ELECTRONIC TECHNOLOGY SERIES

MAGNETISM AND ELECTROMAGNETISM

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MAGNETISM AND ELECTROMAGNETISM

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PREFACE

The wide and varied uses of the properties of magnetism make a study of the theory of magnetism and the principles of magnetic circuits a necessity for the practitioner of electronics. A magnet is an integral essential of navigational and surveying instruments and the very core of most electric motors and instrumentation. Almost all electronic equipment uses features of magnetism either directly or in the supplying circuits. The student of magnetism will find ample opportunity for practical applications of the the theoretical relationships fundamental to magnetic circuits.

The intent of this book, the first in a two-volume treatment of the subject, is to give attention to the major theoretical considerations of magnetism, magnetic circuits, and electromagnetism.*

The mathematical techniques used here are simple but extensive enough to permit the interested student or technician the mastery of typical computations. To further ensure the aim of comprehensive coverage of the subject matter, adequate information is given relating to broad concepts; detailed descriptions of a small number of selected major topics are presented rather than a larger body of less important material; and a sufficient number of practical situations and problems are afforded the reader to allow applications of the principles he has learned.

Specific attention is given the general properties and classifications of magnetism; the molecular theory of magnetism; the magnetic field; lines of force; theory of magnetization; permeability; electromagnetism; relations between magnetic fields and electrical current; rules for deter-

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*The second volume in the series is *Advanced Magnetism and Electromagnetism;* Schure, A., New York: John F. Rider Publisher, Inc., 1959.

mining direction of magnetic fields; magnetic flux; flux density; the magnetic circuit, problems relating to magnetic circuits and electromagnetic fields; the force on a conductor in a magnetic field; magnetization curves; hysteresis; electromagnetic induction; laws of Lenz and Faraday; mutual induction; and inductors and the factors affecting the problems relating to mutual inductors and ignition systems.

The more advanced concepts related to magnetic theory are presented in the second volume dealing with these materials. This book provides a sufficient foundation upon which the more advanced concepts can be built.

Grateful acknowledgment is made to the staff of the New York Institute of Technology for its assistance in the preparation of the manuscript of this book.

New York, New York A. S.
October, 1959

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Chapter 1

INTRODUCTION TO MAGNETISM

1. The Invisible Force

Magnetism is one of the oldest forces known to man. It is likely that primitive man first observed this invisible force that existed between certain lead-colored rocks. By the time of the ancient Greeks, the force was well known and the Greeks gave the name "magnet" to the stone that was capable of attracting similar stones or bits of iron. This particular ore we now call magnetite, and we know it to be an oxide of iron.

2. Early History of Magnetism

In later centuries, the Chinese, and probably others, discovered two other properties of this mystical material. It was observed that a piece of magnetite freely suspended in air tended to line up in an approximately north-south direction. This, of course, led to the primitive compass. A piece of magnetite acted as a "leading stone" for navigators, and hence it became known as "lodestone."

Another property of magnetite, discovered centuries ago, is that the invisible force of magnetism can be passed on to a piece of iron simply by stroking the iron with the lodestone. For hundreds of years, this method was the only known way of making man-made or artificial magnets.

3. Experiment: Attraction of Iron Filings to a Bar Magnet

It was discovered quite early that the magnetism of a natural or artificial magnet is not a uniform thing. Certain parts of the magnet's surface showed relatively strong magnetic effects, other surface areas were weaker, and some areas showed no magnetism at all.

This can be shown readily by taking a "bar" magnet (a magnet cast in the shape of a rectangular bar) and dipping it into a container of fine iron filings. The result will be that shown in Fig. 1. There will be a heavy concentration of filings at the two ends, showing strong magnetism there; the filings begin to taper off from the ends toward the center, indicating progressively weaker magnetism; and finally, in an area around the center of the bar, there will be practically no magnetism indicated.

Let us perform a second experiment with the bar magnet. Suspend the bar from a string so that it can swing freely. If there are no other magnets or large pieces of magnetic material nearby, the bar will line up so that one end will point north and the other end south. If we

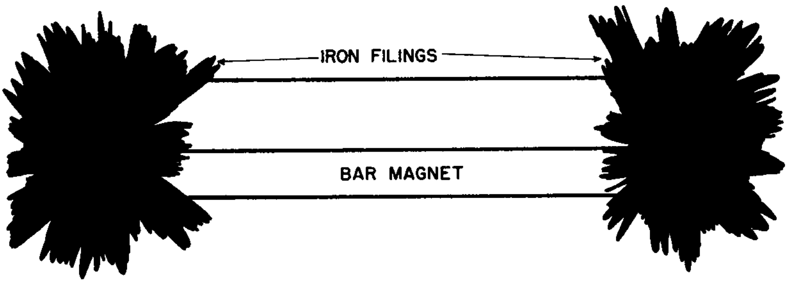


Fig. 1. The attraction of iron filings to a bar magnet.

should swing the magnet around by 180° (by hand) so that it is lined up along the same north-south axis as before but with the ends reversed, the magnet will rotate back to the original position when released. The two ends, both areas of high magnetism, thus seem to have different kinds of magnetism. The bar will line up in only one way.

4. Magnetic Poles

The areas of a magnet where the magnetism seems most concentrated are called "poles." The north-seeking end of the bar magnet is called the north pole of the magnet, and the other end is called the south pole. By careful measurements, it can be shown that the two poles are exactly of the same magnetic strength. Lodestone may have several distinct north and south poles, in which case the total north pole magnetism will be equal to the total south pole magnetic strength.

If two north poles are brought close to each other, it will be readily seen that a force of *repulsion* exists between them. A similar force of repulsion will be found between two south poles. However, when a north and south pole are brought close together, they will *attract* each other. A simple experiment (such as that shown in Fig. 1) will also show that either pole will *attract* unmagnetized iron or steel equally well. These facts can be summarized in three simple statements.

1. Like poles repel.
2. Unlike poles attract.
3. Either pole attracts unmagnetized magnetic material.

5. Artificial Magnets

The method of making artificial magnets by stroking was highly unsatisfactory, but until the discovery of the relationship between electricity and magnetism, it was the only practical method. Some artificial magnets lose their magnetism quickly and are called temporary magnets. Iron is such a temporary magnet. Other materials, such as carbon steels and special alloys, hold their magnetism indefinitely and are

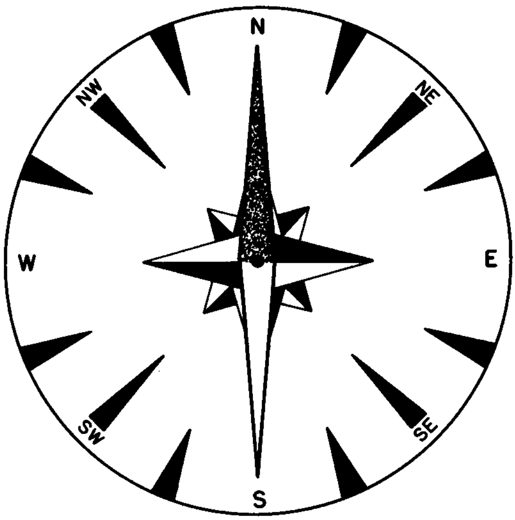


Fig. 2. Typical needle magnetic compass.

called "permanent" magnets. Certain special alloys, such as cobalt steel (containing cobalt, chromium, tungsten, and manganese) and "alnico" (containing aluminum, nickel, and cobalt) make very powerful permanent magnets.

In addition to the bar shape, magnets are often bent into the shape of a horseshoe. This puts both poles close together and increases the magnetic attraction on an unmagnetized substance.

6. Magnetic and Nonmagnetic Substances

By using a magnet, we can classify all substances as either magnetic or nonmagnetic. In Chapter 2, we will make a better classification of the magnetism of materials, but for the time being we will state that only iron, nickel, cobalt, and certain alloys of iron show appreciable magnetic properties (also chromium to a very small degree). All other materials are considered nonmagnetic. It should be noted that a nonmagnetic material, such as aluminum when alloyed with iron and other materials (such as alnico), may contribute to the magnetic strength.

7. Magnetic Compass

We have already mentioned the use of the magnet as a compass. The primitive lodestone is now a thin magnetized steel needle pivoted on a jewel bearing. If set above a graduated scale and placed in a glass covered box to protect it from air currents, it becomes a direction-indicating device. Figure 2 shows a typical magnetic compass.

8. The Earth as a Magnet

The very existence of the magnetic compass leads to the inescapable conclusion that the earth itself is a magnet. Near the geographic north pole, in the vicinity of latitude 70°N, longitude 96°W, there is a magnetic pole that attracts all north poles of magnets. To be consistent with the rules of magnetic attraction, this spot on earth should really be called a south magnetic pole. However, by custom and agreement, it is called the north magnetic pole of the earth. Similarly a south magnetic pole is found in the vicinity of latitude 72°S, longitude 157°E. This pole attracts all south poles of magnets.

The fact that the geographic and magnetic poles of the earth do not coincide means that a compass needle will not necessarily point true north. The angle between true geographic north and the pointing of a compass is called the *magnetic declination* or *magnetic variation* of that position on the earth. For New York City, the magnetic variation is approximately 11°W. This means that a compass needle in that city

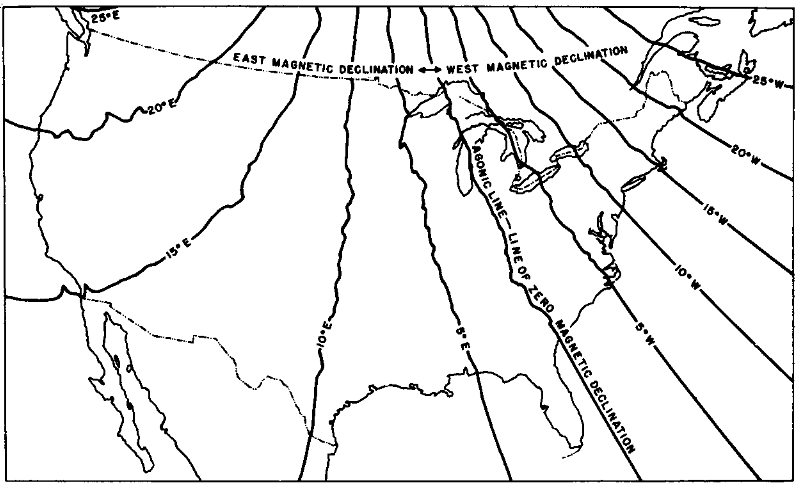


Fig. 3. Map of United States showing isogonic lines.

will point about 11°W of true north. In some places, such as in the vicinity of Savannah, Georgia, a compass points true north. In that case, the magnetic variation is 0°.

If points of equal magnetic variation are connected by lines on a map, such lines are called "isogonic" lines. Figure 3 is a map of the United States showing such isogonic lines. Such charts are prepared by the Department of Commerce and must be revised periodically. These revisions are necessary because the earth's magnetic poles are not stationary but do shift over periods of time. Such shifts change the isogonic lines.

During the Geophysical Year of 1957–1958, a great effort was made to better our understanding of the earth as a magnet. In the course of this study, extensive and highly accurate measurements were made at many points on the earth's surface.

9. Magnetism and Electricity

If magnetism were an isolated phenomenon, involving merely attraction and repulsion of iron bodies, it would have little more interest than its compass applications. However, as we have already hinted, there is a close interrelationship between magnetism and electricity. It is in this relationship that the importance of magnetism exists, and we will speak not only of magnetism, but also of "electromagnetism."

10. Review Questions

1. State the three rules of magnetic attraction and repulsion.
2. You are given an irregular piece of lodestone. Describe an experiment for finding the number and type of poles it has.
3. Name three materials which will make permanent magnets.
4. You have two identical steel bars. How would you determine whether one, both, or neither are magnets? How would you identify the poles as to north or south?
5. A horseshoe magnet is dipped into a container of iron filings. Sketch what you believe would be the resulting pattern.
6. Define magnetic variation.
7. A compass on a ship reads 40° in an area where the magnetic variation is 15°W. What is the true course of the ship?
8. Define an isogonic line.

Chapter 2

THE MAGNETIC FIELD

11. Molecular Theory of Magnetism

If a simple bar magnet were to be sawed in half in such a way that the magnetism were not destroyed, it would be found that both halves had become magnets. The original north pole would have acquired a companion south pole from what was originally the apparently non-magnetic center of the bar. The original south pole would likewise have acquired a companion north pole. Both halves would show all of the characteristics of bar magnets.

Suppose now that this cutting in half were repeated with each of the two parts. We would now find four perfectly normal magnets existing. This process could be continued, and no matter how small each piece became, it would remain a two-pole magnet. This suggests a theory for explaining magnetism called the *Molecular Theory of Magnetism.*

In the molecular theory, it is assumed that if we could continue the cutting process indefinitely, we would finally come to the smallest particle of matter that still has all the properties of the original matter—the molecule. The theory states that the molecules of magnetic materials are also magnets and have definite north and south poles.

In an unmagnetized piece of iron or steel, the molecules are arranged in random order with as many arranged in any one direction as in the opposite direction. As a result of this disorder, the magnetic effects of the molecular magnets cancel out, and the net magnetism of the piece is zero. Figure 4(A) shows this case.

Molecular theory explains magnetism by the positioning or "orientation" of the molecular magnets. In Fig. 4(B), many of the molecules are still in random positions, but some are shown lined up in such a way that the north poles of the molecules face to the right. This will produce a net north pole at the right and a companion south pole at

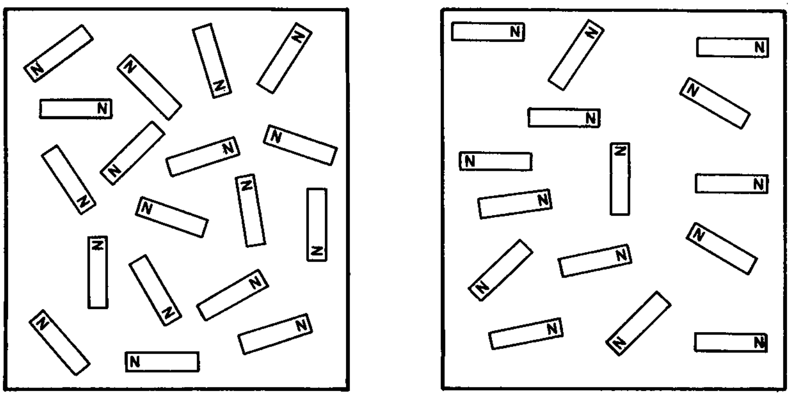


Fig. 4. (A) Random positions of molecular magnets in unmagnetized iron. (B) Orientation of some of the molecular magnets to produce a north pole at the right.

the left. The more molecules that are oriented in this way, the stronger the magnet becomes.

There is much experimental evidence to support the molecular theory of magnetism. If a permanent magnet is strongly jarred by striking or dropping it, some or all of its magnetism may be lost. The explanation? The alignment of the molecules has been upset by the strong jarring. The reverse experiment can also be made. If a bar of unmagnetized steel is held near a strong magnet and is struck hard several times, some molecules will line up and the bar will become a magnet. A fuller explanation of this must wait until the magnetic field is discussed.

If a permanent magnet is heated, the effect of the heat energy is to cause the molecules to move faster. This will result in some molecules going out of alignment and magnetism will be reduced. By heating a magnet to red heat, all traces of magnetism can be destroyed.

12. The Magnetic Field

Whenever it is necessary to describe the action of forces acting at a distance, such as gravitational, electrostatic, and magnetic forces, it is convenient to introduce the concept of a "field." Thus we will define a magnetic field as a region in space where the effects of magnetism can be detected. There is, then, a magnetic field around the earth, since a compass can detect it. Similarly, all magnets must have a magnetic field associated with them.

Let us perform an experiment with a bar magnet and a needle compass. Place the bar on a nonmagnetic surface and explore the area around the magnet with the compass. Make a sketch of the position of the needle at various points around the bar. A "field map" such as the one shown in Fig. 5(A) will be the result. The arrowheads indicate the position of the north pole of the compass.

If we were to connect the obvious lines that are formed by the arrows and use the arrowhead to show the direction of the compass north pole,

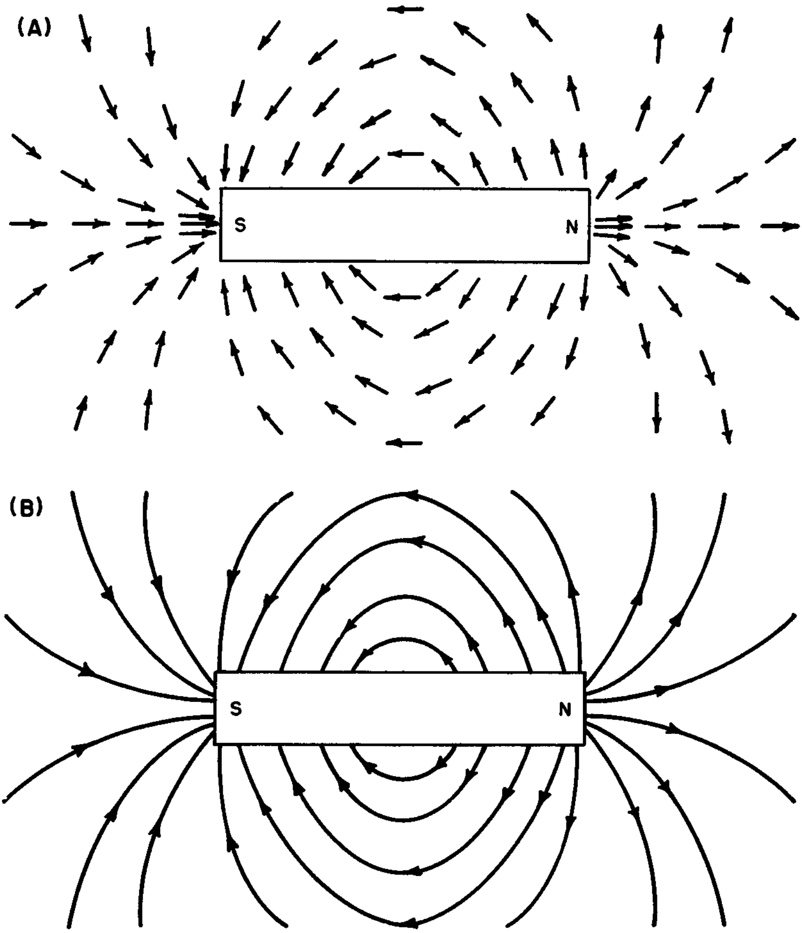


Fig. 5. (A) A "field map" of a bar magnet as mapped by a compass. (B) The lines of force around a bar magnet.

the pattern of Fig. 5(B) would emerge. This shows that a magnetic field has *direction* as well as magnitude. The direction of a magnetic field is defined as the direction toward which a north pole will point when placed in the field.

For a bar magnet, the direction of the field outside of the bar is evidently from the north pole to the south pole. From the action of a compass, we can conclude similarly that the direction of the earth's magnetic field is from the south pole (remember we said that it was really a north magnetic pole) to the north pole (really a south magnetic pole).

13. Lines of Force

In Fig. 5(B) we have shown the magnetic field as a series of directional lines. This turns out to be a very convenient way of considering a magnetic field. Let us call each of those lines a "magnetic line of force." We will consider all magnetic fields to be made up of lines of force, with the number of such lines acting as a measure of the strength of the field. Stronger fields (hence greater magnetism) will contain a larger number of lines. In the next chapter, we will consider the question of the actual number of lines of force in a given field.

As a result of many observations, certain properties of magnetic lines of force have been established. A full understanding of these properties is necessary to understand much of the work that will follow, therefore they must be studied carefully.

1. *Every line of force forms a complete, unbroken loop.* From this, it follows that a line leaving a north pole must return to the *same* north pole. Since outside a magnet, the line goes from the north to the south pole, we must conclude that *inside* the magnet, the line goes from the south to the north pole to complete the loop.

2. *Each line of force is independent of all others.* Lines cannot merge with two becoming one, but must maintain their individual identities.

3. *Lines of force repel each other and cannot cross.* This forms the basis of the repulsion of two like poles. In Fig. 6(A), the lines emerging from each pole are crowded together because of their mutual repulsion. This sets up a force F, as shown, tending to separate the poles. If one or both of the magnets can move, motion in the direction of F will take place.

4. *Lines of force tend to be as short as possible.* In this respect, lines can be thought of as stretched rubber bands trying to shorten. Any process which lengthens a line sets up a counterforce in the line as the

line tries to shorten. This becomes the basis of motor action. It is also the explanation for the attraction of unlike poles, as seen in Fig. 6(B). There are two typical lines shown, going through both magnets to

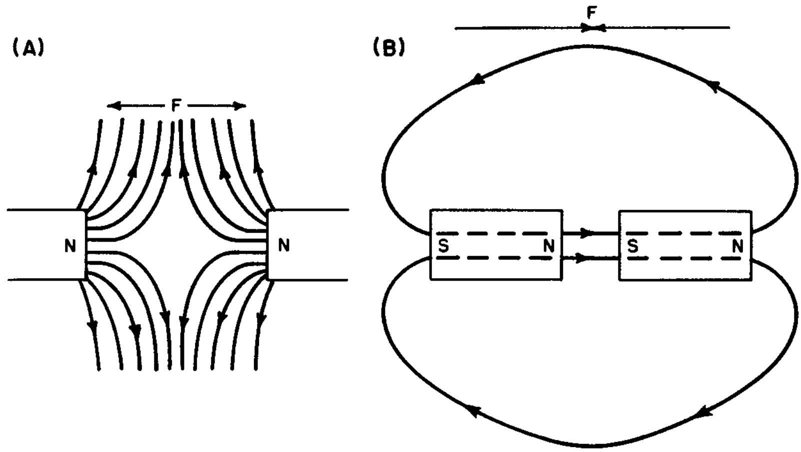


Fig. 6. (A) Repulsion of like poles. (B) Attraction of unlike poles.

form a loop. In order for the lines to shorten, a force F is set up as shown. The magnets will come together if motion is possible.

5. *Lines of force will enter magnetic materials much more readily than nonmagnetic materials.* This is discussed in more detail at the end of the chapter as "permeability." This is the basis of electromagnets and of magnetic shielding. When lines enter a magnetic material, they tend to align the molecular magnets. The material thus becomes a magnet and is attracted to the source of the original lines. This is the basis of attraction of magnets for nonmagnetized iron or steel.

6. *Lines of force pass through nonmagnetic materials with no alteration and no affect on the material.* It is thus impossible to block a magnetic field by a nonmagnetic substance such as air, glass, wood, or aluminum. Magnetic lines of force pass with equal ease through all of these substances.

14. Characteristics of Magnetic Fields

The characteristics of a magnetic field stem from the properties of lines of force. A magnetic field passes through a nonmagnetic material unchanged [as shown in Fig. 7(A)] but is distorted by a magnetic material [as shown in Fig. 7(B)]. Note that in passing through the iron, the field created or "induced" poles in the iron. This is called *induced magnetism*. Contact between a magnet and iron is not needed for induced magnetism, however, the previously mentioned method of making a magnet by stroking with another magnet is another example of induced magnetism.

15. Theory of Magnetization

We see then that a magnetic field has the property of inducing magnetism in a magnetic material. In general, the stronger the field, the stronger the induced magnet. When the field is removed, a material such as soft iron loses most of its induced magnetism. We called this previously a temporary magnet. Steel and special alloys keep most of

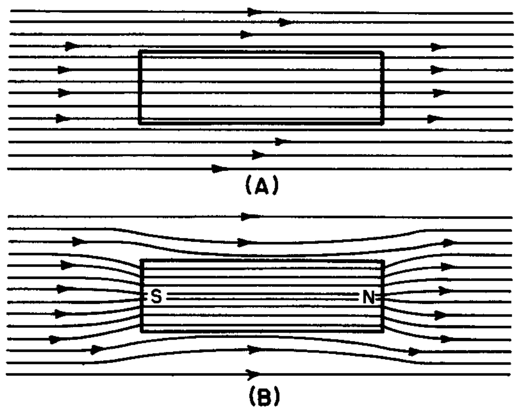


Fig. 7. (A) A magnetic field passing undistorted through a nonmagnetic material. (B) The effect of iron on a magnetic field.

the magnetism and are thus permanent magnets. The ability of a material to retain its magnetism is called its "retentivity." The magnetism remaining after the removal of the field is called "residual magnetism." *All* magnetic materials have some retentivity and, thus, some residual magnetism after exposure to a strong enough field.

16. Permeability

We have stated that lines of force will enter a magnetic material much more readily than a nonmagnetic one. This is illustrated in Fig. 7(B). We can see in the figure that there are more lines per unit area in the iron than in the air around it. The ability of a substance to accommodate lines of force is called its "magnetic permeability" or simply "permeability." The symbol for permeability is the Greek letter mu (μ).

Technically, permeability is a ratio. It is the number of lines of force in a material, divided by the number of lines in a vacuum of the same cross-sectional area, when both are in the same magnetic field strength. The permeability of vacuum is taken to be 1. This is convenient, for now if we say that, for a particular piece of iron, $\mu = 6000$, we mean that the iron can hold 6000 times the number of lines of force that vacuum of the same area can hold.

We can now set up a better classification of the magnetism of materials based on the concept of permeability. We will consider three classes of substances.

1. *Ferromagnetic substances.* These are the familiar iron, steel, special alloys, cobalt, and nickel. They have permeabilities starting above 100 and extending well into the thousands. They are the conventional magnets and electromagnets. As we shall discuss in more detail in later chapters, the permeability of ferromagnetic materials is not a constant. Its value in general *decreases* with *increasing* field strength. Some approximate maximum values of μ for some ferromagnetic substances are given below.

Cobalt	180		
Nickel	300	to	1,000
Iron	5,000	to	8,000
Steel	1,500	to	8,000
Silicon steel		to	10,000
Permalloy		to	85,000

2. *Paramagnetic substances.* These are materials in which μ is 1 or slightly greater than 1. Platinum, aluminum, and air are examples. A typical value of μ for this class is about 1.000015. To test for paramagnetism, place a long thin rod of the substance in a strong magnetic field. If the rod lines up with its long axis in the direction of the field, it is either paramagnetic or ferromagnetic. Its magnetic strength could then separate these classes, since the actual magnetism of a paramagnetic substance is practically nil.

3. *Diamagnetic substances.* These are materials where μ is slightly less than 1. A typical value of μ for a diamagnetic substance is 0.99999. Copper, silver, and bismuth are examples. To test for diamagnetism, place a long thin rod in a strong magnetic field. If the rod lines up with its long axis across the field, the substance is diamagnetic.

17. Handling and Care of Magnets

We have now discussed magnetism from the point of view of poles and from the point of view of fields. Both concepts are useful and we make constant use of them in the remainder of the book. It is evident from the molecular theory of magnetism that some care must be taken in the physical handling of magnets. Dropping or striking magnets or permitting them to get hot will cause a loss of some or even all of the magnetism.

It is also wise when storing magnets to have the path of the lines of force consist of as much iron as possible. A piece of iron called a "keeper" is generally placed between the ends of a horseshoe magnet so that the lines of force have an easy path between the poles. For the same reason, two bar magnets should be placed one on top the other with unlike poles touching for storage.

18. Review Questions

1. Sketch what you believe would be the magnetic field in the vicinity of a horseshoe magnet.
2. Describe the molecular theory of magnetism.
3. Define a magnetic field.
4. Name five characteristics of magnetic lines of force.
5. Define residual magnetism.
6. Sketch the magnetic field that would result if two bar magnets were placed parallel to each other with like poles adjacent and about an inch apart.
7. How would you shield an object from a magnetic field?
8. Define magnetic permeability and write its symbol.
9. Describe the three classes of substances based on magnetism.
10. Describe a series of tests that can be used to separate the three magnetic classes.

Chapter 3

INTRODUCTION TO ELECTROMAGNETISM

19. Oersted's Experiment

The basic discovery relating magnetism to electricity was made by a Danish scientist, Hans Oersted, in 1820. He discovered that a magnetic compass could be deflected from its normal position if a wire carrying electric current were placed near it. This deflection of the compass occurred *only* when current flowed through the wire. When the current was stopped, the compass returned to its usual north-south position.

20. A Magnetic Field is Produced by an Electric Current

From this experiment, it is obvious that there must be a magnetic field produced by a current-carrying conductor. The shape and direction of the field can be determined experimentally.

Put a heavy wire through a hole in a piece of stiff cardboard, as shown in Fig. 8. Pass current through the conductor and sprinkle iron filings on the cardboard. Tap the cardboard gently to permit the filings to arrange themselves in accordance with the magnetic field. The filings will then be found arranged in concentric circles with the conductor as the center. The filings will be densest near the conductor and become less dense with increasing distance from the conductor. Figure 8 shows this circular field pattern.

We can conclude two facts immediately from this observation: These are:

1. The magnetic field around a current-carrying conductor is made up of concentric circular lines of force.
2. The strength of the magnetic field decreases with increasing distance from the conductor.

21. The Relationship between Current and Its Magnetic Field

A third fact can be deduced by varying the current which is flowing through the conductor. The experiment repeated with a larger current flow shows a heavier concentration of filings (hence a stronger magnetic field). Repeating it with a reduced current flow shows a lighter concentration of filings (hence a weaker magnetic field). From this we can state a third relationship for the field around a conductor.

3. The strength of the magnetic field is proportional to the current that flows through the conductor.

It is clear that the strength of a magnetic field at any point some distance from a current-carrying conductor will depend on the amount

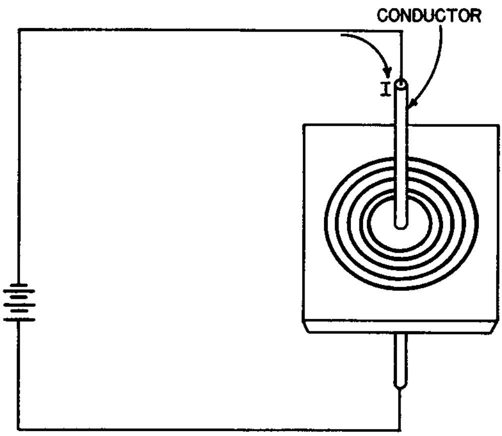


Fig. 8. Pattern of the magnetic field around a current-carrying conductor.

of the current and the distance of the point from the conductor. To replace the clumsy phrase "strength of a magnetic field," we will introduce the term "field intensity," which means exactly the same thing.

Field intensity is given the symbol H and is expressed in a unit called the *oersted*. The oersted was chosen in such a way that the field intensity at a point 1 cm away from the *axis* of a conductor is 1 oersted when a current of 5 amps flows through the conductor. Note that distance is measured from the axis, not the circumference, of the conductor.

Expressed in mathematical form

$$H = \frac{I}{5r} \qquad (1)$$

where H is the field intensity in oersteds, I is the current in amps, and r is the distance from the axis of the wire in cm.

Problem 1. Using equation (1), find the field intensity at a point 6 cm from a conductor carrying 15 amps.

Solution. Here r = 6 cm and I = 15 amps. Therefore,

$$H = \frac{15}{5 \times 6}$$

$$= 0.5 \text{ oersted}$$

Problem 2. Find the current necessary to produce a field intensity of 1.5 oersteds at a distance of 2.5 inches from the conductor.

Solution. Since 1 inch = 2.54 cm, r = 2.5 × 2.54 = 6.35 cm. Solving equation (1) for I, we get

$$I = 5rH$$

Substituting the values for r and H,

$$I = 5 \times 6.35 \times 1.5$$

$$= 47.5 \text{ amps}$$

We have discussed the strength of the magnetic field around a conductor, but we know nothing still of its direction. We have determined that the lines of force in Fig. 8 are circular, but are they clockwise or counterclockwise? To answer that question, we have only to explore the field with a compass and, by noting the direction in which the north pole points, the direction of the field can be established.

Such an exploration of the field in Fig. 8 shows that it goes in a *counterclockwise* direction. If we were to reverse the direction of current flow, the same circular pattern would be seen, but now the direction of the field would be *clockwise.*

These two conditions are shown symbolically in Fig. 9. In Fig. 9(A), the inner circle with the cross symbolizes the cross section of a conductor with current flowing into the paper, or away from us. Think of the cross as the tail of an arrow going away from us. With current in this direction, the magnetic field is counterclockwise as shown. This is the case in Fig. 8, as you can readily determine.

In Fig. 9(B), the inner circle with the dot symbolizes the cross section of a conductor with current flowing out of the paper, or toward us. Think of the dot as the head of an arrow going toward us. With current in this direction, the magnetic field is clockwise as shown.

Figure 9(C) shows a side view of the conductor of Fig. 9(A), and Fig. 9(D) is a side view of the conductor of Fig. 9(B). Note the direction of current and the direction of the magnetic field in these side views, for we shall develop an important and useful rule now concerning the direction of current in a wire and the direction of the magnetic field it produces.

22. A Left-Hand Rule

Consider Fig. 9(C). Imagine now that you are going to grasp the conductor with your *left* hand in such a way that the thumb points in the direction of current flow. Put your left hand in that position now.

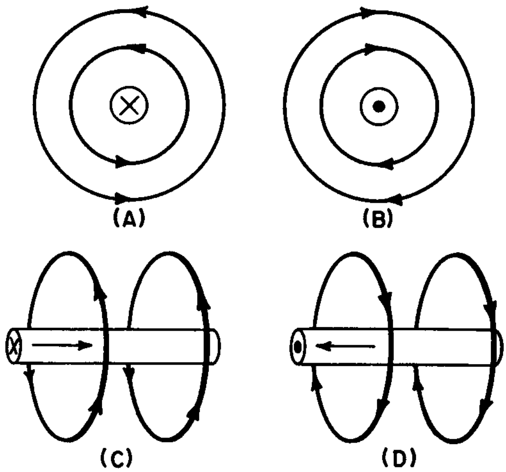


Fig. 9. (A) Current in field counterclockwise. (B) Current out field clockwise. (C) Side view current in field counterclockwise. (D) Side view current out field clockwise.

Notice that your curled fingers point in the direction of the magnetic field. Now try it with Fig. 9(D). Imagine that you are grasping the conductor with your left hand with your thumb in the direction of current flow. Again your fingers are pointing in the direction of the magnetic field. (Throughout this book, we always mean *electron* flow when we say current flow.)

We have thus developed the "Left-Hand Rule for a Current-Carrying Conductor." We will state it in the following manner: when a conductor is grasped with the left hand in such a way that the thumb points in the direction of current flow, the fingers will indicate the direction of the associated magnetic field.

23. Magnetic Field between Conductors Carrying Current

We will now examine the case of two parallel conductors close to each other and both carrying current. Two possibilities exist: the currents in the two conductors can be in the *same* direction, or they can be in *opposite* directions. Figure 10(A) shows the resulting field when both currents are in the same direction. The lines of force encircle both conductors. Since these lines tend to be as short as possible, a force F is set up, which tries to pull the wires together. There is then a force of *attraction* between two such conductors.

If the direction of both currents were changed, the field would be the same except that its direction would be reversed. The force of attraction between the conductors remains the same.

Suppose now that the currents in the two conductors are flowing in opposite directions. No linkage is possible here, for that would mean crossing lines of force, which is impossible. Figure 10(B) shows that each conductor maintains its own field, but the space between the wires becomes crowded with lines of force. Since these lines tend to repel, a force F is set up which acts to separate the conductors. Thus a force of *repulsion* results when currents are in opposite directions.

When large currents flow in wires, these forces become quite large. In the design of power transmission lines, for example, they must be

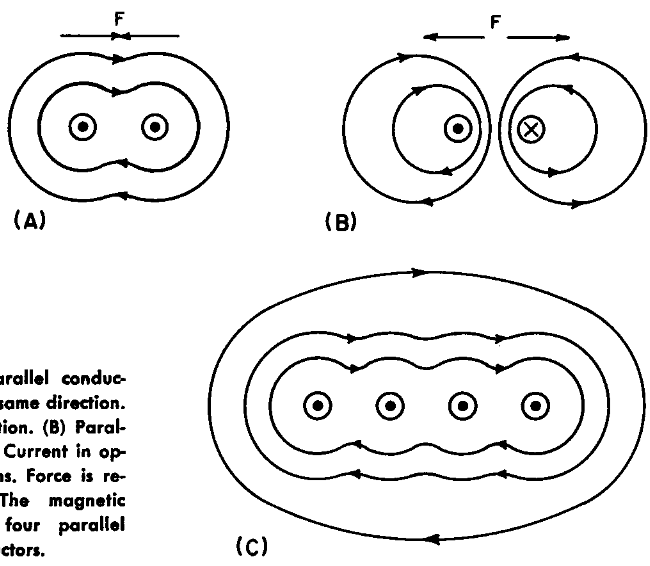


Fig. 10. (A) Parallel conductors current in same direction. Force is attraction. (B) Parallel conductors. Current in opposite directions. Force is repulsive. (C) The magnetic field around four parallel conductors.

taken into account. Under short-circuit conditions, when enormous currents may flow (even though only momentarily), improperly designed equipment may be torn apart by the magnetic forces developed.

If three or more conductors are placed in parallel with current flowing in the same direction in all wires, the lines of force will be found to link all the conductors. Figure 10(C) shows the case of four conductors. Again, if the currents were all reversed, the pattern would remain the same, but the direction of the lines would be reversed.

24. Magnetic Field of a Single Coil

We have covered the case of straight wires carrying current and the fields they set up. Now we will extend the discussion to a coil made by wrapping a wire over a relatively long supporting form. A device like this, where the length of the coil is considerably greater than its diameter, is called a "solenoid."

In Fig. 11(A), the conventional figure symbol of a solenoid connected to a battery is shown. The arrows show the direction of current flow through the turns of wire. Now imagine that we cut this solenoid

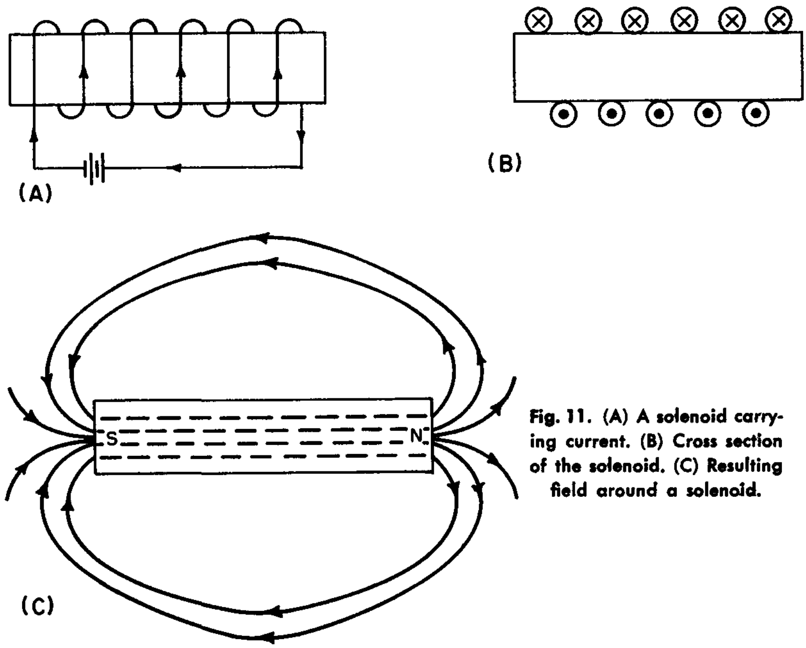


Fig. 11. (A) A solenoid carrying current. (B) Cross section of the solenoid. (C) Resulting field around a solenoid.

along its long axis. This would give us the cross-sectional view of the coil as shown in Fig. 11(B). Study this, and be certain you understand the reason why the current flow in the conductors is as shown.

Observe that, on the top of the coil, each of the wires has current flowing into the paper. On the bottom, each wire has its current coming out of the paper. The wires on top look like the case of parallel conductors with current flow in the same direction, as do the wires on bottom. Compare these with Fig. 10(C); they appear exactly analogous.

We would then expect the bottom wires to set up a magnetic field exactly like the conductors of Fig. 10(C). The top wires would likewise set up a field of the same type, but opposite in direction. The resulting field is shown in Fig. 11(C).

A look at this field shows it to be exactly like that of a bar magnet. Lines of force emerge from the right-hand end of the solenoid, swing around through space; and re-enter the solenoid at its left-hand end. The loops are completed inside the solenoid. Obviously, the right end is, in every respect, a north pole and the left end is a south pole. A solenoid, then, develops a magnetic field with poles like that of a bar magnet. We shall shortly discuss the direction of this field in more detail.

All of the lines of force generated by the solenoid must pass through the center, regardless of where they extend outside the coil. The field intensity in the center of the solenoid is readily calculated, since it is found to depend on the current, the number of turns of wire, and the length of the coil. The formula for the field intensity at the center of a solenoid is

$$H = \frac{kIN}{l} \tag{2}$$

where H is the field intensity in oersteds, k = 0.495 if l is in inches, k = 1.26 if l is in cm, I is the current in amps, N is the number of turns of wire in the solenoid, and l is the length of the solenoid in inches or cm (and then the proper value of k is used). Note that the diameter of the solenoid does not enter into this calculation.

Problem 3. A solenoid has 250 turns wound on a length of 9 in. The diameter of the coil is 2 in. Find the field intensity when a current of 4 amps flows.

Solution. Substituting in equation (2), we get

$$H = \frac{0.495 \times 4 \times 250}{9}$$

$$= 55.0 \text{ oersteds}$$

The diameter of the coil plays no part and was not used.

Problem 4. How many turns of wire are required on a 20-cm solenoid if a field intensity of 80 oersteds is needed and 10 amps are used?

Solution. Solving equation (2) for N, we get

$$N = \frac{Hl}{kI}$$

Substituting the known values,

$$N = \frac{80 \times 20}{1.26 \times 10}$$
$$= 127 \text{ turns}$$

Let us look at equation (2) more closely. For any given solenoid, k and l are constant quantities. The field intensity thus depends on the

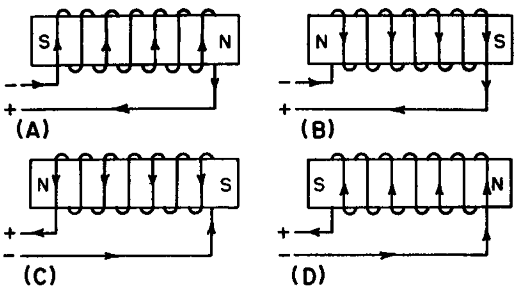


Fig. 12. Magnetic polarity depends upon direction of current flow and direction of winding.

product of I and N, or amperes multiplied by turns. This leads to the concept of "ampere turns" which we shall use later in our discussion of magnetic circuits.

From equation (2), we can also see that field intensity can be expressed in the units "ampere turns per inch" or "ampere turns per centimeter." In the equation, k is thus seen to be simply a conversion factor to convert either of these units to oersteds. We can then state that

1 ampere turn / in = 0.495 oersted
1 ampere turn / cm = 1.26 oersteds
1 ampere turn / m = 0.0126 oersted

The last conversion factor is given because some texts use ampere turns per meter as the unit of field intensity.

We shall now discuss the question of the direction of the magnetic field around a solenoid. This will depend on the direction of current flow and also on the direction of the winding. Figure 12 shows the four possible cases. (A) and (B) have current flow in the same direction, but the windings are reversed. The same is true for (C) and (D). The effect is to reverse the polarity of the solenoid. Similarly, the pairs (A) and (C), (B) and (D) are wound the same way, but the currents are reversed. This too results in a reverse polarity. Note that the effect of changing *both* current and winding direction is to maintain the same polarity. The pairs, (A) and (D), (B) and (C), show this.

A simple left-hand rule will assist us in determining the direction of a solenoid field. Grasp the solenoid with the left hand in such a way that the fingers point in the direction of current flow. The thumb will now point to the north pole.

25. Magnetic Flux

For a given length, the field intensity H around a solenoid depends on the ampere turns. This results in the setting up of a certain number of lines of force. Now if a ferromagnetic material is inserted as a core, it is found that *additional* lines of force come into existence. These additional lines arise from the fact that the core has become magnetized and adds to the total magnetism. The term "magnetic flux" means the *total number* of lines of force that exist.

The symbol for magnetix flux is capital phi (Φ). The unit of flux is the "maxwell," named for the famous English physicist, Clerk Maxwell. By definition, a maxwell is one line of force.

26. Flux Density

Another concept that we must use is the idea of "flux density." Flux density is the number of lines of force per unit area. If we take a unit area of 1 cm^2, then the flux density is expressed in the unit "gauss" (named for Karl Gauss, the German mathematician). In that case, 1 gauss equals 1 maxwell per cm^2. Using the symbol B for flux density, we may write

$$B = \frac{\Phi}{A} \qquad (3)$$

where B is the flux density of a region in gauss, Φ is the flux in maxwells, A is the area of the region in cm^2. This equation can also be written as

$$\Phi = BA \qquad (3a)$$

We can now return to the matter of permeability, first mentioned in the last chapter. The effect of a ferromagnetic core was to increase the number of lines of force. This means that the B is increased for a given H. But permeability was originally defined as the number of lines of force in a material divided by the number that would have existed if a vacuum had been used. Therefore, permeability must also be the ratio of B to H or, as an equation,

$$\mu = \frac{B}{H} \tag{4}$$

where μ is the permeability of a substance, B is flux density in gauss, and H is field intensity in oersteds. This equation is often seen as

$$B = \mu H \tag{4a}$$

Problem 5. An iron-core solenoid is 9 inches long and has 50 turns of wire. The area of the core in cross section is 3 square inches. When a current of 2 amps flows, a flux density of 9500 gauss is measured. Find the permeability of the core and the total magnetic flux it contains.

Solution. Since we have the ampere turns and length, the field intensity is found.

$$H = \frac{kIN}{l} = \frac{0.495 \times 2 \times 50}{9} = 5.50 \text{ oersteds}$$

Using the field intensity and flux density, we find permeability.

$$\mu = \frac{B}{H} = \frac{9500}{5.50} = 1730$$

The total flux is now found by using the flux density and the area. However, the area is given in square inches and *must* be converted to square centimeters in order to use equation (3a). (1 $in^2 = 2.54 \times 2.54\ cm^2 = 6.45\ cm^2$.)

$$\Phi = BA = 9500 \times 3 \times 6.45 = 184{,}000 \text{ maxwells}$$

Problem 6. The iron field pole of a motor has a square cross section, is 4 inches on a side, and is 12 inches long. A total flux of 700,000 maxwells is needed when a current of 1.5 amps flows through the field winding. The permeability of the iron under these conditions is 850. Find the flux density, the field intensity, and the number of turns needed on the pole.

Solution. Since total flux is given and area can be calculated we can proceed to find flux density.

$$A = 4 \times 4 = 16 \text{ in}^2 = 16 \times 6.45 = 103 \text{ cm}^2$$

$$B = \frac{\Phi}{A} = \frac{700{,}000}{103} = 6800 \text{ gauss}$$

Knowing B and μ, H can be found:

$$H = \frac{B}{\mu} = \frac{6800}{850} = 8.00 \text{ oersted}$$

Now N can be found by our knowledge of H, l, and I.

$$N = \frac{Hl}{kI} = \frac{8.00 \times 12}{0.495 \times 1.5} = 129 \text{ turns}$$

As mentioned previously, the permeability of a magnetic material is not constant. It varies with field intensity, being lower at higher values of H. It also depends on the past magnetic history of the material, whether it has previously been magnetized, and under what conditions it had been magnetized. The next chapter will deal with this more fully.

27. Electromagnets

An iron-core solenoid makes up the electromechanical device called an electromagnet. Inserting the core increases the flux density by μ and correspondingly increases the total flux over the air-core solenoid. With high μ cores and a large value of ampere turns, very powerful magnetic fluxes may be obtained. Electromagnets are easily made that can lift many tons of iron or steel.

Other applications of electromagnets, many using the horseshoe or square type shown in Fig. 13, are telephones, relays, circuit breakers, electric bells, telegraph systems, generators, motors, and many others.

A practical electromagnet must lose most of its magnetism when the current is stopped. Cores of low retentivity are used. However, some residual magnetism always remains.

The principle of the electromagnet is used to make permanent magnets. If a high retentivity core is placed in the solenoid, the magnetism will remain after the current is stopped. The core has thus become a

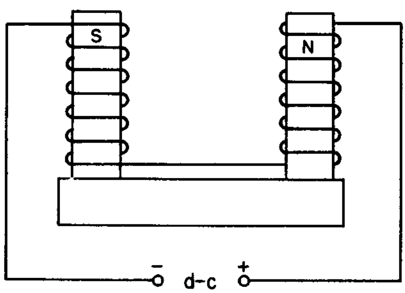


Fig. 13. A horseshoe electromagnet. Correct winding must be used to obtain opposite polarity poles.

permanent magnet. The strength of the magnet will depend on the magnetizing field intensity, the retentivity of the material, and its permeability. An alloy like alnico makes a powerful permanent magnet because of its high permeability and retentivity.

28. Review Questions

1. Describe a method for determining the direction of the magnetic field around a current-carrying conductor.
2. What is the field intensity at a point 4 inches from a conductor carrying a current of 25 amps?
3. Describe the left-hand rule for determining the direction of the field around a conductor. Illustrate the use of this rule with a conductor whose current is coming out as we look at it.
4. Draw the magnetic field around three parallel conductors with current flowing in the same direction.
5. Explain the repulsion of two parallel conductors with oppositely flowing currents.
6. Draw a vertical solenoid so wound and with current in such a direction that the south pole is on top.
7. Describe the left-hand rule for solenoids.
8. A solenoid has 375 turns wound on a length of 125 cm. Find the field intensity when a current of 1.75 amps flows.
9. An 11-inch solenoid has 125 turns. What current must flow if a field intensity of 20 oersteds is needed?
10. How many ampere turns per inch are there in 1 oersted? How many ampere turns per cm are there in 1 oersted?
11. Define and give the symbols and units for magnetic flux and flux density.
12. A certain type steel has a flux density of 16,000 gauss at a field intensity of 6 oersteds. When the field intensity is tripled, the flux density rises to 20,000 gauss. Find the permeability of the material at the two field intensities.
13. The iron field pole of a.generator is 3 inches by 5 inches in cross section and is 14 inches long. There are 150 turns of wire on the pole. A flux density of 4200 gauss is measured. Charts show that the permeability of the iron under these conditions is 700. Find the required field current and the total magnetic flux that will be produced.
14. Explain the greater magnetic strength of an electromagnet over an air-core solenoid.
15. Describe the principle and process of making permanent magnets.

Chapter 4

MAGNETIC CIRCUITS; ELECTROMAGNETIC FIELDS

29. The Magnetic Circuit

Let us imagine that we bend an iron-core solenoid into a circular shape so that it resembles a doughnut. A coil and core in this shape is called a "toroid" and is shown in Fig. 14. The remarkable property of a toroid is that all the lines of force are in the core, with only a negligible amount existing outside the core as "leakage flux." A toroid has no distinct poles.

For a toroid, the path of the lines of force consists only of the material that makes up the core. It is important here to clarify a point about lines of force. When the current through the windings of a coil is direct current, and that is the only kind of current we have considered so far, the magnetic lines of of force are *stationary*. The arrows we put on them indicate only the direction in which the north pole of a compass will point when placed in the field. In no sense do the flux lines actually *move* once they are set up by direct current.

Let us now define a "magnetic circuit." A magnetic circuit consists of the substances through which we must pass as we trace a complete path or loop of the magnetic lines of force. As with an electric circuit, we must have a *complete* path. The difference lies in the fact that there is a motion of electrons in the electric circuit whereas no motion exists in the magnetic circuit. The toroid has been chosen to start this discussion because its magnetic circuit consists only of one material—its core.

The magnetic field of the toroid is shown in Fig. 14(B). The left-hand rule of a solenoid still applies to give us the direction of the field. Equation (2) for field intensity is still valid, so we may write the field intensity of the toroid in the same way.

$$H = \frac{kIN}{l}$$

Let us multiply both sides of the equation by μ. We then get

$$\mu H = \frac{\mu kIN}{l}$$

But the left side of the equation is now the flux density B, and

$$B = \frac{\mu kIN}{l}$$

Now we multiply both sides of the equation by A, giving

$$BA = \frac{\mu kINA}{l}$$

The left member, by equation (3a), is Φ, the total magnetic flux.

$$\Phi = \frac{\mu kINA}{l} \qquad (5)$$

This equation is valid for a toroid, solenoid, or, indeed, any magnetic circuit consisting of a single substance of permeability μ. Let us rewrite

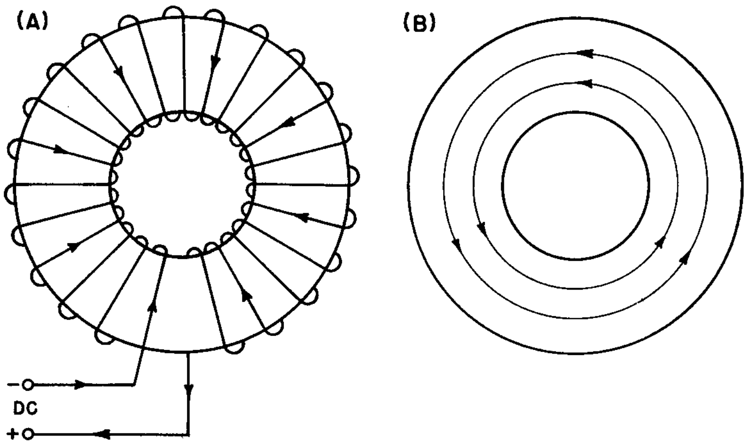


Fig. 14. (A) A toroid (B). Magnetic field of toroid in (A).

equation (5) in a more significant form by dividing both numerator and denominator of the right member by the factor μkA. Thus

$$\Phi = \frac{IN}{\dfrac{l}{\mu kA}} \qquad (5a)$$

30. Magnetomotive Force

Equation (5a) tells us that the magnetic flux is proportional to NI, which we have already called the ampere turns. We will now give ampere turns a name—"magnetomotive force"—and give it a symbol M. In some texts, it is called mmf. It is immediately apparent that it is analogous to the electromotive force or emf of electric circuits.

The denominator of the equation is interesting. We shall call that whole quantity the "reluctance" and give it the symbol $\mathcal{R}$. We then write, by definition,

$$\mathcal{R} = \frac{l}{\mu k A} \tag{6}$$

where $\mathcal{R}$ is the reluctance in "rels," l is the length of the circuit in inches or cm, μ is the permeability, A is the cross-section in cm^2, and $k = 0.495$ in or 1.26 cm.

Note that, as we increase the length l of a magnetic circuit, the reluctance increases. An increase of μ or A decreases the reluctance. Of course, k is simply the conversion factor it has always been. The reluctance of a magnetic circuit thus depends only on the geometry of the circuit and the material of which it is made. The unit "rel" is used for simplicity. We shall shortly develop the physical meaning of this unit.

31. Ohm's Law for a Magnetic Circuit

By using the symbols M and $\mathcal{R}$, we can write equation (5a) in a more meaningful form.

$$\Phi = \frac{M}{\mathcal{R}} \tag{7}$$

where Φ is magnetic flux in maxwells, M is magnetomotive force in ampere turns, and $\mathcal{R}$ is reluctance in rels.

This is *Ohm's Law* of the magnetic circuit. Φ and I, flux and current, are analogous. M and E, magnetomotive force and electromotive force, are analogous. Finally, $\mathcal{R}$ and R, reluctance and resistance, are analogous.

When equation (7) is transposed to read

$$\mathcal{R} = \frac{M}{\Phi} = \frac{\text{ampere turns}}{\text{maxwell}}$$

we see that the unit of reluctance is ampere turns per maxwell. However, the rel will be used for simplicity.

Problem 7. A toroid is 5 cm^2 in area and has a length of 20 cm. The permeability of the core is 1200. There are 60 turns of wire on the toroid and a current of 850 milliamps flows. Find the magnetic flux and flux density.

Solution. M can be found at once from I and N.

$$M = IN = 0.850 \times 60 = 50 \text{ ampere turns}$$

We now find $\mathcal{R}$ from the dimensions and permeability.

$$\mathcal{R} = \frac{l}{\mu k A} = \frac{20}{1200 \times 1.26 \times 5} = 0.00265 \text{ rels}$$

Now Φ and B may be found.

$$\Phi = \frac{M}{\mathcal{R}} = \frac{50}{0.00265} = 18{,}900 \text{ maxwells}$$

$$B = \frac{\Phi}{A} = \frac{18{,}900}{5} = 3780 \text{ gauss}$$

Problem 8. A rectangular iron ring has a length of 30 in and an area of 7.5 in^2. The permeability of the iron is 775. There are 270 turns of wire on the coil. A flux density of 15,000 gauss is needed. What current must be used?

Solution A. This is a longer solution and is used to get practice with Ohm's Law. For complicated magnetic circuits, it becomes the only practical method of solution.

With B and A given, Φ is first found. A is converted to cm^2.

$$\Phi = BA = 15{,}000 \times 7.5 \times 6.45 = 725{,}000 \text{ maxwells}$$

$\mathcal{R}$ is found from the dimensions.

$$\mathcal{R} = \frac{l}{\mu k A} = \frac{30}{775 \times 0.495 \times 7.5 \times 6.45} = 0.00161 \text{ rel}$$

M is now found by Ohm's Law.

$$M = \Phi \mathcal{R} = 725{,}000 \times 0.00161 = 1170 \text{ ampere turns}$$

Now we find I,

$$I = \frac{M}{N} = \frac{1170}{270} = 4.35 \text{ amps}$$

Solution B. This is considerably shorter, but can be used *only* when the magnetic path consists of just one material. We first find H using equation (4).

$$H = \frac{B}{\mu} = \frac{15000}{775} = 19.4 \text{ oersteds}$$

Now, by transposing equation (2), I can be found directly.

$$I = \frac{Hl}{kN} = \frac{19.4 \times 30}{0.495 \times 270} = 4.35 \text{ amps}$$

32. A Magnetic Circuit of More Than One Material

Many magnetic circuits contain more than one substance—iron and air for example. The solution of this type of magnetic circuit is analogous to solving a series electric circuit containing more than one resistor. Consider the horseshoe electromagnet of Fig. 13. The magnetic path may consist of three separate materials with three separate reluctances. The poles may be of one material, the base of a second material, and the third material is the air gap between the poles.

The lines of force around such a system extend through the entire volume of the pole pieces and the base. Through the air gap between the north and south poles, they may extend far out from the magnet. In order to solve for total flux, we must construct an average or median line of force. In general, the practice is to take this line down the center of the windings and the shortest distances through places where there are no windings. Figure 15(A) shows such a median line of force for the electromagnet of Fig. 13.

Let us remember that this median line of force may be only a fiction, but a necessary one if we are to do any calculations. In Fig. 15, the magnetic circuit is thus established as l_1 through the south pole piece,

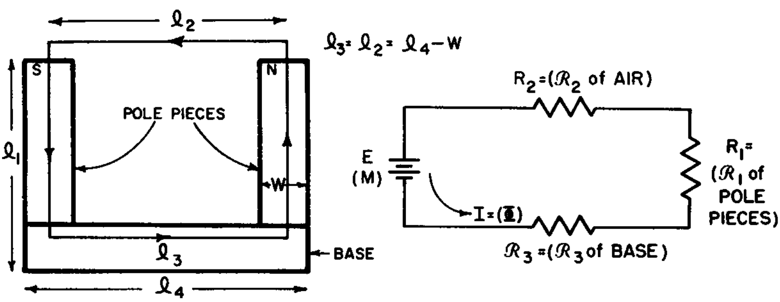


Fig. 15. (A) The median line of force for the horseshoe magnet of Fig. 13. (B) Equivalent electric and magnetic circuit. Total R = $R_1 + R_2 + R_3$. Total $= \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3$.

l_3 through the base, l_1 again through the north pole piece, and l_2 through the air. Note that some approximations are made in these lengths. In this particular circuit, l_2 and l_3 are equal. By inspection, they are also equal to l_{4-w}, where w is the width of the pole piece (or diameter if it is cylindrical in shape).

This magnetic circuit is the equivalent of the series electric circuit of Fig. 15(B). The magnetomotive force M set up by the ampere turns is equivalent to a battery E. The reluctance of the two pole pieces $\mathcal{R}_1$ is analogous to a resistor, R_1. The reluctance of the air $\mathcal{R}_2$, is analogous to R_2, and the reluctance of the base $\mathcal{R}_3$ is equivalent to R_3. It must be recognized that a number of approximations have been made, and more will be made in the actual solution, so that the final solution is, at best, only approximate.

Problem 9. A horseshoe electromagnet of the general shape of that shown in Fig. 15 has dimensions $l_1 = 16$ in, $l_4 = 6$ in. The pole pieces have a width of 4 in and a depth of 5 in. The permeability of the pole pieces is 250, and that of the base is 200. The base has a cross section of 30 in². There are a total of 350 turns of wire and a current of 5 amps is used. Find the total magnetic flux.

Solution. It is necessary to find the total reluctance of the magnetic circuit in order to apply Ohm's Law. To do this, we must find the individual reluctances. To find the reluctance of the pole pieces, we note that the total length of the poles is $2l_1$. In that case,

$$\mathcal{R}_1 = \frac{2l_1}{\mu_1 k A_1} = \frac{2 \times 16}{250 \times 0.495 \times 4 \times 5 \times 6.45} = 0.00201 \text{ rel}$$

To find the reluctance of the base we note that l_3 is 6−4 or 2 in.

$$\mathcal{R}_3 = \frac{l_3}{\mu_3 k A_3} = \frac{2}{200 \times 0.495 \times 30 \times 6.45} = 0.000104 \text{ rel}$$

A difficulty arises in calculating the reluctance of the air gap. What shall we take for its area? Based on long experience, manufacturers of electromagnetic devices have various correction factors that they apply in various situations. For our situation, we will make the approximately valid statement that, when the length of the air gap is small compared with the total length of the rest of the magnetic circuit, we shall consider the area of the gap to be about one and a half times the area of the pole pieces. Using this factor, the area of the gap will be taken as 1.5 × 20 or 30 in².

The reluctance of the gap is then

$$\mathcal{R}_2 = \frac{l_2}{\mu_2 k A_2} = \frac{2}{1 \times 0.495 \times 30 \times 6.45} = 0.209 \text{ rel}$$

The total reluctance of the magnetic circuit is found by adding the individual reluctances

$$\mathcal{R}_T = \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 = 0.00201 + 0.0209 + 0.000104$$
$$= 0.0230 \text{ rel}$$

We now find the magnetomotive force.

$$M = IN = 5 \times 350 = 1750 \text{ ampere turns}$$

The total flux is found by Ohm's Law.

$$\Phi = \frac{M}{\mathcal{R}} = \frac{1750}{0.0230} = 76{,}000 \text{ maxwells}$$

Parallel magnetic circuits also exist, and their solutions are similar to the method used for parallel resistors in electricity. The total reluctance would be the reciprocal of the sum of the reciprocals of the individual reluctances.

33. Force on a Conductor in a Magnetic Field

Let us now consider the situation of a conductor with current flowing through it placed in an external magnetic field. Since the conduc-

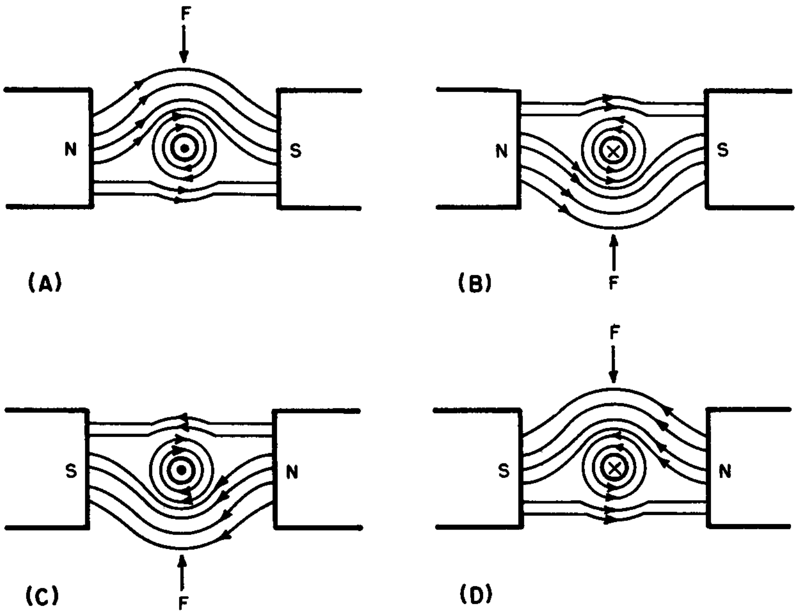


Fig. 16. Direction of force depends upon direction of magnetic polarity and direction of current flow.

tor has its own magnetic field, some interaction between the two fields is to be expected. Figure 16 shows four possible situations. Note that in each case a bending and crowding of the resultant lines of force occur on one side of the conductor. Since lines of force repel each other and tend to become as short as possible, a force F is developed on the conductor in each case. The direction of the force evidently depends on the direction of the fixed magnetic field and the direction of the current flow in the conductor. In Fig. 16, changing only the current direction as form (A) to (B), or (C) to (D) changes the direction of F. Similarly, changing only the direction of the magnetic field as from (A) to (C) or (B) to (D) changes the direction of F. However, changing both factors, current and field, leaves the direction of F unchanged. This is shown from (A) to (D) and (B) to (C).

The effect of a magnetic field on a current-carrying conductor is thus seen to be a force at *right angles* to both the field and the current flow. We can determine the direction of the force by a *left-hand* rule, often called the motor rule (since we have here the basic action of a motor).

The left-hand rule is stated as follows: point the *I*ndex finger in the direction of the current (*I*), and the *M*iddle finger in the direction of the *M*agnetic field. The *TH*umb will then point in the direction of the

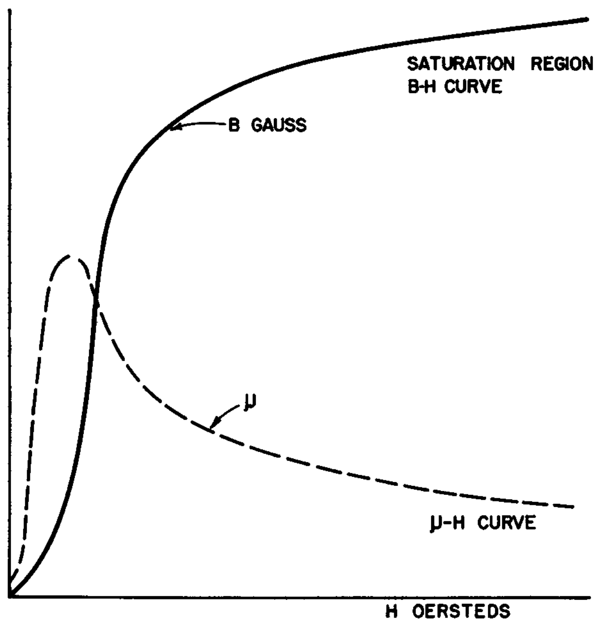


Fig. 17. B-H and µ-H curves.

*TH*rust, or force. The three fingers of the left hand should be held at right angles to each other during this operation. Practice this rule on the four cases of Fig. 16.

The next question is one of the magnitude of the force that is developed. A little thought will show us that the strength of the force should depend on the amount of current flowing and the strength of the magnetic field. The length of the conductor is also a factor. The following formula can be developed from basic mechanics:

$$F = 5.72\ BIl \times 10^{-7} \qquad (8)$$

where F is the force in pounds, B is flux density in gauss, l is length of wire in inches, and I is current in amps.

Problem 10. A conductor carrying a current of 20 amps is at right angles to a magnetic field of flux density equal to 3500 gauss. The conductor is 1½ feet long. Find the force acting upon it.

Solution. The length of the conductor must be changed to inches by multiplying by 12. Then we substitute directly in equation (8).

$$F = 5.72\ BIl \times 10^{-7} = 5.72 \times 3500 \times 20 \times 1.5 \times 12 \times 10^{-7}$$

$$= 0.724 \text{ lb}$$

Problem 11. A 9-inch wire is carrying a current of 35 amps. What must be the flux density of the field to develop a thrust of 1.4 lbs?

Solution. Solving equation (8) for B, we get

$$B = \frac{F}{5.72\ Il \times 10^{-7}} = \frac{1.4}{5.72 \times 35 \times 9 \times 10^{-7}}$$

$$= 7750 \text{ gauss}$$

34. Magnetization Curves

A number of times it has been mentioned that the permeability of a magnetic material is not a fixed quantity but varies with the magnetic field intensity. To illustrate this, we shall draw a magnetization curve, also called a "B-H" curve. A B-H curve plots flux density B on the Y axis against field intensity H on the X axis. A typical curve for a magnetic material starting with *no original magnetism* is shown in Fig. 17. The solid line is the B-H curve. No figures are given, for actual values vary widely, depending on the material. It is the *shape* of the curve with which we are most concerned.

At low values of H, B is low, but rises steeply with further increase of H. As H is increased still further, B rises more slowly and tends to flatten out. The term "core saturation" is applied to the flat regions of the curve.

Since $\mu = B/H$, it is evident that μ cannot be a fixed number for a curve of this shape. If a calculation of B/H were to be carried out point by point on the B-H curve, a series of values of μ would be obtained. The dashed line shows these values of μ plotted against H. Again, no numerical values are considered, as these vary very widely. The shape of the curve is the significant factor.

The μ-H curve shows that a magnetic material has its maximum permeability at a low value of field intensity. The permeability then drops off with increasing field intensity.

35. Hysteresis

The B-H curve of Fig. 17 is for a material that has no original magnetism. Once a magnetic history has been established, an entirely different situation develops. Let us study the curve of Fig. 18. This, too, is a B-H curve, but certainly quite different.

We start the curve at O, where the field intensity and flux density are both zero. We now increase the magnetizing current which increases H and causes a corresponding increase in B. We take this part of the B-H curve to point A where we pause.

We now decrease the magnetizing current, and this, of course, decreases H in proportion. However, the flux density does *not* go down on the same curve that it went up. It goes down more slowly. Finally when I and H have both been reduced to zero, flux density still exists and is represented by the line OB. OB is the residual magnetism or retentivity. For a permanent magnet it would be a high value.

Now what would happen if we reversed the magnetizing current? This would reverse the direction of the field. We can show this by calling it −H on our graph. The negative sign merely means reversal of direction from the original direction. This reduces the flux density until, at point C, it has been reduced to zero. The field intensity OC, in oersteds, necessary to reduce residual magnetism to zero is called the "coercive force."

A further increase of field intensity in the −H direction now reverses the direction of the flux. We show this by marking the flux density as −B. We continue to increase −H, causing an increase of −B until point D is reached.

The path DE is traced out when we reduce magnetizing current and, hence, field intensity to zero once more. The residual magnetism or retentivity is the length OE in gauss. Increasing field intensity in the +H direction takes us first from E to F. At point F, the flux density is zero, so that the length OF in oersteds also represents the coercive force. Further increase in H takes us along the path FA, and the loop is completed.

The curve ABCDEFA is called a "hysteresis loop." Depending on the magnetic material used, the loop may be narrow (materials of low residual magnetism) or wide (materials of high residual magnetism).

It can be shown that the work done in reversing the direction of the magnetic field is proportional to the area enclosed within the hysteresis loop. In a-c applications, where the direction of H reverses at the fre-

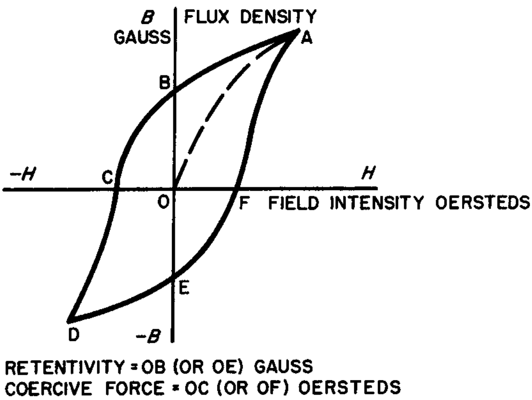


Fig. 18. A hysteresis loop.

quency of the a-c, the energy expended in reversing the field is converted to heat. This heat is a waste of power and is termed "hysteresis loss."

To keep hysteresis losses to a minimum in such devices as transformers and chokes, low-retentivity alloys with narrow hysteresis loops have been developed.

Hysteresis losses evidently also depend on the frequency of the applied a-c. The more times per second we go around a hysteresis loop, the greater must be the work done in reversing the magnetic field. This means greater heat losses as a result. It follows, then, that conventional iron cores can only be used at power and audio frequencies. At radio frequencies, air-core coils or special alloys must be used.

36. The Curie Point

The permeability of a ferromagnetic is also found to depend on the temperature. As the temperature of such a material is raised, the permeability gradually decreases. At a temperature known as the "Curie point," ferromagnetic substances become paramagnetic with a permeability of about 1. The Curie point for soft iron is about 800°C and varies somewhat for other magnetic materials.

37. Relationship of Electric and Magnetic Fields

In the developments of the last chapter and this one, we have seen the intimate relationship that exists between electricity and magnetism. We have seen that an electric current produces a magnetic field. This was Oersted's great contribution. However, an electric current is nothing more than a series of moving charged particles—electrons. Electrons are associated with an electric field, so that the electric current is basically a moving electric field. From the point of view of fields, we can make the statement that a *moving* electric field *generates* a magnetic field.

In the next two chapters, we will explore the converse discoveries made by Faraday and Henry that a *moving* magnetic field *generates* an electric field. Thus there is a definite association among an *electric* field, a *magnetic* field, and *motion.*

38. Energy in Magnetic Fields

When a magnetic field is set up by the action of magnetizing current, a certain amount of energy is required to establish the field. The electric current must perform work to develop the magnetic flux. Now suppose that the current is stopped. Since the Law of Conservation of Energy does not allow the destruction of energy, the energy required to establish the flux must be accounted for. We find that the *collapse* of the lines of force will do an amount of work on the circuit equal to the original work done to develop the flux (assuming that no residual magnetism remains).

A magnetic field requires energy in being established, but returns that energy to the circuit when the field collapses. A magnetic field is thus a storehouse of energy, with the energy stored in the lines of force. From a field point of view, we can look at it in this fashion. A moving electric field (current) generates a magnetic field and transfers a certain amount of energy to it. When the magnetic field moves (collapsing), it generates an electric field and returns its stored energy. Of course this energy transformation is never without losses, one of which we have already mentioned. This is the hysteresis loss where some of the energy is irrecoverably transformed into heat energy.

The energy in a magnetic field is a parallel to the energy stored in an electric field. A capacitor is an example of energy stored in an electric field.

39. Review Questions

1. Define a magnetic circuit.
2. Describe the effects on the reluctance of a material of changes in its length, area, and permeability.
3. Write the three forms of Ohm's Law for a magnetic circuit.
4. What are the units for each of the quantities in Ohm's Law?
5. A toroid 8 cm^2 in area has an air gap 3.5 mm wide. The rest of the toroid is 12 cm in effective length. There are 30 turns of wire and a current of 470 milliamps flows. Find the magnetic flux and the flux density in the iron core. (Note: find the magnetic flux first, treating this as a series magnetic circuit. Then find flux density.)
6. Find the reluctance of an iron core whose permeability is 450 and whose area is 3.5 in^2. There are 40 turns of wire around the core, and a current of 2.3 amps flows. A flux density of 8500 gauss is measured.
7. What is the length of the core in question 6?
8. Name four analogous relationships between magnetic circuits and electric circuits.
9. State the left-hand rule for determining the direction of the force on a current-carrying conductor in a magnetic field.
10. A conductor 1.8 ft. long is perpendicular to a field of flux density equal to 6200 gauss. What is the force on the conductor when 8.35 amps flows through it?
11. A 7.5-inch wire is in a field of 5500 gauss. What current must flow if a thrust of 11 oz. is to be developed?
12. Explain the meaning of the B-H and μ-H curves of Fig. 17.
13. What would be the shape of a B-H for a nonmagnetic material such as air?
14. Explain the different sections of a hysteresis loop.
15. Define retentivity and coercive force from the viewpoint of a hysteresis loop.
16. Define hysteresis loss. Why is it an undesirable thing in most electromagnetic devices?
17. Name two factors that control the amount of hysteresis loss.
18. Define the Curie point.
19. State the general relationships between an electric field and a magnetic field.
20. Describe the energy interchanges that take place in the buildup and collapse of a magnetic field.

Chapter 5

INDUCED ELECTROMOTIVE FORCE

40. Electromagnetic Induction

In the past chapter, we were primarily concerned with the generation of a magnetic field by means of an electric field and motion. In this chapter, we will study the reverse effect—the generation of an electric field (and therefore an electromotive force) by means of a magnetic field and motion.

The generation of an electromotive force, abbreviated "emf," by the action of a magnetic field and motion is called "electromagnetic induction." The emf is called an "induced emf," or "induced voltage."

The basic discovery of electromagnetic induction was made independently and almost as the same time by Joseph Henry, an American, and Michael Faraday, an English physicist. They both announced it in 1831, a year which could well mark the beginning of our electrical and industrial age.

Electromagnetic induction can be illustrated by a simple experiment such as is illustrated in Fig. 19. A galvanometer (a current-indicating device) is connected in a closed loop with a straight length of wire XY. A bar magnet is nearby, as shown. With no motion, the meter will read zero, indicating no current flow and therefore no voltage present. This is the case in Fig. 19(A).

Now imagine that we push the magnet quickly toward the wire, as in Fig. 19(B). During the motion, the needle of the meter would deflect, let us say, to the right. It has indicated a current, and therefore a voltage must have existed during the motion between points X and Y.

As soon as the motion is stopped, the pointer returns to zero, indicating that the voltage no longer exists. Where did the voltage come from?

We say that it was induced by the action of *lines of force cutting the conductor.*

Now let us pull the magnet quickly away from the wire as in Fig. 19(C). Again the pointer will deflect, but this time to the *left.* This shows that the induced emf is of opposite polarity to the emf of Fig. 19(B). Once the motion stops, the pointer returns to zero.

If the magnet were turned around so that the south pole faced the wire, induced emfs would be detected on motion but opposite to those indicated when the north pole was used. Figures 19(D) and 19(E) show these effects.

If the magnet were moved back and forth at a steady rate, a little thought would show that the effect would be to generate an alternating current flow in the circuit. This is indeed the basis of electric power generators.

The experiments shown in Fig. 19 could be repeated, keeping the magnet stationary and moving the wire. We would find that an emf

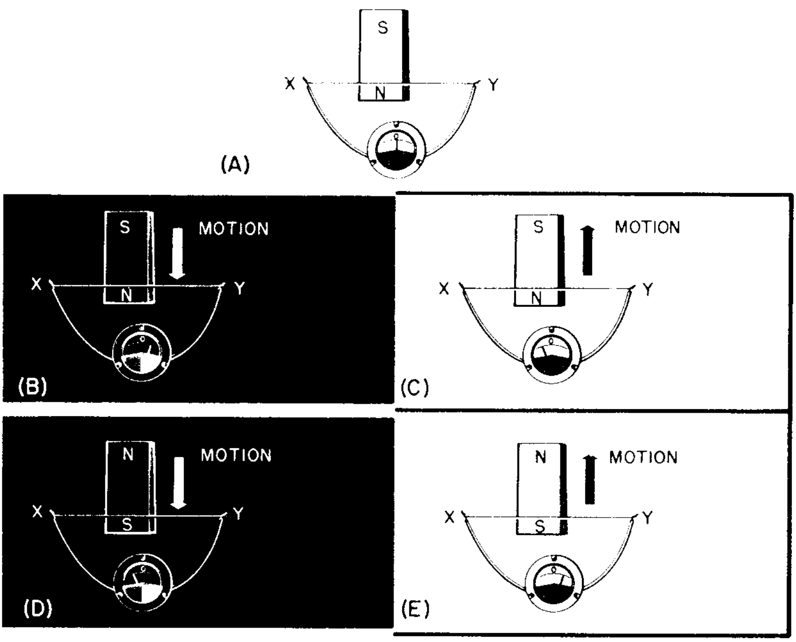


Fig. 19. (A) No motion and no induced emf. (B) An emf is induced between X and Y. (C) When motion is reversed, emf is reversed. (D) When magnetic field is reversed, emf is reversed. (E) When magnetic field is reversed, emf is reversed.

would be generated in each case. As long as lines of force are cut by a conductor, an emf is generated, regardless of whether the conductor, the field, or both are moving.

The fact that electric energy is produced in no way weakens the magnet. The energy comes from whatever is doing the moving—muscular energy, steam turbine, water power, etc.

41. Right-Hand Generator Rule

There is evidently a definite direction associated with the induced emf and, from Fig. 19, we can deduce that it depends on the direction of motion and the direction of the magnetic field.

The direction of the magnetic field is readily established regardless of whether the field comes from a permanent magnet or an electromagnet. Direction of motion, however, is an ambiguous phrase. Since motion is relative, and since either the field or conductor, or both, may be moving (in the case of both moving, the motion may be in the same or opposite directions or at some angle), there must be some definition. We will define the direction of motion as the conductor is moving with respect to the field. This movement may be actual or apparent.

For example, in Fig. 19(B), the magnet is moving downward toward the conductor. The conductor is actually stationary. However, to comply with our definition, the relative motion of the conductor with respect to the field is *upward.* Similarly in Fig. 19(D), the motion is considered to be upward, whereas in (C) and (E), the relative motion of the conductor is downward.

The phrase "direction of the induced emf" must likewise be defined. We will define it as the electric polarity set up by the observed electron current flow in the conductor.

To develop a rule relating emf, motion, and flux, we set up a system as shown in Fig. 20. In 20(A) we see the cross section of a conductor in a magnetic field. In 20(B) we see the result of a downward thrust on the conductor. The result is an electric polarity which drives electrons into the paper. The side of the wire away from us will develop the negative polarity of the induced emf, since the electrons pile up there. The side of the wire toward us will be the positive end of the emf.

Figure 20(C) shows the result of an upward thrust on the wire. The current now comes out of the paper—a reverse polarity of the emf. Figures 20(D) and 20(E) illustrate the effect of reversing the magnetic field. A reversal of the induced emf is, of course, obtained.

A number of rules can be established for determining the direction of the induced voltage. The one we will give is a right-hand rule. This has the virtue of being very similar to the left-hand motor rule and therefore easier to remember. It is called the right-hand *generator* rule and is stated as follows.

Using the right hand, point the *M*iddle finger in the direction of the *M*agnetic field and the *TH*umb in the direction of the *TH*rust or

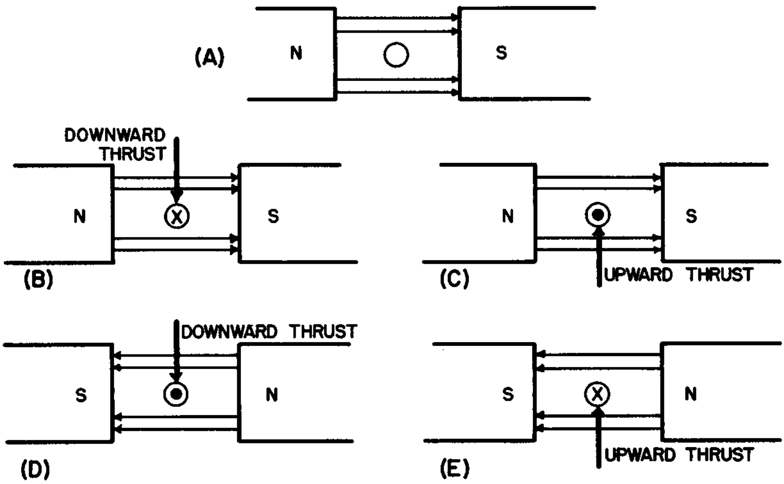


Fig. 20. (A) No motion and no induced emf or current. (B) Direction of induced current as a result of downward motion. (C) Direction of induced current as a result of upward motion. (D) Effect of reversing direction of field. (E) Effect of reversing direction of field.

motion of the conductor with respect to the field. The *I*ndex finger will then point in the direction of the induced current flow (I).

Practice the application of this rule by checking the various parts of Fig. 20.

42. Lenz' Law

We must now consider another effect that occurs whenever a magnetic field induces an emf and an induced current flows. As soon as the induced current starts to flow, a magnetic field is set up around the conductor in accordance with the principles we have already studied.

We now have the familiar situation of the last chapter—two magnetic fields. We have seen that the result of two such fields is a force on the conductor that tends to produce motion. Note that the very act of *moving* a closed-circuit conductor through a magnetic field generates a motion-producing force.

The critical question now arises. Is this new force in the same direction as the original force producing the original motion or is it in the opposite direction? We can answer this question philosophically with a simple logical argument.

Assume that the new force produces motion in the same direction. The conductor would then have a greater downward thrust (if that is the way we started), more emf induced, more induced current flow, still more generated force, more downward thrust, etc. After a time, we could remove the original force and the device would drive itself through the magnetic field forever, generating voltage. This is a perpetual-motion machine, a device frowned upon by nature. Since it violates at least half a dozen physical laws, we must reluctantly concede that it cannot exist.

This compels us to accept the only other alternative, i.e., that the generated force produces motion *opposite* the motion already under way.

This principle was first established by the Russian experimenter, Emil Lenz, and bears his name. Lenz' Law may be stated as follows: the direction of an induced current is always such that its magnetic field tends to oppose the motion that has produced it.

It is to overcome the effect of Lenz' Law that generators must be *driven*. The more current taken from a generator, the greater the opposing force developed [equation (8)] and the harder the machine that drives the generator must work. The opposing force in a generator is often called the motor action of a generator.

Just as generators have motor action, so do motors have generator action which opposes the process that drives the motor. According to Oersted's experiment, a current-carrying conductor in a magnetic field moves. By this very motion, the conductor cuts lines of force, and an induced emf must be established. What is the direction of this generated emf? Right. Lenz' Law tells us that it must apply *opposition* to the current originally flowing in the conductor.

Lenz' Law is, thus, seen to be a generalized rule of opposition in electromagnetic processes.

43. Factors Determining the Magnitude of the Induced emf

Let us now consider the question of the magnitude of the induced emf when a conductor cuts lines of force. Evidently the flux density and the length of the conductor should be controlling factors. This would make the speed or velocity of the conductor important. Faraday investigated the effect of these factors and arrived at a mathematical formulation which relates them to the induced voltage.

44. Faraday's Law

One simple form of Faraday's Law is

$$E = Blv \times 10^{-8} \tag{9}$$

where B is flux density in gauss, l is length of wire in cm, v is the velocity of the wire in cm/sec in a direction perpendicular to the direction of the field, and E is the induced emf in volts.

Problem 12. A conductor 0.55 meter long is moved with a uniform velocity of 2.3 meters per second through a field of flux density of 1200 gauss. Find the induced emf.

Solution. Equation (9) is used directly, but meters must be converted into centimeters by multiplying by 100.

$$E = Blv \times 10^{-8} = 1200 \times 55 \times 230 \times 10^{-8}$$

$$= 0.152 \text{ volt}$$

Problem 13. A conductor 14 in long is moving with a velocity of 810 cm/sec through a magnetic field. What is the flux density if 0.825 volt is generated by this action?

Solution. We must solve equation (9) for B and then put in the known values, remembering to convert inches to centimeters.

$$B = \frac{E}{lv \times 10^{-8}} = \frac{0.825}{14 \times 2.54 \times 810 \times 10^{-8}} = 2860 \text{ gauss}$$

Another form of Faraday's Law, closer to the way he actually proposed it (which was as a differential equation involving calculus), can be readily developed. If we remember that velocity is distance divided by time, or

$$v = d/t$$

we can rewrite equation (9) as

$$E = \frac{Bld \times 10^{-8}}{t}$$

However, the *area* A through which the wire travels is the product of l and d, or

$$A = ld$$

Substituting A for ld in the equation for E, we get

$$E = \frac{BA \times 10^{-8}}{t}$$

However,

$$\Phi = BA$$

Substituting, we get another form of Faraday's Law,

$$E = \frac{\Phi}{t} \times 10^{-8} \qquad (10)$$

where E is induced emf in volts, Φ is total magnetic flux cut in maxwells, and t is the time required to cut the flux in seconds.

Problem 14. A conductor cuts a flux of 60,000 maxwells in 0.2 second. Find the induced voltage.

Solution. Substituting directly,

$$E = \frac{\Phi}{t} \times 10^{-8} = \frac{60{,}000}{0.2} \times 10^{-8} = 0.003 \text{ volt}$$

Problem 15. In what time must a conductor pass through a flux of 150,000 maxwells in order to generate a voltage of 0.250 volt?

Solution. Solving equation (10) for t we get

$$t = \frac{\Phi}{E} \times 10^{-8} = \frac{150{,}000}{0.250} \times 10^{-8} = 0.006 \text{ second}$$

45. Mutual Induction

Another aspect of electromagnetic induction is shown in Fig. 21. This is called "mutual induction" and is the basis of transformers. In Fig.

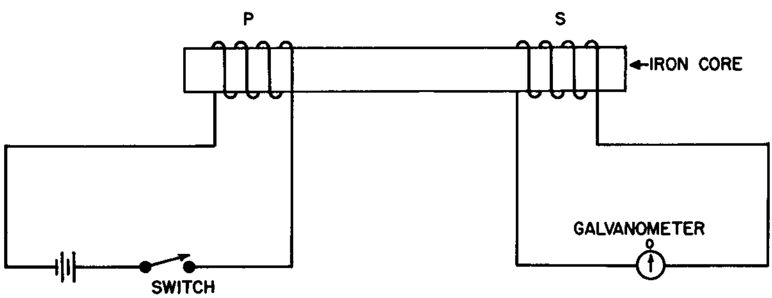


Fig. 21. Mutual induction.

21, with the switch open on coil P, no current flows in P and no current exists in coil S. When the switch is closed, current starts in P and its magnetic field grows to its steady-state value. During the time of growth, the field is moving and its lines of force cut S, inducing an emf. This causes the meter pointer to swing, say, to the right.

As soon as the field reaches its steady value, there is no longer any motion, no lines of force are cut by the wires of S, and the induced

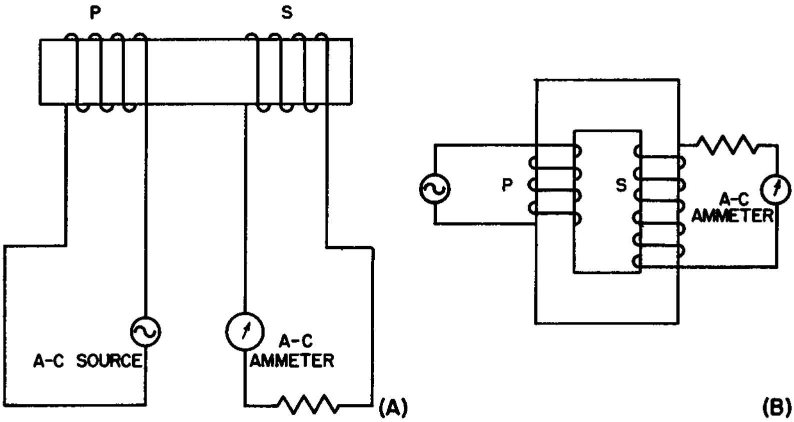


Fig. 22. (A) A transformer based on Fig. 21. (B) Another transformer shape.

emf becomes zero again. The pointer on the meter returns to zero. As long as the current in P remains at the same level, no induced emf will exists in S.

Now the switch is opened. The lines of force collapse and, in so doing, they again cut the conductors in S. The relative motion is now in the opposite direction to that when the lines were expanding. An emf will be induced in S in the opposite direction, and the pointer will deflect to the left.

When all of the lines have collapsed, the induced emf becomes zero, and the pointer returns to the center. It is only during periods of growth or decay of the magnetic field that mutual induction occurs.

The coil P is called the "primary" winding and the coil S the "secondary" winding. When connected as in Fig. 22(A), the device is called a "transformer." In Fig. 22(A), with an a-c applied to the primary, the magnetic field will be constantly growing, decaying, and reversing. There will thus be continual cutting of the lines of force by the secondary windings and an induced alternating emf will be generated.

Figure 22(B) shows another shape that a transformer may take.

46. Review Questions

1. Describe and illustrate an experiment that will demonstrate the existence of electromagnetic induction.
2. Why is there no weakening of magnetic strength during the process of inducing voltages, even when a current flow occurs?
3. If both a conductor and a field were moving in the same direction, with the field moving more rapidly, what direction of motion would be taken for finding the direction of the induced emf? Show your answer by a drawing.
4. Define the phrase "direction of the induced emf."
5. State the right-hand rule for generators. Draw two illustrations of its use.
6. State Lenz' Law as applied to a generator.
7. State Lenz' Law as applied to a motor.
8. Explain the reason why more power must be used to drive a generator that is under load than one not under load.
9. Explain the reason why more current must be fed to a motor when its load increases.
10. A 15-in wire is moved with a speed of 5 m/sec through a flux density of 1750 gauss. Find the induced emf.
11. A conductor moving through a flux density of 2200 gauss with a velocity of 3.6 ft/sec generates a voltage of 0.230 volt. What is the length of the conductor?
12. What emf is generated by a wire cutting a flux of 85,000 maxwells in 1/8 sec?
13. A wire generates a voltage of 0.062 volt in passing through a field in 3/8 sec. What is the total flux passed through?
14. What is meant by mutual induction?
15. Explain the principle of the transformer.

Chapter 6

INDUCTANCE

47. Self Induction

In the last chapter, we discussed an effect that we called mutual induction. We saw that a growing or decaying magnetic field could induce an emf in a nearby conductor. The transformer was developed from that principle.

Let us extend this further. Consider a solenoid connected through a switch to a battery. When the switch is closed, the magnetic field starts to grow. However, in so growing, the lines of force cut the conductors that make up the solenoid. In that case, an induced emf *must* be generated right within the turns of wire of the coil itself.

What about the direction of this new emf? According to Lenz' Law, it must be in such a direction that it *opposes* the change that produced it. What was the change that produced it? In this case, we had closed a switch and the current was trying to rise from zero to some value. The induced emf must, then, be in such a direction as to oppose the rise in current. It is therefore opposite to the applied voltage and is called a "counter voltage" or "counter emf." Counter emf is abbreviated to *cemf.*

This effect can be demonstrated by a circuit such as that shown in Fig. 23. At the moment the switch is closed, the lamp burns brightly. In a short time, it will become dim and will remain that way for as long as the switch stays closed. When the switch was closed, the cemf of the solenoid kept the current through it low. Most of the current then passed through the lamp, lighting it brightly. In a short time, the current in the coil reached a stable value, no further cemf was induced, and the coil became a shunt across the lamp, diverting most of the current from it.

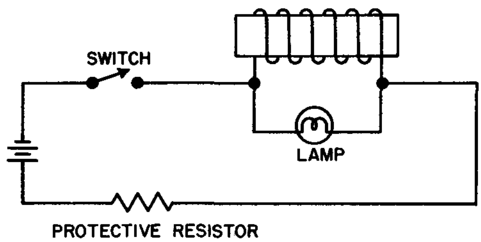
As soon as the current in the solenoid reaches a steady d-c value, the counter emf ceases and there is no further opposition to the current.

The property of inducing a cemf within a device is called "self induction." The property is only present when there is a *changing* current and is absent under d-c conditions.

Returning to Fig. 23, with a steady d-c, the lamp is dim. Now let us open the switch quickly. The lamp will flash brightly for a moment and then go dark.

In opening the switch, we have tried to stop the current flow in the solenoid. This caused the magnetic field around the coil to collapse. In cutting the conductors of the coil, they induced an emf. In what

Fig. 23. A demonstration of self inductance.



direction? Since the change that produced the emf was a reduction in current, the emf must be in such a direction as to try to *continue* the current flow. The collapsing field kept up the current flow, and, since the switch was open, the current flowed through the lamp, lighting it brightly.

The counter emf is always in such a direction as to oppose the change that produces it. The self induction of a device always *opposes the change* in current that produces it.

48. Inductance

It has been shown experimentally and mathematically that the counter emf is proportional to the rate of change of the current and a physical property of the device. Expressed in equation form

$$E = -L \frac{\Delta I}{t} \qquad (11)$$

where E is the self induction in volts. L depends on the nature of the device and is called inductance. Its unit is the henry. ΔI (read delta I) is the *change* in I in amps and t is the time in seconds taken by the current to change.

Let us examine this equation. $\Delta I/t$ represents the *rate* of change of the current. If a current can be made to change very rapidly, as by suddenly opening a switch, very high voltages may be induced. The minus sign in the right side of the equation only serves to remind us that this is an opposing or counter emf. We generally neglect this minus sign in actual computations. We shall do so in our problems.

The quantity L is a new concept. L is the "coefficient of self induction" or simply the "inductance" of a device. It depends on the physics of the coil. The larger the inductance, the greater the cemf built up for the same rate of change of current. Inductance is given the unit *henry.* The henry is defined as follows: a circuit has an inductance of 1 henry if a current changing at a rate of 1 amp/sec induces a cemf of 1 volt.

All circuit elements, including short straight pieces of wire, have some properties of inductance. However, devices where this is the major property, such as chokes, solenoids, and transformers, are called "inductors." An inductor is said to be "inductive" in its circuit, just as resistors are resistive and capacitors are capacitive.

Problem 16. A choke has an inductance of 20 henries and carries a current of 120 milliamps. When the switch is opened, the current drops to zero in 0.05 sec. How large an emf is induced?

Solution. We substitute directly in equation (11), omitting the minus sign as it has no meaning as far as the magnitude of the induced voltage.

$$E = \frac{L\Delta I}{t} = \frac{20 \times 0.120}{0.05} = 48.0 \text{ volts}$$

Problem 17. In the choke of problem 16, the steady current flow was made 700 milliamps. When the switch was opened, the current dropped to zero in 0.015 second. How large is the induced emf?

Solution.

$$E = \frac{L\Delta I}{t} = \frac{20 \times 0.700}{0.015} = 933 \text{ volts}$$

Problem 18. In a flyback power supply (used to develop large voltages), the current drops to zero in a 50-μsec period. What current must be used in order to develop 6000 volts if the inductance is 4.5 henries?

Solution. We must solve equation (11) for ΔI.

$$\Delta I = \frac{Et}{L} = \frac{6 \times 10^3 \times 50 \times 10^{-6}}{4.5} = 66.7 \text{ milliamps}$$

It is inadvisable and often dangerous to open an inductive circuit too quickly. The very high induced voltages that may result can puncture insulation, cause sparks and arcs, and otherwise damage equipment.

49. Growth and Decay of Currents in an Inductive Circuit

When a switch is closed on a d-c circuit containing only resistors, the current may be considered to rise from zero to its maximum value in practically zero time. No physical event can take place in zero time, but, in a "pure" resistive circuit, we can assume that no time elapses. Figure

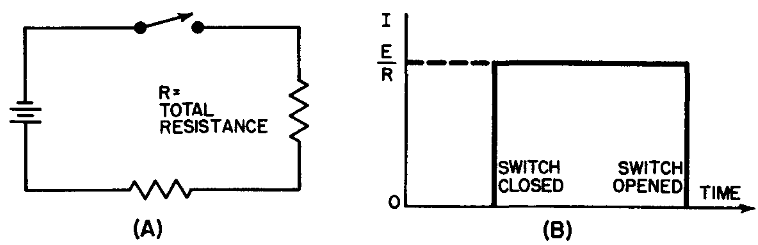


Fig. 24. (A) A "pure" resistive circuit. (B) The graph of I versus Time for a resistive circuit.

24(A) shows such a "pure" resistive circuit, and Fig. 24(B) is a graph of current versus time for this circuit.

When the switch is closed, the current rises to E/R instantly. It stays at that level all during the time the switch stays closed. When the switch is opened, the current instantly drops to zero.

Now let us consider the case where a circuit contains both resistance and inductance. The resistance either may come from a separate resistor or it may simply be the resistance of the wires making up the inductor. Such a circuit is shown in Fig. 25. Note the symbol for an inductor and the fact that its property of inductance is given the symbol L.

Now suppose that the switch is closed on this R-L circuit. The current cannot *instantly* rise to its full value because of the inductance of the circuit. The induced counter emf opposes the current rise so that, instead of an instant jump to full value, there is a relatively slow rise to the full value. Figure 26 shows the shape of this rise.

Once the current has reached its E/R value, L has no effect at all on the circuit. The curve of current versus time for this interval is a straight line.

Now let us open the switch. As explained previously, it is not advisable to try to open a highly inductive circuit. A high induced voltage will form, and the energy in the magnetic field will be relieved by

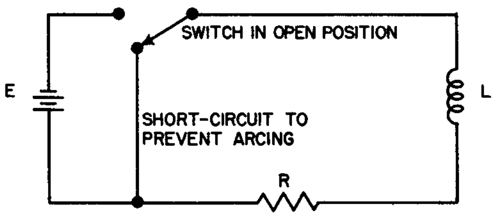


Fig. 25. A series R-L circuit.

arcing or puncturing insulation. Note that, by opening the switch in Fig. 25, we do not open the inductive circuit. We simply remove the battery from the circuit.

On opening the switch, we attempt to stop the current flow in L. This sets up a cemf which attempts to continue the current flow in the same direction. In effect, the energy in the magnetic field is used up as heat (I^2R losses) in the resistor. The current, instead of instantly dropping to zero, decays slowly to zero. Figure 26 shows the shape of this decay. The decay curve and the rise curve are the same shape. One is just the reverse of the other.

This curve is another illustration of the effect of an inductor in a circuit. It acts to oppose any change in the current.

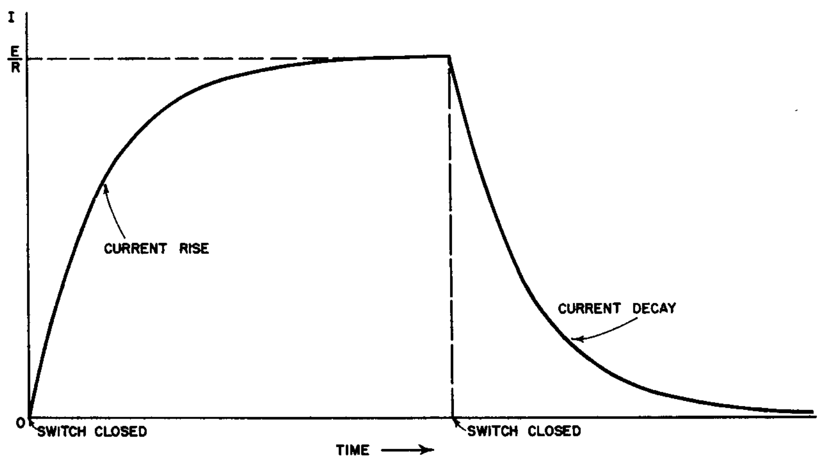


Fig. 26. Curve shows rate of current rise and current decay when switch is closed and open.

50. Concept of Time Constant

An interesting question is that of the actual time consumed for a current in an R-L circuit to rise to maximum and to decay from maximum to zero. Unfortunately, a full analysis involves difficult mathematical procedures. However, certain simplified results can be used to give better insight into the nature of an R-L circuit.

If we were to take the circuit of Fig. 25 and divide the total number of henries by the total number of ohms of resistance, the quotient would be a number called "the time constant" of the circuit. Its unit would be *seconds*. In equation form we may write

$$T = L/R \tag{12}$$

where L is inductance in henries, R is resistance in ohms, and T is time constant in seconds.

Problem 19. Find the time constant of a circuit whose inductance is 5 henries and resistance is 150 ohms.

Solution. $T = L/R = 5/150 = 0.0333$ second

The importance of the time constant is that it enables us to calculate the nature of the rise and decay of the current. After a time equal to one time constant from the closing of the switch, the current in an R-L circuit has risen to approximately 63% of its maximum value. After a time of two time constants, the current has reached about 87% of maximum. In three time constants, about 95% of maximum has been attained. After *five* time constants, it is considered that maximum current has been reached.

During the current decay period, similar calculations can be made. It is only necessary to subtract the percentages for the rise period from 100. During the time of one time constant after opening the switch, the current has fallen to about 37% (100−63) of maximum. After two time constants, the current is down to about 13% of maximum. After three time constants, it is down to about 5% of maximum. The current is considered to have reached zero after five time constants.

Problem 20. In the circuit of problem 19, the applied voltage is 300 volts. What is the value of the current after periods of one, two, three, and five time constants?

Solution. The maximum current of the circuit is given by Ohm's Law.

$$I = E/R = 300/150 = 2 \text{ amps}$$

We have already found the time constant to be 0.0333 second. After one time constant, or 0.0333 second, the current will be

$$I_1 = 0.63\ I = 0.63 \times 2 = 1.26 \text{ amps}$$

After two time constants or 0.0667 second,

$$I_2 = 0.87\ I = 0.87 \times 2 = 1.74 \text{ amps}$$

After three time constants or 0.100 second,

$$I_3 = 0.95\ I = 0.95 \times 2 = 1.90 \text{ amps}$$

After five time constants or 0.167 second, the current is 2 amps.

Problem 21. In the circuit of problem 20, the switch is opened after the current has reached maximum value. Find the values of the current after periods of one, two, three, and five time constants.

Solution. After one time constant or 0.0333 second, the current will be

$$I_1 = 0.37\ I = 0.37 \times 2 = 0.74 \text{ amp}$$

After two time constants or 0.667 second,

$$I_2 = 0.13\ I = 0.13 \times 2 = 0.26 \text{ amp}$$

After three time constants or 0.100 second,

$$I_3 = 0.05\ I = 0.05 \times 2 = 0.10 \text{ amp}$$

After five time constants or 0.167 second, the current has become zero.

The relationship between the rise or decay of current in a circuit containing resistance and inductance in series can be summarized in a graph called a "Universal Time Constant Curve." Such a graph is shown in Fig. 27. The solid line represents the rise of current in such a circuit, and the dashed line indicates the nature of the current decay.

The X axis shows time in L/R units. The number 1 on this axis is, thus, a period of time equal to one time constant. For example, in the problems just worked out, one time constant was equal to 0.0333 second. Where R and L have different values, the number 1 would represent some other period of time, whatever the ratio L/R happened to be. The numbers 2, 3, 4, and 5, of course, stand for 2, 3, 4, or 5 time constant periods.

The Y axis indicates percent maximum current. Maximum current, we have already seen, is E/R.

You will note that the curve shows the various percentages that we have already established for 1, 2, 3, and 5 time constants, both for rise

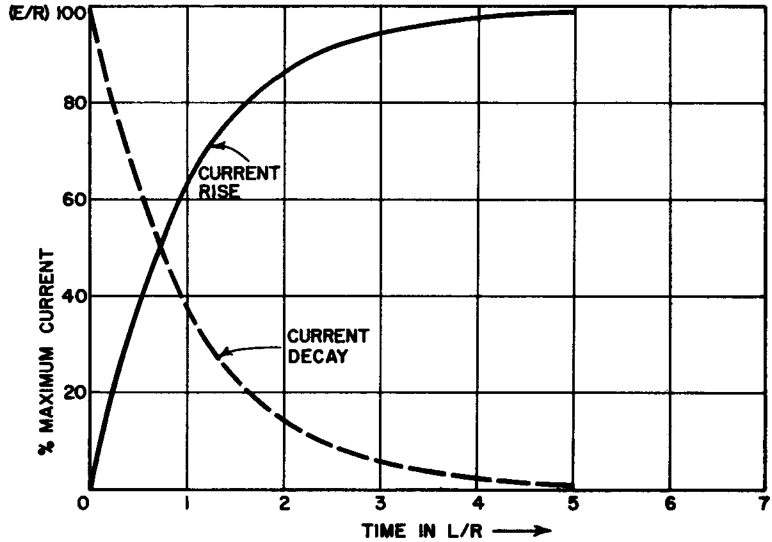


Fig. 27. Curves show current rise and current decay as a function of L/R.

and decay of current. Its usefulness is that it enables us to read in-between values of time and current.

Problem 22. A series circuit is made up of a resistor of 120 ohms and an inductor of 8.5 henries. When a switch is closed, a voltage of 6.3 volts is applied. What is the current at 0.100 second after the closing of the switch?

Solution. Find the time constant of the circuit.

$$T = L/R = 8.5/120 = 0.0708 \text{ second}$$

At a time of 0.100 second, the number of time constants that have elapsed is given by

$$0.100/0.0708 = 1.41 \text{ time constants}$$

Finding this value on the X axis of the rise curve, we see that it corresponds to about 77% of maximum current. The current flowing at this moment will then be

$$I = 0.77\ E/R = 0.77 \times 6.3/120 = 0.0405 \text{ amp}$$

$$= 40.5 \text{ milliamps}$$

Problem 23. In the circuit of problem 22, the steady d-c state had been reached. The battery is then switched out of the circuit, leaving only L and R. How much time will elapse before the current decays to 25 milliamps?

Solution. We first find the maximum or steady d-c current. Call it I_M.

$$I_M = E/R = 6.3/120 = 52.5 \text{ milliamps}$$

Now find the percent of this maximum represented by 25 milliamps.

$$\% = 25/52.5\ (\times\ 100) = 47.6\% \text{ of maximum current}$$

We locate this percentage on the Y axis and read across to the decay curve. We read about 0.77 of one time constant. Since one time constant has already been found to be 0.0708 second, the elapsed time will be

$$t = 0.77 \times 0.0708 = 0.0545 \text{ second}$$

$$= 54.5 \text{ milliseconds}$$

It is interesting to note that the universal time constant curve of Fig. 27 is also applicable to a circuit containing resistance and capacitance in series. The same concepts apply with the difference that the time constant becomes the product of R and C.

51. Energy Stored in an Inductor

In a number of ways now we have seen that the magnetic field around a solenoid, or any inductor, represents stored energy. This energy is supplied by the electric circuit during the rise time of the current. During the steady d-c current flow, the energy is stored. When the current starts to decay, the inductor returns part or all of the energy to the electric circuit.

A simple equation gives us the energy stored in the field around an inductor.

$$W = \frac{1}{2} LI^2 \qquad (13)$$

where W is the energy in joules, L is the inductance in henries, and I is the steady d-c (maximum) current in amps.

Problem 24. An inductance of 10 henries is in series with a resistance of 5 ohms. Two seconds after voltage is applied to the circuit, a current of 2.25 amps is measured. How much energy will be stored by the coil at steady state?

Solution. First we find the time constant

$$T = L/R = 10/5 = 2 \text{ seconds}$$

The 2-second period in the problem is thus one time constant. For a current rise, we have learned that in one time constant period, the current rises to about 63% of maximum current I. Therefore,

$$0.63\ I = 2.25$$

$$I = 2.25/0.63 = 3.58 \text{ amps}$$

Now the energy stored is found,

$$W = \frac{1}{2} LI^2 = 0.5 \times 10 \times (3.58)^2 = 64.1 \text{ joules}$$

Inductors are made commercially either as air-core or as iron-core inductors. We have already discussed one disadvantage of an iron core —the hysteresis loss. This made the use of conventional iron cores impractical at higher frequencies.

52. Eddy Currents

Another disadvantage will be discussed now. Since iron is an electrical conductor, the iron core inside an inductor is effectively the secondary of a transformer. The coil winding acts as the primary. The lines of force from the coil cut the iron of the core and induce a voltage. This voltage causes a current flow within the core itself. This current is called an "eddy current." It circulates, or "eddies" around and around the core, producing I^2R losses. These losses subtract from the efficiency of the inductor, and, at higher frequencies, become intolerable.

Iron cores are generally made of thin strips called "laminations." In this way, the resistance of the core is increased, and eddy currents are reduced. Powdered iron, where each grain is electrically insulated from every other grain by a coating, may be used as core material, even up to the i-f and r-f frequencies.

In general, however, iron-core coils are used at power and audio frequencies, whereas air-core coils are used at the higher frequencies.

53. Types of Inductors

Some commonly used inductor symbols are shown in Fig. 28. Variable inductors of air-core design may have fixed taps which may be wired to individually or selected by a switch, or may have a slider which rides on the wire. Iron-core inductors may be further varied by sliding the core in and out of the solenoid. This type of variation is called "permeability tuning" and is commonly used in i-f tuning devices in radio and television.

A name often used for an inductor in electronics is the word "choke." Thus we speak of an "audio choke" for an inductor used at audio frequencies. Similarly, "r-f choke" is used for coils for r-f applications. Radio-frequency chokes are generally in the microhenry and millihenry range. Audio chokes run in the fractional-henry and henry range. Chokes used as filters in power supplies may go into the tens-of-henries range.

54. Factors Affecting Inductance

A number of factors affect the inductance of coils. No simple formulas exist for calculating it from the physical makeup of the coil. Tables,

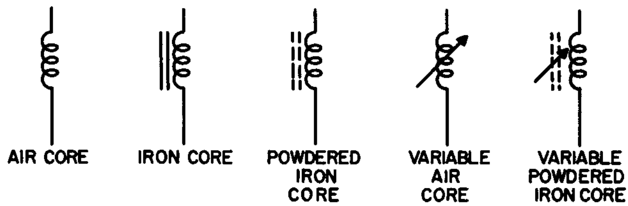


Fig. 28. Schematic symbols for various types of inductors.

based on experience, are most often used for the construction of commercial inductors. However, some general principles do hold concerning some factors that affect inductance.

In general, the inductance of a coil varies as the square of the number of turns. Doubling the number of turns will increase the inductance

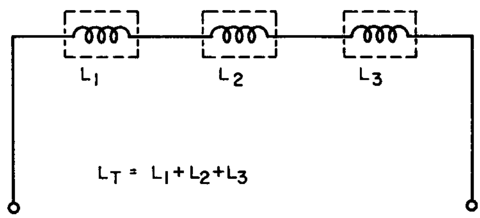


Fig. 29. Three inductors in series. No mutual inductance.

by four times. Taking off one-third the number of turns of a coil would decrease its inductance to one-ninth its original value.

Inductance depends on the permeability of the core. This factor is obvious from our discussion of the meaning of inductance and permeability.

Inductance increases with the square of the diameter of the coil. This can also be stated by saying that inductance increases with the cross-sectional area of the coil.

Inductance decreases with the length of the coil. Coil lengths primarily depend on the size of the wire with which the coil is wound. Using thinner wire (if it can carry the current) will give a shorter coil for the same number of turns and, hence, a greater inductance.

55. Inductors in Series and in Parallel

It is often necessary to use more than one inductor in a circuit. In that case, we may find series and parallel connections of inductors. Several formulas exist that enable us to calculate the total inductance when more than one inductor is used.

Let us take first the case of inductors in series. Let us say, further, that the magnetic field of no inductor cuts any other inductor. Figure 29 shows three such inductors. Magnetic shields are shown around each inductor. In the last chapter, we used a term—mutual inductance—to describe the condition when the field of one coil cut a second coil and induced a voltage in the second coil. Following this idea, we can say that Fig. 29 shows three inductors with no mutual inductance.

In such a case, inductances add in exactly the same fashion as series resistances. This effect is written as a simple equation.

$$L_T = L_1 + L_2 + L_3 \tag{14}$$

Any additional series inductor would simply be added on in the same fashion.

56. Coupling

More often, there is not complete magnetic isolation between inductors in the same circuit. When some lines of force from one coil link a

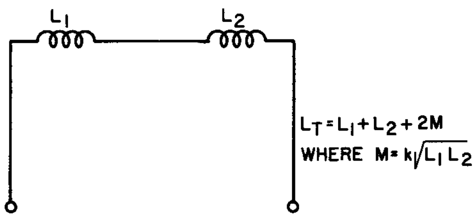


Fig. 30. Two inductors in series with mutual inductance and fields aiding.

second coil, we say that the coils are "coupled." To be able to use the idea of coupling mathematically, we define a concept called "the coefficient of coupling"—symbol k. We say $k = 1$ when *all* the lines of force from one inductor link the second inductor. If only half the lines link, k = 0.5. If a quarter of the lines link, k = 0.25, etc. The ratio of the number of linked lines to the total number of lines available is represented by k.

Since we know k and know the inductances of two coils, we can find the mutual inductance between the coils. The relation is

$$M = k\sqrt{L_1 L_2} \qquad (15)$$

where M is mutual inductance in henries, k is the coefficient of coupling (no unit), and L_1 and L_2 are inductances in henries.

Problem 25. Two coils of 80 and 120 millihenries have a coefficient of coupling of 0.15. Find the mutual inductance of the coils.

Solution. Substitute directly into formula (15).

$$M = k\sqrt{L_1 L_2} = 0.15\sqrt{80 \times 10^{-3} \times 120 \times 10^{-3}}$$

$$= 0.15\sqrt{9600 \times 10^{-6}} = 0.15 \times 98 \times 10^{-3}$$

$$= 14.7 \text{ millihenries}$$

Now let us consider the circuit shown in Fig. 30. Two inductors are in series, and mutual inductance exists between them. In addition, their magnetic fields are in the same direction and are aiding each other. This arrangement is often called "series-aiding." In that case, the total inductance must take the mutual inductance into account. The equation for two inductors in a series-aiding connection is

$$L_T = L_1 + L_2 + 2M \qquad (16)$$

Problem 26. Find the total inductance of the two coils of problem 25, if they are in series-aiding connection.

Solution.

$$L_T = L_1 + L_2 + 2M = 80 + 120 + (2 \times 14.7)$$

$$= 200 + 29.4 = 229.4 \text{ millihenries}$$

It is also possible to connect two inductors in series in such a way that their magnetic fields oppose each other. This connection is called "series-opposing." The equation for the total inductance in a series-opposing circuit for two inductors is

$$L_T = L_1 + L_2 - 2M \qquad (17)$$

Problem 27. Find the total inductance of the two coils of problem 25, if they are connected series-opposing.

Solution.

$$L_T = L_1 + L_2 - 2M = 80 + 120 - (2 \times 14.7)$$
$$= 200 - 29.4 = 170 \text{ millihenries}$$

Figures 31 shows a series-opposing connection for two inductors. When we get to three or more inductors in series-aiding or series-opposing, the computations become very difficult and involve difficult equations.

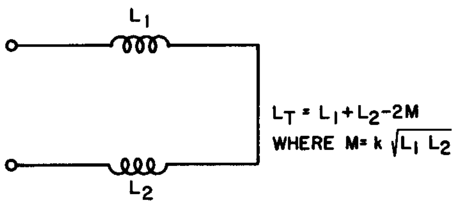


Fig. 31. Two inductors in series with mutual inductance and fields opposing.

Inductors may also be connected in parallel, as shown in Fig. 32. No mutual inductance is considered in this type of circuit, since such calculations are again quite complicated and beyond the scope of this book. Inductances in parallel follow the same type of equation for total inductance as we find in parallel resistances. A reciprocal relation holds and is given by

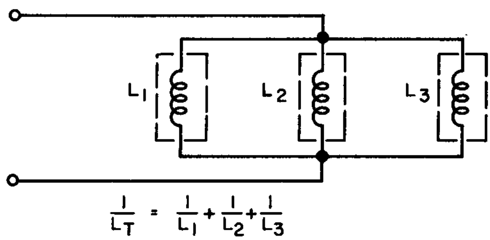


Fig. 32. Three inductors in parallel. No mutual inductance.

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \tag{18}$$

Problem 28. Three inductors of 4, 6, and 8 henries, respectively, are connected in parallel with no mutual inductance. Find the total inductance of the circuit.

Solution.

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} = \frac{1}{4} + \frac{1}{6} + \frac{1}{8}$$

Finding the least common denominator of the numerical fractions as 24, we write each fraction with a denominator of 24.

$$\frac{1}{L_T} = \frac{6}{24} + \frac{4}{24} + \frac{3}{24} = \frac{13}{24}$$

Taking the reciprocal of both sides we get

$$L_T = \frac{24}{13} = 1.85 \text{ henries}$$

57. Determination of Mutual Inductance

The calculation of the coefficient of coupling by direct means is usually not an easy matter. However, by using the principles of series-aiding and series-opposing, mutual inductance can be measured. Call L_a the total inductance of two coils in series-aiding. Then, by equation (16),

$$L_a = L_1 + L_2 + 2M$$

Similarly, if L_o is the total inductance of the same two coils in series-opposing, we can write, by equation (17),

$$L_o = L_1 + L_2 - 2M$$

Now subtract L_o from L_a and we get

$$L_a - L_o = 4M$$

Solving for M,

$$M = \frac{L_a - L_o}{4} \qquad (19)$$

This gives us a direct technique for measuring M and then calculating k. We need first to measure the total inductance of two coils in series-aiding. We then reverse the connections to the coils (now series-opposing) and again measure inductance. Equation (19) gives M. By knowing the individual inductances, we can then calculate k by equation (15).

Problem 29. The total inductance of two coils in series is found to be 650 millihenries. Reversing the leads to one coil gives a total inductance of 610 millihenries. The inductance of one of the coils is known to be 425 millihenries. Find the mutual inductance, the inductance of the second coil, and the coefficient of coupling.

Solution. First find M by equation (19).

$$M = \frac{L_a - L_o}{4} = \frac{650 - 610}{4} = \frac{40}{4} = 10 \text{ millihenries}$$

To find the second inductance, we solve equation (16) for L_2. Writing L_a instead of L_T, we get,

$$L_2 = L_a - L_1 - 2M = 650 - 425 - 20 = 205 \text{ millihenries}$$

Now to find the coefficient of coupling, we solve equation (15) for k. This gives us (expressing all values in henries)

$$k = M/\sqrt{L_1L_2} = 0.010/\sqrt{0.425 \times 0.205}$$
$$= 0.010/\sqrt{0.0870}$$
$$= 0.010/0.295 = 0.0339$$

58. Ignition Systems

A commonplace application of induction is in the ignition systems for gasoline engines in automobiles. The sudden collapse of a magnetic field is used to generate a high induced voltage. This induced voltage is high enough to cause a spark in a gap containing gasoline vapor and air. The spark ignites the mixture.

A simplified schematic of an automobile ignition system is shown in Fig. 33. One side of the car battery is connected to the engine frame

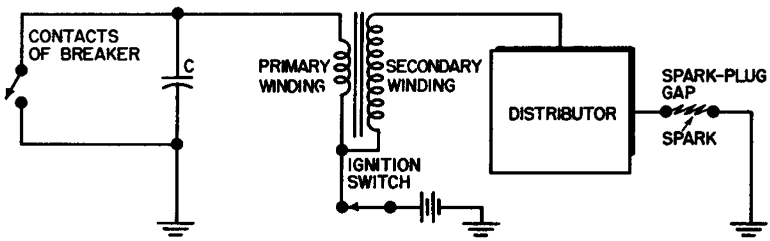


Fig. 33. A simplified automobile ignition system.

and is grounded. The other side of the battery goes to the ignition switch. The act of turning on the ignition closes this switch.

A circuit-breaker mechanism, not shown, contains a rotating cam which closes and opens the contacts of the circuit breaker. When the breaker contacts close, shown as a switch on the diagram, the primary winding of the ignition coil is across the battery through the ground connection. Now what happens when the contacts of the circuit breaker are opened suddenly?

The magnetic field around the primary suddenly collapses. To prevent sparking (and quick destruction) of the breaker contacts, a capacitor C, called a condenser in auto parlance, is placed across the contacts. The condenser charges and absorbs the energy of the primary. Later, when the breaker contacts close, the condenser discharges.

The rapid collapse of the magnetic field of the primary induces a high voltage in the secondary. The secondary winding is made up of a very large number of turns, giving us an additional voltage step-up action. As a result, the induced voltage in the secondary coil is 10,000 to 15,000 volts. The distributor delivers this high voltage to the correct spark plug. The voltage is high enough to produce a spark in the gap of the plug, and the gasoline vapor mixture is ignited.

59. Review Questions

1. Define self induction.
2. In what direction is the self-induced voltage of a coil?
3. Express formula (11) in words.
4. Define the unit, 1 henry, of inductance.
5. A coil has an inductance of 6.75 henries. A steady d-c of 1.55 amps flows through it. The current is then reduced to zero in 0.022 second. How large an emf is induced?
6. Why cannot a current rise instantly to its full value when a switch is closed on an inductive circuit?
7. Why is it poor technique to open an inductive circuit suddenly?
8. Show dimensionally that the time constant has the unit "second." Hint: go back to the basic definition of inductance in terms of units.
9. A circuit has a time constant of 0.00833 second. The resistance in the circuit is 1200 ohms. Find the inductance.
10. In a series R-L circuit, L = 950 millihenries and R = 20 ohms. A switch applies 5 volts across the circuit. What is the current after a period of two time constants? How long a period of time does this represent?
11. The voltage is removed from the circuit of question 10 and the current decays from its steady-state value. What is the current after three time constants?
12. Use the Universal Time Constant Curve to find the current for the circuit of question 10 at a time 0.025 second after the voltage is applied.
13. Use the Universal Time Constant Curve to find the current for the circuit of question 10 at a time 100 milliseconds after the voltage is removed.
14. When is energy supplied to the magnetic field of an inductor?
15. An inductance of 3.50 henries is in series with a resistance of 180 ohms. After a period of one time constant, a current of 0.925 amp is measured. What will be the energy in the magnetic field at steady state?
16. Describe the nature of eddy-current losses in cores. What may be done to reduce these losses?
17. Describe the nature of inductors for use at low and high frequencies.
18. Describe the factors that determine the inductance of a coil.
19. Two inductors with inductances of 2.65 and 7.72 henries, respectively, are connected in series-aiding with a coefficient of coupling of 0.335. Find the total inductance of the pair.

20. Find the total inductance of question 19, if the connections to one inductor were reversed.
21. Find the total parallel inductance of three inductors of 5, 15, and 25 henries, respectively.
22. The total inductance of two coils in series is found to be 5.35 henries. When the lead to one coil is reversed, the total inductance measures 6.17 henries. One of the coils is found to be 1.85 henries. Find the mutual inductance, the inductance of the second inductor, and the coefficient of coupling.
23. Explain the action of the ignition system of an automobile gasoline engine.

APPENDIX 1

Quantity	Symbol	CGS Unit	MKS Unit
Flux	Φ	maxwell	weber
		1 weber = 10^8 maxwells	
Flux density	B	gauss (maxwell/cm^2)	weber/meter^2
		1 weber/meter^2 = 10^4 gauss	
Magnetomotive force	M	ampere turns (or gilbert)	ampere turns
Field intensity	H	oersted (ampere turns/cm)	ampere turns/meter
		1 ampere turn/meter = 100 oersteds	
Reluctance	$\mathcal{R}$	rel	no unit
Permeability	μ	Relative permeability is generally used. This is the ratio of the permeability of a material to the permeability of air or space. No dimensions exist for relative permeability. The permeability of air is taken as 1.	webers/ampere-meter The permeability of space μ_0 is equal to 1.257×10^{-6} webers/ampere-meter

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