MICROWAVE TRANSMISSION DESIGN DATA

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INTRODUCTION

Coaxial lines and hollow pipe wave guides designed to propagate ultra-high frequency radio waves have come into widespread use in recent years. Much equipment utilizing these microwave transmission lines has been and is being designed. In designing such equipment, it is convenient to have for ready reference a considerable amount of information relating to these two types of transmission lines. Unfortunately, much of this useful information has been gathered during wartime, and for secrecy reasons the distribution has necessarily been on a limited scale. Other material was printed before and during the war, but at present it is widely scattered throughout the literature. Still other information of a generally useful nature has been obtained by different groups and individuals, but never passed on to others who would be interested. As a result, any person who is doing work with microwave systems must be prepared to investigate many sources of information in order to locate some particular item.

For these reasons this reference handbook has been prepared. It contains as much pertinent information as could be collected from a number of sources. One of these sources was the microwave research done in the Research Laboratories of the Sperry Gyroscope Company. Other material came from unpublished notes of W. W. Hansen. Information was also obtained from textbooks and periodicals. Much of the material was taken from sources which for reasons of wartime security cannot be identified. It is regretted that credit cannot be given for this material when much credit is due. Throughout the text, references have been made wherever possible to the original sources, so that the reader may refer to them for more complete information if he so desires.

The material in this handbook was collected, prepared and edited by members of the Measurements Development Laboratory of the Sperry Gyroscope Company.

T. Moreno,
May, 1944.

GLOSSARY OF SYMBOLS

General

α = attenuation constant
β = phase constant
γ = propagation constant
δ = skin depth
ε = dielectric constant = unity in free space
ε_1 = dielectric constant of the propagating medium (coaxial lines)
ε = $\varepsilon' - j\varepsilon''$ for complex dielectrics
ε = measure of eccentricity for eccentric lines
$\tan\delta$ = loss tangent of dielectrics = $\frac{\varepsilon''}{\varepsilon'}$
r_p = standing wave ratio in power = $\left(\frac{V_{max}}{V_{min}}\right)^2$
λ = wave length in free space
λ_1 = wave length in the line (coaxial lines)
μ = permeability = unity in free space
μ_1 = permeability of the propagating medium (coaxial lines)
ρ = resistivity
σ = conductivity
σ = unit area of wall (cavity resonators)
ω = angular frequency = $2\pi f$
c = velocity of light = 3×10^{10} cm/sec
f = frequency
p = power factor (dielectric) = $\frac{\varepsilon''}{\varepsilon} \cong \frac{\varepsilon''}{\varepsilon'}$
v = velocity of propagation

Ccaxial Lines

a = radius of inner conductor
b = radius of outer conductor
R = resistance per unit length
L = inductance per unit length
G = conductance per unit length
C = capacity per unit length
Z_i = characteristic impedance = $\sqrt{\frac{R + j\omega L}{G + j\omega C}}$
Z_o = surge impedance = $\sqrt{\frac{L}{C}}$
Y_o = surge admittance = $\frac{1}{Z_o}$

GLOSSARY OF SYMBOLS—*Continued*

Coaxial Lines—Continued

Z_L = load impedance

Y_L = load admittance $= \dfrac{1}{Z_L}$

V_1 = magnitude of incident wave of voltage

V_2 = magnitude of reflected wave of voltage

I_1 = magnitude of incident wave of current

I_2 = magnitude of reflected wave of current

V_{max} = magnitude of maximum voltage (voltage loop) in line

V_{min} = magnitude of minimum voltage (voltage node) in line

I_{max} = magnitude of maximum current (current loop) in line

I_{min} = magnitude of minimum current (current node) in line

λ_1 = wavelength in the line

Wave Guides

f = frequency

f_c = frequency at cutoff

λ = wavelength in free space

λ_c = wavelength at cutoff (air-filled guide)

λ'_c = wavelength at cutoff (dielectric-filled guide)

λ_g = wave length in the guide—general

λ_{go} = wavelength in the guide—air-filled

v_p = phase velocity

v_g = group velocity

B = normalized susceptance

Cavity Resonators

f_o = resonant frequency

λ_o = resonant wavelength

τ = unit volume

σ = unit area of cavity wall

E = electric field

B = magnetic field

R_{SH} = shunt resistance

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Part I

CONCENTRIC LINES

CHAPTER I

GENERAL FORMULAE FOR CONCENTRIC LINES

Concentric lines are transmission systems in which the electromagnetic wave is propagated through a dielectric medium bounded by two coaxial conducting cylinders. The skin depth at microwave frequencies is small enough so that in all cases the conducting medium can be considered of infinite thickness.

The principal symbols used in this chapter are defined as follows:

λ = wave length
f = frequency
ω = angular frequency = $2\pi f$
v = velocity of propagation
c = velocity of light = 3×10^{10} cm/sec
a = outer radius of inner conductor
b = inner radius of outer conductor
ε = dielectric constant = unity in free space
ε_1 = dielectric constant of propagating medium
μ = permeability = unity in free space
μ_1 = permeability of medium separating the conductors
R = resistance per unit length
L = inductance per unit length
G = conductance per unit length
C = capacity per unit length
Z_i = characteristic impedance
Z_o = $\sqrt{L/C}$

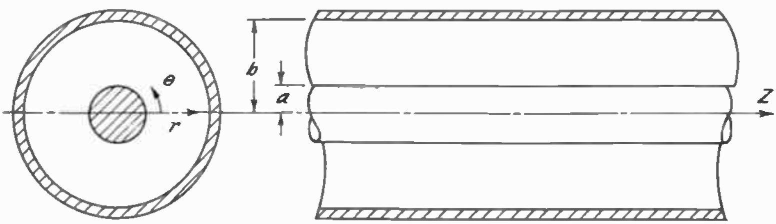


FIG. 1-1

MICROWAVE TRANSMISSION DESIGN DATA

A. *Line Parameters of Concentric Lines*

Inductance $= .4605\mu_1 \log_{10} b/a \times 10^{-6}$ henries/meter

This formula neglects the inductance caused by the current penetrating a finite distance into the conductors.

$$\text{Capacity} = \frac{.241\,\varepsilon_1}{\log_{10}\ b/a} \times 10^{-10} \text{ farad/meter}$$

B. *Characteristic Impedance*

The characteristic impedance Z_i of a transmission line is given by the formula

$$Z_i = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Where losses in the line are small, this formula reduces to

$$Z_i \cong Z_o = \sqrt{\frac{L}{C}}$$

For a coaxial line with low losses, the characteristic impedance is

$$Z_i \cong Z_o = 138\sqrt{\frac{\mu_1}{\varepsilon_1}} \log_{10} b/a = 60\sqrt{\frac{\mu_1}{\varepsilon_1}} \ln b/a$$

The characteristic impedance of a coaxial line as a function of diametric ratio (assuming $\mu_1 = \varepsilon_1 = 1$) is shown in Fig. I–2.

C. *Propagation Constant*

The propagation constant γ of a transmission line is given by the formula

$$\gamma = \sqrt{(R + j\omega L)\ (G + j\omega C)}$$

The propagation constant can be separated into real and imaginary components

$$\gamma = \alpha + j\beta$$

where

$$\alpha = \text{attenuation constant}$$
$$\beta = \text{phase constant}$$

The phase constant is related to the wavelength in the system by the formula

$$\lambda_1 = \frac{2\pi}{\beta}$$

The velocity of propagation v in the transmission line is given by

$$v = \frac{\omega}{\beta}$$

Where losses are small, this velocity of propagation becomes

$$v = \frac{c}{\sqrt{\mu_1 \varepsilon_1}}$$

and the wavelength in the line λ_1 is related to λ by

$$\lambda_1 = \frac{\lambda}{\sqrt{\mu_1 \varepsilon_1}}$$

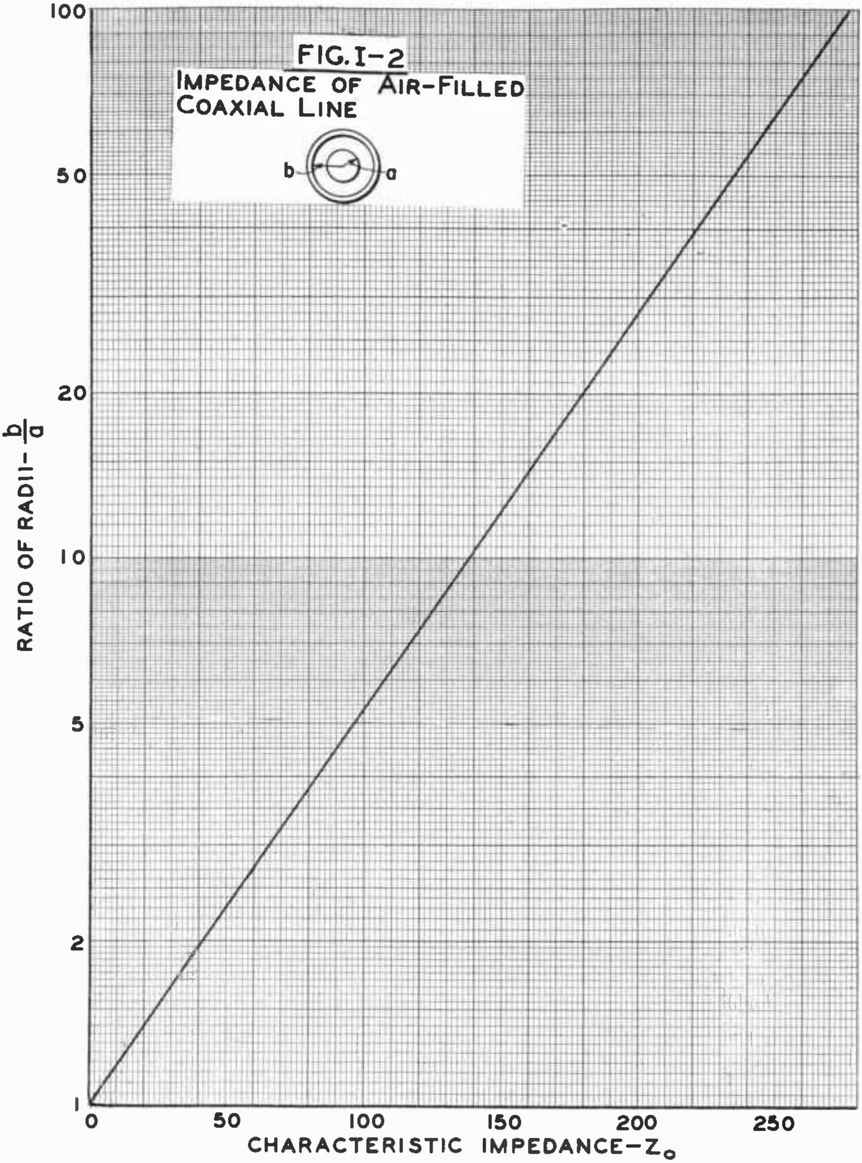


FIG. I-2
IMPEDANCE OF AIR-FILLED COAXIAL LINE
b
a
100
50
20
10
5
2
1
RATIO OF RADII - b/a
0
50
100
150
200
250
CHARACTERISTIC IMPEDANCE - Z_0

D. Voltage and Current Equations

Given a transmission line of characteristic impedance Z_i, terminated by a load impedance Z_r, and knowing the voltage V_r and the current I_r at the load, the voltage and current along the line are given by the following equations:

$$V = V_r \cosh \gamma l + I_r Z_i \sinh \gamma l$$

$$I = I_r \cosh \gamma l + \frac{V_r}{Z_i} \sinh \gamma l$$

Neglecting losses, these equations reduce to

$$V = V_r \cos \beta l + j I_r Z_o \sin \beta l$$

$$I = I_r \cos \beta l + j \frac{V_r}{Z_o} \sin \beta l$$

In the above equations l is the distance from the load to the point where the voltage and current are being measured.

E. Field Configurations

The principal mode of transmission in a coaxial line is the one nearly always used for transmission of energy. The field configurations of this mode are as indicated in Fig. I–3.

In Fig. I–3, solid lines are used to indicate the electric field and broken lines the magnetic field. Where I_o is the conduction current amplitude along the inner conductor, the following equations give the field distributions throughout the dielectric medium.

$$H_r = 0 \qquad E_r = 377 \sqrt{\frac{\mu_1}{\varepsilon_1}} \frac{I_o}{2\pi r} e^{j\omega t - \gamma z}$$

$$H_\theta = \frac{I_o}{2\pi r} e^{j\omega t - \gamma z} \qquad E_\theta = 0$$

$$H_z = 0 \qquad E_z = 0$$

With I_o in amperes, the magnetic field H is in ampere turns per meter, the electric field E in volts per meter, and the radius r in meters. It will be seen that the electric and magnetic field intensities decrease inversely with radial distance from the axis of the line.

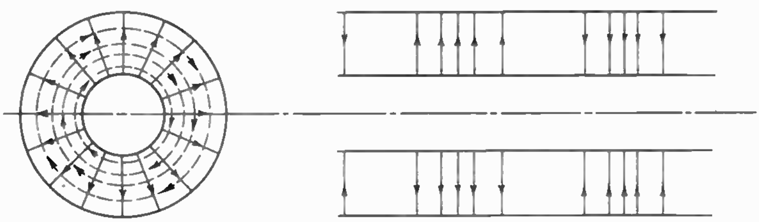


FIG. I - 3

F. Breakdown

Breakdown will occur in a coaxial line when the maximum voltage gradient exceeds a limiting value. The maximum voltage gradient is always found at the inner conductor. No experimental data is available concerning breakdown at ultra high frequencies. At ordinary frequencies under ordinary atmospheric conditions, the breakdown gradient is approximately 30,000 volts/cm.

The electric field intensity at any point in the region between the inner and outer conductors of a coaxial pair is given by

$$E = \frac{V}{r \ln b/a}$$

The gradient is seen to be a maximum when the radius r is equal to the radius of the inner conductor a. For a specified outer diameter b and maximum field strength E_m, the allowed potential difference between conductors is

$$V = E_m\, b \frac{\ln b/a}{b/a}$$

For maximum voltage between conductors, the optimum ratio b/a is 2.718. This corresponds to a characteristic impedance of 60 ohms.

G. Maximum Power Transfer for a Given Maximum Voltage Gradient

The power transferred by a line will depend upon the ratio V^2/Z_i. Hence the maximum power that can be transferred by a given line is

$$P = \frac{E_m{}^2\, b^2}{60} \frac{\ln b/a}{(b/a)^2}$$

For a specified outer diameter b, the maximum power can be transferred by a line when the ratio $b/a = 1.65$. This gives a characteristic impedance of 30 ohms.

This is the theoretical maximum power carrying capacity, but in actual practice it is necessary to limit maximum powers to values considerably less than the theoretical limit. For one thing, gradients are higher at stub supports, if these are used, and there is also more likelihood of breakdown in a rotating joint, or a choke connection. The breakdown in a choke connector is most likely to occur at the center conductor. Referring to Fig. I–4, the maximum gradient in the line will be increased as the ratio a/r_1, and the power carrying capacity is therefore reduced by a factor $(a/r_1)^2$.

H. Variation of Maximum Power Transfer with Altitude

The potential gradient at which breakdown will occur decreases with altitude because of the change in atmospheric pressure, and the maximum power which can be transmitted through the line will decrease accordingly. In Fig. I–5, the ratio P/P_o is plotted against the altitude H, where P_o is

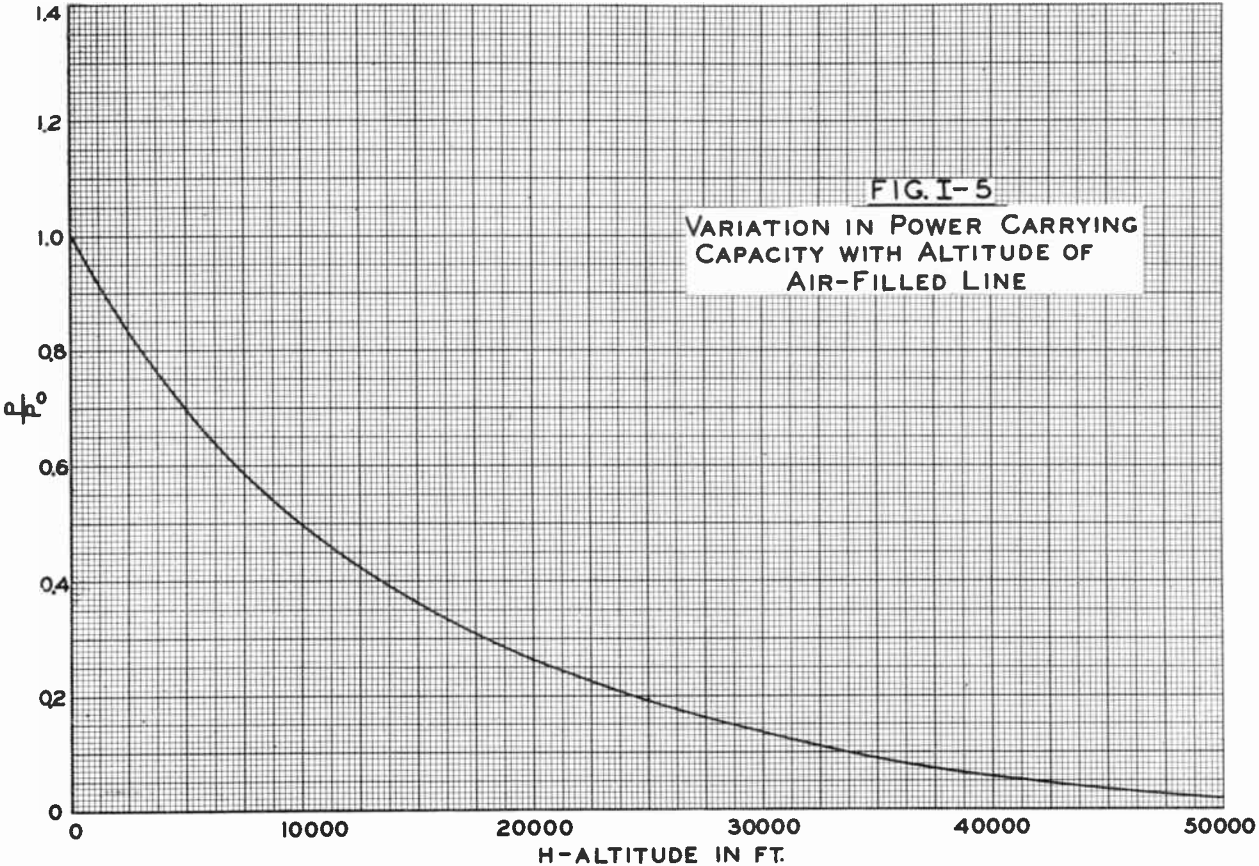


FIG. I-5

VARIATION IN POWER CARRYING CAPACITY WITH ALTITUDE OF AIR-FILLED LINE

GENERAL FORMULAE FOR CONCENTRIC LINES

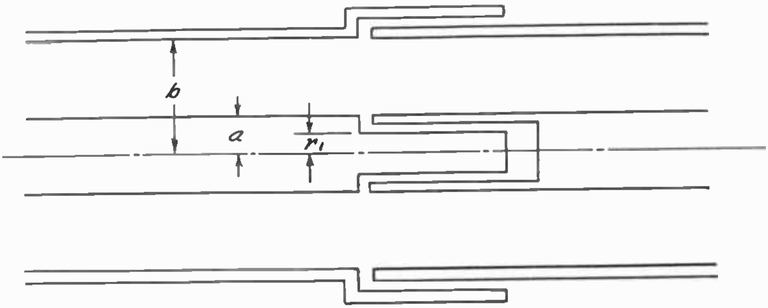


FIG. I-4

the maximum power that can be carried at sea level pressure and P is the maximum power that can be carried in the same line at an altitude H. This curve assumes that Paschen's law holds at ultra high frequencies, and takes a seasonal average air density.

I. Higher Modes of Transmission

If the concentric line is made sufficiently large, propagation of energy becomes possible in modes other than the customarily used principal mode. Normally the line is restricted in size so that there will be no propagation of energy in any of these higher modes, and any of the higher modes that are excited by junctions, discontinuities, etc. will diminish exponentially with distance, and draw no real power. The first higher mode encountered for most lines is that having a field configuration shown in Fig. I–6.

Within seven percent, the limiting wave length at which this mode becomes possible is equal to the circumference at the arithmetic mean diameter; that is, the mode becomes possible when

$$\lambda = 2\pi \frac{b + a}{2}$$

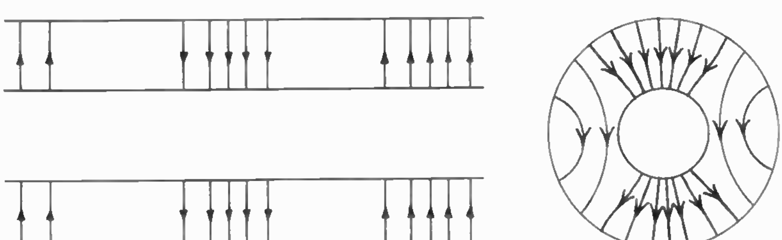


FIG. I-6

J. *Two-wire Line*

For a balanced two-wire line at ultra-high frequencies the parameters are:

Inductance per unit length

$$L = .921 \log_{10} \frac{D}{a} \times 10^{-6} \quad \text{henry/meter}$$

Capacity per unit length

$$C = \frac{.120}{\log_{10} \frac{D}{a}} \times 10^{-10} \quad \text{farad/meter}$$

Resistance per unit length (copper wire)

$$R = \frac{8.4 \sqrt{f}}{\text{radius in cm.}} \times 10^{-6} \quad \text{ohms/meter}$$

Characteristic impedance

$$Z_o = 276 \log_{10} \frac{D}{a} \quad \text{ohms.}$$

In these formulae D is the distance between centers of the two parallel wires, each having a radius a. D and a are measured in the same units.

CHAPTER II

ATTENUATION IN CONCENTRIC LINES

A. *General Formulae for Attenuation*

The propagation constant of a concentric line is defined as

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

where R, L, G, and C, are the line parameters per unit length. The propagation constant is resolvable into real and imaginary components

$$\gamma = \alpha + j\beta$$

where the real component α is known as the attenuation constant. Derived from the above formula, α will be in nepers/unit length. For the attenuation to be expressed in the more common units of decibels per unit length, α in nepers must be multiplied by the constant 8.69. (Attenuation in db/unit length = 8.69 α.)

When the attenuation is small, it may be expressed by the approximate formula

$$\alpha = \frac{R}{2Z_o} + \frac{G}{2Y_o} \text{ nepers/unit length}$$

where $Z_o = \sqrt{L/C}$ and $Y_o = 1/Z_o$.

1. Special Cases Where One or the Other Parameter is Supplying Losses.[1]

Case I: The series resistance R only is not zero. $R \neq O$. $G = O$.

Defining $Q = \dfrac{\omega L}{R}$, the following formulas hold approximately:

a. The loss per wave length is small (R is small).

$$\alpha = \beta/2Q = R/2Z_o$$

b. The loss per wave length is large (R is large).

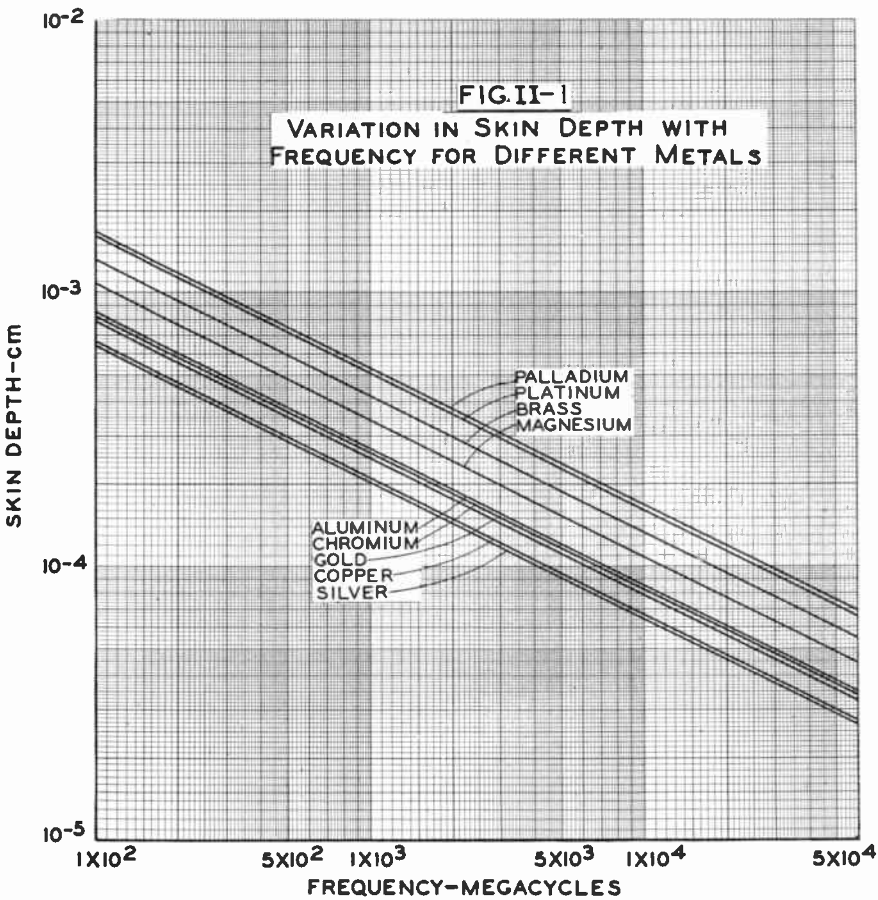
$$\alpha = \frac{\omega\sqrt{LC}}{\sqrt{2Q}}\left(1 - \frac{Q}{2}\right) \cong \frac{\omega\sqrt{LC}}{\sqrt{2Q}}$$

The assumption *a* is generally valid for microwave propagation, but assumption *b* is useful for attenuators.

Case II: The shunt losses only are not zero. $G \neq O$, $R = O$.

[1]From unpublished notes by W. W. Hansen.

ATTENUATION IN CONCENTRIC LINES



This is infrequently true in practice, but in most solid dielectric lines, the shunt losses greatly exceed the series losses at microwave frequencies. Q is defined here as $Q = \frac{\omega C}{G}$.

a. The shunt losses are small (G is small).

$$\alpha = \beta/2Q = \frac{G}{2Y_o}$$

b. The shunt losses are large (G is large).

$$\alpha = \frac{\omega\sqrt{LC}}{\sqrt{2Q}}\left(1 - \frac{Q}{2}\right) \cong \frac{\omega\sqrt{LC}}{\sqrt{2Q}}$$

B. *Causes of Attenuation and Numerical Formulae*

There are two sources of attenuation in coaxial lines: 1. Ohmic losses resulting from currents flowing in the conductors. 2. Dielectric losses resulting from the imperfections in the dielectric medium separating the two conductors.

1. Conductor Losses. At microwave frequencies, the conduction current flowing in the conductor is concentrated in the surface layer. The current density is a maximum at the surface, and decreases exponentially with depth into the conductor. The depth at which the current density has fallen to $1/e$ of its surface value is known as the skin depth δ. This skin depth is a function of frequency, also of the conductor material. It is given by the formula

$$\delta = \sqrt{\frac{\rho}{2\pi\omega\mu}}$$

where ρ is the resistivity of the conductor in abohm-cm., μ is the permeability of the conductor, and δ the skin depth in cm. For copper the permeability is unity, the resistivity is $\rho = 1.72 \times 10^{-6}$ ohm-cm. $= 1.72 \times 10^{3}$ abohm-cm., and the skin depth is $\delta = 1.2 \times 10^{-4}$ cm. for $\lambda = 10$ cm.

The losses resulting from the current being concentrated near the surface are the same as if the total current were of uniform density to a depth δ.

The skin depth δ as a function of frequency is plotted in Fig. II–1 for a number of different metals.

Resistance of a Concentric Line. The resistance per unit length of a concentric line is given by the formula

$$R = \frac{1}{2\pi\delta}\left(\frac{\rho_a}{a} + \frac{\rho_b}{b}\right)$$

In this formula ρ = resistivity in *ohm-cm*, when the skin depth and radii are in cm. ρ_a is the resistivity of the inner conductor, and ρ_b the resistivity of the outer conductor. When inner and outer conductor are of the same resistivity, the expression for resistance reduces to

$$R = \frac{\rho}{2\pi\delta}\left(\frac{1}{b} + \frac{1}{a}\right)$$

Attenuation in a Concentric Line as a Result of Conductor Losses. If the inner and outer conductors of a concentric line are of different materials, the attenuation is given by

$$\alpha = \frac{\pi}{2\lambda}\left(\frac{\delta_a}{a} + \frac{\delta_b}{b}\right)\frac{\sqrt{\varepsilon_1}}{ln\,\frac{b}{a}} \quad \text{nepers/unit length}$$

where δ_b is the skin depth of the outer conductor, δ_a the skin depth of the inner conductor, and λ the wavelength in air of the signal being transmitted. If the same material is used for both inner and outer conductors, the attenuation formula reduces to

$$\alpha = \frac{\pi}{2}\frac{\delta}{b\lambda}\left(1 + \frac{b}{a}\right)\frac{\sqrt{\varepsilon_1}}{ln\,b/a} \quad \text{nepers/unit length}$$

$$= 13.6\,\frac{\delta}{b\lambda}\left(1 + \frac{b}{a}\right)\frac{\sqrt{\varepsilon_1}}{ln\,b/a} \quad \text{db/unit length}$$

The attenuation increases as the square root of frequency, assuming that ε_1 is independent of frequency, and also varies as the square root of the resistivity of the conductors. Table II–I gives the resistivity of a number of conducting materials, and also the attenuation as compared to the attenuation of copper.

TABLE II–I

Resistivity of Metals and Their Relative Attenuation

Metal	Resistivity	Relative Attenuation
Copper	1.72×10^{-6} ohm. cm.	1.00
Aluminum	2.83	1.28
Brass	7	2.02
Chromium	2.6	1.23
Gold	2.44	1.19
Magnesium	4.6	1.63
Palladium	11	2.53
Platinum	10	2.41
Rhodium	5.1	1.72
Silver	1.63	0.97
Tin	11.5	2.58
Tungsten	5.51	1.79
Zinc	6.2	1.90

An optimum ratio b/a exists for a fixed value of b, and minimum attenuation occurs when the ratio b/a is 3.6. This corresponds to a characteristic impedance of 77 ohms for a line with air dielectric. The minimum is rather broad, and impedance values can vary considerably about this value without a marked change in the attenuation.

2. Dielectric Losses. Solid dielectric coaxial lines are not used at microwave frequencies when low attenuation is desired because even a very good dielectric has losses that are very appreciable at these frequencies. To express these losses mathematically, it is convenient to consider the dielectric constant as complex and of the form

$$\varepsilon = \varepsilon' - j\varepsilon''$$

The loss tangent of the dielectric is defined as

$$\tan \zeta = \frac{\varepsilon''}{\varepsilon'}$$

Usually ζ is small, and we can then say that the loss tangent is equal to the power factor of a condenser using the dielectric. The true power factor is defined as

$$\text{power factor} = \cos \theta$$

with

$$\theta = 90° - \zeta$$

The attenuation in a coaxial line resulting from dielectric losses is given by

$$\alpha = \pi \frac{\sqrt{\varepsilon_1'}}{\lambda} \tan \zeta \quad \text{nepers/unit length}$$

$$= 27.3 \frac{\sqrt{\varepsilon_1'}}{\lambda} \tan \zeta \quad \text{db/unit length}$$

3. Attenuation in a Coaxial Line with Both Conductor and Dielectric Losses. The total attenuation in a coaxial line is the sum of the attenuation resulting from the conductor losses and the dielectric losses; that is

$$\alpha_T = \alpha_C + \alpha_D$$

where α_T is the total attenutation, α_C is the attenuation resulting from conductor losses, and α_D is the attenuation resulting from dielectric losses.

It will be seen from an inspection of the attenuation formulae that if the dielectric constant and power factor are independent of frequency, the following is true:

1. The conductor losses are proportional to the square root of frequency.
2. The dielectric losses are linearly proportional to frequency.

Hence at higher frequencies the dielectric losses become increasingly important.

C. Measured Cable Losses

The attenuation in a number of types of microwave cable has been measured with the results given in Table II–II.[2]

These measurements are all at a wavelength of 10 cm. As the dielectric losses are much greater than the conductor losses in this region, the loss will be approximately a linear function of frequency. Cables in which the

[2]Measured at the Naval Research Laboratories.

MICROWAVE TRANSMISSION DESIGN DATA

TABLE II–II

Attenuation in Coaxial Cables

Mfg.	Type	Impedance Ohms	3000 MC DB/100′
P. D.	RG-12/U	75	15.7
G. E.	RG-12/U	75	16.7
Amphenol	RG-11/U	75	17.1
Amphenol	RG-12/U	75	18.0
Okonite	RG-11/U	75	19.7
B. I. W.	RG-11/U	75	50.8
B. I. W.	RG-12/U	75	70.7
Anaconda	RG-9/U	52	16.1
F. T. R.	RG-10/U	52	16.3
G. E.	RG-10/U	52	17.0
P. D.	RG-10/U	52	18.0
F. T. R.	RG-9/U	52	18.2
Gen. Cable	RG-10/U	52	18.8
P. D.	RG-14/U	51	12.0
G. E.	RG-14/U	51	13.6
G. E.	RG-14/U	51	14.8
F. T. R.	RG-17/U	51	8.1
F. T. R.	RG-17/U	51	8.2
G. E.	RG-18/U	51	10.4
P. D.	RG-18/U	51	13.0
P. D.	RG-20/U	51	7.8
G. E.	RG-34/U	72	16.5
Amphenol	RG-5/U	51	22.4
Amphenol	RG-6/U	75	22.4
Amphenol	RG-54/U	58	25.2
Amphenol	RG-4/U	51	28.9
Amphenol	RG-55/U	51	29.7
Amphenol	RG-58/U	51	32.4
Simplex	RG-39/U	70	75.5
Simplex	RG-42/U	76	95.7

dielectric is beaded are not recommended because the power transfer varies with movement of the cable. Solid dielectric cables are to be preferred in this respect, although their measured loss may be a bit higher.

D. Impedance of Resonant Lines

At microwave frequencies, it is often desirable to use concentric lines as circuit elements, particularly as resonant circuits in which the line presents a minimum or maximum resonant impedance.

Case I: Minimum Impedance of an Open-Circuited Line.

Neglecting losses, the input impedance of an open-circuited transmission line of length l is given by

$$Z = -j Z_o \cot \beta l$$

This would indicate that the input impedance passed through zero at the odd quarter wave points. Actually, because of the presence of conductor losses, the input impedance does not reach zero at these points, but is given by the expression

$$Z = 60 \pi \frac{\delta}{\lambda} \frac{l}{b} \left(1 + \frac{b}{a} \right)$$

as the reactive component approaches zero. In this expression, Z is in ohms and the other terms are dimensionally alike. The input impedance is a function of the diametric ratio, reaching a minimum when the ratio b/a is equal to unity.

Case II: Maximum Impedance of a Short-Circuited Line.

Neglecting losses, the input impedance of a short-circuited line is given by the equation

$$Z = j Z_o \tan \beta l$$

indicating that the input impedance passes through infinity at odd quarter wave lengths. Because of the presence of conductor losses, the impedance, although high, is not infinite, and is given by the equation

$$Z = \frac{120}{\pi} \frac{\lambda}{\delta} \frac{b}{l} \frac{ln^2 \left(\frac{b}{a} \right)}{1 + \frac{b}{a}}$$

This neglects losses in the shorting plug or plate. The optimum ratio of b/a for high resonant impedance occurs when $b/a = 9.2$, corresponding to a line of $Z_o = 133$ ohms, but the maximum is quite broad.

E. Losses in Shorting Plugs

The losses in various types of concentric line shorting plugs were measured with the following results.[3] The types of shorting plugs tested are sketched

[3] Measured at the Research Laboratories of the Sperry Gyroscope Co.

in Fig. II–2, and the loss measured is in excess of that introduced by a plate soldered on the end of the line.

Type of Plug	*Measured Loss — db*
Blunt End	.20
Quarter Wave Fingers	.13 — .21
Half Wave Trap	.007

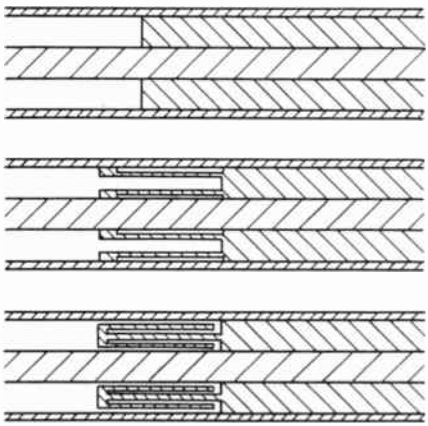


FIG. II - 2

These measurements were all at a wavelength $\lambda = 10$ cm. in solid coaxial line with inner and outer diameters .250″ and .875″ respectively. All shorting plugs were unplated brass. The loss in the plug with the quarter wave fingers is believed to vary over a much wider range than measured, depending upon the alignment and surface finish of the fingers.

F. Summary of Line Properties

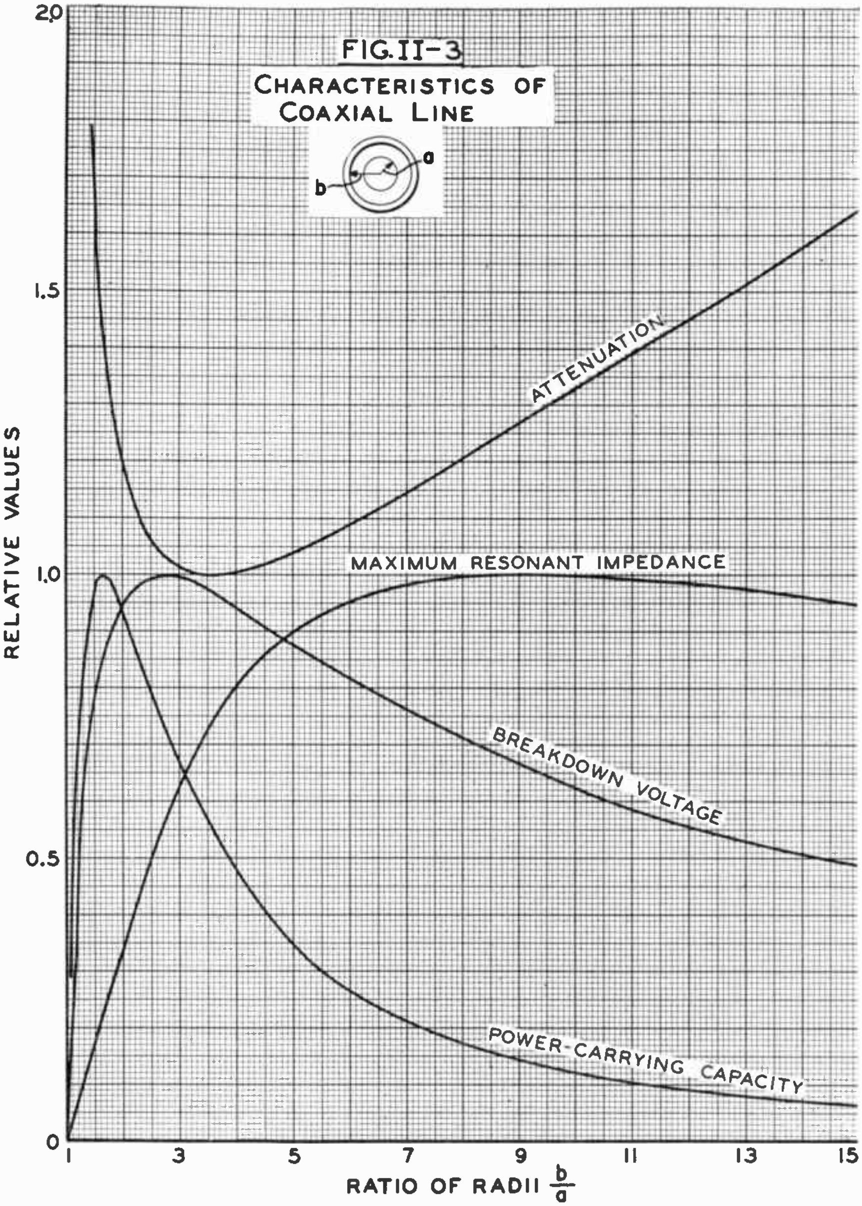
The diametric ratio of coaxial line corresponding to various optimum conditions is given in Table II–III. These are for a fixed diameter of outer conductor.

TABLE II—III

	Diametric Ratio $\frac{b}{a}$	Impedance of Air-Filled Line
Maximum voltage between conductors	2.718	60 ohms
Maximum power transfer	1.65	30
Minimum attenuation	3.6	77
Minimum resonant impedance	1.0	0
Maximum resonant impedance	9.2	133

The variation with diametric ratio of these various parameters is plotted in Fig. II–3, all curves being referred to unity as an arbitrary minimum or maximum.

ATTENUATION IN CONCENTRIC LINES



CHAPTER III

REFLECTIONS AND IMPEDANCE MATCHING

A. Power Transfer Between System Components—General Relations

An arbitrary microwave system is composed of a number of component parts, and associated with these components will be lengths of transmission line that are likely to be electrically "long" although physically short. To know the power transfer between the component parts, the impedance of the components must be known, and also the laws governing the power transfer between components as a function of their impedance.

In most microwave systems, the impedance of many component parts is made equal to the impedance of the transmission line between the components. When this is done the component or "load" is said to be "matched" to the line. There are several reasons for matching components to the interconnecting transmission line. Some of these are as follows:

1. If the generator and load are both matched to the transmission line between them, the generator will always deliver maximum power to the load whatever the length of line. If the load and generator do not match the line, the power delivered to the load by the generator will then depend upon the length of line between them, and in general will be less than the maximum power that the generator is capable of delivering. This does not take into account line losses.

2. Having the load match the transmission line will minimize losses in the transmission line.

3. Having the load match the transmission line keeps the possibility of breakdown in the line at a minimum.

4. It is easy to tell when the load impedance is equal to the line impedance by measuring the standing waves in the input line.

In the discussion that follows, it shall be assumed that $R = G = O$ in the transmission line, i.e., the attenuation in the line will be neglected.

The definitions of some terms introduced and used in this chapter are as follows:

V_1 = magnitude of incident wave of voltage
V_2 = magnitude of reflected wave of voltage
I_1 = magnitude of incident wave of current
I_2 = magnitude of reflected wave of current
$V_{max} = V_1 + V_2$ voltage loop or maximum
$V_{min} = V_1 - V_2$ voltage node or minimum
$I_{max} = I_1 + I_2$ current loop or maximum
$I_{min} = I_1 - I_2$ current node or minimum
$\eta = \left(\frac{V_{max}}{V_{min}}\right)^2$ = standing wave ratio in power
$Z_o = \sqrt{L/C}$ = characteristic impedance of the line
Z_L = load impedance of the line

REFLECTIONS AND IMPEDANCE MATCHING

1. **Percent Power Reflected From an Arbitrary Load.** Looking at a transmission system from the traveling wave point of view, we may regard power as being carried down the line by an electro-magnetic wave incident upon the load. If this traveling wave encounters some terminating impedance other than the impedance of the line on which it travels, part of the incident wave will reflect back toward the source of power, and the remainder will be absorbed in the terminating impedance. The interaction of incident and reflected waves on the line input to the terminating impedance will result in standing waves of voltage and current on the input line; that is, the voltage and current at various points on the line will vary between certain maximum and minimum values, depending upon the point of measurement.

These standing waves of voltage and current are shown in Fig. III–1 for a few typical load impedances. It will be seen that the magnitude and phase of the standing waves are determined by the load impedance, but that voltage loops always coincide with current nodes, and vice versa.

If an arbitrary load of impedance Z_L is used to terminate a line of impedance Z_o, the ratio of incident to reflected wave will be given by the formula

$$\frac{V_2}{V_1} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

This is an equation involving complex quantities, and the ratio V_2/V_1 will give not only the relative magnitude but also the phase difference at the load of the incident and reflected waves. All of the incident wave will be absorbed in the load impedance when $Z_L = Z_o$.

As power varies with the square of the voltage, the impedance being held constant, the percent power reflected from an arbitrary load is given by

$$\% \text{ power reflected} = \left| \frac{V_2}{V_1} \right|^2$$

2. **Relation Between Reflected Power and Standing Wave Ratio.** The extent to which the line and the load are mismatched is usually determined by measuring the standing wave ratio in the input line. The relation between percent power reflected and the standing wave ratio in power r_i is given by

$$\% \text{ power reflected} = \left(\frac{\sqrt{r_i} - 1}{\sqrt{r_i} + 1} \right)^2$$

The relationship expressed in this formula is plotted in Fig. III–2.

3. **Decrease of Power Transfer With Mismatch Between Generator and Load.** Another way of looking at this problem is to consider the generator as having an internal impedance equal to the impedance of the line, and therefore delivering its maximum power output to a load whose impedance is that of the line. Terminating the line with a mis-matched load

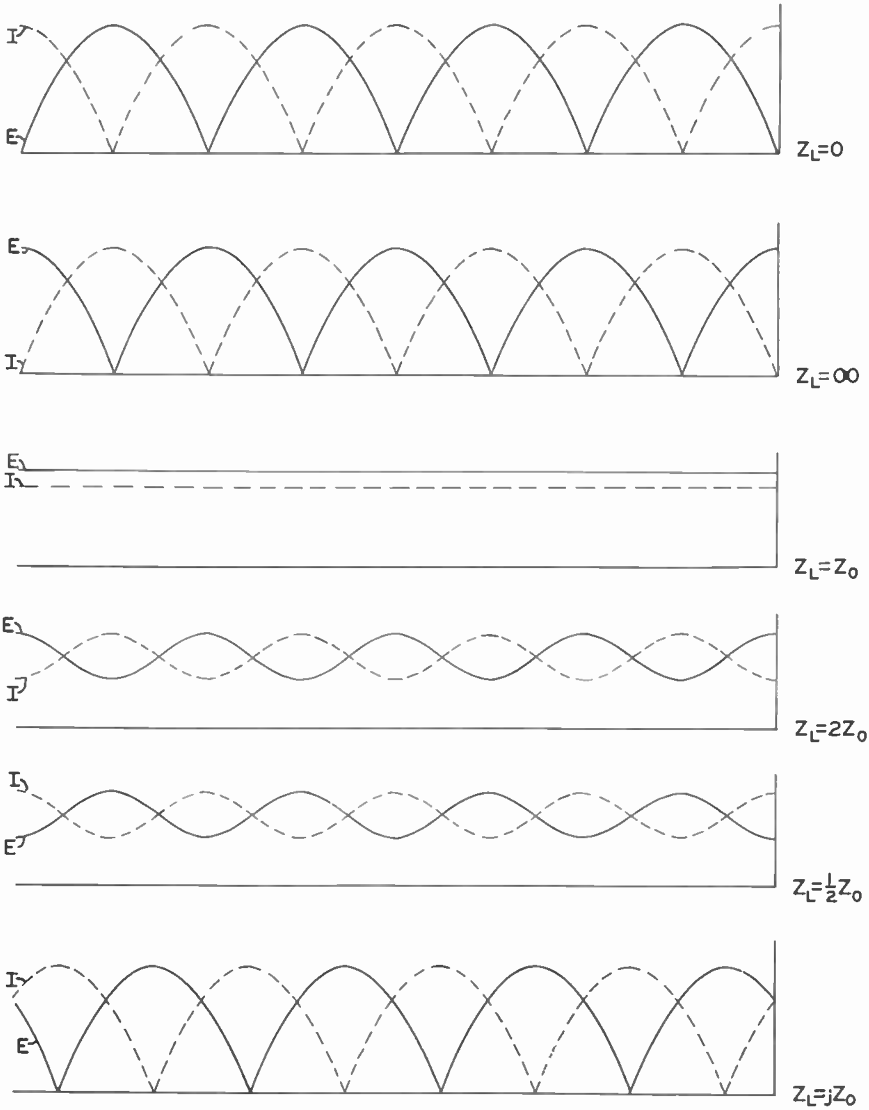


FIG. III - 1

STANDING WAVES ON A TRANSMISSION LINE

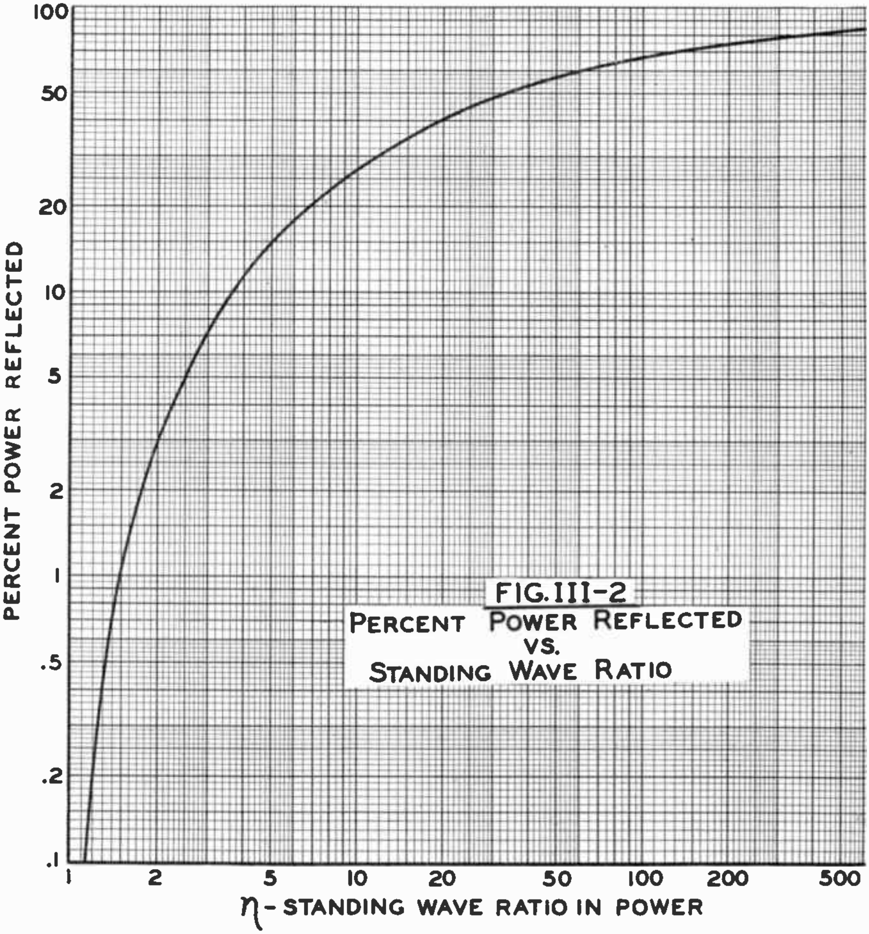


FIG. III-2
PERCENT POWER REFLECTED
VS.
STANDING WAVE RATIO

will result in some impedance other than that of the line being presented to the generator, with a corresponding decrease in power output.

If we say that the generator will deliver its maximum possible output to the impedance Z_o, the percent of this maximum possible power that will be delivered to an arbitrary load Z_L is given by

$$\% \text{ of maximum possible power delivered} = \frac{4RR_o}{|R_o + R + jX|^2}$$

In this expression, Z_o being a pure resistance is called R_o, and $Z_L = R + jX$. This gives the same results for net power transfer as the equations of the previous paragraph.

For a load that is purely resistive, the percent of maximum possible power delivered to the load by a generator matched to the line is given by Fig. III–3. For a load whose resistance is equal to the line impedance, but whose reactance is finite, the percent of maximum possible power delivered to the load is given in Fig. III–4.

4. Power Delivered When Neither Generator nor Load is Matched to the Transmission Line. A more general case than that considered previously is encountered when neither the generator nor the load is matched to the transmission line. When this is true, the power delivered by the generator to the load will depend upon the length of line between the generator and load, as well as the impedance of the load and the internal impedance of the generator.

At microwave frequencies, the impedance of the load is usually found by measuring the standing wave ratio in the line feeding the load, while the internal impedance of the generator is frequently found by removing the source of power and measuring the standing wave ratio in the line with a signal being fed into the generator from the load end of the line. The generator will deliver maximum power into a load that is the complex conjugate of its internal impedance, and its power output will fall off with other load impedances. The extent to which the output will fall is indicated by Fig. III–5. In this figure the ordinate is γ_i looking into the load, while the abscissa is γ_i looking into the generator. The solid lines indicate the greatest decrease in power delivered from the generator that will be found with given standing wave ratios, while the broken lines indicate the least possible decrease of power from the generator. The actual decrease in generator output will fall somewhere between these limits.

For example, if a 5 : 1 standing wave ratio were seen looking into the load, and a 3 : 1 standing wave ratio were seen looking into the generator, the position of the intersection of these values with respect to the diagonal lines determines the loss in output from the generator. In this example, the output will decrease a maximum of 1.8 db and a minimum of slightly less

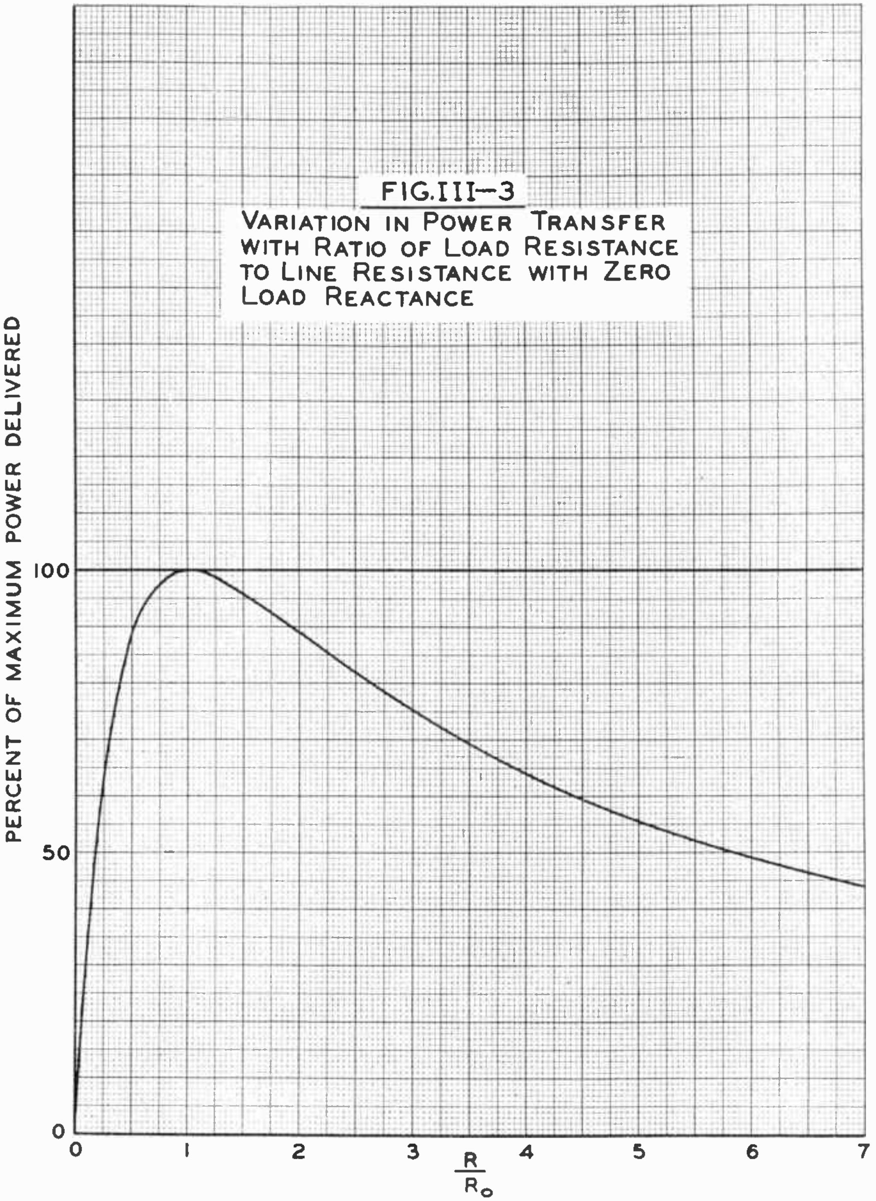


FIG. III—3
VARIATION IN POWER TRANSFER WITH RATIO OF LOAD RESISTANCE TO LINE RESISTANCE WITH ZERO LOAD REACTANCE

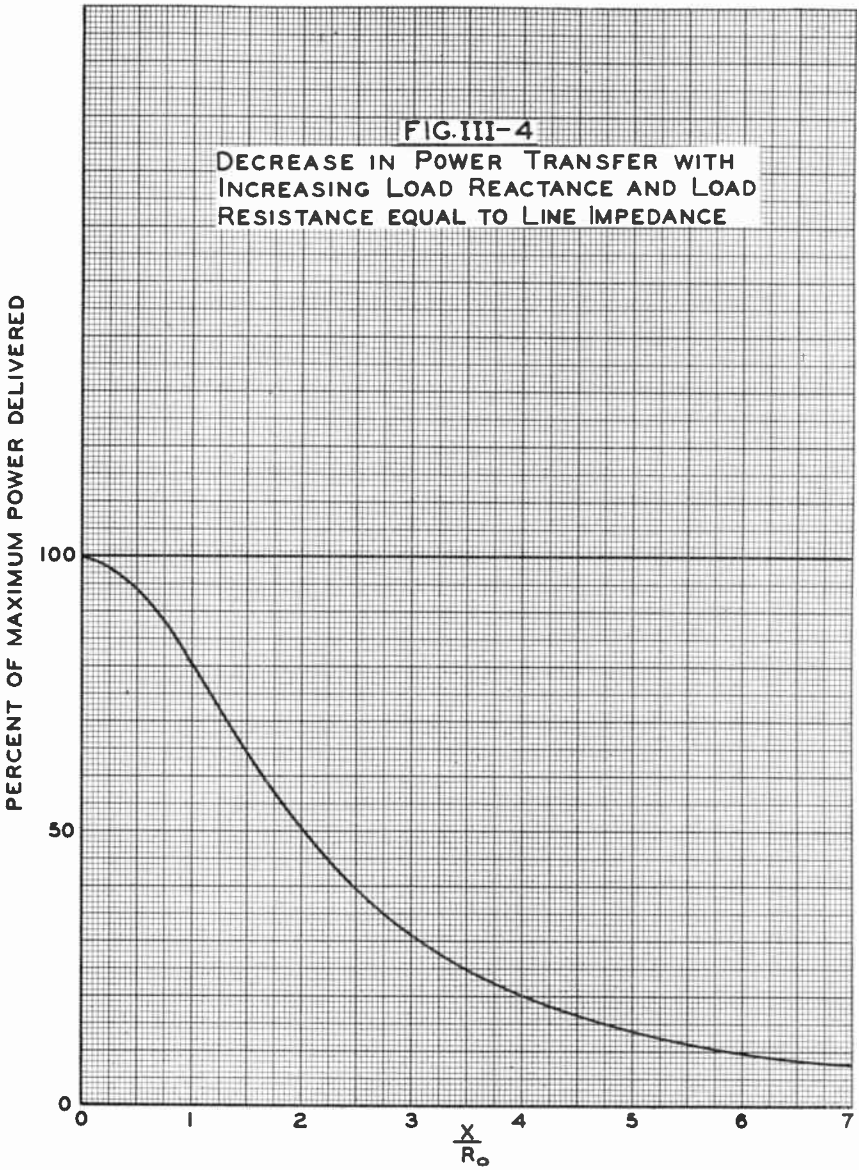


FIG. III-4
DECREASE IN POWER TRANSFER WITH INCREASING LOAD REACTANCE AND LOAD RESISTANCE EQUAL TO LINE IMPEDANCE
PERCENT OF MAXIMUM POWER DELIVERED
100
50
0
0
1
2
3
4
5
6
7
X / Ro

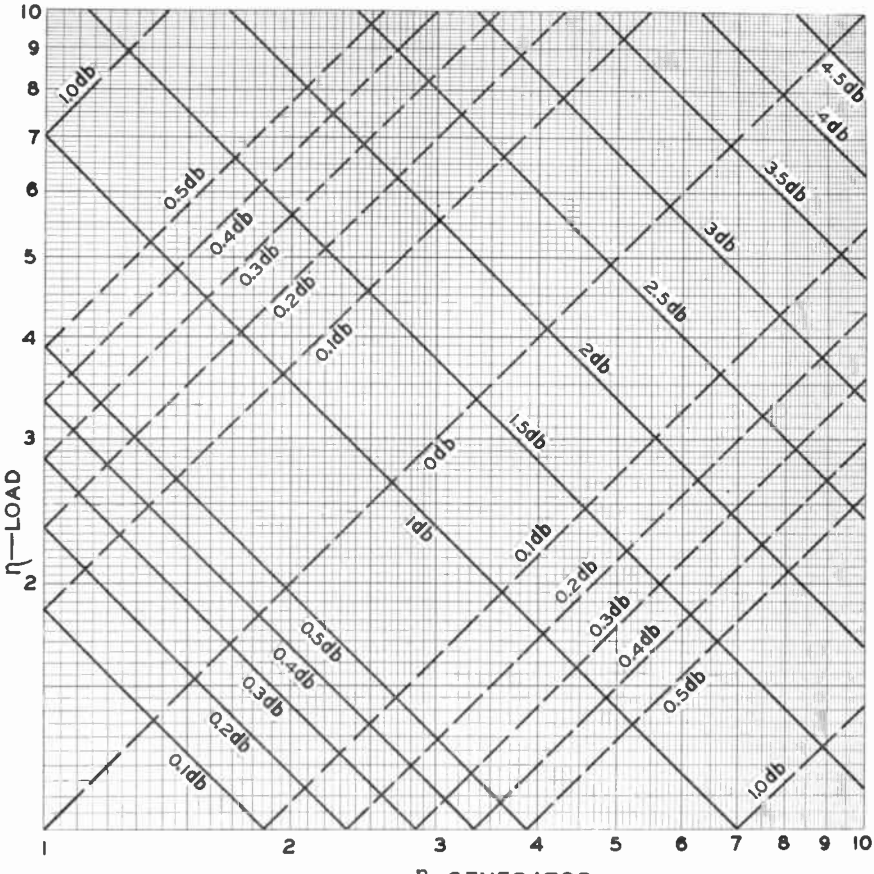


FIG.III-5

LOSS IN OUTPUT WHEN NEITHER GENERATOR NOR LOAD IS MATCHED TO THE INTERCONNECTING LINE

– – – – – – – MINIMUM LOSS

——————— MAXIMUM LOSS

than 0.1 db. The actual loss will fall somewhere between these limiting values.

5. Impedance and Admittance Diagrams. The sending end impedance Z_s of a lossless transmission line is given by

$$Z_s = Z_o \frac{Z_L + j\,Z_o \tan \beta l}{Z_o + j\,Z_L \tan \beta l}$$

while the sending end admittance Y_s is given by

$$Y_s = Y_o \frac{Y_L + j\,Y_o \tan \beta l}{Y_o + j\,Y_L \tan \beta l}$$

The similarity of these two equations is apparent, and it is possible to express the relationships contained in them in a single diagram, which can be used for either admittance or impedance calculations with only a change in the labeling of the co-ordinate axes.

Figs. III–6 and III–7 are graphs which express the above relationships, and each of these diagrams may be used as either an impedance or an admittance diagram. As an impedance diagram, the ordinate is a measure of reactance and the abscissa a measure of resistance, both normalized with respect to the characteristic impedance of the transmission line. Knowing the line impedance and the load impedance, we may locate the point on the chart that represents the load impedance $Z_L = R_L + jX_L$. The ordinate of the point will be X_L/Z_o, and the abscissa R_L/Z_o. The sending end impedance of the line will always fall on the circle that passes through the point Z_L, and links the point 1,0. The location of the sending end impedance on this circle will depend upon the length of line between the load and the sending end. The other, orthogonal family of circles affords a measure of electrical distance, and is calibrated in terms of βl, or electrical degrees of length. The values of sending end impedance repeat every 180 degrees or half wave length. Traveling away from the load and toward the sending end, we must proceed in a clockwise direction around the first family of circles.

Example: A 50 ohm. transmission line is terminated by an impedance $Z_L = 65 + j37.5$ ohm. What is the sending end impedance one-twelfth wavelength from the load?

$R/Z_o = 65/50 = 1.3$, and $X/Z_o = 37.5/50 = 0.75$. This locates the load impedance on the impedance diagram. This point is seen to fall on the intersection of two circles, one being the circle that links the point 1,0 and passes through the point 2,0, and the other being the circle that passes through the point 1,0 and is labled $\beta l = 25°$. The sending end impedance will fall on the first of these circles.

The sending end is a twelfth wavelength, or 30 degrees from the load, and we must therefore proceed around the first circle in a clockwise direction a distance of 30 degrees as measured by the second family of circles.

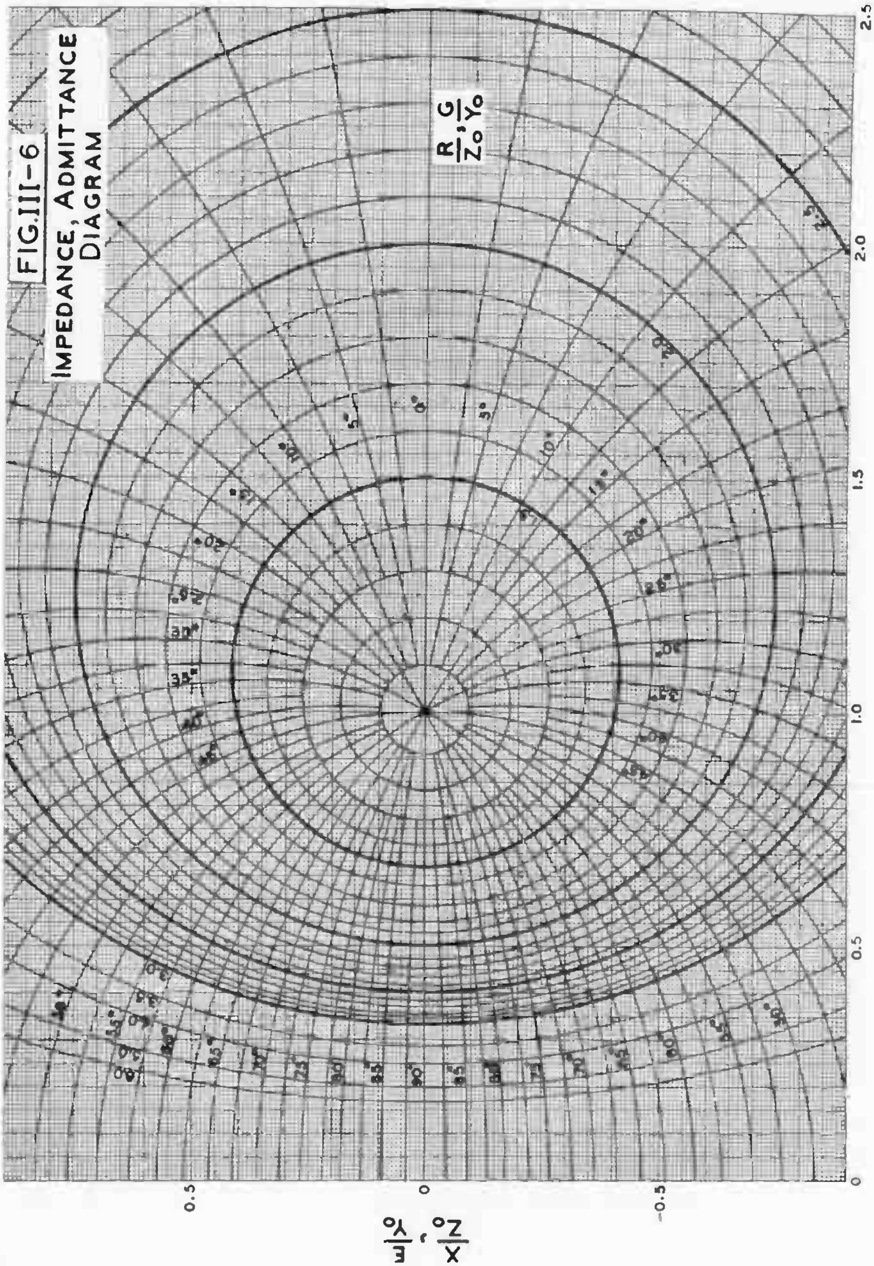


FIG. III-6
IMPEDANCE, ADMITTANCE DIAGRAM

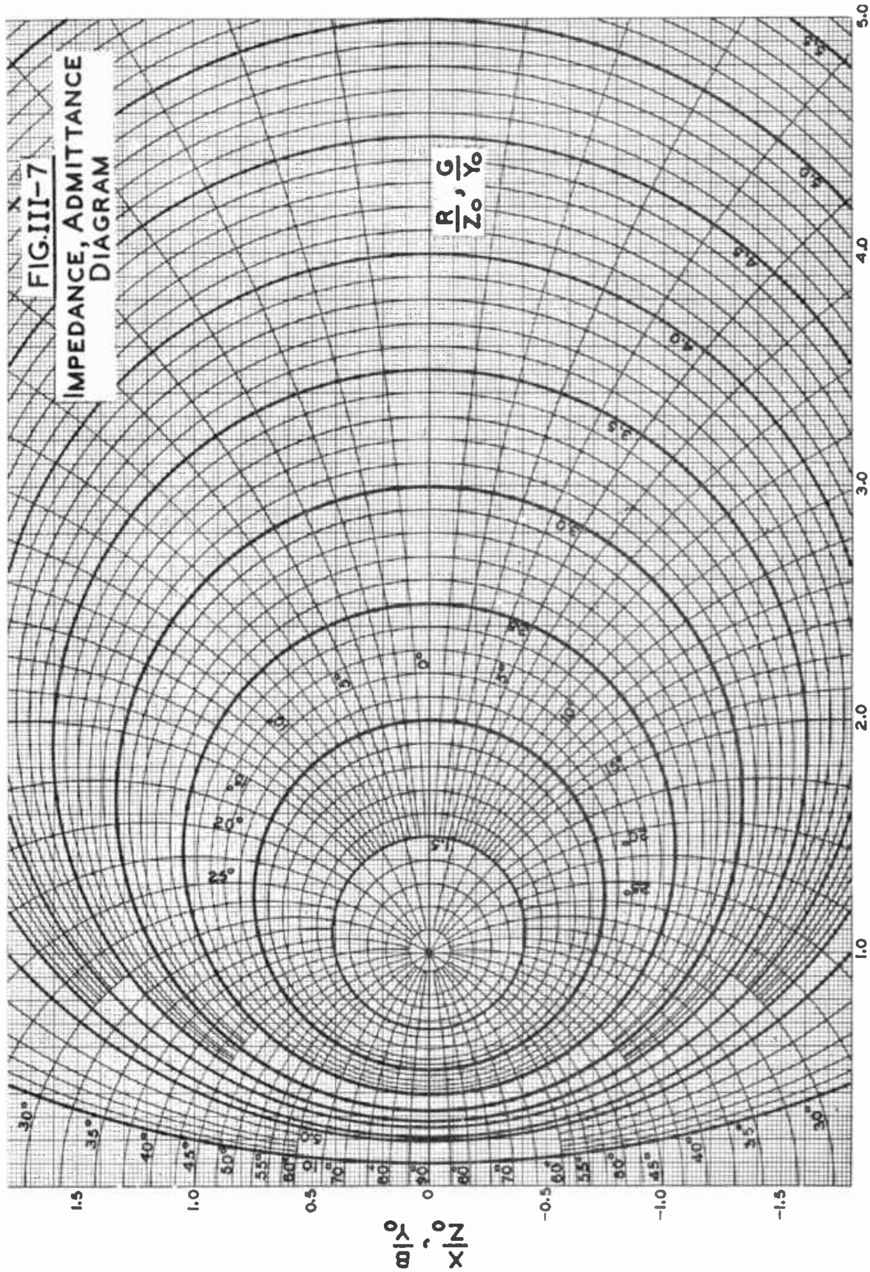


FIG. III-7
IMPEDANCE, ADMITTANCE DIAGRAM

This will bring us to the curve below the horizontal axis that passes through the point 1,0 and is labeled $\beta l = 5°$. The intersection of this curve with our first circle is at the point $R/Z_o = 1.95$, $X/Z_o = -0.25$, and this point determines the desired sending end impedance. This will be $Z_s = 50(1.95 - j0.25) = 97.5 - j12.5$ ohms. The above procedure is illustrated in Fig. III–8.

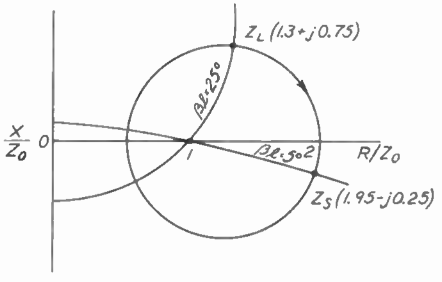


FIG. III - 8

The charts may be used in the same manner as admittance diagrams to determine the sending end admittance of a transmission line at a known distance from a known load. In this case, the ordinate is a measure of susceptance and the abscissa of conductance, both normalized with respect to the transmission line admittance. Traveling away from the load, we still proceed in a clockwise direction, but it must be kept in mind that inductive reactances are of positive sign while inductive susceptances are of negative sign.

B. *Impedance Transformers*

To realize maximum power transfer between components of a system, the impedances of the components should be matched. This matching is often accomplished by impedance transformers. At microwave frequencies, the elements of these transformers are frequently short lengths of transmission line.

1. Sleeves

a. Quarter Wave Sleeves. A generator of internal resistance R_1 may be matched to a load resistance R_2 by inserting between the generator and load a quarter wave length transmission line whose characteristic impedance Z_o' is the geometric mean of the two values of resistance, that is $Z_o' = \sqrt{R_1R_2}$. This is illustrated in Fig. III–9. The generator will then see a load resistance that is equal to its internal resistance, and deliver the maximum possible power output to the load.

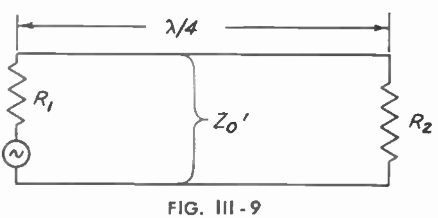


FIG. III - 9

At microwave frequencies, the generator resistance is frequently the characteristic impedance of a transmission line, Z_o, and the quarter wave section will match the resistive load R_2 to the line impedance Z_o.

If the load impedance Z_L of the transmission line is not a pure resistance, it can still be matched to the transmission line. There must be, however, an additional length of line between Z_L and the quarter wave section, of such length that the impedance looking into the added length is a pure resistance R_2. This means that the input to the additional length of line must be at a voltage minimum or maximum. The various impedances must then satisfy the relation

$$Z_o' = \sqrt{Z_o R_2}$$

This is illustrated in Fig. III–10.

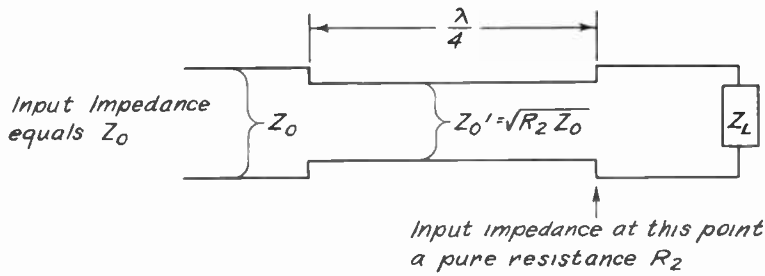


FIG. III - 10

In microwave coaxial line, the quarter wave transformer section is usually obtained by placing a sleeve $\lambda/4$ long over the inner conductor or inside the outer conductor, as in Fig. III–11.

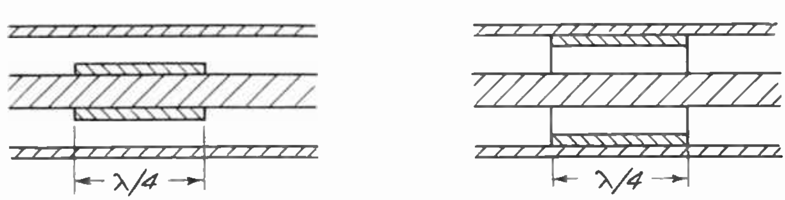


FIG. III - 11

The impedance of the transformer is then always less than the impedance of the line, and R_2 must be less than Z_o. This means that the end of the sleeve facing the load must be placed at a voltage minimum in the line. Chosen thus, $R_2 = Z_o/\sqrt{r_i}$. The following procedure will then determine the size and location of the sleeve necessary to match the load to the line.

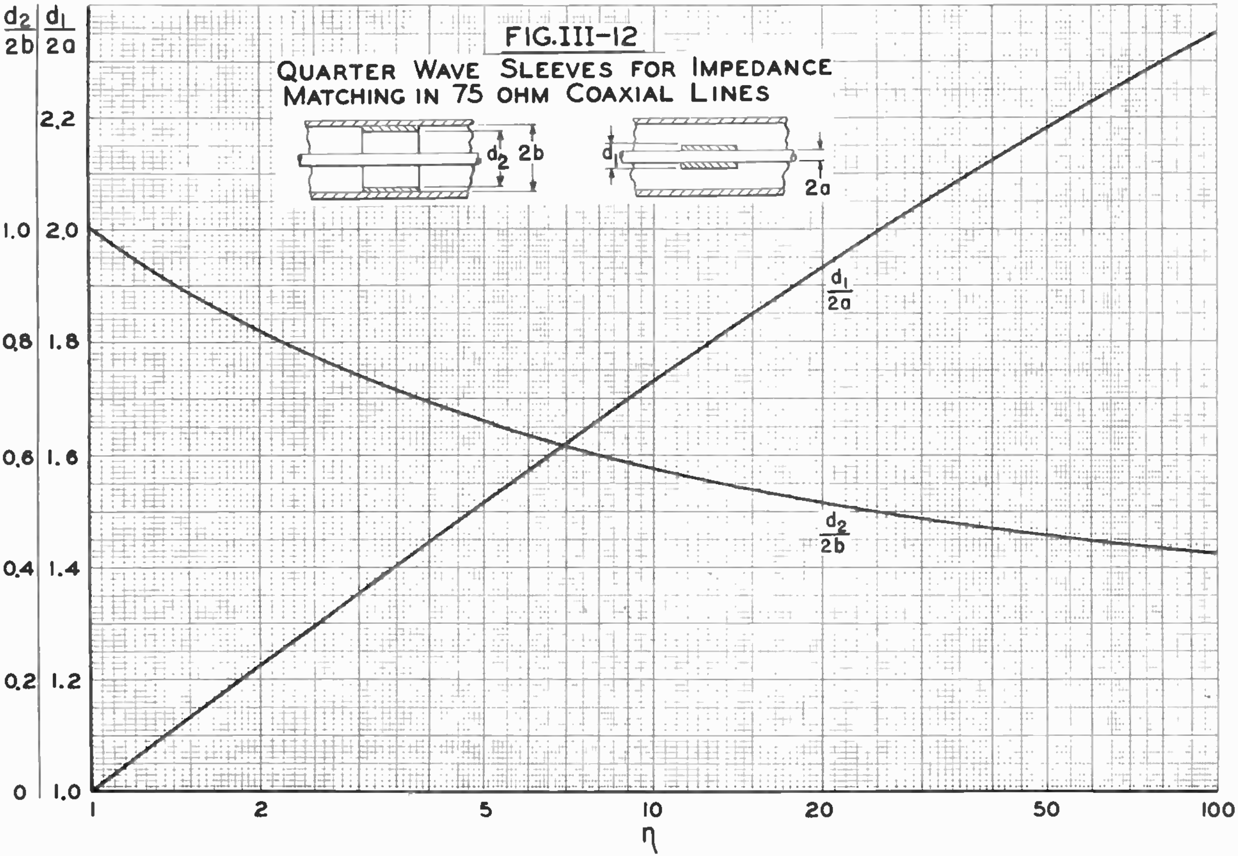


FIG. III-12
QUARTER WAVE SLEEVES FOR IMPEDANCE MATCHING IN 75 OHM COAXIAL LINES

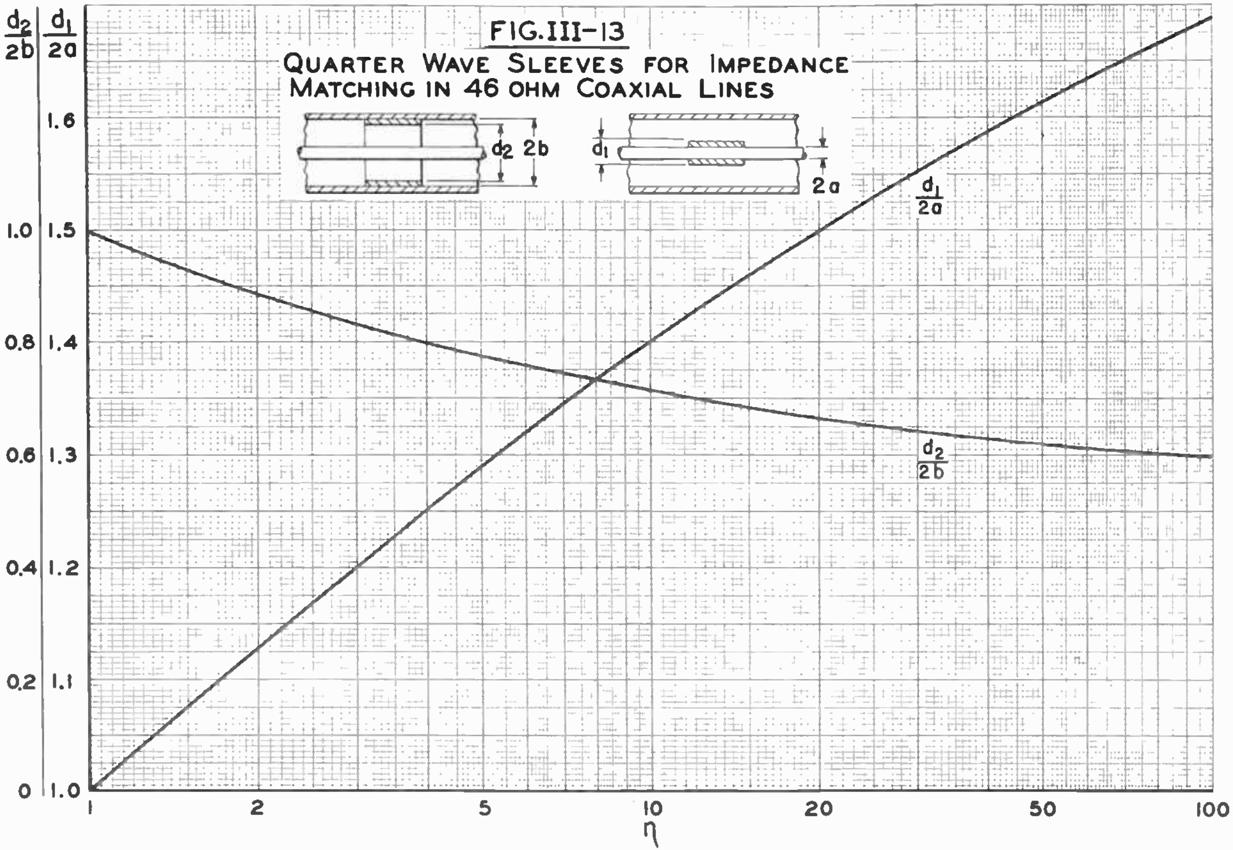


FIG. III-13
QUARTER WAVE SLEEVES FOR IMPEDANCE MATCHING IN 46 OHM COAXIAL LINES

Procedure for matching with $\lambda/4$ sleeve:

1. Measure the standing wave ratio η in the input, and determine the location of a minimum voltage point.

2. Choose a sleeve $\lambda/4$ long whose dimensions are determined by the following equations:

a. If the sleeve is on the inner conductor, its outer diameter d_1 should be

$$d_1 = 2b/\left(\frac{b}{a}\right)^{\eta^{-1/4}}$$

b. If the sleeve is inside the outer conductor, its inner diameter d_2 should be

$$d_2 = 2a\left(\frac{b}{a}\right)^{\eta^{-1/4}}$$

3. Insert this sleeve in the line at a position where the end of the sleeve facing the load is at the point previously determined to be a voltage minimum, or an integral number of half waves from this position.

The ratios $d_1/2a$ and $d_2/2b$ are plotted as functions of the power standing wave ratio η in Fig. III–12 for 75 ohm coaxial line and in Fig. III–13 for 46 ohm coaxial line.

b. Multiple Quarter Wave Sleeves. A single quarter wave sleeve used as a transformer has the disadvantage that it is resonant, i.e., it matches perfectly at only one frequency. The bandwidth over which the match is good can be extended by using two or more quarter wave sleeves, placed together and properly chosen in size. This is illustrated in Fig. III–14. The design equation for such a transformer is as follows:

$$2\,(\log Z_o - \log Z_o') = (\log Z_o' - \log Z_o'') = 2(\log Z_o'' - \log R_2)$$

where, as before, R_2 is the purely resistive impedance seen at a point of minimum voltage in the input line ($R_2 = Z_o/\sqrt{\eta}$).

Devices using multiple quarter wave sleeves can be made less and less frequency sensitive by using more and more sections. The increment in the logarithm of the impedance between succeeding sections should follow the binomial coefficients:

1	2	1		
1	3	3	1	
1	4	6	4	1

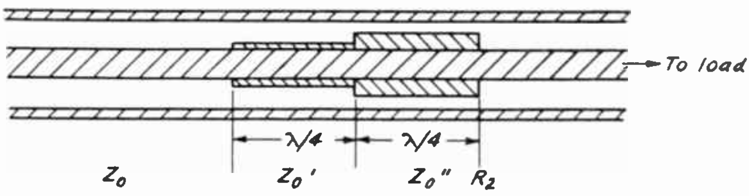


FIG. III - 14

The first of these is the double sleeve described above, the second corresponds to three sleeves, etc.

c. Tapers. It would appear from the above discussion that a frequency insensitive match between two impedances could be obtained by using a length of tapered transmission line whose impedance varied continuously but slowly throughout its length, and whose impedance at each end was equal to the impedance to be matched at that end. This is true to a first approximation, provided that the change in line parameters per wavelength is small. Such a tapered section is illustrated in Fig. III–15.

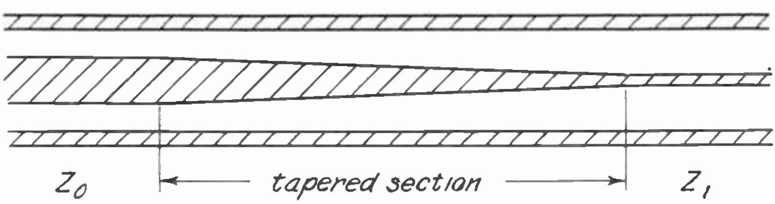


FIG. III-15

To a second approximation, the reflection introduced by the tapered section is given by

$$\frac{V_2}{V_1} = \frac{1}{4\gamma_o}\left(\frac{d\,ln\,Z}{dx}\right)_o - \frac{1}{4\gamma_1}\left(\frac{d\,ln\,Z}{dx}\right)_1 e^{-2\int_o^d \gamma dx}$$

where V_1 is the incident wave of voltage and V_2 the reflected wave. In this expression, the subscript *o* means the value for the taper at point x_o, while the subscript 1 means the value for the taper at the point x_1. The terms $d\,ln\,Z/dx$ are discontinuous at the points x_1 and x_o, so the values that are approached as a limit at the ends of the taper should be used. To the extent for which the above expression is valid, that is for small reflections, the standing wave ratio introduced by the taper is

$$r_i = \left(1 + 2\left|\frac{V_2}{V_1}\right|\right)^2$$

The optimum condition corresponding to minimum reflection occurs when no discontinuity exists in the function $f(Z) = lnZ$ or any of its derivatives. This means that the impedance variation between Z_1 and Z_o should be such that[1]

$$ln\,\frac{Z}{Z_o} = \frac{h}{\sqrt{\pi}}\,ln\,\frac{Z_1}{Z_o}\int_{-\infty}^{x} e^{-h^2x^2}\,dx$$

In this expression x is the distance as measured from the center of the taper, Z is the impedance at the point x, and h an arbitrary constant.

[1]From unpublished notes by W. W. Hansen.

REFLECTIONS AND IMPEDANCE MATCHING

Example: Consider the case where $f(Z) = ln\ Z$ varies linearly with distance throughout the region x_o to x_1. This is illustrated in Fig. III–16.

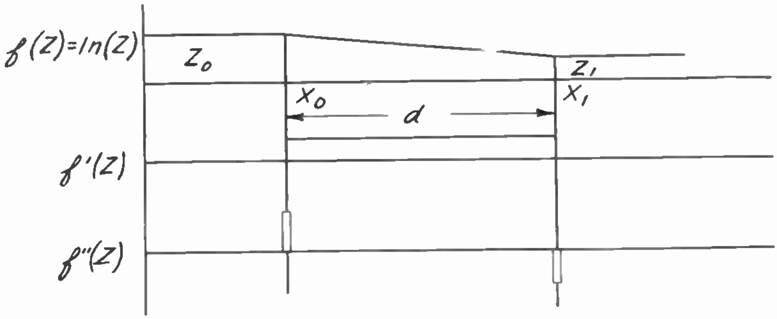


FIG. III - 16

The reflection introduced by the taper is given by

$$\frac{V_2}{V_1} = \frac{\lambda}{8\pi jd}\, ln\, \frac{Z_1}{Z_o}\left(1 - e^{-\frac{4\pi jd}{\lambda}}\right)$$

The variation of reflection with length of taper as calculated by this formula is illustrated in Fig. III–18 for a taper from 46 ohm to 75 ohm line.

2. Single Stub Transformer. Shorted stub sections of line in shunt with the main transmission line act as shunting reactances. Depending upon the length of the line, the reactance may be either inductive or capacitive, and have any value between zero and infinity (neglecting losses). The input impedance of a shorted section of transmission line is given by

$$Z = j\, Z_o \tan \beta l$$

Within limits set by losses in the line, a single stub line, tunable both in position and length, is capable of matching any admittance not purely reactive to the admittance of the transmission line. To do this, the stub must be located at a point in the line where the conductance component of the input admittance is equal to the line admittance. The stub susceptance should then be equal and opposite in sign to the input susceptance at that point.

Example: Referring to the Admittance Diagram of Section A-5 of this chapter, it is desired to match a load of admittance Y_1 to a line of admittance Y_0. This is illustrated in Fig. III–17.

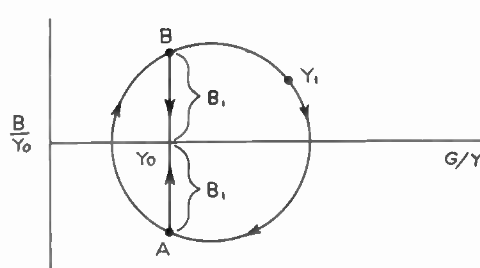


FIG. III - 17

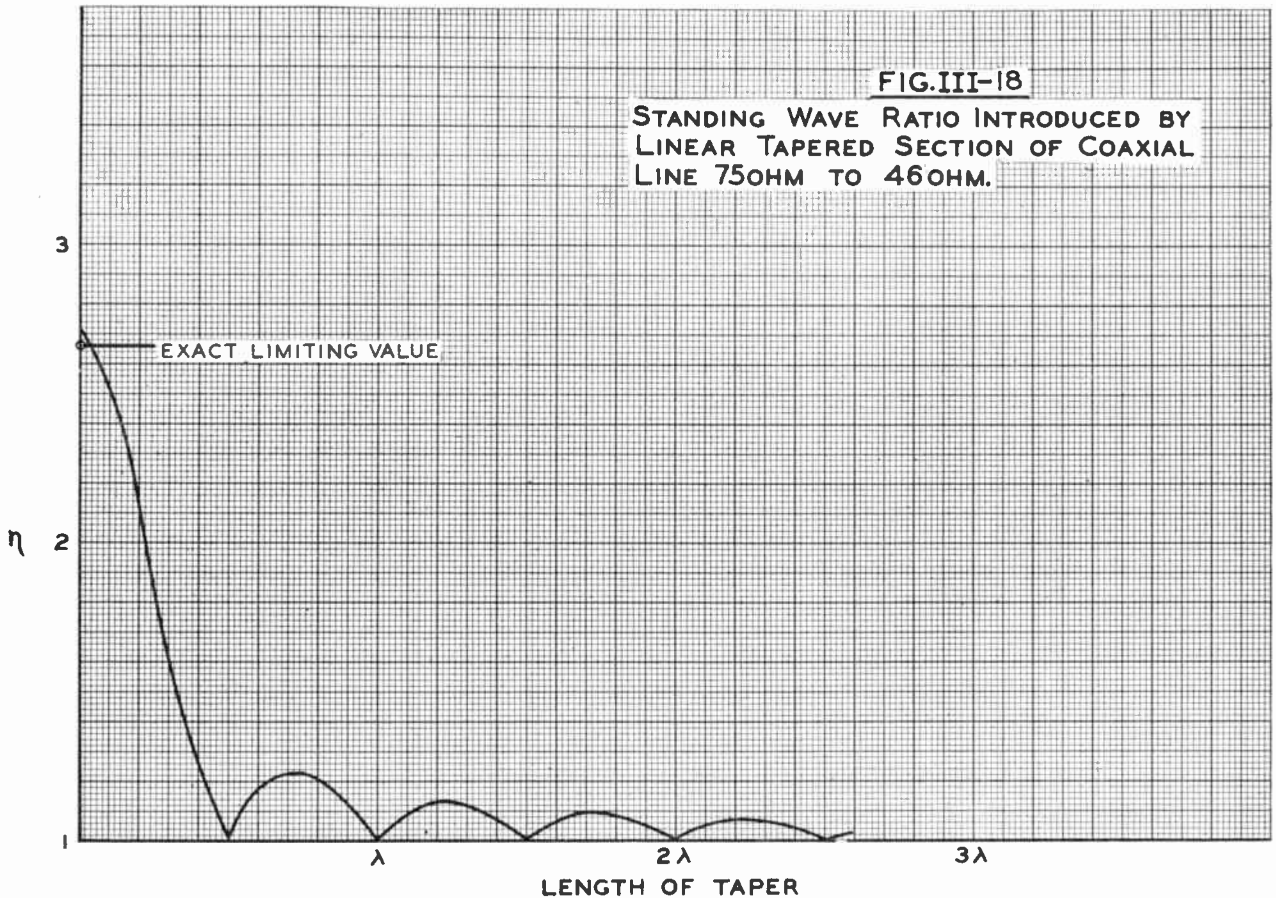


FIG.III-18

STANDING WAVE RATIO INTRODUCED BY LINEAR TAPERED SECTION OF COAXIAL LINE 75OHM TO 46OHM.

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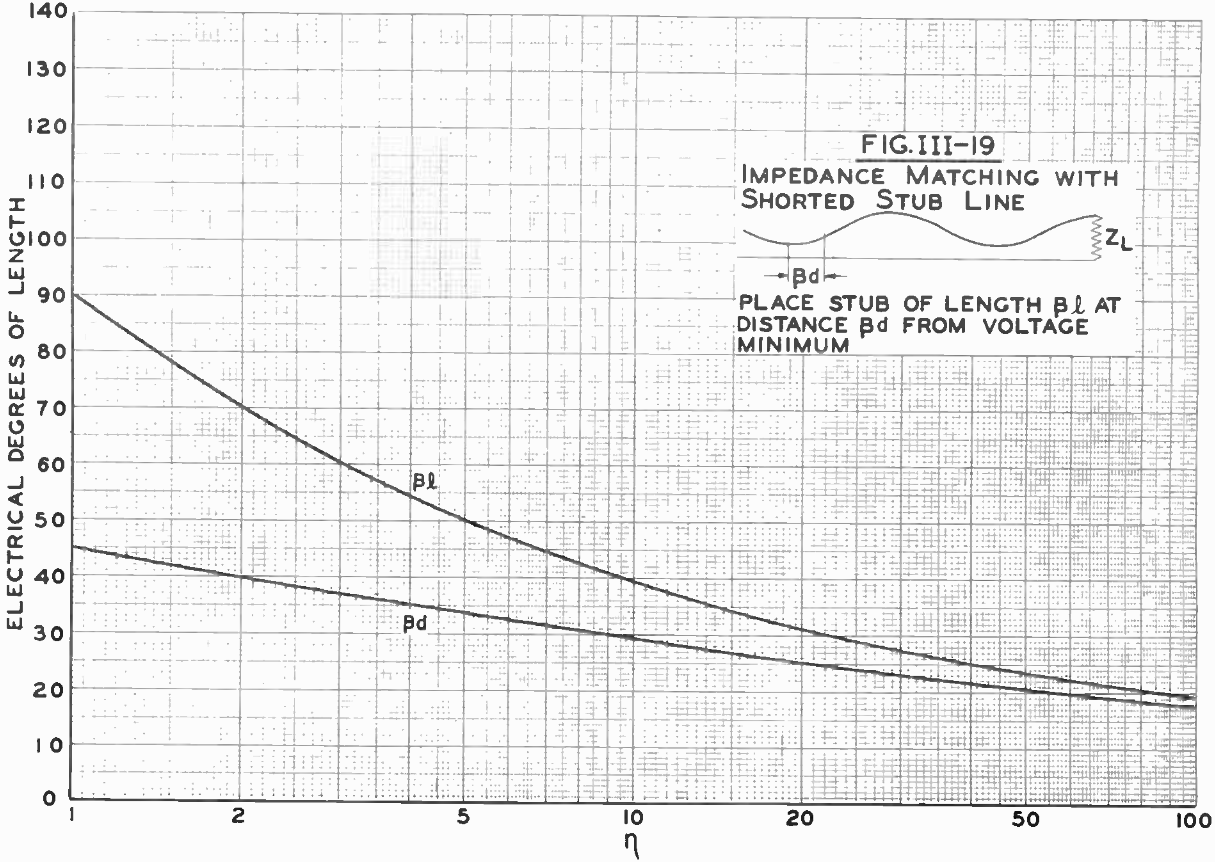

FIG. III-19
IMPEDANCE MATCHING WITH SHORTED STUB LINE

PLACE STUB OF LENGTH $\beta \ell$ AT DISTANCE βd FROM VOLTAGE MINIMUM

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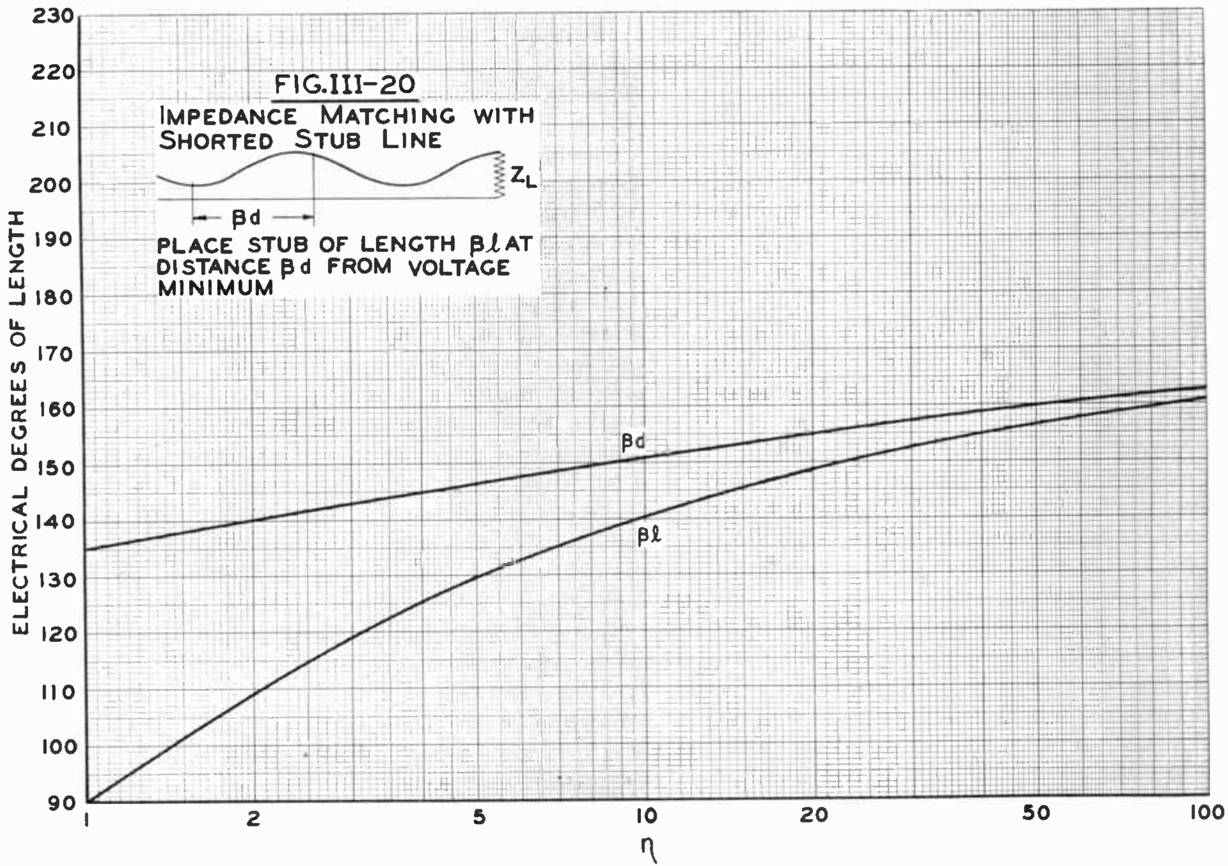

FIG.III-20
IMPEDANCE MATCHING WITH SHORTED STUB LINE

REFLECTIONS AND IMPEDANCE MATCHING

The stub for matching should be located at either point A or point B. If at point A, the input admittance of the stub should be capacitive and of a value B_1, to balance out the inductive component of the input admittance of the line. This will bring the total input admittance at point A to Y_o. A stub located at B should be inductive and have an input susceptance of $-B_1$.

The length and position of the shorting stub necessary to correct a given standing wave ratio is plotted in Fig. III–19 for the inductive shorted stub and in Fig. III–20 for the capacitive shorted stub. These curves represent two alternate solutions to the same problem. The length βl of the stub is in electrical degrees. The distance βd between the required position of the stub and a previously determined voltage minimum is also in electrical degrees. As indicated on the curves, the distance βd should be measured from a minimum toward the load.

3. Double Stub Transformer. Another type of transformer used for impedance matching consists of two tunable shorting stubs, fixed in position. This is illustrated in Fig. III–21.

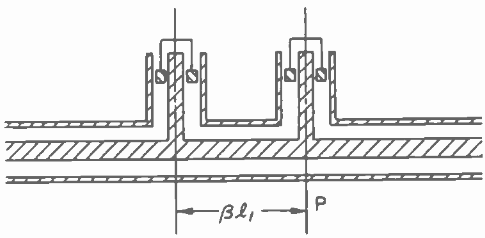

FIG. III - 21

The range of impedances seen by the transformer at the point P which can be matched to the impedance of the line Z_o depends upon the length of line between the two shorting stubs. Specifically, a transformer with two shorting stubs spaced βl_1, electrical degrees apart will match to the line admittance Y_o any load whose conductance component G_1 at the point P is less than $Y_o(1 + \cot^2\beta l_1)$. This is illustrated in Fig. III–22, where the maximum allowable conductance component $G_{1(max)}$ of the admittance at point P is plotted as a function of the distance between stubs. It would appear from Fig. III–22 that the double stub transformer could match the greatest range of impedances if the spacing between the stubs were very nearly some multiple of a half wave-length. But such a transformer would require that the stubs present a very low admittance across the line, which may not be obtainable in practice because of losses in the stub. Also the tuning will be very critical, and the match very sensitive to slight changes in frequency.

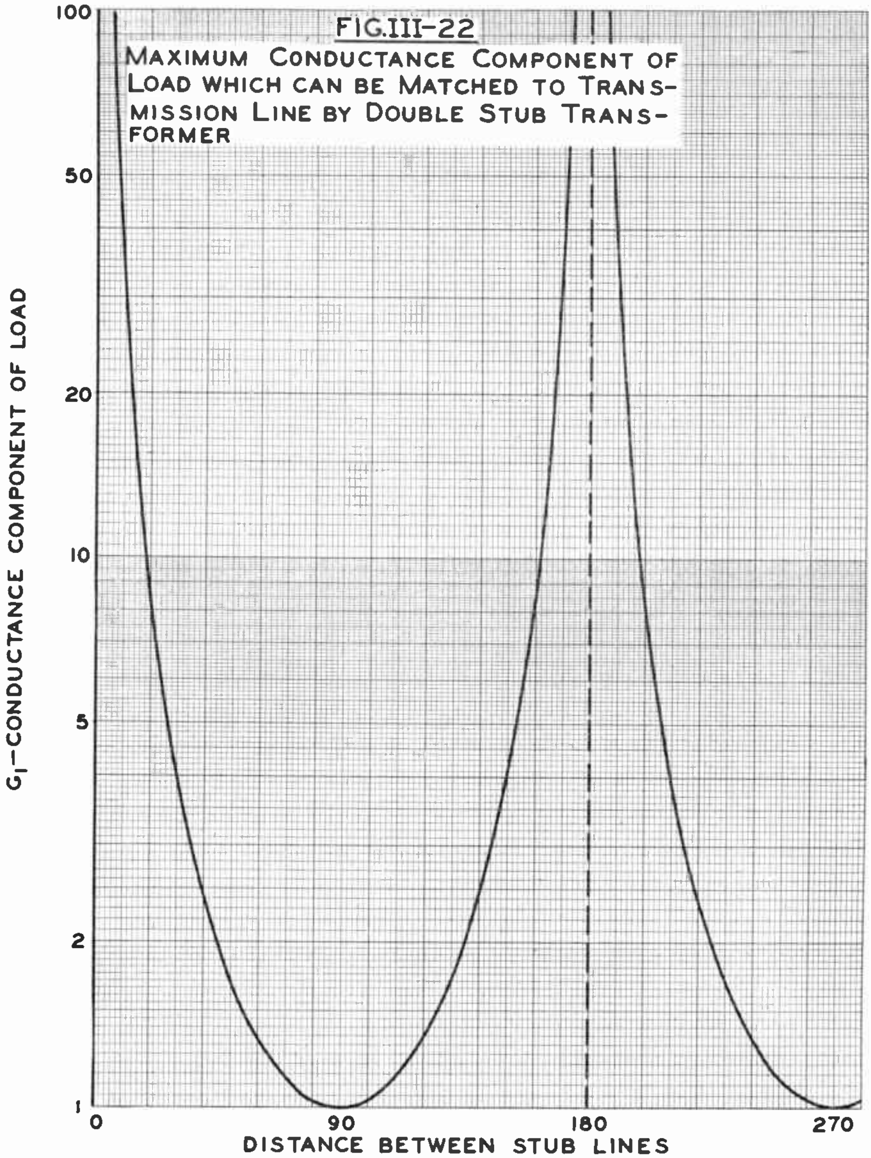
FIG.III-22
MAXIMUM CONDUCTANCE COMPONENT OF LOAD WHICH CAN BE MATCHED TO TRANSMISSION LINE BY DOUBLE STUB TRANSFORMER
100
50
20
10
5
2
1
G_1–CONDUCTANCE COMPONENT OF LOAD
0
90
180
270
DISTANCE BETWEEN STUB LINES

In actual practice, a compromise must be made between the range of impedances which the transformer is capable of matching, and the ease of obtaining a match. An odd multiple eighth wave spacing is frequently used; this will match to Y_o any admittance whose conductance component is less than $2Y_o$ at the point P.

An identical analysis holds when the stubs are in series rather than in shunt except that in the above expressions, impedance should be substituted for admittance, reactance for susceptance, etc.

A double stub transformer will match any impedance to the line impedance if provision is made for inserting an additional quarter wave length of line between the transformer and the load when required.

4. Triple Stub Transformer. A transformer suitable for matching any impedance to any impedance can be constructed by placing three adjustable shorting stubs in shunt with the line, spaced a quarter wave apart, and ganging the first and third together. See Fig. III–23. This transformer has only two adjustments, and has an advantage over the single, movable stub in that there is no joint in the main line requiring sliding contacts.

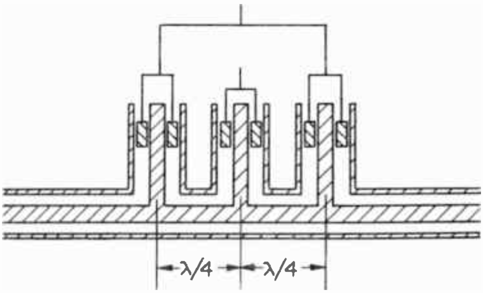

FIG. III - 23

5. Probes as Impedance Transformers. A mechanically convenient impedance transformer may be constructed utilizing probes in coaxial line. The probe introduces a reflection whose magnitude increases with increasing depth of the probe. A transformer may be constructed by using two or more fixed probes spaced a distance apart or a single probe whose position along the line can be varied.

Experimental data for a typical probe are presented in Fig. III–24.[2] The standing wave ratio introduced into a matched line is plotted as a function of the depth of the probe, and is seen to approach infinity as the probe nears contact with the center conductor. The slight deviation from unity standing wave ratio at zero probe depth results from a termination not properly matched. This data is at a wavelength of $\lambda = 10$ cm. in co-axial line whose inner diameter was .250″ and whose outer diameter was .875″. The probe diameter was .250″.

[2]Measured at the Research Laboratories, Sperry Gyroscope Co.

6. Eccentric Line Transformer. To construct an adjustable quarter wave transformer, it is necessary to have the characteristic impedance of the quarter wave section continuously variable. The simplest way to do this is to leave the diametric ratio fixed in the quarter wave section, but to have the eccentricity between inner and outer conductors variable. Making the conductors eccentric increases their capacity and so reduces the characteristic impedance of the line. For the transformer to cover the entire range of impedances, the eccentricity should be continuously adjustable from a point where the two conductors are coaxial to where they are just grazing. The grazing limit can not be approached too closely if the transformer is to carry any considerable amount of power because of the enhanced possibility of breakdown with closer spacing.

If the position as well as the eccentricity of the quarter wave section is made variable, this type of transformer will match any impedance to the characteristic impedance of the line.

The characteristic impedance of an eccentric line is given by

$$Z_o = 60 \cosh^{-1}\left[\frac{b}{2a}(1 - \varepsilon^2) + \frac{a}{2b}\right]$$

where the displacement of the axis of the inner conductor from the axis of the outer conductor is εb. Curves which give the variation in characteristic impedance as a function of the eccentricity of the center conductor are given in Fig. III–25. The curves are self-explanatory.

7. Dielectric Transformer. A short section of dielectric material inserted in a transmission line will act as an impedance transformer. Its performance may be readily calculated because both the characteristic impedance of the line and the wavelength in the line are inversely proportional to the square root of the dielectric constant. These transformers are not widely used because of the small selection of good dielectrics at microwave frequencies.

Example: It is desired to correct a standing wave ratio of 4 : 1 in a line. The procedure is as follows:

1. Select a low loss dielectric material whose dielectric constant is the square root of the measured standing wave ratio. In this case a dielectric constant of two would be required.

2. Cut a bead of this material to fit inside the line whose length is given by

$$\text{length of bead} = \frac{\lambda/4 \text{ in air}}{\sqrt{\varepsilon}}$$

where ε is the dielectric constant of the bead material.

3. Insert this bead in the line so that the face nearest the load is at a voltage minimum. This is illustrated in Fig. III–26.

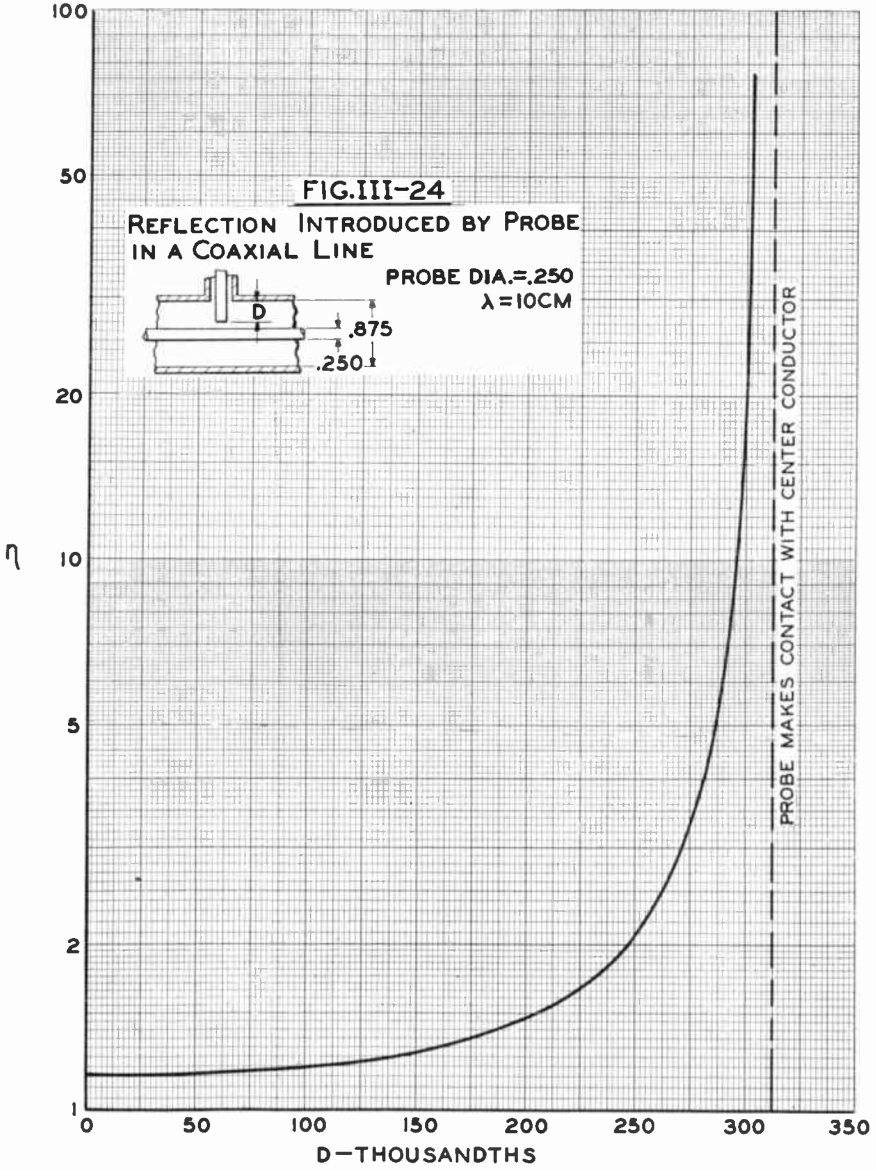
FIG.III-24
REFLECTION INTRODUCED BY PROBE IN A COAXIAL LINE
PROBE DIA.=.250
λ=10CM
D
.875
.250
PROBE MAKES CONTACT WITH CENTER CONDUCTOR
η
100
50
20
10
5
2
1
0
50
100
150
200
250
300
350
D—THOUSANDTHS

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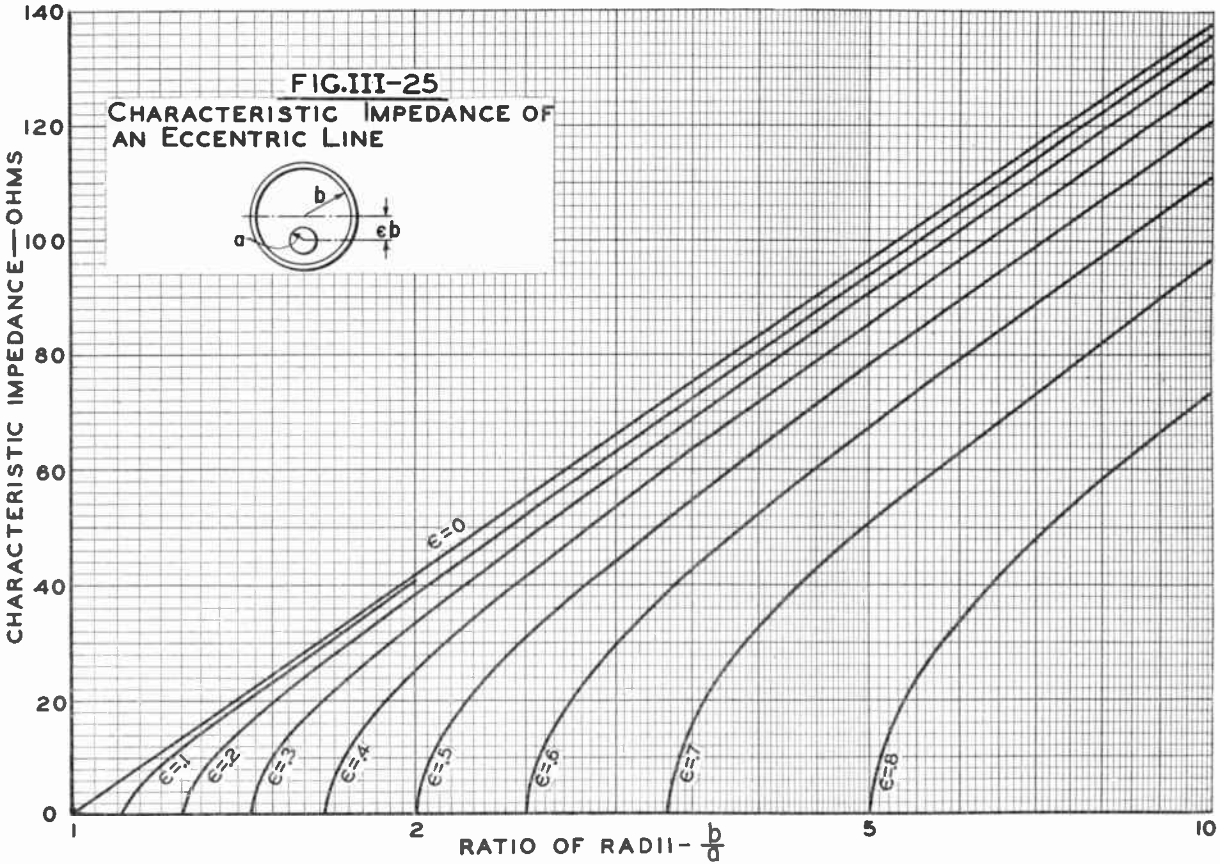


FIG.III-25
CHARACTERISTIC IMPEDANCE OF AN ECCENTRIC LINE

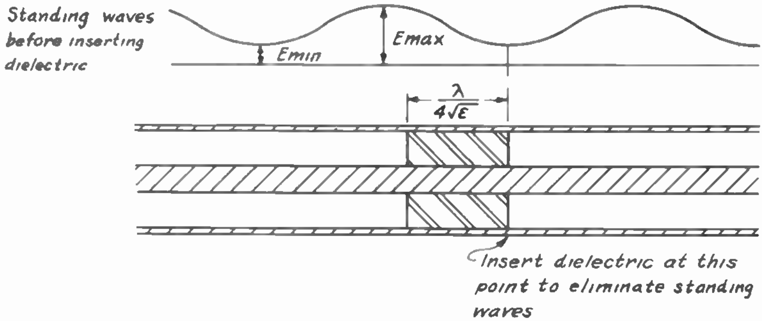


FIG. III - 26

8. Double Slug Transformer. A convenient impedance transformer may be constructed utilizing two sections of line, electrically a quarter wave long, whose impedance is different from that of the main line. These sections are usually obtained by placing slugs of metal or dielectric material within the line that are electrically a quarter wave long. A typical transformer using dielectric slugs is shown in Fig. III–27. Two controls are provided for these

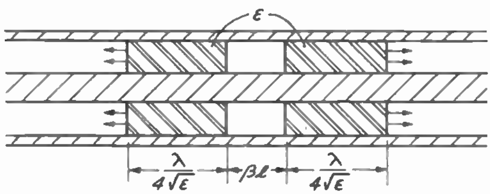


FIG. III - 27

slugs: 1. With the spacing between held constant, the slugs are moved along the line. 2. The spacing between the slugs is varied by moving each slug an equal amount in opposite directions. The advantage of this transformer is that the first control changes the phase of the reflection without affecting the magnitude, while the second control varies the magnitude of the reflection with little effect on the phase.

The following analysis assumes that the slugs are made of dielectric material whose constant is ε, but the results are equally applicable for any sort of slug that reduces the characteristic impedance of the line by a factor $\sqrt{\varepsilon}$.

The magnitude of reflection introduced into a matched line by the transformer is given by

$$\left|\frac{V_2}{V_1}\right| = \left|\frac{j\left(\frac{1}{\varepsilon} - \varepsilon\right)\tan\beta l}{2 + j\left(\frac{1}{\varepsilon} + \varepsilon\right)\tan\beta l}\right|$$

where βl is the electrical distance between the two quarter wave sections. The standing wave ratio as a function of βl is plotted in Fig. III–28 for various values of ε.

The shift in phase $\Delta\varphi$ of the standing waves with increasing βl is given by

$$\Delta\varphi = \frac{1}{2}\beta l - \frac{1}{2}\tan^{-1}\left[\frac{\left(\frac{1}{\varepsilon}+\varepsilon\right)}{2}\tan\beta l\right]$$

This shift in phase is a measure of the extent to which the phase of the standing waves is dependent upon the second control, which primarily affects amplitude. The shift is arbitrarily chosen as zero when βl is zero. Curves are given in Fig. III–29 of this shift as a function of βl for various values of ε. Maximum shift in phase is given by

$$(\Delta\varphi)_{max} = \frac{1}{2}\tan^{-1}\sqrt{\frac{2}{\frac{1}{\varepsilon}+\varepsilon}} - \frac{1}{2}\tan^{-1}\sqrt{\frac{\frac{1}{\varepsilon}+\varepsilon}{2}}$$

9. Line Stretcher Transformer. The power delivery from a generator to a load, neither of which is matched to the line impedance, can be varied by varying the length of line between them. This is frequently done with a "line-stretcher" or "trombone". Although line-stretchers in general will not produce a perfect match, they are widely used because of their simplicity and because they usually increase the power transfer over what would otherwise be obtained with an arbitrary length of line.

C. Beads in Lines[3]

The center conductor of a concentric line must be supported clear of the outside conductor. This is done in cables by using a solid dielectric material between the conductors. But at microwave frequencies, even the best solid dielectrics introduce such a high amount of attenuation that solid dielectric cables cannot be used for transmitting power over any very large distances.

An air dielectric is required when low attenuation is desired, and the center conductor must be supported by either quarter wave stubs or short beads of dielectric. The following material deals with the use of dielectric beads for supporting the inner conductor, and that following with the design of supporting stubs.

1. Reflections from a Single Bead. A single bead of dielectric material introduced into an air-filled concentric line will cause a partial reflection of an incident electromagnetic wave. This results from the changed im-

[3]From unpublished notes by W. W. Hansen.

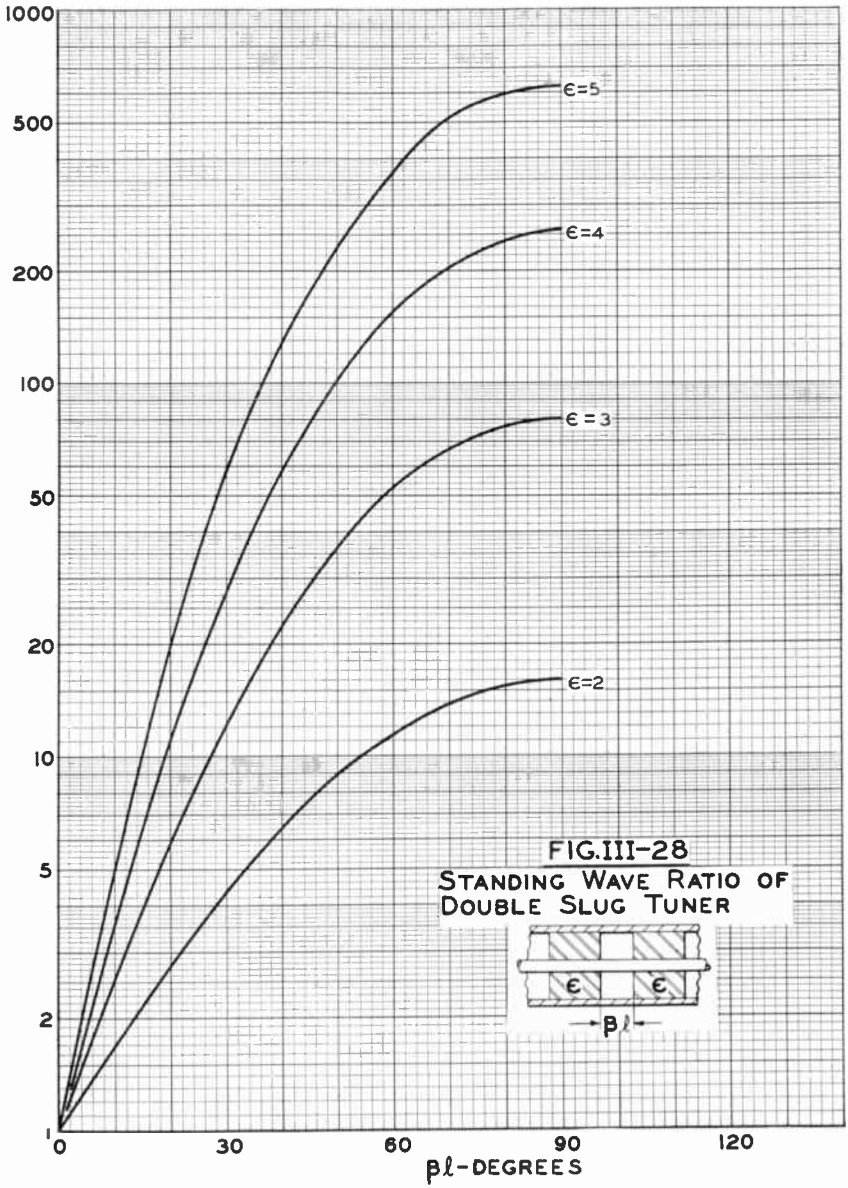


FIG.III-28
STANDING WAVE RATIO OF DOUBLE SLUG TUNER

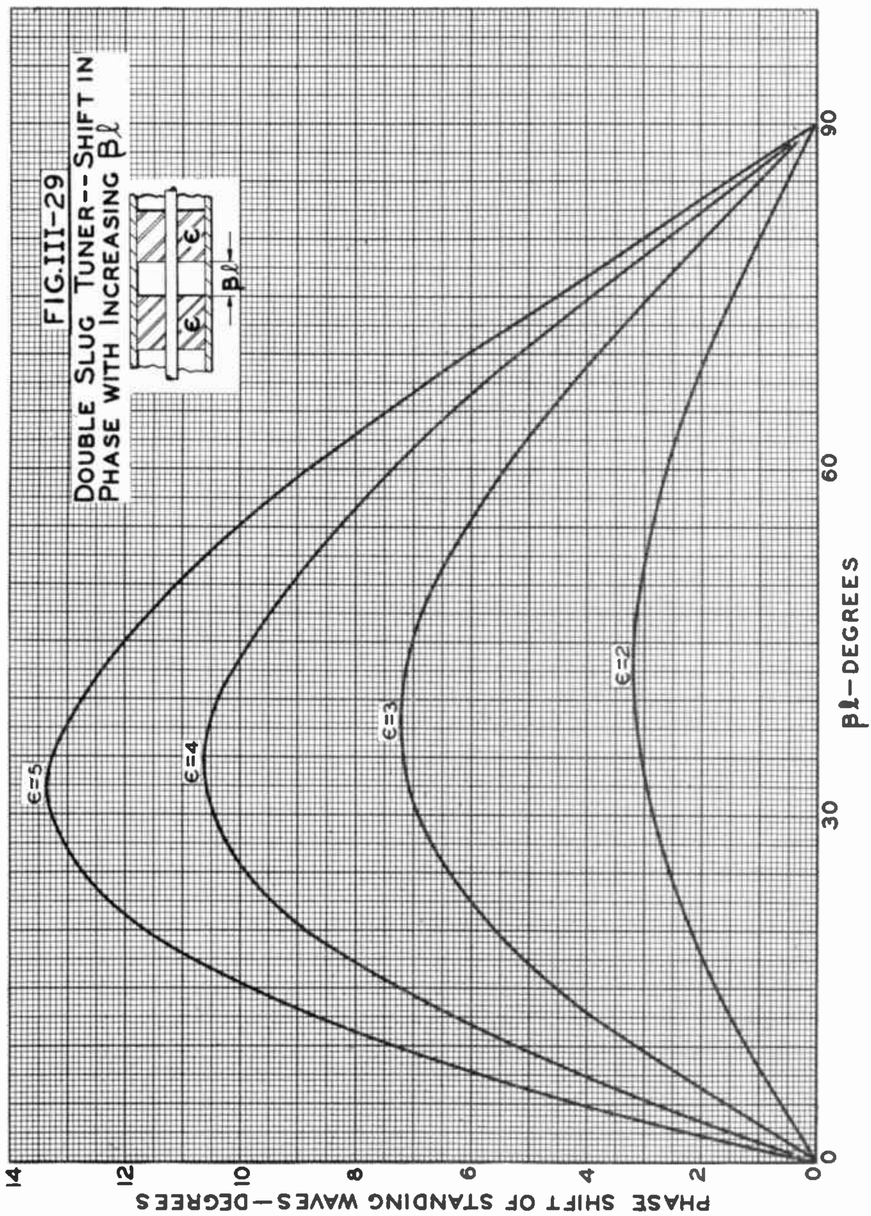


FIG. III-29
DOUBLE SLUG TUNER -- SHIFT IN PHASE WITH INCREASING Bℓ
ε
ε
Bℓ
ε=5
ε=4
ε=3
ε=2
0
30
60
90
βℓ—DEGREES
0
2
4
6
8
10
12
14
PHASE SHIFT OF STANDING WAVES—DEGREES

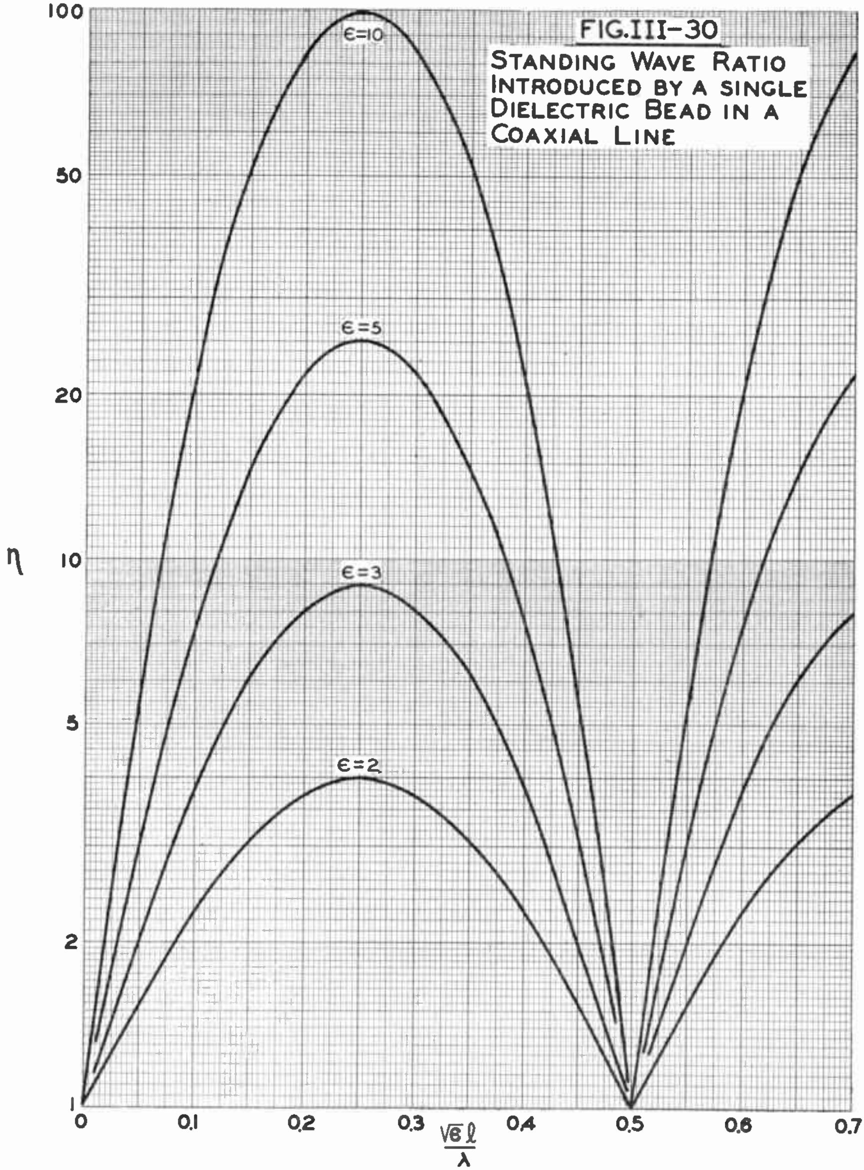


FIG.III-30
STANDING WAVE RATIO INTRODUCED BY A SINGLE DIELECTRIC BEAD IN A COAXIAL LINE
ε=10
ε=5
ε=3
ε=2
η
100
50
20
10
5
2
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
√ε ℓ / λ

pedance of the transmission line in the presence of the dielectric material. The magnitude of this reflection will therefore depend upon the length of the bead.

Neglecting losses, the exact reflection coefficient from a single bead is given by

$$\frac{V_2}{V_1} = \frac{-j\left(\sqrt{\varepsilon} - \frac{1}{\sqrt{\varepsilon}}\right)\tan\sqrt{\varepsilon}\,\beta l}{2 + j\left(\sqrt{\varepsilon} + \frac{1}{\sqrt{\varepsilon}}\right)\tan\sqrt{\varepsilon}\,\beta l}$$

If the value of $\tan\sqrt{\varepsilon}\,\beta l$ is small, this reduces to

$$\frac{V_2}{V_1} = -\frac{j}{2}(\varepsilon - 1)\,\beta l$$

The absolute value of this expression is

$$\left|\frac{V_2}{V_1}\right| = \frac{\pi l}{\lambda}(\varepsilon - 1)$$

The magnitude of the standing wave ratio introduced by a bead is given graphically in Fig. III–30. In this figure, η is plotted against $\frac{\sqrt{\varepsilon}\,l}{\lambda}$, which is the electrical length of the bead in wavelengths. Curves are given for several values of dielectric constant. These curves assume no loss in the dielectric, and it is seen that a half-wave bead introduces no net reflection.

2. **Undercut Beads.** The reflection from a dielectric bead can be minimized by adjusting the diametric ratio at the bead so as to hold constant the characteristic impedance. This is illustrated in Fig. III–31. To maintain the characteristic impedance constant, the radius a' of the inner conductor at the bead should be

$$a' = \frac{b}{\left(\frac{b}{a}\right)^{\sqrt{\varepsilon}}}$$

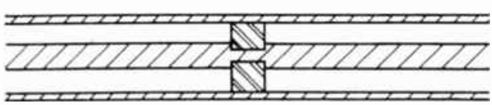


FIG. III - 31

This formula is derived from simple theory, and neglects fringing effects, but it has been checked experimentally and found to agree with measured results. The thinner the bead, the less critical the undercut.

Undercut beads such as this are not frequency selective, and will introduce no great reflection at any frequency, but they are mechanically somewhat difficult to handle in long lines, where split beads seem to offer the best solution.

3. Pairs of Beads. The reflection introduced by a single bead can be cancelled by inserting another similar bead which will introduce a reflection of such magnitude and phase as to cancel the reflection of the single bead. For the two reflections to cancel, the spacing between the beads should be approximately a quarter wave. Referring to Fig. III–32 the exact formula

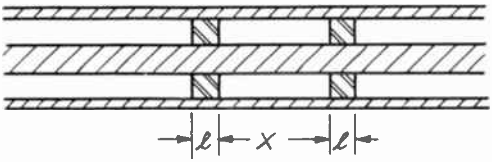


FIG. III-32

for proper spacing between beads to cancel reflections is

$$X = \frac{1}{\beta} \tan^{-1} \left[\frac{2\sqrt{\varepsilon}}{1+\varepsilon} \cot\left(\sqrt{\varepsilon}\, \beta l\right) \right]$$

where ε is the dielectric constant of the bead material. A disadvantage of pairing beads to cancel reflections is that the device is resonant, and will introduce a reflection at frequencies near the designed frequency. For example, consider a concentric line in which the inner conductor is supported by two ¼″ beads of dielectric constant $\varepsilon = 2.6$. If the beads are properly spaced for no reflection at $\lambda = 10$ cm., ($x = 1.391''$) the standing wave ratio introduced into the line as a function of wavelength is given in Fig. III–34.

4. Multiple Beads. If it is desired to use a long coaxial line for transmission of microwave signals, the problem arises of how best to support the center conductor. When this is done by multiple beads of dielectric, some care must be taken in choosing the spacing between the beads.

Consider a large number of beads, of thickness d and uniformly spaced a distance l apart. This is illustrated in Fig. III–33.

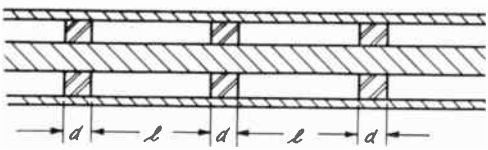


FIG. III-33

It is readily apparent that if the distance l were approximately a half wave, the reflections introduced by each bead would add in phase, and the line would have very high attenuation, even if the dielectric is perfect and absorbs no power. Furthermore, the presence of this reflected wave makes the input impedance highly reactive and prevents power from entering the line.

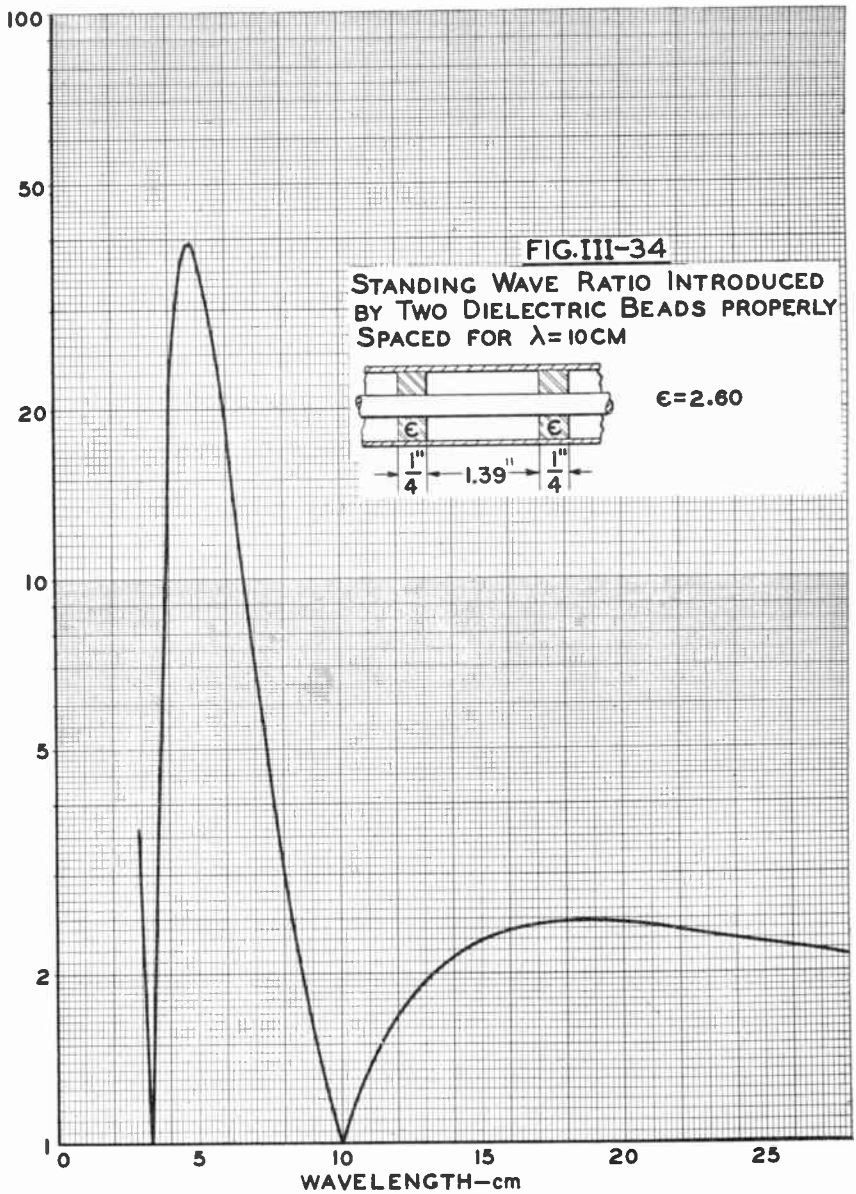


FIG. III-34
STANDING WAVE RATIO INTRODUCED BY TWO DIELECTRIC BEADS PROPERLY SPACED FOR λ = 10CM
ε
ε
ε=2.60
1"/4
1.39"
1"/4
100
50
20
10
5
2
1
0
5
10
15
20
25
WAVELENGTH—cm

An approximate calculation based on filter theory and assuming that $d \ll \lambda/2$ shows that this region of high attenuation is found for values of l between $\lambda/2 - d\varepsilon$ and $\lambda/2 - d$, as illustrated in Fig. III–35.

The presence of losses will modify this curve, and leads to the general form shown in Fig. III–36.

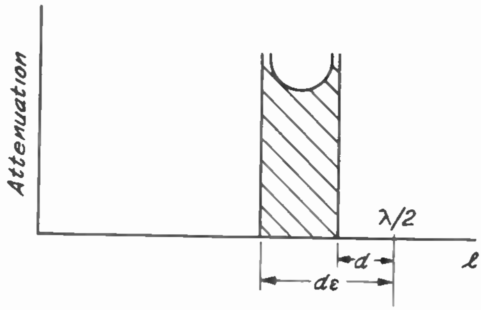


FIG. III - 35

A surprising distance exists between $l = \lambda/2$ and the region of high attenuation, and with losses present, a $\lambda/2$ spacing actually gives less attenuation than a $\lambda/4$ spacing. But the $\lambda/2$ spacing is critical enough to frequency that it is very seldom used—a slight change in frequency resulting in a marked increase in attenuation. The $\lambda/4$ spacing has proven most satisfactory for general use.

Another method of bead support that has given satisfactory results is the use of beads that are electrically a half wave long, and hence introduce no reflection. The actual physical length should be $\lambda/2\sqrt{\varepsilon}$. Such a device is, of course, somewhat frequency sensitive.

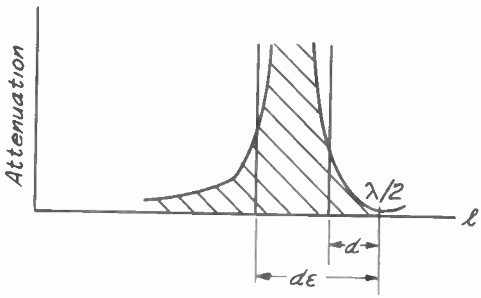


FIG. III - 36

Still another approach to the long line has been developed by Lawson. This starts with two beads, properly spaced so as to minimize reflections. Then two of these units are placed with their centers an odd number of quarter wave lengths apart. This means that in the neighborhood of the correct frequency, the reflection will be a quadratic instead of a linear function of frequency. If the process of adding beads is continued by doubling sections and keeping section centers an odd number of quarter wave lengths apart, the reflections in the immediate vicinity of the correct frequency become less, but for slightly greater deviations of frequency become greater. As the number of sections increases the "pass band" becomes narrower, until for an infinitely large number of sections, there is an infinitely narrow pass band.

D. *Stub-supported Lines*

A quarter wave, shorted section of transmission line has an infinite input impedance, and can therefore be used as a stub support for the center conductor of a coaxial line. Such a support has considerable frequency sensitivity; as at different frequencies, the stub section is no longer a quarter wave long, and introduces a reactance which shunts the transmission line. If the stub support is designed as shown in Fig. III–37, it will introduce a negligible reflection over a much wider range of wavelengths than the simple stub support.

A transformer on the "straight through" section of the center conductor, providing an impedance Z_1, is made $\lambda_o/2$ long, where λ_o is the center wavelength for a given band. Such a transformer will, of course, cause no reflections at λ_o.

The right angle stub of impedance Z_o, shown centered on the transformer, can be adjusted until it is effectively $\lambda_o/4$ long. As illustrated in Fig. III–38, on an admittance diagram of the type shown in Figs. III–6

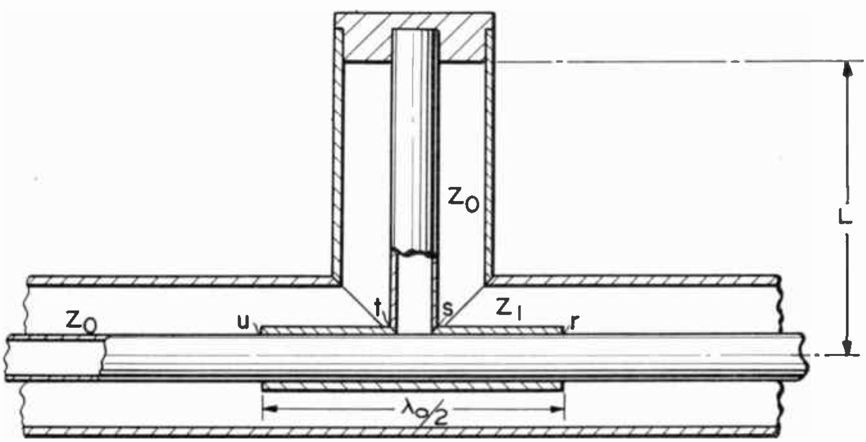


FIG. III - 37

TYPICAL TEE STUB SUPPORT

and III–7, the whole combination will introduce no reflection at λ_o. If the load is matched to the line, the admittance at point r in Fig. III–37 will be Y_o. From r to s is a quarter wave, bringing the admittance at point s back to the G axis. The stub is exactly a quarter wave long, so the admittance at point t will be the same as at point s. From t to u is another quarter wave, and the admittance at u is again Y_o.

There are two other wavelengths, λ_1 and λ_2, one less and one greater than λ_o, at which no reflections occur. At $\lambda_1 < \lambda_o$, illustrated in Fig. III–39, from r to s is greater than $\lambda_1/4$, and the admittance at s will therefore fall below the G axis. But the stub is now longer than $\lambda_1/4$, and introduces a

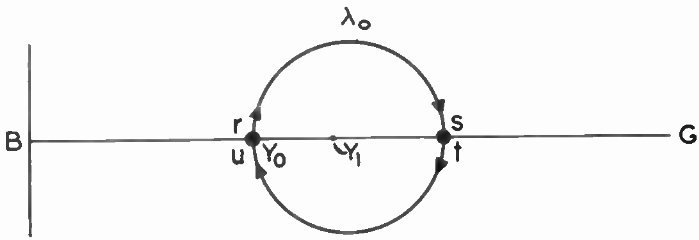


FIG. III-38

STUB IS EQUAL TO $\frac{\lambda_o}{4}$ AND ADDS NOTHING

capacitive or positive susceptance which brings the admittance at t above the G axis. From t to u is greater than $\lambda_1/4$, bringing the admittance at u around to Y_o.

At $\lambda_2 > \lambda_o$, illustrated in Fig. III–40, the section from r to s is less than $\lambda_2/4$, and the admittance at s therefore falls above the G axis. The stub is now inductive, and adds a negative susceptance which brings the admittance at t below the G axis. The section from t to u is now less than $\lambda_2/4$, but brings the admittance at u back again to Y_o.

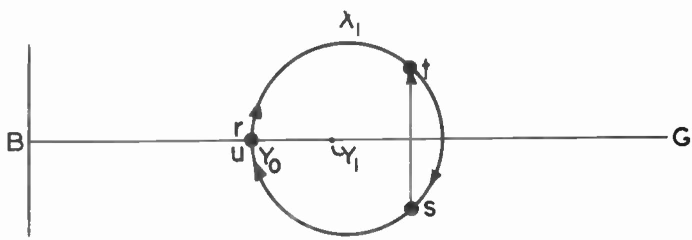


FIG. III-39

STUB IS GREATER THAN $\frac{\lambda_1}{4}$ AND ADDS CAPACITY

The relation between the characteristic impedance of a line (Z_o) and the impedance of the $\lambda_o/2$ transformer section (Z_1), obtained from transmission line theory, is as follows:

$$\left(\frac{Z_1}{Z_o}\right)^3 + 2\left(\frac{Z_1}{Z_o}\right)^2 + \frac{1}{p^2}\frac{Z_1}{Z_o} - 2 = 0$$

Where $$p = \tan\frac{\pi\lambda_o}{2\lambda_1} \text{ and } p = \tan\frac{\pi\lambda_o}{2\lambda_2}$$

MICROWAVE TRANSMISSION DESIGN DATA

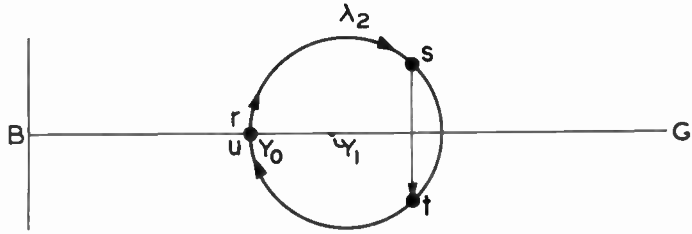


FIG. III - 40

STUB IS LESS THAN $\frac{\lambda_2}{4}$ AND ADDS INDUCTANCE

IMPEDANCE MATCH AT λ_0, λ_1 & λ_2, DEPICTED ON ADMITTANCE DIAGRAMS

For a 7/8″ O.D., 44 ohm coaxial line let $\lambda_1 = 9.1$ cm., $\lambda_0 = 9.9$ cm., and $\lambda_2 = 10.7$ cm. The two equations for p yield values of $+ 8.15$ and $- 7$. As a compromise let $p^2 \cong 50$, which gives $\frac{Z_1}{Z_o} = .835$.

With proper adjustment of the Z_o stub length this type of support centered about $\lambda_o = 10.0$ cm., will exhibit the standing wave ratio-wavelength relation shown in Fig. III–41.

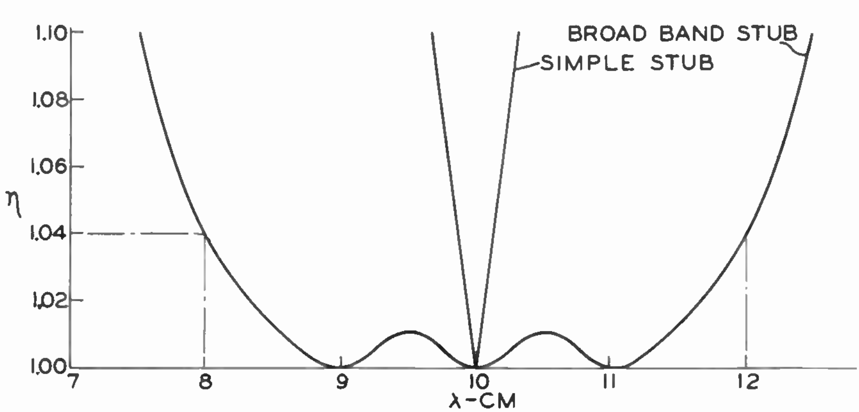


FIG. III - 41

STANDING WAVE RATIO—WAVELENGTH RELATION FOR STUB SUPPORT SHOWN IN FIG.III–37

REFLECTIONS AND IMPEDANCE MATCHING

As indicated in Fig. III–41, this stub support introduces standing wave ratios no greater than 1.04 (power) for any wavelength between 8 and 12 cm. inclusive.

Due to the complexity of line conditions at the junction of the stub and straight through section, no attempt has been made to derive theoretical formulae for the exact length of the stub. The standard stub-length measurement is made between the stub short and the center line of the straight through section, although this is generally not equal to $\lambda_o/4$.

For any given size line it is usually much simpler to determine this length experimentally the first time. Table III–I below gives all available stub lengths and other dimensions for the various size lines in use at present. In referring to the "size" of coaxial lines the dimension given is the outer diameter of the outer conductor. The center wave length for each stub may be determined from dimensions given for the $\lambda_o/2$ transformer.

TABLE III–I
TEE STUB SUPPORTS
(all dimensions in inches)

Line Size	Z_o	$\lambda_o/2$ Transformer		Conductor		Length of Stub Short to Center Line
		Length	Diameter	O.D. Inner	I.D. Outer	
½	51	.315 × 2	.218	.187	.437	.516
⅞	46	.975 × 2	.425	.374	.811	1.450
1	75	.985 × 2	.327	.250	.875	1.311
1⅝	53.3	.975 × 2	.725	.624	1.527	1.750

Phase distortion caused by stub supports, while small, should be considered if the application requires great accuracy. In stub supported coaxial impedance meters, for instance, it will be necessary to determine the increase in electrical over mechanical length for λ_2 and the decrease for λ_1.

Universal Stub for Right Angle Turns in Coaxial Lines. An adaptation of the tee stub principle involves the universal stub shown in Fig. III–42, which permits transmission of microwaves around right angle bends without appreciable reflections over a broad band.

If a short is located properly on the stub section (dotted lines), a tee support results which functions like the one described above. By placing a short at one end of the $\lambda_o/2$ transformer and leaving the stub open, however, a right angle bend is formed. Power may then be introduced through the opposite end of the transformer or through the stub. Careful analysis of the impedance at the vertex of the right angle by means of circle diagrams enables one to place a $\lambda_o/4$ transformer on the stub line which cancels reflections at wavelengths λ_1 and λ_2 as above. If symmetry about the stub is maintained a short may be placed at either end of the $\lambda_o/2$ transformer and a "universal" broad band unit is obtained which permits introduction of microwave energy into any of the three arms.

The diameter of the $\lambda_o/2$ transformer may be obtained from the same equation as that for the tee stub. Location of the short must again be determined experimentally for each size line.

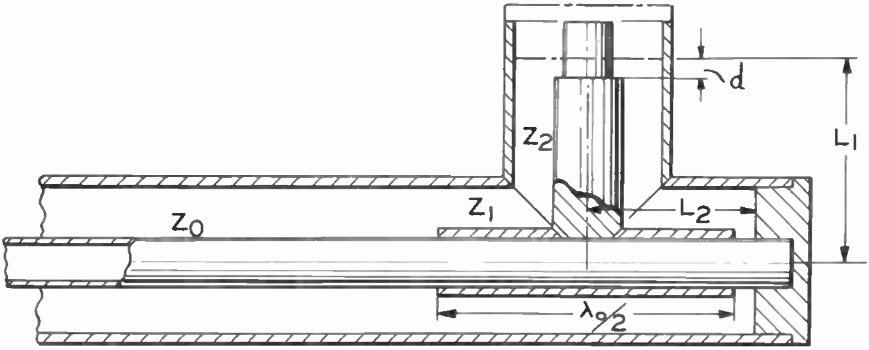


FIG. III - 42

UNIVERSAL STUB

Proper dimensions for a ⅞″ and 1″ universal stub are given in Table III–II below.

TABLE III–II

Line Size	Z_0 Ohms	$\lambda_0/2$ Transformer		Conductor		L_1	L_2	d	Stub Diameter
		Length	Diameter	I.D. Outer	O.D. Inner				
⅞	46	.975 × 2	.425	.811	.374	1.250	1.094	.11	.445
1″	75	.985 × 2	.327	.875	.250	1.410	1.10	.10	.342

Part 2

WAVE GUIDES

CHAPTER IV

GENERAL FORMULAE FOR WAVE GUIDES

Electromagnetic waves will propagate through hollow conducting pipes filled with air or some other dielectric material, even though no center conductor is present. These hollow pipes are customarily termed wave guides. The electromagnetic energy will be transmitted through the pipe, although there is no go and return circuit similar to those used at ordinary frequencies.

There are an infinite number of modes, or field configurations, in which energy can be transmitted through wave guides and concentric lines. In concentric lines, the mode of propagation commonly used is the principal one, in which both the electric and magnetic fields have only components that are normal to the direction of propagation, and the line is restricted in size so that the higher modes cannot propagate. This principal mode requires two separated conductors, and therefore cannot exist in a wave guide. Energy is propagated only in the higher modes, and this requires that the guide exceed certain minimum dimensions to allow transmission of signals of a given wave length.

The various modes of propagation can be divided into two classes. In one of these the electric field has only components that are normal to the direction of propagation; these waves are therefore termed Transverse Electric, or TE waves. TE waves are sometimes called *H* waves, as the magnetic field has components in the direction of propagation. Similarly, TM or Transverse Magnetic waves are waves in which the magnetic field has only components normal to the direction of propagation, and these are sometimes called *E* waves, as the electric field has a component along the axis of the guide.

This section will deal with the ideal properties of wave guides, i.e., losses and attenuation will be neglected, the pipe considered of infinite conductivity, and the dielectric a perfect insulator.

The principal definitions introduced in this chapter are as follows:

f = frequency
f_c = frequency at cutoff
λ = wavelength in free space
λ_g = wavelength in the wave guide
λ_c = wavelength at cutoff
v_p = phase velocity
v_g = group velocity

A. *Field Distribution in Wave Guides*

1. Rectangular Wave Guide. The notation applied to rectangular wave guides will be that illustrated in Fig. IV-1.

The dominant mode of transmission in a rectangular wave guide is the $TE_{1,0}$ wave. The field distribution of this mode is given in Fig. IV–2, and the field equations are as follows:

$$E_z = E_x = H_y = O$$

$$E_y = B\,\frac{\mu\omega\pi}{k^2 a}\sin\frac{\pi x}{a}\sin(\omega t - \beta z)$$

$$H_z = -B\cos\frac{\pi x}{a}\cos(\omega t - \beta z)$$

$$H_x = B\,\frac{\beta\pi}{k^2 a}\sin\frac{\pi x}{a}\sin(\omega t - \beta z)$$

where B is an arbitrary constant of amplitude. The phase constant β, in general given by

$$\beta_{m,n}{}^2 = \frac{\omega^2}{c^2} - \pi^2\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)$$

in this case reduces to

$$\beta^2 = \frac{\omega^2}{c^2} - \frac{\pi^2}{a^2}$$

The quantity k is defined by the expression

$$k^2 = \frac{\omega^2}{c^2} - \beta^2$$

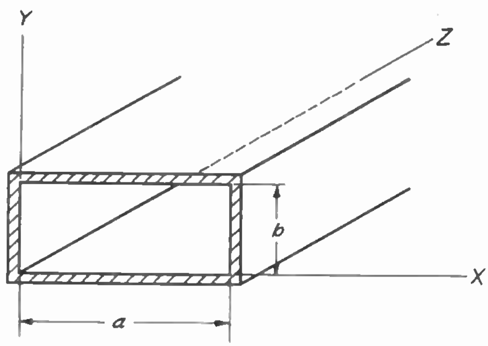
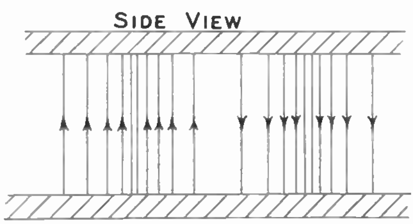
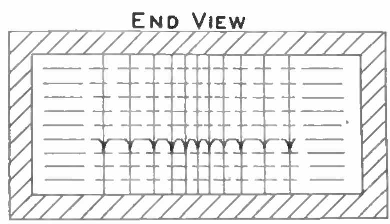


FIG. IV - 1





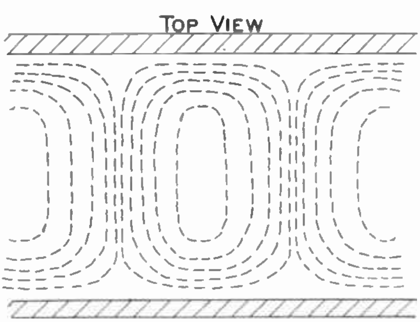


FIG.IV–2

$TE_{1,0}$ MODE IN RECTANGULAR WAVE GUIDE

——— ELECTRIC FIELD

– – – – MAGNETIC FIELD

GENERAL FORMULAE FOR WAVE GUIDES

By dominant mode is meant the mode with the longest cutoff wave length. In rectangular wave guides, the first and second subscript are a measure of the number of half wave variations in transverse field intensity along the x and y dimensions of the guide, respectively. For example, it will be seen that in the $TE_{1,0}$ wave there is a half wave variation of transverse field along the x axis, and no variation of transverse field along the y axis.

Field configurations for a few of the higher modes of transmission are given in Fig. IV-3, and the corresponding field equations are:

$TM_{m,n}$ wave

$$E_z = A \sin\frac{n\pi y}{b} \sin\frac{m\pi x}{a} \cos(\omega t - \beta z)$$

$$E_y = A \frac{\beta}{k^2}\frac{n\pi}{b} \cos\frac{n\pi y}{b} \sin\frac{m\pi x}{a} \sin(\omega t - \beta z)$$

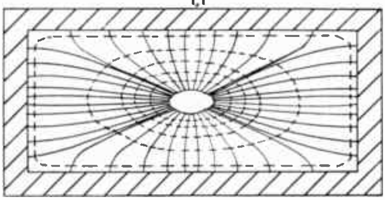
$$E_x = A \frac{\beta}{k^2}\frac{m\pi}{a} \sin\frac{n\pi y}{b} \cos\frac{m\pi x}{a} \sin(\omega t - \beta z)$$

$$H_z = O$$

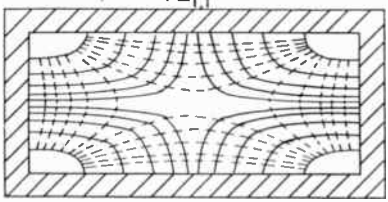
$$H_y = -A \frac{\varepsilon\omega}{k^2}\frac{m\pi}{a} \sin\frac{n\pi y}{b} \cos\frac{m\pi x}{a} \sin(\omega t - \beta z)$$

$$H_x = A \frac{\varepsilon\omega}{k^2}\frac{n\pi}{b} \cos\frac{n\pi y}{b} \sin\frac{m\pi x}{a} \sin(\omega t - \beta z)$$

$TM_{1,1}$



$TE_{1,1}$



$TE_{2,1}$

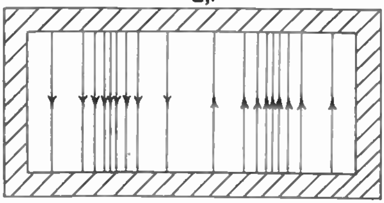


FIG. IV-3
MODES IN RECTANGULAR WAVE GUIDE

In these equations, A is an arbitrary constant of amplitude, β and k have been defined previously.

$TE_{m,n}$ wave

$$E_z = O$$

$$E_y = -B\frac{\omega\mu}{k^2}\frac{m\pi}{a}\cos\frac{n\pi y}{b}\sin\frac{m\pi x}{a}\sin(\omega t - \beta z)$$

$$E_x = B\frac{\omega\mu}{k^2}\frac{n\pi}{b}\sin\frac{n\pi y}{b}\cos\frac{m\pi x}{a}\sin(\omega t - \beta z)$$

$$H_z = -B\cos\frac{n\pi y}{b}\cos\frac{m\pi x}{a}\cos(\omega t - \beta z)$$

$$H_y = B\frac{\beta}{k^2}\frac{n\pi}{b}\sin\frac{n\pi y}{b}\cos\frac{m\pi x}{a}\sin(\omega t - \beta z)$$

$$H_x = B\frac{\beta}{k^2}\frac{m\pi}{a}\cos\frac{n\pi y}{b}\sin\frac{m\pi x}{a}\sin(\omega t - \beta z)$$

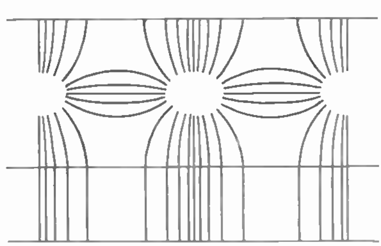
In these equations, B is an arbitrary constant of amplitude, β and k have been defined previously. The higher modes are not so frequently used as the dominant mode, because of the difficulty of exciting one of the higher modes without exciting others. If the dominant mode is used, the pipe can be chosen of such a size that all higher modes will attenuate rapidly with distance and draw no real power.

Current Flow With Dominant Mode of Transmission

The conduction current in a wave guide is confined to the inner surface of the guide, the depth of penetration being determined by skin depth considerations, as outlined in Chap. II—Sec. B-1. The lines of current flow in the walls of the pipe are illustrated in Fig. IV-4. The direction of current flow is orthogonal to the direction of the magnetic field at the inner surfaces of the guide.

FIG. IV-4

CURRENT FLOW IN WALLS OF RECTANGULAR WAVE GUIDE PROPAGATING $TE_{1,0}$ MODE



2. Circular Wave Guides. The notation applied to circular wave guides will be that illustrated in Fig. IV–5.

A number of possible modes of transmission in circular wave guides are illustrated in Fig. IV–6. The dominant mode in circular guide—that with the longest cutoff wave length—is the $TE_{1,1}$ mode, which will be seen to correspond to the $TE_{1,0}$ mode in rectangular pipe. The field equations corresponding to some of these possible modes are:

$TM_{0,1}$ wave (Circular Magnetic Mode)

$$E_z = A \quad J_0\left(u\frac{r}{a}\right)\cos(\omega t - \beta z)$$

$$E_r = -A\frac{\beta a}{u}J_1\left(u\frac{r}{a}\right)\sin(\omega t - \beta z)$$

$$E_\theta = H_z = H_r = O$$

$$H_\theta = -A\frac{\omega a}{u}J_1\left(u\frac{r}{a}\right)\sin(\omega t - \beta z)$$

$$u = 2.405 \qquad \beta = \left(\frac{\omega}{c}\right)^2 - \left(\frac{u}{a}\right)^2$$

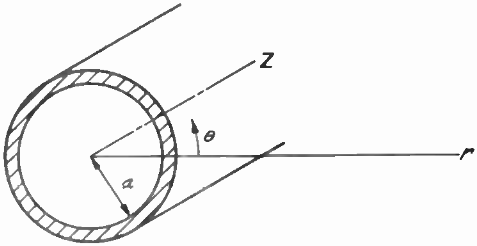


FIG. IV - 5

$TE_{1,1}$ wave (Dominant Mode)

$$H_z = B \quad J_1\left(u'\frac{r}{a}\right)\cos\theta\cos(\omega t - \beta z)$$

$$H_r = B\frac{\beta a}{2u'}\left[J_0\left(u'\frac{r}{a}\right) - J_2\left(u'\frac{r}{a}\right)\right]\cos\theta\sin(\omega t - \beta z)$$

$$H_\theta = -B\frac{\beta a^2}{u_1 r}J_1\left(u'\frac{r}{a}\right)\sin\theta\sin(\omega t - \beta z)$$

$$E_z = O$$

$$E_r = -\mu\frac{\omega}{\beta}H_\theta$$

$$E_\theta = \mu\frac{\omega}{\beta}H_r$$

$$u' = 1.841 \qquad u_1 = 3.39 \qquad \beta = \left(\frac{\omega}{c}\right)^2 - \left(\frac{u'}{a}\right)^2$$

$TE_{0,1}$ wave (Circular Electric Mode)

$$H_z = -B\,J_0\left(u'\frac{r}{a}\right)\cos(\omega t - \beta z)$$

$$H_r = B\frac{\beta a}{u'}J_1\left(u'\frac{r}{a}\right)\sin(\omega t - \beta z)$$

$$H_\theta = E_z = E_r = O$$

$$E_\theta = \mu\frac{\omega}{\beta}H_r$$

$$u = 3.832 \qquad \beta = \left(\frac{\omega}{c}\right)^2 - \left(\frac{u'}{a}\right)^2$$

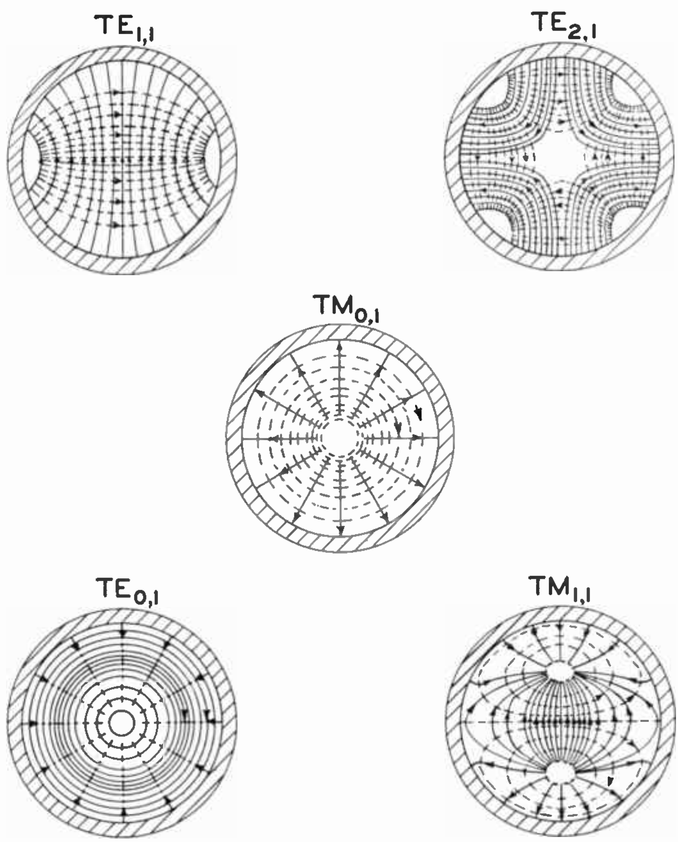


FIG. IV-6

MODES IN CIRCULAR WAVE GUIDE

GENERAL FORMULAE FOR WAVE GUIDES

In these equations A and B are arbitrary constants determining amplitude. The various subscripts have a different connotation when applied to modes in round pipe. The first subscript denotes the order of the Bessel function (or its derivative) which specifies the component of electric or magnetic field parallel to the axis, and the second subscript indicates which root of this Bessel function satisfies the expression $J_n\ (ka) = O$. The first subscript also indicates the number of full-period variations of radial component of field along angular co-ordinates, and the second subscript indicates the number of half-period variations of angular component of field along radial co-ordinates.

3. Elliptical Wave Guide. A knowledge of the behavior of elliptical wave guides is of some importance because the inevitable deformations encountered in a round guide will result in an equivalent ellipticity. A few of the possible modes in an elliptical wave guide are shown in Fig. IV–7,[1] along with the corresponding mode in round pipe. In general, if a round wave guide is deformed, the wave being propagated will split into two components which proceed down the pipe with different phase velocities and different attenuation. This instability will be found in all cases except when

1. Deformation is along an axis of symmetry of the wave.
2. The wave has circular symmetry (e. g. $TE_{0,1}$ or $TM_{0,1}$ wave).

B. Cutoff—Formulae

1. Rectangular Wave Guide. A wave guide acts as a high pass filter, and will transmit electromagnetic waves in a given mode only when the wavelength of the signal is less than the cutoff wavelength. The cutoff wavelength depends upon the mode of transmission, and is given by the formula

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

In this formula m and n are the subscripts denoting the particular mode under consideration (e.g. $TE_{m,n}$), and λ_c is the limiting wavelength that will propagate down the guide. The equation holds for either TE or TM modes of propagation.

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[1] L. J. Chu, Electromagnetic Waves in Elliptical Hollow Pipes of Metal, Jour. App. Phys., Vol. 9, p. 583, Sept. 1938.

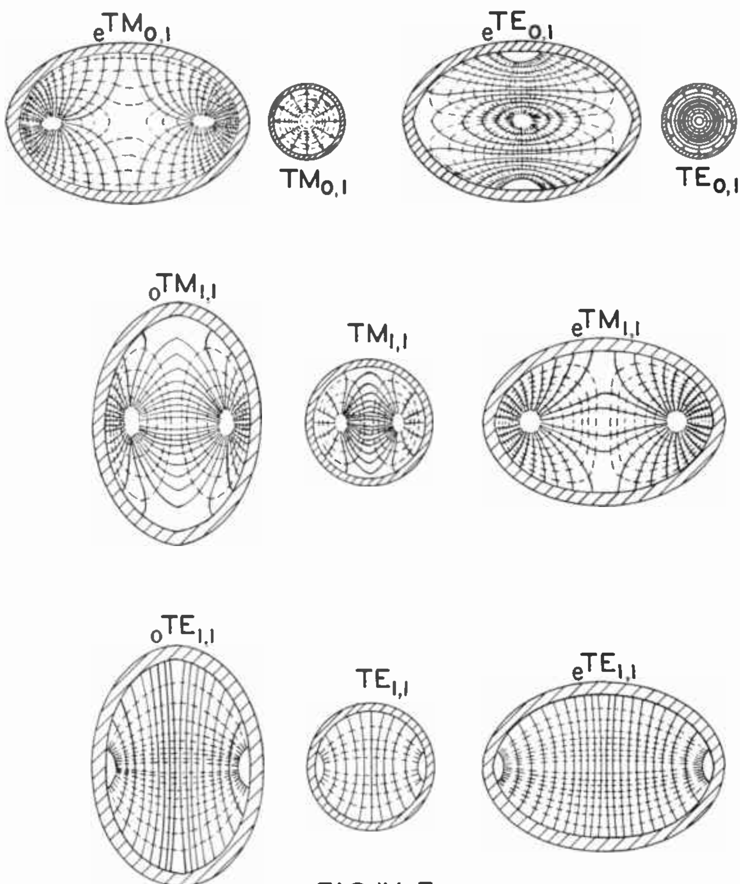


FIG. IV-7
MODES IN ELLIPTICAL WAVE GUIDE

GENERAL FORMULAE FOR WAVE GUIDES

The size of pipe necessary for propagation of some of the lower modes is illustrated in Fig. IV–8. In this figure b/λ is the ordinate, and a/λ the abscissa. The various curves mark the boundary between sizes where the modes will propagate and where they will not. For example, the $TE_{2,0}$ mode will propagate in all sizes of wave guide in which a/λ is greater than unity. To allow only the dominant mode to propagate, one dimension of the guide should not exceed λ and the other should not exceed $\lambda/2$.

2. Circular Wave Guide. The cutoff wavelength in a circular wave guide for a given mode depends on the ratio of diameter to wave length. For the $TE_{n,m}$ wave, the cutoff wavelength is given by the formula

$$\lambda_c = \frac{2\pi a}{u'_{n,m}}$$

where a is the radius of the guide. The constant $u'_{n,m}$ is the mth root of the equation $J_n'(u) = O$. Some of the lower values of $u'_{n,m}$ are

$$u'_{0,1} = 3.832 \qquad u'_{0,2} = 7.016$$
$$u'_{1,1} = 1.841 \qquad u'_{1,2} = 5.33$$
$$u'_{2,1} = 3.05$$

For the $TM_{n,m}$ wave, the cutoff wavelength is given by

$$\lambda_c = \frac{2\pi a}{u_{n,m}}$$

where a is the guide radius and $u_{n,m}$ is the mth root of the equation $J_n(u) = O$. Some of the lower values of $u_{n,m}$ are

$$u_{0,1} = 2.405 \qquad u_{0,2} = 5.520 \qquad u_{0,3} = 8.654$$
$$u_{1,1} = 3.832 \qquad u_{1,2} = 7.016$$
$$u_{2,1} = 5.136$$

In Fig. IV–9 is illustrated the relation between the ratio a/λ and the cutoff wavelength for several of the lowest modes in circular wave guide.

3. Elliptical Wave Guides[2]. The variation in cutoff wavelength with eccentricity for elliptical wave guide is illustrated for some of the lowest modes of transmission in Fig. IV–10. In this figure, the ratio of periphery s to free space wavelength at cutoff λ_c is plotted for some modes as a function of eccentricity resulting in both even and odd waves. The distinction between even and odd waves is illustrated in Fig. IV–7. Technically the distinction between odd and even waves is as follows. The field in an elliptical wave guide is specified in terms of Mathieu functions, which are of two kinds, even and odd. Those waves which are specified by the even functions are termed even waves, and those specified by the odd functions are termed odd waves.

4. Wave Guides Filled with Dielectric Material. The cutoff wavelength λ_c' in a hollow pipe wave guide filled with dielectric material of constant ε is related to the cutoff wavelength λ_c of the same pipe when air filled by the formula

$$\lambda_c' = \sqrt{\varepsilon}\,\lambda_c$$

[2]Chu, loc. cit.

C. *Wave Length in Wave Guides*

In coaxial lines filled with air dielectric, the wavelength in the line is always equal to the free space wavelength, provided the attenuation is small. In hollow pipe wave guides, filled with air dielectric and neglecting attenuation, the wavelength in the guide always exceeds the free space wavelength. As the frequency is unchanged, this would appear to indicate that the velocity of propagation of waves in a wave guide exceeds the velocity of light. Actually, the phase velocity v_p in an air-filled wave guide, given by the formula

$$v_p = f\lambda_g$$

is greater than the velocity of light by the factor λ_g/λ. But the actual rate of propagation of energy down the guide is the group velocity v_g; this is related to the phase velocity by the equation

$$v_g = \frac{c^2}{v_p}$$

where c is the velocity of propagation in free space, i.e. the velocity of light.

The wavelength λ_g in an air-filled hollow pipe wave guide is related to the cutoff wavelength in the guide λ_c and the free space wavelength of the transmitted radiation λ by the formula

$$\lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}}$$

The wavelength in the guide is always greater than the wavelength in free space, and as the free space wavelength approaches the cutoff wavelength of the guide, the guide wavelength approaches infinity. The relations given in the above expression are plotted graphically in Fig. IV-11. In this figure the ratio λ/λ_g is plotted against the ratio λ/λ_c, and the relationship between the two is seen to be a quarter circle of unity radius.

1. Waveguides Filled with Dielectric.

If the waveguide is filled with a perfect dielectric of constant ε_1, the wavelength in the guide is given by the equation

$$\lambda_g = \frac{\lambda}{\sqrt{\varepsilon_1 - (\lambda/\lambda_c)^2}}$$

where λ is still the free space wavelength in air, and λ_c the cutoff wavelength of the guide when air-filled. If the dielectric is not perfect, but has a complex dielectric constant of the form

$$\varepsilon_1 = \varepsilon_1' - j\,\varepsilon_1''$$

the wavelength in the guide is given by the equation

$$\lambda_g = \frac{\lambda}{\sqrt{\varepsilon_1' - \left(\frac{\lambda}{\lambda_c}\right)^2}} \times \frac{1}{\sqrt{\frac{1}{2} + \frac{1}{2}\left\{\frac{\varepsilon_1''^2}{\left[\varepsilon_1' - \left(\frac{\lambda}{\lambda_c}\right)^2\right]^2} + 1\right\}^{\frac{1}{2}}}}$$

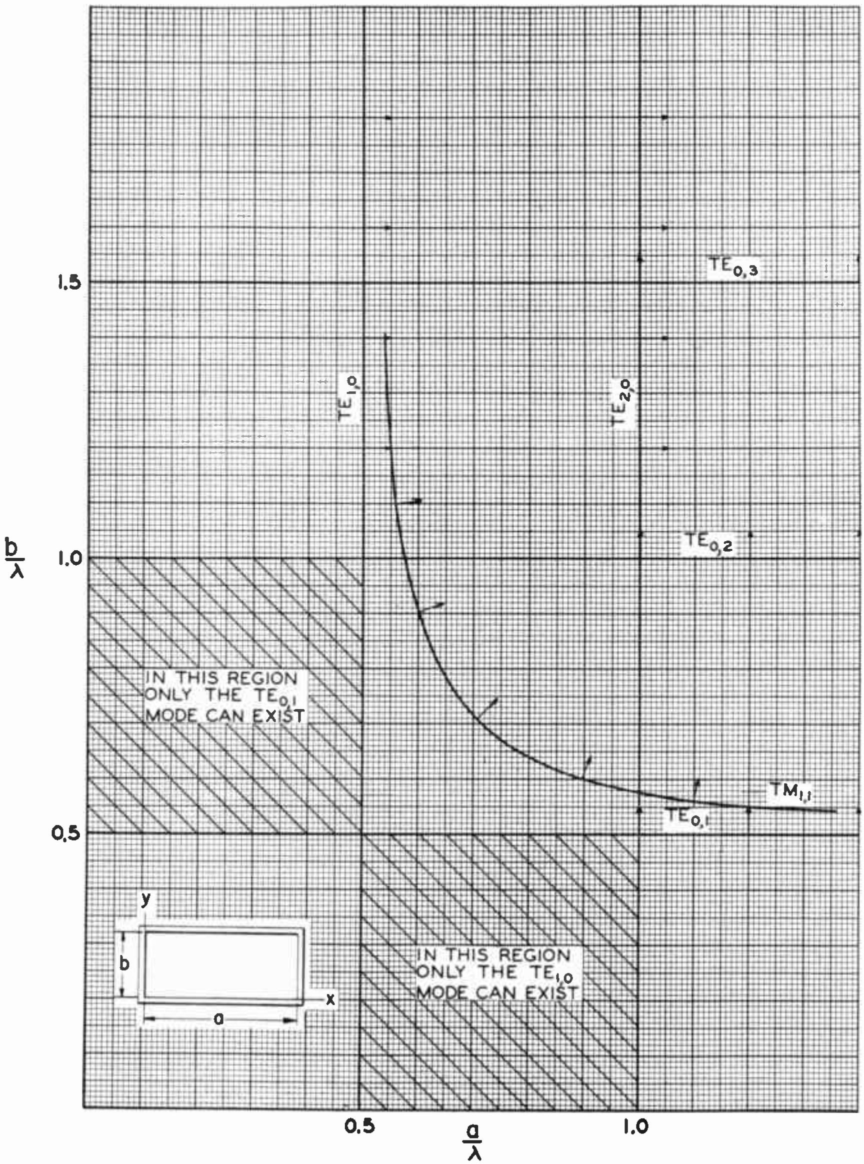


FIG. IV-8

SIZE OF PIPE NECESSARY FOR PROPAGATION OF SOME OF THE LOWER MODES IN RECTANGULAR WAVE GUIDE

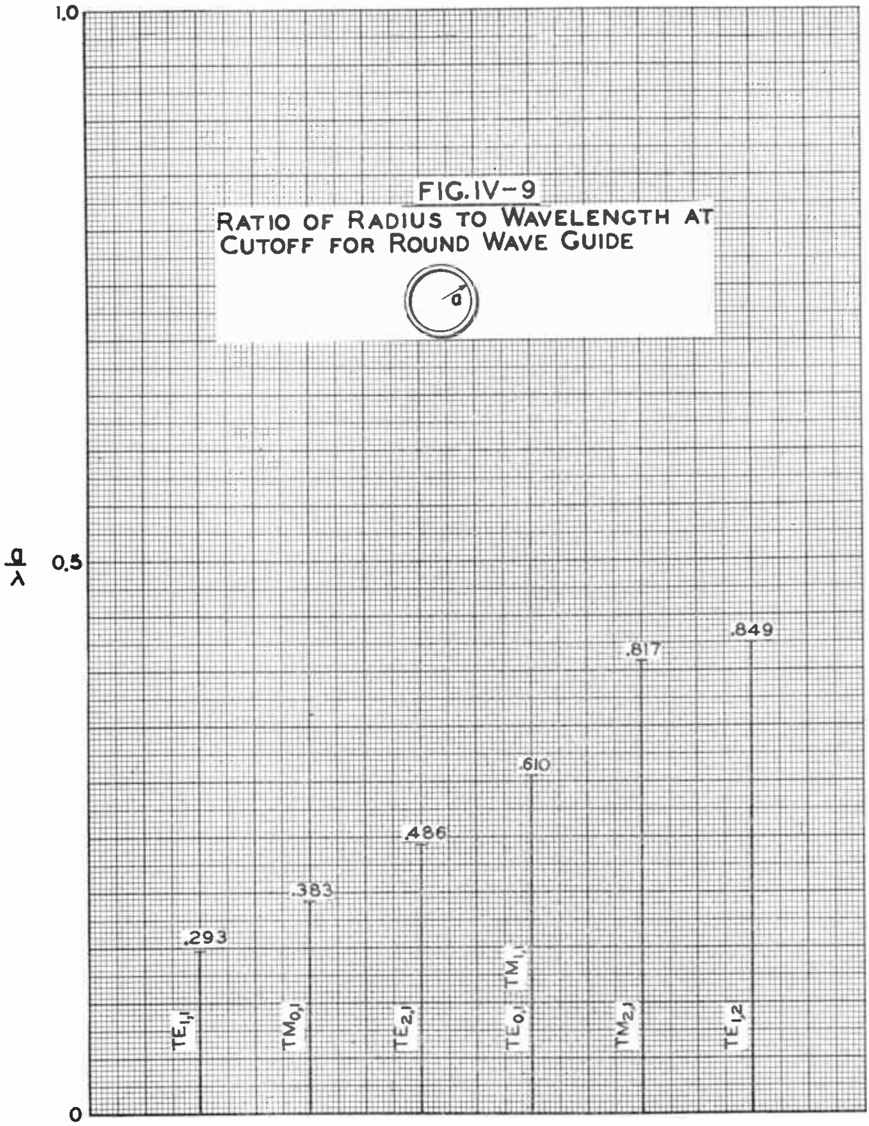


FIG. IV-9
RATIO OF RADIUS TO WAVELENGTH AT CUTOFF FOR ROUND WAVE GUIDE
a
1.0
0.5
0
a/λ
.293
.383
.486
.610
.817
.849
TE1,1
TM0,1
TE2,1
TE0,1 TM1,1
TM2,1
TE1,2

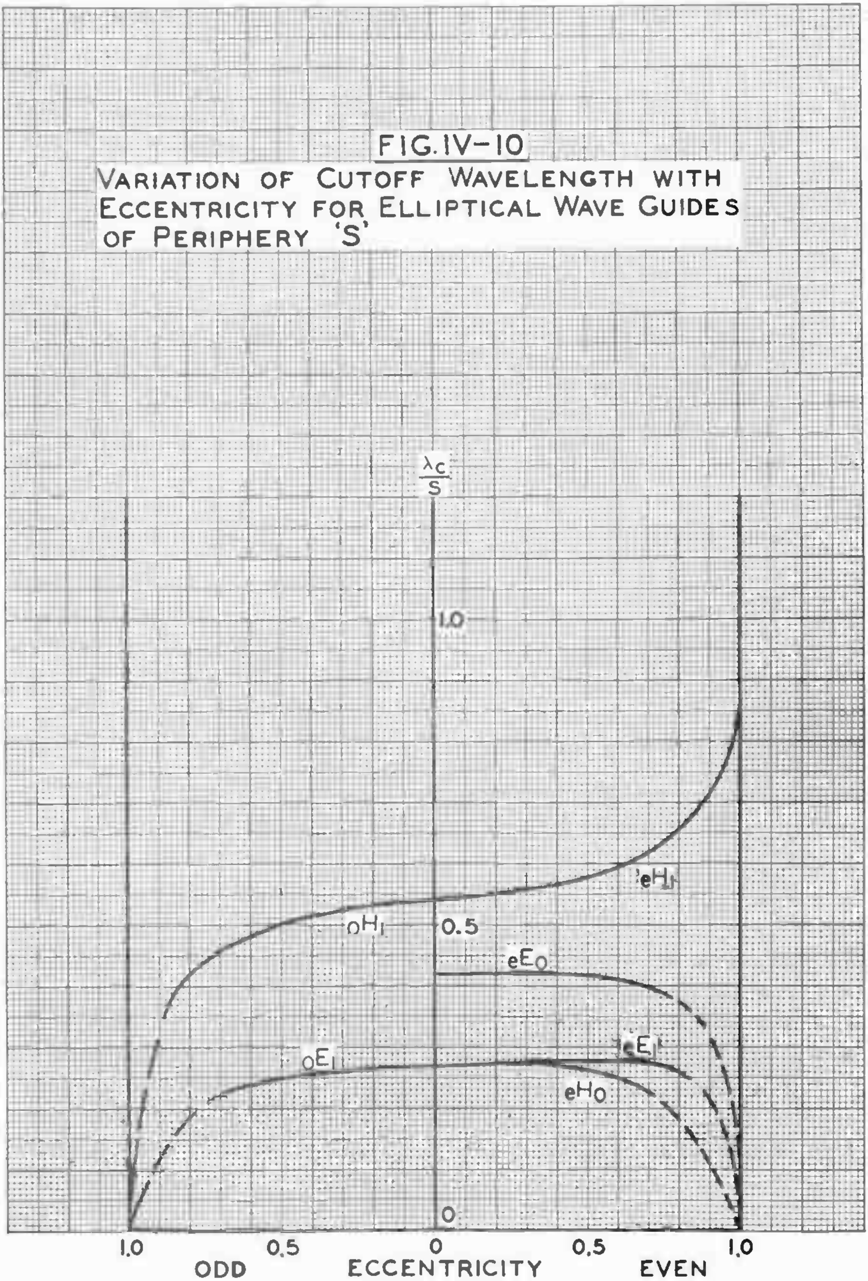


FIG. IV-10
VARIATION OF CUTOFF WAVELENGTH WITH ECCENTRICITY FOR ELLIPTICAL WAVE GUIDES OF PERIPHERY 'S'

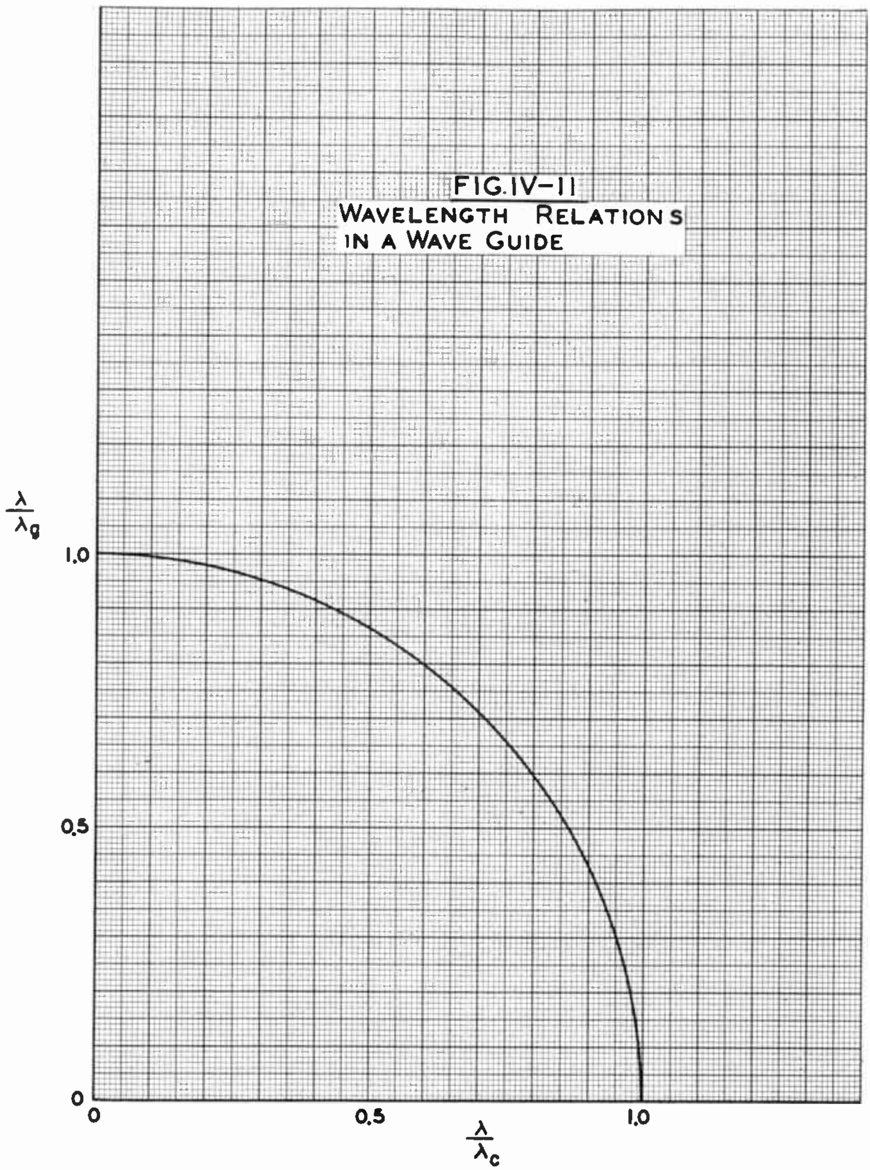


FIG. IV–11
WAVELENGTH RELATIONS IN A WAVE GUIDE
$\frac{\lambda}{\lambda_g}$
1.0
0.5
0
0
0.5
1.0
$\frac{\lambda}{\lambda_c}$

D. *Standard Wave Guides*

Standard waveguides that have been chosen for various wavelengths are given in Table IV–I, along with the guide wavelength associated with each wavelength and wave guide, and the cutoff wavelength for the next highest mode associated with each size of guide.

E. *Recommended Waveguides*

Various standard sizes of round and rectangular pipe suitable for waveguides are listed in Table IV–II, along with the range of wavelength for which each is suitable. These ranges are suggested by the Subcommittee on Wave Guide Connectors of the RMA Committee on H.F. Line Connectors.

F. *Power-Carrying Capacity of Wave Guides*

The maximum power that can be transmitted through an air-filled waveguide will depend upon the maximum electric field strength that can exist without breakdown.

If the maximum allowable breakdown potential is specified, the maximum power carrying capacity of the wave guide can then be expressed when the wavelength and the size of the guide are known. The following formulae give the maximum power in terms of the maximum allowable field strength. Experimental data on allowable field strengths at ultra high frequencies are not available. A value of 30,000 volts/cm. is frequently used at ordinary frequencies as the breakdown voltage under standard conditions of temperature, pressure, and humidity.

The values given by the equations below are the theoretical maximum powers, and do not take into account increases in potential gradient caused by standing waves or discontinuities in the guide. In actual practice, breakdown will occur at considerably lower powers than expressed by the formulas unless extraordinary precautions are taken.

1. Rectangular Waveguide. The maximum power that can be carried by a rectangular waveguide operating in the dominant or $TE_{1,0}$ mode with $a > b$ is

$$\frac{P}{E_{max}^2} = 6.63 \times 10^{-4}\, ab \left(\frac{\lambda}{\lambda_g} \right)$$

If the potential gradient E_{max} is expressed in volts/cm., the dimensions of the guide, a and b, should be expressed in centimeters for the power P to be given in watts. The maximum field intensity occurs parallel to the narrower dimension of the guide, midway between the side walls, and is independent of the distance from the wide faces of the guide.

MICROWAVE TRANSMISSION DESIGN DATA

TABLE IV-I
Standard Wave Guides
Rectangular Guides

Size (O.D.)	a (cm.)	b (cm.)	λ(cm.)	$TE_{1,0}$ mode		$T_{2,0}$ mode
				λ_c(cm.)	λ_g(cm.)	λ_c(cm.)
½" × 1" (0.050" wall)	2.29	1.02	3.20 3.30 3.40	4.57	4.48 4.77 5.09	2.29
⅝" × 1¼" (1⁄16" wall)	2.86	1.27	3.20 3.30 3.40	5.72	3.86 4.04 4.23	2.86
1½" × 3" (0.080" wall)	7.22	3.40	9.1 9.8 10.7	14.44	11.70 13.33 15.92	7.22

Circular Guides

Size (I.D.)	D (cm.)	λ (cm.)	$TE_{1,1}$ mode		$TM_{0,1}$ mode		$TE_{2,1}$ mode	
			λ_c(cm.)	λ_g(cm.)	λ_c(cm.)	λ_g(cm.)	λ_c(cm.)	λ_g(cm.)
15⁄16"	2.38	3.20 3.30 3.40	4.06	5.17 5.54 6.15	3.11			
1 3⁄16"	3.02	3.20 3.30 3.40	5.15	4.08 4.30 4.52	3.94	5.49 6.04 6.73	3.11	
2½"	6.35	9.1 9.8 10.7	10.83	16.8 23.0 70.0	8.29			
2¾"	6.99	9.1 9.8 10.7	11.92	14.1 17.2 24.2	9.12	11.80	7.19	
3"	7.62	9.1 9.8 10.7	13.00	12.8 14.9 18.8	9.95	22.4 56.4	7.85	
4"	10.16	9.1 9.8 10.7	17.33	10.7 11.9 13.6	13.27	12.5 14.5 18.1	10.46	18.5 28.0

GENERAL FORMULAE FOR WAVE GUIDES

TABLE IV—II

Recommended Sizes for Wave Guides

Rectangular Guides

Outer Dimensions Inches	Wall Thickness Inches	Suited for Wave Length Range	—	cm.
1 ½ x 3	.081	7.6	—	11.8
1 x 2	.064, .081	5	—	7.6
¾ x 1 ½	.064, .081	3.7	—	5.7
⅝ x 1 ¼	.064	3.0	—	4.7
½ x 1	.050, .064	2.4	—	3.7
⅜ x ¾	.064	1.7	—	2.6

Round Guides

Outer Diameter Inches				
3	.065	9.6	—	10.9
2 ⅝	.062	8.3	—	9.6
2 ¼	.065	7.1	—	8.1
2	.065	6.2	—	7.2
1 ¾	.065	5.4	—	6.2
1 ½	.042	4.7	—	5.4
1 ⅜	.065	4.15	—	4.8
1 ⅛	.032	3.6	—	4.15
1	.032	3.1	—	3.6
⅞	.035	2.7	—	3.1
¾	.032	2.3	—	2.7
⅝	.020	1.95	—	2.3
9/16	.028	1.7	—	2.0

2. Circular Wave Guide. For the dominant, or $TE_{1,1}$ mode in circular wave guide the relation between maximum power and maximum allowable field strength is

$$\frac{P}{E_{max}^2} = 1.99 \times 10^{-3} a^2 \left(\frac{\lambda}{\lambda_g} \right)$$

where a is the radius of the guide, λ the free space wavelength, and λ_g the guide wavelength. Maximum field strength is at the center of the guide.

For the circular magnetic, or $TM_{0,1}$ mode, there are two separate cases:

Case I: $a/\lambda < 0.761$

$$\frac{P}{E_{max}^2} = 7.69 \times 10^{-3} \frac{a^4}{\lambda^2} \left(\frac{\lambda}{\lambda_g} \right)$$

In this case the maximum field intensity is at the center of the guide.

Case II: $a/\lambda > 0.761$

$$\frac{P}{E_{max}^2} = 3.33 \times 10^{-3} a^2 \left(\frac{\lambda_g}{\lambda} \right)$$

In this case the maximum field intensity is at $r = 0.765a$ and is independent of angle.

3. Variation of Power-Carrying Capacity with Altitude. The maximum possible power that can be carried in a waveguide varies with altitude in the same manner as in concentric lines. Refer to Fig. I–5 for the decrease in maximum allowable power with altitude.

CHAPTER V

ATTENUATION IN WAVE GUIDES

In the previous chapter, it was assumed that the wave guide walls were of infinite conductivity, and that the wave guides were filled with a perfect dielectric. If these assumptions were completely justified, power would be carried down the inside of the wave guide with no attenuation, provided the wave guide was not below cutoff dimensions for the wave being transmitted. In physically realizable wave guides, the conductivity of the walls is finite, and as a result the wave is partially attenuated because of losses in the metal. In addition, if some dielectric other than air is used inside the guide, the losses in the dielectric may result in a very rapid attenuation of the transmitted waves.

A. *Conductor Losses*

In calculating conductor losses, the skin depth in the conductor must be considered. The same considerations apply to wave guides as were outlined in Chapter II, Section B-1 for concentric lines. The attenuation in a wave guide resulting from conductor losses will vary with the square root of the resistivity of the conductor, the same variation as is encountered in concentric lines.

1. Rectangular Wave Guides. The notation used here is the same as is outlined at the beginning of Chapter IV. For a rectangular, copper, air-filled waveguide operating in the dominant or $TE_{1,0}$ mode, the attenuation is given by

$$\alpha_C = \frac{.01107}{a^{3/2}}\left[\frac{\frac{1}{2}\frac{a}{b}\left(\frac{f}{f_c}\right)^{3/2} + \left(\frac{f}{f_c}\right)^{-\frac{1}{2}}}{\sqrt{\left(\frac{f}{f_c}\right)^2 - 1}}\right] \text{db per ft.}$$

The dimensions of the guide, a and b, are in inches. If some metal other than copper is used as a conductor, the attenuation given by the above formula should be multiplied by the appropriate loss factor in column 3 of Table II–1.

In Fig. V–1 is plotted the variation of attenuation with wavelength for the various recommended wave guides listed in Chapter IV—Section E over the wavelength range for which each is recommended. It is assumed that the material is copper.

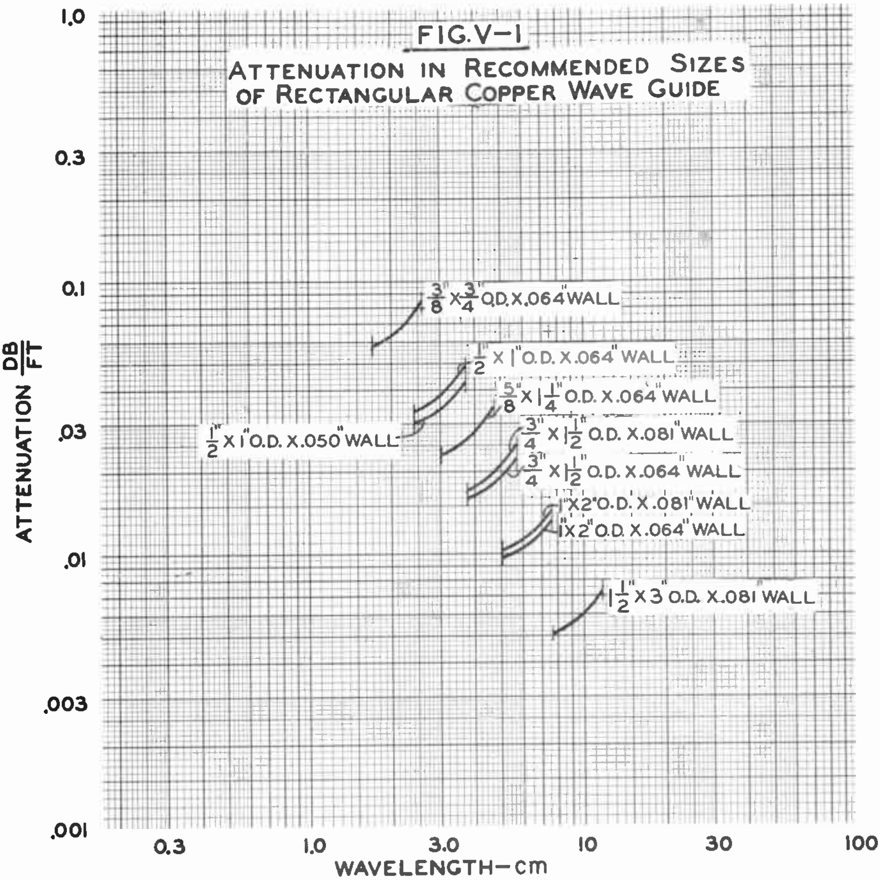


FIG. V-1
ATTENUATION IN RECOMMENDED SIZES OF RECTANGULAR COPPER WAVE GUIDE

Formulae giving attenuation as a function of frequency and guide dimensions for some other modes of propagation in a rectangular copper wave guide are as follows:

$TE_{2,0}$

$$\alpha_C = \frac{.01565}{a^{3/2}} \left[\frac{\frac{a}{2b}\left(\frac{f}{f_c}\right)^{3/2} + \left(\frac{f}{f_c}\right)^{-\frac{1}{2}}}{\sqrt{\left(\frac{f}{f_c}\right)^2 - 1}} \right] \text{db per ft.}$$

$TE_{1,1}$

$$\alpha_C = \frac{.01107}{a^{3/2}} \left[\frac{\frac{a}{b}\left\{1 + \frac{a}{b}\right\}\left(\frac{f}{f_c}\right)^{3/2} + \left\{1 + \left(\frac{a}{b}\right)^3\right\}\left(\frac{f}{f_c}\right)^{-\frac{1}{2}}}{\left[1 + \left(\frac{a}{b}\right)^2\right]^{3/4} \sqrt{\left(\frac{f}{f_c}\right)^2 - 1}} \right] \text{db per ft.}$$

ATTENUATION IN WAVE GUIDES

$$TM_{1,1}\quad \alpha_C = \frac{.01107}{a^{3/2}}\left[\frac{\left[1+\left(\frac{b}{a}\right)^3\right]\left(\frac{f}{f_c}\right)^{3/2}}{\left[1+\left(\frac{b}{a}\right)^2\right]^{3/4}\sqrt{\left(\frac{f}{f_c}\right)^2-1}}\right]\text{db per ft.}$$

As before, the guide dimensions are in inches.

The variation with frequency of the attenuation in a typical rectangular wave guide ($a = 2''$, $b = 1''$) is plotted in Fig. V–2 for some of the lower modes of propagation. The variation with frequency is seen to be similar for all the modes, although the magnitude of attenuation differs with the mode of propagation.

A term L can be defined as the loss length, or length per unit attenuation. If Fig. V–3 the dominant or $TE_{1,0}$ mode of propagation is assumed and

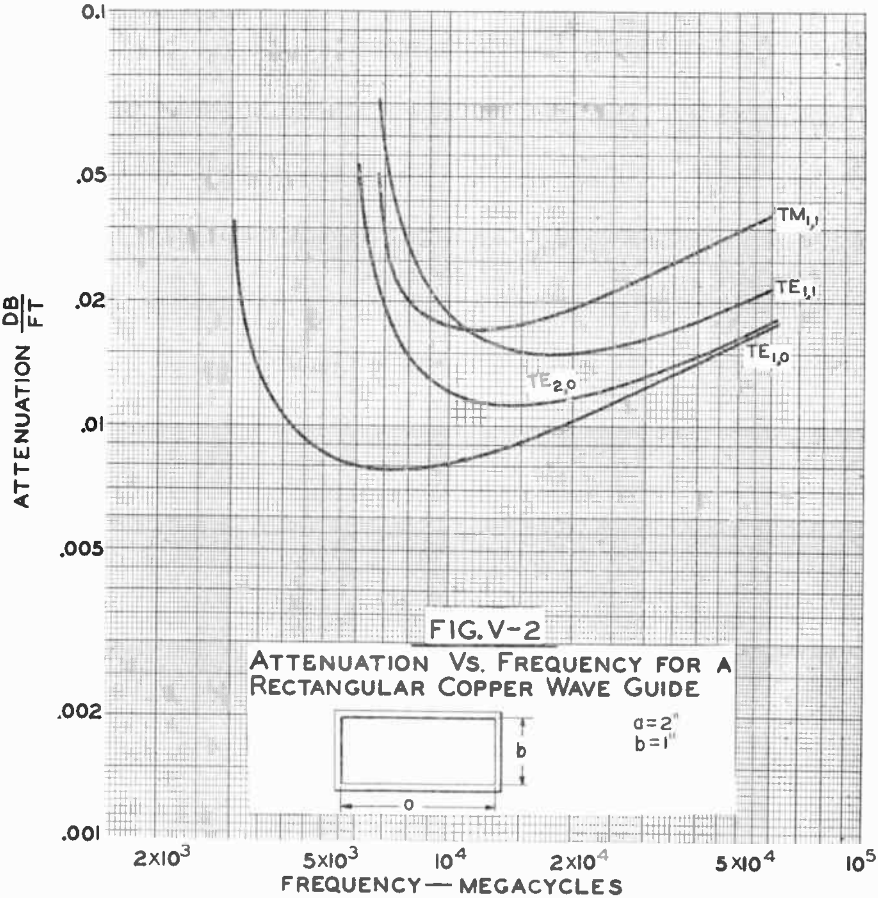
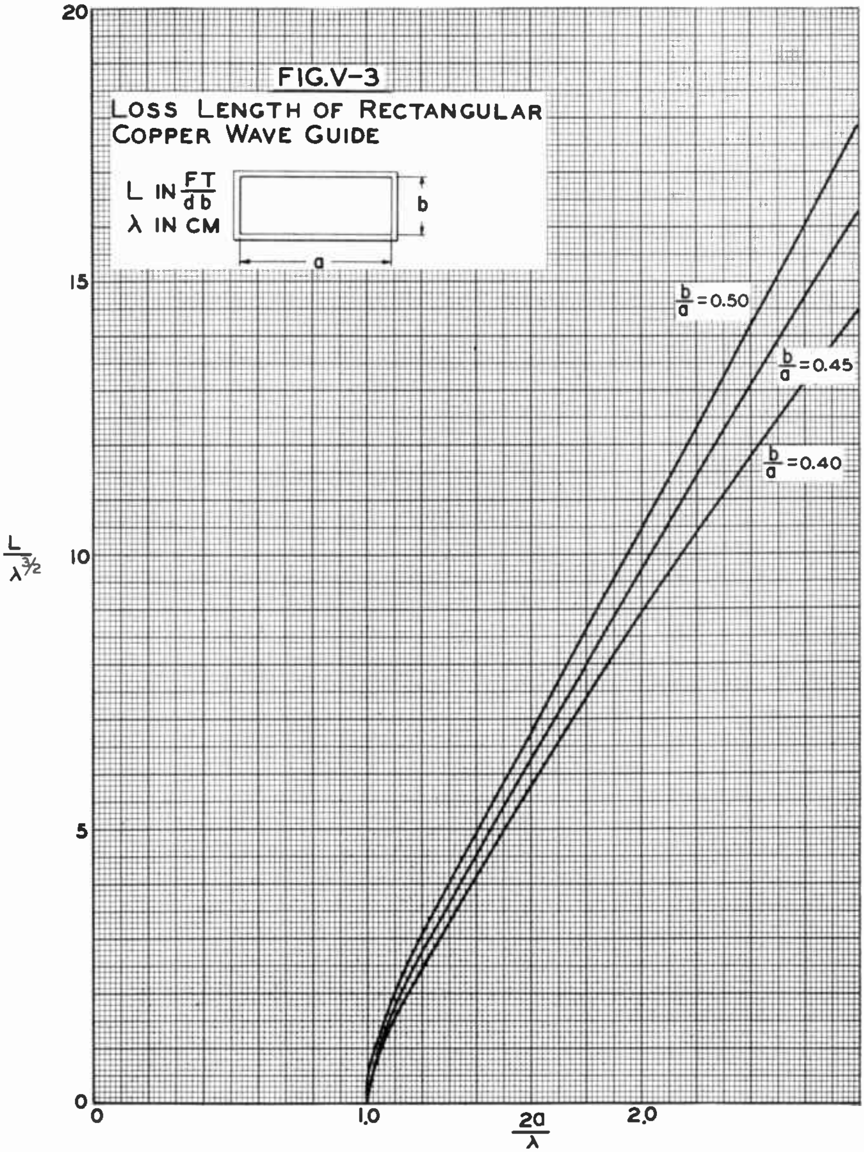


FIG. V-2
ATTENUATION VS. FREQUENCY FOR A RECTANGULAR COPPER WAVE GUIDE

$L/\lambda^{3/2}$ is plotted against $2a/\lambda$ for values of the ratio b/a that cover the range commonly used. In this figure the loss length is in feet/db, and the free space wavelength λ in cm. The conducting material is assumed to be copper. These curves permit rapid calculations of attenuation for any arbitrary wavelength signal in a wave guide chosen within the range commonly used.



2. Circular Wave Guides. The attenuation formulas for conductor losses in circular wave guides are as follows:

For $TE_{1,1}$ mode (Dominant Mode)

$$\alpha_C = \frac{.00423}{a^{3/2}} \frac{\left(\frac{f}{f_c}\right)^{-\frac{1}{2}} + \frac{1}{2.38}\left(\frac{f}{f_c}\right)^{3/2}}{\sqrt{\left(\frac{f}{f_c}\right)^2 - 1}} \text{ db per ft.}$$

For $TM_{0,1}$ mode (Circular Magnetic Mode)

$$\alpha_C = \frac{.00485}{a^{3/2}} \frac{\left(\frac{f}{f_c}\right)^{3/2}}{\sqrt{\left(\frac{f}{f_c}\right)^2 - 1}} \text{ db per ft.}$$

For $TE_{0,1}$ mode (Circular Electric Mode)

$$\alpha_C = \frac{.00611}{a^{3/2}} \frac{\left(\frac{f}{f_c}\right)^{-\frac{1}{2}}}{\sqrt{\left(\frac{f}{f_c}\right)^2 - 1}} \text{ db per ft.}$$

These formulae assume air-filled copper guides with the radius a in inches. For different conductor materials the attenuation should be multiplied by the appropriate loss factor in column 3 of Table II–1.

The variation of attenuation with frequency for a round copper wave guide 2 inches in diameter for each of the above modes is given in Fig. V–4. It is interesting to note that for a given size guide, the attenuation of the $TE_{0,1}$ mode decreases without limit with increasing frequency. Experimental verification of this is lacking, and the anomalous attenuation characteristic is lost if the guide is elliptical. See section A–3 of this chapter.

The variation of attenuation with wavelength for the various recommended circular wave guides of Chapter IV, Section E is plotted in Fig. V–5. This is for the $TE_{1,1}$ mode of transmission in air-filled, copper wave guide. For different conductor materials, the value of attenuation obtained from this figure should be multiplied by the appropriate loss factor given in column 3 of Table II–1.

The loss length L is defined as the length of line per unit attenuation. In Fig. V–6 the term $L/\lambda^{3/2}$ is plotted against $2a/\lambda$ for the above three modes of transmission. In this figure, L is in feet/db, the radius a in inches and the free space wavelength λ in cm. The wave guide is assumed to be copper, air-filled.

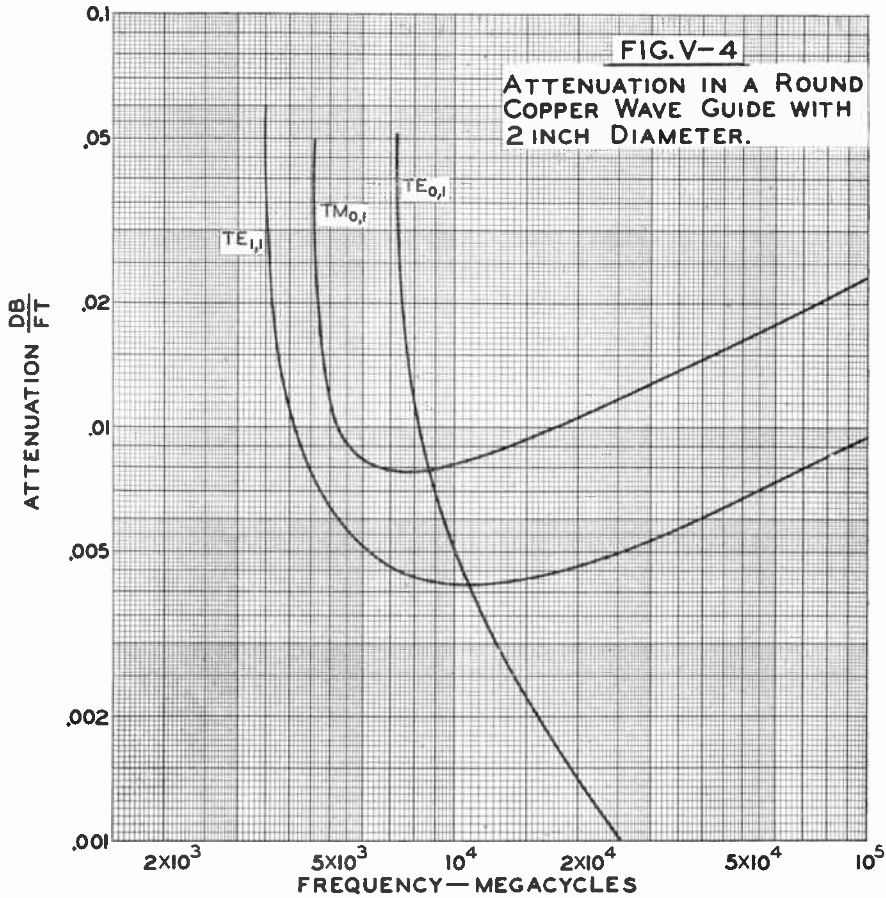


FIG. V-4
ATTENUATION IN A ROUND COPPER WAVE GUIDE WITH 2 INCH DIAMETER.
0.1
.05
.02
.01
.005
.002
.001
ATTENUATION DB/FT
TE1,1
TM0,1
TE0,1
2X10^3
5X10^3
10^4
2X10^4
5X10^4
10^5
FREQUENCY—MEGACYCLES

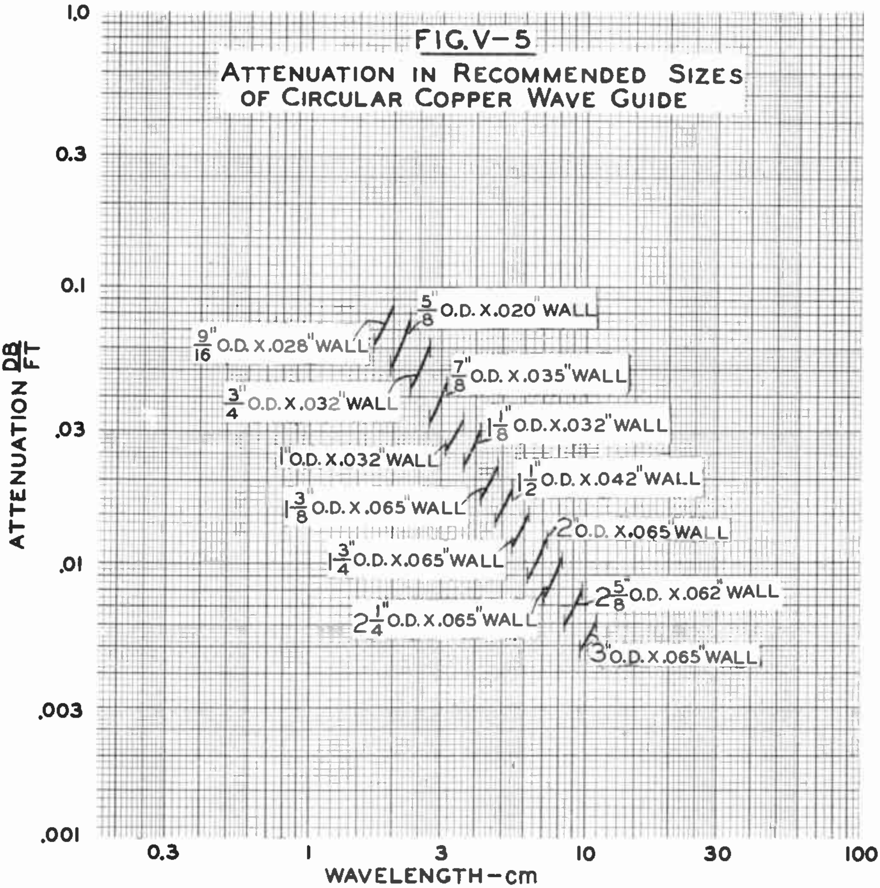


FIG. V-5
ATTENUATION IN RECOMMENDED SIZES OF CIRCULAR COPPER WAVE GUIDE
1.0
0.3
0.1
.03
.01
.003
.001
ATTENUATION DB/FT
0.3
1
3
10
30
100
WAVELENGTH—cm
5/8" O.D. X .020" WALL
9/16" O.D. X .028" WALL
7/8" O.D. X .035" WALL
3/4" O.D. X .032" WALL
1 1/8" O.D. X .032" WALL
1" O.D. X .032" WALL
1 1/2" O.D. X .042" WALL
1 3/8" O.D. X .065" WALL
2" O.D. X .065" WALL
1 3/4" O.D. X .065" WALL
2 5/8" O.D. X .062" WALL
2 1/4" O.D. X .065" WALL
3" O.D. X .065" WALL

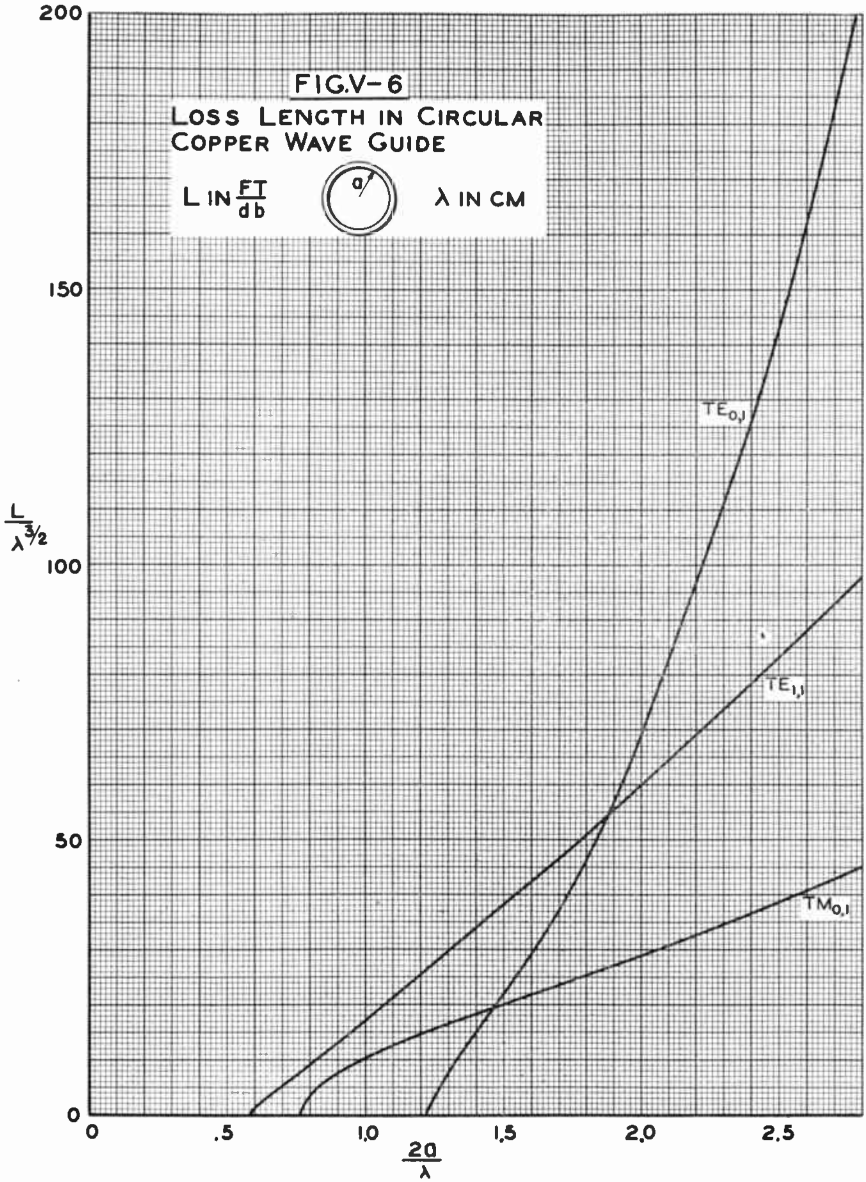


FIG.V-6
LOSS LENGTH IN CIRCULAR COPPER WAVE GUIDE
L IN FT/db
a
λ IN CM
200
150
100
50
0
L/λ^{3/2}
0
.5
1.0
1.5
2.0
2.5
2a/λ
TE0,1
TE1,1
TM0,1

ATTENUATION IN WAVE GUIDES

3. Effect of Ellipticity on Attenuation in Circular Wave Guides.[1] If a circular wave guide is deformed or made elliptical, the propagation will be affected, as outlined in Chapter IV—Sec. A–3. The attenuation of the waves will also be affected. The notation applied here to modes of propagation in elliptical pipe shall be the same as used in Chapter IV. The following conclusions may be drawn regarding attenuation as a function of ellipticity, with the perimeter of the pipe held constant.

1. The attenuation of the ${}_eH_o$ and ${}_eE_o$ waves will always increase as the ellipticity increases. In addition, the ${}_eH_o$ wave will not have an attenuation that decreases indefinitely with increasing frequency unless the ellipticity is zero.

2. The attenuation of the ${}_oH_1$ and ${}_eH_1$ waves will not be appreciably affected by the ellipticity.

3. The attenuation of the ${}_eH_1$ and ${}_oE_1$ waves will be increased with increasing ellipticity except in the case of the ${}_eH_1$ wave very near cutoff.

The minimum attenuation which can be obtained as a function of frequency is plotted in Fig. V–7 as a function of the ellipticity of the guide for the above modes of propagation. The periphery is assumed to be held constant at 40 cm., and the minimum attenuation is given in db/mile for copper, air-filled wave guide. In general, the variations in attenuation are not important except for large deformations.

4. Conductor Losses in Dielectric-Filled Wave Guides. If a wave guide is filled with a dielectric, there will be losses introduced by the dielectric, and in addition, the copper losses will be affected by the presence of the dielectric material. The formulae given in this section for losses in an air-filled guide may be applied to wave guides filled with a dielectric whose constant is ε_1 if some slight corrections are made, as follows:

1. Wherever the cutoff frequency f_c in the air-filled guides appears in the formulae, substitute the cutoff frequency in the dielectric-filled guide f_c'. These terms are related by the equation

$$f_c' = \frac{f_c}{\sqrt{\varepsilon_1}}$$

2. Multiply the modified expression by the factor $\varepsilon_1^{\frac{1}{4}}$. In other words, if the original expression for attenuation in an air-filled guide was

$$\alpha = F(f,a,b,f_c)$$

the modified expression for attenuation in the same wave guide filled with a dielectric whose constant is ε_1 will be

$$\alpha = \varepsilon_1^{\frac{1}{4}}\, F(f,a,b,f_c') = \varepsilon_1^{\frac{1}{4}}\, F(f,a,b,\frac{f_c}{\sqrt{\varepsilon_1}})$$

This expression only takes into consideration the effect upon conductor losses of having a dielectric inside the guide, and does not include losses resulting from the finite power factor of the dielectric.

[1] L. J. Chu, Electromagnetic Waves in Elliptical Hollow Pipes of Metal, Jour. App. Phys., Vol. 9, p. 583, Sept. 1938.

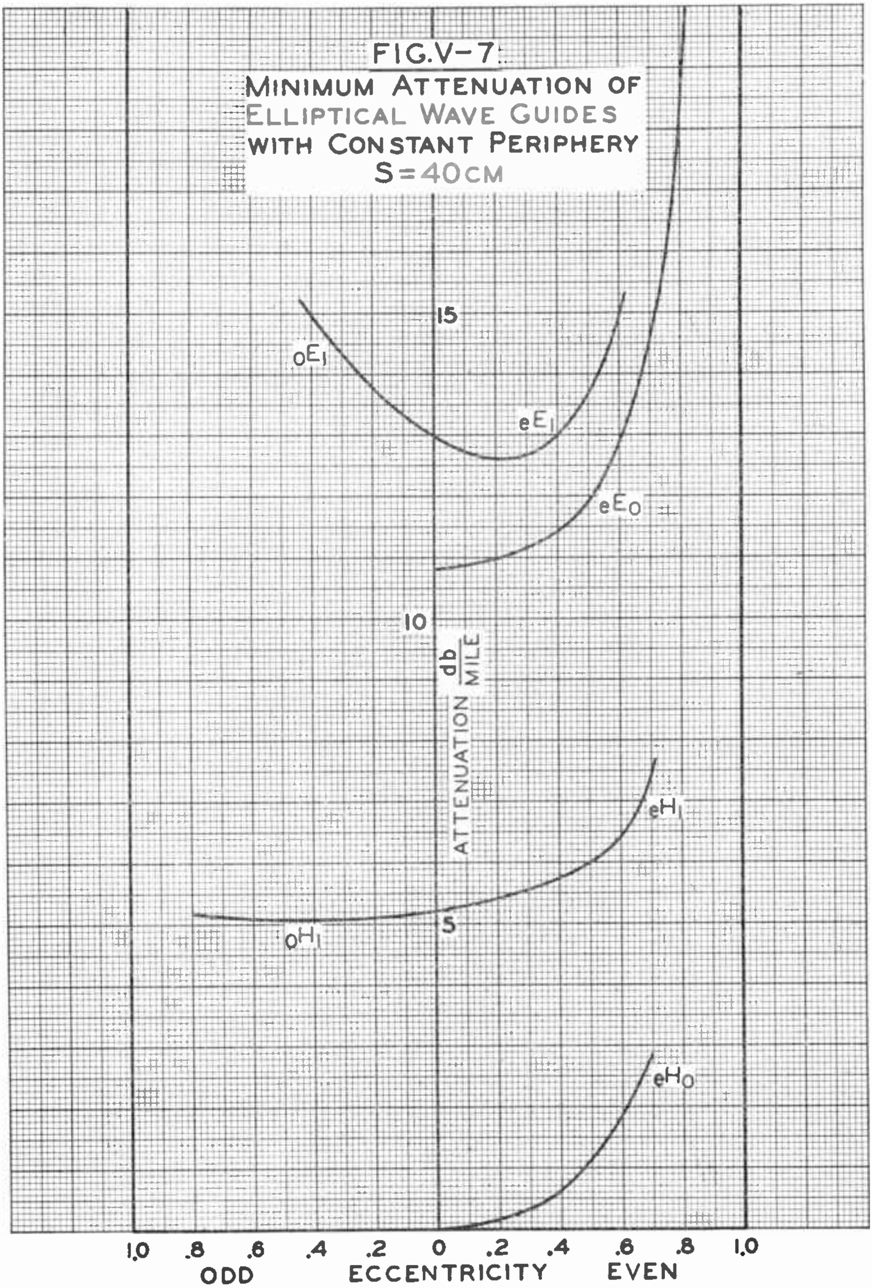


FIG.V-7:
MINIMUM ATTENUATION OF
ELLIPTICAL WAVE GUIDES
WITH CONSTANT PERIPHERY
S=40CM
15
10
5
ATTENUATION db/MILE
oE1
eE1
eE0
eH1
oH1
eH0
1.0 .8 .6 .4 .2 0 .2 .4 .6 .8 1.0
ODD
ECCENTRICITY
EVEN

B. *Dielectric Losses*

In a wave guide filled with dielectric material, there will be losses resulting from the imperfections of the dielectric as well as the finite conductivity of the conducting walls. These dielectric losses may be calculated by assuming the dielectric constant to be complex for actual dielectrics although real for perfect dielectrics. The dielectric constant may therefore be written

$$\varepsilon_1 = \varepsilon_1' - j\,\varepsilon_1''$$

and the loss tangent of the dielectric is defined as

$$\tan \zeta = \frac{\varepsilon_1''}{\varepsilon_1'}$$

The loss tangent of the dielectric is equal to the power factor, defined by

$$\text{power factor} = \cos\,(90^\circ - \zeta)$$

for all but very lossy dielectrics.

If an ideal dielectric whose constant is ε_1 is assumed, the cutoff wavelength λ_c' in the dielectric-filled guide will be

$$\lambda_c' = \sqrt{\varepsilon_1}\,\lambda_c$$

where λ_c is the cutoff wavelength in the same guide when filled with air. Inserting a dielectric with a constant greater than unity will increase the cutoff wavelength of the guide, and permit it to transmit power at lower frequencies than would otherwise be possible. But in general, losses in the dielectric are so high that the saving in space is more than compensated for by the increased attenuation of the power. On the other hand, it is quite feasible to construct satisfactory fixed attenuators by using a wave guide filled with a lossy dielectric.

The wavelength in a guide filled with an ideal dielectric has been given in Chapter IV, Section C–1 as

$$\lambda_g = \frac{\lambda}{\sqrt{\varepsilon_1 - \left(\frac{\lambda}{\lambda_c}\right)^2}}$$

where λ is the free space wavelength and λ_c the cutoff wavelength of the empty guide. If the dielectric is not perfect, the exact wavelength in the guide is

$$\lambda_g = \frac{\lambda}{\sqrt{\varepsilon_1' - \left(\frac{\lambda}{\lambda_c}\right)^2}} \times \frac{1}{\sqrt{1/2 + 1/2\left[\frac{\varepsilon_1''^2}{\left[\varepsilon_1' - \left(\frac{\lambda}{\lambda_c}\right)^2\right]^2} + 1\right]^{\frac{1}{2}}}}$$

which reduces to the approximate formula

$$\lambda_g = \frac{\lambda}{\sqrt{\varepsilon_1' - \left(\frac{\lambda}{\lambda_c}\right)^2}}$$

for all except the most lossy dielectrics.

The exact formula for attenuation in a wave guide resulting from an imperfect dielectric is

$$\alpha_D = 830 \frac{\varepsilon_1''}{\lambda}\left(\frac{\lambda}{\lambda_g}\right) db\ per\ ft.$$

where λ_g, the guide wavelength given above, and the free space wavelength λ are in cm. In all cases where the approximate calculation for wavelength is justified, the attenuation may be expressed as

$$\alpha_D = 830 \frac{\varepsilon_1''}{\lambda} \frac{1}{\sqrt{\varepsilon^1 - \left(\frac{\lambda}{\lambda_c}\right)^2}} db\ per\ ft.$$

C. *Attenuation Resulting from Both Conductor and Dielectric Losses*

To find the total attenuation in a wave guide resulting from both conductor and dielectric losses, the following formula should be applied:

$$\alpha_T = \alpha_D + \alpha_C$$

In this formula α_T, the total attenuation, is seen to be the sum of α_D, the attenuation resulting from dielectric losses, and α_C, the attenuation resulting from conductor losses. When conductor losses are calculated, care should be taken to use the modified formulae which take into account the effect of the dielectric upon the conductor losses.

D. *Losses in Flexible Wave Guide*

The attenuation in various types of flexible wave guides has been measured.[2] The values are given in Table V–1, along with some calculated figures for solid wave guides put in for comparison. The measured values are for guide that is old and well used, and the wavelength was $\lambda = 3.2$ cm. All guides were approximately 1″ × ½″ O. D. rectangular tubing.

TABLE V–I

Losses in Wave Guides

Type of Guide	Attenuation db/ft	Comments
Solid copper	.035	Calculated
Solid brass	.07	Calculated
Flexible copper	.64	Measured—guide very oxidized
Flexible copper—Silver plated	.087	Measured—guide very well used
Flexible aluminum	1.5—4.0	Measured—Loosely wound
Flexible brass	.219	Measured—interlocked and soldered

[2] Measured at Research Laboratories of the Sperry Gyroscope Co.

The attenuation in copper guide is particularly a function of age, for when the copper is clean, its attenuation is very little different from the flexible silver-plated guide.

E. *Losses in Joints in Wave Guides*

The losses in various types of connecting joints for wave guides have been measured at $\lambda = 3.2$ cm. in 1″ × ½″ O.D. × .050″ rectangular wave guide. The conclusions reached were that either the simple butt joint squeezed very tightly, or the choke coupling aligned by pins is satisfactory. The choke has a slightly higher loss but gives more consistent results with reasonable precautions. The following losses were measured in brass guide.

Type of Joint	*Loss in db*
Soldered—soft or silver	.008 to .010
Butt Joint—very tight	.000
Choke and Flange Joint	.009 to .013
Ordinary Butt Joint	.004 to 1

One advantage of a choke coupling is that it permits the ends of the wave guide to be separated without a marked decrease in transmission. The percent transmission as a function of the separation between the ends of the waveguide is given in Fig. V–8 while the standing wave ratio at the input introduced by displacement in various directions is given in Fig. V–9. These measurements are in 1″ × ½″ O.D. × .050″ rectangular wave guide at a wavelength $\lambda = 3.2$ cm.

F. *Losses in Various Types of Shorting Plugs*[3]

The losses in various types of shorting plugs in 1″ × ½″ O.D. × .050″ wall rectangular wave guide at $\lambda = 3.2$ cm. have been measured with the following results. The types of shorting plugs tested are sketched in Fig. V–10, and the loss measured is in excess of the loss introduced by a brass plate soldered on the end of the wave guide.

Type of plug	*Measured loss — db*
A—silver plated	.001
B—silver plated	.005 to .006
C—silver plated	.015 to .025
D—silver plated	.017 to .035
E—brass	.038

The losses in types A to D do not vary greatly with age, but the losses introduced by type E have been observed to increase markedly with age. The above measurement of the type E plug was made with a clean, newly-machined model.

[3]Measured at Research Laboratories of the Sperry Gyroscope Co.

G. *Attenuation in a Wave Guide Below Cutoff*

If a possible mode of propagation is excited in a wave guide that is below cutoff for that particular mode, there will be no real propagation of energy down the guide. The input impedance will be a pure reactance, if losses in the walls are neglected, and the fields that are excited in the guide will diminish exponentially with distance from the point of excitation.

The field as a function of distance down the guide is given by

$$E = E_o\, e^{-\alpha z}$$

where E_0 is the initial amplitude of excitation. The attenuation is given by

$$\alpha = 8.69 \sqrt{\left(\frac{2\pi}{\lambda_c}\right)^2 - \left(\frac{2\pi}{\lambda}\right)^2} \qquad \frac{\text{db}}{\text{unit length}}$$

where λ is the free space wavelength and λ_c the wavelength at cutoff. In Fig. V–11 is plotted attenuation as a function of λ, for a given λ_c, which is in turn a function of the size of the wave guide and the mode of excitation (Chapter IV, Section B). As the free space wavelength becomes much larger than the cutoff wavelength, the attenuation becomes nearly independent of λ, and approaches a limit of

$$\alpha = \frac{54.6}{\lambda_c} \qquad \frac{\text{db}}{\text{unit length}}$$

as λ becomes very large.

It was stated above that there is no real propagation of energy through a wave guide that was below cutoff. This is the idealized case for a perfectly conducting wave guide of infinite length. It is possible to insert a probe or other pick-up device into the guide at a distance from the point of excitation, and abstract power which will be proportional to the square of the field strength at that point. So there will be some propagation of energy through the guide. As the amount of energy abstracted from the guide is proportional to the square of the field strength at the point of pick-up, and this field strength diminishes exponentially with distance from the point of excitation, a wave guide below cutoff can be used as a variable attenuator whose attenuation in decibels is a linear function of distance.

In such an attenuator the $TE_{1,1}$ or $TM_{0,1}$ modes in circular guides are most commonly used. For these modes, the attenuation formula becomes:

For the $TE_{1,1}$ mode

$$\alpha = 8.69 \sqrt{\left(\frac{1.841}{a}\right)^2 - \left(\frac{2\pi}{\lambda}\right)^2} \qquad \text{db/unit length}$$

For the $TM_{0,1}$ mode

$$\alpha = 8.69 \sqrt{\left(\frac{2.405}{a}\right)^2 - \left(\frac{2\pi}{\lambda}\right)^2}$$

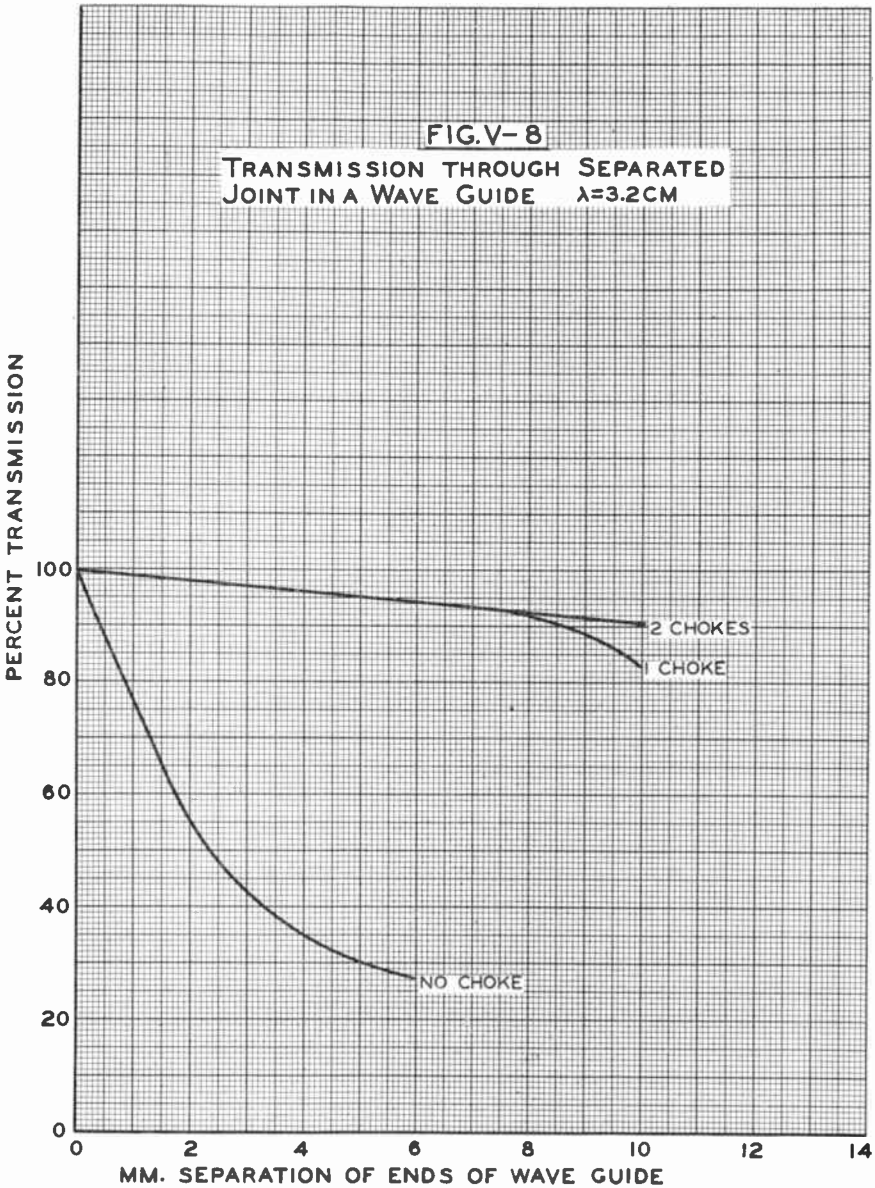


FIG. V-8
TRANSMISSION THROUGH SEPARATED JOINT IN A WAVE GUIDE λ=3.2CM
PERCENT TRANSMISSION
100
80
60
40
20
0
2 CHOKES
1 CHOKE
NO CHOKE
0
2
4
6
8
10
12
14
MM. SEPARATION OF ENDS OF WAVE GUIDE

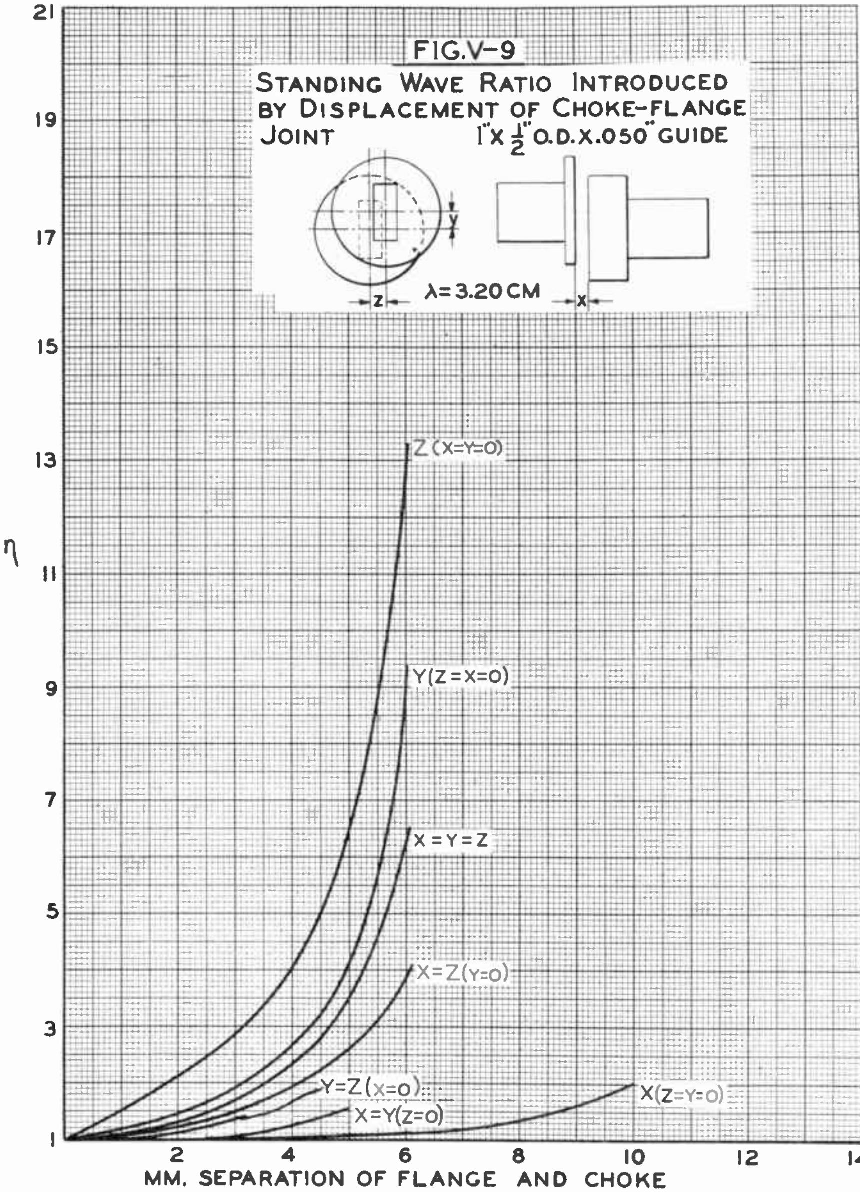


FIG.V-9
STANDING WAVE RATIO INTRODUCED BY DISPLACEMENT OF CHOKE-FLANGE JOINT
1" X 1/2" O.D. X .050" GUIDE
Y
Z
λ=3.20 CM
X
η
Z (X=Y=O)
Y(Z=X=O)
X=Y=Z
X=Z(Y=O)
Y=Z(X=O)
X=Y(Z=O)
X(Z=Y=O)
1
3
5
7
9
11
13
15
17
19
21
2
4
6
8
10
12
14
MM. SEPARATION OF FLANGE AND CHOKE

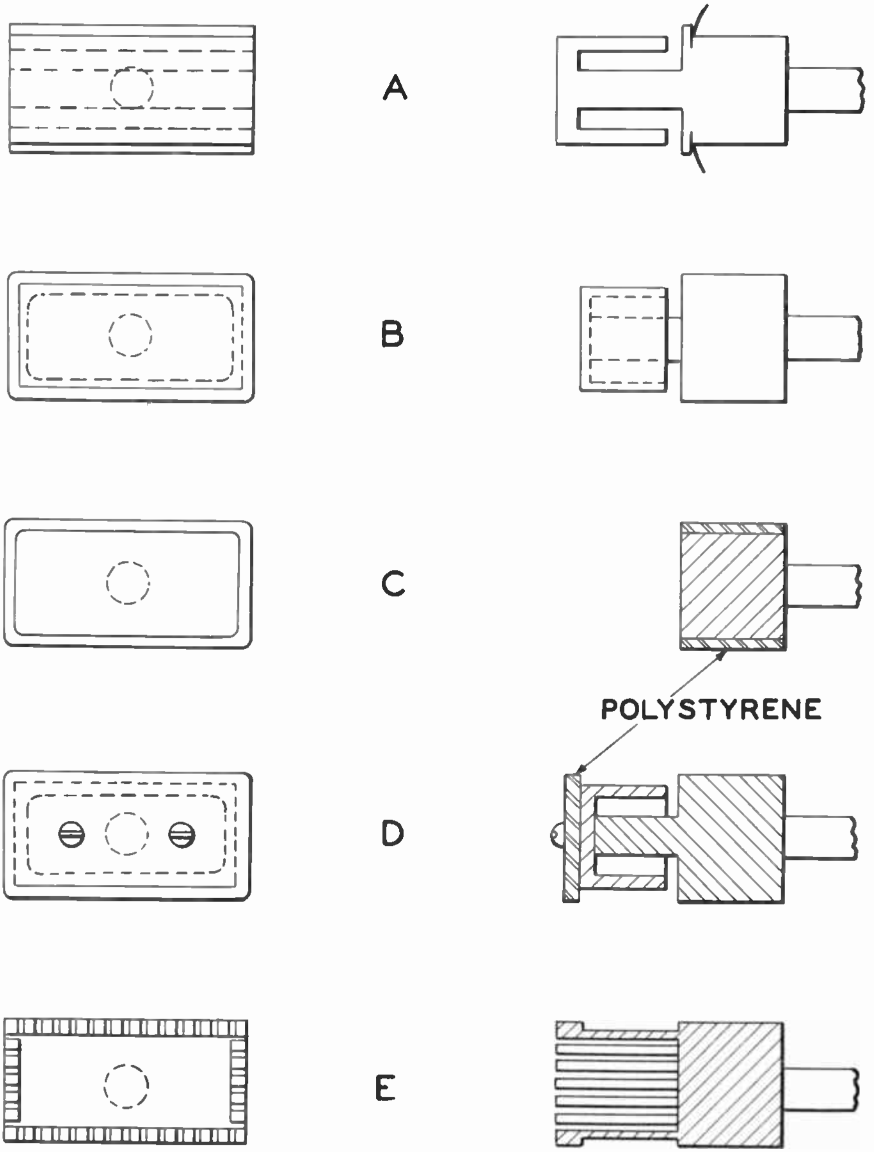


FIG.V-10
WAVE GUIDE SHORTING PLUGS

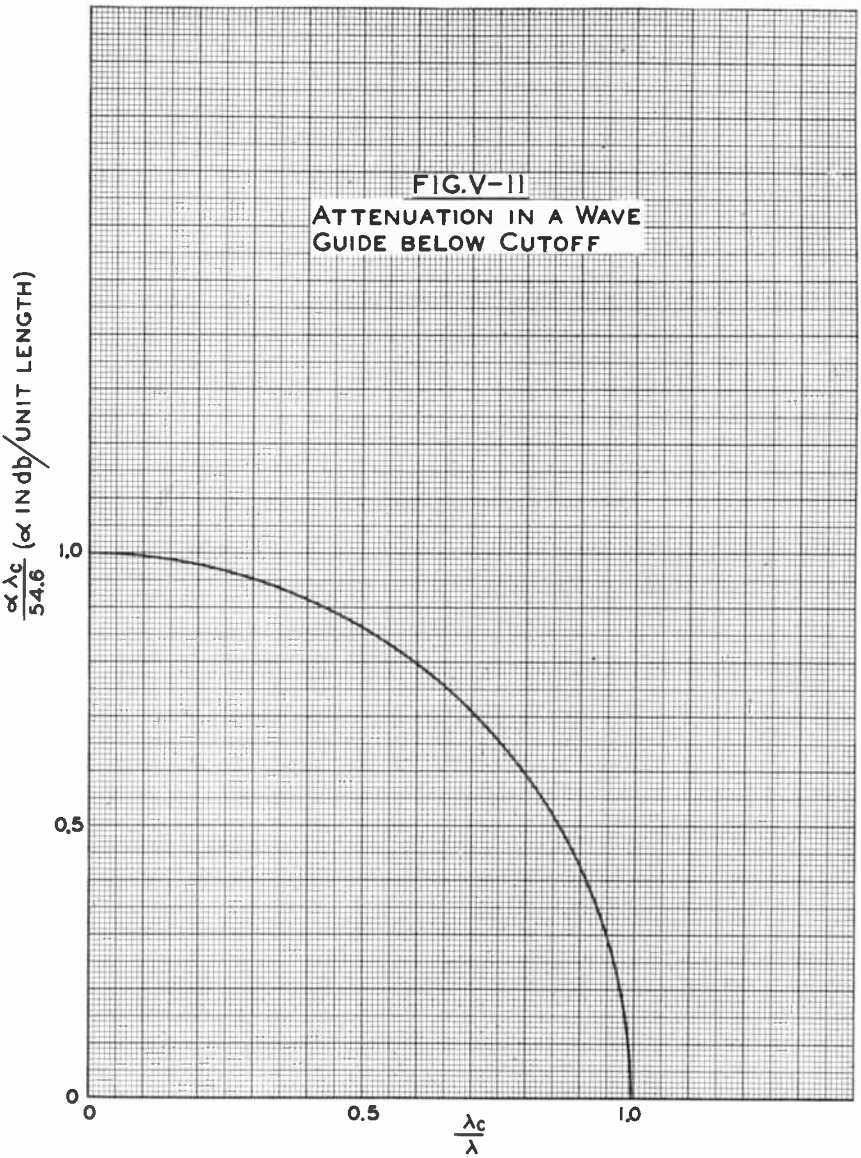


FIG. V-11
ATTENUATION IN A WAVE GUIDE BELOW CUTOFF
$\frac{\alpha \lambda_c}{54.6}$ (α IN db/UNIT LENGTH)
1.0
0.5
0
0
0.5
1.0
$\frac{\lambda_c}{\lambda}$

CHAPTER VI

REFLECTIONS AND IMPEDANCE MATCHING IN WAVE GUIDES

A. *General Discussion*

In an ordinary coaxial transmission line, the principal mode is used for propagation of energy down the line, and the line restricted in size so that the higher modes will not propagate. If there is some discontinuity or reflecting element introduced into a line that is otherwise perfectly matched, part of the incident principal wave will be reflected back toward the generator, and in addition higher order modes may be excited. The line being sufficiently restricted in size (See Chapter I, Section I), these higher order modes will not propagate down the line, but will diminish exponentially with distance from the point of excitation. The higher order fields will vary as

$$E = E_o e^{-\alpha z}$$

where E_0 is their initial amplitude of excitation. The attenuation α is given by

$$\alpha = \sqrt{\left(\frac{2\pi}{\lambda_c}\right)^2 - \left(\frac{2\pi}{\lambda}\right)^2} \qquad \text{nepers/unit length}$$

where λ is the free space wavelength and λ_c the cutoff wavelength for the mode under consideration. If the ratio λ/λ_c is large, the attenuation will be given approximately by

$$\alpha = \frac{2\pi}{\lambda_c} \qquad \text{nepers/unit length}$$

and the fields will have fallen to $1/e$ of their initial amplitude at a distance $\frac{\lambda_c}{2\pi}$ from the point of excitation.

Ordinary transmission line theory may be accurately applied to discontinuities and junctions in the line only if the higher order fields set up by the discontinuity can be neglected. Distortion and fringing of the fields is an indication of the presence of higher order fields, and if the fringing is appreciable, results obtained by ordinary transmission line theory may be in error.

For an ideal line in which these higher order modes can be neglected, the reflection coefficient of the discontinuity, which is the percent of the incident wave that will be reflected from the discontinuity, is given by

$$\frac{E_2}{E_1} = \frac{\dfrac{Z_L}{Z_o} - 1}{\dfrac{Z_L}{Z_o} + 1}$$

where Z_L is the input impedance at the point of discontinuity, Z_o the characteristic impedance of the line, and E_1 and E_2 the magnitudes of the electric fields associated with the incident and reflected waves, respectively.

The ratio Z_L/Z_o in the above expression is termed the normalized impedance at the input to the discontinuity. By measurement of the magnitude and position of the standing waves between the discontinuity and the generator, the normalized impedance may be determined even though the actual line impedance Z_o is not known (See Chapter III, Section A-5). As will be seen, this normalized impedance is a useful concept for transmission systems in which the value of Z_o may be in doubt.

In wave guides which are restricted in size so that only the dominant mode will propagate energy, the effects of a discontinuity are much the same as in a coaxial line, except that the first higher order modes that are excited can never be very far away from cutoff, and can therefore never be neglected. For example, if a rectangular wave guide is carrying the dominant or $TE_{1,0}$ mode, the width a of the guide must always be greater than $\lambda/2$, while cutoff for the next higher mode ($TE_{2,0}$) is given by $\lambda_c = a$.

B. *Impedance in a Wave Guide*

Various definitions have been proposed for characteristic impedance in a wave guide. The one in most general use at present is the "specific wave impedance," which is the ratio of transverse electric to transverse magnetic fields for a given mode in the guide. For a guide of uniform cross section, this impedance is as follows:

For all TE waves:

$$Z_o = \sqrt{\frac{\mu_1}{\varepsilon_1}}\, 377\, \frac{\lambda_g}{\lambda}$$

For all TM waves:

$$Z_o = \sqrt{\frac{\mu_1}{\varepsilon_1}}\, 377\, \frac{\lambda}{\lambda_g}$$

This definition is not exactly the same in physical meaning as the characteristic impedance of a concentric line, for the ratio of transverse electric to magnetic fields in any system operating with the principal mode of transmission is $377\sqrt{\frac{\mu_1}{\varepsilon_1}}$ ohms, regardless of the geometrical configuration of the system. So the exact wave guide analogy of characteristic impedance in a concentric line is doubtful, but the effects of discontinuities, loads, etc. may be expressed in terms of normalized impedances which do not require an exact knowledge of Z_o.

If some reflecting element is introduced into a wave guide, an analogy can be drawn between its action and the action of an impedance discontinuity in an ordinary transmission line. For instance, if the reflecting element is a perfect conductor, one would expect that its action will be equivalent to some combination of series and shunting reactors in an ordinary transmission line which absorb no power. It has been found theoretically and verified experimentally that many reflecting elements whose dimension along the guide axis is small compared to a wave length are equivalent to simple shunting reactances. The normalized input admittance Y to such reflecting elements when introduced into a matched wave guide can be represented by

$$Y = 1 + jB$$

where B is the normalized susceptance of the reflecting element.

C. *Relation Between Normalized Susceptances and Standing Wave Ratio*

Impedance matching in a wave guide—making the load impedance equal to the guide impedance—is usually accomplished with one or more of these reflecting elements. A shunting reactance that introduces a certain standing wave ratio into a matched transmission line will, if properly placed, correct the same standing wave ratio in another system.

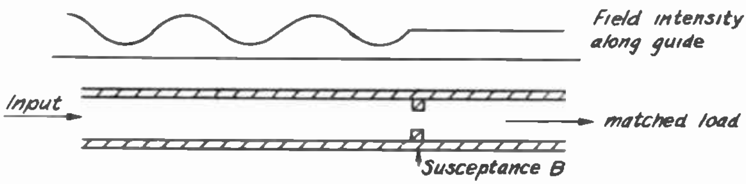


FIG. VI - 1

Consider the system illustrated in Fig. VI–1. The relationship between the shunting susceptance B and the standing wave ratio r_i which it introduces into a matched wave guide or corrects in a mismatched guide is given by

$$r_i = \left(\frac{\sqrt{4 + B^2} + B}{\sqrt{4 + B^2} - B}\right)^2$$

which may also be written as

$$B = \frac{\sqrt{r_i} - 1}{r_i^{1/4}}$$

This relationship between r_i and B is shown graphically in Fig. VI–2.

To correct a given mismatch, the size of the reflecting element should be chosen by the considerations outlined above. Then, to match the load to the line impedance, it should be placed at a position determined by

$$d_1 = \frac{90 - \tan^{-1}\left|\frac{B}{2}\right|}{720}\,\lambda_g$$

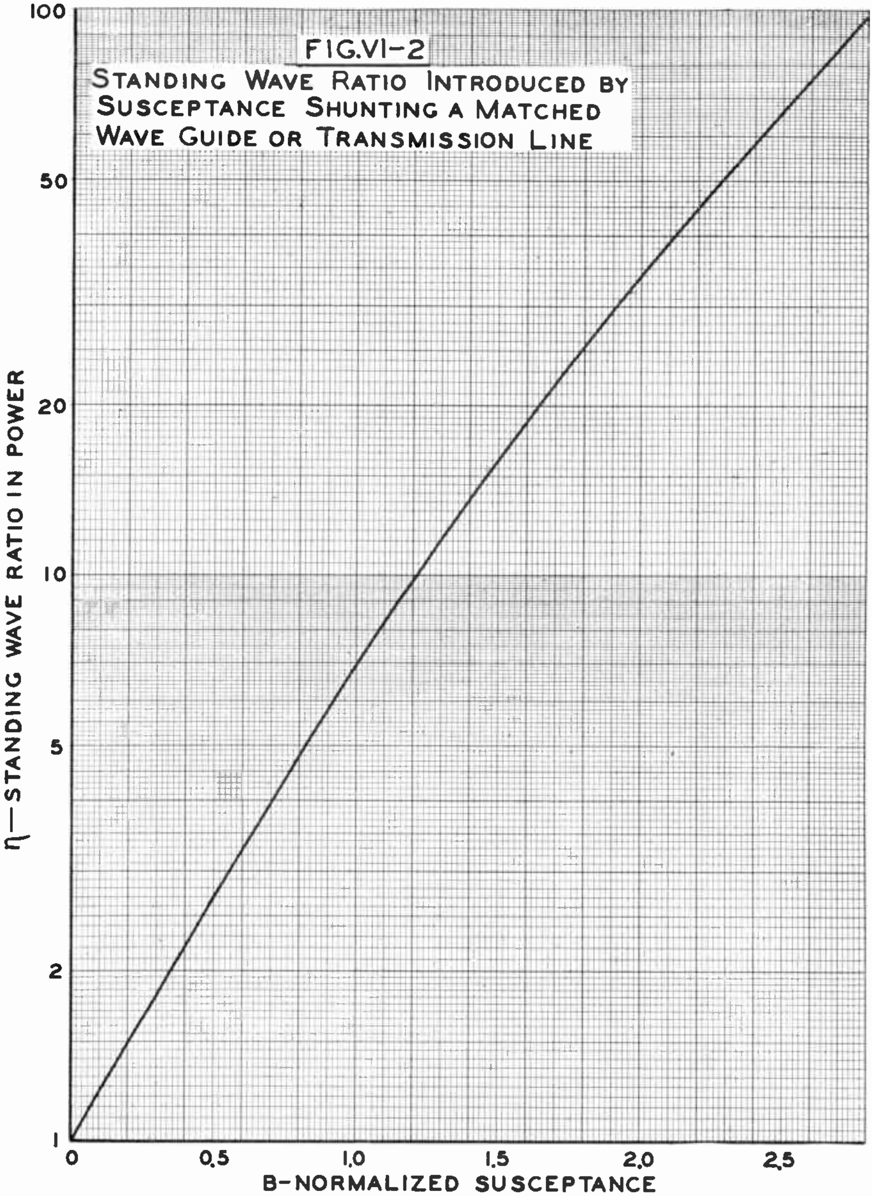


FIG. VI-2
STANDING WAVE RATIO INTRODUCED BY SUSCEPTANCE SHUNTING A MATCHED WAVE GUIDE OR TRANSMISSION LINE
100
50
20
10
5
2
1
η—STANDING WAVE RATIO IN POWER
0
0.5
1.0
1.5
2.0
2.5
B-NORMALIZED SUSCEPTANCE

where d_1 is the distance between the reflecting element and a voltage minimum. If the reflecting element is inductive (B is negative) it should be placed at a distance d_1 on the load side of a minimum, while if capacitive, it should be at a distance d_1 on the generator side of a minimum. The distance d_1 is plotted against $|B|$ in Fig. VI–3.

As a typical example of impedance matching with a reflecting element, consider a 1″ × ½″ O.D. × .050″ wall rectangular wave guide, with $\lambda = 3.2$ cm., feeding a load that gives $\eta = 5.0{:}1$ in the wave guide. With the chosen wave guide and wavelength, the wavelength in the guide $\lambda_g = 1.764''$ and $\lambda_g/a = 1.96$. It is desired to correct this mismatch with a symmetrical inductive window of the type discussed in Section D–1–a of this chapter, illustrated in Fig. VI–4 (an alternative method of matching, suitable for this type of window, is also included in Section D–1–a).

1. From Fig. VI–2, it is seen that $\eta = 5.0{:}1$ requires that the window have a susceptance $B = 0.82$. Therefore $B(a/\lambda_g) = 0.42$.

2. The chosen wave guide and wavelength correspond to one of the curves in Fig. VI–4. From this curve it is seen that to obtain the required value of $B(a/\lambda_g)$, the window opening should be $d/a = .663$, giving $d = .596''$.

3. From Fig. VI–3, this window should be located at a distance $d_1 = .070\lambda_g = .123''$ toward the load (B is negative) from a voltage minimum. For minimum frequency sensitivity, the window should be placed as near the load as possible, yet not so near that there will be interaction of the higher order fields. (See Section D–1 of this Chapter). It will be recalled that voltage minima are spaced a half wave apart in the wave guide, and a reflecting element that produces a certain effect at a given location in a wave guide will produce the identical effect one or more half waves away from that point.

D. *Windows in Wave Guides*

A thin plate of metal placed perpendicular to the axis of a wave guide and partially blocking it acts as a shunt susceptance across the guide. The magnitude of the susceptance, and its sign (whether inductive or capacitive) will depend upon the size and location of the window through the plate.

1. Rectangular Wave Guides

a. Symmetrical Inductive Windows. When the window opening extends completely across the guide and is symmetrically located with the sides parallel to the narrow dimension of the guide (see Figure VI–4) the window

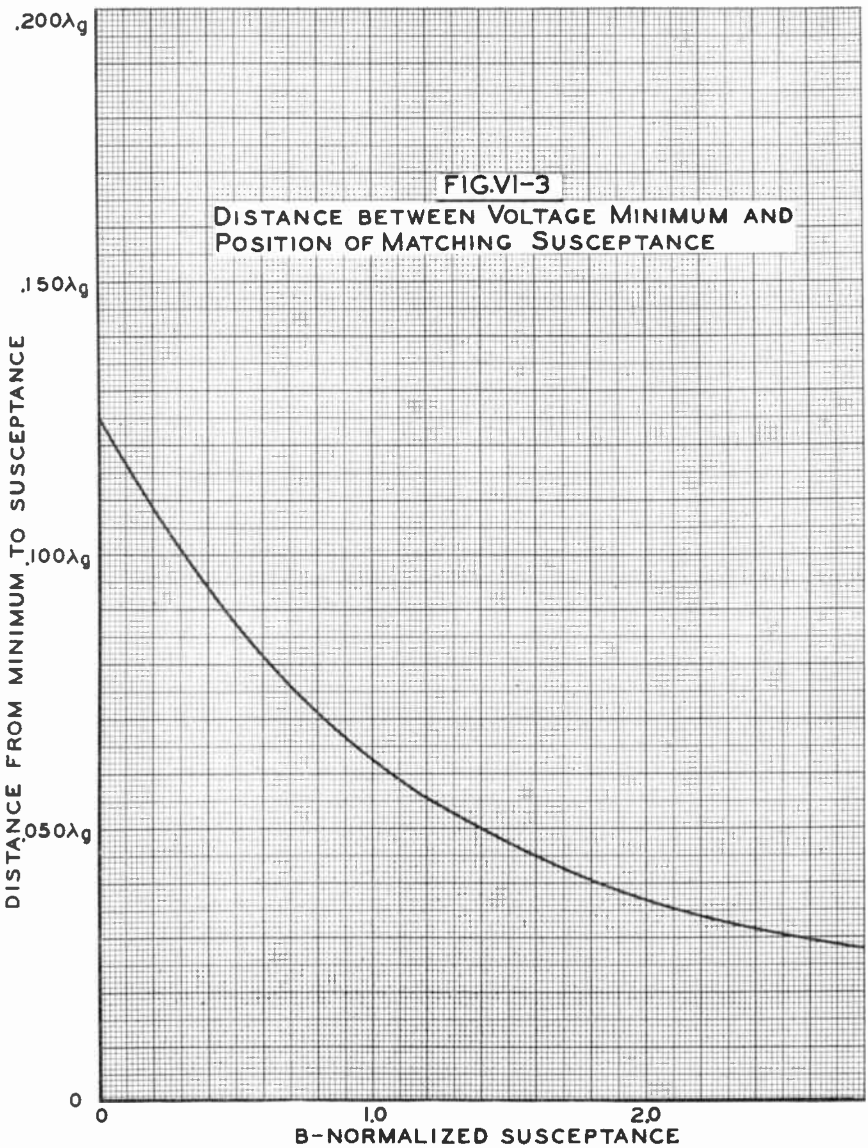


FIG.VI-3
DISTANCE BETWEEN VOLTAGE MINIMUM AND POSITION OF MATCHING SUSCEPTANCE
.200λg
.150λg
.100λg
.050λg
0
DISTANCE FROM MINIMUM TO SUSCEPTANCE
0
1.0
2.0
B-NORMALIZED SUSCEPTANCE

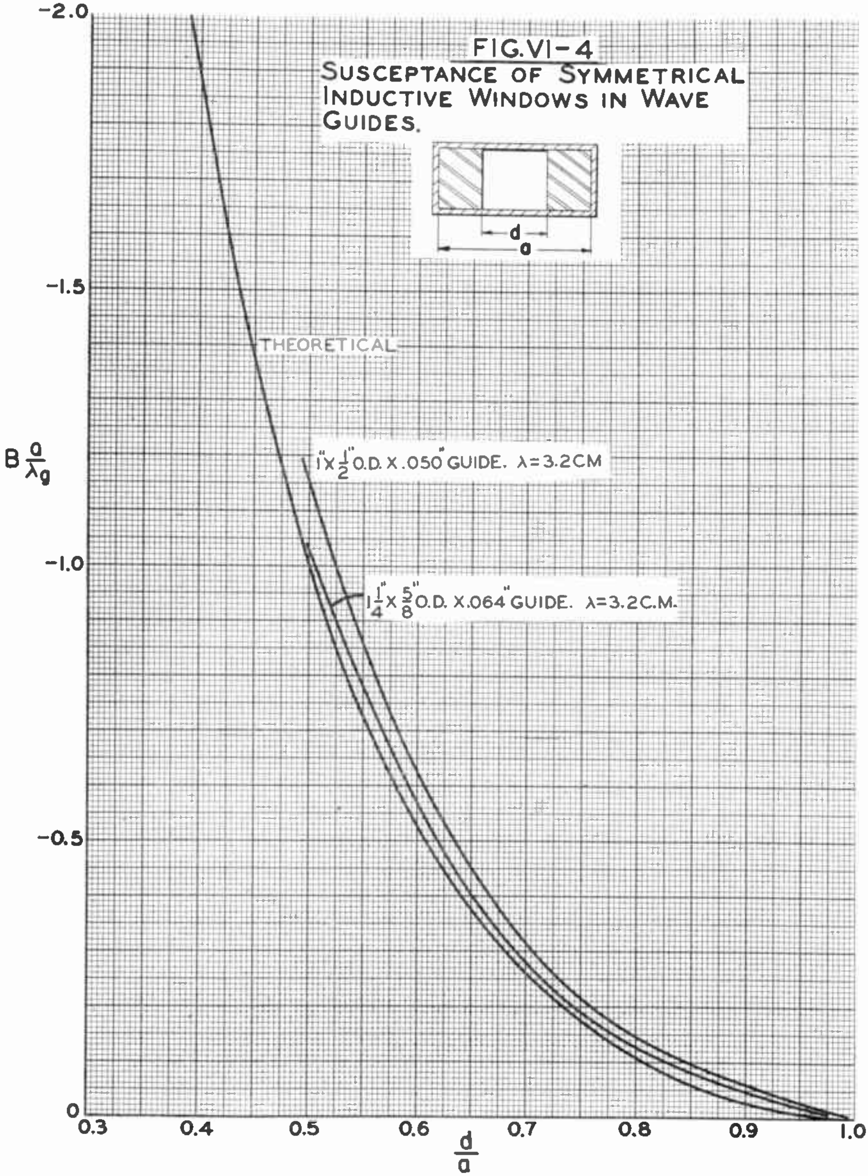


FIG.VI-4
SUSCEPTANCE OF SYMMETRICAL INDUCTIVE WINDOWS IN WAVE GUIDES.
d
a
THEORETICAL
1" x 1/2" O.D. X .050" GUIDE. λ=3.2CM
1 1/4" x 5/8" O.D. X.064" GUIDE. λ=3.2C.M.
-2.0
-1.5
-1.0
-0.5
0
B a/λg
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
d/a

acts like a shunt inductance across the guide. This type of window has the theoretical susceptance

$$B = -\frac{\lambda_g}{a}\cot^2\frac{\pi d}{2a}$$

where λ_g is the guide wavelength, a the guide width, and d the opening of the window.

In Fig. VI–4, $B\frac{a}{\lambda_g}$ is plotted as a function of the ratio d/a. On the same sheet are plotted the experimentally observed results[1] for two different sizes of wave guide at a wavelength of $\lambda = 3.2$ cm. These curves are all seen to lie above the theoretical curve. This is because of the finite thickness of the windows, which is not taken into account by the simple theory, and which effectively increases the susceptance of the window. All of these curves were made with $\frac{1}{32}''$ thick diaphragms.

Special curves for symmetrical inductive windows have been included for one special case: $1'' \times \frac{1''}{2}$ O.D. $\times$ .050″ wall wave guide. $\lambda = 3.2$ cm. For this case, curves are given in Figs. VI–5 and Fig. VI–6 of the standing wave ratio as a function of window opening, and of the distance d_1 (in thousandths of an inch) at which the window must be located on the load side of a minimum. To match impedances using these curves:

1. Measure η and the position of a minimum.
2. From Fig. VI–5 select the proper size window opening.
3. From Fig. VI–6 determine the distance d_1 between a voltage minimum and a proper location of the window, and place the window at this distance on the *load* side of a minimum.

Frequency Sensitivity of Windows. As can be seen from the theoretical formula, the standing wave ratio introduced into a given wave guide by a given window will depend upon the wavelength of the transmitted signal. This effect has been measured[1] for $\frac{1}{32}''$ thick symmetrical inductive windows in $1'' \times \frac{1''}{2}$ O.D. $\times$ .050″ wall wave guide. The experimental results are given in Fig. VI–7, along with the theoretical results.

Coupling Between Windows. Theoretical calculations have been made which give the coupling between symmetrical inductive windows in rectangular wave guide operating in the dominant mode as a function of their susceptance and the distance S between them. This coupling is a result of the interaction of the higher order fields excited by the windows, and modifies the values of susceptance introduced by the individual windows.

[1] Measured at Research Laboratories, Sperry Gyroscope Co.

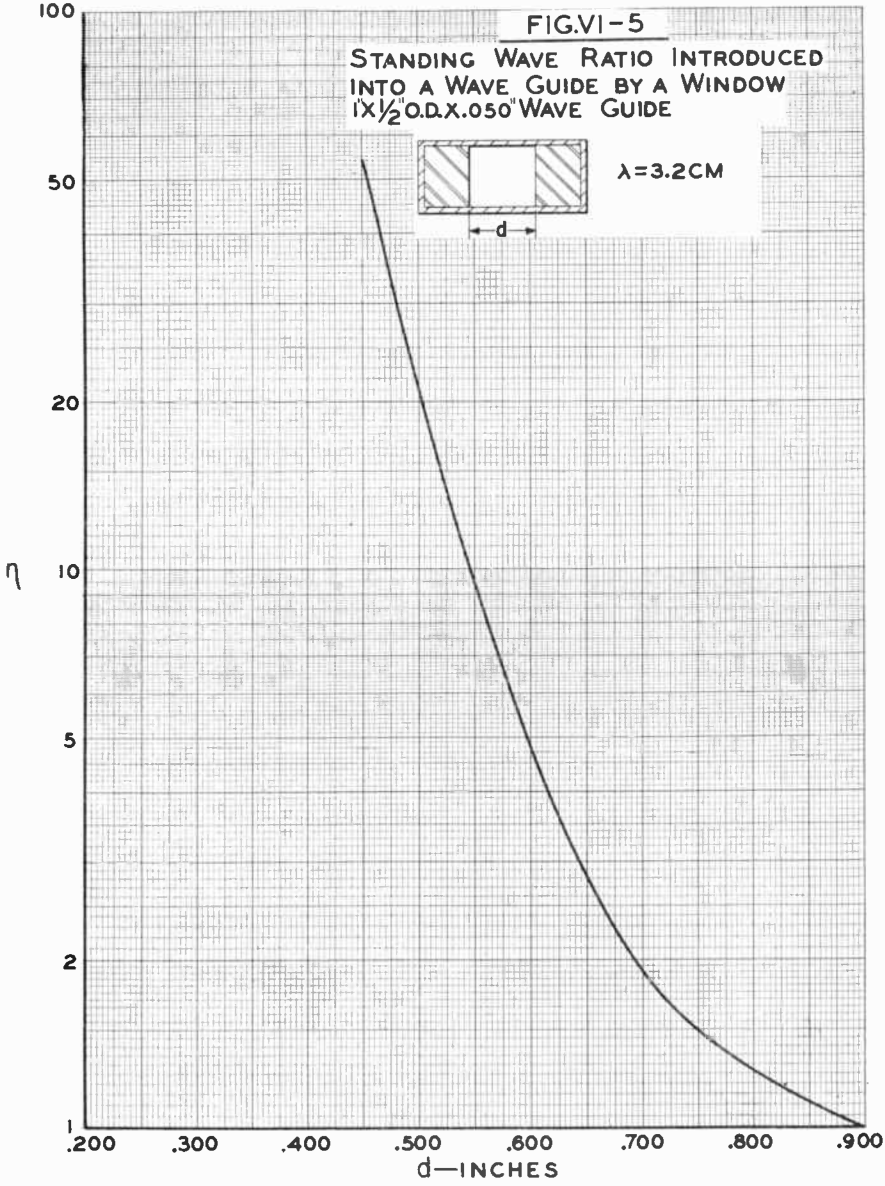


FIG.VI-5
STANDING WAVE RATIO INTRODUCED INTO A WAVE GUIDE BY A WINDOW
1"X½" O.D.X.050" WAVE GUIDE
λ=3.2CM
d
100
50
20
10
5
2
1
η
.200
.300
.400
.500
.600
.700
.800
.900
d—INCHES

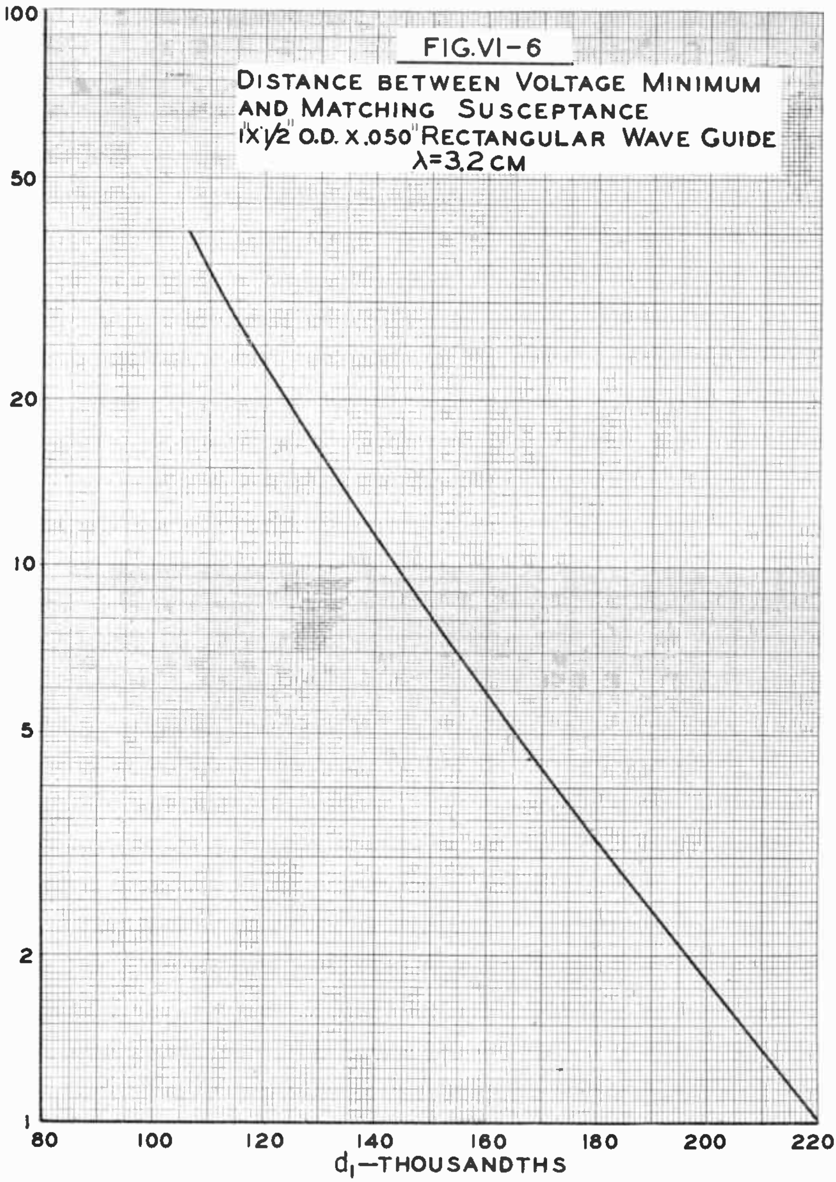


FIG. VI-6
DISTANCE BETWEEN VOLTAGE MINIMUM AND MATCHING SUSCEPTANCE
1" X 1/2" O.D. X .050" RECTANGULAR WAVE GUIDE
λ=3.2 CM
100
50
20
10
5
2
1
80
100
120
140
160
180
200
220
d₁—THOUSANDTHS

The results are conveniently expressed by modifying the admittance of the second window. If Y_1 and Y_2 are the admittances of the windows, then $Y_1 = -jB_1$ and $Y_2 = -jB_2$, with Y_2 the admittance of the window nearest the load. The modified value of Y_2, termed Y_2', is given by

$$Y_2' = -jB_2 - 6\frac{a}{\lambda_g}\left(\frac{B_1}{1 + \frac{a}{\lambda_g}B_1}\right)\left(\frac{B_2}{1 + \frac{a}{\lambda_g}B_2}\right)e^{-\frac{3\pi S}{a}}\, Y_L \sin\frac{2\pi S}{\lambda_g}$$

$$-j\left(B_2 \sin\frac{2\pi S}{\lambda_g} + \cos\frac{2\pi S}{\lambda_g}\right)$$

where a is the width of the guide, S is the distance between the windows, and Y_L is the load admittance referred to the plane of window 2. This equation is valid to the extent that $e^{-\frac{2\pi S}{a}}$ is small compared to unity.

Figs. VI–8a and VI–8b contain plots of the magnitude and phase of the ratio Y_2'/Y_2 as a function of the ratio S/λ_g. These are for the special case of $Y_L = 1$ (matched wave guide), $\lambda_g/a = 1.96$, and two different values of Y_1 and Y_2, specified on the figures.

b. Asymmetrical Inductive Windows. If the window opening extends from top to bottom of the guide with the sides parallel to the electric field, but is not symmetrically located in the center of the guide, the effect will still be that of a shunting inductance, but the magnitude of the inductance will be a function both of the window opening and the degree of displacement from the center. A formula derived for the theoretical susceptance in this case is

$$B = -\frac{\lambda_g}{a}\cot^2\frac{\pi d}{2a}\left(1 + \sec^2\frac{\pi d}{2a}\cot^2\frac{\pi x_o}{a}\right)$$

where x_o is the distance from one side wall to the center line of the window (when the window is centered, $x_o = a/2$). This theoretical susceptance is plotted in Fig. VI–9, along with some experimental results.

For the special case of asymmetry when one side of the window coincides with one wall of the pipe ($x_o = d/2$) the theoretical formula reduces to

$$B = -\frac{\lambda_g}{a}\cot^2\frac{\pi d}{2a}\left(1 + \operatorname{cosec}^2\frac{\pi d}{2a}\right)$$

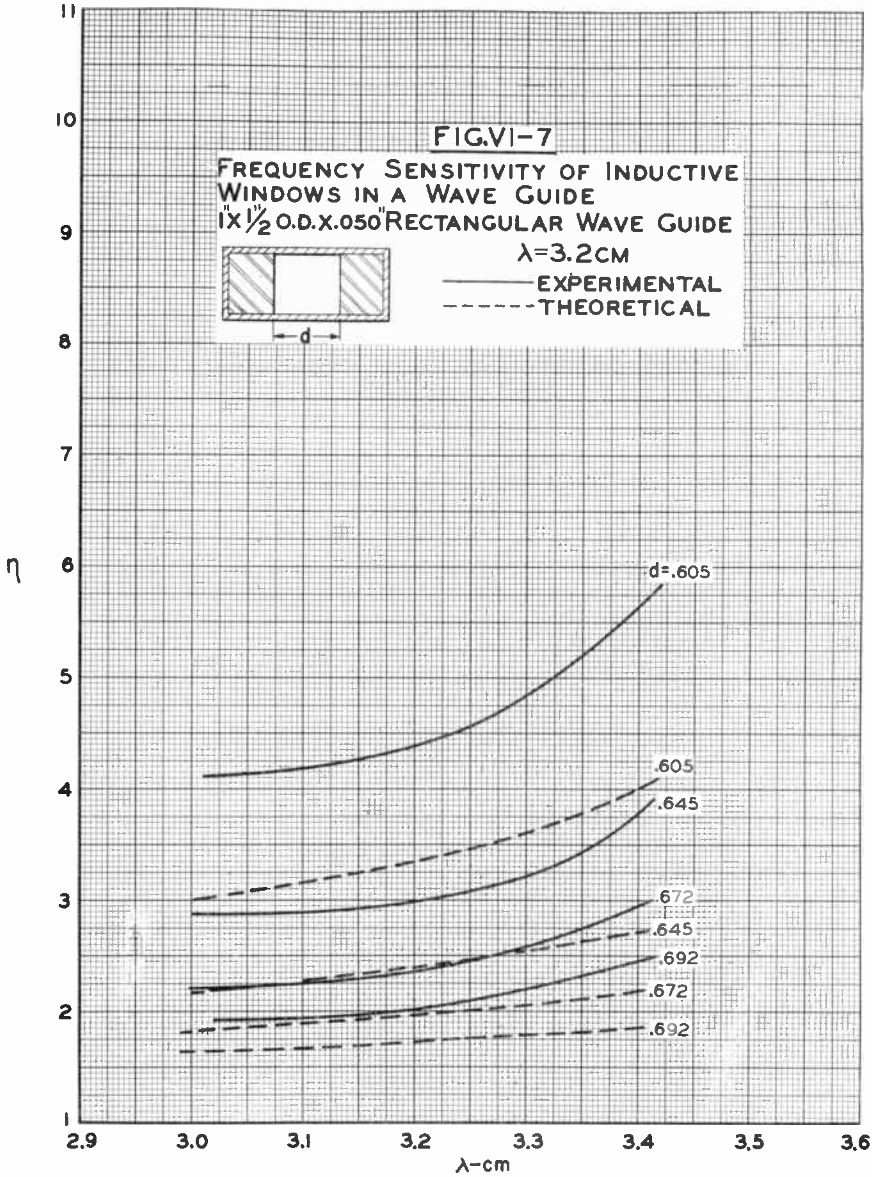
This theoretical formula is plotted in Fig. VI–10, along with some experimental results.[1]

c. Capacitive Windows. When the window opening extends completely across the guide with the sides perpendicular to the electric field, the window acts as a capacitance shunted across the guide. The theoretical value of capacitive susceptance, assuming the window of zero thickness, is

$$B = \frac{4b}{\lambda_g}\log_e \operatorname{cosec}\frac{\pi d}{2b}$$

[1]Measured at Research Laboratories, Sperry Gyroscope Co.

MICROWAVE TRANSMISSION DESIGN DATA



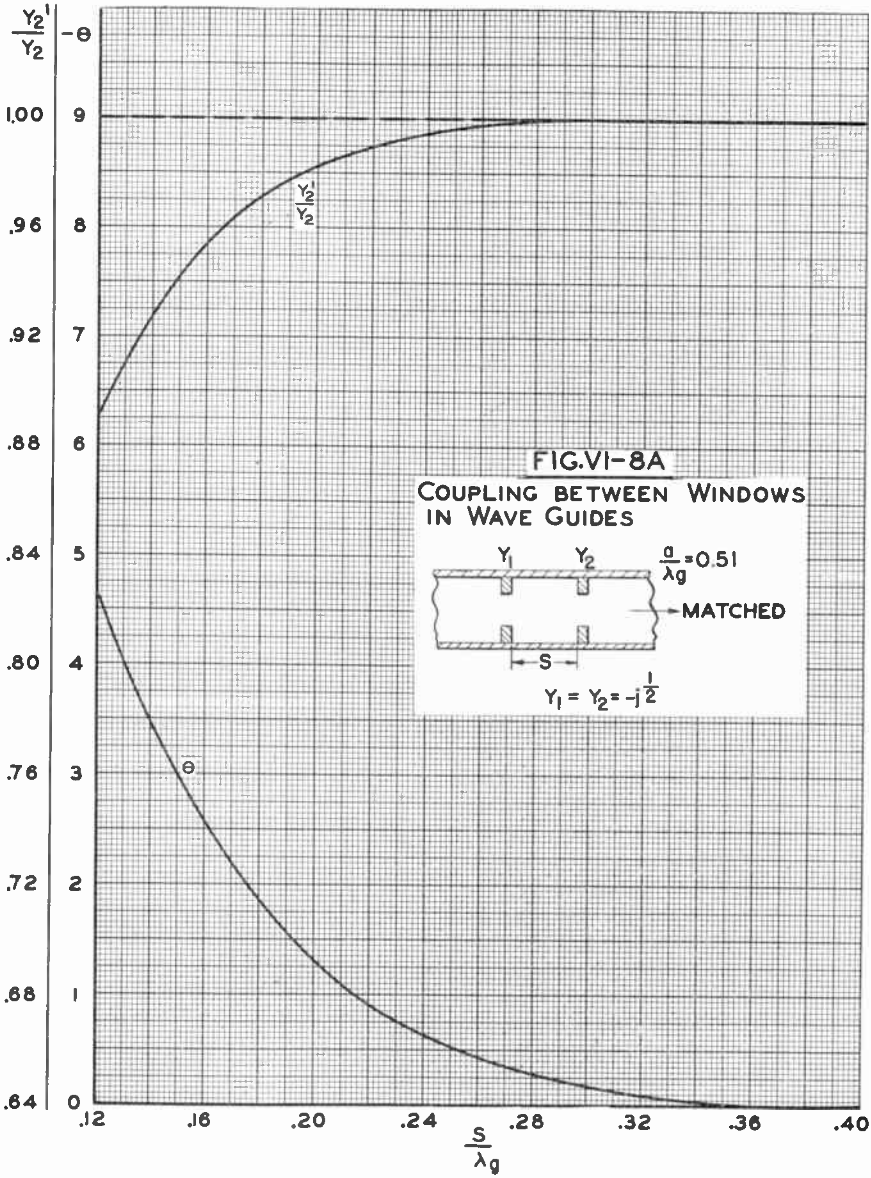

$\frac{Y_2'}{Y_2}$
$-\theta$
1.00
.96
.92
.88
.84
.80
.76
.72
.68
.64
9
8
7
6
5
4
3
2
1
0
$\frac{Y_2'}{Y_2}$
θ
COUPLING BETWEEN WINDOWS IN WAVE GUIDES
Y_1
Y_2
$\frac{a}{\lambda g} = 0.51$
MATCHED
S
$Y_1 = Y_2 = -j\frac{1}{2}$
.12
.16
.20
.24
.28
.32
.36
.40
$\frac{S}{\lambda_g}$

FIG. VI-8A

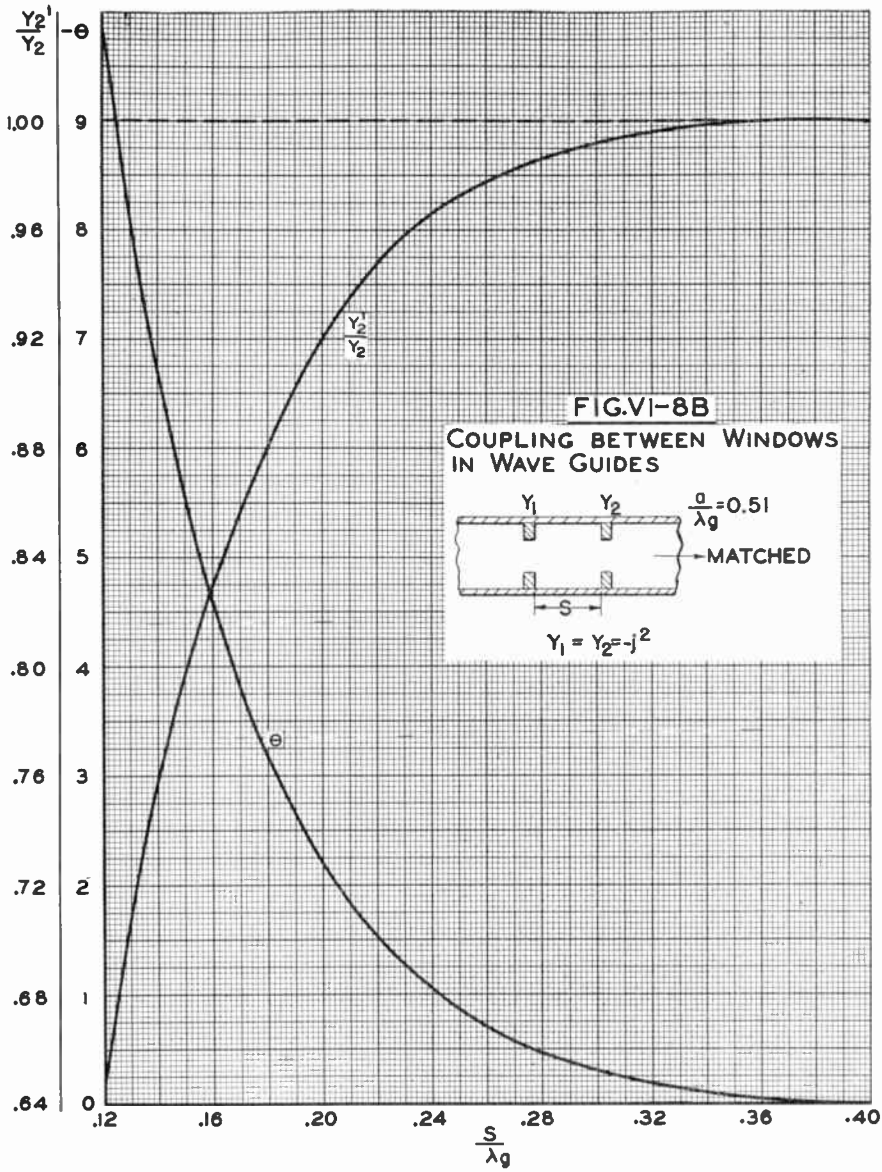
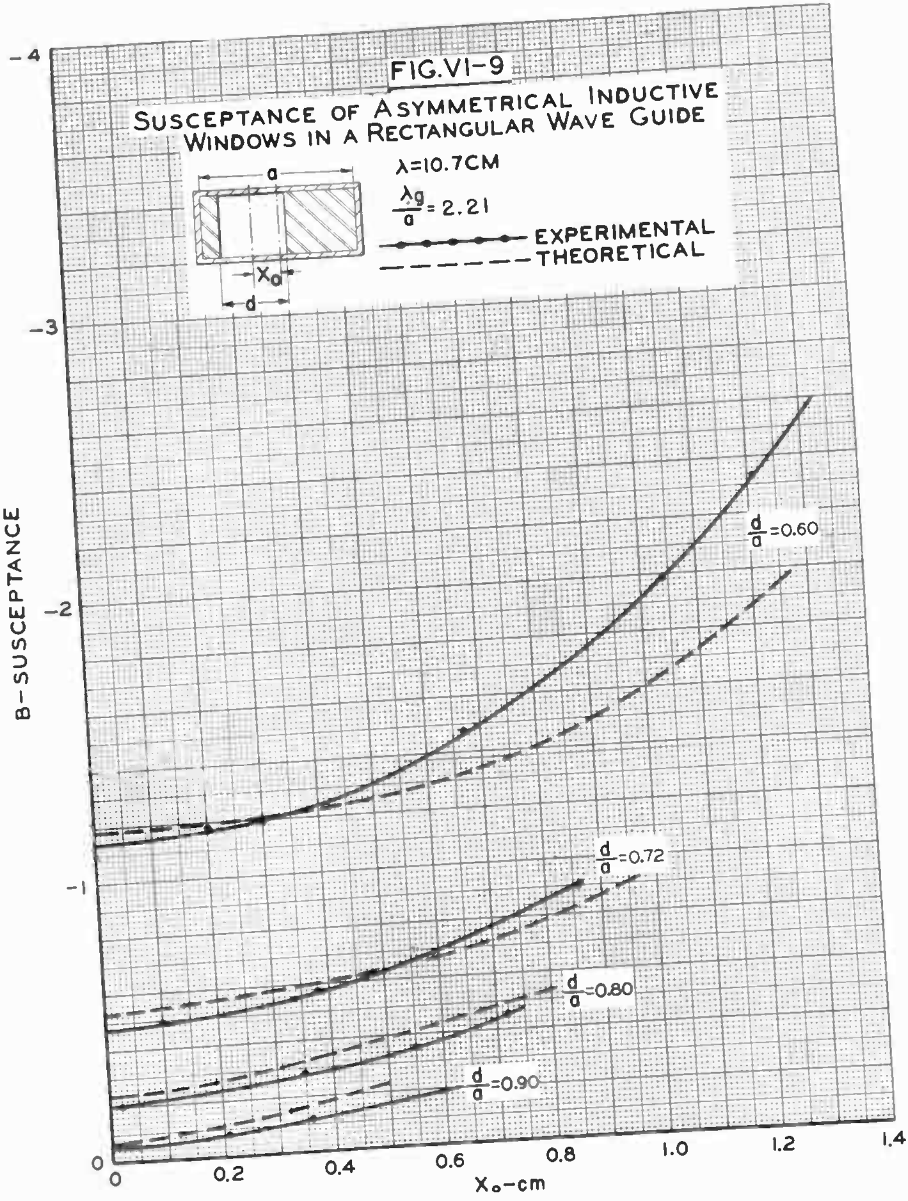


FIG.VI-8B
COUPLING BETWEEN WINDOWS IN WAVE GUIDES
Y_1
Y_2
$\frac{a}{\lambda g}=0.51$
MATCHED
S
$Y_1 = Y_2 = -j2$
$\frac{Y_2'}{Y_2}$
$-\theta$
θ
1.00
.96
.92
.88
.84
.80
.76
.72
.68
.64
9
8
7
6
5
4
3
2
1
0
.12
.16
.20
.24
.28
.32
.36
.40
$\frac{S}{\lambda g}$

FIG. VI-9


SUSCEPTANCE OF ASYMMETRICAL INDUCTIVE
WINDOWS IN A RECTANGULAR WAVE GUIDE
λ=10.7CM
λg/a = 2.21
EXPERIMENTAL
THEORETICAL
a
Xa
d
d/a=0.60
d/a=0.72
d/a=0.80
d/a=0.90
B-SUSCEPTANCE
-4
-3
-2
-1
0
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Xo-cm

where d is the aperture width, b the smaller dimension of the guide, and λ_g the guide wavelength. The thickness of the window has an appreciable effect in the case of the capacitive window. A theoretical expression has been derived giving the susceptance and equivalent shunt conductance of a capacitive window of finite thickness. These expressions are

$$B = B_o + \frac{2\pi W}{\lambda_g}\left(\frac{b}{d} - \frac{d}{b}\right)$$

$$G = \frac{2\pi W}{\lambda_g} B_o \frac{d}{b}$$

where B and G are the shunt susceptance and conductance of a window of thickness W, and B_o is the value given for the window of zero thickness. These values refer to the entrance plane of the window. The two theoretical formulae are plotted in Fig. VI–11 as a function of d/b, along with some experimental results.[1]

The capacitive window is not widely used because the greatly enhanced possibilities of breakdown limit its use to low power systems.

2. Circular Wave Guides

a. Inductive Windows. A circular window centered in a circular wave guide acts as an inductive susceptance for all openings. No theory has been derived as yet for such a window. Some experimental data are presented in Fig. VI–12.

E. *Obstacles in Wave Guides*

1. Tuning Screws. A tuning screw is a cylindrical probe extending into a wave guide parallel to the electric field. The screw acts essentially as a shunting reactance in the guide. The magnitude of the susceptance varies with the depth of the probe in the guide. Short lengths of probe are equivalent to shunting capacities, the susceptance increasing with depth until at a length of approximately a quarter wave in free space a resonance occurs at which substantially all of the incident wave is reflected. For still greater lengths the screw becomes inductive, but in actual applications, it is always used in the capacitive region. The sharpness of the resonance is a function of the diameter of the screw, and higher Q's are found with smaller diameters.

In Fig. VI–13 are plotted the susceptances of two different diameters of tuning screws as a function of the depth that the screw enters into the guide. These measurements were at a wavelength of 3.2 cm., in 1″ x ½″ O.D. x .050″ wave guide.

2. The Inductive Cylindrical Post. If the probe in the above section extends completely across the guide, becoming a cylindrical post, it intro-

[1] Measured at Research Laboratories, Sperry Gyroscope Co.

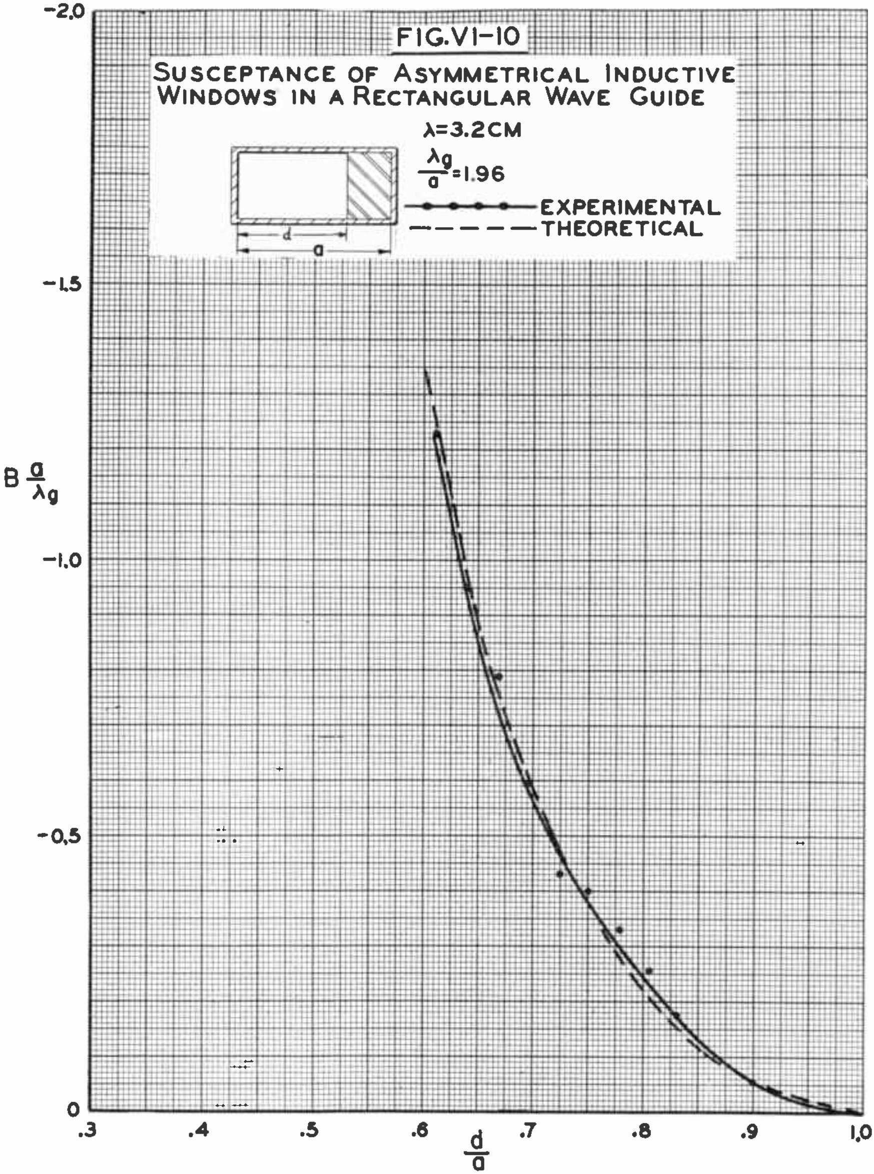


FIG. VI-10
SUSCEPTANCE OF ASYMMETRICAL INDUCTIVE WINDOWS IN A RECTANGULAR WAVE GUIDE
λ=3.2CM
λg/a=1.96
d
a
EXPERIMENTAL
THEORETICAL
-2.0
-1.5
-1.0
-0.5
0
B a/λg
.3
.4
.5
.6
.7
.8
.9
1.0
d/a

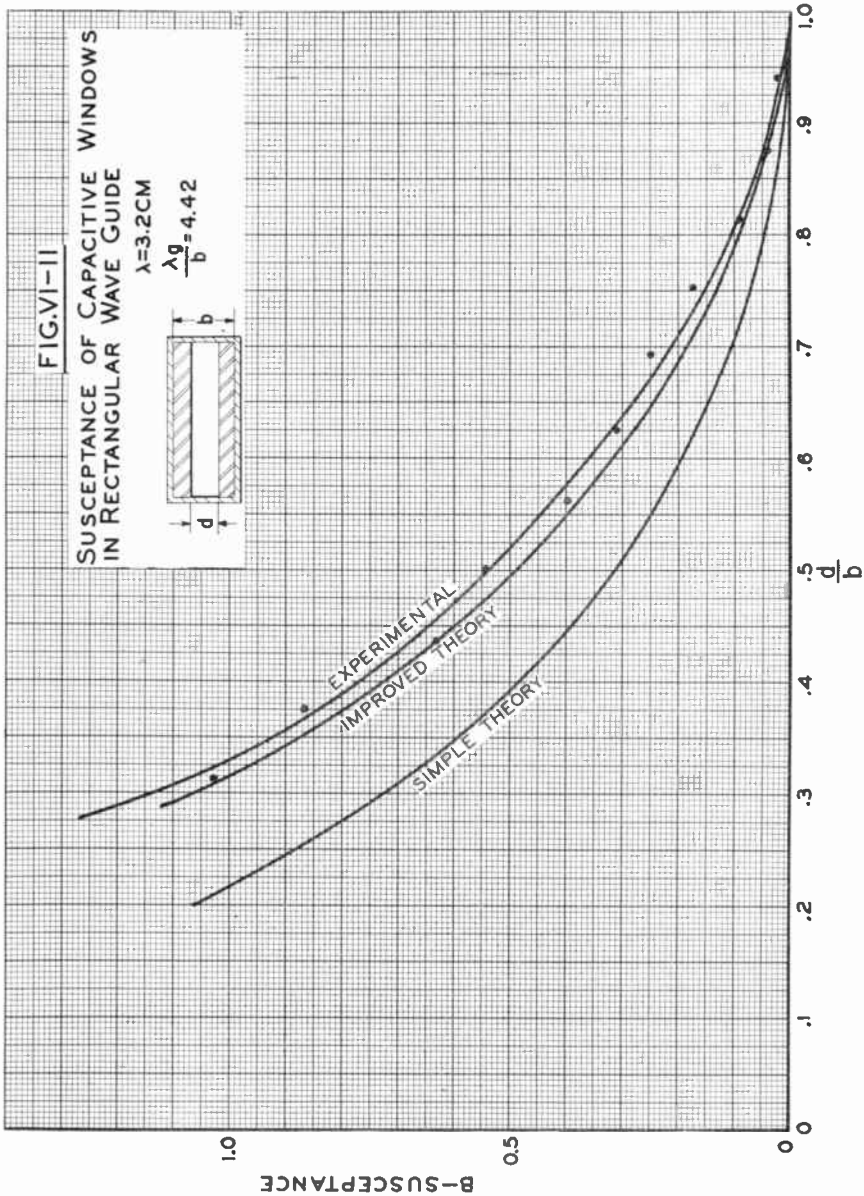


FIG.VI-11
SUSCEPTANCE OF CAPACITIVE WINDOWS IN RECTANGULAR WAVE GUIDE
λ=3.2CM
λg/b = 4.42
b
d
EXPERIMENTAL
IMPROVED THEORY
SIMPLE THEORY
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
d/b
0
0.5
1.0
B-SUSCEPTANCE

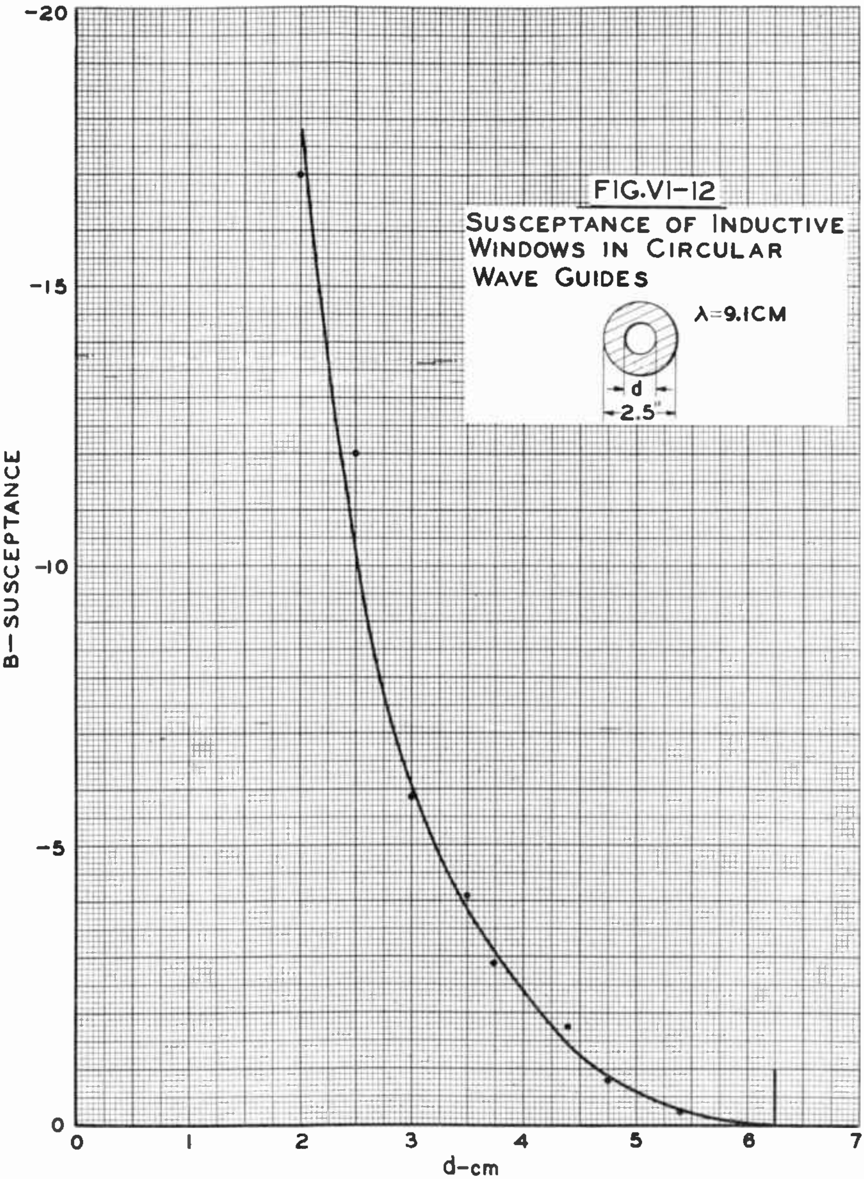


FIG.VI-12
SUSCEPTANCE OF INDUCTIVE WINDOWS IN CIRCULAR WAVE GUIDES
λ=9.1CM
d
2.5"
-20
-15
-10
-5
0
B—SUSCEPTANCE
0
1
2
3
4
5
6
7
d—cm

duces an inductive shunt susceptance whose magnitude depends upon the diameter of the post. A theoretical formula has been derived for this case, giving the susceptance B as

$$B = -\frac{2\lambda_g}{a}\frac{1}{\log_e \dfrac{4a}{\pi d e^2}}$$

where λ_g is the guide wavelength, a the guide width, d the diameter of the post, and e the base of natural logarithms ($e = 2.718$). This formula holds for posts whose diameter is small compared to the width of the guide. This theoretical result is plotted in Fig. VI–14, along with experimental points.

3. The Inductive Strip. If a thin metal strip extends across a rectangular wave guide and is centered with the plane of the strip perpendicular to the axis of the guide, the strip acts as a shunting inductive susceptance. A theoretical expression has been derived for the magnitude of the susceptance as a function of the width of the strip, when the width is small compared to the guide width, as follows:

$$B - 2\frac{\lambda_g}{a}\frac{1}{\log_e \dfrac{8a}{\pi d e^2}}$$

In this formula, λ_g is the guide wavelength, a the larger dimension of the guide, d the width of the strip, and e the base of natural logarithms ($e = 2.718$). A similarity to the formula for the inductive cylindrical post will be noted.

4. The Capacitive Strip. If the strip extends across the guide parallel to the larger dimension of the guide, it acts like a shunt capacitive susceptance. A theoretical formula has been derived for the susceptance of this strip, which holds when the width of the strip is small compared to the small dimension b of the guide. The shunt susceptance is given by

$$B = \frac{\pi^2}{2}\frac{\lambda_g d^2}{\lambda^2 b}$$

where d is the strip width, b the small dimension of the guide, and λ the free space wavelength.

5. The Capacitive Disk in Circular Guide. A thin metal disk that is mounted in the center of a circular wave guide perpendicular to the axis of the guide acts as a shunt capacity across the guide. No theoretical formula has been derived for this disk, but some experimental results are plotted in Fig. VI–15, which give the susceptance as a function of the diameter of the disk.

F. Resonant Structures in Wave Guides

Certain types of structures when placed in a wave guide exhibit resonance phenomena. For example, a diaphragm with an opening of the proper

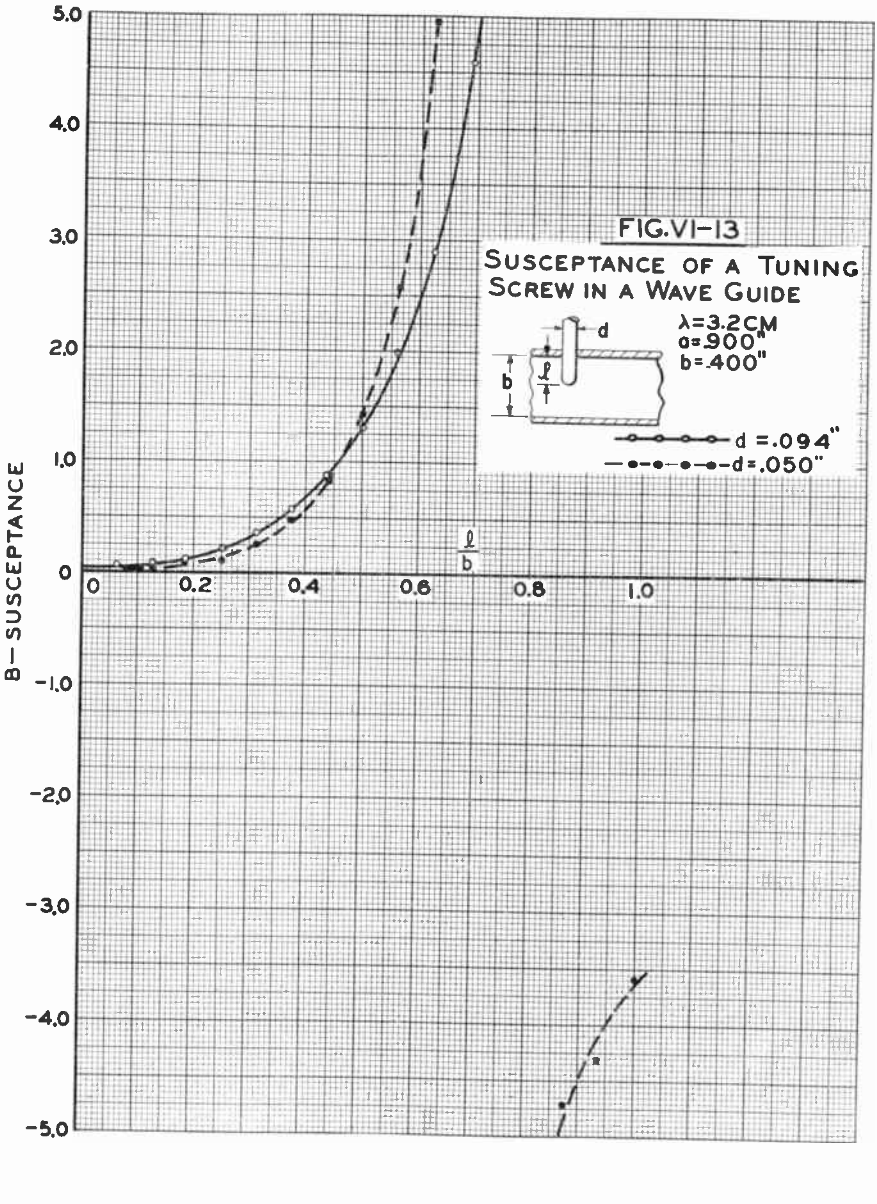


FIG. VI-13
SUSCEPTANCE OF A TUNING SCREW IN A WAVE GUIDE
λ=3.2CM
a=.900"
b=.400"
d
ℓ
b
d =.094"
d=.050"
B – SUSCEPTANCE
ℓ/b
5.0
4.0
3.0
2.0
1.0
0
-1.0
-2.0
-3.0
-4.0
-5.0
0.2
0.4
0.6
0.8
1.0

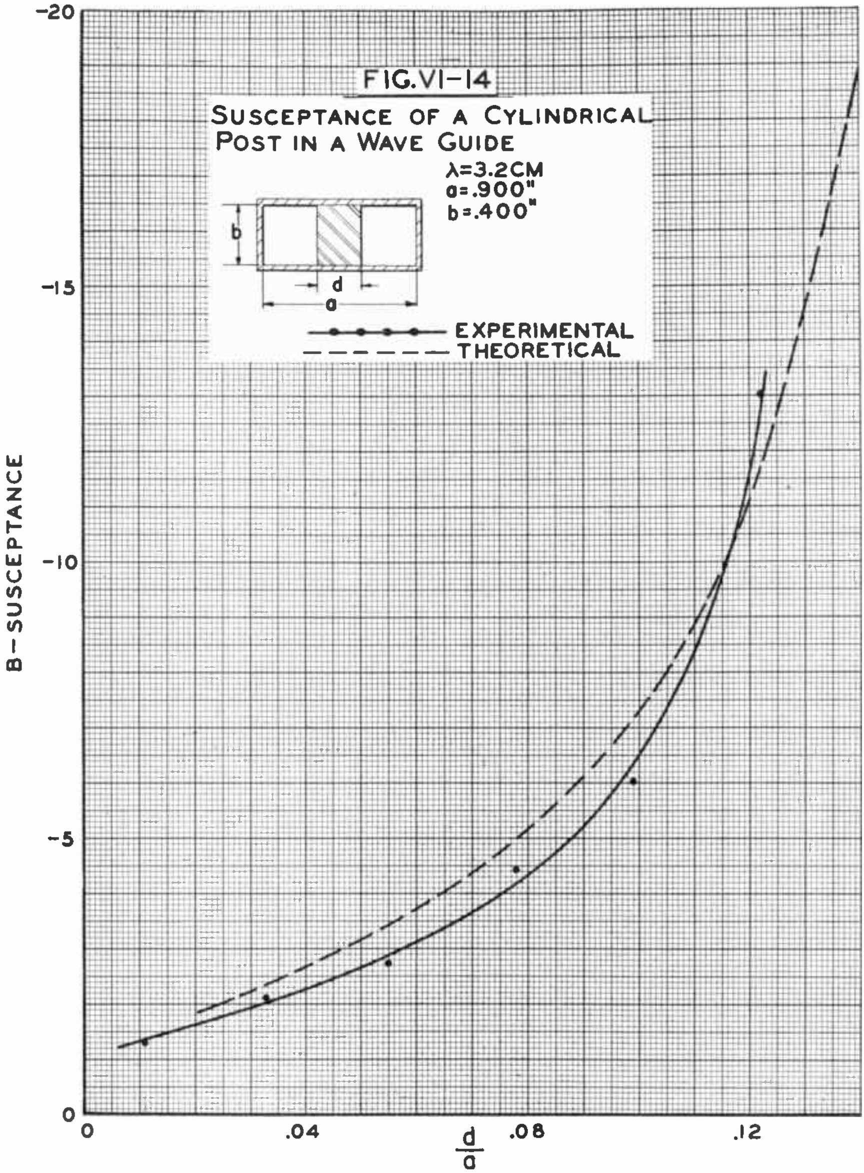


FIG. VI-14
SUSCEPTANCE OF A CYLINDRICAL POST IN A WAVE GUIDE
λ=3.2CM
a=.900"
b=.400"
b
d
a
EXPERIMENTAL
THEORETICAL
-20
-15
-10
-5
0
B-SUSCEPTANCE
0
.04
.08
.12
d/a

size and shape will transmit an incident wave with no reflection at a given frequency, while for openings slightly larger or smaller, the obstacle will reflect part of the incident wave and act as either an inductive or capacitive shunting susceptance. This corresponds to a parallel resonant circuit shunting an ordinary transmission line, which will not disturb the traveling waves when it is at resonance, but will have some effect if slightly off resonance.

Correspondingly, certain obstacles of the proper size and shape when placed in a wave guide will reflect substantially all of an incident wave at a given frequency. An example of this has already been discussed in the case of the tuning screw (Chap. VI—Section E–1) which reached resonance at a length of approximately a quarter of the free space wavelength, and was capacitive if shorter and inductive if longer.

Obstacles of this general type correspond to series resonant circuits shunted across a transmission line, which reflect substantially all of the incident wave at resonance. In many cases, a series resonant obstacle in a wave guide will be similar in size and shape to the opening in a transmitting screen or diaphragm.

1. Resonant Rings. An example of a series resonant structure in a wave guide is the resonant ring. No theoretical formulas for this ring are available, but a number of experimental measurements have been made. A resonant ring is simply a conducting ring of metal which exhibits resonant properties when placed perpendicular to the axis of the wave guide. Both round and rectangular rings exhibit resonance phenomena.

The measured shunt susceptance of a circular ring of square cross section mounted centrally in a circular wave guide is shown as a function of s/λ in Fig. VI–16. In this figure s is the circumference of the ring and λ the free space wavelength, in this case 3.2 cm. Resonance occurs when the circumference is decidedly longer than a free space wavelength.

For a fixed mean diameter $d = .49''$ and a fixed wavelength $\lambda = 3.2$ cm., the susceptance of a resonant ring as a function of the cross sectional dimensions is shown in Fig. VI–17.

The data in Fig. VI–18 give the susceptance of a ring of circular cross-section centrally located in a circular wave guide. The susceptance is given as a function of the diameter. The wavelength was not stated in the original report, but is believed to be about 9.8 cm.

The susceptance of a rectangular resonant ring is plotted in Fig. VI-19 as a function of the mean perimeter of the ring. These measurements were made at a wavelength of $\lambda = 3.2$ cm., in $1'' \times \frac{1}{2}''$ O.D. $\times$ .050″ wall rectangular wave guide. Here again, as in the case of the circular rings, the resonance occurs at a mean perimeter in excess of the free space wavelength.

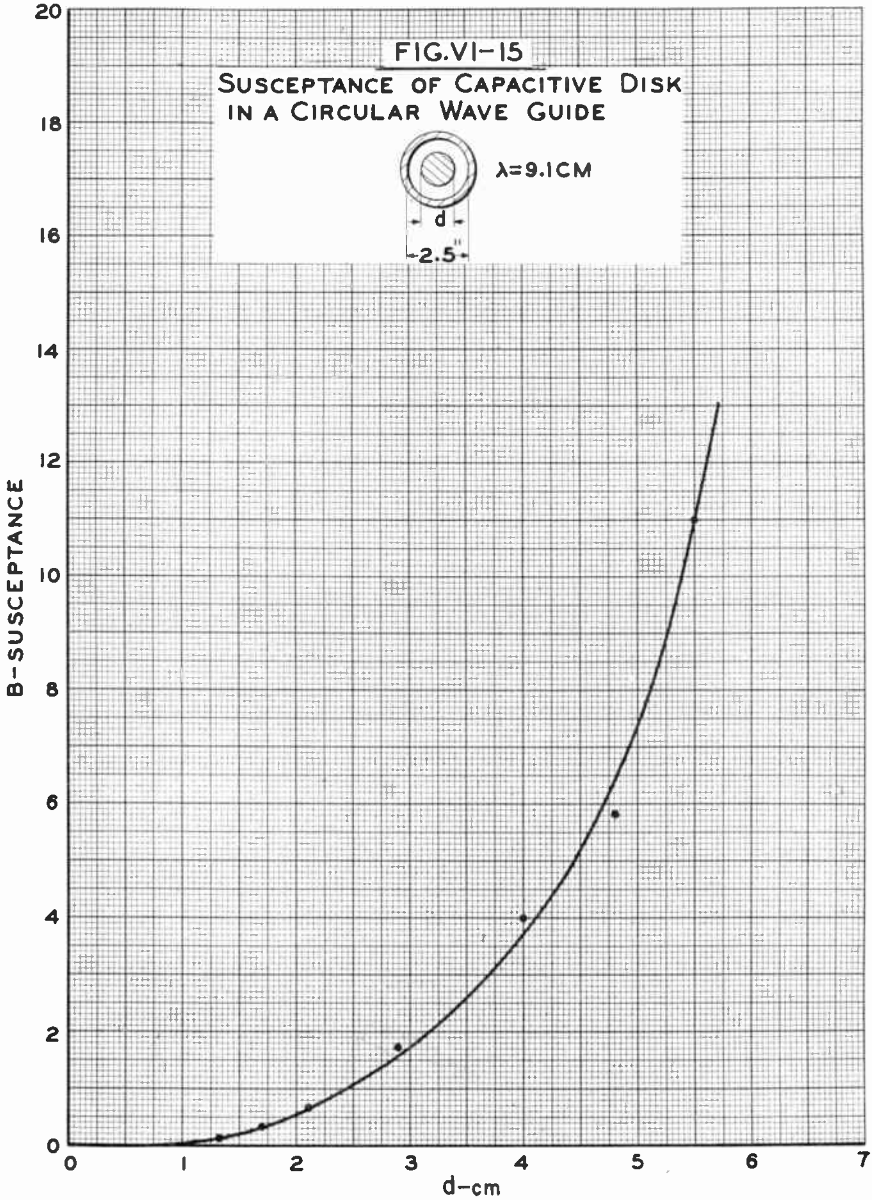


FIG. VI-15
SUSCEPTANCE OF CAPACITIVE DISK IN A CIRCULAR WAVE GUIDE
λ=9.1CM
d
2.5"
B-SUSCEPTANCE
20
18
16
14
12
10
8
6
4
2
0
0
1
2
3
4
5
6
7
d-cm

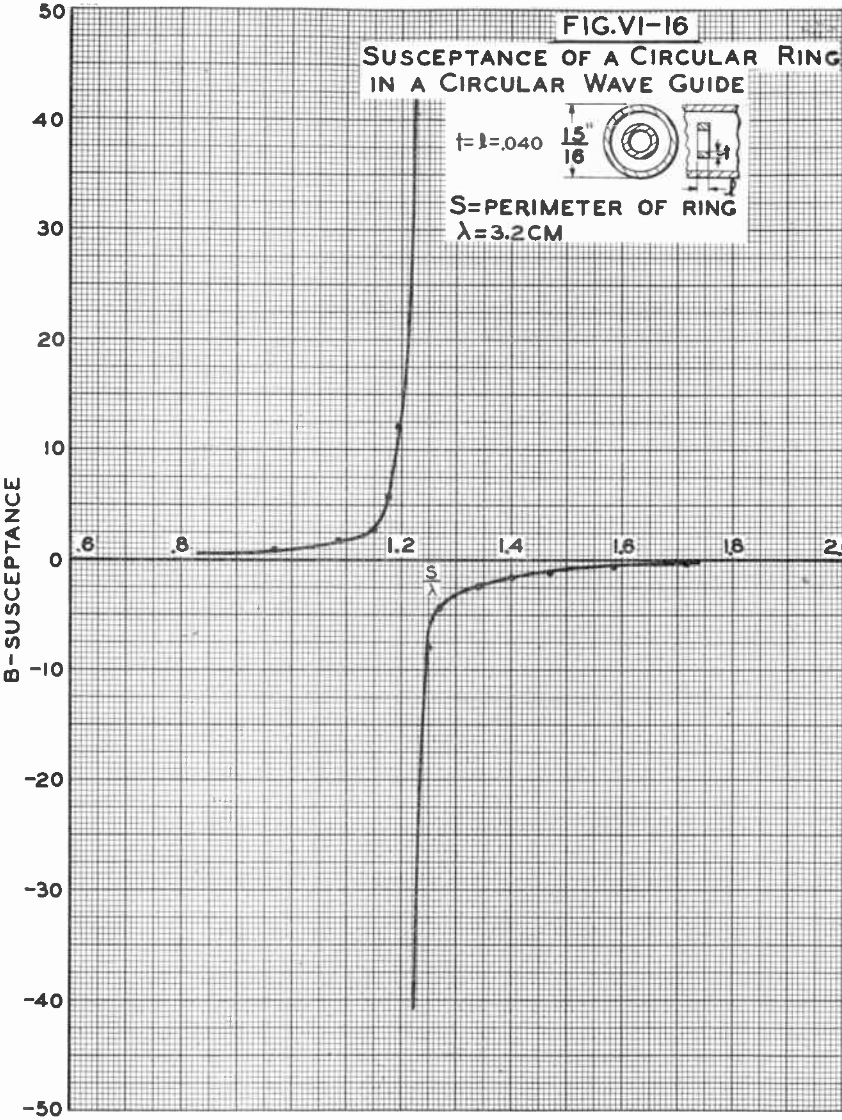


FIG.VI-16
SUSCEPTANCE OF A CIRCULAR RING IN A CIRCULAR WAVE GUIDE
t=l=.040
15/16"
S=PERIMETER OF RING
λ=3.2CM
B-SUSCEPTANCE
S/λ
50
40
30
20
10
0
-10
-20
-30
-40
-50
.6
.8
1.2
1.4
1.6
1.8
2.0

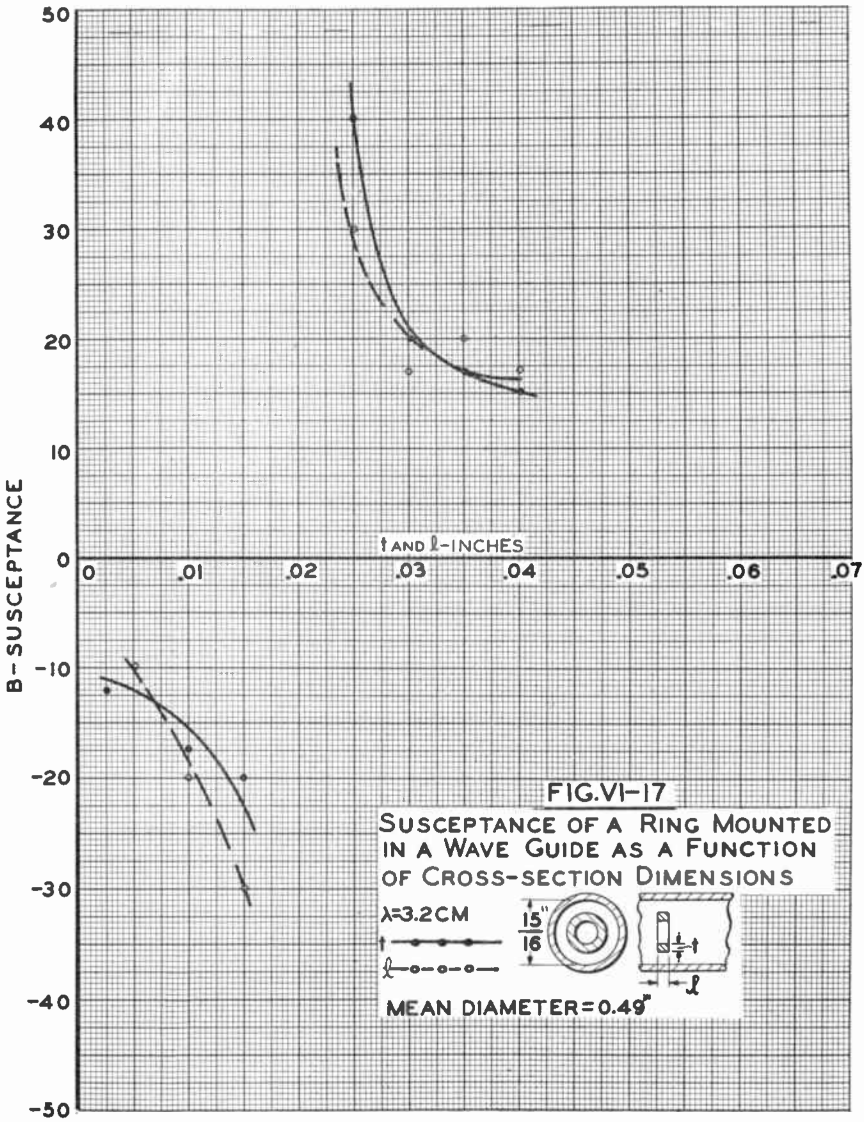


FIG.VI-17
SUSCEPTANCE OF A RING MOUNTED IN A WAVE GUIDE AS A FUNCTION OF CROSS-SECTION DIMENSIONS

2. Transmitting Diaphragms With Rectangular Window Openings. A number of different kinds of openings in diaphragms in wave guides exhibit the properties of a resonant transmitting screen, corresponding to a parallel resonant circuit shunted across a transmission line. The type of screen for which most data are available is a rectangular opening in a thin diaphragm across a rectangular guide. It has been found empirically that the approximate dimensions for resonance for the $TE_{1,0}$ mode are obtainable from the relation

$$\frac{a}{b}\sqrt{1-\left(\frac{\lambda}{2a}\right)^2} = \frac{a'}{b'}\sqrt{1-\left(\frac{\lambda}{2a'}\right)^2}$$

where a and b are the guide dimensions, a' and b' are the dimensions of the opening, with a' being measured parallel to a and b' parallel to b. The free space wavelength is λ. The results expressed in this equation can be represented by the following geometrical construction. In the center of the cross section of the wave guide lay out a line of length $\lambda/2$, parallel to the large dimension of the guide and centered with respect to the walls. Draw a hyperbola passing through the ends of this line and also through the corners of the wave guide. This is illustrated in Fig. VI–20. The approximate dimensions of a resonant rectangular opening are then any rectangle whose corners lie on the hyperbola and whose sides are parallel to the walls of the guide. The experimental results given in Fig. VI–21 provide a check on the validity of the empirical design illustrated in Fig. VI–20. A group of resonant slots were constructed by this design, for $\lambda = 10.7$ cm., the wavelength was then varied about this value until the shunt susceptance of the window was zero. In Fig. VI–21, $\lambda/2a$ is plotted as a function of $\frac{a'}{b'}$, and the experimental points are shown along with the straight line on which they would have fallen had the design been correct. All points fall near but below the line, indicating that the resonant wavelength was less than $\lambda = 10.7$ cm.

3. Resonant Screens and Corresponding Reflectors. A number of differently shaped resonant apertures in diaphragms and the corresponding resonant obstacles have been investigated in round wave guide which can support only the dominant or $TE_{1,1}$ mode. A number of these are illustrated in Fig. VI–22 in which the transmitting screens are placed opposite the corresponding reflecting obstacles.

These measurements are all at a free space wavelength of $\lambda = 9.1$ cm., in a round pipe of inner diameter $d = 2.5''$. A "radiation Q" has been defined for the transmitting screens as

$$Q = \tfrac{1}{2}\frac{\lambda}{\Delta\lambda}$$

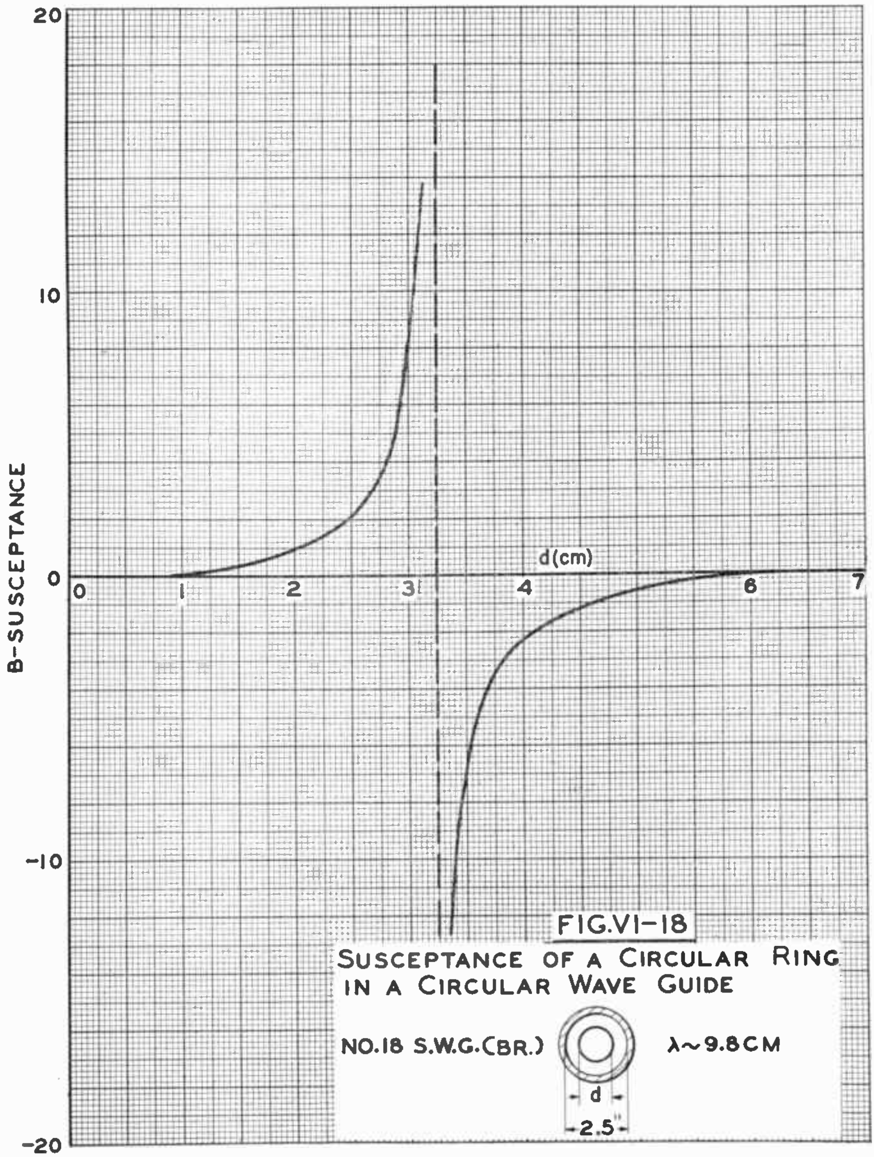


FIG.VI-18
SUSCEPTANCE OF A CIRCULAR RING IN A CIRCULAR WAVE GUIDE

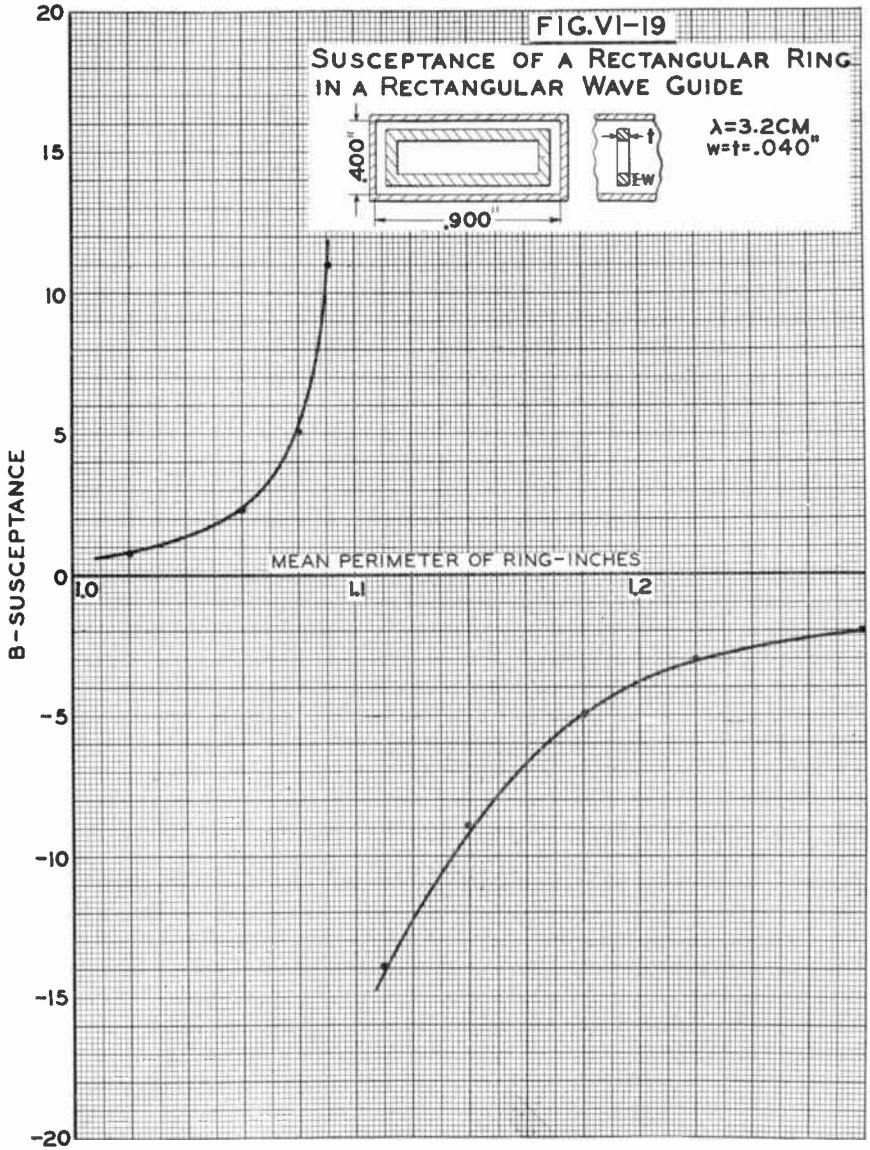


FIG. VI-19
SUSCEPTANCE OF A RECTANGULAR RING IN A RECTANGULAR WAVE GUIDE
.400"
.900"
t
W
λ=3.2CM
w=t=.040"
B-SUSCEPTANCE
MEAN PERIMETER OF RING-INCHES
20
15
10
5
0
-5
-10
-15
-20
1.0
1.1
1.2

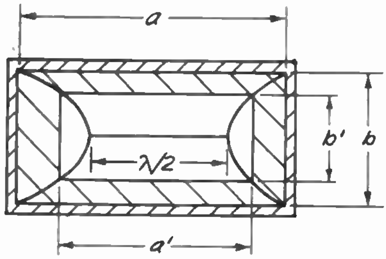


FIG. VI.-20

where λ is the resonant wavelength and $\lambda + \Delta\lambda$ the wavelength at which the susceptance of the screen is equal to unity. The following observations have been made on the screens illustrated in Fig. VI–22.

a. Straight Slit. The slit width was 4% less than λ/2 for resonance.

The *Q*'s were about 25 and 50 for slits ½ mm., wide and .1 mm., wide, respectively.

b. Crescent Slit. The arc length was about λ/2 for resonance and the *Q* for a slit width of .5 mm., was approximately 140.

c. Dumbbell Slit. For a length of straight section equal to 4.2 mm., and radii of the end circles equal to 1.4 mm., the measured *Q* was roughly 9.

d. Circular Slit. The inner circumference of the slit was very nearly λ for resonance.

The variation of *Q* with slit width is shown below:

Slit Width (mm.)	*Q*
0.1	40
0.5	20
0.8	16

In all the above observations, the thickness of the foil used to construct the resonant screens was .004″. Using brass ⅛″ thick increased the *Q*'s by a factor of about 2. For the circular slit the outer diameter had to be increased more than the inner diameter had to be decreased to obtain resonance with slits of various widths. As a typical example, the following values are quoted: 29 mm., I.D.—29.5 mm., O.D.; and 26.5 mm., I.D.—40 mm., O.D.

G. Probe Antennas in Wave Guides[1]

A transition is frequently made from coaxial line to wave guide by having the center conductor of the coaxial line terminate in a probe antenna which radiates energy down the wave guide. The effectiveness of such a transformer is determined by the impedance which the probe termination

[1]See Sperry Gyroscope Company Report 5220-112, Measurements of Impedance of Antennas in Wave Guides.

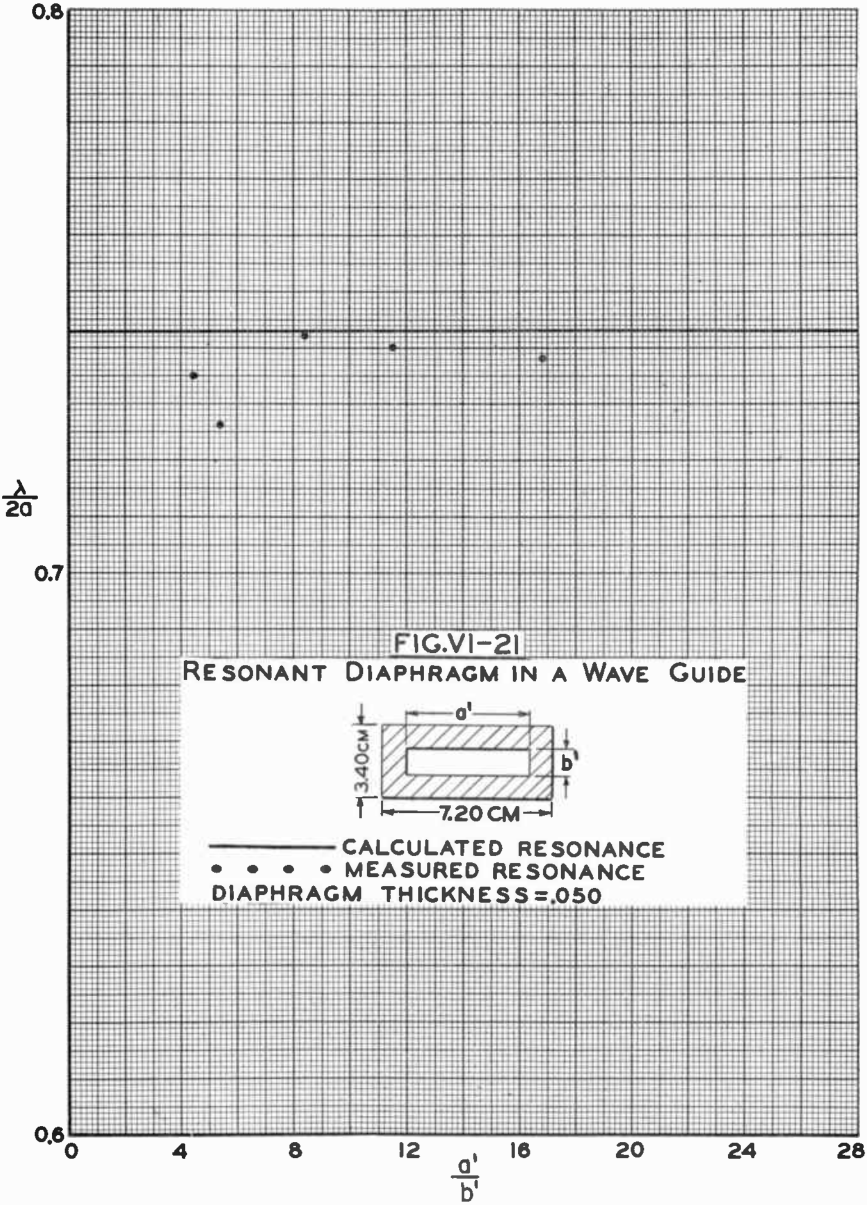


FIG. VI-21
RESONANT DIAPHRAGM IN A WAVE GUIDE

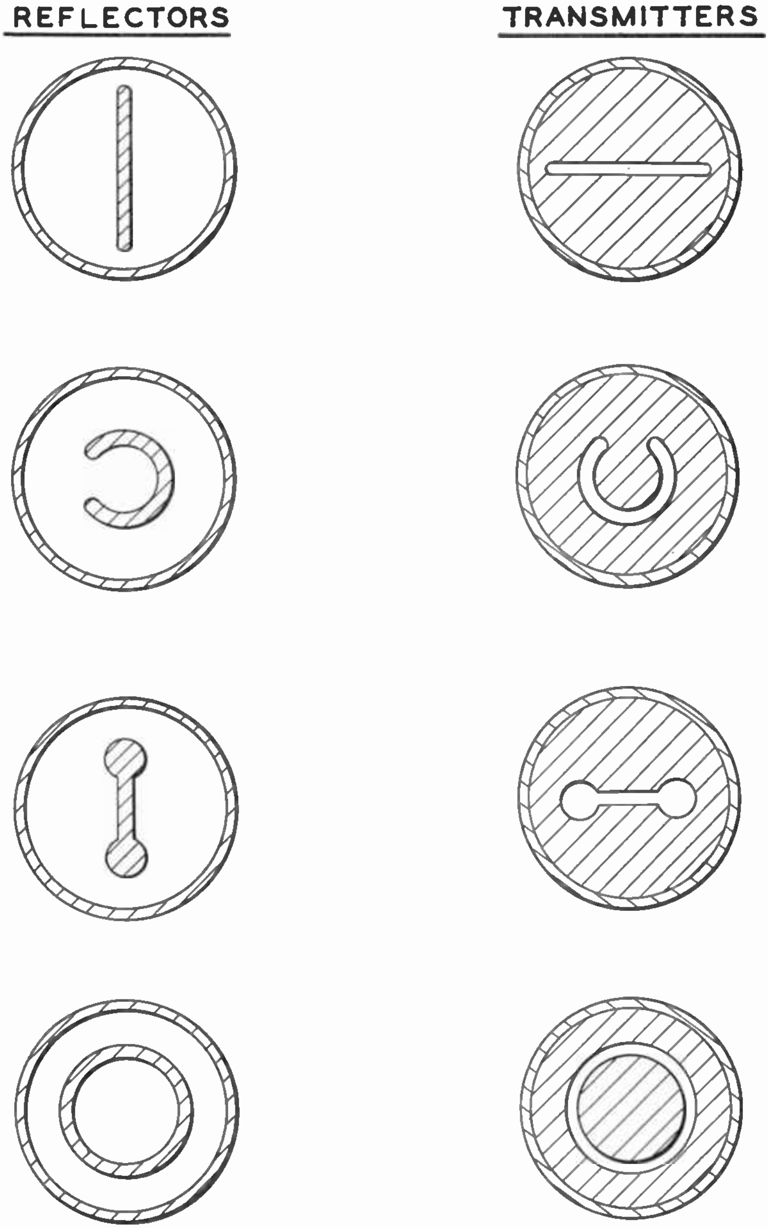


FIG.VI-22

RESONANT STRUCTURES IN CIRCULAR WAVE GUIDE
ELECTRIC FIELD VERTICALLY POLARIZED

of the coaxial line offers to the coaxial line. The impedance of this antenna will be determined by its size and position in the guide, and also the configuration of the guide and the standing waves in the guide.

Measurements have been made in two special cases which are of interest. In both of these, the antenna was an extension into the wave guide of the center conductor of a 70 ohm coaxial line. Two sizes of 70 ohm line were tested: 1. The probe was .040″ in diameter, an extension of the center conductor of a .125″ × 0.40″ coaxial line. 2. The probe was 5/32″ in diameter, an extension of the center conductor of a 1/2″ × 5/32″ coaxial line. All measurements were at a wavelength $\lambda = 3.2$ cm., and the wave guide was 1″ × 1/2″ O.D. × 0.50″ wall. For both lines the impedance was measured with respect to the plane of entry of the antenna into the wave guide. Two types of wave guide loading were investigated.

Case I: The wave guide was terminated at both ends in its characteristic impedance. The antenna impedance was measured as a function of the distance that the antenna extended into the wave guide. The measured reactance and resistance are plotted as a function of antenna length in Fig. VI–23 for the two diameters of antenna.

Case II: One end of the wave guide was terminated in its characteristic impedance, and the other was fitted with a short whose position was variable. The impedance of the antenna was measured as a function of the distance from the antenna to the shorting plug for each of a number of lengths of antenna. The measured resistance and reactance are plotted against the distance from the antenna to the short in Fig. VI–24 for the .040″ diameter antenna and in Fig. VI–25 for the 5/32″ diameter antenna.

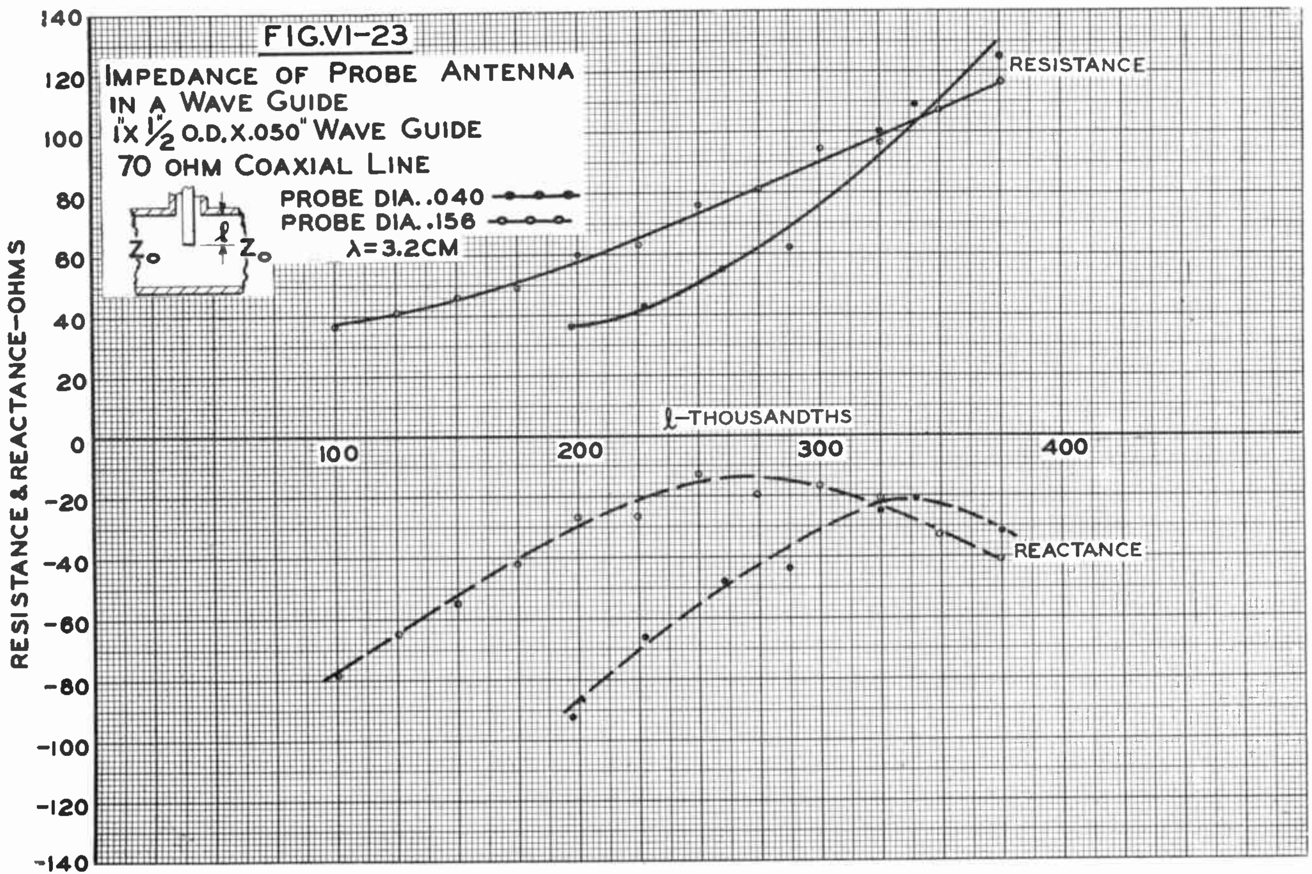


FIG.VI-23
IMPEDANCE OF PROBE ANTENNA IN A WAVE GUIDE
1"X ½" O.D.X.050" WAVE GUIDE
70 OHM COAXIAL LINE

CONFIDENTIAL

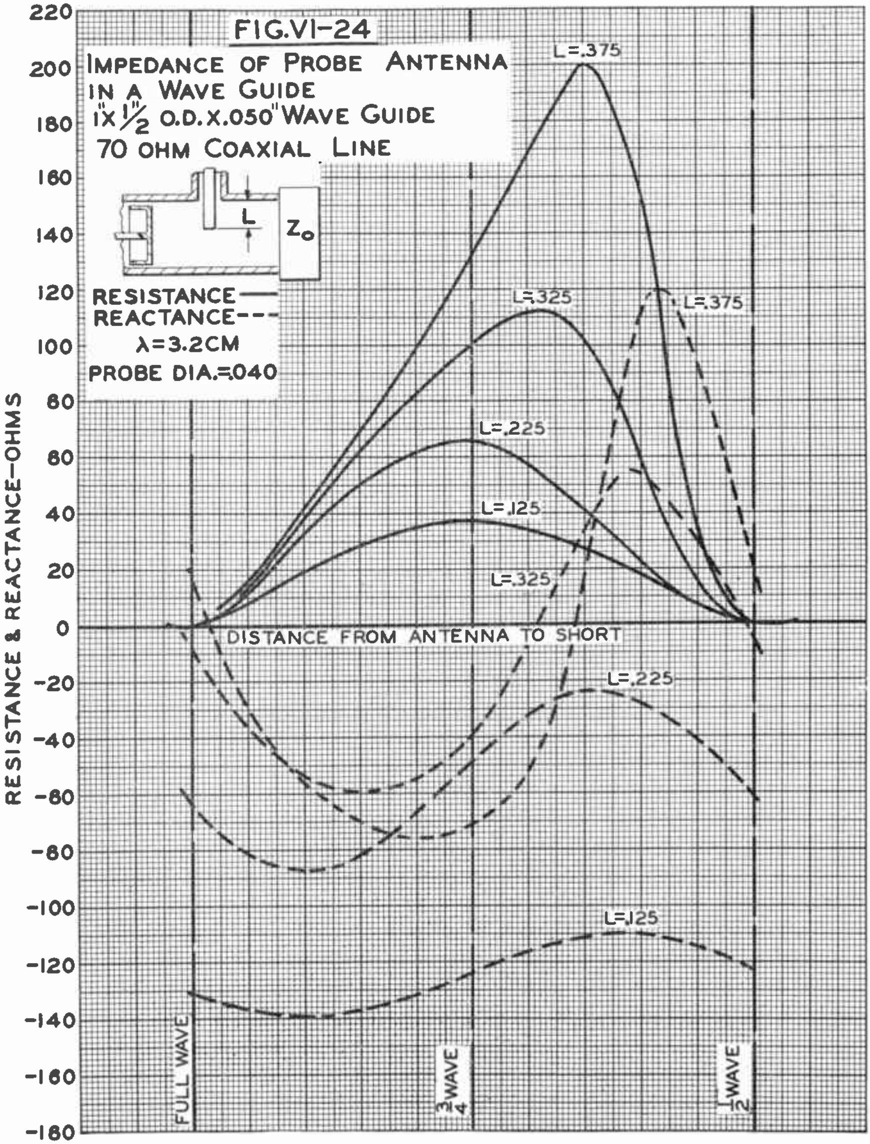


FIG. VI-24
IMPEDANCE OF PROBE ANTENNA IN A WAVE GUIDE
1" X 1/2" O.D. X .050" WAVE GUIDE
70 OHM COAXIAL LINE
L
Zo
RESISTANCE ——
REACTANCE - - -
λ=3.2CM
PROBE DIA.=.040
L=.375
L=.325
L=.225
L=.125
L=.375
L=.325
L=.225
L=.125
DISTANCE FROM ANTENNA TO SHORT
RESISTANCE & REACTANCE—OHMS
220
200
180
160
140
120
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
FULL WAVE
3/4 WAVE
1/2 WAVE

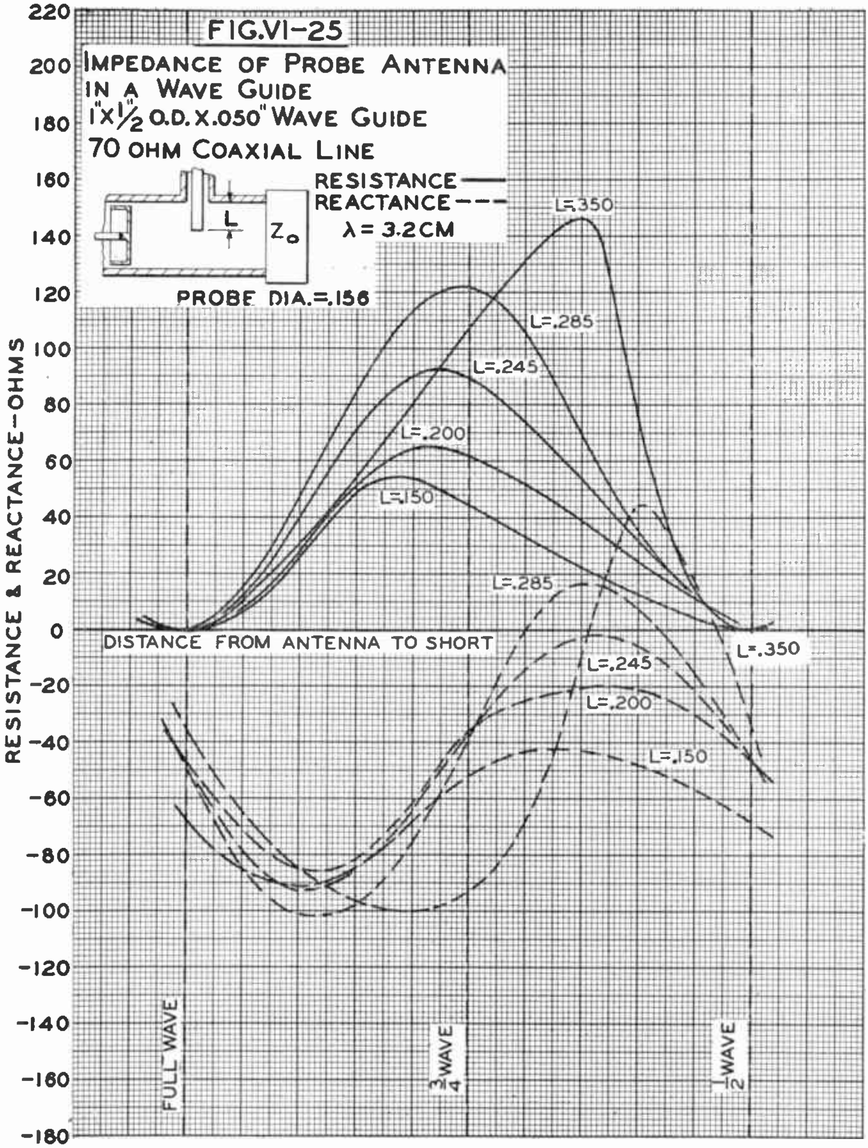


FIG.VI-25
IMPEDANCE OF PROBE ANTENNA
IN A WAVE GUIDE
1"X1½ O.D. X.050" WAVE GUIDE
70 OHM COAXIAL LINE
RESISTANCE ——
REACTANCE - - -
λ = 3.2CM
L
Z₀
PROBE DIA.=.156
L=.350
L=.285
L=.245
L=.200
L=.150
L=.285
L=.245
L=.200
L=.150
L=.350
DISTANCE FROM ANTENNA TO SHORT
RESISTANCE & REACTANCE–OHMS
220
200
180
160
140
120
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
FULL WAVE
¾ WAVE
½ WAVE

CHAPTER VII
WAVE GUIDE TEES AND BENDS

A. *Circular Bends*

The term circular bend as applied here means a wave guide which maintains its internal dimensions as it bends through an angle ϕ on a constant radius. There are three types of wave guide circular bends as shown in Fig. VII–1. Fig. VII–1A shows an E bend in rectangular wave guide, so called because the electric vector is rotated through the angle ϕ in passing through the bend. Fig. VII–1B shows an H bend in rectangular wave guide. This derives its name from the fact that the magnetic vector is rotated. Fig. VII–1C shows a circular bend in circular wave guide. In this case the vector rotated depends on the orientation of the field in the guide.

Theoretically the worst reflection from these bends will be obtained when $\phi = 45°$. Calculations on an E bend with $a = .900''$, $b = .400''$, $r = 1''$, $\lambda = 3.20$, and $\phi = 45°$ give $\eta = 1.03$ where η is the standing wave ratio in power. On a similar H bend at the same wavelength $\eta = 1.02$.

Bends of both types with $a = .900''$, $b = .400''$, $\phi = 90°$ and $r = 1''$ have been tested at $\lambda = 3.20$ cm. On these tests the values obtained were $\eta = 1.1$ for both the E bend and the H bend. Radii of 2″, 3″ and 4″ were also tried and in each case the larger radius was slightly better than the next smaller value.

To obtain values as good as these it is necessary to obtain smooth bends. This is easily accomplished by filling the guide with a low melting alloy before bending and melting this out after the bend is made.

Bends can also be made in circular wave guide but here the wave emerges from the bend elliptically polarized. The phase difference between the field vector parallel to the plane of the bend and the field vector normal to this plane is

$$\theta = \phi \frac{\lambda_g}{r + a} \left[3.64 \left(\frac{a}{\lambda_g} \right)^4 + 0.046 \left(\frac{a}{\lambda_g} \right)^2 - 0.020 \right]$$

where ϕ = angle of bend, r = radius of the inside of the bend, a = radius of guide and λ_g = wavelength in the guide.

For a given angle ϕ, the ellipticity of polarization becomes smaller as the radius of curvature is increased.

The following methods can be used to eliminate the elliptical polarization.

1. In the above equation if we consider only the first term, then $\theta \sim \dfrac{\phi}{r + a}$ where ϕ is the angle the bend goes through and $r + a$ is the radius of curvature of the center line of the wave guide. If two bends are used in

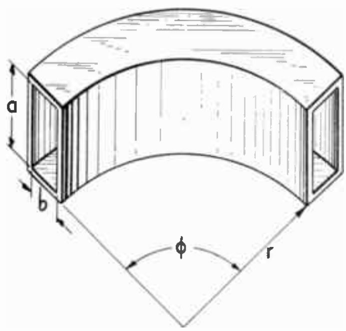


FIG.VII-1A

E-BEND

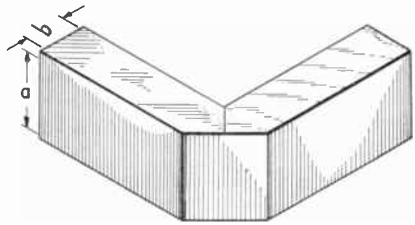


FIG.VII-2A

E-BEND CORNER

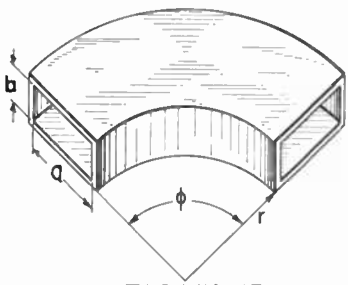


FIG.VII-1B

H-BEND

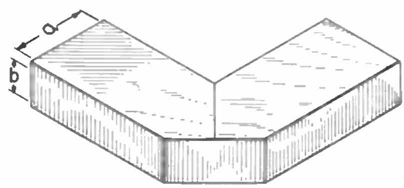


FIG.VII-2B

H-BEND CORNER

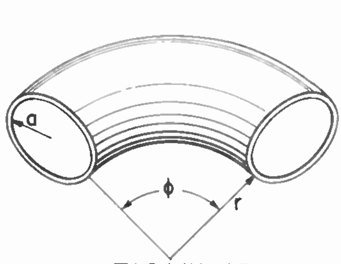


FIG.VII-1C

BEND IN CIRCULAR WAVE GUIDE

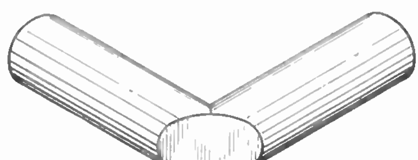


FIG.VII-2C

CORNER IN CIRCULAR WAVE GUIDE

series the polarization caused by the first will be cancelled by the second if

$$\frac{\phi_1}{r_1 + a} = \frac{\phi_2}{r_2 + a}$$

If r_1 and r_2 are different, then ϕ_1 and ϕ_2 will be different and there will be a net curvature of $\phi_1 - \phi_2$ without elliptic polarization.

2. Two identical bends can be placed in different planes so as to cancel the phase differences while at the same time changing the direction of the wave.

3. The ellipticity caused by a bend can be compensated by deforming the pipe to produce an elliptical cross section oriented in the proper manner.

4. A piece of dielectric material may be inserted in the wave guide to correct the elliptical polarization. This is in the form of a strip extending lengthwise in the wave guide. The length and thickness as well as the angle at which this sits in the wave guide must all be correct to neutralize the phase differences.

Although the above methods are all theoretically possible it is evident that successful results with any one of them will be hard to obtain in actual practice.

B. *Corners*

As with the circular bends, wave guide corners also consist of three types; *E*-bend in rectangular wave guide (Fig. VII–2A), *H*-bend in rectangular wave guide (Fig. VII–2B) and corner in circular wave guide (Fig. VII–2C).

In contrast to bends which follow a smooth curve, corners change direction abruptly. It is evident that any such sharp change in direction causes reflections and consequent standing waves. By inserting a plane directly at the corner as in Fig. VII–2 it is possible to eliminate these reflections. Fig. VII–3 and Fig. VII–4 give data for the positioning of this plane to give a standing wave ratio of 1.0. The particular data shown come from measurements made at $\lambda = 10.84$ cm. in rectangular wave guide 7.0 cm. by 3.25 cm. Using $\lambda = 3.20$ cm. in a .900″ × .400″ guide the optimum dimensions for $\phi = 90°$ are $d = .325''$ for the *E* bend and $d = .830''$ for the *H* bend. These give values of $\dfrac{d}{d_o} = .57$ and .61 respectively.

No data is available at the present time on the circular guide corner. Reflections could be eliminated by a similar plane at the corner but undoubtably elliptical polarization would occur. This, theoretically, could be corrected by some method such as deforming the pipe but in actual practice good results would be hard to obtain.

C. *Twists*

Extensive tests have not been carried out on wave guide twists because no difficulty has been experienced with these units. If one wishes to keep η less than 1.1, all that is necessary is to keep the length l over 2λ for a 90°

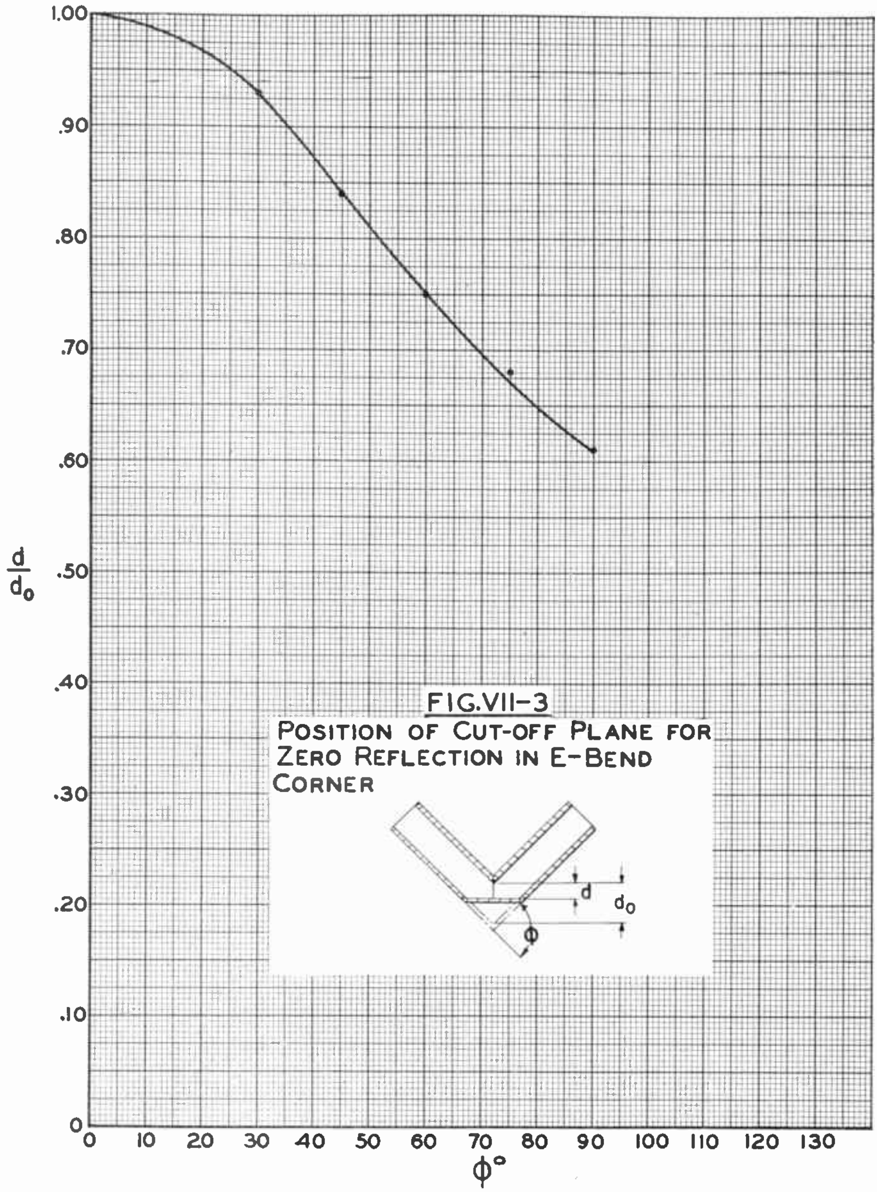


FIG.VII-3
POSITION OF CUT-OFF PLANE FOR ZERO REFLECTION IN E-BEND CORNER

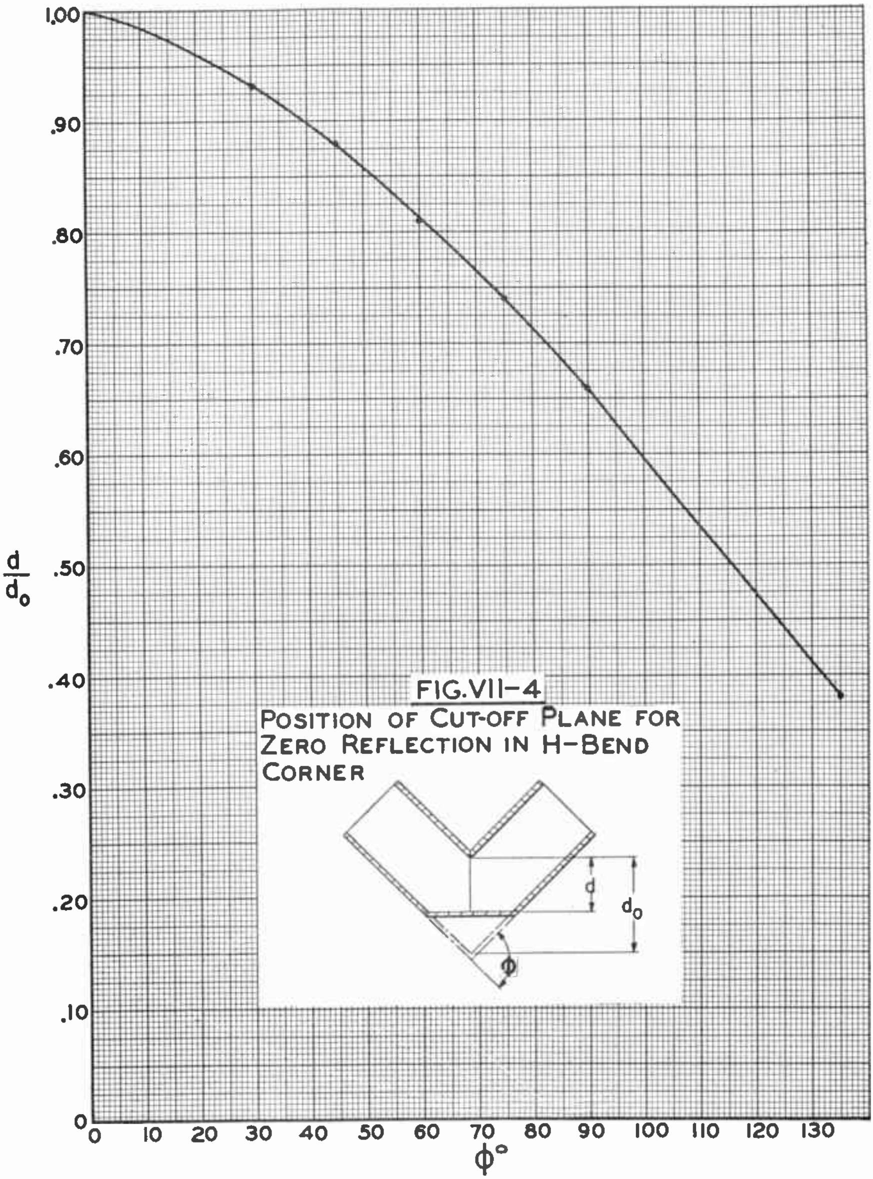


FIG. VII-4
POSITION OF CUT-OFF PLANE FOR ZERO REFLECTION IN H-BEND CORNER

twist (Fig. VII–5). Shorter twists are not only harder to make but cause more reflections.

D. *Tee Sections*

A tee section as considered here consists of a straight section of wave guide from which there branches at right angles another section of wave guide. This may be either rectangular or round wave guide. Only the rectangular type is discussed here. In the rectangular wave guide there are two types of tees. The series tee has the right angle leg branching off the larger dimension (Fig. VII–6A). The shunt tee has the right angle leg branching off the smaller dimension (Fig. VII–6B).

Data are included (Figs. 7 through 24) showing the impedance looking into both series and shunt tees made up of .900″ × .400″ I.D. wave guide operating at $\lambda = 3.20$ cm. This was obtained looking into the tee in the direction of the arrow with a termination Z_o in one leg and a short in the third leg. The impedance is given with respect to the center line of the branching leg. Two types of curves are given. The first (Figs. VII–7 through VII–12) are circle diagrams of the type discussed in Chapter III Section A–5. The values on one family of circles equal the standing wave ratio in voltage ($\sqrt{\eta}$). The values on the other, orthogonal family of circles are the distance in electrical degrees that the center line of the branching leg is from a voltage node (for admittance diagrams) or a voltage loop (for impedance diagrams). The actual impedances (taken from the circle diagrams) are shown in Figs. 13 through 18. These are plotted as a function of the distance between the short and the inside of the wave guide.

The curves are self-explanatory (see Chapter III, Section A) and from them the impedance may be determined for any position of the short. The experimental data on series tees shown here agree very closely with theory that has been developed. It will be noted from the circle diagrams that only the cases shown in Figs. VII–8 and VII–11 give a good match without screw or window matching of some type.

The data shown in Figs. 19 through 24 were obtained in .900″ × .400″ I.D. wave guide by varying the wavelength from 3.00 to 3.60 cm. In this case the short was moved until the minimum standing wave ratio was obtained and the position of the node was measured. η shows the minimum standing wave ratio in power. $\frac{L}{\lambda_g}$ gives the position of the short necessary to obtain the minimum standing wave ratio and $\frac{D}{\lambda_g}$ shows the node position with respect to the center line of the branching leg. In the cases shown in Figs. VII–19 and VII–22 the minimum $\eta = 1.0$ so no standing wave is present.

WAVE GUIDE TEES AND BENDS

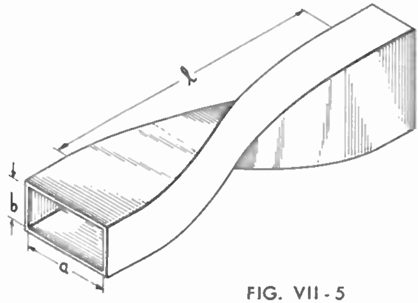


FIG. VII - 5
WAVE GUIDE TWIST

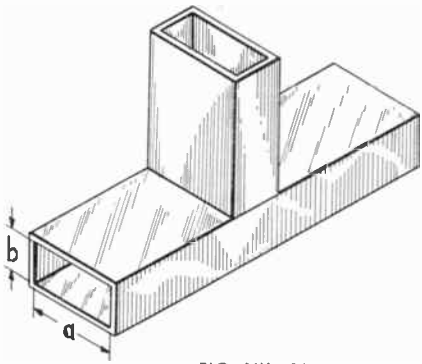


FIG. VII - 6A
SERIES TEE

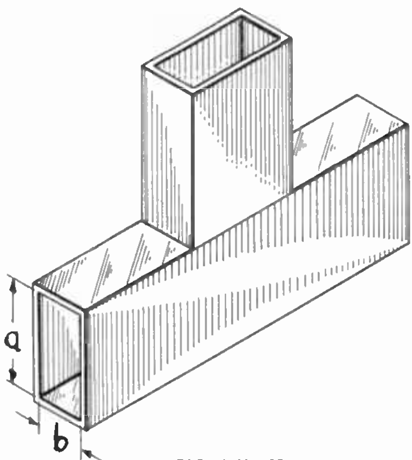


FIG. VII - 6B
SHUNT TEE

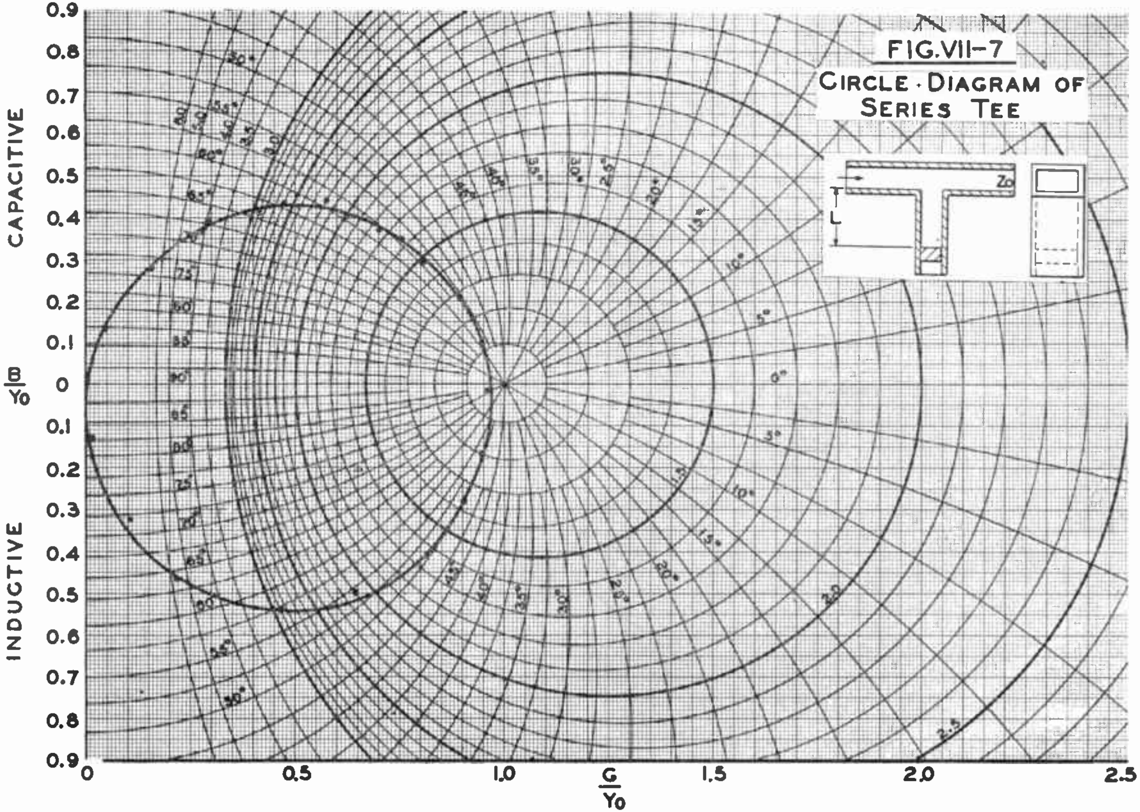


FIG.VII-7
CIRCLE · DIAGRAM OF SERIES TEE

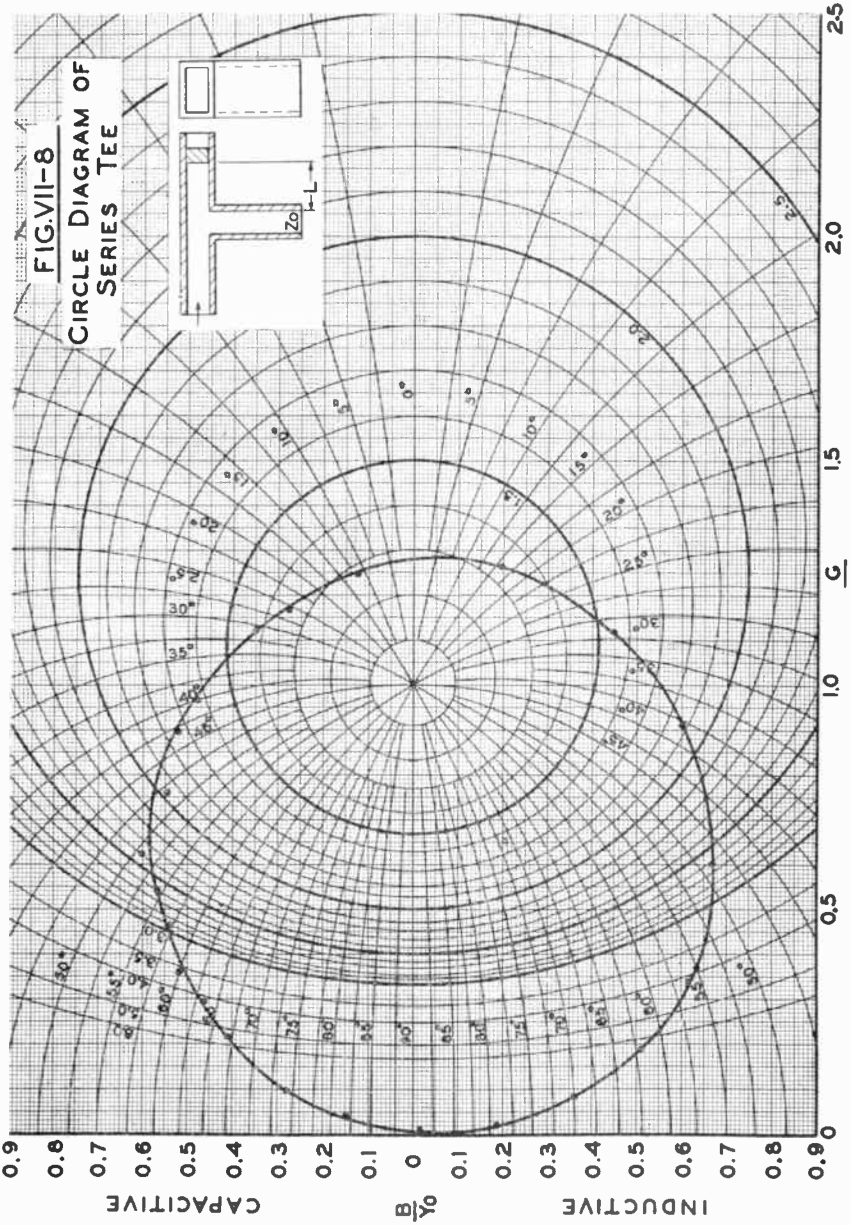


FIG.VII-8
CIRCLE DIAGRAM OF
SERIES TEE
Zo
L
$\frac{G}{Y_0}$
0
0.5
1.0
1.5
2.0
2.5
$\frac{B}{Y_0}$
CAPACITIVE
INDUCTIVE
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0

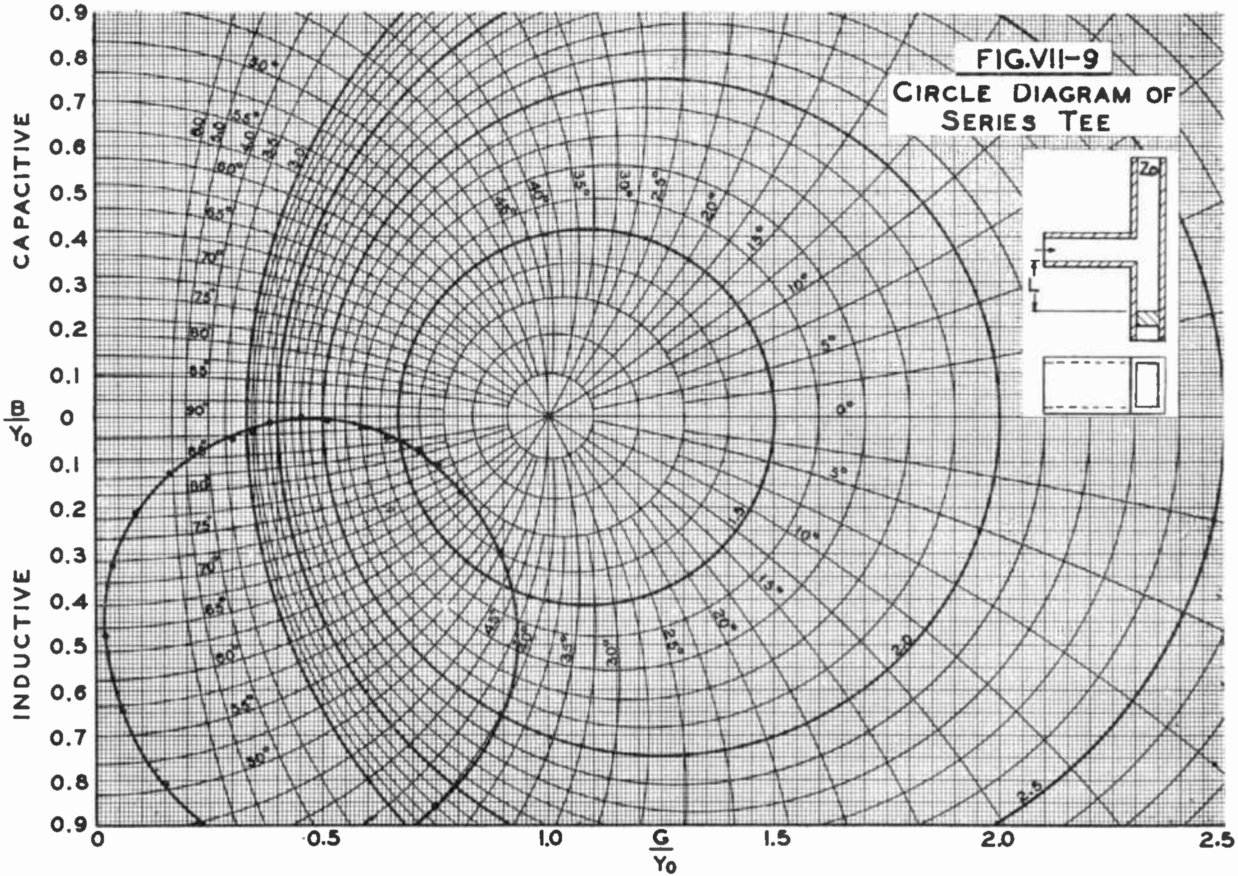


FIG.VII-9
CIRCLE DIAGRAM OF SERIES TEE

CONFIDENTIAL

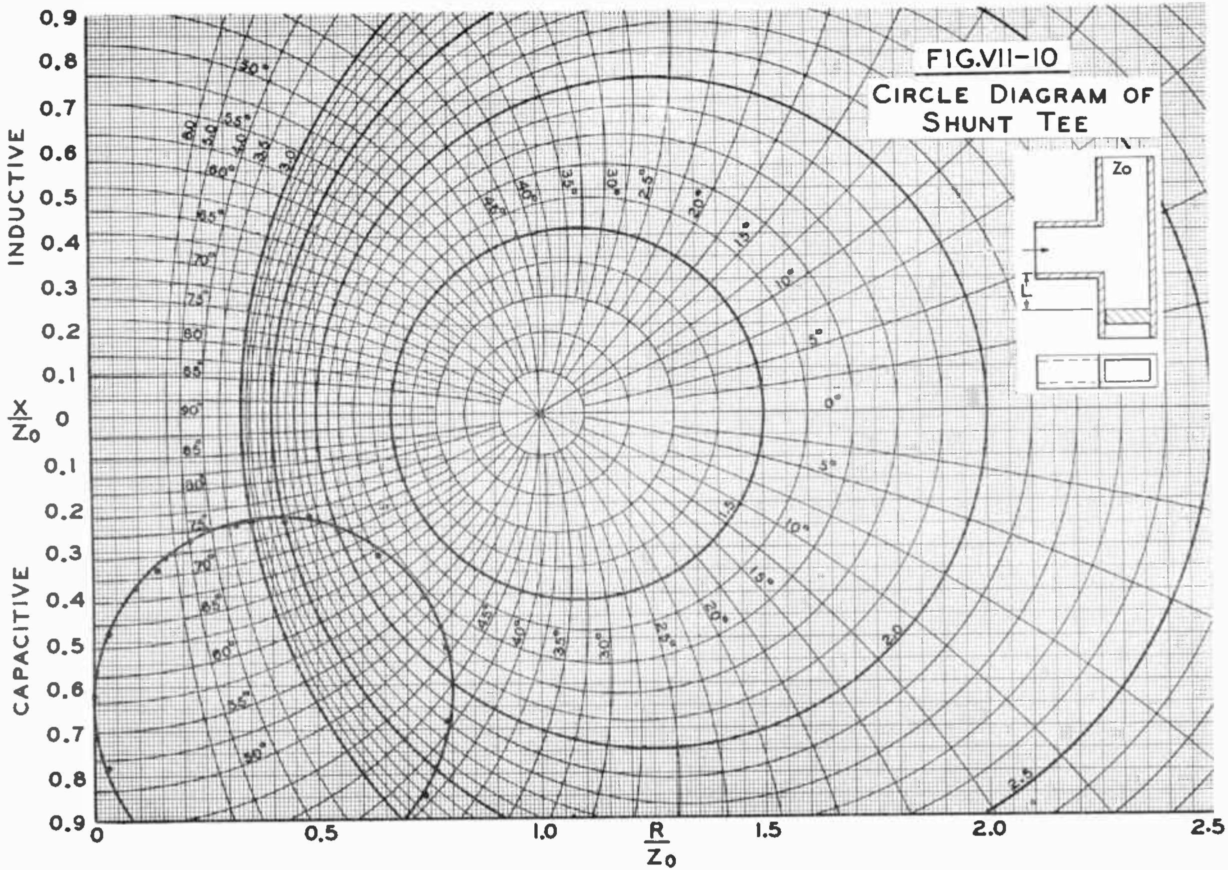


FIG. VII-10
CIRCLE DIAGRAM OF SHUNT TEE

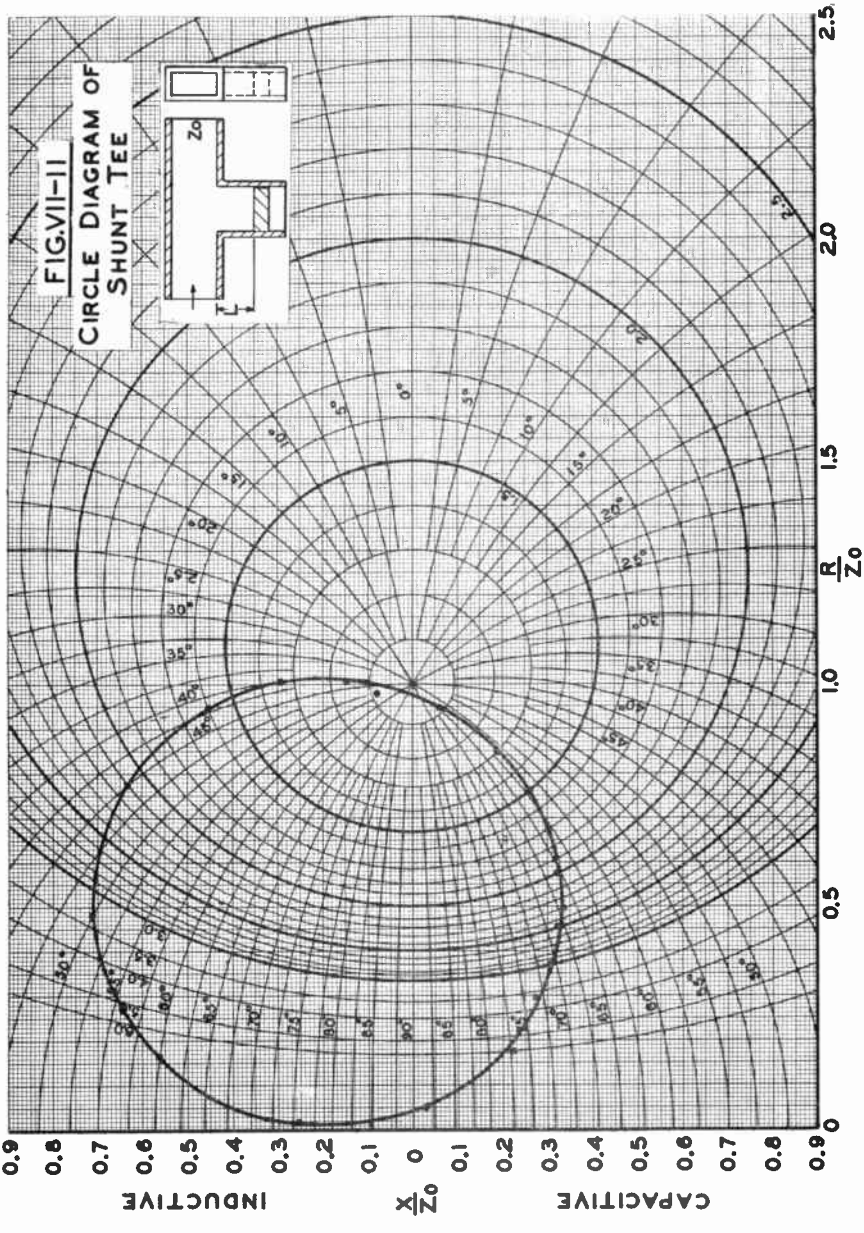


FIG.VII-11
CIRCLE DIAGRAM OF SHUNT TEE
Zo
L
R/Zo
X/Zo
INDUCTIVE
CAPACITIVE
0
0.5
1.0
1.5
2.0
2.5
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

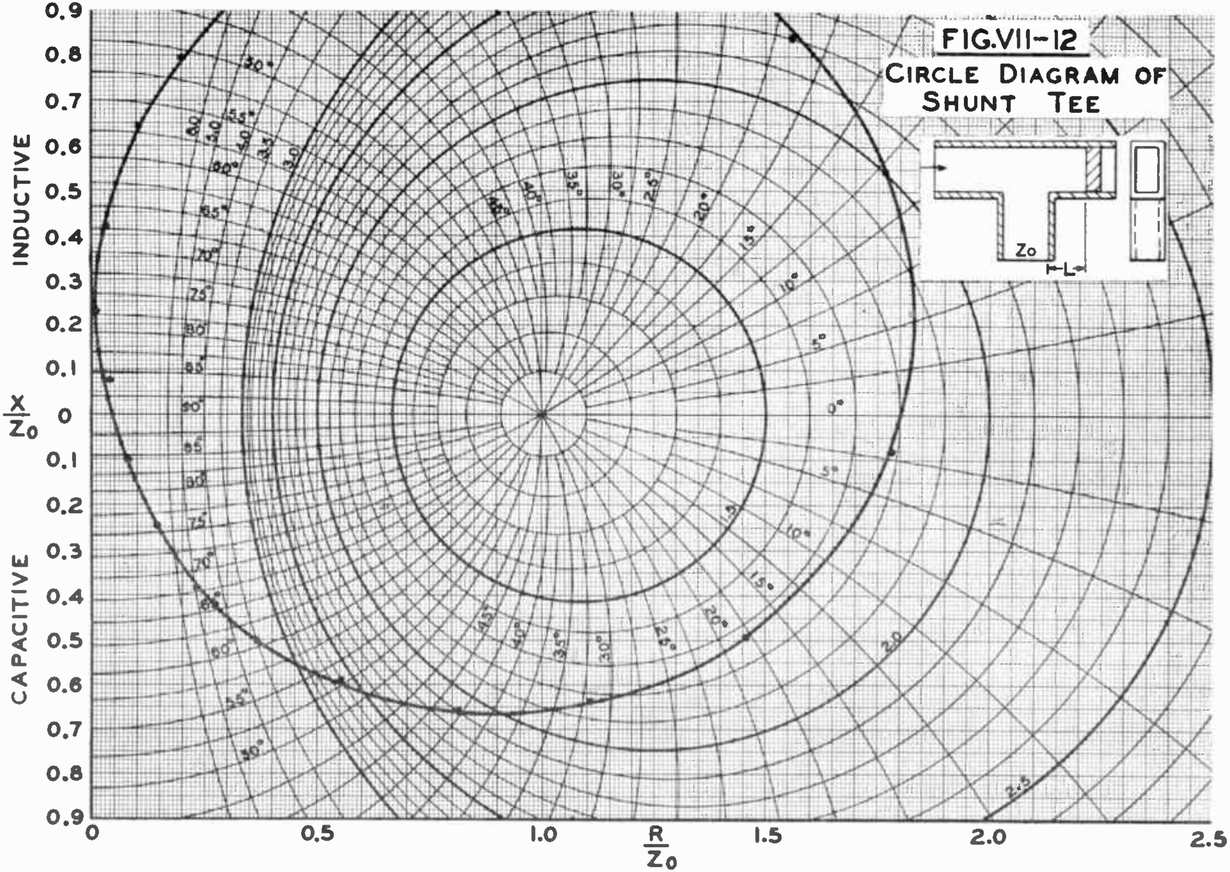


FIG. VII-12
CIRCLE DIAGRAM OF SHUNT TEE

CONFIDENTIAL

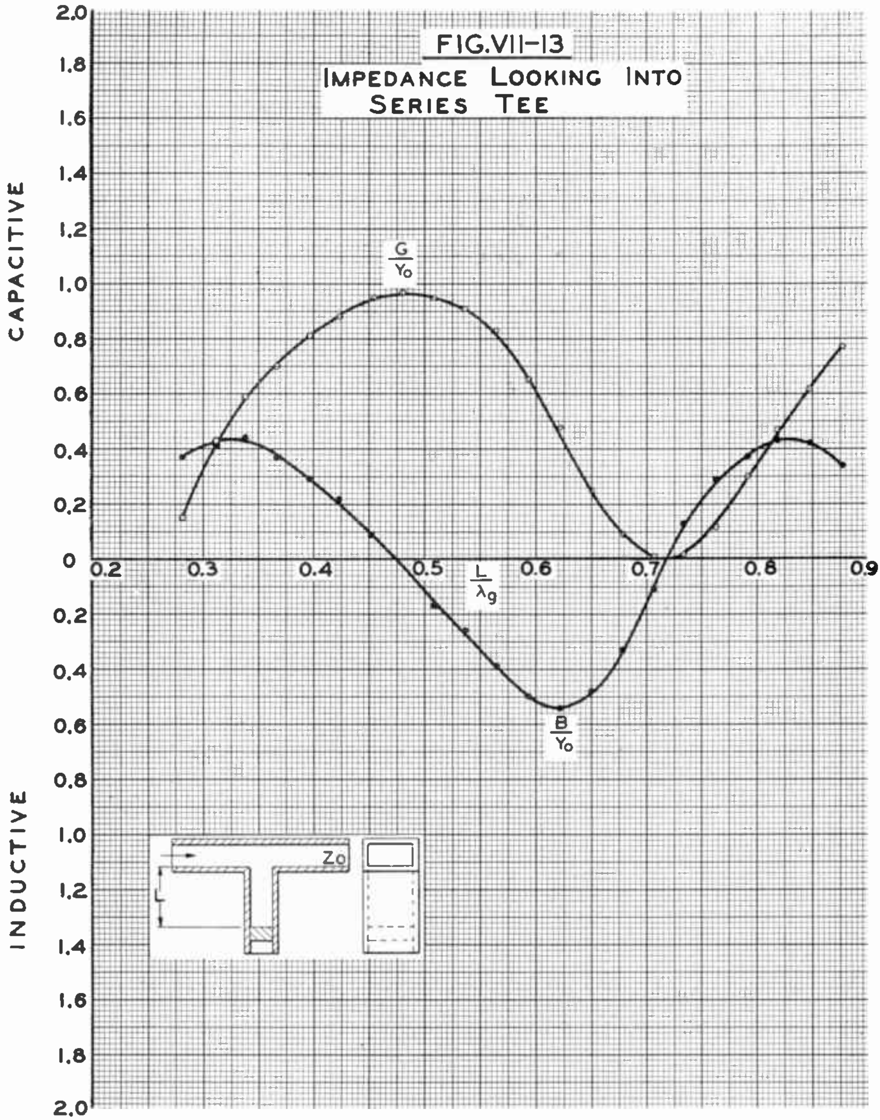


FIG.VII-13
IMPEDANCE LOOKING INTO SERIES TEE
CAPACITIVE
INDUCTIVE
$\frac{G}{Y_0}$
$\frac{B}{Y_0}$
$\frac{L}{\lambda_g}$
Z_0
L
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

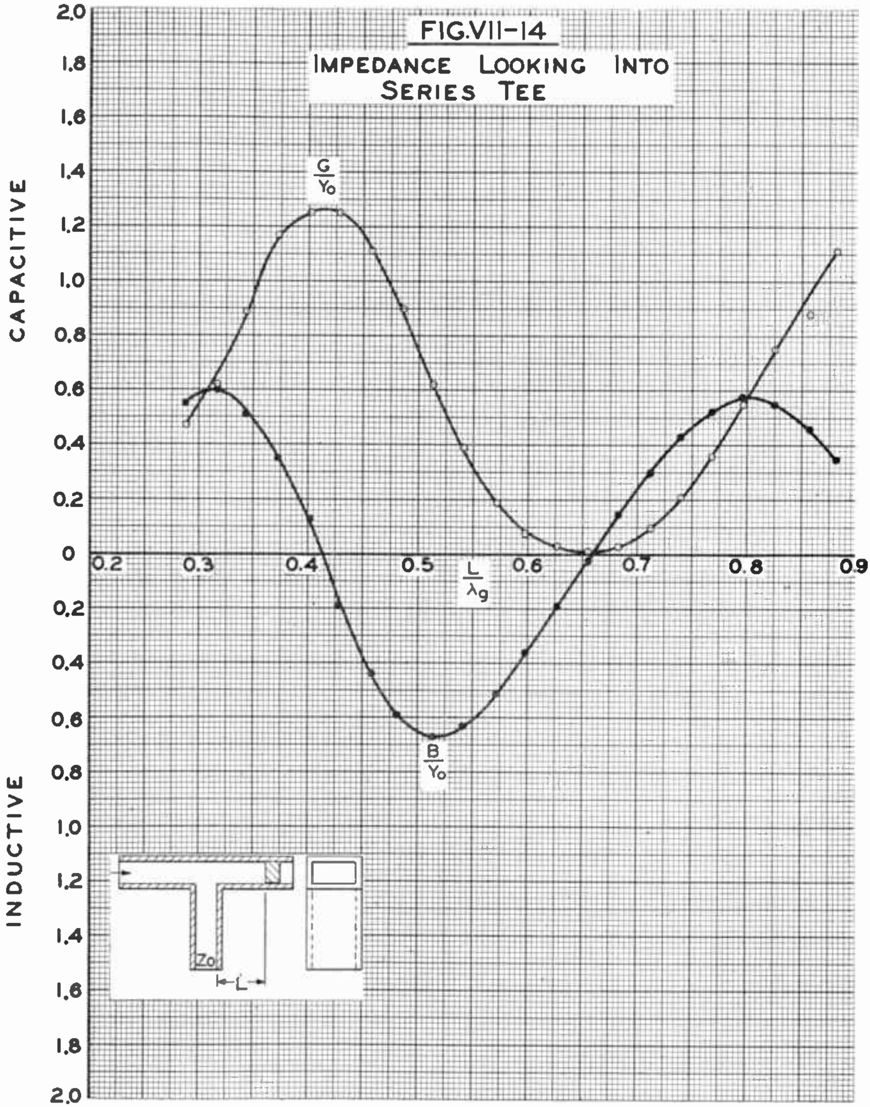


FIG.VII-14
IMPEDANCE LOOKING INTO SERIES TEE
CAPACITIVE
INDUCTIVE
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
L/λg
G/Yo
B/Yo
Zo
L

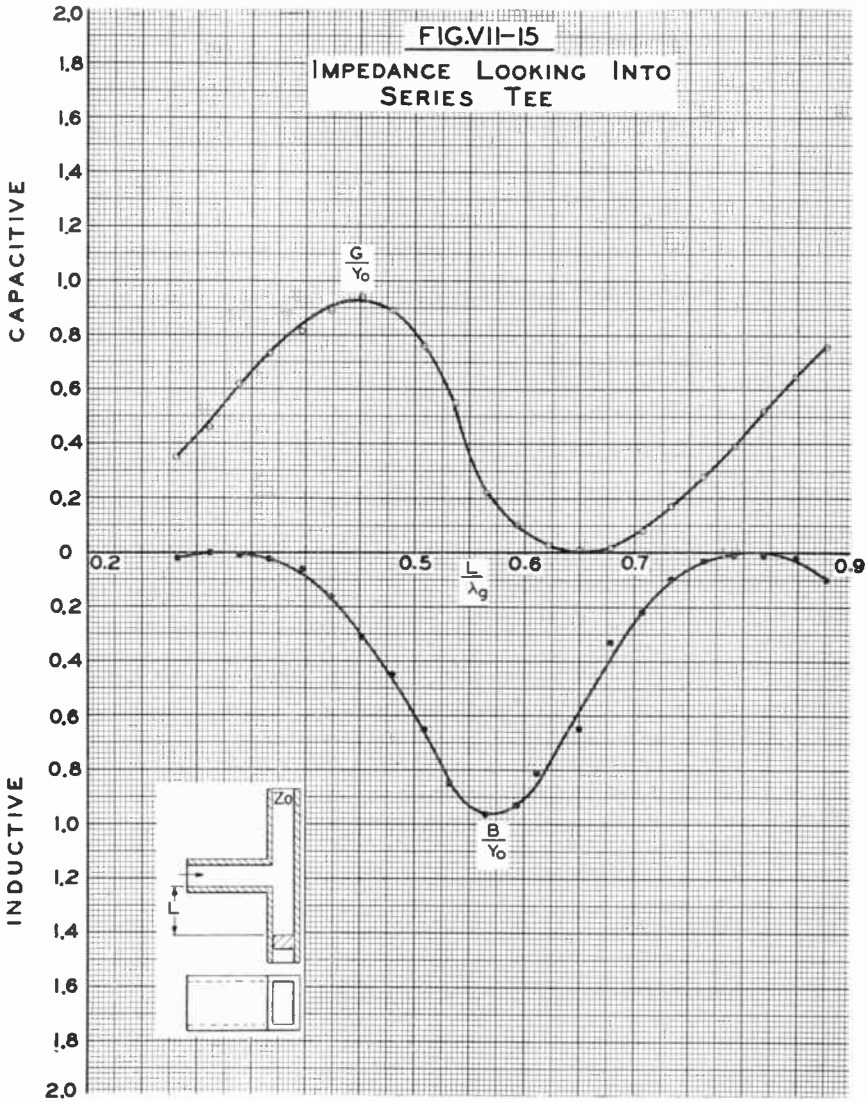
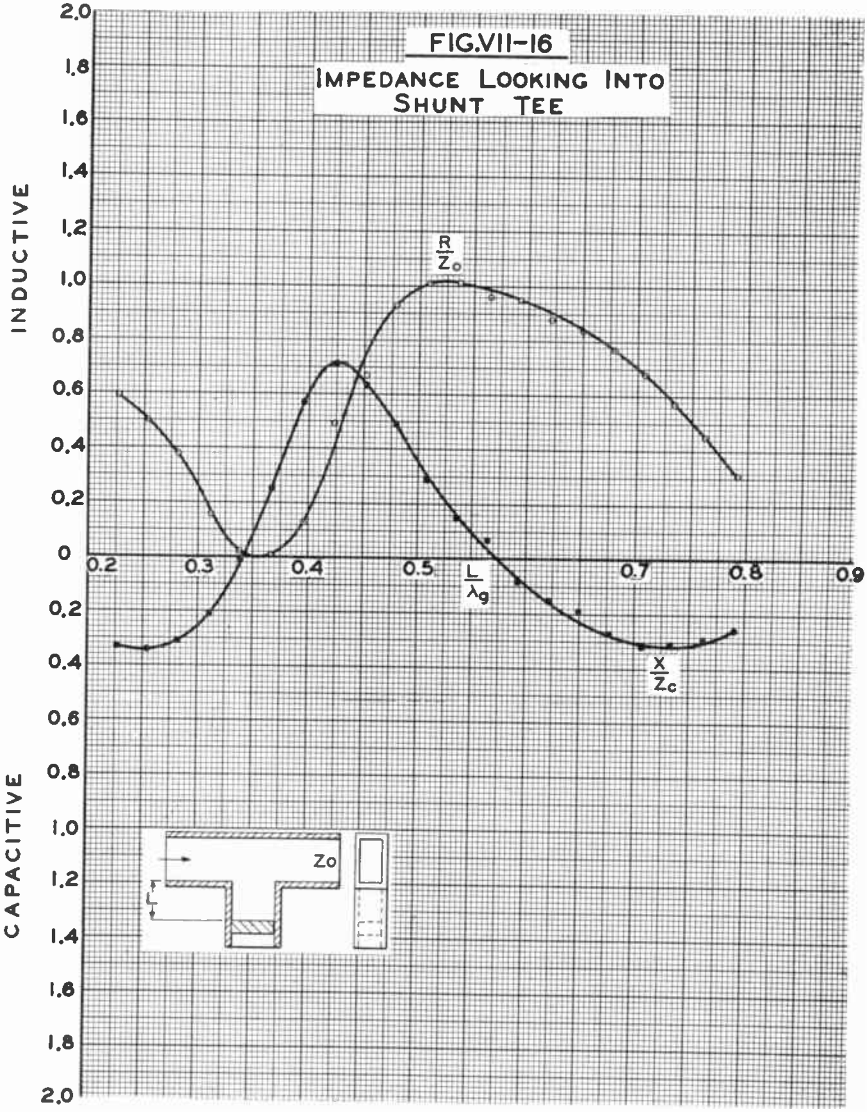


FIG.VII-15
IMPEDANCE LOOKING INTO SERIES TEE
CAPACITIVE
INDUCTIVE
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
0.2
0.5
0.6
0.7
0.9
L/λg
G/Yo
B/Yo
Zo
L

FIG.VII-16

IMPEDANCE LOOKING INTO SHUNT TEE



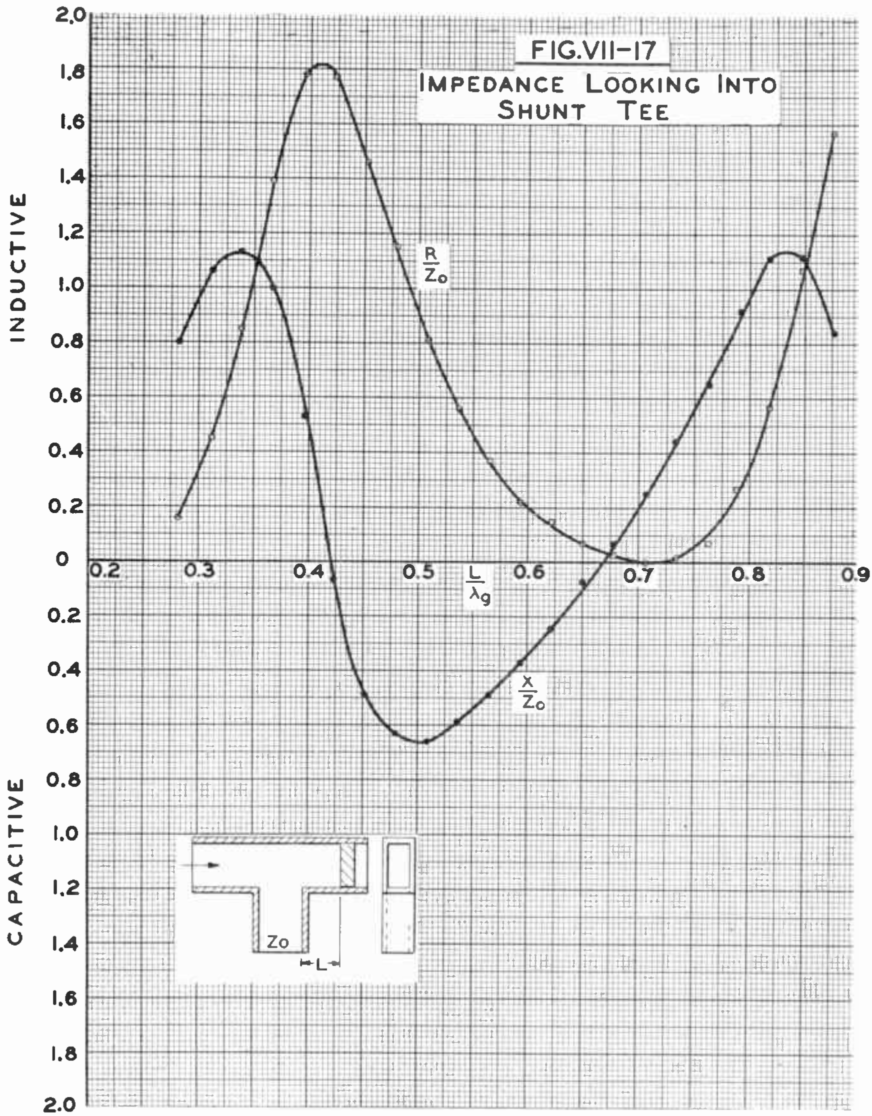


FIG.VII-17
IMPEDANCE LOOKING INTO SHUNT TEE
INDUCTIVE
CAPACITIVE
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
L/λg
R/Zo
X/Zo
Zo
L

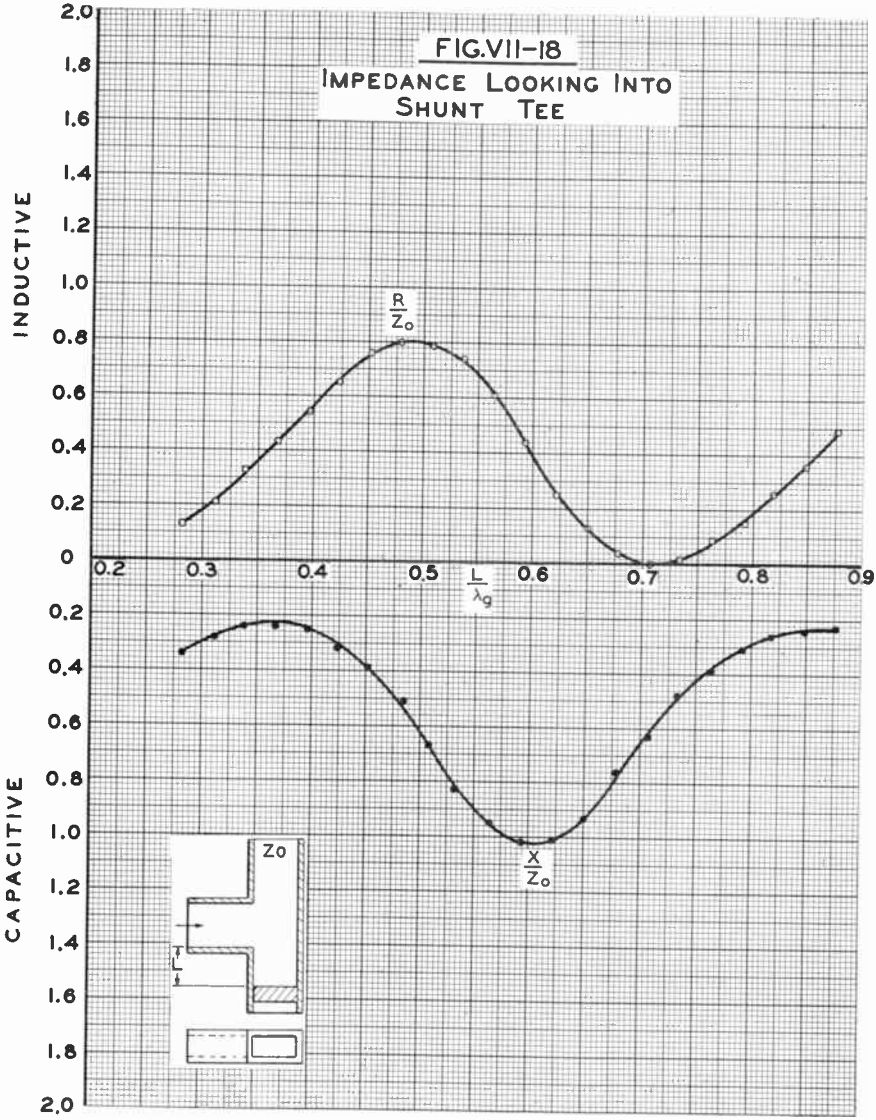


FIG.VII-18
IMPEDANCE LOOKING INTO SHUNT TEE
INDUCTIVE
CAPACITIVE
R/Zo
X/Zo
L/λg
Zo
L
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

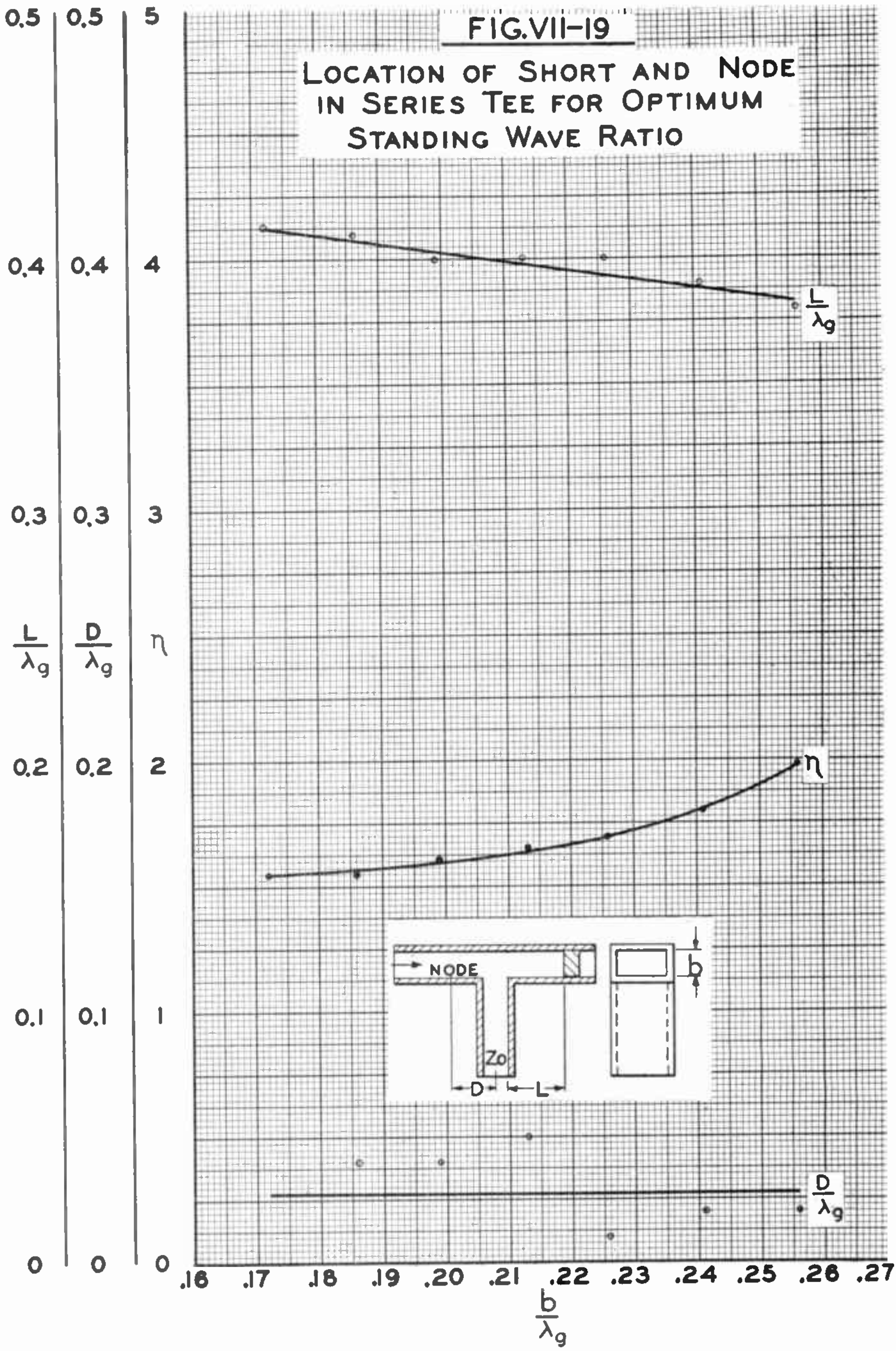


FIG.VII-19
LOCATION OF SHORT AND NODE IN SERIES TEE FOR OPTIMUM STANDING WAVE RATIO
0.5 0.5 5
0.4 0.4 4
0.3 0.3 3
L/λg D/λg η
0.2 0.2 2
0.1 0.1 1
0 0 0
.16 .17 .18 .19 .20 .21 .22 .23 .24 .25 .26 .27
b/λg
L/λg
η
D/λg
NODE
Zo
D
L
b

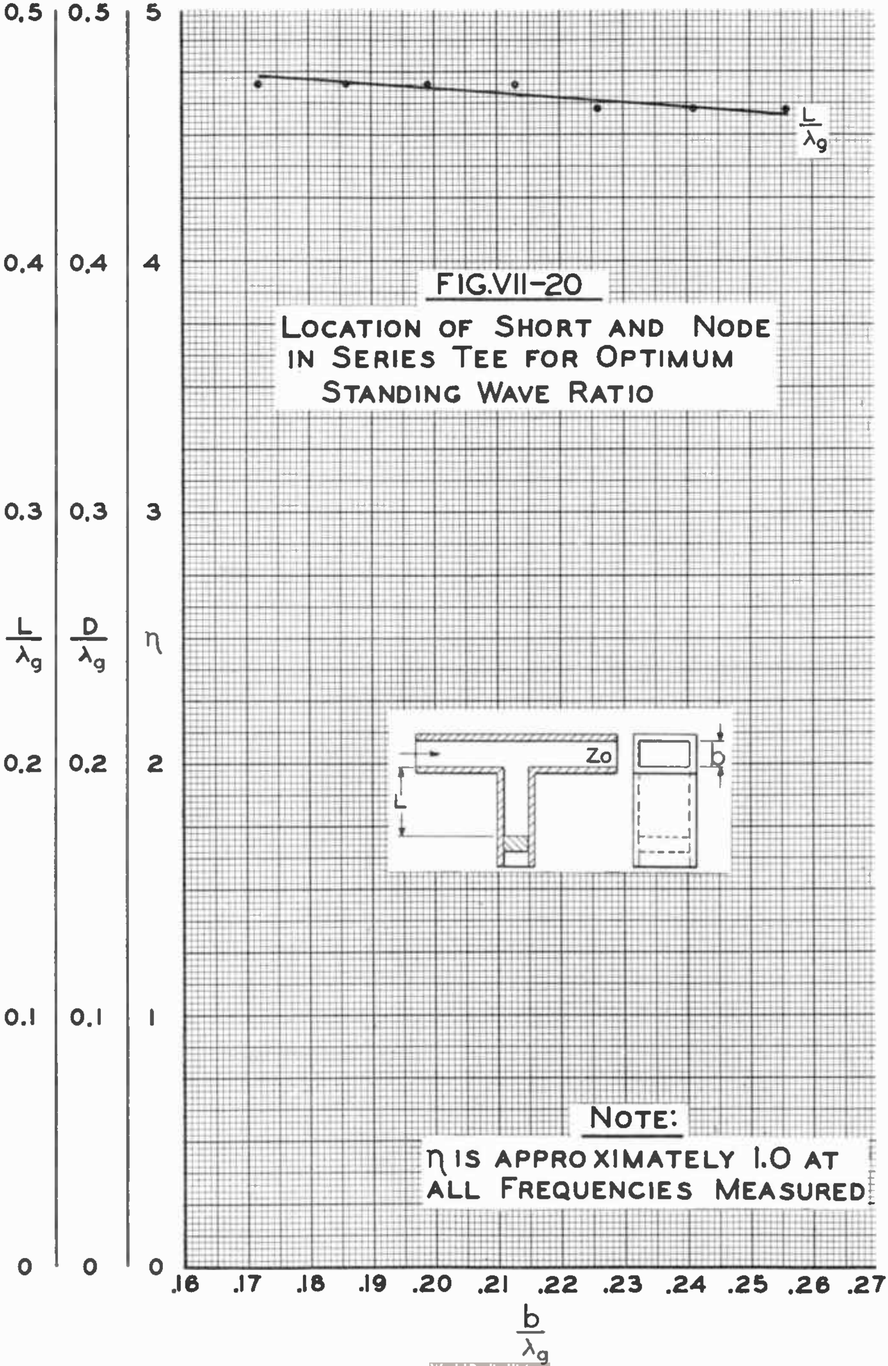


FIG. VII-20

LOCATION OF SHORT AND NODE IN SERIES TEE FOR OPTIMUM STANDING WAVE RATIO

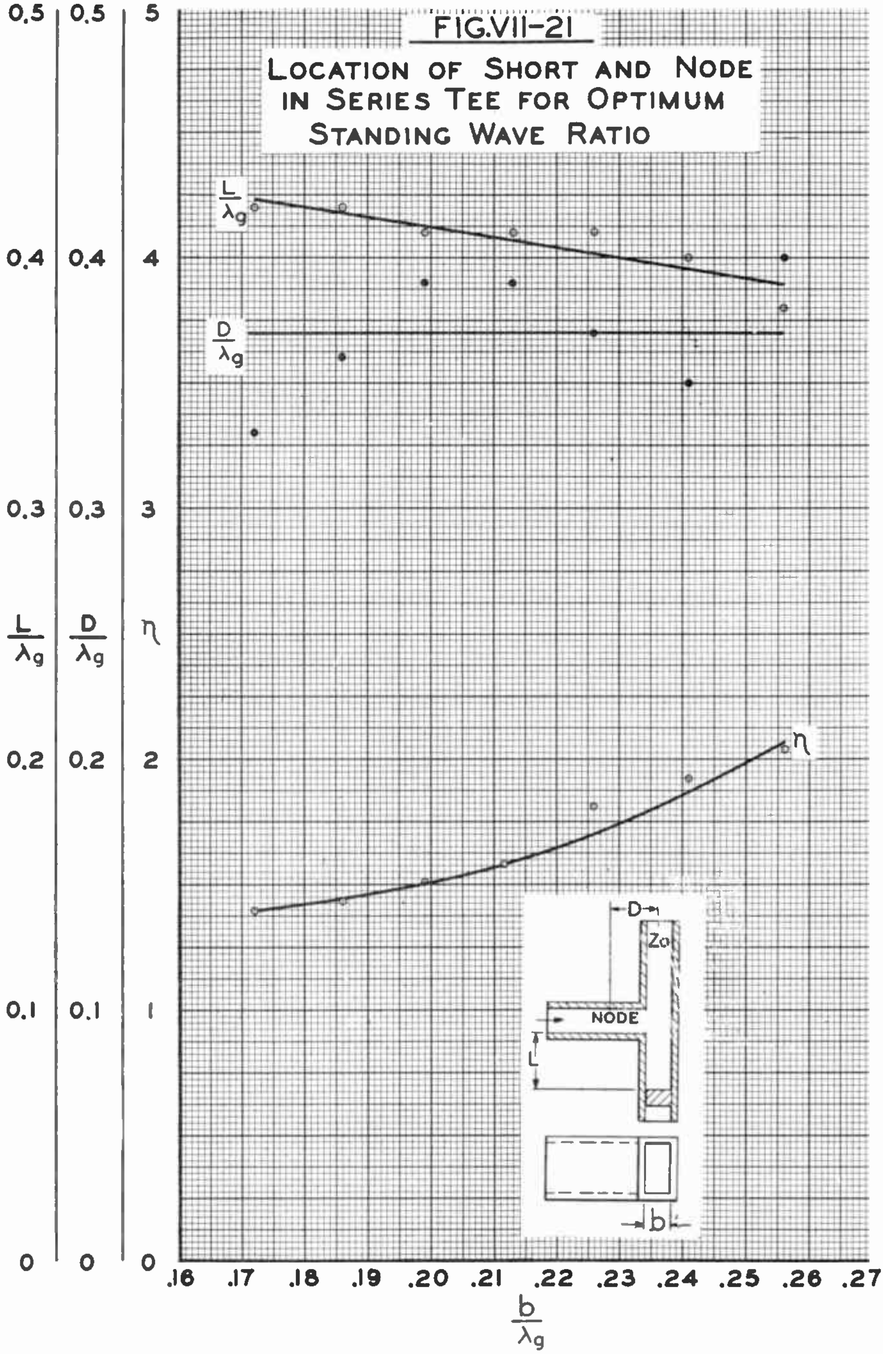


FIG.VII-21
LOCATION OF SHORT AND NODE IN SERIES TEE FOR OPTIMUM STANDING WAVE RATIO
L/λg
D/λg
η
D
Zo
NODE
L
b
0.5 0.4 0.3 0.2 0.1 0
5 4 3 2 1 0
.16 .17 .18 .19 .20 .21 .22 .23 .24 .25 .26 .27
b/λg

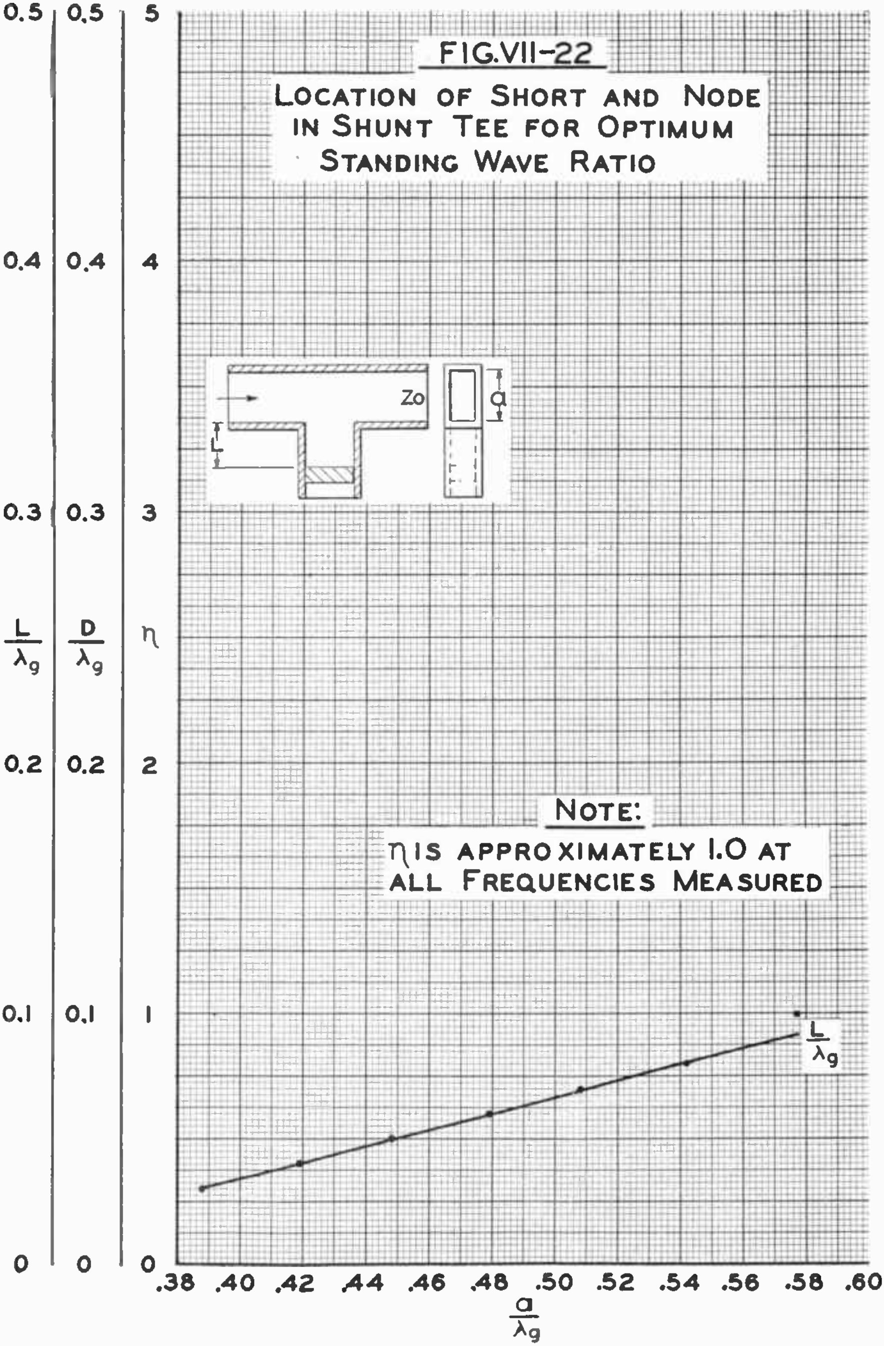


FIG. VII-22

LOCATION OF SHORT AND NODE IN SHUNT TEE FOR OPTIMUM STANDING WAVE RATIO

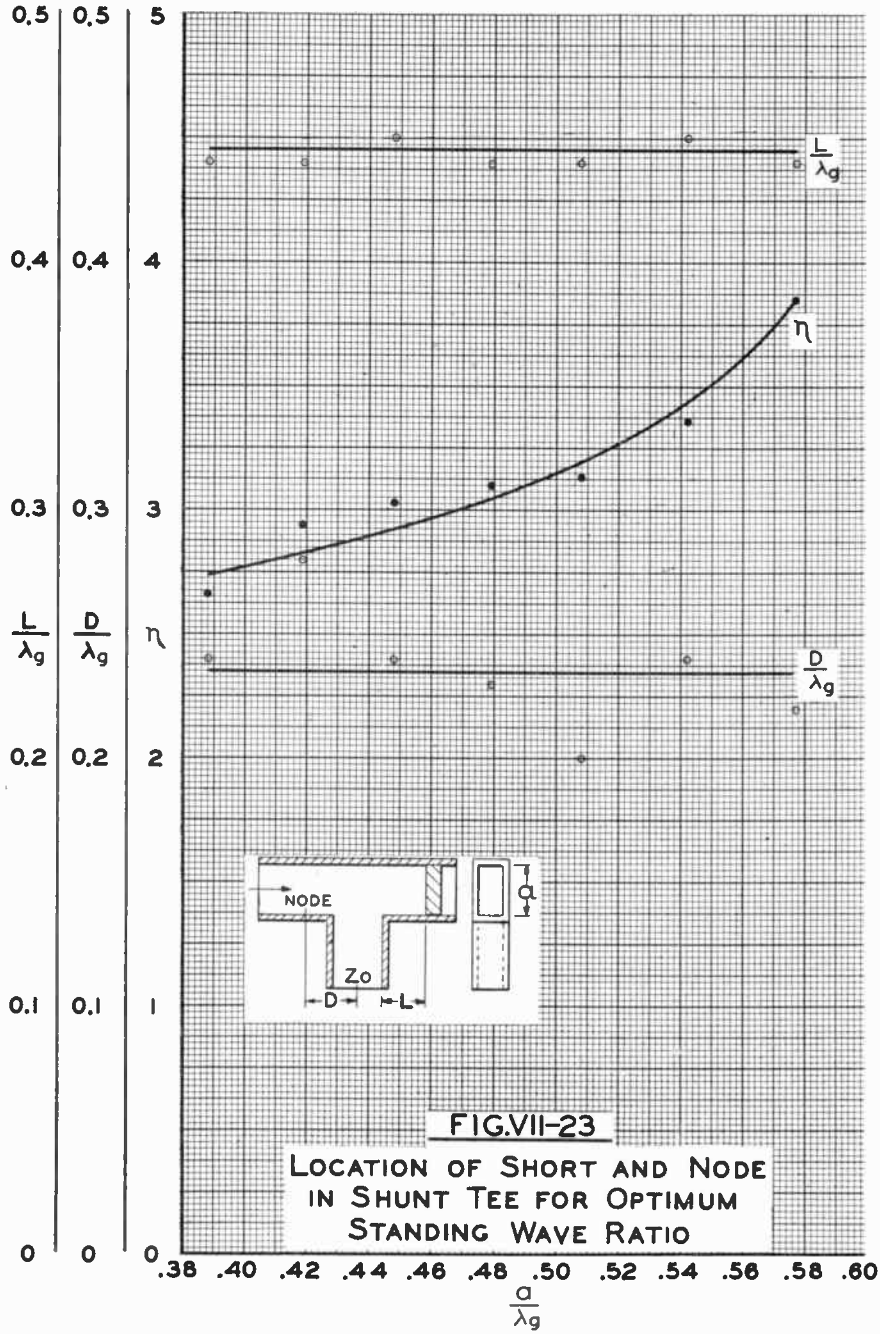


FIG.VII-23

LOCATION OF SHORT AND NODE IN SHUNT TEE FOR OPTIMUM STANDING WAVE RATIO

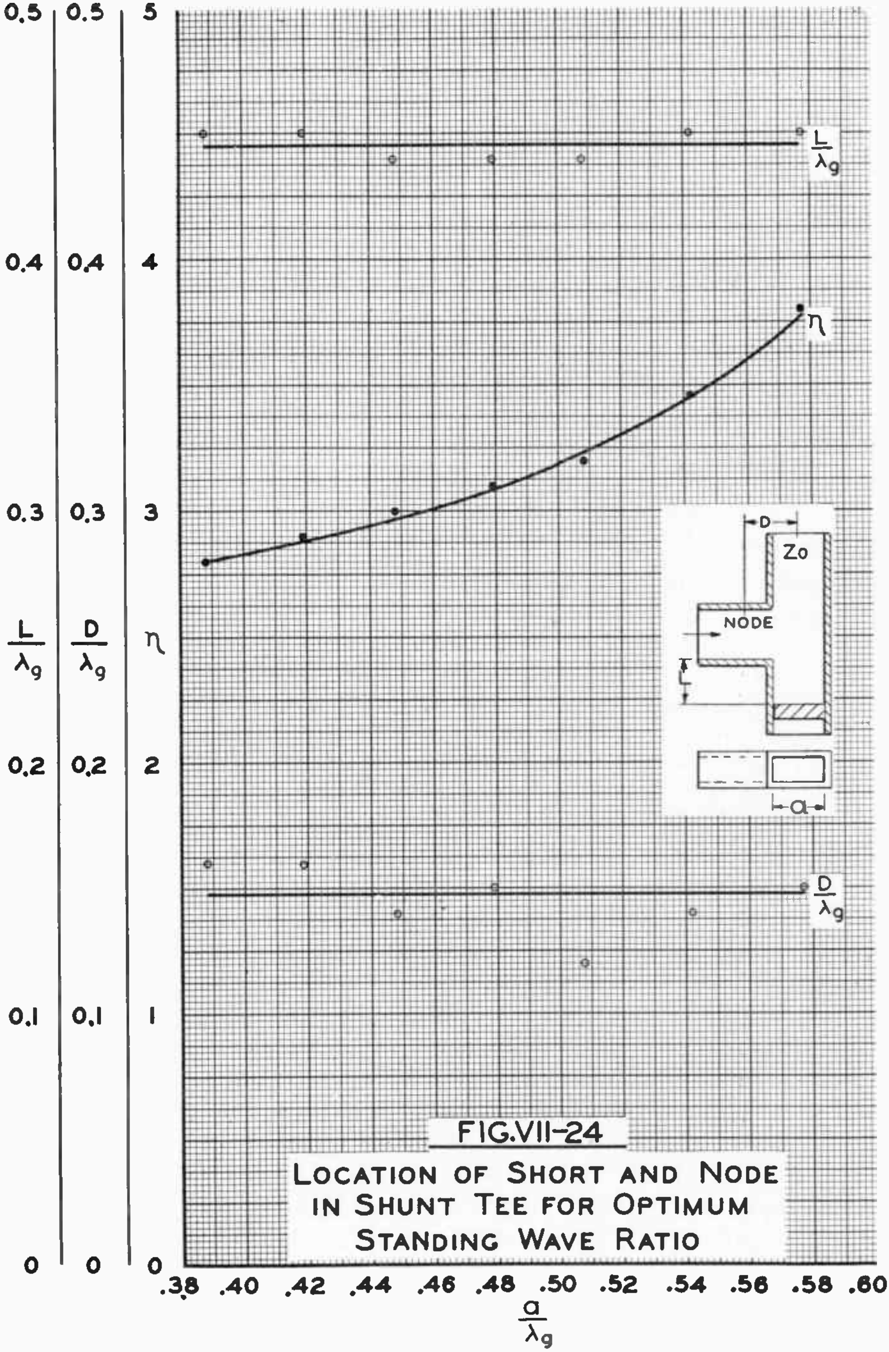


FIG. VII-24

LOCATION OF SHORT AND NODE IN SHUNT TEE FOR OPTIMUM STANDING WAVE RATIO

CHAPTER VIII

WAVE GUIDES FILLED WITH DIELECTRIC MATERIAL

The electric and magnetic fields along the axis of a wave guide vary as $e^{-\gamma z} = e^{-(\alpha+j\beta)z}$, where γ is the propagation constant, α the attenuation constant, and β the phase constant. If the wave guide is partially or completely filled with some dielectric material with a dielectric constant given by $\varepsilon = \varepsilon' - j\varepsilon''$, the propagation constant of the guide will be affected, and the impedance of the guide and its cutoff wavelength will also change.

A. *Propagation Constant*

If the guide is completely filled with a dielectric, the propagation constant of the guide becomes

$$\gamma = \frac{2\pi}{\lambda}\sqrt{(\lambda/\lambda_c)^2 - \varepsilon' + j\varepsilon''}$$

where λ is the wavelength in air and λ_c the cutoff wavelength in the air-filled guide. If the losses in the dielectric are small ($\varepsilon''/\varepsilon' \ll 1$), this formula reduces to

$$\gamma = \frac{\pi}{\lambda}\,\frac{\varepsilon'' + 2j\,[\,\varepsilon' - (\lambda/\lambda_c)^2\,]}{\sqrt{\varepsilon' - (\lambda/\lambda_c)^2}}$$

and for a perfect dielectric

$$\gamma = j\frac{2\pi}{\lambda}\sqrt{\varepsilon' - \left(\frac{\lambda}{\lambda_c}\right)^2}$$

B. *Wave Guide Impedance*

The "specific wave impedance" of a wave guide has been defined in Chapter VI, Section B as the ratio of transverse electric to transverse magnetic field strength. We now define the "normalized impedance" Z of the guide as the ratio of specific wave impedance when the guide is filled with dielectric to specific wave impedance when the guide is air-filled. The following formulas can then be applied;

For all *TE* waves:

It can be shown that for all *TE* waves in wave guides, the normalized impedance Z_{TE} of a dielectric-filled guide is given by

$$Z_{TE} = \frac{\gamma_o}{\gamma}$$

where γ_o is the propagation constant in the air-filled guide and γ the propagation constant in the dielectric-filled guide. This leads to the following formula for normalized impedance in a dielectric-filled guide

$$Z_{TE} = \sqrt{\frac{(\lambda/\lambda_c)^2 - 1}{(\lambda/\lambda_c)^2 - \varepsilon' + j\varepsilon''}}$$

When the dielectric is low power factor, this may be re-written

$$Z_{TE} = \frac{\lambda_g}{\lambda_{go}}\left[1 + \frac{j}{2}\frac{\varepsilon''}{\varepsilon' - (\lambda/\lambda_c)^2}\right]$$

where λ_{go} is the guide wavelength in the air-filled guide and λ_g the guide wavelength in the dielectric-filled guide. For an ideal dielectric

$$Z_{TE} = \frac{\lambda_g}{\lambda_{go}} = \sqrt{\frac{1 - (\lambda/\lambda_c)^2}{\varepsilon' - (\lambda/\lambda_c)^2}}$$

The normalized impedance of a wave guide operating in a *TE* mode and filled with a dielectric is always less than unity, and decreases with increasing dielectric constant.

For all *TM* waves:

It can be shown that for all *TM* waves in wave guides, the normalized impedance Z_{TM} of a dielectric-filled guide is

$$Z_{TM} = \frac{\gamma}{\varepsilon\gamma_o}$$

This leads to the following formula for normalized impedance in a dielectric-filled guide:

$$Z_{TM} = \frac{1}{\varepsilon' - j\varepsilon''}\sqrt{\frac{(\lambda/\lambda_c)^2 - \varepsilon' + j\varepsilon''}{(\lambda/\lambda_c)^2 - 1}}$$

For a low-loss dielectric this becomes

$$Z_{TM} = \frac{\lambda_{go}}{\varepsilon'\lambda_g}\left\{1 + j\frac{\varepsilon''}{\varepsilon'}\left[1 - \frac{\varepsilon'/2}{\varepsilon' - (\lambda/\lambda_c)^2}\right]\right\}$$

and for a perfect dielectric

$$Z_{TM} = \frac{1}{\varepsilon'}\sqrt{\frac{\varepsilon' - (\lambda/\lambda_c)^2}{1 - (\lambda/\lambda_c)^2}}$$

For *TM* waves, if $(\lambda/\lambda_c)^2 < \frac{1}{2}$, the normalized impedance will always be less than unity, and will decrease continuously as the dielectric constant increases. However, if $(\lambda/\lambda_c)^2 > \frac{1}{2}$ the normalized impedance will first increase to a value greater than unity and then decrease to zero, with increasing dielectric constant. It will pass through the value unity at a value of ε' given by

$$\varepsilon' = \frac{(\lambda/\lambda_c)^2}{1 - (\lambda/\lambda_c)^2}$$

C. *Reflections From Dielectric Plugs in Wave Guides*

If there is a sudden change in the dielectric material inside a wave guide, a wave that is incident upon the boundary will be partially reflected because of the discontinuity in the medium of propagation. If the interface between dielectrics is normal to the axis of the wave guide, one might hope that the reflection could be determined by a consideration of the impedance of the system on both sides of the discontinuity. The question then to be asked is:

What property of a wave guide can be considered as analogous to the characteristic impedance of an ordinary transmission line in making these calculations? We have already considered the "specific wave impedance" of a wave guide (Chapter VI, Section B) and have found it in part analogous and in part not analogous to the characteristic impedance of a coaxial or two-wire line. It can be shown, however, that the analogy is good for the particular problem under consideration. If the interface is then the boundary in a given wave guide between air-filled and dielectric-filled regions, the reflection from this boundary will be given by

$$\frac{E_2}{E_1} = \frac{Z - 1}{Z + 1}$$

In this equation E_1 is the magnitude of the electric field of the incident wave, E_2 is the magnitude of the electric field of the reflected wave, and Z is the normalized impedance of the dielectric-filled guide. The standing wave ratio input to the interface, if the dielectric-filled section is terminated in its characteristic impedance, will be

$$\eta = \left[\frac{1 + \left|\frac{E_2}{E_1}\right|}{1 - \left|\frac{E_2}{E_1}\right|}\right]^2$$

or

$$\eta = Z^2 \text{ if } Z > 1 \quad \eta = \left(\frac{1}{Z}\right)^2 \text{ if } Z < 1$$

and the losses in the dielectric are small.

If the portion of wave guide that is filled with dielectric material is finite in length, there will be a reflection at both the incoming and out-going faces of the dielectric region. The phase relationship between the reflections will then depend upon the length of the dielectric region, unless the attenuation in that region is so high that the wave which is reflected from the far face of the dielectric is negligible in comparison with the wave reflected from the near face. The problem is best treated by considering the dielectric-filled region as a length of transmission line with a characteristic impedance and propagation constant different from the air-filled guide. These calculations yield a reflection coefficient of

$$\frac{E_2}{E_1} = -\frac{\sinh \gamma l}{\sinh (\gamma l + \phi)}$$

where l is the length of the dielectric section, γ is the propagation constant of the dielectric section, and

$$\phi = ln\left(\frac{\frac{1}{Z} + 1}{\frac{1}{Z} - 1}\right)$$

WAVE GUIDES FILLED WITH DIELECTRIC MATERIAL

The net wave transmitted, E_3, will be related to the incident wave E_1, by

$$\frac{E_3}{E_1} = \frac{e^{\gamma_o l} \sinh \phi}{\sinh (\gamma l + \phi)}$$

The difference is accounted for partially by the wave that is reflected and partially by the wave that is absorbed in the dielectric. For a rectangular air-filled guide operating in the dominant or $TE_{1,0}$ mode with a plug of perfect dielectric, the percentages of the input power that will be reflected and transmitted are given by

$$\% \text{ P Refl.} = \frac{\frac{1}{4}\left(\frac{\lambda_{go}}{\lambda_g} - \frac{\lambda_g}{\lambda_{go}}\right)^2 \sin^2 \frac{2\pi l}{\lambda_g}}{1 + \frac{1}{4}\left(\frac{\lambda_{go}}{\lambda_g} - \frac{\lambda_g}{\lambda_{go}}\right)^2 \sin^2 \frac{2\pi l}{\lambda_g}}$$

$$\% \text{ P Trans.} = \frac{1}{1 + \frac{1}{4}\left(\frac{\lambda_{go}}{\lambda_g} - \frac{\lambda_g}{\lambda_{go}}\right)^2 \sin^2 \frac{2\pi l}{\lambda_g}}$$

where λ_{go} is the wavelength in the air-filled guide and λ_g the wavelength in the dielectric-filled guide. These formulas have been checked experimentally for polystyrene at $\lambda = 9.0$ cm., with the results shown in Fig. VIII–1. The discrepancy can be accounted for by experimental error, as the measured total power transmitted and reflected sometimes exceeds 100% of the total input power.

D. *Wave Guides Partially Filled with Dielectric Material*

Consider a wave guide of indefinite length in which only part of the cross-sectional area of the wave guide is filled with dielectric material, and the rest is filled with air. In such a wave guide, the fields will be pulled into the dielectric material, and one effect will be to reduce the phase velocity and the wavelength in the guide below the air-filled values. When the guide wavelength λ_g is equal to the free space wavelength λ, there is total internal reflection inside the dielectric at the air-dielectric boundary, and most of the energy is carried in the dielectric-filled portion of the wave guide.

A dielectric cylinder bounded by another dielectric of lower dielectric constant is capable of acting as a wave guide, and will guide electromagnetic radiation although there is no conducting material present. The fields will not be completely enclosed within the dielectric cylinder, however, and will extend to infinity in a direction normal to the axis of the dielectric guide. In this respect, they will be analogous to the fields being guided by an ordinary two-wire transmission line. If the dielectric cylinder is in turn enclosed by a larger, hollow cylinder of conducting material, this cylinder will disturb somewhat the field configurations inside and outside the dielectric cylinder, but its major effect will be to enclose completely all of the energy being guided by the dielectric cylinder. This shielded dielectric wave guide is the limit that is approached when enough dielectric material is

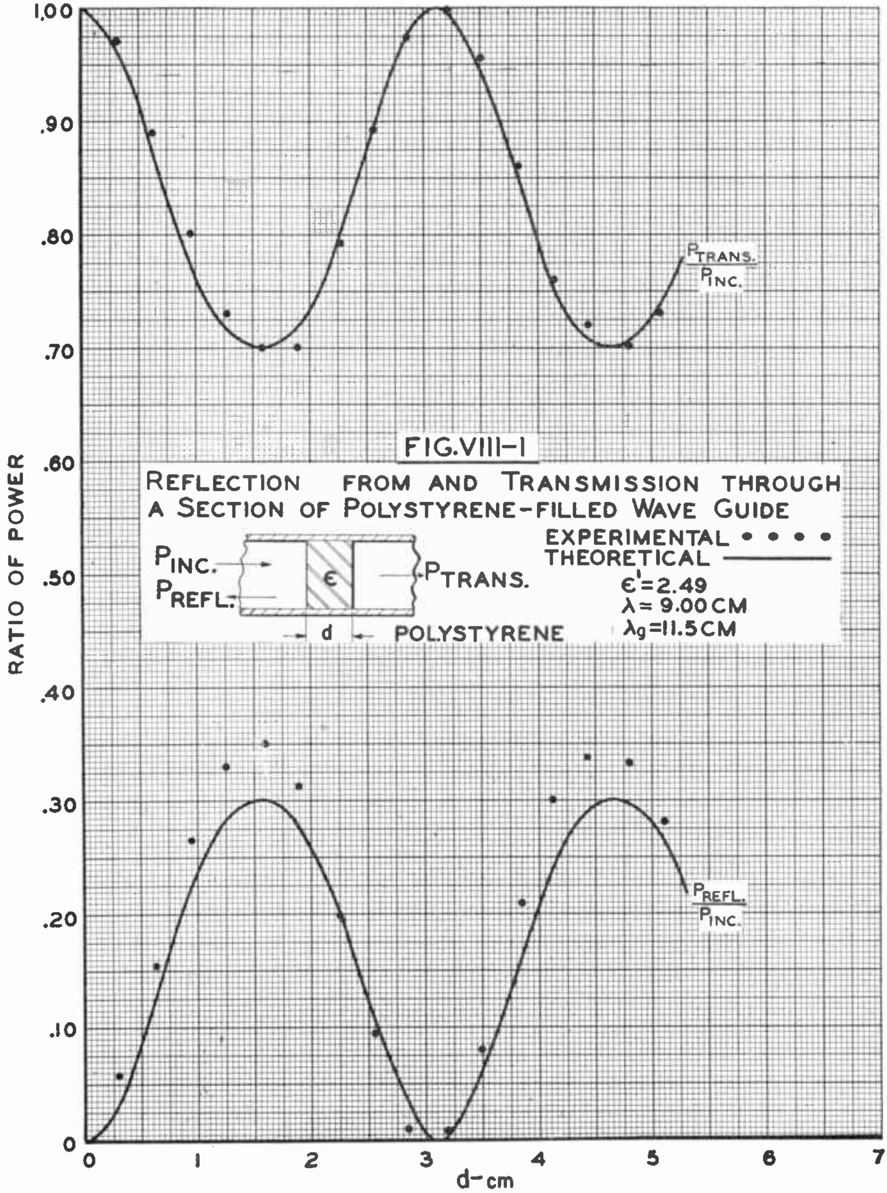


FIG. VIII-1

REFLECTION FROM AND TRANSMISSION THROUGH A SECTION OF POLYSTYRENE-FILLED WAVE GUIDE put into an ordinary hollow pipe wave guide to bring the guide wavelength below the free space wavelength.

Case I: If a rectangular wave guide operating in the $TE_{1,0}$ mode is partially filled with some perfect dielectric material in the manner indicated in Fig. VIII–2, the field configuration will depend upon the ratio λ/λ_g, which in turn will depend upon the size of the wave guide and the amount of wave guide that is filled with the dielectric. Three cases are illustrated in Fig. VIII–2, which are drawn for a typical wave guide and low-loss dielectric. The dielectric constant is assumed in this and subsequent cases to be 2.45, a generally accepted value for polystyrene in the neighborhood of $\lambda = 3$ cm. In the first case $\lambda_g > \lambda$ and the presence of the dielectric somewhat modifies the fields that are present in the air-filled guide. In the second, $\lambda_g = \lambda$; this is the point at which total internal reflection in the dielectric is reached. In the third, $\lambda_g < \lambda$, and this is similar to the shielded dielectric guide. The cutoff wavelength of the wave guide will also be a function of the percentage of the wave guide that is filled with dielectric, and in Fig. VIII–3, a/λ_c is plotted as a function of d/a, where a is the width of the wave guide and d is the width of the dielectric, as illustrated. Also shown on this figure is the variation of a/λ_c as a function of d/a for the next higher, or $TE_{2,0}$ mode. The maxima in the slope of these curves will occur when the transverse field maxima are passing through the interface between the dielectric and air. For the $TE_{1,0}$ mode the maximum slope is at a value of $\frac{d}{a} < 0.5$, indicating that the field is being pulled into the material with the higher dielectric constant.

In Fig. VIII–4, the ratio λ/λ_g is plotted as a function of d/a for various values of a/λ; these curves illustrate the effect of the dielectric upon the guide wavelength. The maxima in the slope of these curves will occur when the increment of d/a is in a region of maximum transverse field strength, as the effect of the dielectric will then be greatest. Here again the maximum slope is at a value $d/a < 0.5$, also illustrative of how the field is pulled into the dielectric material. Fig. VIII–5 contains much the same information as Fig. VIII–4, but here the ratio λ/λ_g is plotted against a/λ for a number of different values of d/a. Also shown on these curves are the points where propagation of the $TE_{2,0}$ mode becomes possible. All of the curves except that labeled $d/a = O$ will eventually become tangent to the $d/a = 1.0$ curve, for at sufficiently high values of a/λ, the dielectric cylinder will in every case act as a dielectric wave guide and contain practically all the energy.

Case II: When the dielectric is inserted at the center of the guide, instead of at the edges, the effect upon cutoff wavelength for the $TE_{1,0}$ and $TE_{2,0}$ modes will be as illustrated in Fig. VIII–6. For the $TE_{1,0}$ mode the maximum slope is at a value of $d/a = O$, this is because the dielectric is being inserted in a region of maximum field strength when it is placed in the center of the guide. A similarity will be observed between the curve for the $TE_{2,0}$ mode in Fig. VIII–6 and the curve for the $TE_{1,0}$ mode in Fig. VIII–3. This arises from the fact that the dielectric is inserted in corresponding regions of field in the two cases.

In Fig. VIII–7, the ratio λ/λ_g is plotted against the ratio d/a for various values of a/λ. This figure should be compared with Fig. VIII–4, which is the corresponding figure for Case I. Maxima of slope occur at $d/a = O$, for the same reason as in Fig. VIII–6. Fig. VIII–8 is comparable to Fig. VIII–5 for Case I. In this case it will be noted that the curves are crowded more toward the curve labeled $d/a = 1.0$, illustrating the greater effect of the dielectric when placed in the strong fields at the center of the guide.

Case III: When the dielectric is against both side walls of the wave guide, as illustrated in Fig. VIII–9, the effect of a given amount of dielectric will be even less than in Case I, because here the dielectric is concentrated even more in a region of low field. The variation of cut-off wavelength with increasing d/a is shown in Fig. VIII–9 for the $TE_{1,0}$ and $TE_{2,0}$ modes. The curve for the $TE_{2,0}$ mode is similar to the curve for the $TE_{1,0}$ mode in Fig. VIII–3, except for a scale factor.

The variation in λ/λ_g as a function of a/λ is given in Fig. VIII–10 for a number of values of d/a. In this case the curves are crowded more toward the curve for $d/a = O$, which indicates the relative ineffectiveness of the dielectric when concentrated in a region of low field.

Case IV: When the dielectric partially fills the wave guide in the manner indicated in Fig. VIII–11, the situation is more complicated than in the preceding cases. The field structure which goes continuously into the $TE_{1,0}$ mode as the dielectric thickness approaches zero has five non-vanishing field components, E_x, E_y, E_z, H_x, and H_z. This field has both electric and magnetic components along the axis of the guide, and is transverse magnetic to the direction normal to the interface between the air and the dielectric. As the thickness of the dielectric approaches zero, the E_z and E_x components will vanish, leaving only the E_y, H_x, and H_z components that are found in an air-filled guide propagating the $TE_{1,0}$ mode.

In this case the properties of the guide are a function of an additional parameter, the ratio of the guide cross-section dimensions. Therefore no single family of curves can be applied to show the variation of λ/λ_g with increasing values of dielectric thickness d. For this reason, only a single, typical curve of λ/λ_g vs. b/λ is shown in Fig. VIII–11 for a wave guide in which $b/a = 0.45$ and $d/b=0.50$. In Fig. VIII–12 are given two curves of λ/λ_g vs. d/b for two typical values of b/λ. These are for wave guides in which $b/a = 0.45$.

E. *Reflections From Tapered Sections of Dielectric*

The general problem of reflections from tapered discontinuities in line impedance have been discussed for coaxial lines in Chapter III, Section B–1C, and the same principles may with reservations be applied to tapered changes of impedance in wave guides. To a second order of approximation, the reflection from a tapered dielectric section of length l in a rectangular wave guide supporting the $TE_{1,0}$ mode only is found to be

$$\frac{E_2}{E_1} = -\frac{j}{8\pi}\left[\left(\frac{d\lambda_g}{dz}\right)_{z=O} - \left(\frac{d\lambda_g}{dz}\right)_{z=l} e^{-4\pi j \int_o^l \frac{dz}{\lambda_g}}\right]$$

If the taper is sufficiently gradual, it will be seen by the above formula that the magnitude of the reflection depends upon the discontinuity in the derivative of the guide wavelength at the beginning and at the end of the taper. A taper which starts at one side of the wave guide and ends at the other should then be very good, as the rate of change of wavelength at the ends of the taper will be small. But there will be oscillations in the curve of reflection as a function of the length of the taper, because of the variations with length of the term $e^{-4\pi j \int_o^l \frac{dz}{\lambda_g}}$. If the taper starts in the center of the guide, the reflection is likely to be larger for a similar taper of the same length, but the curve of reflection vs. length of taper will have smaller oscillations. The standing wave ratio introduced as a function of the length of taper is given in Fig. VIII–13 for a taper that starts on the long dimension of the guide and extends to the other side. That is, a cross-section in the tapered region would appear as Case IV of the partially filled guide.

F. *Effect of a Dielectric Post in a Wave Guide*

A dielectric post which extends across the center of a rectangular wave guide operating in the $TE_{1,0}$ mode, parallel to the lesser dimension of the guide, will act as a shunt admittance provided the diameter of the post is small compared to a wavelength, that is if

$$\left(\frac{2\pi R}{\lambda}\right)^2 << 1$$

where R is the radius of the post. If this assumption is valid, the admittance of the post will be given by

$$\frac{1}{Y} = j\frac{a}{2\lambda_g}\left[\log_e\frac{\lambda}{R} - f\left(\frac{a}{\lambda}\right) - \frac{1}{2\varepsilon}\left(\frac{\lambda}{\pi R}\right)^2 - 1.47\right]$$

where a is the width of the guide, λ_g the guide wavelength, λ the wavelength in free space, ε the complex dielectric constant of the substance forming the post, and $f\left(\frac{a}{\lambda}\right)$ is as given in Fig. VIII–14.

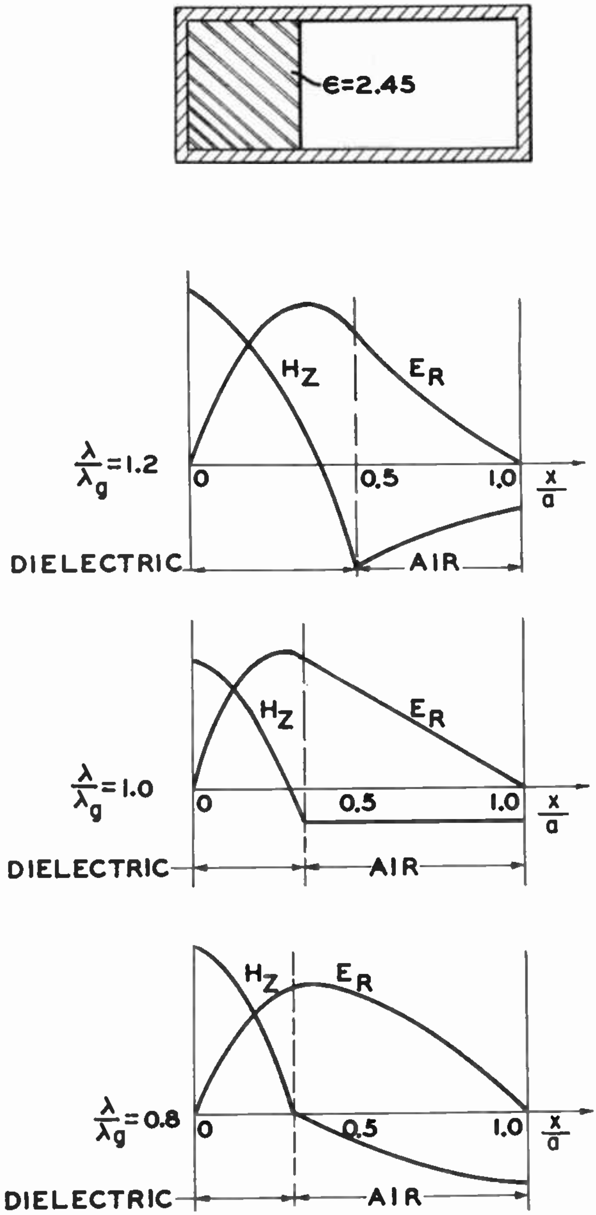
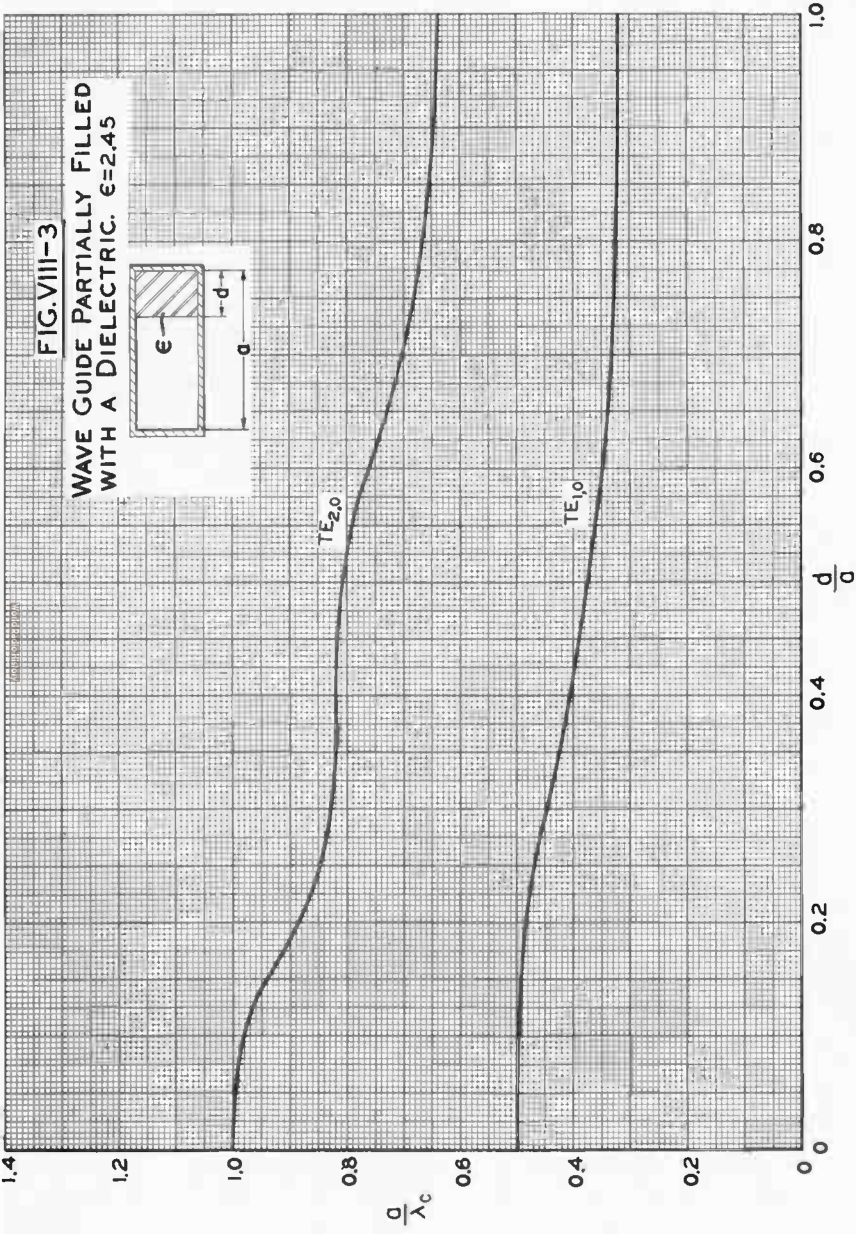


FIG. VIII - 2

FIELD INTENSITY IN A WAVE GUIDE
PARTIALLY FILLED WITH A DIELECTRIC

WAVE GUIDES FILLED WITH DIELECTRIC MATERIAL



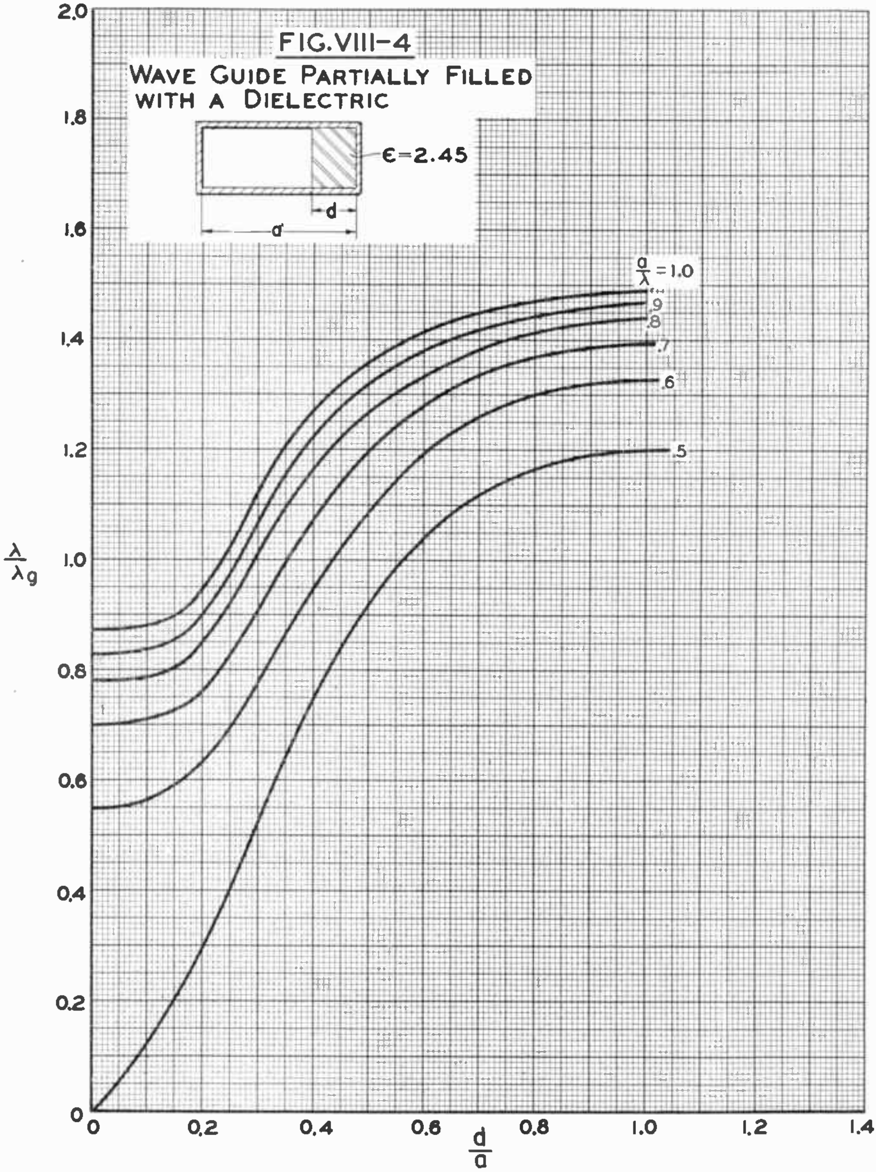
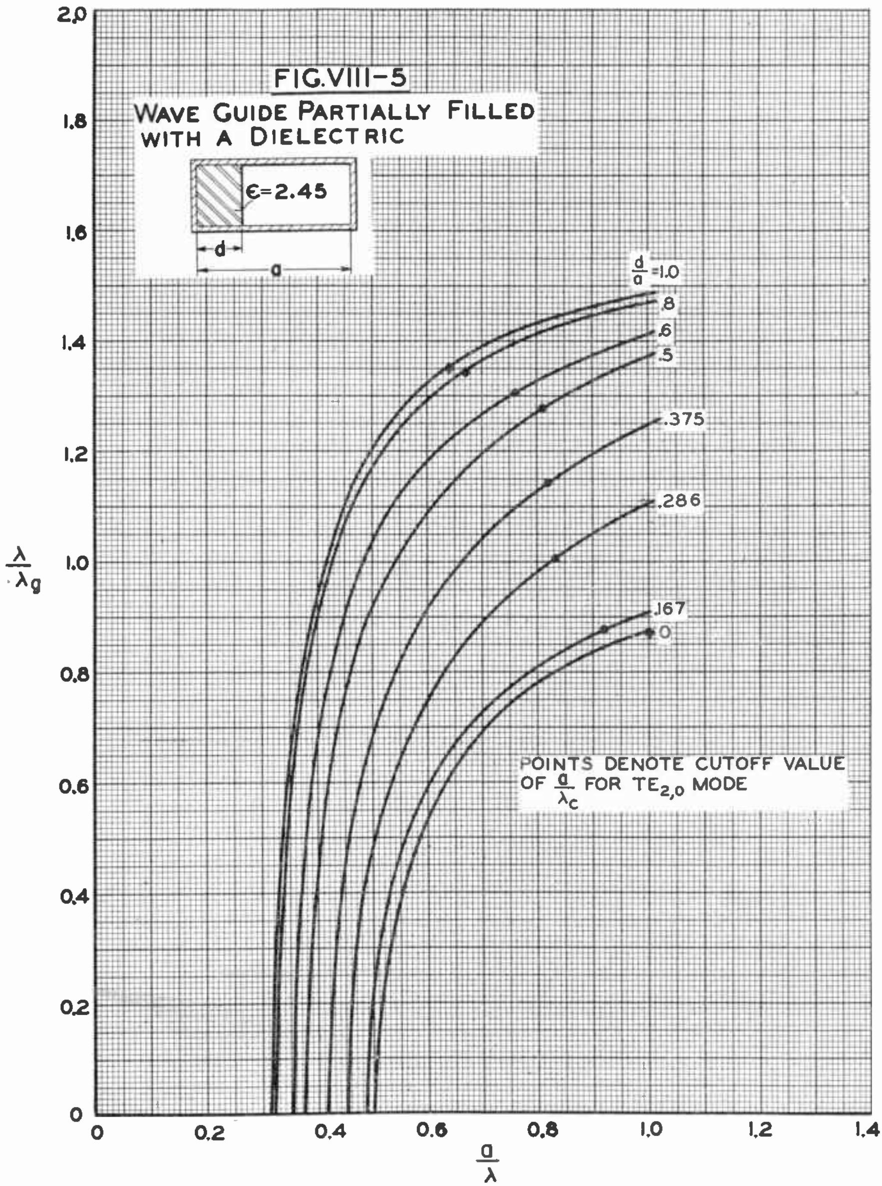


FIG. VIII-4
WAVE GUIDE PARTIALLY FILLED WITH A DIELECTRIC
ϵ=2.45
d
a
a/λ = 1.0
.9
.8
.7
.6
.5
λ/λg
d/a
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

WAVE GUIDES FILLED WITH DIELECTRIC MATERIAL



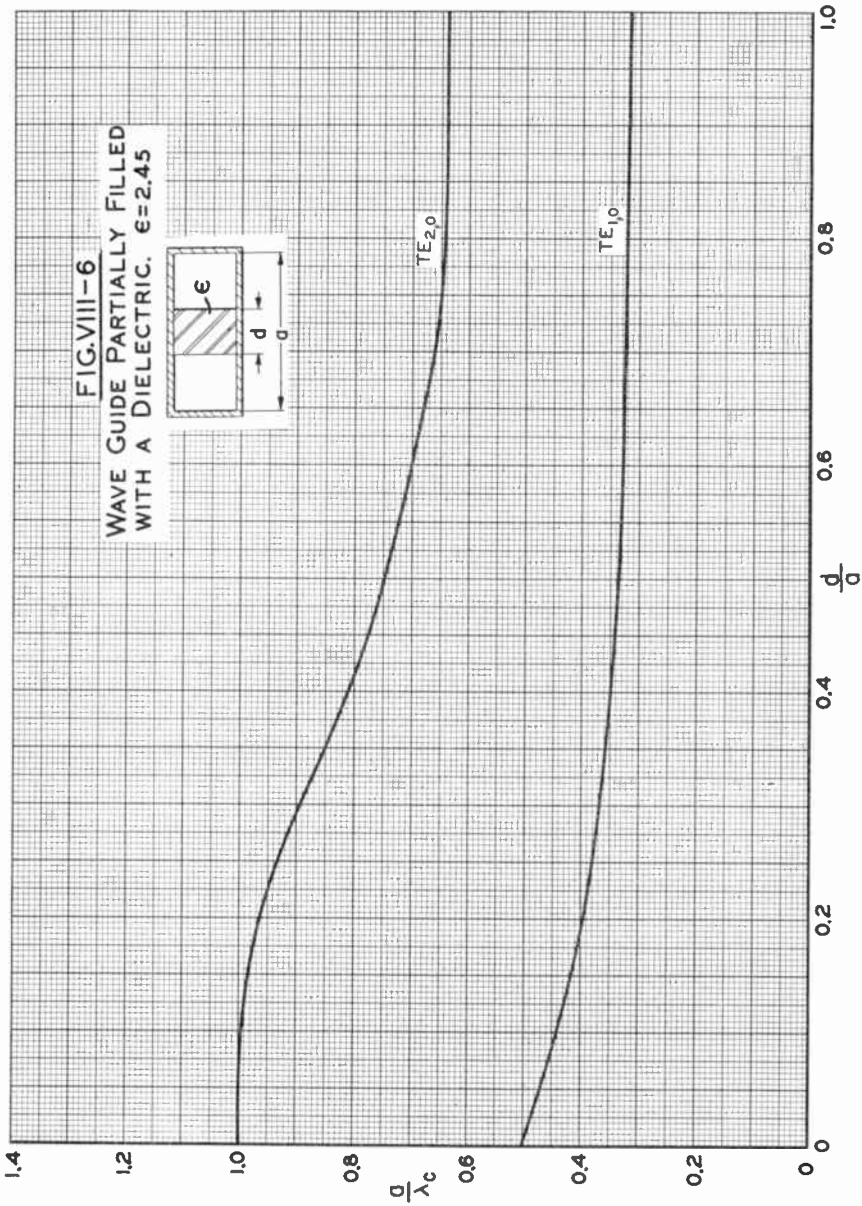


FIG. VIII-6
WAVE GUIDE PARTIALLY FILLED WITH A DIELECTRIC. ε=2.45
ε
d
a
TE2,0
TE1,0
d/a
a/λc
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

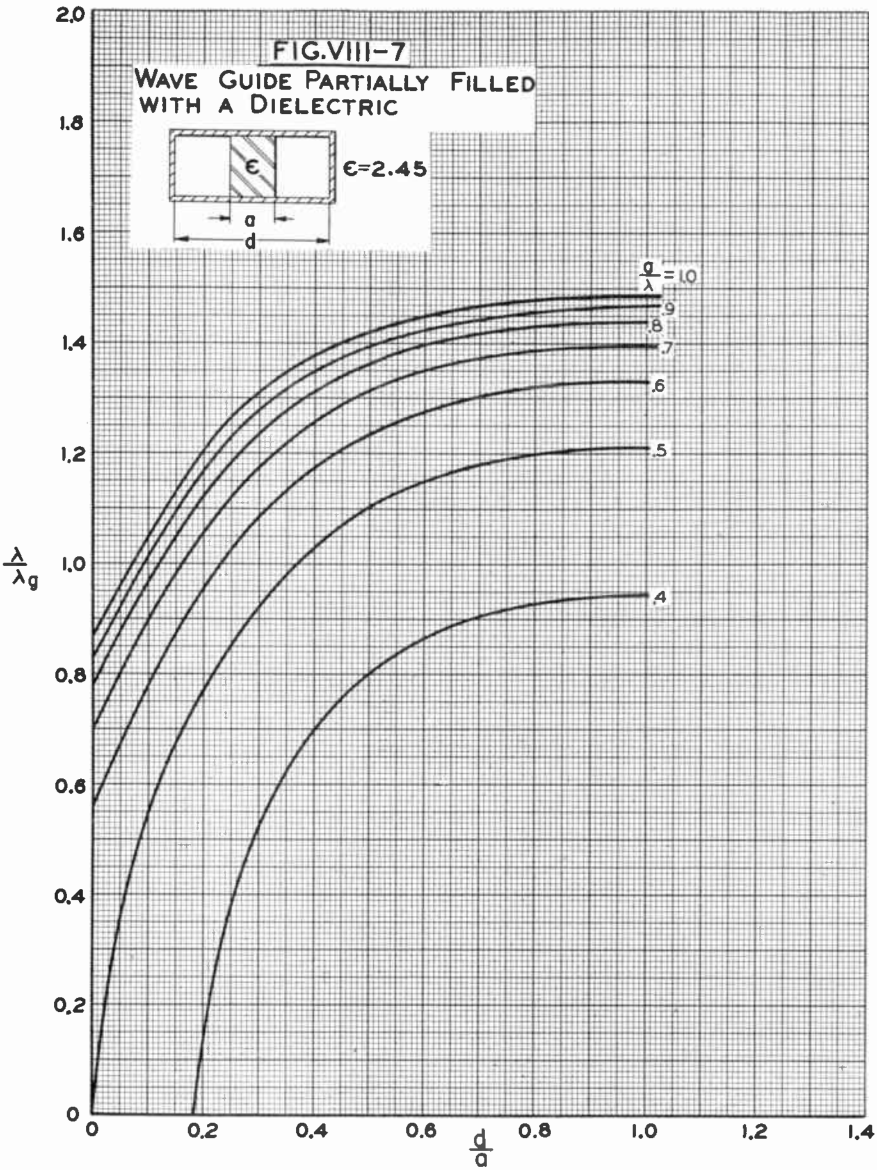


FIG.VIII-7
WAVE GUIDE PARTIALLY FILLED WITH A DIELECTRIC
ε
ε=2.45
a
d
$\frac{a}{\lambda}$ = 1.0
.9
.8
.7
.6
.5
.4
$\frac{\lambda}{\lambda_g}$
$\frac{d}{a}$
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

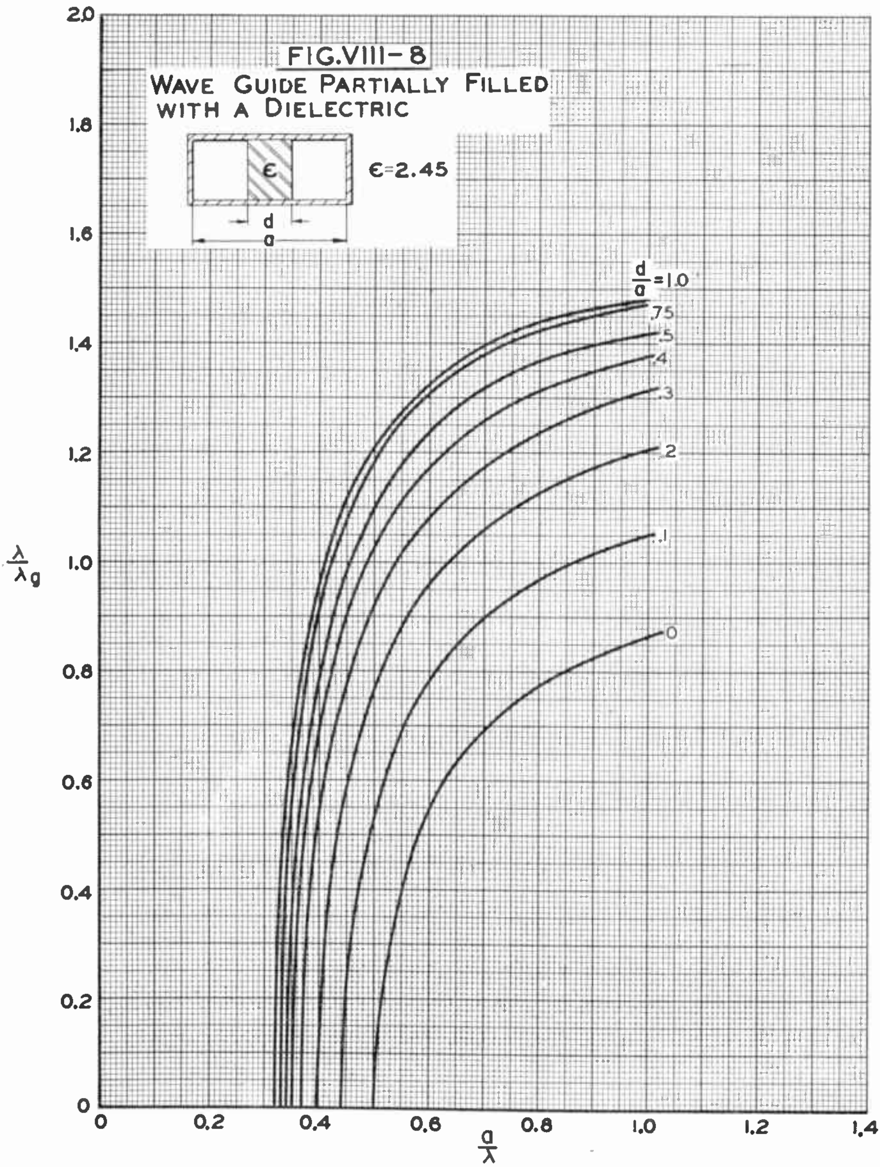


FIG.VIII-8
WAVE GUIDE PARTIALLY FILLED WITH A DIELECTRIC
ε
d
a
ε=2.45
d/a=1.0
.75
.5
.4
.3
.2
.1
0
λ/λg
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
a/λ

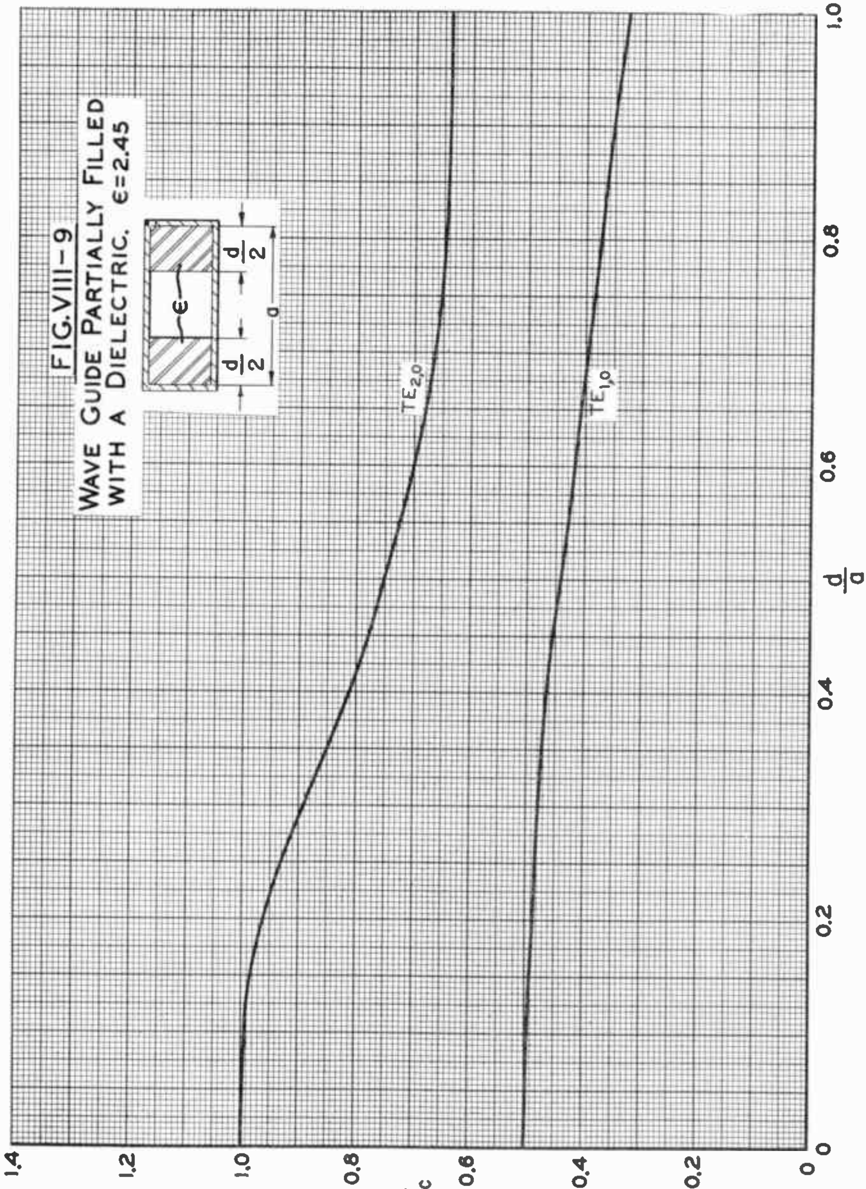


FIG. VIII-9
WAVE GUIDE PARTIALLY FILLED WITH A DIELECTRIC, ϵ=2.45
d/2
ϵ
a
d/2
TE2,0
TE1,0
a/λc
d/a
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0
0.2
0.4
0.6
0.8
1.0

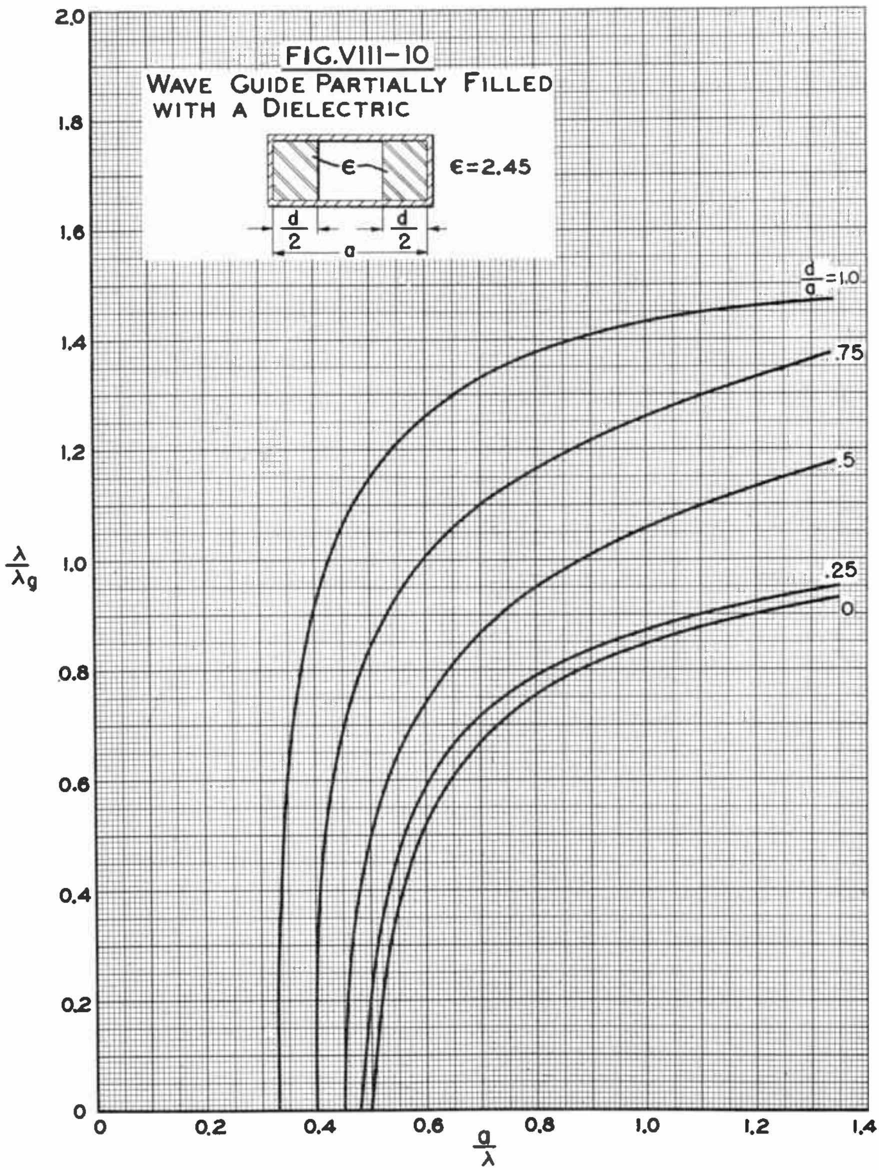


FIG.VIII-10
WAVE GUIDE PARTIALLY FILLED WITH A DIELECTRIC
ε
ε=2.45
d/2
d/2
a
d/a=1.0
.75
.5
.25
0
λ/λg
a/λ
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

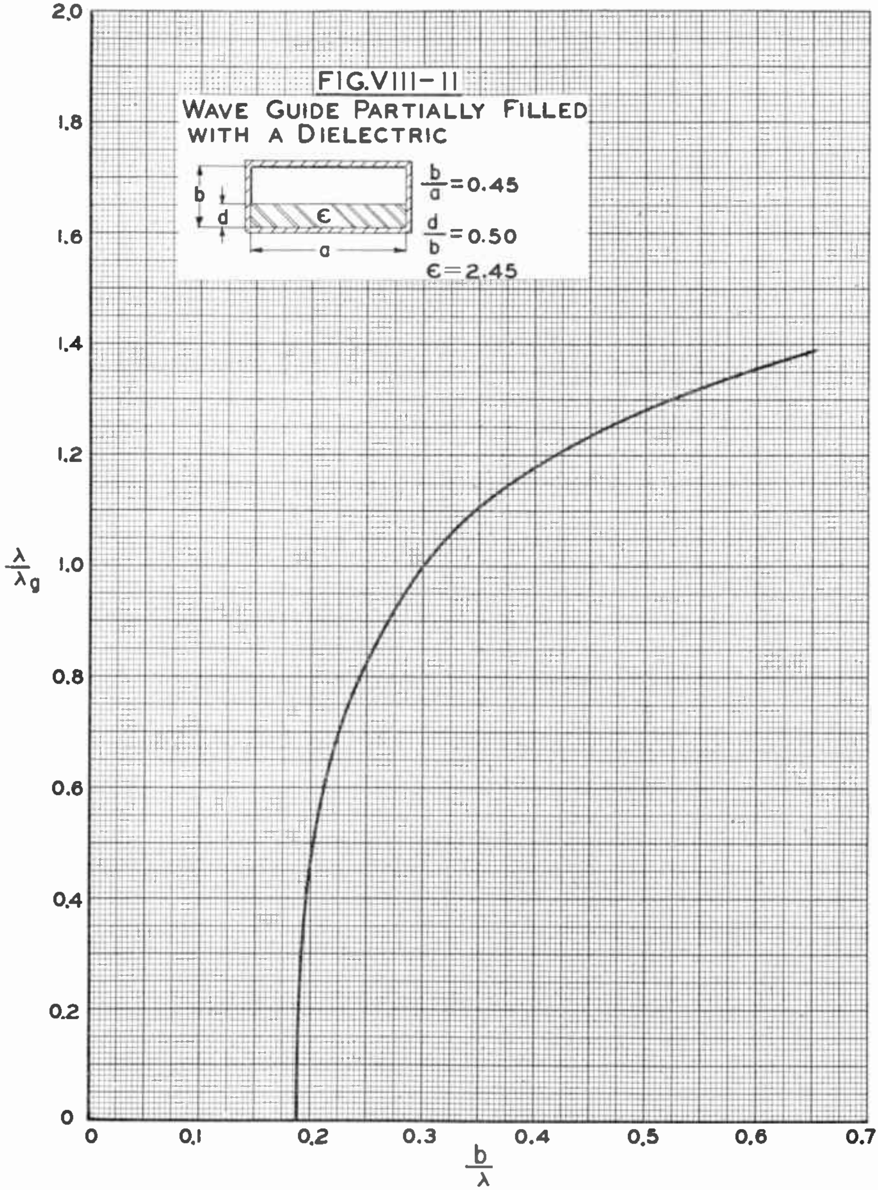


FIG.VIII-11
WAVE GUIDE PARTIALLY FILLED WITH A DIELECTRIC
b
d
ϵ
a
$\frac{b}{a}=0.45$
$\frac{d}{b}=0.50$
$\epsilon=2.45$
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
$\frac{\lambda}{\lambda_g}$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
$\frac{b}{\lambda}$

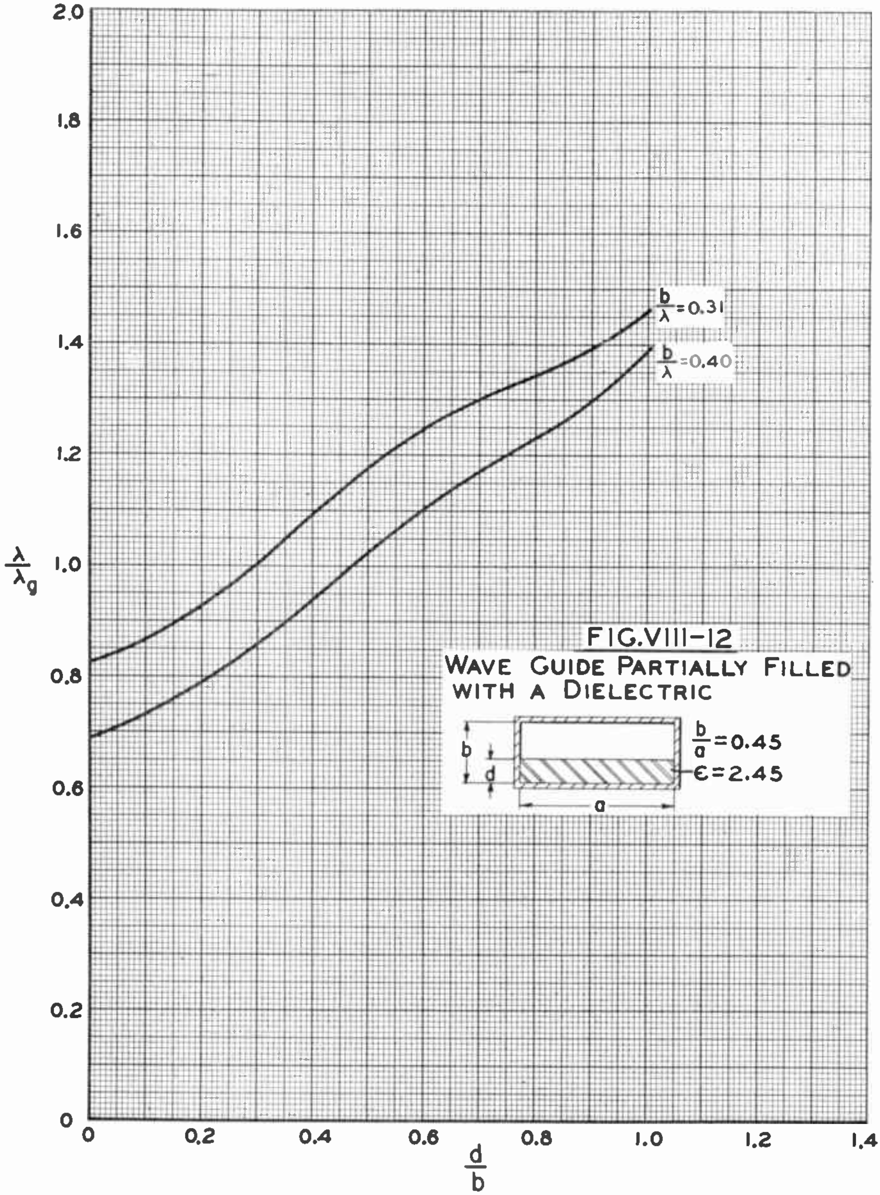


FIG. VIII-12
WAVE GUIDE PARTIALLY FILLED WITH A DIELECTRIC

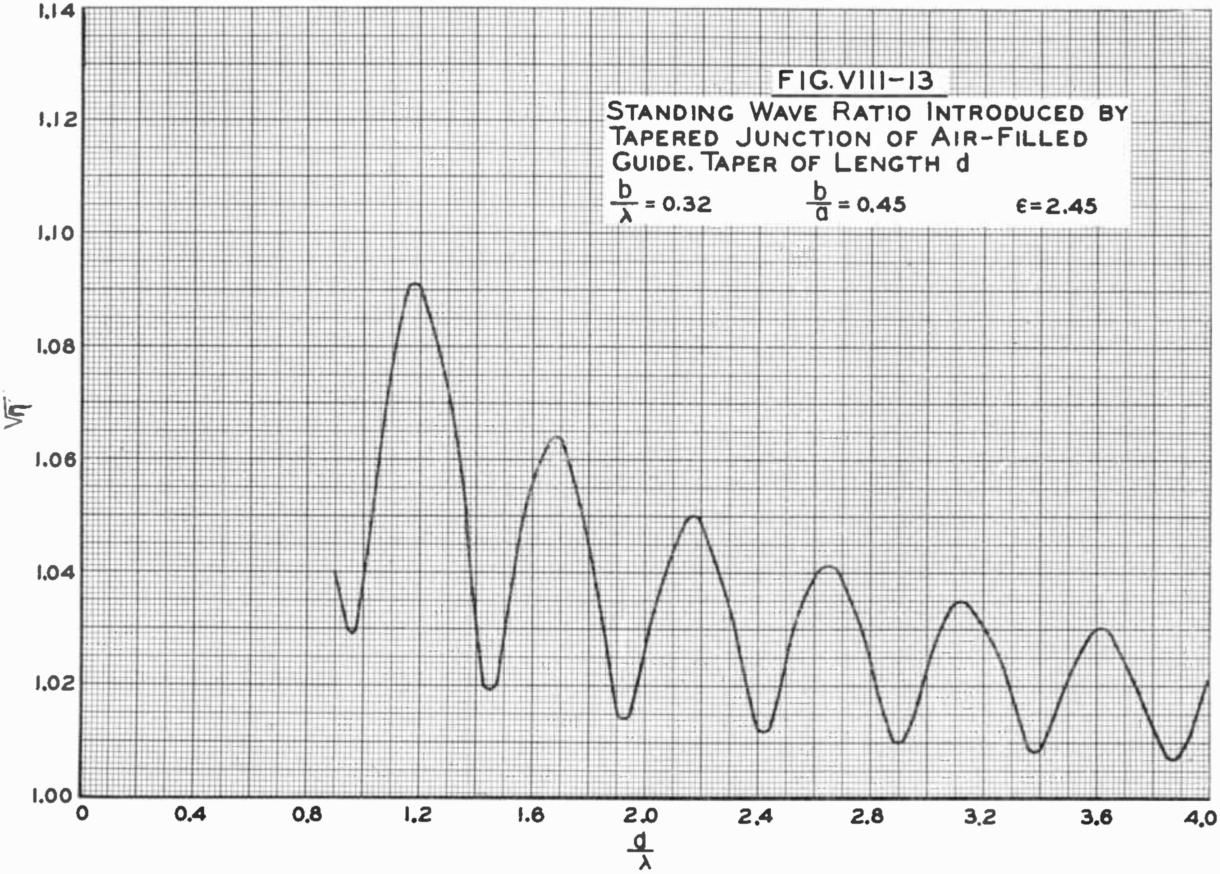


FIG. VIII-13

STANDING WAVE RATIO INTRODUCED BY TAPERED JUNCTION OF AIR-FILLED GUIDE. TAPER OF LENGTH d

$\frac{b}{\lambda} = 0.32$ $\frac{b}{a} = 0.45$ $\epsilon = 2.45$

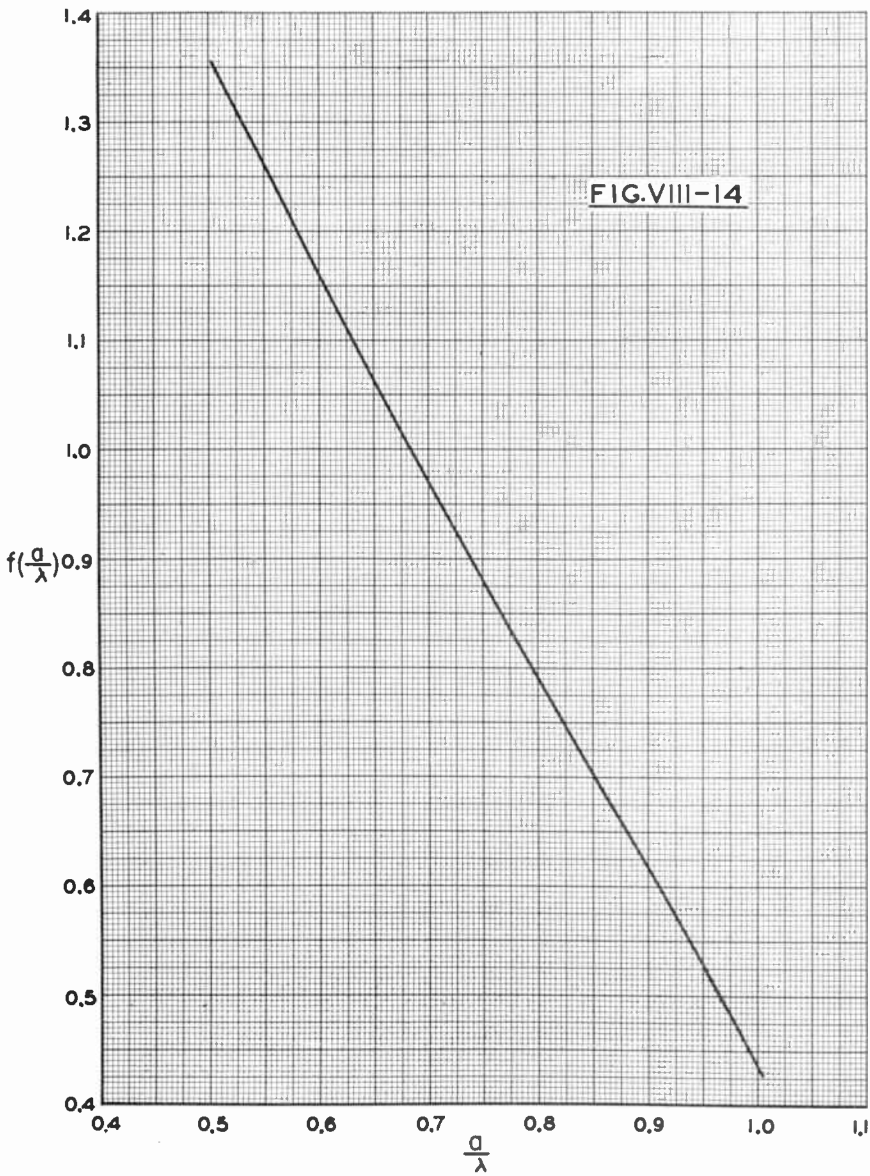

1.4
1.3
1.2
1.1
1.0
$f(\frac{a}{\lambda})$ 0.9
0.8
0.7
0.6
0.5
0.4
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
$\frac{a}{\lambda}$

FIG.VIII-14

Part 3

MISCELLANEOUS

CHAPTER IX

DIELECTRIC MATERIAL

Consider the impedance of a parallel plate condenser, first in air and second with an insulating material between the plates. If the capacity of the air-dielectric condenser is C_o, its impedance will be given by $\frac{-j}{\omega C_o}$. When the condenser is filled with insulating material, its impedance becomes $\frac{-j}{\varepsilon\omega C_o}$, where ε is the dielectric constant of the insulating material.

Losses in the dielectric can be taken into account by considering the dielectric constant as complex and of the form

$$\varepsilon = \varepsilon' - j\varepsilon''$$

The loss tangent of the dielectric is then defined by

$$\tan \zeta = \frac{\varepsilon''}{\varepsilon'}$$

In general ζ is a small angle, and nearly equal to the power factor p. The power factor is

$$p = \frac{\varepsilon''}{\varepsilon} = \cos \theta$$

where

$$\theta = 90° - \zeta$$

At microwave frequencies, the dielectric constant and power factor of insulating materials may be different from the values found for the same materials at lower frequencies. A considerable number of measurements have been made at centimeter wavelengths, with the results given in Table IX–I. Discrepancies that exist between values obtained by different observers may be accounted for in part by inaccuracies in measurement, and in part by the use of different samples.

The value ε' can usually be measured with good accuracy, except for very lossy materials, and the results given are in general accurate within one or two percent. The loss tangent, tan ζ, is much more difficult to measure with great accuracy, especially for low loss dielectrics, but the majority of values given are probably accurate within ten percent for the samples measured. The wide discrepancy in values obtained by different observers for certain substances, notably paraffin, ceresin and linen bakelite, is not fully understood. It may be that different precautions were taken for the presence of moisture in the samples. All values are at a temperature of approximately twenty degrees centigrade.

MICROWAVE TRANSMISSION DESIGN DATA

TABLE IX–I

Material	λ cm.	ε'	Tan ζ	Observer
Barium Titanate* Ba Ti O_3...	10.0	1760	.17	M.I.T.
Titanium Dioxide*..........	10.0	92.5	.016	M.I.T.
Titanium Dioxide*..........	10.0	104	.0006	M.I.T.
Glass (Code#)				
704..................	10.0	4.60	.0044	M.I.T.
705..................	10.0	4.90	.0053	M.I.T.
7052..................	10.0	5.10	.0061	M.I.T.
707..................	10.0	4.00	.0016	M.I.T.
772..................	10.0	4.40	.0051	M.I.T.
774..................	10.0	4.89	.0089	M.I.T.
775..................	10.0	4.26	.0043	M.I.T.
790..................	10.0	3.84	.0008	M.I.T.
012..................	10.0	6.70	.0040	M.I.T.
3320..................	10.0	4.60	.0064	M.I.T.
Benzene..................	10.0	2.27	.0021	M.I.T.
Cable Oil #5314..........	10.0	2.23	.0018	M.I.T.
Castor Oil................	10.0	2.68	.087	M.I.T.
Dow Corning				
Fluid 190**.............	10.0	2.72	.015	M.I.T.
Fluid 200**.............	10.0	2.73	.0095	M.I.T.
Ethylene Glycol............	10.0	12.5	1.1	M.I.T.
Glycerol.................	10.0	5.40	.57	M.I.T.
Nujol....................	10.0	2.14	.0008	M.I.T.
Propylene Glycol..........	10.0	5.4	.90	M.I.T.
Pyranol..................	10.0	2.74	.0026	M.I.T.
Salt Solution .00992 Molal pure Na Cl.............	10.0	77.5	.16	M.I.T.
Salt Solution .1010 Molal pure Na Cl.............	10.0	75.0	.22	M.I.T.
Styrene, N–100...........	10.0	2.42	.0017	M.I.T.
Transil Oil 10–C...........	10.0	2.16	.0028	M.I.T.
Tung Oil..................	10.0	2.61	.039	M.I.T.
Water....................	10.0	77.1	.15	M.I.T.

*The measurements reported are provisional and may be characteristic only of these samples, which were prepared from commercial raw material by the laboratory for Insulation Research, M.I.T.

**These were laboratory samples that may not be representative of plant production.

DIELECTRIC MATERIAL

TABLE IX – I — Continued

Material	λ cm.	ε′	Tan ζ	Observer
Bakelite				
Black Linen................	10.0	4.36	.031	M.I.T.
	10.0	3.98	.073	Sperry
	6.0	3.60	.07	M.I.T.
	3.2	3.79	.080	Sperry
Natural Linen..............	10.0	4.58	.012	M.I.T.
	10.0	4.15	.086	Sperry
	3.2	3.98	.093	Sperry
Black Paper................	10.0	4.05	.070	Sperry
	3.2	3.70	.085	Sperry
Cellulose Acetate...........	10.0	3.42	.064	M.I.T.
Cibanite..................	10.0	3.52	.0064	M.I.T.
Dielectene #100...........	10.0	3.40	.0030	M.I.T.
Dielectene #160...........	10.0	3.28	.0029	M.I.T.
Durite #221–X.............	10.0	3.65	.035	M.I.T.
Lucite....................	10.0	2.53	.0068	M.I.T.
	10.0	2.60	.0084	Sperry
Methyl Methacrylate**...... Polymer............	10.0	2.60	.0063	M.I.T.
Lucite....................	3.2	2.70	.016	Sperry
Plexiglass	6.0	2.70	.014	M.I.T.
Polystyrene...............	10.0	2.55	.0005	M.I.T.
Polystyrene...............	10.0	2.51	.0008	Sperry
Polystyrene...............	3.2	2.52	———	Sperry
Polystyrene Ba Ti O_3**......	10.0	154	.017	M.I.T.
Polythene #80–A**.........	10.0	2.26	.0005	M.I.T.
Polysulfone #111**.........	10.0	3.50	.020	M.I.T.
Polyvinylcarbazole.........	10.0	2.94	.0040	M.I.T.
Styramic..................	10.0	2.65	.0004	M.I.T.
Styrene Copolymer** (#1421) GE............	10.0	2.47	.0008	M.I.T.
Styrene Copolymer.........	10.0	2.52	.001	Sperry
Thiokol Powder............	10.0	4.59	.29	M.I.T.

**These were laboratory samples that may not be representative of plant production.

TABLE IX – I — Continued

Material	λ cm.	ε′	Tan ζ	Observer
Acrawax	10.0	2.92	.090	M.I.T.
Armour Wax AD1180–C	10.0	2.50	.007	M.I.T.
Armour Wax N180	10.0	2.81	.074	M.I.T.
Armour Wax SNAM-1180–A	10.0	2.61	.014	M.I.T.
Beeswax	10.0	2.30	.011	M.I.T.
	10.0	2.32	.014	Sperry
	3.2	2.33	.013	Sperry
Beeswax & Rosin 50:50	3.2	2.43	.009	Sperry
Candellilla	10.0	2.36	.0024	M.I.T.
Carnauba	10.0	2.48	.0060	M.I.T.
Ceraflux	10.0	2.15	.0010	M.I.T.
Ceresin, White	10.0	2.26	.0006	M.I.T.
	10.0	2.28	.003	Sperry
	3.2	2.18	.004	Sperry
Ceresin, Yellow	10.0	2.25	.0006	M.I.T.
	10.0	2.29	.003	Sperry
	3.2	2.18	.004	Sperry
Celowax	10.0	2.27	.0009	M.I.T.
Glycowax A	10.0	2.42	.0060	M.I.T.
Monton	10.0	2.38	.0070	M.I.T.
Nipocer N	10.0	2.45	.0095	M.I.T.
Paraffin	10.0	2.20	.0002	M.I.T.
	10.0	2.19	.002	Sperry
	3.2	2.17	.011	Sperry
Rezo Wox A	10.0	2.63	.0074	M.I.T.
Rezo Wax B	10.0	2.59	.0025	M.I.T.
Shelloc Wax	10.0	2.47	.011	M.I.T.
Wax Kote	10.0	2.46	.0045	M.I.T.
Wax S324 (Glyco Prods, Bklyn, N. Y.)	10.0	2.67	.024	M.I.T.
Italian Lava—unfired	3.2	5.46	.008	Sperry
Italian Lova—fired	3.2	5.87	.021	Sperry
Mahogany	10.0	1.91	.078	Sperry
	3.2	1.95	.094	Sperry

DIELECTRIC MATERIAL

TABLE IX – I — Continued

Material	λ cm.	ε′	Tan ζ	Observer
Oak	10.0	2.49	.070	Sperry
	3.2	2.21	.103	Sperry
Rosin	10.0	2.60	.....	Sperry
	3.2	2.45	.....	Sperry
Rubber (hard)	10.0	2.89	.007	Sperry
	3.2	2.79	.007	Sperry
Sulfur	10.0	3.83	.003	Sperry
	3.2	3.85	.002	Sperry
Transite	10.0	{5.42 5.51}	.072	Sperry
	3.2	{5.25 5.55}	.084	Sperry

CHAPTER X

CAVITY RESONATORS

A. *General Discussion*[1]

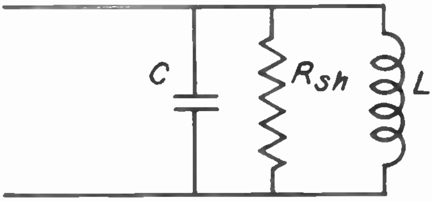


FIG. X-1

At ordinary radio frequencies, a resonant circuit consists of a coil and condenser. Associated with these circuit elements are losses which may be lumped together into an equivalent resistance, as indicated in Figure X–1, where L is the inductance, C the capacity, and R_{sh} an equivalent resistance which takes care of circuit losses. A knowledge of these three parameters permits a complete description of the behavior of the circuit in response to an impressed voltage. The resonant frequency f_o of the circuit is given by the equation

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

and the input impedance Z to the circuit at a frequency f is given by

$$\frac{1}{Z} = \frac{1}{R_{sh}} + j\left(\frac{f}{f_o} - \frac{f_o}{f}\right)\sqrt{\frac{C}{L}}$$

The characteristic impedance R_o of the circuit is defined as

$$R_o = \sqrt{\frac{L}{C}}$$

and the customary definition of Q is

$$Q = R_{sh}/\omega L$$

Ordinary resonant circuits are usually discussed in terms of R_{sh}, L, and C, and equations describing circuit behavior are usually written with these terms involved. But the behavior of a resonant circuit could be described equally well in terms of the three parameters R_o, Q, and f_o.

At microwave frequencies, the components of an ordinary resonant circuit become so small that they are physically not practical to use, and cavity resonators are required because they are physically large and because they are highly efficient. Any hollow, conducting cavity has associated with it an infinite number of resonant frequencies, each corresponding to a different configuration of electromagnetic fields which can be excited in the interior of the cavity. That a cavity resonator is like an ordinary resonant circuit in many waysc an be readily shown, but an exact parallel

[1]From unpublished notes by W. W. Hansen, also W. W. Hansen, A Type of Electrical Resonator, Jour. of App. Phys., Vol. 9, p. 654, October 1938.

cannot be drawn between the two. For example, there is no unique definition of inductance in a cavity resonator. The inductance L of a lumped circuit may be described in any of a number of ways

$$a)\ L = 2\left(\frac{\frac{1}{2}LI^2}{I^2}\right) = 2 \times \frac{\text{energy stored}}{\text{current}^2}$$

$$b)\ L = \frac{LI}{I} = \frac{\text{flux linkages}}{\text{current}}$$

$c)$ Calculate L from the equation $f_o = \dfrac{1}{2\pi\sqrt{LC}}$

with $C = 2 \times \dfrac{\text{energy stored}}{\text{voltage}^2}$

In an ordinary resonant circuit each of these definitions leads to the same calculated value of inductance. This is not true for a cavity resonator, where in the general case a different answer will be obtained by each method of calculation. A similar ambiguity will be encountered in any attempt to find a unique definition of capacity.

But in a cavity resonator there is a unique value of resonant frequency corresponding to a given mode of oscillation; for a given shape of cavity and given mode of oscillation, this resonant frequency will depend only upon the size of the cavity.

The Q of a cavity operating in a given mode of oscillation may also be uniquely defined. We have previously defined Q as

$$Q = \frac{R_{sh}}{\omega L}$$

and one might assume that as no unique definition of L has been found, there is also no unique definition of Q. But Q may also be defined as

$$Q = 2\pi \frac{\text{energy stored}}{\text{energy lost/cycle}}$$

and the Q obtained by this definition is a unique quantity for a given mode of oscillation.

No unique definition may be given for the shunt impedance of a cavity resonator; this is a fundamental shortcoming and cannot be circumvented by a good choice of definition. In fact the losses may be expressed either in terms of an equivalent series resistance, defined by the relation

$$\text{average energy lost/sec} = \frac{I^2R_{se}}{2}$$

or in terms of an equivalent shunt resistance, defined by the relation

$$\text{average energy lost/sec} = \frac{V^2}{2\ R_{sh}}$$

If these definitions were unique, the ratio of shunt to series resistance would be Q^2, as in ordinary resonant circuits. But in general, this ratio is not equal to Q^2 for cavity resonators. Cavity resonators are nearly always voltage fed, so the shunt resistance is usually considered. But this in no way makes less valid the definition of series resistance; there is just an inherent ambiguity and a choice must be made.

Resonant Frequency of a Cavity Resonator. To obtain the resonant frequency of a cavity resonator, solutions to Maxwell's equations must be found which satisfy the boundary conditions imposed by the resonator. It is nearly always assumed that the cavity is made of a perfect conductor, which means that for calculations of resonant frequency, penetrations of the fields into the walls of the resonator are neglected. The boundary conditions which must then be met are that no tangential electric field and no normal magnetic field exist at the surface of the cavity walls.

The electric and magnetic fields are derivable from some sort of potential which satisfies a wave equation, and the calculation usually leads to an attempt to find a suitable potential function which is applicable to the problem in hand. An analytical solution is only possible for a limited number of cavity shapes, shapes which can be readily defined in terms of one of the standard co-ordinate systems. A number of approximate methods of calculation have been developed which give more or less accurate answers for many cavity shapes that cannot be solved by analytical methods. Two cavities that are identical in shape but different in size will have resonant wavelengths that are proportional to the linear dimensions of the cavity.

Q of a Resonator. The Q of a resonator has previously been defined by

$$Q = 2\pi \frac{\text{energy stored}}{\text{energy lost/cycle}}$$

This is a unique definition for a given mode in a given cavity. The quantity Q is frequently used as a figure of merit for a resonant circuit, for it is a measure of the damping of a freely oscillating circuit. It can be shown that the total field energy in a freely oscillating circuit varies with time according to the equation

$$W = W_o e^{-\frac{\omega t}{Q}}$$

where W is the energy at a time t and W_o the initial energy at $t = O$. The Q of a resonator is also a measure of the sharpness of the resonant circuit, as the bandwidth between 70% response points Δf is related to the resonant frequency f_o by

$$\frac{\Delta f}{f_o} = \frac{1}{Q}$$

To calculate the Q of a resonator, the relation must be found between energy stored in the cavity and losses in the cavity. If dielectric losses are neglected, and only losses resulting from conduction currents in the resonator walls considered, these losses are

$$\text{energy lost/cycle} = \frac{\delta}{8}\int B^2 \left| d\sigma \right|$$

where δ is the skin depth, B the magnetic field at the wall of the cavity, and $d\sigma$ an element of area in the cavity wall. The integral is carried out over the interior surface of the cavity. The energy stored in the cavity is

$$\text{energy stored} = \frac{1}{8\pi}\int B^2\, d\tau$$

with $d\tau$ an element of volume, and the integral carried out over the volume of the cavity. The Q of the cavity is then

$$Q = \frac{\lambda}{\delta}\,\frac{2\int B^2\; d\tau}{\lambda\int B^2\,|d\sigma|}$$

It will be seen that Q is a dimensionless quantity. If this equation is rewritten as

$$Q\,\frac{\delta}{\lambda} = \frac{2\int B^2\; d\tau}{\lambda\int B^2\,|d\sigma|}$$

the right hand side is independent of the wavelength and depends only upon the size and shape of the cavity. It is therefore sometimes known as the form factor of the cavity. To a first approximation this becomes

$$Q\,\frac{\delta}{\lambda} \sim \frac{2}{\lambda}\,\frac{\int d\tau}{\int|d\sigma|}$$

as the form factor is not a rapidly varying function of the flux distribution. Also, because the magnetic field is a maximum at or near the surface of the resonator, the mean surface value of B^2 will be twice the mean value throughout the volume, and it can be said approximately that

$$Q\,\frac{\delta}{\lambda} \cong \frac{1}{\lambda}\,\frac{V}{S}$$

where V is the volume and S the bounding area of the resonator. A substitution of typical values will show that Q's greater than 1000 are easily obtainable at microwave frequencies. The cavity Q is also proportional to the volume to surface ratio. Large cavities will therefore have high Q's in general, and cavities that are highly re-entrant are likely to have Q's that are somewhat lower than the average. Two cavities of the same shape but different in size will have Q's that are proportional to the square root of the resonant wavelength.

Shunt Impedance of a Cavity Resonator. The shunt impedance, or more properly the shunt resistance, of a resonator is not a quantity that can be uniquely defined. The shunt resistance of an ordinary resonant circuit can be defined as

$$R_{sh} = \frac{\text{voltage}^2}{2\times\text{energy lost/sec}}$$

This definition is in many ways the most useful one that can be applied to a cavity resonator, as it is a factor of the resonator which tells the amount of input power that must be supplied to the resonator to maintain a given voltage across whatever path may be chosen. It may be calculated by the following conditions. The energy lost per second is given by

$$\text{energy lost/sec} = \frac{\delta f}{8} \int B^2 |d\sigma|$$

where δ is the skin depth, f the frequency, B the magnetic field at the surface, and $d\sigma$ an element of surface in the cavity. The integration is carried out over the enclosing area of the cavity.

The voltage is defined as the line integral of the electric field, which by Stokes' theorem may be set equal to

$$\int E \cdot ds_1 = -\frac{1}{c} \int \dot{B} \cdot d\sigma_1$$

where ds_1 is an element of length, and $d\sigma_1$ an element of area. The value of this integral will depend upon the path of integration chosen. Usually the path chosen is one which gives a maximum voltage difference without an extreme path being chosen. For example, in the resonator illustrated in Fig. X-2, the path of integration is chosen along the axis of the resonator, with the path being closed around outside the cavity. The element of length ds_1 is along the axis of the resonator as indicated. The element of area $d\sigma_1$ lies in a cross section containing the axis and the integration is carried out over the portion of that cross section enclosed by the path of the line integral. If the resonator is excited by a beam of electrons passing through, the voltage used in shunt impedance calculations is obtained by integrating the electric field along the path of the electron beam.

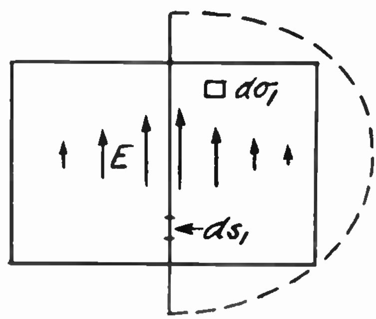


FIG. X-2

The shunt resistance obtained by this method of calculation is then given by

$$R = 16\pi^2 \frac{[\int B \cdot d\sigma_1]^2}{\lambda^2 \int B^2 |d\sigma|} \frac{\lambda c}{\delta} \; emu$$

The factor

$$16\pi^2 \frac{[\int B \cdot d\sigma_1]^2}{\lambda^2 \int B^2 |d\sigma|}$$

is independent of the frequency and has to do only with the shape of the resonator. It then follows that for two resonators of the same shape but of different size, the shunt impedance will be proportional to the square root of the resonant wavelength.

CAVITY RESONATORS

To get some idea of the magnitudes involved, for copper at 10 cm.

$$\frac{\lambda c}{\delta} = 2.5 \times 10^6 \text{ ohms}$$

and the shape factor is approximately unity, so the shunt resistance is of the order of 1 megohm.

That a cavity has a high Q in no way implies that it also has a high value of shunt resistance, in fact it is possible to find cavities with any combination of Q and R_{sh}.

B. *Characteristics of Various Cavity Resonators*

1. Rectangular Resonators

The characteristics of a rectangular prism resonator, such as illustrated in Fig. X–3, are readily calculated by analytical methods.

A resonant wavelength λ_o will be found in such a resonator when

$$\lambda_o = \frac{4}{\sqrt{\left(\frac{l}{a}\right)^2 + \left(\frac{m}{b}\right)^2 + \left(\frac{n}{z_o}\right)^2}}$$

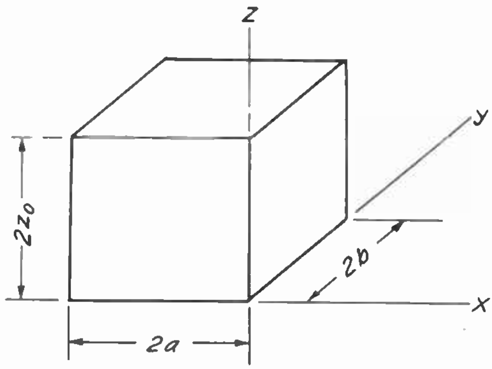


FIG. X-3

where l = number of half wave variations of field along the x axis
m = number of half wave variations of field along the y axis
n = number of half wave variations of field along the z axis
l, m, n = 0,1,2,3, . . . but not more than one may equal zero for fields to exist.

For large resonators of this type, the number of modes dN in a range of wavelength $d\lambda$ is

$$dN = 8\pi \frac{V}{\lambda_1^4} d\lambda$$

where V is the volume of the resonator and λ_1 is the center of the wavelength band $d\lambda$.

The number of resonant states N in such a resonator with wavelengths greater than some minimum wavelength λ_2 is given approximately by

$$N = \frac{8\pi}{3} \frac{V}{\lambda_2^3}$$

This approximate formula is quite accurate even for low N.

Consider a rectangular resonator in which $a = b$, and where $l = m = 1$, and $n = 0$. This is illustrated in Fig. X–4.

The resonant wavelength in such a resonator is given by

$$\lambda_o = 2\sqrt{2}\, a$$

The Q of this resonator is given by

$$Q = .353 \frac{\lambda_o}{\delta} \frac{1}{1 + \frac{a}{2 z_o}}$$

and the shunt impedance by

$$R = 120 \frac{\lambda_o}{\delta} \frac{z_o}{a} \frac{1}{1 + \frac{a}{2 z_o}}$$

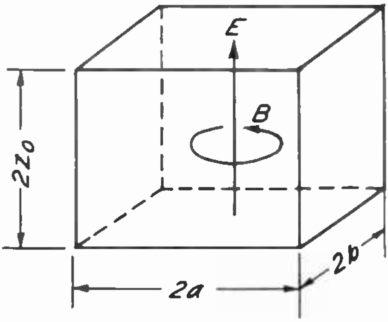


FIG. X-4

where δ is the skin depth. For large cubical resonators of this type in which $a = b = z_0$, resonators operating in a high mode of oscillation, the Q is approximately

$$Q = \frac{\lambda_o}{\delta} \frac{a}{2 \lambda_o}$$

2. Cylindrical Resonators. The infinite number of modes that exist in cylindrical resonators can be divided into two general types: 1. Those in which the electric field is purely transverse to the axis of the cylinder and 2. Those in which the magnetic field is purely transverse to the axis of the cylinder. The notation that will be applied to cylindrical resonators is illustrated in Fig. X–5.

TE Modes. Considering first those modes in which the electric field has no component along the axis, the resonant wavelength λ_o is given by

$$\lambda_o = \frac{4}{\sqrt{\left(\frac{l}{z_o}\right)^2 + \left(\frac{2u'_{n,m}}{\pi a}\right)^2}}$$

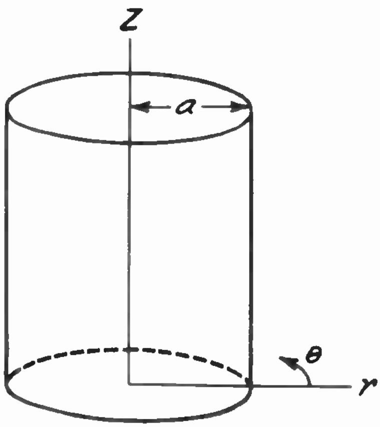


FIG. X-5

Each of these modes occurs when the resonator is effectively a section of cylindrical wave guide that is an integral number of half wavelengths long for some TE mode of transmission in the wave guide. The term l gives the number of half wavelengths contained in the resonator, and must therefore be an integral number, i.e., 1,2,3, . . . No mode exists in which $l = 0$. If the $TE_{n,m}$ mode in a wave guide is the mode being excited in the resonator, $u'_{n,m}$ is the mth root of the equation

$$J_n'(u') = 0$$

A tabulation of some of the lower roots of this equation is given in Chapter IV, Section B–2.

The Q of the resonator when a half wave long will be given by

$$Q = \frac{\lambda_o}{\delta}\frac{1}{\lambda_o}\frac{z_o\left[u'_{n,m} + \frac{\pi a}{2z_o}\right]^2\left[1 - \left(\frac{n}{u'_{n,m}}\right)^2\right]}{\left[\frac{z_o}{a}u'_{n,m} + \frac{a^2\,\pi^2}{4z_o^2} + \frac{a\,(z_o - a)}{4z_o^2}\frac{\pi^2\,n^2}{u'^2_{n,m}}\right]}$$

The Q is seen to decrease with increasing order of excitation. In the case where $n = 0$, and $l = m = 1$, the Q is given by

$$Q = \frac{\lambda_o}{\delta}\,.610\sqrt{1 + \left(.410\,\frac{a}{z_o}\right)}\;\frac{1 + .168\left(\frac{a}{z_o}\right)^2}{1 + .168\left(\frac{a}{z_o}\right)^3}$$

Resonators operating in this mode of oscillation, which corresponds to the $TE_{0,1}$ mode in a wave guide, have an exceptionally high Q, and are ideal for precision wavemeters except that some sort of damping system must be used to eliminate the other modes.

TM Modes. For the other class of resonant modes in which the magnetic field has no component along the axis, the resonant wavelengths are given by

$$\lambda_o = \frac{4}{\sqrt{\left(\frac{l}{z_o}\right)^2 + \left(\frac{2u_{n,m}}{\pi a}\right)^2}}$$

As before, each of these modes occurs when the resonator is effectively a section of cylindrical wave guide that is an integral number of half wavelengths long, this time for a TM mode of propagation. As before, l is an integer (1,2,3,4, . . .) which gives the number of half waves in the resonator. In addition, modes exist when $l = 0$, these modes have an electric field that is purely axial and do not represent a possible mode of propagation in a wave guide.

If the $TM_{n,m}$ mode in a wave guide is the mode excited in the resonator, $u_{n,m}$ is the mth root of the equation

$$J_n(u) = 0$$

When $l = 0$, the axial electric field will still be given by an nth order Bessel function, which will have its mth root at the radius a. As before the term $u_{n,m}$ will be the mth root of the nth order Bessel function which determines the axial field. A tabulation of some of the lower roots of this equation is given in Chapter IV Section B–2.

The Q of the TM modes for a resonator a half wave in length when $n \neq 0$ is given by

$$Q = \frac{\lambda_o}{\delta}\frac{a}{\lambda_o}\frac{1}{1 + \frac{a}{z_o}}$$

If $n = 0$ the Q is given by

$$Q = \frac{\lambda_o}{\delta} \frac{a}{\lambda_o} \frac{1}{1 + \frac{a}{2z_o}}$$

With this exception, for these modes the Q is not a function of the order of excitation, this is in contrast to the other class of modes.

The shunt resistance of the mode in which $l = n = 0$, and $m = 1$, i.e., the lowest mode in which the electric field is purely axial, is given by

$$R = 144 \frac{\lambda_o}{\delta} \frac{z_o}{a} \frac{1}{1 + \frac{a}{2z_o}}$$

and the resonant wavelength λ_o by

$$\lambda_o = 2.61\, a$$

3. Spherical Resonators. The first resonance will occur in a spherical cavity of radius a when

$$\lambda_o = 2.28\, a$$

and the second resonance when

$$\lambda_o = 1.4\, a$$

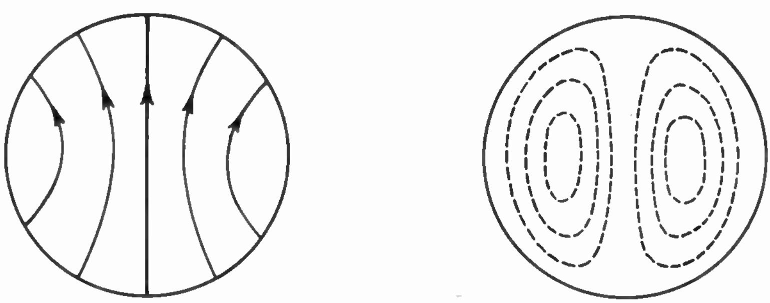
The field configuration for these first two modes is shown in Fig. X-6.

The Q of a spherical cavity operating in the dominant mode is

$$Q = .318 \frac{\lambda_o}{\delta}$$

and the shunt impedance given by

$$R = 104.4 \frac{\lambda_o}{\delta}$$



$\lambda = 2.28a$
Electric field

$\lambda = 1.4a$
Magnetic field

FIG. X-6

CAVITY RESONATORS

4. Spherical Resonators With Reentrant Cones.[2] A resonator that can be solved by analytical methods consists of part of a sphere of radius a, and two cones whose apex is at the center of the sphere and whose half angle is θ_o. Such a resonator is sketched in Fig. X–7, and the field corresponding to the fundamental mode of oscillation is illustrated.

The resonant wavelength of this cavity is given by

$$\lambda_o = 4a$$

and is not a function of the angle θ_o. The Q of the resonator does vary with the angle θ_o, and in Fig. X–8, $Q \frac{\delta}{\lambda_o}$ is plotted as a function of θ_o. The maximum value of Q is found at an angle of $\theta_o = 34°$, and is equal to

$$Q = .1095 \frac{\lambda_o}{\delta}$$

The shunt impedance is also a function of the angle θ_o, and Fig. X–9 gives the variation of $R \frac{\delta}{\lambda_o}$ with θ_o. The maximum value of R is reached at an angle of $\theta_o = 9°$. At this angle R is given by

$$R = 32.04 \frac{\lambda_o}{\delta}$$

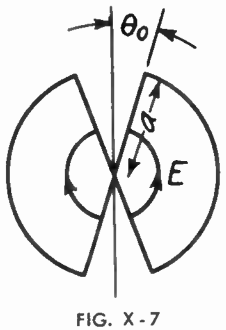


FIG. X-7

5. Ellipsoid-Hyperboloid Resonators.[3] Another type of resonator that has been solved is the ellipsoid-hyperboloid shown in Fig. X–10. The resonator is a figure of revolution about the axis passing through the foci. The resonant wavelength λ_o of this resonator may be determined from Fig. X–11. In this figure, the distance a between the foci is held constant, and also the hyperboloid that determines part of the resonator. The equatorial radius x_o is varied, and λ_o/x_o is plotted as a function of the shape factor σ_o, defined by $\sigma_o = 2\pi x_o/a$. Also shown on the curve are the resonator shapes which correspond to various values of σ_o; these resonators are scaled in the drawing so as to maintain constant the resonant wavelength λ_o.

[2]See W. W. Hansen and R. D. Richtmeyer, On Resonators Suitable for Klystron Oscillators, Jour. of App. Phys., Vol. 10, p. 189, March 1939.
[3]Hansen and Richtmeyer, loc. cit.

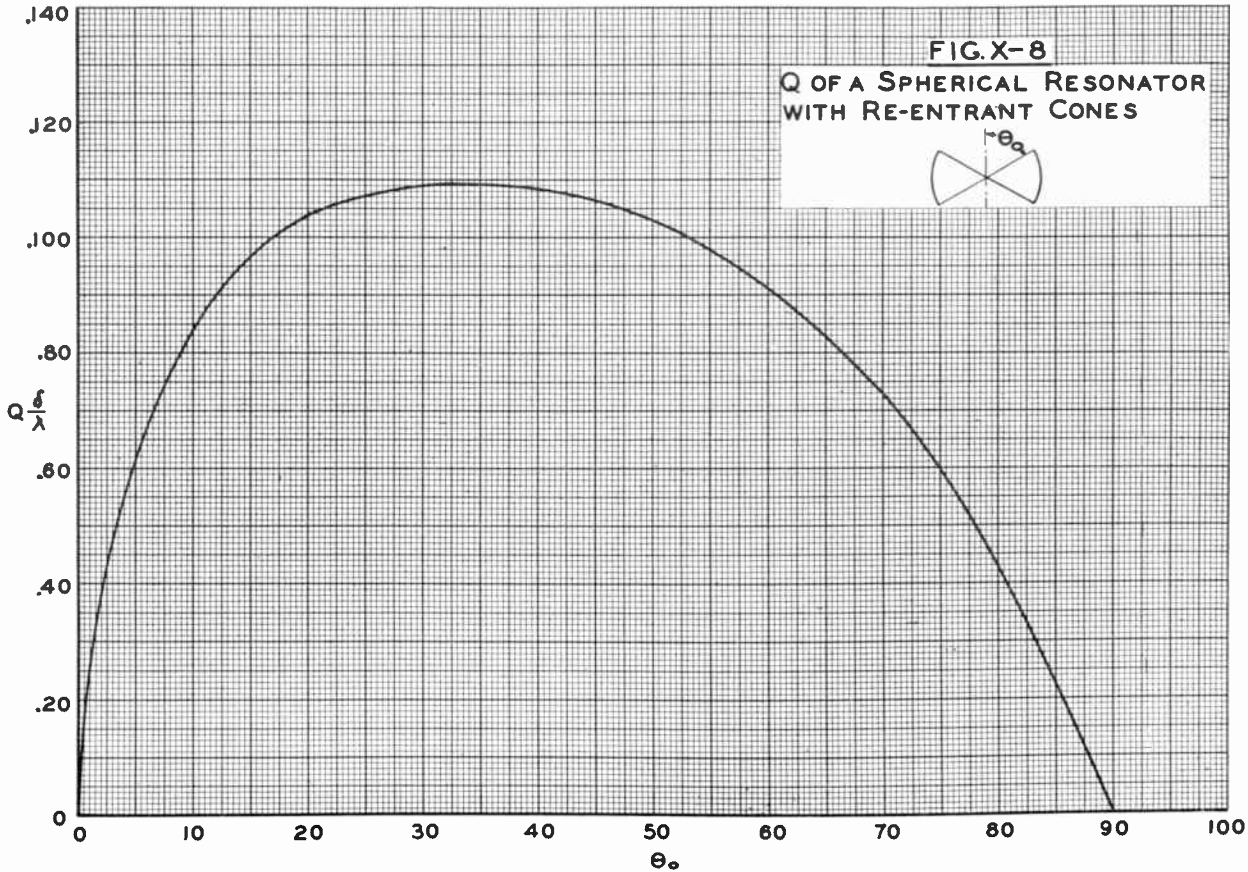


FIG. X-8
Q OF A SPHERICAL RESONATOR WITH RE-ENTRANT CONES

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CAVITY RESONATORS

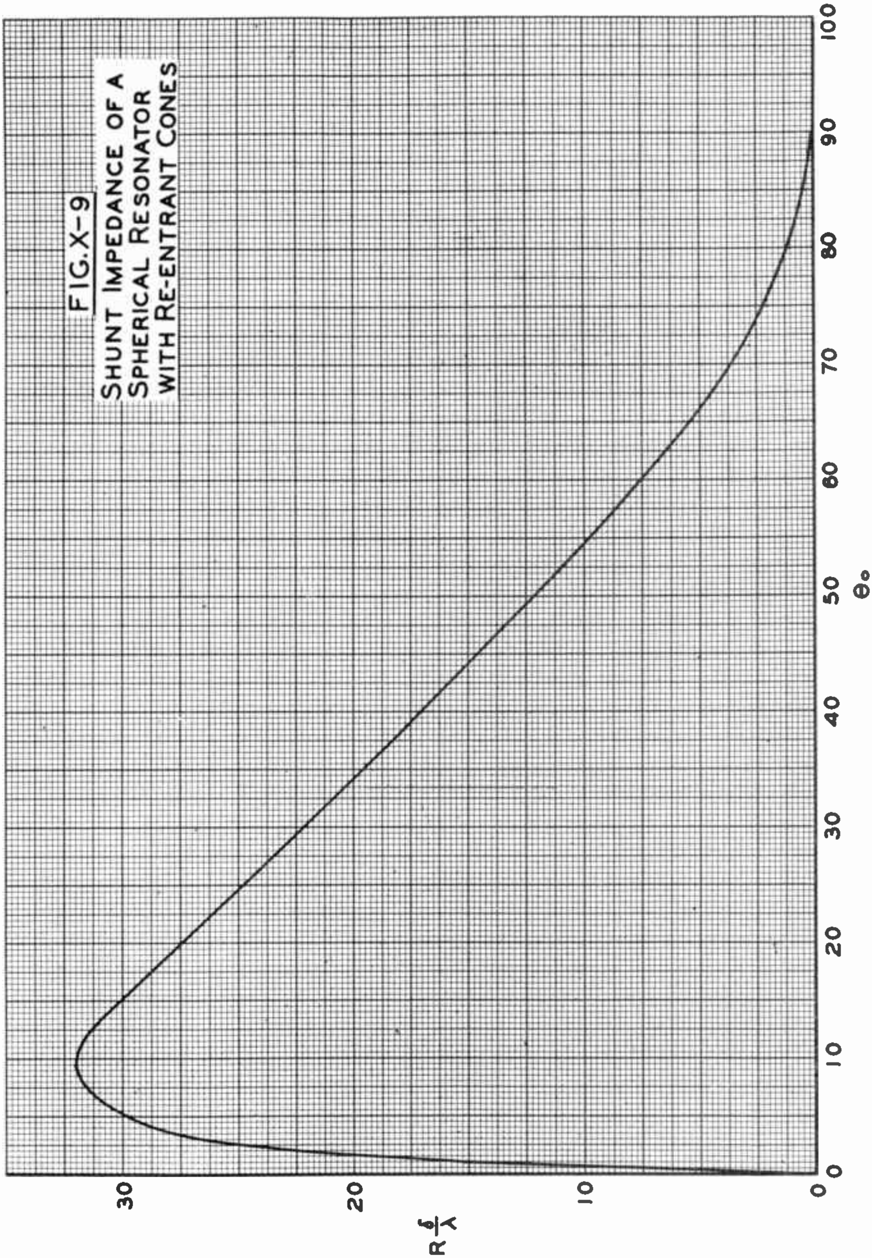

FIG. X-9
SHUNT IMPEDANCE OF A SPHERICAL RESONATOR WITH RE-ENTRANT CONES

MICROWAVE TRANSMISSION DESIGN DATA

The Q of the resonator is also a function of the shape factor σ_o, and in Fig. X–12, $\frac{Q\delta}{\lambda_o}$ is plotted against σ_o. The variation in shunt impedance with the shape can be obtained from Fig. X–13, where $\frac{R\delta}{\lambda_o}$ is plotted against σ_o.

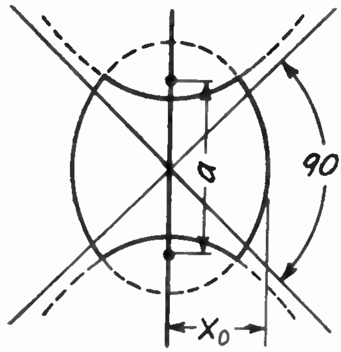

FIG. X - 10

6. Concentric Line Resonator. One type of concentric line resonator is that shown in Fig. X–14. The field equations of the dominant mode in this resonator are

$$E_r = \frac{A}{r} \sin \frac{\pi}{2\, z_o}\, z$$
$$E_\theta = E_z = 0$$
$$B_r = B_z = 0$$
$$B_\theta = \frac{A}{r} \cos \frac{\pi}{2\, z_o}\, z$$

The resonant wavelength of this resonator is

$$\lambda_o = 4\, z_o$$

The Q is given by

$$Q = \frac{\lambda_o}{\delta} \frac{1}{4 + \frac{2\, z_o}{b} \frac{1 + b/a}{\log_e b/a}}$$

The optimum diametric ratio for lowest losses is $b/a = 3.6$, this leads to a formula for Q of

$$Q = \frac{\lambda_o}{\delta} \frac{1}{4 + 7.2 \frac{z_o}{b}}$$

The shunt impedance of this resonator is

$$R = \frac{60}{\pi} \frac{\lambda_o}{\delta} \frac{b}{z_o} \frac{\log_e^2 b/a}{1 + b/a} \frac{1}{1 + 2 \frac{b}{z_o} \frac{\log_e b/a}{1 + b/a}}$$

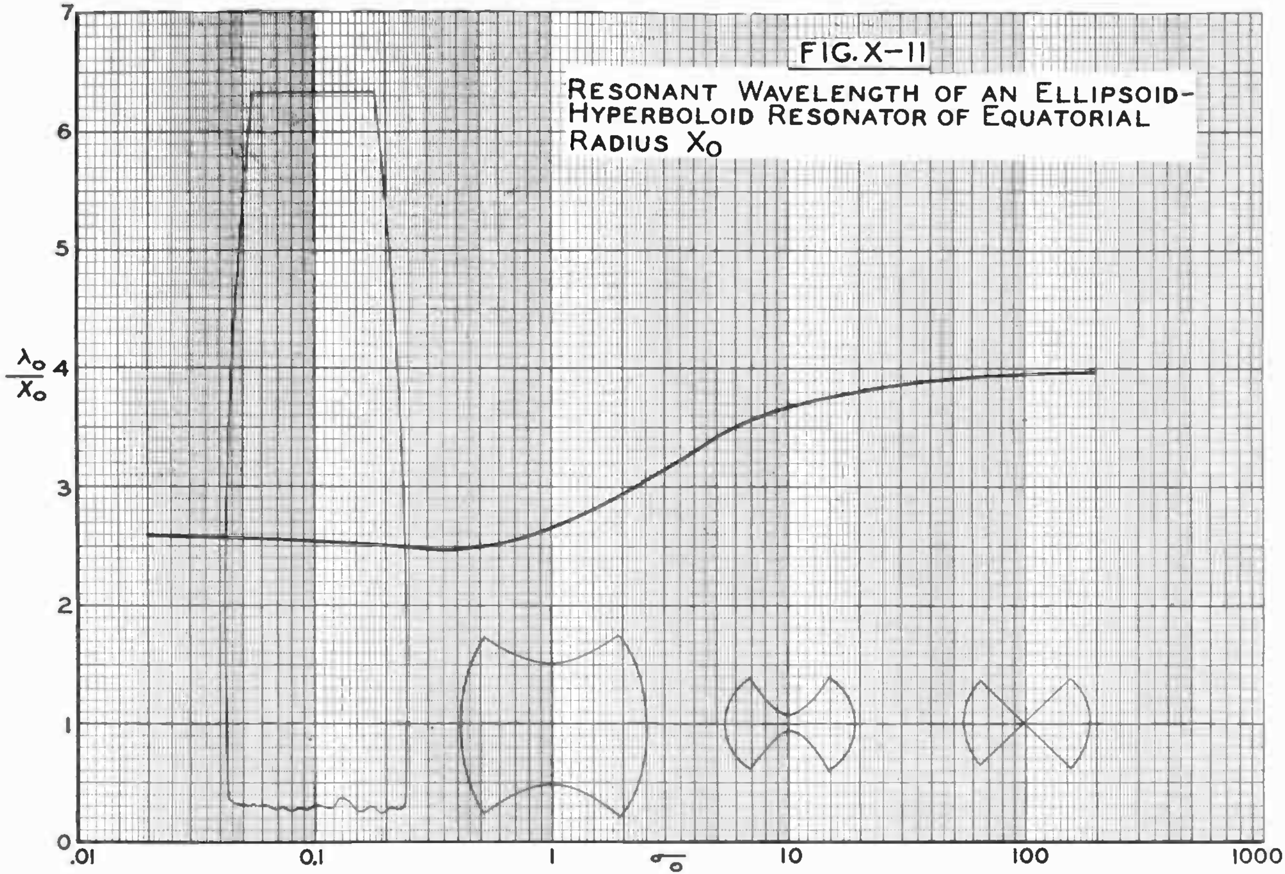

FIG. X-11

RESONANT WAVELENGTH OF AN ELLIPSOID-HYPERBOLOID RESONATOR OF EQUATORIAL RADIUS X_0

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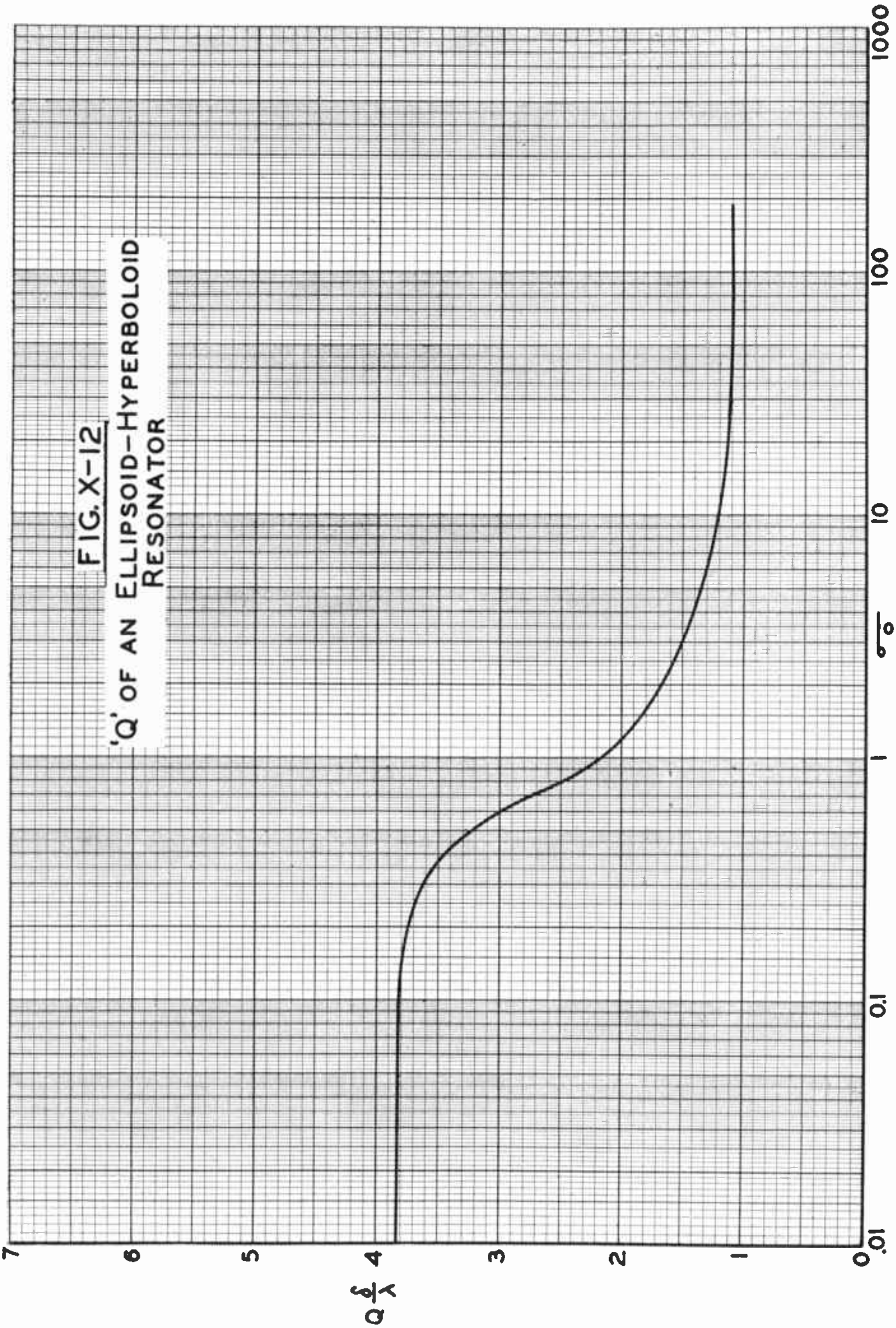


FIG. X-12

'Q' OF AN ELLIPSOID-HYPERBOLOID RESONATOR

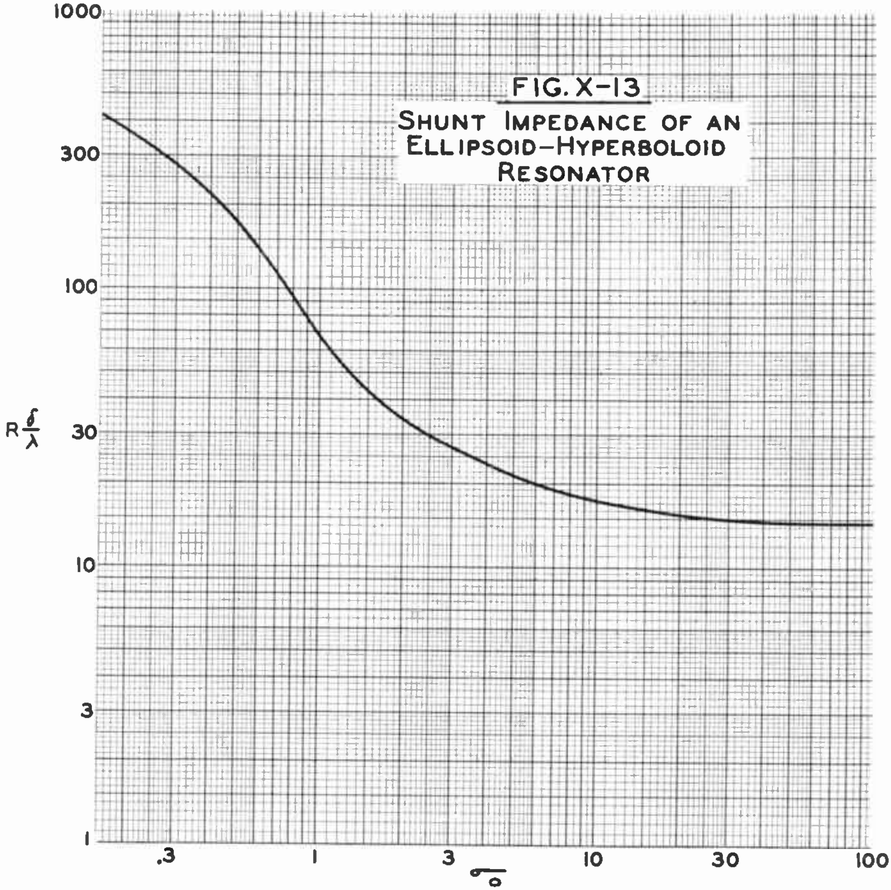


FIG. X-13
SHUNT IMPEDANCE OF AN
ELLIPSOID-HYPERBOLOID
RESONATOR
1000
300
100
30
10
3
1
.3
1
3
10
30
100

When $b \ll \lambda$, the maximum R is found when $b/a = 9.2$. For this value, the above formula reduces to

$$R = 30 \frac{\lambda_o}{\delta} \frac{1}{1.41 + 3.24 \frac{z_o}{b}}$$

and for $b/\lambda_o/\ll/1$, this reduces to

$$R \cong 9.25 \frac{\lambda_o}{\delta} \frac{b}{z_o}$$

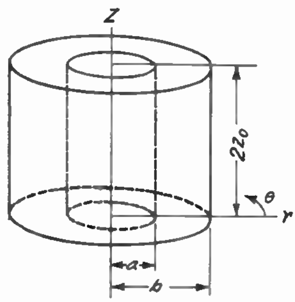


FIG. X - 14

It will be observed that optimum Q occurs at $b/a = 3.6$, and optimum R at $b/a = 9.2$. But at $b/a = 3.6$, R has fallen only to 74% of its maximum value, and at $b/a = 9.2$, Q has fallen only to 78% of its maximum. So the values are not critical with diametric ratio.

The most important results of this section are collected and compared in Table X–I.

7. Quarter Wave Concentric Line Resonators. One widely used type of resonator is the quarter-wave concentric line cavity illustrated in Fig. X–15.

If the length of the cavity z_1 is much greater than the radii, resonance will occur when $\lambda_0 = 4z_0$. If the dimension δ is sufficiently small to introduce an appreciable capacity at the end of the line, the length for resonance must be correspondingly modified. An approximate formula that considers the resonator as a length of concentric line terminated by a lumped capacity is

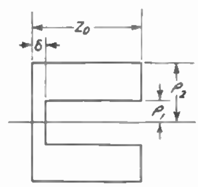


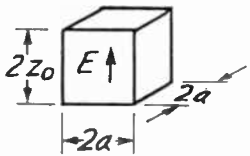
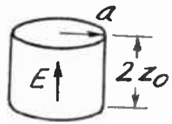
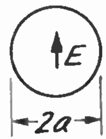
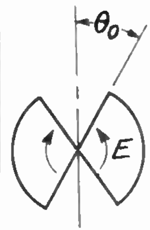
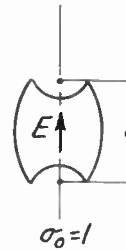
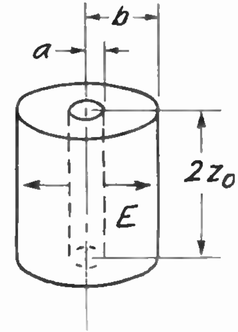
FIG. X - 15

$$\lambda_o = 2\pi \left(\frac{2z_o \, \rho_1^2}{\delta} \log_e \frac{\rho_2}{\rho_1} \right)^{\frac{1}{2}}$$

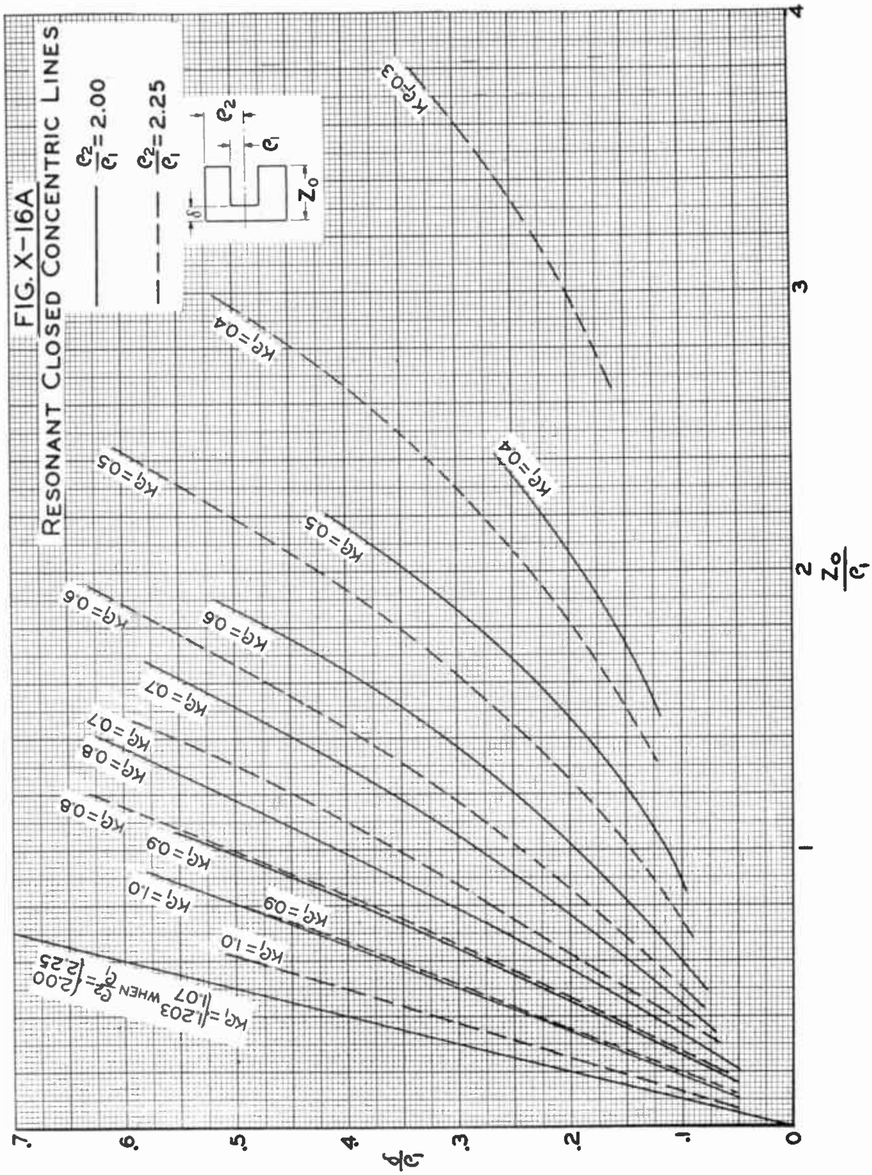
Even this approximation falls far short of the truth when the length of the resonator, z_o, becomes small. A better approximation has been worked out for this case, with the results given in Figs. X–16. These charts make it possible to determine within a few percent the resonant wavelength of a cavity of given dimensions. On all of the charts, the ratio $\frac{\delta}{\rho_1}$ is plotted against the ratio $\frac{z_o}{\rho_1}$. There are several families of curves, each family for a given ratio $\frac{\rho_2}{\rho_1}$. Each curve in the family is for a given value of $k\rho_1$, where k is related to the resonant wavelength λ_o by the formula

$$k = \frac{2\pi}{\lambda_o}$$

TABLE X–I
Characteristics of Resonators

Type of Resonator	Rectangular Prism	Cylinder	Sphere	Dimpled Sphere	Ellipsoid-Hyperboloid	Coaxial
						
λ_0	$2.828a$	$2.61a$	$2.28a$	$4a$	$1.3a$	$4\ z_0$
$Q\frac{\delta}{\lambda}$	$.353\ \dfrac{1}{1+\dfrac{a}{2z_0}}$	$.383\ \dfrac{1}{1+\dfrac{a}{2z_0}}$	.318	.1095 for $\theta_0 = 34°$	.22	$\dfrac{1}{4+7.2\,\dfrac{z_0}{b}}$ for $\dfrac{b}{a}$ 3.6
$R\frac{\delta}{\lambda}$	$120\ \dfrac{z_0}{a}\ \dfrac{1}{1+\dfrac{a}{2z_0}}$	$144\ \dfrac{z_0}{a}\ \dfrac{1}{1+\dfrac{a}{2z_0}}$	104.4	32.04 for $\theta_0 = 9°$	76.4	$30\ \dfrac{1}{1.41+3.24\,\dfrac{z_0}{b}}$ for $\dfrac{b}{a} = 9.2$

MICROWAVE TRANSMISSION DESIGN DATA



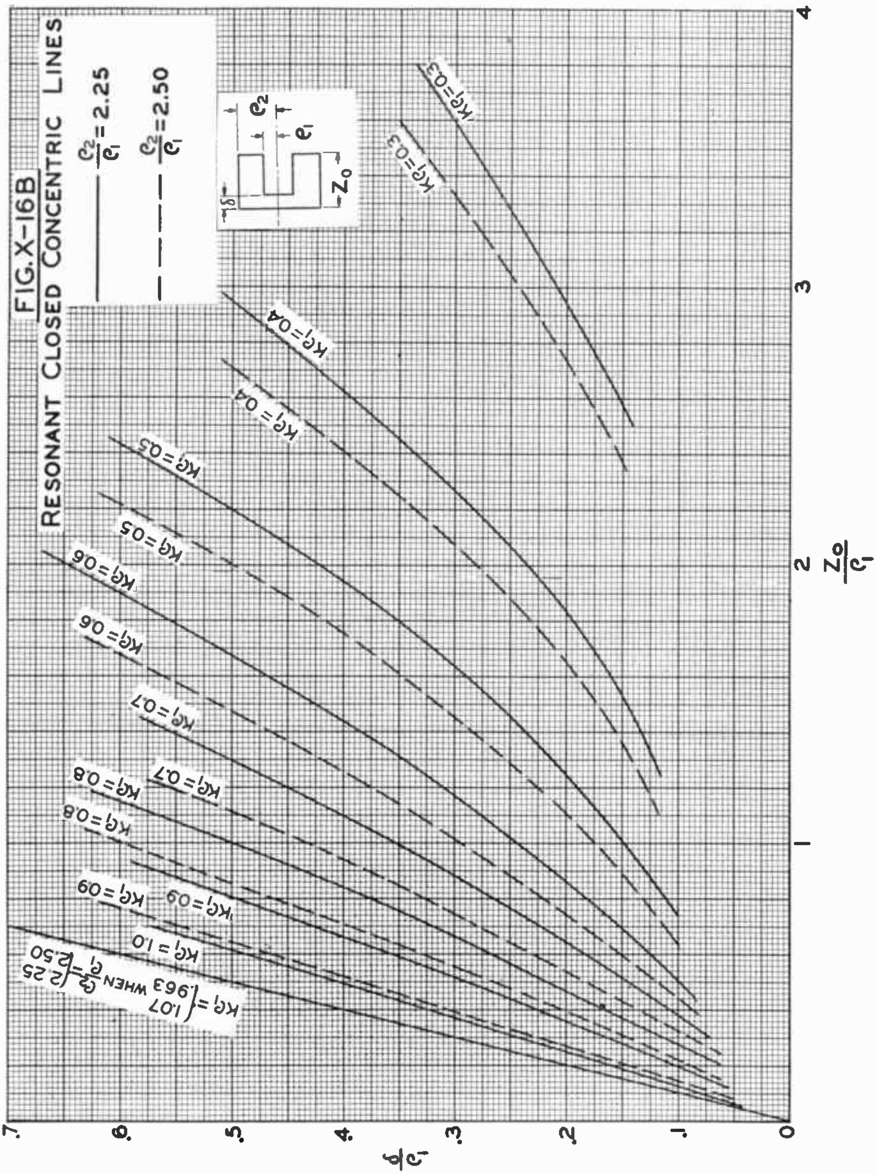


FIG. X-16B
RESONANT CLOSED CONCENTRIC LINES
$\frac{e_2}{e_1} = 2.25$
$\frac{e_2}{e_1} = 2.50$
e_2
e_1
Z_0
δ
$Kq = 0.3$
$Kq = 0.4$
$Kq = 0.5$
$Kq = 0.6$
$Kq = 0.7$
$Kq = 0.8$
$Kq = 0.9$
$Kq = 1.0$
$Kq = 1.07$ / $.963$ WHEN $\frac{e_2}{e_1} = 2.25$ / 2.50
δ / e_1
Z_0 / e_1
0 .1 .2 .3 .4 .5 .6 .7
1 2 3 4

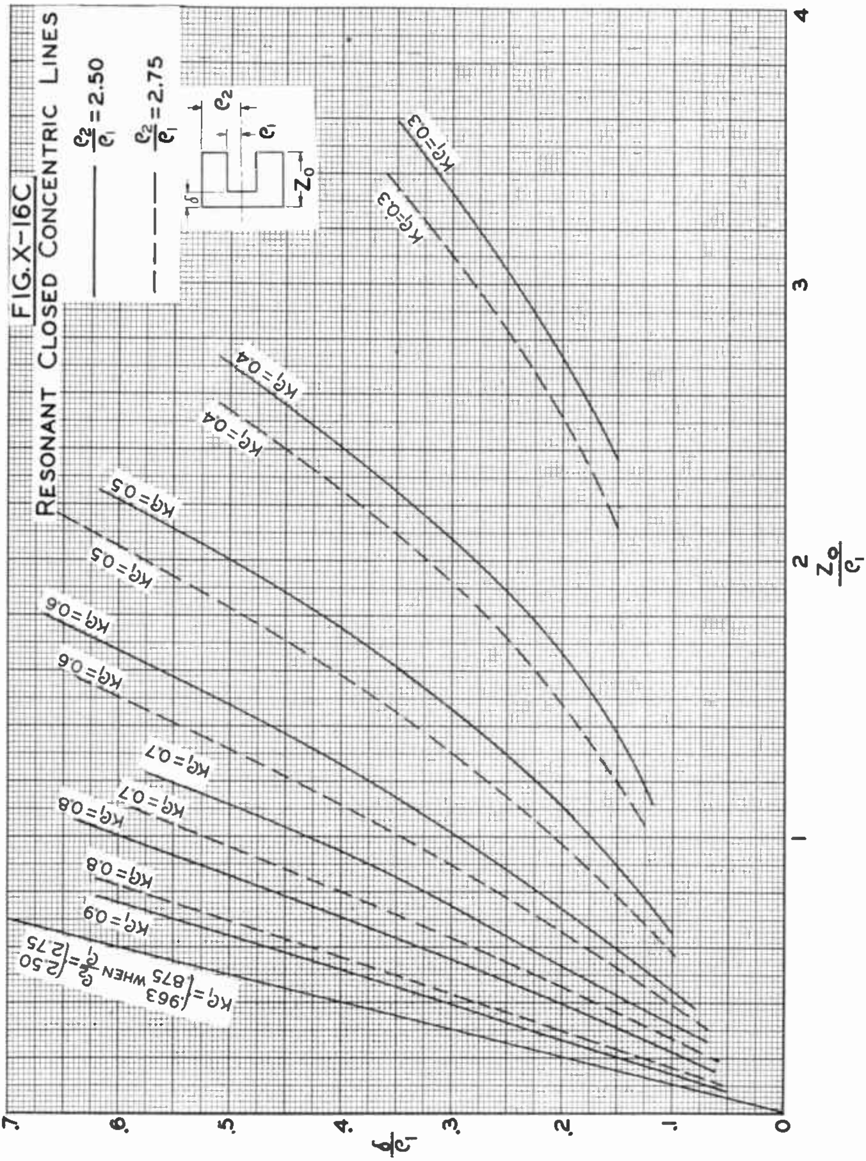


FIG. X-16C
RESONANT CLOSED CONCENTRIC LINES
$\frac{e_2}{e_1} = 2.50$
$\frac{e_2}{e_1} = 2.75$
e_2
e_1
Z_0
δ
$K\mathcal{G} = 0.3$
$K\mathcal{G} = 0.3$
$K\mathcal{G} = 0.4$
$K\mathcal{G} = 0.4$
$K\mathcal{G} = 0.5$
$K\mathcal{G} = 0.5$
$K\mathcal{G} = 0.6$
$K\mathcal{G} = 0.6$
$K\mathcal{G} = 0.7$
$K\mathcal{G} = 0.7$
$K\mathcal{G} = 0.8$
$K\mathcal{G} = 0.8$
$K\mathcal{G} = 0.9$
$K\mathcal{G} = \{.963, .875$ WHEN $\frac{e_2}{e_1} = \{2.50, 2.75$
0
1
2
3
4
$\frac{Z_0}{e_1}$
.1
.2
.3
.4
.5
.6
.7
δ / e_1

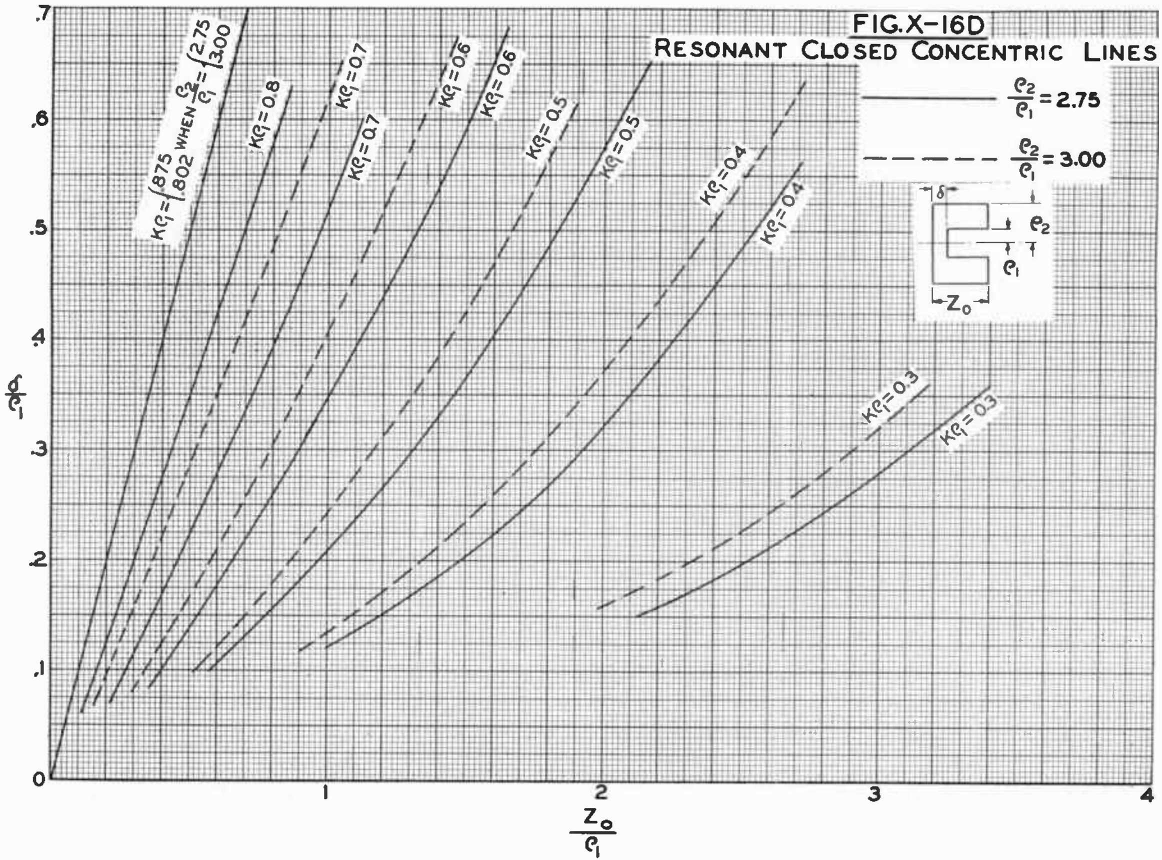
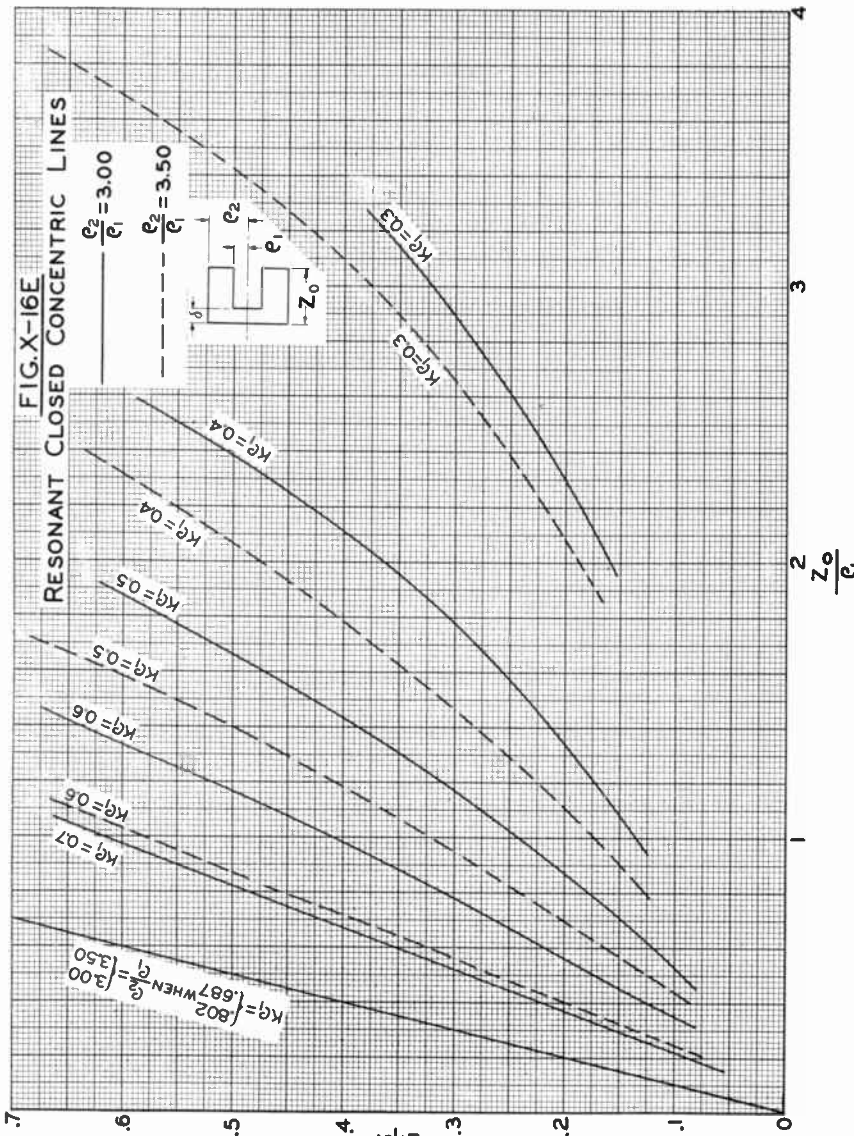


FIG. X-16D
RESONANT CLOSED CONCENTRIC LINES

MICROWAVE TRANSMISSION DESIGN DATA



Two families of curves are plotted on each chart, to aid in interpolation.

If the four dimensions of the cavity are known, λ_o can be determined, or if three dimensions and λ_o are known, the fourth dimension of the cavity is obtainable. Shunt impedance and Q can be determined from Figs. X–17.[4] In these figures the resonant wavelength is assumed constant at $\lambda_o = 3.2$ cm., and the figures are drawn to scale The resonator material is assumed to be copper. Conversion can be made to other wavelengths by scaling the dimensions up or down in proportion to the wavelength. The Q and shunt resistance will vary as the square root of the wavelength.

The Q and R of each resonator are given below the resonator, the R values should be multiplied by 10^5 to give ohms, while the Q values should be multiplied by 10^3.

In each group of figures, the distance δ is held constant, as specified. For each column the dimension z_o is held constant, while for each row the dimension ρ_1 is specified. The table included in each group gives the value of ρ_2 that will then be required for resonance at the specified wavelength.

The values of R and Q given by these figures are not so accurate as the values of λ_o. The accuracy for R and Q will be within 10% for the resonators most nearly resembling a quarter-wave line, and for shorter resonators the errors may be up to 25% or more. But the values are exact in the extreme case of the flat cylinders.

C. *Cavity Resonators as Filters*

Cavity resonators are used in microwave systems as single tuned circuits, and as such find numerous applications as wavemeters, filters, etc. Many of the expressions that have been derived for tuned circuits at ordinary radio frequencies can be applied with equal accuracy to cavity resonators at microwave frequencies. The bandwidth of a cavity resonator between 70.7% response points is given by

$$Q = \frac{f_o}{\Delta f}$$

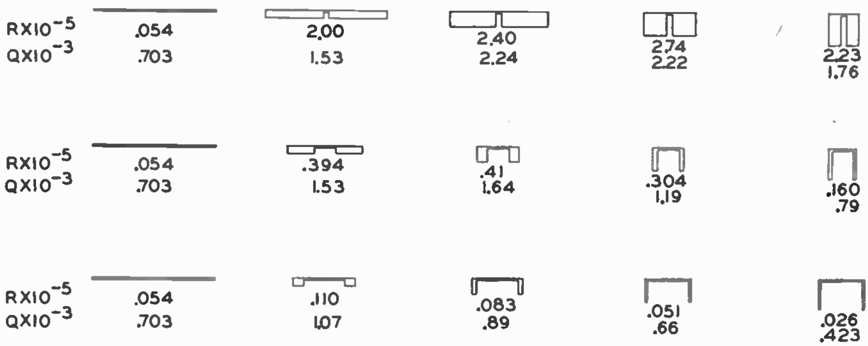
where Δf if the bandwidth and f_o the resonant frequency. When used as a filter, the loss in the cavity is given by

$$Db \text{ loss} = 10 \log \frac{Q_u}{Q_u - Q_L}$$

where Q_u is the unloaded Q of the cavity and Q_L the loaded Q.

[4]See Westinghouse Research Report SR-127, Design Characteristics of Resonant Cavities.

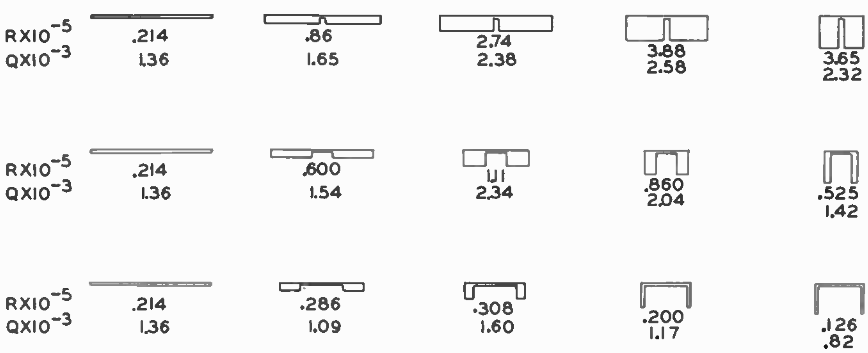
MICROWAVE TRANSMISSION DESIGN DATA



c_2–INCHES

c_1–IN. \ Z_o–INCHES	.010	.060	.120	.180	.240
.020	.485	.472	.378	.204	.113
.080	.485	.294	.163	.120	.102
.120	.485	.242	.198	.179	.172

δ =.010 INCHES



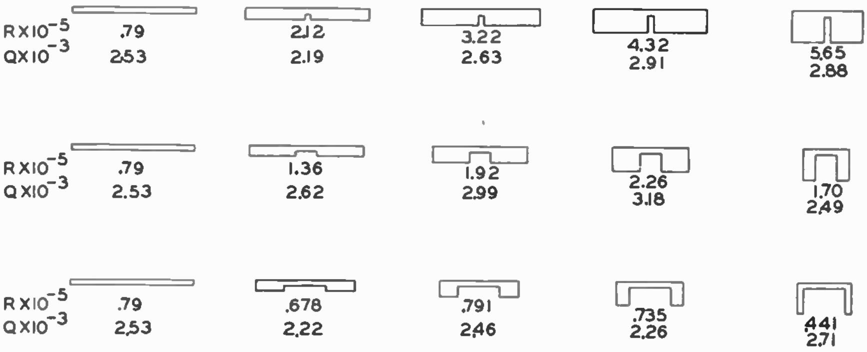
c_2–INCHES

c_1–IN. \ Z_o–INCHES	.020	.060	.120	.180	.240
.020	.485	.456	.436	.321	.172
.080	.485	.404	.251	.167	.127
.160	.485	.328	.233	.199	.183

δ =.020 INCHES

FIG. X-17A

CAVITY RESONATORS

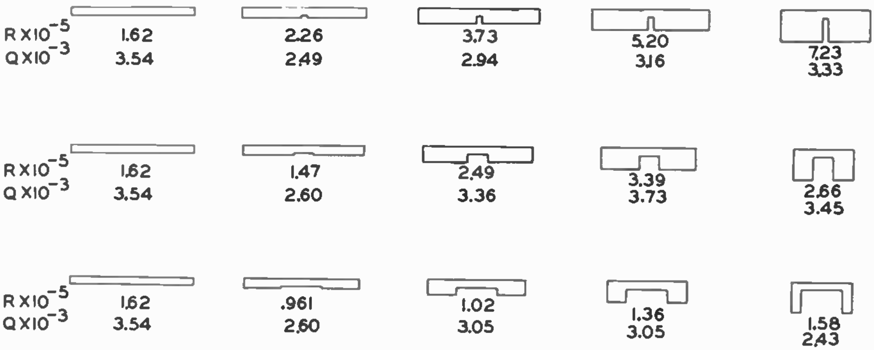


$R \times 10^{-5}$	.79	2.12	3.22	4.32	5.65
$Q \times 10^{-3}$	2.53	2.19	2.63	2.91	2.88
$R \times 10^{-5}$	.79	1.36	1.92	2.26	1.70
$Q \times 10^{-3}$	2.53	2.62	2.99	3.18	2.49
$R \times 10^{-5}$	.79	.678	.791	.735	.441
$Q \times 10^{-3}$	2.53	2.22	2.46	2.26	2.71

ℓ_2-INCHES

ℓ_1-Z_i					
.020	.485	.478	.465	.442	.274
.080	.485	.443	.371	.294	.177
.160	.485	.385	.309	.258	.209
Z_o-INCHES	.040	.080	.120	.180	.240

δ =.040 INCHES



$R \times 10^{-5}$	1.62	2.26	3.73	5.20	7.23
$Q \times 10^{-3}$	3.54	2.49	2.94	3.16	3.33
$R \times 10^{-5}$	1.62	1.47	2.49	3.39	2.66
$Q \times 10^{-3}$	3.54	2.60	3.36	3.73	3.45
$R \times 10^{-5}$	1.62	.961	1.02	1.36	1.58
$Q \times 10^{-3}$	3.54	2.60	3.05	3.05	2.43

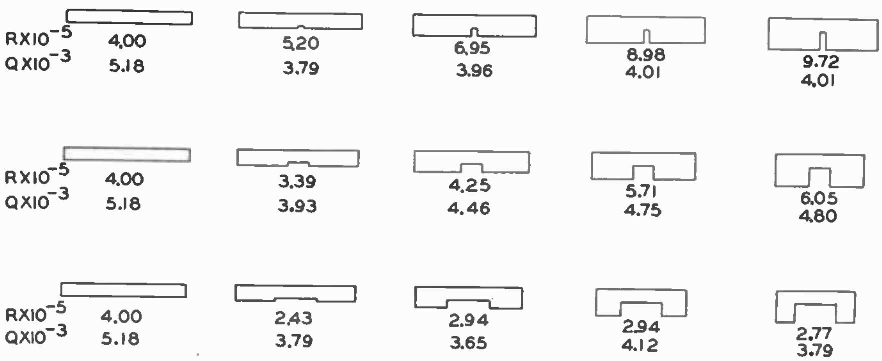
ℓ_2-INCHES

ℓ_1-Z_i					
.020	.485	.480	.474	.457	.350
.080	.485	.470	.430	.370	.235
.160	.485	.452	.378	.312	.238
Z_o-INCHES	.060	.080	.120	.160	.240

δ =.060 INCHES

FIG. X-17B

MICROWAVE TRANSMISSION DESIGN DATA



c_2-INCHES

c_1-IN.					
.020	.485	.481	.474	.461	.425
.080	.485	.475	.449	.403	.342
.160	.485	.465	.412	.352	.306
	.100	.120	.160	.200	.240

Z_o-INCHES

δ =100 INCHES

FIG.X-17C

CHAPTER XI

MEASUREMENT TECHNIQUES

This chapter is written to provide useful information for people working in research laboratories, and to point out some precautions which should be taken to obtain accurate data. The equipment discussed is generally that which is in most common use at Sperry, and for that reason and others, no attempt has been made to provide a truly comprehensive treatise on measurement techniques.

A. *Frequency Measurement*

For rough measurements of frequency, it is permissible to connect the wavemeter directly to the signal source. For accurate measurements, this should not be done when the signal source is an oscillator whose frequency depends somewhat upon the load impedance. This impedance may change when the wavemeter is replaced by other equipment, and most certainly will change as the wavemeter is tuned through resonance. The frequency of magnetron and Klystron oscillators is usually affected by the load impedance; if an amplifier or buffer stage is used, the frequency is much more stable.

Detuning effects of a changing load are reduced if there is attenuation between the oscillator and the load; the changes in impedance are reduced by the attenuation and are not so noticeable at the signal source. Another method is to extract a small fraction of the signal with a power divider and feed this to the wavemeter; tuning effects of the wavemeter are then not so strongly felt at the generator. For the frequency to remain unchanged, the power divider and wavemeter should be left connected, or replaced by another load of identical impedance.

B. *Attenuators*

Calibrated attenuators are used for measuring high powers with low power wattmeters, and for cutting down signals to low power levels. These attenuators can be either fixed or variable, and may be designed for either wave guides or coaxial lines.

An attenuator, to be most useful, should in general have its impedance at both input and output matched to the impedance of the line or wave guide in which it is used. If the match is imperfect, reflections from the attenuator will affect its apparent attenuation, which will then depend in part on the associated circuit elements (see Chapter III). For this reason, when two or more poorly matched attenuators are used in cascade, the measured total attenuation is usually not equal to the sum of the measured values of the individual attenuators.

MICROWAVE TRANSMISSION DESIGN DATA

The match that a variable attenuator presents to the line may depend upon the attenuation and it is sometimes desirable to check this match and see that it is constant over the range of attenuation used.

Fixed attenuators are usually designed to have a given attenuation and to present a matched load at a certain wavelength. If used at a different wavelength, both the attenuation and the match may be changed. Fig. XI–1A shows a fixed wave guide attenuator designed for a wavelength $\lambda = 3.20$ cm. The wave guide is filled with linen bakelite, and there are end steps for matching. Power is absorbed because of the high power factor of the bakelite. Fig. XI–1B shows a similar construction used in coaxial line at a wavelength $\lambda = 10.0$ cm. Attenuators may assume many forms—these examples have standing wave ratios better than $\eta = 1.1:1$ at the designed wavelengths.

When these and similar attenuators are used, care should be taken that they are not required to dissipate so much power as to affect their operation. Their characteristics may also be changed by varying the ambient temperature and humidity.

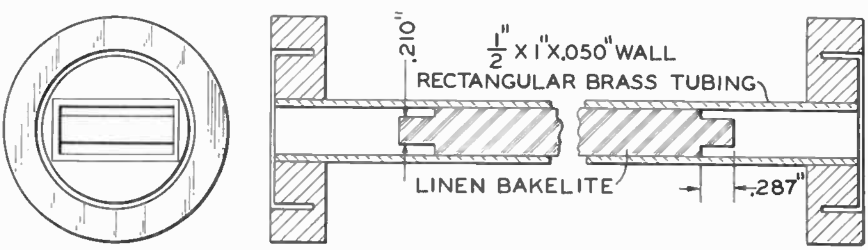


FIG. XI - 1A

WAVE GUIDE ATTENUATOR

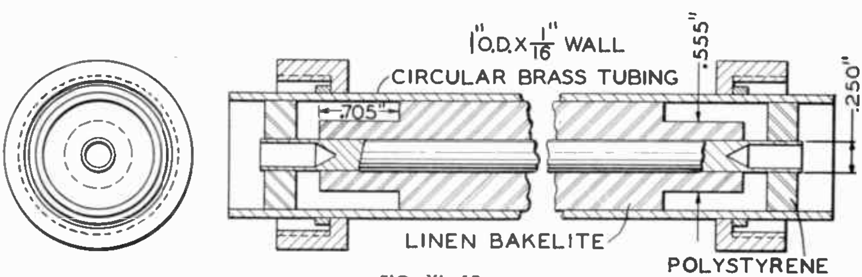


FIG. XI - 1B

COAXIAL ATTENUATOR

C. *Joints*

Two pieces of rigid coaxial line are usually connected with a butt joint on the outer pipe and a pin on the center conductor. A closely-fitting sleeve will hold the outer conductors in line—tolerances should be such that the pin will never buckle the center conductor. These joints are satisfactory if securely fastened, and several such joints in a line will not appreciably affect the standing wave ratio.

Wave guides are joined by butt connectors or choke joints. If the butt joint is carefully aligned and the two guides tightly pulled together, the loss and reflections will be less than in a choke joint. But any misalignment or gap between the adjoining pieces may cause sizeable reflections. Losses in a choke coupling are slightly higher than in a good butt joint, but their use is generally preferred because they are more reliable and give consistent results. Choke joints may be separated much farther than butt joints for the same loss in power. Sidewise misalignment of chokes is more critical than separation, and for a given misalignment, the transmission may actually be improved by separating the chokes (see Chapter V—Section E). A standing wave ratio $\eta = 1.1:1$ may be maintained if one choke is rotated with respect to the other, provided the angle of rotation is less than 5°.

Two chokes are sometimes used at a coupling, or a choke and flange may be paired together. The use of two chokes is more costly, but is often preferable for laboratory use as equipment may be reversed in direction. A disadvantage that arises when two chokes are used is that there may be resonance effects in the slot, resulting in appreciable power losses and a worse standing wave ratio than is found with a choke-flange joint.

D. *Power Measurement*

Basically, most microwave power measurements now involve a conversion of R.F. energy to some other form of energy whose effects can be more readily measured. When R.F. power is absorbed by a resistive element it is converted to heat, and any of several convenient devices may be used to indicate the temperature rise of the element. The microwave power measurement method in most general use today employs a hot wire or thermistor as the absorbing element and a D.C. operated Wheatstone bridge as an indicator. In the paragraphs which follow, specific precautions are presented which, if heeded, will insure maximum accuracy when using equipment now available.

One of the first considerations in making accurate power measurement involves the establishment of conditions for maximum power transfer between the generator and the absorbing element. This means that the impedance presented to the generator must be that into which the generator will deliver its maximum power output. For some generators, including those in which a matched attenuator follows the power source, this optimum load will be the transmission line impedance. But this is not necessarily true for other power generators, including Klystron oscillators.

Matched transmission lines maintain a fixed impedance which is determined by their physical design. When it is desired to have a power source such as a *Klystron deliver its maximum output to the transmission line impedance, it may be necessary to place between the matched line and the power source an impedance transformer which will enable the generator to work into its optimum load impedance.

If the transformer is tuned so that the generator delivers maximum power to the transmission line impedance, the section of line which contains the power-absorbing element must provide a matched termination for the line. If the standing wave ratio of the wattmeter is not unity, part of the input power will be reflected.

The transformer between generator and wattmeter can be adjusted for maximum power output from the generator even if the wattmeter is not matched to the transmission line, and this is sometimes done when the transformer is an integral part of the wattmeter. But the presence of standing waves in the transmission line will then result in increased losses in the line. These can become appreciable if the wattmeter is badly mismatched.

A further precaution involves insertion of "lossy" lines or material into any part of the line between the generator and absorbing element. Only that power which arrives at the hot wire is indicated by the bridge. Steel screws, paper, wood, water, or carbon in any form in the line may absorb an appreciable fraction of power which can then never be measured.

In measuring powers in the lowest range (.5 microwatt to 5 milliwatts) it should be realized that the temperature rise of the absorbing element is very slight. After applying D.C. power to the bridge, five to ten minutes should be allowed for the hot wire (more for a thermistor) temperature to stabilize before R.F. power measurements are attempted. At the end of this time it will be necessary to readjust the bridge current slightly to return the bridge to balance.

Microwattmeters should have casings built of bakelite or other heat-insulating material to protect temperature sensitive components from the heat of the operator's hands. In making any adjustment of these wattmeters care should be taken to see that only such insulated portions are handled. Drafts from fans or other causes over the power measuring unit are also to be avoided, as they are likely to cause erratic drifting of the null indicator.

In using attenuators, it is often necessary at present to make certain

*Reg. Trade-Mark of Sperry Gyroscope Co.

that their calibration is correct and that varying temperatures do not alter this calibration (Section B of this Chapter).

Power dividers have a "built in" dividing ratio which depends on their design and use under fixed conditions. When the impedances of dissipating antennas and wattmeters differ from each other or from the divider itself, erratic dividing ratios will be obtained. It is necessary in every case, therefore, to make certain that standing wave ratios of the antenna and wattmeter used with a power divider are unity.

E. *Measuring Standing Wave Ratios*

1. General Information. The standing wave ratio η as used throughout this handbook is in power, defined by

$$\eta = \left(\frac{E_{max}}{E_{min}}\right)^2$$

where E_{max} and E_{min} are the R.F. fields at the loops and nodes respectively in the line being measured.

Various types of indicating equipment may be used for measuring standing wave ratios. One of the best types uses as a signal source a Klystron which is 100% modulated by a square wave inserted in series with the beam or reflector supply. This square wave is easily generated by a multivibrator and clipper circuit. A square wave is used rather than a sine wave to prevent frequency modulation of the Klystron. The probe crystal output is fed into a tuned audio amplifier with a vacuum tube voltmeter to read the output. This amplifier can be made very sensitive, and it does not respond to stray signals induced in the crystal lead by outside sources unless they are modulated at the frequency to which the amplifier is tuned.

Another type of standing wave detector uses a calibrated variable attenuator between the pick-up probe and the crystal detector, or between the generator and the probe. This can be used to measure very high standing wave ratios with considerable accuracy, as the signal delivered to the crystal is held constant by varying the attenuator while the probe is moved along. The standing wave ratio may then be determined by the difference between maximum and minimum settings on the attenuator. Because of the loss of power in the variable attenuator, a super-heterodyne receiver may be required to detect the signal at the crystal.

The particular equipment discussed below uses a continuous wave signal. The probe crystal feeds into a Rubicon galvanometer with a sensitivity of .004 microamps per millimeter deflection. If other equipment is used the procedure will be similar to that described, with variations to suit the individual.

To measure standing wave ratios, it is convenient to have an attenuator between the crystal and the galvanometer. This may be calibrated either in power ratios or in *db*. It should be remembered that the galvanometer reads D.C. output of the crystal, which is proportional to the square of the R.F. voltage input to the crystal. The crystal should be checked to see that its square law characteristic holds over the range of powers that is used.

In general, standing wave ratios may be measued in the following three ways.

a. For low standing wave ratios (up to 5 : 1 in power) the maximum and minimum may be read directly on the galvanometer with the attenuator setting at a constant value. The zero setting of the galvanometer should be checked periodically for drift if the readings are taken for more than a quarter of an hour. The attenuator should be set so that the maximum is read as near full scale as possible. This is done so that any error due to mis-setting the zero will be minimized, e.g., in measuring a standing wave ratio of 5 : 1, when the zero setting is off 1 division out of 100; there is an error of 4% when the maximum is measured at 100, and of 9% when it is measured at 50. This method is not recommended for accurate measurements when η is greater than 5 : 1 as any shift in the zero setting or any mis-reading of the minimum causes an error which is increasingly large with lower minimums.

b. For standing wave ratios between 5 : 1 and 100 : 1, the attenuator between the crystal and the galvanometer may be used to advantage. This attenuator should preferably be set so that readings over half scale are obtained for both maximum and minimum values.

If the attenuator is calibrated in power ratio

$$\eta = \frac{I'_{DCmax}}{I'_{DCmin}} \cdot \frac{P_{max}}{P_{min}}$$

where η is the standing wave ratio in power. I'_{DCmax} and I'_{DCmin} are the galvanometer readings at E_{max} and E_{min} respectively, and P_{max} and P_{min} are the corresponding readings on the attenuator.
If the attenuator is calibrated in *db*

$$\eta = \frac{I'_{DCmax}}{I'_{DCmin}} \text{ antilog } \frac{db_{max} - db_{min}}{20}$$

As before, I'_{DCmax} and I'_{DCmin} are the galvanometer readings at E_{max} and E_{min} respectively, and db_{max} and db_{min} the corresponding readings on the attenuator.

In both instances, it is not necessary that I'_{DCmax} be greater than I'_{DCmin}.

Standing wave ratios greater than 100 : 1 may be measured by this method if the crystal being used is square law over that range.

c. For standing wave ratios over 100 the double minimum method is satisfactory. To apply this method one needs to be able to measure the movement of the probe as well as the intensity of the field. The value of I'_{DCmin} is determined where I'_{DCmin} is the galvanometer reading at a node ($I'_{DCmin} \alpha E_{min}^2$). Next $2I'_{DCmin}$ is located on either side of the node and the distance between these latter two points is measured. This is shown in Fig. XI–2 as $2X_1$.

The standing wave ratio is now given by

$$\eta = \left(\frac{\lambda_g}{2X_0\pi} \right)^2$$

where λ_1 is the wavelength in the line (either coaxial or wave guide).

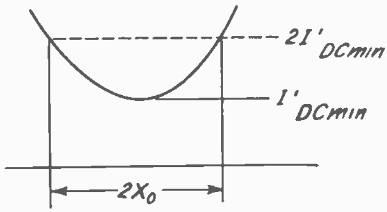


FIG. XI - 2

If enough power is available, it is well to set the attenuator so that the minimum is read just below the mid-point of the scale. Twice the minimum will then occur near the top of the scale, thus giving the most accurate measurements.

2. Node Location. Locating a node by hunting for the minimum is often inaccurate as the minimum is usually broad and the position is hard to determine exactly. A more accurate method is to locate the position of the probe at two points of equal galvanometer reading on either side of the minimum. The average of the two probe positions should be the position of the node. If the power has changed between taking these two values the result obtained will be to one side of the true position. It is well to take several sets of readings at various heights on the galvanometer and see that the values obtained by averaging them agree.

3. Drift. Drift in a galvanometer reading is usually caused by change in power input to the standing wave detector or by pick-up of stray signals. The first of these may be corrected by proper tuning of the oscillator, or if due to voltage changes by stabilizing the power supply. Stray signals cause trouble because they are picked up by the indicator (galvanometer or other), fed through the crystal cable, rectified and returned as D. C. This difficulty may be overcome by installing a line between the crystal and the indicator which passes D. C. but attenuates or filters out the bothersome signal. It is also a good plan to shield the indicator and its cables.