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IRE PROFESSIONAL GROUP ON AUDIO

The Professional Group on Audio is an organization, within the framework of the IRE, of members with principal professional interest in Audio Technology. All members of the IRE are eligible for membership in the Group and will receive all Group publications upon payment of an annual fee of $2.00.

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Message from the New Chairman

In his message as incoming *Chairman* one year ago, my predecessor emphasized the importance of our evaluating the needs of the PGA membership and activating a program to fulfill these needs. During the past several months, we have considered carefully various steps that will provide greater participation by our members. Also, we have considered suggestions for expanding activities that will result in greater coverage of the field of audio into those aspects not now being met by this Professional Group, other Professional Groups, or other organizations. Accordingly, we have decided to devote the entire program of the PGA meeting this fall to the topic of audio in military and space-vehicle communication. We have appointed Michel Copel as its Chairman. This meeting, which is to be held jointly with the Acoustical Society of America, presently is scheduled for November 9, 1961. There is every indication that this will be the most successful fall meeting our Professional Group has held yet.

Since the inception of the TRANSACTIONS ON AUDIO, the *Editor* has been assisted by an Editorial Board. This arrangement often has placed a considerable work load on the *Editor*. To remedy this situation and to make use of the considerable talent available in our Group, a change has been made recently—the appointment of a staff of *Associate Editors*. You will be pleased to learn that Marvin Camras has agreed to remain on as the *Editor* for the coming year.

Your *Chairman* is looking forward to a year of increasingly stimulating activity in our Group and the increased activity of its members in PGA affairs. We hope all of you will communicate your suggestions in this regard.

—CYRIL M. HARRIS

The Editor's Corner

PROGRESS IN AUDIO

THE other day my youngster explained how a television set works. "You turn this knob here, and it makes the picture come out of the tubes and wires." He was quite matter-of-fact about it. It was reasonable and logical to him that turning the knob should produce a picture; just as reasonable as the law of gravity and other everyday facts.

I remembered that when I was his age I was fascinated by a miracle called radio. The older kids who had electric-shop in school used to make crystal sets out of oatmeal boxes. They would attach these to long aerials on roofs and trees. The crystals had cats-whiskers which needed a lot of fooling around with until you got a sensitive spot. Then we would take turns wearing the earphones, and exclaiming to each other how crystal-clear it sounded.

We used to visit our wealthy uncle, who had just bought an RCA Radiola superheterodyne. He showed us how the tubes lit up; he could make them brighter with a rheostat, but you were supposed to keep them dim to save the battery. They were 199 or "peanut tubes" and cost about five dollars apiece, and could burn out if you weren't careful. I remember the strong phenol odor of bakelite when he opened the cabinet doors. People associated this with new radios, in the same way that the inside of a new automobile "smells new."

Grownups spent entire evenings talking about their radio sets, and the dx they could tune in, and how someone on their block stayed up all night and got London. One night a week was "silent night," when the local radio stations would be off, and everyone would light his tubes as bright as he dared to try for distant reception.

My uncle got tired of buying batteries all the time, so he replaced his set with an Atwater-Kent that had five 201A tubes, powered by a storage battery with a trickle charger and a Balkite B-Eliminator. (You could choose RCA 201A's or Cunningham 301A's for the same price.) Eventually I got the old superheterodyne, but I didn't have any money for batteries. I wanted to use the parts, but here too I was baffled because the whole works was potted in a solid block of resin. It did give me a lifetime supply of solder flux, however.

High-fidelity lifelike natural sound quality of those days was attained with a horn-type Utah loudspeaker driven by a 201A in the audio output stage. If you wanted something really advanced you could get a power tube, the 112A, for the output socket. I remember when we tried it; to me it sounded worse, but the grownups kept changing back and forth and saying that a real musician could feel the difference.

The experts didn't like small horn loudspeakers, and they would add a large paper cone that was vibrated by a speaker unit at the center, attached to the cone with a thin wire. Medium-sized cone speakers up to two feet in diameter were on a base which could be set on top of the radio. The jumbo sizes of four or five feet diameter were designed to hang on a wall. Kits for building your own cone were very popular. Those who had the elaborate large-size homemade rigs felt quite secure for a long time. They would listen condescendingly to a store-bought outfit of their friends, remark politely that it sounded exceptionally good for a commercial job, and go home firmly convinced of their own superiority.

There was a slight cause for worry, though, because the radio industry seemed to be moving too fast for its own good. It didn't stick with time-tested neutrodyne and superheterodyne circuits and with the familiar tubes, but was trying such things as ac on the filaments, which was unnatural because hum was put right into the most sensitive part of the set. Also, the number of tube types was getting out of hand—226, 227A, 171A, the 280 rectifier, the 224A screen grid, and the super-power tubes such as the 250, the 210, and the 281—actually transmitting tubes, used in push-pull, besides! How could one engineer or serviceman be expected to keep track of all these?

The real revolution was caused by the dynamic loudspeaker. Commercial versions were so much better than anything previously available that even the die-hards had to change over, albeit grudgingly. Many threw up their hands in despair as life became more complicated by power pentodes and duplex-diode-triode detectors.

These innovations were for the younger generation who were born into the confusing world; who didn't seem bothered by literally dozens of tubes; and who even acted as if they understood modern radio. We old-timers felt that it was at best a superficial, imperfect understanding. They could never really know and grasp the true meaning of the subject as we did in the old days of radio.

MARVIN CAMRAS, *Editor*

PGA News

NATIONAL OFFICERS OF THE PGA, 1961–1962

C. M. Harris
Chairman, 1961–1962

H. E. Roys
Vice Chairman, 1961–1962

Cyril M. Harris (SM'50–F'61) was born in Detroit, Mich., on June 20, 1917. He received the B.A. degree in mathematics in 1938 from the University of California, Berkeley, and the M.A. and Ph.D. degrees in physics in 1940 and 1945, respectively, from the Massachusetts Institute of Technology, Cambridge.

In 1939 he joined the staff of the University of California at Los Angeles as a Teaching Assistant. He then did research at M.I.T. from 1940 to 1941, and war research at Carnegie Institute in Washington, D. C. He did further war research under NDRC Division 17.3 from 1941 to 1945 at M.I.T., where he also worked as a Teaching Fellow.

He was employed by Bell Telephone Laboratories as a Research Engineer from 1945 to 1951. In that year he became affiliated with the Office of Naval Research, London Branch, England, as a Scientific Consultant. He has participated in the Fulbright Exchange Program twice, from 1951 to 1952, as a Visiting Lecturer to the University of Delft, The Netherlands, and in 1960, as Visiting Professor to the University of Tokyo, Japan. Since 1952 he has been Associate Professor of Electrical Engineering at Columbia University, New York, N. Y.

His activities include Vice President of the Acoustical Society of America, Editorial Board of *Physics Today*, published by the American Institute of Physics, Associate Editor of the *Journal of the Acoustical Society of America*, and Past Chairman of the IRE Professional Group on Ultrasonics Engineering. He is also noted for the many books and papers he has written in the field of audio.

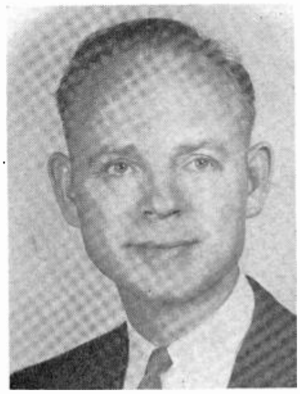
Dr. Harris is a Fellow of the Acoustical Society of America and the Audio Engineering Society. He is a member of the Physical Society of London, Acoustics Group, Groupment des Acousticiens de Lange Francais, Sigma Xi, and Tau Beta Pi.

H. E. Roys (A'27–SM'47–F'55) was born in Beaver Falls, Pa., on January 7, 1902. He received the B.S.E.E. degree from the University of Colorado, Boulder, in 1925.

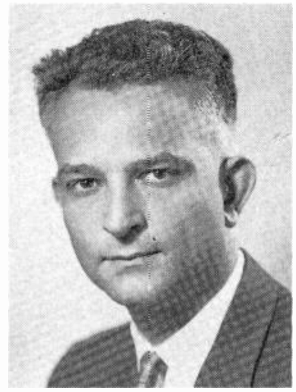
From 1925 to 1930 he was employed by the General Electric Company, Schenectady, N. Y. He has been employed by the Radio Corporation of America since 1930. He began at the Camden Branch doing development work, mainly on phonographs. He relocated in 1941 to the Indianapolis Branch, where he stayed until 1946. While there he did development work on disk and magnetic recording. He returned to Camden, N. J., in 1946 and continued development on recording, including data processing and video recording on magnetic tape. At present he is Manager of Record Engineering at RCA Victor Record Division, Indianapolis, Ind.

He has served as Chairman of the IRE Recording and Reproducing Committee, and is presently Chairman of EIA P-8 Sound System Components Committee and the ASA Section Committee Z-57 on Sound Recording. He has written numerous papers on disk recording.

Mr. Roys is a Fellow of the Audio Engineering Society and the Acoustical Society of America. He is a recipient of the PGA Achievement Award and the Emile Berliner Award.



D. E. Brinkerhoff
Administrative Committee, 1961–1964



F. A. Comerci
Administrative Committee, 1961–1964

W. C. Wayne
Administrative Committee, 1961–1963

Donald E. Brinkerhoff (A'54–M'60) was born in Bryant, Ind., on December 6, 1921. He received the B.S. degree in electrical engineering from Purdue University, Lafayette, Ind., in 1943. He has also taken graduate work in mathematics and physics under the Purdue University Off-Campus Graduate Program at Delco Radio.

During World War II he served as an Officer with the U. S. Army Signal Corps. After completing the Army Officers Electronics School at Harvard University and the Massachusetts Institute of Technology Radar School, both in Cambridge, he was appointed Instructor in Electrical Communication Engineering at the M.I.T. Radar School. He also served as a member of the Army Ground Force Board II where he was responsible for new radar equipment evaluation.

In 1945 he joined Delco Radio, as a specifications engineer, and in 1952 he assumed his present position of Engineer in Charge of the Acoustical Engineering Laboratory, Delco Radio Division, General Motors Corporation, Kokomo, Ind. His responsibilities include loudspeaker and audio-acoustical system design and development.

Mr. Brinkerhoff is a member of the Acoustical Society of America, the Audio Engineering Society, and the Kokomo Engineering Society. He is Chairman of the Subcommittee on Automotive Acoustics under the Committee on Electroacoustics and Audio Devices in the Acoustical Society of America.

Frank A. Comerci (SM'55) was born in Newark, N. J., on January 18, 1920. He received the B.S.E.E. degree from Newark College of Engineering, in 1943.

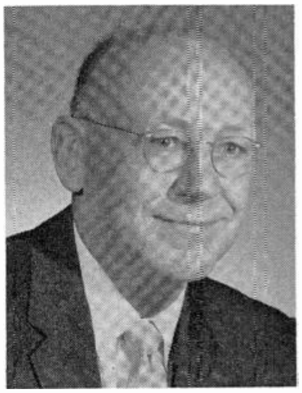
From 1943 to 1946 he served in the U. S. Army as a Communications Officer, installing and maintaining cryptographic speech communications systems. He joined the Rangertone Corporation in 1946, where he worked on the design of the first high-quality magnetic-tape recorder built in the United States. In 1947 he became affiliated with the Navy Material Laboratory, Brooklyn, N. Y., and was in charge of their Acoustics and Communications Section from 1950 to 1959. He was later employed by Audio Devices, Inc., Glen Brook, Conn., as Senior Electronic Engineer. At present he is Manager of the Magnetics Department of CBS Laboratories, Stamford, Conn.

Mr. Comerci is a member of the Acoustical Society of America and the Audio Engineering Society, serving on the Editorial Board of the *Journal of the Audio Engineering Society* for several years.

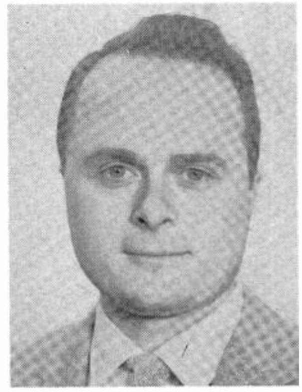
William C. Wayne (A'52–M'57) was born in Edwardsville, Ill., on May 10, 1927. He received the B.S. degree in electrical engineering from the University of Illinois, Urbana, in 1950, and is pursuing graduate studies at the University of Cincinnati, Ohio.

In 1950 he joined the Engineering Department of the Baldwin Piano Company, Cincinnati, Ohio. His duties in his present job as Supervisory Research Engineer concern advanced research on the Baldwin electric organ.

Mr. Wayne is a member of the Acoustical Society of America, the Audio Engineering Society, and the American Guild of Organists. He is Past Chairman of the Cincinnati PGA Chapter.



W. B. Snow
Achievement Award, 1960



D. F. Eldridge
PGA Senior Award, 1960

W. D. Roehr
PGA Award, 1960

William B. Snow (A'26–VA'39–SM'50) was born in San Francisco, Calif., on May 16, 1903. He received the B.S. and E.E. degrees from Stanford University, Calif., in 1923 and 1925, respectively.

From 1925 to 1941 he was employed by Bell Telephone Laboratories, Inc., New York, N. Y., first working on the development of speech testing methods for telephone equipment, determining the effects of distortion on wire transmission, and measuring characteristics of hearing. Bell Laboratories carried out a thorough study of very-high-fidelity transmission, recording, and reproduction of stereophonic sound from 1931 through 1941. He was active throughout this program, being responsible for field installations, tests, and demonstrations, as well as considerable circuit design. During the years 1941 to 1945 he was granted a leave of absence in order to join the U. S. Navy Underwater Sound Laboratory, New London, Conn., operated by Columbia University Division of War Research. Here he headed the Technical Services Department and was active in the scientific work, especially in tests of transmission and detection, noise measurement, and electronic design; he was made Assistant Director in 1943. He joined the Kellex Corporation later Vitro Corporation of America, in 1946; in 1950 he became Director of Physical Research and Development. While there he worked on a variety of classified government projects. In 1952 he established a consulting practice in acoustics and electronics in Santa Monica, Calif. During 1960 he was at Ramo-Wooldridge working on sonar problems. Since 1961 he has been with Bissett-Berman Corporation, Santa Monica, Calif., where his activities center around acoustics and electronic system design. He holds four patents in the field of stereophonic-sound reproduction, and is the author of many papers on electronic and acoustical subjects.

Mr. Snow is a Fellow of the Acoustical Society of America and the Audio Engineering Society, and a member of AIEE, SMPTE, and AAAS. For his wartime activities he received the Army-Navy Certificate of Appreciation. In 1956 he received the PGA Senior Award for his paper on stereophonic sound.

Donald F. Eldridge (A'50–M'55–SM'60) was born in Passaic, N. J., on January 30, 1929. He received the B.S.E.E. degree from Lehigh University, Bethlehem, Pa., in 1949.

He then joined the Boeing Airplane Company, Seattle, Wash., where he engaged in work covering many phases of dynamic data acquisition and reduction. In 1956 he became affiliated with the Research Division of Ampex Corporation, Redwood City, Calif., where he did research on many aspects of magnetic recording. His last position there was as head of the Magnetics Department of the Ampex Corporate Research Division, from which he resigned in December, 1960. He is presently Vice President and Technical Director of Memorex Corporation, Palo Alto, Calif.

William D. Roehr (S'57–M'58) was born in San Jose, Calif., June 11, 1930. From September, 1950, until July, 1954, he served in the United States Navy as an electronics technician. Upon completion of his military service, he attended San Jose State College, Calif., and received the B.S. degree in electrical engineering in 1957.

While in school, he worked during his senior year as an electronics technician at Stanford Research Institute, Menlo Park, Calif. In July, 1957, he came to Motorola Inc., Semiconductor Products Division, Phoenix, Ariz., as an applications engineer.

PGA AWARDS FOR 1960

The Awards Committee of the Professional Group on Audio has announced the award winners for the year 1960.

IRE-PGA Achievement Award

William B. Snow—To honor a member of the PGA, who, over a period of years, has made outstanding contributions to audio technology documented by papers in IRE publications. A certificate and $200 award have been presented.

IRE-PGA Senior Award

Donald F. Eldridge—For the paper "Magnetic Recording and Reproduction of Pulses" which appeared in IRE TRANSACTIONS ON AUDIO, vol. AU-8, pp. 42–57; March-April, 1960. A certificate and $100 award have been presented.

IRE-PGA Award

William D. Roehr—For his papers "A Two-Watt Transistor Audio Amplifier," vol. AU-7, pp. 125–128; September-October, 1959, and "Characteristics of Degenerative Amplifiers Having a Base-Emitter Shunt Impedance," vol. AU-7, pp. 165–169; November-December, 1959, which appeared in IRE TRANSACTIONS ON AUDIO. A certificate and $100 award have been presented.

CHAPTER NEWS

Chicago

"Transient Distortion in Loudspeakers," by Robert J. Larson and Anthony J. Adducci of Jensen Manufacturing Co., Chicago, Ill., was presented at the March 10, 1961, meeting at the Western Society of Engineers Bldg. in Chicago, Ill. A summary of their talk appeared in the March issue of *Scanfax:*

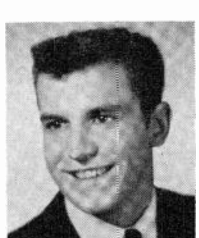
The reponse of a loudspeaker to non-recurring sudden changes in the input signal level will be discussed, together with various ways to measure such transient response.

Waveforms of loudspeaker response to various input signals and a method of plotting a continuous transient response curve will be shown. The authors also will demonstrate some of the psychoacoustic factors involved in listening tests.

Robert J. Larson, chairman of the 1961 IRE Solid-State Lecture Series and past chairman of the Chicago PGA, was educated at Northwestern University, graduating in 1951 with the B.S. degree in electrical engineering. After serving in the Navy during the Korean War, he returned to Jensen where he now is senior development engineer in charge of loudspeaker development.

Co-author Anthony J. Adducci received the B.S. degree in physics in 1959 from St. Mary's College in Winona, Minn. He joined Jensen about a year ago and has been working in the loudspeaker development department.

R. J. LARSON



A. J. ADDUCCI

Milwaukee

"An Improvement in Simulated Three-Channel Stereo," was presented by Peter W. Tappan, Senior Project Electrical Engineer of Warwick Mfg. Co., Chicago, Ill., on March 14, 1961, at the ESM Bldg., 3112 W. Highland Blvd., Milwaukee, Wis.

"The How, Why, and What of Stereo," was the subject of the April 11, 1961, meeting. The speaker was Eugene Carrington, Educational Director of Allied Radio Corp., Chicago, Ill.

SPEECH COMMUNICATION EQUIPMENT SESSIONS SCHEDULED FOR ACOUSTICAL SOCIETY OF AMERICA FALL MEETING

The IRE Professional Group on Audio has cooperated with the Technical Committees on Electroacoustics and Speech Communication of the Acoustical Society of America in arranging a special group of sessions on Speech Communication Equipment at the fall meeting of the Acoustical Society of America. The meeting will be held at the Netherland Hilton Hotel, Cincinnati, Ohio, November 8–11, 1961. A round trip by chartered bus is scheduled for November 9 to Wright Field, Dayton, Ohio, for the special group of sessions on Communication Equipment. Other Acoustical Society sessions of particular interest to PGA members are a Wednesday morning session on Music and Electroacoustics, a Friday morning session on Speech Devices and Tests and a Saturcay morning session on Speech Characteristics. There will also be sessions on Psychoacoustics and on Semiconductor Transducers.

For this meeting the regular Acoustical Society member registration fee of two dollars will apply also to PGA members, instead of the non-member registration fee. Registration both for the meeting and for the trip to Wright-Patterson Air Force Base will take place in the fourth floor foyer of the hotel beginning at 8:30 A.M., Wednesday, November 8. This will make it possible for an adequate number of buses to be reserved. Participants arriving at WPAFB separately can register there by reporting to the reception center, Area B, but individual registration procedures are expected to be time consuming. No hotel commitment has been made by the PGA, members wishing to attend should make their own arrangements for housing well in advance.

The program for the sessions on Speech Communication Equipment follows:

Thursday, November 9

10:00 A.M. Aeronautical Systems Division Auditorium, Building 680T

Joint Session G: Communication (in cooperation with PGA).

Cyril Harris, *Chairman*

Opening Welcome.

Invited Papers (25 minutes)

G1 **Non-Acoustical Means of Communication,** John R. Pierce, Bell Telephone Labs., Inc., Murray Hill, N. J.

G2 **Voice Communication in Aircraft and Aerospace Systems,** Paul S. Veneklasen, Western Electro-Acoustic Lab., Inc., Los Angeles, Calif.

11:00 A.M. Aeronautical Research Laboratory Auditorium, Building 450

Session H: Speech Communication Equipment (in cooperation with PGA).

B. B. Bauer, *Chairman*

Symposium Papers (20 minutes)

H1 **General Layout and Performance Characteristics of a Military Voice Communication System.**

H2 **Transducers for Voice Communication,** William B. Snow, Bissett-Berman Corp., Santa Monica, Calif.

H3 **Noise Environment and Control, Evaluation of Projection and Reception Systems,** Paul S. Veneklasen, Western Electro-Acoustic Lab., Los Angeles, Calif.

H4 **Transducer Developments, Characteristics, Prospects,**

(*Continued on page 132*)

More on Nonlinear Distortion Correction*

J. ROSS MACDONALD†, FELLOW, IRE

Summary—Further consideration is given to basic amplitude limitations which may apply to the complementary distortion method of nonlinear distortion correction. It is found, in disagreement with others, that points at which the differential gain is zero or infinite do not limit the amplitude over which complete correction is possible but that relative maxima, minima, gain zeros, and infinite-gain points in the characteristic do set limitations when the usual simply connected tandem configuration is employed. When the characteristic to be corrected is multiple valued or passes through points of zero or infinite gain within a given amplitude range, a multiply connected correction circuit must be used for perfect correction of distortion over the amplitude range in question.

THERE has recently been a certain amount of controversy concerning amplitude limitations in the complementary distortion method of nonlinear distortion reduction. Two such limitations, which will be further discussed herein, were pointed out in the original paper[1] and Waldhauer[2] later suggested a specific configuration for complete distortion correction which is stated to be limited to the amplitude range over which $|de_1/de_0|>0$, where $e_0=A\cos\omega t$ is an input signal and $e_1\equiv f(e_0)$ is the output signal obtained when e_0 is applied to the input of a predistortion network which is to correct the distortion of a given black box whose input is e_1 and whose output, as shown in Fig. 1(a), is $e_2\equiv g(e_1)$. Perfect distortion correction only occurs when $e_2=Ke_0=g\{f(e_0)\}$, where K is the over-all amplification factor. As pointed out by Pritchard,[3] perfect correction is only achieved in Waldhauer's configuration provided the two amplifiers he uses are assumed to have zero input and infinite output impedance respectively. These conditions, which cannot be met in practice over a nonzero amplitude range, can still be well approximated over a limited range. Within this range, the important advantage of Waldhauer's approach is that the same elements and circuit configuration appearing in the black box whose characteristic is to be corrected appear also in the pre- or postdistortion correcting network.

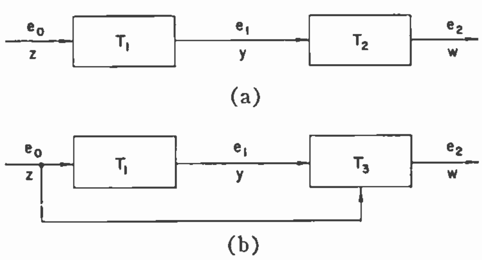


Fig. 1—(a) Usual configuration for correcting nonlinear distortion by complementary distortion. z, y, and w are normalized signal variables. (b) A multiply connected configuration for correcting nonlinear distortion generated in the left-hand circuit.

In the rest of this paper, we shall be concerned with amplitude restrictions for complete distortion correction. In the limit of complete correction, the distinction between pre- and postdistortion vanishes.[2,3] Therefore, we shall consider two nonlinear black boxes connected in tandem as shown in Fig. 1(a) and shall make no distinction between which represents the correcting circuit and which the circuit to be corrected. No significant generality will be lost if we take $K=1$, making the final output equal to the input when complete correction is achieved. In the simplest case, the transfer functions $e_1/e_0\equiv T_1=f(e_0)/e_0$ and $e_2/e_1\equiv T_2=g(e_1)/e_1$ may be considered as real, single-valued operators which operate on a single input to give a single output. Then, the condition

$$T_1T_2 = I \tag{1}$$

where I is the identity operator, leads to complete correction. In this case, the boxes may clearly be interchanged and $T_2T_1=I$ as well. Thus, the operators commute, a result which may also readily be established formally.

In the latter part of the Appendix of the author's paper,[1] a method of complete distortion correction was described which depends on the condition $T_1T_2=I$. This method was later generalized by Waldhauer[2] and has been recently mentioned again by Holbrook and Todosiev.[4] Further discussion of any amplitude restriction applying to this method is needed since the conclusions in the author's paper,[1] those of Waldhauer, and those of Holbrook and Todosiev are inconsistent with each other in some cases. Note that this method is more general than, and is distinct from, Waldhauer's approximate configuration for obtaining response inversion.

In order to discuss amplitude limitations, it will be convenient to consider various classes of transfer functions for the left black box and to ask over what input-signal amplitude range the nonlinearity introduced by

* Received by the PGA, March 16, 1961; revised manuscript received, April 3, 1961.

† Texas Instruments Incorporated, Dallas, Texas.

[1] J. R. Macdonald, "Nonlinear distortion reduction by complementary distortion," IRE TRANS. ON AUDIO, vol. AU-7, pp. 128–133; September–October, 1959.

[2] F. D. Waldhauer, "Comments on 'nonlinear distortion reduction by complementary distortion,'" IRE TRANS. ON AUDIO (Correspondence), vol. AU-8, p. 103; May–June, 1960.

[3] J. R. Macdonald, "Reply to comments on 'nonlinear distortion reduction by complementary distortion,'" IRE TRANS. ON AUDIO (Correspondence), vol. AU--8, pp. 104–105; May–June, 1960.

[4] G. W. Holbrook and E. P. Todosiev, "Amplitude limitations in nonlinear distortion correction," IRE TRANS. ON AUDIO (Correspondence), vol. AU-8, p. 235; November–December, 1960.

the left box of Fig. 1(a) can be completely corrected by the right box. Let $e_1 = a_1e_0 + a_2e_0^2 + a_3e_0^3$, a sufficiently general expression for illustrative purposes. For simplicity, we shall restrict attention to the upper right-hand e_1–e_0 quadrant; thus e_1 and e_0 will both be positive. The behavior in the other quadrants can be easily specified in practical cases. The above expression for e_1 is, in general, asymmetric. If a nonlinear push-pull characteristic is to be represented, e_1 must be made an odd or antisymmetric function of e_0, while a symmetric or even dependence of e_1 on e_0 would give a kind of rectifier characteristic. For a nonlinear amplifier, the over-all response characteristic will fall entirely in the first and third quadrants in cases of practical importance. If the curve is antisymmetric in e_0, the following treatment for a given curve in quadrant 1 can be applied without a significant change to quadrant 3. If the curve is asymmetric, however, the amplitude limitations (if any) which follow from the first and third quadrant responses may be different. Since the input signal is assumed sinusoidal, and hence changes sign, the largest input amplitude which can still be used with complete distortion correction will be determined by the smaller of the two amplitude limitations provided the input ac signal is zero-biased.

It is desirable to express the e_1 vs e_0 characteristic in terms of normalized variables. First, take $a_1 = 1$, consistent with the choice $K = 1$ for the over-all gain. Then, let $a_2 = \epsilon a$; $\epsilon = \operatorname{sgn} a_2$, the sign of a_2. Define $z \equiv ae_0$, $y \equiv ae_1$, $w \equiv ae_2$, and let $a_3/a^2 \equiv \delta$. Then the cubic characteristic becomes

$$y = z + \epsilon z^2 + \delta z^3. \tag{2}$$

A number of cases of this characteristic are plotted in Fig. 2 and will be discussed in terms of amplitude limitations.

In Fig. 2(a), δ has been set to zero and ϵ taken as $+1$. The resulting square-law-distortion characteristic will be completely linearized if the normalized output, $w(y)$ of the second black box in Fig. 1(a) is identically equal to z, the normalized input to the system. When this is the case, the right box has an inverse or complementary characteristic to the left one and the solution of the quadratic yields the single-valued response

$$w = z = \tfrac{1}{2}[\sqrt{1+4y} - 1]. \tag{3}$$

Substitution of $y = z + z^2$ yields an identity as it should. Here there is no ideal, or mathematical, input amplitude limitation when the right side of (3) is synthesized exactly. In practice, the inversion of $y = z + z^2$ cannot be carried out exactly, and Holbrook and Todosiev's[4] reference to *merely* obtaining the inverse is an undue simplification. Waldhauer's[2] specific configuration for distortion correction mentioned earlier represents a useful method of achieving approximate inversion, but it will only be approximate in any real circuit. Thus, in

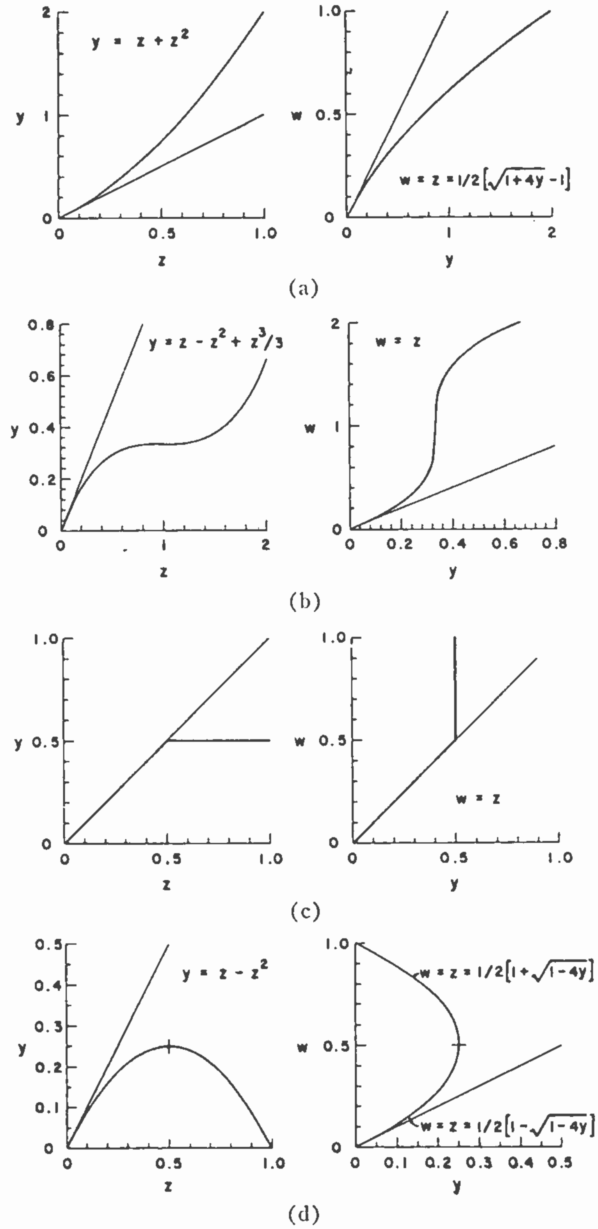


Fig. 2—Various illustrative nonlinear response functions. For each part, the right-hand curve shows the response complementary to that of the left-hand curve.

the present case, there are no mathematical amplitude limitations but there will usually be practical physical ones.[3,4]

In the treatment of the present case in the latter part of the Appendix of reference 1 it was stated that complete correction could be obtained if the characteristic (3) were realized using an analog computer, a general term for passive and active components. Since the Waldhauer method of approximate inversion requires an additional circuit having exactly the same nonlinear response as that to be corrected, it will not be appropriate in all cases.[3] Usually, given a nonlinear characteristic in terms of a e_1–e_0 response curve or its power-law representation, one must synthesize its inverse, such as that in (3), using passive and possibly active components. Such synthesis will often be difficult, but there are no intrinsic physical or mathematical prohibitions to the synthesis of a characteristic such as that in (3). Thus, while power and voltage ratings are always limited in practice, basic laws such as the second law of thermodynamics are not contravened by the synthesis.

Fig. 2(b) shows curves a degree more complicated. Here $\epsilon=-1$ and $\delta=1/3$. Both the direct and inverse characteristics are shown, and it will be noted that the form of the direct characteristic has been selected to give an inflection point in the $y-z$ curve at $z=1$ where the differential gain, dy/dz, is zero. The exact inverse characteristic correspondingly shows $dw/dy=dz/dy=\infty$ at this one point. This result does not require that the gain e_2/e_1 of the right black box be infinite at this point, as Waldhauer[2] has stated, but only that the differential gain be infinite at one point, a condition which can be achieved in a practical circuit using active elements. Here again, the inverse characteristic is still single valued, the conditions $d^2y/dz^2=dy/dz=0$ do not define an amplitude limitation, and there are only practical obstacles to the realization of complete correction over an arbitrary amplitude range.

In Fig. 2(c), a characteristic showing complete saturation for $z\geq 1/2$ is depicted. This nonlinearity can only be corrected for $z>1/2$ with infinite direct gain, not differential gain, in the complementary box. There is here a definite mathematical and practical amplitude limitation. Note that complete correction could be achieved, however, with the circuit of Fig. 1(b). Here the transfer operator T_3 is a function of two separate inputs, one of which is the original input e_0. The complete system is multiply connected, not simply connected as it is in the configuration of Fig. 1(a), the only situation originally considered in complementary distortion correction.[1] In many cases of practical interest, the original signal e_0 is unavailable at the second black box, and sequential or tandem correction such as that shown in Fig. 1(a) is the best that is possible.

Finally, Fig. 2(d) presents a case where the complementary or inverse characteristic is multiple valued. Here $\delta=0$ and $\epsilon=-1$. As shown, the inverse characteristic is single valued up to $y=1/4$, a point where again the differential gain de_2/de_1 is infinite. Here the point $(z, y)=(1/2, 1/4)$ is a relative maximum in y rather than an inflection point, and $d^2y/dz^2\neq 0$. In the region $0\leq w=z\leq 1/2$, the inverse characteristic is single valued and given by

$$w = z = \tfrac{1}{2}[1 - \sqrt{1-4y}], \qquad (4)$$

a response which can be realized approximately by Waldhauer's inversion technique or by other methods. For $1/2\leq z\leq 1$, the necessary inverse characteristic for complete correction is

$$w = z = \tfrac{1}{2}[1 + \sqrt{1-4y}]. \qquad (5)$$

In order to achieve perfect correction over the entire range $0\leq z\leq 1$, the right black box must automatically switch its characteristic from (4) to (5) when z passes the branch point 1/2. Such a logical decision requires knowledge of the original variation of z at the input and can only be made with a circuit such as that of Fig. 1(b). In addition, it is evident that the gain e_2/e_1 is infinite when $z=1$; thus, realization of the characteristic (5) itself up to or beyond $z=1$ can only be achieved if a portion of the original input is available at the right-hand box as in Fig. 1(b). We may conclude from Fig. 2(c) and (d) that whenever a relative maximum or minimum occurs in the response function $e_1(e_0)$ or $y(z)$, as in the case of complete saturation or multiple-valued characteristics, perfect correction using the configuration of Fig. 1(a) cannot be achieved for input amplitudes greater than that which yields the first maximum or minimum. In the present case, the amplitude is thus limited to $z\leq 1/2$, equivalent to the condition $x\leq 1/2$ given in the Appendix of reference 1. This is a mathematical and physical or configurational limitation and disagrees with the conclusions of Holbrook and Todosiev.[4] It should also be mentioned that in the Appendix[1] another amplitude limitation, $x<0.207$, was given which applies to the square-law distortion case. This is a purely mathematical limitation on the inversion-of-series method[1] of determining the complementary or inverse characteristic. This method yields an infinite number of correction terms which, practically, must be realized with a finite number of correcting elements. When $x>0.207$, the power series in question does not converge and complete correction of square-law distortion by this method will be impossible.

In summary, we may conclude that, aside from practical considerations, complete correction of a given nonlinear input-output characteristic is possible using a tandem arrangement for an unlimited input amplitude range provided the initial characteristic always either monotonically increases or decreases and no zero or infinite gain points are reached. On the other hand, whenever the characteristic is multiple valued, a simply-connected tandem correcting circuit can yield complete correction only over an amplitude range extending up to the first relative maximum or minimum or zero or infinite gain point of the characteristic. In the multiple-valued case, a multiply connected correction circuit must be used to achieve complete correction. The number of logical decisions (or separate signal paths) which must be made in such a circuit is equal to the number of relative maxima, minima, gain zeros, and infinite-gain points which occur within the amplitude range over which complete correction is desired.

Acknowledgment

The author is grateful to J. P. Pritchard, Jr., for helpful discussions and criticisms.

A Simplified Noise Theory and Its Application to the Design of Low-Noise Amplifiers*

A. E. SANDERSON†, MEMBER, IRE, AND R. G. FULKS†, MEMBER, IRE

Summary—**Any noisy amplifier can be represented by an equivalent noiseless amplifier plus two noise generators either at the input or the output of the amplifier. The choice of two particular noise generators (the equivalent short-circuit noise voltage and the open-circuit noise current) to characterize a noisy amplifier has a number of advantages over the concept of noise figure. The noise generators can easily be measured separately from the source noise, and the optimum source impedance and the noise figure at any source impedance can then be calculated. Since the amplifier noise is measured separately from the source noise, low noise figures can be easily measured. The optimum source impedance equals the quotient of the two noise generators, and the noise figure depends upon their product. Neither feedback nor input impedance is a consideration in determining noise figure and optimum source impedance.**

Several transistor noise diagrams show how the two noise generators are affected by emitter current, collector voltage, and frequency. Noise diagrams can be used to select the most suitable amplifying devices and optimum operating conditions for various applications.

THE design of low-noise amplifiers can be made much simpler than has been previously realized by the use of equivalent short-circuit and open-circuit noise generators as the measure of the *noisiness* of an amplifier, rather than the noise figure as such. This approach has two advantages: the magnitudes of the two equivalent noise inputs can be measured easily, and rigorous formulas for noise figure and optimum source resistance are most concisely expressed in terms of the two noise-generator parameters.

Basic noise theory[1] states that a noisy amplifier can be represented by an equivalent noiseless amplifier plus a constant-current noise generator in parallel with the input, and a constant-voltage noise generator in series with the input (Fig. 1). The magnitudes of these two generators can be determined independently as follows: with the input terminals shorted, e_n is responsible for the entire noise output of the amplifier. To determine the value of e_n, the short-circuit noise output is compared with the output produced by a small known input voltage large enough to mask the noise. To determine i_n, the noise output of the amplifier with the input terminals open-circuited is compared with that produced by a small known current at the input.

The equations in Fig. 2 show that the minimum noise figure of the amplifier depends primarily upon the *product* of e_n and i_n, while the optimum source resistance depends upon the *quotient* of e_n and i_n. This is an important simplification, because the effects of circuit changes, feedback, operating conditions, and other variables upon e_n and i_n are easily assessed, while their effects upon F_0 and R_0 can be obscure.

For example, the bias resistors of a transistor amplifier are in parallel with the input, and can obviously have no effect upon e_n. However, they will increase the value of i_n, and this will raise F_0 and lower R_0. Resistors in parallel with the input must be large with respect to R_0 (not necessarily with respect to the input impedance) so that their effect on F_0 will be negligible. Resistors in series with the input must be small with respect to R_0 for the same reason.

To specify the noise performance of the amplifier exactly, values are needed for γ (the correlation coefficient between the e_n and i_n generators) and for X_0, the reactive part of the optimum source impedance. To determine X_0, a curve of e_n vs source reactance may be plotted; X_0 then equals the source reactance that gives minimum e_n. To determine γ, noise figure need be measured only once, at the optimum source impedance; then γ is the only remaining unknown in the equation for F_0. However, X_0 is usually negligible at low frequencies, and γ is bounded $(0 \leq \gamma \leq 1)$ and usually lies near 1, so that the upper limit on F_0 (calculated as-

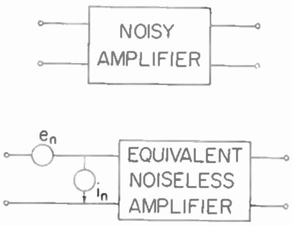


Fig. 1—A noisy amplifier can be represented by an equivalent noiseless amplifier plus a constant-voltage noise generator in series with the input and a constant-current generator in parallel with the input.

$$|Z_0| = \frac{e_n}{i_n}$$

$$F_0 = 1 + (1+\gamma)\frac{e_n\, i_n}{2kT\Delta f}\cdot\frac{R_0}{|Z_0|}$$

USUALLY $X_0 = 0$ AND THEN $|Z_0| = R_0$:

$$R_0 = \frac{e_n}{i_n}$$

$$F_0 = 1 + (1+\gamma)\frac{e_n\, i_n}{2kT\Delta f}$$

Fig. 2—Minimum noise figure F_0 and optimum source impedance $Z_0 = R_0 + jX_0$ can be written concisely in terms of the equivalent noise generators. The correlation coefficient between the two generators γ lies between 0 and 1.

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† General Radio Co., West Concord, Mass.

[1] H. A. Haus, *et al.*, "Representation of noise in linear twoports," PROC. IRE, vol. 48, pp. 69–74; January, 1960.

suming $X_0=0$ and $\gamma=1$) is quite accurate, and may actually exceed the accuracy with which the noise figure may be measured at very low values of F_0.

Both e_n and i_n are independent of feedback, and may be taken outside of the feedback loop with no change in value. The proof is as follows: current feedback to the input obviously will not affect e_n because it will not change the voltage gain of the amplifier or the noise output with the input short-circuited. Since the i_n generator is outside of a current feedback loop, it remains unchanged. An analogous proof shows that e_n and i_n are independent of voltage feedback as well. Since the noise generators are independent of feedback, so also are F_0 and R_0. Also the noise generators are approximately the same for a given device in any of the three amplifier configurations. The generators appear to be a property of the device and independent of the way it is used.

At the optimum source resistance both noise generators contribute equally to the noise output of the amplifier. At other source resistances the noise figure is given by

$$F = 1 + \frac{1}{4kT\Delta f}\left(i_n{}^2 R_s + \frac{e_n{}^2}{R_s} + 2\gamma e_n i_n\right).$$

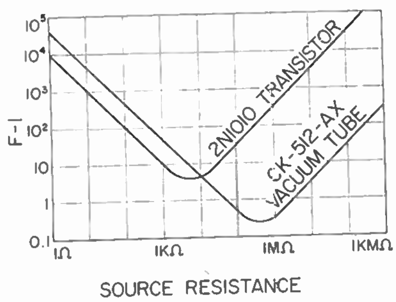

Fig. 3—Comparison of a low-noise transistor with a low-noise vacuum tube. The transistor has a lower noise figure for source resistances below 10,000 ohms, while the vacuum tube is superior for source resistances above 10,000 ohms.

This dependence of noise figure on source resistance is illustrated in Fig. 3. For source resistance much larger or much smaller than the optimum, the noise figure depends upon only one of the generators, and changes linearly with source resistance. For such source resistances, as well as for all reactive sources, it is more meaningful to rate an amplifier in terms of its noise generators (in μv/cycle$^{1/2}$ and $\mu\mu$a/c$^{1/2}$), since these numbers are independent of source resistance and source temperature. The SNR is the ratio of the signal to the appropriate noise generator when the source impedance is much larger or smaller than e_n/i_n.

Both the e_n and the i_n generators vary widely between vacuum tubes and transistors, among the different types in each category, and somewhat with the operating conditions of a particular device. Since nothing can be done in the circuit to affect the generators, it is important to choose the device whose noise performance is best in the region of the intended source impedance, and then to choose the optimum operating conditions for that source impedance. A convenient way of presenting information about the noise generators is the ***noise diagram*** shown in Figs. 4–7. Both e_n and i_n are plotted

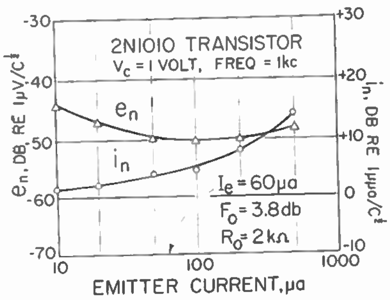

Fig. 4—Noise diagram of a typical low-noise transistor showing e_n and i_n as functions of emitter current. Minimum F_0 occurs at the emitter current which minimizes the product of e_n and i_n.

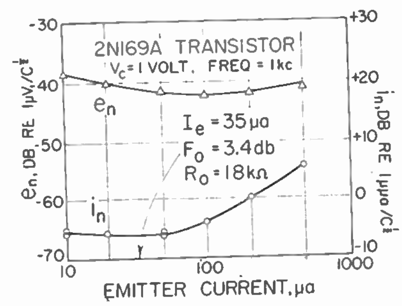

Fig. 5—Noise diagram of a transistor for which F_0 minimizes at a high source resistance. Conventional check of noise figure at 1000-ohm source resistance would not reveal this transistor's low-noise potential.

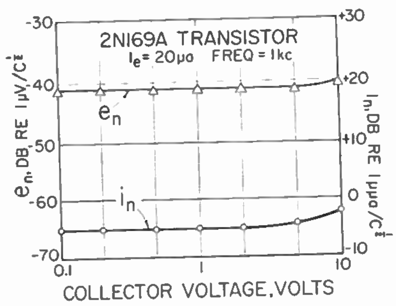

Fig. 6—Noise diagram of a transistor with collector voltage as the independent parameter. Both e_n and i_n seem to be relatively independent of collector voltage up to several volts.

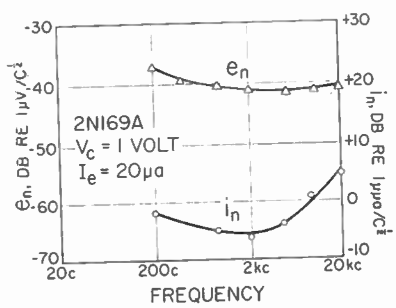

Fig. 7—Noise diagram showing frequency spectrum of the e_n and i_n generators. The two generators do not necessarily have the same frequency spectrum.

on a logarithmic scale against some independent parameter, such as emitter current or collector voltage. Because of the log scale, minimum F_0 is indicated by the minimum *sum* of the e_n and i_n *curves*, while R_0 is proportional to the *difference* between the two curves. A survey of different devices can be made quickly because a minimum of information is required on each device tested.

Manufacturers could perform a service by publishing noise diagrams of their devices for several independent variables such as current, voltage, temperature and frequency, and thus facilitate the choice of the proper amplifying device for each application. The present method of rating by noise figure is at best cumbersome, and can be incomplete when the source resistance at which the noise figure is measured is not both specified and equal to R_0. The general use of noise generators could considerably simplify the representation and application of amplifying devices where noise performance is an important factor.

Average vs RMS Meters for Measuring Noise*

JAMES J. DAVIDSON†, MEMBER, IRE

Summary—**It appears that the controversy is still alive over whether average reading or root-mean-square reading meters should be specified as standard for the measurement of noise. This being the case, it is worthwhile to consider the entire subject from the standpoint of basic fundamentals, to determine what are the significant quantities involved, and then proceed to investigate which type of meter yields the most significant results.**

The following is the result of such an investigation. The entire discussion rests on 1) an axiom, that energy transfer is the fundamental interaction within the universe, and 2) a premise, that for the type of measurements under discussion (audio), all significant processes are linear. Given these two starting points, the conclusion is reached that the meaningful quantities are found by rms measurements.

It is shown further, by concrete example, that measurements made with average reading meters can depart widely from those made with an rms meter. This being the case, it is necessary that *measurement standards* specify the use of rms meters. Those who elect to use average meters, then, bear the responsibility of determining the accuracy of their results in terms of the fundamentally important quantities.

THE FIRST point which must be investigated in any discussion of measurement techniques is, what are the significant factors? Once these are known, it is then possible to determine how best to specify standards of measurement which will yield significant results, and the degree of approximation involved in alternative methods.

The Starting Point

Whenever one considers measurements *of any kind whatever*, the starting point must be the interchange or transfer of energy. The universe runs by energy transfer, and all processes—mechanical, chemical, atomic, electrical, or any other, including communication, imply and require the interchange of energy between one body and another. All measurements, then, have essentially one purpose: to determine the total amount of transferred energy, and/or the *rate* at which it is transferred.

Power

Since to life in general and to human life in particular, time is of the essence, it is usually the rate of energy transfer which assumes predominance. That fact is recognized implicitly by virtue of assigning a special name to the time rate of energy flow, namely power. It must be remembered, however, that the idea of power is arrived at from the fact of energy transfer, and therefore, power is a derivative concept.

To repeat, *power is the time rate of energy transfer*.

In the specific case of electrical phenomena, power is often supposed to be the starting point. It isn't, but provided its origins are kept clearly in mind, it can be convenient to so assume. The reason for the assumption is that "energy pile-up," in the form of heat particularly, is one of the fundamental problems. Further, since energy must be transferred to be useful, and since the rate of transfer determines how much work will be done in a given time, the concept of power becomes a potent tool for judging the *utility* of a given process.

The Measurement of Power

All energy measurements proceed on one common basis; transformation. Whenever energy content or flow is to be determined, some portion of it is converted into another form, the representation of which can be observed by human beings. (Energy, as such, is not di-

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† RCA Victor Record Div., Indianapolis, Ind.

rectly observable; only its effects are.) The most common form of transformation is into mechanical energy with work being done to cause a displacement of a body. When *power* is to be determined, the time rate of displacement (velocity) of the body can be measured. More conveniently still, a restoring force can be added whose force is proportional to displacement (such as a spring), thus giving a static indication of how much energy is present at a given instant. Such is the basis of meter movements.

Now it is possible, for measurement purposes, to transform energy in a number of ways. In general there are two approaches: measuring the *effects* of an increase in energy, or measuring the *causes* of an increase in energy. Thus in a purely mechanical system, for instance, the amount of energy that *was* supplied in the *previous* small time interval can be determined (knowing the mass) by measuring the resulting change in velocity. The amount of energy that *will be* supplied in the *following* small time interval can be determined by measuring the applied force. (The amount of *power being* supplied can, ideally, be found either way, since the time intervals go to zero and, discontinuities excepted, the instant "before" is essentially the same as the instant "after.") Although either method can be used satisfactorily depending on circumstances, it is often important (particularly in the case of power) to keep in mind which one was chosen.

The reason for this is that an "effect" measurement, by its nature, takes into account *all* the causes. It says (if properly performed), "This is what happened." A "cause" measurement, on the other hand, says, "This is what will happen, *provided* nothing else that has not been measured interferes." Thus in the mechanical case, measuring the change in velocity of a body accounts automatically for *all* the forces acting on the body. In measuring forces, however, the experimenter carries the burden of making sure *he* has accounted for all of them.

Now since electricity is a means of *transporting* energy, nearly all electrical measurements fall into the cause category. The observer, then, carries the responsibility of knowing *what* he is measuring in order to obtain *predictable* results.

RMS VS AVERAGE

In Appendix I, the well-known fact is derived that the rms value of a time varying current determines the average rate of energy transport of that current. Whenever significant (*i.e.*, energy or power) measurements are to be made, therefore, it is rms which must be used.[1]

[1] At this point it may be objected that other measurements, peak, or peak-to-peak voltage measurements in particular, are also significant. This is true, but not from the standpoint adopted here. Such measurements are applicable only to the field of nonlinear response, such as dielectric breakdown, etc., and although such phenomena do involve energy transfer and interaction, they are beyond the scope of the immediate discussion. We are here concerned only with *linear* relationships, and it is well to state that point explicitly.

Average reading meters, though, have become widespread throughout the electrical and electronic industries. The sole (albeit important) reason for this is economy. Average meters are comparatively simple, easy to build, rugged, and reliable. It has been suggested that on this basis, average meters already constitute a "de facto" standard, and that, therefore, they should be adopted as *the* measurement standard.

Now the entire justification for the use of average reading meters lies *solely* in the fact that they give a *reasonably close* indication of true rms values under many circumstances. If this were not the case, all their economy, ease of construction, ruggedness, and simplicity would be of academic interest; they would not be used. The admission is explicitly made by the manufacturers of many such instruments, that rms is really the value of importance; the scales are calibrated to *read* the rms value of a sine wave.

The question then becomes, how good is the approximation? Is it good enough, under *all* normal (and enough abnormal) conditions to make the specification of average meters justifiable *in a measurement standard?* To answer correctly, something must be known about performance under varying conditions.

First, one must look at some figures. The rms value of a sine wave is 0.707 times the peak value. The full-wave rectified average is 0.636 times peak. An average reading meter calibrated to indicate the rms value of a sine wave, therefore, adds $0.707/0.636 = 1.11$ times, or 0.91 db to whatever it is measuring.

Consider now the case of random noise.[2] The rms current for a random wave is given the symbol σ_0. Without going through the mathematics, the rectified average current is equal to $0.798\,\sigma_0$. The ratio of rms to average, therefore, is 1.25 times, or 1.96 db. Subtracting the previously mentioned 0.91 db, the average meter reads 1.05 db *lower* than would an rms meter.

Another case of interest (from the signal standpoint) is a square wave. Here the rms and average currents are the same, and equal to the peak current. The 0.91 db factor means, however, that the average meter reads 0.91 db *high*.

Things get worse for more unusual circumstances. On a sine wave with 100 per cent third harmonic, for example, the average meter can read from 0.53 db low to 6.53 db low depending on the phase relation.

In the case of a train of pulses (such as might be found in multiple tics and pops on a record), the average meter gives unduly optimistic results. If the "duty cycle" is 10 per cent, the average meter reads 3.5 db low. If extremely short pulses are measured, for instance a 1 per cent duty cycle, the average meter reads *13.1* db low. These, and a few other examples are summarized in Table I, for waveforms as shown in Figs. 1–5.

[2] This information is taken from: L. L. Beranek, "Acoustic Measurements," John Wiley & Sons, Inc., New York, N. Y., ch. 10; 1949.

TABLE I

Waveform	Duty cycle	i_{rms}	i_{av}	rms meter indicates	Average meter indicates	Average meter error (db)
Sine wave		0.707	0.636	0.707	0.707	0
Sine wave+ 100 per cent 3rd harmonic	0° phase	1.000	0.849	1.000	0.944	− 0.53
	180° phase	1.000	0.425	1.000	0.472	− 6.53
Square wave		1.000	1.000	1.000	1.111	+ 0.91
Random noise		σ_0	$0.798\sigma_0$	σ_0	$0.887\sigma_0$	− 1.05
Pulse train	10%	0.333	0.200	0.333	0.222	− 3.52
	1%	0.995	0.020	0.0995	0.0222	−13.10
	*p%	$\sqrt{\frac{p}{1-p}}$	$2p$	$\sqrt{\frac{p}{1-p}}$	$2.222p$	
Doublet train	10%	0.448	0.200	0.448	0.222	− 6.06
	1%	0.141	0.020	0.141	0.0222	−16.08
	p%	$\sqrt{2p}$	$2p$	$\sqrt{2p}$	$2.22p$	

* See Appendix II.

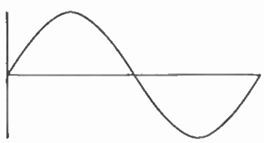
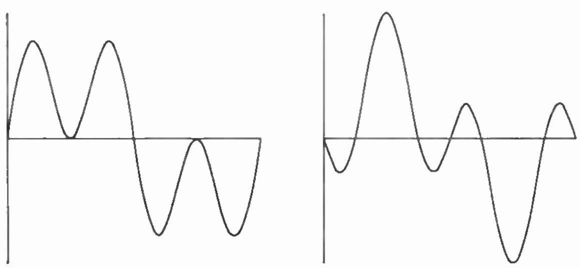


Fig. 1—Sine wave.



(a) (b)

Fig. 2—Sine wave with 100 per cent third harmonic. (a) 0° phase. (b) 180° phase.

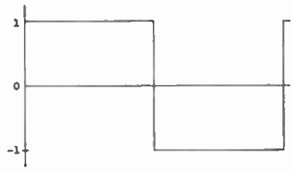


Fig. 3—Square wave.

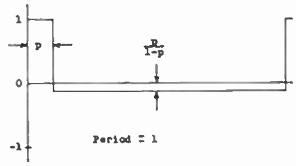


Fig. 4—Pulse train.

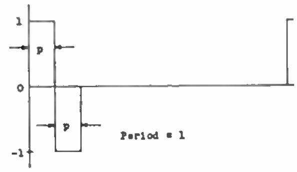


Fig. 5—Doublet train.

Conclusions

From the above data, it should be concluded that average reading meters are not consistent enough to be specified as *the* measurement standard. While such meters are quite adequate for most measurement purposes, the *standards* must always refer back to significant quantities, and the techniques specified should be reliable and unequivocal. The fact that most of the industry is presently equipped with average meters is not sufficient reason for defining measurements whose error is "built in" and variable. If the relationship is known between what the average meter is reading and the true rms value (such as in the case of random noise), then the use of average meters is quite permissible. If it is *not* known, however, it is scarcely proper to write a standard saying, in effect, "The SNR is whatever ratio is measured by this kind of meter." To have meaning, all standards must first be unequivocal, and second must always specify measurement of fundamentally significant quantities. Average reading meters are neither unequivocal, nor do they measure rms.

Appendix I

Derivation of the Effective Value of a Current[3]

Starting from the energy concept, the *potential energy* (PE) of a test charge (q) at a point in an electric field is defined as *the work done against the force exerted on it by the field, when the charge is brought from infinity to the point.* (The potential energy at infinity is defined as zero.)

A related concept, *potential* (abbreviated V), is defined as the ratio of the potential energy of the test charge to the magnitude of the charge, or as *the potential energy per unit charge.*

From these definitions, the potential energy at point a can be written as

$$PE_a = qV_a.$$

Similarly, at point b

$$PE_b = qV_b.$$

The *change* in potential energy in passing from point a to point b

$$\Delta PE = qV_a - qV_b = q(V_a - V_b) = qV_{ab}.$$

If the change in potential energy is transformed to heat (as in a pure resitor), then

$$\Delta PE = H = q(V_a - V_b) = qV_{ab}.$$

Now for a pure resistor ab, through which an infinitesimal charge dq ($=idt$) flows in time dt, the energy given up by the charge in the form of heat

$$dH = dqV_{ab} = idtV_{ab}.\text{[4]}$$

The *rate* of energy dissipation (power) is

$$\frac{dH}{dt} = iV_{ab}.$$

But for a pure resistor; by Ohm's law (not derived)

$$V_{ab} = iR.$$

So

$$\frac{dH}{dt} = i^2R, \qquad \text{or } dH = i^2Rdt \text{ (Joule's law).}$$

Integrating,

$$H_1 = \int_0^t Ri^2dt.$$

Now for a time-varying wave, the instantenous power clearly varies from moment to moment. Nonetheless, it would be very convenient to have a single value which represents the *average rate of energy transport*, regardless of the particular form of time variation. Such a value can be assigned by hypothesizing an unvarying wave with an *effective* value of current, which delivers an equal amount of energy within a specified time. Given such a wave,

$$H_2 = i_{\text{eff}}^2Rt.$$

By the conditions defined, $H_2 = H_1$, and

$$i_{\text{eff}}^2Rt = \int_0^t Ri^2dt.$$

Since R is constant

$$i_{\text{eff}}^2 = \frac{1}{t}\int_0^t i^2dt$$

or

$$i_{\text{eff}} = \sqrt{\frac{1}{t}\int_0^t i^2dt}.$$

Because of the process gone through to find i_{eff} (the square root of the mean value of the square of the current), the effective value of a time varying current is abbreviated i_{rms}.

Appendix II

Derivation of the RMS to Average Ratio for a Pulse Train

For an ac pulse train with duty cycle p/P, maximum amplitude$=1$, and period $P=1$ (Fig. 4).

$$i_{\text{rms}} = \sqrt{\int_0^p dt + \int_p^1 \left(\frac{p}{1-p}\right)^2 dt} = \sqrt{p + \frac{p^2}{1-p}} = \sqrt{\frac{p}{1-p}}$$

and

$$i_{\text{av}} = \int_0^p dt + \left(\frac{p}{1-p}\right)\int_p^1 dt = p + \frac{p}{1-p}(1-p) = 2p.$$

The ratio

$$\frac{i_{\text{rms}}}{i_{\text{av}}} = \sqrt{\frac{1}{4p^2} \times \frac{p}{1-p}} = \frac{1}{2}\sqrt{\frac{1}{p(1-p)}}\,.$$

For very short duty cycles, this reduces to

$$\frac{i_{\text{rms}}}{i_{\text{av}}} = \frac{1}{2\sqrt{p}}\,.$$

[3] After F. W. Sears, "Principles of Physics II," Addison-Wesley Publishing Co., Inc., Reading, Mass.; 1947.

[4] This particular form of energy transformation is chosen purely on the basis of convenience. Since *all* of the potential energy of the moving charge is converted to heat in a pure resistor, the various components of energy are easy to visualize and keep track of. Other types of transformations could be used with equal validity, but at the expense of greater complexity.

An Adjustable Shelf-Type Equalizer with Separate Control of Frequency and Limiting Attenuation or Amplification*

ROBERT H. ROSE†, MEMBER, IRE

Summary—This device provides for control of the high-frequency content of an audio program, allowing the operator to choose independently:

1) the frequency above which equalization is to occur, and
2) the maximum correction in signal strength which results in the range of frequencies under correction.

Three operating controls are used. One control determines whether the high-frequency signals shall be increased, left unchanged, or decreased in strength relative to the low-frequency signals. A second control is calibrated in terms of the frequency above which correction shall occur. The third control adjusts the asymptote which represents the maximum correction which shall occur for frequencies considerably higher than that chosen by the frequency control described above.

The circuit can be described briefly as follows: for high-frequency attenuation, the operation employs a negative feedback amplifier whose output may have high-frequency loss inserted by an RC network following the amplifier; for high-frequency boost, the RC network is inserted in the amplifier feedback path so that the amplifier output (which now becomes the system output) has a rising high-frequency response which corresponds to the attenuation of the RC network. All three controls are continuously adjustable within their ranges of operation.

Introduction

The attenuation function of this system had been realized in a simple unit built several years ago, using the circuit shown in Fig. 1. The output voltage equals the input voltage for all frequencies at which the reactance of the shunt capacitor is appreciably larger

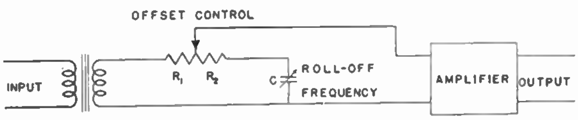


Fig. 1—Treble equalizer.

than the resistance of the series resistor. At higher frequencies, however, the output voltage is attenuated in an amount which increases with frequency, but which cannot exceed a limit set by the voltage divider action of R_1 and R_2. The frequency at which attenuation starts is controlled by adjusting the capacitance, if the total resistance is kept constant, and the maximum amount of attenuation is controlled by the position of the output tap along the resistance.

* Received by the PGA, March 9, 1961. Presented at the IRE Internatl. Conv., New York, N. Y., March 21, 1961.
† Newark College of Engrg., Newark, N. J.

Let us call the maximum attenuation the "offset," and we can easily determine its value as well as define the "rolloff frequency." (When we consider the final equalizer design, we should refer to this frequency as the "inflection frequency.")

$$\text{Offset,} \qquad A = \frac{R_2}{R_1 + R_2}, \tag{1}$$

$$\text{rolloff frequency,} \qquad f_2 = \frac{1}{2\pi C(R_1 + R_2)}. \tag{2}$$

The transmission through the network can readily be expressed in terms of the offset and the ratio of the actual frequency to the rolloff frequency (See Appendix I)

$$\frac{E_{\text{out}}}{E_{\text{in}}} = \frac{1 + jA\dfrac{f_2}{f}}{1 + j\dfrac{f_2}{f}}. \tag{3}$$

This transmission is plotted in Fig. 2, showing the terminology as applied to the characteristic obtained for some arbitrary setting of the controls. When the offset exceeds 10 db, the true half-power frequency is within 10 per cent of the defined rolloff frequency.

The curves in Fig. 3 illustrate the effect of changing the capacitance when the potentiometer setting is kept constant. As long as there is resistance between the slider and the capacitor, the attenuation approaches a constant value as signal frequency increases. Thus, controlled attenuation of high frequencies can be initiated at any reasonable frequency by adjustment of the capacitor.

The effect of the potentiometer, or offset control, for a case in which the rolloff frequency is held constant at 2.5 kc is shown in Fig. 4. The maximum offset produced in this early version of the equalizer was about 22 db.

We may complete the story of this early model with Fig. 5 showing a family of curves for different rolloff frequencies when the equalizer is operated with maximum offset.

Equipment possessing the flexibility shown in the curves in Fig. 5 provides great freedom in the control of the high-frequency spectrum of an audio signal, but restriction of the function to the attenuation mode produces obvious shortcomings.

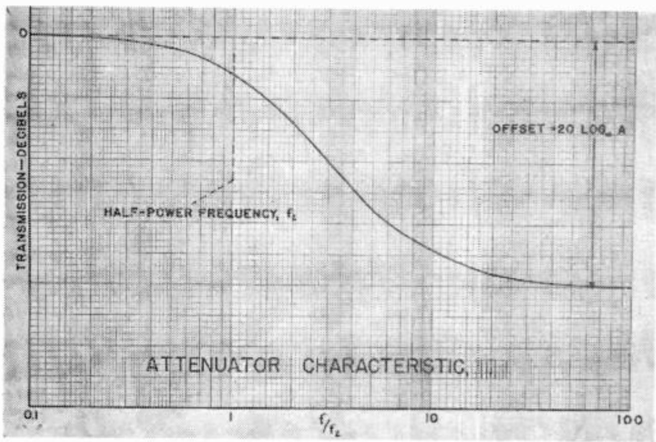


Fig. 2.

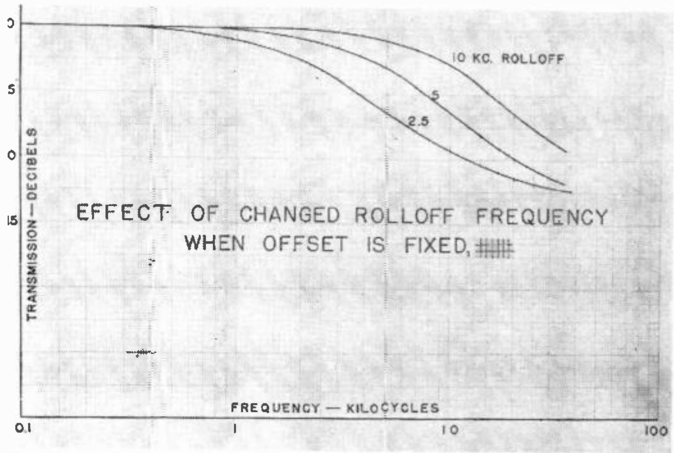


Fig. 3.

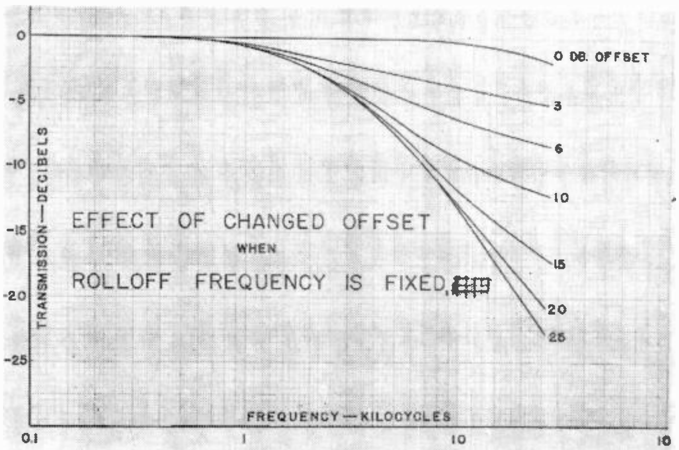


Fig. 4.

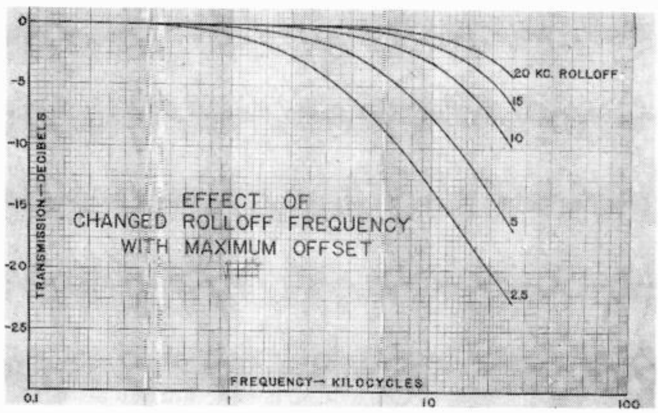


Fig. 5.

Design Objective

Experience with treble-boost equalizers of the resonant-peak variety (about the only type available in professional equipment), led to the feeling that, although such equalizers have a useful function, it would be desirable to have a unit with properties similar to those shown above except that relative boosting of the high frequencies would be provided.

Operational considerations point to a device which can either boost or attenuate the high frequencies. Such a unit should use the same calibrated controls for setting the offset and the inflection frequency in either functional mode. The transition from the attenuation mode through "flat" response and on into the boosting mode should be accomplished smoothly by the use of variable resistances, rather than by the discontinuous steps associated with switches as control devices.

System Concept

The design finally chosen is based on the fact that equalization placed in the negative-feedback loop of a high-gain amplifier produces a system-transfer function which is the inverse of that of the equalizing network.[1] In this instance, the inclusion of a network having high-frequency attenuation in the feedback path provides a system having a rising high-frequency response.

A block diagram of the system is shown in Fig. 6. When the function control is in the position shown, unequalized feedback around the amplifier maintains its response flat throughout the audio spectrum. The equalizer following the amplifier attenuates the high-frequency end of the spectrum, and this signal is selected by the function control to become the output. The performance in this mode is essentially the same as that of the model referred to earlier.

When the function control is shifted to the position opposite that shown in Fig. 6, the signal from the equalizer is used for negative feedback. Since the amplifier now has reduced feedback for those frequencies which fall in the attenuation range of the equalizer, the net transmission through the amplifier rises in the high-frequency end. This boosted signal appears at the intput of the equalizer, and is selected by the function control for use as the system output.

[1] J. D. Ryder, "Electronic Fundamentals and Applications," Prentice-Hall, Inc., New York, N. Y., 2nd ed., p. 321; 1959.

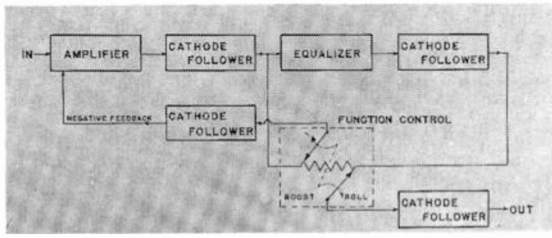


Fig. 6—System block diagram.

Presumably, when the function control is centered, the output signal can be flat even though the equalizer network may be set for its greatest effect. Tests on the finished unit confirmed this.

Stability Problem

Since the offset desired falls in the range of 20–25 db, the amplifier should have approximately 40 db of amplification without feedback, so that for extreme boost settings sufficient loop gain will exist to maintain control of the performance at high frequencies. 100 per cent feedback exists for all frequencies not attenuated by the equalization network; hence, careful control of the gain-phase characteristic of the system is required considerably beyond the audio spectrum, in order to maintain system stability.[2]

Investigation of the stability problem shows that, allowing for extreme conditions of the equalizer circuit settings, control of the gain-feedback loop performance must be maintained from about 5 cps to beyond 10 Mc. Control of the low-frequency end of the system is relatively simple since there are no changes in the low-frequency response resulting from adjustments of the equalizer. When the equalizer is set to be 3 db down at 30 kc, and to provide maximum offset, the resulting changes in gain and phase do not become constant until a frequency of 1 Mc is approached.

Amplifier Design

Although the need for a good broad-band amplifier has been demonstrated, the use of peaking coils is viewed as somewhat of a last resort. Standard triode circuits using RC coupling cannot approach the bandwidth requirement because of the shunt capacitance involved. Most pentodes are eliminated either because they are too nonlinear, or because they cannot drive the low plate load resistances desired for this application without requiring excessive plate current.

An amplifier configuration which meets the several requirements is the cathode-coupled circuit using triodes. It provides a low equivalent-source impedance, a low input capacitance, and good linearity for large signals. Disadvantages of this circuit include the lack of phase inversion of the signal, which causes large in-phase current demands on the power supply and the need for fairly high quiescent currents in order to provide linearity and low tube plate resistance.

Although the equivalent-source resistance of these stages is low (about 2700 Ω), it is desirable to use cathode followers for two functions in the system, 1) to provide still lower impedance to drive the equalizer network, and 2) to allow the equalizer network and the function control to work into low-capacitance loads. This use of cathode followers is shown in Fig. 6.

Amplifier Details

Considerable study preceded the choice of an amplifier comprising two stages, each having the configuration shown in Fig. 7. This amplifier stage delivers an output of 32 v at 1.7 per cent intermodulation distortion, has a voltage amplification of 10, and an equivalent source resistance of 2700 Ω. The biggest penalty is its 30 ma drain from the 500-v plate supply.

As has been noted, two stages of this basic form are used in the amplifier section of the device. Four cathode followers, used for their impedance properties, round out the active portion of the electronics. The complete circuit of the electronic equalizer is shown in Fig. 8.

We note that the circuit has been complicated by the stability problem. High-frequency control networks appear in three places. A fixed RC network shunts the plate load of each of the two amplifier stages, and a variable network located in the feedback pickoff from the mode control is adjusted simultaneously with the offset control to compensate for changes in the ultrasonic-transmission characteristic of the equalizer network.

The loads for two of the cathode followers have strange taps from which leads carry the signals to the mode control. These complications were tolerated in order to 1) minimize any dc potential which might be applied across the mode control, and 2) provide equal signal voltages to the mode control under a condition of zero offset.

The negative feedback signal is applied in the first stage to what would normally be the grounded grid of the cathode-coupled amplifier.

The power supply used with this system provides regulated dc potentials in addition to several separate filament sources which "float" so as to minimize heater-cathode voltages in the tubes.

[2] *Ibid.*, pp. 331–335.

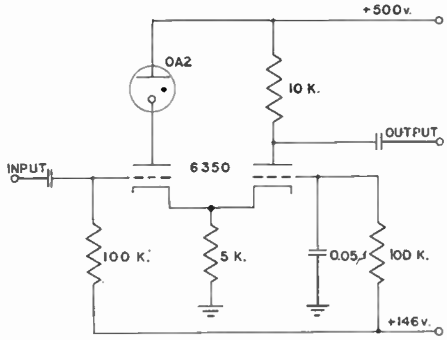


Fig. 7—Basic amplifier stage.

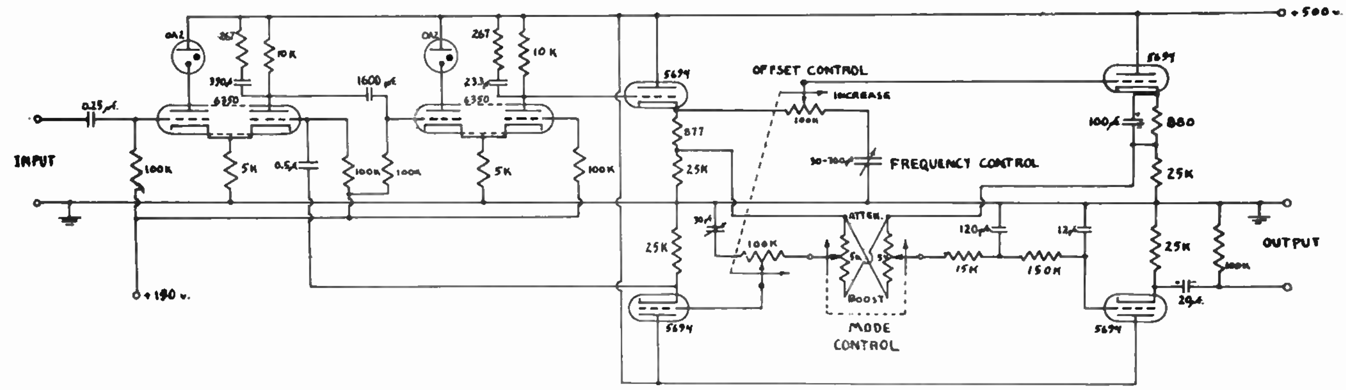


Fig. 8—Equalizer circuit.

Performance

A family of transmission curves showing the effect of changes in the offset for an inflection frequency of 2.5 kc is plotted in Fig. 9. Both the boost and the attenuation, or roll, modes are shown in this figure. Note that there is a slight attenuation which causes a loss of two db to occur at 25 kc, most easily observed on the 0 db curve. This results from a two-stage RC network inserted in the grid of the output-cathode follower, in order to avoid transmission of high-frequency components above the audible band.

The other aspect of the transmission is shown in Fig. 10, where maximum offset is maintained as the inflection frequency is set at several points in the range. Again, both the boost and roll modes are shown.

These curves can but hint at the variety of characteristics available from this system. Since the adjustments are stepless, the equalizer settings can be changed during program transmission with much less chance of disturbing transients than if switches controlled the settings. (Perhaps this is too much temptation to place in the hands of *any* audio engineer!)

An intermodulation test was made with the equalizer operating in the "flat" condition. Normal operating level was considered to fall in the range of 1–2 v, to be compatible with the broadcast and recording industries. Using a 1:1 ratio of 60 and 12,000 cycles, the unit

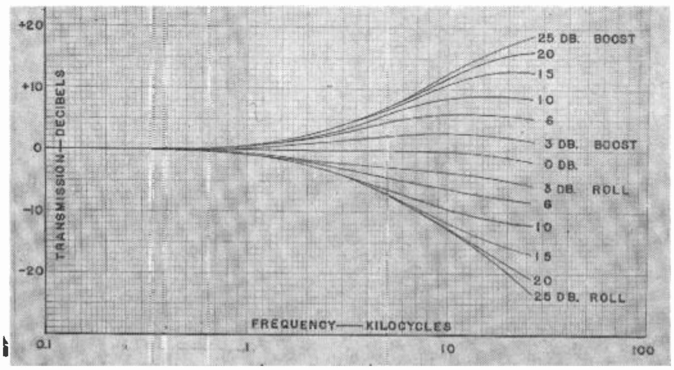


Fig. 9—Effect of offset in boost or roll mode with 2.5 kc inflection frequency.

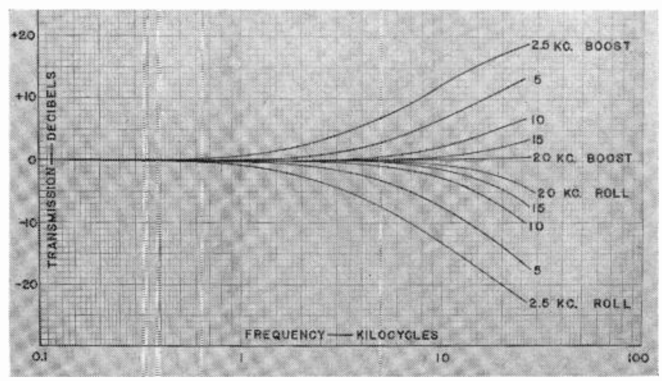


Fig. 10—Effect of inflection frequency in boost or roll mode with maximum offset.

delivers 2 v at 0.1 per cent intermodulation distortion. The distortion at 10 v is 0.6 per cent, and at 30 v, the distortion has risen to 2.7 per cent.

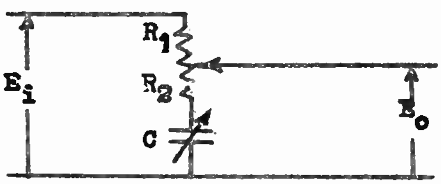
Conclusion

The device described provides exceptional flexibility for trimming the high-frequency balance of an audio program. The basic concept of the design can be extended to the low-frequency end of the spectrum, although the frequency control network would be more complex.

The main disadvantages revolve around two problems: 1) the need for a broad-band amplifier with a carefully trimmed response, and 2) the relatively high amount of energy consumed by a "simple voltage amplifier" in conjunction with the need for the regulated power supply.

Appendix

A. Derivation of the Transmission Equation for the Passive Equalizer Network



The transmission through the network shown above can be written by inspection when one considers the combination of $R_2 + C$ to be the impedance across which the output of a voltage divider is taken. Thus,

$$\frac{E_o}{E_i} = \frac{R_2 + \dfrac{1}{j\omega C}}{R_1 + R_2 + \dfrac{1}{j\omega C}} \tag{4}$$

$$\frac{E_o}{E_i} = \frac{\dfrac{R_2}{R_1 + R_2} + \dfrac{1}{j\omega C(R_1 + R_2)}}{1 + \dfrac{1}{j\omega C(R_1 + R_2)}}. \tag{5}$$

We now need to define the offset and the inflection frequency:

$$\text{offset,} \qquad A = \frac{R_2}{R_1 + R_2}, \tag{6}$$

$$\text{inflection frequency,} \qquad f_2 = \frac{1}{2\pi C(R_1 + R_2)}. \tag{7}$$

Returning to the transmission equation,

$$\frac{E_o}{E_i} = \frac{A - j\dfrac{f_2}{f}}{1 - j\dfrac{f_2}{f}}, \tag{8}$$

$$\frac{E_o}{E_i} = \frac{1 + jA\dfrac{f}{f_2}}{1 + j\dfrac{f}{f_2}}, \tag{9}$$

$$\frac{E_o}{E_i} = \sqrt{\frac{1 + A\left(\dfrac{f}{f_2}\right)^2}{1 + \left(\dfrac{f}{f_2}\right)^2}} \angle \tan^{-1} A\frac{f}{f_2} - \tan^{-1}\frac{f}{f_2}. \tag{10}$$

B. Relationship Between Inflection Frequency and Actual Frequency at which Transmission Differs from Midband Transmission by 3 db

When the magnitude of the transmission is 3 db down from midband, let us call this frequency f_3, and we can write,

$$\left|\frac{E_o}{E_i}\right| = \frac{1}{\sqrt{2}} = \sqrt{\frac{1 + \left(A\dfrac{f_3}{f_2}\right)^2}{1 + \left(\dfrac{f_3}{f_2}\right)^2}}, \tag{11}$$

$$\frac{1}{2} = \frac{1 + \left(A\dfrac{f_3}{f_2}\right)^2}{1 + \left(\dfrac{f_3}{f_2}\right)^2}, \tag{12}$$

$$\left(\frac{f_3}{f_2}\right)^2 (1 + 2A^2) = 1, \tag{13}$$

$$\frac{f_3}{f_2} = \frac{1}{\sqrt{1 - 2A^2}}. \tag{14}$$

This relationship has been calculated for a range of offsets, and is shown in Table I.

TABLE I

Offset, A	f_3/f_2	Offset, db
1/2	1.41	6
1/3	1.13	9.6
1/4	1.07	12
1/5	1.04	14
1/10	1.01	20
1/20	1.00	26

C. Maximum Phase Shift of Passive Equalizer Network

Since the network is within the feedback loop for the boost mode, the maximum phase shift and the frequency at which it occurs are of interest as a preliminary step to any study of stability

$$\theta = \tan^{-1} A\frac{f}{f_2} - \tan^{-1}\frac{f}{f_2}. \tag{15}$$

We can find the frequency for maximum phase shift by taking the first derivative of the above equation, then setting this derivative equal to zero and solving for the frequency.

$$\frac{d\theta}{df} = \frac{\dfrac{A}{f_2}}{1+\left(A\dfrac{f}{f_2}\right)^2} - \frac{\dfrac{1}{f_2}}{1+\left(\dfrac{f}{f_2}\right)^2} = 0, \qquad (16)$$

$$\frac{A}{1+\left(A\dfrac{f}{f_2}\right)^2} = \frac{1}{1+\left(\dfrac{f}{f_2}\right)^2}, \qquad (17)$$

$$\frac{f}{f_2} = \sqrt{\frac{\dfrac{1}{A}-1}{1-A}}. \qquad (18)$$

A few values for maximum phase shift, and the frequency at which maximum phase shift occurs, as a function of the offset, are shown in Table II.

TABLE II

Offset, A	f/f_2	θ_{max}
1/2	1.41	−14.4°
1/5	2.24	−41.8°
1/10	3.16	−54.8°
1/20	4.47	−64.2°

Acknowledgment

The author wishes to express his appreciation to the Newark College of Engineering Research Foundation, Newark, N. J., for sponsoring the project to develop the equalizer system described herein.

"Apparent Bass" and Nonlinear Distortion*

JOHN D. GRIFFITHS†, MEMBER, IRE

Summary—**A discrepancy between the "apparent bass" response heard by the average music listener and anechoic-chamber measurements has been noted for some small loudspeaker systems. This may be caused by the psychoacoustic response to the generation of harmonic distortion by the nonlinear suspension and the inhomogeneous flux gap density in a small speaker. An electrical analog, with controllable distortion, of such a speaker has been subjected to listener tests and evaluation to determine if this is the cause of the apparent bass effect. An analysis of the listener reactions to various music stimuli through the system indicates that this is the case.**

Introduction

WITHIN the past few years several home loudspeaker systems have been marketed which sound very good, and yet, when subjected to measurements in an anechoic chamber, apparently have shown a deficient bass response. This lack of bass is not usually apparent to the average listener, however, at least at louder listening levels.[1]

Several theories have been advanced to account for this discrepancy. An attempt has been made to find a correlation between equal-loudness contours, the sound power spectrum of music, and masking-level curves. It was reasoned that the outer ends of the music spectrum, containing comparatively little sound power, would be either masked or be below the limits of audibility if reproduced. Experiments tend to disprove this.

Another explanation is based on the psychoacoustic phenomenon, "The Case of the Missing Fundamental." Here a pulse train, or series of pulses, is set to a periodic repetition rate, lying within the audible band. The series of pulses is then passed through a high-pass filter, with a cutoff frequency higher than the frequency corresponding to the pulse repetition period. Subjects listening to the pulse-train report being able to hear a frequency corresponding to the pulse-repetition frequency.

Biophysicists, noticing that the effect of simulated bass is present only at loud levels, have felt that this could perhaps be attributed to neural and middle-ear distortion, although little work has been done in quantizing this effect.[2]

Reinhard has noted that loudspeaker voice coils, when moving out of the gap of uniform flux density of the magnet, exhibit a clipping action on the sound wave.[3] This clipping action is not linear with respect to the frequency of the impressed rms voltage. Rather, the clipping is seen to be a function of the displacement of

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† Capt. USAF, Rome Air Dev. Ctr., Griffiss AFB, N. Y.

[1] L. L. Beranek, "Acoustics," McGraw Hill Book Co., Inc., New York, N. Y.; 1954.

[2] H. F. Olson, "Elements of Acoustical Engineering," D. Van Nostrand Co., Inc., New York, N. Y.; 1947.

[3] W. Reinhard, "The inhomogeneity of the magnetic field of a dynamic loudspeaker," *Akust. Z.*, vol. 4; 1939.

the voice coil from its rest position. As long as the entire voice coil is within the uniform flux density gap [within the limits of L in Fig. 1(a)] or as long as the flux density gap is bounded by the voice coil at either end [Fig. 1(b)], no distortion will be present due to variant flux across the voice coil. Inexpensive loudspeakers are rarely made like those in Fig. 1(a) and (b) but rather like the one in Fig. 1(c) for economy reasons. It is seen from Fig. 1(c) that if the speaker cone displaces very much from its rest position, the total flux across the voice coil will change. The voice-coil (and cone) displacement is related to the rms impressed current as follows:

$$m \frac{d^2u}{dt^2} + \beta \frac{du}{dt} + ku = ilB$$

where i = current in voice coil, l = length of voice coil, B = flux across coil, u = cone displacement, m = mass of cone, k = restoring force of suspension, and β = viscous damping coefficient.

The solution for $U_{\max}$ is

$$\frac{I_{\max} lB}{\omega\left[\beta^2 + \left(\omega m - \frac{k}{\omega}\right)^2\right]^{1/2}}$$

where $i = I_{\max} \cos \omega t$, and ω = frequency.

In the low-frequency approximation, and with the speaker in an enclosure, $\beta^2 \gg (\omega m - k/\omega)^2$. Regarding the function. $(1/A)$ as a constant,

$$U_{\max} = \frac{A}{\omega} E_{\max} lB.$$

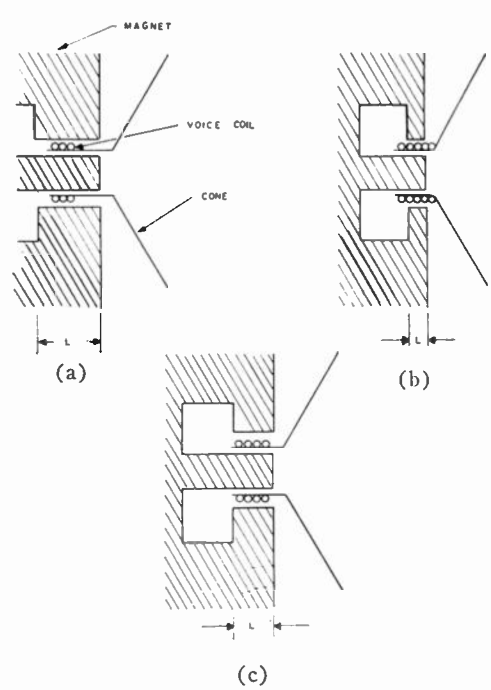

Fig. 1—Loudspeaker flux gaps.

We see from the A/ω term that the displacement function (U) slopes at a (−) 6 db per octave increase in frequency when compared with the voltage function. Electrically, the displacement function can be obtained by feeding the voltage wave into an "integrator" circuit. (An integration circuit is normally a series resistance with its output shunted by a capacitance.)

If the time constant of the circuit is so adjusted that the cutoff frequency, $f = 1/(2\pi RC)$, is below the lowest audible frequency, one can obtain an electrical analog of the displacement function. E output can then be operated on by an electrical peak clipping circuit to simulate loudspeaker clipping (overloading). If the clipped wave is then passed through a "differentiator" circuit the resultant wave is returned to its original form with "displacement clipping" superimposed. (A differentiator is a series capacitance with its output shunted by a resistance.) If the resultant wave is now fed into a "distortionless" electroacoustic transducer, and the degree of electrical clipping varied, various degrees of loudspeaker clipping can be simulated. To achieve a realistic loudspeaker model, however, it is necessary to insert a high-pass filter between the resultant voltage wave and the transducer, since most loudspeakers, especially those exhibiting clipping action at listening levels, have their fundamental resonance frequency well up in the audible spectrum. A loudspeaker which has its rear wave isolated from the front wave will have its response curve drop at 12 db per octave below the resonance frequency, while bass reflex enclosures and others which utilize the rear-wave drop off at 18 db per octave below resonance.[1] The high-pass filter, then, should have its cutoff frequency set at the resonance frequency of the speaker to be simulated, and the filter should have an attenuation characteristic of either 12 or 18 db per octave, depending on which type of enclosure is to be simulated.

An analysis of listener comments and comparisons should yield a correlation between simulated bass response and the degree of displacement clipping. With infinite peak clipping, distortion will be intolerable, whereas with no clipping there will be no apparent bass.

It should be noted that the above analog simulates the most drastic possible case of displacement clipping. For this type of clipping to exist in reality there would have to be an abrupt transition from full flux to no flux linking the voice coil as the voice coil moves out of the magnet flux gap. In practice there is a gradual transition from maximum flux to no flux linking the voice coil, and the higher harmonics generated by the clipping in the analog would not be present in an actual speaker output. It should also be noted that this type of distortion (displacement clipping) will also result from a nonlinear suspension of a loudspeaker cone.[2] Here the analog simulates a physical constraint on the maximum excursion of the loudspeaker cone.

The Experiment

High-quality tapes recorded from live music performances were used as test material. The selections used represented different types of music with various contents of bass. The material was played through the system shown in Fig. 2. The clipper circuit had an adjustable bias control which was manipulated by the subject. There was also an A-B switch controlled by the subject. In the A position an undistorted full-range (including bass) signal was presented to the subject's headphones. In the B position the test signal was "integrated," clipped, "differentiated," and high-pass filtered ($f_c = 200$ cps) with 18 db per octave attenuation below f_c. The peak reproduced signal (0 V.U.) resulted in a maximum sound-pressure level of 90 db at the listener's ear in both A and B positions. Western Electric 711A headphones were used. A dc voltmeter across the diode bias indicated the amount of relative clipping present in the B position. See Fig. 3 for distortion vs bias calibration. Decreasing the bias increases the distortion. The system was low-pass filtered with a cutoff frequency of 5 kc to eliminate dynamic-range problems in the integrator and differentiator circuitry. (A dynamic range of approximately 110 db would be necessary between the integrator and differentiator for an input and output signal of 55 db dynamic range and 9 octave frequency range.)

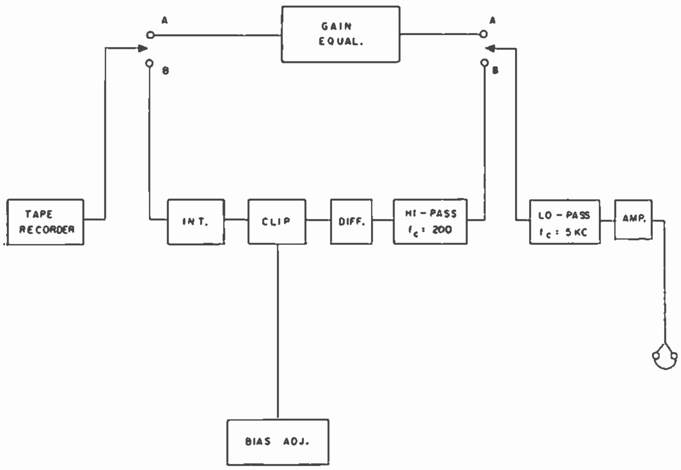

Fig. 2—Loudspeaker electrical analog.

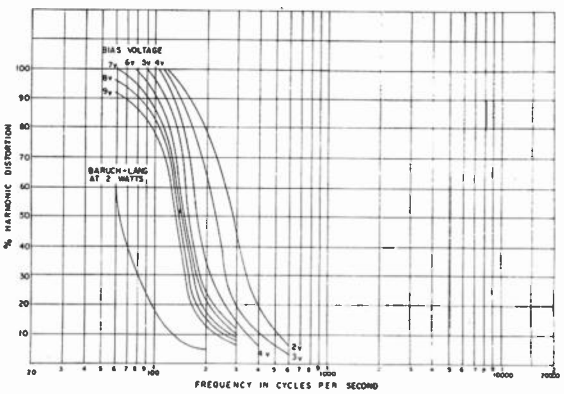

Fig. 3—Distortion vs frequency.

Five selections of nine minutes duration each were presented to the listener. The selections were a full orchestral passage, a full choral, a harpsichord solo, an organ solo with predominant bass, and a piano concerto. The subject was instructed to listen to the selection with the switch in the A position and then attempt to adjust the bias knob, with the switch in the B position, so that the music, especially the bass, was the same in each switch position. The subject was allowed to switch between A and B positions as often as desired. None of the subjects were aware of the experimental circuitry. At the end of each passage, or when the subject felt he had a correct match between the A and B positions, the final bias voltage on the clipping circuit was recorded.

A few of the subjects commented after the first selection that there seemed to be a lot of distortion when the switch was in the B position and the bass control in the extreme clipping range. The subject was then instructed to strive for as low distortion as possible, but, above all, to try to achieve the same amount of bass in each switch position.

Results

Twenty subjects were used during the course of the survey. Of these twenty, two were professional acousticians, four were muscians, five were high-fidelity enthusiasts and the remainder were average music listeners. All were college students or graduates. Initially, it has been planned that the four groups would be treated separately. It resulted, however, that in actual decisions there was little difference among the latter three groups.

The acousticians were completely unable to do the experiment, probably due to prior conditioning in listening to and recognizing distortion as such. Their comments indicated that as long as the bias voltage was above 6 volts, any change in bias caused no audible change in quality, although they were unable to detect the apparent bass effect in this range. They noted that when the bias voltage was reduced below 6 volts, the quality of the music degenerated rapidly. They did not observe that the "bass" control had any effect on the amount of bass present.

One difference among the three groups that were able to do the experiment became obvious while the subjects were actually performing the test. The speed and ease with which a subject was able to make a decision was found to be inversely proportional to the music listening background of the individual. The average music listeners quickly and easily (relatively) reached decisions, while the musicians, and especially the high-fidelity enthusiasts, floundered a great deal while attempting to make a decision. Also, eight of the nine average listeners were quite sure of their decisions, stating that they had achieved a perfect match, while the more experienced subjects occasionally expressed some doubt as to the "correctness" of their choices.

The total time of the experiment was about forty-five minutes for each subject, with approximately equal segments required for each selection. As can be seen from the time necessary to perform each test, the subjects were not able to reach their decisions in a matter of seconds, but required several minutes of what appeared to be intense concentration for each decision. In the case of the more experienced subjects, it was frequently necessary to play a portion of the selection a second time. Eight of the subjects commented that they felt that the test was extremely subtle as to controls and evaluation, while three subjects felt that the range of control was not great enough. Two of the high-fidelity enthusiasts commented that the controls did not make a great deal of difference as regards bass response.

It should be noted here that comments regarding specific phenomena were not solicited from the subjects. Note was made of any comments which the subjects freely offered during the course of the experiment, usually between selections. At the conclusion of the experiment the blanket question, "Do you have any comments or observations to make regarding the test and the relative signals?" was asked. When the subject made a decision before the end of a selection, the bias voltage was recorded and the subject was requested to listen through to the end of the selection and check his decision. No change greater than 0.1 volt was made in the bias voltage.

Eight listeners spotted the distortion for lower bias voltages, though it was not always referred to as distortion *per se*, having been called "raspiness" and "overloading" by some subjects. The two acousticians and one high-fidelity enthusiast considered the distortion intolerable below the 6-v bias level, while the remainder of the subjects usually noticed nothing amiss above the 3- or 4-v level. Two musicians and one average listener stated that they felt they should advance the bass control further, but were stopped by the distortion present.

Two very interesting types of comments were made by some of the subjects, one of which seems to validate quite well the theory that the apparent bass effect is caused by displacement clipping.

First, six subjects reported that they were unable to reproduce the organ with complete faithfulness in the "adjust" position. The particular organ passage used in the experiment was selected because of a series of low background pedal notes throughout the passage. The fundamental frequencies of this series of notes ranged between 40 and 100 cycles, and the notes were relatively free of harmonics, due to the organ stop used. All but one of the subjects reported being able to adjust the clipper to get a semblance of these notes, but not a faithful reproduction. The eighth subject, a musician, stated that to get the pedal notes required advancing the bass control to the point where the rest of the music was "hash." Incidentally, it should be noted that all the subjects except the acousticians indicated that reducing the bias increased the bass response.

Secondly, six subjects stated that while they were able to adjust for equal bass for a few bars, a change in the level of the music or a change in the bass content of the music necessitated a change in the bass control to preserve the equality of the bass response in the two switch positions. The remainder of the subjects indicated this same phenomena by their actions during the course of the experiment. (The actions of the subjects were monitored visually and aurally throughout the experiment.) Since none of the subjects was aware of what was occurring electrically, it must be assumed that the apparent bass effect is a function of amplitude. The only parameters in the system which are functions of signal amplitude are loudness in the earphones and the degree of clipping imposed upon the signal. The subjects were able to match the bass response for different amplitudes by varying the bias control, however, and this seems to rule out the possibility that the apparent bass is due entirely to neural, middle-ear, or earphone distortion.

It should be added that all the subjects were young, under 25, and without hearing loss or impairment. One subject performed the test several times, at one week intervals, in order to determine any learning effect. None was apparent, except for a very slight tendency towards greater distortion values with successive tests.

The experimental data was plotted in order to determine the distribution functions (see Figs. 4–8). The distribution was determined by plotting the number of subject responses (or choices) for each range of bias voltages vs the value of bias voltage. The bias-voltage values were scanned in units of two volts, with the ranges overlapping by one volt in each direction. Each value of bias voltage, therefore, is represented by two points on the function curve.

There is probably some question as to the reason why bias voltage has been used above so often as a parameter. It would be more meaningful, both from an engineering and from a psychological viewpoint, to use percentage harmonic distortion as a parameter, rather than the clipper bias voltage. As can be seen from Fig. 3, the per cent of harmonic distortion is not a linear function of frequency, or any other simple function for that matter, while per cent clipping is an inverse function of frequency. To translate per cent clipping into per cent distortion requires a Fourier series summation, a process somewhat tedious when required for several frequencies for each of several values of bias voltage. The results, at best, are only an approximation to the real case, since a Fourier series summation makes several assumptions which are only partially realizable in actuality. Furthermore, the bias voltage is a single number which represents a contour on Fig. 3. To represent this contour in terms of per cent harmonic distor-

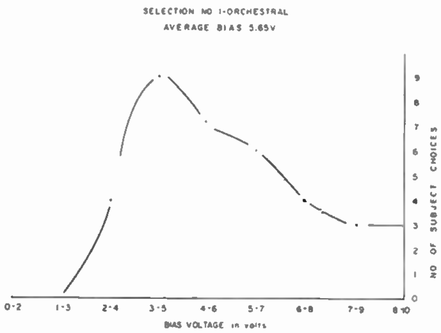

Fig. 4—Distribution of responses for orchestral selection.

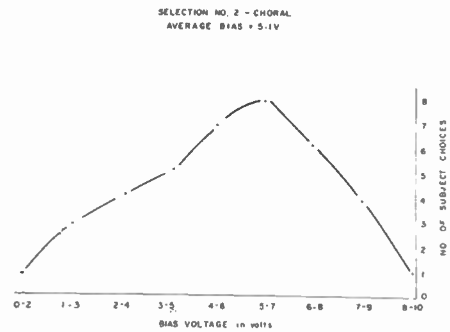

Fig. 5—Distribution of responses for choral selection.

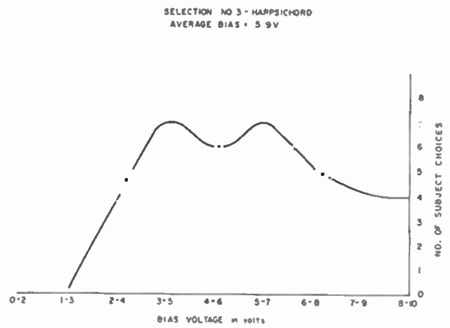

Fig. 6—Distribution of responses for harpsichord selection.

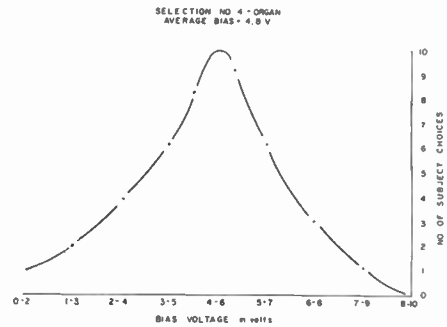

Fig. 7—Distribution of responses for organ selection.

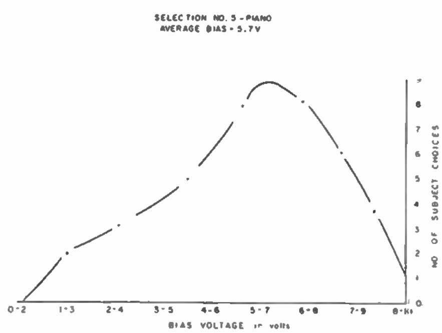

Fig. 8—Distribution of responses for piano selection.

tion, it would be advisable to give values for several different frequencies, to prevent confusion with linear harmonic distortion, while by using Fig. 3 and the bias voltage, values are found immediately for per cent harmonic distortion at any frequency at which the circuit operates.

The degree of randomness of the distribution functions is inversely proportional to the amount of bass present in the music. The curve for the organ solo has a very pronounced peak, whereas the harpsichord solo is very nearly random, bounded on the left by excessive distortion. The two curves for selections having prominent low-frequency bass (orchestral and organ) had peaks corresponding to the greater distortion necessary to generate apparent bass.

It would be easy to overinterpret the data shown and hypothesize a system incorporating controlled distortion. At present too little is known, and less severe methods of generating harmonic distortion than peak clipping would have to be investigated.

Conclusions

It is fairly clear that the apparent bass effect does exist, and is due to some degree to displacement limiting. However, one must not construe the above results to mean that this is the only phenomenon which will give the illusion of bass. One must not discount neural and middle-ear distortion as being able to produce this effect, at least not until this field has been investigated further. It should be remembered that the auditory mechanism is a nonlinear device. Also, it is fairly apparent that a resonance "hump" will augment this effect.

It should also be apparent that this effect is not an adequate substitute for a full-range reproduction system.

Acknowledgment

The author is indebted to Prof. L. L. Beranek for his continued advice and assistance.

The Concept of Linear Interpolation in Spectral Compensation*

C. E. MAKI†, MEMBER, IRE, AND J. M. CHIRNITCH‡, MEMBER, IRE

Summary—Spectral compensation is usually achieved with equalizers cascaded so as to generate a desired response. In the concept of linear interpolation, a series of points are located along the desired response and linear interpolation provided between adjacent points. This is accomplished with contiguous band-pass filters arranged so as to minimize the effect of filter crossover. The performance limitations depend upon the filter characteristics, crossover ripple, and the type of response to be equalized. This approach to compensation has two advantages. First, automatic control of the spectrum can be achieved when the input signal is a random noise voltage. Second, discreet points on the frequency axis permit control using digital techniques. Data is presented for some cases of peak-notch resonances encountered in acoustic and vibration systems.

Introduction

FREQUENTLY in an audio system some means of shaping is used to compensate for disturbing resonances which occur in the frequency response characteristic. These disturbances can occur in either electrical or mechanical form and sometimes in combinations of both. The task of the compensating device is to provide a response which is the inverse characteristic of this system. Many applications require additional shaping requirements such as rolloff which must be introduced into the equalization networks.

In systems driven by monosinusoidal excitation or by a number of finite signals of fixed frequencies, the resulting line spectra can be equalized at any given point. On the other hand, if many frequencies are present simultaneously, as in the case of random excitation, continuous compensation at all points of the spectrum is necessary.

Classical Solutions

Methods of compensation commonly employ electrical or mechanical schemes which produce combinations of integral slopes. Peaks or notches in the spectrum are generated with second-order devices designed such that the resonant frequency and damping can be adjusted for a particular requirement. For example, the frequency response of a loudspeaker can be extended by using tuned ports or acoustically resonant chambers. Rolloff characteristics can be controlled with electrical RC networks chosen to provide the desired characteristics. In most cases, the disturbing resonances introduced by the speaker are neglected.

In vibration test systems the problem still exists, but to a greater degree. The nature of the excitation, and the complexity of the specimen under test present problems which do not usually occur in an unloaded sinusoidally excited system.

For example, the testing of space vehicles is performed in a simulated environment. This provides the packaging engineer with realistic data on performance prior to actual flight. The excitation is random in nature, and the structure is mechanically complex. The compensating device must, therefore, provide equalization for many resonances occurring in the specimen. This is necessary so that the power spectrum applied will be accurately known.

Classical approaches to this problem employ analog computer-type peak-notch equalizers for compensation. Shaping is produced with electrical RC networks. Inherent difficulties with this approach include a tedious set-up procedure and no possibility of extension to automatic control.

Characteristics of the Audio System

Five different types of resonances can occur in the frequency response of an audio system [1]. At least two of these are generally inherent in the transducer, while the other three are associated with the attached load. Loudspeakers and vibration generators exhibit electrical resonance (driving-coil inductance resonating with the driven mass) and axial resonance (driving-coil mass and driven mass decoupling through a connecting member). The first characteristic is highly damped and presents no real problem in equlization. On the other hand, axial resonance presents a high Q characteristic at the upper end of the response. Loudspeakers are designed in such a fashion that the driven mass (cone) cannot be represented by a lumped parameter and as the frequency is increased, part of the cone decouples concentrically. At each decoupling frequency, peak-notch perturbations occur in the frequency response resulting in a jagged characteristic throughout the operating range. Since all of the cone mass finally decouples, no definite axial resonant frequency appears, and the output finally vanishes as the radiating area becomes small.

On the other hand, the table of a vibration generator exhibits a somewhat different characteristic. Since it is designed to be as rigid as possible, a clearly defined axial resonant frequency occurs. Beyond this frequency, the table diaphragms, higher-resonant modes occur, and the machine output becomes difficult to control. Axial reso-

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† Minneapolis-Honeywell Co., Minneapolis, Minn. The work described in this paper was done while Mr. Maki was with MB Electronics, New Haven, Conn.
‡ Lessells and Associates, Inc., Boston, Mass. The work described in this paper was done while Mr. Chirnitch was with MB Electronics, New Haven, Conn.

nance, although undamped, usually is not troublesome from the standpoint of equalization. The symmetrical nature, and lack of steep slopes in the response, make compensation easier. Often, axial resonance occurs above the operating range, and therefore, compensation is unnecessary.

Serious resonances occur in the spectral response when the system output is coupled to a resonant load. These appear as peak-notch pairs, the notch appearing at approximately the natural frequency, of the resonating system. The antiresonant peak is a result of interaction between the resonant load and the output parameters of the transducer.

The resulting peak-notch resonance represents the most severe and most common characteristic which requires equalization in a vibration system. Analysis of the dynamics of the vibration system shows that the equation describing the transfer function of this characteristic can be written as:

$$H_1(S) = \frac{1 + \dfrac{1}{Q_n\omega_n} S + \dfrac{1}{\omega_n{}^2} S^2}{1 + \dfrac{1}{Q_p\omega_p} S + \dfrac{1}{\omega_p{}^2} S^2} \tag{1}$$

where

$H_1(S)$ = is the Laplace transform of the factor modifying the response.
Q_n = the Q of the notch.
ω_n = the notch frequency.
Q_p = the Q of the peak.
ω_p = the peak frequency.
$S = j\omega$.

Each load resonance contributes one peak-notch characteristic. If the configuration of the load is complex and contains many degrees of freedom, interaction of the parameters occurs and the orientation of peak and notch frequencies may appear in any order. Fig. 1 shows the unequalized frequency response of a vibration exciter used in environmental testing. Below this the same response is shown except with the vibration machine resonances equalized. The nature of the peak-notch is clearly evident. Two resonances are oriented with the notch frequency preceding that of the peak; the third shows the inverse relationship.

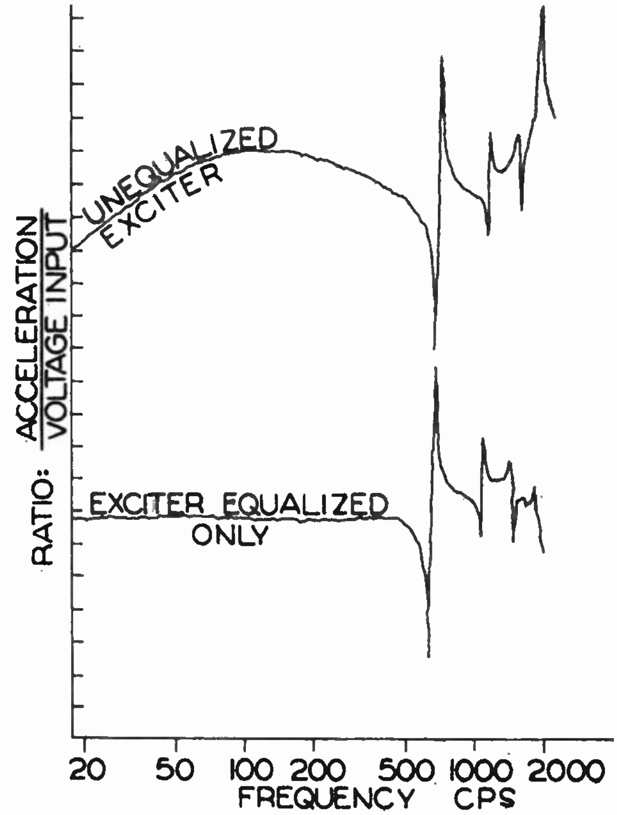

Fig. 1—Equalization of a vibration exciter.

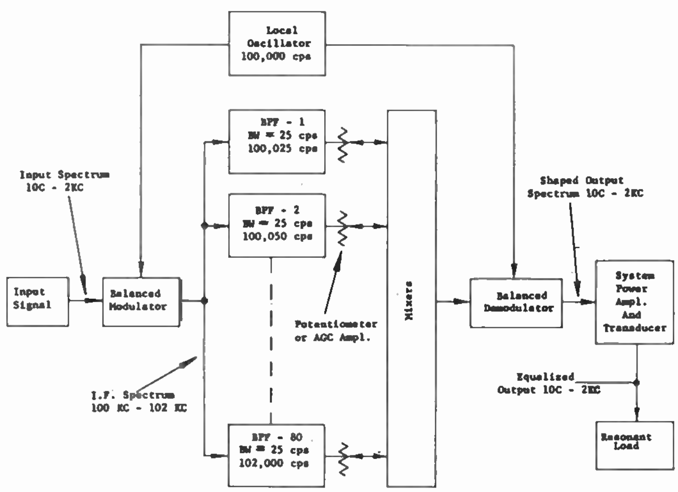

Fig. 2—Multiband equalizer.

Multiband Equalization

Recent developments in the field have yielded an equalizer which not only provides a greater degree of control, but unlike previous systems, can be extended to allow even more versatility. This new approach to the equalization problem employs contiguous narrow-band filters, tuned in such a fashion that the center frequencies of adjacent filters are separated by one bandwidth. Output control of each filter is achieved by providing a potentiometer in the manual system or an AGC amplifier in the automatic equipment. After the level has been properly adjusted, the output signals are recombined and the shaped spectrum signal is introduced at the system input. Fig. 2 shows the block diagram of the basic configuration.

The Filters

The multiband approach depends primarily upon the filters used. Magnetostrictive mechanical filters are employed since they provide a number of advantages. They can be properly summed in parallel, are low-impedance devices, rugged, economical, and extremely temperature stable. They can be easily used to cover the entire audio range. The philosophy of this equalization technique differs from the usual concept of analog compensation and can be considered from the standpoint of linear interpolation.

These magnetostrictive filters employ mechanically resonant elements which are electrically excited. They operate in the vicinity of 100 kc and have a bandwidth of 25 cps. The filter is, therefore, capable of producing an extremely high Q which is in the order of 4000. To utilize these filters in the audio range, the spectrum is translated to 100 kc by means of a carrier system.

The transfer function of the basic filter is given by the following relationship:

$$H_2(S) = \frac{2\zeta\omega_0 S}{S^2 + 2\zeta\omega_0 S + \omega_0^2} \qquad (2)$$

where:

$H_2(S)$ = Laplace transform of the filter response.
ω_0 = resonant frequency of the filter.
ζ = damping factor.
$S = j\omega$.

Experimental curves, shown in Fig. 3, describe the amplitude and phase characteristics of a typical filter. The resonant frequency is 100,100 cps, but the curves were recorded with respect to the corresponding translated frequency of 100 cps. Note that the slope is 72 db/octave.

The response shown in Fig. 4 describes the two sections comprising a composite filter. Near its resonant frequency of 100,075 cps, it has a slope of 72,000 db/octave. This curve is the demodulated response which is at 75 cps. The bandwidth is 25 cps, and the slope is 54 db/octave. Note that each of the two constituent filters exhibits a narrower bandwidth than the resulting composite curve.

The filter array used in the equalizer is composed of 80 such filters, each having a 25 cps bandwidth. The first filter has a resonant frequency of 100,025 cps and successive filters are placed at 25 cps intervals throughout the spectrum, with the 80th filter having a frequency of 102,000 cps.

A 100,000 cps carrier is used as a local oscillator in conjunction with a balanced modulator. The resulting modulated wave then contains two sidebands centering about 100 kc. When this signal is applied to the filter array the upper sideband is filtered out between 100,025 cps and 102,000 cps. The signal is then demodulated and yields a spectrum between 25 cps and 2 kc with filters placed at 25 cps intervals. This equivalent array of filters has the characteristic that the slope increases with frequency. It ranges from 18 db/octave for the 25 cps filter to 1440 db/octave for the 2000 cps filter.

Fig. 5 shows experimental response curves of the filters corresponding to various portions of the audio spectrum. Note that the slope increases with frequency while the bandwidth remains constant.

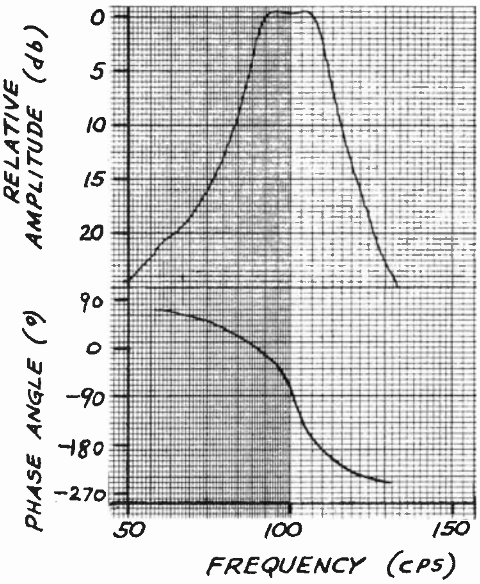

Fig. 3—Characteristics of a typical filter.

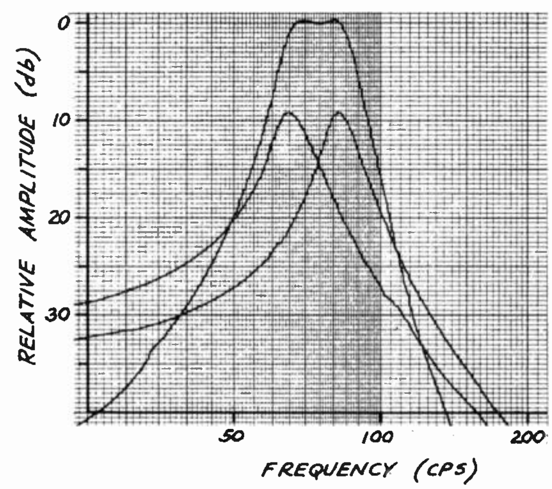

Fig. 4—The two-section filter.

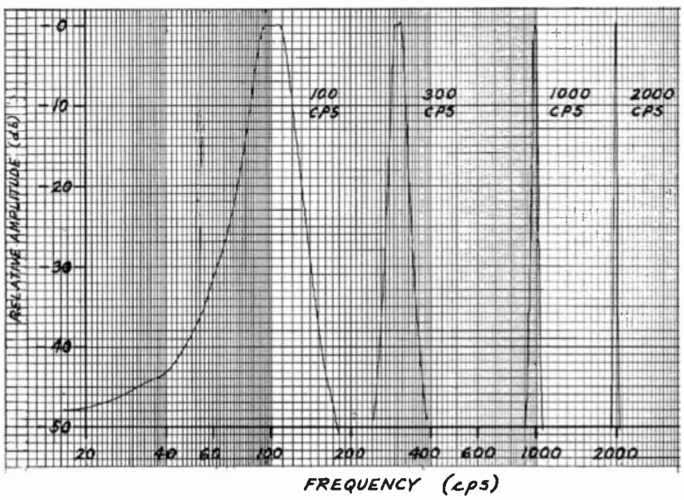

Fig. 5—Response of various filters in the system.

Concept of Linear Interpolation

The philosophy of this multiband equalizer technique differs from the usual concept of analog compensation, and can be considered from the standpoint of linear interpolation.

Compensation of the spectrum is a complicated problem in a vibration system. The basic response covers a wide dynamic amplitude range; many complicated linear and nonlinear resonances can occur within the operating frequency range. Output characteristics, dictated by test specifications, can vary over wide limits resulting in a difficult set-up procedure. Often the equalization equipment is designed in such a fashion that an adjustment to correct the response at midband will influence the characteristics in another part of the spectrum. Multiband equipment is designed to eliminate problems of this nature.

Given a desired equalization curve as shown in Fig. 6, a series of points defined by the intersection of this response with the center frequencies of the contiguous band-pass filters can be established. With this present concept, the equalization system provides a linear interpolation between adjacent points on the desired curve; thus, the desired response is approximated with a series of straight-line segments.

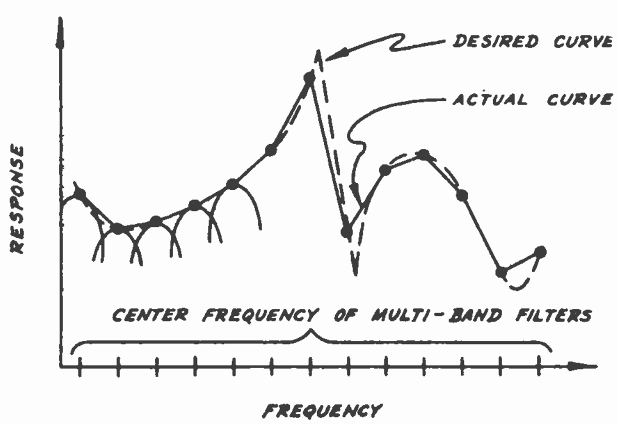

Fig. 6—Linear interpolation.

Accuracy of compensation depends upon four factors. First, the number of points appearing on the desired curve will affect the accuracy. More filters result in better compensation. Analogous to this factor is the accuracy achieved with a digital computer capable of handling a finite number of binary digits. Increased accuracy of computation is possible by increasing the number of binary digits which can be handled in any given word.

Another important consideration in multiband compensation involves the slope of the filter. Since the transition of the response from peak-to-notch frequencies is rapid, sufficient slope capability must be provided in the individual compensating filters. High-filter density is not sufficient if the slope characteristic prevents insufficient resolution.

Since multiple signals must be recombined, it is necessary to insure that adjacent filter output signals exhibit the proper phase relationship to one another. Improper phase characteristics will produce serious voids in the spectrum. It is theoretically possible to combine the filter-signal outputs in such a fashion so as to provide a smooth transition from one contiguous filter to the next. However, production tolerances dictated by the state of the art prevent perfect addition and output "ripple" will exist. Thus, the straight-line interpolation must be modified to include the ripple factor of ± 1 db.

Filter shape will also affect the interpolation concept, the ideal rectangular response providing the greatest deviation from linearity.

These four factors must be considered separately.

Filter Density

Specimen resonances occur on an octave-band basis. Therefore, it is important to establish the number of points required to provide adequate compensation for a peak-notch response of given characteristics. Obviously, each specimen resonance is unique and presents a response which is dependent upon the peak-notch frequency spread and the Q or damping associated with the resonance. The task of establishing performance for all possible situations is formidable and will not be attempted here.

It is more convenient to define a hypothetical response where the ratio of the higher-frequency resonance to the lower one is 1.05 and the Q assumed to be 25.

Permitting the filter-density factor to contribute a deviation of ± 1 db from flatness, it is possible to graphically determine the number of points necessary to compensate this hypothetical resonance. If the number of filters exceeds the number of points required, the expected compensation due to the filter density factor will result in a deviation of less than ± 1 db. In the vibration system, the hypothetical resonance as defined results when 10 per cent of the mass decouples and a filter density of 12 filters per octave is required.

Next it is necessary to establish the filter density available to provide compensation. The computation is trivial and depends only upon the filter bandwidth and its location in the audio spectrum. Thus, it is convenient to plot the filter-density factor as a function of frequency as shown in Fig. 7. If the number of points required for compensation is below the line specifying the filter-density factor of the system, the deviation from flatness due to this factor is less than 1 db. Notice that for the hypothetical resonance, adequate compensation exists above 400 cps when the filter bandwidth is 25 cps.

Filter Slope

A similar analysis applies to the slope requirement of the compensation filters. Steep slopes appear between peak-notch frequencies and it is necessary for the filter slope at the skirt to exceed that of the resonance. Again since the peak-notch resonance occurs on an octave

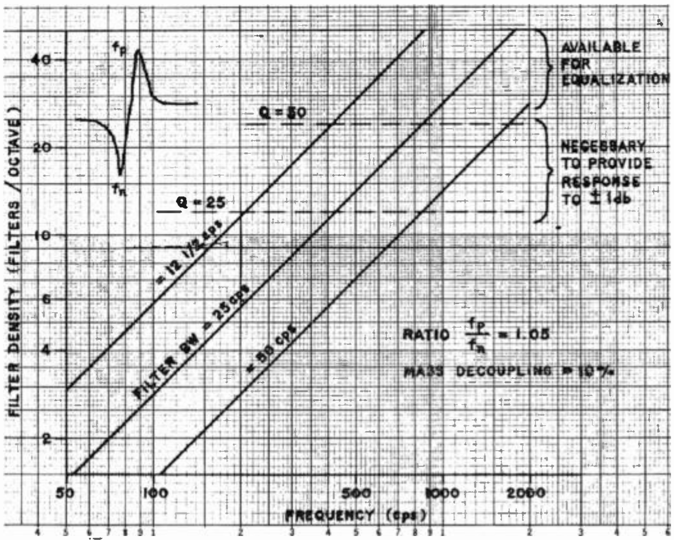

Fig. 7—Number of filters available for equalization and number required for a response of ±1 db.

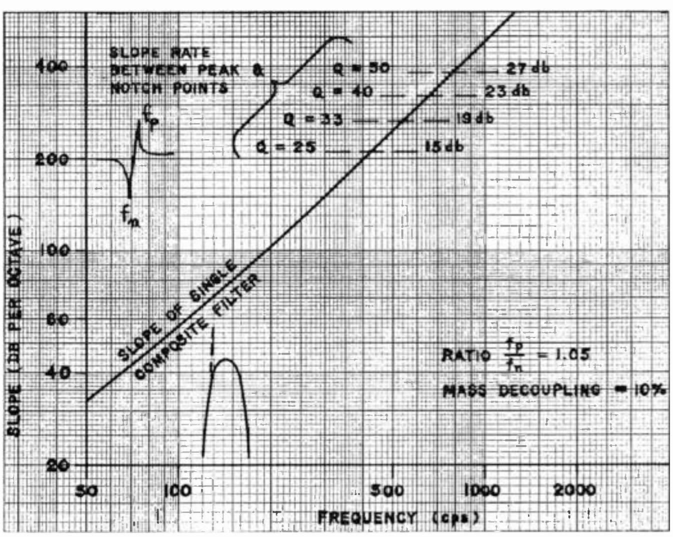

Fig. 8—Relationship between slope rate of a peak-notch and a single filter.

basis, the slope requirement is fixed as a function of frequency. Analyzing the hypothetical resonance shows that the slope between peak and notch frequencies is 220 db/octave. If the filter slope exceeds this requirement, no deviation from this factor occurs. If the filter slope characteristic is less than the peak-notch slope, additional perturbations from a flat response are expected.

This can be represented graphically as shown in Fig. 8. A peak-notch slope requirement appearing below the filter slope factor line indicates that no deviation from flatness due to this factor is introduced by the compensation. Notice that the intersection point of the hypothetical resonance requirement and the line associated with the 25 cps filter occurs at 400 cps.

Double section composite filters are provided in the system described as has been mentioned previously. Thus, two points separated by a single bandwidth in frequency can be separated by 12 db in amplitude. Filter efficiency is greatest when the requirements of the slope and density factors coincide as indicated in this system.

If the slope factor were increased by using a triple-section composite filter, the filter cost would increase by more than 50 per cent, yet the performance would not be measurably increased since the filter density would still dictate the low-limit requirements.

Ripple Factor

An earlier paper provides a detailed mathematical analysis of the summation problem which contributes to the ripple factor of the compensation system [2]. Multiple signals must be summed in a mixing circuit and both real and imaginary parts of each voltage must be considered. Improper summation at the point where adjacent filters cross over (midway between filter-center frequencies) will result in serious notches in the spectrum.

The possibility of this occurrence is eliminated by using the two-section filter configuration and driving adjacent filters out of phase. Phase response of a two-section filter deviates by ±180° from the response at resonance. Internal characteristics of the filter are such that the phase of the component at resonance is plus or minus 90° depending upon whether the filter is driven in or out of phase. Thus, phase of one filter varies from +270° to −90° whereas the adjacent ones change from +90° to −270°. At the crossover point the voltage from each adjacent filter is nearly in phase and cancellation is impossible. Fig. 9 demonstrates the principle when two adjacent filters are adjusted for equal output. Note that a linear transition is possible from the center of one filter to the adjacent ones.

Manufacturing tolerances of the filters cause a depar-

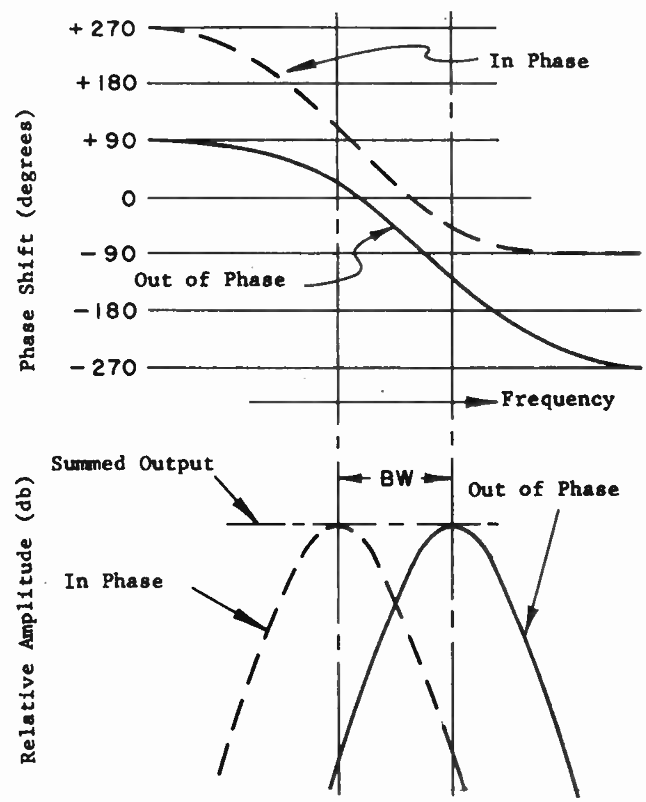

Fig. 9—Phase and amplitude response of two adjacent filters.

ture from this linear interpolation. However, the parameters are chosen such that with known tolerances, only 1 db of ripple is introduced by the ripple factor.

Filter Shape

The fourth factor which contributes to the ability of the system to perform according to the concept of linear interpolation is the filter shape. Deviation introduced by this factor depends upon the amplitude separation of adjacent frequency points. If the filter shape were triangular, the deviation would be minimum.

A similar array of filters is required for the spectrum analyzer which is an integral part of the automatic equalization system. Preferably these filters should approach the ideal band-pass filter response for analyzation, while for equalization finite slopes are desirable. The ideal band-pass filter, if used for equalization, would introduce the greatest deviation from a flat response and, therefore, a greater number of filters would be required for compensation. Since it is economical to use the same type of filter for both equalizer and analyzer, a compromise is necessary to simultaneously fulfill both requirements.

In Fig. 5 the response of four filters in different parts of the frequency range is shown. The frequency axis is plotted on a logarithmic scale, thus, the low-frequency filters appear wider. However, all four filters are 25 cps wide at the half-power points (-3 db down). It can be determined by inspection that the maximum deviation due to the filter shape is $+2.8$ db, whereas the minimum deviation is zero.

Measurement of Ripple Factor

The transfer equation of the array is given by

$$H_3(S) = \sum_{n=1}^{k} (-1)^n \frac{\overline{\Delta\omega}\alpha_n S}{S^2 + 2\zeta_n S + \omega_n^2} \tag{3}$$

where

$H_3(S)$ = Laplace transform of filter array response.
$\overline{\Delta\omega}$ = bandwidth.
α_n = attenuation of filter n.
ζ_n = damping factor of filter n.
ω_n = frequency of filter n.
k = number of filters in the array.

Fig. 10 shows a portion of the array from 75 cps to 400 cps where the demodulated filter responses of 14 adjacent filters are plotted. Above these is the composite characteristic of their summed output. Below is the phase characteristic of the combined array. The ripple in this combined response curve is due to three factors. First, the amplitudes of the individual filters have a variation of $\pm\frac{1}{2}$ db. Second, the bandwidths vary ± 1 cps. Finally, the center frequency has a ± 1 cps variation. The total effect, due to these three properties, can be observed by noting the composite curve. Neglecting end effects, this portion of the array is flat within $\pm\frac{3}{4}$ db.

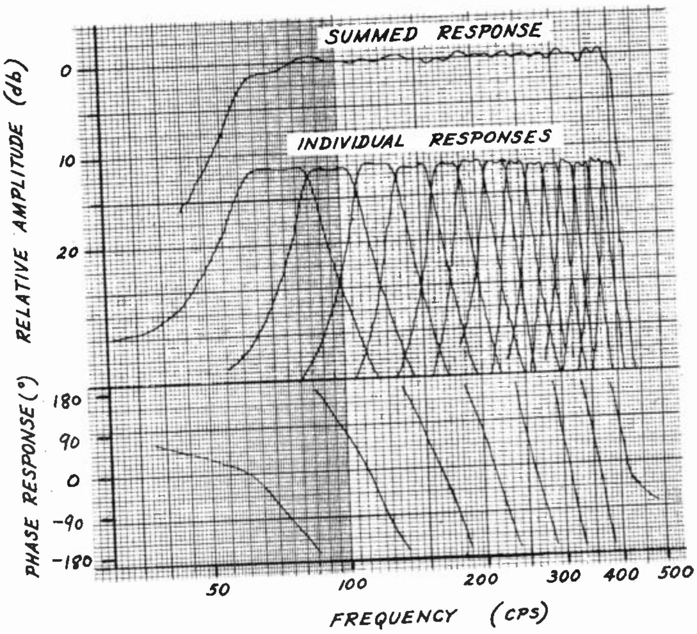

Fig. 10—Summing of the contiguous filter array.

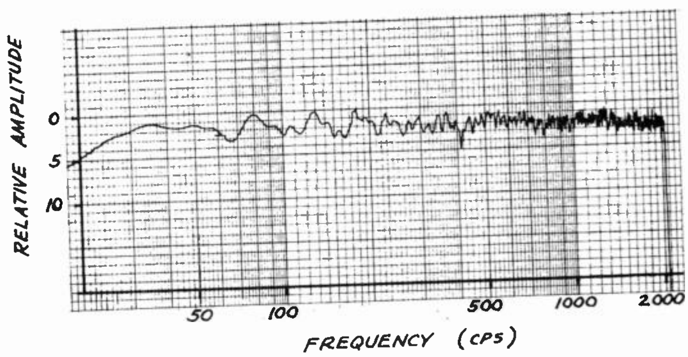

Fig. 11—Combined amplitude response of 80 contiguous filters.

The complete summed array of 80 filters is shown in Fig. 11. Note that the combined ripple is ± 2 db due to frequency bandwidth and amplitude variations.

Factors Affecting the Unequalized Response

Two factors contribute to the shape of the peak-notch characteristic: Q of the resonance; and in the vibration or acoustic systems, the ratio of masses which decouple at each resonance. If the decoupling masses are small, the ratio of peak-to-notch frequencies is also small. The ratio of peak-to-notch frequency in a simple one-degree freedom system is

$$\frac{f_p}{f_n} = \sqrt{1 + \frac{m_r}{m_f}} \tag{4}$$

where

f_p = peak frequency.
f_n = notch frequency.
m_r = resonating mass.
m_f = fixed mass.

Peak-notch amplitude, on the other hand, depends upon both the mass ratio and the Q. If a system of given Q resonates, the amplitude deviation will depend upon the proximity of the peak-notch frequencies. On the other hand, for a given frequency ratio, a higher Q system results in sharper resonances and greater amplitude ratio. Damping at the higher frequencies is dependent primarily upon the molecular structure of the mechanical configuration and it is difficult to control except by choice of materials. Resonances occurring in the low-frequency region are influenced considerably by the output impedance of the vibration generator and associated power amplifier. For example, it is noted that in the vibration system only a notch occurs in the response whenever resonance occurs in the region below exciter electrical resonance (100–400 cps) where the machine is "velocity limited." Thus, the equalization requirements are not as severe in the low-frequency range since the steep slope is eliminated by damping provided by the driving system.

Predicting the unequalized response is impossible since every system is unique. However, some general conclusions can be observed.

1) Resonances occurring in the low-frequency range result in only a notch in the response.
2) Above exciter electrical resonance both the peak and notch exist.
3) Q values in the frequency range below 600 cps are usually less than 25.
4) Q values above 600 cps increase, but the maximum value is usually less than 50.
5) In the high-frequency range the mass which decouples is small, resulting in a small peak-to-notch frequency ratio.
6) The majority of resonances occur above 300 cps.

Results of Equalization

To illustrate the ability of the system to perform a linear interpolation, consider the curve of Fig. 12. This disturbance has a peak Q of 23 and a notch Q of 20. The lower set of curves are responses of the individual filters used to compensate this resonance before adjustments were made. In Fig. 13, the lower curves illustrate the degree of attenuation necessary for each filter in order to achieve compensation. The combined response is shown above. Note that this upper curve should be the inverse of the original disturbance for accurate compensation (see Fig. 12). Finally, in Fig. 14, the combined response of the disturbance and the equalizer is plotted. Note that compensation is flat within $\pm 2\frac{1}{2}$ db.

Although this disturbance does not present a severe challenge to the equalizer, it does illustrate the principle of linear interpolation. A more critical adjustment of the attenuators would yield a better compensation of the resonance.

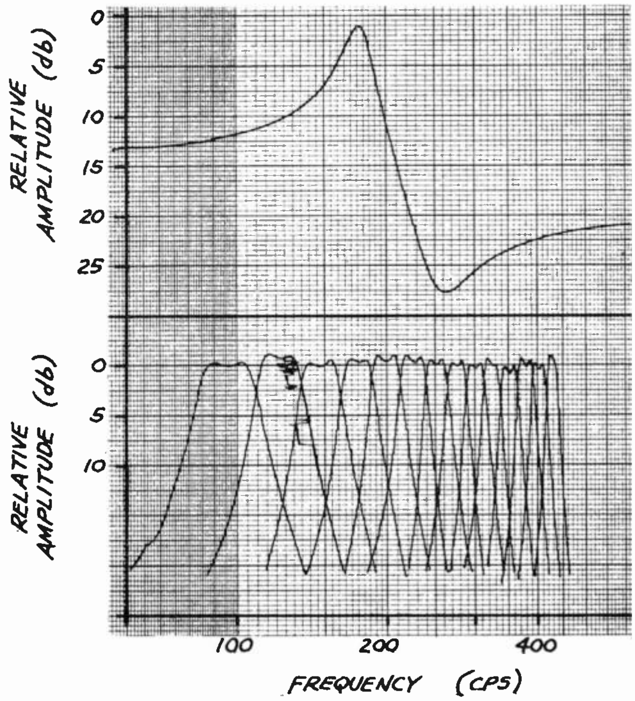

Fig. 12—Filters available to equalize a disturbance.

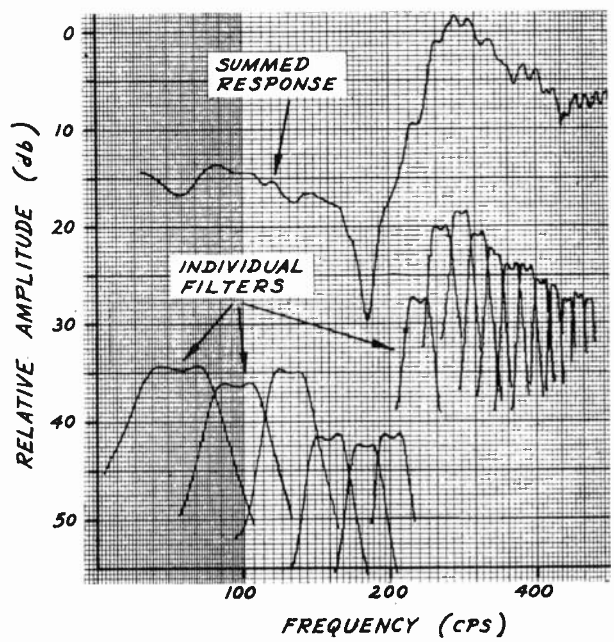

Fig. 13—Equalizer response necessary to equalize disturbance of Fig. 12.

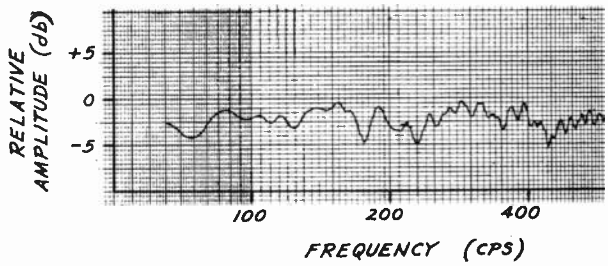

Fig. 14—Disturbance of Fig. 12 equalized.

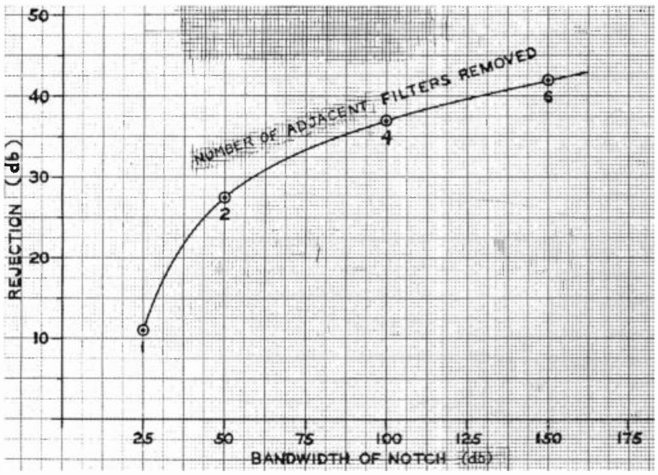

Fig. 15—Bandwidth of notch produced in the response by removal of adjacent filters.

Ability to Produce Notches

Since adding these filters in the multiple array is a function of both amplitude and phase, the rejection due to the removal of filters is not intuitively understood. Removing one filter from the center of the array does not attenuate the response at that frequency to the full range of the instrument since the two adjacent filters will also have a contribution. The resulting notch will, in fact, be 12 db and the bandwidth will be 25 cps. Removal of filters adjacent to this will result in greater bandwidths and attenuations. Responses were taken at various frequencies with successive removal of filters. The resulting bandwidths and attenuation are plotted in Fig. 15.

Peaks can be produced as shown in Fig. 5. The bandwidth will increase in 25 cps increments while the amplitude will remain at 45 db as successive filters are added.

Over-All Performance

The present system provides linear interpolation in vibration testing where random excitation is used. Analysis can be made by means of a sweeping-type analyzer using a single narrow filter. Performance of the system with a typical specimen was examined in this manner as shown in Figs. 16–18. Fig. 16 is the combined response of a vibration exciter and typical resonant load. Fig. 17 is the response of the equalizer after adjustment of the attenuators was made. Note that it is the inverse of the response in Fig. 16. Fig. 18 is the equalized spectrum showing the combined response of the equalizer and vibration exciter with resonant load. Note that compensation was accomplished within $\pm 1\frac{1}{2}$ db.

Automatic Control

The use of this multiple contiguous filter array lends itself easily to automatic control. The attenuators at the output of each filter can be replaced by AGC amplifiers, and an identical set of filters placed in the signal path returning from the vibration exciter (see Fig. 19). Attenuators are then placed in the output of these control filters, and finally the signal is applied to the control input of the AGC amplifier. Thus 80 closed-loop control systems are formed; each one controlling a 25 cps segment of the spectrum.

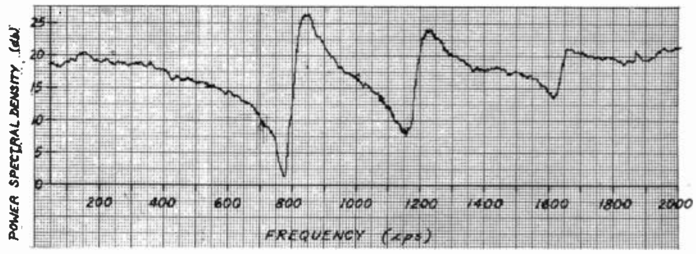

Fig. 16—Unequalized power spectral density (G^2/cps), of a vibration exciter.

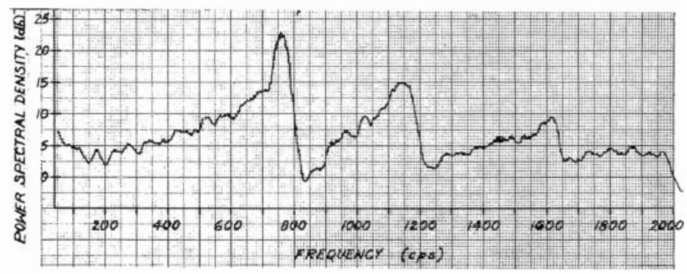

Fig. 17—Equalizer power spectral density (e^2/cps), necessary to compensate response of Fig. 16.

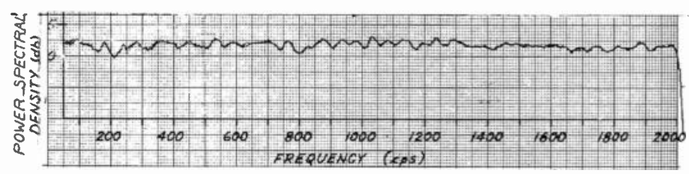

Fig. 18—Equalized power spectral density (G^2/cps) of unequalized system shown in Fig. 16.

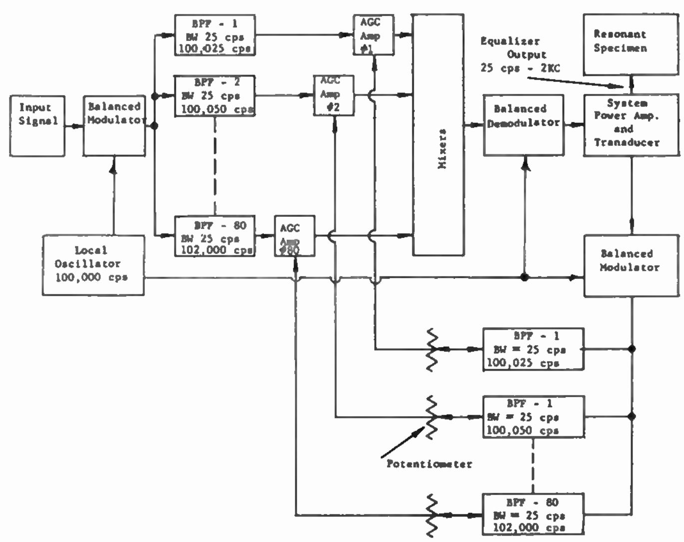

Fig. 19—Block diagram of the automatic equalizer.

The degree of flatness obtained by manual operation is somewhat dependent on the skill of the operator. However, in an automatically-controlled system it is not so. The AGC amplifier is analogous to an extremely skillful operator who anticipates the demands on the equalizer almost as fast as the need arises. Thus, manual and automatic control both have the same potential capabilities, but an operator with the same skill as the 80 AGC amplifiers does not exist. This method of equalization is, therefore, extremely conservative of the operator's time. Equalization is accomplished in seconds—a task which could take considerably longer in a manually-controlled system, depending upon the degree of compensation required and the skill of the operator.

Some types of resonances found in nature are such that their frequency is a function of the drive level. These are termed nonlinear resonances. In a manual equalizer, the equalization must be readjusted for each different excitation level. The automatic equalizer will compensate for changes in equalization automatically as the need arises.

In the course of a test it is a common occurrence for faults to appear in the specimen under test, resulting in a decoupling or partial decoupling of mass. This will normally result in a distortion of the spectrum applied. The automatic system will keep the spectrum constant within the limit of its dynamic range.

Equalization of a typical resonant system by means of this automatic equalizer is shown in a similar set of curves as was used to demonstrate the manual equalizer. A random signal was used to excite the system and experimental curves were made from the sweeping analyzer. Fig. 20 is the response of the vibration exciter and resonant load. Fig. 21 is the inverse response provided by the automatic equalizer. Fig. 22 shows the combined response of the equalizer and resonant system. Note that equalization is obtained within $+1\frac{1}{2}$.

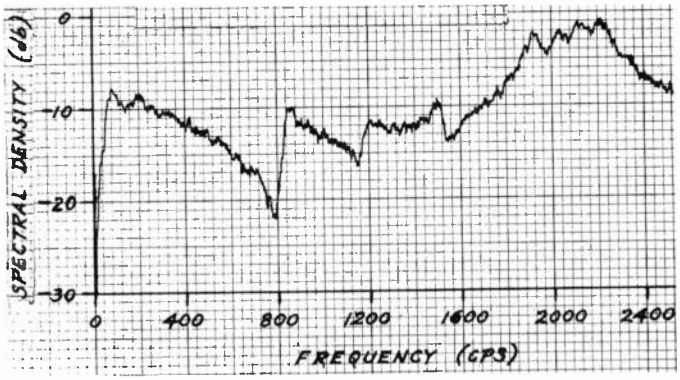

Fig. 20—Power spectral density (G^2/cps) of an unequalized system.

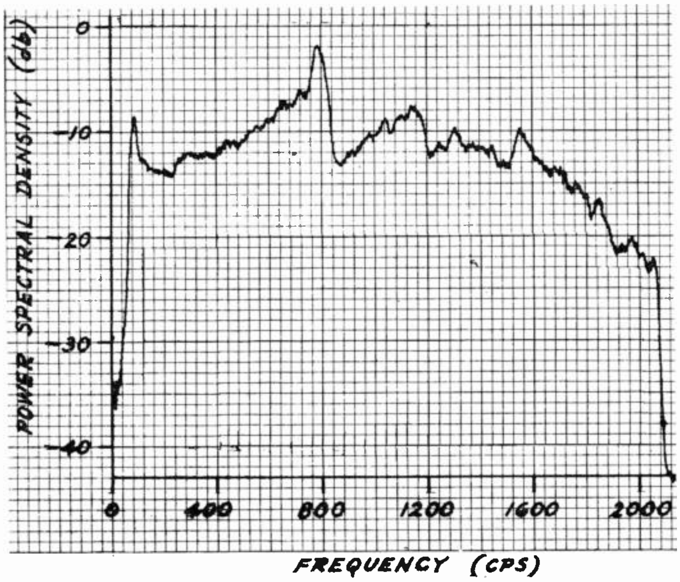

Fig. 21—Spectral density of automatic equalizer (e^2/cps), necessary to equalize disturbance of Fig. 20.

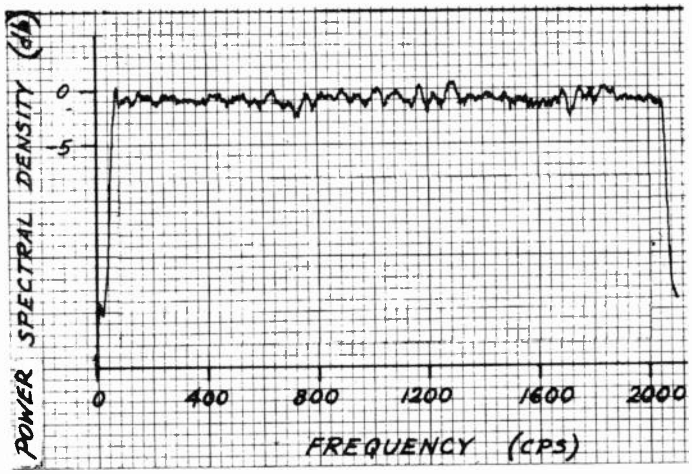

Fig. 22—Equalized power spectral density (e^2/cps), of disturbance shown unequalized in Fig. 20. Equalized by the automatic equalizer.

Extension of the Concept

Extension of the system to programmed tests is also possible with the automatic equipment. Some natural environments such as missile blasts produce a time-varying spectrum. This spectrum can be reproduced in the laboratory, while maintaining equalization by programming the attenuators in the control circuit.

Other extensions of the present system involve decreasing the filter bandwidth at the low end of the spectrum so that improved compensation can be accomplished in that region.

Phase Compensation

Notice that it is impossible to compensate for phase in a multiband equalization system. Parallel signal channels result in a nonminimum phase characteristic and the phase response cannot be determined by the amplitude characteristic. The phase characteristic is independent of amplitude and changes by 360° in exactly two filter bandwidths. This limitation is unimportant if the input is a random-noise voltage or a monosinusoidal signal. However, a square wave will be distorted if applied to the equalized system since the phase response is not linear with frequency.

Conclusions

This system of multiple contiguous filters is offered as a means of compensating for amplitude variations in an audio spectrum. Compensation is a function of two properties, the filter density or number of filters per octave and the slope of the filters. In this present system, constant bandwidth filters dictate that both filter density and slope characteristic increase with frequency. Fig. 7 and 8 indicate the severity of the resonance which can be handled by the array. A typical equalization curve is shown in Fig. 18, where compensation was achieved within ± 1.5 db by use of the system.

Extension to automatic control is easily accomplished by means of AGC amplifiers. Equalization by this method is extremely valuable in saving of time and degree of equalization. It will be superior to the manual system in most applications. Fig. 22 illustrates that a flatness of $\pm 1\frac{1}{2}$ db can be realized by this method.

The present equalizer is used as a means of compensating resonant mechanical systems used in random vibration tests. Present dynamic range of the system is 45 db over which control of amplitude and bandwidth can be accomplished. Extension of these techniques to provide higher resolution in the lower frequency range can be accomplished by providing narrower bandwidth filters.

Use of the linear interpolation concept in equalization problems provides the vibration engineer with a new and powerful tool. The concept can be extended to other areas of control so as to provide functions which have not been available heretofore.

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[3] R. Boynton, "A magnetostrictive filter random wave analyzer," 1960 International Convention Record, pt. 9, pp. 217–226.

Contributors

J. M. Chirnitch

J. M. Chirnitch (S'56–M'59) was born in New Haven, Conn., on February 9, 1929. He received the B.S. degree in electrical engineering from the University of Connecticut, Storrs, in 1958.

In that year he joined MB Electronics, New Haven, Conn., as a Project Engineer working on the design and development of instrumentation used in vibration testing. While there, he was engaged in the design and development of automatic spectral equalizers employing narrow band-pass filters and subcarrier techniques. At present, he is with Lessells and Associates, Inc., Boston, Mass.

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J. J. Davidson

James J. Davidson (S'49–A'52–M'57) was born in New York, N. Y., on July 2, 1930. He received the B.S. degree in electrical engineering from Massachusetts Institute of Technology, Cambridge, in 1952.

Upon graduation he joined RCA, first as a Specialized Trainee, then as a member of the Home Instruments Advanced Development Section in Camden, N. J., where he worked on transistor applications and circuit development of television receivers. In 1954, when the Television and Radio Divisions were split, he joined the Radio Division Advanced Development Section to work on audio and transistor circuits. Transistor noise studies were an important and continuing part of this work.

In 1957, he received the IRE Professional Group on Audio Award for his paper "Low Noise Transistor Microphone Amplifier." Recently he received an award and gold medal from *Semiconductor Products* Magazine for the winning paper in their Circuit Design Award Contest.

In 1960 he transferred to the RCA Victor Record Division, in Indianapolis, Ind., where he has been engaged with problems of disc and tape recording, and instrumentation for magnetic-tape production. He is presently studying music, to gain a fuller understanding and appreciation of all the aspects of music reproduction. Also, he is working on a complete revision of the "IRE Standard on Methods of Measurement of Noise".

Mr. Davidson is a member of the Audio Engineering Society.

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R. G. Fulks

Robert G. Fulks (S'56–M'60) was born in Kansas City, Mo., on April 8, 1936. He received the B.S. and M. S. degrees in electrical engineering from Massachusetts Institute of Technology, Cambridge, in 1959.

He joined the Impedance Group of General Radio Company, West Concord, Mass., in 1959 where he is engaged in the design of low-noise amplifiers for impedance measuring systems.

Mr. Fulks is a member of Sigma Xi and Eta Kappa Nu.

John D. Griffiths (A'58–M'60) was born in Lansing, Mich., on January 23, 1933. He received the S.B. degree from the Massachusetts Institute of Technology, Cambridge, in 1954, and the M.E.E. degree from Syracuse University, Syracuse, N. Y., in 1961.

J. D. Griffiths

From 1954 to 1957, he was with the U. S. Air Force in London, England, where he was Operations Officer, and later Commander of the organization which engineered and installed the ground-based electronics for the USAF in the United Kingdom. This included control towers, ground to air stations, radar air traffic control centers, and navigational aids. Since 1958, he has been with the Rome Air Development Center, AFSC, Griffiss Air Force Base, N. Y., where he was in charge of the speech communications research program in the Advanced Development Laboratory, Directorate of Communications, until 1961. In May, 1961, he became a member of the Advanced Studies Office, DCS/Plans and Operations, where he is concerned with the analysis of future AF system requirements for the development of research planning.

Capt. Griffiths is a member of the Acoustical Society of America.

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J. Ross Macdonald (S'44–A'48–SM'54–F'59) was born in Savannah, Ga., on February 27, 1923. He received the B.A. degree in physics from Williams College, Williamstown, Mass., and the B.S. and S.M. degrees in electrical engineering from the Massachusetts Institute of Technology, Cambridge, in 1944 and 1947, respectively. He was awarded a Rhodes Scholarship from Massachusetts for study at Oxford University, England, where in 1950 he received the D. Phil. degree in physics for theoretical and experimental work in ferromagnetic phenomena.

After war service he returned to M.I.T. for further study and to carry out research on storage tubes for the M.I.T. digital computer project.

J. R. Macdonald

In 1950 he joined the Physics Department of the Armour Research Foundation, and there carried out and directed work in theoretical and experimental physics until 1952. He then spent a year's leave of absence at the Argonne National Laboratory of the A.E.C. working on solid-state physics problems. He is presently Director of the Solid-State Physics Research Department at Texas Instruments, Inc., Dallas. In addition, he is also serving as Clinical Associate Professor of medical electronics at Southwestern Medical School of the University of Texas, Dallas.

Dr. Macdonald is a Fellow of the American Physical Society and AAAS, and a member of Phi Beta Kappa and Sigma Xi.

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C. E. Maki (S'50–A'52–M'53), for a photograph and biography, please see page 236 of the November–December, 1960, issue of these Transactions.

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Robert H. Rose (S'43–A'45–M'54) was born in New York, N. Y., on February 6, 1922. He received the B.S.E.E. degree from Newark College of Engineering, N. J., in 1944, and the M.S.E.E. degree from Stevens Institute of Technology, Hoboken, N. J., in 1949.

From 1944 to 1947, at the I.T. and T. Laboratory, New York, N. Y., he worked on test equipment for remote-control radio, and later on a microwave receiver for a pulsed multiplex system. He joined the Electrical Engineering Department of Newark College of Engineering in 1947. In addition to various undergraduate courses, he has taught electroacoustics, sound recording, and reproducing systems on the graduate level. He took a two-year leave of absence to work as Chief Engineer in a small electronics plant, returning to Newark College in 1958.

R. H. Rose

Mr. Rose is a member of the Audio Engineering Society, the American Rocket Society, the Acoustical Society of America, Eta Kappa Nu, and Tau Beta Pi. He is Chairman of IRE Subcommittee on Definitions (A and E).

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Albert E. Sanderson (A'50–M'54) was born in Bethlehem, Pa., on August 8, 1928. He received the A.B. and A.M. degrees in 1949 and 1950, respectively, from Harvard University, Cambridge, Mass.

A. E. Sanderson

From 1950 to 1957 he was with Aircraft Radio Corporation, Boonton, N. J., working on the development of airborne communication receivers, transmitters, and antennas. In 1957 he joined the Impedance Group of General Radio Company, West Concord, Mass., and has been engaged in the design of low-noise transistorized amplifiers for use as null detectors in impedance bridge measurements.

(*Continued from page 102*)

Jerry P. Christoff, Western Electro-Acoustical Lab., Los Angeles, Calif.

Summary—Veneklasen.

2:00 p.m. Aeronautical Research Laboratory Auditorium, Building 450

Session K: Speech Compression Systems (in cooperation with PGA)
Michel Copel, *Chairman*

Invited Symposium Papers (20 minutes)

K1 **Low Bit Rate Digital Speech Communication,** L. G. Stead, E. T. Jones, and R. C. Weston (presented by Walter Lawrence), Signals Res. and Dev. Est., Christchurch, England.

K2 **Bandwidth Compression of Speech by Spectrum Sampling,** Karl D. Kryter, Bolt, Beranek and Newman, Inc., Cambridge, Mass

K3 **Design vs Performance Factors for Some Speech Compression Systems,** Caldwell P. Smith, AF Cambridge Res. Labs., Bedford, Mass.

K4 **An Analog Semi-Vocoder for Military Use,** Arthur S. Howell, Theodore Stump, and G. O. K. Schneider, General Dynamics/Electronics, Rochester, N. Y.

K5 **Analog Multiplexing of a Semi-Vocoder,** Arthur S. Howell, Theodore Stump, and G. O. K. Schneider, General Dynamics/Electronics, Rochester, N. Y.

Panel Discussion (20 minutes)
Karl Kryter, Moderator

Panel members: Major Nowakoski (Air Force), Joseph De Clerk (Signal Corps), John Swaffield (England), and the previous speakers.

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