—Frequency characteristics of electric circuit, giving relative outputs for a constant input. Curve A includes the average-amplitude meter, while curve B includes the peak-amplitude meter.

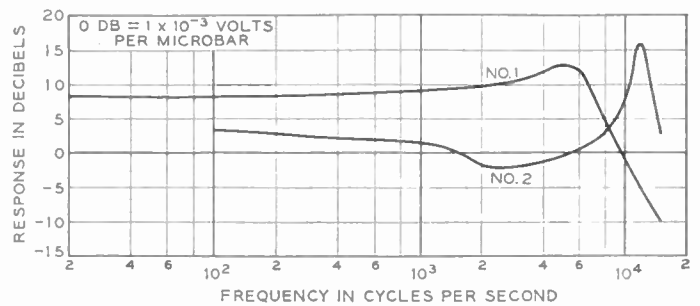


Fig. 6—Pressure-voltage calibrations of the two condenser transmitters.

mately one second for frequencies between 60 and 4000 cps. The condenser microphone was placed at a distance of 10 feet or more from the nearest wall and at a height from the floor about equal to that of the instrument under test, varying from about 2 feet for the bass drum to about 5 feet for the tuba. The instrument, in most cases, was placed 3 feet in front of the microphone, though in some cases it was as much as 5 feet, and for the piano, 10 feet. With the orchestras of fifteen and eighteen pieces, the microphone was placed near the conductor, its distance from the different pieces varying from 6 to 20 feet, with the violins closest. The measuring apparatus, except for the microphone and one stage of amplification, was located in a separate room.

The organ and seventy-five-piece orchestra were picked up in a large motion picture theater. For the orchestra, the regular theater programs were used, but the organ was played particularly for the tests. The microphone was fastened to the conductor's stand, where it served to pick up both orchestra and organ, since the latter opened directly into the orchestra pit.

Only professional musicians were employed for the tests. In each case the musician was instructed to play a selection representative of his instrument, usually including extremes of range and volume, and different types of playing. Selections were made at least two minutes long (without long rests such as the instrument might encounter in an orchestra), so that at least four average amplitude readings and at least 480 peak amplitude readings were obtained during the selection. With the large orchestra the selection sometimes ran to 2000 or more peak measurements.

While it would be desirable to make measurements in all frequency bands during one playing of a selection, this would require a large multiplication of apparatus and observers, and was not found to be practicable. The selection was therefore repeated, once to obtain the distribution of peaks in the unfiltered music, and once for each of the frequency bands. In all cases, readings of total average amplitude were taken simultaneously with the peak and average band amplitudes. Musicians were instructed to make repetitions as nearly alike as possible. In almost all cases, they sounded closely alike to the ear, and in some cases, the musician was able to make fifteen repetitions with a total variation in the total average amplitude of only 2 db. In the case of the piano, a reproducing mechanism with selections on punched paper rolls was used, and the selection could be repeated at will. The orchestras were not under our control, and the curves shown for them are not for the same selection in all bands. However, for the smaller orchestras, all selections used were similar in that they covered a large frequency range and used all instruments most of the time. For the theater orchestra, use was made of a number of repetitions of the same program during a week, and all bands presented together in our curves were at least for very similar types of music.

Where bands are found missing from the results, it is sometimes because opportunity was lacking for completion of the tests (*e.g.*, the 18-piece orchestra), at other times the instrument had no measurable energy in the band, above noise level (*e.g.*, flute and piccolo in low bands), or the tests were deliberately shortened by hitting only the high spots (*e.g.*, organ B).

REDUCTION OF DATA

Calibration of the entire electrical circuit is accomplished by means of sine-wave voltages, introduced into the circuit in series with the condenser transmitter. The rms voltage of the sine wave is measured by means of a vacuum thermocouple, followed by a known amount of attenuation. The rms value may be reduced to peak or average, as desired. The calibration is made in such a way that, for all frequency bands and all gain adjustments, the peak input required to strike any tube of the peak-amplitude meter is known, and the average input corresponding to any 15-second fluxmeter deflection is also known. With these calibrations, it is believed that the error involved in measuring the average voltage generated in the circuit by sound falling upon the diaphragm of the transmitter is not greater than 2 db. While the voltage just required to strike a given tube of the peak-amplitude meter is accurately known, a single peak measurement must be uncertain to the extent of the 6-db interval between tubes. When, however, a large number of measurements are made, and numbers above the different tube levels are plotted, the numbers above any given peak level can be fairly accurately estimated by interpolation.

A greater uncertainty is involved in converting these voltages into sound pressures. In the first place, the frequency characteristic of the apparatus is much less regular when that of the transmitter is included. A rapidly changing characteristic within a frequency band makes it impossible to choose a calibrating frequency for that band which will give accurate results for all possible distributions of sound amplitudes. This is more particularly the case with the bands above 5000 cps. However, the errors due to this cause are not likely to amount to more than a few db, and in most cases are much less than this.

In the second place, while the thermophone calibration of the transmitter, given in Fig. 6, shows accurately the sound pressure on the diaphragm for a given voltage measured in the circuit, this is not the pressure that would exist in the sound field at the same point were the transmitter absent. We have corrected our results for this distortion of the field, using measurements made at various frequencies and angles of incidence, by R. K. Bonell and E. M. Little in these laboratories. The measurements were made with a transmitter having the same geometry as our own. In these corrections, normal incidence of the sound waves on the transmitter diaphragm was assumed. This is a correct assumption

only when combined with further corrections explained in the following paragraph.

A third uncertainty in the conversion of measured voltages to sound pressures arises in connection with reflections of sound waves within the room. While the walls and ceiling were covered with sound-absorbing material, still it was found that reflections began to have an appreciable effect when the source was 2 feet or more distant from the transmitter. As all our measurements were made at distances of 3 or more feet, the effect of reflections must be considered. These reflections distort the spectrum, both by selective absorption of the walls, and by the introduction of sound waves striking the diaphragm at other angles than normal. They also tend to raise the average total sound pressure. We have determined corrections for all of these effects at once by a series of auxiliary measurements, in which a complex sound source of constant composition was used. This was placed first at a distance of 1 foot directly in front of the transmitter, and an analysis of the sound pressures made using the total and band average-amplitude meters. At this distance, it is safe to assume that reflections had no appreciable effect. The source was then moved to other distances and the measurements repeated. The changes in pressure found were somewhat different from those predicted by the inverse-square law of decrease of intensity, and were not uniform with frequency. The same changes could be expected with musical instruments at the same distances. To take care of instruments having different directional properties, the complex source (a loud-speaking receiver) was equipped with horns of different sizes. A one-half inch tube was used as a point source, then horns with openings of $6\frac{7}{8}$ inches, and of 22 inches. The measurements with these horns were applied to the trombone and tuba, respectively. For the piano, a special set of measurements was made, using the piano itself as the source. A 15-second section of a piano roll was used, and measurements made at different distances, care being taken to include exactly the same section of the roll in each playing. For orchestras playing in the laboratory room, there is no definitely assignable distance. Measurements were made with the complex source at a distance of 10 feet in front of the transmitter, and also at 12 feet, 90° from the normal to the diaphragm. The two results are nearly alike, indicating that at these distances the sound from a nondirectional source becomes almost completely diffused. An average of the two results was used for correcting the orchestra data.

The net result of the two corrections explained in the preceding two paragraphs is to give a spectrum of the sound pressures that would exist in the field with no transmitter present and with no reflecting surfaces near. While these corrections have been found on the basis of average amplitude measurements only, the same corrections are assumed to hold for peak amplitudes. A further reduction has been made to put all instruments

on a comparable basis. This is to change distances from those actually used to a uniform 20 feet, on an assumption of inverse-square spreading. With widely spread sources like the piano, organ, and orchestras, virtual point-source distances were estimated, as will be described.

For measurements in the theater, the corrections for reflection of waves have not been determined directly. That there were appreciable reflections seems certain. This is more particularly true for the organ, for which the orchestra platform was lowered to the bottom of the pit, and a semicircular reflecting wall about 10 feet high thereby introduced behind the transmitter. In addition, no audience was present during the organ tests. The best available data would seem to be those used for correcting the orchestra music in the laboratory room; hence, these same corrections have been applied to the orchestra and organ in the theater.

Another possible source of error is that in some way an output may be obtained in certain bands where no frequency within these bands is present in the original sound. This might arise from modulation due to non-linearity in some part of the measuring system, or it might arise from lack of sufficiently high attenuation in the filters, for frequencies outside their transmitted bands. A test has been made in which either of these conditions would have been discovered if it existed. A very pure sine wave was introduced into the system, and the outputs in all other bands compared with that in the band containing this frequency. The peak amplitude of the wave was made equal to the largest peak amplitude found in the music measurements. The results showed definitely that no account need be taken of this effect in any of our music data. •

METHODS OF PRESENTING DATA

The results of the measurements and corrections are shown for average amplitudes in Figs. 7-32, and for peak amplitudes in Figs. 33-58.

The average-amplitude curves are plotted in the following manner. The four or more average amplitude readings taken in a given band are averaged, as are also the readings of the total (unfiltered) average amplitude taken simultaneously. The average of all such total average amplitude figures, obtained for an instrument as the different bands are explored, is also taken. Then band amplitudes are corrected, the small amount necessary to bring the corresponding total to the average of all totals. The band figure is reduced to a distance of 20 feet, on an assumption of inverse-square-law spreading from the distance of actual measurement, and expressed in decibels above the reference level of 0.0002 microbar. A further reduction to a one-cycle band is made by subtracting the bandwidth in decibels, on a power basis ($10 \log_{10} B$, where B = bandwidth in cps). This is equivalent to dividing pressure-squared by bandwidth, and recognizes that, although average amplitudes in

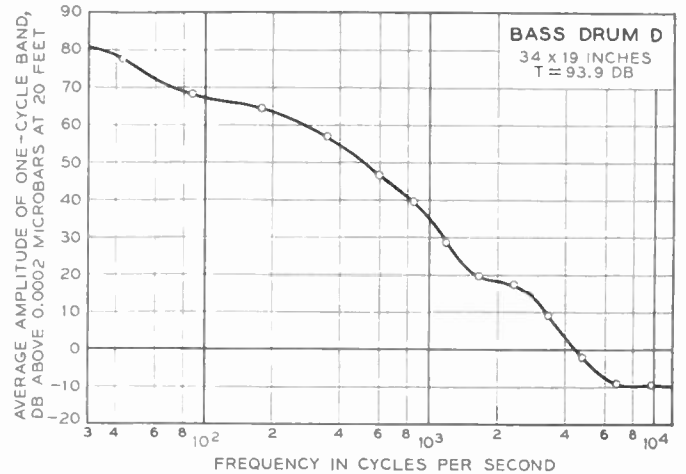
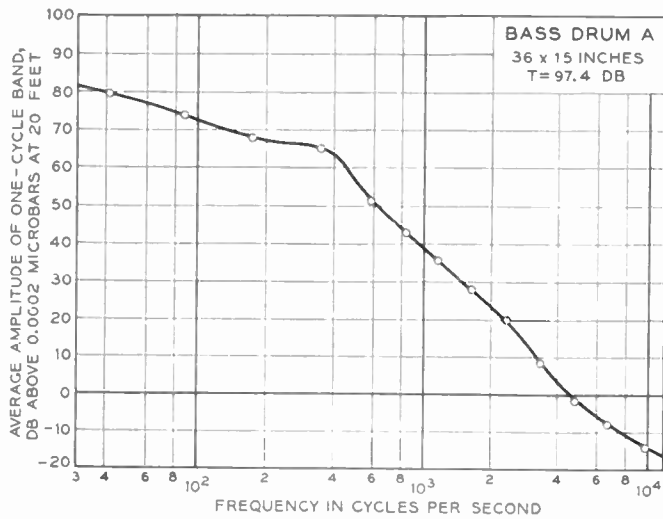


Fig. 7—Frequency characteristic of the per-cycle long average amplitude of a bass drum, as measured in four or more 15-second intervals. T is the total, or unfiltered average amplitude in the same intervals, in db above 0.0002 microbar, at 20 feet. Figs. 8–32 show similar curves for other instruments.

Fig. 10.

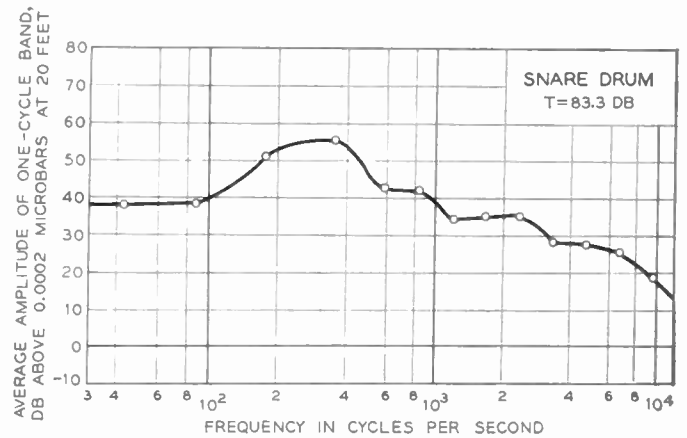
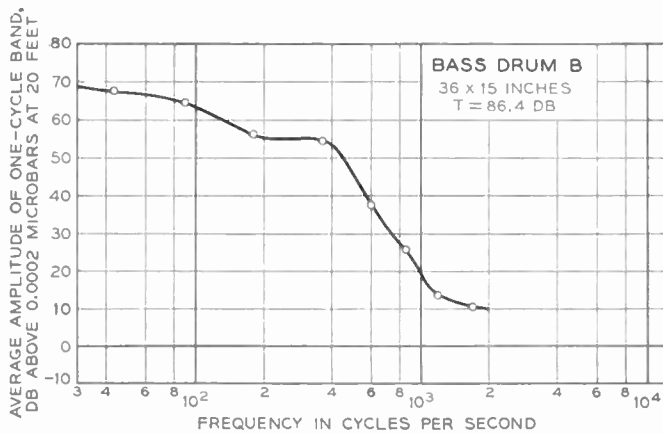


Fig. 8.

Fig. 11.

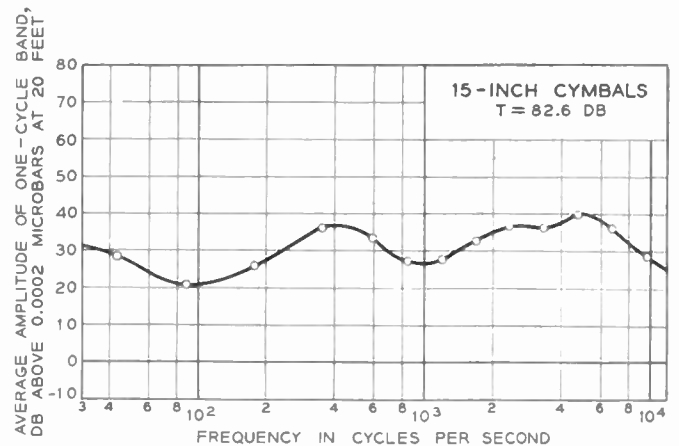
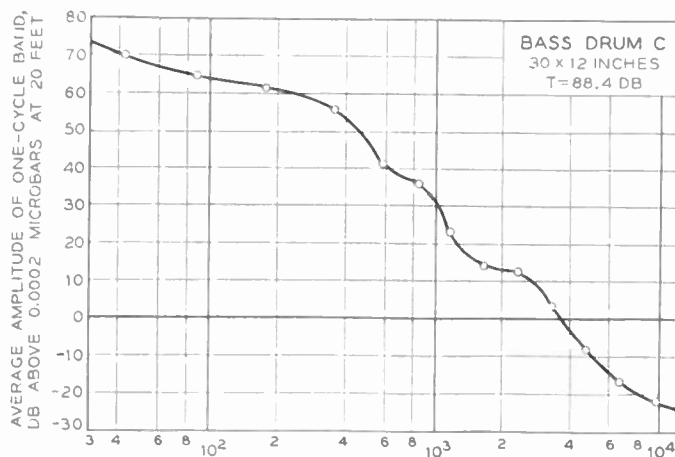


Fig. 9.

Fig. 12.

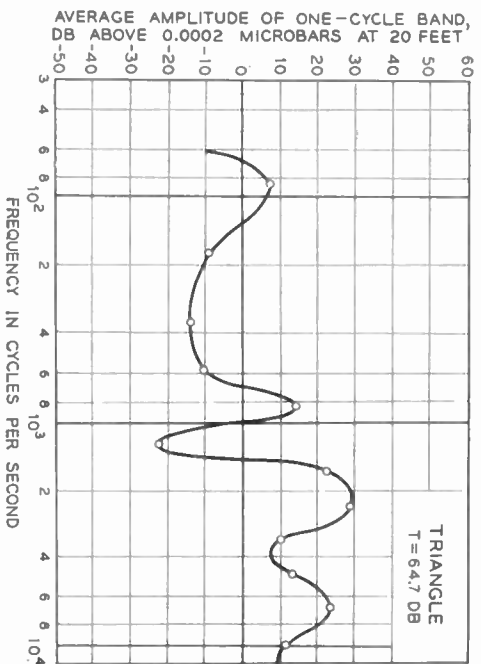


Fig. 13.

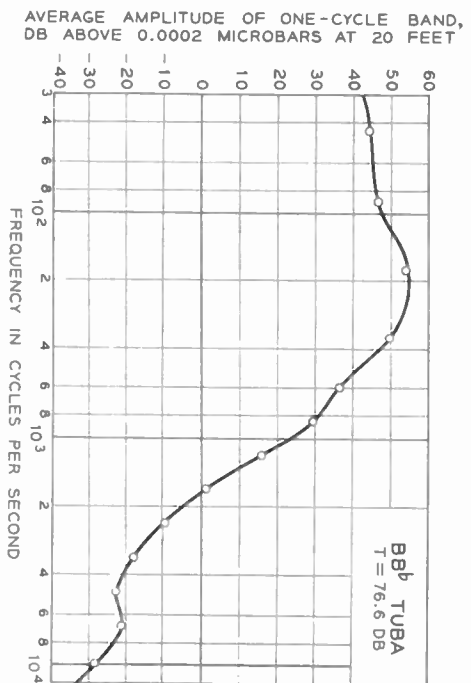


Fig. 16.

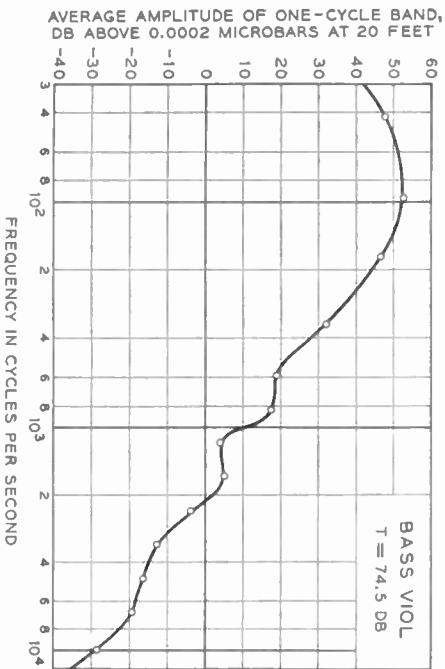


Fig. 14.

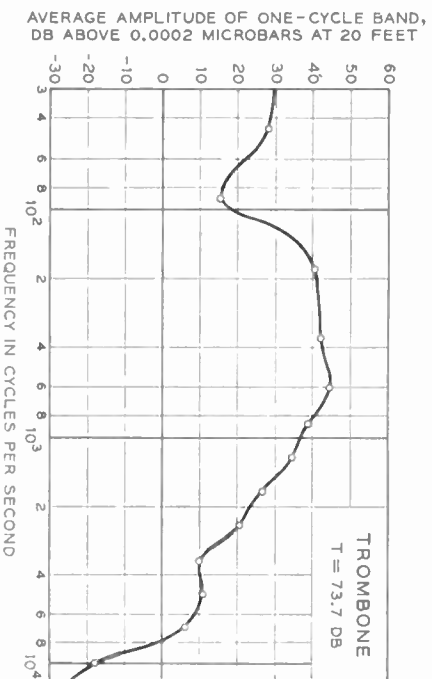


Fig. 17.

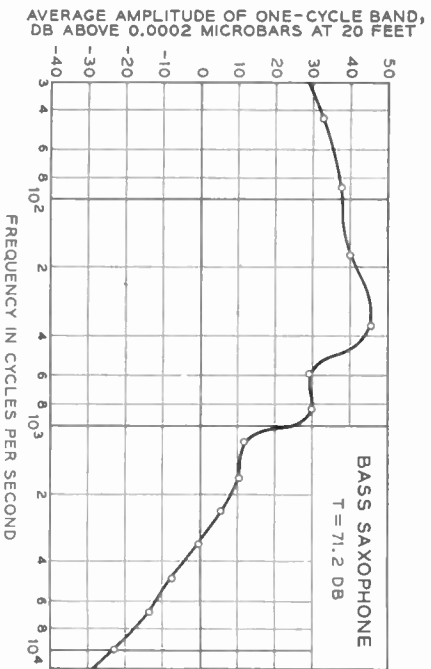


Fig. 15.

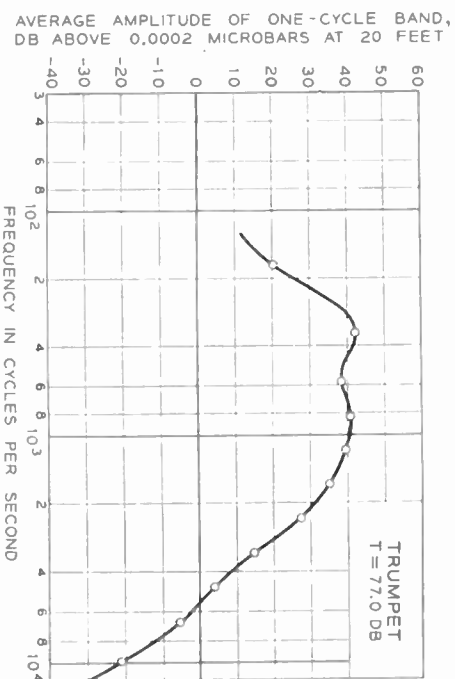


Fig. 18.

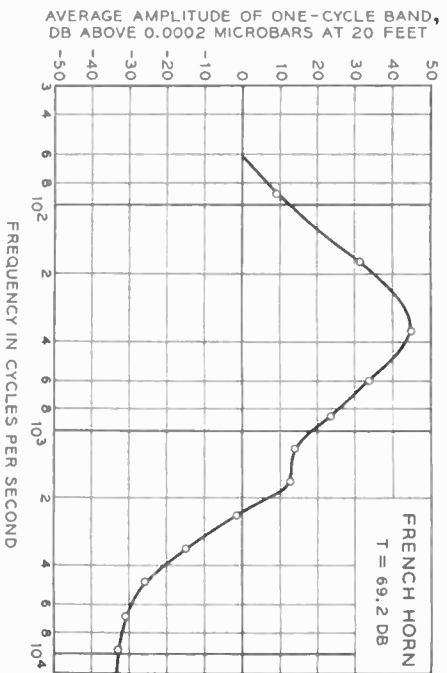


Fig. 19.

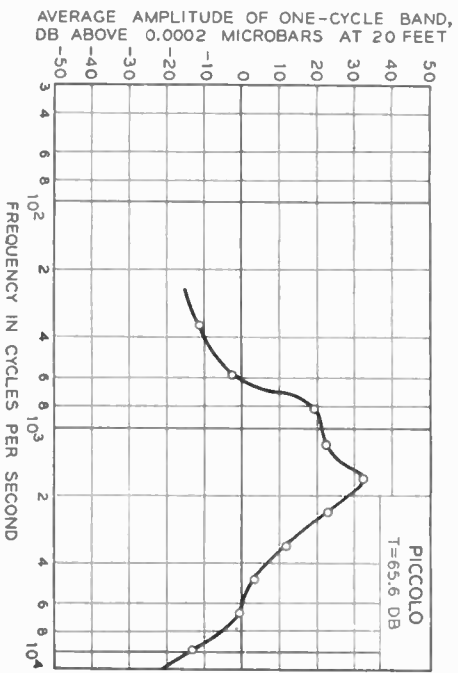


Fig. 22.

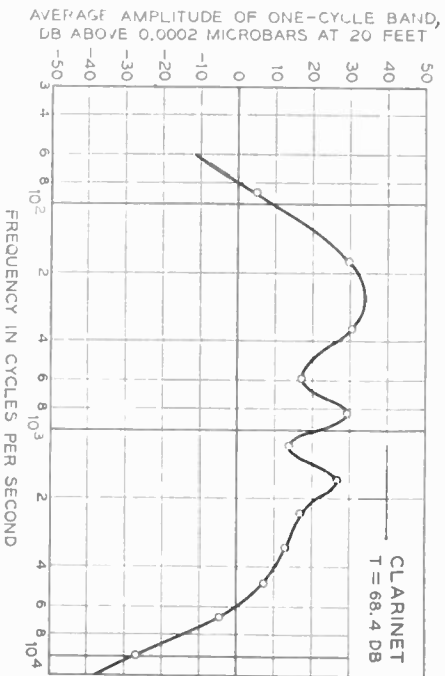


Fig. 20.

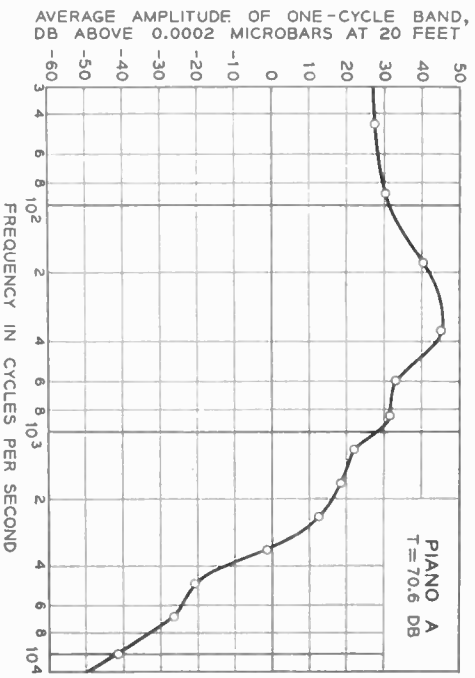


Fig. 23.

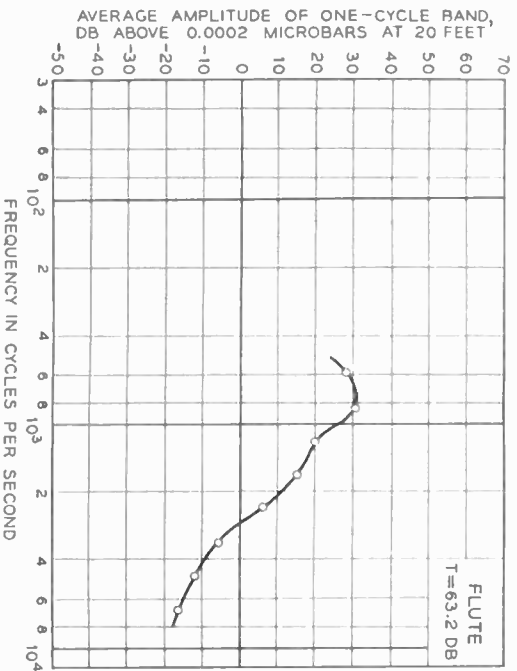


Fig. 21.

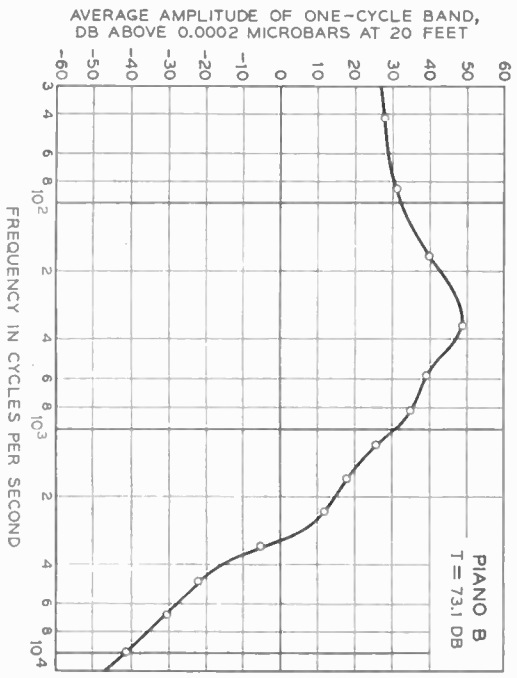


Fig. 24.

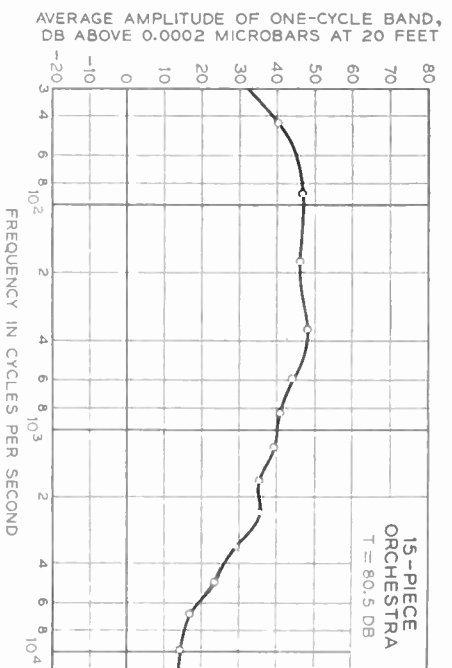


Fig. 25.

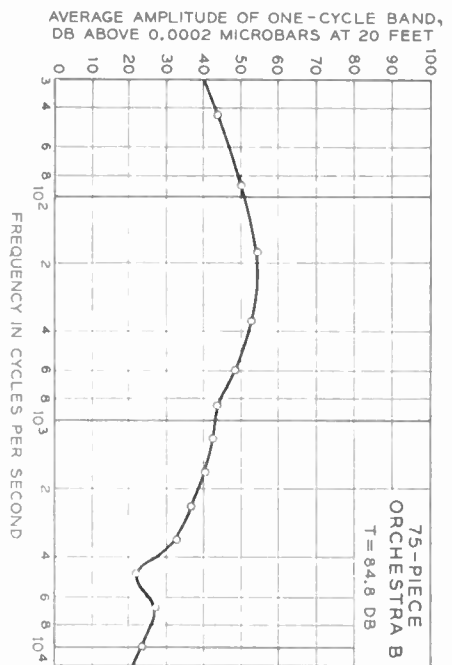


Fig. 28.

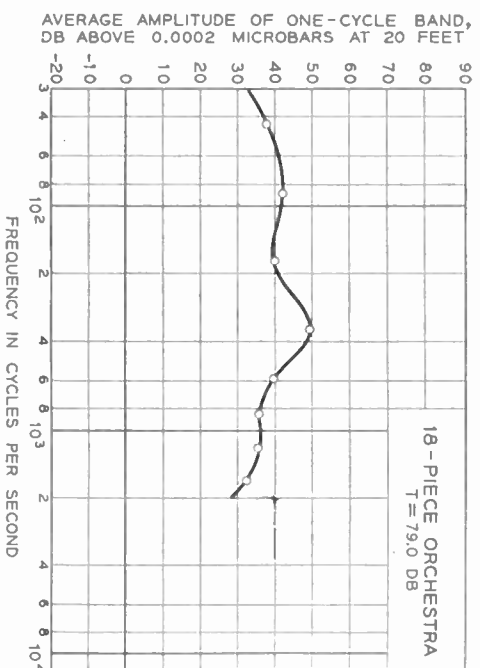


Fig. 26.

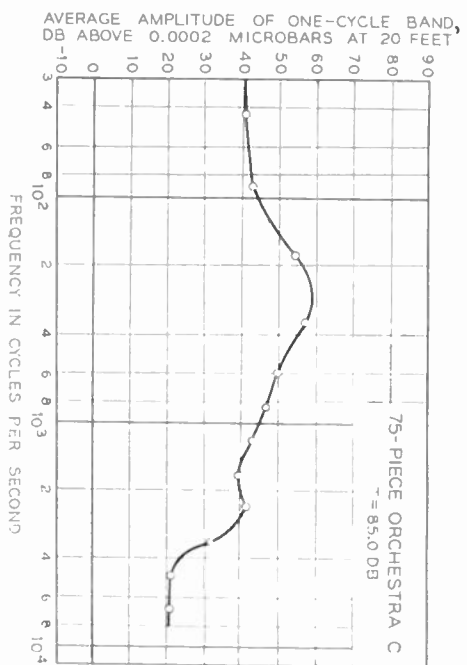


Fig. 29.

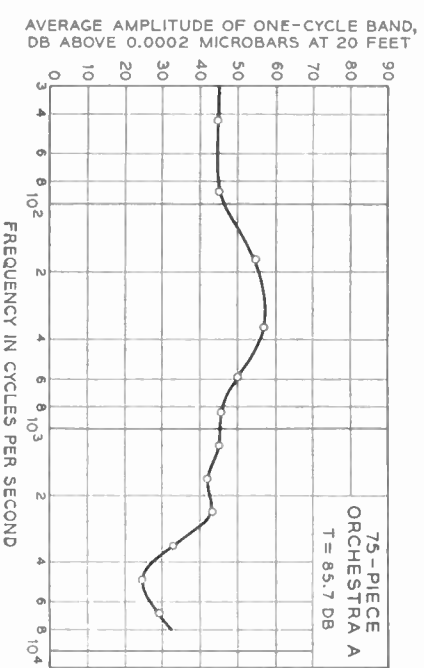


Fig. 27.

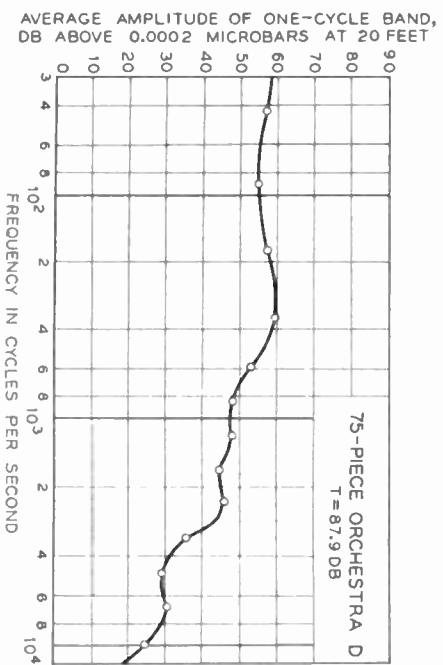


Fig. 30.

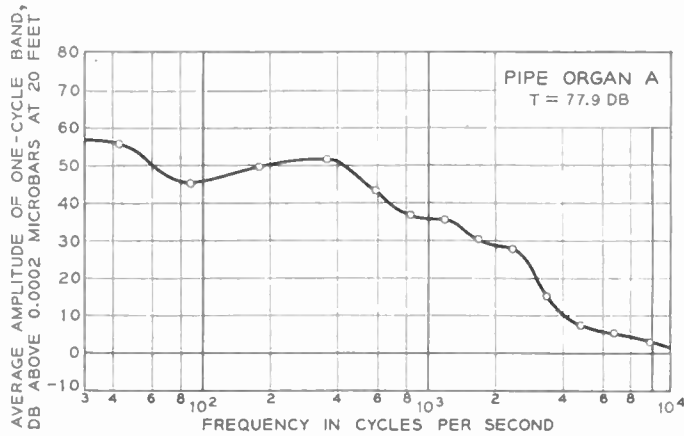


Fig. 31.

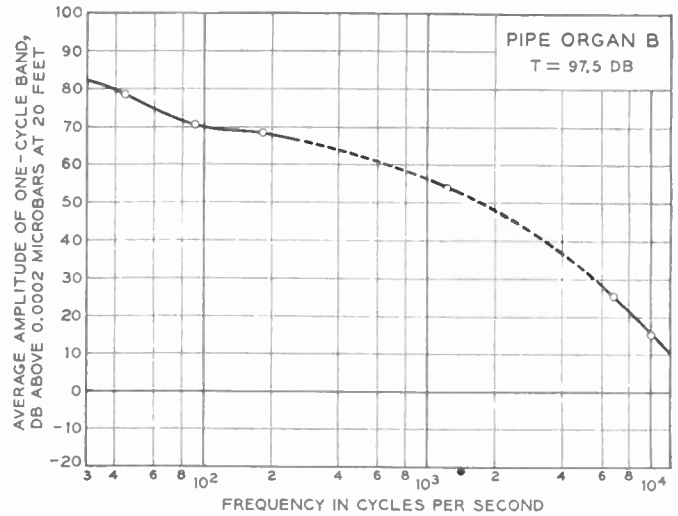


Fig. 32.

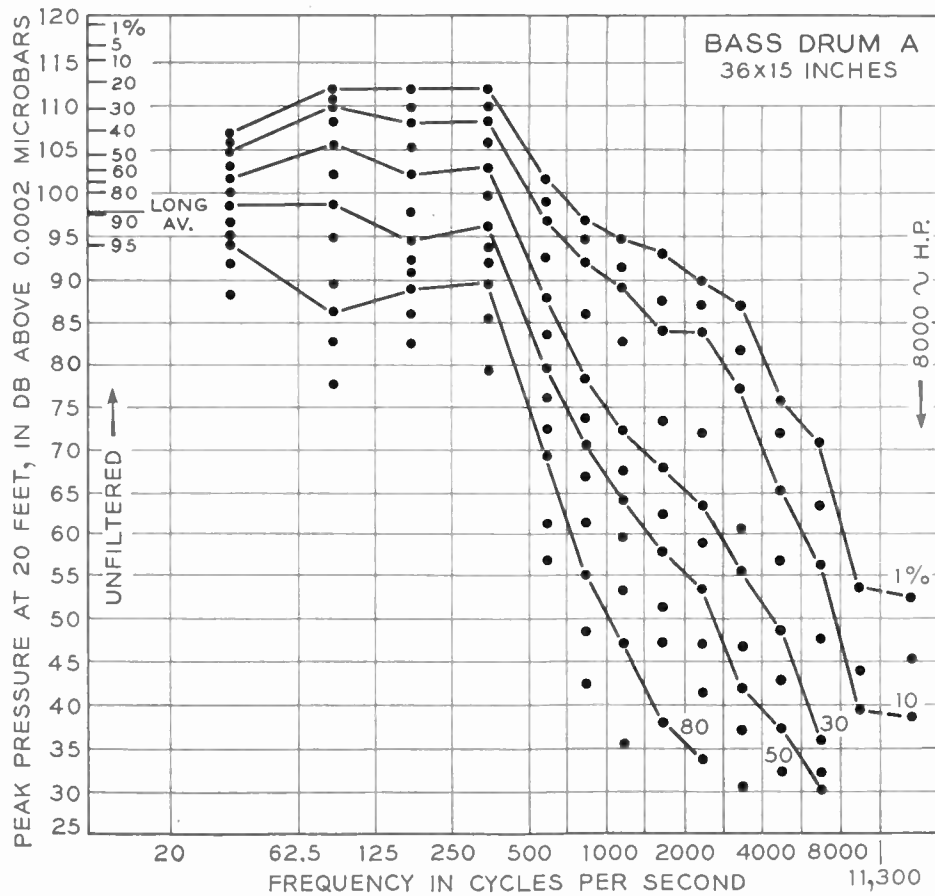


Fig. 33—Distribution of peaks of a bass drum, in one-eighth-second intervals. Each vertical row of points is a set of measurements in the band shown by the divisions of the frequency scale, and each point represents a certain percentage of intervals during which peak pressures rise higher than the ordinate of the point. Certain percentage points in different bands are joined by lines, without any bandwidth adjustments. A run without filters is shown at the left, and an unfiltered average pressure in a long interval (15 seconds) is also shown. Figs. 34-58 show similar data for other instruments.

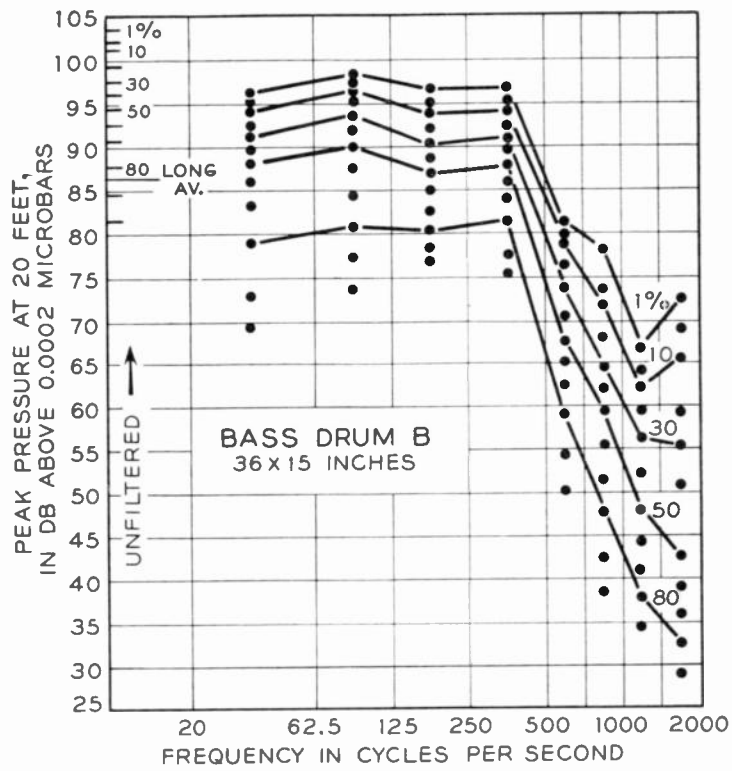


Fig. 34.

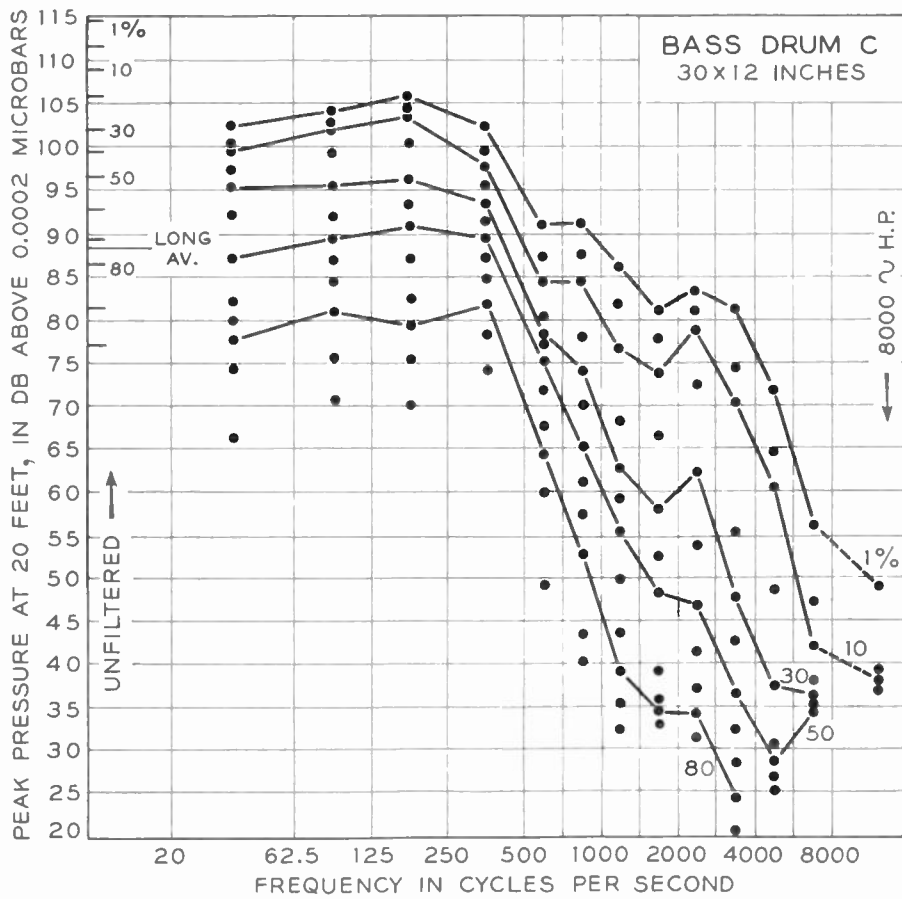


Fig. 35.

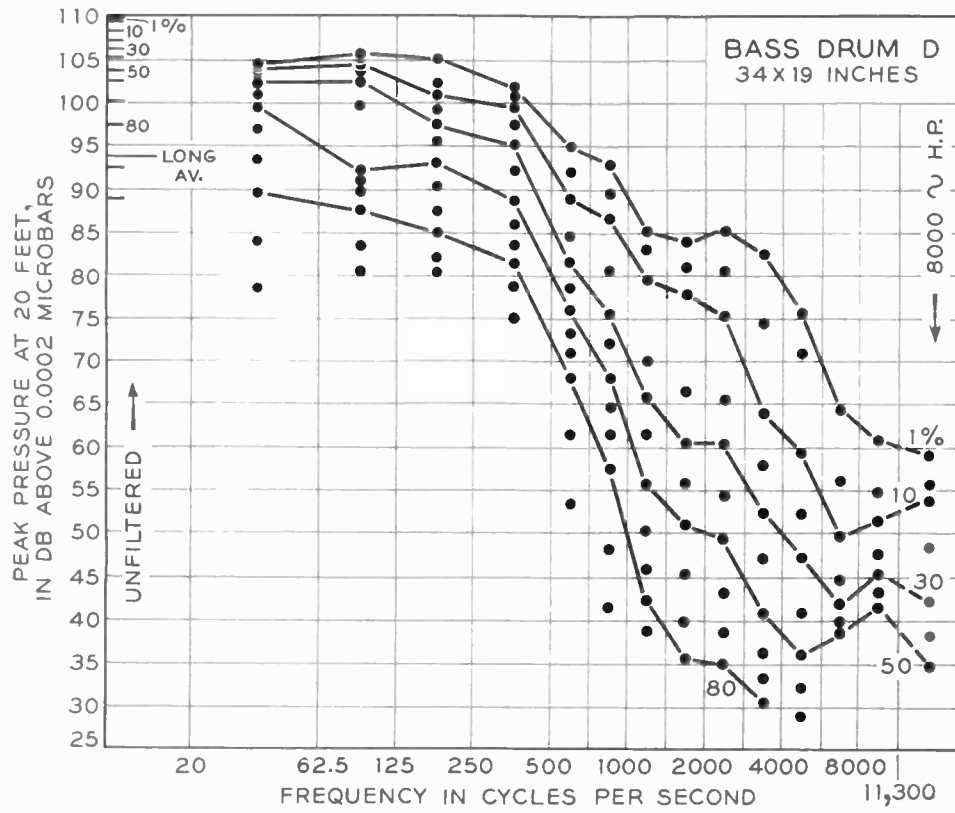


Fig. 36.

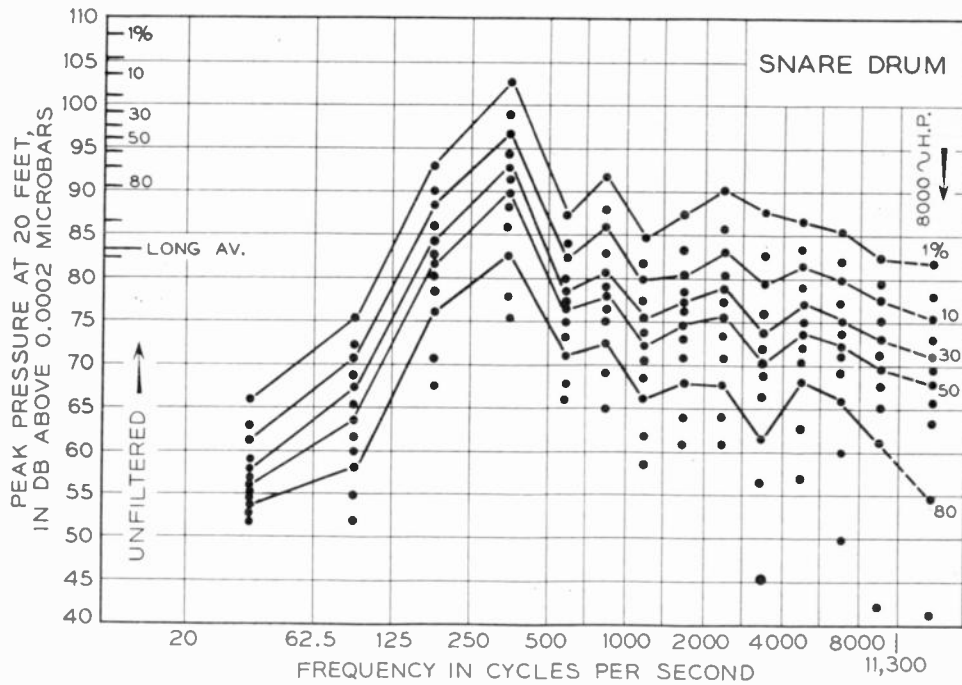


Fig. 37.

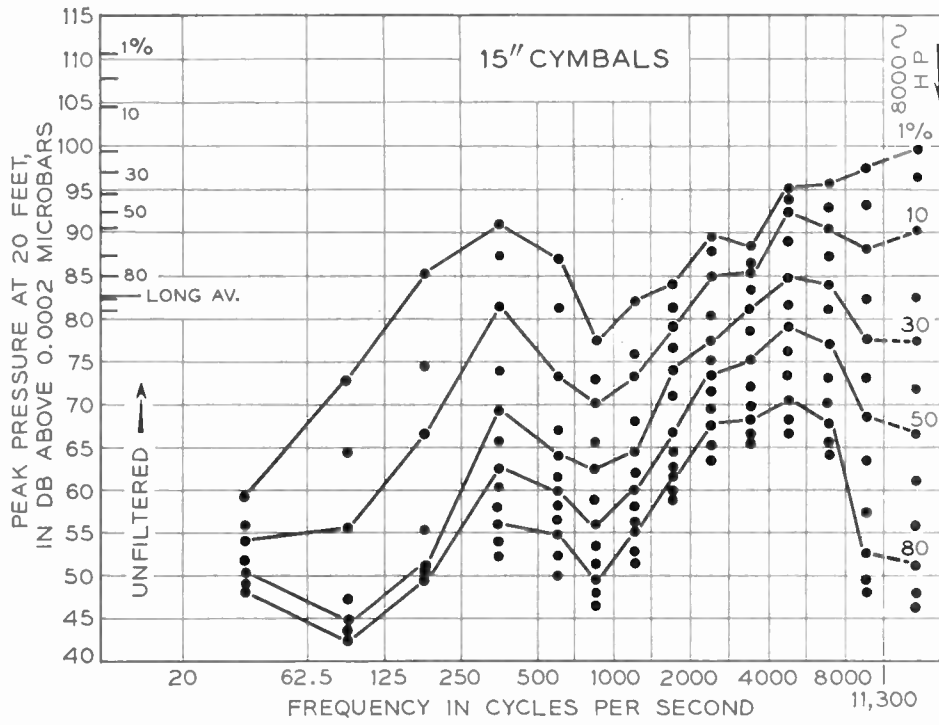


Fig. 38.

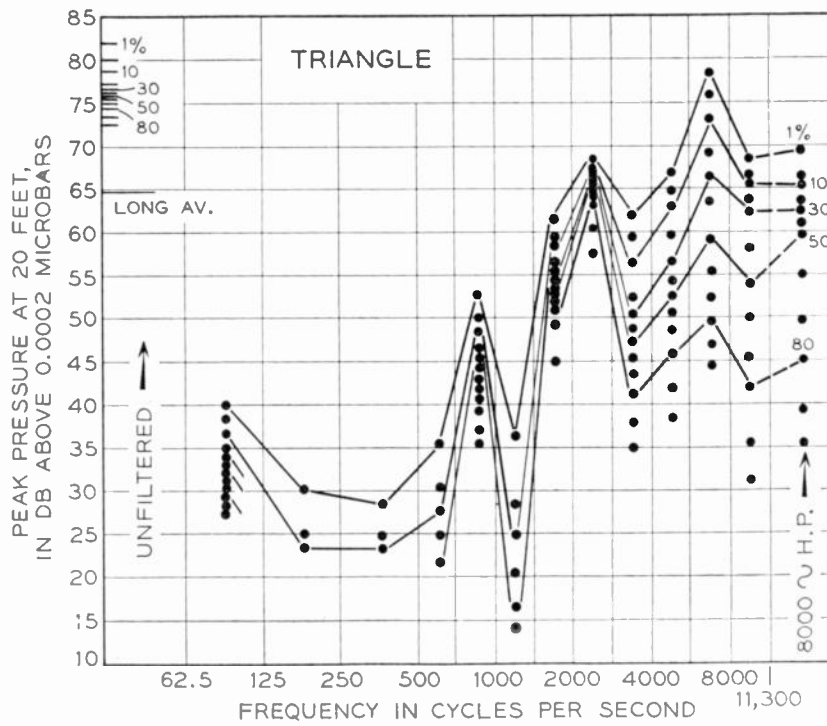


Fig. 39.

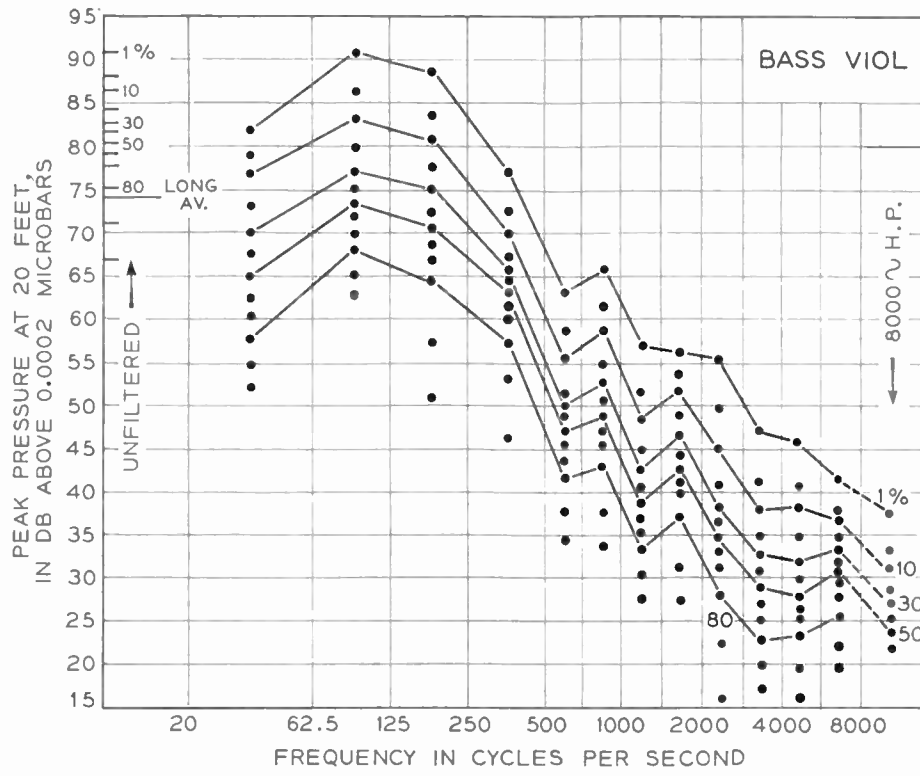


Fig. 40.

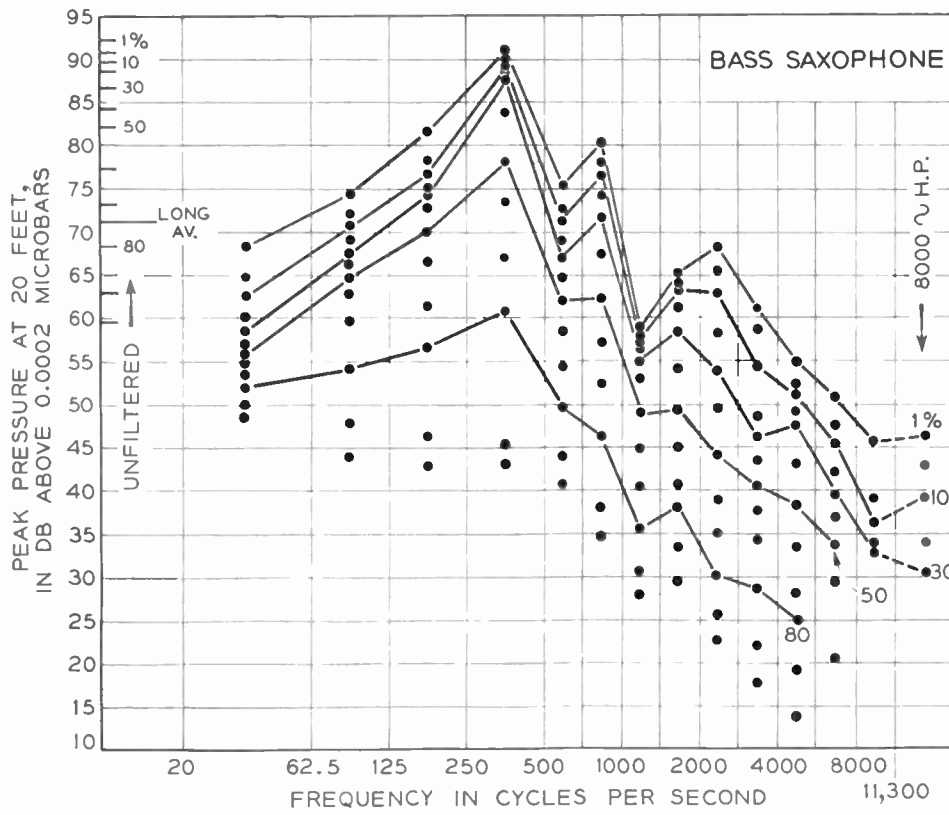


Fig. 41.

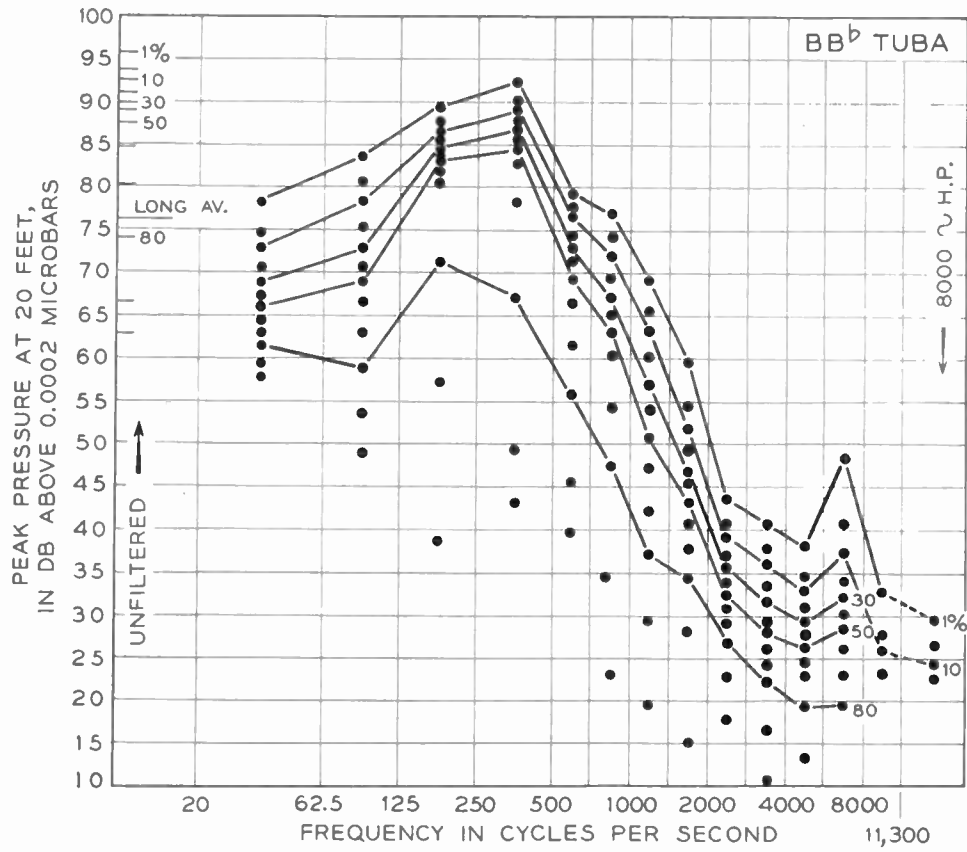


Fig. 42.

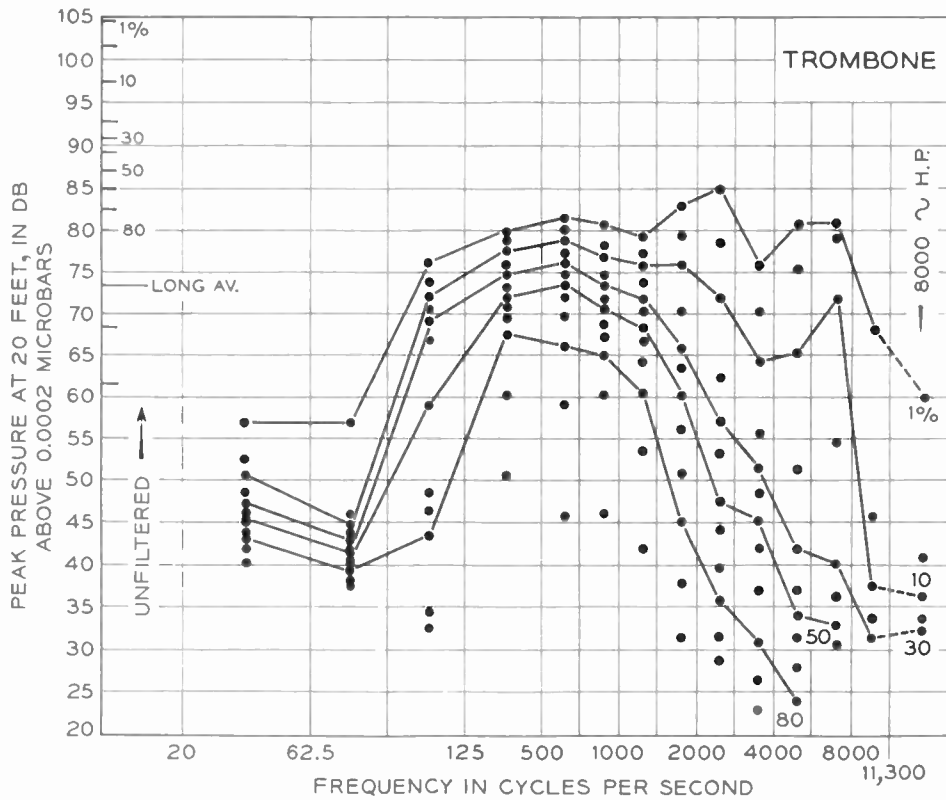


Fig. 43.

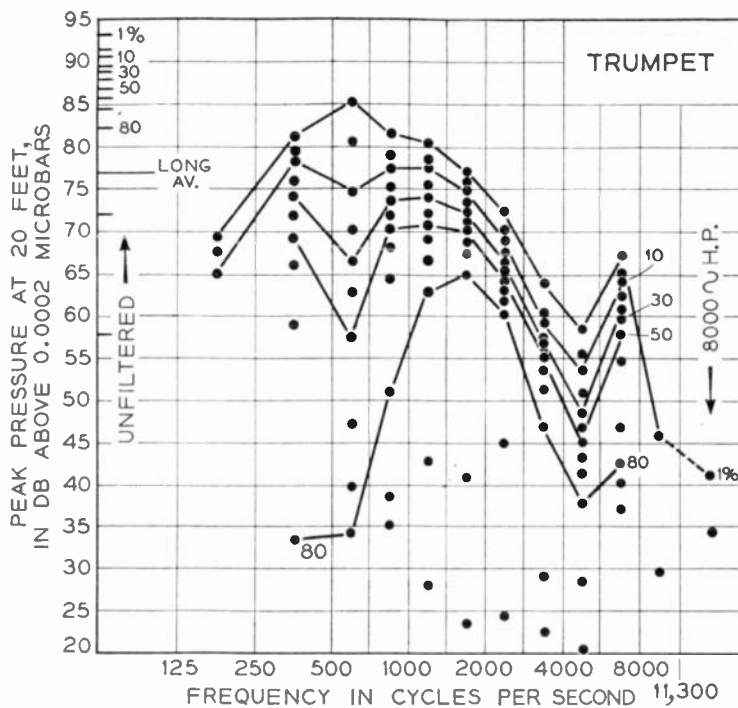


Fig. 44.

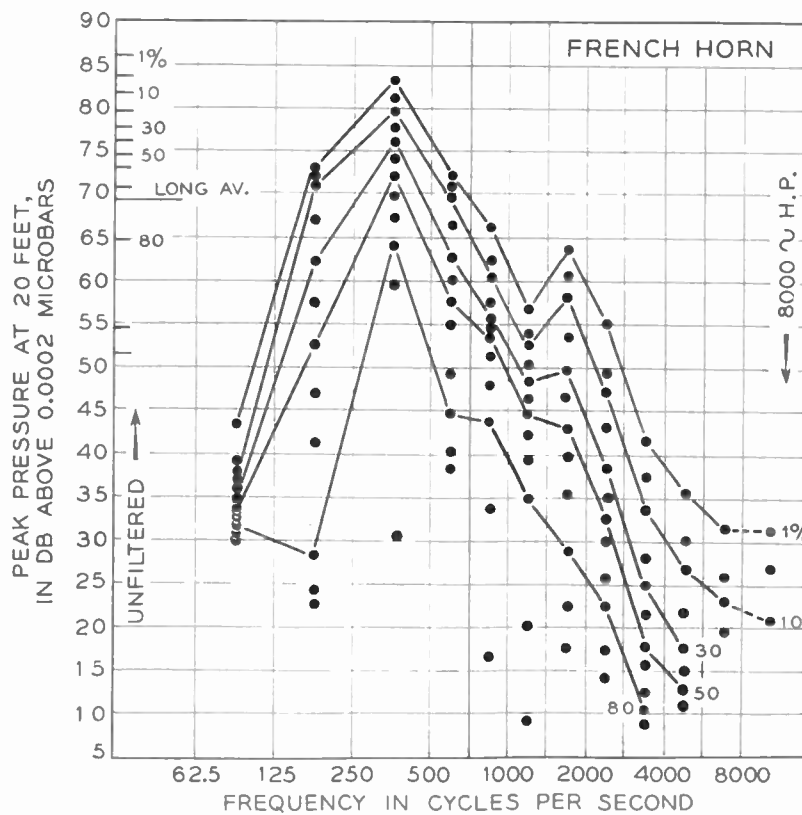


Fig. 45.

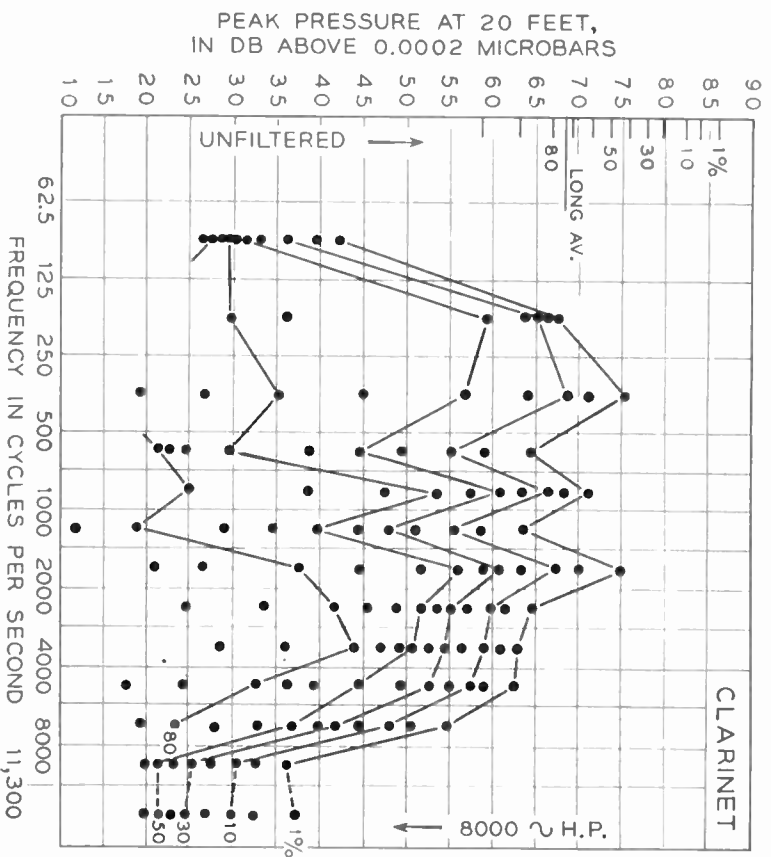


Fig. 46.

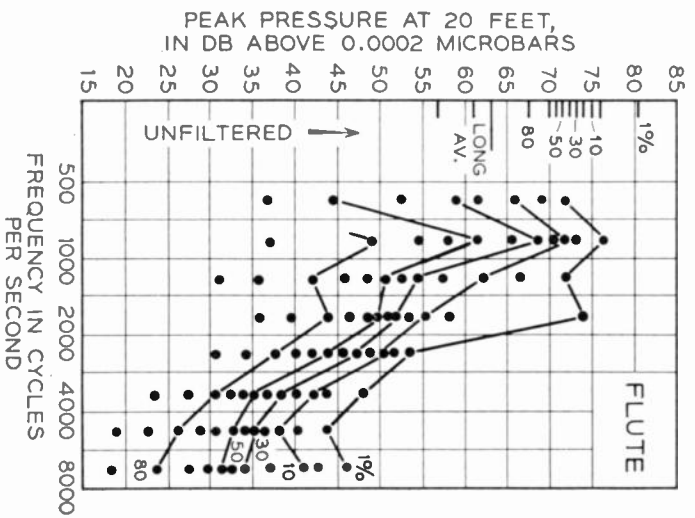


Fig. 47.

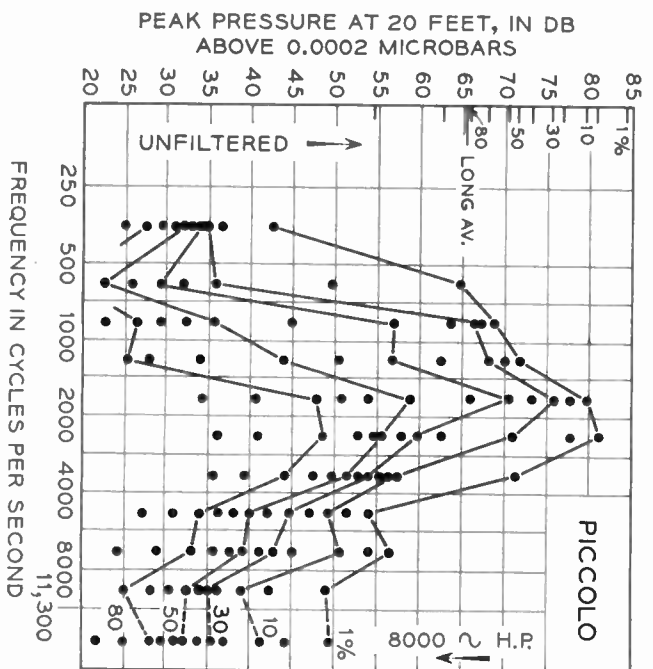


Fig. 48.

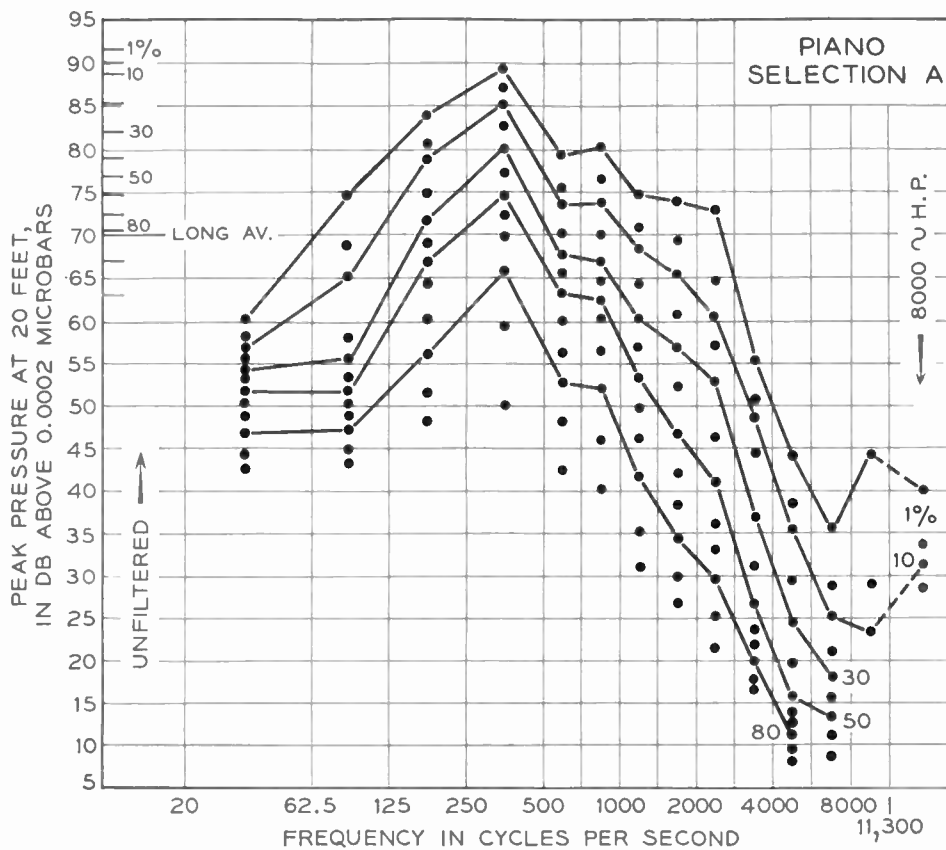


Fig. 49.

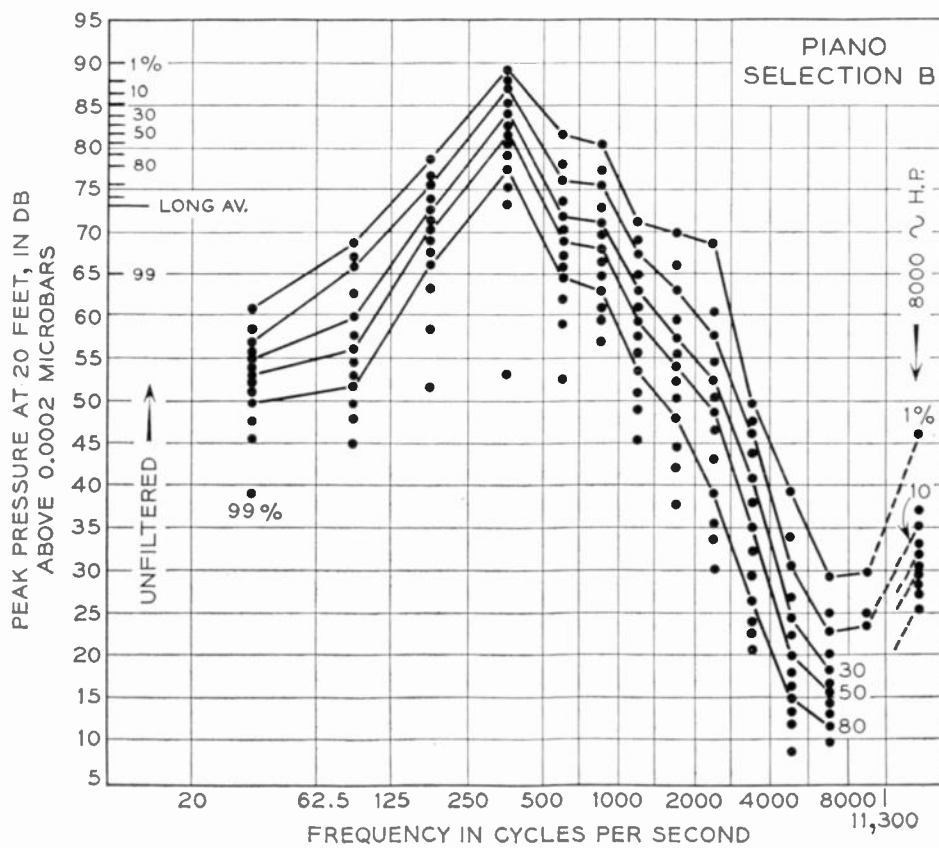


Fig. 50.

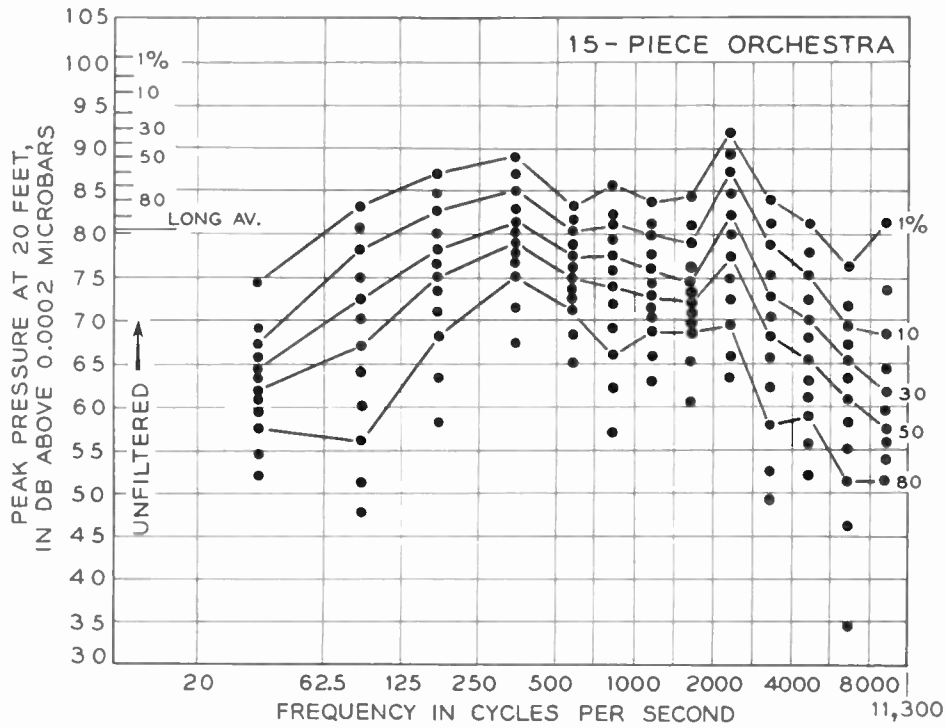


Fig. 51.

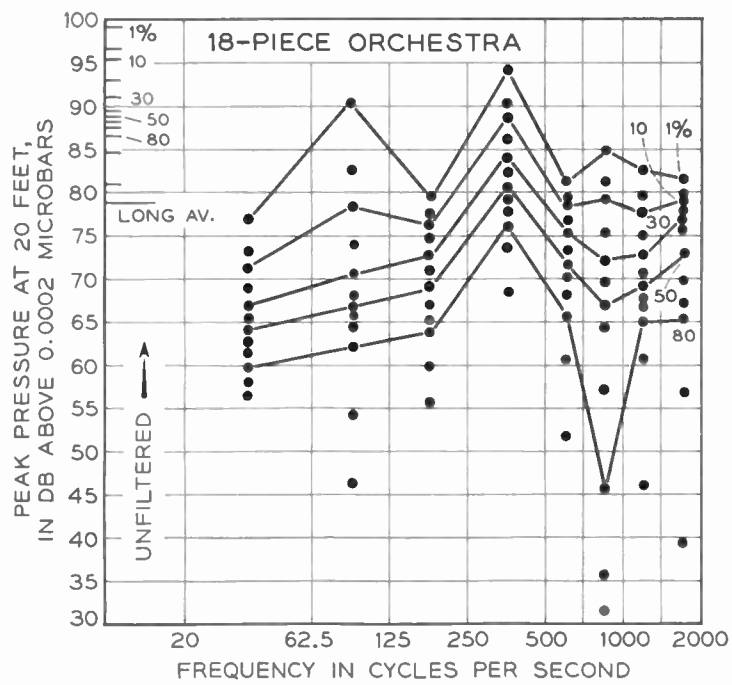


Fig. 52.

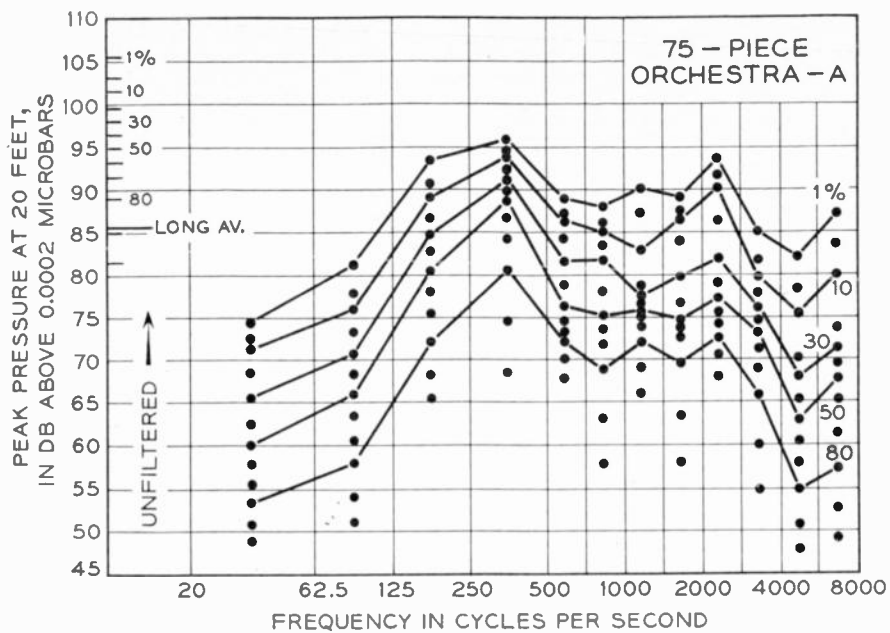


Fig. 53.

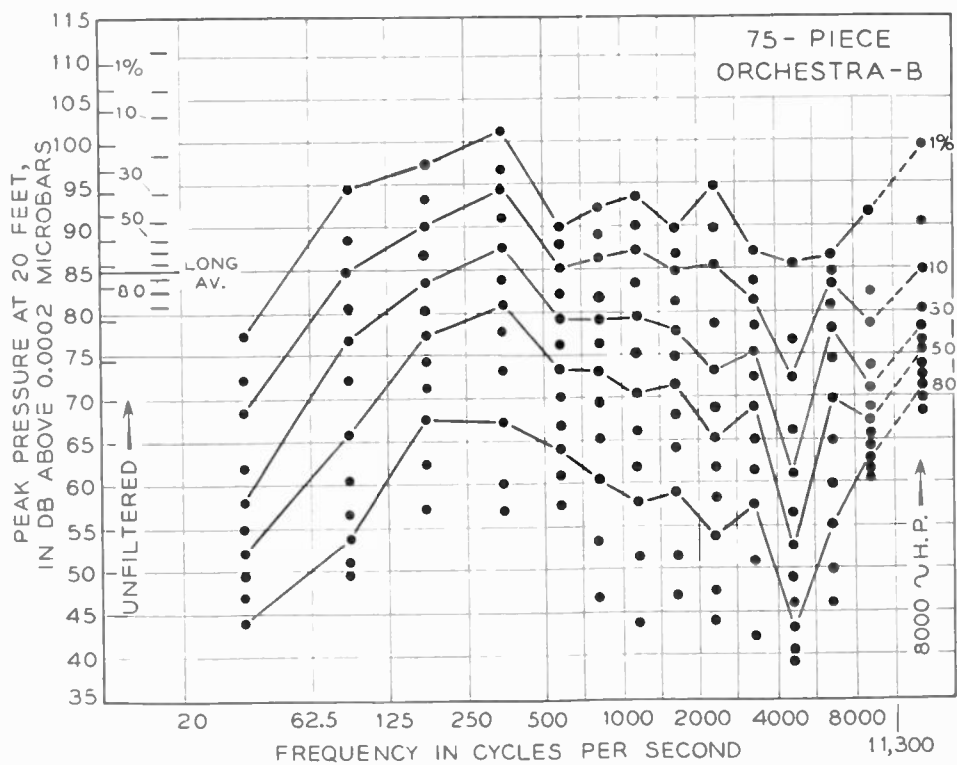


Fig. 54.

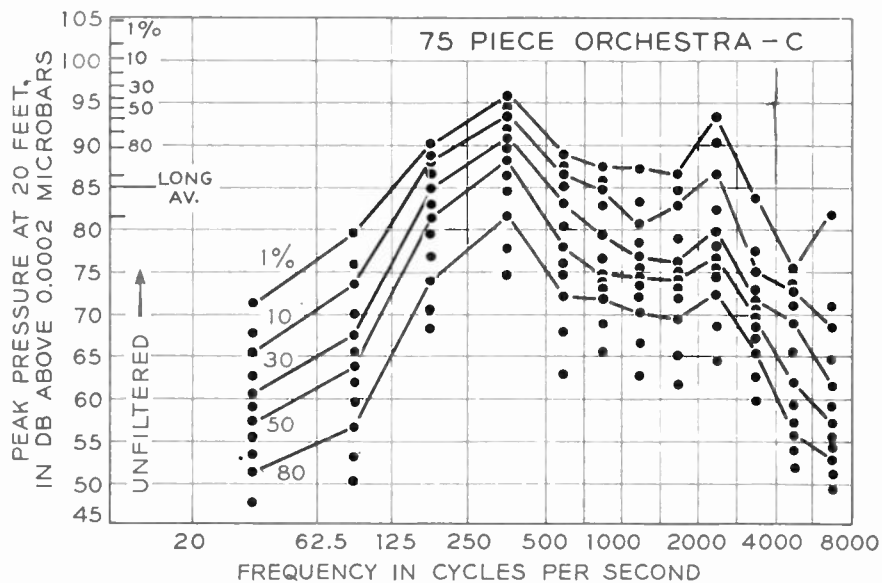


Fig. 55.

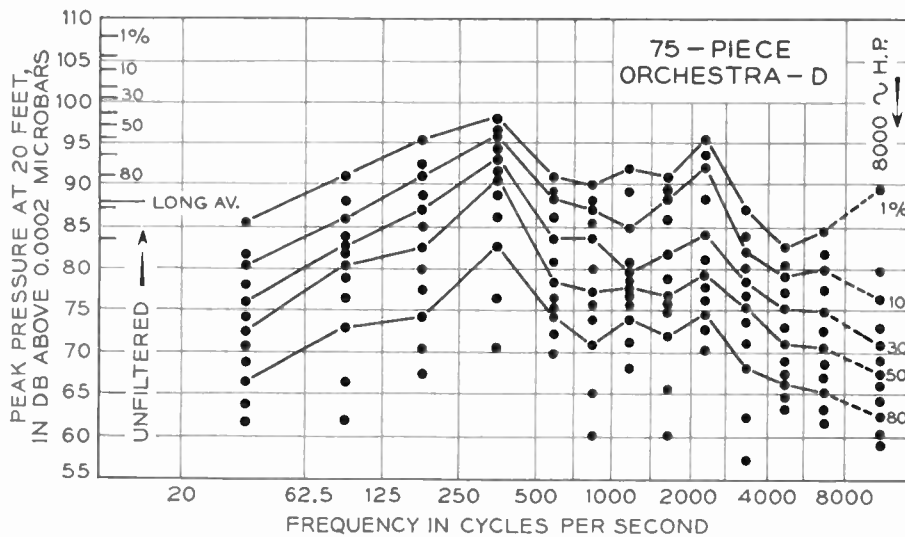


Fig. 56.

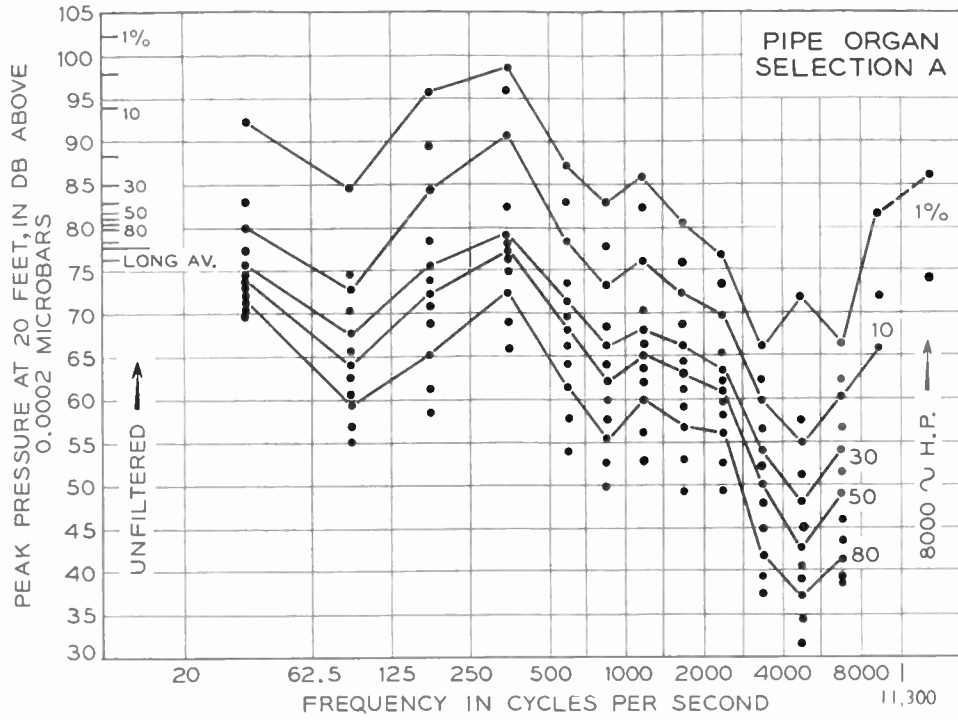


Fig. 57.

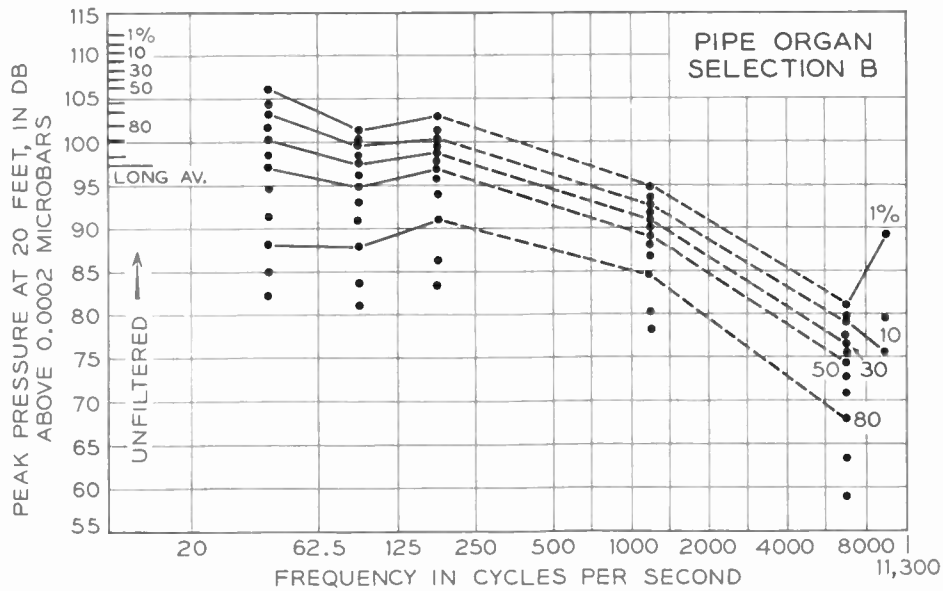


Fig. 58.

the band are measured, the distribution through the band is most probably that of power. The value obtained is plotted as a point in the middle of the band, and a smooth curve is drawn through the points for the different bands. It should be mentioned that, for the bandwidth reduction, the 62.5-cps low-pass band is treated as having a practical lower limit at 30 cps, while the 8000-cps high-pass band is given an upper limit of 12,000 cps. The average of all total average amplitudes is given with each curve, in db above 0.0002 microbar, at 20 feet.

The ordinates of Figs. 7 to 32 purport to give average amplitudes in one-cycle wide bands, through the spectrum. If rms amplitudes were measured, they would be appreciably higher, by different amounts for different instruments and in different bands.⁸

Peak amplitudes are presented in a different manner. The observations in a given band show the percentage of one-eighth-second intervals having peaks in that band higher than the striking level of each of the ten thyatron. By plotting these percentages against levels, a smooth curve can be drawn which permits interpolation to be made within the 6-db intervals between tubes. Such a curve was drawn for each band of each instrument, and levels were then read off at exact selected percentages. The percentages chosen were 1, 5, 10, 20, 30, 40, 50, 60, 70, 80, 90, and 95 per cent. A slight extrapolation was sometimes made for the 1 per cent point. Again reducing to a 20-foot distance, and putting in db above 0.0002 microbar, the above percentage points in a given band are plotted in a vertical row on the figure (e.g., Fig. 33). The lines of the frequency scale in these figures are drawn at the band limits, with the rows of points in the band centers. The practical lower limit of the first band is set arbitrarily at 20 cps.

The highest point in all cases is 1 per cent. The sequence given above is then followed, with the lowest point usually 95 per cent (in Fig. 50 an extra 99 per cent point is added). For example, in the 250-500-cps band of Fig. 33, 30 per cent of one-eighth-second intervals have peak pressures higher than 102.5 db above 0.0002 microbar, at 20 feet. Where points are missing, they are at the lowest levels (highest percentages), and were beyond the measuring range of the apparatus as set.

In a separate space at the left of each figure the points for the unfiltered music are given, in the same set of percentages. Along with these is given the long-interval

measurement of average pressure for the selection. Where the 8000-cps high-pass run was made, these points are shown in a separate space at the right.

For convenience in identifying the percentages, the 1, 10, 30, 50, and 80 per cent points of the different bands are connected by lines. These lines then have the appearance of frequency characteristics, but care must be taken in interpreting them as such. The sharp drop from the band below 500 cps to the one next above, in many cases (Fig. 33, for example), is due in part to the change from octave to half-octave bands. Because of the greater uncertainties involved, no correction for bandwidth has been made for the peaks, such as was done in the average-amplitude curves. With a flat frequency characteristic, changing from half- to one-octave bands might raise levels 6 db if added components were in phase at the peak moment. This might be expected in percussion instruments. A 3-db rise in peaks would be expected on the average, but an actual reduction can be imagined. On a sloping characteristic, uncertainties in such a conversion become larger. It is also very uncertain how a change to one-cps bands would be made. For these reasons, it is left to the reader to make what bandwidth corrections he deems suitable.

COMMENTS ON THE DATA

It is obvious that different instruments, different selections, or different players or conductors, would yield data different from those shown in our curves. These curves cannot be taken, therefore, as typical or average for the instruments, but as examples, tending in many cases to the extreme side in amplitude.

The measurements give two distinct sets of data: 1) average amplitudes; 2) peak amplitudes. The former are obtained by integrating the amplitudes over a period of 15 seconds. This type of average probably has no direct bearing upon the performance of the apparatus intervening between the sound source and the ear. So long an interval as 15 seconds is directly significant only for such effects as heating of circuit elements, breathing and packing of carbon transmitters, etc., unimportant in systems now used for the reproduction of music. The long averages were measured chiefly because they furnished an easy means of checking on the constancy of a musician's performance as he repeated the same selection over and over again while the several frequency bands were being gone over.

The peak data are taken so that each individual measurement gives the maximum amplitude occurring in a one-eighth-second interval. This interval approximately represents the duration of a short musical note. In this sense it is an element of music similar to the syllable as an element of speech. For string and wind instruments, the peak amplitude measured usually is accompanied by many others of substantially equal amplitude occurring in the same one-eighth-second interval. On the other hand, for the most important percussion instruments (drums and cymbals) the sound

⁸ At a later date, a distribution of instantaneous amplitudes was measured in the unfiltered music from the seventy-five-piece orchestra, and was reported in Fig. 14 of footnote 1. Both rms and average amplitudes can be calculated from this distribution, and the difference turns out to be 4.8 db. No such difference for music in bands has been measured, but it would be expected to be greater than in the total because of the higher intermittency of energy in bands, during a selection. Limited band data on speech exist, and have been used in connection with the total difference given above and the data of this paper to deduce a power spectrum for an orchestra. See R. W. Young and H. K. Dunn, "On the interpretation of certain sound spectra of musical instruments," *J. Acoust. Soc. Amer.*, vol. 29, pp. 1070-1073; October, 1957.

wave is highly damped and the peak measured probably is decidedly higher than the other peaks occurring throughout that particular interval. It is thought that in all cases the peaks as here measured represent a quantity which is individually perceived by a musical ear. If this is correct, the overload capacity of apparatus carrying music must be based on these peak quantities.

The frequency analyses for individual instruments agree with those obtained on orchestras in indicating that the sound spectra generated extend over the entire audio range, with components (peak values, of course) in any part of that range so large that if present alone, they would be well above the threshold for people with normal hearing. This certainly applies to a person situated in the conductor's usual position. It is not clear that the above components, say in the very highest frequency range, 10,000 to 15,000 cps, are still very audible in the presence of the much larger low-frequency components. The answer to this depends on the particular masking conditions which obtain in orchestral music. Further, not enough is known about the simultaneity of occurrence of weak and strong components situated in different frequency positions. The results of listening tests indicate that the normal ear appreciates the contributions made to orchestral music by all the frequency bands used.⁹

The quantities directly measured were electrical. They are of general interest insofar as they permit the deduction of corresponding acoustical quantities. The total sound output of an instrument or of an orchestra probably is of more interest than any other single quantity. The conversion from electric to acoustic values involves much uncertainty, and the few figures given below must be viewed as rough estimates. The causes of this uncertainty are discussed above, and are briefly mentioned here. From the voltage output of the transmitter, the pressure in the sound field can be computed with the aid of the "field" calibration of the instrument. But this is definite only provided the angle of incidence of the sound wave upon the transmitter is known. In general, due to reflections, sound arrives at the transmitter from many directions and with unknown relative amplitudes. The measures taken to obtain pressure values as they would exist in a direct wave from the instrument, have been explained above. They are the first source of uncertainty. Secondly, it is necessary to pass from pressure to power. The computation is straightforward only when reflected waves are negligible. For other cases, it is necessary to make a further assumption as to the relation between potential and total energy, and the absorption coefficients of the room must be known. Thirdly, the measurements are made at one place in the room, and it is necessary to estimate the power distribution in the rest of the space. Finally,

there are many minor uncertainties, such as the effect upon peak values of the unknown phase relations of the waves arriving from different directions.

With these reservations, we proceed to a discussion of the peak powers obtained from the different instruments and orchestras. The results are also shown in table I.

TABLE I
PEAK POWER OF MUSICAL INSTRUMENTS
(PEAKS REACHED IN ONE PER CENT OF ONE-EIGHTH-SECOND INTERVALS)

| Instrument | Unfiltered peaks, watts | Band having maximum peaks, cps | Peaks in this band, watts |
|----------------------------|-------------------------|--------------------------------|---------------------------|
| 36×15-inch Bass Drum—A | 25. | 250-500 | 4.0 |
| 36×15-inch Bass Drum—B | 0.63 | 62.5-125 | 0.20 |
| 30×12-inch Bass Drum—C | 6.0 | 125-250 | 0.76 |
| 34×19-inch Bass Drum—D | 2.3 | 62.5-125 | 0.95 |
| Snare Drum | 6.6 | 250-500 | 1.9 |
| 15-inch Cymbals | 15.0 | 8000-11300 | 0.74 |
| Triangle | 0.020 | 5600-8000 | 0.0093 |
| Bass Viol | 0.089 | 62.5-125 | 0.087 |
| Bass Saxophone | 0.37 | 250-500 | 0.12 |
| BB ^b Tuba | 0.28 | 250-500 | 0.16 |
| Trombone | 6.6 | 2000-2800 | 0.060 |
| Trumpet | 0.28 | 500-700 | 0.038 |
| French Horn | 0.051 | 250-500 | 0.024 |
| Clarinet | 0.051 | 250-500 | 0.0042 |
| Flute | 0.013 | 700-1000 | 0.0046 |
| Piccolo | 0.032 | 2000-2800 | 0.013 |
| Piano A | 1st Method | 250-500 | 0.15 |
| | 2nd Method | | 0.40 |
| | 3rd Method | | 0.18 |
| | Average | | 0.24 |
| Piano B Average | 0.29 | 250-500 | 0.24 |
| 15-Piece Orchestra—Average | 4.1 | 2000-2800 | 0.47 |
| 18-Piece Orchestra—Average | 2.9 | 250-500 | 0.74 |
| 75-Piece Orchestra—A | 8.1 | 250-500 | 0.85 |
| 75-Piece Orchestra—B | 27. | 250-500 | 3.2 |
| 75-Piece Orchestra—C | 7.1 | 250-500 | 0.87 |
| 75-Piece Orchestra—D | 13. | 250-500 | 1.4 |
| Pipe Organ A | 2.0 | 250-500 | 0.81 |
| Pipe Organ B | 20. | 20-62.5 | 4.7 |

The 36×15-Inch Bass Drum A (Fig. 33)

The microphone was on the drum axis, in front of and 3 feet distant from the side played. The 119.5-db level shown in Fig. 33 for 1 per cent of intervals corresponds to a pressure of 1260 microbars actually measured at the microphone position. Assume that the radiation out to this 3-foot distance is confined to a cylinder of 3-foot diameter (the same as the drumhead diameter). Then the total sound power of this 1 per cent peak is 25 watts. This is believed to be a conservative estimate. For a realistic reproduction of this drum, a system would probably need to be capable of delivering this amount of acoustic power with reasonably low distortion. Of course, this refers to instantaneous power and to a fortissimo manner of playing. Note also that the 1 per cent of intervals, for the drum playing continuously, would correspond to a much smaller percentage in a lengthy orchestral selection in which the drum had long periods of rest.

The above assumption of no spreading to 3 feet, combined with the assumption of normal spreading from 3 to 20 feet, is equivalent to saying that the levels given

⁹ W. B. Snow, "Audible frequency ranges of music, speech and noise," *J. Acoust. Soc. Amer.*, vol. 3, pp. 155-166; July, 1931.

in Fig. 33 would be found only in the cone which a 3-foot diameter circle would subtend at a point 3 feet distant. This is no doubt unrealistic, but it should also be noted that a free field is assumed. In most actual listening situations, the effects of reflections would have to be taken into account in estimating levels at 20 feet. It should be kept in mind that the bass drum levels given in Figs. 7-10 and 33-36 are "on the beam" of a directive instrument.

This bass drum, *A*, gives the highest peaks of all single instruments studied. In frequency, the drum is very strong in all bands below 500 cps. It gives the highest peak pressures of all instruments in these bands, and also in the two bands between 500 and 1000 cps, and the 1400-2000-cps band. The pipe organ, selection *B* (Fig. 58), was less than 1 db lower in the lowest band. It exceeded the drum in the 1000-1400-cps band, and might have done so in bands neighboring the latter if they had been measured. The 1 per cent peak power of Drum *A* in the 250-500-cps band is estimated at 4 watts.

The 36×15-Inch Bass Drum B (Fig. 34)

The drum dimensions are the same as in *A*, but the drum is a different one and the player is different. Much lower levels than in *A* are found, with the 1 per cent peak only about 0.6 watt unfiltered, about 0.2 watt in the 62.5-125-cps band, which was strongest.

The 30×12-Inch Bass Drum C (Fig. 35)

Although a smaller drum than *A* and *B*, pressure was only 5 db lower than *A* in the 1 per cent unfiltered peaks, and only 4 to 7 db lower than *A* in the three lowest bands. It may be significant that the player was the same as with Drum *A*. The unfiltered 1 per cent peak power estimated is 6 watts.

The 34×19-Inch Bass Drum D (Fig. 36)

The same player as in *A* and *C* also played this drum. Although the unfiltered peaks are about 5 db less than *C*, the peaks of the lowest band are 2 db stronger than *C* but 2 db weaker than *A*.

Snare Drum (Fig. 37)

The distance is 4 feet, with the microphone nearly 90° off the axis of the drumhead. Reflections in the room have an appreciable effect under these conditions. Measurements made with a constant complex source in the same room (see above) show that the average total pressure falls off 8.5 db in going from 1 foot to 4 feet. That at 1 foot may be safely assumed to be due to the direct wave. Taking this into account, and assuming the radiation to be distributed over a hemisphere, the 1 per cent peak power is about 7 watts. On account of the position of the microphone and the directive properties of the drumhead, this is probably an underestimate.

The energy of the snare drum is widely distributed in frequency. It has low but measurable peaks in the lowest band. Between 125 and 2000 cps only the bass drum and pipe organ (of single instruments) are higher, and from 2800 cps up, only the cymbals and organ are higher. It is slightly higher than either bass drum or cymbals in the 2000-2800-cps band.

15-Inch Cymbals (Fig. 38)

The distance is 3 feet, at which the total average pressure has been found to fall 7.2 db below that at 1 foot. Again assuming distribution over a hemisphere, the 1 per cent peak power is 15 watts. This is only about 2 db lower than Bass Drum *A*. Although the peaks are quite high beginning at 125 cps, the cymbals reach their highest peaks in the bands above 8000 cps. They are higher than other instruments in all bands above 2800 cps.

Triangle (Fig. 39)

Calculating on the same basis as the cymbals, the 1 per cent peak power of the triangle is 20 milliwatts. Although no extreme pressures are found for this instrument, its spectrum is unusual in having very low-level bands between higher ones. This is due to the single tone plus harmonics which the instrument emits. It is quite high in level in the bands above 5600 cps, only the snare drum, cymbals, and pipe organ being higher above 8000 cps. The 1 per cent peak power in the 5600-8000-cps band is 9 milliwatts.

In this case, and others following, it may be thought that the assumption of hemispherical distribution of radiation is too conservative. It is thought best to use it, however, since the player's body does act to some extent as a reflector on one side of the instrument, and since it is not known to just what extent phase distortion in the measuring apparatus may change the maximum height of peaks.

Bass Viol (Fig. 40)

Here the instrument is large enough to have pronounced directive properties at the higher frequencies. Since, however, most of the output lies below 300 cps, the assumption of uniform distribution over a hemisphere is probably safe. This leads to about 90 milliwatts peak power in 1 per cent of one-eighth-second intervals. The peaks in the 62.5-125-cps band are almost as high as the unfiltered, and in the 125-250-cps band only 2 db down. There is a high concentration of energy in the low bands. Only the bass drums and the pipe organ have higher peaks in the two lowest bands.

Bass Saxophone (Fig. 41)

The instrument is not extreme, but it is strong in the 250-500-cps band, where the 1 per cent peaks have a power of 120 milliwatts. Unfiltered, the peaks run to 370 milliwatts.

BB^b Tuba (Fig. 42)

The 1 per cent unfiltered peaks are 280 milliwatts, those in the 250–500-cps band 160 milliwatts. Only the bass drum, snare drum, and organ are higher in the bands between 125 and 500 cps. The bell of the horn is large and has directive properties. Calculations are made on the basis of the falling off in pressure with distance observed in a complex source using a horn of similar size.

Trombone (Fig. 43)

Here, also, a horn of similar size was used for finding the rate of pressure decrease with distance. The calculation leads to a peak power of about 7 watts. There is a greater spread between powers calculated for trombone and tuba, than between the pressures measured, because of the greater directivity of the tuba. The long time average pressure of the tuba is slightly higher than that of the trombone, but the difference in 1 per cent peaks is very much in the other direction. This higher peak factor for the trombone is undoubtedly due to the manner of playing. Many notes of the trombone selection were “attacked” in a sharp explosive manner. The many high peaks observed at high frequencies, even above 8000 cps, are also due to this method of attack. This latter point was directly observed by watching the performance of the peak-amplitude meter at high frequencies, while listening at the same time to the playing of the instrument. That the high unfiltered peaks are due to the combination of peaks of several bands in phase is indicated by the fact that no single band has peaks closer than 20 db to those of the unfiltered case. The 2000–2800-cps band is highest, with peaks of 60 milliwatts. From 1400 to 4000 cps, only bass drum, snare drum, and cymbals have higher peaks, and from 2000 to 8000 cps only snare drum and cymbals (plus organ in the 5600–8000-cps band). The player probably used the sharp attack to a greater extent than would ordinarily be the case, so that the sample of trombone music may be extreme, both in height of peaks and in frequency range; yet these figures must be considered if all types of playing are to be perfectly reproduced.

Trumpet (Fig. 44)

The peak power is 280 milliwatts, comparable with bass saxophone and tuba. The 500–700-cps band is strongest, with 38 milliwatts. Between 500 and 1400 cps only bass drum, snare drum, cymbals, and organ have higher peaks (it exceeds the cymbals in the 700–1000-cps band).

French Horn (Fig. 45)

The peak power is about 50 milliwatts, with the 250–500-cps band much the strongest at 24 milliwatts.

Clarinet (Fig. 46)

The unfiltered 1 per cent peaks are about 50 milliwatts—the same as the French horn. The same band is highest, 250–500 cps, but with only about 4 milliwatts. In the clarinet, much more energy appears at higher frequencies than with the horn.

Flute (Fig. 47)

Unfiltered peak power is 13 milliwatts, that in the 700–1000-cps band about 5 milliwatts. Some of the lowest and highest bands were omitted, but qualitative observations showed that the omitted bands were weaker than those shown.

Piccolo (Fig. 48)

The peak power is 32 milliwatts. As expected, its highest energy is higher in frequency than the flute, with peaks of 13 milliwatts in the 2000–2800-cps band.

Piano A (Fig. 49)

The nature of the source makes it more difficult to calculate total acoustic power from the observed pressures. Three methods are given.

First Method: The microphone is 10 feet from the piano, and the room is comparatively small, so that it may be assumed that the sound pressures measured are those of completely diffused sound in the room. The reverberation time of the room is known, and the power may therefore be calculated. The peak power obtained in this manner is 260 milliwatts.

Second Method: A single measurement with a complex point source, with the microphone 12 feet away and turned 90° away from the direction of the source, showed that the average pressure was 14.6 db below that with the source at one foot, directly in front of the transmitter. If we assume the former position corresponds to completely diffused sound, and the latter to a direct wave, it is easily shown that the pressure of diffused sound in the room is the same as if the entire radiation were distributed over a sphere with a radius of 5.37 feet. These assumptions are probably not far from right. Making the calculation for the piano on this basis, the peak power is 690 milliwatts. This is probably high, since even at 10 feet the directive properties of the raised lid of the piano may cause a greater concentration of sound than would be found in completely diffused radiation.

Third Method: Auxiliary piano measurements have been made using a part of a certain mechanical roll. The microphone was placed in the same position as with the more general measurements, and again in a position very close to the open piano. The average pressure was 9.6 db less in the more distant position. The closer position was in the opening of the dihedral angle between sounding board and raised lid, where it might be as-

sumed the sound was distributed over an area roughly 3×6 feet in size. This is equivalent to a hemisphere 1.69 feet in radius, and 9.6 db down from this would place the microphone at an equivalent distance of 5.12 feet. This was the distance assumed in putting the piano data on a 20-foot basis, in the curves. Using it for peak power gives 320 milliwatts.

The remarkable thing about these three results is their agreement, rather than their disagreement, in view of the assumptions involved. An average of the three gives 420 milliwatts. The strongest band is 250–500 cps, with a peak power (average of the three methods) of 240 milliwatts. The instrument is a high grade grand piano with a self-playing mechanism, and the selection in this case was Liszt's "Second Hungarian Rhapsody."

The selection being played mechanically, the question arises as to whether peaks are as high as an artist can produce on the same instrument. A special roll was prepared, and an artist employed. The results showed that he could strike single notes with an average peak 9 db higher than the highest possible with the roll, but that when he played chords involving a number of notes, his peaks were 4 db lower than those with the roll. The highest possible peaks on this piano, with an artist playing, would seem to be about 3 watts.

Piano B (Fig. 50)

The same piano is used, but with a different roll, in this case a four-handed jazz record. The peak power calculated (average of the three methods) is 290 milliwatts, slightly less than with *A*. The 250–500-cps band peaks were 240 milliwatts, the same as with *A*.

Fifteen-Piece Orchestra in Laboratory (Fig. 51)

The room is the same as that for the piano measurements. The first and second methods used with the piano may be applied here, except that with Method 1 the added absorption due to the presence of fifteen men is taken into account. The first method gives 2.4 watts peak power, the second method 5.8 watts. The average is 4.1 watts. No doubt the same orchestra, playing in a larger room, would do so at a somewhat higher level. In frequency the 2000–2800-cps band is highest with 470 milliwatts, but other bands over a wide range are also quite strong.

This orchestra consisted of violin, cello, bass viol, flute, oboe, two alto saxophones, two trumpets, French horn, trombone, tuba, piano, bass and snare drums. The oboe and saxophone were replaced by clarinets part of the time. The reduction to 20 feet was made from the effective radius of 5.37 feet found under Method 2 (see Piano *A*).

Eighteen-Piece Orchestra in Laboratory (Fig. 52)

The difference from the fifteen-piece orchestra was that there were five violins, but no piano. The average

of the two methods of computing peak power gave 2.9 watts. This orchestra was not available long enough to complete all bands. Of those measured, 250–500 cps was strongest with 740 milliwatts. This is nearly 5 db higher than the same band in the fifteen-piece orchestra, although the unfiltered peaks are 1.5 db lower.

Seventy-Five-Piece Orchestra in Theater A (Fig. 53)

The difficulties connected with measurements made on sound picked up in the theater have already been mentioned. They are less certain than measurements made entirely in the laboratory, chiefly because of the added amplifiers and the long line between theater and laboratory. In addition, no data on reverberation time, or on the decrease of sound pressure with distance, are available for the theater. Power calculations are based on the following considerations. The players are distributed over about 210° of a circle approximately 50 feet in diameter. The microphone is mounted near the conductor's desk, approximately in the center of the circle. It is reasonable to take direct sound waves as predominating. The assignment of a single distance between microphone and source is impossible in this case. Considering the location of the various instruments, an effective distance of about 15 feet seems low rather than high. This distance has been used in the reduction to 20 feet, for the curves. Assuming a uniform distribution over a hemisphere of 15 feet radius, the peak power is about 8 watts. The 250–500-cps band is strongest, with 850 milliwatts. The 125–250 and the 2000–2800-cps band are only a little lower. The music in this case is of a lively nature, with brasses prominent, and a moderate amount of drums. Two very similar selections were used. One of them is shown in bands Nos. 1, 3, 5, 7, 9, and 11 (counting the 20–62.6-cps band as No. 1); the other selection is shown in "unfiltered" and in bands Nos. 2, 4, 6, 8, 10, and 12. No measurements were made above 8000 cps.

Seventy-Five-Piece Orchestra B (Fig. 54)

The music here is that of an overture of about ten minutes length. Extremes of volume are wider than with *A*. There are parts in which strings or wood-wind predominate, but also instants where brass, drums, and cymbals are used. Three selections appear, all of them fitting the above description. The first is an arrangement of Bizet music, and is shown in the first unfiltered run, and in bands 1, 3, 5, 8, 10, and 12. The second selection is the overture to "Mignon," and is shown in the second unfiltered and in bands 2, 4, 6, 7, 9, and 11. The third selection is an arrangement of Puccini music, and is shown only in the two bands above 8000 cps. Average pressures in the three selections fell within a range of 1 db, so that very little correction was involved in reducing all to the common average pressure of 84.8 db above 0.0002 microbar at 20 feet. Power calculations show 1 per cent peaks of 21 watts in the first unfiltered, 27 watts

in the second. The 250–500-cps band is highest with about 3 watts, the 8000-cps high-pass next with over 2 watts. These high-frequency peaks are due to a small percentage of intervals in which the cymbals were very prominent.

The fact that the full orchestra does not give as strong 1 per cent peaks in the unfiltered and in some of the bands, as some of the individual instruments, may be explained in several ways. As mentioned before, 1 per cent of intervals for an individual instrument may correspond to a much smaller percentage of the time of an orchestral selection. The microphone used with the orchestra may not be in the most favorable direction from a directional instrument such as the bass drum. Then of course the orchestral selections of the tests may not use the individual instruments in as fortissimo a manner as in the individual tests.

Seventy-Five-Piece Orchestra C (Fig. 55)

This is ballet music, accompanying a scene called "Autumn Leaves." Two parts of the same selection are used, the first part appearing in the unfiltered and in bands 1, 4, 6, 7, 9, and 11; the second part in bands 2, 3, 5, 8, 10, and 12. The bands above 8000 cps are omitted. The peak power is 7 watts, with about 0.9 watt in the 250–500-cps band.

Seventy-Five-Piece Orchestra D (Fig. 56)

The two lowest bands and the bands above 4000 cps are for a selection strong in bass drums and cymbals. The intermediate bands and the unfiltered are repetitions of those in selection *A*. A comparison with *A* will indicate how the low end of the spectrum is raised by the prominence of the bass drum.

Theater Pipe Organ A (Fig. 57)

The opening of the organ into the theater is about 60 feet long. The microphone is centrally in front of it, about 12 feet from the middle. Since the microphone is on the orchestra platform which is in a lowered position, a semicircular reflecting wall about 10 feet high is introduced behind the microphone, 12 feet away at its nearest point. Obviously the situation is even more uncertain than for the orchestra. As a guess, an effective distance of 15 feet, and uniform radiation over a quarter of a sphere, are assumed. This gives a peak power of 2 watts. The 250–500-cps band is strongest, with 0.8 watt. The selection was Kreisler's "Liebesleid," modified so as to make use of the different classes of pipes in the organ, and to include a wide range of volume.

Pipe Organ B (Fig. 58)

The selection is "Hail, Columbia" played fortissimo throughout. The full organ is used, with swells open wide. Several 16-foot pipes are included, and also "re-

sultant 32's," the latter being combinations which give difference tones corresponding to the tone of an open pipe 32 feet long. The purpose was to obtain extreme values for the unfiltered, and for the two ends of the frequency range. Measurements were made unfiltered, in three bands at the low end of the spectrum, in two bands above 5600 cps, and in one central band at 1000–1400 cps. The peak power is 20 watts unfiltered, a little lower than Bass Drum *A*. The lowest band is the strongest with about 5 watts, which is higher than the same band in the drum. This reversal in the order found for peak pressures is due to the greater directivity assumed for the drum. The other low bands and the high bands fall below the bass drum or cymbals in level. The 1000–1400-cps band is higher than any other instrument, and other intermediate bands might also have proved high if they had been measured.

LOWEST LEVELS IN MUSIC

In all of the foregoing, the upper limit of amplitude has been sought. The lowest level that it may be necessary to reproduce is also important because of the bearing it may have on permissible extraneous noises. Some data are available on this point, though they are very incomplete.

The highest pressures obtained in the laboratory were from a 36×15-inch bass drum. It was thought that the lowest level it might be desirable to reproduce would be that of a violin played very softly. A violin player was obtained and placed at a distance of 3 feet from the microphone, the same distance as that used with the drum. He was told to play at the lowest level that could be used with an audience. The resulting unfiltered average pressure, in a 15-second interval of uniform playing, was 0.52 microbar. The highest average pressure from the drum in an equal interval was 133 microbars. The range is 48 db. This is not sufficient, however, for reproduction must go at least as low as the lowest average amplitude of the violin, and at least as high as highest peak amplitude of the drum. The 1 per cent peak of Bass Drum *A* was about 1250 microbars, and the range is increased to 68 db. There still remains doubt whether the two music samples were extremes.

Data from the theater orchestra alone, during the course of the tests, do not cover so wide a range. The lowest 15-second average pressure observed was 0.27 microbar. This was undoubtedly considerably higher than the lowest levels it might be wished to reproduce. The 1 per cent peak pressure of Orchestra *B*, with the same microphone position, was 92 microbars, but it would probably be desirable to reproduce peaks for a smaller percentage than this. Thus the indicated range of about 51 db is considerably too small.

It is concluded that a system reproducing a large orchestra, the music ranging from a fortissimo ensemble to a pianissimo violin solo, might be called upon to handle a range as great as 70 db.

Magnetic Tape Recording with Longitudinal or Transverse Oxide Orientation*

RICHARD F. DUBBE†

Summary—A comparison is made of the performance of magnetic tape when the recording field is in the same direction as, or perpendicular to, the oxide particles as encountered in magnetic disc or video recording.

INTRODUCTION

IN 1951, Minnesota Mining and Manufacturing Company introduced oriented magnetic oxide coatings on their number 111 magnetic tape. Since that time, oriented magnetic tapes have won almost universal acceptance due to their superior recording characteristics. The gamma-ferric oxide particles employed are acicular or needle shaped, being approximately a micron in length and 0.2 micron in width. A common method of aligning these particles is to pass a wet or fluid magnetic coating, consisting of the iron oxide particles suspended in a plastic binder system, through a magnetic field. When the coating is dried, the particles are immobilized and remain in their aligned state.

When an oriented magnetic tape is recorded by a magnetic field that magnetizes the particles in their lengthwise direction, a considerable improvement in remanent induction is obtained compared to magnetizing the particles in a crosswise direction as shown in Fig. 1.

When the particles are randomly oriented, the remanent induction falls between that obtained by crosswise or lengthwise magnetization.

The improvement in remanent induction along with an increased slope of the magnetization curve makes possible better low- and high-frequency output on oriented tapes with lengthwise recording; however, at the present time, at least two magnetic recording applications do not permit magnetizing the particles in their lengthwise direction. The applications are 1) magnetic disc recording, and 2) the sound track on video tape recording.

Magnetic disc recording generally uses sheets of magnetically coated paper or plastic cut into disc form, or metal discs coated with magnetic dispersion. Applications include rapid access memory in computers (IBM Ramac system), geophysical recording (many parallel tracks recorded for a disc revolution), or even some sound recording applications where accessibility is more important than recording time. While it would un-

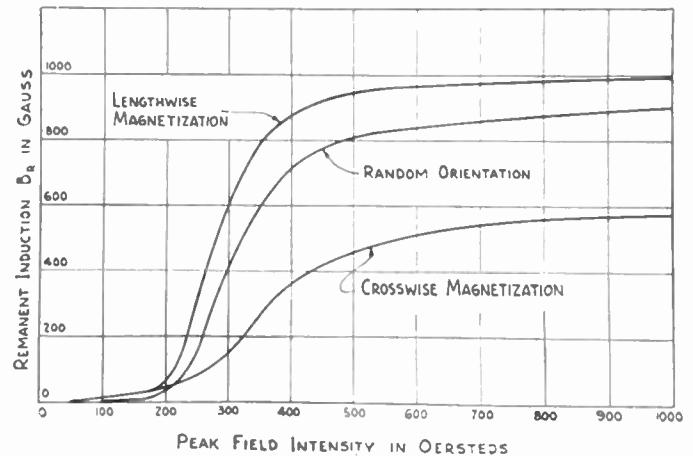


Fig. 1—Remanent induction curves of magnetic tape showing the effect of particle magnetization.

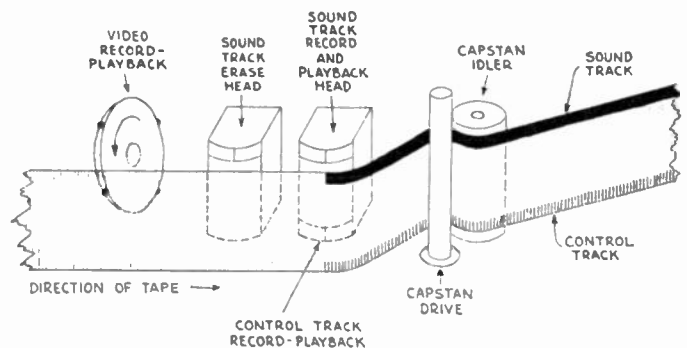


Fig. 2—Commonly used method of recording video information on 2-inch wide magnetic tape.

doubtedly be possible to align the oxide particles in a concentric fashion on disc coatings, to this writer's knowledge, the only practice now employed is to use randomly oriented particles in magnetic disc applications. While the magnetic performance is lower than that possible with oriented coatings, the applications are generally noncritical and the over-all performance is satisfactory.

In video recording, the utmost capabilities of the tape are required, especially at high frequencies (short wavelengths). Fig. 2 shows the general configuration of the video recorders now in use throughout the country. The video signal is recorded by four magnetic heads mounted on a drum rotating at 14,400 rpm. The drum scans or records the tape across the tape's width. The control track and sound track are recorded along the tape's length as in a conventional recorder. To obtain the maximum performance of the video signal, the oxide

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particles are oriented *transversely* on Scotch Brand number 179 Video tape (or perpendicular to the tape's edge). The orientation employed on sound recording tape is *longitudinal*, with the oxide particles aligned parallel to the tape's edge. Since the sound signal is recorded separately along the video tape's edge, it will record the particles crosswise and the sound quality will be somewhat sacrificed for the benefit of the video signal. The remainder of this paper will attempt to evaluate the performance of this sound channel. To reduce confusion, the oxide orientation will be referred to as *longitudinal* or *transverse*, with respect to the tape. The recording direction with respect to the oxide particle will be *lengthwise* or *crosswise*.

TEST METHODS AND RESULTS

The tape used for the following tests is Scotch Brand number 179 Video tape with transverse oxide orientation. The tape was slit from its normal 2-inch width to $\frac{1}{4}$ -inch so it could be tested on conventional professional recording equipment. When tested in this manner, the recording is crosswise on the oxide particles and gives results similar to the performance of the sound track on the video tape recorders.

For comparison purposes, $\frac{1}{4}$ -inch samples were slit across the coated web (originally 2 feet in width) and were spliced together. These samples then will record lengthwise on the oxide particles and give information on the video recording characteristics of the tape. The basic magnetic information was obtained in an hysteresis curve tracer or B-II meter.

Sensitivity

Sensitivity is a measure of the tape's output for a given fixed input level. It is usually measured at a relatively low recording level to insure recording on the magnetic tape's linear characteristic and to avoid effects of distortion. Sensitivity measured for a range of frequencies as a function of bias current will predict the tape's performance under most recording conditions. This information for the number 179 Video tape is given in Figs. 3, 4, and 7.

Low Frequency Sensitivity: The 500-cps sensitivity (30-mil wavelength) of the sound track (crosswise) is shown on Fig. 3 to be 6 db lower than the sensitivity in the lengthwise or video direction at peak bias.¹ One would expect a reduction in the oxides sensitivity when recorded in the crosswise direction due to the decreased slope of the magnetization curve (Fig. 1). Since the slope in the crosswise direction is approximately half that in the lengthwise direction, the 6-db loss in sensitivity is well verified.

Also, this curve shows the center of the linear magnetization region to occur at a higher peak field intensity for the crosswise recording which would indicate the

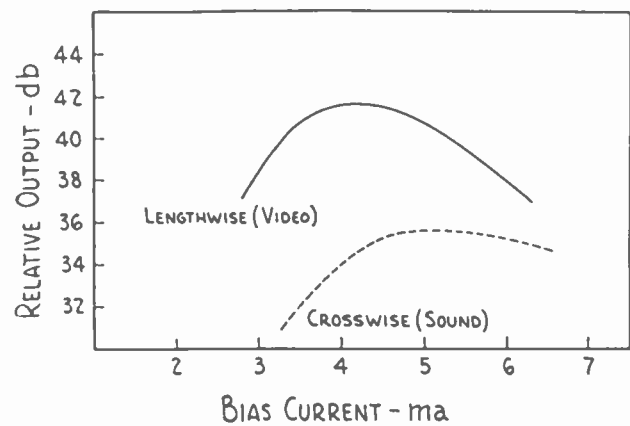


Fig. 3—Output vs bias at 500 cps (30-mil wavelength).

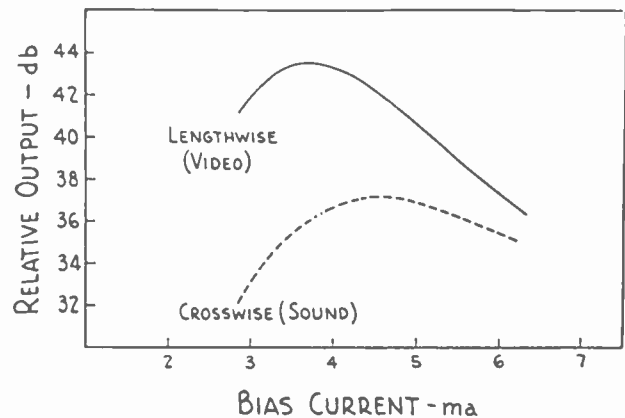


Fig. 4—Output vs bias at 15 kc (1.0-mil wavelength).

need for greater bias current during recording. The center of this region for the lengthwise direction occurs at about 280 oersteds; whereas, for the crosswise direction, it is at 350 oersteds. This ratio is almost identical to the increase of from 4.25 ma to 5.25 ma in peak bias current shown in Fig. 3. Occasionally, writers in the field have proclaimed the advantages of a "broad bias peak" such as shown by the sound track's curve; however, if one plots the two curves as a percentage of peak bias, they will be almost identical except for their amplitude displacement.

High Frequency Sensitivity: Several factors that affect the sensitivity and output of a magnetic tape at high frequencies (Fig. 4) are: surface smoothness, coercivity, slope of the magnetization curve, and oxide orientation. Since the surface of Scotch Brand number 179 Video tape is extremely smooth and is identical on the samples tested, its effect on the sensitivity in lengthwise vs crosswise recording can be neglected. Surprisingly, there is not a great change in coercivity for the two recording directions—255 oersteds in the lengthwise, and 235 oersteds in the crosswise (sound) direction. While a high coercivity is desirable, to reduce the demagnetization effects from close proximity of adjacent poles, higher coercivity tapes require higher bias currents which broaden the region in which the recording takes place

¹ Peak bias is that bias that gives the maximum output for a fixed input level.

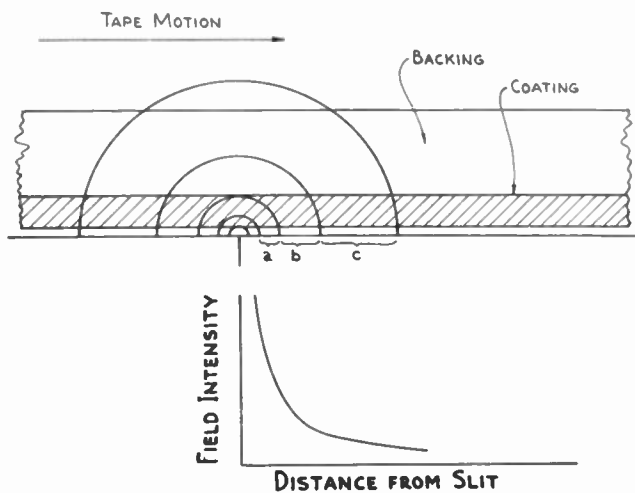


Fig. 5—Magnetic field intensity about an infinitesimal recording slit.

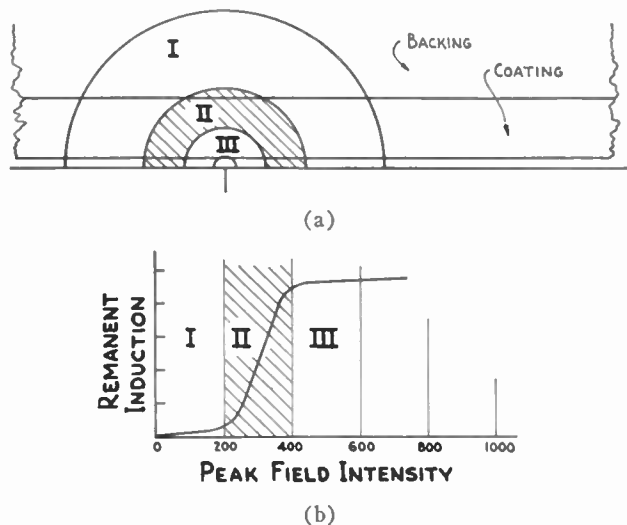


Fig. 6—(a) Enlarged view of recording field; (b) induction vs peak magnetizing field for a typical tape coating.

as shown in Fig. 5. We can thus expect a short wavelength loss in output in the crosswise direction due to its higher bias requirement.

Probably the greatest loss at short wavelength is due to the low slope of the magnetization curve in the crosswise direction. From Fig. 6 it is desirable to have the magnetization of the tape take place in the shortest time possible. To do this, Area II in which magnetization occurs should be as short as possible. Area I has insufficient field strength to magnetize the tape. In Area III the tape is saturated. If Area II is broad, the recording signal will change phase and even reverse polarity before the tape is moved from this magnetizing region. When recording in the lengthwise direction, the magnetization curve is quite steep and Area II is quite short. The tape is then rapidly magnetized permitting it to leave the reversing field before demagnetization takes place.

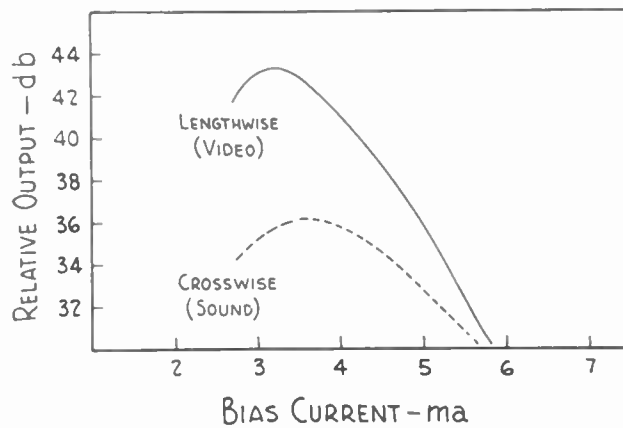


Fig. 7—Output vs bias at 0.5-mil wavelength.

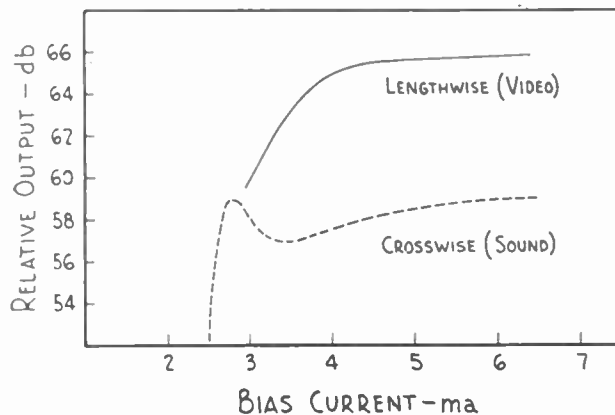


Fig. 8—Output for 1 per cent harmonic distortion at 500 cps (30-mil wavelength).

An interesting phenomenon that occurs at short wavelengths is the effect (or lack of effect) of oxide orientation. Even though there is over 7 db difference at the 0.5-mil wavelength output (Fig. 7) between lengthwise and crosswise recording at low bias fields, the curves almost coincide at high bias currents. This effect is shown to a lesser degree in Fig. 4. If we examine our model recording head and field (Fig. 5), we note the direction of magnetization becomes more and more vertical with increasing field intensity. If the oxide particles are all oriented in planes parallel to the tape's surface, all recording will be crosswise to the particles at high bias fields, regardless of the direction of the particles in their plane.

If the particles are recorded in the lengthwise direction, and are not aligned parallel to the tape's surface but are tipped, one will obtain better high-frequency performance when recording the tape in one direction than in the other even though both directions record lengthwise. Several magnetic recording tapes have shown imperfect alignment of the particles by exhibiting different frequency response depending on which direction the tape is recorded. This effect is again due to the recording field magnetizing the tape at an angle, especially at short wavelengths.

Output

The output for the two recording directions is given in Fig. 8 as a function of bias. The output for 1 per cent harmonic distortion in the lengthwise direction is about 7 db greater than in the crosswise direction. The output for a given distortion is probably largely determined by two characteristics: 1) the tape's total remanence (Φ_r) and, 2) the length of the linear region of the tape's magnetization curve. The Φ_r in the lengthwise direction is 0.68 maxwells; whereas, in the crosswise direction it is 0.44 maxwells. The linear recording region of the magnetization curve for the crosswise direction is quite short (Fig. 1). If the magnetization curve is used to plot a transfer characteristic in a manner similar to class "B" amplifier analysis (this method is only a crude approximation, at best, and neglects the geometric effects of recording and playback), one obtains a difference in output of approximately 6 db. Also, one obtains a bend in the slope of the crosswise direction transfer curve which could account for the peculiar rise in output at low bias currents which does not occur for the lengthwise direction.

Noise

The noise for the two recording directions was measured in three ways:

| | Lengthwise | Crosswise |
|--|------------|-----------|
| DC modulation noise signal-to-noise ratio | 56.5 db | 55.5 db |
| Machine erased and biased tape signal-to-noise ratio | 60.5 db | 55.5 db |
| Bulk erased tape signal-to-noise ratio | 63.5 db | 56.5 db |

The signal level for these tests was taken as 3 per cent odd order harmonic distortion. The noise was measured using a 500-cps high pass filter to eliminate hum components. The dc noise test uses a direct current in the record head equal to the rms value of the ac current required to produce 3 per cent distortion. While the output in the lengthwise direction was 7 db higher, its modulation noise was 6 db higher than in the crosswise direction. This similarity in SNR should be expected since the surface irregularities of both samples are identical and the modulation noise should be proportional to the signal. However, in a typical recording situation, where background noise is the main problem, the SNR of lengthwise recording is seen to be at least 5 to 6 db better than in the crosswise direction.

Layer-to-Layer Signal Transfer

The signal-to-print ratio was measured for the two recording directions:

| | Lengthwise | Crosswise |
|--|------------|-----------|
| Relative signal level at 3 per cent distortion | 61.5 db | 55.5 db |
| Relative print level | 12.0 db | 6.0 db |
| Signal-to-print ratio | 49.5 db | 49.5 db |

The measurements were made at 150°F for four hours according to the Navy Bureau of Ships WT0061 specification.

SUMMARY

While the performance of the tape's oxide when recorded in the crosswise direction is definitely not as good as when recorded lengthwise, the quality of the sound channel on video tape recorders is comparable to standard broadcast quality equipment if properly used. The high-frequency output in the unfavorable sound direction of number 179 Video tape is approximately the same as in the lengthwise (favorable) direction of standard sound recording tapes due to the video tape's extremely smooth surface. Since the video tape recorder operates at 15 ips, excellent quality can be obtained to beyond 15 kc without excessive equalization or special magnetic heads.

The signal-to-noise ratio is sacrificed by approximately 6 db when compared to standard tape recording; however, it has proven adequate for telecasting. Due to the precision of the video tape's surface, dropouts and high intensity modulation noise are practically non-existent in the sound channel.

It is interesting that in video recording tape, probably the most highly perfected tape in use today, the sound channel is used in such a manner that it cannot use the main features of this unusual tape, but must rely on more or less conventional performance.

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On the Response and Approximation of Gaussian Filters*

J. KLAPPER† AND C. M. HARRIS‡

Summary—This paper discusses a filter whose amplitude response characteristic is $e^{-k\gamma^2}$ where γ is a function of frequency and k is a constant related to the filter bandwidth. The amplitude response curve of this filter has the shape of the Gaussian probability function and it is shown that the phase response curve may be assumed to be linear. Previous investigators have shown that such a filter has excellent transient characteristics and that in some sense, it is the optimum filter for transient signals. The responses to impulse and step functions are discussed in this paper. A method for designing an approximate Gaussian filter is given, and the measured responses of such an approximate Gaussian filter are presented.

I. INTRODUCTION

A NETWORK may be said to have an ideal transient response when the waveshape at the output of the network is identical to that of the input, for any input waveshape. Only a resistive network has such a response. A filter, which by the very nature of its application must be frequency discriminative, cannot have such an ideal response. The narrower the bandwidth of a filter, the more distorted will its transient response be. Hence, a filter which has a minimal combination of amplitude-response spread and distortion of the waveshape may be called an "optimum" filter for transients.

The impulse- and the step-functions are selected here as representative input waveshapes for describing the characteristics of the Gaussian filter because: 1) these are the most commonly discussed functions in the literature of transients, 2) the response of a linear filter to any waveshape may be obtained by means of the Superposition Theorem, once the impulse or step response is known.¹ If a bandpass filter has a low-pass analog and a bandwidth which is relatively narrow with respect to its center frequency, it may be assumed that the response of the bandpass filter to the envelope of a wave at the center frequency of the filter is the same as that of the analogous low-pass filter to a wave having the shape of the envelope.² For convenience, this paper primarily considers the responses of a low-pass Gaussian filter. However, the results apply also to the bandpass Gaussian filter under the conditions stated above. The design of a practical Gaussian bandpass filter is described in Appendix II.

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¹ S. Goldman, "Transformation Calculus and Electrical Transients," Prentice Hall, Inc., New York, N. Y., Sect. 4.6; 1949.

² G. E. Valley and H. Wallman, "Vacuum Tube Amplifiers," RLS 18, Sect. 7.1, McGraw-Hill Book Co., Inc., New York, N. Y.; 1948.

A low-pass Gaussian filter may be defined as a filter whose frequency response characteristic is given by

$$G(j\omega) = e^{-k\omega^2} \quad (1)$$

where k is a constant related to the bandwidth. The shape of the frequency-response curve for such a filter, given in Fig. 1, is the same as the Gaussian probability curve. As shown below, Gaussian filters have excellent transient response characteristics and may be approximated relatively easily.

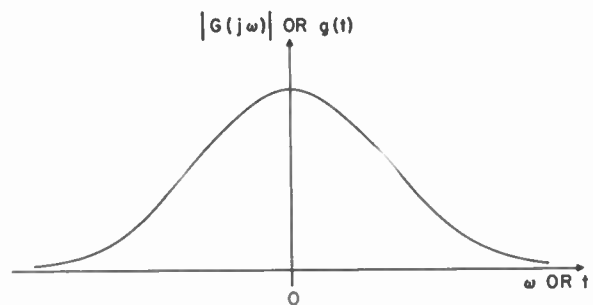


Fig. 1—Gaussian response curve.

II. RESPONSE TO AN IMPULSE

If the input signal to the low-pass Gaussian filter of (1) is a unit impulse, the output is given by³

$$g(t) = \mathcal{F}^{-1}[e^{-k\omega^2}] = \frac{1}{\sqrt{4\pi k}} e^{-t^2/4k} \quad (2)$$

which has the Gaussian shape shown in Fig. 1, the same as the filter's amplitude-response characteristic. This is illustrated by the oscillogram shown in Fig. 2 of the response to an impulse of a bandpass filter which is approximately Gaussian in shape, the design of which is described in Section V. Note that the envelope has the shape of Fig. 1.

Let us define a) the distortion of an impulse due to the filter, by the mean-square-value of the time-response spread $(\Delta t)^2$

$$(\Delta t)^2 = \frac{\int_{-\infty}^{\infty} t^2 |f(t)|^2 dt}{\int_{-\infty}^{\infty} |f(t)|^2 dt}, \quad (3)$$

³ W. Magnus, "Formeln und Sätze für die Speziellen Funktionen der Mathematischen Physik," Springer Verlag, Berlin, Ger., p. 161; 1948.

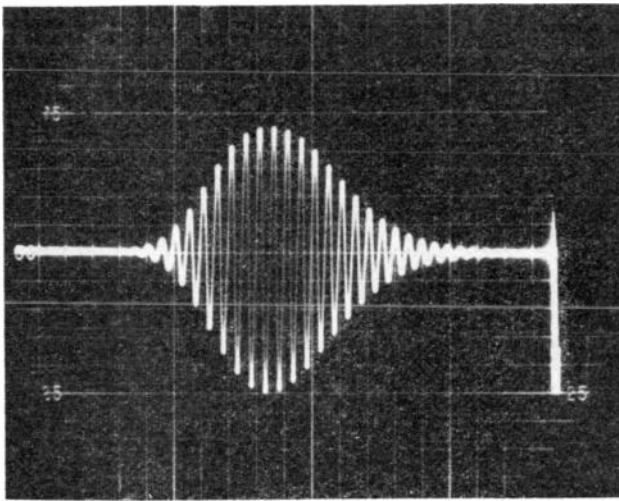


Fig. 2.—Oscillogram of the impulse response of the Gaussian bandpass filter shown in Fig. 8.

and b) the amplitude-response spread of the filter by its mean-square-value, $(\Delta\omega)^2$

$$(\Delta\omega)^2 = \frac{\int_{-\infty}^{\infty} \omega^2 |f(\omega)|^2 d\omega}{\int_{-\infty}^{\infty} |f(\omega)|^2 d\omega} \quad (4)$$

An optimum filter for transients should have a minimal combination of time and amplitude-response spreads. The optimum filter will be Gaussian if the criterion for optimality is that either

$$(\Delta t)^2 + K(\Delta\omega)^2 = \text{minimum} \quad (5)$$

or

$$(\Delta t)(\Delta\omega) = \text{minimum} \quad (6)$$

where K is a weighting constant. An equation having the form of (5) has been solved by N. Wiener,⁴ and an equation having the form of (6) was solved by A. Gabor.⁵

Many pulses occurring in practice are closely Gaussian in shape. A Gaussian pulse after passing through a Gaussian filter will retain its Gaussian shape, but will spread out in time. This may be shown as follows: Let the Gaussian input pulse be given by

$$e_i(t) = e^{-t^2/4b} \quad (7)$$

where b is a constant which determines the pulse width. Then

$$\mathfrak{F}[e_i(t)] = \sqrt{4\pi b} e^{-b\omega^2} \quad (8)$$

From (1) and (8) it follows that

$$\mathfrak{F}[e_0(t)] = \sqrt{4\pi b} e^{-h\omega^2} e^{-k\omega^2} = \sqrt{4\pi b} e^{-(b+k)\omega^2} \quad (9)$$

Then

$$e_0(t) = \mathfrak{F}^{-1}[\sqrt{4\pi b} e^{-(b+k)\omega^2}] = \sqrt{\frac{b}{b+k}} e^{-t^2/4(b+k)} \quad (10)$$

It is apparent that any number of Gaussian filters in tandem will have a combined transfer function that is Gaussian.

III. RESPONSE TO A STEP

The step function is the integral of the impulse function. In view of the assumed linearity of the system, the response of a Gaussian filter to a unit step, $a(t)$, is the integral of the response of such a filter to a unit impulse which was given in (2),

$$\begin{aligned} a(t) &= \int_{-\infty}^t g(x) dx = \int_{-\infty}^t \frac{1}{\sqrt{4\pi k}} e^{-x^2/4k} dx \\ &= 0.5 + \frac{1}{\sqrt{\pi}} \int_0^{t/\sqrt{4k}} e^{-u^2} du \\ &= 0.5 \left[1 + \text{ERF} \left(\frac{t}{\sqrt{4k}} \right) \right] \end{aligned} \quad (11)$$

This equation is plotted in Fig. 3. An oscillogram of the response of a Gaussian bandpass filter, which we have constructed, to a center-frequency modulated step function is shown in Fig. 4. Note that the envelope has the shape of Fig. 3. The response characteristic of Fig. 3 shows no overshoot, no ringing, no smear,⁶ and is perfectly symmetrical with respect to its half-value point. The symmetry is a result of the linear phase response of the Gaussian filter. Furthermore, it implies that the transition is steepest.⁷ The (bandwidth \times rise-time) product⁸ of a low-pass Gaussian filter is 0.343, which is very near the lowest value possible if the response has little or no overshoot. The calculation of this (bandwidth \times rise-time) product is given in Appendix I. The simple RC low-pass network has a (bandwidth \times rise-time) product very close to that of the Gaussian filter, *i.e.*, 0.35. However, the Gaussian filter has a much higher rate of cutoff of its amplitude response curve. As a matter of fact, it would be necessary to employ an infinite number of RC networks in cascade to obtain a Gaussian response curve.

A rectangular filter having zero attenuation in the pass band and infinite attenuation in the stop band is sometimes referred to as the "ideal" filter. The step response of such a low-pass filter is the $S_i(t)$ function shown

⁶ A response characteristic is said to have *smear* if, after the response has approached very close to the steady-state value, it requires an abnormally long time to cover the remaining separation from the steady-state value.

⁷ H. E. Kallmann, R. E. Spencer, and C. P. Singer, "Transient response," Proc. IRE, vol. 33, pp. 169-195; March, 1945.

⁸ The bandwidth between the half-power points; the rise-time from 0.1 to 0.9 of the step.

⁴ N. Wiener, "Extrapolation, Interpolation and Smoothing of Stationary Time Series," John Wiley and Sons, Inc., New York, N. Y., pp. 95-97; 1949.

⁵ A. Gabor, "Theory of communication," JIEE (London), pt. III, pp. 93, 429; 1946.

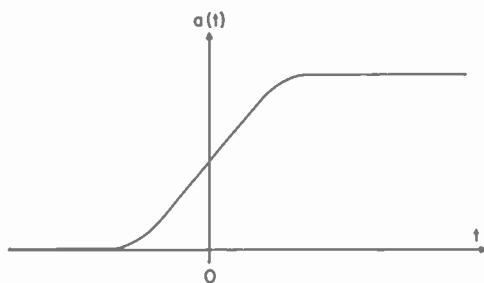


Fig. 3—Gaussian filter step response.

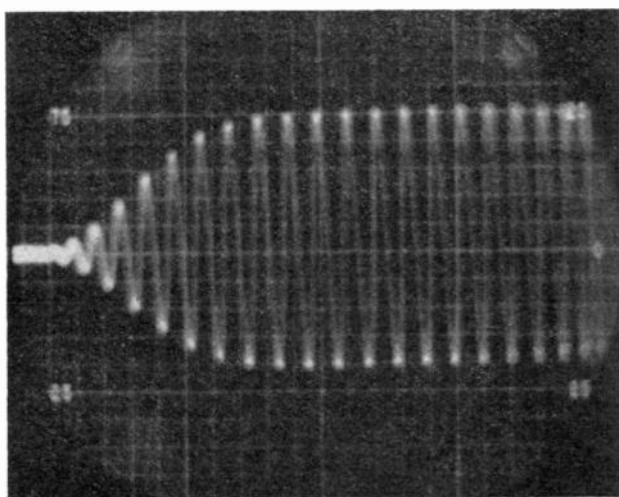


Fig. 4—Oscillogram of the step response of the Gaussian bandpass filter shown in Fig. 8.

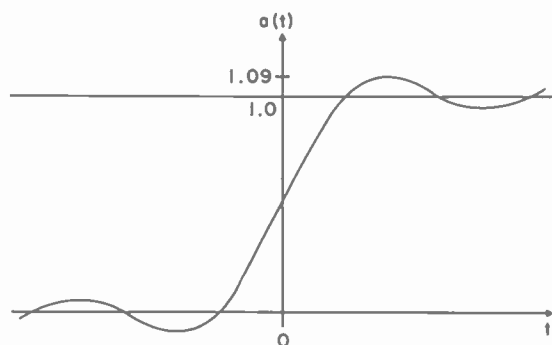


Fig. 5—Step response of a rectangular filter.

in Fig. 5. It has an overshoot of 9 per cent and a (bandwidth \times rise-time) product of 0.445.⁹ Thus, while such a filter has a sharper amplitude response cutoff than a Gaussian filter, it is not an ideal filter for transients. In addition, the rectangular response characteristic is much more difficult to approximate than the Gaussian. The approximations to the rectangular response characteristic by means of the Butterworth, Tchebycheff, or L polynomials¹⁰ result in filters having a greater rate of

⁹ D. K. Cheng, "A note on the reproduction of pulses," *Proc. IRE*, vol. 40, pp. 962-965; August, 1952.

¹⁰ A. Papoulis, "Optimum filters with monotonic response," *Proc. IRE*, vol. 46, pp. 606-609; March, 1958.

amplitude response cutoff than the Gaussian filter for the same number of stages, but such filters inevitably have a response with an appreciable amount of overshoot, primarily due to the nonlinearity of their phase response characteristics.

A problem which is closely related to the transient response of a filter is the filter's response to a frequency varying signal. For a signal whose frequency is varying linearly with time, the Gaussian filter may be swept at about eight times the speed with which the rectangular filter may be swept before its performance is noticeably altered. The condition that the Gaussian filter not alter its performance noticeably is that $q \ll 14(BW)^2$, where q is the sweep-rate in rps/sec and BW is the filter (3 db) bandwidth in radians per second.¹¹

IV. PHASE RESPONSE

We must differentiate here between a "theoretical" Gaussian filter and a "realizable" Gaussian filter obtained by means of physical components. A theoretical low-pass Gaussian filter is one whose frequency response is given by (1) which may be rewritten as

$$G(j\omega) = e^{-k\omega^2} e^{j0 \cdot \omega}, \quad (12)$$

i.e., the theoretical low-pass Gaussian filter has zero phase shift for all frequencies. Since we cannot, in general, prescribe both the phase and amplitude of a physically realizable transfer function, we can at most set

$$|G(j\omega)| = e^{-k\omega^2} \quad (13)$$

and examine whether a phase function $\phi(\omega)$ exists such that

$$G(j\omega) = |G(j\omega)| e^{j\phi(\omega)} \quad (14)$$

will be physically realizable.

It can be shown that no phase function may be attached to the amplitude response function of (13), since (13) does not meet the Paley-Wiener criterion:¹²

$$\int_{-\infty}^{\infty} \frac{|\log e^{-\omega^2}| d\omega}{1 + \omega^2} < \infty. \quad (15)$$

Figs. 1 and 3 also show why no such phase function exists. Note that the time-response of a Gaussian filter to an impulse or step function applied at $t=0$ begins at $t = -\infty$. Obviously, a physical network cannot be a predictor, *i.e.*, it may not have a response before the excitation is applied. Thus, it is necessary to delay the response by an infinite amount of time if the response is to begin at $t=0$ instead of at $t = -\infty$. This same problem

¹¹ S. S. L. Chang, "On the filter problem of the power spectrum analyzer," *Proc. IRE*, vol. 42, pp. 1278-1282; August, 1954.

¹² R. E. A. C. Paley and N. Wiener, "Fourier transforms in the complex domain," *Am. Math. Soc. Colloq. Pub.*, vol. 19, pp. 16-17; 1934.

occurs in any sharp cutoff filter, including the rectangular type of Fig. 5. However, it will be shown below that the phase response of a Gaussian filter may be assumed to be linear.

One method of approximating a low-pass Gaussian filter is by the use of a large number of low-pass RC networks with amplifiers in cascade. The phase function of such a series of identical stages (with the 3-db point for the filter at $\omega = 1$) is

$$\phi_n(\omega) = -n \tan^{-1} \omega \sqrt{\frac{\ln 2}{n}} \quad (16)$$

where n is an integer representing the number of stages. For

$$\omega \ll \sqrt{\frac{n}{\ln 2}}, \quad \tan^{-1} \omega \sqrt{\frac{\ln 2}{n}} \cong \omega \sqrt{\frac{\ln 2}{n}}$$

and (16) becomes

$$\phi_n(\omega) = -\sqrt{n \ln 2} \omega, \quad (17)$$

i.e., the phase response is linear and has the slope of $-\sqrt{n \ln 2}$. In the limit when $n \rightarrow \infty$, the amplitude response approaches the Gaussian function (13) and the phase response approaches perfect linearity and an infinite slope.

A similar result is obtained when Bode's attenuation-vs-phase relation is used. The phase angle ϕ_a at any frequency ω_a is given by¹³

$$\phi_a = \frac{2\omega_a}{\pi} \int_0^\infty \frac{A - A_a}{\omega^2 - \omega_a^2} d\omega \quad (18)$$

where A is the attenuation in nepers corresponding to ω .

Here

$$A = \log |G(j\omega)| = -k\omega^2, \quad (19)$$

and therefore,

$$\phi_a = \frac{-2k\omega_a}{\pi} [\omega]_0^\infty \quad (20)$$

which is unrealizable. The frequency response of any filter constructed by means of lumped physical components may be expressed as a ratio of polynomials in ω .

$$G(j\omega) = \frac{a_n \omega^n + a_{n-1} \omega^{n-1} + \dots + a_0}{b_m \omega^m + b_{m-1} \omega^{m-1} + \dots + b_0}$$

At extremely high frequencies, only the terms having the largest exponents will be effective, *i.e.*, for extremely high frequencies the amplitude response is given by

$$|G(j\omega)| \cong \left| \frac{a_n}{b_m} \right| \omega^{n-m}$$

$$\log |G(j\omega)| \cong \log \left| \frac{a_n}{b_m} \right| + (n-m) \log \omega = B - b \log \omega \quad (21)$$

where b is the number of effective components in the high-frequency limit.

The amplitude response of a Gaussian filter realized by means of lumped physical components, no matter how closely it is approximated, will be given by (21) in the high frequency limit and not by (19). Now assume that the amplitude response of a realizable low-pass Gaussian filter is given by (19) up to a frequency ω_0 , which is much higher than any frequency of interest ω_a , and is given by (21) for all frequencies beyond ω_0 . Neglecting the transition region, (18) may be written as

$$\begin{aligned} \phi_a &= \frac{2\omega_a}{\pi} \left[\int_0^{\omega_0} \frac{-k\omega^2 + k\omega_a^2 d\omega}{\omega^2 - \omega_a^2} \right. \\ &\quad \left. + \int_{\omega_0}^\infty \frac{B - b \log \omega + k\omega_a^2}{\omega^2 - \omega_a^2} d\omega \right] \\ &\cong \frac{-2}{\pi} \left(k\omega_0 + \frac{b + b \log \omega_0}{\omega_0} \right) \omega_a. \end{aligned} \quad (22)$$

Thus as the region over which the filter approximates a Gaussian filter increases, the phase-response linearity region and its slope increase.

A linear phase response is desirable because for a given amplitude-response characteristic, it yields a minimum amount of transient distortion since linear phase response implies an equal amount of time delay for each frequency component. In general, it is desirable that the phase-response linearity extend over more than 90 per cent of the area under the amplitude-response curve, which is about a 12-db range for the Gaussian curve.

Fig. 6 shows the measured phase response of a filter whose amplitude response is shown. It can be seen that the phase linearity extends over the more important low-attenuation region. The oscillograms of the impulse and step responses of this filter are shown in Figs. 2 and 4. The symmetry of these responses verifies that the phase-response linearity extends over an appreciable portion of the frequency response characteristic.

V. APPROXIMATION AND REALIZATION

As indicated in Section IV, a Gaussian filter may be realized only at the cost of infinite delay, and therefore, by an infinite number of components. This is also the case with other sharp cutoff filters, *e.g.*, the rectangular type. The Gaussian filter, however, is relatively easily approximated over the most important region, *i.e.*, the low-attenuation frequency band. Various investigators^{4,14}

¹³ H. W. Bode, "Network Analysis and Feedback Amplifier Design," D. Van Nostrand Co., Inc., New York, N. Y. Sect. 14.3, pp. 305-309; 1945.

¹⁴ C. Cherry, "Pulses and Transients in Communication Circuits," Dover Publ., Inc., New York, N. Y., p. 311; 1950.

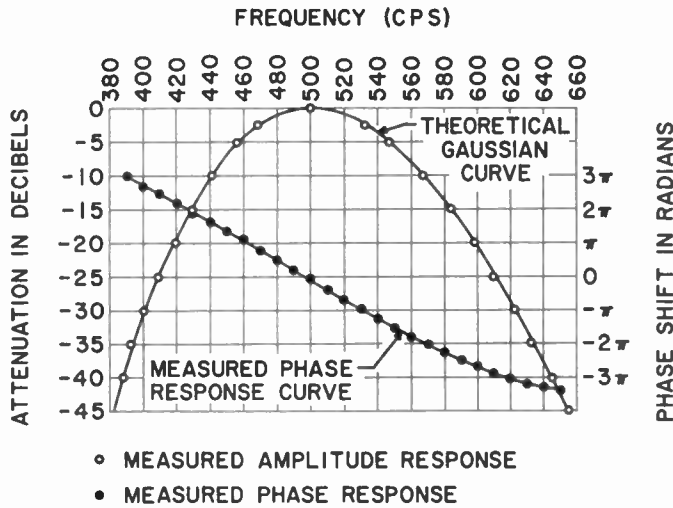


Fig. 6—Amplitude and phase response of the filter of Fig. 8.

have suggested the use of a series of RC low-pass networks and amplifier stages in cascade, to approximate the Gaussian shape, but this does not represent the most efficient use of amplifier stages. It is possible to approximate the Gaussian filter with fewer stages if more effective interstage coupling networks are used. A convenient scheme, of the series-peaking type shown in Fig. 7, has been suggested by Wente.¹⁵ Among other advantages, it has a simple low-pass analog. Assume that an approximation by means of 5 stages is desired. Then the network component values may best be obtained by means of a power-series expansion of the Gaussian function, as follows:

1) Let the amplitude response of the Gaussian filter to be realized be

$$|G(j\gamma)| = e^{-a^2\gamma^2/2} \quad (23)$$

where $\gamma = \omega$ for the low-pass filter,

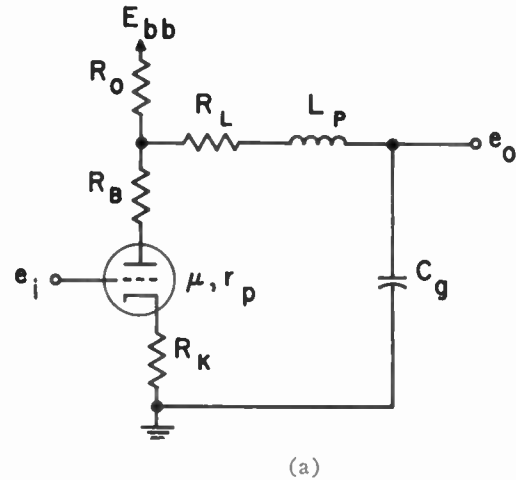
$$\gamma \equiv \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)$$

for the bandpass filter, and a is a constant related to the bandwidth.

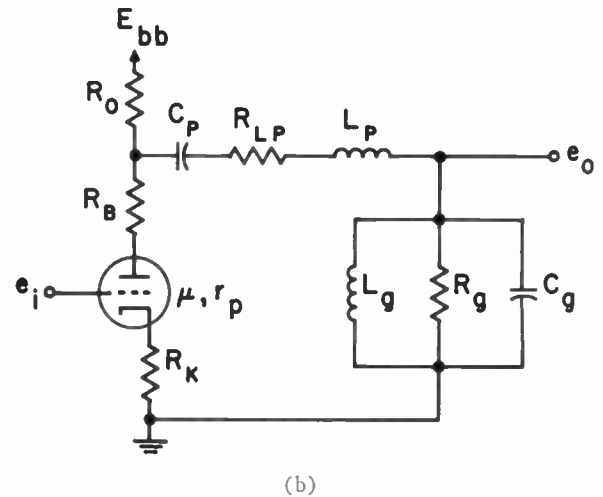
2) Expand in a power series the square of the Gaussian function up to the 20th power:

$$\begin{aligned} |G(j\gamma)|^2 &= G(j\gamma)G(-j\gamma) = e^{-a^2\gamma^2} = \frac{1}{e^{a^2\gamma^2}} \\ &\cong \frac{1}{1 + (a\gamma)^2 + \frac{(a\gamma)^4}{2!} + \dots + \frac{(a\gamma)^{20}}{10!}} \\ &= \frac{1}{P(j\gamma)P(-j\gamma)}, \end{aligned} \quad (24)$$

¹⁵ E. C. Wente, *Acoust. Soc. Amer.*, vol. 26, p. 952; September, 1954.



(a)



(b)

Fig. 7—Series-peaking network as suggested by Wente; (a) low-pass, (b) bandpass.

$G(j\gamma)$ is the realizable function. It has as its denominator $P(j\gamma)$, and all its poles in the left-half plane.

3) Make use of the following relation¹⁶ to find $P(j\gamma)$:

$$\begin{aligned} 10! \left(1 + z + \frac{z^2}{2!} + \dots + \frac{z^{10}}{10!} \right) \\ = (z + 3.55 \pm j0.789)(z + 3.01 \pm j2.33) \\ (z + 1.87 \pm j3.77)(z - 0.0662 \pm j4.97) \\ (z - 3.37 \pm j5.63). \end{aligned} \quad (25)$$

Substituting $z = a^2\gamma^2$, and using the factors of the left half of the γ plane only, it follows that

$$G(j\gamma) = \frac{1}{\prod_{n=1}^5 \left(1 + \frac{j a \gamma}{Q'_{Ln}} - \alpha'_n a^2 \gamma^2 \right)}. \quad (26)$$

The values of Q'_{Ln} and α'_n may be obtained from (24) and (25). The Gaussian function is approximated by the realization of $G(j\gamma)$ as given in (26), i.e., the product of

¹⁶ K. E. Iverson, "The zeros of the partial sums of e^z ," *Math. Tables and other Aids to Comp.*, vol. 7, pp. 165-168; July, 1953.

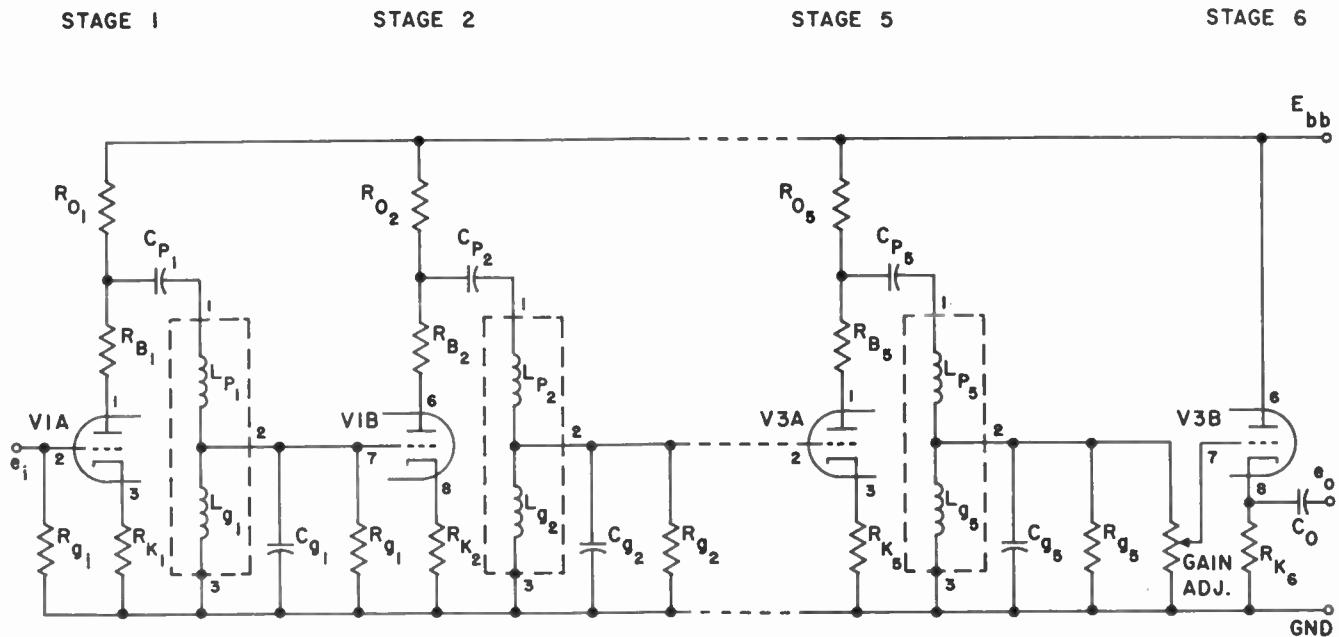


Fig. 8—Schematic diagram of a 5-stage approximate Gaussian bandpass filter.

the 5 left-half plane polynomials by means of 5 stages of the type shown in Fig. 7.

4) Obtain the values of the components for each inter-stage network by comparing each polynomial factor of (26) with the transmission equation of the network of Fig. 7.

It may be deduced from (25) that stages 1-3 have a monotonously drooping amplitude response, while stages 4 and 5 have a double-peaked amplitude response. The design equations of a 5-stage bandpass approximate Gaussian filter are given in Appendix II. (The schematic diagram of this filter is shown in Fig. 8, its measured frequency and phase responses are plotted in Fig. 6.) The above power-series approximation is shown as curve (d) of Fig. 9. Curve (a) is the theoretical Gaussian curve. Curve (b) is due to Thomson,¹⁷ his approximation is based on a maximally-flat time-delay response. Curve (c) is due to Wente¹⁵ who used 5 stages of the type shown in Fig. 7(a), each with a *Q* of 0.56. The power-series expansion gives the best approximation to the Gaussian shape, as shown in Fig. 9.

VI. CONCLUSION

It has been shown that the Gaussian filter has excellent transient characteristics as evidenced by its impulse and step function responses and by its response to a variable frequency signal. It also has a linear phase response, a reasonably sharp amplitude response cutoff, and it is relatively easily approximated, as evidenced by the fact that 5 interstage coupling networks result in a close approximation down to about 45 db.

¹⁷ W. E. Thomson, "Networks with maximally flat delay," *Wireless Engr.* (London), vol. 29, pp. 256-263; October, 1952.

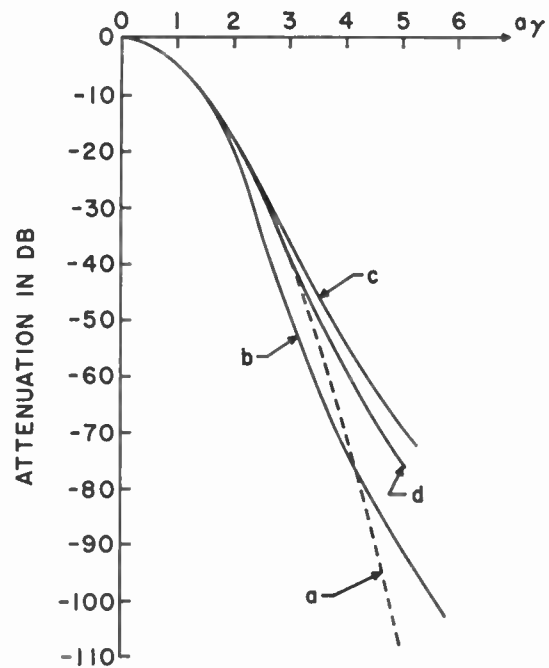


Fig. 9—Approximations to the Gaussian response curve; (a) theoretical Gaussian curve, (b) Thomson approximation, (c) Wente approximation, and (d) power-series approximation.

APPENDIX I

THE (BANDWIDTH RISE-TIME) PRODUCT OF A GAUSSIAN FILTER

The low-pass Gaussian filter frequency response is given by

$$G(j\omega) = e^{-k\omega^2}.$$

Solving for k in terms of the 3-db bandwidth, ω_3 , one obtains:

$$k = \frac{1}{2.89\omega_3^2} \tag{27}$$

For the lowpass Gaussian filter, the unit step response is given by (11)

$$a(t) = \int_{-\infty}^t \frac{1}{\sqrt{4\pi k}} e^{-x^2/4k} dx.$$

This function is tabulated in the form¹⁸

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-t^2/2} dt. \tag{28}$$

If we let $t^2/4k = y^2/2$, we obtain (11) in the same form as (28), namely

$$a(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t/\sqrt{2k}} e^{-y^2/2} dy. \tag{29}$$

The (0.1-to-0.9) rise-time in terms of $t/\sqrt{2k}$ is 2.6¹⁸

and
$$t_r = 2.6\sqrt{2k} = \frac{2.6\sqrt{2}}{\sqrt{2.89\omega_3^2}}.$$

The (bandwidth \times rise-time) product is then

$$f_{3\text{dB}} t_r = 0.343. \tag{30}$$

APPENDIX II

DESIGN EQUATIONS FOR AN APPROXIMATE BANDPASS FILTER

The approximation is given by (26)

$$G(j\gamma) = \frac{1}{\prod_{n=1}^5 \left(1 + j \frac{a\gamma}{Q' L_n} - \alpha_n' a^2 \gamma^2 \right)}$$

a is obtained in terms of the 3-db $\gamma(\gamma_3)$ as follows:
From (23)

$$\begin{aligned} |G(j\gamma_3)| &= e^{-a^2 \gamma_3^2 / 2} = 2^{-1/2} \\ -\frac{1}{2} a^2 \gamma_3^2 &= -\frac{1}{2} \ln 2 \end{aligned} \tag{31}$$

$$\therefore a = \frac{\sqrt{\ln 2}}{\gamma_3} = \frac{f_0 \sqrt{\ln 2}}{BW} \tag{32}$$

where f_0 is the center frequency of the bandpass filter and BW is the 3-db bandwidth.

Eq. (26) can now be written in the form

$$G(j\gamma) = \frac{1}{\prod_{n=1}^5 \left[1 + j \frac{1}{Q' L_n} \frac{f_0}{BW} \gamma - \alpha_n \left(\frac{f_0}{BW} \right)^2 \gamma^2 \right]} \tag{33}$$

¹⁸ H. Cramer, "Math. Methods of Statistics," Princeton University Press, Princeton, N. J., p. 557; 1951.

The values of Q_L and a as a function of n (the stage number) are obtained from (24), (25), and (32) and are tabulated below:

| Stage Number | α | Q_L |
|--------------|----------|--------|
| 1 | 0.1904 | 1.1527 |
| 2 | 0.1817 | 1.2395 |
| 3 | 0.1647 | 1.4497 |
| 4 | 0.1395 | 1.9059 |
| 5 | 0.1057 | 3.1221 |

The equivalent circuit of the network of Fig. 7(b) is given in Fig. 10, after the following transformations are made:

$$R_1 \equiv R_{LP} + \frac{R_0 [R_B + r_P + (\mu + 1)R_k]}{R_0 + R_B + r_P + (\mu + 1)R_k} \tag{34}$$

$$A \equiv \frac{\mu R_0}{R_0 + R_B + r_P + (\mu + 1)R_k} \tag{35}$$

The transfer function of the network of Fig. 10 is given by

$$\begin{aligned} K(j\gamma) &= \frac{e_o}{e_i} \\ &= \frac{A}{\left(1 + \frac{R_1}{R_2} \right) \left(1 + j \frac{Q_1 + Q_2}{1 + r^2 Q_1 Q_2} \gamma - \frac{Q_1 Q_2}{1 + r^2 Q_1 Q_2} \gamma^2 \right)} \end{aligned} \tag{36}$$

where $r^2 = L_g/L_P$ and Q_1 and Q_2 are as usually defined and refer to z_1 and z_2 , respectively (Fig. 10).

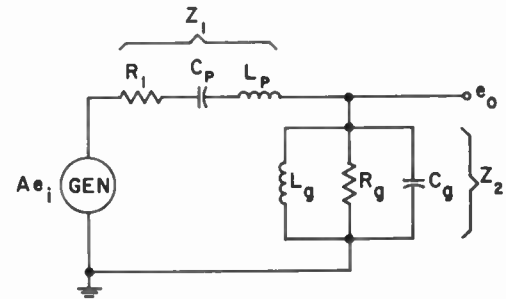


Fig. 10—Equivalent circuit of the network of Fig. 7(b).

Now equate the coefficients of (33) and (36), namely

$$\begin{aligned} \frac{1}{Q_L} \frac{f_0}{BW} &= \frac{Q_1 + Q_2}{1 + r^2 Q_1 Q_2} \\ \text{and } \alpha \left(\frac{f_0}{BW} \right)^2 &= \frac{Q_1 Q_2}{1 + r^2 Q_1 Q_2} \end{aligned} \tag{37}$$

If the parameters of the tube shown in Fig. 7 are those of the 12AY7 at a plate current of 3 mA, the gain per stage at f_0 is taken as 1.1, and the above equations are solved simultaneously, the necessary design equations are:

$$R_k = \frac{0.4544R_0\alpha\left(\frac{f_0}{BW}\right)^2}{L_p/L_g} \tag{38}$$

$$R_B = 43.028R_k - R_0 - 30,860 \tag{39}$$

$$Q_2 = \frac{\left(\frac{f_0}{BW}\right)\left[1 \pm \sqrt{-4\alpha Q_L\left[Q_L - \alpha Q_L \frac{L_g}{L_p}\left(\frac{f_0}{BW}\right)^2\right]}\right]}{2\left[Q_L - \alpha Q_L \frac{L_g}{L_p}\left(\frac{f_0}{BW}\right)^2\right]} \tag{40}$$

$$Q_{LP} = \frac{1}{\frac{Q_2}{\alpha\left(\frac{f_0}{BW}\right)^2} - \frac{Q_2}{L_p/L_g} - \frac{R}{\omega_0 L_p}} \tag{41}$$

where

$$R = \left[1 - \frac{L_p/L_g}{40\alpha\left(\frac{f_0}{BW}\right)^2}\right]R_0 \tag{42}$$

where Q_{LP} is the Q of L_P at f_0 .

f_0 , BW , Q_L , and a are given. We now have 9 unknowns: R_k , R_B , R_0 , L_p , L_g , Q_{LP} , Q_2 , C_p , and C_g . C_p and C_g are chosen so that the tuned circuits containing L_p and L_g resonate at f_0 . Of the 7 remaining unknowns, 3

may be chosen by the designer and the other 4 obtained from (38) through (42).

It is suggested that R_0 , L_p , and L_g be chosen instead of the other variables. L_p and L_g are the most expensive elements while R_0 is an element critically responsible for the gain of the stage. It is possible and desirable, for example, to select the same R_0 , L_p , and L_g throughout the filter (or throughout a bank of filters, as in a spectrum analyzer). The required Q_{LP} and Q_2 of each stage will, in general, be different, but these may easily be adjusted by additional series or parallel resistors.

Contributors

Richard F. Dubbe (S'50-A'54-M'59) was born in Minneapolis, Minn., on January 9, 1929. He received the B.E.E. degree in electrical engineering in 1953 from the University of Minnesota, Minneapolis.



R. F. DUBBE

He served as the chief recording engineer at the Film Productions Company in Minneapolis from 1948 to 1953. Since that time, he has been a technical service engineer for the Magnetic Products Laboratory, Minnesota Mining and Manufacturing Company, where he has been concerned with applications and development of magnetic recording tape. He has presented several papers to the Society of Motion Picture and Television Engineers on laminating magnetic tracks to motion picture film.

Mr. Dubbe is a member of the Society of Motion Picture and Television Engineers.



Hugh K. Dunn was born in Shawneetown, Ill., on October 31, 1897. He received the B.A. degree from Miami University, Oxford, Ohio, in 1918.



H. K. DUNN

After four years on the Miami faculty, he went to California Institute of Technology, Pasadena, where he received the Ph.D. degree in physics in 1925. He then joined Bell Telephone Laboratories in New York.

For a number of years he was engaged in statistical studies of amplitudes and spectra in music and speech, including the distribution of speech in space, and the characteristics of telephone instruments and circuits in terms of real speech. He took part in the early work on the sound spectrograph.

In World War II, he assisted in the development of an acoustic torpedo for use against submarines, but after the war returned to speech studies. He originated the transmission-line analog of the vocal tract, and showed how it leads to a prediction of vowel formant positions. From 1951 to 1957, he was again engaged in military work, and more recently has been concerned with improvement of the artificial larynx.

Dr. Dunn is a Fellow of the Acoustical Society of America and of A.A.A.S., and a member of the American Physical Society, Phi Beta Kappa, and Sigma Xi.

Cyril M. Harris (SM'50) was born in Detroit, Mich., on June 20, 1917. He received the B.A. and M.A. degrees in mathematics and physics, respectively, from the University of California at Los Angeles in 1938 and 1940, and the Ph.D. degree in physics from the Massachusetts Institute of Technology, Cambridge, in 1945.



C. M. HARRIS

In addition to teaching in the Department of Physics at M.I.T. from 1940 to 1945, he engaged in defense research under the NDRC. From 1945 to 1951, he was in the Acoustics Research Department at the Bell Telephone Laboratories. In 1951, he worked with the Office of Naval Research, London Branch, as a Scientific Consultant; following this, he spent the academic year 1951-1952 as a visiting Fulbright Lecturer at the Technical University of Delft, Netherlands. Since 1952, he has been an associate professor of electrical engineering at Columbia University and in charge of acoustics research at the Electronics Research Laboratories.

Dr. Harris is a member of Sigma Xi, Tau Beta Pi, and a Fellow of the Acoustical Society.



Jacob Klapper (A'54-M'56) was born in Ulanow, Poland, on September 17, 1930. Since his arrival in the United States in 1949, he has attended Cooper Union, Brooklyn College, Brooklyn Polytechnic Institute, all in New York, N. Y., and received the B.E.E. degree from the College of the City of New York in 1956, and the M.S. degree in electrical engineering from Columbia University, New York, N. Y., in 1958.



J. KLAPPER

He has held various engineering positions with The Austin Company, The Mark Simpson Manufacturing Company, and others. From 1952 to 1956, he was an electronics engineer with CBS-Columbia working on television and military electronics circuitry. Since 1956, he has been a lecturer in electrical engineering at the College of the City of New York. In May 1957, he joined the Electronics Research Laboratories of Columbia University.

Mr. Klapper is a member of Eta Kappa Nu, Tau Beta Pi and the National Society of Professional Engineers.

Leon J. Sivian was born in Russia in 1894. He received the B.A. degree in electrical engineering at Cornell University, Ithaca, N. Y., in 1916. After a year as instructor at Cornell, he joined the Engineering Department of the Western Electric Company which later became Bell Telephone Laboratories.



L. J. SIVIAN

He engaged in radio research during World War I, and subsequently worked on ship-to-shore telephony. After 1921, his work was mostly in acoustics, especially in the development of a telephone transmission reference system, and methods of microphone calibration. He performed studies on minimum audible sound fields, and his methods of statistical measurements on music have also been applied to speech. His contributions to other problems in acoustics include work on time-delay circuits, diffraction, large amplitudes, sound measurement in liquids, and room acoustics. During World War II he again engaged in defense research, and was associated with the U. S. Navy Radio and Sound Lab. in San Diego, Calif. He died on September 23, 1947.

Mr. Sivian was a Fellow of the Acoustical Society of America, and served on its Executive Council (1939-1942). He was a member of the IRE and was on its Board of Editors. He also served on technical committees of the American Standards Association.



Samuel D. White (A'38-M'44-SM'53) was born December 4, 1905, in New York, N. Y. He received the B.S. degree, with honors, in electrical engineering from Rutgers University, New Brunswick, N. J., in 1927, and the E.E. degree in 1932.



S. D. WHITE

In 1927, he joined Bell Telephone Laboratories where for some years he was engaged in research on acoustic delay systems, measurement of sound pressures in speech and music, hearing thresholds, and instruments and techniques for acoustic measurements. In 1939, he turned to development of switching apparatus but this was interrupted by work on underwater sound devices during World War II. After the war, he continued work on relays, vibrating reed selectors for mobile and microwave radio signaling, packaging of electronic circuits, and classified projects.

Mr. White is a charter member of the Acoustical Society of America, and is also a member of Phi Beta Kappa and Sigma Xi.

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JAMES B. LANSING SOUND, INC., 3249 Casitas Ave., Los Angeles 39, California
Loudspeakers and Transducers of All Types

UNITED TRANSFORMER COMPANY, 150 Varick St., New York, New York
Manufacturers of Transformers, Filters, Chokes, Reactors

UNIVERSITY LOUDSPEAKERS, INC., 80 South Kensico Ave., White Plains, New York
High Fidelity, Commercial-Industrial, and Military Speakers and Accessories

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