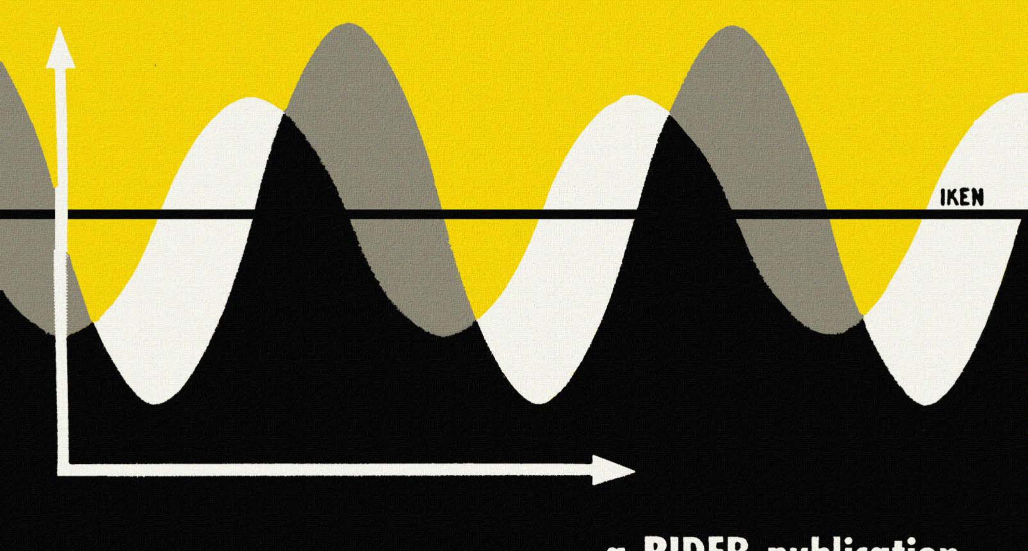
No. 103
$1.25

UNDERSTANDING vectors and phase

by JOHN F. RIDER and SEYMOUR D. USLAN



a RIDER publication

Understanding VECTORS AND PHASE

By

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JOHN F. RIDER PUBLISHER, Inc.

116 West 14th Street, New York 11, N. Y.

4th Printing ______________________1958

Printed in the United States of America

INTRODUCTION

The casual reader is apt to ask: Why such a book as this? . . . Why a book covering a very limited field of application of such a broadly useful tool as a vector? . . . Why a book on phase as applied to a very limited field, when that subject is so closely associated with all actions in electric communication systems? . . . Strangely enough the answers to these questions are simple, as is the basic purpose of this book.

Admittedly, vectors is a branch of mathematics and phase is a relationship. Both have been dealt with in numerous texts in a manner far more expansive than is attempted in this text. Yet we feel that such a book as this has its function. It is intended for the man who has not had an engineering background, yet is thrown in contact with both vectors and phase in his effort to keep abreast of electronic developments by reading books and magazines. We are speaking about the radio serviceman, the radio technician, the radio amateur, or the radio student who is not seeking an engineering training.

It is granted that a radio serviceman need not understand vectors or phase relationships in order to repair a radio receiver or a transmitter. The same applies to any one of the other groups mentioned. A general understanding of radio theory as it applies to vacuum tubes, coupled circuits, d-c and a-c voltage networks in general, radio circuit structure, is sufficient. Continued operation on radio equipment breeds greater familiarity, so that where the theory is not fully understood, a sort of instinctive understanding is developed. This enables the individual to continue functioning in his chosen field.

The assimilation of practical details is not difficult, but the comprehension of theory as presented by the engineer type of writer poses many problems. Not that any fault can be found with the manner in which the new techniques are described — for they are above reproach in every respect. . . . They are prepared with great pains, are well written, and well illustrated. . . . But the man without the engineering background experiences many obstacles — he can understand some of what is said, but cannot hurdle the gap created by the use of a vectorial presentation and the conclusions based thereon. . . .

To expect the engineer to write for the man who does not have the background is unreasonable; there is much more to be said than room in which to say it. . . . Therefore it is only natural that the most direct method of reaching the end will be employed. . . . Yet it is this same article or paper, multipled manifold and appearing in many radio magazines, which is the granary of technical radio knowledge. . . . To make these articles — or at least the greatest number of these papers — most understandable is the reason this book on vectors and phase was written.

Some may wonder why we chose vectors and phase for discussion, rather than the mathematics on which the vector is founded. The answers are numerous. First, because a vectorial presentation is a picture of conditions as they exist, and a picture always is more understandable than an equation. Second, because recognition of conditions as depicted by vectors can be accomplished more readily than by equations. Finally, because an analysis by means of vectors is simpler than by any other means.

Naturally, an understanding of vectors is not a substitute for a background in radio theory. In this respect the authors assume a general knowledge of theory, such as might be gained from attendance at residence schools teaching below the engineering level, correspondence courses, Armed Forces schools, home study, and other methods of acquiring knowledge. . . . Admittedly total lack of theoretical radio knowledge makes the vector meaningless when applied to radio, since that which it explains is not recognized. With an appreciation of general radio principles and alternating quantities as a background, the vector will be found of inestimable value in connection with the assimilation of the contents of papers in radio engineering publications and textbooks.

In summarizing, it is the hope of the authors that this comparatively short text will prove of value to all who have a strong desire to know more, but who, due to circumstances beyond their control, were unable to acquire that background which is so essential to fruitful operation in such a widely diversified field as radio communication.

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CHAPTER 1

WHAT IS A VECTOR?

As to the exact definition of a vector, it would be difficult to include all interpretive meanings in a single sentence. So far as this book and its intended purpose are concerned, we can define a vector as follows:

> A vector is a segment (part) of a straight line that is oriented (pointed) in a specific direction, and which straight line has a starting point and a terminal point.

You will agree that the sentence is definite, but most certainly deserving of much more explanation. If you will examine the illustrations of such vectors in Fig. 1-1, and devote a little time to the association of the illustrations (1), (2), and (3), with the definition, certain pertinent facts will present themselves. It would not be too wild a guess for a person to assume, according to the common understanding of an arrowhead on an arrow, that the point identified as B is the terminal end of the vector. Obviously then, the other end of each line or vector

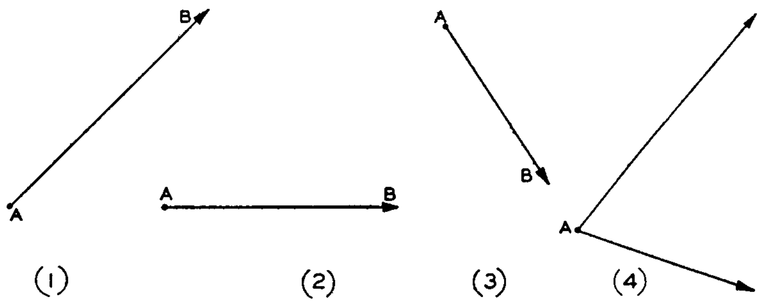


Fig. 1-1—Typical vectors illustrating different magnitudes and directions.

identified as A, is the other boundary. Such an assumption is correct.

Practice has established that the arrowhead is located on the *terminal* point, which makes the other end the *starting* point. As to the letters A and B associated with the two boundaries of the vector, they are purely illustrative, in that there is no set rule which states that identification of the starting point must be A, and the terminal point of a vector must be the letter *B*. They can be any pair of letters dictated by the subject under discussion.

The next significant detail noted in Fig. 1-1 is that vectors (1), (2), and (3) are of different lengths. This is symbolic of the fact that *they represent mathematical details of different amounts*. Just what these details are is of no consequence at the moment, it being sufficient for you to appreciate that whatever the nature of the mathematical detail being represented by a vector, *the length of the line can be correlated with the amount or magnitude of the mathematical detail;* the greater the magnitude of the mathematical detail, the longer will be the length of the vector representation. The converse is naturally true.

The third significant detail is that, if we associated these vector illustrations with road signs, ordinary maps, or other directional indicators, it would be a safe assumption to consider that the purpose of the arrowhead is to point or orient the vector in a certain direction. In other words, whatever the mathematical detail represented by the vector, it is in some way associated with *direction.*

A fourth detail in this illustration is that the three vectors (1), (2), and (3) are shown disassociated from each other. This was done in order to bring out more clearly the fact that they can be of *any* length representing *any* amount of the mathematical detail; also that they can point in any direction. Beyond this, their lack of association has no meaning, for actually, when in use in radio circuit analysis, the vectors frequently originate at the same starting point, as shown in illustration (4) of Fig. 1-1.

Directed Quantities

In speaking about mathematical details shown by vectors, it seems more fitting to consider what they display as *directed quantities,* for in accordance with the definition of the vector, that which it represents is associated with direction. Stated differently, we might say that a vector depicts a directed quantity, which is a mathematical detail consisting of two parts or components; one of these is the amount or *magnitude,* and the other is *direction.*

The association of a vector with a mathematical detail which is made up of two parts, magnitude and direction, is not a casual choice; rather it is extremely important, for it stipulates what can and what cannot be represented by vectors. For example, let us consider the meaning of magnitude, and its association with a vector.

Magnitude

We stated in the definition of the vector that it was a straight line with a starting point and a terminal point. This means that each vector has a definite length representation of a quantity. Since a vector can be used to represent a multitude of things, depending upon application, instead of saying that a vector has length, it is more fitting, so as to encompass all applications, to say that a vector possesses *magnitude*. Thus in Fig. 1-1, the *length of any vector* is indicative of the magnitude of the directed quantity represented by the vector.

The use of the word magnitude encompasses many fields in which the relationship is purely statistical — that is, it embodies strict numbers. If for the moment we think of things which contain magnitude, we find them to be extremely numerous, almost too numerous to record. In fact everything has magnitude regardless of the part it plays in the daily life of man. It may be in the form of weight, dimension, or quantity. However, as you will soon see, all kinds of magnitudes are not expressible by means of vectors; those which are not will be dismissed from further discussion. The deciding factor is whether the quantity which contains the magnitude, is or is not, completely definable by a single number.

Suppose we consider some examples of magnitude. For instance, temperature, the number of automobiles in the nation, the number of radio sets in a home, the attendance at a prize fight, the number of hats on a rack, the books on a shelf, the number of peas in a pod are quantities which are wholly definable in terms of magnitude by a single number. The temperature may be 68 F, or 26 C; the fact that the temperature may rise or fall is of no consequence, for at any one time the temperature can be definitely identified by a single number indicating magnitude. In like manner, the number of cars in the nation can be stated as 26,000,000, and it is a complete answer to the question; the number of radio sets in a home may be six, which is a definite and complete answer; the attendance at a prize fight is stated completely when the number 45,346 is given. The number of hats on a rack, the number of books on a shelf, or the number of peas in a pod likewise are completely definable by quoting a single number representative of magnitude. Quantities which are completely definable by a single number indicating magnitude are known as *scalar quantities* and play no part in vector representations.

On the other hand, there are other kinds of quantities which involve magnitude but which are not completely described when a single number is mentioned — that is, by simple reference to magnitude. For example instantaneous values of alternating current or voltage cannot be described completely by a simple reference to magnitude, that is, a number of volts or a number of amperes, although such is common practice in lay life. Since the relative direction of this voltage or current is changing continuously, proper technical identification must take into account the *direction as well as the magnitude.*

We may speak about the speed of an airplane as being 200 miles per hour, but that is not complete identification of the motion of that plane. To be correct we must also state the direction in which the plane is moving. It is like speaking about the wind. To say that the wind at a certain time has a speed of 10 miles per hour is meaningless to those who are concerned with the wind. They desire to know its *direction* as well. It is only by mentioning both speed and direction, the two combined being the *velocity* that the complete identification is provided.[1] In the case of the plane and the wind, the speed is the magnitude and the direction of motion is the other component, both of which combine to make up the mathematical detail corresponding to a *directed* quantity.

Force is another example of a directed quantity. A mass may be subjected simultaneously to a downward pressure and a pull to one side. To define the action, we state not only the magnitude of the pressure and the pull, but also the direction in which they are acting.

[1]A common misconception is that *speed* and *velocity* have the same meaning; speed is a scalar quantity, whereas velocity is a vector quantity. If a plane flies a circular course at 400 mph, its *speed* is 400 mph, but its *velocity* is continually varying due to its constantly changing direction. Consequently, in speaking of the velocity of the plane at any instant of time both the speed of 400 mph and its instantaneous direction should be taken into account.

Vector Quantities

Such quantities which require both magnitude and direction for complete definition are known as *vector quantities* and can be illustrated by a vector. A scalar quantity can be shown by a straight line, as is done in many graphs, wherein the length of the line is representative of magnitude, but in all of these cases, the direction in which the line is pointing has no significance. On the other hand in the vector representation of a vectorial quantity, the length of the line indicates magnitude, but the direction in which the line is pointing is an extremely important detail. As you can see it is this matter of direction which distinguishes between a scalar and a vectorial quantity.

Direction

Direction, like magnitude, presents some very interesting points for discussion. In its most simple case direction to many readers usually means orientation of some quantity with respect to north, south, east, or west; in other words towards the points of the compass. In some instances such a definition may suffice, but there are many where such identification will not fulfill the requirements, since the term direction is employed in many different ways. In this book we are interested in its application in connection with electrical phenomena. For instance, when discussing the direction of movement of a ship, it may be expressed as ENE (east by northeast) or some other point on the compass. The same is true of a plane, or in both cases, the direction may be identified as being a number of degrees away from a reference direction. Similar identification may be employed in the pointing of a coastal defense gun with respect to its direction along a horizontal plane usually referred to as azimuth.

Accordingly there are three methods of describing direction so that it does seem advisable to establish a standard reference. Such has been done as far as engineering is concerned. This standard is in terms of the degrees in a circle.

The Meaning of Degree

Like direction, the term degree has numerous meanings. One of these is in connection with temperature, another is in connection with educational levels identifying the attainment of knowledge or accomplishments in certain fields. Several other meanings are associated with mathematics, one of which is of interest to us. This is the angular measurement of the subdivisions of a circle. Starting back in the days of Babylon and advancing through Ptolemy, the famous Greco-Egyptian mathematician, there came into being the condition that if a circle,

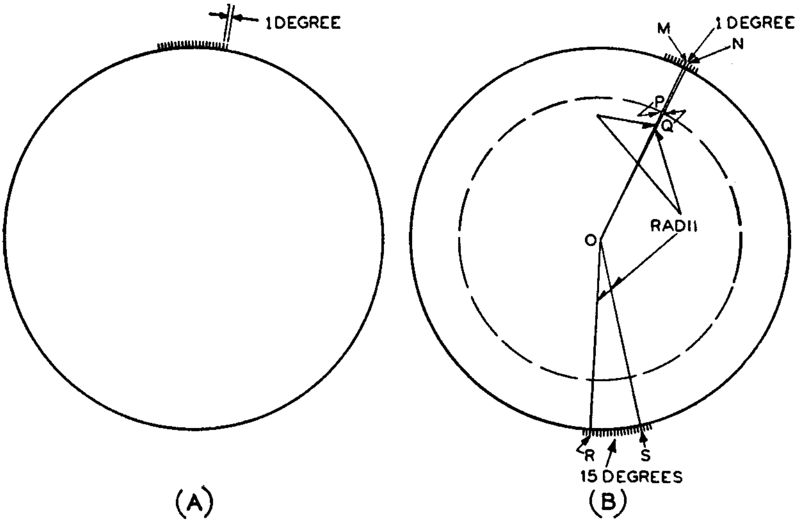


Fig. 1-2—(A) A circle is usually divided into 360 divisions of 1 degree each. (B) Any line from the center to the circumference of the circle is called a radius. The angle embraced by radii *OP* and *OQ* is the same as that embraced by radii *OM* and *ON*.

regardless of its dimensions, is divided into 360 equal divisions, each of the divisions is equal to 1 degree, which is usually written as 1°. If you can imagine a circle such as shown in Fig. 1-2 (A), shrinking in size until it is a mere pin point, similar division of the periphery of the dot into 360 equal divisions would result in 1 degree of the arc being embraced by each segment. In other words, all circles contain 360 degrees, regardless of how large or small they may be. Incidentally, the circumference of our earth is divided into 360 degrees, these being the longitude lines.

Technically, 360 such divisions are not the smallest divisions of a circle. Each of these 1-degree segments — that is, the portion of the circumference between the individual degree division markers — is subject to further subdivision. Each degree can be divided into 60 equal parts known as "minutes," here the word minute meaning space rather than time, and being shown as 1′. In turn each minute of arc is subdivisible into 60 additional equal segments, each of which is known as a "second" and shown as 1″. Thus the most precise reference to some point in terms of degrees might be 26 degrees, 14 minutes, and 47 seconds. For our purposes, however, such fractional breakdown of the circumference of a circle, for that matter the degree, is not necessary, it being sufficient for our purpose to speak in terms of whole numbers of degrees.

The Radius and the Angle

Referring again to Fig. 1-2 (B), imagine a straight line drawn from one of these degree markings, *M* on the circumference, to the center of the circle, designated by *O*. This is a "radius" of the circle and represents the distance from the center of the circle to the circumference. If this radius is revolved with the center of the circle as the pivot point, the circle will be inscribed. The longer the radius, the larger the circle — the shorter the radius, the smaller the circle. Since the circle is the only figure which is equidistant at all points from the center, all the radii of a circle are of equal length, no matter where they join the circumference. To state this differently, such straight lines may be drawn from any point on the circumference to the center of the circle, and all these radii will be of equal length. Thus a circle may have as many radii as may be desired, but for any one circle, all radii will be equal. This is an axiom in geometry.

If two radii are drawn from two adjacent degree markings on the circumference, such as *MO* and *NO* in Fig. 1-2 (B), a figure is formed by the junction of the two lines, a sort of V, which is identified as an "angle," in this case embracing 1 degree. This angular dimension of 1 degree is embraced by the figure *all along its length*. It is significant to note that the actual dimension of the arc of 1 degree is a function of the *length* of the *sides* of the angle, as can be seen by the separation between points M-N on the circle in Fig. 1-2 (B) and the points P and Q, representing 1 degree on the smaller circle shown by the dotted lines. Thus whatever is the angular dimension embraced by the sides of an angle at the circumference of a circle, the same angular dimension is embraced by the sides of the angle all along their lengths.

If, instead of forming an angle of 1 degree by having two radii

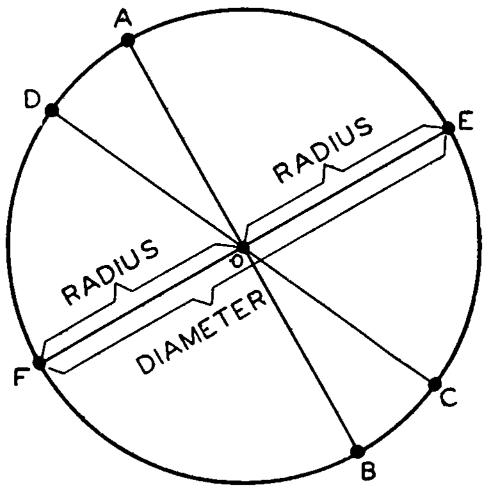


Fig. 1-3. Any radius prolonged through the center to the opposite side of a circle, becomes a diameter. The radius *AO*, for example, when extended to *B*, becomes the diameter *AB*.

drawn from adjacent 1 degree points on the circumference, these radii were drawn so that they embraced 15 degrees at the circumference, the angle formed by the radii would be 15 degrees, as shown by angle *ROS* in Fig. 1-2 (B).

From what has been said, you can readily see that if a radius is revolved so as to describe a complete circle, it will inscribe 360 degrees; in turn the motion of a radius in describing an arc less than a complete circle can be also spoken of as having inscribed a certain number of degrees. For example, if the radius so revolved to the indicated position of *RO*, it would inscribe an angle of 15 degrees; if it revolved around one-quarter of the circle, it would inscribe 360 ÷ 4, or 90 degrees.

Diameter of a Circle

In order to demonstrate that direction can be completely represented by the measurement of degrees, let us continue with the discussion of the circle. We have established the radius in connection with Fig. 1-2 (B). Now, if any one radius is continued through the center of the circle in a straight line to the circumference of the circle on the opposite side, this complete line, consisting of two radii in a straight line, becomes the *diameter* of the circle, as shown in Fig. 1-3. Thus, line *AO,* a radius of the circle, added to line *BO,* also a radius of the circle, results in the diameter *BOA;* radii *CO* and *OD* joined to form a straight line form diameter *COD.* In like manner, radii *FO* and *OE* joined to make a straight line form the diameter *FOE.* From this it follows that the definition of a diameter of a circle is "any continuous straight line that passes through the center of the circle from one part of the circumference to another part of the circumference."

Let us now examine one of these diameters. Since it passes through the center of the circle and joins opposite sides of the circumference,

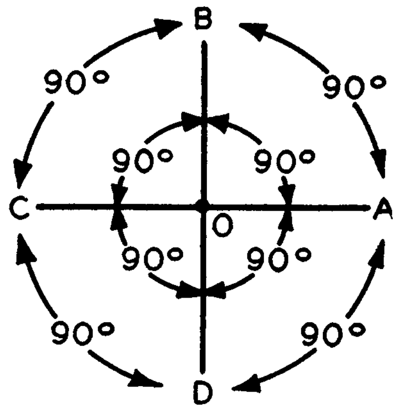


Fig. 1-4—Two diameters *AC* and *BD* perpendicular to each other divide a circle into quarters, each of which embraces 90 degrees.

any diameter divides the circle into two equal parts. As a circle consists of 360°, any one diameter can be said to divide a circle into two equal parts, each of which embraces 180°. From the definition stated and Fig. 1-3, it is readily evident that an infinite number of diameters can be drawn through the center of the circle to the opposite side of the circumference. Since the diameter is equal to the sum of two radii in a straight line, whatever has been said concerning the relationship between the dimension of the radius applies to the dimension of the diameter, fundamentally, the shorter the length of a diameter, the smaller is the circle and the greater the length of a diameter the larger is the circle. But in all cases, regardless of the length of a diameter, it divides the circle into two equal parts, each of which embraces 180°.

Straight Angle

If for the moment we consider the two radii which comprise the diameter, for example *FO* and *OE,* as being the sides of an angle formed by two radii, this angle embraces 180°. Compare this with the 1° angle *MON* and the 15° angle *ROS* in Fig. 1-2 (B). You may more readily appreciate how a diameter forms an angle of 180°, if you will visualize radius *SO* in Fig. 1-2 (B) as being stationary and radius *RO* as rotating until it forms a straight line with *SO*. To accomplish this, radius *RO* was turned through 165° from its initial position, so that it now embraces the original 15° plus the 165° through which it was turned, or a total of 180°. Such an angle is called a *straight angle,* since the sides of the angle form a straight line.

Perpendicular Diameters

Let us now advance to a more extensive division of a circle. Instead of dividing the circle into two equal parts, we desire to divide the circle into four equal parts. As a complete circle consists of 360°, and a diameter divides a circle into two equal parts of 180° each, division of the circle into four equal parts would require some means whereby each part would contain 90°. Since any one single diameter divides a circle into two equal parts of 180°, it stands to reason that two such diameters drawn perpendicularly to each other, would divide the circle into four parts, each of which would embrace 90°. This is shown in Fig. 1-4. The circumference is divided into four equal parts of 90° each, and the four radii which combine to produce the two diameters form four right angles *AOB, BOC, COD,* and *DOA,* each of 90°.

Right Angles

When any line is drawn to another line, not necessarily through it, such that the contact of the two lines produces two right angles, these two lines are said to be perpendicular to each other. This is shown in Figs. 1-5 (A), (B), and (C). In all of these instances, the lines are perpendicular to each other, forming angles of 90°, and they are shown

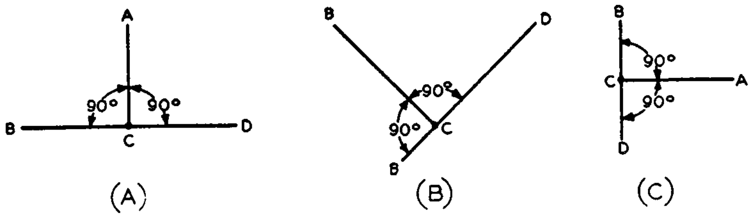


Fig. 1-5 (A), (B), (C)—Three examples of perpendicular lines. When two lines meet and form right angles, they are said to be perpendicular to each other.

in various positions in order to familiarize you with a few possible placements of such angles upon a printed page. Yet each of them contains 90°.

If line *AC* were extended through the contact point *C* on line *BD*, the perpendicularity is extended to the other side, and the two intersecting lines then would form four right or 90° angles, similar to that accomplished with the two perpendicular diameters.

It is upon the use of two such perpendiculars that the entire system of vector representation is based.

CHAPTER 2

THE COORDINATE SYSTEM

We said that two lines perpendicular to each other organize or coordinate the features of the vector system. When used as the basis of vectorial or graphical representation, two such perpendicular lines are referred to as the *coordinate system* or as *coordinate lines*, or finally, as *coordinate axes*. The vertical line is usually referred to as the "vertical" or "Y" axis, and the horizontal line as the "horizontal" or "X" axis. Many everyday problems with which we come into contact, as well as many instruments, use this coordinate system as the basis of measurement.

One of the simple examples is the conventional magnetic field compass. The markings on the compass which correspond to these coordinate lines are the indications of north, south, east, and west, as shown in Fig. 2-1. It is true that upon the conventional compass these perpendicularly arranged axes do not appear as lines, but their existence can easily be imagined and the use of the coordinate system as the basis of

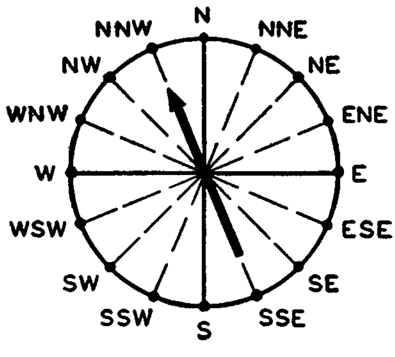


Fig. 2-1—The divisions of the magnetic field compass present a familiar example of diagrammatic representation based on the coordinate system.

orientation or direction indication is readily recognized. In Fig. 2-1 the vertical north and south axis is represented by line NS and the horizontal east and west axis by line EW. Between these axes are other directional divisions which represent points between the four major directions. With the compass needle mounted at the center of the compass, the similarity between the general nature of this pattern and the structure shown in Fig. 1-4 is clearly evident.

The dotted line from the center of the circle to NW indicates the northwest direction; the dotted line from the center of the circle to the ESE point indicates a direction known as east by southeast; and the direction where the magnetic needle in Fig. 2-1 is pointing is known as north by northwest. The other compass designations on the circumference of the circle are other intermediate points between north, south, west, and east.

As can be seen, the northwest direction point divides the right angle of 90° existing between north and west, in half; consequently, the northwest position represents an angle of 45° between north and northwest and between west and northwest. Similarly, NNW divides the 45° of angle between N and NNW into half, forming angles of 22.5° each between N and NNW and between NNW and NW. If the compass needle points anywhere but on any of the lines shown, its exact direction can be expressed as being a certain number of degrees from the nearest main direction line. It is unnecessary for us at this time to divide further the 22.5° angles such as exist between lines shown on the face of the compass, since this is only one illustration of how the coordinate system is used as the basis of instrument measurement.

Degree Assignment

In the majority of systems where the coordinate scheme is employed, only the two perpendicular lines are used as a means of representation, and the actual circle, such as is shown in connection with the compass, is omitted. However, although the circle is omitted, the indication of direction is made with the complete use of the 360 degrees that a circle contains, and, whatever assignment of degrees is made, is in accordance with the 360 degrees in a circle.

If we now correlate this fact with the compass face which was shown in Fig. 2-1, two possible ways of indicating direction are evident. One of these is by utilizing north, east, south, and west and its intermediate points as basic references and to indicate direction between north and east or between any two other main points as being a certain number of degrees with a maximum of 90° from any one of these main directions. In other words, if we wish to indicate points between east and north and use north as the reference, we might say that the desired point is 47° east of north; or, if we wanted to use east as the reference point, we might say that the desired direction is 33° north of east. In like manner we could treat the remainder of the compass, always working within a 90-degree arc existing between any two adjacent main divisions or directions on the compass. Such a method recognizes the existence of 360 degrees in a circle, but accomplishes the necessary indications of direction within 90-degree segments.

An alternate method of expressing direction makes use of the 360 degrees in a circle and also a main compass direction as the reference point. For example, north may be selected as the sole reference point; therefore, it is assigned the 0° designation which also is the 360° point, inasmuch as the starting point and terminal point of a circle occupies the same spot on the circumference. Then all points are indicated as being a certain number of degrees from north.

If a clockwise rotation were selected for the assignment of the degrees of direction, then east would correspond to 90°, south to 180°, west to 270°, and north would be 360° or 0°. To say that an object were located east of the reference point, it would be identified as being 90° away. If something were south of the reference point, it would be stated as being 180° away; if it were located northwest of the reference point, it would be 315° away. In this way the precise location of a direction could be given as a certain number of degrees, minutes, and even seconds of the arc, from the reference north direction.

It is also conceivable that the assignment of the 0° starting point would be some reference direction other than north; it could be east, south, or west as desired. Moreover, the direction of rotation employed in the assignment of the degrees likewise could be the reverse of that used. Obviously no matter what the reference direction or the direction of rotation in the assignment of degrees, the full 360 degrees of a circle could be embraced. Incidentally, the relation between the coordinate lines and the assignment of the 0° starting point is a very important detail in vectorial presentation.

In this connection, let us refer to Figs. 2-2 (A) and 2-2 (B). In illustration (A), the coordinate lines *AC* and *BD* are drawn within a circle which is divided into four segments of 90° each. But how will we assign the degree designations with respect to points *A*, *B*, *C*, and *D*? Will point *B*, *A*, *D*, or *C* be the 0° and 360° starting and terminating point? In fact, without a standard procedure we are really at a loss to know where to start. Some may choose point *A* as the starting and fin-

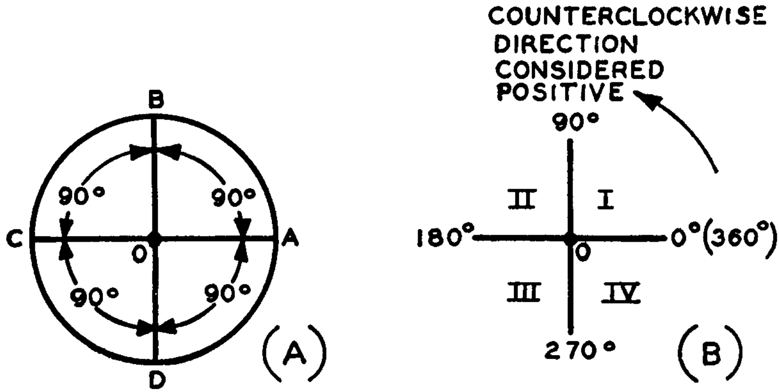


Fig. 2-2 (A)—Coordinate lines within a circle divide it into four segments of 90 degrees each. (B)—In the coordinate system the direction of rotation and the quadrant designations have been standardized, so that counterclockwise rotation is considered positive.

ishing point and work clockwise; others may select the same starting point put rotate in the counterclockwise direction. Someone else may select points *B*, *C*, or *D* as the starting point and work either clockwise or counterclockwise. It is, therefore, evident that some rule must be set to guide all who have occasion to work with vectors. There is such a standard procedure, and it is accepted throughout the United States and many foreign countries, thus enabling a common understanding of what is being shown in engineering circles. This is illustrated in Fig. 2-2 (B).

Counterclockwise Direction Positive

In this illustration we have omitted the circle since its presence is not essential to the use of the coordinate system, nor is it essential to the choice of either the direction of rotation or the assignment of degrees. Concerning the starting point, this is customarily the position occupied by *A* in Fig. 2-2 (A) or east on the compass face. As you note, letter designations are no longer used, instead the 0° identifies the starting point. The *A* termination on line *AO* of Fig. 2-2 (A) corresponds with the 0° starting point of degree reference in Fig. 2-2 (B). As a matter of fact, this line will be deferred to as the 0° reference line throughout this book. Bearing in mind what was said previously about the correspondence between the 0° and 360° points on a circle, the 360° designation in parenthesis is placed adjacent to the 0° designation.

Further examination of Fig. 2-2 (B) discloses that the assignment of degrees, which establishes the direction of rotation with respect to the 0° point is in the *counterclockwise* direction. This selection is not arbitrary, but rather conforms with the accepted standard practice and in engineering circles is identified as *positive*. At first glance the meaning of the word positive may not seem clear, but it will become so as we advance in the text. For the present it should be accepted as stated; let it suffice to say that it is associated with the nature of the quantity. Continuing, the 90° line corresponds to *OB* line of Fig. 2-2 (A); the 180° line corresponds to the line *OC*, and the 270° line corresponds to the line *OD*. In order to complete the 360° of rotation in the counterclockwise direction, the 0° line now becomes the 360° line.

Another way of correlating what has been said about Figs. 2-2 (A) and (B) is to imagine line *AO* of Fig. 2-2 (A) being revolved in the counterclockwise direction until it occupies the positions shown for *BO, CO, DO,* and *AO* respectively. In each of these stages of rotation it passes through or inscribes a number of degrees, 90° at position *BO,* 180°, at position *CO,* 270°, at position *DO,* and 360° when it arrives at its original starting position. Naturally intermediate positions of this line will inscribe intermediate numbers of degrees such as 315°, if the line finally arrives at a point midway between the positions occupied by *DO* and *AO* in Fig. 2-2 (A), remembering that the line is moving in a counterclockwise direction. Designating the coordinate lines in degrees removes all necessity for the presence of the circle or the need for letter designations.

Quadrant Designations

At this time you are wondering about the four Roman numerals shown in Fig. 2-2 (B) but which have not been referred to. What are they? These are *quadrant* designations and mean the following. When a circle is divided into four equal parts by two perpendicular diameters, there are formed four quadrants each containing 90 degrees. If now you compare Figs. 2-2 (A) and (B), you will note that the actual circumference of a circle need not be illustrated in order that we indicate the total number of degrees in a circle, a coordinate system within the circle, the two perpendicular diameters, or the four quadrants.

As to the identification of these four quarter sections or quadrants, the direction of rotation is the determining factor, and, since we consider counterclockwise rotation as the positive direction, the numerical identification of the quadrants conforms with this direction of rotation. Accordingly, the quadrant bordered by the 0° and 90° lines is called the first quadrant and indicated by the Roman numeral I. Continuing in the counterclockwise direction, the quadrant bordered by the 90° and 180° lines is known as the second quadrant and indicated by the Roman numeral II. Still continuing in the same direction, we note the third and fourth quadrants, III and IV respectively.

Such division of the coordinate system into four quadrants simplifies the discussion of many subjects. As you may have noted, identification of parts of these illustrations has been somewhat cumbersome, because it was necessary always to speak about the boundaries which enclosed the portion of the illustrations to which we referred. With the quadrant designations established, it becomes a relatively simple matter to identify the space bounded by any pair or coordinate lines by simply stating the quadrant. Thus if we are interested in discussing action in the space bounded by the 90° and 180° lines, we say "quadrant II" and the identification is positive. The other quadrants are treated similarly.

Negative Angle Direction

In a previous paragraph we established the counterclockwise direction of rotation as being the equivalent of a positive direction. Actually this is better defined as being a positive angle direction. Now we introduce another standard of rotation which may be experienced; this is a clockwise direction of rotation resulting in negative angles. The difference is twofold. First, the direction in which the angles are assigned is clockwise instead of counterclockwise — although the starting point or 0° line is the same as before. The coordinate line previously identified as 270° now is —90° as shown in Fig. 2-3, the minus sign (—) being

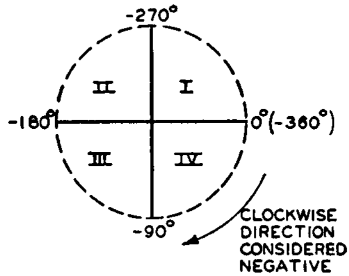


Fig. 2-3—Clockwise rotation from the 0° axis in the coordinate system is considered a negative direction as shown by the negative angle signs. The quadrant designations remain the same as in Fig. 2-2 (B).

placed in front of the angle designation to show that it is negative. It is in this manner that standardization is accomplished for the clockwise direction of angle assignment; that is, if a minus sign precedes the angle designation it indicates clockwise rotation. Accordingly continuing with Fig. 2-2 and comparing it with Fig. 2-2 (B), we note that what was positive 90° previously now is —270°, what was positive 180° previously, is —180°. The 0° starting 360° starting and finishing points are unchanged, although we show the 360° finishing point in Fig. 2-3 preceded by a minus sign to indicate that rotation is in the negative (clockwise) direction between the —270° point and the finish line.

As to the quadrant references, you will note that they do not change with the direction of angle rotation, remaining the same at all times. Thus identification of a vector as being between certain coordinate boundaries can still be made without any complications, because at no time are both arrangements of angle rotation employed simultaneously, moreover the angle designations are either positive or negative, depending upon the conditions being shown and this is indicated.

CHAPTER 3

SINGLE VECTOR REPRESENTATION

Now we are ready to take the first deep plunge, the presentation of single line vectors, but before doing so let us devote a few minutes to a general review. As a matter of convenience below is a list of the basic conditions which we have discussed in some detail.

1. Any quantity which has both magnitude and direction can be represented by a vector.
2. A vector is shown by a straight line which has a length corresponding to the magnitude of the quantity and is oriented in accordance with the direction component of the quantity.
3. The vector is shown in accordance with the coordinate system, which makes use of two perpendiculars.
4. Direction of the vector is indicated in terms of the 360 degrees in a circle, with a 0° reference point.
5. The direction of rotation is shown by either positive or negative angle designations.

With these basic details behind us, we can consider the first single-line vector.

Development of an A-C Voltage

You may recall several references to the fact that an alternating voltage or an alternating current is a quantity possessed of two components, magnitude and direction. Since these voltages and currents are paramount in electronic work, regardless of the manner in which they are generated or used, it seems only fitting and proper to employ them as the basic examples of vector presentation. Such will be done utilizing the rotating generator as the source of the a-c voltage. This device is selected because it permits ideal association with the vector. Inasmuch as a general understanding of electrical and radio principles is assumed in this text, we shall devote only so much attention to the operating features of the rotating generator as is essential to make its association with the vector understood.

Accordingly in Fig. 3-1 (A) is shown the basic a-c generator as a two-pole device with a single-turn armature. The two coil sides are indicated as *AB* and *CD;* these sides rotate past the pole pieces *N* and *S* in a *counterclockwise* direction with a constant velocity and in doing so cut the magnetic lines of force (flux lines) which exist between the faces of the pole pieces. (This magnetic field is assumed to be such that a single-turn armature will produce a sine wave of voltage.) Conforming to adopted convention, when the armature moves past the *N*

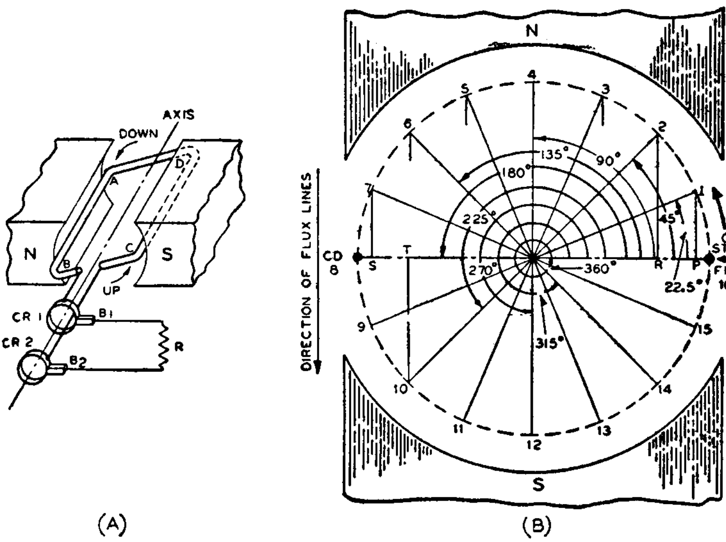


Fig. 3-1 (A)—A basic a-c generator in which the coils rotate past the pole pieces in a counterclockwise direction with an alternately positive and negative polarity. (B)—A head-on view of the generator reveals how the coil sides in conjunction with the parallel lines of flux provide a zero-voltage reference line. During the armature rotation a varying vertical displacement also occurs. The center of the circle is point *O*.

and *S* poles in the counterclockwise direction, the polarity of the voltage at the terminal CR_1 is alternately positive and negative with respect to terminal CR_2.

As to the fundamental conditions which determine the development of a voltage when a moving conductor cuts a stationary field, the maximum voltage is induced when the conductor cuts the field at right angles, and zero voltage is induced when the conductor moves parallel to the field; in other words, the voltage developed depends upon the rate of cutting flux lines.

Let us now look at this fundamental generator from a different angle, a head-on view, as illustrated in Fig. 3-1 (B). The two coil sides of the armature are *AB* and *CD*, and the two pole pieces are *N* and *S*. As illustrated, the coil sides are in the position of zero voltage, since their motion is parallel to the direction of the flux lines. Thus the imaginary line *8-16* is the zero voltage reference line.

Any motion of the armature from this zero voltage point will result in the development of voltage. Using coil side *AB* as the reference side, we note two points of maximum or peak voltage development during armature rotation. These are points *4* and *12* on the circle inscribed by the reference coil side, these being the two positions when the conductor is cutting the flux lines at right angles or at the maximum rate. In one of these positions *(4)*, the terminal connected to coil side *AB* would be positive and in the other maximum voltage position *(12)*, it would be negative. All other positions of the coil side result in output voltages less than maximum, with a minimum of zero.

Another way of analyzing Fig. 3-1 (B), would be to consider the motion of the reference coil side in terms of angular displacement from its zero voltage or 0° position. As the coil side rotates, it inscribes a gradually increasing angle until the full 360 degrees are covered when the cycle of voltage is completed and the coil side returns to its starting position. These gradually increasing angles are shown by the dotted line positions of the imaginary radius of the circle *O-AB* corresponding to the 0° position. We have selected 16 such positions, each indicating a change of 22.5°.

At the same time, concurrently with the angular displacement, a vertical displacement of the rotating coil side also takes place with respect to the horizontal zero voltage reference line. These are shown by the perpendiculars *1-P, 2-R, 7-S,* and *10-T,* drawn to the 0° reference line from the various angular displacement positions. Only four of these are illustrated, although each of the points indicating a momentary position of the rotating coil side could have a perpendicular drawn to the horizontal zero voltage reference line, and each of these would show the vertical displacement of the coil side with respect to the zero voltage reference line at the indicated instants.

The amount of vertical displacement can be expressed in another manner, namely, as the sine of the angle formed between the zero voltage reference line *O-AB* and the momentary position of the coil side on the circle it inscribes during its rotation. These angles for each position of the coil side are shown by the associated arcs and angle designations.

With respect to the vertical displacement, you can readily see that the two points of maximum displacement conform with the two points of maximum voltage, namely points *4* and *12*. This is not a coincidental condition, but rather a fundamental condition associated with the development of the voltage. All other positions of the coil side involve less vertical displacement, consequently less output voltage. The relationship between the maximum output voltage and maximum vertical displacement at 90° and 270°, and the other voltages and vertical displacements bears the same relationship as the sine of the angles formed by the momentary positions of the coil side and the zero voltage reference line at those voltage positions.

For instance, the sine of a 90° angle is +1, which means unity in a positive direction, and for a 270° angle, it is —1, which again means unity, but this time in a negative direction. (The plus and minus signs are indicators of angle direction.) Thus proper identification of the momentary maximum output voltage for the 90° and 270° positions of the soil side would be $E_m \sin 90°$ and $E_m \sin 270°$ respectively. In both instances the multipliers are unity, and the plus and minus signs indicate the relative polarity of the output voltage. If for the moment we assume a maximum voltage of 100 volts as the output of the generator, $E_m \sin 90°$ would be a positive voltage of 100 volts, whereas $E_m \sin 270°$ would be equal to a negative voltage of 100 volts.

Using the two points of maximum voltage as the basis of the discussion, all other momentary voltages obtainable for various positions of the coil side are less than maximum. Moreover, since the vertical displacement is the maximum at the two moments of maximum voltage, all other possible vertical displacements of the coil side must be less than maximum, with zero conforming with the moments of zero voltage. Accordingly, there must be some relationship between the maximum and less than maximum values of voltage; also between the maximum and less than maximum vertical displacements. There is such a relationship, and it is in accordance with the sine of the angles traversed by the moving coil side.

For instance if the coil side moves to position 2 in Fig. 3-1 (B), it has advanced through 45°. Obviously the vertical displacement is less than the maximum, therefore the instantaneous output voltage is less than maximum. Its amount then is expressed as

$$E_2 = E_m \sin 45°$$

and since the sine of an angle of 45° is 0.707, the instantaneous voltage when the coil side is at point *2*, is 100 × 0.707 or 70.7 volts. That this voltage is positive is indicated by the fact that during counterclockwise rotation the sines of all the angles from 0° to 180° are positive, whereas the sines of all angles between 180° and 360° are negative. If the instantaneous voltage at point *3* is desired, it is expressed as

$$E_3 = E_m \sin 67.5°$$

and since the sine of 67.5° is 0.924, the instantaneous value of the voltage is 100 × 0.924 or 92.4 volts. Again it is positive in its relative polarity. For an angle of 225 degrees, corresponding to position *10*, the expression would be

$$E_{10} = E_m \sin 225°$$

and since the sine of 225 degrees is the same as 45°, except that it is negative in character, it is —0.707, which means that the instantaneous voltage would be 100 × (—0.707) or —70.7 volts, and that it would have a negative relative polarity.

The sine of the angle relationship expressed between the instantaneous maximum voltage and less than maximum voltages also exists between the maximum vertical displacement and the less than maximum vertical displacements. Thus the vertical displacement or the length of the perpendicular *2R* corresponding to coil side position *2* or 45° bears the same relationship to the maximum displacement at 90° or position *4*, as the sine of the two respective angles, namely 0.707 for 45° and 1.0 or unity for 90°.

At first glance, these references to the sine of an angle may seem confusing to the man who has not studied trigonometry. Actually it is not difficult, because it is given in tabular form in mathematical texts and is nothing more than a mathematical means of expressing the development of a curve which we have learned to recognize as a sine wave.

The sine wave is the basic wave of either current or voltage, and the sine table is the means of all a-c computations, establishing the value of voltage at any instant of time. For the sake of convenience we list an abridged sine table in steps of 22.5°, and if the maximum or peak value of a voltage is known, the instantaneous values along the cycle can be computed.

Angle	*Sine Value*	*Angle*	*Sine Value*
0°	0	202.5°	—0.383
22.5°	0.383	225°	—0.707
45°	0.707	247.5°	—0.924
67.5°	0.924	270°	—1.000
90°	1.000	292.5°	—0.924
112.5°	0.924	315°	—0.707
135°	0.707	337.5°	—0.383
157.5°	0.383	360°	0
180°	0		

In order to illustrate how such a rotating armature develops a sine wave of voltage, we have combined the illustration of the generator with the sine wave of voltage in Fig. 3-2. The 16 points on the wave correspond to the 16 indicated positions of the reference coil side. The horizontal time baseline bears the angular displacements of the rotating coil side, which also represent the relative amplitudes of the wave with respect to time stated in degrees. The relative direction or polarity of an a-c wave of voltage or current is clearly evident in Fig. 3-2, as is the change in magnitude, so that future reference to an alternating voltage or current as being a vector quantity will require no elaboration.

Vector of an A-C Voltage

When dealing with sinusoidal quantities, for example, sine waves of voltage or current, one method of presentation is an illustration of the curve on a rectangular coordinate, such as was done in Fig. 3-2. However, there are numerous instances when more than one such wave is involved, that is, more than one quantity must be considered. Even then it is possible to show the curves of each of the waves, but such type of illustration is more laborious in many respects, especially the explanation of the action involved. Vectorial representation is by far simpler, as you will see as you progress through these pages.

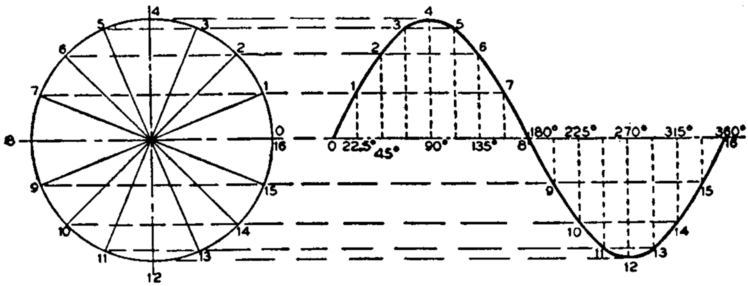


Fig. 3-2—Illustration of how the rotating armature of the two-pole a-c generator of Fig. 3-1 develops a sine wave output during one revolution of the armature.

To present a single a-c voltage in vector form rather than the curve form of Fig. 3-2, does not involve anything new. As a matter of fact, the reader with imagination, recalling the information given in the first two chapters of this text, by this time may have come to the realization that the graphical representation of the movement of the coil side in the generator illustration of Fig. 3-1 (B), repeated here (Fig. 3-3) for the sake of convenience, is the equivalent of a vectorial presentation. As a matter of fact there is such a close relationship between the vectorial presentation and the position of the rotating armature with respect to its zero voltage position, that it is possible to describe the vector of an alternating voltage as being a picture of the position of the armature in the rotating generator which produces the voltage. Let us see if this is so by comparing part (A) of Fig. 3-3, with parts (B), (C), (D), (E), (F), and (G), each of which is a vectorial representation of the instantaneous voltage developed by the generator when the armature is in different positions.

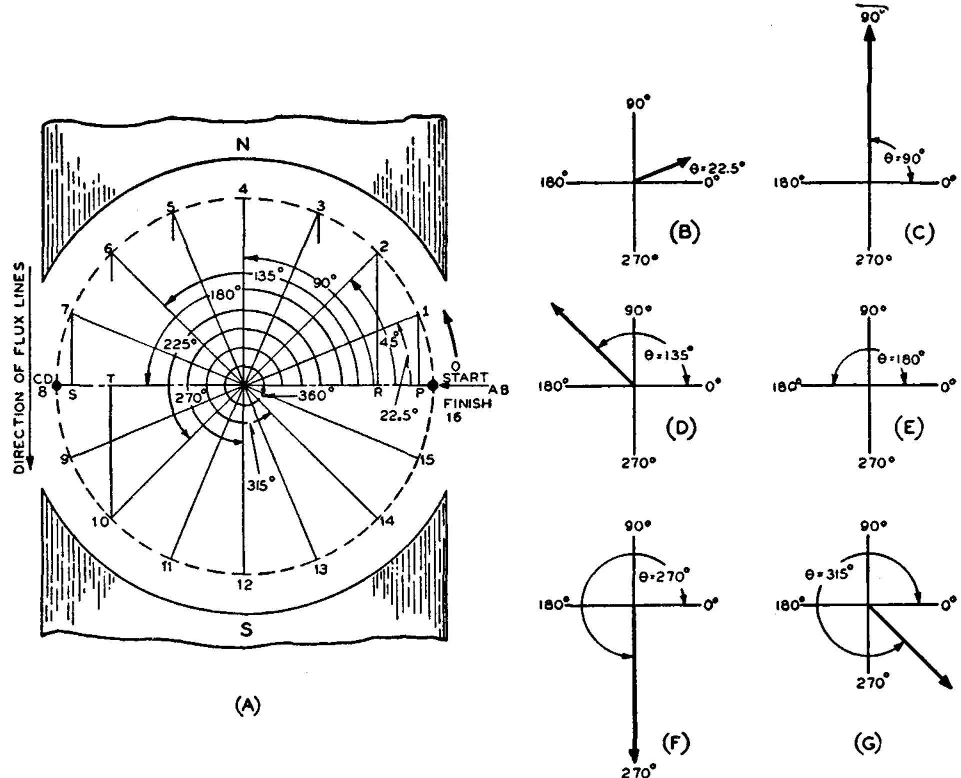


Fig. 3-3—In (A) is the graphical representation of the coil side movement in the generator of Fig. 3-1 (B). Vectorial representations of the instantaneous voltage developed by an a-c generator at six different angular positions of the armature between 22.5 degrees and 315 degrees are shown in Figs. 3-3 (B) to (G) inclusive.

Take as the first example the instantaneous voltage developed when the armature has moved through 22.5°. This voltage can be expressed as

$$e = E_m \sin 22.5°$$

Properly to display this vectorially, we must do certain things; first, provide the required coordinates; second, set an arbitrary length for the vector, in this case 1 inch, as the equivalent of the 100 volts maximum ($E_m = 100$ volts); third, indicate the angles on the horizontal and vertical axes; and fourth, position the vector in accordance with the description of the voltage. All of this is shown in Fig. 3-3 (B), and anyone familiar with vectorial presentation will, upon looking at this illustration recognize certain facts, namely that

1. An instantaneous value of voltage is being shown.
2. This voltage has a relative phase of 22.5°.
3. The voltage is in the positive alternation of the cycle.
4. Value of voltage is 38.27 volts as indicated by the length of the vector.

In like manner Figs. 3-3 (C), (D), (E), (F), and (G) illustrate instantaneous values of

(C) $E_m \sin 90° = 100 \times 1 = 100$
(D) $E_m \sin 135° = 100 \times 0.707 = 70.7$
(E) $E_m \sin 180° = 100 \times 0 = 0$
(F) $E_m \sin 270° = 100 \times -1 = -100$
(G) $E_m \sin 315° = 100 \times -0.707 = -70.7$

all of which are determinable directly from the vector dimensions.

At this point we have reached the stage of vector development where we can forget the rotating machine as the generator of the sine wave and think solely in terms of sine waves and vectors without any regard to the device which generated the voltage wave — for that matter, the device which produced the voltage which caused current to flow through a system — that is, if we are interested in representing current vectorially.

It is a well-known fact that numerous types of devices are capable of producing sine waves of voltage or any types of waves; in electronic circles, the vacuum tube is more frequently the generator of sine waves than the rotating machine. But no matter what type of device produces the voltage or current, the vector representation is unchanged; it is as we have shown in Figs. 3-3 (B) through (G).

Relative Phase

Concerning the vector representations, they shall be used henceforth to describe in full the character of the wave, whether it be of voltage or current. To make this clear we feel that it might be well to devote a few moments to the meaning of "relative phase," a term we used in connection with the vector illustrations of the generator output. If that sine voltage had been produced by a vacuum tube and represented vectorially, the same reference to relative phase would have been made. In other words "relative phase" is a term which is applied to a *single quantity* to express its character at any one instant with respect to *its zero-degree reference point.* Thus every point on a sine wave curve has a relative phase with respect to its zero reference point, which is expressed in degrees.

As to the functions of the "relative phase" of a sine wave (current or voltage), it is a means of expressing the instantaneous magnitude of the wave with respect to its maximum. Thus, if for one wave the relative phase is said to be 90°, we immediately know that the wave is at its positive peak value; if it is 30°, we know that at the instant stated the magnitude of the wave is half of its peak value ($\sin 30° = 0.500$) and positive in its direction. If the relative phase is 270°, we immediately know that the voltage is at its negative peak. As a matter of fact any value of relative phase between 0° and less than 90°, and between 90° and 180° represents a fraction of the positive peak value. Likewise a relative phase between 180° and less than 270°, and between 270° and 360° represents a fractional value of the negative peak.

It is important to remember that "relative phase" and "phase difference" are two different terms with different meanings. One should not be confused with the other, in that *relative phase is used with a single quantity,* whereas *phase difference is applied when more than one quantity* is involved, as will be shown later in this book. In passing we might mention that the phase difference between two waves is the difference between the relative phases of the individual waves. This will be explained in more detail in chapter 4.

CHAPTER 4

MULTIPLE VECTOR PRESENTATION

Having described some basic examples of single-line vectors, although not necessarily all quantities which may be so illustrated, nor the possible resolution of the single-line vectors, we are ready to consider the basic quantities which require more than a single line for vector representation. But in view of where this discussion will lead, it is necessary to digress for a moment and to discuss the elementary concepts of phase difference.

Phase

Phase, sometimes called "phase difference," "phase displacement," "phase relation," or just simply "phase angle" is a very important condition in radio systems. Fundamentally it denotes a certain relation between two or more periodic quantities, such as two or more voltages, two or more currents, or two or more voltages and currents. This relationship is very important not only because it is necessary in connection with the analysis of a-c circuits, but also because the magnitude of several alternating currents or voltages in a circuit cannot be determined without taking the phase relationships into account. The reason for this is that all alternating currents in a circuit are not necessarily additive at every instant; rather they may buck each other at one moment and aid each other at the next to a greater or lesser degree. The same, of course, is true in the case of voltage. Thus the direction of the voltage or current, whichever is being considered, is important.

Moreover the elements of inductance, capacitance, and resistance, which usually comprise an a-c circuit, do not behave in like manner under the influence of an alternating voltage; that is, the current caused to flow by the application of a voltage does not reach its maximum and pass through its minimum simultaneously through these elements, that is, keep in step. Consequently, to appreciate what is happening in the circuit, it is imperative to recognize the phase relationships existing between the current and the voltage through the elements as well as in the circuit as a whole. Therefore, we shall give a brief review of phase relationship as applied to alternating currents and voltages under different conditions.

As a starting point, let us assume two sources of alternating voltage. Just what they are makes no difference at the moment; all that matters is that both are capable of producing sine waves of voltage. Now we start one in operation and Fig. 4-1 (curve *A*) illustrates the wave of voltage being produced. For the sake of clarity, we shall consider this voltage wave at a frequency of one megacycle (1,000,000 cycles); as shown along a rectangular coordinate. Each cycle represents a lapse of one one-millionth of a second, but more important, represents 360 time degrees. As you probably know, when speaking in terms of time degrees, the actual frequency of the wave need not be mentioned. If

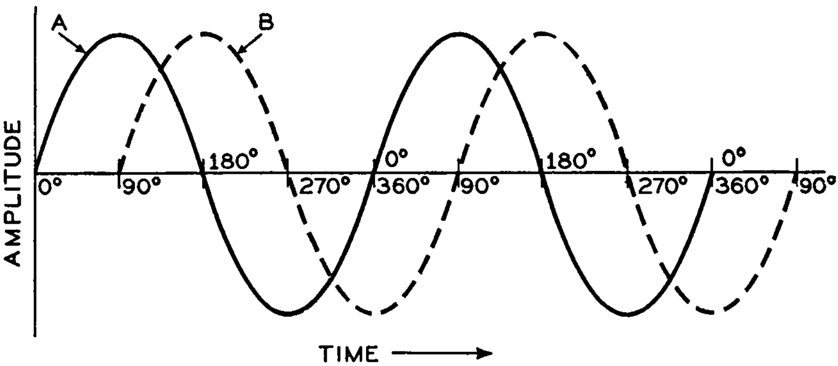


Fig. 4-1—Sine waves *A* and *B* maintain a constant phase difference of 90 degrees set when voltage source *B* started functioning one-quarter cycle later than voltage source *A*. Here *A* is said to be leading *B* or, conversely, *B* is lagging *A*.

you wish to consider this wave of voltage as originating in a rotating generator, you can do so, but recall that there are no rotating devices capable of producing voltages of this high frequency. When desired they must be produced by utilizing vacuum tubes, with or without piezo-effect crystals.

Let us now imagine that one four-millionth of a second after the first voltage source has started in operation, voltage source *B* starts functioning. Since one cycle requires a lapse of one one-millionth of a second, a time interval of one four-millionth of a second is equal to one quarter of a cycle; in other words, voltage source *B* starts functioning one-quarter cycle after voltage source *A*, or 90° later.

Having started 90 degrees apart and being constant in operation, the two voltage sources are generating voltages which are of constant frequency, so that the *phase difference* of 90° which existed between voltage *A* and voltage *B* the instant voltage source *B* started functioning is maintained throughout each cycle, as long as the two sources remain constant in their operation. Regardless of what point you may select on wave *A*, the corresponding point on wave *B* will differ by 90°.

Lag and Lead

To establish the phase difference between two voltages which may be shown graphically or which may appear on an oscilloscope screen, some sort of standard point of reference must be chosen or indicated. As it happens, the determination of phase difference does not really require a reference point, for just as long as the two waves are constant in frequency the selection of corresponding points anywhere along the two waves will result in the same conclusion relative to the phase difference. However, to determine which of the two waves is *leading* or *lagging* the other by an amount equal to the phase difference does require the reference point.

This brings up the subject of direction once more. We stated in connection with the sine wave output of the generator shown in Fig. 3-2 that, while polarity is relative, the half cycle or alternation, shown above the zero-voltage axis or time baseline on which the time degrees are indicated, is considered positive, and the alternation or half cycle below the time axis is considered negative. Thus it is possible to speak about a wave as *positive going* or *negative going,* by which is meant the direction in which it is going. The wave is positive going the instant that it decreases from its maximum negative peak and continues positive going until it has reached its maximum positive peak. The instant it passes that point it is negative going until it reaches its maximum negative point; when it passes that peak, it again is positive going, and this is repeated during each cycle.

To determine lag or lead, between which there is direct correspondence, the matter of direction must be taken into account after the reference point has been chosen. As to the latter, either the peak or the zero voltage points are usually selected as reference. Moreover, you must remember that in all representation, unless otherwise stated, the beginning of the time elapsed in degrees is at the left end of the time baseline. Now combining the selected reference point on the wave with the time degrees shown, the lead is determined by whichever voltage first passes through *zero in the negative going direction.* As is evident in Fig. 4-1, wave *A* passes through zero in the negative going direction earlier (in time degrees) than wave *B;* therefore, wave *A leads* wave *B,* or conversely wave *B lags* wave *A,* both by 90°.

The fact that we selected 90° phase difference as our first example has no special significance. It is not the standard value; the standard is 0°, for as you can see all phase difference is with respect to that condition when the waves being compared are in step, that is, passing through their maxima and minima IN THE SAME DIRECTION at the same instant.

In connection with the phase difference between two such waves — periodic quantities — one important detail deserves further mention. This is the fact that when considering the phase difference between any two waves the angular displacement of each wave by itself is of no significance. In other words, two waves may differ by 90° when one has a relative phase of 270° and the other either 180° or 360° (0°); or when one has a relative phase of 14° and the other, 104°. The corresponding points on each wave selected as representing a phase difference of 90° may have any relative phase between 0° and 360°. This should be remembered because it will receive further mention when phase difference is shown vectorially.

Phase Difference and Relative Phase

A 90° phase difference is just one relationship, it being possible to attain any phase difference between 0° and 180°. The last-named figure is the maximum, despite the fact that time axes may show angular displacements (time degrees) as high as 360°. If you will recall the organization of the coordinate system and the assignment of the degrees bounding the four quadrants, you will remember that the 270° axis was actually 90° away from the 0° axis. This is evident in Fig. 4-1. If you will locate a relative phase of 270° on wave *A*, you will note that it corresponds with a relative phase of 180° on wave *B*, the difference between the two being 90°; the same phase difference will be found if you compare the point on wave *A* which corresponds to

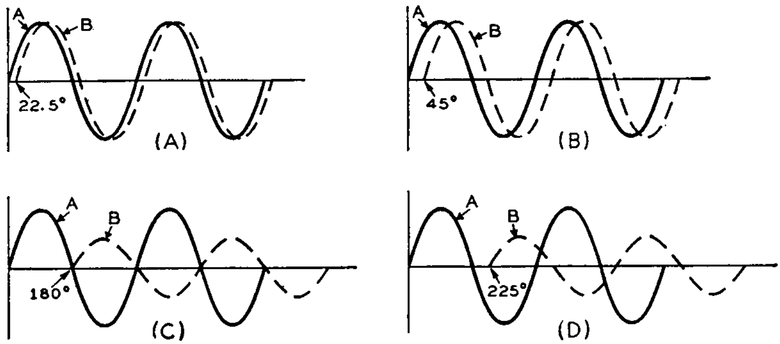


Fig. 4-2—Sine waves of the same amplitude are illustrated in (A) and (B). (A) indicates a phase difference of 22.5 degrees and (B) indicates a phase difference of 45 degrees. In each case *A* is leading *B*, *or* B is lagging *A*. In (C) and (D) wave *A* is still leading wave *B*, even though the amplitude of wave *A* is twice that of wave *B*. The phase difference of 180 degrees in (C) and 225 degrees in (D) remain constant throughout.

180° and the 90° point on wave *B;* or the 90° point on wave *A* with the 0° point on wave *B*. We make these comments because of the manner in which the vectorial presentation of the phase difference between two waves takes the relative phase of each into account.

Concerning values of phase difference other than 90°, several are shown in Figs. 4-2 (A), (B), (C), and (D). These are 22.5°, 45°, 180°, and 225° respectively. In each case wave *A* leads wave *B*, or wave *B* lags wave *A*. In order to illustrate that the relative amplitudes of the respective waves have no bearing on the phase difference, the leading wave is shown as twice the amplitude of the lagging wave in Figs. 4-2 (C) and (D).

Up to this time we have been speaking about phase difference between two voltages. Whatever has been said is true in all respects about two currents, or for that matter about more than two currents, or between voltage and current. This conforms with that portion of the definition of phase which refers to "any two or more periodic quantities." Now we are ready to display these relationships by means of vector diagrams.

Vectors Representing Two Quantities

The previous chapter discussed illustrations of single vectors in which their relation to the coordinate axes establishes their direction, and the length of the vector denotes the magnitude. What is done when two or more quantities, regardless of their character, naturally comparable, are to be examined vectorially? Suppose that it is desired

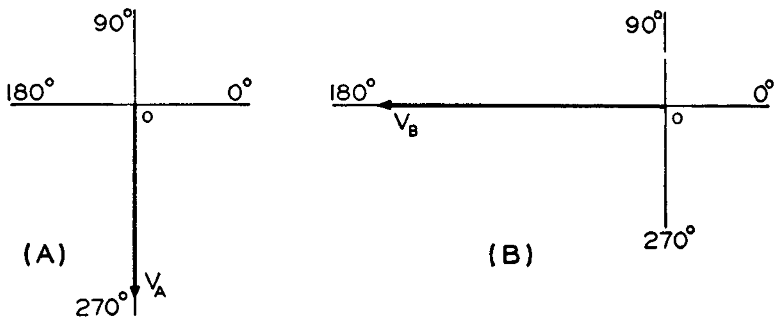


Fig. 4-3—The scale for both voltage vectors is 10 volts to the inch. Voltage V_A being 10 volts is a vector one inch long as indicated in (A). Voltage V_B being 15 volts is a vector 1½ inches long as indicated in (B). Both vectors are drawn with respect to their relative phase.

to compare two voltages relative to phase and magnitude. In order to appreciate vector comparison, it might be best to develop multiple presentation by means of a series of basic steps.

Let us assume two basic sine waves of voltage, A and B, the former of 10 volts maximum and the latter of 15 volts peak. The relative phase of voltage A is 270° and that of voltage B is 180°. . . . The phase difference between these two voltages is similar to that of Fig. 4-1. Now we shall show each of these voltages as a separate vector on its own coordinate system. In order to set the length of the vector to correspond with the magnitude of the quantity, we shall arbitrarily establish that 10 volts equals 1 inch.

Accordingly, voltage *A* is shown as a vector V_A 1 inch long and drawn on the 270° reference line in accordance with its relative phase. This is shown in Fig. 4-3 (A). Voltage *B*, having a maximum value of 15 volts would be a vector 1.5 inches long and since its relative phase is 180 degrees, it is drawn as a vector of that length on the 180° coordinate in Fig. 4-3 (B) as V_B. No doubt you remember that the positioning of these vectors is in accordance with their relative phase which is with respect to 0°. Since counterclockwise or positive angular rotation is assumed, the vector line is placed in that position on the coordinate system which corresponds to its relative phase.

Vector Notation

Examining the two vector pictures in Figs. 4-3 (A) and (B), we note certain details. First, we have two separate coordinate systems, each with its pair of axes. Second, upon each is shown a single vector with its initial or starting point being located at the intersection of the axes, which is as a rule the location of the starting point of multiple vectors which differ in phase. The exceptions to this statement will be discussed as they are introduced. Third, the intersection of the axes bears the letter *O*. This is a reference notation and should not be confused with the zero-degree (0°) designation on this reference line. Thus a vector may bear a notation at its initial point and at its terminal point, which in Fig. 4-3 (A) would be OV_A, these notations corresponding to those associated with the initial and terminal points of the vector. Naturally there is no rigid rule which dictates that the crossover point of the axes be indicated by the letter *O*; any letter which is suitable may be used at the discretion of the author or illustrator. The same is true in connection with the identification associated with the terminal point; any symbol or letter which fits the occasion may be used. In our case V_A is voltage A and I_A is current A, assuming that the vectors show current instead of voltage. As you can see, the identification of what the vector shows is accomplished by means of the notation *at the terminal point* and not the starting point.

There are many ways in which vector notations are written. The more common ones in use are $\overline{OA}$, that is letters corresponding to the starting and terminal points, with a short dash above the letters. Others may be the letters in boldface type in contrast to medium or light-faced type used elsewhere in the book, and sometimes the letter at the starting point is omitted and only the letter at the terminal point mentioned. In contrast we shall mention both the starting and the terminal letters. When this notation is used, the first letter should be understood as being the initial point and the second letter, with or without a subscript or prime letter or number, is the terminal point. Therefore, vector *OA* means that the direction of the vector is towards the letter A, that is, the pointing of the arrowhead is in that direction. On the other hand, if this vector in Fig. 4-3 (A) were reversed in its direction, that is pointing towards the intersection, it would be *AO*.

Continuing with Figs. 4-3 (A) and (B), if for the moment we forget that we have already established the maxima, the relative magnitudes of these voltages are determinable by measurement of the resepective vector lengths, knowing the base as being 1 inch equals 10 volts. Also upon examination, looking first at one vector and then at the other, even without the degree markings written on the diagram, vector *A* is seen to have a greater relative phase than vector *B*. Moreover, with the 0° coordinate as the basis, it is easy to see that voltage *A* is ahead of voltage *B*, since the angular designation of voltage *A* (with respect to the 0° reference point) is greater than that of voltage *B*. To determine the exact extent of the *lead*, the relative phase of the lesser is subtracted from the greater, that is 270° — 180° = 90°, which is the *phase difference* between the two voltages.

One Axis Representation

To compare the two vectors as we have just done is quite easy to understand, but entirely too cumbersome. What makes this so is the use of separate coordinate systems for each vector. In order to compare the two voltages most rapidly, that is, if they differed by small amounts in magnitude or angular displacement, the use of separate coordinate systems would make the operation quite tedious. The situation becomes even more difficult if more than two vector quantities are involved. All of this is alleviated by the use of a single coordinate system for multiple vector presentation.

The use of a single coordinate system facilitates comparison of magnitude and phase regardless of the number of quantities involved.

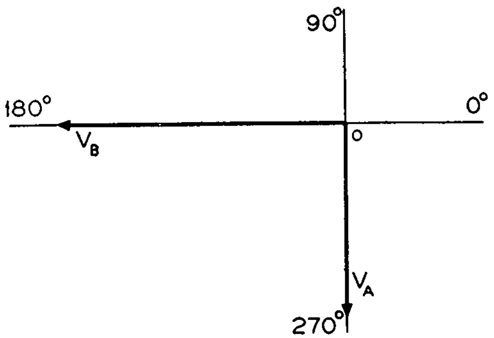


Fig. 4-4—To facilitate the comparison of magnitude and phase, vectors OV_A and OV_B of the two separate coordinate systems of Fig. 4-3 are shown on a single coordinate system.

To show how this is done, let us imagine that the complete coordinate system, including the vector — shown in Fig. 4-3 (B) — is superimposed upon the coordinate system and vector of Fig. 4-3 (A). The final result is Fig. 4-4. Such superimposition is in order since the coordinate system is a fixed structure and no matter how many examples are employed, the basic coordinate arrangement is the same. The length of the coordinate lines has no significance; in fact they need not even be shown, because they can be imagined once the basic 0° reference position is known. The same is true of the angular designations. Showing them is simply a matter of convenience; they too could be omitted if the angle of each vector is indicated, or if the process of solution calls for physical determination of the angle by suitable means.

With the two coordinate systems combined, only one set of lines appears, with the two vectors in their proper places. From this illustration of two vectors on a single set of coordinate axes, we can readily compare the two quantities. A glance is sufficient to show that vector OV_A (voltage A) is shorter than vector OV_B (voltage B), which means the magnitude of the former is less than that of the latter; also the phase difference between the two voltages, as well as the leading or lagging voltage — whichever is of concern — can be seen without any trouble. Other examples of two voltage vectors on a single set of axes are shown in Figs. 4-5 (A), (B), and (C). The conditions associated with the representation are shown in the respective captions. If for

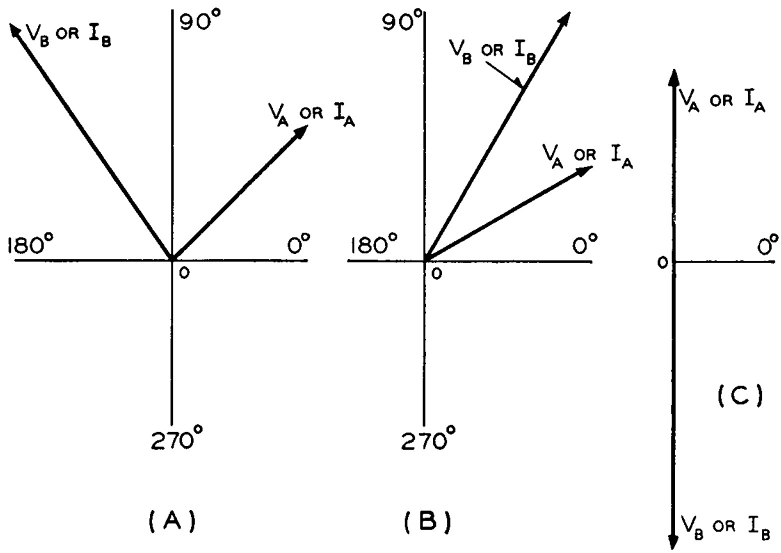


Fig. 4-5—Examples of two voltage vectors on a single set of axes. In (A), (B), and (C) the vectors can represent voltage or current. In all parts vector OV_B (or OI_B) is greater in magnitude and leads vector OV_A (or OI_A). Though the four coordinate lines are not shown in (C), the relative phase of each vector and the phase difference between them are readily apparent.

the sake of a change, you wish to consider these quantities as being current instead of voltage, all that is necessary is to think in terms of amperes or milliamperes of current instead of volts.

Special attention should be paid to the vector illustration in Fig. 4-5 (C). If you will compare this figure with its associated illustrations, the absence of the complete set of axes will be noted. The figure shows two voltages 180° apart, and although the four coordinate lines forming the coordinate system are not shown, except the 0° reference line, nor the angle designations marked on the illustration, the relative phase of 90° for vector OV_A and the relative phase of 270° for vector OV_B are readily apparent. It might be well to mention in passing that such types of presentations are quite common; every author who has occasion to utilize vectors does not as a rule show the full set of coordinates, although they will be used in all examples discussed in this text. So do not be confused if you see vector pictures without the coordinates. Their meaning is the same as if the full set of coordinates, each with its angle designation, were shown.

Quantities in Phase

In all of the dual vectors shown thus far we have consistently and deliberately selected those which involved a phase difference. Let us consider two quantities, either voltage or current, which have zero phase difference, or are *in phase*. Accordingly, we illustrate in Fig. 4-6 (A), two waves of different amplitude but in phase with each other. As you can see the two waves pass through their positive and negative peaks, as well as their zero points in step with each other. When such a condition prevails, regardless of the character of the two quantities, whether they are voltage and current, current and current, voltage and voltage, or anything else, the relative phase of each wave is the same as that of the other wave, so that the difference between these relative phases being zero, the phase difference is designated as zero or 0°

However, it is significant to note that, while the phase difference between the two quantities is zero, each quantity possesses its relative phase at any instant, so that if two such quantities are to be displayed vectorially, the location of the vectors on the coordinate system is determined by the relative phase at the instant of time selected.

Suppose that we select the maximum voltage point as the instant of time. As you know this can be either the 90° or 270° points. We will choose the former. Since the two voltages are in phase, the two quantities have the same direction, consequently the vectors must be

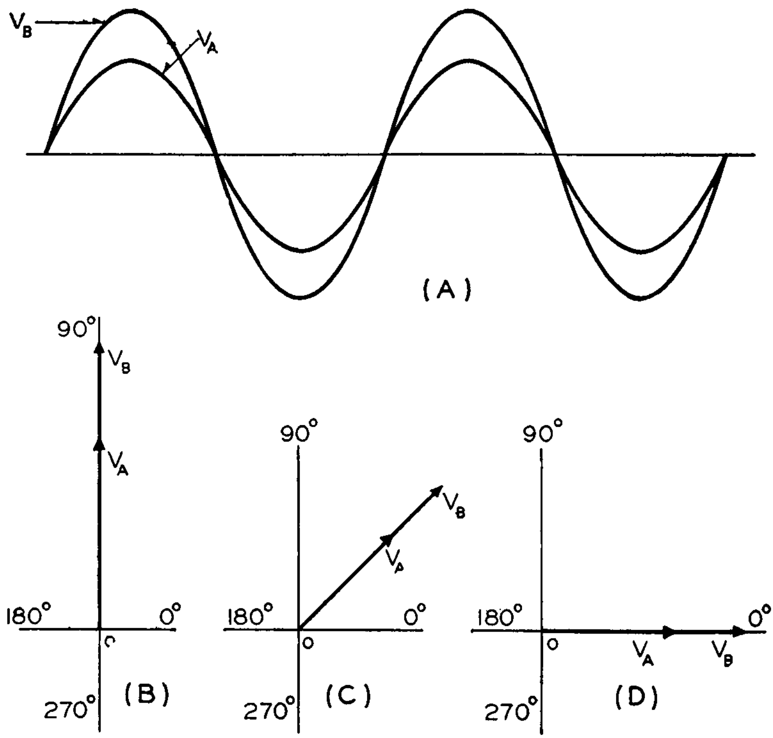


Fig. 4-6—V_B and V_A (A) represent two waves which are of different amplitude but of the same phase. In vector representations (B) and (C) of this relationship, one vector must be superimposed on the other, and because of their different magnitudes, one is longer. Each vector diagram represents a different instant of time. (D) shows the common method of expressing the practical values of curves *A* and *B* vectorially.

pointed in the same direction. To do this means that the one vector must be superimposed on the other. Since the two quantities have different magnitude, the two vectors will be of different lengths, as shown in Fig. 4-6 (B). An example of vector illustration of two such quantities in phase, but with a different relative phase, that is a different instant of time chosen, 45° to be exact, is shown in Fig. 4-6 (C).

Recognition of the existence of a relative phase in the case of two or more quantities which have zero phase difference is important, because in some instances some other quantity may differ in phase from the pair which have zero phase difference, and placing these quantities on the coordinate axes is determined by this condition.

Practical Values of Quantities

At this stage in the development of understanding of the vector, we reach a point where we must introduce some variations of the statements which have been made. Technically speaking, these variations are not contradictions of what has been said, but rather interpretation of the theoretical into the practical, so do not be confused by the new thoughts which are injected.

So far in this text we always have spoken of instantaneous values of the quantities, and when we mentioned voltage and current we always talked in terms of what conditions existed at an instant of time. Now we shall speak in terms of practical or useful values. In other words, it is simple to understand that a voltage which has a peak of 100 volts and an effective value of 70.7 volts are one and the same voltage. Also that a current with a peak value of 1 ampere and an effective value of 0.707 ampere is one and the same thing. As a matter of fact the usual method of identifying the magnitude of voltage and current, with respect to its utility to do a job, is in terms of its effective values, sometimes the average value, but in most cases the root mean square (rms) or effective value. A comparison of phase difference between two quantities, two voltages or voltage and current, for example, can just as readily be made in terms of the effective values as the instantaneous values, inasmuch as a constant phase difference between two quantities remains unchanged when the quantities are spoken of in terms of effective values, or when the quantities are expressed as instantaneous values.

At first thought you might interpret what we are saying as affecting the length of the vector. Such is not the case; it is a matter of what is general practice in the vector illustration. Changing the length of a vector to correspond with the effective rather than the peak value means nothing, because the relationship between the peak and effective values is fixed, so that one is determinable from the other. The matter we are anxious to bring to your attention pertains to the positioning of the vector.

You will recall that we discussed the 0° (360°) reference axis as that which corresponds to the zero voltage point, in which case the vector would have no magnitude. At the same time we also mentioned the fact that this reference line was the start and the finish of a rotating vector which could describe a complete cycle of a periodic quantity such as alternating voltage or current. However, there is no rigid rule which dictates that the beginning and completion of 360 time degrees must start and end at the 0° line. It is conceivable that, because of the nature of a system capable of generating a voltage, the instant the voltage starts may correspond to a relative phase of 90° or some other angular value, in which case a complete cycle of that quantity will be completed when 360° have been covered, but this will not coincide with another quantity which may have actually started its cycle at the 0° point. In other words, the 0° and 360° axis in the coordinate system is more of a *reference line* than actually the starting point of each cycle. This leads up to this final thought.

In practice you may find that, when illustrating phase relationships between quantities, the reference quantity may be shown on the 0° reference line, totally disregarding the relative phase of this reference quantity; also that it does have magnitude. By ordinary reasoning you would expect that any quantity shown on the 0° reference axis would represent zero voltage, which is correct when we construct the sine wave by projecting the vertical displacement of the rotating vector with respect to the zero voltage reference line. But in many vector representations we do not concern ourselves with the development of the sine wave. Instead we are picturing a relationship *between* quantities, so that the function of the 0° axis is purely a reference axis, and it is quite in order to project the reference quantity, or for that matter any other identified quantity, along this axis. This is done in Fig. 4-6 (D), and this, rather than the vector illustrations in Figs. 4-6 (B) and (C), is the commonplace method of showing the reference quantity in texts and technical periodicals.

In the light of the foregoing it is evident that conditions at any one instant of time along a cycle do not matter very much, since whatever constant phase relationship is being shown requires reference to neither the relative phase of the quantities nor the instantaneous value. It is entirely correct to illustrate two voltages in phase with vector lengths equal to the effective values, and these voltages may then be compared with still a third or fourth voltage, by suitably positioning the other voltages in accordance with their difference in phase with the reference voltages.

In connection with the vector presentation of the two in-phase voltages in Fig. 4-6 (D), we want to call attention to the fact that from now on all reference to zero voltage axis will be omitted; — that which was called both the zero voltage and the 0° axis, will henceforth be known only as the 0° reference axis. Again we repeat that this condition is brought about by the fact that we shall no longer deal with instantaneous values, but instead with the useful or effective values. Because in the general application of vectors in radio circuits nstantaneous values, whether zero, or peak, or intermediate, are ignored, and the discussion centers about the phase difference between the effective values of current and voltage existing in the circuit.

Speaking about coordinates, we have shown that to illustrate more than one vector for comparison purposes, a single set of axes is sufficient. It should be remembered, however, that sometimes, because of the nature of the problem involved, so many vectors are shown in one picture that for a truly clear comparison, which may involve numerics, it may be advisable to employ more than one set of axes and to assign certain vectors to one coordinate system and other vectors to another. Or perhaps, as we have done in this text, to utilize a series of coordinate systems, each containing the next step in the progress of resolving the story shown vectorially. Examples of this as applied to radio problems will be given later in this text.

VOLTAGE AND CURRENT RELATIONS

Voltage and current are the paramount considerations in the operation of radio systems, for they represent not only the signal, but also the means of accomplishing and indicating the proper operating conditions. Consequently, the analysis of operation of either the elements of a circuit, or the circuit as a whole is nothing more than an analysis of the current and voltage relationships in the circuit. As a matter of fact, all analysis of radio systems, with the possible exception of the physical motion of components, is the analysis of current and voltage. Accordingly, it seems only fitting and proper, since we are intent upon

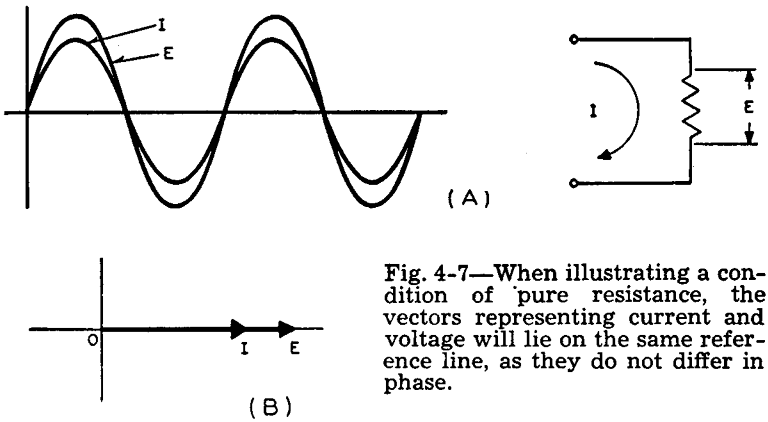


Fig. 4-7—When illustrating a condition of pure resistance, the vectors representing current and voltage will lie on the same reference line, as they do not differ in phase.

explaining vector analysis of conditions found in radio systems, to found our explanations upon the current and voltage relationships associated with resistance, inductance, and capacitance.

Resistance

It is known that a pure resistance, that is, one with negligible inductance and capacitance, causes no change in phase when an alternating voltage is applied, that is, zero phase difference exists between the current through the resistance and the voltage across it. This is shown by the two sine wave curves in Fig. 4-7 (A). The *E* curve is the voltage, and the *I* curve is the current. Consequently, when illustrating this condition vectorially, the voltage and current vectors will lie on the same line. This is illustrated in Fig. 4-7 (B), wherein the two vectors *OE* and *OI* are shown on the 0° reference line.

Vector *OE* represents the magnitude of the voltage across the resistance, and vector *OI* represents the magnitude of the current through the element. The scale to which these vectors are drawn is of no consequence, because phase is the important aspect of the problem.

Inductance

A pure inductance causes constant 90° phase difference between the voltage across its terminals and the current through the element, the voltage leading the current by that amount. This is shown by means of sine-wave curves in Fig. 4-8 (A), wherein the *E* curve is the voltage and the *I* curve that of the current. This phase relationship can be

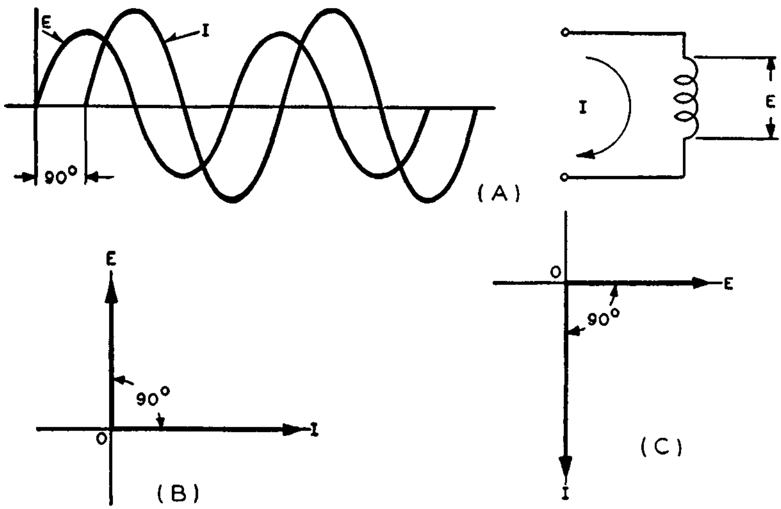


Fig. 4-8—The voltage and current curves for a pure inductance (A) can be shown vectorially in two ways, (B) and (C). In the former, the current *I* is considered the reference quantity and in the latter the voltage *E* is located on the reference line. In both diagrams the voltage is leading the current by 90 degrees.

shown in two ways when presented vectorially. One arrangement of the vectors is shown in Fig. 4-8 (B), and the other is shown in Fig. 4-8 (C). The difference between them is the choice of the reference quantity located on the 0° reference line. In Fig. 4-8 (B), the current

I is considered the reference quantity, so that the voltage vector is displaced *ahead* of the current vector by 90°, by being located on the 90° axis, in accordance with the counterclockwise basis of rotation. There is another way of showing a 90° phase difference with the current vector being used as the reference vector; this would locate the voltage vector on the 270° line. However, this is an erroneous repre-

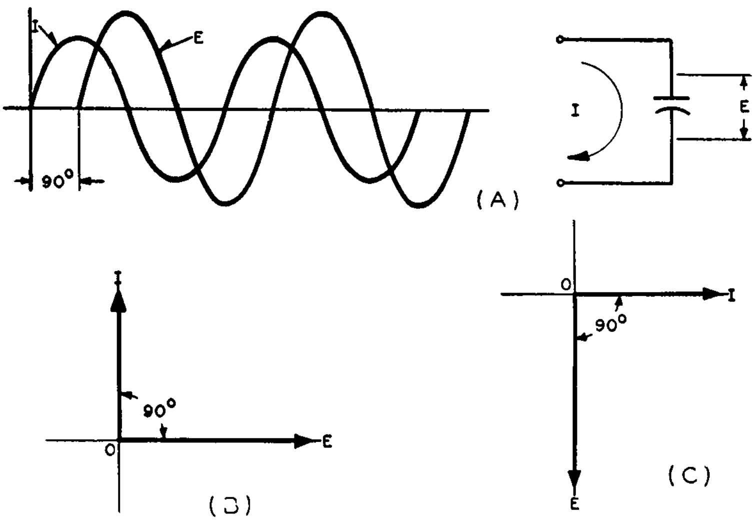


Fig. 4-9—Voltage and current sine waves for a pure capacitance (A). For vector representation two diagrams can be drawn, as in (B) and (C). In (B) the voltage *E* is the reference quantity, and in (C) current *I* is the reference quantity. In each case the current is leading the voltage by 90 degrees.

sentation, for in accordance with the counterclockwise basis of rotation, such a vector presentation would place the voltage behind the current or lagging the current.

The other correct method of showing the 90° phase difference between the voltage and current in an inductive system is as shown in Fig. 4-8 (C), wherein the voltage vector *OE* is located on the 0° reference line and the current vector *OI* is placed on the 270° axis. According to the counterclockwise rotation convention, the voltage vector *OE* is ahead of the current vector *OI*, so that the voltage still leads the current.

Apparently there is a choice in which of these two correct presentations will be used. Such is the case and both are employed, although the most predominant form is with the current vector being placed on the 0° reference line. This is the system we shall employ in this text.

Capacitance

A pure capacitance behaves in a manner exactly opposite to a pure inductance, in that the current *leads* the voltage by 90°. For a certain set of conditions which can be considered general theory, the sine-wave curves of Fig. 4-9 (A), show this phase relationship, and the corresponding vector displays are shown in Figs. 4-9 (B) and (C). Again two vector arrangements are given, illustrating the choice of the reference quantity placed along the 0° reference axis.

Rotation of Vectors

Although the subject to be discussed is fairly obvious, we feel that a brief résumé is worth while to bring the condition more forcefully to your attention. If you will again examine the vectors in Figs. 4-8 (B) and (C), and imagine that shown in the former as being *rotated* simultaneously, you will find that Fig. 4-8 (B) is really Fig. 4-8 (C), for if both vectors are rotated through 270 degrees, the vector arrangement in Fig. 4-8 (B) becomes the arrangement in Fig. 4-8 (C).

The same is true of the vectors shown in Figs. 4-9 (B) and (C). If the two vectors shown in Fig. 4-9 (B) are rotated through 270 degrees, they reach the positions shown in Fig. 4-9 (C). Again the two vectors are one and the same since they indicate the condition of the voltage lagging the current by 90 degrees.

This tells us that we can *rotate all the vectors of a vector diagram in any direction and, as long as the angle between them remains the same, their lead or lag phase relationships do not change.*

Series Circuits

We have just discussed the individual vector illustrations of voltage and current relations in a resistor, capacitor, and inductor. The vector diagrams were shown in association together with their approximate sine waves in order to show how much simpler the vector diagrams are to portray amplitude and phase relationships. Aside from the vacuum tube, resistance, inductance, and capacitance are the basic components of all circuits. In order to illustrate vectorially some of the more elaborate radio circuits it seems best to discuss beforehand some of the simpler types of combinations of these elements of R, L,

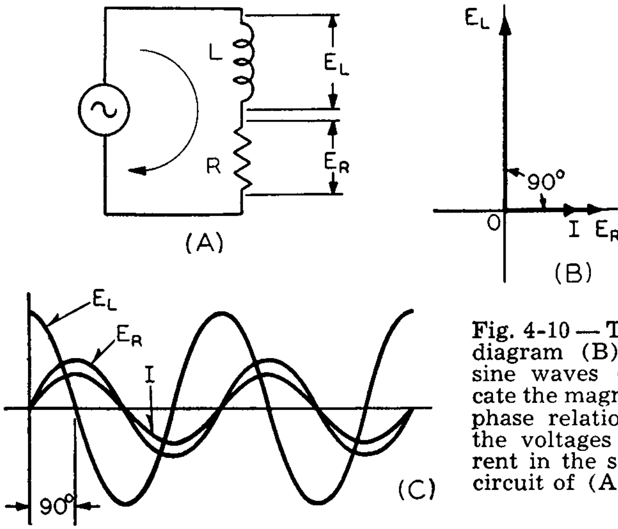


Fig. 4-10 — The vector diagram (B) and the sine waves (C) indicate the magnitude and phase relationships of the voltages and current in the series R-L circuit of (A).

and C. The simplest circuit would then be either a series or parallel combination of any two elements. In this section we will analyze the different series combinations and illustrate their vector diagrams. This will be done in conjunction with the different sine waves for each case for comparison purposes in order to show that the vector system is easier to recognize and understand.

Resistance-Inductance (R-L)

Let us now consider a simple series circuit consisting of only inductance *L* and resistance *R*. This is shown in Fig. 4-10 (A). The voltage

E_R across the resistor R is equal to the current I times the value of R (by Ohm's law), and the voltage drop E_L across the inductor L is equal to the current I times the inductive reactance X_L. (This inductive reactance, measured in ohms and designated as X_L, is equal to $2\pi fL$, where f is the frequency of the applied voltage, L the inductance in henrys, and π a numerical quantity equal to 3.14.) Since the resistance alone offers no phase change to either voltage or current, the voltage drop, E_R, across the resistance can be represented as a vector along the 0° reference line. The same line also carries the current vector OI.

To make the situation more conclusive let us assign values to the problem, so that the comparison will stand out. If we assume that the amount of current I flowing through the circuit is equal to 1 ampere, the inductive reactance is equal to 20 ohms, and the resistance equal to 10 ohms, then the voltage drops will be such that across the inductance we have 1×20 or 20 volts and across the resistance we will have 1×10 or 10 volts. If we then assign a scale of measurement such that 1 inch is equivalent to 20 volts then a vector representing E_R of 10 volts will be ½ inch long. This is illustrated in Fig. 4-10 (B) where vector OE_R is that representing the drop across R. Since we are dealing with voltage relations, it is known that the voltage across an inductance leads the current through it by 90 degrees, and if considered alone the voltage vector representing E_L would be drawn on the 90° reference line as indicated in Fig. 4-10 (B). In other words, the voltage across the inductance of Fig. 4-10 (A) leads the voltage across the resistance by 90°, the latter voltage being in phase with the circuit current.

The sine-wave diagram depicting this phase and magnitude relationship is illustrated in Fig. 4-10 (C). This is another method to establish the fact that in a circuit containing only inductance and resistance the total voltage is always *leading* the current by some phase angle between 0 and 90 degrees. This will be seen later on in the chapter on addition of vectors.

Resistance-Capacitance (R-C)

In a simple circuit containing capacitance and resistance alone the voltage situation is somewhat different. In Fig. 4-11 (A) we have a series circuit containing capacitance C and resistance R. The voltage E_R across the resistor is equal to the current I flowing in the circuit multipled by the value of resistance, (by Ohm's law) and the voltage drop E_C across the capacitor is equal to the current I multipled by the capacitive reactance of C. (This capacitive reactance, measured in ohms and designated as X_C is equal to $1/2\pi fC$, where π and f are as

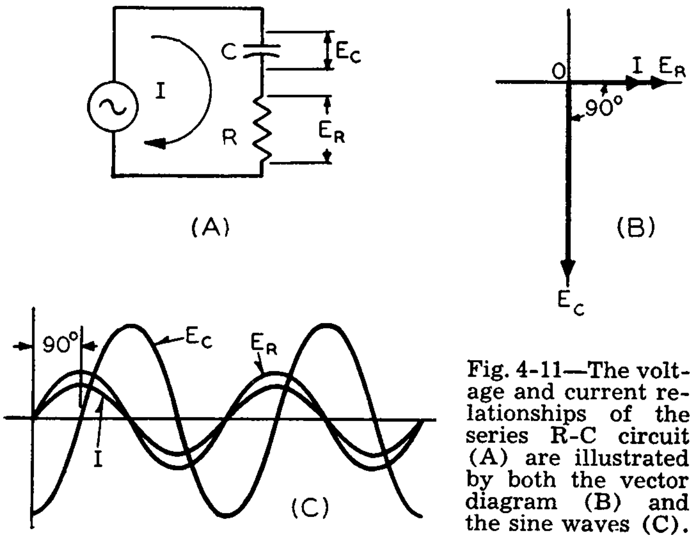


Fig. 4-11—The voltage and current relationships of the series R-C circuit (A) are illustrated by both the vector diagram (B) and the sine waves (C).

explained before and C is equal to the value of capacitance in farads.) If we let the current also equal 1 ampere, the resistance equal 10 ohms, and the capacitive reactance equal 20 ohms, then the voltage drop across the resistance E_R is equal to 1×10 or 10 volts and that across the capacitor 1×20 or 20 volts.

If the same vectorial scale is used in this problem as in the last, the vector representing the voltage drop across the resistor is drawn along the 0° reference line and is designated as OE_R as seen in Fig. 4-11 (B). The current vector OI is on the same line as vector OE_R because the current through, and the voltage across the resistor R, are in phase.

We know that the voltage across a capacitor lags the current flowing through it by 90 degrees; therefore, the voltage vector representing E_C is drawn on the 270° reference line as shown in Fig. 4-11 (B). This is the same as saying that the voltage across the capacitor lags that across the resistor by 90 degrees. The sine-wave diagrams illustrating the phase and magnitude of these voltages are shown in Fig. 4-11 (C). If we wanted to illustrate vectorially the resultant voltage across

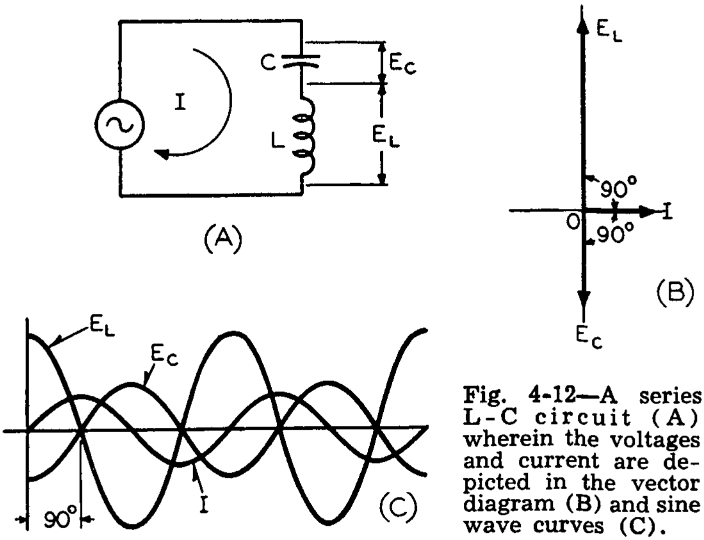


Fig. 4-12—A series L-C circuit (A) wherein the voltages and current are depicted in the vector diagram (B) and sine wave curves (C).

the series combination of the capacitor and the resistor, *vector addition* of the two separate vectors would have to be made. This type of addition is not just plain addition of 10 plus 20 equaling 30 volts, but is completely different as will be seen in chapter 6.

Inductance-Capacitance (L-C)

We know that the current and voltage conditions with respect to any one type of pure reactance are 90 degrees out of phase with each other. We have also seen these conditions vectorially and sinusoidally illustrated with just capacitive reactance alone and also with inductive reactance alone. What would the conditions be in a series circuit, as seen in Fig. 4-12 (A), which contains both inductive and capacitive reactances? It is known that the voltage across an inductance leads the current through it by 90 degrees. If this voltage is designated as E_L and the current as I, then we can represent these two quantities vectorially with the current vector being represented on the 0° reference

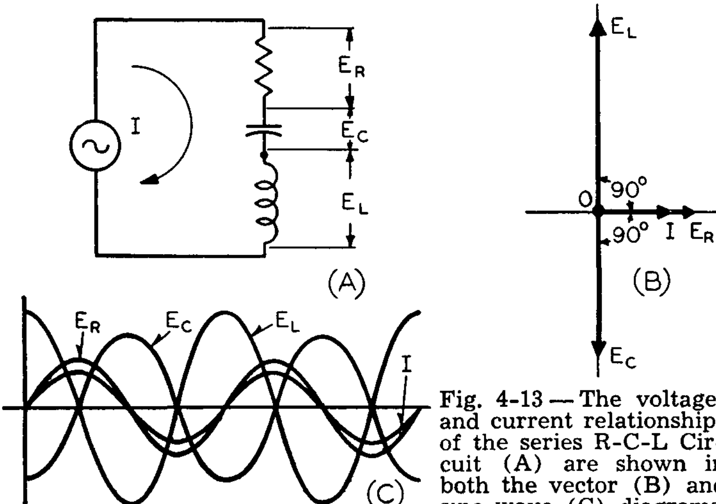


Fig. 4-13 — The voltages and current relationships of the series R-C-L Circuit (A) are shown in both the vector (B) and sine wave (C) diagrams.

line. This is shown in Fig. 4-12 (B). Since in the same circuit a capacitance is present and the same current flows through the capacitor, then the voltage drop E_C across the capacitance is *lagging* this current by 90 degrees. This is illustrated in the vector diagram of Fig. 4-12 (B), where vector OE_C is the capacitive voltage drop. It is seen that this voltage vector lags the current vectors by 90 degrees. The sinusoidal diagram depicting the magnitude and phase relations of the voltage and current in this circuit is shown in Fig. 4-12 (C).

From the vector diagram of Fig. 4-12 (B) a very important fundamental relation between capacitive reactance and inductive reactance is seen to exist, namely that *the voltage drops across a pure capacitor and an inductor are 180 degrees out of phase with each other.*

Resistance-Capacitance-Inductance (R-C-L)

So far we have shown how easy it is to illustrate some of the basic concepts concerning two-element series circuits by means of vector diagrams. Let us now examine a series circuit which contains R, L, and C. A schematic diagram of this series circuit is illustrated in Fig. 4-13 (A) where E_R, E_C, and E_L are the respective voltage drops across the resistance, capacitance, and inductance as caused by the current I flowing through each of them. We already know how to represent vectorially the current I and voltage drop, E_R across a resistance. [See Fig. 4-7 (B)]. The vector diagram involving current and voltage in the inductive circuit was shown in Fig. 4-8 (B) and that in the capacitive circuit was shown in Fig. 4-9 (C). If all these three components are combined in series, we develop the circuit in Fig. 4-13 (A). If all the vector diagrams are combined, the result contains four vectors; one of circuit current and the other three of the respective voltage drops across the individual R, L, and C elements. If E_R equals 10 volts, E_L equals 20 volts, and if E_C equals 15 volts, then with a scale of 1 inch equaling 20 volts the vector diagram is shown in Fig. 4-13 (B). The relative complexity of the equivalent sinusoidal presentation illustrated in Fig. 4-13 (C) is clearly evident. By comparing this latter figure with that of the vector diagram, it is readily seen how much easier it is to *"read"* the vector diagram than the sinusoidal diagram.

PARALLEL CIRCUITS

A fundamental relationship exists between voltage and current in a series circuit. In such a circuit the same current flows through all the elements in the circuit no matter how many are present. This means that once the current flowing through any one element is determined, we know the current flowing through all the elements. In other words we deal with only one current; that is why the current vector is usually drawn on the zero degree reference line, and the different voltage drops are shown in phase relationship to this current vector. The voltages in a series circuit, however, represent a different problem. The total voltage in a series circuit is equal to the *vector sum* of the individual voltage drops across the individual elements. How this final value is determined vectorially is the subject of the next chapter.

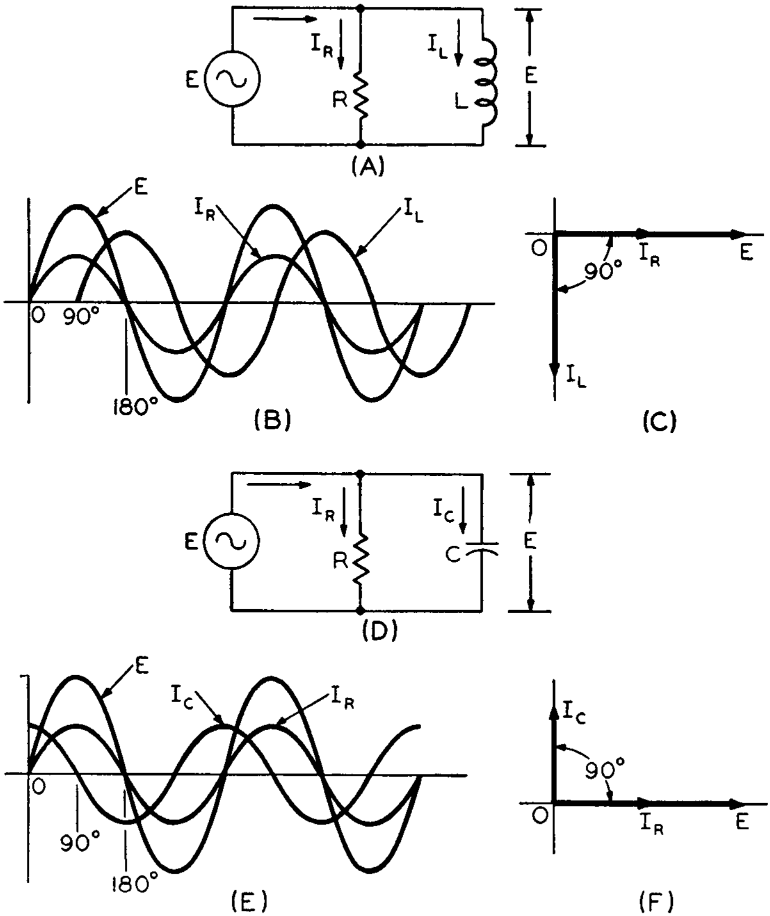
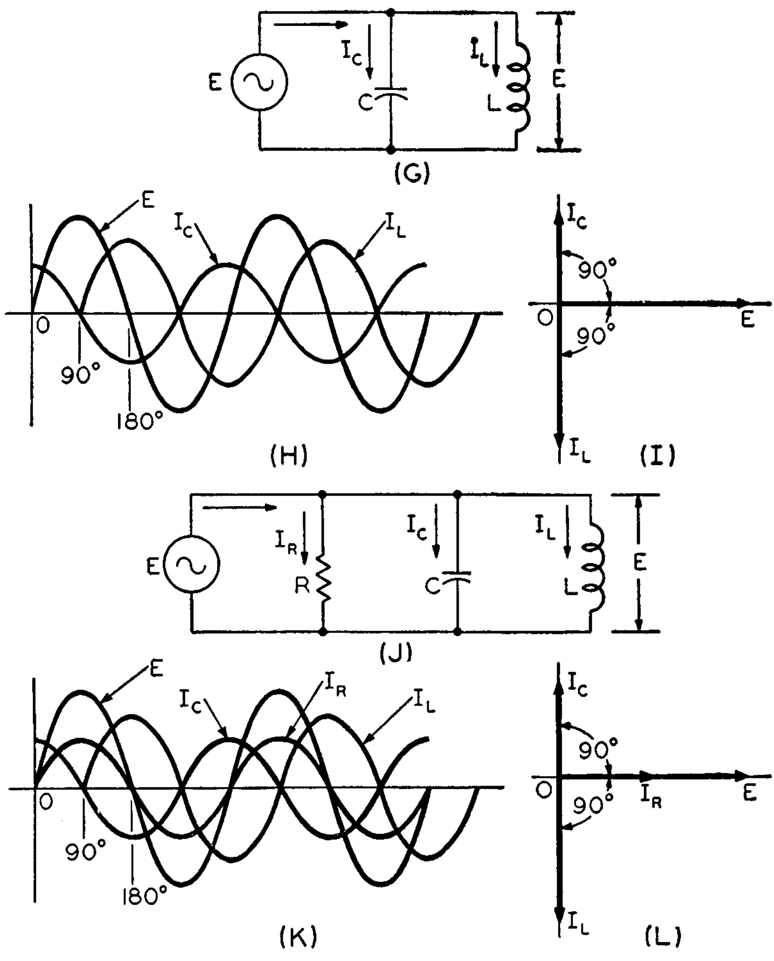


Fig. 4-14 (A) to (L)—The vector diagrams and accompanying sine waves of the different parallel circuits are included to show the magnitude and phase relations of their voltages and currents. A quick comparison of



these sinusoidal diagrams with their vectorial equivalents supports the argument that vector diagrams are visually simpler.

In a parallel circuit the situation is exactly reversed. The total voltage in a parallel circuit is equal to the voltage across any one of the parallel elements. In other words, the same voltage drop exists across each of the parallel elements. This is similar to the current analysis in a series circuit. The current analysis in a parallel circuit is, however, different. In a parallel circuit the current divides among the branches and the total current is equal to the vector sum of the individual currents that flow in the different branches. Stated differently, individual currents flow in each of the parallel branches and then combine to form the total current.

Because of these actions in a parallel circuit, the voltage vector (being the common vector) is located on the zero degree reference line, and the individual phase relationships of the different currents are drawn with respect to this vector. It can be said that in a parallel circuit the voltage vector is the reference vector and in a series circuit the current vector is the reference vector.

Vector Comparison of Parallel Circuits

The fundamental relationships between voltage and current with respect to a resistor, capacitor, or inductor hold true no matter what type of circuit (series or parallel) is being considered. In Fig. 4-14 we see a number of individual parallel circuits together with their respective sine-wave and vector diagrams. In all these cases the applied alternating voltage is the same and, since all the circuits are in parallel, this same value of voltage appears across each of the components in the individual circuits. The values of the components were chosen so that the amount of current that flows through the resistance element is the same in all of the circuits. Likewise a constant current flows through all the inductive branches of the individual circuits and a constant value current flows through all the capacitive branches of the individual circuits. This is done to facilitate preparation of the drawings and understanding of the comparison between the sine-wave diagrams and the vector diagrams.

When the individual series circuits were analyzed, it was stated that the vector diagram method was much simpler to understand than the sine wave diagram with respect to phase relations and comparison of magnitudes. A quick glance at Fig. 4-14, wherein the sinusoidal diagrams contain more curves, will strengthen the realization that the vector diagrams are much simpler to the eye as well as to the mind. For example, in Fig. 4-14 (K) four different sine waves are represented, and the attempt to resolve their phase relations would require more than a reasonable amount of time. However, in Fig. 4-14 (L) the vectors indicate exactly the same things as the sine curves do, and the latter diagram is much quicker to interpret. In fact, this is a vivid example of one basic reason for the use of vectors. That is, vectors are primarily used as a shorthand diagram method of circuit analysis.

Referring again to the different parallel circuits of Fig. 4-14, it is found that the current I_R, flowing through resistor R of the individual circuits (A), (D), and (J) is in phase with the voltage across it. This is readily seen in the vector diagrams of Fig. 4-14, parts (C), (F), and (L), wherein the current vector OI_R coincides with the voltage vector OE_R. The exact amount of current and voltage that the individual vectors represent is of no consequence to us at the moment, but the phase relationships are.

The current I_L, that flows through the inductance L in circuits (A), (G), and (J) *lags* the voltage E across it by 90 degrees. This is indicated by the 90° phase difference between these two quantities in the sine curves of figures (B), (H), and (K). This is much more evident in the vector diagrams of Figs. 4-14 (C), (I), and (L).

The current I_C that flows through the different capacitors C in circuits (D), (G), and (J) *leads* the voltage E across these capacitors by 90 degrees. This is recognized in the vector diagrams of figures (F), (I), and (L) much more easily than in the respective sine curve of figures (E), (H), and (K) representing the same circuits. In vector diagrams of Figs. 4-14 (I) and 4-14 (L) you will note that in a parallel circuit the current through the capacitive part of the circuit is out of phase by 180 degrees with the current flowing through the inductive branch of the circuit.

Other Vector Relations

From all of the preceding discussions in this chapter it appears that in vector analysis of different circuits the only phase relationship that seems to exist between any two quantities is 90 degrees. Of course this is not so. The series and parallel circuits were chosen because they represented the very basic circuits from which vectors can be studied. Numerous cases do exist wherein the phase difference between voltage and current, voltage and voltage, and current and current, are anything but equal to 90 degrees or 180 degrees. However, in these cases the voltage and/or current vector that is illustrated is a combined

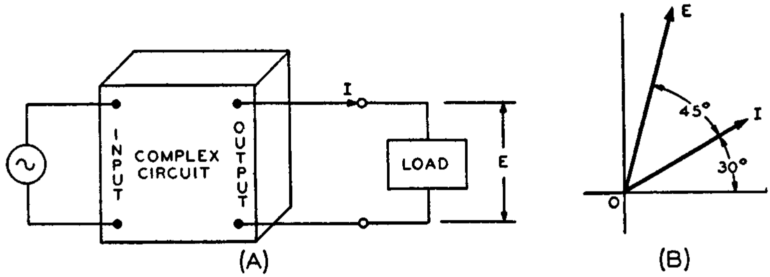


Fig. 4-15—The current *I* flowing out of the complex network (A) is a combination of the individual currents within the network. The impedance load has an effective reactance that is predominantly inductive and equals the resistive component of the load. Therefore the voltage *E* across the load leads the current *I* flowing through it by 45°, as seen in the vector diagram (B).

value of a number of different voltages and/or currents. The following example will make this clear.

In Fig. 4-15 (A) we see a four-terminal network, a "black box." Across the output terminals is connected an impedance load. This four-terminal network is shown in a block form because it is a complex electronic circuit whose internal construction does not concern us in this problem. Due to the complex network the current *I* flowing out of the circuit is really a combined current of the individual currents involved within the complex network. It is found that in representing the current vectorially it is 30° away from the 0° reference line in a positive direction. This is shown in Fig. 4-15 (B) where the current vector *OI* is located 30° away from the 0° reference line.

The impedance load across the output is such that the voltage drop across it *leads* the current *I* through it by 45 degrees. In other words, the impedance load consists of resistance and reactance, and the effective reactance is predominantly inductive; and in any inductive circuit the voltage across it *leads* the current flowing through it. The amount of lead is determined by the magnitude of resistance and reactance, and it can be anywhere from zero to 90 degrees. The vector *OE* in Fig. 4-15 (B) is really a combined voltage of all the individual voltage drops that appear across the resistive and reactive components of the load impedance.

We can take vectors such as the one which appears in Fig. 4-15 (B) and *resolve* them into their respective component parts. In other words, a vector that is the result of combining two other vectors, can be broken down in a manner that will indicate the relative magnitudes of the original two vectors, which composed it. This will be discussed in the following chapter.

Advanced Thoughts

In the vector discussion of series circuits many readers will wonder about the phase relationship of the total voltage (the combined voltage drops across all the individual series circuit components) with respect to the current flowing in the circuit. Likewise others would like to know the phase relationship between the total current (the combined currents from all the parallel branches) and the voltage across the components in a parallel circuit. These thoughts are well founded and are so very important that separate chapters are devoted to the development of the resultant vector.

Later it will be seen that through the basic knowledge of vectors we will be able to solve for the total *impedance* of a circuit which contains any number of resistances and reactances. This will be accomplished by means of two methods. One of the methods is the simple use of basic mathematics, and the other by the use of vector additions. These discussions will illustrate how impedance quantities of circuits can be calculated without the use of higher mathematics or complex numbers.

CHAPTER 5

RESOLUTION OF VECTORS

In the vector illustrations which have been analyzed so far, the need for some system of reference axis (coordinate system) to represent the vectors in question was evident. However, all they have meant to us was that they helped in quickly establishing phase relationships between certain vectors. These axes do, however, serve more use than that at first imagined. These other uses of the axis will now become apparent.

It is therefore seen from this foregoing discussion that the coordinate axis can be put to further use besides the angular reference line. They can actually be used to represent the magnitudes of certain quantities even while being used as a vector reference.

Vector Components

In order to understand fully how the axes can be used in the manner mentioned, it would be best to know a bit more about vectors. In the previous chapter we mentioned that, if a vector is not represented directly on any one of the reference lines, it really is a combined quantity. Stated differently, this type of a vector actually has component parts. In order to find out what these component parts are the use of the coordinate axis comes in very handy. Let us refer to Fig. 5-1. The vector *OA* of this curve can represent any quantity, the actual value of which is of no interest at the moment. The length of the vector is predetermined by a scale already selected. If we were to draw a vertical line from the terminal point of vector *OA* such that it meets the zero-degree reference line perpendicularly, which is point *B* in Fig.

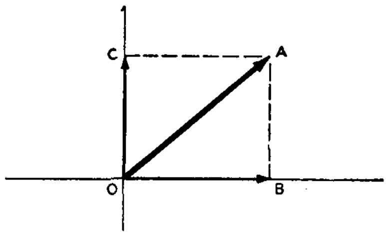


Fig. 5-1—Any vector which does not coincide with one of the reference lines is a combined quantity and may be resolved into its component parts by drawing parallel lines from the terminal point of the vector to the coordinate axes. *OB* is the horizontal component and *OC* the vertical component of the vector *OA*.

5-1, then the line OB is called the *horizontal component* of the vector. Likewise if we draw a line parallel to the zero-degree reference line from the terminal point of the vector OA to the vertical axis at point C, then line OC is called the *vertical component* of the vector. In this manner we show that *every* vector has a vertical and a horizontal component. If a vector coincides exactly with any reference line, for instance, the zero-degree reference line, it is said to have a zero vertical component. In other words, the vertical component is said not to exist under those conditions and the horizontal component is the vector itself.

This method of breaking down a vector into its component parts is called the *resolution of a vector*. The resolution of a vector and the addition of vectors are in a certain sense interchangeably used, so that one helps bring about the other. This will be discussed in the next chapter.

In the meantime let us refer back to the components of the vector of Fig. 5-1 for a moment. The components OB and OC actually represent something specific. They bear a definite magnitude relation to the magnitude of the vector in question. In other words, the lengths of these vector components have a definite meaning. As far as scale readings are concerned, the scale used for the vector OA is used for its component parts. This will be seen in the following section.

Resolution of Series Impedance

Let us consider an impedance of some particular quantity. It is known that impedance takes into account two quantities, namely resistance and reactance. In other words, impedance is the vector combination of resistance and reactance, and these individual quantities are considered as components of impedance. However, we also know that if we have a resistance value of 10 ohms in series with a reactive value of 20 ohms, we cannot simply add these values together and say that the total impedance is 30 ohms. Impedance values must be added algebraically (vectorially) unless they are pure resistances and do not contain a reactive component.

Now let us refer to a typical impedance vector of a series circuit as seen in Fig. 5-2. The impedance in question has a magnitude of 50 ohms and a phase angle of 36.85° (away from the zero-degree reference line). If we set a scale of 1 inch equal to 40 ohms, then the length of the vector, *OZ* being 50 ohms in magnitude, equals 1¼ inches. If we draw perpendicular lines from the terminal point of the vector *OZ* to the 0° line and to the 90° reference line (which in this case may be called the abscissa and the ordinate, or the x and y axis respectively), then the lines *OR* and *OX* are called the component parts of the im-

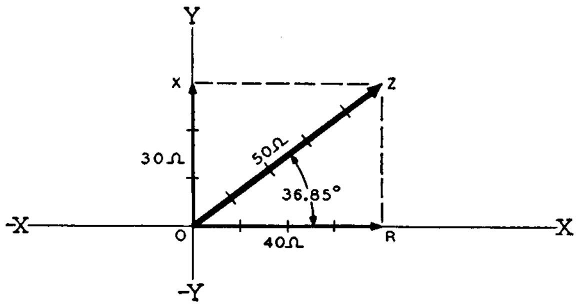


Fig. 5-2—A typical impedance vector of a series circuit shown being resolved into its resistive and reactive components, the reactive component being predominantly inductive.

pedance vector. Line *OR* is the magnitude of the resistive component of impedance, and line *OX* is the magnitude of the reactive component of impedance. If we measure the length of the *OR* component, it is

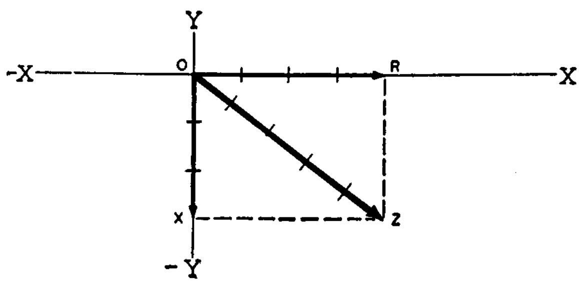


Fig. 5-3—When a series impedance vector is resolved into its component parts, the resistive component always lies on the horizontal axis, and the reactive component on the vertical axis. The reactive component here is predominantly capacitive.

found to be equal to 1 inch. This means that the magnitude of resistance is equal to 40 ohms since the original scale was set for 1 inch to equal 40 ohms. Similarly the length of *OX* is found to be equal to ¾ inch, which means that the magnitude of the reactive component under the same scale, is equal to 30 ohms.

Now it is possible to make certain statements concerning the resolution of series impedance vectors: the resistive component always lies on the horizontal axis, whereas the reactive component always lies on the vertical axis. For example Fig. 5-3 shows the equivalent conditions when the impedance vector has a different phase angle.

Nature of Reactive Component

The effective reactance in Fig. 5-2 is considered to be inductive. The reason for this is, if the circuit is predominantly inductive, the phase angle would be positive and between zero and 90 degrees. This is the situation here where the phase angle is a positive 36.85°. If the circuit was predominately capacitive, the impedance phase angle would be predominately negative and the impedance vector would lie in the fourth quadrant as shown in Fig. 5-3. This means that the reactive component of the impedance, after resolution, would lie on the 270° (or —90°) reference line instead of the 90° reference line. This is the same as saying, in graphical terms, that a capacitive reactance component of impedance would be represented by the negative ordinate (or —y axis) and an inductive reactive component would lie on the positive ordinate (or y axis). In either case, the resistive part is always represented by the positive abscissa (or x axis). When representing impedances vectorially, if the resistive components are real (that is, pure resistance), then the impedance vector will fall into either the first or fourth quadrant, depending upon which reactance dominates the circuit. This will become more evident in the next chapter dealing with the addition of vectors.

Resolution of Parallel Impedance

By this time you have a fair idea of what is meant by the resolution of a vector, for in conventional radio operations such resolution in general consists of plotting the horizontal and vertical component lines. The analysis of the components of a series impedance was quite simple, but the development of the components of a parallel impedance is somewhat more elaborate — perhaps better expressed as involving a departure from the exact method used for resolving the series impedance.

Let us picture a parallel impedance shown by *OZ* in Fig. 5-4. In order to find its exact component parts, all that has to be done is to draw a line between the vertical axis and the horizontal axis so that the terminal point of the vector *OZ* and this line meet perpendicularly. In other words they should meet in such a manner that 90-degree

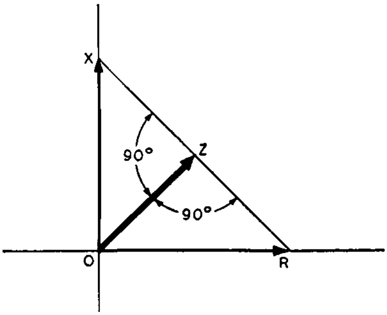


Fig. 5-4—Component parts of parallel impedance vector *OZ* can be found by drawing a straight line from the vertical to the horizontal axis that is perpendicular to vector *OZ* at *Z*. Line *OR* is the resistive component, and *OX* the reactive component.

angles exist on either side of the junction point, as indicated in the Fig. 5-4. Where this line crosses the zero-degree and 90-degree reference lines determines the parallel circuit components. Thus line *OR* is the resistive component and line *OX* is the reactive component. The magnitudes of these components are determined from the same scale as that used for vector *OZ*.

So far we have shown the use of vectors from a strictly graphical point of view. That is, the primary concern has been the explanation of vector diagrams so as to develop an understanding which might be helpful when encountering vectors accompanied by discussion in some text or periodical. The *exact* magnitudes and phase angles selected are of a secondary importance as far as the usefulness of this book is concerned. Their selection was simply a matter of more lucid presentation.

CHAPTER 6

ADDITION OF VECTORS

We now come to the heart of vector analysis — namely, the addition of vectors to form a resultant vector. The addition of vectors is considered the most important type of vector combination, because in radio circuit analysis the addition of vector quantities occurs very frequently. There are other types of vector combinations — subtraction, multiplication, and division — all of which are discussed in the next chapter. All vector combinations are algebraic in nature and do not involve simple arithmetical calculations. When vectors are added it does not necessarily mean that the resultant vector will always be greater than either of the individual vectors; the determining factor is the phase of the vector quantities.

There are three principal methods by which addition of vectors may be performed. They are known as the *parallelogram* method of addition, the *resolution* method of addition, and the *head-to-tail* method of addition. These three methods are used when at least two or more of the vectors involved in the addition have *different* phase angles which are other than 180 degrees *apart.* In other words vectors with the same phase angle and vectors with different phase angles are added in different ways. The method of adding two or more vectors of the same phase angle and of opposite phase angles (180°) is much simpler than the other three types of addition mentioned, although these methods of addition are not at all difficult to comprehend.

In the addition of vectors only those vector quantities bearing the same dimensional relation can be added. That is, a *voltage vector* can be added to *another voltage* vector, but *not* to a current or an impedance vector. Similarly, a *current* vector can only be added to some other *current* vector and not to a voltage or an impedance vector. The reason for this is very obvious if one considers the fact that volts added to amperes or ohms added to volts is as meaningless as apples added to oranges.

The same holds true for the subtraction of vectors, as will be seen in the next chapter.

Addition of Vectors With Same Phase Angle

The simplest type of vector addition is that of two vectors that have the same phase angle. In adding such vectors all that one has to do is to add their magnitudes arithmetically with the angle remaining the same. This is indicated by the vector diagrams of Fig. 6-1 (A), (B), and (C).

In Fig. 6-1 (A) the voltage vector OE_1 has the same phase angle (—60°) as the other voltage vector OE_2. The magnitude of voltage OE_1 is added to that of OE_2 to give the resultant vector OE_R.

In other words, since the magnitudes of both voltage vectors OE_1 and OE_2 are drawn to the same prearranged scale, then all we are doing is adding the length of vector OE_1 to the terminal point of vector OE_2, with the new over-all length, OE_R, representing the resultant

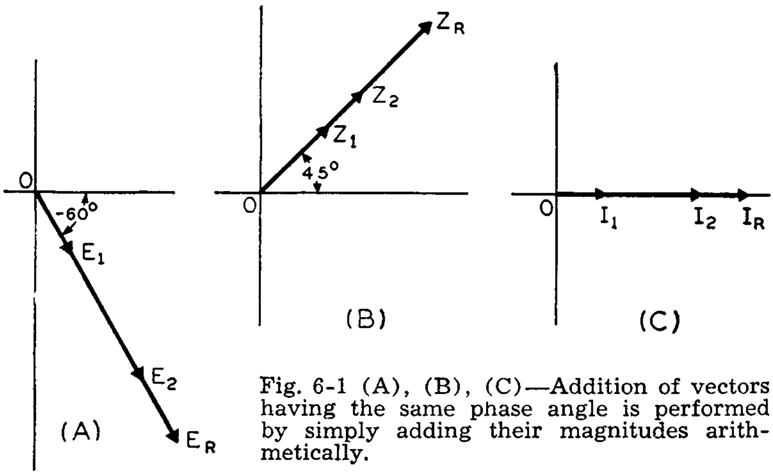


Fig. 6-1 (A), (B), (C)—Addition of vectors having the same phase angle is performed by simply adding their magnitudes arithmetically.

voltage. The same type of vector addition is accomplished in the impedance vector diagram of Fig. 6-1 (B) and in the current vector diagram of Fig. 6-1 (C). In the impedance vector diagram, vectors OZ_1 and OZ_2, each with a positive phase angle of 45°, are added end on end to give the resultant impedance vector OZ_R. In the current vector diagram, vectors OI_1 and OI_2 each having a 0° phase angle are added end on end to give the resultant current vector OI_R. These additions can be better understood if numerical values are given to the vectors.

In order to represent a vector algebraically with respect to its magnitude and phase angle, it is usually written in the following form:

$$10\underline{/60^\circ} \quad \text{or} \quad 10\underline{/+60^\circ}$$

where the number 10 stands for the magnitude of the quantity under discussion (volts, amperes, ohms, etc.) and the number in the bracket (called an angle bracket) depicts the size of the phase angle. If the phase angle has no polarity sign in front of it, or if it has a plus sign, then the angle is considered to be represented in the positive or counterclockwise direction on the vector diagram. If there is a negative sign in front of the angle, then the phase angle is a negative angle, and it will be drawn in clockwise direction from the zero-degree reference line.

If the voltage vectors of Fig. 6-1 (A) have the following numerical values: $E_1 = 6\underline{/-60^\circ}$ volts and $E_2 = 18\underline{/-60^\circ}$ volts, to add them vectorially all that we do is to add only their magnitudes with the phase angles remaining the same. Thus: $6\underline{/-60^\circ}$ volts added vectorially to $18\underline{/-60^\circ}$ volts produces a resultant vector equal to $24\underline{/-60^\circ}$ volts. This type of addition can *only* be done when the *phase angles of the vectors in question are equal* in value. In Fig. 6-1 (A) the vector scale is so chosen that one inch is equal to 16 volts. Therefore, to be graphically accurate, the resultant voltage vector OE_R is equal to 24/16 or 1½ inches.

From the vector additions of Fig. 6-1 (A), (B), and (C) we see that the resultant vector in each instance is greater in magnitude than either one of the individual components that were added. This is a fundamental fact that holds true for the addition of all in-phase vectors — namely that when two or more vectors, all having the *same* phase angle are vectorially added, the resultant vector will *always* be greater than each of the individual vectors that were added.

Addition of Vectors With Opposite Phase Angle

When two voltages or currents are *opposite in phase,* we mean that the difference in phase between them is 180°. In the preceding section it was seen how the addition of in-phase vectors resulted in a final vector greater in magnitude than either of its added parts. In the addition of vectors of opposite phase angles the reverse is true: the resultant vector is smaller in magnitude than either of its additive component vectors, and the resultant vector has the phase angle of the larger of the two vectors.

Let us refer to the vector diagram of Fig. 6-2 (A) to visualize how this type of addition comes about. In this vector diagram we are dealing with two individual voltage vectors, namely vector OE_1 and OE_2. Voltage OE_1 of 0° phase angle is added to voltage vector OE_2 which is 180° out of phase with it. In order to secure the proper resultant vec-

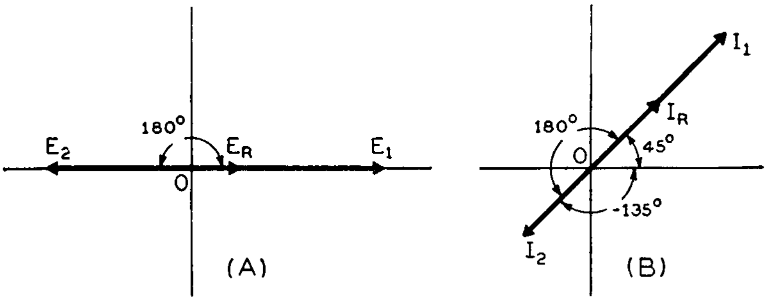


Fig. 6-2 (A), (B)—When vectors opposite in phase are added, the resultant vector is smaller in magnitude than either vector being added and has the phase angle of the larger of the two.

tor it is necessary to *subtract* the magnitude of the smaller vector from that of the larger, and the difference will be the resultant vector of the combination. If voltage $E_1 = 10\underline{/0°}$ volts and voltage $E_2 = 7.5\underline{/180°}$ volts, then the vector addition of the two gives a resultant voltage vector OE_R equal to $2.5\underline{/0°}$ volts. That is, the resultant vector has a magnitude of 2.5 volts at 0° phase angle.

Another out-of-phase vector addition is shown in Fig. 6-2 (B) where the two vectors in question are current vectors represented by OI_1 and OI_2 and where vector OI_1 is twice the magnitude of vector OI_2 and has a phase angle of 45° and where vector OI_2 has a phase angle of 225°, which is the equivalent of —135°. Since vector OI_2 is

180 degrees out of phase with vector OI_1, then the resultant vector OI_R will be equal to half the magnitude of vector OI_1 and it will have the same phase angle.

The addition of in-phase and 180° out-of-phase vectors is very important in the analysis of certain radio problems, namely that of feedback. It is known that inverse feedback, sometimes called negative feedback or degeneration, causes a reduction in the final input signal. The primary reason for this is the fact that the two signals, that is, the feedback signal and the original input signal to the circuit in question, are approximately 180 degrees out of phase. From the illustration of the combination of 180 degrees out-of-phase vectors, it is readily seen that inverse feedback does cause a reduction in the final input signal.

In circuits dealing with regeneration or positive feedback it is known that the signal fed back aids the original input signal existing at the point of feedback and that their result is a larger input signal. In regeneration, the feedback signal is approximately in phase with the original signal at the point where it is fed back, and from the knowledge of vector addition of in-phase signals it is readily seen that the final signal will be increased in magnitude.

The analysis of a typical problem wherein 180° out-of-phase vectors are discussed is included in the push-pull amplifiers in the last chapter.

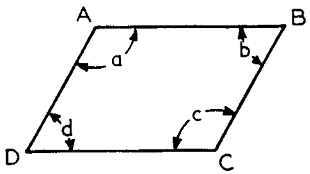
The Parallelogram Method of Addition

It is quite easy to see how vectors in phase and out of phase are added. The procedure is very simple, merely arithmetical addition of one vector magnitude to the other in the in-phase situation, and arithmetical subtraction of the magnitude of the smaller vector from the larger vector in the out-of-phase situation. However, if the phase relationship *between* vectors is such that the phase angle between them is between zero and 180 degrees, the vector method of addition involved is different. In order to add such vectors that differ in phase there are three methods from which to choose, of which the *parallelogram method* is used the most.

Before we discuss this method let us understand what is meant by a *parallelogram*. By definition a *parallelogram is a four-sided straight-*

line figure that has its opposite sides parallel to each other and equal in length, and the opposite angles equal to each other. A typical parallelogram is shown in Fig. 6-3. In this diagram sides *AB* and *CD* are parallel to each other and equal in length and sides *AD* and *BC* are also parallel to each other and equal in length. Opposite angles *d* and *b* are equal to each other and angles *a* and *c* are also equal to each other. (A square is one form of a parallelogram where all the four sides are equal to each other and the four angles involved are all right angles, and therefore equal to each other.)

A straight line joining the opposite corners of any four-sided figure is called a diagonal of the figure. Obviously, only two diagonals can exist in any four-sided figure. If the four-sided figure happens to be a parallelogram, as seen in Fig. 6-4, the diagonal joining the points *D*



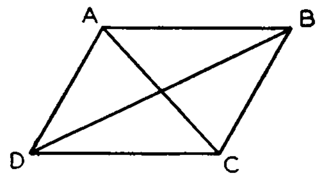


Fig. 6-3 (left)—A four-sided figure that has its opposite sides equal and parallel to each other and its opposite angles equal is called a parallelogram. Fig. 6-4 (right)—In a parallelogram the diagonal joining the smaller angles, *B* and *D*, is longer than the diagonal joining the larger angles, *A* and *C*.

and *B* or cutting the *smaller* angles of the figure is longer than the other diagonal *AC*. If the parallelogram happens to be a square then the lengths of the diagonals are equal.

An important property of *any* figure having four straight sides, no matter what shape it has, is that exactly four angles are involved, which all add up to a total of 360 degrees. In contrast a triangle, which is a three-sided figure, contains only three angles, but they total 180 degrees.

In Fig. 6-5 the vectors *OA* and *OB* are to be added vectorially by the parallelogram method. To do this it is necessary to form a parallelogram out of these vectors, using them as adjacent sides of a parallelogram. This is simply done as follows: Draw a line parallel to vector

OB, and of the same length as the vector, starting from the terminal point of vector *OA*. This parallel line is shown dotted in the figure and is represented by the line *AC*. Likewise draw another line parallel to vector *OA*, and of the same length as this vector, starting from the terminal point of vector *OB*. This parallel line is also shown dotted in Fig. 6-5 and is indicated by the line *BC*. These two dotted lines *AC* and

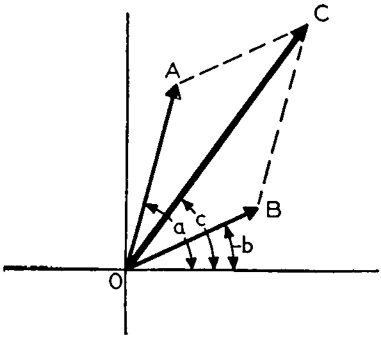


Fig. 6-5—The addition of two vectors *OA* and *OB* by the parallelogram method. These two vectors are chosen as adjacent sides of a parallelogram and *OACB* is drawn. The diagonal from the origin O of the coordinate system to the opposite angle of the parallelogram (C) is the resultant vector *OC* of the addition.

BC join together at the point *C* thus forming a parallelogram, with the corners *O, A, B,* and *C*. If we now draw a diagonal from point *O* to point *C*, *this diagonal will represent the resultant vector of the addition of the other two vectors.* The resultant vector always has its initial point starting from point *O*. The phase angle of vector *OB* is designated as *b* and the phase angle of vector *OA* is designated as *a*. Since both angles are unequal, by the proper vector addition the phase angle of the new resultant vector will be different from either of the other two angles. This is shown in Fig. 6-5 where the phase angle of the resultant vector *OC* is equal to *c*. This phase angle *c* is smaller in value than angle *a* but larger than angle *b*; therefore, it is said that the resultant vector *OC* *leads* vector *OB* by an angle equal to angle *c* less angle *b* but lags vector *OA* by an angle equal to angle *a* less angle *c*.

Another interesting thing is the magnitude of the resultant vector. In this problem of Fig. 6-5 the magnitude is seen to be greater than either of the other two vectors, but if length measurements were made of all the vectors it will be found that the addition is such that the magnitude of the resultant vector *does not equal* the numerical sum of the magnitudes of the other vectors.

In Fig. 6-6 appear a number of other illustrations of the addition of two vectors, using the parallelogram method. The purpose of these six diagrams is to show that, when two vectors are added, the resultant vector will not always be greater in magnitude than either of the two vectors being added. From these vector diagrams it will be noticed that the resultant vectors, designated as vector *OR* in all of the diagrams, vary in magnitude with respect to the two vectors each resultant vector is representing. Some of the resultant vectors may be smaller in magnitude than either of the vectors from which they were added; greater in magnitude than either one; greater than one and less than the other; or equal to the length of one of the vectors being added; the determining factor in all cases is the phase angle of the component vectors.

In all of the vector diagrams wherein the parallelogram method of addition has been used so far, only two vectors were added in each instance. This by no means limits the use of the parallelogram method of addition. Three or more vectors can also be added by this method on the same sets of coordinate axes. However, in practice very seldom are more than three vectors added by this method because the vector diagram becomes somewhat complicated in appearance although really not complicated in construction.

When three or more vectors are added by this process, any two vectors are first added by the use of a parallelogram, and the resultant vector found. This resultant vector is then added with one of the other vectors by the same parallelogram method and a new resultant vector found. This latter resultant vector is added with another vector, and still a new resultant vector is found. This process continues until all the original vectors to be added have been exhausted by the parallelogram method and only the final resultant vector remains; which is the answer to the vector addition of all of the vectors involved in the problem.

Head-to-Tail Method of Addition

Another system of vector addition that can be used instead of the parallelogram method is called the *head-to-tail* method of addition.

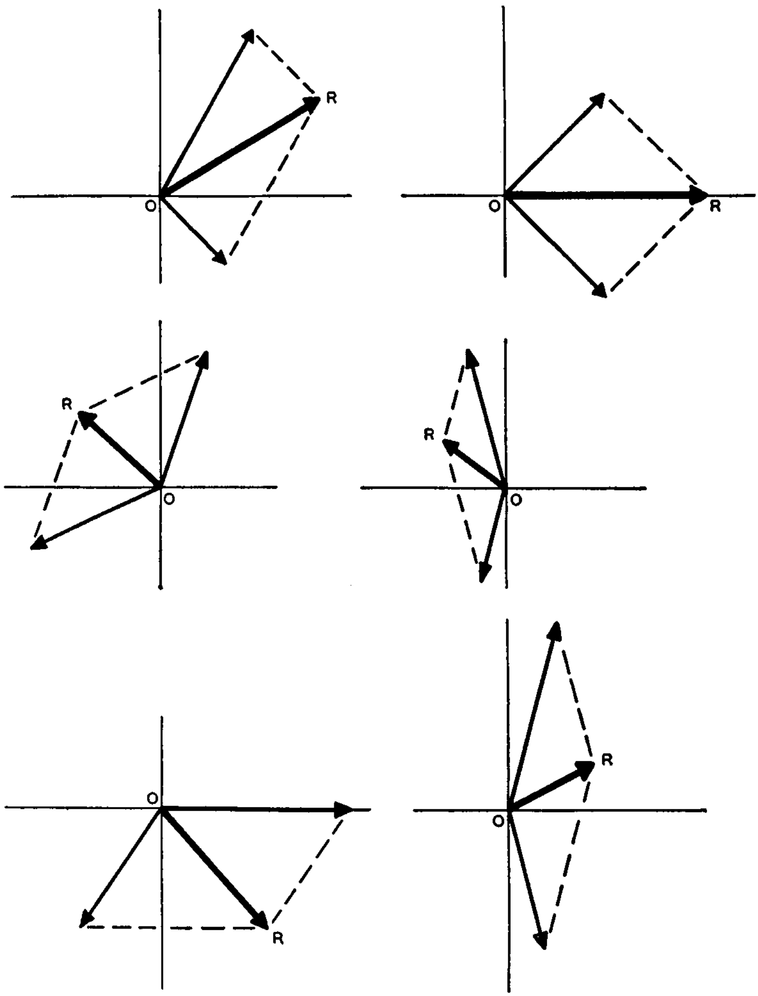


Fig. 6-6—A number of examples of the addition of two vectors by the parallelogram method. The magnitude of the resultant vector relative to that of its components will vary, depending upon the phase angle of the component vectors.

This method usually finds its greatest application where four or more vectors are to be added together.

When two vectors are to be added by this new method, this procedure is followed: First lay out the coordinate axes and draw either one of the vectors to be added in direct accordance with its magnitude and phase angle. Then join the initial point of the other vector to the terminal point of the first vector drawn. The direction of this second vector is so chosen that the terminal point or *arrowhead* of the first vector appears to be the intersecting point of an imaginary coordinate axis. This is shown in Fig. 6-7 (A). In order to make a comparison

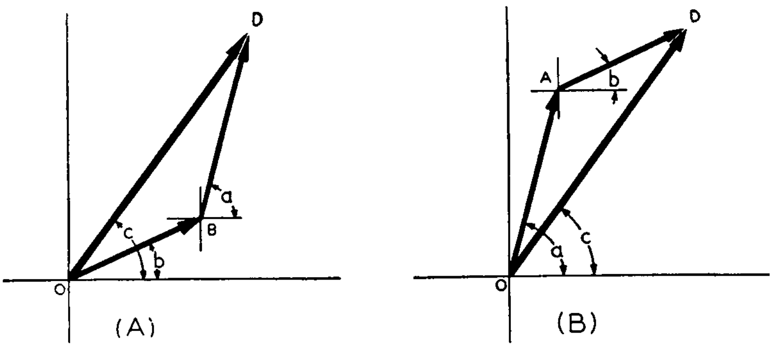


Fig. 6-7—The head-to-tail method of vector addition may be started with either of the two vectors to be added. In (A) the supplementary set of coordinates is drawn at the terminal point of vector *OB* and in (B) at the terminal point of *OA*. The resultant vector in either case is *OD*.

the vectors *OA* and *OB* used in Fig. 6-5 are shown in the illustration of Fig. 6-7. Since we can arbitrarily choose either one of the two vectors to draw first, in Fig. 6-7 (A) vector *OB* is drawn from the origin in accordance with its proper phase angle. We then assume the arrowhead of vector *OB* to be the representative point of another *origin* (indicated by a small coordinate system at point *B*) and draw the other vector from this point. In other words vector *OA* of Fig. 6-5 is redrawn in Fig. 6-7 (A) as vector *BD* at the same phase angle *a* taken with respect to the supplementary coordinate system. (Vector *BD* in this drawing is the same as vector *OA* in that of Fig. 6-5.) The next step is to draw a line connecting point *O* to point *D* ending with vector *OD* which is the resultant vector of the addition. The phase angle of this resultant vector *OD* is designated as c, and, if a comparison between the resultant vectors of Figs. 6-5 and 6-7 (A) is made, it will be seen that both these vectors are equal in magnitude and phase.

In order to show that the arbitrary choice of starting vectors does not alter the final result, we have shown in Fig. 6-7 (B) the vector addition of the same two vectors by the *head-to-tail* method, but this time we have started with vector *OA* of Fig. 6-5 which is the same as *BD* of Fig. 6-7 (A). This time the supplementary coordinate axis is at point *A*, and the other vector is drawn from the origin of the small axes, which is the arrowhead of vector *OA*. The resultant vector of Fig. 6-7 (B) is *OD*, and, if a comparison is made between all three resultant vectors of Figs. 6-5, 6-7 (A), and 6-7 (B), they will be found to be exactly the same in magnitude and in phase.

Upon further comparison of these three figures it will be noted that the triangles formed by the head-to-tail methods of addition of Figs. 6-7 (A) and (B) are each *one-half the parallelogram of Fig. 6-5 wherein the resultant vector is the diagonal* of the parallelogram. From this comparison we can conclude that the head-to-tail method of vector addition is indirectly based upon the parallelogram method.

The terminal point of a vector is also referred to as the (arrow) "head" of the vector and the initial point also referred to as the "tail" of the vector. In adding one vector to the other to find the resultant vector, what we are actually doing is connecting the *head* of one vector to the *tail* of another, hence the name *head-to-tail* method of vector addition. In Fig. 6-7 (A) the tail of vector *BD* is connected to the head of vector *OB*. The resultant vector *OD*, resulting from the addition of these two vectors, is *drawn from the tail of the starting vector to the head of the last vector.*

This latter statement concerning finding the resultant vector, applies no matter how many vectors are added by this method. That is, after all the vectors have been accounted for by the head-to-tail method of addition, the resultant vector is found by drawing a line from the origin of the main coordinate system to the head or terminal point of the last vector. This line is the resultant vector of the complete addition. This will be shown later in this text. Adding only two vectors by this method is not employed to any great extent since the parallelogram method is easier to use. However, when three or more vectors are to be added, this method is more practical.

Let us consider the following problem. Four different voltages of varying phase and magnitude are simultaneously applied across the input of some electric circuit. It is desired to know the magnitude and phase of the total effective voltage input. This can be determined by vectorially adding all four voltages. To do this by the parallelogram method would make the drawing appear somewhat complicated, so the head-to-tail method is used.

The four different voltages in question have values as follows: 10 volts at a phase of 30 degrees, 5 volts at zero phase, 7.5 volts at a phase of 120 degrees, and 7.5 volts at a phase of 150 degrees. The

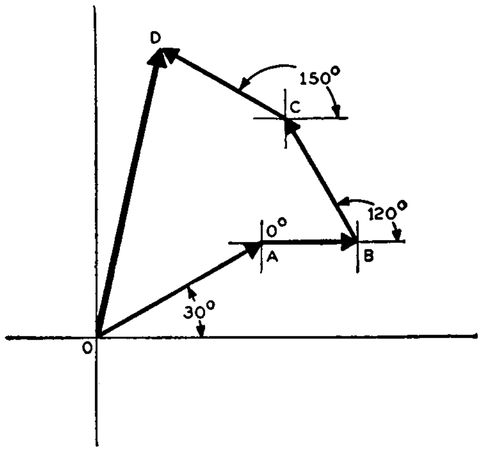


Fig. 6-8—When three or more vectors are to be added, as the four voltage vectors of this figure, the less cumbersome head-to-tail method of addition is preferred. Note the three supplementary sets of coordinate axes.

vector diagram for their addition is illustrated in Fig. 6-8, where the vectors are all drawn to a scale of 1 inch equal to 10 volts. Vector *OA* is first drawn at a phase angle of 30 degrees with its initial point starting at the origin of the main coordinate system. Then the tail of vector *AB* is drawn from the head of vector *OA* at an angle of zero degrees from point *A*. Then from the head of vector *AB* the tail of vector BC is drawn at an angle of 120 degrees. Then from the head of vector BC the tail of vector CD is fixed and drawn at an angle of 150 degrees. This completes the drawing of the individual vectors.

Now in order to find the resultant vector all we do is draw a line from the origin of the main coordinate system, which is the tail of the first vector drawn, namely vector OA, to the head or terminal point of the last vector, CD. The resultant vector is therefore line OD. The magnitude and phase angle of this resultant vector can be determined by measuring the length of the vector and its phase angle with respect to the 0° reference line of the main coordinate axes. A quick inspection of this figure reveals the resultant vector to be greater in magnitude than any of the individual vectors.

The supplementary coordinate systems included in the vector diagrams of the vector additions by the head-to-tail method were drawn only to provide a complete explanation of the method. In most texts or technical periodicals vector diagrams of the head-to-tail method will not contain any supplementary coordinate axes, which are understood to exist nevertheless. This is similar to instances where the main coordinate axes are omitted and are also understood to exist.

Addition by Resolution

There is still a third method by which vectors can be added. This type of vector addition takes into account the knowledge gained from the resolution of a vector. This method of vector addition, involving as many vectors as desired, is accomplished as follows:

Break each vector up into its individual horizontal and vertical components; add vectorially all the vertical components to obtain a resultant vertical component. Thus, a final horizontal and vertical component remain. The next and final step is to *add vectorially* these two components to form the final resultant vector.

A few simple examples of the addition of two vectors by the resolution method will make this type of addition much clearer. In Fig. 6-9 (A) two vectors, OC and OD, are to be added by the resolution method. The first step is to drop perpendicular lines from the heads of the two vectors to the respective horizontal and vertical axis. Vertical component OF and horizontal component OB are from vector OC and vertical component *OE* and horizontal component *OA* are from vector *OD*.

The next step is to add vectorially the respective vertical and horizontal components. Since both groups of components are in the same direction (that is, in phase), the resultant components of each will be greater than either of the individual components in the respective group. Thus vertical component *OF* is added to *OE* to give a resultant vertical component *OH*. The horizontal components are similarly added together, with component *OA* being added to *OB* to give a resultant horizontal component equal to *OJ*.

Next, these two resultant vertical and horizontal components are added in vectorial fashion by the parallelogram method, as if they themselves (the resultant components) had been resolved from some vector. To perform this addition all that is required is to draw line *HG*

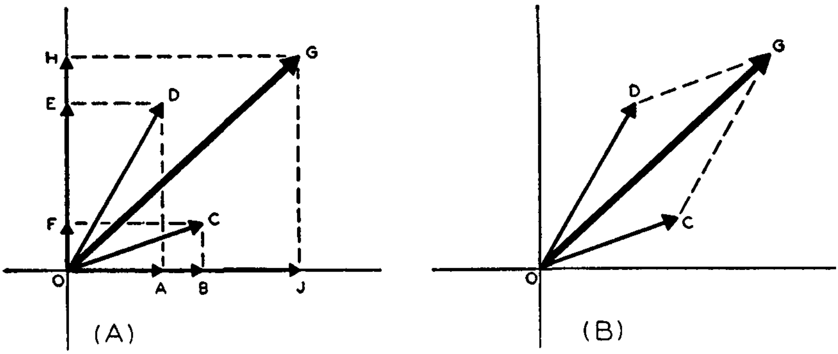


Fig. 6-9—Two vectors *OC* and *OD* are added by the resolution method in (A). Horizontal component *OJ* and vertical component *OH* combine to give resultant vector *OG*. As a check, the vectors are added by the parallelogram method (B) with the same result.

equal and parallel to the resultant horizontal component *OJ* and draw line *JG* equal and parallel to the resultant vertical component *OH*. The diagonal drawn from the origin *O*, to this point *G* is the resultant of the addition of vectors *OC* and *OD*. This is the same as saying lines *OH* and *OJ* are the components due to the resolution of vector *OG*. In order to prove that vector *OG* is correctly the resultant vector of the addition of vectors *OC* and *OD*, the same two vectors can be added together by the parallelogram method as shown in Fig. 6-9 (B). In both vector diagrams the resultant vector *OG* is exactly the same in magnitude and phase.

In order to show that a *resultant* horizontal or vertical component due to resolution can be smaller than the largest single component making up the resultant, let us refer to the vector diagram of Fig. 6-10 (A). Upon resolution of the vectors *OC* and *OD* in this diagram,

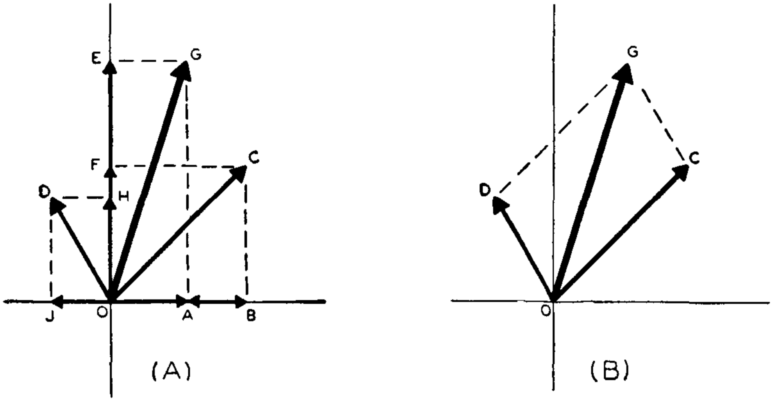


Fig. 6-10—Addition of two vectors by the resolution method (A) showing that the horizontal resultant component *OA* can be smaller than the largest single component *OB* making up the resultant. In (B) the vectors are added by the parallelogram method, and the same resultant vector is obtained.

it will be seen that the horizontal component *OJ* of vector *OD* is in an opposite direction from the horizontal component *OB* of vector *OC*. Since they are in opposite directions, we subtract the smaller component from the larger, as in the addition of 180 degree out-of-phase vectors. This is illustrated by subtracting line *AB*, which is equal to component *OJ*, from component *OB* with the resultant horizontal component equal to *OA*. Note that resultant *OA* is smaller than its component *OB*. The vertical components, since they are in the *same* direction, are additive the same way the components were in the preceding problem, and in this case the resultant vertical component is equal to *OE*. If the resultant horizontal and vertical components of Fig. 6-10 (A) are vectorially added as was done in the preceding problem, the resultant vector will be *OG*. In other words vector *OG* is the resultant of addition of vectors *OC* and *OD*. In Fig. 6-10 (B) the same two

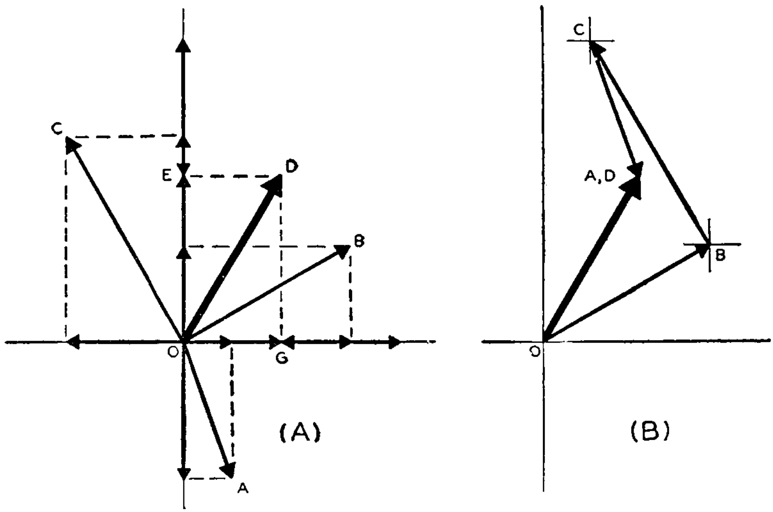


Fig. 6-11—Three vectors *OA*, *OB*, and *OC* are added by the resolution method in (A) to form resultant vector *OD*. This resultant vector checks with the results of the head-to-tail method in (B).

vectors *OC* and *OD* are added by the parallelogram method for comparison purposes to show that the resultant vector *OG* is exactly the same as that in the resolution method of Fig. 6-10 (A).

To find the resultant horizontal and vertical components for the addition of *any* number of vectors by the resolution method, the following simple rules should be applied:

1. Resolve each vector into its respective horizontal and vertical components.
2. Add all those components that are in the same direction.

3. On the axes that contain oppositely directed components subtract the components of smaller magnitude from the larger ones.
4. Add vectorially the final vertical and horizontal components to obtain the resultant vector of the addition.

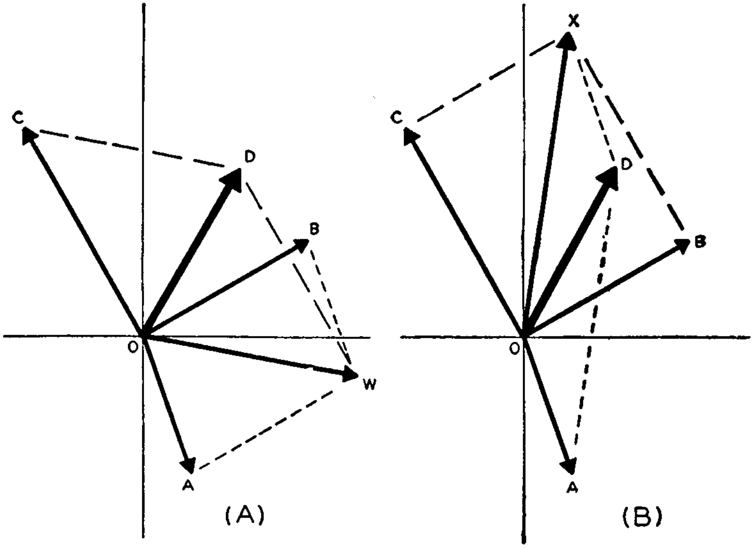
A typical example where these rules are followed is shown in Fig. 6-11 (A) where three vectors, OA, OB, and OC, are added by the resolution method to give a resultant horizontal component *OG* and a resultant vertical component *OE* which when added give the resultant vector *OD*. In Fig. 6-11 (B) the head-to-tail method of addition is illustrated using the same three vectors of Fig. 6-11 (A) for the sake of comparison and in order to show that the resultant vectors in both cases are the same.

Addition of Three Vectors — Parallelogram Method

In order to add three or more vectors by the parallelogram method a number of separate additions must be made. As stated previously, the resultant vector formed by the parallelogram combination of two individual vectors is added by the parallelogram method again to another vector, and the resultant from this combination is further added with another vector and so on.

Let us take a typical example in order to illustrate the way three vectors are added together by the parallelogram method. To simplify the discussion we will use the same three vectors OA, OB, and OC as in Fig. 6-11 (A) and see if we end up with the same resultant vector. These three vectors can be representative of many things as far as electronics are concerned. They may all be voltages, currents, impedances, or any electrical quantity which may have a phase relation.

Since three vectors are involved, any two of the three may be added first. This makes three possible types of vector diagrams which should give the same resultant answer. In Fig. 6-12 (A), (B), and (C) all of these three possible methods of parallelogram addition are shown. In Fig. 6-12 (A) vectors *OA* and *OB* are added first to give the resultant vector *OW;* in Fig. 6-12 (B) vectors *OB* and *OC* are added first to give the resultant vector *OX;* and in Fig. 6-12 (C) vectors *OA* and *OC* are added first to give the resultant vector *OY*. In all of these first additions the respective resultant vectors *OW, OX,* and *OY* have no rela-



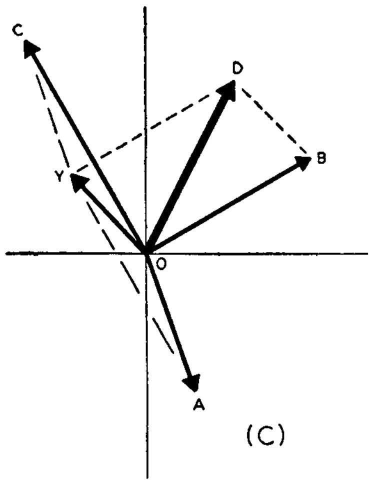


Fig. 6-12—Addition of three vectors by the parallelogram method can be started by first adding any two of the vectors by the parallelogram method and then adding the resultant to the third vector. This is demonstrated in (A), (B), and (C), in each of which vectors *OA*, *OB*, and *OC* are being added. A different set of vectors is used to start the process each time, but the final resultant vector *OD* is the same in all additions.

tion to each other whatsoever. However, if these resultants are added by the same method to the only remaining unused vector in each diagram, the final resultant vector in all these cases will be exactly the same in magnitude and phase. The resultant vector in each diagram is equal to vector *OD*. In Fig. 6-12 (A) resultant vector *OW* is added with vector *OC* to produce the resultant vector *OD*. In Fig. 6-12 (B) resultant vector *OX* is added with vector *OA* to produce the resultant vector *OD*, and in Fig. 6-12 (C) resultant vector *OY* is added with vector *OB* to produce the resultant vector *OD*. In each of these cases the vector *OD* is the final resultant vector for the addition of the three vectors *OA*, *OB*, and *OC* by the parallelogram method. If comparison is made between the final resultant vectors *OD* in Fig. 6-12 (A), (B), and (C) with those resultant vectors *OD* in Fig. 6-11 (A) and (B), they will all be seen to be identical in both magnitude and phase.

If there were a fourth vector to be added to the other three vectors in Fig. 6-12, it would be necessary only to use the parallelogram method of addition between this new vector and the resultant vector *OD* to give us a new final resultant vector. If five vectors were to be added, the same process would continue throughout, utilizing the last resultant vector as part of the final addition.

In most instances, the parallelogram method is that method of vector addition that will be found in texts and technical periodicals that use vectors as a means of radio circuit analysis. Some of these books take it for granted that the parallelogram method is understood, and so the actual formation of the parallelogram (using the dotted lines as we have done) is not carried out, and only the resultant vector, in conjunction with the vectors that are needed, is shown.

In the last chapter, which deals with the vector analysis of radio problems, the parallelogram method of vector addition will predominate, because this method is the most common one used today.

The Impedance Triangle

One of the most important and most common problems in radio circuit analysis is to calculate the magnitude of the impedance of a circuit knowing its combined effective reactance (whether capacitive or inductive) and its total resistance. With the basic knowledge of vectors that we have acquired up to this point and by the use of some very simple mathematics, the analysis of impedance will be easy to visualize.

In the chapter dealing with the resolution of vectors, the problem chosen to represent the method of *resolution* was one dealing with

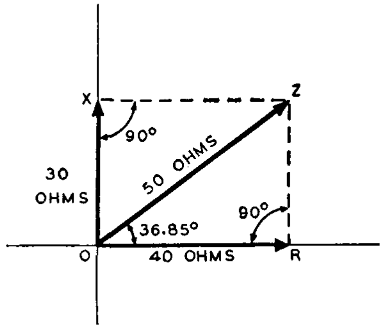


Fig. 6-13—If a resultant series impedance vector is resolved, the vertical component is its effective reactance and the horizontal component is its effective resistance.

impedance (see Fig. 5-2). It was shown that, if a resultant series impedance vector is resolved, the vertical component would be its effective reactance and the horizontal component its effective resistance. For the sake of simplicity and comparison let us redraw Fig. 5-2 as illustrated in Fig. 6-13. From what we know about the resolution method and parallelograms we can readily understand that line *OX* equals line *RZ* and that line *OR* equals line *XZ*. In order to resolve the vector *OZ* draw two perpendicular lines from point *Z* to the ordinate and abscissa, which meet these axes at points *X* and *R* respectively. Consequently, line *ZR* makes a right angle with line *OR* as well as line *ZX* making a right angle with line *OX*, as indicated by the 90° markings in Fig. 6-13.

A parallelogram that has each of its angles a right angle (equal to 90 degrees), as in the diagram *ORZX* of Fig. 6-13, is specifically known as a *rectangle*. The resultant vector *OZ* in Fig. 6-13 is a diagonal of the rectangle *ORZX*, and *any diagonal of a rectangle divides the rectangle into two similar right triangles*. A triangle, as many know, is just an enclosed figure having three straight sides which contains three and only three angles. When one of these three angles of the triangle is a right angle the triangle is known as a *right triangle.*

If we split the rectangle of Fig. 6-13 into two right triangles by means of diagonal *OZ*, the two triangles will appear as seen in Fig. 6-14 (A) and (B) with the same numerical values. In both of these triangles *OZ* is equal to an impedance of 50 ohms at a phase angle equal to 36.85°. In Fig. 6-14 (A) *OX* is the reactive component of impedance and equal to 30 ohms and *XZ* is the resistive component equal to 40 ohms. In Fig. 6-14 (B) the 30-ohm and 40-ohm sides are likewise equal to the reactive and resistive components respectively.

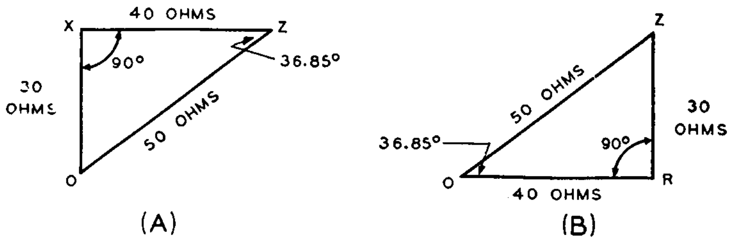


Fig. 6-14—The rectangle of Fig. 6-13 can be split into two right triangles, each of which has *OZ* equal to an impedance of 50 ohms at a phase angle of 36.85 degrees and the 30- and 40-ohm sides equal to the reactive and resistive components, respectively.

From the foregoing discussion it is readily seen that any series impedance can be represented by a right triangle where the components are the series resistance and reactance.

It is on such a right triangle that we can base the simple computations necessary to determine the magnitude of impedance (and the phase angle if necessary) from the reactance and resistance values. In the problem to be discussed the reactance may either be capacitive or inductive, because either one will still give the same magnitude of impedance. The phase angles, of course, will differ, depending upon whether the reactance is capacitive or inductive.

For the problem to be illustrated let us take the triangle of Fig. 6-14 (B) with the same numerical figures but with different letter designations and show how from the values of resistance and reactance we can derive the magnitude of impedance. This triangle designated with letters *A, B,* and *C* is shown in Fig. 6-15. Given any two of the values of either resistance, reactance, or impedance the third one can be simply computed. Though mathematical computations are being kept to a minimum in this book, the use of the impedance triangle is important enough to justify this mathematical discussion.

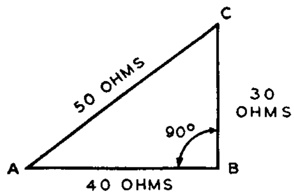


Fig. 6-15—Computations to determine the magnitude of the impedance or components of a series circuit are based on this triangle: given any two sides, the third one can be simply computed.

In any right triangle the two sides that form the right angle, as sides *AB* and *BC* in Fig. 6-15, are called *the legs* of the triangle. The side connecting the two legs, side *AC,* is called the *hypotenuse* of the triangle. The hypotenuse of any right triangle is always larger than either one of its legs but smaller than the arithmetical sum of the two legs.

An old geometrical theorem* about right triangles is that the square of the hypotenuse is equal to the sum of the squares of the two legs. Referring to Fig. 6-15 this theorem is expressed symbolically as follows:

$$(AC)^2 = (AB)^2 + (BC)^2$$

This can also be written as follows:

$$AC = \sqrt{(AB)^2 + (BC)^2}$$

which is the same as the foregoing except that this latter relation is in terms of side *AC* alone and not *AC* squared.

Before we go any further, it may be well to explain what is meant by some of the terms used in the above relations. The term $(AC)^2$ is read as "*AC* squared" which numerically means the value of *AC* times itself. Thus if *AC* equals 50, $(AC)^2$ equals 50×50 or 2500. In the second relation above the $\sqrt{\ }$ is called a square root or radical sign. In order to solve for the resultant number under this sign, we must

*Called the Pythagorean theorem after a famous Greek philosopher and mathematician who lived about the year 500 B.C.

know what number, when multiplied by itself, will equal the amount under this square root sign. Thus if $\sqrt{16}$ is to be solved we must know what number when multiplied by itself will equal 16. This number is 4. If there is a series of numbers underneath the radical sign, then the mathematics relating to these numbers must be carried out first until only one final number remains. After this is done then the answer to the square root can be solved.

When a square root number is multiplied by itself the answer will be the number under the radical sign. This is proved by taking the equation

$$(AC) = \sqrt{(AB)^2 + (BC)^2}$$

and squaring both sides. (Remember squaring means multiplying by itself). Thus:

$$(AC)^2 = (AB)^2 + (BC)^2$$

which is what we started out with.

In the illustration of the impedance triangle of Fig. 6-15, side *AC* equals the impedance, so let us use the impedance sign of Z to represent this side. Likewise let the letter R represent the resistance component, line *AB*, and the letter X the reactive component of the impedance, line *BC*. With these new letter designations the *magnitude* of impedance of a series circuit is

$$Z^2 = R^2 + X^2 \qquad \text{or}$$
$$Z = \sqrt{R^2 + X^2}$$

If we substitute the values for R and X as taken from Fig. 6-15, it will be found that the computed value of Z will be the same as that in the diagram. Hence

$$Z = \sqrt{R^2 + X^2}$$
$$Z = \sqrt{(40)^2 + (30)^2}$$
$$Z = \sqrt{(1600) + (900)}$$
$$Z = \sqrt{2500}$$
$$Z = 50 \text{ ohms}$$

It is, therefore, seen that both values of impedance are the same. This method is quick and easy; one from which either R, X, or Z can be found if any two of the three values are known.

Though the phase angle of the impedance can also be calculated, the computation requires a knowledge of basic trigonometry and is beyond the scope of this book.

Addition of the Vectors in Chapter 4

In Chapter 4 a number of multiple vector diagrams were illustrated. In the problems shown we did not find any resultant vectors because at that point in the chapter the addition of vectors was presumably unknown. However, we now can perform the vector additions.

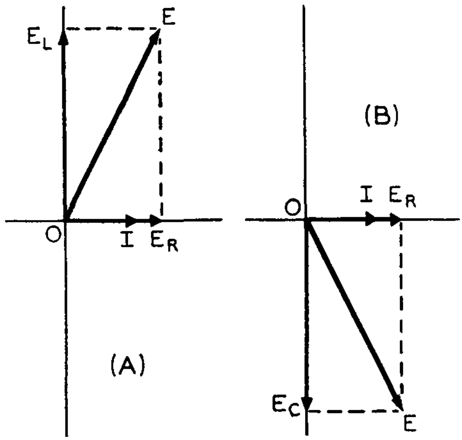


Fig. 6-16 (A)—Vector addition of the voltage across the resistive and inductive branches of the series R-L circuit of Fig. 4-10 yields a resultant vector *OE* representative of the total voltage across the circuit.

(B)—Vector addition of the voltages across the individual branches of the series R-C circuit of Fig. 4-11 yields a resultant vector *OE* representative of the total voltage across the R-C circuit.

Series R-L Circuit

In Fig. 4-10 (B) the vector diagram represents the different voltages across the resistive and inductive branches of a series R-L circuit in conjunction with the current flowing through the circuit. If the voltage vectors OE_R and OE_L representing the voltage drops across the resistance and inductance components respectively are vectorially added together, the resultant vector will be that representative of the total voltage across the series R-L circuit. This vector addition by the parallelogram method is illustrated in Fig. 6-16 (A) where the resultant voltage is represented by vector *OE*. From this diagram it is readily seen that in an inductive series circuit which contains some resistance, the voltage across the whole circuit leads the current through it by some phase angle between 0° and 90°, the exact amount depending upon the magnitude of the inductance and resistance.

Series R-C Circuit

In Fig. 4-11 (B) the vector diagram represented the voltages across the individual branches of a series R-C circuit with respect to the current flowing through the circuit. As in the series R-L circuit, if the voltage drop across the capacitance, vector OE_C, and the resistance, vector OE_R, are vectorially added, the resultant voltage vector will be that across the whole circuit. This is indicated in the vector diagram of Fig. 6-16 (B) where vector OE is the resultant voltage across the series R-C circuit. From this vector diagram it is seen that the

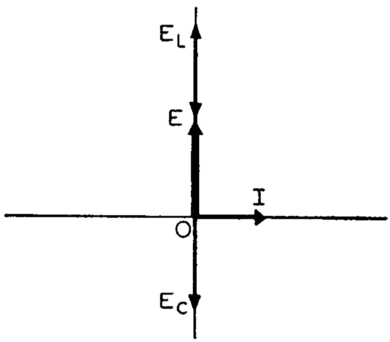


Fig. 6-17—Vector addition of the voltage components across a series L-C circuit of Fig. 4-12 yields a resultant voltage vector OE which is predominantly inductive since the inductive reactance is larger than the capacitive. Consequently, the resultant voltage leads the current through the circuit by 90°.

total voltage across the R-C circuit vector, OE, lags the current, vector OI, flowing through it. In any capacitive circuit containing some resistance the voltage across the circuit lags the current flowing through it (or the current leads the voltage) somewhere in between 0° and 90°. The actual amount of degree lag of the voltage is determined by the ratio of the magnitude of the reactance of C and the resistance of R.

Series L-C Circuit

For the series L-C circuit of Fig. 4-12 (A) one can see in the vector diagram for this circuit in Fig. 4-12 (B) that the voltage drop vector, OE_L, across the inductance is greater than the voltage drop, vector OE_C, across the capacitance. This indicates that the inductive reactance is greater than the capacitive reactance at the frequency of operation.

since the same current flows through both. If the inductive reactance is greater, then the circuit should be predominately inductive, and the voltage across the whole series circuit should lead the current through it by 90°, since it is assumed that no resistance is present in the circuit. This is all indicated in the vector diagram of Fig. 6-17 which is the same as Fig. 4-12 (B) except that the vector addition of the voltage components is done in the diagram of Fig. 6-17. Since the inductive voltage drop, vector OE_L, is exactly opposite in phase (that is, 180 degrees) from the capacitive voltage drop vector, OE_C, for the vector addition of the two we subtract the smaller magnitude from the larger and the difference is the resultant voltage vector, *OE*. The resultant voltage vector, *OE*, is seen to be inductive and leading the current vector *OI* by 90 degrees.

Series R-L-C Circuit

In order to see what the resultant voltage vector will look like in a series R-L-C circuit in which the inductive reactance is greater than

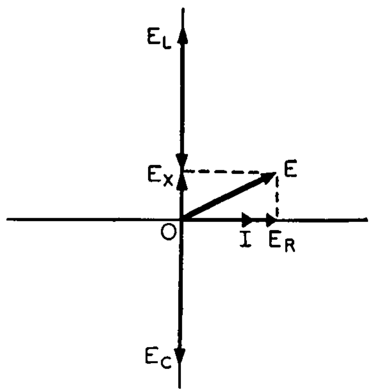


Fig. 6-18—The resultant voltage vector *OE* for the series R-L-C circuit of Fig. 4-13, in which the inductive reactance is greater than the capacitive reactance, is obtained by the vector addition of the three individual voltages E_C, E_L, and E_R.

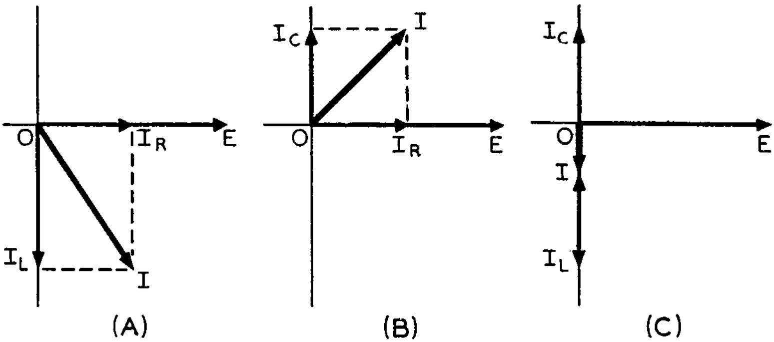
the capacitive reactance let us refer to the vector diagram of Fig. 6-18. This vector diagram is exactly the same as that one of Fig. 4-13. The process of vector addition of the three individual voltages is quite simple. Since three voltages are present, two separate vector additions have to be made. We can start with any two voltage vectors we desire, but since two of the voltage vectors are 180° out of phase, it would be much simpler to start with these. Therefore, we vectorially add inductive voltage vector, OE_L, and capacitive voltage vector, OE_C, by subtracting the magnitude of the smaller vector, OE_C, from the larger vector, OE_L, resulting in vector, OE_X. Since this addition is only for two reactances, we call the final vector OE_X the *resultant reactive voltage drop.* The next vectorial addition to be carried out is between this reactive voltage, vector OE_X and the resistive voltage, vector OE_R. The addition is by the parallelogram method, and the final resultant voltage vector across the whole series R-L-C circuit is vector *OE.* Since the reactive part of the circuit is predominately inductive, this final voltage, vector *OE,* leads the current, vector *OI,* flowing through the circuit.

From the series circuit vector diagrams containing inductance and capacitance a very interesting phenomenon is evident. In Figs. 6-17 and 6-18 it will be noticed that the magnitudes of the resultant voltage across the complete series circuit is *less* than either one or both of the reactive voltage drops. In other words it is possible in a series circuit to have the voltage drop across either one or both of the reactances *greater* in magnitude than the total voltage drop across the *whole* series circuit. This is often the case in series resonant circuits wnere it is often stated that a "resonant rise in voltage" has occurred across either reactive component. This is especially so when there is a minimum amount of resistance in the series circuit.

Parallel Circuits

In Fig. 6-19 are shown four vector diagrams. Each one represents a different type of parallel circuit based upon those of Fig. 4-14. Fig. 6-19 (A) and Fig. 4-14 (C) are the same vector diagrams, except that in the former illustration vector addition is included. This vector diagram represents a parallel R-L circuit in which the inductive branch current lags the resistive branch current by 90 degrees. Vectorially adding these two separate branch currents as shown in Fig. 6-19 (A) results in vector *OI,* which is the total current flowing the parallel R-L circuit. This total current lags the voltage across the circuit, indicating that the circuit is inductive.

Fig. 6-19 (B) is the same as Fig. 4-14 (F) but in the vector diagram of the former there is vectorial addition of the current components. The circuit represented is a pure R-C parallel network. The vector addition of the capacitive current, vector OI_C, and the resistive current, vector OI_R, results in the total line current of the R-C circuit, and it is designated as vector *OI* in Fig. 6-19 (B). This resultant cur-



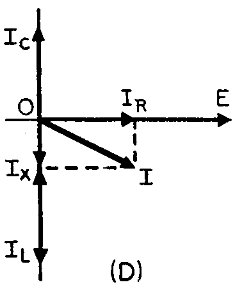


Fig. 6-19 (A), (B), (C), (D)—Vector addition of the current components of the parallel circuits shown in Fig. 4-14. Except for the vector additions, the vector diagrams are the same: Fig. 6-19 (A) and Fig. 4-14 (C), Fig. 6-19 (B) and Fig. 4-14 (F), Fig. 6-19 (C) and Fig. 4-14 (I), Fig. 6-19 (D) and Fig. 4-14 (L).

rent vector *OI* indicates that the circuit is capacitive in nature, because this resultant current vector leads the voltage, vector *OE*, across the circuit.

In both the parallel R-L circuit and the parallel R-C circuit the phase angle between the resultant current and the voltage is such that it will lie somewhere in between 0° and 90° in the R-C circuit and between 0° and —90° in the R-L circuit because of the resistive component of current in both circuits.

Figs. 6-19 (C) and 4-14 (I) are the same in that they both represent the same parallel L-C circuit, but in Fig. 6-19 (C) the vector addition of the branch currents is shown. The inductive reactance is smaller than the capacitive reactance at the frequency of operation, so that when the main circuit current divides into the two branch currents, the current in the capacitive branch will be smaller than that in the inductive branch. Consequently, after the vector addition of these two branch currents, which are 180 degrees out of phase, the resultant current will be found to be inductive in nature. This is shown in Fig. 6-19 (C) where vector *OI*, is the total line current or resultant current of the complete parallel system, and it lags the voltage across the circuit by exactly 90 degrees because there is no resistance in the network.

In Fig. 6-19 (D) is shown the vector addition of the R-L-C parallel circuit vectors of Fig. 4-14 (L). In this R-L-C circuit the inductive reactance is still smaller than the capacitive reactance, but there is also a resistive branch, which means that the resultant current will still lag the voltage across the circuit but by some angle less than 90 degrees. This is all shown in Fig. 6-19 (D). There are three separate current vectors here, consequently two separate vector additions are made. This is similar to the series R-L-C network. First the vector addition of the reactive (180 degrees out of phase) components are made, yielding a resultant reactive current vector OI_X. Next this reactive vector OI_X and the resistive current vector OI_R are vectorially added, and the final resultant current vector is *OI*. Since the circuit is primarily inductive in nature, this resultant current lags the voltage across the circuit by some phase angle in between 0° and 90°, the exact amount depending upon the magnitude of vectors OI_X and OI_R.

A very interesting phenomenon regarding parallel circuits that contain both inductive and capacitive branches is evident from the vector diagrams Fig. 6-19 (C) and (D). From these diagrams it will be noticed that the final resultant vector *OI* is *smaller* than either current flowing through the reactive branches of the circuits. This of course is not absolutely true of all parallel circuits that contain inductance and capacitance. However, in parallel resonant circuits that contain a minimum amount of resistance, this difference in current definitely manifests itself. Under these conditions it is often stated that a "resonant rise in current" has occurred through either the inductive or capacitive branch of the circuit.

CHAPTER 7

SUBTRACTION, MULTIPLICATION, AND DIVISION OF VECTORS

In problems dealing with the subtraction, multiplication, and division of vector quantities of such related topics as voltage, current, impedance, power, and the like, vector diagrams can also be used to supplement mathematical computations. For instance, if a current that has a magnitude of 12.5 amperes at a phase angle of 20 degrees is to be subtracted from another current that has a magnitude of 15 amperes at a phase angle of 60 degrees, the subtraction of magnitudes and phase angles *cannot* be made directly by simple arithmetical subtraction. The answer to this problem *would not be* 2.5 amperes at a phase angle of 40 degrees.

Since it is not the purpose of this book to delve into much mathematical computations but rather to illustrate the use of vectors, an actual quantitative analysis is really not necessary. However, some of the mathematics involved is not complex, so that in certain problems that will be analyzed mathematics may be included as a comparative check of a particular vector diagram under discussion.

Subtraction by the Parallelogram Method

In the subtraction of vectors the parallelogram method is also used but in a different manner from that employed in the addition of vectors. As an example, suppose that we desired to subtract one alternating current from another. The vectors representing these two alternating currents are *OA* and *OB* as illustrated in Fig. 7-1. In order to subtract vector *OA* from vector *OB* we draw a parallelogram using these two vectors, but since vector *OA* is to be subtracted *from* vector *OB* we use *vector OB as a diagonal of the parallelogram* and not as a side of the parallelogram, as was done with the addition of vectors.

Thus in Fig. 7-1 a line is drawn from the terminal point of vector *OB*, parallel and equal to the magnitude of vector *OA*. This line is designated as *BC* in Fig. 7-1. Next connect points *A* and *B* by a straight line and then points *O* and *C* to complete the parallelogram. From this final diagram the parallelogram is seen to be the four-sided figure surrounded by points *OABC*. The resultant vector from this type of subtraction is equal to the line drawn from the origin to point *C*, namely vector *OC*. This example demonstrates how simple this method of vector subtraction really is. It is not essential to draw line *AB*, since line *OC*, the resultant, can be drawn without the use of line *AB*. The drawing of line *AB* was suggested to show that the method of subtraction indirectly involves the use of a parallelogram.

If we wanted to subtract vector *OB* from vector *OA* we would likewise use both vectors *OA* and *OB* to form a parallelogram, but in this instance vector *OA* is used as the *diagonal* and vector *OB* as one of the

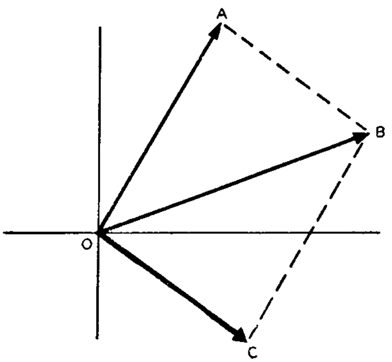


Fig. 7-1—A parallelogram is constructed to subtract one vector from another, but the minuend, in this case *OB*, is used as a diagonal and not as a side of the parallelogram, as both quantities are in addition. Vector *OC* is the resultant of this subtraction.

sides of the parallelogram to be constructed. This vector subtraction is shown in Fig. 7-2. From the terminal point of vector *OA* a line is drawn parallel and equal to vector *OB*. This line is designated as *AD*. Next points *A* and *B* as well as points *O* and *D* are joined by straight lines to complete the remaining parallel and equal sides of the parallelogram represented by letters *OBAD*. It is readily seen that vector *OA* is the diagonal of the parallelogram and that the resultant vector is *OD*. If the two resultant vectors *OC* and *OD*, of Figs. 7-1 and 7-2 respectively, are examined, it will be seen that they are exactly equal in magnitude but opposite in phase (that is, 180 degrees out of phase).

In both of these vector diagrams the magnitude of the resultant vector of subtraction can be readily determined by the length of line *AB*, since this line is equal in length to the resultant vector in both diagrams. In other words, in both of these vector subtractions the line *AB*, which is parallel and equal to the resultant vectors, is situated in exactly the same physical location. If we extend resultant vector *OC* of Fig. 7-1, wherein vector *OA* is subtracted from vector *OB*, by a

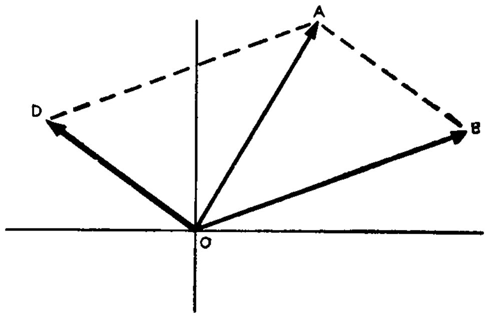


Fig. 7-2—The minuend and subtrahend of Fig. 7-1 have been reversed, and here *OB* is subtracted from *OA* to give the resultant vector *OD*.

line of equal length *from* the initial point of the vector, then this new line will represent the resultant vector of the two vectors *OA* and *OB* when the subtraction is reversed; that is when vector *OB* is subtracted from vector *OA*. In other words this extended vector will be the same as the resultant vector *OD* in Fig. 7-2.

Subtraction by Resolution

Subtraction of vectors, like their addition, can be accomplished by a number of different methods other than the parallelogram arrangement. In this section we will study the subtraction of vectors by the *resolution method*. For comparison purposes we will use the same two vectors *OA* and *OB* as employed in Figs. 7-1 and 7-2. The subtraction of these two vectors by the resolution method is illustrated in Figs. 7-3 (A) and (B). The subtraction in Fig. 7-3 (A) corresponds to that in Fig. 7-1 and in Fig. 7-3 (B) corresponds to that in Fig. 7-2. Let us look at these two vector diagrams of Fig. 7-3 and analyze them according to what we know about the resolution of vectors.

In either case the first step is to resolve both vectors *OA* and *OB* into their respective vertical and horizontal components. In Fig. 7-3 (A) where we are subtracting vector *OA* from vector *OB*, we subtract the vertical and horizontal components of vector OA from the vertical and horizontal components of vector OB respectively. The vertical component of vector OA is designated as OE and the horizontal component as OG; the vertical component of vector OB is designated as

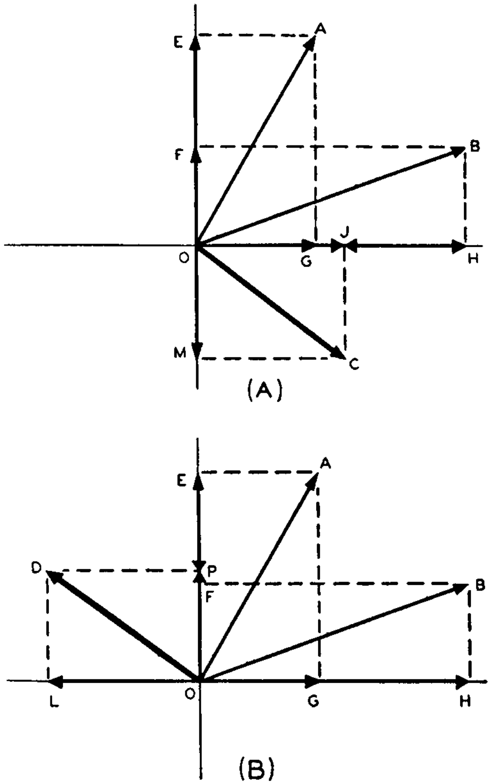


Fig. 7-3—In (A) we have subtraction by the resolution method of the vectors of Fig. 7-1. The resultant vector is OC which checks with vector OC in Fig. 7-1. In (B) subtraction by the resolution method of the vectors of Fig. 7-2 yields resultant vector OD which checks with vector OD of Fig. 7-2.

OF and the horizontal component as OH. Next we *subtract* these individual resultant components. The subtraction of components lying directly on the reference axes of the coordinate system is accomplished arithmetically since no phase-angle differences are involved. In subtracting one vertical component from another, as component

OE from OF in Fig. 7-3 (A), even though the magnitude of *OE* is greater, we take a line equal to the length of component *OE* and starting out from the terminal point of component *OF* we draw this line in the opposite direction from which the terminal point of vector *OE* is facing. Accordingly we end up at point *M* and the resultant vertical component of subtraction is equal to *OM*. In subtracting the horizontal components of the same vector subtraction of Fig. 7-3 (A), the horizontal component *OG* of vector *OA* is subtracted from horizontal component *OH* of vector *OB*. Following the method used in subtracting the vertical components, we end up with a resultant horizontal component equal to line *OJ*.

These two resultant vertical and horizontal components are the respective component parts of the resultant vector which is obtained from subtracting vector *OA* from vector *OB*. Since these two components are resultant components, adding them by the parallelogram method gives us the resultant vector *OC*, representing the subtraction of vector *OA* from *OB*. Upon comparison of this resultant vector *OC* of Fig. 7-3 (A) with that of Fig. 7-1, it can be seen that they are identical both in magnitude and phase.

In Fig. 7-3 (B) the subtraction of vector *OB* from vector *OA* is carried out in a similar manner—first resolving the individual vectors into their respective component parts and then subtracting the vertical and horizontal component of vector *OB* from the respective vertical and horizontal components of vector *OA*. The resultant components from this subtraction are the component parts of the resultant vector. The resultant vector of this subtraction is vector *OD* in Fig. 7-3 (B), and, when it is compared with the resultant vector obtained by the parallelogram method of subtraction of Fig. 7-2, it is found to be exactly the same in both magnitude and phase.

It should be remembered that in the subtraction of vectors, only those vector quantities having the same dimensional characteristics can be subtracted from each other; that is voltage from voltage, current from current, etc.

It is very seldom that the vector subtraction of three or more vectors is used in the vector analysis of radio circuit problems; consequently we are not including it in this book. However, such subtraction is similar to the addition of more than two vectors.

Multiplication of Vectors

In the algebraic method of vector multiplication, if we know the magnitudes and phase angles of the individual vectors, all that need be done is to multiply their magnitudes arithmetically by each other and add their phase angles to get the resultant vector. This is a very simple mathematical process for the multiplication of vectors, because the arithmetic involved can be done quickly and the resultant vector value can be easily plotted on a vector diagram.

For instance, if we have any two vectors, one with a magnitude designated by the letter P at a positive angle designated by the Greek letter Θ (theta) and the other with a magnitude M at a positive phase angle designated by the Greek letter Φ (phi), (both angle signs having been used interchangeably), then they would be mathematically represented as $P\underline{/\Theta}$ and $M\underline{/\Phi}$. Following the above rule for multiplication then:

$$P\underline{/\Theta} \times M\underline{/\Phi} = P \times M\underline{/\Theta + \Phi}$$

This means that if P equals 10, M equals 6, Θ equals 30 degrees, and Φ equals 15 degrees, then when both vectors are multipled, the resultant vector would be:

$$10\underline{/30^\circ} \times 6\underline{/15^\circ} = 10 \times 6\underline{/30^\circ + 15^\circ} = 60\underline{/45^\circ}$$

which means that the resultant vector would have a magnitude of 60 at a phase angle of 45°. In order to illustrate this by some typical electrical problem, let us consider the simple application of Ohm's law as applied to a-c theory.

In basic mathematical form Ohm's law states that $E = IZ$, where E is the voltage, I the current, and Z the impedance of the circuit in question. If in some electric circuit the current is equal to 1.33 amperes at a phase angle of 15 degrees and the impedance is equal to 2 ohms at a phase angle of 45 degrees, the amount of voltage across this circuit would be $1.33 \times 2\underline{/15^\circ + 45^\circ} = 2.66\underline{/60^\circ}$ volts. A simple means of showing this by a vector diagram is first to draw the current and impedance vector in direct accordance with their magnitudes and phase angles, then compute the voltage vector as previously shown, and after the computation, draw the final voltage vector in the vector diagram. In doing this we are not multiplying the vectors by straight geometrical means, but instead we are involving some mathematics. The method involved for the true geometric multiplication of two vectors is somewhat more difficult than in the subtraction and addition of vectors as previously discussed. The geometric method of vector multiplication will now be considered.

The problem to be illustrated will, as above, involve the multiplication of a current vector of $1.33\underline{/15^\circ}$ amperes and an impedance vector of $2\underline{/45^\circ}$ ohms to give us a resultant voltage vector. *In multiplying vectors in true geometric fashion the two vectors involved have to be drawn exactly to the same scale!* In other words, if we set some scale for impedance, say 10 ohms equal to 1 inch, then 10 amperes will also have to equal 1 inch, so that the resultant voltage vector of multiplication will have 1 inch equal to 10 volts. In addition or subtraction the vectors involved were all of the same type, either all voltage vectors, all current vectors, etc., because one cannot subtract say 10 volts from 6 amperes or add 10 volts to 6 amperes, because the answers would not mean anything. In multiplication the situation is different, because we *can* multiply two different kinds of quantities to give us a new third quantity and that is the reason why the vectors multiplied *although dimensionally different* have to be drawn to the same scale.

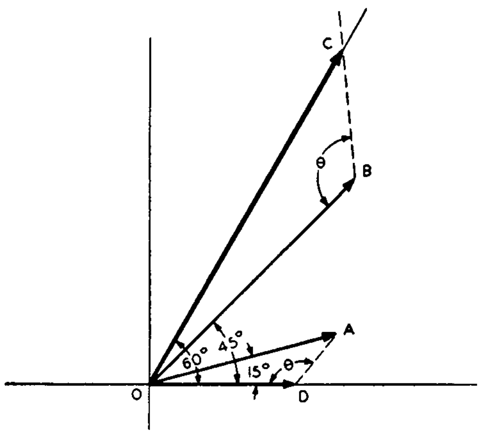


Fig. 7-4—Multiplication of current vector *OA* and impedance vector *OB* results in voltage vector *OC*. The method employed in multiplying vectors is discussed in the text.

In Fig. 7-4 the multiplications of the impedance and current vectors are shown. Vector OA is the current vector of 1.33 amperes at a phase angle of 15 degrees, and vector OB is the impedance vector that has a magnitude of 2 ohms and a phase angle of 45 degrees. The scale chosen for this problem is such that 1 inch is equal to 1.33 ohms and 1.33 amperes. This means that the length of the current vector is just 1 inch long, the 2-ohm impedance vector is 1.5 inches long, and the resultant voltage vector will be such that 1 inch will equal 1.33 volts.

Referring to Fig. 7-4, the first operation is to measure an angle of 60° from the zero reference line, which angle represents the addition of the respective phase angles of the vectors (15° + 45°) and then draw a straight line from the origin of the coordinate system at this angle of 60 degrees. In doing this we establish the line along which the resultant vector will lie. The next step is to lay off on the 0° reference line a line of *unit length*. By *unit length* we mean that length equivalent to 1 ampere or 1 ohm of the vectors involved. Since the scale involved for both vectors is the same, we can refer to such a dimension as unit length.

Since the scale of measurement is 1 inch to 1.33 (ohms or amperes), the unit length is equal to 1/1.33 or ¾ inch. Consequently, a line equal to ¾ inch and designated as OD, is drawn on the 0° reference line. The next step is to draw a straight line from the terminal point of vector OA to point D, giving line AD, which makes an angle designated as Θ with line OD at point D. Next measure, from the terminal point of vector OB, an angle equal to Θ, (the angle formed by sides OD-AD) in the direction indicated in Fig. 7-4, and from this angle of measurement draw a straight line from the terminal point of vector OB. This line will intersect the 60-degree line, which was drawn from the origin, at point, C. This intersection point C determines the length of the resultant vector of the vector multiplication. Therefore, line OC is the resultant vector of the multiplication of vector OA and OB.

By mathematics we have shown the value of the resultant vector to have a magnitude of 2.66 volts at a phase angle of 60 degrees. The phase angle for the resultant vector in the vector diagram is readily seen to be equal to 60 degrees since we established this previously, but the thing to check is whether the resultant vector OC has the same magnitude as the mathematical answer. To determine this, measure the length of the resultant vector OC, and upon measurement it is found to equal exactly 2 inches. Inasmuch as the scale originally established is the same for the unit lengths of *all* vectors involved, then the same scale applies to the resultant vector. Therefore, since the scale is 1 inch equals 1.33 units, the 2 inches of magnitude of the resultant vector OC is equal to 2×1.33 or 2.66 units. As the resultant vector is a voltage vector, then its magnitude is equal to 2.66 volts. This answer checks the mathematical computation indicating that the geometrical vector method of multiplication is correct.

In order to multiply more than two vectors, we first multiply two of the vectors as shown in Fig. 7-4 and then we use the resultant of this multiplication as a new vector to be vectorially multiplied with one of the original vectors not originally utilized. The resultant of this vector multiplication is further multiplied with another unused original vector and so on until all the original vectors to be multiplied have been utilized in which case the *final resultant* vector is the answer to the complete vector multiplication.

Multiplication With a Negative Phase Angle

In the foregoing method of vector multiplication only positive phase angles were involved. This is not a requirement of vector multiplication because any combination of phase angles can be employed utilizing the same method.

For example, let us assume that the phase angle of the impedance vector in the preceding problem instead of being a positive 45° is a negative 45°. This means the new impedance vector is 2 ohms at a negative angle of 45° or written as $2\underline{/-45^\circ}$. The mathematical multiplication of the current and new impedance is:

$$1.33\underline{/15^\circ} \times 2\underline{/-45^\circ}$$

and since we have to multiply the magnitudes and add the phase angles we find the mathematical solution to the foregoing as

$$1.33 \times 2\underline{/15^\circ + (-45^\circ)} = 2.66\underline{/-30^\circ}.$$

The resultant voltage vector is still at a magnitude of 2.66 volts but at a negative phase angle of —30 degrees. When "adding" a positive and a negative phase angle, the resultant phase angle is the difference between the two angles and bears the sign of the larger angle.

To show this multiplication geometrically is quite easy, since we already know how to multiply from the previous vector diagram problem. This multiplication is shown in Fig. 7-5. First, both vectors *OA* and *OB* are drawn according to the same scale of 1 inch equaling 1.33 units, as in the previous problem with respect to their proper magnitudes and phase angles. Next, at an angle of —30° from the 0° reference line, draw a straight line, extending away from the origin in the fourth quadrant. This —30° angle is geometrically obtained by measuring in a counterclockwise direction from vector *OB* the positive angle of 15°. Next lay off a unit length on the 0° reference

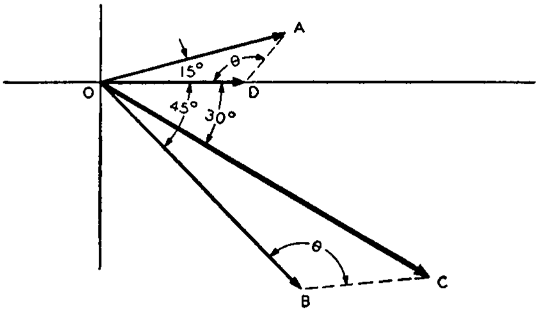


Fig. 7-5—Vector multiplication involving current vector *OA* with a positive phase angle and impedance vector *OB* with a negative phase angle. The resultant voltage vector is seen to have a negative phase angle.

line as was done in Fig. 7-4. This length is still equal to ¾ inch due to the same scale proportions and is designated as line *OD* in Fig. 7-5. To point *D* connect a line from point *A*. Measure the angle Θ indicated at *D* and lay off this same exact angle at point *B*, as shown in Fig. 7-5. From point *B*, and at this angle Θ, draw a line from *B* until it reaches point *C* on the —30° line previously drawn. The line *OC* represents the resultant vector of multiplying vector *OA* ($1.33\underline{/15^\circ}$ amperes) by vector *OB* ($2\underline{/-45^\circ}$ ohms). If the length of the resultant vector *OC* is measured, it will be found to equal 2 inches, which is equivalent to 2.66 volts in accordance with the scale chosen. The phase angle, of course, is readily seen to be a negative 30 degrees as determined previously.

Division of Vectors

The division of vectors is quite easy to perform by simple algebraic means as in the multiplication of vectors. All that has to be known is the magnitude and phase angle of the vector quantities involved. The *algebraic* division of any two vector quantities is quite simple: compute as in the case of multiplying two vector quantities. All one does is to *divide* their respective magnitudes and *subtract* their phase angles. For example let us assume two vectors, such that one vector has a magnitude equal to the letter A and a phase angle of Θ and the other vector has a magnitude equal to B at a phase angle of Φ. If we wanted to divide the former by the latter we would write, according to the above rule:

$$\frac{A \angle \Theta}{B \angle \Phi} = \frac{A}{B} \angle \Theta - \Phi$$

If A equals 10 units, Θ equals 45°, B equals 5 units, and Φ equals 30°, the resultant division would be:

$$\frac{10 \angle 45^\circ}{5 \angle 30^\circ} = \frac{10}{5} \angle 45^\circ - 30^\circ = 2 \angle 15^\circ$$

If the phase angles are such that Φ equals —30 degrees instead of a positive 30°, then the division would be:

$$\frac{10 \angle 45^\circ}{5 \angle -30^\circ} = \frac{10}{5} \angle 45^\circ - (-30^\circ) = \frac{10}{5} \angle 45^\circ + 30^\circ = 2 \angle 75^\circ \ *$$

Or if both phase angles were negative, we would get:

$$\frac{10 \angle -45^\circ}{5 \angle -30^\circ} = \frac{10}{5} \angle -45^\circ - (-30^\circ) = \frac{10}{5} \angle -45^\circ + 30^\circ = 2 \angle -15^\circ \ *$$

Or if Θ is negative and Φ is positive, we would get:

$$\frac{10 \angle -45^\circ}{5 \angle 30^\circ} = \frac{10}{5} \angle -45^\circ - 30^\circ = 2 \angle -75^\circ \ *$$

*To subtract one quantity from another, the sign of the quantity to be subtracted (the subtrahend) is changed, and the quantities are then added algebraically. When a negative quantity, such as (—30°), is to be subtracted, its sign must be changed (made positive), and the quantity is then *added* algebraically. This holds true whether the original negative quantity is subtracted from a positive or a negative quantity.

When the minus sign is included in parentheses, (—30°), it indicates that the quantity is negative. When subtracting such a negative quantity, indicated as —(—30°), what occurs is that the two minus signs effectively become equal to a plus sign, therefore —(—30°) is the same as +30°.

From these simple relations we can easily see how to divide two vectors algebraically no matter what the polarity of their phase angles may be.

Let us consider a basic mathematical problem in division as applied to Ohm's law before we discuss the division of vector by geometrical means, namely by the vector diagram method. For example, if we know the voltage and current in a circuit, how can we determine the effective impedance of the circuit? If the voltage E equals 4 volts at a phase angle of 110° and the current I equals 2 amperes at a phase angle of 40°, then the impedance Z as given by Ohm's law is:

$$Z = \frac{E}{I} = \frac{4\underline{/110^\circ}}{2\underline{/40^\circ}} = \frac{4}{2}\underline{/110^\circ - 40^\circ} = 2\underline{/70^\circ}$$

Thus we can see that the resultant value of impedance is equal to 2 ohms at a positive phase angle of 70°. If the polarity of the phase angles is changed in any way whatsoever, we would follow the simple rules previously illustrated.

To illustrate this impedance problem by means of a vector diagram we proceed in the following manner:

First, knowing the two vectors to be added, we draw them on the vector diagram as shown in Fig. 7-6 with respect to their magnitude and phase. Here vector OA represents the voltage vector of $4\underline{/110^\circ}$ volts, and OB represents the current vector of $2\underline{/40^\circ}$ amperes. In this diagram the vectors all have to be drawn to the same unit scale, as was done in the multiplication problem. In Fig. 7-6 the scale used is 1 inch equals 2 units, thus the 2-ampere magnitude of vector OB is 1 inch in length and the 4-volt magnitude of vector OA is 2 inches in length. The process of vector division is as follows:

Having drawn each individual vector and knowing the phase angles of the vectors, we *subtract* the value of the phase angle of the denominator from that of the numerator and at this point draw a straight line from the origin. Since the phase angle of the numerator is 110° and that of the denominator is 40°, then after subtraction the resultant phase angle is 70° with respect to the 0° reference line, and a line corresponding to 70° line is drawn from this point extending out into the first quadrant. The next step is to lay off a unit length along the 0° reference line. Since the scale used is 2 units per inch, then 1 unit is equal to ½ inch, so the unit length is equal to ½ and is represented by line *OD* on the 0° reference line. Next draw a line from the terminal point of that vector which is considered to be in the denominator in regular division, (vector *OB*) to the point *D*. This

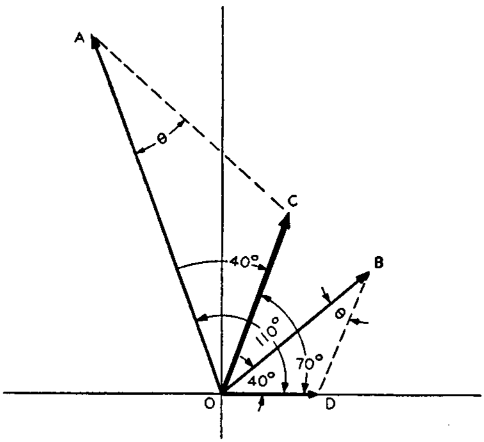


Fig. 7-6—Vector division of voltage vector *OA* by current vector *OB* results in impedance vector *OC*. The scale used is one inch to two units. The geometric method employed is discussed in the text.

construction is represented by line *BD*. The angle Θ formed by line *OB* and *BD* is then measured and from the terminal point of vector *OA* we measure off an angle equal to this same angle, and then draw a straight line from point *A* at this angle Θ until it reaches the line drawn in the first quadrant at an angle of 70°. These two lines meet at point *C*, and the length of the line *OC* determines the magnitude of the resultant impedance vector after division. In other words, vector *OC* is the resultant vector, and has a phase angle of 70 degrees. If this vector *OC* is measured, it will be found to be one inch long and from the scale set up for this problem of 2 units equal to 1 inch, the magnitude of the impedance vector is, therefore, exactly equal to 2 ohms. Thus we see that the vector division by geometric means gives us the same answer as by algebraic methods.

In order to prove the answers that we have found for the impedance of the preceding problem, let us assume that the impedance answer is correct and then solve for the current. In other words, from Ohm's law the current will be equal to the algebraic division of voltage by the impedance. Thus algebraically:

$$I = \frac{E}{Z} = \frac{4\angle 110^\circ}{2\angle 70^\circ} = \frac{4}{2}\angle 110^\circ - 70^\circ = 2\angle 40^\circ$$

which proves that the value of impedance is correct, because the value of current computed is exactly the same as that given in the first problem.

To prove this by the aid of a vector diagram is quite easy, because we already know how to divide vectors from the previous analysis of Fig. 7-6. We assume that we know the impedance vector of $2\angle 70^\circ$ ohms and the voltage vector of $4\angle 110^\circ$ volts and that we have to find the current vector by geometric division. For comparison we chose

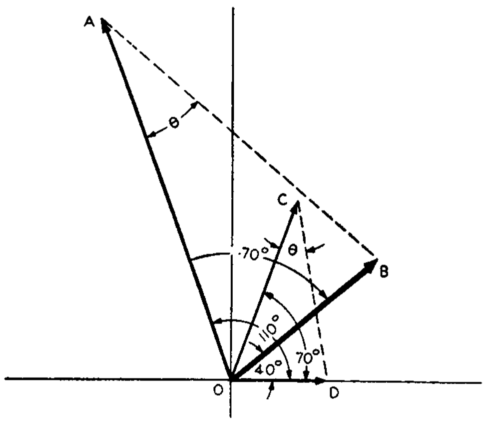


Fig. 7-7—To check the accuracy of the diagram of Fig. 7-6, it is assumed that the impedance vector *OC* and voltage vector *OA* are known, and the current vector *OB* is found by vector division.

the same scale of 2 units to 1 inch as in the preceding problem. Next we draw the voltage vector *OA* and the impedance vector *OC* in accordance with their proper magnitudes and phase angles as seen in Fig. 7-7. Next *subtract* the phase angle of the impedance vector, 70° in a clockwise direction from the 110° phase angle of the voltage vector and we get an angle of 40° with respect to the 0° reference line. From this 40° point we draw a straight line in the first quadrant extending from the origin. Next lay off on the zero degree reference line a unit length, which is equal to ½ inch as in the preceding problem. This line is represented as *OD* in Fig. 7-7. Since we are to divide the voltage vector *OA* by the impedance vector *OC* (that is, *OA*/*OC*), we connect the terminal point of the vector in the *denominator* of the division with point *D* of the unit length. Then measure angle Θ between vector *OC* and line *CD*. Knowing the value of this angle we then draw an angle of the same value from the terminal point of vector *OA*; and from this angle measurement we draw a line extending from the terminal point of vector *OA* until it reaches the 40-degree line drawn from the origin. The point where it contacts this line is designated as point *B*, and this point determines the magnitude of the resultant vector of division. From the vector diagram this resultant vector is seen to be line *OB* and from measurement its length is found to be equal to 1 inch and thus its magnitude is equal to 2 amperes.

Since the phase angle of the resultant vector is already known to be 40 degrees, the complete resultant current vector is then equal to $2\underline{/40^\circ}$ amperes which is the same as in the preceding problem. Compare all the vectors in Figs. 7-6 and 7-7 as to magnitudes and phase angles, and it will be seen that they are all alike.

CHAPTER 8

RADIO CIRCUIT PROBLEMS

By now you should have a fairly good idea of what vectors are and how they are used to portray certain relationships existing in simple circuit problems.

The addition of vectors is the most important type of vector combination, because it is used in practice more often than any other type. Although subtraction, multiplication, and division of vectors play an important role in vector analysis, they are seldom used (so far as vector diagrams are concerned) in the analysis of radio circuit problems. In fact, throughout this chapter, in all the different circuit problems discussed the main type of vector combination which will be carried out is addition. With respect to the addition of vectors, the principal method that will be used is the parallelogram, the reason being that this is the most common method of vector addition found in texts and other technical literature.

Series Resonance

As a starter let us analyze a typical series circuit during the condition of resonance. In Fig. 8-1 (A) is shown a typical series resonant circuit consisting of *R, L,* and *C*. The resistance *R* may be just the inherent resistance of the coil, it may be a separate resistor, or it can be considered a combination of both. Such series tuned circuits may be found in many parts of radio communication systems, in r-f, i-f, and a-f circuits, serving as traps, filters, peaking circuits, and other special applications.

In a series tuned circuit, for that matter in all types of tuned circuits containing L and C, at resonance, the capacitive reactance equals the inductive reactance. Since the current flowing in a series circuit is the same throughout the whole circuit, the voltage drop across the inductance, designated as IX_L, equals the voltage drop across the capacitance designated as IX_C. The voltage across the inductance *leads* the current through it by 90° and the voltage across the capacitance *lags* the current through it by 90°. This is illustrated by the respective voltage and current vectors in Fig. 8-1 (B). On the other hand the voltage drop across the resistance designated as *IR* is in phase with the current flowing through it. This also is shown in Fig. 8-1 (B). Thus, this vector diagram shows three voltage and one current vector corresponding to the circuit conditions.

To find the resultant voltage across the complete series circuit, we have to add vectorially all three voltages. In the case of the series

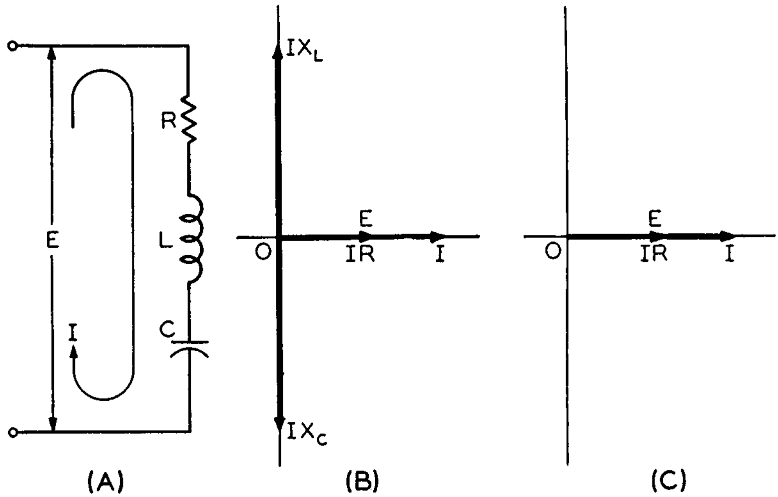


Fig. 8-1—The three voltages and the current of the series resonant circuit in (A) are laid out in vector form (B) to correspond to the circuit conditions, and by vector addition of the voltages the resultant voltage vector *OE*, which equals the voltage drop across the resistance, is obtained (C).

resonant circuit this is easy. First, since the two reactive voltages are exactly 180° out of phase with each other, the magnitude of the smaller can be subtracted from the larger. However, since at resonance the reactances are equal, the addition of these two voltage vectors results in zero voltage, and the only voltage we have left is the *IR* voltage drop across the resistance. This is readily seen in the resultant vector diagram of Fig. 8-1 (C), wherein the total voltage *E* across the series circuit at resonance is equal to the *IR* voltage drop across the resistance.

From the resultant vector diagram Fig. 8-1 (C) a number of things about series resonant circuits are determinable. First, as we have already mentioned, the voltages across the reactances are equal and 180° out of phase. Second, the current in a series resonant circuit is in phase with the voltage across the complete circuit. Since the current is in phase with the voltage across the complete circuit, the impedance offered by a series tuned circuit is *purely resistive* at resonance, and is equal to the *resistance* of the series circuit. Third, since the reactances cancel each other, the current at resonance is primarily determined *by the resistance of the circuit* (from Ohm's law $I = E/R$), and the smaller the resistance, the greater the series resonant current. Speaking about current conditions in such a circuit, the conditions which determine the current also determine the sharpness or broadness of the resonance curve; the less the resistance, the sharper the resonance curve and IX_L and IX_C present across the reactive elements will be greater than the total voltage across the whole circuit.

If a series circuit contains only pure inductance and capacitance, then the current at resonance would be infinite. However, since every inductance contains some resistance, the current can never be infinite at resonance, but the current at resonance in a series tuned circuit is said to be a maximum, and the impedance at resonance is said to be a *minimum* and *resistive*. If the reactances are not equal, then whichever reactance is greater will predominate, and the circuit is then said to be inductive or capacitive in nature. If the inductance is greater than the capacitance, the voltage across the whole series circuit will *lead* the current, but if the capacitance is greater the voltage will lag the current. Under such conditions IX_L will not be equal to IX_C.

Parallel Resonance

The conditions of parallel resonance are not so easy to explain as those during series resonance. The addition of the vectors for parallel tuned circuits was illustrated in Fig. 6-19 in chapter 6, but those parallel tuned circuits were of the simplest type, that is did not involve resonance. Suppose we analyze first the *ideal* type of parallel resonant circuit, one containing only so-called pure inductance and capacitance. The circuit is illustrated once more in Fig. 8-2 (A), and its accompanying vector diagram is illustrated in Fig. 8-2 (B).

Parallel resonance occurs when the capacitive reactance equals the inductive reactance. This means that the current I divides equally in

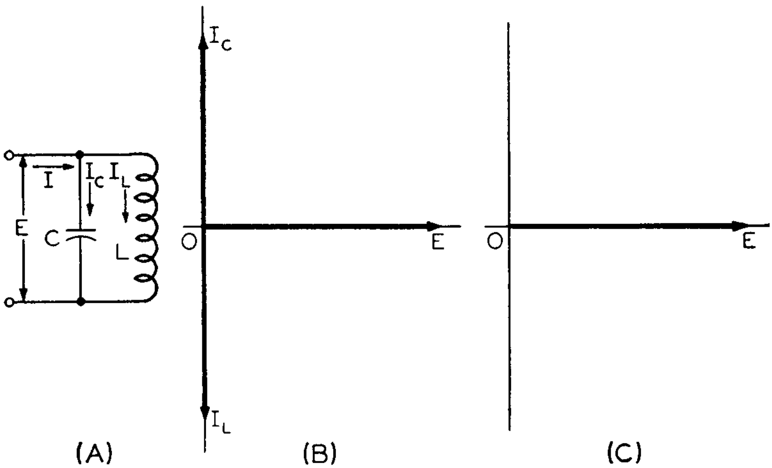


Fig. 8-2—(A) The ideal parallel L-C resonant circuit has the two branch currents I_C and I_L equal. They are shown by equal but 180° out-of-phase current vectors OI_C and OI_L in (B) with respect to the voltage E across the circuit. Addition of these two currents (C) ideally results in zero current.

both branches, so that inductive current I_L equals the capacitive current I_C. The current I_C, through the capacitance leads the voltage across it by 90°, and the current I_L through the inductance lags the voltage by 90°. This is indicated in the vector diagram of Fig. 8-2 (B). The unique thing about this ideal circuit is that if the two vector currents are added vectorially the effective resultant current becomes zero, since the two branch currents are equal but 180° out of phase. Under such circumstances the impedance at resonance would *appear to be* infinite since the resultant current is zero as in the diagram of Fig. 8-2 (C) which is the vector addition of Fig. 8-2 (B) — hence the statement that a parallel resonant circuit is a high impedance circuit.

The foregoing case is ideal, that is, the inductance and capacitance are pure and contain no resistance. However, every inductance contains some inherent resistance, therefore, the resultant current can never be zero, and the total impedance of the circuit can never be infinite. However, the current in a parallel tuned circuit is a minimum at resonance, and the impedance a maximum. Such circuits are very common in radio networks and are found in r-f and i-f tuned systems, filters, and in coupling networks where a certain degree of selectivity is desired, either for acceptance or for rejection of specific frequencies.

The Q of a coil is said to be high when its inductive reactance is much greater than its series resistance. Consequently, as the Q of the coil used in parallel tuned circuits is increased, the resultant current at resonance is decreased proportionally and the impedance increased proportionally. Another very interesting highlight about the parallel tuned circuits is that its impedance at resonance is resistive. The value of this resistive impedance is not, however, equal to just the resistance of the circuit as it is in a series tuned circuit. For those circuits where the resistance is small compared with the value of inductive reactance (high Q coils), the value of impedance at resonance, which is resistive, is equal to the Q of the coil (or circuit) multiplied by the inductive reactance. It follows that the resistive impedance of such a parallel resonant circuit is also equal to the square of the inductive reactance divided by the resistance of the coil. Written in mathematical form we have

$$\text{Impedance} = QX_L$$

or

$$\text{Impedance} = \frac{(X_L)^2}{R}$$

where X_L is the inductive reactance of the coil, R the resistance of the coil, and Q equal to

$$\frac{X_L}{R}.$$

The true representation of a parallel circuit is illustrated in Fig. 8-3 (A). Actually the circuit consists of two branches one of which is a capacitance and the other a series circuit of inductance and resistance, with the resistance being the inherent resistance of the coil. Since this is the more practical case for parallel resonance, let us study the vector diagram for this circuit as shown in Fig. 8-3 (B) and then compare it with Fig. 8-2 (C). Despite its apparent complexity, this vector diagram is not complicated at all. Everything discussed in the previous chapters can be applied here for a com-

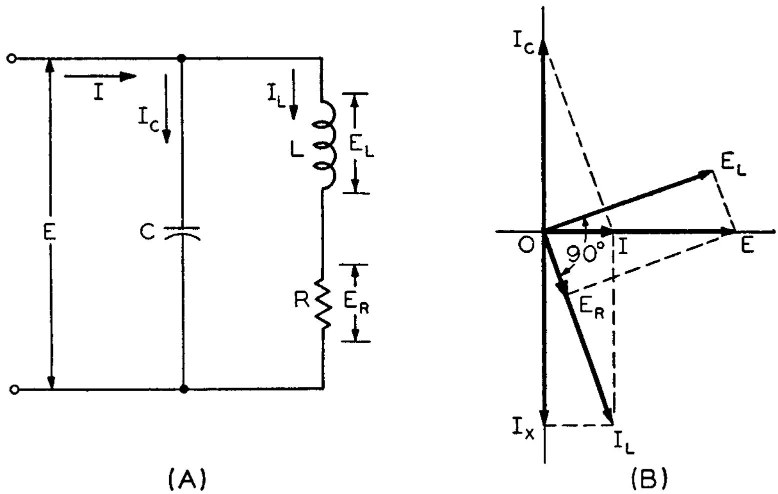


Fig. 8-3 (A)—A true parallel L-C circuit has inherent resistance in the inductance. (B) The vector addition of the different voltages and currents, where vector *OE* is the resultant voltage vector and vector *OI* the resultant current vector.

plete understanding of the current and voltage relations in the parallel circuit of Fig. 8-3 (A).

First of all the voltage E exists across both parallel branches and this is shown as voltage vector OE along the 0° reference line. Current I_C flowing through the capacitor leads the voltage E across it by 90°. This is indicated by the current vector OI_C being drawn 90° in the counterclockwise position from voltage vector OE. Since there is resistance in the inductive branch, the current I_L flowing through it cannot be lagging the voltage across the branch by 90°. The inductive current I_L must lag the voltage across it by some angle between 0 and 90° depending upon the magnitude of the resistance R. If a high Q coil is used, the resistance would be small, and the current I_L would be lagging voltage E by almost 90°. The larger R becomes with respect to the inductive reactance (that is, Q becomes smaller), the smaller the angle of lag becomes.

In the case of Fig. 8-3 (A) the coil is assumed to be a high Q one, and the angle of lag of the current I_L with respect to voltage E is seen to be near 90° in Fig. 8-3 (B). Since this current flows through the inductance L, the voltage E_L across this inductance leads the current through it by 90°. This is also shown in the vector diagram, where voltage vector OE_L is seen to be leading the current vector OI_L by 90°. The voltage drop E_R across the resistance is in phase with the current I_L flowing through it. Consequently, in the vector diagram the voltage vector OE_R, representing that across the resistance, coincides with the current vector OI_L. It is readily seen from Fig. 8-3 (A) that the voltage drop across the inductive branch is equal to E, and that the separate voltage drops E_L and E_R across the inductance and resistance respectively should add up to give the value of this voltage.

However, this addition has to be made in vector form. In other words, if we add vectors OE_L and OE_R vectorially by the parallelogram method, the resultant vector should be vector OE as shown in Fig. 8-3 (B). Since a resistive component does exist in this parallel circuit, the resultant current I cannot be zero. In order to find the resultant current we add the branch current vectors OI_C and OI_L by the parallelogram method and draw the necessary diagonal to obtain the resultant vector. This resultant vector is designated as OI in Fig. 8-3 (B). Since the resistance is small, the resultant current is also seen to be very small and will be in phase with the voltage across the circuit.

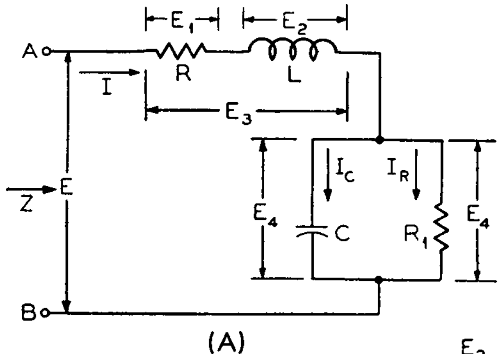
Up to this point nothing has been said about the effects of resonance in such a circuit. For example, how does parallel resonance manifest itself in this circuit of Fig. 8-3 (A) and in the accompanying vector diagram of Fig. 8-3 (B)? As mentioned, when the values of capacitive and inductive reactances are equal, the circuit will be in resonance. To display this on a vector diagram it is necessary to resolve the individual vectors representing the branch currents into their vertical components. These vertical components should be equal in magnitude and 180° out of phase with each other for true parallel resonance. In Fig. 8-3 (B) this is found to be true. Current vector OI_C is its own vertical component and OI_X is the vertical component of current vector OI_L, and it is seen that OI_C and OI_X are equal and opposite in phase. Therefore, if the vertical components are added, they will cancel each other out and supposedly only the horizontal components will remain. This also is proved true by the vector diagram wherein the horizontal component of vector OI_L and the resultant current vector *OI* are one and the same.

Another very interesting thing about parallel resonance is further proved by the vector diagram of Fig. 8-3 (B) — the fact that at resonance the circuit is purely resistive. This means that the total current *I* should be in phase with the voltage *E* across the complete circuit. This is shown by the resultant current vector *OI* coinciding with the voltage vector *OE*, thereby indicating the in-phase relationship. Since the resultant current at resonance is a minimum, the impedance of the circuit at resonance is, from Ohm's law of $Z = E/I$, a maximum.

Series-Parallel Circuit

In many types of communicator systems the networks encountered often combine series and parallel circuits. Many of these circuits are critical in their design, especially as to the capacitive or inductive behavior of the complete circuit. To show that series-parallel circuits can be portrayed by the means of vector diagrams, an analysis of a typical series-parallel arrangement will be included here. The circuit illustrated in Fig. 8-4 (A) can be considered as representative. The problem is to find the phase relationships between the resultant voltage *E* across the whole circuit and the total line current *I*.

Before studying the vector diagram let us discuss the circuit a little. It consists of an inductance *L* in series with a resistance *R*, and this arrangement is further in series with a parallel circuit of capacitance *C* and resistance R_1. We are concerned with five voltages and three currents present in this circuit. The voltages are as follows: the voltage E across the complete circuit, voltage E_1 across resistance R, voltage E_2 across inductance L, voltage E_3 across the series combination of inductance L and resistance R, and lastly voltage E_4 across either the capacitor C or resistor R_1, since they are in parallel and the voltage across each is the same. Voltages E_1 and E_2 when added vec-



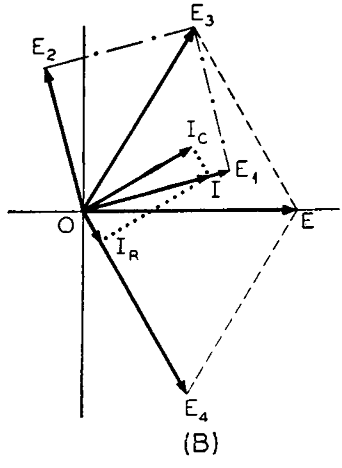


Fig. 8-4 (A)—A typical series-parallel circuit arrangement involving three currents and five voltages. The resultant current is I, and the resultant voltage E. (B)—Vector diagram for this circuit illustrating the magnitudes and phase relations of the vectors. Resultant current vector OI is leading resultant voltage vector OE, indicating that the reactance of the circuit is predominantly capacitive.

torially should give us voltage E_3. Voltages E_3 and E_4 when added vectorially should give us the final resultant voltage E.

The currents involved in the circuit are as follows: the final resultant current I, the branch current I_C flowing through the capacitive branch of the parallel part of the network, and the current I_R

flowing through the resistive branch of the parallel network. Since the line current splits up into branch current I_C and I_R, the vector addition of I_C and I_R should yield the resultant current I.

Without any knowledge of the values of the respective circuit elements and the frequency of the applied voltage, we cannot draw the vector diagram. We do not know whether impedance Z looking into terminals A and B is primarily inductive, capacitive, or purely resistive in nature. Once the impedance is established, the phase relationship between the resultant voltage E and the resultant current I will be known. For example, if the impedance is primarily inductive, the resultant voltage will lead the current; if it is capacitive, the resultant voltage will lag the current. In either case the lead or lag will be between zero and 90 degrees, the actual amount depending upon the relation between the magnitude of the resistive and reactive components, making up the over-all impedance.

If we assume that the capacitive reactance of C is greater than the inductive reactance of L making the over-all reactance of the circuit capacitive, the resultant current should lead the resultant voltage. This is shown in the vector diagram of Fig. 8-4 (B).

Let us now analyze the vector diagram and corresponding circuit. In drawing such a vector diagram it is always best to choose the 0° reference line as the starting point for one of the principal vectors of the circuit it represents. In this vector diagram the resultant voltage E across the whole circuit is designated by the voltage vector OE and drawn on the 0° reference line. As it has been assumed that this circuit has an impedance in which the reactive part is mostly capacitive, the line current I leads the voltage E. This means that voltage drop E_3 across the series R and L circuit, in which the reactance is inductive, leads this same current I and hence leads the voltage E. This is shown vectorially in Fig. 8-4 (B) from which vector OE_3 is seen to be leading voltage vector OE. It was stated previously that voltage E is the algebraic addition of voltages E_3 and E_4. Since we already have the representative vectors of voltages E and E_3 drawn on the vector diagram of Fig. 8-4 (B), we need only form a parallelogram out of these two vectors with vector OE as the diagonal. Since voltage E_4 equals voltage E minus voltage E_3 we are fundamentally carrying out some vector subtraction by the parallelogram method. Completing the vector subtraction the resultant vector for voltage E_4 is designated as vector OE_4 in Fig. 8-4 (B). This resultant vector represents that voltage across the parallel branch of the circuit. Since the parallel branch is more capacitive than the impedance of the complete circuit, voltage vector OE_4 lags voltage vector OE.

The analysis of the circuit up to this point enables us to construct the rest of the vector diagram easily. Current I_R flowing through resistance R_1 of the parallel branch is in phase with the voltage across it, and in the vector diagram, current vector OI_R is in phase with voltage vector OE_4. The current I_C flowing through the purely capacitive circuit should lead the current I_R by 90 degrees. This is shown in the vector diagram where current vector OI_C leads the current vector OI_R by 90°. That is to say, the current I_C flowing through the capacitance C is leading the voltage E_4 across the capacitance, by 90°.

We have thus established both current vectors within the parallel branch. If these two current vectors, namely OI_C and OI_R, are added, their resultant vector should be that of the total current in the circuit. This is done in the vector diagram by the parallelogram method, and the diagonal OI is found to be the resultant current vector of addition. It is seen to be leading the voltage vector OE, by some small phase angle, indicating that the reactive part of the impedance of the circuit is predominately capacitive. Since the current I flows through resistor R then the voltage drop E_1 across this resistance is in phase with the current. This is shown in the vector diagram wherein voltage vector OE_1 and current vector OI are seen to be in phase. However, this same current I flows also through inductance L, in which case the voltage drop E_2 across this inductance leads the current I flowing through it by 90°. This is illustrated vectorially by voltage vector OE_2 leading the current vector OI by 90°, which is to say the voltage drop across the inductance is in quadrature (90°), leading the voltage drop across the resistance of the same series circuit. If these two individual voltage drops E_1 and E_2 are added vectorially, they should result in voltage E_3. In the vector diagram this is found to be true where voltage vectors OE_1 and OE_2, representing voltages E_1 and E_2, respectively, are combined by the parallelogram method, and the resultant voltage vector is vector OE_3, which is as it should be since vector OE_3 represents voltage E_3.

From this vector diagram we can learn a lot of things. We can readily see the magnitude and phase relationship between any two or more vectors. We can also quickly determine what can be done to the circuit in order to accomplish certain desired results. For instance, if we desire to resonate the circuit at the frequency of operation now in effect, what would have to be done to the circuit to accomplish this resonance?

For resonance, the resultant current I and resultant voltage E must be in phase with each other making the total impedance resistive in nature. One method of accomplishing this is to move current vector OI in a clockwise direction to coincide with voltage vector OE. This can be done by a number of means. First the current vector OI_R can be increased in magnitude by decreasing the value of R_1 to the point where the vector addition of OI_R and OI_C makes the resultant vector OI coincide with vector OE. Another method would be just to decrease the magnitude of the current vector OI_C, by decreasing the capacity of C, until the current OI_C reaches a point where, when it is vectorially added with current vector OI_R, the resultant of the combination will coincide with voltage vector OE. The series components of L and R also can be changed to yield this resonance.

Double Tuned Transformers

One of the most common yet most important types of coupling in radio circuits is the i-f and r-f tuned transformers. These circuits are highly selective to certain frequencies due to their tuned circuit properties. Although many are familiar with the functions of such tuned transformer networks, not many are quite familiar with the voltage and current relations existing in such circuits. In Fig. 8-5 (A) is illustrated a typical schematic diagram of a double tuned transformer circuit. Accompanying this circuit are the respective voltages and currents involved. Voltage E_P is that voltage existing across the primary circuit, and voltage E_S that voltage existing across the secondary circuit. The current flowing in the inductive branch of the primary is designated as I_P and that flowing in the inductive half of the secondary is designated as I_S.

It is often misunderstood that the voltage E_S across the secondary circuit is the voltage induced into the secondary coil from the primary coil. This is not true. What actually happens is that the current flowing through the primary inductance causes a magnetic field to link the turns about this inductance. Part of the magnetic lines of flux from this magnetic field also encircle the inductance L_S in the secondary, thereby inducing a voltage into that secondary circuit. This

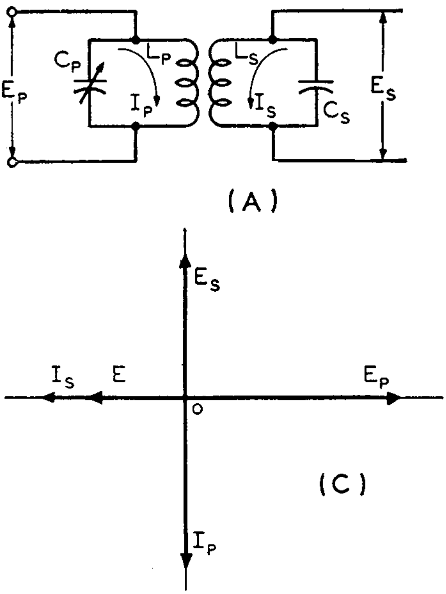


Fig. 8-5—A typical schematic diagram of a double tuned transformer circuit (A) and the equivalent diagram for the secondary tuned circuit (B), where *E* represents the induced voltage. The vector diagram (C) illustrates the phase relationships between the various voltages and currents of the circuit.

induced voltage, however, is the same as if a generator were connected in series with the coil L_S and the capacitor C_S of the secondary circuit. This is better illustrated in the equivalent diagram for the secondary tuned circuit as shown in Fig. 8-5 (B). The generator voltage *E* represents the induced voltage in the secondary caused by the current flowing in the primary inductance.

In i-f and r-f transformers, the circuits are specially tuned, so that the frequency of the applied voltage or signal to the primary is exactly equal to the resonant frequency of the tuned circuits. In the analysis of this circuit high Q coils are assumed to be used, such that the resistive part of the inductances are considered to be negligible as compared with the inductive reactances. Consequently, the resistive parts are said to have little or no effect on phase change within the circuit.

The vector diagram depicting this case of double tuned transformers is shown in Fig. 8-5 (C). The most important relation that exists between just two coupled coils (no loading in the secondary) is that the voltage across the primary is 180° out of phase with the voltage across the secondary, the latter being the induced voltage in this instance. If the secondary circuit has some load or other elements attached to it, as a capacitor, besides the inductance, the induced voltage is still 180° out of phase with that voltage existing across the primary, but the induced voltage into the secondary circuit is now considered to be in series with the elements, as shown in Fig. 8-5 (B).

Now let us refer back to the circuit of Fig. 8-5 (A) and discuss the voltage and current relations existing there. The primary voltage E_P, shown as vector OE_P in the vector diagram of Fig. 8-5 (C), is drawn along the 0° reference line because it is the initial voltage. The induced voltage E, smaller in magnitude than E_P and designated as vector OE, is drawn 180° out of phase with vector E_P. The current I_P flowing in the inductive arm of the primary is lagging the voltage E_P across it by 90°, as indicated by vector OI_P being drawn 90° in clockwise position from vector OE_P. From the vector diagram, we see that the current vector OI_P, which causes the magnetic field about L_P and in turn causes the induced voltage vector OE, is leading this induced voltage by 90°.

It is known that for a parallel L-C circuit at resonance the reactances cancel each other, and the circuit is purely resistive in nature, making the current I_S in the equivalent circuit of Fig. 8-5 (B) in phase with the induced voltage E. Examining the vector diagram of Fig. 8-5 (C), we find that the current vector OI_S and the induced voltage vector OE coincide with each other, indicating this in-phase relationship. Current I_S in Fig. 8-5 (B), caused by the induced voltage E, when flowing through capacitor C produces a voltage drop E_S, across this capacitor which is lagging the current through it by 90°. This is shown in the vector diagram of Fig. 8-5 (C) by voltage vector OE_S lagging the current vector OI_S by 90°.

Looking at the vector diagram once more we can conclude that for a double tuned transformer tuned to resonance, the current in the primary tuned circuit leads that in the secondary tuned circuit by 90°, that the voltage across the secondary leads the voltage across the primary by 90°, that the current in the primary is 180° out of phase with the voltage across the secondary and that the current flowing in the secondary is 180° out of phase with the voltage across the primary.

Push-Pull Amplifiers

An analysis of push-pull amplifiers can be quite confusing and often is misunderstood. The most important thing for the correct operation of push-pull amplifiers is the need for a good, balanced system, that is, one in which both tubes used for the push-pull arrangement are identical in their characteristics and the voltages applied to the elements of each of the tubes are identical for the proper symmetry and balance of the system. The main criterion for the proper operation of push-pull circuits *is that the a-c voltages on the grids of the tubes should be, at any one instant of time, 180° out of phase with each other but equal in magnitude.* To discuss fully and to analyze all the different types of push-pull circuits commonly used in the communication field is beyond the scope of this book,* but two typical push-pull circuits, such as are commonly used in radio receivers of today, are shown in Fig. 8-6 and 8-7.

One method of obtaining the necessary equal, but 180° out-of-phase voltages applied to the grids of the push-pull tubes is to use an input push-pull transformer; that is, one that has its secondary center tapped, with both ends going to the grids of the push-pull tubes and the primary receiving the signal input. Another method, most commonly used in the radio receivers of today, is to use the inherent phase inversion quality of amplifying vacuum tubes to obtain the necessary 180° out-of-phase voltages on the grids of the push-pull tubes. The latter type of circuit will be discussed here. This so-called

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*For a more detailed analysis of push-pull circuits see "How It Works" book of Vol. XV of the "Perpetual Trouble Shooters Manual" published by John F. Rider, Publisher, Inc., New York, N. Y.

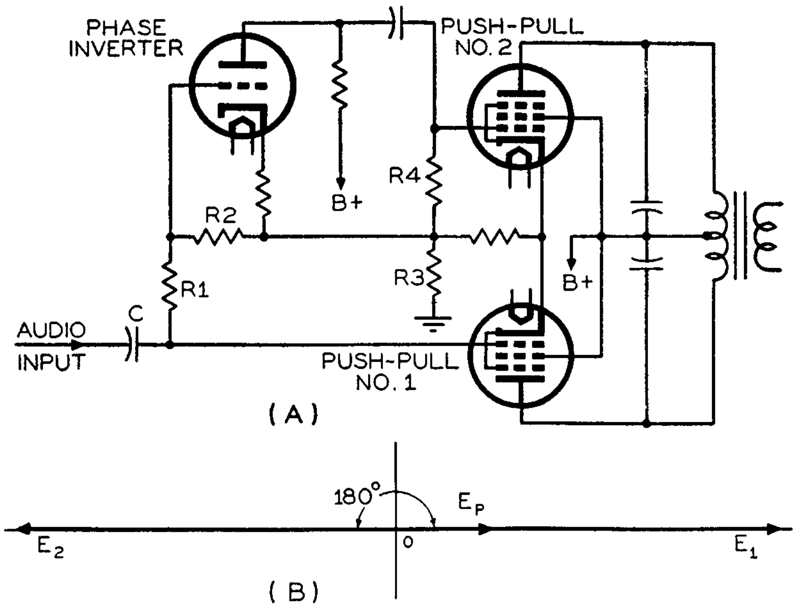


Fig. 8-6—(A) is a typical push-pull circuit using a separate tube as a phase inverter. The vector diagram (B) illustrates that voltage vector OE_1 of push-pull tube No. 1 is equal in magnitude but 180° out of phase with voltage vector OE_2.

phase inversion quality of a vacuum tube is briefly described as follows:

When there is an *increase* in the signal applied to the grid of a tube, the plate voltage *decreases* by a proportionate amount, and, when the grid signal *decreases*, the plate voltage *increases* by a proportionate amount. This means that the voltages existing on the plate and grid of a tube are opposite in phase, or 180° out of phase. With this idea foremost, let us now study the push-pull circuit of Fig. 8-6 (A).

An audio input signal is impressed across the series resistor network R_1, R_2, and R_3 to ground. Since the control grid of the No. 1 push-pull tube is connected directly to the audio input signal coupling

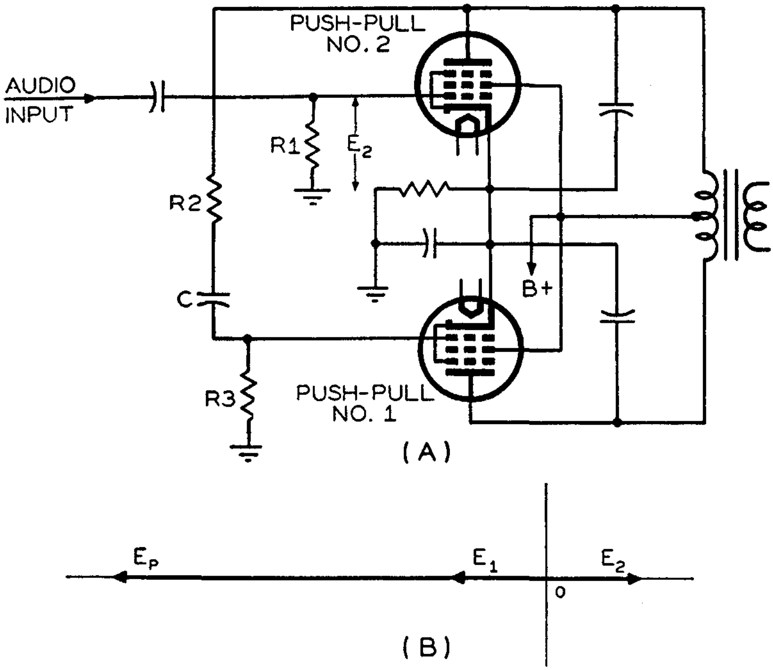


Fig. 8-7—The push-pull circuit (A) uses push-pull tube No. 2 as the phase inverter. Here the audio is impressed directly across the grid of tube No. 2 (B) illustrates the equal but 180° out-of-phase voltages E_1 and E_2 which are applied to tubes No. 1 and No. 2, respectively.

capacitor *C*, the audio voltage impressed across the grid-to-ground circuit of this tube is the same as the total available audio voltage. Calling this voltage drop across the resistor network or the grid-to-ground network of push-pull tube No. 1, E_1, we can represent this voltage by a vector, as indicated by vector OE_1 along the 0° reference line in Fig. 8-6 (B).

It will be noticed that in Fig. 8-6 (A) the grid of the phase inverter tube is connected to the high side of R_2, so that the voltage drop across series resistors R_2, R_3, and ground is equal to only part of that audio voltage drop across the complete three resistor network and ground.

In other words this voltage applied between the grid and ground of the phase inverter tube is equal to

$$\frac{R_2 + R_3}{R_1 + R_2 + R_3} \times E_1$$

and is also in phase with voltage E_1. This voltage drop across the grid to ground circuit of the phase inverter tube is designated as E_P, and the voltage vector OE_P in Fig. 8-6 (B) represents this voltage drop. Vector OE_P coincides with vector OE_1 since they are of the same phase, but the former vector is smaller in magnitude since its voltage representation is a fraction of voltage E_1.

Voltage E_P being applied to the grid of the phase inverter tube is amplified, and due to the phase inversion qualities of the tube the voltage appearing in its output circuit, call it E_2, is 180 degrees out of phase with that at the grid of the same tube. This voltage is effectively impressed across the control grid-to-ground circuit (across resistors R_3 and R_4) of push-pull tube No. 2. Since this voltage E_2 is 180° out of phase with that appearing on the grid of the phase inverter, its representative vector designated as OE_2 in Fig. 8-6 (B) is drawn 180° out of phase with vector OE_P. From this vector diagram it is readily seen that vector OE_1 applied to the grid-to-ground circuit of push-pull tube No. 1 and vector OE_2 applied to the grid-to-ground circuit of push-pull tube No. 2 are 180° out of phase with each other. The amplification of the phase inverter tube is such that the input voltage is increased in magnitude to the amount needed to equal the magnitude of the voltage across the grid-to-ground circuit of push-pull tube No. 1. This means the phase inverter tube, besides accomplishing the necessary phase reversal also helps make the two voltages on the push-pull grids equal in magnitude. This is indicated by vectors OE_1 and OE_2 of Fig. 8-6 (B) being equal in length.

The amplification of the phase inverter tube can be easily computed by dividing its input voltage by its output voltage. Since the vectors are representative of these voltages, we can divide the length of vector OE_2 by vector OE_P and the amplification will be known. Thus for the vector diagram of Fig. 8-6 (B) vector OE_2 equals 2 inches and vector OE_P equals 0.5 inch, indicating that the amplification of the phase inverter tube is equal to 2/0.5 or 4. It should be remembered that for this to be true the vectors must all be drawn to the same scale.

The grid to cathode voltages, besides the grid to ground voltages, of the push-pull tubes are also equal in magnitude and 180° out of phase with each other. The reason for this is that the cathode resistor R_3 is common to both the grid to ground circuits of both push-pull tubes and therefore can be neglected. In fact, in practice its resistance as compared with resistors R_1 and R_2 is so small as to have negligible effect on the voltage dividing qualities of the input audio voltage E_1.

A different phase inversion push-pull circuit from that just discussed is illustrated in Fig. 8-7 (A). The result is similar to the circuit just discussed; that is, equal but out-of-phase voltages are applied to the grid of both push-pull tubes, but the manner in which it is obtained is somewhat different. The vector diagram for this circuit appears in Fig. 8-7 (B).

The total audio input voltage to the system is impressed directly across resistor R_1 and hence upon the grid of the No. 2 push-pull tube. This voltage, designated as E_2, is represented by vector OE_2 on the 0° reference axis of Fig. 8-7 (B). By virtue of the amplification and phase inversion quality of this No. 2 push-pull tube, there appears in its plate circuit an audio signal greater in amplitude and 180° out of phase with that existing on the grid of the same tube. In other words tube No. 2 acts as both a push-pull amplifier and phase inverter. This amplified, but out-of-phase voltage, appears between the plate of push-pull tube No. 2 and ground and thus appears across the series network of R_2, C, and R_3 to ground. This voltage designated by the letter E_P is represented by vector OE_P in Fig. 8-7 (B), and it is seen to be greater in magnitude than vector OE_2, but also 180 degrees out of phase with it.

The reactance of capacitor C at the audio frequencies is very small in comparison with the resistances of R_2 and R_3, so that very little signal voltage from the plate of the No. 2 push-pull tube appears across capacitor C. Practically all of this voltage is dropped across resistors R_2 and R_3. The portion of the voltage appearing across resistor R_3 is, therefore, equal to the output voltage from the No. 2 push-pull tube multiplied by the quotient of $R_3/(R_2 + R_3)$. The values of R_2 and R_3 are so chosen that the voltage drop across R_3 is *equal* in magnitude to that voltage applied to the grid of the No. 2 push-pull tube. The voltage across resistor R_3 is effectively applied across the grid of the No. 1 push-pull tube. This voltage, designated as E_1, is represented by vector OE_1 in Fig. 8-7 (B). From the vector diagram it is seen that this voltage vector OE_1 is in phase with vector OE_P and also that it is 180° out of phase but equal in magnitude to vector OE_2. Consequently, the primary requisites of equal but 180° out-of-phase voltages for push-pull operation are seen to be applied to the circuit of Fig. 8-7 (A).

The amplification of the No. 2 push-pull tube, which also acts as the phase inverter, is found by dividing the magnitude of vector OE_2 (or OE_1), since they are both equal in magnitude, into the magnitude of vector OE_P. Thus by measurement we find OE_2 equals 0.5 inch and OE_P equals 2.25 inches, hence the amplification equals 2.25/0.5 or 4.5.

The Discriminator Circuit

The discriminator circuit as used in automatic frequency control circuits and as f-m signal demodulators is very important to the oper-

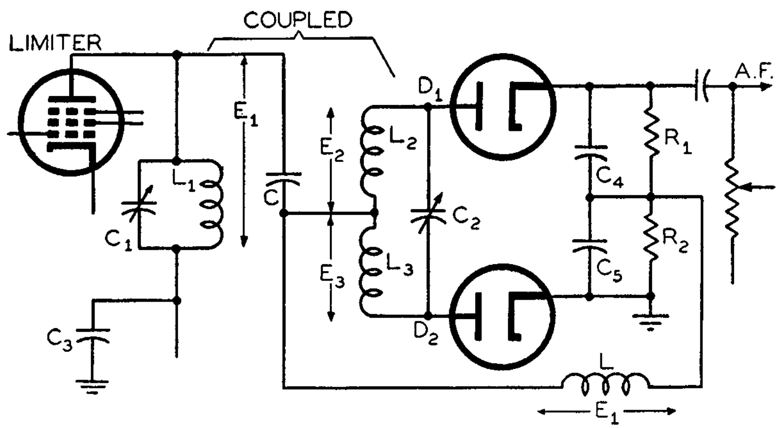


Fig. 8-8—A typical discriminator circuit as used in f-m receivers where the two diodes can be separate tubes or a duo-diode tube.

ation of these systems. The function of the discriminator in either case is for detection of a modulated signal, whether it is amplitude, frequency, or phase modulated. The type discriminator to be discussed is the Foster Seeley circuit often referred to as the center-tuned or phase discriminator. The discussion will take into account the use of this discriminator circuit in f-m receivers. The analysis of this f-m discriminator network as well as other f-m demodulators can be quite simply explained through the expedient use of vector diagrams.

In brief, the function of this phase discriminator is to convert the audio-frequency variations of the f-m signal into amplitude variations, so that when this converted signal is applied to the proper audio system and loudspeaker, the intelligence transmitted is made audible.

A typical discriminator circuit as used in f-m receivers is shown in Fig. 8-8 along with the respective voltage drops across certain parts of the circuit. The circuit consists of double tuned transformer network in which the secondary has a special center tap connection. The primary tuned circuit is the load circuit for a limiter tube, and the secondary tuned circuit is connected to a differentially connected rectifier system consisting of two diodes or a duo-diode tube. There is nothing different about this so-called discriminator transformer, inasmuch as the primary circuit of L_1C_1 and the secondary circuit, consisting of the secondary inductance (L_2 and L_3) and capacitor C_2, both are tuned to the i-f peak of the set for proper operation. Voltage E_1 is that i-f signal voltage developed across the primary tuned circuit at resonance.

Two methods of coupling the signal from the primary to the secondary circuit are used in this system. The resonant primary is inductively coupled to the resonant secondary winding; at the same time the signal voltage E_1 across the primary is fed to the r-f winding L via the coupling capacitor C. If the circuit of C, L, and C_5 is traced, it will be seen that L is in shunt with the tuned primary, the latter being grounded through C_3. Neither C, L, C_4 or C_5 are of such magnitude as to alter the resonant conditions of C_1 and L_1, the resonant primary. Thus we can set up immediately the condition that, whatever signal voltage exists across C_1-L_1, the same signal voltage with respect to magnitude and phase exists across L. The direct connection between the coupling capacitor C and the mid-point of the secondary winding is of no consequence with respect to the signal transfer between the primary and the secondary circuits: it happens to be the common junction between the means of feeding the signal to the choke L, and the point to which the choke L must be connected to complete the differential rectifier circuit.

Thus the secondary system receives signal voltages in two ways: the resonant secondary receives its signal voltage by inductive coupling, and the r-f choke derives its signal voltage by means of direct coupling through the fixed capacitor C. Returning to the two coils which comprise the secondary winding and the associated signal voltages, the latter come about in the following manner. When a winding is tapped at the mid-point and a voltage is induced in that winding by means of a varying magnetic field, the total voltage developed across the entire winding divides between the two halves. This is readily evident when it is realized that half the total number of turns exists between the center tap and one end, and between the center tap and the other end. So, whatever the nature of the signal voltage developed across the tuned secondary circuit C_2-L_2-L_3, it is possible to show this voltage divided into two parts: that is, across each half of the winding. These are designated as E_2 and E_3.

Resonance Conditions

Let us examine this circuit at resonance. In this examination the understanding of double tuned transformer action with respect to induced voltages and currents as previously analyzed in conjunction with Fig. 8-5 should be well established in our minds. At resonance the frequency of the applied signal and the resonant frequency of the tuned circuit are both the same. Since the inductances and capacitances of a tuned circuit effectively cancel each other at resonance, the circuit behaves like a resistance. In a resistive circuit the current is in phase with the voltage, so in the secondary tuned circuit the induced current, call it I, caused to flow by the induced voltage, call it E, is in phase with this induced voltage. It should be remembered that this induced voltage is effectively in series with the inductance and capacitance of the secondary tuned circuit.

The in-phase relationships between E and I are indicated in the vector diagram of Fig. 8-9 where vectors OE and OI are the respective induced voltage and current vectors, and they are seen to be in phase with each other. The voltage across the primary circuit, designated as

E_1, is, if you recall, 180° out of phase with the induced voltage. This voltage E_1 is the main voltage upon which all the other voltages are based. Consequently, this voltage is drawn as vector OE_1 along the 0° reference line.

Examining the vector diagram of Fig. 8-9, vector OE_1 is seen to be 180° out of phase with induced voltage vector OE. Since the voltage

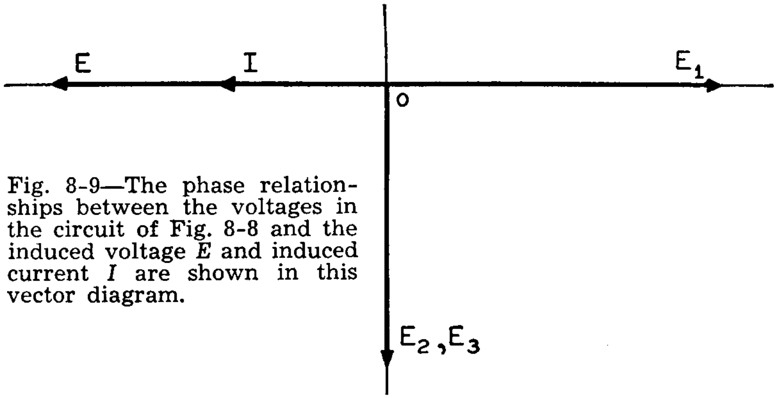


Fig. 8-9—The phase relationships between the voltages in the circuit of Fig. 8-8 and the induced voltage E and induced current I are shown in this vector diagram.

across a pure inductance leads the current through it by 90 degrees, then the voltage drops E_2 and E_3 across the secondary coil (called reactive voltage drops because the inductance is considered to be a pure inductance containing negligible resistance) lead the current I flowing through it. This is indicated in the vector diagram where vectors OE_2 or OE_3 are leading the induced current by 90°.

This also means that the reactive voltage drop across the secondary coil is lagging the primary voltage E_1 by 90°, which is evident in the vector diagram where vector OE_1 is seen to be leading vectors OE_2 and OE_3 by 90°. (This was also shown in the vector diagram of Fig. 8-5 (C) of the double tuned transformer previously discussed.) This detail of a 90° phase difference between these two voltages is very important to the operation of the discriminator. It should still be remembered that this voltage E_1 also exists across coil L of Fig. 8-8 in the same phase and magnitude as that existing across the primary tuned circuit.

Considering the secondary as being center tapped means that it is in a push-pull arrangement, and hence voltages E_2 and E_3 are equal in

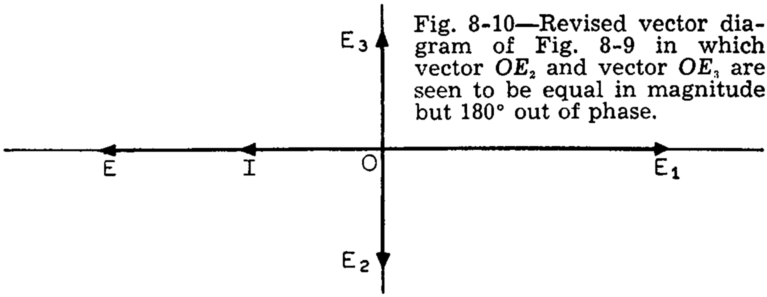


Fig. 8-10—Revised vector diagram of Fig. 8-9 in which vector OE_2 and vector OE_3 are seen to be equal in magnitude but 180° out of phase.

magnitude but 180° out of phase with each other. However, the same current flows through both parts of the secondary coil, so that a 90° phase relation must still exist between each voltage and current; but

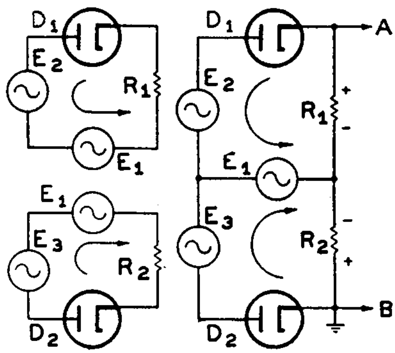


Fig. 8-11—The duo-diode circuit of Fig. 8-8 is redrawn to show the individual voltages that act on each diode. Two separate diode circuits are drawn and recombined to show how they function together. Note the directions of current flow and the polarity across the load resistors.

in one case one of the voltages is leading the current and in the other case the voltage is lagging the current by 90°. This also means that one half of the secondary voltage drop is leading voltage E_1 by 90° and the other half lagging voltage E_1 by 90°. All of which is simply indicated in the revised vector diagram of Fig. 8-9 as appearing in Fig. 8-10, in which vector OE_3 of the previous vector diagram has been shifted 180°.

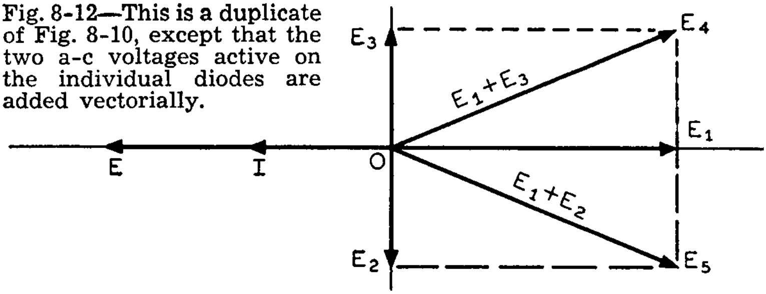
Now in order to understand how all these voltages affect the duo-diode circuit, we redraw in simple form that part of Fig. 8-8 appearing to the right of the secondary of the transformer as illustrated in Fig. 8-11. In this figure we have made two separate circuits of the diodes, showing the respective voltages that act upon each diode. These two simple circuits then are combined to show how they actually work together. From this figure and Fig. 8-8 it is readily seen that voltage E_1 is common to both diodes since it exists across the inductance L. Also, since voltage E_2 is, at the instant of time illustrated, active on diode D_1 and voltage E_3 active on diode D_2, it is readily seen from Fig. 8-11 that voltage E_2 and E_1 are active on diode D_1 and voltages E_3 and E_1 are active on diode D_2.

If this simplified circuit is examined, it will be seen that the rectified current flowing through the individual diode circuits puts a certain polarity on their individual load resistors. Since the current in a rectifier flows from cathode to plate, the current in the diode circuits will be flowing in opposite directions, and the polarities across the individual load resistors will be bucking each other. This means that between points A and B a voltage will exist which will be the difference between the voltage drops across resistors R_1 and R_2. If the voltages E_2 and E_3 have the *same phase angle* with respect to the voltage E_1, both diode currents will be equal in value, and the same voltage drop will appear across each load resistor R_1 and R_2.

Since each resistance voltage drop is opposite in polarity but equal in value, the total voltage measured between points A and B will be zero. Thus, under these circumstances the output of the differential rectifier circuit is zero. If, however, the phase relationships of E_2 to E_1 and E_3 to E_1 differ, a differential voltage will exist between point A and B, because there will be different voltage drops across resistors R_1 and R_2 due to different diode currents.

A zero voltage exists across points A and B when the resonant frequency of the tuned discriminator i-f transformer is exactly equal to the applied frequency. This is simply indicated by the vector diagram of Fig. 8-12. This diagram is a duplicate of Fig. 8-10 with the exception that the two voltages active on the individual diodes are added vectorially. In other words, since all the voltages involved are a-c voltages and have phase angles as well as magnitude, vector addition is needed and not strict arithmetical addition. Thus, in Fig. 8-12 the parallelogram method of vector addition is used, and vector OE_4 represents the resultant vector of the vector addition of voltages E_3

Fig. 8-12—This is a duplicate of Fig. 8-10, except that the two a-c voltages active on the individual diodes are added vectorially.



and E_1 across diode D_2 and vector OE_5 represents the resultant vector of the vector addition of E_2 and E_1. Since side E_1 is common to both rectangles formed by the vector additions and since E_3 and E_2 are equal in magnitude and E_3 is 90° leading and E_2 90° lagging E_1, from resultant vectors OE_4 and OE_5 are equal in magnitude, causing the same current to flow in each diode circuit.

Applied Frequency Greater Than Resonant Frequency

Since an f-m signal is varying in frequency above and below its center frequency component, then at any one instant during one cycle of frequency deviation the instantaneous frequency will be different. If the i-f transformer remains fixed (that is, usually tuned to the center frequency component of the i-f f-m signal), then, when the instantaneous value of the f-m signal is equal to its center frequency component, we have the case of applied frequency equaling resonant frequency. The situation for this was discussed in the preceding section. At either side of the center frequency component of the f-m signal, the instantaneous frequency is different from the i-f transformers resonant frequency. Under these conditions it is the same as saying the discriminator transformer is tuned below or above the intermediate frequency.

Let us now consider the case where the instantaneous value of the f-m signal is greater than the intermediate frequency. This is the same as saying that the discriminator transformer is tuned below the incoming intermediate frequency. The circuit is still the same as in Fig. 8-8 and the nonresonant conditions do not alter the fundamental rules of the action of the primary circuit, so that the voltage E_1 that exists across this circuit also exists across L in both the same phase and magnitude.

The induced voltage E in the secondary remains 180° out of phase with the primary signal, for this too is a fundamental condition which is not altered by resonance conditions. However, the relationship

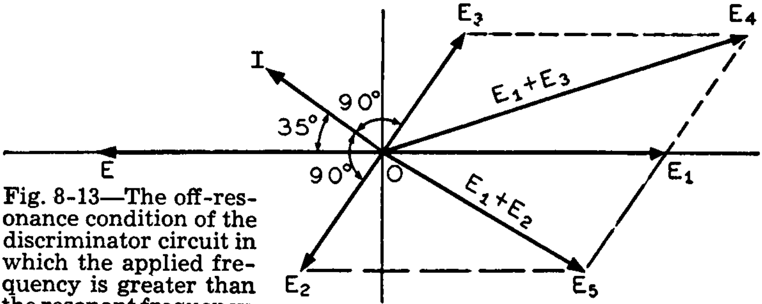


Fig. 8-13—The off-resonance condition of the discriminator circuit in which the applied frequency is greater than the resonant frequency.

between the induced voltage E and the current I, which it causes to appear in the secondary circuit, is affected by the state of resonance, and in turn alters related conditions.

When the applied frequency is higher than the resonant frequency, the reactance of the coil, even though it is fixed in inductance, becomes greater than the reactance of the capacitor. This is in accordance with the fundamental law that inductive reactance varies in proportion to frequency and capacitive reactance varies inversely with frequency. Accordingly, a portion of the inductive reactance will offset the capacitance reactance, but a certain amount of inductive reactance will remain to display a control on the induced current. In other words, the circuit as a whole now appears as an inductance and resistance in series, rather than as a resistance alone, which is the case at resonance.

Under this circumstance the induced current I will no longer be in phase with the induced voltage E but rather will lag this voltage by a certain amount, depending upon the extent to which the instantaneous f-m signal is greater than the tuned frequency of the transformer. This is all indicated in the vector diagram of Fig. 8-13 for the off-resonance condition now being considered. Voltages E and E_1 are still seen to be 180° out of phase but the phase relationships of the other component voltages differ from the vector diagram of Fig. 8-12.

Let us say that the difference in frequency between the instantaneous frequency of the f-m signal and the tuned i-f transformer is such that the amount of inductive reactance remaining is sufficient to cause the induced current I to lag the induced voltage E by 35°. This is indicated in Fig. 8-13 where the current vector OI is seen to lag the voltage vector OE by 35°.

No matter what the phase relationship between the induced voltage and induced current, the two voltages, E_2 and E_3 across the secondary are still 180° out of phase with each other and equal in magnitude. The induced current flowing through this secondary still bears the same phase relationship to these secondary voltages. In other words, regardless of the phase difference between E and I, secondary voltage E_3 will still lag current I by 90° and secondary voltage E_2 will still lead current I by 90°. This is indicated in the vector diagram of Fig. 8-13 and, if this vector diagram and that of Fig. 8-12 are compared, these phase relations will be seen to hold true.

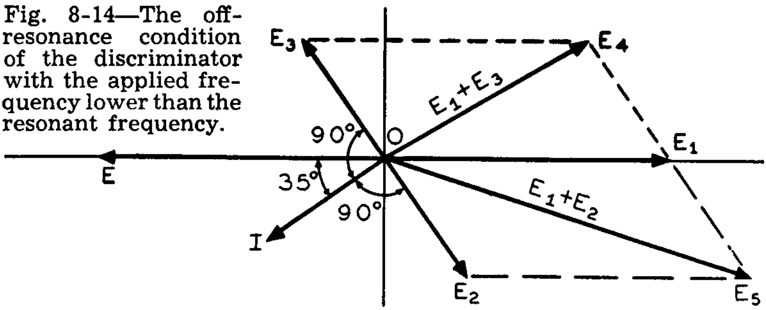
Let us further compare these two vector diagrams. In order to keep the 90° phase relations between voltages E_2, E_3, and current I intact when current I lags induced voltage E by 35 degrees, voltage vectors OE_2 and OE_3 are both shifted 35° clockwise. This is the same as saying the complete vector line E_2 to E_3 is shifted 35° in a negative direction. When the respective voltages existing across the individual diodes are added under these circumstances, it will be seen from the vector diagram of Fig. 8-13 that the resultant vector OE_4 representing that voltage across diode D_2 and resultant vector OE_5 representing that across diode D_1 are no longer equal since vector OE_4 is greater than vector OE_5. In this instance diode D_2 will draw the greater current, and in Fig. 8-11 load resistor R_2 will have a greater voltage drop than resistor R_1, and hence there will exist a differential voltage across point

A to *B*, with point *B* being more positive than point *A*. This is the same as saying point *A* is negative with respect to point *B*.

Applied Frequency Lower Than Resonant Frequency

When the instantaneous value of the f-m signal input to the discriminator circuit is such that it is less than the resonant frequency of the discriminator transformer, a differential voltage will still

Fig. 8-14—The off-resonance condition of the discriminator with the applied frequency lower than the resonant frequency.



exist across the diode loads, but the polarities will be reversed. Let us see how this happens.

We still are at off-resonance conditions, even though we are on the lower side of the resonant frequency and the same 180° phase relationship between *E* and E_1 exists. Under these conditions the applied frequency is lower than that of the resonant frequency of the i-f transformer; the impedance of the secondary of the i-f transformer is such that the capacitance reactance more than balances out the inductive reactance and the secondary is primarily capacitive. Since this circuit is capacitive, the induced current *I* leads the induced voltage *E*. If the off-resonance conditions are such that a phase angle of 35° still exists between *I* and *E*, *I* will be *leading* *E* by 35°, as seen in the vector diagram of Fig. 8-14. Since the 90° phase relations between voltages E_2, E_3, and current *I* must still exist, these two voltages are effectively shifted in phase 35° in a counterclockwise or positive direction. This is indicated in Fig. 8-14 where vectors OE_2 and OE_3 are still 180° out of phase with each other but no longer 90° out of phase with vector OE_1. Now when the individual voltages across the diode circuits are combined vectorially (as shown by the parallelogram method in Fig. 8-14), it will be seen that resultant vector OE_5 across diode D_1 is greater in magnitude than resultant vector OE_4 across diode D_2. Therefore, the current in the circuit of diode D_1 is greater than the other diode current. This means a greater voltage drop exists across R_1, the load resistor of diode D_1, and a differential voltage exists across point A to B of the diode circuit of Fig. 8-11. However, under these conditions the polarity of point A will be more positive than point B which is the same as saying point B is negative with respect to point A.

Summarizing the action described, you can readily see that if a varying frequency signal, one which varies in frequency around a mean, is applied to the discriminator network — provided that the range of frequencies covered is not beyond the acceptance bandwidth of the discriminator transformer — a signal which changes in amplitude and polarity will be obtained. The output signal or audio signal amplitude, as you have seen, is determined by the frequency deviation; for the less the frequency deviation, the less the departure from a 90° phase relationship between the reactive voltages E_2 and E_3 and E_1. The greater the frequency deviation, the greater is the difference in angular displacement between E_2 and E_1 and E_3 and E_1, so that the differential voltage obtained from the diodes is greater. When viewed from the angle of audio intensity, the greater the differential voltage from the rectifiers, the louder the audio signal, since the extent of deviation at the transmitter is a function of modulating voltage level. The greater the modulating voltage level within prescribed limits, the greater the deviation frequency.

A Simple Phase Modulator

When the amplitude of a (carrier) signal is changed in accordance with the amplitude of some external signal, this carrier signal is said to be amplitude modulated (a. m.). If the frequency or the phase of the carrier is changed in accordance with the amplitude of some external signal, then two new types of modulation manifest themselves, namely, frequency modulation (f. m.) and phase modulation (p. m.).

Between these last two types of modulation, f. m. and p. m. the former is quite well known compared with the latter. In order to produce a direct f-m signal reactance tubes are invariably used. They have the ability to reflect a varying reactance, due to a varying audio input, upon an oscillator tank circuit, changing its resonant frequency in accordance with the varying reactance and thereby frequency-modulating the oscillator signal. This type of modulation is well known and can be found in most books dealing with the subject of f. m. It is the procedure of producing a p-m signal that is very seldom discussed. In this section we will analyze a simple phase modulator through the use of a vector diagram.

When a carrier is frequency modulated, its instantaneous frequency is deviated away from the center or original resting frequency of the oscillator, the amount of deviation being determined by the amplitude of the modulating signal. In p.m. a similar situation exists.

When the carrier signal is phase modulated, the instantaneous phase of the carrier is deviated away from its original resting position before modulation. It can be easily understood that such a p-m signal can be had if the carrier is passed through some time delay network which

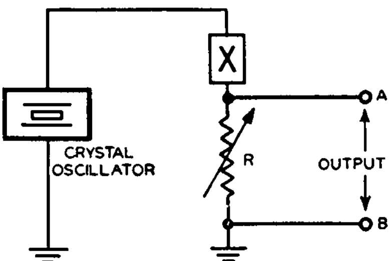


Fig. 8-15—A simple phase modulator circuit in which the reactance X and the resistance R represent a time delay network that imparts a phase change to the crystal oscillator frequency. The variable resistance R can alter the phase of the crystal frequency.

will make the carrier change in phase. If this time delay network is made to vary in accordance with the amplitude of some audio signal, this network would change the phase of the carrier in conformity with this audio signal, and the resultant output from the time delay network will be a p-m signal.

In phase modulators a crystal oscillator circuit is invariably used for the source of carrier signal, because crystal oscillators are very stable and practically remain constant in frequency. There are numerous methods of producing p-m signals. Though a great many of them are practical, many are not.

In Fig. 8-15 a simple p-m circuit is shown. This circuit is the basic one used in certain indirect f-m (or p-m) transmitters of today. It essentially consists of a crystal oscillator in parallel with a series com-

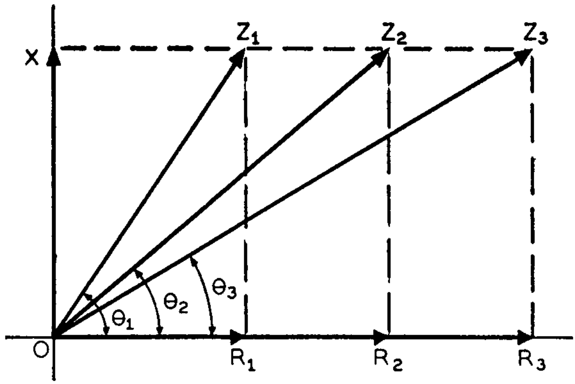


Fig. 8-16—From this vector diagram it is readily seen that by varying values of resistance (R_1, R_2, *and* R_3) the phase angle of the system changes (Θ_1, Θ_2, Θ_3).

bination of a reactance X and a resistance R. This series combination of X and R is the time delay network that imparts a phase change to the crystal oscillator frequency. The output voltage of this circuit is taken across terminals A, B. The system works as follows.

Without the time delay network, the frequency of the oscillator has a certain phase relation of its own. With the time delay network wired across the oscillator circuit, as shown, the resistive and reactive components together are so chosen (at fixed values) that they offer a phase between 0 and 90 degrees, the exact amount determined by the fixed values of X and R. If the resistance R is made variable, then the phase of the oscillator frequency is likewise made variable, and the oscillator output voltage across terminals A and B is then said to vary in phase.

The way the XR network causes a phase shift is easily explained by the vector diagram of Fig. 8-16, in which X represents the fixed value of reactance and the R's represent different values of the variable resistance R. Since there is a 90° phase difference between a pure reactance and a pure resistance, the reactance and resistive components are drawn vectorially at right angles from each other.

If the initial value of resistance is equal to R_2, the total impedance of the series reactance-resistance network can be found by simple vector addition of these reactive and resistive components. From Fig. 8-16 the resultant vector OZ_2 is the impedance for the case where the resistance is equal to R_2. The angle Θ_2 that vector OZ_2 makes with vector OR_2 is the phase angle of the system.

If the value of resistance is increased from R_2 to R_3, the magnitude of the impedance will also increase, as witnessed by the vector addition of the constant value of X and the new value of resistance R_3 as indicated in Fig. 8-16, where vector OZ_3 is the new resultant impedance vector. It will be noted that the phase angle has changed from Θ_2 to a smaller value equal to Θ_3. If the resistance is decreased from the starting value of R_2 to a new value R_1, the total impedance of the series reactive-resistive circuit will likewise decrease. This new impedance is represented by the impedance vector OZ_1 in the vector diagram. Under these circumstances the phase angle changes again, but this time it increases to a value equal to Θ_1. (It is to be noted that all the vector additions are done by the parallelogram method.)

It is thus seen that the phase angle offered by the impedance network to the oscillator circuit of Fig. 8-15 changes with change in resistance, and consequently the crystal oscillator frequency output is varied in phase and is said to be phase modulated. If the vector diagram is studied further, it will be noticed that the larger the value of R the greater will be the value of impedance, but the phase angle will approach zero degrees, which means that the reactance X has negligible effect in the impedance and phase angle. However, if the resistance gradually decreases to a smaller and smaller value, the impedance will also decrease accordingly, and the phase angle of the impedance will approach 90° but will never be greater. Under this circumstance the resistance will have negligible effect and the impedance will be almost a pure reactance.

From this analysis it is seen that the phase changes in the circuit of Fig. 8-15 will always lie between 0 and 90 degrees. This is tantamount to saying that the impedance vector (that resultant vector caused by the vector addition of X and R) of Fig. 8-16 can rotate anywhere between the horizontal resistance reference line and the vertical reactance line of X. Since these two lines are at right angles to each other, that is, 90° apart, the impedance vector can only have phase angles which are between 0 and 90 degrees, the exact value determined by the instantaneous setting of the variable resistance R of Fig. 8-16.

We have just seen how a simple circuit like that of Fig. 8-15 can produce a p-m frequency output. This circuit can be further advanced to the point where the resistance R is made to vary in accordance with an input audio voltage. The circuit for such a system is illus-

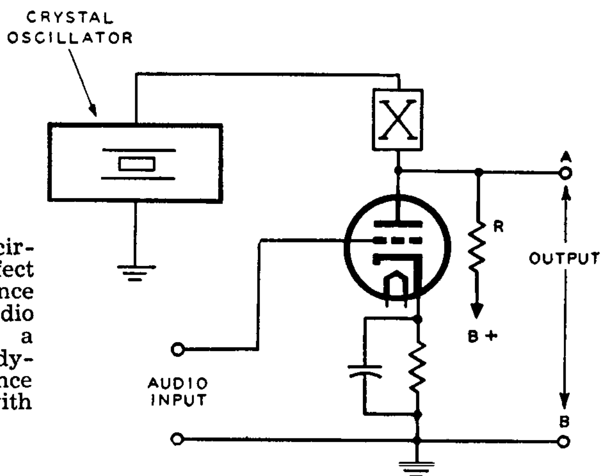


Fig. 8-17—A p-m circuit in which the effect of a variable resistance is made by an audio input signal and a triode tube. The dynamic plate resistance of the tube varies with audio input.

trated in Fig. 8-17. The only difference between the two circuits of Figs. 8-15 and 8-17 is that the variable resistor in the former circuit is replaced by the triode tube circuit in the latter circuit. It must then follow, in order for the two systems to function in the same manner, that the triode tube and its associated network act as a variable resistance. This is exactly what happens. The dynamic plate resistance of the tube is the effective resistance between the low side of the reactance X and ground. The plate resistance is made to vary in accordance with the audio input signal across its grid, and this plate resistance variation causes p.m. of the crystal oscillator frequency. It is fully explained as follows:

Without any audio signal input to the triode, a definite amount of plate resistance exists which is determined by the fixed cathode bias and the plate voltage. This system, as it now exists, offers a *fixed* phase relation to the oscillator frequency. However, if an audio input signal is impressed across the grid, such that the tube is still operative on the linear portion of its e_b-i_b curve, the plate resistance will vary in accordance with the grid voltage, and this variation of resistance is therefore linear. In other words, if the instantaneous grid voltage is changed, then the plate current i_b will change accordingly.

Since the plate resistance is dependent upon the change in the ratio of r-f plate voltage to r-f plate current, then for any one instantaneous change of grid voltage this ratio will change due to the changing r-f plate current. In effect then, the varying grid voltage, due to an audio signal, changes the existing value of the plate resistance of the triode, which then changes the instantaneous phase of the oscillator frequency in accordance with the audio signal. Thus it is said that the oscillator frequency is phase modulated in accordance with the audio input signal. In the case just illustrated the time delay network is the reactance X in series with the triode amplifier.

A. M. TO P. M. TO F. M.

The first so-called f-m transmitter allowed to operate commercially by the Federal Communications Commission (FCC) was that of Major E. H. Armstrong. The name of Major Armstrong is synonymous with frequency modulation. Although the final output signal from the Armstrong transmitter is effectively an f-m wave, the initial method is not one of direct f.m. Instead it is the type which has become known as indirect f.m. There are, however, many different types of indirect f.m., but the method used in the Armstrong transmitter is of unique interest. Basically an a-m signal is converted to a p-m signal, and, due to prearranged circuit constants, this p.m. is directly equivalent to an f-m signal. That is, the system essentially consists of going from a.m. to p.m. to f.m In this section we will not analyze the modulator circuit of the Armstrong transmitter nor will we discuss any particular circuit. Through the use of vector diagrams we are going to discuss only how we can go from a.m. to p.m. to f.m.

In considering the method of shifting from a.m. to p.m. to f.m., a few important facts about these modulated signals should be known. In this respect the relationships between the center frequency components and the sidebands of the modulated signals are the important factors.

In amplitude modulation no matter what the percentage of modulation there are only two sidebands present. That is the a-m signal consists of three components, namely, the center frequency or carrier component and the upper and lower sideband components. The frequency of the upper sideband is equal to the frequency of the carrier signal that is modulated plus the frequency of the (audio) modulating signal. The frequency of the lower sideband is equal to the frequency of the carrier minus the modulating frequency. Thus in the frequency spectrum for an a-m wave just three lines would be seen, two of small amplitude representing the sidebands on either side of the center frequency component.

In frequency modulation, the situation is completely different. Virtually an infinite number of sidebands can be present in an f-m wave. Some of these sidebands are so very small in amplitude that they have little or no effect on the signal. Those sidebands that have appreciable amplitude and those that fall within the required bandwidth are known as *effective sidebands.* It is the frequency of the modulating signal and the amount of peak frequency deviation that determines the number of effective sidebands and effective bandwidth. If the ratio of peak deviation to audio modulating frequency (often called the *degree of modulation*) is kept small enough, the number of effective sidebands of the resulting f-m signal can be made equal to two, the same as in a-m; in fact it is possible to have the frequency and amplitude of the sidebands and center frequency component of an f-m wave equal to those components of an a-m wave.

However, there is a difference, and that is the phase relation between the respective sidebands and the center frequency components. If the two sidebands of the f-m wave are added together and if the two sidebands of the a-m wave are also added together, a new wave will appear in both instances which we will call the *double sideband* signal. These double sideband waves represent the difference between the modulated waves and their respective center frequency components. On an oscilloscope both double sideband waves appear exactly the same when the individual sidebands are equal in frequency and amplitude. But in a.m. the double sideband component is in phase with the center frequency component, whereas in f.m. the double sideband component is 90° out of phase with its center frequency component.

From the afore-mentioned facts we have a ready means of obtaining an f-m wave from an a-m wave. First of all, we slightly amplitude-modulate a carrier signal and then separate the double sideband component from the center frequency component. The next step is to take this double sideband and shift it in phase by 90°. After this phase shift is accomplished, the next and final step is to combine this phase-shifted double sideband with the original unmodulated carrier signal. This combination will result indirectly in an f-m signal. In the process of going from a.m. to f.m what really results in the combination of the phase-shifted double sideband is phase modulation. It is important to understand that, when a signal is modulated directly in phase, it indirectly changes the frequency of the signal also. Likewise when a signal is frequency modulated, it produces an equivalent phase modulation. The following vector diagrams will make this process of a.m. to p.m. to f.m. more readily realizable.

It was stated that in a.m. the double sideband component of the modulated signal is *in phase* with the center frequency component. Therefore, the resultant amplitude of the a-m signal is a simple process of addition. In Fig. 8-18 three vector diagrams representing amplitude modulation are illustrated. In each case two separate vector additions are made. In all of these vector diagrams vectors *AB* and *AC* represent the upper and lower sideband components of an a-m signal, vector *OA* represents the center frequency component of the a-m wave, vector *AD* is the double sideband component from the vector addition of vectors *AC* and *AB*, and vector *OD* is that representing the final a-m wave itself. The three different vector diagrams each represents some instantaneous part of the a-m signal. Fig. 8-18 (A) and (B) is that part of the modulated signal which is greater than the amplitude of the carrier (that is, on the positive crests of the a-m signal) and Fig. 8-18 (C) is a part of the modulated signal which is less in amplitude than that of the carrier (that is, in the negative part of the a-m signal.)

Vectors *AB* and *AC* were drawn head-to-tail fashion because the functioning of the a-m signal components can be readily seen; that is, instead of drawing the initial points of the sideband vectors *AB* and *AC* from the origin, they are drawn (at the same phase angle, of

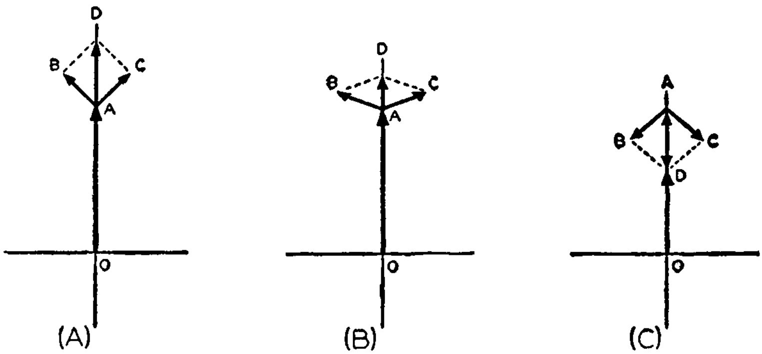


Fig. 8-18—Vector diagrams of an a-m signal. In (A) and (B) the double sideband component vector *AD* is in phase with the center frequency component vector *OA*, making the resultant vector *OD* greater than *OA*. In (C) vector *AD* is 180° out of phase with vector *OA*, making the resultant vector *OD* smaller than *OA*.

course) from the terminal point of vector *OA*, the center frequency component. This is permissible, and it also portrays the fact that the sidebands of the a-m signal rotate about the vector *OA*, as witnessed by the different positions of the sidebands in all three drawings. In all diagrams the vector *AD* results from the vector addition (by the parallelogram method) of the sideband vectors *AB* and *AC*, and hence this resultant vector *AD* is the double sideband.

In Figs. 8-18 (A) and (B) this double sideband component, vector *AD*, is seen to be in phase with the center frequency component, vector *OA*. Consequently, the final resultant a-m signal is the vector addition of the center frequency component, vector *OA*, and the double sideband component, vector *AD* which simply results in vector *OD*. In Fig. 8-18 (C) the phase relationships between the individual sideband vectors *AB* and *AC* is such that, when vectorially added, by the parallelogram method, the double sideband vector *AD* which results is 180° out of phase with the center frequency component, vector *OA*. Consequently, the addition of these two out of phase vectors results in the final vector *OD*, which is the resultant a-m signal for that particular instant of time represented by the vector diagram under discussion.

Now if the amount of modulation of the a-m signal is considered slight, the resulting double sideband will be small as compared with the center frequency component, so that this double sideband can be

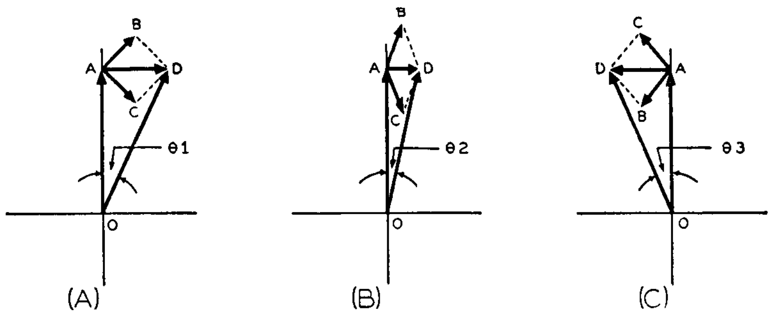


Fig. 8-19—The double sideband vectors *AD* of the a-m signals of Fig. 8-18 have been shifted in phase by 90°. The final resultant vectors, *OD* in this case, are seen to be varying in phase with respect to vector *OA*.

used for producing the final f-m signal. Let us assume that in the vector diagrams of Fig. 8-18 this double sideband vector is small, but for the purposes of illustration only it was drawn somewhat larger. If this double sideband, vector *AD*, is separated from the center frequency component, shifted in phase by 90°, and then recombined with the center frequency component, the new resultant signal will be equivalent to an f-m wave. This f-m wave would be equivalent to a low ratio of frequency deviation to audio modulating frequency, in the case of directly frequency-modulating a carrier signal, in order that only two sidebands appear.

To see this better, let us refer to the vector diagrams of Fig. 8-19 which are the vector diagrams of Fig. 8-18 with the sidebands, and hence the double sideband, shifted in phase (in a clockwise direction) by 90°. To achieve this the double sideband was separated from the center frequency component, shifted in phase by 90°, and then recombined with the center frequency. (In the actual system, it is recombined with a part of the original unmodulated carrier which is at the same frequency as the center frequency component of the modulated wave and only differs in amplitude.) The vector designations in these vector diagrams are the same as in the preceding one, except that here they refer to an f-m wave. From these diagrams it is seen that just by shifting the sidebands 90° a completely new type of resultant vector is the outcome. In the vector diagrams for a.m. the final resultant a-m vector *OD* for each case was in phase with its center frequency component vector *OA*. Though in the vector diagrams for Fig. 8-19 the resultant vector is still *OD,* it will be noticed that *this vector has a different phase relationship to vector OA in each case.* In Figs. 8-19 (A) and (B) resultant vector *OD* has phase angles Θ_1 and Θ_2 respectively that lag the center frequency component vector *OA*. In Fig. 8-19 (C) the phase angle of vector *OD* is leading vector *OA* by an angle equal to Θ_3. Thus we see that the resultant vector *OD* is varying in phase, which means that the a-m signal by the special process mentioned is changed to a p-m signal However, the conditions of an f-m signal under a slight degree of modulation is effectively represented by the vector diagrams of Fig. 8-19. This is the same as saying that a p-m signal produces an equivalent f-m signal.

In order that the resulting p-m signal be equivalent to an f-m signal, the audio modulating frequency of the original a-m wave is to have an inverse frequency effect. This is accomplished by passing the audio signal before modulation, through an inverse frequency network, also called an audio correction network. The reason for this inverse audio frequency effect is that in a p-m signal, as well as in an f-m signal, the amount of deviation should be dependent only upon the amplitude of the audio signal and not upon the frequency of the audio signal.

Another very important point about the p-m signal is that the p-m wave should vary only in phase and remain constant in amplitude, because if amplitude variations appear in the p-m signal, a certain amount of distortion in the output of a loudspeaker will result when the p-m signal is demodulated and the intelligence reproduced. From the vector diagrams of Fig. 8-19 the resultant p-m vectors *OD* also vary in amplitude. This is readily noticed by knowing that resultant vectors *OD* are the hypotenuses of right triangles bounded by vectors *OA* and *AD* which are considered the legs of the triangle. As vector *AD* (the double sideband) is changing in magnitude in each case, so is the resultant vector *OD* changing in magnitude.

As mentioned, these vector diagrams were drawn to show the a.m. to p.m. to f.m relationships, and the magnitudes of the vectors used in no way is supposed to represent the actual or typical values employed for the correct operation of such a system. In fact the magnitude of the double sideband as used in a typical indirect f-m system is very small as compared with that of the signal with which it is combined. *It is made small enough so that the difference in magnitude of the resultant p-m signal and the center frequency component is so minute that the amount of a.m. introduced will cause a negligible amount of distortion.*

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