## Cyclopedia of

# Applied Electricity 

## A General Reference Work on

Direct-Current Generators and Motors, Alternating-Current Machinery, Armature Winding, Storage Batteries, Interior Electric Wiring, Electric Lighting, Meters, Industrial Controllers, Electric Railways, Railway Signaling and Car Lighting, Power

Stations, Switchboards, Transmission and Distribution
Systems, Electro-Chemistry, Welding, Etc.

## Illustrated With Over Two Thousand Engravings EIGHT VOLUMES




## Authors and Collaborators

FRANCIS B. CROCKER, M.E., Ph.D.<br>Formerly Vice-President, Crocker-Wheeler Company, Ampere, New Jersey<br>Formerly Professor of Electrical Engineering, Columbia University, New York<br>Past-President, American Institute of Electrical Engineers<br>Joint Author of "Management of Electrical Machinery," "Electric Motors," "Direct- and Alternating-Current Machine Design"

## f

ROBERT ANDREWS MILLIKAN, Ph.D., Sc.D.
Director, Norman Bridge Laboratory of Physics, California Institute of Technology, Pasadena, California
Author of "Mechanics, Molecular Physics, and Heat"
Co-Author of "First Course in Physics," "Electricity, Sound, and Light," "A Study Course in Practical Physics"
Member, American Physical Society, American Institute of Electrical Engineers, National Academy of Sciences

## $;$

WILLIAM ESTY, S.B., M.A.
Professor of Electrical Engineering, Lehigh University Joint Author of "The Elements of Electrical Engineering" Fellow, American Institute of Electrical Engineers

## ¢

HARRY BARNES GEAR, A.B., M.E.
Assistant to Vice-President, Commonwealth Edison Company, Chicago Fellow, American Institute of Flectrical Engineers, Western Society of Engineers, Illuminating Engineers Society
Joint Author of "Electric Central Station Distribution Systems"; "Electric Motors"'

## $\$$

## DAVID P. MORETON

Associate Professor of Electrical Engineering, Armour Institute of Technology
Associate Member, American Institute of Electrical Engineers
Author of "Practical Applied Electricity"; "Electrical Measurements and Meter Testing"; "Electrical Equipment for the Motor Car"

## $\%$

## ARTHUR L. NELSON

Superintendent, Interior Wiring Department, Commonwealth Edison Company, Chicago
Member, Western Society of Engineers

## ;

LOUIS DERR, S.B., A.M.
Professor of Physics, Massachusetts Institute of Technology

## Authors and Collaborators-Continued

## CHARLES F. BURGESS, E.E.

President, Northern Chemical Engineering Laboratories
Formerly Professor of Chemical Engineering and Applied Electrochemistry, University of Wisconsin
Past-President, American Electrochemical Society
Member, American Institute of Electrical Engineers, American Chemical Society, Society of Chemical Industry, Western Society of Engineers

MORTON ARENDT, E.E.
Assistant Professor of Electrical Engineering, Columbia University, New York
Fellow, American Institute of Electrical Engineers
Co-Author of "Electric Motors, Control and Application"; "Electricity as Applied to Submarines'

## $\%$

DANA PIERCE, B.A.
Vice-President, Underwriters' Laboratories, Inc.
F. E. AUSTIN, B.S., E.E.

Professor of Electrical Engineering and Mechanics, Thayer School of Engineering, Dartmouth College, Hanover, New Hampshire
Member, Association for Promotion of Engineering Education
Author of "How to Make High Pressure Transformers"; "Examples in Alternating Currents"; "Examples in Battery Engineering"; "Induction Coils, in Theory and Practice"

## HENRY H. NORRIS, M.E.

Engineering Editor, Electrical Railway Journal, New York Formerly Professor of Electrical Engineering, Cornell University

GEORGE C. SHAAD, E.E.
Professor of Electrical Engineering, University of Kansas

## WARD HARRISON

Illuminating Engineer, National Lamp Works of General Electric Company
Special Lecturer at Case School of Applied Science
Lecturer, Illuminating Engineering Course, Joint Auspices of University of Pennsylvania and Illuminating Engineering Society
Member and Past Vice-President of Illuminating Engineering Society Member, National Electric Light Association, American Gas Association

## GLENN M. HOBBS, Ph.D.



Manager, Testing Laboratories, Sears, Roebuck and Company
Formerly Instructor in Physics, University of Chicago
American Physical Society
Member, Sigma Xi Society
C. C. ADAMS, B.S.

District Switchboard Specialist. General Electric Company
Member, American Institute of Flectrical Engineers

## Authors and Collaborators-Continued

GEORGE W. CRAVENS
President, Climax Engineering Company, Clinton, Iowa
Formerly Automotive Engineer, U. S. Army
Member, Society of Automotive Engineers, American Society of Me-chanical Engineers, American Institute of Electrical Engineers,American Electrochemical Society
8H. M. STOLLER, B.E., M.S.Electrical Engineer, Western Electric CompanyMember, American Institute of Electrical Engineers, Sigma Xi Society,New York Telephone Society, New York Flectrical Society
O. J. BUSHNELLSuperintendent, Meter Department, Commonwealth Edison CompanyMember, American Institute of Electrical Engineers$\dagger$
WINFIELD DEXTER BEARCE, B.S., E.E.
Engineer, Railway Engineering Department, General Electric Company Associate Member, American Institute of Electrical EngineersGEORGE J. KIRKGASSER, E.E.Electrical Engineer, The Cutler-Hammer Manufacturing CompanyFormerly Assistant Engineer, Underwriters' Laboratories, Inc.Associate Member, American Institute of Electrical EngineersAuthor of "Electric Lighting and Motor Wiring"
E. W. SEEGER, M.E.
Electrical Engineer, The Cutler-Hammer Manufacturing Company Member, American Institute of Electrical Engineers ..... 8
E. S. BISHOP, Ph.D.
Formerly Head, Physics Department, University High School, Univer- sity of Chicago
STACEY E. DENNY, C.E.
Consulting Civil Engineer, American School
Formerly Signal Engineer, Chicago and Northwestern Railway
Formerly Assistant Engineer, Union Switch and Signal Company-
JOHN H. JALLINGSMechanical Engineer, ChicagoFormerly Superintendent and Chief Constructor for J. W. Reedy Ele-vator CompanyCARL H. DUNLAPHead, Electrical Engineering Department, American SchoolAssociate Member, American Institute of Electrical Engineers
\$
A. G. TURNBULL
Engineer, Meter Department, Commonwealth Edison Company$\$$
JESSIE M. SHEPHERD, A.B.
Head, Publication Department, American Technical Society

## Authorities Consulted

THE editors have freely consulted the standard technical literature of America and Europe in the preparation of these volumes. They desire to express their indebtedness particularly to the following eminent authorities, whose well-known works should be in the library of every electrician and engineer.

Grateful acknowledgment is here made also for the valuable cooperation of the foremost engineering firms and manufacturers in making these volumes thoroughly representative of the very best and latest practice in the design, construction, and operation of electrical machinery and instruments; also for the valuable drawings, data, suggestions, criticisms, and other courtesies.

## FRANCIS B. CROCKER, M.E., Ph.D.

Formerly Vice-President, Crocker-Wheeler Company, Ampere, New
Formerly Professor of Electrical Engineering, Columbia University; Past-President, American Institute of Electrical Engineers
Joint Author of "Management of Electrical Machinery," "Electric Motors," Direct- and Alternating-Current Machine Iesign

## WM. HENRY TIMBIE

Associate Professor, Electrical Engineering, Massachusetts Institute of Technology
Formerly Head of Applied Science Department, Wentworth Institute Member, American Institute of Electrical Engineers
Author of "Elements of Electricity,", "Essentials of Electricity," "Essentials of Alternating Currents," "Electrical Measurements, Alternating Current and Direct Current"; Co-Author of "Alternating Current Electricity and Its Application to Industry"

WILLIAM ESTY, S.B., M.A.<br>Professor of Electrical Engineering, Lehigh University<br>Fellow, American Institute of Electrical Engineers<br>Joint Author of "The Elements of Electrical Engineering"

## CHARLES P. STEINMETZ

Chief Consulting Engineer, General Electric Company
Fellow and Past-President, American Institute of Electrical Engineers Author of ", "Engineering Mathematics," "Alternating-Current Phenomena," "Radiation, Light and Illumination," "Theory and Calculation of Transient Electric Phenomena and Oscillations," "Electric Discharges, Waves and Impulses and Other Transients," "Theory and Calculation of Electrical Circuits," "Theoretical Elements of Electrical Engineering," and "General Lectures on Electrical Engineering'"

## Authorities Consulted-Continued

## TERRELL CROFT

Directing Engineer, Terrell Croft Engineering Company
Member, American Institute of Electrical Engineers
Author of ""American Electricians" Handbook," "Library of Practical Electricity"

## DUGALD C. JACKSON, C.E. nology Institute of Electrical Engineers Dynamos" chinery," "Street Railway Fares" <br> J. J. THOMSON, D.Sc., LL.D., Ph.D., F.R.S.

Professor of Electrical Engineering, Massachusetts Institute of Tech-
Member, American Society of Mechanical Engineers; Fellow, American
Author of "A Textbook on Electromagnetism and the Construction of
Joint Author of "Alternating Currents and Alternating-Current Ma-

Fellow of Trinity College, Cambridge University; Cavendish Professor of Experimental Physics, Cambridge University
Author of "The Conduction of Electricity Through Gases," "Electricity and Matter"

## ;

ROBERT ANDREWS MILLIKAN, Ph.D., Sc.D.
Director, Norman Bridge Laboratory of Physics, California Institute of Technology, Pasadena, California
Joint Author of "A First Course in Physics," "Electricity, Sound, and Light," "A Study Course in Practical Physics"

## JOHN PRICE JACKSON, M.E.

Director, Joint Fuel Committee, National Electric Light Association, American Gas Association, and American Electric Railway Association
Formerly Professor of Electrical Engineering, Pennsylvania State College
Fellow, American Institute of Electrical Engineers
Joint Author of "Alternating Currents and Alternating-Current Machinery"

## $\%$

MICHAĖL IDVORSKY PUPIN, A.B., Sc.D., Ph.D.
Professor of Electro-Mechanics, Columbia University Fellow, American Institute of Electrical Engineers
Author of "Propagation of Long Electric Waves," and "Wave Transmission Over Non-Uniform Cables and Long-Distance Air Lines"

## VLADIMIR KARAPETOFF

Professor of Electrical Engineering, Cornell University
Fellow, American Institute of Electrical Engineers
Author of "Magnetic Circuit," "The Electric Circuit," "Engineering Appliances of Higher Magnetics," "Elementary Electrical Testing," "Experimental Electrical Engineering and Manual for Electrical

## Authorities Consulted-Continued

LAMAR LYNDON, B.E., M.E.

Consulting Electrical Engineer, New York
Fellow, American Institute of Electrical Engineers; Member, American Electrochemical Soclety
Author of "Storage Battery Engineering," Hydroelectric Power Rate Making for Public Utilities
$\$$
FRANK L. FOWLE
Consulting Electrical Engineer Editor-in-Chief, "Standard Handbook for Electrical Engineers" Member, American Institute of Electrical Engineers
;
ARTHUR E. KENNELLY, A.M., D.Sc.
Professor of Electrical Engineering, Harvard University and Massachusetts Institute of Technology
Fellow and Past-President, American Institute of Electrical Engineers
Joint Author of "Alternating Currents," "Arc Lighting," "Electric Heating,", "Electric Motors," "Electric Railways," "Incandescent Lighting"

SILVANUS P. THOMPSON, D.Sc., B.A., F.R.S., F.R.A.S.
Late Principal and Professor of Physics in the City and Guilds of London Technical College; Past-President, Institution of Electrical Engineers
Author of "Electricity and Magnetism," "Dynamo-Electric Machinery," "Polyphase Electric Currents," "The Electromagnet"

## NICOLA TESLA

Inventor of Original Induction Motor and Numerous Other Electrical Appliances
Fellow, American Institute of Electrical Engineers
Author of "Experiments with Alternate Currents of High Potential and High Frequency"

## ANDRE E. BLONDEL

Professor, Ecole National des Pontes et Chausses, Paris
Honorary Member, American Institute of Electrical Engineers
Author of "Synchronous Motors and Converters"
MAURICE A. OUDIN, M.S.
Vice-President, International General Electric Company, Schenectady, New York
Member, American Institute of Electrical Engineers
Author of "Standard Polyphase Apparatus and Systems"

## FREDERICK BEDELL, Ph.D.

Professor of Applied Electricity, Cornell University
Author of "The Principles of the Transformer"; Joint Author of "Alternating Currents"

## M. LUCKIESH

Director of Applied Science, Nela Research Laboratories, National Lamp Works of General Electric Company, Cleveland, Ohio
Member, American Institute of Electrical Engineers
Author of "The Lighting Art," "Color and Its Application," Light and Shade"

## Authorities Consulted-Continued

ALEXANDER MILLER GRAY
Head, Department Electrical Engineering, Cornell University
Member, American Institute of Electrical Engineers
Author of "Electrical Machine Design," "Principles and Practice ofElectrical Engineering"
Associate Professor of Electrical Engineering, University of Minnesota
Associate Member, American Institute of Electrical Engineers
Author of "Electric Meters," "Elementary Magnetism and Electricity," "Theory and Operation of Direct-Current Machinery," "Principles of Radiotelegraphy"

## -

F. A. C. PERRINE, A.M., D.Sc.
Consulting Engineer
Formerly President, Stanley Electric Manufacturing Company
Formerly Manager, Insulated Wire Department, John A. Roebling Sons' Company
Author of "Conductors for Electrical Distribution"

## HARRY B. GEAR

Assistant to Vice-President, Commonwealth Edison Company Fellow, American Institute of Electrical Engineers
Co-Author of "Electrical Central Station Distributing Systems," "Power Transmission," "Distribution Systems," American School of Correspondence

## RALPH D. MERSHON

Consulting Electrical and Mechanical Engineer
Fellow and Past-President, American Institute of Electrical Engineers Author of "Mershon Diagram," and Numerous Technical Transmission Line Papers
COMFORT A. ADAMS
Lawrence Professor of Electrical Engineering, Harvard University Fellow and Past-President, American Institute of Electrical Engineers Author of "Dynamo Design Schedules," and Many Other Technical Papers

## JAMES RALEY CRAVATH

President, Pioneer Electric Company, Richmond, California Fellow, American Institute of Electrical Engineers Author of "Practical Illumination'"

## HOBART MASON, B.S., E.E.

Assistant in Electrical Engineering, Polytechnic Institute of Brooklyn Associate Member, American Institute of Electrical Engineers
Author of "Static Electricity"
Joint Author, of "Dynamo-Electric Machinery," "Alternating-Current Machines''
H. M. HOBART, B.Sc.
Consulting Engineer, General Electric Company Fellow, American Institute of Electrical Engineers Member, Institution of Civil Engineers Author of "Design of Polyphase Generators and Motors'" Joint Author of "Armature Construction"


## Foreword

ELECTRICITY, during the last hundred years, has advanced from an interesting yet mysterious phenomenon to the leading source of energy on the globe. Thousands of master minds have studied experimental data and have verified theories, while more practical heads have utilized the principles discovered. The applications of the electric current are numberless and are to be found in every home, even unto the highways and byways of our less thickly populated districts. The visible results are on every side. Think of the important parts the generator and motor play in our industrial life; call to mind the immense power plants which are to be found all over the country; stop and realize the convenience of the electric light when we turn it on in our home, or the electric car when we take it at a near-by corner.
(1. Think again of the telegraph and the recent discoveries in the radio field and the thousand other applications of electric energy which contribute to our daily comfort. It is not too much to expect that our scientists and inventors may ultimately find a way of transmitting electricity without wires from generating stations located at the original sources of power to the point of application so that the burden of the world's labor shall be shouldered by electricity.
I. This Cyclopedia of Applied Electricity was created to meet the demand for a complete and practical working treatise on the generation and application of electrical energy. For twenty years this reference library has held an enviable place in the field of electrical literature. By repeated revisions the publishers have kept pace with the rapid developments in the field. In this Cyclopedia one will find a logical discussion on such subjects as Armature Winding, Storage Batteries, and Transformers. Methods of Distribution and Transmission are given adequate treatment. The articles on Electric Wiring, Electric

Signal Systems, and Electric Household Appliances present the subject in a thorough and practical manner. Commercial uses, such as lighting, welding, transportation, and communication, are exhaustively treated. The discussion includes the construction as well as the management of devices, instruments, and machines in practical use, while the treatment of operating troubles is complete.

IL Owing to the use of many special words and terms, a glossary has been included in Volume VIII. The definitions are given in simple language and, where possible, reference is made to the volume and page where added information may be found.
(1. Throughout, the Cyclopedia is as scientifically correct as any work could be, and yet the treatment of the various subjects is as free as possible from abstruse mathematics and unnecessary technical phrasing, particular attention being given to the careful explanation of involved but necessary formulas. Diagrams, curves, and practical examples are given whenever they may be helpful in explaining the subject. Blueprints present information with a real shop atmosphere, while the numerous illustrations and inserts furnish complete pictorial aid to the text.

IL Books on electrical topics, if all gathered in a common library, would contain so much duplicate material that anyone trying to keep up with electrical progress would lose a great deal of time. To overcome this difficulty the publishers of this Cyclopedia have gone to original sources and have secured, as writers of the various sections, men of wide practical experience and thorough technical training. Each writer is an acknowledged authority on the subject which he covers. The contributions of these men have been correlated by our Board of Editors into the logical and unified Cyclopedia here presented.
(1. Acknowledgment is due the staff of Authors and Collaborators whose hearty cooperation has made this work possible.

# Table of Contents 

## VOLUME II

Alternating-Curbent Machinery<br>By William Esty $\dagger$<br>Page *11

Introduction: Electric Power Systems-Physical Nature of Alternating Current-Alternating Electromotive Forces and Currents-Advantages and Disadvantages of Alternating Currents-Comparison of Direct- and Alternating-Current Problems-Physical Basis for Difference between Di-rect-Current and Alternating-Current Calculations-Graphical Representation of Alternating Electromotive Forces and Currents-Harmonic Electromotive Forces and Currents-Relation between Maximum and Effective Values-Inductance-Fundamental Equations of AlternatingCurrent Circuit-Electrical Resonance-Miscellaneous ConsiderationsAlternators: Fundamental Equation of Alternator-Armature Reaction and Inductance-Field Excitation-Polyphase Alternators and Systems: Single-Phase, Two-Phase, and Three-Phase Systems-Y- and Delta-Connected Armatures-Measurement of Power: Balanced and Unbalanced Systems-Armature Windings: Classification, Single-, Two-, and ThreePhase Windings-Commercial Types of Machines-Revolving-Armature Alternators: Fort Wayne Single-Fhase-Westinghouse Armatures-General Electric Three-Phase Alternator-Revolving-Field Aiternators: Con-struction-Water-Wheel-Driven Alternators-Steam Turbine-Driven Al-ternators-Economy Factors in Alternators: Conditions Affecting CostPower Losses-Efficlency-Rating and Overload Capacities-AiternatingCurrent Testing: Insulation Testing, Characteristic Curves-Determination of Resistance of Armature-Regulation-Conditions for Tests of Regulation-Tests and Computation of Regulation of Alternating-Current Generators-Example of Method-Potier Method-Intermittent Method of Muking a Heat rest-Calculation of Effelency-Synchronous Motors-Torque and Power Output-Use as a Condenser-Motor TestingTransformer: Physical Action-Maximum Core Flux-Ideal Transformer Action Graphically Represented-Infiuence of Coil Resistances and Magnetic Leakage-Transformer Connections-Parallel-Constant-Voltage Transformers-Sertes-Current Transformers-Transformers in Polyphase Systems-Phase Transformation-Practical Considerations-Transformer Losses and Efficlency-Commerclal Types of Transformers: Core, Shell-Three-Phase Transformers-Cooling of Transformers-Series or Current Transformers-Constant-Current Transformers-Transformer Fuse Blocks -Inspection and Maintenance of Distributing Transformers-Low-Voltage Secondary Circuits Should Be Grounded-Transformer Tests-Impedance -Regulation-Efficiency Calculation-Conversion of Alternating into Direct Current: Aluminum Valve Rectifier-Mercury-Vapor Arc Rectifier -Rotary or Synchronous Converter: Comparison with Direct-Current Dynamo-Changing Direct-Current Dynamo into Rotary Converter.E.M.F. Relations for Rotary Converter-Current Relations for Rotary Converter-Rotary Converters in Practice-Starting Rotary ConvertersOscillators for Rotary Converters-Characteristic Types of Rotary Con-verters-Hunting of Rotary Converter-Inverted Rotaries-Control of Direct-Current Voltage-Field Excitation-Rotary Converter with Edison Three-Wire System-Six-Phase Converter-Transformer Connections for Rotary Converters-Testing Rotary Converters-Motor-Generators: Comparison with Rotary Converter-Uses and Types of Motor-Generators -Induction Motor: Constructive Elements-Stator Windings and Their Action-Action of Induction Motor-Ratio of Rotor Voltages to Stator Voltages-Structural Details of Typica. Induction Motor-Types of Rotors for Constant and Variable Speed-Behavior at Starting and in Operation -Typical Induction Motor Installations-Single-Phase Induction MotorFrequency Changer-Comparison of Synchronous Motor and Induction Motor-Induction Motor Tests: Core Loss, Impedance, Efficiency, SlipPerformance Curves of Induction Motor-Heyland Diagram-Reconnection of Induction Motors: Changes in Voltage-Comparison of Motor Voltages with Various Two- and Three-Phase Connections-Effect of Incorrect Voltage on Performance-Changes of Frequency-Effect of Incorrect Frequency on Performance-Changes of Voltage, Frequency, and Phase-Change of Poles-Switchboard and Station Appliances
lndex
Page 515
*For page numbers, see foot of pages.
$\dagger$ For professional standing of authors, see list of Authors and Collaborators at front of volume.

alternating-Current generator of the engine type

# ALTERNATING-CURRENT MACHINERY 

## PART I

## INTRODUCTION

Relation of Power Supply to Industry. Before beginning the study of alternating currents and the description of the various types of alternating-current apparatus, it will be more interesting and profitable to get a bird's-eye view of the general plan of an alternating-current power system. But to fully grasp the plan of any particular power system, it is helpful to get first an appreciation of the relation of a power supply to the development of industry and to the promotion of the convenience and welfare of mankind.

One of the measures of the degree of civilization attained by a community, state, or nation is the extent and universality of its use of power. Among primitive peoples practically all work was done by hand. Manual labor was the rule; labor-saving machinery was all but unknown. Under modern conditions of civilization, however, some form or other of power is needed in almost every building, whether in town or country. Although the amount of power needed at any one place may be considered insignificant, yet the total amount used by a factory, a city block, or a community may be surprisingly large. Take, for example, a steel works. It needs power for a great many purposes: for hauling coal, coke, ore, limestone, and other raw materials; for driving rolling milis; and for operating elevators, hoists, fans, air compressors, and machine tools. It needs power for lighting offices, shops, and yards, and for operating various labor-saving devices too numerous to mention. The location of this host of power-consuming machines being widely scattered over different buildings and throughout different floors in any one building, it would, therefore, not be practicable or economical to install a source of power, like a steam or gas engine, at each point where power is wanted. The
proper solution of the problem is to generate at one centrally located power station all the power needed for the entire plant, and then to distribute this power by suitable means to the various floors and buildings where it is to be utilized.

Creation and Utilization of Power. The power station contains the prime movers-such as steam engines or turbines, gas or oil engines, and water wheels which generate mechanical powerusing as sources of energy coal, gas, oil, or water, as the case may be. In some small power plants and factories the prime movers still transmit their mechanical power through ropes or belts to a main line shaft equipped with pulleys which, by means of smaller belts, transmit the power to counter-shafts. By another series of pulleys and belts the counter-shaft is made to drive either individual machines or groups of machines. In the case of an electric power plant the prime movers are used to drive electric generators (dynamos), which generate electric power either in the form of direct currents or alternating currents. To prevent any possible misunderstanding, it should be emphasized that an electric generator is not of itself a source of power. It is a machine which merely serves to convert the energy of mechanical motion, imparted to it by a prime mover, into energy in the form of electric currents.

Tendency toward Large Power Plants. The same reasons, which, in the case of a mill, department store, or hotel, lead to a concentration of power generation at a power station for each, can in general be urged in favor of combining many individual isolated power plants into one or more large central power stations. Thus even in our largest cities, like New York, Chicago, Philadelphia, Boston, and many others, the great bulk of the power needed for municipal and private lighting, for manufacturing, traction, etc., is generated at a few huge central stations, some of them having a maximum output of over 100,000 kilowatts. The power so generated is distributed to consumers at a price so low that it oftentimes becomes cheaper to buy power than to generate it in small private plants. The tendency toward doing away with small uneconomical plants and buying power from the large central station is increasing. The economies possible in fuel consumed, in labor and materials costs, and in management, due to the use of
large generating units operating under more favorable load conditions, are enormous and are not fully realized by the public.

Interlinking Power Lines. As the next logical step in the centralization of generating stations comes the plan recently proposed by the Secretary of the Interior-to link up existing central electric generating stations all along the Atlantic seaboard from Virginia up through the New England states by means of tie lines. The general scheme of interlinking the power lines of several electric-power companies has been for some years in operation on the Pacific coast and is highly successful. If this plan be carried out, as it should be, it would mean connecting together by copper feed wires the existing transmission lines of a large number of electric-power companies each operating now in a limited territory. The plan aims to create an almost continuous network of electric transmission wires paralleling the coast, making what has been called a "river of electric power."

The chief advantage of this proposal is that it would materially reduce the cost of electric power to the consumer, increase facilities for transportation, manufactures, and housekeeping, and greatly encourage and stimulate a more general use of electric power. It would mean saving of time and lessening of drudgery, and would add to the comfort and well-being of communities.

Methods of Transmitting Power. Having outlined the development of a power supply originally planned to meet the needs of a manufacturing firm or a group up to a general supply adequate for the needs of whole communities or even states, let us now consider the various methods available for transmitting power.

There are four common ways of transmitting power for considerable distances, as follows: (1) by mechanical means, using rope drives, belts, and shafting; (2) by compressed air conducted in pipes or flexible hose; (3) by steam under pressure flowing in pipes; (4) by electricity in the form of direct currents or alternating currents flowing in metallic conductors. Each of these methods has its peculiar advantages under certain limiting conditions, and where the distances involved are relatively short. When, however, the distance over which the power is to be transmitted becomes considerable, the first three methods become impracticable and prohibitively costly.

For a general system of power transmission electricity stands without a rival. The electric current passes over wires which are stationary, and which may be easily bent in turning corners and in avoiding obstructions. Wires are easily supported in various ways. Most important of all, the cost of electric transmission lines is relatively small, the power lost is small, and the depreciation and maintenance charges against the lines are also relatively small. Moreover, electric power may be turned on, shut off, and controlled by devices that are more positive in action, more rapid and satisfactory in their operation, than those which must be used when other methods of transmitting power are employed. The apparatus for the control and protection of electrical machinery is simpler, more compact, and more reliable on the whole than that required for other systems of power transmission. Electrical transmission over long distances is thus more economical, more flexible, more easily adapted to a variety of uses, and withal more convenient, than any other system.

Electric Power Systems. An electric power system involves three main features or processes: generation, transmission, and distribution, the utilization in each of which involves the use of either direct currents or alternating currents. There are in general, therefore, three kinds of electric power systems, as follows:
(1) A system in which direct-current power is generated, transmitted, and utilized.
(2) A system in which direct-current power is generated and transmitted, but is then converted into alternating current for use.
(3) A system in which alternating-current power is generated and transmitted, but utilized either in the form of alternating current, or in the form of converted direct current.

The earliest electric-power stations in this country were of the first type, and today some of the smaller stations are still furnishing direct current for incandescent and arc lighting, and for supplying direct-current motors over limited distances. The second system has not met with favor in the United States, although the Thury direct-current system has been in successful use for many years in European countries. The third system is almost exclusively used in this country today, most of the electric power used for lighting, industrial purposes, and traction being generated and
transmitted in the form of alternating currents at high voltages. The alternating-current power delivered by the transmission lines to step-down, or reducing, transformers located at substations may then be distributed in the form of alternating currents, or may be converted into direct currents at voltages suited to those industries requiring it-such, for example, as electrochemical works and electric railways.

IIigh Voltage Necessary. Long-distance electric power transmission to be economical or even commercially possible must be accomplished by using high voltages. Since electric power is the product of two factors-voltage and current-it follows that to be able to use small, inexpensive transmission wires, and to reduce the heating losses therein, the current should be reduced to a minimum. This for a given amount of power transmitted means using a voltage as high as practicable.

Advantages of Alternating Currents. for Transmission. The transmission of direct currents on a large scale at high voltages has so many disadvantages that it has never been adopted in this country. The principal advantages of alternating currents for long-distance power transmission are as follows:

1. The generators for alternating currents, called alternators, can be built in larger sizes and more cheaply than those for direct currents. Thus alternators driven by steam turbines have been built to deliver 60,000 kilowatts, while the largest direct-current generators are limited to an output of about 2500 kilowatts and are inherently not adapted for high speeds.
2. Alternating-current power can more easily and cheaply be converted into power at either a higher or a lower voltage through the agency of a very simple, cheap, and efficient apparatus-the transformer. Transformers cannot be used on direct-current circuits.
3. The alternating-current system is much more flexible, more easily controlled, and more adaptable to various kinds of service.
4. By combining two or three alternating-current circuits into a polyphase (a two- or a three-phase) system it becomes possible to convert electrical into mechanical power through the use of motors of simple, compact, and rugged construction, such as induction and synchronous motors.

Physical Nature of Alternating Current. The flow of electricity in the form of an electric current, may take place in any one of three ways:
(1) The flow may be continuous, never changing in value or direction, in which case it is called a steady direct current.
(2) The flow may always be in the same direction, but its rate of flow may change, in which case it is called a pulsating direct current.
(3) The flow may change in both magnitude and direction in a periodic manner, in which case it is called an alternating current.

Steady Direct Current. The first case, that of a steady direct flow, is analogous to the flow of water in a river, or the flow of


Fig. 1. Curve Showing Rate of Flow of Tide
water in a pipe connected to a centrifugal water pump. In either of these cases the water flows steadily in the same direction.

Pulsating Direct Current. The second case, that of a pulsating current, is analogous to the flow of blood through the arteries. Here the pumping action of the heart causes the flow of blood to vary periodically in value, but the direction of flow is never reversed. In a similar manner a piston type of pump forces water through a pipe; the flow is pulsating but always in the same direction.

Periodic Alternating Current. The third type of flow, the alternating flow, is analogous to the coming in and going out of tides. In this case there is a distinct and periodic reversal in direction of flow.

Analogy of Tides. Suppose that the tide is lowest at 12 o'clock noon, Fig. 1. At this instant the water is neither going
out nor coming in, that is, its rate of flow is zero; an instant later the water begins to flow in, and the rate at which it flows in continually increases until. 3 p.m. After 3 P.m. the water continues to flow in but at a lesser rate until 6 p.M., when the tide is highest. But at this instant the water is neither flowing in nor out; that is, its rate of flow is again zero.

The curve, Fig. 1, is plotted to show how the rate of flow, not the height of the tide, changes during the day or with time. After 6 p.м. the water begins to flow out at an increasing rate until 9 р.м., when it is flowing out at its greatest rate. From 9 p.m. to 12 midnight the water continues to flow out, but at a decreasing rate. At 12 midnight the water has ceased completely to flow out; the tide is at its lowest ebb. At this point, therefore, the tide has


Fig. 2. Curve Showing Height of Tide
gone through a complete cycle and is ready to start a new cycle of movements in endless repetition.

The time taken to complete the full cycle was 12 hours, which is called the periodic time, or simply the period. Thus the tide goes through about two cycles per day of 24 hours. The number of cycles, two in this case, completed in a unit of time would be called the frequency.

In a similar way an alternating current periodically flows back and forth in a circuit, but the time it takes to go through a complete set of positive and negative values, that is, its period, is much less than for the tidal flow. The number of cycles through which the current passes in the same unit of time is therefore very much larger. The frequency of an alternating current is expressed in cycles per second.

Phase Relation. If, as in Fig. 2, we plot the height of the water or tide for the different values of time taken in Fig. 1, we find that the height of the tide likewise goes through a complete cycle in the same time as was taken by the flow of water. It should be observed, however, that when the tide is lowest the water is not moving at all, and likewise when the tide is highest the rate of flow is zero. In alternating-current terms it would be said that the height of the tide is out of phase with the rate of flow of water. In this case, calling one cycle 360 degrees, it is out of phase by one quarter of a cycle, or 90 degrees.

## ALTERNATING ELECTROMOTIVE FORCES AND CURRENTS

Simple Alternator. The alternator is an arrangement by means of which mechanical energy is used to cause the magnetic flux from a magnet to pass through the opening of a coil of wire first in one and then in the opposite direction. This varying magnetic flux induces in the coil, first in one direction and then in the other, what is called an alternating electromotive force, which in turn produces an alternating current in the coil, and in the circuit which is connected to the terminals of the coil.

In the common type of alternator, the above-mentioned magnet and coil move relatively to each other. Fig. 3 shows the essential features of such an alternator. The poles $N, S, N, S$, etc., of a multipolar magnet called the field magnet, project radially inward toward the passing teeth $a a a$ of a rotating mass $A$ of laminated iron; and upon these teeth are wound coils of wire $c c$, in which the alternating electromotive force is induced. The rotating mass of iron with its windings of wire is called the armature. At one end of the armature (not shown in the figure) are mounted two insulated metal rings $r$, called collccting rings. These metal rings are connected to the ends of the armature winding, and metal brushes $b b$ rub on these rings, thus keeping the ends of the armature winding in continuous contact with the terminals of the external circuit to which the alternator supplies alternating current. No external circuit is shown in the figure.

The electromotive forces induced in adjacent armature coils are in opposite directions at each instant, and the coils are so connected together that these electromotive forces do not oppose each
other. This is done by reversing the connections of every alternate coil, as indicated by the dotted lines connecting the coils in Fig. 3. The electromagnetic action of this type of alternator depends only upon the relative motion of field magnet and armature, and large machines are usually built with stationary armature and revolving field magnet. In the type of machine illustrated in Fig. 3, the armature revolves while the field magnet is stationary. This type, called the revolving-armature type, is generally adopted in small alternators.

The field magnet of an alternator is usually an electromagnet which is excited by a continuous electric current supplied by an independent generator, generally by an auxiliary continuous-current dynamo, called the exciter. The exciting current flows through coils

of wire wound on the projecting poles $N, S, N, S$ of the field magnet. These coils are not shown in Fig. 3.

The type of armature core shown in Fig. 3 is called the toothed armature core; and the armature winding is said to be concentrated, that is, the armature conductors are grouped in a few heavy bunches. Armature cores are also made with many small slots, in which the armature conductors are grouped in small bunches. This type of core is called a multi-toothed core, and the winding is said to be distributed.

In some of the earlier types of alternators the armature core consisted of a smooth, cylindrical mass of laminated iron, upon
the face of which the conductors were arranged in bands side by side, one layer or more in depth. This type of armature is called the smooth-core armature; it has been superseded by the toothed core type.

Variations of Electromotive Force. Cycle. The electromotive force of an alternator passes through a set of positive values while a given coil of the armature is passing from a south to a north pole of the field magnet, and through a similar set of negative values while the coil is passing from a north to a south pole. The complete set of values, including positive and negative is called a cycle.

Frequency. Frequency is equal to the number of cycles per second; it is sometimes expressed by stating the number of alternations or reversals per minute. For example, an alternator having a frequency of 133 cycles per second has 266 reversals or alternations per second, or 15,960 alternations per minute. Frequencies are sometimes specified in alternations per minute, but specification in cycles per second is the more usual practice and is preferable.

$$
\text { cycles per sec. }=\frac{\text { alternations per min. }}{2 \times 60}
$$

Period. The fractional part of a second occupied by one cycle is called the periodic time, or period, of the alternating electromotive force or current.

Let $f$ be the frequency in cycles per second, and $T$ the period expressed as a fraction of a second. Then

$$
\begin{equation*}
f=\frac{1}{T} \tag{1}
\end{equation*}
$$

Therefore, if an alternating current has a frequency of 60 cycles per second, the period $T$ ' of one cycle is one-sixtieth of a second.

Relations Between Speed and Frequency. Let $p$ be the number of poles of the field magnet of an alternating-current machine; let $n$ be the speed of its armature in revolutions per minute; and let $f$ be the frequency of its electromotive force in cycles per second. Then

$$
\begin{equation*}
f=\frac{p}{2} \times \frac{n}{60} \tag{2}
\end{equation*}
$$

Examples. 1. A certain alternator has 10 poles, and runs at 1,500 revelutions per minute. What is its frequency?

Soldtion. Substituting 10 for $p$, and 1,500 for $n$, in equation (2), we have

$$
f=\frac{10}{2} \times \frac{1500}{60}=125 \text { cycles per second }
$$

2. An alternator is to run at 600 revolutions per minute and is to give a frequency of 60 cycles per second. What number of poles is required?

Solution. Solving equation (2) for $p$, we have

$$
p=\frac{2 \times 60 \times f}{n}
$$

from which, substituting $f=60$, and $n=600$, we have

$$
p=\frac{2 \times 60 \times 60}{600}=12 \text { poles }
$$

Advantages and Disadvantages of Alternating Currents. The electric transmission of a given amount of power may be accomplished by a large current at low electromotive force, or by a small current at high electromotive force. In the first case very large and expensive transmission wires must be used, or the loss of power in the transmission line will be excessive. In the second case comparatively small and inexpensive transmission wires may be used. Thus it is a practical necessity to employ high electromotive forces in long-distance transmission of power.

Example. It is desired to transmit 1,000 kilowatts of power over a distance of 10 miles, supposing that a loss in the line of 10 per cent of the power delivered is considered permissible. This corresponds to a loss of 100 kilowatts.

Solution.-Case 1. Suppose that the electromotive force at the receiving end of the line is to be 100 volts. Then the current would be $1,000,000$ watts divided by 100 volts, or 10,000 amperes. The resistance of the line must be such that the watts lost in the line-namely 100,000 watts-would be equal to $I^{2} R$, so that the resistance $R$ of the line must be $\frac{W}{I^{2}}$, or 0.001 ohm . This would require two transmission wires each 10 miles long and 33 inches in diameter, or a total weight of 175,000 tons of copper, which would cost about $\$ 52,500,000$.

Case 2. Suppose that the electromotive force at the receiving end of the line is to be 1,000 volts. Then the current would be $1,000,000$ watts divided by 1,000 volts, or 1,000 amperes. The resistance of the line must be such that the watts lost in the line-namely, 100,000 watts-would be equal to $I^{2} R$, so that the resistance $R$ of the line must be 0.1 ohm . This would require two transmission wires, each 10 miles long and 3.3 inches in diameter, or a total weight of 1,750 tons of copper, which would cost about $\$ 525,000$.

## TABLE I

## Size and Cost of Copper Wire-Two=Wire System

To transmit 1,000 kilowatts a distance of $10 \cdot$ miles (one way) with a line loss equal to 10 per cent of the power delivered, for three different values of electromotive force

| Volts at <br> Receiving End <br> of Line <br> $E$Amperes <br> In <br> Line <br> $I$ | Ohms <br> in <br> Line <br> $R$ | Diameter <br> of <br> ofe, <br> in Inches | Weight <br> of <br> in Tire, | Cost of <br> Line Cons <br> Coper. <br> in <br> Dollars |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 100 | 10,000 | 0.001 | 33 | 175,000 | $52,500,000$ |
| 1,000 | 1,000 | 0.1 | 3.3 | 1,750 | 525,000 |
| 10,000 | 100 | 10.0 | 0.33 | 17.5 | 5,250 |

Case 3. Suppose that the electromotive force at the receiving end of the line is to be 10,000 volts. Then the current would be $1,000,000$ watts divided by 10,000 volts, or 100 amperes. The resistance of the line must be such that the watts lost in the line-namely 100,000 watts-would be equal to $I^{2} R$, so that the resistance $R$ of the line must be 10 ohms. This would require two transmission wires, each 10 miles long and 0.33 inch in diameter, or a total weight of 17.5 tons of copper, which would cost about $\$ 5,250$.

These results are summarized in Table I.
Transformation of High E. M. F.'s. High electromotive forces are dangerous under the conditions that ordinarily obtain among users of electric light and power; and many types of apparatus, such as incandescent lamps, operate satisfactorily only with medium or low electromotive forces. Therefore, means must be provided, at a receiving station, for transforming the power delivered, from high electromotive force and small current to low electromotive force and large current, if long-distance transmission is to be successful. This is called step-down transformation. The advantage of the alternating current over the direct current lies almost wholly in the cheapness of construction and of operation, and in the high efficiency of the alternating-current apparatus as compared with the direct-current apparatus that is required for transformation.

In step-down transformation of direct current, a motor takes a small current from the high-electromotive-force .transmission mains, and drives a dynamo which delivers large current to service mains at low electromotive force. This apparatus, or its equivalent, the dynamotor, is expensive to construct; it requires attention in operation; and its efficiency is never, perhaps, above 90 per cent.

The step-down transformation of alternating currents is accomplished by means of the alternating-current transformer, which is described later on. The alternating-current transformer is very much cheaper than a dynamo and motor of the same output; it requires no attention in operation; and its efficiency under full load is usually greater than 97 per cent, especially in large sizes.

Simple Construction of A. C. Machines. The alternating current has some minor advantages over the direct current on account of the fact that alternating-current machines are frequently simpler in construction than direct-current machines. In particular, the commutator is not an essential part of an alternating-current generator. Again, in the case of the inductor alternator and the induction motor, the rotating part may not have any sliding electrical contacts whatever.

Miscellaneous A. C. Machines. The simple single-phase alternating current is not well adapted to general power service. The single-phase alternating-current induction motor does not start satisfactorily under load, in the case of large machines, although self-starting single-phase motors up to perhaps 20 horse-power are in commercial use, where neither direct-current nor polyphase alternating-current machines are available.

The single-phase series commutator motor within a few years has been developed especially for electric railway service both for trolley cars and for electric locomotives. Its operating characteristics, resembling closely those of the direct-current series motor, however, are not suitable for general power requirements. This type of motor is used on the electric locomotives of the New York, New Haven and Hartford Railroad and others.

For uninterrupted service the synchronous motor is frequently used, the starting being effected by an auxiliary engine or other independent mover. The synchronous motor is not satisfactory when frequent starting is necessary, for such service the induction motor being used. The simple induction motor, to start satisfactorily, must be supplied with two or more distinct alternating currents transmitted to the motor over separate lines. This is called the polyphase system of transmission, and is reserved for full treatment in later pages.

For some purposes, especially for the electrolytic processes
used on a large scale in electro-chemical works, only direct current can be used. When power transmitted by alternating current is to be delivered in the form of direct current, the conversion is effected by means of the rotary converter, motor generator, or mercury rectifier.

Comparison of Direct= and Alternating=Current Problems. Direct Current. In direct-current work the electrical engineer is concerned with the relations between electromotive force, resistance, current, and power. These relations are determined by the applica. tions of the following laws:

Power Law for direct-current circuits

$$
\begin{equation*}
P=E I \tag{3}
\end{equation*}
$$

in which $P$ is the total power in watts delivered to a circuit by a generator of which the terminal electromotive force is $E$ volts, when it produces a current of $I$ amperes.

Onm's Law for direct-current circuits

$$
\begin{equation*}
I=\frac{E}{R} \tag{4}
\end{equation*}
$$

in which $I$ amperes is the steady current produced by $E$ volts acting on a circuit of $R$ ohms resistance.

Joule's Law for direct-current circuits

$$
\begin{equation*}
P=I^{2} R \tag{5}
\end{equation*}
$$

in which $P$ is the power in watts expended in heating a circuit of $R$ ohms resistance, when a current of $I$ amperes is forced through the circuit.

Kirchoff's Laws for direct-current circuits.

1. When a circuit branches, the current in the main circuit is equal to the sum of the currents in the separate branches.
2. (a) When two or more sources of electromotive force are connected in series, the total electromotive force is the sum of the individual electromotive forces.
3. (b) When an electromotive force acts on a number of elements or things in series, it is subdivided into parts, each of which acts upon one of the elements, and the sum of these parts is equal to the total electromotive force. For example, an arc-light dynamo of which the terminal electromotive force is 3,000 volts,
acts on 60 similar arc lamps connected in series. Neglecting the resistance of the connecting wires, each lamp is acted upon by onesixtieth of the total electromotive force, or by 50 volts.

Alternating Current. In alternating-current work the electrical engineer is likewise concerned with the relations between electromotive force, resistance, current, and power. These relations are determined by the application of the same fundamental laws as in the case of direct currents, but in more or less modified forms.* A summary of the fundamental laws of alternating currents is here given simply for purposes of comparison.

Power Law for alternating-current circuits

$$
\begin{equation*}
P=E I \cos \theta \tag{6}
\end{equation*}
$$

in which $P$ is the power in watts delivered to a circuit by an alternator of which the "effective" terminal electromotive force is $E$ volts, when it produces an "effective" current of $I$ amperes in a circuit, and $\cos \theta$ is what is called the "power factor" of the circuit.

Ohm's Law for alternating-current circuits

$$
\begin{equation*}
I=\frac{E}{\sqrt{R^{2}+X^{2}}} \tag{7}
\end{equation*}
$$

in which $I$ is the "effective" current in amperes produced by an "effective" electromotive force of $E$ volts acting on a circuit of which the resistance is $R$ ohms, and the "reactance" is $X$ ohms. The expression $\sqrt{R^{2}+\mathrm{I}^{2}}$ in equation (7) is called the impedance, and it is expressed in ohms.

Joule's Law for alternating-current circuits

$$
\begin{equation*}
P=I^{2} R \tag{5}
\end{equation*}
$$

in which $P$ is the power in watts expended in heating a circuit of $R$ ohms resistance when an "effective" alternating current of $I$ amperes is forced through the circuit.

Kirchoff's Laws for alternating-current circuits

1. When an alternating-current circuit branches, the "effective" current in the main circuit is the "geometric" $\dagger$ (or "vector") sum of the "effective" currents in the separate branches.
[^0]Example. An alternator $A$, Fig. 4, supplies an effective current of $I$ amperes in the main circuit, which divides into two branches. The effective currents in the two branches are $I_{1}$ and $I_{2}$ amperes, respectively. The relation between $I, I_{1}$, and $I_{2}$ is shown in Fig. 5. The angles $\theta_{1}$ and $\theta_{2}$ depend


Fig. 4. Diagram of a Branched Alternating Circuit


Fig. 5. Vector Diagram of Branched Circuit
upon the relative values of the resistance and reactance of the respective branches, as is explained later. It is to be particularly noticed that the arithmetical sum of $I_{1}$ and $I_{2}$ is in general greater than $I$.
2. (a) When two or more alternators (or transformer secondaries) are connected in series, the total effective electromotive force is the "geometric" (or "vector") sum of the effective electromotive forces of the individual alternators.


Fig. 6. Two Alternators in Series


Fig. 7. Vector Diagram of E. M. F.s for Two Alternators in Series

Example. Two alternators. $A_{1}$ and $A_{2}$, Fig. 6 , of which the effective electromotive forces are $E_{1}$ and $E_{2}$, respectively, are connected in series to supply mains. Then the effective electromotive force $E$, between mains, is the geometric sum of $E_{1}$ and $E_{2}$, as shown in Fig. 7. The angle $\theta$ depends upon the positions, relatively to the field magnets, of the armature coils on the respective machines.
(b) When an alternating electromotive force $E$ acts upon a number of elements or things in series, it is subdivided into parts, each of which acts upon one of the elements, and the "geometric" (or "vector") sum of these parts is equal to $E$.

Example. Two coils $b$ and c, Fig. 8, are connected in series between mains supplied from an alternator, of which the effective electromotive force is $E$. Then the total effective electromotive force $E$ is subdivided into two parts $E_{1}$ and $E_{2}$, which act upon the respective coils, as indicated in Fig. 8; and the "geometric," or "vector" sum of $E_{1}$ and $E_{2}$ is equal to $E$, as shown in Fig. 9. The angles $\theta_{1}$ and $\theta_{2}$ depend upon the relative resistance and reactance of the respective coils. It is to be particularly noticed that the arithmetical sum of $E_{1}$ and $E_{2}$ is in general greater than $E$.

Physical Basis for the Differences between D.C. and A.C. Calculations. The above mentioned differences between directcurrent and alternating-current calculations are due to the fact that an alternating current changes rapidly in value from instant to instant, while a direct current is more or less steady and does


Fig. 8. Alternating Current Through Two Coils in Series


Fig. 9. Vector Diagram of E. M. F. for Conditions Shown in Fig. 8
not change its direction of flow. A clear idea of the effects of the rapid changes of an alternating current may be obtained as follows:

Fig. 10 represents an alternator producing alternating current in a circuit of wire; and Fig. 11 represents a valveless pump, of which the piston oscillates rapidly up and down, producing an alternating current of water in a circuit of pipe. The current of water is called alternating because it is periodically changing its direction of flow according to whether the pump piston is pushing the column of water up or down. The e.m.f. of the alternator $A$ not only has to overcome the resistance of the wire in order to cause an alternating current to surge back and forth through the circuit, but it also has to overcome the electrical inertia of the circuitfirst, in getting a pulse of current started: and second, in stopping
this pulse of current and starting another in the reverse direction. The pressure developed by the pump $P$ not only has to overcome the frictional resistance of the pipe in order to cause an alternating current of water to surge back and forth through the pipe, but


Fig. 10. Simple Alternating Circuit


Fig. 11. Water Analogy for an Alternating Circuit
it also has to overcome the inertia of the water in the pipe-first, in getting a pulse of water current started; and second, in stopping this pulse and starting another in the reverse direction.

Fig. 12 represents an alternator producing alternating current in a circuit of wire which contains an electric condenser $C$; and Fig.' 13 represents a valveless pump producing an alternating current of water in a circuit of pipe, which leads to a chamber $H H$, across which is stretched an elastic diaphragm $D D$. In this case the pressure developed by the pump has to overcome the frictional resistance of the pipe, the inertia of the water, and the elastic reaction of the diaphragm $D D$. Similarly the alternating electro-


Fig. 12. Alternating Circuit Containing Condenser


Fig. 13. Water Analogy for Alternating Circuit with Condenser
motive force of the alternator $A$, Fig. 12, has to overcome the electrical resistance of the wire, the electrical inertia of the wire circuit, and the electro-elastic reaction of the dielectric, between the condenser plates.

The electrical inertia of a circuit is called its inductance and the electro-elasticity of a co ndenser is called its capacity, and it is to inductance and capacity that the peculiar features of alternatingcurrent calculations are due.

The effect of capacity is strikingly shown by the fact that an alternating current may be made to flow through a circuit which for direct currents would be an open circuit like that shown in Fig. 12. Thus an alternator connected to long transmission lines, which are disconnected at the distant end and perfectly insulated from the ground, will send a considerable alternating current into the lines, which current can be measured by an alternating-current ammeter. In such a case the current is called the charging current of the line;


Fig. 14. Development of Three Field Magnet Poles and E. M. F. Curve for One Cycle
and it may amount to many amperes, according to the length of the line, the distance apart and size of wires, and the electromotive force of the alternator.

Graphical Representation of Alternating Electromotive Forces and Currents. When an armature conductor of an alternator approaches a north pole of the field magnet, the electromotive force of the machine rises in value as the conductor enters the strong field under the pole; and the electromotive force falls in value as the conductor passes from under the pole. As the conductor passes the point midway between two adjacent poles, the electromotive force of the machine falls to zero, since no lines of the force are cut
at this point. As the armature continues to revolve and the conductor approaches the next (the south) pole of the field magnet, the electromotive force of the machine again increases in value, but in a direction opposite to that of the previous electromotive force; and it falls again to zero as the conductor passes from under the south pole and reaches the point midway between the next pair of poles.

In Fig. 14 are represented the development of three successive poles $S, N, S$ of the field magnet of an alternator, from which the lines of magnetic flux are emanating, and spreading out more or less as they enter the armature core. The armature core, also a developed view, is shown as having only one slot $A$, which contains a number of armature conductors. The ordinates of the curve $E E E E E E$ represent the successive instantaneous values of the electromotive force induced in the armature conductors as the slot moves from left to right.

The duration of one cycle is indicated in the figure, and this cycle repeats itself as the conductors pass by successive pairs of field poles. Thus, in a ten-pole alternator, there would be five complete cycles, or five complete waves of the electromotive curve for each


Fig. 15. Typical E. MI. F. Curve for Alternator- revolution. When a wave repeats itself after a definite time interval, it is called a periodic wave.

The curve $E E E E E E$ is called the electromotive curve or electromotive force wave of the alternator.

A curve of which the ordinates represent the successive instantaneous values of the alternating current and of which the abscissas represent time, is called an alternating-current curve or alternatingcurrent wave.

Figs. 15, 16, and 17 show typical forms of electromotive force curves given by commercial alternators. Fig. 15 shows what is called a "peaked" wave; Fig. 16 shows a "flat-topped" wave; and Fig. 17 shows a "sine" or "sinusoidal" wave. All three waves are of
course periodic. The exact shape of the electromotive force wave given by an alternator depends upon the relations between pole pitch (distance from center to center of adjacent poles), width, and shape of pole faces, width


Fig. 16. Typical E. M. F. Curve for Alternator"Flat Topped" Wave of armature coils, and distribution of coils on the armature. Alternators which give electromotive force waves approximating a sine wave, are preferred for power transmission.
Average and Effective Values of E. M. F. The average value of an alternating electromotive force or current during a complete cycle is zero, inasmuch as similar sets of positive and negative values occur.

The average value of an electromotive force or current during the positive (or negative) part of a cycle is usually spoken of briefly as the "average value" or "mean value," and is not zero.

Consider now an alternating current, of which the instantaneous value is $i$. The rate at which heat is generated in a circuit through which the current flows is $i^{2} R$, where $R$ is the resistance of the circuit; and the average rate at which heat is generated in the circuit is $R$ multiplied by the average value of $i^{2}$.
A. continuous current which would produce the same heating effect would be one of which the square is equal to the average value of $i^{2}$, or of which the actual value is equal to $v \overline{\text { average } i^{2}}$. This square root of the average square of an alternating current is called the effective value of the alternating current. Similarly, the square


Fig. 17. Typical $\underset{\text { " }}{\text { ESine" }}$ M. Wave . Curve for Alternator - root of the average square of an alternating electromotive force is called the effective value of the alternatingelectromotiveforce. Voltmeters and ammeters used for measuring alternating electromotive force or current always give effective values irrespective of wave form; and in specifying an alternating electromotive force or current, its effective value is always used.

Example. Ten successive instantaneous values of an alternating electromotive force during half a cycle are $0,30,60,80,90,95,90,80,60$, and 30 volts. The sum of these values is 615 volts, which, divided by the number of values, namely ten, gives 61.5 volts, which is the average value of this electromotive force during half a cycle.

Squaring each of the above values, adding the squares together, and dividing their sum by their number, namely ten, gives the


Fig. 18. Rectangular Form of E. M. F. Curve average value of the square of the electromotive force, which is $4,702.5$ volts $^{2}$; and the square root of this average square is 68.57 volts, which is the effective value of the given electromotive force.

Form Factor. The ratio effective value - average value, depends upon the shape of the electromotive force wave, and is called the "form factor" of the wave.

Example. The form factor in the above case is $\frac{68.57}{61.5}$, or 1.115 . The form factor of the electromotive force curve given in Fig. 17, which is a sine wave, is 1.11 . The more peaked the wave the greater the value of its form factor. The rectangular electromotive force shown in Fig. 18 has a form factor equal to unity, which is the least possible value of the form factor. This rectangular wave, however, is never realized in commercial alternators.

Instantaneous and Average Power. Let $e$ be the value, at a given instant, of the electromotive force of an alternator and let $i$ be the value of the current at the same instant. Then $e i$ is the power in watts which is delivered by the alternator at the given instant; and the average value of $e i$ during a complete cycle is the average power delivered by the alternator.


Fig. 19. E. M. F., Current, and Power Curves for an Alternator in a Circuit Containing Inductance

In Fig. 19 the full-line curve represents the electromotive force of an alternator and the heavy-dotted curve represents the current delivered by the alternator to a receiving circuit having
inductance, such as an induction motor, for instance. The ordinates of the light-dotted curve represent the successive instantaneous values of the power $c i$. As shown in the figure, the power has both


Fig. ©0. E. M. F., Current, and Power Curves for Alternating Circuit with Large Inductance
positive and negative values; the alternator does work on the circuit when $e i$ is positive, or is above the horizontal axis of time; and the circuit returns power to the alternator when $c i$ is negative, or is below the horizontal axis of time; and this means of course, that while $e i$ is negative, the dynamo is momentarily a motor and will be for the moment returning power to the fly whetl of the driving engine or turbine.

When the inductance of the receiving circuit is very large, the electromotive force and current curves are related as shown in Fig. 20; the instantaneous power ei passes through approximately similar sets of positive and negative values, as shown by the light-dotted curve; and the average power is approx-


Fig. 21. Diagram of Harmonic E. M. F.s imately zero. This case would be very closely exemplified by an alternator connected to a transformer whose secondary was open-circuited, that is, supplying no current.

Harmonic Electromotive Forces and Currents. A line $O P$, Fig. 21, revolves, at a uniform rate, $f$ revolutions per second about a point $O$, in the direction of the arrow $g h$. Since the length of $O P$ is fixed, the path or locus of the point $P$ will be a circle about $O$ as a center. Consider the projection $O b$ of this rotating line upon the fixed line $A B$, this projection being considered positive when above $O$ and negative when below $O$.

A harmonic electromotive force (or current) is an electromotive force (or current) which is at each instant proportional to the line Ob.

The line $O b$ represents at each instant the actual value $e$ of the harmonic electromotive force to a definite scale, and the length of the line $O P$, which is the maximum length of $O b$, represents the maximum value $E$ of the harmonic electromotive force to the same scale. The line $O b$ passes through a complete cycle of values during one revolution of $O P$, and so also does the harmonic electromotive force $e$. Therefore, the revolutions per second $f$ of the line $O P$ is the frequency of the harmonic electromotive force $e$. The rotating lines $E$ and $I$, Fig. 22, of which the projections on a fixed line (not shown in the figure) represent the actual instantaneous values $e$ and $i$ of a harmonic electromotive force and a harmonic current, are said to "represent" the harmonic electromotive force and current, respectively. Of course, the rotation of the lines $E$ and $I$ is a thing merely to be imagined. The rotation is understood to be in a counter-clockwise direction, as indicated in Fig. 21.

Clock Diagram Representation. A diagram in which a number of electromotive forces or currents, or both, are represented by lines imagined to be revolving, is called a clock diagram. Simple problems involving relations between a number of harmonic electromotive forces and currents of the same frequency, are most easily treated by means of the clock diagram. The


Fig. 22. Clock Diagram proper representation of alternating electromotive forces and currents in a clock diagram, requires that
(a) The given electromotive forces and currents be harmonic, and be of the same frequency.
(b) The lengths of the lines represent their maximum value to a suitable scale although the scales chosen for volts and for amperes may be different.
(c) The direction of the electromotive forces and currents be indicated by arrow heads. -
(d) The relative position or phase of the electromotive forces and currents be constant and indicated by the angle between the various lines representing the given quantities.

When an electromotive force (or current) wave is not a sine curve, the electromotive force (or current) is not harmonic, and can-
not properly be represented by a line in a clock diagram, because the projection of the rotating line is not at each instant proportional to the electromotive force (or current). The approximate represen-


Fig. : 3 . Method of Plotling Sine E. M. F. Curve
tation, by lines in a clock diagram, of non-harmonic electromotive forces (or currents)-such, for example, as those represented by the curves in Figs. 15, 16, and 18, depends upon the finding of harmonic electromotive forces (or currents) which for the particular purpose in view are approximately equivalent to the actual given electromotive forces (or currents).

Graphical Representation. A harmonic electromotive force or current is represented by a sine wave as shown in Fig. 17. The relation between the rotating line $O P$ in Fig. 19 and the sine-wave curve of electromotive force is shown as follows:


Fig. 24. Simple Dynamo Diagram

Divide the circumference of the circle in Fig. 23 into equal parts, and lay off a horizontal line divided into the same number of equal parts.

Draw horizontal dotted lines through each division on the circumference of the circle, and vertical dotted lines through the corresponding divisions on the horizontal line. The points of the intersection of these pairs of dotted lines are points on a curve which is a curve of sines.

A flat loop of wire with its,terminals connected to two collecting rings gives a harmonic electromotive force when it is rotated at constant speed in a uniform magnetic field. This arrangement is shown in Fig. 24.

Algebraic Representation. The line OP, Fig. 21, revolves uniformly $f$ revolutions per second and, therefore, it turns through $2 \pi f$ radians* per second, since there are $2 \pi$ radians in a revolution; that is

$$
\begin{equation*}
\omega=2 \pi f \tag{8}
\end{equation*}
$$

in which $\omega$ is the angular velocity of the line $O P$ in radians per second. Let time be reckoned from the instant that $O P$ coincides with $O a$; then, after $t$ seconds, $O P$ will have turned through the angle $\beta$ ( $=\omega t$ ); and from Fig. 21 we have

$$
O b=O P \sin \beta=O P \sin \omega t
$$

since $O b$ is the projection of $O P$ on the line $A B$. But $O b$ represents the actual value $e$ of the harmonic electromotive force at the time $t$, and $O P$ represents its maximum value $E$; therefore

$$
\begin{equation*}
e=E \sin \omega t \tag{9}
\end{equation*}
$$

is an algebraic expression for the actual value $e$ of a harmonic electromotive force at time $t, E$ being the maximum value of $\rho$, and $\frac{()}{2 \pi}$ being the frequency according to equation (8).
Similarly

$$
\begin{equation*}
i=I \sin \omega t \tag{10}
\end{equation*}
$$

is an algebraic expression for the actual value $i$ of a harmonic current at time $t, I$ being the maximum value of $i$.

If time is reckoned from the instant that $O P$, Fig. 21, coincides with the line $O b$, then equations ( 9 ) and (10) become

$$
\begin{aligned}
& e=E \cos \omega t \\
& i=I \cos \omega t
\end{aligned}
$$

Synchronism. Two alternating electromotive forces or currents are said to be in synchronism when they have the same frequency. Two alternators are said to run in synchronism when their electromotive forces and frequencies are similar.

Phase Difference. Consider two harmonic electromotive forces represented by the ordinates of the curves $E_{1}$ and $E_{2}$, Fig. 25. The electromotive force represented by the curve $E_{1}$ reaches its maximum value before the electromotive force represented by the curve $E_{2}$. The electromotive force $E_{1}$ is said to lead, or to be ahead of, the

[^1]electromotive force $E_{2}$ in phase. Conversely, the electromotive force $E_{2}$ is said to lag behind or to follow the electromptive force $E_{1}$ in phase. The same two


Fig. 25. Curves of Two Related E. M. F.s electromotive forces $E_{1}$ and $E_{2}$ are also represented by the lines $O E_{1}$ and $O E_{2}$ in the clock diagram, Fig. 26. Here the line $O E_{2}$ is behind $O E_{1}$, since the imagined rotation about $O$ as a center is counter-clock-
wise. The phase difference is the time interval $\theta$ in Fig. 25, or the angle $\theta$ between $O E_{1}$ and $O E_{2}$ in Fig. 26. If according to equation (9) the actual value of the harmonic electromotive force $E_{1}$ is $e_{1}=E_{1} \sin \omega t$, then the actual value of the electromotive force $E_{2}$, which lags $\theta$ degrees behind $E_{1}$, is $e_{2}=E_{2} \sin (\omega t-\theta)$. Similarly, if $E_{2}$ were taken as the reference line in the diagram, its actual value would be $e_{2}$ $=E_{2} \sin \omega t$, and the value of $E_{1}$ would then be $e_{1}=E_{1} \sin (\omega t+\theta)$.

When the angle $\theta$, Fig. 26, is zero, as shown in Fig. 27, the electromotive forces $E_{1}$ and $E_{2}$ are said to be in phase. In this case the electromotive forces increase together and decrease together;


Fig. 26. Clock Diagram of E. M. F.s for Fig. 25


Fig. 27. Clock Diagram for Curves in Fig. 25 with E. M. F.s in Phase
that is, when $E_{1}$ is zero, $E_{2}$ is also zero; and when $E_{1}$ is at its maximum value, so also is $E_{2}$, etc. Therefore, $e_{1}=E_{1} \sin \omega t$ and $e_{2}=E_{2} \sin \omega t$.

When $\theta=90^{\circ}$, as shown in Fig. 28, the two electromotive forces are said to be in quadrature. In this case one electromotive force is zero when the other is a maximum, etc., or $e_{1}=E_{1} \sin \omega t$ and $e_{2}=E_{2} \sin$ ( $\omega t-\frac{\pi}{2}$ ).

When $\theta=180^{\circ}$, as shown in Fig. 29, the two electromotive forces are said to be in opposition. In this case they are at each instant opposite in sign; and when one is at its positive maximum, the other is at its negative maximum, etc. In this case $e_{1}=E_{1} \sin \omega t$ and $e_{2}=E_{2} \sin (\omega t \pm \pi)$. It is to be particularly noted that the principle of phase difference which has been illustrated in Figs. 24-27 for the case of two harmonic electromotive forces $E_{1}$ and $E_{2}$, applies equally to the case of two harmonic currents $I_{1}$ and $I_{2}$ and to the case of an electromotive force $E_{1}$ and a current $I_{1}$. Thus if in the clock diagram, Fig. 22, $E$ represented a harmonic electromotive force having a maximum value


Fig. 28. Diagram of E. M. F.s of Fig. 25 in Quadrature of 1,000 volts, and $I$ represented a harmonic current having a maximum value of 10 amperes with a phase difference of $\theta=30^{\circ}$ between them, the instantaneous values of $E$ and $I$ would be

$$
\begin{aligned}
& e=1000 \sin \omega t \\
& i=10 \sin \left(\omega t-\frac{\pi}{6}\right)
\end{aligned}
$$

Addition of Harmonic Electromotive

Fig. 29. Diapram of E. M. F.s of Fig. 25 When in Opposition Forces and Currents. Consider two harmonic electromotive forces of which the successive instantaneous values $e_{1}$ and $e_{2}$ are represented by the projections of the lines $E_{1}$ and $E_{2}$, Fig. 30, which are imagined to be revolving about the point $O$. These electromotive forces being of the same frequency, the lines $E_{1}$ and $E_{2}$ revolve at the same speed, so that the angle between $E_{1}$ and $E_{2}$ remains unchanged in value. The ordinary arithmetical sum of $e_{1}$ and $e_{2}$, namely $e_{1}+e_{2}$, is a harmonic electromotive force of the same frequency as $e_{1}$ and


Fig. 30. Vector Diagram Showing Two Harmonic E. M. F.s $e_{2}$; and this electromotive force ( $e_{1}+e_{2}$ ) is represented by the projection of the line $E$, Fig. 30 , which revolves at the same speed as $E_{1}$ and $E_{2}$.

Th s is evident when we consider that the projection, on any line, of the diagonal of a parallelogram is equal to the sum of the
projections of two adjacent sides of the parallelogram, as shown in Fig. 31. The projection of $E_{1}$ is $O c$, which represents $e_{1}$; and the projection of $E_{2}$ is equal to $c d$,


Fig. 31. Diagram Showing Addition of Harmonic E. M. F.s which represents $e_{2}$. The projection of the diagonal $O b$ of the parallelogram is $O d$, which is the sum of $O c$ and $c d$. The two lines marked $E_{2}$ in Fig. 31 are equal and parallel and have, therefore, the same projected length on the vertical line.

As a corollary to the above, it may be stated that the ordinary arithmetical sum ( $e_{1}+e_{2}+e_{3}+$ etc.) of the instantaneous values of any number of harmonic electromotive forces (or currents) is another harmonic electromotive force (or current) of the same frequency; it is represented in magnitude and phase by a line that is the geometric (or vector) sum of the lines representing the given individual electromotive forces (or currents). This is evident when we consider that $e_{1}+e_{2}$ is a harmonic electromotive force (or current) according to the above discussion; and this, added to $e_{3}$, gives an electromotive force (or current) which is harmonic and of the same


Fig. 32. Addition of E. M. F.s by Vector Polygon frequency as $e_{1}, e_{2}$, and $e_{3}$.

The geometric or vector sum of a number of lines is obtained as follows: Given three lines $O E_{1}, O E_{2}$, and $O E_{3}$, Fig. 32. Find the diagonal $O A$ of the parallelo. gram constructed on $O E_{1}^{\prime}$ and $O E_{2}$ as sides. This gives the vector sum of $O E_{1}$ and $O E_{2}$. Next construct a parallelogram on $O A$ and $O E_{2}$ as sides; the diagonal $O E$ of this parallelogram is the vector sum of the three given lines. This line $O E$ is the closing side of the polygon formed by drawing $O E_{1}$, then drawing $O E_{2}$ from the extremity of $O E_{1}$ (this giving the point $A$ ), and then drawing $O E_{3}$ from $A$. This method is called addition by means of the vector polygon. For example, two alternators $A$ and $B$ running in synchronisth
are connected in series between mains as shown in Fig. 33. If the electromotive forces of $A$ and $B$ are in phase, the electromotive force between the mains will be simply the numerical sum of the electromotive forces of $A$ and $B$. If, on the other hand, the electromotive forces of $A$ and $B$ differ in phase, the state of affairs will be as represented in Fig. 34, in which the lines $A$ and $B$ represent the


Fig. 34. Vector Diagram of Conn ditıons in Fig. 33 if E. M. F.s Differ in Phase
electromotive forces of the alternators $A$ and $B$, respectively, $\theta$ is the phase difference of $A$ and $B$, and the line $E$ represents the electromotive force between the mains. The line $E$ is the vector sum or resultant of $A$ and $B$, and as shown is the diagonal of a parallelogram constructed on $A$ and $B$ as sides.

Again, alternators $A$ and $B$ running in synchronism are connected in parallel between the mains as shown in Fig. 35. Let the lines $A$ and $B$, Fig. 36, represent the currents given by the alternators $A$ and $B$, respectively, the phase difference between the currents being $\theta$; then the current in the main line is represented by $I$.

Consider the case of two circuits $A$ and $B$, Fig. 37, connected in


Fig. 35. Two Alternators Running in Synchronism Connected in Parallel


Fig. 36. Vector Diagram of Currents for the Condition of Fig. $3 \overline{5}$
series between the mains of an alternator. The line E, Fig. 38, represents the electromotive force between the mains; the line $A$
represents the electromotive force between the terminals of the circuit $A$; and the line $B$ represents the electromotive force becween the terminals of the circuit $B$. The circuits $A$ and $B$ are


Fig. 37. Diagram of Two Coils in Series with an Alternator


Fig. 38. Diagram of E. M. F.s for Conditions Shown in Fig. 37
supposed to have inductance. If either of the circuits contains a condenser, then the electromotive forces $A$ and $B$, Fig. 38, may be nearly opposite to each other in phase, and $A$ and $B$ may each be indefinitely greater than the electromotive force $E$ between the mains.

Again, two circuits $A$ and $B$, Fig. 39, are connected in parallel across the terminals of an alternator as shown. The current $I$ from the alternator is related to the currents $A$ and $B$ as shown in Fig. 40. If either of the circuits $A$ or $B$ contains a condenser, then the currents $A$ and $B$ may be nearly opposite to each other in phase, and the currents $A$ and $B$ may each be indefinitely greater than the current $I$ from the alternator.


Fig. 39. Coils in Parallel with an Alternator

Subtraciion of Harmonic Electromotive Forces and Currents. One harmonic electromotive force (or current) is subtracted from another by reversing the direction of the line that represents it in the clock diagram, and then adding the reversed line (or vector) to the other. An example of the subtraction of harmonic electromotive forces will be given in connection with the discussion of three-phase electromotive force.
Relation between Maximum and Effective Values. The effective value $E$ or $I$ of a harmonic, electromotive force, or current-that is,
one whose graph is a sine wave-is equal to the maximum value E or I divided by the square root of 2 . That is

$$
\begin{gather*}
E=\frac{\mathrm{E}}{\sqrt{2}}  \tag{11}\\
I=\frac{\mathrm{I}}{\sqrt{2}} \tag{12}
\end{gather*}
$$

This may be shown as follows: Let $e$ (=E $\sin \omega t$ ) be a harmonic electromotive force. To find the average value of $e^{2}$ ( $=\mathrm{E}^{2} \sin ^{2} \omega t$ ), it is necessary to find the average value of the square of the sine of the uniformly variable angle $\omega t$. We have the general relation


Iig. 40. Current Diagram ot the Conditions Shown in Fig. 39

$$
\text { (a) } \sin ^{2} \omega t+\cos ^{2} \omega t=1
$$

so that

$$
\text { (b) Av. } \sin ^{2} \omega t+\mathrm{Av} \cdot \cos ^{2} \omega t=1
$$

Now, during a cycle, the cosine of a uniformly variable angle passes similarly through the same set of values as the sine; hence Av. $\sin ^{2} \omega t$ and $\mathrm{Av} . \cos ^{2} \omega t$ are equal, so that from equation (b) above, we have

$$
2 \mathrm{Av} \cdot \sin ^{2} \omega t=1
$$

or

$$
\text { Av. } \sin ^{2} \omega t=\frac{1}{2}
$$

The average value of $e^{2}$ is

$$
\text { Av. } e^{2}=\mathrm{E}^{2} \mathrm{Av} \cdot \sin ^{2} \omega t
$$

or

$$
\text { Av. } e^{2}=\frac{\dot{\mathrm{E}}^{2}}{2}
$$

and

$$
\sqrt{\text { Av. } e^{2}}=\frac{\mathrm{E}}{\sqrt{2}}
$$

In Fig. 21 the length of the revolving line $O P$ was understood to represent the maximum value of the harmonic electromotive force (or current). When, however, a number of harmonic electromotive forces '(or currents) are represented to scale by lines in a clock diagram, the lengths of the lines may be interpreted as giving
not maximum but effective values, since there is a constant ratio $\sqrt{2}$ between the effective and the maximum values of each of the electromotive forces (or currents) represented in the diagram.

Note. It is desirable to interpret the lines in a clock diagram in terms of effective values rather than maximum values, because effective values are always given by measuring instruments and are nearly always used in numerical calculation. Therefore, unless it is expressly stated to the contrary, the lines in clock diagrams are always understood to represent effective values.

For example, a certain harmonic alternating current gives a reading of 100 amperes on an alternating-current ammeter, and its effective value is, therefore, 100 amperes. This harmonic current actually pulsates between zero and a maximum value of $\pm \sqrt{2}$ $\times 100$ amperes, or $\pm 141.4$ amperes.

Again, a certain harmonic alternating electromotive force gives a reading of 1,000 volts on an alternating-current voltmeter, and its effective value is, therefore 1,000 volts. This electromotive force actually varies between zero and a maximum value of $\pm \sqrt{2} \times$ 1,000 volts, or $\pm 1,414$ volts.

The above simple relation between maximum and effective values is true only for harmonic (that is, sine-wave) electromotive forces and currents. In general, the maximum values of alternating electromotive forces or currents cannot be inferred from effective values as measured by voltmeters or ammeters. Thus, an alternating electromotive force which is known to have a peaked-wave form might have a maximum value very greatly in excess of $\sqrt{2}$ times its effective value.

Expression for Power. (a) When the current is in phase with the electromotive force, that is, when the circuit is non-inductive, then the power (average $e i$, see page 22) is

$$
\begin{equation*}
P=E I \tag{13}
\end{equation*}
$$

in which $P$ is the power in watts, $E$ is the effective value of the electromotive force in volts, and $I$ is the effective value of the current in amperes. Equation (13) is identical with the power equation for direct-current circuits.
(b) If the phase difference between current and electromotive force in a given circuit were 90 degrees, which can never actually occur, then the power (average value of $e i$ ) would be equal to zcro, as explained on page 23.
(c) When the phase difference between current and electromotive force is $\theta^{\circ}$, as shown in Fig. 41, then

$$
\begin{equation*}
P=E I \cos \theta \tag{14}
\end{equation*}
$$

in which $P$ is the power (average $e i$ ) in watts, $E$ is the effective value of the electromotive force in volts, and $I$ is the effective value of the current in amperes.

For example, the given current $I$ shown in Fig. 41 may be thought of as resolved into two components, as shown in Fig. 42. One of these components $I \cos \theta$ is parallel to (that is, in phase with) $E$; and the other $I \sin \theta$ is at right angles to $E$. The power corresponding to the actual current $I$ may be thought of as the sum of the powers corresponding to its two components, respectively. But the component $I \sin \theta$ is at right angles to $E$, as in (b) above; hence the power corresponding to it is zero. This component is, therefore, often called the watless component or, more correctly, the reactive component of the given current.

On the other hand, the component $I \cos \theta$ is parallel to (that is, in phase with) $E$, as in (a) above; hence the power corresponding to this component is equal to $E \times I \cos \theta$. The component $I \cos \theta$ of the given current $I$ is called the power compo-


Fig. 41. Diagram Showing Phase Difference Between E. M. F. and Current nent or, more correctly, the active component of $I$, and the factor $\cos \theta$ the power factor of the circuit.

Inductance. It has been pointed out, page 18, that an electric circuit has a certain kind of inertia analogous to the inertia of water in a circuit of pipe, and it was there noted that this inertia of an electric circuit is called inductance. If an electric current in a circuit is made to change in value, a portion of the electromotive force acting upon the circuit must be used to cause the current to change.

In the same way a force over and above that required to overcome frictional resistance must act upon a moving body to accelerate it, that is, to make its speed increase. The inertia of a body is measured by the force required to accelerate it at the rate of unit change in speed per second; and the inductance of a circuit is measured by the electromotive force required to cause a current in the circuit
to change at the rate of one ampere per second. A circuit is said to have an inductance of one henry* when one volt (over and above the electromotive force required to overcome the electrical resistance) will cause the current to change at the rate of one ampere per second.

Let $x$ be the rate in amperes per second at which the current in a circuit is increasing in value. Then the electromotive force $E$ (over and above that required to overcome the resistance of the circuit) required to cause the current to increase at this rate is

$$
\begin{equation*}
E=L x \tag{15}
\end{equation*}
$$

in which $L$ is the inductance of the circuit in henrys, $E$ being expressed in volts.


Fig. 42. Resolution of the I of Fig. 41 into Two Components

Example. The coil of a certain large electromagnet has 2.5 henrys of inductance and 5 ohms of resistance. At a given instant this coil is connected to 110 -volt direct-current mains. At the instant of connecting the coil, the current is zero, and all of the 110 volts is used to cause the current in the coil to increase, so that, according to equation (15), $x$ is equal to $\frac{E}{L}$, or 110 volts $\div 2.5$ henrys, or 44 amperes per second. That is, the current in the magnet coil begins to increase at the rate of 44 amperes per second. When the current in the magnet coil has reached the value of 10 amperes, 50 volts ( $=5$ ohms $\times 10$ amperes) of the total 110 volts are used in overcoming the resistance of the coil, so that 60 volts ( $=110$ volts -50 volts) are used to make the current increase. Therefore, as the current in the coil passes the value of 10 amperes, it is increasing at the rate $\frac{E}{L}$, or 60 volts $\div 2.5$ henrys, or 24 amperes per second.

The inductance of a coil wound on a given spool is proportional to the square of the number of turns $N$ of wire. For example, a given spool wound with No. 16 wire has 500 turns and an inductance of, say, 0.0025 henry; the same spool wound with No. 28 wire would have about eight times as many turns, and its inductance would be about 64 times as great, or 0.16 henry.

[^2]The inductance of a coil of given shape is proportional to its linear dimensions, the number of turns of wire being unchanged. For example, a given coil has an inductance of 0.022 henry; and a coil three times as large in length, diameter, etc., but having the same number of turns of wire, has an inductance of 0.066 henry.

To Prove $L=\frac{\Phi N}{I}$. In Fig. 43 is shown a coil of $N$ turns containing negligible resistance and carrying a direct current of $I$ amperes which sets up a flux of $\Phi$ lines of force. If the number of turns remains fixed and there is no iron in the circuit, the value of $\Phi$ varies directly with the current $I$. Thus for every change of one ampere the value of $\Phi$ will change in a corresponding manner by an amount $\frac{\Phi}{I}$ (flux per ampere) which is a constant depending on the size, shape, number, and arrangement of the

coils if no iron is present. If for any reason the current flowing from $a$ to $b$ should increase at the rate of 1 ampere per second, it is obvious that the flux through the coil will increase at the rate of $\frac{\Phi}{I}$ lines per second and in each turn of the coil there will be induced $\frac{\Phi}{I} a b$ volts, or $\frac{\Phi}{10^{8} I}$ volts, in such a direction as to oppose the increase in current; or, according to Lenz's law, from $b$ to $a$. Since the foregoing value of voltage is induced in each of the $N$ turns and the turns are in series, the total induced voltage from $b$ to $a$ will be $\frac{N \Phi}{10^{8} I}$ volts. If for any rcason the current $I$ from $a$ to $b$ should decrease at the rate of 1 ampere per second, a voltage of $\frac{N \Phi}{10^{8} I}$ volts would be induced in a direction to oppose the decrease, or from $a$ to $b$.

Should the current change at any other rate than 1 ampere per second, the induced voltage would be numerically equal to $\frac{N \Phi}{10^{8} I}$ times the rate of change of current in amperes per second. Let us choose the positive direction of current from $a$ to $b$. Then for a positive change, or increase, in current the induced voltage is negative and for a negative change, or decrease, in current the induced voltage is positive, so that to properly represent the induced voltage in magnitude and direction it is necessary to write
$e($ induced $)=\frac{N \Phi}{10^{8} I}$ times the rate of change of $I$
It so happens that $\frac{N \Phi}{10^{8} I}$ is a constant for any one particular coil and is represented by the symbol $L$ commonly known as the inductance. Thus

$$
e=-L \text { times the rate of change of } I
$$

From what has been said in preceding paragraphs it might be inferred that inductance has a tendency to keep the current at a constant value, and this is correct; for, should the terminals $a$ and $b$ be joined by a conductor of zero resistance, the current $I$ would flow forever. To make $I$ increase in the direction of $a$ to $b$ would require a voltage in this direction, and the rate of increase of $I$ caused by this applied voltage would be such a value as to make the induced voltage exactly equal and opposite to the applied voltage. This same balancing effect should and will exist if a voltage is applied from $b$ to $a$ to cause the current to decrease. Thus we can say
$e($ applied $)=L$ times the rate of change of $I$
Thus a positive applied voltage causes a positive rate of change of $I$, and a negative applied voltage causes a negative rate of change of $I$.

Example. A current of 5 amperes is passed through a coil of 1000 turns and produces 100,000 lines of flux. Calculate the inductance in henrys, assuming that no iron is present.

Solution. $\quad L=\frac{\Phi N}{10^{8} I}=\frac{100000 \times 1000}{10^{8} \times 5}=0.2$ henry

Formulas for Inductance. The inductance in henrys of a coil of wire wound in a thin layer on a long wooden cylinder having a length of $l$ centimeters and a radius of $r$ centimeters may be proved to be

$$
\begin{equation*}
L=\frac{4 \pi^{2} r^{2} N^{2}}{l \times 10^{9}} \tag{16}
\end{equation*}
$$

in which $N$ is the total number of turns of wire in the coil. This equation is strictly true for very long coils wound in a thin layer; but the equation is also very useful in calculating the approximate inductance of even short, thick coils. Thus, a coil having a length of 25 centimeters and a mean radius of $2 \frac{1}{2}$ centimeters, containing 150 turns of wire, has from equation (16) an approximate inductance of

$$
L=\frac{4 \pi^{2}(2.5)^{2} \times 150^{2}}{25 \times 10^{9}}=0.00022 \text { henry }
$$

If a coil of $N$ turns of wire is wound on a long rod of iron, instead of wood or other non-magnetic material, the inductance in henrys is given by the equation

$$
\begin{equation*}
L=\frac{4 \pi^{2} r^{2} N^{2} \mu}{l \times 10^{9}} \tag{17}
\end{equation*}
$$

in which $L, r, N$, and $l$ are the same as in equation (16), and $\mu$ is the permeability of the iron core at the particular flux density produced in the iron. This equation applies to any iron rod of length $l$ centimeters and radius $r$ centimeters, wound with $N$ turns of wire, whether in the form of a long, straight rod or bent into a closed ring.

The permeability $\mu$ of iron varies from 500 to 1,000 or more; and, therefore, the effect* of placing an iron core in a coil is to increase greatly the inductance of the coil.

The iron core of an inductance coil to be used with alternating currents should be laminated to reduce eddy currents and the con-

[^3]sequent loss of energy, and to prevent excessive heating of the core.
For example, the inductance of the field coil of a certain shuntwound dynamo is 7.5 henrys. The inductance of a pair of No. 0 , B. \& S. copper line wires carried at a distance of 18 inches apart on a pole line is 0.0035 henry per mile. The inductance of the secondary coil of a large induction coil (X-ray coil) having 200,000 . turns of wire, is 2,000 henrys.

Series and Parallel. The inductance of two or more coils in series is equal to the sum of the individual inductances.

The equivalent inductance of two or more similar coils in parallel, such as the similar coils on an armature, is equal to $\frac{1}{n}$ of the inductance of one coil, $n$ being the number of coils connected in parallel.

Capacity. It has been pointed out, page 18, that a charged condenser has an elastic-like reaction analogous to the elastic reaction of a distorted diaphragm. The elasticity of a diaphragm might be measured by the pressure required to distort it to the extent of producing one unit of increase of volume in the space on one side of the diaphragm and one unit of decrease of volume in the space on the other side of the diaphragm. Similarly, the capacity of a condenser may be, and in fact is, measured by the electromotive force required to force one unit of charge of electricity into one plate of the condenser, and at the same time to withdraw one unit of charge from the other plate. A condenser is said to have a capacity of $1 \mathrm{farad}^{*}$ when one volt of clectromotive force pushes one coulomb of electric charge into the condenser.

The charge $Q$ pushed into a condenser by a steady electromotive force $E$ is

$$
\begin{equation*}
Q=C E \tag{18}
\end{equation*}
$$

in which $C$ is the capacity of the condenser in farads, $Q$ is the charge in coulombs, and $E$ is the electromotive force in volts. The clectromotive force required to hold a given charge $Q$ in the condenser is of course equal to $\frac{Q}{C}$.

Condensers, to have a large capacity (as much as a microfarad), are usually made up of alternate sheets of tinfoil and waxed paper,

[^4]TABLE II
Inductivities of Dielectrics
Referred to Air as Unity

| Glass | 3.00 to 10.00 | Mica |  |
| :--- | :--- | :--- | :--- |
| Vulcanite | 2.50 | Shellac | 4.00 to 8.00 |
| Paraffin | 1.68 to 2.30 | Turpentine | 2.93 to 3.3 |
| Beeswax | 1.86 | Petroleum | 2.15 to 2.43 |
| Gutta-percha | 3.3 to 4.9 | Rubber (pure India) | 2.12 to 2.19 |

or mica, as indicated in Fig. 44. Alternate metal sheets are connected together giving two terminals as shown, thus practically forming two metallic plates of large area. A condenser having a capacity of 1 microfarad contains about 3,600 square inches of tinfoil. Inductivity of Dielectric. The capacity of a condenser of given dimensions depends upon the material used between the plates, called the dielectric. The quotient, capacity of a condenser


Fig. 44. Diagram of Condenser with given dielectric $\div$ its capacity with air as the dielectric, is called the inductivity or the specific inductive capacity of the given dielectric. The values of the inductivity for the most commonly used dielectrics are given in Table II.

Capacity of Condensers. The capacity of a condenser is given by the equation

$$
\begin{equation*}
C=\frac{885 k A}{10^{10} d} \tag{19}
\end{equation*}
$$

in which $C$ is the capacity in microfarads, $k$ is the inductivity of the dielectric used, $d$ is the distance in centimeters between plates, $i . e$., the thickness of the dielectric, and $A$ is the area in square centimeters of both sides of all the imer plates plus the area of the inner surfaces of the two outside plates.

Note. If the total number of plates $=n, A$ will be the area of both sides of ( $n-1$ ) plates; or, in other words, it is necessary to provide one more plate than is necessary, using both sides, to equal an area of $A$ square centimeters.

Example. It is required to design a plate condenser to have a capacity of 1.5 microfarads using a dielectric of oiled paper 0.0043 inch thick and tinfoil 0.0007 inch thick.

Assuming for the oiled paper an inductivity of 2.67, equation (19) gives for the total active plate area
$A=\frac{1.5 \times 10^{10} \times 0.0043 \times 2.54}{885 \times 2.67}=69400$ square centimeters $=10759$ square inches
If 65 plates be used, the area of one side of each plate will be

$$
\frac{10759}{2(65-1)}=84 \text { square inches }
$$

so that the dimensions of one active plate may be $10 \frac{1}{2}^{\prime \prime} \times 8^{\prime \prime}$ and the area of tinfoil needed will be $84 \times 65=5460$ square inches.

The capacity of an ordinary 2-quart Leyden jar is about 0.005 microfarad. The capacity of an average submarine telegraph cable is about 0.4 microfarad per nautical mile. The capacity of a pair of transmission lines of No. 0, B. \& S. wires placed 18 inches apart between centers on poles, is 0.036 microfarad per mile.

Series and Parallel. The capacity of a number of condensers in parallel is equal to the sum of the individual capacities.

The capacity of a number of condensers in series is

$$
\begin{equation*}
C=\frac{1}{\frac{1}{C_{1}}+\frac{1}{C_{2}}+\frac{1}{C_{3}}+\text { etc. }} \tag{20}
\end{equation*}
$$

in which $C_{1}, C_{2}, C_{3}$, etc., are the individual capacities, and $C$ is the joint capacity.

Fundamental Equations of the A. C. Circuit. An alternator $A$, Fig. 45, delivers an alternating current of $I$ amperes, effective, to a circuit consisting of a resistance of $R$


Fig. 45. Alternating Circuit Containing Resistance, Inductance, and Capacity ohms, an inductance of $L$ henrys, and a condenser of which the capacity is $C$ farads, all connected in series. It is to be remembered that any coil having inductance has resistance also; that is, inductance and resistance are practically inseparable. Nevertheless inductance and resistance are essentially different in nature and in their effects, and they are always considered separately, so that it is helpful to think of them as actually separated in a circuit, as indicated in Fig. 45. A resistance is conventionally represented thus, $\sim \sim \sim \sim$; an inductance thus, $-\infty \infty \infty$; and a condenser thus,

The current in the circuit, Fig. 45, is assumed to be harmonic, that is, to be a sine-wave current, and this current is represented by the line $O I$ in Fig. 46.

A portion of the electromotive force of the alternator is used to overcome the resistance of the circuit. The portion of the electromotive force so used is an alternating electromotive force of which the effective value is RI; it is in phase with the current, and is represented by the line RI in Fig. 46.

A portion of the electromotive force of the alternator is used to overcome the inertia or inductance of the circuit in causing


Fig. 46. Vector Diagram of Conditions in Fig. 45 the current to increase and decrease. The portion of the electromotive force so used is an alternating electromotive force of which the effective value is $\omega L I$; it is 90 degrees ahead of $I$ in phase, and is represented by the line $\omega L I$ in Fig. 46. The quantity $\omega$ is equal to $2 \pi$ times the frequency of the current $I$.

A portion of the electromotive force of the alternator is used to overcome what we have previously called the electro-elasticity of the condenser, that is, the force necessary to hold the electric charge on the condenser plates at each instant. The portion of the e.m.f. so used is an alternating electromotive force of which the effective value is $\frac{I}{\omega C}$; it is 90 degrees behind $I$ in phase, and is represented by the line $\frac{I}{\omega C}$ in Fig. 46.

The total electromotive force $E$ of the alternator is equal to the geometric (or vector) sum of the parts $R I, \omega L I$, and $\frac{I}{\omega C}$. This vector sum is formed by subtracting $\frac{I}{\omega C}$ from $\omega L I$, since it is opposite to $\omega L I$ in direction, and then adding $R I$ and $\left(\omega L I-\frac{I}{\omega C}\right)$ geometrically, as shown in Fig. 47, in which the line $O_{a}$ represents $\left(\omega L I-\frac{I}{\omega} \bar{C}\right)$
and the line $E$ represents the geometric sum of $O a$ and $R I$.
From Fig. 47 we have, by geometry:

$$
E^{2}=R^{2} I^{2}+\left(\omega L I-\frac{\mathrm{I}}{\omega C}\right)^{2}
$$

or

$$
E^{2}=I^{2}\left[R^{2}+\left(\omega L-\frac{1}{\omega C}\right)^{2}\right]
$$

or

$$
\begin{equation*}
I=\frac{E}{\sqrt{R^{2}+\left(\omega L-\frac{1}{\omega C}\right)^{2}}} \tag{21}
\end{equation*}
$$

The quantity $\omega L-\frac{1}{\omega \bar{C}}$ is called the reactance of the circuit. The term $\omega L$ is often called inductance reactance; and the term $\frac{1}{\omega C}$ is often called capacity reactance. Inductance reactance is always positive, and capacity reactance is always negative. It is convenient to represent the reactance $\omega L-\frac{1}{\omega C}$ of a circuit by the single letter $X$; that is

$$
\begin{equation*}
X=\omega L-\frac{1}{\omega C} \tag{22}
\end{equation*}
$$

Therefore, writing $X$ for $\omega L-\frac{1}{\omega C}$ in equation 21 , we have

$$
\begin{equation*}
I=\frac{E}{\sqrt{R^{2}+X^{2}}} \tag{23}
\end{equation*}
$$

Furthermore, from the right triangle in Fig. 47 we have

$$
\tan \theta=\frac{\omega L-\frac{1}{\omega C}}{R}
$$

or

$$
\begin{equation*}
\tan \theta=\frac{X}{R} \tag{24}
\end{equation*}
$$

in which $\theta$-is the angle of phase lag of the current $I$ behind the electromotive force $E ; X$ is the reactance of the circuit; and $R$ is the resistance of the circuit.

Resistance, Reactance, and Impedance. Consider a harmonic alternating electromotive force $E$ which produces a harmonic alternating current $I$ in a circuit. This electromotive force may be resolved into two components, one parallel and the other perpendicular to $I$, as shown, for example, in Fig. 46. The component of $E$ parallel to $I$ is equal to $R I$.

The resistance of an al-ternating-current circuit is sometimes defined as that


Fig. 47. Complete Diagrain of Conditions in Fig. 45 factor which, multiplied by the current, gives the component (of the electromotive force) which is parallel to $I$.

The component of $E$ perpendicular to $I$ is equal to $\omega L I-\frac{I}{\omega C}$, or is equal to $X I$.

The reactance of an alternating-current circuit may be defined as that factor which, multiplied by the current, gives the component (of the electromotive force) which is perpendicular to I.

The factor $\sqrt{\overline{R^{2}+X^{2}}}$, which, when multiplied by the current $I$, gives the total value of the electromotive force $E$, is called the impedance (denoted by $Z$ ) of the alternating-current circuit. Of course, $E$ divided by $Z$ gives the value of the current $I$.

Note. Resistance, reactance, and impedance are all expressed in ohms; we may, for example, speak of 10 ohms of resistance, 10 ohms of reactance, or 10 ohms of impedance. Thus, ohms are used in alternating-current work to express the three essentially different things-resistance, reactance, and impedance; and a specification of a certain number of ohms is not intelligible unless it is stated whether it is ohms of resistance, ohms of reactance, or ohms of impedance.

The reactance and the impedance of a circuit depend upon the frequency of the alternating current, as well as upon the physical constants $L$ and $C$ of the circuit, since the factor $\omega$ is equal to $2 \pi$ times the frequency.

The reactance of a circuit may be positive or negative, according as $\omega L$ is larger than or less than $\frac{1}{\omega C}$. When reactance is posi-
tive, the inductance reactance $\omega L$ exceeds the capacity reactance $\frac{1}{\omega C}$, and the current is behind the electromotive force in phase, as shown in Fig. 47. When the total reactance is negative, however, the capacity reactance exceeds the inductance reactance, and the current is ahead of the electromotive force in phase, as shown in Fig. 48. The same results may be obtained from equation (24). If the total reactance $X$ is negative, then $\tan 0$ is negative, which means that 0 is a negative angle, or that the electromotive force is behind the current in phase, or that the current is ahead of the electromotive force.

Special Cases of Electromotive Force and Current Relations. A clear understanding of the following examples as special cases of the general relations of electromotive force and current as discussed on pages 42 and 43 depends upon the following facts:
(a) That the effect of inductance in an alternating-current circuit becomes negligible when the inductance is very small, for then the reactance $\omega L$ due to the inductance is small, and the portion of the electromotive force required to overcome the inductance, namely, $\omega L I$, Fig. 46 , is also small.


Fig. 48. Conditions of Fig. 45 When Total Reactance is Negative
(b) That the effect of a condenser in an alternating-current circuit becomes negligible only when the capacity of the condenser is very large, for then the reactance $-\frac{I}{\omega C}$ due to the condenser is small, and the portion of the electromotive force required to overcome the electro-elasticity of the condenser, namely, $\frac{I}{\omega C}$, Fig 46, is also small.

The effect of an inductance may be rendered negligible by short-circuiting it with a low-resistance wire; and the effect of a
condenser also may be rendered negligible by short-circuiting it with a low-resistance wire.

Case A. Non-inductive or non-reactive circuits. A circuit which does not contain a condenser and does not have any perceptible inductance is called a non-reactive circuit. The term noninductive is frequently used in the sense in which non-reactive is here defined. A non-reactive circuit contains only resistance; and the total electromotive


Fig. 49. Diagram of a Non-Reactive Alternating Circuit force required to produce a given alternating current $I$ in a non-reactive circuit of which the resistance is $R$ ohms, is $R I$ volts,* and the electromotive force and current are in phase with each other. Therefore, the relation between alternating electromotive force and current in a non-reactive circuit is precisely the same as in the case of direct currents. That is

$$
E=R I
$$

or

$$
\begin{equation*}
I=\frac{E}{R} \tag{25}
\end{equation*}
$$

Fig. 49 represents a non-reactive circuit connected to an alternator $A$; and Fig. 50 shows the relation between the electromotive force and current.

Any circuit in which the outgoing and returning wires are very near together, has very small inductance. An ordinary incandescent lamp, for example, has a negligible inductance. An incandescent lamp the resistance of which when hot is 220 ohms, takes half an ampere, effective, when connected to alternating-


Fig. 50. E. M. F. and Current Relations for a Non-Reactive Alternating Circuit current supply mains between which the effective electromotive force is 110 volts; the current is in phase with the electromotive force and the power in watts is equal to the product of effective volts times effective amperes, or 55 watts. Alternating-current voltmeters are always made as nearly as possible non-inductive.

[^5]Case B. Circuits containing resistance and inductance. In this case the reactance $X(=\omega L)$ is positive, and the current lags behind the electromotive force in phase, as before


Fig. 51. Alternating Circuit Containing Resistance and Inductance pointed out. The tangent of the angle of lag is equal to $\frac{X}{R}$, according to equation (24); therefore, the angle of lag of the current is small when $X$ is small compared with $R$, and the angle of lag approaches $90^{\circ}$ when $X$ is very large compared with $R$.

Fig. 51 represents a circuit containing resistance and inductance, connected to an alternator $A$; and Fig. 52 shows the relation between the electromotive force and current.

Examples. 1. A non-inductive resistance takes 10 amperes from 220 volt, 60 -cycle mains. What current will it take: (a) from 220 -volt, 25 -cycle mains; (b) from 110-volt, 60-cycle mains?

Since the resistance is non-inductive, the impedance is equal to the resistance, and $I=\frac{E}{R}$. Solving for $R$ we obtain $R=\frac{220}{10}=22$ ohms.
(a) The current will be $I=\frac{220}{22}=10$ amperes.
(b) Since there is no inductance or capacity in the circuit, the impedance is independent of the frequency. Therefore, $I=\frac{110}{22}=5$ amperes.
2. An impedance coil of negligible resistance takes 3 amperes from 220 -volt, 60 -cycle mains. What current will it take (a) from 220 -volt, 25 cycle mains? (b) from 110 -volt, 60 -cycle mains?

In this case the resistance $R$ is zero, so that the impedance is equal to the reactance, and $I=\frac{E}{X}$. Solving for $X$ we obtain $X=\frac{220}{3}=73.3$ ohms when the frequency is 60 cycles.
(a) When the frequency is reduced from 60 to 25 cycles, the reactance $X$ is reduced in the same ratio since $X=2 \pi f L$. Therefore, the reactance at 25 cycles is $73.3 \times \frac{25}{60}$ ohms. The current is $I=\frac{220 \times 60}{73.3 \times 25}=7.2$ amperes.
(b) Since frequency is again 60 cycles, $X=73.3$, and $I=\frac{110}{73.3}=1.5$ amperes.

A coil of wire usually has a very considerable inductance, especially if it is wound on a laminated iron core. In fact, a coil
wound on a laminated iron core usually has so large a reactance $X(=\omega L)$, that the angle $\theta$, Fig. 52 , is very nearly $90^{\circ}$.

Example. A certain coil has a resistance of 2 ohms and an inductance of 0.3 henry when provided with a laminated-iron core. This coil is connected to an alternator giving 1,000 volts effective electromotive force at a frequency of 133 cycles per second, so that the factor $\omega$ is equal to $2 \pi \times 133$, or 835.7 radians per second; the reactance of the coil is $835.7 \times 0.3$, or 250.7 ohms; the impedance is $\sqrt{2^{2}+250.7^{2}}$ or 250.7 ohms; the current is $\frac{1,000}{250.7}$, or $3.989 \mathrm{am}-$ peres; the current lags about $89 \frac{1}{2}^{\circ}$ behind the electromotive force; and the power delivered to the coil is 1,000 volts $\times 3.989$ amperes $\times \cos 89 \frac{1}{2}^{\circ}(0.008)$, which is equal to 31.9 watts ( $=I^{2} R$ ). The product $E I$, sometimes called $a p$ parent watts, is equal to 3,989 volt amperes.

This example illustrates one remarkable feature of alternating currents-namely, the very small amount of actual power that is delivered to a circuit of large reactance even though the electromotive force is large and the current considerable. In the case of a direct current, 3.989 amperes taken from 1,000-volt mains would mean an actual delivery of 3,989 watts of power, while in the above case the actual power delivered is only 31.9 watts. The ratio true watts $\div$ apparent watts is called the power factor of a circuit; and in case of the coil here under discussion, this ratio is equal


Fig. 52. Diagram of E. M. F.s and Current Relations for Conditions in Fig. 51 to about $0.008(=\cos \theta)$.

One never encounters in practice a circuit in which the reactance is so large compared to the resistance as in the above example; that is, one never encounters one in which the power factor is so small as 0.008 . Cases are often met with, however, where the reactance is from two to ten times as large as the resistance. Thus, one of the primary windings of a certain 110 -volt induction motor has a resistance of 0.7 ohm and a reactance of 4.2 ohms . With zero load this circuit, equation (23), takes $\frac{110}{\sqrt{0.7^{2}+4.2^{2}}}=\frac{110}{4.258}$, or 25.83 amperes; the angle of phase lag of the current, according to equation (24), is about $80 \frac{1}{2}^{\circ}$; and the power factor of the circuit is 0.164 .

A circuit which contains a coil wound on an iron core takes more power than is expended in the mere heating of the wire, namely,


Fig. 53. Circuit Containing Resistance and Capacity $I^{2} R$, for some power is consumed in the iron core on account of magnetic hysteresis and eddy currents. In the above examples this consumption of power in an iron core is neglected for the sake of simplicity.

The term equivalent resisto ance is used to designate a fictitious resistance which when multiplied by $I^{2}$ gives the actual power consumed by such a circuit, including both the power consumed in heating the wire, and that consumed by core loss. The equivalent resistance of such circuits has a value larger than the mere resistance of the copper winding. Case C. Circuits containing resistance and a condenser. In this case the reactance $X\left(=-\frac{1}{\omega C}\right)$ is negative, and the current leads the electromotive force in phase, as before pointed out. The tangent of the angle of lead is equal to $\frac{X}{R}$, according to equation (24). Therefore, the angle of lead of the current is small when $X$ is small


Fig. 54. Diagram if E. M. F. and Current Relations for Comditions of Fig. 53; compared with $R$, that is, when $\omega C$ is large; while the angle of lead of the current approaches $90^{\circ}$ when $X$ is large compared with $R$, that is, when $\omega C$ is small.

Fig. 53 represents a circuit containing resistance and a condenser connected to an alternator $A$; and Fig. 54 shows the relation between the electromotive force and current.

Example. A condenser with a capacity of 2 microfarads (which is large, as condensers go) is connected to alternating-current mains through a resistance coil of 200 ohms. The effective electromotive force between the
mains is 1,000 volts, and the frequency is 133 cycles per second, so that the factor $\omega$ is equal to $2 \pi \times 133$ or 835.7 radians per second; the reactance of the condenser is $\frac{10^{6}}{2 \times 835.7}=598.3$ ohms (negative); the impedance of the circuit is ${\sqrt{\overline{200}^{2}}+\overline{598.3}^{2}}^{2}=630.85 \mathrm{ohms}$; the current, according to equation (23), is 1.585 amperes; the current, according to equation (24), is $71^{\circ} 31^{\prime}$ ahead of the electromotive force in phase; and the power delivered to the circuit is 1,000 volts $\times 1.585$ amperes $\times \cos 71^{\circ} 31^{\prime}$, which is equal to 502.5 watts $\left(=I^{2} R\right)$.

If the above condenser is connected to the 1,000 -volt, 133 -cycle mains through a wire of negligible resistance, then the current will be $\frac{1,000 \text { volts }}{598.3 \text { ohms }}$ or 1.671 amperes; the current will be very nearly $90^{\circ}$ ahead of the electromotive force in phase; the power factor $\cos 0$, will be nearly zero; and of course the power delivered to the condenser will be nearly zero.

A circuit containing a condenser takes a little more power than is expended in the mere heating of the wire, namely, $I^{2} R$, for some power is consumed in the insulating material between the condenser plates. This power consumed in the dielectric is said to be due to dielectric hysteresis. In the above examples, this consumption of power in the insulating material of a condenser is neglected for the sake of simplicity.

Case D. Circuit in which the inductance reactance wL is balanced by the capacity reactance $\frac{1}{\omega C}$. In this case equation (21) reduces to $I=$ $\frac{E}{R}$; that is, the electromotive force acting upon the circuit has to overcome resistance only, as in the case of the non-reactive circuit. This case in which $\omega L-\frac{1}{\omega C}$ is equal to zero is considered again in the following article on resonance.

Electrical Resonance. Consider a circuit, like the one shown in Fig. 45, containing a given resistance $R$, a given induction $L$, and a given capacity $C$. Suppose that the alternator $A$ is at first run at very slow speed so as to give very low frequency, and is then gradually increased in speed so as to cause the frequency to increase. This gradual increase of frequency will cause a gradual increase in the value of the factor $\omega$ (equal to $2 \pi$ times the frequency); and as $\omega$ increases, the following relations between inductance reactance $\omega L$ and capacity reactance $\frac{1}{\omega C}$ will obtain:
(a) At first, when the frequency is very low (few cycles per second), the value of $\omega$ is small. Therefore, the inductance reactance $\omega L$ is small; the capacity reactance $\frac{1}{\omega C}$ is large; and the total net reactance $\omega L-\frac{1}{\omega C}$ is negative, and very nearly the same as $\frac{1}{\omega C}$ alone.
(b) As the frequency increases, the value of $\omega$ increases. Therefore, the inductance reactance $\omega L$ increases; the capacity reactance $\frac{1}{\omega C}$ decreases; and the total net reactance $\omega L-\frac{1}{\omega C}$ increases in value on account of the increase of $\omega L$, and also on account of the decrease of $\frac{1}{\omega C}$. For a certain critical value of the frequency, $\omega L$ becomes equal to $\frac{1}{\omega C}$, so that the total net reactance is then zero. That is

$$
\omega L-\frac{1}{\omega C}=0
$$

or

$$
\omega L=\frac{1}{\omega C}
$$

or

$$
\omega^{2}=\frac{1}{\overline{L C}}
$$

or

$$
\omega=\frac{1}{\sqrt{L C}}
$$

or, since $\omega$ equals $2 \pi f^{\prime}$, we have

$$
\begin{equation*}
f^{\prime}=\frac{1}{2 \pi V} \overline{\overline{L C}} . \tag{26}
\end{equation*}
$$

in which $f^{\prime}$ is the critical value of the frequency for which inductance reactance is balanced by capacity in the given circuit, $L$ being the inductance of the circuit and $C$ the capacity of the condenser. See Fig. 45.
(c) As the frequency increases beyond the critical value $f^{\prime}$, the inductance reactance $\omega L$ continues to increase; the capacity reactance continues to decrease; and the net reactance $\omega L-\frac{1}{\omega C}$, now positive in value, continues to increase in value.

Now, imagine the electromotive force of the alternator to be constant in value, although increasing in frequency as the alternator is speeded up.* The current in the circuit will at first increase with increasing frequency until the critical frequency $f^{\prime}$, equation (26), is reached, and then the current will decrease in value as the frequency increases, a maximum value of current being produced at the critical frequency $f^{\prime}$. This production of a maximum cur-


Fig. 55. Graphical Relation of Current and Frequency; E. M. F. Constant rent at the critical frequency $f^{\prime}$ is called electrical resonance. At critical frequency the reactance $\omega L-$ $\frac{1}{\omega C}$ is zero; and the gencral equation ( $\Omega$ ) reduces to $I=\frac{E}{l}$, as explained in Case 1), page 49. That is, the value of the current at the critical froquency is determined solely by the resistance of the circuit and for a given resistance has its maximum value.

The variation of current in a circuit like that shown in Fig. 45, with increasing frequency, electromotive force being kept constant in value, is shown graphically in Fig. 55, which is calculated from the following data: $E=200$ volts (effective); $R=2$ ohms; $L=0.35$. henry; and $C=20$ microfarads.

The critical frequency corresponding to these values of $L$ ard $C$ is 60 cycles per second, according to equation (26). The raximum point of the curve is not a cusp, as would appear from the figure; but the curve is rounded at the top, the figure being drawn on too small a scale to show it.

It should be remarked that the conditions for complete resonance can be obtained only when $C$ and $L$ are constant and con-

[^6]centrated (not distributed), and when the electromotive force is harmonic. If the electromotive force is non-harmonic, it means that it is composed of a number of sine waves having different frequencies. It is evident, therefore, that $\omega L$ can never be made exactly equal to the fraction $\frac{1}{\omega C}$ unless there is but one fundamental frequency equal to $f=\frac{\omega}{2 \pi}$.

Multiplication of E. M.F. by Resonance. When resonance exists in a circuit containing an inductance and a condenser in series, the alternating electromotive force $\omega L I$ between the terminals of the inductance, and the alternating electromotive force $\frac{I}{\omega C}$ between the terminals of the condenser, may each be much greater than the alternating electromotive force $R I$ which acts upon the circuit. This


Fig. 56. Diagram Showing Multiplication of E. M. F.s by Resonance fact is easily understood by means of the mechanical analogue. If even a very weak periodic force act upon a weight which is suspended from a spiral spring, the weight will be set into violent vibration, provided the frequency of the force is the same as the proper frequency of oscillation of the body. The forces acting on the spring may reach enormously greater values than the periodic force which maintains the motion of the system. Moreover, the forces which act upon the weight to produce its up-and-down acceleration may reach values very much larger than the periodic force which maintains the motion.

Example. A coil having an inductance of 0.352 henry and a resistance of 2 ohms, and a condenser of 20 microfarads capacity, are connected in series between alternating-current mains. The critical frequency of this circuit is 60 cycles per second, according to equation (26): The electromotive force between the mains is 200 volts, and its frequency is 60 cycles per second. The current in the circuit is $\frac{200 \text { volts }}{2 \text { ohms }}$, or 100 amperes, according to equation (21); the effective electromotive force between the condenser terminals is 13,270 volts effective $\left(=\frac{I}{\omega C}\right)$; and the electromotive force between the terminals of the inductance is also 13,270 volts effective ( $=\omega L I$ ).

The multiplication of electromotive force by resonance may be clearly understood with the help of the clock diagram, Fig. 56.


Fig. 57. An Alternator Delivering Current to a Long Transmission Line

The electromotive force $\omega L I$ required to overcome inductance reactance is equal and opposite to the electromotive force $\frac{I}{\omega C}$ required to overcome capacity reactance, as shown in Fig. 56, so that the geometric sum of $\omega L I, \frac{I}{\omega C}$, and $R I$ is, simply, RI. A transmission line has both inductance and capacity and, therefore, electrical resonance may occur on a transmission line. The phenomena of a transmission line, however, are very greatly complicated by the fact that the capacity is distributed; and a simple explanation of line reasonance can be given only by approximation.

For example, an alternator $A$, Fig. 57, delivers current to a long transmission line. The resonance effects are nearly independent


Fig. 58. Diagram of Conditions Shown in Fig. 57 with Transmission
Lines Insulated from Each Other
of whether the receiving apparatus at the end of the line, viz, at $B$, is connected into the circuit or not. We shall, therefore, consider that the receiving apparatus is disconnected and, furthermore, that the ends of the two transmission lines are insulated from each other.

A first approximation to the behavior of the line may be obtained by looking upon the distant end of the line $b b b$ as a condenser purely and simply, while the near end of the line $a a a$ is looked upon simply as an inductance.


Fig. 59. Circuit Showing Condenser and Inductance in Paralle] The transmission line shown in Fig. 57 is then equivalent to the combination shown in Fig. 58, which is identical with the combination shown in Fig. 45. The value of $L$ may be taken as the inductance of, say, half the length of the line; and the capacity $C$ may be taken as the capacity of the distant half of the transmission line. Then, if the alternator $A$ gives a frequency equal to the critical value of these values of $L$ and $C$, as per equation (26), we shall have resonance, and the electromotive force between the lines at the distant end (between the terminals of $C$, Fig. 58) may be greatly in excess of the electromotive force of the alternator $A$. This condition actually occurs in the practical operation of long transmission lines; and it is not an uncommon thing to have as much as 11,000 volts at the receiving end of a long transmission line when the electromotive force of the generator is but 10,000 volts.

Multiplication of Current by Resonance. An alternator A, Fig. 59, delivers current to a circuit which divides at the points $a$ and $b$ into two branches, one branch containing an


Fig. 60. Vector Diagram for Multiplication of Current by Resonance inductance $L$ and the other branch containing a capacity $C$, as shown. The two branches constitute a closed circuit in and of themselves; and if the frequency of the alternator is equal to the critical frequency of the circuit constituted by the two branches, that is, if the frequency of the alternator is equal to $\frac{1}{2 \pi V} \overline{\overline{L C}}$, as per equation (26), then the small current $I$ from the alternator will divide into two currents $I_{1}$ and $I_{2}$ in the respective branches, and the currents $I_{1}$ and $I_{2}$ may each be very much larger in value than the undivided cur-
rent $I$. The fact is that, because of resonance, a very large current is made to surge back and forth around the closed circuit formed by the two branches.

The multiplication of current by resonance may be clearly understood with the help of the clock diagram, Fig. 60. The line $O E$ represents the electromotive force between the branch points and $a b$, Fig. 59; the line $I_{1}$ represents the lagging current which


Fig. 61. Diagram of Circuit Showing Multiplication of Current by Resonance the electromotive force $E$ produces in the branch containing the inductance; the line $I_{2}$ represents the leading current which the electromotive force $E$ produces in the branch containing the condenser; and the line $I$, which is the geometric sum of $I_{1}$ and $I_{2}$, represents the total current in the undivided part of the circuit in Fig. 59.

For example, three similar 32 -candle-power incandescent lamps $A, B$, and $D$, Fig. 61, each having 100 ohms resistance, are connected as shown, to 550 -volt mains; $L$ is an inductance of 0.597 henry; and $C$, a capacity of 2.49 microfarads. Then the current flowing through the lamp $A$ is not quite 0.4 ampere, while one ampere of current flows through each of the lamps $B$ and $D$.

Miscellaneous Considerations. Condenser as Compensator for Lagging Current. An alternator may be designed to develop a certain effective electromotive force $E$, and to deliver a certain effective current $I$, at full load. The permissible power output of such an alternator would be $E I$ watts to a non-reactive circuit having unity power factor $(\cos \theta=1)$; but if the receiving circuit is reactive, the permissible power output of the alternator is only $E I \cos \theta$,


Fig. 62. Diagram of Condenser Compensating for Lagging Current where the power factor $(\cos \theta)$ may in practice have a value of .8 or less. If a condenser $C$ of sufficiently large capacity, Fig. 62, is connected across the terminals of an alternator $A$ in parallel with
an inductive receiving circuit $R L$, the effect of $L$ will be neutralized; the current delivered by $A$ will be in. phase with the electromotive force of $A$; and the permissible power output will be EI. The condenser is said to compensate for the lagging current taken by the inductive receiving circuit.

The compensation produced by a condenser is due to the fact that the alternating current taken by it is ahead of the electromotive force in phase, while that taken by the reactive receiving circuit is behind the electromotive force in phase.

Another advantage, aside from the increase of the permissible power output of the generator, that would result from this compensating of lagging current by means of a condenser, is that the electromotive force of the alternator would not fall off so much with increase of load as is the case when lagging current is not compensated for. The cost, however, of large condensers is so great that their use for compensation of


Fig. 63. Vector Diagram of Conditions Shown in Fig. 62 lagging current is not commercially practicable, as may be seen from the following discussion:

Let $I_{r}$ be the current delivered to the receiving circuit $R L$, Fig. 62. Let $I_{c}$ be the current delivered to the condenser $C$; this current is 90 degrees ahead of $E$ in phase. Let $I_{a}$ be the current delivered by the alternator $A$. It is desired that $I_{a}$ be in phase with $E$, as shown in Fig. 63. Let $\cos \theta$ be the power factor of the receiving circuit $R L$.

From Fig. 63 it is evident that $I_{\sigma}$ is equal and opposite to that component of $I_{r}$ which is at right angles to $E$, namely, $I_{r} \sin \theta$. Therefore

$$
I_{c}=I_{r} \sin \theta
$$

Now, $I_{0}$ is equal to $\frac{E}{\omega C}(=\omega C E)$, that is, is equal to the electromotive force hetween the condenser terminals divided by the reactance of the condenser. The value of $\sin \theta$ is $\frac{\omega L}{1 \cdot \frac{R^{2}+\omega^{2} L^{2}}{}}$; and $I_{r}=$
$-\frac{E}{\sqrt{R^{2}+\omega^{2} L^{2}}}$; so that, substituting the values of $I_{r}$ and $\sin \theta$, the above equation for $I_{c}$ becomes

$$
\omega C E=\frac{\omega L E}{R^{2}+\omega^{2} L^{2}}
$$

whence

$$
\begin{equation*}
C=\frac{L}{R^{2}+\omega^{2} L^{2}} \tag{27}
\end{equation*}
$$

in which $R$ is the resistance (in ohms) of the receiving circuit; $L$ is the inductance (in henrys) of the receiving circuit; $\omega$ is a factor equal to $2 \pi$ times the frequency in cycles per second; and $C$ is the capacity (in farads) of the condenser required to compensate for the lagging current delivered to the receiving circuit. Suppose an alternator having an electromotive force of 1,100 volts and a frequency of 60 cycles per second, delivers 102.4 amperes of current to a receiving circuit of which the power factor is 0.871 ( 9.35 ohms resistance and 0.014 henry inductance). The capacity of a condenser, which will compensate for the lagging current in this case, may be calculated from equation (27) as follows:

$$
\begin{aligned}
C & =\frac{0.014}{9.35^{2}+(2 \pi \times 60)^{2} \times 0.014^{2}} \\
& =0.0001214 \text { farad }=121.4 \text { microfarads }
\end{aligned}
$$

This condenser would take from the mains a current of $50.34 \mathrm{am}-$ peres, which would. be 90 degrees ahead of the e. m.f. in phase, and this current would be equal and opposite to the wattless component of the 102.4 amperes of current delivered to the inductive circuit. Such a condenser would require about 114,000 leaves of tinfoil 8 inches $\times 10$ inches, separated by 114,000 leaves of paraffined paper, each 0.03 inch in thickness. This would give a stack of condenser leaves of about 400 feet total thickness; and the cost of meterial and labor would be at least $\$ 10$ per microfarad.

Such a condenser would be impracticable but a synchronous motor' with over-excited field magnets behaves like a large condenser in that it takes an armature current which is ahead of the electromotive force in phase. The synchronous motor is often used in practice to compensate for lagging line current and when so operated is called a rotary condenser.

Circuits in Series. An alternator A, Fig. 64, delivers current to two coils (or elements) in series as shown. 'Let $R_{1}$ be the resistance and $X_{1}$ the reactance of coil 1 . Let $R_{2}$ be the resistance


Fig. 64. Two Elements in Series in Alternating Circuit


Fig. 65. Diagram of E. M. F. Conditions for Fig. 64
and $X_{2}$ the reactance of coil 2. Let $I$ be the current flowing through the circuit; $E$ the electromotive force between the mains; $E_{1}$ the electromotive force between the terminals of coil 1 ; and $E_{2}$ the electromotive force between the terminals of coil 2. Let $\theta$ be the phase difference between $E$ and $I ; \theta_{1}$ the phase difference between $E_{1}$ and $I$; and $\theta_{2}$ the phase difference between $E_{2}$ and $I$, as shown in Fig. 65.

Of course $\theta_{1}$ is the angle whose tangent is $\frac{X_{1}}{R_{1}} ; \theta_{2}$ is the angle whose tangent is $\frac{X_{2}}{R_{2}}$; and $\theta$ is the angle whose tangent is $\frac{\left(X_{1}+X_{2}\right)}{\left(R_{1}+R_{2}\right)}$, according to equation (24).

Example. Two impedance coils have resistances of 5 and 8 ohms and inductances of 0.01 and 0.2 henry, respectively. If these coils are connected in series across 220 -volt, 60 -cycle mains, find: (a) the current; (b) the voltages impressed across the coils; and (c) the phase relations between the current and the voltages impressed across the coils.

Solution. We have $R_{1}=5, R_{2}=8, E=220, f=60, L_{1}=0.01, L_{2}=0.2$. Then $\omega=2 \pi f=2 \pi \times 60=377$ radians per second. The reactance of coil 1 is $X_{1}=\omega L_{1}=377 \times 0.01=3.77$ ohms, and the reactance of coil 2 is $X_{2}=$ $\omega L_{2}=377 \times 0.2=75.4$ ohms. The total impedance of the circuit is $Z=$ : $\sqrt{\left(R_{1}+R_{2}\right)^{2}+\left(X_{1}+X_{2}\right)^{2}}=\sqrt{(5+8)^{2}+(3.77+75.4)^{2}}=80.23$ ohms, so that the current is
(a)

$$
I=\frac{E}{Z}=\frac{220}{80.23}=2.74 \text { amperes }
$$

(b) To find the voltages $E_{1}$ and $E_{2}$. (See Figs. 64 and 65.) The impedance of coil 1 is $Z_{1}=\sqrt{5^{2}+\overline{3.77^{2}}}=6.262$ ohms and the impedance of coil 2 is $Z_{2}=$ $\sqrt{8^{2}+\overline{75.4^{2}}}=75.82$ ohms. The magnitude of the voltage $E_{1}$ is

$$
E_{1}=I Z_{1}=2.74 \times 6.262=17.16 \text { volts, }
$$

and the magnitude of the voltage $E_{2}$ is

$$
E_{2}=I Z_{2}=2.74 \times 75.82=207.7 \mathrm{volts}
$$

(c) The phase angle $\theta_{1}$ between the current and $E_{1}$ is obtained from the relation $\tan \theta_{1}=\frac{X_{1}}{R_{1}}=\frac{3.77}{5}=0.754$, whence

$$
\theta_{1}=37^{\circ} 1^{\prime}
$$

Similarly the phase angle $\theta_{2}$ between $I$ and $E_{2}$ is found to be $\tan \theta_{2}$ $=\frac{75.4}{8}=9.425$, or

$$
\theta_{2}=83^{\circ} 56^{\prime}
$$

and $\tan \theta=\frac{X_{1}+X_{2}}{R_{1}+R_{2}}=\frac{79.17}{13}=6.090$, whence

$$
\theta=80^{\circ} 41^{\prime}
$$

Referring to Fig. 65 it is evident that by taking the line (or vector) representing the current $I$ as the axis of reference, the lines representing the electromotive forces, $E_{1}, E_{2}$, and $E$ may each be resolved into two components, one parallel to or in phase with $I$ and the other perpendicular to or in quadrature with $I$. Then the sum of the components of $E_{1}$ and $E_{2}$ parallel to $I$ will be equal to the component of $E$ parallel to $I$, and similarly the sum of the components of $E_{1}$ and $E_{2}$ perpendicular to $I$, will be equal to the vertical component of $E$. Expressing these statements in equation form and substituting the values, we have the following:

$$
\begin{aligned}
\left(E_{1} \cos \theta_{1}\right)+\left(E_{2} \cos \theta_{2}\right) & =(E \cos \theta) \\
\left(17.16 \cos 37^{\circ} 1^{\prime}\right)+\left(207.7 \cos 83^{\circ} 56^{\prime}\right) & =\left(220 \cos 80^{\circ} 40.3^{\prime}\right) \\
(13.71)+(21.98) & =(35.69 \text { approx. }) \\
\left(E_{1} \sin \theta_{1}\right)+\left(E_{2} \sin \theta_{2}\right) & =(E \sin \theta) \\
\left(17.16 \sin 37^{\circ} 1^{\prime}\right)+\left(207.7 \sin 83^{\circ} 56^{\prime}\right) & = \\
(10.34)+(206.7) & =\left(217 \text { approx. } 80^{\circ} 40.5^{\prime}\right)
\end{aligned}
$$

and

Circuits in Parallel. An alternator A, Fig. 66, delivers current to two coils (or elements) in parallel as shown.

Let $R_{1}$ be the resistance, and $X_{1}$ the reactance of coil 1 ; let $R_{2}$ be the resistance, and $X_{2}$ the reactance of coil 2; let $E$ be the electromotive force between the coil terminals; let $I_{1}$ be the current in coil $1, I_{2}$ the current in coil 2 , and $I$ the total current; and let $\theta$ be the angle of phase difference between $E$ and $I ; \theta_{1}$ the
phase angle between $I_{1}$ and $E$; and $\theta_{2}$ the angle between $I_{2}$ and $E$, as shown in Fig. 67.

The angles $\theta_{1}, \theta_{2}$, are known from the relations

$$
\tan \theta_{1}=\frac{X_{1}}{R_{1}} \quad \tan \theta_{2}=\frac{X_{2}}{R_{2}}
$$

The branch currents are: $I_{1}=\frac{E}{\sqrt{R_{1}{ }^{2}+X_{1}{ }^{2}}}$ and $I_{2}=\frac{E}{\sqrt{R_{2}{ }^{2}+X_{2}{ }^{2}}}$
but since they are not in phase, they cannot be added algebraically to obtain the resultant current $I$, but must be added geometrically as is done in Fig. 67. By trigonometry the magnitude of $I$ is

$$
I=\sqrt{I_{1}{ }^{2}+I_{2}{ }^{2}+2 I_{1} I_{2} \cos \left(\theta_{2}-\theta_{1}\right)}
$$

In the case of circuits in parallel the voltage across each branch is the same, and it is, therefore, convenient to determine all currents by their magnitudes and their relations with respect


Fig. 66. Two Elements in Parallel in Alternating Circuit
to this common voltage. Thus in Fig. 67 draw the line $O E$ to represent the voltage in magnitude; it is convenient to draw it horizontal since the current vectors in the clock diagram are to be referred to this vector voltage as the axis of reference. The vectors $O I_{1}$ and $O I_{2}$ are then drawn from $O$ to represent the. currents $I_{1}$ and $I_{2}$ in magnitude and with the proper directions as determined by the phase angles $\theta_{1}$ and $\theta_{2}$.

The magnitude and phase of the resultant current $I$ may be obtained graphically from the clock diagram, but this method is not to be recommended where accuracy is sought.

To obtain by calculation the phase angle between the main current $I$ and the voltage $E$, it is necessary to resolve $I$ into its components parallel to and in quadrature with $E$.

The active, or power, component of the current $I_{1}$ is

$$
I_{1} \cos \theta_{1}=\frac{R_{1} I_{1}}{Z_{1}}=\frac{E}{Z_{1}} \times \frac{R_{1}}{Z_{1}}=E g_{1}
$$

where $g_{1}=\frac{R_{1}}{Z_{1}^{2}}$ is called the conductance of circuit 1 .
Similarly the power component of $I_{2}$ is

$$
I_{2} \cos \theta_{2}=\frac{R_{2} I_{2}}{Z_{2}}=E g_{2}
$$

The "reactive" or "wattless" component of the current $l$. is

$$
I_{1} \sin \theta_{1}=I_{1} \frac{\Gamma_{1}}{Z_{1}}=\frac{E}{Z_{1}} \times \frac{Y_{1}}{Z_{1}}=E b_{1}
$$

where $b_{1}=\frac{X_{1}}{Z_{1}^{2}}$ is called the susceptance of circuit 1.


Fig. 67. Vector Diagram of Current Conditions for Fig. 66

Similarly the reactive component of the current $I_{2}$ is

$$
I_{2} \sin \theta_{2}=I_{2} \frac{X_{2}}{Z_{2}}=\frac{E}{Z_{2}}=E b_{2}
$$

Since $I$ is the vector sum of the two branch currents

$$
I=I_{1}+I_{2}
$$

The power component of the resultant current is the sum of the active components of the two branch currents, or

$$
I \cos \theta=I_{1} \cos \theta_{1}+I_{2} \cos \theta_{2}=E g_{1}+E g_{2}=E\left(g_{1}+g_{2}\right)
$$

and the reactive component of $I$ is

$$
I \sin \theta=I_{1} \sin \theta_{1}+I_{2} \sin \theta_{2}=E b_{1}+E b_{2}=E\left(b_{1}+b_{2}\right)
$$

## Therefore

$$
\tan \theta=\frac{I \sin \theta}{I \cos \theta}=\frac{b_{1}+b_{2}}{g_{1}+g_{2}}
$$

and the magnitude of the vector $I$ being the hypotenuse of a rightangle triangle, having a base $I \cos \theta$, and an altitude $I \sin \theta$, is

$$
I=E V \overline{\left(g_{1}+g_{2}\right)^{2}+\left(b_{1}+b_{2}\right)^{2}}
$$

By drawing a semicircle on $O E$ as a diameter and extending the current vectors $I_{1}, I_{2}$, and $I$, till they meet the semicircle at $B, A$, and $C$ in Fig. 67, some important relations may be deduced. Join the points $A, B$, and $C$ with $E$, thus completing the triangles $O A E, O B E$, and $O C E$. They are all right-angle triangles because inscribed in semicircle.

A careful study of Fig. 67 will show that $O A=R_{2} I_{2}, A E=X_{2} I_{2}$, (see page 43), and that $\sqrt{\overline{O A}^{2}+\overline{A E^{2}}}=O E=E$ from which $R_{2}=$ $\stackrel{O A}{I_{2}}, X_{2}=\frac{A E}{I_{2}}$, and $Z_{2}=\frac{O E}{I_{2}}$.

Similarly $O B=R_{1} I_{1}, B E=X_{1} I_{1}$, and $\sqrt{\overline{O B}^{2}+\overline{B E}^{2}}=O E=E$ and $R I=O C, C E=X I$, and $\sqrt{\overline{O C}^{2}+\overline{C E^{2}}}=E$ in which $R$ is called the "equivalent resistance" of the total circuit, and $X$ its "equivalent reactance." In other words, if the branched circuits were replaced by a single coil having a resistance equal to $R$ and a reactance equal to $X$, the coil would take the same current from the mains as before, with the same angle of lag, and would absorb the same total power. The impedance of this single "equivalent coil" is not the sum of the impedances of the separate coils, but is $\frac{E}{I}$.

Example. An impedance coil having a resistance of 19.05 ohms and a reactance (at a frequency of 60 cycles) of 11 ohms is connected in parallel with a second impedance coil having a resistance of 22 ohms and a reactance of 38.1 ohms across 220 -volt, 60 -cycle mains. Find (a) the current in each coil and the total (or line) current; (b) the phase rclation of the currents; (c) the power factor of the entire circuit and the power exyended in it; and (d) the "equivalent" resistance and reactance of the circuit.

## Q1

Solution. (a) To find the currents, we have

$$
I_{1}=\frac{E}{\sqrt{R_{1}^{2}+X_{1}^{2}}}=\frac{220}{\sqrt{(19.05)^{2}+(11)^{2}}}=\frac{220}{22}=10 \text { amperes }
$$

and

$$
\text { - } \quad I_{2}=\frac{E}{\sqrt{R_{2}^{2}+X_{2}^{2}}}=\frac{220}{\sqrt{(22)^{2}+(38.1)^{2}}}=\frac{220}{44}=5 \text { amperes }
$$

The above values are simply the magnitudes of the branch currents. Before they can be represented in a clock diagram, their directions must be found.
(b) $\tan \theta_{1}=\frac{X_{1}}{R_{1}}=\frac{11}{19.05}=0.577$, or

$$
\theta_{1}=-30^{\circ}
$$

The minus sign means a lagging current.

$$
\begin{gathered}
\tan \theta_{2}=\frac{X_{2}}{R_{2}}=\frac{38.1}{22}=1.732, \text { or } \\
\theta_{2}=-60^{\circ}
\end{gathered}
$$

The angle of phase difference between $I_{1}$ and $I_{2}$ is

$$
\theta_{2}-\theta_{1}=-60^{\circ}-(-30)^{\circ}=-30^{\circ}
$$

The vector currents may now be drawn to scale as is done in Fig. 67. The resultant or vector sum of the currents $I_{1}$ and $I_{4}$ may be found by drawing the diagonal of the parallelogram on $I_{1}$ and $I_{2}$ as sides. The main current is then $O I$.

Its magnitude may be found from equation (28)

$$
\begin{aligned}
I & =\sqrt{I_{1}^{2}+I_{2}^{2}+2 I_{1} I_{2} \cos \left(\theta_{2}-\theta_{1}\right)} \\
& =\sqrt{10^{2}+5^{2}+(2 \times 10 \times 5 \times 0.866)}=\sqrt{211.6}=14.55 \text { amperes. }
\end{aligned}
$$

To find the phase of $I$, we have

$$
\tan \theta=\frac{b_{1}+b_{2}}{g_{1}+g_{2}}
$$

where $b_{1}=\frac{X_{1}}{Z_{1}^{2}}=\frac{11}{19.05^{2}+11^{2}}=\frac{11}{22^{2}}=0.0227 ; \quad b_{2}=\frac{X_{2}}{Z_{2}^{2}}=\frac{38.1}{(44)^{2}}=0.01968 ;$
$g_{1}=\frac{R_{1}}{Z_{1}^{2}}=\frac{19.05}{(22)^{2}}=0.0394 ;$ and $g_{2}=\frac{R_{2}}{Z_{2}^{2}}=\frac{22}{(44)^{2}}=0.01136$.

$$
\tan \theta=\frac{0.0227+0.01968}{0.0394+0.0114}=\frac{0.04238}{0.0508}=0.8359
$$

or

$$
\theta=39^{\circ} .53^{\prime}
$$

which is the angle by which $I$ lags behind $E$.
(c) The power factor of coil 1 is $\cos \theta_{1}-\cos 30^{\circ}=0.86603$ and the power expended in it is $I_{1}^{2} R_{1}=10^{2} \times 19.05=1905$ watts, or it is $E^{2} g_{1}=(220)^{2}$ $\times 0.0394=1905$ watts.

The power factor of coil 2 is $\cos \theta_{2}=\cos 60^{\circ}=0.5$ and the power expended in it is $I_{2}^{2} R_{2}=(5)^{2} \times 22=550$ watts, or it is $E^{2} g_{2}=(220)^{2} \times 0.01136=550$ watts.

The power expended in coils 1 and $2=1905+550=2455$ watts.
The power factor of the entire circuit (two branches) is

$$
\cos \theta=\cos 39^{\circ} 53^{\prime}=0.76735
$$

and the power expended in it is

$$
E I \cos \theta=220 \times 14.55 \times 0.76735=2455 \text { watts }
$$

(d) The "equivalent resistance" $R$ is

$$
\frac{O C}{I}=\frac{220 \cos 39^{\circ} 53^{\prime}}{14.55}=\frac{168.81}{14.55}=11.6 \mathrm{ohms}
$$

The power absorbed in $R$ is

$$
I^{2} R=(14.55)^{2} \times 11.6=2455 \text { watts }
$$

The "equivalent reactance" $X$ is

$$
\frac{C E}{I}=\frac{220 \sin 39^{\circ} 53^{\prime}}{14.55}=9.7 \mathrm{ohms}
$$

The current $I$ lags behind the voltage $E$, by an angle $\theta$ given by $\tan \theta=\frac{X}{R}$ or $\tan \theta=\frac{9.7}{11.6}=0.836$ from which $\theta=-39^{\circ} 53^{\prime}$ the same as found above.

Electromotive Force Losses in Transmission Lines. An alternator of which the electromotive force is $E$, delivers current over a transmission line of which the resistance is $R_{1}$ and the reactance (inductance reactance) is $X_{1}$, to a receiving circuit of which the resistance is $R_{2}$ and the reactance is $X_{2}$. The total electromotive
force used to overcome the resistance and the reactance of the transmission line is $E_{1}$, and the electromotive force between the


Fig. 68. Diagram of E. M. F. Losses in Transmission Lines-Receiving Circuit Non-Reactive terminals of the receiving circuit is $E_{2}$. The current delivered is $I$. Then the general relation between $E, E_{1}$, and $E_{2}$ is as shown in Fig. 65, except that $E_{2}$ is usually very much larger than $E_{1}$ in value. There are three interesting and simplespecial cases of electromotive force losses in transmission lines, as follows:*

Case 1. When the receiving circuit is non-reactive. In this case the electromotive force $E_{2}$ between the terminals of the receiving circuit is in phase with $I$, the power factor of the receiving circuit is unity, and the general diagram of Fig. 65 takes the form shown in Fig. 68. The total electromotive force $E_{1}$ consumed in the line is sometimes called the impedance loss or drop and its two components $R_{1} I$ and $X_{1} I$, as shown in Fig. 68, are called the resistance loss and the reactance loss, respectively. Now, a careful inspection of Fig. 68 makes it evident that the numerical difference between the values of $E$ and $E_{2}$ is very nearly equal to the resistance loss in the line $R_{1} I$; and that the reactance loss in the line $X_{1} I$ has little to do with the difference between the values of $E$ and $E_{2}$. Therefore, when the receiving circuit is non-reactive, the difference in value between generator electromotive force $E$ and receiver electromotive force $E_{2}$ is sensibly


Fig. 69. E. M. F. Losses When the Receiving Circuit is Highly Reactive equal to the resistance loss of electromotive force in the line, and sensibly independent of the reactance loss of electromotive force in the line.

[^7]Case 2. When the receiving circuit is highly reactive. In this case the electromotive force $E_{2}$ between the receiving circuit terminals is nearly 90 degrees ahead of $I$ in phase, and the general diagram of Fig. 65 takes the form shown in Fig. 69. A careful inspection of Fig. 63 makes it evident that the difference in value of $E$ and $E_{2}$ is very nearly equal to the reactance loss in the line $X_{1} I$; and that the resistance loss in the line $R_{1} I$ has little to do with the difference between the values of $E$ and $E_{2}$. Therefore, when the receiving circuit is highly reactive, the numerical difference in value between generator electromotive force $E$ and receiver electro-


Fig. 70. E. M. F. Losses with Large Capacity Reactance motive force $E_{2}$ is sensibly equal to the reactance loss of electromotive force in the line, and sensibly independent of the resistance loss of electromotive force in the line.

Case 3. When the receiving circuit has large capacity reactance. In this case the electromotive force $E_{2}$ between the receiving circuit terminals is nearly 90 degrees behind $I$ in phase, and the general diagram of Fig. 65 takes the form shown in Fig. 70. A careful inspection of Fig. 70 makes it evident that the difference between the values of $E$ and $E_{2}$ is very nearly equal to the reactance loss in the line $X_{1} I$, and that this reactance loss is added to the generator electromotive force $E$ to give the receiver electromotive force $E_{2}$. Inspection of Fig. 70 shows furthermore that the resistance loss in the line $R_{1} I$ has little to do with the difference in value of $E$ and $E_{2}$. Therefore, when the receiving circuit has a high capacity reactance, the reactance loss in the line is sensibly equal to the rise in value of the electromotive force between generator and receiver, and this rise in value is sensibly independent of the resistance loss of electromotive force in the line. It is somewhat confusing to speak of the electromotive force $X_{1} I$ as reactance loss when the receiving circuit has a high capacity reactance; it would be better in this case to speak of $X_{1} I$ as the reactance gain of electromotive force in the line.

Examples. 1: Assume $E y$ equals 2000 volts, $Z_{2}$ equals 20 ohms. $R_{1}$ equals 1 ohm, and $X_{1}$ equals 4 ohms. Calculate $E$ for Cases 1, 2, and 3.

Solution. Case 1 (Fig. 68).

$$
\begin{gathered}
R_{2}=Z_{2}, I=\frac{E_{2}}{R_{2}}=100 \text { amperes } \\
R_{1} I=100 \text { volts } \quad X_{1} I=400 \mathrm{volts} \\
E=\sqrt{\left(E_{2}+R_{1} I\right)^{2}+\left(X_{1} I\right)^{2}}=2138 \text { volts }
\end{gathered}
$$

Case 2 (Fig. 69).

$$
\begin{gathered}
X_{2}=Z_{2} I=\frac{E_{2}}{X_{2}}=100 \text { amperes (lagging) } \\
R_{1} I=100 \text { volts } \quad X_{1} I=400 \text { volts } \\
E=\sqrt{\left(E_{2}+X_{1} I\right)^{2}+\left(R_{1} I\right)^{2}}=2402 \text { volts }
\end{gathered}
$$

Case 3 (Fig. 70).

$$
\begin{gathered}
X_{2}=Z_{2} \quad I=\frac{E_{2}}{X_{2}}=100 \text { amperes (leading) } \\
R_{1} I=100 \text { volts } \quad X_{1} I=400 \text { volts } \\
E=\sqrt{\left(E_{2}-X_{1} I\right)^{2}+\left(R_{1} I\right)^{2}}=1603 \text { volts }
\end{gathered}
$$

2. Given $E_{2}$ equals 2000 volts, $R_{2}$ equals 16 ohms, $X_{2}$ equals 12 ohms, $R_{1}$ equals 1 ohm , and $X_{1}$ equals 4 ohms. Find $E$, when (a) $X_{2}$ is an inductive reactance; and when (b) $X_{2}$ is a capacity reactance.

Solution. (a)

$$
I=\frac{E_{2}}{\sqrt{R_{2}^{2}+X_{2}{ }^{2}}}=100 \text { amperes }
$$

From Fig. 71

$$
\begin{aligned}
E & =\sqrt{\left(R_{2} I+R_{1} I\right)^{2}+\left(X_{2} I+X_{1} I\right)^{2}} \\
& =\sqrt{(1600+100)^{2}+(1200-400)^{2}}=2335 \text { volts }
\end{aligned}
$$

(b) As before

$$
I=100 \text { amperes }
$$

From Fig. 72

$$
\begin{aligned}
E & =\sqrt{\left(R_{2} I-R_{1} I\right)^{2}+\left(X_{1} I-X_{2} I\right)^{2}} \\
& =\sqrt{(1600+100)^{2}+(1200-400)^{2}}=1879 \mathrm{volts}
\end{aligned}
$$

3. Given $E$ equals 2200 volts, $R_{2}$ equals $16 \mathrm{ohms}, X_{2}$ equals $12 \mathrm{ohms}, R_{1}$ equals 1 ohm , and $X_{1}$ equals 4 ohms. Find $E_{2}$, when (a) $X_{2}$ is an inductive reactance; and when (b) $X_{2}$ is a capacity reactance.

Solution. (a)


Fig. 71. Vector Diagram, for
Example 2, "a"


Fig. 72. Vector Diagram for Example 2, "b"

$$
\begin{gathered}
I=\frac{E}{\sqrt{\left(R_{2}+R_{1}\right)^{2}+\left(X_{2}-X_{1}\right)^{2}}}=94.44 \text { amperes } \\
E_{2}=\sqrt{\left(R_{2}{ }^{2}+X_{2}{ }^{2}\right) I=20 \times 94.44}=1889 \text { volts }
\end{gathered}
$$

(b)

$$
\begin{gathered}
I=\frac{E}{\sqrt{\left(R_{2}+R_{1}\right)^{2}+\left(X_{1}-X_{2}\right)^{2}}}=117.15 \text { amperes } \\
E_{2}=\sqrt{R_{2}{ }^{2}+X_{2}{ }^{2}} \quad I=2343 \text { volts }
\end{gathered}
$$

Electromotive Force Losses in Alternator Armatures. Let $E$ be the total induced electromotive force in the armature of an alternator. A portion $E_{1}$ of this electromotive force is used to overcome the resistance $R_{1}$ and the reactance $X_{1}$ of the armature; and the remainder $E_{2}$ is available at the terminals of the alternator for producing current in the outside circuit, of which the resistance is $R_{2}$ and the reactance is $X_{2}$ ' $^{\prime}$ The general relation between $E, E_{1}$, and $E_{2}$ is as shown in Fig. 65, except that $E_{2}$ is usually very much larger than $E_{1}$ in value. There are three interesting and simple special cases of electromotive force losses in alternator armatures, as follows:

Case 1. When the receiving circuit is non-reactive. In this case the electromotive force $E_{2}$ between the terminals of the alternator is in phase with the current $I$ delivered by the machine, and the general diagram of Fig. 65 takes the form shown in Fig. 68, from which it is evident that when the receiving circuit is nonreactive, the numerical difference in value between the total induced electromotive force $E$ and the terminal electromotive force $E_{2}$ of the machine is sensibly equal to the resistance loss of electromotive force $R_{1} I$ in the armature, and sensibly independent of the reactance loss of electromotive force $X_{1} I$ in the armature.

Case 2. When the receiving circuit is highly reactive. In this case the electromotive force $E_{2}$ between the terminals of the alternator is nearly 90 degrees ahead of $I$ in phase, and the general diagram of Fig. 65 takes the form shown in Fig. 69, from which it is evident that when the receiving circuit is highly reactive the numerical difference in value between the total induced electromotive force $E$ and the terminal electromotive force $E_{2}$ of the machine is sensibly equal to the reactance loss of electromotive force $X_{1} I$ in the armature, and sensibly independent of the resistance loss of electromotive force $R_{1} I$ in the armature.

Case 3. When the receiving circuit has large capacity reactance. In this case the electromotive force $E_{2}$ between the terminals of the alternator is nearly 90 degrees behind $I$ in phase, and the general diagram of Fig. 65, takes the form shown in Fig. 70, from which it is evident that when the receiving circuit has a high capacity reactance the difference in value between $E$ and $E_{2}$ is sensibly equal to $X_{1} I, E_{2}$ being larger than $E$, and sensibly independent of the resistance loss of electromotive force $R_{1} I$ in the armature.


## ALTERNATING-CURRENT MACHINERY

## PART II

## ALTERNATORS

Fundamental Equation of Alternator. The equation expressing the effective electromotive force of an alternator in terms of the useful magnetic flux per pole, the number of poles, the number of armature conductors, and the speed of the armature, is called, from its importance in calculations in designing, the fundamental equation of the alternator. This equation is

$$
\begin{equation*}
E=\frac{K p \Phi n Z}{10^{8}} \text { volts } \tag{29}
\end{equation*}
$$

in which $E$ is the effective electromotive force of the alternator; $K$ is what we shall call the electromotive force factor of the machine and its value depends upon the ratio of breadth of pole face to pole pitch, and upon the distribution of windings upon the armature core;


Fig. 73. Concentrated or Uni-coil Winding $p$ is the number of poles of the field magnet; $\Phi$ is the useful magnetic flux per pole, that is, the number of lines of magnetic flux that cross the gap from one pole into the armature; $n$ is the speed of the armature in revolutions per second; and $Z$ is the total number of conductors on the surface of the armature.
We shall discuss this equation for the simplest case first, that is, when the armature conductors are concentrated in one slot per pole. This type of winding is called a concentrated, or uni-coil, winding, illustrated in part in Fig. 73.

A given conductor cuts $p \Phi$ lines of force in passing all of the poles in one revolution, and since the armature makes $n$ revolutions per second, the given conductor cuts $n p \Phi$ lines of force per second. Now, by definition, the cutting of one line of force per second induces
in a conductor one c.g.s. (centimeter-gram-second) unit of electromotive force. Therefore, there is an average of $n p \Phi$ c.g.s. units of electromotive force induced in one armature conductor; but since there are $Z$ armature conductors in series between the collector rings, the average electromotive force between collector rings is $Z n p \Phi$ c. g. s. units, or $\frac{Z n p \Phi}{10^{8}}$ volts.

The factor by which the average electromotive force must be multiplied to give the effective electromotive force is called the form factor of the electromotive force curve of the alternator. Therefore, if $K$ is this form factor, we have

$$
\text { effective } E=\frac{K Z n p \Phi}{10^{8}} \text { volts }
$$

Since $\frac{Z}{2}=T$, or $Z=2 T$ and since $p n=2 f$, this equation may be written so as to give the electromotive force (effective) of the alternator in terms of the number of turns $T$ of wire on the armature, and of the frequency $f$, as follows:

$$
\begin{equation*}
E=\frac{4 K \Phi f T}{10^{8}} \text { volts } \tag{30}
\end{equation*}
$$

For example, an alternator has 200 turns of wire on its armature and $1,000,000$ lines of magnetic flux from each field pole. It is run to give a frequency of 125 cycles per second. The value of the factor $K$ is 1.11 , assuming a sine-wave electromotive force curve and concentrated winding. The effective electromotive force of this alternator, therefore, is

$$
\frac{4 \times 1.11 \times 10^{6} \times 125 \times 200}{10^{8}}=1,110 \text { volts }
$$

Electromotive Form Factor $K$ in Equation (29). When the magnetic flux in the air gap is distributed in the ideal way explained below and represented in Fig. 74, the factor $K$ is called the phase constant of the winding.

When the winding is concentrated, the factor $K$ is called the form factor of the electromotive force curve. This factor $K$ depends in general upon the manner in which the magnetic flux is distributed
in the air gap, and upon the manner in which the armature windings are distributed around the armature.

Case 1. When a harmonic electromotive force is induced in each turn of the armature winding. This is the case-never fully realized


Fig. 74. Ideal Distribution of Magnetic Flux


Fig. 75. Armature Winding Diagram,
in practice-when the magnetic flux-density-that is, the field intensity in the gap space between the pole faces and the iron of the armature core-is zero at the points $a$, Fig. 74, and when this field intensity increases to a maximum at $c$ and at $c^{\prime}$ in such a manner that the field intensity at any point $b$ is proportional to the sine of the angle $\beta$.

Consider an armature rotating in a magnetic field distributed in the ideal way above specified. Suppose the winding to be arranged in slots spaced as shown in Fig. 75, four slots per pole. Fig. 76 shows one group $a b c d$ of these slots drawn to a larger scale. Two wires on the armature, at a distance apart equal to the distance between adjacent north poles, and subtending the angle $q$, Fig. 76, have induced in them two electromotive forces. These electromotive forces are to be thought of as differing in phase


Fig. 76. Enlarged Section of_Fig. 75 by $360^{\circ}$, because of the fact that the electromotive force in a given conductor passes through a complete cycle while the conductor moves from the center of a given north pole to the center of the next north pole.

Therefore, the phase difference of the electromotive forces induced in the wires placed in slot $a$, and those induced in the wires placed in slot $b$, is $\frac{s}{q} \times 360$; or, in other words, this angle is:

## width of tonth + width of slot $\times 360$ <br> circumference of armature $\div$ number of pairs of poles $\times 360$

The lines $A$ and $B$ in the clock diagram, Fig. 77, represent in magnitude and phase the electromotive forces induced in the wires in slots $a$ and $b$, respectively,


Fig. 77. Clock Diagram of Induced E. M. F.'s for Fig. 76 Fig. 75. Similarly, the lines $C$ and $D$ in Fig. 77 represent the electromotive forces induced in the wires in slots $c$ and $d$, respectively, Fig. 75. If, now, the windings in the slots $a, b, c$, and d, Fig. 75, are as in practice connected in series, the total electromotive force produced by all the windings will be represented by the line $E$ in Fig. 77. The line $E$ is the closing side of the polygon of which the sides $A, B^{\prime}, C^{\prime}$, and $D^{\prime}$ are drawn respectively, parallel and equal to the electromotive force lines $A, B, C$, and $D$. The value of $K$, which, in the case of the ideally distributed field flux here considered, is called the phase constant of the winding, is equal to the ratio $\frac{E}{4 . A}$; that is, the value of $K$ is equal to the ratio of the length of the line $E$ to four times the length of one of the lines $A, B, C$, or $D$.

Case 2. When a harmonic electromotive force is not induced in each turn of the armature winding. We shall discuss, (a) the case of a concentrated winding in which case $K$ is simply the form factor of the electromotive force wave; and (b) the case of a distributed winding.
(a) Fig. 78 shows a developed view of a four-pole altcrnator having four armature conductors $a, b, c$, and $d$, represented by the symbols $\odot$ and $\oplus$ depending upon whether the induced electromotive forces are directed towards or away from the reader, respectively. Of course, these four conductors are connected in series between the collecting rings (not shown in the figure) and in tracing
the circuit from one collecting ring to the other, one would pass down along conductor $a$, then across to conductor $b$, up $b$, then across to conductor $c$, down $c$, then across to conductor $d$, up $d$, and then to the other collecting ring.

Let time be reckoned from the instant when conductor $a$ is in the position shown; and let time be plotted as abscissas, and successive values of the induced electromotive force as ordinates, in the diagram $A B$, Fig. 78. Suppose that the intensity of the magnetic field in the gap space between pole faces and armature core is uniform, and that it terminates sharply at the pole tips, that is, that there is no spreading of the lines of force such as is shown in Fig. 74; as a matter of fact, however, the field always does spread. The armature conductors move with uniform velocity to the right, and the ratio of breadth of pole face, 6 inches, to pole pitch, 10 inches, is . 6 .

Then, during each cycle the duration of which is 20 units of time, the successive instantaneous values of the induced electromotive force are: constant, positive, and equal to $E$ for 6 units of time;


Fig. 78. Development and E. M. F. Curve for a Four-Pole Alternator
zero for 4 units of time; constant, negative, and equal to $E$ for 5 units of time; and again zero for 4 units of time. The average value of the electromotive force during the first half cycle is, therefore, equal to

$$
\frac{(E \times 6)+(0 \times 4)}{10}=.6 E \text { voilts }
$$

The squares of the successive instantaneous values of the induced electromotive force are: constant, positive, and equal to $E^{2}$ for 6 units of time; zero for 4 units of time; and so on. The average value of the squares of the successive instantaneous values of the induced electromotive force during half a cycle is, therefore, equal to

$$
\frac{\left(E^{2} \times 6\right)+\left(0^{2} \times 4\right)}{10} \text { volts }^{2}=.6 E^{2} \text { volts }^{2}
$$

or
effective value of induced electromotive force $=\sqrt{.6} \times E$ volts
The value of $K$ (form factor), in this particular case of a concentrated winding, is

$$
K=\frac{\text { effective value }}{\text { average value }}=\frac{\sqrt{.6 E}}{.6 E}=1.29
$$

The value of $K$ for a concentrated winding may be calculated, as in the above example, for any breadth of pole face.
(b) The value of $K$, in the case of a distributed winding, is cal-


Fig. 79. Development and E. M. F. Curve of Four-Pole Alternator with Two Conductors for Each Field Pole
culated as follows, assuming, for the sake of clearness, a ratio of pole breadth to pole pitch of $\frac{12}{20}=0.6$.

Fig. 79 shows a four-pole alternator with two armature conductors $a$ and $b$ for each field pole. (Only two of these conductors are shown in the figure.) The curve of the electromotive force induced in all the $a$ conductors is shown by the rectangular waves. $A B$. This electromotive force has a constant, positive value $E$
for 12 units of time; then a zero value for 8 units of time; then a constant, negative value $E$ for 12 units of time; and so on.

The curve of the electromotive force induced in all the $b$ conductors is shown by the rectangular waves $C D$. This electromotive force rises from zero to the full value $E$ at the instant when the conductors $b$ are in the position of $a$, as shown in the figure; that is, the electromotive force induced in conductors $b$ rises from zero to its full value $E, 7$ units of time before the corresponding rise of the electromotive force occurs in conductors $a$.

The total electromotive force curve $E F$ is found by adding corresponding ordinates of the curves $A B$ and $C D$. A careful inspection and comparison of $A B$ and $C D$ shows that the total electromotive force of the alternator is zero for 1 unit of time, equal to $E$ for 7 units of time, equal to $2 E$ for 5 units of time, and equal to $E$ for 7 units of time, during each half cycle of 20 units of time. Therefore, the average value of the electromotive force $E F$ during half a cycle is

$$
\frac{(0 \times 1)+(E \times 7)+(2 E \times 5)+(E \times 7)}{20}=\frac{24}{20} \times E=1.2 E
$$

Referring to the curve $E F$, Fig. 79, it is evident that the squares of the successive instantaneous values of the total electromotive force of the machine are as follows: $0^{2}$ for one unit of time, $E^{2}$ for seven units of time, $(2 E)^{2}$ for five units of time, and $E^{2}$ for seven units of time, during each half cycle of twenty units of time. Therefore, the average value of the squares of the electromotive force $E F$ during half a cycle, is

$$
\frac{\left(0^{2} \times 1\right)+\left(E^{2} \times 7\right)+\left(4 E^{2} \times 5\right)+\left(E^{2} \times 7\right)}{20}=\frac{34}{20} \times E^{2}
$$

and the effective value of the electromotive force $E F$ is

$$
\sqrt{\frac{34}{20}} \times E=1.304 E
$$

The value of $K$ for the special case under consideration, as shown in Fig. 79, is the ratio

$$
\frac{\text { effective electromotive force }}{\text { average electromotive force }}=\frac{1.304}{1.2}=1.087
$$

which is simply the form factor of the electromotive force curve of the alternator. The form factor of a sine wave electromotive force has already been shown to be 1.11.

Note.-The factor $K$ is the same thing as form factor whenever the distance $a b$. Fig. 79, between the remotest conductors of a group of conductors is less than the distance $c d$ between the pole tips, on the assumption that the magnetic lines of force do not spread into the spaces between the pole tips.

Armature Reaction. The current that circulates in an alternator armature has magnetizing action; and the actual useful flux $\Phi$ per pole is due to the combined magnetizing action of the field coils and of the armature coils. This magnetizing action of the armature current with respect to its effect upon the useful flux $\Phi$, is called armature reaction. In case the current in the armature lags behind the electromotive force-when, for example, the outside receiving circuit has inductance, as when the alternator supplies current to induction motors-


Fig. 80. Portion of Armature and Field Coil for SinglePhase Alternator the effect of armature reaction is to reduce the useful flux $\Phi$ from each pole. In case the current in the armature is ahead of the electromotive force in phase (a condition that obtains when the alternator supplies current to an over-excited synchronous motor or to any receiving apparatus over a long transmission line, or, in general, when the receiving apparatus acts like a condenser), the effect of armature reaction is to increase the useful flux $\Phi$ from each pole.
To state the matter in another way, it may be said that the effect of a lagging armature current is to oppose the magnetizing action of the field coils. On the other hand, the effect of a leading current in the armature is to help the magnetizing action of the field coils.

In an alternator the invisible variations in phase difference between the armature current and the electromotive force, due to the varying character of the receiving apparatus, correspond, in their influence on armature reaction, to the visible variations in the position of the brushes of a direct-current generator.

Fig. 80 represents a single-phase alternator of the revolving armature type running in the direction indicated by the curved arrow. The electromotive force induced in the armature coil $A$ is zero at the instant when the armature is in the position shown; and if the armature current is in phase with the induced electromotive force, the current also will be zero at this instant.

In considering armature reaction we shall discuss three cases, as follows:

1. Armature current in phase with the electromotive force
2. Armature current lagging behind the electromotive force

3 . Armature current leading the electromotive force
Case 1. As the armature tooth $A$, Fig. 80, passes by the field pole $N$, the current in the armature coil on $A$ is reversed in direction. If the current is in phase with the electromotive force induced in the armature, this reversal of direction of current occur: at the instant when the tooth $A$ is squarely under $N$. In thi: case the armature current flowing in coil $A$ just previous to the reversal, that is, when the tooth $A$ is approaching $N$, opposes by its magnetizing action the flux from $N$; and after reversal the current in the coil $A$ helps the flux from $N$. Therefore, the former or demagnetizing action of coil $A$ is balanced by the subsequent magnetizing action; and the only effect of the armature current in $A$ is to weaken the one side of the pole $N$, and to strengthen to an equal extent the other side, thus leaving the useful flux $\Phi$ unchanged.

Case 2. When the current lags behind the electromotive force, the reversal of current in the coil $A$ occurs at a later instant than in Case 1, that is, when the coil $A$ has passed heyond the center of the pole $N$. Hence, the demagnetizing action of the current in coil $A$ before reversal lasts for a longer time than the magnetizing action of the current in coil $A$ after reversal, and the domagnetizing action exceeds the magnetizing action. Therefore, the resultant effect of the armature reaction is to decrease the use: ful flux $\Phi$ from the pole $N$.

Case 3. When the current is in advance of the electromotive force in phase, the reversal of current in the coil $A$ occurs at an instant earlier than in Case 1, that is, before the coil $A$ has reached the center of the pole $N$. Hence, the demagnetizing action of the:
current in coil $A$ before reversal lasts for a shorter time than the magnetizing action of the current in coil $A$ after reversal, and the magnetizing action exceeds the demagnetizing action. Therefore, the resultant effect of the armature reaction is to increase the useful flux $\Phi$ from the pole $N$.

Armature Inductance. The value of the inductance of an alternator armature varies with the position of the armature coils with respect to the field-magnet poles, so that the inductance of an armature increases and decreases at a frequency twice* as great as the frequency of the electromotive force of the alternator. It is helpful to remember that inductance is proportional to the product of the magnetic flux into the number of turns threaded by this flux, divided by the amperes passing through the turns. The armature of the alternator shown in Fig. 3, page 9, for example, has about three or four times as much inductance when the armature teeth are squarely under the field poles as it has when the armature teeth are midway between field poles. That is, the magnetic flux produced through the armature teeth by a given current is three or four times as great in the first case as in the second case. This fluctuation of armature inductance makes it very difficult to predetermine the electromotive force and, in general, the behavior of a machine. In the following discussion the armature inductance is assumed to be constant.

The inductance of an alternator armature is proportional to the linear dimensions of the armature, other things being equal; and the inductance of an armature of given size and given total number of turns is much greater when the winding is concentrated than it is when the winding is distributed.

A moderate amount of armature inductance is advantageous in alternators which are to be run in parallel; and in case of a shortcircuit, the armature inductance keeps the current from becoming excessive. On the other hand, armature inductance is more or less objectionable in an alternator which is to be used to supply current at constant electromotive force, on account of the electromotive force that is lost in the armature.

[^8]The inductance of an armature is best determined by sending through it when at rest, from an outside source, a measured alternating current $I$, and measuring the electromotive force $E$ (volts drop) between the collecting rings. Then

$$
E=I V^{\prime} \overline{R^{2}+\omega^{2} L^{2}}
$$

or, solving for $L$, we have

$$
\begin{equation*}
L=\frac{1}{I \omega} \sqrt{E^{2}-(I R)^{2}} \tag{31}
\end{equation*}
$$

Knowing the armature resistance and the frequency $\frac{\omega}{2 \pi}$, we can find $L$ from equation (31). The value of $L$ thus calculated depends greatly upon the position in which the armature is held, as explained above, and also upon the degree of field excitation.

For example, the armature of a certain single-phase alternator has a resistance of 0.2 ohm measured between collector rings.- An electromotive force of 100 volts at a frequency of 125 cycles per second ( $\omega=785$ radians per second) applied to the collecting rings of the armature at rest, produces an effective current of 100 am peres. Therefore, the inductance of the armature, as calculated by use of equation (31), is

$$
L=\frac{1}{100 \times 785} \sqrt{100^{2}-\overline{20^{2}}}=0.00125 \text { henry }
$$

Electromotive Force Lost in Armature Drop. The electromotive force between the collecting rings of an alternator with given load, is less than the electromotive force between rings at zero load, with given field excitation, because of two electromotive force losses that occur, and because of the effect of armature reaction.
(a) The loss of electromotive force, or the drop, is due, in the first place, to the resistance of the armature. This loss is equal to $I R$; it is in phase with $I$; and it is precisely analogous to the electromotive.force lost in a direct-current armature due to the resistance of the armature. This $I R$ drop is of relatively small value and importance.
(b) The loss of electromotive force, or the drop, is due, secondly, to the inductance of the armature. This loss is equal to $\omega L I$, and it is 90 degrees abead of $I$ in phase.
(c) The demagnetizing action of the armature current on the field lessens the useful flux, and thus indirectly causes a falling-off in the induced electromotive force.

The result of the actions (b) and (c) above is to cause a loss of electromotive force in the armature of the same character in each case in so far as phase relations with current are concerned. Therefore, it is convenient to attribute the total effect of (b) and (c) to a fictitious armature inductance $L^{\prime}$, which is, of course, larger in value than the armature inductance $L$ in equation (31). The inductance reactance $\omega L^{\prime}$ corresponding to this equivalent inductance $L^{\prime}$, is called the synchronous reactance of the armature.

Alternator Regulation. Given an alternator, having constant field excitation. It has a certain electromotive force between collecting rings when its current output is zero. As the current output increases, the electromotive force between collector rings generally decreases, because of the actions already described; and, conversely, as the current output decreases, the terminal electromotive force rises. The increase of electromotive force from full load to zero load, with constant full-load field excitation and constant speed of driving, expressed as a percentage of the full-load terminal electromotive force, is called regulation of the alternator.

For example, a certain alternator gives 1,100 volts between its collector rings at full-load current and full-load field excitation. When the current output is decreased to zero by opening the main switch, leaving the field excitation and speed unchanged, the terminal electromotive force rises to 1,166 volts. The regulation is, therefore,

$$
\frac{1,166-1,100}{1,100} \times 100=6 \text { per cent }
$$

The regulation of a given alternator varies greatly with the character of the receiving circuit to which it delivers current When the receiving circuit has large inductance reactance (as in the case of under-loaded transformers and induction motors), the terminal electromotive force, under increasing load, falls off very much more than when the receiving circuit is non-inductive (as, for example, when the receiving circuit consists of incandescent lamps supplied through fully loaded transformers). In other words, the regulation of an alternator is larger (i.e., poorer) for in-
ductive receiving circuits than for non-inductive receiving circuits. If the receiving circuit has large capacity reactance (as, for example, when the receiving circuit consists of over-excited synchronous motors), the terminal electromotive force of the alternator will rise with an increase of the current output; and the regulation of the alternator will be negative. In practice, the receiving circuit never as a whole has capacity reactance.

For example, a given alternator having a regulation of 8 per cent on a non-inductive receiving circuit (unity power factor), has a regulation of about 21 per cent on an inductive receiving circuit having a power factor of 0.9 and a regulation of about 26 per cent at a power factor of 0.8 (lagging).

Field Excitation. In most alternating-current systems the voltage at the points from which current is distributed is kept constant or approximately constant. This requires that the voltage at the terminals of the alternator be somewhat increased as the amount of current (or load) is increased, the amount of the increase in electromotive force depending on the volts lost in the line. If the field excitation of an alternator be kept constant while the current taken from the armature is increased, the voltage at the terminals will decrease, just as in the case of a direct-current shunt generator. Hence, in order to keep the voltage at the terminals constant, or to cause a rise of voltage with increasing current output, it becomes necessary to increase the field excitation with increasing current output.

There are in general three methods in use for accomplishing this voltage control.
(1) By varying the field excitation with the load.
(a) Through control of the field-exciting current of the alternator by a rheostat operated either by hand or automatically from the alternator.
(b) Through control of the exciter itself by the main current from the alternator with or without a rheostat. This is accomplished in one of the three following ways: first, compounding the exciter with rectified current supplied to its field circuit; second, by supplying the armature circuit of the exciter with alternating current from the alternator (compensated field method); and third, by an external regulator for varying the exciter field current by rapidly
short-circuiting its field rheostat, the length of the periods during which the short-circuit is maintained, depending on the terminal voltage of the alternator.
(2) By interaction between the fields of the alternator and its exciter. This is Heyland's method which is used abroad but not in this country. Use is made of the stray flux from the alternator field which is arranged to strengthen the field of the exciter and thus obtain a compounding effect. The reaction on the field of the exciter is proportional to the armature reaction of the alternator and the terminal voltage of the exciter follows closely the variations in the load on the alternator.
(3) By utilizing the magnetic flux due to the armature current so that the armature reaction of the alternator increases the total flux per pole, and thus


Fig. 81. Diagram Showing Mcthod of Separate increases the voltage of the alternator as the load increases. The exciting field current itself is not varied. This is Walker's method and it has been used by the British Westinghouse Company.
Of the above methods the first includes practically all that are used at present in this country. The tendency is to abandon the attempts to design alternators to be inherently self-regulating and to avoid as far as possible all special devices internal to the alternator and its exciter for securing automatic voltage control, and to adopt instead an automatic regulator external to the alternator.

Under method (1) will be described the three commonest ways of voltage control employed at present, namely, separate excitation; composite excitation, and the automatic regulator, external to the alternator.

Case 1. Separate excitation. The simplest method is that illustrated by the diagram, Fig. 81, in which $A$ represents the armature winding, the terminals of which $T_{1}$ and $T_{2}$ are connected to the collector rings, which in turn are connected to the line wires through the brushes.

The field of the alternator is excited by a set of coils on the pole pieces. These coils are represented by $F$; and current is supplied to these coils from a small direct-current dynamo $E$, called the exciter. This exciter is a small direct-current shunt-wound or compound-wound dynamo furnished with an adjustable rheostat $r$ in series with its field $f$. An adjustable rheostat $R$ is placed in the alternator field circuit also. When the electromotive force of the alternator decreases, its field may be strengthened by cutting out resistance in either $R$ or $r$, or in both.

Regulation by $r$ alone is generally used in large machines, since the exciter's field current is relatively small, while the alternator field current is usually large and hence would cause a large $I^{2} R$ loss if passed through a rheostat. Separate excitation is still used in some of the older electric lighting stations.

Case 2. Composite excitation. The electromotive force of an alternator excited as in Case 1 falls off greatly with increasing current output; and to counteract this tendency automatically, an auxiliary field excitation is sometimes provided, which increases with the current output of the machine. For this purpose the whole or a portion of the current


Fig. 82. Diagram Showing Method of Composite Excitation given out by the machine is rectified,* and sent through the auxiliary field coils. This arrangement is shown in Fig. 82.

The field winding of the alternator has two sets of coils $F$ and $C$. The coils $F$ are separately excited as in Case 1. The coils $C$, known as the "series" or "composite coils," are excited by the main current from the alternator. One terminal of the armature winding is connected directly to a collecting ring. The other armature terminal connects to one set of alternate bars of the rectifying

[^9]commutator $B$. From the rectifier the current is led through the winding $C$, thence back to the rectifier, and thence to the other collecting ring. The shunt $S$, within the commutator, may be used when it is desired to rectify only a part of the current. There is also a shunt $S^{\prime}$ which may be used to regulate the amount of current flowing through the coil $C$.

The alternating-current rectifier is an arrangement for reversing the connections of the field circuit with each reversal of the current from the alternator, so that the current may flow always in the same direction in the field circuit. The rectifier is a commutator mounted on the armature shaft. This commutator has as many bars as there are poles on


Fig. 83. Diagram Showing Composite ${ }^{7}$ Excitation with Transformer the field magnet of the alternator. These bars are wide, and are separated by quite narrow spaces filled with mica insulation. Let these bars be numbered in order around the commutator. The even-numbered bars are connected together, and the odd-numbered bars are connected together. The connecting wire leading from one terminal of the alternator armature to one of the collector rings is cut; and the two ends thus formed are connected, one to the even-numbered bars (shown in full black in Fig. 82) of the rectifying commutator, and the other to the odd-numbered bars (shown white in Fig. 82). The field circuit that is to receive the rectified current is connected to two brushes which rub on the rectifying commutator, these brushes being so spaced that one touches an odd-numbered bar when the other touches an even-numbered bar. The brushes are carried in a rocker arm, which is moved forwards or backwards until the brushes are passing from one bar to the next at the instant that the alternating current from the alternator is passing through the value zero. The proper adjustment of the brushes is indicated by a minimum of sparking.

Fig. 83' shows an alternator $A$ 'with two sets of field coils $F$ and $C$ as before. One armature terminal is connected to a collecit-


Fig. 84. Two-Phase Alternator Diagram with Composite Field Excitation with Balanced Receiving Circuits
ing ring; and the other armature terminal connects to the primary of a transformer $T$, and thence to the other collecting ring. The terminals of the secondary coil of $T$ connect to the bars of the rectifying commutator $B$, from which the composite field winding $C$ is supplied. The transformer $T$ is usually placed inside the armature.


Fig. 85. Three-Phase Alternator Diagram with Composite Field Excitation with Balanced Receiving Circuits

Composite field excitation is, however, not satisfactory in case of polyphase alternators, unless the receiving circuits supplied from the alternators are approximately balanced. Unbalancing of the receiv-
ing circuits changes the electromotive forces generated in the different phase windings of the armature, by different amounts; and composite excitation, applied to the magnetic field as a whole, cannot, properly, correct the different electromotive force variations of the several phases.

In cases where the receiver circuits are approximately balanced, the current for the composite field excitation is taken through a rectifying commutator from the secondary coil of a series (or current) transformer which has two or three distinct primary coils, one for each phase. This arrangement applied to a two-phase alternator is shown in Fig. 84, and applied to a three-phase alternator in Fig. 85. The effect of the several primary coils on the series transformer is to balance up the slight differences of the several polyphase currents, in so far as their action upon the composite excitation is concerned. This method has been used by the Westinghouse Company in the case of alternators of small capacity.

Case 3. Automatic regulator. There are on the market a number of automatic devices which are designed to change the field strength of a generator in accordance with a change in generator voltage. The most successful of these devices is the generator voltage regulator manufactured by the General Electric Company. This regulator differs from other types of regulators in that it does not make use of the principle of switching resistances in and out of the field circuit by the step-by-step method. This regulator controls the generator voltage by rapidly opening and closing a shunt circuit connected across the exciter field rheostat, the duration of such periods of short-circuit being varied automatically. The field rheostat is first turned in until the exciter voltage is much reduced and the regulator circuit is then closed. This short-circuits the rheostat through contacts in the regulator, causing the voltage of the exciter and the generator to immediately increase. At a predetermined point the regulator contacts are automatically opened which causes the field current of the exciter to again pass through the rheostat. The resulting decrease in voltage is quickly checked by another closing of the regulator contacts, which continue to vibrate to and fro thus keeping the generator voltage within the desired limits.

Fig. 86 is the front of a "form A2" generator voltage regulator designed for alternators having exciters of small capacities. Fig. 87 is a rear
view of the regulator showing the resistance box and iron brackets for mounting it at the end of the switchboard if desired, although it is recommended that it be mounted directly on the switchboard. A diagram of the electrical connections for a generator voltage regulator with an alternator and its exciter is shown in Fig. 88.

The regulator has a direct-current control magnet, an alternating-current control magnet, and a relay. The direct-current control magnet is connected to the exciter bus bars. This magnet has a fixed stop-core in the bottom and a movable core in the top which is attached to a pivoted lever


Fig. 86. Front View of Generator Voltage Regulator having at the opposite end a flexible contact pulled downward by four spiral springs. For clearness, however, only one spring is shown in the diagram. Opposite the direct-current control magnet is the alternating-current control magnet which has a potential winding connected by means of a potential transformer to the al-ternating-current generator or bus bars. There is an adjustable compensating winding on the alternating-current magnet connected through a current transformer to the principal lighting feeder. The object of this winding is to raise the voltage of the al-ternating-current bus bars as the load increases. The alternatingcurrent control magnet has a movable core and a lever and contacts similar to those of the directcurrent control magnet, and the


Fig. 87. Back View of Generator Voltage Regulator two combined produce what is known as the "floating main contacts."

The relay consists of a $U$-shaped magnet core having a differential winding and a pivoted armature controlling the contacts which open and close the shunt circuit across the exciter field rheostat. One of the differential windings of the relay is permanently connected across the exciter bus bars and tends to keep the contacts open. The other winding is connected to the exciter bus bars through the floating main contacts and when the latter are closed neutralizes the effect of the first winding and allows the relay contacts to shortcircuit the exciter field rheostat. Condensers are connected across the relay contacts to prevent severe arcing and possible injury.

The cycle of operation is as follows: The circuit shunting the exciter field rheostat through the relay contacts is opened by means of a single-pole switch at the bottom of the regulator panel and the


Fig. 88. Diagram of Electrical Connections for Generator Voltage Regulator
rheostat turned in until the alternating-current voltage is reduced 65 per cent below normal, which so weakens both of the control magnets that the floating main contacts are closed. This closes the relay circuit and demagnetizes the relay magnet, releasing the relay armature, and the spring closes the relay contacts. The singlepole switch is then closed and as the exciter field rheostat is shortcircuited, the exciter voltage will at once rise and bring up the voltage of the alternator. This will strengthen the alternating-current and direct-current control magnets, and at the voltage for which the counterweight has been previously adjusted, the main contacts will open. The relay magnet will then attract its armature and by opening the shunt circuit at the relay contacts will throw the full resistance into the exciter field circuit tending to lower the exciter
and the alternator voltage. The main contacts will then be again closed, the exciter field rheostat short-circuited through the relay contacts, and the cycle repeated. This operation is continued at a high rate of vibration due to the sensitiveness of the control magnets and maintains not a constant but a steady exciter voltage.

One of the advantages of this regulator is that in controlling the voltage of the alternator by operating entirely on the field circuit of the exciter, the heating losses are far smaller and the efficiency correspondingly higher than is the case with those regulators which operate directly on the alternator field.

Another advantage is that several alternators may be operated in parallel using but one regulator, if all use the same exciter. On the other hand if, as is more usual, several exciters are used in parallel, one regulator and an equalizer rheostat for each additional exciter are necessary.

If two or more exciters, not operating in parallel, are used, a separate regulator must be installed for each exciter. The Tirrill "form F" regulator is made for large installations, and is furnished with several relays varying from two to twelve, according to the size, the capacity, and the characteristics of the exciters used. While these "form F" regulators differ more or less in detail according to special conditions, the main features of operation are the same as in the "form A2" regulator.

The standard voltage for exciters is 125 volts, and in some cases as high as 250 volts. Tirrill regulators are designed for a range of from 70 to 140 volts in the first case, and for a range of from 140 to 280 volts in the second case.

With the growing use of these automatic regulators external to the alternator, it has been found desirable by manufacturers to minimize their efforts to design alternators of low inherent regulation, especially in the case of turbo-alternators and machines of large rated output. A low inherent regulation is today considered an expensive and unnecessary luxury, for by some sacrifice in this quality a relatively large gain in rated capacity becomes possible, and in many cases a higher efficiency.

The advent of the automatic regulator has thus enabled designers to effect considerable improvement in alternators by relieving them of the troublesome question of low inherent regulation, and
permitting them to give greater weight than ever to the important matters of increasing output and efficiency.

## POLYPHASE ALTERNATORS AND SYSTEMS <br> SINGLE $=$ PHASE SYSTEM

Limitations. As long as alternating current was generated, transmitted, and used for electric lighting only, the single-phase system gave complete satisfaction, simplicity in the generating, transmitting, and receiving apparatus being its most striking and valuable feature.

In the earlier days of the electric lighting industry, there was very little, if any, demand for current to operate motors for power purposes. Since that time, however, there has developed an everincreasing demand for current for power purposes, fully equaling, if not exceeding, that for lighting work. With the advent of this new condition, the great obstacle to the use of the single-phase alternat-ing-current system became manifest. Single-phase constant-speed motors are difficult to make self-starting* under load, especially in units of large size; and hence the use of the single-phase system for general power purposes, with the apparatus now available, is not practicable.

It was in 1888, in Italy, that Ferarris discovered the important principle of the production of a rotating magnetic field by means of two or more alternating currents displaced in phase from one another, and he thus made possible, by means of the induction motor, the use of polyphase currents for power purposes. The most important advantage of polyphase alternating currents over the simple single-phase system is that alternating-current motors can be satisfactorily operated by them. It was mainly the requirements of the induction motor that led to the development of the polyphase system.

## TWO=PHASE SYSTEM

Two-Phase Alternator. The simplest form of polyphase generator consists of two similar and independent single-phase armatures mounted rigidly on one and the same shaft, one beside the other, in such a manner that the electromotive forces at the terminals of

[^10]the respective armatures arrive at their maximum values 90 degrees, or one-fourth of a period, apart. The currents from such a machine are said to have a two-phase relationship. The two separate armatures are supposed to revolve inside the same crown of field magnet poles.

Fig. 89 shows an end view of such an arrangement, but armature $B$ is here shown inside of armature $A$ for the sake of alearness. As will be seen in the figure, armatures $A$ and $B$ are so mounted on the shaft that the slots of $A$ are midway under the poles $N$, $S$ when the slots of $B$ are midway between the same poles. With this arrangement, the electromotive force generated in the armature coils of $A$ and $B$ are so related in their variations that the electromotive force of $A$ is at its maximum when the electromotive force of $B$ is zero. Or in other words, the two electromotive forces are 90 degrees apart in phase, or are in quadrature (at right-angles) to each other.

A careful study of Fig. 89 will show that the electromotive forces induced in armatures $A$ and $B$ are 90 degrees apart in phase. Thus the figure shows the armature $A$ in the position in which its windings (in the slots) are cutting lines of magnetic flux from the field poles at a maximum rate, while


Fig. 89. Diagram of Two-Phase Alternator with Two Scparate Armatures the armature $B$ is shown in the position in which its windings are midway between the field poles where they do not cut any magnetic flux at all. Therefore, the electromotive force in the windings of the armature $A$ is at its maximum value, while the electromotive force in the windings of armature $B$ is zero at the same instant. That these electromotive forces generated in the windings of $A$ and $B$ differ in phase by 90 degrees, may also be shown as follows: The electromotive force generated by a conductor on armature $A$ passes through a complete cycle of changes as it moves from the center of one north pole to the center of the next north pole. The interval between the centers of two adjacent north poles, a "double pole pitch," corresponds then to 360 "electrical" degrees. Therefore, the phase (space) difference between the conductors
on $A$ and $B$ is seen to be $\frac{1}{4}$ of 360 degrees, or 90 electrical degrees.
.The two equal, but distinct and independent electromotive forces generated by such a two-phase alternator are generally used to supply two distinct


Fig. 90. Diagram Showing Method of Combining Separate Armature Windings on One Core and separate currents to two distinct and independent circuits. When so used the system is called a two-phase, four-wire system. We shall see later that it is possible to interconnect the two circuits in such a manner that one of the four line wires may be omitted.

In practice the actual twophase alternator is constructed by placing both the armature windings of $A$ and $B$ upon one and the same armature core, instead of on separate cores. To accomplish this the armature core has twice as many slots as either $A$ or $B$ in Fig. 89. Fig. 90 shows such an armature. The slots marked $a_{1}, a_{2}$, $a_{3}$, etc., contain the conductors comprising phase $A$; whereas the slots marked $b_{1}, b_{2}, b_{3}$, etc., contain the conductors comprising phase $B$. The $A$ winding passes in slot $a_{1}$ from front to back of the armature core; then towards the


Fig. 91. Distributed or Multi-Coil Winding for Two-Phase Alternator reader (that is, from back to front) in slot $a_{2}$; then from front to back in $a_{3}$, from back to front in $a_{4}$, and so on. The various conductors in slots $a_{1}, a_{2}$, $a_{3}$, etc., are joined in series by connectors (at front and back), and the two ends of the final series are connected to two collector rings.

The $B$ winding passes in slot $b_{1}$, from front to back of the armature core; then towards the reader, that is, from back to front, in slot $b_{2}$; then from front to back in $b_{3}$. from back to front in $b_{4}$, and
so on. The various conductors in slots $b_{1}, b_{2}, b_{3}$, etc., are joined in series, and the two ends of the final series are connected to two collector rings, which rings are distinct from the pair of rings to which the $A$ winding is connected.

The armature windings $A$ and $B$ just described are of the concentrated or uni-coil type, page 74 , having only one slot per pole for each winding, i. e., per phase. Distributed or multi-coil windings also are frequently used for two-


Fig. 92. Diagram of Collector Ring System for Two-Phase Alternator phase alternators. Thus, Fig. 91 shows an end view of a portion of a two-phase armature with its $A$ and $B$ windings each distributed in two slots per pole. The coils belonging to winding $A$ are lightly shaded, and those belonging to winding $B$ are darkly shaded in the figure. The connections between the coils of the $A$ winding are shown in the figure by the full lines, while the connections of the $B$ winding are shown by the dotted lines.

Two-phase alternators are usually provided with two sets of collector rings; one ring, however, may be made to serve as a common connection for the two armature windings, as shown in Fig. 92. The lines $A$ and $B$ in the clock diagram,. Fig. 93, represent the generator electromotive forces, $a$ represents the current in main $1, b$ represents the current in main 2 , and $c$, which is the vector sum of $a$ and $b$, represents the current in the common main 3. If $a=b$, it is evident that $c=$ $a \sqrt{2}=b \sqrt{2 .}$

## THREE=PHASE SYSTEM

Three $=$ Phase Alternator. Consider three similar singlephase armatures $A, B$, and $C$,


Fig. 93. Clock Diagram of E. M. F.'s and Currents for Two-Phase Alternator mounted side by side on the same shaft and revolved in the same field, each armature having as many slots as there are field poles. Fix the attention upon a certain armature slot of $A$, and let time be reckoned from the instant that this slot is squarely under an $N$ pole. Let $t$ be the time which elapses as this
armature slot passes from the center of one $N$ pole to the center of the next $N$ pole. The armature $B$ is to be so fixed to the shaft that its slots are squarely under the poles at the instant $\frac{1}{3} t$; and the armature $C$ is to be so fixed that its slots are squarely under the poles at the instant $\frac{2}{3} t$. While a slot passes from the center of one $N$ pole to the center of the next $N$ pole, the electromotive force passes through one complete cycle. Hence, the electromotive forces given by three armatures arranged as above, will

Fig. 94. Clock Diagram of Three-Phase E. M. F.'s and Currents
 be 120 degrees apart in phase, as shown in Fig. 94, in which the lines $A, B$, and $C$ represent the respective electromotive forces. The currents given by the armatures to three similar receiving circuits lag equally behind the respective electromotive forces, and are represented by the dotted lines $a, b$, and $c$. This combination of three single-phase alternators is called a three-phase alternator. In practice the three distinct windings $A, B$, and $C$ are placed upon one


Fig. 95. Arrangement of Slote for ThreePhase Alternstor Armature and the same armature body. For this purpose the armature core has three times as many slots as $A, B$, or $C$.

Fig. 95 shows the arrangement of the slots for such a winding. The slots belonging to phase $A$ are drawn in heavy lines, and are marked $a_{1}, a_{2}$, etc. Those belonging to phase $B$ are shown dotted, and those belonging to phase $C$ are shown in light lines. The $A$ winding would pass up slot $a_{1}$, down $a_{2}$, up $a_{3}$, etc.; the $B$ winding, up $b_{1}$, down $b_{2}$, up $b_{3}$, etc.; and similarly for winding $C$.

The windings $A, B$, and $C$ here described are of the concentrated type, having only one slot per pole for each winding. Dis-
tributed windings also are frequently used for three-phase alternators. Thus Fig. 96 shows a portion of a three-phase armature with its $A, B$, and $C$ windings each distributed in two slots per pole. The coils belonging to windings $A, B$, and $C$, respectively, are differently shaded to distinguish them. The manner of connecting the coils of each winding is described on page 119.

If the three circuits of a three-phase alternator are to be entirely independent, six collector rings must be used, two for each winding; however, the circuits may be kept practically independent by using four col-


Fig. 96. Portion of Diagram of Windings for Three-Phase Alternator Armature lector rings and four mains, as shown in Fig. 97. The main 4 serves as a common return wire for the independent currents, in mains 1 , 2. and 3. When the three receiving circuits are equal in resistance and reactance, that is, when the system is balanced, the three currents are equal, and are 120 degrees apart in phase (each current lagging behind its electromotive force by the same amount as the others); and their sum is at each instant equal to zero. In this case, main 4, Fig. 97, carries no current. Therefore, main 4 and the corresponding collector ring may be dispensed with, the three windings connected together at the point $N$, called the common junction or neutral point. This arrangement, shown in the symmetrical diagram, Fig. 98, is called the " Y " or "star" scheme of connecting the three windings or phases $A, B$, and $C$.

Another scheme-for connecting the three windings $A, B$, and $C$ (also for balanced loads), called the " $\Delta$ " (delta) or "mesh" scheme, is illustrated in Fig. 99. Winding (or phase) $A$ is connected between rings 3 and 1 ; winding (or phase) $B$ between rings 1 and 2; and winding (or phase) $C$ between rings 2 and 3 .


Fig. 97. Collector Ring System for ThreePhase Alternator

The direction in a circuit in which the electromotive force or current is considered as a positive electromotive force or current,
is called the positive direction through the circuit. This direction is chosen arbitrarily. The arrows in Figs. 98 and 99 indicate the positive directions in the mains and through the windings. It must be remembered that these arrows represent not the actual directions of the electromotive forces or currents at any given instant, but merely the directions of positive electromotive forces or currents. Thus, in Fig. 98, the currents are considered positive when flowing from the common junction towards the collecting rings, and the currents are never all of the same sign.

Y=Connected Armatures. Electromotive Force Relations. We shall consider the electromotive force between mains 1 and 2, Fig. 98 , to be positive, when it tends to send current through a receiving circuit from main 1 to main 2 . Similarly, the electromotive force


Fig. 98. The "Y" Scheme of Connecting Three Phases in Three-Phase Alternator


Fig. 99. Diagram of " $\triangle$ " Scheme for Connecting Phases in ThreePhase Alternator
between mains 2 and $S$ is considered positive, from main 2 to main 3 ; and the electromotive force between mains 1 and 3 is considered positive, from main 3 to main 1. Passing through the windings $A$ and $B$ from ring 2 to ring 1 , Fig. 98 (which is the direction in which an electromotive force must be generated to give an electromotive force acting upon a receiving circuit from main 1 to main 2), the winding $A$ is passed through in the positive direction, and the winding $B$ in the negative direction. Therefore, the electromotive force from main 1 to main 2 is $A-B$. Similarly the electromotive force from main 2 to main $B$ is $B-C$, and the electromotive force frcm main 3 to main 1 is $C-A$. These differences are shown in the clock diagram, Fig. 100. The electromotive force between mains 1 and 2 (namely, $A-B$ ) is 30 degrees behind $A$ in phase, and its effective
, value is $2 E \cos 30^{\circ}=\sqrt{3} E$, where $E$ is the common value of each of the electromotive forces $A, B$, and $C$. Similar statements hold concerning the electromotive forces between mains 2 and 3 , and those between mains 3 and 1 . Hence, the electromotive force between any pair of mains leading from a three-phase alternator with a Y-connected armature is equal to the electromotive force generated per phase multiplied by $1 / \overline{3}$

Current Relations. In the Y connection, the currents in the mains are equal to the currents in the respective windings or armature phases, as is evident from Fig. 98.
$\Delta=$ Connected Armatures. Electromotive Force Relations. In $\Delta$-connected armatures the electromotive forces between the mains


Fig. 100. Clock Diagram of E. M. F.s for Three-Phase "Y" Winding


Fig. 101. Clock Diagram for Currents in " $\Delta$ " Connected ThreePhase Armature
or collector rings are equal to the electromotive forces of the respective windings, as is evident from Fig. 99.

Current Relations. Referring to Fig. 99, we see that a positive current in winding $A$ produces a positive current in main 1 , and that a negative current in winding $B$ produces a positive current in main 1 ; therefore, the current in main 1 is $a-b$, where $a$ is the current in winding $A$, and $b$ is the current in winding $B$. Similarly, the current in main 2 is $b-c$, and the current in main 3 is $c-a$. These differences are shown in Fig. 101. The current in main 1 (namely $a-b$ ) is 30 degrees behind $a$ in phase; and its effective value is $\sqrt{3} I$, where $I$ is the common effective value of the current:,
$a, b, c$, in the different phases. Similar statements hold for the currents in mains 2 and 3; so that the current in each main from a $\Delta$-connected armature is $\sqrt{3}$ times the current in each winding. Receiving Circuits to Three=Phase Mains. Dissimilar Circuits (Unbalanced System). When the receiv-


Fig. 102. "Y" Method of Connecting Receiving Circuits ing circuits which take current from three-phase mains are dissimilar, that is, do not each have equal resistance and reactance, four mains should be employed, as indicated in Fig. 97; each receiving circuit being connected from main 4 to one of the other mains. A common example of an unbalanced system is where a mixed load of induction motors and incandescent lamps is connected unsymmetrically to three-phase receiving mains. It is, however, desirable to keep the three windings $A, B$, and $C$ of the alternator almost equally loaded; and in practice the receiving circuits are so disposed as to satisfy this condition as nearly as possible.

Similar Circuits (Balanced System). When three-phase currents are used to drive induction motors, synchronous motors, or rotary converters, each one of these machines takes current equally from the three mains; and since three-phase currents are utilized chiefly in the operation of the machines mentioned, the system is usually balanced. In this case three mains only are employed, and each receiving unit has three similar receiving circuits connected to the mains according to either the $Y$ or the $\Delta$ method. The $Y$ method of connecting receiving circuits is shown in Fig. 102. One terminal


Fig. 103. " $\Delta$ " Method of Connecting Receiving Circuits equal to $\frac{E}{\sqrt{3}}$, where $E$ is the electromotive force between any pair of mains.

TABLE III
$\Delta$ = and $Y$-Connection Data In Mains

|  | E. M. F. between <br> Mains | Current in Earh <br> Main | Power Rating |
| :---: | :---: | :---: | :---: |
| $\mathbf{Y}$ connection..... | $E_{w}$ | $\sqrt{3} I_{w}$ | $\sqrt{3} E_{w} I_{w}$ |
| $\sqrt{3} E_{w}$ | $I_{w}$ | $\sqrt{3} E_{w} I_{w}$ |  |

The $\Delta$ method of connecting receiving circuits is shown in Fig. 103. Here the three receiving circuits are connected between the respective pairs of mains; the electromotive force acting on each receiving circuit is the electromotive force between the mains; and the current in each receiving circuit is $\frac{I}{3}$, where $I$ is the
current in each main.

Examples. 1. The three windings or phases of a three-phase induction motor are $\mathbf{Y}$-connected to three-phase mains. The voltage between mains is 500 , and each main delivers 25 amperes to the motor. It is required to find the current in each phase of the motor, and the electromotive force acting on each phase of the motor.

Solution. Since the windings are $\mathbf{Y}$-connected, the current in each is the same as the current in each main, namely, 25 amperes; and the electromotive force acting on each phase of the motor winding is $\frac{500}{\sqrt{3}}$, or 288.7 volts. The power input is $P=\sqrt{3} \times 500 \times 25=21625$ watts.
2. The three phases of the above three-phase induction motor are $\Delta$-connected to three-phase mains. The voltage between mains in 288.7, and the current in each main is 43.3 amperes. It is required to find the current in each phase of the motor, and the electromotive force acting on each phase of the motor.

Solution. Since the windings are $\Delta$-connected, the electromotive force acting on each phase is the same as the voltage between the mains, namely, 288.7 volts; and current in each phase of the motor is $\frac{43.3}{\sqrt{3}}$ or 25 amperes. The power input is $P=\sqrt{\overline{3}} \times 288.7 \times 43.3=21625$ watts, the same as before.

Summary of Electromotive Force and Current Relations for $\Delta$ and $Y$ Connections. Let $E_{w}$ be the rated electromotive force of each winding, and $I_{w}$ the rated full-load current output of each phase of the winding of a three-phase alternator, then, for a generator with non-inductive load, the data is as given in Table III.

Let $E$ be the electromotive force between mains of a three-

## TABLE IV <br> $\Delta=$ and $Y=$ Connection Data in Receiving Circuits

| $\Delta$ connection..... | E. M. F. between Ter- <br> minals of Each Re- <br> ceiving Circuit | Current in Each <br> Receiving <br> Circuit | Total Power <br> Input |
| :---: | :---: | :---: | :---: |
|  | $E$ | $\frac{I}{\sqrt{3}}$ | $\sqrt{3} E I$ |
|  | $\frac{E}{\sqrt{3}}$ | $I$ | $\sqrt{3} E I$ |

phase system, and let $I$ be the current in each main, then, for three receiving circuits, the data is as given in Table IV.

The permissible power output or rating of a three-phase alternator is the same whether its armature windings are Y -connected or $\Delta$-connected.

The power output of a three-phase generator is $\sqrt{3} \times$ electromotive force between mains $\times$ current in one main $\times$ power factor of the receiving circuits.

## MEASUREMENT OF POWER

In alternating-current circuits, power cannot in general be measured by means of an ammeter and a voltmeter, as in the case of direct current, because the power expended is generally less than the product of effective electromotive force and effective current on account of the difference in phase between the electromotive force and the current.

A well-designed wattmeter is the standard instrument for measuring power in alternating-current circuits.

The current flowing in the circuit is passed through fixed coils and the current which depends upon the potential of the circuit is passed through a movable coil or coils. The resultant effect is directly proportional to the power in watts and is indicated by the scale reading. (See "Electrical Measurements.")

The several circuits of a polyphase system are often entirely separate and independent; and in such cases the total power delivered to a receiving apparatus is found by measuring the power delivered to each separate receiving circuit. The total power delivered is the sum of the amounts delivered to the different receiving circuits.

In order to measure the power delivered to one of the receiving circuits of a polyphase system, the current coil of the wattmeter is to be connected in series with this receiving circuit, and
the pressure (or voltage) coil of the wattmeter is to be connected between the terminals of this receiving circuit. In some cases this connection of the voltage coil cannot be made, because one terminal of the receiving circuit may be out of reach in the interior of the apparatus.

Balanced Systems. In general, the several circuits which receive current and power from polyphase mains are more or less unlike in both resistance and reactance, and take different amounts of current and power from the mains. It is, however, desirable that the several receiving circuits be alike, so that they may take equal currents and equal amounts of power from the mains. When this condition is realized, the system is said to be balanced.

For example, when independent groups of lamps are supplied from polyphase mains, each group taking current directly, or through a transformer, from one phase of the polyphase system, the system is said to be unbalanced when the number of lamps is not the same in the several groups. In general, the supply of power to several separate, independent, and unrelated receiving circuits, such as independent groups of lamps and single-phase motors, leads to the unbalancing of a polyphase system. Apparatus, such as polyphase induction motors, synchronous motors, and rotary converters which are especially designed to take power from polyphase mains, is always provided with two or three similar receiving circuits so as to take equal amounts of current and power from each phase of the system. When such polyphase apparatus takes power from polyphase mains only, the system is always very nearly balanced.

If a polyphase system were exactly balanced it would be sufficient to measure the power delivered by one phase only; but since a balanced condition of a system is seldom exactly realized in practice, there may be considerable error introduced by assuming that a system is balanced, and by calculating the total power from the wattmeter reading of power delivered by one phase only.

In balanced or approximately balanced polyphase systems, the measurement of power by use of a single wattmeter is best accomplished by special arrangement of connections as follows:

Three-Wire Two-Phase Systems. The current coil of the wattmeter should be connected in the middle main as shown in Fig. 104. After a reading is taken with the pressure coil connected between
middle and lower mains, this connection is quickly changed to the upper main, as indicated by the dotted line, and the wattmeter again read. The sum of the two successive readings gives the total power. If this method is used, the system should not only be balanced, but no change in the load should occur between readings.


Fig. 104. Diagram of Power Connection for Three-Wire Two-Phase Induction Motor

For example, the power taken by a two-phase induction motor is measured by a wattmeter connected as shown in Fig. 104. When the wattmeter is connected as shown by the full line in the figure, it reads 9,900 watts. When the wattmeter is connected as shown by the dotted line, it reads 1,415 watts. The total watts delivered are, therefore, 11,315 watts, the two phases of the motor being assumed to be balanced. Each phase of the motor receives, therefore, $\frac{11,315}{2}=5,657$ watts. The current delivered to each phase, as measured by an alternating-current ammeter is 32.14 amperes; and the electromotive force acting on each phase of the motor, that is, between the terminals of each receiving circuit, is 220 volts. The apparent power (volt-amperes) delivered to each phase is 220


Fig. 105. Diagram of Power Connection for Three-Wire Three-Phase System
volts $\times 32.14$ amperes $=7,071$ apparent watts; and the power factor of each receiving circuit is

$$
\frac{5,657 \text { watts }}{7.071 \text { apparent watts }} \text {, or } 0.80
$$

The two readings of a wattmeter, connected as shown in Fig. 104, are unequal, even though the receiving circuits are balanced, because of the effect of lagging currents; or, in other words, because the two receiving circuits are inductive.

It is to be carefully noted that, in general, neither reading of the wattmeter measures the power delivered to either one of the receiving circuits.

Three-Wire Three-Phase Systems. The current coil of the wattmeter should be connected in series with one (any one) of the three mains, as shown in Fig. 105. After one reading of the wattmeter is taken with the voltage coil connected to main 2 , as indicated by the full line, the connection is quickly changed to main 1 , as indicated by the dotted line, and a second reading of the watt-

$\leftarrow$ to supply mains


Fig. 106. Power Connection of Four-Wire Two-Phase System
meter is taken. The total power delivered to the three similar (that is, balanced) phases, is equal to the sum of the two readings.

Unbalanced Systems. In general, any receiving apparatus is sufficiently unbalanced to require the measurement of power to be made on the assumption that the receiving circuits are unbalanced.

Four-Wire Two-Phase. When four mains are used, two for each separate phase, then two wattmeters are required, one for measuring the power delivered by each phase. Each of these wattmeters is connected exactly as in the case of single-phase delivery of power, as shown in Fig. 106. The sum of the readings $W_{1}+W_{2}^{\prime}$ of the two wattmeters gives the total power delivered. Two readings should, of course, be taken as nearly simultaneously as possible.

Three-Wire Two-Phase. When a two-phase system is balanced or unbalanced and has three supply mains, one main acting as the


Fig. 107. Diagram of Power Connection for Three-Wire Two-Phase System
common return for the other two, then the arrangement shown in Fig. 107 gives the best results. The total power delivered to the receiving circuit is the sum $W_{1}+W_{2}$ of the readings of the two wattmeters. The readings should be taken as nearly simultaneously as possible.


Fig. 108. Dfagram of Power Connection for Six-Wire Three-Phase System

Six-Wire Three-Phase. When six mains are used, two for each separate phase, then three wattmeters are required, one for measur-
ing the power delivered by each phase. Each of these wattmeters is connected exactly as in the case of single-phase delivery of power, as shown in Fig. 108.

In practice, six wires are never used for three-phase systems on account of complications and the excessive amount of copper required.

Three-Wire Three-Phase. When three mains are used in a three-phase system, two wattmeters are sufficient for the complete measurement of the power delivered to any three-phase receiving unit, whether the receiving circuit is balanced or unbalanced, or whether it is connected $Y$ or $\Delta$.

Fig. 109 shows two wattmeters connected for measuring the power delivered to a $\Delta$-connected three-phase receiving system. The algebraic sum of the readings of the two wattmeters gives


Fig. 109. Diagram of Power Connection for Three-Wire Three-Phase System
the total power delivered independent of balance or lag. When the current lag in the circuit is less than 60 degrees, $i . e .$, when the power factor is greater than 0.5 , then the arithmetical sum of the readings of the two wattmeters gives the total power. But if the lag is greater than 60 degrees, i.e., when the power factor is less than 0.5 , the relation of the currents in the current coil and pressure coil of one of the wattmeters causes it to give a negative reading; hence the arithmetical difference of the readings of the two instruments gives the power.

There may be a difficulty in determining which condition exists in some cases, especially when the power delivered to partially loaded induction motors whose power factor is low, is to be measured. In such cases one may determine whether the sum or difference of readings is to be taken by temporarily transferring the connection of one of the voltage coils from the middle wire to
the outside wire. For example, the voltage coil of wattmeter $W^{\prime \prime}$, Fig. 109, would be temporarily connected to main 1 instead of to main 3 , as indicated by the dotted line. If the wattmeter, the connections of which have been changed, reverses, that instrument having the lesser indication is reading negatively. The nature of the load usually enables one to judge roughly whether the power factor is less or greater than 0.5.

To prove the accuracy of these deductions, let the positive direction in the mains 1 and $\mathscr{2}$ and in the three receiving circuits be chosen as indicated by the arrows in Fig. 109. These directions are chosen symmetrically with respect to the two wattmeters. Let the instantaneous currents in the receiving circuits be $i^{\prime}, i^{\prime \prime}$, and $i^{\prime \prime \prime}$, as shown in the figure. Let $a$ be the instantaneous current in main 1 , and let $b$ be the instantaneous current in main 2. Then, from the arbitrary choice of signs,

$$
\begin{aligned}
& a=i^{\prime}+i^{\prime \prime \prime} \\
& b=i^{\prime \prime}-i^{\prime \prime \prime}
\end{aligned}
$$

The reading $W^{\prime}$ of the upper wattmeter is equal to the average value of the product of the current $a$, which flows through the current coil of the instrument, and the electromotive force $c^{\prime}$, which is acting upon the pressure coil of the wattmeter. That is

$$
W^{\prime}=\operatorname{average}\left(a e^{\prime}\right)
$$

Similarly, the reading $W^{\prime \prime}$ of the lower wattmeter gives

$$
W^{\prime \prime}=\text { average }\left(b e^{\prime \prime}\right) .
$$

Substituting the above values of $a$ and $b$ in the expressions for $W^{\prime}$ and $W^{\prime \prime}$, and adding results, we have

$$
W^{\prime}+W^{\prime \prime}=\text { average }\left(e^{\prime} i^{\prime}\right)+\text { average }\left(e^{\prime \prime} i^{\prime \prime}\right)+\text { average }\left(e^{\prime}-e^{\prime \prime}\right) i^{\prime \prime \prime}
$$

But, from the figure, $e^{\prime}-e^{\prime \prime}=e^{\prime \prime \prime}$; hence

$$
W^{\prime}+W^{\prime \prime}=\text { average }\left(e^{\prime} i^{\prime}\right)+\text { average }\left(e^{\prime \prime} i^{\prime \prime}\right)+\text { average }\left(e^{\prime \prime \prime} i^{\prime \prime \prime}\right)
$$

Although a formal proof of the principle of the two-wattmeter method has not been given for a Y -connected circuit, it is not necessary to show independently that it holds for both cases. A little consideration will show that if the electromotive forces acting between the three wires, the currents flowing in them, and their phase relations are given, there is then a perfectly definite amount of power transmitted along the three lines, and it is quite immaterial whether this power is being delivered to circuits connected $\Delta$ or Y .

## ARMATURE WINDINGS

In general, any direct-current armature winding may be used for the armature of an alternator; but the desirability of generating comparatively high voltages in the armature so as to avoid the use of step-up transformers, makes it necessary to abandon the styles of winding best suited to direct-current machines, and to use windings specially adapted to the conditions of alternatingcurrent practice.

Comparing the armature windings used for direct-current machines with those for alternators, we find, frst, that all the reentrant (or closed-coil) direct-current windings must necessarily be either two-circuit or multiple-circuit windings, i. e., they must have at least two paths in parallel through the armature between brushes; and second, that the armatures of alternators (and synchronous motors) may, and generally do, from practical considerations, have one-circuit windings, $i$. e., windings having one circuit per phase. It follows, therefore, that any direct-current winding may be used for alternating-current machines; but the converse statement, that any alternating-current winding may be used for direct-current machines, is not true in general. In other words, the windings of alternating-current armatures are essentially non-re-entrant (or open-circuit) windings. The only exceptions are the $\Delta$-connected (or mesh-connected) polyphase windings, and the short-circuited windings of "squirrel-cage" induction motors, both of which are re-entrant (or closed-circuit) windings. The $\Delta$-connected polyphase windings are, therefore, the only windings that can be used for the armatures of rotary converters.

In the type of winding generally employed for alternators, a number of distinct coils are arranged on the armature; in these coils alternating electromotive forces are induced as they pass the fieldmagnet poles, and the several coils are connected in series between the collecting rings.

Classification. According to Shape of Core. Armatures for alternators, just as in the case of direct-current machines, may be divided into drum armatures; ring armatures; and disk armatures. Of these the ring and the disk armatures are seldom used in America although they are to some extent adopted in European practice.

The ring and the disk types of the armature are mechanically less stable than the drum type; and the ring armature, moreover, other things being equal, requires more wire to be wound upon it for a given output than in the case of the drum armature, and possesses, therefore, a greater inductance than the latter type. Drum armatures, whether the alternators are of the revolving or stationary armature type, have laminated iron cores similar in construction to the armature cores for direct-current machines. Disk armatures, on the other hand, are usually made up without iron, thus introducing constructional difficulties.

According to Construction of Core. With reference to the construction of their cores, the armatures of alternators may be classified, as in the case of direct-current machines, into smooth-core armatures and toothed-core armatures.

In the smooth-core armature the conductors, arranged in flat coils, lie on the surface of the armature core, and the coils in some cases are bent down over the ends of the core, where they are fastened by end plates or by blocks of wood or fiber. The spaces in the centers of the coils are filled with wooden blocks either screwed to the cores or held in place by the binding wires. In other cases the coils are flat or "pancake" shaped, and of the same length as the armature core. In the latter case they are laid upon the cylindrical surface of the armature core, and are securely bound with wire bands.

The form of the wave of electromotive force produced by smooth-core armatures is very nearly harmonic (sinusoidal) or slightly flat-topped. The inductance of a smooth-core (or surfacewound) armature is considerably less than that of a toothed-core armature. Although much used in earlier designs, the smoothcore armatures owing largely to their comparatively weak mechanical structure, have been superseded in modern practice by the toothed-core constructions.

One or another of the forms of toothed-core armature is now almost universally used in practice. The conductors are laid in slots or grooves, the sides and bottom of which are first carefully insulated by troughs of mica-canvas, micanite, or other suitable insulating material. The insulated conductors (cotton covered) are generally wound into coils on "formers," each coil being care-
fully taped, and then impregnated with insulating compound (or varnish). The coils are then thoroughly dried by baking in ovens.

The conductors being enclosed in slots between teeth which project more or less over the conductors, the toothed-core type is often called iron-clad. This construction has three great advantages over the smooth-core type:
(a) It allows the length of air gap from iron of pole-face to iron of armature core to be reduced to a minimum; just enough for mechanical clearance. Other things being equal, this means a saving in the copper required to magnetize the field.
(b) It protects the embedded conductors from injury.
(c) It affords an admirable way of supporting and securing the conductors firmly in place against the action of centrifugal force; and, further, it shields the conductors almost completely from the racking action of the magnetic drag due to the magnetic field.

The shape and number of the slots in a toothed armature core have a marked effect on the shape of the electromotive force wave, and upon the regulation of the alternator. The shape of the wave is affected by the distribution of the magnetic flux in the air gap. The regulation is affected by the inductance of the armature coils, and the inductance depends on the number and shape of the armature slots.

Fig. 110 shows a portion of an armaturecore disk or stamping for a 12 -pole, uni-tooth (one slot per pole per phase), three-phase alternator. The armature winding adapted to this uni-tooth core is, of course, the unicoil or concentrated winding. The armature


Fig. 110. Portion of Stamping for Twelve-Pole UniTooth Three-Phase Alternator coils are held in place in the slots by wedges of wood or fiber driven in from the ends of the core and fitting into notches near the tops of the teeth, as shown in the figure. This construction, now almost universally adopted by manufacturers, avoids the necessity for any binding wire on the armature core.

Alternators with uni-tooth armature cores are characterized by large armature inductance and by peaked-wave shapes of the induced electromotive forces; also by marked variations in the shape of the wave of induced electromotive force, according to the magnitude and power factor of the load.

On account of their comparatively large inductance, uni-tooth armature constructions require a relatively large increase in the field-cxciting current in passing from no load to full load output. In other words, regulation is poorer than for multi-tooth armature cores.

According to present practice in design, the great majority of alternators are constructed with armature-core stampings having two or three or more slots per pole per phase. Fig. 111 shows a portion of an armature-core stamping for a 12-pole, three-phase alternator. It has three slots per pole per phase. The slots are open, which, together with the distributed (multi-coil) type of winding, results in a low armature inductance. This means that a relatively small increase in the field-exciting current is required in passing from no load to full load output.

Alternators with multi-tooth armature cores are especially adapted for long-distance transmission where step-up transformers are used. The regulation is better than with the uni-tooth core construction; and the wave shape of the electromotive force generated by the distributed winding approaches a sine wave, which is the best wave shape for the long-distance transmission of power. This is because of the fact that the nearer a


Fig. 111. Portion of Stamping for TwelvePole Three-Phase Alternator given curve of electromotive force approaches a sine curve, the less the likelihood of a dangerous rise of voltage (resonance) occuring at the distant end of a long transmission line because of the capacity (condenser) effect. A long transmission line has a series of frequencies of electrical oscillation just as a stretched violin string has a series of frequencies of mechanical vibration. If the frequency of the current delivered by an alternator happens to coincide with any of these "proper" frequencies of the transmission line, the line will have violent electrical oscillations set up in it, and excessive voltages will occur at certain points along the line. A sine wave of electromotive force has only one frequency. Any other kind of an alternating electromotive force is composed of a series of sine waves of ascending frequencies (the harmonics or over-tones in music), all combined into a resultant wave form. There is, therefore, more danger that one out of all
the frequencies may coincide with one of the "proper" line frequencies than that the single frequency of a sine wave may so coincide.

According to Progression of Winding. With reference to the progress of the winding from slot to slot, armature windings may be divided into spiral or ring windings; lap windings; and wave windings. These terms have the same meaning when applied to alternator windings as they do when applied to the windings of direct-current machines.

According to Disposition of Coils. With reference to the disposition of the coils around the periphery of the core, we have to distinguish between two general classes, viz, concentrated, or uni-coil, uindings, and distributed, or multi-coil, windings.

Concentrated, or uni-coil, windings, as the name implies, consist of one coil per pole per phase. The armature conductors are thus grouped in bundles, and usually placed in slots, there being one slot per pole for each phase.

Examples of concentrated windings are illustrated in Figs. 3 and 73 , in which the armature conductors are shown as lying in one slot per pole. Fig. 3 shows adjacent sides of two different armature coils lying in one slot. In some cases, each slot is filled by one side of a single armature coil, giving one slot per phase per pair of poles. Such windings are sometimes called "half-coiled" or "hemi-tropic."

Distributed, or multi-coil, windings consist of several coils per pole per phase. The armature conductors are distributed in two or more slots per pole per phase.

Examples of distributed windings are shown in Figs. 91 and 96. Fig. 91 shows an end view of a two-phase winding distributed in two slots per pole per phase. Fig. 96 shows an end view of a three-phase winding distributed in two slots per pole per phase.

Concentrated windings are less expensive to make; and they give a greater effective electromotive force (at zero load) for a given number of conductors, other things being equal, than distributed windings. This is on account of the fact that all the conductors of a concentrated winding cut the field flux simultaneously, while the various conductors of a distributed winding do not cut the field flux simultaneously.

Concentrated windings have greater inductance than distributed windings for the same total number of conductors, and also cause a greater armature reaction for a given current than distributed windings do. Consequently the terminal electromotive force of an alternator falls off more with a concentrated winding than with a distributed winding, when the current output is increased.


Fig. 112. Completed Armature with Strap Winding, Four Slots Per Phase
Therefore, an alternator with a concentrated winding has a poorer (higher) regulation than an alternator with a distributed winding; and although a concentrated winding may give a higher electromotive force at zero load, it may actually give a lower electromotive force at full load.

According to Form of Conductor. According to the form of the conductors used, armature windings may be divided into three classes, viz, wire winding; strap uinding; and bar winding.


Fig. 113. Portion of Bar-Wound Armature, One Bar Per Slot
Wire winding, which is usually emplojed in high-voltage machines of low-current output, consists of machine-wound coils, which are entirely formed and insulated before being placed in the armature slots.

Strap winding is used for machines of lower voltage and of greater current output, and it consists of copper strap, forged into the required shape and carefully insulated.

Both the wire and strap windings are placed in the slots without any mechanical bending, thus preventing damage to the insulation. In armature cores having the slots partially closed, the winding is slipped in from the end; but in cores having open slots, wedges of hard fiber secure the coils in place.

Fig. 112 is an illustration of strap winding distributed in four slots per pole per phase. The completed armature, ready for direct connection to a steam engine, is shown in the figure, and is intended to revolve inside of a stationary field-magnet structure. The four collector rings indicate that the armature is wound for two phases. It is manufactured by the Westinghouse Electric Company.

Bar windings are held in place by the overhanging tips of the teeth. The bars, after being carefully insulated, are slipped into the slots from one end of the


Fig. 114. Concentrated Single-Phase Armature Winding armature. The end connections of the bar winding are bolted and soldered to the bars after the conductors are in place.

Bar windings are usually arranged with either one or two bars per slot. There are no band wires on the armature core.

Fig. 113 shows a portion of a barwound revolving armature having one bar per slot.
Single=Phase Windings. Fig. 3 shows a common type of single-phase winding having one coil per pole. Fig. 114 shows another type of concentrated single-phase winding, having one coil to each pair of poles or one slot per pole. The sketch $b$ is a sectional view of a portion of the armature core, showing one of the slots containing the conductors forming one side of a single armature coil and standing opposite to a field pole. In the main diagram the heavy sector-shaped figures represent the coils, and the light lines represent the connections between the terminals of the coils. The radial parts of the sector-shaped figures represent the portions
of the coils that lie in the slots, and the curved parts represent the ends of the coils. The circles at the center of the figure represent the collecting rings, one being shown inside the other for clearness. The arrows represent the direction of the current at a given instant. All electromotive forces induced under $N$ poles are in one direction, and all electromotive forces induced under $S$ poles are in the opposite direction. These remarks apply to Figs. 114 to 121 inclusive.

Fig. 115 represents a single-


Fig. 115. Single-Phase Armature Winding, Two Slots Per Pole phase winding distributed in two slots per pole, all the coils being connected in series. The sketch $b$, is a sectional view of a portion of the armature core, showing two slots.

Two-Phase Windings. The two-phase winding consists of two independent single-phase windings on the same armature, each being connected to a separate pair of collecting rings, as shown in Figs. 116 and 117. Fig. 116 shows a two-phase concentrated winding, one slot per pole for each phase. Fig. 117 shows a two-phase winding distributed in two slots per pole for each phase. In each of the figures (116 and 117), the winding of one of the phases is shown by dotted lines, to distinguish it from that of the other.


Fig. 116. Two-Phase Concentrated Armature Winding


Fig. 117. Two-Phase Armature Winding, Two Slots Per Pole Per Phase

Three=Phase Windings. The three-phase winding consists of three independent single-phase windings on the same armature,
the terminals of the individual windings being connected according to the Y scheme or $\Delta$ scheme, as explained on page 119. Fig.


Fig. 118. Three-Phase Concentrated Winding Y-Connected


Fig. 119. Three-Phase Concentrated Winding $\triangle$-Connected

118 shows a three-phase concentrated winding (one slot per pole for each phase), Y-connected. Fig. 119 shows the same winding $\Delta$ connected. In Figs. 118 and 119 the winding for phase $A$ is shown by heavy full lines; the winding for phase $B$ is shown by light full lines; and the winding for phase $C$ is shown by dotted lines.

The Y connection gives $1^{\prime \prime} \overline{3}$ times as much electromotive force between collecting rings as the $\Delta$ comection for the same total number of conductors per phase, and is the more suitable for high electromotive force machines. The $\Delta$ connection, on the other hand, is especially adapted for


Fig. 120. Three-Phase Bar Winding, Two Slots Per Pole Per Phase machines for large current output. The line current is $\sqrt{3}$ times as great as the current in each winding in a $\Delta$-connected armature.

Fig. 120 shows a three-phase bar winding distributed in two slots per pole for each phase. The sketch $\zeta$, is a sectional view of one slot containing a single conductor in the form of a rectangular bar.
Fig. 121 shows a three-phase coil winding distributed in two slots per pole for each phase and arranged in two layers, there being as many coils on the armature as there are slots, so that portions of
two coils lie in each slot, one above the other. The portions of the coils represented by full lines lie in the upper parts of the slots, and the adjacent dotted portions lie in the bottoms of the same slots. The sketch $b$ is a sectional view of one slot showing two half-coils one above the other, thus constituting a two-layer winding.

The method of connecting up the separate windings of a threephase alternator is as follows:

Y Connection. The terminals of the individual windings which are to be connected to the common junction and to the collecting rings, may be determined as follows: Consider the instant when winding $A$ is squarely under the poles, as shown in Fig. 118. The electromotive foree in this winding (and the current also, if the circuit is non-inductive) is a maximum, and the currents in the other two phases $B$ and $C$ are each half as great. If winding $A$ is connected so that its current is flowing away from $k$, windings $B$ and ( $r$ must be connected so that their currents flow towards $k$.
$\Delta$ Connecrion. The three windings form a closed circuit when $\Delta$ connected. The total electromotive force around this circuit at any instant must be zero. Consider the instant when winding $A$ is squarely uncler the poles, as shown in Fig. 119. The electromotive force in this winding is a maximum, and the electromotive forces in the other two windings are each half as great. Then winding $A$ is connected to 2ny pair of rings, say 1 and 2; winding $B$ is connected to ring 3 and ring 1 (or 2); and winding $C$ is connected to ring 3 and ring $\mathcal{Z}$ (or 1 ); these connections are


Fig. 121. Three-Phase Coil Winding, Two Slots Per Pole Per Phase made so that the electromotive forces at the given instant are in the directions indicated by the arrows in Fig. 119.

## COMMERCIAL TYPES OF MACHINES

Alteruators are of three types, differing in the means employed for causing the conductors to cut the magnetic flux from the field magnet, viz, revolving-armature type; revolving-field type; and inductor type.
(Of these three types the one which experience has proven the fittest to survive is the revolving field type and it is today generally adopted by manufacturers as the standard. Alternators of the revolving armature type are still manufactured to supply special
demands and for small isolated electric plants, and they may be found in satisfactory operation in many of the older and smaller electric central stations.

The inductor type is rarely if ever built today. In this type both the field winding and the armature (core and winding) are stationary, and the magnetic flux produced by the field winding is caused to move past the armature conductors by means of a revolving iron structure furnished with polar projections called the inductor. The advantages of this type, viz, absence of moving wires, collecting devices and brushes, with the consequent minimum cost of attendance are outweighed by the disadvantages of increased size, weight, and cost, and by the lower efficiency and poorer (higher) regulation especially when supplying current to inductive loads such as induction motors or other apparatus having a power factor less than unity. On account of these disadvantages the manufacture and use of inductor alternators have been virtually discontinued.

The revolving-armature type of alternator is limited to a general power and lighting distribution where only a moderate voltage is required. Machines of this type are comparatively cheap to build. They can be automatically compounded by the use of composite field windings without any complication of parts, which is not the case with the revolving field type. They can be furnished with an auxiliary armature winding and small commutator for exciting their fields, thus dispensing with any external exciter.

The revolving-armature type, on the other hand, is not suitable for generating either high or low voltages, on account of the difficulties of insulating the armature conductors and collecting rings in the first case (high voltage), and of collecting a large armature current in the second case (low voltage).

The advantages of the revolving field type over the revolving armature type are as follows:
(1) The revolving field type gives more space for the armature winding, and thus permits the stationary armature to be easily insulated to withstand a testing pressure of over 30,000 volts.

Large alternators with stationary armatures have been built to generate voltages up to 20,000 ; but it is doubtful whether, on the whole, it is economical to build them for voltages greater than about 13,000.
(2) The insulation of the armature is relieved from the strains due to centrifugal force at high speeds. Furthermore, a revolving field can be made stronger and more compact than a revolving armature and, therefore, the revolving-field type of alternator is much the better suited to the high speeds employed in alternators driven by steam turbines.
(3) The number of collecting rings is reduced to a minimum, viz, two, and the amount of electrical energy transmitted through them is only about two per cent of that which would have to be transmitted through the collector rings of a revolving armature alternator of the same capacity. The voltage between the collector rings is also relatively small, being either 125 volts, or 250 volts in the larger machines.

## REVOLVING=ARMATURE ALTERNATORS

In this type of alternator, the field magnet is stationary, while the armature is mounted on a shaft and is driven (by means of a belt or mechanical coupling) by the prime mover, which may be either a steam engine, a steam turbine, or a water wheel. The current induced in the armature conductors is delivered to the external circuit through collector rings on which brushes rub. The armature may be wound for a single-phase current (having two collector rings), for two-phase currents (having four collector rings), for three-phase currents (having three collector rings), etc. The revolving-armature type is used almost exclusively for alternators of small output and moderate voltage.

Although single-phase alternators, and especially those of the revolving-armature type have been virtually superseded by polyphase machines of the revolving-field type, still there are today a number of the older fashioned alternators in regular use in some of the smaller electric lighting stations. For this reason some of the typical features of these machines will be described before considering the more modern designs.

Fort Wayne Single=Phase. Fig. 122 shows a 90 -kilowatt 1,100 volt 8 -pole single-phase belt-driven alternator manufactured by the Fort Wayne Electric Company, of Fort Wayne, Indiana. It is designed to be driven at a speed of 900 revolutions per minute; hence the frequency of its electromotive force is $\frac{\frac{8}{2} \times 900}{60}=60$ cycles per sec-
ond. The figure shows the two collector rings adjacent to the armature, also the rectifying commutator with its brushes for supplying

uni-directional current to the coarse wire coils of the composite field winding.

The exciter is a shunt-wound 4-pole 2-kilowatt direct-current generator running at a speed of 1,400 revolutions per minute. It is shown belted to a pulley on the alternator shaft. The current from this exciter is led to the fine wire coils of the composite field winding. Each field pole of the alternator is provided with two coils wound on one and the same spool-one of coarse wire, supplied with current from the rectifying commutator; and the other of fine wire, supplied with current from the exciter.

The field structure, base-plate, and pedestals are cast in one piece, and the whole machine rests upon a cast-iron sub-base. This sub-base is provided with slide rails along which the machine may be moved by means of a screw turned by a ratchet and lever, as shown-this for the purpose of adjusting the tension of the main driving belt. The field poles are "built up" of sheet-iron stampings and are held together by long bolts. These field poles are arranged in the mould in which the field frame is cast, and are thus cast-welded to the frame. The field coils are machine wound on spools, with insulated copper wire, the coarse wire coils being wound on top of the fine wire coils. The field Fig. 123. German Silver Resistance Coils in Shunt with spools are held in position
 on the field poles by brass collars fixed to the outer ends of the poles.

The coarse wire coils, connected in series, are supplied with uni-directional current from the rectifying commutator; and the entire set is shunted with resistance consisting of a strip of German silver wound on an insulated form, Fig. 123, and mounted on an insulated block inside the hollow pedestal on the collector end of the machine.

The armature is of the iron-clad ring type built up of small overlapping sheet-steel punchings, annealed and japanned to reduce the loss caused by hysteresis and eddy currents.

The armature winding is of the concentrated or uni-coil type. The coils are wound by hand directly on each armature tooth, insulated copper ribbon being used. Armature coils are generally wound on formers, and afterwards sprung into place in the slots on the armature core.

Westinghouse Uni=Coil Armature. Figs. 124 and 125 show a Westinghouse single-phase uni-coil armature very much like the one used in the alternator shown in Fig. 122. The coils of the West-


Fig. 124. Weatinghouse Single-Phase Uni-Coil Armature Core
inghouse alternator, however, are machine-wound on formers; and, after being taped, varnished, and baked, they are spread out slightly so as to pass over the teeth, and are forced into place in the deep slots, being securely held there by wooden wedges. Fig. 124 shows the core unwound, and Fig. 125 the method of placing coils in this type of machine. The armature teeth are $\mathbf{T}$ shaped, and partially overhang the armature coils, thus protecting the coils from injury. There are, of course, the same number of armature teeth as field poles in the uni-coil armature. Thus Figs. 115 and 116 show an armature core for an eight-pole field.

Fig. 126 shows the completed armature of which the core and coils are shown in Figs. 124 and 125. At the ends of the armature are two brass shields for protecting the ends of the armature coils.


Fig. 125. Westinghouse Single-Phase Armature with Coils in Place
The ring-oiling, spherical-seated, self-aligning bearings are shown on the shaft in Fig. 126; and at the end of the shaft are shown the two
collecting rings and the rectifying commutator. The wires leading to the collector rings and to the rectifying commutator are led through the shaft, which is made hollow.


Fig. 126. Westinghouse Single-Phase Uni-Coil Armature Complete


Fig. 127. Westinghouse Single-Phase Armature Core and Coils. Distributed Winding


Fig. 128. Westinghouse Single-Phase Distributed Winding Armature Complete
Westinghouse Armature with Distributed Winding. Figs. 127 and 128 show a single-phase Westinghouse alternator armature with distributed winding. Three armature coils are shown in Fig. 127
separately and also grouped together. The manner of grouping shown is carried out when the armature coils are assembled on the armature core; and there are as many of these groups of armature coils as there are poles in the field. Fig. 128 shows the finished armature with end shields, collecting rings, and rectifying commutator.

In assembling the stampings of an armature core a stiff, corrugated stamping (or its equivalent) is inserted at intervals between groups of the flat sheet-iron stampings, thus leaving radial air ducts from the inside to the outside of the armature core. These are known as ventilating ducts, and their object is to permit of a free circulation of


Fig. 129. Field Structure of a Westinghouse $180-\mathrm{Kw}$. Alternator cool air through the armature core and coils, thus preventing excessive rise of temperature. The motion of the armature causes air to be drawn in through the end shields to the interior of the armature, whence it is thrown out through the radial ducts, as in the case of a ventilating fan. Four of these spaces between stampings can be seen in Figs. 127 and 128; and the armature "spider" and end shields at each end, as shown, are provided with apertures for admitting cool air.

Field Structure for Westinghouse $180=\mathrm{Kw}$. Alternator. Fig. 129 shows the field structure of a Westinghouse 180 -kilowatt alternator before the field coils have been placed upon it. The field poles (laminated) are shown projecting radially inwards from the castiron, ring-shaped yoke. The yoke is cast in two pieces, and the upper half is bolted to the lower halt, as shown in the figure. The yoke is divided in this way in order that the upper half may be unbolted and may be lifted off by means of the eye-bolt on the top of the machine, thus giving easy access to the armature for inspection and repairs.

General Electric Three=Phase Alternator. Fig. 130 shows a view of a 25 -kilowatt three-phase revolving-armature type of alternator
built by the General Electric Company for use in small isolated plants. The alternator is belt-driven at 1,800 r.p.m. and, therefore, requires four poles to give a frequency of 60 cycles.

The view shows three collector rings mounted on the armature shaft for the collection of the three-phase currents, and a commutator between the rings and the armature core. The special feature of this machine is that it requires no separate exciter, for the armature is wound with two distinct windings, one of which is con-


Fig. 130. General Electric 25-Kw. Three-Phase Alternator
nected to the commutator and supplies the exciting current to the field coils. The main armature winding generates the three-phase alternating currents and is connected to the three collector rings. The field structure consists of laminated pole pieces cast into a rigid stationary frame. The field-magnet cores are wound with coils which furnish a magnetic flux which is common to both alternator and exciter armature windings. These machines are built in three sizes and rated at $7.5,15$, and 25 kw . at unity power factor, and at

6,12 , and $20 \mathrm{kw} .$, respectively, at 0.8 power factor. Standard voltages are $120,240,480$, and 600 volts at 60 cycles, and their ratings are based on either two- or three-phase operation. For single-phase operation any one of the phases may be used, but the rating then is only 70 per cent of the polyphase rating on account of the heating.

## REVOLVING=FIELD ALTERNATORS

In this type of alternator, the armature is stationary, and the armature windings are arranged in slots on the inner face of the armature structure, the latter forming a closed ring inside of which the multipolar field magnet revolves. The armature structure consists of an external frame of cast iron or steel supported on a bedplate. The armature core proper consists of a ring built up of relatively small stampings of sheet iron dovetailed into the external frame and pressed together between two flanges by bolts. The external frame in this type of alternator does not to any perceptible extent carry lines of force, that is, it does not form a part of the magnetic circuit, but serves only as a support for the laminated armature core.

Construction. Frame. When a given type of alternator has been standardized by a manufacturer, it is customary to lay out a complete line of machines, which differ in weight by fifteen or twenty per cent between consecutive sizes. The capacity of a given frame is dependent upon speed, voltage, and specified performances.

Frames are made in two general styles, one called the box type and the other the skeleton type. The box type consists of a single casting for the smaller sizes until an outside diameter of about 8 feet is reached. Above this diameter the frame castings are usually divided into upper and lower sections, split construction being necessary on account of the limitation imposed in handling and shipping.

The skeleton type consists of two side castings between which substantial spacing rods are set at suitable intervals. For manufacturing reasons the skeleton type is preferred for certain sizes of machines, as its construction readily permits of changes being made in the assembling of the armature-core laminations without necessitating a change in the patterns.

The main point to be looked after in alternator frames is a design that will give the maximum of rigidity. The only function
of the frames of rotating-field alternators is to hold rigidly in place the parts composing the stationary element. The frame is further designed with openings at the back of the laminations to give thorough ventilation. The frames contain dovetailed slots into which the laminated iron for the stationary part is pressed. The laminated


Fig. 131. Allis-Chalmers Revolving Field Alternator with Rotor Removed
iron, however, carries all the magnetic flux and the frame is simply for the purpose of holding it in position.

Armature. The armature (stator) core consists of sheet-iron laminations carefully selected and annealed before assembling in order to reduce core loss. The purchings are stacked together and held rigidly in place by heavy steel clamping fingers, the outer circumference of the laminations being dovetailed for fastening to the
frame and the"inner circumference being slotted to receive the armature windings. The punchings are separated at intervals by ventilating plates, which give opportunity for air to circulate freely through the ducts thereby created and out at the open back of the frame. Heavy' end plates are supplied at both ends of the laminations so as to prevent their bulging out at the ends.


Fig. 132. Allis-Chalmers Revolving Field Alternator with One Bearing Removed and Rotor in Foreground
Bed Plate and Bearing Pedestals. Alternators may be divided into three types depending upon the method of driving, viz, engine type, coupled type, and belted type. In the engine type the revolving element is mounted directly on an extended engine shaft. In the coupled type, as the name indicates, the alternator is entirely selfcontained and is designed to be coupled directly to a prime mover, usually a water wheel. In the belted type the alternator is self-
contained and mounted entirely separate from the prime mover and connected therewith by a belt.

The almost universal practice is to make the bed plate, the individual bearing pedestals and the frame`all separate. The parts are properly machined and bolted together. This practice enables the manufacturer to use the same parts for the various types of machines, and to vary the combinations of bed plate, bearing pedestal, and frames to suit any case that may arise. For some of the smaller beltedmachines abracketbearing support instead of pedestal on a bed plate is used. This allows the size of the bed plate to be reduced to a minimum.

Fig. 131 shows a frame of the box type with the laminations and windings placed therein complete. Fig. 132 shows the same alternator with one of the end brackets containing a bearing removed and the rotor (revolving field) in the foreground. This alternator is for belt driving and is called "Type AB" by the Allis-Chalmers Company who manufacture it.

Fig. 133 is a section of an


Fig. 133. Portion of Genfral Electric Alternator Showing Method of Dovetailing Core Laminations alternator showing the method of dovetailing the core laminations to the stator frame. Heavy clamping rings or end plates are mounted on both sides of the core by means of bolts, and supporting fingers extend along the teeth on either side of the slots, as shown in Figs. 134 and 135.

An ample circulation of air for cooling is provided by means of ducts formed by suitable spacing blocks inserted at intervals between the laminations, as may be seen in Figs. 134 and 136.

Armature Coils. The armature coils are wound on formers and, the slots being open, the coils can be easily removed and replaced in case of injury. They are taped and impregnated with an insulat-
ing compound. After being tested, the coils are inserted in the armature slots which are lined with horn fiber and retaining wedges of wood are dovetailed into $\mathbf{V}$-shaped notches on either side of the slot, as shown in Figs. 134 and 136.

Supporting Ring. Where heavy windings project beyond the laminations, an additional support is provided by means of an insulated metal ring, to which the outer ends of the coils are fastened; the coils are thereby protected from mechanical displacement, or distortion due to the magnetic disturbances caused by violent fluctua-


Fig. 134. Armature Coil Support Construction-General Electric Alternator
tions of the load or short-circuits. Fig. 136 shows a section of a supporting ring of this type and indicates the method of connecting the coils to it. In order to admit of the prompt replacement of damaged coils, sufficient space is usually provided between the alternator bearings to allow ample movement of the stator to permit of ready access to both stator and rotor coils. Where space economy necessitates the use of a short shaft, access to the windings may be procured by disconnecting some of the coils and lifting the upper half of the stator.

Terminals. Flexible leads are brought through the frame of the machine near the bottom and connected directly to the line or to terminal blocks which are mounted on the frame. The former arrangement is usually employed.

Rotating Field. The construction of the rotating field is very similar to that of the stationary armature except that the punchings are dovetailed into a rotating spider instead of a stationary frame.

Fig. 137 is a view of a laminated field core in process of construction, showing how the laminations are assembled with overlapping


Fig. 135. Armature Coil Support Construction-General Electric Alternator
joints, and are fastened to the supporting structure. Three individual stampings are shown separate in the figure.

In assembling the stampings that form the built-up structure of field poles and yokes, the stampings break joints in order to enable the structure to resist centrifugal force without bringing undue stress upon the central supporting structure. The stampings are perforated with a number of holes, as shown. These holes register in the built-up structure, and bolts are passed through them in order


Fig. 136. Portion of General Eleptric Stationary Armature Showing Method of Connecting Armature Coils


Fig. 137. Method of Constructing Laminated Field Cores
to clamp the laminations together. At intervals of about 3 inches, the laminations are separated by a corrugated lamination (or its equiv. alent), which serves to form ventilating ducts that extend inwardly to large openings in the cast-iron segments. These ventilating ducts in the revolving field register or coincide with corresponding ducts in the external stationary armature.

The field-pole tips are beveled so as to produce a distribution of magnetic flux which will give approximately a sine wave electromotive force at no load (under load conditions the wave would be somewhat distorted).

For alternators of relatively small rated output, from


Fig. 138. Field Spider Showing Laminated Construction about 30 to 200 kilowatts, and designed for belt driving at a high speed, another type of construction is used by the Westinghouse Company in their "Type G" generators.

The central portion of therevolving part, shown in Fig. 138, is a laminated spider, built up of thin steel plates assembled upon a mandrel and firmly riveted together under hydraulic power.

The core is accurately bored and the spider is pressed upon the shaft in the same manner as a cast-steel spider. The poles, one of which is shown in Fig. 139, are also built up of


Fig. 139. One of the Field Poles Which Fits into the Core of Fig. 138 steel laminations of the same thickness as those of the spider, and riveted together. Each pole is dovetailed into the spider and retained by two taper steel keys. Fig. 140 shows the rotor core with pole pieces assembled on the spider.

The pole pieces are thereby securely held in place during operation; but poles and coils may be easily taken out when desired, by knocking out the steel keys.


Fig. 140. Rotor Core with Pole Pieces Assembled on Spider


Fig. 141. Rotor of Fig. 140 with Field Coils in Place The field coils of these "type G" alternators are wound with wire, and the complete rotor or revolving field for the 30 - and 50 -kilowatt machines is shown in Fig. 141. The pole pieces of this type of alternator rated above 50 kilowatts up to 200 kilowatts are provided with practically closed slots in the pole face for the copper bars of a "squirrelcage" winding. The slots are plainly shown in Fig. 138, and the rotor with the cage winding in place is shown in Fig. 142. This winding, also called an "amortisseur" winding, acts as an effective damper to prevent hunting between machines operated in parallel. The collector rings for conveying through brushes the direct current for exciting the field coils, shown to the right in Fig. 142 , are of cast iron, insulated from the iron bushing over which they are shrunk by V -shaped mica. Another rozor construction is often enployed in the case of large alternators designed for engine drivirig. In such machines the revolving field structure consists of laminated pole pieces bolted
to-a cast-steel or iron ring, which is connected to the hub by arms of suitable section, as shown in Fig. 143. The pole pieces are built of sheet iron, spreading at the pole face so as to secure not only a large sectional area for the magnetic flux passing from the poles into the steel ring, but also to hold the field coils in place.

In Fig. 143 is shown the revolving field of an alternator built by the General Electric Company for direct connection to reciprocating engines. In Fig. 144 is shown the laminated pole piece and one of the strip-wound field coils, and in Fig. 145 is shown a complete pole piece with its coil ready for bolting to the face of the supporting structure.

Field Coils. The field coils are wound on spools or in moulds. Wire is used for the smaller generators and in those of larger capacity a single strip of flat copper bent on edge is usually employed, as shown in Fig. 144, so that the edge of every turn is exposed to the


Fig. 142. Rotor with Squirrel-Cage Winding
air to facilitate radiation, and hence cooling. The strap-wound coils are insulated between turns with ashestos strip forming a fireproof coil which is practically indestructible. The wire coils are wound dry and treated with insulating varnishes. In every case the insulation provided is easily sufficient for the low voltage to which the field winding is subjected.

Excitation. The direct current for exciting the field magnet is carried in to the revolving-field structure by means of brushes rubbing on collector rings to which are connected the terminals of the field coils. The latter are all connected in series.

Alternators are usually separately excited by 125 -volt direct current. This voltage is generally regarded as standard, is easily


Fig. 143. Rotor Construction with Laminated Pole Pieces Bolted to Cast-Iron or Steel Ring
handled, and lends itself readily to the operation of lights and small auxiliary machines in power stations. For large alternators a voltage


Fig. 144 Laminated Pole-Piece and Strip-Wound Field Coil for Rotor of Fig. 143
of 250 has frequently been employed and field coils can be wound for this voltage when required. The lower voltage is generally
to be preferred, principally because it permits the use of strap-wound field coils. For generators large enough to employ field coils of this type at the higher voltage, 250 -volt excitation is not objectionable. The limitation is based on the fact that very thin copper strap cannot be successfully bent edgewise.

Water=Wheel=Driven Alternators. Alternating-current generators which are designed to be directly connected to water-wheels are usually driven at speeds of from about 150 to 600 revolutions per minute, depending upon the size and type of water wheel used. The customary range of speeds in water-wheel-driven alternators is, therefore, intermediate between the slow-speed range of the enginedriven type and the high-speed range of the steam turbine-driven type. Alternators of the water-wheel type are built both with horizontal and with vertical shafts according to conditions. The form of foundation used varies with the service for which they are intended. For the machines with horizontal shafts, cast-iron or channel iron bases are used, and in some cases simple foundation plates are provided for the stator with separate sole plates for the bearing standard. For vertical shaft alternators the base is constructed either for mounting directly on the turbine casing or on separate foundations above the


Fig. 145. Completed Field Coil for Rotor of Fig. 143 wheel pit. The shafts are keyed to the rotor and are arranged for coupling to the waterwheel shaft.

The style of bearings adopted depends upon the size of the alternator. Some of the smaller machines are furnished with end shield bearings but the standard form for horizontal shaft alternators is a pedestal bearing arranged for oil-ring lubrication. Large machines are often provided with water-cooled bearings which consist of a number of short tubes extending horizontally through the oil well
below the bearing through which the cooling water is conducted, thereby reducing the temperature of the oil.

Vertical shaft alternators are arranged for direct connection to the water-wheel shaft and are usually provided with one or two guide bearings. Step or suspension bearings arranged for forced oil circulation are also standard. The standard water-wheel type of alternator is wound for three phases, but it can be adapted for two phases without change, except in the armature coils and punchings. Where single-phase operation is required, the three-phase winding


Fig. 146. View of General Electric Water-Wheel-Driven Alternators
is furnished, the single-phase current being taken off from any two of the three armature terminals. When thus used the alternator full-load rating at unity power factor is only about 70 per cent of the full-load three-phase rating.

Where transformers are used to step-up the voltage for longdistance power transmission, it is considered good practice to install alternators wound for the standard voltages of 2,300 or for 6,600 volts, but where current is to be transmitted at the generator voltage, armature windings for 11,000 volts, 60 cycles, or for 13,200 volts, 25 cycles, are considered standard.

Fig. 146 is a general view of three water-wheel-driven alternators manufactured by the General Electric Company and installed in a hydroelectric power plant at Spokane, Washington. The alternators are each rated at 2,250 kilowatts, 60 cycles, 2,300 volts, 138 revolutions per minute and are wound with three phases. Each machine has $\frac{60 \times 2 \times 60}{138}=52$ poles. The view shows the exciters directly mounted on an extunsion of the main shaft, and supported on a bracket which is bolted to the iron base.

These alternators may be furnished either with or without direct-connected exciters. When arranged for direct connection, the armature of the exciter is carried on the generator shaft. In


Fig. 147. General Electric Alternator with Direct-Connected Exciter Bracketed to the Same Base
the smaller sizes the magnet frame is bolted to the bearing bracket, as shown in Fig. 147, but in the larger sizes special construction is used, depending upon the conditions of the particular installation. Fig. 147 also shows clearly the coupling for direct connection to the water wheel at the right, and the box type of frame with ventilating apertures. This General Electric alternator is rated at 3,000 kilowatts, 2,300 volts, and 514 revolutions per minute.

Fig. 148 is a general interior view of the new hydroelectric plant of the Connecticut River Power Company at Vernon, Vermont. Five vertical shaft water-wheel-driven alternators are used, each
being rated at 2,500 kilowatts 2,300 volts, 60 cycles, and 133 revolutions per minute. The two small vertical shaft generators, located between the first and second large units, are exciters, each driven independently by a small water wheel installed below the main floor level. This practice of using exciters driven independently of the main water wheels is to be recommended, as it is a safeguard against a general shutting down of the plant due to accident to one or more of the main machines.


Fig. 148. Interior View of Connecticut River Power Company's Hydroelectric Plant
Fig. 149 shows an armature structure for a 600 -kilowatt AllisChalmers "water-wheel type" alternator, with a three-phase winding in place. This winding is distributed in two slots per pole per phase. The coils that constitute one of the phases are those of which the ends show most distinctly. The sides of the coils belonging to the other two phases lie in the remaining slots, four of which are surrounded by each pair of coils belonging to the first phase. The manner of connecting the coils belonging to each phase is explained on page 119. This armature structure, Fig. 149, has two extensions cast as part of the frame, which are intended to rest upon a foun-
dation on a level with the floor. The lower part of the ring is designed to extend below the level of the floor into a pit. In the same figure is shown also a metal shield which is screwed to the frame and serves to protect the ends of the armature coils.


Fig. 149. Stationary Armature for 600-Kw. Allis-Chalmers "Water-Wheel Type"Alternator
Steam Turbine=Driven Alternators. This type of machine, often called a turbo-alternator, has been in the past few years greatly improved and developed. The steam-turbine has already largely supplanted the reciprocating engine in all large steam-electric stations and the present general tendency is away from the low-speed engine-

TABLE V
Turbine Speeds for Alternators

| PoLes | 2 | 4 | 6 | 8 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 60 cycles <br> 25 <br> cycles | 3600 <br> 1500 | 1800 <br> 750 | 1200 | 900 | 720 |

driven alternators and towards the high-speed turbine-driven alternator. The steam turbine operates most efficiently at high speeds, so in order to accommodate the generators to these conditions with the standard frequencies of 25 to 60 cycles, the number of field poles must be reduced to a minimum. For this reason 60 -cycle generators have been built with two poles for 3,600 revolutions per minute in capacitics up to 2,500 kilowatts, and with four poles for 1,800 revolutions per minute up to 10,000 kilowatts.

25 -cycle alternators are necessarily limited to a maximum speed of 1,500 revolutions per minute, even with only two poles. Twopole alternators have been built in sizes up to 7,500 kilowatts, and even larger machines are possible. Table V gives the revolutions per minute which have been used for turbo-alternators.

The steam turbine is an essentially high-speed machine and the alternator designed to be driven by it must be constructed with special attention to the effects of high speed. This requirement has developed many interesting though difficult problems in design, such as constructions able to withstand the enormous centrifugal forces developed in the rotor, amounting to 4,000 pounds on each pound of material near the surface of the rotor. The important matters of securing perfect balance and freedom from vibration at high speeds, of adequate ventilation, and lubrication of bearings, have been successfully met.

Advantages. The most important advantages of steam turbines over reciprocating steam engines are:
(1) High economy at all loads.
(2) Economy in floor space and building material.
(3) Moderate initial cost and low maintenance expense.
(4) Simplicity of construction; absence of all small clearances; absence of thrust balancing pistons with their heavy and uncertain leakages.
(5) Maintenance of efficiency and general durability.
(6) Ability to effectively utilize the large increase of available energy incident to the use of high steam pressure and high vacuum
(7) Ability to use high superheat without mechanical difficulties.

Fig. 150 is a general view of a Westinghouse-Parsons threephase four-pole turbo-alternator rated at 1,000 kilowatts, 60 cycles, 4,400 volts, and 1,800 revolutions per minute, as installed for the Public Service Corporation of Columbus, Ohio. The steam turbine is shown at the right and is directly connected through a horizontal shaft to the alternator at the left. This alternator is of the revolving field type as is the universal practice with turbine-driven machines.


Fig. 150. General View of Westinghouse-Parsons Three-Phase Four-Pole Turbo-Alternator
Stator. Except in the largest sizes, the stator frames are made in one piece. The large machines are divided horizontally, the two halves being bolted together, and the stator laminations are assembled as if the frame were solid. The frame consists of a heavy casting with internal strengthening ribs and is bored out to receive the laminated iron core. The internal ribs are provided with dovetail slots into which the laminations are fitted. The frame is designed to provide a rigid mechanical support for the stator core. The castiron frame does not form a part of the magnetic circuit. The two-
part frames have faced joints and the two parts are secured and keyed together so that they form practically a single piece. The armature or stator is built up of punchings of soft sheet steel of a high magnetic quality. Ventilating plates are provided at suitable intervals, forming air ducts in the core. The core is slotted to receive the armature winding, the shape of the slot depending upon the capacity of the machine and the character of the winding. Either open or partly closed slots are used, the edges of the former being grooved at the top to receive the retaining wedges holding the coils in place. At the


Fig. 151. Method of Bracing Armature Coils ends, the teeth are supported by finger plates and heavy retaining plates which are pressed into place and keyed. Closed end bells are provided which cover the ends of the armature coils and the moving parts of the machine between the frame and ventilating system. The end bells close each end of the generator and form a duct through which cool air is drawn into the machine and forced out through ventilating ducts in the stator into the interior of the frame, from which the air passes down through the bed plate and escapes. In the large generators the air also escapes through openings in the top of the frame. The rotor of the machine is provided with a fan at each end to draw the air into the machine. Formwound armature coils are used and the winding is of wire, strap, or bar copper, depending upon the capacity and voltage of the machines. The coils are insulated before they are assembled on the machine. The slots are provided with a lining of fibrous material and the coils are wedged into the slots by wedges fitted in grooves in the sides of the slots or below the overhanging tips of the teeth. The armature coils are firmly braced at the ends of the frame in such a
manner as to insure them against displacement, as shown in Fig. 151.

Rotor. The revolving field or rotors are constructed with two, four, or six poles, according to the frequency and capacity of the


Fig. 152. Four-Pole Rotor for $10,000-\mathrm{Kw}$. Machine
machine. The fields are of small diameter and are designed with special care to avoid windage losses and to facilitate ventilation. The poles of the two-pole and four-pole rotors are machined from the disk or disks forming the central body, and the slots to receive the field coils and the grooves for the binding wedges are milled. This construction is illustrated in Fig. 152, which is a view of a four-pole rotor for a 10,000 -kilowatt machine designed for a speed of 750 revolutions per minute. The six-pole rotors are built up by bolting poles to a central body. The rotors are carefully balanced after


Fig. 153. Two-Pole Rotor Shaft
they are wound. In some designs the rotor is pressed and keyed on to the shaft, and in others the shaft is formed of steel, cast or forged integral with the rotor core. For two-pole machines the
rotor is generally made from a solid cylinder and the shaft is made in two portions and secured to each end of the rotor by heavy bronze flanges and suitable bolts, as shown in Fig. 153.


Fig. 154. Curtis Vertical-Shaft Turbo-Alternator of 5,000-Kw. Capacity
The field consists of copper strap embedded in slots cut in the poles, as shown in Fig. 152. The coils are wound directly in place under a heavy tension. A groove is cat in both sides of the slots and brass wedges are driven in to hold the coils in place. The coils are heavily insulated with material of high dielectric and mechanical strength, applied in several layers. The winding and insulation are tightly wedged into place.

Curtis Turbo-Alternator. A general view of a Curtis steam turbine alternator, built by the General Electric Company, is shown in Fig. 154. This unit is rated at 5,000 kilowatts, 13,000 volts, 25 cycles, and 750 revolutions per minute and is of the vertical type. The generator is located at the top of the unit and the turbine wheels are located below. The stationary armature with


Fig. 155. Two Views of Stationary Armature of Curtis Alternator
the clamping devices for rigidly supporting the coils is clearly shown in Fig. 155.

Fig. 156 is a view of a smooth core rotor for a 9,000 -kilowatt vertical type of Curtis turbo-alternator. A feature of interest is the method employed for the attachment of the shields for the end windings. The outer retaining cylinders are secured to the inner shell by a large number of short bolts which remove a part of the stress of the end windings from the outer shell. The rotor is so
designed that it acts as a powerful fan, forcing air throughout the parts of the generator requiring ventilation.

The advantages claimed for the vertical shaft type of Curtis turbine for large machines are:
(1) The relative positions of revolving and stationary parts are definitely fixed by the step-bearing.
(2) The stationary part is symmetrical, and free from distortion by heat.
(3) The shaft bearings are relieved from all strain, and friction is practically eliminated.
(4) The shaft is free from deflection, and can be made of any size without reference to bearings.


Fig. 156. Smooth Core Rotor for $9,000-\mathrm{Kw}$. Curtis Turbo-Alternator
(5) The turbine structure affords support and foundation for the generator.
(6) The cost of foundations is small, and their support is naturally simple.
(7) Much floor space is saved.
(8) All parts of the machine are accessible.

For high speed machines and turbines connected to directcurrent generators, the horizontal shaft has some advantages. The fact that many electric generators of the horizontal type have already been developed for such conditions is frequently responsible for
the selection of the horizontal type of unit. In the smaller sizes this type is standard. The characteristics of the Curtis turbine specially adapt it for driving horizontal shaft generators. The shaft is very short, of small diameter, and has a comparatively low surface speed in the bearings, resulting with the light weight of the revolving parts in low bearing friction and small tendency to wear.


## ALTERNATING-CURRENT MACHINERY

PART III

## ECONOMY FACTORS IN ALTERNATORS

## CONDITIONS AFFECTING COST

Speed. The most important factor in determining the cost of an alternator for a given rated output, is its speed, inasmuch as a low-speed machine must be much larger than a high-speed machine of the same rated output. Belt-driven machines are always run at the highest speed compatible with safety to the alternator itself; while, on the other hand, the speed of a direct-connected alternator, being determined by the speed of the engine or water wheel, is usually less than is necessary for safe running. Therefore, a belt-driven machine is usually cheaper than a direct-connected machine of the same rated output. Very large machines must be direct-connected, inasmuch as belt driving is out of the question for large machines on account of the excessive cost of very large belts, the great amount of floor space required, the power lost in the belt, the expense of attendance and maintenance, and the noise. Direct-connected alternators, especially machines of large rated output, except turbo-alternators, are usually designed with the rotating member (armature or field) of large diameter, in order that the permissible speed of the alternator may be approximately the same as the proper speed of the driving engine or water wheel. The steam turbine being inherently a high-speed machine requires an alternator built for high speed, which means a rotor of small diameter.

Voltage. A second factor affecting cost is the voltage that is to be developed in the armature. A machine for high voltage must have a large number of armature conductors, and these conductors must be highly insulated, that is, the insulation must occupy a
relatively large portion of the winding space in the slots. This requires a large machine for a given power output, on account of the space wasted, as it were, in insulation. To offset this disadvantage, a high-voltage alternator does not require the use of step-up transformers, the voltage generated in the alternator being suited for the transmission of power to moderate distances, that is, the in the cost of the step-up transformers. Alternators are rarely built to generate voltages higher than 3000 volts.

Regulation. A third factor affecting cost is found in the re quirements of close regulation and high efficiency. Thus an alternator of given power output may be made smaller in size and, therefore, cheaper, if high efficiency and low regulation are not demanded. High efficiency and low regulation mean a liberal use of iron and copper in order to secure a minimum loss of power and electromotive force in a machine. Furthermore, the efficiency of a given size of alternator for given output may be increased at the expense of regulation, or vice versâ. The increasing adoption of a satisfactory automatic voltage regulator for alternators makes it unnecessary as well as expensive to specify a low inherent regulation, as explained on page 91.

Frequency. A fourth factor affecting cost is the frequency required. With given speed an increase of frequency means an increase in the number of field poles and in the number of armature coils and, therefore, an increase in the cost of construction. This. element of cost is most prominent in very slow-speed direct connected alternators. For example, a 60 -cycle alternator, direct-connected to an engine running at 300 r . p. m., must have 24 poles to give the required frequency. To reduce the frequency to 25 cycles would require only 10 poles, with a corresponding reduction in the cost of the field-magnet copper and of labor. On the other hand, lowering the frequency of an alternator, while keeping the speed constant, would require an increase in the useful flux per pole and a corresponding increase in the cross-sectional areas of the field yoke, the field poles, and the armature core. This would mean an increase in the amount of iron to be used in the machine. The frequencies in most general use today are 60 cycles and 25 cycles. 60-cycle apparatus is generally lower in price and should always be chosen for general lighting and power service.

## POWER LOSSES

The power lossesin an alternator consist of the following parts:
(a) Loss due to brush, journal, and air friction. Air friction is usually called windage.
(b) Power consumed in heating the field windings by the exciting current.
(c) Power loss in heating the armature windings by the armature current or currents.
(d) Hysteresis loss in all iron that is subject to variations of magnetization and eddy currents in all metal parts subject to such variations.

Friction and windage loss can be determined only by experiments upon the finished machine.

The power consumed in the field windings is $I^{2} R$, where $I$ is the field current and $R$ is the resistance of the field circuit. The field rheostat is properly a part of the machine, and the losses occurring in it are, therefore, a part of the machine losses. This same formula may be used to calculate power consumed in each field winding of a composite-field alternator.

The power lost in heating the armature windings is $I^{2} R \times$ the number of phases, where $I$ is the current in each phase and $R$ is the resistance of each phase.

The power lost by eddy currents and hysteresis may be approximately calculated by the method employed for the corresponding calculation in the case of a transformer.

## EFFICIENCY

The efficiency of an alternator is the ratio output of power $\div$ input of power. Since the mechanical input of power is equal to the output of power plus all the losses of power, we have also

$$
\text { efficiency }=\frac{\text { output }}{\text { output }+ \text { losses }}
$$

At zero load (zero output), the efficiency of an alternator is, therefore, zero; the efficiency increases with increasing load, reaches a maximum, and falls off for large loads. An alternator may be designed to give its maximum efficiency at any prescribed fraction of full load; it is generally desirable, however, to design the machine to give its maximum efficiency at approximately full load.

The efficiency of a large alternator at full load is usually greater than the efficiency of a small alternator. For example, the large alternators in the great power stations at Niagara Falls have efficiencies of over 98 per cent. A well-designed 50 -kilowatt alternator has an efficiency of about 90 per cent.

Practical and Ultimate Limits of Output. The dotted curve, Fig. 157, is the characteristic curve of a given alternator. This curve shows the relation between the current output (plotted as abscissas) and the electromotive force between the collecting rings (plotted as ordinates, using scale to the left), the field excitation being kept constant. The ordinates of the full-line curve (scale shown to the right in the figure) represent the power outputs (in kilowatts) corresponding to the different current outputs assuming


Fig. 157. Characteristic Curves of an Alternator
a non-inductive receiving circuit. The maximum output of the alternator, in this case, is 68 kilowatts when the current output is 38 amperes and the corresponding electromotive force is 1,790 volts. In practice the allowable power output of an alternator is limited to a smaller value than this maximum output by one or the other of the following considerations:
(a) Electric lighting and power service usually demands an approx- . imately constant electromotive force; and it is not permissible to take from an alternator a current so large as greatly to reduce its electromotive force. This difficulty may be largely overcome by providing for an increase of field excitation of the alternator with increase of load, as is done in the alternator with a composite field winding, or by means of an automatic regulator.
(b) The current delivered by an alternator generates heat in the armature of the alternator; and the temperature of the armature rises until it radiates heat as fast as heat is generated in it by the current. Excessive heating of the armature endangers the insulation of the windings by charring, and it is not permissible to take from an alternator a current so large as to heat its insulation more than $55^{\circ}$ to $85^{\circ} \mathrm{C}$. above the temperature of the air; this heating, therefore, fixes the allowable output and rating of an alternator.

Influence of Power Factor upon Output. Alternators are rated according to the power they can deliver steadily to a non-inductive receiving circuit without overheating. The amount of power which an alternator can satisfactorily deliver to an inductive receiving circuit is less than that which it can deliver to a non-inductive receiving circuit, because of the phase difference of electromotive force and current. The cosine of the angle of phase difference $(\cos \theta)$ is called the "power factor" of the receiving circuit, as before pointed out. The power factor of incandescent lighting circuits is very nearly unity if the transformers are all operating at approximately full load.

A transformer having its primary coil connected to alternatingcurrent mains, but furnishing no current from its secondary coil, has a power factor of from about 0.3 with a frequency of 25 cycles to about 0.7 with 60 cycles. The power factor increases with the secondary lamp load, until, in the neighborhood of full load, the power factor is nearly unity.

Induction motors, like transformers, have a maximum power factor at full load, which rarely exceeds 0.96 ; this factor under partial loads falls to 0.7 or even less; and at starting it is a minimum around 0.3.

A mixed load consisting of transformers and induction motors more or less fully loaded, has an average power factor of from 0.8 to 0.85 .

It should be carefully noted that alternators are rated in kilovoltampere output, that is, a 100 -k.v.a. alternator should deliver its full energy output of 100 kilowatts at unity power factor; but if the power factor should be only 0.8 , the true power output would be reduced to 80 kilowatts, although the armature current and the $I^{2} R$ loss would be about the same as if the machine were delivering 100 kilowatts at unity power factor. To find the actual power output of an alternator the "apparent power," or kilovolt-amperes, must he multiplied by the power factor of the load.

## RATING AND OVERLOAD CAPACITIES

Previous to the definite recommendations of the American Institute of Electrical Engineers in the matter of overload capacities as given below, there were great differences among different manu-
facturers in the rating of alternators. Thus, one maker might have sold a certain alternator as a 50-kilowatt alternator, although the machine might have been capable of delivering 50 per cent overload (or 75 kilowatts) for several hours without dangerous rise of temperature; whereas another maker might have sold an exactly similar machine as a 75 -kilowatt alternator.

As an illustration of the difference of permissible rating of a given alternator according to conditions of service, the following is taken from the practice of one of the large American manufacturing companies.

A certain alternator has twenty poles, and runs at 150 r.p.m. When this machine is rated at 300 kilowatts, its maximum rise of temperature will not exceed $35^{\circ} \mathrm{C}$. (by thermometer) when it is operated continuously with noninductive full load; its maximum rise of temperature will not exceed $55^{\circ} \mathrm{C}$. when it is run for two hours at 50 per cent overload non-inductive. Its full-load regulation will be 6 per cent with non-inductive load, and 18 per cent with 80 per cent power factor.

When this same machine is rated at 360 kilowatts, its maximum rise of temperature will not exceed $40^{\circ} \mathrm{C}$. (by thermometer) when it is operated continuously with non-inductive full load; its maximum rise of temperature will not exceed $55^{\circ} \mathrm{C}$. when it is run for two hours at 25 per cent overload non-inductive. Its full load regulation will be 8 per cent with non-inductive load, and 22 per cent with 80 per cent power factor.

In general the rated power output of any alternator may be 20 per cent higher with a permissible rise of temperature of $40^{\circ} \mathrm{C}$. and a regulation of 8 per cent on non-inductive full load, than with a permissible rise of temperature of $35^{\circ} \mathrm{C}$. and a regulation of 6 per cent on non-inductive full load.

American Institute Rules. In order to establish a definite and uniform basis for rating electrical machinery and for guaranteeing its performance in service under normal full load as well as overload, the following standardization rules have been adopted by the American Institute of Electrical Engineers. The numbers on the left are the paragraph numbers of the revised Standardization Rules of the Institute.

## STANDARDS FOR ELECTRICAL MACHINERY

250. The expressions "machinery" and "machines" are here employed in a general sense, in order to obviate the constant repetition of the words "machinery or induction apparatus."
251. All temperatures are to be understood as centigrade.
252. The expression "capacity" is to be understood as indicating "capability," except where specifically qualified, as, for instance, in the case of allusions to electrostatic capacity, i. e., capacitance.
253. Wherever special rules are given for any particular type of machinery or apparalus (such as switches, railway motors, railway substation machinery, etc., these special rules shall be followed, notwithstanding any apparent confficl with the provisions of the more general sections. In the absence of special rules on any particular poinl, the general rules on this point shall be followed.
254. Objects of Standardization. To ensure satisfactory results, electrical machinery should be specified to conform to the Institute Standardization Rules, in order that it shall comply, in operation, with approved limitations in the following respects, so far as they are applicable.

| Operating temperature | Efficiency |
| :--- | :--- |
| Mechanical strength | Power factor |
| Commutation | Wave shape |
| Dielectric strength | Regulation |
| Insulation resistance |  |

261. Capacity or Available Output of an Electrical Machine. So far as relates to the purposes of these Standardization Rules, the Institute defines the Capacity of an Electrical Machine as the load which it is capable of carrying for a specified time (or continuously), without exceeding in any respect the limitations herein set forth.

Except where otherwise specified, the capacity of an electrical machine shall be expressed in terms of its available output.
262. Rating of an Electrical Machine. Capacity should be distinguished from Rating. The Rating of a machine is the output marked on the Rating Plate, and shall be based on, but shall not exceed, the maximum* load which can be taken from the machine under prescribed conditions of test. This is also called the rated output.
263. Principle of Machine Ratings. The principle upon which machine ratings are based, so far as relates to thermal characteristics, is that the rated load, applied continuously or for a stated period, shall produce a temperature rise which, superimposed upon a standard ambient temperature, will not exceed the maximum safe operating temperature of the insulation.
264. A. I. E. E. and I. E. C. Ratings. When the preseribed conditions of test are those of the A. I. E. E. Standardization Rules, the rating of the machine is the Institute Rating. When the prescribed conditions of the test are those of the I. E. C. $\dagger$ Rules, the rating of the machine is the I. E. C. rating. A machine so rated in either case may bear a distinctive sign upon its rating plate.
265. Standard Temperature and Barometric Pressure for Institute Rating. The Institute Rating (Sce section 262) of a machine shall be its

[^11]capacity when operating with a cooling medium of the ambient temperature of reference ( $40^{\circ}$ for air or $25^{\circ}$ for water, see sections 305 and 309) and with barometric conditions within the range given in section 308. See sections 305A, 307, 320 , and 321.
266. Temperature Rises Apply to Ambient Temperatures. The temperature rises specified in these rules apply to all ambient temperatures up to and including, but not exceeding, $40^{\circ} \mathrm{C}$. for air and $25^{\circ} \mathrm{C}$. for water. (For definition of ambient temperature see section 303.)
267. Machinery Guarantee. Any machinery destined for use with higher ambient temperatures of cooling mediums, and also any machinery for operation at altitudes for which no provision is made in section 308, should be the subject of special guarantee by the manufacturer. The methods of test and performance set forth in these rules will, however, afford guidance in such cases.

## UNITS IN WHICH RATING SHALL BE EXPRESSED

274. Rating of Direct-Current Generators. Generator ratings shall be expressed in kilowatts ( kw .) available at the terminals at a specified voltage.
275. Rating of Alternators and Transformers. Alternator and transformer ratings shall be expressed in kilovolt-amperes (kv-a.) available at the output terminals, at a specified voltage and power factor.
276. Rating Expressed in Kilowatts. It is strongly recommended that the rating of motors shall be expressed in kilowatts* (kw.) available at the shaft. (An exception to this rule is made in the case of Railway motors, which, for some purposes, are also rated by their input.)
277. Auxiliary Machinery. Regulators, resistors, reactors, balancer sets, stationary and synchronous condensers, etc., shall have their ratings appropriately expressed. It is essential to specify also the voltage (and frequency, if a.c.), of the circuits on which the machinery may appropriately be used.

## KINDS OF RATING

There are various kinds of rating such as:
281. Continuous Rating. A machine rated for continuous service shall be able to operate continuously at its rated output, without exceeding any of the limitations referred to in section 260.

[^12]282. Short-Time Rating. A machine rated for short-time service (i.e., service including runs alternating with stops of sufficient duration to ensure substantial cooling), shall be able to operate at its rated output during a limited period, to be specified in each case, without exceeding any of the limitations referred to in section 260. Such a rating is a short-time rating.
283. Nominal Ratings. For railway motors and sometimes for railway substation machinery, certain nominal ratings are employed. Nominal ratings for automobile propulsion motors and generators are not recommended.
284. Duty=Cycle Operation. Many machines are operated on a cycle of duty which repeats itself with more or less regularity. For purposes of rating, either a continuous or a short-time equivalent load may be selected, which shall simulate as nearly as possible the thermal conditions of the actual duty cycle.
285. Standard Durations of Equivalent Tests. These shall be for machines operating under specified duty-cycles as follows:

| 5 minutes | 30 minutes |
| ---: | ---: |
| 10 minutes | 60 minutes |
| 15 minutes | 120 minutes |

and continuous.
Of these the first six are short-time ratings, selected as being thermally equivalent to the specified duty cycle.

When, for example, a short-time rating of 10 minutes duration is adopted, and the thermally equivalent load is 25 kw . for that period, then such a machine shall be stated to have a $10-$ minute rating of 25 kw .
286. Commencement of Equivalent Short=Time Test. In every case the equivalent short-time test shall commence only when the windings and other parts of the machine are within $5^{\circ} \mathrm{C}$. of the ambient temperature at the time of starting the test.
287. Continuous Rating Understood. In the absence of any specification as to the kind of rating, the continuous rating shall be understood.*
288. Continuous Ratings. Machines marked in accordance with section 264 shall be understood to have a continuous rating, unless otherwise marked in accordance with section 285.

## HEATING AND TEMPERATURE

300. Temperature Limitations of the Capacity of Electrical Machin= ery. The capacity, so far as relates to temperature, is usually limited by the maximum temperature at which the materials in the machine, especially those employed for insulation, may be operated for long periods without deterioration. When the safe limits are exceeded, deterioration is rapid. The insulating material becomes permanently damaged by excessive temperature, the damage increasing with the length of time that the excessive temperature is maintained, and with the amount of excess temperature, until finally the insulation breaks down.

[^13]301. Operating at Temperatures in Excess of the Safe Limit. The result of operating at temperatures in excess of the safe limit is to shorten the life of the insulating material. This shortening of life is, in certain special cases, warranted, when necessary for obtaining some other desirable result, as, for example, in some instances of railway and other motors for propelling vehicles, in providing greater power within a limited space. The design of coils used in control and switching apparatus, such as contactors, controllers, induction starters, and in other operating mechanisms as arc lamps, magnet windings, etc., is such that the effect of higher temperature in the coils is less serious owing to the fact that these devices usually have shorter life than electrical machinery in general.
302. Life of Insulation Unaffected by Lower Temperature. There does not appear to be any advantage in operating at lower temperatures than the safe limits, so far as the life of the insulation is concerned. Insulation may break down from various causes, and when these breakdowns occur, it is not usually due to the temperature at which the insulation has been operated, provided the safe limits have not been exceeded.
303. The Ambient Temperature. This is the temperature of the air or water which, coming into contact with the heated parts of a machine, carries off its heat. See sections 309,310 , and 314.
304. The Cooling Fluid. This may cither be led to the machine through ducts, or through pipes, or merely surround the machine freely. In the former case the ambient temperature is to be measured at the intake of the machine itself. In the latter case see section 314.
305. Ambient Temperature of Reference for Air. The standard ambient temperature of reference, when the cooling medium is air, shall be $40^{\circ} \mathrm{C}$.

305A. Danger to Insulation of Exceeding Limits of Temperature Rise. Whatever may be the ambient temperature when the machine is in service, the limits of the maximum observable temperature or of temperature rise specified in the rules should not be exceeded in service; for, if the maximum temperature be exceeded, the insulation may be endangered, and if the rise be exceeded, the excess load may lead to injury, by exceeding limits other than those of temperature; such as commutation, stalling load and mechanical strength. For similar rcasons, loads in excess of the rating should not be taken from a machine.
306. Basis for Temperature Rises. The permissible rises in temperature given in column 2 of Table VII following Rule 352 have been salculated on the basis of the standard ambient temperature of reference, by subtracting $40^{\circ}$ from the highest temperatures permissible, which are given in column 1 of the same table.
307. Permissible Temperature Rises. A machine may be tested at any convenient anbient temperature, preferably not below $10^{\circ} \mathrm{C}$., but whatever be the value of this ambient temperature, the permissible rises of temperature must not exceed those given in column 2 of Table VII.
308. Altitude. Increased altitude has the effect of increasing the temperature rise of some types of machinery. In the absence of information in regard to the height above sea level at which a machine is intended to work in ordinary service, this height is assumed not to exceed 1000 meters ( 3300 feet). For machinery operating at an altitude of 1000 meters or less, a test at any altitude less than 1000 meters is satisfactory, and no correction shall be applied to the observed temperatures. Machines intended for operation at higher altitudes shall be regarded as special. (See section 267.) It is recommended that when a machine is intended for service at altitudes above 1000 meters ( 3300 ft .) the permissible temperature rise at sea level, until more nearly accurate information is available, shall be reduced by 1 per cent for each 100 meters ( 330 ft .) by which the altitude exceeds 1000 meters. Water cooled oil transformers are exempt from this reduction.
309. Ambient Temperature of Reference for Water-Cooled Machinery. For water-cooled machinery, the standard temperature of reference for incoming cooling water shall be $25^{\circ} \mathrm{C}$. measured at the intake of the machine.
310. Testing of Water=cooled Transformers. In testing watercooled transformers it is not necessary to take into account the surrounding air temperature, except where the cooling effect of the air is 15 per cent or more of the total cooling effect, referred to the standard ambient temperature of reference of $25^{\circ} \mathrm{C}$. for water and $40^{\circ} \mathrm{C}$. for air. When the effect of the cooling air is 15 per cent. or more of the total, the temperature of the cooling water should be maintained within $5^{\circ} \mathrm{C}$. of the surrounding air. Where this is impractical, the ambient temperature should be determined from the change in the resistance of the windings, using a disconnected transformer, supplied with the normal amount of cooling water, until the temperature of the windings has become constant.
311. Rotating Machines, Cooled by Forced Draught. In the case of such machines a conventional weighted mean shall be employed, a weight of four being given to the temperature of the circulating air supplied through ducts (see section 304), and a weight of one to the surrounding room air. In the case of air-cooled transformers, see "exception" section 321.
312. Machines Cooled by Other Means. Machines cooled by means other than air or water shall receive special consideration.
313. Outdoor Machinery Exposed to Sun's Rays. Outdoor machinery not protected from the sun's rays at times of heavy load, shall receive special consideration.
314. Measurement of the Ambient Temperature During Tests of Machinery. The ambient temperature is to be measured by means of several thermometers placed at different points around and half-way up the machine, at a distance of 1 to 2 meters ( 3 to 6 feet), and protected from drafts, and abnormal heat radiation, preferably as in section 316 .
315. Value To Be Adopted for the Ambient Temperature. During a test the value to be adopted for ambient temperature is the mean of the readings of the thermometers (placed as above), taken at equal intervals of time during the last quarter of the duration of the test.
316. To Avoid Errors Due To the Time Lag. Errors due to the time lag between the temperature of large machines and the variations in the ambient air, can be avoided only when every reasonable precaution is taken to reduce these variations and the errors arising therefrom. Thus, the thermometer for determining the ambient temperature shall be immersed in a suitable liquid, such as oil, in a suitable heavy metal cup. This can be made to respond to various rates of change, by proportioning the amount of oil to the metal in the containing cup. A convenient form for such an oil-cup consists of a massive metal cylinder, with a hole drilled partly through it. This hole is filled with oil and the thermometer is placed therein with its bulb well immersed. The larger the machine under test, the larger should be the metal cylinder employed as an oil-cup in the determination of the ambient temperature. The smallest size of oil-cup employed in any case shall consist of a metal cylinder 25 mm . in diameter and 50 mm . high ( 1 inch in diameter and 2 inches high).
317. Thermometers Used for Taking Temperatures. Thermometers for taking the temperatures of machinery shall be covered by felt pads 3 mm . ( $\frac{1}{8}$ inch) thick and $4 \times 5 \mathrm{~cm}$. wide ( $1 \frac{1}{2}^{\prime \prime} \times 2^{\prime \prime}$ ), cemented on; oil putty may be used for stationary and small apparatus.
318. Transformer Testing. In such testing, and sometimes in testing other machines, it may be desirable to avoid errors due to time lag in temperature changes, by employing an idle unit of the same size and subjected to the same conditions of cooling as the unit under test, for obtaining the ambient temperature as described in section 310.
319. Underground Machinery. Where machines are partly below the floor line in pits, the temperature of the rotor shall be referred to a weighted mean of the pit and room temperatures, the weight of each being based on the relative proportions of the rotor in and above the pit. Parts of the stator constantly in the pit shall be referred to the ambient temperature in the pit.
320. Correction for the Deviation of the Ambient Temperature of the Cooling Medium, at the Time of the Heat Test, from the Standard Ambient Temperature of Reference. Numerous experiments have shown that deviation of the temperature of the cooling medium from that of the standard of reference, at the time of the heat run, has a negligible effect upon the temperature rise of the apparatus; therefore, no correction shall be applied for this deviation. It is, however, desirable that tests should be conducted at ambient temperatures not lower than $10^{\circ} \mathrm{C}$.
321. Exception. A correction shall be applied to the observed temperature rise of the windings of air-blast transformers, due to difference in resistance, when the temperature of the ingoing cooling air differs from that of the standard of reference. This correction shall be the ratio of the inferred absolute ambient temperature of reference to the inferred absolute temperature of the ingoing cooling air, i.e., the ratio $274.5 /(234.5+t)$; where $t$ is the ingoing cooling-air temperature.

Thus, a cooling-air room temperature of $30^{\circ} \mathrm{C}$. would correspond to an inferred absolute temperature of $264.5^{\circ}$ on the scale of copper resistivity, and the correction to $40^{\circ} \mathrm{C}$. ( $274.5^{\circ}$ inferred absolute temperature) would be $274.5 / 264.5$ $=1.04$, making the correction factor 1.04 ; so that an observed temperature rise
of say $50^{\circ} \mathrm{C}$. at the testing ambient temperature of $30^{\circ} \mathrm{C}$. would be corrected to $50 \times 1.04=52^{\circ} \mathrm{C}$. this being the temperature rise which would have occurred had the test been made with the standard ingoing cooling-air temperature of $40^{\circ} \mathrm{C}$.
322. Duration of Temperature Test of Machine for Continuous Service. The temperature test shall be continued until sufficient evidence is available to show that the maximum temperature and temperature rise would not exceed the requirements of the rules, if the test were prolonged until a steady final temperature was reached.
323. Duration of Temperature Test of Machine with a Short=Time Rating. The duration of the temperature test of a machine with a short-time rating shall be the time required by the rating. (See sections 285 and 286).
324. Duration of Temperature Test for Machine having more than One Rating. The duration of the temperature test for a machine with more than one rating shall be the time required by that rating which produces the greatest temperature rise. In cases where this cannot be determined beforehand, the machine shall be tested separately under each rating.
325. Temperature Measurements during Heat Run. Temperature measurements, when possible, shall be taken during operation, as well as when the machine is stopped. The highest figures thus obtained shall be adopted. In order to abridge the long heating period, in the case of large machines, reasonable overloads of current, during the preliminary period, are suggested for them.

## TEMPERATURE MEASUREMENTS

340. Life of the Insulation of a Machine. This depends in great measure upon the actual temperatures attained by the different parts, rather than on the rises of temperature in those parts.
341. Maximum Temperatures. The temperatures in the different parts of a machine which it would be desirable to ascertain, are the maximum temperatures reached in those parts.
342. Observable Temperature in Insulated Windings. As it is usually impossible to determine the maximum temperature attained in insulated windings, it is convenient to apply a correction to the observable temperature, which approximates the difference between the actual maximum temperature and the observable temperature by the method used. This correction, or margin of security, is provided to cover the errors due to fallibility in the location of the measuring devices, as well as inherent inaccuracies in measurement and methods.
343. Methods of Determining Temperature. In determining the temperature of different parts of a machine three methods are provided.
344. Method a.-Thermometer Method. This method consists in the determination of the temperature, by mercury or alcohol thermometers, by resistance thermometers, or by thermocouples, any of these instruments being applied to the hottest accessible part of the completed machine, as distinguished from the thermocouples or resistance coils embedded in the machine as described under Method c.

# TABLE VI <br> Temperature Coefficients of Copper Resistance 

| Temperature of the winding, <br> in degrees <br> dhe initial enesistanice <br> is measured | Increase in resistance of cop- <br> per per dearea ce <br> ohm of initial <br> resistance |
| :---: | :---: |
| 0 | 0.00427 |
| 5 | 0.00418 |
| 10 | 0.00409 |
| 15 | 0.00401 |
| 20 | 0.00393 |
| 25 | 0.00385 |
| 30 | 0.00378 |
| 35 | 0.00371 |
| 40 | 0.00364 |

346. Hottest=Spot Temperature. When Method a is used, the hottestspot temperature for windings shall be estimated by adding a hottest-spot correction of $15^{\circ} \mathrm{C}$. to the highest temperature observed, in order to allow for the practical impossibility of locating any of the thermometers at the hottest spot.
347. Exception. When the thermometers are applied directly to the surfaces of bare windings, such as an edgewise strip conductor, or a cast copper winding, a hottest-spot correction of $5^{\circ} \mathrm{C}$., instead of $15^{\circ} \mathrm{C}$.; shall be made. For commutators, collector rings, bare metallic surfaces not forming part of a winding, or for oil in which apparatus is immersed, no correction is to be applied.
348. Method b.-Resistance Method. This method consists in the measurement of the temperature of windings by their increase in resistance, corrected* to the instant of shut-down when necessary. In the application of this method, thermometer measurements shall also be made whenever practicable without disassembling the machine $\dagger$ in order to increase the probability of revealing the
[^14]TABLE VII
Permissible Temperatures and Temperature Rises for Insulating Materials

|  |  | 1 | 2 |
| :---: | :---: | :---: | :---: |
| Class | Description of Material | Maximum Temperature to which the Material May be Subjected | $\underset{\text { Temperature }}{\substack{\text { Maximum } \\ \text { Rise }}}$ |
| A | Cotton, silk, paper, and similar materials, when so treated or impregnated as to increase the thermal limit, or when permanently immersed in oil; also enameled wire* | $105^{\circ} \mathrm{C}$. | ${ }^{6} 65^{\circ} \mathrm{C}$. |
| B | Mica, asbestos, and other materials capable of resisting high temperatures, in which any Class A material or binder is used for structural purposes only, and may be destroyed without impairing $\dagger$ the insulating or mechanical qualities of the insulation | $125^{\circ} \mathrm{C}$. | $85^{\circ} \mathrm{C}$. |
| C | Materials capable of resisting higher temperatures than Class B, such as pure mica, porcelain, quartz, etc. | No limits | specified |

highest observable temperature. Whichever measurement yields the higher temperature, that temperature shall be taken as the "highest observable" temperature and a hottest-spot correction of $10^{\circ} \mathrm{C}$. added thereto.
349. The Temperature Coefficient of Copper. This shall be deduced from the formula $1 /(234.5+t)$. Thus, at an initial temperature $t=40^{\circ} \mathrm{C}$., the temperature coefficient of increase in resistance per degree centigrade rise, is $1 /(274.5)=0: 00364$. Table VI, deduced from the formula, is given for convenience of reference.
350. In Circuits of Low Resistance. In low-resistance circuits other than transformer windings (see section 351), where joints and connections form a considerable part of the total resistance, the measurement of temperature by the resistance method shall not be used.
351. To Ascertain Winding Temperatures. The temperature of the windings of transformers is always to be ascertained by "method b." In the case of air-blast transformers, it is especially important to place thermometers on the coils near the air outlet. See section 348.

[^15]352. Method c.-Embedded Temperature-Detector Method. This method consists in the use of thermocouples or resistance temperature detectors, located as nearly as possible at the estimated hottest spot. When "Method $c$ " is used, it shall, when required, be checked by "Method b"; the hottest spot shall then be taken to be the highest value by either method, the required correction factors (sections 348 and 356) being applied in each case.
353. Temperature Detectors. By building into the machine suitably placed temperature detectors, a temperature not much less than that of the hottest spot will probably be disclosed. When these devices are adopted for such temperature determinations, a liberal number shall be employed, and all reasonable efforts, consistent with safety, shall be made to locate them at the various places where the highest temperatures are likely to occur.
354. Location of Temperature=Detectors. Temperature detectors should be placed in at least two sets of locations. One of these should be between a coil-side ${ }^{*}$ and the core, and one between the top and bottom coil-sides where two coil-sides per slot are used. Where only one coil-side per slot is used, one set of detectors shall be placed between coil-side and core, and one set between coil-side and wedge.
355. Application of "Method c." "Method c " should be applied to all stators of machines with cores having a width of 50 cm . ( 20 in .) and over. It should also be applied to all machines of 5000 volts and over, if over 500 kv-a., regardless of core width. This method is not required for inductionregulators, which shall be tested as transformers.
356. Correction Factor for "Method c." In the case of two-layer windings, with detectors between coil-sides, and between coil-side and core, add $5^{\circ} \mathrm{C}$. to the highest reading. In single-layer windings, with detectors between coil-side and core and between coil-side and wedge, add to the highest reading $10^{\circ} \mathrm{C}$. plus $1^{\circ} \mathrm{C}$. per 1000 volts above 5000 volts of terminal pressure.

## TEMPERATURE LIMITS

375. Limits for Hottest Spots in Insulations. Table VII gives the limits for the hottest-spot temperatures of insulations. The permissible limits are indicated in column 1 of Table VII. The limits of temperature rise permitted under rated-load conditions are given in column 2, and are found by subtracting $40^{\circ} \mathrm{C}$. from the figures in column 1 . Whatever be the ambient temperature at the time of the test, the rise of temperature must never exceed the limits in column 2 of the table. The highest temperatures and temperature rises, attained in any machine at the output for which it is rated, must not exceed the values indicated in Table VII and clauses following.
376. Permissible Temperatures and Temperature Rises For Insu= lating Materials. Table VII gives the highest temperatures and temperature rises to which various classes of insulating materials may be subjected, based on a standard ambient temperature of reference of $40^{\circ} \mathrm{C}$.
377. Note. The Institute recognizes the ability of manufacturers to employ Class B insulation successfully at maximum temperatures of $150^{\circ} \mathrm{C}$. and even higher. However, as sufficient data covering experience over a period of years at such temperatures are at present unavailable, the Institute adopts
[^16]TABLE VIII
Permissible Hottest-Spot Temperatures and Limiting Observable Temperature Rises in Other Than Water=Cooled Machinery*

$125^{\circ} \mathrm{C}$. as a conservative limit for this class of insulation, and any increase above this figure should be the subject of special guarantee by the manufacturer.
378. Lower=Temperature Class Material. When a lower-temperature class material is comprised in a completed product to such an extent, or in such ways, that its subjection to the temperature limits allowed for the higher-temperature class material with which it is associated would affect the integrity of the insulation either mechanically or electrically, the permissible temperature shall be fixed at such a value as shall afford ample assurance that no part of the lowertemperature class material shall be subjected to temperatures higher than those approved by the Institute and set forth above.

[^17]
## ALTERNATING=CURRENT TESTING

Object and Scope of Testing. In the commercial testing of alternating-current apparatus, as in that of direct-current apparatus, the object of the tests is to determine the performance of the apparatus under normal working conditions. Care must be taken, therefore, to carry out each test under the normal working conditions with respect to speed, voltage, frequency, etc. Errors of observation may be greatly reduced by taking a series of observations instead of a single observation or a single set of observations. The observations of this series should be plotted point for point, and a smooth curve drawn through the points in such a way that the points will be equally distributed on both sides of the curve.

Different classes of apparatus require different tests; moreover, the same test may be performed differently on different kinds of machines. For the sake of convenience the tests necessary for each class of alternating-current apparatus will be considered under its own heading.

In general every piece of electrical apparatus must satisfy two vital requirements, namely, (a) it must have insulation of sufficient strength to stand safely the voltage at which it is intended to be operated; and (b) it must not overheat under normal working conditions.

Faulty insulation must be carefully guarded against, since, by the breaking down of the insulation, the apparatus must be put out of service. For example, in the case of transformers for lighting service, a breakdown of the insulation between the primary and the secondary coils endangers the life of persons coming in contact with fixtures on the secondary circuit. Overheating a machine causes a gradual deterioration of the insulation, which may finally result in a complete breakdown of the apparatus. Overheating must, therefore, be avoided.

Insulation Testing. There are three distinct kinds of insulation tests: (a) the determination of the electrical resistance of the insulation in ohms; (b) the subjecting of the insulation of an apparatus to a prescribed voltage in excess of the rated voltage of the apparatus. This test is intended to insure that the apparatus will operate safely at its rated voltage. It is frequently called the "test of dielectric
strength;" and (c) the subjecting of the insulation to a voltage which is increased until the insulation is punctured or breaks down. This is called the "break-down test."

Insulation Resistance. The insulation resistance test, or in other words, the determination, in ohms, of the resistance of an insulating coating, cover, material, or support is usually made by measuring with a sensitive galvanometer the very small current that is forced through the insulation by a known direct or steady electromotive force. The value of the resistance is equal to the impressed electromotive force divided by the current.

The resistance in ohms of the insulation is of only secondary importance as compared with the dielectric strength or the resistance to rupture by high voltage. Insulation resistance tests should, if possible, be made using the electromotive force for which the apparatus is designed.

The insulation resistance of the complete apparatus should be such that the rated voltage of the apparatus will not send more than $10 \frac{1}{0} \sigma \delta \sigma$ of the full load current, at the rated terminal voltage, through the insulation. Where the value found in this way is more than 1 megohm, it is usually sufficient.

Dielectric Strength. The test of dielectric strength may be made as follows: The terminals of the secondary coil of a step-up "high potential" transformer are connected to the terminals of a spark gauge and to the terminals of the insulation to be tested, as shown in Fig. 158. The figure shows that the apparatus under test and the spark gap are connected in parallel between the terminals of the secondary (high voltage) coil of the testing transformer. The spark gauge is set at a certain sparking distance corresponding to the voltage to be used in the test; and the voltage is increased gradually by adjusting the water rheostat in series with the primary (low-voltage) coil of the transformer, and is then to be continuously applied for a prescribed period, usually one minute, until either the insulation is punctured or the desired voltage is reached, as will be indicated by the sparking across the gap between the needle points.

Fig. 158 shows the electrical connections for carrying out a test of dielectric strength on a certain commercial transformer, marked in the figure "apparatus under test." It will be seen that
one terminal of the high-voltage coil of the testing transformer is connected to one terminal of the primary coil (at the left) of the transformer under test, and the other terminal of the high-voltage coil of the testing transformer is connected to one of the four terminals of the secondary coils of the transformer under test. The high voltage is, therefore, applied to the insulation between the primary and secondary coils of the transformer under test.


Fig. I58. Diagram of Connections for Dielectric Strength Test with High Potential Transiormer

The use of a water rheostat in series with the primary (low voltage) coil of the testing transformer, as shown in Fig. 158, for varying the impressed voltage, is open to some objection. Such a resistance is liable to seriously affect the wave form of the electromotive force, thereby causing its maximum value to bear a different. and unknown ratio to its effective value. Thè most approved way of securing voltage control is by adjusting the field excitation of the alternator supplying the testing transformer with current. Another method, though not as satisfactory, is by using a transformer with a variable ratio of primary to secondary turns.

Tests of dielectric strength are made with voltages ranging from $1 \frac{1}{2}$ to 10 times the rated terminal voltage of a piece of apparatus, according to the rated voltage and output of the apparatus. For example, a transformer of any: output whose rated terminal voltage is 20,000 would, according to the recommendations of the American Institute of Electrical Engineers, be tested with 40,000 volts, whereas a 1,000 -volt transformer would be tested with 3,500 volts. An induction motor under 10 h . p. rated at 110 volts would be tested with 1,000 volts.

Break-Down. The break-down test is frequently applied in the


Fig. 159. Curve Showing Relation between Sparking Distance and Voltage
testing of small samples of insulating material. For example, a sheet of fuller board, mica-canvas, oiled linen or cloth, would be clamped between sheets of metal connected to the terminals of the highvoltage coil of the testing transformer, and the voltage would be increased until the insulation was punctured, the voltage causing puncture being recorded. A basis is thus obtained for the acceptance or rejection by the purchaser of a lot of insulating material.

Table IX gives the sparking distances in air between opposed sharp No. 00 sewing needle points, corresponding to various effective sinusoidal voltages. The voltages are expressed in kilovolts (1 kilovolt $=1,000$ volts). Fig. 159 is a curve plotted from the data given

TABLE IX
Needle-Gap Distances for Various Spark-over Voltages
(At $25^{\circ} \mathrm{C} ., 760 \mathrm{MM}$. barometer, relative humidity of 80 per cent)

| Kilovolts Square Rool of Mean Squar | Distance |  | Kilovolts Square Root of Mean Square | Distance |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Inches | Cms. |  | Inches | Cms. |
| 5 | 0.225 | 0.57 | 140 | 13.95 | 35.4 |
| 10 | 0.47 | 1.19 | 150 | 15.0 | 38.1 |
| 15 | 0.725 | 1.84 | 160 | 16.05 | 40.7 |
| 20 | 1.0 | 2.54 | 170 | 17.10 | 43.4 |
| 25 | 1.3 | 3.3 | 180 | 18.15 | 46.1 |
| 30 | 1.625 | 4.1 | 190 | 19.20 | 48.8 |
| 35 | 2.0 | 5.1 | 200 | 20.25 | 51.4 |
| 40 | 2.45 | 6.2 | 210 | 21.30 | 54.1 |
| 45 | 2.95 | 7.5 | 220 | 22.35 | 56.8 |
| 50 | 3.55 | 9.0 | 230 | 23.40 | 59.4 |
| 60 | 4.65 | 11.8 | 240 | 24.45 | 62.1 |
| 70 | 5.85 | 14.9 | 250 | 25.50 | 64.7 |
| 80 | 7.1 | 18.0 | 260 | 26.50 | 67.3 |
| 90 | 8.35 | 21.2 | 270 | 27.50 | 69.8 |
| 100 | 9.6 | 24.4 | 280 | 28.50 | 72.4 |
| 110 | 10.75 | 27.3 | 290 | 29.50 | 74.9 |
| 120 | 11.85 | 30.1 | 300 | 30.50 | 77.4 |
| 130 | 12.90 | 32.8 |  |  |  |

in the table and shows graphically the relation between sparking distance and voltage. The voltage corresponding to a given sparking distance varies greatly with the sharpness of the needle points, and with the shape of the electromotive force wave. It also varies with the relative humidity.

Sometimes the value of the voltage applied to the apparatus is not measured by the spark gauge, but is inferred from the reading of a low-reading voltmeter connected between the points $A$ and $B$ in Fig. 158. Thus, if there are 100 times as many turns of wire in the secondary (high-voltage) coil of the testing transformer as in the primary ccil, then the readings of the voltmeter connected as specified must be multiplied by 100 . The kest practice is to connect an electrostatic voltmeter in parallel with the spark gap and to check its readings against the sparking distances in accordance with Table IX. However, the voltmeter gives effective values while the spark gap gives maximum values of the voltage.

Characteristic Curves. Saturation Curve. The saturation curve of a generator shows the relation between the volts generated in the armature and the amperes of field current (or ampere-turns of the field) for a constant armature current. The armature current may be zero, in which case the curve is called the no-load saturation curve, or sometimes the open-circuit characteristic curve. Observations for a saturation curve may be taken with full-load current in the armature; but this is rarely done, except in alternators of comparatively small output. If a full-load saturation curve is desired, it can be approximately calculated from the no-load saturation curve and the curve for synchronous impedance, as will be explained later.


Fig. 160. Diagram of Connections for Saturation Curve Test of Alternator

The diagram of connections for determining tne no-load saturation curve is shown in Fig. 160. The alternator is represented as a three-phase machine of the revolving-field type (armature is stationary). The field winding is comnected through slip-rings $d$ and $e$, and brushes $f$ and $g$, to an ammeter $A$, and through an adjustable resistance $r$, to the exciter or other direct-current source. A voltmeter $V_{1}$ is connected across the field-winding terminals to measure the voltage applied to the field winding. The voltmeter $V_{2}$, for measuring the voltage generated by the machine, is connected directly to two of the armature terminals, $a$ and $c$, in the case of a low-voltage machine. If the voltage generated is greater than the capacity of the voltmeter, a multiplying coil or a
step-down potential transformer may be used to reduce the electromotive force to be measured. For very high voltages a potential transformer must be used.

A series of observations of the electromotive force between the terminals of one of the phases, such as $a$ and $c$, is made for different values of the


Fig. 161. Saturation Curve for $2000-\mathrm{Kw}$. Alternator field current. Eight or nine points along the curve are usually sufficient, the series extending from zero electromotive force to about fifty per cent above normal rated voltage. The points should be taken more closely together in the vicinity of normal voltage than at other portions of the curve. Care must be taken that the generator is run at its rated speed, and this speed must be kept constant. Deviations from constant speed may be most easily detected by the use of a tachometer. If the machine is two-phase or three-phase, the voltmeter may be connected to any one phase throughout a complete series of observations. The voltage of all the phases should be observed for normal full-load excitation by connecting the voltmeter to each phase successively, keeping the field current constant at normal voltage. This is done in order to see how closely the voltage of the different phases agree.

The observations required for the determination of the saturation curve are volts at armature terminals, $V_{2}$; amperes in field, $A$; volts at field terminals, $V_{1}$; and speed.

Fig. 161 shows a saturation curve taken from a 2,000-kilowatt three-phase alternator of the revolving-field type, having 16 poles, and generating 2,000 volts and 576 amperes per phase, when run at 300 revolutions per minute.


Fig. 162. Set of Curves for Two-Phase $135-\mathrm{Kw}$. Inductor Alternator
Fig. 162 gives a number of curves for a two-phase 135 -kilowatt 2,400 -volt 60 -cycle inductor alternator. In particular, the no-load saturation curve is shown giving the relation of the field current to the voltage between armature terminals of one phase. This curve shows that nearly 7 amperes of field current is required to give rated voltage, namely 2,400 , at no load. The field current required to give rated voltage at full non-inductive load is $8.8 \mathrm{am}-$
peres. From the saturation curve given in Fig. 162 it is evident that this full-load field current will produce about 2,625 volts at no load.

Synchronous Impedance C'urve.* The synchronous impedance curve shows the relation between armature voltage and armature current, the armature being short-circuited so that the only condition that limits the


Fig. 163. Curve Showing Synchronous Impedance Drop for $2,000-\mathrm{Kw}$. Alternator current for a given voltage generated is the synchronous impedance of the armature. This is materially different from the impedance of the armature when the machine is standing still.

The connections for this test are similar to those for the saturation curve, except that the voltmeter (or potential transformer) connected to the armature is replaced by an ammeter. If the current is beyond the capacity of the ammeters at hand, a current transformer may be connected in place of the ammeter, and the ammeter may be connected to its secondary.

A series of observations is to be taken of the current in the armature, with the latter short-circuited through the ammeter, for different field currents, commencing at a very low value, and increasing the field current by successive steps until the armature current has reached a value of 100 per cent above its rated fullload value. The last few readings must be made quickly to pre-
vent undue heating of the armature. The armature winding should be at approximately normal temperature when the test is made. The speed should be kept approximately at the rated speed of the machine. It is not as essential to keep the speed constantly at rated value as when observations are being made for the determination of the saturation curve. The observations to be recorded are: amperes in the armature; amperes in field; volts at field terminals; and speed.

Fig. 163 shows a curve giving the relation between electromotive force induced in the armature and the current in the armature when short-circuited through an ammeter. This figure relates to the same 2,000 -kilowatt alternator whose saturation curve was given in Fig. 161. The electromotive forces plotted in this figure are not observed values, but the field excitations required to produce them are observed, and the electromotive forces corresponding to these field excitations are taken from the saturation curve.

The total electromotive force induced in the armature for a given value of the field current may be read off from the no-load saturation curve of the machine, obtained as previously described. A curve may then be plotted with the electromotive force induced in the armature as ordinates, and the observed armature currents (on short-circuit) as abscissas. This curve is sometimes called the synchronous impedance curve, although it does not explicitly show the values of the synchronous impedance of the armature. The synchronous impedance of the armature for a given value of armature current may, however, be derived from this curve by dividing the total electromotive force induced in the armature (ordinate) by the corresponding value of the short-circuited armature current (abscissa). For example, the synchronous impedance corresponding to 576 amperes (which is the full-load current per phase of the machine) is

$$
\frac{1,145 \text { volts }}{576 \text { amperes }}=1.99 \mathrm{ohms}
$$

Synchronous impedance is sometimes used as a basis to predetermine the regulation of the machine, thereby avoiding the trouble and expense of an actual test of regulation under full load. The synchronous impedance of an alternator armature is very nearly equal to the synchronous reactance of the armature, inas-
much as the armature resistance is usually small. Moreover the electromotive force required to overcome synchronous reactance is very nearly equal to the electromotive force required to overcome synchronous impedance.

Determination of Resistance of Armature. In the case of a three-phase armature, the resistance per phase cannot be measured directly between collector rings, since


Fig. 164. Diagram for Armature when Y-connected there are two phases in series between collector rings in a Y -connected armature; while in case of a $\Delta$-connected armature, two phases in series are in parallel with the third phase between any two collector rings.

The resistance per phase can be measured either directly by Wheatstone bridge and galvanometer, or by the direct-current voltammeter. The latter method consists in passing a measured value of direct current through the armature of the alternating-current machine while the armature is warm, and reading the volts drop across the terminals $(A B$, for example, Fig. 164). The measurement is taken with the machine stationary and the field unexcited. The resistance between the two terminals used is then given by the volts drop divided by the value of direct current, according to Ohm's law. By taking a number of readings an average value of $R_{A B}$ may be obtained.

In the case of a Y -connected armature, the resistance per phase is one-half the resistance between terminals, provided that the resistance of all the phases are alike. In case the resistances of the phases are unequal, the resistance of any phase may be deduced as follows:

The resistance between terminals $A$ and $B$, Fig. 164, is

$$
\begin{equation*}
R_{A B}=a+b \tag{i}
\end{equation*}
$$

The resistance between terminals $B$ and $C$ is

$$
\begin{equation*}
R_{B C}=b+c \tag{ii}
\end{equation*}
$$

The resistance between terminals $C$ and $A$ is

$$
\begin{equation*}
R_{C A}=c+a \tag{iii}
\end{equation*}
$$

Then

$$
\begin{align*}
a & =R_{A B}-b  \tag{iv}\\
b & =R_{B C}-c  \tag{v}\\
c & =R_{C A}-a \tag{vi}
\end{align*}
$$

Substituting ( $v i$ ) in ( $v$ ) we obtain

$$
\begin{equation*}
b=R_{B C}-R_{C A}+a \tag{vii}
\end{equation*}
$$

Substituting (vii) in (iv) we obtain

$$
a=R_{A B}-R_{B C}+R_{C A}-a
$$

from which

$$
a=\frac{R_{A B}-R_{B C}+R_{C A}}{2}
$$

Similarly we find

$$
b=\frac{R_{B C}-R_{C A}+R_{A B}}{2}
$$

and

$$
c=\frac{R_{C A}-R_{A B}+R_{B C}}{2}
$$

If $R_{A B}=R_{B C}=R_{C A}$, then $a=b=c=\frac{R_{A B}}{2}$.
For a $\Delta$-connected armature with equal resistances per phase, the resistance per phase equals $\frac{3}{2}$ times the resistance between terminals. The general expression for the resistance of any phase can be deduced as in the above case for $Y$ connection. In this case there are two circuits between $A$ and $B$, one through the phase $a$, and the other through phases $b$ and $c$ in series, as shown in Fig. 165. Remembering that the joint resistance of two (or more) circuits in parallel is the reciprocal of


Fig. 165. Resistance Diagram for $\Delta$-Connected Armature the sum of the reciprocals of the resistances of the several branches, we have

$$
R_{A B}=\frac{1}{\frac{1}{a}+\frac{1}{b+c}}
$$

$$
\begin{aligned}
& R_{B C}=\frac{1}{\frac{1}{b}+\frac{1}{c+a}} \\
& R_{C A}=\frac{1}{\frac{1}{c}+\frac{1}{a+b}}
\end{aligned}
$$

From these three equations the three unknown quantities $a, b$, and $c$ may be found by algebraic elimination as in the case of the Y connection.

Regulation.* The direct determination of the regulation of an alternator would be as follows:

The alternator would be run at normal rated speed, delivering rated fullload current with rated full-load electromotive force at its terminals. The main circuit would then be opencd, thus reducing the current ouput to zero. The excitation would be left unchanged, and the rise of terminal electromotive force would be observed. Then

$$
\text { regulation in per cent }=\frac{\text { rise of terminal electromotive force }}{\text { rated full-load terminal electromotive force }} \times 100
$$

Regulation Curve. Fig. 162 shows the regulation curves of a two-phase, 135 -kilowatt, 2,400 -volt, 60 -cycle inductor alternator. The lower regulation curve shows the regulation of the alternator at 100 per cent power factor, that is, on non-inductive load; and the upper regulation curve shows the regulation of the alternator at 70 per cent power factor. The abscissa of a given point on one of these regulation curves represents a given current output per phase of the machine; and the ordinate of the point represents the voltage obtained at the armature terminals when this armature current is reduced to zero, the field current being kept constant at that value which gives the rated voltage of 2,400 volts with the given current output per phase.

For example, with full-load current output, namely, 28.1 amperes per phase and 100 per cent power factor, the voltage rises from 2,400 to 2,625 (the ordinate of the regulation curve for 100 per cent power factor corresponding to the abscissa representing 28.1 amperes of armature current) when the load is thrown off (armature current reduced to zero), the field current remaining

[^18]unchanged. From these data the full load regulation of the machine on 100 per cent power factor is found to be
$$
\frac{225}{2,400} \times 100=9.4 \text { per cent }
$$

With half-load current output, namely 14.05 amperes per phase, and 70 per cent power factor, the voltage rises from 2,400 to 2,650 when the load is thrown off, the field current remaining unchanged. From these data the half-load regulation of the machine at 70 per cent power factor is found to be

$$
\frac{250}{2,400} \times 100=10.4 \text { per cent }
$$

These curves show that the regulation of the machine is higher on inductive loads than on non-inductive loads.

This direct determination of regulation by observation is not feasible with large machines, on account of the large amount of power required. In all practical testing the regulation may be determined indirectly by calculation.

American Institute Rules. The method used in predetermining the regulation is standardized by the American Institute of Electrical Engineers. The following are the rules of the A.I.E.E. concerning the regulation of electrical apparatus and prime movers:

## REGULATION

## DEFINITIONS

560. Regulation. The regulation of a machine in regard to some characteristic quantity (such as terminal voltage or speed) is the change in that quantity occurring between any two loads. Unless otherwise specified, the two loads considered shall be zero load and rated load, and at the temperature attained under normal operation. The regulation may be expressed by stating the numerical values of the quantity at the two loads, or it may be expressed by the "percentage regulation," which is the percentage ratio of the change in the quantity occurring between the two loads, to the value of the quantity at either one or the other load, taken as the normal value. It is assumed that all parts of the machine affecting the regulation maintain constant temperature between the two loads, and where the influence of temperature is of consequence, a reference temperature of $75^{\circ} \mathrm{C}$. shall be considered as standard. If change of temperature should occur during the tests, the results shall be corrected to the reference temperature of $75^{\circ} \mathrm{C}$.

The normal value may be either the no-load value, as the no-load speed of induction motors; or it may be the rated-load value, as in the voltage of a.c. generators.

It is usual to state the regulation of d.c. generators by giving the numerical values of the voltage at no load and rated load, and in some cases it is advisable to state regulation at intermediate loads.
562. Constant-Potential $\mathbf{A}=\mathbf{C}$. Generators. In these generators the regulation is the rise in voltage (when the specified load at specified power factor is reduced to zero) expressed in per cent of normal rated-load voltage.
563. Constant-Current Machines. The regulation in constant-current machines is the ratio of the maximum difference of current from the ratedload value (occurring in the range from rated-load to short-circuit, or minimum limit of operation), to the rated-load current.
564. Constant=Speed Direct=Current Motors. In these and also induction motors, the regulation is the ratio of the difference between full-load and no-load speeds to the no-load speed.
565. Constant=Potential Transformers. The regulation in constantpotential transformers is the difference between the no-load and rated-load values of the secondary terminal voltage, at the specified power factor (with constant primary impressed terminal voltage) expressed in per cent of the rated-load secondary voltage, the primary voltage being adjusted to such a value that the apparatus delivers rated output at rated secondary voltage.
566. Converters, Dynamotors, Motor=generators and Frequency Converters. In such machines the regulation is the change in the terminal voltage of the output side between the two specified loads. This may be expressed by giving the numerical values, or as the percentage of the terminal voltage at rated load.
567. Transmission Lines, Feeders, etc. The regulation is the change in the voltage at the receiving and between rated non-inductive load and no load, with constant impressed voltage upon the sending end. The percentage regulation is the percentage change in voltage to the normal rated voltage at the receiving end.
568. Steam Engines, Steam Turbines and Internal Combustion Engines. In these machines the percentage speed regulation is usually expressed as the percentage ratio of the maximum variation of speed, to the rated-load specd in passing slowly from rated load to no load (with constant conditions at the supply).
569. Fluctuation. If the test is made by passing suddenly from rated load to no load, the immediate percentage speed regulation so derived shall be termed the fluctuation.
570. Regulation in Hydraulic Turbine. In a hydraulic turbine or other water motor, the percentage speed regulation is expressed as the percentage ratio of the maximum variation in speed in passing slowly from rated load to no load (at constant head of water), to the rated-load speed.
571. Regulation in a Generator Unit. In a unit consisting of" a generator combined with a prime mover, the speed or voltage regulation should be determined at constant conditions of the prime mover; i.e., constant steampressure, head, etc. It includes the inherent speed variations of the prime mover. For this reason, the regulation of a generator unit is to be distinguished from the regulation of either the prime mover, or of the generator combined with it, when taken separately.

## CONDITIONS FOR TESTS OF REGULATION

580. Speed and Frequency. The regulation of generators is to be determined at constant speed, and of alternating-current apparatus at constant frequency.
581. Power Factor. In apparatus generating, transforming or transmitting alternating currents, the power factor of the load to which the regulation refers should be specified. Unless otherwise specified, it shall be understood as referring to non-inductive load, that is to a load in which the current is in phase with the e.m.f. at the output side of the apparatus.
582. Wave Form. In the regulation of alternating-current machinery receiving electric power, a sine wave of voltage is assumed, except where expressly specified otherwise.
583. Excitation. In commutating machines, rectifying machines, and synchronous machines, such as direct-current generators and motors, as well as in alternating-current generators, the regulation is to be determined under such conditions as to maintain the field adjustment constant at that which gives rated-load voltage at rated-load current, as follows:
(1) In the case of separately excited field magnets-constant excitation.
(2) In the case of shunt machines, constant resistance in the shunt-field circuit.
(3) In the case of series or compound machines, constant resistance shunting the series-field windinge.
584. Tests and Computation of Regulation of $\mathrm{A}-\mathrm{C}$. Generators.


Fig. 166. Saturation Curves (Method b) Any one of the three following methods may be used. They are given in the order of preference.

Method a. The regulation can be measured directly, by loading the generator at the specified load and power factor, then reducing the load to zero, and measuring the terminal voltage, with speed and excitation adjusted to the same values as before the change. This method is not generally applicable for shop tests, particularly on large generators, and it becomes necessary to determine the regulation from such other tests as can be readily made.
585. Method $b$. This consists in computing the regulation from experimental data of the open-circuit saturation curve and the zero power-factor saturation curve. The latter curve, or one approximating very closely to it, can be obtained by running the generator with over excitation on a load of idlerunning under-excited synchronous motors. The power factor under these conditions is very low and the load saturation curve approximates very closely the zero power factor saturation curve. From this curve and the open-circuit curve, points for the load saturation curve, for any power factor, can be obtained by means of vector diagrams.

To apply "Method b," it is necessary to obtain from test, the open-circuit saturation curve $O A$, Fig. 166, and the saturation curve $B C$ at zero power factor
and rated-load current. At any given excitation $O c$, the voltage that would be induced on open circuit is ac, the terminal voltage at zero power factor is $b c$, and the apparent internal drop is $a b$. The terminal voltage $d c$ at any other power factor can then be found by drawing an e.m.f. diagram* as in Fig. 167, where $\phi$ is an angle such that $\cos \phi$ is the power factor of the load, be the resistance drop ( $I R$ ) in the stator winding, $b a$ the total internal drop, and $a c$ the total induccd voltage; $b a$ and $a c$ being laid off to correspond with the values obtained from Fig. 166. The terminal voltage at power factor $\cos \phi$, is then $c b$ of Fig. 167, which, laid off in Fig. 166, gives point d. By finding a number of such points, the curve $B d d^{\prime}$ for power factor $\cos \phi$ is obtained and the regulation at this power factor (expressed in per cent) is $\frac{100 \times a^{\prime} d^{\prime}}{d^{\prime} c^{\prime}}$, since $a^{\prime} d^{\prime}$ is the rise in voltage when the load at power factor $\cos \phi$ is thrown off at normal voltage $\boldsymbol{c}^{\prime} d^{\prime}$.


Fig. 167. Terminal Voltage with Reactance


Fig. 168. Terminal Voltage without Reactance

Generally, the ohmic drop can be neglected, as it has very little influence on the regulation, except in very low-speed machines where the armature resistance is relatively high, or in some cases where regulation at unity power-factor is being estimated. For low power factors, its effect is negligible in practically all cases. If resistance is neglected, the simpler e.m.f. diagram, Fig. 168, may be used to obtain points on the load saturation curve for the power factor under consideration.
586. Method $c$. Where it is not possible to obtain by test a zero powerfactor saturation curve as in "Method b," this curve can be estimated closely from open-circuit and short-circuit curves, by reference to tests at zero power factor on other machines of similar magnetic circuit. Having obtained the estimated zero power-factor curve, the load saturation for any other power factor "s obtained as in "Method b."

Thus "Method $c$ " is the same as "Mcthod b;" except that the zero powerfactor curve must be estimated. This may be done as follows. In Fig. 169, $O A$ is the open-circuit saturation curve and $O E$ the short-circuit line as shown

[^19]by test. The zero power-factor curve corresponding to any given current $B F$ will start from point $B$, and for machines designed with low saturation and low reactance, will follow parallel to $O A$, as shown by the dotted curve $B D$, which is $O A$ shifted horizontally parallel to itself by the distance $O B$. In high-speed machines, or in others having low reactance and a low degree of saturation in the magnetic circuit the zero power-factor curve will lie quite close to $B D$, particularly in those parts that are used for determining the regulation. This is the case with many turbo-gencrators and high-speed waterwheel generators. In many cases, however, the zero power-factor curve will deviate from $B D$, as shown by


Fig. 169. Saturation Curves (Method $c$ )


Fig. 170. Saturation and Short Circuit Characteristics
$B C$, and the deviation will be most pronounced in machines of high reactance, high saturation, and large magnetic leakage. The position of the actual curve $B C$
with relation to $B D$, can be approximated with sufficient exactness by investigating the corresponding relation as obtained by test at zero power-factor on machines of similar characteristics and magnctic circuit. Or curve $B C$ can be calculated by methods based on the results of tests at zero power-factor. After $B C$ has been


Fig. 171. Application of Terminal Voltage Rule obtained, the saturation curve and regulation for any other power factor can be derived as in "Method b."

Example of "Method a"A.I.E.E., Rule 584. The following test data were obtained on a three-phase, 15 -kv.-a., 220 -volt, 60 -cycle, $\Delta$-connected alternator. As a more severe test only one phase of the armature was used, the other two windings carrying no current.
(1) Careful determination of the opencircuit saturation characteristic of the machine yielded the curve OA, Fig. 170.
(2) With the armature short-circuited the field current ( $O B$ ) required to produce fullload armature current (23 amperes) was 0.83 amperes.
(3) While the alternator was loaded with an under-excited synchronous motor, the armature current was maintained constant at fullload valuc ( 23 amperes) and the field was varied. Data were obtained as follows:

| $I$ (field) in amperes | 1.9 | 2.82 | 4.18 | 5.26 | 6.3 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $E_{l}$ (terminal) in volts | 8.2 | 123 | 172 | 206 | 233 |

From this and the short-circuit data (2), the zero power-factor characteristic $B C$ was constructed.
(4) With the machine loaded with resistance and variable reactance in series, the power factor was adjusted to two different values, the terminal voltage being held constant at 220 volts for full-load current in each case. The load was then disconnected and the rise in voltage noted. The results were as follows:

| $\operatorname{Cos} \phi$ | $E_{i}$ | $I_{a}$ | $E_{g}$ <br> (noload) | Regulation <br> (per cent) |
| :---: | :---: | :---: | :---: | :---: |
| 0.5 | 220 | 23 | 251 | 14.1 |
| 0.87 | 220 | 23 | 237 | 7.7 |

Example of "Method b" A.I.E.E., Rule 585. From the data in "Method a" and the curves, Fig. 170, plot the load-saturation curves for power factors of 0.5 and 0.87 using the A.I.E.E. method, and from these curves obtain the voltage regulation.

## Solution:

(1) Assume values of ac, Fig. 168, A.I.E.E., Rule 585. For example, let $a c=220$ volts.
(2) From the curves, Fig. 170, $a b=40$ volts.
(3) Draw $a b$ vertical and to scale, Fig. 171. Draw the line $b c$ making an angle $\phi$ with the horizontal such that $\cos \phi=0.5$ or $\phi=60$ degrees. From $a$ as a center strike an are with $a c=220$ volts as a radius. This locates the point $c$. $b c$ is then determined to scale. $b c=184.5$ volts.
(4) Locate the point $d$ on the load-saturation curve for $\cos \phi=0.5$, by making $d c$ equal to the value $b c$ taken from the diagram, Fig. 171.
(5) In a similar manner other points can be located and the load-saturation curve plotted for $\cos \phi=0.5$.

The following table shows the calculated values of $b c$, or $c d$, for different values of field current.

| Field Amperes | ${ }^{a c} E_{a}$ | $a b$ <br> $Z I_{a}$ drop | $b c$ <br> $\operatorname{Cos} \phi=0.5$ | $b c$ <br> $\operatorname{Cos} \phi=0.87$ <br> (See Fig. 172) |
| :---: | :---: | :---: | :---: | :---: |
| 3.0 | 175 | 47 | 132.5 | 146 |
| 3.5 | 194 | 44 | 154.5 | 168 |
| 4.0 | 210 | 42 | 172.5 | 185 |
| 4.4 | 220 | 40 | 184.5 | 197 |
| 4.5 | 223 | 39 | 188.5 | 201 |
| 5.0 | 233 | 35 | 202 | 213.5 |
| 5.5 | 243 | 31 | 216 | 226 |

(6) Corresponding to a terminal voltage of 220 volts, the field currents are 5.25 amperes for $\cos \phi=0.87$, and 5.68 amperes for $\cos \phi=0.5$. The voltages at zero load corresponding to these values of field current are: 238 volts for $\cos \phi=0.87$, and 247 volts for $\cos \phi=0.5$.
Voltage regulation when $\cos \phi$ is $0.87=\frac{238-220}{220} \times 100$ per cent $=8.2$ per cent Voltage regulation when $\cos \phi$ is $0.5=\frac{247-220}{220} \times 100$ per cent $=12.3$ per cent

Potier Method of Determining Voltage Regulation. This method is an attempt to separate the synchronous impedance drop, $Z I_{a}$ of the armature into its two parts, that part which is a pure reactance drop $X_{a} I_{a}$ and that which is caused by the demagnetizing action of the field of the armature current. It is often referred to as simply armature reaction. In this method, as in other methods, the armature resistance is so small that its effect can usually be neglected. The Potier construction is as follows:
(1) In Fig. 170 take a point $P_{2}$ well up on the knee of the magnetization curve. Through $P_{2}$ draw $P_{2} M$ equal and parallel to $O B$. From $M$ draw $M L$ parallel to the open circuit characteristic
at the origin. A perpendicular dropped from the intersection of $M L$ with the open-circuit curve gives $P_{1} F\left(=X_{a} I_{a}\right)$, and $P_{2} F$ $\left(=F_{1}\right)$ the armature reaction in terms of field current.


Fig. 172. Potier Method for Determining Terminal Voltage
(2) In Fig. 173 add $X_{a} I_{a}$ vectorially to $E_{t}$ (the terminal volts under full load). The field current $F_{a}$ corresponding to the resultant voltage $E_{a}$ (Fig. 173) is then obtained from the opencircuit saturation curve.
(3) $\mathrm{F}_{1}$, the armature reaction, is then drawn as a vector parallel to $X_{a} I_{a}$ and when added to $F_{a}$ gives a resultant field


Fig. 173. Potier Construction Diagram
excitation $F_{q}$. This is the field current necessary to maintain the terminal voltage $E_{t}$ at full load, and the corresponding voltage $E_{\sigma}$

Table X
Regulation Values for Three=Phase $\Delta$-Connected Alternator by Three Methods

| Method | 0.5 Power Factor |  |  | 0.87 Power Factor |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{array}{\|c\|} E_{t} \\ \text { Full-Load } \\ \text { Voltage } \end{array}$ | $\begin{array}{\|c} E_{g} \\ \text { No-Load } \\ \text { Voltage } \end{array}$ | $\begin{gathered} \text { Regulas- } \\ \text { tion } \\ \% \end{gathered}$ | $\begin{gathered} E_{t} \\ \text { Full-Load } \\ \text { Voltage } \end{gathered}$ | $\left\lvert\, \begin{gathered} E_{g} \\ \text { No--Load } \\ \text { Voltage } \end{gathered}\right.$ | $\begin{gathered} \text { Regula- } \\ \text { tion } \\ \% \end{gathered}$ |
| (a) A.I.E.E. Rule 584 | 220 | 251 | 14.1 | 220 | 237 | 7.7 |
| (b) A.I.E.E. Rule 585 | 220 | 247 | 12.3 | 220 | 238 | 8.2 |
| Potier | 220 | 249.5 | 13.4 | 220 | 238 | 8.2 |

on the open-circuit saturation curve is the voltage to which the machine will rise when the load is removed.

$$
\text { Regulation }=\frac{E_{g}-E_{t}}{E_{t}} \times 100 \text { per cent }
$$

Example. Determine the regulation of the given machine for a powerfactor of 0.87 , using the Potier method.

Solution. (1) From the saturation curves and the Potier construction,

$$
\begin{gathered}
X_{a} I_{a}=P_{1} F=24 \text { volts } \\
F_{1}=P_{2} F=0.45 \text { amperes }
\end{gathered}
$$

(2)

$$
\begin{aligned}
E_{a} & =\sqrt{\left(E_{t}+X_{a} I_{a} \sin \phi\right)^{2}+\left(X_{a} I_{a} \cos \phi\right)^{2}} \\
& =\sqrt{(220+24 \times 0.5)^{2}+(24 \times 0.87)^{2}=233 \text { volts }}
\end{aligned}
$$

(3)

$$
\begin{gathered}
\left.F_{a} \text { (corresponding to } E_{a}=233 \text { volts }\right)=5.0 \text { amperes } \\
\begin{array}{c}
F_{1}=0.45 \text { amperes } \\
\begin{aligned}
F_{g}(\text { approximately }) & =\sqrt{(5.0+0.45 \sin \phi)^{2}+(0.45 \cos \phi)^{2}} \\
& =5.24 \text { amperes }
\end{aligned} \\
\left.E_{g} \text { (corresponding to } F_{g}=5.24\right)=238 \text { volts } \\
\text { Regulation }=\frac{238-220}{220} \times 100 \text { per cent }=8.2 \text { per cent }
\end{array}
\end{gathered}
$$

The values of the regulation of the three-phase, $15-\mathrm{kv} .-\mathrm{a}$. , 220 -volt, 60 -cycle, $\Delta$-connected alternator, calculated by the three different methods described above, are brought together in Table X for comparison.

Heat Test. The heat test is made by running the generator under full-load conditions until a constant temperature has been reached. When this condition has been attained the machine is shut down, and the temperature of the various parts is taken by thermometers placed against the heated surfaces. The resistances of the armature and field windings are also measured while hot. By comparing these "hot" resistances with the same resistances previously measured at room temperature, the temperature rise of the armature and field coils can be computed. The method based on the temperature coefficient of copper prescribed by the Standardization Rules of the American Institute of Electrical Engineers for making these computations is as follows:

The fundamental relation between the increase of resistance in copper and the rise of temperature is taken as

$$
R_{t}=R_{0}(1+0.0042 t)
$$

where $R_{0}$ is the resistance of the copper conductor at $0^{\circ} \mathrm{C}$. and $R_{t}$ is the corresponding resistance at $t^{\circ} \mathrm{C}$. This is equivalent to taking a temperature coefficient of 0.42 per cent per degree C . temperature rise above $0^{\circ} \mathrm{C}$. For initial temperatures other than $0^{\circ} \mathrm{C}$., a similar formula may be used substituting the proper coefficient corresponding to the actual initial temperature. The formula thus becomes at $25^{\circ} \mathrm{C}$.

$$
R_{i+r}=R_{i}\left(1+\frac{0.3853 r}{100}\right)
$$

where $R_{i}$ is the initial resistance at $25^{\circ} \mathrm{C} . R_{i+r}$ the final resistance, and $r$ the temperature rise above $25^{\circ} \mathrm{C}$.

In order to find the temperature rise in degrees centigrade from the initial resistance $R_{i}$ at the initial temperature $i^{\circ} \mathrm{C}$. and the final resistance $R_{i+r}$, we may use the formula

$$
r=(234.5+i)\left(\frac{R_{i}+r}{R_{i}}-1\right) \text { degrees } \mathrm{C} .
$$

Example. The "cold" resistance of the armature of a generator is measured and found to be 0.046 ohm at a temperature of $20.5^{\circ} \mathrm{C}$. After a heat run under full load the resistance is found to be 0.052 ohm. What is the rise in temperature?

In this case the initial temperature $i$ is $20.5^{\circ} \mathrm{C}$., the final resistance $R_{i+r}$ is 0.052 ohm , and the initial resistance $R_{i}$ is 0.046 ohm . The temperature rise is, therefore,

$$
r=\left(234.5+20.5\left(\frac{0.052}{0.046}-1\right)=33.15\right. \text { centigrade degrees }
$$

which means that the final temperature of the armature is

$$
i+r=20.5+33.15=53.65^{\circ} \mathrm{C}
$$

The temperatures of the following generator parts are usually recorded:

| Armature coils | Field coils |
| :--- | :--- |
| Armature laminations | Pole tip (leading) |
| Armature ventilating ducts | Pole tip (trailing) |
| Frame | Field yoke |
| Bearings | Room |

Small generators are usually run at full load, delivering their output to water rheostats. During the heat run, thermometers, placed on different parts of the machine, are read regularly. If the generator is of the revolving-armature type, a thermometer is placed against the field winding, and another thermometer is so placed that the hot air issuing from the ventilating ducts will come in contact with it. If the generator be of the revolvingfield type, thermometers are placed against the armature coil, on the armature laminations, and in the ventilating ducts. When these thermometers do not show an increasing temperature, and the resistance of the field, as determined from the field ammeter and voltmeter, has become constant, the machine is considered as having attained its ultimate temperature, and is shut down for the application of thermometers, which should be ready for immediate application, as the machine cools rapidly. The time taken by a machine to reach constant temperature varies from 3 to 4 hours in the case of a small machine, and from 12 to 18 hours for very large ones. The highest permissible limits of temperature and temperature rises according to the Standardization Rules of the A.I.E.E. have been given on pages 161 to 164 . In heat tests the rules governing temperature measurements, as well as the precautions to be taken (given on pages 165 to 169 ) should be fully observed.

The temperature should be measured after a run of sufficient duration to reach practical constancy. This is usually from 6 to 18 hours, according to the size and construction of the apparatus. It is permissible, however, to shorten the time of the test by running the generator for a lesser time on an overload in both current and voltage, then reducing the load to normal, and maintaining it at this load until the temperature has become constant.

In making a heat test of a large alternator it is customary to imitate full-load conditions, electrically and magnetically, so as to produce all the heating effects that would occur under actual full load, but without taking any actual electrical power from the alternator. To be able to do this is of great advantage from two standpoints: first, that of convenience; and second, that of economy. To accomplish this desirable result, the generator is in certain cases run on short-circuit with a number of the field spools connected in opposition to (or "bucked" against) the remaining field spools or, in other words, with the effective number of poles reduced.

In order to determine the proper number of field-magnet spools which should be connected in opposition, we proceed in the following manner:

It was previously explained that from the saturation and synchronous impedance curves of a generator, we could determine the field current required to produce rated voltage at full load. In the case of the $2,000-\mathrm{kw}$. generator, of which the curves are given in Figs. 161 and 163, the normal full-load field current was found to be 93.8 amperes. This field current would, if the armature were short-circuited, produce, according to Fig. 163, an armature current of more than 1,050 amperes, which is greatly in excess of rated full-load current. If the field current were reduced to 43 amperes, the armature current would be normal; but, of course, the field current would be very much below the normal value mentioned above; therefore, the magnetic density in the field magnet and the armature core, and the consequent iron losses in the armature, would also be below normal.

We may reduce the excessive current in the short-circuited armature to its normal full-load value by reducing the effective number of field poles in the ratio of 93.8 to 43 instead of by reducing the field current in this ratio. The alternator has 16 field poles, and to reduce this number in the ratio of 93.8 to 43 , would give $\frac{16 \times 43}{93.8}=7.34$ poles; but inasmuch as there must be an integral and even number of field poles, the closest approximation to the desired number is 8 effective poles. If, therefore, we reverse the connections of any four (adjacent) field coils, the four
pȯtes produced by these reversed field coils will be reversed in polarity; and the electromotive forces induced in the armature conductors under these reversed poles will be reversed, and will balance the electromotive forces induced in the armature conductors under four of the unreversed field poles, so that the electromotive forces induced in the armature conductors under the remaining eight poles only, will be effective in producing current in the armature; that is, only eight of the sixteen field poles will be effective.

If, under these conditions, we short-circuit the armature of the alternator, it will take approximately the full-load field current of 93.8 amperes to produce full-load current in the armature, and the heat test can then be made under full-load conditions, namely, full-load armature current, full-load field current, and full-load iron losses, while the machine is running on short-circuit and, therefore, delivering no power. Only the power represented by the losses occurring in the machine need be supplied to drive it. If the normal field current does not produce a normal armature current, after the field spools are connected in opposition as described, the effect of a slight excess of armature current may be approximately balanced by a slight deficiency of field current, or vice versa.

Intermittent Method of Making a Heat Test. The foregoing compromise method of conducting a heating test, while much used in the past, is open to several objections and has been largely superseded by what is known as the "intermittent short-circuit and open-circuit" method, or, briefly, the "intermittent" method. Unlike the preceding method, it involves no change in the connections of the machine. It requires no expenditure of energy save what is needed to supply the losses of the machine. Yet every part of the alternator reaches the same temperature as it would after a heating test on actual full load. The method presupposes that the separate losses of the machine are known, as they would be in practice, from previous tests.

The simplest way of describing the method will be to follow through a concrete case. Suppose, then, a certain machine has at full load losses as follows: a friction loss of 5 kw ., an armature copper loss of 10 kw ., and an iron loss of 50 kw . If the machine were
actually run at full load for one hour, the energy losses would be:
$\begin{array}{lr}\text { Friction } & 5 \times 60=300 \mathrm{kw} \text {--minutes } \\ \text { Armature copper } & 10 \times 60=600 \mathrm{kw} \text {-minutes } \\ \text { Iron loss } & 50 \times 60=3000 \mathrm{kw} \text {--minutes }\end{array}$
giving a total energy loss of 3900 kw .-minutes.
Let the machine run for 5 minutes with its armature shortcircuited, and at such a current that the armature copper loss is 40 kw ., which corresponds to an armature current equal to $\sqrt{3}$ times the normal full-load value. Now let the machine run for the next ten minutes with the armature circuit open, but with over-excited field so as to produce an iron loss of 75 kw . This adjustment is made according to the indications of the wattmeter connected to the input side of the motor used to drive the alternator under test. Allowances for other losses, including those in the motor itself as well as the friction of the alternator, must of course be made.

If this cycle of operations, namely five minutes on short circuit with 3 times normal armature copper loss followed by a period of ten minutes with $1 \frac{1}{2}$ times normal iron loss, then another five minutes on short circuit, and so on, is repeated regularly throughout the time of the test, we find that:

$$
\begin{array}{ll}
\text { Loss in friction per hour } & =5 \times 60=300 \mathrm{kw} \text {.-minutes } \\
\text { Loss in armature copper per hour } & =4 \times 30 \times 5=600 \mathrm{kw} \cdot-\text {-minutes } \\
\text { Loss in iron per hour } & =4 \times 75 \times 10=3000 \mathrm{kw} .- \text {-minutes }
\end{array}
$$

thus giving a total energy loss of 3900 kw .-minutes per hour and the same loss in each part as would have occurred under normal full load in the same time.

The above discussion, however, did not include the loss in the magnet windings. It will be less than normal during the shortcircuit test, and greater than normal during the open-circuit test; so that on the average it does not differ greatly from its normal value. If greater precision is demanded, corrections are easily applied to determine the temperature rise of the field coils at the correct full-load loss in the coils themselves.

Core Loss and Friction Test. For this test the generator is driven by a motor at normal speed on open circuit. The power required to drive it, its field current being zero, is measured. The power so measured is the power lost in friction and windage. Then
the field current is increased, step by step, until the terminal electromotive force has increased to from 25 to 50 per cent above its normal rated full-load value; the terminal voltage is observed ${ }^{\circ}$ at each step, the armature being on open circuit (main switch open). The power required to drive the machine at each of these observed voltages is determined. This power is used to supply friction and windage loss and iron (or core) loss at the observed voltage.

The difference, therefore, between the power required to drive the machine at a given voltage on open circuit, and the amount of power required to drive the machine without-field excitation, that is, to supply the friction and windage loss, represents the iron loss, or core loss, at the given voltage. Fig. 174 shows the no-load core-loss curve for the $2,000-\mathrm{kw}$. alternator of which the saturation curve and the synchronous impedance curve are given in Figs. 161 and 163 , respectively.


Fig. 174. No-Load Core-Loss Curve for $2,000-\mathrm{Kw}$.

The most convenient way to measure the power delivered to the alternator is to drive it by a motor and measure the electrical input to the motor. If we determine the losses occurring in the motor the difference between the input to the motor and these losses is the power delivered to the alternator. Fig. 175 shows a diagram of complete connections for the carrying out of the core loss test. $A$ is the alternator under test. Its voltage is measured by the voltmeter $V_{1}$ connected to the secondary of the potential transformer $T^{\prime}$. Its field current, supplied by the exciter $E$, is measured by the ammeter $A_{1}$ and controlled by the resistance $R_{E}$ in the field of the exciter. The power is supplied to the motor $M$ by the direct-current generator $G$. The power is measured by the ammeter $A_{2}$ and the voltmeter $V_{2}$. The motor $M$ is separately excited, its field current being measured by the ammeter $A_{3}$. This field current is kept constant by adjusting the rheostat $R_{F}$.

As the field current and the voltage of the generator under test are increased, the load on the driving motor $M$ is increased also, since the hysteresis and eddy current loss in the generator, which must be supplied by the motor, are increased. As the load on the motor increases, it will slow down if its field current and the voltage between its brushes remain constant. Ilowever, the alternator to be tested must be run at constant speed, and its speed is controlled by varying the voltage at the motor terminals by means of the field rheostat $R_{G}$ of the direct-current generator $G$. In the figure the field rheostat $R_{G}$ of the generator $G$, is shown beside the alternator $A$. It is placed at this point for convenience, so that the observer stationed at the alternator may readily keep the speed under control.


Fig. 175. Diagram of Connections for Core-Loss Test of Alternators
The input to the motor armature in watts is equal to the product of the readings $V_{2}$ and $A_{2}$. Part of this input is consumed in supplying the various losses in the motor armature, including friction and windage; and the remainder is converted into useful mechanical power, and is transmitted by the belt (not shown in Fig. 175) to the pulley of the alternator $A$, which is being tested. In other words, the watts input to $A$ is equal to the input to the armature of $M$ minus the various losses in the armature of $M$. The losses in the armature of $M$ consist of hysteresis, eddy current,
friction, windage, and $I^{2} R$ losses. Since the alternator is to be run at constant speed, the motor will also run at constant speed if there is no belt slip, which should be the case; and since the motor runs at constant speed and at a constant field excitation, all the losses in the motor armature will be constant in value, whatever its load, except the $I^{2} R$ loss. This $I^{2} R$ loss varies because the current $I$ varies for different loads on the motor. If the speed of the motor were controlled by varying its field current, instead of varying the voltage applied at its terminals, the above-mentioned losses in the motor armature would not remain constant; hence the latter method is adopted.

If the motor be run free, i.e., with belt off, the output of the motor will be zero and, therefore, the power input must be used up entirely in supplying the losses. Let $E$ be the voltage at motor brushes, and $I$ the current in the motor armature when it is running free (or unloaded), then $E I=$ stray power $+I^{2} R$, where "stray power" equals the sum of all the constant losses in the motor armature, i.e., hysteresis, eddy current, friction, and windage losses. Hence, stray power $=E I-I^{2} R$.

The mechanical or useful output of the motor at a given load, the voltage applied to its brushes being $E_{1}$, and its armature current being $I_{1}$, is

$$
\text { output }=E_{1} I_{1}-I^{2}{ }_{1} R_{1}-\text { stray power }
$$

That is, the mechanical output is equal to the total electrical input minus the total losses.

The stray power can be determined once for all by running the motor free, or at no load, at the speed and with the field current used in the test when $E$ and $I$ were observed. Thus, we may determine the power required to drive $A$ (the machine to be tested) with zero field excitation of $A$, in which case we obtain its friction and windage losses; and we may determine the power required to drive $A$ at any given field excitation and voltage, in which case we obtain the sum of the friction, windage, and core (or iron) losses.

In carrying out these tests, therefore, the following order is found most convenient:
(1) The motor is made to drive the alternator with zero field current in the latter; and a number of observations are taken, as indicated in the tabular arrangement below. This is to determine the friction and windage loss of the alternator.
(2) The alternator is excited to produce a series of values of terminal voltage, for each of which a similar set of observations is taken; and so on, until the voltage of the alternator has been increased to about 25 per cent above normal rated voltage.
(3) The motor is now shut down and its belt thrown off, and it is then run unloaded in order to determine its own stray-power losses.
(4) Finally the motor is shut down, and the resistance of its armature is measured.

For each set of rcadings the following observations should be recorded:

$$
\text { Motors }\left\{\begin{array} { l } 
{ \text { Volts at brushes } } \\
{ \text { Amperes in armaturc } } \\
{ \text { Amperes in field } } \\
{ \text { Speed } }
\end{array} \quad \text { Alternator } \left\{\begin{array}{l}
\text { Volts at terminals } \\
\text { Amperes in field } \\
\text { Speed }
\end{array}\right.\right.
$$

The amperes in the field of the motor, and the speed of the motor and generator are recorded merely to show whether they have been kept constant during the test. For the best results the motor should have a rated capacity of from 15 to 20 per cent of the rated capacity of the machine to be tested.

Example. In a core-loss test of the $2,000-\mathrm{kw}$. alternator above referred to, the following observations were taken: With zero field excitation of the alternator, the voltage applied to the motor armature and the current in the motor armature were 510 volts and 48 amperes, respectively. When the alternator was excited to give 2,070 volts between its terminals, the volts applied to, and the current in, the motor armature were 511 volts and 117 amperes, respectively. With the motor running unloaded the voltage applied to, and the current in, the motor armature were 509 volts and 11.5 amperes, respectively. The resistance of the armature of the motor was 0.056 ohm . Find:
(a) The stray-power loss of the driving motor, from the readings taken above when the motor was running unloaded.

Solution. Motor input was 509 volts $\times 11.5$ amperes $=5,853$ watts. But by definition, stray power $=E I-I^{2} R$. Hence

$$
\text { stray power }=5,853-0.056 \times(11.5)^{2}=5,846 \text { watts }
$$

(b) The friction and windage loss of the alternator.

Solution. Power supplied to motor when driving alternator at full speed with zero field excitation was 510 volts $\times 48$ amperes $=24,480$ watts; and the useful output of the motor or the power used to drive the alternator was the power input to the motor minus stray-power loss and minus $I^{2} R$ loss in its armature. Therefore
friction and windage loss $=24,480-5,846-\left[(48)^{2} \times 0.056\right]=18,505$ watts
(c) The core (or iron) loss of the alternator.

Solution. Power supplied to motor when driving the alternator at full speed, field excited to give 2,070 volts between alternator terminals, was 511 volts $\times 117$ amperes $=59,787$ watts. The useful output of the motor in this case was 59,787 watts $-5,846$ watts $-\left[(117)^{2} \times 0.056\right]=53,175$ watts; and this is equal to the sum of friction and windage loss and iron loss. Therefore
core loss of alternator $=53,175-18,505=34,670$ watts

The core loss of the alternator was calculated for each value of the field excitation of the alternator in a manner similar to the above. These various core losses are plotted as ordinates of the curve in Fig. 174, the abscissas being the electromotive forces between alternator terminals corresponding to the various field excitations. This curve shows that at the rated full-load voltage, namely, 2,000 volts, the core loss of the alternator is 30.5 kilowatts or 1.525 per cent of the rated full-load output of the alternator.

Calculation of Efficiency. 2,000-Kw.. Alternator. Since the efficiency of an alternator is the ratio of the output to the output plus the losses, as explained on page 155 , we may now calculate the efficiency of the alternator, having made all necessary tests. In the case of the $2,000-\mathrm{kw}$. alternator referred to above, for which the curves have been given, the following losses occur:
(1) Friction and windage loss. The loss was found under (b) in the example just given to be 18,505 watts.
(2) $I^{2} R$ loss in armature at full load. The resistance of each phase of the armature winding of the given alternator was measured and found to be 0.00924 ohm , and the full-load rated current is 576 amperes per phase. Therefore, the $I^{2} R$ loss per phase is

$$
(576)^{2} \times 0.00924=3,065 \mathrm{watts}
$$

Hence, the total $I^{2} R$ loss for all three phases is

$$
3 \times 3,065 \text { watts }=9,195 \text { watts }
$$

(3) $I^{2} R$ loss in the field. The resistance of the field winding of the given alternator is 0.966 ohm , and the full-load field current is 93.8 amperes. Hence, $I^{2} R$ loss in the field is 8,500 watts.
(4) Core loss. The core loss on open circuit at full-load excitation was found to be 30,500 watts. This is somewhat less than the core loss would be with full-load excitation and with full-load armature current.

Therefore, the sum of all the losses is

| Friction and windage loss | 18,505 |
| :--- | ---: |
| $I^{2} R$ loss in armature | 9,195 |
| $I^{2} R$ loss in the field | 8,500 |
| Core loss | 30,500 |
| Total loss | $\underline{66,700}$ watts |

The rated full-load output being 2,000 kilowatts, we have
efficiency at full load $=\frac{2,000,000}{2,000,000+66,700}=96.8$ per cent
If the efficiency at any fractional part of full load is desired, it inay be calculated as follows:
$\frac{1}{2}$ rated full-load output
$\frac{1}{2}$ rated full-load output + losses at half load
(1) Friction and windage loss is approximately the same at half load as at full load.
(2) $I^{2} R$ loss in the armature is one-fourth as great as at full load since the armature current is half as great.
(3) $I^{2} R$ loss in the field is slightly less at half load than at full load, inasmuch as field current is slightly less.
(4) Core loss at half load is slightly less than core loss at full load.

1955-Kw. Inductor Alternator. Fig. 162 shows the efficiency curve of a two-phase, 135 -kilowatt, 2,400 -volt, 60 -cy cle inductor alternator. The ordinates of this efficiency curve represent the efficiencies of this alternator corresponding to different current outputs per phase, the latter being plotted as abscissas. The efficiencies shown by this curve apply to the machine when it delivers power to noninductive receiving circuits ( 100 per cent power factor). When the power factor is less than 100 per cent, the efficiencies are less than the efficiencies shown by the curve.

At full-load current output of 28.1 amperes per phase and 100 per cent power factor, the efficiency as represented by the ordinate of the curve is 93 per cent. At half-load current output of 14.05 amperes per phase, the efficiency is $90 \frac{1}{2}$ per cent. At full load the several losses in this machine are as follows:
(1) Friction and windage loss (obtained by experiment), 2,000 watts.
(2) $I^{2} R$ loss in the armature (calculated from hot resistance of armature 1.46 ohm per phase, and full-load armature current 28.1 amperes per phase) is

$$
2 \times(28.1)^{2} \times 1.46=2,305 \text { watts }
$$

(3) $I^{2} R$ loss in the field (calculated from hot resistance of field coil 9.35 ohms, and full-load field current 8.8 amperes) is

$$
9.35 \times(8.8)^{2}=724 \text { watts }
$$

(4) Core loss (determined by experiment using the method described on page 209), 5,000 watts.

The total loss is, therefore

| Friction and windage loss | 2,000 watts |
| :--- | ---: |
| $I^{2} R$ loss in armature | 2,305 watts |
| $I^{2} R$ loss in field | 724 watts |
| Core loss | $\frac{5,000}{}$ watts |
| Total loss | 10,029 watts |

Therefore
efficiency at full-load $=\frac{135,000}{135,000+10,029}=93$ per cent

## SYNCHRONOUS MOTORS

Motors designed to be operated with alternating currents may be divided into three classes: synchronous motors, induction motors, and alternating-current commutator motors. There are a few special types which do not come under the above classification, yet by far the larger part of all the motors run with alternating currents belong to one or another of these classes. Of the three classes the induction motor is the most commonly used, because it has properties which adapt it to a much wider field of application than is possible with either the synchronous motor or the commutator motor.

Any Alternator a Synchronous Motor. Any alternatingcurrent generator may be operated as a motor, and become a source of mechanical energy, when its armature is supplied with alternating current and its field is excited with direct current. It then becomes a synchronous motor and runs at a constant speed regardless of its load, provided that the frequency of the alternating current supplied to its armature is constant. The armature may be of the stationary or revolving type and the winding may be single phase or polyphase.

The single-phase synchronous motor is not self-starting, but must be started, brought up to synchronous speed, and have its
counter-voltage brought into proper phase relations with the supply voltage before the armature of the motor may be connected to the alternating-current supply mains and receive power from them. This process of "synchronizing" the motor with the supply main must be gone through each time the motor is to be put in operation.

The polyphase synchronous motor is self-starting provided its mechanical load is not too large. To be self-starting, polyphase currents must be supplied to the armature at a fraction of the normal voltage, and the field must be unexcited. The action of the polyphase currents in the armature is to set up a rotating field, just as in the polyphase induction motor explained in a later discussion. This rotating magnetic field gives rise to a small torque which brings the armature up to nearly synchronous speed, and when the field of the motor is excited the armature is drawn into synchronism.

Synchronous Speed. The term "being in synchronism" means that the rotating member of the motor has a definite speed and relative position with respect to the rotor of the generator driving it, such that the motor voltage will oppose the generator voltage and be of exactly the same frequency. Under these conditions the force action of the currents flowing in the armature of the motor will always cause rotation in the same direction. This becomes obvious if we consider a conductor carrying its maximum positive current when directly under a north pole. The force action on this conductor is a maximum and has a definite direction, i.e., the direction in which the motor is running. Half a period later the current will have its maximum negative value, and the force will again be a maximum, but it would be in a reverse direction if it were not for the fact that while the current was changing from positive to negative the conductor was moving with respect to the poles at just the proper speed to bring it directly under a south pole when the current in the conductor became a maximum negative value, thus keeping the direction of the force the same as before. The speed at which the motor must run to develop a counter-e.m.f. of the same frequency as the supply is called the "synchronous speed." That the motor cannot run at anything but synchronous speed is also evident, for if it
should run at a slightly greater or less speed there would necessarily be some instant when the conductor carrying its maximum positive current would be under a south pole and the force action, being reversed, would stop the motor. When a synchronous motor stops for this reason it is said to have fallen out of synchronism.

While the synchronous motor runs at the speed required to generate a frequency equal to the frequency of the generator driving it, it is not necessarily true that it runs at the same speed as the generator, for the motor might not have the same number of poles as the generator. The speed of the motor will be the same as that of the generator only when the motor happens to have the same number of poles as the generator. The speed at which a synchronous motor will run when connected to an alternator supplying current at a frequency $f$, is

$$
s=\frac{2 \times f \times 60}{p}
$$

where $s$ is the speed of the motor in revolutions per minute; $f$ is the frequency of the alternating-current supply; and $p$ is the number of poles of the motor field. For example, if a 10 -pole motor were run from a 60 -cycle alternator, the speed of the motor would be

$$
s=\frac{2 \times 60 \times 60}{10}=720 \text { r.p.m. }
$$

Moreover, it follows that, if the generator had 10 poles, it too would run at $720 \mathrm{r} . \mathrm{p} . \mathrm{m}$. If the generator had 20 poles it would run at 360 r.p.m., or just half the speed of the motor it was driving. Thus the ratio of generator speed to motor speed is in inverse ratio to the number of their poles. Any change in the speed of the generator would change its frequency and therefore change the motor speed in a proportional manner. While the motor speed always bears a simple ratio to the generator speed, the cyclic speeds must be the same. That is to say, the angular velocity ( $\omega$ ) expressed in electrical radians per second is the same for both motor and generator, and is the same for all synchronous machines of the same frequency.

Synchronizing. The process of bringing a motor into synchronism with a generator is called "synchronizing." A simple
understanding of just what conditions must be met and what takes place in synchronizing can be obtained by considering both the motor and the generator to be single-phase, 2 -pole, revolvingarmature types having one slot per pole. Two machines of this construction are shown diagrammatically in Fig. 176, where $A$ represents the generator and $B$ the motor.

Consider the motor $B$ to be driven by a small engine or an auxiliary motor so that the following conditions are satisfied:
(1) The e.m.f. of $B$ is the sume as that of $A$.
(2) The frequency of $B$ is the same as that of $A$. This is equivalent to saying that their angular velocities in electrical radians per second are the same.


Fig. 176. Diagram Showing Principle of Synchronizing Motor and Generator
(3) A conductor $b$ of the motor $B$ is in the same relative position with respect to the center of the $N$ pole of $B$ as a similar conductor $a$ of the generator $A$ is with respect to the $N$ pole of $A$. This is equivalent to saying that the electromotive force of $B$ is in phase with that of $A$.

The first condition is obtained by adjusting the field current of $B$ until a voltmeter shows the voltage of $B$ to be the same as that of the generator $A$.

The second condition is obtained by adjusting the speed of the small engine or auxiliary motor driving the motor $B$ until the rapidity with which the lamp $L$ grows light and dark becomes very slow. The cause of the flickering of the light is that the voltage of $B$ is at one instant adding to and the next instant subtracting from the voltage of $A$. When the frequencies of $A$ and $B$ are exactly the same the light will not flicker at all, but may either stay bright or grow dark, depending on whether or not the third condition is satisfied.

The third condition is satisfied when the light is dark. When all of the stated conditions are satisfied it is evident that, as far as the circulation of current through the two armatures in series is concerned, the e.m.f. of $B$ is at every instant equal and opposite to that of $A$. Thus no current will flow through the lamp $L$ and none will flow through the armatures if the switch across the lamp is closed. The switch $S$ may then be closed and the motor will be in synchronism with the generator. With respect to the closed circuit the voltage relationships are given by Fig. 177. The machine driving the motor $B$ may now be disconnected, and $B$ will be driven electrically by generator $A$.

All single-phase synchronous motors require an outside source of power to bring them up to synchronous speed. This is not necessarily true of polyphase synchronous motors, although large


Fig. 177. Synchronized Motor and Generator
polyphase synchronous motors are nearly always brought up to speed the same as single-phase synchronous motors. Whenever a polyphase motor is synchronized, each phase must be made to meet the above-mentioned phase conditions.

Torque and Power Output. Suppose the synchronous motor $B$ in Fig. 176 to be running in synchronism with the generator $A$, and the auxiliary starting motor disconnected. The motor running idle is of no value. It must be able to develop a torque and deliver mechanical power. The torque of a synchronous motor is developed in the following manner: Suppose a mechanical load to be placed on the motor. The motor will try to stop and for a small interval of time will run at a less speed than the generator $A$, such that the conductor $b$ will fall behind the position corresponding to the conductor $a$ of generator $A$ by a small angle of
$\theta$ radians, and now occupies a position $b^{\prime}$ as shown in Fig. 178. This means that the voltage of $B$ is no longer opposite that of $A$, and the phase relationship is now expressed by Fig. 178.

Since the two voltages $A$ and $B$ do not oppose each other at every instant, there is a resultant voltage $e$ acting on the two armatures in series and causing a current of $I$ amperes to flow. This current lags behind the voltage $e$, which produced it, by approximately 90 degrees, because the impedance of the armatures is mostly reactance, the ohmic resistance being comparatively negligible. The further the rotor of $B$ falls behind that of $A$, or in other words the greater the angle $\theta$ becomes, the greater will $e$ and $I$ become. The rotor of $B$ will be retarded until the current, which flows as a result of the phase difference of the voltages $A$ and $B$, is large enough to overcome the retarding torque. The


Fig. 178. Voltage Conditions of Motor under Load
motor will then run at the same angular velocity as before but always keeping an angle $\theta$ behind its zero load position until the load is removed, when it will accelerate until it occupies the same relative position that it did before the load was put on.

Thus for every different torque or load there is a different relative position for the armature of the motor with respect to the armature of the generator. It is also evident that there is a limit to the load that can be placed on the motor, for there is a certain value for the angle $\theta$ beyond which the torque produced by the additional flow of current will be less, the greater the angle becomes, and therefore cause the motor to stop.

A given synchronous motor operating with given applied voltage is therefore capable of developing a definite maximum torque, or of delivering a definite maximum amount of power. An attempt to take more than this maximum power from the machine
causes it to fall out of step, that is, out of proper phase relation with respect to the supply voltage; and the motor, accordingly, stops. A properly designed synchronous motor will, however, carry a reasonable overload, perhaps 200 per cent, before reaching the above-mentioned maximum at which the machine stops.

Examples. 1. A synchronous motor B, connected across an 1100 -volt line, has an armature resistance of 1 ohm and a reactance of 5.98 ohms . In synchronizing, its field current is so adjusted as to make its induced voltage equal to the line voltage. Suppose the motor to be loaded to such a degree that its armature falls 30 degrees behind its zero load position. ( $\phi=30$ degrees) Find (a) the resultant voltage acting on the armature; (b) the armature current; (c) the angle between the armature current and the resultant voltage producing it; (d) the angle between the line voltage and armature current, $\theta_{a}$, and the power factor, $\cos \theta_{a}$; (e) the angle, $\theta_{b}$, between the armature current and the part of the line voltage which overcomes the motor induced voltage ( $-B$ ), and $\cos \theta_{b}$; (f) the power input to the motor; (g) the armature $R I^{2}$ loss; (h) the mechanical power developed at the armature conductors, or the electrical . power converted into mechanical power; (i) the efficiency of conversion.

Solution. (a) To calculate e.

$$
\begin{aligned}
& \left\langle\overline{e A}(\text { between } e \text { and } A)=\frac{180^{\circ}-\phi}{2}=\frac{180^{\circ}-30^{\circ}}{2}=75^{\circ}\right. \\
& e=B \frac{\sin \dot{\phi}}{\sin e A}=1100 \times \frac{.5000}{.9659}=569 \text { volts (law of sines) }
\end{aligned}
$$

(b) To calculate $I$.

$$
I=\frac{e}{\sqrt{R^{2}+X^{2}}}=\frac{569}{\sqrt{{1^{2}+\overline{5.98}}^{2}}}=94 \text { amperes }
$$

(c) To calculate $<\overrightarrow{e ̨ I}$.

$$
<\overline{e I}(\text { between } e \text { and } I)=\tan ^{-1} \frac{X}{R}=\tan ^{-1} \frac{5.98}{1}=80^{\circ} 30^{\prime}
$$

(d) Tu calculate $\theta_{a}$ and $\cos \theta_{a}$.
$\theta_{a}(<$ between $I$ and $A)=80^{\circ} 30^{\prime}-<\overline{e A}=80^{\circ} 30^{\prime}-75^{\circ}=5^{\circ} 30^{\prime}$
$\operatorname{Cos} \theta_{a}=\operatorname{Cos} 5^{\circ} 30^{\prime}=0.9954$
(e) To calculate $\theta_{b}$ and $\operatorname{Cos} \theta_{b}$. From Fig. 179

$$
\begin{gathered}
\theta_{b}(\text { <between } I \text { and }-B)=\phi-\theta_{a}=30^{\circ}-5^{\circ} 30^{\prime}=24^{\circ} 30^{\prime} \\
\operatorname{Cos} \theta_{b}=\operatorname{Cos} 24^{\circ} 30^{\prime}=0.9100
\end{gathered}
$$

(f) To calculate the power input.

$$
\text { Power input }=\frac{A I}{1000} \operatorname{Cos} \theta_{a}=\frac{1100 \times 94 \times .9954}{1000} \Rightarrow 102.9 \mathrm{kw} .
$$

(g) To calculate the armature $R I^{2}$ loss.

$$
\text { Armature copper loss }=\frac{R I^{2}}{1000}=\frac{1 \times 94^{2}}{1000}=8.8 \mathrm{kw}
$$



Fig. 179. Calculation of $\theta_{b}$
(h) To calculate the mechanical power developed at the armature conductors.
First method

$$
\begin{aligned}
\text { Mechanical power } & =\frac{B I \cos \theta_{b}}{1000} \\
& =\frac{1100 \times 94 \times 0.91}{1000}=94.1 \mathrm{kw}
\end{aligned}
$$

Second method

$$
\begin{aligned}
\text { Mechanical power } & =\text { input }-R I^{2} \text { loss } \\
& =102.9-8.8=94.1 \mathrm{kw} .
\end{aligned}
$$

(i) To calculate the efficiency of conversion.

Efficiency $=\frac{\text { output }}{\text { input }} \times 100=\frac{94.1}{102.9} \times 100=91.5$ per cent
2. Determine the armature current, line power factor, and mechanical power developed for different values of the angle $\phi$ in Example 1. Plot the results and determine the break-down load with the value of $\phi$ corresponding. (See Fig. 180.)
Solution. Procced as in Example 1.

| Values of $\phi$ | Armature Amperes | $\operatorname{Cos} \theta$ | Mech. HP. |
| :---: | :---: | :---: | :---: |
| 10 | 31.7 | .997 | 45.3 |
| 20 | 63.1 | 1.00 | 87.6 |
| 30 | 94.0 | .995 | 126.1 |
| 40 | 124.3 | .983 | 159.5 |
| 50 | 151.7 | .964 | 184.6 |
| 60 | 181.5 | .937 | 206.8 |
| 70 | 208.3 | .903 | 219.5 |
| 80 | 233.3 | .812 | 223.5 |
| 90 | 256.7 | .814 | 220.4 |

The motor develops its maximum power when $\phi=86$ degrees (approx.), and further increase of load will make the motor stop (fall out of synchronism).
3. If the synchronous motor in Example 1 had been synchronized with a 10 per cent less line voltage, what mechanical power would it develop ( $\phi=30$ degrees)? Express the result as a per cent of the former value.

Solution.

$$
\begin{gathered}
A=B=1100 \text { volts } \times(1.00-.10)=990 \text { volts } \\
\qquad e=990 \times \frac{.5000}{.9659}=512 \text { volts }
\end{gathered}
$$

From Example 1, (a)

$$
I=\frac{e}{Z}=\frac{512}{\sqrt{1^{2}+5.98^{2}}}=84.6 \text { amperes }
$$

Mechanical power $=B I \cos \theta_{b}=990=\times 84.6 \times .910076 .2 \mathrm{kw}$.
Expressed as per cent of its former value in Example 1, it is-

$$
\text { Mechanical power }=\frac{76.2}{94.1} \times 100 \text { per cent }=81 \text { per cent }
$$



Fig. 180. Angular Displacement in Electrical Degrees of the Motor Rotor
The maximum power that can be delivered by a synchronous motor is greatly increased by increasing the applied voltage, and greatly decreased by decreasing the applied voltage. In fact the maximum power output is proportional to the square of the
applied voltage; therefore the voltage of supply should never be allowed to fall much below the normal or rated voltage.

Hunting Action. Supposing the load in the preceding discussion to be suddenly thrown on the motor, the motor would slow down momentarily and fall behind the generator in phase. When the motor has fallen behind sufficiently to take in power enough to enable it to carry its load, it is still running slightly below synchronism; it therefore falls still further behind, and takes an excess of current from the generator which quickly speeds it above syn. chronism. It then gains on the generator in phase until it is taking less current than is required for its load, when it again slows down, and so on. This oscillation of speed above and below synchronism, called hunting, is accompanied by great changes in the current supplied to the synchronous motor, and by rapid rise and fall of e.m.f. between the terminals of the motor. It is frequently a source of great annoyance, especially where several synchronous motors, or rotary converters, are run in parallel from the same mains.

Hunting is frequently produced by the periodic changes in the speed of the engine which drives the generator. Thus the engine momentarily increases its speed as the steam acts upon the piston at each stroke, and diminishes its speed at intervals between the strokes.

The hunting of a synchronous motor is a phenomenon of the same nature as the hunting of a steam engine having an oversensitive governor. When the load on the engine is suddenly increased the engine slows down momentarily, causing the governor to admit more steam than is needed for the increased load. The result is that the engine quickly speeds up, causing the oversensitive governor to shut off too much steam, so that the engine slows down again, and so on. Hunting is associated with more or less violent shifting of the resultant magnetic flux in the air gap under the poles. This is due to the fluctuations in the armature current, caused by the flow of "corrective" current between the generator and motor when hunting occurs.

Reduction by Use of Dampers. Hunting is greatly reduced or entirely eliminated by the use of heavy copper frames or dampers in the neighborhood of the poles. One form of damper is shown
-in Fig. 181, which is a view of a portion of the revolving field of an alternator. The dampers consist of rectangular copper frames driven into place under the overhanging tips of two adjacent poles. A damper is provided between each pair of adjacent poles, all around the field, both in the alternator and in the synchronous motor.

Another form of damper that has been found very effective is called the "squirrel-cage" damper. Heavy bars of copper are placed in slots at the surface of the poles and their ends bolted to two closed copper rings so as to short-circuit all the bars. This cage damper applied to a rotor for use as a synẹhronous motor is illustrated in Fig. 182, which relates: to a 200-kilowatt Type E machine of the Westinghouse Company.

The principle of damping action is that the shifting mag-


Fig. 181. Portion of Revolving Field Showing netic field sets up induced currents in the short-circuited frames or bars of the damper, and these currents react on the magnetic field so as to oppose the shifting of the flux and thereby dampen the hunting oscillations. The electrical effect of dampers is analogous to the mechanical effect produced by immersing the bob of a swinging pendulum in a heavy oil which resists the motion of the bob.

Flywheels are sometimes used to dampen the hunting action, their effect being to keep the speed more uniform and make it less susceptible to sudden changes caused by the throwing off and on of load.

Self-Starting of Polyphase Synchronous Motor. As stated before, no auxiliary motor is needed to bring the polyphase synchronous motor up to speed. It will start itself if connected to the line without exciting its field. This starting torque is fairly large in value and depends for its action upon two things, namely, the production of a rotating magnetic field by the polyphase cur-
rents in the armature and the presence of short-circuited copper bars placed in slots in the pole faces of the field structure, just the same as the bars introduced for the purpose of damping the hunting action. As a matter of fact these bars perform both functions.

Production of the Rotating Magnetic Field. The production of a rotating magnetic field by the polyphase currents in the armature is very easily understood by considering the instantane-


Fig. 182. Rotor of a Synchronous Motor Provided with "SquirrelCage" Form of Damper
ous magnetic effects of these currents in the armature. For example, take a two-pole, two-phase, rotating-field type of synchronous motor, having one slot per pole per phase. Fig. 183 illustrates this structure. Being a two-phase machine the currents in phase $A$ are displaced a quarter of a cycle ( 90 degrees) from the currents in phase $B$. Let us assume that $A$ lags behind $B$ when the positive directions of currents in the armature are as
indicated by the conventional signs in the diagram, Fig. 183. Consider the separate and combined effects of the currents $A$ and $B$ at each of the instants indicated. At the instant $1, A$ has its maximum negative value and produces a magnetic field in the direction indicated by the arrow 1 . At the same instant the current in $B$ is zero and it produces no magnetic field. At the instant 2, one-eighth of a cycle ( 45 degrees) later, $A$ is negative and $B$ is positive, both being 0.707 of their maximum value. The current $A$ produces a horizontal magnetic field 0.707 of its maximum strength, while $B$ produces a vertical field 0.707 of its maximum strength. The combined effect of these two fields is a field of maximum strength and in the direction indicated by


Fig. 183. Two-Phase Two-Pole Synchronous Motor and Magnetic Field Produced by Polyphase Current
arrow 2. If the separate and combined effects of the two-phase currents $A$ and $B$ are considered for the other instants $3,4,5,6$, 7 , and 8 , it will be seen that the magnetic field is always of the same strength but changes in direction, making one complete revolution in a period just the same as if north and south poles, located diametrically opposite each other, were rotated at synchronous speed about the region occupied by the rotating magnetic field. In the case considered the field rotates counterclockwise. If one of the phases be reversed so that the current in phase $A$ leads the current in phase $B$, the rotation will be in just the opposite direction, or clockwise. The direction of rotation of polyphase synchronous motors, which enables them to be synchronized with a polyphase generator, is reversed by the reversal
of one phase of its armature winding. This is true for three-phase as well as two-phase motors.

Starting Torque. Suppose the rotating field structure of the above motor, with its short-circuited copper bars, is placed in the region occupied by the rotating magnetic field. The field poles, which under running conditions are excited with direct current, are not excited when the machine starts. The rotating magnetic field cuts across the short-circuited copper bars inducing currents in them which oppose the motion of the field, with the result that a torque is produced on the bars, causing the field structure to rotate in the direction of the rotating magnetic field. In order to get the most favorable value of starting torque the resistance of the bars should be large in comparison with their reactance, so that the induced currents are more nearly in phase with the induced voltage; in other words, the currents should be strongest where the rotating field is strongest.

Pull=In Torque. When the synchronous motor has been brought up to the highest speed possible by the inherent inductionmotor action, it still does not run at synchronous speed. There must necessarily be a difference in speed between the rotating magnetic field and the short-circuited copper bars placed in the field poles, in order that currents may be induced in these bars. This difference in speed is called the slip. For example, a 720 r.p.m. polyphase synchronous motor might be brought up to only 690 r.p.m. by inherent induction-motor action. Upon reaching the highest speed possible, the field must be excited and the rotor is then immediately pulled into synchronism by the force action between the revolving field and the definite magnetic-field poles of the motor. The amount of load in pounds at one foot radius which the motor can pull into step from slip speed to synchronous speed is called the "pull-in" torque.

High-resistance, short-circuited copper bars give a high starting torque but a large slip and therefore a small pull-in torque. The pull-in period is the critical instant in the starting of polyphase synchronous motors and it is generally the load that can be pulled into synchronism that determines the starting torque. Synchronous motors have been designed to start under from 30 to 50 per cent of full-load torque.

Starting Compensators. The objection to this mode of starting is that the machine takes excessively large lagging currents at starting; and this. generally causes a drop in the supply voltage great enough to disturb seriously the general system of distributing mains from which the synchronous motor receives its currents. This excessive demand for current at starting is objectionable when the motor takes a large portion of the generator output, or is used in connection with a lighting service, especially when the motor is started and stopped at frequent intervals.

To avoid an excessive demand for current at starting, an autostarter or compensator is frequently used. The starting compensator for a two-phase synchronous motor consists of two transformers (three transformers for a three-phase machine) having their primaries connected across the respective phases of the supply mains, their secondaries being provided with a number of taps so that, at starting, a fraction of the full supply voltage can be applied to the armature terminals of the synchronous motor. This fraction is usually from 40 to 60 per cent of the full voltage; and a switching device is provided by means of which the change from fraction to full voltage can be quickly made when the synchronous motor reaches full speed. This starting compensator is also used in connection with induction motors. The transformers used in the starting compensator are always autotransformers.

Excitation. At the time of starting, the armature and field windings of a synchronous motor are related to each other as are the primary and secondary of an alternating-current transformer. The result is that when the field coils have many turns of wire a dangerously high e.m.f. may be induced in them.

This production of high voltages in the field coils of a selfstarted polyphase synchronous motor may be to a great extent avoided by using few turns of large wire in the field winding, thus necessitating the use of a low-voltage exciter. For this reason exciters giving an e.m.f. as low as 50 volts are frequently used. Another method of obviating the danger referred to is to provide short-circuited metal rings around the field poles. These rings limit the changes in magnetism in the pole pieces and thereby prevent the formation of excessively high induced voltages in the field coils.

In synchronous motors of the stationary-field type the field circuit may be broken up into many separate parts so as to divide the induced e.m.f. Thus Fig. 184 shows the stationary field of an alternator (synchronous motor) with the terminals of each field spool brought out to convenient switches on the frame of the machine. During starting these switches are open, and when the machine has reached synchronous speed they are all closed, thus connecting all field spools in series to the exciter.

Field Excitation and Power Factor. While the power factor of a non-synchronous (induction) alternating-current motor is fixed by its design, and its current is always lagging behind the


Fig. 184. Diagrammatic View of Stationary Field of a Synchronous Motor
applied e.m.f., the current delivered to a synchronous motor may be made either lagging or leading at will. This remarkable control of phase of the current is accomplished by varying the strength of the field excitation, and is illustrated by the vector diagrams in Figs. 185, 186, and 187.

The resultant voltage $e$, the vector sum of the line voltage $A$ and the induced voltage $B$ of the motor, in the case of all three figures is the voltage which causes the armature current $I$ to flow. The armature current $I$ lags almost 90 degrees behind the resultant voltage $e$ because the armature impedance is mostly reactance. A change in the voltage $B$ changes the relative position of $e$ with respect to the line voltage $A$ and also changes the relative position
of $I$ with respect to the line voltage, making the phase angle $\theta$ and the power factor of the line almost anything desired.

Fig. 185 shows that an increase in the excitation of a synchronous motor will cause the line to deliver a leading current. Fig. 186 shows the opposite effect. When the synchronous motor is under-excited it causes a current to flow which lags behind the line voltage. By properly adjusting the motor field excitation the line current and voltage may be brought in phase with each other, making the power factor of the line unity and at the same time obtaining a minimum current for a given power input to the


Fig. 185. Effect of Motor Over-Excited, Current Leading


Fig. 186. Effect of Motor Under-Excited, Current Lagging


Fig. 187. Effect of Normal Excitation, Power-Factor Normal
motor. This is shown in Fig. 187. Under these conditions the efficiency of transmission will be a maximum because the $R I^{2}$ loss , will be a minimum.

Considered simply as a motor, without reference to the transmission system as a whole, the most efficient point of operation is with unity power factor, or, in other words, with field excitation which will make the armature current a minimum.

The effect upon the armature current, produced by varying the field excitation, is shown by the curves in Fig. 188. Up to a certain point, as the excitation is increased, the armature current
is lagging, and decreases to a minimum value. Further increase of the exciting current causes the armature to take more current, which is now ahead of the applied e.m.f. in phase, that is, is now leading. There is one value of exciting current for which the


Fig. 188. Curve Showing Variation of Armature Current with Field Excitation armature current is a minimum. In motors of good regulation this value of the exciting current varies but slightly with different loads. Use As a Condenser. A synchronous motor with its field magnet over-excited takes. a current which is ahead of the applied voltage in phase. Such a machine may, therefore, be connected across the terminals of an inductive receiving circuit, as shown in Fig. 189, so as to compensate for the lagging current delivered to the receiving circuit, thus reducing the line current to the lowest value that will suffice to transmit the power taken by the receiving circuit. Fig. 190 shows how an over-excited synchronous motor, running idle, takes a current which leads the line voltage by almost 90 degrees, and in this respect acts like a condenser.


Fig. 189. Diagram Showing Synchronous Motor Used as a Condenser
In Fig. 191 is shown by means of a vector, or clock diagram, how the leading current $I_{2}$ taken by the synchronous motor $M$ in Fig. 189 gives, when combined with the current $I_{1}$ delivered to the receiving circuit, a resultant line current $I$, which is in phase with the e.m.f. $E$ between the mains.

A synchronous motor used primarily for compensating the lagging current delivered to a receiving circuit is called a rotary condenser or a synchronous compensator. The rotary condenser is especially useful when induction motors or lightly loaded transformers or both are supplied over a long transmission line. In such cases the reduction of the line current to the smallest possible value effects considerable saving in the matter of power losses in the transmission line. The use of a rotary condenser is also an advantage in that the regulation of the voltage at the receiving end of the transmission line is improved.


Fig. 190. Effect of Over-Excitation on Synchronous Motor
In making calculations to determine the rating of a synchronous motor required to correct for low power factor, the following fundamental points should be borne in mind:
(1) The total current of a receiving circuit may either lead or lag behind the voltage, depending on whether the circuit contains capacity or inductance.
(2) The current of any receiving circuit is made up of two component currents, one in phase with the applied voltage and available for doing work, and a second either 90 degrees ahead of or 90 degrees behind the applied voltage and not available for doing work. The former is called the power component and the latter the reactive component of the current.
(3) The term "power factor" serves to indicate the amount by which the current leads or lags behind the voltage and may be
expressed as a ratio of the power current to the total current, or, what is equivalent, the ratio of kilowatts (kw.) to kilovolt-amperes (kv.-a.).
(4) The total or combined current of two or more parallel circuits is the "vector" or "geometric" sum of the currents in each branch circuit. The total power factor of the combined circuits, therefore, depends on the respective components of the currents in each branch. Hence, to


Fig. 191. Clock Diagram of Current Relations in Synchronous Motor determine the resultant power factor, the respective branch currents are resolved into their power and reactive components, these are then added to give the total of each component, and finally the resultant current is determined by taking the square root of the sum of the squares of the total power and reactive components. The resultant power factor then is the ratio of the power component of the total current to the total current.

Example. A 550-volt, single-phase line delivers a lagging current of 100 amperes to a receiving circuit of 80 per cent power factor. A synchronous


Fig. 192. Calculation of Leading Current motor is connected in parallel with the receiving circuit and the total power factor increased to 100 per cent. Determine (a) the reactive and leading component of the synchronousmotor current; (b) the kv.-a. rating of the motor if it requires no current to drive it; (c) the kv.-a. rating of the motor if at the same time it is delivering a load such that the power component of its current is 80 amperes.
Solution. (a)

> Power component of $I=100 \times .80=80$ amperes reactive component of $I=\sqrt{100^{2}-80^{2}}=60$ amperes

The synchronous motor must take a leading current $I_{c}$ equal to the lagging reactive current of 60 amperes (Fig. 192).
(b) Kv.-a. rating of the synchronous motor running idle.

$$
\mathrm{kv} .-\mathrm{a} .=\frac{550 \text { volts } \times 60 \mathrm{amperes}}{100}=33 \mathrm{kv} .-\mathrm{a} .
$$

(c) Kv.-a. rating of the synchronous motor when delivering a load in addition.

$$
\text { Total current }=\sqrt{(\text { power component })^{2}+(\text { reactive component })^{2}}
$$

$$
\begin{aligned}
& =\sqrt{\overline{80}^{2}+\overrightarrow{60}^{2}=100 \text { amperes }} \\
\text { kv.-a. rating } & =\frac{550 \text { volts } \times 100 \text { amperes }}{1000}=55 \mathrm{kv} .-\mathrm{a} .
\end{aligned}
$$

A synchronous motor may deliver a considerable portion of its rated mechanical output and at the same time supply a large


Fig. 193. Ratio of Reactive Power to True Power
corrective current without overheating its armature. For example, a synchronous motor taking a total current of 100 amperes could have a power component of current equal to 70.7 amperes and at the same time supply 70.7 amperes of leading reactive current.

$$
\left(100=\sqrt{\left.(70.7)^{2}+(70.7)^{2}\right)}\right.
$$

In the case of polyphase circuits it is sometimes more convenient to consider the kv.-a. of the receiving circuit as divided into true power, kw., and reactive power, where

$$
\mathrm{kv} .-\mathrm{a} .=\sqrt{(\text { true power })^{2}+(\text { reactive power })^{2}}
$$

In making computations on this basis it is very convenient to know the ratio of reactive power to true power or reactive power to kv .-a. for different values of power factor. These ratios are plotted in Fig. 193.

Example. A 500-hp., 2200-volt, three-phase induction motor has an efficiency of 90 per cent and an average power factor of 80 per cent. Determine (a) the kv.-a. rating of the induction motor; (b) the reactive power delivered; (c) the power factor of an over-excited synchronous motor which will compensate for the reactive component of the current and at the same time deliver a mechanical load such that its real or true power input is 300 kw ; (d) the kv.-a. rating of the over-excited synchronous motor.

Solution. (a)

$$
\begin{aligned}
\text { Kv.-a. rating } & =\frac{\mathrm{kw} . \text { output }}{\text { efficiency }} \times \frac{1}{\text { power factor }} \\
& =\frac{500 \times .746}{.90} \times \frac{1}{.80}=518 \mathrm{kv} .-\mathrm{a} .
\end{aligned}
$$

(b) From curve No. 1 the ratio of reactive power to kv.-a. for 80 per cent power factor $=.60$
Therefore
reactive power taken by the induction motor $=\mathrm{kv} .-\mathrm{a} . \times .60$

$$
=518 \times .60=310.8 \mathrm{kv} \cdot-\mathrm{a} .
$$

(c) The synchronous motor must furnish the reactive power of the induction motor by taking a leading current.

$$
\text { The ratio of reactive power to } \mathrm{kw} .=\frac{310.8}{300.0}=1.036
$$

and from curve No. 2 the corresponding power factor is 69.5 per cent.
(d) From curve No. 1 corresponding to a power factor of 69.5 per cent the ratio of reactive power to $\mathrm{kv} .-\mathrm{a}$. is . 719 .

$$
\mathrm{kv.-a} .=\frac{\text { reactive power }}{.719}=432 \mathrm{kv} .-\mathrm{a} .
$$

Advantages. The synchronous motor, especially in units of large output, possesses a number of features which make its use at times preferable to that of the induction motor. Its advantages . may be briefly summed up as follows:
(1) Unvarying speed at all loads.
(2) Power factor, variable at will by change of exciter current, can be made approximately unity at any load.
(3) The current in the armature can be made to lead the e.m.f. by over-exciting the field magnets, thus producing the same effect as a large condenser. The leading current in the armature can be made to neutralize the unfa'rorable effects of inductance (which causes lagging currents) in other parts of the system.
(4) The synchronous motor is cheaper to build, especially for low speeds, than the induction motor.
(5) Its efficiency is generally higher than that of the induction motor.
(6) It is especially adapted to high-voltage winding.

Disadvantages. The synchronous motor, on the other hand, has several disadvantages, as follows:
(1) It is not adapted to work requirin 0 .aiable speeds, as no independent speed regulation is possible.
(2) Its starting torque is only a fraction of its full load torque; hence it is not suitable for work requiring large starting torque, or frequent starting of the load.
(3) It has a tendency to "hunt."
(4) It requires an exciting current which must be supplied by an outside source.
(5) It requires the most skillful and intelligent attention.

Applications. Synchronous motors are used where power is required in large amounts, and where the motor does not have to be started and stopped frequently.

For work requiring constant and low speeds, large units of 100 hp . or more have been direct-connected to the following classes of machines: compressors, rolls, conveyors, crushers, fans, grinders, mixers, tube mills, line shafts, etc.

For work requiring constant but higher speeds, synchronous motors have been direct-connected to pumps, blowers, fans, frequency changers, motor-generator sets, etc.

In regard to power factor correction it has been used in three ways: as a synchronous condenser operating at leading power factor; as a synchronous motor taking a mechanical load and operating at unity or slightly leading power factor; or as a com-
bination of the two, that is, taking a load and correcting for lagging power factor at the same time.

## MOTOR TESTING

The difference between a synchronous motor and an alternating. current generator consists mainly in the method of operating; ans alternator will run as a synchronous motor, and vice versa. Therefore, all the tests described on pages 170 to 203, with reference to alternators, may be applied in a similar manner to synchronous motors.

In making a heat test on a small synchronous motor, it is usually run at full load as a motor.

In the case of large synchronous motors, the "heat-run" or test may be made by rumning the machine as a generator on short-circuit, with a portion of its field coils connected in opposition, as described on page 192 on the heat test of alternators. The "intermittent snor-circuit and open-circuit" method described in the case of an alternator on page 195 may be applied to the synchronous motor also.

The efficiency of a synchronous motor is calculated in the same manner as that of an alternator, page 201.

Phase Characteristic. Regulation for alternators, as described on page 182, is not calculated in the case of a synchronous motor; but the determination of the "phase characteristic" of a synchronous motor corresponds to, and is substituted for, the regulation test. A "phase characteristic" is a curve showing the relation between the armature current and the field current of a synchronous motor, the test being carried out under constant voltage, frequency, and load conditions. Phase characteristics are shown in Fig. 188.

Phase characteristics are usually taken at no load, although it not infrequently happens that it is desired to obtain a phase characteristic at full load. To make the test at no load, it is simply necessary to run the motor unloaded, supplying alternating currents of the proper voltage and frequency to the armature terminals of the motor. The field current is then varied by successive steps, both above and below its normal value, until the armature current has attained rated full-load value, both for leading and for lagging current. For each value of the field current, simultaneous observations are made of the amperes in the armature, the volts supplied
to the armature terminals, the amperes in the field, and the speed. The normal field current is that value for which the armature current has the lowest value.

If a phase characteristic at full load is desired, the most convenient way of loading the motor is to cause it to drive a directcurrent generator of known efficiency, by means of a belt connecting their respective pulleys. The power output of the direct-current generator can be accurately measured by means of an ammeter and a voltmeter. Knowing the output, and the efficiency of the direct-current generator at any output, the mechanical input, to the generator, that is, $\frac{\text { output }}{\text { efficiency }}$, can be calculated. This input to the generator is evidently equal to the mechanical output of the synchronous motor, the power lost in the belting being negligible.

The load on the motor must be kept constant throughout the test. The armature current for a full-load phase characteristic cannot be varied through such a wide range of values as at no load, owing to the inability of the armature windings of the motor to carry the excessive current without over-heating.

Pulsation Test. This test is to determine whether or not the synchronous motor has a decided tendency to hunt. For this test the synchronous motor is supplied with alternating current or currents from a very steadily driven alternator over an "artificial transmission line." This "artificial line" consists simply of a resistance equal to the resistance* of the transmission line over which the synchronous motor is to be supplied with current when finally installed.

The synchronous motor is driven at zero load, taking current through the artificial line, the tendency to hunt being greatest at zero load. The connections are the same as in the test for phase characteristic. The hunting action is indicated by the swinging to and fro of the pointers of the ammeters and voltmeters. The field current of the motor is varied from considerably below to considerably above rated full-load field current. For each value of the field current, the indications of the instruments are carefully observed; the pulsations of the instruments are noted.

[^20]Note. During this test, care must be taken that the alternator supplying the power to drive the motor under test, does not pulsate, for, if it does, the pulsations of the instrument pointers would not then give a reliable indication of the performance of the motor itself under normal working conditions.

Break=Down Test. As its name implies, this test is to determine the maximum power that a synchronous motor will deliver at its pulley, before falling out of synchronism and stopping. As in the case of the full-load phase characteristic, the power output of the motor is most conveniently absorbed and measured by belting the motor to a direct-current generator, and measuring the electrical output of this generator. For further description and details of the break-down test, the reader is referred to the article on the breakdown test for induction motors.

Self=Starting Test. The object of a starting test on a synchronous motor is to determine:
(a) The voltage and current required to start the motor.
(b) The time required for the motor to reach synchronous speed (synchronism).
(c) The electromotive force induced in the field-magnet windings at the instant of starting.

The starting test is made on polyphase motors only, for, as previously stated, single-phase synchronous motors are not inherently self-starting, but must, even in the smaller sizes, be provided with special starting devices.

The synchronous motor to be tested is connected to mains supplying alternating currents of the proper frequency. Arrangement is made to adjust the voltage applied to the armature terminals of the motor by means of potential (voltage) regulators connected in the armature circuits between the supply mains and the armature terminals. An ammeter is connected in series with each phase winding of the armature, that is, two ammeters are needed for a two-phase motor, and three anmeters for a three-phase motor. A voltmeter is connected to the terminals of the secondary coil of a small step-down potential transformer, the primary coil of which is connected to the terminals of the field winding of the motor. A voltmeter is also connected between the supply mains.

The test is made with no current in the field and the field circuit open.* The voltage applied to the armature terminals is slowly

[^21]increased until the motor starts to revolve. At the instant that the motor starts, the readings of all the instruments are recorded as follows: volts between armature terminals; amperes in the armature (each phase); and volts induced in field windings.

After the motor starts, the voltage applied to the armature terminals is kept constant; and the time required for the motor to attain synchronous speed, reckoned from the instant it starts, is observed. The exact instant that synchronism is reached, is indicated by violent swings of the pointers of the ammeters as well as those of the voltmeters connected to the armature terminals. At synchronism, another set of observations is taken.

In order to find the most unfavorable position of the armature and the corresponding starting current, and the time required to reach synchronous speed from this most unfavorable position, the above procedure is repeated for a series of initial positions of the armature, chosen as follows:

The circumference of the armature between the centers of two adjacent field poles is divided into a number of equal parts, this number not being a multiple of the number of phases, nor of the number of slots per pole per phase; usually there are seven parts. The starting positions thus chosen will include every possible position that a magnet pole may have relative to an armature slot. These parts are marked by chalked lines, and the various starting positions of the armature relative to the field-magnet poles are determined by setting each of the marks in succession into coincidence with a given field pole tip.


# ALTERNATING-CURRENT MACHINERY 

PART IV

## TRANSFORMER

Description. The transformer consists of two separate and distinct coils of wire insulated from each other, and wound upon one and the same laminated iron core. Fig. 232, page 272, shows a sectional view of a commercial type of transformer. In practice, one of the coils receives alternating current from a high (or low) voltage source of supply; and the other coil delivers alternating current to a receiving system at a low (or high) voltage. When the transformer receives alternating current at high voltage and delivers it at low voltage, we have what is called step-down transformation; when the transformer receives alternating current at low voltage and delivers it at high voltage, we have what is called step-up transformation.

The coil of a transformer which receives alternating current from a source of supply, is called the primary coil; and the coil which delivers alternating current is called the secondary coil. In Fig. 232, each limb of the core is wound with half of the secondary coil (coarse wire) next to the core, and with half of the primary coil (fine wire) over the secondary.

The alternator, as we have already seen, is a machine in which an alternating electromotive force is produced by the cutting of a permanently established magnetic flux by wires on account of the motion of the flux relative to the wires.

The transformer, on the other hand, is an arrangement whereby an alternating electromotive force is produced in a stationary coil of wire (secondary) by reversals of magnetic flux through a stationary iron core, these reversals of flux being produced by alternating current supplied to the primary coil of the transformer.

Physical Action. Without Load. When the secondary of a transformer is on open circuit, it can of course, deliver no current; and the transformer is said to be operating at zero load. Under these conditions, only a small amount of alternating current flows through the primary coil. This current causes repeated reversals of magnetic flux through the iron core. These reversals of magnetic flux induce electromotive forces in both coils. The electromotive force thus induced in the secondary coil is opposite in direction, and very nearly equal, to the electromotive force applied to the primary coil. Only the difference between the applied electromotive force and the opposing induced electromotive force is available for producing current through the primary coil; and since this difference is small, the primary current is small at zero load. The primary current at zero load is called the no-load current of the transformer.

I'ith Load. Whether the secondary coil of a transformer is delivering current or not, the reversals of magnetic flux in the transformer core always induce an alternating electromotive force in the coil. When alternating current is taken from the secondary of a transformer, the transformer is said to be loaded. The action of this secondary current as it flows through the secondary coil, is to oppose the magnetizing action of the slight current already flowing in the primary coil, thus decreasing the maximum value reached by the alternating magnetic flux in the core, and thereby decreasing the induced electromotive forces in both coils. The amount of this decrease, however, is very small, inasmuch as a very small decrease of the induced electromotive force in the primary coil greatly increases the difference between electromotive force applied to the primary coil and the opposing electromotive force induced in the primary coil, so that the primary current is greatly increased. In fact, the increase of primary current due to the loading of the transformer is just great enough (or very nearly) to exactly balance the magnetizing action of the current in the secondary coil; that is, the flux in the core must be maintained approxinately constant by the primary current whatever value the secondary current may have.

Electromotive Force Relations. The electromotive forces induced in the respective coils of a transformer are proportional to the number of turns of wire in each; and from the above discussion, it is evident that the electromotive force induced in the primary coil
is seusibly equal to the supply electromotive force, whether the transformer is loaded or not. Therefore

$$
\begin{equation*}
\frac{E^{\prime}}{E^{\prime \prime}}=\frac{N^{\prime}}{N^{\prime \prime}} \tag{32}
\end{equation*}
$$

in which $E^{\prime}$ is the electromotive force applied to the primary, $E^{\prime \prime}$ is the electromotive force at which the secondary coil delivers alternating current, $N^{\prime}$ is the number of turns of wire in the primary coil, and $N^{\prime \prime}$ is the number of turns of wire in the secondary coil.

Current Relations. The magnetizing action of the primary current $I^{\prime}$ of a transformer having $N^{\prime}$ turns of wire in its primary coil, may be expressed by the product $N^{\prime} I^{\prime}$, that is, by ampere-turns; and similarly, the magnetizing action of the secondary current $I^{\prime \prime}$ may be expressed by the product $N^{\prime \prime} I^{\prime \prime}$, where $N^{\prime \prime}$ is the number of turns of wire in the secondary coil. Therefore, since the magnetizing actions of the two coils are equal (and opposite), we have

$$
N^{\prime} I^{\prime}=N^{\prime \prime} I^{\prime \prime}
$$

or

$$
\begin{equation*}
\frac{I^{\prime}}{I^{\prime \prime}}=\frac{N^{\prime \prime}}{N^{\prime}} \tag{33}
\end{equation*}
$$

in which $N^{\prime}$ and $N^{\prime \prime}$ are the turns of wire in the respective coils; $I^{\prime \prime}$ is the current delivered by the secondary coil; and $I^{\prime}$ is the increase of current taken by the primary coil over and above the no-load current, due to the fact that the secondary coil is delivering current.

Now in most commercial transformers the no-load current is quite small; and, neglecting this current entirely, the only current in the primary coil would be the increase of primary current due to the fact that the secondary coil is delivering current. Therefore, equation (32) expresses, with sufficient accuracy for most purposes, the relation between the actual primary current $I^{\prime}$ and the secondary current $I^{\prime \prime}$.

Summary of Electromotive Force and Current Relations. A transformer which delivers current $I^{\prime \prime}$ to a receiving circuit, takes an amount of current equal to $I^{\prime \prime} \times \frac{N^{\prime \prime}}{N^{\prime}}\left(=I^{\prime}\right)$ from the source of supply. The electromotive force of the source of supply is $E^{\prime}$, and the electromotive force at which the secondary delivers current is equal to $E^{\prime} \times \frac{N^{\prime \prime}}{N^{\prime}}\left(=E^{\prime \prime}\right)$.

Example. A certain transformer rated at $5 \frac{1}{2}$ kilowatts has 660 turns of wire in its primary coil and 66 turns of wire in its secondary coil. The primary coil is connected between 1,100 -volt supply mains. Therefore, the secondary electromotive force, by equation (32) is

$$
E^{\prime \prime}=\frac{N^{\prime \prime}}{N^{\prime}} \times E^{\prime}=\frac{66}{660} \times 1,100=110 \mathrm{volts}
$$

If ten lamps, each taking half an ampere, are connected to the secondary coil, the secondary current will be 5 amperes; and the primary current, by equation (33) will be

$$
I^{\prime}=\frac{N^{\prime \prime}}{N^{\prime}} \times I^{\prime \prime}=\frac{66}{660} \times 5=.5 \mathrm{ampere}
$$

The power delivered to the lamps by the secondary coil is equal to $E^{\prime \prime} I^{\prime \prime}$ since the lamp circuit is non-inductive. That is:

Power delivered to lamps $=110$ volts $\times 5$ amperes $=550$ watts.
The power delivered to the primary coil in this case (non-induetive secondary circuit), is equal to $E^{\prime} I^{\prime}$. That is:

Power delivered to primary $=1,100$ volts $\times 0.5$ amperes $=550$ watts.
If 100 lamps, each taking half an ampere, are connected to the secondary coil, the secondary current will be 50 amperes, and the primary current will be 5 amperes; the power delivered to the lamps will be 5,500 watts, and the power delivered to the primary coil will also be 5,500 watts.

The above calculations ignore the following actions which take place in an actual transformer: (a) losses of electromotive force in overcoming the resistances of primary and secondary coils; (b) losses of power ( $I^{2} R$ ) in the primary and secondary coil; and (c) loss of power in the iron core, due to hysteresis and eddy currents.

Automatic Action of the Transformer. When the load on a transformer is increased, the primary of the transformer automatically takes additional current and power from the supply mains in direct proportion to the load on the secondary. When the load on the secondary is reduced, for example, by turning off lamps, the power taken from the supply mains by the primary coil is automatically reduced in proportion to the decrease in the load. This automatic action of the transformer due to the balanced magnetizing action of the primary and secondary currents is illustrated in the above example.

Ideal and Practical Transformer. The foregoing discussion of electromotive force and current relations in a transformor is based upon the following assumptions:
(a) That the no-load current of the transformer is negligible, and that it represents no power taken from the supply mains; or, in other words, that eddy-current and hysteresis losses are absent.
(b) That the resistance of the coils is negligible, so that the electromotive force applied to the primary coil is wholly balanced by the opposing electromotive force in the primary coil, and so that the whole of the electromotive force induced in the secondary coil is available at the terminals of that coil.
(c) That all the magnetic flux which passes through the primary coil passes through the secondary coil also; or, in other words, that there is no magnetic leakage.

A transformer that would meet these conditions would be an ideal transformer. A well-designed transformer operating on moderate load does approximate quite closely to the ideal transformer in its action; and equations (32) and (33) are much used in practical calculations. For some purposes, however, it is desirable to consider the action of the transformer, taking account of coil resistances, of eddy currents and hysteresis, and of the fact that some lines of magnetic flux pass through one coil without passing through the other (magnetic leakage). The extent to which a well-designed transformer deviates from an ideal, is exemplified by the following actual results obtained with the $5 \frac{1}{2}$-kilowatt transformer used in the example on the preceding page.

At no load, the value of $E^{\prime \prime}$ is 109.8 volts; the no-load current is 0.129 amperes; and the power taken from the mains by this no-load current (core loss) is 100 watts. This core loss is nearly constant at all loads.

When 100 lamps, taking 50 amperes of current, are connected to the secondary, then $E^{\prime \prime}$ is 107.2 volts. The $I^{2} R$ loss in the primary coil is 65 watts; and the $I^{2} R$ loss in the secondary is 65 watts. Therefore, the power delivercd to the lamps is 5.36 kilowatts; the power taken from the supply mains is $5,360+100+65+65$, which is equal to 5,590 watts; and the full-load efficiency of the transformer is 96 per cent.

Maximum Core Flux. In the designing of transformers and in the predetermination of core loss, it is necessary to calculate the maximum value reached by the alternating magnetic flux through the transformer core. This maximum value of the core flux may be easily and accurately calculated in the following manner:

Let $\Phi$ be the maximum value of the core flux (equal to the product $B A$ of maximum flux density and sectional area of the core) $E$ the effective value of the electromotive force applied to the primary coil, $N$ the number of turns of wire in the primary coil, and $f$ the frequency of the applied electromotive force, in cycles per second. Now consider the instant when the core flux is at its maximum posi-
tive value $\Phi$. After a quarter of a cycle, or after $\frac{1}{4 f}$ second, the flux is reduced to zero. The average rate of change of the flux during this quarter of a cycle is equal to

$$
\frac{\text { total change of flux }}{\text { elapsed time }}=\frac{\Phi}{\frac{1}{4 f}}=4 f \Phi
$$

which is equal to the average electromotive force (in c.g.s. units) induced in each turn of wire in the primary coil. Therefore, the total electromotive force induced in the $N$ turns of wire in the primary coil is $4 f \Phi N$ c.g.s. units, or $4 f \Phi N \div 10^{8}$ volts, since $10^{8}$ c.g.s. units equal 1 volt.

The average value of the induced electromotive force is equal (very nearly) to the average of the electromotive force applied to the primary coil; and it must be multiplied by the form factor* of the electromotive force curve to give the effective value $E$ of the applied electromotive force. The form factor of a sine-wave electromotive force is 1.11. Therefore

$$
E=1.11 \times \text { average value }=4.44 f \Phi N \div 10^{8} \text { volts }
$$

Hence, solving for $\Phi$, we obtain the equation

$$
\begin{equation*}
\Phi=\frac{E \times 10^{8}}{4.44 \times N \times f} \tag{34}
\end{equation*}
$$

Example. In the $5 \frac{1}{2}$-kilowatt transformer used as an illustration on page 234, there are 660 turns of wire in the primary coil, that is $N=660$. This primary coil is connected to alternating-current supply mains so that the electromotive force applied to the primary coil is 1,100 volts (effective). The frequency of the electromotive force is 125 cycles per second ( $=f$ ). On the assumption that the electromotive force wave is a sine curve, we have, using equation (34),

$$
\Phi=\frac{1,100 \times 10^{8}}{4.44 \times 660 \times 125}=300,000 \text { lines of magnetic flux }
$$

This is the maximum value of the alternating magnetic flux in the transformer core.

The cross-sectional area $A$ of the transformer core is $15 \frac{1}{2}$ square inches, so that the maximum value of the magnetic flux-density in the core is

$$
\frac{\Phi}{A}=\frac{300,000}{15 \frac{1}{2}}=19,350 \text { lines per square inch }
$$

This is equivalent to a flux-density of 3,000 lines per square centimeter.

[^22]Ideal Transformer Action Graphically Represented. sents the alternating magnetic flux in the core of a transformer, the line $O E^{\prime}$ represents the electromotive force applied to the primary coil, and the line $O E^{\prime \prime}$ represents the electromotive force induced in the secondary coil. When the transformer is at zero load, the current in the primary coil (the no-load current) lags greatly behind the applied electromotive force $E^{\prime}$, as shown in the figure, in which the line $O I_{0}$ represents the no-load current. The electromotive forces induced in both primary and secondary coils are 90 degrees behind the core


Fig. 194. Clock Diagram for Transformer without Load flux $O \Phi$ in phase, and the electromotive force $O E^{\prime}$ applied to the primary coil, being at each instant opposite to the electromotive force induced in the primary, is 90 degrees ahead of $O \Phi$ in phase.

Case (b) With Load, Receiving Circuit Nearly Non-Inductive. The lines $G \bar{\Phi}, O E^{\prime} O E^{\prime \prime}$, and $O I_{0}$ in Fig. 195 represent alternating core flux, primary applied electromotive force, secondary induced electromotive force, and no-load current, exactly as in Fig. 194. The line $O I^{\prime \prime}$ represents the secondary current lagging slightly behind the secondary electromotive force $O E^{\prime \prime}$; and the line $O A$ represents the increase of primary current due to the loading of the transformer. The total primary current is represented by the line $O I^{\prime}$, which is the vector (or geometric) sum of $O .1$ and $O I_{0}$. The current $O A$ is exactly opposite to $O I^{\prime \prime}$ in phase; and the product of this current $O A$ and the primary turns $N^{\prime}$ balances the magnetizing action $N^{\prime \prime} I^{\prime \prime}$ of the secondary current. As is evident from Fig. 195, the loading of a transformer (noninductive load) not only increases the value of the primary current, but reduces its angle


Fig. 195. Clock Diagram for Transformer with Load
of lag behind the primary applied electromotive force. Thus, at zero-
load the primary current is $O I_{0}$; and when the transformer is loaded (non-inductive load) the primary current becomes $O I^{\prime}$.

Case (c) With Load, Receiving


Fig. 196. Clock Diagram for Transformer with Load-highly Inductive Receiving Circuit Circuit IIighly Inductive. In Fig. 196 the line $O I^{\prime \prime}$ represents the current delivered by the secondary coil of a transformer to a highly inductive receiving circuit; the line $O A$ represents the increase of primary current due to the load; and $O I^{\prime}$ represents the total primary current. In this case, also, the part $O A$ of the primary current is exactly opposite in phase to the secondary current $O I^{\prime \prime}$.
Influence of Coil Resistances and Magnetic Leakage. The foregoing discussion takes account of the no-load current of a transformer. This no-load current is the only factor that affects the ideal relation, equation (33), between primary and secondary currents in a transformer. Coil resistances and magnetic leakage are, on the other hand, the only things that affect perceptibly the ideal relations, equation (32), of primary and secondary electromotive forces.

Magnetic leakage is equivalent, in its effect upon the action of a transformer, to an outside inductance (a choke coil) connected in series with the primary.coil. Let $L^{\prime}$ be this inductance (in henrys) which is equivalent to the magnetic leakage of a transformer. Then $\omega L^{\prime}$ ( $\omega$ equals $2 \pi$ times the frequency) is the reactance (in ohms) of this inductance.

The effects of coil resistances and magnetic leakage upon the ideal relation between $E^{\prime}$ and $E^{\prime \prime}$ are shown in the clock diagram, Fig. 197.

The total electromotive force $O E^{\prime}$ applied to the primary coil is used (a) to overcome the resistance $R^{\prime}$ of the
primary coil; (b) to overcome the electromotive force induced in the primary coil by the leakage flux; and (c) to balance the electromotive force induced in the primary coil by the magnetic flux $O \Phi$, which passes through both coils.

The part (a) of $O E^{\prime}$ is equal to $I^{\prime} R^{\prime}$, and it is in phase with $I^{\prime}$. The part (b) of $O E^{\prime}$ is equal to $\omega L^{\prime} I^{\prime}$, and it is 90 degrees ahead of $I^{\prime}$ in phase. The part (c) of $O E^{\prime}$ is represented by the line $O A^{\prime}$. The total electromotive force induced in the secondary coil is equal to $O A^{\prime} \times \frac{N^{\prime \prime}}{N^{\prime}}$. This electromotive force is represented by the line $O B$. A portion of this total induced electromotive force $O B$ is used to overcome the resistance $R^{\prime \prime}$ of the secondary coil; and the remainder $O E^{\prime \prime}$ is available at the terminals of the secondary coil to force current through the secondary receiving circuit.

From Fig. 197 it is evident that the ratio $\frac{E^{\prime \prime}}{E^{\prime}}$ is less than its ideal value $\frac{N^{\prime \prime}}{N^{\prime}}\left(=\frac{O B}{O A A^{\prime}}\right)$ because of the 'resistance loss" $I^{\prime} R^{\prime}$ of electromotive force in the primary coil, because of the "leakage loss" $\omega L^{\prime} I^{\prime}$ of electromotive force in the primary coil, and because of the "resistance loss" $I^{\prime \prime} R^{\prime \prime}$ of electromotive force in the secondary coil.

Performance. With Non-Inductive Load. When a transformer secondary delivers current to a non-inductive circuit, then $O I^{\prime \prime}$, Fig. 197, is parallel to $O B$, and $O I^{\prime}$ is nearly parailel to $O E^{\prime}$, so that $R^{\prime} I^{\prime}$ is nearly parallel to $O E^{\prime}$, and $\omega L^{\prime} I^{\prime}$ is nearly at right angles to $O E^{\prime}$. Therefore, the difference in value between $O E^{\prime}$ and $O A^{\prime}$ is nearly equal to $I^{\prime} R^{\prime}$, and nearly independent of $\omega L^{\prime} I^{\prime}$. Therefore, the falling off of the secondary voltage $E^{\prime \prime}$, with increase of load, is due almost wholly to $I^{\prime} R^{\prime}$ and to $I^{\prime \prime} R^{\prime \prime}$ when the receiving circuit is non-inductive, and is not due, to any perceptible extent, to magnetic leakage.

With IIighly Inductive Load. When a transformer secondary delivers current to a highly inductive circuit, then $O I^{\prime \prime}$, Fig. 197, is nearly at right angles to $O B$, and $O I^{\prime}$ is nearly at right angles to $O E^{\prime}$, so that $I^{\prime} R^{\prime}$ is nearly at right angles to $O E^{\prime} ; \omega L^{\prime} I^{\prime}$ is nearly parallel to $O E^{\prime}$; and further, $I^{\prime \prime} R^{\prime \prime}$ is nearly at right angles to $O B$. Therefore, the difference in value between $O E^{\prime}$ and $O A^{\prime}$ is nearly ${ }^{`}$ equal to $\omega L^{\prime} I^{\prime}$, and nearly independent of $R^{\prime} I^{\prime}$, while $O B$ is nearly equal to $O E^{\prime \prime}$. Therefore, the falling off of the secondary voltage $E^{\prime \prime}$, with
increase of load, is due chiefly to $\omega L^{\prime} I^{\prime}$, that is, to magnetic leakage, when the receiving circuit is highly inductive, and is not due to any great extent to coil resistances.

Example. The primary of a certain 10 -kilowatt ( 1,000 -volt : 100 -volt) transformer has a resistance of 1.5 ohms, and the secondary coil has a resistance of 0.015 ohms. The secondary coil delivers 100 amperes to a non-inductive receiving circuit; and, ignoring no-load current, the primary takes 10 amperes from 1,000 -volt mains. The $I R$ loss of electromotive force in the primary coil is, therefore, 1.5 ohms $\times 10$ amperes $=15$ volts, so that the portion $O A^{\prime}$, Fig. 197, of the primary applied voltage is very nearly $1,000-15=985$ volts. Therefore, the total electromotive force $O B$ induced in the secondary coil is $\frac{N^{\prime \prime}}{N^{\prime}} \times 985$ volts $=98.5$ volts. The $I R$ loss of electromotive force in the secondary coil is 0.015 ohms $\times 100$ amperes $=1.5$ volts, so that the electromotive force between the terminals of the secondary coil is 98.5 volts -1.5 volts, or 97 volts. In this case, the secondary receiving circuit is non-inductive, and the portion $\omega L^{\prime} I^{\prime}$ of the primary applied voltage is nearly at right angles to $E^{\prime}$. This loss of voltage $\omega L^{\prime} I^{\prime}$ has, therefore, no appreciable effect in lessening the value of the available part $O A^{\prime}$ of the primary applied voltage $E^{\prime}$.

The leakage reactance $\omega L^{\prime}$ of the above transformer is 5 ohms. If the secondary coil delivers 100 amperes of current to a very highly inductive receiving circuit, then this 100 amperes is nearly 90 degrees behind $O B$, Fig. 197, in phase; and the primary current of 10 amperes is nearly 90 degrees behind $O A^{\prime}$ in phase. Therefore, the leakage voltage loss $\omega L^{\prime} I^{\prime}$, which is equal to 50 volts, is nearly parallel to $O A^{\prime}$, so that $O A^{\prime}$ is very nearly equal to 1,000 volts -50 volts, or 950 volts, and $E^{\prime \prime}$ is very nearly equal to $\frac{N^{\prime \prime}}{N^{\prime}} \times 950$ volts, or 95 volts. In this case, $I^{\prime} R^{\prime}$ and $I^{\prime \prime} R^{\prime \prime}$ are nearly at right angles to $O A^{\prime}$ and $O B$, and these resistance losses of voltage do not have an appreciable effect in lessening the secondary terminal voltage $E^{\prime \prime}$.

The above discussion of the effects of coil resistances and of magnetic leakage shows that the ratio of $E^{\prime}$ and $E^{\prime \prime}$ is very nearly equal to its ideal value $\frac{N^{\prime}}{N^{\prime \prime}}$ when the primary and secondary currents of a transformer are small, that is, when the load on the transformer is zero.

## TRANSFORMER CONNECTIONS

Parallel-Constant=Voltage Transformers. In systems of distribution where alternating currents are delivered to a number of units (groups of lamps, for example) all at constant voltage, each unit (each group of lamps) is supplied from the secondary of a separate and distinct transformer; and the primaries of the respective transformers are connected in parallel across the constant-voltage
mains that lead out from the supply alternator. This arrangement is shown in Fig. 198, in which $P, P^{\prime}, P^{\prime \prime}$, etc., are the transformer primaries; $S, S^{\prime}, S^{\prime \prime}$, etc., are the corresponding secondaries; and $A, A^{\prime}, A^{\prime \prime}$, etc., are the separate receiving units or groups of lamps. For this kind of service, where the primary of a transformer is supplied at constant voltage, and it is desired that the transformer shall deliver current to a receiving unit at sensibly constant voltage irrespective of load, i. e., irrespective of the number of lamps, the transformer must be designed so that a very slight decrease of induced electromotive force in the primary will permit the necessary current to flow through the primary coil. This requires that the coils of the transformer shall have as little resistance as possible, and that the primary and secondary coils shall be wound close together, so that no perceptible portion of the magnetic flux that is forced through the core by the magnetizing current may flow out of the core between the coils, instead of passing through the secondary coil as well as through the primary coil. A transformer


Fig. 198. Diagram of Connections of Constant. Voltage
Transformers in Parallel
specially designed to realize these two conditions is sometimes called a constant-potential or constant-voltage transformer.

A constant-voltage transformer is necessary if it is desired to transform a given voltage in a determinate ratio so as to be able to infer the value of the given voltage from the measured value of the transformed voltage. Specially designed transformers, however, are not always necessary for this purpose, for the reason that the voltmeter used for measuring the transformed voltage usually takes very little current, and it is the currents in a transformer that disturb the ideal ratio of voltage transformation.

Multi-Coil Type. Most commercial transformers are now made with two (or more) primary coils, and with two (or more) secondary coils. Each of the primary coils of such a transformer may be
adapted to direct connection to 1,100 -volt mains, and be wound with wire large enough to carry, say, 10 amperes without undue heating. In this case, if these two primary coils are properly connected in parallel they constitute in effect a single primary coil, suited for direct connection to 1,100 -volt mains, and capable of taking 20 amperes without undue heating. On the other hand, if these two


Fig. 199. Proper Parallel Connections for Transformers


Fig. 200. Improper Parallel Connections for Transformers
primary coils are properly connected in series, they constitute in effect a single primary coil suited for direct connection to 2,200 -volt mains, and capable of taking 10 amperes of current without undue heating. Each of the secondary coils of such a transformer may likewise be adapted to deliver 100 amperes at 110 volts, in which case the two secondaries, if properly connected in parallel, constitute in effect a single secondary coil adapted to deliver 200 amperes at 110 volts; whereas, if the two secondaries are properly connected in series, they constitute in effect a single secondary adapted to deliver 100 amperes of current at 220 volts.

Two coils of a transformer (primary or secondary) are properly connected in parallel when the current which divides between them flows around the core in the same direction in both coils, that


Fig. 201. Proper Series Connections for Transformers


Fig. 202. Improper Series Connections for Transformers
is, so that both coils magnetize the core in the same direction. Proper and improper connections in parallel are shown diagrammatically in Figs. 199 and 200. See also "Polarity Test," page 321.

Two coils of a transformer (primary or secondary) are properly connected in series when the current which flows through them flows around the core in the same direction in both coils. Proper and improper connections in series are shown diagrammatically in Figs. 201 and 202.

When two primaries of a transformer improperly connected in parallel are connected to the supply mains, the currents in the coils oppose each other in their magnetizing action on the core. The result is that the core is not perceptibly magnetized; but little opposing electromotive force is induced in the coils (by leakage flux); and the two improperly connected coils constitute a short circuit when they are connected to the supply mains.

When two primaries of a transformer, improperly connected in series, are connected to the supply mains, the current which


Fig. 203. Proper Transformer Connections to a Three-Wire System
flows through the two coils will have equal and opposite magnetizing actions in the two coils. The core flux will, therefore, be practically zero, and no counter-electromotive force will be induced in the windings to balance the applied electromotive force. The flow of current is, therefore, hindered only by the coil resistances and by the electromotive forces induced by the leakage flux. The result is that two coils improperly connected in series constitute a short-circuit when they are connected to the supply mains.

Two secondary coils improperly connected in parallel give rise to short-circuit conditions. Two secondary coils improperly connected in series do not lead to short-circuit conditions, but give zero electromotive force between their terminals.

Edison Three-Wire System—Single-Phase. The Edison threewire system, extensively used in direct-current distribution, is commonly used in alternating-current distribution. Fig. 203 shows two transformers properly connected for supplying current to a threewire system; and Fig. 204 shows two transformers improperly connected for supplying current to a three-wire system. In the proper connection, the middle secondary main carries only the difference of the currents in the outside mains $a$ and $b$; and in the improper connection, the current in the middle secondary main is the sum of the currents in the outside mains. The proper connection gives


Fig. 204. Improper Transformer Connections to a Three-Wire System
double voltage between the outside secondary mains and the improper connection gives zero voltage between outside secondary mains.

The Edison three-wire system must not be confused with the two-phase and three-phase three-wire systems. The advantages of the Edison three-wire system when used for the distribution of single-phase alternating currents, are exactly the same as the advantages of this system when used for the distribution of direct currents, namely, a great saving in the copper required in the distributing mains. This saving, in general, amounts to five-eighths of the copper that would be required for a two-wire system using
one-half the total voltage between outside mains. As explained on page 12 , the amount of copper required varies inversely as the square of the voltage used.

Single-phase current may be supplied to Edison three-wire distributing mains by a single transformer having two secondary coils properly connected in series to the outside mains, the middle main being connected to the junction of the two secondary coils. An "autotransformer" or "balance coil" having a tap at the middle point of its single winding is often used. (See page 250.)


Fig. 205. Connections for "Banked" Transformers
Banking of Transformers. Two service mains may be supplied with current by two or more transformers with their primaries connected in parallel between the supply mains, and with their secondaries properly connected in parallel to the service mains. Fig. 205 shows two transformers properly "banked" so as to supply current to the two service mains $a$ and $b$. In many large transmission systems, such as the Niagara-Buffalo, a dozen or more large transformers may be banked (on each phase) for step-up or step-down transformation.

It is very important to note, however, that proper connections alone will not insure satisfactory parallel operations. Perfect oper-
ation of two or more transformers in parallel means that each of the separate units contributes to the total load an amount of power proportional to its rated output, and that the numerical sum of the currents in the separate units is equal to the line (total) current. To secure this desirable result, two conditions must be fulfilled: (a) the ratio of primary turns to secondary turns must be the same in all the units, and (b) the voltage drop from no load to full load must be the same, both in magnitude and phase, for all the units.

If condition (a) is not fulfilled, there will be a difference at all loads in the secondary voltages, and the transformer having the highest voltage will carry the largest load. Condition (b) is fulfilled if all the transformers have the same impedance volts ( $Z I$ ) and the same ratio of resistance to impedance. If the transformers in parallel have the same impedance volts, it follows that the magnitude of the current furnished by each will be inversely as its impending upon the magnetic quality of the iron. For carbon sheet steel used for transformer cores the value of $\eta$ is about 0.0021 . same phase, and the total current will then be the numerical sum of the individual currents.

Given two or more transformers, having the same ratio of primary to secondary turns and having relatively small magnetizing currents, to be banked so as to operate in parallel; the division of the total load between them depends chiefly upon the total impedances between the bus bars, including with each transformer its connecting wires and any meters or relays through which the current may pass. In any case where transformers banked in parallel do not divide the total load according to their rated capacities, a proper division of the current may be effected by increasing the impedance in the circuit of the transformer which delivers more than its share of the total current. This may be easily done by inserting a suitable choke coil (a coil of wire wound on a laminated iron core) in series with either its primary or secondary lead wires.

When the ratio of reactance to resistance is unequal in the transformers, the phases of the secondary currents are not the same, so that the transformers may deliver equal currents, and still not deliver equal amounts of power to the circuit. It follows that a wattmeter connected with its series (current) coil in circuit with only one of, say, two transformers of equal rating will not measure
half of the total power. It will measure more than half or less than half according to the value of $\cos \theta$ in the expression $E I \cos \theta$, where $\cos \theta=\frac{\text { resistance }}{\text { impedance }}$, or $\tan \theta=\frac{\text { reactance }}{\text { resistance }}$.

From the above discussion it is evident that special care must be taken in banking transformers for parallel operation to see that the several units are delivering their proper share of the total current output. Furthermore, it is not safe to assume that transformers even of the same rated capacity and of the same make will share a given load equally when operated in parallel. The only safe procedure in such cases is to measure the voltage, current, and watts of the several transformers.

Series=Current Transformers. In some of the older systems of distribution by alternating currents, it was desired that each


Fig. 206. Connections for Transformers in Series
receiving unit (arc lamp, for example) should receive a constant current equal to the whole, or to a definite fractional part, of the constant current delivered by an alternator. This condition can be realized by supplying each unit or group of units from the secondary of a separate and distinct transformer, the primaries of all the transformers being connected in series as shown in Fig. 206, in which $P, P^{\prime}, P^{\prime \prime}, P^{\prime \prime \prime}$, etc., are the primaries of the respective transformers; $S, S^{\prime}, S^{\prime \prime}, S^{\prime \prime \prime}$, etc., are the transformci secondaries; and $A, A^{\prime}, A^{\prime \prime}$, $A^{\prime \prime \prime}$, etc., are the receiving units. For this kind of service, where the primary of a transformer is supplied with a definite current, and it is desired that the transformer shall deliver to a receiving unit a current which is equal to an invariable fractional part of the primary current, irrespective of variations of resistance in this unit, the transformer must be designed to take as small a magnetizing current as possible, for, according to the discussion on page 233, it is the magnetizing current that disturbs the ideal relation of primary
to secondary current in a transformer. A transformer which is specially designed to realize this condition is sometimes called a current transformer.

The transformer used in connection with the composite field excitation of an alternator, as shown in Fig. 83, is a current transformer which delivers a current equal to a definite fraction of the current output of the alternator to the rectifying commutator.

The current transformer is frequently used for sending current equal to a definite fractional part of an alternating current through an ammeter from the reading of which, together with the known ratio of current transformation, the value of the whole alternating current is deduced.

The current transformer here described must not be confused with the so-called constant-current transformer which receives variable


Fig. 207. Diagram of Connections for Transformer in Series with a Number of Lamps
current from a constant voltage supply, and delivers a constant current to a group of receiving units connected in series, the delivered current being constant irrespective of increase or decrease in the number of receiving units.

The connection of a transformer primary in series in a circuit containing many other elements (lamps) so that the current passing through the primary does not vary much with the varying resistance of the circuit to which the secondary of the transformer delivers current, gives rise to actions which are not very familiar to electrical engineers, for the reason that this arrangement is now seldom used in practice. The actions, however, which are interesting, are as follows:

Fig. 207 represents a transformer primary $P$ connected in series in a circuit containing many elements $e^{\prime}, e^{\prime \prime}, e^{\prime \prime \prime}$, etc. (lamps).

The action will be described in two steps, viz, (a) on the assumption that the magnetizing current of the transformer is always negligible, and (b) without the aid of this simplifying assumption.
(a) If the magnetizing current is always negligibly small, then the current in $A$ is equal to a fixed fractional part of the sensibly constant current in the main circuit, so that any increase of the resistance or reactance of $A$ must be accompanied by a corresponding increase of the voltage $E^{\prime \prime}$, which is pushing current through $A$; and this must be accompanied by a corresponding increase of the voltage $E^{\prime}$ between the terminals of the primary coil $P$. Thus, if $A$ has zero resistance and zero reactance, then $E^{\prime \prime}$ is zero, and $E^{\prime}$ is zero. That is, the current in the main circuit flows through $P$ without any opposition at all, just as if $P$ were a connection of zero resistance.

If the resistance or reactance of $A$ is increased, $E^{\prime \prime}$ (and also $E^{\prime}$ ) must increase, which means that the current in the main circuit encounters greater and greater opposition in flowing through $P$.

If the circuit of $A$ is opened (infinite resistance), then, on the above assumption of negligible magnetizing current, the opposition to the flow of current in $P$ becomes infinite, so that breaking the circuit of $A$ is equivalent to breaking the main circuit.
(b) As a matter of fact, as the resistance or reactance of $A$ is increased, causing an increase of $E^{\prime \prime}$ and $E^{\prime}$, the magnetism of the transformer core must increase proportionally with $E^{\prime \prime}$ and $E^{\prime}$ in order that these increased voltages may actually be induced in the transformer coils; and this increase of magnetism of the core requires more and more magnetizing current.* The magnetizing current, therefore, is not always negligible irrespective of the resistance of $A$. In fact, when the resistance of $A$ is infinite (open circuit), there is no secondary current; all the current in the coil $P$ is magnetizing current; and the voltage $E^{\prime}$, which opposes the flow of current through $P$, rises only to that value which corresponds to the degree of magnetism of the core that can be produced by the magnetizing action of the whole primary current. The transformer then becomes simply a choke coil.

Autotransformer. Given a source of supply of alternating

[^23]current. This current may be delivered to a receiving unit in three ways:
(a) By connecting the unit directly to the supply mains.
(b) By connecting the unit to the secondary of a transformer of which the primary is connected to the supply mains.
(c) By a combination of methods (a) and (b).

The combination method may be realized with any ordinary transformer; which, when so used, is called an autotransformer, or compensator.

Note. It will appear in the following discussion that when the voltage of the supply differs but little from the desired service voltage, the combination method is much preferable to the method in which the transformer alone is used, because of the fact that a smaller transformer suffices, and because the combination method involves less energy loss. This combination or autotransformer method is quite simple of treatment when attention is confined to a particular case; but it is complicated when attempt is made to give it a general discussion, that is, when attempt is made to discuss all the theoretically possible ways in which a given ordinary transformer may be used as an autotransformer.

In Fig. $208 A$ and $B$ are alternating-current supply mains, between which the voltage is, say $100 ; C$ and $D$ are service mains, to which it is desired to deliver alternating current at 90 or 110 volts; $P$ and $S$ are the primary and secondary coils of an ordinary transformer. The primary $P$ is connected to $A$ and $B$ as shown. The secondary $S$ has one-tenth as many turns as $P$, therefore, the voltage induced in $S$ is one-tenth of the voltage acting on $P$, or 10 volts. Let the dotted arrows represent the directions of the induced electromotive forces in the two coils at a given instant. Then the long heavy arrow will represent the direction of the voltage between the mains at the same instant, inasmuch as the induced voltage in the primary coil of a transformer is always opposed to the supply voltage.

Auto Step-Up Transformation. The 10 volts induced in the coil $S$, Fig. 208, will help push current into the service mains if we connect from supply main $A$, out of which the current at the given instant is tending to flow, to terminal $g$ of coil $S$; connect from
terminal $f$ to service main $C$; and connect from service main $D$ to supply main $B$, as shown by the dotted lines. In this case, the voltage induced in the coil $S$ is added to the supply voltage, and the service voltage is, therefore, 110 volts.

Auto Step-Down Transformation. The 10 volts induced in the coil $S$ will oppose the flow of current into the service mains if we connect from supply main $A$ to terminal $f$, connect from terminal $g$ to service main $C$, and connect from service main $D$ to supply main $B$, as shown by the dotted lines in Fig. 209. In this case the induced voltage in the coil $S$ is subtracted from the supply voltage, and the service voltage is now 90 .

Current Relations. In Figs. 208 and 209 the directions of the induced voltages in $P$ and $S$ are shown by the dotted arrows. These induced voltages are in the same direction in the two coils. The currents in the two coils of a transformer are, on the other hand, always in opposite directions, inasmuch as they balance the magnetizing action of each other. Suppose for the sake of concreteness that 10 amperes are delivered to the service mains. Then we have the following relations:
(a) Ten amperes flow through $S$, Fig. 208, in the same direction as the induced electromotive force of ten volts; so that one ampere (one-tenth as much current, since there are ten times as many turns in $P$ as in $S$ ) flows through $P$ in opposition to the induced or counterelectromotive force of 100 volts. Therefore, the coil $P$ takes 100 watts from


Fig. 209. Connections for an Auto Step-Down Transformer the supply mains, which power is transferred to the coil $S$ by ordinary transformer action, whence it is given out (ten volts pushing ten amperes) in assisting the flow of current to the service mains. The total power delivered to $C D$ is evidently 1,100 watts ( 10 amperes at 110 volts), and of course the total power taken from the supply mains is 1,100 watts ( 11 amperes at 100 volts).
(b) Ten amperes flow through $S$, Fig. 209, in a direction opposite to the ten volts of induced electromotive force, so that one ampere flows through $P$ in the same direction as the induced electromotive force of 100 volts. Therefore, of the 1,000 watts delivered by the supply mains in forcing the ten amperes through $S$ and through the service mains, 100 watts are delivered to the coil $S$, and 900 watts are delivered to the service mains ( 10 amperes at 90 volts). The 100 watts delivered to $S$ are transferred by transformer action to the coil $P$, and delivered by $P$ back to the supply mains.

It is to be particularly noted that only 100 watts are involved in the
above as genuine transformer action, although 900 or 1,000 watts are actually delivered to the service mains.

The autotransformer is used commercially for many purposes. A common use is as a "balance coil" in a three-wire distribution from a two-wire supply, and it is also used as a balancer in connection with rotary converters for supplying a three-wire direct-current service as explained on page 361. Compensators are also used for the operation of low-voltage tungsten lamps in parallel for house lighting, for electric signs, for arc lamps, as voltage regulators for mercury-vapor rectifiers, as explained on page 330 . Autotransformers are also widely used as "starting compensators" for alter-nating-current motors. In such cases they supply a reduced voltage (usually half voltage) to the motor circuits while the armature is accelerating from rest. Each autotransformer is usually provided with several taps so that a number of low voltages may be obtained at will.

Autotransformers are used for supplying a varying voltage to single-phase commutator motors used on electric cars and locomotives, and thus for controlling the speed of the motors.

## TRANSFORMERS IN POLYPHASE SYSTEMS

A polyphase transmission system is essentially the utilization of two or three entirely separate and distinct single-phase transmis-


Fig. 210. Transformer Connections on a Two-Phase System


Fig. 211. Transformer Connections on a Three-Wire Two-Phase System
sion systems of which the separate and distinct electromotive forces or currents are maintained in definite phase relations with each other by mechanical comnections in the generator. Step-up or step-down transformation in a polyphase system is accomplished, in general, by a separate and distinct transformer of the ordinary type for each phase.

Two=Phase System. Fig. 210 shows a two-phase system in which the current of each phase $A$ and $B$ is transmitted over an


Fig. 212. Connections for Three Transformers with Primaries $\Delta$-Conneeted to Three-Wire Three-Phase Mains
entirely independent circuit, and the electromotive force of each phase is stepped down (or up) by an ordinary transformer $P^{\prime} S^{\prime}$ and $P^{\prime \prime} S^{\prime \prime}$, respectively. Fig. 211 shows what is called the three-wire, tuo-phase system, in which one line wire is used as a common return wire for both phases, and where two ordinary transformers $P^{\prime} S^{\prime}$ and $P^{\prime \prime} S^{\prime \prime}$ are used for stepping the voltage down (or up). These two figures contain all that is essential in the step-down or step-up transformation of a two-phase system.


Fig. 213. System Shown in Fig. 212 with One Transformer Removed Giving V-Connections

Three=Phase System. 'The usual transmission line for a threephase system consists of three wires, each wire being in effect a com-
mon return for currents that pass out over the other two. In this case, the usual arrangement for step-down (or step-up) transformation is to connect the


Fig. 214. Diagrammatic View of Two Separate Transformers for Step-Up or Step-Down Transforma-tion-Two-Phase System three primaries of thrce ordinary transformers to the supply mains, using either Y or $\Delta$ connections; and to connect the three secondaries to the service mains, using either $Y$ or $\Delta$ connections. Furthermore, the primaries may be Y -connected and the secondaries $\Delta$-connected, or vice versa. The $\Delta$ connections of both primaries and secondaries are preferred in practice, inasmuch as with this parrangement the complete three-phase, step-down (or step-up) transformation is still effected even though one transformer may be entirely disconnected because of burn-out or breakdown. In this case the two remaining transformers are said to be connected in V. In such a case, however, the two remaining transformers do not have two-thirds of the transforming capacity of all three, but only $\frac{8,5}{10}$ of $\frac{2}{3}=0.507$ as much, or a little over one-half the capacity.

Fig. 212 shows three ordinary transformers $P^{\prime} S^{\prime}, P^{\prime \prime} S^{\prime \prime}$, and $P^{\prime \prime \prime} S^{\prime \prime \prime}$, with their primaries $\Delta$-connected to three-wire, three-phase supply mains 1,2 , and 3 ; and with their secondaries $\Delta$-connected to three-wire, three-phase service mains $a, b$, and $c$.


Fig. 215. Two Transformers for Two-Phase System with Common Magnetic Return Circuit

Fig. 213 shows the arrangement of Fig. 212 with one of the transformers $P^{\prime \prime \prime} S^{\prime \prime \prime}$ omitted (any one of the three may be omitted), thus giving the V connection. This arrangement is operative for threephase, three-wire step-up or step-down transformation, except that its power capacity is only about 57 per cent of the power capacity of the arrangement in which three similar transformers are used.

Transformers with Compound Magnetic Circuits. Let $A$ and $B$, Fig. 214, be the two separate transformers to be used for step-up or step-down transformation on a two-phase system. Each iron core may have its own return circuit for the magnetic flux; or a single magnetic return $C$ may be used for the two, as shown in Fig. 215. In the latter case only 1.4


Fig. 216. Three Transformers for Three-Phase System with Common Magnetic Return Circuit as much iron need be used for the common return as would have to be used for each single magnetic return.

Similarly, three transformers $A, B$, and $C$, Fig. 216, used for step-up or step-down transformation on a three-phase system, may be combined magnetically so that each transformer core is the magnetic return for the other two, as shown. Commercial examples of polyphase transformers are described on pages 291 and 292.

Phase Transformation. If a two-phase* supply is connected to the similar primarics of two transformers, an electromotive force of any desired value and of any desired phase may be produced. Fig. 217 shows the two-phase supply mains connected to the similar primaries $P^{\prime}$ and $P^{\prime \prime}$ of two separate transformers. On core $A$ is wound a secondary coil $a$, and on core $B$ is wound a secondary coil $b$; these two secondary coils are connected in series, and the desired electromotive force $E$ is produced by the two secondary coils in conjunction. In order that the desired electromotive force $E$ may


Fig. 217. Connections for Phase Transformation from Two-Phase Supply Mains be produced by coils $a$ and $b$ jointly, the following conditions must be fulfilled:

[^24]Let the vectors $A$ and $B$ in the clock diagram, Fig. 218, represent the two two-phase electromotive forces; and let $E$ represent the desired electromotive force. Then coil


Fig. 218. Vector Diagram of E.M.F. Relations for Fig. 217 a, Fig. 217, must be wound with a sufficient number of turns of wire to produce the component a of $E$; and coil $b$ must be wound with a sufficient number of turns of wire to produce the component $b$ of $E$. See Fig. 218.

If it is desired that the resultant electromotive force $E$ be in the second, third, or fourth quadrant, the coil $a$ or the coil $b$, or both coils $a$ and $b$ must be reversed as indicated in Fig. 219. It, therefore, follows that by correctly proportioning and connecting the coils $a$ and $b$, any desired value and position of the resultant electromotive $E$ may be produced.

The general two-phase, three-phase transformer consists of two separate transformers with similar primary coils $P^{\prime}$ and $P^{\prime \prime}$ which are connected to the respective phases of the two-phase supply mains, as shown in Fig. 217; and each of the three-phase electromotive forces is produced by a pair of properly proportioned and properly connected secondary coils (one coil on each transformer) connected in series. Each pair of secondary coils constitutes one unit of the three-phase system, and these three units may be either Y -connected or $\Delta$-connected to three-wire, three-phase service mains.

The general two-phase-three-phase transformer is greatly


Fig. 219. Vector Diagram of E.M.F. Relations for $E$ in any Quadrant simplified if we choose to have one of the threc-phase electromotive forces in pliase with one of the two-phcise electromotive forces. Thus, if $E$, Fig. 218, is in phase with $B$, then $E$ may be produced by a single secondary coil on core $B$, instead of being produced by a pair of secondary ccils, one on each core.

Scott Transformer, or T-Connection. The two-phase- three-phase transformer permits of still further simplification if the $\Delta$-connection of the three-phase units is excluded, that is, if the three-phase units are to be adapted only for Y -connection to the three-phase mains. This simplification is the Scott transformer.

The transformer shown in Fig. 217 may be converted into a Scott transformer by replacing its windings with the windings shown in Fig. 224. The two similar primary coils $A$ and $B$, which are connected


Fig. 220. Vector Diagram of E.M.F. Relations in Three Secondaries of Scott Transformer to the respective phases of a two-phase system, are placed on cores $A$ and $B$, respectively. Coils $b, a$, and $c$ are the three secondaries


Fig. 221. Diagram of Divided Secondary in Scott Transformer which are Y -connected to the three-phase mains. $b$ is placed on core $B$ and gives the electromotive force $b$, Fig. 220. It is to be noticed that coils $a$ and $c$ are formed by bringing out a connection at the mid-point of one large coil, Fig. 221. $a$ and $c$ are placed on core A, Fig. 217, and give the electromotive forces $a$ and $c$, Fig. 220.

The number of turns on $a$ and $c$ are equal. The number of turns on $b$ are made to equal $\sqrt{3}$ times the turns on either $a$ or $c$, i.e., if $a$ and $c$ each have 50 turns, $b$ will have approximately 87 turns.

The points 1, 2, and 3, Fig. 220 , are at the angles of an equilateral triangle. The point $O$ represents the common junction point of the three terminals, one from each of the coils $a, b$, and $c$. The other terminals of these coils are connected to the three-phase mains represented by the figures


Fig. 222. Vector Diagram of E.M.F. Relations in Scott Transformer 1, 2, and 3, Figs. 220 and 224.

Review page 98, Book II, to get a clear idea of the electromotive force and current relations in three-phase systems Y -connected.

From these relations and keeping in mind the ratios of the windings of the coils in Fig. 224, it may be seen that the electromotive force $a$ helps to push current in a receiving circuit from main 1 to main 2; while electromotive force $b$ opposes $a$


Fig. 223. Y Connections for Three Secondaries of Scott Transformer for Two-Phase System in this respect. Therefore, the electromotive force from main 1 to main $\mathscr{2}$ is $a-b$, as shown in Fig. 222. Similarly the electromotive force $b$ helps to push current in a receiving circuit from main 2 to main 3 , while electromotive force $c$ opposes $b$ in this respect. Therefore, the electromotive force from main 2 to main $\mathcal{S}$ is $b-c$, as shown in Fig. 222. Lastly the electromotive force $c$ helps to push current in a receiving circuit from main 3 to main 1 , while the electromotive force $a$ opposes $c$ in this respect. Therefore, the electromotive force from main 3 to main 1 is $c-a$, as shown in Fig. 222.

An inspection of Fig. 222 will show that $a-b=b-c=c-a$. Therefore, if $E$ is allowed to represent the value of each, this discussion shows that the electromotive forces between mains 1 to 2 , 2 to 3 , and 3 to 1 have the common value $E$ volts, and are 120 degrees apart in phase, provided

$$
b=E \cos 30^{\circ}
$$

and

$$
a=c=E \sin 30^{\circ}
$$

But the ratios of the windings were so proportioned that this is true. Hence the e.m.f.s are 120 degrees


Fig. 224. Complete Connections of Scott Transformer apart and result in a balanced three-phase circuit.

A clear idea of the Scott transformer may now be obtained as follows: Two similar cores have similar primary coils, which are connected to the respective phases of a two-phase system. One of these cores has a secondary winding $b$, one end of which is connected to one of the three-phase mains (main 2, as shown in

Fig. 223), and the other end of which is connected to the middle point of the secondary winding $a c$, which is wound on the other core. The terminals of the winding $a c$ are connected to the remaining two of the three-phase mains (mains 1 and 3 , as shown in Fig. 223). The entire secondary winding $a c$ has $\frac{2}{\sqrt{3}}$ times as many turns of wire as the coil $b$, that is, 1.16 times as many turns. The complete connections of the Scott transformer are shown in Fig. 224.

The three-phase system requires less line copper than either the single-phase or the two-phase system to transmit a given amount of power with a given line voltage and with a given loss. Hence, for the long-distance transmission of electric power, the threephase system is universally adopted in this country. For the local distribution of electric power, on the other hand, the two-phase system offers certain advantages. It is often the case, therefore, that two-phase alternators are used to generate alternating currents at a central station, and that two-phase currents are used for power and lighting purposes in the neighborhood of the station. When, however, power is to be transmitted to points fifteen or more miles distant, it becomes desirable, as explained above, to use the threephase system. It is in such cases, especially, that phase transformation is used. The Niagara-Buffalo transmission is the earliest as well as one of the most extensive examples of this practice. Power is generated by large two-phase alternators at 2,200 volts. A large part of this power is distributed to factories and chemical works in the vicinity of the central power plant. A large amount of power also is transmitted to Buffalo, a distance of eighteen miles, by means of three-phase alternating currents derived from the two-phase alternators by two-phase, three-phase transformation. Scott transformers are used; and the two-phase currents at a voltage of 2,200 are stepped up to 22,000 volts, and at the same time are transformed to three-phase currents. At Buffalo, the three-phase, highvoltage currents are stepped down to about 2,200 volts, and are transformed back into two-phase currents, also by Scott three-phase two-phase transformers. The 2,200 -volt two-phase currents are then distributed by feeders and service mains for lighting and power purposes throughout the city.

TABLE XI
Kilovolt-Ampere Ratings of Transformers for Scott Connection (Three-phase to Two-phase) to Serve a Given Horsepower Load

The following table gives the ratings recommended by the Westinghouse Electric and Manufacturing Company for transformers serving squirrel-cage induction motors and indicates the efficiency of the installation. The temperature guarantee with performances as shown is a $50^{\circ} \mathrm{C}$. rise.

| Horsepower of Motor | Number of Transformers | $\underset{\text { Transformer }}{\text { Ki. }}$ | Total Kv.-A. on Both Transformers when Motor ls Full Load | Efficiency of Bank of Transformers with Full Load on Motor |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{1}{2}$ | 2 | $\frac{1}{2}$ | 0.75 | 92.8 |
| 1 | 2 | $\frac{1}{2}$ | 1.35 | 94.5 |
| 2 | 2 | 1 | 2.40 | 95.2 |
| 3 | 2 | $1 \frac{1}{2}$ | 3.4 | 95.9 |
| 5 | 2 | $2{ }^{\frac{1}{2}}$ | 5.5 | 96.4 |
| $7 \frac{1}{2}$ | 2 | 4 | 8.1 | 97.0 |
| 10 | 2 | 5 | 10.7 | 97.2 |
| 15 | 2 | $7{ }^{1}$ | 15.7 | 97.4 |
| 20 | 2 | $10^{2}$ | 20.9 | 97.6 |
| 30 | 2 | 15 | 31.5 | 97.8 |
| 40 | 2 | 20 | 42.0 | 98.0 |
| 50 | 2 | 25 | 51.0 | 98.0 |
| 75 | 2 | $37 \frac{1}{2}$ | 77.0 | 98.3 |

Kv.-a. on each transformer $=\frac{\text { hp. } \times 746 \times 1.08}{\text { (eff. } \times \text { p.f. of motor) }}$
Efficiency $=\frac{\mathrm{kv} .-\mathrm{a} \text {. on transformer }}{\mathrm{kv} .-\mathrm{a} . \text { transformer loss }+\mathrm{kv} \text {. } \mathrm{a} \text {. on transformer }}$

Comparison of Phase and Voltage Transformers. In comparing Scott connections just discussed with open $\Delta$ or $V$ connections and also with three transformers $\Delta$ and $Y$ connected, the following points should be carefully noted:

1. All connections effect the voltage transformation without noticeable distortion of phase relations.
2. Neutral point is easily reached on T, V, and Y.
3. $\Delta$ connection is the only one capable of transforming in emergencies with one disabled transformer.
4. T connection gives much better inherent regulation than V connection and its iron losses are 15 per cent less.
5. The initial cost of transformers makes the T or V connections very desirable for small three-phase power service.

## PRACTICAL CONSIDERATIONS

Transformer Losses. The power output of a transformer is less than its power intake because of the losses in the transformer. These losses are: (a) the iron or core losses due to eddy currents and hysteresis; and (b) the copper losses due to the resistances of the primary and secondary coils.
(a) Iron Losses. The iron losses are practically the same in amount at all loads. They depend upon the frequency and range of the flux density B, upon the quality and volume of the iron, and upon the thickness of the laminations. Dr. C. P. Steinmetz has • found by exhaustive experiments that for ordinary sheet steel the hysteresis loss may be expressed in watts as

$$
\begin{equation*}
W_{h}=\eta V f \mathrm{~B}^{1.6} \times 10^{-7} \tag{35}
\end{equation*}
$$

in which $f$ is the frequency in cycles per second; B is the maximum flux density in the iron core, in lines per square centimeter; $V$ is the volume of the iron, in cubic centimeters; and $\eta$ is a coefficient depending on the magnetic quality of the iron. The value of this constant for soft annealed sheet steel used for transformer cores is about 0.0027 . For silicon-steel, now much used in constructing the cores of transformers, the value of $\eta$ may be taken as about 0.00093 .

The eddy current loss, in watts, is

$$
\begin{equation*}
W_{e}=b V f^{2} t^{2} \mathrm{~B}^{2} \times 10^{-7} \tag{36}
\end{equation*}
$$

in which $t$ is the thickness of the laminations, in centimeters; and $b$ is a constant depending upon the specific electrical resistance of the steel. For ordinary sheet steel the value of $b$ is $1.65 \times 10^{-11}$, and for silicon-steel $0.57 \times 10^{-11}$. Insufficient insulation between laminations causes excessive eddy current loss, and results in a much higher loss than equation (36) indicates. The equation is derived on the condition of perfect insulation between laminations, a condition which is hardly ever realized in practice.

Equations (35) and (36) may be used for calculating the approximate hysteresis and eddy-current losses in any mass of laminated iron subjected to periodic magnetization, such as alternator armatures and the rotor and stator iron in an induction motor, but the losses thus calculated are usually smaller than the actual losses. It is preferable when possible to find by a wattmeter test the actual total core loss per pound of steel at different flux densities and

TABLE XII
Transformer Efficiencies, Losses, Etc.

| KV-A. | $\underset{\text { Wats }}{\text { Loss }}$ |  | Per Cent Efficiency |  |  |  | Per Cent Requlation |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Iron | $\begin{gathered} \text { Cop- } \\ \text { per } \end{gathered}$ | $\begin{aligned} & \text { Full } \\ & \text { Load } \end{aligned}$ | 3/4 Load | 1/2 Load | 1/4Load | ${ }_{\text {P }}^{100 \%}$ P. $\%$. | $\begin{aligned} & 90 \% \\ & \text { P. F. } \end{aligned}$ | $\stackrel{80 \%}{\text { P.F. }}$ | $60 \%$ P. F. \% |  |
| $\frac{1}{2}$ | 15 | 13 | 94.7 | 94.4 | 93.2 | 88.7 | 2.62 | 3.21 | 3.28 | 3.16 | 8.0 |
| 1 | 20 | 24 | 95.8 | 95.7 | 95.1 | 92.0 | 2.42 | 3.03 | 3.12 | 3.04 | 5.5 |
| $1{ }^{\frac{1}{2}}$ | 25 | 35 | 96.0 | 96.0 | 95.5 | 92.7 | 2.36 | 2.96 | 3.07 | 3.00 | 4.0 |
| 2 | 30 | 42 | 96.5 | 96.5 | 96.2 | 93.8 | 2.12 | 2.76 | 2.88 | 2.86 | 3.6 |
| 21 | 33 | 51 | 96.8 | 96.8 | 96.5 | 94.5 | 2.08 | 2.71 | 2.83 | 2.83 | 3.3 |
| 3 | 34 | 64 | 96.8 | 97.0 | 96.8 | 95.2 | 2.16 | 2.79 | 2.91 | 2.88 | 3.0 |
| 4 | 40 | 75 | 97.2 | 97.3 | 97.1 | 95.7 | 1.90 | 2.77 | 3.00 | 3.12 | 2.5 |
| 5 | 45 | 93 | 97.3 | 97.5 | 97.3 | 96.1 | 1.90 | 2.76 | 2.99 | 3.11 | 2.3 |
| $7^{\frac{1}{2}}$ | 62 | 125 | 97.6 | 97.7 | 97.6 | 96.4 | 1.70 | 2.60 | 2.84 | 3.00 | 2.2 |
| 10 | 80 | 148 | 97.8 | 97.9 | 97.7 | 96.5 | 1.51 | 2.42 | 2.68 | 2.89 | 1.9 |
| 15 | 105 | 212 | 97.9 | 98.0 | 97.9 | 97.0 | 1.44 | 2.36 | 2.63 | 2.85 | 1.6 |
| 20 | 131 | 268 | 98.0 | 98.1 | 98.0 | 97.1 | 1.39 | 2.51 | 2.87 | 3.21 | 1.5 |
| 25 | 147 | 319 | 98.2 | 98.3 | 98.2 | 97.4 | 1.33 | 2.45 | 2.82 | 3.17 | 1.3 |
| 30 | 163 | 374 | 98.2 | 98.4 | 98.3 | 97.6 | 1.32 | 2.45 | 2.82 | 3.16 | 1.2 |
| $37 \frac{1}{2}$ | 197 | 433 | 98.3 | 98.4 | 98.4 | 97.7 | 1.20 | 2.34 | 2.72 | 3.09 | 1.2 |
| 50 | 240 | 550 | 98.4 | 98.6 | 98.5 | 97.9 | 1.15 | 2.29 | 2.68 | 3.07 | 1.0 |

*In per cent of full load current.
frequencies. Curves may then be plotted one for each frequency, using total core loss in watts per pound as abscissas, and $B$ as ordinates.
(b) Copper, Losses. The copper losses, in watts, are

$$
\begin{equation*}
W_{o}=R^{\prime} I^{\prime 2}+R^{\prime \prime} I^{\prime \prime} \tag{37}
\end{equation*}
$$

This loss is nearly zero when the transformer is not loaded; it increases with the square of the current; and becomes excessive when the transformer is greatly overloaded.

Transformer Efficiency. The ratio power output $\div$ power intake is called the efficiency of a transformer.

Table XII, shows the efficiencies, losses, and regulation of a recent series of combined core- and shell-type transformers designed and manufactured by a large American company. These transformers are designed for a frequency of 60 cycles per second and primary voitages of 1,100 or 2,200 volts, according to whether the two
halves of the primary coil are connected in parallel or in series. The secondary voltages are 110 or 220 volts.

Fig. 225 shows graphically the relation between the efficiency and the output for a 7.5 -kilowatt, core-type transformer designed for primary voltages of 1,040 and 2,080 , and secondary voltages of 104 and 208, and a frequency of 60 cycles per second. The core loss is 86.5 watts and the total copper loss at full load is 117.8 watts. The high efficiency throughout a wide range of load is worthy of note and is typical of all well-designed transformers.

Fig. 226 shows graphically the various losses and the efficiency of a Westinghouse air-blast transformer rated at 550 kilowatts used to step up from 500 volts


Fig. 225. ' Graphic Relation between Efficiency and Output for a Core Type Transformer to 10,500 volts at a frequency of 25 cycles per second (3,000 alternations per minute).

In Fig. 226, the curve representing the iron loss is plotted as a horizontal straight line because the iron loss for a given transformer is practically constant for all loads. The curve representing the copper loss is a parabola. The efficiency of a given transformer is a maximum at that load for which the iron loss is equal to the copper loss. This load is evidently the abscissa of the point at which the iron-loss line intersects the cop-per-loss curve. As seen in Fig. 226, the maximum efficiency occurs at about 101 per cent of full load; at any other load (for the given transformer) the efficiency will be less than at 101 per cent of full load.

The transformer output (non-inductive receiving circuit) is $E^{\prime \prime} I^{\prime \prime}$. The internal loss is $W_{h}+W_{e}+W_{c}$, so that the intake is $E^{\prime \prime} I^{\prime \prime}$ $+W_{h}+W_{e}+W_{c}$, and the efficiency is

$$
\begin{equation*}
\text { efficiency }=\frac{E^{\prime \prime} I^{\prime \prime}}{E^{\prime \prime} I^{\prime \prime}+W_{h}+W_{e}+W_{c}} \tag{38}
\end{equation*}
$$

A complete calculation of efficiency is worked out on page 320.

All-day Efficiency. Usually a transformer is connected to the mains continuously, and current is taken from the secondary for a few hours only, each day. In this case the iron loss is incessant and the copper loss is intermittent. The total work given to the transformer during the day may greatly exceed the total work given out by it, especially if the continuous iron losses are not reduced to as


Fig. 226. Efficiency Curves for a Westinghouse Air-Blast Transformer
low a value as possible. The ratio total energy given out by the transformer $\div$ total energy received by the transformer during the day is called the "all-day efficiency" of the transformer. In other words it is the ratio $\frac{\text { total watt-hours output }}{\text { total watt-hours input }}$ during the day. The all-day efficiency is given by the formula
all-day efficiency $=\frac{E^{\prime \prime} I^{\prime \prime} \times t}{E^{\prime \prime} I^{\prime \prime} \times t+\left(W_{h}+W_{e}\right) \times 24+W_{c} \times t}$
in which $t$ is the number of hours during the day of 24 hours that the transformer is loaded, and $I^{\prime \prime}$ is the average current delivered by the secondary while the transformer is loaded. The other symbols have the same significance as on pages 261-263.

Since the iron loss of a given transformer is continuous as long as the transformer is connected to the primary supply mains, it follows that to obtain a high all-day efficiency, it is necessary to use a transformer whose iron loss is as small as possible. In general, if a transformer is to be operated at light loads the greater part of the day, as is the case in electric lighting service, it is much more economical to use a transformer designed for a small iron loss than for a small full-load copper loss. In very small transformers, the iron and the copper losses are made about equal, but for outputs of about 5 kilowatts and upwards, the iron loss is often made only about one-half as great as the full-load copper loss. (See columns 2 and 3, Table XII.)

On the other hand, in cases where transformers are to be used to supply a load of induction motors, and the conditions are such that the transformers are operated at or near full load during most of the day, it is more economical to use transformers designed to give equal iron and copper losses at full load. In general, a given transformer works at the maximum efficiency when it is operating at a load such that its iron loss is equal to its total copper loss.

Transformer Regulation. The secondary terminal voltage of a transformer falls off in value with increasing load, and rises with decreasing load. The Standardization Rules of the A.I.E. define regulation as follows: In constant-potential transformers, the regulation is the difference between the no-load and rated-load values of the secondary terminal voltage, at the specified power factor (with constant primary impressed terminal voitage) expressed in per cent of the rated-load secondary voltage, the primary voltage being adjusted so that the rated output is delivered at rated secondary voltage.

Example. A certain 5 -kw. transformer gave a secondary terminal voltage of 200 volts at rated non-inductive load; when the load was reduced to zero, the voltage rose to 203 volts.

According to the above definition, the regulation of this transformer is

$$
[(203-200) \div 200] \times 100=1.5 \text { per cent }
$$

The regulation of the average distributing transformers on the market at present varies from about 1 per cent to 3.5 per cent, and when operated on power factors as low as 60 per cent, the regulation in some cases is as high as 4 per cent. In incandescent lighting it is especially important to have transformers with a low regulation, otherwise the voltage at the lamps will fluctuate excessively as the
load changes. This means unsatisfactory illumination for the consumer, and more frequent lamp renewals for the lighting company. Thus, for lighting work it is important to specify a low regulation.

The regulation of a transformer is lower when used for supplying a non-inductive load (such as incandescent lamps), than when supplying an inductive load (such as induction motors). For a given kind of load the regulation of large transformers is lower than for small sizes. These matters are clearly brought out by a careful study of the values of regulation given in Table XII. The method of calculating regulation is explained on page 317.

Practical and Ultimate Limits of Output. When the secondary current of a transformer is increased, the secondary electromotive force drops off, and the power output increases with the current, reaching a maximum as in the case of the alternator. This maximum power output is the ultimate limit of output of the transformer. Practically, the output of a transformer is limited to a much smaller value than this maximum output, (a) because of the necessity of cool running; (b) because in most cases it is necessary that the secondary electromotive force be nearly constant; and (c) because the efficiency of a transformer is low at excessive outputs.

Small transformers have relatively large radiating surfaces; and in such transformers the requirements of a small (good) regulation, as a rule, determine the allowable output.

Large transformers, on the other hand, have relatively small radiating surfaces, and their allowable output is limited by the permissible rise in temperature. Some transformers ranging in rated output from 100 to 1,250 kilowatts are provided with air passages through which air is made to circulate by a fan. Transformers which are not cooled by an air blast up to about 500 kilowatts capacity are submerged in oil, which, by convection, carries heat from the transformer to its enclosing case, where it is radiated. Very large transformers are not only submerged in oil, but are also water-cooled.

Rating of Transformers. A transformer is rated according to the power it can deliver continuously to a non-inductive receiving circuit without undue heating; and the ratio of transformation, together with a specification of the frequency and effective value of the primary electromotive force to which the transformer is adapted, are also given by the manufacturer.

In order to secure uniformity in the rating of transformers by manufacturers, the American Institute of Electrical Engineers has formulated Standardization Rules relating to permissible temperatures and temperature rises. The Rules, with special application to the rating of transformers are given on pages' 317 and 319 .

Abnormal Conditions of Operation. Transformers are frequently used under conditions departing more or less widely from the conditions specified by the manufacturer in regard to values of primary and secondary voltages, frequency, and current output. Thus, if a transformer is used with a primary applied voltage in excess of the rated value (frequency being unchanged), the core flux will be increased according to equation (34) and the core losses will be increased according to equations (35) and (36).

If, on the other hand, a transformer is used with a frequency less than the rated frequency (the voltage applied to the primary being unchanged), the core flux will be increased according to equation (34), and the hysteresis loss in the core will be increased according to equation (35); whereas, the eddy-current loss is unchanged.

The effect on the core losses of varying the line voltage of a 60 -cycle transformer is illustrated in the tabular matter below. This relation will be dependent upon the quality of iron used in the transformers and on how near it is being worked at normal voltage to the knee of the magnetization curve as well as upon the relation between hysteresis and eddy current losses.

| Per Cent of <br> Rated Voltage | Per Cent of <br> Core Los at <br> Rated Voltage | Per Cent of <br> Rated Voltage | Per Cent of <br> Core L Loss at <br> Rated Voltage |
| :---: | :---: | :---: | :---: |
| 80.8 | 66.0 | 104.0 | 109.0 |
| 84.0 | 71.0 | 105.0 | 111.0 |
| 88.0 | 77.0 | 110.0 | 123.0 |
| 92.0 | 84.0 | 115.0 | 137.0 |
| 96.0 | 92.0 | 119.0 | 149.0 |
| 100.0 | 100.0 |  |  |

The increase of core loss due to increase of primary applied voltage, or to decrease of frequency, or to both, may be compensated for by reducing the allowable current output, and thereby reducing the copper loss, in order that the total heating of the transformer may not exceed the normal amount.

Examples. A given transformer is rated at 5 kilowatts, and is designed to take current from 1,100 -volt mains at a frequency of 60 cycles per second. Under these conditions hysteresis loss $W_{h}$, eddy current loss $W_{e}$, and copper loss $W_{\mathrm{c}}$, will be called normal.
(a) The transformer is loaded so that the output is 6 kilowatts at the rated electromotive force and frequency. Find $W_{c}$ in terms of normal.
(b) The transformer is used at rated electromotive force, but at a frequency of 75 cycles per second. Find $W_{h}$ and $W_{e}$ each in terms of normal.
(c) The transformer is used at rated frequency, but with primary electromotive force of 1,500 volts. Find $W_{h}$ and $W_{e}$ each in terms of normal.
(d) The transformer is used on primary electromotive force of 1,500 volts. Find $f$ for which $W_{h}$ is normal.
(e) With primary electromotive force of 1,500 volts, what load would give normal $W_{c}$ ?

Solutions. (a) Increasing the output in the ratio 5 to 6, increases both primary current and secondary current in the same ratio and, therefore, increases $I^{\prime 2} R^{\prime}$ and $I^{\prime 2} R^{\prime \prime}$ in the ratio of $5^{2}$ to $6^{2}$. Therefore, the total copper loss becomes $\frac{36}{25}=1.44$ times the normal copper loss.
(b) Increasing the frequency in the ratio 60 to 75 decreases the fluxdensity B in the same ratio, namely, 75 to 60 . Hence, the hysteresis loss per cycle is decreased in the ratio $75^{1.8}$ to $60^{1.6}$, and the total hysteresis loss is $\frac{75}{60} \times \frac{60^{1.6}}{75^{1.6}}$ or $\left(\frac{60}{75}\right)$, or 0.87 . Therefore, the total hysteresis loss at the increased frequency is 0.87 times the normal hysteresis loss.

From equation (36), the eddy current loss is proportional to $f^{2} \mathbf{B}^{\mathbf{2}}$. In the case under consideration, $f$ is increased and $\mathbf{B}$ is decreased in the same ratio, so that the product $f^{2} B^{2}$ remains unchanged. Hence, the eddy-current loss in a given transformer is independent of the frequency with given primary applied voltage.
(c) Increasing the primary voltage in the ratio 11 to 15 increases the flux-density B in the same ratio. Therefore, the hysteresis loss is increased in the ratio $\left(\frac{15}{11}\right)^{1.6}=1.64$; and the eddy current loss is increased in the ratio $\left(\frac{15}{11}\right)^{2}=1.86$. That is, the hysteresis loss becomes 1.64 times its normal value; and the eddy current loss becomes 1.86 times its normal value.
(d) The hysteresis loss is proportional to $f \mathbf{B}^{1.6}$; and $\mathbf{B}$ is proportional to $\frac{E^{\prime}}{f}$; therefore, the hysteresis loss is proportional to $f \times\left(\frac{E^{\prime}}{f}\right)^{1.6}$ or to $\frac{E^{\prime 1.6}}{f^{0.6}}$. This ratio $\frac{E^{\prime .1 .6}}{f^{0.6}}$ must have the same value under the normal conditions as under
the new conditions, if the hysteresis loss is to be the same. That is

$$
\frac{(1,100)^{1.6}}{(60)^{0.6}}=\frac{(1,500)^{1.6}}{x^{0.6}}
$$

or

$$
(1,100)^{1.6} \times x^{0.6}=(60)^{0.6} \times(1,500)^{1.6}
$$

or taking logarithms of both sides, we have
$(1.6 \times \log 1,100)+(0.6 \times \log x)=(0.6 \times \log 60)+(1.6 \times \log 1,500) ;$ or, $(0.6 \times \log x)=(0.6 \times \log 60)+(1.6 \times \log 1,500)-(1.6 \times \log 1,100) ;$ from which we find $x$ equal to 137 cycles per second.
(e) The copper loss $W_{c}$ has normal value when the primary and secondary currents have normal value. Therefore, when the primary applied voltage is increased in the ratio 11 to 15 , the output increases in the same ratio, the current and $W_{c}$ being normal. Therefore, the output is $\frac{15}{11} \times 5 \mathrm{kw}$., or 6.82 kilowatts, to give normal $W_{c}$ with a primary applied voltage of 1,500 volts.

## COMMERCIAL TYPES OF TRANSFORMERS

Transformers may be classified, according to the relative disposition of the iron and copper, into core-type transformers and shelltype transformers. The distinc-


Fig. 227. Core-Type of Transformer Assembled with Case Removed tion between these two types of transformer may be understood by referring to Figs. 227 and 257. In the core type, Fig. 227, it will be seen that the iron core is almost entirely surrounded by the copper windings, while in the shell type, Fig. 257, the coils are almost entirely surrounded by the sheet-iron laminations, forming a shell.

There has been much discussion as to the relative merits of the shell and core types, some manufacturers going so far as to claim exclusive advantages for one or the other. The fact is that no general conclusion can be drawn as to which type is better, for each possesses inherent characteristics which specially adapt it to certain conditions. A brief comparison of these characteristics will aid in determining which type to use for specified conditions of service, size, voltage, and the like.

The core type has relatively a lighter core of smaller sectional area but a greater length of magnetic circuit, while the copper is relatively heavier, containing more turns, although of shorter mean length. The core type is more easily wound as cylindrical formed coils may be used, and the coils are more accessible and expose more surface to radiation. The core type, with its relatively large winding space, is better adapted for high voltages which require many turns and large space for insulation, smaller currents and, therefore, small wires, and higher frequencies with low magnetic flux densities.

The shell type, on the other hand, is particularly suited for transformers of moderate voltage, requiring few turns and little insulation, large currents, and low frequency with corresponding mag-


Fig. 228. Standard Form of Laminated Transformer Core netic flux.

The net result is that manufacturers generally adopt the core type for transformers of small capacity and high voltage, and the shell type for large transformers, even up to 150,000 volts.

Substantial advancement in transformer design has been made in the past few years, notably in the combination of the advantages of both the core and the shell types in one transformer, in the use of silicon-steel for the laminations, thus greatly reducing the core loss, in the use of better insulating materials and meth-

Fig. 229. Core with Upper Yoke Removed, Showing One Coil_in Position
 ods, and in more effective methods of cooling. With these improved materials and methods it is even feasible to build reliable transformers with outputs up to 14,000 kilovolt amperes, and for voltages up to 150,000, or higher.

Core Type. Fig. 227 shows a complete core-type transformer without its case; Fig. 228 shows the complete core built up of thin
sheet steel strips, or stampings. The two upright portions of the core upon which the wire coils are placed, as shown in Figs. 227, 229,


Fig. 230. Both Transformer Coils in Position, Showing Laminated Strips Which Form the Upper Yoke of Core
and 230, are called the limbs of the core. The short horizontal parts of the core that do not have windings of wire upon them are called yokes. These yokes serve to complete the magnetic circuit, and they are made just long enough to give room for the coils, as shown in Fig. 231. Fig. 229 shows the core with its upper connecting yoke removed to permit of the slipping of the coils into position on the core; the left-hand limb of the core is shown wrapped with a thick layer of insulating material in order to prevent electrical contact between the wire of the coil and the iron of the core. Fig. 230 shows the coils in place, and a pile of loose sheet-iron


Fig. 231. Section Through CoreType of Transformer stampings which are used to form the upper connecting yoke.

Fig. 231 is a sectional view of the completed core and coils; and Fig. 232 is a sectional view of the complete transformer enclosed
in a cast-iron case, which is usually filled with oil. Half of the primary coil and half of the secondary coil also is placed upon each limb of the core, as shown by the sectional view, Fig. 231. The terminals of each half of the primary (fine-wire) coil are connected to binding screws on a porcelain connection board mounted on top of the transformers inside the case, as shown at the right in Fig, 232. The terminals of each half of the secondary (coarse-wire) coil are passed through porcelain bushings to the outside of the case, as shown at the right in Fig. 232. The half coils of the primary may be connected in parallel or in series according


Fig. 232. Section Through Commercial Core-Type of Tranaformer to the value of the voltage applied to the primary; and the half coils of the secondary may be connected in parallel or in series according to the desired value of the secondary voltage, as explained on page 242.


Fig. 233. Plan of Transformer Showing Strap Connections for Arranging Primaries in Series or Parallel

Fig. 233 is a top view of the transformer with the cover of its case removed showing the terminals of the secondary coils, and the
porcelain connection-board to which the terminals of the primary coil are connected. The change from series to parallel connection of the halves of the primary coils, is effected by means of the copper connecting straps $S S$.

The style of core-type transformer illustrated in Figs. 227 to 232 is adopted by several manufacturers for transformers of small output. The style of core-type transformer adopted by the General Electric Company for larger outputs up to 350 kilowatts, is illustrated in Figs. 234 to 238, which show a high voltage core-type


Fig. 234. General Electric CoreType Transformer Partly Assembled transformer designed for 60 cycles. These transformers are submerged in tanks of oil, one of which is shown in Fig. 238, and which are made with a cast-iron base and sides of heavy steel with deep corrugations to facilitate cooling of the tank by radiation. Fig. 236 shows the arrangement of the coils on the core; it also shows the passages between the core and the inside coil, and between the two coils, for the circulation of the oil.

Oil is a better heat-conducting medium than air; it carries heat from a transformer to the containing case much better than air, so that a transformer in oil will show a much lower temperature. The use of oil, moreover, preserves the insulation, keeping it soft and pliable, and preventing oxidation by air; consequently, its use is advantageous in producing proper conditions to maintain uniform core loss and a superior insulation. Furthermore, oil is itself a very good insulator having the valuable property common to all liquid insulators, that it is not permanently damaged by a puncture caused by lightning; for example, in this case the resistance of the oil is only momentarily broken down, the oil immediately flowing into the break and sealing the insulation.

Another variety of the core type is that adopted by the CrockerWheeler Company in their "remek-type" transformer. The core punchings are shown in Fig. 239. Each lamination consists of a
core and yoke punching, which are punched simultaneously from the whole sheet by a compound die. In assembling the transformer, care is taken to get the core and yoke punchings back into the same


Fig. 235. General Electric Core-Type Transformer with Tank Removed
relative position as before punching, in order to secure the lowest possible magnetic reluctance of the joints, and hence a small magnetizing current. The laminations when assembled are held to-
gether by iron end plates bolted together and the core portions are securely clamped together and to the yokes, as shown in Fig. 240. The core, as seen, constitutes a double mag-


Fig. 236. Coils and Core of General Electric Transformer Showing Oil Ducts netic circuit, the magnetic flux due to the windings on the central core dividing on passing into the yokes, half on each side. The yoke laminations are made wider than onehalf the width of the central core, in order to reduce the flux density in them, thereby reducing the core loss and the magnetizing current.

The windings consist of former-wound coils which surround the core as illustrated in Fig. 241. The high voltage winding is placed between the two halves of the low voltage winding in order to reduce magnetic leakage and thus improve regulation.

Vertical ducts are provided between the windings, and between the core and windings, as may be seen in Fig. 241. This arrangement is to insure a free circulation of the oil in which the transformer is immersed and facilitates dissipation of the heat by radiation and convection.

The assembled transformer core and coils ready to be placed in the oil-filled tank are shown in Fig. 242. The high voltage termi-


Fig. 237. Specimen Coils for General Electric Core-Type Transformer
nals are shown connected to a terminal board in front, and the four secondary terminals, two for each coil, are shown in the rear.

These "remek-type" transformers are wound for a primary voltage of 2,200 , secondary voltages of 220 and 110 , and are built in sixteen sizes ranging from 0.6 to 50 kilowatts.

Fig. 243 shows a $10-\mathrm{kw}$. combination core- and shell-type


Fig. 238. General Electric Core-Type Transformer Completely Assembled Showing Deep Corrugations to Facilitate Cooling
transiormer removed from its tank. It is called the distributed core type (type H form K) by its makers, the General Electric Company, although it resembles the shell rather than the core type.

The primary and the secondary coils are wound on formers of cylindrical shape, and placed on the center limb only. They are separated from each other and from the core by spacing blocks, thus forming ducts and passages for free circulation of the oil with which the containing tank is filled. Fig. 244 shows clearly the disposition of the coils, mica insulation, and oil ducts, with respect to the central limb.

The core, as shown in Fig. 245, contains four magnetic circuits in parallel, each circuit consisting of a separate core similar in general outline to that used in the simple core type. One limb of each magnetic circuit is built up of two different widths of punchings, forming a cross-section such that when the four circuits are assem-


Fig. 239. Core Punchings for Crocker-Wheeler "Remek-Type" Transformer
bled together they interlock to form a common central limb upon which the primary and the secondary coils are-placed.

The four remaining legs consist of punchings of equal width. These occupy a position surrounding the coil at equal distances from the center, on the four sides, forming a channel between each leg and coil, thereby presenting large surfaces to the oil and allowing it free access to all parts of the winding.

The punchings of each size of transformer are all of the same length, assembled alternately, and forming two lap joints equally distributed in the four corners of the core, thereby giving a magnetic circuit of very low reluctance.

This type of construction combines the best features of both shell and core types, namely a short mean length of turn in the windings, and a short length of magnetic circuit in the core.


Fig. 240. Assembled Core Showing Method of Clamping


Fig. 241. Method of Assembling Coils Showing Oil Ducts


Fig. 242. Assembled "Remek" Core and Coils


Fig. 243. General Electric Combination Core and Shell Type Transformer


Fig. 244. Details of Core and Coils Showing Oil Ducts.


Fig. 245. Plan of Assembled Core and Coils Showing Four Magnetic Circuits in Parallel


Fig. 246. Westinghouse Combination Shell and Core Type


Fig. 247. Assembled Coils for Westinghouse Transformer

Another example of the combined shell- and core-type of transformer is the distributing transformer (types $S$ and SA ) of the West-


Fig. 248. Section through Westinghouse "Type S" Transformer


Fig. 250. Pancake Coil for Shell-Type Transformer


Fig. 249 . Part Section through Westinghouse Transformer
 Wound with Tape
inghouse Electric Company. These transformers are almost identical in form and design with the "type H form K" transformers just described.

Fig. 246 shows the core and coils and terminals of the "type S" transformers removed from the case, and a general view of the case. Fig. 247 shows the appearance of the assembled coils and leads of 10 - and $15-\mathrm{kw}$. transformers, with the ventilating ducts. The figure shows the high voltage winding mounted concentrically between two low voltage windings, which arrangement reduces magnetic leakage and thus improves "regulation."

As illustrated in Fig. 246, separate high and low voltage porcelain terminal blocks are mounted upon extensions of the upper end frame, and are thus kept well apart to prevent mistakes in making electrical connection between the high and low voltage coils.

Fig. 248 is a section through a $\frac{1}{2}$-kw. "type $S$ " transformer, and shows the distribution of the windings in layers, the high voltage coils


Fig. 252. Pancake Coil Showing Use of Insulated Flat Strips being placed between two sections of low voltage coils. Fig. 249 is a section through a $50-\mathrm{kw}$. transformer and shows the large oil ducts arranged between sections of the high voltage winding. The angle irons used to clamp the laminations together are shown at the top and bottom. These distributing "type S" transformers are built in sixteen sizes from $\frac{1}{2}$ to 50 kilowatts, and for the standard primary voltages of 2,200 and 1,100 , and secondary voltages of 220 and 110. Standard frequencies are 25,40 , and 60 cycles.

Shell Type. In shell-type transformers the coils, both primary and secondary, are usually wound in pancake form on formers, as shown in Figs. 250 to 253 . The coils in small transformers are wound with round wire, but in the larger sizes flat rectangular copper strip is used with one turn per layer, in many layers, Figs.


Fig. 253. Method of Binding up Pancake Strip Coils 250 and 252. The insulation between turns consists of paper, mica, or varnished cambric, or all three together, according to the voltage
and other conditions. The thin pancake coils are then treated with an insulating compound and wound with a number of layers of tape according to the voltage for which they are designed, each layer being given several coats of insulating varnish baked on in ovens. The coils are then assembled into groups of two or more sections, Fig. 253, and the groups into complete windings, the primary and the secondary being intermixed or sandwiched in order to


Fig. 254. Partially Assembled Air Blast Shell-Type of Transformer
reduce magnetic leakage. Between the various groups suitable insulating barriers are interposed.

In transformers for high voltages used for transmission of power, the insulation of a considerable length of the conductor nearest the terminal leads is heavily reinforced. This is a very important precaution, as the extra dielectric strength of the insulation of these end turns is a safeguard against break-downs which might otherwise occur due to the excessive voltages from lightning discharges or other surges to which transmission lines are unfortunately subjected.

The various groups of windings are encased in a box-like structure which, while serving as an electrical and a mechanical protection, is so arranged that it does not obstruct the air or oil ducts which are provided between the various coil sections. The windings are then set up vertically in the bottom frame and the magnetic circuit is built up piece by piece around them in the form of rectangular sheets of steel, Fig. 254. In building up the core out of the stampings care is taken to break joints in successive layers. The top frame is then put on and tightly clamped to the bottom, thus compressing and holding the core. After the addition of connection board, leads, insulating bushings, and the like, the transformer is ready for its


Fig. 255. Sectional View of Fort Wayne ShellType Transformer casing or tank.

Fig. 254 shows a large General Electric air-blast, shell-type transformer at that stage of construction where the core stampings are being built up around the coils, the coils being protected by thick strips of insulating material. In this air-blast transformer, air passages or ducts are provided between the layers of the coils, and at intervals, between the core stampings, as shown in the figure. The air for cooling the transformer is admitted at the base, and passes vertically through the ducts in the coils. Air is also admitted to the air ducts in the core through a damper on one


Fig. 256. Sample Coils for Fort Wayne Transformer side of the transformer, and escapes through the perforations in the casing on the opposite side

Figs. 255 to 258 give a general view and structural details of a $15-\mathrm{kw}$. shell-type transformer manufactured by the Fort Wayne Electric Works. The shape of the core stampings is shown in Fig. 255 , which is a section through the transformer. The secondary winding, as seen in the figure, is subdivided into four sections, and the primary into two. The order of the sections as arranged in the "window" of the core is: One-fourth total secondary turns, one-half total primary turns, one-fourth secondary turns, one-fourth


Fig. 257. Assembled Core and Coils for Fort Wayne Transformer
secondary turns, one-half primary turns, and one-fourth secondary turns. This intermixing the various sections reduces greatly magnetic leakage, and thus improves regulation.

Fig. 256 shows a group of pancake coils insulated and taped ready to be assembled.

Fig. 257 shows the core and coils assembled. The four leads on the left are from the secondary coils, and the six leads on the right are the four primary terminals to which are added two extra taps.

Hig. 258 is a top view of the transformer with the cover re-
moved and shows how the leads are brought up to the connection board and out of the case. Each of the two primary sections is provided with what is called a ten per cent tap. That is, while the normal voltage for which each primary section is wound is 1,100 volts, by using the ten per cent taps, it is possible to change the ratio of transformation, and obtain ratios of $20: 1,19: 1,18: 1,10: 1$, $9.5: 1,9: 1,5: 1$, and $4.5: 1$, according to the connections of the primary and the secondary sections, whether in series or in parallel. The advantage of these ten per cent taps is that it permits a transformer to be used near the station where the voltage is high or at the end of a feeder where the voltage may be five or ten per cent lower, by simply changing the electrical connections at the terminal board shown in Fig. 258.


Fig. 258. Top View of Fort Wayne Transformer with Cover Removed Showing Arrangement of Lead Wires

Figs. 259 to 261 show constructive details peculiar to very large shell-type transformers which are usually of the water-cooled type. Fig. 259 represents a General Electric transformer with its containing tank removed to show the method of suspending the core and cooling coils from the cover. The cooling coils are of lap-welded wrought-iron pipe with electrically welded joints. Water is kept circulating through these coils in order that the oil filling the case and surrounding the transformer may be kept cool while the transformer is in operation. The core and the coils are first assembled after which they are tightly clamped and suspended from the cover
by means of heavy bolts, as shown in Fig. 259. The completed transformer filled with oil may be easily lifted and moved about by means of the heavy lifting lugs on the upper side of the cover.

Some of the advantages of this cover suspension construction are: (a) all coil terminals are brought out through the cover and,


Fig. 259. General Electric Shell-Type Water-Cooled Transformer-Tank Removed and Cooling Coils Shown in Section
therefore, are not interfered with when the transformer is removed from its tank. (b) The terminal board at the top is accessible through openings in the cover and changes in electrical connection an be made by simply raising the transformer a few feet out of the tank by a crane, without drawing off any oil. Inspection of all parts is also easily and quickly made. (c) There being but a few bolts to loosen, it is easy to remove the transformer from the tank,
or to lift tank and all by means of the lugs on the cover. (d) With this construction there is no need of lowering crane hooks or chains into the tank, thereby avoiding possible danger to insulators and to coil insulation.

The construction of water-cooled transformers is, in mechanical


Fig. 260. General Electric Water-Cooled Transformer Showing Coils in Position
and electrical design, similar to the oil-cooled transformers, the only difference being in the tank. In the water-cooled type the tank is built up of heavy boiler plate iron riveted to a cast-iron base. All joints on the tank are heavily riveted and thoroughly caulked to make them oil tight.

Three=Phase Transformers. During the past few years the size of transformer units has been steadily increasing in response
to the demand from the power companies engaged in generating and transmitting electrical power on an enormous scale. Longdistance transmission of electrical power is most economically carried out by the three-phase system, which is the standard prac-


Fig. 261. General Electric Water-Cooled Transformer Completely Assembled
tice. These conditions have caused a demand for large three-phase transformers, which under certain conditions are to be preferred to three single-phase transformers of the same aggregate capacity, but no general rule can be given as to the relative value of the two types.

The advantages of the three-phase transformer over a group of three single-phase transformers of the same total capacity are:
(1) Lower cost.
(2) Higher efficiency
(3) Requires less floor space.
(4) Has less weight.
(5). Connections and outside wiring very much simplified as only three primary and three secondary leads are usually brought out.
(6) Lower transportation charges and cost of installation.
(7) Presents a symmetrical and compact appearance.

The disadvantages of the three-phase type are:
(1) Greater cost of spare units.
(2) Greater derangement of service in case of break-down.
(3) Greater cost of repairs.
(4) Reduced capacity obtainable in self-cooling units.
(5) Greater difficulty in bringing out taps for a large number of voltages.

In general single-phase transformers are preferable where only one transformer is installed and where the expense of a spare transformer would not be warranted. In such installations the burnout of one phase of a three-phase unit would cause considerable inconvenience for the reason that the whole transformer would have to be disconnected from the circuit before repairs could be made. If, however, single-phase transformers are used, the damaged transformer can be cut out with a minimum amount of trouble, and the other two transformers can be operated at normal temperature open $\Delta$ - (or $\mathbf{V}$-)connected at 58 per cent of the normal capacity of the group of three transformers, until the third unit can be replaced.

With a three-phase shell-type transformer, if both the primary and the secondary are $\Delta$-connected, trouble in one phase will not prevent the use of the other two phases in open delta. By shortcircuiting both primary and secondary of the defective phase and cutting it out of circuit, the magnetic flux in that section is entirely neutralized. This cannot be done, however, with any but $\Delta$-connected transformers. Where a large number of three-phase transformers can be used, it is generally advisable to install three-phase units.

Three-phase transformers are made both of the core and the shell types, according to circumstances. Thus, Fig. 262 shows a three-phase core-type transformer with its case removed, made by the General Electric Company. A three-legged core is used, each leg being wound with the primary and the secondary coils of one
phase. The magnetic circuit, arranged as shown in Fig. 216, is explained on page 47 . Since the weight of the core and the coils is greater than in the single-phase transformers, the core-clamps and other mechanical parts are made larger and stronger while two bolts on each side, instead of one, are used to support it from the cover. Three leads only are brought out of the cover on the primary and the secondary sides, all connections being made on the inside of the tank.

The General Electric Company has built three-phase transformers of the shell type up to 10,000 kilowatts capacity, for 100,000


Fig. 262. General Electric Three-Phase Core-Type Transformer-Case Removed volts primary and 11,000 volts secondary, and a frequency of 60 cycles. They are now building them for outputs of more than 14,000 kilowatts.

Fig. 263 shows a three-phase shell-type of transformer removed from its tank. It, is rated at 1,800 kilovolt-amperes, 48,000 volts and 25 cycles. This transformer, which is built by the Westinghouse Company, is designed to be placed in a tank filled with oil which is kept cool by water circulating in a coil of brass tubing surrounding the transformer and below the surface of the coil.

Cooling of Transformers. Transformers of moderate size have large radiating surface compared with their losses. Such transformers, therefore, can radiate the heat due to core losses and copper losses without excessive rise of temperature.

A transformer which is twice as large as a given transformer in length, in breadth, and in thickness, has eight times as much volume but only four times as much radiating surface as the latter. The large-transformer having nearly eight times the losses and faur times the radiating surface of the smaller transformer would rise to a much higher temperature than the smaller transformer in
order to radiate the heat due to its losses. Large transformers must, therefore, be provided with special means for cooling. It is mue'n cheaper to provide special cooling devices than to attempi to make the transformer large enough to keep cool by natural radiation.


Fig. 263. Westinghouse Three-Phase Shell-Type Transformer-Tank Removed

Various methods of cooling are adopted in practice, according to which the following classification of transformers may be made:

Self-cooling dry transformers.
Self-cooling oil-filled transformers.
Transformers cooled by forced current of air.
Transformers oil-filled cooled by forced current of water.
Self-Cooling Dry Transformers. These transformers are usually of small output, and no special means of cooling is provided, the
natural radiation being depended upon for cooling. Some larger ones up to 5 - or 10 -kilowatt capacity have been made in this way, but they are heavier and more expensive than if oil-cooled.

Self-Cooling, Oil-Filled Transformers. Transformers of this type are very generally employed, the entire core and coils being immersed in oil. Transformers are practically always enclosed in a cast-iron or sheet-steel case, and this is simply filled with a special high-grade mineral oil. No increase in cooling surface is thereby secured, but the natural circulation of the oil tends to equalize the temperature of the various parts, and carries the heat to the case, from which it is radiated. In most self-cooling types, the case is made with external ribs or corrugations to increase its radiating surface. The large volume of oil also absorbs considerable heat, so that the temperature rises more slowly. Hence, for moderate periods of operation, up to 3 or 4 hours-which is ordinarily sufficient in electric lighting-the maximum temperature would not be reached. Another advantage gained by this arrangement is an improvement in insulation. This is due to the high insulating qualities of the oil itself, and to the fact that a disruptive discharge takes place through it much less readily than through the air that it displaces, distances being the same. This arrangement possesses, moreover, the power of self-repairing any break in the insulation. If ordinary materials, such as cloth or mica, become punctured, they lose their insulating properties, and the apparatus cannot be used until the fault is repaired, which ordinarily involves considerable time and expense. On the other hand, if oil is punctured, it tends to close in and repair the break, unless the discharge lasts so long that a charring occurs, which may make a permanent conducting path.

The chief objection to the use of oil is the danger of fire. If a short-circuit occurs inside the transformer, the oil may be thrown out and ignited at the same time; or a fire started in any other way might be made far more disastrous than it would otherwise be owing to the presence of a large quantity of oil. In this way, several power plants have been destroyed by fire with large loss of property. There is no special precaution that will entirely eliminate this risk; but care in locating such transformers, in avoiding overheating, and in protecting the machines by effective lightning arrosters, will reduce the hazard.

Oil-cooled transformers can be built for any voltage and to almost any size, although the economical maximum limit in output is reached at about 500 kilowatts.


Fig. 264. Bank of Air-Blast Transformers Showing Arrangement of Air Chamber
Air-Blast Transformers. These transformers are now commonly employed, and have advantages over those of the oil-cooled type, in that the danger of fire is avoided, and the cooling effect can be regulated in accordance with working conditions. They are so constructed that air can circulate through and around the core and coils, the ventilation being forced by a blower driven by a motor. A transformer of $100-\mathrm{kw}$. capacity requires about 450 cubic feet of air per minute at a pressure of 0.5 ounce per square inch, the power consumed by the blower set being less than one per cent of the full-load output of the transformer. The flow of air is controlled by dampers; and the proper amount of air can be determined from its temperature as it issues from the top; ordinarily this temperature should not be more than $20^{\circ} \mathrm{C}$. above the temperature of the room.


Fig. 265. Section of Air-Blast Transformer Showing Circulating Currents

Fig. 264 shows a bank of six large air-blast transformers supported on I beams over an air chamber supplied with air from a fan blower. The air passes in at the buttom of each transformer
case, penetrates through ducts in the core and coils, and passes out at the top of the case, as shown in Fig. 265.

The process of building up the core of a shell-type air-blast transformer is illustrated in Fig. 254. The air-blast type is used for moderate voltages where cooling water is expensive or not available, and is built for voltages up to 33,000 in sizes up to 5,000 kilowatts. The limiting voltage for this type is determined by the excessive thickness of the solid insulation needed and the consequent difficulty in radiating heat from the copper. For voltages above 33,000 , the oil-insulated water-cooled type is, therefore, recommended.

Water-Cooled Transformers. Transformers in which water is used for cooling are always immersed in oil. In an oil-insulated water-cooled transformer, all the heat generated in the iron and copper, except a small amount radiated from the surface of the case, is dissipated by means of water flowing through coils of pipe placed under the oil near the surface. As the copper and iron become heated, the heat is transferred to the oil coming into contact with their surfaces. As the oil is heated up, convection currents are produced by the hot oil rising from the transformer and flowing over towards the sides of the case; here it comes into contact with the surface of the cooling coils, giving up its heat to them and then sinks along the sides of the case to the bottom where it is ready to be heated up again and repeat the cycle. This method of water cooling is so effective that very little heat is dissipated from the tank and there is nothing to be gained by corrugations. From this it is seen that the cooling coils form a very important part of the oil-insulated water-cooled transformer and that, if for any reason they should fail to perform their work of carrying away the heat or should develop a leak, the transformer may be seriously damaged or even destroycd.

Water cooling is at present the most effective method for dissipating the heat generated in very large units and whereever the voltages exceed 33,000 volts. It is very convenient for water-power plants, the supply of water being at hand; but where a natural flow is not available, and pumps or city water mains have to be utilized, the expense may be prohibitive.*

[^25]Series or Current Transformers. The chief difference between the series (current) and the shunt (voltage) transformer is, as the name implies, in the manner of connecting it in the circuit. The series transformer has its primary coil in series with the line, while the shunt transformer has its primary shunted across the line wires. In the latter the primary voltage is determined by that of the main circuit, and the primary current is determined by the impedance of the transformer, varying according to the secondary load. In the series transformer, on the other hand, the primary current is determined by the current in the main circuit, and the primary voltage is merely the drop across the primary terminals due to the primary impedance. The total impedance of the secondary circuit being normally constant, the change in load is due to a simultaneous change in the primary current and voltage. In the shunt (voltage) transformer the actual ratio of the primary to the secondary current is of minor consequence, whereas a constant ratio of voltages (especially in the so-called potential transformers used in connection with voltmeters and wattmeters) is of the highest importance. In the design of a series transformer, the questions of


Fig. 266. Portable Current Transformer Showing Primary Winding Terminals coustant-voltage ratio and of high efficiency receive no attention, whereas the matter of securing a definite ratio of secondary to primary amperes receives the most careful attention.

It was pointed out on page 249 that a transformer intended to give an accurately fixed ratio between primary and secondary currents must be so designed as to have a very small magnetizing current. This is accomplished by using a very low magnetic flux density in the iron core, so as to reduce the watts of core loss to a minimum. The magnetizing current is also reduced by using ironcore plates of high permeability and with no breaks or joints in the magnetic circuit.

The series transformer is mostly employed for insulating an ammeter, a current relay, or series coil of a wattmeter or watt-hour meter from a high voltage circuit, or for reducing the line current to a value suited for these instruments. The secondary coils of such current transformers are usually wound for five amperes.

Figs. 266 and 267 show two commercial types of portable current transformers made by the General Electric Company. Both types have cores built up of closed ring-shaped stampings of thin sheet steel. The primary winding of the transformer shown in Fig. 266 is divided into four coils, both ends of each coil being brought out at one end and connected to suitable terminal blocks. By connecting these coils in series, series-parallel, or in parallel, three different current ratios are obtained, the standard ratings being 200-100-50 amperes primary with 5


Fig. 267. Portable Current Transformer without Regular Primary Windings amperes secondary. The secondary is wound as one coil, the terminals of which are brought out at the top.

The type shown in Fig. 267 has no regular primary winding, but the core is provided with an opening through which a cable carrying the current to be measured may be passed one or more times to make the primary winding. The ratio of transformation depends upon the number of times the cable is made to pass through this opening. This form of current transformer is designed to have 1,000 ampere turns at full-load rating. The standard transformer has a 5 -ampere secondary winding giving a ratio with one primary turn passing through the center of the core of $100: 5$, or $20: 1$. If the cable is passed through the opening twice, the ratio is $50: 5$, or $10: 1$, etc. Both types are rated at 40 watts, and may be used on circuits the voltage of which does not exceed 2,500 volts.

The instrument, such as an alternating-current ammeter, to be used with these transformers, is connected directly in series with the secondary coil. If the ratio of transformation is $200: 5$ and the ammeter indicates 5 amperes, the actual value of the current passing
in the line and through the primary will be 200 amperes. In Fig. 207 the coil $A$, in series with the secondary coil $S^{*}$, may represent the switchboard ammeter.

The question may arise as to why the switchboard ammeter is not connected as a shunt across the terminals of a low-resistance link inserted in the main circuit whose current is to be measured, exactly as in the case of switchboard ammeters for direct currents. There are two reasons why this arrangement would not be permissible:
(a) An alternating current does not divide between two branches of a circuit in inverse proportion to the resistances of the branches, when either branch has inductance. Therefore, the reading of a shunted alternating-current ammeter cannọt be multiplied by a constant factor to give the total current in the main circuit.
(b) It is objectionable to have the ammeter in electrical connection with high-voltage mains. The use of the series transformer is, therefore, preferable on the grounds of both accuracy and safety.

Constant=Current Transform= ers. The constant-current transformer is a transformer specially designed to take a nearly


Fig. 268. Magnetic Circuit in ConstantCurrent Transformer constant current at varying angles of lag from constant-voltage mains, and to deliver a constant current from its secondary coil to a receiving circuit of variable resistance. The action of this transformer is as follows: The primary coil $P$ and the secondary coil $S$ of the transformer surround a long, laminated-iron core, as shown in Fig. 268. This core, and the yokes at the top, bottom, and sides, form a double magnetic circuit. The magnetic flux $\Phi$, which at a given instant passes through the primary coil, flows partly through the secondary coil as the useful flux $U$, and partly leaks across between the primary and secondary coils as the leakage flux $L$.

The leakage flux $L$, in flowing across the air spaces from core to

[^26]TABLE XIII
Constant=Current Transformer Data*

|  | Number of Lamps | $\underset{\text { LAMP }}{\text { Volts per }}$ | $\begin{aligned} & \text { SEcondary } \\ & \text { Vouts } \end{aligned}$ | Secondary Amperes |
| :---: | :---: | :---: | :---: | :---: |
| 2,200 | 50 | 76.6 | 3,830 | 6.60 |
| 2,200 | 40 | 77.6 | 3,105 | 6.70 |
| 2,200 | 30 | 77.2 | 2,315 | 6.67 |
| 2,200 | 25 | 81.4 | 2,035 | 6.65 |
| 2,200 | 20 | 84.2 | 1,685 | 6.65 |
| 2,200 | 15 | 86.0 | 1,290 | 6.65 |

*Test of a 50 -light, 6.6 -ampere constant-current transformer, frequency 60 cycies per second.
yokes, constitutes an intense magnetic field which pushes up on the secondary coil. The secondary coil is suspended, and partly counterbalanced by a weight, so that the upward push of the leakage flux just suffices to sustain the coil. If the resistance of the secondary receiving circuit is increased, the immediate result is to reduce the secondary current below its normal value, which lessens the upward push of the leakage flux on the secondary coil. The secondary coil then, owing to the unbalanced action of the weight, moves down towards $P$; the leakage flux is lessened in amount and the useful flux $U$ is increased in amount. This increase of useful flux increases the induced electromotive force in $S$; and the downward movement of $S$ continues until the induced electromotive force in $S$ is large enough to produce the normal value of the current through the increased secondary resistance.

Similarly, a decrease of resistance of the secondary receiving circuit causes a momentary increase of secondary current which increases the upward push on the secondary coil. This coil moves upwards until the secondary current is reduced to the normal value.

Table XIII shows the approximate constancy of secondary current in a constant-current transformer supplying current to a varying number of arc lamps connected in series to its secondary coil.

With 50 lamps, the primary current is 14.83 amperes; and with 20 lamps, is 14.78 amperes. In the latter case, the primary current lags 71 degrees behind the primary applied voltage; the power factor corresponding to this angle is 0.326 ; and the power received from the mains is $2,200 \times 14.78 \times 0.326=10,600$ watts, or

530 watts per lamp. With 40 lamps, the primary current lags 55 degrees behind the primary applied voltage; the power factor corresponding to this angle of lag is 0.574 ; and the power received from the mains is $2,200 \times 14.81 \times 0.574$ which is equal to 18,800 watts, or 470 watts per lamp. The efficiency of the transformer with a 50 -lamp load was 93.9 per cent, and with a 20 -lamp load it was 85.7 per cent. The power factor of the system as a whole varies


Fig. 269. Mechanism of General Electric Two-Coil Constant-Current Transformer
from 72 per cent to 76 per cent at full load, decreasing considerably at light loads.

Fig. 269 is a general view of the mechanism of the two-coil (one primary, and one secondary), constant-current transformer of the General Electric Company; and Fig. 270 shows the transformer in its containing case.

The mechanism of the transformer is surrounded by a corrugated sheet iron casing designed primarily for the protection of the
coils. The casing is enclosed by a cast-iron base and top which are provided with ample openings for the proper ventilation of the transformer.

Within the working limits, the magnetic repulsion between the fixed and moving coils of the system for a given position is proportional to the current flowing in the coils, which makes the


Fig. 270. General Electric Two-Coil Transformer Completely Assembled
transformer capable of being adjusted, therefore, so as to main-- tain any current, simply by changing the amount of counterweight.

In transformers up to and including 50 -lamp capacity having but one movable coil-the secondary, as in Fig. 269-the counterweight is equal to the weight of the coil less the electrical repulsion; and a reduction in the counterweight will produce an increase
in the current. A lever is supported by knife-edge bearings on hardened steel tables which are clamped to the top of the core. To one end of this lever are secured two fixed arcs to which are attached two cables which support the movable secondary coil. At the outer end of the lever an adjustable arc carries a counterweight suspended by a cable.

In transformers designed to supply 75 and 100 lights, having two primary and two secondary coils, the movable secondary coils are balanced one against the other by a system of double-rocker arms supported on knife edges. The weight necessary to balance the repulsion between the primary and the secondary coils is carried on a small auxiliary lever. In this case, a decrease in the counterweight is followed by a decrease in the current.

The arc on the counterweight lever is made adjustable because the repulsion exerted by a given current flowing in the coils is not the same for all positions of the coils, being greater when the primaries and secondaries are close together and less when the primaries are separated. By means of the adjustable arc, the effective radius of the balancing weight is made to change as the coils move through their working range. When the primary and the secondary coils are separated by the maximum distance, the resultant force which tends to attract them to each other should be less than when they are close together.

Regulation. When current flows in the primary and secondary coils, the mutual repelling forces separate the coils until equilibrium is restored. The current corresponding to the position of equilibrium may be adjusted by changes in the counterweights, and the coils will then always take such a position as will maintain that current constant in the secondary coils, regardless of the external resistances to which the coils are connected. With any current less than normal, the repelling force diminishes, and the primary and secondary coils approach each other, thus restoring normal current. As soon as the secondary current exceeds normal, the resultant pull exerted by the counterweight and coils is overcome, and the secondary coil moves away from the primary, again restoring normal current. Transformers of this design can be made to maintain constant current even more accurately than the constant-voltage transformer maintains uniform voltage.

In Fig. 271 on the left is shown the diagram of electrical connections for a constant-current transformer for either 5,35 , or 50 lights, and on the right, the diagram for the 75 - or 100 -light transformer. The 75 - and 100 -light transformers are furnished with two primary coils which may be connected in series for 2,200 volts or in parallel for 1,100 volts; for example, connecting $B$ to $C$ puts the two primary coils in series, and connecting $B$ to $D$ and $C$ to $A$ puts them in parallel for 1,100 volts. In these larger transformers there are two secondary coils also, and leads are arranged so that the


Fig. 271. Connections for Constant-Current Transformer for 5, 35, 50, 75, and 100 Lights
lamps can be divided between two circuits by means of multicircuit connections, as shown in Fig. 271 on the right.

Open-circuiting plug switches are used to disconnect the line from the secondary of the transformer when testing for a ground or an open circuit. These are also used to disconnect one of the circuits of a multi-circuit transformer in order that it may be repaired without interrupting the other circuit. The ammeter jacks, which are provided, are used for the purpose of enabling one ammeter to measure the current in more than one circuit. By
inserting the plug in any ammeter jack, that particular ammeter is connected in series with that circuit.

The primary windings are provided with fuses which are made part of the primary plug switch and are mounted on the back of


Fig. 272. Westinghouse Single-Pole Primary Fuse Block
the panel. They are of the tube expulsion type, depending on the expulsive force exerted by the gases formed by melting of the fuse. This force is sufficient to blow out any are which tends to form within the narrow tube in which the fuse is located.

Transformer Fuse Blocks. The safety fuse links designed to protect a transformer from burn-out in case of short-circuit, are usually placed in circuit with both primary and secondary coils. The fuse links connected in circuit with the high-voltage coil are usually encased in a porcelain tube which encloses the arc that is formed when the fuse melts; and the expansion of the highly heated vapors in the tube extinguishes the arc by what is called "expulsive" action.

The tubes containing the fuses are usually provided with brass terminals for the fuses, the terminals projecting as blades from one side of the tube. With fuse and termi-


Fig. 273. Fort Wayne Sin-gle-Pole Primary Fuse Block nals complete, the tube is pushed home in a receptacle containing metal spring clips that receive the fuse terminals somewhat after the manner of an ordinary knife-switch.

When the fuse melts or "blows," the tube carrying the fuse and terminals is withdrawn from the receptacle. A new fuse may


Fig. 274. Method of Connecting Wires to Primary Fuse Block on Poles Before Current Passes to Transformer
then be put in place without danger to the attendant, after which the tube and fuse may be replaced in the receptacle.

In small transformers for moderate voltages, the fuse receptacles sometimes form part of the containing case; but in large transformers the fuse receptacles are, as a general rule, entirely
separate from the transformer, and are mounted at any convenient point near the transformer, for example, on a cross-arm or on the pole where the transformer is placed. In the case of transformers for use out-of-doors, the fuse receptacles always consist of waterproof cases of cast iron or porcelain, usually the latter.

Fig. 272 shows the type of single-pole primary fuse block furnished by the Westinghouse Electric Company. The block is made of porcelain, finished in black, and is weatherproof. The upper portion of the cut-out contains the stationary contacts which are deeply recessed in the porcelain and are well separated from each other. The contacts are so constructed that the plug is held securely in place by giving it a partial turn after inserting it. When the plug is in position, the fuse is in sight, so that its condition can be easily noted without incurring the danger of opening the primary circuit by pulling out the fuse plug while the fuse is still intact and the transformer is under load.

Fig. 273 shows a single-pole fuse-plug block made by the Fort Wayne Electric Works for protecting primary circuits. The block and plug are all porcelain, finished in black, and serve as both a primary switch and a cut-


Fig. 275. Sample Mounting for Out-Door Transformers out. The block is mounted on the upper cross-arm of the pole and is shaped so as to be used as an insulator. The wires may be brought directly from the line to the cut-out, and thence to the primary winding of the transformer without other support, as shown in Fig. 274.

The plug and cavity in the fuse block are elliptical in section. The plug, shown at the bottom of the block in Fig. 273, is inserted
concentrically with the cavity and by a quarter turn the contact blade is forced between bronze spring terminal clips, and the major axis of the elliptical section of the plug brought into line with the minor axis of the cavity. The result is that the cavity is divided into two parts separated by the plug which tightly fits the cavity. The fuse lies in a groove in the plug with its ends held under screws on the plug blades. The groove is long enough to prevent arcing, and danger of shock is prevented by the insulated rim of the plug. The plug is firmly held in place by the spring terminal clips.

## INSTALLATION AND MAINTENANCE OF TRANSFORMERS

Mounting of Outdoor Transformers. Fig. 275 shows a transformer in its water-tight containing case, mounted on a pole, with its primary coil connected through a double-pole fuse block to 1,100 -volt mains, and its two-coil secondary connected to threewire mains for supplying incandescent lamps in a near-by building. Two suspension hooks $A$, made of heavy strap iron, are attached to the back of the transformer case by slipping the bolt-heads $B$ into the sockets $C^{\prime}$, after which the nuts are screwed tight.

The transformer is hoisted to its position on the building or cross-arm by means of a rope or chain, which is slipped over the two hoisting lugs $D$. When the transformer is hoisted into position, the suspension hooks are slipped over the cross-arm, and screwed fast by means of lag screws. The lead wires from the primary coil are connected to the lower fuse-box leads, and the upper fuse-box leads are connected to the mains. The wires from the secondary coils are led into the building where the secondary fuses are placed. It is now more usual to install two single-pole fuse blocks as illustrated in Fig. 274, than one double-pole block as shown in Fig. 275.

## Inspection and Maintenance of Distributing Transformers.

 There are various ways of determining whether a transformer is in working order before taking it down from a pole. Every lineman is familiar with the simple test, using an incandescent lamp and a fuse, checking the voltage ratio, and the like. Many transformers, however, show all the symptoms of being burned out when they are merely short-circuited temporarily, a condition that may in many cases be easily remedied.Causes of Trouble. By looking inside a distributing transformer it is often possible with a little experience to tell the nature of the trouble - whether or not the coils have been burned out, and, if they have been, whether the burn-out was caused by lightning or by an overload in service. In an examination of hundreds of distributing transformers alleged to have been burned out, it was found that many were actually burned out by overloading, and the coils were so badly damaged as to require complete rewinding. Many others were found to have their coils punctured between turns and at numerous places. These also required rewinding. A third group, however, proved to be merely short-circuited at their terminals, and all that was needed to put them into serviceable condition was the removal of the offending cause of the short-circuit. One of the commonest causes of these short-circuits is the formation of a metallic film or bridge across the surface of the porcelain terminal block between the high-voltage terminals. The conducting film is produced by a lightning discharge coming in over the line, entering the high-tension coils, and starting an arc at the porcelain block, on its way through the core and case to the ground. A portion of the porcelain block may thus be melted as well as a part of the copper or brass coil terminals. The metallic vapor thus formed condenses on the porcelain, leaving a conducting film between the high-tension terminals and virtually short-circuiting the transformer. When the blowing of the high-tension fuses or other tests indicate the presence of such a short-circuit, it is often possible by means of a hammer or a pair of pliers to break the porcelain block between terminals and thus remove the conducting film, leaving the transformer in fit condition for resuming service.

Burn-out Due to Overload. The odor and color of the oil contained in the case of the transformer will usually indicate whether or not the burn-out is due to sustained overload. If this is the case, the smell of the charred oil is pronounced and lingers for weeks at a time. It does not necessarily follow, however, that such oil has lost its insulating properties. In many cases the oil may be renovated and made suitable for use again. In the interest of economy it is well, in doubtful cases, to refer the matter to the electrical engineer before condemning the oil as worthless.

Inspection May Save Trouble. In case of trouble with a distributing transformer it is advisable to remove the cover and make at least a cursory inspection in order to find out whether or not the transformer must be taken off the pole and replaced by a new one. Thus a preliminary inspection and the exercise of a little judgment will often save time, money, and prolonged service interruptions.

Testing for Punctured Insulation. A lightning discharge entering the transformer from the line may escape to ground, after puncturing the insulation of the high-tension coils. The oil in the case may partially seal the puncture and enable the transformer to continue giving service with no direct indication that the insulation has been impaired. In such cases the weakness exists, although not apparent with the normal working voltages.

Increasing Impressed Voltage. The best way of ascertaining the actual condition of the insulation in these cases is to subject the distributing transformer to a gradual increase of the impressed primary voltage up to about twice the normal value and note carefully its behavior. This test will often reveal concealed defects in the insulation of the coils. But even though a transformer has passed successfully the foregoing test, there is no guarantee that it may not suffer a puncture of the coil insulation from lightning the next day or week. After all, the necessary and sufficient condition for safe operation is to properlv ground the low-voltage secondary circuit, as explained below.

Punctures of Both High- and Low-Tension Coils. Let us suppose that punctures of the insulation of both high- and lowtension coils have taken place. This condition is a dangerous one and may later cause the low-tension coils to come in contact with the high-tension coils either directly or by both being grounded on the iron core. The low-tension house mains may then without warning be carrying high-tension current, a condition which is likely to cause severe if not fatal shocks to persons and increased risk of fires. Lnder these conditions if a person touches an incandescent lamp socket or fixture while being more or less grounded through the feet on damp flooring or through zontact with a radiator, gas pipe, or bath tub, the high-tension etirrent may through this imperfect line-wire insulation pass into the secondary circuit through the person, out to the ground, and
thence back to the other side of the high-tension. line. In this connection it should be remembered that the ordinary braided weatherproof insulation so commonly used for 2200 -volt distributing circuits is really not very effective as insulation, especially after long exposure to the elements, and its insulating qualities should never be taken seriously. Again, the primary 2200 -volt mains are often strung through foliage and the swaying of limbs and branches against the insulation may wear it off and thus ground one or both of the line wires, especially in wet weather.

It is thus evident how a circuit may be completed through the body of a person who is grounded, even though the person touch but one side of the low-tension circuit. This is possible because of the accidental grounding due to poor insulation of one or the other wires of the high-tension mains leading to the transformer.

Low=Voltage Secondary Circuits Should Be Grounded. Grounding of low-voltage secondary circuits has been urged for many years by the National Electric Light Association. It is recommended in the National Electrical Code and is the practice of progressive central-station companies. Grounding prevents accidents to persons, and damage by fire to property. If some point of a low-voltage secondary circuit is grounded, no point of the circuit (except under unusual conditions) can rise above its normal voltage in case of a breakdown between primary and secondary windings of the transformer, or of other accidental connection between the primary and secondary circuits.

If the secondary is not grounded and the transformer breaks down, the primary voltage is impressed on the secondary circuit. A person touching any bare part of the secondary circuit would probably receive the primary voltage if he were grounded by contact with a radiator, pipe, or gas fixture. Moreover, the secondary not being grounded and there being a ground on the primary circuit, the primary voltage impressed on the low-voltage fittings of the secondary circuit might cause a fire.

With the secondary grounded, a transformer breakdown will often reveal itself through the blowing of the primary fuses. Where a normal voltage in excess of 250 volts is possible between any wire of a secondary circuit and ground, it is doubtful whether
the secondary should be grounded, as shocks to ground from such a system might cause death.

Methods of Making Ground Connections. Ground connections can be made in many ways. They may be made inside of buildings by connecting to pipes or may be installed at the poles which support the transformers or the secondary networks. Central-station practice favors grounds at poles. A galvanized iron pipe $\frac{1}{2}$ inch or $\frac{3}{4}$ inch in diameter is driven into the earth next to the pole to a depth of 8 feet or more to ensure its reaching permanently moist soil. The pipe is left projecting about a foot above the earth and is not usually protected. The ground wire, of either No. 4 or No. 6 gage bare copper, is soldered or preferably wedged between the upper end of the pipe and the


Fig. 276. Theoretical Diagrams of Secondary Ground Connections
inner surface of a cylindrical iron pipe cap, which is driven on over the pipe end to ensure good electrical contact. The ground wire is supported upon the pole by cleats or straps, not by staples. Ground wires should be encased in wooden molding for a distance of at least 7 feet from the earth to protect passers-by from possible shocks. Some companies encase the entire length of the ground wire in molding to protect the linemen.

The National Electrical Code requires that for three-phase systems the ground wire be of the same carrying capacity as any one of the three mains. There should be a ground for each transformer or group of transformers, and when transformers feed a network with a neutral wire there should be, in addition, a ground at least every 500 feet.

Ground-wire connections to transformer secondaries should be made to the neutral point or wire if one is accessible. Where no neutral point is accessible one side of the secondary circuit may be grounded, provided the maximum voltage between the grounded point and any other point in the circuit does not exceed 250 volts. In Fig. 276 are shown diagrams of transformer secondaries with the proper location of ground connection indicated in each of the several cases.

In Fig. 277 is illustrated the arrangement of some of these connections with commercial transformers. The neutral point of each transformer feeding a two-phase, four-wire secondary should be grounded, unless the motors taking energy from the secondary


Fig. 277. Ground Connections to Secondaries of Commercial Transformers
have interconnected windings. Where they are interconnected, the center or neutral point of only one transformer is grounded. No primary (high-tension) coils are shown in Figs. 276 and 277. In Fig. 277 the secondary winding of each transformer is shown divided into two sections, as it is in commercial transformers.

## TRANSFORMER TESTS

Heat Test. The simplest method of performing this test is to connect the primary coil of the transformer to mains giving the rated voltage and frequency of the transformer, and to load the secondary with a bank of lamps or a water rheostat, adjusting the resistance so as to get rated full-load current from the transformer.

The run should be continued until an approximately constant temperature is reached.

The objection to loading the transformer in the way described is that it requires taking the full rated power from the transformer, which power, therefore, is usually wasted. If two transformers of the same voltage and rated capacity are available, the test may be made on the two simultaneously by what is known as the loading-back method, as follows:

The two secondaries, Fig. 278, connected in parallel, are excited from a low-voltage circuit $A$ at normal voltage and frequency; consequently normal voltage is induced in each primary winding. The two primaries are connected in series, but in such a way as to oppose each other. The resultant voltage


Fig. 278. Connections for Transformer Heat Test
between the points $a$ and $b$ will then be zero, notwithstanding the fact that full voltage exists between the terminals of each transformer secondary. Therefore, if the points $a$ and $b$ be joined together no current will flow. If, however, instead of being joined, these terminals are connected to the terminals of the circuit $C$, any voltage impressed at $C$ will produce a current in the circuit of the primary coils independent of the voltage existing in each of the primary coils. Since each transformer is in effect short-circuited by the other, it follows that approximately twice the impedance voltage* of one transformer impressed at $C$ will cause full-load current to flow through the primaries and secondaries of both. Under these conditions, the transformers will run at full load, while the total energy required for the test amounts to

[^27]merely the losses in the two. The circuit $A$ supplies the excitation current and core losses, the circuit $C$ the full-load current and copper losses.

The auxiliary electromotive force impressed at $C$ may be derived from the same source as the electromotive force at $A$, by means of a transformer. A regulating resistance must be connected in series with it to allow adjustment of the electromotive force at $C$ until the ammeter registers full-load current.

The important temperatures to be observed are those of the coils, core, and room. The temperature of the case and of the oil may be observed as checks. The determination of the temperature of the coil may be made by thermometer or by measurement of resistance. If a transformer has remained in a room of constant temperature many hours, so that the temperature is approximately uniform throughout, thermometer measurement indicates quite accurately the temperature of the windings. If, however, the transformer is radiating heat, as during the heat run, the actual temperature of the copper coils will be much greater than the temperature of surface insulation.

If we know the "cold" resistance, as measured under the first of the above conditions, and the temperature of the coil at the time of measurement, we have a means of finding the "hot" temperature of the coil by measuring its "hot" resistance. The rise in temperature above the temperature at which the "cold" resistance was measured, may be determined from the equation

$$
r=(234.5+i)\left(\frac{R_{i+r}}{R_{i}}-1\right)
$$

in which $R_{i}$ is the "cold" resistance, $R_{i+r}$ is the "hot" resistance, $i$ is the initial temperature, and $r$ is the rise in the temperature expressed in degrees centigrade. The temperature coefficient for commercial copper wire is taken at 0.0042 .7

Further information concerning methods of loading transformers for temperature tests is found in the following Standardization Rules of the American Institute of Electrical Engineers:
393. Conditions for Testing. Whenever practicable, transformers should be tested under conditions that will give losses approximating as nearly as possible to those obtained under normal or specified load conditions, maintained for the required time. The maximum temperature rises measured during this test should be considered as the observable temperature rises for the given load. An approved method of making these tests is the "loadingback" method. The principal variations of this method are the following:
394. With Duplicate Single-Phase Transformers. Duplicate single-phase transformers may be tested in banks of two, with both primary and secondary windings connected in parallel. Normal magnetizing voltage should then be applied and the required current circulated from an auxiliary source. One transformer can be held under normal voltage and current conditions, while the other may be operating under slightly abnormal conditions.
395. With One Three-Phase Transformer. One three-phase transformer may be tested in a manner similar to section 394, provided the primary and
secondary windings are each connected in delta for the test. Normal threephase magnetizing voltage should be applied and the required current circulated from an auxiliary single-phase source.
390. With Three Single-Phase Transformers. Duplicate single-phase transformers may be tested in banks of three, in a manner similar to section 395 by connecting both primary and secondary windings in delta, and applying normal three-phase magnetizing voltage and circulating the required current from an auxiliary single-phase source.
397. Among other methods that have a limited application and can be used only under special conditions the following may be mentioned:
(1) Applying dead load by means of some form of rheostat.
(2) Running alternately for certain short intervals of time on open circuit and then on short circuit, alternating in this way until the transformer reaches steady temperature. In this test, the voltage for the open-circuit


Fig. 279. Wiring Diagram for Core Loss and Exciting Current Test
interval and the current for the short-circuit interval shall be such as to give the same integrated core loss, and the same integrated copper loss, as in normal operation.

Core=Loss and Exciting=Current Test. For this test, the transformer is connected as shown in the diagram, Fig. 279, the primary being left on open circuit. Theoretically the test may be carried out with either coil connected to mains of the proper voltage and frequency. In practice, however, it is better, from the standpoint both of convenience and of safety, to connect the secondary coil.

The electromotive force is adjusted by means of the variable resistance until the voltmeter indicates rated secondary voltage. The ammeter then indicates the exciting or no-load current; and the wattmeter indicates, very closely, the core loss.

Resistance of Coils. The resistance of a coil of a transformer may be measured by the ordinary drop-of-voltage method. This method consists in passing through the coil a direct current, the
value of which is noted by an ammeter, while the drop of voltage across the coil is measured by a voltmeter. Then the resistance is $R=\frac{E}{I}$. Knowing the resistance of the coils, we can determine the drop in voltage due to resistance, under load. This loss of electromotive force is usually expressed in per cent of the electromotive force supplied to the primary.

Impedance. One of the important constants of a transformer is its impedance ratio, that is, the ratio of the voltage consumed by its total internal impedance at full-load current to its rated fullload voltage. The impedance of a transformer is measured by shortcircuiting one of its windings, impressing an alternating electromotive force on the other winding and making simultaneous measurements


Fig. 280. Diagram of Connections for Making Transformer Impedance Test
of current and impressed voltage. $E$ and $I$ being thus observed, the impedance is found as $Z=\frac{E}{I}$ ohms. The value thus obtained includes the impedance of both primary and secondary windings.

Impedance may be considered as constant at all loads. It is usually measured at full-load current, and the impressed voltage is then called the impedance volts, and when expressed in per cent of the rated voltage of the transformer, it is called the per cent impedance drop. It is evident that the "impedance ratio" as defined above is simply the per cent impedance drop divided by 100 .

The impedance of a transformer is an important factor in determining the regulation by calculation. To determine the impedance voltage, the transformer is connected as shown in Fig. 280. As the impedance voltage is not very large-varying from 2 per
cent to 6 per cent of rated primary voltage in standard transform-ers-a much more accurately readable deflection of the voltmeter will be obtained if the primary coil is connected to the mains. As will be seen by referring to Fig. 280, the secondary coil is shortcircuited. The primary coil is connected in series with an adjustable resistance to the low-voltage mains. The resistance is slowly cut out, until full-load current flows in the coil, as indicated by the ammeter. Then the voltmeter indicates the impedance voltage. This should be expressed in per cent of the normal voltage of the coil. From the equation

$$
\begin{equation*}
I=\frac{E}{\sqrt{R^{2}+(2 \pi f L)^{2}}} \tag{40}
\end{equation*}
$$

the total impedance, inductance, and reactance can be computed, provided $R$ and the frequency $f$ are known.

Example. In a test on a $7.5-\mathrm{kw}$. transformer with secondary shortcircuited and primary connected to 2,080 -volt mains, the impedance voltage was 61.1 volts at full-load current of 3.6 amperes in the primary, at a frequency of 60 cycles. The impedance drop being 61.1 , the per cent impedance drop is $\frac{61.1}{2080}=2.935$ per cent, and the impedance is $Z=\frac{61.1}{3.6}=16.95$ ohms.

Since impedance $=Z=\sqrt{R^{2}+X^{2}}, X=\sqrt{Z^{2}-R^{2}}$ and the total reactive drop expressed in per cent is

$$
\% X I=\sqrt{(\% \text { impedance drop })^{2}-(\% R I)^{2}}
$$

If $\% R I=1.57$ from test, and $\% Z I=2.935$ as calculated above, then

$$
\% X I=\sqrt{\overline{2.935}^{2}-\overline{1.57^{2}}}=2.48
$$

or

$$
X=\frac{2.48}{3.6} \times \frac{2080}{100}=14.3 \mathrm{ohms}
$$

Regulation. The definition of regulation given on page 265 will be repeated here. In constant-potential transformers, the regulation is the difference between the no-load and rated-load values of the secondary terminal voltage, at the specified power factor (with constant primary impressed terminal voltage) expressed in per cent of the rated-load secondary voltage, the primary voltage being adjusted to such a value that the apparatus delivers rated output at rated secondary voltage.

The regulation of a transformer may be determined directly by exciting the transformer at rated frequency and with a primary
voltage such that rated secondary voltage is obtained at full load, using lamps or a water rheostat as load. The increase in secondary terminal voltage from full load to no load is then observed, the primary voltage and frequency being kept constant throughout the test. This increase in secondary voltage divided by the secondary full load voltage is then the regulation expressed in per cent. This method is, however, unsatisfactory, because of the small difference between the full load and no load values, and the liability of error in measuring either of them. Moreover, this direct method is not generally applicable for shop tests, particularly on large transformers. Much more reliance can be placed on results calculated for any specified load and power factor from separate measurements of the impedance watts, impedance volts drop, and resistance than on actual measurement of regulation.

A number of methods have been proposed for the calculation of transformer regulation, but the following from the Standardization Rules of the American Institute of Electrical Engineers is simple and practically correct.
587. Tests and Computation of Regulation for Constant-Potential Transformers. The regulation can be determined by loading the transformer and measuring the change in voltage with change in load, at the specified power factor. This method is not generally applicable for shop tests, particularly on large transformers.

The regulation for any specified load and power factor can be computerl from the measured impedance watts and impedance volts, as follows: Let:
$P=$ impedance watts, as measured in the short-circuit test and corrected to $75^{\circ} \mathrm{C}$.
$E_{s}=$ impedance volts, as measured in the short-circuit test.
$I X=$ Reactance Drop in Volts.
$I=$ Rated Primary Current.
$E=$ Rated Primary Voltage.
$q_{r}=$ per cent drop in phase with current.
$q_{x}=$ per cent drop in quadrature with current.

$$
\begin{gathered}
I X=\sqrt{E_{z}^{2}-\left(\frac{P}{I}\right)^{2}} \\
q_{r}=100 \frac{P}{E I} \\
q_{x}=100 \frac{I X}{E}
\end{gathered}
$$

588. Then-
589. For unity power factor, we have approximately:

$$
\text { Per cent regulation }=q_{r}+\frac{q^{2} x}{200}
$$

589. 2. For inductive loads of power-factor $m$ and reactive-factor $n$,

$$
\text { Per cent regulation }=m q_{r}+n q_{x}+\frac{\left(m q_{x}-n q_{r}\right)^{2}}{200}
$$

Example. A $500-\mathrm{kv} .-\mathrm{a} ., 60$-cycle, 11,000 - to 2200 -volt transformer is to be tested, and its regulation calculated: (a) at full-rated load and voltage at unity power factor; (b) at full-rated load and voltage, with a power factor of 0.8 .

Solutions. For the preliminary test the low-voltage winding is shortcircuited through a suitable ammeter and the following measurements are made on the high-voltage side:

Impedance (primary) volts $E_{s}=347$; full-rated load primary current $I=45.4$ amperes; power input $P=3380$ watts. The total resistance of the transformer reduced to primary is given by $R=\frac{P}{l^{2}}$, and the total resistance drop is $R I=\frac{P}{I}$.

Therefore, total reactance drop (reduced to primary) is

$$
\begin{aligned}
I X & =\sqrt{(\text { total impedance drop })^{2}-(\text { total resistance drop })^{2}} \\
& =\sqrt{E_{s}^{2}-\left(\frac{P}{I}\right)^{2}} \\
& =\sqrt{347^{2}-\left(\frac{3380}{45.4}\right)^{2}}=338.9 \text { volts }
\end{aligned}
$$

Here

$$
\begin{aligned}
q_{r} & =100 \times \frac{3380}{11000 \times 45.4} \\
& =0.678(\text { per cent } R I \text { drop })
\end{aligned}
$$

and

$$
\begin{aligned}
q_{x} & =100 \times \frac{339}{11000} \\
& =3.08(\text { per cent } X I \text { drop })
\end{aligned}
$$

Therefore (a) for unity power factor, that is. when $m=1$, and $n=0$, we have approximately

$$
\text { Per cent regulation }=0.678+\frac{\overrightarrow{3.08}^{2}}{200}=0.725
$$

and (b) for 0.8 power factor, that is, when, $m=0.8$, and $n=\sqrt{1-\overline{0.8_{2}^{2}}}=0.6$

Per cent regulation $=(0.8 \times 0.678)+(0.6 \times 3.08)+$

$$
\begin{aligned}
& \frac{[(0.8 \times 3.08)-(0.6 \times 0.678)]^{2}}{200} \\
= & 2.41
\end{aligned}
$$

The regulation for any other power factor $m$, or load, may be calculated in a similar manner.

Efficiency Calculation. The efficiency of any piece of apparatus at a given load is equal to the output divided by the input. The input is equal to the output plus the losses. The efficiency may then be defined as the ratio of the output to the output plus the losses. In nearly every case the efficiency can be determined more accurately by measuring the losses, and then computing the effi-


Fig. 281. Transformer Connected for Polarity Test


Fig. 282. Wiring Diagram for Transformer Polarity Test
ciency according to the second definition, than by attempting to measure the total output and input, and then taking their ratio.

Example. A given 5-kw. transformer is rated at 2,000 volts primary, and 200 volts secondary, at a frequency of 60 cycles per second. The coil resistances are found by measurement to be:

Primary coil resistance. . . . . . . . . . . . . . . . . . . . . 10.1 ohms
Secondary coil resistance. . . . . . . . . . . . . . . . . . 0.067 ohms
At full load, full-load currents are:
Primary current. . . . . . . . . . . . . . . . . . . . . . . . . . 2.5 amperes
Secondary current....... . . . . . . . . . . . . . . . . . . 25.0 amperes
Core loss, as determined by test. . . . . . . . . . . . . . . . 70 watts j
Copper losses at full-load are:
Primary loss $=I^{\prime 2} R^{\prime}=10.1 \times(2.5)^{2}=\ldots \ldots . .63$ watts
Secondary loss $=I^{\prime 2} R^{\prime \prime}=0.067 \times(25)^{2}=\ldots \ldots .42$ watts
Total loss at full-load ..... 175 wattsFull-load output. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 5,000 watts
Full-load intake. ..... $.5,175$ watts
Full-load efficiency $5,000 \div 5,175=$ ..... 96.6 per cent
At half load:Total $I^{2} R$ loss................................................. . . . . 26 watts
Core loss. ..... 70 watts
Total loss ..... 96 watts
Half-load output ..... 2,500 watts
Half-load intake. ..... 2,596 watts
Half-load efficiency, $2,500 \div 2,596=$ 96.3 per cent

The all-day efficiency of a transformer is the ratio of the output of work (watt-hours) during the day to the total input of work (watt-hours). The usual conditions of practice will be met if the calculation is based upon 5 hours at full load, and 19 hours at no load. See page 264.
Output:
5 hours at full load $=5$ hours $\times 5,000$ watts $=25,000$ watt-hours
19 hours at zero load $=0$ watt-hours
Total output in 24 hours $=25,000$ watt-hours
Input:
5 hours at full load $=5$ hours $\times 5,175$ watts $=25,875$ watt-hours
19 hours at zero load $=19$ hours $\times 70$ watts $=1,330$ watt-hours
The zero load intake is but very little more than core loss, since $I^{\prime 2} R^{\prime}$ is negligible at zero load.

$$
\begin{aligned}
& \text { Total intake in } 24 \text { hours } \\
& \text { All-day efficiency } 25,000 \div 27,205,205 \text { watt-hours } \\
& =\quad 91.9 \text { per cent }
\end{aligned}
$$

Polarity Test. Transformers are generally designed so that the instantaneous direction of flow of the current in certain selected leads is the same in all transformers of the same type. For example, the transformer shown in Fig. 281 is designed so that the current at any instant flows into lead $A$ and out of lead $C$. Such transformers run properly in parallel when similar primary and secondary leads on different transformers are connected together. The primaries of two transformers $A$ and $B$ are connected to supply mains, the connections of one primary being made without reference to the connections of the other. It is c.as.red to determine first, how the secondaries are to be connected in farallel to supply current to one and the same receiving circuit; and second, how the secondaries are to be connected in series to supply current to a receiving circuit. This test is made as follows:

Connect a terminal, say a, Fig. 282, of the secondary of one transformer to one terminal $c$ of the secondary of the other transformer. Then connect
two 110-volt lamps in series (or a voltmeter) to the other two terminals $b$ and $d$. If the lamps do not light (or if the voltmeter gives no deflection), then first, the terminals $a$ and $c$ are the proper ones to connect together to one service main; and the terminals $b$ and $d$ are to be connected together to the other service main; and second, the terminals $a$ and $c$ are not the proper ones to be connected together, but $b$ and $c$ are properly connected together in order to connect the two coils in series, the other two terminals $a$ and $d$ being connected to the service mains.


GROUP OF FOUR-AMPERE SERIES RECTIFIER SETS AT PLUM STREET STATION, CINCINNATI, OHO
Courtesy of General Electric Company

## ALTERNATING-CURRENT MACHINERY

## PART V

## CONVERSION OF ALTERNATING INTO DIRECT CURRENT

In spite of the increasing use of alternating currents, there is always a demand for devices of various kinds for converting the power received from an alternating-current circuit into power in the form of direct current. Thus, many electro-chemical processes, electrolytic refining of silver and copper, the charging of storage batteries, and direct-current motors all require direct current. Electric power, as is well known, can be transmitted long distances most economically by the three-phase alternating-current system employing high voltages. The demand for direct-current power arises when the power is to be utilized at the receiving end of the line.

The conversion from alternating current to direct current is accomplished in practice in the following ways: (a) by the rectifyingcommutator; (b) by the aluminum valve rectifier; (c) by the mercuryvapor rectifier; (d) by the rotary (or synchronous) converter; (e) by the motor-generator.

Rectifying Commutator. The rectifying commutator is a commutator driven at a speed synchronous with the alternating current supplied to its brushes and it reverses the connections of the armature windings of an alternator as a whole. The rectifying action consists in reversing alternate (negative) half waves of the current delivered by the alternator so that the current in the receiving circuit is always in one direction. The application of this device to the compounding of alternators is explained on page 85.

The rectifying commutator is limited to the rectifying of comparatively small currents at moderate voltages on account of the prohibitive sparking which occurs at the brushes at high voltages and large outputs.

Aluminum Valve Rectifier. The aluminum valve rectifier is an electrolytic cell which depends on the property of aluminum electrodes in certain electrolytes to let electric current pass in one direction (when the aluminum electrode is cathode) but not in the other. The simplest arrangement of a single-cell valve rectifier is to connect the secondary of the transformer whose current is to be converted into direct current to the positive plate (iron) of the cell and the negative plate (aluminum) to the positive terminal of the storage battery to be charged. To complete the series circuit the negative terminal of the storage battery is connected to the other end of the transformer secondary. A common glass battery jar, containing an approximately neutral solution of pure ammonium phosphate in which an iron plate (the positive) and an aluminum


Fig. 283. Diagram of Connections for Aluminum Valve Rectifier
plate (the negative) are immersed, constitutes the rectifier. With such a single cell only one-half of the complete alternating-current wave is utilized, the other half of the current wave of reversed sign being suppressed by the aluminum valve action. The reversed current which would tend to flow from the aluminum plate through the electrolyte to the iron plate is stopped because of the formation of a thin insulating film of aluminum oxide over the aluminum plate which resists the passage of current unless the voltage exceeds about 150 volts.

A better plan of connecting a single cell so that both halves of the alternating-current wave will be rectified is given in Fig. 283. The choke coil $C$ is an autotransformer, tapped at its middle point by a connection to the direct-current load $B$, such as a storage
battery to be charged. The other terminal of the battery is connected to the iron plate in the cell. The coil $C$ offers a high impedance to the alternating current, but very little impedance to the intermittent direct current flowing out from its middle tap, because an equal amount of current comes from each end and flows in opposite directions around the core of the choke coil. Assuming that at a given instant the terminal $A$ of the secondary of the supply transformer is positive, the path of the current will be as shown by the arrowheads. Since current cannot enter the solution of the cell by way of an aluminum electrode, the 5 amperes have to pass through one-half of the chokecoil winding and out of the middle tap into the battery $B$, from which it enters the rectifier cell through the iron electrode and returns through the aluminum electrode at the right, back to the other terminal $D$ of the transformer secondary. Since the tap point of the autotransformer $C$ is at its middle point, the energy delivered to its lower half acting as a primary coil is transferred by transformer action to its upper half, so that the 10 -ampere pulsating direct current is furnished by the combined currents of 5 amperes in each of the two halves of the coil $C$. It follows that at any instant there is twice as much current through the direct-current circuit $B$ as through the alternating-current circuit, but at half the voltage.

Thus, if both the alternating-current and the direct-current voltages are measured by alternating-current voltmeters, which give effective values, the alternating-current voltage would be, say, 100 volts, and the direct-current voltage would be 50 volts. In practice the direct-current voltage would be less than 50 volts on account of voltage drops and leakage in the cell.

With each reversal of the alternating-current voltage, the direction of the current in the alternating-current circuit, including the coil $C$, is reversed and each aluminum electrode alternates with the other in becoming an active negative terminal and then an inactive positive, but the direction of flow of the direct current through the battery $B$ will remain unchanged.

The aluminum valve rectifier is adapted for rectifying relatively small currents (up to about 25 amperes direct current), its capacity being limited by the heating of the cells. The efficiency is low, ranging from 50 to 60 per cent in practice. The power factor is never over 90 per cent, even on full load. The highest effective value of
the alternating-current voltage that can be used with the cell at normal operating temperatures is about 175 volts, which means that about 55 volts will be obtained on the direct-current side.

As electrolytes, ammonium phosphate or sodium phosphate are considered best. The cell should be artificially cooled to work satisfactorily. The voltage may be regulated by means of resistance in either the alternating-current or the direct-current circuits, but a better method is by adjusting the alternating-current voltage by a small autotransformer having a number of taps. The greatest advantage of the aluminum valve rectifier is its cheapness and sim-


Fig. 284. Mercury-Vapor Arc Rectifiers
plicity. On account of its small direct-current output, its poor efficiency, and comparatively high cost of maintenance, it is not as yet of great commercial importance.

Mercury=Vapor Arc Rectifier. The mercury-vapor arc rectifier, like the aluminum valve rectifier, operates by the action of electric valves. The rectifier bulb consists of a closed glass vessel provided with four electrodes; those marked $A A$, Fig. 284, called anodes (or positives) are of graphite, and the other two $B$ and $C$, called cathodes, are of mercury. The air is exhausted from the bulb which contains only mercury vapor. This like other metal vapors is an electrical
conductor under some conditions, and the graphite or positive electrodes are immersed in this vapor. A pool of mercury in the bottom of the bulb forms the negative electrode or cathode $B$, and the small electrode $C$ is used merely for starting the mercury arc between $A$ and $B$.

The rectifying action depends on the properties of mercury in the presence of mercury vapor, as follows: Current can readily pass from either of the graphite electrodes through the mercury vapor, but if it is attempted to reverse the direction of the current, a very high apparent resistance is developed at the surface of the mercury electrode which prevents the flow of current from $B$ to $A$. The diagram of electrical connections for a mercury rectifier is given in Fig. 285. The alter-nating-current supply circuit is connected to the two positive electrodes $A$ and $A^{\prime}$ of the bulb and also to two reactance coils $F$ and $E$. On account of the check-valve action just described, the pulsations of the alternating current pass alternately from $A$ and $A^{\prime}$ into the mercury (negative) electrode $B$, from which


Fig. 285. Connections for Mercury-Vapor Arc Rectifier they are delivered as an uni-directional current for charging the storage battery $J$. When the bulb starts to rectify there is a high resistance at the surface of the mercury which must be broken down before current can pass. This cathode resistance acts like an insulat. ing film over the surface of the mercury, but when a current is once started it will continue to flow, meeting with but small resistance as long as the current is uninterrupted. The briefest interruption, however, permits the cathode resistance to increase enormously, and thus stop the action of the bulb.

In order to break down the cathode resistance, on starting the rectifier, the bulb is tilted or shaken so that the starting cathode $C$, Fig. 285, is brought into contact with the cathode $B$ by a mercury bridge. Current then passes between $C$ and $B$ and the little stream of mercury which bridges the space between the electrodes breaks with a spark when the bulb is returned to its normal vertical position. This spark or initial arc breaks down the cathode resistance by forming mercury vapor which enables the graphite anodes to become active and the rectifying action to start. After the tube is in operation, the circuit through $C$ is opened by a switch.

The action of the rectifier may be followed in detail with the aid of Fig. 285. Let us assume an instant when the terminal $I I$ of the supply transformer is positive, then the path of the current will be shown by the small arrowheads, the electrode $A$ will be positive, and the current is free to flow from $A$ across the mercury vapor to $B$, the mercury negative electrode (cathode). From $B$ the current passes through the storage battery $J$, through the reactance coil $E$, and back to the negative terminal $G$ of the transformer. A moment later when the impressed voltage falls below a value sufficient to maintain the arc against the counter-electromotive force of the arc and storage battery, the current would be interrupted at the end of the first half cycle before the current from anode $A^{\prime}$ could be established.

In order to prevent this interruption two reactance (choke) coils $F$ and $E$ are connected in the circuit, as shown. The inductance of these coils delays the decreasing current from one anode, as $A$, until the voltage of the transformer has passed through zero, reverses, and builds up to such a value as to cause $A^{\prime}$ to start an arc between it and the mercury cathode $B$. The path of the current is now from $G$ down to $A^{\prime}$, across the mercury vapor to $B$, out through $J$, the coil $F$, and back to $H$, which is now the negative terminal. The new path is indicated by the arrows enclosed in circles. A moment later $A$ again becomes active, and the path of the current is again indicated by the plain arrowheads as at first. Mercury-vapor rectifiers as made for charging storage batteries, operating arc lamps, small direct-current motors, etc., are furnished in four sizes, suitable for $10,20,30$, and 40 amperes of direct current, respectively. They are designed for a frequency of 60 cycles, but can be adapted to com-
mercial frequencies from 25 to 140 cycles per second. They are made for the standard secondary alternating voltages of 110 and 220 volts.


In Fig. 284 are shown two types of the 30 -ampere size of tube. The one on the left is for 25 to 40 volts, the one on the right for 90 to 250 volts. The average life of a tube under
normal operating conditions is 600 hours, with an efficiency of from 75 to 80 per cent.

The rectifier outfit, Figs. 286 and 287, consists of a rectifier tube, reactance, regulating compensator and panel. The panel simply requires connections to be made to the secondaries of the supply transformer and to the direct-current load circuit.

The double-pole "line switch," Figs. 286 and 287, connects to the alternating supply circuit. The circuit-breaker protects


Fig. 287 Diagrammatic Views of Front, Back, and Side of Rectifier Equipment
the direct-current circuit from excessive current. The starting switch is a single-pole double-throw spring switch used to start the tube on a resistance on the back of the panel. The anode switch is an auxiliary spring switch mechanically operated by the starting switch, and automatically opens the anode circuit when the handle of the starting switch is released.

The dial switch is provided with a double set of contact buttons, one set for rough and the other for fine regulation of the direct-current voltage. These contact buttons are connected to taps
brought out from the regulating compensator mounted at the back of the panel.

Another important application of the mercury-vapor rectifier is in connection with street and out-door lighting by series, constant, direct-current arc and incandescent lamps. The arc lamps commonly used for this service are known as "luminous arc" and "flame arc" lamps. The series incandescent lamps for this service have filaments of tungsten, and are made for candle powers ranging from 25 to 80 , and for currents of 4 and 6.6 amperes.

This system of series, constant, direct-current lighting involves an alternator delivering alternating current at 2,200 volts to the primary of a constant-current transformer whose secondary is designed to give a constant alternating current of either 4 or 6.6 amperes at a voltage depending upon the number of arc lamps in series to be supplied. A mercury-vapor rectifier is then used to convert the constant alternating current into a constant direct current suitable for use in the arc lamps.

The rectifiers constructed for this service differ from those described above only in certain details. The rectifier tubes are operated in oil-filled tanks, for cooling purposes, and the voltages of the rectified current range from 1,200 volts in the 12 -light outfit up to 6,450 volts in the 75 -light outfit. Rectifier tubes can be built for voltages up to 13,000 .

## ROTARY OR SYNCHRONOUS CONVERTER

The rotary or synchronous converter is a machine for converting alternating current into direct current, or vice versa. The importance that such machines have assumed in the electrical industry is due to several causes:
(a) It is necessary, for economic reasons, to cse alternating current at high voltages in long-distance transmission, as explained on page 11. Therefore, rotary converters are required for changing the alternating current into direct current for use in electric railway motors, which must be supplied with direct current from the trolley wire at points at a distance from the power house.
(b) Rotary converters are needed for charging storage batteries in places where the central station supplies alternating current, and inverted rotaries are necessary for factory driving with
alternating-current motors in cases where direct current only is supplied by central stations.
(c) Direct current is necessary in many of the chemical and electro-metallurgical industries such as the electrolytic reduction of aluminum from its ores, the electrolytic refining of copper, etc. If alternating current is generated and transmitted to these establishments, it must be converted into direct current before it can be utilized.

The rotary converter is chiefly used to convert polyphase alternating currents into direct current on a large scale.

Comparison with Direct=Current Dynamo. In general appearance and construction, the rotary converter resembles the directcurrent generator very closely. The chief outward difference is the addition of a number of collector rings concentric with the shaft on one side of the armature, and the commutator is very much larger than in the ordinary direct-current generator. Another point of difference is in the relative dimensions of the magnetic circuit, including yoke and magnet cores, which are smaller than would be usual or desirable in ordinary direct-current generators.

Under the usual condition of running, the armature is driven, as in a simple synchronous motor, by alternating current supplied to the collector rings from an external source. While so revolving direct current can be taken from brushes bearing upon the commutator.

The current in the armature of a rotary converter may be thought of as the difference between the inflowing alternating currents and the outflowing direct current. The average value of the current in a given armature conductor is, therefore, smaller in value than in the corresponding direct-current generator, and the heating effect $I^{2} R$ is correspondingly less. Furthermore, the magnetizing action of the inflowing alternating current upon the armature is almost completely neutralized by the magnetizing action of the outflowing direct current. Therefore, a larger number of smaller conductors may be wound upon a given armature core if the armature is to be used for a rotary converter, than would be permissible if the armature were to be used for a direct-current generator. That is, the allowable power output of a machine of given size is not limited to so small a value if the machine is to be used as

TABLE XIV
Power Ratings of Rotary Converters in Kilowatts

| Continuous- | Single- | Three- | Four- | Six- |
| :---: | :---: | :---: | :---: | :---: |
| Current | Phase | Phase | Ring | Phase |
| Dynamo | Converter | Converter | Converter | Converter |
| 100 | 85 | 132 | 162 | 192 |

a polyphase rotary converter, as it would be if the machine were to be used as a direct-current generator.

Table XIV gives the power ratings which a machine that would be rated at 100 kw . if used as a direct-current generator has, when it is used as a single-phase, three-phase, two-phase (four-ring), and six-phase converter, respectively.

To Make a Direct=Current Dynamo into a Rotary Converter. Consider an, ordinary bipolar* direct-current dynamo. Imagine two opposite commutator bars of the machine to be marked $a$ and $b$, respectively, Fig. 288. Let the field magnet of the machine be excited, and the armature be driven at a speed of $n$ revolutions per second in a counterclockwise direction, as shown by the arrow. At a given instant the marked bars will be midway between the direct-current brushes, as shown in the figure. Let us call the position of the armature at this instant the position A, Fig. 289, and let us consider the way in which the electromotive force


Fig. 288. Commutator and Brush Diagram for D. C. Dynamo as a Rotary Converter between the given pair of commutator bars $a$ and $b$ changes as the armature rotates.
(1) While the armature is making the first quarter of a revolution from the $A$ position, the bars will move until bar $a$ touches the + brush and bar $b$ touches the - brush; and the electromotive force between the bars will grow from zero to the full value $E$ of the direct electromotive force between the brushes.

[^28](2) While the armature is making the second quarter of a revolution from the $A$ position, the bars will move until they are again midway beween the direct-current brushes; and the electromotive force between the bars will drop from the value $E$ to zero.
(3) While the armature is making the third quarter of a revolution from the $A$ position, the bars will move until bar $a$ touches the - brush, and bar $b$ touches the + brush; and the electromotive force between the bars, which must now be considered as negative, will grow from zero to the value $E$.
(4) While the armature is making the fourth quarter of a revolution from the $A$ position, the bars will move until they are again midway between the direct-current brushes; and the electromotive force between the bars, which is still to be considered as negative, will drop from the value $E$ to zero.


Fig. 289. E.M.F. Curve for One Revolution of D. C. Dynamo
These successive changes of electromotive force, between the given pair $a b$ of commutator bars, which occur during one complete revolution of the armature, are shown graphically in Fig. 289.

It is at once evident that the electromotive force between the bars $a$ and $b$ is an alternating electromotive force, and that this alternating electromotive force passes through a cycle of values during each revolution of the armature, so that its frequency is equal to the revolutions per second of the armature in the case of a bipolar machine.

Furthermore, it is clear that the alternating electromotive force between a given pair of commutator bars $a b$ on a direct-current dynamo may be utilized for the production of alternating current; or the direct-current dynamo may be made into an alternator by providing a pair of insulated metal collecting rings connected permanently to the bars $a$ and $b$, respectively, and which are kept in
continuous connection with an outside circuit by means of an auxiliary pair of brushes, that is, brushes entirely separate and distinct from the direct-current brushes before mentioned.

A direct-current dynamo made into an alternator in this manner but with its direct-current brushes and commutator kept intact, and provided with two collecting rings only, is called a single-phase rotary converter. When the machine is provided, as explained below, with three collecting rings, four collecting rings, or six collecting rings, it is called a polyphase rotary converter.

Three=Ring Converter. Three equi-


Fig. 290. Brush and Commutator Diagram for ThreeRing Converter distant commutator bars $a, b$, and $c$, Fig. 290, of a direct-current dynamo are connected to three collector rings. It is shown later in this discussion that the electromotive force between bars $a$ and $b$ is, in phase, 120 degrees ahead of the electromotive force between bars $b$ and $c$, and 240 degrees ahead of the electromotive force between bars $c$ and $a$. Three electromotive forces related in this way are called three-phase electromotive forces; and a direct-current dynamo provided with three slip-rings as specified, is called a three-phase, or three-ring, rotary converter.

Four=Ring Converter. Four equidistant commutator bars $a, b, a^{\prime}, b^{\prime}$, Fig. 291, of a direct-current dynamo are connected to four collecting rings. Then the electromotive force between the rings $a$ and $a^{\prime}$ is at its zero value when the electromotive force between rings $b$ and $b^{\prime}$ is at its greatest value, or vice versa. Two electromotive forces related in this way are


Fig. 291. Brush and Commutator Diagram for FourRing Converter called two-phase electromotive forces; and a direct-current dynamo with four collecting rings, as specified, is called a two-phase, or more accurately, a four-ring rotary converter.

Six=Ring Converter. Six equidistant commutator bars $a, b$, c, $a^{\prime}, b^{\prime}, c^{\prime}$, Fig. 292, of a direct-current dynamo, are connected to six slip-rings. Such a machine is called a six-phase or six-ring rotary converter. The electromotive force between the rings $a$ and $a^{\prime}$ is, in phase, 120 degrees ahead of the electromotive force between rings $b$ and $b^{\prime}$, and 240 degrees ahead of the electromotive force between rings $c$ and $c^{\prime}$. Three electromotive forces so related are called threephase electromotive forces; and a six-ring rotary converter may, under certain conditions, be supplied with three-phase alternating currents.

Multipolar Rotary Converters. The discussion already given on converters refers, for the sake of simplicity, to a bipolar machine. In case of a multipolar d. c. dynamo, having $p$ field poles, the connections of the $n$ rings of an $n$-ring converter are as follows:

Ring 1 is connected to the $\frac{p}{2}$ equidistant commutator bars which, for a given position of the armature, are squarely opposite the centers of, say, the north poles of the field magnet. Let $d$ be the distance between the commutator bars to which ring 1 is connected. Ring $\mathscr{2}$ is connected to the $\frac{p}{2}$ equidistant commutator bars which are $\frac{1}{n}$ of $d$ ahead of the bars connected to ring 1 ; ring 3 is connected to the $\frac{p}{2}$ equidistant commutator bars which are $\frac{2}{n}$ of $d$ ahead of the bars connected to ring 1 ; and so on.
For example, a 6 -pole direct-current dynamo with 72 commutator bars, to be made into a three-ring converter, would have ring 1 connected to commutator bars 1,25 , and 49; ring ? connected to commutator bars 9,33 , and 57 ; and ring 3 connected to commutator bars 17,41 , and 65 .
E. M. F. Relations for Rotary Converter. Let $E$ be the value of the steady electromotive force between the direct-current brushes of a rotary converter; and let $E_{2}, E_{3}, E_{4}, E_{6}$, be the effective values
of the alternating electromotive force between adjacent collecting rings of a two-ring, three-ring, four-ring, and six-ring rotary converter, respectively. It is desired to find the relationship between these various electromotive forces.

Relationship between $E$ and $E_{2}$. The maximum value of the alternating electromotive force between the collecting rings of a two-ring converter occurs at the instant when the commutator bars to which the collecting rings are connected are in contact with the direct-current brushes, and this maximum value is, of course, equal to $E$. Therefore, the effective value $E_{2}$ of the alternating electromotive force between the slip-rings of a two-ring converter is equal to $\frac{E}{\sqrt{2}}$. That is

$$
\begin{equation*}
E_{2}=\frac{E}{\sqrt{2}} \tag{41}
\end{equation*}
$$

Relationship between $E_{2}$ and $E_{3}$. The discussion of the relation. ship between $E_{2}$ and $E_{3}$ will be carried out for a very special case, namely, where the armature has 18 conductors; and this special discussion will lead to a very simple geometrical construction which will easily give the relationship between $E_{2}, E_{3}, E_{4}$, and $E_{6}$, and will also show the phase relations of the various electromotive forces of a threering, of a four-ring, or of a six-ring converter.

Fig. 293 represents a directcurrent ring-wound armature having


Fig. 293. $\underset{\text { Wiagram of a D. C. Ring. }}{\text { Wound Armature }}$ 18 conductors, each conductor representing a turn of wire. Consider the conductor $a$. This conductor has an alternating electromotive force induced in it as the armature rotates. Let this alternating electromotive force (effective value) be represented by the short line $a$ in Fig. 294.

Consider the next following conductor $b$. The alternating electromotive force induced in this conductor has the same value as that
induced in conductor $a$, but is behind it in phase by the angle $\frac{360^{\circ}}{18}$ $=20^{\circ}$, where 18 is the total number of armature conductors. Let the electromotive force induced in conductor $b$ be represented by the short line $b$ in Fig. 294.

Similarly, the short lines $c, d, e, f, g$, etc., in Fig. 294, represent the alternating electromotive forces (effective values) induced in the conductors $c, d, e, f$,
Fig. 294. E.M.F.
$\underset{\text { Fig. } 293}{\text { Diagram for }} g$, etc.
Consider first the two-ring converter. Suppose that its slip-ring 1 is connected to the commutator bar which is between conductors $r$ and $a$, as shown in Fig. 293; then its other slip-ring will be connected to the bar which is between conductors $i$ and $j$, and the alternating electromotive force $E_{2}$ between these two slip-rings will be the vector sum of the electromotive forces $a$, $b, c, d, e, f, g, h$, and $i$ (Fig. 294), as shown in Fig. 295. That is, $E_{2}$ (effective value) is represented by the diameter of the polygon (circle) in Fig. 295.

Consider now the three-ring converter. Suppose its three rings are connected as shown by the numbers 1, 2, and 3 in Fig. 293. Then the electromotive force $E^{\prime}$, between rings 1 and 2 is the vector sum of the electromotive forces $a, b, c, d, e$, and $f$, as shown in Fig. 295; the electromotive force $E^{\prime \prime}{ }_{3}$, between rings 2 and 3 , is the vector


Fig. 295. E.M.F. Relations in Two- and Three-Ring Converter sum of the electromotive forces $g, h, i$, $j, k$, and $l$, as shown in Fig. 295 and the electromotive force $E^{\prime \prime \prime}{ }_{3}$, between rings 3 and 1 , is the vector sum of the electromotive forces $m, n, o, p, q$, and $r$, as shown in Fig. 295.

Therefore, the effective value of the electromotive force $E_{2}$ between the two rings of a two-ring converter, being represented by the diameter of a circle, the effective value of the electromotive force $E_{3}$, between any two rings of a three-ring converter is represented by a 120 -degree chord of the same circle. Therefore

$$
E_{3}=\frac{\sqrt{3}}{2} \times E_{2}
$$

or, using the value of $E_{2}$ from equation (41), we have

$$
\begin{equation*}
E_{3}=\frac{v \overline{3}}{2} \times \frac{E}{v^{2}}=0.612 E \tag{42}
\end{equation*}
$$

It is to be noted that in order to make a direct-current machine into a three-, four-, or six-ring converter, the number of armature conductors must be divisible by the number of rings. Therefore, the armature shown in Fig. 293 is not suitable for a four-ring converter, although it is suitable for a three-ring converter.

The foregoing discussion shows that if the effective value of $E_{2}$ is represented by the diameter of a circle, then the effective value of $E_{3}$ is represented by a 120 -degree chord, the effective value of $E_{4}$


Fig. 296. E.M.F. Relations in a FourRing Converter


Fig. 297. E.M.F. Relations in a Six-
is represented by a 90 -degree chord, and the effective value of $E_{6}$ is represented by a 60 -degree chord of the same circle. This is shown in Figs. 295, 296, and 297.

From Fig. 296 we have

$$
E_{4}=\frac{E_{2}}{\sqrt{2}}
$$

or, substituting the value of $E_{2}$ from equation (41), we have

$$
\begin{equation*}
E_{4}=\frac{1}{\sqrt{2}} \times \frac{E}{\sqrt{2}}=\frac{E}{2} \tag{43}
\end{equation*}
$$

From Fig. 297 we have

$$
E_{6}=\frac{1}{2} E_{2}
$$

or, substituting the value of $E_{2}$ from equation (41), we have

$$
\begin{equation*}
E_{6}=\frac{1}{2} \times \frac{E}{\sqrt{2}}=0.354 E \tag{44}
\end{equation*}
$$

Summary of E. M. F. Relations for the Rotary Converter. Let E be the electromotive force between the direct-current brushes of a rotary converter; then,

$$
\left.\begin{array}{l}
E_{2}=0.707 \mathrm{E}  \tag{45}\\
E_{3}=0.612 \mathrm{E} \\
E_{4}=0.500 \mathrm{E} \\
E_{6}=0.354 \mathrm{E}
\end{array}\right\}
$$

in which $E_{2}, E_{3}, E_{4}$, and $E_{6}$, are the effective values of the alternating electromotive force between adjacent collecting rings on a tworing, three-ring, four-ring, and six-ring converter, respectively, and $E$ is the steady value of the electromotive force between the directcurrent brushes in each case.

Examples. If a rotary converter is to deliver direct current at 500 volts:
(a) It must be supplied with single-phase alternating current at 353.5 volts effective if it is a two-ring converter.
(b) It must be supplied with three-phase currents at 306 volts effective between each pair of the three supply inains, if the converter is a three-ring converter.
(c) It must be supplied with two-phase currents over four-wire supply mains with 250 volts effective between mains connected to adjacent collector rings, or 353.5 volts effective between mains connected to opposite collector rings, if the converter is a four-ring converter.
(d) It must be supplied with six-phase currents over six-wire supply mains, with 177 volts effective between the mains connected to adjacent collector rings; or with 306 volts effective between the mains connected to rings 1 and 9 ; or with 353.5 volts effective between the mains connected to opposite collector rings.

Modification of Theoretical Voltage Ratios in Actual Machines. The theoretical ratios of alternating to direct-current voltage are not always found to hold good in practice. This is owing to a variety of causes, chief among which are: the deviation of the generator voltage from a sine wave; the voltage drop in the armature wind-

TABLE XV
Voltage Ratios of Rotary Converters

| Percentage | Pole Arc | 50 Per Cent | 67 Per Cent | 74 Per Cent | 80 Per Cent |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Three-phase | [550-volt | 67 | 63 | 62 | 61.5 |
|  | 250-volt | 67.5 | 63.5 | 62.5 | 62 |
|  | 125-volt | 68 | 63.8 | 63 | 62.5 |
| Two-phase | 550-volt | 78 | 73.5 | 72.5 | 72 |
|  | 250-volt | 79 | 74 | 73 | 72.5 |
|  | ( 125 -volt | 79.5 | 74.5 | 73.5 | 73 |

ings; the position of the direct-current brushes on the commutator; and the degree of field excitation.

As the direct-current voltage at the commutator brushes, neglecting the resistance drop $I R$ in the converter, is equal to the maximum instantaneous voltage between opposite collector rings, a flat-top wave gives a higher ratio, i.e., lower direct-current voltage, and a peaked wave a lower ratio, i.e., higher direct-current voltage, for the same impressed alternating-current voltage. Moreover, the shape of the electromotive force wave impressed by the generator upon the converter, is modified by the form of the counterelectromotive force wave of the converter. Hence, a short pole arc of the converter, producing a peaked wave of counter-electromotive force, tends to lower the direct-current voltage, and a long pole arc tends to raise the direct-current voltage, for the same impressed alternating voltage.

A displacement of the brushes from the neutral point decreases the direct-current voltage for a given alternating-current voltage, the variation in extreme cases amounting to several per cent.

Over-excitation of the field magnet may increase the directcurrent voltage one or two per cent; while with under-excitation, i.e., with lagging current in the armature, the direct-current voltage may be decreased one or two per cent for a given alternating-current voltage.

Under average conditions of full-load operation, the standard types of converters have ratios alternating-current voltage $\div$ directcurrent voltage, approximately as given in Table XV.

In the normal operation of the rotary converter, namely, when furnishing direct current, the drop in the armature reduces the direct-current voltage. When run as an inverted converter, i.e.,
when delivering alternating current, direct current being fed to the brushes, the drop is on the alternating-current side; consequently the voltage ratio of a converter is lower when it is run inverted.

For preliminary calculations where the data of operation is not known, the following voltage ratios may be used with most standard converters:

|  | No load | Full load |
| :--- | :---: | :---: |
| For single-phase | 71.5 | 73 |
| For two-phase | 71.5 | 73 |
| For three-phase | 61 | 62.5 |
| For six-phase <br> $\quad$ (measured on diameter) | 71.5 | 73 |
| For six-phase <br> $\quad$ (measured on alternate rings) | 61 | 62.5 |

In operating rotary converters, it is customary to make allowance for the departure of the actual voltage ratio from the normal ratio. The amount of the allowance to be made cannot always be predetermined; but any ordinary departure from the theoretical voltage may be easily compensated for by using transformers provided with taps on the secondary windings which will permit a voltage change of about 5 per cent.

Current Relations for Rotary Converter. The rotary converter, as ordinarily used to convert alternating current to direct current, behaves as a synchronous motor so far as its intake of alternating current or currents is concerned. In the case of the synchronous motor with a given belt load, the intake of alternating current varies with the degree of field excitation, the intake of current being a minimum for a certain field excitation, and the power factor nearly unity (see page 226). In the case of the rotary converter also, the intake of alternating current or currents is a minimum, and its power factor is unity for a certain field excitation, the direct-current output of power being given. Under these conditions there is a definite relation between the direct-current $I$ delivered by the converter, and the effective value of the alternating current flowing in at each collecting ring. In fact

$$
\left.\begin{array}{l}
I_{2}=1.414 I  \tag{46}\\
I_{3}=0.943 I \\
I_{4}=0.707 I \\
I_{6}=0.471 I
\end{array}\right\}
$$

in which $I_{2}, I_{3}, I_{4}$, and $I_{6}$ are the effective values of the alternating current entering at each collector ring of a two-ring, three-ring, four-ring, or six-ring converter, respectively; and $I$ is the direct current delivered by the converter. These equations (46) are based on the assumption that the converter has unity power factor as above pointed out, and that the intake of power is equal to the output of power, i.e., the losses of power in the machine are ignored.

Derivation of the Equations for $I_{2}$ and $I_{3}$. The method of deriving equations (46) will be sufficiently indicated by deriving the first two, namely, the equations for $I_{2}$ and $I_{3}$ :

The direct-current output of power from the converter is $E I, E$ being the electromotive force between the direct-current brushes, and $I$ being the direct current delivered. The intake of power is $E_{2} I_{2} \times$ power factor; but since the power factor is supposed to be unity, the intake of power is simply $E_{2} I_{2}$. Therefore, ignoring losses of power, we have

$$
E_{2} I_{2}=E I
$$

But $E_{2}=0.707 E$, by the first of equations (45), so that $E_{2} I_{2}=$ $0.707 E \times I_{2}=E I$. Hence

$$
0.707 I_{2}=I
$$

or, in final form, the equation for $I_{2}$ is

$$
I_{2}=1.414 I
$$

The direct-current output of power is $E I$, and the intake of power is $\sqrt{3} E_{3} I_{3}$, the power factor being unity; that is, the power delivered by three-phase supply mains is equal to $1-\overline{3}$ times the voltage between mains times the current in each main, as explained on page 128. Therefore, ignoring power losses in the machine, we have

$$
\sqrt{3} E_{3} I_{3}=E I
$$

But $E_{3}=0.612 E$, according to the second of equations (45), so that

$$
\sqrt{3} E_{3} I_{3}=\sqrt{3} \times 0.612 \times E I_{3}=E I . \text { Hence }
$$

$$
\sqrt{3} \times 0.612 \quad I_{3}=I
$$

or, in final form, the equation for $I_{3}$ is

$$
I_{3}=0.943 I
$$

Example. A rotary converter delivers 500 amperes of dircct current, the field excitation being adjusted so that the intake of alternating current may be a minimum.
(a) If this converter is a two-ring converter, it must be supplied with 707 amperes of alternating current from single-phase mains.
(b) If this converter is a three-ring converter, it must be supplied with three-phase currents from three-wire mains, with 471.5 amperes effective in each main.
(c) If this converter is a four-ring converter, it must be supplied with two-phase currents from four-wire supply mains, with 353.5 amperes effective in each main.
(d) If this converter is a six-ring converter, it must be supplied with six-phase currents from six-wire mains, with 235.5 amperes effective in each main.

## ROTARY CONVERTERS IN PRACTICB

Uses of the Rotary Converter. The uses of the rotary converter are as follows:
(a) Direct-Current Generator or Motor. The rotary converter may be used as a direct-current generator or motor, in which cases the collector rings are not used, but the machine must be provided with a pulley.
(b) Alternating-Current Generator or Synchronous Motor. The machine may be driven by belt and used to deliver alternating currents from its collector rings as an ordinary alternator; or the machine may be supplied through its collector rings with alternating currents, and may be driven as an ordinary synchronous motor delivering mechanical power to drive machinery. In either case the direct current for exciting the field magnet may be taken from the commutator of the machine, or from a separate direct-current dynamo (exciter).
(c) Double-Current Generator. The rotary converter may be driven by belt, and used to deliver both direct current and alternating current from commutator and collector rings, respectively. When so used, the machine is called a double-current generator.
(d) Regular Rotary Converter. The machine may be driven as a synchronous motor taking alternating current from supply mains, and delivering, not mechanical power, but electrical power in the form of direct current from its commutator. When so used, the machine is called a rotary converter. This is the most frequent use of the machine. The rotary converter does not require a pulley. Under these conditions, the machine exhibits all of the peculiarities of the synchronous motor. Thus, with under-excited field magnet, the rotary converter takes an unduly large amount of alternating
current from the supply mains at a low power factor. As the field excitation is increased, the intake of alternating current decreases (for given output of direct-current power), and the power factor increases. For a certain degree of field excitation the power factor is nearly unity, and the alternating current or currents delivered to the machine are in phase with the alternating electromotive forces between the supply mains. When the field magnet is over-excited,the alternating currents supplied to the converter are ahead of the alternating electromotive forces in phase, and the power factor is less than unity.
(e) Inverted Rotary Converter. The machine may be driven as a direct-current motor taking current from direct-current supply mains through its commutator, and delivering, not mechanical power, but electrical power in the form of alternating currents from its collector rings. When so used, the machine is called an inverted rotary converter; it does not require a pulley.

Starting Rotary Converters. There are three methods in common use for starting rotary converters, namely, (a) from the alternatingcurrent side, as a synchronous motor; (b) from the direct-current side, as a shunt motor; or (c) by means of a special direct-connected starting motor, usually of the squirrel-cage induction type.
(a) As an Induction Motor. The first method is one of the most common and has been fully discussed on page 214. It consists in supplying polyphase currents at a suitable voltage to the collector rings. The resulting torque developed by the armature will bring the converter up to synchronous speed in from one to two minutes. The proper value of impressed voltage for starting is usually obtained from taps on the main transformers. This method of starting has the advantage of making synchronizing unnecessary, but on the other hand it requires a large starting current which, if the converter is relatively large as compared with the alternator supplying it, causes an excessive drop of voltage throughout the circuit.

A difficulty sometimes met in this method of starting is, that a particular direct-current brush or set of brushes may be positive or negative according to the direction of the last pulse of alternating current just before the machine jumps into synchronism. Therefore, when several rotary converters are to supply direct current to common bus bars, special care must be taken to see that the polarity
of a given machine is correct before it is connected to the bus bars. If the machine comes up to the speed with the wrong polarity, the field "break-up" switch, described on pages 217-218 and shown in Fig. 184, must be thrown from one position to the other, thus reversing the field. Self-starting converters are also generally equipped with a switch for disconnecting the shunt across the series coils from the series field winding, during starting, so as to prevent the circulation of a large induced alternating current in both the shunt and the series field winding. The induced current would not only cause excessive heating, but would also hinder starting by its braking effect.
(b) As a Shunt Motor. When a direct-current supply is available, the converter may be started as a direct-current shunt motor, by supplying direct current to the commutator side of the machine, the alternating-current main switch being open. This method is convenient and is used in many installations. In some cases the direct current is obtained from another converter already in operation, or from a small storage battery which may be at hand. Sometimes a small motor-generator set, consisting of an induction motor coupled to a direct-current generator, is installed to supply current for the starting of one or more converters in a station.

In the direct-current method of starting, the fields should be fully excited by closing the field switch first, and there should be a resistance in series with the armature when the motor switch is closed. Failure to excite the field may cause the converter to increase its speed to a dangerous extent, just as in the case of a directcurrent shunt motor with excessively weak field, for in starting the converter from the direct-current side, it is not running as a synchronous motor but as a simple shunt direct-current motor. If the converter is compound wound, the series field circuit must be opened, otherwise the current flowing through it will magnetize the field poles in opposition to the shunt field windings (differential compounding) and may even prevent the machine from starting.

The operation of starting is as follows:
(1) See that the alternating-current main switch is open.
(2) Close the field switch.
(3) Leave the starting resistance in circuit with the armature, and then close the direct-current main switch.
(4) When normal speed is reached, cut out the starting rheostat,
and vary the field strength until the synchronizing device shows that the converter is in synchronism with the generator.
(5) Close the main alternating-current switch when the synchronizing device shows that the rotary converter is in step with the alternating-current supply.

If the converter is furnished with a shunt winding only, adjust the field to give minimum alternating-current input.

If the converter has a series winding also, the shunt field should be adjusted to give at no load the direct-current no-load voltage at which it is rated.

When a rotary converter is started as a direct-current motor,


Fig. 298. Westinghouse Rotary Converter Equipped with Separate Starting Motor on Same Shaft
it is easy to bring the machine into operation with a particular direct-current brush or set of brushes positive.
(c) By Separate Motor. The third method of starting rotary converters, by means of a separate-starting motor, is often used. In such cases the starting motor is usually supported on the converter either on one end of the base plate or by the pillow block. The rotor is mounted on the armature shaft of the converter just outside of the bearing, as shown in Fig. 298. The starting motor is usually of the induction type with squirrel-cage rotor, and is
furnished with a number of poles which is two less than the number of poles on the converter. This enables the motor to bring the converter up to and above the synchronous speed, and speed regulating devices are arranged for synchronizing the converter with the alter-nating-current supply mains.

This method of starting has the great advantage of requiring a relatively small starting current, and is used especially when because of limited capacity of generators or transmission system it is essential to keep the starting current as low as possible.

The disadvantages of this method are: that it requires time and skilled attendants to synchronize properly, and that the auxiliary motor adds to the cost and requires additional space.

A modification of the last method is sometimes employed in which the converter is started from rest by means of a separate starting motor and then thrown directly on to the alternatingcurrent busses in series with a suitable reactance which limits the instantaneous rush of current to a safe value. This method combines the advantages of separate motor starting and self-starting from the alternating-current side, in that it obviates the necessity of synchronizing and insures the rotary coming in with the right polarity; on the other hand, it requires somewhat more time than the self-starting method and a heavier line current than with the induction starting motor alone. An example of this method is shown in Fig. 298, a 300-kilowatt three-phase rotary converter manufactured by the Westinghouse Company. It has 10 poles and when supplied with alternating three-phase, 60 -cycle currents, at 367 volts per phase, delivers a direct current of 500 amperes at 600 volts from the commutator. This converter is built for electric railway substation service, and is provided with an induction motor for separate starting. At the right-hand end of the shaft may be seen the mechanical oscillator described below.

Oscillators for Rotary Converters. The armature of a belted dynamo or motor is always caused by the belt to shift slowly to and fro endwise in its bearings, thus largely reducing the uneven wearing away of the commutator in grooves where the brushes rub. The rotary converter, however, not being mechanically driven tends to run without end-play, and some special end-play device, or oscillator, is necessary. An effective device much used is an electro-
magnet mounted opposite to the end of the rotary-converter shaft. This electromagnet is excited about ten times per minute, and at each time gives an endwise pull on the shaft, causing the desired endwise movement of the latter. A mechanical end-play device regularly used by the Westinghouse Company consists of a steel plate with a grooved ball race and ball backed by a spring, and mounted at one end of the shaft. As the grooved plate is not quite parallel to the end of the shaft, when the converter is prop-


Fig. 299. General Electric 300-Kilowatt Three-phase Rotary Converter
erly installed with the armature slightly inclined towards the oscillator, the hardened steel ball is caught at the lowest point between the race and the end of the shaft. The ball is carried upward as the armature revolves and the spring is compressed. The reaction of the spring now forces the shaft away and the ball falls back to its normal position.

Characteristic Types of Rotary Converters. Fig. 299 shows a 300-kilowatt, three-phase (three-ring) rotary converter built by the General Electric Company. It has a six-pole field magnet and six sets of direct-current brushes, each set having eight single brushes.

Its rated speed is 500 revolutions per minute, which, with a six-pole field, gives a frequency of 25 cycles per second. The three collector rings are mounted on the armature shaft on the end opposite to the commutator. It can be seen from the figure that the commutator is larger in comparison with the size of the machine than is usual in an ordinary direct-current generator. Each collector ring has three brushes bearing upon it in order to permit of the delivery to the machine of the very large alternating currents from the supply


Fig. 300. General Electric Three-Phase Rotary Converter for Electric Light Service
mains. The use of three brushes on each collector ring is preferable to the use of one broad brush, inasmuch as it is desirable to make the rings narrow to save space. This machine is rated at 550 volts between its direct-current brushes, so that the full-load direct-current output is 300,000 watts divided by 550 volts, or 546 amperes. Therefore, on the assumption of unity power factor and 100 per cent efficiency, as explained on page 336, the alternating current entering at each collector ring is 546 amperes $\times 0.943=515$ amperes effective; and the effective voltage between collector rings is 550 volts $\times 0.612^{*}$ $=336$ volts. The alternating-current power supplied to the machine is

$$
\sqrt{3} \times 336 \text { volts } \times 515 \text { amperes }=300,000 \text { watts }
$$

[^29]Fig. 300 shows a three-phase rotary converter manufactured by the General Electric Company for electric lighting service at 250 volts. The characteristic features which distinguish the rotary converter from the direct-current generator are here especially prominent, namely, the large commutator and great brush contact area, the comparatively large collector rings, and the relatively small magnetic system, including yoke and field-magnet cores.

Hunting of Rotary Converter. A rotary converter (regular), being a synchronous motor in relation to its alternating-current supply, has a tendency to hunt, as explained on page 212.

A rotary converter, however, having no pulley and not being mechanically connected to machinery, is much more sensitive in responding to the pulsations of an engine or to other causes of hunting than is a synchronous motor delivering mechanical power.

The hunting oscillations of a synchronous motor are always due to external disturbances, that is, to disturbances originating outside of the alternating-current generator, the line, and the synchronous motor.

A sudden change of load on the synchronous motor, for example, is followed by a series of oscillations. Whether or not the oscillations due to a certain change of load give rise to trouble, depends largely upon the resistance and the reactance of the transmission line, and upon the frequency. The greater the resistance and the reactance of the transmission line, the greater the trouble from hunting; and at high frequencies the trouble from hunting is much greater than at low frequencies. Thus a 25 -cycle rotary converter gives no serious trouble from hunting if the line resistance and reactance are not excessively high, whereas a 60 -cycle rotary converter is more likely to give trouble unless special provision is made to diminish hunting, as explained below.

A periodic variation in the speed of the engine driving the alternator from which a rotary is supplied with alternating current, produces very troublesome hunting when this variation of speed is in rhythm with the hunting oscillations. This class of hunting is obviated by increasing the fly-wheel capacity of the engine, or by changing the resistance or the reactance of the transmission lines. The latter method changes the rhythm of the hunting oscillations and thereby does away with the coincidence of rhythm, which is the chief cause of excessive hunting oscillations due to engine pulsations.

The hunting oscillations, once started, usually reach their maximum under given conditions in a few minutes of time, so that serious trouble due to hunting, such as the dropping out of step of the rotary, or excessive sparking at the commutator, usually occurs soon after the hunting begins.

Hunting which is approximately the same at all loads, from noload to over-load, is more troublesome when the field of the rotary converter is over-excited so as to take leading currents, than when the excitation is such as to give either unity power factor or lagging currents.

Hunting is also greater when several rotaries are supplied from an alternating-current generator, than when a single rotary converter is supplied, unless there are short lengths of alternatingcurrent mains between them, or unless the converters supply direct current in parallel to the same direct-current mains.

Dampers. Hunting, whether due to engine pulsations or to momentary outside disturbance, such as sudden change of load or momentary short-circuit, is greatly reduced by the use of massive


Fig. 301. Diagram Showing Copper Dampers in Place in Field Poles copper bridges or frames extending partly over the pole faces, as shown in Fig. 181. A more effective arrangement is shown in Figs. 142 and 301. A number of holes or channels are provided in each pole tip. In these holes or channels heavy copper conductors $A$ and $B$ are placed, and these conductors are short-circuited by being connected at the ends by the bars $C C$. These copper frames or "dampers" diminish hunting oscillations because, when a machine is hunting, the magnetic flux from pole-face to armature core is shifted forwards and backwards over the pole-face, and this shifting flux induces electromotive forces in the copper frames, which produce currents tending to oppose the shifting of the flux, and thereby oppose the hunting oscillations.

A rotary converter having solid cast-steel pole pieces has little or no tendency to hunt. The action of the solid poles is the same as the action of the massive copper conductors shown in Fig. 301.

The use of solid poles, however, leads to excessive eddy-current losses; hence solid poles are not considered desirable.

Inverted Rotaries. When a rotary converter is used to convert a direct current into an alternating current, taking direct current in at the commutator and delivering alternating current at the collector rings, it is called an inverted rotary converter. While the rotary converter is generally used to convert alternating current into direct current, it sometimes happens that inverted converters are desirable. For example, in a low-tension, direct-current system, a district remote from the central station may be supplied with current by converting direct current to alternating. current at the station (by means of inverted rotaries); then, by step-up transformers, raising the voltage to a high value, and transmitting it as high-tension alternating current; and finally, at the distant point, reconverting it (using step-down transformers) to direct current. Again, in a station containing direct-current generators for shortdistance supply, and alternators for long-distance supply, the converter may be used as the connecting link to shift the load from the direct to the alternating generators, or conversely. The machine which is used for shifting the load in this way is caused to operate as a regular rotary or as an inverted rotary according to the demand for direct current or alternating current.

The behavior of an inverted rotary converter is different in many respects from the performance of the same machine when used as a regular rotary converter. When converting from alternating current to direct current, the speed of the converter is rigidly fixed by the frequency of the alternating current supplied to it, and cannot be varied by altering its field excitation. Varying the field excitation would merely change the phase difference between the alternating electromotive force and current supplied to the machine, and hence the power factor, as in the case of the synchronous motor. When converting from direct current to alternating current, however, the speed of the converter, as in a direct-current motor, will be proportional to the applied direct-current voltage, and will also depend upon the field excitation. The effect of weakening the field is to increase the speed, and the effect of strengthening the field is to decrease the speed. It is evident, therefore, that there should be littie or no series field winding provided on an inverted rotary con-
verter, as it will change in speed under load and deliver alternating currents at a variable frequency. If the field becomes greatly weakened, an inverted rotary converter may reach a dangerously high speed before the attendant has time to prevent it, and the armature of the machine may be torn to pieces by the excessively large centrifugal forces.

Changing the field excitation of an inverted rotary converter will not change the voltage of the alternating current, because the ratio of transformation in a given converter is fixed; changing the field strength merely causes a change in the speed of the rotary.

The voltage of the alternating current may be changed by changing the voltage of the applied direct current, or it may be varied by using alternating-current potential regulators, page 448.

Speed-Limiting Devices. An inverted rotary being an alternatingcurrent generator, its field strength depends upon the intensity and the phase relation of the alternating current supplied by its armature; thus a lagging current in the armature reduces the field strength, and hence increases the speed and the frequency; whereas, a leading current increases the field strength, and thus decreases the speed and the frequency. Again, if the alternating-current side of an inverted rotary converter delivers large lagging currents to inductive receiving circuits, such as induction motors, the demagnetizing action of the lagging armature currents on the field may result in a dangerously high speed. In operating inverted rotary converters, therefore, especially when they are liable to be overloaded on the alternating-current side, as in the starting of synchronous or induction motors supplied from the inverted rotary, great care should be taken to see that the field excitation is always great enough to prevent excessive speeds. When used for the above purpose, special speed-limiting devices should be used.

A method used by the Westinghouse Company to prevent this tendency of the inverted rotary converter to race, is as follows:

[^30]increase in its voltage. If then the speed of the inverted rotary converter increases, the voltage of the exciter immediately increases and strengthens the field of the converter, thus checking its tendency to race.

The same result is attained by the General Electric Company, but in a different manner, as follows:

A kind of centrifugal governor is attached to the shaft of the inverted rotary and revolves with it. If the speed of the rotary converter increases above a certain value, the governor acts, thus closing an electric circuit which automatically throws off the power supplied to the rotary.

Control of Direct=Current Voltage. The relations between the alternating-current voltages supplied to a rotary converter and the direct-currẹnt voltage of the machine, as explained on page 331, and as expressed in equation (45), apply to the case in which the field excitation of the rotary converter is such that the alternating currents delivered to the machine are in phase with the applied alter-nating-current voltages; that is, these relations apply to the case in which the power factor of the machine is unity. The directcurrent voltage of a rotary converter may be slightly greater or less than the ideal value given in equations (45), but the fact remains that the ratio of the alternating-current voltage to the directcurrent voltage of a given converter is nearly constant. The fixed ratio of these voltages is a serious handicap in electric railway and similar work where it is desirable to have the station voltage increase as the demand for current increases, in order that the voltage at a distant point of the line shall be kept up in spite of the increase in line drop. In order, therefore, to increase the voltage of the directcurrent output of the rotary converter it is necessary to increase also the alternating-current voltage supplied to its collector rings. This increase of voltage may be secured by the following methods:
(a) Variable ratio step-down transformers
(b) Variable ratio low-voltage autotransformers
(c) Voltage regulators (see page 448)
(d) Synchronous regulators or boosters
(e) Series field winding properly proportioned in connection with series inductive reactance (compounding)

Methods (a), (b), (c), and (d) are non-automatic, and are used where the load is fairly constant over considerable periods. Method (e) is entirely automatic within a range of 10 to 15 per cent and is frequently used where the load is rapidly fluctuating, as in electric
railway service. With the necessary auxiliary apparatus, methods (c) and (d) can be made to operate automatically.

In using method (a) the transformers for rotary converter service are designed for the normal secondary voltage which will give the required direct-current voltage. They" are provided with taps on both primary and secondary windings which allow compensation to be made for line drop or for any small variation from the desired standard voltage. These taps, however, cannot be used for controlling the voltage while the apparatus is in service. Method (d) requires a synchronous booster which is merely an auxiliary alternatingcurrent generator, with the same number of poles as the converter, and whose armature is mounted on the shaft of the converter. The armature winding of this generator is connected in series between the armature and the collector rings of the converter. The field


Fig. 302. Rotor of Westinghouse Six-Phase Rotary Converter with Synchronous Regultaor magnets of the booster are separately excited so that its armature voltage may be varied from zero to a maximum. Thus, since the armature windings of both the converter and the booster are in series, the alternating-current voltage supplied to the collector rings may be varied at will, and the voltage on the direct-current side changed accordingly.

If necessary the excitation and the polarity of the regulator may be reversed, so that the voltage derived from the regulator may be subtracted from the normal value of the impressed alternat-ing-current voltage instead of added to it, and a corresponding reduction of the direct-current voltage obtained. The regulator may be arranged for either automatic or manual control.

With a synchronous regulator having 15 per cent of the rotary converter capacity, the direct-current voltage can be varied 30 per
cent at will; or, if desired, a constant voltage can be maintained. Moreover, the regulator field can be so controlled that when operating in parallel with other rotary converters, or storage batteries, the converter will take its proper share of the total load.

Fig. 302 shows the rotor of a 770 -kw., six-phase rotary converter provided with a synchronous regulator or booster, made by the Westinghouse Electric Company. Passing from left to right in the figure, there are: the commutator of the converter, the armature of the converter, the armature of the synchronous regulator, the six collector


Fig. 303. Westinghouse Three-Phase Converter with Syachronous Regulator
rings, and on the extreme right the squirrel-cage rotor of the auxiliary starting motor.

Fig. 303 is a view of a Westinghouse three-phase 1,000 -kw. converter with a synchronous regulator. The regulator frame, with its field poles and windings, is supported by brackets attached to the converter frame. The converter itself is the same as other standard converters, the main field windings being free from all regulating devices.

Method (e) for voltage control depends on the action of the series field winding in connection with series reactance. As already
shown on page 226 , the field excitation of a synchronous motor may be varied through quite a range above or below that corresponding to unity power factor, the machine taking leading currents when its field is over-excited, and lagging currents when its field is underexcited. This is especially the case when the synchronous motor has considerable armature inductance, and when the transmission line also has considerable inductance. This remark applies to the rotary converter also; and, when the transmission line has considerable reactance, the alternating-current voltages between the collector rings of a rotary converter and the direct-current voltage between its direct-current brushes vary with the field excitation of the converter.

Where there is both inductance and resistance drop in the feeders and a considerable variation in the alternating voltage supply, the converter, if provided with series field winding, can be made to regulate automatically for constant direct-current voltage within reasonable limits, as explained below.

As the ratio of the alternating current to the direct-current voltage of a converter is nearly constant, the impressed alternating voltage must be varied in order to vary the direct-current voltage. This can be done by taking advantage of the fact that an alternating current passing over an inductive circuit will decrease in voltage if lagging in phase behind its electromotive force, and will increase in voltage if leading. Just as in the case of a synchronous motor, a certain field excitation in any converter will give a minimum armature current. If the excitation be decreased, the armature current will be increased but will be lagging. By providing, therefore, sufficient reactance in the alternating-current circuit connecting a converter with its source of power, the alternating-current voltage at the converter terminals may be varied by means of the field excitation of the converter, and without altering the generator voltage.

When it is desired to control the direct-current voltage of a rotary converter independently of the voltage of the alternatingcurrent generator that supplies the alternating currents, the transmission line is frequently given an artificial reactance by connecting inductance (reactance) coils in series with the alternating-current supply mains. Thus, the alternating currents delivered to the converter are caused to flow through these reactance coils. When,
therefore, a rotary converter has a compound field winding (series and shunt), as described below, the use of reactance coils in the supply mains is necessary if the transmission lines do not of themselves have sufficient reactance.

Field Excitation. Various methods are employed for exciting the field magnet of rotary converters, as follows:
(a) By Armature Reaction. When an alternating-current generator delivers leading current to a receiving circuit, the magnetizing action of the armature currents tends to strengthen the field magnet poles, as explained on page 84. If an alternating-current generator were always used to deliver leading currents, it would be possible to depend upon the magnetizing action of the armature currents entirely for exciting the field magnet, without using any direct current whatever in the field windings; in fact, the field windings could be dispensed with altogether.

When a synchronous motor, or rotary converter, takes lagging currents from alternating-current supply mains, the magnetizing action of these currents in the motor armature is to strengthen the field magnetism of the motor, or rotary converter. This action alone may be utilized for exciting the field magnet of a synchronous motor, or rotary converter, and rotary converters have been designed and commercially operated with this mode of field excitation.
(b) Self-Excitation by Direct-Current Taken from the Commutator of the Machine. The usual method of exciting the field of a rotary converter is by means of direct current taken from the commutator of the machine itself. There are three schemes for carrying out this method of field excitation, exactly as in the case of ordinary direct-current generators, as follows:
(1) Series excilation. This scheme, in which the entire direct-current output flows through the field winding (coarse wire), gives a field excitation which is zero when the direct-current output is zero, and which rises to fullrated excitation when full-load output of direct current is reached. This scheme of field excitation is not suitable for rotary converters, inasmuch as a rotary converter should have an approximately constant-field excitation, or a field excitation which changes through a comparatively narrow range only.
(2) Shunt excitation. In this scheme the field winding is made of comparatively fine wire. Its resistance, therefore, is comparatively high, and it is connected directly between the direct-current brushes with an adjustable field rheostat in its circuit, exactly as in the ordinary shunt-wound direct-
current dynamo. This scheme gives an approximately constant field excitation, and it is much used in rotary converters. The variation of field excitation for the purpose of controlling the power factor of the converter is accomplished by means of the adjustable field rheostat.
(3) Compound excitation. The combination of series and shunt excitation is frequently used in rotary converters so as to provide for slightly increasing field excitation (by means of the series winding) with increasing directcurrent output. This scheme of field excitation is, however, more limited when applied to a rotary converter than when applied to an ordinary directcurrent dynamo, for the reason that too great an increase of field excitation in a rotary converter (as in the case of a synchronous motor) causes the converter to fall out of synchronism and stop, or "break down," as it is termed.

Compound-wound rotary converters are used to advantage for supplying current which is constantly fluctuating (and where the generators supplying the converters do not greatly exceed the latter in kilowatt capacity) as in railway service, and in cases where it is necessary to maintain constant or increasing voltage with increasing load. More or less prominence can be given to shunt or series windings as may be required.

The regulation is made automatic by a series field winding on the converter, but the inductance of the transmission lines and generator must frequently be increased by introducing reactive coils.

The amount of raising ("boosting") or lowering of the voltage is proportional to the reactance in circuit, for a given series field. Considering, however, that the maximum output of the converter and its stability are affected by too much reactance, the introduction of reactance should not be carried too far.

In compound-wound converters the shunt excitation is generally adjusted to give a lagging current of from 20 per cent to 30 per cent of full-load current at no load by under-excitation, and the series field is adjusted to give a slightly leading current at full load. This arrangement lowers the impressed voltage at the converter at no load and raises it at full load enough (with constant voltage at the generator) to compensate for all the losses of voltage in the system, thus making possible the delivery of a constant direct-current voltage at all loads.

It has been found in practice that the compound winding distinctly diminishes the stability of running when the tendency to hunt is present to any extent. The series winding should be cut out
when starting up from the direct-current side. This is conveniently accomplished by a double-throw switch which in one position connects the junction of the series winding and the negative brushes to the starting rheostat, and in the other position connects this junction with the equalizing bar.
(c) Separate Excitation. The use of a small auxiliary directcurrent dynamo to supply direct current for exciting the field of a rotary converter has been mentioned on page 348, where it was pointed out that this method of field excitation is especially suited to inverted rotaries.

Rotary Converter with Edison Three=Wire System. The Edison three-wire system must ordinarily be supplied with current from two direct-current generators connected in series between the outside mains, and with the neutral main connected to the junction of the two machines.

To operate a three-wire system from a single direct-current generator would give rise to great differences of voltage on the two sides of the system when the number of lamps on one side differs greatly from the number of lamps on the other side; in fact, it would not be allowable to turn off a lamp on one side without turning off a lamp on the other side at the same time. To avoid this difficulty some arrangement which is equivalent to the use of two generators is necessary.

A rotary converter may be used to deliver direct current to an Edison three-wire system giving every advantage ordinarily obtained by the use of two direct-current generators connected in series. For such service the third wire must be connected to the neutral point of the transformer group in such a manner that the current in the neutral will flow equally in both directions through the transformer windings and, therefore, will not change the effective magnetic flux in the core of the transformer. When three-phase distribution is used the direct current neutral is brought out from the common junction point of the inter-connected $Y$ secondaries. With two-phase distribution the direct current neutral can be connected to the middle point of the secondary coils of the transformer.

The connections of a three-phase rotary converter for supplying direct current to an Edison three-wire system are shown in

Fig. 304. The three primaries $A, B$, and $C$ of the step-down transformers for supplying alternating currents to the rotary converter


Fig. 304. Connections for a Three-Phase Rotary Converter on Edison Three-Wire System are either Y - or $\Delta$-connected to the high voltage supply mains. The three secondaries $a, b$, and $c$ are $Y$-connected to the collector rings $r^{\prime}, r^{\prime \prime}, r^{\prime \prime \prime}$, of the rotary converter. The two outside direct-current mains 1 and 3 are connected to the direct-current brushes of the converter as shown, and the middle main (neutral main) 2 is connected to the common junction $N$, or neutral point, of the Y -connected secondaries $a, b$, and $c$. With these connections, the voltage between mains 1 and 2 is a steady direct-current voltage, as is also the voltage between mains 2 and 3 , and each of these voltages is equal to half the voltage between mains 1 and 3 . When the number of lamps connected between mains 1 and 2 is different from the number connected between mains 2 and 3 , or vice vers $\hat{a}$, the neutral main must carry a direct current equal to the difference of the direct currents in the mains 1 and 3 . This direct current in the neutral main 2 is actually supplied through the secondary coils $a, b$, and $c$ from the collector rings. The rotary converter may also be


Fig. 305. Connections for D. C. Generator on Edison Three-Wire System with Two-Phase Rotary Converter Connected as a Balancer
ased to take the place of two machines, a generator and a motor, used in combination to form a "balancer" for keeping the voltages on the two sides of an Edison three-wire system the same.

Fig. 305 shows an ordinary direct-current generator $G$, sup-
plying current to the outside mains 1 and 3 , of an Edison three-wire system, and a two-phase rotary converter $R$, connected as a "balancer" to supply the necessary direct current to the middle main 2. The armature only of the rotary converter is shown in the figure, its direct-current brushes being connected to the outside mains 1 and 3. One pair of opposite collector rings of the converter $R$ is connected to an inductance coil $a a$, wound on an iron core, and the other pair is connected to an inductance coil $b b$. The middle points of these two inductance coils are connected together and to the middle or neutral main 2 of the three-wire system.

Six=Phase Converter. The large rating of a six-phase converter as shown in Table XIV, together with other advantages enumerated below, make this machine the standard converter for large outputs. The advantages of the six-phase converter are as follows:
(a) The high rating, namely, 1.92 times the rating of the same machine as a direct-current generator, or 1.45 times the rating of the same machine as a three-phase converter, means that the machine may be smaller and, therefore, cheaper for a given output. The high rating of the six-phase converter is due to the fact that its armature winding is tapped at six points (for a twopole machine), and that alternating currents enter the armature winding at six points, so that the length and the resistance of the paths from collector rings to commutator are less, and the heating of the armature windings is less than it is in a three-phase converter, for example, for the same directcurrent output.
(b) The six-phase converter runs more stably than a converter having a smaller number of collector rings, and has less tendency to hunt.
(c) The magnetizing actions of the alternating and direct currents in the armature are more nearly balanced in the six-phase converter than in a converter having a fewer number of collector rings, and commutation is freer from sparking and flashing.

In spite of the above advantages, the greater complication of the transformer and collector connections outweighs them in the smaller sizes. Six-phase converters, therefore, are rarely built in units of less than 500 kilovolt amperes. Above this output the sixphase is usually preferable to the three-phase machine.

Transformer Connections for Rotary Converters. Figs. 306 to 313 show the connections which are commonly employed between transformers and rotary converters. The circular spiral winding at the bottom of each figure represents the armature winding of a bipolar converter, the collector rings being omitted to avoid confusing the diagram. Figs. 306, 308, 310, and 312 show the two-phase
and three-phase connections, while Figs. 307, 309, 311, and 313 are for six-phase connection.


Two-Phase Diametrical
Fig. 306. Transformer Connections for Two-Phase Rotary Converter


Fig. 307. Transformer Connections for SixPhase Rotary Converter

Each pair of figures, 306 and 307, 308 and 309, 310 and 311, 312 and 313 are closely related. Thus in Figs. 306 and 307, the two terminals of each transformer secondary coil are joined to the armature winding of the converter at points 180 degrees apart. Such connection is called the diametrical connection and it is possible when the rotary has an even number of rings. In any diametrical connection of the secondaries of the step-down transformers (one trans-


Three-Phase $\triangle$ Connection
Fig. 308. Transformer $\Delta$ Connections for Three-Phase Rotary Converter


Six-Phase Double $\triangle$ Connection
Fig. 309. Transformer Double $\Delta$ Connections for Six-Phase Rotary Converter
former for a two-ring converter, two transformers for a four-ring converter, and three transformers for a six-ring converter), the voitage
in each secondary coil is the same in value, no matter how many rings the converter may have.


Fig. 310. Transformer Y Connections for 'Three-Phase Rotary; Converter


Fig. 311. Transformer Double Y Connections for Six-Phase Rotary Converter

The diametrical connection of the transformer secondaries to the converter rings is simpler than the connections shown in Figs. 309,311 , and 313 , inasmuch as the diametrical connection requires only one secondary coil on each of the step-down transformers and, therefore, but two secondary leads are brought out from each transformer; whereas the connections shown in Figs. 309, 311, and 313 require two secondaries on each transformer and, therefore, four secondary leads from each transformer. The switching arrangements for the diametrical connection are, therefore, simpler than they are for the connections shown in Figs. 309, 311, and 313.

Fig. 308 shows three step-down transformers receiving three-phase currents and delivering three-phase currents to a rotary converter, the secondaries being connected to the collector rings. Fig. 310 shows the same arrangement except that the secondaries are Y -con-


Three-Phase T-Connection
Fig, 312. Transformer T Connections for Three-Phase Rotary

Converter nected to the collector rings. Fig. 312 shows a Scott* trans-

[^31]former receiving currents from two-phase supply mains and delivering three-phase currents to a three-ring converter. This Scott transformer arrangement is often called the $\mathbf{T}$ connection, and it may be adapted with slight modification to three-phase supply, that is, to transform three-phase alternating currents into two-phase alternating currents.

For three-phase rotary converters, the transformers should preferably be connected in $\Delta$, as this permits the system to be operated with only two transformers, in case the third has to be cut out of the circuit temporarily for repairs.

The $T$ connection as shown in Figs. 312 and 313 requires only two transformers, and it can be used to change from either two-phase or three-phase to three-phase, as shown in Fig. 312, or from two-phase or three-phase to six-phase, as shown in Fig. 313. The Y or T arrangement of transformers is particularly advantageous where a rotary is to be used to supply direct current to an Edison three-wire system. Of course, two converters can be used, one on each branch of the three-wire system, and this is the preferable method where the branches are liable to be greatly unbalanced. With the Y or T connection of transformers a single converter can be connected across the outside wires of the three-wire system, the neutral wire can then be joined to the neutral point of the $Y$ connection, or to a tap, in one of the transformer windings, which tap corresponds to the neutral point of the Y connection.

## TESTING ROTARY CONVERTERS

Standard Tests. Fgr a rotary converter, the saturation and core loss curves are obtained in the same manner as in the case of an alternating-current generator, with the exception that the directcurrent voltage is also recorded. The phase characteristic is determined and the pulsation test is made in the same manner as for a synchronous motor, except that the direct-current voltage should be
recorded. The machine, of course, is run as a synchronous motor, being supplied with alternating current through its collecting rings. The pulsation test should be made with the converter self-excited.

Heat Run. To make a heat run on a rotary converter it may be run either as a synchronous motor taking alternating currents through its collector rings, or as a direct-current motor supplied with direct curmnt through its brushes and commutator. if driven as a synchronous aiternating-current motor, the full load output is taken from the commutator end of its armature, or vice versa, and is delivered to a water rheostat or other receiver. During the run the following readings should be recorded every half hour:

| Volts | Volts | Amperes | Amperes | Amperes | Volts | Speed | Room |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| D.C. | A. C. | D. C. | A.C. | Field | Field |  | Temp. |

When the rotary converter has reached a constant temperature, after running continuously a number of hours under rated full-load conditions, the machine may be shut down, and thermometers quickly applied to measure the temperature of the following parts:

| Armature laminations | Field spools |
| :--- | :--- |
| Armature ventilating ducts | Pole tip, leading |
| Armature coils front end | Pole tip, trailing |
| Armature coils back end | Frame |
| Armature binding wires | Bearings |
| Commutator | Room |

The condition of constant temperature is indicated when the voltage applied to the terminals of the field winding in order to keep the current in the field winding constant, no longer increases. In other words, the resistance of the field winding increases for a time due to increasing temperature of the winding, and it takes an increasing voltage at the field terminals to maintain the field current constant. But when the temperature of the field windings becomes constant, the voltage required at the field terminals no longer increases.

Motor-Generator Method. If two similar rotary converters are at hand, the heat test may be made by the "motor-generator" method,
in a manner somewhat similar to the method described under transformers. This method applied to rotary converters involves running one machine as an inverted rotary taking. power through its brushes and commutator from direct-current mains. The second machine is made to run as a regular rotary converter, taking its power from the collector rings (that is, alternating-current side) of machine No. 1. Machine No. 2 then delivers its power in the form of direct current from its commutator back to the direct-current supply mains, or to the commutator of machine No. 1. Machines Nos. 1 and 2 are now said to be "tied together in multiple on both the direct-current


Fig. 314. Wiring Diagram for Motor-Generator Method of Testing Rotary Converters
and the alternating-current sides." If the machines are similar in every respect, no appreciable current will flow between the two under these circumstances. Each will run from the direct-current side, the two together taking only enough power from the direct-current mains to supply the no-load losses in both machines. If, however, an auxiliary electromotive force is applied in the circuit between the machines on the alternating-current side, it will immediately cause current to flow between the machines precisely as in the case of the transformer test above referred to.

Fig. 314 is a diagram of the electrical connections for this method of running. $R_{1}$ and $R_{2}$ are the two three-phase rotaries. They are shown connected together electrically on the alternating-current
side (that is, the collector rings of $R_{1}$ are connected to the corresponding collector rings of $R_{2}$ ) through an induction regulator $R$. This regulator supplies to the circuit of each phase the auxiliary electromotive force necessary to cause the current to flow between the two machines. The electromotive forces supplied by the regulator are adjustable. When the machines are first thrown together the regulator is in such a position that its electromotive forces are zero.

On the direct-current side (commutator ends) both machines are shown electrically connected to the direct-current supply mains. The method of procedure in starting the two machines is as follows:

First, $\dot{R}_{1}$ is started as a direct-current motor from the directcurrent side (commutator). Its field current is adjusted until it runs at the right speed when the normal voltage is applied to its armature terminals; $R_{1}$ is at this time entirely disconnected from $R_{2}$. Next $R_{2}$ is started in the same manner. Then $R_{1}$ and $R_{2}$ are synchronized and connected together, as shown in the diagram, by closing switches $S_{1}, S_{2}$, and $S_{3}$ on the alternating-current side.*

The most convenient way to synchronize the machines for testing is to connect up a series of incandescent lamps across each of the switches $S_{1}, S_{2}$, and $S_{3}$. The number of incandescent lamps in each series should be such that they can stand twice the normal alternating voltage of each machine.

The field current of $R_{2}$ is varied until it is almost in synchronism with $R_{1}$, as shown by the slow pulsations of the synchronizing lamps. When the lamps are dark, there is zero voltage across the terminals of the switches, and then they can be safely closed. If the lamps connected across the switches do not all go out at the same instant, the machines cannot be synchronized. In this case the direction of rotation of one of the machines must be reversed, or two of the leads on the alternating-current side of one of the machines must be interchanged.

Up to this time the two machines have been running independently of each other. Each has taken sufficient current from the mains to supply the losses occurring in it while running unloaded. When the two machines are connected together on the

[^32]alternating-current side by closing the switches $S_{1}, S_{2}$, and $S_{3}$, they run in synchronism. If the regulator $R$ is in its zero position, no appreciable current will flow between the two machines. As the voltage of $R$ is increased, however, a current will circulate between the two machines. If the voltage of $R$ is in such a direction that $V_{1}$ is greater than $V_{2}$, then $R_{2}$ will run as a regular rotary converter, and $R_{1}$ as an inverted rotary. The power will pass from $R_{1}$ to $R_{2}$ on the alternating-current side, and from $R_{2}$ to $R_{1}$ on the directcurrent side. $R_{1}$ will run as an inverted rotary, driving $R_{2}$ on the alternating-current side, and $R_{2}$ will run as an ordinary rotary converter driving $R_{1}$ on the direct-current side.

By sufficiently increasing the voltage of $R$ we can cause fullload or even over-load conditions to obtain. The direct-current mains deliver merely the power to supply the losses in the entire system. Ammeter $A$ indicates the current supplied to overcome the losses in the system. Under these circumstances, $R_{1}$ will have a load slightly greater than $R_{2}$. Ammeter $A_{1}$ should give a reading equal to the sum of the readings of the two ammeters $A$ and $A_{2}$.

The chief advantage of this method is that full-load conditions are obtained without actually supplying full-load current from the direct-current supply mains. The mains supply simply the power losses in both machines. Another great advantage is the ease with which the load can be controlled. By this method, moreover, two machines can be tested more easily than a single machine by the ordinary method. If it is desired to test one machine only, the load may be put on in the same way by connecting it up with a second machine. It is not necessary in this case that the second machine be of the same size as the first. It is only necessary that it be of the same voltage and frequency, and that it can carry without excessive heating the full-load current of the machine to be tested. The current, during the run, is adjusted to the normal fullload value of the machine to be tested.

## MOTOR=GENERATORS

The last method of obtaining direct current from alternatingcurrent circuits is by a motor-generator, which consists of an alter-nating-current motor driving one or more direct-current generators.

The motor may be of the induction or of the synchronous type. In either case the motor is usually mounted on the same base with, and mechanically coupled to, the generator. The smaller sets up to about 150 kilowatts are usually driven by induction motors for 25 , 40 , or 60 cycles, and are customarily used for charging storage batteries or for furnishing the exciting current for the fields of alternators or synchronous motors. The larger sets, from 200 kilowatts upwards, are used for the supply of lighting, railway, or power systems, and are usually driven by synchronous motors, which have the advantage of permitting the control of the power factor of the circuit by varying the field excitation, but have the disadvantage of requiring a direct-current field excitation. Synchronous motors have an advantage over induction motors in that large sizes may be wound for 11,000 or 13,200 volts, whereas induction motors of the same size are limited to about 2200 or 6600 volts.

Comparison with the Rotary Converter. A motor-generator employing a synchronous motor does not seem to possess any essential advantages over the rotary converter, except in some cases where independent control of the direct-current voltage is desired. The use of the synchronous motor does not remove the objections to the rotary converter which are based on the fact that it is a synchronous machine.

A motor-generator employing an induction motor has the advantage of using induction instead of synchronous apparatus, thereby securing many of the advantages summarized later in the comparison between synchronous and induction motors, page 401.

Circuits which are supplied by alternating-current generators the speed of which has rapid and periodic fluctuations, or in which for any other reason the conditions are such as to cause "hunting" in synchronous machines, may, however, satisfactorily supply an induction motor driving a generator. The various characteristics of the induction motor under emergency conditions, such as a sudden overload, momentary interruption, or lowering of the voltage of the supply circuit, may cause little or no inconvenience if the induction motor is used, whereas it might cause serious interruption to a rotary converter or a synchronous motor. The induction motor driving a generator is also to be preferred where the size of the units is quite small and the attendance is unskilled. It is also preferable for single-
phase work. The armature for the induction motor should in general be of the squirrel-cage type as the required starting torque does not cxceed 20 per cent of full-load torque, and the required starting torque can be obtained with about full-load current.

The rotary converter, like the synchronous motor, is not suitable for general distribution in small units. It has the advantage over the motor-generator in point of cost, there being but one machine instead of two; in point of efficiency, there being the loss in one machine instead of two; and in its effect upon the voltage of the transmission system as a whole, as it may be compounded to overcome the drop which would otherwise occur in generator and transmission circuit.

On the other hand, the electromotive force of the direct current delivered by the converter has a more or less fixed relation to the electromotive force of the alternating currents supplied; whereas the electromotive force of a motor-driven generator is independent of the electromotive force of the supply circuit, and it may be adjusted or compounded as desired. In practice, however, it is found that the voltage delivered by a rotary converter can be satisfactorily adjusted and controlled by regulating devices or by compounding, so that usually the close relation between the electromotive forces at the two ends of the converter is not disadvantageous, provided the electromotive force of the supply circuit is reasonably constant. This statement applies to those cases where a practically constant direct-current voltage is desired. There are, of course, special cases, in which the voltage is to be adjusted over a wide range, or where, for other reasons, the motor-generator is to be preferred. In many casses the motor may be operated without requiring step-down transformers, whereas they would be necessary with rotary converters.

Uses of Motor=Generatcrs. Motor-generator sets are often used in central stations in connection with arc-lighting service. In these cases alternating-current motors are used for driving arclighting generators, as in the large plant of the Buffalo General Electric Company. They are also used in some places for supplying three-wire direct-current lighting circuits from alternating-current mains.

Motor-generators are often used in central stations for fur-
nishing direct current to the field-magnet windings of alternators, the direct-current generator part of the set acting as the exciter for one or more large alternators. The alternating current supplied to the (synchronous or induction) motor end of the motor-generator set is taken from the main station bus bars, and may or may not be stepped-down by means of reducing transformers.

Examples of this use of motor-generator sets are to be found in the immense power stations of the Interborough Rapid Transit Company, and the New York Edison Company, of New York City, and in many other places.

Another use for motor-generators is as frequency changers (or converters). In such cases they are used for transforming an


Fig. 315. General Electric Motor-Generator Set with D. C. Exciter
alternating current of one frequency to another alternating current of higher or lower frequency, usually the former. The current may be transformed from three-phase to two-phase, or vice versa, at the same time. Thus it might be required to supply a two-rhase, 60 -cycle lighting system from a 25 -cycle, three-phase transmission system, with or without change of voltage.

Types. A motor-generator set built by the General Electric Company is shown in Fig. 323. It consists of a combination of three distinct machines, all mechanically coupled to the same shaft, viz, a three-phase synchronous motor of the revolving-field type; a three-phase alternator also of the revolving-field type; and a direct-
current generator acting as an exciter for the fields of both the alternator and the synchronous motor.

Summary of Data for Fig. 315

|  | Synchronous Motor | Alternator | Exciter |
| :---: | :---: | :---: | :---: |
| No. of poles | 8 | 12 | 10 |
| Rated output (kw.) | 275 | 250 | 10 |
| Speed. | 570 | 570 | 570 |
| Frequency | 38 | 57 |  |
| Volts. |  |  | 60 |

This particular motor-generator set is designed to be used as a frequency-changer to change from 38 to 57 cycles per second.


Fig. 316. General Electric Motor-Generator Set-Induction Motor Driving Compound-Wound D. C. Generator

The three-phase alternating currents having a frequency of 38 cycles per second are led into the stationary armature terminals of the synchronous motor (shown next to the pulley in Fig. 315). The motor having eight field poles, its synchronous speed is 570 r. p. m.; and since both the alternator and the exciter are coupled to the same shaft, all the machines run at 570 r. p. m.

The alternator having 12 field poles, the frequency of its induced electromotive forces is 57 cycles per second. Hence, the
three-phase alternating electromotive forces induced in the stationary armature of the alternator have a frequency 50 per cent higher

than those of the alternating currents which are supplied to the driving synchronous motor.

Fig. 316 is a view of a motor-generator set built by the General Electric Company. It consists of a compound-wound direct-current generator driven by an induction motor, both machines being mounted on the same bed-plate and having a common shaft.

Fig. 317 shows two motor-generator sets, each consisting of two Brush arc-light dynamos directly coupled to an induction motor. In some of the stations of the Brooklyn Edison Company Brush arc-light dynamos are driven in pairs by being directly coupled to three-phase, 25 -cycle synchronous motors. The constancy of the speed of synchronous motors makes them especially well suited for driving electric generators.


RICHMOND POLYPHASE MOTOR BELTED TO A PUMF
Courtesy of Richmond Electric Company

## ALTERNATING-CURRENT MACHINERY

## PART VI

## INDUCTION MOTOR

Constructive Elements. It has already been pointed out that the successful use of alternating current for power purposes depends largely upon the use of the induction motor supplied with polyphase currents. This machine consists of a primary member and a secondary member, each with a winding of bars or wire. The primary


Fig. 318. Diagram of Squirrel-Cage Type of Rotor


Fig. 319. Diagram Showing Slots in Stator of Induction Motor
member is usually stationary, and is often called the stator. The secondary member is usually the rotating member, and is often called the rotor. Fig. 318 shows a rotor of the squirrel-cage type. It consists of a drum $A$ built up of thin circular sheet-iron disks. Near the periphery of this drum are a number of holes parallel to the axis of the drum. In these holes heavy copper rods $b$ are placed, and the projecting ends of these rods are screwed and soldered or welded to massive copper rings $r$, one at each end of the drum.

The stator is a laminated iron ring $F F$, Fig. 319, closely surrounding the rotor. .This ring is slotted on its inner face, as shown; windings are arranged in these slots, and these windings receive
current from polyphase supply mains. These polyphase currents produce in the stator iron a rotating state of magnetism, the action


Fig. 320. Squirrel-Cage Rotor Mounted in Ordinary Four-Pole Field


Fig. 321. Diagram of Four-Pole TwoPhase Induction Motor
of which on the rotor is the same as the action of an ordinary field magnet mechanically revolved. Thus Fig. 320 shows a squirrelcage rotor $A$ surrounded by an ordinary field magnet revolving in the direction of the arrows. This motion of the field magnet induces currents in the short-circuited copper rods of the rotor; the field magnet exerts a dragging force on these currents, and causes the rotor to revolve. No electrical con-


Fig. 322. Diagram of One-Half the Stator Conductors of Fig. 321 nections of any kind are made to the rotor.

Stator Windings and Their Action. The stator windings are arranged in the slots $s$, Fig. 319, in a manner exactly similar to the arrangement of the windings of the two-phase or three-phase alternator armature according as the motor is to be supplied with two-phase or three-phase currents.

Two-Phase. Fig. 321 shows an end view of a four-pole twophase induction motor. In this figure, the outline only of the rotor (or secondary) is shown. The stator conductors are represented in section by the small circles, the slots being omitted for the sake of clearness, and the end con-
nections of half the stator conductors are shown in Fig. 322. In this figure the straight radial lines represent the conductors which lie in the slots of the stator, and the curved lines represent the end connections. The stator conductors are arranged in two distinct


Fig. 323. Magnetic Action of Conductors Carrying Currents between Iron Poles


Fig. 324. Magnetic Action of Conductors Carrying Currents between Iron Poles
circuits. One of these circuits includes all of the conductors marked $A$, and this circuit receives current from one phase of a two-phase system. The other circuit includes all of the conductors marked $B$ and this circuit receives current from the other phase of the twophase system. The terminals of the $B$ circuit are shown at $t t^{\prime}$, Fig. 322. The conductors which constitute one circuit are so connected that the current flows in opposite directions in adjacent groups of conductors as indicated by the arrows in Fig. 322.

The action of a band of conductors between the two masses of iron is shown in Figs. 323 and 324. The small circles represent the conductors in section; conductors carrying down-flowing currents


Fig. 325. Instantaneous Effect of Alternating Current for a Given Position of Rotor


Fig. 326. Magnetic Effect of Alternating Current $\frac{1}{8}$ of a Cycle Later than in Fig. 325
are marked with crosses and those carrying up-flowing currents with dots. The action of the currents in these bands of conductors is to
produce magnetic flux along the dotted lines in the direction of the arrows.

The lines $A^{\prime}$ and $B^{\prime}$ in the clock diagrams of Figs. 325, 326, and 327 are supposed to rotate, and their projections on the fixed


Fig. 327. Magnetic Effect of Alternating Current $\frac{1}{4}$ of Cycle Later Than in Fig. 325 line $e f$ represent the instantaneous values of the alternating currents in the $A$ and $B$ conductors, respectively. These conductors are represented in section by the small circles. These small circles are marked with crosses when they carry down-flowing currents, with dots when they carry up-flowing currents, and they are left blank when they carry no current.
Fig. 325 shows the state of affairs when the current in conductors $A$ is a maximum and the current in conductors $B$ is zero, and the dotted lines indicate the paths of the magnetic flux. This flux enters the rotor from the stator at the points marked $N$ and leaves the rotor at the points marked $S$.

Fig. 326 shows the state of affairs, one-eighth of a cycle later, when the current in the $B$ conductors has increased and the current in the $A$ conductors has de-


Fig. 328. Complete Connections for Four Poles of One Circuit of Three-Phase Induction Motor creased to the same value, so that equal currents flow in the $A$ and in the $B$ conductors. The points $N$ and $S$ have moved over one-sixteenth of the circumference of the stator ring, from the positions they occupied in Fig. 325.

Fig. 327 shows the state of affairs, after another eighth of a cycle, when the current in the $B$ conductors has reached its maximum value, and the current in the $A$ conductors has dropped to zero. The points $N$ and $S$ have moved acain over one-sixteenth
of the circumference of the stator ring. This motion of the points $N$ and $S$ is continuous, and these points make one complete revolution (in a four-pole motor) during two complete revolutions of the vectors $A^{\prime}$ and $B^{\prime}$, that is, while the alternating currents supplied to the stator windings are passing through two cycles. In general:

$$
\begin{equation*}
n=\frac{2 f}{p} \tag{47}
\end{equation*}
$$

in which $n$ is the revolutions per second of the stator-magnetism, $p$ is the number of poles, $N$ and $S$, and $f$ is the frequency of the alternating currents supplied to the stator windings.

Three-Phase. When an induction motor is driven by threephase currents, the stator conductors are arranged in three distinct circuits $A, B$, and $C$, which are either $\Delta$-connected or $Y$-connected to the supply mains. Fig. 328 shows the complete connections, for four poles, of the $A$ circuit with its terminals $t t^{\prime}$. The $B$ and $C$ circuits are similarly connected.

In general, the $q$-phase stator winding for $p$ poles has $p q$ equidistant bands of conductors. The first, $(q+1)$ th, $(2 q+1)$ th, etc., bands are connected in one circuit, so that currents flow oppositely in adjacent bands, and this circuit takes current from one phase of the $q$-phase system. The second, $(q+2)$ th, $(2 q+2)$ th, etc., bands are similarly connected in another circuit and take current from the second phase of the $q$-phase system. The third, $(q+3)$ th, $(2 q$ $+3)$ th, etc., bands are similarly connected in another circuit and take current from the third phase of the $q$-phase system; and so on.

## ACTION OF INDUCTION MOTOR

Many important details of the action of the induction motor are most easily explained by looking upon the induction motor as a rotor influenced by an ordinary field magnet, mechanically revolved. The complete theory of the action of the induction motor is, however, similar to the theory of the alternating-current transformer.

Torque and Speed. Let $n$ be the number of revolutions per second of the field, and $n^{\prime}$ the revolutions per second of the rotor. When $n=n^{\prime}$, the rotor and field revolve at the same speed, so that their relative motion is zero; no electromotive force is then induced in the rotor conductors and no current flows and, therefore, the revolving field exerts no torque upon the rotor. As the speed of the
rotor decreases, the difference of the speeds of rotor and field $\left(n-n^{\prime}\right)$, increases and, therefore, the electromotive force induced in the rotor conductors, the currents in the conductors, and the torque with which the field drags the rotor, all increase. If the whole of the field flux were to pass into the rotor and out again in spite of the demagnetizing action of the current in the rotor conductors, then the torque would increase in strict proportion to $\left(n-n^{\prime}\right)$. As a matter of fact, because of the demagnetizing action of the rotor currents, a larger and larger portion of the field flux passes through the space between the stator and rotor conductors as the speed of the rotor decreases, and this magnetic leakage causes the torque to increase more and more slowly as $\left(n-n^{\prime}\right)$ increases. The torque usually reaches a maximum value, and then decreases with further increase of $\left(n-n^{\prime}\right)$.

Fig. 329 shows the typical relation between torque and speed of an induction motor. Ordinates of the curve represent torque, and abscissas measured from 0 represent rotor speeds. The rotor is said to run above synchronism when it is driven so that $n^{\prime}$ is greater than $n$. The rotor


Fig. 329. Graphical Relation of Torque and Speed of Induction Motor never actually reaches synchronous speed, but approaches it very nearly when the induction motor is running unloaded. In order to cause the rotor to run above synchronism, that is, to make $n^{\prime}$ greater than $n$, or in order to cause the rotor to run backwards, that is, in a direction opposite to that of the revolving magnetism in the stator iron, the rotor must be driven mechanically from an outside source of power.

Starting Resistance in the Rotor Windings. The speed of the rotor for which the maximum torque occurs, depends upon the resistance of the rotor windings, and it is advantageous under certain conditions of operation to provide at starting such resistance in these windings as to at once produce the maximum torque, this resistance being cut out as the motor approaches full speed.

Efficiency and Speed. For the sake of simplicity, let us assume that the only opposition to motion of the revolving field magnet
is the reaction of the torque which it exerts on the rotor. Let this torque be represented by $T$. Then the power expended in driving the field magnet is $2 \pi n T$, and the mechanical power delivered to the rotor is $2 \pi n^{\prime} T$, and this power $2 \pi n^{\prime} T$ is available at the pulley of the motor, except for slight losses due to friction in the bearings and to air friction. Therefore, ignoring friction losses, $2 \pi n T$ is the input of power in driving the revolving field, and $2 \pi n^{\prime} T$ is the output of power, so that the efficiency of the induction motor is $\frac{n^{\prime}}{n}$.

This expression for efficiency ignores all the losses of power in the revolving field magnet* and the friction and windage losses in the rotor, and shows that the efficiency of an induction motor is zero when the rotor stands still, that it increases as the rotor speeds up, and approaches 100 per cent, ignoring field losses and friction, as the rotor speed approaches the speed of the revolving field. The ratio $\frac{n^{\prime}}{n}$ ranges from 0.85 to 0.95 or more, in commercial induction motors under full load, but the actual full load efficiencies of induction motors range from 75 per cent, or even less for small motors, to about 95 per cent for very large motors.

Ratio of Mechanical to Electrical Energy in Rotor. The total power delivered to the rotor is equal to $2 \pi n T$ where $n$ and $T$ have the meanings above specified. That is, all of the power used to drive the field magnet (ignoring losses in the field) is delivered to the rotor. Now, the mechanical power delivered to the rotor is equal to $2 \pi n^{\prime} T$, as already explained; therefore, the difference $2 \pi n T-2 \pi n^{\prime} T \dagger$ is electrical power used to force the rotor currents through the rotor windings.

Therefore, when the field speed is $n$ and the rotor speed is $n^{\prime}$, the total power delivered to the rotor, the mechanical power developed in turning the rotor, and the electrical power developed in the rotor windings are to each other as, $n, n^{\prime}$, and $\left(n-n^{\prime}\right)$, respectively.

[^33]Ratio of Rotor Voltages to Stator Voltages. When the rotor is wound with the same number of conductors as the stator, then when the rotor is standing still, the rotating stator magnetism induces in the rotor windings electromotive forces of the same value and of the same frequency as the electromotive forces induced in the stator windings by this rotating stator magnetism (neglecting magnetic leakage). Moreover, the electromotive forces induced in the stator windings are very nearly equal and opposite to the voltages applied to the stator windings. When the difference of the speeds of the rotor and the stator magnetism is $\left(n-n^{\prime}\right)$, the electromotive forces induced in the rotor windings are the fractional part, $\left[\frac{n-n^{\prime}}{n}\right]$, of the voltages applied to the stator windings, and the frequency of the electromotive forces induced in the rotor windings is the fractional part, $\left[\frac{n-n^{\prime}}{n}\right]$, of the frequency of the voltages applied to the stator windings.

Example. Let a certain three-phase induction motor having a stator wound for 6 poles, and taking three-phase alternating currents at a frequency of 60 cycles and a voltage of 220 between any two of the three supply mains, have a rotor furnished with the same number of conductors as the stator. Further, let the no-load speed of the rotor be 1194 r.p.m., and its full-load speed be 1,143 r. p. m.

Assuming that the magnetic leakage may be considered negligible, it is required to find:
(a) The synchronous speed.
(b) The electromotive forces (three-phase) induced in the rotor windings at no-load and at full-load.
(c) The frequency of the electromotive forces induced in the rotor windings at no-load and at full-load.

Solution: (a) The synchronous speed of the rotating magnetism in the stato: is, according to equation (47),

$$
n=\frac{2 \times 60}{6}=20 \text { revolutions per second, or } 1,200 \text { r.p.m. }
$$

(b) The electromotive forces (three-phase) induced in the rotor windings at no-load are

$$
\frac{\left(n-n^{\prime}\right)}{n} \times \text { voltage applied to stator }
$$

or

$$
\left(\frac{1200-1194}{1200}\right) \times 220=1.10 \text { volts }
$$

The voltages induced in the rotor windings at full-load are

$$
\left(\frac{1200-1143}{1200}\right) \times 220=10.45 \text { volts }
$$

It is interesting to note that if the slip of the rotor at no load were zero, or in other words, if $n^{\prime}$ were equal to $n$ there would be zero electromotive forces induced in the rotor windings at no-load.
(c) The frequency of the electromotive forces induced in the rotor windings at no-load is

$$
\frac{\left(n-n^{\prime}\right)}{n} \times \text { frequency of stator voltage }
$$

or

$$
\left(\frac{1200-1194}{1200}\right) \times 60=0.3 \text { cycle per second }
$$

The frequency $y$ of the electromotive forces induced in rotor windings at full-load is

$$
\left(\frac{1200-1143}{1200}\right) \times 60=2.85 \text { cycles per second }
$$

When the rotor is at rest, the frequency of the electromotive forces induced in the rotor windings is the same as the frequency of the stator voltage, namely, 60 cycles per second.
 value $\left(n-n^{\prime}\right)$ or for a large value of $\frac{n^{\prime}}{n}$ (efficiency) if its rotor resistance is small. High efficiency depends, therefore, upon low rotor resistance.

The necessity of high rotor resistance to give large torque at starting has nothing to do with the necessity of making the rotor
resistance small in order to secure full load torque at as nearly synchronous speed as possible. These two conflicting conditions may be realized in one motor by an arrangement whereby a resistance which is in circuit with the rotor conductors at starting, may be short-circuited when the motor nearly reaches its rated speed.

Structural Details of a Typical Induction Motor. Figs. 330 to 333 show the structural details of a typical induction motor manufactured by the Westinghouse Electric and Manufacturing Company. It has a stationary primary member (often called the stator or field) and a rotating secondary member (often called the rotor or armature).


Fig. 331. Primary Member of Westinghouse Induction Motor Completely Wound

The primary member is mounted in a hollow cylindrical frame of cast iron shown in Fig. 330. This frame forms a base for the machine, and also supports the two endbrackets which carry the self-oiling bearings. Inside the frame are several lugs that support the stator core laminations far enough from the frame to leave space between the frame and the core for ventilation. The iron core of the primary member consists of a ring built up of sheetsteel stampings slotted on the inside to receive the primary conductors as shown in Fig. 331. The laminations are assembled, clamped, and keyed between stiff end rings inside the lugs on the frame. Steel keys in one or more of the lugs prevent circular movement of the laminations. Fig. 331 shows the primary member completely wound.

The primary conductors are usually grouped in former-wound coils of wire which are thoroughly taped and insulated before being slipped into place in the slots in the stator core. In larger motors
copper strap bent into the proper form is used instead of wire for forming the coils.

The terminals of the primary winding are brought out, usually at one side of the motor, and are clamped in insulated cleats or bushings. The leads which supply alternating currents to the motor are attached to these terminals through suitable connectors.


Fig. 332. Iron Rotor Core for Westinghouse Induction Motor

The iron core of the secondary member (rotor), shown in Fig. 332 , is also built up of ring-shaped stampings of sheet-steel assembled, clamped, and keyed between stiff end plates on the arms of the rotor spider. The spider is pressed on the shaft and keyed. Ventilating plates on the rotor cores of the larger motors act like the blades of a fan and force strong currents of air between the rotor end rings and the core and through all the openings in and around the stator windings and core, thus keeping all parts cool. The secondary conductors consist of rectangular copper bars placed in nearly closed slots, around the periphery of the core. These bars project beyond the laminated core, and they are screwed and soldered or welded at each end to massive rings of copper to form a short-circuited secondary winding, Fig. 332. This type of secondary member is called a squirrelcage rotor. The complete


Fig. 333. Westinghouse Induction Motor Completely Assembled motor is shown in Fig. 333.

Types of Rotors for Constant and Variable Speed. There are three types of secondary members used in commercial induction motors, according to the conditions of service to be met. The start-
ing and running conditions determine which type to adopt in any given case. These types are shown in Figs. 334, 335, and 336.


Fig. 334. Squirrel-Cage Type of Rotor
Fig. 334 is a squirrel-cage rotor. Fig. 335 is a rotor wound with insulated wire, forming what is called a definite ${ }^{*}$, or polar, winding. The terminals of this winding are connected to a starting resistance mounted inside of the rotor. A switch is arranged to short-circuit this starting resistance, and is operated while the motor is running by means of a rod which lies inside of the hollow shaft of


Fig. 335. Rotor with Definite or Polar Winding with Starting Resistance Inside Core
the rotor. This rod terminates in a small handle or knob at the end of the rotor shaft as shown in Fig. 335. Fig. 347 shows a complete three-phase induction motor with the knob and rod for operating the internal starting resistance.

[^34]Fig. 336 is a rotor with a winding similar to the winding of Fig. 335, but instead of connecting the terminals of the rotor winding to an internal starting resistance, these terminals are brought out to collector rings on the end of the shaft as shown in Fig. 336. The circuits of the rotor windings are completed through adjustable external resistances which are connected to the rotor windings by means of brushes rubbing on the collector rings. These adjustable external resistances are regulated by a cylindrical switch or controller similar in general to the ordinary electric street-car controller. Fig. 337 shows a Westinghouse controller for induction motors used for cranes, hoists and similar apparatus.

The cylinder has a set of contacts for making, breaking, and reversing the primary circuit, and another set of contacts for control-


Fig. 336. Rotor with Polar Winding and Collector Rings
ling the speedtof the motor by varying the resistance in the secondary circuits. The two sets of contacts on two drums are mounted on the same shaft and all operations are performed by moving one controller handle. The number of speed steps in each direction of rotation may be $6,9,12$, or 15 , according to the capacity of the controller.

An induction motor provided with a squirrel-cage rotor takes excessive current from the alternating-current supply mains at starting. The squirrel-cage rotor requires from three to four times full-load current to produce at starting a torque equal to the torque developed when running at full-load. Hence, when the motor has to start under a heavy load, or where the taking of excessive currents from the supply mains will interfere with other apparatus supplied
from the same mains by causing excessive drop of voltage, the squirrelcage type of rotor is objectionable, especially in large size motors. On the other hand, the extreme simplicity of the squirrel-cage rotor and its ability to carry enormous currents without injury, largely compensate for the above mentioned disadvantages. Its speed is practically constant, varying only a few per cent from full load to no load. The operating characteristics of the squirrel-cage type of induction motor are such as adapt them to a wide variety of purposes and make them especially suitable for continuous constantspeed service.

An induction motor provided with a rotor like that shown in Fig. 335, with an internal starting resistance, takes at starting only about one and one-half full-load rated


Fig. 337. Westinghouse Controller for Induction Motors current from the supply mains, giving a starting torque of about one and onehalf full-load torque. Such an induction motor, therefore, is used only where a starting torque not greatly in excess of full-load torque is required. The advantage of this type of rotor is that it does not take excessive currents at starting, and it will start, therefore, without producing excessive drop of electromotive force in the alternating-current system from which the motor receives its power. The starting resistance in motors up to about 50 horse-power consists of cast-iron grids enclosed in a triangular frame which is bolted to the end plates holding the rotor laminations together. The whole of this resistance is in series with the secondary winding at starting. As the motor increases in speed, the resistance is short-circuited by sliding spring metal brushes along the inside surface of the grids. The brushes are supported by a metal sleeve sliding or the shaft which is operated by a rod passing through the end of the shaft.

An induction motor having a rotor provided with collector
rings is generally used for cranes, hoists, elevators, and other work where variable speed is required. The starting resistance used in the type of rotor shown in Fig. 335 is designed to carry the rotor current for a short time only, that is, during starting; if kept continuously in circuit for the purpose of speed control, this starting resistance would become excessively hot. For speed control, therefore, an external resistance must be used.

The range of speed control possible in the case of an induction motor provided with a rotor having collector rings connected to external adjustable resistances is about the same as the range of speed control obtainable with a shunt-wound direct-current motor having a regulating rheostat in its armature circuit.

Behavior at Starting and in Operation. When an induction motor is running without load, its speed is nearly equal to the speed of the rotating magnetic field, namely, synchronous speed. Under these conditions the stator takes only sufficient current to force the magnetic flux through the reluctance of the magnetic circuit, and to supply the $I^{2} R$ losses of the stator windings, the core loss, and the friction and windage loss of the rotor.

When the motor is loaded, its speed decreases in nearly direct proportion to the load, from nearly synchronous speed at no-load to about 98 per cent of synchronous speed in the case of large motors, and to about 92 per cent of synchronous speed in small motors at full-load. Therefore, the induction motor is practically a constant-speed motor. The decrease in speed expressed as a percentage of synchronous speed is called the slip of the motor. The slip of large motors is thus about 2 per cent at full-load, and that of small motors is about 8 per cent at full-load.

When an induction motor is overloaded, it takes excessive current from the supply mains, and its torque increases up to a certain value of the slip (a definite value for a given motor). When loaded up to this point the machine is unstable, and the least additional loading causes the machine to "break down" or stop.

This maximum output which a given motor can deliver is usually about one and one-half to two and one-half times as great as its rated full-load output. This maximum output is proportional to the square of the electromotive force of the alternating currents supplied to the motor. Thus a certain induction motor rated at 220
volts has a maximum power output of 1.8 times its rated output. The same motor supplied with currents at $£ 00$ volts would have a maximum output of $\left(\frac{200}{220}\right)^{2} \times 1.8=1.49$ of its rated output.

When an induction motor is operated at slightly less than its rated frequency but with full-rated voltage, the speed of the motor will be decreased in proportion to the frequency, but its power output will not be greatly affected. The efficiency of the motor, and its rise of temperature under full-load, will be approximately


Fig. 338. Three-Phase Starting Compensator Complete ${ }^{-}$


Fig. 339. Three-Phase Starting Compensator with Cover Removed
unchanged, and the maximum power output will be slightly increased.
An induction motor having a squirrel-cage rotor will develop sufficient torque to start satisfactorily with from 40 per cent to 80 per cent of the rated voltage applied to the primary member. Therefore, the current required at starting may be greatly reduced by supplying the primary member, at starting, with current through a step-down transformer which is designed to reduce the supply voltage to $40,50,60$, or 80 per cent of the rated voltage of the motor, and to multiply the delivered current in the same ratio. This
step-down transformer is usually an autotransformer. An autotransformer, with its special switching device for changing the motor connections quickly from the low-starting voltage to the fullrunning voltage, is called an autostarter, or a compensator. The autostarter may be located at any convenient point, either near the motor or at a distance from it.

Figs. 338 and 339 are general views of a three-phase handoperated starting compensator of the wall type, as manufactured by the General Electric Company. The compensator consists of three core-type autotransformers, a cable clamp, and a special switch assembled in a metal case with external handle and release lever. In the wall-suspension type the switch is located at the bottom, as seen in Fig. 339, and is enclosed by an oil-filled tank. Fig.


Fig. 340. Electrical Connections for Three-Phase Starting Compensator
340 is a diagram of the electrical connections. The three coils of the three-phase winding are connected in $Y$, the line to the three free ends of the coil, and the starting connections of the motor to the taps. For motors from 5 to 18 horse-power, these compensators are provided with taps for starting the motor at 50,65 , and 80 per cent of the line voltage, any one of which may be selected after trial for permanent connection to the switch, for starting according to the requirements of any individual case.

The shaft of the switch, as seen in Fig. 339, extends through the sides of the compensator case, and is operated by a lever at the right, being held in the running position by a lever at the left. The switch, provided with heavy wiping contacts, is immersed in oil
and is intended to be used as a line switch as well as for starting the motor. The lever has three positions: "off", "starting", and "running".

To start the motor, the switch is thrown to the "starting position", and is left there until the motor reaches nearly full speed and then the switch is quickly thrown over to the "running position". The time required for bringing an induction motor from rest up to rated speed is about one minute, the time in any given case depending upon the value of the starting voltage used, and the amount of load on the motor at starting. In the "off position" both compensator and motor windings are disconnected from the line. In the


Fig. 341. Diagram of Electrical Connections for a Two-Phase Compensator for Starting Two-Phase Induction Motors
"starting position", Fig. 340, the switch connects the line to the terminals, and the motor to the taps, of the compensator winding, without circuit breakers or fuses in circuit. In the "running position" the compensator winding is cut out and the motor is connected to the line through suitable fuses or overload relays mounted directly above the compensator. For instance, Fig. 339 shows three "cartridge" fuses, one in each of the three line wires. To prevent the attendant from carelessly throwing the motor directly on the line at starting, an automatic latch is provided, so arranged that the lever at the "off position" can be thrown only into the "starting position"
(backward), and can be thrown into the "running position" (forward) only by a quick throw of the lever.

The "no-voltage release," shown in Fig. 340, is an electromagnet whose laminated plunger holds a tripping lever which engages with the lever mounted on the switch shaft. If for any reason the supply voltage is cut off from the motor, the no-voltage release acts promptly to release the tripping lever which in turn, through the action of a strong spring, throws the operating lever to its "off position," where it remains until the supply voltage is restored, and the motor started.

Fig. 341 shows a diagram of connections for a two-phase compensator for starting two-phase induction motors. In this case the line is connected to the ends of each coil, and the starting connections of the motor to one of these ends and the taps.


Fig. 342. Two-Coil Autotransformer with Three-Phase Induction Motor
For starting constant-speed polyphase motors above 5 horsepower, a starting compensator is generally used, but below 5 horsepower polyphase motors are connected directly to the supply mains.

It is not necessary to employ three autotransformers for starting a three-phase induction motor. Two autotransformers connected in open delta, or V are quite satisfactory for this purpose and are commonly used. Fig. 342 shows two V-connected autotransformers arranged for starting a three-phase induction motor, and by the use of four winding taps on the coils permits its operation at four different voltages.

The direction of rotation of a two-phase motor is reversed by reversing the connections of one of the phases, that is, by reversing the connections of the two wires belonging to one-phase supplying current to the stator windings of the motor.

The direction of rotation of a three-phase (three-wire) motor is reversed by interchanging the connections of any two of the three wires used to lead the three-phase currents to the stator windings of the motor.

Typical Induction Motor Installations. Among the advantages of electric driving over mechanical driving are: increased shop production, economy of power, ease of control, better and more convenient arrangement of machines, and saving of floor space. Fig. 343 is a view of a Snyder 21 -inch drill press driven by a constant-speed 2 -hp. induction motor. Fig. 344 shows a 24 -inch Chandler planer driven by a $7 \frac{1}{2}$-hp. induction motor running at $1,200 \mathrm{r} . \mathrm{p} . \mathrm{m}$. The individual motor drive for planers has many advantages, one independent motor being used for planing and one for adjusting the cross-head.

A typical application of constant-speed induction motors to the driving of spindles in a textile mill is illustrated in Fig. 345.


Fig. 343. Snyder Drill Press Driven by Induction Motor A large number of Crocker-Wheeler 5-hp. motors are directly coupled to the shafts which drive the spindles above, thus eliminating all overhead shafting, hangers, and

An application of variable-speed in-


Fig. 344. Chandler Planer Driven by Induction Motor
ing phase-wound rotors with slip rings is given in Fig. 346, which shows several Westinghouse $500-\mathrm{hp}$. "type $\mathrm{HF}^{\prime \prime}$ induction motors operating Worthington pumps in a municipal pumping plant.


Fig. 345. Constant Speed Induction Motors Used for Driving Spindles in a Textile Mill


Fig 346. Westinghouse $500-\mathrm{H} . \mathrm{P}$. Induction Motora Operating Centrifugal Pumps in a Municipal Pumping Plant

The laminations of both the stator and the rotor of large induction motors are provided with ventilating ducts through which air is driven by centrifugal action. Fig. 346 shows a number of holes in the cast-iron casing through which the air flows after passing through the ventilating ducts between the laminations of the rotor and the stator.

Fig. 347 shows an induction motor driving a triplex pump. This induction motor has a starting resistance inside of the rotor,


Fig. 347. Induction Motor Driving Triplex Pump
and the switch rod and knob are shown projecting from the rotor shaft at the left.

Fig. 348 illustrates a very common method of installing induction motors when used for driving line shafting in shops or factories. The motor is shown bolted in an inverted position to the ceiling by means of lag screws, and is furnished with two pulleys, each belted to a different line shaft.

Since induction motors require no adjustment and practically no attention, they may be installed where direct-current motors are
not suitable, with the consequent advantage of saving valuable floor space.

Single=Phase Induction Motor. When an induction motor (two-phase or three-phase) is once started, and is running at full speed, all the phases but one may be disconnected from the primary member and the machine will continue to operate and to carry approximately 70 per cent as much load with the same slip and temperature rise as when all the phases are connected to the supply mains.

An induction motor, however, will not start when one phase only of its primary member is connected to single-phase supply


Fig. 348. Induction Motor Suspended from the Ceiling, Driving Shafting by a Belt
mains. Therefore, when it is to be operated from single-phase mains, special provision for starting must be made. An induction motor designed to operate from single-phase mains and provided with special arrangements for starting, is called a single-phase induction motor.

Three methods of starting single-phase induction motors are in general use, as follows: (a) by hand; (b) by phase splitting; (c) by repulsive action of field on armature obtained by temporary connections during starting.
(a) Hand Starting. Very small induction motors may be
started by giving a vigorous pull on the belt which connects the motor to the machinery which it drives.
(b) Split-Phase Starting. When an alternating current divides between two branches of a circuit, there is a phase difference between the currents in the two branches, if the ratio of resistance to reactance is different in the two branches. This is especially true if one branch contains a condenser. A two-phase motor will start when the cur-


Fig. 349. Stator of Single-Phase Induction Motor-Holtzer-Cabot Company
rents in the two stator circuits are less than 90 degrees apart in phase, although the starting torque grows less and less as the phase difference of the two currents decreases. The dephasing of the two parts of a single alternating current in two dissimilar branches of a given circuit is called phase-splitting, and a single-phase induction motor may be arranged to start as a two-phase motor, by splitting a singlephase current and using the two parts of the split current exactly as one would use two genuine two-phase currents.

Fig. 349 shows the stator of a 2-h.p. single-phase induction motor of the Holtzer-Cabot Electric Company. One set of stator
coils, the "working coils", consist of many turns of coarse wire occupying three-fourths of all the stator slots, and the other set of stator coils, the "starting coils", consist of fewer turns of fine wire occupying one-fourth of all the slots.

At starting, both sets of coils are connected to the single-phase supply mains, and the difference in the resistance and the reactance in the two sets of coils splits the single-phase current supplied, sufficiently to give a slight starting torque. This type of split-phase induction motor cannot start with any considerable load, hence the load, if it is difficult to start, should be thrown on to the motor


Fig. 350. Wagner Single-Phase Induction Motor
by means of a friction clutch after the motor is running at full speed. The rotor used in the Holtzer-Cabot motor is of the squirrel-cage type.

A single-phase motor will run in either direction equally well, depending only upon the direction in which it is started. Therefore, the hand-started motor may be started in either direction. The direction of starting of the split-phase motor may be reversed by reversing the connections of the starting winding.
(c) Repulsion Motor Starting. If an ordinary direct-current dynamo were provided with a laminated field magnet, and if its field magnet were excited by an alternating current, currents would
be induced in the armature windings by the alternating field, provided the brushes of the direct-current machine were set at an angle of about $45^{\circ}$ (for a two-pole machine) from their proper position for collecting a direct current. These currents induced in the armature would be acted upon by the alternating field so as to produce a torque which would cause the armature to rotate. A selfstarting, single-phase, alternating-current motor constructed on this principle is called a repulsion motor. It is not entirely satisfactory in operation, but the repulsion-motor principle furnishes the


Fig. 251. Part Section of Wagner Single-Phase Induction Motor
best means for making a self-starting single-phase induction motor, that is, a motor which is arranged so that it can act as a repulsion motor while starting, and which by changing certain inside connections can be altered into an induction motor when it reaches full speed.

Fig. 350 is a general view of a single-phase induction motor arranged to start as a repulsion motor, and built by the Wagner Electric Manufacturing Company. The motor shown has a fourpole stator winding, the iron stator core being made very much like the core of an ordinary induction motor, namely, in the form of a laminated ring closely surrounding the armature, and slotted on its inner face.

The armature is of the ordinary direct-current drum type provided with a disk commutator with radial commutator bars. Four (for a four-pole machine) short-circuited brushes are pressed against the face of the disk-shaped commutator, as shown in Fig. 350. At starting, the stator winding is connected to the singlephase supply mains, and the machine starts as a repulsion motor. Inside of the armature are two governor weights $V$, Fig. 351, which are thrown outwards by the centrifugal force when the machine reaches full speed, thus pushing the solid copper ring $K$ into contact with the inner ends of the commutator bars $L$, and thus completely short-circuiting the armature winding. At the same time barrel $I$, which is pushed endwise by the governor weights and which carries the short-circuiting copper ring $K$, pushes the brush holder or rocker arm endwise, and lifts the brushes off the commutator.

In starting the Wagner single-phase motor, the supply voltage is usually reduced to a fractional part of the full running voltage. This is accomplished by the use of a small step-down transformer, usually an autotransformer, in much the same way as has been explained in connection with the autostarter or starting compensator for two-phase and three-phase induction motors.

Induction Generator. An induction motor runs as a motor at a speed less than the speed of the rotating magnetism in the stator iron (synchronous speed). When the motor load is decreased, its speed approaches synchronous speed, and the intake of power from the alternating-current mains falls off more and more. If the rotor is driven by an external source of mechanical power, it may be speeded up to synchronism, in which case the intake of power becomes zero, except for core loss in the stator iron. If now the rotor is speeded above synchronism by the external source of power, the stator windings deliver power to the alternating-current mains, provided the alternating-current generator remains connected to the mains to fix the frequency. When an induction motor is so used, it is called an induction generator. The induction motor as an induction generator is not at present of much commercial importance.

Frequency Changer. An induction motor provided with a rotor having a definite winding with terminals brought out to collector rings (see Fig. 336) may be used as a so-called frequency changer. When the rotor stands still, the rotating stator magnetism induces electre-
motive forces at full frequency, that is, of the same frequency as the alternating currents supplied to the stator. If the rotor runs at one-fourth speed, let us say, the relative speed of the rotor and the stator magnetism is three-fourths of the speed of the latter, and hence electromotive forces of three-fourths full frequency are induced in the rotor windings. If the rotor is run backwards at, let us say, one-half of the speed of the stator magnetism, then the relative speed of the rotor and the stator magnetism is one and one-half times the speed of the stator magnetism, and electromotive forces of one and one-half times full frequency are induced in the rotor windings.

Example. A certain induction motor runs at one-third synchronous speed ( $n^{\prime}=\frac{1}{3} n$, page 377), then, ignoring stator losses, all of the power delivered to the stator is transmitted to the rotor, and of this total power onethird appears as mechanical power driving the rotor, and two-thirds appears as electrical power developed in the rotor windings. This electrical power, ignoring the resistance loss in the rotor windings, is delivered to the rotor collecting rings.

Furthermore, if the rotor has the same number of conductors as the stator, then the electromotive forces between collector rings are two-thirds as great as the voltages applied to the stator windings, and their frequency is two-thirds as great.

If the rotor of an induction motor is driven backwards by an external source of power at one-half synchronous speed ( $n^{\prime}=-\frac{1}{2} n$ ), then all of the electrical power delivered to the stator together with the mechanical power used for driving the rotor, appears as electrical power in the rotor windings, and the rotor voltages are one and one-half times as great in value, and one and one-half times as great in frequency as the voltages applied to the stator.

The stator current in an induction motor, or a frequency changer, is sufficient at no-load to magnetize the stator. This stator current is called the no-load current of the machine. When current is taken from the rotor, an equal (and opposite) additional current is taken from the supply mains by the stator windings, exactly as in the case of the transformer.

The above statements are based on the assumption that the rotor windings are exactly like the stator windings, both as to the number of conductors, and as to the grouping of the conductors into separate circuits or phases. If the rotor has half as many conductors as the stator, the rotor voltages are halved and the rotor currents are doubled, other things being equal.

In alternating-current plants, designed primarily for the transmission of power, and hence using a low frequency (e. g., 25 to 40 cycles per second), there is sometimes a need for a limited amount
of current of a higher frequency. To meet such conditions, a frequency suitable for lighting purposes, 60 cycles or more, may be cheaply and easily obtained by means of the frequency-changer. This is essentially an induction motor as explained above, the rotor of which is driven mechanically by an auxiliary synchronous motor in a direction, usually opposite to its natural rotation. The current of lower frequency is fed to the stator windings and the current of higher frequency is taken out of the rotor windings by means of collector rings. The frequency of the motor current will depend on the speed at which the rotor is driven. Thus, if the rotor is driven at its rated speed but in a direction opposite to its natural rotation, the frequency of the current delivered by it to the collector rings will be twice the normal, or if run at half the normal speed in its natural direction, the frequency will be one-half the normal. To change a current with a frequency of 40 cycles into one of 60 , the motor would be run at one-half speed in an opposite direction, while to obtain 60 cycles from a 25 -cycle current, the rotor would run nearly one and one-half times the rated speed in an opposite direction.

The total power delivered to a frequency changer is partly electrical power delivered directly from the low frequency supply mains to the stator of the frequency changer, and partly mechanical power delivered by belt to the rotor of the frequency changer from the auxiliary driving motor. The power output of the frequency changer is wholly electrical and in the form of increasedfrequency alternating currents from the rotor.

The electrical power delivered to the stator of the frequency changer is

$$
P_{e}=\frac{f P}{f^{\prime}}
$$

and the mechanical power delivered by belt to the rotor of the frequency changer is

$$
P_{m}=\frac{\left(f^{\prime}-f\right) P}{f^{\prime}}
$$

where $P$ is the total power delivered to the machine, being a little greater than the total power delivered by the machine at the increased frequency; $f$ is the low frequency of the alternating currents supplied to the stator of the machine; and $f^{\prime}$ is the higher frequency
of the alternating currents delivered by the rotor of the machine.
Therefore, the rotor of the frequency changer must be designed for the total output of power at the higher frequency, and the stator of the frequency changer must be designed for the intake of the amount of power $P_{e}$, which is supplied to it electrically.

For example, a frequency changer rated at 100 kw . to change a 40 -cycle current to one having a frequency of 60 cycles per second would be made up as follows: An auxiliary driving synchronous motor rated at 33.3 kw . designed to take current from 40 -cycle mains at a speed of, say 600 r. p. m.; it would, therefore, have eight poles. Its armature would be direct connected to the rotor of the frequencychanger which would be rated at 100 kw . The stator of the frequencychanger would be rated at 66.7 kw . and would be supplied with current having a frequency of 40 cycles per second. If wound for four poles, the synchronous speed (as an induction motor) of the rotor of the frequency-changer would be normally 1,200 r.p.m. But by driving the rotor (by the auxiliary synchronous motor) at a speed of $600 \mathrm{r} . \mathrm{p} . \mathrm{m}$. in a direction opposite to its natural rotation, the frequency of the rotor currents would be that due to an equivalent speed of $1,200+600=1,800$ r.p.m.; corresponding thus to a frequency of 60 cycles per second.

For the sake of a simple illustration, the ratings as given above are based on the assumption of a 100 per cent efficiency, which, of course, on account of the unavoidable power losses, is never realized in practice.

It is evident that the frequency-changer can at the same time be designed to change the electromotive force by using a suitable number of turns in the stator and rotor windings. It can also be used to change the number of phases of the system by providing a rotor wound for a number of phases different from that of the stator. For instance, it may be designed to convert from three-phase, 6,000 velts, and 25 cycles, to two-phase, 2,500 volts, and 62.5 cycles. On account of this flexibility the frequency changer is sometimes called a "general alternating-current transformer." A number of these frequency changers are in present use, but on account of excessive magnetic leakage, they are not as satisfactory in operation as motorgenerators. One of the large manufacturing companies is now recommending as a frequency changer a motor-generator consist-
ing of a polyphase induction motor of one frequency driving mechanically an alternator designed for the frequency desired.

## COMPARISON OF SYNCHRONOUS MOTOR AND INDUCTION MOTOR

To summarize the characteristic behavior in service of synchronous and induction motors, and to simplify the comparison between them, the following tabular statement in parallel columns, prepared by C. $\mathrm{r}^{\prime}$. Scott, is given.

The induction motor chosen for comparison with the synchronous motor is of the so-called "squirrel-cage" type, started by applying a low electromotive force to the primary winding. The description following will, of course, require modification in some particulars, if the secondary is furnished with adjustable resistance, but these modifications are of minor importance and do not affect the general comparison.

## SYNCHRONOUS MOTOR

## INDUCTION MOTOR

## Auxiliary Apparatus Required

1. A starting motor; or, if selfstarting, some form of resistance or transformer for reducing the voltage.
2. An exciter, driven by the motor or otherwise, with circuits to switchboard and motor.
3. Rheostats for exciter and motor.
4. Instruments for indicating when field current is properly adjusted.
5. Main switch and exciter switches.
6. A friction clutch is required in many cases.
7. A two-way main switch with autotransformers giving a low e.m.f. for starting. This may be at any distance from the motor.
8. No exciter is required.
9. No field rheostats are required.
10. No instruments are required.
11. No exciter switches are required.
12. No friction clutch is required, as the motor starts its load

## Construction

1. Armature winding.
2. Field winding with many turns. Liable to accident from "field discharge" if exciting current is suddenly broken; or from high e.m.f. by induction from the armature if the field circuit is open.
3. Collector rings and brushes.
4. Primary winding.
5. Secondary, short-circuited.
$\dot{3}$. No moving contacts on "squirrel cage" secondary.

## Starting-Normal

1. Motor is brought up to speed without load; if starting motor is used, the main motor must be brought to proper speed and "synchronized"; if self-starting, the starting devices must be cut out of circuit at the proper time.
2. Exciter is made ready for delivering proper current and the motor field must be excited, adjustments being made by rheostats until instruments give proper indication.
3. Load is thrown on by friction clutch or other means.
4. Throw switch to starting and then to running position.
5. There is no exciter. (The motor is magnetized by lagging current from the generator.)
6. The motor starts its own load.

## Starting-Abnormal

1. If the several operations in starting be performed improperly or in wrong order, injury may result. If a starting motor is used, the synchronizing may be attempted at an improper speed or phase; if the motor is self-starting and it is connected to the circuit without the starting devices, a large current will flow which may induce a high e.m.f. in the field circuit; if the field circuit be open, a

- high e.m.f. may be induced in it at other times also.

2. If a load having inertia be applied by closing the friction clutch too quickly the motor may be overloaded and stopped.
3. If motor stops owing to failure of current supply, it is not self-starting when the current returns. An attendant is always required for starting.
4. The only possible error is in starting with the switch in the running or full voltage position, which simply causes the motor to exert a greater torque and consume a greater current than is necessary.
5. The motor starts its own load and requires no friction clutch.
6. The motor will stop if the current is cut off at the power house and then start again when the current is supplied to the circuit.

## Starting and Maximum Running Torque

1. The starting torque of the selfstarting motor is very small and an excessive current is required for developing it. The motor starts as an induction motor, but inefficiently, as the design which is best for synchronous running is not good for starting.
2. The starting torque is adjustable and may be several times full load torque.
3. The maximum torque is several times the full load torque, and occurs at synchronous speed; below this speed the torque is very small; any condition which momentarily lowers the speed causes the motor to stop.

## Speed

1. The motor has a single definite speed; at other speeds its torque is very small, and the current is very large.
2. The maximum torque is usually greater than that of the synchronous motor, but it occurs at a reduced speed and there is a large torque at lower speeds.
3. The motor may be designed for a practically constant speed, with large torque at lower speeds; or for several definite speeds by changing the number of poles; or for variable speed, for cranes, elevators, hoists, and the like.

## Current

1. If there is useful starting torque, the current required for producing it is very great.
2. The running current depends upon the wave form. If the wave form of the motor and of the circuit differ, a corrective current will follow, which cannot be eliminated by adjustment of field excitation.
3. The running current depends upon uniformity of alternations of the current, i.e., upon the uniformity of the speed of the generator and other synchronous motors. The motors attempt to follow the generator speed exactly. If the latter pulsates, the motors pulsate also; they vibrate about a mean position, "hunting" or "pumping." One motor pumping incites others. The current is increased even though the conditions may still be operative.
4. The running current depends upon the relation between the field current (which is adjusted by the attendant) and the e.m.f. of the circuit. The main current may be made leading or lagging or theoretically it may be neither. The e.m.f. of the circuit is an element which is under the partial control of the attendants at every motor, as well as at the generator station.
5. The starting current may be made proportional to the torque, and is $1 \frac{1}{2}$ to $2 \frac{1}{2}$ times that required for the same torque at high speed.
6. The running current is practically independent of the difference in wave form, as it has no wave form of its own.
7. The current is practically independent of fluctuations in generator speed, as there is a slip between the synchronous and the actual speed of the motor.
8. The current is not subject to any adjustments which the motor attendant can make, nor is the e.m.f. of the circuit in any way under his control.

## Power Factor

1. As the power factor is the relation between actual current and energy current, it is dependent upon wave form, hunting, and field current. Under favorable conditions, the motor may have a high power factor; under many actual conditions it may not; under some conditions the highest attainable power factor is less than that of the induction motor.
2. The current may be lagging or leading, depending upon the motor field strength.
3. The power factor varies with load, but is definite and is practically independent of wave form and hunting.
4. The current to the motor is always a lagging current.

## Reaction Upon Generator and Circuit

1. The motor impresses its own wave form on the circuit.
2. A motor may augment the fluctuations in generator speed by the oscillation of its own armature. One motor may increase the disturbance in the circuit so as to interfere with other motors not otherwise seriously affected.
3. As the current may be either lagging or leading, the drop in e.m.f. in the generator, and between generator and motor may be either more or less than that which could be caused by a non-inductive load or by an induction motor.
4. If a short-circuit occurs in the transmission system, the motor acts as a generator, which thereby greatly increases the current and the intensity of the short-circuit.
5. If the circuit is opened, either by a switch, a circuit breaker, a fuse, or the kreaking of the line, the motor speed falls, its e.m.f. is no longer in phase with that of the circuit; the two are thereby added, thus doubling the normal e.m.f. and bringing increased strains on the insulation and the opening devices.
6. The motor has no wave form to impress upon the circuit; its tendency is to smooth out irregularities in a wave not a sine.
7. The motor has a damping action upon fluctuations in frequency; in some cases a synchronous motor which hunts may run smoothly when an induction motor is connected to the same circuit.
8. The drop in e.m.f. is always greater than would be caused by noninductive load.
9. The motor does not generate current when there is a short-circuit.
10. The motor does not generate e.m.f. when it is disconnected from the circuit.

## Causes which May Accidentally Stop a Motor

1. Momentary lowering of e.m.f. caused by short-circuit on the line, or by accident to another motor, or by error in synchronizing a generator, or by the "switching over" of the motor from one circuit to another, is apt to cause the motor, particularly if carrying load, to fall from synchronism and stop.
2. A heavy load, even momentary, may exceed the limiting torque and cause the motor to drop from synchronism, even though the load be removed immediately. The connection between generator and motor is rigid.
3. If the generator speed suddenly increases, a motor carrying a load having inertia may be unable to increase its speed quickly without exceeding the limiting torque, which will cause the motor to stop.
4. Momentary lowering of e.m.f. causes momentary decrease in speed.
5. An excessive load receives the stored energy of the motor and of the load itself as the motor speed falls; when the excess load is removed the motor speed increases again. The connection between generator and motor is elastic.
6. The motor readily follows changes in generator speed.

## Summary

1. The motor is an active element in the system; it acts as a generator in impressing its own wave form, its e.m.f. and its fluctuations upon the circuit. These fluctuations may be caused by an intermittent load.
2. The m .tor is a sensitive element in the system. Its successful operation is dependent upon a proper relation between the design of the motor itself and of other machines in the system. Its successful operation also depends upon the proper adjustment and freedom from speed fluctuation in generators and other motors. It is liable to momentary variations from normal conditions, such as a sudden over-load and sudden increase of generator speed or a momentary fall in e.m.f.
3. The motor requires skill and care on the part of the attendant for starting, for readjusting and for keeping the various brushes and auxiliary apparatus in condition.
4. The motor is a passive element in the system. Each motor attends to its own work and does not try to run the system.
5. The motor is not sensitive to differences in the design of other apparatus operating on the same system.
6. No experience and electrical skill are required of the attendant and there is little or nothing to get out of order either through carelessness or design.
7. The power factor is under the control of the operator and the current may be made leading or lagging. Instruments are necessary in order that proper adjustments may be made by the attendant.
8. The motor and its operation are complex and involve many possibilities of accident.
9. The motor has a definite power factor, depending upon the load; the out-of-phase current does not vary greatly at different loads. The changing load, therefore, has comparatively little effect upon the drop in voltage and in regular service there is little liability that the motor will disturb the e.m.f. of the circuit.
10. The motor and its operation are simple and reliable.

The synchronous motor is obviously not suitable for general distribution of power, owing particularly to its lack of starting torque, the skill required in attendance, and the liability of the motor to stop if the conditions become abnormal. These objectionable features, however, are of much less importance when motors are installed in substations or are of sufficiently large size to justify an attendant.

The characteristic of the synchronous motor which may be particularly advantageous is the fact that the power factor of the current can be varied and that the current may be made leading.

## INDUCTION MOTOR TESTS

Heat Test. The heat test on induction motors is usually carried out by connecting the terminals of the stator windings to mains of the proper voltage and frequency, and taking rated full-load mechanical power from the rotor until a constant temperature has been reached. Then the motor is shut down, and the temperatures of the following parts are taken:
Armature laminations
Armature conductors
Field conductors
Field laminations
Frame
Bearings
Room

If the motor is small, the mechanical power output may be absorbed by a brake. For large power motors, however, a brake becomes troublesome. In any case, the most convenient way to
measure the power is to belt to the motor a direct-current sepa-rately-excited generator, whose losses can be easily determined, and measure the ouput of this generator. The output of the motor is equal to the output of the generator plus the losses in the generator. The losses in the generator are: the $I^{2} R$ loss in the armature, brush and bearing friction, and windage loss, and the core loss due to hysteresis and eddy currents.

The field being separately excited, the field loss need not be considered. The field current must, however, be kept constant.

During the heat run, the following observations are regularly recorded:

| MOTOR |  |  | GENERATOR |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Volts | Amperes | Speed | Volts | Amperes <br> Armature | Amperes <br> Field | Speed |  |

The $I^{2} R$ loss in the generator armature can be calculated from the current flowing, and the resistance of the armature.

To determine the stray-power losses (equal to all the losses except $I^{2} R$ ) proceed as follows:

Disconnect the motor tiom the alternating-current mains, and run the combination of motor and generator from the direct-current end, at the same speed as that recorded during the run, the direct-current machine being now used as a motor. Its field is excited to the same value that it had during the run. The speed is adjusted to the right value by varying the voltage supplied to the direct-current machine used as a motor. When the speed is correct, record the amperes and volts taken by the direct-current machine. Next the belt is thrown off, and the voltage supplied to the armature terminals of the direct-current motor is adjusted until proper speed is attained, the field current remaining as before, and the input of power is again recorded.

Let $W_{1}=$ the watts input to the direct-current motor with the belt off; $W_{2}=$ watts at same speed to drive both machines with belt on; $C=$ core loss of direct-current machine; $F=$ friction of direct-current machine without belt; $F_{1}=$ friction of induction motor without belt; $f=$ increase of bearing friction of direct-current machine due to belt tension; and $f_{1}=$ increase of friction of bearings of induction motor due to belt tension. Then

$$
\begin{equation*}
W_{1}=C+F \tag{i}
\end{equation*}
$$

and

$$
\begin{equation*}
W_{2}=C+F+F_{1}+f_{1}+ \tag{ii}
\end{equation*}
$$

Subtracting
(i) from (ii) we get

$$
W_{2}-W_{1}=F_{1}+f_{1}+f
$$

or

$$
W_{2}-W_{1}-F_{1}=f+f_{1}
$$

$$
\begin{aligned}
& \text { Now, since the two machines are of about the same size, we may assume } \\
& \text { that the increase in friction of each, due to belt tension, is the same, so that } \\
& \qquad f=f_{1}=\frac{W_{2}-W_{1}-F_{1}}{2} \\
& \text { But } F_{1} \text {, the friction of the induction motor, can be determined as de- } \\
& \text { scribed under core-loss test. Hence, we have the following expression for } \\
& \text { the stray-power loss of the generator: } \\
& \text { stray-power loss }=W_{1}+\frac{W_{1}-W_{2}-F_{1}}{2} \\
& \text { total output of motor }=\text { stray-power loss }+E I+I^{2} R
\end{aligned}
$$

This expression gives an exact method of determining the output of any kind of motor, by using a direct-current generator as a load. If $F_{1}$ is unknown, we may neglect the increase of bearing friction due to belt tension, giving results sufficiently accurate for a heat run.

Breakdown Test. The breakdown test, as in the case of a synchronous motor, is to determine the maximum output of the motor, that is, the load which will cause the motor to "break down" and stop. To make this test the load on the motor is increased, until the motor breaks down, the maximum output being noted. It is essential that the alternating currents be supplied to the motor at normal voltage. As the torque and, therefore, the maximum output, varies as the square of the voltage, the results obtained by this test will not be accurate unless the voltages applied to the stator windings are kept constantly at normal value. If for any reason it is impossible to load the motor to the breakdown point, the voltage may be reduced, and the maximum load at normal voltage may be calculated from the value obtained at the reduced voltage by multiplying the observed maximum load by $\left(\frac{\text { normal voltage }}{\text { reduced voltage }}\right)^{2}$.

If, for example, an induction motor rated at 50 horse-power gives a maximum output of 25 horse-power at one-half its rated voltage, its maximum output at normal voltage would be approximately 100 horse-power.

The most convenient way to load the motor in the above test is to belt it to a direct-current generator, as previously explained. To determine the output of the induction motor, from observations on the direct-current generator, proceed in the same manner as described under heat run.

In commercial work it is not customary to run this test to the breakdown point. The usual method is to find out if the motor will stand 50 per cent overload without breaking down. When, however, it is desired to obtain a full set of data on a machine, the test is carried to the breakdown point.

Starting Torque Test. The stationary or starting torque developed by a motor determines the amount of load under which it will start. To perform this test a brake is clamped to the pulley as shown in Fig. 352. $A$ is the center of the pulley; $B$ is the point of suspension of the brake arm from the spring dynamometer $S$; $C$ is a reference pointer, carried on a standard, on a horizontal line through the center of the pulley. The spring $S$ will measure the


Fig. 352. Prony Brake Arrangement for Testing Starting Torque
tangential force exerted by the motor at a radius $A B$ when $B$ has moved down to the point $C$, and the spring dynamometer $S$ is suspended vertically.

The reading of $S$ will be affected by the friction in the motor bearings. The following procedure will eliminate this error: Fasten to the brake arm a weight $W$, sufficient to overcome the friction of the bearings, so that the arm, if left unsupported, will always be carried downward by its weight. By slowly raising and lowering the brake arm by means of the cord which supports the spring dynamometer $S^{\prime}$ (no current passing through the motor), we get two different readings of $S$ when $B$ passes the point $C$. While the arm is being raised the friction in the bearings acts in the same direction
as the weight $W$. Second, as $B$ passes $C$, while the arm is being lowered, the friction of the bearing acts against the direction of the pull due to W.

Let $W=$ pull, exerted by the weight of the brake and arm weight; $F=$ friction of the bearings; $a=$ scale reading when arm is being raised; and $b=$ scale reading when arm is being lowered. Then

$$
W+F=a
$$

and

$$
W-F=b
$$

or

$$
W=\frac{a+b}{2}
$$

Let $T$ be the force exerted by allowing the current to act on the motor, this force being in the same direction as $W$. Then, with current flowing in the motor, let $c=$ scale reading while arm is being raised; and $d=$ scale reading while arm is being lowered. Then

$$
T+W+F=c
$$

and

$$
T+W-F=d
$$

or

$$
T+W=\frac{c+d}{2}
$$

But from above we have

$$
W=\frac{a+b}{2}
$$

substituting this value of $I$, we have

$$
T=\frac{c+d}{2}-\frac{a+b}{2}
$$

By this method both the weight of the brake arm and the friction of rest in the bearings are eliminated.

To carry out this test, this set of observations is repeated for as many values of the current in the motor as desired. The current should range at least from one-half to twice the normal full-load current. To obtain the desired current, the voltage across the motor terminals must be adjusted.

Core Loss Test. The core loss of an induction motor cannot
be measured by the method used in the case of a synchronous motor or alternating-current generator, namely, by driving it by a directcurrent motor and observing the motor input for various voltages generated, since an induction machine will not generate electromotive force unless it is connected to the mains. If it is connected to the mains, the mains may supply power to it, and hence the power supplied by the direct-current motor is not the total power delivered to the machine.

The core loss is measured in the same manner as in the case of a transformer, namely, by connecting it to the alternating-current


Fig. 353. Curve Showing Watts Input to a 1-H. P. Motor at Different Voltages
mains at normal voltage and frequency, and measuring the watts input at no-load. There is this difference, however, between the case of the transformer and that of an induction motor, that, whereas in the former the power input is practically all needed to supply the core loss, in the latter, a part of the power input goes to supply the friction and windage losses of the motor, and the appreciable $I^{2} R$ loss in the stator windings.

When an induction motor is run at no-load, the speed remains practically constant as the voltage is reduced, until the motor "breaks down'" and stops. The power input to the motor (at no-load) at any voltage consists of the core loss at that voltage plus the friction
and windage loss plus the $I^{2} R$ losses in the stator and rotor windings. But the friction and windage loss is nearly constant at constant speed, therefore the watts input to the motor at various voltages consists in part of the constant friction and windage loss, and in part of the variable core loss. Such a series of observations on a 1-h. p., 550 -volt, 3-phase motor is shown plotted in Fig. 353. The voltage can be reduced to the point at which the motor breaks down; beyond this we cannot go. As the ordinates on this curve are equal to the sum of a constant and a variable part, and since at zero voltage the variable part becomes zero, it follows that if we prolong the curve as shown in the dotted portion until it crosses the axis of watts, that is for zero volts, the value intercepted on the axis of watts may be considered, without much error, to be the constant part of the watts, namely, the friction and windage loss. Thus in Fig. 353, 26 watts represents the power lost due to friction and windage in the case of the induction motor above mentioned. The sum of the core loss, friction losses, and $I^{2} R$ loss at normal voltage, viz, 550 volts, is found from the curve to be 109 watts. For each observed value of the volts and the watts, the current per phase to the stator windings must be recorded. Then, the resistance of the stator windings having been measured, the $I^{2} R$ loss corresponding to any given voltage can be calculated. To obtain the core loss corresponding to any voltage, we must, therefore, subtract the constant friction losses plus the $I^{2} R$ loss in the stator windings from the total observed input to the motor (when running unloaded).

For example, in the case of the $1-\mathrm{h} . \mathrm{p}$. induction motor under consideration, the current input per phase at no-load was measured and found to be 0.655 amperes, when the voltage between supply mains was 550 volts. The resistance per phase was also measured and found to be 15.5 ohms. The total $I^{2} R$ loss at no-load was, therefore, $3 \times \overline{0.655}^{2} \times 15.5=20$ watts.

Therefore, at normal voltage

$$
\text { core loss }=109-26-20=63 \text { watts }
$$

Impedance Test. This test is carried out in the same manner as the core-loss test. The motor is connected to the alternatingcurrent supply mains and the amperes flowing, the watts input, and the volts at the terminals of the motor are measured, the watts,
of course, being measured by wattmeters. In this test, however, the motor is not allowed to run free, but its armature is blocked to prevent it from turning. Instead of supplying normal voltage to the motor, the voltage supplied is cut down to only five or ten per cent of the normal value and is then raised carefully until the ammeters show about one-third to one-half of the full-load current. The motor must be supplied with current at normal frequency. The following observations should be recorded:

> Amperes in each line
> Volts for each phase
> Total watts

The observations are repeated with increasing value of the voltages, until the current has reached from one and one-half to over twice the full-load value. Fig. 354 shows curves of observations taken in this way for the 1-h. p., 550-volt, three-phase motor referred to above. This motor takes one ampere of current per phase at full-load with normal voltage of 550 volts. The two curves are plotted with current and volts, and with watts and volts as ordinates and abscissas, respectively. The former curve is the straight line.

Since for full-load current with the armature standing still, the voltage applied to the termi-


Fig. 354. Curve for Impedance Test of Induction Motor nals of the motor is very low, the number of watts supplied to overcome core loss is very low. In fact, practically all the watts supplied are used up in heating the conductors of the stator and rotor. The watts input, therefore, for normal rated full-load current but with rotor blocked, may be taken as a measure of the total $I^{2} R$ losses (primary and secondary) of the entire machine at full-load. For the motor for which the curves are shown, the $I^{2} R$ losses at full-load current are equal to 100 watts.

Efficiency Test. As in the case of the machines previously
considered, the efficiency of an induction motor, at a given load is equal to the output, divided by the output plus the losses, at that load. All the losses can be determined from the core-loss test and the impedance test; the core loss and friction and windage losses being determined from the former, and the copper losses from the latter. For the three-phase, 1-h. p. motor above mentioned, the losses at full load are as follows:

| Friction and windage loss | $=26$ watts |
| :--- | :--- |
| Core loss | $=63$ watts |
| Copper loss $\left(I^{2} R\right)$ | $=100$ watts |
| Total losses | $=189$ watts |

therefore,

$$
\text { efficiency at 1-h. p. output }=\frac{746}{746+189}=79.8 \%
$$

The efficiency of large machines is generally calculated in this way. For smaller machines it is more usual to determine the efficiency by actually measuring input and output. The reason for this is that the actual losses occurring in the motor when it is loaded, are different from the values as calculated from results of tests at no-load. The differences between these calculated and actual losses are comparatively large in a small machine.

To test the efficiency by measuring the total input and output, the most convenient method is to belt the motor to a directcurrent generator, measuring the output of the generator, and the input to the motor. The output of the motor is, of course, equal to the output of the generator plus the generator losses. The losses of the generator may be calculated as described under the heat test.

The output and input of the motor are measured for various loads successively from a very small load to perhaps 50 per cent overload, so that a curve may be constructed showing the relations between efficiency and load. The losses of the generator must be determined at each speed occurring in the series of observations, because the generator losses vary with the speed.

Slip Test. The slip at a given load on the motor is the ratio

$$
\frac{n-n^{\prime}}{n}
$$

where $n$ is the synchronous speed of the motor, and $n^{\prime}$ is the speed
under the given load. Slip is usually expressed in per cent, in which case

$$
\text { per cent slip }=\frac{n-n^{\prime}}{n} \times 100
$$

The synchronous speed in revolutions per second is equal to $\frac{2 f}{p}$, where $f$ is the frequency of the alternating currents supplied to the stator, and $p$ is the number of "poles" of stator magnetism, NS, NS in Figs. 325, 326, and 327. The slip is independent of the number of phases.

The speed of the motor at zero-load is very nearly equal to the synchronous speed, and where $f$ and $p$ are not knowa, the zero-load speed may be used for $n$ without great error.

The determination of the slip of an induction motor at fullload with full-rated voltage applied to the stator windings, is an important test. This slip may be determined by observing, by means of a speed counter or tachometer, the speed $n^{\prime}$ of the motor under full load and with full rated voltage. If $p$ and $f$ are known, the slip may be calculated as explained above.

Various methods have been proposed for measuring the slip of an induction motor directly. A simple piece of apparatus for this purpose consists of a black disk with $p$ white radial lines or sectors painted upon it, where $p$ is the number of poles of the motor. This disk is attached to the motor pulley. If, when the motor is running, the disk is illuminated by an alternating-current arc lamp supplied with current from the same mains as the motor, the white lines on the disk would appear stationary if the motor were running in synchronism. If the inotor were below synchronism, the disk would appear to rotate with a speed equal to the difference between the synchronous and the actual speed. Thus the slip can be measured directly by counting the apparent revolutions per minute of the disk. For example, the synchronous speed of the 1-h. p. motor considered above, is $1,200 \mathrm{r} . \mathrm{p} . \mathrm{m}$. In applying the above test the disk made 70 apparent revolutions per minute when the motor was running at full load and, therefore, the slip at full-load was

$$
\frac{70}{1,200}, \quad \text { or } \quad 5.83 \%
$$

Performance Curves of an Induction Motor. Fig. 355 shows the performance curves of a $175-\mathrm{h} . \mathrm{p}$., three-phase, twelve-pole induction motor having a synchronous speed of $600 \mathrm{r} . \mathrm{p}$. m., when supplied with three-phase currents at 550 volts and 60 cycles per second. The abscissas of all the curves are horse-power output. The dropping of all the curves in the regior between 340 and 366 horse-power shows that the maximum power which the machine can develop is about 350 horse-power. There are three scales of ordinates, namely, amperes input, per cent, and pounds torque.

The ordinates of the curve marked "synchronism" are measured on the per cent scale, and they represent the speeds in per cent of the synchronous speed at the various loads.

The ordinates of the curve marked "amperes input" give the amperes input per phase at the various loads.


Fig. 355. Performance Curves of a $175-\mathrm{H}$. P. Three-Phase Induction Motor
The ordinates of the curve marked "torque" are measured on the pounds torque scale, and they represent the torque developed by the motor at various loads, these torques being expressed in pounds of force acting at one foot radius, that is, in pound-feet.

The ordinates of the curves marked "efficiency", "apparent efficiency", and "power factor", are measured on the per cent scale, and they represent these quantities at various loads. The efficiency is, of course, the ratio, $\frac{\text { output }}{\text { input }}$. The power factor is the factor by which the product of volts times amperes must be multiplied to give the true input of power per phase.

The product volts times amperes is called the apparent power delivered by an alternating-current generator, and the appareni efficiency of an induction motor is the ratio, $\frac{\text { output }}{\text { apparent power input }}$. The apparent efficiency is thus equal to the true efficiency multiplied by the power factor.

The relation between the three factors, efficiency, power factor, and apparent efficiency, will be made clear by the following equa.tions, each based on definition:

$$
\begin{aligned}
\text { efficiency } & =\frac{\text { useful output }}{\text { actual input }} \\
\text { power factor } & =\frac{\text { actual input }}{\text { apparent input }} \\
\text { apparent efficiency } & =\frac{\text { useful output }}{\text { apparent input }}
\end{aligned}
$$

From the above three equations, it is evident that

$$
\text { apparent efficiency }=\text { efficiency } \times \text { power factor }
$$

When the output of a motor under a given load is measured mechanically by observing the useful torque developed at the armature shaft, and the speed of the armature, we have

$$
\begin{aligned}
\text { useful output } & =\frac{2 \pi n^{\prime} T}{33,000} \text { horse-power } \\
& =0.142 n^{\prime} T \text { watts }
\end{aligned}
$$

where $n^{\prime}$ is the speed of the armature in revolutions per minute; and $T$ is the useful torque in pound-feet, developed by the armature.

The actual input to a three-phase alternating-current motor of the induction type may be expressed as follows:

$$
\text { actual input }=\sqrt{3} E I P \text { watts }
$$

where $E$ is the electromotive force (effective value) applied between any two of the stator terminals; $I$ is the current (effective value) in any one of the leads supplying the stator windings; and $P$ is the power factor of the motor.

The apparent input to a three-phase induction motor may be expressed as follows:

$$
\text { apparent input }=\sqrt{3} E I \text { watts }
$$

From the above equations we may easily write the expressions for the efficiency, power factor, and apparent efficiency, as follows:

$$
\begin{aligned}
\text { efficiency } & =\frac{0.142 n^{\prime} T}{\sqrt{3} E I P} \\
\text { power factor }=P & =\frac{\sqrt{3} E I P}{\sqrt{3} E I} \\
\text { apparent efficiency } & =\frac{0.142 n^{\prime} T}{\sqrt{3} E I}
\end{aligned}
$$

Example. A certain 12-pole induction motor whose performance curves are given in Fig. 355, is rated as follows:

Number of phases, 3
Output rated at full load, 175 horse-power
Voltage supplied (between mains) 550 volts
Speed, at full load, 585 r.p.m.
Frequency, cycles per second, 60
A brake test on the motor when running at full load gave the following data (see curves in Fig. 355).

Useful torque on rotor shaft, pound-feet. 1,560
Speed, in revolutions per minute . . . . . . . 585
Amperes input per phase. . . . . . . . . . . . . 170
Power factor, in per cent. . . . . . . . . . . . . . 88
It is required to calculate the following quantities in order to check the accuracy of the curves plotted in Fig. 355.
(a) The synchronous speed in r.p.m.
(b) The slip at full load in per cent.
(c) The useful output in watts and in horse-power
(d) The actual input in watts
(e) The efficiency
(f) The apparent input in watts
(g) The apparent efficiency

Solution.
(a) The synchronous speed, according to equation (47) is

$$
\frac{2 f}{p}=\frac{2 \times 60}{12}=10 \text { revolutions per second }
$$

or

$$
10 \times 60=600 \text { r.p.m. }
$$

(b) The slip as referred to on page 415, is

$$
\left(\frac{n-n^{\prime}}{n}\right) 100=\left(\frac{600-585}{600}\right) 100=2.5 \text { per cent }
$$

## TABLE XVI <br> Capacities of Standard Transformers

| H. P. <br> Capacity Motor | Three-Phase |  | Two-Phase 2 Trangformers |
| :---: | :---: | :---: | :---: |
|  | 2 Trangformers | 3 Transformers |  |
| 1 | . 6 kw . | . 5 kw . | . 6 kw . |
| 2 | 1.5 kw. | 1.0 kw . | 1.0 kw . |
| 3 | 2.0 kw . | 1.5 kw . | 1.5 kw . |
| 5 | 3.0 kw . | 2.0 kw . | 3.0 kw . |
| $7 \frac{1}{3}$ | 4.0 kw . | 2.5 kw. | 4.0 kw . |
| 10 | 5.0 kw . | 3.5 kw . | 5.0 kw . |
| 15 | 7.5 kw . | 5.0 kw . | 7.5 kw . |
| 20 | 10.0 kw. | 7.5 kw . | 10.0 kw. |
| 30 | 15.0 kw. | 10.0 kw. | 15.0 kw. |
| 50 | 25.0 kw. | 15.0 kw. | 25.0 kw. |
| 75 |  | 25.0 kw. | 35.0 kw. |
| 100 |  | 30.0 kw. | 45.0 kw. |

(c) The useful output from the above equations is $0.142 n^{\prime} T$ watts. Substituting the given values of $n^{\prime}$ and $T$, we have

$$
0.142 \times 585 \times 1560=129,600 \text { watts }
$$

or

$$
\frac{129,600}{746}=173 \text { horse-power }
$$

(d) The actual input in watts is $\sqrt{3} E I P$, whence substituting the given values of $E, I$, and $P$ gives

$$
\begin{aligned}
\text { actual output } & =1.732 \times 550 \times 170 \times 0.88 \\
& =142,500 \mathrm{watts}
\end{aligned}
$$

(e) The efficiency (real) is

$$
\frac{\text { useful output }}{\text { actual input }}=\frac{129,600}{142,500}=90.9 \text { per cent }
$$

(f) The apparent input in watts is $\sqrt{3} E I$, or

$$
1.732 \times 550 \times 170=162,000 \mathrm{watts}
$$

(g) The apparent efficiency is

$$
\frac{\text { useful output }}{\text { apparent input }}=\frac{129,600}{162,000}=80 \text { per cent }
$$

A comparison of the above results with the corresponding values obtained from the curves in Fig. 355, shows on the whole a very satisfactory agreement.

The power factors of standard commercial induction motors of American manufacture vary at full load from 0.75 to 0.95 de-
pending upon the size and the frequency of the motor. The efficiencies range from 0.75 to 0.95 . The apparent efficiencies in motors above 5 hp . output will be found, as a rule, not less than 0.75 . This means that the transformers supplying current to induction motors of average sizes, must have an aggregate capacity of about one kilowatt for every horsepower of rated output of the motors.

Table XVI gives approximate capacities of standard transformers that should be used with two-phase and three-phase induction motors.

Heyland Diagram. The performance curves of an induction motor may be predetermined with sufficient accuracy for most purposes by means of what is called the "Heyland Diagram," which is fundamentally a clock or circle diagram of the currents per phase taken by an induction motor on various loads, for constant impressed voltage. In this diagram use is made of the fact that, if the voltage is constant, the current vectors are proportional to apparent power (volt-amperes). The portion of the current which is in phase with the voltage will then be proportional to the true power, and the portion of the current which is in quadrature with the voltage will be proportional to the reactive power. In the construction of this diagram the following test data are necessary:
(1) The normal voltage $E$, the current $I_{0}$, and the power $W_{0}$ are measured for each phase when the motor is running light, as in stray-power tests.
(2) The normal voltage $E$, the current $I_{L}$, and the power $W_{L}$ are measured for each phase when the rotor of the motor is locked, or prevented from rotating, as in the starting-torque tests.
(3) The resistance $R^{\prime}$ of the primary of the induction motor is measured and the $R^{\prime} I^{2}$ loss per phase calculated.

Analysis of Diagram. The Heyland diagram, Plate I, shows the current, voltage, and power relationships per phase. To construct a diagram choose convenient voltage and current scales, representing the voltage $E$ per phase by the vertical line $O E$. The magnetizing, or no-load current $I_{0}$, is represented by the line $O I_{0}$ making an angle $\theta_{0}$ with $O E$, and the current $I_{L}$ (measured when the rotor is prevented from rotating), by the line $O I_{L}$, making an angle $\theta_{L}$ with $O E$. The projection $O_{A} 1$ of the no-load current


$O I_{0}$ upon the voltage line is called the power component of the no-load current, and when multiplied by the voltage $E$ gives the stray-power loss per phase, provided the $R I^{2}$ losses at zero load are comparatively small. The horizontal projection $O B$ of $O I_{0}$ represents the true magnetizing current per phase taken by the primary (stator), and when multiplied by the voltage gives the reactive power at zero load. In a similar manner the vertical portion, $C I_{L}$ of $O I_{L}$, when multiplied by the voltage, will represent the stray-power loss per phase plus the $R I^{2}$ losses of the primary and secondary when the rotor is locked. The reason for this is that, in this case, both the primary and secondary currents are very large, while the stray-power loss remains practically constant because the decrease of stray power, due to the friction and windage losses being zero, is counterbalanced by the increased hysteresis and eddy-current losses of the secondary member (rotor).

Connect the points $I_{0}$ and $I_{L}$ by a straight line and with this line as a chord construct a semi-circle so that its diameter lies along the extension of the horizontal line $A I_{0}$. The diameter will intersect the line $C I_{L}$ at some point $D$. Lay off $D F$ equal to the $R^{\prime} I_{L}{ }^{2}$ loss of the primary and draw the line $I_{0} F$. Then, if $C D$ (equal to $I_{0} B$ ), when multiplied by the voltage, represents the stray-power loss (this would assume that the stray-power loss is practically constant for all values of load and slip), $I_{L} F$ times $O E$ will be the $R^{\prime \prime} I^{\prime \prime 2}$ loss of the secondary.

Now for any total current per phase OI taken by the motor, $I_{0} I$ will represent the additional current per phase taken by the primary, because of the fact that the motor is loaded and the currents in the secondary tend to reduce the flux produced by the currents in the primary. If $I G$, the vertical projection of $O I$, is drawn intersecting $I_{0} I_{L}, I_{0} F, I_{0} D$, and $O C$ at the points $K, J, H$, and $G$ respectively, it can be proved that:

OI, representing the total current per phase, when multiplied by the voltage $O E$, will represent the volt-amperes, or apparent power input, per phase for the particular load on the motor; and in a similar manner,
$G I=$ the power component of the current per phase $G I \times O E=$ the true power input per phase
$G H \times O E=$ the stray-power loss per phase
$I I J \times O E=$ the $R^{\prime} I^{\prime 2}$ loss of the primary per phase
$J I \times() E=$ the power input to the secondary per phase
$J K \times O E=$ the $R^{\prime \prime} I^{\prime 2}$ loss of the secondary per phase
$K I \times O E=$ the mechanical output per phase
$\frac{K I}{J I} \times 100$ per cent $=$ the secondary efficiency
$\frac{K I}{J I} \times n=n^{\prime}$, or the speed of the rotor
$\frac{J K}{J I} \times 100$ per cent $=$ the slip
$\frac{K I}{G I} \times 100$ per cent $=$ the total efficiency of the motor
$\frac{G I}{O I}=$ the power factor of the motor or $\cos \theta$
$\frac{K I \text { (in watts) } \times 33000}{746 \times 2 \pi n^{\prime}}=$ the torque per phase in pound-feet
Example. Construct the Heyland diagram and determine the performance curves of a 200 -volt, 60 -cycle, 4 -pole, 5 -hp., 2 -phase induction motor, having given (a) the running saturation curves, Fig. 356; (b) locked saturation curves, Fig. 357; and (c) $R^{\prime}=1.08$ ohms per phase (armature warm).

Solution. (a) To determine $I_{0}$ and $\theta_{0}$
From the running saturation curve corresponding to 200 volts, Fig. 356, we obtain $I_{0}=4.9$ amperes per phase, and $W_{0}=147$ watts per phase.
Therefore

$$
\begin{aligned}
\cos \theta_{0} & =\frac{W_{0}}{E+I_{0}}=\frac{147}{200+4.9}=0.15 \\
\theta_{0} & =81^{\circ} 20^{\prime}
\end{aligned}
$$

(b) To determine $I_{L}$ and $\theta_{L}$

From the locked saturation curve corresponding to 200 volts, Fig. 357, we obtain $I_{L}=43.1$ amperes per phase, and $W_{L}=4.18 \mathrm{kw}$. per phase.
Therefore

$$
\begin{aligned}
\cos \theta_{L} & =\frac{W_{0}}{E \times I_{L}}=\frac{4.18 \times 1000}{200 \times 43.1}=0.485 \\
\theta_{L} & =61^{\circ}
\end{aligned}
$$

(c) To determine $D F$

$$
\begin{aligned}
& D F \times O E=R^{\prime} L_{L}{ }^{2}=1.08 \times(43.1)^{2}=2.01 \mathrm{kw} \\
& D F=\frac{R^{\prime} I_{L}^{2}}{O E}=\frac{2.01 \times 1000}{200}=10.05 \text { amperes }
\end{aligned}
$$

Having determined the above values, construct the Heyland diagram according to the method just described. Then assume different values of $O I$ and calculate the total kv.-a. input, the kw. input, the power factor $(\cos \theta)$, the speed, the hp. output, the apparent efficiency, the true efficiency, and the total torque developed.

Assuming $O I=15$ amperes we have


Fig. 356. Running Saturation Curves
(1) To determine the kv.-a. input

Apparent power per phase $=\frac{O E \times O I}{1000}=\frac{200 \times 15}{1000}=3 \mathrm{kv} .-\mathrm{a}$.
Total kv.-a. for both phases $=2 \times 3 \mathrm{kv} .-\mathrm{a} .=6 \mathrm{kv} .-\mathrm{a}$.
(2) To determine the kw. input

True power per phase $=\frac{G I \times O E}{1000}=\frac{12.55 \times 200}{1000}=2.51 \mathrm{kw}$.
Total power input $=2 \times 2.51 \mathrm{kw} .=5.02 \mathrm{kw}$.
(3) To determine the power factor $(\cos \theta)$

$$
\operatorname{Cos} \theta=\frac{\text { total kw. }}{\text { total kv.-a. }}=\frac{5.02}{6.00}=0.837
$$

(4) To determine the speed

$$
n^{\prime}=\text { Speed }=\text { Synchronous Speed } \times \frac{I K}{I J}=1800 \times \frac{9.8}{10.85}=1626 \text { r.p.m. }
$$

or

$$
\frac{I K}{I J} \times 100=\frac{9.8}{10.85} \times 100=90.3 \text { per cent of synchronism }
$$



Fig. 357. Locked Saturation Curves
(5) To determine the hp. output

Total hp. output $=\frac{\mathrm{kw} . \text { output }}{0.746}=\frac{2 \times O E \times I K}{1000 \times 0.746}=\frac{2 \times 200 \times 9.8}{746}=5.26 \mathrm{hp}$.
(6) To determine the apparent efficiency

Apparent efficiency $=\frac{\mathrm{kw} . \text { output }}{\mathrm{kv.-a} \text {. input }} \times 100$ per cent $=\frac{I K}{10} \times 100$ per cent

$$
=\frac{9.8}{15.0} \times 100=65.3 \text { per cent }
$$

(7) To determine the true efficiency

True efficiency $=\frac{\mathrm{kw} . \text { output }}{\mathrm{kw} . \text { input }} \times 100$ per cent $=\frac{I K}{I G} \times 100$ per cent $=\frac{9.8}{12.55} \times 100$

$$
\begin{equation*}
=78.1 \text { per cent } \tag{8}
\end{equation*}
$$

To determine the total torque developed
Total torque in $\mathrm{lb} .=\frac{\mathrm{hp} \text {. output } \times 33000}{2 \pi n^{\prime}}=\frac{5.26 \times 33000}{2 \pi \times 1626}=17.0 \mathrm{lb}$.
Assuming other values for $O I$, the values of its various factors can be determined. The results are shown in Table XVII and are platted in Fig. 358.


Fig. 358. Performance Curves of $5-\mathrm{Hp}$. Two-Phase Induction Motor

## RECONNECTION OF INDUCTION MOTORS*

Factors Involved in Changing Induction Motors. An induction motor is designed for a definite phase connection, voltage, frequincy, speed, and horsepower. In other words it is designed to give a certain speed and horsepower when operating on a specified system. It is often desired, however, to operate the same motor on a system other than the one specified and have it develop the same power and speed. In almost every case where a change of supply is considered, it involves a change in the winding of the

[^35]TABLE XVII
Values Calculated from Heyland Diagram for a Two-Ph

| $\begin{gathered} \text { Amperes } \\ \text { Phase } \end{gathered}$ | Input |  | Output |  | Per Cent Efficiency |  | Factor <br> Power | Speed <br> Per Cent of Synchronism | Pounds Torque |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Kw. | Kv.-a. | Kw. | Hp. | App. | True |  |  |  |
| 6.10 | 1.36 | 2.44 | 1.04 | 1.39 | 42.6 | 76.5 | 55.7 | 98.2 | 4.14 |
| 7.95 | 2.32 | 3.18 | 1.90 | 2.55 | 59.8 | 81.9 | 73.0 | 95.8 | 7.76 |
| 9.60 | 3.02 | 3.84 | 2.48 | 3.32 | 64.6 | 82.1 | 78.6 | 94.5 | 10.27 |
| 11.10 | 3.62 | 4.44 | 2.96 | 3.97 | 66.7 | 81.8 | 81.5 | 93.8 | 12.37 |
| 12.95 | 4.31 | 5.18 | 3.46 | 4.64 | 66.8 | 80.2 | 83.2 | 92.0 | 14.63 |
| 15.00 | 5.02 | 6.00 | 3.92 | 5.26 | 65.3 | 78.1 | 83.7 | 90.3 | 17.00 |
| 18.75 | 6.24 | 7.50 | 4.58 | 6.14 | 61.1 | 73.4 | 83.2 | 86.7 | 20.65 |
| 23.80 | 7.63 | 9.52 | 5.04 | 6.76 | 52.9 | 66.0 | 80.2 | 81.2 | 24.28 |
| 29.75 | 8.83 | 11.90 | 4.83 | 6.48 | 40.6 | 54.7 | 74.2 | 72.0 | 26.20 |

motor. The change of system may involve the change of any number of four factors involved in the performance of the motor, namely, the voltage, the number of phases, the frequency, and the number of poles. These changes will therefore be discussed separately and at the same time the relation of each to the other will be brought out.

## Changes in voltage

Conditions to Be Met. Adapting induction-motor windings to a change in the supply voltage is more often desired than any other change. In changing the windings to accommodate a change in voltage, two main conditions have to be met if the operation of the motor is to be kept normal: first, the insulation resistance must be able to stand the new voltage; second, the iron of the magnetic circuit must be saturated to the same degree as before. The latter condition amounts to the same thing as saying that the volts per turn of the windings must remain the same.

In considering the insulation only, if the new voltage is lower than the old no further attention need be given to this except to see that it is mechanically in good condition and free from moisture and dirt. If the new voltage is higher than the old, the insulation must be considered. This can always be settled by insulation tests, but in practice the apparatus necessary to make these tests is not always available, hence it is convenient to have general rules by which to be guided.

Classification of Insulations. While it is generally true that almost any insulation will stand 1000 volts if it will stand any at all, it would not be good practice to make this assumption. The following classification used by many manufacturers would be a better general rule to follow: class 1 , up to and including 500 volts; class 2,500 to 1200 volts; class 3,1200 to 3500 volts; class 4 , 3500 to 6000 volts; class 5,6000 to 8000 volts. This classification will cover the range of voltages used on induction motors. A general statement may then be made regarding the use of these classes as follows: Any machine of a higher voltage class may be operated on a lower voltage, but no machine in a lower voltage class should be operated on a voltage higher than its own class. If the voltage change is such that it involves a change from one
class to another, tests should be made to determine whether or not the insulation is sufficient.

Insulation Tests. Test for Dryness. Two tests are necessary to determine whether or not the insulation is sufficient. The first should be an insulation test to determine the condition of dryness and cleanliness of the insulation. This test is performed by applying a direct e.m.f. of about 500 volts to the insulated winding and the frame of the machine, with a high-resistance voltmeter connected in series in the circuit. The insulation in this case forms part of the electric circuit and scarcely any current will flow. The voltmeter therefore shows but a slight deflection. The resistance of the insulation, in ohms, is then calculated from the equation

$$
R=\frac{r(E-e)}{e}
$$

where $E$ is the line voltage, $e$ the reading of the voltmeter, $r$ the resistance of the voltmeter in ohms, and $R$ the resistance of the insulation in ohms.

According to the American Institute of Electrical Engineers Standardization Rules, the insulation at operating temperature of the machine shall not be less than that given by the equation:

$$
\begin{aligned}
& R \text { in megohms }=\frac{\text { normal terminal voltage }}{\text { rated capacity in kv.-a. }+1000} \\
& (1 \text { megohm }=1000000 \text { ohms })
\end{aligned}
$$

If the insulation resistance is too low, it can sometimes be improved by drying to remove the moisture.

Dielectric Test. The second test is the dielectric test to determine the ability of the material to withstand voltage stresses. It is performed by applying for one minute, between winding and ground, an alternating voltage equal to twice the normal voltage of the circuit plus 1000 volts.

Changes in Winding. Assuming that the insulation will stand the new voltage, we must now consider what change must be made in the winding in order to keep the volts per turn the same. In this respect the induction motor is similar to the transformer, being designed for a certain voltage per coil or group of
coils. As stated before, the operation of the machine will be normal as long as the voltage per turn or per coil remains unchanged, after changing the grouping of the coils made necessary by the change of supply voltage.

The possible changes in grouping of coils may be treated in three classes as follows:
(1) Series to parallel or parallel to series connections, with combinations thereof.
(2) Delta to star, or star to delta.
(3) Combinations of series and parallel, and delta and star, connections.

Series to Parallel or Parallel to Series. An example of the first class is illustrated in Fig. 359. If the machine were rated at


Fig. 359. Typical Series to Parallel Connections
440 volts and its coils connected in series as in (a), then to operate on 220 volts it would be connected as shown in (b) and for 110 volts as in (c).

Delta to Star or Star to Delta. An example of the second class is illustrated in Fig. 360. Suppose a three-phase machine to be star-connected for 440 volts. Then, when the coils are deltaconnected, it could be operated on 254 volts as normal voltage. In a simple change from star to delta the terminal voltage changes in the ratio of $\frac{1}{\sqrt{3}}$, or 57.7 per cent. In a change from delta to star the terminal voltage changes in the ratio of $\sqrt{3}$ to 1 or 173 per cent. In the above example the machine considered could also be operated on 220 volts in place of 254 volts ( 87 per cent of normal), but at the expense of increased heating or copper loss for the same power output.

Combination Series and Parallel, Delta and Star. The third class is a little more complicated and can best be explained by considering typical examples. In making the necessary changes it should always be kept in mind that windings are usually grouped so as to form one coil per pole per phase. These polephase groups, as they are called, must be handled as units and should not be split up into smaller coils. Therefore, if a 6 -pole machine were being considered there would be six pole-phase groups in each phase, and each phase could be connected so as to form one series-2-parallel, 3-parallel, or 6-parallel circuits, but not 5 or 7 parallels because the number of poles, and therefore the number of pole-phase groups in one phase group, are not divisible by these numbers. Take, for example, a 6-pole, 2200 -


Fig. 360. Typical Delta to Star Connections
volt, 3 -phase motor, Plate II. It is desired to connect it to 440 volt mains. The motor is series star-connected. The volts per pole-phase group will then be $\frac{1}{6} \times \frac{2200}{\sqrt{3}}=211.7$ volts; and if two of the pole-phase groups are connected in series, they could be placed across 423 volts. This means that the motor could be delta-connected and have three parallel circuits in each phase, each parallel circuit containing two pole-phase groups, of the same phase, connected in series. The connection is shown in Plate III, and is called a 3 -parallel-delta winding.

Winding Combinations. Changes of this nature can be summarized in convenient form as in Tables XVIII and XIX for three-phase and two-phase motors respectively. If a motor connetted originally as shown in any horizontal column has a voltage


## TABLE XVIII

## Comparison of Motor Voltages with Various Three=Phase Connections

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Series-Star | 100 | 50 | 33 | 25 | 20 | 17 | 58 | 29 | 19 | 15 | 2 |  |
| 2-Parallel Star | 200 | 100 | 67 | 50 | 40 | 33 | 116 | 58 | 39 | 29 | 23 | 19 |
| 3-Parallel Star | 300 | 150 | 100 | 75 | 60 | 50 | 173 | 87 | 58 | 43 | 35 | 29 |
| 4-Parallel Star | 400 | 200 | 133 | 100 | 80 | 67 | 232 | 116 | 75 | 58 | 46 | 39 |
| 5-Parallel Star | 500 | 250 | 167 | 125 | 100 | 83 | 289 | 144 | 96 | 72 | 58 | 48 |
| 6 -Parallel Star | 600 | 300 | 200 | 150 | 120 | 100 | 346 | 173 | 115 | 87 | 69 | 58 |
| Series-Delta | 173 | 86 | 58 | 43 | 35 | 29 | 100 | 50 | 33 | 25 | 20 | 17 |
| 2-Parallel Delta | 346 | 173 | 115 | 85 | 69 | 58 | 200 | 100 | 67 | 50 | 40 | 33 |
| 3 -Parallel Delta | 519 | 259 | 173 | 130 | 104 | 87 | 300 | 1.50 | 100 | 75 | 60 | 50 |
| 4-Parallel Delta | 692 | 346 | 231 | 173 | 138 | 115 | 400 | 200 | 133 | 100 | 80 | 67 |
| 5-Parallel Delta | 865 | 433 | 288 | 216 | 173 | 144 | 500 | 250 | 167 | 125 | 100 | 83 |
| 6-Parallel Delta | 1038 | 519 | 346 | 260 | 208 | 173 | 600 | 300 | 200 | 150 | 120 | 100 |

of 100 , its voltage, when reconnected inf the manner indicated in any vertical column, is shown at the intersection of the two columns. The figures in the tables should be considered as percentages or comparative values rather than actual voltages. For example, the motor just considered was a 2200 -volt motor with a series-star winding to be reconnected for 440 volts. Since 440 is 20 per cent of 2200 volts, the problem resolves itself into how a series-star connection may be changed so that the resulting voltage will be 20 per cent of its original value. Consulting Table XVIII, locate the horizontal line reading "Series-Star," or the existing connection. Since 20 per cent is required, read along the same horizontal line till the figure 20 is reached. This is found under the vertical heading " 5 -Parallel Star." In other words, if the number of poles is divisible by 5 the winding can be put in 5 parallels and operated on 440 volts; since $\frac{2200}{5}=440$. Since 6 poles were assumed, the number of poles is not divisible by 5 and a 5 -parallel connection is not possible. A further search across the table shows the figure 19 under the vertical heading " 3 -Parallel Delta"; 19 per cent of 2200 is 418 volts, which is 95 per cent of 440 . This varies from the value 423 previously men-

TABLE XIX
Comparison of Motor Voltages with Various
Two-Phase Connections

|  | Series | 2-Parallel | 3-Parallel | 4-Parallel | 5-Parallel | 6-Parallel |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Series | 100 | 50 | 33 | 25 | 20 | 17 |
| 2-Parallel | 200 | 100 | 67 | 50 | 40 | 33 |
| 3-Parallel | 300 | 150 | 100 | 75 | 60 | 50 |
| 4-Parallel | 400 | 200 | 133 | 100 | 80 | 67 |
| 5-Parallel | 500 | 250 | 167 | 125 | 100 | 83 |
| 6-Parallel | 600 | 300 | 200 | 150 | 120 | 100 |

tioned for the reason that the table is made to the nearest whole number and $\frac{100}{3 \times \sqrt{3}}=19.2$ per cent. This connection can be used satisfactorily on 440 volts. Similar problems can thus be solved by inspection, making Tables XVIII and XIX very convenient for reference.

Effect of Incorrect Voltage on Performance. In cases where a winding has been reconstructed it is not always convenient to obtain the correct voltage necessary to produce exactly the same volts per turn for which the motor was originally designed. The motor must therefore operate under conditions other than normal, resulting in a change of speed, torque, power factor, and efficiency.

Effect on Speed. The effect on speed is shown by the following tabulation:*

| Per Cent Voltage Applied | 80 |  | 90 | 100 | 110 | 120 | 130 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 140 |  |  |  |  |  |  |  |
| Approximate Per Cent Slip | 1.5 | 1.25 | 1.0 | 0.8 | 0.7 | 0.6 | 0.5 |

This table is for a normal slip of 1 per cent and a normal voltage of 100 per cent. For any other slip the values in the table are proportionate. The results show that the increase in slip for a given drop in voltage is proportionately greater than the decrease in slip caused by a corresponding increase in voltage.

Effect on Torque. The torque of an induction motor will vary as the square of the applied voltage. This is approximately true whether the motor is at rest or running.

[^36]TABLE XX
Results of Tests on a 2-Phase, 8-Pole, 50-Hp., 200-Volt, 60-Cycle Induction Motor

| Factor | 60 Cycles |  |  | 50 Crcles |
| :---: | :---: | :---: | :---: | :---: |
|  | 250 Volts | 220 Volts | 200 Volts | 200 Volts |
| Output (watts) | 37300 | 37300 | 37300 | 37300 |
| Speed (r.p.m.) | 872 | 865 | 856 | 713 |
| Slip (per cent) | 3.1 | 4.0 | 4.8 | 4.8 |
| Pull-out (max. torque) | 4.9 | 3.7 | 3.1 | 3.7 |
| Starting Torque | 3.0 | 2.5 | 2.1 | 2.5 |
| Real Efficiency (per cent) | 88.8 | 88.5 | 87.8 | 87.6 |
| Power Factor (per cent) | 86.8 | 92.0 | 93.2 | 92.0 |
| Apparent Efficiency (per cent) | 77.2 | 81.4 | 81.8 | 80.7 |
| Iron Loss | 1650 | 1300 | 1100 | 1320 |
| Friction and Windage | 620 | 620 | 620 | 430 |
| Prim. Copper | 1230 | 1410 | 1700 | 1750 |
| Sec. Copper | 1180 | 1550 | 1880 | 1880 |
| Total Losses | 4680 | 4880 | 5200 | 5380 |
| Real Input | 42000 | 42200 | 42500 | 42680 |
| Magnetizing (per cent) | 41.0 | 28.5 | 23.6 | 28.5 |
| Leakage (per cent) | 8.8 | 10.7 | 12.9 | 10.9 |
| Apparent Input | 48300 | 45800 | 45700 | 46300 |

Note. Losses are given in watts; pull-out and starting torques are given in terms of fullload torque.

Effect on Power Factor. The power factor decreases with an increase in voltage and increases with a decrease of voltage because of the lower value of magnetizing current on reduced voltage and vice versa.

Effect on Efficiency. The effect on the various losses and efficiency will vary considerably with the design. Table XX gives experimental data* taken from a test of a $50-\mathrm{hp}$., 2 -phase, $200-$ volt, 8 -pole, 60 -cycle induction motor.

## CHANGES OF FREQUENCY

Reasons for Changes. The necessity for operating motors on a frequency differing from that for which they were originally designed may be the result of actually changing the frequency of the power supply and thereby affecting a number of motors in one installation, or it may result from applying used or repurchased motors on new circuits. The installation of these machines

[^37]on new circuits necessitates in many cases a change in frequency as well as a change in voltage and phase. The commonest changes of this kind are from 25 cycles to 60 cycles, and vice versa. There are also some changes from 60 to 50 cycles and a few from 60 to 40 , or the reverse.

Effect of Change on Speed of Motor. The most important and immediately noticeable change in the motor, when the frequency is changed, is that the motor operates at a different speed, the change in speed being proportional to the change in frequency. The so-called synchronous speed, or the number of revolutions per minute made by the magnetic field of the stator, is equal to the expression $\frac{\text { cycles } \times 120}{\text { number of poles }}$. From this it follows that if all other things remain the same, the speed will change in proportion to the change in frequency.

For example, a 4 -pole motor operated on 25 cycles will have a synchronous speed equal to $\frac{25 \times 120}{4}$, or 750 r.p.m. The fullload speed is usually from 3 to 5 per cent less than the synchronous speed. If this motor is operated on 60 cycles the speed becomes $\frac{60 \times 120}{4}=1800$ r.p.m. This immediately brings up two serious mechanical questions. First, is the mechanical design of the rotor such that it will stand this increase in speed amounting to 240 per cent of its original value? The peripheral speed of the rotor (diameter $\times \pi \times$ r.p.m.) should not be permitted to go beyond 7500 feet per minute, without the manufacturer of the machine being consulted. Second, can the belting and gearing be suitably adjusted so that the speed of the driven machine or apparatus will remain practically unchanged? If these two questions cannot be satisfactorily taken care of, it will be necessary the change the number of poles so that the speed can remain nearly the same as before.

In the case considered, the nearest combination would be to wind the motor for 10 poles. This would give a speed of $\frac{60 \times 120}{10}$, or 720 r.p.m. There are, therefore, two conditions to consider when the frequency is changed-the first, when the poles are
unchanged; the second, when the poles are changed to keep the speed the same as before.

Effect of Change on Operating Voltage. The next thing that is affected by the change of frequency is the operating voltage. That is to say, if the frequency of supply is changed, the voltage per turn or per coil should be changed in the same direction and by the same amount. This becomes evident when we consider that the induced voltage in the primary, which opposes the line voltage, is proportional to the flux times the frequency. Therefore, if the frequency is increased, the flux will not have to be so large to induce the required back e.m.f., and the flux will automatically be reduced in the same ratio that the frequency was increased. Since the value of torque developed varies as the square of the flux, the torque and therefore the horsepower will be greatly reduced, for

$$
\mathrm{hp} .=\frac{2 \pi \times \text { torque in lb.-ft. } \times \text { r.p.m. }}{33000}
$$

At the same time it must be remembered that the increase in frequency would cause an increase in speed that would partly make up for the decrease in torque.

Again, there are two cases to be considered-first, when the voltage is changed in proportion to the change in frequency to make the most use of the iron present, and also to keep the torque constant; second, where the horsepower is to be kept constant.

To Keep Torque Constant. The physical conception of the first case is that if the frequency and voltage are varied together and the motor is working against the same torque, the magnetic density of the iron will remain the same, and the current in the copper of the stator and the rotor will remain substantially the same, but the horsepower will rise and fall with the voltage and frequency, since it is proportional to the product of the torque and speed.

To Keep IIorsepower Constant. In the second case it is desired to keep the horsepower the same, and, therefore, the torque must be decreased in the same proportion that the frequency and speed increase. Since the torque varies as the square of the applied voltage, it is evident that approximately the same horse-
power can be kept with a changed frequency by varying the voltage applied to the motor coils as the square root of the change in frequency instead of in direct proportion to the change in frequency. An example of this would appear in the operation of a 440 -volt, 25 -cycle motor on a 550 -volt, 40 -cycle circuit. The square root of $\frac{40}{25}$ is 1.26 . Then if $1.26 \times 440$, or 554 volts be used at 40 cycles, the magnetic density in the iron will be about 80 per cent of its 25 -cycle value and the torque will be $\left(\frac{80}{100}\right)^{2}$, or 64 per cent of its 25 -cycle value. Since the speed will be $\frac{40}{25}$ of that on 25 cycles, the resulting horsepower will be $\frac{40}{25} \times \frac{64}{100}$, or 1.02 times its 25 -cycle value, or practically the same.

The fact that a motor can operate on a higher frequency without changing the voltage leads to a word of caution regarding the converse proposition-that of operating the motor on a lower frequency without changing the voltage. In this case the magnetic flux would have to increase and the iron would become over-saturated with the result that the stray-power loss and also the magnetizing current would be greatly increased, resulting in undue heating.

Effect of Incorrect Frequency on Motor Performance. The synchronous speed of an induction motor changes directly with the frequency while the per cent slip will remain practically unchanged.

The torque, which is proportional to the product of the flux and secondary current or to the square of the flux, is inversely proportional to the square of the frequency, since the flux produced by the applied voltage is directly proportional to the voltage applied and inversely proportional to the frequency.

The power factor will be improved on higher frequency because of the lower flux and therefore the lesser degree of magnetic saturation in the iron, and will be lowered on low frequency because of the increased saturation.

The effect on the losses and efficiency, which differ with the design, can be studied from the example given in Table $\mathbf{X X}$,
which shows the test data of a 50 -hp., 2 -phase, 220 -volt, 8 -pole, 60 -cycle motor when operated on both 60 cycles and 50 cycles.

Changes of Voltage and Frequency. Small variations in voltage and frequency of about 10 per cent from the normal values are usually permissible, but such variations are always accompanied by a change from the normal operation, as explained above. A summary of the more important changes with either voltage or frequency differing from the normal is given below:

|  | Power Factor | Torque | Per Cent Slip |
| :---: | :---: | :---: | :---: |
| Voltage Higher | Lower | Higher | Lower |
| Voltage Lower | Higher | Lower | High |
| Frequency Lower | Lower | Higher | Unchanged |

Where considerable changes in voltage or frequency are involved, a change in winding is always necessitated, with the exception of the case where the voltage and frequency are varied together and in the same ratio. In this case the performance will no longer be the same, although the motor will operate satisfactorily. The following tabular data shows the results of an actual test on a 30 -hp., 2 -phase, 200 -volt, 60 -cycle motor run on 25.8 cycles.*

| $\begin{gathered} \text { Fre- } \\ \text { quency } \end{gathered}$ | Volts | Amperes | Watts | Torque | R.P.M. | R.Plip | Efucienc: | Power latuc | Hp. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 60 | 400 | 73.5 | 26400 | 185 | 850 | 50 | 84.8 | 89.7 | 30 |
| 25.8 | 170 | 78.3 | 12000 | 185 | 335 | 55 | 73.8 | 00.3 | 11.8 |

## CHANGE OF PHASE

Reconnecting Coils for Three $=$ and Two=Phase Circuits. When it becomes desirable to use a three-phase induction motor on a two-phase circuit, or vice versa, it is necessary to reconnect the coils. This is possible with many types of motors, provided the number of coils per pole is equal to six or some multiple thereof. Assume, for example, a motor having six slots and six coils per pole. The phase relations of the induced voltages of

[^38]these coils can be seen from Fig. 361. For the instant represented by the figure, the e.m.f. induced in the conductor $a$ is a positive maximum, that in $b$. less, $c$ still less, etc. It can also be


Fig. 361. Phase Relations of Induced Voltages in Pole Coils
seen that the phase of the e.m.f. in each coil differs by 30 degrees from that of the preceding coil. These e.m.f.'s may be represented by the six vectors in Fig. 362. To connect the motor for three-phase operation, coils $a$ and $b$ are connected in series, thus producing the voltage $a+b$. In


Fig. 362. Vector Diagram of Voltages a like manner the pairs $c d$ and ef produce the voltages $c+d$ and $e+f$ respectively. It must be remembered that there are as many coils $a a^{\prime}, b b^{\prime}, c c^{\prime}$, etc., as the number of poles for which the motor was wound; that all of these are generally connected in series; and that they then constitute what may be called one phase of the winding.

It will be seen that the voltages of the three phases so produced differ in phase by only 60 degrees, while it is necessary
that they be equally spaced 120 degrees. This is easily accomplished by reversing the $c+d$ phase when making the final connection of the winding, namely, star or delta.

To connect the motor for two-phase operation, the groups of coils $a, b$, and $c$ are placed in series, thus producing a resultant voltage $a+b+c$, (Fig. 363). In a like manner, groups $d$, $e$, and $f$ in series produce a resultant voltage $d+e+f$. These two resultant voltages are seen to be 90 degrees apart as required for twophase operation.

Any motor may have its coil connections opened, thus producing the condition assumed at the beginning of this discussion, and the connections may then be made for either two or three phase as desired. It must be observed however that if a motor, originally operating on 220 volts and three-phase delta, had 220 volts applied to the groups of coils $a$ and $b$ in series, then when three groups $a, b$, and $c$ are placed in series, as for two-phase, the voltage per phase should be somewhat higher ( $1.41 \times 220=310$ volts) to produce the same magnetic condition in the motor and thus enable the motor to deliver its original full-load rated


Fig. 363. Vector Diagram of Voltages Connected for Two-Phase Operation output.

When a motor has a phase-wound secondary it will not be necessary to reconnect the winding of the secondary, as the revolving field produced by the new type of primary winding is essentially unchanged; thus a reconnected motor with a twophase primary may be operated using its original three-phase rotor, and a change in the starting or controlling resistances is unnecessary.

## CHANGE OF POLES*

Reasons for Change of Poles. When the frequency of the supply current is changed, the synchronous speed of all induction motors operated from this supply will be changed in proportion to the change of frequency. If this change in frequency results

[^39]in an undesirable motor speed, the number of poles on the motor must be changed. In a similar manner a motor operating on normal frequency may have its poles changed in order to operate at a speed more favorable to the mechanism it drives.

Change of Poles Inversely Proportional to Change of Speed. The synchronous speed of an induction motor is inversely proportional to the number of poles. Thus on 60 cycles a 2 -pole motor has a synchronous speed of 3600 r.p.m.; a 4 -pole motor, 1800 r.p.m.; and an 8 -pole motor, 900 r.p.m. If the speed is to be lowered in a given ratio, the poles will have to be increased in the same ratio.

Some induction motors are designed to operate with two or more ranges of speed; the change of poles is accomplished by having two or more primary windings, each having a different number of poles, or by using a single winding which can be connected so as to form different numbers of poles. The latter arrangement is used on the battleship New Mexico, where the two-phase induction motors driving the propellors are so constructed that the single winding may be connected to give either 24 or 36 poles. This winding, however, is unusual, the preferable ratio of poles for a single winding being $2: 1$.

Analysis of Typical Case. The problem of completely rewinding an induction motor for a different number of poles is not as difficult as that of adapting an old winding to a new connection, and it is the latter problem with which this article deals. As an illustrative example we will take a 25 -horsepower, 4 -pole motor operating on a 40 -cycle, two-phase, 220 -volt current. The machine has 72 slots and 72 coils, the individual coils spanning 14 slots and each phase winding composed of 2 groups of coils in parallel. It is desired to operate the same motor on a 60-cycle, three-phase, 550 -volt circuit at the same speed and horsepower. If a new winding were placed on the motor, the span of the coils could easily be changed to $\frac{72 \text { slots }}{6 \text { poles }}=12$ slots. The new connection would then enable the motor to run at the same speed as before, and if the proper voltage were chosen the same torque could be developed, thus giving the same horsepower. With a slot pitch of 12 for a 6 -pole winding, each inductor directly under
a north pole will have its return inductor directly under a south pole and the e.m.f.'s induced in each will add directly. In neither a 4 -pole nor a 6 -pole connection is this true if the span is 14 slots and the winding is said to be a fractional pitch or chord winding.

Effect of Chord Winding. In making use of the old winding, it is the effect of this chord winding on the terminal voltage allowable that is of greatest importance. In Fig. 364 let $\theta$ be the number of electrical degrees corresponding to the span of the coil. Then, the


Fig. 364. Vector Diagram Showing Effect of Chord Winding voltage $e$ in the inductor under a north pole adds vectorially to the voltage $e$ of the inductor under a south pole. The resultant e.m.f. produced by both is $2 e$ $\sin \frac{\theta}{2}$, which is $\sin \frac{\theta}{2}$ times what would be produced by a full-pitch winding. The ratio $\sin \frac{\theta}{2}$ is called the "chord factor" and must be taken into account in figuring the new voltage necessary to produce normal operation.

Calculating New Voltage. Assuming the old winding is used with a slot pitch of 14 , the voltage to be used with the new connection is calculated by considering the following effects:
(1) The effect on the voltage of changing the phase and distribution factor.
(2) The effect on the voltage of the change in frequency.
(3) The effect on the voltage of the change of poles. This may be taken into account by considering that with a constant field a high speed will generate more counter-e.m.f., and a lower speed less e.m.f.
(4) The effect on the "chord factor."

Before changing the number of phases, the voltage could be increased from 220 volts to 440 volts by connecting the 2-parallel coils in each phase in series. Then if the winding is changed from 2- to 3 -phase the voltage becomes $440 \times \frac{2}{3}$, or $\frac{880}{3}$ volts. The distribution factor for a normal 2-phase winding is 0.905 and
for a normal 3 -phase winding 0.955 . Therefore the allowable e.m.f. on 3 -phase becomes $\frac{880}{3} \times \frac{.955}{.905}$, or 309.5 volts.

If the poles were not changed and the machine operated on 60 cycles, the voltage could be increased in the ratio of $\frac{60 \text { cycles }}{40 \text { cycles }}$. The effect of lowering the speed to its former value by changing the poles, means that the rotating magnetic field is going to rotate slower. This may be thought of as inducing less countere.m.f. so that the voltage must be reduced in the ratio of $\frac{4 \text { poles }}{6 \text { poles }}$. Ignoring the chord factor, the new voltage becomes $309.5 \times \frac{60}{40} \times \frac{4}{6}$, or 309.5.

Last of all, the chord factor affects the e.m.f. in the following manner: In the old winding a slot pitch of $\frac{72 \text { slots }}{4 \text { poles }}$, or 18 slots, corresponded to 180 electrical degrees, so that a pitch of 14 slots corresponded to $\frac{14}{18} \times 180$, or 140 degrees, and the chord factor $\left(\sin \frac{\theta}{2}\right)$ becomes in $70^{\circ}$, or 0.94 . In the new winding a slot pitch of $\frac{72 \text { slots }}{6 \text { poles }}$, or 12 slots, corresponds to 180 degrees, and therefore a pitch of 14 slots corresponds to $\frac{14}{12} \times 180$, or 210 degrees. The chord factor then becomes $\sin 105^{\circ}\left(=\sin 75^{\circ}\right)$, or 0.98 . The voltage per phase for the new connection may therefore be $\frac{0.98}{0.94}$ of the voltage determined by considering everything but the chord factor, and the e.m.f. per phase is equal to $309.5 \times \frac{.98}{.94}$, or 323 volts. By connecting the motor star, a terminal voltage of $\sqrt{3} \times 323$, or 560 volts would become the correct or normal voltage to use. This is almost exactly 550 volts so that the change would be possible.

Operating Precautions After Change. This example is intended to bring out the fundamental points to be considered in
changing the poles of a motor. After a motor is changed in any way, it should be started up slowly and the load gradually thrown on. Careful observations should be made to see that there is no local heating, undue noise, or vibration. The current per phase and speed readings should be taken to see that the motor does not take an undue amount of power and also that the rotor resistance is correct. A slip of 10 per cent or more accompanied by the heating of the rotor would show that the rotor, or secondary winding, was not suitable.

Conclusion. From the preceding discussion it can be seen that all changes, whether of voltage, frequency, phase or poles, involve changes in voltage, so that the most important question is, whether the volts per turn have been kept the same. It is best not to allow the applied voltage to differ more than 10 per cent from normal. In general the effects of high and low voltage may be summarized as follows:

## High Voltage:

(1) Increases the magnetic density
(2) Increases the magnetizing current
(3) Decreases the "leakage current" (leakage reactive component)
(4) Increases the starting torque and maximum torque
(5) Decreases the slip
(6) Decreases the secondary copper loss
(7) Increases the iron loss
(8) Usually decreases the power factor
(9) May increase or decrease the efficiency and heating, depending on the proportions of primary and secondary copper loss and iron loss in the normal machine, and also the degree of saturation of the iron

## Low Voltage:

(1) Decreases the magnetic density
(2) Decreases the magnetizing current
(3) Increases the leakage current
(4) Decreases the starting and maximum torque
(5) Increases the slip
(6) Increases the secondary copper loss
(7) Decreases the iron loss
(8) Usually increases the power factor
(9) May decrease or increase the efficiency and heating, depending upon the proportions of primary and secondary copper loss and iron loss in the normal machine, and also upon the degree of saturation of the iron

From the foregoing discussion it may be concluded that:
(1) Changes in voltages alone are the easiest class of changes and can usually be made.
(2) Changes in the number of phases alone can rarely be made satisfactorily.
(3) Changes in the number of poles are limited, due to the mechanical form of the coils.
(4) Changes in frequency alone, or in combination with changes in voltage or phase, can sometimes be made if the change in speed is not objectionable.
(5) Complicated changes should not be attempted except by persons of some experience, and should be handled with caution.
(6) If the calculated peripheral speed of the rotor (which equals the rotor diameter in feet $\times 3.1416 \times$ r.p.m.) exceeds 7000 feet per minute on any proposed change, the maker of the motor should be consulted before the change is made.
(7) In case of doubt on any point, consult the manufacturer.

## SWITCHBOARD AND STATION APPLIANCES

The switchboard is a frame carrying most of the measuring and controlling devices used in a plant.


Fig. 335. Diagram Showing Switrhboard Control for Large Rheostats

Large apparatus are frequently installed at a distance from the switchboard, and are controlled from the switchboard by systems of levers, by compressed air, or by electrical relays. Thus Fig. 365 shows a large rheostat mounted underneath the floor upon which the switchboard stands. The rheostat is operated by means of sprocketwheels and chain which are moved by a hand-wheel on the switchboard.

## SWITCHBOARDS

A.c. switchboards differ from d.c. switchboards as follows:
(a) On account of the special devices such as frequency meters, transformers, power-factor indicators, and synchronizing devices, which are used in a.c. work, and are not used in d.c. work.
(b) Because of complications associated with the use of an auxiliary direct-current generator for exciting the field magnets of alternators.
(c) Because of the frequent use of extremely high voltages, as high as 100,000 volts or more, in alternating-current systems.

Switchboards are now usually built up of standard panels uniform in size, the style varying with the service required. Large switchboards for handling many generators or many feeder circuits, are built up by placing a number of these standard panels side by side. This method of building large switchboards has the following advantages:
(1) It reduces the necessity of special work to a minimum, and permits the use of standard apparatus, thus reducing cost.
(2) It provides for interchangeability of panels, thus making rearrangement of feeder, generator, and exciter panels easy and convenient.
(3) Theuse ofstandardpanels uniformly equipped with standard apparatus makes it easy and cheap to renew damaged parts.
(4) It enables extensions to be made easily and systematically.

In some large stations as many as ten or more generator panels and fifteen or more feeder panels are erected side by side. The panels of a switchboard are usually erected and wired completely at the factory, and all the instruments are attached. After thorough inspection and testing, they are shipped to their destination, the instruments being detached, and shipped separately.


Fig. 366. Standard Switchboard for Single-Phase Generator
Switchboards are usually constructed of a skєleton frame of angle iron, to which panels of marble or slate are fastened by bolts and nuts. The various instruments are
attached to the marble or slate panels. In many cases the apparatus itself is located behind the board, the hand-wheel or operating lever only, being placed on the front of the board.

Slate, when entirely free from metallic veins, is a fair insulator, but the frequent occurrence of such veins, and the tendency of slate to absorb moisture from the air, render it unreliable, especially for switchboards on which high-voltage apparatus is to be installed.


Fig. 367. Front Elevation and Connections for Board Shown in Fig. 366
Marble is the standard material for switchboard panels, and it only is used for high-voltage panels.

Typical Single=Phase Switchboard. Fig. 366 shows a front view of a standard switchboard for one single-phase generator and two feeder circuits. The front elevation of this board and the complete diagram of electrical connections are shown in Fig. 367. These switchboards are manufactured by the Fort Wayne Electric Works, for electric lighting plants of moderate size.

The standard voltages for this type of panel are 1,150 and 2,300 volts, at frequencies of from 25 to 140 cycles per second.

The kilowatt capacity or rating of switchboard panels depends on the current carrying capacity of its equipment. Thus, the rating of these panels ranges from 15 to 300 kilowatts at 2,300 volts and from $7 \frac{1}{2}$ to 150 kilowatts at 1,150 volts.

The ammeter and voltmeter used on these boards, indeed nearly all switchboard ammeters and voltmeters, are of the electromagnetic type, that is, they consist of a coil of wire ac-


Fig. 368. Details of Switchboard Fuse Holder tuating a movable piece of soft iron to which the pointer is attached. The electromagnetic type is described on page 67.

The essential features of the electrostatic ground detector are also described on page 67 .


Fig. 369. Front and Side Views and Complete Diagram of Connections for Standard Panel for Two or More Single-Phase Alternators in Parallel

The two double-pole quick air-break generator switches with marble barriers are shown in Fig. 366. They are used for connect-
ing the generator terminals to either or both of the feeder circuits at the top of the panel, as may be seen in the diagram of connections shown in Fig. 367. When either switch is opened, the are which is produced is prevented by the marble barrier from flashing across from blade to blade of the switch, and thus short-circuiting the generator.

Instead of these air-break switches, panels may be equipped with non-automatic oil switches, mounted at the back of the board. The type of switch illustrated in Fig. 366, and which breaks the circuit in air is suitable only for moderate voltages up to perhaps 2,300 volts, on account of the danger of arcing.

One of the fuse holders, shown at $A$ in Fig. 367, is illustrated in Fig. 368. The body of the fuse holder consists of an insulated metallic chamber, into which is screwed a fiber tube. That portion of the fuse located in the lower chamber has a smaller cross-section than the portion in the upper tube, which insures that the fuse will melt inside the chamber. When the fuse melts and breaks, the arc is extinguished by the explosive action which accompanies the sudden heating of the air enclosed in the fuse chamber. New fuses can be easily inserted in the holder by removing the screw plug shown at the bottom of the bulb. The holder is connected by means of blades fastened to each end which fit into clips mounted at the top of the panel, as shown in Figs. 366 and 367. The potential transformer has its secondary connected through a high resistance to the voltmeter. If the ratio of transformation of the potential transformer is 20 to 1 , and the voltmeter reads 110 volts, the voltage between the generator terminals is $20 \times 110=2,200$ volts.

Typical Switchboard for Operating Two or More Single-Phase Alternators in Parallel. Fig. 369 shows front and side elevations and a complete diagram of electrical connections of one of the General Electric Company's standard panels for one of several single-phase generators to be operated in parallel. The equipment of this panel is as follows:

1 double-pole generator switch, mounted on the back of the board, and operated by a lever on the front of the board.

2 expulsion fuse blocks complete.
1 generator ammeter.
1 field ammeter.
1 voltmeter and one potential transformer.
1 field switch.

1 synchronizing device, complete.
Tripod and front plate for "generator rheostat" (in the field circuit of the generator) with shaft and hand-wheel.

Tripod and front plate for "exciter rheostat" (in the field circuit of the exciter.)

All necessary framework and connections.
The location of apparatus for this panel, as designated by letter, -is as follows:
$R=$ rheostat in the field circuit of the generator, "generator rheostat."
$a=$ ammeter in the field circuit of the generator "field ammeter."
$V=$ voltmeter between the generator terminals (through potential transformer).
$A=$ ammeter for the main alternating current.
$L=$ synchronizing lamp.
$S=$ "generator switch" in the main circuit.
$r=r h e o s t a t ~ i n ~ t h e ~ e x c i t e r ~ f i e l d ~ c i r c u i t, ~ " e x c i t e r ~ r h e o s t a t . " ~ " ~$
This apparatus is essentially the same as the apparatus already described on page 424 with the exception of the ground.detector, the field switch with discharge resistance, and the synchronizing device. One ground detector is sufficient for a number of machines operated in parallel, and it is usually mounted on a bracket attached to one of the generator panels. The field switch is arranged to short-circuit the field winding of the alternator at, or just before, the instant of disconnecting the exciter from the field windings. This allows the current in the field winding to die away slowly. The opening of a field switch which is not provided with a resistance produces an excessively high electromotive force between the field terminals, which is likely to cause puncture of the insulation of the field windings.

The synchronizing device consists of the synchronizing bus bars connecting the various generator panels; the synchronizing lamps; a small transformer (the same one being used for the voltmeter) for stepping down the voltage to a suitable value for the synchronizing lamps; and connection plugs for connecting the secondary of the potential transformer through the synchronizing lamps to the synchronizing busses. Two types of connecting plugs are used, one of which reverses the connections made by the other. The complete connections of the synchronizing device for two single-phase machines is shown in Fig. 370.

Polyphase Switchboard. In case of polyphase machines, one phase only is connected to the synchronizing device, which is, there-
fore, the same for single-phase and for polyphase alternators. The operation of alternators in parallel is very common in modern central stations.*

The voltmeters and synchronizing device are always connected back of the main generator switch, that is, between the generator and the switch, for the reason that the voltage of the machine must be synchronized before the main switch is closed.

The connections of the two types of synchronizer plugs are shown in Fig. 370. Neither plug is used when one alternator only is operated. When another machine is to be put into operation,


Fig. 370. Complete Connections for Synchronizing Device for Two Single-Phase Machines
either type of plug is used to connect the synchronizer busses to any one of the machines already running, and the other type of plug is used for connecting the machine which is being synchronized to the synchronizer busses. Thus the synchronizer busses are oppositely connected to the two machines, and the synchronizing lamps are bright when the conditions are proper for closing the main switch. This is the common practice of the General Electric Company.

When more than two generators are operated in parallel, and one direct-current generator is used as an exciter for supplying current to the field windings of all the alternators, a separate switch-

[^40]board panel called the exciter panel, is usually installed. Upon this panel the exciter field rheostat and controlling devices are mounted. In this case exciter bus bars are led from the exciter panel to all the main generator panels.

Typical Switchboard Panel for Two-Phase Alternator Operated in Parallel with Other Two-Phase Machines. Fig. 371 shows front and side elevations, and complete wiring diagram (back view), of one of the General Electric Company's switchboard panels for a two-phase alternator which is to be operated in parallel with other two-phase machines.


Fig. 371. Front and Side View and Complete Wiring Diagram of Switchboard Panel for Two-Phase Alternator Operated in Parallel with Other Two-Phase Machines

The potential transformer which supplies reduced voltage to the voltmeter and to the synchronizer busses has its primary connected across one phase of the generator, and the synchronizing device is exactly the same as for the single-phase machines.

The equipment of this two-phase generator panel differs from that of the single-phase panel shown in Fig. 369, in the following particulars:
(a) The main generator switch is a four-pole switch for connecting the four leads from the generator to the four lines which pass out from the top of the board through four fuses.
(b) Two alternating-current ammeters are used, one for each phase

The "generator rheostat" is in the field circuit of the two-phase generator, and the "exciter rheostat" is in the field circuit of the exciter, exactly as in Fig. 369.

The equipment of apparatus for this panel is as follows:
$R=$ rheostat in the field circuit of the generator, "generator rheostat".
$a=$ ammeter in the field circuit of the generator, "field ammeter".
$V=$ voltmeter between the terminals of one phase of the generator (through the potential transformer).
$A_{1}$ and $A_{2}=$ ammeters, one for each phase of the generator.
$L=$ synchronizing lamp.
$r=$ rheostat in the field circuit of the exciter, "exciter rheostat".

## $f, f, f, j=$ fuses.

The four small circles above and below the words "synchronizing busses" are the points of the four-pole main switch.


Fig. 372. Complete Wiring Diagram for Two Two-Phase Generators Running in Parallel
Fig. 372 shows the complete connections of two two-phase generators for parallel running.

Typical Switchboard Panel for Three-Phase Generator Operated in Parallel with Other Three-Phase Machines. Fig. 373 shows front and side elevations, and complete wiring diagram (back view) of one of the General Electric Company's switchboard panels for a threephase alternator, which is to be operated in parallel with other three-phase machines. The potential transformer which supplies reduced voltage to the voltmeter and to the synchronizing busses
has its primary connected across one phase of the generator, and the synchronizing device is exactly the same as for single-phase machines.

The equipment of this three-phase panel differs from that of the single-phase and two-phase panels shown in Figs. 369 and 371 in that (a) the main generator switch is a three-pole switch for connecting the three generator leads to the three lines which pass out from the top of the board through three fuses; and (b) three alter-nating-current ammeters are used, one for each phase.

Fig. 373 also shows the following points, which however, are not characteristic of a thee-phase panel, but might be used on a two-phase panel.


Fig. 373. Front and Side View and Complete Wiring Diagram for a Three-Phase Generator Operated in Parallel with Other Three-P'hase Machines
(a) The two phases not connected to the potential transformer proper, are stepped-down to a reduced voltage by the use of one additional potential transformer T, Fig. 373.

As explained on page 260, two transformers suffice for stepping-down three phases to two phases. Both potential transformers have their secondaries connected to a "voltmeter plug switch", by means of which the voltmeter may be connected so as to indicate, at the will of the operator, the voltage of any one of the three phases.
(b) The lines which pass out at the top of the pancl in Fig. 373 are shown connected to the three main bus bars or rods.
(c) No exciter is shown in Fig. 373, and no exciter field rheostat, but the panel is arranged so that one large exciter may be used for all the generators. For this purpose exciter busses (two of them), connect the one large exciter to all the generator paneis.
(d) The generator field rheostat is geared to the hand wheel by means of sprocket wheels and chain on the back of the switchboard.

The equipment of apparatus for this panel is as follows:
$R=$ "generator field rheostat".
$a=$ ammeter (direct-current) in the field of the generator.
$V=$ voltmeter between the terminals of one phase of the generator (through potential transformer).
$p=$ voltmeter plug switch.
$A_{1}, A_{2}, A_{3}=$ ammeters for alternating currents, one in each phase of the generator.
$L=$ synchronizing lamp.
$S=$ main generator switch.
$f, f, f=$ fuses.
$T=$ additional potential transformer.
Feeder Panels. In large generating stations, the bus bars, to which the various generators are connected in parallel, lead from the generator panels to the feeder panels.


Fig. 374. Front Elevation of Typical Three-Phase The feeders are the separate and distinct circuits which receive current from the bus bars and transmit it outside the station to points more or less remote. Each pair of feeders (or set of three or four in polyphase distribution), as it comes into the station, is protected by lightning arresters. From the lightning arresters the feeders are brought to the feeder panels through fuse blocks, and through ammeters, and connected to bus bars by means of suitable switches.

When there are but few feeder circuits, the feeder switches are mounted on the generator panel as shown in Fig. 366.
Ground detectors are used primarily for detecting grounds on feeder circuits, and where many feeders are connected to the bus bars, tne ground detectors are mounted on the feeder panels. When there are but few feeder circuits the ground detector may be connected to the bus bars, in which case the ground detector may be mounted on a generator panel or on a feeder panel, if feeder panels are used.

It is frequently desirable to control the voltage supplied to a feeder circuit independently of the voltage between the bus bars.

For this purpose voltage or potential regulators are used, and these voltage regulators are either mounted upon the feeder panels or are controlled by levers or hand wheels which are mounted on the feeder panels.


Fig. 375. Front View of Standard High Voltage Switchboard for Central Stations, etc.

When the energy (watt-hours) delivered to a feeder circuit is to be measured, the integrating watt-hour meter is mounted on the feeder panel.


Fig. 376. Rear View of Switchboard Shown in Fig. 375
Circuit breakers, when used, are usually mounted upon the feeder panels and arranged to open the feeder switch when the current delivered to the feeder becomes excessive.

Fig. 377. Diagram of High Voltage Switchboard Showing System of Levers for Operating Switches by "Remote Control"

Fig. 374 shows a front elevation of a typical three-phase feeder nanel manufactured by the Crocker-Wheeler Company. The equipinent of this panel includes the following apparatus:

3 alternating-current ammeters, one for each line of the three-line feeder.
Necessary current transformers.
1 three-pole, single-throw, non-automatic oil switch, mounted back of the panel and operated by a lever on the front of the panel.

3 expulsion fuse blocks.
High=Voltage Panels. In alternating-current generating stations for long-distance transmission, the alternating currents are generated at a medium or low voltage, and are then stepped-up to 10,000 or to 100,000 volts or more for transmission. In such stations, the lowvoltage switches and controlling devices, including exciter switches and rheostats, are mounted on panels separate from the high-voltage switches and devices. Such stations have, therefore, low-voltage panels and high-voltage panels. The high-voltage panels differ from the low-voltage panels in having very much greater distances between the high-voltage parts, in order to avoid the danger of shortcircuit by sparking across through the air, and in having special forms of remote control switches.

Figs. 375 to 377 give views of a standard switchboard designed for central stations and industrial plants for voltages from 2,200 to 13,000 . They are given here to illustrate especially the sc-called "remote control" method of operating switches.

## SPECIAL SWITCHBOARD APPARATUS

Lincoln Synchronizer. When incandescent lamps are used as synchronism indicators in the starting of a synchronous motor, or in the paralleling of alternators, the pulsations of brightness indicate only the difference in frequency of the two machines, and it is in general impossible to tell from the behavior of the lamps which of the machines is running at the greater frequency.

The Lincoln synchronizer is a device for indicating positively the difference in the frequency of two alternators which are being adjusted into synchronism, and also for indicating positively the phase difference of the two machines at each instant.

The Lincoln synchronizer is a very small machine of which the iron parts are exactly like the field magnet and armature core
of a very small two-pole direct-current motor, except that both field magnet and armature core are laminated. The field magnet is excited with alternating cur-


Fig. 378. Lincoln Synchroscope rent by connecting the field winding to the terminals of the alternators $A$ and $B$, which are being synchronized. The armature of the synchronizer is wound with two coils and at right angles to each other. Each of these coils is independently connected through collecting rings to the terminals of alternator $B$. One coil has connected in series with it a non-inductive resistance, and the other coil has connected in series with it a large inductance. The armature of the small machine has attached to it a pointer which moves over a divided circle. When machines $A$ and $B$ have exactly the same frequency, this pointer is stationary and its reading on the circle indicates the phase difference between the two machines. When machine $A$ is


Fig. 379. Wiring Diagram for a Synchroscope for Single-Phase or One Phase of Polyphase Circuit
running at a slightly greater frequency than $B$, the pointer rotates in a certain direction indicating at each instant the changing phasedifference between $A$ and $B$. When machine $A$ is running at a
slightly lower frequency than $B$, the pointer rotates in the opposite direction, indicating at each instant the changing phase-difference between $A$ and $B$. The speed of the pointer in revolutions per second is in each case equal to the difference in frequency of machines $A$ and $B$, in cycles per second.

Fig. 378 is a view of the exterior of a Lincoln synchroscope, "type A," as made by the Westinghouse Electric Company. It has a 9 -inch dial and is intended to be mounted on the switchboard. Its coils are wound for 100 volts so that a voltage transformer is necessary for connecting it to the bus bars, and one voltage transformer for each machine to be synchronized.

Fig. 379 is a diagram of connections for the "type A" synchroscope for single-phase or one phase of polyphase circuits for voltages of 500 and over.

## CIRCUIT-INTERRUPTINO DEVICES

Devices whose sole function is the opening of an electric circuit may be classified into: fuses, switches, and circuit breakers. In the design of any circuit-interrupting device four general requirements are involved:
(a) To provide means for carrying the rated current of the device without excessive drop of voltage or heating, and also such overloads as will occur in practice.
(b) To insulate all live parts for the maximum voltage.
(c) To provide mechanical means for opening the circuit.
(d) To prevent or make harmless any arc that may form.

Circuit-interrupting devices may be automatic or non-automatic. In general, fuses are always automatic, switches always non-automatic, and circuit breakers either one or the other. In general, circuit breakers are so arranged that a spring or gravity tends to open them, and they are held closed by latches or toggle mechanisms It is this feature which makes the difference between a switch and a non-automatic circuit breaker.

Fuses. A fuse is the simplest and cheapest circuit-breaking device. It consists of a wire or link of fusible metal (usually of lead alloyed with zinc and tin) connected electrically in series with the circuit to be protected. When more than the predetermined current passes through it, the heat generated in it due to its resistance is sufficient to melt it, and the fuse is said to "blow", thus opening the circuit. The open or exposed type of wire- or link-fuse is rarely used
on account of its tendency to scatter molten metal and increase the fire hazard. Moreover, open link fuses cannot be accurately rated on account of the exposure and variable length of the fusible strip. It was to overcome these objections that the enclosed type of fuse was developed. The latter type of fuse* consists of a fusible wire or link enclosed within a fiber tube fitted with a fireproof material to exclude the air and suppress the arc formed when the fuse embedded in this material melts and opens the circuit. Suitable terminals are provided so that the fuse may be mounted in a fuse block. Enclosed fuses have been standardized into classes according to the voltage and ampere capacity, and are rated so that they will carry ten per cent overload indefinitely, and will open at twenty-five per cent overload.

A very important difference between the fuse and the circuit breaker is the matter of time element. Thus the over-load circuit breaker depends for its operation on the value of the current only, whereas the biowing of a fuse depends not only on the current but also on the time it flows. That is, the breaker will open immediately at any load in excess of that for which it is set, and will not operate at any smaller current, no matter how long it may continue. Standard fuses, on the other hand, will in time operate at an overload of only 25 per cent, and will open in a proportionately shorter time with greater over loads. Certain conditions, therefore, require the use of fuses, while others are better met by breakers, or by a combination of the two.

Air=Break Switches. Special styles of switches are required in many cases in alternating-current work on account of the excessively high voltages, especially when the switch has to handle large currents at high voltage.

Mention has been made on page 421 of the use of a marble slab between the blades and contact points of a double-pole switch to prevent the arcs, formed when the switch is opened, from flashing across and short-circuiting the points of the switch which are connected to the generator terminals. Such a switch with marble barrier is shown in Fig. 366.

Arcing is reduced to a minimum in every case by opening the switch very quickly, and all of the special alternating-current switches

[^41]described below, are arranged to open with a snap, even though the controlling lever is moved slowly.

Fig. 380 shows a common form of knife switch specially constructed to give a quick break. The switch blade is in two parts $a$ and $b$, to one of which, $a$, is attached the operating handle. The two parts $a$ and $b$ of the blade are connected by strong helical springs


Fig. 380. "Quick Break" Knife Switches
$s s$, one on each side of the blade. When the handle is pulled forward to open the switch, blade $a$ leaves clip $c$, while the springs are put in tension more and more until the switch blade $b$ is pulled from the clip $c$, with a snap, thus causing a sudden breaking of the circuit. Fig. 380 shows two single-pole quick-break switches. The one on the left, designed for 600 amperes, is provided with two massive tubular terminals $t$, into which the terminals of the main circuit are inserted and soldered. The switch on the right, designed to carry 3,600 amperes, is furnished with massive studs $d d$, which project through holes drilled in the switchboard to the back of the board.

Connection to these studs is made by clamping the terminals of the main circuit to them by means of the threaded nuts $n n$. On account of the large current to be carried, extra large contact surfaces are provided between switch blades and clips, as seen in the figure.

Damage to switch-blades and clips, due to arcing, is reduced to a minimum by providing carbon blocks which make a connection in parallel with the switch connection proper. When the switch is opened, the metal connection is broken first, and the carbon connections afterwards, so that the arcing always takes place between the carbon blocks.


Fig. 381. Half Section of Over-Load Type of Circuit Breaker-Closed
Circuit Breakers. In Fig. 381 there is shown in elevation and half section a plain overload type of circuit breaker for either director alternating-current circuits up to 600 volts. Fig. 382 is a general riew of the same single pole breaker as manufactured by the Roller Smith Company. Fig. 381 shows the breaker closed and Fig. 382, open. Referring to Fig. 381, $A$ is a rectangular magnet core journaled on the cylindrical shaft shown, and supported by it between two non-magnetic supporting frames, only one of which, $B$, is shown in the sectional view. To $A$ is secured one of tie terminals of the main
coil $C$, which is made up of a number of hard-rolled copper strips; and also the arm $D$. The other terminal of the laminated winding is secured to the lower stud $P$. Current entering through the stud $P$ passes through the main windings $C$ into the arm $D$, through the contact plate $E$ into the stationary copper brush $F$, and finally out through the upper stud $Q$. When the current through the breaker exceeds the predetermined value for which it is adjusted, the attrac. tion exerted by the core $A$ on the ends $K$ of the pivoted armature causes it to rise with increasing speed until the finger $M$, a part of the armature, strikes $R$. The roller $H$ is thus forced downward past the roller $G$, thus permitting the strong outward pressure of the brush $F$ and that of the coil $C$ to throw the arm outward and thus break the circuit, first between the brush fingers and the contact plate $E$, and finally between the carbons $S$ and $F^{\prime}$. It will be noted that the carbon block $F^{\prime}$ is mounted on a spring support and thus maintains its contact with $S$ after the main contacts between the copper brush $F$ and plate $E$ have been broken. This action effectually prevents arcing and pitting of the main contacts which would


Fig. 382. General View of Roller-Smith Single-Pole Breaker-Open soon wear out the breaker.

To reset the breaker, the handle which the act of opening has raised, is pulled down, thus forcing the roller $H$ up past the roller $G$, which compresses the brush $F$ and the coil $C$, by forcing the arm back into its initial position.

These breakers are also made with under-load, no-voltage, and shunt-trip attachments, and may be operated from any distant point if desired.

The same methods are available for suppressing the arc between the break points of a switch as for suppressing the arc which tends to maintain itself between the terminals of a fuse link when
the link fuses. Thus some of the older switches were designed to break the circuit between the end of a movable copper rod and a stationary copper socket, both of which are surrounded by a porcelain tube, thus utilizing the expulsion principle for suppressing the arc.

The most effective method, however, for suppressing the arc* on a high-voltage switch is to design the switch so as to open quickly under oil, and for very high voltages to provide for several simultaneous breaks in series.


Fig. 383. General Electric Oil Break Switch Mounted on Panel


Fig. 384. General Electric Oil Break Switch -Rear View-Switch Open

Oil=Break Switches. Air-break switches are not now considered reliable or safe for alternating-current circuits above about 600 volts, hence the adoption of oil-break switches for practically all alter-nating-current circuits of 440 volts and over is today almost universal.

Figs. 383 and 384 are views of a typical General Electric oilbreak switch, mounted on a panel. The six insulators which support

[^42]the stationary contacts and studs each consist of a single piece of porcelain, and can be easily detached from the frame when necessary. The contact fingers are flared, are of copper, and are fastened to the studs by heavy springs which insure good contact with the moving blades. The oil tanks are of sheet metal lined with maple, and are designed to be easily removable for inspection of switch contacts and oil. These oil switches are designed with operating mechanism mounted directly on switchboard panel and the oil switch proper mounted:


Fig. 385. Westinghouse Three-Pole Oil Circuit Breaker-"Remote Control"
(a) directly on panel;
(b) on pipe framework directly back of panel;
(c) on pipe framework remote from panel; or
(d) on flat surfaces or in cells remote from panel.

These switches are made single-throw with one, two, three, or four poles; and for voltages of 2,500 up to 15,000 , and with current capacities ranging from 100 to 300 amperes, respectively.

Any of these oil switches may have operating mechanisms which are either non-automatic, or automatic with one, two, or three overload-coil attachments. Fig. 383 shows front view of the switch closed, and Fig. 384 back view of the switch open.

Great improvement has been made in recent years in the design of large circuit breakers for protecting alternating-current circuits up to 110,000 volts and over. Fig. 385 shows a Westinghouse threepole oil circuit-breaker for 44,000 -volt circuits. Its capacity is 300 amperes. It is hand-operated and illustrates what is called "remotecontrol". This type of breaker may also be electrically operated


Fig. 386. "Type GA" Westinghouse Oil Circuit Breaker Removed from Tank Showing Open and Closed Positions
by means of suitable solenoids or coils energized by direct current under the control of the operator. They may also be made automatic in action by the addition of suitable relay attachments.

Fig. 386 shows a single-pole of the "type GA" Westinghouse oil circuit breaker removed from its tank, in both the open and closed positions. The upper or fixed contacts are firmly secured to the lower end of the leads, which are clamped by iron collars, which in
turn are bolted to the tank cover through which the leads pass. This upper contact consists of a circular piece of brass of greater area than the moving contact, which insures the entire surface of the latter bearing upon the stationary contact, the object being to eliminate the necessity of accurately centering the contacts one upon the other.

The lower or movable contacts are carried by a heavy metallic cross bar and consist of pieces of cylindrical brass rod backed by compression springs of sufficient strength to insure good contact and which also render the contacts self-aligning. Copper braid shunts are used to carry the current around the springs in order to prevent any deterioration by the passage of current through them.

It will thus be seen that the contact is of the simple "butt" type between two circular plane surfaces. The high voltage necessitates the unusually long, widely separated, and specially insulated "condenser" type of terminals shown in Fig. 386.

The general type of oil switch illustrated in Fig. 386 is today the standard for high-voltage circuits carrying large alternating currents, and is used extensively in many of the most recent high-voltage generating stations.

Fig. 387 is a general view of a General Electric three-pole oil


Fig. 387. General Electric Three-Pole Oil Switch Operated by Electric Motor switch, or rather three single-pole switches operated by an electric motor, which is controlled from the switchboard. Each singlepole switch in Fig. 387 is mounted in a separate brick compartment. The oil is contained in long metal cylinders in order to reduce the amount of oil to a minimum. The long brass connecting rods pass through holes in the porcelain bushings in the tops of the cylinders and the ends of these rods carry conical lugs which fit into spring sockets. These spring sockets are connected to rods which pass
out through porcelain bushings in the bottoms of the cylinders, and these lower rods are connected together in each compartment, thus making a double-break single-pole switch. The metal cylinders in


Fig. 388. Stillwell Single-Phase VoltageRegulator Complete Fig. 387 are entirely insulated from the switch points proper.

Feeder or Voltage Regulators. When several feeder circuits are supplied from the same bus bars, and when it is desired to control the voltage on each feeder circuit independently of the others, feeder regulators are required.

Single-phase feeder regulators are autotransformers with their primary coils connected across (that is, as a shunt to) the bus bars, and their secondaries connected in series with the feeder circuit. They are of three types, as follows:
(a) A type in which the secondary coil has many leads brought out to points on a dial switch, so that the number of active turns on the secondary coil may be changed at will, thus permitting the adjustment of the feeder voltage to any desired value.
(b) A type in which the primary and secondary coils are wound at right-angles to each other on the inner face of a laminated iron ring very much like the stator ring of an induction


Fig. 389. Dial of Stillwell Single-Phase Voltage Regulator motor. The magnetic flux, due to the primary coil, is made to pass in whole or in part through the secondary coil by turning a laminated core as explained below.
(c) Polyphase feeder-regulators usually consist of several single-phase regulators, one for each phase. The advantage of this arrangement is that the voltage of each phase may be controlled separately. A combined poly-phase feeder regulator is, however, sometimes used.
(a) Stillwell Regulator. Fig. 388 is a general view of a Stillwell single-phase voltage regulaThe dial switch alone is shown in

Fig. 389. The complete internal connections are shown in Fig. 390. The primary coil of the regulator is permanently connected across the bus bars (or generator terminals). One feeder wire passes out directly from the generator and the other passes through few, or many, turns of the secondary coil of the regulator, and thence to the line. The reversing switch $A_{1}$ serves to connect the feeder $f$ to one or the other terminal of the secondary coil, and the arm of the dial switch connects the line wire to any one of the taps which are brought out from the secondary coil. For one position of the reversing switch, the induced voltage in the secondary turns, which are connected in series with the feeder circuit, is added to the generator voltage, thus raising the feeder voltage. For the other position of the reversing switch, the induced voltage in the secondary turns, which


Fig. 390. Complete Internal Connections for Stillwell Voltage Regulator are in series with the feeder circuit, is subtracted from the generator voltage, thus reducing the feeder voltage.

When the arm of the dial switch touches two adjacent contact points (and it must be arranged to always touch one point before it leaves the other), the intervening turn (or turns) of the secondary coil of the regulator is short-circuited. To overcome this difficulty, the arm is made double, that is, two arms $A$ and $B$ move together side by side as shown in Fig. 391. These arms are shown connected


Fig. 391. Diagram of Special Form of Dial Switch for Stillwell Regulator to two contact points, say 15 and 16. C represents a special form of choke coil consisting of two windings on one iron core. These two windings are arranged so that equal currents flowing out from $A$ and $B$ circulate around the core in opposite directions, so that the
core is not magnetized, and the windings of $C$ have no choking action. When, however, the two fingers $A$ and $B$ touch adjacent points of the


Fig. 392. Diagram of Magnetic Voltage Regulator dial switch, the turns of wire $S$ on the regulator secondary tend to send a very large current out on $A$, say, and back on $B$. These oppositely flowing currents circulate around the core of $C$ in the same direction. These currents, therefore, magnetize the core, and the windings have in consequence a very considerable choking action, the effect being to choke down oppositely flowing currents in the fingers $A$ and $B$, and to allow currents in the same direction to flow freely through it.
(b) Magnetic Voltage Regulator. The type of voltage regulator mentioned under (b) is sometimes called the magnetic voltage regulator. A laminated iron ring $R R R R$, Fig. 392, has four large deep slots on its inner face in which the primary coil $P P$ and the secondary coil $S S$ are placed. A laminated core $C C$ mounted on a spindle is arranged to be turned into any desired position by means


Fig. 393. Magnetic Voltage Regulator in Case with_Cover Removed of a hand wheel. In the position indicated by full lines, the core carries the magnetic flux due to the primary coil in one direction through the secondary coil, and in the position indicated by dotted lines, the core carries the magnetic flux due to the primary coil in the other direction through the secondary coil. Therefore, when the core is moved slowly from position 1 to position 2, in the direction indicated by the arrows, the voltage induced by the secondary coil changes gradually from a full positive value to an equal negative value in its relation to the primary voltage. That is, when the core is in position 1 , the induced voltage in the
secondary coil has its greatest value of say, 100 volts which, if the coils are properly connected, is added to the bus-bar voltage $E$, giving a feeder voltage of $E+100$. When the core is midway between the two positions 1 and 2 , the induced voltage in the secondary coil is zero, and the feeder voltage is then equal to the bus-bar voltage $E$. When the core is in position 2 , the induced voltage in the secondary coil is


Fig. 394. General Electric Six-Phase Induction Regulator Complete in Case
again at a maximum of say, 100 volts, but in such a direction as to oppose the bus-bar voltage, so that the feeder voltage is $E-100$ volts.

Fig. 393 is a view of a magnetic voltage regulator with the covering of its containing case removed. The two coils, primary and secondary, at right angles to each other, are clearly shown with their leads passing out to the connection board which occupies the compartment on the back of the case in the figure. The hand wheel
for turning the iron core is also shown. A valuable feature of this type of regulator is that it produces a continuous variation of voltage, whereas the Stillwell regulator produces a step-by-step variation.
(c) Induction Regulator. The combination polyphase induction regulator is called the induction regulator from its similarity to the induction motor. The action of the induction regulator stated in simplest terms is as follows: A regular induction motor stator has its polyphase windings connected across the polyphase bus bars. This produces a rotating magnetism in the stator iron as explained on page 372 . This rotating magnetism rotates in synchronism with the generator or generators which are supplying current to the bus bars. Inside of this induction motor stator (or primary) is placed a polyphase armature which does not revolve, but it is mounted


Fig. 395. Laminated Core of the Stationary Member of an Induction Regulator
on a spindle so that it may be turned through an angle of $60^{\circ}$ or $90^{\circ}$ by means of a hand wheel and worm gear. The rotating stator magnetism induces polyphase electromotive forces in the windings of this polyphase armature; these polyphase electromotive forces are in synchronism with the electromotive forces between the bus bars, and the two (or more) windings of the polyphase armature are connected in series with the two (or more) feeder circuits (constituting of course one set of polyphase feeders) which are to be regulated. The electromotive forces in the stationary armature windings may be in phase with the bus-bar electromotive forces, in which case the regulator raises the voltage by the greatest amount of which it is
capable. By turning the stationary armature by means of the hand wheel and worm gear, the phase difference between the bus-bar voltages and the voltages induced in the stationary armature windings may be gradually changed from coincidence of phase to opposition of phase, during which time the boosting effect of the regulator will gradually drop to zero, become negative, and reach its greatest negative value when opposition of phase is reached. Thus, if the electromotive force induced in each armature winding of the regulator is 100 volts, and if the bus-bar voltage is 1,000 (each phase), then the voltage between the feeders can be varied from 900 volts to 1,100 volts by means of the regulator.

Fig. 394 is a general view of a six-phase induction regulator in its containing case, manufactured by the General Electric Company. This particular machine has the primary windings on the movable member, whereas the armature windings are on the stationary member. It is used for controlling the alternating-current voltages applied to the collector rings of a six-phase rotary converter. The figure shows a small directcurrent motor mounted on the regulator case for turning the movable member of the regulator, this motor being also controlled


Fig. 396. Movable Core of the Primary Member of an Induction Regulator by the hand wheel shown in the figure.

Fig. 395 shows the laminated iron core of the stationary member of an induction regulator, the stampings of which are clamped in a cast-iron shell. Fig. 396 shows the movable core.

Ratings of Voltage Regulators. It was shown, page, 253 that the total amount of power delivered to service mains at a slightly increased voltage produced by an autotransformer is very much greater than the power actually transformed from the primary to the secondary of the transformer. In fact, the power actually transformed is equal to the increase (or decrease) of voltage multiplied by the total current delivered, and the rating of the autotransformer (which determines its size) is based upon the power transformed.

For example, a voltage regulator is to be used for raising the voltage of 2,000 -volt bus bars to a maximum of 2,100 volts, and the


Fig. 397. Diagram of Connections for Voltmeter Compensator
maximum current to be handled is 100 amperes. In this case the transformer rating of the regulator is 100 amperes at 100 volts (or 10 kilowatts), whereas the total power to be delivered to the feeders is, at its maximum, 2,100 volts $\times 100$ amperes or 210 kilowatts.

Since a voltage regulator transforms only a small fraction of the power delivered to the feeders which it controls, the losses of power in the regulator are very small indeed. Thus the 10 -kilowatt regulator might have a total loss of 300 watts, which is 3 per cent of the power actually transformed by the regulator, and only oneseventh of one per cent of the total power delivered to the feeders.

Voltmeter Compensator. The voltmeter compensator is a device by means of which the voltmeter on the switchboard in a station is made to indicate the voltage between the transmission lines at some remote feeding center or receiving station. The essential principles of this instrument are made clear from Fig. 397. An alternatingcurrent generator $G$ delivers alternating current at voltage $E_{0}$ be-


Figs. 398 (left) and 399 (right). Vector Diagrams Showing E.M.F. Relations in the Line and in the Compensator
tween its terminals. This current is transmitted over a line of which the resistance is $R$ (including $r$ ) and the reactance is $X$ (including
$x$ ), and the voltage at the receiver is $E$. The line loss of electromotive force is $X I$ due to reactance, and $I R$ due to resistance, as


Fig. 400. Wiring Diagram for Voltmeter Compensator as Used in Practice in High Voltage Circuits
explained on page 66. The relation between $E_{0}, E, I R$, and $X I$ is shown in Fig. 398. T, Fig. 397, is a transformer which supplies to the voltmeter $V$ a voltage exactly in phase with $E_{0}$, and, say, one-


Fig. 401. Diagram of Circuits and Apparatus in Connection with Westinghouse Compensated Voltmeter
tenth as great. Let $x$ be a reactance one-tenth as great as $X$, and $r$ a resistance one-tenth as great as $R$. Then the voltage between
the points $a$ and $b$ consists of two parts, $r I$ and $x I$, which are in phase with, and one-tenth as great as, $I R$ and $X I$, respectively. Therefore $r I$ and $x I$ subtracted from one-tenth $E_{0}$ give a voltage which is exactly equal to, and in phase with, one-tenth $E$, as shown in Fig. 399 , so that the voltmeter being acted upon by (one-tenth $E-r I-x I$ ) gives a reading which multiplied by 10 is equal to $E$.

In practice, it is not desirable to connect the voltmeter wires to the high-voltage mains, and the essential features of the arrangement shown in Fig. 397 are realized by introducing $r$ and $x$ in series with the secondary of a current transformer $T^{\prime \prime}$, as shown in Fig. 400. More or less of the resistance $r$ and of the reactance $x$ may be included in the voltmeter circuit by means of two dial switches of which the arms are represented by $d d$, Fig. 400. The object of these dial switches is to enable a given compensator to be adjusted to any given transmission line.

Fig. 401 shows all of the apparatus used in connection with the Westinghouse compensated voltmeter; the lines $l l$ are shown at the left, $T$ is the step-down potential transformer, $T^{\prime \prime}$ is the series transformer, $d d$ is the case containing the resistance $r$ and the reactance $x$, and upon which the dial switches are mounted, $V$ is the voltmeter, and $V R$ is the resistance in series with the voltmeter.

## LIGHTNING ARRESTERS

Effects of Lightning. When a lightning stroke occurs in the neighborhood of a transmission line, a sudden rush of electric current takes place over the line due to one or more of the following causes:
(a). Electric charge accumulated on the line is suddenly released and tends to flow to earth.
(b) The magnetic action of the lightning discharge induces a sudden rush of current in the line.
(c) When the lightning discharge actually strikes the line, an enormous rush of current takes place over the line and to earth.

When the sudden rush of current which accompanies a lightning discharge encounters a portion of the circuit which has considerable inductance, very great electromotive forces are created, in the same way that enormous mechanical forces are created when a moving body strikes a heavy obstacle, and the insulation of the circuit, be it air or solid insulation, is likely to be broken down, or
punctured, giving a short path to the earth for the rush of current. Thus, a sudden rush of current coming into a station over a transmission line, encounters the highly inductive windings of wire of a transformer, dynamo, or other apparatus. The electrical inertia (inductance) of the windings dams up the rush of current, as it were, and the current rush is almost sure to break through the insulation at the very entrance to the winding, passing from the copper wire over to any metal which is connected more or less thoroughly to earth, resulting in the coils or apparatus being burned out or badly damaged.

A lightning arrester is a device for shielding a transformer, dynamo, or other piece of apparatus from the rushes of current which come into a station on an overhead line during a thunderstorm and from disturbances due to the static unbalancing of the electrical circuits.

The lightning arrester consists of:
(a) An inductance coil, or choke coil as it is called, for damming up more or less completely the rush of current before it reaches the dynamo or other apparatus.
(b) A weak place, specially arranged in the insulation of the line, that is, a spark gap, just in front of the choke coil through which the dammed up rush of current may break, and flow


Fig. 402. Choke Coil Mounted on Marble Base harmlessly to earth.
(c) A conducting path to earth as straight and direct as possible, and a good earth connection. Turns and bends in the conducting wire connected to earth are to be avoided, inasmuch as they introduce inductance which tends to check the quick flow of current to earth.
(d) A device for extinguishing the electric arc which may be maintained across the spark gap by the regular line current itself, after the rush of current from the lightning discharge has passed and gone.

The choke coil of a lightning arrester must be very highly insulated. Two types of choke coil are commonly used. Fig. 402 shows a cylindrical choke coil mounted on a marble base, and consisting of three layers of large and very highly insulated wire. The
terminals of the coil are at its two ends. Fig. 403 shows a choke coil consisting of an insulated copper strip wound like a roll of tape. One terminal of this choke coil is on the


Fig. 403. Choke Coil made of Insulated Copper Strip outer edge and the other is at the center. The figure shows the method of supporting the coil on a special insulator carried on a bracket. Iron cores do not add to the damming action of a choke coil in the case of the excessively quick rushes of current which accompany lightning discharges, for the reason that the iron does not have time to become magnetized. Therefore, lightning arrester choke coils are always made without iron cores.

Multi=Gap Non=Arcing Arrester. The spark gap of alternating-current lightning arresters is usually made between blocks of metal containing zinc or cadmium. A. J. Wurtz made the discovery that it takes a very high voltage to maintain an alter-nating-current arc between metal electrodes which contain zinc or cadmium. The behavior of these alloys is somewhat analogous to that of the mercury-vapor rectifier described on page 327 in that reversal of the cur-



Fig. 404. Westinghouse Multi-Gap Lightning Arrester rent flow which makes the mercury an anode requires a greatly increased voltage. If this is not supplied the flow could not be maintained. Thus an alternating electromotive force of a frequency of 60 cycles per second, and 500 volts (effective value), cannot maintain an arc across a $\frac{3}{64}$-inch air gap between massive blocks of brass. This non-arcing property of certain alloys is made use of in the multi-gap arrester of the Westinghouse Electric Company, shown in Fig. 404. It consists of seven independent knurled brass cylinders supported by two overhanging
porcelain blocks forming a unit which is mounted in a weatherproof cast-iron case. The two end cylinders are connected, respectively, to the two line wires and the center cylinder is connected to ground. This arrester may also be used single pole, one for each side of a two-wire circuit. Connections for single-phase and for three-phase circuits are given in Fig. 405. For two-phase circuits each phase is to be connected as in the single-phase diagram.

This type of arrester is suited for use within a radius of three miles from the source of power on systems of 200 -kw. capacity or less, and up to 400 kilowatts when connected more than three miles away from the power source.


Fig. 405. Lightning Arrester Connections for Single-Phase and Three-Phase Circuits
This type of arrester is not recommended for protecting circuits liable to low power factors, nor for 25 -cycle circuits, for experience has shown that under these conditions the arcing cannot be suppressed.

Multi=Path Arrester. The multi-path arrester of the Westingbouse Company is single pole and adapted to either alternating- or direct-current circuits of any voltage up to 1,000 . It can be mounted upon a pole. It consists of a spark gap in series with a high resistance block of carborundum. This block offers a very high resistance under normal conditions and moderate voltages, but its resistance to a static discharge is very low, and it readily conducts a lightning
discharge to ground. After the discharge has passed, it resumes its normal high resistance, thus preventing the line voltage from maintaining an arc. Fig. 406 is a view of the arrester in the lower half of its iron casing, the upper half being shown removed.

Electrolytic=Cell Lightning Arrester. This type of arrester, which has proved highly effective, is widely used. A single cell consists of two aluminum plates and an electrolyte, such as ammonium phosphate, which form a condenser that will stand about 340 volts alternating before breaking down. Up to this critical value of the voltage, which varies somewhat with the electrolyte used, a very small current passes through the cell, but as soon as the critical voltage


Fig. 406. Westinghnuse Multi-Path Lightning Arrester -Upper Half of Case Removed
is exceeded, the current increascs very rapidly with even a slight increase in the voltage. As soon, however, as the voltage drops below the critical value, the resistance of the cell increases greatly.

In applying this interesting property of the aluminum cell to the lightning arrester, a large number of cells are made up in series and mounted in standard size units to be connected in series, the number of units chosen depending upon the voltage of the lines to be protected, allowing on the average about 300 volts per cell.

Fig. 407 shows a cross-section of an aluminum cell lightning arrester as made by the General Electric Company. It consists of
a series of concentric inverted cones of aluminum placed one above the other with a vertical spacing of about 0.3 inch. The electrolyte is poured into the cones and partly fills the space between adjacent ones. The pile of cones with the electrolyte between them is then immersed in a tank of oil, which helps to insulate cones from each other except for the electrolyte, and also prevents evaporation of the solution. A cylinder of insulating fiber concentric with the stack of cones is placed between the latter and the steel tank. This arrangement assists the free circulation of the oil and increases the insulation between tank and aluminum cones.

A stack of cones is used for each phase, and up to 7,250 volts all the stacks are placed in a single tank. Where the line voltage exceeds 7,250 , each stack of cones is placed in a separate tank. The connection of the stacks to the line wires through spark gaps and to the ground is different, depending upon whether the circuits have their neutral point purposely grounded or not, and upon the line voltage.

The top cone of each stack is not directly connected between a line wire and the ground, because at normal line voltage some current would flow continuously through the aluminum cells, which


Fig. 407. Cross-Section of Aluminum Cell Lightning Arrester would heat them and evaporate the electrolyte. To prevent this flow of current, the aluminum stacks are in practice connected to the line wires through spark gaps adjusted for a sparking voltage just above that of the normal line voltage. Under normal conditions no current passes from the line wires through the aluminum cells, but a lightning discharge with its accompanying high voltage easily jumps the spark gap and passes through the aluminum cells to ground. As soon, however, as the discharge is over, the resistance or the spark gap in series with the aluminum plates is too large to permit the line voltage to maintain an arc.

Fig. 408 is a view showing a typical installation of an aluminum lightning arrester for a 6,600-volt, three-phase, non-grounded neutral
circuit. As shown the three stacks of cones, one for each phase, are installed in one tank which is placed on a rack insulated from the ground. Two of the stacks are connected directly to two of the


Fig. 408. Typical Installation for Aluminum Lightning Arrester on Three-Phase Non-Grounded Neutral Circuit
three line wires, each through a "horn-gap"; the third is connected to the remaining line wire through a "transfer device" or rotating switch, and a horn gap. The fourth stack is connected to the ground through the transfer device, which may be explained' as follows:

When the stacks of aluminum cones have been disconnected from the line wires for a certain length of time, the insulating film
on the cones deteriorates, and experience has shown that in order to keep the aluminum cells in good working order, they must be connected to the line wires and charged at certain definite intervals. In some cases this type of arrester must be charged daily, and in others weekly. The charging process consists simply of short-circuiting the spark gaps between the stacks and the line wires for a few moments. The transfer device provides a means of interchanging the ground stack with one of the line stacks during the charging operation so that the insulating films of all the cells may be reformed to an equal extent. For arresters up to 27,000 volts, the device is mounted with three insulators on the pipe frame work and is operated by a hand-wheel, as shown in Fig. 408.

Combination of a Condenser with a Choke Coil. The choking action of a coil of wire is due to its electrical inertia or inductance,

and the shielding of the apparatus behind a choke coil from severe electrical strains is precisely analogous to the following mechanical arrangement:

A wall is shielded from the severe strains due to a hammer blow by allowing the hammer to strike against a very heavy ball of iron which rests against the wall. The completeness with which this heavy iron ball shields the wall depends upon the interposing of an elastic substance between the ball and the wall. The ball alone cannot shield the wall unless the wall itself is slightly elastic.

In the same way the completeness with which a choke coil shields the dynamo and other apparatus depends upon the presence of some electro-elasticity*, or capacity in the circuit behind the choke coil. In some cases the circuit behind the choke coil has suf-
ficient capacity for this purpose; it is always best, however, to connect a condenser behind the choke coil as shown in Fig. 409.

The choke coil and condenser are immersed in an oil tank when installed. From Fig. 409 it is seen that three wires $b b b$ connect to choke coil and condenser. This combination of choke coil and condenser is called a static interrupter by the Westinghouse Company.

## Instructions for Installing Lightning Arresters

Location. As regards the location of lightning arresters, electric plants may be divided into two groups:
(a) Plants in which the individual pieces of apparatus such as transformers, motors, arc lights, etc., are many in number and widely scattered. In these cases lightning arresters should be located for the purpose of protecting the whole line. They should be located at a number of points, more numerous on the parts of the line particularly exposed, and fewer in number on the parts that are naturally protected, especially those parts shielded by tall buildings or numerous trees. No definite statement can be made as to the number of arresters needed per mile, as the requirements of different cases vary widely. Under average conditions no point of the circuit should be more than 1,000 feet from an arrester. It is not usual to find distributed apparatus on circuits of over 2,500 volts, but when such cases occur, a lightning arrester should be placed as near as possible to each piece of apparatus.
(b) Plants in which the apparatus is located at a few definite points in the system, as in a high-voltage transmission line. In such cases the arresters should in general be located to protect especially those points where apparatus is situated, that is, should be placed with the object of protecting the apparatus rather than the line as a whole.

The lightning arrester should always be so connected that in passing from the line to the apparatus the arrester is reached first, the choke coil second, and the condenser, if one is used, third.

Insulation. A lightning arrester is naturally exposed to severe voltage strains and, therefore, all active parts must be well insulated. On arresters for low voltages it is not a difficult matter to secure proper insulation, as the construction of the arrester itself affords protection. On high-voltage arresters, however, proper insulation is a more difficult matter. All arresters are marble or porcelain mounted, the marble or porcelain serving as an insulating support for the arrester. On circuits exceeding 6,000 volts, to obtain further insulation, the marble or porcelain bases should be mounted on wooden supports, well dried and shellaced. For 12,000 volts and above, the bases or panels should receive additional insulation in the form of porcelain or glass insulators used as supports.

Two high-voltage arresters attached to different line wires should not be placed side by side without either a barrier or a considerable space between them.

Grounds. Too much importance cannot be attached to the making of proper ground connections which should be as short and straight as pos-
sible. A poor ground connection will render ineffective every effort made with choke coils and lightning arresters to divert static electricity into the earth. It is important, therefore, to not only construct a good ground connection, but to maintain it so.

## Station Ground for Lightning Arresters. The following practice

 is recommended by the General Electric Company:The best method of making a station ground for lightning arresters is to drive a number of 1 -inch iron pipes six or eight feet into the earth surrounding the station. A good way is to first drive a somewhat larger pipe and then insert the smaller pipe, filling the space with salt water. A quantity of salt should be placed around each pipe at the surface of the ground.

These pipes should all be connected together and to the arresters by means of a copper wire, (not less than No. 2 B.\&S.) or, preferably, by a thin copper strip. It is advisable also to connect these pipes to the iron framework of the station, and to any water mains, metal flumes, or trolley rails which are available. Where plates are placed in streams of running water, they should be buried in the mud along the bank in preference to being laid in the stream. Streams with rocky bottoms are to be avoided.

Whenever plates are placed at any distance from the arrester, it is necessary also to drive a pipe into the earth directly beneath the arrester, thus making the ground connection as short as possible. Earth plates at a distance cannot be depended upon. Long ground wires in a station are not dependable unless a lead is carried to the multiple grounding pipes installed as described. As it is advisable occasionally to examine the underground connections to see that they are in proper condition, it is well to keep on file exact plans of location of ground plates, ground wires, and pipes, with a brief description, so that data can be readily referred to.

From time to time the resistance of these ground connections should be measured to determine their condition. This is very easily done when pipe grounds are installed, as the resistance of one pipe can be accurately determined when three or more pipes are used. For example, if there are three pipes $X, Y$, and $Z$, and the resistance of $X+Y=20$ ohms, resistance of $X+Z$ $=15 \mathrm{ohms}$, and the resistance of $Y+Z=20$ ohms, then by solving the equations the resistances of $X, Y$, and $Z$ may be obtained. Thus, eliminating $X$ from the first two equations we have

$$
Y-Z=5
$$

Combining this with the third equation above and eliminating $Z$ we have

$$
\begin{gathered}
2 Y=25 \\
Y=12.5 \text { ohms }
\end{gathered}
$$

Substituting we have

$$
Z=7.5 \mathrm{ohms} \quad X=7.5 \mathrm{ohms}
$$

The resistance of a single pipe ground in good condition has an average value of about 15 ohms . A less exact method of keeping account of the condition of the earth connections is to divide the earth pipes into two groups and connect each group to the 110 -volt lighting circuit with an ammeter in series. If there is a flow of about 20 amperes the conditions are satisfactory, provided the earth pipes are properly distributed around the station.

## APPENDIX

## PARALLEL OPERATING OF ALTERNATORS

Necessary Conditions. In the parallel or multiple operation of alternators, it is necessary that they be similar in three respects in order to insure their working together properly when connected in parallel,* viz, frequency, phase, and voltage.
(a) Frequency. Two generators are of the same frequency when the numbers of alternations or reversals of their electromotive forces in a given time are equal. This requirement is fulfilled when the product of the number of poles by the revolutions per minute is the same for each machine. The frequency of a generator, being dependent upon its speed, may be controlled by the regulation of the speed of its prime mover.
(b) Phase. Two generators are in phase when the positions of their armatures with respect to their field poles are the same, i.e., when similar armature coils are opposite positive field poles at the same instants. When this condition exists the electromotive forces of the machines are both positive at the same time, and their maximum values occur at the same instant; the electromotive forces are said to be coincident in phase.
(c) Voltage. Two generators are of the sume voltage when the pressure measured across the armature terminals is the same for each machine. The voltage for a given speed being dependent upon the field strength may be controlled by means of the rheostat in the field circuit.

Determination of Relative Frequency and Phase Coincidence. If two similar generators are running at exactly the same speed their difference in phase remains constant. This condition, however, does not exist in practice unless the armatures are rigidly connected, as the inevitable fluctuation in engine and water-wheel speeds and in belt slippage causes the position of the armatures with reference to their field poles to be continually changing, and consequently the difference between the phases to be likewise changing. As generators should not be thrown in parallel excepting when their frequencies are practically the same, and at the time their phases

[^43]are in exact coincidence, or nearly so, it is essential to have an accurate means of determining when these conditions exist.

The principle of the most common method of determining when generators are of the same frequency and are coincident in phase, is illustrated in Fig. 410. $A$ and $B$ represent two singlephase generators, the leads of which are connected at the switch $C$, through two series of incandescent lamps $D$ and $E$. It is evident that as the relative positions of the phases of the electromotive forces change from that of exact coincidence to that of exact opposition, the flow of current through the lamps varies from a minimum when the machines oppose each other in forcing current through the lamps as shown by the arrows in Fig. 410, to a maximum when the machines help each other in forcing current through the lamps.* If the electromotive forces of the two machines are exactly equal and in phase, the current through the lamps will be zero, and as the difference in phase increases, the lamps will light up and will increase in brilliancy until the maximum is reached; when the phases are in exact opposition. From this condition they will decrease in brilliancy until completely dark, indicating that the machines are again in phase. The rate of pulsations of the lamps depends upon the difference in frequency, i.e., upon the difference in the speeds of the machines, and by adjustment of the governors of the engine or water-wheel, or the tension of the belt, the rate can generally be reduced to as low as one pulsation in ten seconds, which affords ample time for throwing the switch connecting the generators in parallel.


Fig. 410. Wiring Diagram Showing Method of Determining Coincidence in Phase and Frequency of Iwo Generators

Synchronizer. When the phases of two generators coincide, the machines are said to be "in phase," "in step," or "in synchronism'". The apparatus used for determining when generators are in phase is called a synchronizer. In Fig. 410 the lamps constitute the synchronizer. While a series of lamps, alone, may be used for syn-

[^44]chronizing machines of very low voltage, it is not safe or practical to use this method for machines of high voltage. The most common arrangement for synchronizing alternators, in general, is illustrated in the diagram, Fig. 411. $A$ and $B$ represent two single-phase generators with switches in the main leads. There are two transformers, the primaries of which are connected across the main leads of $A$ and $B$, respectively, the secondaries being connected in series through the lamps $E$. Now, if the transformers are connected similarly in the two circuits, as shown in the diagram, then when the generators $A$ and $B$ are in phase, the electromotive forces in the secondaries will be in phase and no current will fiow through the lamps, but when the generators are out of phase, the electromotive forces in the secondary circuits will be out of phase also, and current will flow through the lamps. The amount of this current and the resultant brilliancy of the lamps depends on the difference in phase. If the connections of either the primary or the


Fig. 411. Wiring Diagram Showing Method of Determining Phase Relation by Synchronizer secondary of either transformer be now reversed from those shown in Fig. 411, the indications of the lamps will be reversed; that is, when the generators are in phase the lamps will burn at maximum brilliancy, and vice versa. It is the common practice of the Westinghouse Company to arrange the transformer connections so that the lamps shall be dark when the generators are in phase.

In order to determine whether the synchronizer lamps will be bright or dark for a given connection of transformers when the generators are coincident in phase, remove the main fuses or raise collector brushes from one machine, and throw in the main switches with the other generator at full voltage. Since both primaries are now connected through the switches to one machine, the lamps will be in the same condition as when the main or paralleling switches
are open and both generators are coincident in phase. If the lamps burn brightly and it is desired that they be dark for an indication of phase coincidence, the connections of one of the primaries or of one of the secondaries of the transformers should be reversed.

The lamps which are used with the synchronizer should be adapted for the highest voltage which they will receive. Thus, if they are placed upon the secondaries of two 100 -volt transformers, there should be two 100 -volt lamps or four 50 -volt lamps in series. If two 200 -volt machines have the lamps applied directly without transformers it will be necessary to use four 100 -volt lamps, or their equivalent.

Phasing=out Three=Phase Circuits.* Before connecting the leads from a polyphase generator, which is to operate in parallel with others, to the generator switch, the circuits must be phased out. That is, the leads must be so arranged that each lead from the generator will, when the generator switch is thrown, connect to the corresponding lead of the other generator. If this is not arranged considerable damage may be done owing to an interchange of current when the two machines are paralleled. After once phasing out, it is necessary to synchronize only one phase of the machine with the corresponding phase of the other machine.

Connections for phasing out three-phase circuits are shown in Fig. 412. If voltage transformers are not used, the sum of the voltages of the lamps in each line should be approximately the same as the voltage of the circuits. On 440 -volt circuits, two 220 -volt or four 110 -volt lamps should be used in each phasing-out lead.

To phase out, run the two machines at about synchronous speed. If the lamps do not all become bright and dark together, interchange any two of the main leads on one side of the switch, leaving the lamps connected to the same switch terminals, after: which the lamps should all fluctuate together and the connections; are correct. The machines are in phase when all the lamps are, dark,

Synchronizing Connections for Three:Phase Generators. The synchronizing connections for three-phase generators are.

[^45]shown in Fig. 413. A synchronizing plug may be used instead of the single-pole synchronizing switch shown. The illustration indicates the connections used where machines are to be synchronized to a bus. Where only two machines are to be synchronized, the connections are the same as shown in Fig. 413 except that the bus transformer and the corresponding lamp are omitted and one plug is required instead of two.

Synchronizing Dark or Light. Synchronizing dark appears to be the preferable method. All the connections shown are for synchronizing dark. When the lamps are dark the machines are in phase and it is necessary to close the switch when the


Fig. 412. Connections for Phasing Out Threc-Phase Circuits
pulsation is the slowest obtainable or ceases altogether, that is, at or just before the middle of the longest dark period.

Should a filament break, the synchronizing lamps would remain dark and thus apparently indicate synchronism and possibly cause an accident. Therefore it is considered desirable by some to reverse the synchronizing circuit connections and thereby synchronize light. Synchronizing light eliminates the danger due to the breaking of a filament, but has the disadvantage that the time of greatest brilliancy is difficult of determination. The light period is relatively long compared with the dark period, so that synchronizing light is usually considered the more difficult;
and were it not that with the synchronizing light method the danger due to filament breakage is eliminated, the method would never be used. The probability of a filament breaking just at the time of approaching synchronism and when the machines are not in phase is remote. If it occurs at any other time in the operation it will be noticed. As a protection against accidents due to breakage, two synchronizing lamps should always be placed in multiple.

Number of Lamps to Use in a Group to Indicate Synchronism. Thé number of lamps to use in a group to indicate synchronism is determined by the voltage of the generators. With highvoltage circuits it is not feasible to use a sufficient number of


Fig. 413. Connections for Synchronizing Three-Phase Generators
lamps, so a transformer that has a voltage sufficient for a 110volt lamp is employed. (See Figs. 412 and 413.) The greatest voltage impressed on the lamps is double that of the voltage transformers or generators. Thus the maximum voltage on the lamps, where two 220 -volt generators are being synchronized, is 440 -volts. The dark period may be shortened by impressing on the lamps a voltage higher than normal. For two 220 -volt machines, for example, three 110 -volt lamps might be used.

Synchroscopes. Synchroscopes are instruments which indicate the difference in phase between two electromotive forces at each instant. They show whether the machine to be synchronized is running fast or slow and indicate the exact instant when the machines are in synchronism.

Rate of Pulsation and Size of Pulley. The difference in speed between two machines may be determined by the rate of pulsation of the synchronizing lamps. It is sometimes convenient to know this difference in speed, especially when two generators are belt-driven from the same shaft. If the speeds are not equal, it may be necessary to turn off one of the pulleys in order to make them equal. One pulsation of the lamps, i.e., the interval between two consecutive occurrences of maximum brilliancy, indicates a gain of one cycle or two alternations of one machine over the other. Thus, if there is one pulsation of brightness per minute, and the number of alternations is 7,200 per minute, then one machine gives 7,202 alternations, while the other gives 7,200 . If the number of pulsations of brightness is 36 per minute, then one machine gives 7,272 alternations, while the other gives 7,200 alternations, and the first machine is, therefore, running 1 per cent faster than the second machine. In order to determine which machine is running the faster, the load may be thrown upon one machine, or its belt may be slackened so as to decrease its speed. If this be done to the machine which attempts to run too fast, the pulsations will become less rapid; while if it be done to the machine which is running slower, the pulsations will become more rapid. If one machine is running 1 per cent faster than the other, it will be necessary to reduce the diameter of the pulley of the other (slower) machine by 1 per cent.

The thickness of the belt, the tightness of the belt, the slippage (dependent upon the kind of belt, the condition of the surfaces of the pulley and belt, and the load) are all factors which affect the speed and must all be kept in mind.

Directions for Connecting One Alternator in Parallel with Another Alternator.
(1) Frequency. The speed of the new machine which is to be connected in parallel must be made such as to give the same frequency as the one already running. If the latter is carrying a load, it may be necessary to reduce the speed of the unloaded machine below that at which it tends to run with no load, by adjusting the engine valve, or the water wheel gate or nozzle, or the belt slippage, in order to secure the proper speed.

The adjustment of the engine should preferably not be by
throttling, as the governor is liable to "hunt" when the throttle is opened. It is desirable to be able to adjust the governor for changing the speed while the engine is running.
(2) Voltage. The field excitation of the new machine should be adjusted so that its voltage is the same as that on the bus bars, the measurement being made by a voltmeter.
(3) Phase Coincidence. Synchronizer lamps indicate by their slow rate of pulsation that the machines are of practically equal frequency. When the synchronizer lamps indicate the proper phase relation, i.e., phase coincidence (preferably when the lamps are dark), all is ready for closing the switch.
(4) Main Switch. Close the main switch a little too soon (when the machines are approaching the proper position) rather than too late (when they are receding from it). If the switch is operated by compressed air or for any other reason does not close the instant the handle is operated, due allowance must be made for the interval.
(5) Equalizing Switch. If the generators are composite wound, close the equalizer switch.
(6) Adjustment of Load. Adjustment may now be made by means of the governor or otherwise so that each machine receives its proper share of load.
(7) Adjustment of Field Currents. The field currents of the several generators should be properly adjusted to eliminate cross currents between the armatures and maintain the proper voltage on the bus bars.

Directions for Cutting Out An Alternator Which Is Running in Parallel with One or More Alternators.
(1) Preferably cut down the driving power until it is just about sufficient to run the generator at zero load. This will automatically reduce the load on the generator.
(2) Adjust the resistance in the field winding of the machine which is to be cut out until its armature current is a minimum.
(3) Open the main switch, then the equalizer switch.

It is usually sufficient, however, to simply disconnect a machine from the bus bars, thereby throwing all the load suddenly on the remaining machines, without having made any special adjustments of the load or the field current. The objec-
tion to this method is that it may cause serious hunting of the remaining machines.

The field circuit of the generator to be disconnected from the bus bars must not be opened before the main switch has been opened; for if the field were opened first, a heavy current would flow between the armatures.

## INDEX

The page numbers of this volume will be found at the bottom of the pages;
the numbers at the top refer only to the section.

Page

## A

A.C. circuit, fundamental equations of 51
Air-break switches 480
All-day efficiency 274
Alternating current, advantages
and disadvantages of 21
a.c. machines 23
high e.m.f.'s, transformation of 22
Alternating-current machinery 11-514
alternating into direct current, conversion of 335
alternating electromotive force
and currents
alternators 81
appendix 506
armature windings 119
commercial types of machines 129
economy factors in alternators 163
induction motor 391
introduction 18
measurement of power 112
motor-generator 382
rotary converter 343
switchboard and station appli-
ances 464
synchronous motors 213
transformer 241
Alternating into direct current,
conversion of
335,382
aluminum valve rectifier 336
mercury-vapor arc rectifier 338
motor-generator 382
rectifying commutator 335
rotary or synchronous converter 343
$\begin{array}{cc}\text { Alternating electromotive forces } \\ \text { and currents } & 15,18\end{array}$
Note.-For page numbers see foot of pages.

Page
Alternating electromotive forces
and currents (continued)
a.c. circuit, fundamental equa-
tions of
advantages of a.c. currents for
transmission
15
alternating current, advantages
and disadvantages of
21
capacity 49
circuits in parallel $\quad 70$
circuits in series $\quad 69$
condenser, capacity of 50
condenser as compensator for
lagging current 66
d.c. and a.c. calculations 27
d.c. and a.c. problems, com-
parison of
electrical resonance 60
electromotive force, average and effective values of 31
$\begin{aligned} & \text { electromotive force losses in al- } \\ & \text { ternator }\end{aligned} 78$
electromotive force losses in
transmission lines
75
electromotive force variations 20
graphical representations of $\quad 29$
harmonic electromotive forces
and currents 33
inductance 44
maximum and effective values,
relation between 4
power, instantaneous and aver- 32 age 32
simple alternator $\quad 18$
speed and frequency, relations
between
Alternator testing 180
armature resistance 190
Page Page
Armature windings
Armature windings ..... 119 ..... 119
classification
classification ..... 119 ..... 119
according to construction of
according to construction of core core ..... 120 ..... 120
according to disposition of
according to disposition of coils coils ..... 123 ..... 123
according to form of con-
according to form of con- ductor ductor ..... 125 ..... 125
according to progression of
according to progression of winding winding ..... 123 ..... 123
according to shape of core
according to shape of core ..... 119 ..... 119
single-phase
single-phase ..... 126 ..... 126
three-phase
three-phase ..... 127 ..... 127
two-phase
two-phase ..... 127 ..... 127
Automatic regulator
Automatic regulator ..... 98 ..... 98
Autotransformer
Autotransformer ..... 259 ..... 259 ..... 188 ..... 181 ..... 192 ..... 88
Autotransformer
B
B
Bar winding
Bar winding ..... 126 ..... 126
Breakdown test
Breakdown test ..... 238 ..... 238
Alternators ..... 81 ..... 81
armature inductance ..... 90 ..... 90
armature reaction ..... 88 ..... 88
armature windings ..... 119 ..... 119
commercial types ..... 129
economy factors ..... 163 ..... 63
electromotive force lost inarmature drop91
field excitation ..... 109
fundamental equation of ..... 81
measurement of power ..... 112
parallel operating of ..... 506
polyphase ..... 102
single-phase system ..... 102
three-phase system ..... 105
two-phase system ..... 102
regulation of ..... 92
Aluminum valve rectifier ..... 336
American Institute rules 168, 193, ..... 202
heat test ..... 202
heating and temperature ..... 171
rating ..... 170
regulation ..... 193
standards for electrical ma- chinery ..... 168
temperature limits ..... 179
temperature measurements ..... 175
Appendix ..... 506
Armature drop, e.m.f. lost in91
Armature inductance90
Armature reaction88
efficiency, calculation of ..... 211 ..... 211


heat test ..... 202 ..... 202
sulation testing ..... 180 ..... 180
breakdown ..... 183 ..... 183
dielectric strength ..... 181 ..... 181
object and scope of testing ..... 180 ..... 180
American Institute rules ..... 193 ..... 193
curve ..... 192 ..... 192
Potier method of determining ..... 199 ..... 199

185
185
characteristic curves
185
185
saturation
206
206
core loss and friction test
C
Circuit-breakers ..... 482
Circuit-interrupting devices ..... 479
air-break switches ..... 480
circuit-breakers ..... 482
feeder or voltage regulators ..... 488
fuses ..... 479
lightning arresters ..... 496
oil-break switches ..... 484
voltmeter compensator ..... 494
Commercial types of machines ..... 129
revolving-armature alternators ..... 131
revolving-field alternators ..... 138
Compromise method of making heat test ..... 202
Condenser ..... 50, 230
Constant-current transformer ..... 308
Cooling of transformers ..... 301
self-cooling dry transformers ..... 302
self-cooling oil-filled transform- ers ..... 303
water-cooled transformers ..... 305
Core-loss and exciting-current test ..... 325
Core-loss and friction test ..... 206
Core type transformer ..... 280
Note.-For page numbers see foot of pages.

| Page |  |  | Page |
| :---: | :---: | :---: | :---: |
| Current relations for rotary con- |  | Electromotive force (continued) |  |
| verter | 354 | variations of | 20 |
| Curtis turbo-alternator | 159 | cycle | 20 |
|  |  | frequency | 20 |
| D |  | period | 20 |
|  |  | Electromotive force relations for rotary converter |  |
| ielectric, inductivity of | 50 |  | 348 |
| Dielectric strength, test of | 181 | Excitation | 227 |
| ielectric test | 448 |  |  |
| Direct current | 16 | Feeder panels | 474 |
| Direct-current and alternatingcurrent problems, comparison of |  | Feeder or voltage regulators induction | 488 |
|  |  |  | 492 |
|  | 24 | magnetic | 490 |
| Distributing transformers, inspection and maintenance of | 367 | ratings of | 493 |
|  | 317 | Stillwell | 488 |
|  |  | Field excitation | 93 |
|  |  | Field excitation and power factor | 228 |
| E |  | Field excitation of rotary con- |  |
| Economy factors in alternators alternator testing |  | ert |  |
|  |  | Form factor | 32 |
|  | 180 | Fort Wayne single-phase alter- |  |
| conditions affecting cost | 163 |  | 131 |
| frequency | 164 | Four-ring rotary converter | 347 |
| regulation | 164 | Frequency, changes of changes of voltage and frequency | 453 |
| speed | 163 |  |  |
| voltage | 163 |  | 457 |
| efficiency influence of power factor | 165 | effect of incorrect frequency |  |
| upon output | 167 | effect on operating voltage | 455 |
| practical and ultimate limits |  | effect on speed of motor | 454 |
| of output | 166 | reasons for changes | 453 |
| power losses | 165 | Frequency changer | 417 |
| rating and overload capacities | 167 | Fuses | 479 |
| American Institute rules | 168 | G |  |
| Efficiency of alternator 165 |  |  |  |
| influence of power factor upon output | 167 | General Electric rotary converter General Electric three-phase alternator |  |
| practical and ultimate limits of output | 166 |  | 136 |
| Efficiency calculation | 330 | H |  |
| Electric power systems | 14 |  |  |
| Electrical resonance | 60 | Harmonic electromotive forces |  |
| multiplication of current by | 65 | and current | 3 |
| multiplication of e.m.f. by | 63 | addition of |  |
| Electrolytic-cell lightning arrester | 500 | algebraic representation |  |
| Electromotive force |  | clock diagram representation | 3 |
| average and effective values of | 31 | graphical representation |  |

[^46]| Page |  |  | Page |
| :---: | :---: | :---: | :---: |
| Harmonic electromotive forces |  | Induction motor (continued) |  |
| and currents (continued) |  | stator windings and their |  |
| phase difference | 36 | actions | 392 |
| subtraction of | 41 | tests | 426 |
| synchronism | 36 | breakdown | 428 |
| Heat run | 379 | core loss | 450 |
| Heat test 202, | 322 | efficiency | 433 |
| Heating and temperature, Amer- |  | heat | 426 |
| ican Institute rules for | 171 | impedance | 432 |
| Hunting action | 222 | performance curves of | 436 |
| Hunting of rotary converter | 363 | slip | 434 |
|  |  | starting torque | 429 |
| I |  | Inductivity of dielectric | 50 |
| Impedance | 326 | Inspection of transformers | 317 |
| Inductance | 44 | Installation of transformers | 317 |
| formulas for | 48 | Insulation classification | 447 |
| series and parallel | 49 | Insulation testing 180,319, | . 448 |
| Induction generator | 417 | Interlinking power lines | 13 |
| Induction motor | 391 | Intermittent method of making |  |
| action of | 395 | heat test | 205 |
| behavior at starting and in |  | Inverted rotaries | 365 |
| operation | 405 | Iron losses | 271 |
| efficiency and rotor-resistance | 399 | J |  |
| efficiency and speed | 396 | Joule's law | 24 |
| trical energy in rotor | 397 | K |  |
| ratio of rotor voltages to |  | Kirchoff's laws | 24 |
| stator voltages | 398 | L |  |
| riable speed | 401 | Lightning arresters | 496 |
| rotor windings, starting resistance in | 396 | condenser with choke coil, combination of | 503 |
| torque and speed | 395 | electrolytic-cell | 500 |
|  |  | installing, instructions for | 504 |
| structural details of | 400 | ground | 504 |
| compared with synchronous |  | insulation | 504 |
| motor | 421 | location | 504 |
| constructive elements | 391 | lightning, effects of | 496 |
| frequency changer | 417 | multi-gap non-arcing | 498 |
| Heyland diagram | 440 | multi-path | 499 |
| induction generator | 417 | Lincoln synchronizer | 477 |
| installations | 410 | Low-voltage secondary circuits, |  |
| reconnection of | 445 | grounding | 320 |
| single-phase | 413 |  |  |
| hand starting | 413 | M |  |
| repulsion motor starting | 415 | Maintenance of transformers | 317 |
| split-phase starting | 414 | Mercury-vapor are rectifier | 338 |

[^47]

[^48]Page Page
Rotary, or synchronous, con-verter (continued)
uses of ..... 356
six-ring ..... 348
testing of ..... 378
heat run ..... 379
standard ..... 378
three-ring ..... 347
Rotating magnetic field, produc- tion of ..... 224
S
Saturation curve ..... 185
Scott transformer ..... 267
Self-starting of polyphase syn- chronous motor ..... 223
Self-starting test ..... 238
Series or current transformer ..... 306
field coils ..... 147 .....
292 .....
292
Shell type transformer
Shell type transformerframerotating field138
supporting ring143 Single-phase alternator18
terminals
142 Single-phase windings ..... 126102143 Six-phase rotary converter
steam-turbine-driven typewater-wheel-driven type153 Six-ring rotary converter375149Rotary, or synchronous, con-verter343
comparison with direct-currentdynamo
current relations for344
direct-current dynamo madeinto rotary converter 345
e.m.f. relations for ..... 348
four-ring ..... 347
multipolar ..... 348
in practice ..... 356
characteristic types of361
direct-current voltage, control ..... of ..... 367
with Edison three-wire system ..... 373
field excitation ..... 371
hunting of ..... 363
inverted ..... 365
oscillators for360
six-phase ..... 365
starting of357
transformer connections for ..... 375f armature determination of 190of190
of coils ..... 325
131field structure for Westing-
house 180-kilowatt ..... 136
Fort Wayne single-phase ..... 131
General Electric three-phase ..... 136
Westinghouse armature with ..... 135
Westinghouse uni-coil armature ..... 134
Revolving-field alternators ..... 138
construction ..... 138
armature141
bed plate and bearing pedes-tals140
354
Speed and frequency, relations between ..... 20
Speed-limiting devices ..... 366
Starting compensators ..... 227
Starting torque ..... 226
Steam-turbine-driven alternators ..... 153
advantages ..... 154
Curtis turbo-alternator ..... 159
rotor ..... 157
stator ..... 155
Strap winding ..... 126
Switchboard apparatus ..... 477
circuit-interrupting devices ..... 479
lightning arresters ..... 496
Lincoln synchronizer ..... 477
Switchboards ..... 464
feeder panels ..... 474
high-voltage panels ..... 477
polyphase ..... 469
typical single-phase ..... 466
Synchronism ..... 36
Synchronizing ..... 215
Synchronous converter ..... 343
Page
Synchronous impedance curve ..... 188
Synchronous motors ..... 213,
advantages ..... 234
applications ..... 235
compared with induction motor ..... 421
disadvantages ..... 235
excitation ..... 227
field excitation and power factor ..... 228
hunting action ..... 222
motor testing ..... 236
production of rotating mag- netic field ..... 224
pull-in torque ..... 226
reduction by use of dampers ..... 222
self-starting of polyphase syn- chronous motor ..... 223
starting compensators ..... 227
starting torque ..... 226
synchronizing ..... 215
synchronous speed ..... 214
torque and power output ..... 217
use as condenser ..... 230
Synchronous speed ..... 214
T
Tables
$\Delta$ - and Y-connection data inmains
$\Delta$ - and Y-connection data in receiving circuits ..... 112capacities of standard trans-
comparison of motor voltagesphase connections451
comparison of motor voltages
with various two-phase connections 452 ..... 452
constant-current transformer data ..... 309
data
inductivities of dielectrics ..... 50
kilovolt-ampere ratings of
transformers for Scott connection ..... 270
needle-gap distances for vari-
ous spark-over voltages needle-gap distances for vari-
ous spark-over voltages ..... 184111
formers ..... 439
with various three-Page
Tables (continued)
permissible hottest spot tem-peratures and limitingobservable temperaturerises in other thanwater-cooled machinery 178
permissible temperatures andtemperature rises for in-sulating materials177
power ratings of rotary con-verters in kilowatts345regulation values for three-phase $\Delta$-connected alter-nator by three methods 201results of tests on inductionmotor453
size and cost of copper wire- two-wire system ..... 22temperature coefficients of cop-per resistance176
transformer efficiencies, losses,etc.272
turbine speeds for alternators ..... 154
values calculated from Hey-land diagram for induc-tion motor446
voltage ratios of rotary con- verters ..... 353
Temperature limits, American In- stitute rules for 171, 178
Temperature measurements,American Institute rulesfor175
Three-phase alternator ..... 105
$\Delta$-connected armatures ..... 109
current relations ..... 109
electromotive force relations ..... 109
Y -connected armatures ..... 108
current relations ..... 109
electromotive force relations ..... 108
receiving circuits ..... 110
dissimilar circuits (unbal- anced system) ..... 110
similar circuits (balanced system) ..... 110
Three-phase transformers ..... 298
Three-phase windings ..... 127

|  | Page |  | Page |
| :---: | :---: | :---: | :---: |
| Three-ring rotary converter | 347 | Transformer (continued) |  |
| Torque and power output | 217 | polyphase systems |  |
| Transformer | 241 | phase transformation | 265 |
| automatic action of | 244 | three-phase | 263 |
| coil resistances and magnetic |  | two-phase | 263 |
| leakage | 248 | practical considerations | 271 |
| commercial types of | 279 | efficiency | 272 |
| constant-current | 308 | rating of | 276 |
| cooling of | 301 | regulation | 275 |
| core | 280 | transformer losses | 271 |
| fuse blocks | 314 | regulation 275, | 327 |
| series or current. | 306 | tests | 322 |
| shell | 292 | core-loss and exciting-current | 325 |
| three-phase | 298 | efficiency calculation | 330 |
| connections | 250 | heat | 322 |
| 8uto-transformer | 259 | impedance | 326 |
| auto step-down trans- |  | polarity | 331 |
| formation | 261 | regulation | 327 |
| auto step-up transforma- |  | resistance of coils | 325 |
| tion | 260 | Transformer connections for ro- |  |
| current relations | 262 | tary converters | 375 |
| parallel constant-voltage |  | Transformer fuse blocks | 314 |
| transformers | 250 | Transmission of power | 13 |
| banking of | 255 | Two-phase alternator | 102 |
| Edison three-wire system -single-phase | 254 | Two-phase windings | 127 |
| multi-coil type | 251 | V |  |
| series-current transformers | 257 |  |  |
| description | 241 | Voltage, changes in | 447 |
| ideal action graphically repre- |  | changes in winding | 448 |
| sented | 247 | classification of insulations | 447 |
| ideal and practical | 244 | conditions to be met | 447 |
| maximum core flux | 245 | effect of incorrect voltage on |  |
| performance | 249 | periormance | 452 |
| with highly inductive load | 249 | insulation tests | 448 |
| with non-inductive load | 249 | Voltmeter compensator | 494 |
| physical action | 242 |  |  |
| current relations | 243 | W |  |
| electromotive force relations | 242 |  |  |
| with load | 242 | Water-wheel-driven alternators | 149 |
| without load | 242 | Westinghouse armature with dis- |  |
| polyphase systems | 262 | tributed winding | 135 |
| with compound magnetic cir- |  | Westinghouse uni-coil armature | 134 |
| cuits | 265 | Wire winding | 125 |


[^0]:    *The student is not expected to understand fully the reasons for the statements here given, until he has completed Parts I and II.
    +Alternating elentromotive forces and alternating currents are added in the same way that forces are added, that is, by means of the principle known as the "parallelogram of forces."

[^1]:    *The unit of angle chiefly used in mechanics and in all theoretical work is the radian. It is the angle of which the are is numerically equal to the radius (of a circle). There are, therefore, $2 \pi$ radians in one circumference.

[^2]:    *The henry is a very large inductance, and the inductances usually met with in practice are expressed in thousandths of a henry, that is, in milli-henrys.

[^3]:    * The permeability $\mu$ of a given sample of iron is not constant, but decreases in value as the magnetizing force incresses. Therefore, the inductance $L$ of a coil having an iron core is not a definite constant quantity as is the inductance of a coil without an iron core but depends upon the magnetizing eurrent,

[^4]:    *The farad is an exceedingly large capacity, and capacities encountered in practice are usually expressed in millionths of a farad, that is, in microfarads.

[^5]:    *Effective values are always understood except where it is distinctly stated to the contrary

[^6]:    *In practice this condition could be realized by adjusting the field rheostat of the alternator, of of the exciter. so as to reduce the exciting current as the speed of the alterpator is increased.

[^7]:    *This discussion applies to comparatively short lines. ten miles or less in length, inssmuch as the capacity of the line is not here taken into account.

[^8]:    *The electromotive force of an alternator passes through a cycle when an armature coil passes from a north pole of the field to the next north pole. The inductance passes through a cycle of values when an armature coil passes from one field pole to the next field pole.

[^9]:    Connections to field coils are reversed with every reversal of main current, so that, in the field coils, the current is uni-directional.

[^10]:    *This statement does not include the single-phase series commutator motor which is especially adapted to railway motors.

[^11]:    *The term "maximum load" does not refer to loads applied solely for mechanical commutation, or similar tests.
    $\dagger$ I.E.C. stands for "International Electrotechnical Commission." This rating has not yet been established.

[^12]:    * Since the input of machinery of this class is measured in electrical units and since the output has a definite relation to the input, it is logical and desirable to measure the delivered power in the same units as are employed for the received power. Therefore, the output of motors should be expressed in kilowatts instead of in horsepower. However, on account of the hitherto prevailing practice of expressing nechanical output in horsepower, it is recommended that for machinery of this class the rating should, for the present, be expressed both in kilowatts and in horsepower; as follows:
    kw. - approx. equiv. hp.
    For the purposes of these Rules the horsepower shall be taken as 746.0 watts.
    In order to lay stress upon the preferred future basis, it is desirable that on Rating plates, the Rating in kilowatts shall be shown in larger and more prominent characters than the rating in horsepower.

[^13]:    * An exception is made in the case of motors for railway service, where in the absence of any specification as to the kind of rating, the "nominal rating" as defined in section 319 shall be understood.

[^14]:    * Whenever a sufficient time has elapsed between the instant of shut-down and the time of the final temperature measurement to permit the temperature to fall, suitable corrections shall be applied, so as to obtain as nearly as practicable the temperature at the instant of shutdown. This can sometimes be approximately effected by plotting a curve, with temperature readings as ordinates and times as abscissas, and extrapolating back to the instant of shut-down. In other instances, acceptable correction factors can be applied. In transformers of $200 \mathrm{kv}-\mathrm{a}$. and less the measured temperature shall be increased one degree for every minute between the instant of shut-down and the time of the final temperature measurement, provided this time does not exceed three minutes.

    In cases where successive measurements show increasing temperatures after shut-down, the highest value shall be taken.

    In testing air blast transformers the air supply should be shut off immediately at the end of the heat run and the air duct closed to prevent a further circulation of air. In checking the temperature ascertained by the Resistance Method, the reading of all the thermometers in contact with coils should be noted carefully and the maximum temperature reached by any of them, if higher than that determined by resistance, should be taken as the maximum observable temperature of the windings. When the above procedure has been followed, a hottest-spot correction of $5^{\circ}$ shall be applied.
    $\dagger$ As one of the few instances in which the thermometer check cannot be applied in " Method $b$," the rotor of a turbo-alternator may be cited.

[^15]:    * For cotton, silk, paper and similar materials, when neither treated, impregnated nor immersed in oil, the highest temperatures and temperature rises shall be $10^{\circ} \mathrm{C}$. below the limits fixed for Class A.
    $\dagger$ The word impair is here used in the sense of causing any chánge which would disqualify the insulation for continuous service.

[^16]:    * A coil is one of the two active sides of the coil lying in a slot.

[^17]:    * For exceptions to these limitations see section 301 .
    $\dagger$ For cotton, silk, paper and similar materials, when neither treated, impregnated nor immersed in oil, the highest temperatures and temperature rises shall be $10^{\circ} \mathrm{C}$. below the limits fixed for class $A$.
    $\ddagger$ In these formulas, $E$ represents the rated pressure between terminals in kilovolts. Thus for a three-phase machine with single-layer winding, and with 11 kilovolts between terminals, the hottest-spot correction to be added to the maximum observable temperature will be $16^{\circ} \mathrm{C}$.

[^18]:    *See page 82.

[^19]:    * "Method b," for deducing the load saturation curve, at any assigned power factor, from no-load and zero power-factor saturation curves obtained by test must be regarded as empirical. Its value depends upon the fact that experience has demonstrated the reasonable correctness of the results obtained by it.

[^20]:    *Usually the transmission line has a resistance such as to give a 10 per cent drop of electromotive force, when full-load current is delivered to the motor.

[^21]:    *The primary of the potential transformer takes but little current, and the field cirsuit, to all intents and purposes, is open.

[^22]:    *See page 22.

[^23]:    *It must be remembered that the magnetizing current is that part of the primary current whose magnetizing action is not balanced or annulled by the secondary current as explained on page 233.

[^24]:    *Or three-phase supply. The discussion of phase transformation is, however, much simpler with a two-phase supply.

[^25]:    *In any of these types of transformers depending upon forced circulation of air or water, it is vitally important to avoid any stoppage of the fow, as this is likely to cause a burnout of the transformer coils.

[^26]:    *The electrical connections for a series transformer have been described on page 249.

[^27]:    * The impedance voltage of a transformer is the electromotive force which must be applied to the primary coil to produce full-load current in both coils when the secondary coil is shortcircuited. This voltage is from 2 per cent to 6 per cent of the rated full-load primary voltage. See page 317.

[^28]:    *The following discussion applies to multipolar machines also, but the statements are much simpler when limited to the bipolar machine.

[^29]:    *See equation (42), page 339.

[^30]:    The converter is separately excited by a small direct-current generator mechanically connected to, and driven by, it. The speed of the exciter will, therefore, change with every change in the speed of the inverted rotary. The magnetic circuit (magnet cores, yoke, etc.,) and magnet coils of the exciter are so designed that its armature can generate normal voltage when the machine is being worked at a point considerably below the "knee" of the saturation curve. Any increase in the speed of the exciter will, therefore, cause a great

[^31]:    *The principle of phase transformation is explained in detail on page 257.

[^32]:    *The apnaratus for determining when the two machines are in synchronism is not shown in the figure.

[^33]:    *In the actual induction motor, it ignores the loss of power due to the heating of the stator windings by, the supplied alternating currents, and the loss of power due to core losses in the stator iron.
    $\dagger$ When torque is expressed as pounds weight on a lever arm of one foot in length the torque is gaid to be expressed in pound-feet, and power in watts is equal to $\frac{2 \pi n T \times 746}{550}$
    $=8.52 n T$ watts, where $n$ is the speed in revolutions per second.

[^34]:    *This winding is identical with the stator winding as described on page 374.

[^35]:    * Based on a valuable series of articles entitled "Reconnecting Induction Motors," by A. M. Dudley, and published in Power, Vol. 46 (1917).

[^36]:    * "Some Points about Induction Motors," by J. W. Welsh, in Electrical Journal, Vol. II.

[^37]:    * "Effert of Voltage and Frequency Variations on Induction Motor Performance," by G. B. Werner, in Electrical Journal, Vol. III.

[^38]:    * "Some Points about Induction Motors," by J. W. Welsh, in Electrical Journal, Vol. II.

[^39]:    *"IReconnecting Induction Motors," by A. M. Dudley, in Electrical Journal, Vol. XIII.

[^40]:    *A detailed explanation of how two or more aiternators are synchronized in order to run in parallel is given in the Appendiz, Part VI.

[^41]:    *The expulsion type of fuse, much used for msuium voltages in alternating-curreut systems, is described on page 304 , in connection with transformer fuse blocks.

[^42]:    * A multiple-break switch which makes two or more breaks (in series) in a circuit simultaneously, is used to lessen the trouble from arcing. Some of the special switches described subsequently are double-break switrhes.

[^43]:    *Furthermore, two alternators, to operste satisfactorily in parallel, must have electromotive force waves of the same shape.

[^44]:    *When the two machines oppose each other in forcing current through the lamps they would help each other, or be in phase with each other, in producing current in the receiving circuit to which the machines are to be connected in parallel.

[^45]:    * From "American Electrician's Handbook."

[^46]:    Note.-For page numbers see foot of pages.

[^47]:    Note.-For page numbers see foot of pages.

[^48]:    Note.-For page numbers see foot of pages.

