IEEE Transactions ON AUDIO

Volume AU-11 SEPTEMBER-OCTOBER, 1963 Number 5

Published Bimonthly

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PROFESSIONAL TECHNICAL GROUP ON AUDIO

IEEE TRANSACTIONS ON AUDIO

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Message from the Chairman

A TUTORIAL SESSION on Audio was held on the afternoon of October 29, 1963, at the National Electronics Conference in Chicago, Ill. The attendance at this session was heartening and plans are being initiated for one or more sessions at NEC in 1964.

During the evening, your Administrative Committee held a business meeting. J. Bell, Chairman, and one Member of the Professional Technical Group on Broadcast and Television Receivers were invited to this meeting for the purpose of merger considerations. Discussion indicated that the Advisory Commissions of both groups were definitely opposed to any mergers. Both groups felt that their membership had specific interests which would not be fulfilled effectively in a mixed group. As a result, discussions of any mergers were tabled indefinitely. However, the possibility of joint meetings with PTGBTR will be considered for the future.

It is planned to have two sessions on Audio at the 1964 IEEE International Convention and one or more sessions at the 1964 National Electronics Conference.

FRANK A. COMERCI

The Editor's Corner

Paperese

IN THE PRESENTATION of papers at technical conventions, authors frequently utter phrases which sound deceptively like English but which are actually in a strange tongue called Paperese. When spoken by an expert, the transition from normal English to Paperese is difficult to recognize. Those less skilled, however, occasionally betray the shift by a moistening of the lips, a cough or a vacant stare.

Some people at these conventions actually listen to some of the papers, and for the benefit of the less experienced among these we present here some of the more common phrases in Paperese together with their English translations. We are indebted to D. von Recklinghausen for suggesting several of these phrases. Supplementary lists would be welcome and will be published in a future issue if enough contributions are received.

Paperese	English Translation
A novvul approwch wuz uzed.	Benjamin F. Meissner did it thirty years ago.
Thuh theery wull bee preezentd furst.	I worked out the math last night.
Obviuslee . . .	I haven't bothered to figure it out, but . . .
Thuh dezine ubjectivz wur az followz:	When we finally got the prototype working, it had these characteristics:
Theez ubjectivz wur mett.	We're having a little trouble with production models, but the prototype works fine.
. . . strateforwurd . . .	. . . brute force . . .
. . . awtomattik . . .	. . . has an on-off switch
. . . sollid stayt . . .	. . . contains a diode . . .
. . . portubbl . . .	. . . has a handle . . .
. . . semmee portubbl . . .	. . . has two handles . . .
Purformans haz bin radicallee improovd.	Last year's model was lousy.
Thiss wurk wuz purformd fowr yeerz uggo.	I ran out of new material for papers, but needed an excuse to go to the convention.
Thiss resurch haz demmunstrated thuh feezi billittee uv thiss approwch.	We couldn't make it work, but we're trying to get the government to extend the contract.

Peter W. Tappan, Editor

National Officers of the PTGA, 1963-64

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Chairman
1963–1964

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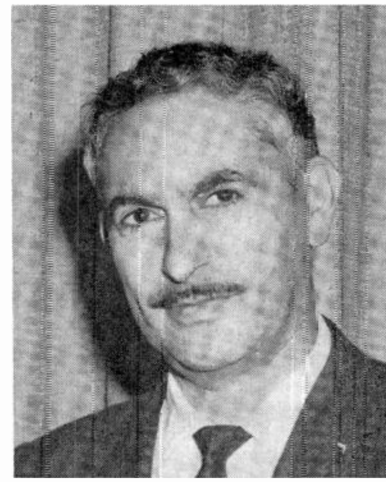
Vice Chairman
1963–1964



M. COPEL
Secretary-Treasurer
1963–1964

Frank A. Comerci (SM'55) was born in Newark, N. J., on January 18, 1920. He received the B.S.E.E. degree from Newark College of Engineering in 1943.

From 1943 to 1946 he served in the U. S. Army as a Communications Officer, installing and maintaining cryptographic speech communications systems. He joined the Rangertone Corporation, in 1946, where he worked on the design of the first high-quality magnetic tape recorder built in the United States. In 1947 he became affiliated with the Navy Material Laboratory, Brooklyn, N. Y., where he was in charge of their Acoustics and Communications Section from 1950 to 1959. He was later employed by Audio Devices, Inc., Glenbrook, Conn., as Senior Electronic Engineer. At present he is Manager of the Magnetics Branch of Columbia Broadcasting System Laboratories, Stamford, Conn., and has responsibility for fundamental and applied research on magnetic materials and magnetic recording techniques. He has written several papers on magnetic recording and flutter.

Mr. Comerci is a member of the Acoustical Society of America and the Audio Engineering Society, serving on the Editorial Board of the *Journal of the Audio Engineering Society* for several years, and he is a member of the Sound Committee of the Society of Motion Picture and Television Engineers. He is Chairman of the IEEE Recording and Reproducing Committee and serves as IEEE representative to ASA Section Committee Z-57 on Sound Recording.

William M. Ihde (M'51—SM'52) was born in Sapporo Japan, on September 29, 1923. He received the B.S. and M.S. degrees in electrical engineering from the Massachusetts Institute of Technology, Cambridge, in 1948.

From 1948 to 1950, he was in research and development, with the General Radio Company, West Concord, Mass., after which he joined their field engineering group. He was transferred to the district office in Chicago, in 1951, and became District Manager in 1955.

Mr. Ihde is a registered professional engineer in the State of Illinois, a member of the Acoustical Society of America, and a Senior Member of the Instrument Society of America. He has served as Chairman of the PTGA, Chicago Section, and Chairman of the Chapters and Membership Committee of PTGA.

Michel Copel (M'53–SM'57) was born in Paris, France, on March 20, 1916. He received the B.S. degree in 1935 from the University of Paris, and the E.E. degree in 1937 from the Conservatoire National des Arts et Metiers in Paris. He also attended New York University, N. Y.

From 1942 to 1946 he was engaged in the design and development of military loudspeaker equipment as Chief Design Engineer of University Loudspeakers. From 1946 to 1948 he was Senior Engineer at Dictograph Products, Inc. Since 1948 he has been engaged in research and development in the field of acoustics and speech communication at the Naval Applied Science

I. Kerney
Administrative Committee
1963–1966

R. H. Rose
Administrative Committee
1963–1966

Laboratory, Brooklyn, N. Y. He currently heads the Acoustics and Interior Communication Group.

Mr. Copel is a member of the Acoustical Society of America. He has served as Chairman of the Ways and Means Committee of PGA and as Organizer of the Audio Sessions at the 1955, 1956 and 1961 IRE Conventions. He is currently participating in Standards work in the IEEE and the American Standards Association, as member and chairman of several technical committees. In 1961 and 1962 he joined the U. S. delegation to the meetings of the International Electrotechnical Commission and the International Standards Organization.

Iden Kerney (SM'54) was born in Iowa and attended Harvard College, Cambridge, Mass., receiving the B.S. degree in communications engineering in 1923.

From 1923 to 1934, he was employed by the Department of Development and Research of the American Telephone and Telegraph Company, and from 1934 until his retirement in 1963, by the Bell Telephone Laboratories. Aside from the war years when he was associated with a secret communications system, he was concerned with systems engineering developments in connection with baseband and carrier-derived facilities and networks for 5, 8 and 15 kc program transmission in the United States and Canada. In 1963 he was temporarily assigned as Assistant Director of Engineering for Systems, U. S. Underseas Cable Corp., Washington, D. C., for work on a contract for the U. S. Air Force on a submarine cable installation for communications and missile tracking, between Grand Turk and Antigua in the Carribean.

Mr. Kerney served as Chairman of the IRE Audio Techniques Technical Committee from 1956 to 1958, and Chairman of the IRE-IEEE Audio and Electroacoustics Technical Committee from 1958 to the present. He is a member of the Awards Committee of the PTGA, a charter member of the Acoustical Society of America, and a member of the Harvard Engineering Society.

Robert H. Rose (S'43–A'45–M'54) was born in New York, N. Y., on February 6, 1922. He received the B.S.E.E. degree from Newark College of Engineering, N. J., in 1949.

From 1944 to 1947, at the I. T. and T. Laboratory, New York, N. Y., he worked on test equipment for remote-control radio, and later on a microwave receiver for a pulsed multiplex system. He joined the Electrical Engineering Department of Newark College of Engineering in 1947. In addition to various undergraduate courses, he has taught electroacoustics, sound recording and reproducing systems on the graduate level. He took a two-year leave of absence to work as Chief Engineer in a small electronics plant, returning to Newark College in 1958.

Mr. Rose is a member of the Audio Engineering Society, the Acoustical Society of America, Eta Kappa Nu and Tau Beta Pi. He has been Chairman of the IEEE Subcommittee on Definitions (A and E) since May, 1960.

Design of Velocity-Feedback Transducer Systems for Stable Low-Frequency Behavior

H. W. HOLDAWAY

Summary—Loudspeaker drive systems employing negative velocity feedback, by directly controlling the voice-coil motion, can improve the over-all linearity and largely suppress the fundamental resonance. These features permit good results using reasonably efficient loudspeakers in small sealed-box enclosures.

In stable systems the degree of control provided is directly related to the loop gain. Instability and overloading problems can arise in partially capacity-coupled amplifiers unless special phase-compensation circuits are employed. For comparable distortion reduction at higher frequencies one must employ the same number of valves in the main amplifier as in conventional high-quality amplifiers. An additional valve stage is needed outside the main feedback loop to provide correction for the loss of 20 db per decade in radiating efficiency below the point of ultimate resistance of the equivalent piston radiator.

The main amplifier is developed from Mullard circuits. Additions include a bridge in the voice-coil circuit and low-frequency phase-compensation elements. Reliable design procedures for these are given in detail. Specifications necessary for satisfactory performance are discussed.

I. Introduction

Historical Background

THE APPLICATION of velocity-derived feedback to a power amplifier driving an electromechanical transducer, such as a loudspeaker, can substantially reduce over-all distortion including that caused by nonlinear behavior of the loudspeaker suspension itself. Since such nonlinearity can produce much greater distortion at the lowest audio frequencies than that due to all other elements in the reproducing chain, the possible improvement can be quite substantial. This is especially true where sealed-box-type loudspeaker enclosures[1] are used. Furthermore, improved dynamic control of the voice-coil motion gives improved transient response.

In one of the earliest references to the principle of velocity feedback, H. F. Olson[2] described two such systems. He mentioned in general terms the problems to be overcome to ensure stability and stressed the necessity of applying frequency response compensation where a direct radiator loudspeaker is driven at constant velocity.

A bridge circuit for extracting a velocity-derived signal for feedback from the voice-coil circuit has been noted.[3–5] Subsequent articles[6–9] have described the combination of "negative voltage feedback" with "positive current feedback" from which there results some component, at least, of velocity-derived feedback.

A relatively comprehensive and fresh discussion of the bridge type of velocity-feedback circuit has been given by Werner[10] and by Werner and Carell[11] of the RCA Transducer Design Group. Here the feedback was regarded as causing the output impedance of the amplifier to become negative to an extent which would cancel out all or part of the blocked voice-coil impedance of the loudspeaker. A practical system was described which provided almost constant velocity drive. Although the basic amplifier lineup was perhaps a little below what would normally be regarded as a high-fidelity arrangement, this was offset by the larger than normal reduction in over-all distortion due to the inclusion of an internal positive feedback loop.[12]

More recent discussions of velocity feedback have been given by Pierce[13] and de Boer.[14] The former used a synthesis approach to the design of a woofer system and the latter re-examined the fundamental principles of velocity feedback.

Problems Restricting Widespread Adoption of Velocity Feedback

The application of velocity feedback has never been very widespread. Some of the accounts have been a

Manuscript received January 10, 1963.
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[1] J. F. Novak, "Performance of enclosures for low-resonance high-compliance loudspeakers," IRE Trans. on Audio, vol. AU-7, pp. 5–13; January–February, 1959. See also Werner.[10]

[2] H. F. Olson, "Elements of Acoustical Engineering," D. Van Nostrand Co., Inc., Princeton, N. J., pp. 135–136; 1940.

[3] F. Langford-Smith, "Loudspeaker damping," *Wireless World* (Letter), vol. 53, pp. 309, August, 1947; (Replies), vol. 53, pp. 343–344, September, 1947; vol. 53, pp. 401–402, October, 1947.

[4] D. T. N. Williamson, "More views on loudspeaker damping," *Wireless World* (Letter), vol. 53, pp. 401–402; October, 1947.

[5] P. G. A. H. Voigt, "Loudspeaker damping," *Wireless World* (Letter), vol. 53, pp. 487–488; December, 1947.

[6] H. H. Lowell, "Motional feedback," *Electronics*, vol. 24, pp. 334, 336; December, 1951.

[7] U. J. Childs, "Loudspeaker damping with dynamic negative feedback," *Audio Eng.*, vol. 36, pp. 11–13, 33; February, 1952.

[8] W. Clements, "It's positive feedback," *Audio Eng.*, vol. 36, p. 20; May, 1952.

[9] U. J. Childs, "Further discussion on positive current feedback," *Audio Eng.*, vol. 36, pp. 20–22; May, 1952.

[10] R. E. Werner, "Effect of a negative impedance source on loudspeaker performance," *J. Acoust. Soc. Am.*, vol. 29, pp. 335–340; March, 1957.

[11] R. E. Werner and R. M. Carell, "Application of Negative Impedance Amplifiers to Loudspeaker Systems," presented at 9th Annual Meeting of the Audio Engineering Society, New York, N. Y.; October 12, 1957.

[12] J. M. Miller, Jr., "Combining positive and negative feedback," *Electronics*, vol. 23, pp. 106–109; March, 1950.

[13] W. H. Pierce, "The use of pole-zero concepts in loudspeaker compensation," IRE Trans. on Audio, vol. AU-8, pp. 229–234; November–December, 1960.

[14] E. de Boer, "Theory of motional feedback," IRE Trans. on Audio, vol. AU-9, pp. 15–21; January–February, 1961.

little vague on important aspects and one suspects that some applications may not have met the essential requirements set forth by Olson and by Werner. This may account for the occasionally expressed contention that this type of feedback would be most likely to give proved performance in medium or low-grade systems.

One inherent source of difficulty is that reliable and economically acceptable amplifiers usually employ at least one capacitative coupling located at the grid circuit of the power stage. Another is that the output transformer places an inductive shunt across the load impedance. Since one result of applying negative velocity feedback is to force the amplifier to supply sufficient signal to maintain constant velocity drive at all frequencies, and since the types of coupling referred to above bring about in each case a 20 db per decade falloff in low-frequency response, a strong possibility exists that the stages preceding either coupling circuit (operating at a relatively high-signal level) may be overloaded at the lower frequencies. *Below the fundamental resonant frequency* of the loudspeaker this problem of "electronic overloading" is further augmented by the need to provide at the voice-coil terminals (for constant velocity) a voltage which rises with falling frequency. A consequence is that some methods for stabilizing compensation, which may be effective at low-signal levels, must be rejected because at high-signal levels they fail to overcome this defect.

Phase shifts at low frequencies can arise from interstage couplings as above, as well as from cathode and screen bypass circuits, and with velocity feedback are attributable to the large reactive component of the voice-coil impedance. *These must be specifically allowed for* in designing the stabilization compensation of the feedback system. By supplying the screen of pentode V_2 of Fig. 2 from a relatively low-resistance divider network without a bypass capacitor, one such source of potentially troublesome phase shift was avoided at the cost of but a slight loss in loop gain.

Other problems of a more psychological nature involve restricted interchangeability of units and ratings of amplifiers as separate units. These are discussed in Section XIV.

Cone Break-Up and Its Minimization

Until comparatively recently, an additional problem has been the failure of conventional loudspeakers, because of cone breakup,[15] to behave as ideal piston radiators over a sufficiently wide range of frequencies. Although the effect of velocity feedback in suppressing resonances in the motion of the voice-coil may be very satisfactory at the fundamental resonant frequency, it may be quite the opposite for certain types of resonance at higher frequencies. For some of these the mode of diaphragm vibration may be such that portions will vibrate in phase and other portions 180° out of phase with the voice-coil motion, thus causing typical troughs in the acoustical output response of the loudspeaker. Evidently velocity feedback on its own would accentuate such troughs.

With conventional loudspeakers one must adopt the compromise solution of combining negative voltage feedback with the negative velocity feedback. Because the latter generally falls off at frequencies removed from the fundamental resonant frequency, the combined feedback tends to provide constant voltage drive to the voice-coil at the higher frequencies. Thus there will be very little degradation in response at cone breakup resonant modes, as compared with a drive using a conventional negative feedback power amplifier.

A substantial improvement in such behavior should be possible using loudspeakers of "sandwich" construction based upon design principles enunciated by D. A. Barlow.[16] One commercially available version[17] appears to meet all the requirements for velocity-feedback systems, including a relatively large $\frac{1}{2}$-inch excursion of the voice-coil. Since the extreme stiffness of the cone (for the normal mass) minimizes cone breakup at frequencies of up to, say, 600 cps, electronic control of the voice-coil motion should also imply full control of the actual acoustic radiator.

Type of Loudspeaker Enclosure

It is possible to provide comparatively simple *response correction* for a *constant velocity-driven* piston radiator only if it is mounted on an infinite baffle or a sealed box. Unless further electronic correction were provided to cancel out some effects of enclosure resonances and port radiation, the audible frequency response could be unsatisfactory if the loudspeaker were mounted on other types of tuned enclosure such as a vented box.

II. Basic Design Principles

The design procedure described here is based upon an amplifier type in which the lineup of main components follows a commonly employed high-quality design.[18] The latter is modified by incorporating (stabilization) compensation circuits which have proved satisfactory. It is considered that this could provide a flexible basis for the more widespread adoption of such servo-type loudspeaker drive systems.

Essential Elements

To give a consistent level of acoustical output, a frequency response correction stage must be provided, in

[15] L. L. Beranek, "Acoustics," McGraw-Hill Book Co., Inc., New York, N. Y., pp. 118–123 and pp. 199–201; 1954.

[16] D. A. Barlow, "Rigidity of loudspeaker diaphragms," *Wireless World*, vol. 64, pp. 564–569; December, 1958.

[17] "Leak 'sandwich' full range loudspeaker," *Gramophone*, vol. 39, pp. 133–134; August, 1961.

[18] "Circuits for Audio Amplifiers," Mullard, Ltd., London, England, pp. 29–38 and pp. 39–52; 1959. Amplifier circuits used as the basis for this paper are due mainly to the late W. A. Ferguson and D. H. W. Bushby.

addition to the main feedback system, to compensate for the 20 db per decade falloff below the point of ultimate resistance of the equivalent piston radiator driven at constant velocity.

Within the negative feedback loop of the main feedback system an internal positive feedback loop was provided over *one* stage.[12] This has the effect of considerably increasing, without incurring risks of instability, the negative feedback over all *other* stages including, in particular, the output stage, the output transformer and the loudspeaker itself.[19]

This positive feedback was applied above a selected asymptotic break-frequency in such a way that this stage, in conjunction with the preceding interstage coupling circuit, gave a response falling in direct proportion to frequency down to about 5 cps. Moreover, the choice of break frequency referred to above modified the gain of the amplifier (see Appendix II). Thus, below about 40 cps, the loop gain and phase shift are largely controlled by this portion of the amplifier, and the stability at very low frequencies can be ensured. An important feature of this stabilizing system is that it tends to prevent overloading of the phase-splitter and output stages.

Further stabilizing compensation was provided by a transitional phase-shift network ("λ network") in the negative feedback return path. By providing a measure of low-frequency boost and phase lag in the negative feedback path, this network further tended to minimize overloading at the higher-level stages, besides increasing the stability margin at low frequencies. By providing a control on the amplitude of the voice-coil motion below a selected low audio frequency (in this case about 30 cps), the λ network also helped to prevent mechanical overloading of the loudspeaker at the lowest frequencies.

Typical Performance Specifications

A satisfactory design, providing an adequate margin of stability at low frequencies, is one in which a sufficiently large damping factor is associated with the two dominant poles of the closed-loop transfer function. Values of the damping factor "ζ" less than 1 should correspond to an oscillation at a subaudible frequency, preferably removed from a turntable rumble frequency. Some designs may lead to critical damping $\zeta=1$, or even to completely aperiodic response. Any of these would be completely satisfactory, the higher values of ζ corresponding to the greater margin of stability.

Apart from the requirement for adequate stability margin a satisfactory design should provide a voice-coil velocity which is constant down to a suitable low frequency (such as 30 cps). The latter depends upon the limiting volume displacement of the loudspeaker diaphragm and the desired midrange level of acoustic output. Below this frequency the velocity response, apart from a small hump which may be associated with the dominant poles, may fall corresponding to asymptotic slopes of 20, 40 and 60 db per decade successively, with decreasing frequency (see Fig. 6). At the high-frequency end the response should be flat up to a frequency, about 600 cps, at which the low-frequency stability analysis, being no longer valid, should be replaced by the method of frequency response design described in another paper.[20]

Pole Locations

An important aspect of the design is that it provides one or two negative poles in the transfer function *well separated from the other poles*, together with some control upon their approximate location. Advantage is taken of this feature which permits quick and precise evaluation of the poles using methods similar to those described by S. N. Lin[21] and P. L. Taylor.[22] A brief outline of suitable methods for polynomial factorization is given in Appendix I.

Use of Pole and Zero Cancellations

In order to reduce the order of the servo system and to permit reasonably simple pole and zero analysis, cancellation techniques have been employed where convenient in the design of the compensation circuits. A more detailed discussion is given later.

Analysis vs Synthesis

Because of the essential restrictions on amplifiers for audio reproduction, the author has adopted an analytic rather than a synthetic approach. In this way the design will always conform to the limitations imposed by the amplifier elements and minor modifications can be made quite quickly to achieve the most satisfactory compromise subject to these constraints.

III. Analysis of Basic Feedback System

Simplified Schematic Feedback Circuit

For the purposes of analysis the circuit to be considered may be represented by the simplified block diagram of Fig. 1.

The feedback bridge is made up of the components Z_1, Z_2, Z_{vc}, Z_M and Z_3. Here Z_{vc} is the blocked voice-coil impedance and Z_M is the reflected motional impedance at the loudspeaker terminals. The voice-coil impedance is $Z_T=Z_{vc}+Z_M$. If the bridge circuit is exactly balanced

[19] Full benefit can be obtained only if the loudspeaker is designed to give a constant BL product. This is achieved either by the voice-coil being long enough to extend outside the fringe of the magnetic field over the whole range of its permissible movement, or by its being short and in all positions lying within a sensibly constant magnetic field.

[20] H. W. Holdaway, "Controlling the upper frequency response characteristics of velocity feedback loudspeaker systems," this issue, page 174.

[21] S. N. Lin, "A method of successive approximations for evaluating the real and complex roots of cubic and higher order equations," *J. Math. Phys.*, vol. 20, pp. 231; 1941.

[22] P. L. Taylor, "Servomechanisms," Longmans, Green & Co., Ltd., Melbourne, Australia, Appendix 1; 1960. (Based on Lin.[21])

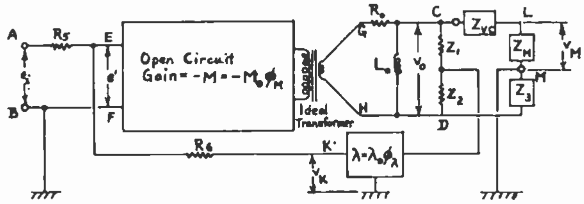


Fig. 1—Schematic diagram of the velocity-feedback system. R_5 includes the output impedance (resistive) of V_1.

for all frequencies with the voice-coil blocked ($Z_M=0$), *i.e.*, if

$$\frac{Z_1}{Z_2} = \frac{Z_{vc}}{Z_3} \tag{1}$$

the only output from the bridge when free to move would be due to the unbalance caused by the presence of Z_M. The voltage developed across Z_M is in fact the back EMF induced by the voice-coil motion and is proportional to the voice-coil velocity. If V is that velocity, B the magnetic induction and L the length of the voice-coil wire which is situated in the magnetic field B (all in mks units), then for a current i in the voice-coil moving with velocity V,

$$V_M = Z_M i = BLV \tag{2}$$

is this induced EMF. From the viewpoint of the electronic circuit this EMF is calculated as $Z_M i$. *Provided the product BL is constant* as a result of a suitable loudspeaker design, the induced EMF v_M is a direct measure of the velocity V. Evidently v_M not is directly accessible for measurement.

If Z_2 is larger or Z_3 smaller than required by (1) when the voice-coil motion is blocked, a proportion of negative voltage feedback will be included. The latter will predominate over the velocity feedback at frequencies well removed from the fundamental resonance and will tend to flatten out the closed-loop *voltage response to the voice-coil terminals*. At low frequencies this increases the stability margin, at the midrange it minimizes troughs in the acoustic response where cone breakup is present, and at high frequencies it may be needed to provide a reasonably controlled level response with some loudspeaker systems. Some degree of such a levelling out in response may also be needed to prevent the amplifier overloading. Furthermore, it is possible that the amount of velocity feedback alone may be too small to provide adequate reduction of harmonic distortion at frequencies well above the fundamental resonance of the loudspeaker, unless it is supplemented by negative voltage feedback.

Bridge Unbalance

It is convenient to refer to the percentage u of bridge unbalance at zero frequency, obtained by first balancing the bridge with Z_2 shunted by an impedance of ($100Z_2 \div u$). On removing such a shunt the impedance Z_2 is u per cent greater than previously. The balancing procedure corresponds to the relationship

$$\frac{(u+100)Z_1}{100Z_2} = \frac{Z_{vc}}{Z_3} \tag{3}$$

(since $Z_M=0$ at zero frequency).

In the following analysis it will be assumed that a certain percentage u of dc unbalance ($u \geqq 0$) will have been provided by such an adjustment. The above method is quite convenient for practically adjusting the bridge circuit components. These may be set up as a dc bridge and brought to balance with the appropriate shunt across Z_2.

Analysis of the Feedback Circuit

In Fig. 1 R_0 is the output impedance as reflected to the secondary and L_0 is the reflected self-inductance of the output transformer. The elements Z_1 and Z_2 will subsequently be replaced by the pure resistances R_1 and R_2, but are shown as impedances for the sake of generality. These are chosen large in absolute magnitude compared with R_0 so that the current through them can either be ignored or allowed for as a minor correction. The latter can be achieved by changing both the output impedance R_0 and the open-circuit gain M of the amplifier by the factor $(Z_1+Z_2)/(R_0+Z_1+Z_2)$.

We write for the open circuit gain $-M=-M_0\phi_M$, where M_0 is a pure number and ϕ_M represents the complex part of the gain function. *The gain $-M$ is taken here to include all tandem compensation components.* Similarly the frequency dependent circuit, or λ network, in the feedback path has a transfer function which may be written $\lambda=\lambda_0\phi_\lambda$, where λ_0 is a pure number and ϕ_λ represents the complex portion of λ. The input impedance of the λ network is made so high that it only negligibly loads the bridge circuit, while at the same time its output impedance is so low that the resistors R_6 and R_5 load it very slightly. Allowing for the shunting effect by L_0 and by the load impedance Z_T+Z_3, the gain of the amplifier from EF to CD is readily deduced to be

(gain transfer function, EF to CD)

$$= \frac{-M\left(\dfrac{L_0 s}{R_0}\right)(Z_T+Z_3)}{(Z_T+Z_3)\left(1+\dfrac{L_0 s}{R_0}\right)+L_0 s} \tag{4}$$

where $s=\sigma+j\omega$ is the complex angular frequency.

Applying the usual methods of feedback circuit analysis, assuming that the current flow in AE is the same as in EK, the transfer function from AB to LM is given by

$$\left(\frac{V_M}{e_i}\right)_s = \frac{-\left(\frac{R_6}{R_5}\right) M T_0 s Z_M}{\left(\frac{R_5 + R_6}{R_5}\right)\{(Z_{vc} + Z_M + Z_3)(1 + T_0 s) + R_0 T_0 s\} + M\lambda\left(\frac{Z_2}{Z_1 + Z_2}\right) T_0 s \left(Z_{vc} + Z_M - \frac{Z_1 Z_3}{Z_2}\right)}, \tag{5}$$

where

$$T_0 = L_0/R_0.$$

IV. Circuit Standardization and Simplifications

Fixed Circuit Parameters

At this stage it is convenient to fix certain aspects of the feedback circuitry and thereby introduce some simplifications. For the low-frequency analysis the blocked "voice-coil" inductance will be neglected, and thus Z_{vc} is replaced by R_{vc}, the voice-coil resistance measured at zero frequency. Also Z_3 is replaced by its zero frequency value R_3 and pure resistances R_1 and R_2 are employed in place of Z_1, Z_2. Thus (3) becomes

$$\left(\frac{u + 100}{100}\right)\frac{R_1}{R_2} = \frac{R_{vc}}{R_3} \tag{6}$$

where the bridge is unbalanced by u per cent. To further fix the design it is decided to make

$$R_3 = \tfrac{1}{8} R_{vc}. \tag{7}$$

This value of R_3 has been found to give a reasonable compromise between providing a sufficiently high feedback voltage and minimizing power loss through dissipation in R_3. The actual power loss of approximately $\frac{1}{2}$ db can evidently be neglected. If a fairly efficient loudspeaker were used a larger value of R_3 could be employed where it was desired to employ a higher proportion of velocity feedback.

It will be observed that the suffix s on the left-hand side of (5) has been employed to emphasize the complex frequency form of the transfer function. In much the same way we can write $\phi_M = \phi_M(s)$ and $\phi_\lambda = \phi_\lambda(s)$. In actual examples $\phi_M(s)$ and $\phi_\lambda(s)$ are rational (polynomial) functions in s.

$$\left(\frac{V_M}{e_i}\right)_s = \frac{-\left(\frac{R_6}{R_5}\right)\left(1+\frac{900}{u}\right)\frac{s\phi_M(s)}{\lambda_0}\left(\frac{Z_M}{R_{vc}}\right)}{\frac{9}{8}\left(\frac{R_5+R_6}{R_5}\right)\left(1+\frac{900}{u}\right)\frac{1}{\lambda_0 M_0}\left\{\left(1+\frac{8Z_M}{9R_{vc}}\right)\left(s+\frac{1}{T_0}\right)+\frac{8R_0 s}{9R_{vc}}\right\}+s\phi_\lambda(s)\phi_M(s)\left\{1+\frac{Z_M}{R_{vc}}\left(1+\frac{100}{u}\right)\right\}}. \tag{8}$$

Frequency Normalization

Further simplifications of more conveniently sized numbers are obtained by normalizing frequencies. To this end we introduce the nondimensional variable

$$p = \left(\frac{s}{\omega_0}\right) \tag{9}$$

where $\omega_0 = 2\pi f_0$ is the fundamental resonant angular frequency of the loudspeaker *in its enclosure, i.e.*, the resonant frequency for Z_M. Then for low frequencies (where cone breakup can be ignored) Z_M can be written as

$$Z_M(p) = \frac{2\zeta_0 R_M p}{p^2 + 2\zeta_0 p + 1}. \tag{10}$$

R_M is the rise in the voice-coil impedance at resonance above the zero frequency resistance and $\zeta_0 = \frac{1}{2}Q_0$ is the natural damping factor of Z_M, while Q_0 is the corresponding Q factor. Eq. (10) can give a good approximation from low frequencies up to about 200 or 300 cps, but may depart progressively from the truth with the onset of cone breakup and as the diaphragm becomes a more effective radiator.

The terms $\phi_M(s)$ and $\phi_\lambda(s)$ when factorized can be arranged so that s appears always multiplied by a time constant T_i or else appears as $(s+1/T_i)$. It is fairly easy then to show that if all T_i are replaced by the pure numbers $T_i' = \omega_0 T_i$, terms such as $\phi_M(s)$ can be written $\phi_M(p)$ in which $p = s/\omega_0$. Then (8) can be rewritten

$$\left(\frac{V_M}{e_i}\right)_p = \frac{-\left(\frac{R_6}{R_5}\right)\left(1+\frac{900}{u}\right)\frac{p\phi_M(p)}{\lambda_0}\left(\frac{Z_M}{R_{vc}}\right)}{\frac{9}{8}\left(\frac{R_5+R_6}{R_5}\right)\left(1+\frac{900}{u}\right)\frac{1}{\lambda_0 M_0}\left\{\left(1+\frac{8Z_M}{9R_{vc}}\right)\left(p+\frac{1}{T_0'}\right)+\frac{8R_0 p}{9R_{vc}}\right\}+p\phi_\lambda(p)\phi_M(p)\left\{1+\frac{Z_M}{R_{vc}}\left(1+\frac{100}{u}\right)\right\}} \tag{11}$$

where $Z_M \equiv Z_M(p)$ is given by (10).

The term $\phi_M(p)$ incorporates suitable tandem compensation elements and $\phi_\lambda(p)$ is designed to assist the compensation and also to limit the amplitude of the voice-coil motion at low frequencies.

V. Low-Frequency Loop Gain

The effective low-frequency loop gain is the ratio of the terms in the denominator of (11) which contain $\phi_M(p)$, to the remaining terms, that is,

$$\text{loop gain} = \frac{8\lambda_0 M_0 R_5 p \phi_\lambda(p)\phi_M(p)\left\{1+\frac{Z_M}{R_{vc}}\left(1+\frac{100}{u}\right)\right\}}{9(R_5+R_6)\left(1+\frac{900}{u}\right)\left\{\left(1+\frac{8Z_M}{9R_{vc}}\right)\left(p+\frac{1}{T_0'}\right)+\frac{8R_0 p}{9R_{vc}}\right\}}. \tag{12}$$

Eq. (12) provides the measure of the amount of feedback at low frequencies applied over all stages other than the positive feedback stage and is evidently strongly frequency dependent. By using a frequency analysis approach with $p=j\ (\omega/\omega_0)$ it is possible to produce curves of loop gain and loop phase shift vs frequency. This is one method for checking stability margin and for suggesting suitable compensation techniques.

VI. Design Through Pole and Zero Techniques

The alternative approach, which will be described here, starts by rationalizing the terms of $\phi_\lambda(p)$ and $\phi_M(p)$ as they appear in (11). The degree of the resulting numerator is then reduced by suitably chosen pole-zero cancellations and the reduced polynominal subsequently factorized.

To aid the discussion we write

$$\phi_\lambda(p) = \frac{N_\lambda}{D_\lambda} \tag{13}$$

and

$$\phi_M(p) = \frac{N_M}{D_M} \tag{14}$$

where N_λ, D_λ, N_M and D_M are simple polynomials in p in which, due to the choice of λ_0 and M_0, the coefficient of the term of highest degree is in all cases unity. We also substitute for Z_M in (12) from (10). On multiplying numerator and denominator of (12) by

$$D_M D_\lambda(p^2+2\zeta_0 p+1)$$

we obtain

$$\left(\frac{V_M}{e_i}\right)_p = \frac{-\left(\frac{R_6}{R_5}\right)\left(1+\frac{900}{u}\right)\left(\frac{1}{\lambda_0}\right)\left(\frac{2\zeta_0 R_M}{R_{vc}}\right)p^2 N_M D_\lambda}{\frac{9}{8}\left(\frac{R_5+R_6}{R_5}\right)\left(1+\frac{900}{u}\right)\left(\frac{1}{\lambda_0 M_0}\right)D_\lambda D_M\left\{\left(p+\frac{1}{T_0'}+\frac{8R_0 p}{9R_{vc}}\right)(p^2+2\zeta_0 p+1)+\frac{16}{9}\frac{R_M}{R_{vc}}\zeta_0 p\left(p+\frac{1}{T_0'}\right)\right\} + N_\lambda N_M p\left[p^2+2\zeta_0 p\left\{1+\frac{R_M}{R_{vc}}\left(1+\frac{100}{u}\right)\right\}+1\right]}. \tag{15}$$

One pole of (15) for the values of circuit parameters found in this application corresponds approximately to the larger root of the equation

$$p^2+2\zeta_0 p\left\{1+\frac{R_M}{R_{vc}}\left(1+\frac{100}{u}\right)\right\}+1=0$$

that is, approximately

$$-2\zeta_0\left\{1+\frac{R_M}{R_{vc}}\left(1-\frac{100}{u}\right)\right\}$$

and is thus relatively removed from the origin of $u \leqq 10$. This feature facilitates the numerical root-finding methods employed subsequently.

VII. Practical Velocity-Feedback System

A servo-type amplifier system which has been found to give a satisfactory margin of low-frequency stability is shown in Fig. 2. Low-frequency compensation in this circuit was designed to suit a loudspeaker A, of which the voice-coil impedance characteristics when mounted on its sealed-box enclosure are given in Fig. 3.

To illustrate that the circuit is not unduly critical, satisfactory performance was also obtained with a second loudspeaker B, whose impedance characteristics, as shown in Fig. 4, are substantially different at low frequencies. The only change made was to adjust the zero frequency resistance of Z_3 to once again equal $\frac{1}{8}$ of the voice-coil resistance, thus giving the same degree of bridge unbalance at zero frequency as for loudspeaker A.

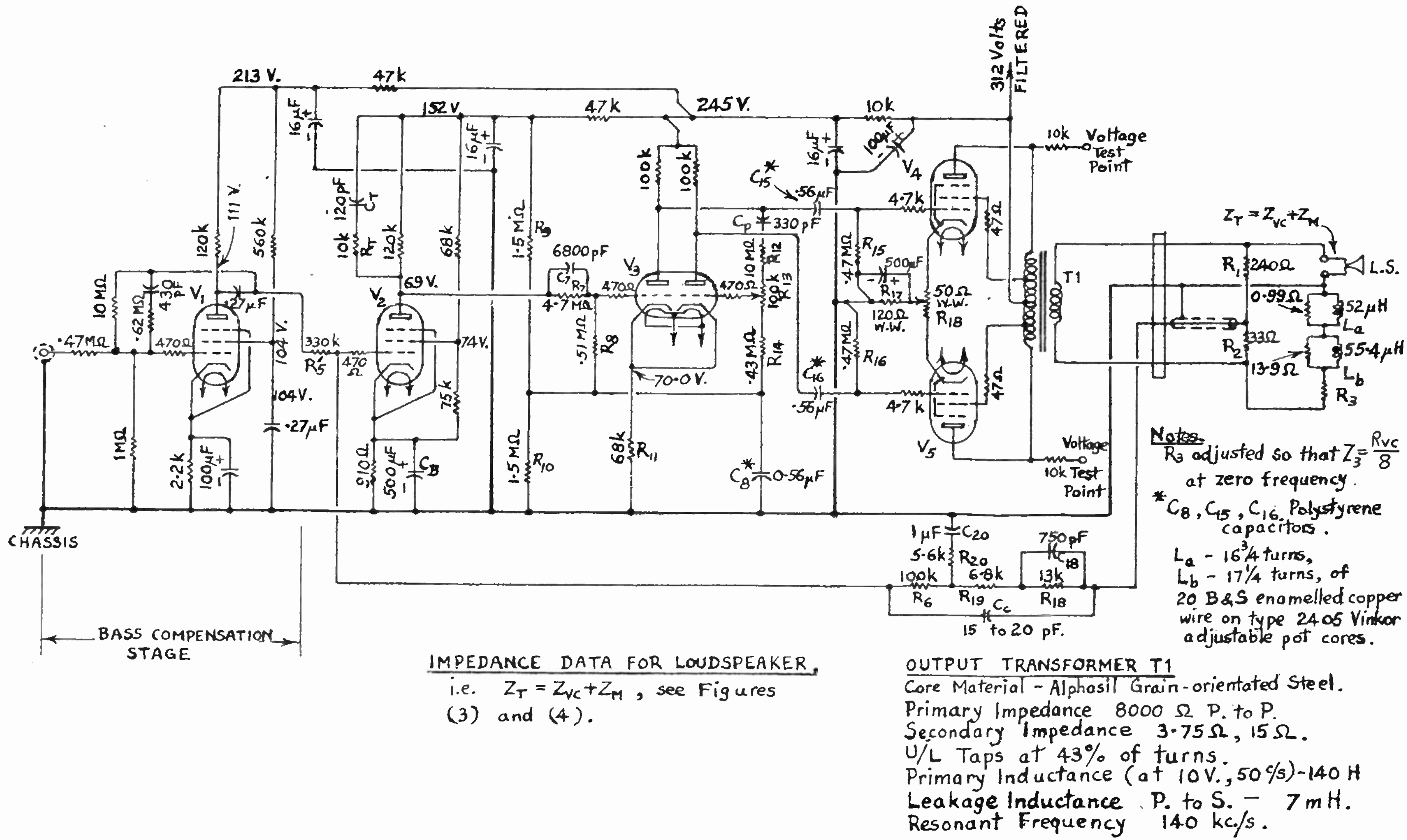


Fig. 2—Practical velocity-feedback system—amplifier circuit diagram.

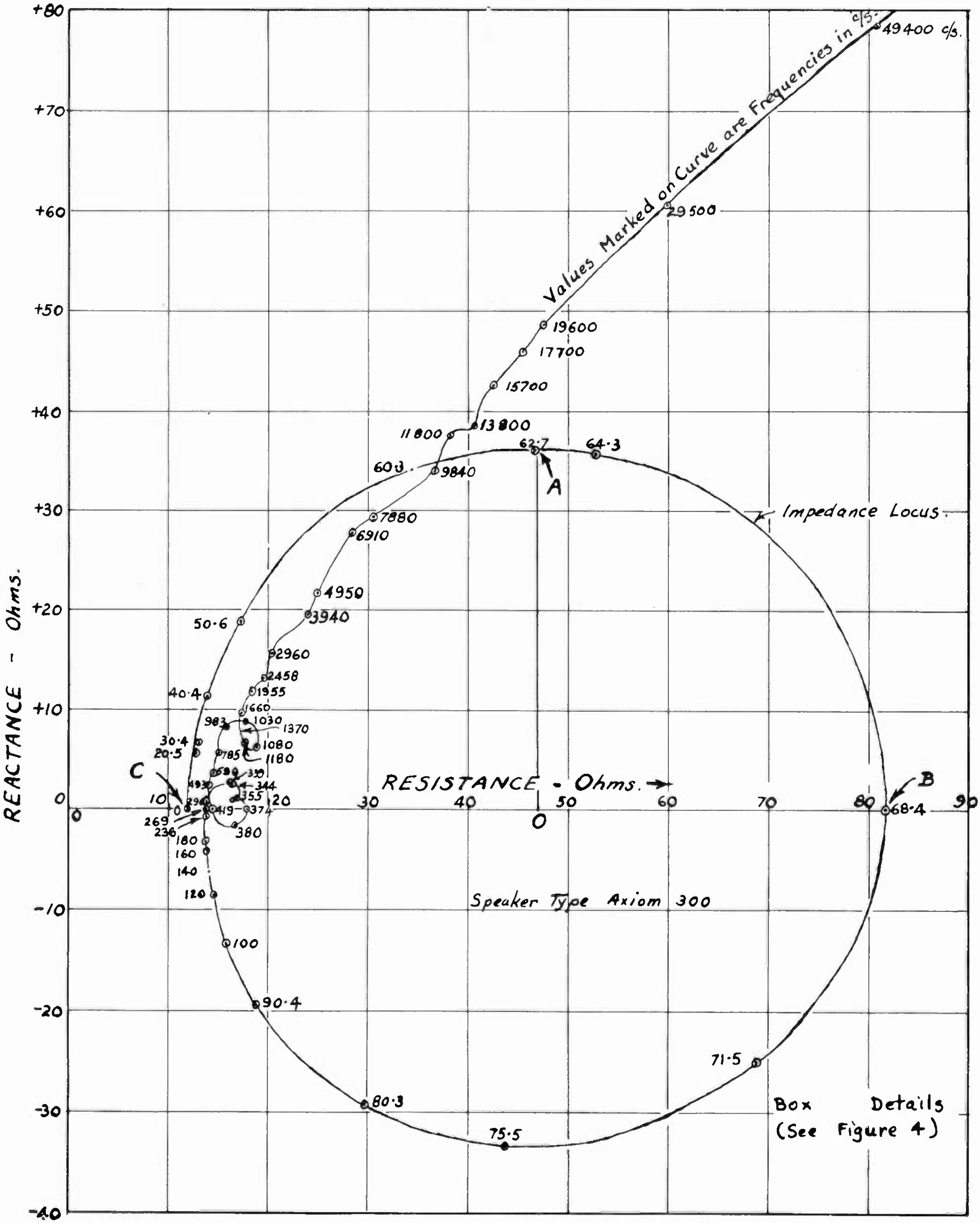


Fig. 3—Voice-coil impedance characteristics with loudspeaker *A* in sealed box enclosure.

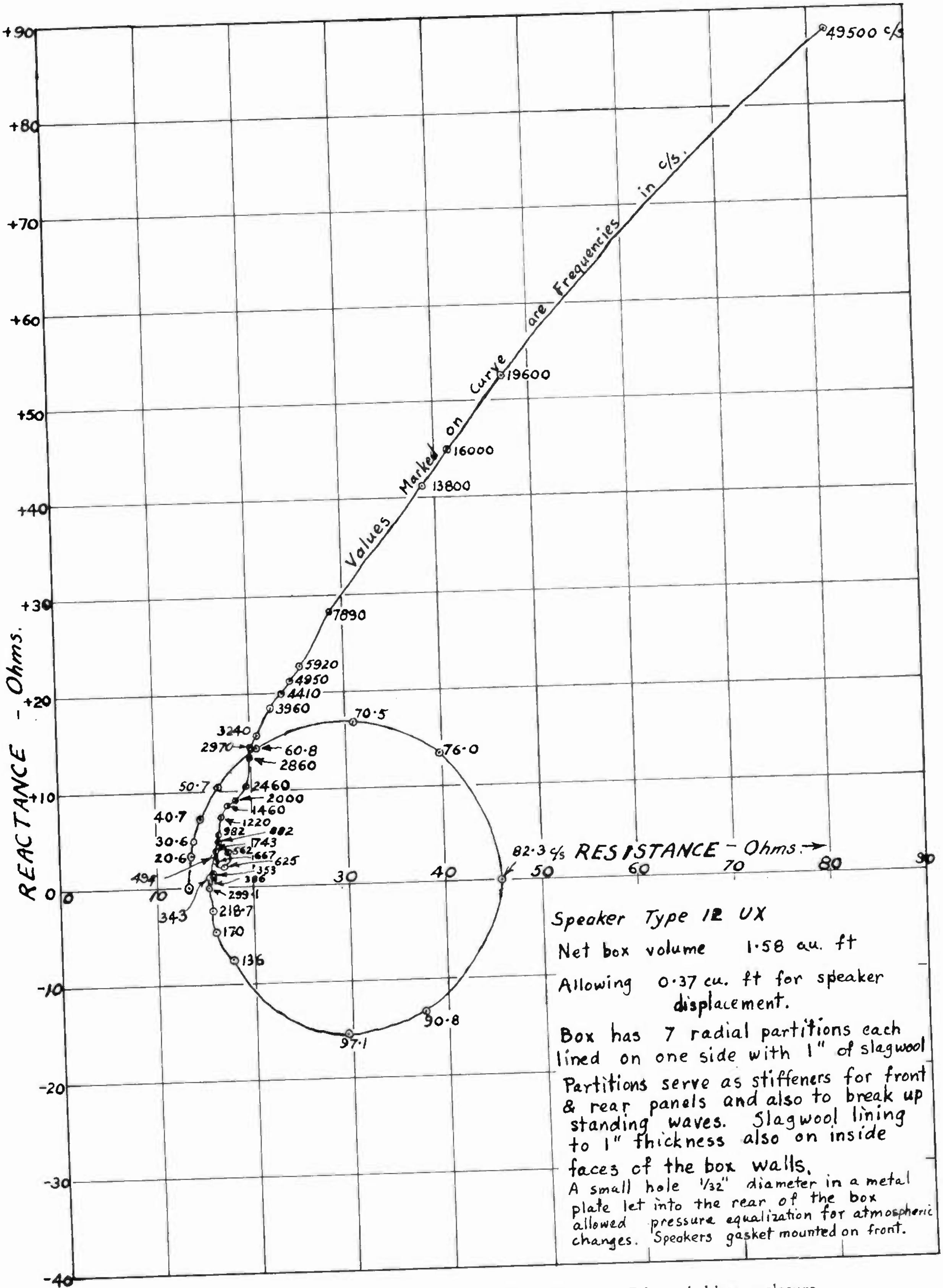


Fig. 4—Voice-coil impedance characteristics with loudspeaker B in sealed box enclosure.

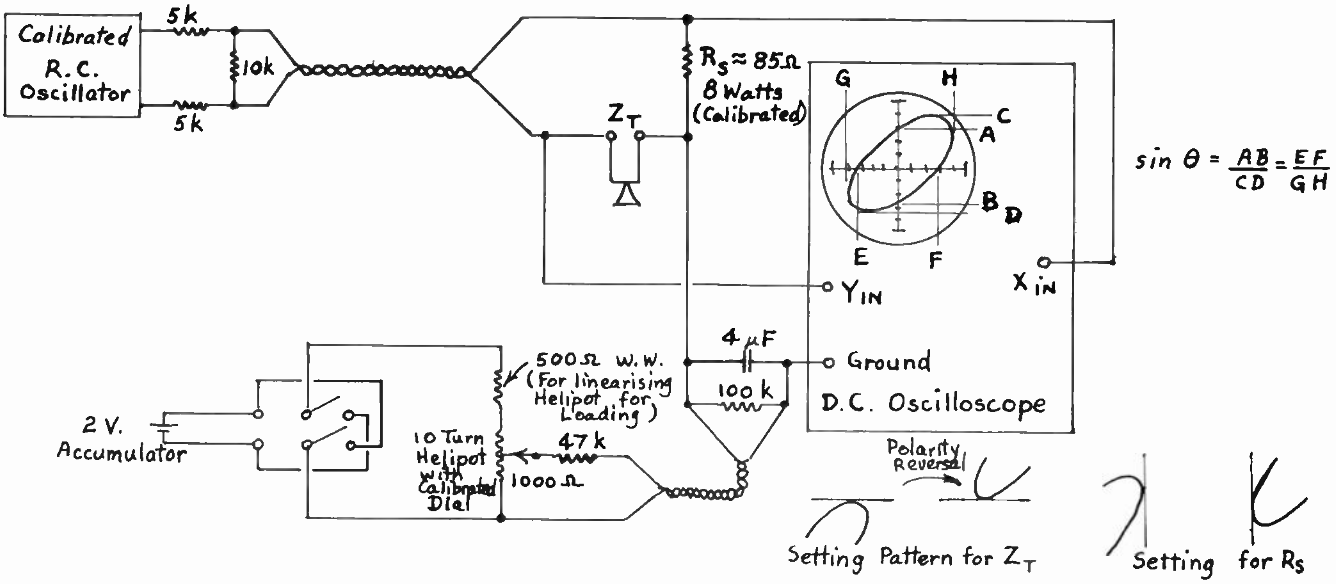


Fig. 5—*Slide-back impedance measuring system with polarity reversal.* Helipot is used to place the pattern tangential to the cross lines. (Complementary shift control on the oscilloscope is used to locate point of tangency approximately near center.) The phase angle is determined from sin θ, together with the counting of all phase reversals to eliminate ambiguities. Repeat readings taken with X, Y inputs are interchanged above 10,000 cps to permit compensation for phase differences within the oscilloscope circuits.

VIII. Loudspeaker Voice-Coil Impedance Measurements

As a preliminary to the following design procedures, it was necessary to measure the impedance at the voice-coil terminals for a selected set of frequencies. However, the measurements need not be as numerous as those indicated in Figs. 3 and 4. The loudspeaker should be mounted on its enclosure and, if the bridge circuit is not to be located at the loudspeaker, the leads connecting the loudspeaker to the amplifier should be included. Fig. 5 shows one suitable arrangement for making the impedance measurements.

It is convenient to plot the reactive vs the resistive component of Z_T in the same way as it is done in Figs. 3 and 4 while the data is being obtained, as this can give guidance as to what further readings may be needed. At frequencies around and below the fundamental resonance, the locus of the points approximates a semicircle fairly well. Groupings of data around the point B (Fig. 3) and just above the point C help to establish these points accurately. It is desirable also to obtain a grouping of points near A, the extremity of the radius OA, which is normal to the diameter BC. In particular, the frequency ω_A at point A should be established fairly accurately in addition to the fundamental resonant frequency ω_0 (at point B). Then, referring to (10), $\frac{1}{2}R_M = OB = OC = OA$, while $2\zeta_0 = \omega_0/\omega_A - \omega_A/\omega_0$. The graphical process of fitting a semicircle tends to give some correction for random errors in measurement. An alternative procedure giving probably a more accurate estimate could be based upon an adaption of a method of complex-curve fitting due to E. C. Levy.[23] In this adaption $2\zeta_0 R_M$ and $2\zeta_0$ would be regarded as unknown constants to be evaluated by a weighted least-squares-fitting of the data.

For subsequent requirements measurements should also be made of Z_T at selected spot frequencies around 300–400 cps, 2500–3000 cps and 12,000–16,000 cps, avoiding frequencies at which noticeable troughs or peaks in the impedance magnitude are evident. The frequencies should also be chosen, bearing in mind that the data will later be used to synthesize Z_3, so as to control the relative level of the voice-cell response of the system at the selected frequencies.

IX. Factor Cancellation Procedures

The use of factor cancellations, by reducing the degree of the denominator polynomial of (15), simplifies and speeds up the analysis. The difference between the actual performance of a system with imperfect cancellation and that of the ideal system, as analyzed, is normally insignificant. Unless very low accuracy has been achieved in cancellation the stability is unaffected, but in most cases it is possible to select a direction of mismatch which will, if anything, give a slightly greater margin of stability. Where the mismatched poles and zeros correspond to a subaudible frequency, the effect of frequency response is of no importance. When the frequencies concerned lie in the audible range, the effect of a 5 per cent mismatch on both transient and frequency

[23] E. C. Levy, "Complex-curve fitting," IRE Trans. on Automatic Control, vol. AC-4, pp. 37–43; May, 1959.

response is normally negligible; this follows from the formulation of the inverse Laplace transform in terms of residues.[24]

With typical selections of hardware, only certain of the naturally occurring factors can be cancelled out satisfactorily or modified so that they cancel out other factors. The physically most suitable factors for cancellation will be apparent from the design example which follows. Whereas in other situations some of these cancellations may be impractical, it may be necessary to accept a somewhat higher degree of complication in the subsequent analysis.

X. Practical Design Sample

Essential Data

Our example will be based upon the following data taken from Figs. 2–4.

1) *Loudspeaker A:*
IN enclosure

$$Z_M = \frac{11.43p}{p^2 + 0.1639p + 1} \text{ ohms.}$$

$$R_M = 69.7 \text{ ohms.}$$

Measured voice-coil resistance $R_{vc} = 11.94$ ohms, resonant frequency is 68.4 cps and $\omega_0 = 429.6$ radians per second.

2) *Loudspeaker B:*

$$Z_M = \frac{10.90p}{p^2 + 0.3333p + 1} \text{ ohms.}$$

$$R_M = 32.7 \text{ ohms.}$$

Measured voice-coil resistance $R_{vc} = 13.11$ ohms, resonant frequency is 82.3 cps and $\omega_0 = 517.1$ radians per second

3) *Output Transformer:* Data is as given on Fig. 2.

4) *Output Stage of the Main Amplifier:* Allowing for the ultralinear screen feedback and a small amount of negative current feedback due to approximately 25 ohms in each cathode lead, we deduce for the output resistance reflected into the secondary of the transformer

$$R_0 = 27.6 \text{ ohms}$$

or, allowing for the shunting effect by R_1 and R_2,

$$R_0 = 25.1 \text{ ohms.}$$

Using loudspeaker A

$$R_0/R_{vc} = 2.106.$$

The "open-circuit" voltage gain of the output stage, referred to the secondary of the output transformer and allowing for the shunting by R_1 and R_2, was estimated to be 2.26, referred to equal but opposite signals applied to the grids of the EL84's.

The time constant

$$T_0 = \frac{L_0}{R_0} = \frac{140 \times 15}{8000} \times \frac{1}{25.1}$$

$$= 0.01046 \text{ second.}$$

Then, using loudspeaker A,

$$T_0' = \omega_0 T_0 = 429.6 \times 0.01046$$

$$= 4.495 \text{ (nondimensional)}$$

and

$$1/T_0' = 0.2225.$$

5) *Complete Amplifier*

a) *Gain constant:* For the amplifier as a whole, from the grid of V_2 to the secondary of the output transformer, it was deduced that the open circuit gain *constant* was

$$M_0 = \frac{4590}{a} \tag{16}$$

where the factor $1/a$ arises from the positive feedback coupling when critically adjusted (see Appendix II).

b) *"Open-circuit" phase characteristics:* With $\omega_0 = 429.6$ radians per second as for loudspeaker A, and a 1000-μf bypass capacitor across the 910-ohm cathode resistor of V_2

$$\phi_M(p) = \underbrace{\frac{(p + 0.002587}{(p + 0.004825}}_{1} \cdot \underbrace{\frac{(p + b)}{(p + a)}}_{2} \cdot \underbrace{(p + a)}_{3} \cdot \underbrace{\frac{p}{(p + 0.00811)}}_{4} \cdot \tag{17}$$

In this expression for $\phi_M(p)$ the fraction marked 1 arises from the cathode bypass of valve V_2, the factor $(p+b)/(p+a)$ marked 2 arises from the interstage coupling between V_2 and the phase-splitter V_3, the factor $(p+a)$ marked 3 arises from the positive feedback coupling, while the term marked 4 arises from the interstage cou-

[24] J. A. Truxal, "Automatic Feedback Control System Synthesis," McGraw-Hill Book Co., Inc., New York, N. Y., p. 291; 1955.

pling between V_3 and the push-pull output stage. The actual gain, with the load and including the shunting effect of L_0, may be deduced from (4). Loading effects are allowed for in (15) by the inclusion of terms such as R_0/R_{vc} and T_0'. The basic derivation of the terms 2 and 3 in (17) is given in Appendix II.

6) *Transfer Function of the Feedback Path:* The low-frequency transfer function of the λ network in the feedback path (see Appendix III) is given by

$$\lambda_0 = 0.2162 \tag{18}$$

and

$$\phi_\lambda(p) = \frac{(p + 0.4157)}{(p + 0.93888)}. \tag{19}$$

Detailed Cancellation Procedures

On checking actual component values in the bridge circuit which was to have a 10 per cent unbalance, the actual unbalance was 9.98 per cent. The denominator term

$$p^2 + 2\zeta_0 p\left\{1 + \frac{R_M}{R_{vc}}\left(1 + \frac{100}{u}\right)\right\} + 1$$

in (15) becomes, with the data given,

$$(p + 10.652)(p + 0.09388).$$

A change of the cathode bypass of V_1 from 1000 μf to 319 μf modifies the factor

$$\frac{p + 0.002587}{(p + 0.004825)} \quad \text{to} \quad \frac{(p + 0.00811)}{(p + 0.01513)}.$$

Two cancellations having been effected in the expression for $\phi_M(p)$ of (17), the latter is now reduced to

$$\phi_M(p) = \frac{p(p + 0.0742)}{(p + 0.01513)}$$

if we assume $b = 0.0742$.

This choice of b was made to affect a further cancellation of a factor of the denominator term

$$\left(p + \frac{1}{T_0'} + \frac{8R_0 p}{9R_{vc}}\right)(p^2 + 2\zeta_0 p + 1) + \frac{16R_M\zeta_0 p}{9R_{vc}}\left(p + \frac{1}{T_0'}\right)$$

of (15). This cubic has one factor $(p+0.0742)$ which was readily evaluated by Lin's method,[21,22] and for the data given here is equal to $(p+0.0742)\ (p^2+0.4643p+1.0444)$.

Making the necessary substitutions in (15), cancelling the factor $(p+0.0742)$ which appears in N_M and the just mentioned cubic, and cancelling the factor $(p+0.09388)$ which appears in D_λ and as a factor of the term

$$p^2 + 2\zeta_0 p\left\{1 + \frac{R_M}{R_{vc}}\left(1 + \frac{100}{u}\right)\right\} + 1$$

we get for (15) after setting $a = 0.632$ in (16)

$$\left(\frac{V_M}{e_i}\right)_p = \frac{-\left(\frac{1}{3\cdot 5}\right)\cdot(91.21)\cdot\left(\frac{1}{0.2162}\right)\cdot(0.960)p^3}{\frac{9}{8}\cdot\left(\frac{4\cdot 5}{3\cdot 5}\right)\cdot(91.21)\cdot\left(\frac{0.632}{0.2162\times 4590}\right)\cdot(2.872)\cdot(p^2 + 0.4643p + 1.0444)(p + 0.01513) + p^2(p + 0.4157)(p + 10.652)} \tag{20}$$

$$= \frac{-115.7p^3}{(p^4 + 11.309p^3 + 4.5437p^2 + 0.25372p + 0.0038131)}. \tag{21}$$

Factorization of the Denominator

The denominator of (21) can be factorized, as in Appendix I, the quadratic factor $(p^2+11.246p+3.8307)$ first being taken out. On further factorizing we finally obtain

$$\left(\frac{V_M}{e_i}\right)_p = \frac{-115.7p^3}{(p+10.89)(p+0.3516)(p+0.03424)(p+0.02907)}. \tag{22}$$

The first factor $(p+10.89)$ can be replaced by the constant value 10.89 over the operative region of this analysis, *i.e.*, below ~200 cps. Effectively then,

$$\left(\frac{V_M}{e_i}\right)_p = \frac{-10.62p^2}{(p + 0.3516)(p + 0.03424)(p + 0.02907)}. \tag{23}$$

Velocity Response (Closed-Loop)

From the point of view of frequency response, the velocity response is evidently completely aresonant with asymptotic break frequencies at 0.3516×67.4, 0.03424×67.4, 0.02907×67.4 cps, that is, 24.05 cps, 2.342 cps and 1.989 cps. The theoretical low-frequency velocity response is given in Fig. 6. In the analysis starting from (17) the effect of C_8 in Fig. 2 has been ignored because 1) being $\gg 25$ times C_p it has a negligible effect on the positive feedback circuit and 2) the time constant R_8C_8 corresponds to 0.56 cps, which is reasonably removed from the critical region around 2 cps. Actually, as the effect of a finite R_8C_8 time constant is to slow down the mean slope of the loop-gain function in this vicinity, it is likely to improve the low-frequency stability, but Fig. 6 will be in error at the very low-frequency end. If preferred, a capacitor of 1 or 2 μf could be employed, but for correct biasing of V_3 a very low leakage condenser is essential for C_8.

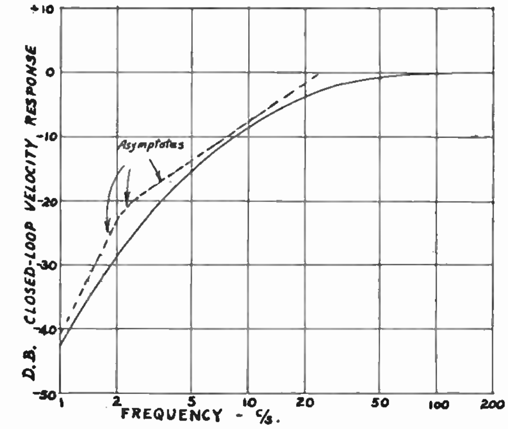


Fig. 6—Theoretical closed-loop velocity response for exact pole and zero cancellations.

System Loop Gain and Phase Shift at Low Frequencies

The loop gain and loop phase shift of the system can be estimated using (12), or calculated numerically by taking the ratio of the corresponding denominator terms in (20) and replacing p by $jx\equiv j(\omega/\omega_0)$.

Loop gain function

$$= \frac{jx(10.652+jx)(0.4157+jx)}{0.24131(0.01513+jx)\left\{0.4643+j\left(x-\dfrac{1.0444}{x}\right)\right\}}. \quad (24)$$

The resulting loop gain and loop phase shift of the system are given in Fig. 7. Evidently the gain margin is more than 15 db and the phase margin 68°. The effective loop gain above 30 cps exceeds 23 db and reaches about 40 db near the fundamental resonant frequency.

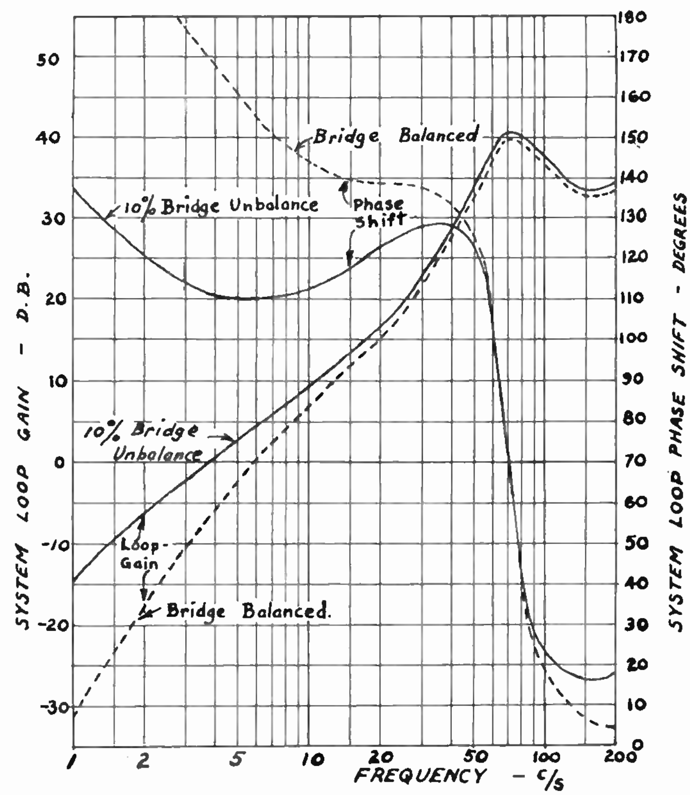


Fig. 7—System loop gain and phase shift for the velocity-feedback systems analyzed.

Low-Frequency Velocity Response with Zero Bridge Unbalance

Before letting u approach zero, we multiply the numerator and denominator of (15) by $0.01u$. In the limit, when u tends to zero, a number of terms disappear. The most important change is that in the last term of the denominator, which becomes $2N_\lambda N_M \zeta_0 p^2 R_M/R_{vc}$. One consequence is that the term $(p+0.09388)$ of D_λ [see (19)] cannot now be cancelled. Also, whereas previously the degree of the denominator polynomial was one more than that of the numerator, it is now of the same order.

Making the appropriate substitutions, the transfer function now becomes

$$\left(\frac{V_M}{e_i}\right)_p = \frac{-\left(\dfrac{1}{3\cdot 5}\right)\cdot(9)\cdot\left(\dfrac{1}{0.2162}\right)\cdot(0.960)p^3(p+0.09388)}{\dfrac{9}{8}\cdot\left(\dfrac{4\cdot 5}{3\cdot 5}\right)\cdot(9)\cdot\left(\dfrac{0.632}{0.2162\times 4590}\right)\cdot(2.872)\cdot(p^2+0.4643p+1.0444)(p+0.09388)(p+0.01513) + (0.1639)\cdot\left(\dfrac{69.7}{11.91}\right)p^3(p+0.4157)}. \quad (25)$$

After expanding and then dividing numerator and denominator by 0.9830, we obtain

$$\left(\frac{V_M}{e_i}\right)_p = \frac{-11.62p^3(p+0.09388)}{(p^4+0.4195p^3+0.02656p^2+0.002774p+0.00003594)}. \tag{26}$$

Attempts to remove a quadratic factor from the denominator of (26) soon establish that the process will not converge because three of the roots are now close together. However, it is quite easy to take out the linear factor $(p+0.3670)$. The remaining cubic must have at least one real root. This is quickly obtained by the method of finding remainders after division by trial factors, followed by interpolation or extrapolation to zero remainder. (See Appendix I.) The next factor $(p+0.01453)$ is thus soon established. We then obtain

$$\left(\frac{V_M}{e_i}\right) = \frac{-11.62p^3(p+0.09388)}{(p+0.3670)(p+0.0145)(p^2+0.03798p+0.006740)}. \tag{27}$$

The quadratic term in the denominator of (27) corresponds to an oscillation at 0.0821 times the bass resonant frequency of the loudspeaker with a damping factor of $\zeta=0.231$. Clearly the system is still reasonably stable and, in view of the relatively low frequency of this oscillation, the small peak in the response would be unimportant. The corresponding loop gain and loop phase-shift characteristics are also given in Fig. 7. It is evident that even at zero bridge unbalance the gain margin is about $11\frac{1}{2}$ db and the phase margin about 24°. Practical tests at $\frac{1}{2}$ per cent bridge unbalance showed that the system was completely stable; a small residual unbalance was allowed to provide a small margin for thermal changes in the resistances.

XI. Voltage Response Measured at the Voice Coil

Low-Frequency Transfer Function

For certain purposes, for example when one knows the free-field acoustic response of the loudspeaker, it is useful to be able to estimate the voltage response of the amplifier as measured at the voice coil of the loudspeaker. For the low-frequency region this may be deduced from (15) if we also make use of the relationship

$$\left(\frac{V_T}{V_M}\right) = \left(\frac{Z_{vc}+Z_M}{Z_M}\right) = \frac{Z_T}{Z_M}. \tag{28}$$

Thus we obtain (with Z_{vc} replaced by R_{vc})

$$\left(\frac{V_T}{e_i}\right)_p = \frac{-\left(\frac{R_6}{R_5}\right)\left(1+\frac{900}{u}\right)\left(\frac{1}{\lambda_0}\right)\left\{p^2+2\zeta_0 p\left(1+\frac{R_M}{R_{VC}}\right)+1\right\}p^2N_MD_\lambda}{\text{same denominator as (15)}}. \tag{29}$$

The quadratic term in the denominator is responsible for a trough in the response at the bass-resonant frequency. It is this trough which just cancels out the normal rise in acoustic ouput around the resonant frequency. Eq. (29) in this form is only valid for frequencies up to ~200 cps.

Behavior at Frequencies Above ~200 cps

Due mainly to the term Z_{vc}, $Z_T=Z_M+Z_{vc}$ takes on, at frequencies higher than 200 cps, values which cannot be too easily represented by simple lumped circuits, although measured impedance values may be readily obtained. On the other hand, due to the internal positive feedback, the effective gain of the amplifier can be quite high. Combining (28) and (5) and neglecting terms which do not contain M, one obtains after cancelling MT_0s

$$\left(\frac{V_T}{e_i}\right) = -\left(\frac{R_6}{R_5}\right)\left(\frac{R_1+R_2}{\lambda R_2}\right)\frac{Z_T}{\left(Z_T-\frac{R_1Z_3}{R_2}\right)} \tag{30}$$

which gives the velocity response transfer characteristic for higher frequencies.

The suffix s has been dropped on the left-hand side to indicate that (30) is to be treated mainly from the point of view of frequency response. Approximate constancy of the ratio Z_1/Z_2 at the low-frequency value R_1/R_2 may be maintained, if necessary, by addition of compensating capacitors or by choosing sufficiently small resistance values for R_1 and R_2 as in Fig. 2. The λ network normally would be shunted by stray capacity plus the input capacity of V_2. This can be offset by introducing the capacitor C_C across λ network. With C_C incorporated and employing a transitional phase-shift network C_TR_T in the plate circuit of V_2 to control loop gain and phase near the resonant frequency of the output transformer, no difficulty was experienced with high-frequency instability. A more detailed discussion of the treatment using (30) to control at high frequencies the voice-coil response will be given in another paper.[20]

XII. Steps in the Low-Frequency Design Procedure

Recapitulation

Methods of factor cancellation have been demonstrated. In the example it was possible to limit by these means the degree of the system to the 4th. The methods described for root determination are quite efficient up to the 4th degree, and still quite practical for 5th- or 6th-degree polynomials. However, it may not always be practical, nor is it essential, to achieve exact cancellation in all cases. For example, one cancellation required that a capacitor C_B of 319 μf be used for the cathode bypass of V_2. This is impractical to achieve with an electrolytic capacitor, and one would in fact employ either 250 μf or 500 μf. Further root evaluations show that progressively greater stability margins are obtained with the larger values of C_B. However, if one wishes to check this point one finds that the denominator polynomials are now of the 5th degree. For the circuit adopted in Fig. 2 a value of 500 μf was chosen.

Similarly the positive feedback coupling capacitor C_p across V_3 should be chosen to match time constants arising from R_7, C_7, R_8 and the output resistance of V_2 at low frequencies (see Appendix II). Here, matching within 5 per cent is quite practical. If a small mismatch does occur it is preferable that C_p should err on the smaller side as this gives a slightly lower average attenuation-slope to the loop-gain vs frequency characteristic.

Sequence of the Design Procedure

Since a number of cancellations have been employed, and to some extent these are interlocking, the appropriate time constants must be chosen in the correct sequence.

1) First, a choice is made of a loudspeaker and enclosure, and impedance measurements are taken at the voice-coil circuit (as presented to the bridge circuit).
2) A decision should be made upon the degree of bridge unbalance. Since this has some bearing on the problem of providing for low-frequency stability, a revision may be needed after a first analysis has been completed.
3) The low-frequency characteristics of the λ network may now be decided. Where practicable this may be chosen so that a cancellation can occur with a factor of

$$p^2 + 2\zeta_0 p\left\{1 + \frac{R_M}{R_{vc}}\left(1 + \frac{100}{u}\right)\right\} + 1$$

but in any case the midfrequency value of $|\lambda|$ should not be less than about 0.2 if "white" noise arising from excessive amplifier sensitivity is to be avoided. In some cases, for example if u is very small, exact cancellation may be impossible. In this case the polynomial to be factored will be of the 5th degree, if all other cancellations are effected.
4) A decision should be made on the time constants of the coupling between the phase-splitter and the output stages. It is inadvisable to use a much shorter time constant than that used in Fig. 2. The value of C_B to give a corresponding cancellation is calculated to simplify the analysis, but the next larger value is adopted in the actual circuit.
5) A decision now has to be made on a suitable value of the constant a in (16) and (17). Since this choice has a decisive influence on the final stability margin, it may be used to control the damping factor associated with the dominant poles of the system.
6) The circuit coupling from V_2 to V_3 may now be adjusted so that one time constant equals $1/\omega_0 a$ (see Appendix II), and the other matches a time constant corresponding to one factor of the cubic term

$$\left(p + \frac{1}{T_0'} + \frac{8R_0 p}{9R_{vc}}\right)(p^2 + 2\zeta_0 p + 1) + \frac{16R_M}{9R_{vc}}\zeta_0 p\left(p + \frac{1}{T_0'}\right)$$

which appears in (15) and is fully determined by the loudspeaker characteristics and the self-inductance of the output transformer at suitable signal amplitudes.

Normally, sufficient variation can be achieved by a choice of R_7 and of the time constant R_7C_7. In fact, $b = 1/\omega_0 R_7 C_7$, where b is shown in (17).
7) The capacitor C_p can be chosen so that $C_p(R_{12} + R_{13} + R_{14})$ (see Fig. 2) matches a time constant arising from the *component values* of R_7 and C_7.

XIII. Discussion

There are various pitfalls which can cause troubles of overloading or instability in high-performance velocity feedback systems. A basic type of compensated amplifier has been described which is designed to avoid these problems and has been found to work satisfactorily in velocity-feedback systems. Methods have been given for studying the low-frequency stability. While use has been made of pole and zero cancellations to simplify the analysis, precise cancellations are not essential and within reasonable limits do not seriously disturb the audible low-frequency behavior. The latter may thus be deduced with sufficient accuracy from the simplified analytical model. The resulting circuits are not unduly critical.

High-frequency performance of the system will be discussed in another paper,[20] but with the compensation components shown in Fig. 2 satisfactory operation was obtained with no sign of instability up to and beyond the self-resonant frequency of the output transformer.

That this result was achieved with the two loudspeakers tested, which were of completely different origin, is not too surprising since the high-frequency impedance characteristics, unlike those at low frequencies, are quite similar.

It is important at the design stage to be able to establish not only whether the amplifier system will be stable but also to be able to see what changes may be needed if the initial arrangement turns out to be unstable. For controlling the low-frequency stability the most important single factor is the parameter a in the present paper (unless the degree of bridge unbalance is varied). An increase in a gives a greater margin of stability but reduces the loop gain at low frequencies. Evidently a should be chosen as small as possible to remain consistent with stability, and preferably should not be chosen so large that af_0 is greater than 30–40 cps.

The most important specifications on closed-loop performance concern

1) the asymptotic break frequency below which the velocity response should fall corresponding to an asymptotic slope of 20 db per decade and
2) the highest acceptable angular frequency ω_n and the minimum value of the damping factor ζ_n corresponding to the dominant poles. The frequency $\omega_n/2\pi$ should be reasonably clear of turntable rumble frequencies, *i.e.*, less than 10 cps, especially if a relatively small value of ζ_n be accepted.

Once a particular analysis has been performed leading to equations such as (20)–(22), it is usually possible to make quite rapid adjustments to obtain a more desirable compromise. For example, if there was no intention of reducing the percentage of bridge unbalance, as described in this paper, it might be considered that the illustrative design gave larger than necessary loop gain and phase margins for stability and that the loop gain of 23 db at 30 cps should be increased to give reduced distortion. A simple estimate shows that a reduction of a to half the previous value would increase the loop gain by about 3 db at 30 cps and up to 6 db at lower frequencies. Revising the denominator of (20) with a changed from 0.632 to 0.316 quickly leads to a new denominator polynomial for (21), *viz.*,

$$p^4 + 11.188p^3 + 4.4859p^2 + 0.12686p + 0.0019066.$$

Factorization of the latter to slide rule accuracy took about 20 minutes to give the factors $(p+0.387)$ $(p+10.773)$ $(p^2+0.0292\ p+0.000457)$. The values of $\omega_n = 0.0214\omega_0$ and $\zeta_n=0.682$ deduced for the quadratic factor are quite satisfactory. Practical realization is approximated by making $R_7=2.0$ megohms, $C_7=0.016\ \mu f$ and $C_p=680$ pf in Fig. 2.

At low frequencies it can be shown that the acoustic pressure response is proportional to the product of voice-coil velocity, diaphragm area and frequency.[1] The response correction centered around V_1 produces a response which approximates a $1/f$ asymptotic characteristic between 35 cps and 580 cps. Thus the amplifier with feedback compensation which gives level *velocity* response should, when combined with this correction stage, give level acoustic response from say 200 cps down to 35 cps.

As Novak has shown,[1] sealed-box enclosures normally exhibit appreciably higher loudspeaker distortion than the nearest vented-box equivalents. Usually the need to keep the resulting resonant frequency as low as possible and to provide sufficient damping leads to relatively low acoustic efficiencies and a corresponding need for amplifiers with higher than normal power capabilities. The velocity-feedback system overcomes most of these problems, since it permits the use of a higher-efficiency loudspeaker by completely suppressing the fundamental resonance and reducing by quite a large factor any nonlinear loudspeaker distortion. Only the back radiation is lost. Werner[10] has shown that the distortion may be reduced to very low values using a bridge circuit which is almost balanced.

Largely because of cone breakup with conventional loudspeakers, an unbalance of some 10–15 per cent would be required in order to maintain reasonably level response at the higher frequencies. This situation would be modified if a loudspeaker with a near-rigid cone were employed. The system could be adapted to suit most loudspeakers, but full benefits would be possible only where the loudspeaker is capable of a large volume displacement and has been designed to have an essentially constant BL product.

The employment of very stiff-cone loudspeakers in velocity feedback systems with $\sim$1 per cent of bridge unbalance seems to be the next step logically. This should lead to a worthwhile advance in the quality of acoustical reproduction, largely correcting the situation where the loudspeaker is accepted as the weakest link in the reproduction chain.

XIV. Other Aspects Affecting This Application

Interchangeability of Units

With velocity feedback, the normally free and independent choice of loudspeaker system and amplifier is no longer possible, since the amplifier and its associated loudspeaker system must be designed as an integral unit, that is, a wide-band servosystem. *Given a suitable type of amplifier and compensation principle* it may, nevertheless, be easier and less critical to provide this compensation, rather than to build an enclosure correctly tuned to match a particular loudspeaker. Certainly nothing could be simpler than a sealed-box enclosure. Moreover, as Werner[10] has shown, satisfactory performance can be obtained with velocity feedback using a box as small as ½ cubic foot. Such a system clearly offers excellent prospects where highly compact speaker systems are required.

Difficulties regarding interchangeability of speaker systems and amplifier could be minimized by:

1) Adoption of a basic type of amplifier with compensation circuits incorporating a minimum number of components, which may be specified, to suit any typical loudspeaker in a selected sealed-box enclosure, in order to give satisfactory and stable operation.

2) Provision by *loudspeaker manufacturers* of complete feedback bridge circuits to suit particular loudspeakers (or other means of supplying a velocity-derived signal, such as a separate magnet and pick-up coil). The most satisfactory location for such elements would be at the loudspeaker or its enclosure. Low-impedance circuits would minimize capacitative shunting. This location, besides facilitating interchangeability, would eliminate problems due to the lead resistances being added to the voice-coil resistance.

3) In a transistorized version the velocity feedback power amplifier could be mounted on the loudspeaker enclosure to form an integral unit.

Amplifier Ratings

It has been customary to provide separate rating of conventional high-grade power amplifiers when operating into a purely resistive load equal to the nominal voice-coil impedance. However, the only completely satisfactory way to rate a velocity-feedback system would be in terms of measured acoustic output, using free-field response and distortion measurements taken with a high-grade calibrated microphone. Nevertheless, a somewhat artificial rating could be made of the amplifier alone by using a purely resistive dummy load and bridge circuit, with resistance ratios made equal to the measured values in the voice-coil bridge circuit at zero frequency.

Possible Future Trends

It seems possible that the next distinct advance in acoustic reproduction using electromagnetic loudspeakers could follow the general employment of velocity-servo, stiff-cone, acoustic, radiator systems, since there appears to be no other way of reducing loudspeaker distortion in compact systems to a level approaching that caused by other components. Such a system would go close to giving controlled acoustic output without introducing further possible instability and other problems (which could be the case if the feedback signal were taken from a fairly close-coupled microphone).

Appendix I

Determination of Polynomial Factors

A useful method for factorizing polynomials is given by Lin[21] and Taylor.[22] A variation of the methods given there will be applied to factorizing the denominator of (21), so

$$p^4 + 11.309p^3 + 4.5437p^2 + 0.25372p + 0.0038131 \quad (31)$$

Because of the relative smallness of the last two terms on the right, it is evident that an approximate quadratic factor would be $(p^2+11p+4)$. From experience it is found that the coefficient of the middle term is only a little less than the coefficient of p^3, *i.e.*, 11.309. The next term differs somewhat more from the coefficient of p^2, *i.e.*, 4.5437. The trial factor is taken as $(p^2+11.25p+4)$. The polynomial (31) is written in reverse order with detached coefficients and a trial division made by $(4+11.25p+p^2)$. Thus

			0.0009533	0.06075			
4.0	11.25	1	)0.0038131	0.25372	4.5437	11.309	1
			0.0038131	0.01072	0.00095		
				0.24300	4.54275	11.309	
				0.24300	0.68332	0.0607	
					3.85943	11.2483.	

Evidently the last division should give the answer unity. One can regard $(3.859+11.2483p+p^2)$ as probably being closer to the true quadratic factor than the first trial factor. When the process converges the coefficients move monotonically towards their final values. It is thus possible by judicious choice of each successive trial factor to speed up the convergence. The figure 0.68332 which appears above is obtained as 0.06075×11.2483 rather than $0.06075 \times$ the trial figure 11.25. This also speeds the convergence slightly. The following is a list of trial factors and corresponding remainders.

Trial Factors			Corresponding Remainders after 2 Division Stages		
3.85	11.2483	1	3.8331	11.2460	1
3.833	11.2460	1	3.8311	11.2457	1
3.831	11.2457	1	3.83075	11.24569	1
3.83074	11.24569	1	3.830735	11.24569	1

It is evident that to the relatively large number of figures given here the quadratic factor is $(p^2+11.24569p+3.83074)$.

The remaining polynomial factor is readily identified from the division process. In the case of the last trial factor given above it is

$$(0.000995395 + 0.0633105p + p^2).$$

Finally we write the factors of (31) as

$$(p^2 + 11.2457p + 3.83074)(p^2 + 0.06331p + 0.0009954).$$

From this point the two quadratic factors may be factorized by conventional methods. The method given above is suited to determining factors corresponding to the two *largest roots* and, as in the above illustration, converges quite rapidly. In most cases slide rule accuracy is probably sufficient and even fewer stages of calculation are needed. If we follow directly the method of Lin[21] and Taylor,[22] we obtain instead the factors corresponding to the *two smallest* roots.

In case only one of the largest two roots is well removed from other roots, exactly the same method can be applied to take out a linear rather than a quadratic factor. One such factor, corresponding to a root sufficiently removed from the others for the process to converge, is normally present for the system designs considered here. It arises from the low-frequency amplitude limiting function of the λ network.

As an example we take the 4th-degree polynomial occurring in (26)

$$p^4 + 0.4195p^3 + 0.02656p^2 + 0.002774p + 0.00003594.$$

It is soon evident, on trial, that there will be no convergence permitting a quadratic factor to be obtained. One stage of determining a first linear factor is illustrated below.

		0.000097655	0.007272	0.05241		
0.368	1	)0.00003594	0.002774	0.02656	0.4195	1
		0.00003594	0.0000977			
			0.0026763	0.02656		
			0.0026763	0.007272		
				0.019288	0.4195	
				0.019288	0.05241	
					0.36709.	

A next trial factor could be $(p+0.3670)$. Convergence can also be speeded by determining the larger root of the quadratic $p^2+0.4195p+0.019288$, taken from the third last line of calculation. Finally, the factor obtained is $(p+0.36701)$ and the conjugate polynomial factor is

$$(p^3 + 0.05250p^2 + 0.007292p + 0.00009972).$$

Since this factor is a cubic it must have at least one real root and, hence, there must be at least one linear factor. Writing the cubic down in order of decreasing powers of p, we can use simple division trials. For example, dividing by $(p+0.015)$ yields $p^2+0.03750p+0.0067294$ with a remainder of -0.00000302.

Similarly, dividing by $(p+0.0145)$ we obtain $p^2 +0.3800\,p+0.0067409$ with the remainder $+0.00000018$.

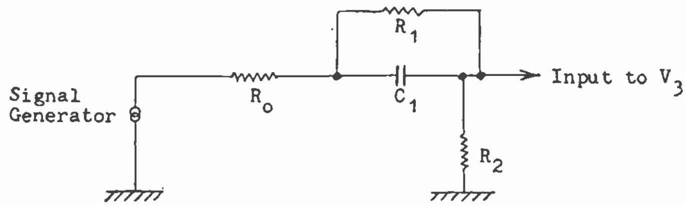
Interpolating to zero remainder we obtain as the next trial factor $(p+0.014528)$. In the present instance, this is found to be the required factor to the accuracy required. Finally the original polynomial can be written as $(p+0.36701)$ $(p+0.14528)$ $(p^2+0.03798p+0.006740)$. The quadratic factor here has no real roots. It corresponds to a natural resonant frequency $\omega_n = (\sqrt{0.006740})\omega_0 = 0.08210\omega_0$, with a damping factor

$$\zeta_n = \frac{0.03798}{2 \times 0.08210} = 0.2313$$

Appendix II

Interstage Coupling from V_2 to V_3

The equivalent circuit is as shown in the following diagram:



where R_0 is the output impedance of V_2.

From the example of Fig. 2, $R_0=0.115$ MΩ, $R_1=4.7$ MΩ, $R_2=0.51$ MΩ, $C_1=0.00667$ μf. (This is valid except at the very lowest frequencies, approximately 1 cps, where the capacitor C_8 of Fig. 2 ceases to be effective.)

The transfer function of the above coupling

$$= \left(\frac{R_2}{R_2 + R_0}\right)\frac{(p + 1/\omega_0 R_1 C_1)}{\{p + (R_1 + R_2 + R_0)/\omega_0 R_1 C_1 (R_2 + R_0)\}}$$

where $p = s/\omega_0$.

For loudspeaker A ω_0 is 429.6 radians per second. Inserting the component values given above,

$$\text{transfer function} = 0.8166\,\frac{(p + 0.0742)}{(p + 0.632)}\,.$$

This is the source of the term $(p+b/p+a)$ in the expression for $\phi_M(p)$ given in (17).

Positive Feedback Over V_3 at Low-to-Medium Frequencies

The gain of the stage can be written

$$\frac{-m}{1 - m\beta}$$

where m is the gain without feedback and β is due to the RC coupling and a potentiometric tapping to give an adjustable value of β. With critical adjustment, the value of β equals $1/m$ at higher frequencies. For subcritical adjustment, suppose β equals $(1-\epsilon)/m$, *i.e.*, suppose

$$\beta = (1 - \epsilon)p/m(p + a)$$

where ϵ is a small real positive number. The term $p/m(p+a)$ arises from the RC coupling (at the very lowest frequencies this is not strictly correct because condenser C_8 of Fig. 2 ceases to be an effective short circuit). The expression for the gain becomes

$$\frac{-m(p + a)}{\epsilon(p + a/\epsilon)}\,.$$

This expression also ceases to be correct at the upper end of the audio range unless m is no longer regarded as a constant but incorporates an allowance for stray capacitative shunting. Where the adjustment of the positive feedback is critical the gain function is

$$\frac{-m(p+a)}{a}.$$

In practical cases ϵ would probably take on values around 0.05 to 0.1, and the more general expression would be needed. Analysis of the system behavior with the type of circuit in Fig. 2 has shown that the over-all closed-loop response is but slightly affected if ϵ is given values ranging from $+0.1$ to 0. Negative values of ϵ imply a reversal of amplifier gain at very high frequencies, but then account must be taken of capacitative shunts. Negative values of ϵ are to be avoided in any case as they lead to instability in the amplifier when the main negative feebback loop *is opened.* In practice the adjustment of ϵ is made with the main loop opened, while the trimmer potentiometer R_{13} is advanced until oscillation starts (the loudspeaker replaced by a dummy load, of course) and then slowly retarded until the amplifier just drops out of oscillation.

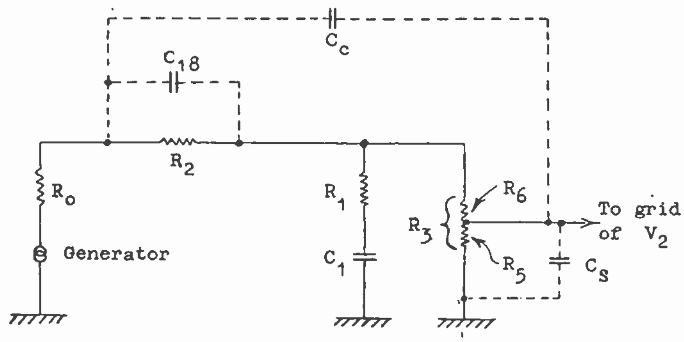
For the reasons given it is convenient in analysis to use the simpler stage gain expression $-m(p+a)/a$. This is the source of the factor $1/a$ in (21) and the factor $(p+a)$ marked 3 in (22).

Appendix III

The Transfer Function of the λ Network for Low-Frequency Amplitude Control and Feedback Compensation

At low frequencies the λ network behaves as a transitional phase-shift network and, being in the feedback path, aids stability as well as modifies the over-all velocity vs frequency response of the system. In terms of frequency analysis it helps to control low-frequency phase shifts by reducing the average attenuation-slope of the loop gain vs frequency diagram. In the illustrative feedback system, this flattening occurs between 28 cps and 6.3 cps and is sufficiently close to the critical frequency region (for instability) to achieve a useful control of phase shift.

A simplified diagram of the λ network follows. *The circuits shown dotted significantly affected performance only at the upper audio-frequency limits and are not included* in the formula for the transfer function.



In this diagram R_0 is the output impedance of the bridge, taken to be purely resistive at low frequencies, and R_3 is the effective load placed on the λ network, *including the output impedance* (assumed to be resistive) *of* V_1. After normalizing, the resulting transfer function is

$$\frac{\left(\frac{R_1R_3}{R_1+R_3}\right)\left(p+\frac{1}{\omega_0R_1C_1}\right)}{\left\{R_0+R_2+\frac{R_1R_3}{R_1+R_3}\right\}\left[p+\frac{1}{\omega_0\left\{R_1+\frac{(R_0+R_2)R_3}{R_0+R_2+R_3}\right\}C_1}\right]}.$$

The noncomplex portion which equals

$$\frac{R_1R_3}{R_1R_3+(R_1+R_3)(R_0+R_2)}$$

gives the value of λ_0. This value should not be too small or the amplifier exhibits audible "white" noise. A value of 0.2 or greater is quite satisfactory. The time constant R_1C_1 approximately corresponds to the asymptotic break-frequency below which the voice-coil motion becomes amplitude-limited (a small shift in the break-frequency results after feedback).

For example, values of circuit components in accordance with Fig. 2 are

$$C_1 = 1\ \mu\text{F}$$
$$R_1 = 0.0056\ \text{M}\Omega,\quad R_1C_1 = 0.0056\ \text{second}$$
$$(R_0+R_2) \approx 0.02005\ \text{M}\Omega$$
$$R_3 \quad 0.45\ \text{M}\Omega.$$

Then

$$\text{transfer function} = \frac{0.2162(p+0.4157)}{(p+0.09388)}.$$

This is the transfer function given in (18) and (19).

Controlling the Upper-Frequency Characteristics of Velocity-Feedback Loudspeaker Systems

H. W. HOLDAWAY

Summary—A treatment already given[1] permits velocity-feedback systems to be designed for adequate stability margin at low frequencies. At the higher frequencies allowance must be made for the modifying effects of the relatively impure blocked voice-coil "inductance."

Because of the nature of this inductance, a design procedure was developed in terms of *closed-loop voltage response at the voice-coil terminals.* A relatively simple compensation could be provided to produce a suitable balance in the acoustic frequency response at the low, middle and upper audio frequencies. Stable operation also was obtained with no signs of parasitic oscillations.

Practical examples, on testing, established the closeness of certain simplifying approximations used in the design. Results indicate how this type of feedback can suppress the fundamental resonance and extend the bass response.

An important feature of the design is that it is based upon measurements of voice-coil impedance of the loudspeaker in its enclosure but without the need to block the voice-coil motion.

Introduction

A PREVIOUS PAPER[1] has described the main features of velocity-feedback systems and has demonstrated a technique for compensating in order to achieve a controlled degree of stability at very low frequencies.

Werner[2] has stressed the importance of providing, in a bridge circuit located at the voice-coil, components such that the "negative" output impedance of the amplifier cancels out the blocked voice-coil "inductance." However, precise cancellation is not easy, since the blocked voice-coil does not behave as a pure inductance because it is modified by eddy current and hysteresis losses.

Werner[2] avoided using inductances by making the impedance corresponding to Z_2 of the present Fig. 1 a resistance shunted by a series RC combination. Although this method can be used to *balance the bridge circuit* over a suitable range of frequencies, examination of (10) in Holdaway[1] shows that for large values of the loop gain the response, being in part proportional to $(Z_1+Z_2)/Z_2$, could exhibit an unintended rise at high frequencies. It is easily demonstrated that this effect is small if the bridge operates into a relatively low impedance circuit (compared with Z_1 and Z_2). Where this

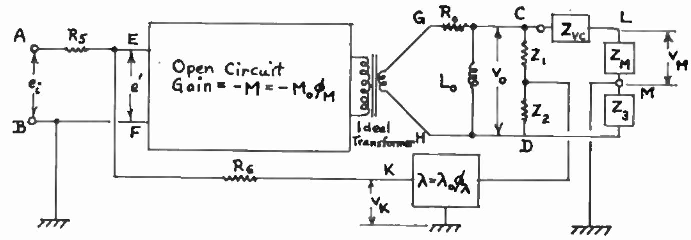


Fig. 1—Schematic diagram of the velocity feedback system. Note that R_5 includes the output impedance (resistive) of V_1.

arrangement cannot be readily affected, as for example with the type of feedback compensation circuit in Holdaway,[1] it seems preferable to locate the high-frequency compensation components in Z_3 rather than in Z_2.

Recently available Ferroxcube pot-core assemblies[3,4] now make it a relatively simple matter to provide inductances which may be accurately trimmed for magnitude and, moreover, are well-shielded from external interaction. Alternatively, it is not difficult to construct sufficiently accurate single-layer inductance coils designed by means of the Esnault-Pelterie formula.[5] Where a pair of these are used they should be mounted to minimize mutual inductive coupling and in some layouts mumetal shielding also may be needed.

If one adopts Werner's approach[2] of considering the system as generating a negative output impedance to fully or partly cancel out the blocked voice-coil impedance, it is not easy to predict from this information *alone* how the system frequency response will come out. It becomes essential to study the system behavior directly in terms of system frequency response if the latter is sought, the actual impedance cancellation being regarded as incidental.

For the higher audio frequencies, it is the author's experience that the most satisfactory approach is to synthesize Z_3 so as to achieve an acceptable degree of control of the *closed-loop response* as measured *at the voice-coil terminals.* This approach is particularly convenient since loudspeaker data is usually available in the

Manuscript received January 10, 1963.
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[1] H. W. Holdaway, "The design of velocity-feedback transducer systems for stable low-frequency behavior," this issue, pp. 5–23.

[2] R. E. Werner, "Effect of a negative impedance source on loudspeaker performance," *J. Acoust. Soc. Am.*, vol. 29, pp. 335–340; March, 1957.

[3] "Magnetic Components," in "Mullard Technical Handbook," vol. 6, Mullard Ltd., London, England; 1963. Ferrite assemblies, data sheets for Type LA2405 Vinkor adjustable pot core. (Other similar types could, of course, be employed.)

[4] C. J. Kunz, Jr., "Pot cores offer design advantages," *Electronics*, vol. 35, pp. 80–83; April, 1962.

[5] F. Langford-Smith, Ed., "Radiotron Designer's Handbook," Amalgamated Wireless Valve Co., Pty. Ltd., Sydney, Australia, 4th ed., p. 432; 1955.

form of acoustic output for a constant voltage input to the voice-coil terminals. Unless the latter shows very pronounced departures from a fairly level average response, it is possible to control within adequate limits the *acoustic* response of the system. Where facilities exist for directly measuring the over-all acoustic response, the results of such tests may be employed for revising the design. However, even here it is usually more convenient to *operate* in terms of voice-coil voltage response.

By working in terms of frequency response, it is fairly easy to take into account the departures in behavior of the blocked voice-coil "inductance" from that of a pure inductance, without making direct measurements on the *blocked* voice-coil inductance. This is fortunate since the latter may be difficult to determine with precision or may lead to the destruction of a loudspeaker.

Basic Principle for Upper Frequency Compensation

For the middle and high audio frequencies, internal positive feedback applied over the phase-splitter stage of Fig. 2 insures the loop gain's being relatively high. A good approximation to the closed-loop voice-coil voltage response can then be derived from (35) in Holdaway[1] in the form

$$\left(\frac{V_T}{e_i}\right) = -\left(\frac{R_6}{R_5}\right)\left(\frac{R_1 + R_2}{\lambda R_2}\right)\left[\frac{Z_T}{Z_T - \dfrac{R_1 Z_3}{R_2}}\right]. \qquad (1)$$

Here e_i is the input to the feedback system, V_T is the voltage developed across the voice coil, R_1 and R_2 are the values assumed by Z_1 and Z_2, respectively, of Fig. 1 in accordance with the practical circuit of Fig. 2.

While the low-frequency compensation of this amplifier system was based upon pole-zero techniques, middle- and high-frequency compensation is most conveniently achieved by frequency-plane analysis based on (1). Depending upon the choice made for Z_3 an adequate degree of control can be exercised upon the closed-loop voltage response at the voice-coil terminals.

Conventional loudspeakers are most probably designed by trial and error, guided by past experience to give a reasonably level acoustic response. In this process, use is made of cone breakup and changes of directivity pattern in achieving a suitable *average* response. When using such loudspeakers the object would be to produce an approximately level voltage response above ~200 cps at the voice-coil terminals, ignoring the rapid fluctuations in response which appear at a finer level of frequency resolution. It has been found that by controlling the closed-loop response so that it is level at three frequencies which were approximately equally spaced on a logarithmic scale of frequencies, the response at intermediate frequencies was maintained reasonably level also. For medium quality requirements it should be possible to achieve an adequate degree of control by equalizing the response for only two selected frequencies. For this case judicious use of Werner's type of circuit could sustain the response to a higher upper frequency because of the effect of the multiplicative factor (R_1+Z_2/Z_2) which appears in (1) when R_2 is replaced by Z_2.

The design procedure given here makes use of data collected to give the magnitude and phase angle of the voice-coil impedance at selected control frequencies. It is evident that the chosen frequencies should avoid places where sharp peaks or troughs occur in the impedance magnitude.

If the closed-loop response is to be the same or nearly the same at the three selected control frequencies, it is convenient to make the closed-loop phase shift zero at these frequencies. Where a fairly large departure from a level response is required, this would be inadvisable and the phase shift should comply with Bode's relationships between gain and phase shift for minimum phase-shift networks.[6] An easy way of doing this would be to set up a simple RC impedance where the magnitude varies with frequency in the same way as the desired closed-loop response. The closed-loop phase shifts can then be matched at the selected frequencies to those of the RC network. For simplicity, this paper will be limited to the zero phase-shift case.

Design for a Specified "Average"[7] Response

General Principles

The impedance Z_3 is regarded as being made up as indicated in Fig. 3. The resistive shunts across the inductors simulate approximately the way in which hysteresis and eddy current effects modify the blocked voice-coil inductance. The magnitude of R_3 is largely determined from previous considerations.[1] The remaining four components may then be adjusted to match closed-loop response and phase shift at two frequencies to values previously determined for a lower frequency, at which both L_a and L_b very nearly behave as short circuits across R_a and R_b, respectively.

For Z_3 as given in Fig. 3

$$Z_3 = R_3\left\{1 + \frac{j\omega L_a/R_3}{1 + j\omega L_a/R_a} + \frac{j\omega L_b/R_3}{1 + j\omega L_b/R_b}\right\}. \qquad (2)$$

Eq. (2) can be written in the more convenient form

$$Z_3 = R_3\left\{1 + \frac{j\omega T_a'}{1 + j\omega T_a} + \frac{j\omega T_b'}{1 + j\omega T_b}\right\} \qquad (3)$$

where

$$T_a' = L_a/R_3, \qquad T_a = L_a/R_a$$

[6] H. W. Bode, "Network Analysis and Feedback Amplifier Design," D. Van Nostrand Company Inc., Princeton, N. J., pp. 312–314; 1945.

[7] "Average" is used here in the sense of a moving average on the frequency scale.

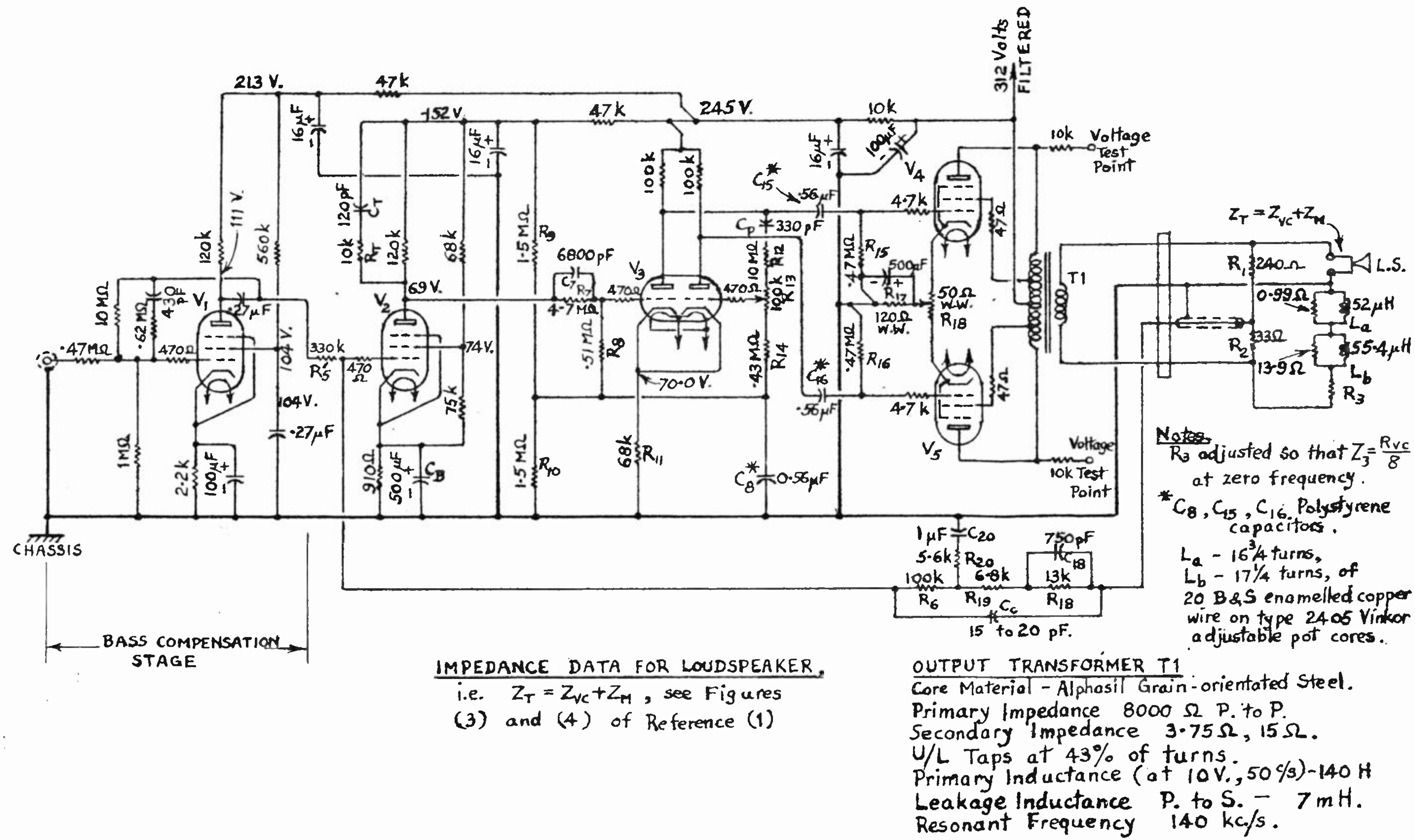

Fig. 2—Practical velocity-feedback system with amplifier circuit diagram.

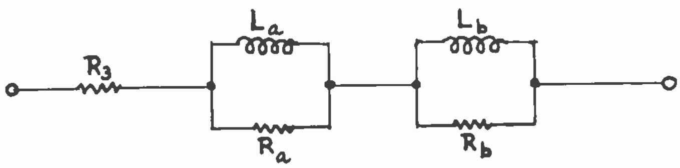

Fig. 3—Arrangement of the impedance Z_3. Note that R_3 represents the zero frequency resistance of the practical circuit.

and

$$T_b' = L_b/R_3, \qquad T_b = L_b/R_b. \tag{4}$$

Following the method of design given in the previous paper,[1] the feedback bridge circuit will be defined as being unbalanced by u per cent at zero frequency if

$$\left(\frac{u+100}{100}\right)\cdot\left(\frac{R_1}{R_2}\right) = \left(\frac{R_{vc}}{R_3}\right). \tag{5}$$

While this design is based upon maintaining a fairly level response at the higher frequencies, the net effect will be to approximately maintain a constant value of u when the right-hand side of (5) is replaced by Z_{vc}/Z_3. The resistance R_3 in (5) is the same as R_3 in (11) of the previous paper,[1] *i.e.*, it is the zero frequency resistance of the impedance Z_3, while R_{vc} is the voice-coil resistance and R_1 and R_2 are the pure resistances replacing Z_1 and Z_2, respectively, in Fig. 1.

As previously, R_3 was fixed at a value of $\frac{1}{8}R_{vc}$. Then the actual degree of bridge unbalance may be determined precisely from the ratio of the resistors R_1 and R_2. Relatively low resistance values insure the constancy of the actual impedance ratio over the audio frequency range by minimizing capacitative shunting.

A positive value of u means that some negative voltage feedback is provided by the bridge circuit in addition to the negative velocity feedback. If the low-frequency compensation has been designed suitably, it may be possible for u to take on zero or even slightly negative values before instability sets in. However, there is evidently no advantage in allowing u to become negative and, even with rigid diaphragm loudspeakers, a positive value of perhaps $\frac{1}{2}$–1 per cent may be desirable to allow for small tolerance on bridge component values and for changes in the value of R_{vc} compared with other bridge components as they warm up in operation. With conventional high-quality loudspeakers a value of about 10 per cent or more for u may be necessary to avoid accentuated peaks and troughs in the acoustic response of the closed-loop system. Once again, it is emphasized that the feedback system controls the *voice-coil motion* but it does not directly control the average diaphragm motion (and hence acoustic output).

Referring to (1), the ratios R_6/R_5 and R_1+R_2/R_2 are constant and over the frequency range here considered λ is also a constant. Thus the voice-coil voltage response is proportional in magnitude and phase to the factor

$$F = \frac{Z_T}{\left(Z_T - \dfrac{R_1 Z_3}{R_2}\right)}. \tag{6}$$

Making use of (3) and (5) we can also write

$$F = \frac{Z_T}{Z_T - \dfrac{R_1 R_3}{R_2}\left\{1 + \dfrac{j\omega T_a'}{j + j\omega T_a} + \dfrac{j\omega T_b'}{1 + j\omega T_b}\right\}}. \tag{7}$$

We consider here the simplest case where F is either constant or very nearly constant at the three control frequencies and it is sufficient to specify the three nearly equal values of F with zero phase angle at these frequencies. Under these conditions, Z_3 should have the same phase angle as Z_T at the control frequencies. Typical control frequencies would be ~300 cps, ~2500 cps and ~16,000 cps.

Since Z_3 is not yet fully determined, at 300 cps the value of F as a first approximation is

$$F_0 = \left|\frac{Z_T}{\left(Z_T - \dfrac{R_1 R_3}{R_2}\right)}\right| \tag{8}$$

which is justified if $\omega T_a'$ and $\omega T_b'$ may both be regarded as small compared with 1. Based upon experience, or else making an allowance and using as a first approximation component values of Z_3 to be given later in this paper, a value more closely approximating the correct value of $|F|$ at 300 cps can be deduced. It is then possible to calculate from the desired voltage response the values F should take at the upper two control frequencies.

It follows that at these two frequencies we can write

$$\frac{R_1 Z_{31}}{R_2} = \mu_1 Z_{T1}$$
$$\frac{R_1 Z_{32}}{R_2} = \mu_2 Z_{T2}, \tag{9}$$

where μ_1 and μ_2 are pure numbers, μ_1 refers to the control frequency around 2500 cps, and μ_2 refers to the control frequency around 16,000 cps. The appropriate values of Z_3 and Z_T are Z_{31}; Z_{T1}; and Z_{32}; Z_{T2}, respectively. Then from (6) we can write

$$F_1 = \frac{1}{1-\mu_1} \quad \text{or} \quad \mu_1 = \frac{F_1 - 1}{F_1}$$
$$F_2 = \frac{1}{1-\mu_2} \quad \text{or} \quad \mu_2 = \frac{F_2 - 1}{F_2} \tag{10}$$

where F_1 and F_2 are the values (pure numbers) of F at the upper two control frequencies (approximately 2500 cps and 16,000 cps, respectively). Since F_1 and F_2 are known values, the corresponding values of μ_1 and μ_2 can be determined.

At ~2500 cps we define

$$x_1 = \omega_1 T_a \quad \text{and} \quad y_1 = \omega_1 T_a'. \tag{11}$$

Also, at ~16,000 cps let

$$x_2 = \omega_2 T_b \quad \text{and} \quad v_2 = \omega_2 T_b'. \tag{12}$$

In addition, define

$$r \triangleq \omega_2/\omega_1. \tag{13}$$

The value of R_1R_3/R_2 is already known from considerations discussed earlier. From measurements of the voice-coil impedance Z_T, the latter's resistive and reactive components are known at the two upper control frequencies. Suppose these values are

$$Z_{T1} = \rho_1 + j\xi_1$$
$$Z_{T2} = \rho_2 + j\xi_2, \tag{14}$$

respectively.

Making the substitutions from (11) and (12) into (3) and (9) and equating the real and imaginary components of (9), we deduce the following set of equations:

$$\frac{x_1y_1}{1+x_1^2} = \frac{R_2\mu_1\rho_1}{R_1R_3} - 1 - \frac{x_2y_2}{r^2+x_2^2} \tag{15}$$

$$\frac{y_1}{1+x_1^2} = \frac{R_2\mu_1\xi_1}{R_1R_3} - \frac{ry_2}{r^2+x_2^2} \tag{16}$$

$$\frac{x_2y_2}{1+x_2^2} = \frac{R_2\mu_2\rho_2}{R_1R_3} - 1 - \frac{x_1y_1}{1/r^2+x_1^2} \tag{17}$$

$$\frac{y_2}{1+x_2^2} = \frac{R_2\mu_2\xi_2}{R_1R_3} - \frac{y_1/r}{1/r^2+x_1^2}. \tag{18}$$

Use of Successive Approximations

Eqs. (15)–(18) may be solved for x_1, y_1, x_2 and y_2 by a method of successive approximations, depending upon the fact that $r \approx 6.5$ and $r^2 \approx 42$. Taking (15) and (16), the terms in x_2 and y_2 are at first neglected. The ratio of (15) divided by (16) gives a first approximation to x_1. Then (16) can be used to calculate a first approximation to y_1.

Using the first approximation for x_1 and y_1, (17) and (18) can now be solved in a similar manner for first approximations to x_2 and y_2.

We now revert to (15) and (16) and, by inserting the first approximations for x_2 and y_2, it is possible again to solve and obtain second approximations for x_1 and y_1. With these values inserted, (17) and (18) are now solved to obtain second approximation values for x_2 and y_2.

The process may be repeated until sufficiently constant values of x_1, y_1, x_2 and y_2 are obtained. These are the required values. After about three cycles of the process it is possible to see how the process is going and to speed up the convergence by using trial approximation values which anticipate the approach to the final values.

Once the values of x_1, y_1, x_2 and y_2 have been found, (11) and (12) enable T_a, T_a', T_b and T_b' to be evaluated. Finally, since we know T_a, T_a', T_b and T_b' and R_3, (4) permits us to calculate L_a and L_b and then R_a and R_b.

Less Stringent Specifications

For less exacting performance requirements the closed-loop response in the vicinity of 300 cps may be related to the response at only one other frequency, say about 8000 cps. We simplify Z_3 by completely omitting L_b and R_b. Then it is possible to calculate x_1 and y_1 directly from (15) and (16) with $x_2 = y_2 = 0$. Eq. (11) then gives T_a and T_a' and, finally, (4) gives L_a and R_a. If, on subsequently testing the response it is found that it is not maintained sufficiently level between the two control frequencies, it may be necessary to reduce the higher control frequency and accept perhaps a more rapid falloff in the upper frequency response.

Illustrative Design Example

Fig. 2 is a reproduction of the same circuit that appears in Fig. 2 in Holdaway.[1] The values shown for the components comprising Z_3 were deduced from the following data.

Taken on loudspeaker B, the following impedance measurements were obtained (the loudspeaker was in a sealed-box enclosure):

at 343 cps $Z_T = 15.48 + j1.04$ ohms, $|Z_T| = 15.50$ ohms;
at 2460 cps $Z_T = 19.30 + j12.38$ ohms, $|Z_T| = 22.94$ ohms;
at 16,000 cps $Z_T = 40.70 + j44.80$ ohms, $|Z_T| = 60.5$ ohms.

The required value of R_3 is deduced to be 1.639 ohms, corresponding to a value for R_{vc} of 13.11 ohms. The measured values of R_1 and R_2 were 243.1 and 33.44 ohms, respectively. The value of $|F_0|$ is 4.42. It was decided to make $F_1 = 4.65$ and $F_2 = 5.21$, corresponding to 2460 cps and 16,000 cps. From (10)

$$\mu_1 = \frac{F_1 - 1}{F_1} = \frac{3.65}{4.65} = 0.7848,$$

$$\mu_2 = \frac{F_2 - 1}{F_2} = \frac{4.21}{5.21} = 0.8082.$$

From (5)

$$\left(\frac{u+100}{100}\right) = \frac{R_2}{R_1}\cdot\frac{R_{vc}}{R_3} = \frac{33.44}{243.1} \times 8.00 = 1.1005.$$

That is, $u = 10.05$ per cent of unbalance. Putting $\rho_1 = 19.30$, $\xi = 12.38$, $\rho_2 = 40.70$, $\xi_2 = 44.80$, (15)–(18) become

$$\frac{x_1y_1}{1+x_1^2} = 0.2713 - \frac{x_2y_2}{r^2+x_2^2}$$

$$\frac{y_1}{1+x_1^2} = 0.8155 - \frac{ry_2}{r^2+x_2^2}$$

$$\frac{x_2y_2}{1+x_2^2} = 1.7589 - \frac{x_1y_1}{1/r^2+x_1^2}$$

$$\frac{y_2}{1+x_2^2} = 3.0391 - \frac{y_1/r}{1/r^2+x_1^2}$$

with

$$r = 16{,}000/2460,$$
$$= 6.514,$$
$$r^2 = 42.44,$$

while $\omega_1 = 2\pi \times 2460 = 15{,}430$ radians/sec and $\omega_2 = 2\pi \times 16{,}000 = 100{,}530$ radians/sec. After several trials we obtain $x_2 = 0.402$, $y_2 = 3.401$. Then applying (15) and (16),

$$\frac{x_1y_1}{1+x_1^2} = 0.2713 - 0.0321 = 0.2392$$

and

$$\frac{y_1}{1+x_1^2} = 0.8155 - 0.5200 = 0.2955.$$

From these we deduce as next approximation $x_1 = 0.8098$, $y_1 = 0.4892$.

With these values of x_1 and y_1 we apply (17) and (18) to obtain

$$\frac{x_2y_2}{1+x_2^2} = 1.7589 - 0.5831 = 1.1758$$

and

$$\frac{y_2}{1+x_2^2} = 3.0391 - 0.1105 = 2.9286.$$

From these we deduce next approximations $x_2 = 0.4015$, $y_2 = 3.4006$.

The final values obtained were $x_1 = 0.8099$, $y_1 = 0.4892$, and $x_2 = 0.4015$, $y_2 = 3.4006$.

To give a clearer picture of the progress of successive approximations four-figure accuracy has been employed. For most purposes slide-rule accuracy would suffice and would require fewer approximation cycles in consequence.

It follows that

$$T_a = \frac{10^6 \times 0.8099}{15{,}430} = 52.48\ \mu\text{sec}$$

$$T_a' = \frac{10^6 \times 0.4892}{15{,}430} = 31.70\ \mu\text{sec}.$$

Similarly,

$$T_b = \frac{10^6 \times 0.4015}{100{,}530} = 3.994\ \mu\text{sec}$$

and

$$T_b' = \frac{10^6 \times 3.4006}{100{,}530} = 33.83\ \mu\text{sec}.$$

Finally,

$$L_a = T_a'R_3 = 52.0\ \mu\text{h},$$

$$R_a = \frac{L_a}{T_a} = 0.99\ \text{ohm},$$

$$L_b = T_b'R_3 = 55.4\ \mu\text{h},$$

$$R_b = \frac{L_b}{T_b} = 13.9\ \text{ohms}.$$

These are the component values of Z_3 as shown on Fig. 2. The 52-μh and 55.4-μh inductors were made up of $16\frac{3}{4}$ and $17\frac{1}{4}$ turns, respectively, of 20 B. and S. enamelled copper wire on Mullard Type LA2405 Ferroxcube pot cores. Precise adjustment was made by comparing them with a 50-μh standard inductor using a Q meter. The 0.99-ohm and 13.88-ohm shunt resistors were made of Eureka wire folded back at the center and wound from the center outwards onto formers made from conventional higher value resistor bodies so as to provide approximately noninductive resistors. Eureka, Constantan and Advance wire have the advantage of being fairly readily soldered to the pig-tail ends of the conventional resistor. After approximate adjustment of the resistance value, the wire was bound onto the resistor body with nylon thread, a coating of epoxy resin was applied and the unit was put into an oven to cure at about 105°F. Subsequently, the point of soldering was adjusted to trim the resistance to its final value.

Since the inductors have a finite rather than a zero resistance, the practical embodiment of the impedance Z_3 was a little different from the theoretical one. The series resistor R_3 was trimmed so that in series with the other assembled components the dc resistance was that of R_3 in the theoretical case. This means that the resistive component of Z_3 will have its correct value at very low frequencies but will be a little low at the highest frequencies. As the value of R_3 was about 0.09 ohm less than in the theoretical configuration, the over-all reduction at the highest frequencies was but a negligible fraction of the total impedance.

Additional High-Frequency Compensation Circuit Elements

Transitional Phase-Shift Network in the Plate Circuit of V_2

A transitional phase-shift network is formed by R_T

and C_T in conjunction with the effective anode load resistance of the pentode V_2 and the shunt capacity resulting from the combined effects of strays and the effective input capacity of V_3 (including the Miller effect). The circuit component values were designed to so control the attenuation slope contributed by this interstage coupling that the gain would have leveled out at about a 20-db lower value, with relatively small phase shift, in the vicinity of 140 kc. This is the frequency at which the large phase shifts associated with a natural resonance of the output transformer could give rise to instability.

Compensating Capacitor C_c

See Fig. 2 and also the diagram of the λ network in Appendix III of Holdaway.[1] The capacitor C_c is designed to operate in conjunction with the effective input capacitance of V_2 in order to maintain λ constant at the same value that it is at 300–500 cps up to the limits of the frequency range in which high-frequency instability could occur. This may be checked by opening the feedback loop at the input to the λ network and connecting an oscillator to the input. The value of C_c may be adjusted so that, using an oscilloscope with compensated probe, there is no phase shift between the input to the λ network and the input at the grid of V_2. The phase shift at V_2 input can, if desired, be made to lead slightly at very high frequencies by choosing a slightly larger value of C_c. The range of values 15 $\mu\mu$f–20 $\mu\mu$f was found to give completely stable operation with the circuit configuration and components used by the author.

Capacitor C_{18}

This capacitor is not essential for purposes of stabilizing the high-frequency response. It has the effect of introducing a 6-db rise in *loop gain* with asymptotic break-frequencies at 12,000 cps and 24,000 cps, approximately. The object of this is mainly to smooth out the bump which appears in the closed-loop response, thus shifting the response to the position of the dotted line on the response curve for loudspeaker A in Fig. 4. The small additional phase lead introduced will have only a minor effect in increasing stability in the critical region and may be offset by the 6-db increase in loop gain. However, no difficulty has been experienced with stability when C_{18} is incorporated. The smoother response curve should insure better transient response, especially as some combinations of pickup cartridge and record surface may tend to produce a resonant peak located at about the same frequency range.

Comparison of Predicted and Observed Behavior

The design method employed is based upon a simplifying approximation which, in turn, depends upon maintaining a sufficiently high value of loop gain over the frequency range of application. It is, therefore, important to make a verification of the method used to establish control of the closed-loop voice-coil voltage response. For this reason, critical components were measured to somewhat higher accuracy than normally would be required (normally a precision of no better than 1/10–1/20 of the adopted percentage of bridge unbalance would suffice). Since the response is directly related to the factor F in (7), under suitable conditions, this equation may be used to predict the closed-loop voice-coil voltage response at frequencies where a measured value, resistive and reactive, of the voice-coil impedance Z_T is available.

The results of such a comparison are shown in Fig. 4, which comprises calculated responses and measured responses for the two alternative loudspeakers employed. From about 100 cps up to 50,000 cps the two sets of results are indistinguishable. Though not shown on Fig. 4, the measured response above 50,000 cps continued to fall off steadily with no suggestion of peaks or instability up to 220,000 cps. No signs of higher frequency oscillations were evident when the output was tested with an oscilloscope.

At the extreme low-frequency end the measured response tended to sag progressively with reduction in frequency compared with the prediction of (7). This would be expected since the λ network no longer has a constant attenuation at low frequencies, and also the effect of phase shift and finite loop gain would start to become apparent. In fact, the falloff can be accounted for largely by the term $p/(p+0.3516)$, which in the low-frequency study[1] leads to a 20 db per decade asymptotic falloff starting from 24 cps. (See also Fig. 6 in Holdaway.[1])

The voice-coil response shows distinct troughs at various resonant frequencies, the most important being at the fundamental resonant frequency of the speaker and enclosure. Some of the other troughs are actually undesirable and can be reduced preferentially by incorporating a higher degree of bridge unbalance, though this will be achieved by sacrificing some of the potential reduction in loudspeaker distortion due to the feedback.

The dashed curves in Fig. 4 are interesting in that they illustrate how velocity feedback correctly applied permits an extension of low-frequency response, subject, of course, to the use of a loudspeaker with a large possible volume displacement. When the frequency compensation for a constant velocity-driven loudspeaker has been included, the resulting voice-coil response shows a pronounced rise below the fundamental resonant frequency. This, of course, is in just the right place to offset the usual falloff in acoustic output which would occur in this region using a conventional feedback amplifier to drive the loudspeaker. It is also quite clear that poor results would be obtained without the compensation stage. Evidently, too, the turntable and pickup employed should introduce only low levels of hum and rumble if the extended bass response is to be obtained in this way.

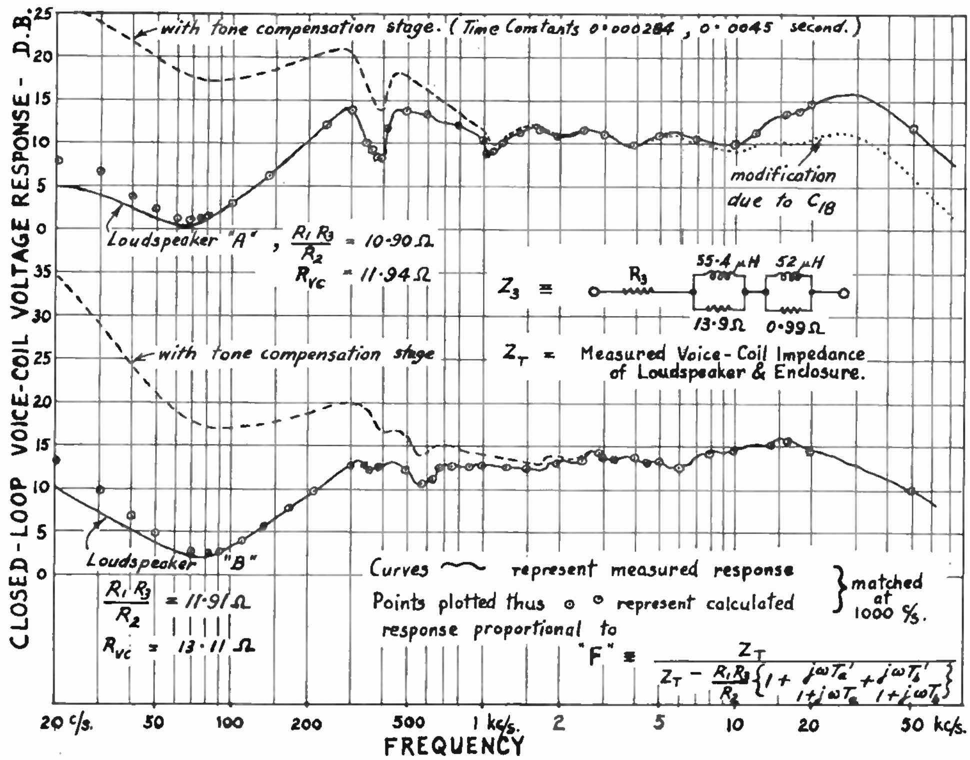

Fig. 4—Closed-loop voice-coil voltage response system with loudspeakers A and B, measured cubic feet and calculated response. (See also Fig. 6 in Holdaway.[1])

Use of "Wide-Range" Loudspeakers

The loudspeakers employed in this study were both of the simple wide-range double-cone type, employing a mechanical crossover. In the course of investigations, it was found that both showed a somewhat unpleasant "coloration" either when installed in the feedback system or when driven directly from an RC oscillator, or a conventional feedback amplifier. Eventually, it was established that this effect was due to "bell-like" resonances occurring particularly at the rim of the tweeter cone. The effect was less marked in loudspeaker A, in which small pieces of sponge material were attached to this rim, evidently intended to partially damp out this effect. Similar effects have been mentioned by various observers and have been detected in experiences of the B.B.C. These have been discussed by Barlow.[8]

If a two-way or three-way loudspeaker system with crossover networks had a reasonably smooth impedance vs frequency characteristic, there seems to be no reason why the methods of these papers could not be applied to such a system, treated as a unit. But, for best damping of the fundamental resonance, it would be desirable for the crossover frequency to be as far removed as practical from the fundamental resonant frequency. Such a situation would certainly be possible if the low-frequency unit were a reasonably efficient version of the *rigid* diaphragm type of loudspeaker.

[8] D. A. Barlow, "Rigidity of loudspeaker diaphragms," *Wireless World*, vol. 64, pp. 564–569; December, 1958.

Discussion

A method has been described for controlling the over-all voice-coil response of velocity-feedback systems for the middle and upper frequencies. The procedure succeeds in controlling the over-all response within reasonable limits by precisely controlling the response at about 2500 cps and 16,000 cps, as compared with the response at about 300 cps. For various reasons, such as minor resonances and cone breakup in the loudspeaker, and the difficulty with only a four-component lumped circuit system of matching the rather impure type of impedance presented by the voice-coil "inductance," some further small irregularities exist in the response curve. These are normally not of any consequence but may be reduced throughout by using a higher percentage of bridge unbalance or, for example, minor modifications, such as the inclusion of C_{18}.

An alternative approach, which has not been developed by the author, might be to provide an impure inductance by using a suitably scaled voice-coil and magnet assembly, possibly from a small (and cheap) loudspeaker. The voice-coil action would need to be blocked by setting it in epoxy resin or something similar.

The results in Fig. 4 illustrate that it is quite practicable not only to equalize the high-frequency response by direct calculations (within reasonable limits at any rate) but that the factor F in (7) gives a direct measure of the response over most of the audio-frequency range. It is thus possible, given impedance data for the loudspeaker, to use (7) for calculating at the design stage an anticipated voice-coil response, since the loop gain of the basic amplifier system presented here is sufficiently high to make this possible.

An alternative to performing the calculations and making *detailed* impedance measurements is to set up the circuit of Fig. 5 in which Z_1, Z_2 and Z_3 are the designed bridge components to be used in conjunction with the loudspeaker enclosure represented by Z_T. The response of the corresponding feedback amplifier is then found to be the same as the ratio of the voltages $v_T \div v_0'$ of Fig. 5. This method works quite satisfactorily in practice and avoids the measurement of phase angles, avoids calculations, and of course avoids delays due to possible instability problems. Also, it can be set up before a prototype amplifier is available. Although shown here with XY connections to the dc oscilloscope, this is not essential. Switching leads from the bridge circuit, or using a double beam dc oscilloscope, is a satisfactory alternative. An ac-coupled oscilloscope could be used if the reversing switch shown were replaced by a double-pole double-throw chopper switch.

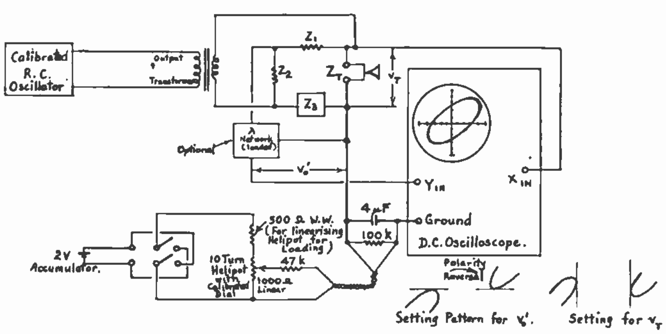

Fig. 5—Slide-back voltage comparison system (with polarity reversal) for predicting closed-loop voice-coil voltage response.

ACKNOWLEDGMENT

Measurements on this system were made with facilities in the laboratories of the Commonwealth Scientific and Industrial Research Organization of Australia. The author is particularly indebted to discussions with Dr. E. F. Denby of the same organization. The first suggestion of making use of the voice-coil response measurements was Dr. Denby's.

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From 1941 to 1945 he was on active duty with the Royal Australian Air Force, serving as a Radar Officer. He was a Lecturer in Physics and Civil Engineering at the New South Wales University of Technology, Sydney, from 1946 to 1948, when he joined the Snowy Mountain Hydro-Electric Authority, Sydney, Australia as an Executive Engineer. In 1952 he returned to the University as a Senior Lecturer in Civil Engineering for a period of two years. After a year in Europe on a Textile Fellowship he joined the Commonwealth Scientific and Industrial Research Organization in February, 1956, as a Senior Research Officer in the Division of Textile Physics, Ryde, Sydney, Australia. Since joining the Organization he has supervised the electronics section at the Division of Textile Physics.

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Correspondence

On the Theories of AC Bias Used in Magnetic Tape Recording

It was quite a surprise to find the following sentence in P. R. Hinrichs' paper above.[1] "Thus, Zenner's theory has been discounted since he assumed that the bias signal was recorded."[2]

I disinterred my lone remaining copy of this paper to see how such an impression might have been given, and I did find an unfortunate sentence: "The recording contains the audio frequency, the bias frequency . . . ," etc. It would have been much better if I had said, "The recording *flux* contains . . . ," etc. However, the very same paragraph does include this more helpful sentence: "Self-demagnetization in the recording medium and limited playback resolution provide a low-pass filter which attenuates undesired (higher than audio) frequencies."

The intent was to get rid of the bias eventually, either by not retaining it in the record or by not playing it back. Usefulness of the bias ends when the audio has been properly recorded.

I do not wish to make great claims for this paper. It was written long ago, but later than and with knowledge of Camras' early work,[3,4] and it was never intended to dispute his graphical analysis. It did shed some further light on such matters as second-and third-harmonic behavior, frequencies of beats, etc.

It was not based upon permanent recording of the bias; neither did it rule out the case wherein the bias is recorded.

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Author's Comment[5]

From an examination of Zenner's paper, it is clear that he is working with the "recorded flux" and not the "recording flux." He states, "The similarity of single exposure and SCMC B_r-H curves permits us to assume that each element of length of the recording medium is subjected to a single instantaneous value of both audio and bias." Use of this assumption led to the result that the ac bias signal appeared as a component of the recorded flux with the same coefficient as the signal; *i.e.*, $B_r = K(X+Y) + \cdots$ where X and Y are defined by Zenner as the audio and bias signals, respectively. If the bias frequency can be recorded, then the modulation scheme illustrated in Fig. 4 of Hinrichs' paper would work. On playback, the low-pass filtering effect would cause the playback equipment to respond only to the net flux or average value. The fact that this modulation scheme does not work implies that the original hypothesis, that the bias signal is recorded, was incorrect. No one was more disappointed than the author with the fact that this method of modulation would not work since it seemed to provide a potential method for removing most of the nonlinearities associated with magnetic tape recording equipment.

The results of four other independent experiments indicated that the bias signal was not recorded:

1) No maximum bias frequency was found that could not be exceeded for a particular input level. If the bias signal had been recorded, a discontinuity would have been found.
2) There was no change in the playback signal even when the bias frequency was increased until there were two complete bias cycles per domain.
3) The bias signal did not appear on the tape after it had been developed by the use of iron oxide although the "expected" results were well within the resolution of the measuring equipment.
4) In Fig. 5 of Hinrichs' paper, it is shown that there is no increase in tape noise when the recording flux consists only of ac bias. The nonhomogeneities of the tape should have produced noise if the bias signal had been recorded.

Thus, Zenner's theory is predicated on an incorrect assumption, *viz.*, that the recorded flux contains the bias frequency. His theoretical results must therefore be discounted, and any correlation between these results and measured phenomena must be regarded as coincidental. For example, it is not difficult to account for the beat frequencies present. The recording flux is a nonlinear function of its input current because of the nonlinear properties of the recording head. Consequently, it is not surprising that Zenner found the frequencies described in his paper. If this recording head nonlinearity is assumed to be of the general form of that used by Zenner, the seven "phenomena consistent with analysis," which Zenner points out to substantiate his theory, are explained.

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Manuscript received September 10, 1963.

[1] P. R. Hinrichs, IEEE Trans. on Audio, vol. AU-11, pp. 78–81; May–June, 1963.

[2] R. E. Zenner, "Magnetic recording with ac bias," Proc. IRE, vol. 39, pp. 141–146; February, 1951.

[3] M. Camras, U. S. Patent No. 2,351,004; May 30, 1944.

[4] M. Camras, "Graphical analysis of linear magnetic recording using high-frequency excitation," Proc. IRE, vol. 37, pp. 569–573; May, 1949.

[5] Received October 3, 1963.

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