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AN ANALYSIS OF COUPLED TUNED CIRCUITS AT RADIO FREQUENCIES

BY

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An analysis of coupled tuned circuits begins properly with a detailed study of the tuned circuit uncoupled followed by consideration of the changes brought about as the second tuned circuit is coupled with it first weakly and then with gradually increasing strength. We are especially interested in the steady state characteristics involving current amplitude, phase and frequency as they are affected by the different degrees of coupling. We shall see that there are no abrupt changes in these characteristics during the process and by emphasizing this viewpoint we obtain the more satisfying mental picture of the characteristics exhibited by coupled tuned circuits which relates them closely to those shown by uncoupled tuned circuits instead of keeping them in separate watertight mental compartments.

Our plan will be to simplify the problem by making certain assumptions and approximations. We shall, for example, take the usual expressions for the current in terms of the impressed e.m.f., resistance, inductance and capacitance and convert them into expressions which do not involve the particular values of resistance, inductance and capacitance. We shall also assume in the case of coupled circuits that the power factor of the primary circuit is equal to the power factor of the secondary circuit. From these derived expressions we shall plot a series of curves, inspection of which will show clearly how the characteristics of coupled tuned circuits may be evolved from those of the uncoupled circuits. Finally, we shall give a graphical method of obtaining the same characteristics.

THE UNCOUPLED TUNED CIRCUIT:

The magnitude of the alternating current which flows in a series tuned circuit when an alternating e.m.f. is impressed is given by the expression,

$$I = \frac{E}{R + j\left(\omega L - \frac{1}{\omega C}\right)} \quad \text{--------- (1)}$$

where,
E = the impressed alternating e.m.f.
R = the resistance
L = the inductance
C = the capacitance
and ω = 2π times the frequency of the impressed e.m.f.

The ωL in the reactance term in the denominator of equation (1) may be factored out and the reactance term rewritten as follows,

$$\left(\omega L - \frac{1}{\omega C}\right) = \omega L \left(1 - \frac{1}{\omega^2 LC}\right)$$

Substituting for 1/LC its value in the expression for resonance, namely, $\omega_0^2 = 1/LC$, the reactance term may again be rewritten,

$$\left(\omega L - \frac{1}{\omega C}\right) = \omega L\left(1 - \frac{\omega_0^2}{\omega^2}\right) = \omega L\left(1 - \frac{f_0^2}{f^2}\right) \quad \text{----- (2)}$$

In the last expression, f_0 is the frequency of resonance and f is any frequency which the impressed e.m.f. may have. Substituting (2) into (1) and dividing both numerator and denominator by R, equation (1) may be transformed to,

$$I = \frac{E}{R} \cdot \frac{1}{1 + j\frac{\omega L}{R}\left(1 - \frac{f_0^2}{f^2}\right)} \quad \text{-------- (3)}$$

The frequency f may be expressed in terms of the resonant frequency f_0 by,

$$f = f_0 + \Delta f_0 \quad \text{---------- (4)}$$

where Δf_0 is the difference between f and f_0. Thus, if the resonant frequency is 1000 KC and the frequency in which we are interested is 1001 KC we may express it as f_0 + 1 KC. Substituting equation (4) in the expression $\left(1 - f_0^2/f^2\right)$ there results,

$$\left(1 - \frac{f_0^2}{f_0^2 + 2\Delta f_0 \cdot f_0 + \Delta f_0^2}\right) = \left(1 - \frac{1}{1 + \frac{2\Delta f_0}{f_0} \; \frac{\Delta f_0^2}{f_0^2}}\right) \quad (5)$$

When the reactance of the tuning coil $\omega_0 L$ at the resonant frequency is very large in comparison with the effective resistance R we find that we need consider only a relatively small range of frequencies to obtain a selectivity curve which is sufficiently complete for

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practical purposes. This is especially true at radio frequencies where the ratio of reactance to resistance, or the Q of the coil as it is sometimes called, of a tuning coil may reasonably be 200, for example. Assuming the value of 200 it may be shown that the amplitude of the current drops to 1/5 of the amplitude at resonance when the frequency of the impressed e.m.f. deviates from the resonant frequency by slightly over one percent. Consequently, the major portion of the selectivity curve does not require consideration of frequencies which deviate more than a few percent from the resonant frequency. Confining our attention to the frequency range comprised in deviations of only a few percent from the resonant frequency is the same as saying that $\Delta f_o/f_o$ will be always very small as compared with unity. Therefore, in the right hand expression of equation (5) we may neglect $\Delta f_o^2/f_o^2$ which is much smaller still. Then if we carry out the operation of division indicated in the expression $\frac{1}{1+\frac{2\Delta f_o}{f_o}}$ there results,

$$1-\frac{2\Delta f_o}{f_o}+\left(\frac{2\Delta f_o}{f_o}\right)^2-\text{etc} \text{-------} (6)$$

where $\left(\frac{2\Delta f_o}{f_o}\right)^2$ and all the remaining terms are, of course, also negligible. The right hand expression of equation (5) becomes by the substitution of (6),

$$\left(1-\frac{f_o^2}{f^2}\right)=\left(1-\frac{1}{1+\frac{2\Delta f_o}{f_o}}\right)=1-\left(1-\frac{2\Delta f_o}{f_o}\right)=\frac{2\Delta f_o}{f_o} \quad (7)$$

Making use of these approximations, equation (3) may be written,

$$I=\frac{E}{R}\cdot\frac{1}{1+j\frac{\omega L}{R}\left(\frac{2\Delta f_o}{f_o}\right)} \text{-------} (8)$$

Finally, if we place $\frac{\omega L}{R}\left(\frac{2\Delta f_o}{f_o}\right)$ equal to v, we obtain,

$$I=\frac{E}{R}\cdot\frac{1}{1+jv} \text{--------} (9) \; *$$

which is equation (1) greatly simplified by the foregoing approximations.

Equation (9) expresses the current as two factors of which E/R is the amplitude of the current at the resonant frequency, and $\frac{1}{1+jv}$ is a function of v, v being defined in the preceding paragraph. Since E/R has a fixed value in any particular case, the whole story of resonance is contained in the second factor $\frac{1}{1+jv}$ which does not explicitly involve the particular values of R, L and C which compose the tuned circuit. Therefore, assigning values to v and plotting various curves for the second factor provides a set of curves which is applicable to any series tuned circuit. To interpret the curves in the light of a particular tuned circuit involves merely the translation of the v scale into a frequency scale, as will be explained in a later paragraph.

It will be instructive to rationalize the denominator of $\frac{1}{1+jv}$ by multiplying both numerator and denominator by 1 - jv, which gives,

$$\frac{1}{1+jv}=\frac{1}{1+v^2}-j\frac{v}{1+v^2} \text{--------} (10)$$

Of this result, $\frac{1}{1+v^2}$ is the term in phase with, and $-j\frac{v}{1+v^2}$ the term in quadrature with, the impressed e.m.f. The two terms are shown plotted in figs. 1a and 1b, respectively. The real term, in phase with the impressed e.m.f., is symmetrical about v = 0, i.e., has equal values for equal positive and negative values assigned to v, and is always positive in sign. The imaginary term, in quadrature with the impressed e.m.f., has values of opposite sign but equal magnitude for equal positive and negative values assigned to v. On comparing the curves, it will be seen that for v = -1 both terms have the same magnitude and sign and for v = +1 they have the same magnitude but opposite signs, the magnitude in each case being one half the value of the real term for v = 0. The phase angle of the resultant current with respect to the impressed e.m.f. is, of course, the angle whose tangent is the imaginary term divided by the real term. For negative values of v the tangent is positive, indicating a leading current, or capacitive effect, while for positive values of v the tangent is negative, indicating a lagging current, or inductive effect. This accords, as it should, with the customary reactance diagram for a series tuned circuit, the net reactance being capacitive below resonance and inductive above.

We may plot equation (10) in still a different way. We may consider the real and imaginary terms to be the co-ordinates of a point corresponding to a value of v in which the real term is the distance along the x-axis and the imaginary term along the y-axis in locating the point. Such a plot is shown in fig. 1c and will be recognized as the familiar circle diagram. We needn't have gone to the trouble of plotting a series of points, as just stated, except, perhaps, to label the points so found with the corresponding value of v. We may determine the construction of the circle by considering equation (10) written in a slightly different form,

$$\frac{1}{1+jv}=\frac{\sqrt{1+v^2}}{1+v^2}\angle\tan^{-1}-v=\frac{1}{\sqrt{1+v^2}}\angle\tan^{-1}-v \quad (11)$$

It is easy to see that the cosine of the angle whose tangent is minus v is equal to $\frac{1}{\sqrt{1+v^2}}$, so that,

$$\frac{1}{1+jv}=\cos\left(\tan^{-1}v\right)\angle\tan^{-1}-v \text{------} (12)$$

This is the equation of a circle with a radius of 1/2 and center on the x-axis at a distance of 1/2 to the right of the origin, the angle designation indicating that positively increasing values of v succeed one another in a clockwise direction.

Finally, we have plotted in fig. 1d the absolute magnitudes of equation (10), i.e., $\frac{1}{\sqrt{1+v^2}}$, against v. This is the current response curve of a series resonant circuit, and corresponds to the characteristic which would be obtained by measurement of the current flowing in the circuit as the e.m.f. of constant amplitude is varied in frequency according to v.

We turn now to the practical question of how to interpret the variable v, which has been defined as equal to $\frac{\omega L}{R}\left(\frac{2\Delta f_o}{f_o}\right)$. This definition

* An alternative derivation leading to the same result as equation (9) may be found in Dr. A. E. Guillemin, 'Communication Networks', Vol. I, page 117 et seq.

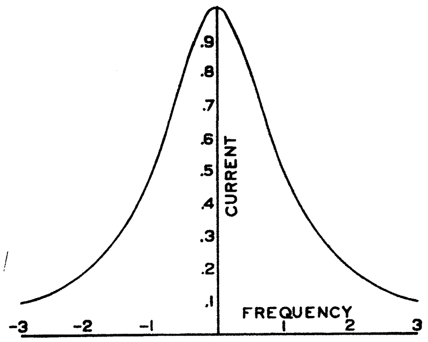


FIG. 1a

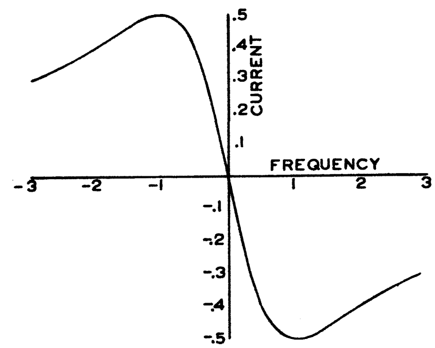


FIG. 1b

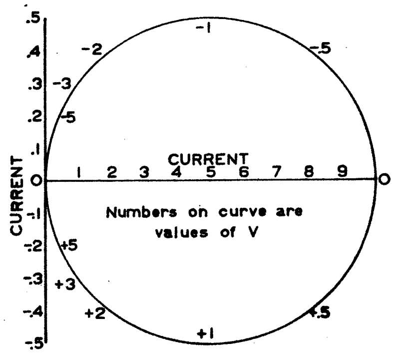


FIG. 1c

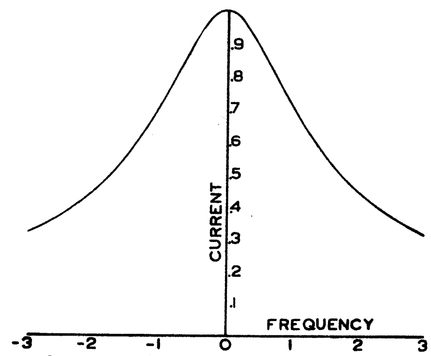


FIG. 1d

was made to simplify equation (8), throwing it into the form of equation (9), and, therefore, appears arbitrary and without any practical significance; let us see if this is so. We have already hinted that the v scale in figs. 1a, 1b, 1c and 1d may be translated into a frequency scale, and we proceed to show how this may be done. The factor $\frac{\omega L}{R}$, or the Q of the tuning coil, is known to be nearly constant over its useful frequency range as a tuning inductance, and is certainly constant enough over the small range of frequency involved in resonance to be considered constant in the definition of v. It goes without saying that the resonant frequency f_o is constant in any particular case, as it is the frequency at which the inductance and capacitance of the circuit resonate. Therefore, the only variable in the definition of v is Δf_o, so that we may say v is proportional to the number of cycles per second off resonance. In view of this, we can convert the v scale into a frequency scale which shows the frequency departure from resonance merely by substituting the value of $\frac{\omega L}{R}$ and f_o for the individual case.

For example, let $\frac{\omega L}{R} = 200$ and $f_o = 1000$ KC.

Substituting these values, we find that $\Delta f_o = 2.5v$ KC, so that for v = 1 on the v scale we put 2.5 KC, for v = 2, 5.0 KC, etc., the minus values of v giving the frequency departure below resonance.

Before leaving the subject, we call attention to the relative unimportance of the larger values of v in describing resonance. What corresponds to our v = -1 and v = +1 have been called quadrantal values by Prof. A. E. Kennelly, the frequency values between which the phenomenon of resonance may be said to occur. These are the frequencies above and below resonance at which the reactance is numerically equal to the resistance, the phase angle of the current being minus or plus 45°, respectively. Referring to the circle diagram of fig. 1c, it will be seen that in going from v = -1 to v = +1 one half the circumference is traversed. In covering the range of v from -5 to +5 nearly 7/8 of the circumference is passed over, so that very little room is left for larger values of v. Reference to the other figures will show that the current amplitudes at these values of v are very small compared with the magnitude of the current at resonance, or v = 0. This is our justification for making the approximations employed in deriving equation (8) from (3), along with the assumption that $\frac{\omega L}{R}$ is reasonably large.

We could have avoided the approximations noted by letting v equal $\frac{\omega L}{R}\left(1 - \frac{f_o^2}{f^2}\right)$ in equation (3), but that would have necessitated a more complicated procedure in converting the v scale to a frequency scale in a particular case. We prefer not to do that because it merely obscures the problem at hand, which is to obtain a simple mental picture of the manner in which the characteristics of the simple tuned circuit are modified as a second tuned circuit is coupled to it.

THE COUPLED TUNED CIRCUIT PRIMARY:

We now propose to investigate the characteristics of a series tuned circuit when a second tuned circuit is coupled to it, that is, we shall direct our attention to what happens to the primary current as a secondary tuned circuit is coupled to it at various strengths of coupling. We shall assume, for definiteness, that the coupling is magnetic, and, to avoid too much complication, that the primary and secondary circuit constants are exactly alike. The latter assumption applies, of course, to cases where the primary and secondary constants are not alike providing the power factor and resonant frequency of the primary and secondary are equal, because referring the primary to the impedance level of the secondary, or vice versa, reduces the problem to the case where the constants are exactly alike.

* In all figures the Frequency Scale is in terms of v and the Current Scale must be multiplied by E/R.

Under these conditions the equation for the primary current is,

$$I_1 = \frac{E}{R + j\left(\omega L - \frac{1}{\omega C}\right) + \frac{\omega^2 M^2}{R + j\left(\omega L - \frac{1}{\omega C}\right)}} \quad ------(13)$$

in which E, R, L, C and ω have the same significance as in equation (1), R, L and C being the same for both primary and secondary, and M is the mutual inductance between the primary and secondary tuning coils.

Equation (13) may be simplified by applying the same approximations that were used to reduce equation (1) to equation (9), which is equivalent to substituting $R(1+jv)$ for $R+j\left(\omega L-\frac{1}{\omega C}\right)$

Making this substitution in (13), we have,

$$I_1 = \frac{E}{R} \cdot \frac{1}{(1+jv) + \frac{\omega^2 M^2}{R^2} \cdot \frac{1}{(1+jv)}} \quad -----(14)$$

We may assume that, for a given value of M, $\omega M/R$ is very nearly constant over the frequency range of interest. Let $\omega M/R = m$. Then (14) becomes,

$$I_1 = \frac{E}{R} \quad \frac{1}{(1+jv) + \frac{m^2}{(1+jv)}} \quad ------(15)$$

Rationalizing denominators and collecting terms, we have,

$$I_1 = \frac{E}{R} \cdot \frac{1}{\left(1+\frac{m^2}{1+v^2}\right) + jv\left(1-\frac{m^2}{1+v^2}\right)}$$

$$= \frac{E}{R} \cdot \frac{\left(1+\frac{m^2}{1+v^2}\right) - jv\left(1-\frac{m^2}{1+v^2}\right)}{\left(1+\frac{m^2}{1+v^2}\right)^2 + v^2\left(1-\frac{m^2}{1+v^2}\right)^2} \quad ----(16)$$

Equation (16) is in the form of two factors, the first of which is the amplitude of the current at resonance of the uncoupled tuned circuit, the second being a function not only of v, as in the case of the single tuned circuit, but also of m which represents the degree of coupling. When m = 0, equation (16) reduces to (9) as it should, because the primary then becomes an uncoupled tuned circuit, and the tuning coil of the secondary is fixed in a position where none of the magnetic flux from the primary coil links with the turns of the secondary coil. When m is increased from zero, which is equivalent to altering the position of the secondary coil so that it links with more and more of the flux from the primary coil, marked, but gradual, changes occur in the phase, amplitude and frequency characteristics of the primary current. In order to show these changes, we have elected to halt at several arbitrary values of m and plot characteristics corresponding to those already given for the single tuned circuit. Figs. 2a, 2b, 2c and 2d show such curves, each figure containing a set of curves for m = 0, 1/2, 1 and 2. That is, m is substituted in equation (16) and a series of values given to v for each value of m. Fig. 2a shows the real term, in phase with the impressed e.m.f., fig. 2b, the imaginary term, in quadrature with the impressed e.m.f., and fig. 2d, the absolute magnitude, all versus v, which has exactly the same significance as explained in connection with the single tuned circuit. Fig. 2c is plotted from equation (16) in the same manner as fig. 1c, so that a straight line connecting the origin with any point on the curves, corresponding to some value of v, shows by its length the amplitude of the current, and, by the angle it makes with the x-axis, the phase with respect to the impressed e.m.f.

The values of m selected, incidentally, also provide a basis for comparing the curves as to critical coupling (m = 1), less than critical (m = 1/2) and greater than critical (m = 2). The significance of critical coupling is treated in detail in most text books dealing with coupled tuned circuits, and we need only mention at this point that it represents a borderline case in the shape of the frequency characteristic of the current in the secondary circuit and will, therefore, be discussed more in detail in a later paragraph.

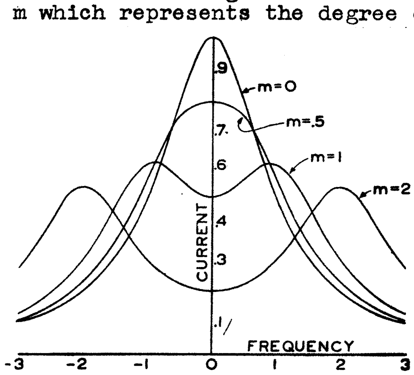


FIG. 2a

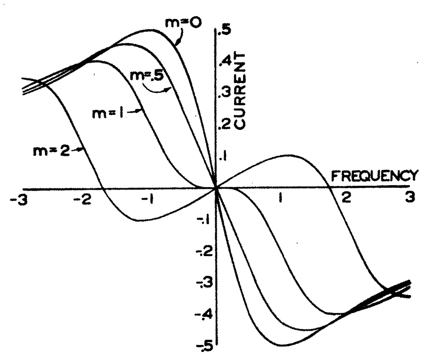
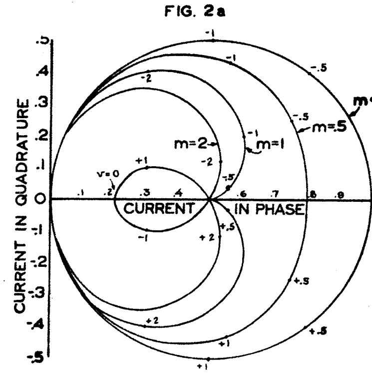


FIG. 2b



Numbers on curves are values of v

FIG. 2c

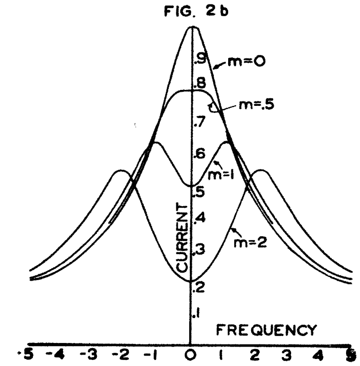


FIG. 2d

* In all figures the Frequency Scale is in terms of v and the Current Scale must be multiplied by E/R.

Referring to fig. 2a, it will be seen that the most rapid changes accompanying an increase in m occur at and close to $v = 0$, the frequency of resonance of the uncoupled tuned circuit. As m goes from zero to 1/2, the peak of the $m = 0$ curve is flattened out while scarcely any changes occur in the legs of the curve. In going to $m = 1$, the rapid decrease in the amplitude at $v = 0$, with the legs of the curve changing but little, causes two peaks to form, one above $v = 0$ and one an equal amount below. Further increase in the coupling, to $m = 2$, causes a deeper depression at $v = 0$ and two peaks with wider separation than in the case of $m = 1$.

We can throw more light on these peaks by resorting to an explanation which is somewhat academic, but, nevertheless, affords a simple way of comprehending them. Let us suppose that we increase m to indefinitely large values and see what happens. A word of caution should be inserted here to the effect that there is an upper limit to the value of m in any practical case because the coefficient of coupling cannot be greater than 100%. However, we are dealing with radio frequency tuned circuits where the coefficient of coupling is not more than a few percent, so there is ample room for increasing m to values large enough to approximate the conditions assumed in this explanation. Now let us go back to the first expression for I_1 given in equation (16). If we place the reactance term equal to zero we find that $v^2 = m^2 - 1$ and, since we are assuming m to be large, $v^2 = m^2$ or $v = \pm m$. In other words, the primary current resonates at $v = +m$ and $-m$ when m is large. With this hint, let us replace v by w where $v = m + w$, that is, so that $w = 0$ when $v = m$ and w varies about m in the same manner that v varies about zero. Now let us make this change of variable in the expression $\frac{m^2}{1+v^2}$. In the first place, assuming w to have only small values as we did in the case of v,

$$v^2 = (m+w)^2 = m^2 + 2mw \quad \text{(approximately)} \quad (17)$$

So that, $$\frac{m^2}{1+v^2} = 1 - \frac{2w}{m} \quad \text{(approximately)} \quad (18)$$

Therefore, $$1 + \frac{m^2}{1+v^2} = 2 \quad \text{(approximately)} \quad (19)$$

and,

$$v\left(1 - \frac{m^2}{1+v^2}\right) = (m+w)\left(1 - 1 + \frac{2w}{m}\right) = 2w \text{ (approx.)} \quad (20)$$

Substituting (19) and (20) into the second expression of equation (16),

$$I_1 = \frac{E}{R} \cdot \frac{2 - j2w}{4 + 4w^2} = \frac{E}{2R} \cdot \frac{1 - jw}{1 + w^2} \text{ ------ } (21)$$

We note that the second factor in (21) is exactly the same as (10) except that it is in terms of w instead of v. Upon comparing it with (9), the second factor of which is rationalized in (10), we observe that the current at resonance factor is E/R in (9) and E/2R in (21), that is, the current at resonance in (21) is 1/2 the amplitude of the current at resonance in the uncoupled tuned circuit. If we had placed $v = -m + w$, so that $w = 0$ when $v = -m$ and w varies about $-m$ in the same manner that v varies about zero, we should arrive at the same equation (21), and in this case it would apply to the resonance conditions about $v = -m$. We may conclude, therefore, that for large values of m, the primary current resonates at two frequencies, $v = +m$ and $-m$, the expansion of the frequency scale about these resonant frequencies being in terms of a new variable w.

How shall we go about expanding the frequency scale in terms of the new variable w? Taking the resonant frequency $v = m$ as an example, we see, first of all, that we ought not assume the approximations which hold only when v is small because now we assume m to be large. We go back, therefore, to equation (3) and let $v = \frac{\omega L}{R}\left(1 - \frac{f_0^2}{f^2}\right)$. Since $v = \frac{\omega L}{R}\left(1 - \frac{f_0^2}{f^2}\right) = m + w = \frac{\omega M}{R} + w$

we solve for $$w = \frac{\omega(L-M)}{R} - \frac{\omega L}{R} \cdot \frac{f_0^2}{f^2} \text{ ------ } (22)$$

Next we replace the resonant frequency f_0 with the new resonant frequency f_m which, of course, is equal to f when $w = 0$, or, $0 = \frac{\omega(L-M)}{R} - \frac{\omega L}{R} \cdot \frac{f_0^2}{f_m^2}$, from which, $\frac{f_0^2}{f_m^2} = \frac{L-M}{L}$. Since $\frac{f_0^2}{f^2} = \frac{f_0^2}{f_m^2} \cdot \frac{f_m^2}{f^2}$ we may substitute it in (22), which gives,

$$w = \frac{\omega(L-M)}{R}\left(1 - \frac{f_m^2}{f^2}\right) \text{ ----------- } (23)$$

Now we may apply the same method of approximation as we did with v and obtain, $\frac{\omega(L-M)}{R}\left(\frac{2\Delta f_m}{f_m}\right)$

It is clear that we are in a position to interpret w in exactly the same manner as explained for v merely by substituting f_m for f_0. The expression (L-M) in place of L simply means that the decreased inductance brings about resonance at a higher frequency.

We are now better able to interpret the trend of the curves plotted by substituting numerical values in equation (16) for successive increasing values of m. Returning to fig. 2a, for example, it is easy to see what is happening as m increases. The curves intermediate $m = 0$ and m equals a large magnitude represent a smooth transition from a single tuned circuit of resistance R to what is the equivalent of two separate single tuned circuits of resistance 2R, the resonant frequencies of the latter being at $v = +m$ and $-m$. The $m = 2$ curve in fig. 2a already shows a close approach to the large m condition. The peaks occur at $v = \pm 2 = \pm m$, the maximum amplitudes are slightly greater than 1/2 the maximum amplitude for $m = 0$, and the shape of the two resonance curves is very nearly the same as the $m = 0$ curve with the ordinates all reduced in proportion. Turning next to fig. 2b, which shows the curves for the second, or imaginary, term of equation (16), we observe that the curve for $m = 0$ crosses the horizontal axis only once, going from positive to negative at $v = 0$. As m increases, it continues to cross the axis, but at a less steep angle, and is accompanied by a flattening effect near $v = 0$. Curve $m = 1$ represents a boundary condition (critical coupling) where the curve flattens out sufficiently to follow along the axis for an appreciable distance on either side of $v = 0$. Further increase in m brings about three crossings of the horizontal axis, positive to negative at a negative value of v, negative to positive at $v = 0$, and finally positive to negative at a positive value of v. The important thing is that it is crossing the axis from positive to negative twice, once above and once below $v = 0$ and if m is made large, according to our analysis, the crossing points will then be at $v = +m$ and $-m$, the curve in the neighborhood of the crossing points being exact duplicates of the $m = 0$ curve with all of the ordinates at half value. This tendency is clearly borne out by the $m = 2$ curve where the crossing points are at ± 1.75, or slightly less than ± 2. In view of what has been said already, the absolute magnitude curves of fig. 2d are self-explanatory.

The polar curves of fig. 2c reveal the trend from a single tuned circuit of resistance R to two separate tuned circuits of resistance 2R in a novel and striking manner. The m = 0 curve is, of course, a duplication of the circle diagram of the uncoupled tuned circuit. As m is made to increase, the circle flattens slightly in the right hand portion where the small values of v occur. The flattened part takes on a decided depression at v = 0 as m is further increased until at m = 1 the curve has assumed a distinct cardioidal shape. This is the case of critical coupling and again we note that it is a borderline case because a greater value of m is accompanied by the formation of a loop in the place of the depression. According to our analysis, this loop should grow in size along with a contraction in the remaining part of the curve until, when m is large, it becomes one of two coincident circles, the remaining portion of the curve forming the other circle, the diameter of which is exactly 1/2 the diameter of the m = 0 circle. Examination of the m = 2 curve indicates that this result is well on the way.

THE COUPLED TUNED CIRCUIT SECONDARY:

It is highly important that we should also investigate what goes on in the secondary of a coupled tuned circuit, because the load, or energy receiver, is located in the secondary circuit, and the primary purpose in using a coupled tuned circuit is to utilize the peculiar characteristics associated with the transfer of energy from a source of oscillations connected to the primary circuit to a load connected to the secondary circuit. That brings us to the main difference between the primary and secondary circuit problems, namely, that in the primary circuit, the impressed e.m.f. and the resulting current are in the same circuit, while in the secondary circuit, the impressed e.m.f. and the resulting current are in different circuits. The current flowing in the secondary circuit with an impressed e.m.f. of value E in the primary circuit is given by;

$$I_2 = \frac{j\omega ME}{\left[R + j\left(\omega L - \frac{1}{\omega C}\right)\right]^2 + \omega^2 M^2} \quad \text{------(24)}$$

where R, L, C, M and ω have the same significance as previously. Upon comparing this with equation (13) for the primary current we observe the following relation,

$$I_2 = \frac{j\omega M I_1}{R + j\left(\omega L - \frac{1}{\omega C}\right)} \quad \text{----------(25)}$$

Equation (25) leads to the conclusion that we may regard the secondary circuit as a single tuned circuit with an impressed voltage equal to $j\omega M I_1$. We see that the secondary is fed by the voltage developed by the primary current flowing through the mutual reactance, a potentiometer effect. Let us follow up this idea, but first let us apply to equation (25) the simplifications already developed, i.e., $R(1+jv) = R + j\left(\omega L - \frac{1}{\omega C}\right)$ and $m = \frac{\omega M}{R}$, so that,

$$I_2 = \frac{jm}{1+jv} \times I_1 = \frac{jm(1-jv)}{1+v^2} \times I_1 \quad \text{-----(26)}$$

We now follow out the multiplication indicated in (26), using the value of I_1 in equation (16), and obtain,

$$I_2 = \frac{E}{R} \cdot \frac{m\left[2v + j(1+m^2-v^2)\right]}{4v^2 + (1+m^2-v^2)^2} \quad \text{---(27)}$$

If the simplifications are applied directly to equation (24), the result may be put in the same form as (27).

The curves shown in figs. 3a, 3b, 3c, and 3d are plotted from equation (27) for the same values of m and the same range of v in each case as in figs. 2a, 2b, 2c and 2d, which were plotted from equation (16) for the primary circuit. We have employed a uniform system of designating the figures in which the numbers 1 refers to a single tuned circuit, 2 to the primary of a coupled tuned circuit, and now 3 to the secondary of a coupled tuned circuit, and of the letters following the numbers, a refers to the real part of the current, b to the imaginary part, c to the polar diagram in which the magnitude versus angle is plotted, and d to the absolute magnitude, or the square root of the sum of the squares of the real and imaginary terms, plotted against v.

Before proceding with a detailed examination of the last set of curves, let us make a general analysis of the effect of giving extreme values to m. In order to make use of our previous similar discussion in connection with the primary current, let us keep in mind equation (26) which expresses the secondary current in the form of two factors, one of which is the primary current. If m = 0, the secondary current is, obviously, zero for all values of v. If, however, m is greater than zero, but still quite small, we have already seen that the primary behaves like a simple tuned circuit and (26) shows that, under this condition, the net result is the same as two simple tuned circuits in tandem, without any complicated reactions being perceptible. Referring again to the discussion of the primary case, we determined that when m is large there are two distinct frequencies of resonance at v = + m and -m, respectively, and now, on account of the relation in (26), we conclude that the same is true for the secondary current. Let us replace the variable v by w such that v = m + w, where m is large and w small, and substitute into equation (27). Neglecting quantities which are relatively small, we obtain,

$$I_2 = \frac{E}{2R} \cdot \frac{1-jw}{1+w^2} \quad \text{------(28)}$$

which is exactly the same as equation (21) for the primary current. This is the equation showing resonance about the frequency v = + m in terms of the frequency variable w. On the other hand, if we define w such that v = -m + w and substitute into (27), we find,

$$I_2 = -\frac{E}{2R} \cdot \frac{1-jw}{1+w^2} \quad \text{---------(29)}$$

which is the same as (21) and (28) with the sign reversed. It will be remembered that the primary current for this case, i.e., resonance about the frequency v = -m, is also expressed by (21) without a change in sign. This signifies that for resonance about v = + m the primary and secondary currents are equal and in phase, while for resonance about the frequency v = -m they are equal but in opposition. The conversion of w into a frequency scale about the resonant frequencies is, of course, the same as explained for the primary circuit. With these general remarks we now pass to a consideration of the plots for the secondary current.

Fig. 3a shows the real component of the secondary current as given by (27) for values of m equal to 1/2, 1 and 2. The first thing that strikes us is the resemblence between these curves and those of fig. 2b for the imaginary component of the primary current, the main difference being that where fig. 2b is positive fig. 3a is negative and vice versa. The multiplier jm in (26) accounts for this because, when the multiplication is carried out the imaginary term of the primary current is made real and the sign reversed by the product of the two j's. The resemblance applies only to the general outlines of the curves, the numerical values being substantially different. A horizontal line coinciding with the x-axis would represent the $m = 0$ curve. As m is made to increase, the characteristic shape of the curves appears, a positive peak above $v = 0$ and a negative peak below, with the crossing point at $v = 0$, the peaks increasing in magnitude and spreading away from $v = 0$ at equal and opposite values of v. Our analysis tells us that at large values of m we should approach resonance about the frequencies $v = +m$ where the primary and secondary currents are equal and in phase and $v = -m$ where the currents are equal but in opposition. This tendency is quite apparent in the trend of the curves as m takes on the values of 1/2, 1 and 2 progressively. How well advanced it is by the time $m = 2$ may be seen by comparing the $m = 2$ curves in fig. 3a and fig. 2a, where the peaks occur at $v = \pm m = \pm 2$. As predicted, they are in phase at $v = +2$ and in opposition at $v = -2$, the peak magnitudes in fig. 2a being slightly above, and in fig. 3a slightly below, the limiting value of 1/2 the value at resonance of the single tuned circuit. Notice, too, in fig. 3a the curvature developing in the line crossing at $v = 0$ as m increases, accomodating its shape to that which it must assume at large values of m, the same as fig. 1a at half amplitude.

The curves of fig. 3b, showing the imaginary component of the secondary current as given by equation (27), have a suggestion of the appearance of those of fig. 2a, the reason being clear from (26). Here, as before, a horizontal line coinciding with the x-axis would correspond to the $m = 0$ curve. With increasing m a positive peak rises up at $v = 0$, see $m = 1/2$ curve, reaching a maximum value of 1/2 for $m = 1$ (critical coupling), then receding back towards zero again for m larger than unity, the legs spreading out all the while and crossing to negative values at values of v approaching $+m$ and $-m$. Eventually, as m is still further increased, the righthand portion of the curve becomes the same as fig. 1b for the single tuned circuit at half amplitude, the crossing being at $v = +m$, while the lefthand portion becomes the negative of fig. 1b at half amplitude, the crossing being at $v = -m$.

The absolute magnitude curves plotted from (27) in fig. 3d show the tendency to be expected from the consideration given the curves of figs. 3a and 3b. They also show the $m = 1$ curve in relation to curves in which m is less than or greater than unity, namely, $m = 1/2$ or 2. The $m = 1$ (critical coupling) curve is of great importance in the practical use of coupled tuned circuits because the current amplitude is nearly constant for an appreciable range of frequencies above and below $v = 0$. It is the borderline case between a curve having a single maximum, as in the $m = 1/2$ curve, and a curve having two maxima, as in the $m = 2$ curve. Once the two maxima appear, the peak values remain constant at 1/2. This is to be compared with the double peaks which develop in fig. 2d as m increases, where the peak values are greater than 1/2 when the peaks first appear, decreasing asmptotically to 1/2, and, eventually, coinciding with the curves of fig. 3d at large values of m.

Now let us examine the polar curves in fig. 3c. A point at the intersection of the coordinate axes is sufficient to represent the case of $m = 0$. We notice, first of all, that the axis of symmetry of these curves is the vertical axis, while it was the horizontal axis for fig. 2c. As m is assigned increasing values, the curves take on a cardioidal shape which is reminiscent of the $m = 1$ curve of fig. 2c. Indeed, the $m = 1$ curves of figs. 2c and 3c have exactly the same dimensions, so that if the $m = 1$ curve of fig. 3c were rotated in the plane of the paper 90° counterclockwise with the center at the origin of axes, then slid bodily to the right along the horizontal axis a distance of 1/2, and finally rotated 180° about the horizontal axis, it would then coincide in every respect with the $m = 1$ curve of fig. 2c. After the cardioidal curve expands to a maximum vertical distance of 1/2 for $m = 1$, a depression

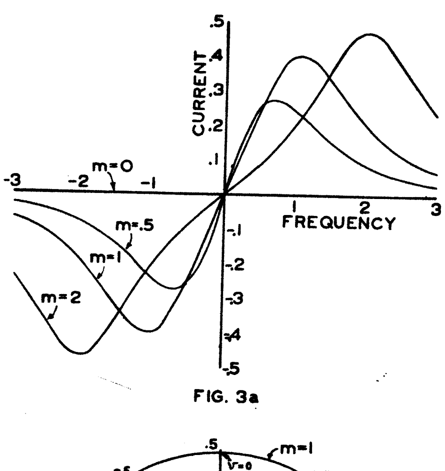


FIG. 3a

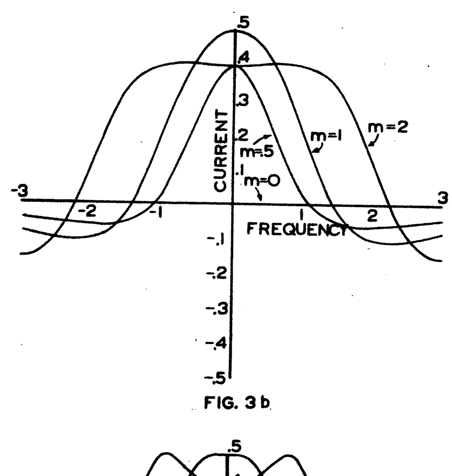
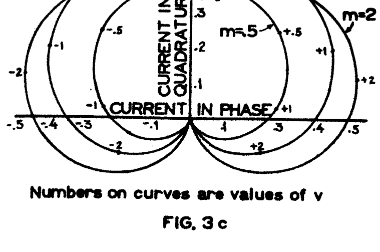


FIG. 3 b



Numbers on curves are values of v

FIG. 3c

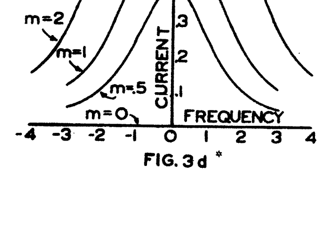


FIG. 3d *

* In all figures the Frequency Scale is in terms of v and the Current Scale must be multiplied by E/R.

appears about the vertical axis where v = 0 which deepens with further increase in m until, finally, at large values of m the curve evolves into two tangent circles of equal diameter, the vertical axis forming the tangent to them at the origin of axes, the horizontal axis from 0 to +1/2 and from 0 to -1/2 being their diameters. The righthand circle coincides with the coincident circles of fig. 2c for large m. One of the primary circles is for resonance about v = +m and should be paired with the righthand secondary circle which is also for resonance about v = +m, and with which it coincides exactly, the other primary circle is for resonance about v = -m and should be paired with the lefthand secondary circle with which it is in opposition.

GRAPHICAL METHOD FOR CONSTRUCTING THE POLAR DIAGRAMS:

As we have seen, the polar diagrams, figs. 1c, 2c and 3c, contain all the information provided by the remaining figures, providing the values of v are marked along the polar curves. By drawing straight lines connecting the origin with successive values of v along the polar curves, the horizontal projections of these lines with the corresponding values of v are all the data we need to plot the (a) curves, the vertical projections and the lengths of the lines against v for the (b) and (d) curves respectively. Therefore, let us see if we cannot construct the polar curves graphically.

The circle diagram for the single resonance circuit is so well known that we shall proceed directly to the problem of the coupled tuned circuit. For this purpose, it will be convenient to alter the form of equation (15) by multiplying the numerator and denominator by 1 + jv, giving the result,

$$I_1 = \frac{E}{R}\cdot\frac{1+jv}{(1+jv)^2+m^2} \quad -(30)$$

Noting that $m^2 = (-jm)^2$ we can factor the denominator of the second factor.

$$I_1 = \frac{E}{R}\cdot\frac{1+jv}{[1+j(v-m)][1+j(v+m)]} \quad ----(31)$$

Now expand the second factor into partial fractions,

$$\frac{1+jv}{[1+j(v-m)][1+j(v+m)]} = \frac{A}{1+j(v-m)} + \frac{B}{1+j(v+m)} \quad (32)$$

in which A and B are constants as yet undetermined. In order to determine them, clear (32) of fractions.

$$1+jv = A[1+j(v+m)] + B[1+j(v-m)] \quad (33)$$

Next equate the coefficients of the j terms,

$$v = A(v+m) + B(v-m) \quad --------(34)$$

If we let m = v in (34), we find that A = 1/2, and, similarly, if we let m = -v, B = 1/2. Consequently,

$$I_1 = \frac{E}{2R}\cdot\left[\frac{1}{1+j(v-m)} + \frac{1}{1+j(v+m)}\right] \quad -(35)$$

We shall now derive a similar expression for I_2. Using the first expression in (26), and the value of I_1 given in (31), we obtain the following expression for I_2,

$$I_2 = \frac{E}{R}\cdot\frac{jm}{[1+j(v-m)][1+j(v+m)]} \quad ------(36)$$

Expanding the second factor into partial fractions,

$$\frac{jm}{[1+j(v-m)][1+j(v+m)]} = \frac{C}{1+j(v-m)} + \frac{D}{1+j(v+m)} \quad (37)$$

Where C and D are constants to be determined. Clear (37) of fractions and equate the j terms,

$$m = C(v+m) + D(v-m) \quad --------(38)$$

When v = m, C is found to be 1/2, and when v = -m, D is found to be -1/2. Therefore,

$$I_2 = \frac{E}{2R}\cdot\left[\frac{1}{1+j(v-m)} - \frac{1}{1+j(v+m)}\right] \quad ------(39)$$

We may consolidate (35) and (39) into one expression,

$$I_1, I_2 = \frac{E}{2R}\cdot\left[\frac{1}{1+j(v-m)} \pm \frac{1}{1+j(v+m)}\right] \quad (40)$$

in which the + sign is used for I_1 and the - sign for I_2.

Equation (40) suggests the graphical method because each of the terms in the brackets may be represented by a circle diagram, the circles of which coincide, the scales being different. Referring to fig. 4, the construction consists

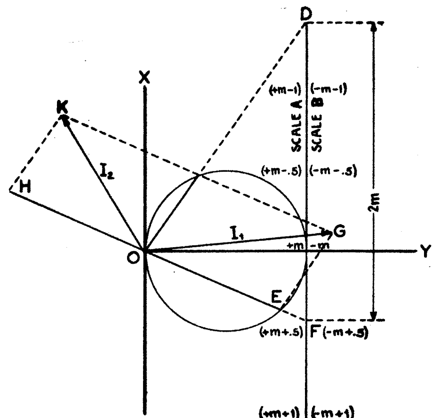


FIG. 4

first in laying out a circle of radius 1/2 with the center on the x-axis at a distance of 1/2 to the right of the origin. The line for the scales of v is next drawn perpendicular to the x-axis at a distance of 1 to the right of the origin, i.e., tangent to the circle. For convenience, let us call the first term in the brackets A and the second term B. As a starting point in marking the A scale, we observe that the phase angle of A is zero when v = +m, so we mark the scale where it touches the extremity of the diameter with the value of +m. Then we mark off the A scale in both directions from the starting point, the units divisions of which upward from the starting point are (m-1), (m-2), etc., and downward from the starting point, (m+1), (m+2), etc., each unit being the length of the diameter of the circle. We find, similarly, that the starting point for the B scale should be labeled with v equal to -m, the units divisions upwards from this starting point being marked with (-m-1), (-m-2), etc., and downward with (-m+1), (-m+2), etc., the length of each unit being, as before, the length of the diameter. The point on the A scale corresponding to a value of v = 0 is (m-m), or m units above the starting point, while on the B scale v = 0 corresponds to (-m+m), or m units below the starting point on the B scale. In going from v = 0 to v = +1, we move down one unit on each scale, and so forth, so that the distance between points on the A and B scales for corresponding values of v remains constant for a given value of m and equal to 2m, the point on the A scale always being above that on the B scale.

To find I_1, assuming the scales to be laid out for a given value of m, locate points D and F, fig. 4, corresponding to the chosen value of v on the A and B scales, respectively. Connect 0 and D by a straight line intersecting the circumference of the circle at C. The line OC is the vector representing the first term in the brackets of equation (40), its magnitude being the length of the line in units of the diameter, and its phase angle being the angle between OC and OX. Connect O and F by a straight line intersecting the circumference at E. The line OE is the vector representing the second term in the brackets of (40). Finally, add the vectors OC and OE by completing the parallelogram of which they are adjacent sides, and the diagonal OG is the vector of I_1. If, now, we wish to find the corresponding vector for I_2, we merely substract the vector OE from the vector OC by extending OE back through O to H such that OH is equal in magnitude to OE, and then add the vectors OH and OD getting vector OK for I_2. Repeating this for a whole range of values of v we obtain a series of vectors for I_1 and I_2. Drawing a curve through the extremities of the primary current vectors results in a curve such as those in fig. 2c, and a similar procedure for the secondary current vectors results in a curve such as those in fig. 3c.

It is instructive to consider our reasoning about extreme values of m in the light of this graphical construction. When $m = 0$, for example, points D and F coincide, and, likewise, vectors OC and OE, so that the curve traced out by the extremities of I_1 is another circle of twice the diameter of the circle OCE, or a single tuned circuit with the current at resonance equal to E/R. Since the vectors to be subtracted to obtain I_2 are always equal under the condition $m = 0$, I_2 is equal to zero for all values of v. This is also apparent, of course, by substituting $m = 0$ into equation (40). On the other hand, when we come to look at the case where m is large we see that points D and F are widely separated for corresponding values of v. Imagine D and F sliding down from large negative values of v, keeping a constant but large distance apart. Line OF will be the first to cut the circle to any great extent, and it will have cut nearly all the way around the circumference before line OD will begin to cut into the circle appreciably. Thus, we have the effect of two single resonant circuits resonating at the frequencies $v = -m$ and $+m$, the amplitude of the current at resonance being E/2R. Note that OF is the first to traverse the circle, of which the chord OE is the second vector in the brackets of (40). Now, this is positive for I_1 and negative for I_2, so that the primary and secondary currents are in opposition when they resonate at $v = -m$. On the other hand, when OD traverses the circle after OF has practically finished, the chord OC is the vector representing the first term in the brackets of (40) and this is seen to be positive for both I_1 and I_2. Therefore, the primary currents are in phase when they resonate at $v = +m$. All of the foregoing checks in an interesting way our previous speculations on extreme values given to m.

CONCLUSION:

We have chosen to emphasize the behavior of coupled tuned circuits in terms of the less complex phenomena of simple series tuned circuits. In this view, the coupling is regarded as increasing gradually from zero to large values accompanied, as we have described, in the primary, by a smooth transition from simple resonance at one frequency to simple resonance at two widely separated frequencies, the current amplitude at resonance for each being half the amplitude of the former, and, in the secondary, from zero current at all frequencies to simple resonance at the same two widely separated frequencies, the current amplitude at resonance for each being the same as for the primary, in phase at one resonant frequency and opposed at the other. The curves in our figures are snapshots on the way. Most practical importance is attached to a limited range in the midst of the transition as, for instance, at and near critical coupling, where certain features of the current versus frequency curve are found desirable, but it is interesting to think of this as an intermediate stage in the merging of two cases of simple resonance.

MEETING NOTES

The meeting held on June 13, 1935 at Columbia University was devoted to papers prepared by Messrs. D.E. Harnett, M.P. Case, and Walter Lyons of the Hazeltine Service Corporation, and demonstrations of two new developments in broadcast radio receiver design, by these gentlemen. The first of these to be shown was their automatic bass compensation system whereby an auxiliary audio-frequency amplifier effective only in the lower register, is added to the conventional radio receiver, with such automatic control as makes it effective only at the lower output levels, and, in fact, automatically makes it increasingly effective as the output level is lowered.

Operational demonstrations of any kind are invariably difficult in a large meeting room such as is used by the Club; and in this case the fact that the arrangements were effective only at low receiver output levels made demonstrations unusually difficult; notwithstanding all of which, the demonstration of the equipment was thoroughly indicative of its interesting characteristics. It was especially interesting to note how the tonal picture of the music remained little unchanged as the general level was lowered or raised, notwithstanding the human ear's lack of sensitivity both to the lower and extreme upper registers.

The second paper concerned itself with the variable selectivity superheterodyne which was demonstrated. This system comprises an intermediate frequency-amplifying system in which the frequency band width is controllable by means of variation in the inter-circuit couplings. This demonstration also was highly successful, in that a great change in fidelity possible through the variation of I F selectivity, as well as possibilities in the direction of avoiding adjacent channel interference, were readily evident.

Radio Club of America Celebrates 25th Anniversary

Dedicated to the "spirit of good fellowship and the free interchange of ideas among all radio enthusiasts" the Radio Club of America, Inc., has issued a special Year Book in commemoration of its 25th anniversary. Among the names appearing in the roster of present and past leaders of this pioneer club (said to be the oldest radio club in the world) are to be found many long since prominent in the world of radio communication as it is known today, and several now appearing on the roster of active members of the A.I.E.E.

Quoting from opening paragraphs of the Year Book:

"The story of the Radio Club of America begins over a quarter of a century ago, during the really dark ages of the radio art, about 1907. . . . Here we find a group of small boys who, according to the true American spirit, were so interested in flying that they formed the Junior Aero Club of U.S.

"In conjunction with their experiments in aviation, these youngsters had, for some time, also been interested in what was then known as wireless. In fact, the new idea of sending messages without wires had proved itself so fascinating, that they found themselves actually devoting most of their spare time to tinkering with wireless apparatus. There were at this time a small number of so-called amateur wireless experimenters in and about New York City, so the boys decided to form a new club with wireless as an object."

"Accordingly, . . . a special meeting of the Aero Club, for the purpose of forming a new club, with wireless telegraphy and telephony as its main interest. . .was held at the Hotel Ansonia in New York City on January 2, 1909. . . . Thus, the Junior Wireless Club Limited was founded," and bore that name until October 21, 1911, when it was changed to the Radio Club of America in recognition of its expanding membership and interests.

The Year Book presents a comprehensive outline of the history of the club, lists the major contributions of its members to communication literature, and includes a roster of members and of past and present officers. Copies of the Year Book are said to be available at the club's executive headquarters, 11 West 42nd Street, New York, N. Y.

ELECTRICAL ENGINEERING

25 Years of Radio

JANUARY 2, 1909 the Junior Aero Club of U. S. met at the Hotel Ansonia at the instance of W. E. D. Stokes, Jr. to consider a new hobby, radio. The members of this group, now the Radio Club of America, were of gentle age—Mr. Stokes was then 12. Today these boys are grown but their interest in wireless survives. The history of their club is the history of radio in America.

In the Year Book will be found photos of Louis Pacent and Harry Sadenwater listening for signals from Europe, Armstrong's regenerative apparatus, radio stations of E. V. Amy and others, Harry Houck's home-made loose coupler, station 1BCG which pumped 200 meter signals to Europe and caused M. I. Pupin and David Sarnoff to go to Greenwich, Connecticut, to see what "the boys are doing."

The entire book brings back memories of the old and glamorous days. Evidently the committee, under George Burghard, is still amateur at heart. In the Book is a history of each of the several hundred members.

ELECTRONICS

Twenty-fifth Anniversary Year Book, Radio Club of America, Incorporated. The oldest radio club in the world was 25 years old in 1934. In commemoration of the event, the club has issued the Anniversary Year Book. The greater part of this book is devoted to a history of the club and its members, richly illustrated with reproductions of letters, newspaper clippings, pictures of apparatus and their constructors.

The first meeting was held on January 2, 1909 in the Hotel Ansonia of New York City. At that meeting there were five members and the Club was then called the "Junior Wireless Club, Limited." The president, Mr. W. E. D. Stokes, Jr., was 14 years old and had made his own wireless set. Apparently, the entrance requirements were that you had to make your own set.

By 1911, the members had become sufficiently numerous to warrant issuing a typewritten list. At this time the name of the Club was changed to "Radio Club of America." The Club fought for the rights of amateurs and succeeded in preventing an unfavorable bill from being passed in Washington.

Many members of the Club have become important figures in the radio industry, contributing many new ideas. A list of papers read before the Club and a list of members are included in the book. The membership is now 320.

RADIO NEWS

"RADIO CLUB" SILVER ANNIVERSARY YEAR BOOK

IT GIVES US a great deal of pleasure to be able to review the Twenty-Fifth Anniversary Year Book of the Radio Club of America, Inc. Dedicated to "The Spirit of Good Fellowship and the Free Interchange of Ideas Among All Radio Enthusiasts," this book contains a wealth of information and inspiration in its many pages. In fact, a comprehensive review would, we feel, require nearly as many pages as this year book contains; and so this review shall consist of only a very brief summary of a few of the many highlights incorporated in its 85 pages.

The preface, which outlines the spirit that has contributed so much to the growth and prestige of the Radio Club of America from the eight charter members of the Junior Wireless Club Limited to its present 320, was written by George Eltz, Jr., who it happens was one of those charter members.

"A History of the Radio Club of America, Inc.," by George E. Burghard, traces in a very interesting and vivid manner the story of the Radio Club from its beginning in 1909. Mr. Burghard has condensed his material into some 38 well-illustrated pages.

Immediately following is a foreword by Lawrence C. F. Horle. This foreword precedes the complete listing of the Proceedings of the Radio Club. To appreciate the value of this listing of the Proceedings, even if only for reference purposes, one needs but glance at names of the engineers who have presented papers.

An enrollment of past officers, condensed history of each member, and the Constitution of the organization completes this year book.

The members of the Twenty-Fifth Anniversary Year Book Committee, namely, George E. Burghard, Ernest V. Amy, Edwin H. Armstrong, George J. Eltz, Jr., John F. Grinan, Lawrence C. F. Horle, Frank King, Robert H. Marriott, Fred Muller, Joseph J. Stantley, and W. E. D. Stokes, Jr., deserve a vote of thanks from the Radio Club.

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