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# HANDBOOK OF OPERATIONAL AMPLIFIER ACTIVE RC NETWORKS 

This handbook has been compiled by the Applications Engineering Section of
Burr-Brown Research Corporation. This department will weicome the opportunity of offering its technical assistance in the design of Active RC Networks or other operational amplifier applications.

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Circuit diagrams in this handbook are included to illustrate typical operational amplifier applications and are not intended as constructural information. Although reasonable care has been taken in preparing this handbook, no responsibility is assumed for inaccuracies or consequences of using information presented. Furthermore, such information does not convey to the purchaser of the amplifier described any license under the patent rights of Burr-Brown Research Corporation or others.

## PREFACE

A little over two years ago, the Applications Engineering Section of BurrBrown prepared a Handbook of Operational Amplifier Applications describing the properties of operational amplifiers and some of their many uses. It was felt that such a handbook would be of interest and value not only to present users of operational amplifiers, but also to those who were considering new applications. The response with which this handbook was received was most rewarding. The original supply of books was quickly exhausted, and several additional printings also rapidly disappeared.

Recently, we have been receiving an ever increasing number of inquiries from our customers concerning the use of operational amplifiers in filtering applications. To satisfy these requests, we undertook the preparation of this second handbook. It contains both theoretical and applied treatments of the application of active RC networks to the more common filtering problems.

This booklet is intended for you, the user. If you should have any questions concerning the material in this handbook, or any other operational amplifier application, please do not hesitate to contact the Applications Engineering Section at Burr-Brown. We will be most happy to talk with you at any time.

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## SECTION I

## ACTIVE RC NETWORK THEORY

The subject of active RC networks is one which has attracted considerable attention in the past few years from network theorists. Many new active devices and many new techniques have been developed. Some of these techniques have been of great theoretical interest, but of little practical value. Others, however, offer great practicality and have great potential for everyday application. In writing this handbook, the goal has been to screen the large volume of literature on this subject, and present only those techniques which are of definite practical value to the working engineer. All of the realization schemes described in Chapters 2 through 5 have been proven on the bench, and full details on their implementation are given in the circuits section of this handbook. In addition, each of these techniques is described in the text, where some of the pertinent theoretical background is given. The reader who is interested in a more detailed theoretical treatment will find that the references listed in Appendix A will give him an excellent introduction into the considerable literature on this subject.

## CHAPTER 1

## INTRODUCTION



This is a handbook on active RC networks. The first question about this subject that one might ask is, "What is an active RC network?" The answer is simple. It is a collection of resistors, capacitors, and an active element (or elements). Viewed in another sense, it is a circuit without induc-
tors. Why leave out inductors? There are many reasons. First of all, the inductor is a relatively large and heavy element. This is especially true at frequencies in the audio range and below. Second, inductors generally have more dissipation associated with them than capacitors of similar size do. In other words, commercially available inductors are not nearly as "ideal" as commercially available capacitors usually are. If you have tried to use network synthesis tech-
 niques you have probably discovered that the dissipation (or resistance) associated with inductors can cause considerable difficulty. For these reasons (and a few others such as non-linearity, saturation, and cost) more and more interest is being shown in circuit design techniques which avoid the use of inductors, namely active RC networks.

Can active RC networks do everything that passive RLC networks can do? Yes, and more'. They can have natural frequencies anyplace in the left half ot the complex frequency (or "s") plane. They can function as oscillators; in other words they can have natural frequencies on the $\mathrm{j} w$ axis. They can provide transformation ratios just like the
 coupled coils of a transformer do (however they can't provide the isolation). They can even provide perfect coupling and thus realize "ideal" transformers, which actual coupled coils cannot do. They can gyrate microfarads of capacitance into hundreds of henries of inductance, etc. There won't be space in this handbook to cover all of the things that active RC networks can do. Instead, we'll try to show you in detail how to use them to do some of your more common filtering tasks. If you are interested in more specialized opplications, some references are given in Appendix A.

How does the tremendous capobility of active RC networks come obout? Certainly not from the passive elements, the resistors and capacitors. Token by themselves, these elements can produce natural frequencies only on the negotive real axis of the complex frequency plane, a relatively uninteresting region for mast filtering applications. Active RC networks, on the other hand, can have natural frequencies anywhere


Natural frequencies for passive RC circuits
on the complex frequency plane. Right half plane natural frequencies, of course, are not useful because they signify unstable network behavior, so we'll just consider the usable active RC natural frequencies as being in the left half plone or on the $\mathrm{j} \omega$ axis. Since it is the "octive" element that gives
active RC networks their potentiality, let's briefly consider such elements in more detail.

There are several types of active elements that can be used in active RC networks First, there is the ideal voltage amplifier of high gain. By "high" here we mean a gain in the order of at least 60 db . By "ideal" we mean infinite input impedance and zero


Stable natural frequencies for active RC circuits output impedance. The operational amplifier is an example of such an active element. Second, there is the ideal voltage amplifier of low gain. By "low" here we mean a gain in the order of 20 db or less. Such an element is sometimes referred to os o controlled source. Third, there is the NIC (negative-immittonce converter, olso sometimes referred to os o negotive-impedonce converter). This is o two-port device (a device with two sets of terminal pairs) with
 the property that an impedance connected ocross one set of terminols oppeors os o negative impedonce at the other set of terminals. Fourth, there is the gyrator, o device that converts copocitonce to inductance and vice verso. An interesting point to be noted here is that ony of the lost three types of octive elements listed obove can olso be reolized very simply and occurately with operationol omplifiers. Thus, we see thot the operational amplifier can be considered as a bosic building block for constructing every type of active RC network. Mony more detoils obout the active elements

introduced above will be given in the sections that follow. The networks which use operational amplifiers to realize these active elements will also be discussed. First, however, let us say a few things about the operational amplifier.

The modern differential input operational amplifier may be simply modeled as an ideal voltcge amplifier of very low output impedance (we'll assume that it is zero), very high input impedance (we'll assume that it is infinite), and very high gain, with the property that the output voltage is proportional to the difference in the voltages ap-


Fig. 1-I. Ideal model for on operational amplifier. lent circuit for such a model is shown in Fig. 1-1. The circuit symbol that will be used in future discussions is shown with the same terminal numbers in Fig. 1-2.

As a result of the properties of the operational amplifier, when it is inserted in circuit configurations, the voltage be-


Fig. 1-2. Circuil symbol for an operational amplifier. tween the input terminals numbered 1 and 2 in Figs. 1-1 and 1-2 is driven to zero. Due to the high input impedance and zero voltage, the current into both of these terminals may be considered as zero. These two characteristics comprise the "virtual ground" concept which is a basic tool for analyzing operational amplifier circuits. For more detailed information on the properties and characteristics of operational amplifiers, you should consult the "Handbook of Operational Amplifier Applications" which is available from Burr-Brown Research Corporation.

In the remainder of this handbook, we shall discuss in detail how the various
types of active elements introduced above may be used to produce the commonest types of network characteristics, namely, the low-pass, the high-pass, and the band-pass characteristic. We shall see that each of the active elements has its own advantages and disadvantages in the different circuit configurations. So, without more delay, let us start our investigation of active RC networks, a world without inductors.


## CHAPTER 2

THE INFINITE-GAIN SINGLE-FEEDBACK CIRCUIT

The first active element that we shall consider for realizing active RC networks is the operational amplifier. In this chapter we shall investigate its use directly as an operational amplifier, in other words we shall not first modify it so that its characteristics approach those of some other active device. It may be helpful at this point to review briefly some of the characteristics of the operational amplifier. Those readers who are familiar with operational amplifiers may skip the next section without loss of continuity.

## The Operational Amplifier

In Fig. 2-1 we have shown a symbolic representation for an operational amplifier which defines the input voltages $E_{1}$ and $E_{2}$ and the output voltage $\mathrm{E}_{\mathrm{o}}$. The terminal numbers 1 ,
 2, and 4 are standard markings on all Burr-Brown operational amplifiers. In terms of


Fig. 2-2. Open-Loop $D C$ Transfer Characteristic of the Operational Amplifier
these voltages we may plot a typical open-loop d-c transfer characteristic as shown in Fig. 2-2. From this figure we see that terminal I may be referred to as the "inverting" input terminal,
while terminal 2 may be referred to as the "non-inverting" input terminal. In a typical solid-state operational amplifier (for example, the Burr-Brown Model 1506) the magnitude of $E_{0}$ is greater than 10 volts at saturation. The open-loop $d-c$ gain of the amplifier is 100,000 , so we see that the magnitude of $E_{5}$, the differential input voltage that produces saturation, is 0.1 millivolts. The open-loop frequency characteristic of a typical operational amplifier is shown in Fig. 2-3. The slope of the roll-off is less


Fig. 2-3. Open-Loop Frequency Characteristic af a Typical Operational Amplifier than $12 \mathrm{db} /$ octave. The location of the break point is determined by a compensation network. In some operational amplifiers (such as the Burr-Brown Model 1506) this network is integral with the operational amplifier circuitry; in others, (such as the BurrBrown Model 1509) it is external to the amplifier packaging, and thus may be readily changed. Stability considerations determine the proper choice of compensation network for a given circuit configuration; however, most operational amplifiers are compensated so as to provide adequate performance for the majority of circuit applications. For additional information on stability, compensation, or other general properties of operational amplifiers, the reader is referred to the "Handbook of Operational Amplifier Applications," published by Burr-Brown.

## The Basic Single Feedback Circuit

The basic circuit that will be considered in this chapter consists of two passive networks, which we will refer to as network $A$ and network $B$, and an operational
amplifier. Network $A$ is connected between the input to the circuit and the input terminal of the operational amplifier. Network B is used as a feedback network from the output to the input of the operational amplifier. The circuit is shown in Fig. 2-4. It

should be noted that the operational amplifier is used in an inverting configuration, i.e., with its non-inverting input terminal (terminal 2) grounded. We shall call this circuit an infinite-gain single-feedback circuit since the operational amplifier which is the active element normally has very high gain, and since the feedback around it is made to a single point.

To characterize the properties of the two passive networks, we shall use their y parameters. For network A we may define voltage and current variables as shown in Fig. 2-5. The relations between these variables and the $y$ parameters of the network are

$$
\begin{align*}
& I_{l a}=y_{1 l a} E_{l a}+y_{12 a} E_{2 a}  \tag{1}\\
& I_{2 a}=y_{12 a} E_{l a}+y_{22 a} E_{2 a}
\end{align*}
$$



Fig. 2-5. The Port Variables for Network A

Similarly, for network $B$ and the variables shown in Fig. 2-6, we may write


Fig. 2-6. The Port Variables for Network 8
$I_{1 b}=y_{11 b} E_{1 b}+y_{12 b} E_{2 b}$
$I_{2 b}=y_{12 b} E_{1 b}+y_{22 b} E_{2 b}$

All of the voltage and current variables and the $y$ parameters defined in equations
(1) and (2) are functions of " s ", the complex frequency variable.

## The Voltage Transfer Function

The basic network configuration for the infinite-gain single-feedback circuit has been redrawn in Fig. 2-7 to indicate the variables of the two passive networks ex-

plicitly. In Chapter 1, it was pointed out that due to the "virtual ground," the voltage between terminals 1 and 2 of the operational amplifier moy be considered to be zero. Thus, the voltage $\mathrm{E}_{2 a}$ shown in Fig. 2-7 is zero. From the second equation of (1), we see that under this condition $I_{2 a}=y_{12 a} E_{1 a}$. In addition, since $E_{1 a}$ and $E_{1}$ are equal, we may write

$$
\begin{equation*}
I_{2 a}=y_{12 a} E_{1} \tag{3}
\end{equation*}
$$

Similarly, for network $B, E_{1 b}$ is zero, and $E_{2 b}=E_{2}$. Thus we see that

$$
\begin{equation*}
I_{1 b}=y_{12 b} E_{2} \tag{4}
\end{equation*}
$$

The virtual ground concept also tells us that the current into terminal 1 of the operational amplifier is negligibly small. Thus we see that $I_{2 a}=-I_{1 b}$. We moy now combine equations (3) and (4) to obtain

$$
\frac{E_{2}}{E_{1}}=\frac{-y_{12 a}}{y_{12 b}}
$$

This is the open-circuit voltage transfer function for the infinite-gain single-feedback active circuit configuration.

Let us examine the voltage transfer function given in equation (5) more closely. If networks $A$ and $B$ are passive $R C$ networks, their natural frequencies will be on the negative real axis of the complex frequency plane. Let us assume that both of the passive networks have the same natural frequencies; then the denominators of the functions $y_{12 a}$ and $y_{12 b}$ will cancel and the locations of these natural frequencies will not affect the voltage transfer function of the overall network. The poles of the voltage transfer function of the active network configuration will then be determined solely by the zeros of the transfer admittance $y_{12 b}$. Since a passive RC network can have the zeros of its transfer admittance anywhere on the complex frequency plane, this says that we can realize complex conjugate poles in our voltage transfer function. Such poles will, of course, be restricted to the left half of the complex frequency plane for reasons of stability. Similarly, the zeros of the voltage transfer function given in equation (5) will be determined by the zeros of $y_{12 a}$, and therefore we can realize any desired real or complex conjugate zeros in our valtage transfer function. Thus we see that an infinite-gain single-feedback active $R C$ network configuration can be used to realize almost any desired pole-zero configuration.

One other property of this circuit should be noted. Suppose that another network with transfer admittance $y_{12}$ is also connected to the input terminal of the operational amplifier. The connection is shown in Fig. 2-8, where the additional network is labeled as network $C$. The input voltages to networks $A$ and $C$ are $E_{l a}$ and $\mathrm{E}_{1 \mathrm{c}}$ respectively. An analysis similar to the one made in the preceding paragraph shows that the output voltage $E_{2}$ for this circuit is given by the relation

$$
\begin{equation*}
E_{2}=-\left[\frac{y_{12 a}}{y_{12 b}} E_{1 a}+\frac{y_{12 c}}{y_{12 b}} E_{1 c}\right] \tag{6}
\end{equation*}
$$



Thus we see that the infinite-gain single-feedback circuit configuration can also be used for summing signals from separate sources. This can be done without any interaction occurring between the sources.

## The Passive Networks

In general, most filter designs require the use of complex conjugate poles.
To produce these by the active RC technique described in this chapter, we thus require passive networks which have transfer admittances with complex conjugate zeros. There are several such network configurations, of which the two most common ones are the bridged-T network and the twin- $T$ network. It is beyond the scope of this handbook to analyze such networks in detail. For completeness, however, we will present a simple design procedure for each type of network in this section.

An example of a bridged-T network is shown in Fig. 2-9. The


Fig. 2-9. Bridged-T RC Nelwork
units for the elements of this network are farads for the capacitors and mhos ( $G=1 / R$ ) for the resistors. For a transfer admittance frequency. normalized to one radian/second, and of the form

$$
\begin{equation*}
-y_{12}=\frac{s^{2}+a s+1}{s+a} \tag{7}
\end{equation*}
$$

the elements will have the following values:

$$
\begin{array}{ll}
C_{1}=1.0 & G_{2}=1 /\left(a-1 / G_{1}\right)  \tag{8}\\
G_{1}=2.5-a & C_{2}=G_{1} G_{2}
\end{array}
$$

Such a network is not useful for producing zeros that lie close to the $j \omega$ axis, i.e., for small values of the constant $a$ in the numerator of equation (7). A useful range of the constant a for this circuit is

$$
\begin{equation*}
1 / 2<a<2 \tag{9}
\end{equation*}
$$

It should be noted that the usual frequency and impedance normalizations must be applied to this circuit to determine the actual element values.

An example of a twin-T network is shown in Fig. 2-10. If the network elements are chosen such that

$$
\begin{align*}
& A_{1}=C_{1}=G_{1}=(2.5-a) \frac{1+a}{2+a} \\
& A_{2}=C_{2}=G_{2}=A_{1} /\left(A_{1}-1\right)  \tag{10}\\
& A_{3}=C_{3}=G_{3}=A_{1} A_{2} /(1+a)
\end{align*}
$$



Fig. 2-10. Twin-T RC Network
the transfer admittance of this network will be

$$
\begin{equation*}
-y_{21}=\frac{(s+1)(s+a s+1)}{\left(s+\sigma_{1}\right)\left(s+\sigma_{2}\right)} \tag{11}
\end{equation*}
$$

Note that in equations (10), the numerical values of the capacitors in farads and the resistors in mhos are equal. For convenience, these values have also been referred to
as $A_{1}, A_{2}$, and $A_{3}$. The constants $\sigma_{1}$ and $\sigma_{2}$ in equation (11) can be found by the relations

$$
\begin{equation*}
\sigma_{1}=\frac{A_{1}+A_{2}}{A_{3}} \quad \sigma_{2}=\frac{1}{\sigma_{1}} \tag{12}
\end{equation*}
$$

This network may be used for producing zeros which are as close to the $j \omega$ axis as desired, i.e., for as small a value of the constant a as may be required. Extremely small values of $a$, however, may lead to oscillations in the overall active circuit. It should be noted that the numerator of the transfer admittance of this network as given in equation (11) is of third degree with a negative real zero at -1 . The poles of the transfer admittance, however, are located with geometrical symmetricity about -1 . This may be seen from the second equation of (12). It is easily shown that as the constant a approaches zero, $\sigma_{1}$ and $\sigma_{2}$ both approach unity. Thus, when a equals zero, cancellation occurs between one of the poles and the negative real zero in the numerator of the transfer admittance. Similarly, for small values of $a$, the numerator zero very nearly cancels one of the poles, and the transfer admittance may be assumed to have the form

$$
\begin{equation*}
-y_{12}=\frac{s^{2}+\text { as }+1}{s+1} \tag{13}
\end{equation*}
$$

without significant loss of accuracy.

## Network Design

Now let us consider the application of the infinite-gain single-feedback active RC circuit to the realization of three common filtering applications - the low pass, high pass, and band pass cases. For the low pass network, the frequency normalized voltage transfer function is of the form

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{-H}{s^{2}+\text { as }+1} \tag{14}
\end{equation*}
$$

where $H$ is a positive real constant giving the mognitude of the poss band gain. A common choice for the constant $a$ is $\sqrt{2}$. This gives a maximally flot (sometimes called a Butterworth) frequency response characteristic. A bridged-T network of the type shown in Fig. 2-9 may be used to produce the complex conjugate poles. The element values can be found from equotions (8). The bridged-T network is used as network B of Fig. 2-4. The transfer admittance for network A must have no finite zeros, but must have a single negative real pole. A network satisfying these requirements is
 shown in Fig. 2-11. This network has the transfer admittance

$$
\begin{equation*}
-y_{21}=\frac{G_{1} G_{2} / C}{s+\left(G_{1}+G_{2}\right) / C} \tag{15}
\end{equation*}
$$

Fig. 2-11. Low Poss Network A
Since this network must have the same pole location as network $B$, the factor $\left(G_{1}+\right.$ $\mathrm{G}_{2}$ )/C must be set equal to the constant $a$. The complete network, together with a summary of the design procedure, and some sample element values is given in the circuit section of this handbook as Circuit No. 1.

For the high pass network, the frequency normalized voltage transfer function will be of the form

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{-\mathrm{Hs}^{2}}{s^{2}+o s+1} \tag{16}
\end{equation*}
$$

where H is a positive real constant giving the magnitude of the gain in the pass band. The same bridged-T network which was used for the low pass cose may be used for network $B$ for this case, since the poles of the transfer function will normally have similar locations. The transfer odmittonce for network $A$ must now have two zeros at the origin and a single pole on the negotive real axis. A network satisfying these requirements
is shown in Fig. 2-12. It has the transfer admittance

$$
\begin{equation*}
-y_{12}=\frac{s^{2} C_{1} C_{2} /\left(C_{1}+C_{2}\right)}{s+G /\left(C_{1}+C_{2}\right)} \tag{17}
\end{equation*}
$$

The element values must be chosen so


Fig. 2-12. High Poss Network $A$ that the factor $G /\left(C_{1}+C_{2}\right)$ equals the constant $a$. The complete network, a design procedure, and a set of typical element values is given in the circuit section as Circuir No. 2.

For a band pass network, the frequency normalized voltage transfer function will be of the form

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{-H_{s}}{s^{2}+a s+1} \tag{18}
\end{equation*}
$$

The magnitude of the gain in the pass band for this function is $H / a$, where $H$ is a positive real constant. Most band pass filter applications require that the constant a be small, i.e., that the poles be close to the $i_{w}$ axis. For this case the bridged-T network is not satisfactory, and it will usually be necessary to use a twin-T network of the type shown in Fig. 2-10 as network B. If we assume that the transfer admittance of the twin-T network is of the form given in equation (13), then the transfer admittance of network A must have a single zero at the origin and a pole at -1. A network satisfying these requirements is shown in Fig. 2-13. It has the transfer admittance



Fig. 2-13. Single Zero - Single Pole
The factor G/C must be set equal to unity. The complete network, a design procedure, and a set of typical element values is given in the circuit section of this handbook as Circuit No. 3.'

It should be noted that, in any realization produced by the method outlined
in this chapter, the gain constant of the complete network realization is easily adjusted by changing the impedance normalization of either of the component passive networks. This is easily seen from equation (5). For example, to raise the overall network gain, one may either lower the impedance level of network $A$ (and thus raise the magnitude of $y_{12 a}$ ) or raise the impedance level of network $B$ (and thus lower the magnitude of Y 12 b ). It should also be noted that all circuits described in this chapter produce a signal inversion in addition to the frequency dependent properties which have been noted.

## Conclusions

At this point we may make some conclusions regarding some of the properties of the infinite-gain single-feedback active RC circuit configuration described in this chapter. These conclusions will assist us in determining the relative merits of this configuration as compared to the other configurations which will be described in the chapters that follow. One of the major disadvantages of this active RC circuit configuration is the large number of passive elements that it requires. For example, in the band pass network described as Circuit No. 3, we see that eight elements are needed. Another difficulty is brought about by the fact that bridged-T and twin-T networks must be used to produce the complex conjugate poles. This means that any adjustment or trimming of the pole locations will be difficult since the passive elements interact to a high degree in such networks. On the positive side of the ledger is the fact that this configuration has its pole locations determined completely by the passive networks. Thus the pole locations will remain relatively stable and independent of changes in the active element. This is a considerable advantage when it is desired to design high- $Q$ networks, where the poles are located close to the $\mathrm{j} w$ axis, since even small pole displacements may produce instability in this case. Another advantage of this configuration is tnat the output impedance of the network is equal to the output impedance of
the operational amplifier, which with high loop gain is very low. Thus, this circuit may be used to drive other networks, without the need for an isolating stage, and without appreciable change in the circuit characteristics due to loading. Yet another desirable feature is the capability of summing signals at the input.

## CHAPTER 3

THE INFINITE-GAIN MULTIPLE-FEEDBACK CIRCUIT

In the preceding chapter an active RC network configuration described as an infinite-gain single-feedback circuit was presented. An operational amplifier was used as the active element, and a single feedback path was provided around it. In this chapter another active RC network configuration will be presented. An operational amplifier will again be used as the active element; however, more than one feedback path will be provided around it. The advantages and disadvantages of the two approaches will be compared.

The Basic Multiple Feedback Circuit

The basic circuit that will be described in this chapter consists of a number of two-terminal passive elements, interconnected so as to form feedback paths around an operational amplifier. The operational amplifier is used in an inverting configuration, i.e., with its non-inverting input terminal (terminal 2) grounded. The general circuit configuration is shown in Fig. 3-1. We shall call this circuit an infinite-gain multiplefeedback circuit. In applying this circuit to the realization of transfer functions, it is practical to restrict each of the passive two-terminal elements to a single resistor or a single capacitor. In addition, if we limit ourselves to the realization of a voltage transfer function with a single pair of complex conjugate pales, and with zeros located only at the origin of the complex frequency plane or at infinity, then a maximum of five


Fig. 3-1. Multiple-Feedback (MFB) Operational Amplifier Circuit
elements is necessary. The low pass, high pass, and band pass cases are included in this class of transfer functions. The extension of the method to other cases will be clear from the discussion that follows.

## The Voltage Transfer Function

The basic circuit that moy be used to realize voltage transfer functions with a single pair of complex conjugate poles and with zeros restricted to the origin or infinity is shown in Fig. 3-2. Each of the elements $Y_{i}$ shown in this figure represents a

single resistor or a single capacitor. Reference currents $I_{i}$ and an interior voltage $E_{0}$ have been defined in the figure to aid in the analysis. From the figure we may write

$$
\begin{equation*}
E_{1}=\frac{1}{Y_{1}} I_{1}+E_{0} \tag{1}
\end{equation*}
$$

It is also obvious that

$$
\begin{equation*}
I_{1}=I_{2}+I_{3}+I_{4} \tag{2}
\end{equation*}
$$

The virtual ground imposed by the operational amplifier requires that the voltage across both $Y_{2}$ and $Y_{3}$ equal Eo. Similarly, the voltage across $Y_{4}$ is the difference between $E_{0}$ and $E_{2}$. Thus we may write expressions for the branch currents as

$$
\begin{align*}
& I_{2}=Y_{2} E_{0} \\
& I_{3}=Y_{3} E_{o}  \tag{3}\\
& I_{4}=Y_{4}\left(E_{o}-E_{2}\right)
\end{align*}
$$

The virtual ground also requires that $I_{3}=I_{5}$, thus we may write

$$
\begin{equation*}
I_{3}=Y_{3} E_{0}=-Y_{5} E_{2}=I_{5} \tag{4}
\end{equation*}
$$

If we substitute the relations of equations (2), (3), and (4) into equation (1), we obtain

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{-Y_{1} Y_{3}}{Y_{5}\left(Y_{1}+Y_{2}+Y_{3}+Y_{4}\right)+Y_{3} Y_{4}} \tag{5}
\end{equation*}
$$

This is the open-circuit voltage transfer function for the infinite-gain multiple-feedback circuit shown in Fig. 3-2. The elements $Y_{i}$ of this network may readily be chosen so as to obtain low pass, high pass, and band pass voltage transfer functions. This will be shown in the next section.

## Network Design

Let us first consider the use of the infinite-gain multiple-feedback configuration to realize a low pass network. It is desired to obtain a frequency normalized voltage transfer function of the form

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{-H}{s^{2}+\text { as }+1} \tag{6}
\end{equation*}
$$

where $H$ is a positive real constant which specifies the gain in the pass band, i.e., the d-c gain. If we compare the above equation with equation (5), we see that in order to have the numerator not be a function of "s", both of the elements $Y_{1}$ and $Y_{3}$ must be resistors. Similarly, in order to generate the $s^{2}$ term in the denominator, $Y_{5}$ must be a capacitor, as must either $Y_{2}$ or $Y_{4} . Y_{4}$, however, must be a resistor; otherwise, it will not be possible to realize the constant term in the denominator (this must come from the product $\mathrm{Y}_{3} \mathrm{Y}_{4}$ ). Thus, we must make the following choices for the elements of the circuit shown in Fig. 3-2:

$$
\begin{array}{lll}
Y_{1}=G_{1} & & Y_{3}=G_{3} \\
Y_{2}=s C_{2} & & Y_{4}=G_{4} \\
& Y_{5}=s C_{5} &
\end{array}
$$

The circuit with these elements is shown in Fig. 3-3. The voltage transfer function for

this circuit is

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{-G_{1} G_{3}}{s^{2} C_{2} C_{5}+{ }_{s} C_{5}\left(G_{1}+G_{3}+G_{4}\right)+G_{3} G_{4}} \tag{8}
\end{equation*}
$$

It should be noted that this circuit produces a signal inversion, as will be true for all the circuits realized by this technique. The specific solutions for the element values
in terms of the constonts a ond H moy be found by equoting corresponding coefficients in equotions (6) ond (8). Such o process leads to o simultoneous set of equotions which ore, unfortunotely, non-lineor. The noture of the set of equotions is such, however, thot constroints may be opplied to develop o set of solutions. Such o set of solutions, together with other design informotion, is given in the circuit section of this hondbook os Circuit No. 4. It should be noted thot olthough the solutions given hove been found to give good experimentol results, they ore not unique; i.e., other sets of solutions olso exist.

The high poss network con be considered in o monner similor to the low poss network. The frequency normolized high poss voltoge tronsfer function will be of the form

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{-H s^{2}}{s^{2}+a s+1} \tag{9}
\end{equation*}
$$

where H is o positive real constont which specifies the goin in the poss bond, i.e., the high frequency goin. The network elements shown in Fig. 3-2 must be chosen os follows:

$$
\begin{array}{lll}
Y_{1}={ }_{s} C_{1} & & Y_{3}={ }_{s} C_{3} \\
Y_{2}=G_{2} & & Y_{4}={ }_{s} C_{4}  \tag{10}\\
& Y_{5}=G_{5} &
\end{array}
$$

The resulting voltoge tronsfer function may be expressed in terms of the network elements os

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{-s^{2} C_{1} C_{3}}{s^{2} C_{3} C_{4}+s G_{5}\left(C_{1}+C_{3}+C_{4}\right)+G_{2} G_{5}} \tag{11}
\end{equation*}
$$

The circuit configurotion is shown in Fig. 3-4. The some comments thot were made

with respect to solving for the element values in the low pass network also apply in this case. A set of design equations and a summary of other information on this circuit is given in the circuits section of this handbook as Circuit No. 5 .

There are several configurations of five elements which can be used to realize a band pass network with a frequency normalized transfer function of the form

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{-H_{s}}{s^{2}+a s+1} \tag{12}
\end{equation*}
$$

where $H$ is a positive real constant and $H / a$ is the magnitude of gain in the pass band. One of the most practical configurations is the one defined by the following choice of elements:

$$
\begin{array}{lll}
Y_{1}=G_{1} & & Y_{3}=s C_{3} \\
Y_{2}=G_{2} & & Y_{4}=s C_{4}  \tag{13}\\
& Y_{5}=G_{5} &
\end{array}
$$

For such a choice, the voltage transfer function may be written in terms of the elements as

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{-s G_{1} C_{3}}{s^{2} C_{3} C_{4}+s G_{5}\left(C_{3}+C_{4}\right)+G_{5}\left(G_{1}+G_{2}\right)} \tag{14}
\end{equation*}
$$

The circuit configuration is shown in Fig. 3-5. A set of non-linear equations must be

solved for the element values. The solutions and the circuit design information is given in the circuits section of this handbook as Circuit No. 6.

Conclusions

In a manner similar to that of the last chapter, we may make some conclusions regarding the characteristics of the infinite-gain multiple-feedback circuit configuration. One advantage we see is that the number of elements is greatly reduced from the number required for the infinite-gain single-feedback circuits of the preceding chapter. For example, for the low pass and high pass circuits, a maximum of five elements is required, compared with seven for the single-feedback circuit. As may be seen from the design equations given in the circuits section of the handbook, there are also cases in which one of the five elements may be eliminated. In the band pass case, four or five elements are required rather than eight, a considerable saving. The same advantage of low output impedance which was pointed out for the single-feedback circuit holds true for the multiple-feedback circuit, since the output impedance of the circuit is just the closed loop output impedance of the operational amplifier. Thus this circuit may be used to drive other circuits without degradation of performance due ta loading effects.

The multiple-feedback circuit, hawever, has some disadvantages which the
single-feedback circuit does not. For example, it is not possible to obtain high $Q$ band pass realizations with the multiple-feedback configuration without resorting to large spreads of element values. Another disadvantage is the fact that since feedback is made to two points, there is no one single point in the circuit which can be used to sum separate signals as could be done in the single-feedback configuration. Also, if it is desired to realize transfer functions with zeros other than at the origin or infinity, the networks and the design procedure for the multiple-feedback case become considerably more complicated. Finally, this approach, in general, cannot be used to achieve as large a value of gain constant as may be obtained by the single-feedback configuration. Articles have appeared in the literature discussing the application of the infin-ite-gain multiple-feedback circuit configuration to the realization of more complicated transfer functions. For the reader who wishes to pursue this topic further, some references are given in Appendix $A$.

## CHAPTER 4

## the CONTROLLED SOURCE CIRCUIT

In the preceding chapters, some general properties of active $R C$ circuits were introduced, and two types of active RC circuit configurations were analyzed. Both of these configurations required an active element with a high value of gain. Thus, they were referred to as "infinite-gain" realizations, and an operational amplifier was used to provide the gain. In this chapter a quite different circuit configuration will be presented. It requires an active element with a relatively low value of gain, which we shall refer to as a controlled source.

In general, a controlled source is an active network element which has an output voltage or current which is a function of some single input voltage or current, but is unaffected by any of the other voltages or currents in the network. There are four types of controlled sources: the voltage-controlled voltage source, the currentcontrolled voltage source, the voltage-controlled current source, and the current-controlled current source. Certain physical devices have characteristics which make them act in a manner similar to some of these sources. For example, a transistor acts somewhat like a current-controlled current source. Similarly, a pentode acts very nearly like a voltage-controlled current source.

The Voltage-Controlled Voltage Source

In our discussion of the use of controlled sources in active RC circuits, we
will limit ourselves to a single type of source, an ideal voltage amplifier with infinite input impedance, zero output impedance, and an output voltage which is equal to the input voltage multiplied by some positive or negative constant. This device we shall refer to as a VCVS (voltage-controlled voltage source). A circuit model for it is shown in Fig. 4-1. From this figure we see that $E_{2}=K E_{1}$, where the constant $K$ is usually referred to as the "gain." It may be either positive or negative. The circuit symbol which we shall use for the VCVS is shown in Fig. 4-2.


Fig. 4-1. VCVS Circuir Model


Fig. 4-2. VCVS Circuit Symbol

Now let us see how we con realize a VCVS with an operational amplifier. For the values of. $K$ that are positive and greater thon unity, the circuit shown in Fig. 4-3 may be used. Note that in this circuit, the non-inverting terminal of the operational amplifier (terminal 2) is used as the input terminal. Using the virtual ground concept introduced in Chapter 1 , we see that terminals 1 and 2 must be at the same potential. Since the voltage at terminal 1 is equal to $R_{a} l_{1}$, the voltage at terminal 2 must be the
 same; therefore, $E_{1}=R_{a} l_{1}$. No current flows into the operational amplifier terminals; therefore, the currents $I_{1}$ and $I_{2}$ are equal. Thus $E_{2}=\left(R_{a}+R_{b}\right) I_{1}$. Combining these relations we obtain

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{R_{a}+R_{b}}{R_{a}} \tag{1}
\end{equation*}
$$

Since the input impedance at terminal 2 (as well as that at terminal 1) of the operational amplifier is very large, the circuit shown in Fig. 4-3 may be considered as an ideal VCVS with a value of $K$ equal to $\left(R_{a}+R_{b}\right) / R_{a}$. It should be noted that the idealness of this VCVS realization is relatively independent of the impedance levels which are chosen for $R_{a}$ and $R_{b}$. in practice, values of $R_{a}$ in the range of 100 k ohms will usually prove satisfactory.

## Network Design

The controlled source described in the last section moy be used, in connection with passive RC networks, to obtain various network functions. The first such function that we shall consider is the transfer function for the low pass network. The frequency normalized voltage transfer function is

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{H}{s^{2}+a s+1} \tag{2}
\end{equation*}
$$

where $H$ is a positive real constant giving the value of the gain in the pass band. A network configuration which will produce this function is shown in Fig. 4-4. In terms

of the elements shown in this figure, we may write the voltage transfer function as

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{K G_{1} G_{2}}{{ }_{s} 2 C_{1} C_{2}+s\left(C_{2} G_{1}+C_{2} G_{2}+C_{1} G_{2}-K C_{1} G_{2}\right)+G_{1} G_{2}} \tag{3}
\end{equation*}
$$

From the preceding equations we see that no signal inversion is introduced by this network configuration.

The transfer function given in equation (3) points out some interesting properties of this circuit realization. First of all, the low frequency gain, i.e., the gain in the pass band, may be found by evaluating equation (3) in the limit as "s" approaches zero. It is readily seen that this gain is simply equal to $K$. In other words, the overall gain specified for the circuit directly determines the gain of the VCVS. Second, if it is desired to change the value of the cutoff frequency for this circuit, this may be done by changing the values either of the resistors or the capacitors. Such changes will not affect the gain in the pass band. If, in addition, the changes are made in such a way that the ratio of the two elements changed remains the same, i.e., if the same percentage change is made to each of them, then the relative shape of the magnitude and phase characteristics of the network will remain unchanged. Thus this filter has the property that its cutoff frequency can be readily shifted.

The values of the network elements must be found by simultaneously equating the coefficients of equations (2) and (3). The resulting set of equations is non-linear, but solutions for the element values in terms of the constants a and $H$ are easily found. Such a set of solutions, together with other design information for the circuit is given in the circuits section of the handbook as Circuit No. 7.

The low pass circuit described above is sometimes modified to the configuration shown in Fig. 4-5. If this figure and Fig. 4-4 are compared, it is easily seen that the right terminal of the capacitor $C_{p}$ has been moved from the output of the operational amplifier (which is also the output of the controlled source) to the junction of the two resistors $R_{a}$ and $R_{b}$ whose values determine the gain of the controlled source. If the magnitude of the impedance of the capacitor $\mathrm{C}_{1}$ is much larger than the values of the resistors $R_{a}$ and $R_{b}$, so that the capacitor does not load the resistor network, then the voltage transfer function for the circuit shown in Fig. 4-5 is


The above equation illustrates one advantage of this configuration; i.e., none of the coefficients in the denominator are a function of $K$, the gain of the controlled source. Thus K may be varied, changing the gain of the network, without changing the frequency characteristics of the network. Such a change of gain may, of course, be accomplished by varying either $R_{a}$ or $R_{b}$ in Fig. 4-5. It should be noted that this circuit also has a disadvantage. Since there is no subtraction of terms in the coefficient of the first degree term in the denominator of equation (4), it is not possible to realize transfer functions of the type given in equation (2) in which the constant a has very small values.

The frequency normalized voltage transfer function for a high pass network is

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{H s^{2}}{s^{2}+a s+1} \tag{5}
\end{equation*}
$$

A realization for such a transfer function using a VCVS as the active element is shown in Fig, 4-6. The voltage transfer function is

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{K s^{2} C_{1} C_{2}}{s^{2} C_{1} C_{2}+s\left(C_{1} G_{2}+C_{2} G_{2}+C_{2} G_{1}-K C_{2} G_{1}\right)+G_{1} G_{2}} \tag{6}
\end{equation*}
$$



The same comments that were made with respect to the low pass network also apply here; namely, H is positive, and the pass band gain (in this case the high frequency gain) is equal to the gain $K$ of the VCVS and is not a function of the passive elements of the network. Similarly, the cutoff frequency can be changed by changing the values of the resistors or the capacitors, and, if equal percentage changes of the elements are made, the relative shape of the frequency characteristics of the network will remain unchanged. A set of formulas for determining the values of the network elements, together with other design information, is given in the circuits section of the handbook as Circuit No. 8.

In a manner similar to that which was done for the low pass network, the high pass configuration shown in Fig. 4-6 may be modified to the configuration shown in 4-7, by moving the right terminal of the resistor labeled $G_{1}$ from the output of the con-

trolled source to the junction of the two resistors $R_{a}$ and $R_{b}$. If the magnitude of the
resistance of the element $G_{1}$ is much larger than the values of the resistors $R_{a}$ and $R_{b}$, so that the gain-determining resistor network is not loaded, then the voltage transfer function for the circuit shown in Fig. 4-7 is

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{K_{s}{ }^{2} C_{1} C_{2}}{s^{2} C_{1} C_{2}+s\left(C_{1} G_{2}+C_{2} G_{2}\right)+G_{1} G_{2}} \tag{7}
\end{equation*}
$$

Since none of the denominator coefficients in equation (7) are functions of $K$, the gain of the controlled source, this gain may be varied, changing the gain of the network, without changing the frequency characteristics of the network. As in the low pass case, it is not possible to realize a voltage transfer function of the type given in equation (5) in which the constant a has very small values.

The frequency normalized voltage transfer function for a band pass network is

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{H_{s}}{s^{2}+a s+1} \tag{8}
\end{equation*}
$$

where $H$ is a positive real constant and $H / a$ is the gain in the pass band. There are several network configurations using a VCVS as the active element which may be used to realize such a transfer function. One such configuration which has been found to give good experimental results is shown in Fig. 4-8. The voltage transfer function in

terms of the elements is

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{s K C_{1} G_{1}}{s^{2} C_{1} C_{2}+s\left(C_{1} G_{3}+C_{2} G_{1}+C_{2} G_{2}+C_{1} G_{1}+C_{1} G_{2}-K C_{1} G_{2}\right)+G_{3}\left(G_{1}+G_{2}\right)} \tag{9}
\end{equation*}
$$

It should be noted that in the denominator, $K$, the gain of the controlled source, appears only in the coefficient of the first degree term. Thus, in high $Q$ realizations, we may adjust the real part of the pole locations by varying $K$ without appreciably affecting the resonant frequency. Since this adjustment may be made by varying the valve of either of the resistors associated with the operational amplifier reolization for the VCVS (see Fig. 4-3), this is a very convenient means of adjusting the $Q$ of the network. From equation (9) we see that such adjustments will also change the overall gain constant $H$, but this is usually a minor effect. The design information for this circuit is given in the circuits section of the handbook as Circuit No. 9.

## Other Realizations with Voltage-Controlled Voltage Sources

Up to this point, in this chapter we have described active RC circuit configurations which have required a non-inverting VCVS. There are also many network configurations which require the use of an inverting VCVS, i.e., one in which K, the gain constant, is negative. Such a source may be produced by an operational amplifier and


Fig. 4-9. Inverting Operational Amplifier VCVS"
the circuit shown in Fig. 4-9. Note that in this circuit the non-inverting terminal (terminal 2) of the operational amplifier is grounded. If we apply the virtual ground concept to this circuit we see that $E_{1}=I_{1} R_{a}$. Similarly,
$I_{2}=I_{1}$, and therefore $E_{2}=-I_{1} R_{b}$. Thus we, may write

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{-R_{b}}{R_{a}} \tag{10}
\end{equation*}
$$

For large values of $R_{a}$, this circuit may be considered as an ideal VCVS with a vaiue
of $K$ equal to $-R_{b} / R_{a}$. Since the resistor $R_{a}$ must be chosen large, this places a limit on the maximum gain value that $K$ may have, as well as on the frequency range over which the circuit will effectively model the ideal VCVS. Actual values of $R_{a}$ to be used will necessarily depend on the impedance level of the rest of the circuit, but values of the general order of 100 k ohms to 1 Megohm are not uncommon.

An example of the use of an inverting VCVS to realize a low pass transfer function as given in equation (2) is shown in Fig. 4-10. The voltage transfer function

for this network is

$$
\frac{E_{2}}{E_{1}}=\frac{-|K| G_{1} G_{2}}{s^{2} C_{1} C_{2}+s\left[C_{2}\left(G_{1}+G_{2}+G_{3}\right)+C_{1} G_{2}\right]+G_{2}\left(G_{1}+G_{3}+|K| G_{3}\right)}
$$

To avoid misinterpretation, the negative sign associated with the constant $K$ has been written into the equation, and the gain is expressed as a magnitude. In a similar manner the inverting VCVS may be used to realize a high pass network function of the type given in equation (5). The circuit is shown in Fig. 4-11. The voltage transfer function


$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{-|K| s^{2} C_{1} C_{2}}{s^{2} C_{2}\left(C_{1}+C_{3}+|K| C_{3}\right)+s\left[C_{2} G_{1}+G_{2}\left(C_{1}+C_{2}+C_{3}\right)\right]+G_{1} G_{2}} \tag{12}
\end{equation*}
$$

The gain $K$ has been shawn as a magnitude in the same manner that was dane in equatian (11). It is also thearetically possible to use the inverting VCVS to realize a band pass transfer function. However, the value of the gain that is required for the source for even a moderately high value of $Q$ is usually excessively high. Thus, such an application is af only limited value.

## Conclusions

The active RC circuits using controlled sources which have been described in this chapter have several advantages and disadvantages when compared to the circuits of the preceding chapters. First, a new and different variable appears in the transfer function equations, namely K, the gain of the controlled source. In the infinite-gain realizations given in Chapters 2 and 3, the network functions were almost completely unaffected by changes in the gain. In the realizations of this chapter, however, not only is there a strong dependence on the gain of the source, but this effect may actually be used to vary the properties of the network. Thus, the presence of the gain as a variable is both an advantage and a disadvantage, and the potential user of these circuits must be aware of both.

The high and low pass networks that use the controlled source present another new and useful characteristic; i.e., the pass band gain is independent of the element values. This has considerable application for the case where the cutaff frequencies are to be varied without changing the gain. It should also be noted that the circuits presented in this chapter have the same low output impedance that the infinite-gain realizations had. Thus, they may be used to drive other circuits without using isolating amplifier stages between them and the circuits that follow.

## CHAPTER 5

THE NIC IN ACTIVE RC CIRCUITS

In the first chapter of this handbook, the concept of an active device called an NIC (negative-immittance converter) was introduced. It was pointed out that an NIC could be used in connection with resistors and capacitors to realize a wide range of network functions. In this chapter we shall explore some of the properties of this device and see in detail how it can be used in active RC circuits.

The NIC (Negative-Immittance Converter)

Basically, the NIC is a two-port device which has the property that the impedance seen at either of its ports is the negative of the impedance connected to the other port. This "negative" action can come about in either of two ways. As a first way, the NIC can invert the direction of current flow with respect to that which would normally occur in a passive network, without disturbing the relative polarity of the input and output voltages. For example, consider a two-port network with a load impedance $Z$ as shown in Fig. 5-1. If a current $\mathrm{I}_{\mathrm{Z}}$ flows out of port 2 as shown, then we would expect that a current $I_{1}$ would flow into port 1 (assuming that the


Fig. 5-1. Two-Port Nelwork With Load output voltage $\mathbf{E}_{\mathbf{Z}}$ and the input voltage $\mathbf{E}_{\mathbf{1}}$ have the same polarity). If the two-port device somehow inverts one of these currents, then we have the situation wherein the
application of voltage $E_{\mathbf{l}}$ to port 1 causes a current to flow in a direction opposite to that shown for $l_{1} ;$ i.e., it opposes the applied voltage. In other words, the input impedance is negative. More formally, such a device can be defined in terms of the two-port variables


Fig. 5-2. The Port Variables for a
Fig. 5-2. The Port Variables for
Two-Port Network
shown in Fig. 5-2, and the equations

$$
\begin{align*}
& E_{1}=E_{2} \\
& I_{1}=K I_{2} \tag{1}
\end{align*}
$$

These equations may be said to define an ideal current-inversion negative-immittance converter or INIC for short. The constant $K$ is usually referred to as the "gain" of the INIC. In the next section we shall show that such a device can be easily realized by a single differential-input operational amplifier.

Let us investigate some properties of the INIC. First of all, consider the case where an impedance $Z_{2}$ is connected across the terminals of port 2 as shown in Fig. 5-3. The variables of port 2 are then constrained by the relation $E_{2}=-Z_{2} I_{2}$. Substituting this relation into equations (1) we see that

$$
\begin{equation*}
Z_{I N}=\frac{E_{1}}{I_{1}}=-\frac{Z_{2}}{K} \tag{2}
\end{equation*}
$$

Thus the input impedance at port 1 is


Fig. 5-3. INIC Input Circuit $1 / K$ times the negative of the impedance connected across port 2 . Thus we see that the magnitude of this negative impedance may be easily varied by changing the gain K of the INIC. Similarly, if an impedance $Z_{1}$ is connected across port 1 of the INIC


Fig. 5-4. INIC Output Circuil as shown in Fig. 5-4, then the impedance ${ }^{2}$ OUT ${ }^{\text {seen at port } 2 \text { may be shown }}$

$$
\begin{equation*}
Z_{O U T}=\frac{E_{2}}{I_{2}}=-K Z_{1} \tag{3}
\end{equation*}
$$

It should be noted that again a negative impedance is produced, but that in this case
the gain constant $K$ has the opposite effect to the one it had in equation (2). For the cose where $K=1$, the INIC will theoreticolly give the some results in either direction. Procticolly, however, stability considerotions bosed on the network configuration in which the INIC is used usuolly do not permit interchonging ports 1 and 2 of the INIC, even for the unity goin cose. More will be said obout this when discussing the reolization of the INIC.

A second way in which the "negotive" oction of an NIC can be brought about is by inverting the voltage while keeping the direction of current flow through the twoport device unchanged. In terms of the variables shown in Fig. 5-2, this type of NIC is defined by the relations

$$
\begin{align*}
& I_{1}=-I_{2}  \tag{4}\\
& E_{1}=-K E_{2}
\end{align*}
$$

Such a device may be referred to as an ideal voltage-inversion negative-immittance converter or VNIC for short. The constant $K$ is called the "gain" of the VNIC.

Space does not permit developing some of the other properties of the NIC, such as power relationships, impedance transformations, etc. The reader who is interested in learning more about this device should consult the references listed in Appen$\operatorname{dix} A$.

A Realization for the INIC

An INIC (ideal-current-inversion negative immittance converter) may be reolized by using a differential-input operational amplifier as the active element. The circuit is shown in Fig. 5-5. We may analyze this circuit by means of the virtual ground concept introduced in Chapter 1. This concept tells us that the voltage between terminals 1 and 2 of the operational omplifier is zero; thus we see that the voltages at the
two ports of the overall network shown in Fig. 5-5 are equal. Similarly, we know that no current flows into either of the amplifier terminals 1 and 2. Since the voltage across the two resistors $R_{1}$ and $R_{2}$ must be the same, we see that $I_{1} R_{1}=I_{2} R_{2}$; in other words, the ratio
 of the currents at the two ports is determined by the ratio of the resistors. We
 of the INIC may write the above relationships as

$$
\begin{align*}
E_{1} & =E_{2} \\
I_{1} & =\frac{R_{2}}{R_{1}} I_{2} \tag{5}
\end{align*}
$$

These are the same relations given in equations (1), with $K=R_{2} / R_{1}$. Thus we see that the circuit shown in Fig. 5-5 has the properties of an INIC, and that the gain constant $K$ may be easily adiusted by changing the values of either of the resistors $R_{1}$ or $R_{2}$.

Stability of the INIC

In the discussion given above, it was assumed that the voltage between terminals 1 and 2 of the operational amplifier was zero. This assumption greatly simplified the analysis of the circuit. Actually, there will always be a small voltage present between these terminals. In the INIC circuit this small voltage becomes a significant factor in determining whether the circuit will be stable. To see this, consider the case where a resistor $R_{a}$ is connected across port 1 of the INIC shown in Fig. 5-5, and another resistor $\mathrm{R}_{\mathrm{b}}$ is connected across port 2. The circuit may be redrawn using the model for the operational amplifier shown in Fig. 1-1 of Chapter 1. The result is shown in


Fig. 5-6. The constant $K$ shown in this figure represents the gain of the operational amplifier (not the gain of the INIC). In order for this circuit to be stable, the voltage $E_{2}-E_{1}$, even though it is small, must never be positive. It if should go positive, the feedback provided by the resistor networks will drive the operational amplifier into saturation. Therefore, the condition for stability is

$$
\begin{equation*}
E_{1} \geq E_{2} \tag{6}
\end{equation*}
$$

The voltages $E_{1}$ and $E_{2}$, however, if the currents $I_{1}$ and $I_{2}$ are zero, moy be expressed in terms of the four resistors shown in Fig. 5-6. Thus the inequality given in equation (6) may also be expressed as

$$
\begin{equation*}
\frac{R_{a}}{R_{1}+R_{a}} \geq \frac{R_{b}}{R_{2}+R_{b}} \tag{7}
\end{equation*}
$$

This may be reduced to

$$
\begin{equation*}
R_{a} R_{2}=R_{b} R_{1} \tag{8}
\end{equation*}
$$

For the case where $R_{1}$ and $R_{2}$ are equal, corresponding with an INIC gain of unity, we see that

$$
R_{a} \geq R_{b}
$$

is a necessary condition for stable operation of the INIC. To indicate this, it is customary to refer to port 1 as the open-circuit-stable (OCS) port of this INIC realization, and port 2 as the short-circuit stable (SCS) port. Similar restrictions may be developed for the situation where the INIC is imbedded in networks containing capacitors as well
as resistors, to ensure that the operational amplifier is not driven into saturation.

## The Basic INIC Circuit

There are several methods that have been proposed whereby a voltage transfer function may be realized by passive networks and an INIC. We shall discuss only one of these here, some others may be found in the references listed in Appendix A. The basic circuit configuration consists of two passive $R C$ networks labeled $A$ and $B$, and an INIC, and is shown in Fig. 5-7. First let us consider the cascade connection of the B


Fig. 5-7. Basic Voltage Transfer Circuit Using the INIC
network and the INIC. If we describe the properties of the B network by its y parameters, then the effect of the INIC is to invert the current at the output port of network $B$ and also multiply it by a constont. The y parameters of the cascaded connection of the B network and the INIC may thus be shown to be

$$
\begin{array}{ll}
y_{11}=y_{11 b} & y_{12}=y_{12 b} \\
y_{21}=-K_{12 b} & y_{22}=-K_{22 b} \tag{10}
\end{array}
$$

where $y_{11 b^{\prime}} y_{12 b}$, and $y_{22 b}$ are the $y$ parameters of the $B$ network by itself, and $K$ is the gain of the INIC. Note that for the cascade connection, $y_{12}$ is not equal to $y_{21}$, os would be the case for a passive network. When the A network is connected in parallel, its y parameters add to those given in equations (10). Thus, the y porameters for
the overall network are

$$
\begin{array}{ll}
y_{11}=y_{11} a+y_{11 b} & y_{12}=y_{12 a}+y_{12 b}  \tag{11}\\
y_{21}=y_{12 a}-K_{12 b} & y_{22}=y_{22 a}-K_{y_{22} b}
\end{array}
$$

Since the open-circuit voltage transfer function for an arbitrary network is simply the ratio $-y_{21} / y_{22}$, we may write the voltage transfer function of the basic circuit as

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{-y_{12 a}+K_{y_{12} b}}{y_{22 a}-K_{y_{22} b}} \tag{12}
\end{equation*}
$$

In the references, some specific rules are developed for determining the parameters of the component passive networks $A$ and $B$ in such a woy that they can be realized by passive RC networks for a given desired vol tage transfer function. We shall not develop these rules here. In the next section, however, we shall show the results of applying such developments to obtain the same three types of transfer functions that we realized in earlier chapters.

## Network Design

The basic circuit configuration described in the preceding section may be used to obtain various networks. The first network that we shall consider here is the low pass network having a frequency normalized voltage transfer function of the form

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{H}{s^{2}+a s+1} \tag{13}
\end{equation*}
$$

A network configuration which realizes this voltage transfer function is shown in Fig. 5-8. The elements $C_{2}$ and $G_{3}$ form the $A$ network of Fig . 5-7. For this network we have the following $y_{12}$ and $y_{22}$ parameters:

$$
\begin{align*}
& y_{12 a}=-G_{3}  \tag{14}\\
& y_{22 a}=s C_{2}+G_{3}
\end{align*}
$$



The elements $C_{1}, G_{1}$, and $G_{2}$ shown in the figure form network $B$. This network has the $y$ parameters

$$
\begin{gather*}
y_{12 b}=\frac{-s C_{1} G_{1}}{s C_{1}+G_{1}} \\
y_{22 b}=\frac{s C_{1}\left(G_{1}+G_{2}\right)+G_{1} G_{2}}{s C_{1}+G_{1}} \tag{15}
\end{gather*}
$$

Substituting the expressions from equations (14) and (15) into equation (12), and rearranging terms, we find that the voltage transfer for the circuit shown in Fig. 5-8 is

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{s C_{1}\left(G_{3}-K G_{1}\right)+G_{1} G_{3}}{s^{2} C_{1} C_{2}+s\left[C_{2} G_{1}-K C_{1} G_{2}+C_{1}\left(G_{3}-K G_{1}\right)\right]+G_{1}\left(G_{3}-K G_{2}\right)} \tag{16}
\end{equation*}
$$

If we constrain the elements of this network so that $G_{3}$ equals $K G_{1}$, then equation (16) reduces to

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{G_{1} G_{3}}{s^{2} C_{1} C_{2}+s\left(C_{2} G_{1}-K C_{1} G_{2}\right)+G_{1}\left(G_{3}-K G_{2}\right)} \tag{17}
\end{equation*}
$$

If we compare this equation with equation (13) we see that the constant $H$ is positive; i.e., no signal inversion is provided by this circuit. Equations relating the values of the network elements to the constants $a$ and $H$ of equation (13) have been tabulated, and, together with other design information on this circuit, they are presented in the circuits section of this handbook as Circuit No. 10.

In a similar fashion we may use passive RC networks and an INIC to realize a
high pass network. Such a network will have a voltage transfer function of the form

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{s^{2} H}{s^{2}+a s+1} \tag{18}
\end{equation*}
$$

A network configuration which realizes this function is shown in Fig. 5-9. The ele-


Fig. 5-9. INIC High Pass Active Filter
ments $C_{2}$ and $G_{2}$ shown in this figure constitute network $A$ of Fig. 5-7. The $y$ parameters for this network are

$$
\begin{align*}
& y_{12 a}=-s C_{2}  \tag{19}\\
& y_{22 a}=s C_{2}+G_{2}
\end{align*}
$$

The elements $C_{1}$ and $G_{1}$ constitute network $B$. Its y parameters are

$$
\begin{equation*}
y_{22 b}=-y_{12 b}=\frac{{ }^{s} C_{1} G_{1}}{{ }_{5} C_{1}+G_{1}} \tag{20}
\end{equation*}
$$

If we substitute the expressions from equations (19) and (20) into equation (12), after rearranging terms, we find that the voltage transfer function for the overall network is

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{s^{2} C_{1} C_{2}+s G_{1}\left(C_{2}-K C_{1}\right)}{s^{2} C_{1} C_{2}+s\left[C_{1} G_{2}+G_{1}\left(C_{2}-K C_{1}\right)\right]+G_{1} G_{2}} \tag{21}
\end{equation*}
$$

If we constrain the elements of this network so that $C_{2}$ equals $K C_{1}$, then equation (21) reduces to

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{s^{2} C_{1} C_{2}}{s^{2} C_{1} C_{2}+s C_{1} G_{2}+G_{1} G_{2}} \tag{22}
\end{equation*}
$$

We see that the constant H of equation (18) is pasitive. The design information for this circuit is given in the circuits section of the handbook as Circuit No. 11.

As a final example of circuit design using an INIC, consider the band pass network with a frequency normalized voltage transfer function of the form

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{-s H}{s^{2}+a s+1} \tag{23}
\end{equation*}
$$

A circuit which realizes this function is shown in Fig. 5-10. The elements $C_{2}$ and $G_{2}$


Fig. 5-10. INIC Band Pass Active Filter
comprise network A of Fig. 5-7. The $y$ parameters are

$$
\begin{align*}
& y_{12 a}=0 \\
& y_{22 a}=s C_{2}+G_{2} \tag{24}
\end{align*}
$$

The elements $C_{1}$ and $G_{1}$ comprise network $B$. Its y parameters are

$$
\begin{equation*}
y_{22 b}=-y_{12 b}=\frac{s C_{1} G_{1}}{{ }_{s C_{1}}+G_{1}} \tag{25}
\end{equation*}
$$

The overall voltage transfer function for the circuit moy be found by inserting the relations of (24) and (25) into equation (12). After rearranging terms we obtain

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{-s K C_{1} G_{1}}{s^{2} C_{1} C_{2}+s\left(C_{1} G_{2}+C_{2} G_{1}-K C_{1} G_{1}\right)+G_{1} G_{2}} \tag{26}
\end{equation*}
$$

Note that the constant H of equation (23) is positive for this circuit, and that the circuit provides a signal inversion.

The voltage transfer function given in equation (26) points out some of the
interesting properties that the INIC realization of the band pass network possesses. In the denominator, it should be noted that the INIC gain constant appears only in the coefficient of the first degree term. Thus, in high $Q$ realizations, it is possible to adjust the real part of the pole positions, i.e., the $Q$ of the circuit, without significantly changing the magnitude of the pole positions, i.e., without significantly changing the resonant frequency. At a given frequency, therefore, we can control the bandwidth of the network by changing the gain of the INIC, i.e., by varying the values of either of the resistors associated with the operational amplifier INIC realization. A second interesting feature of this network is that by choosing a non-unity INIC gain constant, it is possible to design the network so that both resistive elements have the same value, and both capacitive elements have the same value. Thus, the problem of obtaining accurately specified passive element values is considerably minimized. Finally, if both resistors have the same value and are varied the same amount, the resonant frequency of the network is changed, although the $Q$ of the network remains invariant. The same is true if the values of the capacitors are changed in the indicated manner. Thus we have a circuit where a single resistor may be changed to vary the bandwidth, and a pair of resistors may be changed to vary the resonant frequency, and the two effects do not interact. The design equations and other information for this circuit are given in the circuits section of the handbook as Circuit No. 12.

## Conclusions

In this chapter, an entirely new "breed" of circuits has been presented, namely, circuits which use an INIC as the active element. The INIC can, of course, also be used to produce single negative-valued elements which in turn can be used to compensate for dissipation, to reduce input capacitance, etc. We have used the INIC as an integral portion of circuits realizing low pass, high pass, and band pass voltage
transfer functions. The band pass circuit is an especially attractive one because af the minimum number of elements used, the fact that all elements of a given kind have the same value, and the ease with which the $Q$ and the resonant frequency of the network may be adiusted.

It shauld be noted that all the circuits given in this chapter share a common disadvantage; namely, their output impedance is not zero. Therefore, if such networks are cascaded, suitable isolating stages must be used to separate them. The advantage of ease of adjustment provided by some of these networks must therefore be weighed against the disodvantage of the requirement for the extra circuitry involved in isolating the filtering stages. A more detailed comparison af the advantages and disadvantages of the various realization schemes that have been presented in Chapters 2 through 5 is given in Chapter 7.

## CHAPTER 6

## ANOTHER ACTIVE DEVICE - THE GYRATOR

In addition to the active elements which have been introduced in the preceding chapters, there is another one that deserves mention. It is called a gyrator. In this chapter we shall give a short introduction to the properties and potential uses of this element.

## Definition of a Gyrator

A gyrator is a non-reciprocal two-port device defined by the equations

$$
\begin{align*}
& I_{1}=G E_{2}  \tag{1}\\
& I_{2}=-G E_{1}
\end{align*}
$$

This two-port device is usually considered to have a common ground, and, in this case, the gyrator is represented by the symbol shown in Fig. 6-I. The constant $G$ is called the gyration conductance. The reference arrow drawn inside the circle in the figure indicates that the gyration action from terminal 1 to terminal 2 with terminal 3 common (as shown), is


Fig-6-1. Gyrator Symbol the same as that which would occur from terminal 2 to terminal 3 if terminal 1 was used as the common terminal. Similarly, it is the same as that which would occur from terminal 3 to terminal 1 if terminal 2 was used as the common terminal. We shall see what
this gyration action consists of in the paragraphs that follow.

## Properties of the Gyrator

From equations (1) we see that the $y$ parameters of a gyrator are

$$
\begin{array}{ll}
y_{11}=0 & y_{12}=G \\
y_{21}=-G & y_{22}=0 \tag{2}
\end{array}
$$

If we calculate the input admittance $\mathrm{Y}_{\mathrm{IN}}$ of a two-port network, defined by its y parameters, when an admittance $Y_{2}$ is connected across the terminals of port 2, as shown in Fig. 6-2 we obrain

$$
\begin{equation*}
Y_{1 N}=y_{11}-\frac{y_{12} y_{21}}{y_{22}+Y_{2}} \tag{3}
\end{equation*}
$$

For the case where the two-port is a gy-


Fig. 6-2. The Input Admitronce of o Terminated Two-Port Network

$$
\begin{equation*}
Y_{I N}=G^{2} / Y_{2} \tag{4}
\end{equation*}
$$

This is the input admittance of a gyrator terminated in an admittance $\mathrm{Y}_{2}$. Equation (4) tells us that if $Y_{2}$ is a capacitor, then, at part 1 of the gyrator we see a two-terminal behavior exactly paralleling that of an inductor. In other words, a gyrator can be used to "gyrate" a capacitor into an inductor. Thus, with resistors, capacitors, and gyrators, we can achieve any network realization which can be achieved with resistors, capacitors, and inductors. If we can obtain a gyrator with a small value of $G$, then we can gyrate very small capacitors into very large inductors, a most useful feat:

One other property of the gyrator may be of interest. This concerns the power
relationships at the terminals of the gyrator. For any two-port device, the total instantaneous power consumed by the device is

$$
\begin{equation*}
p(t)=e_{1}(t) i_{1}(t)+e_{2}(t) i_{2}(t) \tag{5}
\end{equation*}
$$

Substituting relations equivalent to those given in equation (1), but in terms of functions of time, into the above equation, we see that

$$
\begin{equation*}
p(t)=e_{1}(t) i_{1}(t)-e_{1}(t) i_{1}(t)=0 \tag{6}
\end{equation*}
$$

Thus we see that the gyrator neither adds energy to the circuit in which it is used, nor consumes it. As such, its terminal properties are those of a lossless passive network component. We shall see, however, that its realization inevitably requires the use of active elements.

## A Gyrator Realization

There are several ways of realizing a gyrator. One of the methods uses two
INICs. Consider the circuit shown in Fig. 6-3. It is easily shown that this circuit has


Fig. 6-3. Gyrator Realization Using Two INIC's.
the $y$ parameters given in equation (2), and thus functions as a gyrator. The values of the resistors (in mhos) shown in the figure determine the value of the gyration conductance. References to some other methods for realizing gyrators are given in Appendix A.

## Circuit Realizations

There are considerably fewer results available in the literature regarding the use of gyrators and RC circuits for the realization of transfer functions than there are for any of the other classes of networks that have been discussed in this handbaok. Since the state of the art is relatively new, and since the active elements are considerably more complicated than any which have been discussed in the previous chapters, we shall not present general circuits for the realization of low pass, high pass and band pass circuits as was done in those chapters. To give an example of one form that such realizations may take, however, consider the circuit shown in Fig. 6-4.


Fig. 6-4. Gyrotor Band Poss Active Filter
The voltage transfer function for this circuit is

$$
\begin{equation*}
\frac{E_{2}}{E_{1}}=\frac{s}{s^{2}+0.2 s+1.01} \tag{7}
\end{equation*}
$$

Thus, the circuit realizes a band pass voltage transfer function.

## Conclusions

Due to the complicated nature of the realizations for the gyrator, this network element has not achieved a wide usage at this time. In addition to the disadvantage of complexity, realizations which use it as the active element have two other disadvantages in that their output impedance will not be zero, and they will be capable only of
the gain that a passive RLC circuit is capable of. Despite these disadvantages, the gyrator also has some potential advantages. First, its lossless nature provides a theoretical bar to circuit instability, since if no power is being supplied to the circuit, instability cannot occur. Second, since capacitors in general have a higher quality factor (lower dissipation) than inductors, gyration of a capacitor may produce a better inductor than those which are readily available. Finally, the possibility of using a gyrator for impedance multiplication implies the ability to realize very low frequency circuits without the need of relatively large-valued reactive elements.

## CHAPTER 7

## A SUMMARY

In Chapters 2 through 5 of this handbook four different active RC synthesis techniques were presented. These were the infinite-gain single-feedback technique, the infinite-gain multiple-feedback technique, a technique using controlled sources, and a technique using negative-immittance converters. The various methods were all applied to the realization of low pass, high pass, and band pass voltage transfer functions. At this point the reader may well ask, "If I want to realize a low pass function (or a high pass or a band pass one) which of the methods is best?" The answer to the question, of course, depends on how the word "best" is defined. If "best" means fewest elements, then the infinite-gain single feedback technique is certainly eliminated. If the network is to be cascaded with other networks, then the NIC approach is probably not a good one. Thus, the answer to such a question depends on the details of the application, which will vary considerably from one situation to another.

The purpose of this handbook has been to give the prospective user several different techniques for each of the filter realizations, in order to permit him flexibility in selecting the technique that more nearly meets his specific application. To provide a further guide to such a choice, some of the odvantages and disadvantages of the various realization techniques are summarized in Table I. A study of this table will provide a good review of the material which has gone before.

Table :
Summary of the Advantages and Disadvantages
of the Various Realization Techniques

|  |  | Realization | Technique |  |
| :---: | :---: | :---: | :---: | :---: |
| Property | InfiniteGain SingleFeedback | InfiniteGain MultipleFeedback | Controlled Source | NegativeImmittance Converter |
| Minimal number of network elements | - | + | + | + |
| Ease of adjustment of characteristics | - | 0 | 0 | + |
| Stability of characteristics | + | + | - | - |
| Low output impedance | + | + | + | - |
| Presence of summing input | + | - | - | - |
| Relatively high gain available | + | - | + | + |
| Low spread of element values | + | - | + | + |
| High- $Q$ realizations possible | + | - | + | + |
| + indicates the realization is superior for the indicated property |  |  |  |  |
| 0 indicates the rea <br> - indicates the realiz | tion is aver | for the indi for the indi | d property |  |

This concludes our introduction to the wonderful world of active RC networks, a world without inductors. It is hoped that the reader will find the techniques which have been presented in this handbook useful, and that he will be able to apply them to his own filtering problems. Needless to say, the Applications Engineering Section of Burr-Brown would welcome any questions or comments that the reader might have on the material of this booklet or on any other operational amplifier application. Feel free to call us at any time.


# SECTION II <br> CIRCUITS 

## INTRODUCTION

All of the examples shown in this section have been proven on the bench with the results shown. In addition, the transfer functions are given along with methods for determining component values and comments on the nominal range of values. We trust that these "hints and kinks" will enable you to readily modify the circuits given for your application.

Two possible points of confusion, however, deserve mention. In the theory section we have, for simplicity, worked with frequency normalized transfer functions $\omega_{0}\left(=2 \pi f_{0}\right)$ and conductance $G(=1 / R)$. In moving on to the real world of circuits, we find it convenient to "un-normalize" the transfer functions $f_{0}\left(=\omega_{0} / 2 \pi\right)$ and deal with resistance $R(=1 / G)$. Having made this shift explicit, we trust it will not create problems.

Please note: since the design formulas give correct values to within an arbitrary constant, an impedance denormalization constant, $k$, is included and is to be chosen for convenience.

It is our hope that the circuits presented will trigger the idea that develops into your circuit. Sharing your circuit with us will enable us to share it with other engineers. Similarly, we would welcome the opportunity to share our experience and the latest advancements in network theory and amplifiers with you.

CIRCUIT 1
Single Feedback


FORM OF TRANSFER FUNCTION

$$
\begin{gathered}
\frac{E_{2}}{E_{1}}=\frac{-H \omega_{0}^{2}}{s^{2}+a \omega_{0} s+\omega_{0}^{2}} \\
\text { where: } \omega_{0}=2 \pi f_{0} \\
A_{0}=H
\end{gathered}
$$

Choose:
a (= $\sqrt{2}$ for "maximally flat" , 'Butterworth' response)
Let:

$$
b=(2.5-a)
$$

Calculate: $\quad C_{a}=\frac{4 H}{a^{2}} \frac{k}{2 f_{0}}$

$$
R_{\mathrm{a}}=\frac{\mathrm{a}}{2 \mathrm{Hk}} \text { (two such resistors) }
$$

$C_{1 b}=\frac{k}{2 \pi f_{0}}$

$$
\mathrm{R}_{1 b}=\frac{1}{b} \cdot \frac{1}{k}
$$

$$
c_{2 b}=\frac{b^{2}}{a b-} \frac{k}{2 \pi f_{0}}
$$

$$
R_{2 b}=\left(a-\frac{1}{b}\right) \cdot \frac{1}{k}
$$


$\left|\frac{E_{2}}{\mathrm{E}_{1}}\right| \begin{gathered}(\mathrm{db}) \\ 20\end{gathered}$

EXAMPLE

$$
\begin{array}{ll}
\text { Want: } & f_{0}=200 \mathrm{c} / \mathrm{s} \\
& \left|A_{o}\right|=100(40 \mathrm{db}) \\
& a=\sqrt{2} \\
\text { Calculate: } & \\
& C_{a}=1.59 \mu \mathrm{f} \\
& C_{1 b}=.00796 \mu \mathrm{f} \\
& C_{2 b}=.0175 \mu \mathrm{f} \\
& R_{a}(2)=707 \Omega \\
& R_{1 b}=92.1 \mathrm{~K} \\
& R_{2 b}=49.3 \mathrm{~K}
\end{array}
$$

## CIRCUIT 2

Network B
Single Feedback
High Pass



Choose
$a(=\sqrt{2}$ for "maximally flat" response, $40 \mathrm{db} /$ decade rolloff) $a>\sqrt{2}$, slower rolloff $a<\sqrt{2}$, peaking, faster rolloff

Let:
$b=(2.5-a)$
Calculate: $\quad C_{1 a}=C_{2 a}=2 H \frac{k}{2 \pi f_{a}}$

$$
R_{a}=\frac{1}{4 H_{\alpha}} \cdot \frac{1}{k}
$$

$C_{1 b}=\frac{k}{2 \pi f_{0}}$
$R_{1 b}=\frac{1}{b} \cdot \frac{1}{k}$
$C_{2 b}=\frac{b^{2}}{a b-1} \frac{k}{2 \pi f_{o}}$
$R_{2 b}=\left(a-\frac{1}{b}\right) \cdot \frac{1}{k}$


## EXAMPLE

$$
\begin{array}{ll}
\text { Want: } & f o=20 \mathrm{c} / \mathrm{s} \\
& \left|A_{0}\right|=10(20 \mathrm{db}) \\
& a=\sqrt{2} \\
\text { Choose: } & k=10^{-5} \\
\text { Calculate: } & C_{1 a}=C_{2 a}=1.592 \mu \mathrm{f} \\
& C_{1 b}=.0796 \mu \mathrm{f} \\
& C_{2 b}=.175 \mu \mathrm{f} \\
& R_{a}=1.77 \mathrm{~K} \Omega \\
& R_{1 b}=92.1 \mathrm{~K} \\
& R_{2 b}=49.3 \mathrm{~K}
\end{array}
$$

CIRCUIT 3
Network B
Single Feedback
Band Pass



FORM OF TRANSFER FUNCTION

$$
\begin{gathered}
\frac{E_{2}}{E_{1}}=\frac{-H \omega_{0} s}{s^{2}+\alpha \omega_{0}+\omega_{0}^{2}} \\
\text { where: } \omega_{0}=2 \pi f_{0} \\
A_{0}=H / a \\
Q=1 / a
\end{gathered}
$$

Let:

## Calculate:

$$
\begin{array}{lll}
a=1 / Q & C_{a}=H \frac{k}{2 \pi f_{0}} & R_{a}=\frac{1}{H} \cdot \frac{1}{k} \\
b=(2.5-a) \frac{1+a}{2+a} & C_{1 b}=b \frac{k}{2 \pi f_{0}} & R_{1 b}=\frac{1}{b} \cdot \frac{1}{k} \\
H=A_{o} / Q & C_{2 b}=\frac{b}{b-1} \cdot \frac{k}{2 \pi f_{0}} & R_{2 b}=\frac{(b-1)}{b} \cdot \frac{1}{k} \\
& C_{3 b}=\frac{b^{2}}{(b-1)(1+a)} \cdot \frac{k}{2 \pi f_{0}} & R_{3 b}=\frac{(b-1)(a+1)}{b^{2}} \cdot \frac{1}{k}
\end{array}
$$

Note: except for frequency denormalizing, corresponding R's and C's are inversely related.


EXAMPLE

$$
\begin{array}{ll}
\text { Want: } & f_{o}=100 \mathrm{c} / \mathrm{s} \\
& \left|A_{o}\right|=10(20 \mathrm{db}) \\
\text { Choose: } & \mathrm{k}=10^{-4} \\
\text { Calculate: } & \mathrm{C}_{\mathrm{a}}=.159 \mu \mathrm{f} \\
& \mathrm{C}_{1 b}=.200 \mu \mathrm{f} \\
& \mathrm{C}_{2 b}=.778 \mu \mathrm{f} \\
& \mathrm{C}_{3 b}=.888 \mu \mathrm{f} \\
& \mathrm{R}_{\mathrm{a}}=10 \mathrm{~K} \Omega \\
& \mathrm{R}_{1 \mathrm{~b}}=7.96 \mathrm{~K} \\
& \mathrm{R}_{2 b}=2.04 \mathrm{~K} \\
& \mathrm{R}_{3 b}=1.79 \mathrm{~K}
\end{array}
$$

CIRCUIT 4
Multiple Feedback Low Pass


$$
\frac{E_{2}}{E_{1}}=\frac{-H \omega_{0}^{2}}{s^{2}+a \omega_{0} s+\omega_{0}{ }^{2}}
$$

$$
A_{0}=H
$$

$$
\omega_{0}=2 \pi f_{0}
$$



TRANSFER FUNCTION

$$
\frac{E_{2}}{E_{1}}=\frac{-1 / R_{1} R_{3}}{s^{2} C_{2} C_{5}+s C_{5}\left(1 / R_{1}+1 / R_{3}+1 / R_{4}\right)+1 / R_{3} R_{4}}
$$

Choose:
$C=\frac{k}{2 \pi f_{0}},($ defines $k)$
Calculate: $\quad C_{5}=C=\frac{k}{2 \pi f_{0}}$
$C_{2}=\frac{4}{a^{2}}(H+1) \frac{k}{2 \pi f_{0}}$
a ( $=\sqrt{2}$ for "maximally flat" response, $40 \mathrm{db} /$ decade rolloff)
$R_{1}=\frac{a}{2 H k}$
$R_{3}=\frac{a}{2(H+1) k}$
$R_{4}=\frac{a}{2 k}=H R_{1}$


EXAMPLE
Want :

$$
\begin{aligned}
& f_{0}=100 \mathrm{c} / \mathrm{s} \\
& A_{0}=10(20 \mathrm{db}) \\
& a=\sqrt{2}
\end{aligned}
$$

Choose: $\quad C=0.1 \mu f$
Calculate: $k=6.28 \times 10^{-5}$
$C_{5}=.1 \mu \mathrm{f}$
$C_{2}=2.2 \mu f$
$R_{1}=1125 \Omega$
$R_{3}=1020 \Omega$
$R_{4}=11.25 K$


TRANSFER FUNCTION

$$
\frac{E_{2}}{E_{1}}=\frac{-s^{2} C_{1} C_{3}}{s^{2} C_{3} C_{4}+s\left(C_{1}+C_{3}+C_{4}\right) / R_{5}+1 / R_{2} R_{5}}
$$

Choose:

$$
\begin{aligned}
& C=\frac{k}{2 \pi f_{0}},(\text { defines } k) \\
& H=\left|A_{0}\right| \\
& a \quad(=\sqrt{2} \text { for "maximally flat" } \\
& \quad \text { response, } 40 \mathrm{db} / \text { decade rolloff })
\end{aligned}
$$

$$
\text { Calculate: } \quad C_{1}=C_{3}=\frac{k}{2 \pi F_{0}}=C
$$

$$
C_{4}=C / H
$$

$$
\begin{aligned}
& R_{2}=\frac{a}{k(2+1 / H)} \\
& R_{5}=\frac{H(2+1 / H)}{a k}
\end{aligned}
$$



EXAMPLE

$$
\begin{array}{ll}
\text { Want: } & f_{0}=0.1 \mathrm{c} / \mathrm{s} \\
& A_{0}=1(0 \mathrm{db}) \\
& a=\sqrt{2} \\
\text { Choose: } & \epsilon=1 Q_{\mu} \mathrm{f} \\
\text { Calculate: } & k=6.28 \times 10^{-6} \\
& C_{1}=C_{3}=10 \mu \mathrm{f} \\
& C_{4}=10_{\mu} \\
& R_{2}=75.2 \mathrm{~K} \\
& R_{5}=338 \mathrm{~K}
\end{array}
$$

CIRCUIT 6
Multiple Feedback Band Pass


$$
\frac{E_{2}}{E_{1}}=\frac{-H \omega_{0} s}{s^{2}+a \omega_{0} s+\omega_{0}^{2}}
$$

$$
\begin{aligned}
& A_{0}=H / a \\
& Q=1 / a \\
& \omega_{0}=2 \pi f_{0}
\end{aligned}
$$



TRANSFER FUNCTION

$$
\frac{E_{2}}{E_{1}}=\frac{-s C_{3} / R_{1}}{s^{2} C_{3} C_{4}+s \quad 1 / R_{5}\left(C_{3}+C_{4}\right)+1 / R_{5}\left(1 / R_{1}+1 / R_{2}\right)}
$$

Choose:

$$
\left.C=\frac{k}{2_{\pi} f_{0}}, \text { (defines } k\right)
$$

$$
a=1 / Q
$$

$$
H=\left|A_{0}\right| / Q
$$

Calculate: $\quad C_{3}=C_{4}=\frac{k}{2 \pi f_{o}}=C$

$$
\begin{aligned}
& R_{1}=1 / H k \\
& R_{2}=\frac{1}{(2 Q-H)} \\
& R_{5}=\frac{2 Q}{k}
\end{aligned}
$$




Note: 100 sec. count used to determine frequency

EXAMPLE

$$
\begin{array}{ll}
\text { Want: } & f_{0}=1.6 \mathrm{c} / \mathrm{s} \\
& Q=10 \\
\text { Choose: } & C=10 \mu \mathrm{f} \\
\text { Calculate: } & k=10-4 \\
& A_{0}=10(20 \mathrm{db}) \\
& \\
& H=C_{4}=10 \mu \mathrm{f} \\
& R_{1}=10 \mathrm{~K} \Omega \\
& R_{2}=527 \Omega \\
R_{5}=200 \mathrm{~K}
\end{array}
$$

## CIRCUIT 1



$$
\frac{E_{2}}{E_{1}}=\frac{+H \omega_{0}^{2}}{s^{2}+\alpha \omega_{0} s+\omega_{0}^{2}} \quad A_{0}=H=K
$$



TRANSFER FUNCTION

$$
\frac{E_{2}}{E_{1}}=\frac{K / R_{1} R_{2}}{s^{2} C_{1} C_{2}+s C_{2} / R_{1}+C_{2} / R_{2}+C_{1} / R_{2}(1-K)+1 / R_{1} R_{2}}
$$

Choose: $\quad C_{1}=\frac{k}{2 \pi f_{0}} \quad$ (defines $k$ )
Calculate: $m=\frac{a^{2}}{4}+(K-1)$
$K=A_{0}$
$C_{2}=m C_{1}=\frac{m k}{2 \pi_{o}}$
a ( $=\sqrt{2}$ for "maximally flat" response)

$$
R_{1}=\frac{2}{a k}
$$

$$
R_{2}=\frac{\alpha}{2 m k}
$$

Note: choose $R$ and $(K-1) R$ such that $R+(K-1) R=100 K$,or use a 100 K potentiometer.



EXAMPLE

$$
\begin{aligned}
& \text { Want: } \quad f_{0}=30 \mathrm{c} / \mathrm{s} \\
& a=\sqrt{2} \\
& C_{1}=.1 \mu \mathrm{f} \\
& A_{o}=10(20 \mathrm{db}) \\
& \text { Calculate: } m=9.5 \\
& k=1.89 \times 10^{-5} \\
& C_{2}=.95 \mu \mathrm{f} \\
& R_{1}=75 K \\
& R_{2}=3.94 \mathrm{~K} \\
& K=10
\end{aligned}
$$

CIRCUIT 8


$$
\frac{E_{2}}{E_{1}}=\frac{H_{s}^{2}}{s^{2}+a \omega_{0} s+\omega_{0}^{2}}
$$

where: $A_{0}=K$
$\omega_{o}=2 \pi f_{0}$


TRANSFER FUNCTION
$\frac{E_{2}}{E_{1}}=\frac{K s^{2} C_{1} C_{2}}{s^{2} C_{1} C_{2}+s\left[C_{2} / R_{2}+C_{1} / R_{2}+(1-K) C_{2} / R_{1}\right]+1 / R_{1} R_{2}}$

Choose: a (= $\sqrt{2}$ for "maximally flat", 'Butterworth' response)

$$
\begin{aligned}
& C_{1}=\frac{k}{2 \pi f_{0}} \quad \text { (defines } k \text { ) } \\
& c_{2}=C_{1}
\end{aligned}
$$

Calculate: $R_{1}=\frac{a+\sqrt{a^{2}+8(K-1)}}{4 k}$

$$
R_{2}=\frac{4}{a+\sqrt{a^{2}+8(K-1)}} \cdot \frac{1}{k}
$$




## EXAMPLE

$$
\begin{array}{ll}
\text { Want: } & f_{0}=300 \mathrm{c} / \mathrm{s} \\
& \\
& =\sqrt{2} \\
& A_{0}=100(40 \mathrm{db}) \\
& C_{1}=.1 \mu \mathrm{f}=\mathrm{C}_{2} \\
\text { Calculate: } & k=1.884 \times 10^{-4} \\
& R_{1}=39.3 \mathrm{~K} \\
R_{2}=717 \Omega \\
K & =100
\end{array}
$$



Note: choose $R$ and $(K-1) R$ such that $R+(K-1) R=100 K$ or use a 100 K potentiometer.


EXAMPLE

$$
\begin{aligned}
& \text { Want: } \quad f_{0}=300 \mathrm{c} / \mathrm{s} \\
& Q=10 \\
& C_{1}=.02 \mu f
\end{aligned}
$$

Calculate: $K=2.133$
$A_{0}=21.33 \quad(26.6 \mathrm{db})$
$C_{1}=.02 \mu \mathrm{f}$
$\mathrm{C}_{2}=.01 \mu \mathrm{f}$
$k=37.8 \times 10^{-6}$
$\mathrm{R}_{1}=53 \mathrm{~K}$
$\mathrm{R}_{2}=17.7 \mathrm{~K}$
$R_{3}=106 \mathrm{~K}$

CIRCUIT 10

$\frac{\text { TRANSFER FUNCTION }}{E_{1}}=\frac{s C_{1}\left(1 / R_{3}-K / R_{1}\right)^{0}}{E_{1}}+1 / R_{1} R_{3}-1 / R_{1}\left(1 / R_{3}-K / R_{2}\right)$
Choose: $\quad$ a ( $=\sqrt{2}$ for"maximally flat" response)
Let: $\quad K=1$ and $R_{1}=R_{3}$
Calculate: $\quad b=\frac{a}{2}+\sqrt{\frac{a 2}{4}+\left(A_{o}-1\right)}$
$C_{2}=\frac{k}{2 \pi f_{0}} \quad$ (defines $k$ )
$C_{1}=\frac{A_{0}}{b^{2}} \frac{k}{2 \pi f_{0}}=\frac{A_{0}}{b^{2}} C_{2}$
$R_{1}=R_{3}=\frac{b}{A_{0} k}$
$R_{2}=\frac{b}{\left(A_{o}-1\right) k}$


## EXAMPLE

$$
\begin{array}{ll}
\text { Want: } & f_{0}=4 \mathrm{c} / \mathrm{s} \\
& a=\sqrt{2} \\
& A_{0}=10(20 \mathrm{db}) \\
& C_{2}=1 \mu \mathrm{f} \\
\text { Calculate: } & b=3.79 \\
& k=2.51 \times 10^{-5} \\
& C_{1}=.696 \mu \mathrm{f} \\
& R_{1}=R_{3}=15.1 \mathrm{~K} \\
& R_{2}=16.8 \mathrm{~K}
\end{array}
$$

INIC
High Pass



## TRANSFER FUNCTION

$$
\frac{E_{2}}{E_{1}}=\frac{s^{2} C_{1} C_{2}+s\left(C_{2}-K C_{1}\right) / R_{1}}{s^{2} C_{1} C_{2}+s\left[C_{1} / R_{2}+\left(C_{2}-K C_{1}\right) / R_{1}\right]+1 / R_{1} R_{2}}
$$

Note: Only unity gain possible with this configuration,

Choose: a ( $=\sqrt{2}$ for "maximally flat" 'Butterworth' response)
Calculate: $\mathrm{C}_{1}=\mathrm{C}_{2}=\frac{\mathrm{k}}{2 \pi f_{\mathrm{o}}} \quad$ (defines $k$ )
$R_{1}=\frac{\alpha}{k}$
$R_{2}=\frac{1}{a k}$
$K=1$


EXAMPLE

$$
\begin{array}{ll}
\text { Want: } & f_{0}=160 \mathrm{c} / \mathrm{s} \\
& a=\sqrt{2} \\
& C_{1}=.1 \mu \mathrm{f}=\mathrm{C}_{2} \\
\text { Calculate: } & \mathrm{k}=1.007 \times 10^{-4} \\
& \mathrm{R}_{1}=14.1 \mathrm{~K} \\
& \mathrm{R}_{2}=7.02 \mathrm{~K}
\end{array}
$$

CIRCUIT 12
INIC
Band Pass

TRANSFER FUNCTION

$$
\frac{E_{2}}{E_{1}}=\frac{-K_{s} C_{1} / R_{1}}{s^{2} C_{1} C_{2}+s\left(C_{2} / R_{1}+C_{1} / R_{2}-K C_{1} / R_{1}\right)+1 / R_{1} R_{2}}
$$

Choose:

$$
\begin{aligned}
& C_{1}=\frac{k}{2 f_{0}} \text { (defines } k \text { ) } \\
& \text { Note: } \quad A_{0}=\frac{K}{2-K} \\
& C_{2}=C_{1} \\
& Q=\frac{1}{2-K} \\
& R_{1}=\frac{1}{k} \\
& R_{2}=R_{1} \\
& \begin{array}{l}
f_{o}=\frac{1}{2 C_{1} R_{1}} \\
K=2-1 / Q
\end{array}
\end{aligned}
$$

Note: The choice of $R$, in the INIC is relatively arbitrary, but it should be near 10K for best results. ( $R+K R$ can be a single trimming potentiometer of size 20 K to 30 K .)

(Frequency determined by 10 sec count)

EXAMPLE
Want: $\quad Q=100$
$f_{0}=80 \mathrm{c} / \mathrm{s}$
$C_{1}=.1 \mu f$
Calculate: $K=1.99$
$A_{0}=199 \quad(46 \mathrm{db})$
$C_{1}=C_{2}=.1 \mu f$
$k=50.2 \times 10^{-6}$
$R_{1}=R_{2}=19.9 \mathrm{~K}$

## APPENDIX A

## REFERENCES

The literature has many references to the topics in the field of active RC circuits which have been introduced in this handbook. Some of these are given in the list that follows. No attempt has been made to make this an all inclusive list; rather it is presented to give the interested reader a starting place in his pursuit of more detailed information on this subject. The references are arranged by chapter.

## Chapter 1

A general theoretical treatment of the subject of active networks, together with many additional realization techniques may be found in the book by K. L. Su, Active Network Synthesis, Mc-Graw-Hill Book Co., Inc., New York, 1965.

## Chapter 2

General design formulas for several types of bridged-T and twin-T networks, as well as a general discussion of the synthesis of passive RC networks may be found in the book by N. Balabanian, Network Synthesis, Chap. 7, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1958. A more detailed presentation of several cases, together with several design charts may be found in Appendix VI of the book by C. J. Savant, Jr., Control System Design, 2nd Ed., McGrow-Hill Book Co., Inc., 1964.

A simplified method for the design of twin-T networks with transmission zeros
in the right-half of the complex frequency plane may be found in the article by $B$. $A$. Shenoi, "A New Technique for Twin-T RC Network Synthesis," IEEE Transactions on Circuit Theory, Vol. CT-11, No. 3, pp. 435-436, Sept. 1964.

A discussion of the odjustment problem in twin-T networks may be found in the article by K. Posel, "A New Treatment of the RC Parallel-T Network," Proceedings of the Institution of Electrical Engineers (England), Vol. 110, No.I, pp. 126-138, Jan. 1963.

Chapter 3

A tabular method which may be used to determine the voltage transfer function of a network using a fairly arbitrary feedback network is given in the article by A. G. J. Holt and J. I. Sewell, "Table for the Voltage Transfer Functions of SingleAmplifier Double-Ladder Feedback Systems," Electronics Letters (published by the Institution of Electrical Engineers, England), Vol. 1, No. 3, pp. 70-71, May 1965.

A design method for a third order filter using a single operational amplifier is given in the article by L. K. Wadhwa, "Simulation of Third-Order Systems with DoubleLead Using One Operational Amplifier," Proceedings of the IRE, Vol. 50, No. 6, pp. 1538-1539, June 1962. Articles on similar filters with different zeros moy be found in the February and April Proceedings issues of the same year.

## Chapter 4

The classic article in this area is the one by R. P. Sallen and E. L. Key, "A Practical Method of Designing RC Active Filters," IRE Transactions on Circuit Theary, Vol. CT-2, No. 1, pp. 74-85, March 1955. Some odditional circuits are given in the article by N. Balabanian and B. Patel, "Active Realization of Complex Zeros," 1EEE Transactions on Circuit Theory, Vol. CT-10, No. 2, pp. 299-300, June 1963.

## Chapter 5

The properties of a negative-immittance converter as a two-port device were defined by A.I. Larky, "Negative-Impedance Converters," IRE Transactions on Circuit Theory, Vol. CT-4, No. 3, pp. 124-131, September 1957. In addition to the circuit for an NIC given in the Larky paper, several other circuit realizations have appeared in the literature. See, for example, D. P. Franklin, "Direct-Coupled Negative-Impedance Converter," Electronics Letters, Vol. 1, No. 1, p. 1, March 1965. Power and impedance transformation properties are discussed by L. P. Huelsman, "A Fundamental Classification of Negative-Immittance Converters," 1965 IEEE International Convention Record, Part 7, pp. 113-118, March 1965.

The basic circuit for the realization of a voltage transfer with an INIC presented in this chapter is described in more detail by T. Yanagisawa, "RC Active Networks Using Current Inversion Type Negative Impedance Converters," IRE Thansactions on Circuit Theory, Vol. CT-4, No. 3, pp. 140-144, Sept. 1957. Another basic approach that uses VNICs is given by J. G. Linvill, "RC Active Filters," Proceedings of the IRE, Vol. 42, No. 3, pp. 555-564, March 1954.

Books which cover the topics of this chapter are the ones by Su (see Chapter 1 reference) and L. P. Huelsman, Circuits, Matrices, and Linear Vector Spaces, McGraw-Hill Book Co., Inc., New York, 1962.

# APPENDIX B 

## SPECIFICATIONS

Typical Burr-Brown Operational Amplifiers
The specifications shown in Table 1 and Table 2 (on the following page) are indicative of: (1) the complexity of operationat amplifiers, (2) the state of the art in aperational amptifiers as of the publishing date of this handbook, and (3) the extensive line of operational amplifiers manufoctured by Burr-Brown Research Corporation. Complete specifications are available on request.

TABLE
Performance at $25^{\circ} \mathrm{C}$ and with rated supply.

| $\begin{aligned} & \text { STANDARD MODULE TYPE } \\ & 1500 \\ & 1300 \\ & 1600 \\ & 1900 \end{aligned}$ | $\begin{aligned} & 1506 \\ & 1901 \end{aligned}$ | $\begin{aligned} & 1507 \\ & 1902 \end{aligned}$ | $\begin{aligned} & 1509 \\ & 1305 \mathrm{~A} \\ & 1606 \mathrm{~A} \end{aligned}$ | $\begin{aligned} & 1510^{(3)} \\ & 1607 B \end{aligned}$ | $1514^{(3)}$ | 1517 | Units |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| RATED OUTPUT <br> Voltoge (min) <br> Current (min) | $\pm 10$ $\pm 20$ | $\pm 10$ $\pm \quad 2$ | $\pm 10$ $\pm 20$ | $\begin{aligned} & \pm 10 \\ & \pm 30 \end{aligned}$ | $\begin{aligned} & \pm 10^{(2)} \\ & \pm 20 \end{aligned}$ | $\begin{aligned} & \pm 10 \\ & \pm \quad 2 \end{aligned}$ | $\begin{aligned} & V \\ & m A \end{aligned}$ |
| DC GAIN (typ) <br> UNITY GAIN CROSSOVER (typ) <br> FULL POWER RESPONSE (min) <br> StEWING RATE (ryp) | $\begin{array}{r} 106 \\ 1.5 \\ 20 \\ 2 \end{array}$ | $\begin{array}{r} 96 \\ 1.5 \\ 20 \\ 2 \end{array}$ | $\qquad$ <br> Broadband, Note 1 | $\begin{array}{r} 90 \\ 30 \\ 1000 \\ 100 \end{array}$ | $\begin{array}{r} 106 \\ 0.75 \\ 10 \\ 1.0 \end{array}$ | $\begin{array}{r} 86 \\ 1.0 \\ 10 \\ 1.0 \end{array}$ | $d B$ <br> $\mathrm{Mc} / \mathrm{s}$ <br> $\mathrm{kc} / \mathrm{s}$ <br> $\mathrm{V} / \mathrm{\mu s}^{\mathrm{s}}$ |
| INPUT VOLTAGE <br> Offset, $25^{\circ} \mathrm{C}$ (typ) ${ }^{(6)}$ <br> Drift, $-25^{\circ} \mathrm{C}$ to $+85^{\circ} \mathrm{C}$ (typ) <br> (mox) <br> INPUT CURRENT <br> Offset, $25^{\circ} \mathrm{C}$ (ryp) <br> Drift, $-25^{\circ} \mathrm{C}$ to $+85^{\circ} \mathrm{C}$ (typ) <br> (mox) | $\begin{aligned} & \pm 0.3 \\ & \pm 5 \\ & \pm 10 \\ & \pm \quad 5 \\ & \pm 0.3 \\ & \pm 0.5 \end{aligned}$ | $\begin{aligned} & \pm 0.3 \\ & \pm 5 \\ & \pm \end{aligned} 10$ | $\begin{aligned} & \pm 0.3 \\ & \pm 5 \\ & \pm 15 \end{aligned}$ | $\begin{aligned} & \pm 0.3 \\ & \pm 10 \\ & \pm 25 \\ & \pm 10 \\ & \pm 0.5 \\ & \pm 1.0 \end{aligned}$ | $\begin{aligned} & \pm 0.3 \\ & \pm 5 \\ & \pm 15 \\ & \pm 10 \\ & \pm 0.3 \\ & \pm 0.5 \end{aligned}$ | $\begin{aligned} & \pm 0.5 \\ & \pm 10 \\ & \pm 30 \\ & \pm 10 \\ & \pm 0.5 \\ & \pm 3.0 \end{aligned}$ | mV <br> $\mu \vee /{ }^{\circ} \mathrm{C}$ <br> $\mu \mathrm{V} /{ }^{\circ} \mathrm{C}$ <br> $n A$ <br> $n A{ }^{\circ} \mathrm{C}$ <br> $n A{ }^{\circ} \mathrm{C}$ |
| INPUT NOISE 10 $10 \mathrm{KC} / \mathrm{S}$ ('yp) <br> INPUT IMPEDANCE <br> Differential (typ) <br> Common Mode (ryp) <br> INPUT VOLTAGE LIMITS <br> Cammon Mode (max) <br> Absolute Moximum <br> OUTPUT IMPEDANCE (IyP) | $\begin{array}{r} 6 \\ 0.5 \\ 25 \\ \pm 10 \\ \pm 15 \\ 5.0 \end{array}$ | $\begin{array}{r} 6 \\ 0.5 \\ 25 \\ * 10 \\ * 15 \\ 1.5 \end{array}$ | $\begin{array}{r} 0.5 \\ 25 \end{array}$ $\begin{aligned} & \pm 10 \\ & \pm 15 \\ & 0.5 \end{aligned}$ | $\begin{gathered} 10 \\ 0.5 \\ - \\ \pm \\ - \\ 0.15 \end{gathered}$ | $\begin{array}{r} 6 \\ 0.5 \\ 25 \\ \hline 10 \\ \pm 15 \\ 0.5 \end{array}$ | $\begin{aligned} & 10 \\ & 0.2 \\ & 20 \\ & \pm 10 \\ & \pm 15 \\ & 2.0 \end{aligned}$ | $\mu \vee \mathrm{rms}$ <br> $M \Omega$ <br> $\mathrm{M} \Omega$ <br> $V$ <br> $V$ <br> $k \Omega$ |
| OPERATING TEMPERATURE RANGE <br> Minimum <br> Maximum | $\begin{array}{r} -40 \\ +85 \end{array}$ | $\begin{array}{r} -40 \\ +85 \end{array}$ | $\begin{array}{r} -40 \\ +85 \end{array}$ | $\begin{array}{r} -40 \\ +85 \end{array}$ | $\begin{array}{r} -40 \\ +85 \end{array}$ | $\begin{aligned} & -25 \\ & +85 \end{aligned}$ | ${ }^{\circ} \mathrm{C}$ |
| POWER SUPPLY <br> Rated (typ) ${ }^{(4)}$ <br> Quiescent (ryp) (5) | $\begin{aligned} & \pm 15 \\ & \pm \quad 5 \end{aligned}$ | $\begin{aligned} & \pm 15 \\ & \pm 10 \end{aligned}$ | $\begin{aligned} & \pm \\ & \pm \quad 5 \\ & \pm \quad 5 \end{aligned}$ | $\begin{aligned} & \pm 15 \\ & \pm 12 \end{aligned}$ | $\begin{aligned} & \pm 15 \\ & +10 \end{aligned}$ | $\begin{aligned} & \pm 15 \\ & \pm 10 \end{aligned}$ | Vde <br> mA |
| UNIT PRICE IN DOLLARS, U.S. (see current price list) <br> 1500 <br> 1300 <br> 1600 <br> 1900 | $\begin{aligned} & 95 \\ & 125 \end{aligned}$ | $\begin{aligned} & 75 \\ & 110 \end{aligned}$ | $\begin{array}{r} 95 \\ 115 \\ 125 \end{array}$ | 135 <br> 165 | 125 | 39 | $\begin{aligned} & \$ \\ & \$ \\ & \$ \end{aligned}$ |

Notes: Price and performance subject to chonge.
(1) Through apprapriate selection of phase campensation, the user can achieve gain=bandwidth products as high os 50 Mc/s, full power response to $50 \mathrm{kc} / \mathrm{s}$, and slewing rates to $5 \mathrm{~V} / \mu \mathrm{s}$.
(2) Each output. Can be used to swing 40 volts peak-to-peak into floating loods or to provide equal outpuis of opposite polority.
(3) Models 1510 and 1514 measure $1.8^{\prime \prime} \times 2.4^{\prime \prime} \times 0.6^{\prime \prime}$ maximum ( $1500-68$ madule).
(4) Ronge: $\pm 12 \mathrm{Vdc}$ to $\pm 18 \mathrm{Vdc}$.
(5) Total current demonds opproximotely equol to quiescent plus ouiput current.
(6) All 1300 and 1600 Series units feoture internol voltage offset odiustment.

## APPENOIX B (continued)

TABLE 2

Performonce at $25^{\circ} \mathrm{C}$ with roted supply.

| STANDARD MODULE TYPE 1500 1300 1600 1900 | $\begin{aligned} & 1538 A^{(2)} \\ & 1838 \end{aligned}$ | $\begin{aligned} & 1540 \\ & 1940 \end{aligned}$ | $\begin{aligned} & 1541^{(2)} \\ & 1341 \\ & 1641 \end{aligned}$ | $\begin{aligned} & 1542(2) \\ & 1342 \\ & 1642 \end{aligned}$ | 1701 | $\begin{aligned} & 1552 \\ & 1952 \end{aligned}$ | Units |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| RATED OUTPUT Voltoge (min) Current (min) | $\begin{aligned} & \pm 10 \\ & \pm 20 \end{aligned}$ | $\begin{aligned} & \pm 20 \\ & \pm 10 \end{aligned}$ | $\begin{aligned} & \pm 50 \\ & \pm 10 \end{aligned}$ | $\begin{aligned} & \pm 100 \\ & \pm 10 \end{aligned}$ | $\begin{aligned} & \pm 10 \\ & \pm 10 \end{aligned}$ | $\begin{aligned} & \pm 10 \\ & \pm 20 \end{aligned}$ | $\begin{aligned} & V \\ & m A \end{aligned}$ |
| DC GAIN (ryp) UNITY GAIN CROSSOVER ('YP) FULL POWER RESPONSE (min) SLEWING RATE (typ) | $\begin{array}{r} 160 \\ 15 \\ 300 \\ 30 \end{array}$ | $\begin{aligned} & 106 \\ & 1.5 \\ & 10 \\ & 2.0 \end{aligned}$ | $\begin{array}{r} 110 \\ 0.4 \\ 3 \\ 1.0 \end{array}$ | $\begin{array}{r} 110 \\ 0.4 \\ 3 \\ 2.0 \end{array}$ | $\begin{gathered} 92 \\ 3 \\ 30 \\ 3 \end{gathered}$ | $\begin{aligned} & 106 \\ & 1.5 \\ & 100 \\ & 10 \end{aligned}$ | $d B$ <br> $\mathrm{Mc} / \mathrm{s}$ <br> $\mathrm{kg} / \mathrm{s}$ <br> $\mathrm{V} / \mathrm{\mu s}$ |
| INPUT VOLTAGE $\begin{aligned} & \text { Offset, } 25^{\circ} \mathrm{C}(\text { typ }) \\ & \text { Drift, }-25^{\circ} \mathrm{C} \text { to }+85^{\circ} \mathrm{C} \text { (typ) } \end{aligned}$ <br> INPUT CURRENT Offset, $25^{\circ} \mathrm{C}$ (typ) Orift, $-25^{\circ} \mathrm{C}$ to $+85^{\circ} \mathrm{C}$ (typ) (max) | $\begin{aligned} & \pm 0.03 \\ & \pm 0.5 \\ & \pm 1 \\ & \pm .03 \\ & \pm .001 \\ & \pm .003 \end{aligned}$ | $\begin{aligned} & \pm 0.3 \\ & \pm 5 \\ & \pm 15 \\ & \pm 10 \\ & \pm 0.3 \\ & \pm 0.5 \end{aligned}$ | $\begin{aligned} & \pm 0.3 \\ & \pm 10 \\ & \pm 25 \\ & \pm 10 \\ & \pm 0.5 \\ & \pm 1.0 \end{aligned}$ | $\begin{aligned} & \pm 0.5 \\ & \pm 10 \\ & \pm 25 \\ & \pm 10 \\ & \pm 0.5 \\ & \pm 1.0 \end{aligned}$ | $\begin{aligned} & \pm 0.5 \\ & \pm 5 \\ & \pm 15 \\ & \\ & \pm 20 \\ & \pm 5 \\ & \pm 15 \end{aligned}$ | $\begin{aligned} & \pm 0.5 \\ & \pm 5 \\ & \pm 15 \end{aligned}$ <br> 10.1 <br> Note 6 | $m V$ <br> $\mu V /{ }^{\circ} \mathrm{C}$ <br> $\mu V /{ }^{\circ} \mathrm{C}$ <br> nA <br> ${ }^{n} A^{\circ} \mathrm{C}$ <br> ${ }_{n} A^{\circ} \mathrm{C}$ |
| INPUT NOISE TO $10 \mathrm{KC} / \mathrm{S}$ (ryp) <br> INPUT IMPEDANCE <br> Differentiol (ryp) <br> Common Mode (ryp) <br> INPUT VOLTAGE LIMITS <br> Common Mode (max) <br> Absolute Maximum <br> OUTPUT IMPEDANCE (ryp) | $\begin{gathered} 8 \\ 0.5 \\ - \\ \pm 15 \\ 5 \end{gathered}$ | $\begin{array}{r} 6 \\ 0.5 \\ 50 \\ \pm 15 \\ \pm 26 \\ 7 \end{array}$ | $\begin{array}{r} 10 \\ 0.5 \\ 50 \\ \\ \pm 20 \\ \pm 60 \\ 5 \end{array}$ | $\begin{array}{r} 10 \\ 0.5 \\ 50 \\ \pm 20 \\ \pm 120 \\ 10 \end{array}$ | $\begin{gathered} 3 \\ 0.3 \\ 20 \\ \pm 10 \\ \pm 15 \\ 0.2 \end{gathered}$ | $10$ $\begin{aligned} & 10^{4} \\ & 10^{4} \end{aligned}$ $\begin{aligned} & \pm 10 \\ & \pm 15 \end{aligned}$ | $\mu \vee \mathrm{rms}$ <br> $M \Omega$ <br> $M \Omega$ <br> V <br> v <br> k $\Omega$ |
| OPERATING TEMPERATURE RANGE Minimum Maximum | $\begin{aligned} & -40 \\ & +85 \end{aligned}$ | $\begin{aligned} & -40 \\ & +85 \end{aligned}$ | $\begin{aligned} & =40 \\ & .85 \end{aligned}$ | $\begin{array}{r} -40 \\ +85 \end{array}$ | $\begin{array}{r} -40 \\ +85 \end{array}$ | $\begin{array}{r} -40 \\ +85 \end{array}$ | $\begin{aligned} & { }^{\circ} \mathrm{C} \\ & { }^{\circ} \mathrm{C} \end{aligned}$ |
| POWER SUPPLY <br> Roted (ryp) (4) <br> Quiescent (typ) (s) | $\begin{aligned} & \pm 15 \\ & \pm 8 \end{aligned}$ | $\begin{aligned} & \pm 28 \\ & \pm \quad 5 \end{aligned}$ | $\begin{aligned} & \pm 60 \\ & \pm 10 \end{aligned}$ | $\begin{aligned} & \pm 120 \\ & \pm 10 \end{aligned}$ | $\begin{aligned} & \pm 15 \\ & \pm 4 \end{aligned}$ | $\begin{aligned} & \pm 15 \\ & \pm 7 \end{aligned}$ | Vde mA |
| UNIT PRICE IN DOLLARS, U.S. (see current price list) 1500 <br> 1300 <br> 1600 <br> 1700 <br> 1900 | 175 210 | $95$ $125$ | $\begin{aligned} & 145 \\ & 165 \\ & 175 \end{aligned}$ | $\begin{aligned} & 165 \\ & 185 \\ & 195 \end{aligned}$ | 135 | $145$ $165$ | $\begin{aligned} & \$ \\ & \$ \\ & \$ \\ & \$ \end{aligned}$ |

NOTES: Price and performance subject to chonge.
(1) Through oppropriote selection of phose compensotion, the user con achieve gain-bandwidth products as high as $50 \mathrm{Mc} / \mathrm{s}$, full power response to $50 \mathrm{kc} / \mathrm{s}$, and slewing rotes to $10 \mathrm{valt} / \mathrm{l} / \mathrm{s}$.
(2) Models 1541 ond 1542 meosure $1.8^{\prime \prime} \times 2.4^{\prime \prime} \times 0.6^{\prime \prime}$ max. ( $1500-68$ module).
(3) All 1300 and 1600 Series units feoture internal valtage offset adiustment.
(4) Range: $\pm 3$ volts of rypical for $\pm 26$ volt supply, $\pm 5$ volts of typical for $\pm 60$ volp and $\pm 120$ valr supplies.
(5) Total current opproximately equal to quiescent plus output current.
(8) Input current doubles every $10^{\circ} \mathrm{C}$.

## APPENDIX B (continued)

TABLE 3

MODEL 1520 Power Booster

| Rated <br> Oulput |  | $D C$ <br> Gain O.L. | Operoting <br> Temperature Range |  | Power Supply |  | Unít Price |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
|  |  | roted |  |  | quies |  |
| volts | ma |  | 'yp | min | $\max$ | Pyp | typ |  |
| min | min | db | ${ }^{\circ} \mathrm{C}$ | ${ }^{\circ} \mathrm{C}$ | volis | mo |  |
| $\pm 10$ | $\pm 100$ | 0 | -40 | $+85$ | $\pm 15$ | $\pm 15$ | \$55 |

TABLE 4

| Roted Output |  | Input Volitoge |  |  | Unit Price |
| :---: | :---: | :---: | :---: | :---: | :---: |
| volis | mo |  |  |  |  |
| nom | min | nominal | $\min$ | max | (U.S. Dollors) |
| $\pm 15$ | $\pm 100$ | $\pm 24$ | $\pm 20$ | $\pm 25$ | $\$ 95$ |

TABLE 5
MODEL 501 Dual Power Supply

| Type | Dimensions (max) | Rated Output | Unir Price <br> (U.S. Dollors) |
| :---: | :---: | :---: | :---: |
| modular | $31 / 2^{\prime \prime} \times 3^{\prime \prime} \times 2^{\prime \prime}$ | $\pm 15 \mathrm{~V}(0 \pm 100 \mathrm{mo}$ | $\$ 148$ |

## 1500 SERIES

MATING CONNECTOR occommadates all 1500 Series units for plug-in imstollation or test. Model 1500 MC : $\$ 2$

JACK SET consists of eight individual jacks. Model 1500JS: $\$ 2$

FEEDBACK BOARD provides solder terminols for feedbock components. Model 1500 FB ; $\$ 10$

CIRCUIT SIMULATOR is a porch ponel with
$3 / 4$ " spaced jacks for feedbock elements. Model 1500C5: $\$ 25$

## 1300 SERIES

MATING CONNECTOR (Burndy 4535) is furnished with unit. Extra connectors: $\mathbf{\$ 2}$

BLANK MODULE is some packoge os the 1300
Series amolifiers. Teminals for each pin ore mounted next to the connector. Model 1300 M : $\$ 15$

FEEDBACK BOARD is similor to the 1300 M
except thot rerminals are spoced over the
board. Model 1300F B : $\$ 15$
CIRCUIT SIMULATOR is similor to the 1500 CS
with 1300 connector. Model I300CS : $\$ 35$

## 1600 SERIES

RACK ADAPTER holds 16 units in :
$3-1 / 2^{\prime \prime} \times 19^{\prime \prime}$ rack spoce. Model
1600-18R : 580

WIRED RACK ADAPTER is identical to the 1600-18R but includes mating connectors ond power buss wire. Model 1600-18RW: $\$ 100$

HALF RACK ADAPTER holds 7 units in a 3-1/2" $\times 9-1 / 2^{\text {"1 }}$ rock space. Mordel 1600-7R:550

POWERED RACK ADAPTER holds 10 units. See Power Supplies. Model 1600-10R.

POWERED RACK ADAPTER holds 12 units. See Power Supplies. Model 1600-12R.

MATING CONNECTOR (Burndy 4535)
is furnished with unit. Extra conmecfors: $\mathbf{\$ 2}$

BLANK PANEL provides unifarm
appearance of the rock. Model
16008P: $\$ 2$
CARD EXIENDER allows testing without
disconnecting unit from rack. Model 1800CE: $\$ 30$

1900 SERIES
MATING CONNEC TOR is similar to the Model 1500 MC . Model 1900 MC : $\$ 2$

JACK SET consists of five mating jacks similar to 1500 J . Model 1900 JS : $\$ 1$

ADAPTER PLUG allows use of 1500 CS
Circuit Simulator. Model 1900AP: $\$ 3$

## MECHANICAL DATA

1500 SERIES


1600 SERIES


1600-1


1600-2
meChanical data (continued)

1300 SERIES


Typical Mounting
Connector Burndy EC 4535 Applies to 1300 Series and 1600 Series


1700 SERIES


1900 SERIES


2900 SERIES


REACTANCE CHART


