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PROFESSIONAL TECHNICAL GROUP ON AUDIO

IEEE PROFESSIONAL TECHNICAL GROUP ON AUDIO

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IEEE TRANSACTIONS ON AUDIO

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The Editor's Corner

WHEN we hinted that universities may be neglecting certain areas in their curricula[1] we were hastily reassured by a letter from Charles A. Ranous, Associate Professor in the Electrical Engineering Department of the University of Wisconsin, Madison, as follows:

"I am sure it will be pleasing to you to learn that, in a small way, we at Wisconsin are covering some of those generally neglected areas.

"We have a course we entitle *Engineering Professional Expression 99*. We include a quick review of current methods for eliminating key parameters from catalog descriptions (Poetry 307); the students themselves supply, without aid, excellent examples of the ambiguous report (Fiction 409); during a proposal project, ample opportunity is made available and fully utilized for optimum degradation of design (Creative Engineering 524); in accord with the late Mr. Dewey's dictums we learn committee inaction by doing (Political Science 404); and we practice the art of making design calculations completely incomprehensible to nontechnical management (Business 574).

"Our graduates have indeed indicated that the course eased their adaptation to the corporate environment. Other areas have been suggested from time to time, but our staff still retains some old-fashioned prejudices about training our graduating seniors to speak and write informatively, so we have not as yet been able to incorporate them. A couple of these suggestions still in abeyance are the following:

[1] IRE TRANS. ON AUDIO, vol. AU-10, p. 155; November–December, 1963.

"ENGINEERING SERVICES

(*Business Management, 333*)

Everyone knows that there are techniques for delaying the delivery of manuscripts from Typing, drawings from Drafting, reports from the Laboratory, and parts from Purchasing and Incoming Inspection. The state of the art is now such, however, that these techniques can be codified and simplified to that *one best way* which education constantly seeks and tenaciously retains.

"INFORMATION CHANNELS

(*Secretarial Science, 419*)

By judicious use and adaptation of the instrument known to the social scientist as the *sociogram* it is now possible to control the grapevine. Consequently, no senior should be allowed to leave the ivy halls without mastery of this technique. The engineering graduate learns without formal instruction, and it is true that he must cultivate the boss' secretary. Without the guidance of theory, however, he may miss the finer points such as the probability that the secretary with whom the boss' secretary has her coffee break may be more approachable.

True, the courses you suggest and ones like these are being pretty well covered in-plant, but it is quite clear that such coverage is slow and unsystematic. If we are to educate for living, our best minds must formulate theory for these practices and arrange them in logically-ordered course syllabi."

MARVIN CAMRAS, *Editor*

PTGA News

CHAPTER NEWS

Boston

On December 12, 1962, Daniel von Reiklinghausen of H. H. Scott and Company, Maynard, Mass., presented a paper to the members entitled "Design Principles of Small, Wide-Range, Direct-Radiator Loudspeaker Systems."

Chicago

On January 16, 1963, Adelore Petrie, of General Electric Company, Decatur, Ill., presented a paper entitled "GE Phono System Test Record."

"The Measurement and Interpretation of Reverberation Times in Auditoriums"was presented on March 8, 1963 by Don Davis of Altec-Lansing Corporation, Anaheim, Calif. As reported in the March issue of *Scanfax and Spice:*

Illustrated with data taken during 20 acoustical surveys during the past years, Davis showed samples of the "best" and the "worst."

Don Davis has specialized in the use of high-fidelity equipment since attending Purdue University. He has owned a chain of high fidelity shops; acted as manufacturer's representative for firms such as Klipsch, Marantz, Grado, and Audiophile; and (during the 1958 World's Fair) organized a series of special high-fidelity demonstrations in Brussels at the request of the State Department.

In 1959, Don and his wife were appointed special consultants to the American National Exhibition in Moscow and were in charge of all audio exhibits at the exhibition. After spending the summer in Russia, Don became regional sales manager, Chicago area, for Altec-Lansing Corp.

Photography, ham radio, writing and sports cars are among Don's hobbies. He has owned 13 different foreign cars and has had experience with major racing events such as the Grand Prix. He is the author of numerous articles on both high fidelity and sports cars. He is a member of the Acoustical Society of America, the Audio Engineering Society, and the IEEE.

Detroit

We have received from the Michigan Chapter of the Acoustical Society of America a notice that on April 10, 1963, Dr. Leo L. Beranek, of Bolt Beranek and Newman Inc., Cambridge, Mass., presented a paper entitled "Recent Experiences in the Acoustical Design of Concert Halls." This Chapter often meets in conjunction with the PTGA Chapter in the Detroit area and so this paper is reported here. It is recommended that many chapters pool efforts to obtain a speaker. This assures the speaker greater exposure and possibility of greater attendance at his paper. Also, it eases the burden of the program committees.

Philadelphia

On November 16, 1962, Sidney Lidz of Dynaco Company, Philadelphia, presented a paper to the Chapter entitled "FM Stereo Multiplex."

Philadelphia also reports that on January 25, 1963, a paper entitled "Design Aspects of the Franklin Hall Sound Reinforcement System" by Lloyd Williams of Bolt Beranek and Newman Inc., and Leon Slawasky of General Sound Company, was presented.

San Francisco

San Francisco reports that Myron Ferguson and Ralph Brown, of Lenkurt Electric Company, San Carlos, Calif., presented a paper entitled "Communication Satellite Problems and Solutions," on September 25, 1962.

Twin Cities

In the Twin Cities of St. Paul-Minneapolis, Karl Kramer of Jensen Manufacturing Company, Chicago, Ill., on December 11, 1962, presented a paper entitled "Trends in Use and Design of Loudspeakers."

Washington

The Washington, D.C., Chapter reported that on December 4, 1962, Richard M. Siefkin, of Delco Radio Division, General Motors Corporation, Kokomo, Ind., presented a paper entitled, "Engineering Considerations in the Design of an FM-AM Automobile Broadcast Receiver."

Note:

It is felt that these are but a few of the papers showing the activities of the local chapters. If any of the local chairmen want anything special reported in the News column, send the article to the *Editor* of these TRANSACTIONS, or at least report your meetings to headquarters.

Headquarters reports that there have been three new sections formed as of February 15, 1963. The PTGA chapters are in Indianapolis, Ind., and Ottawa, Ontario, Canada. The third chapter is in St. Louis, Mo., and is joint with the Chapter on Broadcasting. Congratulations to the men who were instrumental in the forming of these Chapters and welcome to the fold. When your programs are formed let the TRANSACTIONS know and we will be glad to report your activities.

WILLIAM M. IHDE

ANNOUNCEMENTS

PTGA ELECTION RESULTS

Frank A. Comerci, Chairman of the Administrative Committee 1963–1964

William M. Ihde, Vice Chairman 1963–1964, and Chairman-Elect 1964–1965

Iden Kerney, Robert H. Rose, and **John G. McKnight,** Members of the Administrative Committee

SUMMER CONFERENCE OFFERED BY UNIVERSITY OF MICHIGAN

Automatic Speech Recognition, July 8–19, 1963 (10 Class Days) Fee: $275. Gordon E. Peterson, *Chairman. Lecturers:* H. K. Dunn, J. L. Flanagan, J. W. Forgie, D. B. Fry, A. S. House, H. F. Olson, and others.

This course will be an introduction to the control of machines by means of speech. Major subjects to be considered include: the instrumental analysis of speech waves, the interpretation of the information-bearing parameters of speech, and storage and computational procedures which can be used in deriving print-outs for automatic speech recognition. The linguistic elements which must be recognized will be defined, and their relation to acoustical data will be examined. The course will include a foundation in phonetic, phonemic, and grammatical theories which can be applied to the problems of automatic speech recognition. The acoustical characteristics of phonemes and other linguistic units will be discussed. Logical procedures in the interpretation of speech spectra, and digital computer techniques in the interpretation of sequences of linguistic units will also be considered. Basic problems in automatic speech recognition will be analyzed. Various approaches to implementing automatic speech recognition and various automatic speech recognition systems will also be reviewed.

Applicants should have a degree in engineering, mathematics, or a physical science, and a broad interest in language processing. Some experience in research on natural language is desirable.

For complete information write:

Conferences Secretary
Engineering Summer Conferences
West Engineering Building
University of Michigan
Ann Arbor, Mich.

Loudspeaker Testing in Reverberant Rooms*

J. M. EARGLE†, MEMBER, IEEE AND R. J. LARSON‡, MEMBER, IEEE

Summary—In the diffuse sound field of a reverberant room acoustical intensity is directly proportional to the acoustical power emitted by a source and inversely proportional to absorption in the room. If the room and microphone characteristics are known, pressure response curves of loudspeakers can be plotted which are proportional to the total acoustical power radiated by the loudspeakers. The instrumentation procedures used in mean energy density (MED) testing are described in detail with particular emphasis on the facility at the Jensen Manufacturing Company, and the role of MED measurements in the evaluation of loudspeakers is discussed.

Symbols

V = volume of an enclosure.
S = area of enclosure boundaries.
α = average absorption coefficient of enclosure boundaries.
a = total absorption in enclosure in equivalent area of totally absorptive surface.
p = rms acoustic pressure (force per unit area).
p_r = reverberant rms acoustic pressure (force per unit area).
P = acoustic power emitted from a source.
I = acoustic intensity (power per unit area).
I_r = reverberant acoustic intensity (power per unit area).
ρ = density of air (mass per unit volume).
c = velocity of sound in air.
T_{60} = reverberation time (sec); time required for reverberant sound pressure to decay 60 db or one millionth of its initial value.
ρc = acoustic impedance of air.

Introduction

LOUDSPEAKER design engineers have always preferred to test their products in rooms whose acoustical behavior may be described in simple mathematical terms over their useful frequency ranges. This has led to the almost universal acceptance of the anechoic chamber, and although a good anechoic chamber may be a very expensive proposition, its mathematical description is indeed simple. It is absorptive enough so that there is no significant reverberant field; pressure at the microphone is assumed to arise from the direct field only. Over its useful frequency range the room obeys the "inverse square law." This is to say that sound intensity varies inversely as the square of the distance from the source. At low frequencies, depending on size and treatment of the room, the inverse square relationship will not hold due to the setting up of standing waves, and thus the lower frequency limit of the room is set.

A reverberant room on the other hand is one in which the energy in the reverberant field is far greater than that in the direct field. Such a room is made with extremely hard nonabsorptive surfaces so that sound energy must go through numerous reflections before it is effectively attenuated. A reverberant or diffuse field tends to set up a constant energy density throughout the room (except close to the source where the direct field may be significant), and the simple mathematical law obeyed by the reverberant field is that the square of the sound pressure in the room is directly proportional to the energy density. Thus, using a pressure detecting device, it is possible with proper equalization to obtain directly a graph of the total power output of a source as a function of frequency in very much the same manner that sound pressure graphs are obtained in anechoic rooms.

Let us consider the useful properties of each kind of room. The anechoic room gives the design engineer detailed information on the directional properties of a loudspeaker. It tells him how "flat" or "ragged" the response may be in any direction from the speaker. The mean energy density (MED), measurement made in the reverberant room on the other hand gives the engineer an immediate picture of the loudspeaker's total energy output and consequently a direct measure of its efficiency. Furthermore, it is the only practical way of examining the reinforcements and cancellations which may take place at the crossover frequencies in two- and three-way systems. Most listening is done of course in rooms where both the direct and reverberant fields are significant, and it follows that information from both anechoic and MED measurements will be of value to the design engineer.

In addition to MED vs frequency measurements reverberant rooms can be put to other uses. With the aid of a high-speed graphic recorder, absorption characteristics of materials may be measured, and with the aid of a noise generator and band-pass filters, the acoustic sealing characteristics of headphones may be examined. These are problems not ordinarily encountered in a loudspeaker laboratory, but they certainly show the wide range of usefulness of a reverberant room.

Having described briefly the nature and usefulness of MED testing we now turn to a study of the physics involved in the workings of a reverberant room with particular emphasis on the reverberation room at the Jensen Manufacturing Company and its attendant instrumentation.

* Received September 19, 1962; revised manuscript received October 19, 1962. This paper was presented to the Chicago Chapter of IRE-PGA on December 14, 1962.
† Victor Record Division, RCA, New York, N. Y.
‡ Jensen Manufacturing Company, Chicago, Ill.

ACOUSTICAL BEHAVIOR OF REVERBERANT ENCLOSURES

The problem is to show how pressure in a reverberant room is related to the acoustical power being emitted to the room. What we are doing in effect is evaluating acoustically the following integral:

$$P = 1/\rho c \int p_r^2 dA \tag{1}$$

where P is the acoustic power emitted by the source, ρc the impedance of air, p_r the reverberant sound pressure and dA the incremental area of a sphere surrounding the source. This integral relates the acoustic pressure and intensity of a source of sound with the acoustical impedance of the medium. (It is analogous to the electrical expression Power $= E^2/R$.) Its direct solution would involve summing and averaging the square of the sound pressure at incremental areas on a sphere located in the far field surrounding a source of sound. If the source exhibited symmetry in its radiation pattern the task would be lessened, but in any case the computations would be tedious and involved. The reader's intuition may now give him some insight into how the room may be able to perform the integration for us. It is easy to see how energy emitted from the source in *all* directions hits highly reflective surfaces and contributes to an over-all energy density level in the room whose actual value depends upon the rate of energy emission as well as the rate of absorption.

At this point we turn to the work which W. C. Sabine did years ago in architectural acoustics. After long study of the reverberation characteristics of large rooms Sabine arrived at an empirical equation relating the volume of a room, the total absorption present and the time required for reverberant sound to decrease to one millionth of its steady-state value after interruption of the source. The equation is of the form

$$T_{60} = \frac{kV}{a} \tag{2}$$

where V is the volume of the room, a the total absorption in equivalent area of completely absorptive surface and k a constant of proportionality depending upon the system of units employed. In British units the equation is

$$T_{60} = \frac{0.049V}{a}. \tag{3}$$

The reader is referred to a rigorous derivation of (3) due to Knudsen.[1]

Although there are more accurate equations relating absorption, volume and reverberation time when the reverberation time is short, Sabine's equation holds very accurately at middle and high frequencies in fairly large rooms where absorptive material is evenly distributed—in short, in rooms where the sound field is sufficiently diffuse. The equation leads to erroneous results only when there is excessive absorption in the room.

In Sabine's work (which was usually at 512 cps), absorption was computed as αS where α is the average absorption coefficient for the boundaries of the room and S the area of the boundaries. The absorption coefficient α is the ratio of absorbed to randomly incident sound characteristic of a material. In British units the unit of absorption is the sabin and and has the dimensions of area. At the frequency used by Sabine it could be assumed that all absorption took place at the boundaries of a room, but later studies showed that there was a component of absorption due to the air in the room which varies substantially with frequency and rather drastically with relative humidity. Thus, the denominator of (3) is properly composed of two terms, one representing boundary absorption and the other representing atmospheric absorption,

$$\text{Total absorption (sabins)} = a = \alpha S + 4mV. \tag{4}$$

The atmospheric absorption term $4mV$ introduces the atmospheric attenuation constant m which is best gotten from graphs appearing in standard acoustics references.[2] Atmospheric absorption is a phenomenon not entirely understood, and further inquiry into its nature is beyond the scope of this paper. It should be carefully noted that atmospheric absorption is proportional to the volume of air in a room whereas the boundary absorption is proportional to the area of its boundaries. Thus, if we double the dimensions of a room, the boundary absorption term will increase by four while the atmospheric absorption term will increase by eight. Thus, in large rooms with highly reflective surfaces, the atmospheric absorption term can exceed the boundary absorption term at high frequencies. In Table I the total absorption of the Jensen live room is broken down into both its components giving the reader an idea of relative magnitudes of the two terms at different frequencies.

By making measurements of the reverberation time in a live room at frequencies at which the field is fairly diffuse, (3) can be solved for the total absorption present in the room at that frequency. In practice, single frequencies are not used in reverberation time measurements. Narrow noise bands or warbled frequency bands (± 10 per cent) are used since they provide more even decay characteristics than do single frequencies. There are numerous ways of examining sound decay characteristics, but perhaps the most direct is with a high-speed graphic recorder. If the recorder has logarithmic response, the exponential decay is plotted as a straight line whose slope can be easily measured. The reverberation time can then be computed from the slope. Using a Sound Apparatus Model SL-4 graphic recorder rever-

[1] V. O. Knudsen [1], pp. 120–128.

[2] L. L. Beranek [2], pp. 309–310.

beration times were measured in the live room in the Engineering Division of the Jensen Manufacturing Company. Total absorption in sabins was computed directly from (3), and the total absorption was further broken into its two components by means of standard graphs for this purpose. All measured and computed values pertinent to the room are shown in Table I.

The Jensen live room (see Fig. 1) has dimensions of 24.8 ft×14.5 ft×10.9 ft. The walls are of sand-filled concrete block covered with hard plaster, and the floor is of poured concrete. Total volume is 3940 ft^3, and the boundary area is 1576 ft^2. Sacrifices had to be made in the construction of the room because of budget limitations, and the result is a ceiling that is not as rigid structurally as the other boundaries. Its resonances however are in the frequency range of the lowest room modes and are consequently out of the useful range of the room thus causing no concern.

TABLE I

Characteristics of Jensen Reverberation Room at 66 per cent Relative Humidity

Frequency (cps) (Warbled ± 10 per cent)	T_{60} (sec)	Absorption in Sabins			α Boundary
		a_{Total}	$a_{Boundary}$	$a_{Atmosphere}$	
10,000	1.20	160	42	118	0.027
9,000	1.40	137	31	106	0.020
8,000	1.53	126	32	94	0.020
7,000	1.87	103	27	76	0.017
6,000	2.06	93.5	33.5	60	0.021
5,000	2.54	75.4	31.4	44	0.020
4,000	2.60	74.2	39.2	35	0.026
3,000	3.53	54.6	34.6	20	0.022
2,500	3.67	52.6	36.6	16	0.023
2,000	4.00	48.2	37.2	11	0.024
1,500	4.53	42.5	34.5	8	0.022
1,000	4.80	40.2	35.2	5	0.022
800	5.20	37.1	37.1	—	0.023
600	5.68	34.0	34.0	—	0.022
400	4.46	43.2	43.2	—	0.027
200	4.80	40.2	40.2	—	0.025
100	3.12	62.0	62.0	—	0.040

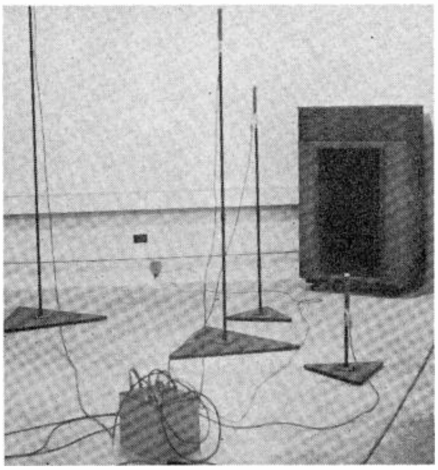

Fig. 1—Photo of Jensen live room showing commutator and part of 8 microphone array.

Equalization and Calibration of the Live Room

If the reverberant sound pressure in the live room is recorded as a function of frequency, two corrections must be added to the readings if they are to be proportional to energy density at all frequencies. These corrections are for the random incidence diffraction characteristics of the microphone and for the absorption characteristics of the room. Correction curves for a variety of sound field conditions are usually available for microphones commonly used for acoustical measurements. The correction curve for the room absorption is of course different for each room and is calculated from the relation

$$I_r = P/a \tag{5}$$

where P is the power emitted by the source, I_r the diffuse intensity in the room and a the total absorption. Thus, for a constant power input to the room the intensity will vary inversely as the absorption. If the absorption is doubled the intensity will drop 3 db. Therefore, the necessary absorption equalization will be proportional to a plot of 10 log a vs frequency. Fig. 2 shows the equalization which must be added to a Western Electric 640AA microphone that has already been equalized for perpendicular incidence so that its output in a diffuse field will be proportional to pressure. Fig. 2-B is the equalization demanded by the room absorption characteristics and Fig. 2-C is the sum of the two curves. If the correction shown in Fig. 2-C is added to the output of the Western Electric 640AA then its output on a decibel basis is in fact proportional to the energy density in the room over the useful frequency range.

Since the absorption equalization curve depends upon the total absorption present, the equalization requirements will certainly vary if there are significant changes in the atmospheric absorption term due to fluctuations in relative humidity. Most live rooms are in

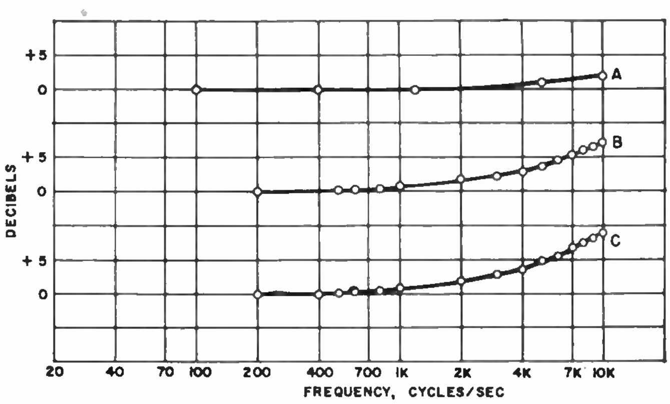

Fig. 2—A, perpendicular incidence to random incidence correction curve for 640AA microphone. B, room absorption correction curve. C, sum of curves A and B.

fairly controlled environments, and humidity variations can easily be kept within reasonable limits. The data of Table I were taken at a relative humidity of 66 per cent, and the atmospheric absorption is seen to be 118 sabins at 10,000 cps. At a relative humidity of 40 per cent the atmospheric absorption term would be 159 sabins. This difference is sufficient to lower the reverberation time from 1.20 sec to 0.96 sec and corresponds to a change in the absorption equalization characteristic of about 1 db. If humidity variations cannot be adequately controlled, corrections could easily be plotted for the humidity range likely to be encountered.

The calibration of the measuring system is concerned with the determination of just what pressure level in the room corresponds to what acoustical power output of the source. This can be done indirectly with a standard speaker, usually a horn-loaded midrange unit whose efficiency is accurately known. Since a known electrical power input to the horn will provide a known acoustical power output, the gain of the measuring system may then be set so that some convenient ordinate on the graph of the recorder corresponds to the acoustical level in the room. Another method, one perhaps more in keeping with the other instrumentation techniques employed so far, is to solve the expression

$$I_r = \frac{kp_r^2}{\rho c} \tag{6}$$

where I_r and p_r are respectively the reverberant intensity and pressure in the room and ρc the acoustic impedance of air. Eq. (5) is substituted into the above expression which then reduces in decibel notation to

$$\text{Acoustic Power Level} = \text{Reverberant Pressure} + 10 \log a - 106.5 \tag{7}$$

where the acoustic power level is expressed in db relative to 1 mw, the pressure expressed in db relative to 0.0002 microbar and a the total absorption in sabins.[3] There should be reasonably good correlation between these two methods if the original efficiency measurements on the standard were accurate and if the reverberant pressure levels were carefully made. In practice the first method is used in setting the gain of the system. The standard source is usually mounted in the room at some appropriate position, and is excited to produce a standard power level. The gain of the measuring system is set accordingly.

Extending the Usefulness of the Room at Low Frequencies

The response of a live room at low frequencies is erratic since the foregoing assumptions, valid as they are for middle and high frequencies, simply do not hold any longer. We must depart from Sabine's approach and consider what happens to sound in a "small" enclosure, a room whose dimensions are comparable to the wavelengths being considered.

A rectangular enclosure has normal modes of vibration given by the following equation:

$$f_{\text{normal}} = c/2\sqrt{\left(\frac{n_x}{l_x}\right)^2 + \left(\frac{n_y}{l_y}\right)^2 + \left(\frac{n_z}{l_z}\right)^2}$$

where c is the speed of sound, $l_{x,y,z}$ the dimensions of the room and $n_{x,y,z}$ separately chosen integers from zero to infinity. At low frequencies these modes may be spaced appreciably apart, whereas at high frequencies they are so closely spaced as to form almost a continuum. It is of course at these high frequencies where the room becomes effectively "large" and where Sabine's assumptions are valid. Because of low absorption, the lower frequency modes may be very sharply tuned thus giving rise to pronounced peaks and dips in the reverberant sound pressure even though a source with constant power output is used to excite the room. If there are structural resonances connected with the room, energy will be absorbed at their characteristic frequencies giving rise to further fluctuations in sound pressures.

Attempts to search out the low-frequency characteristics of a room by means of reverberation time measurements are usually futile since the decays tend to take place only at natural modes regardless of the frequency of excitation. Thus, if the room is excited at some frequency halfway between two natural modes, the decay will show the effects of beats between the adjacent modes. It is even likely that the boundary absorption may vary significantly between two adjacent modes so as to give rise to different reverberation times. For these reasons it is often difficult to secure recorder traces whose average slope is meaningful or even measurable.

A convenient rule seems to be that a live room produces fairly diffuse fields above a frequency about ten times that of the lowest natural mode for the smallest room dimension. This mode of the Jensen live room is approximately 40 cps, and it should be possible above 400 cps to sample the pressure in the room with a single microphone. If a ± 10 per cent warble of the frequency is used this is certainly true. Below 400 cps something more is needed in addition to warbling to give accurate pressure sampling. The microphone may be slowly rotated and its output averaged by a well-damped meter. Another method employs a number of randomly placed microphones whose outputs are commutated and averaged; this gives good results down to about 100 cps. Below 100 cps the commutated microphone arrangement is not satisfactory, even with frequency warbling. Large rotating vanes which continuously change the

[3] H. C. Hardy, H. H. Hall, and L. G. Ramer [6], pp. 99–107. See also, the Appendix.

room dimensions are often used to extend the lower cutoff frequency. If the frequency sweep rate is slow compared to the rotation of the vane, then at each warble band the recording system will register an average of the pressures characteristic of all the dimensional changes produced by the vane rotation. This method, depending on the vane size, will lower the cutoff frequency down to the range of the lowest natural modes. Another means of promoting diffusion at all frequencies is the placement of hard cylinders of various sizes about the room. Finally, it should be said that parallel surfaces in live rooms are undesirable, they may be unavoidable in many cases where existing structures are modified.

There are no diffusing cylinders or vanes in the Jensen live room, but there are plans to incorporate them eventually. One Western Electric 640AA microphone is used to explore the frequency range above 400 cps, and 8 commutated Electro-Voice 654 microphones are used from 400 cps down to 100 cps. The commutated set will eventually be used for sampling at lower frequencies when a rotating vane is added to the system.

Technique of Measurement

After the system is readied for operation, a known voltage is applied to a standard horn mounted in the room. The efficiency of the horn has been accurately determined from motional impedance measurements, and consequently the reverberant power level in the room is known. The gain of the recording system is then adjusted so that the stylus of the plotter is set at an ordinate corresponding to the acoustic power level. See Fig. 3 for a block diagram of the measuring system. The speaker to be tested is then placed in a corner of the room and a Western Electric 640AA microphone placed on the opposite side of the room. A known voltage is applied to the terminals of the speaker, and a frequency run is made from 400 cps to 10,000 cps. The frequency is warbled at a rate of 5 cps with a frequency deviation of about ±10 per cent. The time taken for the run is about 5 min. The run results in a rather jagged curve (see Fig. 4 for a sample curve on a full-range system) which may be easily averaged. To this averaged curve is added the microphone and absorption corrections discussed earlier.

Another curve is run on the system over the range of 100 cps to 400 cps using the 8 commutated microphones. It is averaged and joined to the first curve for a composite picture of the system's response (see Fig. 5).

The technique which has been described is quite rigorous; it is used when the power output of a loudspeaker must be accurately plotted over the entire frequency range. The method is cumbersome since it necessitates making two runs which must then be matched together for complete graph. Often in design work it is desirable to know the *difference* in response caused by small changes in design. What the engineer then looks for are the differences between two response curves rather than the absolute value of either curve. This statement applies equally to anechoic and MED measurements, and with this consideration in mind a variant technique for running MED curves has been devised which uses the 8 commutated microphones through the entire range. Their output is fed through an equalizer which corrects in part for their response as well as for the room absorption characteristic. Commutation tends to smooth out the response variations in the 8 microphones since the peaks and dips in one micro-

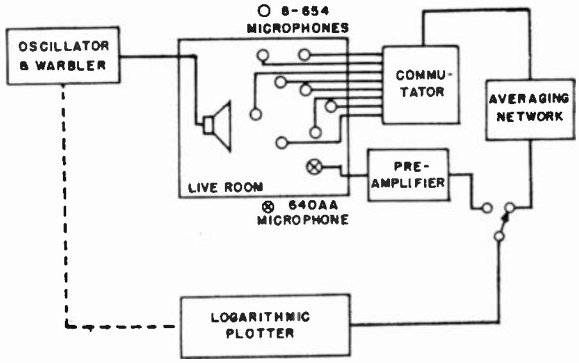

Fig.3—Block diagram of MED system.

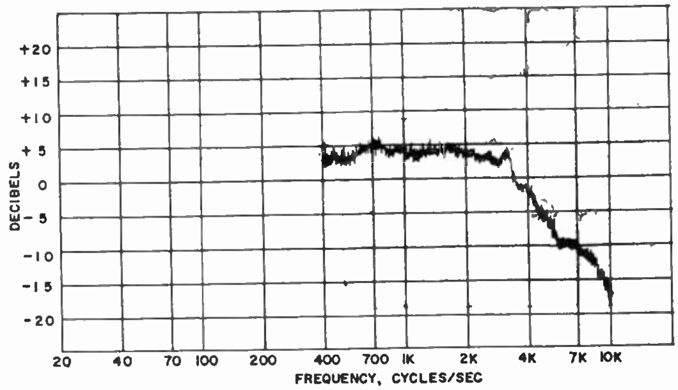

Fig. 4—Raw curve (unequalized) of MED vs frequency for bookshelf speaker system from 400 to 10,000 cycles.

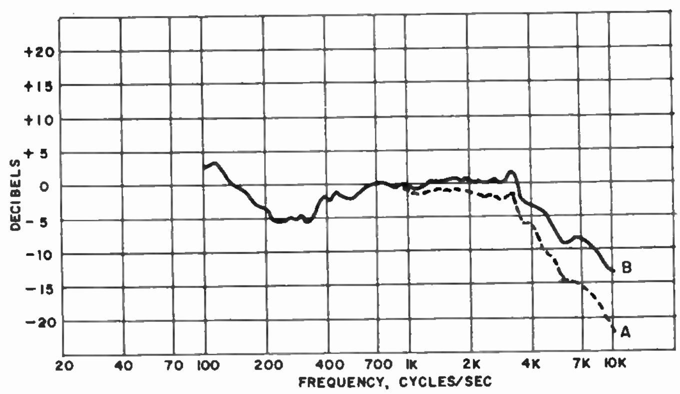

Fig. 5—MED of bookshelf speaker showing *A*, smoothed curve, and *B*, with correction for room and microphone. 0 db corresponds to 4.6 acoustical mw. Input to speaker was 1 available watt.

phone will not necessarily correspond to those of another. The output is not entirely smooth relative to the Western Electric 640AA, but its deviations are certainly within reasonable limits.

With this technique a laboratory calibrated noise source is often used to set the gain of the recording system. This is a modified squirrel-cage exhaust fan minus its outer case rotated at twice its normal speed. The device, manufactured by the ILG Ventilating Company, Chicago, Ill., was developed by Howard C. Hardy and Associates, consultants in acoustics. Its noise output, which is characterized by fairly equal energy per octave from 75 to 10,000 cps, is remarkably constant over a wide range of line voltage and temperature.[4]

Evaluation of Measurements

It was mentioned earlier that MED measurements provided a ready method of determining the efficiency of a speaker as a function of frequency. Actually, if a well regulated voltage is maintained at the terminals, the resulting curve is more properly referred to as the "power available efficiency vs frequency." The power available efficiency is the ratio of acoustical power radiated to the maximum power which the source can supply to the loudspeaker under test. It is a notion which involves the impedance of the generator and the complex impedance of the loudspeaker. This is more meaningful than a curve showing the absolute efficiency since loudspeakers are customarily driven by amplifiers of low source impedance. If the regulation of the amplifier as well as the resistive and reactive components of the loudspeaker impedance as a function of frequency are known, then the power available efficiency can be easily corrected to absolute efficiency.

In commerical loudspeaker manufacturing exact efficiency measurements are of academic interest only. They enter into this discussion only in that they provide an excellent test of how accurately the room and microphone characteristics have been compensated for. Efficiency measurements in the Jensen MED room are usually within 1 db of those made by motional impedance measurements made in a vacuum tank.

The MED room finds great utility in conjunction with anechoic measurements in getting a quick idea of the extent of polar sharpening of response at high frequencies without the labor and time involved in making polar plots in the anechoic room. It is possible for a speaker to exhibit beautifully flat response on axis and yet sound quite dull under normal listening conditions due to a rapid fall-off of energy output with rising frequency. Fig. 6 shows this dramatically. Fig. 6 shows the on-axis anechoic response of a 6 inch×9 inch single cone speaker which sells for perhaps $10.00. It is certainly an "extended range" speaker in one sense. Fig. 6 also shows the response of the unit in the MED room, and it

[4] H. C. Hardy [9].

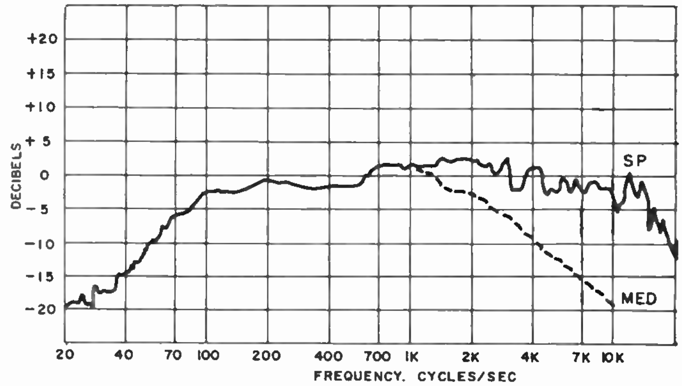

Fig. 6—Comparison of anechoic chamber on-axis sound pressure with MED of an "extended range" 6 inch×9 inch speaker.

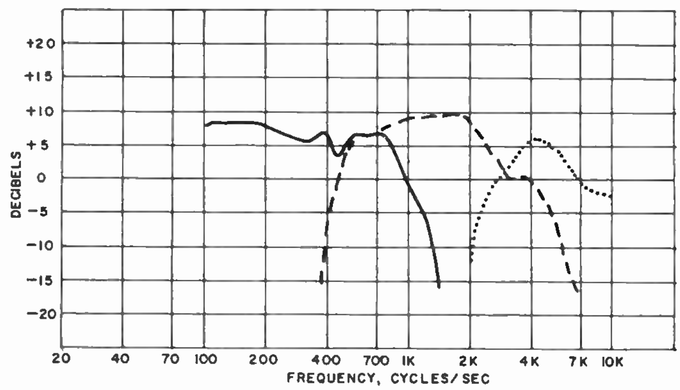

Fig. 7—MED curves for the three channels in a Jensen Imperial.

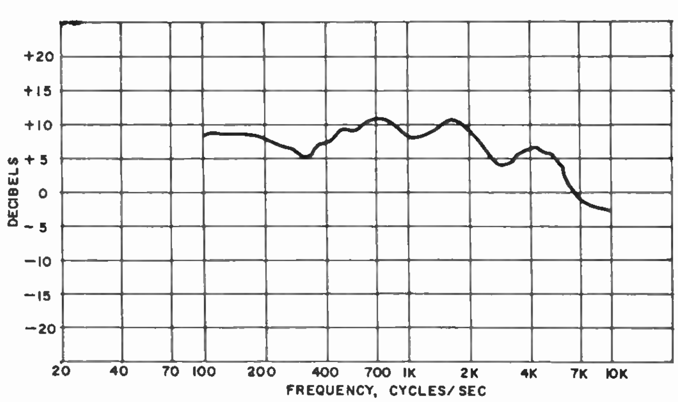

Fig. 8—MED curve for complete Jensen Imperial system.

shows a steadily decreasing energy output as frequency rises characteristic of a progressively narrowing radiation pattern. It is suggested that this speaker would be excellent for monitoring purposes where it would be listened to on axis at close range and at fairly low power levels. Under these conditions the reverberant field would be only a small part of what the listener hears, and the falling off of energy at high frequencies would not be apparent.

Perhaps the most important application of MED measuring techniques is in examining crossover phenomena in 2- and 3-way systems. This is the only way that the response of a multispeaker system can be measured since anechoic measurements of the entire system would give rise to complex interferences between units. Fig. 7 shows MED curves run separately on the components of a Jensen Imperial Laboratory reference standard system, each component fed through its appropriate crossover network. Fig. 8 shows the MED response of the complete system. Note that the transition from midrange to tweeter takes place very naturally; the difference in absolute level at the crossover point is very close to 3 db implying that the two units are acoustically in phase through the transition range. This is possible with a 12-db/octave network. The woofer-midrange transition is essentially the same; however, there appear to be some slight but irreconcilable differences between the two curves, notably the dip at 1100 cps and the rise at 450 cps in Fig. 8.

Perhaps a comment is in order on the shape of MED curves in general. A number of full-range systems have been tested and almost without fail they show a drop in high-frequency output above about 4000 cps. This statement implies nothing about the on-axis pressure response of these speakers made under anechoic conditions. It implies only that polar sharpening at the high end is almost inevitable in tweeter design. This may appear undesirable at first, but a moment's reflection leads to a different conclusion. The decrease in high-frequency energy output may be adequately compensated for by the unnaturally high treble preemphasis which finds its way into many records and by the accented treble which is inherent in the close microphone placement so popular today in the recording industry.

Conclusion

An MED vs frequency measuring system, its instrumentation, and application to loudspeaker engineering have been described in detail. With the aid of observations made in this paper and in the cited references, loudspeaker engineers can easily design and build such facilities for their own work.

At the present time MED measuring of loudspeakers has not been generally accepted as a necessary tool in design work, but there are indications of a shift of interest in this direction. Loudspeaker design philosophy is certainly not a static thing, and the advent of stereo seems to have brought a heightened awareness of dispersion, directionality and balance in loudspeakers which did not exist before. Certainly in this regard MED testing can take its place as a valuable and time saving tool.

Appendix

Derivation of (7)

In a diffuse field the following relation holds:

$$I_r = P/a$$

where I_r is the reverberant intensity, P is the acoustic power absorbed in the room and a is the total absorption in the room.

In general

$$I_r = \frac{kp_r^2}{\rho c}$$

where I_r is the reverberant intensity, p_r is the reverberant sound pressure, ρc is the acoustic impedance of air and k is a constant of proportionality.

Substituting, we get in cgs units

$$P = \frac{p_r^2 a}{4\rho c}$$

where P is power in ergs/sec, p_r is pressure in dynes/cm², a is the total absorption in cm² and ρc is the acoustic impedance of air in grams/cm² sec.

Since power ratios in decibels are defined as

$$\text{db ratio} = 10 \log \frac{P}{P_0}$$

we have

$$\text{Acoustic Power Level} = 10 \log \left[\frac{p_r^2 a}{4\rho c P_0}\right].$$

Since we want our reference power P_0 to be 1 mw we recall that

$$1 \text{ mw} = 10^4 \text{ erg/sec.}$$

Also, absorption is usually measured in ft² (sabins) rather than cm², so we recall

$$1 \text{ ft}^2 = 929 \text{ cm}^2.$$

Finally, we wish to measure pressure relative to 0.0002 dynes/cm².

Substituting,

$$\text{Acoustic Power Level} = 10 \log \left[\frac{p_r^2 a(929)}{164 \times 10^4 \times (5 \times 10^3)^2}\right].$$

Therefore,

$$\text{Acoustic Power Level} = \text{Reverberant Pressure Level} + 10 \log a - 106.5.$$

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Tracking Distortion as Phase Modulation*

DUANE H. COOPER†

Summary—**In support of the urgency for establishing a standard for the vertical tracking angle in stereo disc recording, the phase perturbing character of tracking error distortion is shown, and used to calculate the phase and amplitude intermodulation distortion (PIM and AIM) and harmonic distortion arising from tracking errors. These results are compared with those of other authors. Phase cross modulation (PXM) between the stereo channels is demonstrated and offered as the basis for measurement of tracking errors. Psychoacoustic consequences are considered.**

Introduction

Although mathematical analyses of the distortion arising from tracking-angle error in phonograph reproduction have been given[1,2] and are rather well known,[3,4] it does not appear to have been so widely noticed that this distortion arises from PM. The PM character is nevertheless easily demonstrated, and it has always provided the basis for the calculation of harmonic content of a single tone, as well as the intermodulation products resulting when two tones are present.[1] Such a conceptually simple view of the process makes it easy to devise means of measuring the tracking distortion using Lissajous figures on an oscilloscope, for example, distinguishing the resulting PXM (phase cross-modulation)[5] from distortion leading to AIM (amplitude intermodulation), such as tracing distortion.[6]

A fuller appreciation of tracking-angle-error distortion has become of greater importance since the advent of stereo recording. There has been no universally accepted standard for the vertical tracking angle, so that we have been presented with tracking errors running to dozens of degrees. Stereo cutters are designed for vertical angles ranging from true vertical, or zero degrees (Decca-London)[3] to 23 degrees (Westrex),[4] but, because of lacquer spring back, these same angles are measured to range from 18 degrees negative to a small positive angle near 3 degrees.[7] Cartridge vertical tracking angles seem to range from 0 to 40 degrees.[3] The corresponding distortion percentages are startlingly large. Our ears have sometimes been offended, but not, apparently, to the degree the figures would suggest. Can it be that the blessings of stereo override the offense, or is it the case that our ears are less troubled by PIM than by AIM?

In either case, it is clearly urgent that a standard be established for vertical tracking angle on an international basis. It is hoped that the analyses given in the present article will help in this matter, if they only serve to emphasize the urgency.

Mathematical Structure of Tracking Distortion

Consider an arbitrary wave form

$$y = f(x).$$

The y values may be thought of as the displacements of the groove from its quiescent position, depending upon the position x along the groove. The form of the wave is given by the function f. Let it be supposed that this waveform has been "plotted" by the recording cutter in rectangular (for the present) coordinates, but that this same waveform is to be traced by an ideal stylus in skew coordinates, with skew angle ϕ. In Fig. 1, the skew angle is shown; it is the tracking error.

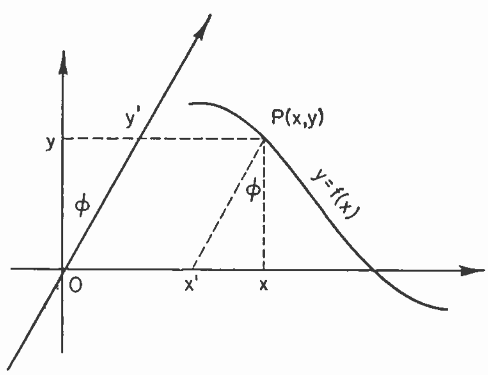

Fig. 1—The transformation from rectangular coordinates to skew coordinates.

In skew coordinates, each displacement y' corresponds to a position along the quiescent groove axis x', connected by some new function which is to be expressed in terms of f. It is observed that

$$y' = y/\cos\phi,$$

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and that

$$x = x' + y \tan \phi.$$

Thus, since

$$y = f(x) = f(x' + y \tan \phi) = f(x' + y' \sin \phi),$$

the result is

$$y' = f(x' + y' \sin \phi)/\cos \phi, \qquad (1)$$

which is to say that y' depends upon a shifted variable. The shift is modulated by the y' value itself. As Baerwald observed, it is just this modulated shift which accounts for the modulated phase.[1]

Phase Intermodulation Effects

With the help of these preliminaries, it is easy to calculate the degree of PIM. Suppose the cutter had engraved a small-amplitude "carrier" (amplitude a_c, wavelength λ_c)

$$a_c \cos 2\pi x/\lambda_c, \qquad (2)$$

together with a large-amplitude low-frequency sinusoid

$$a_m \sin 2\pi x/\lambda_m. \qquad (3)$$

The choice between sine and cosine is arbitrary and does not affect the indexes to be calculated. The sum of (2) and (3) is y. If the low-frequency term in y' is not written, there remains from (1)

$$\frac{a_c}{\cos \phi} \cos \frac{2\pi}{\lambda_c} \left\{ x' + \left[a_m \sin \frac{2\pi}{\lambda_m} (x' + \cdots) \right.\right.$$
$$\left.\left. + a_c \cos \frac{2\pi}{\lambda_c} (x' + \cdots) \right] \tan \phi \right\},$$

and supposing, as assumed, that a_c is very small compared to a_m, then there remains only

$$\frac{a_c}{\cos \phi} \cos \left\{ \frac{2\pi x'}{\lambda_c} + \frac{2\pi a_m \tan \phi}{\lambda_c} \right.$$
$$\left. \cdot \sin \left[\frac{2\pi x'}{\lambda_m} + \frac{2\pi a_m \tan \phi}{\lambda_m} \sin \frac{2\pi}{\lambda_m} (x' + \cdots) \right] \right\}.$$

In this expression, as in the preceding, the ellipsis (. . .) has been used to stand for the filling out of the rest of the argument of the trigonometric functions, continuing in the same manner already started.

From this expression, it is seen that the carrier term is phase modulated by a distorted version of the low-frequency wave. The distortion of low-frequency wave itself is the harmonic distortion already analyzed elsewhere.[1] Its analysis by the present methods will be discussed later. Setting that distortion aside for the moment, the remaining high-frequency term is

$$\frac{a_c}{\cos \phi} \cos \left[\frac{2\pi x'}{\lambda_c} + \frac{2\pi a_m \tan \phi}{\lambda_c} \sin \left(\frac{2\pi x'}{\lambda_m} \right) \right],$$

the classical form for PM. The modulation index is

$$\beta_m = (2\pi/\lambda_c) a_m \tan \phi. \qquad (4)$$

In terms of this index, the phase-modulated wave has the form

$$\cos \left[(2\pi/\lambda_c) x' + \beta_m \sin (2\pi x'/\lambda_m) \right]. \qquad (5)$$

It is known that the PM sidebands each have strength $\beta/2$, since there are only two for small β, and that this corresponds to the AM case in which the two sidebands have strengths $m/2$ for 100-m per cent modulation.[8] The only difference in the two cases is that the lower sideband is reversed in sign for the PM case relative to the AM case. (Alternatively, it may be said that the PM carrier is in phase quadrature with respect to the AM case for small β.) The result is that for β not so large as one radian, the percentage PIM should be taken to be

$$100\beta_m = \text{percentage PIM}. \qquad (6)$$

Large values of β (*i.e.*, larger than unity) need a more elaborate interpretation. This is indicated later.

It is instructive to consider a numerical example. Let ϕ be 15 degrees, then $\tan \phi$ is 0.27. For a groove speed of 30 cm/sec, a 1000-cycle carrier has a wavelength of 0.03 cm. Let the modulating tone have a peak amplitude of 0.01 cm, corresponding for a 100-cycle tone to a peak recording velocity of about 6.3 cm/sec. This is a high level, to be accommodated with larger than normal groove spacing, as is sometimes provided. The modulation index is

$$\beta_m = \frac{2\pi}{3} \times 0.27 = 0.56,$$

or 56 per cent. This is a peak phase shift of 0.56 radian or 32 degrees. The sidebands resulting from this modulation are at 900 cps and 1100 cps and have an amplitude which is 28 per cent of the original carrier amplitude. The amplitude of the carrier is 92 per cent of its unmodulated value because of the large value of the modulating index. There is an FM corresponding to this PM. The peak frequency deviation is $\beta \times 100$ cps or 56 cps; this is 5.6 per cent of the carrier frequency, corresponding to a peak flutter index of 5.6 per cent. Also, the harmonic distortion of the 100-cps tone, to be discussed shortly, is mostly second harmonic, amounting to 2.8 per cent. Further, there is an AIM to be discussed in a latter section, which is just 5.6 per cent.

The formula for β_m can be expressed in terms of the peak slope $\tan \alpha_m$ of the low-frequency tone. This slope is

$$\tan \alpha_m = 2\pi(a_m/\lambda_m) = v_m/s, \qquad (7)$$

where v_m is the peak recording velocity, and s is the groove speed. The peak PM index is

$$\beta_m = (\lambda_m/\lambda_c) \tan \alpha_m \tan \phi = (\nu_c/\nu_m) \tan \alpha_m \tan \phi, \qquad (8)$$

[8] F. E. Terman, "Radio Engineering," McGraw-Hill Book Co., Inc., New York, N. Y., 3rd ed., pp. 483–493; 1947.

in which ν is used to denote frequency in cps. PM produces FM whose peak deviation is

$$\Delta\nu = \nu_m \beta_m. \tag{9}$$

This deviation, expressed as a fraction of the carrier frequency, is $\tan \alpha_m \tan \phi$. When expressed in per cent, it corresponds to a peak flutter index.

Harmonic Distortion Effects

For a single tone the present formulation yields for the distorted waveform

$$\sin\{(2\pi/\lambda)x' + (2\pi a/\lambda)\tan\phi \sin[(2\pi/\lambda)x' + \cdots]\}. \tag{10}$$

In this, it is seen that the PM comes at the same frequency as the carrier. The modulating waveform even has the same distortion as the resulting effect upon the carrier wave. Here, the automodulation index is

$$\beta = (2\pi a/\lambda)\tan\phi = \tan\alpha \tan\phi. \tag{11}$$

Again, the upper and lower sidebands have amplitude $\beta/2$. The lower sideband cancels out, in a manner to be seen later, and the upper sideband lies at the second harmonic frequency. Thus the percentage second harmonic is

$$100\,\beta/2 = 50 \tan\alpha \tan\phi, \quad = 50\,(2\pi a/\lambda)\tan\phi. \tag{12}$$

This same result was given by Bauer,[2] following Baerwald,[1] under the approximation $\tan\phi \approx \phi$ radians. He argued that since λ is proportional to radius, more tracking error can be tolerated at the greater radii. He also pointed out that for the case of velocity response, there is twice as much sensitivity to the amplitude of the second harmonic, so that the factor $\frac{1}{2}$ should be removed. The above formula stands in the case of amplitude response, and the alternate formula $100(2\pi a/\lambda)\tan\phi$ is for velocity response. With RIAA equalization, however, all but the middle frequencies near 1000 cps are covered with a nearly uniform amplitude response.

The worst case of harmonic distortion from tracking error does come in the low treble region near 1000 cps. Not only does the RIAA curve make for a velocity response there, but $\tan\alpha$ is also largest in that neighborhood. At extreme treble, tracing distortion imposes a maximum-curvature limitation; while in the bass, groove spacing imposes a maximum-amplitude limitation on recording level. In the neighborhood of 1000 cps, one can have $\tan\alpha$ near 1, along with velocity sensitivity; thus, a 15-degree tracking error can result in a second-harmonic-distortion content near 27 per cent ($100 \tan 15° = 27$).

The Case of Large Modulating Index

The analyses given are valid for β not too large. When β is large, accurate Fourier analysis of phase modulated waves requires the use of Bessel coefficients $J_n(\beta)$ giving the relative strength of the nth-order sidebands.[8]

We have used the approximation

$$J_1(\beta) \approx \beta/2 \tag{13}$$

in discussing the strength of the first-order sidebands. In the case of intermodulation, we compared $J_1(\beta)$ with $m/2$, the strength of the corresponding AM sidebands for modulation index m, to derive the corresponding percentage of phase intermodulation $100\,\beta$. There is less meaning in drawing the same correspondence for large β, however. For example, AM above 100 per cent has little meaning, but there is no limit to the value the PM index can have. At very large indexes the departure from the AM picture is striking; very high-order sidebands become important, and, for certain values of β, the carrier vanishes. The vanishing of the carrier (J_0) happens for the first time at $\beta = 2.4048$. Thus, while the calculation of distortion content requires the use of Bessel coefficients, the index β can still be used to characterize the situation, and it gives the percentages directly when small. These coefficients are plotted in Fig. 2, for β up to 1. It may be seen that even for β near 1, $2J_1(\beta)$ is very nearly equal to β and that the errors committed in using β itself are not outrageous.

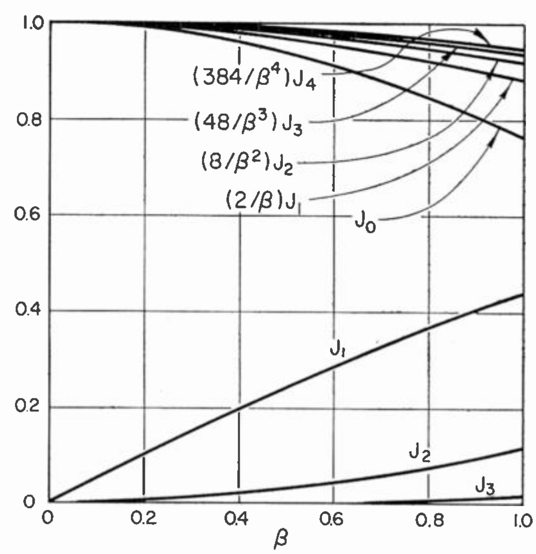

Fig. 2—Bessel coefficients for $0 \le \beta \le 1$.

The errors resulting from neglecting the fact that the modulating waveform was itself distorted can be calculated by a process of convolution. We will not go into that here, except to say that the errors so introduced can be shown to be smaller than those already mentioned.

In the analysis of the harmonic distortion of a single tone, we find that for larger values of β the more exact expressions are, for the relative strengths of the harmonics, just

$$\begin{aligned} \text{fundamental:}\ & J_0(\beta) - J_2(\beta), \\ \text{second:}\ & J_1(\beta) + J_3(\beta), \\ \text{third:}\ & J_2(\beta) - J_4(\beta), \end{aligned} \tag{14}$$

etc. Again, we have left out considerations turning around the modulating waveform being itself distorted. All these errors are no worse than those already discussed. The extra terms given above arise from sidebands around the carrier at "negative frequencies" extending into the domain of "positive frequencies," those sidebands at zero frequency cancelling exactly.

The Case for Nonzero Cutter Tracking Angles

The assumption was made that the recording cutter "plotted" the original waveform in rectangular coordinates. Let it now be supposed that instead, the cutter operated at a skew angle ψ, the cutter tracking angle. We still denote the tracking-angle error as ϕ. In this case we must replace $\tan \phi$ by

$$\sin \phi / \cos (\phi + \psi) \tag{15}$$

every place that $\tan \phi$ appears. It must be remembered that if ϕ and ψ have not the same sense, their opposing sense must be taken into account in calculating $\phi+\psi$. Tracking errors in the same sense as the cutter tracking angle are slightly more serious than those in the opposite sense.

Amplitude Intermodulation Effects

It is remarkable that tracking distortion, introducing in essence only phase perturbations, should give rise to effects involving AIM. Some hint of this is already seen in phase automodulation producing harmonic content, much as an amplitude perturbation would.

When the high-frequency terms had been written out, a simple-minded approach would have been to start from (5), writing $\omega = 2\pi\nu$, and changing to time, for the independent variable

$$a_c \cos (\omega_c t + \beta_m \sin \omega_m t)$$

and then expand the cosine of the sum to write

$$a_c \cos \omega_c t - a_c \beta_m \sin \omega_m t \sin \omega_c t$$

by systematically neglecting all but the *zero*th- and first-order terms in the power expansions of $\cos (\beta \sin \omega t)$ and $\sin (\beta \sin \omega t)$, as would be justified for β small. Then, from the same addition formula for the cosine, the result would be

$$a_c \cos \omega_c t + a_c(\beta_m/2) \cos (\omega_c + \omega_m)t - a_c(\beta_m/2) \cos (\omega_c - \omega_m)t, \tag{16}$$

showing the first-order PIM sidebands explicitly. Similarly, the low-frequency term

$$a_m \sin (\omega_m t + \beta_c \cos \omega_c t)$$

can be written as

$$a_m \sin \omega_m t + a_m(\beta_c/2) \cos (\omega_c + \omega_m)t + a_m(\beta_c/2) \cos (\omega_c - \omega_m)t. \tag{17}$$

In this case the justification β_c small is given by the hypothesis of a_c being small.

Upon comparing the expansions (16) and (17) it is seen that the sidebands for the low-frequency term stand very far from the frequency of that oscillation. Indeed, they are at the same frequencies as the first-order sidebands grouped around the high-frequency carrier. Interpreting these as "belonging" to the high-frequency term requires interpreting them as representing cosine AM of that carrier. Thus, there is simultaneous AM and PM which can be represented, conveniently as

$$a_c(1 + m \cos \omega_m t) \cos (\omega_c t + \beta_m \sin \omega_m t). \tag{18}$$

The AIM is not necessarily small, simply because β_c is small, since the modulation index is

$$m = a_m \beta_c / a_c = \tan \alpha_m \tan \phi. \tag{19}$$

The AIM is greater if a velocity-sensitive pickup is used. The 6-db/octave slope invokes slope detection of the PM components, producing first-order sidebands representative of additional AM of the same magnitude and phase as those AM sidebands already present. The result is that, for the velocity mode, the AM index is twice that given in (19). Bauer also gives this result.[7]

Madsen derives[3] in a manner which I do not understand, the AIM index

$$\left[\frac{\cos (\alpha_m - \phi)}{\cos (\alpha_m + \phi)}\right]^{1/2} - \left[\frac{\cos (\alpha_m + \phi)}{\cos (\alpha_m - \phi)}\right]^{1/2}$$

for the velocity mode. When small, this index agrees with that given here. I am not able to account for his finding that this index depends on the harmonic relation between ω_m and ω_c.

Baerwald's analysis[1] systematically enumerates the beat tones generated by the phase perturbation from two tones initially present, without assuming one or the other to be weak, and without sorting the beats into AIM and PIM groups. There is substantial justice in his point of view, especially if one were not contemplating intermodulation testing, but, rather, thinking of the spurious signals the phase perturbation would introduce into complex musical passages. On the other hand, one may wonder if the possibility of sorting some of the beat tones into PM groups could offer the hope that those tones would be any less audible. A later section reviews some of the psychoacoustic evidence concerning this question.

Measurement of Tracking Angle Error

The general design of the stereo disc offers a special convenience in the intermodulation measurement of tracking error, in that the PM and AM effects need not appear in the same channel. Their separation is achieved by exploiting PXM. This means that in the presence of a tracking-angle error for one channel, the large amplitude signal in that channel produces a PM in the other channel of the stereo groove. The expression for this crossmodulation is the same as that for intermodulation, so that it is sufficient to demonstrate it graphically.

Fig. 3 shows a diagram of an oblique (45 degrees) side view of a stereo groove. The line of sight is grazing to the wall bearing a large-amplitude long wavelength signal, and normal to the wall bearing a short wavelength signal. On the latter wall are drawn lines along the crests of the waves. The dashed lines are downward projections of the direction the stylus is constrained to move because of a tracking-angle error, in response to the long wavelength signal as the pattern goes sliding by to the left.

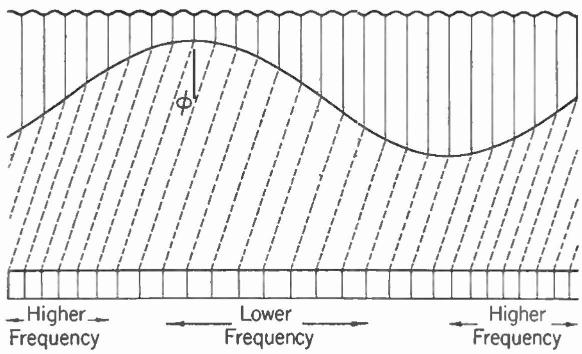

Fig. 3—Oblique view of stereo groove. One wall is engraved with a large-amplitude long wavelength oscillation, the other with a low-amplitude short wavelength one. The varying spacing of the slanted dashed lines demonstrates PXM.

In climbing up the "hill," the stylus is urged to the right, increasing the effective groove velocity. On the other hand, when sliding "downhill," the stylus is dragged somewhat to the left, decreasing the effective groove velocity. These dashed lines demonstrate the closer spacing between crests in the short wavelength signal arising from the uphill parts of the long wavelength signal, and the greater spacing between crests from the downhill parts. The shorter wavelength region is one of advancing phase. The phase achieves its maximum accumulated advance at the crest of the hill. Going downhill the lower frequency portion corresponds to the phase lagging behind, and it accumulates its greatest lag at the bottom of the hill. This diagram also demonstrates PIM.

There is a close analog in the generation of warp wow. A spurious very long wavelength vertical signal(warp) causes FM (wow) both in the vertical channel (PIM), and in the lateral channel (PXM). In this instance, the stylus cantilever is effectively pivoted at the arm pivot for vertical motion. For an 8-in arm, with its pivot $\frac{1}{4}$-in above the record surface, the analog of tan ϕ is $\frac{1}{32}$. For a sinusoidal warp, showing two oscillations per revolution at a 4-in radius, and having a peak amplitude of $\frac{1}{16}$ in, the peak slope corresponding to tan α is also $\frac{1}{32}$. Under these conditions the resulting wow is a peak frequency shift of one part in 1024, or 0.10 per cent, an acceptable figure.

The proposed measurement of tracking-angle error requires a special test record, an audio oscillator, and an oscilloscope for displaying Lissajous patterns. It is proposed that a band on the record have a low-amplitude high-frequency signal say 2000 cps, on both channels (pure lateral) for calibration purposes. The next band would have a large-amplitude low-frequency signal, say 100 cps, on one channel L and the low-amplitude 2000-cycle signal on the other, R. The high-frequency signal from the R channel of the reproducing cartridge, equalized for an amplitude response and filtered to remove any cross-talk of the low-frequency signal, is used to drive the y deflection of the oscilloscope. An audio oscillator, set to 2000 cycles to get a nearly stationary pattern, drives the x deflection.

The Lissajous pattern will slowly drift among the three patterns shown in Fig. 4. The sine of the peak phase excursion is given in

$$y_{\max}(0) = y_{\max} \sin \beta_m, \qquad (20)$$

conveniently determined in the first or the third of these patterns. This measures the L tracking-angle error, if the amplitude (in cm) of the low-frequency signal and the wavelength (in cm) if the high-frequency signal be known.

For the measurement of the R tracking-angle error, the R channel on the groove in a third band should bear the large-amplitude low-frequency signal, and the L channel the high-frequency signal. The y axis of the scope should be driven by the L signal from the cartridge.

If the two tracking errors give the same indication, then one or the other of the vertical or horizontal (usually the latter) is making a negligible contribution. Changing the overhang (lateral error), or using shims to change the vertical tracking error (or adjusting the height of the arm pivot without changing the overhang) will reveal which is making the larger contribution. If all the tracking error is vertical, it will be greater than the right or left errors by the factor $\sqrt{2}$.

The measurement of tracking error by PIM in monosystems is more difficult. Both signals need to be present in one purely lateral channel. The difficulty comes about because of the presence of simultaneous AIM and because of the need to filter a large-amplitude low-frequency signal out of the scope signal, since one cannot use channel separation to do most of this filtering, as in the stereo case.

However, one could use a stereo cartridge to get a purely lateral measurement by PXM, if the large-amplitude low-frequency signal were purely lateral and the low-amplitude high-frequency signal were purely vertical. The cartridge is connected to provide a difference output. Similarly, one could measure purely vertical tracking error by reversing the roles of the lateral and vertical channels and selecting the sum output from the cartridge. The necessary bands to do these things should be located at a variety of radii, so that the variation with radius could be plotted.

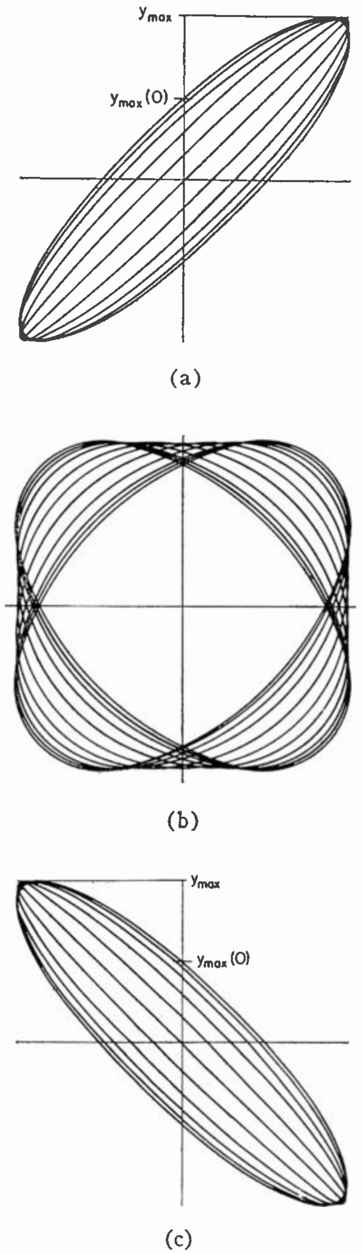

Fig. 4—Lissajous patterns showing phase variations. (a) About the in-phase oscillator signal. (b) About the quadrature-phase oscillator signal. (c) About the out-of-phase oscillator signal. The oscillator signal drives the x deflection.

The pitch of the high-frequency tone is to be chosen low enough to minimize the effects of tracing distortion and to avoid the wall-stylus resonance. The requisite low amplitude is also helpful in the former. If this frequency is too low, however, the crossmodulation will be smaller and more difficult to measure. Also, the filtering needed for display will be more difficult if the pitches of the two tones are not sufficiently separated. The pitch of the low-frequency tone should be well above the cartridge-arm resonance, so that the motion of the cartridge body at this frequency will be negligible.

Psychoacoustic Effects

Listening tests with these single channels, into which PXM had been introduced, would provide a means of determining the subjective distress caused by such distortion in the absence of other kinds of distortion. Little is known about such effects, except through experience in dealing with the subject of flutter.

Ordinarily, flutter is thought to arise from a vibration generated in the mechanism of playback or recording equipment, the vibration typically occurring at 30 cps or lower frequencies. The physical effect is one of FM, in which the peak flutter index is $F=\Delta\nu/\nu_c$; the corresponding PM index is $\beta=(\nu_c/\nu_m)F$. There is abundant psychoacoustical evidence for the effect of flutter, for sustained tones, music and speech.[9]

It appears that flutter is more noticeable at low modulating frequencies near 3 cps, than near 30 cps, and that above 80 cps it becomes more noticeable again. The effect increases with modulating frequency. The absolute threshold is for a flutter index of about 0.15 per cent (rms.) for a modulating frequency near 100 cps, when headphones are used for listening. For a 1000-cps carrier tone, the corresponding PM index is 1.5 per cent.

The audibility of PM is enhanced in a multipath acoustical environment (live room). The absolute threshold in such circumstances can be for a flutter index as low as 0.015 per cent, for a modulating frequency near 100 cps. It may be supposed that this enhancement is caused by a conversion of the PM sideband tones to those having an AM character. The sound, as heard, has a quality like that of AM. Interference among paths can selectively alter the strength of one of the sidebands, or even reverse its sign relative to the others, so that the phase relations corresponding to AM sidebands obtain for some fraction of the signal.

Klipsch has demonstrated the objective effect[10] which is the acoustical analog of the multipath distortion problem well known in radio reception. He points out that the same intermodulation exists as Doppler distortion in loudspeakers.

Acknowledgment

The author is indebted to B. B. Bauer for pointing out to him in private correspondence the full status of the AIM effect.

[9] F. A. Comerci, "Perceptability of flutter in speech and music," *J. SMPTE*, vol. 64, pp. 117–122; March, 1955.

[10] P. Klipsch, "Subjective effects of frequency modulation distortion," *J. Audio Engrg. Soc.* vol. 6, p. 143(L); April, 1958.

The Vertical Tracking Angle Problem in Stereophonic Record Reproduction*

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Summary—At the time of the widespread introduction of stereophonic records in 1957, two cutter systems became available—one embodying a 23° vertical recording angle and the other a 0° angle. In 1961 the Engineering Committee of the RIAA recommended a 15° vertical tracking angle standard for stereophonic pickups. In the intervening years, however, the matter of vertical angles appears to have been neglected by most pickup makers inasmuch as a range of angles of 0° to over 40° has been reported, in the extreme cases audible distortion being noticed. A new factor has been added by recent CBS Laboratories' discovery that, because of lacquer and stylus springback, the actual slant of the recorded wave is considerably smaller than the recording angle. With 23° cutter normally used, for example, the actual modulation angle is near 0°. A special mounting and special recording stylus have been developed for providing the cutter with an added inclination of 14° which produces the desired 15° effective modulation slant.

Modulation slants can be measured by microscopic measurements of dissymmetry of square-wave traces, by measurements of the shift of optical patterns, by measurement of distortion with the forward and reverse pickup orientation on a test record, and by placing the pickup in a normal and an inclined playing position. This latter method of measurement also yields the effective value of pickup tracking angle including the effect of any existing longitudinal elasticity of the stylus mounting.

Introduction

A STEREOPHONIC disc cutter differs from a monophonic cutter by its ability to execute vertical as well as lateral motions. At the time of commercial introduction of stero discs in 1957, and thereafter, several cutter systems were offered to record makers. Among them, the Westrex 3C cutter directed its vertical components of stylus motions along a line inclined forwardly at 23°. The alternative angle most frequently mentioned was 0° attributed to cutters of European manufacture. Thus the designers of pickups were confronted with an uncertainty as to the vertical tracking angle to be used in stereophonic pickups.

This uncertainty did not result in hesitation. New pickup designs were introduced rapidly to fill the requirements of high fidelity enthusiasts and home phonograph makers. The popularity of various pickups did not appear to bear a patent relationship to the vertical tracking angles. Lately, however, the whole matter of tracking angles has been in the limelight. Therefore, we have attempted to bring together the known facts about it, including some recent findings from CBS Laboratories. It is now possible to appraise, measure, adjust, and bring into control the vertical tracking angles of pickups and the vertical modulation slant in records. The already excellent quality of stereophonic record reproduction is expected to improve further as a result of a better undertsanding of all these factors.

The Engineering Committee of the Record Industries Association of America (RIAA) deserves a great deal of credit for standardizing the type of modulation and the essential dimensions of stereophonic records, so that interchangeability could be achieved, prior to mass production of records and players. The engineers correctly appraised the vertical modulation slant as an important, if secondary, characteristic of stereophonic records. There was not sufficient agreement about the merits of the alternative vertical cutter angles and the matter was left unsettled but not forgotten. Thus, in 1961, the Committee recommended a vertical tracking angle of 15° for stereophonic pickup. In the intervening years, however, the matter of vertical tracking angles seemingly vanished from the list of important design objectives of the pickup makers. The wide disparity of existing vertical tracking angles was summarized by E. R. Madsen.[1] He lists 10 typical pickups with a range of angles from 0° to 40°. It is significant that 7 out of 10 exhibit angles of 23° and above.

A similar but somewhat more extensive set of measurements has been performed at CBS Laboratories with the result paralleling those of Madsen. Fourteen pickups were measured and the vertical tracking angles were found to vary from 10° to 42°. Interestingly, seven of the pickups were of the piezoelectric variety intended primarily for home phonograph use; their angles varied from 10° to 25° with an average of 16.5°. The remaining seven units were principally of the magnetic type intended for components sales; they had angles varying from 24.5° to 42° with an average of 31°. The typical arrangement of transducers and styli in piezoelectric and magnetic pickups suggests that the angles of pickups are likely to have been influenced in many cases by the convenience in arranging the transducer parts, rather than by attempting to conform with any particular set of cutter angles.

Vertical tracking angles have been referred to above as important even though secondary attributes of sterophonic pickups. The generally excellent quality of reproduction obtained from stereophonic discs in the face

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[1] E. R. Madsen, "Vertical tracking angle—a source of I.M. distortion," *Audio Magazine*, vol. 46, pp. 21–24; November, 1962.

of the disparity of angles attests to this view. It has been shown that the distortions resulting from the differences in angles normally to be expected are small compared to other distortions in the reproducing system.[2] Nevertheless, under extreme conditions of pickup tracking angle disconformity with the vertical modulation slant (called vertical tracking error), distortions may reach significant proportions, especially at the inside radii of the disc where the wavelengths are short. As only the off-center sounds are affected by vertical tracking error distortion while the important central sounds are not, the listener tends to be less aware of it than he might have been had he been listening to monophonic sound. One finds a counterpart in photography where central sharpness of an image tends to override edge aberrations. The dramatic space effect of stereo additionally tends to make the listener less critical of equipment defects, just as the presence of color serves to diminish the critique of lens quality in photography.

Vertical tracking error distortion is believed to have become noticeable in FM stereo broadcasting where the monophonic listener hears only the sum signal. The effect is further accentuated by purposeful selection of material with extreme left-right separation and little center sound. It is suggested that care be exercised in the choice of pickups and records for FM stereo broadcasting to insure high-quality compatible monophonic reception.

Another source of vertical tracking error inherent in the nature of the recording process has been discovered and placed under control at CBS Laboratories.[3] This effect stems from springback of the variable longitudinal strains set up in the cutter and the lacquer by vertical modulation. Thus, in normal use, the 23° vertical recording angle of the Westrex cutter is found to produce a modulation slant of $0 \pm 2.5°$, depending upon the method of measurement. The cutter requires an added 14° inclination to generate a 15° modulation slant suggested by the RIAA standard. This adjustment can be achieved readily with the Westrex cutter, but a special stylus is required. This stylus has been developed at CBS Laboratories. There has not been an exhaustive study made of the behavior of 0° angle cutters, but at least one set of measurements showed that negative modulation slants in the amount of some 15° may be expected.

A further finding of the above mentioned study is that a simple geometrical measurement of the vertical pickup tracking angles is not sufficient. A variable longitudinal strain of the stylus suspension stemming from varying frictional forces between the stylus and the record during the playback process sometimes appears to cause the effective tracking angle to be in excess of that measured by optical means.

Thus, the whole problem of vertical tracking error presents a rather confused picture and requires a concerted effort on part of the industry to bring about a sensible solution. At the same time it is important to place this problem into proper perspective. Stereophonic records played on reasonable playback equipment provide superb sound. The important but secondary factor of vertical tracking angles is understood at the present time and is capable of measurement and control. Since it costs no more to do the job right, it would appear to be beneficial for all concerned to bring the tracking angles to the RIAA 15° standard with all deliberate haste. The great majority of listeners will notice no improvement, but there will be a significant improvement in those cases where extreme conditions of pickup tracking angles and modulation slant have in the past produced perceptible distortion.

Vertical Tracking Angles

The principal information content in a stereo record resides in the sum signal of the 45–45° modulation which appears as a lateral cut, and is not affected by the vertical tracking angles. However, the stereo information is identified with the difference signal which is contained in the vertical mode. Therefore, proper tracking of vertical modulation assumes increasing importance with records having considerable channel separation, especially toward the end of the record where wavelengths are short.

Since the cutter and the pickup suspensions are pivoted above the record surface, the stylus tip motions are contained in a plane slanted away from the point of suspension (Fig. 1). The slant angle of the pickup is called the vertical tracking angle A, and the slant angle of the cutter is called the vertical recording angle B. Assuming that a sinusoidal vertical signal is applied to the cutter, the modulation actually cut can be expected to be contained in a coordinate system with inclined ordinates. The inclination angle of the modulation ordinate is called the vertical modulation slant C, and the difference between the vertical tracking angle and the vertical modulation slant is a vertical tracking error angle D. If D is zero then the pickup will reproduce an undistorted signal. It had been assumed from the beginning of sterophonic recording that vertical modulation slant was equal to the vertical recording angle, but as stated before, it has been discovered at CBS Laboratories that the real modulation slant is substantially smaller than the recording angle.

This discovery occurred quite accidentally. We were trying to measure pickup distortion in the vertical mode and for this purpose an intermodulation test record was needed. At the same time, recognizing a need for transient response measurements, we decided to place a square-wave modulation on the same record in the

[2] C. R. Bastiaans, "Further thoughts on geometric conditions in the cutting and playing of stereo discs," *J. Audio Engrg. Soc.*, vol. 11, pp. 6–15; January, 1963.

[3] B. B. Bauer, "Vertical tracking improvements in stereo recording," *Audio Magazine*, vol. 47, pp. 19, 20, 46, 68, 69; February, 1963.

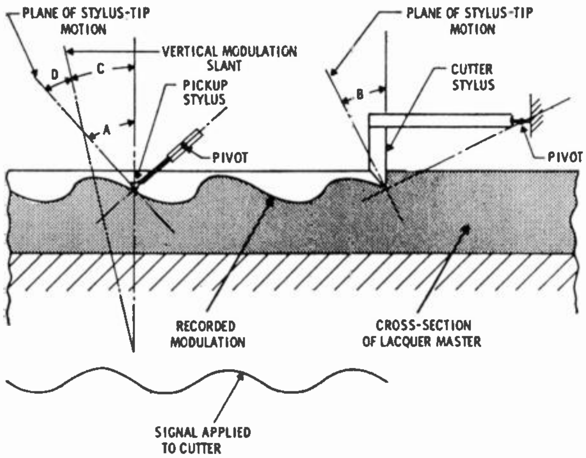

Fig. 1—Diagrammatic representation of vertical tracking angle, A, vertical recording angle, B, and vertical modulation slant, C. Because of lacquer springback, C is smaller than B.

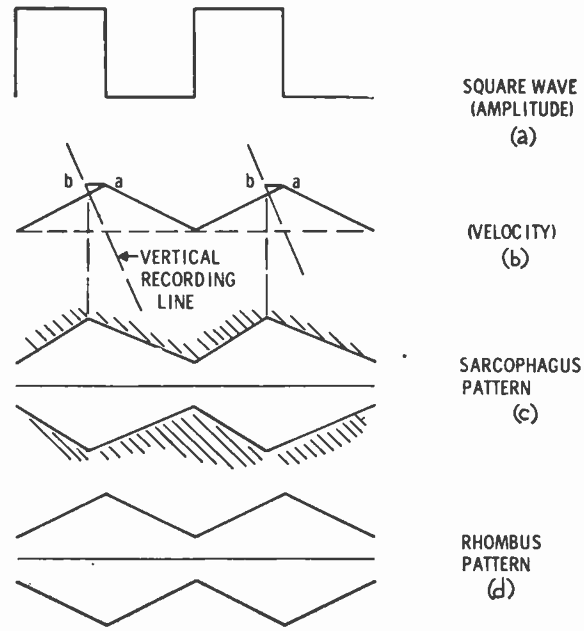

Fig. 2—(a) Square wave applied to the cutter amplifier. (b) Motion of recording stylus. (c) Sarcophagus pattern produced by slanted cut. (d) Rhombus pattern with 0° modulation slant.

lateral, left, right and vertical modes. This record became the CBS Laboratories Stereophonic Test Record STR-110. The STR-110 indeed represents the type of modulation obtained with the standard Westrex cutter system. Upon replay a number of anomalies were observed. Firstly, the second-order harmonic distortion and the IM distortion were substantially greater in the vertical than in the lateral modes, and significantly greater than the theoretically expected values. Also, in replaying the vertical square wave, it was noticed on the oscilloscope that the lengths of the two halves of the reproduced wave were slightly unequal. Now, as shown in Fig. 2, a square wave (a) becomes a triangular wave (b) on a velocity basis, and when such a wave is recorded in the vertical mode with a cutter having a recording angle other than 0°, the peaks of the wave are shifted along the recording line from a to b, so that the recorded groove presents a sarcophagus pattern (c). Of course, when such modulation is played with a pickup having an appropriate vertical tracking angle, symmetry is restored and a perfect square wave is reproduced. However, microscopic examinations of the modulation of the STR-110 revealed a rhombus pattern (d) which implied that the 23° recording angle produced near 0°-slant modulation.

Longitudinal Springback

It was deduced that a heretofore unsuspected effect existed in the vertical recording process. This effect could be attributed to the longitudinal elasticity of the lacquer or the transverse elasticity of the recording stylus in combination with the longitudinal alternating stresses which occur in vertical recording. The former is difficult to calculate but may be readily observed under a microscope while disturbing the surface of the lacquer with a pin or stylus: the lacquer is seen to move as a rubber mass. The latter is less readily detected but may be calculated from physical data. The role of the spring of the stylus is easily visualized: in cutting a deeper groove more force is needed in the direction tangent to the record surface; this increases the stylus strain, thus diminishing the angle of effective vertical stylus-tip motion.

The action of lacquer elasticity is a bit more difficult to visualize but it may be explained as follows [Fig. 3(a)]. Say a cutting stylus moves with a recording angle B to cut a triangular velocity pattern, similar to that in Fig. 2. Initially the lacquer is in an unstressed condition. As the disc advances to the left so that successive horizontal intervals pass by the cutter, the tip of the stylus moves along the slant lines, and cuts the shape a-b-c. While the cutting action takes place, the shearing forces push the material forward, as shown by the slanted dash lines, the slant being proportional to the depth of cut, and greatest at b. As the cutter emerges at c, the stress again is diminished. After the cutter has passed by any point, the lacquer springs back taking on the form shown in Fig. 3(b). It is seen that the triangle a'-b'-c' has an equivalent vertical modulation slant C which is smaller than the recording angle B by the amount of a springback angle S. For convenience, S is assumed to include both the stylus and the lacquer springback. With the conventional Westrex cutter and stylus system, S of the order of 23°. Thus the normal 23° recording angle of the Westrex cutter produces a modulation slant of near zero.

To study the stresses involved in the springback process a magnified view of the lacquer surface at the instant of being cut is shown in Fig. 4. The curved coordinates suggest the type of deformation which takes place in the lacquer. The stylus deformation in the opposite direction is not shown. At right is a cross section of the material at the point of contact with the stylus. The cutting force F may be expressed as a power

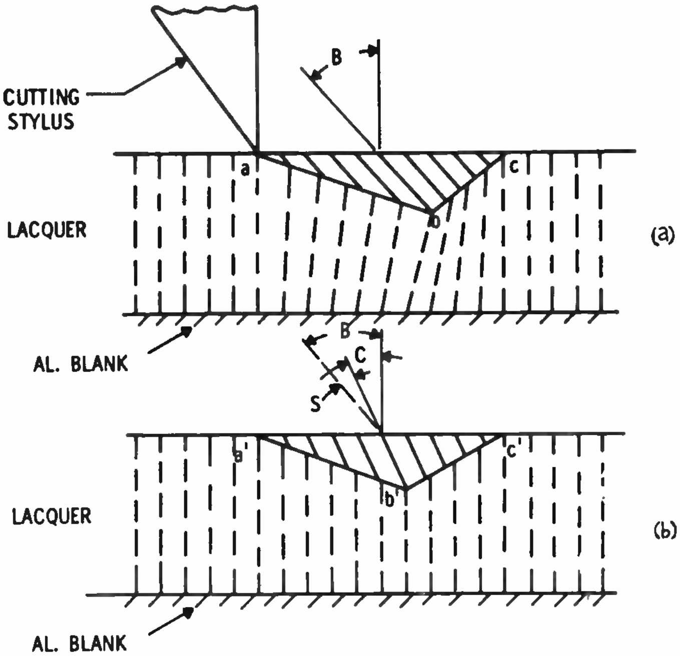

Fig. 3—Stressing of lacquer during cut. (a) Followed by springback after cut. (b) The resulting modulation a'-b'-c' is less slanted than the line of cutter motion a-b-c.

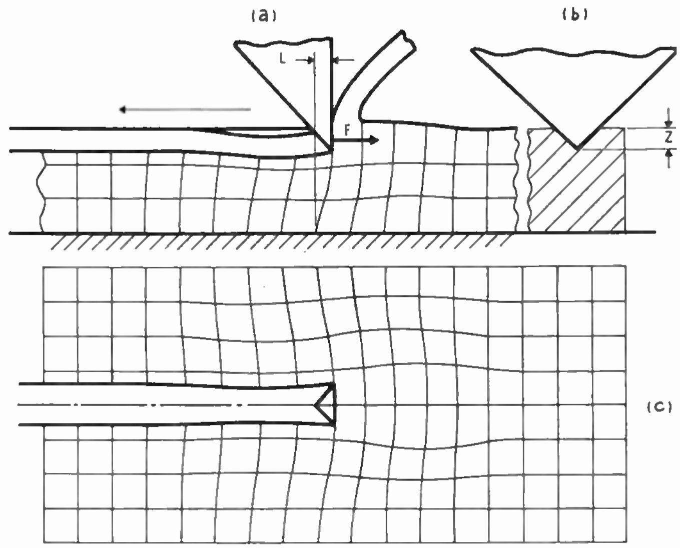

Fig. 4—Stresses in lacquer during recording.

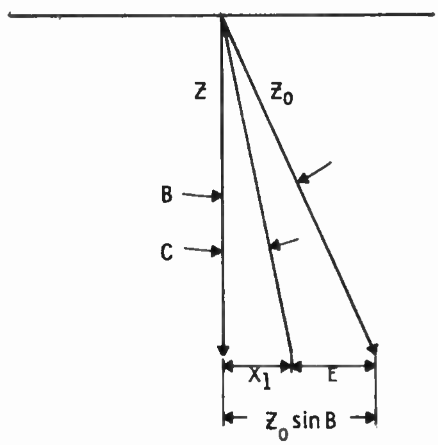

Fig. 5—Springback relationship during vertical modulation.

series in terms of depth-of-cut Z and cutting velocity V. To a first approximation, however, we assume that the cutting force depends principally upon the perimeter of shear which is proportional to the depth of cut by a factor of proportionality a,

$$F = aZ. \tag{1}$$

Let the cutter displacement along the geometrical cutting angle B as shown in Fig. 5 be Z_0. We write,

$$Z = Z_0 \cos B \tag{2}$$

and substituting into (1),

$$F = aZ_0 \cos B. \tag{3}$$

The longitudinal geometrical displacement of modulation ordinate without the stress is $Z_0 \sin B$. Because of the stylus compliance the actual displacement is lessened. Including the springback of the lacquer a total diminution E takes place which, for simplicity, is assumed to be a linear function of F, say $E = c_m F$, where c_m is the total mechanical compliance. Substituting the value of F from (3),

$$E = c_m a Z_0 \cos B. \tag{4}$$

Thus, instead of being displaced horizontally by the quantity $X = Z_0 \sin B$, the modulation after springback is found to be displaced by the quantity $X_1 = Z_0 \sin B - E$. Substituting E from (4), and factoring,

$$X_1 = Z_0(\sin B - c_m a \cos B) \tag{5}$$

to obtain the modulation slant angle, we divide by Z, from (2),

$$\tan C = \tan B - c_m a. \tag{6}$$

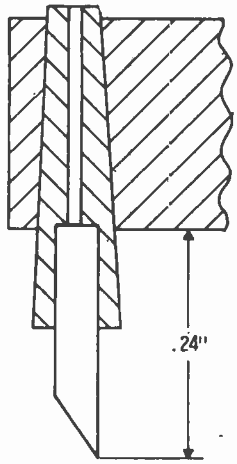

Fig. 6—Normal cutting stylus arrangement.

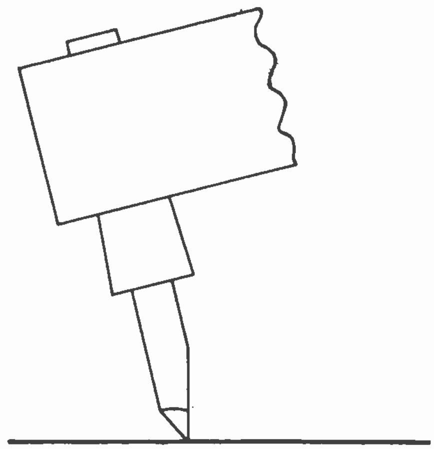

Fig. 7—Inclined cutting stylus arrangement.

In the Westrex cutter $B=23°$ and C is approximately 0°; solving for $c_m a$ we find,

$$c_m a = \tan 23° - \tan 0° = 0.42. \quad (7)$$

As a next step we attempt to separate $c_m a$ into its factors. For this it is necessary to determine the longitudinal force required to cut the groove as a function of the depth of cut. This has been done by mounting the stylus in a pivoted mount and measuring the force needed to hold it in position during the cutting process. A groove 0.004 cm deep required a force of 40 grams (40,000 dynes). Therefore, the a in (1) is 40,000/0.004-10^7 dynes per cm of depth. Solving for c_m,

$$c_m = c_m a/a = .42/10^7 = .042 \times 10^{-6} \text{ cm per dyne.} \quad (8)$$

At this point we can attempt to separate c_m into the individual contributions of the lacquer and the stylus. The conventional stylus for a Westrex cutter consists of a 0.040-inch diameter sapphire or ruby rod which is embedded into a tapered aluminum shank pressed into the cutter bar, as shown in Fig. 6. The portion of the rod protruding outside the bar is 0.24 inch long. This length is needed to accommodate the heating coil and the suction pipe for the thread being cut. The shank strengthens the cutting rod but at the same time adds an element of elasticity at the base of the cantilver beam which is difficult to calculate. The stiffness of the rod c_s was determined from the following simple experiment: A 6-inch sapphire rod 0.040 inch in diameter was clamped at one end so that a 12-cm-long cantilever beam was formed. A transverse 10,000-dyne force applied at the end produced a 0.32-cm deflection. Transferring this by the cube-of-length function to obtain the compliance of a 0.24-inch (0.6 cm) rod,

$$c_s = (0.32/10{,}000)/(0.6/12)^3$$
$$= 0.004 \times 10^{-6} \text{ cm per dyne.}$$

Thus c_s would appear to be only 10 per cent of c_m. However, the total stylus compliance is 2–4 times greater because the whole cutter bar-stylus structure is subjected to the stress. From direct measurements it has been estimated that the mounting and sylus compliance may be responsible for 20 per cent to 40 per cent of the springback with the remainder being attributable to the lacquer elasticity. Some frequency, velocity, and amplitude dependence also appears to exist. A more precise study of the phenomenon is in progress.[4]

Next, we can calculate the cutting angle which produces the desired modulation slant C of 15° with the Westrex cutter system. This is readily obtained from (3).

$$\tan B = \tan C + c_m a$$
$$= \tan 15° + 0.42 = 0.69.$$

Therefore, $B=35°$. Experimentally it has been determined that a 37° angle provides a better approximation. A simple way of obtaining this angle is by tilting the cutter 14° and employing a special stylus to preserve the desired normal relationship between the cutting face and the surface of the lacquer. The type of stylus developed at CBS Laboratories for this purpose is shown in Fig. 7. Ruby is used instead of sapphire because it permits closer inspection of the surface for the quality of lapping and freedom from microscopic flaws which weaken the tip structure. The type of shank and stylus dimensions are carefully controlled to promote uniformity of springback.

A test record similar to the STR-110 was recorded with the inclined cutter system, becoming CBS Laboratories STR-111 test record.

[4] During the discussion following the presentation of this paper J. G. Woodward of RCA Laboratories reported on measurements of cutting force F as a function of depth Z proposing a relationship in the form $F=kZ^n$. This led us to observe that, if the slope of the force-depth curve is $dF/dZ=nkZ^{n-1}$, the springback could be expected to vary with the average depth of cut. By actual measurements, however, the springback appears to be relatively independent of the average depth of cut. One might conjecture that Woodward's nonlinear F-vs-Z function combines with a complementary lacquer compliance-vs-depth-of-cut function to produce a relatively constant springback-vs-depth relationship.

CBS LABORATORIES

A Division of Columbia Broadcasting System, Inc.

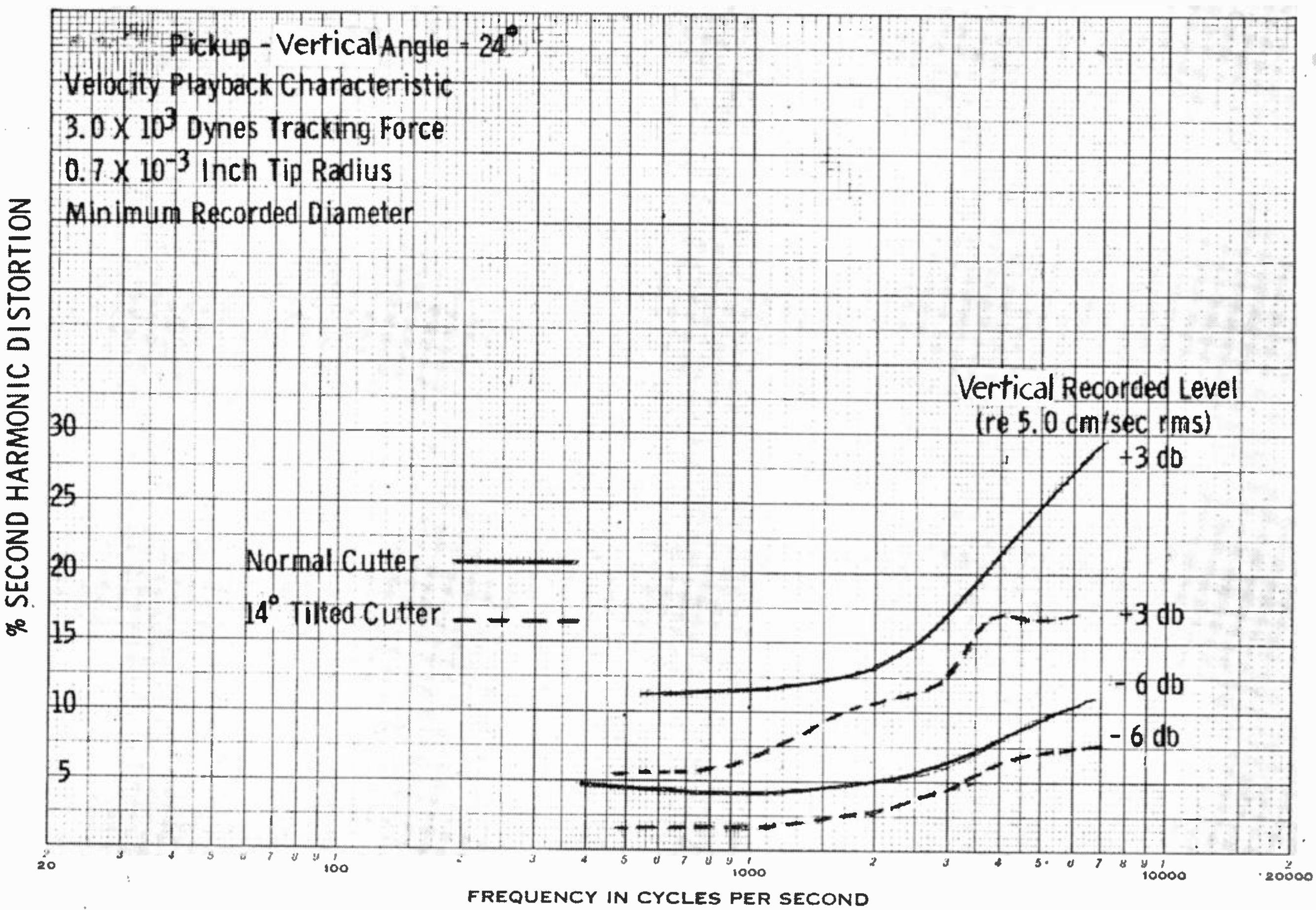

Fig. 8—Distortion improvement with inclined cutter.

The effectiveness of the improved recording system is shown in Fig. 8, in which the second harmonic distortion content in a vertical recording is plotted, as a function of frequency, with the conventional and the inclined cutter methods. The pickup used in these measurements is a moving-coil unit with a vertical tracking angle of 24° and this in part accounts for the distortion remaining with the improved system of recording. Tracing distortion accounts for the rest.

Modulation Slant of Square Waves

Having established the theory of modulation springback, it becomes necessary to determine the real modulation slant angles. This has turned out to be not an insignificant problem; some of the solutions being outlined below. The examples given are merely by the way of illustration.

The simplest method stems from the geometry of the square-wave sarcophagus pattern. In Fig. 9(a) the square-wave modulation is shown in cross section. If the modulation slant were 0°, the tip of the stylus would follow the path p-q-r. With an effective slant C, the path will be p-t-r. C is determined by the angle between the line s-t and the perpendicular o-t to the surface of the record, where s lies midway between p and r. We locate p, s, t and r on the plan view in Fig. 9(b). Because the cut is made with a stylus with a 90° included angle, the intersection between the groove wall and the record surface p'-t'-r' is identical to the vertical intersection p-t-r, and the point s' is found midway between p' and r'. Thus, by taking a plan-view photograph of the pattern and performing the operations indicated, the modulation slant may be determined. Unfortunately, the intersection between the groove and the record surface is somewhat uncertain because of the presence of horns, so that this measurement is difficult to perform with precision. A modulation slant of some 18° has been estimated for the STR-111 record vertical square wave, by this type of measurement.

Another possibility is to use optical patterns. Consider a square-wave modulation with zero slant (Fig. 10). When a collimated beam of light L is reflected from it and the width b between the brightest reflections is

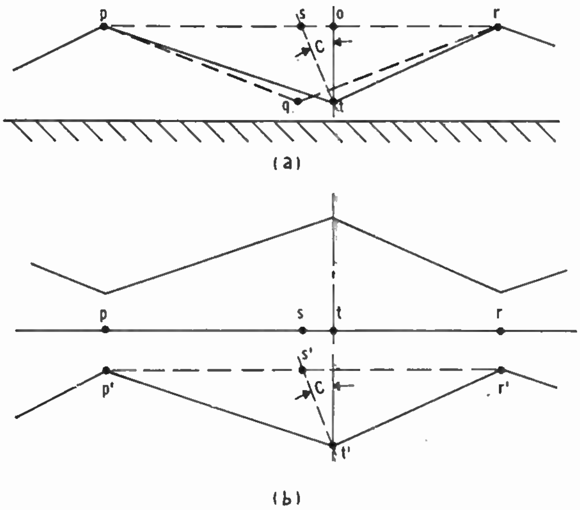

Fig. 9—Geometric measurement of modulation slant.

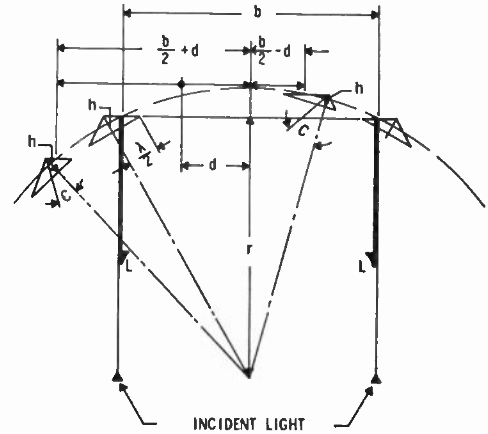

Fig. 10—Optical pattern measurement of modulation slant.

measured from a distance, it is easy to see that

$$h/(\lambda/2) = b/2r \tag{9}$$

where

h = height of the wave
λ = wavelength
b = pattern width
r = groove radius.

If the modulation is slanted by an angle C, then one-half wavelength is diminished by $h \tan C$ and the other one is augmented by the same amount. As suggested by Fig. 10, the distance of reflection from the center line will increase in the first case and diminish in the second. Let this first distance be $(b/2)+d$ and the second distance $(b/2)-d$. We can write

$$h \Big/ \left(\frac{\lambda}{2} - h \tan C\right) = (b/2 + d)/r \tag{10}$$

and

$$h \Big/ \left(\frac{\lambda}{2} + h \tan C\right) = (b/2 - d)/r. \tag{11}$$

Solving for tan C,

$$\tan C = rd/((b^2/4) - d^2) \tag{12}$$

or, since d^2 is small compared with $b^2/4$,

$$\tan C = 4rd/b^2, \text{ approximately.} \tag{13}$$

For example, in the STR-111 the vertical square wave having a groove radius of 5.2 inches under a certain condition of viewing sustained a pattern width b of 1.6 inches, which was displaced 0.040 inch off-center. Substituting in (13), $\tan C = 4 \times 5.2 \times 0.040/1.6^2 = 0.33$; therefore $C = 18°$. It is evident that precision of the angle thus determined depends entirely upon the care with which the pattern is measured. A small band of unmodulated grooves of normal pitch should be provided near the modulated band to be measured to assist with determining the location of the line of symmetry of the pattern. Displacement to the right of the line of symmetry denotes positive modulation slant, displacement to the left, negative slant.

From the above measurements, it is concluded that the springback for a 1000-cps square wave is of the order of 19°.

Modulation Slant of Sine Waves

The modulation slant of a vertically modulated sine wave may be observed at the intersection between the groove wall and the surface of the record; however, this portrayal does not lend itself to a measurement. The light-pattern measurement method appears to be applicable, and we have used it to good advantage.

For example, the 300-cps vertical modulation in the STR-111 record, located at the 3.6-inch radius groove, has a width of 2.15 inches and a displacement from the line of symmetry of 0.078 mil. Using (13), we find $\tan C = 4 \times 3.6 \times 0.078/2.15^2 = 0.22$. This corresponds to a slant of 12.5°. A springback of 24.5° for this type of modulation is expected from this measurement.

Another way of performing sine-wave slant measurements is by means of a special turntable and pickup arm shown in Fig. 11. The turntable can rotate in either direction. The arm may be placed at either side of the turntable so that the frictional force is always forward of the arm regardless of the direction of rotation. (Some pickups undergo a change in response when played "against" the stylus.) The arm support slides on a rail so that a straight pickup arm may be placed tangent to the grooves to avoid side trust. The vertical post allows the arm pivot to be raised to any desired degree so as to add known increments of vertical tracking angle, as desired.

The basic principle involved is to measure the second harmonic distortion of the sine wave in the forward and backwards modes with the arm which is initially horizontal, or is inclined at an arbitrary initial angle, and then, with an incremental angle θ added. θ con-

Fig. 11—Turntable and arm arrangement for playing a record forward, backward, at various tracking angles.

veniently may be 10°. Then the following measurements are performed:

1) Distortion in the forward mode of rotation with the arm at the initial angle: H_1
2) Distortion in the reverse mode with the arm at the initial angle: H_2
3) Distortion in the forward mode of rotation with θ added: H_3
4) Distortion in the reverse mode with θ added: H_4

The second harmonic distortion is very nearly proportional to the difference between the vertical tracking angle A and the vertical modulation slant angle C; therefore,

$$H_1 = k(A - C) \tag{14}$$

$$H_2 = k(A + C) \tag{15}$$

$$H_3 = k(A + \theta - C) \tag{16}$$

$$H_4 = k(A + \theta + C) \tag{17}$$

where k is $v/2V$ for the amplitude pickup mode and v/V for the velocity mode, if the angles are expressed in radians; v is the maximum modulation velocity and V is the linear groove velocity. However, the actual value of k is not needed, as shown by the following equations: Adding (14) and (15), and (16) and (17),

$$(H_2 + H_1)/2 = kA \tag{18}$$

$$(H_4 + H_3)/2 = k(A + \theta). \tag{19}$$

Dividing (19) by (18),

$$(H_4 + H_3)/(H_2 + H_1) = (A + \theta)/A. \tag{20}$$

Solving for A,

$$A = \theta \Big/ \left(\frac{H_4 + H_3}{H_2 + H_1} - 1\right). \tag{21}$$

Also, from (14) and (15),

$$(H_2 - H_1)/2 = kC. \tag{22}$$

Dividing (22) by (18), and solving for C,

$$C = A(H_2 - H_1)/(H_2 + H_1). \tag{23}$$

Thus, the set of 4 measurements yields C and A. For more careful work readings will be taken at various levels of modulation and the points plotted on a graph giving distortion as a function of v/V for each of the 4 conditions mentioned. "Best" straight lines are plotted through the distortion points and the origin. Thence, the four values of H may be read along any arbitrary ordinate of the graph. By the way, the value of A obtained in this manner includes the effects of longitudinal elasticity of the pickup mounting discussed elsewhere.[3]

The user must be careful to apply obvious tests to insure that the initial angle is sufficiently high so that $A > C$. The actual vertical tracking angle of the cartridge is determined by subtracting from A this initial angle.

Measurement of the 300-cps modulation slant of the STR-111 record by means of the above method shows a slant of 10°. From this and previous measurement it has been deduced that under some conditions at low frequency the springback may be greater than anticipated. To determine the effect of frequency and groove radius upon modulation slant, vertical sine-wave test cuts were made with the 14° inclined cutter system at the outermost and innermost radii of the disc at 300 and 1000 cps, with the following results:

	Modulation Slant	
	300 cps	1000 cps
Outside Radius	8°	15°
Inside Radius	15°	15°

The implication of these figures is discussed in a later section.

Modulation Slant for Two-Frequency Waves

Intermodulation tests are accepted by many workers in acoustics as among the most significant measures of distortion. Therefore, considerable effort was extended toward a study of IM reduction by means of the inclined-cutter system.

For IM tests, lateral and vertical bands are provided on the STR-110 and STR-111 records with 200/4000 and 400/4000 tones, and therefore, an IM analyzer can be used with great saving of time and labor. The values of IM for pickup vertical tracking angle A and modulation slant C is given here without proof.

$$\mathrm{IM} = (v/V)\cos C(\tan A - \tan C) \quad \text{(for the amplitude mode)} \tag{24}$$

$$\mathrm{IM} = 2\frac{v}{V}\frac{(\tan A - \tan C)}{\dfrac{1}{\cos C} + (v/V)^2(\tan A - \tan C)^2 \cos C} \tag{25}$$

(for the velocity mode).

For reasonably small values of distortion, these equations become approximately

$$\mathrm{IM\ (Amplitude)} = (v/V)(A - C) \tag{26}$$

$$\mathrm{IM\ (Velocity)} = 2(v/V)(A - C). \tag{27}$$

It is seen that the form of these equations is precisely the same as that for the harmonic distortion (14) and (15) except for a factor of 2. Therefore, the analysis performed with (14)–(23) is applicable; (21) and (23) may be used to calculate the vertical tracking angle A of pickups and the modulation slant C of records, respectively, when values of IM are used in place of the corresponding values of H.

With the use of this type of analysis, the IM bands on the STR-111 record have been measured as having a modulation slant angle of 15–16.5°. It should not be surprising that these readings may be at slight variance with those obtained by other methods. The IM bands contain low-frequency and high-frequency waves combined, which may be subject to springback in varying manners, so that precise correspondence cannot be *a priori* expected.

Review of Effects of Modulation Slant

A review of the figures in the preceding sections shows that the inclined-cutter system produces a vertical modulation slant which, for the important IM readings, 1000-cps second harmonic distortions at all radii and 300-cps second harmonic distortions at the inner radii, varies from 15° to 16.5°. At low frequency and large radii there is a greater disparity; fortunately the distortions corresponding to these longer wavelengths are generally negligible. Therefore, we conclude that the inclined-cutter scheme well meets the effective 15° modulation slant prescribed by the RIAA.

At this juncture, one can obtain an idea of the amount of distortions that might have been produced under conditions of extreme modulation in a stereophonic record with infinite channel separation using 0° and 15° modulation slants. Let the repeated loud passages during recording reach a 0–"V.U." velocity of 5 cm/sec rms, and assume a 9-db peak factor. Then the peak velocity on a lateral basis is 20 cm/sec or 14.1 cm/sec for each of the two channels acting independently. The vertical component of this peak modulation is 10 cm/sec/channel.

Assuming a linear groove speed of 50 cm/sec at the outer radius and 20 cm/sec at the inner radii, and using (14) and (26), for the amplitude mode (because the RIAA playback curve approximates this mode), the set of values in Table I is obtained.

TABLE I
PICKUP ANGLE

Effective Vertical Tracking Angle	Distortion Per Cent					
	15°		23°		35°	
Effective Modulation Slant	0°	15°	0°	15°	0°	15°
Second Harmonic						
Outer radii	2.6	0	4	1.4	6	3.5
Inner radii	6.5	0	10	3.5	15	8.7
Intermodulation						
Outer radii	5.2	0	8	2.8	12	7.0
Inner radii	13.0	0	20	7.0	24	17.4

From Table I some interesting information can be gleaned. Pickups which at the present time have vertical tracking angles of 15° produce a small but not negligible amount of distortion with the type of modulation currently available on stereophonic records. With the 15° modulation slant, any distortion stemming from this latter cause is reduced to 0.

A pickup with 23° effective tracking angle used with conventional records under extreme conditions of modulation may produce a significant amount of distortion at the inner radii. With the 15° modulation, this distortion drops by a factor of $\frac{1}{3}$ becoming almost entirely negligible.

With a 35° vertical tracking angle, distortion will be considerable with either the 0° or the 15° modulation, although the improvements in the latter case will be of the order of $\frac{1}{2}$.

The new method for producing 15° modulation provides improved performance with all pickups and is especially beneficial with those which have been designed with the known existing cutter angles in mind. It would appear to be highly advisable for pickup designers in the future to adhere to the 15° tracking angle figure. In this manner the ultimate quality of which the stereophonic discs are capable will be achieved.

Acknowledgment

The author is grateful to his colleagues at CBS Laboratories and Columbia Records: Dr. P. C. Goldmark and W. S. Bachman, for stimulating interchange of ideas and support; A. Schwartz and A. Gust for valuable discussion and assistance with the experiments; and E. Porterfield for his excellent contributions to the practical application of the new technology to manufacturing processes.

A Study of Tracking-Angle Errors in Stereodisk Recording*

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Summary—**Lateral tracking-angle errors are easily controlled in stereodisk systems and present no serious problems. Vertical tracking-angle errors are much less easily controlled and frequently are the cause of serious harmonic and intermodulation distortion in playback. The mechanisms producing vertical tracking-angle distortion are outlined. Among other things, it is shown that IM products due to frequency-modulation effects in a multicomponent signal are particularly serious. It is shown, also, that the effective tilt angle in a recorder, while cutting a groove, may be considerably different from the nominal tilt angle determined by the internal geometrical configuration of the cutter. This difference can be attributed to bending of the recording stylus as a result of the drag force of the record material being removed from the groove. With present understanding it should be possible to reduce the distortion due to vertical tracking-angle errors to negligibly-low values in stereodisk systems.**

Introduction

In the years prior to the advent of stereodisk recording the effects of an error in the lateral tracking angle of a pickup were investigated thoroughly. Baerwald[1] made a comprehensive mathematical analysis of the problem and was able to calculate both the harmonic and the two-tone intermodulation distortion products arising from tracking-angle errors. Bauer[2] offered a simplified, approximate version of Baerwald's analysis and provided convenient formulas for calculating the optimum lateral tracking angle. Since each channel in a stereodisk record has a lateral component of modulation it is important that the stereo pickup have the correct lateral tracking angle if distortion in playback is to be minimized. Since each channel also has a vertical component of modulation, the vertical tracking angle of the pickup must also have the proper value. While the importance of the vertical tracking angle has been recognized for some time, only recently has the problem begun to receive the attention which it deserves.[3]

The lateral tracking angle presents no problem in the design of a pickup and its mounting arrangement, and it will not be considered further in this study. On the other hand, the design of a pickup having a correct vertical tracking angle involves the configuration of all parts of the pickup and requires, as well, a standardization of the effective tilt angle of stereo-disk recorders. The term, "effective" tilt angle, is used because during the course of this study certain anomalies were found which could be explained only by assuming a cutter tilt angle considerably different from the nominal tilt angle based on the geometrical configuration of the moving system of the cutter. In this report the nature of the anaomalous behavior will be discussed, and the experiments leading to a quantitative explanation of the observed behavior will be described. It is believed, on the basis of present understanding, that playback distortion due to vertical tracking-angle error can now be reduced to very low values.

The Nature of Vertical Tracking-Angle Distortion

In a stereo pickup the stylus tip may be, and usually is, constrained to move along a line which makes an angle ϕ with the normal to the record surface when the stylus is driven by a vertically-modulated groove. Such a configuration is illustrated in Fig. 1(a). The angle ϕ is the vertical tracking angle. A corresponding situation may occur in a stereodisk recorder if either the driving system or the cutter mounting arrangement is such as to produce a longitudinal (*i.e.*, in the direction of groove travel) component of motion of the stylus along with a vertical component when a vertical driving force is applied. In such a case, the tip of the recording stylus moves along a line making an angle θ with the normal to the record surface, as illustrated schematically in Fig. 1(b). The angle θ is called the tilt angle.

Unless the vertical tracking angle and the tilt angle are the same, the playback signal from the pickup will be a distorted version of the signal fed to the cutter during recording. The form of this distortion is illustrated for a sinusoidal signal in Fig. 2, where a 0° tilt angle and pure vertical modulation has been assumed. Since the pickup stylus follows the sinusoidal modulation $y = y_0 \sin 2\pi x/\lambda$, while being constrained to move along a line making angle ϕ with the vertical, an instantaneous phase shift of $\Delta = y \tan \phi$ occurs. In these equations y is the instantaneous value of the recorded vertical displacement of amplitude y_0, λ is the wavelength, and x is measured along the direction of groove travel. During positive displacements Δ has positive values representing instantaneous delays. During negative displace-

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[1] H. G. Baerwald, "Analytic treatment of tracking error and notes on optimal pickup design," *J. Soc. Motion Picture and Television Engrs.*, vol. 37, pp. 591–622; December, 1941.

[2] B. B. Bauer, "Tracking angle in phonograph pickups," *Electronics*, vol. 18, pp. 110–115; March, 1945.

[3] *cf.* C. C. Davis and J. G. Frayne, "The westrex stereo disk system," Proc. IRE, vol. 46, pp. 1686–1693; October, 1958.

E. P. Skov, "Stereo disk problems," *J. Aud. Engrg. Soc.*, vol. 8, pp. 12–18; January, 1960.

E. R. Madsen, "Problems Concerning the Influence of the Vertical Tracking Angle on the Reproduction Quality of Stereo Records," presented at the Thirteenth Annual Convention of the Audio Engineering Society, New York, N. Y.,; October 12, 1961.

C. R. Bastiaans, "Further thoughts on geometric conditions in the cutting and playing of stereo discs," *J. Aud. Engrg. Soc.*, vol. 11, pp. 6–15; January, 1963.

ments Δ has negative values representing phase advances. As a result the output of a displacement-sensitive pickup will have the waveform indicated in Fig. 2.

The analysis of Baerwald[1] for lateral tracking-angle errors may, with a minor modification, be used to calculate the amplitudes of the harmonic distortion components due to vertical tracking-angle errors. However, it does not appear possible to describe the distorted waveform by mathematical expressions in closed form. Nevertheless, a useful qualitative and semiquantitative understanding of the effects of vertical tracking-angle errors is possible without resort to involved mathematics. As was shown above, a vertical tracking-angle error leads to instantaneous phase delays or advances. The phase delays may also be thought of as variations in the longitudinal velocity of the groove relative to the stylus. For example, referring to the case depicted in Fig. 2, on the ascending slopes of the modulation the pickup stylus is pulled forward along with the groove, thus lowering the groove-stylus velocity, while on descending slopes the groove-stylus velocity is increased. The distorted output signal of the pickup is a wave which is frequency modulated by itself. The sidebands due to the frequency modulation are harmonics of the sinusoidal signal.

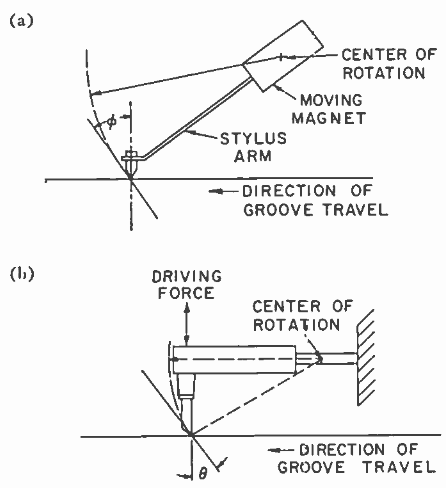

Fig. 1—(a) Diagram showing the vertical tracking angle of a moving-magnet pickup. (b) Diagram showing the tilt angle in a stylized stereodisk recorder mechanism.

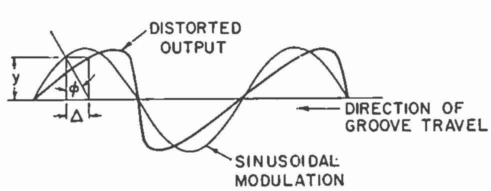

Fig. 2—Distorted waveform of a sinusoidal vertical modulation played back with a pickup having a vertical tracking angle, ϕ.

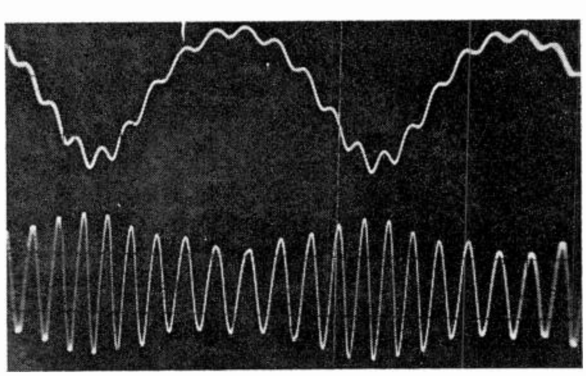

Fig. 3—Upper: Playback waveform of a 400+4000 cps vertical recording with a vertical tracking angle error. Lower: The 4000-cps component, alone, of the upper trace. Note the presence of both amplitude modulation and frequency modulation.

The frequency-modulation concept is especially useful in the consideration of intermodulation products generated in a two-component signal, such as the 400+4000-cps signal which was found to give useful results in the present study. The playback waveform of such a signal as displayed on an oscilloscope screen is shown in Fig. 3. The upper trace is the reproduced 400+4000-cps signal recorded with a 4:1 velocity ratio. The lower trace shows the 4000-cps component after filtering out the 400-cps component. A velocity-sensitive pickup and a flat playback-amplifier response were used in reproducing these signals. Both amplitude modulation and frequency modulation of the 4000-cps component by the 400-cps component are evident in the lower trace. AM and FM both give rise to intermodulation products, and both will be considered in following paragraphs.

Frequency-modulation theory may be applied to a recorded 400+4000-cps signal to permit estimates to be made of the magnitudes of the intermodulation products due to tracking-angle errors. If the peak velocity of the 400-cps component is v_{400}, the net peak longitudinal stylus-groove velocity leading to IM products in playback is approximately

$$v_x = v_{400}(\tan\theta - \tan\phi).$$

The peak frequency deviation of the 4000-cps component due to the 400-cps component is, therefore,

$$\Delta f = 4000 \frac{v_x}{V_G}$$

where V_G is the linear velocity of an unmodulated groove. The modulation index is defined as

$$m_f = \frac{\text{frequency deviation}}{\text{modulating frequency}}$$

from which we derive the relation

$$m_f = \frac{4000\,(\tan\theta - \tan\phi)\cdot v_{400}}{400\,V_G}.$$

Plots and tables of the amplitudes of an FM carrier and its sidebands as a function of the modulation index are available.[4] As a specific example, take $v_{400} = 8$ cm/sec, $\theta = 15°$, $\phi = 30°$, and the signal recorded at a 5-in diameter on a disk rotating at $33\frac{1}{3}$ rpm. In this case the

[4] *cf.* F. E. Terman, "Radio Engineering," McGraw-Hill Book Co., Inc., New York, N. Y., 3rd ed., p. 487 ff.; 1947.

frequency deviation of the 4000-cps component is 447 cps, and the modulation index is 1.11. The amplitude of the 4000-cps carrier is $A_0=0.78$. The amplitude of the first-order sidebands at 3.6 and 4.4 kcs is $A_1=0.47$. The amplitude of the second-order sidebands at 3.2 and 4.8 kcs is $A_2=0.14$. It should be borne in mind that the preceding calculations are only approximate. The equations used are valid only for small values of ϕ and θ, and the second-order effects of distortion components were neglected. In an exact calculation, the sideband amplitudes would be somewhat larger than those found here.

It should be evident from this example that the frequency deviation and the relative power carried by the FM intermodulation sidebands due to even a modest tracking-angle error are not inconsequential. Moreover, in addition to the IM products calculated above, harmonics of the 400-cps component will be produced in the amount of several per cent. The percentages of the IM and harmonic distortion products vary approximately inversely with the groove diameter for very small tracking-angle errors and/or recorded levels. For larger errors and levels the percentages of the distortion products increase more rapidly than the first power of the diameter, but in a complex way determined by the Bessel functions which describe the carrier and its sidebands. For certain conditions, which may be encountered in practice, the IM percentages become infinite. That is, all of the high-frequency power is carried in the sidebands and none in the carrier.

The amplitude modulation observed along with the frequency modulation in Fig. 3 is due partly to the fact that a velocity-sensitive pickup and a flat playback amplifier were used in the tests. With such a pickup and amplifier combination, the voltage output is proportional to frequency for a constant displacement. Thus, the frequency modulation arising from the vertical tracking-angle error will give rise to amplitude modulation in the output of the playback amplifier. In addition, another mechanism exists which produces amplitude modulation on a displacement basis. For reasons of space this mechanism will not be considered further here beyond stating that the AM which it produces is equal to and in phase with the AM due to frequency modulation. In the example used above, the amplitude of each AM sideband relative to the carrier is 11 per cent. This may be compared with the relative amplitude of 60 per cent calculated earlier for each first-order FM sideband. In the case of a displacement sensitive pickup or of a velocity-sensitivity pickup used with an integrating network, the AM will be reduced to one-half the above value, while the FM remains unchanged.

Two other points may be noted here. First, the usual instrumentation for measuring IM distortion is sensitive only to amplitude modulation of the high-frequency component of the two-component test signal. Hence, IM measurements of tracking-angle error distortion by means of such instruments will give a much too optimistic picture of the situation. Second, tracing errors caused by the fact that the recorded modulation is being traced in playback by a spherical-tipped stylus of finite radius also give rise to harmonic and IM distortion products. It can be shown that the IM products due to tracing distortion are in quadrature with the IM products due to tracking-angle distortion. In general, both tracing and tracking-angle distortion occur together in playback although for certain conditions and certain types of signal one or the other may predominate. This coexistence of two sources of distortion sometimes complicates the interpretation of experimental measurements of either source.

Distortion Measurements

Stereodisk recorders in current commercial use are reported to have tilt angles ranging from 0° to 23°. As a part of the present study, the vertical tracking angles of a number of pickups were measured. These included all types in current use and all models which could be considered for use in high-quality phonograph systems. The measuring technique is described in Appendix I. In the various models the measured angles ranged from 5° to 50°. In view of the tracking-angle errors which can and do occur in stereophonic phonograph systems, it should be instructive to make experimental measurements of the distortion in playback as a function of the vertical tracking angle. For this purpose a vertically-

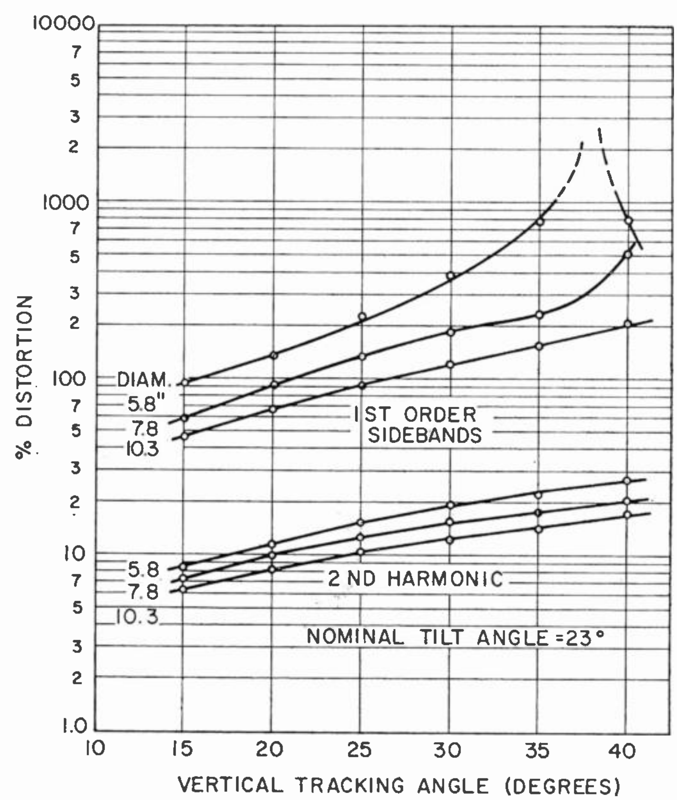

Fig. 4—Playback distortion measurements of a 400+4000-cps test signal recorded with a nominal 23° tilt angle.

modulated test record was made using the 400+4000-cps signal with a 4:1 velocity ratio, and a peak vertical velocity of 8.8 cm/sec. This signal was recorded in bands at various diameters on a disk rotating at $33\frac{1}{3}$ rpm. A Westrex 3-C recorder was used in the conventional manner in making the record master.

In playing back the IM test record, a pickup having a high-quality moving-magnet transducer was used. The pickup was mounted on the pickup arm by means of a head which permitted adjustment of the vertical tracking angle over the range from 10° to 40°. The center of rotation of the angular adjustment was located at the stylus, so that adjustment of the vertical angle did not alter the lateral tracking angle or the overhang. The output of the pickup was fed to an amplifier having a flat response. A wave analyzer was used to measure the magnitudes of the signal and distortion components. A set of playback measurements of the IM test record is shown in Fig. 4. The right- and left-hand channels were measured individually. Results for the two channels were nearly equal, and their average values have been used in plotting the curves of Fig. 4. The percentage of the 2nd harmonic was determined by measuring the voltages of the 400-cps fundamental and of the harmonic at 800 cps. The intermodulation products were determined by measuring the voltages of the 4000-cps carrier and of each of the sidebands at 3600 and 4400 cps. The two sideband voltages were added, and their sum divided by the carrier voltage and multiplied by 100 gave the percentage first-order sidebands, as plotted.

The results shown in Fig. 4 exhibit an unexpected form of behavior. It is known that the Westrex 3-C recorder has a tilt angle of 23°. Consequently, one expects the distortion in playback to be a minimum when the vertical tracking angle is also 23°. Instead, the distortion is seen to increase with increasing angle, with no indication whatever of a minimum at 23°. Similar results were obtained when the vertical tracking angle was varied by using different pickups. Regardless of other differences in the performance of the pickups, the lower the value of the vertical tracking angle, the lower was the distortion when playing back the test record described above.

In order to investigate this anomalous behavior more extensively, two additional IM test records were made. One of these was recorded with a Neumann recorder having a nominal tilt angle of 0°. The other was recorded with the Westrex 3-C recorder rotated through a 15° angle to give a tilt angle of 38°. Both records contained the same signal as the first test record. The playback results for these two records are shown in Figs. 5 and 6. In the case of the 0° tilt angle, not only is no minimum observed in the distortion vs angle curves of Fig. 5, but the distortions measured are considerably higher than in the case of the 23° tilt angle. Two "infinities" are found in the first-order sideband curves whereas only one infinity occurred in the case of the 23° data. More will be said about these infinities in a later

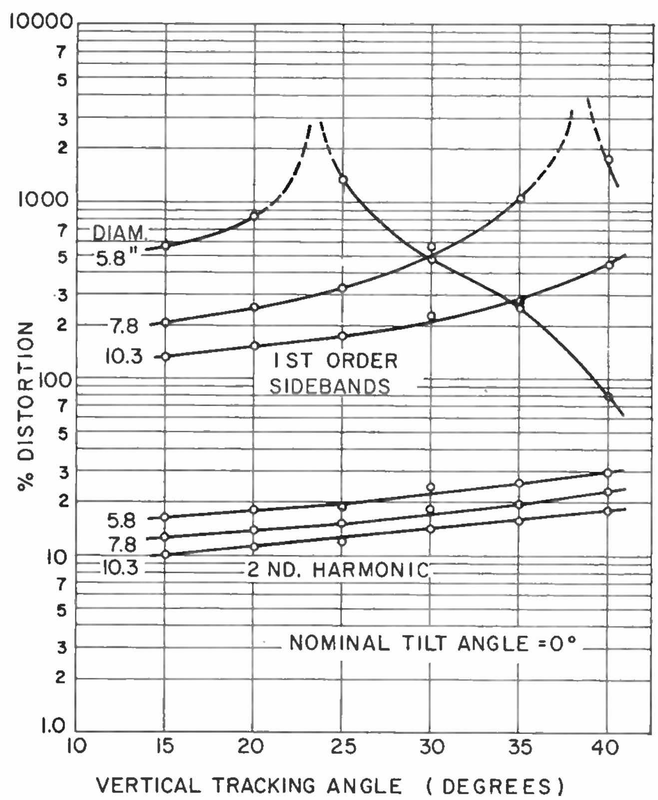

Fig. 5—Playback distortion measurements of a 400+4000-cps test signal recorded with a nominal 0° tilt angle.

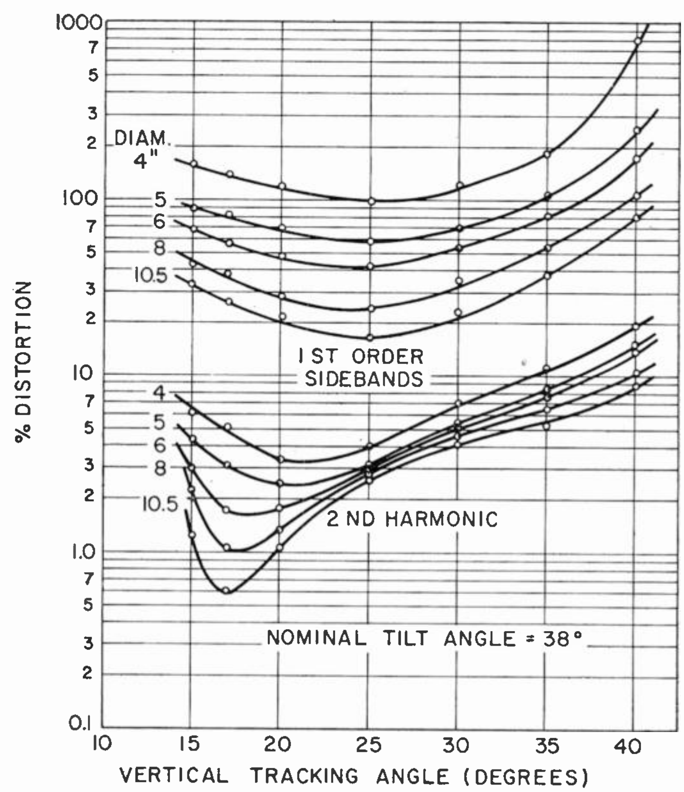

Fig. 6—Playback distortion measurements of a 400+4000-cps test signal recorded with a nominal 38° tilt angle.

paragraph. The data of Fig. 6 for the 38° tilt angle not only show markedly lower distortions than were found in the other cases, but now minima occur in the curves. However, the minima occur for vertical tracking angles some 13° to 20° lower than the 38° tilt angle. The minima of the curves of Fig. 6 do not all occur for the same tracking angle. While it cannot now be stated with complete certainty, it is likely that these differences are due in large part to the presence of tracing distortion which has a greater effect on the IM products than on the low-frequency harmonics, and which has a greater dependency on diameter than does tracking-angle distortion under the conditions of this test.

The results of the tests depicted in Figs. 4–6 indicate that the supposed relationship exists between the tilt angle and the vertical tracking angle, but that one or the other angle has an effective value which is considerably different from the nominal value. In the next section it will be shown that a mechanism exists in the recording process which is sufficient to account for the anomalous behavior. The same conclusion was reached by Bauer and his co-workers.[5] Before moving to that discussion, let us consider more carefully the "infinities" noted in Figs. 4 and 5. The plotted points of the per cent-sideband vs tracking-angle curves would not, by themselves, be sufficient to justify the extrapolations of any of the curves to indicate distortions approaching infinity. However, examination of the raw data on which the curves are based shows that, as the critical angles are approached and exceeded, the amplitudes of the sidebands increase gradually while the amplitude of the carrier goes through a minimum. This is in accordance with FM theory which shows that the Bessel function giving the amplitude of the carrier goes through its first zero for a modulation index of 2.4. The Bessel functions giving the sideband amplitudes are still increasing for this value of index.

The vertical tracking angles at which infinities occur can be used in making an approximate calculation of effective tilt angles. Remembering that the modulation index for this condition is $m_f=2.4$, and using the expression derived earlier to relate the index and the angles, we can write

$$\tan\theta_{\text{eff}} = \tan\phi_\infty - 0.151D$$

for $\phi>\theta$, where D is the groove diameter in inches, and for the peak recorded velocity of 8.8 cm/sec used in the measurements. The four occurrences of infinities in Figs. 4–6 (including an extrapolation of the upper-most curve in Fig. 6) lead to the values tabulated below.

ϕ_∞	D (in)	$\tan\phi_\infty$	$\tan\theta_{\text{eff}}$	θ_{eff}	θ_{nom}	$\theta_{\text{nom}}-\theta_{\text{eff}}$
24°	5.8	0.445	−0.430	−24°	0°	24°
38°	7.8	0.781	−0.399	−22°	0°	22°
38°	5.8	0.781	−0.094	−5°	23°	28°
43°	4.0	0.932	0.330	18°	38°	20°

[5] B. B. Bauer, "Vertical tracking improvements in stereo recording," *Audio* vol. 47, pp. 19–22; February, 1963.

As was pointed out earlier, the expressions used in these calculations are valid only when ϕ and θ are small, so that the values of θ_{eff} as tabulated are only approximate. The true values of θ_{eff} must be somewhat larger (*i.e.*, more positive) than those shown. Even with this limitation the differences between nominal and effective tilt angles found here agree reasonably well among themselves. The value of the effective tilt angle shown in the last row is also in reasonable agreement with the angle for minimum distortion in Fig. 6.

A Basis for Tilt-Angle Discrepancies

The playback data reported here indicate that both types of recorder used in these tests can have effective tilt angles quite different from their nominal tilt angles. Only the Westrex 3-C recorder was available for careful study in this work. Measurements of this recorder show that the discrepancy between effective and nominal tilt angles can be attributed almost entirely to bending of the recording stylus and rocking of the stylus shank in its tapered-hole mounting while cutting vertically-modulated grooves. The experiments on which this conclusion is based will now be outlined.

When the Westrex 3-C recorder is fed a signal phased for vertical motion of the stylus, and the stylus tip moving freely in air is observed by means of a microscope, the tip motion corresponds very closely to that expected for a 23° tilt angle. Likewise, the tip motion corresponds to a 23° angle when the stylus is deflected by a vertical force applied to the tip of the stylus. However, a quite different result is observed when a longitudinal force is applied to the stylus tip. A rigid fixture was constructed to hold the recorder while various longitudinal forces were applied by means of a wire cemented to the stylus tip. A microscope was used to measure both longitudinal (x) and vertical (y) deflections of the stylus tip for each application of the longitudinal force. A typical plot of x and y deflections as a function of longitudinal force is shown in Fig. 7. Each point is the average value of several measurements. If the moving system of the cutter, including the stylus, were perfectly rigid the stylus tip would be constrained to move along a line making a 23° angle with the Y axis, and the longitudinal deflection would be $x=y\tan 23°=0.424\,y$. By contrast, the data of Fig. 7 show that the longitudinal deflections are considerably greater than the vertical deflections, indicating at once that the cutter system is not perfectly rigid. Examination and measurement of the motion of various points along the length of the stylus and at other points on the moving system show that an excessively large longitudinal component of deflection was present only on the stylus. Results of measurements of the bending compliance of other styli mounted in tapered noles in a large solid block lent support to the conclusion that stylus bending and rocking in the mounting hole were responsible for the observed longitudinal deflections. Greater over-all exposed lengths of stylus resulted in greater deflections for a given longitudinal force. Styli with dural shanks

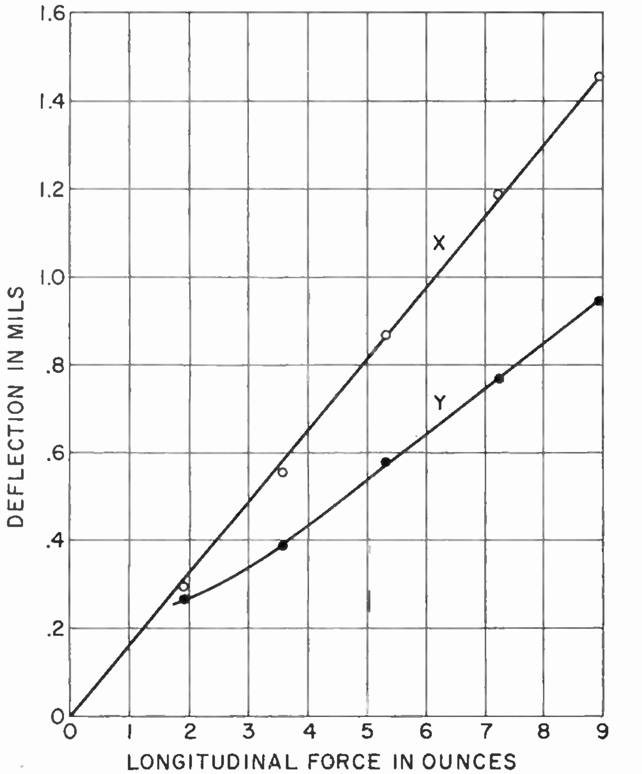

Fig. 7—Longitudinal deflection (x) and vertical deflection (y) of the stylus tip of a Westrex 3-C recorder for a longitudinal force applied to the stylus tip.

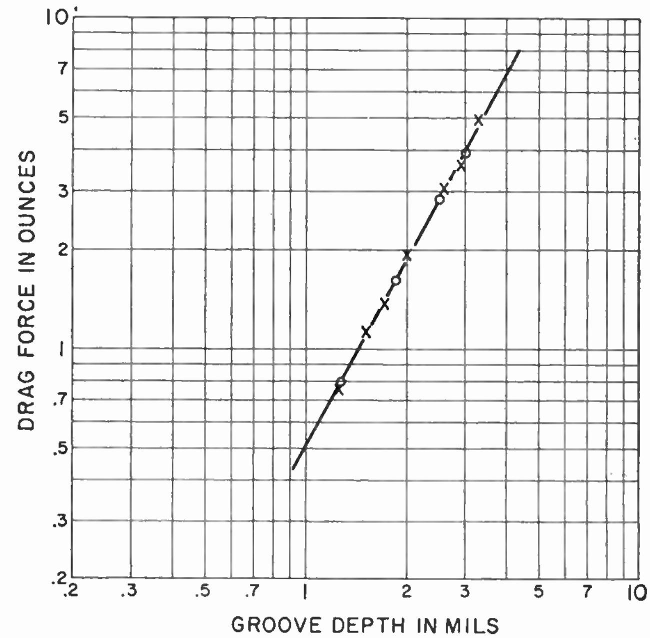

Fig. 8—Drag force at tip of recording stylus as a function of groove depth during recording.

were found to be more compliant than those with brass shanks, as is to be expected.

It now remains to establish whether or not longitudinal forces of sufficient magnitude to produce the required deflections are present during the recording process. To this end the drag force exerted on the stylus by the record material being removed from the groove was measured, using the techniques described in Appendix II. The results of this measurement are presented in Fig. 8, where the drag force is plotted as a function of the groove depth on a log-log chart. When the data of Figs. 7 and 8 are taken together, it is seen that the drag force is sufficient to produce significant longitudinal displacements of the tip of the recording stylus. The value of the drag force at any instant depends on the groove depth at that instant. When vertical modulation is being recorded, the drag force will be modulated and hence, also, the longitudinal component of the motion of the stylus tip will be modulated. The displacement of the stylus tip in the direction of groove travel becomes greater as the stylus moves downward to cut a deeper groove. On the other hand, the displacement of the stylus tip due to the true tilt angle of the cutter becomes greater as the stylus moves upward. The longitudinal displacements due to the two sources are out of phase and will partially or totally cancel one another, thus resulting in a smaller net longitudinal displacement which corresponds to a smaller effective tilt angle.

When the groove dimensions of the record used in obtaining the playback data of Fig. 4 are taken in conjunction with the data of Figs. 7 and 8, an effective tilt angle of 1° is calculated for the Westrex 3-C recorder and its stylus as used in these tests. In view of the uncertainties associated with the stylus deflection measurements due to using different styli in different parts of the work, and in view of the approximate nature of the analysis, the agreement between the present value of 1° for the effective tilt angle and the −5° value calculated earlier from IM measurements is remarkably good. It is possible that the remaining discrepancy can be attributed to lacquer springback as suggested.[5]

Conclusions

Among the many interesting results of this study, the following major conclusions may be listed: 1) Vertical-tracking-angle errors can, and frequently do, produce serious distortion in 45/45 stereodisk systems. 2) A very serious form of such distortion is the intermodulation of higher- by lower-frequency components of the program. 3) This type of intermodulation results from both amplitude modulation and frequency modulation of the upper- by the lower-frequency components. 4) Since the intermodulation products caused by the frequency modulation are much greater in magnitude than those caused by the amplitude modulation, the conventional methods of measuring IM distortion, which are based on a measurement of amplitude modulation, do not give a correct indication of vertical tracking-angle distortion. 5) Vertical tracking-angle distortion may be reduced to negligible values, in principle, if the vertical tracking angle of the pickup is made equal to the tilt angle of the recorder. 6) In practice, there may be a considerable discrepancy between the effective tilt angle of a recorder and the nominal tilt angle based on the geometrical configuration of the cutter. 7) It appears that the greatest part of the tilt-angle discrepancy can be attributed to bending and rocking of the recording stylus in response to the action of the drag force of the record material at the stylus tip.

Some obvious and difficult problems must be solved if an industry-wide standard tilt angle and vertical tracking angle is to be proposed and accepted. However, the benefit to be gained in terms of improved quality in stereodisk reproduction makes the move toward standardization well worthwhile.

Appendix I

The Measurement of Vertical Tracking Angles

The fixture sketched in Fig. 9 was used in measuring the vertical tracking angles of pickups. The pickup is mounted on one side of the fixture. On the opposite side of the fixture is mounted a micrometer screw with its axis perpendicular to the mounting flanges of the pickup. The end of the micrometer spindle is brought into contact with the stylus and is advanced by a measured amount Δy. At the same time, a microscope is used to view the stylus, looking down into the plane of the drawing, and to measure the stylus deflection Δy in the direction perpendicular to the micrometer axis. The vertical tracking angle is given by $\tan \phi = \Delta x / \Delta y$. In normal use of a pickup in a phonograph the stylus arm has a static deflection due to the tracking force. The values of Δx and Δy should be measured for deflections close to the static deflection which is normal for the pickup under test.

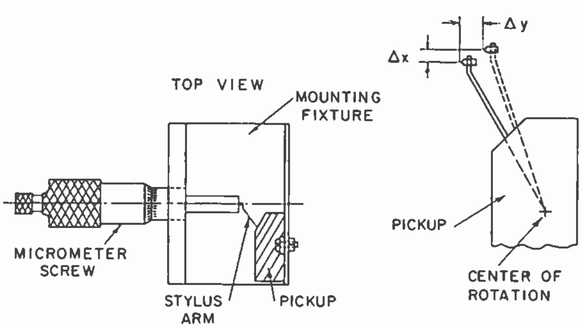

Fig. 9—Technique for the measurement of the vertical tracking angle of a pickup.

Appendix II

Measurement of the Drag Force on a Recording Stylus

The recorder was mounted on a Neumann recording lathe, and the fluid-coupling feature of the turntable drive was used to advantage in the measurement of the drag force on the stylus. The stops and springs which normally provide a compliant mechanical link between the turntable and the drive motor were removed so that the fluid coupling was the only link between motor shaft and turntable. When the turntable without a cutting load was driven in this manner it rotated at the normal $33\frac{1}{3}$ rpm rate as indicated by the stationary stroboscope pattern when the strobe marks on the turntable rim were illuminated by a power-line-synchronized strobe lamp. When the recording stylus was cutting a groove in a lacquer surface of a disk on the turntable, the rotational speed was reduced by a small but readily measurable amount. The reduced speed was measured by noting the time required for ten strobe marks to progress past a fixed point in front of the line-synchronized strobe lamp. The rotational speed was measured for a series of groove depths and for grooves at various diameters.

The drag force required to reduce the speed of the turntable by various amounts was determined by means of the Prony brake arrangement sketched in Fig. 10. A cord making a wrap somewhat more than 180° at a 16-in diameter around the rim of the turntable was attached to two spring balances. Various tensions could be applied to the cord and could be read on the balances. Tension in the cord caused a friction torque at the turntable rim, resulting in a speed reduction. The turntable speed was measured by measuring the time required for ten strobe marks to pass a point in front of the strobe lamp as before. Data obtained in this way were plotted to give a calibration curve of turntable speed vs braking force at a 16-in diameter. With this curve at hand, the drag forces associated with various groove cuts at various diameters and corresponding speed reductions could be found. The results of drag force vs groove depth are plotted in Fig. 8. Grooves cut near a 10-in diameter are indicated by circles on the graph. The crosses correspond to grooves near a 6-in diameter. It is apparent that the drag force is not a strong function of linear groove velocity in the range of the present measurements.

As a matter of passing interest, removal of the stylus heat during recording resulted in a small (5 per cent—20 per cent) *decrease* in the drag force on the stylus. The presence of modulation did not make a measurable difference in the drag force.

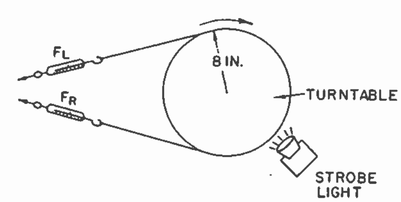

Fig. 10—Prony brake used in the measurement of turntable speed as a function of braking torque.

Acknowledgment

The authors wish to express their appreciation for assistance received in various portions of this study from J. B. Halter, M. Pradervand and T. A. Spitzo, and for the continuing support and encouragement of Dr. H. F. Olson, Director of the Acoustical and Electromechanical Research Laboratory of RCA.

Speech Compression CODIMEX System*

J. L. DAGUET†

Summary—A relatively simple installation is sufficient to obtain a four-to-one reduction of the frequency bandwidth occupied by a telephonic signal. The speech is decomposed into its three principal formants which constitute the basic information carrying elements. Each of these formants is compressed both in amplitude and in frequency following a definite mathematical law, and in accordance with a specified formula for transforming the frequency spectrum. The speech obtained is highly intelligible and the spectrum of the reconstituted signal is a satisfactory reproduction of that of the initial telephonic signal. The installation enables additional telephonic communications to be transmitted in the existing channels. CODIMEX means: COmpression, DIvision, Multiplication, EXpansion.

I. Introduction

ONE of the fundamental problems in telecommunications is making the best use of the transmission media, *i.e.*, placing more and more telephone conversation on a single pair of wires, in a cable or in a given RF bandwidth allocated to a radiotelephonic network.

A method offering means of increasing the capacity of a channel consists in compressing the speech frequency spectrum. The vocoder system, conceived by Homer Dudley, is a well-known example. The reduction of the bandwidth occupied by a voice signal is made possible by the redundancy existing in speech. A particular form of this redundancy is characterized by the existence of formants which appear as zones of concentrated energy in the graphic representation of vocal sound spectra in respect to time. This approach was used by Potter in his work entitled "Visible Speech."[1] The notion of formants was dealt with in a study made by Flanagan.[2]

The purpose of this treatise is to define the transformations applied to each of these signals and to describe the manner in which these operations were achieved in the installation.

The CODIMEX system falls into the category of "formant tracking" devices which likewise include the VOBANC system described in an article by Bogert.[3] These two processes are the same in that the instantaneous frequency of a signal transposed into a single sideband without carrier undergoes a dividing operation, but differ as to the operation effected on the instantaneous amplitude of the same transposed signal.

In the VOBANC system the instantaneous amplitude, as defined above, is maintained, whereas with the CODIMEX system this amplitude undergoes an operation linked to that applied to the frequency. An algebraic operation determined in respect to the analytic signal makes possible the division of the logarithm of the amplitude by the same factor used in dividing the frequency. The related divisions result in the compression of the dynamic. This relationship may be likened to the similarity which exists in Shannon's equation for determining the capacity of a channel, $C=W \log (1+S/N)$, between the frequency bandwidth and the logarithm of the signal-to-noise ratio. It is theoretically possible, for a given capacity, to reduce the frequency bandwidth of a channel by increasing the logarithm of the signal-to-noise ratio.

The spectral analyses carried out on the compressed formants of the CODIMEX system reveal the following effects:

1) Reduction by a factor of 8 in the frequency scale excursion,
2) Concentration of the spectrum about an average frequency,
3) Increase of the average amplitude level which shows only slight variations.

In the VOBANC system there is a reduction by a factor of 2 in the excursion of the formants[3] which keep the same spectral form. With the Codimex system effects 2) and 3) result in the transmission of signals representing the compressed formants into the channel at nearly similar and slightly varying levels. The energy of these signals is concentrated in a very narrow frequency zone.

Intelligibility measurements give proof of the excellent protection against noise existing in the transmission channel as long as such noise is beneath a threshold of approximately 15 db below the signal.

Demultiplexing operations are effected by means of compressed formant selection filters which, by virtue of the compandor effect, require less attenuation in the bands to be eliminated. These filters are the critical elements of the system and account for the greater part of the deterioration likely to occur in the reconstituted voice signal. The less stringent the characteristics of the filters, the less the deterioration is of effect. They must be carefully studied to avoid overshoot phenomena manifested by effects analogous to those described by Bogert[3] as burbles.

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[1] R. K. Potter, Kopp, and Green, "Visible Speech," D. Van Nostrand Company, Inc., Princeton, N. J.; 1947.

[2] James L. Flanagan, "Automatic extraction of formant frequencies from continuous speech," *J. Acoust. Soc. Am.*, vol. 28, pp. 110–118; January, 1956.

[3] B. P. Bogert, "The VOBANC—a two to one speech band with reduction system," *J. Acoust. Soc. Am.*, vol. 31, pp. 399, 404; May, 1956.

II. Operation

The voice signal is, to begin with, separated into three parts corresponding to the frequency bands 300–700 Hz, 700–2000 Hz and 2000–3400 Hz. Each of these bands contains one of the three main formants or basic elements in the transmission of vocal information. Frequency analyzers clearly show the existence of three zones of maximum energy characterizing each of the formants. A separate operation of bandwidth reduction is effected on each formant isolated by an appropriate filter network.

A. Analytic signal

The starting point is the breakdown of the signal into elements: amplitude $a(t)$ in respect to time and the cosine of an angle in respect to time $\cos\phi(t)$ in such way as to represent a real signal which is a function of time $s(t)$

$$s(t) = a(t)\cos\phi(t).$$

Let us consider a signal $s(t)$ occupying a limited frequency band of finite energy, *i.e.*, where $s(t)$ is a summable square. Using the Hilbert transformation, the signal may be made to correspond to a quadrature signal,

$$\sigma(t) = \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{s(\tau)}{\tau - t}\,d\tau.$$

We know that the reverse is likewise true,

$$s(t) = -\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{\sigma(\tau)}{\tau - t}\,d\tau.$$

The functions $s(t)$ and $\sigma(t)$ are orthogonal and constitute the two components of a vector represented by the complex function of time

$$\psi(t) = s(t) + j\sigma(t)$$

introduced by D Gabor.[5] The function $\psi(t)$ was named, by Ville, the "analytic signal."[6] The modulus and argument will be rendered evident by writing

$$\psi(t) = a(t)e^{j\phi(t)}$$

where $a(t)$ is an essentially positive function.

The actual signal is the real part of the analytic signal. The functions $a(t)$ and $\cos\phi(t)$ are unique and entirely determined from $s(t)$.

$\psi(t)$ is the affix of a vector situated in the complex plane. This vector is of length $a(t)$ and of angle $\phi(t)$ in respect to the time axis. It rotates about the origin with an angular velocity $d\,\phi(t)/dt$ which may serve to define the instantaneous frequency. In the case of a sinusoidal frequency $\cos\omega t$ the analytic signal vector is $e^{j\omega t}$, which will be recognized as Fresnel's vector.

[5] D. Gabor, "Theory of communication," *J. IEE.*, pt. III GB, vol. 93, pp. 429–457; November, 1945.

[6] J. Ville, "Théorie de la notion signal analytique Cables et Transmission," vol. 1, pp. 61–74; January, 1948.

B. Reduction of the Signal Bandwidth

The process of frequency compression consists in subjecting the signal, by appropriate technological methods, to transformations equivalent to a mathematical operation on the analytic signal. In the frequency compression system examined, the analytic signal is made to undergo a nonlinear operation characterized by the extraction of the root,

$$\sqrt{\psi(t)} = \sqrt{a(t)}\cdot e\,\frac{j\phi(t)}{2}.$$

It may be shown that, under these conditions, the spectrum of the signal obtained, in complex notation, will be given in function of the spectrum of the original signal, before extracting the root, by the integral equation,

$$\int_0^F \phi_{\frac{1}{2}}(f)\times\phi_{\frac{1}{2}}(F-f)df - \phi(F) = 0.$$

We have, by definition of the complex spectrum

$$\psi(t) = \int_{-\infty}^{+\infty}\phi(F)e^{2\pi jFt}dF.$$

The complex spectrum $\phi(F)$ is obtained from the spectrum of the real signal $s(t)$ by suppressing the negative frequencies and doubling the amplitude of the positive frequencies.

$$\phi(F) = 0 \quad \text{for} \quad F < 0.$$

For example, with a sinusoidal signal,

$$\cos 2\pi ft = \frac{e^{2\pi jft} + e^{-2\pi jft}}{2} = \mathrm{Re}\left[e^{2\pi jft}\right]$$

$$\psi(t) = e^{2\pi jft}.$$

Using

$$\psi_{\frac{1}{2}}(t) = \sqrt{x(t)} = \int_{-\infty}^{+\infty}\phi_{\frac{1}{2}}(f)e^{2\pi jft}df$$

the product $\psi_{\frac{1}{2}}(t)\times\psi_{\frac{1}{2}}(t) = \psi(t)$ becomes the convolution integral in the Fourier transformation. The image function is

$$\phi(F) = \int_{-\infty}^{+\infty}\phi_{\frac{1}{2}}(f)\times\phi_{\frac{1}{2}}(F-f)df$$

since

$$\phi_{\frac{1}{2}}(f) = 0 \quad \text{for } f < 0$$

$$\phi_{\frac{1}{2}}(F-f) = 0 \quad \text{for } f > F.$$

We obtain

$$\phi(F) = \int_0^F \phi_{\frac{1}{2}}(f)x\phi_{\frac{1}{2}}(F-f)df$$

by a change of variable. The equation may be written more symmetrically

$$\phi(F) = \int_{-F/2}^{+F/2} \phi\tfrac{1}{2}\left(\frac{F}{2} - f\right) \times \phi\frac{1}{2}\left(\frac{F}{2} + f\right) df.$$

This correspondence enables $\phi\frac{1}{2}(F/2)$ to the found when $\phi(F)$ is known. Inversely $\phi(F)$ may be found when $\phi\frac{1}{2}$ $(F/2)$ is known. If $\phi\frac{1}{2}$ $(F/2)$ occupies a limited band, $\phi(F)$ will occupy a band twice as wide with double frequencies. The results of the transformation may be studied either mathematically or graphically for simple functions.

In the case of a square-wave signal the width of the spectrum is infinite and the decrease of the spectral amplitude when the frequency increases, is slow. It is by applying the integral equation that the spectrum is maintained. This is only a check as the squaring operation does not change the rectangular form of the wave.

When $\phi(F)$ is not limited in bandwidth and is represented by a Gauss curve, the transform $\phi\frac{1}{2}$ $(F/2)$ is a homothetic Gaussian curve of $1/\sqrt{2}$ ratio. In the case of frequency modulation, typical frequency compression is obtained by frequency division.

For signals corresponding to the speech formants, the spectral analyses show that the operation corresponds to a reduction in bandwidth proportional to the compression. The square root extraction may be repeated. In practice, three successive operations are carried out with the resulting signal

$$[\psi(t)]^{1/8} = [a(t)]^{1/8}\, e^{i\phi(t)/8}.$$

Following transmission, the signal undergoes the reverse operations at the receiving end.

C. The Technology Means Applied

The amplitude $a(t)$ and the phase $\phi(t)$ of the analytic signal $\psi(t)$ associated with a real signal are bought into evidence by subjecting the real signal to a single sideband translation with elimination of the carrier wave. If Ω is the carrier angular frequency, the signal $a(t)$ cos $[\Omega t+\phi(t)]$ is obtained from $s(t) = a(t)$ cos $\phi(t)$. This results from the definition of the analytic signal, which implicitly contains the quadrature signal $\sigma(t)$ of the signal $s(t)$.

It is known that single sideband may be obtained by two modulations using carriers in quadrature of the signal $s(t)$ and the quadrature signal $\sigma(t)$

$$\begin{aligned} s(t) \cos \Omega t - \sigma(t) \sin \Omega t &= a(t) \cos \phi(t) \cos \Omega t - a(t) \sin \phi(t) \sin \Omega t \\ &= a(t) \cos [\Omega t + \phi(t)]. \end{aligned}$$

The single sideband with suppressed carrier therefore produces an envelope $a(t)$ and the cosine of a variable phase $\Omega t+\phi(t)$.

The technological operations corresponding to the extraction of the square root are directly carried out on the transposed signal (Fig. 1).

1) The envelope $a(t)$ is detected. This is possible due to separation of the spectra of $a(t)$ and cos $[\Omega t+\phi(t)]$ [Fig. 1(a)].
2) The envelope $a(t)$ is added to the signal giving:

$$a(t)\{1 + \cos [\Omega t + \phi(t)]\} \text{ [Fig. 1(b)].}$$

3) This signal is fed into a network in which the output voltage is proportional to the square root of the input voltage [Fig. 1(c)]

$$(a(t)\{1 + \cos [\Omega t + \phi(t)]\})^{1/2} = \sqrt{2a(t)}\left|\cos \frac{\Omega t + \phi(t)}{2}\right|.$$

4) At the points where the signal $|\cos [\Omega t+\phi(t)]/^2|$ passes through zero, a trigger switches a scale of two, thus producing a division by 2. [Fig. 1(*d*)].
5) At each triggering, the counter tube output operates a commutator which reverses the sign of

$$\cos \frac{\Omega t + \phi(t)}{2}$$

thus giving

$$\sqrt{2a(t)} \cos \frac{\Omega t + \phi(t)}{2} \qquad \text{[(Fig. 1(e)].}$$

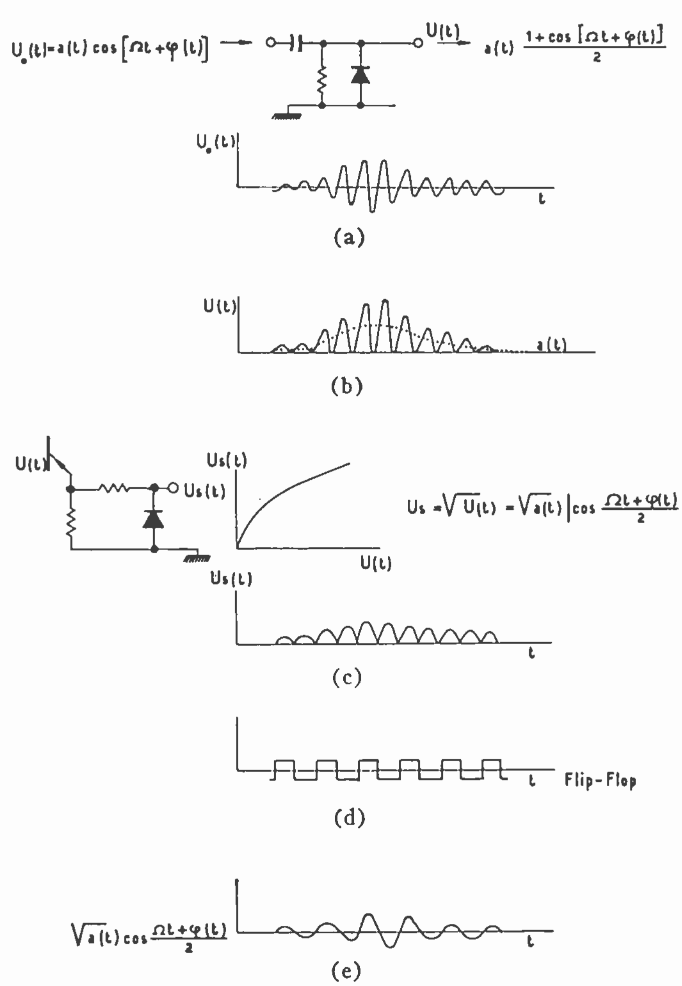


Fig. 1—Coder.

In this manner the square root is extracted.

The preceding operations are effected three successive times on each signal. For carrying out the transmission, the transpositions are made in order to constitute a frequency multiplex. At the receiving end the following operations take place:

1) Demultiplexing and selection of the compressed signals.
2) Squaring by means of a full-wave rectifier operating along a parabolic characteristic (Fig. 2).

$$\left[\sqrt{2a(t)}\ \cos\frac{\Omega t+\phi(t)}{2}\right]^2 = 2a(t)\cos^2\frac{\Omega t+\phi(t)}{2}\ 2a(t)\left\{\frac{1+\cos\left[\Omega t+\phi(t)\right]}{2}\right\}.$$

The low-frequency signal $a(t)$ is eliminated by a high pass filter and we have

$$a(t)\cos\left[\Omega t+\phi(t)\right].$$

3) A demodulation by means of a carrier of angular frequency Ω restores the signal

$$s(t) = a(t)\cos\phi(t).$$

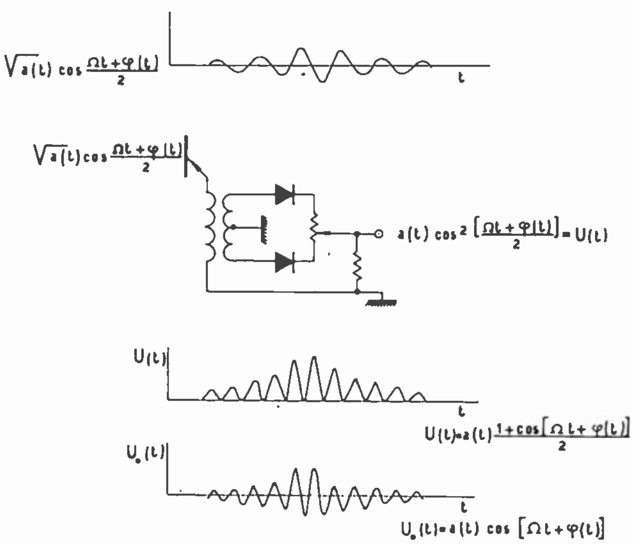


Fig. 2—Decoder.

III. Design

The principle of the various operations to be effected has already been described. In the practical fulfillment of these operations, the above principles were adhered to for the sake of reliability.

The device for reducing bandwidth includes, in the transmitter installation, a circuit designed to select the three speech formants and translate them into a single sideband with suppression of the carrier. The enclosed wiring diagram (Fig. 3) shows the formant selector filters 1, 2 and 3 corresponding respectively to formants 1, 2 and 3. The bands chosen are

Filter 1	300 to 700 Hz
Filter 2	700 to 2000 Hz
Filter 3	2000 to 3400 Hz.

The translation in the neighborhood of 100 kHz is effected in two steps for formants 1 and 2, in such way as to sufficiently attenuate the carrier and the image band. Formant 3, the frequency band of which is situated higher in the audio spectrum, can be translated by a single modulation. After passing through the dividing compressor, each formant is channeled into a frequency band of reduced width (around 12 kHz). The three compressed formants are then translated into audio frequency for juxtaposition into a 300–1200-Hz band corresponding to the band utilized for the transmission of a telephone channel. Each communication of reduced width can be transmitted in a frequency multiplex or any other multiplex system. At the other end of the transmission hook-up the reverse operations are effected (Fig. 4). The three compressed formants are separated by selective filters and translated to approximately 12 kHz. Each formant passes through a multiplier-expander device restoring the dynamic and the 100-Hz (approx.) bandwidth and is relocated by demodulation in its initial position in the audio range. The two essential elements of the system are the compressor-divider and the multiplier-expander which carry out the nonlinear operations.

Compressor-Divider

The purpose of this element is to transform the signal

$$a(t)\cos\left[\Omega t+\phi(t)\right]$$

into

$$\sqrt[8]{a(t)}\ \cos\left[\frac{\Omega t+\phi(t)}{8}\right].$$

In an initial operation an instantaneous compressor provides

$$\sqrt[8]{a(t)}\ \cos\left[\Omega t+\phi(t)\right].$$

This compressor is composed of three identical stages (Fig. 5) each of which include a current injector charged through a capacitor by two germanium junction diodes mounted back to back and working in the parabolic portion of their characteristic. The input signal is the current supplied by the injector, *i.e.*, an amplifier of very high internal impedance. The output signal is the voltage across the dipole formed by two cascade mounted diodes.

To insure correct operation in the parabolic portion of the characteristic the diodes must be properly biased. This bias must be adjustable in function of the ambient temperature.

In examining the characteristics of germanium

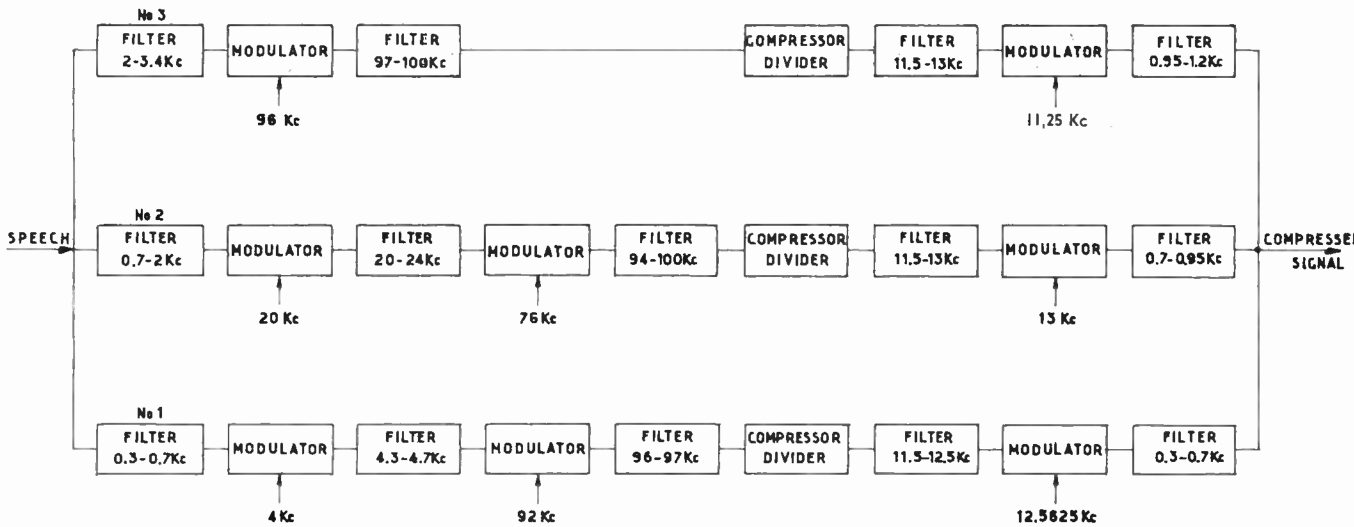


Fig. 3—Emission.

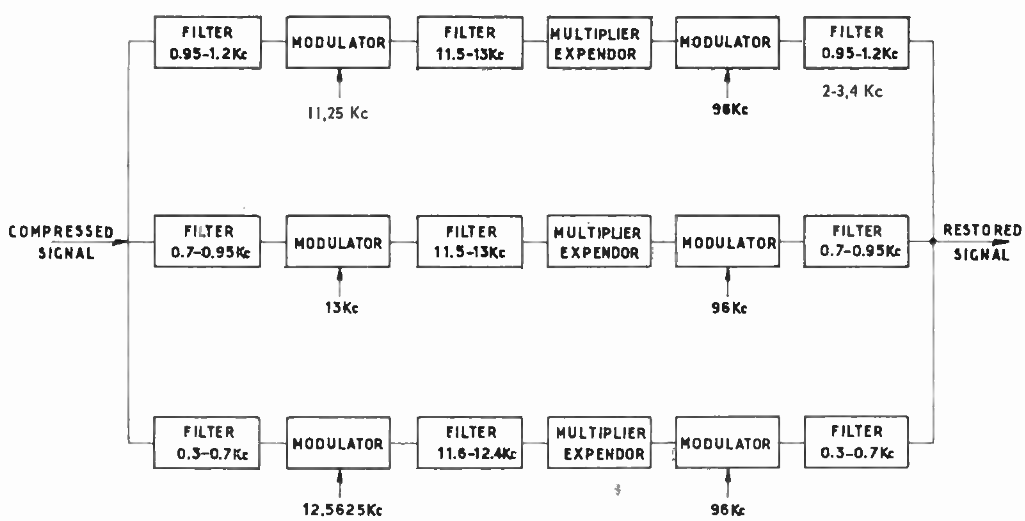


Fig. 4—Reception.

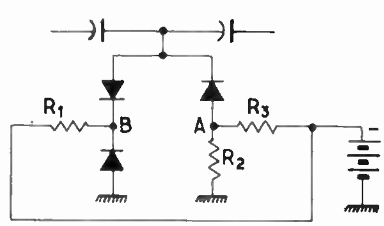


Fig. 5—Compressor.

diodes it will be seen first that the curved regions separate from each other, in function of the temperature, by a displacement parallel to the voltage axis and second, that the rectilinear portions tend to diverge. Moreover, a part of the rectilinear zone undergoes a displacement double that of the curved region.

The following bias circuit has been adopted. The lower part of the compression dipole formed by the two diodes back to back is open circuited to permit biasing by means of a dc voltage applied in series with the diodes. A negative voltage is applied to the lower leg of the diode directed downwards in order to bring a diode of the same type, the other leg of which is grounded, into the linear zone. The current flow in this compensation diode is adjusted to the value corresponding to a temperature vs voltage displacement double that corresponding to the parabolic region.

The compensation diode being biased on offers slight resistance (around 100 ohms) to alternating currents. A 100-ohm resistor is placed in the leg of the other compression diode. By means of a resistor connected to a negative voltage the bias applied in series to the two compression diodes is adjusted, at ambient temperature, to obtain a parabolic compression of the dynamic of 48 db with an approximation of ± 1 db.

In function of the temperature variations, the compensation diode corrects the bias over a range -40 to $+55°$C.

The signal obtained at the output of the third compression stage is sent in two directions. In the first direction a two-stage limiting circuit effective to over 50 db extracts at the zero points the pulses constituting the signal $\cos\,[\phi(t)+\Omega t]$. These pulses actuate a three stage counter which divides the signal by eight.

In the case where a sinusoidal frequency $A \cos \omega t$ is applied to the audio input of the channel, the final counter delivers a square-wave signal with a fundamental of

$$\cos\left[\frac{\omega t + \Omega t}{8}\right].$$

In taking a zero point of the signal for the time axis origin the Fourier series applied is

$$\frac{4}{\pi}\left[\sin\frac{\omega t + \Omega t}{8} + \frac{1}{3}\sin\frac{3}{8}(\omega t + \Omega t) + \frac{1}{5}\sin\frac{5}{8}(\omega t + \Omega t) + \frac{1}{7}\sin\frac{7}{8}(\omega t + \Omega t) + \cdots\right].$$

This rectangular signal is sent into a ring modulator. At the other input the signal coming from the amplitude compressor is applied directly. This corresponds to the second direction indicated at the compressor output.

The time axis origin which was chosen for the rectangular signal is also a zero point for the signal furnished by the compressor. With this time origin, the latter is of the form $\sqrt[8]{A}\sin(\omega+\Omega)t$. With the aid of a filter, a frequency band near $\frac{1}{8}\,\Omega/2\pi$ is selected. The rectangular signal is eliminated by symmetry at the modulator output. The result supplied by the ring modulator is the product of the two signals which it received. The only useful products are given by the expression

$$\sqrt[8]{A}\sin(\omega + \Omega)t \times \frac{4}{\pi}\left[\frac{1}{7}\sin\frac{7}{8}(\omega + \Omega)t + \frac{1}{9}\sin\frac{9}{8}(\omega + \Omega)t\right]$$

from which we obtain

$$\frac{2}{\pi}\sqrt[8]{A}\left[\frac{1}{7}\cos\frac{\omega + \Omega}{8}t + \frac{1}{9}\cos\frac{\omega + \Omega}{8}t\right].$$

This provides, with the approximation of a numerical factor equal to

$$\frac{2}{\pi}\left(\frac{1}{7} + \frac{1}{9}\right),$$

the signal

$$\sqrt[8]{A}\cos\frac{\omega + \Omega}{8}t.$$

For each frequency applied we obtain, with a constant phase shift of $\pi/2$, a signal of compressed amplitude, the frequency of which is divided by eight. We know that, in single sideband transmission, a systematic phase shift could be eliminated during demodulation but, in reality, this is of no practical effect.

The operation described for a sinusoidal signal is extended when a complex telephonic signal is applied. It is hereby assumed that the translation to a single sideband was effected at a frequency high enough that the relative bandwidth be small in respect to the frequency of transposition. Under these conditions, the signal to be divided (in frequency) has the form $\sqrt{a(t)}\cos[\Omega t+\phi(t)]$ at the output of the instantaneous compressor.

After the limiting operation, we obtain the zero points of the signal $\cos[\Omega t+\phi(t)]$ the frequency of which varies as is the case with typical frequency modulation. The limiting circuit enables the zero points to be registered without the latter being displaced in function of the amplitude variations of the applied signal. The triggering time of the counters is sufficiently short to avoid any alteration of the zero points supplied by the limiting circuit.

Although the useful variations of the dynamic at the limiter input is reduced by 6 db, we have found that, with a fixed frequency signal subjected to a 40 db amplitude modulation, the signal obtained at the divider output is strictly constant with no frequency variation in function of amplitude modulation. With a 50-db sensitivity, no interference line could be detected by means of a selective analyzer.

The foregoing design has been adopted in preference to that described earlier in chapter 2—Operation, in order to avoid the direct use of the signal envelope which contains a dc component offering a difficult problem of stability in function of temperature.

Expander-Multiplier

The role of this unit, at the receiving end, is the reverse of that of the compressor divider before transmission. It consists of three stages each of which extends the dynamic by a factor of 2 and doubles the frequency. Thus from

$$\sqrt[8]{a(t)}\cos\frac{\Omega t+\phi(t)}{8}$$

we obtain in three successive steps

1) $$\sqrt[4]{a(t)}\cos\frac{\Omega t+\phi(t)}{4}$$

2) $$\sqrt{a(t)}\cos\frac{\Omega t+\phi(t)}{2}$$

and finally

3) $$a(t)\cos\left[\Omega t+\phi(t)\right].$$

In order to benefit from the current gain provided by transistors, rectification is accomplished by means of the base-emitter diodes of two transistors symmetrically connected through a transformer to a low impedance source (Fig. 6).

Each base-emitter diode allows current to pass only during the negative swing. By choosing a suitable dc bias for the base and for the emitter of each transistor, a current vs applied voltage characteristic of parabolic form is obtained for the lower levels. A tuned transformer, connected as a common load for the two collectors, furnishes a voltage proportional to the total current and therefore proportional to the current alternatively traversing the base-emitter diodes, with the gain of the transistors. The source impedance being very low, the voltage applied to the bases is independant of the relatively high (and variable) impedance presented by the bases.

If the voltage applied to the primary of the input transformer is $\sqrt{a(t)}\cos\left[\Omega t+\phi(t)\right]$ the current circulating in the collectors is proportional to

$$a(t)\cos^2\left[\Omega t+\phi(t)\right]=\frac{a(t)}{2}\left\{1+\cos\left[\Omega t+\phi(t)\right]\right\}.$$

The output transformer does not transmit the low-frequency band $a(t)/2$ which represents the detected envelope. The primary is however tuned to correctly pass

$$\frac{a(t)}{2}\cos\left[\Omega t+\phi(t)\right].$$

The principal difficulty with nonlinear circuits requiring a well defined curve form is caused by temperature drift in the characteristics of the semiconductor components. The results obtained by using diodes in the amplitude compressor show that it is possible to obtain an excellent correction in acting on a single parameter: bias.

The same applies to transistors. It is possible to choose a bias, for the base-emitter diodes, providing the maximum level for the usable signal at a frequency double that of the signal at the input, and functioning in accordance with an exact parabolic law for all applied voltages below a set value (Fig. 7).

In order to maintain the base-emitter bias voltage for transistors (2) and (3) at the correct operating value for all temperatures, use is made of a semiconductor component, the characteristic of which varies inversely with that of the above transistors. The bias applied to the midpoint of the input transformer secondary is supplied from a resistor bridge through transistor (1) which is *npn* if transistors (2) and (3) are *pnp*. Transistor (1) operates in class A.

The impedance offered between the emitter and ground to alternating current is very low. This impedance, in series with the source and the midpoint of the transformer, is approximately equal to the resistance between the base and ground divided by the gain of the transistor.

Transistor (1) provides the means of obtaining an excellent temperature compensation without any notable increase in source impedance. This is, however, the disadvantage of a low dc drain. An over-all correction, by means of a thermistor, of the complete expander-multiplier unit ensures an accuracy of around one db in the −40 to +55°C temperature range.

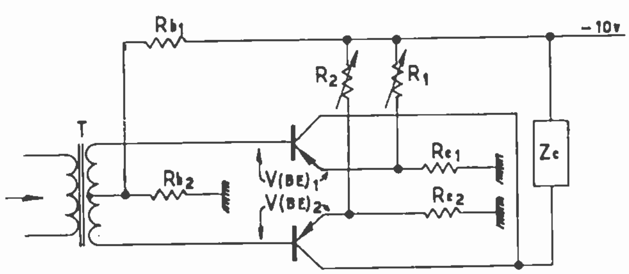


Fig. 6—Expander-multiplier.

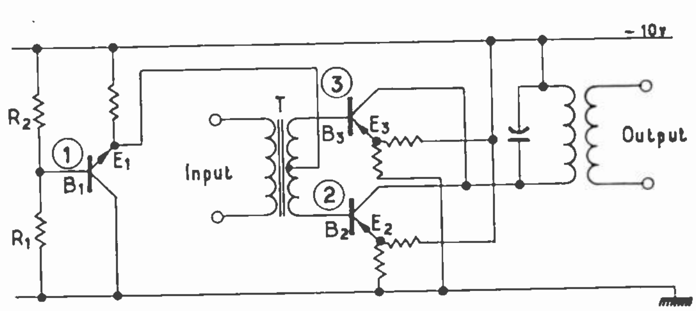
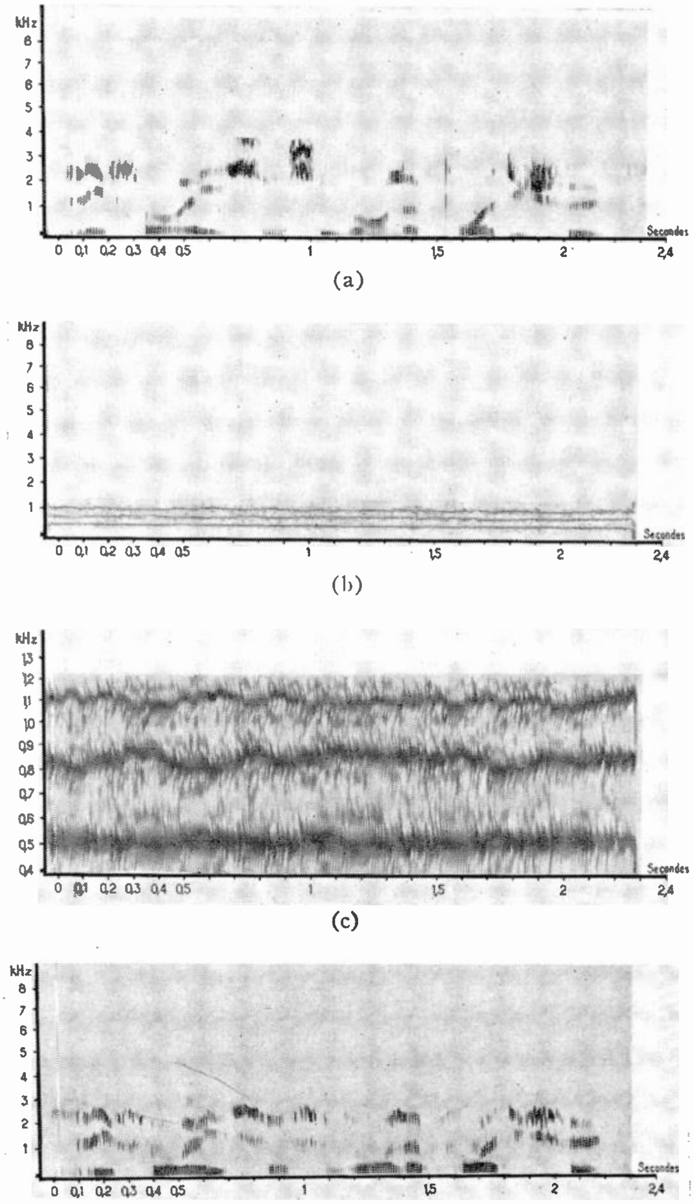


Fig. 7—Temperature compensation of the expander-multiplier.



(a)

(b)

(c)

(d)

Fig. 8—Sonagrams.

IV. Spectral Analysis and Telephonic Quality

A. Results Obtained

The photographs included in the present report illustrate the different frequency analyses effected with the Kay Electronic Company Sonagraph. The spoken sentence used for the analysis was recorded on a wide-band tape recorder. The signal duration is 2–4 seconds and the frequency band analyzed 4 kHz. A sonagram representing the original signal at the tape recorder output is shown in Fig. 8(a). Fig. 8(b) shows a sonagram illustrating, on the same scale, the compressed formants in the line. The sonagram in Fig. 8(c) shows these formants with an $\times 10$ expansion of the frequency scale. Finally, in Fig. 8(d) we see the reconstituted signal at the low-frequency output of the system, at the original scale.

B. Telephonic Quality Obtained

Table I outlines the results of the telephonometric tests carried out by the acoustical division of the National Telecommunications Center (Centre National des Télécommunications) during the investigation of the system. The intelligibility for logatomes measured with the aid of a high quality microphone was around 75 per cent. Bisyllabic words showed an intelligibility of approximately 95 per cent. The results are given in per cent. At each test were received

- 900 logatomes
- 300 words
- 120 sentences

TABLE I

SPEECH COMPRESSION SYSTEM TELEPHONOMETRIC TESTS

Test Conditions	Intelligiblity (logatomes)		Bisyllabic Words Intelligibility	Sentence Intelligibility†
	Logatomes	Sounds*		
U43				
Without noise	72.3% (2)	89.9%		A=98.3%
With noise 40 db SNR	74.7% (3)	90.6%		B=1.7%
With noise 20 db SNR	72.5% (4)	89.8%		C=0
With noise 15 db SNR	72.1% (5)	89.1%		D=0
With noise 6 db SNR	64.7% (7)	86.4%		E=0
SFERT				
Without noise	75.7% (1)	90.8%	94.7%	
With noise 15 db	67.5% (6)	87.3%		

* The sounds are the constituent elements of logatomes
† Correctness of sentence transmission (in per cent).
A—Entire sentence correctly received.
B—Over-all sense understood with several errors in detail.
C—One or several key words badly interpreted.
D—Over-all sense not understood due to important gaps.
E—Reception unintelligible.
Note: A single U43 hook-up, normally utilized in the conditions presented by the calibration circuit and with a normal listening level, provides a sharpness of 93 per cent for logatomes and 97 per cent for sounds.

The telephonic equipment used was as follows:

1) Type U43 set with carbon microphone used in the French government services,
2) High quality laboratory transmitter-receiver (French reference system for SFERT transmission).

CONCLUSION

The method employed ensures, for each speech formant, a concentration of the spectrum and an equalization of amplitude, brought about by the increase of the average level in respect to the peak level. In the transmission channel, the dynamic of each formant is reduced from 48 db to 6 db. During conversation there is little variation in the level of the compressed formants, as is borne out by the sonagrams. It follows from this that the narrow-band filters, serving in the selection of the compressed formants after their juxtaposition in the transmission channel, must meet stringent requirements as concerns the curves defining loss and phase in function of frequency.

The characteristics of this are analogous to those of the compandors used in carrier current systems. Adequate protection is provided against intermodulation and inherent noise in the transmission channel. A threshold exists below which noise has no effect. On the contrary, when the noise level exceeds this threshold there is capture of the vocal signal. This only occurs in the case of extremely disturbed transmission conditions.

ACKNOWLEDGMENT

The author wishes to associate the memory of Colonel P. Marcou, deceased in September 1956, with the culmination of this work, the basis of which was established through the common effort of Colonel Marcou and the author.

Correspondence

Comments on "A New Automatic Level Control"*

The author would like to comment on a fine and timely paper by Kaiser and Bauer.[1] In the introduction and under the heading, "Stereophonic Operation," the authors state that modulation percentage (of a frequency-modulated signal) is determined by the sum of the left and right channels. This, however, is not correct.

The modulation percentage is determined by the *stronger* channel. This may be the left or the right channel and under program conditions control may rapidly alternate between both channels. If one would apply a +10-db signal to only the left channel causing 100 per cent of modulation, any signal up to +10 db applied to the right channel will not alter the modulation percentage.

Therefore, aside from other considerations, a level control device should under stereophonic conditions be controlled by the magnitude of the stronger channel yet act upon both channels to preserve their ratio.

A. H. Bott
300 Garwood Place
Cherry Hill, N. J.

* Received April 4, 1963.

[1] A. Kaiser and B. B. Bauer, "A new automatic level control for monophonic and stereophonic broadcasting," IRE Trans. on Audio, vol. AU-10 pp. 171–176; November–December, 1962.

Contributors

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Duane H. Cooper was born in Gibson City, Ill. on August 21, 1923. He received the B.S. degree in 1950, and the Ph.D. degree in 1955, from the California Institute of Technology, Pasadena.

Since 1954 he has been with the Control Systems Laboratory (now Coordinated Science Laboratory) of the University of Illinois, Urbana, where he is Research Associate Professor of Physics and Electrical Engineering. In recent years his principal interests have been statistical detection, information, and decision theory, with applications to pattern recognition, and the applications of digital computers to data analysis, as well as Monte Carlo simulation of physical systems.

Dr. Cooper is a member of the American Physical Society.

Jacques L. Daguet was born in Bourg, France, on March 15, 1917. He entered the Ecole Polytechnique in Paris in 1938, and received a degree in Telecommunication from the Ecole Nationale Supérieure des Télécommunications in 1947.

From 1949 to 1957 he was employed as a Postal and Telegraph engineer at the Centre National d'Etudes des Télécummunications, Paris. Since 1957, he has been responsible for advanced studies at Télécommunications Radioélectriques et Téléphoniques (TRT), Paris.

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He has worked for the Defense Research Laboratory of the University of Texas and the Austin Recording Company, both in Austin and for Universal Recording Corp., Chicago, Ill. In the field of electroacoustics he has worked for Klipsch and Associates, Incorporated, Hope, Ark., and for the Jensen Manufacturing Company, Chicago, Ill. While at Jensen he did development work in horn loudspeakers and in mean energy density measuring techniques. He is presently working for the RCA Victor Record Division of the Radio Corporation of America, New York, N. Y.

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Robert J. Larson (S'48–M'62) was born in Galesburg, Ill., on January 7, 1926. He received the B.S. degree in electrical engineering from Northwestern University, Evanston, Ill., in 1951.

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