RADIO ENGINEERING

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ELECTRICAL ENGINEERS' DATA BOOKS

VOLUME THREE

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RADIO ENGINEERING

With Special Sections on

TELEGRAPHY & TELEPHONY

The series planned and edited by

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AUTHOR'S PREFACE

The existing information on matters of radio technique, particularly in respect of quantitative data, is somewhat difficult of access, and can only be obtained by reference to expensive textbooks and publications of the various learned societies.

Such details, moreover, are often highly mathematical in form, and it is necessary to read a considerable quantity of extraneous matter before the desired information can be extracted.

In order to overcome these defects, Mr. E. B. Wedmore, Director of the British Electrical and Allied Research Association, conceived the happy idea of a series of data books dealing with each phase of Electrical Engineering.

The information in the present volume (which is concerned exclusively with radio engineering, and to a small extent telegraphy and telephony) has been collated as a result of a careful study of the principal source of data, and has been presented in a concise form with due reference to the practical value of the various points, while in view of the rapid expansion of radio engineering a certain descriptive matter is included to make the data applicable to new developments.

The primary object of the work, however, is the production, not of another textbook, but of a handy volume of reference which will be indispensable to everyone concerned with the technique of radio.

J. H. R.

London,
September, 1925.

RADIO ENGINEERING

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RADIO COMMUNICATION

SECTION I
RADIO COMMUNICATION

RADIO CALCULATIONS AND MEASUREMENTS.

Calculation of Inductance and Coil Design.

THE inductance of a circuit is defined as the linkage per unit current, the term "linkage" signifying the product of the flux associated with the circuit, the number of turns of conductor, and a factor which allows for the fact that all the flux does not link with all the turns—*i.e.*, there is a certain flux leakage. The flux is proportional to the product of the area and the number of turns, so that, neglecting the third factor, the inductance varies directly as the area enclosed and as the square of the number of turns. The somewhat involved formulæ which follow are all based on this fundamental law with suitable corrections to allow for the third factor.

The flux in a circuit is distributed both in and around the wire. In the case of single wires and simple configurations the flux distribution can be calculated fairly accurately, and, moreover, the flux inside the wire is of appreciable importance. As the frequency varies the flux distribution changes, and consequently the inductance in such cases is dependent on frequency. With coils, on the other hand, the variation in inductance with frequency from this cause is hardly appreciable, because the flux outside the wire is the preponderating factor, and also because the variation, if any, is less than the error inherent in the formulæ themselves.

The unit of inductance is the *Henry*, which is defined in terms of the E.M.F. of self-induction (see Appendix A). This unit, however, is inconveniently large for radio circuits, and it is more usual to express the small inductances encountered in radio practice in terms of microhenries (μH).

In the formulæ which follow all inductances are given in microhenries, the linear dimensions being in centimetres unless otherwise stated.

Long, Straight, Round Wire.—The inductance of such a wire is given by—

$$L = 0{\cdot}002l \left[\log_e \frac{4l}{d} - 1 + \mu\delta \right],$$

where l = length of wire,
d = diameter of wire,
μ = permeability of wire.

If $2l/d$ is less than 1,000, the term $d/2l$ should be added inside the brackets.

δ is a term involving frequency. Its value varies from $\frac{1}{4}$ at low frequency to zero at infinite frequency. A table is appended giving the values of δ for a range of frequencies.

It should be observed that the inductance of a wire has no physical meaning unless the wire is part of a circuit. The formula given above is for a long wire so far removed from its return circuit that any effects of mutual inductance between the go and return leads may be neglected.

There is a slight difference between the values of the inductance at high frequencies according to whether the coil is carrying damped or continuous waves, but the correction only affects the fourth decimal place, and is therefore negligible.

TABLE I.

Value of Frequency Correction Factor δ.

First find $x = 0{\cdot}1405 d \sqrt{\frac{\mu f}{\rho}}$,

where d = diameter of wire,
μ = permeability of material,
ρ = specific resistance of material,
f = frequency.

Then δ may be obtained in terms of x.

x.	δ.	x.	δ.
0·0	·250	12	·059
0·5	·250	14	·050
1·0	·249	16	·044
1·5	·247	18	·039
2·0	·240	20	·035
2·5	·228	25	·028
3·0	·211	30	·024
3·5	·191	40	·0175
4·0	·172	50	·014
4·5	·154	60	·012
5·0	·139	70	·010
6·0	·116	80	·009
7·0	·100	90	·008
8·0	·088	100	·007
9·0	·078	∞	·000
10·0	·070		

Long, Straight, Rectangular Wire:

$$L = 0{\cdot}002l \left[\log_e \frac{2l}{t+w} + \tfrac{1}{2} + 0{\cdot}2235 \left(\frac{t+w}{l}\right)\right], \qquad \text{(B.S.)}$$

where l = length of wire,
t = thickness of wire,
w = width of wire.

If l is greater than $50\,(t+w)$, the last term may be neglected. This formula assumes unit permeability of the wire; no frequency correction is practicable with rectangular wire.

The above formula and many of the others which follow are taken from the Circular No. 74 of the Bureau of Standards, Washington (Radio Instruments and Measurements). Where such is the case, acknowledgment is made by the letters B.S. in the margin.

Two Parallel Wires—Go and Return:

Round wire: $$L = 0{\cdot}004l \left[\log_e \frac{2D}{d} - \frac{D}{l} + \mu\delta\right], \qquad \text{(B.S.)}$$

where D = spacing between the wires from centre to centre,
l and d are the length and diameter of the wire,
μ and δ are as before.

Rectangular wire:

$$L = 0{\cdot}004l \left[\log_e \frac{D}{t+w} + \frac{3}{2} - \frac{D}{l} + 0{\cdot}2235\left(\frac{t+w}{l}\right)\right], \qquad \text{(B.S.)}$$

where D = spacing of wires from centre to centre.
l = length of wire,
t and w are the thickness and width of the wire.
Unit permeability is assumed, and, as before, the last term may be neglected if $l > 50\ (t+w)$.

These formulæ assume that the wires are of the same dimensions. If this is not the case, the inductance must be worked out from the expression $L = L_1 + L_2 + 2M$, utilising the formulæ for mutual inductance which are given later. The effect of end connections is also neglected. If this is not permissible, the formula for a rectangle should be employed.

Rectangle:

Round wire:

$$L = 0{\cdot}004 \left[(a+b)\log_e \frac{4ab}{d} - a\log_e (a+g) - b\log_e (b+g) + 2\left(g + \frac{d}{2}\right) - (a+b)(2 - \mu\delta)\right]$$

Rectangular wire:

$$L = 0{\cdot}004 \left[(a+b)\log_e \frac{2ab}{t+w} - a\log_e (a+g) - b\log_e (b+g) + 2g - \frac{a+b}{2} + 0{\cdot}447\,(t+w)\right] \qquad \text{(B.S.)}$$

Here a and b are the sides of the rectangle, and $g = \sqrt{a^2 + b^2}$.

The other symbols have the same meanings as before. Unit permeability is assumed in the last formula.

Circle:

Round wire: $$L = 0{\cdot}01257a \left[\log_e \frac{16a}{d} - 2 + \mu\delta\right], \qquad \text{(Kirchhoff.)}$$

where a = mean radius of circle,
d = diameter of wire.

Only valid when $d/2a < 0{\cdot}2$.

Rectangular wire: $$L = 0{\cdot}01257a \left[\log_e \frac{35{\cdot}8a}{tw} - 2\right],$$

where a = mean radius of circle,
t and w are thickness and width of wire.

Thin tape of width w:

$$L = 0{\cdot}01257a \left[\log_e \frac{35{\cdot}85a}{w} - 2\right].$$

Tube Bent into Circle.—Where r_1 and r_2 are approximately equal (as is usually the case)—

$$L = 0{\cdot}01257 \left[\left(1 + \frac{r^2}{4a^2}\right) \log_e \frac{8a}{r} - 2\right],$$

where a = mean radius of circle,
r = mean of r_1 and r_2.

Long, Straight, Thin Tape:

$$L = 0{\cdot}002l \left[\log_e \frac{2l}{w} + \tfrac{1}{2} - \frac{t}{w}\right], \qquad \text{(Eccles.)}$$

where l = length of tape,
t = thickness of tape,
w = width of tape.

Horizontal Earthed Wires.—This case is of frequent occurrence in radio work, the earth providing the return circuit, as in the case of an aerial. For a single wire—

$$L = 0{\cdot}002 \left[l \log_e \frac{4h}{d} + l \log_e \left\{\frac{l + \sqrt{l^2 + d^2/4}}{l + \sqrt{l^2 + 4h^2}}\right\} + \sqrt{l^2 + 4h^2} - \sqrt{l^2 + d^2/4} + \mu l \delta - 2h + \frac{d}{2}\right],$$

where h = height of the wire above ground,
l = length of wire,
d = diameter of wire,
μ is the permeability of the material,
δ is the frequency correction factor.

If d/l is negligible, the expression may be recast into a more simple form —viz.:

If

$$\frac{2h}{l} \lesseqgtr 1, \quad L = 0{\cdot}002l \left[\log_e \frac{4h}{d} - P + \mu\delta\right],$$

$$\frac{l}{2h} \lesseqgtr 1, \quad L = 0{\cdot}002l \left[\log_e \frac{4l}{d} - Q + \mu\delta\right], \qquad \text{(B.S.)}$$

where P and Q are constants, the values of which are given below.

TABLE II.

VALUE OF CONSTANTS P AND Q IN EARTHED WIRE FORMULÆ.

$2h/l$ ..	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
P ..	0	0·098	0·190	0·278	0·361	0·439	0·514	0·584	0·651	0·714	0·774
$l/2h$ ..	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
Q ..	1·0	1·05	1·10	1·15	1·20	1·25	1·29	1·34	1·38	1·43	1·47

Inductance of N-Earthed Wires in Parallel.—This is a case of some practical importance in the design of aerials. The calculation involves the knowledge of the mutual inductance between two horizontal earthed wires. This is given by—

$$M = 0{\cdot}002l\left[\log_e \frac{2h}{D} - P + \frac{D}{l}\right] \text{ if } 2h/l \geqq 1,$$

$$M = 0{\cdot}002l\left[\log_e \frac{2l}{D} - Q + \frac{D}{l}\right] \text{ if } l/2h \leqq 1, \qquad \text{(B.S.)}$$

D being the spacing between the wires,
l the length,
P and Q as in the last formulæ.

Where the spacing of all the wires is the same, and the wires are of the same diameter, the inductance of the combination may be obtained from the following formula:

$$L = l\left\{\frac{L_1 + (n-1)\,M_1}{n} - 0{\cdot}001\,k\right\}, \qquad \text{(B.S.)}$$

where L_1 = inductance per unit length,
M_1 = mutual inductance between two adjacent wires as determined by the formulæ just given,
k is a spacing factor, values of which are appended.

In all other cases the required inductance can be worked out from the data given by a process of summation, considering the effect on each wire in turn of all the other wires.

TABLE III.

VALUE OF SPACING FACTOR FOR PARALLEL EARTHED WIRES.

n=2	3	4	5	6	7	8	9	10	12	14	16	18	20
k=0	·308	·621	·906	1·18	1·43	1·66	1·86	2·05	2·37	2·63	2·85	3·04	3·24

Inductance of Coils.—Except in the case of single turn loops, either circular or rectangular, the inductances of which have already been given, the inductance of coils cannot be computed with accuracy.

In the case of an infinitely long solenoid an exact formula is available, since all the flux produced must link with the coil at some portion of its length. Hence the linkage is simply—

$$= \text{flux} \times \text{number of turns},$$

$$= \frac{4\pi}{10} \cdot \frac{In}{l} \cdot \pi a^2 \times n, \ a \text{ being the radius of the coil.}$$

And hence the inductance for such a coil, being the linkage per unit current,

$$= \frac{4\pi^2 n^2 a^2}{10l}.$$

All the coils employed in practice, however, fall short of this ideal condition, and consequently corrections have to be introduced.

All the formulæ which follow are current sheet formulæ. That is to say, they are deduced on the assumption that the coil is wound with infinitely thin tape with infinitely thin insulation between the turns (or in the case of multilayer coils the whole cross-section is assumed to be filled with conductor).

Corrections are introduced, therefore, to allow for the configuration of the coil and also a small fraction ΔL is added or subtracted to allow for the difference between the actual winding and the current sheet assumption.

This latter correction is specified with each formula, but it will be as well to explain here the general principles involved.

(*a*) *For Single Layer Coils.*—(i.) Wound with round wire, the inductance is first worked out by the appropriate current sheet formula, and a correction, due to Rosa, is applied. This correction is of the form $\Delta L = kan\ (A + B)$, where k, A, and B are constants, this quantity ΔL being *subtracted* from the current sheet inductance.

(ii.) With rectangular wire, the inductance is worked out from a suitable *multilayer* formula, and a similar correction applied. The correction in this case, however, is very complicated, and since the value of the correction is rarely more than 1 per cent., the labour involved is hardly commensurate with the result gained. A sufficiently accurate estimate may be arrived at by *adding* a small quantity $\Delta L = kan\ Q$, where k and Q are constants.

(*b*) *For Multilayer Coils.*—Here a comparatively simple correction is available, a small quantity $\Delta L = kan\ Q'$ being added.

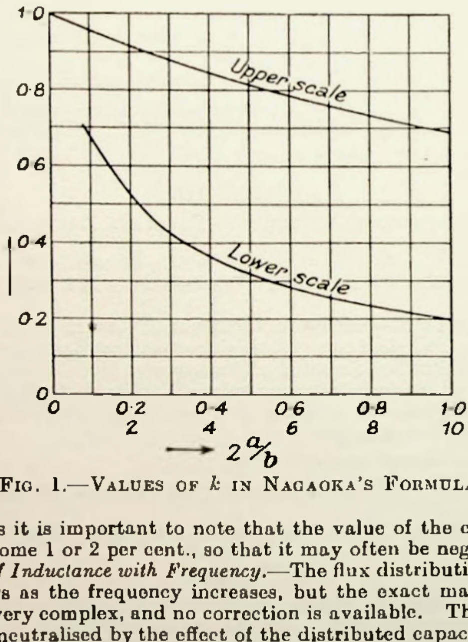

Fig. 1.—Values of k in Nagaoka's Formula.

In all cases it is important to note that the value of the correction only amounts to some 1 or 2 per cent., so that it may often be neglected.

Variation of Inductance with Frequency.—The flux distribution throughout the coil alters as the frequency increases, but the exact manner in which it does so is very complex, and no correction is available. The effect is to a large extent neutralised by the effect of the distributed capacity of the coil. The former tends to reduce the inductance of the coil as the frequency is raised, while the latter effect increases the apparent inductance, and becomes increasingly important as the frequency increases. The actual increase at any particular frequency due to this latter cause is given by—

$$L = L_0\ (1 + \omega^2 LC \times 10^{-18})\ \text{(L and C being in } \mu\text{H and } \mu\mu\text{F).}$$

Single Layer Coils.—For single layer solenoids, the best formula is that due to Nagaoka—viz.:

$$L_s = 0{\cdot}03948\ \frac{a^2 n^2}{b} k,$$

where a = radius of coil,

b = length of winding = nD (D being the pitch),

n = number of turns,

k is a factor allowing for the configuration of the coil. Values of k are plotted against the ratio $2a/b$ in Fig. 1, the actual figures being given in Table IV.

TABLE IV.

VALUES OF k IN NAGAOKA'S FORMULA.

$2a/b$.	k.	$2a/b$.	k.	$2a/b$.	k.
·00	1·000	1·0	·683	4·0	·365
·05	·979	1·1	·667	4·5	·341
·10	·959	1·2	·648	5·0	·320
·15	·939	1·3	·629	5·5	·301
·20	·920	1·4	·612	6·0	·285
·25	·902	1·5	·595	6·5	·271
·30	·884	1·6	·579	7·0	·258
·35	·867	1·7	·565	7·5	·247
·40	·850	1·8	·551	8·0	·237
·45	·834	1·9	·538	9·0	·219
·50	·818	2·0	·526	10	·203
·55	·803	2·2	·503	12	·179
·60	·789	2·4	·482	15	·153
·65	·775	2·6	·463	20	·124
·70	·761	2·8	·445	25	·105
·75	·748	3·0	·429	30	·0910
·80	·735	3·2	·415	35	·0808
·85	·723	3·4	·401	40	·0728
·90	·711	3·6	·388	45	·0664
·95	·700	3·8	·376	50	·0611
1·00	·688	4·0	·365	100	·0350

This is a current sheet formula, and applies for round wire wound without any appreciable spacing between the turns. If there is any spacing, a correction must be applied, as has previously been pointed out, and—

$$L' = L - \Delta L,$$

where

$$\Delta L = 0{\cdot}01257an\,(A + B).$$

Here A is a constant depending in the ratio d/D, and B depends only on the number of turns. Values of A and B are given in Figs. 2 and 3.

If the coil is wound with rectangular wire—

$$L = L_s - 0{\cdot}01257\,\frac{n^2aw}{b}\,S,$$

where w = radial depth of wire,

t = axial thickness,

a and b are as above,

S is a constant involving the ratio b/w, values of which are given in Fig. 6.

Correction for Spacing:

$$L' = L + \Delta L,$$

where $$\Delta L = 0{\cdot}01257an\left(0{\cdot}6\log_e \frac{w+D}{w+t}\right);$$

where ω and t are as above,

D = pitch of winding.

Polygonal coils may be treated as coils of radius equal to the mean of the inscribed and escribed circles.

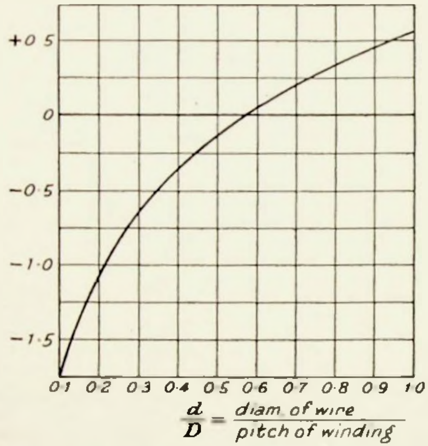

Fig. 2.—Value of Constant A in Rosa's Correction.

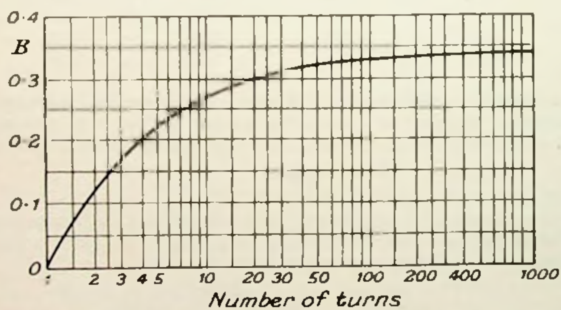

Fig. 3.—Value of B in Rosa's Correction.

Torus with Single Layer Winding.—A torus is a ring of circular cross-section (see Fig. 4). Here—

If R = radius of torus to centre of cross-section,

a = radius of winding,

n = number of turns,

$$L = 0{\cdot}01257n^2\,[R - \sqrt{R^2 - a^2}].$$

Toroid of Rectangular Section:

$$L = 0{\cdot}002\, n^2 h \log_e \frac{r_2}{r_1},$$

where r_1 = inner radius of toroid,
r_2 = outer radius of toroid,
h = axial depth of toroid.

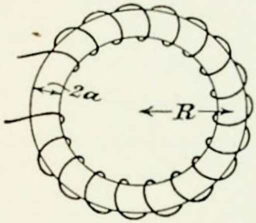

FIG. 4.—TOROID WITH SINGLE LAYER WINDING.

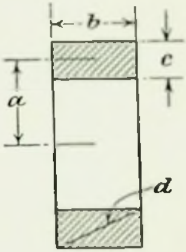

FIG. 5.—MULTILAYER COIL

Multilayer Coils.—Several formulæ are available here, some being more suitable than others according to the type of coil.

For long coils of few layers, Rosa has derived the following formula:

$$L = L_s - 0{\cdot}01257 \frac{n^2 ac}{b} S,$$

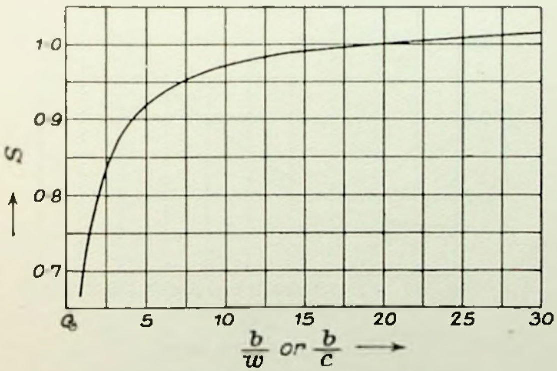

FIG. 6.—VALUE OF CONSTANT S IN TERMS OF b/c.

where a = mean radius of coil,
b = axial length of coil, See Fig. 5.
c = radial depth of coil,
L_s = inductance calculated by Nagaoka's formula.

S is a correction dependent on the ratio b/c, values of which are plotted in Fig. 6.

This formula is accurate to within 0·1 per cent. for long coils, but becomes less accurate as c/a becomes $\gtreqless$ 0·25, and $b/a \gtreqless 5$.

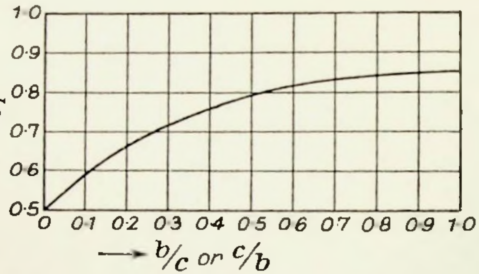

FIG. 7.—VALUES OF y_1.

For short coils where b and c are small compared with a, Stefan's formulæ are more suitable.

If $b>c$, $L_0=0{\cdot}01257an^2\left[\left(1+\frac{b^2}{32a^2}+\frac{c^2}{96a^2}\right)\log_\epsilon\frac{8a}{d}-y_1+\frac{b^2}{16a^2}y_2\right]$.

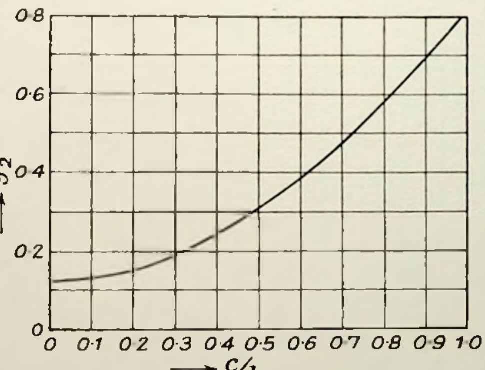

FIG. 8.—VALUES OF y_2.

where $d=\sqrt{b^2+c^2}$, and the other terms are as before.

If $b<c$, substitute $\frac{c^2}{16a^2}y_3$ for the last term.

y_1, y_2, and y_3 are constants given in Table V. below, the values also being plotted in Figs. 7, 8, and 9.

Correction for Spacing of Windings :

$$L = L_0 + 0{\cdot}01257an\left[\log_e \frac{D}{d} + 0{\cdot}155\right].$$

where D = pitch of winding,
d = diameter of wire.

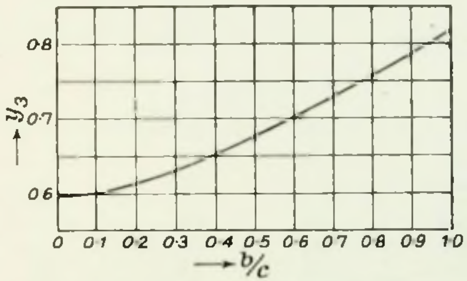

FIG. 9.—VALUES OF y_3.

Multilayer coils wound with rectangular wire are rarely used except for transmitting inductances, in which case the spacing is considerable. In such cases the rectangular wire or strip may be translated into equivalent round wire of equal area, and a value of D/d obtained.

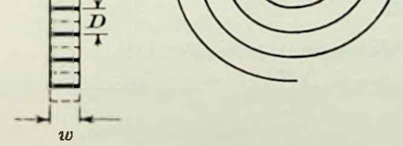

FIG. 10.—SPIRAL OF FLAT STRIP.

Flat Spirals.—If wound with round wire—

$$L_0 = 0{\cdot}01257an^2\left[\log_e \frac{8a}{c} - \tfrac{1}{2} + \frac{c^2}{96a^2}\left(\log_e \frac{8a}{c} + \frac{43}{12}\right)\right]. \quad \text{(B.S.}$$

where $c = n$D,
$a = a_1 + \frac{1}{2}(n-1)$ D, and a_1 is as shown in Fig. 10.

If wound with rectangular wire, treat as a disc coil, using Stefan's formula where

$b = w$,
$c = nD$,
$d = \sqrt{b^2 + c^2}$
$a = a_1 + \frac{1}{2}(n - 1)D$. (See Fig. 10.)

Correction for Spacing:

$L = L_0 - 0.01257an(A + B)$ for round wire,
$= L_0 + 0.01257an\,Q$ for rectangular wire,

A, B, and Q being correction factors previously cited.

TABLE V.

VALUES OF y_1, y_2, AND y_3 FOR USE WITH STEFAN'S FORMULÆ.

b/c or *c/b*.	y_1.	*c/b*.	y_2.	*b/c*.	y_3.
00	·500	·00	·125	·00	·597
·025	·525	—	—	—	—
·05	·549	·05	·127	·05	·599
·10	·592	10	·132	·10	·602
·15	·631	15	·142	·15	·608
·20	·665	·20	·155	·20	·615
·25	·695	·25	·171	·25	·624
·30	·722	·30	·192	·30	·633
·35	·745	·35	·215	·35	·643
·40	·765	·40	·242	·40	·654
·45	·782	·45	·273	·45	·665
·50	·796	·50	·307	·50	·677
·55	·808	·55	·344	·55	·690
·60	·818	·60	·384	·60	·702
·65	·827	·65	·427	·65	·715
·70	·833	·70	·474	·70	·729
·75	·838	·75	·523	·75	·742
·80	·842	·80	·576	·80	·756
·85	·845	·85	·632	·85	·771
·90	·847	·90	·690	·90	·786
·95	·848	·95	·752	·95	·801
1·00	·848	1·00	·816	1·00	·816

Square Coils—*Multilayer Coils* (see Fig. 11):

$$L = 0.008an^2\left[\log_e \frac{a}{b+c} + 0.2235\frac{b+c}{a} + 0.726\right]. \qquad \text{(B.S.)}$$

where

a = mean side of square,
b = axial width,
c = radial depth,
n = number of turns.

If $b = c$, this becomes—

$$L_0 = 0.008an^2\left[\log_e \frac{a}{b} + 0.447\frac{b}{a} + 0.033\right]. \qquad \text{(B.S.)}$$

Correction for Spacing:

$$L = L_0 + 0.008an\left[\log_e \frac{D}{d} + 0.155\right].$$

Single Layer Coils:

Round wire: Put $c = 0$.
Rectangular wire: Put $b = n\text{D}$,
c = radial thickness = w.

In this case, the correction for spacing becomes—

$$\text{L} = \text{L}_0 - 0{\cdot}008an\ (\text{A}+\text{B}) \text{ for round wire,}$$
$$= \text{L}_0 + 0{\cdot}008an\ \text{Q} \text{ for rectangular wire.}$$

Rectangular Coils:

$$\text{L}_0 = 0{\cdot}004\ (a+a_1)\ n^2 \left[\log_e \frac{2aa_1}{b+c} - \frac{a}{a+a_1}\log_e (a+g) - \frac{a_1}{a+a_1}\log_e (a_1+g) + \frac{2g}{a+a_1} - \tfrac{1}{2} + 0{\cdot}447\frac{b+c}{a+a_1}\right],$$

where a and a_1 are the mean sides of the rectangle, b, c, and n being as above.

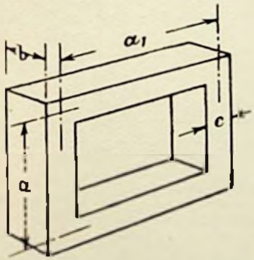

FIG. 11.—MULTILAYER RECTANGULAR COIL.

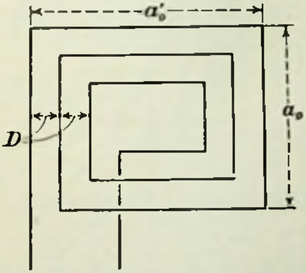

FIG. 12.—FLAT RECTANGULAR COIL.

Correction for Spacing:

$$\text{L} = \text{L}_0 + 0{\cdot}004\ (a+a_1)\ n \left[\log_e \frac{\text{D}}{d} + 0{\cdot}155\right].$$

Single Layer Coils:

Round wire: Put $c = 0$.
Rectangular wire: Put $b = n\text{D}$,
c = radial thickness.

Flat Rectangular or Square Coils:

Put $a = a_0 - (n-1)\ \text{D}$,
$a_1 = a_0' - (n-1)\ \text{D}$
$c = n\text{D}$ (see Fig. 12).

For round wire: Put $b = 0$.
Rectangular wire: Put b = width of strip.

Corrections for spacing in both the above groups of formulæ:

$$\text{L} = \text{L}_0 - 0{\cdot}004\ (a+a_1)\ n\ (\text{A}+\text{B}) \text{ for round wire,}$$
$$\text{L} = \text{L}_0 + 0{\cdot}004\ (a+a_1)\ n\text{Q} \text{ for rectangular wire.}$$

Approximate Formulæ.—For many purposes the accuracy of the formulæ already given is greater than necessary, and formulæ giving results correct to

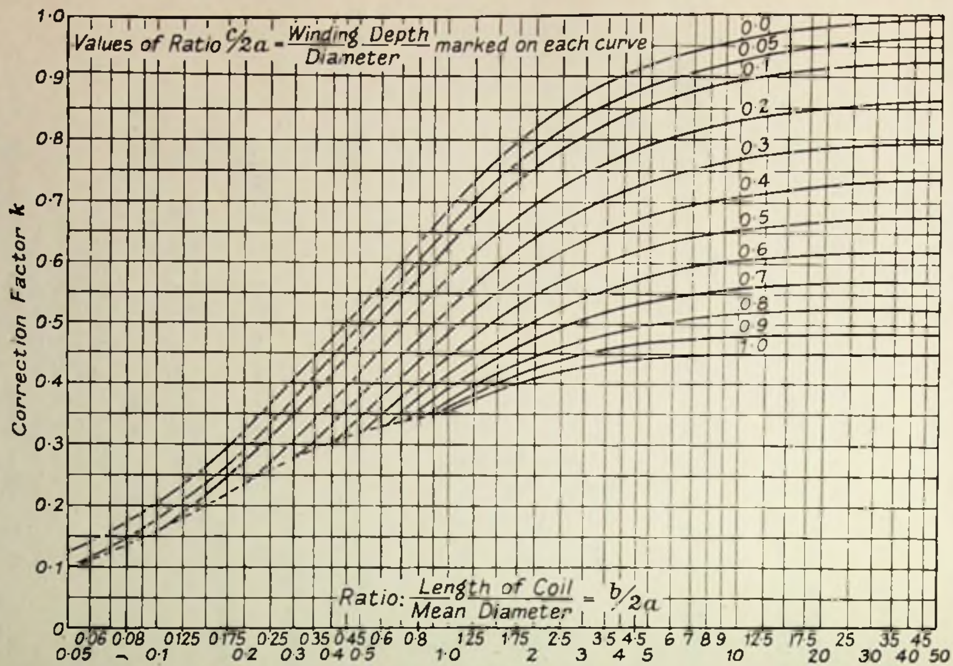

FIG. 13.—CURVES OF FACTOR k′ FOR ALL COILS (THICK OR THIN).

about 2 per cent. are quite adequate. Several such formulæ are given below.

General Formula:

$$L = 0{\cdot}2\,\frac{n^2 D_1^2}{3{\cdot}5D_1 + 8b} \times \frac{D_1 - 2{\cdot}25c}{D_1}, \qquad \text{(Reyner.)}$$

where L = inductance in μH,
D_1 = *outside* diameter,
b = length of coil,
c = radial depth of coil, } Dimensions in *inches*.

This formula is intended for rough-and-ready calculations. It is easily committed to memory, and involves no correction curves. It is accurate to within 2 or 3 per cent. for close wound coils within the limits $c/D_1 < 0{\cdot}3$, $b/D_1 < 2{\cdot}0$. The first portion only is employed for single layer coils, the second term being a correction to allow for the depth of the winding when multilayer coils are used.

Coursey has also evolved a series of correction curves, based on Nagaoka's formula, whereby this simple expression may be employed with any type of coil—*i.e.*:

$$L = 0{\cdot}03948\,\frac{a^2 n^2}{b}\,k',$$

where L = inductance of coil in μH,
a = mean radius (cms.),
b = axial length (cms.),
k' is a constant depending on the ratio of $\frac{b}{2a}$ and $\frac{c}{2a}$, where c = radial thickness of the coil.

For single layer coils k' is the same as Nagaoka's factor, values of which were given in Table IV., Fig. 1.

For multilayer coils, the value of k' is given in Figs. 13 and 14, the latter being an enlarged portion of Fig. 13, suitable for coils of very small section.

These formulæ, again, only apply to close windings. For spaced windings the accurate formulæ should be employed.

Pancake (Disc) Coils.—The inductance of disc coils may be found from Reyner's formula directly. Coursey's curves, however, do not extend below $\frac{c}{2a} = \frac{b}{2a}$—*i.e.*, coils of square section.

In order to use these curves, therefore, the inductance may be obtained of an equivalent coil having the same mean diameter, but with b and c interchanged.

This assumption is tolerably accurate where $\frac{c}{b}$ is not too large, but is somewhat in error for very thin coils. For everyday work, however, this method is satisfactory.

Fig. 15 illustrates some types of equivalent coils.

Rectangular Coils.—As a practical approximation, the inductance of a rectangular or square coil may be taken as equal to that of a circular coil of equal area—*i.e.*:

$$D_{\text{equiv}} = \sqrt{\frac{4ab}{\pi}} = 1{\cdot}128\sqrt{ab},$$

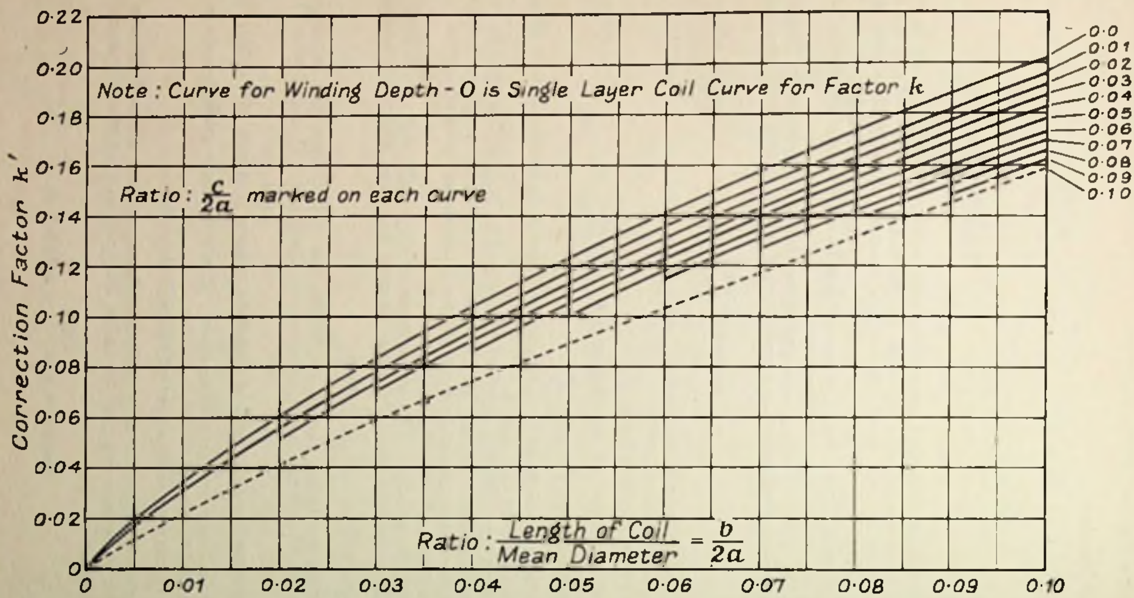

Fig. 14.—Curves of k' for Coils of Very Small Section.

where a and b are the mean dimensions of the rectangular coil, D_{equiv} is the mean diameter of the equivalent circular coil. If c is the radial thickness of the coil, $D_{external} = D_{equiv} + c$.

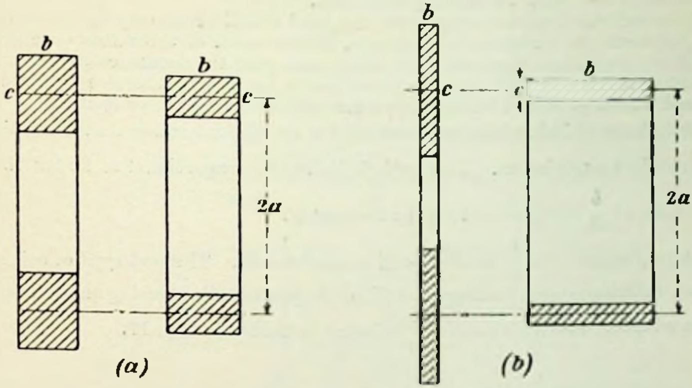

Fig. 15.—Types of Coil having Approximately Equal Inductances.

It can be shown that when rectangular coils are employed, the greatest inductance is obtained when the coil is square ($a = b$). Professor Howe has worked out curves showing the ratio of the inductance obtainable

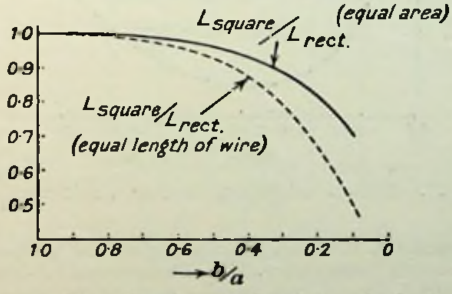

Fig. 16.—Ratio of Inductances of Square and Rectangular Coils.

with a square coil to that of a rectangular coil, first when the area of the two coils is the same, and secondly when the length of wire is the same. Fig. 16 shows the ratios for the two cases in terms of b/a, from which it will be seen that the square coil always gives maximum inductance.

Design of Inductance Coils.—In designing an inductance, the approximate formula may be employed at first. A certain configuration is assumed, and the number of turns necessary obtained. If this involves the use of too small a wire, or if in any other respect the result is not satisfactory, the dimensions may then be suitably modified.

If spaced windings are employed, the trial calculations may be based on the approximate formulæ, taking, say, 70 per cent. of the value obtained to allow for spacing, the final work being done with the accurate formulæ.

Excessive distributed capacity gives rise to losses, and therefore suitable spaced windings should be employed for coils desired to have low loss.

Best Shape of Coil.—The requirement for an efficient coil is that the ratio $\frac{R}{L}$ should be a minimum. This ratio is inversely proportional to D, but the best ratio of $\frac{b}{D}$ is dependent on the frequency.

At low frequencies, $\frac{b}{D}$ should be of the order 0·43. The best type of coil is then a multilayer one, having $c = b$ (*i.e.*, of square section), using an ordinary close winding, the variation of $\frac{R}{L}$ being as indicated in Fig. 17.

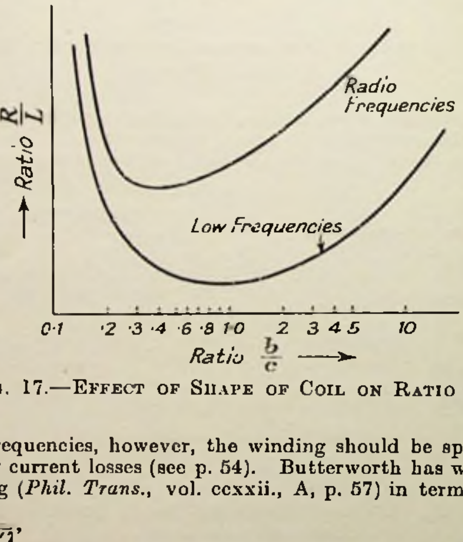

FIG. 17.—EFFECT OF SHAPE OF COIL ON RATIO $\frac{R}{L}$.

At higher frequencies, however, the winding should be spaced to avoid excessive eddy current losses (see p. 54). Butterworth has worked out the correct spacing (*Phil. Trans.*, vol. ccxxii., A, p. 57) in terms of an argument $z = \frac{830}{\sqrt{R_0'\lambda}}$,

where R_0' = D.C. resistance of wire per 1,000 yards,

λ = wave length (metres).

The spacing should then be as indicated in Fig. 18; 20 per cent. departure from this curve produces 5 per cent. increase of R/L.

If this correct spacing is adopted, then—

$$\frac{b}{D} \text{ should } = 0{\cdot}35 \text{ for medium frequencies } (z < 2),$$
$$= 0{\cdot}31 \text{ for high frequencies } (z > 2),$$

with a single layer winding.

These conditions, however, may involve an unduly large diameter, in which case a multilayer winding may be adopted. The condition for $\frac{b}{D}$ remains unaltered, although, if $z > 1$, the resistance of the coil will be greater than that obtainable with a single layer.

When multilayer coils are adopted, the best ratio of b/c is of the order 0·3 to 0·5, as shown in Fig. 17.

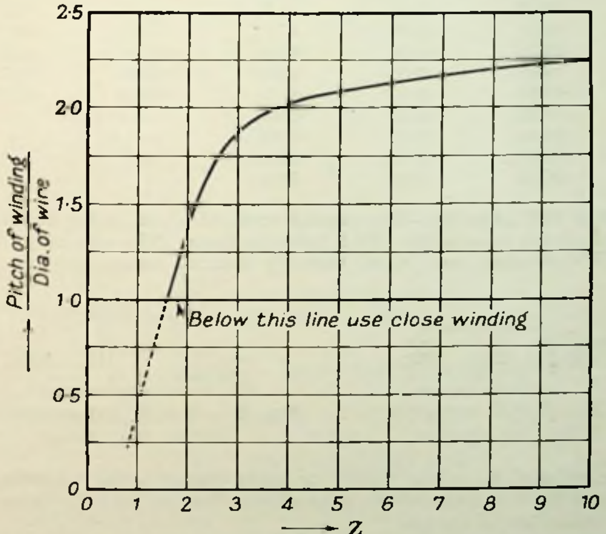

Fig. 18.—Correct Spacing for High Frequency Inductances.

For receiving inductances, Table VI. will be of service. With close windings the number of turns may be taken as $\frac{\text{area of winding}}{\text{diameter of wire (covered)}}$.

For transmitting inductances these remarks should be considered in conjunction with those on p. 55.

TABLE VI.

WIRE TABLE (FINE WIRES).

Overall Diameter of Wire in Inches.

SWG.	*Bare.*	*SSC.*	*DSC.*	*SCC.*	*DCC.*
20	·036	·038	·0395	·042	·046
21	·032	·034	·0355	·038	·042
22	·028	·030	·0315	·034	·038
23*	·024	·026	·0275	·030	·034
24	·022	·0238	·025	·028	·032
25*	·020	·0218	·023	·026	·030
26	·018	·0198	·021	·024	·028
28	·0148	·0166	·0178	·0208	·0248
30	·0124	·0139	·0149	·0184	·0224
32	·0108	·0123	·0133	·0158	·0198
34	·0092	·0107	·0117	·0142	·0182
36	·0076	·0091	·0099	·0116	·0156
38	·0060	·0075	·0083	·0100	·0140
40	·0048	·0063	·0071	·0088	·0128
41*	·0044	·0057	·0064	—	—
42*	·0040	·0053	·0060	—	—
43	·0036	·0049	·0056	—	—
47	·0020	·0033	·0040	—	—

Effect of Coil Capacity.—The several turns of a coil, being at differing potentials, have a capacity effect between them. The sum of all these effects is equivalent to a small capacity shunted across the coil. This

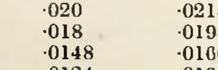

FIG. 19.—E.M.F. INTRODUCED INTO COIL ITSELF.

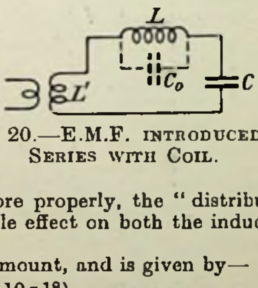

FIG. 20.—E.M.F. INTRODUCED IN SERIES WITH COIL.

capacity, referred to as the "self" or, more properly, the "distributed" capacity of the coil, exercises an appreciable effect on both the inductance and the resistance of the coil.

The inductance is increased by a small amount, and is given by—

$$L = L_0 (1 + L_0 C_0 \omega^2 \times 10^{-18}),$$

where L_0 = true inductance of the coil in μH,
L = effective inductance of the coil in μH,
C_0 = distributed capacity of coil in $\mu\mu$F.

The effect on the resistance of the coil, however, depends upon its relative position in the circuit. There are two possible cases.

In the first case the E.M.F. introduced into the circuit is introduced into the coil itself (see Fig. 19). Here the distributed capacity is simply in parallel with the tuning capacity, and consequently increases the wave length of the circuit.

* Non-stock sizes.

In the second case the E.M.F. is introduced into the circuit in series with the coil; the equivalent circuit is then as shown in Fig. 20. Now the effective resistance of a coil with a parallel condenser is dependent upon the frequency of the current, and approaches infinity when $f=\frac{1}{2\pi\sqrt{LC}}$. The effective resistance of a coil is thus increased by the distributed capacity by an amount dependent upon the frequency. Fig. 21 is a curve illustrating

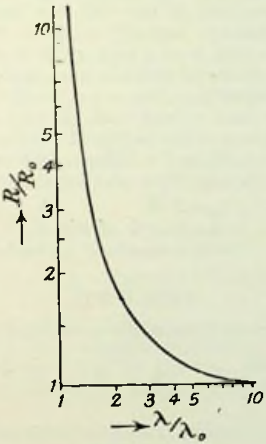

FIG. 21.—EFFECT OF DISTRIBUTED CAPACITY ON RESISTANCE OF COIL.

this point. It is the resistance itself which is increased (see p. 68), and not the reactance.

The effective resistance is given by—

$$R_{eff}=\frac{R}{\omega^2C_0^2R^2+(1-\omega^2L_0C_0)^2},$$

L_0, C_0, and R being in henries, farads, and ohms.

If the frequency at which the coil is being used is not near the resonant frequency of the coil (which is, of course, the general case)—*i.e.*, if $(1-\omega^2L_0C_0)$ is not near zero—this expression reduces to—

$$R_{eff}=\frac{R}{(1-\omega^2L_0C_0)^2}.$$

The effective inductance is given by—

$$L_{eff}=\frac{L(1-\omega^2LC_0)-C_0R^2}{\omega^2C_0^2R^2+(1-\omega^2L_0C_0)^2}.$$

The expression may be written $L_{eff}=L_0\ (1+\omega^2L_0C_0)$ approximately $=L_0\left(1+3{\cdot}553\ \frac{L_0C_0}{\lambda^2}\right)$, provided $(1-\omega^2L_0C_0)$ is not near zero.

Dead end effects are occasioned by utilising a portion of the coil only, without isolating the turns not in use. The circuit then develops into a species of coupled circuit (see Fig. 22), and will tune to two frequencies. Such a case may be treated by the appropriate methods employed for coupled circuits.

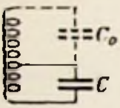

FIG. 22.—EFFECT OF DEAD ENDS.

In addition to this effect, however, which may not be very serious, there is the loss in the unused portion of the coil (see below), and there are also leakage capacity currents from the end of the coil which is at a high radio frequency potential. These effects all detract from the efficiency.

Value of Distributed Capacity.—The capacity of a coil is in any case very small. For single layer close wound coils Howe has shown that C_0 in $\mu\mu$F is of the order of $0 \cdot 5r$, where r is the radius of the coil in centimetres. It will be seen that this is very small, and is independent of the number of turns.

Drude has given the following data also for single layer coils:

Natural wave length: $\lambda_0 = 2\ kl$ metres,
where l = length of wire
k is a constant given below.

TABLE VII.

WAVE LENGTH CONSTANT FOR AIR-CORE COILS.

$\frac{b}{D} = \frac{\text{length of coil}}{\text{winding pitch}}$. $\frac{D}{d} = \frac{\text{winding pitch}}{\text{diameter of wire}}$.	6	4	3	2	1	0·7	0·3	0·1
1·09	·68	·76	·84	1·00	1·33	1·56	2·08	2·79
1·24	·67	·75	·83	·97	1·27	1·46	1·91	2·57
2·40	·66	·74	·81	·96	1·23	1·41	1·79	2·28

For multilayer coils C_0 is somewhat larger, but not very much more so; the chief cause of loss in a multilayer coil is the dielectric loss. The wires and dielectric in the middle of the coil are in a comparatively concentrated field, and unless spaced windings are employed, heavy losses may result.

Dielectric Losses in Coils.—This is an important feature in the design of coils. The dielectric of the "distributed" condenser is, of course, the insulation between the turns, and this as a rule is highly absorbing. That is to say, the loss due to dielectric absorption is large. Particular care must be taken, therefore, in the design of coils required to have low loss. If an ordinary winding is employed, the winding should be left unvarnished, although a thin coat of pure wax may be given in some cases. Even so the loss in the cotton or silk insulation is not negligible, and it is often desirable to space the winding. For receiving purposes several varieties of patent winding have come into use; in one form the wires cross each other at an angle, and successive layers are thus kept apart a distance equal to the thickness of the wire. In another form small fibre spacers are used to keep the successive layers adequately spaced.

For transmitting purposes spaced windings are always employed, the spacing factor being of the order 0·2 or less, as has already been stated.

The magnitude of the loss due to absorption may be gathered from the following example: Assume that the distributed capacity of a given coil is 50 $\mu\mu$F, the phase angle being 2° (see p. 29). If the coil is tuned with a capacity of 950 $\mu\mu$F to a frequency of 10^5 (=3,000 metres), phase angle ψ of C and C_0 in parallel $=\frac{50}{950+50}\times 2°=6'$.

Added resistance $=\frac{\tan\psi}{\omega(C+C_0)}=\frac{\tan 6'}{2\pi\times 10^5\times 10^{-9}}=2{\cdot}79$ ohms.

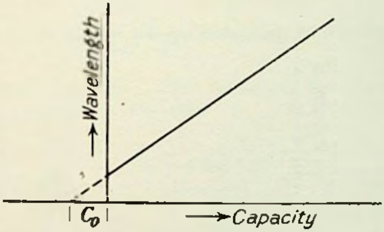

Fig. 23.—Illustrating Method of Determining C_0.

Fig. 23 indicates a method of determining the distributed capacity of a coil. The coil is shunted by a variable condenser, and the wave length observed at various settings of C. By drawing a graph and producing it as shown in the figure, the value of C_0 may be obtained.

Calculation of Mutual Inductance.

The calculation of mutual inductance is a rather more complicated process than the calculation of the self inductance. For certain simple cases the mutual inductance can be evaluated with a fair degree of accuracy, but in the majority of cases the labour involved is very considerable.

Certain formulæ are given here which cover most of the cases met in practice.

Two Parallel Wires:

$$M=0{\cdot}002l\left[\log_e\frac{2l}{D}-1+\frac{D}{l}\right], \qquad \text{(B.S.)}$$

where l = length of wires (both equal),
D = spacing.

In the above formula and all that follow, M is given in microhenries, the linear dimensions being measured in centimetres.

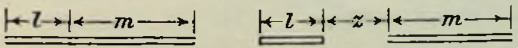

Fig. 24.—Two Wires, Axes in Line.

Two Wires, Axes in Line.—(*a*) Ends touching (Fig. 24):

$$M=0{\cdot}001\left[l\log_e\frac{l+m}{l}+m\log_e\frac{l+m}{m}\right],$$

where l and m are the lengths of the two wires.

(*b*) Ends separated by a distance z:

$$M = 0{\cdot}001\,[(l+m+z)\log_\epsilon(l+m+z) + z\log_\epsilon z - (l+z)\log_\epsilon(l+z) - (m+z)\log_\epsilon(m+z)]. \quad \text{(B.S.)}$$

Two Wires with Axes Parallel:

$$M = 0{\cdot}001\left[l\log_\epsilon\frac{AD+AD'}{AC+AC'} + m\log_\epsilon\frac{AD+AD'}{BD+BD'} + z\log_\epsilon\frac{(AD+AD')\,(BC+BC')}{(AC+AC')\,(BD+BD')} + AC + BD - AD - BC\right]. \quad \text{(Eccles.)}$$

where the dimensions are as indicated on the accompanying figure (Fig. 25).

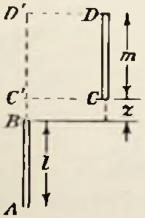

FIG. 25.—TWO WIRES WITH AXES PARALLEL.

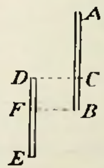

FIG. 26.—TWO WIRES OVERLAPPING.

Special cases of this formula arise from time to time, and a few of these are appended herewith.

1. Where the two wires overlap, as in Fig. 26, the mutual inductance is given by—

$$M = M_{EFBA} + M_{FDBC} + M_{FDCA}.$$

2. Where $z = 0$, and the two wires are of equal length (Fig. 27):

$$M = 0{\cdot}001\left[l\log_\epsilon\left(\frac{2l+\sqrt{D^2+4l^2}}{(l+\sqrt{D^2+l^2}}\right) - \sqrt{D^2+4l^2} + 2\sqrt{D^2+l^2} - D\right].$$

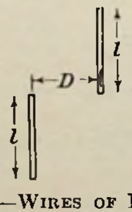

FIG. 27.—WIRES OF EQUAL LENGTH.

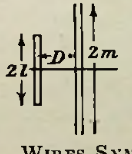

FIG. 28.—WIRES SYMMETRICALLY PLACED.

3. Where the two wires are symmetrically placed (Fig. 28):

$$M = 0{\cdot}002\left[2l\log_\epsilon\left\{\frac{l+m+\sqrt{(l+m)^2+D^2}}{D}\right\} + (l+m)\log_\epsilon\left\{\frac{l+m+\sqrt{(l+m)^2+D^2}}{m-l+\sqrt{(m-l)^2+D^2}}\right\} + \sqrt{(m-l)^2+D^2} - \sqrt{(l+m)^2+D^2}\right].$$

Two Equal Parallel Rectangles :

$$M = 0{\cdot}004\left\{a \log_e \frac{(a+x)y}{(a+z)d} + a_1 \log_e \frac{(a_1+y)x}{(a_1+z)d} + 2(z+d-x-y)\right\},$$

(Neumann.)

where a and a_1 are the sides of the rectangles,

d = distance apart,

$x = \sqrt{a^2+d^2}$,

$y = \sqrt{a_1^{\,2}+d^2}$,

$z = \sqrt{a^2+a_1^{\,2}+d^2}$.

Two Parallel Co-axial Circles.—Let the radii be a and b (a being the smaller), and let the planes be d apart.

Find distances r_1 and r_2—

where $r_1 = \sqrt{(b+a)^2+d^2}$,

$r_2 = \sqrt{(b-a)^2+d^2}$. (B.S.)

Then $M = F\sqrt{ab}$,

where F is a constant depending on the ratio r_2/r_1, values of which are given in Table X. below.

Coils—Co-axial Solenoids (not Concentric) :

$$M = 0{\cdot}00987 \frac{a^2b^2n_1n_2}{2l \times 2m}[k_1q_1 + k_3q_3],$$

(Rosa Grover.)

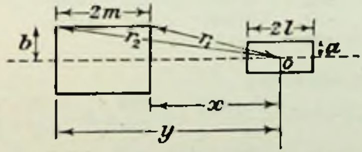

FIG. 29.—TWO CO-AXIAL SOLENOIDS NOT CONCENTRIC.

where a and b are the radii of the coils (a being the smaller), $2l$ and $2m$ are the lengths,

$$\left.\begin{aligned} r_1 &= \sqrt{x^2+b^2} \\ r_2 &= \sqrt{y^2+b^2} \end{aligned}\right\}\text{See Fig. 29.}$$

$$k_1 = \frac{2}{b^2}\left(\frac{y}{r_2} - \frac{x}{r_1}\right) \quad q_1 = 2l;$$

$$k_3 = \tfrac{1}{2}\left(\frac{x}{r_1^{\,5}} - \frac{y}{r_2^{\,5}}\right) \quad q_3 = a^2l\left(3 - 4\frac{l^2}{a^2}\right).$$

Other terms negligible in engineering practice.

TABLE VIII.

VALUE OF F IN FORMULA FOR MUTUAL INDUCTANCE BETWEEN TWO CIRCLES.

$\frac{r_2}{r_1}$	F	$\frac{r_2}{r_1}$	F	$\frac{r_2}{r_1}$	F	$\frac{r_2}{r_1}$	F
1·00	0·00	·74	1·150	·48	4·318	·23	1·17
·99	$0{\cdot}70 \times 10^{-5}$	·73	1·228	·47	4·501	·22	1·22
·98	2·0	·72	1·310	·46	4·69	·21	1·27
·97	3·7	·71	1·394	·45	4·89	·20	1·33
·96	5·8	·70	1·491	·44	5·09	·19	1·39
·95	8·1	·69	1·571	·43	5·30	·18	1·45
·94	$1{\cdot}07 \times 10^{-4}$	·68	1·664	·42	5·51	·17	1·52
·93	1·36	·67	1·760	·41	5·74	·16	1·59
·92	1·68	·66	1·859	·40	5·97	·15	1·66
·91	2·02	·65	1·962	·39	6·21	·14	1·74
·90	2·39	·64	2·068	·38	6·46	·13	1·83
·89	2·78	·63	2·177	·37	6·72	·12	1·93
·88	3·19	·62	2·290	·36	7·00	·11	2·03
·87	3·63	·61	2·407	·35	7·27	·10	2·15
·86	4·09	·60	2·527	·34	7·56	·09	2·28
·85	4·57	·59	2·652	·33	7·86	·08	2·42
·84	5·08	·58	2·780	·32	8·18	·07	2·58
·83	5·61	·57	2·913	·31	8·50	·06	2·78
·82	6·16	·56	3·050	·30	8·85	·05	3·00
·81	6·74	·55	3·191	·29	9·20	·04	3·28
·80	7·35	·54	3·337	·28	9·57	·03	3·64
·79	7·97	·53	3·487	·27	9·96	·025	3·87
·78	8·63	·52	3·643	·26	$1{\cdot}04 \times 10^{-2}$	·020	4·15
·77	9·31	·51	3·803	·25	1·08	·015	4·51
·76	$1{\cdot}002 \times 10^{-3}$	·50	3·969	·24	1·12	·010	5·02
·75	1·074	·49	4·140				

Co-axial Concentric Solenoids:

$$M = 0{\cdot}01974\,\frac{a^2 n_1 n_2}{g}\left[1+\frac{a^2b^2}{8g^4}\left(3-4\,\frac{l^2}{a^2}\right)\right],$$

where $2l$ and $2m$ are the lengths of the coils, *l being the shorter.*
a and b are the radii, *a being the smaller.*
$g = \sqrt{b^2+m^2}$ (see Fig. 30).

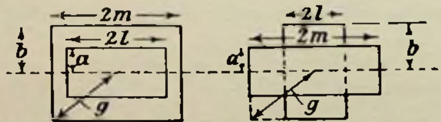

FIG. 30.—CO-AXIAL CONCENTRIC SOLENOIDS.

The second term is often negligible in comparison with unity. It is often more convenient in a case like this to know the coupling factor $k = \frac{M}{\sqrt{L_1 L_2}}$. This is given approximately by $k = \frac{a^2 l}{b^2 m}$.

Coils of Rectangular Section:

$$M = n_1 n_2 M_0,$$

where M_0 is the mutual inductance between two co-axial circles (or rectangles) located at the centre of the cross-section of the coil (see Fig. 31).

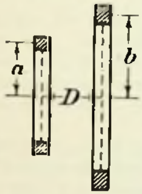

FIG. 31.—COILS OF RECTANGULAR SECTION.

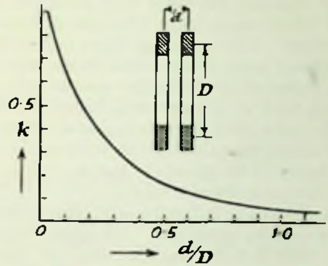

FIG. 32.—VARIATION OF k WITH AXIAL DISPLACEMENT.

This is accurate to about 1 per cent. Further accuracy can only be obtained at expense of an incommensurate amount of labour.

Flat Rectangular or Spiral Coils:

$$M = n_1 n_2 M_0,$$

where M_0 is the mutual inductance between two co-axial rectangles or circles for which—

$$\left.\begin{aligned} a &= a_0 - (n-1)\,D \\ a_1 &= a'_0 - (n-1)\,D \end{aligned}\right\}\text{for rectangular coils,}$$

$$a = a_1 + \tfrac{1}{2}(n-1)\,D \text{ for circular coils (see Figs. 10 and 12).}$$

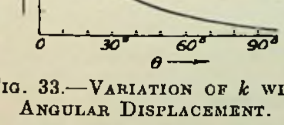

FIG. 33.—VARIATION OF k WITH ANGULAR DISPLACEMENT.

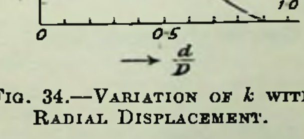

FIG. 34.—VARIATION OF k WITH RADIAL DISPLACEMENT.

Variable Couplings.—Since pancake coils are very common, the variation of coupling factor with the relative positions of the coils is appended.

From these curves it is possible to obtain the mutual inductance, given the self inductances of the individual coils. Or if the coils are to form a variometer, then the inductance of the coils in series is $L = L_1 + L_2 \pm 2M$.

If the coils are connected in parallel, $L = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 \pm 2M}$.

Fig. 32 gives the variation of k with the axial distance between the coils, while Figs. 33 and 34 give the variation of k for angular and radial displacements. For simple co-axial rotation, k is proportional to sin θ.

Calculation of Capacity and Condenser Design.

The capacity between two points is the quantity of electricity which has to be transferred from one point to the other to produce unit potential difference. The capacity of an isolated body is the capacity obtained when the second point is removed to an infinite distance.

All parts of a conductor are at the same potential when no current is flowing, and the charge will distribute itself so that this is so. It is only in simple cases, such as spheres and ellipsoids, that the charge also is uniform, but it simplifies calculations to consider the charge uniform and the potential as varying from point to point. This point is referred to later in dealing with the capacity of aerials.

The following formulæ give the capacities of some of the more common bodies met in practice. All dimensions are in cms., and the capacities in micro-microfarads ($\mu\mu$F). An air dielectric is assumed; for any other dielectric the formulæ should be multiplied by the dielectric constant ε.

Isolated Sphere: $C = 1{\cdot}112r$, where r is the radius.

Concentric Spheres: $C = 1{\cdot}112 \dfrac{r_1 r_2}{r_1 - r_2}$,

where r_1 is the larger, and r_2 the smaller radius.

Two Spheres close together:

Joint capacity:

$$C = 1{\cdot}112r\left(1+\frac{x}{6r}\right)\left(0{\cdot}3863 - \frac{x}{12r}\right).$$

Capacity between the two:

$$C = \frac{1{\cdot}112r}{2}\left(1+\frac{x}{6r}\right)\left(1{\cdot}2074+\tfrac{1}{2}\log_e\frac{r}{x}+\frac{x}{18r}\right). \quad \text{(Russell.)}$$

where $x =$ distance between nearest points.

If the spheres are so far apart that it is immaterial whether x is measured from the nearest points or between the centres, then the capacity is given by—

$$C = \frac{1{\cdot}112r_1r_2x}{(r_1+r_2)x - 2r_1r_2}.$$

Capacity of Sphere to Earth:

$$C = 1{\cdot}112hr/h - r,$$

where r is the radius,
h is the height above earth.

Isolated Disc:

$C = 0{\cdot}708r$, where r is the radius.

Co-axial Cylinders:

$$C = \frac{1{\cdot}112l}{2\log_e\dfrac{r_1}{r_2}},$$

where r_1 is the larger and r_2 the smaller radius.

Two Long Parallel Cylinders far Apart:

$$C = \frac{1 \cdot 112 l}{2 \log_\epsilon \dfrac{d^2}{r_1 r_2}},$$

where l is the length,
r_1 and r_2 are the radii,
d is the distance apart.

Long Parallel Strips:

$$C = \frac{1 \cdot 112}{4\pi^2}\left[1 + x + \log_\epsilon\ (1 + x + \log_\epsilon\ [1 + x])\right],$$

where
$$x = \frac{\pi b}{d},$$
b is the breadth of the strip,
d is the distance apart.

If x is more than 100—

$$C = 0 \cdot 0255\ (x - 4 \cdot 62). \qquad \text{(J. J. Thompson.)}$$

Plate Condensers:

$$C = 0 \cdot 0885\ \frac{\varepsilon A(n-1)}{d},$$

where A = area of plate,
n = number of plates,
K = dielectric constant,
d = distance between plates.

If several dielectrics are employed—

$$C = 0 \cdot 0885\ \left(\frac{A\ (n-1)}{d_1/\varepsilon_1 + d_2/\varepsilon_2 + \ .\ .\ .}\right).$$

Edge effect is the slight increase of capacity obtained due to the non-uniform distribution of the field at the edges of the plates. If d is small compared with the dimensions of the plates, the edge effect may be allowed for by increasing the size of the plates, adding an "additional strip,"

$$w = 0 \cdot 11d \text{ for plates with straight edges,}$$
$$= 0 \cdot 44d \text{ for circular plates.}$$

Variable Condensers:

$$C = \frac{0 \cdot 1390\varepsilon\ (n-1)\ (r_1^2 - r_2^2)}{d},$$

where r_1 is the radius of the moving plate,
r_2 is the small radius on the fixed plates (see Fig. 37),
d is the distance between the plates,
n is the number of plates,
ε is the dielectric constant.

Losses in Condensers.—Condenser losses are of two varieties—conductor loss and dielectric loss. The former is due to the resistance of the plates and leads of the condenser, and is thus equivalent to a series resistance. The power factor, therefore (see Fig. 35), is $\cos\theta = \sin\psi$. It will be seen that $\tan\psi = R\omega C$, which increases directly with the frequency. The magnitude of the effect may be gathered from the following example: $C = 0 \cdot 01\,\mu$ F, R = 1 ohm. At a frequency of 50 cycles, the power factor is $3 \cdot 8 \times 10^{-6}$, which is negligible. At 10^6 cycles, on the other hand, the power factor is 6·3 per cent., which is a serious loss. Hence it is most desirable to keep down the resistance of the leads and plates in a radio condenser as far as is practicable.

The second source of loss, dielectric loss, is more complex. There is first of all the ordinary leakage through the dielectric, which is in effect a resistance

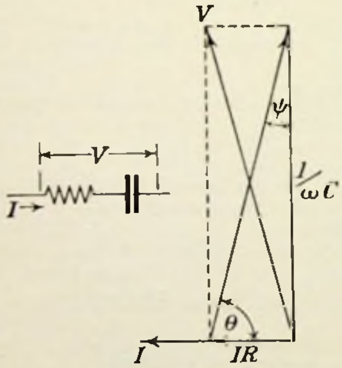

FIG. 35.—CONDENSER WITH SERIES RESISTANCE.

shunt across the condenser (Fig. 36). Here if ψ is the phase angle as before, then tan $\psi = \frac{1}{R\omega C}$. Here the power factor is inversely proportional to the frequency, so that the effect, though appreciable at low frequency, is negligible at high frequency.

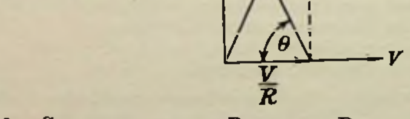

FIG. 36.—CONDENSER WITH PARALLEL RESISTANCE.

More important is the phenomenon known as dielectric absorption. When a condenser is charged, the initial rush of current is followed by a relatively small more gradual current which appears to soak into the dielectric in some manner.

Similarly, if a condenser is discharged and left for a few minutes, it is found that a further discharge can be obtained, which indicates that a portion of the charge is stored in the dielectric in some manner quite different from the simple electron displacement which constitutes the ordinary charge. If the condenser is charged and discharged periodically, this dielectric absorption causes heat to be generated in the dielectric. Mathematically, therefore, the loss is considered as due to a fictitious resistance in series with the condenser, and this is often termed the "equivalent series resistance" of the condenser (although this is not a good term, since the power factor is constant and does not increase with frequency, as is the case with a series resistance).

The variation of this loss with frequency is not accurately known, but is approximately inversely proportional to the frequency; this is to be expected, since at the higher frequencies the time available for the dielectric absorption is less.

As an approximation, therefore, the power factor of a condenser due to this cause is constant. This holds for all forms of concentrated capacity, and also applies to aerials above the radiation band (see discussion of aerial resistances, p. 97).

The Bureau of Standards, Washington, gives the following figures for the power factor of various types of condenser measured at 14,500 volts:

Compressed air	0·001*
Moulded (Murdock)	0·004
Glass plates in oil	0·005
Glass plate (Moscicki)	0·006
Micanite	0·023
Paper	0·024

It will be seen that the loss due to dielectric absorption only occurs in solid or liquid dielectrics. In air or gases, the loss is negligibly small. When brushing sets in, however, a very rapid increase of loss takes place, and this, of course, at once limits the voltage at which the particular condenser may be employed.

Design of Condensers.—Condensers, as used in transmitting circuits, which have to handle large quantities of energy, are sometimes termed power condensers.

The energy stored in such a condenser is given by $W=\frac{1}{2}CV^2$, and hence the power handled is $\frac{1}{2}V^2CN$, where N is the number of sparks discharged per second. For a continuous wave system, the power is simply $V \times I$.

If R is the total resistance of the condenser, then the power lost is given by $P=\omega CV^2 \sin\varphi$, where $\sin\varphi, =\cos\theta$, is the power factor.

Receiving Condensers.—The design of receiving condensers is a relatively simple matter. Adequate insulation coupled with tolerably low loss is the chief factor in the design. Consequently, for standard condensers, air dielectric should be employed, and the space occupied by the supports should be as small as possible.

Variable condensers are very commonly employed for tuning the various circuits involved. Such a condenser comprises two sets of plates; one set is fixed, while the other set is mounted on a spindle and arranged to be rotated into the spaces between the fixed plates. Hence in the maximum position the fixed plates completely overlap the moving, while in the

* Due to distance pieces.

minimum position there is no overlap, and consequently the capacity is a minimum. It is not quite zero, since there is a small capacity existing between the edges of the two banks of plates, but with good design the minimum value can be reduced to about 3 to 4 per cent. of the maximum.

The most common form of variable condenser employs semicircular plates (see Fig. 37). The capacity of such a condenser has been given on p. 29; the expression may be rewritten in terms of polar co-ordinates, in which case the equation is of the form:

$$C = a\theta + b.$$

The increase of capacity is thus directly proportional to the increase in the angle. Now in tuning receiving circuits, the wave length obtained with a constant inductance is proportional to the square root of the capacity in the circuit. Using the ordinary form of condenser, therefore, the change in wave length produced by the rotation of the moving plates will be proportional to $\sqrt{\theta}$. It is often advantageous to have a condenser which gives a uniform variation of wave length, and it will be obvious that with such an instrument the capacity must vary as the square of the angle of

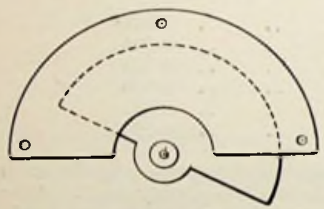

Fig. 37.—Variable Condenser.

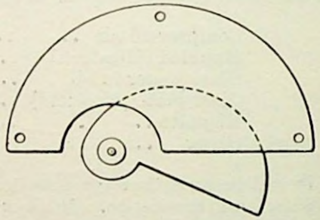

Fig. 38.—Square-Law Condenser.

rotation. The moving plates are therefore made of the shape shown in Fig. 38, the radius at each point being obtained from the expression:

$$r = \sqrt{4a\theta + r_1^2},$$

where r_1 is the radius of the centre boss carrying the spindle.

A third type of variable condenser, of somewhat more limited application, is the type used on decremeters. Here the requirements for a uniform scale are that the percentage change in capacity for a given variation of angle shall be the same at all points of the scale (see p. 73). The condition for this is—

$$r = \sqrt{2C_0\varepsilon^{a\theta} + r_1^2},$$

where C_0 is the capacity when $\theta = 0$, r_1 is the radius of the spindle boss as before, and a is a constant.

Variation of Capacity with Frequency.—The capacity of a condenser varies with the frequency due to two causes. The first is inherent in the condenser itself, it being found that the current flowing into a condenser in a very short time is slightly less than that obtained when ample time is afforded for the operation. The phenomenon may be a secondary effect of the direct conduction of current through the dielectric, but in any case the apparent capacity gradually decreases as the frequency is raised, tending to a limit C_0 at infinite frequency. This capacity C_0 is called the geometric capacity, and corresponds to the calculated value.

On the assumption that the variation with frequency is due to conduction, the capacity at any frequency is given by—

$$C = C_0\,(1 + \psi^2),$$

where C_0 is the geometric capacity, and $\psi = \dfrac{10^6}{R\omega C_0}$.

The order of the effect may be gauged from the following example: If $C_0 = 0{\cdot}001\mu F$, $R = 10M\Omega$, then the capacity at $60\sim = 0{\cdot}00107\ \mu F$.

For all radio frequencies the effective capacity $= C_0$.

The second cause of variation is the inductance in the leads of the condenser. Due to this cause the apparent capacity varies in accordance with the expression:

$$C_a = C\,[1 + \omega^2 CL \times 10^{-18}],$$

C and L being in μH and $\mu\mu$F.

Table IX. gives the values of the capacities of air condensers in terms of the dimensions and number of plates. This table is extracted from *Popular Wireless*, vol. iv., No. 68.

The figures given in Table IX. are calculated on the assumption that No. 22 S.W.G. metal plates are used. If other gauges of metal are employed, the following corrections must be made:

Thickness of Spacing Washer.	*Thickness of Plates (S.W.G.).*			
	18	20	22	24
	Percentage Increase or Decrease.			
$\frac{3}{32}$ in.	+43	+13	—	−8
$\frac{1}{8}$ in.	+26	+9	—	−6
$\frac{5}{32}$ in.	+18	+7	—	−4
2 mm.	+65	+18	—	−11
3 mm.	+28	+9	—	−6

Capacity of Aerials.

This is a rather specialised subject, and in the case of multi-wire aerials is apt to involve rather laborious calculation.

Professor Howe has devoted considerable time and energy to the solution of the problem, and has succeeded in evolving methods whereby exceedingly accurate results can be obtained, under suitable conditions.

The capacity of an aerial is composed mainly of the capacity of the wires themselves, and the calculation of this quantity may be effected with reasonable celerity. The figure thus obtained, however, is always too small, and may in extreme cases be only some 50 per cent. of the actual capacity. The proximity of the earth, of masts and buildings, all tend to increase the capacity, and it is the calculation of the effect of these subsidiary factors which entails the extra and somewhat tedious calculations.

These effects cannot, unfortunately, be neglected, but numerical examples are given in the data which follow which will serve to indicate the relative importance of the several corrections.

TABLE IX.—Capacities of Variable

Diameter of Moving Plates in Inches.	*Thickness of Spacing Washers.*	3 *Plates.*	4 *Plates.*	5 *Plates.*	10 *Plates.*	15 *Plates.*
2	3/32 in.	·000018	·000028	·000037	·000083	·00013
	1/8 in.	·000013	·000019	·000025	·000056	·000088
	5/32 in.	·0000094	·000014	·000019	·000043	·000066
	2 mm.	·000024	·000036	·000048	·00011	·00017
	3 mm.	·000013	·00002	·000027	00006	000094
2½	3/32 in.	·000031	·000046	·000061	·00014	·00021
	1/8 in.	·000021	·000031	·000041	·000093	·00014
	5/32 in.	·000016	·000023	·000031	·00007	·00011
	2 mm.	·00004	·000059	·000079	·00018	·00028
	3 mm.	·000022	·000033	·000044	·0001	·00016
3	3/32 in.	·000045	·000068	·00009	·0002	·00032
	1/8 in.	·000031	·000046	·000061	00014	·00021
	5/32 in.	·000023	·000035	·000046	00011	·00016
	2 mm.	·000059	·000088	·00012	·00026	·00041
	3 mm.	·000033	·00005	·000066	·00015	00023
3½	3/32 in.	·000063	·000094	·00013	·00028	·00044
	1/8 in.	·000042	·000064	·000085	·00019	·0003
	5/32 in.	·000032	·000048	·000064	·00015	·00023
	2 mm.	·000081	·00012	·00016	·00037	·00057
	3 mm.	·000046	·000068	·000091	·00021	·00032
4	3/32 in.	·000083	·00012	·00017	·00037	·00058
	1/8 in.	·000056	·000084	·00011	·00025	·00039
	5/32 in.	·000043	·000064	·000085	·00019	·0003
	2 mm.	·00011	·00016	·00021	·00048	·00075
	3 mm.	·00006	·000091	·00012	·00027	·00042
4½	3/32 in.	·00011	·00016	·00021	·00048	·00074
	1/8 in.	·000072	·00011	·00014	·00032	·0005
	5/32 in.	·000054	·000082	·00011	·00024	·00038
	2 mm.	·00014	·00021	·00027	·00062	·00096
	3 mm.	·000077	·00012	·00015	·00035	·00054
5	3/32 in.	·00013	·0002	·00026	·00059	·00092
	1/8 in.	·000089	·00013	·00018	·0004	·00062
	5/32 in.	·000067	·0001	·00013	·0003	·00047
	2 mm.	·00017	·00026	·00034	·00077	·0012
	3 mm.	·000096	·00014	·00019	·0004	·00067

Condensers (Microfarads). (Air Dielectric.)

20 *Plates.*	25 *Plates.*	30 *Plates.*	40 *Plates.*	50 *Plates.*	60 *Plates.*
·00018	·00022	·00027	·00036	·00045	·00054
·00012	·00015	·00018	·00024	·00031	·00037
·00009	·00011	·00014	·00018	·00023	·00028
·00023	·00029	·00035	·00046	·00058	·0007
·00013	·00016	·0002	·00026	·00033	·0004
·00029	·00037	·00045	·0006	·00075	·00091
·0002	·00025	·0003	·0004	·00051	·00061
·00015	·00019	·00023	·0003	·00038	·00046
·00038	·00047	·00058	·00077	·00097	·0012
·00021	·00027	·00032	·00043	·00055	·00066
·00043	·00054	·00066	·00088	·0011	·0013
·00029	·00037	·00045	·0006	·00075	·0009
·00022	·00028	·00034	·00045	·00057	·00069
·00056	·0007	·00085	·0011	·0014	·0017
·00031	·0004	·00048	·00064	·00081	·00097
·0006	·00076	·00091	·0012	·0015	·0019
·0004	·00051	·00062	·00083	·001	·0012
·00031	·00039	·00047	·00063	·00079	·00095
·00077	·00097	·0012	·0016	·002	·0024
·00043	·00055	·00066	·00089	·0011	·0013
·00079	·00099	·0012	·0016	·002	·0025
·00053	·00068	·00082	·0011	·0014	·0017
·0004	·00051	·00062	·00083	·001	·0013
·001	·0013	·0016	·0021	·0026	·0032
·00057	·00073	·00087	·0012	·0015	·0018
·001	·0013	·0015	·0021	·0026	·0031
·00068	·00086	·001	·0014	·0018	·0021
·00052	·00065	·00079	·0011	·0013	·0016
·0013	·0017	·002	·0026	·0034	·0041
·00074	·00093	·0011	·0015	·0019	·0023
0013	·0016	·0019	·0026	·0032	·0039
·00085	·0011	·0013	·0017	·0022	·0027
·00064	·00081	·00098	·0013	·0016	·002
·0016	·002	·0025	·0033	·0042	·005
00091	·0011	·0014	·0019	·0023	·0028

The method will be described with a fair amount of detail, so that the principles may be applied if desired to more complex cases not specifically referred to here. The method, in brief, is as follows:

The capacity under consideration is the static capacity, the effective capacity under working conditions being obtained from this as indicated on p. 83. If an aerial system is charged to a given static potential, the charge will not be uniform, but will distribute itself so that the potential at each and every point is the same. It is more convenient for the purposes of calculation, however, to assume the charge to be uniform and the potential to be varying from point to point. The average potential may then be determined, and this potential is assumed to be the same as would be acquired by the aerial under actual physical conditions (*i.e.*, with a non-uniform distribution of charge). The error involved in this assumption is very small and is quite negligible.

If now the aerial is assumed to carry a charge q per unit length, and if the total length of wire in the aerial is l, then the total charge on the aerial $= lq$.

If the average potential has been determined to be V_{av}, then capacity $= \frac{lq}{V_{av}}$, in electrostatic units (multiply by 1·112 to convert to $\mu\mu$F).

The data which follow, therefore, are chiefly concerned with the determination of V_{av}, and with the additions and subtractions necessitated by the proximity of the earth and foreign bodies.

(1) Capacity of the Aerial itself—*Single Wire:*

$$V_{av} = 2\left(\log_e \frac{l}{r} - 0{\cdot}309\right),$$

where l = length of aerial in centimetres,
r = radius of wire in centimetres,

and the wire is assumed to carry unit charge per centimetre length.

Hence the capacity $C = \dfrac{l}{2\left(\log_e \frac{l}{r} - 0{\cdot}309\right)}$ centimetres,

$$= \frac{1{\cdot}112 l}{2\left(\log_e \frac{l}{r} - 0{\cdot}309\right)} \ \mu\mu\text{F}.$$

The values of $\left(2 \log_e \frac{l}{r} - 0{\cdot}31\right)$ may readily be obtained from the log curves given in Figs. 39 and 40.

Flat Multiple Wire Aerials.—In this case the potential at any point in the aerial is due to the charge on that particular wire plus the effect of the charges on all the other wires. If there are n wires—

$$V_{av} = 2\left\{n\left(\log_e \frac{l}{d} - 0{\cdot}309\right) + \log_e \frac{d}{r} - B\right\},$$

where d = spacing of the wires,
l = length of aerial,
r = radius of wire,
B is a constant depending on n, all dimensions being in centimetres.

This formula is not strictly accurate, since it is calculated on the assumption that V_{av} is the average of the potentials of the middle points of the several wires, but the error involved is small.

Values of the constant B are given below. Values of $\log_\epsilon \frac{l}{d}$ and $\log_\epsilon \frac{d}{r}$ may be obtained from the curves given.

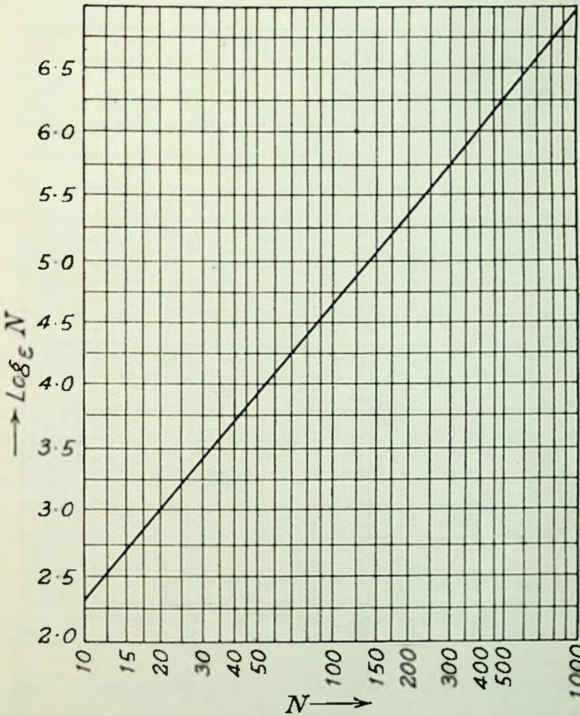

FIG. 39.—VALUES OF LOG$_\epsilon$ N FOR N = 10 TO N = 1,000.

The capacity of such an aerial, therefore, is given by—

$$C = \frac{1 \cdot 112\, nl}{2\left\{ n\left(\log_\epsilon \frac{l}{d} - 0 \cdot 309\right) + \log_\epsilon \frac{d}{r} - B \right\}} \mu\mu F.$$

The numerator involves n since, in determining the total charge, the total length of wire in the aerial must be taken into account.

TABLE X.

VALUES OF CONSTANT B IN FORMULA FOR CAPACITY OF MULTIWIRE AERIALS.

n	2	3	4	5	6	7	8	9	10	11	12
B	0	0·46	1·24	2·26	3·48	4·85	6·40	8·06	9·80	11·65	13·58

Above $n = 12$, $B = 2{\cdot}44\,(n - 6{\cdot}7)$.

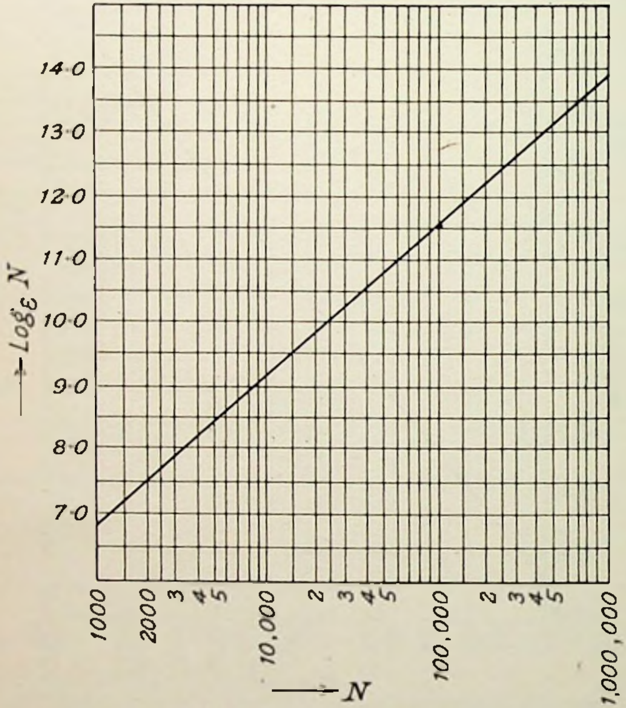

FIG. 40.—VALUES OF LOG N FOR $N = 10^3$ TO $N = 10^6$.

Four-Wire Box Type Aerial.—By a similar process of reasoning, the average potential is found to be—

$$V_{av} = 2\left\{\log_e \frac{l}{r} + Y\right\},$$

where d = spacing of the wires,
l = length of aerial,
r = radius of wires,
Y = a constant depending upon l/d.

Values of Y are given below. In this case, since the total length of wire is $4l$, the capacity is:

$$C = \frac{4 \cdot 448 l}{2 \log_\epsilon \frac{l}{r} + Y} \mu\mu F,$$

Values of Y above.

l/d	20	50	100	150	200
Y	7·58	10·22	12·26	13·48	14·33

Values of $\log_\epsilon \frac{l}{r}$ can be obtained from the log curves given in Fig. 40.

Capacity of Multiwire Cage Aerials.—The formula for the four-wire box type aerial is obtained by considering the potential of one wire on the system due:

(*a*) To its own-charge.
(*b*) To the charges on the other wires.

Since the system is symmetrical, the average potential of any one wire will be the same as that of the whole aerial.

Assuming unit charge per centimetre, the potential due to its own charge will be:

$$V = 2 \left(\log_\epsilon \frac{l}{r} - 0 \cdot 309 \right).$$

The potential due to a wire at a distance d is given by—

$$V = 2 \left(\sinh^{-1} \frac{l}{d} - \sqrt{1 + \frac{d^2}{l^2}} + \frac{d}{l} \right),$$

and from this the average potential of the wire due to all other wires in the aerial may be obtained.

In the case of a four-wire cage—

$$V = 2 \left(\log_\epsilon \frac{l}{r} - 0 \cdot 309 \right), \text{ due to itself,}$$

$$+ 4 \left(\sinh^{-1} \frac{l}{d} - \sqrt{1 + \frac{d^2}{l^2}} + \frac{d}{l} \right), \text{ due to the two nearest wires,}$$

$$+ 2 \left(\sinh^{-1} \frac{l}{\sqrt{2}d} - \sqrt{1 + \frac{2d^2}{l^2}} + \frac{\sqrt{2}d}{l} \right), \text{ due to fourth wire.}$$

The factor Y just given is the evaluation of the quantity (− 0·309 + last two terms) for different values of $\frac{l}{d}$.

This method may be extended to cages containing more than four wires. For a six-wire cage, the average potential is given by—

$$V_{av} = 2 \left(\log_\epsilon \frac{l}{r} - 0 \cdot 307 \right)$$

$$+ 4 \left(\sinh^{-1} \frac{l}{d} - \sqrt{1 + \frac{d^2}{l^2}} + \frac{d}{l} \right)$$

$$+ 4 \left(\sinh^{-1} \frac{l}{\sqrt{3}d} - \sqrt{1 + \frac{3d^2}{l^2}} + \frac{\sqrt{3}d}{l} \right)$$

$$+2\left(\sinh^{-1}\frac{l}{2d}+\sqrt{1+\frac{4d^2}{l^2}}+\frac{2d}{l}\right),$$

where, as before, d is the spacing of two adjacent wires (see Fig. 41).

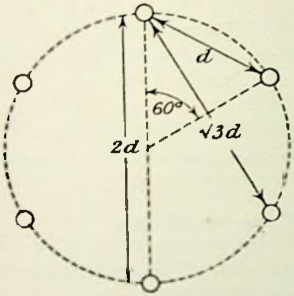

FIG. 41.—SIX-WIRE CAGE.

The potential may obviously be written in the form:

$$V=2\left[\log_e\frac{l}{r}-0{\cdot}309+f\left(n,\frac{l}{d}\right)\right]$$

the last two terms being the factor Y. $C=1{\cdot}112\ nl/Y$.

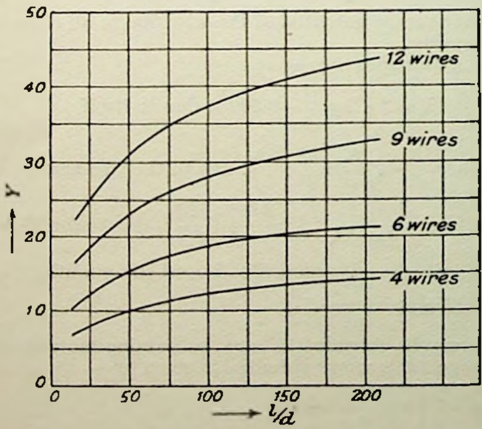

FIG. 42.—VALUES OF Y IN FORMULA FOR CAPACITY OF CAGE AERIALS

The author has evaluated Y for cages comprising 6, 9, and 12 wires in terms of l/d, as with the four-wire box aerial. The results are given in the table appended, and are also plotted in Fig. 42.

Values of Y for other numbers of wires may be obtained by extrapolation or from the formula:

$$Y = (Y_4 + 0{\cdot}31)\frac{n}{4} - 0{\cdot}31,$$

where Y_4 is the value of Y for a four-wire cage and n = number of wires.

Values of sinh x are given in Appendix A, and also in Fig. 293.

TABLE XI.

VALUES OF Y FOR CAGE AERIALS.

Number of Wires:	l/d = 20	50	100	150	200
4	7·58	10·22	12·26	13·48	14·33
6	11·53	15·49	18·59	20·38	21·65
9	17·44	23·38	28·00	30·72	32·63
12	23·36	31·28	37·40	41·00	43·61

d = spacing between *adjacent* wires.

It does not pay to increase n indefinitely, unless the spacing is kept constant (*i.e.*, the diameter of the cage is increased).

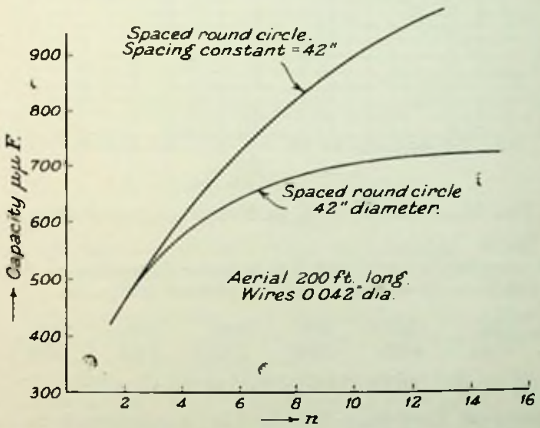

FIG. 43.—VARIATION OF CAPACITY OF CAGE AERIAL WITH THE NUMBER OF WIRES.

Fig. 43 gives the capacity of a cage aerial 200 feet long with wires 0·042 inch diameter:

(*a*) With the wires spaced evenly round the circumference of a circle of 42 inches diameter.

(*b*) With the spacing kept constant at 42 inches.

It will be seen that in the first case a limit is very quickly reached; the capacity in the second case continues to increase with n, but it is obvious that a limit will shortly be attained.

(2) **Effect of Earth.**—The formulæ given so far refer only to the capacity of the aerial in space. The proximity of the earth increases the capacity, or, in other words, decreases the value of V_{av}. This decrease may be regarded as due to the effect of an "image" of the actual aerial situated at a distance below the ground equal to the height of the actual aerial above the ground.

To allow for the effect of the earth, therefore, it is necessary to subtract from the value of V_{av} previously determined a quantity nE, where n is the

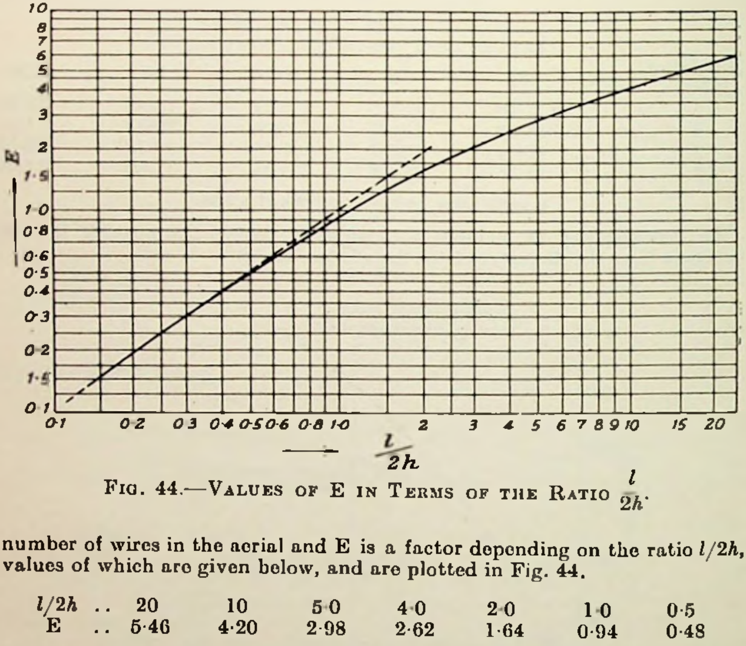

FIG. 44.—VALUES OF E IN TERMS OF THE RATIO $\frac{l}{2h}$.

number of wires in the aerial and E is a factor depending on the ratio $l/2h$, values of which are given below, and are plotted in Fig. 44.

$l/2h$ ..	20	10	5·0	4·0	2·0	1·0	0·5
E ..	5·46	4·20	2·98	2·62	1·64	0·94	0·48

For values of $l/2h$ less than 0·5, E $= l/2h$.

(3) **Capacity of Lead-In.**—The lead-in is considered in conjunction with the remainder of the aerial. There is first of all the capacity of the lead-in by itself (making due allowance for the effect of the earth), and secondly there is the effect of the lead-in on the aerial, and *vice versa*.

The capacity of the lead-in itself is calculated, from the expression for a single wire.

The potential due to the image in the earth is found to be simply equal to the charge on the wire, provided the end of the wire is more than 3 or 4 feet from the ground. Consequently—

$$V_{av} = 2\left\{\log_e \frac{l}{r} - 0{\cdot}809\right\}.$$

If the wire runs down to the ground, however, the potential due to the earth is equal to 1·38 times the charge on the wire, and the above formula must be modified accordingly.

(4) **Effect of Lead-In on Aerial, and vice versa.**—The average potential of the aerial is increased by the presence of the lead-in, and *vice versa*. Where the two are at right angles, as is usually the case, this increase can be determined by the following formula:

Let l = length of the wire under consideration (uncharged).
l' = length of wire at right angles (assumed to carry unit charge per centimetre).
$m = l/l'$.

Then potential of l due to l' is given by—

$$V = \sinh^{-1}\frac{1}{m} - \frac{\sinh^{-1} m}{m}.$$

The value of this correction is given in Fig. 45. It should be borne in mind that this curve is calculated on the assumption that the wire at right angles

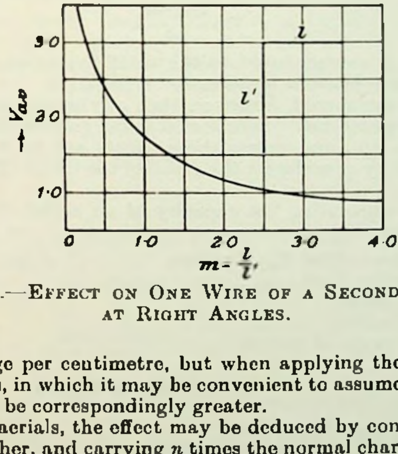

FIG. 45.—EFFECT ON ONE WIRE OF A SECOND WIRE AT RIGHT ANGLES.

carries unit charge per centimetre, but when applying the method to more complicated cases, in which it may be convenient to assume a greater charge, the potential will be correspondingly greater.

For multiwire aerials, the effect may be deduced by considering the wires all bunched together, and carrying n times the normal charge. This method is not quite accurate, a small correction being necessary owing to the spacing of the wires. This correction is not serious unless the ratio m is small, and the wires widely spaced. A reasonably accurate correction may be obtained by multiplying the value of V thus obtained by a correction factor—

$$X = 1 - 0{\cdot}85n'\sqrt{\frac{d^2}{ll'}};$$

where d = spacing of the wires,
l and l' are the lengths of the wires,
n' is a factor depending on the number of wires n.

n	2	3	4	5	10
n'	0·5	0·89	1·25	1·54	3·29

If the lead-in is not at right angles to the aerial, the curves employed in the calculations on umbrella aerials should be utilised.

Where the lead-in is composed of wires spaced at the top and tapering to a point at the base, Professor Howe has shown that if the angle between the wires is a reasonably acute (*i.e.*, less than about 60°), then the lead-in may be replaced by a series of parallel wires at a distance $d' = 0{\cdot}36d$, where d is the spacing of the wires at the top of the aerial.

This formula is also useful in considering fan-shaped aerials.

In the case of T aerials only half of the horizontal portion is considered. When finding the effect of the lead-in on the aerial, l is taken as the *half length* of the horizontal top. When finding the effect of the aerial on the lead-in, the same procedure is adopted, but the result is doubled.

(5) **Effect of the Image of the Lead-In on the Aerial, and vice versa.**—The image of the lead-in affects the aerial, reducing the average potential, and *vice versa*. This effect is not large, being of the order of 2 or 3 per cent. in ordinary cases, and it is fortunately readily calculable.

The charge on the image is assumed concentrated at its mid-point. If, then, d is the distance of the aerial from this mid-point, the potential is given by—

$$V = \frac{\text{charge}}{d}.$$

The charge is, of course, equal to the total charge on the aerial, but is opposite in sign, and hence this correction is negative.

It should be remembered, however, that any masts or buildings in the proximity of the aerial may produce effects far greater than the effect just considered (see p. 46), and unless these effects are to be considered the accuracy obtained by considering the effect of the image of the lead-in is not justifiable.

Summary.—In calculating the capacity of an aerial, therefore, the procedure is as follows:

Horizontal Portion.—Find V_{av} due to—

Charge on wire itself	a
Charge on image	$-b$
Charge on lead-in	c
Charge on image of lead-in	$-d$

Average potential $= a - b + c - d$
$= \text{M}$

Lead-In.—Find V_{av} due to—

Charge on wire itself, allowing for earth	e
Charge on horizontal portion	f
Charge on image of horizontal portion	$-g$

Average potential $= e + f - g$
$= \text{N}$

If l is the length of the horizontal portion, and h is the length of the lead in, then—

$$V_{av} = \frac{\text{M}l + \text{N}h}{l + h},$$

and

$$C = \frac{h + l}{V_{av}}.$$

When dealing with multiwire aerials, h and l are the total lengths—*i.e.*, n times the geometrical lengths.

Sample Calculation (this example is taken from Professor Howe's original paper).—Find the capacity of an aerial consisting of ten parallel wires 4 feet apart, 600 feet long, and 200 feet high, with ten leading-in wires at the centre converging to a point near the earth. Radius of wire $= 0{\cdot}048$ inch, $l/d = 150$, $d/r = 1{,}000$.

Potential of horizontal portion:	
(*a*) Due to charge on wire itself	88·2
(*b*) Due to image ($l/2h = 1{\cdot}5$; see Fig. 44) ..	− 13·2
(*c*) Due to charge on lead-in: $m = 1{\cdot}5$, $n = 10$, $V = 14{\cdot}2$ (see Fig. 45).	
Correction for spacing: $V = 14{\cdot}2\,(1 - 0{\cdot}046)$ ..	13·6
(*d*) Due to image of lead-in (mean distance = 336 feet) $= \frac{10 \times 200}{336}$	− 6·0 (approx.).
Total	82·6
Potential of lead-in:	
(*e*) Due to its own charge (here the wires converge; hence they are treated as parallel and $0{\cdot}36d$ apart $= 1{\cdot}44$ feet) $l/d = 1{\cdot}39$, $d/r = 360$, $V = 84{\cdot}8 - 10$ (due to earth) ..	74·8
(*f*) Due to horizontal portion: $m = 0{\cdot}66$, $V = 21{\cdot}34 \times 2$ (for the two halves).	
Correcting for spacing: $V = 42{\cdot}7\,(1 - 0{\cdot}044)$ ..	40·9
(*g*) Due to horizontal image: $\frac{10 \times 600}{336}$	− 17·9
Total	97·8

Hence we have 10×600 feet at an average potential of 82·6, and 10×200 feet at a potential of 97·8.

$$V = \frac{82{\cdot}6 \times 6000 + 97{\cdot}8 \times 2000}{8000} = 86{\cdot}35,$$

$$C = \frac{8000 \times 30{\cdot}5}{86{\cdot}35}$$

$$= 2825 \text{ electrostatic units,}$$

$$= 2825 \times 1{\cdot}112 = 3140 \text{ micro-microfarads.}$$

Umbrella Type Aerials.—Here the general procedure is the same as before, but there are one or two extra factors to be taken into account.

First of all, the capacity of each rib is considered, and the total capacity of the top portion is taken as n times this value, where n is the number of ribs. The capacity of the lead-in may then be determined, due to its own charge and due to the charges on the ribs.

Potential of a Rib.—This is made up of the potentials due to—

1. Its own charge.
2. Charges on the other ribs.
3. Charge on vertical wire.
4. Image of ribs.
5. Image of vertical wire.

Potential of Lead-In.—This is made up of the potentials due to—

6. Its own charge.
7. Charges on the ribs.
8. Its own image.
9. Image of the ribs.

Of the above factors, all except Nos. 2, 3, and 7 are straightforward. Nos. 3 and 7 are calculated from the curves given below.

Here, if, as before—

$$m = \frac{l}{l'} = \frac{\text{length of wire under consideration}}{\text{length of wire at an angle } \gamma \text{ (charged)}},$$

then the average potential of the wire under consideration, due to a unit charge on the wire at an angle, can be determined in terms of both m and γ from Fig. 46.

The potential of the lead-in due to the ribs is, of course, taken as n times the potential due to one rib calculated by the above process.

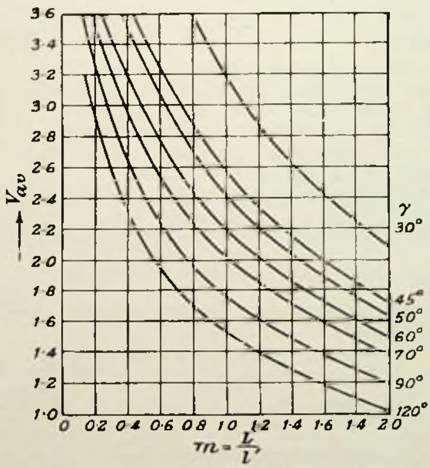

FIG. 46.—POTENTIAL OF ONE WIRE DUE TO A SECOND WIRE AT AN ANGLE γ.

The potential of one rib due to the effect of all the other ribs (No. 2) can be obtained from Fig. 46 by placing $m = 1$. Then, by considering all the ribs in turn and finding their relative angles, the effect of all the ribs may be obtained by a simple addition.

Where the number of ribs, however, does not exceed six, the effect on one rib of all the others may be taken directly from Fig. 47, which gives the average potential in terms of the number of ribs and the angle made with the vertical.

Effect of Masts and Buildings.—A lead-in near an earthed mast will induce a charge on the mast, and this in its turn will affect the average potential of the lead-in. Assume a charge q per centimetre length on the mast.

Then (see Fig. 48) the average potential of the mast will be made up of the potential—

Due to itself $= aq$
Due to its image $= -1{\cdot}38q$ (since mast is actually earthed).
Due to wire $= c$
Due to image of wire $= -d$

Total V of mast $= (a - 1{\cdot}38)\,q + c - d$.

Since the mast is earthed this must $= 0$, and hence q can be found.

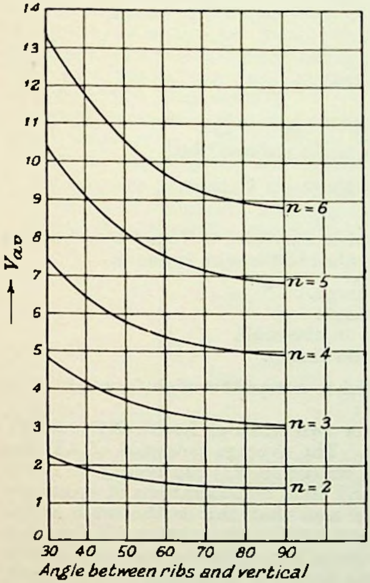

Fig. 47.—Effect on One Rib of all the Other Ribs.

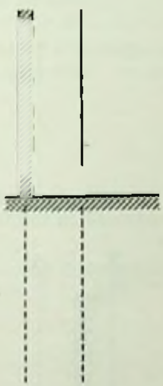

Fig. 48.—Effect of Mast on Lead-In.

Then the average potential of the wire—

Due to itself $= x$
Due to its own image $= -1$
Due to charge on mast $= -cq$
Due to image of mast $= +dq$

where c and d have the same values as above. Hence the effect of the mast may be readily determined. It should be noted that this method only applies if the mast is earthed, and, in general, the capacity is increased by from 5 to 20 per cent. according to the proximity of the lead-in to the mast. If the mast is insulated, however, as is becoming increasingly common to-day, the effect is indeterminate.

The increase of capacity varies inversely as the distance between the mast and the lead-in. Fig. 49 shows the increase for a particular case in which the mast was 200 feet high.

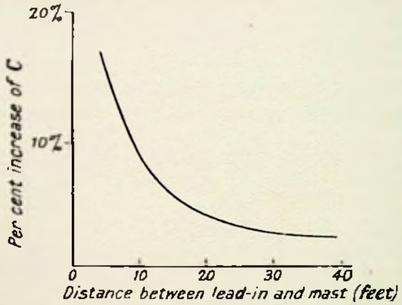

Fig. 49.—Effect of Mast on Capacity.

The effect of a building near the lead-in may also be found by a method of images. Here (see Fig. 50) the potential of the wire is due to—

1. Its own charge.
2. Its own image.
3. The image in the wall.
4. The diagonal image.

Of these, the last is the only one which is not quite straightforward. It is best obtained in the following manner:

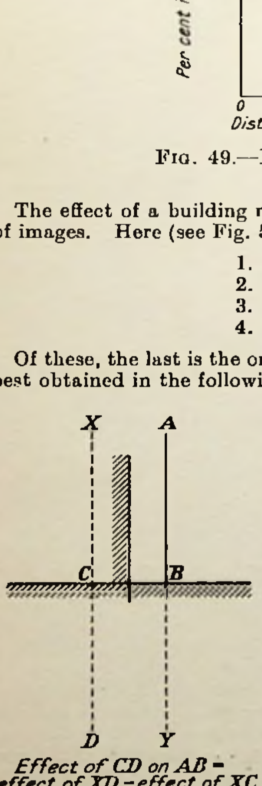

Fig. 50.—Effect of Building on Lead-In.

Consider two wires of length XD and AY (Fig. 50). The average potential of AY due to a unit charge on XD can readily be determined, and from considerations of symmetry it can be seen that this is the same as the average potential of the portion AB due to a unit charge on XD. Now the average potential of AB due to XC is also readily determined, so that the effect of CD on AB is obtained by subtraction.

It should be observed that the effect of the masts and buildings in the neighbourhood of an aerial is very considerable, and may cause an increase of as much as 30 per cent. or more in the capacity of the aerial, which would swamp the comparatively small corrections due to the effect of the earth. Consequently, before commencing any calculations on an aerial, the effect of such foreign bodies should be reviewed generally to determine first whether the effect is likely to be serious, and if so whether it is accurately determinable.

General Data.—When considering complicated networks wherein varying cross-sections may be involved, the surface density of the charge should be assumed constant (since this is the fundamental assumption on which the formulæ are based). For example, if the lead-in wires are twice the diameter of the aerial wires, then they should be assumed to have a charge of 2 units per centimetre length if the aerial wires are assumed to carry unit charge.

If the aerial is composed of a network of cages or sausages, the capacity of the cages themselves should be worked out by the usual formulæ, each wire being assumed to carry unit charge. In calculating the effect of one cage on the other, however, they may be replaced by single wires, each carrying a charge of n units, where n is the number of wires.

Capacity of a Circular Wire.—This is sometimes required. If PQ is an arc of the circle which is small enough to be considered linear, and if θ is the angle subtended at the centre by this arc, then average potential = potential of length PQ $- 2 \log_e \tan \theta/4$.

It may be observed that this average potential is usually about 5 per cent. higher than that of a straight wire of the same length as the circumference, so that the capacity of a wire is reduced by bending it into a circular form.

Approximate Formulæ.—The following approximate formulæ are given by the Bureau of Standards, Washington, and are convenient in many cases.

Single Wire Parallel to the Ground:

$$C = \frac{0{\cdot}2416l}{\log_{10}\frac{4h}{d} - k_1}\ \mu\mu\text{F, where } 4h/l \leqq 1;$$

$$C = \frac{0{\cdot}2416l}{\log_{10}\frac{2l}{d} - k_2}\ \mu\mu\text{F, where } l/4h \leqq 1.$$

Here l = length of wire,
d = diameter of wire,
h = height of wire above ground (all dimensions in centimetres).
k_1 and k_2 are constants which are plotted below.

Two Horizontal Wires Parallel to the Ground:

$$C = \frac{0{\cdot}4831l}{\log_{10}\frac{4h}{d} + \log_{10}\frac{2h}{D} - 2k_1}\ \mu\mu\text{F, where } 4h/l \leqq 1;$$

$$C = \frac{0{\cdot}4831l}{\log_{10}\frac{2l}{d} + \log_{10}\frac{l}{D} - 2k_2}\ \mu\mu\text{F, where } l/4h \leqq 1.$$

Here D is the spacing of the wires; the other terms are as above.

Single Vertical Wire (lower end several metres from the ground):

$$C = \frac{0{\cdot}2416l}{\log_{10}\frac{2l}{d}}\ \mu\mu\text{F } (l \text{ and } d \text{ as above}).$$

N Wires in Parallel:

$$C = C_1\sqrt{N-1}, \text{ where } C_1 = \text{capacity of a single wire.}$$

This formula is approximate only.

More accurately—

$$C = \frac{0{\cdot}2416\, ln}{p_{11} + (n-1)\, p_{12} - nk}\, \mu\mu F,$$

where n = number of wires,
D = spacing of wires,
l = length of wires,

$$p_{11} = \log_{10}\frac{4h}{d} - k_1 \text{ where } 4h/l \leqq 1,$$

$$= \log_{10}\frac{2l}{d} - k_2 \text{ where } l/4h \leqq 1,$$

$$p_{12} = \log_{10}\frac{2h}{D} - k_1 \text{ where } 4h/l \leqq 1,$$

$$= \log_{10}\frac{l}{D} - k_2 \text{ where } l/4h \leqq 1.$$

k is a constant plotted below,
k_1 and k_2 are as before.

TABLE XII.

Values of k.

n = 2	3	4	5	6	8	10	12	14	16	18	20
k = ·0	·067	·135	·197	·256	·361	·445	·515	·571	·619	·660	·704

TABLE XIII.

Values of k_1 and k_2.

$l/4h$, $4h/l$ =	0	·01	·2	·3	·4	·5	·6	·7	·8	·9	1·0
k_1 =	0	·001	·004	·009	·016	·025	·035	·045	·057	·069	·082
k_2 =	0	·043	·086	·128	·169	·209	·247	·283	·318	·351	·383

Austin has given the following empirical formula for flat-topped aerials not too elongated nor having wires too widely spaced:

$$C = \left(4\sqrt{a} + 0{\cdot}885\,\frac{a}{h}\right) 10^{-5}\, \mu\mu F,$$

where a = area in metres^2,
h = height in metres.

If $l > 8b$, where b is the breadth, the value given above should be multiplied by a correction factor $k = \left(1 + 0{\cdot}015\,\frac{l}{b}\right)$.

Capacity between Two Horizontal Wires.—If the wires are at the same height (h)—

$$C = \frac{0{\cdot}1208 l}{\log_{10}\frac{2D}{d} - \frac{D^2}{8h^2}}\, \mu\mu F,$$

where D is the spacing between the wires.

If the wires are in a vertical plane, use the same formula with h equal to the mean height. All dimensions in centimetres.

High Frequency Resistance.

A conductor carrying a current has associated with it a magnetic field which is distributed both in and around the conductor.

If the flux is varying, the portion inside the conductor produces eddy currents in the material of the conductor itself which are in such a direction as to oppose the flow of current.

The current, therefore, is unevenly distributed throughout the conductor, tending to flow more in the outside layers, so that the effective resistance of the conductor will increase with the frequency.

The problem, for all but the simplest cases, is not amenable to any rigid mathematical treatment, but the general laws may be derived and formulæ obtained, part mathematical, part empirical, which have a certain application in radio engineering.

The increase of resistance is found to depend on the actual dimensions of the wire, the permeability thereof, and the frequency of the current.

For straight wires fairly simple formulæ are available which give the ratio of the resistance at frequency f to that of frequency $=0$ (*i.e.*, for direct current). This ratio is written R_f/R_o.

The problem is still further complicated if the conductor is wound in a coil, in which case the external flux exercises a considerable influence. The formulæ available here are very limited in both application and accuracy.

Straight Round Wire:

The ratio $\frac{R_f}{R_o}$ depends on the parameter $x = 0{\cdot}0995d\sqrt{\frac{2\mu f}{\rho}}$,

where d = diameter of wire (cms.),
f = frequency of the current,
μ = permeability of wire,
ρ = specific resistance of wire (microhm-cms.).

Having evaluated this parameter, the value of $\frac{R_f}{R_o}$ may be obtained from Table XIV. below.

For copper wire at 20° C. $x = 0{\cdot}107d\sqrt{f}$.

The formula only applies to circuits in which the wires are far apart, so that the mutual inductance is negligible. Any mutual inductance increases the skin effect by an amount which is not readily calculable, but the effect is only serious if the wires are very close.

TABLE XIV.

VALUE OF $\frac{R_f}{R_o}$ IN TERMS OF x.

x.	$\frac{R_f}{R_o}$.	x.	$\frac{R_f}{R_o}$.
·0	1·000	1·3	1·015
·6	1·001	1·4	1·020
·8	1·002	1·5	1·026
·9	1·003	1·6	1·033
1·0	1·005	1·7	1·042
1·1	1·008	1·8	1·052
1·2	1·011	1·9	1·064

TABLE XIV.—*Continued.*

x.	$\frac{R_f}{R_o}$.	x.	$\frac{R_f}{R_o}$.
2·0	1·078	5·0	2·043
2·2	1·111	5·2	2·114
2·4	1·152	5·4	2·184
2·6	1·201	5·6	2·254
2·8	1·256	5·8	2·354
3·0	1·318	6·0	2·394
3·2	1·385	6·2	2·463
3·4	1·456	6·4	2·533
3·6	1·529	6·6	2·603
3·8	1·603	6·8	2·673
4·0	1·678	7·0	2·743
4·2	1·752	7·2	2·813
4·4	1·826	7·4	2·884
4·6	1·899	7·6	2·954
4·8	1·971	7 8	3·024

Above $x = 8$ use formula $\frac{R_f}{R_o} = 0{\cdot}353x + 0{\cdot}27$.

It is sometimes desired to construct a resistance or other piece of apparatus of which the high frequency resistance is very little different from that at low frequencies. For this purpose the following table has been compiled by the Bureau of Standards, in which are given the largest permissible diameters of various kinds of wire, in order that the ratio $\frac{R_f}{R_o}$ shall not exceed 1·01:

TABLE XV.

MAXIMUM DIAMETER OF WIRES (CM.) FOR A RATIO $\frac{R_f}{R_o} = 1{\cdot}01$.

λ	3000	1500	500	300	150	100
Material:						
Copper ..	·0356	·0251	·0145	·0112	·0079	·0065
Silver ..	·0345	·0244	·0141	·0109	·0077	·0063
Platinum ..	·1120	·0793	·0457	·0354	·0250	·0205
Manganin ..	·1784	·1261	·0729	·0564	·0399	·0325
Constantan ..	·1892	·1337	·0772	·0598	·0423	·0345
German silver	·1942	·1372	·0792	·0614	·0434	·0354
Carbon ..	1·601	1·131	·6541	·5060	·3580	·2920
Iron μ = 1,000	·0026	·0019	·0011	·0008	·0006	·0005
= 500	·0037	·0026	·0015	·0012	·0008	·0007
= 100	·0084	·0059	·0034	·0026	·0019	·0015

If the ratio $\frac{R_f}{R_o}$ is required to be 1·1, the values given in the table should be multiplied by 1·78, while for $\frac{R_f}{R_o} = 1{\cdot}001$ the values are multiplied by 0·55. The table, as before, only applies to wires appreciably spaced from any other wires.

Tubular Conductor.—Here it is necessary to evaluate the parameter $\beta = xt\sqrt{2}$, where t is the thickness of the tube and x is the parameter already employed for wires.

d here is the outside diameter of the tube.

Values of $\frac{R_f}{R_o}$ in terms of β are given in Table XVI.

Strip Conductors.—Where the two conductors of a circuit are in the form of narrow strips with their wide faces adjacent (see Fig. 51), the ratio R_f/R_o

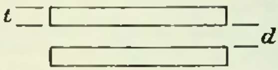

Fig. 51.—Two Strips Face to Face.

may be calculated from the same formula as is employed for tubes, t in this case being the thickness of the strips, and d the distance apart.

The formula only holds provided the strips are close together; the greater the distance d, the greater becomes the ratio R_f/R_o, and in the limiting case of a single strip the ratio may be several times as great as that given by the above formula.

For the case where the strips are edge to edge no formula is available.

A single strip of approximately square section may be treated as a round strip of equal area.

TABLE XVI.

Value of $\frac{R_f}{R_o}$ in Terms of β

β.	$\frac{R_f}{R_o}$.	β.	$\frac{R_f}{R_o}$.
·0	1·000	2·2	2·132
·3	1·001	2·3	2·248
·4	1·002	2·4	2·364
·5	1·006	2·5	2·477
·6	1·012	2·6	2·588
·7	1·021	2·7	2·697
·8	1·036	2·8	2·803
·9	1·057	2·9	2·907
1·0	1·086	3·0	3·010
1·1	1·123	3·1	3·111
1·2	1·170	3·2	3·212
1·3	1·229	3·3	3·311
1·4	1·298	3·4	3·410
1·5	1·378	3·5	3·509
1·6	1·468	3·6	3·608
1·7	1·566	3·7	3·706
1·8	1·672	3·8	3·804
1·9	1·783	3·9	3·902
2·0	1·898	4·0	4·000
2·1	2·015	—	—

For values above 4, $\frac{R_f}{R_o} = \beta$

Concentric Main with Solid Inner Conductor:

$$R_f = \sqrt{\mu\rho f}\left(\frac{1}{a}+\frac{1}{b}\right) \text{ per unit length,} \qquad \text{(Russell.)}$$

where a is the radius of the solid conductor,
b is the internal radius of the outer conductor.

High Frequency Resistance of Coils.—The skin effect in coils is a very much more complicated matter. The distribution of the current in the wire depends not only on the flux due to the wire itself, but is also considerably affected by the main coil flux.

The only case in which the skin effect is accurately determinable is that of a *long* single layer solenoid wound with wire of rectangular cross-section. $\frac{R_f}{R_o}$ is then obtained from Table XVI. in terms of $\beta = 2{\cdot}803\tau\sqrt{\frac{\mu f}{\rho}}$, τ in this case being the radial thickness of the wire; μ, f, and ρ are as before.

Spaced Windings.—If the ratio $\frac{\text{pitch of winding}}{\text{width of strip}} = \frac{D}{w}$ is greater than unity (*i.e.*, a spaced winding), but is less than 3, then β may be replaced by $\beta' = \beta\sqrt{\frac{w}{D}}$, where w is the axial width of the wire, and D is the pitch.

Circular Wire.—An approximate solution for round wire may be obtained by the following method: First find R_f'/R_o' for a coil wound with square wire of side equal to the diameter of the round wire.

Then $$\frac{R_f}{R_o} = 1 + 0{\cdot}59\,[R_f'/R_o' - 1] \text{ approximately.}$$

This formula may be 10 per cent. or more in error.

Sommerfield has given the formula:

$$\frac{R_f}{R_o} = \sqrt{\frac{\pi\omega}{2\rho}}\left\{1 + 0{\cdot}276\left(\frac{2\pi r}{D}\right)^2\right\},$$

where r = radius of wire,
$\omega = 2\pi f$,
D = pitch of winding.

Multilayer Coils.—Here the problem is wellnigh incalculable. The eddy current losses in the dielectric become of increasing importance, such losses being proportional to f^3.

Max Wien has given the following formulæ, which, however, do not allow for dielectric loss:

1. If $\frac{b}{a} > 16$:

$$\frac{R_f}{R_o} = 1 + \frac{\pi^4 r^6 n^2 \omega^2}{b^2\rho^2}\left\{1 - \left(\frac{2a}{b}\right)^2\right\},$$

where a = radius of coil,
b = length of coil,
r = radius of wire,
n = number of turns,
$\omega = 2\pi f$,
ρ = specific resistance.

2. Flat coils:

$$\frac{R_f}{R_o} = 1 + \frac{\pi^4 r^6 n^2 \omega^2}{\rho^2 D_1^2}\left\{1 + \frac{3D_2^2}{4D_1^2}\right\}^2,$$

where D_1 = outside diameter,
D_2 = inside diameter,
the other symbols being as before.

Stranded Wire.—Where multilayer coils have to be employed, stranded wire may sometimes be employed with advantage. This consists of a cable made up of a large number of fine wires, all insulated and so stranded that each wire in turn comes to the outside of the conductor, and so carries its share of the current. The connections to a solid conductor must be made round the circumference, so that the wires collect the current, which is, of course, flowing only on the surface of the solid conductor.

Ordinary close stranding, however, suffers from serious dielectric loss, and at high frequencies (low wave lengths) the solid conductor is better. The two are equal in the region of 1,000 to 2,000 metres. The most effective conductor is a hollow tube of loosely stranded basket weave, but this is too expensive for ordinary purposes.

Design of Inductance Coils.—As has previously been stated, the design of inductance coils for transmitting purposes involves the reduction of the resistance as far as possible. Professor Fortescue (*Journ. I.E.E.*, vol. lxi., p. 933) has investigated the problem of the design of coils having a minimum ratio of R/L, based, however, on considerations of conductor resistance only. He shows that the ratio of R/L is inversely proportional to the diameter of the coil, but that the shape has little effect within fairly wide limits.

For stranded conductor the best number of strands is given as $n = 0{\cdot}27 \left(\frac{\rho\lambda}{d_1^{\,3}}\right)\left(\frac{D_1}{KN}\right)$, and for solid conductor the best diameter is $d = 0{\cdot}286 \frac{KN}{D_1}$ centimetre for comparatively short wave lengths, such that d, when obtained, is $< 10\sqrt{\lambda\rho}$,

where D_1 = outside diameter of coil (cms.),
d_1 = diameter of single strand (cms.),
ρ = specific resistance of conductor,
λ = wave length in metres,
K is a constant, values of which are given in Table XVII. below.

With these values the average value of R/L for coils not too long or too deep is given by—

$$\frac{2400}{D_1} \cdot \frac{d_1}{\lambda} \text{ for stranded conductor;}$$

$$\frac{2000}{D_1} \sqrt{\frac{\rho}{\lambda}} \text{ for solid conductor.}$$

These formulæ only apply providing the windings are adequately spaced, the space factor, defined as $\frac{\text{area of conductor (total)}}{\text{area of cross-section of coil}}$, being not $> 0{\cdot}2$, such a low value being necessary to avoid excessive dielectric losses and sparking between turns.

It is possible, therefore, to make a rough calculation to find the approximate value of D_1, and hence to find the value of n or d above, and thence to design the coil to have the minimum resistance with the proper space factor.

Practical experience seems to indicate that the effective resistance of an inductance, allowing for dielectric loss, is about twice that due to conductor resistance only, if reasonable precautions are taken to reduce such loss as far as possible. In the absence of definite data on the subject, the design of such coils must be largely a matter of practical experience. (See also p. 18.)

TABLE XVII.

VALUE OF WINDING FACTOR K.

c/D_1: b/D_1.	·5	·75	1·0	1·25	1·5
·0	·99	·68	·52	·43	·37
·1	·77	·55	·44	·37	·31
·2	·65	·48	·38	·32	·27
·3	·57	·42	·33	·28	·24
·4	·52	·37	·29	·24	·21

b = axial length of coil,
c = radial depth of coil,
D_1 = overall diameter of coil.

Radio Frequency Measurements.

The chief measurements required at radio frequencies are those of inductance, capacity, resistance, and frequency (wave length).

Inductance and Capacity.—The inductance of a coil may conveniently be measured at radio frequencies by connecting a condenser of known capacity in shunt and finding the wave length of the circuit. The inductance may then be obtained from the expression—

$$\lambda = 1884\sqrt{LC},$$

where
λ = wave length in metres,
L = inductance in microhenries,
C = capacity in microfarads.

If the condenser is variable, the effective inductance may be determined at the actual wave length to be employed; while if a series of readings are taken, the distributed capacity of the coil may be determined by the method shown on p. 23.

Capacity measurements are usually made by a substitution method since a standard capacity is a more commercial proposition than a standard inductance at radio frequencies. A coil is taken and tuned with the given condenser, the wave length being observed. A standard variable condenser is then substituted for the unknown capacity, and the circuit is adjusted to the same wave length as before, when $C_s = C_x$.

Variations of these methods to suit particular conditions will readily suggest themselves.

Wave Length.—The measurement of wave length is accomplished by means of an accurately calibrated tuned circuit known as a wave meter. The construction of such instruments is relatively simple, the chief factor in the design being the avoidance of overhanging turns which may give rise to discontinuities in the calibration curve. Fig. 52 shows such a case in which the curve obtained as the condenser is increased is different from that obtained as the condenser is reduced. This effect is due to the fact that the circuit behaves as a coupled circuit, and consequently tunes at two frequencies.

Wave meters may be of two types—radiating and detecting. In the first case a buzzer circuit is suitably coupled to the tuned circuit, so giving rise to the radiation of damped wave trains which may be detected by the circuit under test. One form of circuit is that shown in Fig. 53, in which the buzzer is direct coupled to the tuned circuit. A second circuit is as shown in Fig. 54. Here the buzzer sets up oscillations in L_1C_1, which in turn set the circuit L_2C_2 oscillating. If C_1 is made large (several microfarads), and L_1 is composed of one or two turns only, then this circuit is

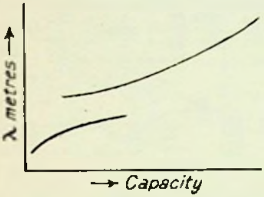

Fig. 52.—Discontinuity in Wave-Meter Calibration due to Overhanging Turns.

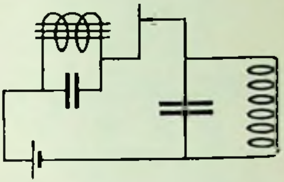

Fig. 53.—Simple Buzzer Wave-Meter Circuit.

very rapidly damped, and the oscillations in L_2C_2 are simply the free oscillations in the circuit, and are independent of L_1C_1. This method, known as "impact excitation," is thus very useful for accurate measurements in that the presence of the buzzer does not affect the wave length of the circuit.

In the second type of wave meter, a detecting circuit is placed in shunt across the condenser. This affects both the tune and the decrement of the wave-meter circuit, which should therefore be calibrated with its appropriate detecting circuit in position. In particular, the crystal should be of

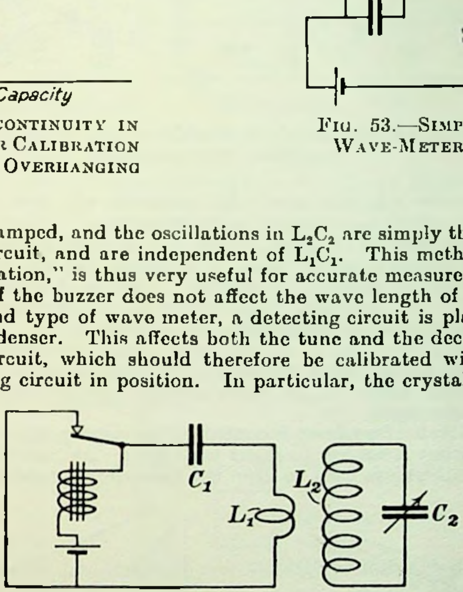

Fig. 54.—Buzzer Wave-Meter Circuit arranged for Impact Excitation.

a pattern having a rigid contact (such as carborundum) to avoid variations of decrement and tune due to variable resistance effects.

Fig. 55 shows several forms of detecting wave-meter circuits with their relative audibility, while Fig. 56 shows the effect on tuning and decrement for representative examples of the circuits enumerated in Fig. 55. These data are taken from the Circular No. 74 of the Bureau of Standards, Washington.

Standard wave meters may be so designed that the inductance and capacity are accurately calculable or measurable, but it is more usual to build a reasonably accurate instrument and calibrate it from a standard of frequency. One of the principal methods of producing a standard frequency is to design a valve generator which is very rich in harmonics. This generator is then arranged to oscillate at a fundamental frequency of 1,000 cycles, or some such figure, which is accurately controlled by a calibrated tuning-fork. The radio circuits are then tuned to a high harmonic

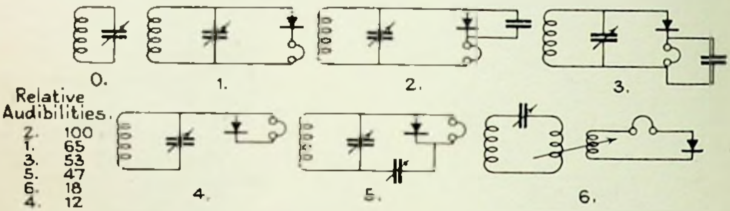

Fig. 55.—Some Detecting Wave-Meter Circuits with their Relative Audibilities.

of this frequency—*e.g.*, the hundredth harmonic will give a frequency of 100,000 cycles, which corresponds to 3,000 metres.

Such an arrangement is shown in Fig. 57. The principal portion of the circuit is the "multivibrator," due to Abraham and Bloch. Assume the anode current of one valve i_{a1} to increase. Then v_{a1} decreases, and with it v_{g2}, to which it is connected. This causes i_{a2} to decrease; v_{a2} increases, causing v_{g1} to increase, so augmenting the increase of i_{a1}. There is thus a sudden rush of current till i_{a2} is zero, after which the charges on the grid condensers leak away, and a reverse current rush occurs. The result is a series of current rushes in the valve circuits, giving a flat-topped wave very rich in harmonics.

The multivibrator frequency is controlled by varying the grid condensers, but this frequency is subject to slight variations. An accurately calibrated tuning-fork is therefore used to keep the frequency constant. The fork is

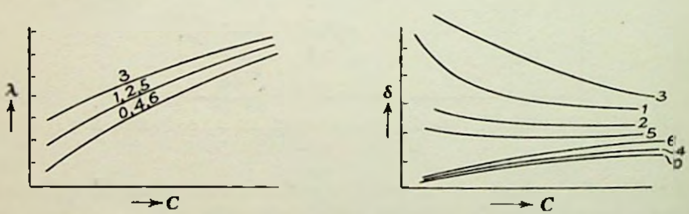

Fig. 56.—Effect on Wave Length and Decrement of the Addition of a Detecting Circuit to the Simple Circuit in Fig. 55.

mounted with two coils up against the prongs. The vibration of the fork induces currents in the grid coil which causes augmented currents to flow in the anode coil, and the magnetic effect of these currents sustains the vibration of the fork. There is thus generated a constant (low) frequency, and if the multivibrator is tuned to the same frequency, the tuning-fork circuits will control the current rushes, so maintaining the fundamental of the multivibrator absolutely constant.

Resistance.—The measurement of the radio-frequency resistance of a circuit is of considerable importance. The measurement of decrement is also in essence a measurement of resistance.

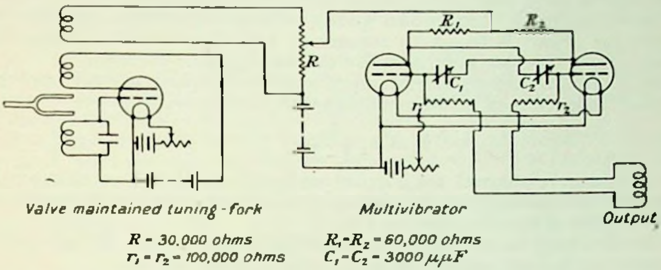

FIG. 57.—CIRCUIT FOR PRODUCING A STANDARD FREQUENCY.

One simple method of obtaining R_f is known as the reactance variation method. A suitable indicating device is inserted in the circuit, and a source of undamped high-frequency E.M.F. is coupled to the inductance, as in

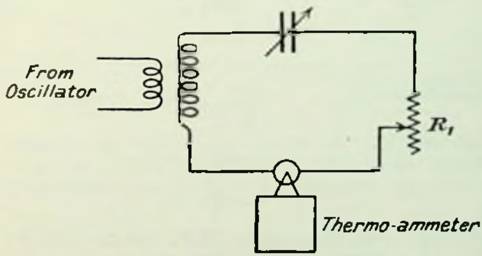

FIG. 58.—RESISTANCE VARIATION METHOD.

Fig. 58. The circuit is tuned to resonance, and the current I noted. L or C is then varied, and a different current I_1 is obtained.

Then

$$R = X\sqrt{\frac{I_1^2}{I^2 - I_1^2}},$$

where X is the reactance of the circuit under the mistuned condition. When the circuit is tuned X is, of course, zero.

If the reactance is varied by a change of the capacity C—

$$R = \frac{C \sim C_1}{\omega C C_1}\sqrt{\frac{I_1^2}{I^2 - I_1^2}}.$$

If L is varied—

$$R = \omega(L \sim L_1)\sqrt{\frac{I_1^2}{I^2 - I_1^2}}.$$

These formulæ are obtained on the assumption that the E.M.F. induced in the secondary circuit is the same in each case; hence the coupling must be kept weak. Due allowance must be made for the resistance of the measuring device, and any other extraneous resistance.

Precautions must also be taken to avoid stray capacity couplings, and it is advisable to earth the low potential side of the condenser.

A second method is that known as the resistance variation method. In this case the circuit is tuned to resonance, and the current I noted. A resistance R_1 is then inserted, and the current I_1 again observed.

If the E.M.F., as before, is undamped and the coupling is weak, so that the E.M.F. induced may be assumed equal in both cases, then—

$$R = R_1 \frac{I_1}{I - I_1}$$

The resistance employed must be of such a type that its resistance is substantially independent of frequency, and should therefore be made of straight lengths of fine wires (see p. 52).

The method may be used with damped E.M.F.'s if desired. If the circuit is energised by impact excitation, free oscillations are set up.

The condition is then that the power dissipated in the circuit is the same in each case.

Hence
$$I^2R = I_1^2(R + R_1), \text{ whence } R = R_1 \frac{I_1^2}{I^2 - I_1^2}.$$

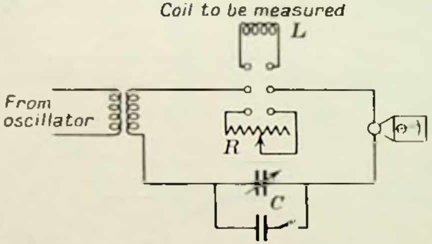

FIG. 59.—CIRCUIT FOR MEASURING H.F. RESISTANCE BY THE SUBSTITUTION METHOD.

If, however, the applied E.M.F. itself is damped, the problem becomes the same as that referred to on p. 73, and may be treated in the same way.

A third method, which can only be applied to a portion of a circuit, is that known as the substitution method. The circuit under consideration is coupled to a source of undamped E.M.F., and is tuned to the requisite frequency.

The portion of the circuit of which the resistance is required is then removed and replaced with a simple variable resistance. The circuit is retuned, and the resistance adjusted to give the same current as before. This resistance is then equal to that of the portion of the circuit which was removed.

Retuning must be done by means of a condenser of negligible or known resistance.

A circuit for measuring the resistance of a coil by this means is shown in Fig. 59.

Current Measurement.—Radio-frequency instruments are all constructed on thermal principles, any other form possessing appreciable inductance and capacity, which is undesirable, and gives rise to variable errors depending on frequency.

For currents of 0·1 up to 10 amperes a simple hot wire ammeter may be employed. The wire used must not change its resistance with frequency to any appreciable extent, and must therefore be designed in accordance with the table on p. 52.

Hot wire ammeters are very prone to zero shift. For accurate measurements, therefore, the instrument should pass a certain current first. The current may then be cut off, and the zero adjusted, due allowance being made for the time taken by the wire to cool.

For larger currents, shunts must be employed. A simple shunt cannot be used, since the shunting effect is dependent on both R and ωL. Hence either R must be very much greater than L, or *vice versa*.

The first type of shunt consists of a series of fine wires arranged round the circumference of a disc. With such an arrangement all wires carry an equal current. Hence an ammeter inserted in one of the wires will read $\frac{1}{n}$ of the true current, n being the number of wires.

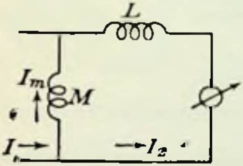

FIG. 60.—INDUCTANCE SHUNT.

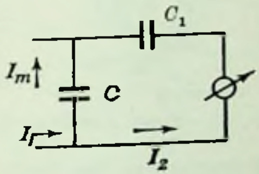

FIG. 61.—CAPACITY SHUNT.

On the other hand, inductance shunts of negligible resistance may be employed, as shown in Fig. 60. Then, provided the inductance and resistance of the ammeter are negligible compared with the radio-frequency reactances—

$$\frac{I_1}{I_2}=\frac{L+M}{M}.$$

Similarly, a capacity shunt may be employed as indicated in Fig. 61. In this case—

$$\frac{I_1}{I_2}=\frac{C+C_1}{C_1}.$$

Current Transformers.—A more usual method of measuring large currents is to use a current transformer, the arrangement being shown in Fig. 62. With such an instrument—

$$\frac{I_1}{I_2}=\frac{L_2}{M}\left(1+\frac{R_2^2}{2\omega^2L_2^2}\right),$$

R_2 being the resistance of the secondary circuit, including the ammeter, and is assumed small compared with ωL_2.

This applies only to undamped waves, or waves of decrement less than 0 03 or so.

Such instruments may be air cored or iron cored. In the former case the two windings may both be wound on a cylindrical former suitably spaced. In certain circumstances, however, it is desirable to employ an iron core. In such a case the secondary is toroidal in form, the primary consisting of a few turns of wire linking with it. Assuming that there is no magnetic leakage (which means that the inductance of the ammeter must be negligibly small), it can be shown that—

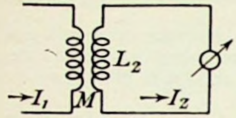

Fig. 62.—Transformer Shunt.

$$\frac{I_1}{I_2}=\frac{n_2}{n_1}\left(1+\frac{aR_2}{\omega L_2}\right),$$

a being a constant depending on the quality of the iron employed (usually slightly less than unity). Since L_2 is now very large, this correction term is practically negligible, being of the order of 0 2 per cent. at radio frequencies.

Some forms of current transformer are shown in Fig. 63.

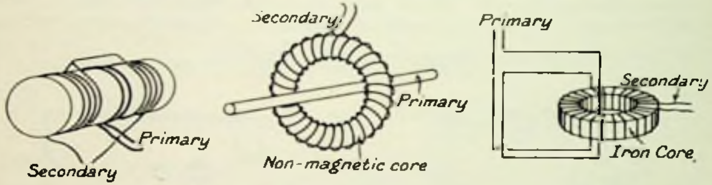

Fig. 63.—Some Forms of Radio-Frequency Current Transformer.

Measurement of Very Small Currents.—For small currents a thermocouple may be employed. This is a device in which the heat generated by the passage of the current effects a junction of two dissimilar metals, and an E.M.F. is set up which can be detected with a sensitive galvanometer. Such a device

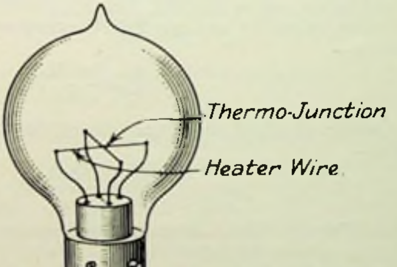

Fig. 64.—Vacuo-Junction Thermocouple.

is shown in Fig. 64. The sensitivity depends on the air pressure, so that it is usual to mount these couples in a glass bulb from which the air is exhausted. A reduction of pressure to 0·01 mm. mercury will increase the sensitivity by about twenty-five times. This is due to the reduction of convection of heat away from the heater wire. Currents as low as 0·1 milliampere may be measured by employing a suitable couple.

If T (°C.) is the temperature rise at the junction, the E.M.F. (μV) produced is given approximately by the equations below :

Copper-constantan	$\text{Log}_{10}\,E = 1{\cdot}14 \log_{10} T + 1{\cdot}34.$
Platinum-plat. iridium (10 per cent.) ..	$\text{Log}_{10}\,E = 1{\cdot}10 \log_{10} T + 0{\cdot}89.$
Platinum-plat. rhodium (10 per cent.) ..	$\text{Log}_{10}\,E = 1{\cdot}19 \log_{10} T + 0{\cdot}52.$

Austin has devised a couple of the *self-heated type*, using tellurium and platinum. The radio-frequency current, in passing through this junction, sets up an E.M.F., which is measured with a galvanometer. The impedance of the galvanometer prevents any high-frequency current from passing through it. This couple is said to be very sensitive.

A bridge method of measuring small currents employs a device known as a "bolometer." This consists of a simple Wheatstone bridge, as shown in Fig. 65, in one of the arms of which is a short length of fine wire.

The radio-frequency current is passed through this strip and heats it up, so causing a change in resistance, which throws the bridge out of balance.

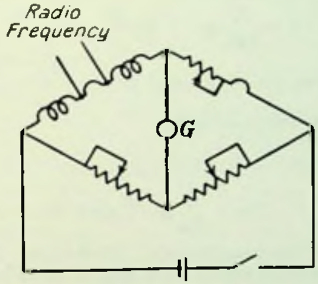

FIG. 65.—BOLOMETER.

Choke coils are provided to confine the AC to the bolometer wire, and a similar wire is inserted in the adjacent arm to balance the effect of the DC current flowing in the bridge.

Voltage Measurements.—Medium and high voltages are usually measured by means of an electrostatic voltmeter of any convenient pattern.

A very useful device for small voltages is the Moullin voltmeter, which consists essentially of a rectifying valve. The applied E.M.F. produces a change of anode current, which, under suitable conditions (such as constancy of filament and anode voltages), is proportional to the voltage applied.

Two types of instrument are made, one employing anode rectification, which can only be used with continuous circuits (*e.g.*, it could not be used to obtain the voltage across one of two condensers in series), while the other, employing cumulative grid rectification, is somewhat less sensitive, but may be used for any type of circuit. The first type will read to 0·02 volt, while the second will read from 0·5 volt to 10 volts.

TUNING AND RADIATION.

Radio communication is very largely concerned with the questions of tuning and resonance. These problems fall into two classes:

(*a*) Transient phenomena.
(*b*) Steady state phenomena.

Owing to the increasing use of C.W., many circuit problems are considerably simplified as they fall into class (*b*). The transient phenomena, however, maintain their importance, and are considered below.

Laws of Oscillatory Circuits.—Consider a condenser C charged to a voltage E, and then discharged through an inductance and resistance in series. The differential equation to the discharge is—

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=0, \text{ where } q=\text{charge on condenser.}$$

The solution of this equation is—

$$i=\frac{E}{2\beta L}\varepsilon^{-\alpha t}\left(\varepsilon^{\beta t}-\varepsilon^{-\beta t}\right).$$

$$\text{where } \alpha=R/2L \text{ and } \beta=\sqrt{\alpha^2-\frac{1}{LC}}$$

There are three possible conditions:

1. $\alpha^2>\frac{1}{LC}$.

Then $$i=-\frac{E}{\beta L}\varepsilon^{-\alpha t}\sinh\beta t.$$

This is known as a "non-oscillatory" discharge. The current rises to a maximum and falls away again to zero, as indicated in Fig. 66 (*a*).

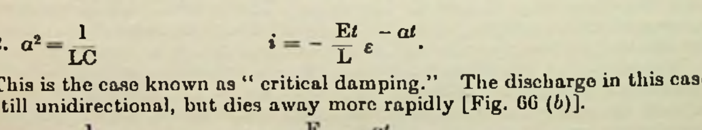

FIG. 66.—TYPES OF DISCHARGE.

2. $\alpha^2=\frac{1}{LC}$ $$i=-\frac{Et}{L}\varepsilon^{-\alpha t}.$$

This is the case known as "critical damping." The discharge in this case is still unidirectional, but dies away more rapidly [Fig. 66 (*b*)].

3. $\alpha^2<\frac{1}{LC}$. $$i=-\frac{E}{\omega L}\varepsilon^{-\alpha t}\sin\omega t,$$

where $$\omega=j\beta\sqrt{\frac{1}{LC}-\alpha^2}.$$

This is the oscillatory condition, each successive oscillation being smaller than the preceding one owing to the losses in the circuit. The term $\varepsilon^{\alpha t}$ indicates this gradual decrease or "damping." Fig. 66 (*c*) shows the form of the current in this case.

Translating these formulæ into terms of the actual constants of the circuit, we have—

Non-Oscillatory Discharge:

$$R^2 > 4L/C;$$

$$\text{Current: } i = \frac{E}{\beta L}\varepsilon^{-\frac{R}{2L}t}\sinh \beta t,$$

$$\text{Condenser voltage: } v = \frac{E}{2\beta}\varepsilon^{-\frac{R}{2L}t}\left\{(\beta+\alpha)\varepsilon^{\beta t} + (\beta-\alpha)\varepsilon^{-\beta t}\right\}.$$

Dead-Beat Discharge (Critical Damping):

$$R^2 = 4L/C;$$

$$i = \frac{E}{L}t\varepsilon^{-\frac{R}{2L}t},$$

$$v = -E\,\varepsilon^{-\frac{R}{2L}t}\left(1 + \frac{t}{\sqrt{LC}}\right).$$

The maximum current occurs when $t = 2L/R = \sqrt{LC}$.

Oscillatory Discharge:

$$R^2 < 4L/C;$$

$$i = \frac{E}{\omega L}\varepsilon^{-\frac{R}{2L}t}\sin \omega t,$$

$$v = -E\,\varepsilon^{-\frac{R}{2L}t}\sin\left(\omega t + \frac{\omega}{\alpha}\right).$$

The frequency of the oscillations is—

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC - \frac{R^2}{4L^2}}}.$$

This frequency is the *natural frequency* of the circuit. It differs slightly from the resonant frequency (see p. 67) owing to the resistance of the circuit, which is responsible for the term $\frac{R^2}{4L^2}$. In all practical circuits, however, the resistance is small, so that this difference is hardly appreciable.

The term $\alpha = R/2L$ is termed the *damping factor* or *decay coefficient* of the oscillation. This quantity is referred to later (p. 73).

Effect of Condenser Leakage.—If the condenser has a leakage $g = \frac{1}{r}$, where r is the insulation resistance, the equations are slightly modified.

$$a = \frac{R}{2L} + \frac{g}{2C},$$

$$\beta = \sqrt{\frac{1}{4}\left(\frac{R}{L} - \frac{g}{C}\right)^2 - \frac{1}{LC}}.$$

The effect, therefore, is to increase the damping, but the permissible resistance for oscillations is greater, the condition being—

$$\left(\frac{R}{2L} - \frac{g}{2C}\right) < \frac{1}{\sqrt{LC}}.$$

It is interesting to note that if $R/L = g/C$, then the natural frequency and the resonant frequency are equal.

Forced Oscillations.—If instead of charging the condenser to a given value and permitting it to discharge, an alternating E.M.F. is introduced into the circuit, the differential equation becomes—

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E\varepsilon^{-mt}\sin(pt + \varphi).$$

The solution of this is—

$$i = \frac{E}{Q}\left[\varepsilon^{-mt}\sin(pt + \varphi - \theta) - P\varepsilon^{-at}\cos(\omega t - \psi)\right];$$

where $$P = \frac{\sqrt{p^2 + Q\sin\varphi\sin(\varphi - \theta)}}{\omega},$$

$$Q = \sqrt{[(a-m)^2 + \omega^2 - p^2]^2 + 4(a-m)^2p^2},$$

$$\tan\theta = \frac{2(a-m)p}{(a-m)^2 + \omega^2 - p^2},$$

$$\tan\psi = \frac{p\cot(\varphi - \theta) + a - m}{\omega},$$

a = decay coefficient, $\omega = 2\pi \times$ frequency } of free oscillation as determined by the laws already given.

The first term in the solution is the forced oscillation, the second term being the free oscillation. Under normal circumstances the second term dies away very rapidly. If the circuit is adjusted, however, so that the frequency of the free oscillation is the same as that of the forced, the expression reduces to the form:

$$i = \frac{E}{Z}\varepsilon^{-(m+a)t}\sin(\omega t - \gamma).$$

Tuning and Resonance.

Consider now the steady state conditions.

In any circuit containing L, C, and R, the impedance is given by $\sqrt{(X^2 + R^2)}$. Under suitable conditions it is possible to adjust the values of L and C so that the reactance $X = 0$. This condition is called *resonance*, and when such a condition obtains the current in the circuit is a maximum, and is in phase with the applied E.M.F.

Series Resonance.—In the case of a simple circuit containing L, C, and R in series (Fig. 67), the reactance is given by $X = \left(L\omega - \frac{1}{C\omega}\right)$.

The condition for resonance is thus that $L\omega = \frac{1}{C\omega}$; in other words, $\omega^2 LC = 1$. This corresponds to a frequency $f = \frac{1}{2\pi\sqrt{LC}}$, which is the same as the natural frequency of the circuit, neglecting the effect of resistance. When the constants of a circuit are so adjusted that the natural frequency of the circuit is the same as that of the applied E.M.F., the circuit is said to be "tuned" to the particular frequency. In all ordinary work the difference between the natural frequency and the resonant frequency is so small that the tuning calculations are all based in the resonant frequency,

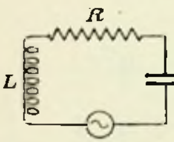

FIG. 67.—SERIES RESONANT CIRCUIT.

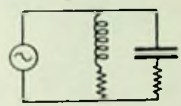

FIG. 68.—PARALLEL RESONANT CIRCUIT.

and the data which follow refer to the tuning properties of various classes of circuits deduced from the consideration of their resonant frequencies.

Parallel Resonance.—This is a very common case in practice, and is illustrated in Fig. 68. The impedance is given by—

$$Z = \frac{1}{\sqrt{\left(\omega C - \frac{\omega L}{R^2 + \omega^2 L^2}\right)^2 + \left(\frac{R}{R^2 + \omega^2 L^2}\right)^2}}.$$

The condition for resonance is that—

$$\omega C = \frac{\omega L}{R^2 + \omega^2 L^2}.$$

This corresponds to a minimum current round the whole circuit, but a maximum current circulating in the portion LC.

This circuit is therefore substantially different from the series circuit.

When this resonant condition obtains, the current is in phase with the voltage, and is given by $I_r = \frac{ER}{R^2 + \omega^2 L^2}$.

The frequency at which resonance occurs depends on the resistance in both the inductive and capacitative arms of the combination, and may be obtained from—

$$\omega = 2\pi f = \sqrt{\frac{L - R_L{}^2 C}{L^2 C - R_O{}^2 C^2 L}}.$$

If Rc is small, as is usually the case—

$$\omega = \sqrt{\frac{1}{LC} - \frac{R_L{}^2}{L^2}},$$

and if the total resistance in the circuit is negligible—

$$\omega = \sqrt{\frac{1}{LC}},$$

which is the same as for series resonance.

The frequency, however, under normal conditions does depend to a small extent on the resistance in the circuit, whereas with a series circuit it is independent of R. This should not be confused with the natural frequency, which is the same for both series and parallel connections.

Fig. 69 gives the vector diagram for a parallel circuit.

General Case of Parallel Resonance.—The circuit shown in Fig. 70 is the general case of a parallel resonant circuit in series with a simple series circuit.

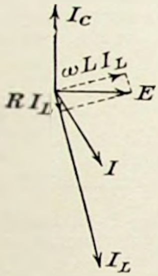

Fig. 69.—Vector Diagram of Parallel Circuit.

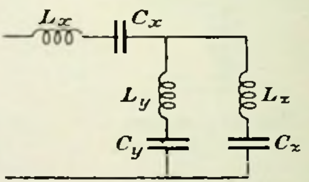

Fig. 70.—General Case of Parallel Resonance.

Here, if X is the reactance of L_xC_x, and P is the reactance of the parallel combination, then when the circuit as a whole is tuned to the incoming frequency, $X + P = 0$.

If it is desired to eliminate some other frequency, then the parallel combination must be so adjusted that $Y + Z = 0$ for this frequency, where Y and Z are the impedances of the two arms of the parallel combination respectively.

The desired result is accomplished by tuning L_xC_x and L_yC_y to the accepted frequency. The impedance of the circuit is then $R_x + R_y$, the latter term being small in comparison with the impedance of L_zC_z. At the undesired frequency, however, L_zC_z is so adjusted that $Y + Z = 0$. Under these conditions—

$$R_p = \frac{Z^2}{R_y + R_z},$$

$$X_p = \frac{(R_y - R_z)Z}{R_y + R_z},$$

R_p and X_p being the resistance and reactance of the complete parallel combination. It should be noted that it is actually the resistance which is high, and not the reactance, which is zero if $R_y = R_z$.

The suppressed frequency is obtained from—

$$\omega_z^2 = \frac{C_y + C_z}{C_yC_z(L_y + L_z)},$$

which is independent of L_xC_x.

Parallel circuits have some peculiar properties. If the circuit LC_1C_2 is in resonance (Fig. 71), then the impedance across the points AB is purely resistive. If the E.M.F. is supplied across any other two points on the circuit, however, the circuit remains in resonance. That is to say, Z_{CB}, Z_{BD}, and Z_{CD} are all purely resistive.

The fact that the effective inductance and resistance of a coil depend upon the frequency of the current flowing through the coil has already been referred to (p. 21).

The following formulæ will probably be useful: If f is the resonant frequency of the coil and condenser combination, and f' is the frequency at which the circuit is being run—

$$R_{eff} = \frac{R}{m^2R^2\frac{C}{L} + (m^2 - 1)^2}$$

$$L_{eff} = L\frac{R^2\frac{C}{L} + (m^2 - 1)^2}{m^2R^2\frac{C}{L} + (m^2 - 1)^2}$$

where $$m = \frac{f'}{f}.$$

Note that at resonance $R_{eff} = \frac{L}{CR}$.

Another peculiar property of parallel circuits is that if $R_L = R_C = \sqrt{\frac{L}{C}}$, the reactance of the circuit is zero for all frequencies, and $R = R_L$.

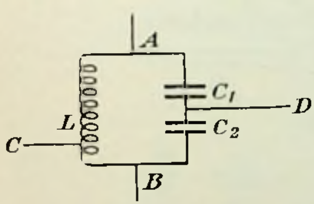

FIG. 71.—PARALLEL CIRCUIT.

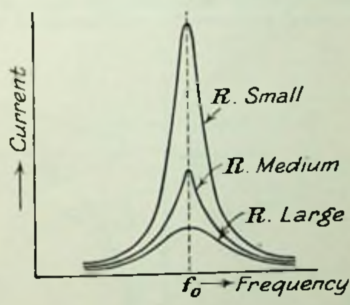

FIG. 72.—RESONANCE CURVES FOR SIMPLE SERIES CIRCUIT.

Resonance Curves.—If the current in a circuit is obtained over a steadily increasing range of frequencies, including the resonant frequency, a curve may be plotted showing the variation of current with frequency. Such a curve is called a resonance curve, and the representative curves for series and parallel circuits are given in Figs. 72 and 73. (See also p. 160.)

These curves illustrate the necessity for keeping the resistance low if sharp resonance is required, particularly in the case of parallel resonance.

Reactance Diagrams.—The tuning point or points of a circuit may be obtained by splitting the impedance into its reactive and resistive components in the usual manner. The condition for resonance is then that the

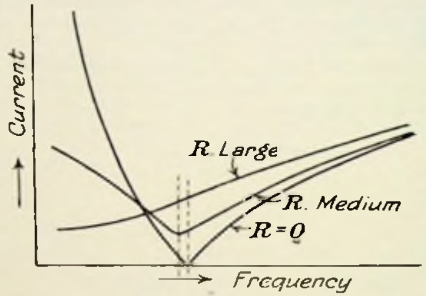

FIG. 73.—RESONANCE CURVES FOR SIMPLE PARALLEL CIRCUIT.

reactive component shall vanish. In this connection the use of the symbol j is extremely useful, since it facilitates the separation of the real (resistive) and imaginary (reactive) components.

Where the resistance of the circuit is small, however, the investigation of the properties of any particular circuit may be considerably simplified by the use of reactance diagrams. This method, which is very ably dealt with in the Circular No. 74 of the Bureau of Standards, Washington (Radio Instruments and Measurements), consists in splitting the circuit up into several portions, the reactances of which are known. By drawing a curve

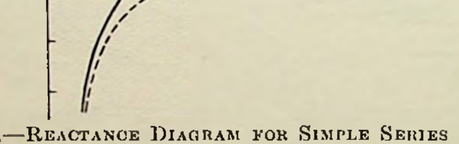

FIG. 74.—REACTANCE DIAGRAM FOR SIMPLE SERIES CIRCUIT.

of the variation of reactance with frequency for each of the several portions and adding them together, the reactance of the whole circuit may readily be obtained.

Fig. 74 shows the reactance diagram drawn out for a simple series circuit. The reactance of the inductance is ωL, and is represented by the straight line rising in value as the frequency increases. The reactance of the condenser is $\frac{1}{\omega C}$, and is represented by one branch of a hyperbola. The total reactance of the combination is the sum of these two curves. It will be observed that this curve crosses the zero line at one point. This is a tuning point, since the reactance is zero, and it can readily be seen from the figure that at this point $\omega L = \frac{1}{\omega C}$.

Fig 75 shows the reactance diagram for a simple parallel circuit. In this case it is not the reactances of the component parts which have to be added

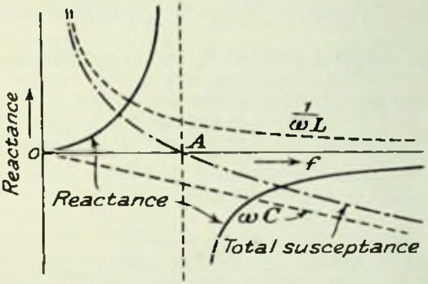

FIG. 75.—REACTANCE DIAGRAM FOR SIMPLE PARALLEL CIRCUIT.

together, since the components are in parallel. The susceptances, therefore, are obtained, the susceptance being the reciprocal of the reactance.

The primary curves plotted are thus ωC and $\frac{1}{\omega L}$, and the total susceptance of the circuit is the sum of these two, and is plotted in the chain dotted line. The full line is the reciprocal of the total susceptance, and thus is the total reactance of the circuit.

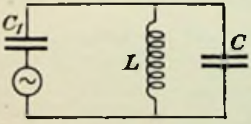

FIG. 76.—SIMPLE REJECTOR CIRCUIT.

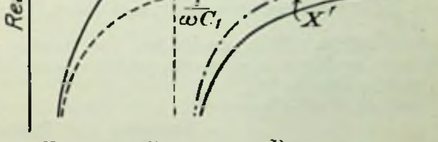

FIG. 77.—REACTANCE DIAGRAM FOR CIRCUIT SHOWN IN FIG. 76.

Any form of circuit, no matter how complex, may be treated in a similar manner, provided the resistance is negligible. For example, consider the case shown in Figs. 76 and 77, which comprises a condenser in series with parallel LC combination. X′ is the reactance of the parallel combination. The dotted line is the reactance of the condenser C_1 and the full line is the sum of the two, and gives the total reactance of the circuit.

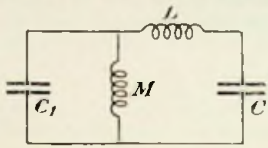

FIG. 78.—SIMPLE FORM OF COUPLED CIRCUIT.

This circuit has a tuning point at A, and an infinite impedance point at B. This property is often useful when it is desired to tune to one frequency and eliminate some other frequency near to the first. By proper design the points A and B may be obtained at any desired frequencies. It should be remembered, however, that the impedance at B is only infinite when R = 0.

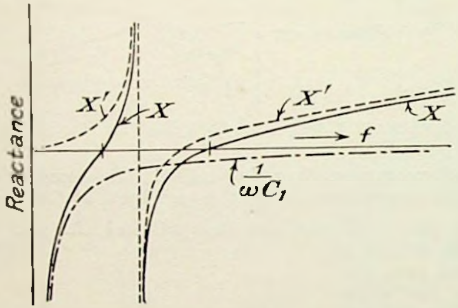

FIG. 79.—REACTANCE DIAGRAM FOR CIRCUIT SHOWN IN FIG. 78.

As a further example, the circuit shown in Fig. 78 may be considered. X′ is the reactance of the combination of L and C in series combined with M in parallel.

$\frac{1}{\omega C_1}$ is the reactance of C_1, and the full line is the reactance of the whole combination. This circuit has two tuning points and one infinite point. It is in reality a species of coupled circuit which accounts for the two frequencies (see p. 74).

Damping and Decrement.

The damping of a circuit controls the decay of current with a free oscillation, and also affects the growth or decay of current wherever transient conditions are brought into play.

This factor is measured in terms of the "decrement" of the oscillation, which is the ratio of the peak value of one oscillation to the peak value of the preceding oscillation.

From the equation to a free oscillation it follows that this ratio $=\varepsilon^{\frac{R}{2L}T}$,

where T is the time of one oscillation $=1/f$.

It is more convenient to work with the logarithm of this quantity, which is known as the "logarithmic decrement":

$$\delta=\frac{R}{2fL}=\pi\frac{R}{\omega L}=\pi R\omega C=\pi R\sqrt{\frac{C}{L}}.$$

The importance of this quantity in radio circuits is considerable. In spark transmitters it is obviously desirable to reduce the decrement as far as possible, in order to radiate as much energy as possible. In C.W. transmitters the question of decrement becomes merged into the simple considerations of the resistance of the circuit. It is important, however, in high speed and telephonic working, since the decrement not only controls the decay of the current, but the building up as well.

These points are referred to in detail on pp. 160, 203, and 212.

Measurement of Decrement.—This really consists in measuring the resistance and finding the decrement from one of the formulæ already given.

Provided a continuous oscillation is employed for the measurement, this method is quite accurate. Any convenient method of finding the resistance may be employed (see p. 59).

It is sometimes desired to find the decrement of one of a system of two circuits, one of which is radiating damped waves, and the other is tuned to the same frequency. Here both forced and free oscillations exist, having the same frequencies but different decrements. Bjerknes has shown that at resonance—

$$I^2=\frac{NE_0^{\,2}}{16L^2aa'(a+a')},$$

where E_0 = maximum value of impressed E.M.F.,
a = damping exponent of impressed E.M.F.,
a' = damping exponent of receiving circuit,
N = number of wave trains per second.

This formula only applies when the coupling between the circuits is very weak. The two circuits, if desired, may be a transmitting and receiving station.

A resistance R is then added, and the decrement of the receiving circuit is thus increased to a''.

Then
$$I_1^{\,2}=\frac{NE_0^{\,2}}{16L^2aa''(a+a'')}.$$

Rewriting the expression in terms of decrements, letting δ and δ' be the initial decrements of the transmitting and receiving circuits respectively, and δ_1 the added decrement due to R (so that $\delta_1+\delta'=\delta''$), then it may be shown that—

$$\frac{I^2}{I_1^{\,2}}=\frac{\delta''(\delta+\delta'')}{\delta'(\delta+\delta')}=\frac{(\delta'+\delta_1)(\delta+\delta'+\delta_1)}{\delta'(\delta+\delta')}.$$

This can be solved if either δ or δ'' is known. If not, the problem is complex, but the application is so limited that the solution is not given here.

Decremeter.—When measuring decrement by the reactance variation method, an instrument known as a decremeter is sometimes employed.

This is simply a suitably calibrated condenser, the plates of which are so designed that the percentage change of capacity for a given scale reading is the same throughout the whole range of the instrument. Hence the value of $\frac{C \sim C_1}{CC_1}$ may be read off directly. (See p. 32.)

Coupled Circuits.

It is frequently necessary in radio engineering practice to transfer the energy from one circuit to another. This may be accomplished by coupling together two suitable portions of the circuits either electromagnetically or electrostatically. The laws governing the use of such coupled circuits are of considerable importance. There are two types of coupled circuit to be considered:

(*a*) Two oscillatory circuits coupled together, one of which is set oscillating, the energy being transferred via the coupling to the secondary circuit. The oscillations in this case are free.

(*b*) Two circuits coupled together, in one of which there is a steady source of alternating E.M.F. The oscillations in this case are forced.

It is found that the two circuits interact, one upon the other, producing a complex tuning system. In order to gauge the extent of this interaction it is necessary to be able to define the degree of coupling between the two circuits. This is measured in terms of a " coupling factor "—

$$k = \frac{X_m}{\sqrt{X_1 X_2}},$$

where X_m = mutual or common reactance,
X_1 and X_2 are the total *similar* reactances in the primary and secondary circuits respectively.

The laws of these circuits will now be considered, the case of the free oscillations being considered first.

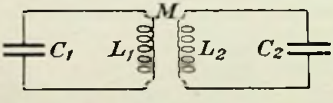

Fig. 80.—Magnetic Coupling.

(*a*) **Coupled Circuits with Free Oscillations**—*Magnetic Coupling.*—In the case shown in Fig. 80, the coupling is magnetic, and—

$$k = \frac{M}{\sqrt{L_1 L_2}}.$$

The effect of one circuit on the other will be appreciated from the following considerations: Let the two circuits be oscillating and each carrying a unit current.

The flux in the coil L_1 is in two portions, one due to itself and the other due to the effect of the other coil. If the two currents in the primary and secondary respectively are in phase, these two fluxes are additive, while if the currents are 180° out of phase the fluxes are in opposition.

For any other phase relationships the total flux linked with the primary coil lies between these two extreme values.

Hence the effective inductance of the primary coil varies between $L_1 + M$ and $L_1 - M$. Obviously, therefore, the natural frequency of the primary circuit will be affected, since this is defined by $f = \frac{1}{2\pi\sqrt{L_{eff}C_1}}$.

In practice it is found that two frequencies are set up in the circuit, one corresponding to an effective inductance $L + M$ and the other to $L - M$. These two frequencies beat with each other in the usual manner, the net result being a frequency $\frac{1}{2}(f_1 + f_2)$, modulated by a frequency $\frac{1}{2}(f_1 - f_2)$.

The current in the secondary is of the same form, and Fig. 81 shows the two currents. It will be observed that the current envelopes are 90° out of

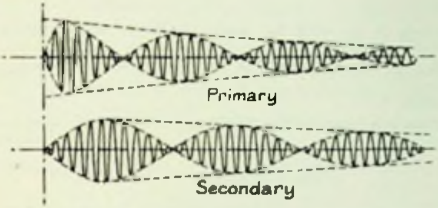

FIG. 81.—ILLUSTRATING BEATING OF CURRENTS IN TIGHTLY COUPLED CIRCUITS.

phase. Hence the physical explanation of the effect is that the energy is transferred from one circuit to the other, and then back again, the operation continuing until all the energy in the two circuits has been dissipated.

The effect is not marked as long as the coupling is kept small, the transfer of energy then being too slow for any appreciable surging to take place.

The effect may be investigated mathematically, the values of the frequencies set up being obtainable in terms of the constants of the circuit. The problem is complex unless the resistance is neglected, which is generally justifiable.

In this case—

$$E_1 = \left(\omega L_1 - \frac{1}{\omega C_1}\right) I_1 + M\omega I_2,$$

$$M\omega I_1 = \left(\omega L_2 - \frac{1}{\omega C_2}\right) I_2,$$

$$\therefore \frac{E_1}{I_1} = \left(\omega L_1 - \frac{1}{\omega C_1}\right) + \frac{\omega^2 M^2}{\omega L_2 - \frac{1}{\omega C_2}}.$$

The tuning points occur when the reactance is zero. Hence, rewriting we have—

$$\left(\omega L_1 - \frac{1}{\omega C_1}\right)\left(\omega L_2 - \frac{1}{\omega C_2}\right) + \omega^2 M^2 = 0;$$

$$\therefore \omega^4(L_1L_2 + M^2) - \omega^2\left(\frac{L_2}{C_1} + \frac{L_1}{C_2}\right) + \frac{1}{C_1C_2} = 0.$$

This is a quadratic in ω, the roots of which are rather complex if expressed explicitly. They may more readily be obtained, however, in terms of—

$$\omega_1 = \frac{1}{\sqrt{L_1C_1}}, \quad \omega_2 = \frac{1}{\sqrt{L_2C_2}}, \quad \text{and } k = \frac{M}{\sqrt{L_1L_2}}.$$

Then ω is given by—

$$\omega = \sqrt{\frac{\omega_1^2 + \omega_2^2 \pm \sqrt{(\omega_1^2 - \omega_2^2)^2 + 4k^2\omega_1^2\omega_2^2}}{2(1-k^2)}}.$$

When $\omega_1 = \omega_2$, as is usually the case—

$$\omega = \frac{\omega_1}{\sqrt{1 \pm k}}.$$

As k approaches 0, it will be seen that ω approaches ω_1; while as k approaches 1, one of the values of ω approaches $\frac{\omega_1}{\sqrt{2}}$, and the other approaches infinity.

A further interesting case arises when $\omega_1 = \omega_2$, and $L_1 = L_2$. Then—

$$\omega' = \frac{1}{\sqrt{(L_1 + 2M)C_1}},$$

$$\omega'' = \frac{1}{\sqrt{L_1C_1}}, \text{ which is independent of M.}$$

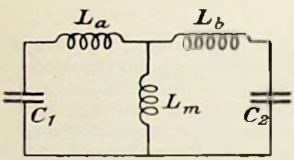

Fig. 82.—Direct Coupling.

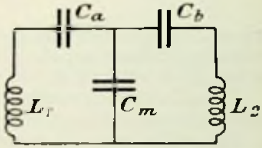

Fig. 83.—Capacity Coupling.

Direct Coupling.—In Fig. 82 a circuit is shown having a portion of the inductance common to both primary and secondary. This is termed direct coupling. It may be treated in exactly the same manner as magnetic coupling, putting—

$$L_1 = L_m + L_a,$$
$$L_2 = L_m + L_b,$$
$$M = L_m,$$

whence

$$k = \frac{L_m}{\sqrt{(L_m + L_a)(L_m + L_b)}}.$$

Electrostatic Coupling.—Fig. 83 shows one form of capacity coupling, in which part of the capacity is common to both circuits. By obtaining the equation to the primary impedance, and equating this to zero, as with the case of inductive coupling, a similar expression may be obtained for ω—*i.e.*, if—

$$\omega_1 = \sqrt{\frac{C_a + C_m}{L_1C_aC_m}}, \quad \omega_2 = \sqrt{\frac{C_b + C_m}{L_2C_bC_m}}, \quad k = \sqrt{\frac{C_aC_b}{(C_a + C_m)(C_b + C_m)}},$$

then $$\omega = \sqrt{\frac{\omega_1^2 + \omega_2^2 \pm \sqrt{(\omega_1^2 - \omega_2^2)^2 + 4k^2\omega_1^2\omega_2^2}}{2}},$$

When $$\omega_1 = \omega_2, \qquad \omega = \omega_1\sqrt{1 \pm k}.$$

As k approaches 0, ω approaches ω_1; while as k approaches 1 (*i.e.*, when C_m is small), the values of ω are $\sqrt{2}\,\omega_1$ and 0.

If $\omega_1 = \omega_2$, and $L_1 = L_2$, then—

$$\omega' = \sqrt{\frac{2C_a + C_m}{L_1C_aC_m}},$$

$$\omega'' = \sqrt{\frac{1}{L_1C_a}},$$ which is independent of C_m.

Other forms of capacity coupling may be treated by the use of the same formula, the appropriate values being substituted for ω_1, ω_2, and k.

Values of k, ω_1, and ω_2 for several of the more common types of coupled circuit in use to-day are appended.

Magnetic Coupling:

Mutual coupling (Fig. 80):

$$k = \frac{M}{\sqrt{L_1L_2}},$$

$$\omega_1 = \frac{1}{\sqrt{L_1C_1}},$$

$$\omega_2 = \frac{1}{\sqrt{L_2C_2}}.$$

Direct coupling (Fig. 82):

$$k = \frac{L_m}{\sqrt{(L_m + L_a)(L_m + L_b)}},$$

$$\omega_1 = \frac{1}{\sqrt{(L_a + L_m)C_1}},$$

$$\omega_2 = \frac{1}{\sqrt{(L_b + L_m)C_2}},$$

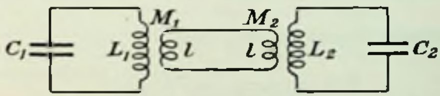

FIG. 84.—COUPLING WITH SEPARATE CIRCUIT.

Separate coupling circuit (Fig. 84):

$$k = \sqrt{\frac{M_1^2}{lL_1} + \frac{M_2^2}{lL_2}},$$

$$\omega_1 = \frac{1}{\sqrt{L_1C_1}},$$

$$\omega_2 = \frac{1}{\sqrt{L_2C_2}}.$$

Combination of mutual and direct couplings (Fig. 85):

$$k = \frac{L - M}{\sqrt{L_1 L_2}},$$

where L is the common portion,
L_1 and L_2 are the total inductances in primary and secondary.

$$\omega_1 = \frac{1}{\sqrt{L_1 C_1}} \qquad \omega_2 = \frac{1}{\sqrt{L_2 C_2}}.$$

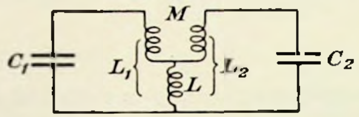

FIG. 85.—COMBINATION OF DIRECT AND MUTUAL COUPLING.

Electrostatic Coupling:

(*a*) With common capacity (Fig. 83):

$$k = \sqrt{\frac{C_a C_b}{(C_a + C_m)(C_b + C_m)}},$$

$$\omega_1 = \sqrt{\frac{C_a + C_m}{L_1 C_a C_m}},$$

$$\omega_2 = \sqrt{\frac{C_b + C_m}{L_2 C_b C_m}}.$$

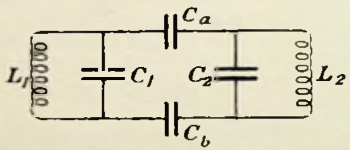

FIG. 86.—CAPACITY COUPLING WITH SEPARATE CIRCUITS.

(*b*) With separate capacities (Fig. 86):

$$k = \frac{C_s}{\sqrt{(C_1 + C_s)(C_2 + C_s)}},$$

where

$$C_s = \frac{C_a C_b}{C_a + C_b},$$

$$\omega_1 = \frac{1}{\sqrt{L_1 C_1}},$$

$$\omega_2 = \frac{1}{\sqrt{L_2 C_2}}.$$

If $C_b = \infty$ (*i.e.*, is short circuited), $C_s = C_a$.

It may be observed that if the primary and secondary of this circuit are tuned to the same frequency, so that $L_1C_1 = L_2C_2 = LC$, the system is practically monofrequency. For—

$$f' = \frac{1}{2\pi\sqrt{LC}};$$

$$f'' = \frac{1}{2\pi\sqrt{LC\left(1 + \frac{C_s}{C_1} + \frac{C_s}{C_2}\right)}}.$$

Now for maximum energy transfer C_s/C is small ($\frac{1}{200}$ or less), so that $f' = f''$ nearly.

General Case (Fig. 87):

$$k = \pm\frac{m \pm s}{1 \pm ms},$$

where

$$m = \frac{M}{\sqrt{(L_1 + L_a)(L_2 + L_b)}};$$

$$s = \sqrt{\frac{C_1C_2}{C}}.$$

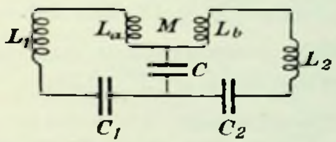

FIG. 87.—GENERAL CASE OF COMBINED MAGNETIC AND ELECTROSTATIC COUPLING.

(*b*) **Coupled Circuits with Forced Oscillations.**—In the second case, where there is a continuous source of alternating E.M.F. in the primary, the phenomena are somewhat simpler. In the first few moments after switching on there exist both forced and free oscillations. The latter, however, rapidly die away, and the two circuits then behave as a simple transformer, which may be treated according to the ordinary laws of alternating currents. Some of the more useful formulæ are summarised below.

Inductive Coupling.—The presence of the secondary affects the constants of the primary circuit. If R_1' and X_1' are the equivalent primary resistance and reactance respectively, then—

$$R_1' = R_1 + \frac{M^2\omega^2}{Z_2^2} R_2;$$

$$X_1' = X_1 - \frac{M^2\omega^2}{Z_2^2} X_2.$$

Exactly similar expressions may be obtained for the equivalent secondary constants.

If the coupling between the two circuits is very close and there is no leakage, then the equivalent inductance of the primary circuit is zero. In practice this condition is never attained, because there is always some leakage, apart from the fact that there is often a portion of the primary inductance which is not coupled to the secondary. The equivalent inductance of the primary is thus often known as the " leakage " inductance, and the value of this is of some importance in the design of a circuit. The leakage inductance L_s is given by—

$$L_s = L_1 - \frac{M^2\omega^2}{Z_2^2} L_2.$$

It is often more convenient to express this in terms of the coupling factor

$$k = \frac{M}{\sqrt{L_1 L_2}}.$$

Then
$$L_s = L_1 \left[1 - k^2 \left(\frac{L_2\omega}{Z_2} \right)^2 \right].$$

It is customary in radio work to tune the various circuits involved. Either the primary or the secondary, or both, may be tuned. If the primary is tuned and the secondary kept constant, then the maximum current is obtained by making $X_1' = 0$. If the secondary is tuned, then X_2' should $= 0$.

Hence, for resonance—

$$X_1 = \frac{M^2\omega^2}{Z_2^2} X_2 \text{ } (X_2 \text{ constant});$$

$$X_2 = \frac{M^2\omega^2}{Z_1^2} X_1 \text{ } (X_1 \text{ constant}).$$

The optimum condition occurs when both these conditions hold simultaneously. This gives that $X_1 \left(R_2 - \frac{M^2\omega^2 R_1}{Z_1^2} \right) = 0.$

There are two possible conditions:

1. $M^2\omega^2 < R_1R_2$. This requires an imaginary value of X_1 to make $R_2 - \frac{M^2\omega^2 R_1}{Z_1^2} = 0$, so that the condition in this case is that $X_1 = 0$.

2. $M^2\omega^2 > R_1R_2$. The condition is then that $R_2 - \frac{M^2\omega^2 R_1}{Z_1} = 0$.

Pierce calls these two cases *deficient* and *sufficient* coupling respectively. The value of the secondary current in the two cases is given by—

1. Deficient coupling:

$$I_2 = \frac{\omega M Z_1}{R_1R_2 + M^2\omega^2} I_1.$$

2. Sufficient coupling:

$$I_2 = \frac{Z_1}{2\sqrt{R_1R_2}} I_1.$$

Note that in the second case I_2 is independent of M.

Capacity Coupling.—The above remarks apply to capacity couplings of the type shown in Fig. 83, if $\frac{1}{C_m^2\omega^2}$ is substituted for $M^2\omega^2$.

Then
$$R_1' = R_1 + \left(\frac{1}{C_m^2\omega^2 Z_2^2} \right) R_2,$$

$$X_1' = X_1 - \left(\frac{1}{C_m^2\omega^2 Z_2^2} \right) X_2.$$

The resonance relations are as before, with the appropriate substitution.

Resistance Coupling.—Cases sometimes arise where two circuits are coupled by a resistance which is common to both circuits. Such a circuit would have a resistance in place of C_m in Fig. 83. Here—

$$R_1' = R_1 - \frac{R_m^2}{Z_2^2} R_2;$$

$$X_1' = X_1 + \frac{R_m^2}{Z_2^2} X_2;$$

where R_1 and R_2 are the total resistances in the primary and secondary (including R_m),
R_m is the common resistance.

The only possible complete resonant condition here is one of deficient coupling, so that $X_1 = X_2$ must be zero.

$$I_2 \text{ then} = \frac{R_m Z_1}{R_1 R_2 - R_m^2} I_1.$$

Use of Reactance Diagrams.—The tuning properties of any particular form of coupled circuit may be obtained very simply by drawing a reactance diagram.

The procedure is to find the reactance of the primary circuit, allowing for the effect of the secondary. Tuning or infinite impedance points may then be detected in the usual manner.

Consider first the case of direct coupling (Fig. 82). The circuit here resolves itself into a condenser C_1 in series with an inductance L_a, and a combination of an inductance L_m and the circuit $L_b C_2$ in parallel. Each of these components may be evaluated in the usual way, the complete reactance diagram being as shown in Fig. 88.

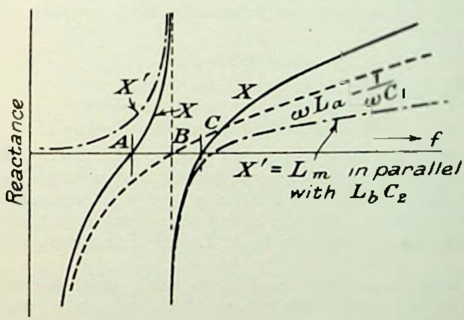

Fig. 88.—Reactance Diagram for Circuit shown in Fig. 82.

It will be observed that there are two tuning points on either side of an infinite impedance point. This is a property which is common to all coupled circuits. The infinite impedance point corresponds to the natural frequency of the secondary circuit by itself $= \frac{1}{2\pi\sqrt{L_2 C_2}}$.

Magnetically coupled circuits may be treated in a similar manner by use of the following substitutions:

$$L_1 = L_m + L_a,$$
$$L_2 = L_m + L_b.$$

Capacity couplings may be treated in the same way. Consider the case shown in Fig. 89. X′ is the reactance of C_m in parallel with C_b and L_2 in

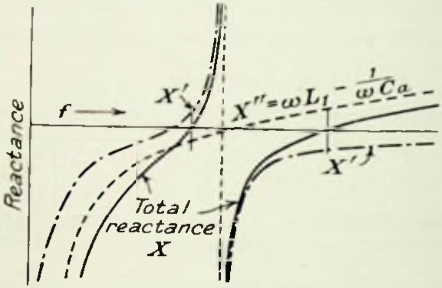

Fig. 89.—Reactance Diagram for Circuit shown in Fig. 83.

series. X″ is the reactance of L_1 and C_a in series, and the total reactance is thus as indicated by the full line. Here, again, it will be seen that there are two tuning points on either side of an infinite impedance point.

Tuning Calculations.—The resonant frequency of a series circuit has been seen to be given by $f = 2\pi\sqrt{LC}$.

Since $f \times \lambda = c$, the velocity of light $= 3 \times 10^{10}$ cms./sec., we may deduce the expression—

$$\lambda = 1{\cdot}884\sqrt{LC},$$

where λ = wave length (metres),
L = inductance (microhenries),
C = capacity (micro-microfarads).

For rapid design the chart given in Fig. 90 will prove useful. Choosing a given capacity and inductance as abscissa and ordinate respectively, the intersection of the two will give the wave length as referred to the diagonal scale—*e.g.*, 200 $\mu\mu$F and 3,200 μH give a wave length of 1,500 metres. This chart is a modification of one due to A. J. Gill of the Post Office Engineering Department.

It is often more convenient and is certainly more scientific to refer to the frequency of the oscillation rather than the wave length. This may be obtained by utilising the expressions given above. For convenience a frequency-wave length conversion chart (Fig. 91) is appended.

Aerials and their Characteristics—Radiation.

There are two main divisions of this subject. First, there are the tuning characteristics of any given aerial system; and, secondly, there are the radiative properties, involving a consideration of the resistances of the aerial system and the losses in the surroundings.

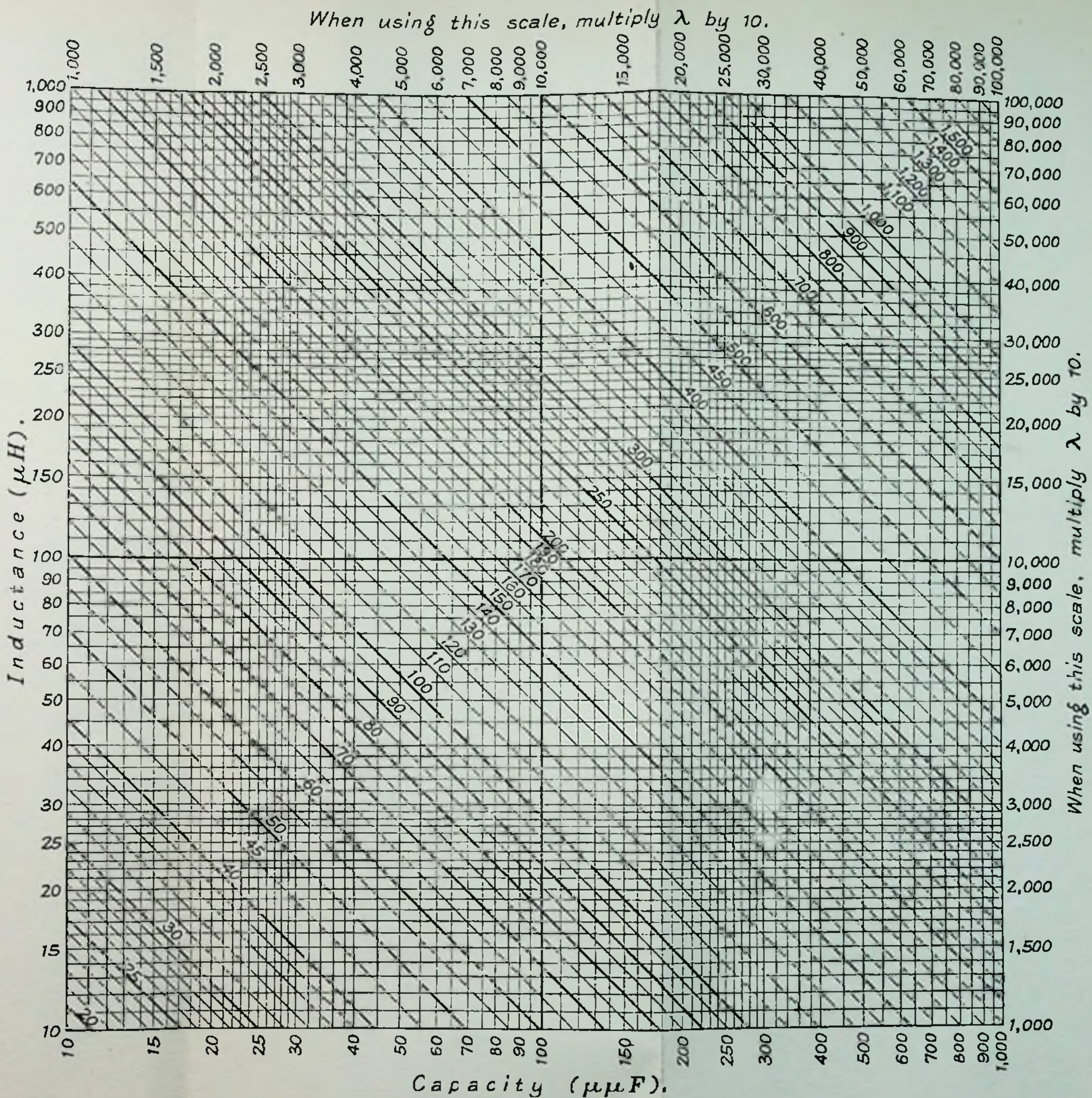

Fig. 90.—Wave Length Diagram.

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Tuning Characteristics.—The inductance and capacity of an aerial system are not concentrated at one point, but are distributed throughout the whole length of the aerial (apart from any loading coils which may be inserted, the effect of which will be considered later).

The current or voltage, therefore, will not be uniform over the aerial, but will vary in a sinoidal manner. Fig. 92 shows two possible methods of

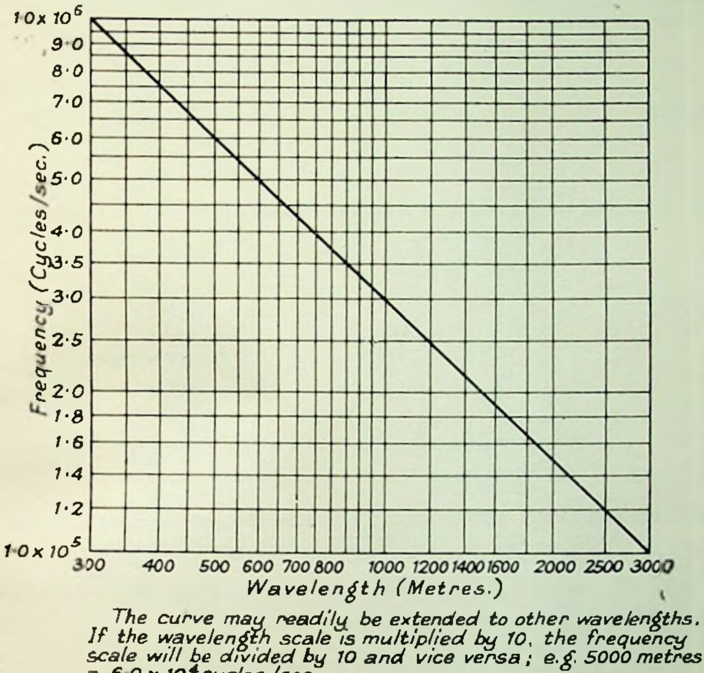


FIG. 91.—FREQUENCY-WAVE LENGTH CONVERSION CHART.

distribution, one corresponding to the fundamental of the aerial, and the other to the third harmonic.

In order to find what the effective values of the inductance and capacity of an aerial are, let L_o and C_o be the static values calculated from the usual formulæ. It has been shown by several investigators that if an E.M.F. is introduced into a system having distributed inductance and capacity, the reactance of the system is given by—

$$X = -\sqrt{\frac{L_o}{C_o}} \cot \omega\sqrt{C_oL_o}.$$

The reactance diagram for such a system is given in Fig. 93 (*a*). It will be seen that there are recurring tuning points, and also intermediate points of infinite impedance.

Aerial Oscillating naturally.—These tuning points occur when—

$$\omega\sqrt{C_o L_o} = m\pi/2,$$

where $m = 1, 3, 5,$ etc.

Since $f = \omega/2\pi$,

$$f = \frac{m}{4\sqrt{C_o L_o}},$$

whence

$$\lambda_o = \frac{4c}{m}\sqrt{C_o L_o}, \text{ where } c = \text{velocity of light,}$$

$$= \frac{1 \cdot 2}{m}\sqrt{L_o C_o} \text{ metres.}$$

This is termed the *natural wave length* of the aerial.

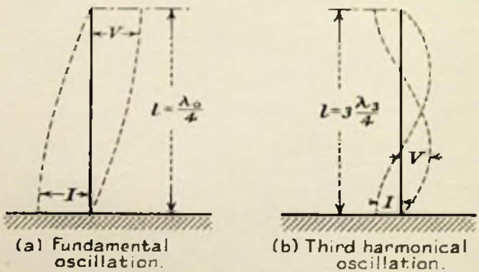


FIG. 92.—CURRENT AND VOLTAGE DISTRIBUTION ON A SIMPLE AERIAL OSCILLATING FUNDAMENTALLY AND HARMONICALLY.

At high frequencies $\sqrt{L_o C_o}$ approximates to l/c, l being the total length of the aerial, and hence, neglecting the end effect, $\lambda_o = \frac{4l}{m}$.

Referring now to the expression for the reactance—

$$X = -\sqrt{\frac{L_o}{C_o}}\cot \omega\sqrt{C_o L_o},$$

this may be expanded in the usual manner—*i.e.*:

$$X = -\sqrt{\frac{L_o}{C_o}}\left(\frac{1}{\omega\sqrt{C_o L_o}} - \frac{\omega\sqrt{C_o L_o}}{3} + \ldots\right).$$

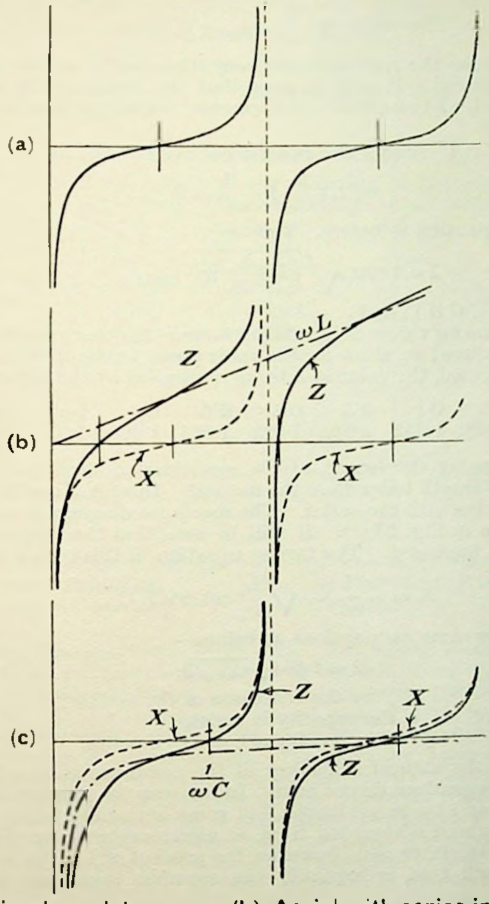
If $\omega\sqrt{C_o L_o}$ is small—*i.e.*, provided the aerial is not operating at or near its natural wave length—the remaining terms may be neglected so that—

$$X = -\frac{1}{\omega C_o} + \frac{\omega L_o}{3}.$$

Hence, provided λ is not near λ_o—

$$C_{eff} = C_o \text{ and } L_{eff} = L_o/3.$$

Loaded Aerial.—From practical considerations, aerials are usually operated at wave lengths several times the natural wave length. (See p. 97.)



(a) Simple aerial. (b) Aerial with series inductance.
(c) Aerial with series condenser.

FIG. 93.—REACTANCE DIAGRAMS FOR SIMPLE AND LOADED AERIALS.

In order to tune the aerial, a "loading" inductance is inserted in the aerial at the base of the lead-in. This inductance also serves to couple the aerial to the transmitter.

To tune the aerial to a given frequency, therefore, the value of the loading coil must be such as to satisfy the equation—

$$X = \omega L - \sqrt{\frac{L_0}{C_0}} \cot \omega\sqrt{C_0 L_0} = 0.$$

Fig. 93 (*b*) shows the reactance diagram for a loaded aerial such as has just been considered. It will be seen that the frequency at the tuning points is reduced, and also that the harmonics are not integral multiples of the fundamental.

If $\lambda \neq \lambda_0$ ($\omega\sqrt{C_0 L_0}$ small), the equation above reduces to—

$$\omega\left(L + \frac{L_0}{3}\right) - \frac{1}{\omega C_0} = 0$$

by the same expansion as before. Hence—

$$\lambda = 1{\cdot}884 \sqrt{\left(L + \frac{L_0}{3}\right) C_0} \text{ metres,}$$

when L is in μH, C is in $\mu\mu$F.

This is only true for values of $L = 2L_0$ or more. In other cases a correction should be introduced to allow for the subsequent terms of the expansion, which were neglected, the value of λ being multiplied by the factor k below:

L/L_0	0	0·1	0·2	0·3	0·4	0·6	0·8	1·0	1·5	2·0
k	1·103	1·063	1·042	1·030	1·023	1·014	1·009	1·007	1·004	1·003

Use of Shortening Condenser.—It is sometimes desired to operate an aerial at a wave length lower than the natural. In such a case a condenser is inserted in series with the aerial. The reactance diagram of such a combination is given in Fig. 93 (*c*). It will be seen that the frequency at the tuning points is increased. The tuning equation in this case is—

$$X = -\frac{1}{\omega C} - \sqrt{\frac{L_0}{C_0}} \cot \omega\sqrt{L_0 C_0},$$

so that, with the same assumptions as before—

$$\lambda = 1{\cdot}884\sqrt{C_s L_0 / 3},$$

where L_0, C_0 are the constants of the aerial,
C is the capacity in series,
C_s is the capacity of C and C_0 in series.

At or around the natural frequency of the aerial the assumptions made in the foregoing equations do not hold. In this case the effective inductance and capacity may also be evaluated, but some ambiguity arises according to whether the aerial is regarded from an electromagnetic or electrostatic point of view. In either case, however, the product of $L_{eff}\, C_{eff}$ is the same, and as this is all that is required, the separate values are not given. When $\lambda = \lambda_0$—

$$L_{eff}\, C_{eff} = \frac{4}{\pi^2} L_0 C_0,$$

$$\lambda = 1{\cdot}884 \frac{2}{\pi} \sqrt{L_0 C_0},$$

$$= 1{\cdot}2\sqrt{L_0 C_0},$$

which is the same result as was obtained on p. 84.

Measurement of Effective L and C.—The effective inductance and capacity may readily be measured by measuring the wave length of the aerial with two loading coils of different values in circuit. Then—

$$\lambda_1 = 1{\cdot}884k\sqrt{(L_1 + L_o/3)C_o},$$
$$\lambda_2 = 1{\cdot}884k\sqrt{(L_2 + L_o/3)C_o},$$

k being the correction on p. 86. L_o and C_o are thus both obtained by solving the two equations.

Radiation from an Aerial.

The fundamental conception of the radiation of electromagnetic waves was obtained by Hertz from the consideration of an oscillating dipole or doublet. This consists of two small charges, one positive and the other negative, separated by a distance h. These charges are assumed to oscillate about their middle point in a straight line, the motion of the two charges being in opposite directions at any particular instant (see Fig. 94).

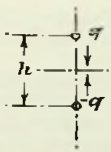


FIG. 94.—SIMPLE DIPOLE OR DOUBLET.

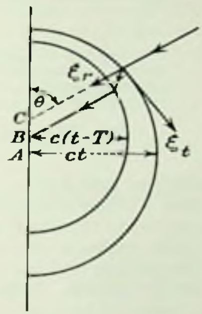


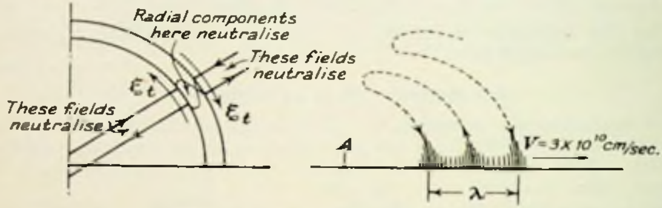
FIG. 95.—KINK PRODUCED IN LINE OF FORCE BY DECELERATION OF CHARGE.

Considering first of all one charge only. This charge has associated with it lines of force, one of which is shown in Fig. 95. Now suppose that this charge is moving upwards with a velocity v, carrying with it the associated lines of force. Any electromagnetic disturbance will be propagated through the ether with a definite velocity, which velocity is found to be 3×10^{10} centimetres per second, the speed of light. (To make the case more general, the velocity may be assumed to have some other value v', but it is found that in order completely to satisfy the several equations which are deduced, this velocity must be the same as the velocity of light.)

At a given time $t = 0$, therefore, the charge q is moving upwards with a velocity v. Let the charge now decelerate and come to rest at B. If it had continued to move uniformly upwards, it would, in time t have reached the point C. Because of the finite velocity of the propagation of the disturbance in the ether, it follows that outside a sphere of radius ct the lines of force emanate from a centre C, since the disturbance resulting from the deceleration of the charge has only travelled a distance ct.

Again, if T is the time occupied by the charge in decelerating from A to B, it follows that inside a sphere of radius $c\,(t - T)$ the lines of force will radiate from B. In between these two spheres the line of force will be kinked in some manner in order that it may change its centre, and this kink is the disturbance which is radiated into space caused by the deceleration of the charge q.

Fig. 96 shows the same diagram considered for the complete doublet, each of the charges accelerating from rest, reaching a maximum velocity, and



(a) Showing how all fields except the tangential components of the kink neutralize each other.

(b) Resultant fields at a distance. (near earth's surface.)

FIG. 96.—PRODUCTION OF ELECTRIC WAVES.

decelerating to rest again. The lines of force outside the shell neutralise each other. Those within the shell may be resolved into two components, one tangential to the shell, and the other radial.

The radial components cancel out, leaving two tangential components, one on the outside and the other on the inside of the shell, these two being in opposite directions. If the total effect of all the lines of force is considered, the net result will be a series of bands of electric field, first in one direction, and then in the other.

Further, since the doublet continues to oscillate, the first pair of bands will be followed by a second pair, and so on, the distance between successive bands being cT.

Now the time of one complete oscillation of the doublet $= 2T = 1/f$, where f is the frequency of oscillation of the charges.

Consequently, the distance between any two bands of electric field *in the same direction* $= 2cT = c/f = \lambda$.

This distance λ is termed the " wave length " of the wave, and it will be seen that the wave length and frequency are connected by the simple relation $\lambda \times f = c$.

Associated with the electric field by virtue of its motion is a magnetic field, horizontal and in phase with the electric field. It should be noted that the one is simply another manifestation of the other. *The electric and magnetic fields do not exist separately, but according to whether the phenomenon is regarded from an electrostatic or electromagnetic point of view, so does one obtain a series of vertical bands of electric field or horizontal bands of magnetic field respectively.*

Another important point to note is that the fields considered are totally separate from the ordinary electric and magnetic fields near the aerial. These fields are 90° out of phase, and, moreover, die away as the square of the distance, whereas it will be found that the true radiative fields are inversely proportional to the first power of the distance away. In the immediate neighbourhood of the aerial, within a distance of a few wave lengths, the two fields exist together, but at any appreciable distance the primary fields are absolutely negligible.

Application of Doublet Theory to Aerials.—A simple Hertzian oscillator consists of two plates separated by a wire containing inductance, all the capacity being assumed to be concentrated in the plates themselves. Such a system may readily be investigated by use of the doublet theory.

The more practical case is that of an aerial system, as shown in Fig. 97 (*b*). This may be taken as the upper half of a Hertzian oscillator, and the doublet theory applied, *on the assumption that the capacity is all concentrated in the upper end of the aerial.*

This assumption is not always justifiable, as will be seen later, but some of the results obtained on this theory are appended, as they give useful information on the mechanism of the propagation of electric waves.

Simple Hertzian Oscillator.—For a simple oscillator of length $2h$, having all the capacity concentrated at the ends [Fig. 97 (*a*)], the field strengths are as below:

$$\text{Electric field } \mathcal{E} = 4\pi\sqrt{\frac{\mu}{k}} \cdot \frac{Ih}{\lambda d} \sin\theta \text{ electrostatic units.}$$

$$\text{Magnetic field } \mathcal{H} = 4\pi\frac{Ih}{\lambda d} \sin\theta \text{ electromagnetic units.}$$

where I = current in transmitting aerial, h = height of transmitting aerial, λ = wave length } in C.G.S. units,
d = distance from transmitter,
θ = angle with vertical (Fig. 95),
μ and k are the magnetic and electric permeabilities of the medium.

Thus for air ($\mu = k = 1$) the electric field in E.S.U. is equal to the magnetic field in E.M.U. (1 E.M.U. $= c \times 1$ E.S.U.).

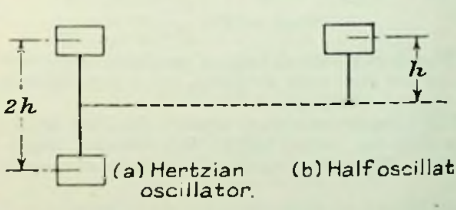


FIG. 97.—SIMPLE OSCILLATOR.

Putting I in amperes, and h, λ, and d in metres, we have for the field strength at the earth's surface ($\sin\theta = 1$):

$$\mathcal{E} = 377\frac{Ih}{\lambda d} \text{ volts per metre.}$$

Power Radiated.—The power radiated is given by—

$$P = \frac{2}{3} \cdot (4\pi)^2 \sqrt{\frac{\mu}{k}} \frac{h^2}{\lambda^2} I^2.$$

For transmission through air, putting I in amperes (R.M.S.) and h and λ in similar units (metres), this reduces to—

$$P = 3168 \frac{h^2}{\lambda^2} I^2.$$

Practical Form of Aerial.—The simple Hertzian oscillator has a limited application to aeroplane work, but the more general type of aerial is one utilising the earth as one plate of the condenser. This is, then, equivalent to the upper half only of a simple oscillator [see Fig. 97 (*b*)].

For such a case the expressions for $\mathcal{E}$ and $\mathcal{H}$ remain unchanged, h now being the actual height of the aerial, while the power radiated is one-half of its previous value, since it is only the power radiated in a hemisphere above the equatorial plane which is effective. Hence—

$$P = 1584 \frac{h^2}{\lambda^2} I^2.$$

These values assume that the capacity is concentrated at the upper end of the aerial.

Radiation Resistance.—When an oscillating current is set up in an aerial, a certain expenditure of power is required in order to maintain the oscillations, owing to the losses set up by the resistance of the conductors, the dielectric losses, etc.; there is, in addition, a portion of the power which produces electromagnetic waves, and is effective in producing useful radiation. The conductor and other losses can all be expressed in terms of an equivalent resistance, so that it is found convenient to express the actual radiation in the same manner. This is done in terms of a fictitious "radiation resistance," which is defined by the relation—

$$\text{Power radiated} = \text{radiation resistance} \times (\text{current})^2.$$

In the case of an aerial with concentrated capacity, as just considered, the radiation resistance is obviously—

$$R_{rad} = 1584 \frac{h^2}{\lambda^2}.$$

Effective Height.—In a practical form of oscillator the current distribution is not uniform, as has just been assumed, since the capacity is no longer concentrated in the end of the aerial.

To allow for this it is customary to assume that the height of the aerial is somewhat less than the actual height, this reduced height being termed the effective height. Values for the effective heights of some forms of aerials are appended.

Single Vertical Wire:

At natural wave length: $h = 2l/\pi$.
At a considerably longer wave length: $h = 0{\cdot}7l$.

Umbrella Aerial:

$\lambda >> \lambda_0$: $h = 1{\cdot}414h'$, where h' is the height to the middle of the ribs.

Flat-topped Aerials.—Pierce has shown ("Electric Oscillations and Electric Waves," G. W. Pierce) that the doublet formula is not applicable to practical aerials, owing to the fact that the current is not uniformly distributed. He shows that the ordinary correction introduced by the assumption of an effective height is not adequate, and he has accordingly obtained the expressions for $\mathcal{E}$, $\mathcal{H}$, and the radiated power by summing the effect of an infinite number of doublets situated all along the aerial, due allowance being made for the gradually diminishing current as the far end of the aerial is approached. These results have only been applied to single wires and flat-top aerials, but even so they constitute a very valuable contribution to the data on the subject.

The current is assumed to be—

$$i = \mathrm{I} \sin \frac{2\pi c}{\lambda} t \sin \frac{2\pi}{\lambda}\left(\frac{\lambda_o}{4} - l\right).$$

The first term is the time variation, the second being the space variation over the length of the aerial [see Fig. 92 (*a*)]. The current at the base ($l = 0$) is—

$$\mathrm{I}_o = \mathrm{I} \sin \frac{\pi\lambda_o}{2\lambda}.$$

On this assumption the values of $\mathcal{E}$, $\mathcal{H}$, and P are found to be—

$$\mathcal{E}_{\text{vert}} = \mathcal{H}_{\text{hor}} = \frac{2\mathrm{I}_o}{cd \sin \theta \sin \frac{\pi\lambda_o}{2\lambda}} \cos \frac{2\pi}{\lambda}(ct - d) \times$$

$$[\cos \mathrm{B} \cos (\mathrm{A} \cos \theta) - \sin \mathrm{B} \sin (\mathrm{A} \cos \theta) \cos \theta - \cos (\mathrm{A} + \mathrm{B})],$$

where $\mathrm{A} = \frac{2\pi h}{\lambda}$,

$\mathrm{B} = \frac{2\pi b}{\lambda}$,

b = length of flat top,
h = height of aerial,
λ = wave length.
θ = angle with vertical,
d = distance away from aerial,
I_o = current at base of aerial,
$\mathcal{E}$ = electric field strength (E.S.U.),
$\mathcal{H}$ = magnetic field strength (E.M.U.).

When $\theta = 90°$—*i.e.*, in the horizontal plane—

$$\mathcal{E} = \mathcal{H} = \frac{2\mathrm{I}_o}{cd} \cos \frac{2\pi}{\lambda}(ct - d)\left[\frac{\cos \mathrm{B} - \cos \overline{\mathrm{A}+\mathrm{B}}}{\sin \frac{\pi\lambda_o}{2\lambda}}\right].$$

The last term is a function of λ/λ_o, the other terms being constant. It is thus possible to compare the value of the received field for a given transmitted current I_o in terms of λ/λ_o. Fig. 98 shows this relation for several values of $\gamma = \frac{b}{h+b}$.

It will be seen that there is a large increase in the field strength around λ_o, but that if, as is usual, practical conditions necessitate the use of a wave length several times the natural, there is little to choose. It also indicates that the flat top hinders the radiation from the aerial, its sole use in practice being to increase the capacity of the aerial so that I_o may be increased.

These curves do not take into account the absorption of the waves during transmission (see p. 93).

Power Radiated.—The expressions for radiated power, involving as they do the summation of several series, are rather complicated, but Professor Pierce has worked out the radiation resistance of single wire and flat-top

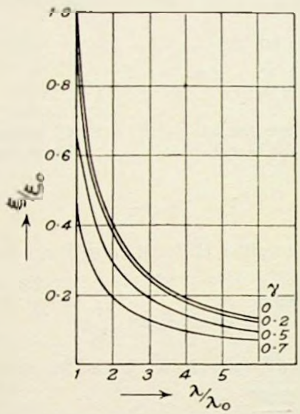


FIG. 98.—FIELD STRENGTHS AT GIVEN DISTANCE FROM AERIAL IN TERMS OF λ/λ_0.

aerials in terms of λ/λ_0 and γ. Since $\lambda_0 = 4l$, these two factors completely define any particular aerial, and Table XVIII. appended is thus of considerable value. Some of the values in the table have been plotted against λ/λ_0 in Fig. 99.

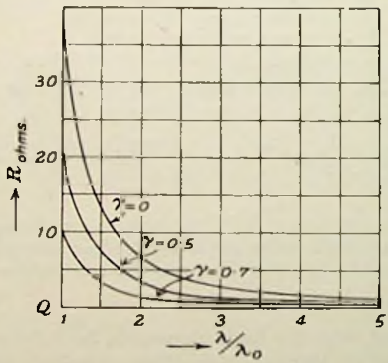


FIG. 99.—RADIATION RESISTANCE OF FLAT-TOP AERIALS.

The values for power radiated are considerably less than those obtained by the doublet formulæ, the discrepancy being most marked in the case of single vertical wires ($\gamma = 0$), which bears out Professor Pierce's contention that the ordinary assumptions of effective height are inadequate for the purpose.

TABLE XVIII.

RADIATION RESISTANCE OF FLAT-TOP AERIALS.

λ/λ_0.	$\gamma=0$	·2	·3	·4	·5	·6	·7	·8	·9
1·0	36·6	33·3	29·7	25·5	20·3	14·7	9·7	4·9	1·2
1·2	21·8	20·2	18·8	15·8	12·4	9·0	6·0	2·9	·94
1·4	15·1	14·0	12·2	10·5	8·6	6·1	4·0	2·0	·70
1·6	11·0	10·0	9·0	7·8	6·3	4·4	2·8	1·4	·50
1·8	8·3	7·7	6·7	6·0	4·7	3·2	2·2	1·1	·33
2·0	6·5	6·1	5·5	4·8	3·8	2·7	1·7	·75	·18
2·2	5·2	5·0	4·6	3·9	3·0	2·2	1·4	·57	·16
2·4	4·4	4·2	3·8	3·2	2·5	1·8	1·2	·48	·14
2·6	3·8	3·5	3·1	2·7	2·1	1·5	1·0	·42	·12
2·8	3·3	3·0	2·6	2·3	1·8	1·3	·86	·37	·10
3·0	2·8	2·5	2·2	1·9	1·5	1·1	·74	·33	·09
3·2	2·5	2·3	2·0	1·7	1·3	·92	·64	·29	·08
3·4	2·2	2·0	1·8	1·6	1·1	·84	·55	·25	·072
3·6	2·0	1·9	1·6	1·4	1·0	·77	·47	·22	·066
3·8	1·75	1·7	1·4	1·3	·94	·71	·39	·19	·060
4·0	1·62	1·5	1·3	1·1	·88	·66	·31	·16	·055
4·5	1·30	1·21	1·05	·89	·75	·54	·26	·12	·042
5·0	1·00	·92	·80	·68	·63	·42	·22	·09	·032
5·5	·78	·73	·65	·56	·53	·36	·19	·08	·025
6·0	·61	·54	·49	·44	·43	·29	·16	07	·019
7·0	·38	·36	·33	·32	·28	·22	·12	·06	·013
10·0	·22	·18	·17	·15	·13	·11	·07	·04	·011

In calculating the power radiated, however, an allowance should be made for any ineffective current in the aerial. There are two effects which reduce the effective radiation. The first is that of the masts and stays, in which currents are induced, which in turn radiate waves 180° out of phase with the radiation from the main aerial. To overcome this it is becoming increasingly common to employ insulated masts or to construct the masts of wood.

The second effect is that of simple capacity leakage to earth, the current leaking away in places where it is not effective in producing radiation.

Received Current—Absorption.—If h_2 is the height of the receiving aerial, then the E.M.F. induced therein by a given wave is $\mathcal{E} \times h_2$, where $\mathcal{E}$ is the electric field strength as determined by the formulæ already given.

The current may be obtained by dividing this value of E by the impedance of the aerial. When the aerial is tuned, as is usually the case, the impedance is simply R_2, so that—

$$I_2 = \frac{\mathcal{E}h_2}{R_2}.$$

This formula is correct over short distances only. At longer distances from the transmitter, the waves are attenuated by various processes, and this effect has to be allowed for.

It will be as well to consider at this juncture the propagation of the waves over the earth's surface.

An electric wave is propagated in a straight line. Obviously, therefore, if the receiving point is far distant from the transmitting point, the direct ray from the transmitter would normally never reach the receiver owing to the curvature of the earth. The conducting nature of the earth would permit of a certain refraction, but this in itself is not sufficient to account for the long-distance transmission known to be possible.

Now investigations have shown the existence of a layer of ionised gas at a considerable height above the earth's surface, and it is assumed that certain of the waves, leaving the transmitter in an upward direction, are reflected at this layer, and the waves are thus enabled to travel round the curved surface of the earth.

This theory was first investigated by Oliver Heaviside, and the ionised layer of gas is known in consequence as the Heaviside layer.

During the day the sun's rays ionise the air to a considerable extent, the ionisation increasing rapidly as the height above the ground increases. At night, however, the ionisation near the earth disappears, since the ionisation under normal circumstances is a function of the pressure, and the edge of the Heaviside layer thus becomes fairly sharply defined.

During the day a certain absorption takes place, due to the ionisation of the atmosphere, but at night this absorption is considerably less, resulting in distinctly greater received field strength.

Austin and Cohen of the U.S. Navy conducted experiments to obtain data on the attenuation of waves at distances up to 2,000 kilometres, as a result of which they proposed the well-known Austin-Cohen factor:

$$\mathcal{E}_r = 377 \frac{Ih}{\lambda d} \cdot \varepsilon^{-0{\cdot}0015 d/\sqrt{\lambda}},$$

where $\mathcal{E}_r$ = received field strength (microvolts per metre),
h = height of transmitting aerial (metres),
λ = wave length (kilometres),
d = distance from transmitter (kilometres).

If λ is expressed in metres, the expression becomes—

$$\mathcal{E}_r = 377 \frac{Ih}{\lambda d} \times 10^3 \,.\, \varepsilon^{-0{\cdot}048 d/\sqrt{\lambda}}.$$

This formula only applies over water by daylight. It becomes increasingly inaccurate if d is increased beyond 2,000 kilometres.

At considerably greater distances (5,000 to 15,000 kilometres) the field strength is found to be much greater than would be anticipated from this formula. Fuller has suggested that the correction factor at these ranges should be $\varepsilon^{-0{\cdot}0045 d/\lambda^{1{\cdot}4}}$, d and λ being in kilometres as before.

The most satisfactory theory, however, is that propounded by Professor Howe (*Electrician*, September 12 and October 10, 1924), who has suggested that at great distances the energy is transmitted in a thin shell between the earth and the Heaviside layer, which then function as a simple transmission line, and may be treated accordingly.

At a distance d from the transmitter there will be a certain quantity of energy contained in a circular zone of radius $r \sin \theta$ (Fig. 100). As the distance d increases, $r \sin \theta$ increases, and with it the area of the zone, so that, assuming the energy the same as before, the field strength will be diminished. Beyond $\theta = 90°$, however, the zone contracts again, and hence the field strength increases, and at the antipodes would be considerably strengthened.

There is, however, the attenuation due to the propagation of the energy over the "transmission line," so that this increase of field strength is counteracted to a large extent, but actually at distances of 19,000 kilometres the field strength does increase.

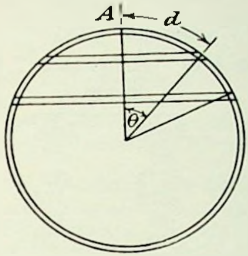


FIG. 100.—ILLUSTRATING THE DEPENDENCE OF $\mathcal{E}$ ON θ.

Assuming a height of 100 kilometres for the Heaviside layer, Howe obtains the formula:

$$\mathcal{E} = 386 \frac{Ih}{\lambda} \cdot \gamma \cdot 10^{-8} \frac{\varepsilon^{-\beta d}}{\sqrt{\sin \theta}} \text{ volts/centimetres,}$$

where I, h, and λ have the usual significance.

$\gamma^2 = \dfrac{\text{power transmitted over line.}}{\text{power radiated at transmitter}}$. The power does not commence to be transmitted round the line until the waves strike the Heaviside layer at a certain critical angle. At this point the power is less than that originally radiated.

$\beta = \dfrac{r}{240\pi h_t}$, where r = effective resistance per square centimetre of line (including earth and Heaviside layer); h_t = height of Heaviside layer.

d = distance round surface of earth (centimetres).

The s.s. *Aldebaran* carried out certain experiments at ranges up to 20,000 km., and found that the field strength could be expressed in the form—

$$\mathcal{E} = 377 \frac{Ih}{\lambda d} \cdot k,$$

where k is the factor shown in Table XIX. below.

Howe has shown that these results may be obtained from his theory by giving reasonable values to γ and r. The average values are:

$$\gamma = 0{\cdot}15 \text{ by day,}$$
$$= 0{\cdot}25 \text{ by night,}$$

$$r = 25 \text{ ohms by day,}$$
$$15 \text{ ohms by night.}$$

The formula shows that small changes in r or h_t have enormous effect. If r is doubled (or h_t halved), the field strength is reduced to 0·005 of its former value. This may account for the large variations experienced in practice.

The effect of earth resistance on range has been determined by Zenneck, relative figures being given in Table XX.

TABLE XIX.

VALUES OF k IN "ALDÉBARAN" FORMULA.

d (*Kilometres*).	9,000 λ.		11,000 λ.	
	Night.	*Day.*	*Night.*	*Day.*
1,000	·86	·37	·89	·49
2,000	·775	·26	·78	·35
4,000	·615	·175	·63	·25
6,000	·49	·11	·52	·20
8,000	·40	·065	·45	·16
12,000	·285	—	·35	—
18,000	·20	—	·27	—

TABLE XX.

EFFECT OF EARTH ON RANGE.

Nature of Earth.	*Range of Transmission (Kilometres).*
Perfect conductor	1,000
Sea water	920
Fresh water or marsh	700
Wet soil	560
Damp soil	270
Dry soil	150
Very dry soil	55

Standard Values of Field Strength.—A value of the received field strength of 50 microvolts per metre will give a good signal under good conditions. It will not be adequate, however, if jamming or atmospherics are heavy. To allow for working under adverse conditions, values of $\mathcal{E}_r$, as below, are taken as standard.

Ship and shore communication: 95 mV/m.
Reliable point-to-point communication: 150 mV/m.

In view of this definition of satisfactory working conditions it is becoming the practice to rate a transmitter in terms of the "metre-amperes" of the aerial. This is simply the product of the effective height and the current at the base of the aerial.

$$Ih = \frac{\mathcal{E}_r \lambda d \times 10^{-3}}{377} \epsilon^{0 \cdot 048 d/\sqrt{\lambda}},$$

where Ih = metre amperes (written m × A),
$\mathcal{E}_r$ = received field strength (microvolts/metre).
λ = wave length (metres).
d = distance from transmitter to receiver (kilometres).
h = effective height (metres).

Thus, given the m × A of an aerial system, the range of a station may be estimated, and *vice versa*.

Penetration of Waves in a Conducting Medium.—The waves are very rapidly attenuated in a conducting medium such as sea water. In such a case the current at a depth x may be determined from the formula:

$$i = I_0 \varepsilon^{-\sqrt{\frac{2\pi\omega\mu}{\rho}}x} \sin\left(\omega t - \frac{2\pi\omega\mu}{\rho} x\right).$$

where i = current at depth x,
I_0 = current at surface,
μ = permeability of material,
ρ = specific resistance of material.

Aerial Resistance and Losses.

The importance of the loss resistances in an aerial may be judged from the fact that an aerial 800 feet high working on a wave length of 20,000 metres has a radiation resistance of the order of $\frac{1}{4}$ ohm. The total resistance of the aerial, however, may be as much as 2 ohms, giving an aerial efficiency only of the order of 12 per cent.

The resistance of an aerial may be resolved into four factors:

(*a*) Radiation resistance: $\propto \frac{1}{\lambda^2}$.

(*b*) Eddy current loss and conductor resistance including earth resistance: $\propto \frac{1}{\lambda^{\frac{1}{2}}}$.

(*c*) Dielectric loss: $\propto \lambda$.

(*d*) Leakage loss: $\propto \lambda^2$.

The resistance of an aerial may be determined for a range of wave lengths around the working point, and a curve plotted as shown in Fig. 101. This is

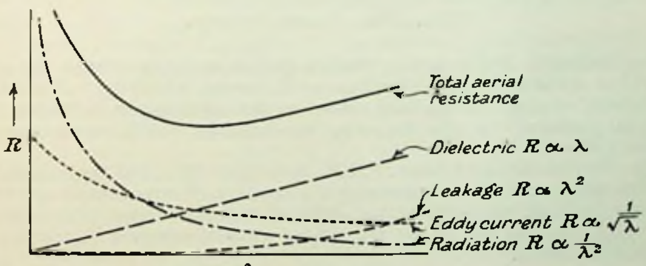


FIG. 101.—TYPICAL AERIAL RESISTANCE CURVE ANALYSED INTO ITS COMPONENTS.

usually done for any new aerial, and it is fairly simple to fit a curve of the form—

$$A/\lambda^2 + \beta/\lambda^{\frac{1}{2}} + C\lambda + D\lambda^2$$

to the actual curve. The magnitude of the several components may thus be determined, and should any one term be excessively high, the matter can be investigated. Note that the total resistance has a minimum value which usually occurs when λ = 2 or 3 times λ_0.

Radiation Resistance.—This question has already been discussed on p. 90, where it was shown to be proportional to $1/\lambda^2$.

Eddy Current and Conductor Loss.—The conductor resistance may readily be obtained at the particular frequency employed.

Eddy current losses in the earth, masts, and buildings are responsible for a fairly heavy proportion of the losses. Masts and buildings should thus be kept out of the field of the aerial where possible; the earth loss is more important, and is, in fact, the chief source of loss in many aerials.

The electric field produced by an aerial is in two portions, one vertical and the other horizontal, due to the flat top. This horizontal field produces eddy current losses in the earth.

There is also a downward radiation from the flat top, which, being ineffective in assisting the main radiation, is to be regarded as a loss. This also gives rise to earth losses.

The loss due to the horizontal field may be reduced by an earth screen. This is a system of wires, as shown in Fig. 102, which extends underneath

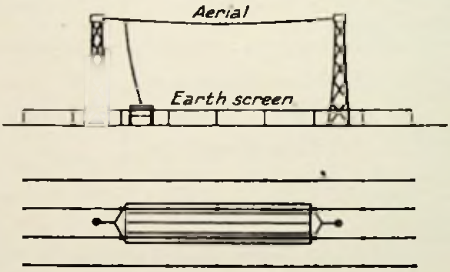


Fig. 102.—Earth Screen.

and on each side of the aerial. Such a system not only reduces the actual resistance of the return path for the aerial current, which in itself effects an appreciable saving, but the field set up by the earth screen is in opposition to that produced by the flat top, so that the earth losses are still further reduced.

Eckersley has shown (*Journ. I.E.E.*, vol. lx, p. 581) that the loss may be continuously reduced by increasing the number of screen wires down to a spacing $d = \frac{1}{4}$ W, where W is the width of the main aerial. At or around this point the field set up by the screen wires themselves begins to cause losses in the earth.

The screen should extend about half the width of the aerial to each side, and at the open end or ends of the aerial.

Dielectric Loss.—This is due to the presence in the field of the aerial of highly absorbing dielectrics, such as trees. The grass under the aerial is an offender in this respect to a considerable extent. A screen is effective here also, inasmuch as it screens the ground from the vertical field, which is responsible for this loss.

Other attempts have been made from time to time to reduce the earth loss. The Alexanderson multiple earth system, which has been tried out in America, employs an aerial several miles long having tuning points at intervals. At these points a loading coil is connected between the earth and the aerial, the whole system being tuned to the wave length employed. Leads are taken back to the transmitting building from each of the tuning points. This system reduces the earth resistance itself, but does not affect the other losses.

A system which appears to be a compromise between the two systems just described is the Meissner system, in which earth systems are laid out at a number of points under the aerial, if possible at points of maximum field strength, these several systems being led in by overhead wires to the transmitting building. The currents in the several earth leads are regulated by chokes or condensers. This system is said to give very good results.

Earth screens have a tendency to oscillate at their own natural frequency, and care must be taken to avoid trouble of this nature. This question is dealt with in Eckersley's paper.

Leakage Loss.—This loss is very rarely encountered, and is solely due to bad insulation, or to running the aerial at too high a voltage.

Model Aerials.—Model aerials may be made to determine the performance of a particular aerial and earth system. If λ is the wave length at which the full-size aerial is to be employed, and if the model is made $1/s$ full size, then Eckersley has shown that if the resistance is observed at a wave length λ/s, the eddy current loss on the full-size aerial will be $1/\sqrt{s}$ of that observed on the model, the dielectric loss $1/s$ times as great, while the leakage loss will be the same.

General Considerations.

The design of an aerial is dependent upon a large number of considerations, the cost usually being the deciding factor. The power radiated is proportional to h^2, I^2, and λ^{-2}. λ is decided by the range of the station, and since considerations of resistance require λ to be a small multiple of λ_0, the natural wave length of the aerial is determined within certain limits. The power radiated thus depends upon the product I^2h^2.

The aerial current is proportional to the capacity, which in turn is inversely proportional to the height. I^2h^2 is thus to a large extent independent of the height. C, however, does not decrease linearly as h increases, so that this statement is only approximately correct. Moreover, the greater the current the larger are the losses due to conductor and other losses, and since these are two or three times as great as the radiation resistance, this factor is important. Where an earth screen is employed, however, the effect of this latter is reduced, and the product I^2h^2 is appreciably more constant.

Finally, the power input is limited by the maximum voltage which the aerial will stand, $V = \sqrt{\frac{2W}{C}}$, and since the insulation of an aerial is an expensive business, this limit is comparatively quickly reached. Hence, to increase the power input, the capacity must be increased.

Generally speaking, therefore, it pays to obtain the requisite capacity at as great a height as possible, the limit being reached when a gain in aerial efficiency is more expensive than the extra power required to run on a less efficient aerial.

VALVE CHARACTERISTICS AND DESIGN.

The three electrode thermionic valve or triode comprises: (1) A suitable receptacle (usually a glass bulb), which is exhausted to a high degree of vacuum; (2) an incandescent filament of wire which emits electrons; (3) a metal anode surrounding the filament, raised to a positive potential, thereby attracting the electrons emitted from the filament; (4) a wire grid or mesh interposed between filament and anode. By virtue of its position, variations of grid potential have considerable effect on the electron stream flowing from filament to anode. Due to the relatively small area, however, it does not act to an appreciable extent as a collective electrode.

There are two principal characteristics of a triode. One is the variation of the anode current (emission) with anode voltage at a constant grid potential. The second is the variation of anode current with grid voltage at a constant anode potential.

The form of these two curves is shown in Fig. 103 (*a*) and (*b*). The former, which is taken for $v_g = 0$, is of the form $i_a = Av_a^{3/2}$ for the lower values of v_a.

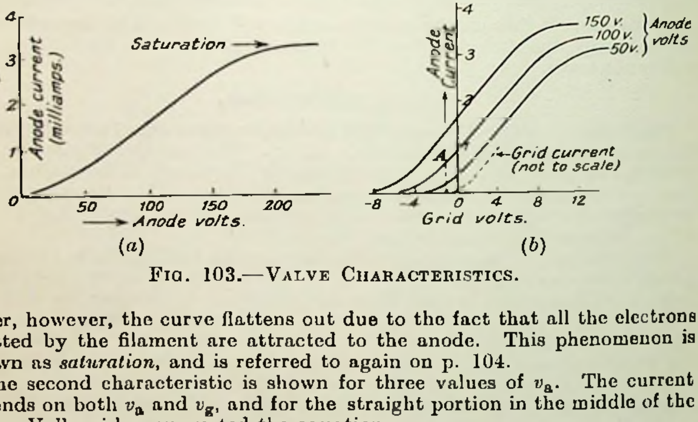


FIG. 103.—VALVE CHARACTERISTICS.

Later, however, the curve flattens out due to the fact that all the electrons emitted by the filament are attracted to the anode. This phenomenon is known as *saturation*, and is referred to again on p. 104.

The second characteristic is shown for three values of v_a. The current depends on both v_a and v_g, and for the straight portion in the middle of the curves Vallauri has suggested the equation:

$$i_a = av_g + bv_a + c,$$

where v_g = grid voltage,
v_a = anode voltage,
a, *b*, and *c* are constants.

Now *a* is obviously the slope of the anode current-grid voltage curve, while *b* is the slope of the anode current-anode voltage curve. Hence—

$$a = \frac{\partial i_a}{\partial v_g}\,{}_{v_a \text{ const}} \quad \text{and} \quad b = \frac{\partial i_a}{\partial v_a}\,{}_{v_g \text{ const}}$$

The internal anode-filament impedance $r_i = \dfrac{\partial v_a}{\partial i_a} = \dfrac{1}{b}$.

Amplification Factor.—It will be seen that a given change in v_g produces a greater change in i_a than would be produced by a similar change in v_a. This leads to the conception of the voltage amplification factor of the valve μ_o:

$$\mu_o = \frac{\text{change in } v_a}{\text{change in } v_g} \text{ to produce a given change in } i_a,$$

$$= \frac{\partial v_a}{\partial v_g}_{i_a \text{ const}} = \frac{\partial v_a}{\partial i_a} \cdot \frac{\partial i_a}{\partial v_g} = \frac{1}{b} \cdot a = \frac{a}{b}.$$

Hence the equation to the current may be written—

$$i_a = \frac{1}{r_i}(v_a + \mu_o v_g) + c.$$

Any of the foregoing parameters of the valve may readily be found from the characteristics. a is usually tolerably constant, but b depends to a large extent on the grid voltage at which the characteristic is taken.

It is customary to specify, therefore, the average value of $r_i = \frac{1}{b}$ when $v_g = 0$, and the value of μ_o deduced from this value of b.

The actual amplification of a valve is limited by the impedance in the anode circuit. The effect of this and the several calculations on valve circuits involving these factors μ_o and r_i are referred to in the chapters dealing with the particular applications of the valve to practical conditions.

In order to assist in the design of circuits including valves a series of characteristics of the chief valves in use at the present day is given at the end of this chapter, together with the values of r_i and μ_o, and other operating data.

Grid Current.—When the grid of a valve becomes positive, a certain current will be attracted to the grid itself.

This current is small (of the order $\frac{1}{20}$ to $\frac{1}{50}$ of the anode current) owing to the small area of the grid, but it is nevertheless to be avoided, since it introduces undesirable effects into the circuits connected across the grid, and reduces the effectiveness of the valve. These points will be referred to later. The point at which the grid current commences to flow occurs near zero grid potential, but is often slightly negative.

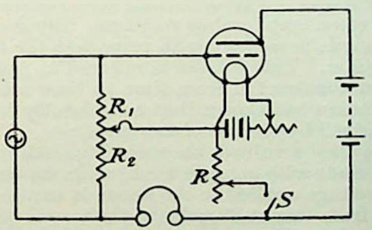


Fig. 104.—Circuit for Finding μ_o and r_i.

For any particular valve the quantities μ_o and r_i may be determined experimentally by the use of the circuit shown in Fig. 104. In the first case the switch S is open, and R_1 and R_2 are adjusted till no sound is heard in the telephones. Then—

$$\mu_o = \frac{R_2}{R_1}.$$

In the second case R_1 and R_2 remain fixed at any convenient values, and the switch S is closed. If R is then varied till no sound is heard in the telephones—

$$r_1 = R\left(\frac{R_1}{R_2}\mu_o - 1\right).$$

R, R_1, and R_2 should be of the order of 1,000 to 5,000 ohms.

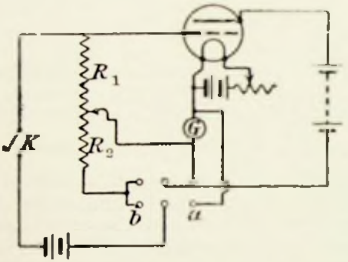


FIG. 105.—APPLETON SLOPEMETER.

Another circuit of use in this connection is the Appleton slopemeter. This is shown in Fig. 105. If the change-over switch is in the position (*a*) and R_1 is adjusted till there is no change of current when the key K is closed, then—

$$\frac{\partial i_a}{\partial v_g} = a = \frac{1}{R_1}.$$

If the switch is in position (*b*) and the ratio R_1/R_2 is varied till there is no deflection on closing K, then—

$$\mu_o = \frac{R_2}{R_1}.$$

Special Forms of Valve.—Valves having two electrodes only (the grid being omitted) are often employed as rectifiers. Since a valve is only conducting when the anode is positive with respect to the filament, it forms a highly efficient rectifier. This subject is referred to again on p. 153.

Valves have been constructed from time to time with more than three electrodes. Barkhausen has shown that considerably higher amplification factors may be obtained by the use of two grids.

A circuit utilising such a valve is shown in Fig. 106.

The inner grid is made with an open mesh, while the outer grid has a close mesh. The total voltage applied to the anode is small—say 30 volts—and a tapping is taken from this at 20 volts to the second grid.

The combination of the filament, g_1 and g_2 act as a simple valve having a low μ_o = say $3\frac{1}{3}$. The current flowing to the anode is, however, controlled by g_2, which, being of close mesh, gives an amplification of, say, 30. The total amplification is thus $3\frac{1}{3} \times 30 = 100$, which could only be obtained with a single grid valve by having a very close grid, necessitating an anode voltage of the order of 600, and also a considerable negative potential to avoid excessive grid current.

Similarly, a valve of this type may operate on 1 or 2 volts high tension, giving results equivalent to 20 to 40 volts H.T. with a simple three-electrode valve.

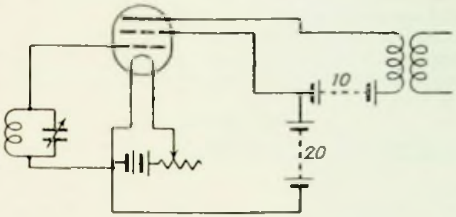


FIG. 106.—DOUBLE GRID VALVE.

The current in such a valve is given by—

$$i = f\left[v_{g1} + \frac{v_{g2}}{\mu_1} + \frac{v_a}{\mu_1\mu_2}\right].$$

Another device, due to Scott-Taggart, employs two anodes, and is known as a negatron. One anode is controlled by a grid which is connected to the main anode, the circuit being as shown in Fig. 107.

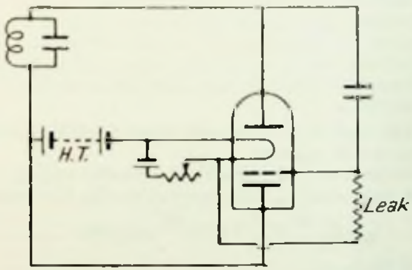


FIG. 107.—NEGATRON VALVE.

If the potential of the main anode increases, so also does that of the grid, which causes a current to flow to the second or diversion anode. The current to the main anode therefore falls instead of increasing, which gives a negative resistance effect which may be utilised in any convenient manner.

Design of Valves.

The design of a valve to comply with a given specification is a somewhat complicated matter, and is dependent to a large extent upon empirical formulæ. The following remarks, however, will serve to indicate the methods employed to determine the principal dimensions.

Emission.—The most important property of a valve is the emission of electrons from the filament.

Richardson has shown that the actual emission from a hot filament is given by—

$$i = A\sqrt{T}\,\varepsilon^{-b/T} \qquad (1)$$

where A and b are constants,
T = temperature in degrees absolute,
i = emission in amperes per square centimetre of cathode surface.
Values of A and b for various substances are appended.

TABLE XXI.

VALUES OF A AND b IN RICHARDSON'S EQUATION.

Material.	A.	b.
Carbon (untreated)*	10^{34}(?)	$4{\cdot}8 \times 10^4$
Platinum*	$7{\cdot}5 \times 10^{25}$(?)	$4{\cdot}93 \times 10^4$
Thorium	20×10^7	$3{\cdot}9 \times 10^4$
Tungsten†	$2{\cdot}36 \times 10^7$	$5{\cdot}26 \times 10^4$
Molybdenum	$2{\cdot}1 \times 10^7$	$5{\cdot}0 \times 10^4$
Tantalum	$1{\cdot}12 \times 10^7$	$5{\cdot}0 \times 10^4$
Oxide (coated platinum)	$8 - 24 \times 10^4$	$1{\cdot}94 - 2{\cdot}38 \times 10^4$

Space Charge.—Richardson's equation gives the total emissivity or capability for emitting electrons at any given temperature. In an actual filament, however, the electrons which are emitted cluster round the filament and form a negatively charged "cloud," known as the "space charge," which repels all further electrons back into the filament.

The insertion of an anode raised to a suitable positive potential causes some of these electrons to be attracted across the gap, thus reducing the space charge and permitting more electrons to be emitted, the result being a steady flow of current.

Saturation.—The value of this current depends on the configuration of the electrodes and the anode voltage.

With a wire filament and a cylindrical co-axial anode of radius r centimetres raised to a potential V_a with respect to the filament—

$$i_a = \frac{14{\cdot}69}{r} V_a^{3/2} \times 10^{-6} \text{ amperes} \qquad (2)$$

per unit length of filament.

For a plane anode at a distance d centimetres from a filament system also substantially plane, the equation becomes—

$$i_a = \frac{2{\cdot}33}{d^2} V_a^{3/2} \times 10^{-6} \text{ amperes} \qquad 2\,(a)$$

per unit length of filament.

With a given configuration, therefore, the anode current will increase as $V_a^{3/2}$, until a point is reached where all the electrons emitted by the filament are attracted to the anode. Further increase can then only be obtained by increasing the temperature of the filament. This total emission or saturation current may be determined from Richardson's equation. In practice

* J. J. Thomson. † Langmuir.

the total emission of a valve is the saturation anode current obtained with grid and anode connected together (see p. 108).

Design of Filament.—The design of a filament to produce a given emission is complicated by the fact that valve filaments are so short that appreciable end-cooling occurs due to the supports, and the temperature is not uniform over the whole filament. The effect of this and the proper allowance to be made is discussed fully in a paper by Stead (*Journ. I.E.E.*, vol. lviii., p. 107).

An approximate method is given in the paper, which applies to tungsten filaments of diameters from 0·0035 to 0·012 centimetre, and operating at temperatures of 2,000° K. to 2,500° K., which is the normal range of bright emitting filaments. The procedure is:

1. Assume a value of T_m, the temperature of the hottest portion of the filament.

2. Find voltage correction factor—

$$\delta E_f = A - B,$$

where

$$A = \frac{T_m - 350}{2500};$$

$$B = 0{\cdot}88 - 100 d_f,$$

where d_f = filament diameter in centimetres.

3. Add δE_f to the voltage E at which the filament is to be run—

$$E_f = E + \delta E_f.$$

4. Find $l_f = E_f\sqrt{d_f}/K$, where K is a constant, values of which are given in Fig. 108.

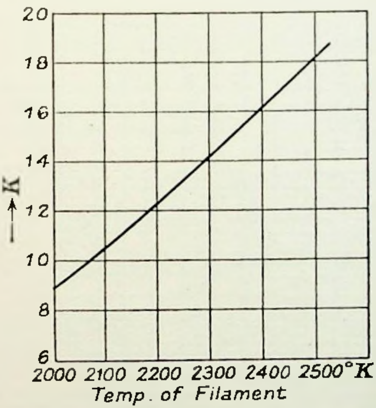


FIG. 108.—VALUE OF K IN EQUATION FOR l_f.

This determines the filament length. To obtain the emission it is necessary to obtain the length l_f' of an equivalent filament having a uniform temperature $= T_m$.

5. Find $l_f' = l_f - \delta l$, where $\delta l = K_1\sqrt{d_f} - 0{\cdot}30$. Values of K_1 are given in Fig. 109.

6. The emission may now be found from $i = K_2 l_f' d_f$ amperes, where K_2 is a constant given in Fig. 110.

The design of the filament is thus a matter of trial and error. Values of T_m and d_f are assumed and the emission obtained. If this is not what is required, suitable modifications of the assumptions are made and the calculation repeated.

The temperature is determined to a large extent by the life. No very definite figures can be given in this connection. A tungsten filament at 2,300° K. would have a probable life of 1,000 hours or more, while at a temperature of 2,450° K the life would be of the order of 300 hours only.

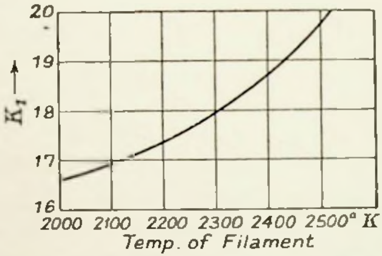


FIG. 109.—VALUE OF K_1 IN END CORRECTION FORMULA.

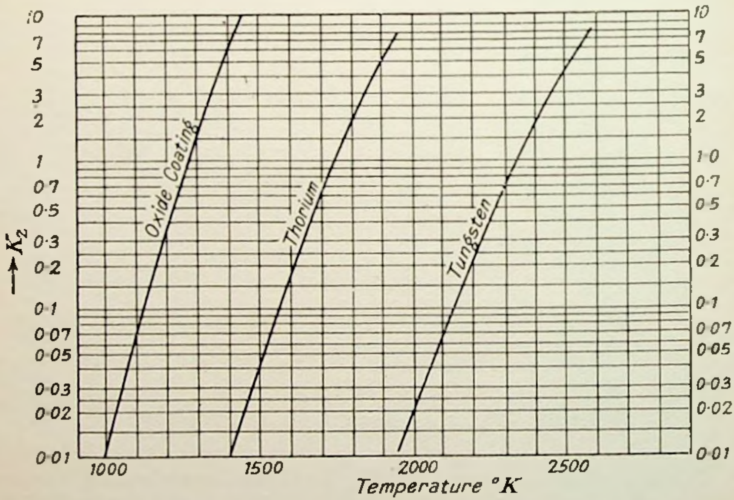


FIG. 110.—VALUES OF K_2 IN EMISSION FORMULA.

Filament Current.—The filament current depends upon the 3/2 power of the diameter, and also upon the temperature.

Having settled the diameter and temperature to produce a given emission, the current may be determined from the equation—

$$I_f = K_3 \, d_f^{\,3/2},$$

where K_3 is the constant given in Fig. 111.

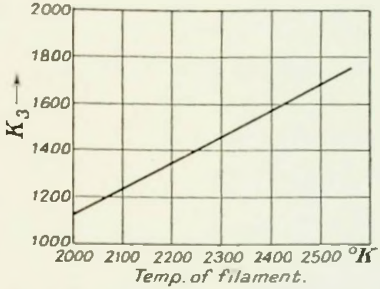


FIG. 111.—VALUE OF K_3 IN FILAMENT CURRENT FORMULA.

The curves as obtained by Stead apply only to tungsten filaments.

For purposes of comparison, however, values of K_3 have been plotted in Fig. 110 for thoriated and oxide-coated filaments. The construction of these is described later. A dull emitter filament designed on the lines given will be approximately correct only, since the end-cooling is not necessarily the same.

If the anode circuit is connected to the positive leg of the filament, the emission current itself will heat the filament to a small extent, and this may increase the emission by as much as 25 to 35 per cent.

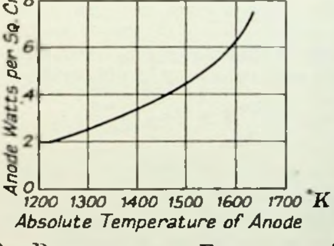


FIG. 112.—DISSIPATION OF ENERGY BY ANODE.

Design of Anode.—The design of the anode resolves itself into two portions:

1. The satisfying of Langmuir's space charge equation in order that the necessary emission may be obtained at the working voltage. This limits the radius of the anode.

2. The provision of adequate surface to dissipate the heat caused by the bombardment of the anode by the electrons. The watts dissipated per square centimetre of a nickel anode in terms of the temperature are given in Fig. 112, which is taken from a paper by Stead (*Journ. I.E.E.*, vol. lix., p. 427). The watts dissipation is the product of $v_a i_a$, and the dissipation per square centimetre is calculated on the external surface of the anode only. The melting-point of nickel is 1,720° K., a red heat being obtained at about 1,300° K.

Design of Three Electrode Valve.—The provision of a grid between the anode and filament modifies the strength of the field overcoming the space charge.

The total emission may then be determined by considering the grid and anode as a cylinder of the same radius as the grid at a potential $\frac{v_a+\mu_o v_g}{1+\mu_o}$.

Hence Langmuir's equation becomes—

$$i_e = 14{\cdot}69 \frac{l_f'}{r_g}\left(\frac{v_a+\mu_o v_g}{1+\mu_o}\right)^{3/2} \times 10^{-6} \text{ amperes} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

where l_f' = equivalent length of uniformly hot filament,
r_g = radius of grid.

Amplification Factor.—Gossling has discussed the subject of the design of valves in a paper before the I.E.E. (*Journ. I.E.E.*, vol. lviii., p. 670), wherein the amplification factor is given as—

$$\mu_o = \frac{2\pi n\, r_g \log \frac{r_a}{r_g}}{\log \frac{1}{\pi n d}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where n = number of grid wires per centimetre,
r_a = radius of anode,
r_g = radius of grid,
d = diameter of grid wires. } cms.

This formula applies to spiral grids, and also to the longitudinal supports of such grids. By a combination of the two a mesh grid may be considered.

It should also be noted that putting $v_g = 0$ in equation (4) and rewriting, the value of μ_o at zero grid volts may be obtained from—

$$(\mu_o + 1)^{3/2} = \frac{14{\cdot}69\, l_f'}{r_g\, i_o} \mathrm{E}^{3/2} \times 10^{-6} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

where l_f' = equivalent length of filament,
r_g = radius of grid,
E = anode voltage,
i_o = anode current at $v_g = 0$.

Internal Impedance.—As long as the grid current is not appreciable, the anode current is proportional to $V^{3/2}$. Under these conditions—

$$r_i = \frac{2}{3}\,\frac{v_a+\mu_o v_g}{i_a}.$$

Design of Receiving Valves.—The above formula and data apply to valves where the anode voltage is ten or more times that used for the filament. Where this is not so, as is often the case in receiving valves, the design is somewhat modified by the non-uniformity of the field occasioned by the appreciable difference of potential between the anode and the positive and negative ends of the filament. This causes a reduction in the slope of the curves, the effect being shown in Fig. 113, which will indicate the extent of the

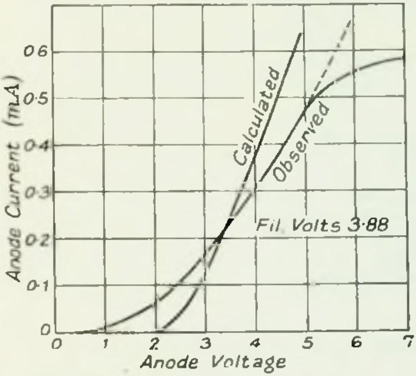


FIG. 113.—ILLUSTRATING VARIATION IN CHARACTERISTIC WITH SMALL ANODE VOLTAGES.

variation. For further details of the corrections necessary, the reader is referred to the paper by Gossling previously mentioned.

General.—The following figures may be useful:

Charge on an electron:

$$e = 4{\cdot}774 \times 10^{-10}\text{E.S.U.},$$
$$= 1{\cdot}591 \times 10^{-19}\text{Coulomb.}$$

Mass of electron (for infinitely small velocities):

$$m_0 = 8{\cdot}995 \times 10^{-28} \text{ gm.}$$

Ratio of charge to mass of electron:

$$e/m_0 = 1{\cdot}77 \times 10^7 \text{ E.M.U./gm.}$$

The *electron evaporation constant* or the *electron affinity* is a measure of the amount of work which an electron must do to escape from the surface of a body. If—

w = work done,
e = charge on electron.

Then the electron affinity $\varphi = \frac{w}{e}$.

TABLE XXII.

Values of Electron Affinities of Various Substances.

Substance.	*φ in Terms of Equivalent Volts.*	*Substance.*	*φ in Terms of Equivalent Volts.*
W	4·52	Bi	3·7
Pt	4·4	Zn	3·4
Hg	4·4	Th	3·4
Mo	4·3	Ca	3·4
C	4·1	Al	3·0
Ag	4·1	Mg	2·7
Cu	4·0	Li	2·35
Sn	3·8	Na	1·82
Fe	3·7	Oxide-coated Pt	1·55 to 1·9

φ can also be expressed as—

$$\varphi = 8{\cdot}6b \times 10^5 \text{ volts,}$$

where b is the constant in Richardson's equation.

The total emission from a filament at a given temperature may be obtained if the value of φ is known. The table below gives the emission in terms of T and φ, and this table, in conjunction with the preceding one, will serve to indicate the thermionic properties of any particular substance.

TABLE XXIII.

Values of Emission in Terms of φ and T.

Temperature (Degrees K.).	*Total Emission in Amperes/Square Centimetres of Cathode Surface*			
	$\varphi = 2$ *Volts.*	3	4	5
1,000	25×10^{-3}	20×10^{-8}	20×10^{-13}	20×10^{-18}
1,500	72	30×10^{-3}	13×10^{-6}	60×10^{-10}
2,000	40×10^2	12	36×10^{-2}	11×10^{-5}
2,500	46×10^3	43	42×10^{-1}	40×10^{-3}

Valve Manufacture.

The manufacture of a valve is similar in general principles to that of an ordinary electric lamp. The various components are made up and fixed in position on a glass stem. The whole is then mounted in a glass bulb and the air inside is pumped out.

The exhaustion in a valve, however, is carried to a very high degree. Any gas molecules left inside the bulb may be ionised by collision with the rapidly moving electrons, and the positively charged nucleus drifts slowly towards the cathode. This causes a variation of the anode current, which gives rise to a variable characteristic. This is unsatisfactory for commercial working where a constant characteristic is very desirable. The relatively heavy positive ions also will bombard the filament, and may in extreme cases cause disintegration thereof.

Not only is it necessary to remove all free gas, but also all "occluded" gas has to be removed as far as possible. The walls of the bulb and the electrodes themselves absorb small quantities of gas, and as the valve warms up in use these gases escape and cause "softening" of the valve. It is

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customary, therefore, to apply a high potential to the anode and grid, and run the filament at a temperature somewhat above normal. This causes a heavy electron bombardment of the electrodes, which thus are raised to a red heat and deliver up the occluded gases. The bulb also is heated in an oven at the same time.

Dull Emitter Valves.—Recent developments have been in the direction of reducing the filament power. Special filaments are employed which give the necessary emission at temperatures of 1,200° to 2,000° K. only. It is more than ever desirable with this class of valve that the vacuum shall be very high, because the presence of gas in the valve causes chemical action to take place in the filament, which destroys the dull emitting properties. The successful production of such valves is largely due to the introduction in the bulb of a "getter," which cleans up the gas, not only during exhaustion, but afterwards, when the valve is in operation. One such getter very largely employed is metallic magnesium. A small piece of this metal is fused on to the anode during construction, and the valve during exhaustion need only be heated to a sufficient temperature to volatilise the magnesium, which then combines with any free gas in the bulb. The most dangerous gases are water vapour, nitrogen, and oxygen, with all of which the magnesium readily combines.

There are two chief forms of dull emitter filaments. One form consists of a thin strip of platinum coated with oxides of calcium, barium, strontium, etc. This type of filament runs at a dull red heat (1,000° to 1,400° K.), and is used in the Wecovalve and similar types.

This type of filament is described in a paper by Arnold (*Physical Review*, 1920, vol. xvi., p. 76).

The standard filament is made up by preparing solutions of barium carbonate and strontium carbonate or hydroxide in a suitable carrier, such as resin or paraffin. A thin platinum strip is then coated with four coats of each mixture alternately; the process is repeated twice, making sixteen coats in all. The filament is then heated to 1,200° to burn off all the carrier, and the coating so formed is found to be extremely durable. Arnold gives the following figures:

$$A = 8\text{–}24 \times 10^4$$
$$b = 1{\cdot}94\text{–}2{\cdot}38 \times 10^4$$
$$\varphi = 1{\cdot}55\text{–}1{\cdot}9.$$

A comparison of the emission of this type of filament with a pure tungsten filament may be obtained by referring to Fig. 110.

Arnold also gives the following details of other oxide coatings:

	φ.
BaO 50 per cent., SrO 50 per cent.	1·97–2·28
BaO 50 per cent., SrO 25 per cent., CaO 25 per cent.	2·39–2·54
CaO	3·22–3·51

The pure CaO coating is unstable, however, and peels off in use.

The second type of filament (as used in the DER and DE3 types) is the thoriated type. In the manufacture of tungsten filaments it has long been the practice to mix a small percentage of thorium oxide with the tungsten, as this gives a more robust filament. It is found that by heating the filament to 2,900° K. for one to two minutes, followed by a period of a few minutes at 2,250° K., a filament is obtained which will give as large an emission at 1,380° K. as a pure tungsten filament at 2,000° K.

The heating at 2,900° is supposed to drive off all gaseous and solid impurities, after which the heating at 2,250° causes the thorium to diffuse to the surface of the filament, constituting a very thin, almost atomic layer of pure thorium.

This will give a satisfactory emission at a much reduced temperature, and it is found that the wastage of the thorium layer due to emission is made up by further diffusion from the inside of the filament. The property is lost if the filament is run at too high a temperature (above 2,250° K.), and the thorium layer is destroyed at once by the presence of any appreciable quantity of free gas, and, consequently, the pressure is not allowed to exceed 0·00001 millimetre.

The bulb size is also important. As the anode voltage is increased, stray electrons reach the bulb and ionise the occluded gases therein. Hence, for

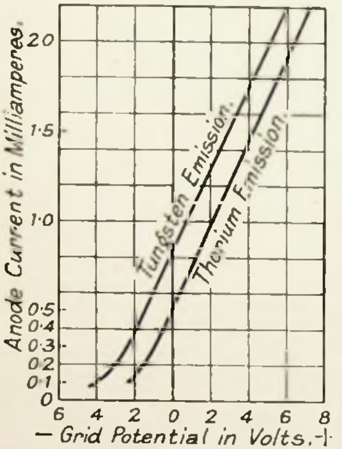

Fig. 114.—Effect on Characteristics of Use of Thoriated Filament.

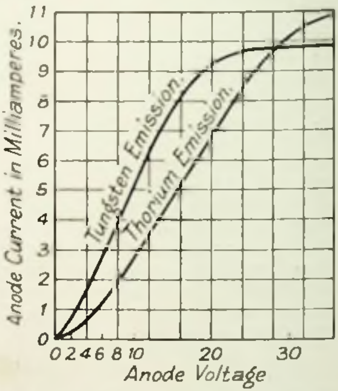

Fig. 115.—Showing Decreased Slope obtained with Thorium Emission.

a given bulb size there is a limiting anode voltage, which increases with the size of bulb, and is usually specified on the valve.

A very complete description of this type of valve will be found in a paper by Thompson and Bartlett (*Journ. I.E.E.*, vol. lxii., p. 689), from which the foregoing data have been extracted.

It is found with this type of filament that, owing, it is thought, to some contact potential effect, the anode current at a particular grid voltage is less than that obtained with a tungsten filament.

The slope of the anode-current anode-voltage curve is also somewhat less, and the saturation is more gradual. These effects are fully described in the paper referred to, and are illustrated in Figs. 114 and 115.

Valve Characteristics.

There are on the market a large number of valves of various makes and designations. Some indication is given in the following pages of the characteristics of the most common types. In many cases several valves made by different firms, but intended to be put to the same use, have substantially similar characteristics, and in such case a representative curve has been drawn. It will be appreciated that in any case the values given are only average figures, but they serve to indicate the suitability of a valve for a particular purpose. The author is indebted to the various firms mentioned below for their courtesy in providing the very complete details given.

TABLE XXIV.

General Purpose Valves.

Type.	*Maker.*	*Fil. Volts.*	*Fil. Amperes.*	*H.T. Volts.*	*Internal Impedance (Ohms).*	μ_o.	*Representative Characteristic.*
Ora	Mullard	3·5–4	0·65–0·75	30–90	30,000	8·5	Ora
AR	Ediswan	—	—	20–50	36,000	6·0	Ora
R	B.T.H.	—	—	30–80	27,000	9·0	Ora
P1	Cossor	—	—	20–80	20,000	6·6	Ora
R	Ediswan	—	—	50–100	25,000	7·5	B5
R	M.O.V. Co.	—	—	70–100	40,000	9·0	DER
RA	Mullard	—	—	50–100	30,000	7·0	DER
P2 (red top)	Cossor	—	—	60–80	40,000	11·0	DER
DER	M.O.V. Co.	1·8	0·3–0·4	30–70	45,000	9·0	DER
LF Ora	Mullard	—	—	20–100	30,000	5·0	ORA
ARDE	Ediswan	—	—	20–50	35,000	9·0	DER
B3	B.T.H.	—	—	20–80	27,000	7·5	ORA
B5	B.T.H.	2·5–3	0·06	30–80	17,000	6·0	B5
DE3	M.O.V. Co.	—	—	20–80	20,000	5·0	B5
DF Ora	Mullard	—	—	20–100	20,000	5·5	B5
AR06	Ediswan	—	—	20–50	37,000	10·5	DER
Weco-valve	Mullard	0·8–1·1	0·25	20–50	18,000	4·7	Weco-valve
Weco-valve	W.E. Co.	—	—	20–50	25,000	6·0	Weco-valve
P3 (green top)	Cossor	—	0·22	20–60	20,000	6·6	Weco-valve
P4 (blue top)	Cossor	—	—	20–60	40,000	11·0	DER
R5V	M.O.V. Co.	5	0 7	30–100	40,000	8·0	DER

Note concerning Characteristic Curves.—The curves given are all static characteristics obtained with zero external impedance in the anode circuit. The dynamic or working characteristic is different, as is explained on p. 162.

The figures in brackets under the several characteristics are the values of grid voltage at which grid current commences to flow. The three chief parameters are:

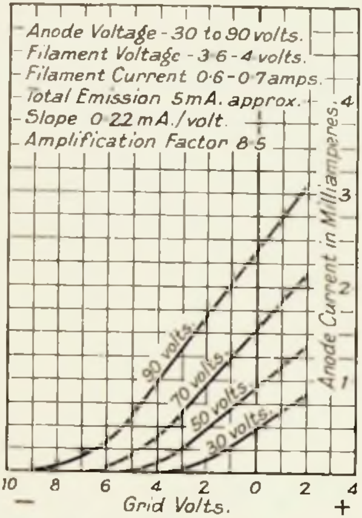

Fig. 116.—Ora (Mullard). (−1.)

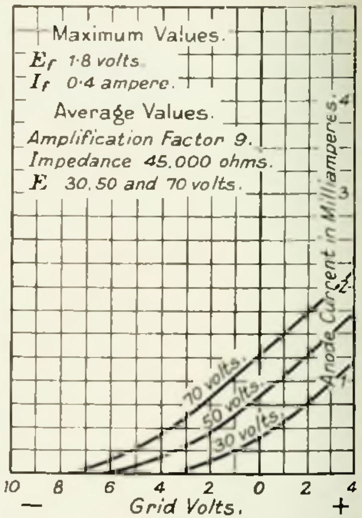

Fig. 117.—DER (M.O.V. Co.). (+0·7.)

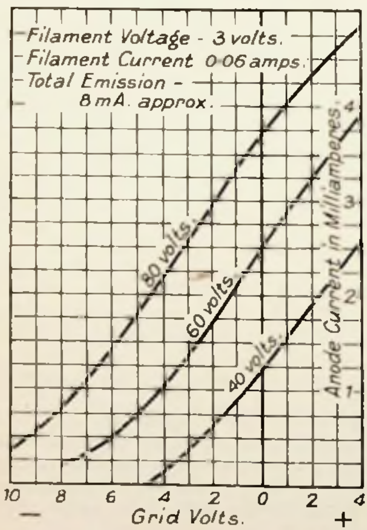

Fig. 118.—B5 (B.T.H.). (−0·3.)

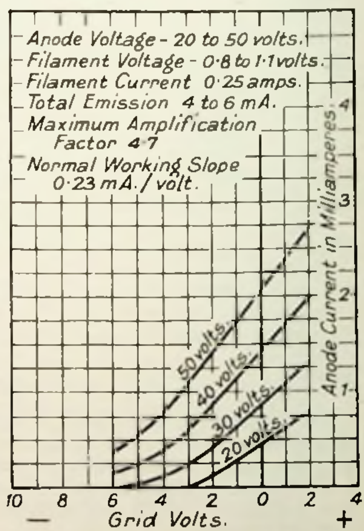

Fig. 119.—Wecovalve (W.E. Co.).

1. $a = \frac{\delta i_a}{\delta v_g}$ = slope of the anode current-grid voltage characteristic at the particular grid and anode voltages employed.

2. $r_i = \frac{1}{b}$. This is really the slope of the anode-current anode-voltage characteristic. Its value may be estimated by taking currents (from the curves given) at two values of anode voltage, one on each side of the actual voltage being used, both readings being taken for the value of v_g at which the valve is to be operated. Then—

$$r_i = \frac{v_a - v_a'}{i_a - i_a'}.$$

This parameter varies considerably with grid voltage, and also with anode voltage at low values. Hence it is important to obtain the value under appropriate working conditions. The value of v_a should be estimated with due allowance for the voltage drop in the external anode circuit. The values given in the accompanying tables are all taken at $v_g = 0$.

3. $\mu_o = \frac{a}{b}$ = voltage amplification factor. The actual amplification factor may be obtained from this as described on p. 162.

TABLE XXV.

Low Impedance Valves.

Type.	*Maker.*	*Fil. Volts.*	*Fil. Amperes.*	*H.T. Volts.*	*Internal Impedance (Ohms).*	μ_o.	*Representative Characteristic.*
V24	M.O.V. Co.	5·0	·75	20–50	22,000	6·0	V24
S3	Mullard	3·4–3·8	·65	15–50	10,000	4·0	V24
DEV	M.O.V. Co.	3·0	·2	20–40	21,000	5·5	V24
DE6	M.O.V. Co.	1·8	·4	30–110	12,000	5·0	DE6
D·3LF	Mullard	2·0	·3	30–100	15,000	7·0	DE6
D·06LF	Mullard	3·0	·06	30–100	15,000	7·0	DE6

TABLE XXVI.

High Impedance Valves.

(The first four valves are particularly suitable as rectifiers for *anode* rectification. The remainder are suitable for H.F. amplication.)

Type.	*Maker.*	*Fil. Volts.*	*Fil. Amperes.*	*H.T. Volts.*	*Internal Impedance (Ohms).*	μ_o.	*Representative Characteristic.*
Q	M.O.V. Co.	5·0	·45	50–150	150,000	45	Q
DEQ	M.O.V. Co.	3·0	·2	30–50	100,000	25	DEQ
QX	M.O.V. Co.	5·0	·71	30–100	71,500	18	QX
S5	Mullard	3·6	·65	20–50	170,000	50	Q
R4	M.O.V. Co.	3·8	1·2	40–80	50,000	12	R4
R4B	M.O.V. Co.	3·7	·65	40–80	75,000	14	QX
D·3HF	Mullard	2·0	·3	40–100	60,000	17	QX
D·06HF	Mullard	3·0	·06	40–100	60,000	17	QX

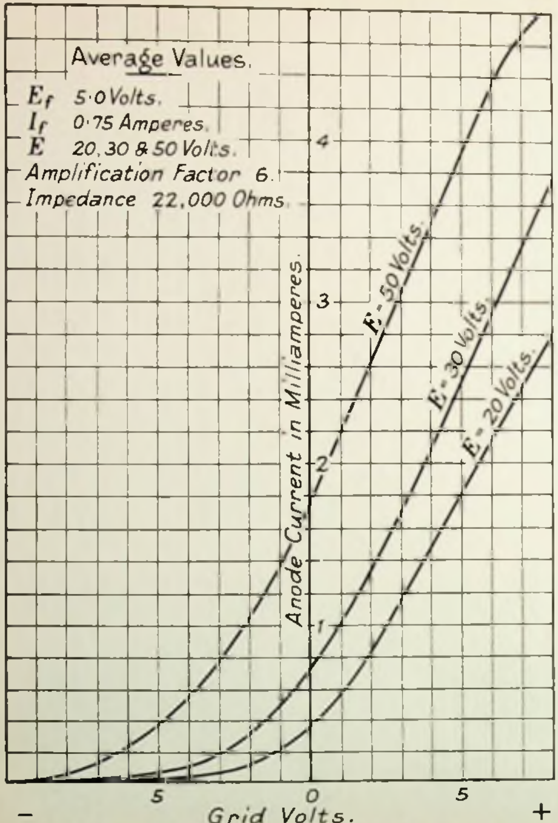

FIG. 120.—V24 (M.O.V. Co.). (−0·5.)

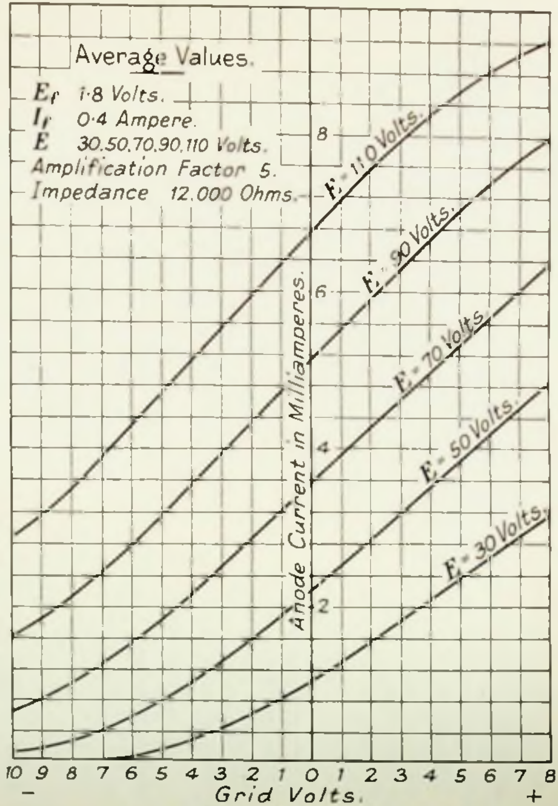

FIG. 121.—DE6 (M.O.V. Co.). (+0·7.)

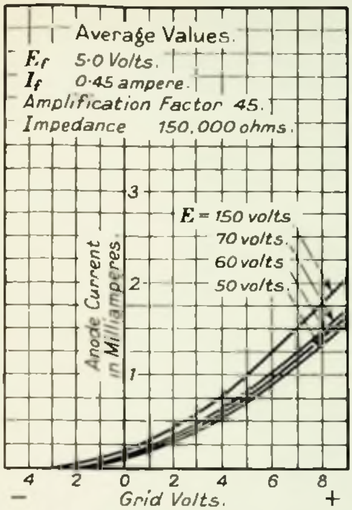

FIG. 122.—Q (M.O.V. Co.). (−1·5.)

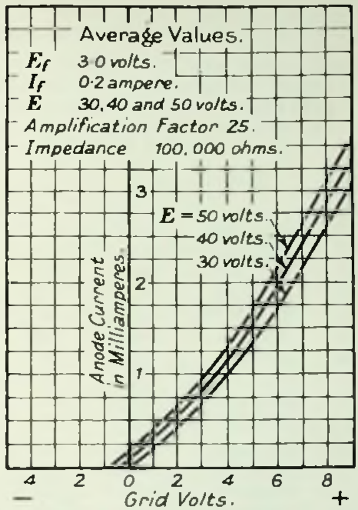

FIG. 123.—DEQ (M.O.V. Co.). (−0·5.)

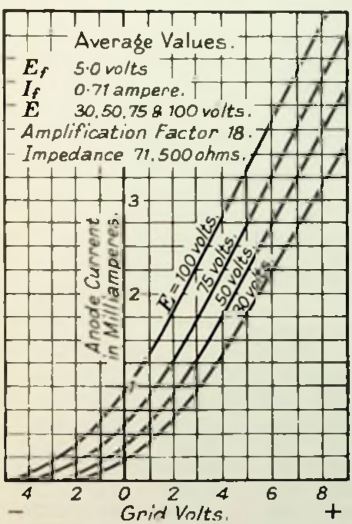

FIG. 124.—QX (M.O.V. Co.). (−1·0.)

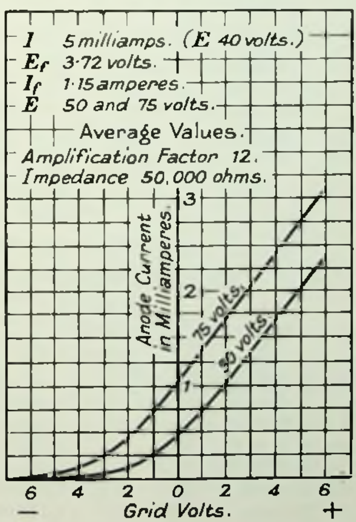

FIG. 125.—R4 (M.O.V. Co.).

TABLE XXVII.

Power Valves.

(These valves are intended for low frequency amplification, using high anode voltages with a considerable negative grid voltage to avoid grid current and consequent distortion. The last two valves are particularly suitable for resistance capacity amplification.)

Type.	*Maker.*	*Fil. Volts.*	*Fil. Amperes.*	*H.T. Volts.*	*Internal Impedance (Ohms).*	μ_o.	*Representative Characteristic.*
PA1	Mullard	6·0	1·5	200–400	8,500	7·5	PA1
PV2	Ediswan	6·0	1·5	200–400	11,000	4·5	PA1
LS2	M.O.V. Co.	6·0	1·5	200–400	8,000	5·0	PA1
PA2	Mullard	5·5	·85	100–200	7,000	4·0	PA2
LS5	M.O.V. Co.	4·5	·8	100–200	6,000	5·2	PA2
PA3	Mullard	4·0	·75	70–150	10,000	4·0	PA3
PV3	Ediswan	4·0	·7	70–110	25,000	5·0	PA3
LS3	M.O.V. Co.	4·0	·7	100–200	10,000	4·5	PA3
PV1	Ediswan	6·0	1·5	300–600	20,000	11·0	LS1
LS1	M.O.V. Co.	6·0	1·5	200–400	15,000	11·0	LS1
P5DE	Ediswan	5·0	·25	50–150	9,000	5·0	PA2
DE4	M.O.V. Co.	3·6	·3	50–150	10,000	7·0	PA2
DFA0	Mullard	3·5	·35	50–100	5,500	4·5	DFA0
DFA1	Mullard	5·5	·20	50–100	5,500	4·5	DFA0
DFA2	Mullard	3·5	·25	50–100	7,500	4·5	PA2
DE5	M.O.V. Co.	5·0	·22	40–150	7,000	7·0	PA2
B4	B.T.H.	6·0	·2	40–100	7,500	4·5	PA2
Note.—The DE6 (Fig. 121) is also suitable for this class of work.							
DFA3	Mullard	5·5	·06	50–150	13,000	7·5	PA3
B6	B.T.H.	3·0	·12	50–150	9,000	6·3	PA3
B7	B.T.H.	6·0	·06	50–150	9,000	6·3	PA3
DFA4	Mullard	5·5	·2	100–300	27,000	20	DE5B
DE5B	M.O.V. Co.	5·5	·25	100–300	30,000	20	DE5B

TABLE XXVIII.

Transmitting Valves.

(A few representative curves only are given.)

Type.	*Maker.*	*Fil. Volts.*	*Fil. Amperes.*	*Anode Volts.*	*Dissipation Watts.*	*Internal Impedance (Ohms).*	μ_o.	*Representative Characteristic.*
T15	M.O.V. Co.	6·0	1·0	400–800	15	50,000	25	T50
T30	M.O.V. Co.	7·0	1·75	1,000–1,500	30	70,000	45	T30
T50	M.O.V. Co.	7·0	2·5	1,000–2,000	50	35,000	30	T50
T100	M.O.V. Co.	10·0	3·5	1,000–2,000	100	50,000	55	T100
T250	M.O.V. Co.	12·5	5·25	1,000–2,000	250	25,000	25	T250
O150	Mullard	10·0	3·3	1,500–2,500	175	30,000	35	T250
O250	Mullard	12·0	5·2	1,500–3,500	300	8,500	14	O250
O500	Mullard	19·0	5·4	2,500–5,000	600	12,000	30	O500

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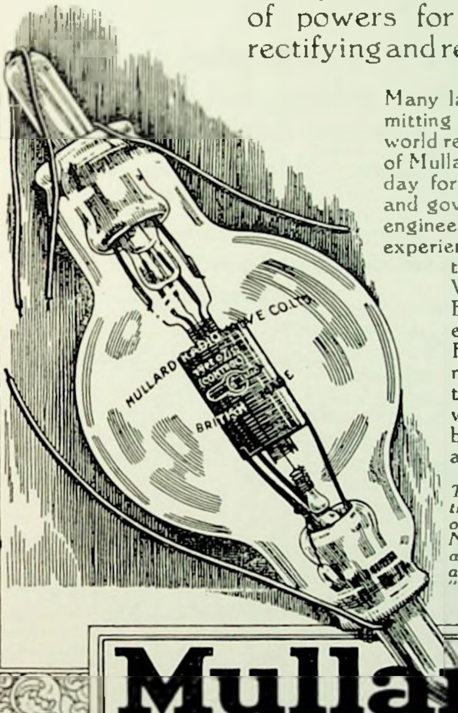

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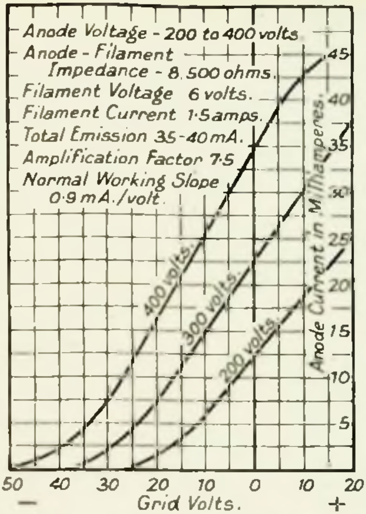


Fig. 126.—PA1 (Mullard). (0.)

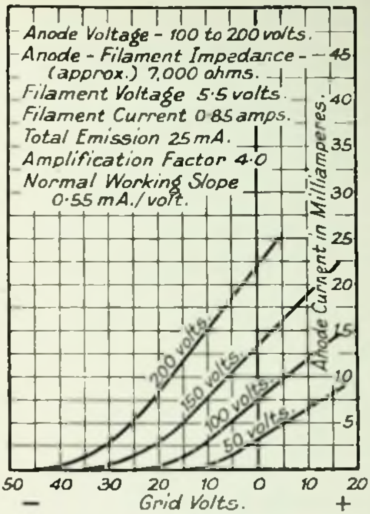


Fig. 127.—PA2 (Mullard). (0.)

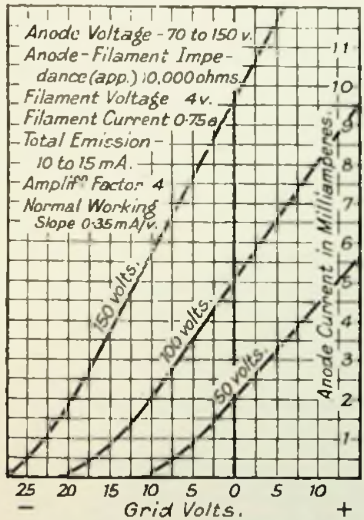


Fig. 128.—PA3 (Mullard). (–1·5.)

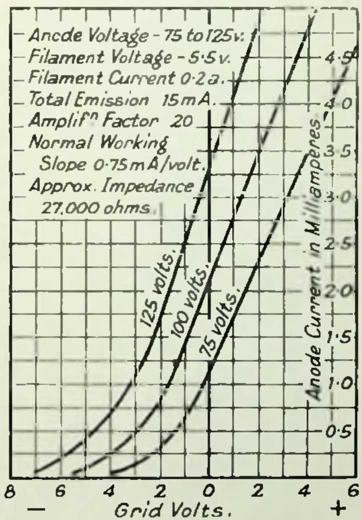


Fig. 129.—DE5B (M.O.V. Co.). (0.)

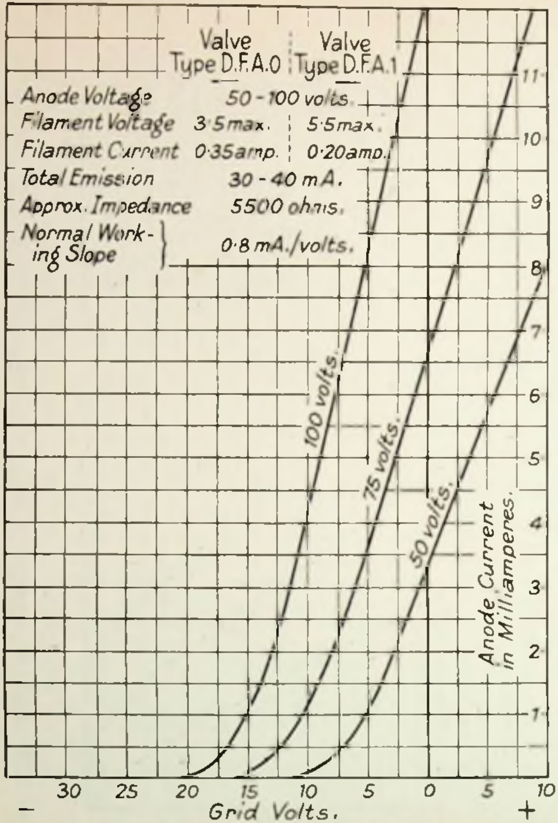


Fig. 130.—DFA0 (Mullard). (0.)

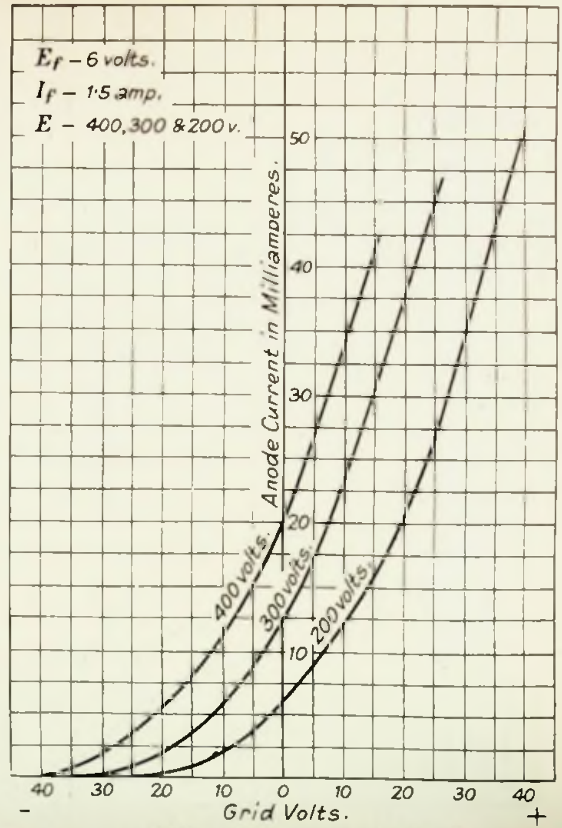


Fig. 131.—LS1 (M.O.V. Co.). (– 0.)

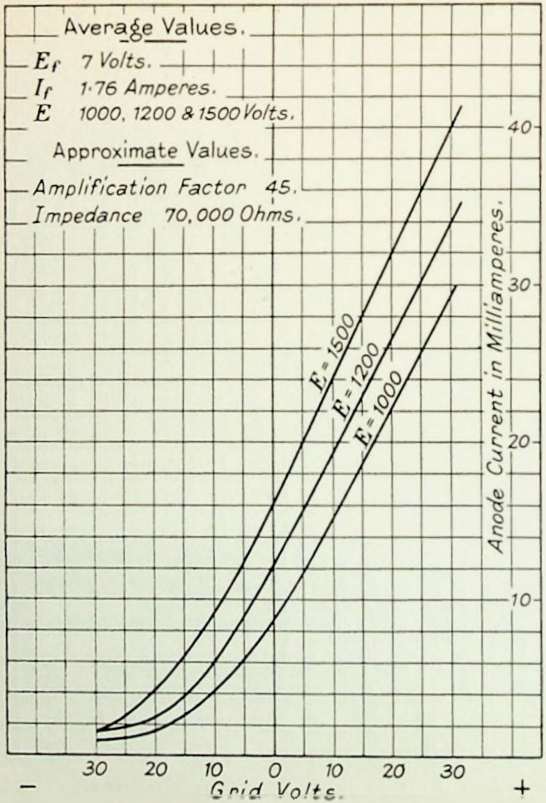


FIG. 132.—T30 (M.O.V. Co.).

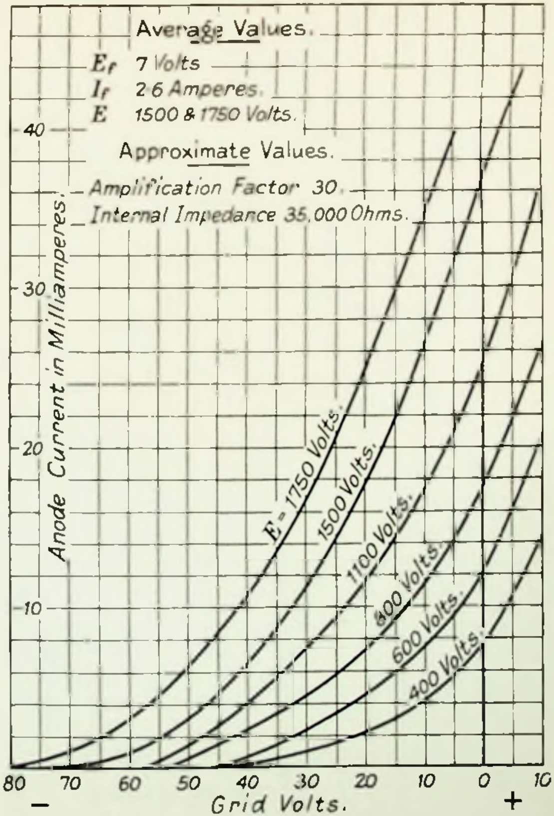


FIG. 133.—T50 (M.O.V. Co.).

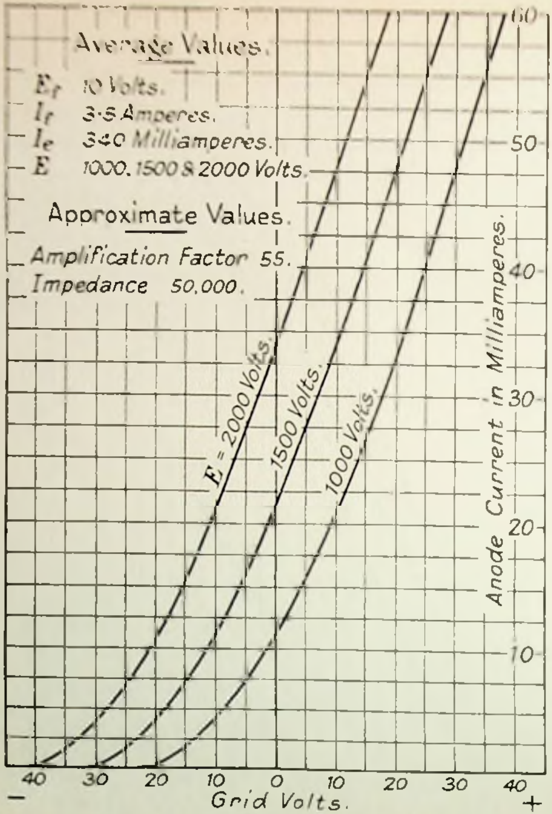


FIG. 134.—T100 (M.O.V. Co.).

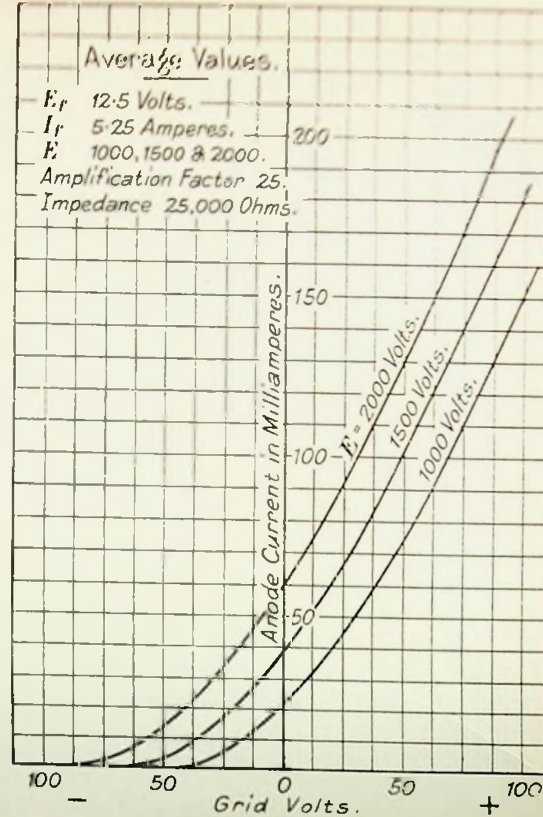


FIG. 135.—T250 (M.O.V. Co.).

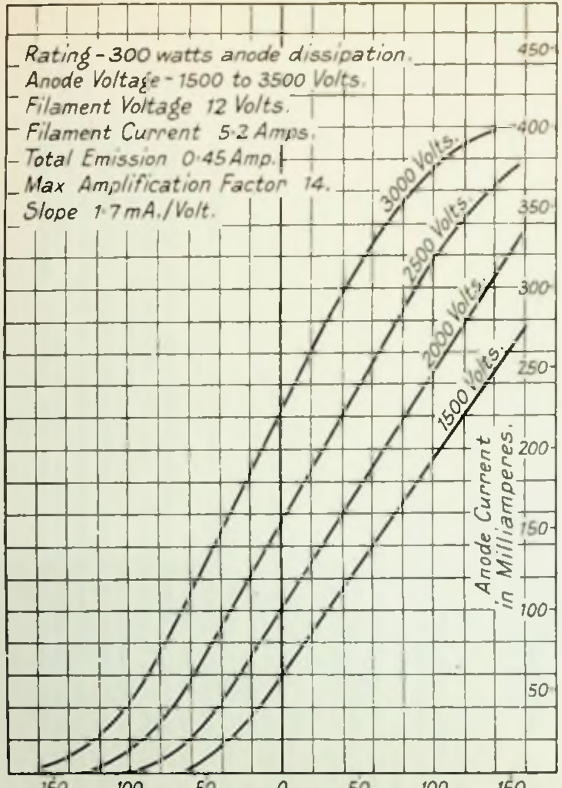


FIG. 136.—O250 (MULLARD).

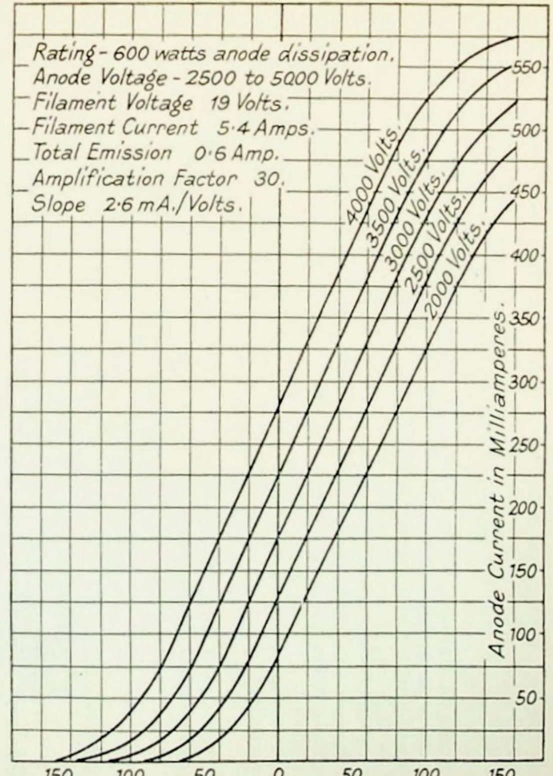


FIG. 137.—O500 (MULLARD).

RADIO TRANSMITTERS.

Radio transmitters fall into two main classes. In the first, an oscillating circuit is made up having a condenser in series with an inductance and a spark gap. The condenser is charged to a high potential which breaks down the gap, with the result that an oscillatory discharge takes place. This system is known as the spark system.

The second system utilises suitable means for supplying energy to the circuit continuously, this system being known as the continuous wave system.

The two systems are essentially different, and will therefore be considered separately.

Spark Transmitters.

The simplest form of spark transmitter is shown in Fig. 138. The aerial constitutes the condenser, which is charged to a high potential by the induction coil, and the voltage thus rises until the spark gap breaks down. The condenser then discharges across the gap in an oscillatory fashion, provided that the resistance of the circuit and gap combined is not too high, so radiating a train of waves. These waves will be heavily damped, and will die away after a comparatively few oscillations, after which the gap will become non-conducting again, and the process will be repeated. Since the oscillations will die away in about 1/100,000 second, there is a considerable interval between the successive sparks, which only occur at a frequency of about 50 per second with an induction coil, or 500 to 1,500 per second if an alternator is employed, as is more usual.

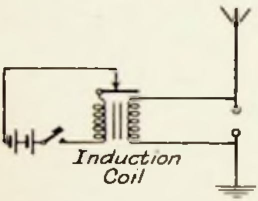


Fig. 138.—Precharged Aerial.

A system such as this, however, employing what is known as a precharged aerial, is prohibited by international convention, because the aerial can oscillate at an infinite number of frequencies from the fundamental ($\lambda = 4l$ approximately) upwards. This would give rise to considerable and unnecessary interference, so that the system is not permitted. The difficulty is overcome by employing a primary tuned circuit containing the spark gap, which is coupled to the aerial in a suitable manner. The wave length is then determined by the closed circuit, and, moreover, since the spark does not occur in the aerial circuit, the decrement of the system is lower, and the waves are not so rapidly damped.

The coupling, however, must not be too tight, as otherwise beats will be set up, as was described on p. 75.

The amount of coupling permissible depends on the type of spark gap employed. It is very desirable that the successive sparks should occur regularly, for under such conditions a clear musical note is obtained in the receiver, which is of great assistance in reading through atmospheric disturbances. To ensure regular sparking it is essential that after each spark the hot gas should be dissipated as rapidly and completely as possible, so that the spark may always occur with the same voltage across the gap.

Plain spark gaps, therefore, are made of non-arcing material, such as zinc or aluminium—not copper—and it is usual to construct them with parallel faces to permit the spark to " wander," and so avoid pitting as far as possible. Oil enclosed gaps are sometimes employed to increase the cooling of the gap after each spark.

Plain gaps, however, are little used, except for small powers. The more usual form of gap is the synchronous type. Here current is supplied by an alternator to a transformer, which steps up the voltage to a suitable value. The spark gap takes the form of a series of spokes on a disc mounted on the alternator shaft. These spokes pass within a small distance of two fixed electrodes, and at these points a spark occurs. The arrangement is so designed that the spokes are in the correct sparking position just at the moment when the spark is due, which, as will be seen later, is just before the peak of the E.M.F. wave of the alternator. The oscillation takes place, and the condenser commences to build up again, usually acquiring its

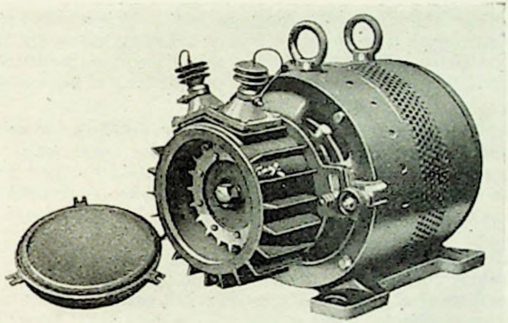


FIG. 139.—1½-KILOWATT DISC DISCHARGER.
(By courtesy of Radio Communication Company.)

sparking potential again one half-cycle or one cycle later. Since the disc is mounted on the alternator shaft, there will be a set of electrodes ready to take the spark every time. The spark thus always occurs at a definite point in the cycle, and the rotary motion of the disc cools the electrodes, and removes any ionised air, so further assisting the regularity of the sparking. The successive sparks take place between different electrodes on the disc, so that the same electrode is only used once in every ten sparks or so. Fig. 139 shows the disc discharger on a 1½-kilowatt set made by the Radio Communication Company.

Quenched Spark Gaps.—The surging of the energy between the primary and secondary circuits, which occurs when the coupling between the two is too tight, could obviously be prevented if by some means the primary circuit were definitely broken after the first surge—*i.e.*, after the energy had been completely transferred to the aerial, and before any retransfer took place. This is accomplished by using a special form of gap built up of a number of sections in series. The individual gaps are very close together, the spacing being of the order of 0·01 inch, and large cooling fins are provided. Fig. 140 shows the construction of such a gap. With such a system the cooling facilities are considerably increased, with consequent reduction of ionisation, so that when the spark is extinguished for the first time it cannot rekindle, and no transfer of energy is possible. With a gap of this kind the coupling may be made as high as 15 to 18 per cent., whereas with the ordinary rotary gap it seldom exceeds 5 per cent., a plain gap being still less efficient. Since the transformation efficiency affects the design of the primary circuit, the use of a quenched gap enables a somewhat cheaper construction to be employed.

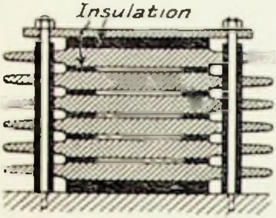


FIG. 140.—DIAGRAM OF QUENCHED SPARK GAP.

It is important that the individual gaps should be air-tight. The sparking across the gaps ionises the air, and separates the nitrogen and oxygen, which latter combines with the metal. The remaining nitrogen is less readily ionised, and this, combined with the rapid cooling facilities of the construction, give the gap its quenching properties. The voltage across an individual gap is about 1,200 volts.

The Physics of the Spark.—When the voltage on the condenser rises above a certain limit determined by the length of the spark gap, an oscillatory discharge takes place, the current jumping across the gap and forming a spark. This discharge has been investigated by various scientists, and appears to be intermediate between a glow discharge and a simple arc discharge.

The equation to the spark is of the form: $v = a + b/i$.

The resistance, therefore, is of the form: $R = a/i + b/i^2$.

The decrement of a spark train is thus not constant, the waves tending to die away according to a linear law (the second term usually being small compared with the first), instead of in an exponential manner.

Effective Value of Current in Spark Circuit.— If the current is assumed to be of the form $i = I_0 \varepsilon^{-at} \sin \omega t$, then it may be shown that the R.M.S. current is given by—

$$I^2 = \frac{NI_0^2}{4f\delta} \cdot \frac{1}{1+\left(\frac{\delta}{2\pi}\right)^2} = \frac{NI_0^2}{4f\delta} \text{ if } \delta \text{ is small;}$$

where N = number of sparks per second,
f = frequency of oscillations,
δ = logarithmic decrement.

Since $I_0^2 = E_0^2 \frac{C}{L}$, the expression may be written: $I^2 = \frac{NE_0^2C}{2R}$, where C is the capacity of the condenser, and E_0 is the voltage to which it is charged.

Sparking Voltage.—The actual voltage necessary to produce a spark depends upon (1) the size and shape of the spark surfaces; (2) the nature of the surfaces; (3) the nature and pressure of the gas; (4) the nature of the light illuminating the gap, ionisation being produced by any ultraviolet light present, with corresponding reduction of the sparking voltage.

The lowest sparking voltage for a given length of gap, is obtained when the two electrodes are needle points, and Fig. 141 below gives the sparking voltages in air between needle points at atmospheric pressure.

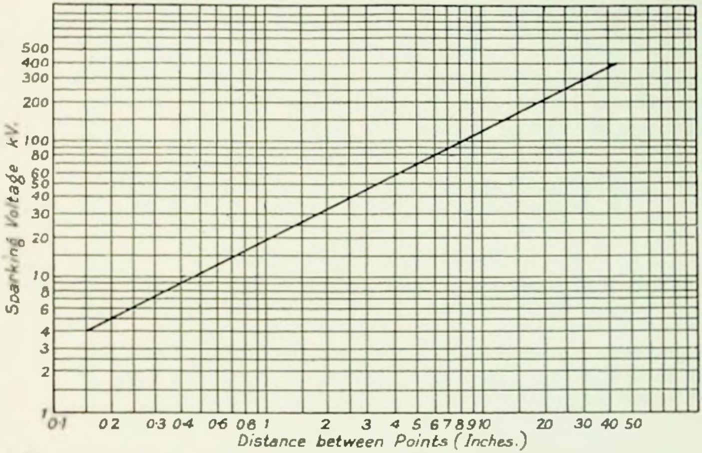


FIG. 141.—SPARKING VOLTAGE BETWEEN NEEDLE POINTS (IN AIR).

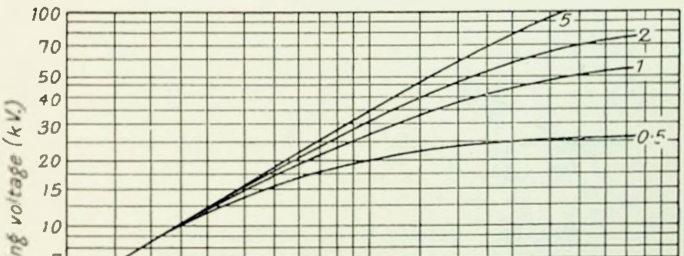


FIG. 142.—SPARKING VOLTAGE BETWEEN BALLS IN AIR.

For simple spark gaps employing spherical electrodes the voltage depends upon the diameter of the balls, as well as the distance apart. Fig. 142

gives the sparking voltages for various sizes of balls and for various lengths of gap. These data are due to Kaye and Laby.

Plane Disc Electrodes.—This is a case of some practical importance, since, as has been seen, spark gaps are usually made with parallel faces. Baille has given the following formula:

$$V = 30s + 1{\cdot}35 \text{ kilovolts},$$

where s is the length of the gap (cms.). This formula only applies to gaps of the order of 1 millimetre.

Variation of Sparking Voltage with Pressure:

$$E = E_0\ (0{\cdot}2 + 0{\cdot}8P/760) \text{ at } 17° \text{ C},$$

where E is the voltage at a pressure P millimetres, E_0 is the voltage at the normal pressure (760 millimetres).

Design of Spark Transmitter.

The design of a spark transmitter resolves itself into two portions: the high frequency circuits, and the low frequency circuits supplying the voltage to charge the condenser.

High Frequency Circuits.—A representative circuit for a spark transmitter is given in Fig. 143. There are two high frequency circuits: (*a*) the aerial circuit, and (*b*) the closed circuit L_1L_2C and the spark gap.

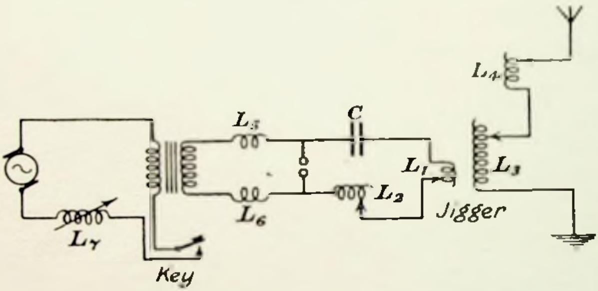


Fig. 143.—Circuit of Spark Transmitter.

The first essential is that the condenser C shall be of sufficient size to handle the energy required. Let η be the efficiency of transformation between the closed circuit and the aerial; the energy stored in the condenser C is $\frac{1}{2}NCV^2$, N being the number of sparks per second. If the aerial energy required is W watts, then—

$$C = \sqrt{\frac{2W}{\eta NV}}.$$

V is determined by the size and type of gap to be employed, so that C is thus obtainable.

The design of the remainder of the circuit then follows from the considerations of tuning. The efficiency of transformation η is the ratio of the power supplied to the aerial circuit to that supplied to the primary circuit, and thus allows for the losses in the primary. For the 1½-kilowatt spark set at North Foreland the value of this efficiency is 60 per cent.

Jigger—Degree of Coupling.—The aerial coupling transformer, usually known as the "jigger," is designed generally in accordance with the laws of coupled circuits already discussed (p. 75).

Drude has suggested that the correct value of the coupling factor may be determined in terms of the degree of coupling, which is given by—

$$k'^2 = k^2 \sim \left(\frac{\delta_1 \sim \delta_2}{2\pi}^2\right),$$

where k' = degree of coupling,
k = coupling factor,

δ_1 and δ_2 are the decrements of the primary and secondary circuits.

The coupling is loose if $k < \frac{\delta_1 - \delta_2}{2\pi}$, and is tight if $k > \frac{\delta_1 - \delta_2}{2\pi}$. The optimum condition for a single wave coupling without any surging occurs when $k = \frac{\delta_1 - \delta_2}{2\pi}$. In this case the voltage on the secondary (maximum) is one eighth of the maximum voltage in the primary circuit.

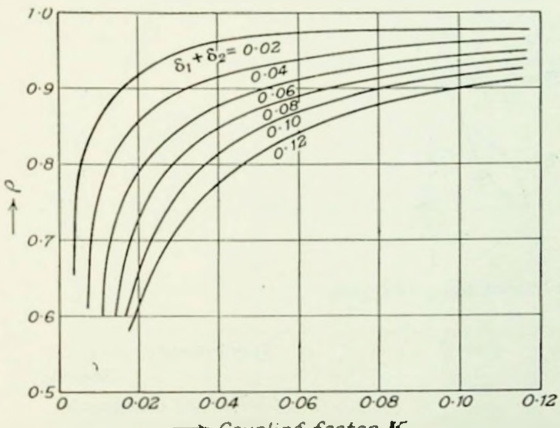


Fig. 144.—Values of Factor ρ in Terms of k.

In general the voltage in the aerial (secondary) circuit depends upon the damping in each circuit, the respective capacities, and the coupling between the two circuits. Taylor has shown that the voltage may be expressed in the form:

$$V_2 = \rho \sqrt{\frac{C_1}{C_2}}\, V_1,$$

and Fig. 144 shows the variation of the constant ρ with the coupling. It will be seen that little advantage is gained by increasing k beyond 0·1. The energy transferred from primary to secondary is proportional to ρ^2.

Choke Coils.—The choke coils L_5 and L_6 are for the purpose of preventing the high frequency oscillations from flowing back through the transformer secondary, and causing damage to the windings.

The value of these chokes is about 10,000 μH for a set working over a range of 300 to 800 metres. As an additional precaution, the insulation of the end turns of the secondary of the transformer is specially reinforced.

Low Frequency (Charging) Circuit.—When a spark takes place, the secondary of the transformer is momentarily short-circuited. This causes an arc to take place across the spark gap, which causes undesirable burning of the electrodes, apart from the effect of the short circuit on the transformer itself.

In order to overcome this defect, it is customary to tune the charging circuit to the frequency of the alternator. The spark is then extinguished at the point where the alternator voltage is zero, and hence does not reignite. It may be remarked in passing that if the low frequency circuit is correctly adjusted, the spark sounds like the crack of a whip, whereas arcing is immediately distinguishable by a hissing noise.

The method of tuning varies with the conditions. With small sets the reactance of the circuit may be varied by employing a leaky transformer,

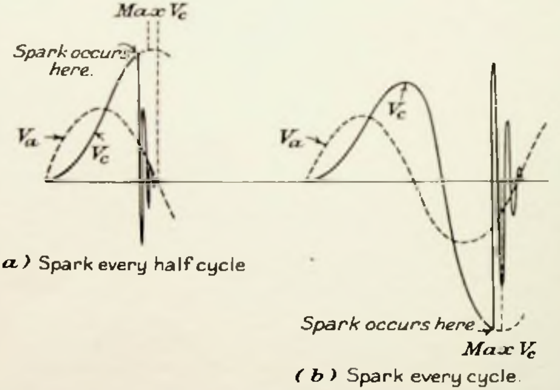


Fig. 145.—Illustrating Build-up and Discharge of Condenser Voltage.

the leakage being variable, but the more usual method, used on larger sets, is to provide a separate variable choke coil in series with the primary of the transformer.

The tune of the circuit may readily be obtained from the ordinary transformer laws. If T is the transformation ratio of the instrument, then the transformer with the condenser C across the secondary may be replaced by an equivalent condenser of value CT^2.

This neglects the effect of the inductances L_1, L_2, L_5, and L_6, which are all very small compared with that of the alternator and choke coil. The circuit is thus designed so that the inductance of the alternator and the external choke coil tune with the equivalent condenser CT^2 to the alternator frequency. It may be observed, however, that although the radio frequency inductances may be neglected in determining the size of the low frequency choke, the low frequency tune is very sensitive to the smallest changes in these inductances. A change of wave length thus usually necessitates a readjustment in the value of the low frequency choke.

Fig. 145 shows the rise of the voltage on the condenser, under correctly adjusted operating conditions, with the spark taking place every half-cycle and every cycle. It will be observed that the tuning is slightly sharp—*i.e.*, the condenser voltage reaches its maximum just before the alternator voltage is zero. This is so that at the end of the spark train, when the gap opens, the alternator voltage shall be zero. If this is not the case, arcing may be produced.

Design of Transformer.—When the circuit is tuned and the alternator voltage is E sin pt, the voltage across the equivalent condenser is given by—

$$V_c=\frac{E}{pCR}\left\{\frac{R}{2pL}\,\varepsilon^{-\frac{R}{2L}t}\sin pt-\left(1-\varepsilon^{-\frac{R}{2L}t}\right)\cos pt\right\}.$$

The first term is the transient term, which rapidly dies away, the steady state being represented by the second term. In this case, however, the transient term is of importance, since the spark is usually arranged to occur

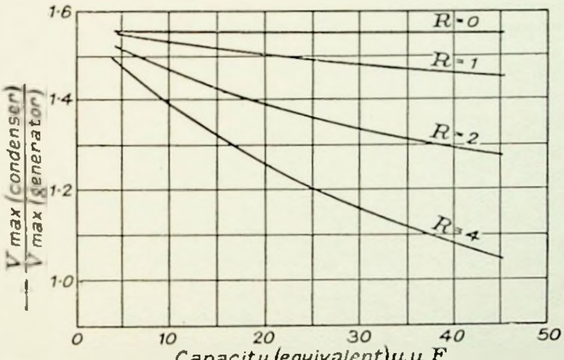


FIG. 146.—VALUE OF $\frac{\text{V CONDENSER}}{\text{V GENERATOR}}$ WITH SPARK EVERY HALF-CYCLE.

every half-cycle or every complete cycle. The maximum value of the voltage on the condenser is therefore obtained from this expression in terms of the alternator voltage.

For most practical cases, assuming a spark every half-cycle, this ratio is of the order 1·5. Curves are given in Fig. 146, which show the variation of this ratio with the values of both C and R.

The voltage on the actual condenser C will, of course, be T times this value, and this actual voltage must equal that originally assumed for the spark gap. Hence the transformation ratio T may be calculated.

From the ordinary transformer laws, the primary must be designed to fulfil the condition—

$$V_{eff} = 4{\cdot}44\ ABn_1 f \times 10^{-8},$$

where

A = area of core (sq. cms.),
B = flux density,
n_1 = number of turns,
f = frequency.

V_{eff} in this case is not $\dfrac{V_{max}}{\sqrt{2}}$, since the voltage is not sinusoidal. The following values may be used:

Spark every half-cycle: $V = 0{\cdot}5\ V_{max}$.
Spark every cycle: $V = 0{\cdot}55\ V_{max}$.

Power Factor in Charging Circuit.—If $\dfrac{Lp}{R} > 5$ and $\dfrac{p}{\pi N} < 3$, then Baillie has shown that the power factor is given by—

$$\cos\varphi = \frac{\left(1 - \varepsilon^{-\beta}\right)^2}{1 + \frac{1}{2\beta}\left(1 - \varepsilon^{-2\beta}\right) - \frac{2}{\beta}\left(1 - \varepsilon^{-\beta}\right)},$$

where $\beta = \dfrac{R}{2NL}$, N being the number of sparks per second.

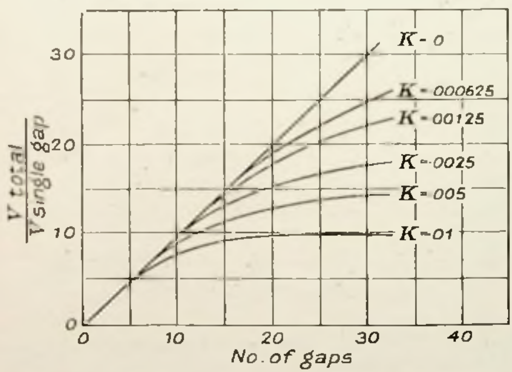


FIG. 147.—RATIO OF TOTAL VOLTAGE TO VOLTAGE PER GAP WITH QUENCHED SPARK.

Spark Voltage.—For a plain or rotary gap the spark voltage may be calculated from the formulæ and tables already given.

In the case of quenched gaps, however, the voltage across the whole gap is not n times the voltage across a single gap owing to the leakage capacity to earth. Schuleikin and Freeman have shown that if—

$$k = \frac{\text{leakage capacity to earth}}{\text{capacity between adjacent plates}},$$

then the ratio of the total voltage to that across a single gap is as shown in the curves in Fig. 147. These curves refer to the case where one end of the gap is earthed. If the mid-point of the transformers is earthed, the value of k should be multiplied by 2.

The voltage across a single gap may be calculated from the formulæ for plane discs given on p. 128.

Continuous Wave Transmitters.

The spark system of generating electromagnetic waves suffers from the disadvantages that it is only radiating energy for a small fraction of the total time, and also, due to the damped character of the wave, tuning is not sharp and considerable interference to other services is caused.

Moreover, the voltage on the aerial for the same power is less for a C.W. than a spark system. Since this is one of the limiting factors in aerial design, a higher power input is obtainable with a given aerial system using C.W. A fourth advantage lies in the fact that the absorption of continuous waves is somewhat less than with spark working.

The C.W. system has therefore completely superseded the spark system for long-distance and point-to-point communication. Spark working is, however, still extensively used for ship and shore communication, mainly for two reasons. Firstly, the prime and running costs are smaller, particularly in respect of the receiver, which need only be a simple crystal apparatus. Secondly, the broad band of a spark transmitter facilitates tuning in, and also enables SOS and similar calls to be heard even if the ship is not accurately tuned to the correct 600-metre wave length.

There are three main systems in use to-day for generating C.W. Firstly, there is the high frequency alternator, in which, by suitable design, the frequency generated is either that required or some integral fraction thereof. Secondly, there is the arc system, in which an electric arc suitably connected may be made to sustain oscillations continuously in an ordinary oscillatory circuit. While, thirdly, there is the valve system, employing the regenerative properties of the thermionic valve. These systems will now be considered in detail.

The High-Frequency Alternator System.

There are two main types of high frequency alternator. In the first, *the Alexanderson alternator*, the principle of the ordinary inductor alternator is extended to the limit. Fig. 148 gives a diagrammatic view of such a machine. The field system is in the form of a hollow ring having a slot on the inside. Inside the ring is a circular field coil, which produces a flux round the cross-section, as shown in Fig. 148, this flux flowing across the gap or slot previously mentioned.

The rotor is in the form of a thin disc mounted co-axially with the field system, the edge of the disc running in the slot in the field system. The rotor disc is not solid, but is toothed, while the inside of the slot in the field system is also provided with corresponding pole pieces round which are wound the armature coils.

The rotation of the disc, therefore, alters the distribution of the flux in the gap, the flux in any particular pole piece being a maximum when a tooth of the rotor is in the gap, and a minimum when in the intermediate position between two teeth. In practice, to avoid air friction, the spaces between the teeth are filled with gun-metal.

By suitable design, very high frequencies may be obtained by this means. Assuming that there are 250 slots on the disc, which is rotating at a speed of 200 r.p.sec. (12,000 r.p.m.), the frequency would be $250 \times 200 = 50,000$ cycles per second.

This corresponds to a wave length of 6,000 metres. By careful design frequencies of 100,000 may be obtained, but these machines are generally employed for rather lower frequencies. It is not economical to build these

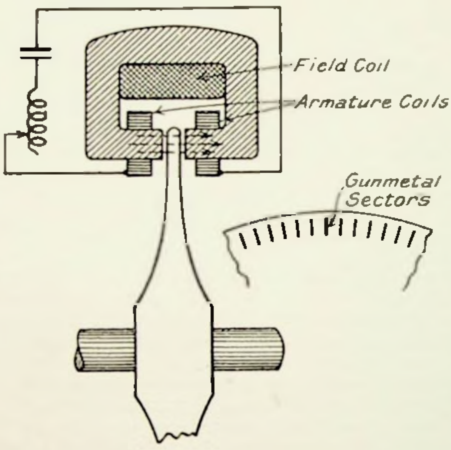


FIG. 148.—SECTION OF ALEXANDERSON ALTERNATOR.

machines for small powers, and their chief use is for long-distance communication, requiring powers of 50 to 250 kilowatts, and operating on wave lengths of 6,000 metres and upwards.

On large machines, the machine is totally enclosed and the rotor is run in a vacuum to reduce the air friction, which, at the very high speeds employed, is a considerable source of loss.

The inductance of the armature windings, which are all connected in series, is so designed that it will tune to the particular frequency with the capacity of the aerial, a small external choke coil being inserted for fine adjustment purposes.

Goldschmidt Alternator.—Another very ingenious machine, which is becoming obsolete, is the Goldschmidt alternator. It is well known that the field system of an ordinary alternator has induced therein small E.M.F.'s of double the frequency of the armature. In the Goldschmidt machine these E.M.F.'s are allowed to produce large currents by tuning the field system to the requisite frequency, and consequently they in turn induce E.M.F.'s of triple frequency in the armature. The process may be repeated indefinitely, but the losses in the machine begin to increase rapidly after four or five frequency transformations, so the process is not carried any farther.

Fig. 149 illustrates the principle involved. The initial D.C. field current is supplied by a battery or auxiliary generator through the choke L_0. This choke is essential in order to prevent the double frequency currents from leaking through the supply circuit. The rotor is tuned to the frequency of the alternator *f*, which is usually of the order of 10,000 cycles per second. These currents induce E.M.F.'s of frequency 2*f* in the field system, which pass through the circuit $C_2L_2'C_2'$, which is tuned with the stator inductance to accept this frequency. These currents in turn set up frequencies 3*f* in the rotor, which are accepted by the circuit $L_{rotor}\,C_1C_3$, and these currents

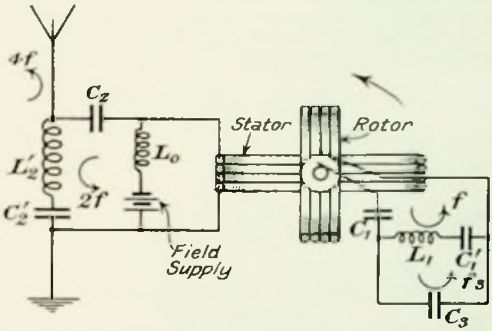


FIG. 149.—DIAGRAM OF GOLDSCHMIDT ALTERNATOR.

finally react on the stator, producing currents of frequency 4*f* in the circuit $L_{stator}\,C_1C_{aerial}$, the final frequency thus being 40,000 cycles, corresponding to 7,500 metres.

A machine which is in use to a large extent in France is the Bethenod-Latour alternator. This machine operates on the same principle, but the transformations are effected in separate machines on the same shaft. The field of the second machine is thus supplied with current of frequency *f*, so generating currents of double frequency in the rotor, and so on.

One of the chief difficulties in designing high-frequency alternators is the very small space available for each pole. Latour overcomes this difficulty by arranging the poles in several layers in the armature and staggering successive layers. Thus, with three layers the successive poles would each be staggered 120° (electrical).

Fig. 150 illustrates the system. For further details the reader is referred to a paper by Latour (*Proc. I.R.E.*, vol. viii., p. 220).

Speed Regulation.—It is essential with machines of this type to maintain the speed absolutely constant, as the slightest variation will cause variations in the wave length transmitted. In practice, the variation in frequency is found to be from 10 to 20 cycles in 50,000, which means that the speed is kept constant to within 0·05 per cent. This is a remarkably fine performance, which can only be attained by the use of very elaborate regulators.

The starting of such machines is also a very skilled business, since the rotor runs at such a high speed that several critical speeds have to be passed through in the running-up period.

Keying.—The keying on high-frequency alternator may be effected in a variety of ways. One method is to break the field circuit. A more usual method is to short circuit the alternator. Since the alternator is normally tuned, this calls for less expenditure of power than when the short circuit is removed.

A third method, due to Alexanderson, employs a device known as a magnetic key. This arrangement consists of a choke coil having two windings. One of these windings is connected across the alternator terminals; owing to its high inductance at the radio frequency, it does not affect the alternator. The depression of the key, however, sends a steady D.C. current round the second winding, which saturates the iron, causing the inductance of the main coil to drop to a value of the same order as that without any iron core. This produces a considerable short-circuiting effect on the alternator, and so

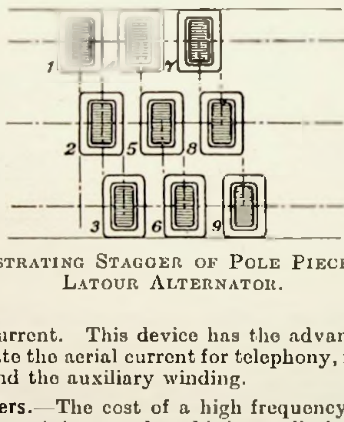


Fig. 150.—Illustrating Stagger of Pole Pieces on Bethenod-Latour Alternator.

reduces the aerial current. This device has the advantage that it may be employed to modulate the aerial current for telephony, if desired, by passing speech currents round the auxiliary winding.

Frequency Changers.—The cost of a high frequency alternator increases rapidly as the frequency is increased, and it is usually found more economical in practice to generate a somewhat lower frequency and to double or treble the frequency by means of certain special forms of static transformer, known as frequency changers.

One such arrangement is shown in Fig. 151, known as Joly's doubler. The D.C. winding on the centre limb produces a flux round the instrument, as indicated by the arrows. The current from the alternator or other source of supply passes round the two primary windings in series, the effect being to increase the flux in one limb and to decrease it in the other. The D.C. polarising current is arranged to magnetise the iron at the knee of the B-H curve, as shown in Fig. 152. Owing to this asymmetry, the decrease of flux in the bottom limb is greater than the corresponding increase in the top limb. The secondary windings have an E.M.F. induced in them by the flux pulses, these two E.M.F.'s being in the opposite directions, owing to the directions of the windings. Since the values of the flux changes are not the same, however, there will be a resultant E.M.F. induced as a result of the flux pulse.

Since there will be one flux pulse per *half-cycle* of the applied E.M.F., the E.M.F. induced in the secondary will be of double frequency. This effect will be clear from Fig. 152. It will be seen from Fig. 151 that the primary

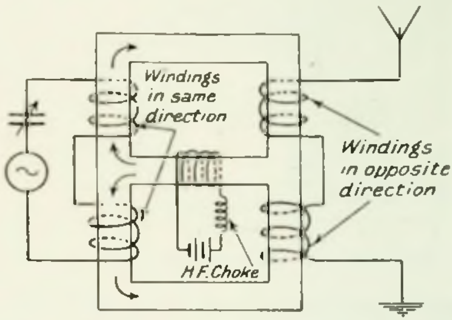


FIG. 151.—JOLY'S FREQUENCY DOUBLER.

circuit is tuned. This is essential, as there is a double frequency term in the primary back E.M.F. which has to be suppressed.

Plohl's Doubler.—A somewhat similar arrangement, due to Plohl, is shown in Fig. 153. Here two transformers are employed, and the two circuits are

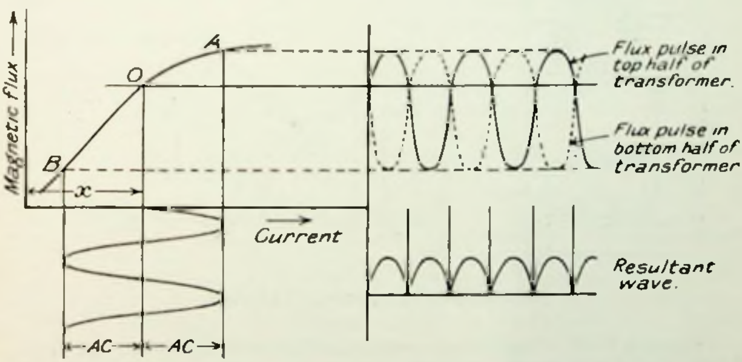


FIG. 152.—ILLUSTRATING OPERATION OF JOLY FREQUENCY DOUBLER.

placed in parallel across the supply. Hence the back E.M.F. of each circuit must be the same. If the transformer A is again run at the saturation point, the flux pulses in the two halves of the transformer are different. Hence the currents in the two circuits must also be different, in order that the back E.M.F., which is proportional to the product of these two quantities, may be the same. The flux in the second transformer B will thus be due to the resultant of these two current pulses. Since the current pulses are unequal, and the windings in the opposite directions, there will be a resultant pulse,

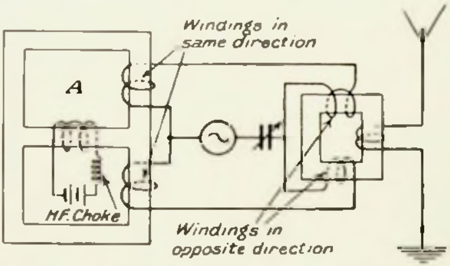


FIG. 153.—PLOHL'S FREQUENCY DOUBLER.

giving rise to an E.M.F. in the aerial circuit every half-wave, which produces a double frequency as before.

Two doublers may, if desired, be used in series, and in practice this is often done. Owing to the high frequency of the currents handled by these instruments, the losses are rather heavy, although specially thin sheets are employed for the core stampings. The efficiency is usually of the order of 70 per cent.

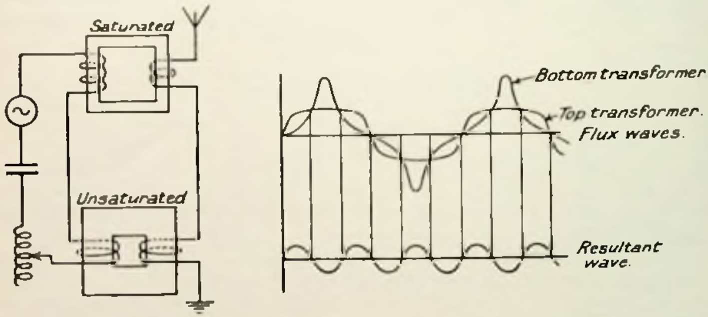


FIG. 154.—FREQUENCY TREBLER.

Frequency Treblers.—An instrument, ascribed to both Clinker and Taylor, has been designed by means of which the frequency may be trebled.

The arrangement is shown in Fig. 154. Two transformers are employed, one being so designed that the current taken by the primary will saturate the iron, while the other is designed never to reach the knee of the curve. Owing to the saturation, the flux in the first transformer will have a flat top, while that in the other transformer will be of a peaked nature.

The two secondary coils are wound in opposition, so that the resultant E.M.F. in the secondary will be due to the difference of these two fluxes. This produces an E.M.F. of three times the applied frequency, as is shown in Fig. 154.

The Oscillating Arc.

The second system for the production of C.W. is the oscillating arc. Duddell discovered that the ordinary carbon arc could be made to sustain oscillations in a low frequency tuned circuit placed in shunt across the arc, as shown in Fig. 155. Investigation showed this to be due to the peculiar

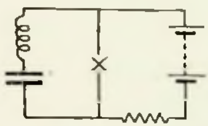


Fig. 155.—Simple Arc-Maintained Oscillatory Circuit.

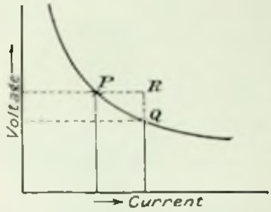


Fig. 156.—Characteristic of Carbon Arc.

falling characteristic of the arc (Fig. 156). It will be seen that if the arc current increases, the voltage across the arc decreases, and *vice versa*. This is the reverse of the effect obtained with a simple resistance; in other words, as far as *variations* of voltage or current are concerned, the arc may be considered as having a *negative* resistance. If this negative resistance effect is made equal to the positive resistance in the circuit, the damping in the circuit will be reduced to zero, and the oscillations will be maintained indefinitely.

The conductivity of the arc depends on the ionisation of the gases therein, which depends upon the temperature. This, again, depends on the current through the arc, so that the conductivity increases with the current and hence the peculiar falling characteristic is obtained.

The simple arc, as discovered by Duddell, however, will only oscillate over a very limited range of frequency. In order to sustain oscillations at radio frequencies, it is necessary to provide means of "scavenging" the arc during the period of extinction, removing as far as possible all ionisation of the gas in between the electrodes. To do this, several modifications were introduced by Professor Poulsen, with subsequent improvements by Pederson, Elwell, and others, with the result that frequencies up to 100,000 may be generated by this means. The chief modifications are as follows:

The arc is burned in an enclosed chamber in an atmosphere of hydrocarbon vapour, the thermal conductivity of such gases being greater than that of air. In order further to assist the dissipation of the heat from the arc, the chamber itself is water-cooled. A second modification is the provision of a strong magnetic field at right angles to the arc itself for the purpose of blowing out the arc rapidly as soon as the voltage across it falls to zero. While, finally, in order to obtain as steady an arc as possible, a copper anode is employed, which has to be water-cooled because of the intense heat evolved. The cathode is of non-cored carbon, and is slowly rotated to ensure even burning.

A general idea of the construction may be obtained from Fig. 157, which illustrates the various components.

The cycle of operations may be considered in detail. There are two operations involved in each cycle. During the first half-cycle oscillating current

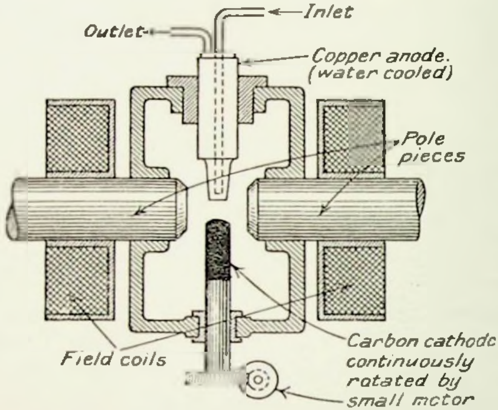


FIG. 157.—DIAGRAM OF RADIO-FREQUENCY ARC GENERATOR.

is set up by the energy stored in the L.C. circuit. During the second or charging period, energy is drawn from the D.C. supply circuit at the expense of the arc.

Consider the complete circuit shown in Fig. 158. The oscillating circuit LCR is shunted across the arc. L_2 is a choke coil, while R_2 is the starting resistance, which is usually cut out when the arc is running. Assume

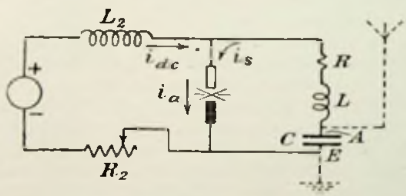


FIG. 158.—CIRCUIT DIAGRAM OF RADIO-FREQUENCY ARC GENERATOR.

that the condenser C is charged so that the terminal A is positive. It will then discharge through the arc. The current through the arc, therefore, is $i_a = i_{dc} + i_s$, and the voltage across the arc will be *reduced*, owing to the falling characteristic of the arc. This effect continues until the condenser is fully discharged, when both i_s and i_a are at a maximum, the voltage across the arc being a minimum.

The current then begins to decrease, charging the condenser C in the reverse direction. The arc current will decrease, and the voltage across it will rise accordingly until the condenser is completely charged in the opposite direction, when $i_s = 0$, $i_a = i_{dc}$, and $v_a = v_{dc}$. This completes the first half-cycle.

The condenser then discharges a second time, the current this time being in the reverse direction. i_s is thus negative, so that the arc current is reduced. The arc voltage accordingly rises, and hence the current drawn from the D.C. mains is less.

The total D.C. current, however, cannot alter appreciably, owing to the choke coil L_2, so that the surplus current flows round the oscillatory circuit, assisting the discharge and subsequent charge of the condenser. Hence a certain quantity of energy is extracted from the D.C. supply during this portion of the cycle.

When the arc current reaches a certain minimum value, the magnetic field extinguishes the arc. This occurs normally just before the oscillating current reaches its maximum. As soon as the oscillating current commences to decrease, however, the D.C. supply decreases as well, and this causes a high voltage surge on the choke coil L_2 which re-ignites the arc. The cycle of operations then continues as before.

The cycle of events is illustrated in Fig. 159. It will be seen that the re-ignition voltage is little greater than the extinction voltage. This is due to

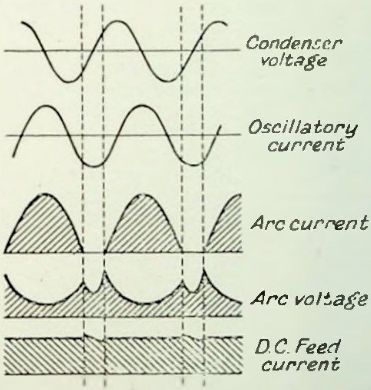


FIG. 159.—CYCLE OF OPERATIONS IN AN ARC TRANSMITTER.

the fact that the arc length increases (due to the blowing-out effect of the magnetic field) as the current decreases, whereas the re-ignition length is the shortest distance between the electrodes.

It will be observed that the success of the cycle depends on the maintenance of a steady D.C. supply current, and the choke coil L_2 must be so designed that this condition is fulfilled. It also serves the purpose of preventing the high frequency oscillations from leaking back through the generator, and is thus usually air-cored.

The theory just propounded is based on the assumption that the oscillatory current is just sufficient to extinguish the arc at each oscillation. That is, the maximum value of $i_s = i_{dc}$. Hence, if I_s is the R.M.S. value of the oscillating current, assumed sinusoidal—

$$I_s = \frac{I_{DC}}{\sqrt{2}} = 0{\cdot}707\ I_{DC}.$$

This is the most efficient condition of operation, and arcs are always adjusted in practice to run under such conditions.

An arc system is usually connected direct to the aerial, which then *replaces* the condenser C, as indicated by the dotted lines in Fig. 158.

Keying—Spacing Wave.—The arc cannot be started and stopped instantaneously. Hence, in order to "key" the signals generated, a "marking" wave is radiated when the key is depressed, and a slightly different "spacing" wave when the key is up. These two wave lengths are comparatively close together, being about 1 per cent. different on high-power equipments, so that the interference with other traffic is not as great as would appear at first sight. Moreover, if the modulation of a continuous wave by Morse signals at a moderate speed is investigated, it will be found that the usual system whereby the radiation is started and stopped by the key actually causes more interference than that produced by a spacing wave system, due to the fact that an oscillating system will transfer from one frequency to another without any transient oscillations which cause interference.

The fact that power is being consumed all the time an arc is in operation is not a serious disadvantage, since the cost of the power supplied to the aerial is a very small proportion of the total cost of running a station, most of which consists of capital charges and staffing expenses.

The actual wave length change is brought about by short-circuiting part of the aerial tuning inductance, or, more usually, by shorting a coil or coils coupled to a portion of the A.T.I., since this method isolates the key from the high voltages on the aerial.

The power radiated during the spacing period may be absorbed by a dummy aerial having the same constants as the actual aerial, but this use of a "back shunt" is little, if ever, employed except for quite small powers.

An arc may also be keyed by using the Alexanderson magnetic key described on p. 136, particularly if it is desired to transmit telephony.

Operation of Arc.—If the characteristic of an arc is plotted for several different arc lengths, it is found that the slope of the characteristic decreases rapidly as the arc length decreases. There is thus a critical value of the slope below which the arc is unable to sustain the oscillations, and the arc length must thus be greater than that corresponding to this critical value. In starting up an arc, therefore, the arc is first struck (all the auxiliaries, such as water, circulation pumps, etc., having first been started). The arc length is then gradually increased until the arc suddenly commences to oscillate, after which it is adjusted until the oscillating current is 0·7 times the D.C. feed current, as previously described.

Strength of Magnetic Field.—The magnetic field serves to blow out the arc at the extinction point, and it also controls the ionisaton of the gas in the inter-electrode space, so controlling the re-ignition point to a large extent. The operation of the arc is thus somewhat sensitive to changes in the strength of the magnetic field. If the field is too weak, the extinction and re-ignition voltages are low, and the effective pulse of D.C. energy per oscillation is reduced. The stronger the magnetic field becomes, the less is the ionisation, and the higher are the extinction and re-ignition voltages, with consequent increase of efficiency, until the pulses become too peaked and far apart, at which point harmonics begin to appear in the generated oscillations.

The time available for the scavenging effect is proportional to $1/f$, where f is the frequency of the oscillations. Hence B is proportional to $\frac{1}{\lambda}$. Again, the effect of the field is to increase the velocity of ion stream to a point where the continuity is broken. The field strength required to do this obviously depends on the initial velocity of the ions, which in turn is proportional to the square root of the temperature of the arc. Hence $B \propto \frac{1}{\sqrt{EI}}$.

B is also dependent on the voltage tending to maintain the arc, and also on the number of ions in the arc (in other words, the current), so that B is proportional to EI.

The total of all these effects, therefore, is that—

$$B = \frac{k \cdot EI}{\lambda\sqrt{EI}} = k\frac{\sqrt{EI}}{\lambda},$$

k being a constant depending on the nature of the gas in which the arc is burning.

For kerosene: $k = 4{\cdot}25$.
For ethyl alcohol: $k = 8{\cdot}5$. (Fuller.)

Fig. 160 shows the arc equipment in use at the Lyons Radio Station. The large coils are the field coils providing the magnetic field. The control

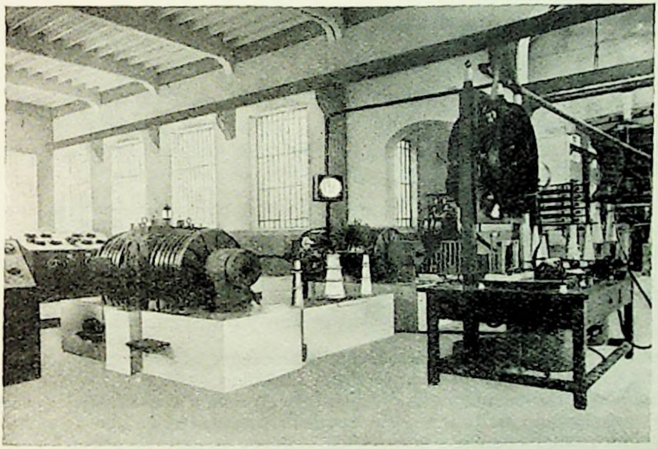


Fig. 160.—View of Lyons Radio Station.

panel may be seen on the left of the arc, while the main aerial tuning inductance is on the extreme right. These arcs deliver over 300 amperes to the aerial.

Arc equipment in general is chiefly of service for long-distance communication, employing wave lengths of 5,000 metres upwards. They may be constructed to handle powers of from 5 to 1,000 Kilowatts. Lower powered equipments may be designed, but the valve is a more convenient unit at lower powers.

The design of arc equipments is a highly specialised branch of the subject, and can hardly be considered here. The reader is referred for further details to " The Poulsen Arc Generator," by C. F. Elwell (Benn Bros., Ltd.).

Coupled Arc Circuits.—The plain aerial arc system suffers from the disadvantage that the aerial is not a mono-frequency system, and hence the radiated wave may contain a large proportion of harmonic oscillations. These harmonics, and also the undesirable " mush " usually radiated by high power arc stations, may be reduced to a negligible amount by employing a coupled circuit system. Such a circuit has been installed at the Post Office Radio Stations, at Northolt and Leafield, and the following information relating to the Northolt Station is extracted from a paper by A. C. Warren (" Northolt Radio Station," *Journ. I.E.E.*, vol. lxii., p. 967), to whom the compiler is indebted for assistance in preparing this section.

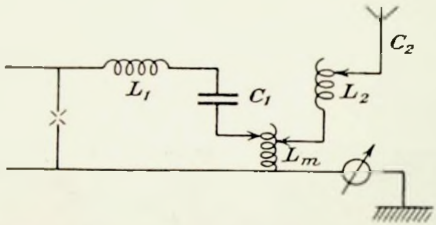


FIG. 161.—COUPLED CIRCUIT ARC TRANSMITTER.

The circuit employed is shown in Fig. 161, the values of the various constants being:

$L_1 = 2{,}800\,\mu$H.	$C_1 = 5{,}000\,\mu\mu$F.	$R_1 = 2$ ohms.
$L_2 = 2{,}000\,\mu$H.	$C_2 = 7{,}000\,\mu\mu$F.	$R_2 = 3{\cdot}4$ ohms.

If a secondary circuit is coupled to the primary circuit, it will be equivalent to an impedance Z_1' in the primary circuit, where—

$$Z_1' = \frac{\omega^2 L_m^2}{Z_2^2}(R_2 + jX_2)$$ by the ordinary transformer laws,

where R_2 and X_2 are the resistance and reactance of the secondary circuit, and L_m is the common portion of the inductance.

Now, since the frequency is obtained by equating the effective reactance of the primary circuit to zero, the frequency generated will be a function of Z_1'. The actual equation is a cubic. Below a certain critical coupling there is only one real root, but above this value there are three real roots.

This may be made clear by referring to Fig. 162. The equation may be put in the form—

$$\frac{x + A}{B^2} - \frac{x}{1 + x^2} = 0,$$

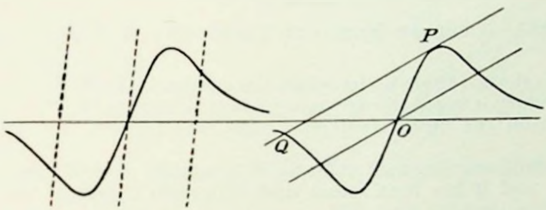
where A is a constant depending on the amount of mistuning between the primary and secondary, and $= (\omega_1 - \omega_2)\frac{R_2}{2L_2}$, ω_1 and ω_2 being the resonant frequencies $= \frac{1}{\sqrt{L_1C_1}}$ and $\frac{1}{\sqrt{L_2C_2}}$ respectively.

B is a constant depending on the coupling $= \frac{k\omega L_2}{R_2}$.

The first expression is a straight line of which the slope is inversely proportional to the coupling, and the distance from the point O is dependent on the mistune of the secondary.

The second expression gives a curve as shown, and the roots are obtained by the points of intersection of the two curves. It will be seen that if the slope of the straight line is greater than the tangent to the curve at O, there can never be more than one root, whatever the value of A. As the coupling is increased, however, the slope of the line decreases until beyond a certain value there are three real roots at values of A not too far removed from O.

Ziehen Effect.—For any particular value of the coupling, however, greater than the critical value, there is an unstable condition where the straight line is tangential to the curve. At this point any further increase in the mistune



(*a*) Coupling less than critical value. (*b*) Coupling greater than critical value.

FIG. 162.—ILLUSTRATING ZIEHEN EFFECT.

will cause two of the roots suddenly to become imaginary. If the circuit was oscillating, therefore, at a frequency as at P [Fig. 162 (*b*)] it will suddenly change to a frequency given by the root Q.

This effect has been termed "Ziehen" (to draw out) by Moller, who has investigated the action in some detail (*Jahrbuch für Drahtlose Telegraphie*, December, 1920). The phenomenon is observed with any form of coupled circuit, whether arc or valve maintained, provided the frequency of the oscillation is controlled by the constants of the circuit itself. The change of frequency is accompanied by a sudden current change, as will be seen from Fig. 163.

It will be observed that the higher the coupling the greater is the mistuning permissible before Ziehen occurs. The critical coupling below which Ziehen is impossible is given by $k = \frac{R_2}{\omega L_2}$. The quantity B in the frequency equation is thus simply $\frac{k \text{ actual}}{k \text{ critical}}$.

It is desirable to work above the critical coupling, as below that coupling the resonance curve for the secondary circuit is extremely sharp, and thus small variations in tune (due to aerial sway, rain, etc.) seriously affect the aerial current.

Beyond the critical coupling the ratio of $\frac{I_2}{I_1}$ (and thus the efficiency) increases with the coupling. However, this is to some extent balanced by an increased harmonic radiation. It is found that the limiting position of Ziehen is quite definite, and it is possible to key (marking and spacing) in the region of Ziehen, but clear of the limiting position. This is shown in Fig. 163,

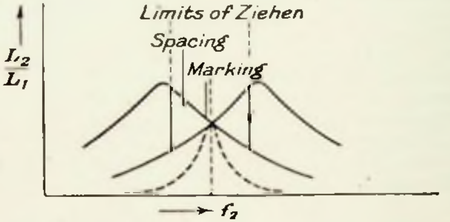


FIG. 163.—CURRENT RATIOS IN TERMS OF THE SECONDARY TUNE.

and shows the condition under which the station at Northolt actually works. The secondary is tuned to a frequency lower than the primary (but tuned by coming from the higher frequency side), the frequency generated being *below* f_1.

The actual coupling used at Northolt is actually several times the critical coupling, and it has been found that with such couplings and a ratio of $\frac{L_2}{L_1} = 3$, the efficiency, using a coupled circuit, exceeds the efficiency obtained with the arc direct in the aerial circuit. These conditions, however, necessitate a high voltage on the arc, which is not always practicable.

Valve Transmitters.

In the third form of C.W. transmitter the oscillations are maintained by the use of a thermionic valve, and this type of transmitter is rapidly becoming the most important of the three. The theory of operation is, in essence, very simple. The oscillations in a simple tuned circuit die away owing to the damping in the circuit, each oscillation being smaller than the preceding one. If the voltage variations on the condenser of an oscillating circuit are impressed across the grid and filament of a valve, similar but considerably magnified variations will be produced in the anode circuit. If these anode current variations are suitably coupled to the original oscillating circuit, they may be caused to induce in the said circuit E.M.F.'s in such a direction as to equal, at each oscillation, the voltage lost owing to the damping in the circuit, and the oscillations will thus be continuously sustained. In order to accomplish this result it is only necessary that the coupling shall be in the correct direction, and shall exceed a certain critical value. There are thus three possible cases:

1. Coupling in wrong direction. Damping of oscillating circuit increased.

2. Coupling in right direction, but insufficient. Damping of circuit reduced. This process may be continued indefinitely up to the limiting condition when the coupling reaches the critical value.

3. Coupling in right direction and above the critical value. Damping of circuit negative, so that the oscillation builds up instead of decreasing. The increase of current, however, is accompanied by an increase in the losses, which are proportional to I^2R, so that a limit is ultimately reached when the losses are equal to the total power supplied by the valve.

The design of a valve oscillator, however, involves rather more than the simple consideration of the "feedback" or "reaction" coupling, and the problem will be investigated in more detail.

Simple Valve Oscillator.—One of the simplest forms of valve oscillator is that shown in Fig. 164. Here LC is the tuned circuit connected in the anode circuit of the valve, the coil L being coupled to a second coil L′ connected across the grid and filament.

Assuming now that M has been adjusted to some value slightly greater than the minimum required for oscillation, consider what happens. The grid and

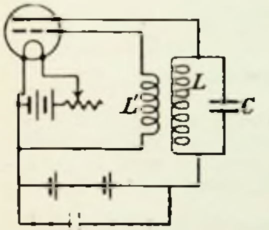


FIG. 164.—SIMPLE VALVE OSCILLATOR.

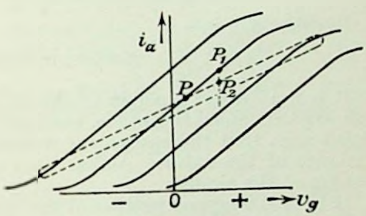


FIG. 165.—OSCILLATION ELLIPSE.

anode voltages at any instant may be considered as being such that the valve is working on a point P of the characteristic (see Fig. 165). If v_g increases, i_a will increase. This will cause an increased voltage drop in the inductance L, which will reduce the effective anode voltage. The current i_a will thus not increase to a value as at P_1, but will take up a position as at P_2, on a different characteristic curve drawn for the reduced value of v_a. The complete cycle of the current during one oscillation, therefore, will follow the ellipse shown dotted in Fig. 165.

The effect of this is to increase the effective length of the straight portion of the characteristic, the larger the inductance (and consequently the voltage drop) the more horizontal being the ellipse, and the greater being the effective straight portion. The oscillation will continue to increase until the voltage variations begin to include the curved portion of the characteristic, at which point any further increase of v_g will produce little increase of i_a, and the oscillation will automatically limit itself.

A limit to the increase of L is soon reached, however, when the voltage drop is so heavy that the variations of v_a are never large enough to sweep over the whole characteristic.

Oscillatory Current.—The current in the oscillating circuit, assuming that the mean value of the anode current i_o is half the saturation current, is given by—

$$i = \frac{i_o}{R}\sqrt{\frac{L}{C}}\sin \omega t.$$

This is the upper limit of current under favourable conditions. There is a second limit, however, which is that the maximum value of the oscillating voltage can never exceed the voltage of the battery $B = v_o$. Accepting this as a criterion, the current is given by—

$$i = v_o\sqrt{\frac{C}{L}}\sin \omega t\cdot$$

The actual value of the current is whichever of the two happens to be the less. If the former limit applies, then the current is inversely proportional to the resistance in the oscillatory circuit. If the second condition is the criterion, then the current is independent of R.

The maximum value of the current is obtained when the two conditions occur together. Hence—

$$\frac{i_o}{R}\sqrt{\frac{L}{C}} = v_o\sqrt{\frac{C}{L}}. \quad \therefore \frac{C}{L} = \frac{i_o}{Rv_o} = \frac{1}{Rr}.$$

In other words—

$$r_1 = \frac{L}{CR} = \text{effective resistance of oscillating circuit.}$$

It is not always possible to obtain this condition in practice, so it is customary to connect the anode, not to the end of the inductance L, but to some point such that the equivalent resistance of the circuit is equal to the impedance of the valve. As has been seen (p. 69), this does not affect the tuning of the circuit.

If L_1 is the tapped portion of the inductance, then the optimum condition is given by—

$$L_1 = \frac{\sqrt{Rr_1}}{\omega}.$$

As an approximation, using single layer coils, if l' is the length of the tapped portion, and l is the total length, then $l'/l = \frac{\sqrt{Rr_1C}}{L}$ approximately.

Feed Current.—Under the simple conditions assumed, the feed current flowing in the anode circuit is given by—

$$i_a = \frac{\hat{I}}{r_1}\sqrt{R^2 + (L - \mu_o M)^2\,\omega^2}\,\sin(\omega t + a),$$

where $\hat{I}$ = maximum value of oscillating current,

$$a = \frac{\pi}{2} + \tan^{-1}\omega C\,r_1.$$

Power in Oscillating Circuit.—The oscillating circuit is usually an aerial, and the useful power is then I^2R', where R' is the resistance of the aerial. For maximum efficiency this resistance should be equal to that of the coil = R. Under these conditions the power input *to the aerial* is—

$$P_a = \frac{V^2 R}{2(R^2 + \omega^2 L_1^2)},$$

where V = R.M.S. value of oscillating voltage.

This, as has been seen, is a maximum when—

$$L_1 = \frac{\sqrt{Rr_1}}{\omega}.$$

Hence $$P_a = \frac{V^2}{2(R+r_1)} = \frac{V^2}{2r_1}, \text{ since } R << r_1.$$

It is interesting to note that this is independent of μ_o, and is limited by r_1. This, again, is limited by the distance between the anode and the grid, which must be sufficient to withstand the potential difference between them, and this constitutes one of the limits to the power which can be handled by a given valve. To increase the power output, several valves may be run in parallel. This is referred to later.

High Efficiency Working.—The total power is $I^2 R = \frac{\hat{I}^2 R}{2}$.

If $$\hat{I} = \frac{i_o}{R}\sqrt{\frac{L}{C}} = v_o\sqrt{\frac{C}{L}}, \text{ then } \hat{I}^2 = \frac{i_o v_o}{R}.$$

Hence $$\text{Power} = \frac{i_o v_o}{2}.$$

The power input to the valve is obviously $i_o v_o$, so that the maximum efficiency under the conditions stated (sinusoidal voltage and current variations) is 50 per cent.

It is possible, however, to obtain higher efficiencies than this under suitable operating conditions. Fig. 166 shows the relation of the currents and

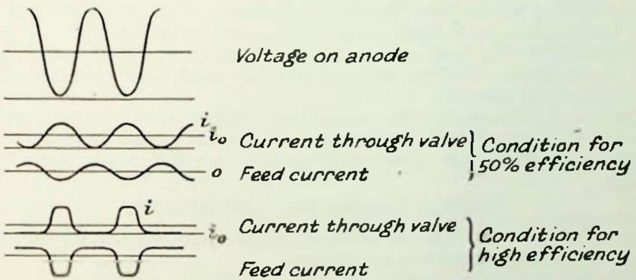


FIG. 166.—ILLUSTRATING CONDITIONS FOR HIGH EFFICIENCY WORKING.

voltages concerned in the cycle of operations. It will be seen that the alternating energy passing to the oscillating circuit is alternately extracted from and returned to the valve each half-cycle.

It is, however, extracted when the voltage is high, and returned when the voltage is low, so that there is a net extraction of energy from the valve each half-cycle, which is supplied by the steady D.C. current.

Now if the steady voltage on the grid is made more negative, the extraction of energy will be less in quantity, but will last for a longer time, the return being greater for a shorter time. So long as the two quantities are equal, the oscillating current will remain the same. In the second case, however, the mean anode current is less, so that the energy supplied to the valve is less, with consequent increase of efficiency.

This will be clear from a reference to Fig. 166; the maximum efficiency obtainable in this way is of the order of 85 per cent.

It may be observed, referring to Fig. 166, that the valve is required to supply the maximum current when the voltage on the anode is a minimum. Transmitting valves, therefore, must be designed to give full emission at low values of anode voltage. Consequently, if a fault occurs on the circuit, and the valve has to stand the full anode voltage, the emission rises to a value many times in excess of the normal value, and the valve will be destroyed, the anode melting under the excessive electron bombardment. Steps have to be taken, therefore, to protect the valve from such a contingency.

Grid Control.—It will be clear that the adequate control of the grid potential is an important factor in the operation of a valve transmitter. As the grid is often at a negative potential of 200 volts, a battery control is inconvenient. A very common method utilises the well-known grid-blocking condenser. The effect of inserting a condenser in the grid circuit of a valve is discussed on p. 175, where it is shown that small grid currents flow into the condenser, which thus acquires a negative charge. If a high-resistance leak is shunted across the condenser, this charge will slowly leak away, and the grid automatically acquires a potential such that, at each oscillation, a small grid current flows, sufficient to make up for this loss. This is the best condition for the operation of the valve. For it is obviously desirable that the grid shall not be positive for more than a small fraction of each cycle, since the presence of any appreciable grid current introduces damping into the oscillating circuit, with consequent loss of efficiency.

Transmitting valves are therefore designed so that, under normal conditions, the maximum voltage variation on the grid required to produce saturation shall only just cause grid current to flow. Even so, with large valve transmitters of 30 to 40 kilowatts, as much as 3 kilowatts may be expended in grid energy.

In adjusting the valve for maximum efficiency, the most convenient method is to increase M slightly; this automatically causes more grid current to flow, and reduces the steady grid potential until a steady state is again reached.

Coupling required for Self-Oscillation.

The value of the mutual inductance M necessary to sustain oscillations may be found approximately as follows:

The anode oscillating current—

$$i_a = \frac{V_a}{r_1 + \frac{L}{CR}} = \frac{\mu_o V_g}{r_1 + \frac{L}{CR}}.$$

Hence power in anode circuit—

$$= V_a I_a = \frac{\mu_o V_g V_a}{r_1 + \frac{L}{CR}} = \frac{\mu_o}{r_1 + \frac{L}{CR}} (IM\omega)(IL\omega)$$

(where I = R.M.S. value of oscillating current),

$$= \frac{\mu_o}{r_1 + \frac{L}{CR}} \cdot \frac{M}{C} I^2 \quad \text{since } \omega^2 = \frac{1}{LC}.$$

This must equal the power in the oscillating circuit $= I^2R$,

whence
$$M = \frac{L + CRr_1}{\mu_0}.$$

Other circuits may be treated in a similar manner.

These equations are deduced on the assumption that the resistance of the battery B is small compared with r_1, which is a legitimate assumption in practice. The effect of any grid current is also neglected. Since valves are operated in practice under conditions in which appreciable grid current does flow; the value of M thus obtained is a little too low.

A more usual form of circuit for high-power transmitters is that shown in Fig. 167. Here the generator supplying the necessary high tension voltage

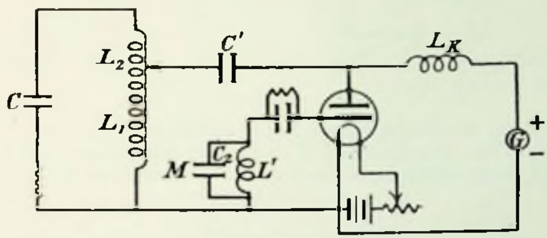


FIG. 167.—VALVE OSCILLATOR WITH CHOKE FEED.

is connected directly across the anode and filament of the valve through a radio-frequency choke coil L_k. The oscillating circuits are connected in parallel with the supply circuits. The circuit is thus to some extent similar to that employed with an arc, the D.C. supply remaining sensibly constant, variations in the anode voltage of the valve itself being set up by the oscillatory feed current which is superposed upon this steady value. The steady value of the applied voltage, however, is reflected across the anode blocking condenser C′, so that if the inductance L_k is made infinite, the circuit resolves itself into the simpler type of oscillator already considered. A condenser C_2 is often inserted across the coil L′, not for tuning purposes, but as a phase compensator to ensure that v_g is exactly 180° out of phase with v_a. The condition for oscillation with this circuit is given by—

$$M = \frac{L_1}{\mu_0} + \frac{(L_1 + L_2)RCr_1}{\mu_0 L_1}\left(1 + \frac{L_1}{L_k}\right).$$

Tuned Grid Circuit.—Another form of oscillating circuit is shown in Fig. 168. This type of circuit is principally employed for small oscillators, such as are used in receiving circuits. It is unsuitable for power circuits, because to obtain reasonable output the grid voltage variations would be so heavy as to cause saturation long before the anode circuit had developed its full power. The condition for oscillation is given by—

$$M = \frac{CRr_1}{\mu_0},$$

neglecting, as before, the resistance of the battery B.

Electrostatic Coupling.—Instead of a magnetic feedback, the necessary reaction may be obtained by direct or electrostatic coupling. One case of this is the well-known "Hartley" circuit, which is shown in Fig. 169. This circuit may be analysed in the same way as the previous ones; the filament tapping point must be near the centre point of the inductance L.

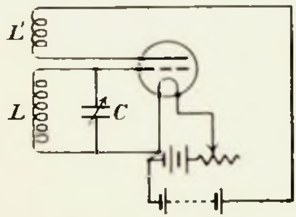


FIG. 168.—VALVE OSCILLATOR WITH TUNED GRID.

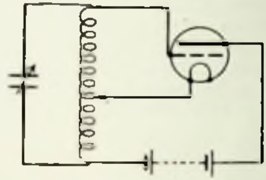


FIG. 169.—HARTLEY CIRCUIT.

Another form of circuit, employing so-called capacity coupling, is that shown in Fig. 170. The operation of this circuit is really due to a direct coupling. The voltage across L_1 is divided into two portions by the circuit L_3C_1

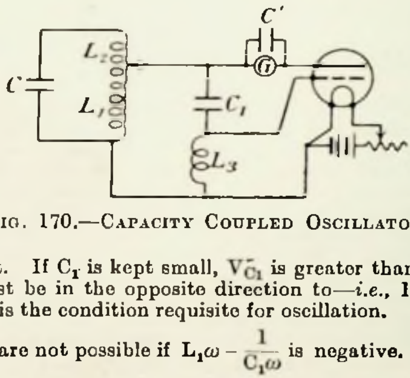


FIG. 170.—CAPACITY COUPLED OSCILLATOR.

shunted across it. If C_1 is kept small, V_{C_1} is greater than V_{L_1}, and consequently V_{L_3} must be in the opposite direction to—*i.e.*, 180° out of phase with V_{L_1}, which is the condition requisite for oscillation. Obviously, therefore, oscillations are not possible if $L_1\omega - \frac{1}{C_1\omega}$ is negative.

The condition for oscillations is thus that—

$$\frac{R + (\mu_0 - 1)\,\omega L_1 L_3 - \dfrac{L_1}{\omega C}}{r_1 C\left\{\omega(L_1 + L_3) - \dfrac{1}{\omega C}\right\}} < 0.$$

Coupled Valve Circuits.—The Ziehen effect described on p. 145 is also present in the case of a coupled circuit the primary of which is energised by a valve. There is a second condition to be considered here, however. In the case of an arc, the maintenance of the oscillation is to a large extent independent of the value of the inductance. With a valve oscillator, on the other hand, the oscillations can be maintained only if the coupling between the anode and grid circuits is adequate.

The effective value of the primary inductance depends upon which of the two possible frequencies the circuit is generating. Under certain conditions the circuit will suddenly jump from one frequency to the other, and this is a phenomenon which must be guarded against in valve transmitters employing coupled circuits.

Consider the curves shown in Fig. 171. The two full-line curves represent the variations of current in the secondary circuit, as the capacity of that circuit is varied, for the two possible frequencies.

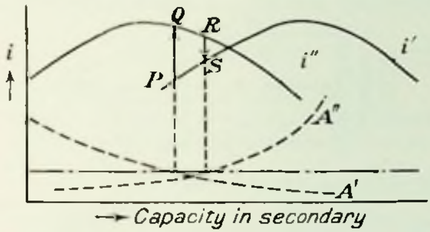


Fig. 171.—Illustrating Double Frequency Effect obtained with Coupled Circuits.

The dotted curves represent the value of $\mu_0 M$, which, in order that the oscillations may be sustained, must be greater than $L + CRr_1$, this critical value being represented by the chain dotted line.

Now assume that the circuit is oscillating at the lower frequency; the current will be as given by the curve i'. If the capacity is reduced, the current will fall and A′ will gradually rise until at the point P, A′ becomes greater than $\mu_0 M$; this oscillation can therefore no longer be sustained. A″, however, is less than the critical value, so that the system suddenly changes over to the other frequency, the current jumping from P to Q. If C is increased again, the system continues to oscillate at the lower frequency until this point R is reached, when A″ becomes greater than the limit. A′ is now less than the limit, so the system jumps to the lower frequency again.

This frequency jump is perfectly definite, the frequency jumping up as the capacity is increased, and *vice versa*.

The critical period where sudden frequency jumps take place may occur at various values of the secondary capacity according to the constants of the circuit, and under certain conditions might occur at the point X. At such a point the change in frequency would occur with little or no circuit change, so that there would be no indication that anything was amiss.

Provision of High Tension Voltage.—As the production of steady potentials of the order of 5,000 to 10,000 volts by D.C. machines is a somewhat expensive proposition, it is customary to provide the necessary high tension voltage for valve transmitters by employing alternating current which is transformed up to a suitable voltage, and is then rectified by a suitable means. The simple thermionic valve, provided with two electrodes only, provides an admirable means of rectifying the high voltage alternating current.

The circuit employed is as shown in Fig. 172, the current flowing through the top valve when the point A is positive, and through the bottom valve when the point B is positive (*i.e.*, the next half-cycle). Both halves of the wave are thus rectified, such a process being known as double-wave rectification. If only one valve is employed, only one half of the wave is rectified, this being known as single-wave rectification.

The currents passing through the valves charge the condenser C_1 in the same direction, and the required D.C. supply is drawn off from this condenser, which thus acts as a reservoir.

The design of a rectifying circuit consists primarily in finding the capacity of this condenser. As the load is taken from the condenser the voltage falls,

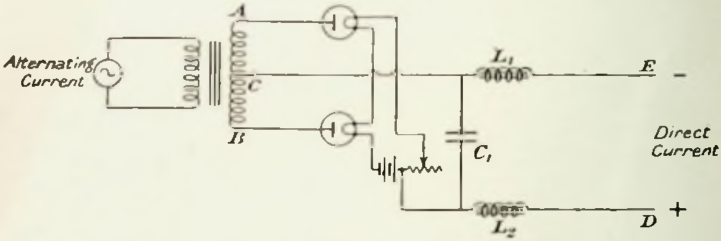


FIG. 172.—PROVISION OF H.T. SUPPLY FROM RECTIFIED A.C.

being replenished in a series of pulses, so that the voltage is actually slightly pulsating. The larger the condenser the less the ripple, while choke coils L_1 and L_2 are inserted to assist still further in the production of a comparatively steady voltage.

The action is analysed in more detail in Fig. 173.

Here the full line represents the voltage delivered by the secondary of the transformer. No current will flow through the rectifying valves until the voltage of the transformer is greater than that of the condenser C_1.

FIG. 173.—CURRENT AND VOLTAGE RELATIONS IN RECTIFYING VALVES.

At this point, however, a sudden rush of current will flow through the particular valve, which will continue until the voltage has again fallen below that of the condenser, and the effect of this will be to increase the condenser voltage by a small amount. The voltage will then fall gradually, due to the current taken out of the condenser by the load, and will continue to fall until the next rush of current takes place, either through the same valve if single-wave rectification is employed, or through the other valve with the more usual double-wave rectification.

Fig. 173 is drawn on the latter assumption, all the current pulses being shown on the same side of the axis for clearness.

Now it will be obvious from the figure that the times of charging and discharging the condenser are approximately the same, and equal to $1/4f$, where f is the frequency of the alternator supply.

If V is the initial voltage on the condenser, and R is the resistance of the load circuit, then the decrease of voltage in time $1/4f$ is given by—

$$v = V\left(1 - \varepsilon^{-\frac{1}{4fCR}}\right),$$

V being the average voltage on the condenser.

This is the maximum variation of voltage, which must be kept within certain limits according to the ripple permissible. The size of the condenser C may thus be determined.

The effect of the choke coils may be determined by the application of the usual alternating current laws.

Several rectifying valves may be, and are, employed in parallel if the power to be handled is greater than the dissipation of a single valve.

Where a bank of valves is to be employed it is often convenient to use a three-phase supply. By use of suitable connections, as shown in Fig. 174,

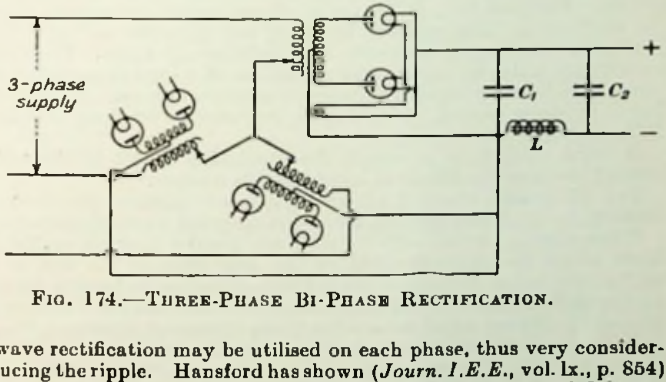


FIG. 174.—THREE-PHASE BI-PHASE RECTIFICATION.

double-wave rectification may be utilised on each phase, thus very considerably reducing the ripple. Hansford has shown (*Journ. I.E.E.*, vol. lx., p. 854) that under suitable conditions the ripple is reduced to zero. If θ is the electrical angle elapsing between the beginning of the cycle and the beginning of the pulse of current through the rectifier, then the critical angles when no ripple occurs are—

Two-phase (double-wave rectification) $\theta = \frac{\pi}{4}$.

Three-phase (double-wave rectification) $\theta = \frac{\pi}{3}$ or $\frac{\pi}{6}$.

Since $\frac{V_{average}}{V} = \sin\theta$, these points can be expressed in terms of

V_{av}/V_{max}. Fig. 175 shows the percentage ripple obtained in terms of this ratio for single, two-phase, and three-phase voltages.

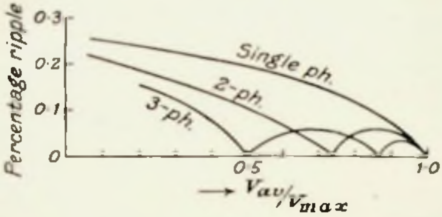


FIG. 175.—PERCENTAGE RIPPLE FOR VARIOUS RECTIFIER SYSTEMS (ALL DOUBLE WAVE).

With the circuit given on Fig. 174, a 300 ∼ ripple is obtained, the supply being 50 ∼. In an actual case C_1 and C_2 were 0·25 μF each, and L was 20 henries.

Keying.—Keying on a valve oscillator may be effected in a variety of ways. One method is to break the H.T. supply, or to reduce it by inserting a high resistance in the feed circuit.

A second method, very often used, is to disconnect the leak in the grid circuit. The grid then builds up to a large negative potential, which stops the oscillation. This method, however, may give rise to "grid tick," the condenser discharging suddenly and building up again. Each discharge permits the valve to oscillate for a fraction of a second, so that a series of ticks are radiated, occurring at intervals of two or three seconds up to twenty-five or more per second. This may be overcome by breaking or reducing the H.T. supply simultaneously.

A third method is to couple the oscillating circuit to the aerial, and control the coupling circuit in some suitable manner.

For all powers above 3 kilowatts, it is advisable to permit the set to oscillate feebly during spacing, instead of stopping the oscillation completely.

Compensating Chokes.—Choke coils are usually inserted in the filament leads which are short-circuited by the depression of the key, so causing an increase in the filament current, which compensates for any voltage drop on the mains when the load is applied, and also allows for the diminution of filament brilliancy which occurs due to the sudden electron emission.

Some details are appended relating to the valve transmitter at the Northolt Radio Station. These details are taken from a paper by A. C. Warren (*Journ. I.E.E.*, vol. lxii., p. 967).

The valve panel is capable of delivering 40 kilowatts to the aerial. Three water-cooled metal valves are used, each capable of dissipating 5 to 10 kilowatts. Alternatively, six silica valves, dissipating 2½ kilowatts each, may be employed.

The H.T. supply is obtained from a three-phase bi-phase arrangement, as shown in Fig. 174, which is arranged to deliver a voltage of 8,000 to 18,000 in 1,000-volt steps.

" In order to run valves in parallel it is essential that they shall all have the same characteristics. Inequalities between valves introduce intervalve oscillations or ' spurious ' circuit oscillations. To this end all valves should have separate filament regulation, and by this means it is possible to equalise the loads carried by each valve.

"The circuit finally employed is shown in Fig. 176. For efficiency the oscillatory grid voltage should be 180° out of phase with the oscillatory anode voltage. This result will be obtained if the impedance of the anode blocking condenser C_4 is very low compared with the effective impedance of the aerial circuit. The proportions of the circuit for working with three water-cooled metal valves are as follows:

$L_1 = 2{,}000\ \mu H.$	$C_2 = 7{,}000\ \mu\mu F.$	$R_2 = 1{,}000$ ohms.
$L_2 = 10{,}000\ \mu H.$	$C_1 = 0{\cdot}3\ \mu F.$	$R_5 =$ carbon lamp.
$L_4 = 100{,}000\ \mu H.$	$C_4 = 0{\cdot}1\ \mu F.$	
	$C_5 = 5{,}000\ \mu\mu F.$	

"If the best conditions have been determined for any given circuit constants, then, as this circuit is altered to retain best conditions, we should keep

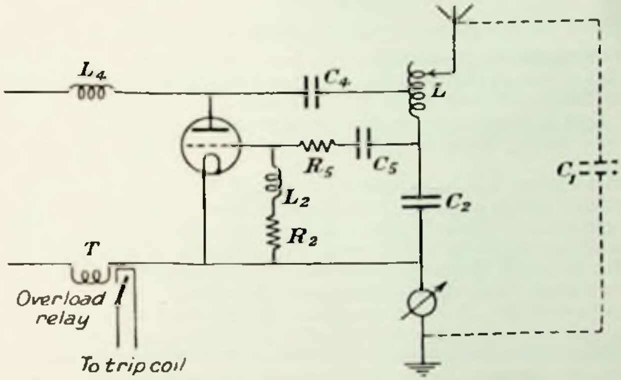


FIG. 176.—SKELETON CIRCUIT AT NORTHOLT VALVE PANEL.

constant the anode voltage and grid voltage both D.C. and H.F. As the number of valves (N) is increased, the aerial current (I) increases in proportion to the square root of the number of valves. Thus the effective anode tap must be decreased in the same ratio and the capacity of C_2 must be increased.

"The adjustment of the anode tap does not affect the efficiency within reasonable limits, but determines the output of the set, the anode tap being reduced to increase the output. The value of the grid coupling condenser is determined by the type of valve used, the aerial current, and the value of the grid leak.

"Referring to Fig. 176, C_2 is the grid coupling condenser, R_2 the grid leak, and L_2 a choke to prevent high-frequency current passing through the grid leak circuit. C_5R_5 is a protective circuit in case the anode blocking condenser C_4 should break down (short circuit), and prevents the instantaneous current flow from throwing the main H.T. voltage on the valve grids before the overload relay T operates and makes the set 'dead.'"

For high-powered valve sets, however, the modern tendency is to generate the oscillations with what is known as a master valve. The currents so obtained are then amplified to any required degree by a bank of high-power valves. The advantage of such a system is that the frequency is independent of the aerial, and is thus unaffected by swaying in a high wind, etc.

The frequency of the master valve may be controlled with very great precision; thus the wave length radiated remains absolutely constant, which enables very selective receiving devices to be employed.

RADIO RECEIVING APPARATUS.

The design of radio receiving apparatus falls into three main classes:

(*a*) The aerial system.
(*b*) The tuning system.
(*c*) The amplifying system.

Of these, (*b*) and (*c*) are to a large extent interdependent.

The Principles of Reception.—The first stage in the reception of electromagnetic waves is the erection of a circuit which will be influenced by the electric fields radiated from the transmitting station. In general, it is found that a good radiator is a good receiver, so that a receiving aerial may, and usually does, consist of a simple vertical wire, with or without a flat top.

Such a system will have induced therein E.M.F.'s due to the electric field of the wave, and in order to obtain the maximum current from these E.M.F.'s the aerial system is tuned to the frequency of the incoming wave, so reducing the aerial impedance to a minimum. This process of tuning is also essential in order to select the particular station required from the whole gamut of electric waves constantly sweeping past the aerial.

Detection of the Received Currents.—These currents, however, are very feeble, and the only instrument capable of detecting such small currents is a telephone receiver, which will respond to currents as low as 5×10^{-9} amperes. Neither the telephone nor the ear, however, will respond to frequencies of the high order used for radio communication, and it is necessary, therefore, first to modulate the radio-frequency currents at an audible frequency, and, secondly, to rectify these currents, so that they are converted into unidirectional pulsating currents. These currents will then give rise to vibrations of the telephone diaphragm in accordance with the impressed modulations.

The modulation may be effected either at the transmitting or the receiving end. In the case of spark or telephony transmitters, the transmitted wave is suitably modulated. A spark transmitter radiates trains of waves at a musical frequency, while with a telephony transmitter the radio-frequency currents are modulated in accordance with speech vibrations.

Fig. 177 illustrates the effect of rectification for a spark train. The average value of the currents, even when modulated, is zero until rectification takes place, producing a unidirectional pulse. The aggregate of these pulses, occurring at a musical frequency, produces a musical note in the telephone.

Heterodyning.—The most general system in use to-day, however, particularly for long distances, employs unmodulated continuous waves, and in order to render the received currents audible, some form of modulation has to be introduced at the receiving end. This is accomplished by the process known

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as *heterodyning*, which consists in introducing into the circuit a second oscillation at a frequency nearly equal to that of the incoming signal. The two oscillations will then combine and produce beats in the usual way, and these beats may be arranged to occur with any desired frequency. If f_1 and f_2

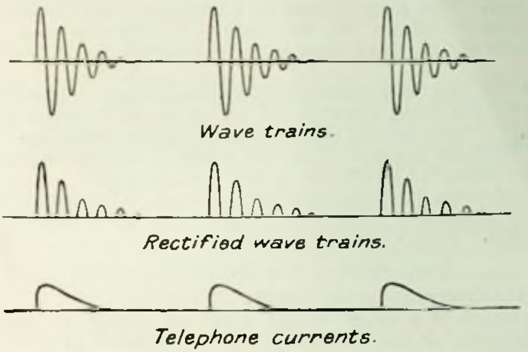


FIG. 177.—ILLUSTRATING CONVERSION OF HIGH FREQUENCY CURRENTS TO UNIDIRECTIONAL PULSES BY MEANS OF RECTIFICATION.

are the frequencies of the signal and local currents respectively, the combination of the two will be a frequency $\frac{f_1+f_2}{2}$ modulated at a frequency $\frac{f_1-f_2}{2}$.

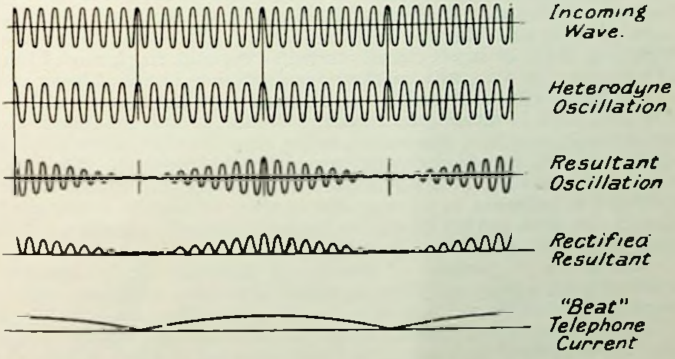


FIG. 178.—ILLUSTRATING PRINCIPLE OF HETERODYNING.

This, when rectified, will produce currents in the telephone pulsating with a frequency $f_1 - f_2$. The effect is demonstrated in Fig. 178.

Other methods of modulating the received wave have been employed from time to time, but the heterodyne system is by far the most common.

Design of Tuner.

The reactance of a tuned circuit is zero at a particular frequency, but is also comparatively small for a band of frequencies on either side of the resonant point. This may be seen by referring to the resonance curve of a simple series circuit (p. 69), in which it will be seen that appreciably large currents are obtained even when the circuit is not quite in resonance.

Consequently, any transmitting stations working on wave lengths close to that of the station being received will set up currents comparable with the actual signal current, and will thus interfere with the reception. The extent of such interference depends upon the width of the resonance curve, the narrower the curve the greater being the "selectivity."

It is thus of value in designing a tuning circuit to be able to estimate the width of the resonance curve at a point where the current is $1/n$ of the resonant value. This is given by the expression—

$$\text{Band width} = W = \delta\sqrt{n^2 - 1}/\pi \text{ cycles,}$$

where δ is the decrement of the circuit.

The decrement of a single circuit, however, cannot be reduced below $\delta = 120/f$, where f is the frequency, or the signals lack definiteness and "ringing" occurs. Hence to increase the selectivity the first circuit is loosely coupled to a second circuit tuned to the same frequency.

The resonance curve of two coupled circuits is not only narrower than that of a single circuit, but is also more flat topped. This is due to the effect of one circuit on the other (see p. 80). By employing a chain of such circuits, a "band filter" is obtained, having a flat top and a sharp cut off (*i.e.*, steep sides).

If all the circuits are similar the band width of such a filter is given by putting $n^{2/p}$ for n^2 in the above formula, p being the number of circuits.

In addition to tuning the high frequency oscillations, it is possible when receiving C.W. to insert circuits tuned to the note frequency of the signal. (This cannot be done with spark systems, the note obtained being composed of a series of pulses which is by no means a pure sine wave, and hence does not lend itself to filtering by resonant circuits.) This practice is becoming increasingly common, one reason being that low frequency amplifiers are more efficient, a second reason being the cheaper cost. The problem is simply one of convenience and economics of design. Provided the signal strength is sufficient to operate the rectifier efficiently, experiments have shown that high and low frequency tuning are equally effective, but that low frequency tuning is distinctly cheaper.

Economical Proportions of Tuner.—A detecting device operates more efficiently on a reasonably strong signal than on a weak one, and hence it is advisable to have a certain high frequency amplification. (But see p. 175.)

Moreover, with the heterodyne system there are two different frequencies, one on each side of the heterodyne frequency, each of which will give the same heterodyne note, and a simple note tuned amplifier cannot discriminate between these two.

It is customary, therefore, to provide sufficient high frequency tuning to eliminate one side of the heterodyne band, and then to adopt low frequency tuning to obtain the remaining selectivity desired. The design of tuning circuits to have the requisite characteristics may be carried out in accordance with the principles laid down in the first two sections. The condition for selectivity is simply that $\frac{R}{L}$ shall be as low as is practicable.

In some cases where particular interference on a definite wave length is experienced, circuits possessing infinite impedance points (rejectors) may be utilised.

Design of Amplifiers.

The next point to be considered is the design of the necessary amplifier. Under this heading also comes the detector, which is considered later. In many cases the tuning circuits are made an integral part of the amplifier; in such circumstances, a judicious application of the principles just considered, coupled with the essential features of amplifier design, should produce a satisfactory receiver.

The valve may be regarded as an amplifying repeating device, and the design of an amplifier consists in determining the requisite impedances of the input and output circuits in order to obtain the maximum efficiency.

Input Circuit.—The impedance of the input circuit should theoretically be of the same order as the internal impedance between the grid and filament of the valve.

The grid filament impedance of a valve comprises two factors:

(*a*) The internal resistance between grid and filament.
(*b*) The effective capacity between grid and filament.

The resistance depends to a large extent upon the grid potential. With a negative grid, r_g is practically infinite, but as soon as the grid becomes positive, grid current starts to flow and the resistance falls rapidly.

Since, however, the grid potential is nearly always zero or negative, the resistance may be taken as being high, and consequently should be matched with a high impedance input circuit. The actual design of the input circuit, however, is to some extent controlled by other factors, as will be seen, and the actual condition to be complied with in practice is that the alternating voltage applied to the grid should be as high as possible.

There is still the effect of the second factor to be considered.

Morecroft has shown that for ordinary receiving valves the capacity between grid and filament with the anode free is from 6 to 10 $\mu\mu$F, while that between grid and anode, with the filament free, lies between 3 and 14 $\mu\mu$F. Morecroft calls these the *geometric* capacities, in distinction to the *effective*

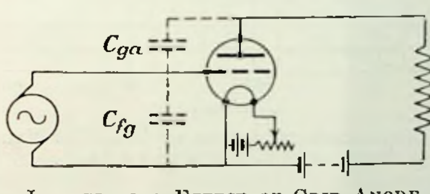


FIG. 179.—ILLUSTRATING EFFECT OF GRID ANODE CAPACITY.

capacity, which is considerably greater. For, referring to Fig. 179, it will be seen that the alternating voltage charges the condenser C_{fg} to a voltage v_g. It also, however, charges the condenser C_{ga} through the anode circuit.

and since the anode circuit contains an E.M.F. μv_g, where μ is the amplification factor of the valve, the total voltage to which C_{ga} is charged is $(\mu+1)\, v_g$.

The current taken is thus the same as would be taken by an equivalent condenser—

$$C_{eff} = C_{fg} + (\mu + 1)\, C_{ga}.$$

Now it will be seen later that μ depends upon the impedance in the anode circuit, varying from 0 to μ_0 where μ_0 is the voltage amplification factor obtained from the characteristic. Hence C_{eff} depends on the conditions under which the valve is operating. If—

$$\mu = 6, \text{ say, and } C_{fg} = 7\ \mu\mu\text{F},$$
$$C_{ga} = 9\ \mu\mu\text{F},$$
$$C_{eff} = 7 + (7 \times 9) = 70\ \mu\mu\text{F}.$$

The impedance of this at low frequencies is negligibly high—3 megohms at 800 cycles, but at a frequency of 500,000 (600 metres) the impedance is only 7,000 ohms, which exercises a very appreciable shunting effect. Hence for high frequency amplification it is desirable to use valves having a low interelectrode capacity and also a low amplification factor.

Actually, the effective capacity is only about 80 per cent. of the value given above, due to mutual action between the electrodes.

Working Point on Characteristic—Effective Amplification.—Before considering the output circuit it will be as well to consider the effect of impedance in the anode circuit upon the characteristic and the amplification factor.

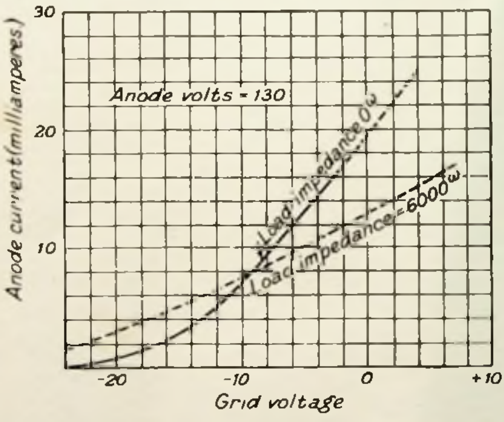


Fig. 180.—Characteristic Curve of W.E. Co. Valve under Different Conditions of Load.

Fig. 180 shows the characteristic curves of a Western Electric Power valve. The full line is the characteristic as usually taken, with no external impedance in the anode circuit. At -9 volts on the grid and 130 volts on the anode, the anode current is 7·5 milliamperes. If the grid voltage is increased to -5 volts, the anode current increases to 13 milliamperes.

If, however, the anode circuit contains an impedance of 6,000 ohms, the voltage on the anode will be reduced, which will limit the rise of the anode current to 10 milliamperes. Similarly, a decrease in grid volts causes a decrease of current, which is counteracted to some extent by a rise in anode voltage due to a reduced voltage drop on the external impedance. The net result is that to alternating voltages the characteristic with an external impedance of 6,000 ohms in the anode circuit is as shown by the dotted line.

It will be observed that this characteristic is substantially a straight line, but of considerably less slope. Hence the effective amplification is less. Actually, the effective value of the amplification factor is given by—

$$\mu = \frac{\sqrt{R^2 + X^2}}{\sqrt{(R + r_1)^2 + X^2}} \mu_o,$$

where R and X are the resistance and reactance of the external impedance,

r_1 is the internal impedance of the valve,
μ_o = voltage amplification factor.

If the external impedance is a pure resistance—

$$\mu = \frac{R}{R + r_1} \mu_o.$$

If the external impedance is a pure reactance—

$$\mu = \frac{\omega L}{\sqrt{r_1^2 + \omega^2 L^2}} \mu_o,$$

assuming $R << \omega L$, which is usually valid.

In the above formulæ it is important to note that the value of r_1 must be taken under the correct working conditions—*i.e.*, v_a must be the actual voltage on the anode, allowing for the resistance drop in the impedance.

The value of the grid voltage also affects r_1, which increases as the voltage on the grid is made negative, and this effect must also be taken into consideration.

Output Circuit.—The output circuit is again designed in accordance with the principle that the external and internal impedances of the valve should be of the same order, but the question of the efficient repeating of the signal between one valve and the next has also to be considered. It will be best to treat the several possible cases separately.

There are three main types of amplifier:

(*a*) Resistance coupled, in which the output circuit is a high resistance, the voltage across which is transferred direct to the grid through a condenser.

(*b*) Reactance coupled, which is similar to (*a*), except that a reactance is employed.

(*c*) Transformer coupled, which is self-explanatory.

Resistance Coupling.—An amplifier of this type is shown in Fig. 181. The variations of anode current produce varying voltages across the resistances R_3 and R_4, which are transferred to the grid circuit through the condensers C_1 and C_2. These condensers are necessary because the steady anode current produces a steady positive potential across the resistances, and if such were applied to the grid the valve would become inoperative. The grids, therefore, are isolated by means of the condensers shown. A condenser in such a position, however, gradually acquires a negative potential under the influence of an incoming signal. This phenomenon is discussed on p. 175, but it will suffice to remark here that to avoid this negative bias from becoming excessive, a high resistance leak is connected across the grid and filament.

Considering now the general case where R is the resistance in the anode circuit, the anode current—

$$i_a = \frac{\mu_o v_g}{R + r_l},$$

$$v_a = i_a R = \frac{\mu_o v_g R}{R + r_l},$$

whence $$\mu = \frac{v_a}{v_g} = \frac{R}{R + r_l} \mu_o.$$

Hence μ increases towards μ_o as R is increased. But the H.T. voltage must be increased at the same time, otherwise r_l increases and μ falls off.

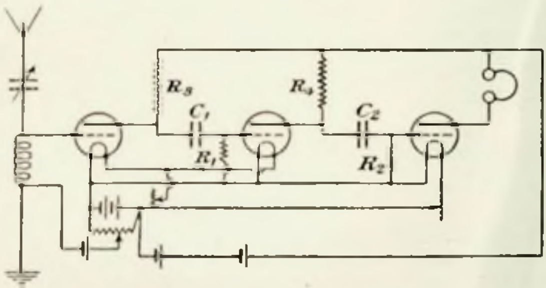


Fig. 181.—Resistance Coupled Amplifier.

It is easy to show that for a given H.T. voltage the maximum amplification occurs when $R = r_l$, when $\mu = \frac{\mu_o}{2}$.

This value may be exceeded by making R greater than r_l, provided the H.T. voltage is increased, so that the effective anode voltage, allowing for the drop in the resistance, remains the same.

The economical limit is when R is two or three times r_l. R must be so designed as to dissipate i_o^2R watts continuously, where i_o is the mean value of the anode current.

The size of the transfer condensers depends upon the frequency. The impedance of the condenser should be small compared with the grid filament impedance. At low frequencies this requires a condenser of the order of 0·05 μF, but a condenser of 200 to 400 $\mu\mu$F is satisfactory at radio-frequencies (see also p. 176). The leak should vary from 0·25 MΩ at low frequencies to 2–3 MΩ at radio-frequencies.

Subject to the limit imposed upon the net amplification, and the high H.T. voltage required, resistance amplification is a very satisfactory method at low frequencies and for radio-frequencies up to about 300,000 cycles (1,000 metres). Beyond this point the shunting effect of the valve capacity (anode-filament) begins to exercise an appreciable effect.

Reactance Coupling.—Here, as has been seen, $\mu = \mu_o \frac{X}{\sqrt{(r_l^2 + X^2)}}$. The problem is thus similar to that just considered, but has the advantage that the H.T. does not have to be increased as X is increased, since the resistance drop in the reactance is negligible.

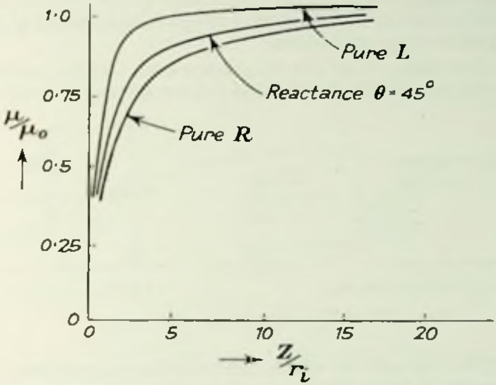


Fig. 182.—Showing Effect of Anode Impedance.

The value of μ/μ_o in terms of X/r_i is given in Fig. 182, which also shows the resistance coupled condition. It will be seen that if $X = 2r_i$, μ practically $= \mu_o$. This type of coupling, therefore, is considerably more efficient, but the fact that the amplification can never exceed μ_o is a serious disadvantage.

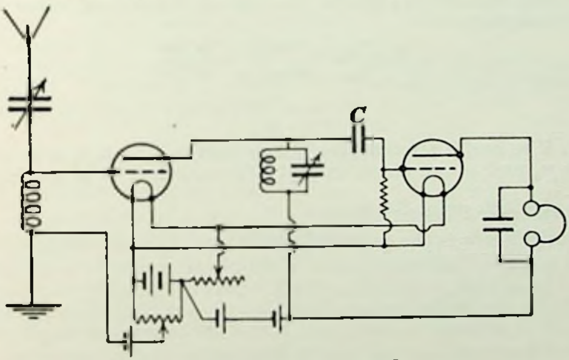


Fig. 183.—The Tuned Anode Circuit.

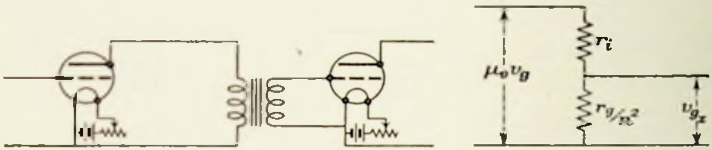
The same remarks concerning frequency apply here as with resistance coupling, while at radio-frequencies the reactance is further shunted by the distributed capacity of the coil.

This leads to a very common form of coupling known as the tuned anode system. A condenser is inserted in parallel with the reactance as shown in

Fig. 183. By varying the condenser the coil can be tuned to the incoming signal, at which point the impedance of the tuned anode combination is a maximum (tending to infinity if R is low), and hence the maximum voltage is applied to the grid.

It must be remembered, however, that μ can never exceed μ_0, no matter how large the grid voltage variations.

Transformer Coupling.—The low frequency case will be considered first. The actual circuit is as shown in Fig. 184 (*a*), neglecting the valve batteries,



(*a*) Actual circuit. (*b*) Equivalent circuit.

FIG. 184.—TRANSFORMER COUPLING.

while the equivalent circuit, in which the secondary impedance is reflected into the primary circuit by the usual transformer laws, is shown in Fig. 184 (*b*).

This assumes no magnetising current in the transformer, zero internal impedance (leakage reactance), and a resistive secondary load. Then—

$$v_{g_2} = \frac{r_g/n^2}{r_i + r_g/n^2}\,\mu_0\, v_g = \frac{\mu_0\, r_g}{n^2 r_i + r_g},$$

where n is the transformation ratio of the transformer.

The actual voltage applied to the grid of the second valve is n times this value, so that—

$$S = \frac{v_{g_2}}{v_{g_1}} = \frac{n\mu_0 r_g}{n^2 r_i + r_g} = \frac{\mu_0 n a}{n^2 + a},$$

where
$$a = \frac{r_g}{r_i}.$$

Hence, if μ_0 and n are constant, S increases to a limit $\mu_0\, n$ as a increases, while, if μ_0 and a are constant and n is varied, S is a maximum when $n = \sqrt{a}$, at which point $S = \mu_0 \frac{n}{2}$.

This second case is the practical condition. It is not always practicable to make $n = \sqrt{a}$, but in order to obtain best results it will be seen that r_g should be as high as possible, which is accomplished by making v_{g_0} negative. This is also desirable from the point of view of distortionless amplification, which is referred to later, and hence low frequency amplifying valves are always designed to work with a large negative grid potential.

Now in a practical type of low frequency transformer the leakage reactance and conductor resistance are negligible, as assumed, but the magnetising current is not. In order to keep this factor low, the no-load reactance (secondary open) should be high. This reactance is in parallel with r_g/n^2 in Fig. 184 (*b*), and thus reduces v.

Fig. 185 shows the values of S for a transformer having $n = 4$ coupled to a valve of which $\mu_0 = 6$ and $r_i = 10{,}000$ in terms of X_0, the no-load reactance.

It will be seen that there is little advantage in increasing X_0 above 20,000 ohms, but that S is very low if X_0 is too small.

It also shows that although the value of $n = 4$ is about right for $r_g = 25{,}000$ and is much too low for $r_g = 10^6$, yet S in the latter case is distinctly higher, indicating that it is very important to maintain r_g as high as possible.

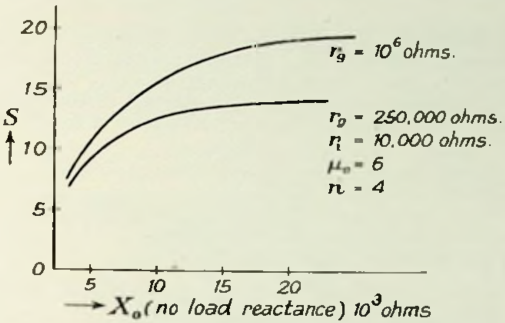


Fig. 185.—Effect of Primary Reactance.

In a practical transformer, the higher the number of secondary turns the greater the leakage reactance, and also the greater the distributed capacity of the winding which reduces the effective transformation ratio. Hence a practical limit is reached with $n = 4$ or 5, this limit being even lower if the value of $a = r_g/r_i$ is small.

In practice X_0 is made about twice r_i, which for ordinary frequencies gives a no-load inductance of the order of 10 to 20 henries.

High Frequency Transformers.—At high frequencies the problem is somewhat modified. Here there is considerable leakage between primary and secondary. If k is the coupling factor, then—

$$S = \frac{\mu_0 \omega k \sqrt{L_1 L_2}}{\sqrt{\omega^2 L_1^2 + r_i^2}}$$

neglecting the resistance of the coils.

S thus varies directly as k and $\sqrt{L_2}$, but in a complex manner with L_1. Other things being equal, S is a maximum when $\omega L_1 = r_i$, but this usually gives a value of L_1 which brings the natural period of the coil within the working range.

It is therefore usual to tune L_1, under which conditions ωL_1 becomes $>> r_i$ and the expression reduces to—

$$S = \frac{\mu_0 k \sqrt{L_2}}{\sqrt{L_1}}.$$

Hence L_1 should be made as small as possible, down to the limit where the effective resistance of the circuit (L_1 and C in parallel) $= \frac{L}{CR}$ becomes comparable with r_i. L_2 may then be as large as possible up to the limit when the coil and valve capacities begin to be troublesome.

This limit occurs somewhat early, so that high frequency amplification cannot be made highly efficient, particularly as a valve with a low value of μ_0 should be used, as previously explained.

The trouble may be overcome by tuning L_2, but then k must be small to avoid double frequency effects, which immediately reduces S again.

Such an arrangement, however, is very selective, but requires careful handling.

Telephone Receivers.—The impedance of a telephone receiver varies in a complex manner with frequency. If the diaphragm is clamped, the effective resistance and reactance increase with the frequency. But if the diaphragm is free to move, there is superposed the "motional" resistance and reactance, which becomes very large near the resonant points of the diaphragm. It is

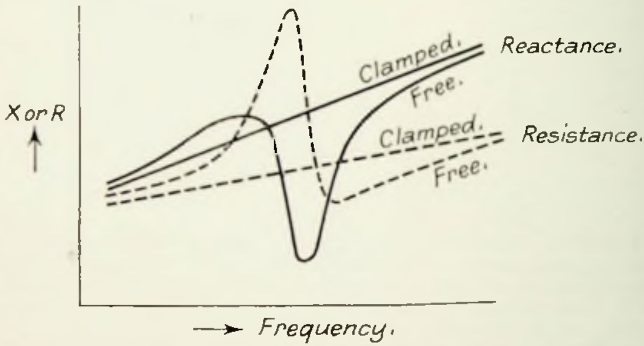


FIG. 186.—IMPEDANCE AND RESISTANCE CURVES OF A TELEPHONE RECEIVER.

the motional impedance that is of importance, since it is a measure of the energy being transformed into useful work. Fig. 186 illustrates this point.

The design of a telephone is somewhat complex, but empirically it is found best to make the total impedance of the same order as r_l.

A high resistance instrument has many turns of fine wire, with a high R and Z, and is thus suitable for inclusion direct in a valve circuit. It is not so robust, however, as a low resistance telephone, which has fewer turns of thicker wire. Such an instrument has a lower impedance, and thus requires to be coupled to the valve circuit through a suitable transformer, which may be designed on the principles just laid down, substituting Z for r_g.

This method of treating transformer coupled amplifiers is due to Morecroft, and for further information the reader is referred to "The Principles of Radio Communication," by that author.

Distortionless Amplification.—The first essential for the faithful reproduction in the output circuit of the E.M.F.'s applied to the valve is that the working portion of the characteristic shall be substantially linear. If this is not the case then distortion occurs.

Assume that the characteristic obeys a square law. Then—

$$i_a = A\,[v_a + \mu_0(v_g + V\sin\omega t)]^2,$$
$$= A\,(v_a + \mu_0 v_g)^2 + 2A\mu_0\,(v_a + \mu_0 v_g)\,V\sin\omega t$$
$$+ \mu_0^2\,\frac{AV^2}{2}\cos(2\,\omega t + \pi) + \mu_0^2\,\frac{AV^2}{2},$$

where V_a and V_g are the steady values.

The third term is of double frequency, and thus introduces distortion. The fourth term is a constant term which will cause an increase of i_a. Hence, as a test for distortionless working the applied E.M.F. should be increased. If the anode current increases then distortion is present. With a linear law this fourth term is not present, and hence the average anode current is unaffected by V.

The process of detection also introduces distortion. (See p. 203.)

A second cause of distortion is due to the presence of grid current. As soon as the grid becomes positive the resistance r_g becomes comparatively low, introducing heavy damping during part of the cycle.

To avoid these two causes of distortion, the grid is operated at a negative potential, such that the maximum variation of v_g never causes any appreciable

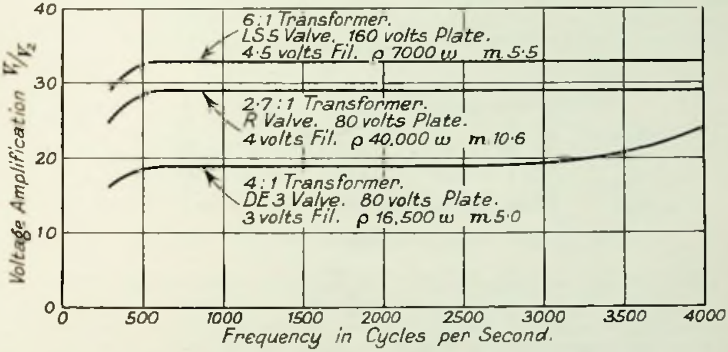


FIG. 187.—AMPLIFICATION CURVES OF MARCONI IDEAL TRANSFORMER.

grid current. This involves working well down the characteristic, where the law is not linear unless the external impedance is of the same order or greater than r_1. Moreover, the anode voltage must be so chosen that with the requisite negative grid voltage the working point on the characteristic falls about *midway between zero grid volts and the voltage at which the anode current is reduced to zero*. This point occurs at − 9 volts in Fig. 180, which has already been referred to.

A third source of distortion is due to the transformers used to couple successive valves together. Here it is essential to maintain the amplification sensibly constant over the working range of frequency. (This, of course, only applies to speech or music amplification.)

S has been seen to be $= f\,(n_1\,X_o)$. As the frequency increases, n falls off owing to the greater effect of distributed capacity. X_o, however, rises, and these two effects tend to neutralise each other.

Hence, with a well-designed transformer S can be maintained constant over a very wide range of frequency. Fig. 187 illustrates a practical case of good design.

The range of frequency over which amplification should be maintained for faithful speech reproduction is about 500 to 2,000. Music requires a rather larger band.

Dye has discussed the question of transformer design very thoroughly (*Experimental Wireless*, vol. ii., Nos. 12, 13, and 14). He shows that—

1. The addition of a shunt resistance across the secondary tends to equalise the amplification at various frequencies. No serious loss of amplification results if this shunt is not less than $0{\cdot}2n^2$ megohms.

2. A shunt capacity across the secondary of a value about $\frac{1{,}000}{n^2}$ $\mu\mu$F will improve the effective amplification ratio. The distributed secondary capacity should be included in this shunt.

Limitations of High Frequency Amplifiers.—Apart from the limitations just referred to, the design of high frequency amplifiers is further complicated by the fact that the capacity between grid and anode of the valves

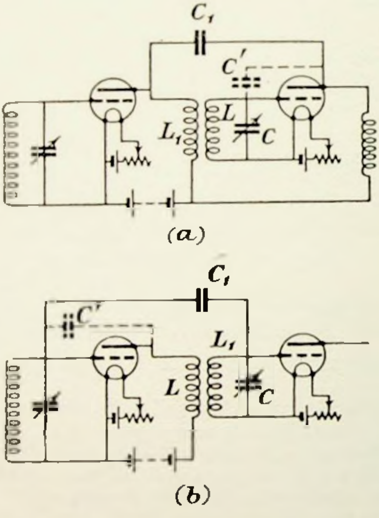


FIG. 188.—METHODS OF NEUTRALISING VALVE CAPACITY.

causes a certain reaction effect which tends to cause the associated circuits to generate continuous oscillations. This is highly undesirable, since it is not under control.

Blatterman has shown (*Radio Review*, October, 1920) that no reaction due to internal capacity is possible where the input and output impedances are pure resistances.

In other cases oscillation may occur under suitable conditions.

(*a*) Resistance R_g in grid circuit; inductance L_a in anode circuit.

Oscillation occurs if $\omega^2 L_a C_m R_g < r_t$, where C_m is the capacity from grid to anode.

(*b*) If the grid circuit is inductive $= L_g$, then no matter what the anode impedance, oscillations will result if $\omega^2 L_g C_m < 1$.

This is the dangerous case because the frequency of the oscillation is independent of the anode tune, and therefore may be at a totally different frequency.

Various circuits have been devised for overcoming this difficulty of self-oscillation. One, due to Hazeltine, known as the neutrodyne, consists in neutralising the effect of the valve capacity by a suitably connected very small external capacity. Consider Fig. 188 (*a*). Some of the energy in the anode circuit of the second valve is transferred through the anode-filament capacity to the grid circuit, so causing self-oscillations.

To neutralise this a small capacity C_1 is connected between the anodes of the valves. This permits a small current to flow through the coil L_1, which induces an E.M.F. in the coil L equal and opposite to that due to the feed-back through the valve. The condition for this is that $L_1C_1 = LC^1$.

This requires a very small value of C_1. Several methods may be adopted to produce this small capacity. One form of neutralising condenser comprises two plain wires about ¼ inch apart over which slides a metal tube

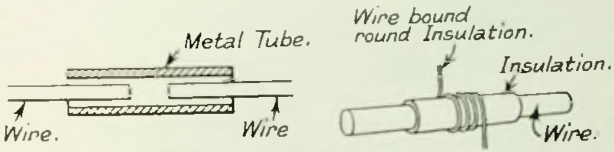


FIG. 189.—FORMS OF NEUTRALISING CONDENSER.

(insulated from the wires), as in Fig. 189. The tube may be varied in position till the requisite capacity is obtained. A simpler method still is to run the respective anode leads parallel for a short distance. With such neutralising arrangements it is most important to keep the wiring well spaced, so that all intercircuit capacities are under control.

Fig. 188 (*b*) shows a method of neutralising the self-capacity of the first valve. The same condition applies—viz., $L_1C_1 = LC^1$.

Use of Reaction.—Deliberate reaction may be used in certain cases, the grid and anode circuits of a valve being magnetically coupled together. Then, if the coupling is in the right direction, but is less than the critical value, the decrement of the circuits is reduced, with consequent sharpening of tune and increase of signal strength.

This process cannot be used to its fullest advantage because, if the reaction is excessive, atmospheric shocks will cause the circuits to oscillate and the signal may temporarily be obscured.

A further difficulty lies in the fact that the variation of the reaction coupling also alters the tune of the circuits, which must be carefully retuned after each variation.

A circuit which has been used to some extent employs a condenser to control the H.F. output of the valve. The anode of the valve is provided with high tension through a choke coil L_K (Fig. 190), while the high frequency oscillations pass through the circuit C_1L.

If C_1 is made small (about 100 $\mu\mu$F maximum), the high frequency current can be controlled in amplitude by simple variation of this capacity. Any tendency to oscillate is thus under perfect control.

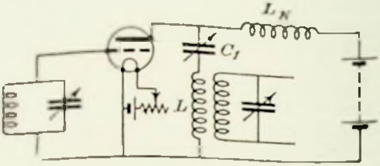


FIG. 190.—VALVE CIRCUIT WITH CONDENSER FEED CONTROL.

Filters.—It is sometimes desirable to cut off all frequencies above or below a given frequency, and for this purpose networks of inductance and capacity or resistance and capacity, known as filters, are employed.

If it is desired to cut off all frequencies above a certain value, a *low-pass* filter is employed, as shown in Fig. 191.

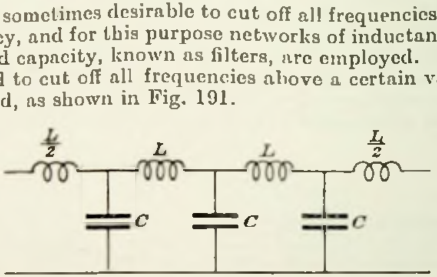


FIG. 191.—LOW-PASS FILTER.

This cuts off all frequencies above $f = \frac{1}{\pi\sqrt{LC}}$. The impedance of the network is low at any frequency below this frequency, but rises rapidly as the critical point is passed. It should be noted that the arrangement is symmetrical, the two inductances at the end being L/2.

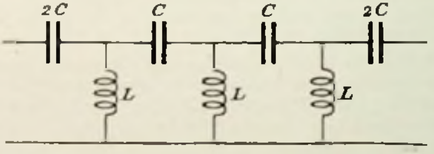


FIG. 192.—HIGH-PASS FILTER.

Fig. 192 shows a *high-pass* filter which cuts off all frequencies below $f = \frac{1}{4\pi\sqrt{LC}}$.

The conditions already given, however, are inadequate for the complete design of a filter; it is further necessary that the frequencies which are passed through the filter shall be transmitted along the line without reflection. This is accomplished by making the surge impedance of the filter equal to that of the input and output circuits, which should therefore be equal.

For a resistance load, this condition demands that $R_{load} = \sqrt{\frac{L}{C}}$ for a low-pass filter.

This condition only applies below the critical frequency. In conjunction with the equation for critical frequency, therefore, L and C may thus be solved.

For a high-pass filter, the condition (which in this case only applies above the critical frequency) is that—

$$R_{load} = \sqrt{\frac{C}{L}}.$$

For further information the reader is referred to "Electric Oscillations and Electric Waves" (Pierce), where the subject is very fully treated.

Detectors.

The simplest form of detector in use to-day is the crystal. Certain combinations of two crystals or a crystal and a metal exhibit unequal conductivities according to the direction of the voltage applied across them.

Hence, by applying a sinusoidal voltage to such a device, the resultant current is asymmetrical, giving a resultant unidirectional current.

Obviously, the more complete the suppression of current in one direction the more efficient the rectification. The actual rectification, however, depends on the curvature of the voltage-current characteristic at the working point. The question is discussed under the question of valve rectification, the phenomena of rectification with a crystal and a valve (utilising anode rectification) being parallel.

Characteristics of two types of crystal are appended. Fig. 193 shows the variation of current with voltage applied for a Perikon crystal. This comprises a crystal of zincite in contact with a chalcopyrites crystal. It will be seen that a very sharp bend in the curve occurs at zero applied voltage. Such a combination will thus act as an efficient rectifier without any steady applied E.M.F. This type is known as the "double-contact," "non-polarised" type. Other types of detector employ one crystal and a light metallic contact. This type is known as the "single-contact" type, the contact usually being in the form of a light spring of fine wire.

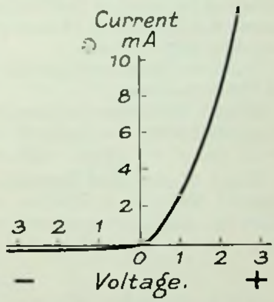


FIG. 193.—CHARACTERISTIC OF PERIKON DETECTOR.

Such combinations are very sensitive to contact pressure, and in many cases are subject to loss of sensitivity due to oxidation of the contacts. A certain amount of searching for a sensitive spot is also necessary, though in many cases this occupies very little time.

The most reliable crystal is the carborundum-steel combination. The characteristics of this crystal are shown in Fig. 194.

It will be seen that in order to operate at a sensitive spot a polarising voltage of the order of 1 to 1·2 volts is required. Carborundum is an impure silicon carbide, the contact being made by a flat steel spring. This combination becomes more sensitive as the pressure increases, which renders it very stable.

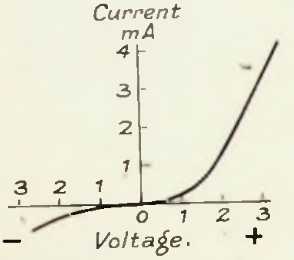


FIG. 194.—CHARACTERISTIC OF CARBORUNDUM DETECTOR.

The crystal is rather more expensive than the simple types, owing to the fact that the crystal has to be selected from a block of carborundum as delivered from the furnace, and only a comparatively few crystals out of the whole block are of any use. But, when obtained, it constitutes a reliable and sensitive detector, and is the only crystal detector used commercially.

Valve Rectification.—The ordinary valve forms a very convenient detector. There are two methods of employing such a device. The first is to adjust the grid potential so that the valve is operating at the base of the characteristic, where a variation of v_g produces asymmetrical variations of i_a, with consequent rectification, as with a crystal. This method is known as *anode rectification*. It must be remembered that the effective characteristic is not the simple curve obtained with the external anode impedance $=0$. Hence the critical grid voltage will be somewhat more negative than that indicated from the simple characteristic.

It is customary, therefore, when using such a system to employ a low external impedance in the anode circuit, and to utilise a valve having a high internal impedance of the order of 100,000 ohms, and possessing a high amplification factor.

Further, the characteristic is designed to lie well towards the positive side of the zero line.

Hence, when the effective characteristic is drawn out, the slope, though less than that with $Z=0$, is still large, and the current falls to zero with a sharp bend and at a voltage which is not too negative.

Value of Rectified Current.—For small values of the applied voltage the current through a detector obeys a square law. Hence if $v_g = V \sin \omega t$, then the change in anode current for a change in grid voltage v_g is given by—

$$\Delta i_a = \frac{V^2}{4} \cdot \frac{d^2 i_a}{d v_g^2} \sin \omega t.$$

If Z is the radius of curvature of the characteristic—

$$Z = \frac{\left(1+\frac{di_a}{dv_g}\right)^{3/2}}{\frac{d^2 i_a}{dv_g^2}}.$$

Then the expression may be written :

$$\Delta i_a = \frac{V^2}{4} \frac{\left(1+\frac{di_a}{dv_g}\right)^{3/2}}{Z} \sin \omega t.$$

Hence, if Z is not varying rapidly, the most sensitive position is that where the slope is greatest—*i.e.*, where the curve is just beginning to straighten out.

Hence, for weak signals a certain high frequency amplification should be employed, to enable the detector to function efficiently.

Rectification of Heterodyne Signals.—This conclusion, however, does not apply to heterodyne reception. Here the applied voltage $v = S \sin \omega_1 t + H \sin \omega_2 t$. The detector current is proportional to v^2, but the only term which is of interest is the audio-frequency component involving $(\omega_1 - \omega_2)$, which is of the form k SH $\cos (\omega_1 - \omega_2)\, t$.

This is a linear function, the current being proportional to the signal, *and to the heterodyne.*

Hence the stronger the heterodyne, the stronger will be the beat note, up to a limit where the characteristic ceases to obey a square law (*e.g.*, in a valve, saturation effects commence).

In this case Appleton and Mary Taylor have shown (*Proc. I.R.E.*, vol. xii., p. 277) that there is an optimum value of heterodyne. Most detector characteristics are of the form $i_d = a_0 + av + \beta v^2 + \gamma v^3 - \delta v^4$.

If H $>>$ S (the practical case), then the optimum heterodyne strength is independent of S, and is given by $H^2 = 2\beta/9\delta$.

For ordinary receiving valves this is of the order of 2 to 3 volts, a crystal requiring 4 to 5.

Instead of analysing the characteristic, the optimum value of H may be found experimentally by plotting the *mean* detector current for gradually increasing low frequency voltages. A curve similar to a valve characteristic is obtained, exhibiting a gradual saturation effect. The optimum value of H is that at which the curve has the greatest slope.

Grid Rectification.—A second method of rectification by means of a valve utilises the curvature of the grid current characteristic. To do this a condenser is inserted in the grid circuit, and the oscillations of grid voltage cause this condenser to build up to a negative potential. The current flowing in the anode circuit is correspondingly reduced, so producing a pulse of current in the telephones. This method, known as grid rectification, is very largely employed owing to its simplicity and the cumulative action of the building up. The phenomena involved will therefore be considered in greater detail.

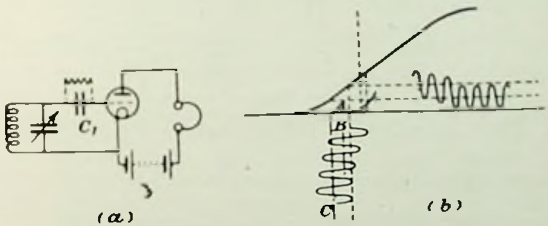


FIG. 195.—ILLUSTRATIVE ACTION OF VALVE AS CUMULATIVE RECTIFIER.

Consider Fig. 195, in which a condenser C_1 has been inserted in the grid filament circuit. The oscillating voltage is transferred through the condenser to the grid, which is thus subjected to voltage variations, and the anode current will fluctuate about its mean value.

Now, due to the condenser in the grid circuit, the valve will adjust itself till it is working at a point A on the characteristic [Fig. 195 (*b*)] at which no grid current is flowing. This point occurs at different potentials with various types of valves, but usually at a small negative grid voltage.

When the grid is positive, a small current will flow from the filament to the grid. Due to the blocking condenser, however, this current cannot complete its circuit back to the filament, but remains as a charge on the condenser C_1. The grid therefore becomes slightly negative relative to its original potential, and takes up a position as at B. During the next half-cycle the grid becomes negative, but returns to the point B, since no further current has flowed into the grid condenser nor has any leaked away if the valve is hard.

The next positive half-cycle will commence from the point B, and as soon as the grid voltage reaches the point A, grid current will flow and will cause an increase in the negative charge on the condenser. The negative half-cycle will have no effect, as before. Each succeeding oscillation, therefore, will cause the grid to acquire an increasingly negative potential until a point C is reached, where the voltage variation never makes the grid sufficiently positive to allow any grid current to flow. The action then ceases, and the grid is left charged to a steady negative potential.

The variations of anode current, therefore, do not take place about a steady point, but about a mean value which is decreasing in a series of jerks, and finishes appreciably less than the original value.

The change is detected in the telephones, which are placed in the anode circuit. It is, however, necessary to reset the device after each train of waves, and to this effect a high resistance is shunted across the condenser (or to one side of the filament), which allows the charge to leak away during the comparatively long interval between successive trains of waves.

Size of Condenser.—In considering this the circuit may be redrawn, as shown in Fig. 196. Here G is the source of alternating voltage, C_1 is the grid condenser, and C_2 is the grid filament capacity of the valve.

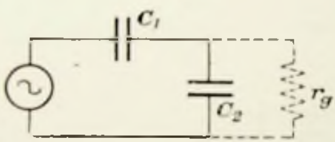


FIG. 196.—EQUIVALENT CIRCUIT TO FIG. 195 (*a*).

The voltage applied to the grid is that across C_2—

$$v_g = I/C_2\omega = v\omega\left(\frac{C_1 C_2}{C_1+C_2}\right)/C_2\omega = \frac{C_1}{C_1+C_2}\,v.$$

Hence the voltage on the grid depends on the ratio of C_1 to C_2, *independent of frequency*, and to obtain best results C_1 should be large compared with C_2, so making $\frac{C_1}{C_1+C_2}$ approach unity.

The effective value of C_2 is of the order of 50 to 100 $\mu\mu$F, and hence C_1 should not be lower than 200 to 300 $\mu\mu$F.

There is, however, a contrary effect to be considered. The value of the condenser must be such that it will build up to the full voltage required in the time available. Now the time taken to build up depends on both the capacity C and the leak resistance R.

The full mathematical treatment is distinctly complex, because the building up depends on the grid-current characteristic of the valve, and the treatment involves both the first and the second differential coefficients.

Fortunately, however, certain simplifications can be made in the treatment. There are two operations to be considered:

(*a*) The building up of the condenser.
(*b*) The leak away, resetting the device for the next signal.

Fig 197 shows the charging of the grid condenser both with and without a leak. As has been previously explained, it is only when the grid voltage rises above the point where the grid current commences to flow that any charging of the condenser takes place.

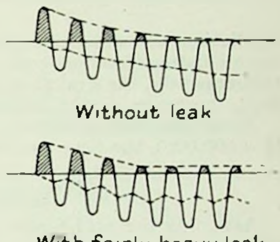


Fig. 197.—Illustrating Effect of Grid Leak.

The effective portions of the voltage cycle are accordingly shown shaded in Fig. 197.

Without a leak, the voltage builds up rapidly at first, and less quickly afterwards, gradually acquiring a steady value equal to the maximum amplitude of the applied voltage.

With a leak, on the other hand, the initial rate of building up is not so rapid, due to the leak, and the grid potential falls again during the idle half-cycle from the same cause. This means, however, that in the next half-cycle the shaded area is larger and more charge is acquired by the grid condenser. The net result is that the building up, although initially slower, continues at a more rapid rate than in the first case, and the total time required to build up is found to be very little different, whatever the value of the leak, within fairly wide limits. Mathematically it may be shown that for this condition to apply $\frac{1}{R}$ must be small compared with $\frac{1}{r_g}$, where r_g is the grid-filament resistance of the valve, which means that the grid leak cannot be reduced much below 2 megohms.

The building up of the condenser, therefore, is controlled almost entirely by the capacity thereof, the sole function of the grid leak being to arrange for the dissipation of the charge in good time for the next impulse.

The values of C and R may now be investigated.

The method which is adopted here is not strictly correct from a mathematical point of view, but gives a very fair approximation, which agrees with the results obtained by more complex methods. It consists of estimating the time available for charging, and then working out the value of the capacity which will acquire, say, 90 per cent. of its full voltage in the time available when a resistance r_g is in series.

The capacity of the valve is ignored, because it takes no part in the rectifying action.

The charging time is the time elapsing between the beginning of the oscillation and the point where the condenser has acquired its maximum voltage, and this is determined empirically.

1. **Damped Waves.**—Fig. 198 shows the building up of the grid voltage when receiving a spark train. It will be seen that the first three or four

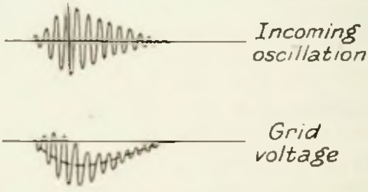


FIG. 198.—BUILDING-UP OF CONDENSER WITH SPARK SIGNALS.

oscillations are sufficient to build up the voltage on the condenser, the succeeding oscillations having no effect owing to their rapidly diminishing amplitude. Consequently, the time t available for building up is very small, and is dependent, moreover, on the frequency.

Assuming four effective oscillations and a wave length of 600 metres ($f=500{,}000$), the time available is—

$$t=\frac{1}{500{,}000}\times 4=8\times 10^{-6}\text{ seconds.}$$

Assuming the grid filament resistance of the valve to be 300,000 ohms, which is a reasonable average value, the capacity required can be worked out in order that the voltage may build up to a given fraction of the full value during the time available. For a trial calculation the condenser will be assumed to charge up to 90 per cent. of the applied voltage.

$$v=\text{V}\left(1-\varepsilon^{-\frac{t}{Cr_g}}\right)=\text{V}\,(1-0{\cdot}1).$$

Therefore
$$\varepsilon^{-\frac{t}{Cr_g}}=0{\cdot}1$$

$$\log_\varepsilon \varepsilon^{-\frac{t}{Cr_g}}=\log_\varepsilon 0{\cdot}1,$$

$$-\frac{t}{Cr_g}=-2{\cdot}3026.$$

$$\text{C}=\frac{8\times 10^{-6}}{2{\cdot}3\times 300{,}000}\text{ approximately.}$$

$$=11{\cdot}6\text{ micro-microfarads.}$$

This is, of course, ridiculously small, and with such a value the major part of the voltage drop would occur on the external condenser. Assuming an effective internal capacity of 80 $\mu\mu$F, the ratio of the voltage actually applied across the grid v_g to the signal voltage v would be—

$$v_g/v=\frac{11{\cdot}6}{81{\cdot}6}=0{\cdot}142.$$

The voltage acquired by the condenser is only 90 per cent. of this.

Hence $v_c/v = 0{\cdot}142 \times 0{\cdot}9 = 12{\cdot}8$ per cent.

As C_1 is increased the build-up voltage will fall off, but v_g/v will increase, and the two effects tend to balance one another.

If the net ratio of v_c/v is worked out for various values of C_1, the curve shown in Fig. 199 is obtained.

The maximum efficiency occurs at $C_1 = 40\ \mu\mu F$, where $v_c/v = 17$ per cent., while with present-day values of 200 to 300 $\mu\mu F$ the ratio of v_c/v is only about

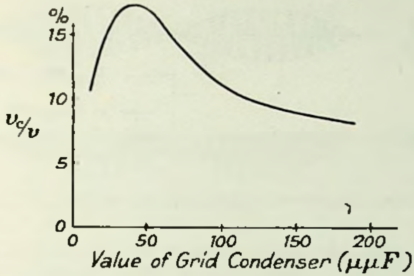


FIG. 199.—RECTIFICATION EFFICIENCY WITH SPARK SIGNALS.

6 per cent. The method is thus very poor for spark reception, and, moreover, becomes increasingly poor as the wave length is reduced.

The size of the leak depends on the condenser. The time available for the leak away to occur is very nearly $\frac{1}{1000}$ second, assuming a note frequency of 1,000 for convenience. If the condenser is assumed to lose 99 per cent. of its charge in this time, then—

$$\varepsilon^{-\frac{t}{CR}} = 0{\cdot}01$$

$$-\frac{t}{CR} = \log_e 0{\cdot}01 = -4{\cdot}6 \text{ approximately.}$$

$$R = \frac{10^{-3}}{4{\cdot}6\ C}.$$

If $\qquad C = 35\ \mu\mu F,\ R = 6{\cdot}2$ megohms.

$\qquad C = 350\ \mu\mu F,\ R = 0{\cdot}62$ megohm.

This last value, as has previously been shown, is too low for efficient working. It will be seen that the commonly accepted values of to-day are by no means the best as far as spark reception is concerned.

2. **Undamped Waves.**—With a C.W. signal the time available for charging depends on the grid leak, although in a somewhat indirect manner. Consider Fig. 200, which indicates the building-up process with a C.W. signal. A heterodyne of the same strength as the signal is assumed, but the reasoning is still valid for considerably larger values of heterodyne. It will be observed that, as the amplitude of the signal is steadily increasing for half the heterodyne modulation, the grid has a considerably longer time to build up than in the case of a spark train. Having built up, however,

the charge must leak away during the remainder of the modulation, so leaving the condenser ready to build up again on the next modulation.

Now it will be found that, with the values in common use to-day, the leak away is not sufficiently rapid. Consequently, as indicated in Fig. 200, the grid does not start to build up until several oscillations have elapsed, and this at once limits the time available for building up. From Fig. 200 it will be

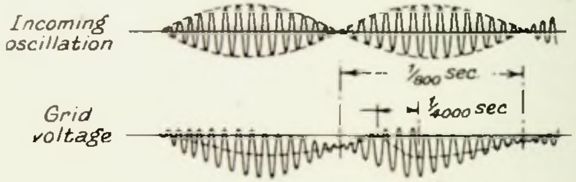


FIG. 200.—BUILDING-UP OF CONDENSER WITH C.W. SIGNALS.

seen that the time available for charging is about one-fifth of the time of one heterodyne modulation. Assuming an 800-cycle note (the average frequency employed), t is thus $\frac{1}{4000}$ second.

If, as before, $$\varepsilon^{-\frac{t}{CR}} = 0{\cdot}1,$$

$$C = \frac{1}{4000} \cdot \frac{10^{12}}{2{\cdot}3 \times 300{,}000} = 362\ \mu\mu\text{F}.$$

With this value $$v_c/v_g = \frac{362}{362+80} = 0{\cdot}82.$$

Hence $v_c/v = 0{\cdot}82 \times 0{\cdot}9 = 74$ per cent., which indicates that the method is distinctly more efficient for C.W. working.

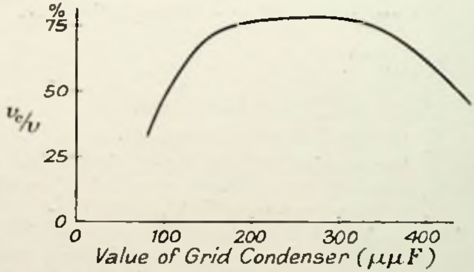


FIG. 201.—RECTIFICATION EFFICIENCY WITH C.W. SIGNALS.

The values of v_c/v in terms of C have again been worked out, the curve being given in Fig. 201. Here it will be seen that any capacity between 150 and 350 $\mu\mu$F is suitable.

The grid leak is usually made about 3 megohms. This means that in the time available, which is $\frac{4}{5} \times \frac{1}{800}$ second, the voltage will have dropped to—

$$v = V\varepsilon^{-\frac{t}{CR}}$$

$$= V\varepsilon^{-\frac{10^{12}}{1,000 \times 3,000,000 \times 250}}$$

$$= V\varepsilon^{-1 \cdot 33}$$

$$= 0 \cdot 26 \text{ V},$$

assuming C to be 250 $\mu\mu$F, which is the maximum of Fig. 201.

This shows that the charge does not completely leak away in the time available, and hence some such action as was indicated in Fig. 200 will take place. The succeeding heterodyne modulations will thus not cause the grid to build up negative again until the amplitude has risen above the value of 0·26 V quoted above. The maximum value to which the grid builds up is still the maximum value of the applied voltage, and consequently the effective reduction of grid voltage is only some 75 per cent. of the full voltage.

This further reduces the efficiency of the operation from 74 to 55 per cent. The only remedy is to reduce the leak, which, as has been seen, cannot be done, because for values below 2 or 3 megohms the leak begins to exercise an appreciable effect on the charging of the condenser. It will be seen, therefore, that the leak, though necessary, is a distinct evil.

Telephony.—Telephony reception is similar to C.W. reception, except that for clarity of tone efficient rectification must take place with note frequencies as high as 2,000 cycles per second or more. For this purpose it is customary to use a smaller condenser (200 $\mu\mu$F or less in place of the usual 350), which, of course, permits a more rapid build-up. It appears probable that still lower values could be employed with advantage.

The grid leak is often reduced at the same time, but since the grid leak controls the working point on the characteristic, its value depends to some extent on the type of valve with which it is employed.

Working Point on the Characteristic.—Owing to the presence of the grid leak, the valve does not work at the point of zero grid current, but at some other point which is determined by the value of the grid leak itself.

To elucidate this point reference may be made to Fig. 202, which shows a grid current-grid voltage characteristic. If the filament end of the leak is connected to the negative end of the filament, then a line may be drawn from O having a slope such that $\cot \theta = R$ (to the same scale as the curve), where R is the resistance of the leak. At any point on this line the voltage and current will be connected by the relation $\frac{V}{I} = R = \cot \theta$.

Obviously, therefore, where this line intersects the grid-current characteristic will be the working point of the valve.

It will be seen that the smaller the leak the steeper is the line and the farther up the characteristic is the working point. In some cases the leak is connected to the positive filament lead, in which case the leak line originates at B, which for the same value of the leak gives a working point farther up the characteristic.

Fig. 202 also indicates the true nature of the building-up process. There is a permanent grid current flowing, and the incoming oscillation causes variations

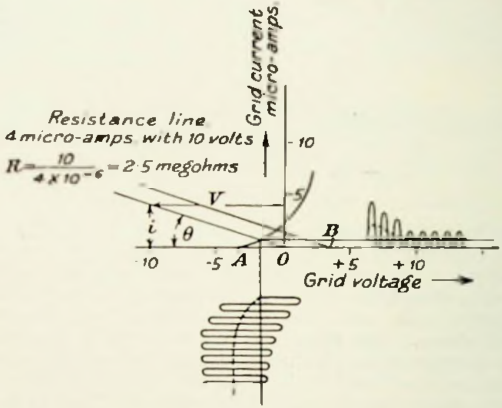


FIG. 202.—CONTROL OF WORKING POINT BY GRID LEAK.

of this current. Owing to the curvature of the characteristic, however, these variations are not symmetrical, and there is an increase in the average grid current, which causes the condenser to build up to a negative potential. In

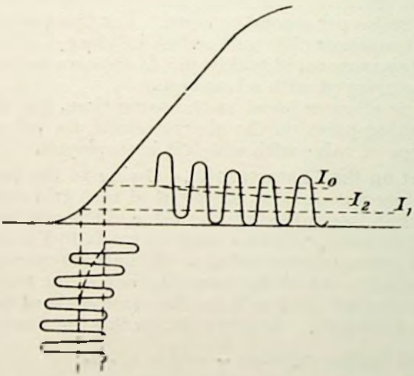


FIG. 203.—LOSS OF RECTIFICATION WITH WORKING POINT TOO LOW DOWN.

the steady state the increase of grid current is just a little more than the decrease, to make up for the loss due to leakage.

Effect of Anode Voltage.—The working point on the grid characteristic is also important from its effect on the anode current.

Referring back to Fig. 195, it will be seen that the variations of anode current are perfectly symmetrical, but that owing to the grid condenser action the mean value steadily falls, finishing appreciably lower than the original value I_0.

Fig. 203, however, indicates the state of affairs when the working point on the characteristic is too low down. Here, due to the curvature of the characteristic, the variations of anode current become asymmetrical. The average value of the current is then no longer I_1, but I_2, which is not appreciably different from I_0. In other words, the rectification effect is seriously impaired, and there will be a critical value of the applied voltage at which there is no change whatever in the mean anode current, and no rectification will result.

The remedy is to increase the H.T., while the connection of the grid leak to the positive side of the filament also obviates the trouble to some extent, because this shifts the working point well up the characteristic.

If the H.T. is increased so far that the working point occurs on the upper bend of the characteristic, as in Fig. 204, then the rectification is increased,

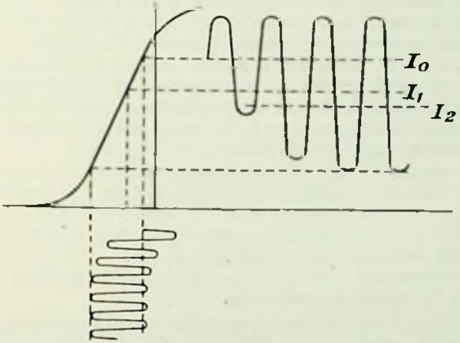


FIG. 204.—INCREASED RECTIFICATION AT TOP OF CHARACTERISTIC.

and not reduced. In this case the final value of the anode current I_2 is appreciably lower than I_1, so giving increased signal strength for the same applied voltage.

Value of Rectified Current.—For unheterodyned weak signals the change in anode current is given by—

$$\Delta i_a = \frac{\frac{V^2}{4} \frac{di_a}{dv_g}}{\left(\frac{1}{R} + \frac{di_g}{dv_g}\right) \frac{d^2 i_g}{dv_g^2}},$$

where R is the leak resistance. Hence this form of detector also obeys a square law.

Optimum Heterodyne.—The remarks on p. 175 referring to heterodyne reception apply also to this method of rectification, in that the detector obeys a linear law with respect to heterodyned signals. As the strength of the heterodyne is increased, however, a condition similar to that indicated in Fig. 203 is obtained, so limiting the strength of the beat note.

Appleton (*loc. cit.*) has shown that the optimum heterodyne strength occurs just before this limiting effect becomes apparent. The actual optimum value is obtainable in terms of the grid current and anode current characteristics, but this is usually not a practical proposition.

The more practical method is to plot, as before, the *mean* anode current for various values of applied voltage (low frequency A.C. will suffice). The curve so obtained falls rapidly at first, as v_g is increased, reaching a minimum where the anode rectification and the grid rectification effects neutralise each other (Fig. 203) and the rectification is nil, after which the curve gradually rises again. The optimum heterodyne strength is that at which the slope of the curve is a maximum, which occurs just before the minimum point, at a value of about 1 to 1½ volts.

Receiving Aerial Systems.

The aerial system is becoming recognised as one of the most important defences against jamming and atmospheric interference. The simple aerial is usually in the form of a horizontal wire at a height of 50 to 200 feet above the ground, according to circumstances. The height is the important factor, since $E_r = \mathcal{E} \times h$, the horizontal top serving to provide the necessary capacity, and to a small extent to increase the effective height.

The natural wave length should always be less than that of the station to be received, since a certain loading inductance must be inserted at the base to which the receiver may be coupled.

This limit, however, is only of importance at short wave lengths.

For simple circuits the voltage developed across the loading coil may be tapped across the grid and filament of the first valve (or connected to the detector), but for a circuit in which any selectivity is desired the aerial is tuned and loosely coupled to a secondary circuit.

This may be connected direct to an amplifier or to further tuned circuits, according to the design of the receiver.

Aperiodic Aerial.—A type of aerial sometimes employed comprises a simple aerial having a high resistance (10,000 to 50,000 ohms) inserted instead of an inductance. The voltage variations across this resistance are amplified by a valve, and the currents obtained are then filtered, tuned, and amplified in the usual way. Several different stations may be received on the same aerial if desired.

Frame Aerials.—Another very useful receiving system employs a loop of wire in place of the usual aerial. The electric field in the advancing wave induces E.M.F.'s first in one side of the loop and then in the other. These two E.M.F.'s will be in opposition round the loop, but due to the fact that one side of the loop is affected before the other there will be a slight phase difference, and the result will be that the two E.M.F.'s do not cancel out, but combine to form a resultant.

If the loop is tuned, appreciable currents will be set up by this E.M.F., which may be coupled to an amplifier in the usual manner.

Obviously, the above reasoning does not hold if the plane of the loop is at right angles to the direction of the received wave, for the electric field then affects both sides of the loop together, and there is no phase difference between the two E.M.F.'s.

Moreover, as the loop is rotated from this zero position, the resultant

E.M.F. will increase to a maximum when the plane of the loop is in the direction of reception, when it will commence to decrease again. The loop is thus directional in its reception and this property is very valuable.

A frame aerial consists of a loop of several turns, in which case the total E.M.F. is the aggregate of the resultant E.M.F.'s in each turn.

A frame aerial is equivalent to a vertical aerial of height—

$$h = 2\pi NH \frac{L}{\lambda} \cos \theta,$$

where N = number of turns,
H = height of frame,
L = width of frame,
λ = wave length being received,
θ = angle between frame and line of reception.

This neglects the axial thickness of the frame.

A frame, however, is not usually so efficient as an aerial. For maximum efficiency, the internal resistance of the aerial should equal the radiation resistance.

This in practice is difficult of achievement, but the total resistance of a receiving aerial can be reduced to within four or five times the radiation resistance.

With a frame, on the other hand, the radiation resistance is very small, and hence for a reasonable efficiency R would have to be impracticably small.

$$R_{rad} \text{ for simple aerial} = \frac{4\omega^2 h^2}{3c}.$$

h = efficient height.
c = velocity of light.

$$R_{rad} \text{ for frame} = \frac{4\omega^4 A^2 N^2}{3c^2}.$$

A = area.
N = number of turns.

The directional properties of a frame have been seen to be proportional to cos θ. It is often convenient to draw a polar diagram of the reception of a given system, wherein the length of the radius vector at any angle represents the strength of the received signal when the frame is at that angle.

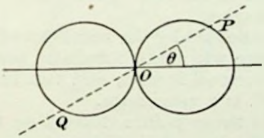


Fig. 205.—Polar Diagram of Simple Frame.

For a simple frame the polar diagram is a " figure of eight," as shown in Fig. 205.

The best size of loop and the spacing of the turns depends upon the wave length, and some indication of the appropriate size for any particular condition may be gained from the tables attached. The strength of signals received is proportional to $\frac{NAL}{\lambda^2 R}$.

TABLE XXIX.

BEST SIZE OF SQUARE FRAME FOR VARIOUS WAVE LENGTHS.

λ Metres.	*Side of Square (Feet).*	*Number of Turns.*	*λ Metres.*	*Side of Square (Feet).*	*Number of Turns.*
50 to 150	4	1	1,200	8	12
	3	1		6	14
200	8	1		4	20
	6	2	1,600	8	16
300	8	2		6	20
	6	4		4	30
600	8	4	2,500	8	30
	6	7		6	40
	4	10		4	60
800	8	7	3,500	8	45
	6	10		6	65
	4	15			

(D. S. Brown.)

TABLE XXX.

BEST SPACING WITH GIVEN SIZE OF FRAME.

Side of Square (Feet).	*Spacing of Turns (Inches).*
4	$\frac{1}{4}$
6	$\frac{5}{16}$
8	$\frac{5}{16}$
10	$\frac{3}{8}$
12	$\frac{15}{16}$

A system which is in considerable use to-day is the Marconi Bellini-Tosi system, in which two large frames are erected at right angles to each other. The connections from the frames are led to the field coils of a "radiogoniometer." This instrument consists of two rectangular coils placed symmetrically at right angles, with a third (search) coil rotating inside.

The field coils are connected one to each loop. Each loop will have E.M.F.'s induced in it by waves passing the system, the relative values depending on the direction of the signal. These E.M.F.'s will set up currents, which in turn will produce in the radiogoniometer a complex system of two magnetic fields at right angles.

The search coil will be affected by the resultant of these two fields, so that as this coil is rotated there will be two definite positions 180° apart where the resultant E.M.F. in the coil is zero, and two further positions 90° from the first where the E.M.F. is a maximum.

The actual positions of these points depend on the relative strengths of the two fields, which again depend in the first place on the direction from which the particular signal is coming. Hence the rotation of the centre coil of the radiogoniometer has the same effect as rotating one of the large external loops.

This system therefore permits the use of large aerials, the rotation being electrical instead of mechanical. Signals can be received on such a system of a strength comparable with that on an ordinary aerial, thus obviating one of the chief disadvantages of a frame.

A diagram of the system is given in Fig. 206.

Originally it was customary to tune each external loop and employ a weak coupling to the rotating coil, which formed the secondary. The more modern system, however, uses a tight coupling in the goniometer ($k = 50$ to 80 per cent.), and the secondary only is tuned. The loss due to the loops not being tuned is to some extent compensated for by the tighter coupling, but the resultant signal strength is weaker than with the tuned system. This disadvantage is more than outweighed by the simplicity of the arrangement.

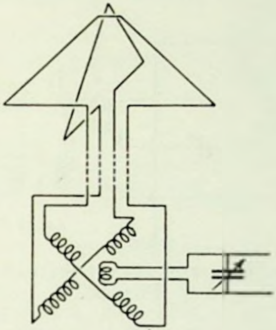


FIG. 206.—BELLINI-TOSI SYSTEM.

The design of the secondary is modified by the tight coupling, and the effective inductance has to be obtained by the usual transformer laws. The leakage inductance of the stator coils should be of the same order as the inductances of the respective loops (which should, of course, be equal for reasons of symmetry). Having decided this, the design of the rotor coil simply consists in arranging that the effective inductance is such as to tune with the condenser employed.

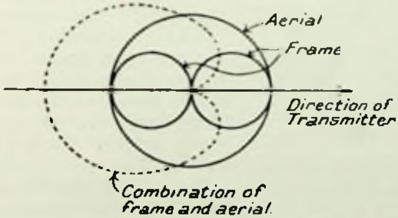


FIG. 207.—HEART-SHAPED DIAGRAM.

Heart-Shaped Diagram.—It is sometimes desired to eliminate reception from one direction and to receive signals coming from the opposite direction. This may be accomplished by employing a combination of a frame and an aerial. The aerial receives equally well in all directions. A frame has a figure of eight polar diagram, but the current set up when the frame is pointing in a given direction is 180° out of phase with that which flows when the frame is rotated through 180°.

This fact is utilised in the "heart-shaped balance," or "barrage" reception. The currents set up in the frame and in the aerial are both allowed to affect a common secondary circuit. In one position of the frame the two currents assist each other, but for a signal from the reciprocal direction the

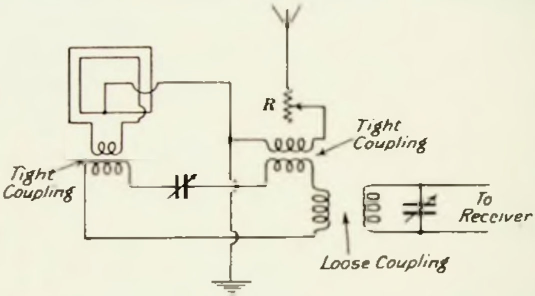


FIG. 208.—CONNECTIONS FOR HEART-SHAPED DIAGRAM (UNTUNED FRAME).

currents are in opposition. If the two currents are adjusted to the same strength, they will cancel out and give a zero position. The resulting polar diagram is then a cardioid, as shown in Fig. 207, the reception from one direction being cut off over a wide sector.

It is necessary to adjust not only the strengths, but the phases of the two currents. If the aerial and frame circuits are both tuned, the balance then only holds for the particular wave length to which they are tuned. This is a disadvantage, and it is more usual to employ an untuned frame tightly coupled to a tuned secondary to which the aerial is also coupled.

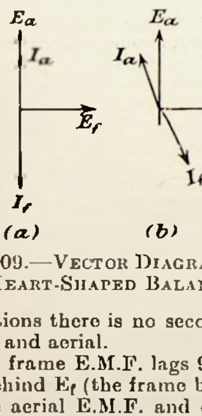


FIG. 209.—VECTOR DIAGRAMS FOR HEART-SHAPED BALANCE.

The considerably reduced strength on the untuned frame enables resistance to be inserted in the aerial circuit, which broadens the tune and maintains the balance over an appreciable band of wave length. Such a circuit is shown in Fig. 208.

Fig. 209 shows the vector diagram for the circuit when balanced. Under such conditions there is no secondary current, and thus no interaction between frame and aerial.

The frame E.M.F. lags 90° behind the electric field, and the current lags 90° behind E_f (the frame being untuned and inductive).

The aerial E.M.F. and current are in phase with the electric field, and hence by equating I_a and I_f a complete balance is obtained.

Actually, owing to resistance in the frame, I_f does not lag by quite 90°, so that I_a is made slightly leading [Fig. 209 (*b*)].

The Beverage Aerial.—This is a form of aerial which has very marked directive properties. It consists of a very long, low, horizontal wire, the length being comparable with the wave length being received, and the height from 10 to 30 feet. It depends for its action primarily on the fact that the electric field at the receiving point is not quite vertical, but is slightly tilted, the end near the earth lagging behind that at a height owing to the fact that the earth is not a perfect conductor.

A wave reaching the aerial induces therein a small E.M.F., which propagates a wave along the wire, as with ordinary telephone transmission (see p. 253).

The velocity of this wave, however, is less than the velocity of the radio wave, so that it lags behind to a gradually increasing extent. Due to this lag, it absorbs energy from the wave, and as the lag increases the energy absorption increases, and the E.M.F. induced in the aerial is thus cumulative.

It is necessary, however, to earth the far end of the line through a network having a surge impedance equal to that of the line, as otherwise reflection will occur. The length of the line may be increased until the line wave is 180° behind the radio wave, after which the E.M.F. begins to decrease.

The length at which this condition occurs depends on the constants of the line, which determine the wave velocity.

The principal advantage of this type of aerial is the marked directivity obtained under suitable conditions. If the receiver is situated at A (Fig. 210),

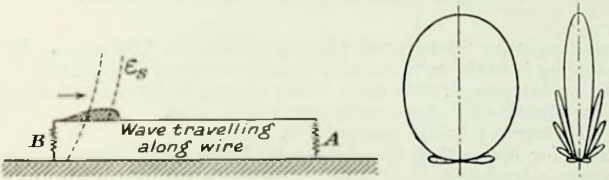


FIG. 210.—ARRANGEMENT AND POLAR DIAGRAMS OF BEVERAGE AERIAL.

the reception will be a maximum for waves in the direction BA, but will be zero for waves in the direction AB, provided the line is adjusted to have no reflection at the ends. By suitable adjustments the polar diagram can be made very narrow indeed, some representative diagrams being shown in Fig. 210.

Examples of Commercial Receivers.—Fig. 211 is a skeleton diagram of a receiver suitable for ship and shore communication, where rapid searching on a variety of wave lengths is desirable, and only a comparatively short range of reception is required. The wave length range of the receiver may be increased by using large coils with tappings for the lower wave lengths, but dead-end switches should be employed, particularly in the anode circuit, to obviate troubles due to distributed capacity, as has been previously mentioned.

Fig. 212 is a diagram of a Marconi type RC1A receiver used for long-distance traffic.

The various components are as follows:

1. Directional selector—a Bellini-Tosi arrangement.

2. Phasing panel for operating the aerial system as a simple frame, simple aerial, or combination of both (heart shape).

3. H.F. tuning and filtering arrangements.

4. Heterodyne oscillator, which is adjusted to give a beat note of 2,500 cycles second.

5. High frequency rectifier employing anode rectification. The point A is connected to the filament through a suitable negative battery.

6. Limiter for reducing atmospherics to the same strength as the signal.

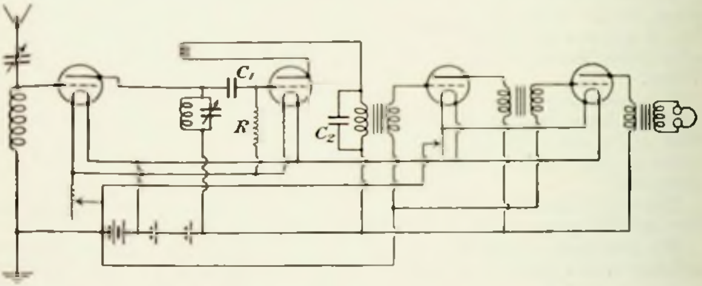


Fig. 211.—Simple Short Wave Receiver.

7. Low frequency filters tuned with a fixed tune to 2,500 note. The intervalve coupling is variable to increase the selectivity if desired, while a resistance can be introduced into each tuned circuit to broaden the resonance band, thus allowing for slight variations in the note.

8. Double current bridge (see p. 210).

9. Relay for converting the signals to telegraphic impulses for transmission over a land line.

Instead of 9 and 10, a second (low frequency) oscillator may be introduced, which heterodynes the 2,500 cycle current to 1,000 or 800 cycles suitable for aural reception.

DESIGN OF MASTS AND AERIALS.

(*Prepared by* G. H. Farnes *and* W. F. Smith.)

The mast system of a large transmitting station is the most expensive part of the whole equipment, and the design of the masts constitutes a highly specialised branch of radio engineering. The problems involved are comparatively simple problems in structural design, and it is not proposed to enter into great detail. The operations involved in the design of various classes of mast will be outlined, however, after which the application of the principles should present no difficulty.

Radio masts are of two types:

(*a*) Self-supporting towers.
(*b*) Stayed masts.

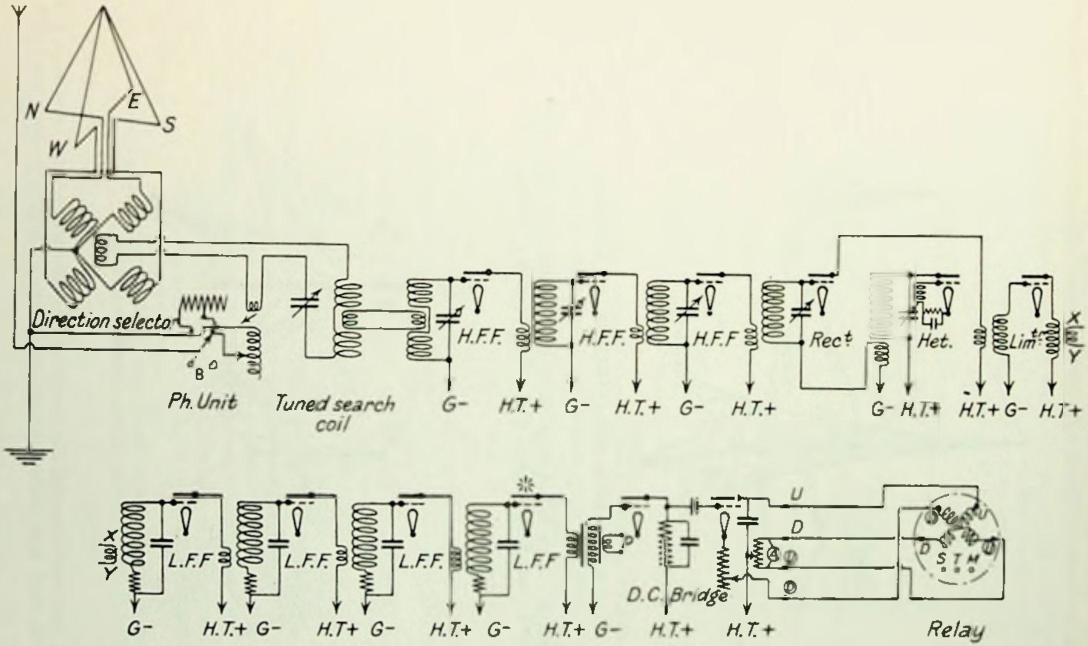


FIG. 212.—DIAGRAM OF MARCONI TYPE RC1A RECEIVER.

(By courtesy of M.W.T. Co.)

An example of the first type is shown in Fig. 213. The structure is capable of standing the loads involved without any stays. Fig. 214 is an example of

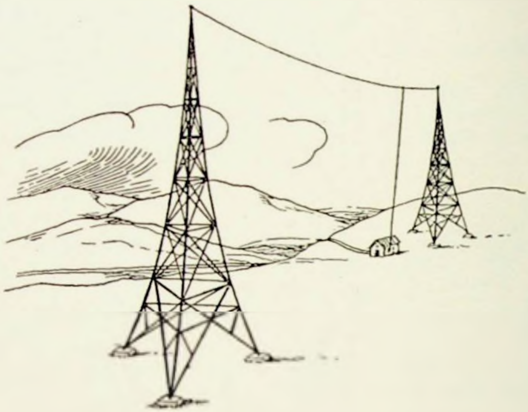


Fig. 213.—Self-Supporting Towers.

a stayed mast. This latter type is distinctly cheaper, the cost being only one quarter to one-fifth of that of a self-supporting tower.

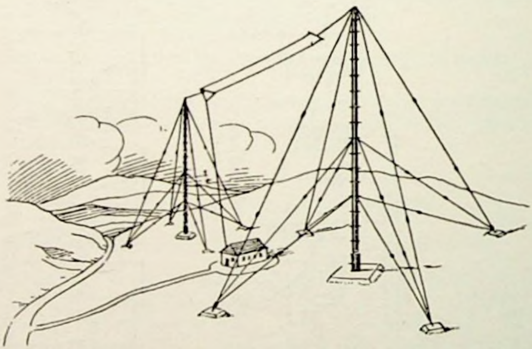


Fig. 214.—Stayed Mast.

Against this must be set the considerably greater ground area required for a stayed mast, but this consideration is usually negligible compared with cost.

Design of Self-Supporting Tower.

A self-supporting tower is simply a cantilever having a distributed wind load and a concentrated pull due to the aerial at the extreme end (Fig. 215).

The worst condition occurs when these two forces act in the same direction, and the mast may thus be designed according to the ordinary theory of structures.

The wind load is not uniform, but increases with the height (see p. 198). This factor is the weakest link in the design of masts owing to the incomplete information available. This renders mast design very difficult, and necessitates the use of factors of safety rather larger than the normal, a figure of 3 to 5 usually being assumed in such cases.

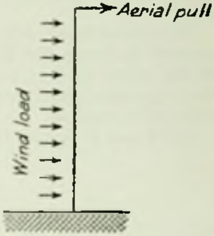


FIG. 215.—FORCES ON SELF-SUPPORTING TOWER.

Design of Stayed Masts.

Stayed masts are of two types:

1. Base rigidly fixed; this involves a maximum bending moment at the ground level.
2. Base pivoted on a ball-and-socket or equivalent joint.

The chief factors in the design of such masts are the determination of the stay stresses and dimensions, and the determination of the size of structure necessary in order that the requisite rigidity may be obtained.

The second type of mast, with the ball-and-socket joint, is assumed to deflect under load as a whole—*i.e.*, still retaining its linear formation; and if the stays are correctly designed this assumption is valid.

The first type of mast, however, is equivalent to a cantilever with supports at varying distances, and thus definitely bends.

Assuming that the top of the mast deflects a given amount, then the deflections at the centre stay points would be somewhat less with the built-in mast than with the ball-and-socket joint type.

Both cases are susceptible to rigid mathematical treatment if the loading is accurately known. The wind pressure, however, is not known with any certainty, so that approximations have to be resorted to, and it is usual, therefore, to consider the mast as deflecting as a whole without bending.

Calculation of Stay Tensions.—The first step is to assume a suitable structure for the mast, which may conveniently be done by referring to similar designs. Now the worst condition which the mast has to stand is that when the wind load and the aerial pull are in the same direction. Let—

A_h = horizontal component of aerial tension.

V_k = horizontal component of wind load on a section k at a height h_k.

$F_1, F_2, F_3 \ldots$ = horizontal components of tensions in stays 1, 2, 3 . . .

$f_1, f_2, f_3 \ldots$ = distances from ground of stay points 1, 2, 3 . . .

All these forces are assumed to be in the same plane.

Then $$A_h + V_1 + V_2 + \ldots V_n = F_1 + F_2 + \ldots F_n.$$

Also, taking moments about the base—

$$A_h \times h + V_1 h_1 + V_2 h_2 + \ldots V_n h_n = F_1 f_1 + F_2 f_2 + \ldots F_n f_n.$$

From these equations F_1, F_2, etc., the *resultant horizontal forces* at each stay point may be obtained.

The wind load is obtained from the data given on p. 198. The aerial pull is calculable from the weight of the wire by the ordinary catenary laws, due allowance being made for the wind load on the aerial wires.

It is now necessary to calculate the cross-sectional area and initial tensions of the stays required to produce these forces F_1, F_2, etc., under the required wind load.

The aerial tension may be in the same plane as one set of stays, but more usually is along the bisector of the stays (*i.e.*, at 45° with a four-stay mast). If the wind load is assumed to be in the same direction as the aerial pull, then the windward stays will tighten when the mast deflects and the leeward stays will slacken.

Let l and l' be the initial and final lengths of the stays.

a = radius of stay anchorage.
ε = deflection.
h = height of stay point.

Then $$l' = l\left\{1 \pm \frac{a\varepsilon}{h^2 + a^2}\right\}$$ if the aerial pull is in the plane of the stays;

$$= l\left\{1 \pm \frac{a\varepsilon}{\sqrt{2}\left(h^2 + \frac{a^2}{2}\right)}\right\}$$ if the aerial pull is at 45° to the plane of the stays.

The + and − signs refer to the tight and slack stays respectively.

Again, if T_t and T_s are the tensions in the tight and slack stays respectively, it can be shown that—

$$T_t - T_s = F\frac{\sqrt{h^2 + a^2}}{a}$$

where the aerial pull is in the plane of the stays;

$$= F\frac{\sqrt{h^2 + a^2}}{a\sqrt{2}}$$

where the aerial pull is at 45°, F being the resultant horizontal force at the stay point as determined by the original equation.

A value of T_t is now assumed. The elongation of the stay due to the deflection of the mast is known, and hence T_o, the normal tension in the stay, can be found. Knowing the decrease in length of the slack stays, T_s can then be found, which enables $T_t - T_s$ to be evaluated.

This value must satisfy the equations just given above, and a process of trial and error is adopted till the correct tensions are found.

If three stays only are used, the same method is employed with suitable modifications to the formulæ, which may readily be deduced.

Design of Structure.—The next step is the design of the structure itself. This really involves the checking of the structure already assumed. Each section of the mast is treated separately, and the total stress due to bending and compression is calculated. If it transpires that the structure is inadequate, then the whole mast has to be redesigned with a new structure.

Bending.—With rigid masts the bending moment at any section may be obtained by considering the mast as a whole, and employing the Theorem of

Three Moments. The wind load is considered as uniformly distributed on each section. The deflection at each stay point may be neglected, the effect being that the stress obtained is slightly too high.

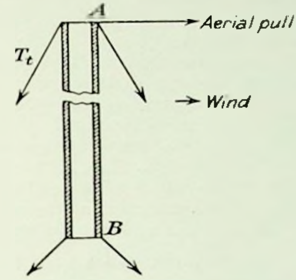


FIG. 216.—SECTION OF TUBULAR MAST.

Compressive Stress—Case A : Tubular Mast (with Four Stays).—Consider the top section. The maximum compressive stress will occur at the bottom of the section. The direct compressive load at B (Fig. 216) will be that due to—

1. Dead weight of top section AB.
2. Vertical component of aerial pull.
3. Dead weight of four stays.
4. Vertical component of stay tensions as just determined.

The stress due to bending is given by $\frac{M}{Z}$ when—

$$M = \text{bending moment at section B,}$$
$$Z = \text{section modulus} = \frac{\text{moment of inertia}}{\text{least radius of gyration}}.$$

Then if L is the total vertical load, and A is the C.S. area of the structure, the total stress is given by—

$$S = \frac{\text{L (tons)}}{\text{A (square inches)}} + \frac{\text{M (inch tons)}}{\text{Z (inch}^3)}.$$

The allowable stress is given by—

$$S = 18{,}000 - 80\frac{l}{k} \text{ pounds/square inches,}$$

where l = length of section,
k = least radius of gyration.

This is an empirical formula based on practical experience, and only applies when l/k is >50.

If $l/k<50$, a fixed safe stress of 14,000 pounds/square inches or 6·25 tons/square inches should be allowed.

In considering the next section, the direct compressive stress calculated for the first section is increased by—

1. Dead weight of this section.
2. Dead weight of next set of stays.
3. Vertical component of tensions in next set of stays.

The procedure in finding the stress is then the same as above.

Proceeding in this manner, the stresses at various sections of the structure can be determined, and compared with the allowable stress. If in any case the allowable stress is exceeded, then a new section must be chosen, and the whole of the calculations repeated.

Case B : Lattice Masts.—With latticed structures there is another condition to allow for in the design.

Suppose that the mast consists of three main posts with horizontal and diagonal members forming a triangular section mast. The direct compressive stress per single post will be one-third of the total compressive load. The bending stress, however, is calculated in a slightly different manner. Fig. 217 shows a skeleton plan and elevation of a section of a triangular mast. It is assumed that the wind causes the mast to bend over about the point A (as a fulcrum). This adds a compressive load to this member $= \frac{M}{d}$, where

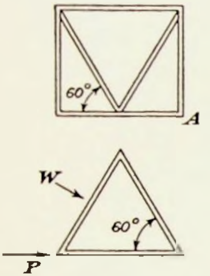


Fig. 217.—Triangular Structure.

M = bending moment over section considered, and
d = distance from centre of area of post considered to a line joining the C.A.'s of other two posts.

From these two direct stresses and the sectional area of one post it is possible, as before, to determine the stress in that post. This stress may or may not agree with that calculated by considering the mast as a rigid structure, but in the design the higher stress must be taken, and subsequent calculations based on it.

End Shear.—When the mast is subjected to a horizontal wind load, there is shear across the section. This will be a maximum at the stay points, and the worst condition obtains when the mast is rigidly fixed at those points. This condition does not occur in practice, the actual stress being something less than that calculated on this assumption. The shear stress will be the total wind load on a mast section divided by the cross-sectional area at the stay points.

Besides the end shear due to the wind there is a slight end shear due to the compressive load, and this is usually accounted for by increasing the end shear by an amount in accordance with the empirical formula—

$$S = \frac{200\,L}{18{,}000 - 80\frac{l}{k}},$$

where S = shear in pounds/square inches,
L = total compressive load in pounds,
l = length of section in inches,
k = least radius of gyration of cross-section in inches.

Design of Small Masts.—In the majority of cases it will be found that the top section of the mast could be made of a smaller moment of inertia than the lower section; this has the advantage with high masts that the wind load is reduced. For smaller masts, however, opinion is divided as to the advisability of varying the sections. With low masts the chief difficulty arises in the erection, and the flexibility of the top portion becomes an important

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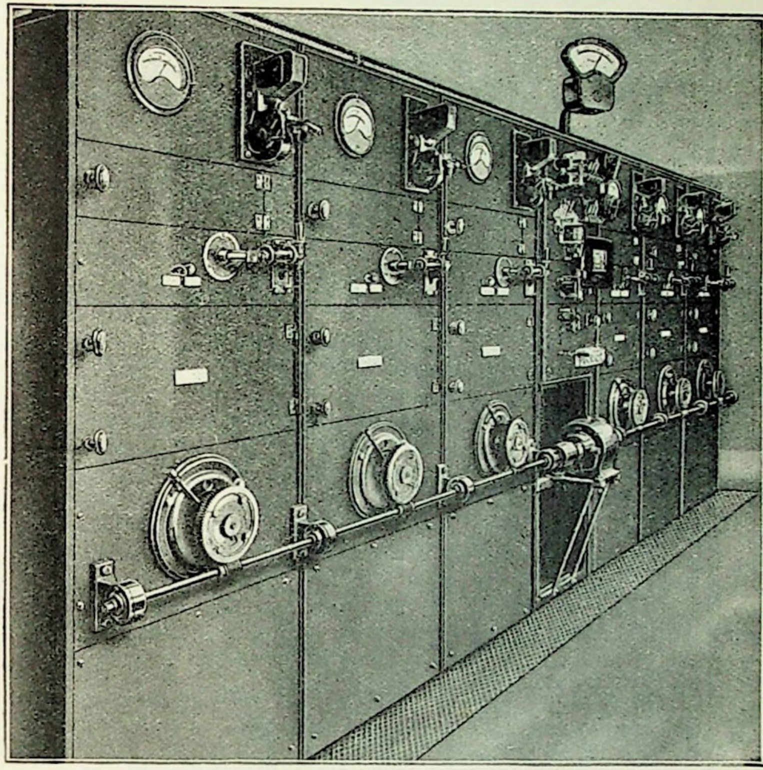

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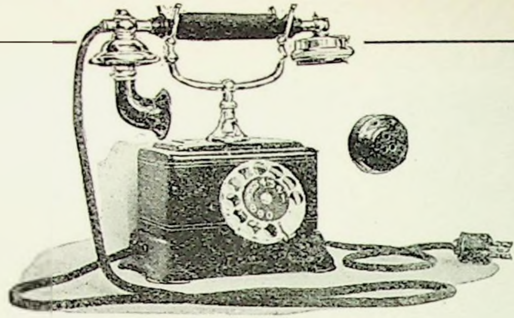

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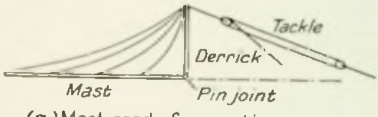
factor. Masts up to approximately 150 feet are usually raised by means of a derrick (see Fig. 218), and the stays are made off before erection.



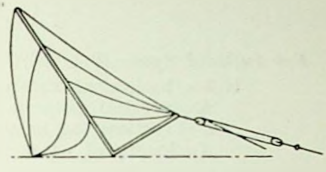


FIG. 218.—ERECTION OF SMALL MASTS WITH DERRICK.

The initial tension of the stays is thus rather indeterminate, and unequal stay tensions may bring in unknown forces to produce large bending moments. Hence a section which is ample when the mast is in position may not withstand the loads during erection, and due allowance must be made for this factor.

Tension in Wires.

The relation between the sag and the span of a wire hanging under its own weight (or acted upon by a uniform load) is given by the well-known catenary laws. If b is span, then the dip or sag is given by—

$$y = a\left(\cosh\frac{l}{2a} - 1\right),$$

where a is the minimum height of the wire (see Fig. 293 for values of cosh x).

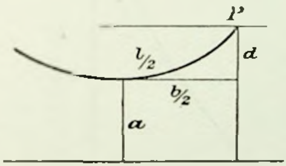


FIG. 219.—CATENARY CURVE.

The actual length of the wire is—

$$l = 2\sqrt{y(y+2a)}.$$

The tension at P is given by $T = w(y+a)$, where w = weight of wire per unit length.

To allow for wind, let w_1 be the wind load per unit length, and w' the resultant thrust per unit length due to both wind and weight.

Then, since the wind acts horizontally, $w' = \sqrt{w^2 + w_1^2}$.

Approximate Formulæ.—For flat arcs (dip $< \frac{1}{4}$ span) Eccles gives the following formulæ:

$$a = \frac{b^2}{8d^2}\left(1 + \frac{4}{3}\cdot\frac{d^2}{b^2 - 4d^2}\right) = \frac{b^2}{8d^2} \text{ nearly,}$$

where b = span; d = dip.

Then length $l = b + \frac{1}{24} \cdot \frac{b^3}{a^2}$,

$$T = \omega(a+d) = \frac{wb^2}{8d} \text{ approx.}$$

For Inclined Spans ($\theta < 45°$):

If b = horizontal distance between the ends of the wires,
h = vertical distance between the ends of the wires,
d = vertical sag in centre of wire,
l = length of wire.

(*a*) Wire leaving both supports with downward slope:

a and l are as above,

$$T = w\left\{a + d\left(1 + \frac{2ha}{b^2}\right)\right\}.$$

(*b*) Wire leaving one support with an upward slope:

$$a = \frac{b^2}{8d^2}\sqrt{1 + \left(\frac{h - 4d}{b}\right)^2},$$

$$l = 8ad/b + b\,(h - 4d)/2a,$$

$$T = 8a^2dw/b^2.$$

A formula which is sometimes useful gives the tension in terms of the slope at the ground level. If a stay of a mast is already in position, by looking

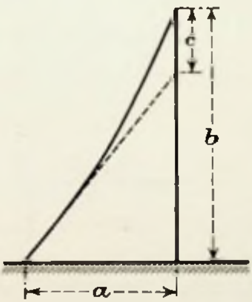


FIG. 220.—ILLUSTRATING METHOD OF ESTIMATING STAY TENSION.

along the stay the distance c (Fig. 220) may be estimated. (This is usually possible, since masts are built in equal sections.) Then the tension is given by—

$$T = w\,\frac{a^2 + (b - c)^2}{2c},$$

where T = tension in pounds,
w = weight in pounds per unit length,
a, b, and c are as shown in Fig. 220.

Wind Pressure Assumptions.—The most important factor in mast design is the wind pressure, which is assumed. By taking a value which is unnecessarily high a very expensive structure will result.

It is fairly certain that the velocity of the wind, and hence the pressure exerted, is greater at a point some distance above the earth than near the earth.

Several equations have been determined to fix this increase, though none of them are claimed to be accurate.

In the *Electrician* of July 1, 1921, S. P. Wing deduces the following formulæ from experimental results:

$$P = (0.00126h + 1.16)\, P_g,$$

where P = pressure in pounds per square foot at a point h feet above ground,
P_g = pressure at ground level.

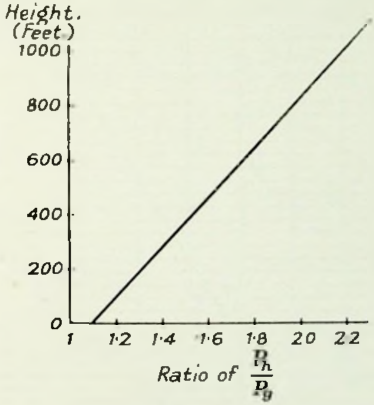


Fig. 221.—Variation of Wind Pressure with Altitude.

This expression is plotted in Fig. 221, which gives P_h/P_g in terms of the height.

Eccles, however, has suggested that owing to wind eddies near the ground it is safer to assume a uniform wind pressure (equal to that at the top) over the whole mast. A pressure of 30 to 60 pounds per square foot appears to be the usually accepted value to-day.

In calculating the pressure on stay and aerial wires the curves given in Fig. 222 will probably be useful. These figures were obtained by the National Physical Laboratory.

Calculation of Wind Load.—The wind load is calculated by multiplying the wind pressure by the projected area of the mast.

In the case of lattice masts the back of the mast also offers a certain area to the wind, and to allow for this the effective area is taken as about 1·6 times the projected area of the front face for triangular sections, and 1·8 for square sections.

In calculating the wind load on wires the projected area may be taken as 0·8 times the diameter of the wire × length.

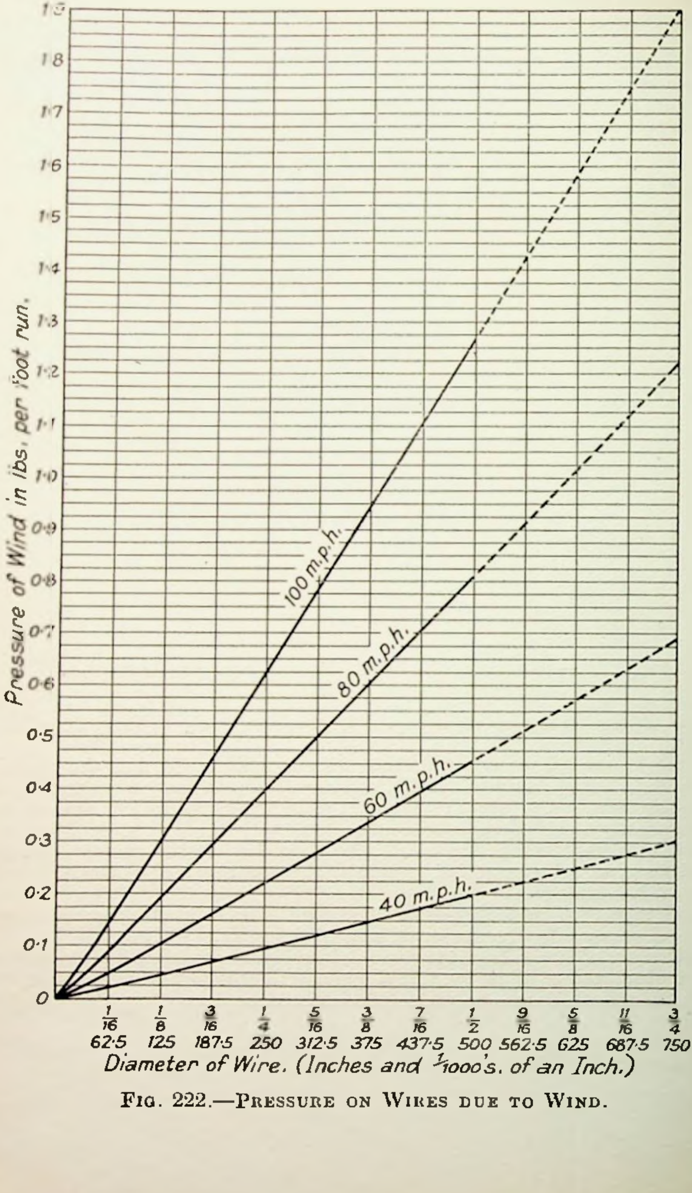


Fig. 222.—Pressure on Wires due to Wind.

Aerial Design.

The design of the aerial itself is a comparatively simple matter. The electrical problems concerned are dealt with elsewhere. Having decided the form that the antenna is to take, the mechanical details may readily be worked out from the usual catenary laws.

The most important feature in aerial design is that of the insulation. Great difficulty is experienced in obtaining suitable insulating materials. The cause of breakdown is not so much the high voltages as the high frequency of the voltages employed.

Porcelain is the only substance which has given satisfaction for this class of work, but even here the smallest crack will allow moisture to enter, and this sets up eddy currents in the insulator which rapidly cause complete breakdown. One type of aerial insulator is illustrated in Fig. 223. The centre porcelain tube is hollow to enable the material to be well moulded and thoroughly dried before glazing. The metallic guard ring at the end is for

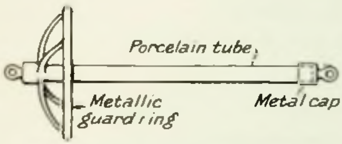


FIG. 223.—TYPE OF AERIAL INSULATOR.

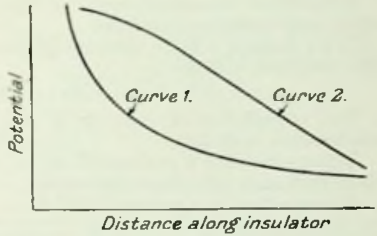


FIG. 224.—ILLUSTRATING EFFECT OF GUARD RING ON POTENTIAL GRADIENT.

the purpose of reducing the potential gradient. Experience shows that a steep potential gradient will cause breakdown even at a comparatively low voltage. From Fig. 224 it will be seen that the guard ring (Curve 2) reduces the gradient. Moreover, any brush discharge will take place from the ring, and not from the porcelain tube, so minimising risk of breakdown.

Insulators may be used in series, but the breakdown voltage for two insulators is only about 50 per cent. greater than that for one, and so in proportion. There is, however, a certain gain in mechanical strength.

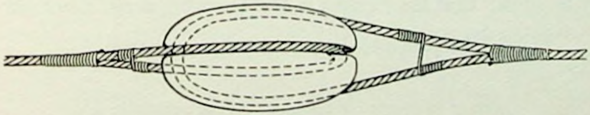


FIG. 225.—EGG INSULATOR, SHOWING METHOD OF BINDING-IN.

To avoid wasteful oscillations being set up in the stays, these are broken up into short lengths by means of insulators. The egg type of insulator is almost universally used for this purpose. It possesses the advantage that the porcelain is in compression, in which state it is very much stronger than in tension. Moreover, should one break, the stay does not break with it (Fig. 225). The leakage path, however, is small, so that this system of insulation is not suitable for very high voltages.

The safe stresses for porcelain vary with the quality, but the following average figures will serve as an indication:

Breaking stress (tension)	= 3,000 to 6,000	pounds per square inch.
,, ,, (compression)	= 45,000 to 65,000	,, ,,
Coefficient of expansion	$= 4$ to 9×10^{-6}	per degree Centigrade.

MISCELLANEOUS.

Modulation.

Modulation consists in varying the amplitude of a wave in accordance with some predetermined law.

If a voltage $e = \mathrm{E} \sin \omega t$ is modulated with a sine wave of frequency $p/2\pi$, the resultant voltage is—

$$e = \mathrm{A} \sin \omega t + \mathrm{B} \sin pt \sin \omega t.$$

The modulation is complete if A = B, and, if not, then the percentage modulation is B/A.

When receiving a modulated wave, however, the processes of tuning and detection each introduce distortion.

Assume that the received E.M.F.—

$$e_r = \mathrm{A}_1 \sin \omega t + \mathrm{B}_1 \sin pt \sin \omega t.$$

This may be written:

$$e_r = \mathrm{A}_1 \sin \omega t + \frac{\mathrm{B}_1}{2} \cos (\omega - p)\, t - \frac{\mathrm{B}_1}{2} \cos (\omega + p) t.$$

In other words, the modulated wave is equivalent to a fundamental of frequency $\omega/2\pi$, and two "side bands" of frequency $\frac{\omega - p}{2\pi}$ and $\frac{\omega + p}{2\pi}$ respectively.

The impedance of the tuned aerial circuit will be purely resistive to the fundamental frequency, but the two "side bands" will encounter a reactive impedance, which will thus cause attenuation and phase distortion.

It may be shown that the current in the receiving aerial is actually—

$$i_r = \frac{1}{\mathrm{R}} \left\{ \mathrm{A}_1 \sin \omega t + \mathrm{B}_1 \cos \varphi \sin (pt - \varphi) \sin \omega t \right\},$$

where
$$\cos \varphi = \frac{1}{\sqrt{1 + \frac{4p^2 \mathrm{L}^2}{\mathrm{R}^2}}} = \frac{1}{\sqrt{1 + \frac{4\pi^2}{\delta^2} \cdot \frac{p^2}{\omega^2}}}.$$

This shows that the modulation is less complete than that of the transmitted wave, and there is a phase shift. Moreover, this latter varies with p, so that in speech transmission the phase shift is variable, and distortion

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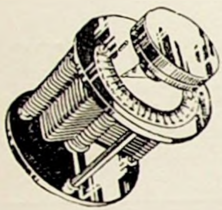

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occurs. It will be seen that the higher the decrement δ the less the distortion.

This effect is demonstrated by the following figures:

δ	..	0·005	0·01	0·02	0·04	0·10
$\cos\varphi$	..	0·369	0·62	0·85	0·955	0·99

Hence low decrement receivers should not be employed for speech reception.

A further source of distortion lies in the detector. The detector current is given by—

$$i_d = a_1 e + a_2 e^2 + \ . \ . \ .$$

Assuming a voltage $e = A \sin \omega t + B \sin pt \sin \omega t$ to be applied across the detector, the current will thus be—

$$i = \frac{a_2}{2}(A^2 + 2AB \sin pt + \frac{B^2}{2} - \frac{B^2}{2} \cos 2pt) + \text{radio-frequency terms.}$$

There is thus a double frequency term introduced by the action of the detector.

Blatterman (*Radio Review*, March, 1921) has worked out the actual detector current for a series of different types of modulation.

Five types of modulation are considered, and to facilitate comparison of the results a wave length of 600 metres is assumed with a note frequency of 1,000 cycles per second, and δ_r assumed $= 0·02$.

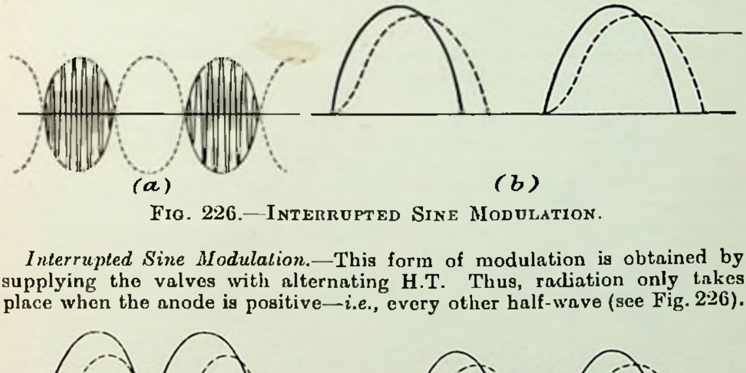

FIG. 226.—INTERRUPTED SINE MODULATION.

Interrupted Sine Modulation.—This form of modulation is obtained by supplying the valves with alternating H.T. Thus, radiation only takes place when the anode is positive—*i.e.*, every other half-wave (see Fig. 226).

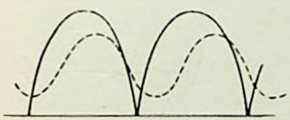

FIG. 227.—TONIC TRAIN.

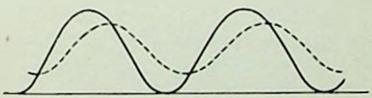

FIG. 228.—PURE SINE MODULATION.

Note that the modulation of the received current in these two cases is incomplete.

The envelope of the received current is shown in Fig. 226 (*b*), and indicates the distortion relative to the full line, which represents the current which would be obtained without attenuation or distortion.

Double Sine Modulation.—This is similar to the previous case, but double wave rectification is employed. f here $=500$, in order to produce a note of 1,000 frequency.

The distortion is indicated in Fig. 227.

Pure Sine Modulation.—This is self-explanatory. The transmitted and received currents are shown in Fig. 228.

Chopped C.W.—This is obtained by definite interruption of the radiation by some suitable means; assuming equal lengths of active and quiescent periods, the received current is as in Fig. 229.

Spark Modulation.—A spark transmitter may be considered as a modulated C.W., the expression being of the form:

$$i_s = \mathrm{I}[\mathrm{S}_0 + \mathrm{S}_1 \sin(pt+\theta_1) + \mathrm{S}_2 \sin(2pt+\theta_2) + \ . \ . \ .] \sin \omega t.$$

This expression is not rapidly convergent, but fortunately the terms of the received current become small after ten terms.

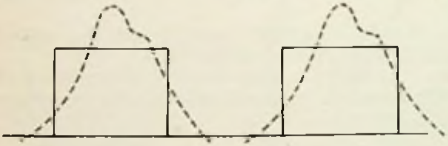

FIG. 229.—CHOPPED C.W.
This is a case where the received wave is over-modulated.

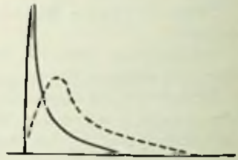

FIG. 230.—SPARK MODULATION.

It is necessary to assume a transmitter decrement, and this is taken as 0·1. On this assumption, with $f = 1{,}000$, $\lambda = 600$, $\delta_r = 0{\cdot}02$, the distortion and attenuation are as shown in Fig. 230.

These results may be summarised for purposes of comparison on the assumption that the audibility is proportional to the R.M.S. value of the detector current. So far the maximum values of the currents in the transmitter have been assumed equal.

This is hardly a fair comparison, however, since it is the R.M.S. aerial current which is measured in any transmitting system.

Comparative figures are given, therefore, for both cases.

TABLE XXXI.

RELATIVE AUDIBILITY OF DIFFERENT TYPES OF MODULATION.

Type of Modulation.	*Audibility.* Equal Maximum Currents in Transmitting Aerial.	Equal R.M.S. Currents in Transmitting Aerial.
Pure sine	1·0	1·0
Spark	0·341	2·96
Interrupted C.W.:		
Half wave	0·936	1·46
Double wave	0·802	0·697
Chopped C.W.	1·014	1·25

The results in the last column represent fairly the conditions obtained in practice with a crystal receiver. With a valve receiver, however, employing cumulative grid rectification the longer duration of the currents obtained with pure sine and I.C.W. double wave increases the audibility, while the short duration of the spark trains causes inefficient detection (see p. 179).

The marked superiority of spark transmission vanishes, therefore, when valve reception is employed, and there is little to choose between the various systems.

Radio Telephony.

This is a branch of the science of radio communication which has developed very rapidly in recent years.

As far as reception is concerned, the general remarks just made with regard to modulation still apply. To avoid distortion in the tuning circuits, high decrement circuits should be employed, which, of course, renders efficient and selective amplification difficult.

The best form of tuner for this class of work employs a series of filter circuits of the type described on p. 160, giving a uniform amplification over a given band of frequency with a sharp cut off on either side.

Another disadvantage of radio telephony from a commercial standpoint is the comparatively wide band of wave length necessary. The modulation of the carrier wave produces side bands, so that a band of frequency of 3,000 to 6,000 cycles is required. At a medium or long wave length this would require a wide band of wave length, and would cause considerable interference. Hence, telephony is confined to short waves, which at once limits the range for commercial working.

The Western Electric Company have devised a system which overcomes this defect to a large extent. In this system the carrier wave and one side band are filtered out, and only the second side band is radiated. In order to render the speech intelligible, however, the carrier wave must be re-introduced at the receiving point, and this is accomplished by employing an oscillating detector, the frequency of the heterodyne oscillation being the same as that of the carrier wave suppressed at the transmitting point.

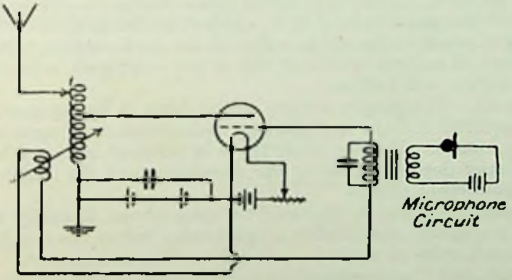

FIG. 231.—GRID CONTROL CIRCUIT FOR RADIO TELEPHONY.

Since the side band amplitude is only one-half that of the carrier wave, the total power to be radiated is only one-quarter of that required with the ordinary system, while the band of wave length required is considerably reduced.

Methods of Modulation.—In all systems, however, the fundamental oscillation has to be modulated in accordance with the speech or music vibrations obtained from a microphone.

There are two main methods of effecting this. In the first the speech currents, suitably amplified, are introduced into the grid circuit of the oscillating valve through a transformer. This system is illustrated in Fig. 231. The secondary of the transformer is shunted with a condenser, which forms a low impedance path for the high frequency oscillations.

The second method, known as the " choke control " method, employs a separate modulating valve. Both the oscillating and modulating valves are fed from a common source of H.T. through a choke coil L_1 (Fig. 232). Speech

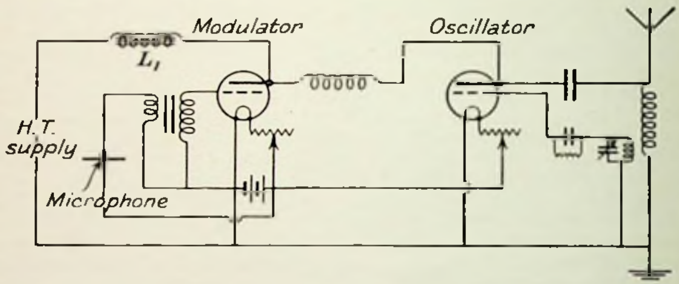

FIG. 232.—CHOKE CONTROL RADIO-TELEPHONE TRANSMITTER.

variations are impressed on the grid of the modulating valve, which causes variations in the anode current. The current through the choke coil must remain sensibly constant, and hence the feed current to the oscillating valve will be reduced, with consequent reduction of the oscillating current in the aerial. The aerial current is thus modulated in accordance with the speech currents as required. The modulating valve should thus be so designed and operated that the maximum E.M.F. applied to the grid should produce an anode current equal to the mean value of the feed current to the oscillator valve. Hence, if several oscillator valves are employed, a fewer number of modulator valves will suffice.

Microphones.—The simple carbon microphone is by no means up to the standard required for the satisfactory transmission of speech and, more particularly, music. Various other types of microphone have been devised, one of the chief desiderata being the elimination of the diaphragm, which introduces resonance effects.

For simplicity, however, the carbon microphone is easily the best, all other types requiring considerable amplification before the resultant currents are of the same order as those in a simple carbon instrument.

The " push-pull " microphone is a development of the carbon transmitter. It consists of two transmitters coupled one on each side of a light diaphragm. In this way a balanced action is obtained which eliminates resonance effects to some extent.

A device employing no diaphragm at all is the flame transmitter. Two platinum electrodes are suitably inserted in a gas flame rendered conducting by means of salts. Sound waves impinging on the flame alter its conductivity, and cause a current of varying intensity in the local circuit. This device gives faithful reproduction, but requires careful shielding from draughts.

A similar device is the Tucker microphone, employing very fine, short platinum wires heated to a dull red heat. The air waves then vary the conductivity. The sensitiveness of the device is greatly increased by passing a gentle stream of air over the hot wires.

An instrument which is very successfully employed by the B.B.C. is shown in Fig. 233. It consists of a thin, flat coil freely suspended in a powerful magnetic field. The minute movements of this coil produced by the sound waves set up currents which, when amplified, are of exceedingly good quality. The microphone is so sensitive that the instrument itself as well as the amplifier have to be mounted on rubber suspensions to eliminate extraneous noises due to vibration.

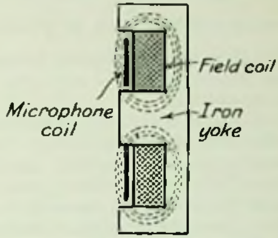

FIG. 233.—DIAGRAM OF MAGNETOPHONE.

The valves, in particular, are suspended by light rubber cords.

All microphones have a varying response at different frequencies. By suitable design of the amplifying circuits this effect can be compensated for. the result being that the combination is equally sensitive to all frequencies.

Atmospherics.

One of the greatest obstacles to the development of the science of radio communication is the presence of parasitic electrical disturbances known as " atmospherics " or " X's." These disturbances cause loud crackling and roaring noises in the receiver under bad conditions, so much so that the signal being received may be completely swamped.

Until recently very little was known of these disturbances, but the Radio Research Board has tackled the problem, and oscillographic studies have been made. The result of these investigations was published by Watson Watt (*Wireless World*, August 1, 1923), and shows that atmospherics are aperiodic or semiperiodic pulses of comparatively low frequency, some of the more

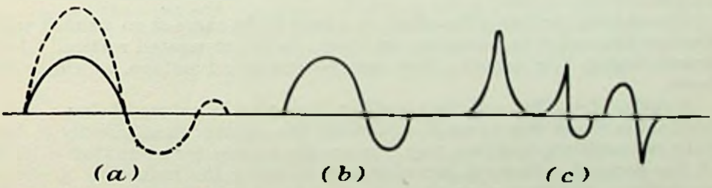

FIG. 234.—TYPES OF ATMOSPHERIC.

general types being shown in Fig. 234. About 30 per cent. are type (*a*), 30 per cent. type (*b*), and the remainder one of the various types shown in (*c*).

Moullin (*Journ. I.E.E.*, vol. lxii., p. 353) has investigated the effect on the receiver for the various forms of atmospheric pulse, and has shown it to be twofold. One is the simple effect of the low frequency pulse, which is negligible; the second is the transient oscillation set up by the beginning and end of the pulse. He shows that the effect produced is proportional to the initial rate of increase of the atmospheric E.M.F., and that the worst offender is type (*a*), in which both the commencement and the termination are very abrupt. He shows further that, mathematically, a damped sine wave $e = A\epsilon^{-\frac{pt}{2}} \sin pt$ produces an equivalent effect, where $p = \frac{\pi}{T}$, T being the time from beginning to end of the pulse. He then investigates the effect on several types of receiver, using this expression as representative of an atmospheric.

Watson Watt and Appleton found that the mean value of p was $3{\cdot}6 \times 10^3$, which is of comparatively low frequency. Hence it is only the transient terms produced which are troublesome. These oscillations are at the same frequency as the signal—*i.e.*, the frequency to which the set is tuned—and hence tuning, as such, is useless against this type of interference. The decrement of the circuit, however, is of prime importance.

Tuned Aerial.—For a simple tuned aerial Moullin shows that if the E.M.F. induced in the aerial is $e = E \sin \omega t$, then—

$$\frac{\text{signal P.D.}}{\text{atmospheric P.D.}} = \frac{E\pi\omega}{A\delta p},$$

where A and p are as above, and δ = decrement of aerial.

The signal strength is inversely proportional to δ, while the atmospheric P.D. is independent of δ. Hence the lower the decrement of the aerial the greater the signal/X ratio.

Aperiodic Aerial.—This type of aerial was described on p. 184. Here the rate of increase of voltage due to an atmospheric is dependent on RC, where R is the resistance in the aerial circuit and C is the capacity. R should be small enough to make the aerial behave to an X as a simple capacity, while at the considerably higher signal frequency it behaves as a simple resistance, the P.D. developed thereon being applied to the valve.

For such a condition RC should be of the order of $3/\omega$. Below this value the P.D. due to the signal falls off, while above this value the P.D. due to the atmospheric rises rapidly. This criterion gives values somewhat lower than those usually employed in practice, R being 5,000 to 10,000 ohms according to this theory.

Tuned Loop Aerial.—The effect on a loop is the same as on a tuned aerial. Certain atmospherics, however, are known to be propagated vertically downwards, and in this respect a loop may possess an advantage, as will be seen later.

Effect of Amplifier.—If the amplifier decrement is reduced below a certain value, the shock due to an X will cause the receiver momentarily to burst into self-oscillation, which may obscure the signals for some time after the X has finished. Hence it is undesirable to carry the reduction of receiver decrement, by reaction or otherwise, too far. Any efforts in this direction should be directed to the aerial. The product of frequency and decrement should not be <100 to avoid this ringing effect.

Owing to the fact that the X shocks the aerial at its natural frequency, it would appear that a reduction of interference would result from mistuning the aerial slightly. The drop in signal, however, more than balances the increased readability unless the decrement is high, which has been seen to be undesirable.

Effect of Duration of Signal.—The above formulæ assume that the signal persists long enough to build up to its full value. At high speeds, however, and on long wave lengths this is rarely the case.

Since valve reception is usually employed, the most important ratio is that of the quantities of electricity rectified:

$$\frac{\text{Q signal}}{\text{Q atmospheric}} = \frac{E\pi\omega}{A\delta p}(1+f\delta T),$$

where f = frequency,
δ = decrement,
T = duration of signal (seconds).

Hence the ratio of signal energy to atmospheric energy is always greater than that given by the original simple formula, and the greater $f\delta T$ the greater the immunity from atmospherics. Thus the higher the speed of working the greater the interference.

Where X's are strongly directional, as they are in some parts of the world, ordinary directional receivers may be employed with good effect.

In any case, a frame may be useful in eliminating " static "—*i.e.*, vertically propagated—disturbances. If the coupling coil to the receiver is made in two equal portions, the mid-point being connected to earth, then any vertical disturbance will affect both sides of the frame simultaneously, and the two E.M.F.'s will flow through the two halves of the coupling coil to earth in opposite directions, and so balance out.

Limiting may also be employed with some effect. Under suitable conditions of anode voltage a rectifying valve may be arranged to saturate with a signal very little stronger than that being received. The effect of an X is thus reduced to the same order as the signal.

High Speed Reception—Remote Control.

In order to record the signals received it is simply necessary to produce sufficient current change in an amplifier to operate a mechanical relay. The local circuit of this relay may be caused to operate a Wheatstone receiver, a creed printer, or any other device. The modern tendency in reception of medium or long distance traffic is to equip a receiving station in the best site available, complete with suitable directional equipment if necessary, and to amplify the received signals sufficiently to operate a relay. The signals are then relayed telegraphically to a central receiving office, which also controls the transmitting station by remote control. The transmitting and receiving stations thus become simple repeater stations, all traffic being handled at the central office. The advantages of such an arrangement are obvious.

If atmospheric conditions are such as to preclude satisfactory recording, then the signals are relayed telephonically to the C.R.O. The personal element then is available, an experienced operator being able to discriminate between X's and signals to the extent of being able to read signals, in extreme cases, through interference so fierce that the signal cannot be heard by an ordinary person.

In order to operate a relay, the signals, which are alternating at a frequency of 500 to 2,500 cycles per second, must be again rectified, so producing a unidirectional current throughout the duration of the signal.

The modern practice is to use relays of a robust construction, though more sensitive than telegraph relays, capable of operating on 1 to 2 milliamperes.

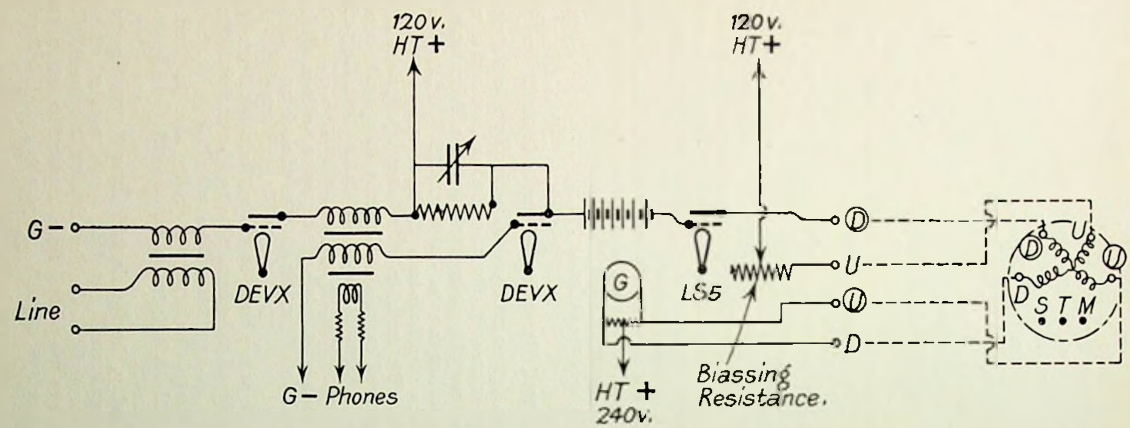

FIG. 235.—DOUBLE CURRENT BRIDGE (MARCONI).

(By courtesy of M.W.T. Co.)

As the speed is increased, however, higher currents of from 10 to 20 milliamperes are required. Balancing devices are sometimes employed, using two valves, in one of which the current increases, while that in the other decreases, thus producing a double current effect, enabling a differential relay to be employed. Such a device is shown in Fig. 235. There are three valves. The first is a simple power amplifier, the second is normally adjusted to supply a current of about 15 milliamperes, while the current from the third is passed round the relay in the opposite direction. During a space the third valve supplies about 30 milliamperes, so giving an effective spacing current of 15 milliamperes. On the arrival of a signal, however, the current in the third valve is reduced to zero; current in the second valve remains steady, so that a 15-milliampere marking current is produced.

The sensitivity of the relay is capable of mechanical control, and the instrument is adjusted to operate at a current only slightly less than that available (which should, however, be several times the minimum required to work the relay). By this means any interference or jamming of a strength appreciably less than the signal is eliminated.

One form of relay is known as the Carpenter relay, a diagram of which is given in Fig. 236. The armature is pivoted in the centre of a magnetic field

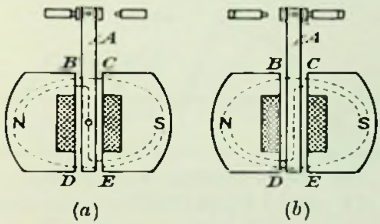

Fig. 236.—Carpenter Relay.

supplied by a powerful permanent magnet. A spacing current passed round the armature coil produces a flux distribution through the armature as at (*a*) in Fig. 236. There is thus a greater flux at points B and E, and the armature moves to the left. With a marking current the position is reversed. This form of construction gives a robust relay which moves from contact to contact with a snap. It will operate on currents of 0·2 to 0·5 milliampere.

Other forms of relay are employed which are modifications of the ordinary telegraph relay.

A device which is capable of recording the received signals directly is the McLachlan magnetic drum recorder. This consists of an iron drum carrying a field coil inside. On the drum slides a shoe attached to the recording mechanism. It is found that when current is passed round the field coil, the magnetisation of the drum produces a pull on the shoe fifty times in excess of that which is to be expected theoretically, and this force is adequate to record the signals received directly with an operating current of 2 to 10 milliamperes only.

Turner has shown (*Journ. I.E.E.*, vol. lxii., p. 192) that the decrement of the circuit exercises considerable effect on the shaping of the signals. The value of the E.M.F. at time T is $e = E(1 - \varepsilon^{-f\delta T})$. If $f\delta T$ is large, the signal rises rapidly and is flat-topped.

The relay operates when the current reaches a certain value, and if $f\delta T$ is large the shaping is good. As $f\delta T$ decreases, however, the signal current becomes peaky, and it is impossible to find a value of i_o (the current

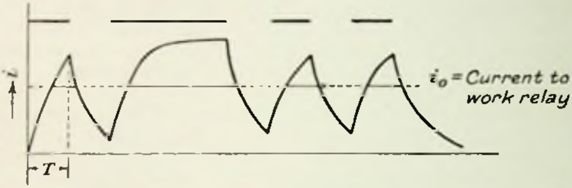

FIG. 237.—SHAPING OF SIGNALS WITH $f\delta T = 2$.

required to operate the relay) for which the shaping is satisfactory. The limit occurs around $f\delta T = 2$.

Figs. 237 and 238 illustrate this effect for the letter *l*. In the first $f\delta T = 2$, and the shaping with $i_o = 0{\cdot}7 i_{dot}$ is good. In the second $f\delta T = 1$, and no value of i_o can be found to give good spacing of the signals.

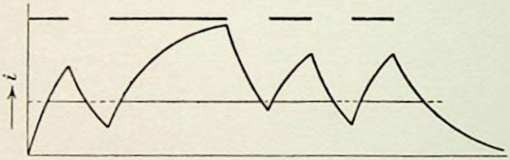

FIG. 238.—SHAPING WITH $f\delta T = 1$.

No value of i_o can be found which will give good spacing.

Now f is constant, T is inversely proportional to the speed of working, and hence an increase of speed should be accompanied by a decrease of decrement.

Since ringing sets in if $f\delta > 100$, there is an automatic limit to the speed for a given wave length, and it is impossible to work at high speeds on very long wave lengths.

Screening.

Screening is of two kinds—natural and artificial. Natural screening is the effect sometimes observed where, in certain localities, the signals from a distant transmitting station are abnormally weak or non-existent, while in other localities even farther away good readable signals are obtainable.

This type of screening is usually due to one of two causes. The first is illustrated by Fig. 239. The penetration of the wave into the earth is small, the major part of the field following the contour of the hill as shown. At the top the electric field continues to travel onwards in a straight line, but the end near the earth is not vertical, but nearly horizontal.

An ordinary aerial will thus not be affected, and it is only the small vertical component of the field which produces signals. This type of screening vanishes as the receiving point is removed from under the lee of the hill.

This effect should not be confused with a true shadow effect which is observed if the dimensions of the obstruction are of the same order as the wave length. The wave then travels round on each side of the obstruction

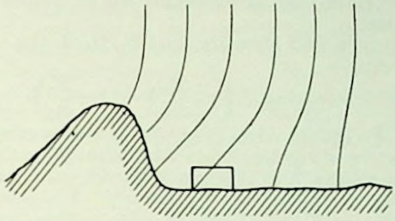

FIG. 239.—ILLUSTRATING SCREENING DUE TO A HILL.

(Fig. 240), and the two waves interfere on the far side of the obstruction, and produce a shadow as with light. This effect is usually only observed with short waves, and with very short waves is very marked.

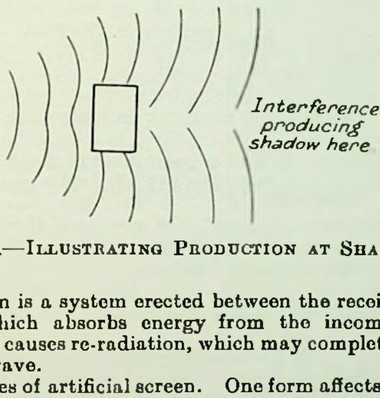

FIG. 240.—ILLUSTRATING PRODUCTION AT SHADOWS.

An artificial screen is a system erected between the receiving system and the transmitter, which absorbs energy from the incoming wave. The current so produced causes re-radiation, which may completely wipe out the effect of the main wave.

There are two types of artificial screen. One form affects the electrostatic field, and the other the electromagnetic field.*

Consider a simple loop circuit, the mid-point of the loop being earthed. There are two ways in which a loop may respond to an electromagnetic wave. The first is the pure loop action, which may be considered as due to the linkage of the loop with the electromagnetic field, while there is also the E.M.F. induced by the electrostatic field in the frame acting as a simple aerial. These two effects respond to different types of screen.

* In view of the fact that these are manifestations of the same phenomenon, this statement may seem fallacious. The fact, however, is that receiving systems such as loops and coils which may be conveniently considered as being influenced by the magnetic field of the wave, can be considered from an electrostatic point of view if desired (and on p. 184 the theory of loop aerials is deduced from such a view-point). Hence it is simply a matter of convenience.

Electrostatic Screens.—To screen the electrostatic field it is simply necessary to erect a series of simple aerials all round the receiving system.

Barfield (*Journ. I.E.E.*, vol. lxii., p. 249) has investigated the effect of various types of screen on a small frame aerial 2 feet 6 inches square.

The screening was estimated by observing the reduction of field strength produced by the screen.

If $E_1(H_1)$ is the normal field strength, and $E_2(H_2)$ is the strength with the screen in position, then—

$$S = \text{screening} = \left(1 - \frac{E_2}{E_1}\right) = \left(1 - \frac{H_2}{H_1}\right).$$

A simple screen of vertical wires [Fig. 241 (*a*)] gave a screening effect = 80 per cent. A system of earthed horizontal wires [Fig. 241 (*b*)] above the coil gave still better results, S in this case being 88 per cent.

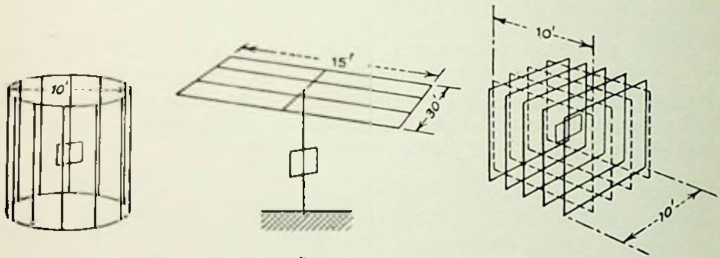

FIG. 241.—TYPES OF ELECTROSTATIC SCREEN (ALL 10 FEET HIGH).

The most efficient screen was a combination of (*a*) and (*b*)—a series of non-closed loops at right angles [Fig. 241 (*c*)]. This gave a screening of well over 90 per cent.

The above results were obtained with screens of the dimensions shown in the figure, the spacing of the wires being 6 inches. The closer the wires the more effective does the screening become.

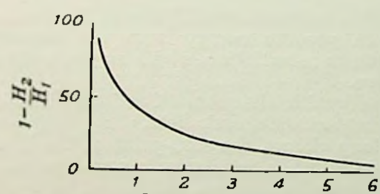

FIG. 242.—EFFECT OF SPACING ON SCREENING RATIO.

The ratio of $\frac{S_d}{S_o} = \frac{\text{screening with spacing } d}{\text{screening with no spacing}}$ is shown in Fig. 242. The over-all length of screen was kept constant, so that as the spacing was increased the number of wires decreased.

The essentials of an electrostatic screen are simply that each side of the screen shall be connected to the opposite side by a good conducting path.

Electromagnetic Screens.—To screen the magnetic field a series of *closed* loops were erected. These loops have induced therein currents which produce secondary fields in opposition to that of the electric wave. The screening effect thus produced depends on the number and spacing of the loops in a similar manner to that for an electrostatic screen, as shown in Fig. 242.

It will be obvious that since each side of the screen acts as an aerial, a magnetic screen, such as is described above, will also screen the electric field. The converse, however, is not true. In order to screen the magnetic field only, leaving the electric field unaffected, Professor Howe has suggested the system shown in Fig. 243, such a screen being discontinuous in the direction of the electric field.

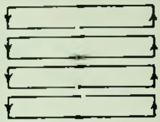

Fig. 243.—Arrangement to Screen Electromagnetic Field only.

Wire netting makes a very efficient screen, a 2-inch mesh giving 89 per cent. screening, while a 1-inch mesh gives 96 per cent.

It should be observed that the screening effect depends on the provision of closed conducting paths round the object to be screened, and the effect so produced only affects the component of the magnetic field perpendicular to the plane of such paths. A magnetic screen is therefore directional, and to be complete must provide conducting paths in three planes mutually at right angles.

Complete Screening.—This is difficult to obtain; it is often desirable to protect certain portions of a receiver from all external influences, and to do so the receiver must be enclosed completely in a metal box electrically continuous in every direction, and with no gaps whatever.

Care must be taken that the dimensions of the screen are large compared with the coil or other apparatus to be screened, as otherwise secondary reactions will be set up. Any coils must be at least 3 inches axially and 1½ inches radially from the screen, or the decrement will be seriously increased.

Direction Finding.

The directional properties of frame aerials may be utilised to determine the direction of any desired signal. In particular, the bearings of ships at sea may be obtained, and this branch of navigation is becoming of increasing importance.

The direction finding apparatus simply consists of a frame aerial (suitably calibrated in terms of the cardinal points), together with a suitable amplifier. The set may be installed on board ship or on land. In the former case bearings may be taken on any fixed stations of which the true position is known, and the position of the ship deduced therefrom. This method suffers from several disadvantages, however—viz.:

(*a*) The bearing obtained on the direction finder is relative to the length of the ship, and to obtain the actual bearing the compass bearing of the ship must also be obtained. In rough weather the rolling of the ship's compass may introduce errors of several degrees.

(*b*) The various masses of metal in a ship distort the bearings and necessitate a correction curve. Any redistribution of such metallic masses (*e.g.*, after reloading with cargo, etc.) will render the correction curve inaccurate.

If the D.F. apparatus is placed on land, these two disadvantages are obviated, an accurate and constant calibration being possible, giving the bearing relative to true north. Such a bearing, however, must be charged for by the land station, whereas a ship having D.F. gear installed can obtain bearings as often as it chooses.

A single bearing does not fix the position of the ship, and two bearings should be obtained. The position is then given by the intersection of the two. Unless the two bearings intersect at an angle greater than 45°, however, the intersection will be indeterminate, and liable to error.

In such a case a third bearing should be obtained from another direction, as shown in Fig. 244. The true position is in the centre of the "cocked hat" so obtained, the smaller the hat the more accurate being the position.

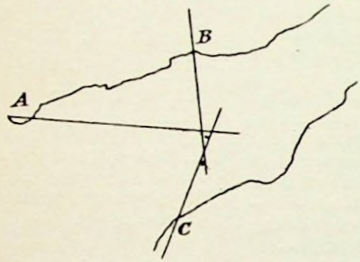

Fig. 244.—Method of obtaining Cocked Hat from Three D.F. Stations.

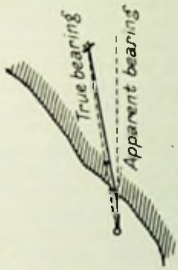

Fig. 245.—Illustrating Error due to Coast Refraction.

Errors in Bearings.—Errors in D.F. bearings arise from two main causes: (*a*) Geographical factors; (*b*) transmissional errors (night effect).

Hilly or broken country in the path of the wave will cause distortion, while masses of metal, trees, etc., will also produce errors. If there are any such objects behind the D.F. station, but at a comparatively short distance, errors will be introduced by re-radiation from these objects. Trees are especially undesirable, as the errors are variable, and a belt of trees near the station may give rise to an unreliable sector over which bearings cannot be given.

In the case of land stations situated on the coast, any bearings making an angle of less than 20° with the coast line are subject to variable error due to coast effect. While if the station is inland, the bearing is subject to refraction when crossing the coast line, as indicated in Fig. 245.

With a Bellini-Tosi system, if one of the loops has a larger capacity to earth than the other (due to the proximity of buildings, etc.), the bearings will be distorted towards the direction of this loop. This error thus changes each quadrant, and is known as "quadrantal error." It may be eliminated by reducing the size of the appropriate loop.

For further information for all matters relating to direction finding, the reader is referred to an excellent treatise by R. Keen ("Radio Direction and Position Finding").

Night Effect.—Bearings by day are usually reliable to $\pm 1°$. At night time, however, and particularly round the periods of sunrise and sunset, large errors are experienced.

Eckersley has shown this to be due to the Heaviside layer, and consequently only experienced at distances above 15 to 20 miles.

The electric waves are normally vertically polarised, the electric field being vertical and the magnetic field horizontal. Reflection from the Heaviside layer, however, may be shown to produce a horizontally polarised wave in which the electric field is horizontal, and this affects the frame in its zero position, so causing a distorted bearing. In some cases it is possible to obtain a zero position by a further rotation of the frame past its true zero position, while in others the minimum is so bad that no definite bearing is obtainable.

The effect will be clear from Fig. 246. The direct normally polarised wave does not affect the frame, nor does the reflected wave, since it does

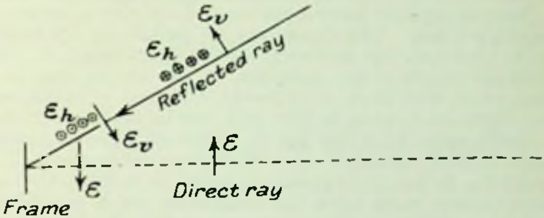

Fig. 246.—Illustrating Response of Frame to Horizontally Polarised Reflected Ray.

not link with the top and bottom. The horizontally polarised reflected ray does link with the top and bottom, however, and as it is arriving at an angle there is a phase difference between the E.M.F.'s produced in the top and bottom, which gives rise to a resultant E.M.F.

This effect is only obtained by reflection from a sharply defined Heaviside layer, such as is obtained at night. At sunrise and sunset the layer is in a state of violent turmoil, due to the ionisation or deionisation of the upper atmosphere, and extraordinary variations of bearing may be produced at such periods. The bearing has been known to appear to rotate six times in five minutes.

Eckersley has also shown that the heart-shape balance is upset by this night effect, one side of the heart being distorted, sometimes to the extent

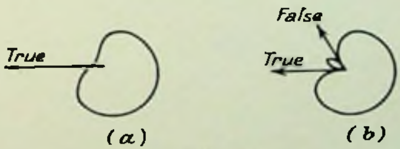

Fig. 247.—Heart-Shaped Diagram Distorted by Night Effect.

of producing a second zero, but the *true zero always remains*, due to the fact that the frame is in its *maximum* position at balance. Fig. 247 shows some distorted heart shapes.

After the bearings have settled down (after the sunset period) the results obtained are tolerably reliable, because it is found that any error due to night effect is always accompanied by some abnormal feature, such as very strong signals, flat minima, etc. Hence a normal bearing at night may be assumed to be tolerably correct.

Short Wave Transmission.

Development at the present moment is in the direction of the production of short waves of the order of 15 to 100 metres. These waves are so short that reflectors of dimensions comparable with the wave length can be constructed, and the radiation can thus be concentrated in a single beam. By the use of such reflectors at both transmitting and receiving points, the power required can be reduced to something like $\frac{1}{85}$ of that normally required.

Beacon stations have been erected on this principle, which send out a rotating beam. This beam can be picked up by ships fitted with short wave receivers, and according to the signal received, which is different for different points of the compass, so can the bearing be determined. The absorption with such short waves, however, is enormous, so much so that two receivers have to be provided, one on each side of the ship.

Quite apart from the use of reflectors, however, enormous ranges have been covered on short waves with very low powers. No reliable data are available as yet, but experience seems to indicate that communication will depend very much upon the conditions which prevail in the upper atmosphere, and that continual changes of wave length will be required.

Another difficulty is the production of any appreciable power in the aerial, due to the very small capacity thereof, but the Marconi Company have succeeded in obtaining powers of 100 kilowatts and more, and satisfactory communication has been established over long distances. It remains to be seen whether such communication can be maintained for a reasonably large proportion of the twenty-four hours.

Perhaps the chief advantage of the system lies in the entire absence of any atmospheric disturbances on these wave lengths.

TELEGRAPHY AND TELEPHONY

SECTION II

TELEGRAPHY AND TELEPHONY

NOTE.—*Since the telegraphs and telephones in this country are a government monopoly, the information given in this section is limited to data likely to be of use to electrical engineers generally, and radio-engineers in particular. Reference is made to sources of more detailed information should such be desired.*

TELEGRAPHY.

(*Prepared by* A. G. KING, ESQ.)

TELEGRAPHY is the science of communicating intelligence by means of suitably timed impulses, the various letters and symbols being represented by definite combinations of such impulses.

The simpler systems of telegraphy employ the Morse code. The various letters and other symbols to be signalled are represented by combinations of currents of short or long duration, known respectively as dots and dashes.

The Direct Sounder System.

The simplest system which is employed to any extent is the direct sounder system.

Fig. 248 shows the connections of an up and down station working on this principle. The up station is sending a current out to line; this passes through

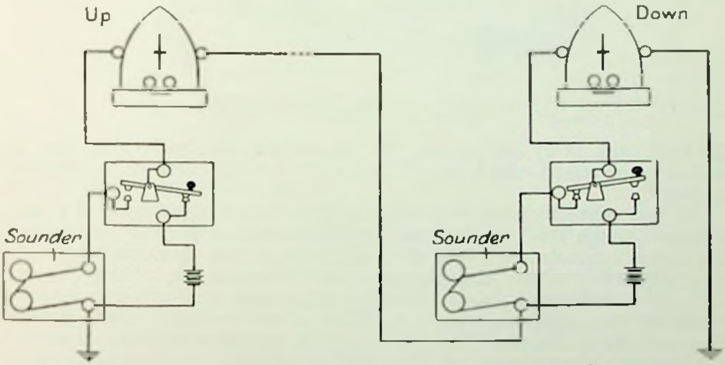

FIG. 248.—DIRECT SOUNDER WORKING (UP-STATION SENDING).

the sounder at the down station, thus producing an audible signal. There is no current in the line when the key is up, and for this reason the system is known as the single current system. The several component parts of this circuit are described below.

Fig. 249 shows the connections for three stations working on this principle, an intermediate station being interposed between the up and down stations.

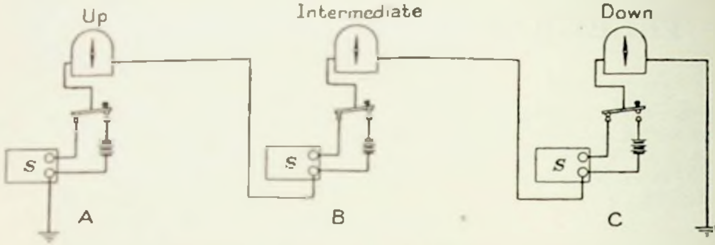

FIG. 249.—SOUNDER WORKING WITH INTERMEDIATE STATION.

Of any two stations, the "up" station is the one nearer London. Hence the intermediate station (Fig. 249) is an up station with respect to C, but a down station with respect to A.

The Single Current Key.—The single current key (Fig. 250) consists of a metallic lever AA, which is pivoted at B. The key is normally held against

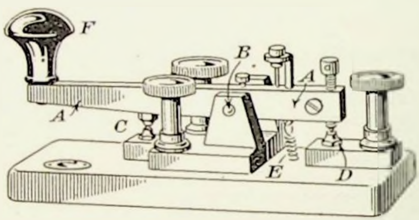

FIG. 250.—SINGLE CURRENT KEY.

the back stop D by the spring E. To connect the battery to line, the knob F is depressed, which opens the back contact D and closes the front contact C.

The top contact of the back stop at D is formed by means of a screw passing through the lever, so enabling the amount of play to be regulated. The actual contacts are tipped with platinum or gold-silver, an alloy which does not readily oxidise, so minimising possible trouble due to dirty contacts.

Sounders.—Fig. 251 shows the ordinary pattern of Post Office sounder. There are two coils, AA, wound on soft iron cores and connected in series. The cores are made of soft tube iron, split to minimise residual magnetism, and rest on a piece of soft iron H called the yoke, thus forming a horse-shoe magnet. A brass lever K is pivoted between two screws E, and is normally held up against stop C by a spring, the tension of which can be varied by means of a milled headed screw F. B is a soft iron armature fixed at right angles to the lever above the cores. When a current flows through the coils the cores become magnetised, attract the armature, and pull down the lever, thus producing a sharp metallic sound. If correctly adjusted, the instrument will operate with a current of 55 milliamperes, but the working current in practice is usually between 60 and 80 milliamperes. This pattern is only used for direct working, and has a resistance of 20 ohms.

Polarised Sounder.—With the ordinary type of Post Office sounder a current in either direction will operate the instrument, but with the polarised type the movement of the armature is determined by the direction of the current. Polarised sounders are more sensitive than non-polarised instruments.

The polarised sounder is similar in appearance to the ordinary sounder. The soft iron yoke of the latter, however, is replaced by a strong horse-shoe magnet. This produces a polarity in the core equal in value to the magnetism produced by 40 milliamperes. The magnetism produced in the cores and the tension of an opposing spring exert opposite forces upon the lever of the sounder. According to the direction of the current the magnetism of the core is either augmented or weakened; in the former case the armature is attracted to the bottom stop, and in the latter case the

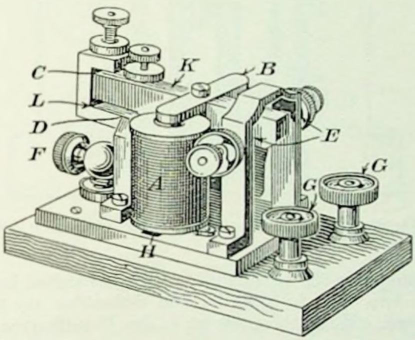

FIG. 251.—SOUNDER (POST OFFICE PATTERN).

tension in the spring overcomes the decreased magnetic pull and the armature is attracted to the upper stop.

The sounder is wound differentially, each winding having a resistance of 500 ohms. The instrument will operate at twenty words per minute with 3 milliamperes.

Galvanometer.—A galvanometer is a simple form of ammeter. It is inserted in telegraph circuits to indicate the direction and, to a certain extent, the strength of the current at the particular point. The most general type of instrument, known as the "differential galvo," is shown diagrammatically in Fig. 252. This instrument consists of two bobbins, BB, fixed side by side about $\frac{1}{4}$ inch apart. On a horizontal axis between them swings a soft iron needle A, of U shape, pivoted near the lower end of the U. This needle is magnetised by a pair of permanent magnets, M, placed below it, and swings with its free end uppermost, being kept in that position by the weight of the lower end of the pointer attached to it, and swinging in front of the dial.

The bobbins are wound with two coils, half of the coil being wound on each bobbin, as indicated in Fig. 252. The windings are identical, each having a resistance of 50 ohms, with a 300 ohm coil in shunt. If equal currents are passed through the windings in opposite directions, the resultant deflection of the needle is nil. This property is utilised in duplex working.

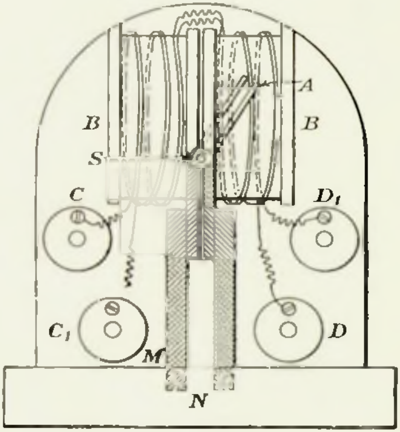

FIG. 252.—DIFFERENTIAL GALVANOMETER (SCHEMATIC DIAGRAM).

For ordinary purposes the two windings are connected in series or parallel. With one coil or two coils in parallel the deflection up to about 30° is 1° per 0·4 milliampere. With the coils in series 1° indicates 0·2 milliampere.

Relays.—The direct sounder system can only be operated over comparatively short distances; for working over longer distances a relay is inserted in the circuit. This is a delicately adjusted instrument which can be actuated by currents too small to operate a sounder. The relay does not itself give readable signals, but its action brings into play a " local " battery, which supplies the current necessary to operate a sounder.

There are two general types of relay used for Morse telegraph circuits—the polarised relay, and the non-polarised relay. In the former the direction of the current through the relay determines the direction in which the tongue will move, whilst in the latter the movement of the tongue is always toward the same stop, independent of the direction of the current by which it is actuated.

Post Office Standard Relay.—The Post Office standard relay is illustrated in Fig. 253. It contains two electromagnets with soft iron cores. These cores are polarised by a permanent horse-shoe magnet, which is bent round the electromagnets in order to make the whole more compact. The south pole of the permanent magnet is at the top, so that the two ends of the adjacent cores therefore have north polarity, while the two bottom ends have south polarity. Two short soft iron armatures are fixed to a vertical brass spindle, and oscillate between the cores. A light German silver armature, called the tongue, is attached to the spindle, and moves with the armatures, so making contact with two studs called the marking and spacing stops, and represented by the letters M and S in Fig. 253. The studs and the tongue

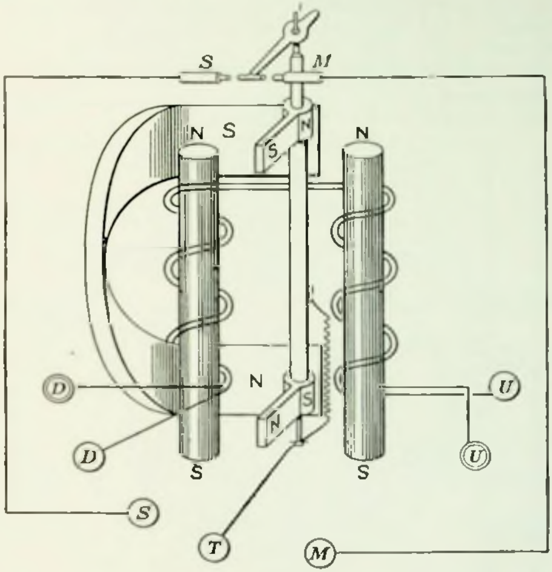

FIG. 253.—SKELETON DIAGRAM OF POST OFFICE STANDARD RELAY.

are tipped with platinum or gold-silver alloy. The instrument has differential windings, the ends of one coil being indicated by D and U, and the ends of the other coil are indicated by (D) and (U).

Currents required with coils in parallel:

Standard A	.. .. ..	20 to 30 milliamperes.
Standard B	.. .. ..	30 to 40 ,,

Currents required with coils in series:

Standard A	.. .. ..	10 to 15 milliamperes.
Standard B	.. .. ..	15 to 20 ,,

The Single Current System, with Relay.—The connections for single current working are shown in Fig. 254. The relay takes the place of the sounder in direct working, and a local circuit is added.

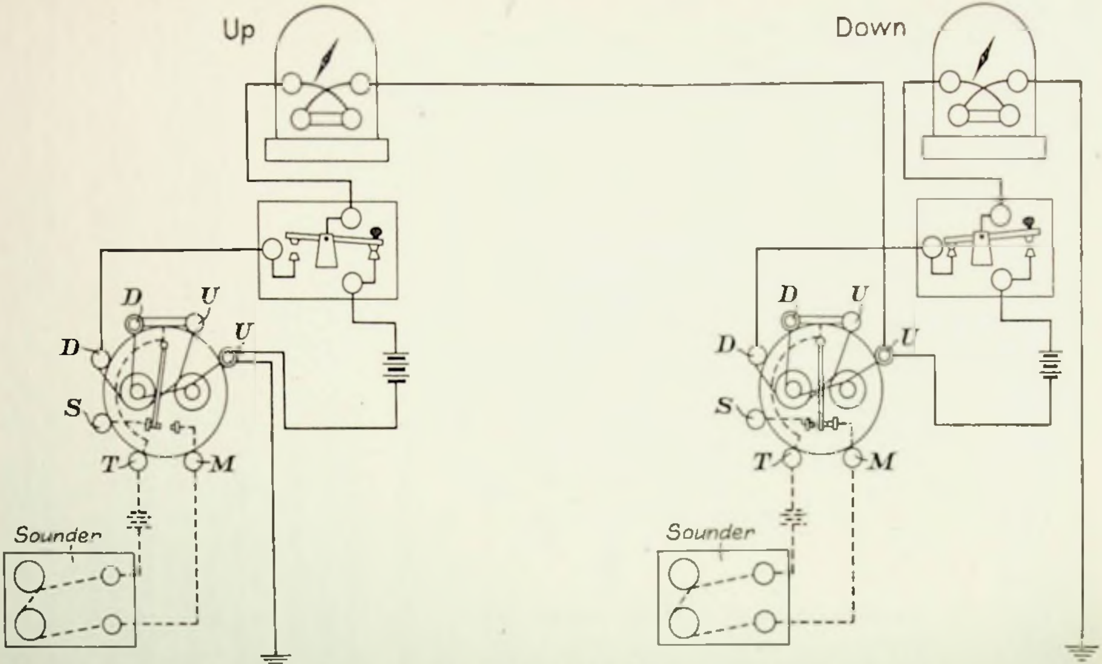

Fig. 254.—Single Current Working with Relays (Up-Stations Sending).

Double Current Working.

When a relay is used for single current working the tongue is returned to the normal position when the current ceases either by a magnetic bias or, in the case of non-polarised relays, by a spring.

In the double current system a polarised relay is invariably used, but in order to bring the tongue back to the spacing stop after the marking current has ceased a reverse or spacing current is used. This current flows in the opposite direction round the circuit as soon as the marking current ceases. It passes through the coils of the relay from D to U, thus producing

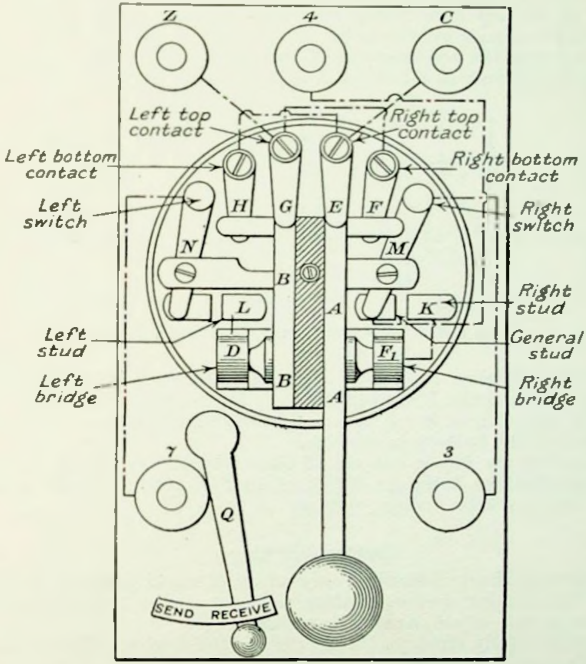

Fig. 255.—Double Current Key.

an effect opposite to the marking current, so bringing the tongue over to the spacing stop. No bias is necessary, and the tongue can work centrally between the poles. Double current working has the further advantage that the range of the relay is increased. Changes of adjustment are required in single current working when varying currents are received from stations at different distances, or due to varying leakage on the line; such alterations are unnecessary in double current working, since, if the marking current varies for any reason, the reversing current will change to the same extent.

Double current working also counteracts the effects of capacity on long lines or cables. In single current working the capacity of the line delays the rise of current at the receiving station at the beginning of a signal, and prolongs it at the end, so interfering with the correct shaping of the signals.

Consequently the speed with a single current system is considerably reduced if the line is of a high capacity. With double current working the time taken by the line to reverse its charge is always the same, so that, although a time lag may be present, the shaping of the signals is preserved.

Double Current Key.—The simple make and break key used for single current working cannot be used for double current circuits. Here it is necessary to employ a reversing key, and the instrument employed is in effect a simple change-over switch.

Fig. 255 shows the construction of such a key. The line terminals are connected to the two metal strips A and B, which make contact with the

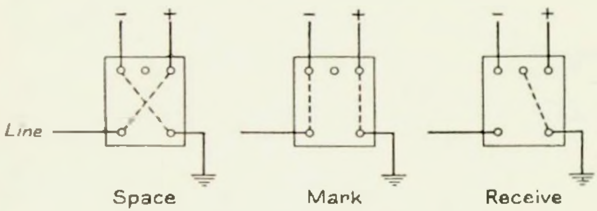

FIG. 256.—CONNECTIONS WITH D.C. KEY.

springs G and E when the key is depressed, and with H and F when the key is up. Since G and F are connected to the negative of the battery, and E and H to the positive, it will be seen that the depression of the key reverses the position of the battery in the line.

For receiving the key is cut out of circuit by the switch Q, the several possible connections being as indicated in Fig. 256. Fig. 257 gives the connections of a double current system.

Duplex Working.

The systems hitherto described only admit of working in one direction at a time. The duplex system enables messages to be transmitted over a single wire *in both directions* simultaneously.

The system chiefly employed is the differential duplex. This is based on a relatively simple principle.

The currents sent out from the station are fed to the middle point of the galvanometer, the two coils being connected in series. The current thus splits. Half travels along the line to the distant station, and thence to earth, returning to the battery through one coil of the relay. The other half traverses the relay in the opposite direction, having first passed through a network R. If this network is so adjusted as to have the same characteristics as the line, the two currents will be equal, and the relay will be unaffected.

Received signals, however, only traverse one coil of the relay which is thus operated.

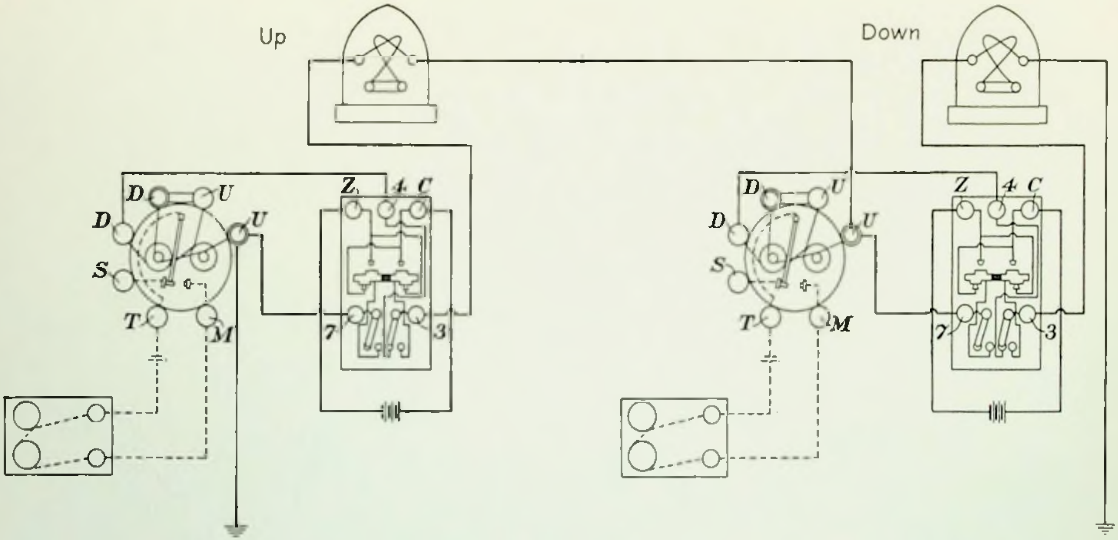

FIG. 257.—DOUBLE CURRENT SYSTEM USING D.C. KEYS.

Keys shown in "receive" position.

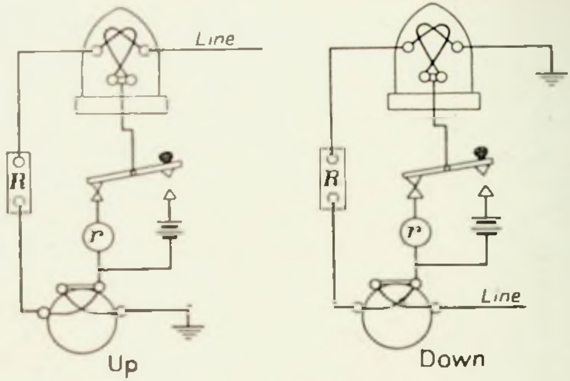

FIG. 258.—SINGLE CURRENT DUPLEX.

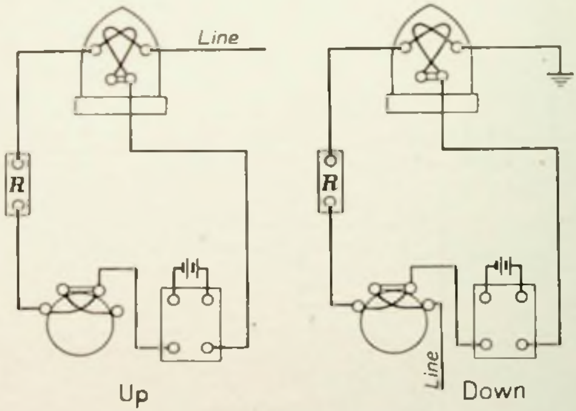

FIG. 259.—DOUBLE CURRENT DUPLEX.

Bearing this principle in mind, it will be easy to follow the arrangements in Figs. 258 and 259, which show the circuits for duplex working on the single current and double current systems respectively.

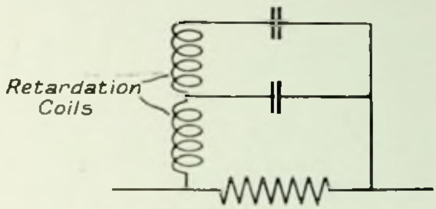

FIG. 260.—TYPE OF BALANCING NETWORK.

The balancing network is simply a resistance with a capacity shunt to balance the resistance and capacity of the line. An arrangement as shown in Fig. 260 is usually employed, since with this system it is possible to obtain a network having appreciably the same time constant as the line, which is of importance.

Diplex Working.

The diplex system enables two messages to be sent over the same wire *in the same direction* simultaneously.

The principle again is simple. One circuit is a plain D.C. duplex arrangement. In series with the line, however, is a non-polarised relay which is biassed to be insensitive to the normal currents. The second message is sent by increasing the voltage of the battery. This does not

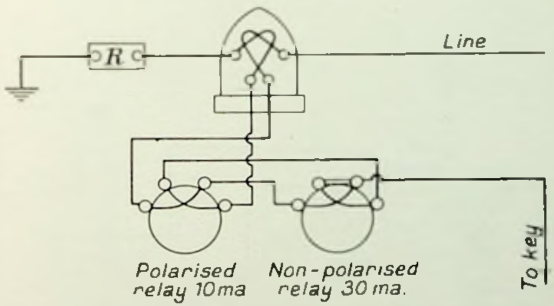

FIG. 261.—DIPLEX CIRCUIT: RECEIVING ARRANGEMENTS.

affect the polarised relay, which is already operative, and depends for its signals on a change of direction. The non-polarised relay, however, previously inoperative, now responds to the increased current, and records the signal.

Fig. 261 shows the receiving arrangements for a diplex circuit. The

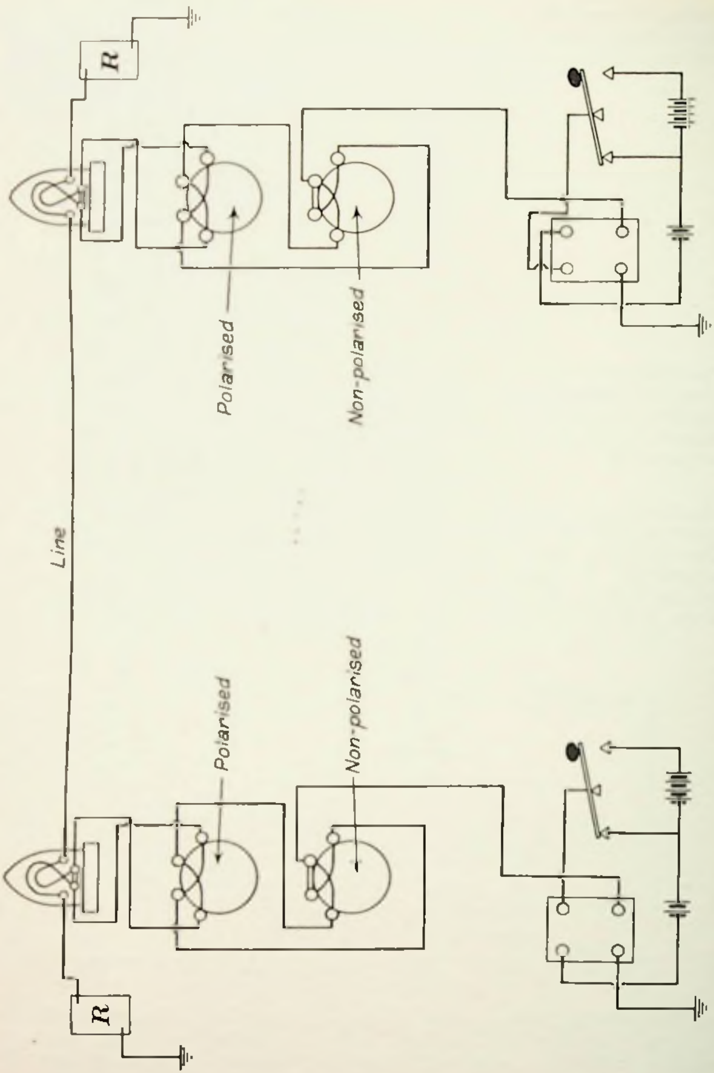

FIG. 262.—SKELETON DIAGRAM OF QUADRUPLE CIRCUIT.

transmitting arrangements are indicated in Fig. 262. The ratio of the voltages for diplex working should be 3/1.

Quadruplex Working.—If the diplex condition is arranged at both ends, four messages may be transmitted simultaneously over the same wire. This circuit is known as the quadruplex, and is illustrated in Fig. 262.

Battery Voltages.

The voltage of the battery required may be calculated from the resistance of the line, the resistances of the instruments, and the received current required. The voltage ranges from 24 for local lines up to 120 for long-distance routes, 40 and 80 volts being employed for intermediate distances.

For large towns it is feasible to employ a single battery for all circuits, provided the resistances of the individual lines is not widely different. The limit of discrepancy is about 25 per cent. Secondary cells are employed in such cases.

For D.C. working the battery cannot be reversed, however, since there is more than one circuit on the battery, and a short circuit would occur. Hence

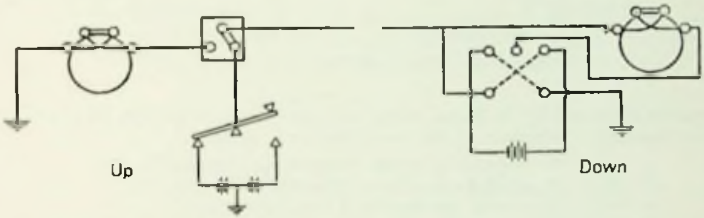

Fig. 263.—Common Battery System.

a split battery is used of double the required voltage, the centre point being earthed, and a simple SC key being employed. Fig. 263 shows a skeleton D.C. simplex set, the up station employing a common battery.

Speed of Working—Repeaters.

The speed of working depends on the time taken by the signal to build up to the voltage necessary to operate the relay. This depends on $\frac{1}{CR}$, where C and R are the capacity and resistance of the line.

Hence— Speed $\propto \frac{1}{CR}$

$$\propto \frac{1}{l^2}, \text{ since C and R both depend on } l.$$

Thus the speed of working may be quadrupled by halving the length of the line. On long lines, therefore, it is the practice to insert a "repeater." This is simply a relay, the local circuit of which retransmits the message over the remainder of the line.

Fig. 264 shows a two-way repeater—*i.e.*, one which repeats the messages on both directions.

The question of the received current is dealt with on p. 237. The speed of working over a given line may, however, be obtained from certain empirical

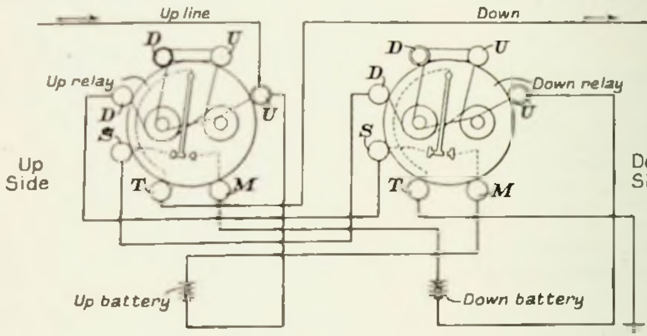

FIG. 264.—DOUBLE CURRENT REPEATER.

formulæ obtained by A. Eden, which are applicable to British lines working with shunted condensers at the receiving end. Let—

W = speed in words per minute obtainable.
C = total capacity of the line (microfarads).
R = total resistance of the line (ohms).

Then $$W = \frac{10,000,000}{CR} \text{ for iron aerial line,}$$

$$= \frac{12,000,000}{CR} \text{ for copper aerial line,}$$

$$= \frac{15,000,000}{CR} \text{ for copper G.P. covered cable.}$$

The High Speed Wheatstone Automatic System.

In the automatic system the hand-worked key is replaced by a machine capable of sending the dots and dashes at a maximum rate of 200 to 300 words per minute, and the messages are recorded at the receiving station on a tape. The dots and dashes composing the message to be forwarded must be represented in a particular way by holes punched in a paper slip before they can be transmitted. A portion of the slip is represented in Fig. 265.

The electrical mechanism of the transmitter is shown in Fig. 266. The double current key is replaced by a compound lever DU, the portions D and U being insulated from each other. When the transmitter is started the arm V is caused to oscillate, either by a clockwork train or other suitable drive.

If the pin P rises, the arm A rises also, being pulled up by the spring S_1. This motion is transmitted to the rod H. Simultaneously the pin P′

descends, depressing A′ and moving H′ to the right. The lever DU is thus moved in a counter-clockwise direction, connecting the battery to line.

The next instant the position is reversed, the lever DU moving clockwise, thus again connecting the battery to line, but in the reverse direction. Thus periodic reversals of current are sent along the line.

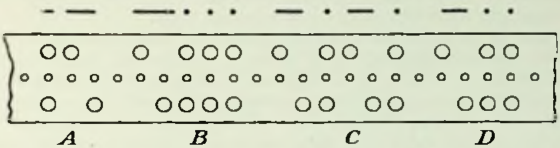

Fig. 265.—Wheatstone Slip.

Consider now the effect of inserting a piece of punched slip, as shown in Fig. 265. The first two combinations, one vertical and one diagonal, represent the letter A (. —).

When the pin P′ rises, the rod M will be free to pass through the first upper hole, and the lever UD will be moved, and will send a "marking" current; when the reverse movement of the rocking arm Y takes place, rod S will be

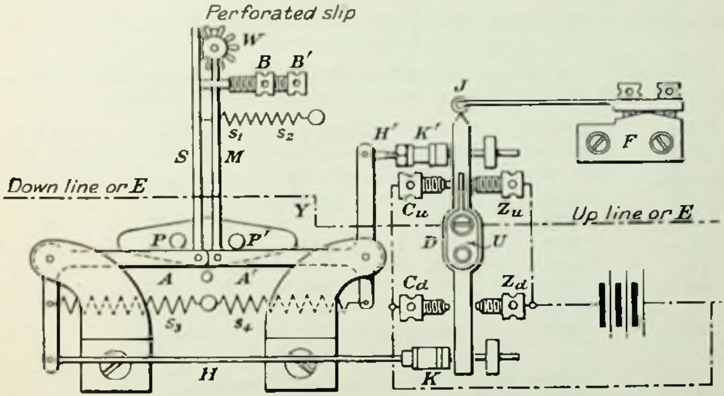

Fig. 266.—Mechanism of Wheatstone Automatic Transmitter.

free to pass through the first lower hole, and the current sent by DU will be reversed; a dot will therefore have been sent. On the next movement of the rocking arm M will be free to pass through the second upper hole, giving a marking current again. The length of the "spacing" current is thus equal to that of the previous "marking" current. When the rocking arm leaves S free to rise, however, it is prevented from so doing by the paper tape, which is not perforated below the second upper hole. In this case, therefore, the marking current is kept on until the rod S is again free to rise, which it can do through the second lower hole, and the current is then reversed. It will be seen that the marking current is kept on during movements equal to two dots and the space between, and this is equal to the length of a dash. Thus the letter A (. —) has been transmitted. Similarly any desired signal may be sent by running suitably perforated tape through the transmitter. The tape is pulled through continuously by the wheel W, which engages with the centre holes in the tape.

The receiver is in effect an ordinary polarised relay worked by the direct line current, and surmounted by a train of clockwork, which is driven by means of a weight or small electric motor. Its speed of running is regulated by a fly expanding through a rotary motion. Current in a spacing direction through the relay swings the printing wheel against the ink feeding wheel, which dips into an ink reservoir, whilst a reverse current swings the printing wheel into contact with the moving slip, thus printing the signals.

The coils of the "receiver" are each wound with two wires, each having a resistance of 200 ohms. The diagram of connections for a Wheatstone set is shown in Fig. 267. One station only is detailed, and a hand key is shown in parallel with the Wheatstone transmitter.

It is often desired to ascertain the speed of transmission with a Wheatstone transmitter. The following table will prove useful in this connection:

TABLE XXXII.

For ascertaining the Actual Speed of Transmission on Wheatstone Automatic Circuits.

Direction.—Pass 10 feet of perforated slip (representing fifty average words) through the transmitter, and observe the time occupied.

Time Occupied. *Seconds.*	*Number of Words per Minute.*	*Time Occupied.* *Seconds.*	*Number of Words per Minute.*	*Time Occupied.* *Min.*	*Sec.*	*Number of Words per Minute.*
7½	400	30	100	1	7	45
8½	353	33	91	1	15	40
10	300	37	81	1	25	35
12	250	43	70	1	40	30
15	200	50	60	2	0	25
20	150	55	55	2	20	20
24	125	60	50			

The Hughes Type Printing Telegraph.

The Hughes is a synchronous type printing telegraph, its action being largely mechanical. It is unique in its action in that only one current is required for each character printed. The currents are all of the same duration, but are separated by unequal intervals of time.

The operating keyboard consists of fourteen black and fourteen white keys arranged alternately, the arrangement being similar to the keyboard of a piano.

Each transmitting key is connected to one of twenty-eight pins which work in radial slots, the latter being regularly spaced in a horizontal plate. A pivoted lever which is carried on a radial arm passes over the pins, and when a key is depressed it causes the corresponding pin to project above the

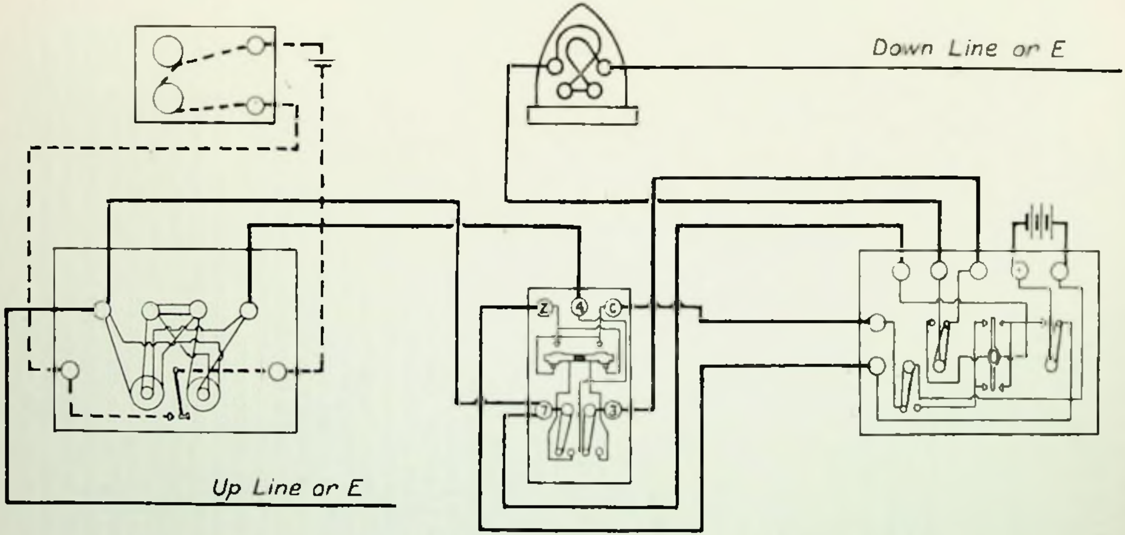

Fig. 267.—Connections of Wheatstone Transmitter.

surface of the plate, thus causing the lever to move upward when it passes. In this way a current is sent, via the line, to the distant station.

Each station is provided with an identical type wheel, these two type wheels being caused to revolve in synchronism with each other and with the radial contact arm, and at any instant the same letter occupies the same position in relation to the tape.

The passage of the current attracts the type wheel on to the tape, so printing the particular letter at both the sending and receiving stations. Which letter is printed depends on the particular key depressed.

At each signal a mechanical device is brought into operation, which rectifies any slight lack of synchronism between the sending and receiving stations.

When a letter has been printed, the time elapsing before the next operation depends on the order of the letters. If the next letter is proper to a pin some distance away, it can be printed straight away. If it is near the first letter, however, it cannot be printed till one complete revolution has elapsed. The operator thus sends his message in a series of jumps, an experienced operator being able to get three or even four letters printed per revolution if the run of the letters permits. There are frequent cases, however, where a pause of one complete revolution must be made, and as the rotating arm only makes 120 revolutions per second, this at once limits the speed on the Hughes system to about thirty words per minute.

The Baudôt System.

This system is extensively employed on British and Continental lines, and is of considerable importance. The principle of the Baudôt system depends on the use of a five-unit code. Each station is provided with a keyboard fitted with five keys. When the operator desires to signal any particular letter, he depresses the appropriate combination of keys. The five keys are connected in turn to the line, and if a particular key is depressed, a current impulse is sent along the line. The currents thus sent to line operate electromagnets at the receiving end, five such relays being connected to line in turn. Thus the position of the armatures of the receiving relays is determined by the position of the sending keys. According to the particular combination, so are certain local circuits operated which select and print on a tape the appropriate character, move the tape forward, and return the armatures to normal. The nerve centre of the system is the "distributor," which comprises a rotating brush sweeping over a series of segments. The five transmitting keys are wired up to five such segments, each key thus being connected to the line in turn. A similar distributor is provided at the receiving point which rotates in synchronism with the first, so that when a particular key is depressed at the transmitting point, the appropriate relay is connected to line at the receiver.

As it is imperative that the operator should know the precise moment at which to depress the keys, a time-tapper or "cadence" is operated by the distributor giving an audible click. Moreover, to prevent the operator from allowing the keys to rise before the distributor branch has passed the last of the five segments, the keys are each locked until the passage of the cadence current, which releases the keys. Since relatively few contacts are required on the distributor for any one set, it is obviously possible to connect several such sets to the same line. One of the most common instruments is arranged for four complete sets of contacts on the distributor. With suitable arrangements it is possible to transmit in both directions simultaneously, and hence, with four-way distributors, eight messages may be transmitted simultaneously over one line. The maximum speed of the Baudôt is thirty words per minute.

Submarine Cable Telegraphy.

The problem of telegraphic transmission through submarine cables is more complex, owing to the fact that the cable has a very large capacity. The growth of current when the key is depressed is thus small, and ordinary Morse signalling is impracticable. On cables, therefore, simple reversals of current are utilised, a current in one direction representing a dot, while the reverse current represents a dash.

Sensitive electromagnetic relays, known as siphon recorders or undulators, are employed for receiving. These instruments consist of a modified form of d'Arsonval galvanometer, in which a light moving coil is suspended in a strong magnetic field. The received currents cause deflections of this coil, which in turn cause movements of a light silver siphon. This siphon dips into an inkwell at one end, the other resting on a moving paper tape, so making a continuous ink line.

The received currents deflect the siphon, and cause humps above or below the line, so producing Morse signals.

In practice the current changes in the line are often so sluggish that the siphon never returns to zero between two marks in the same direction, the

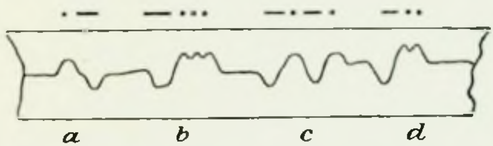

FIG. 268.—UNDULATOR SLIP (CABLE).

tape being as indicated in Fig. 268, but an experienced operator is able to read the tape without difficulty.

Growth of Current.—The actual growth of the current in the circuit is of small importance in land line circuits, except over very long lines at high speeds. In submarine work, however, it is of prime importance.

The inductance and leakage in a cable are usually negligible in comparison with the capacity and resistance. On this assumption (see p. 253)—

$$\frac{\delta^2 V}{\delta x^2} = \mathrm{CR}\,\frac{\delta V}{\delta t},$$ C and R being per unit length.

Fourier has solved this equation for a cable of length l, showing that the current i_2 at the receiving end after a time t has elapsed is given by—

$$i_2 = \frac{\mathrm{V}_1}{\mathrm{R}l}\left\{1 - 2(\eta - \eta^4 + \eta^9 - \eta^{16} + \ldots)\right\},$$

where

$$\eta = \varepsilon^{-\frac{\pi^2 t}{\mathrm{CR}l^2}}.$$

The series inside the bracket is peculiar in that its value is ½ when $t = 0$, and remains nearly so until η has fallen to about ¾. The value of i_2 is thus practically zero until t is such that $\eta = \frac{3}{4}$. Fig. 269 shows the growth of i_2 with time t. Let—

$$\tau = \text{value of } t \text{ when } \eta = \tfrac{3}{4}.$$

Then

$$\varepsilon^{-\frac{\pi^2\tau}{CRl^2}} = \frac{3}{4},$$

$$\tau = 2{\cdot}915\ CRl^2 \times 10^{-8} \text{ seconds},$$

where

C = capacity per unit length in μF.
R = resistance per unit length in ohms.

τ is usually 0·05 to 0·2 second, which limits the speed of working to about forty words per minute, even on short cables.

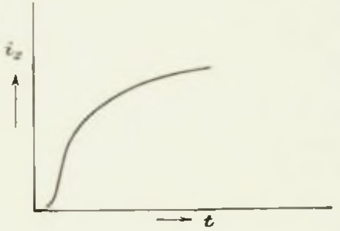

Fig. 269.—Arrival Curve.

For fuller treatment concerning telegraphy the reader is referred to H. W. Pendry's "Elementary Telegraphy," and for a detailed exposition of the telegraph systems of the British Post Office to T. E. Herbert's "Telegraphy."

TELEPHONY.

The essential principles involved in telephony are simple. The air vibrations occasioned by the speech are caused to affect a device which translates them into electrical vibrations. These currents are transmitted over the line to the receiver, where they cause vibrations of a diaphragm, which in turn set up air waves, and so reproduce the speech. The study of telephony chiefly involves the consideration of the several different methods of effecting the simple processes outlined above, and for this purpose several excellent books have been published, such as Poole's "Practical Telephone Handbook," from which much of the data following has been extracted.

Microphone.—The device used for converting the air waves to electrical vibrations is known as a microphone. For commercial line telephony the carbon microphone is always employed. This depends for its action on the variation of contact resistance between carbon granules according to the pressure. The instrument consists of a small receptacle containing carbon granules, and the speech waves impinge on a diaphragm connected to one side of the receptacle, thus producing variations of pressure, with consequent variation of the current through the instrument. There are two main types in use at the present day. One, the solid back transmitter, consists of a small brass cylinder closed at one end. The inside of the base is fitted with a thin carbon disc, which is electroplated on one side, and soldered to the case. The cover is a thin mica diaphragm, to which is attached a second carbon disc fitted in the same manner as the first. The intervening space is filled with carbon granules. The microphone itself, which is ⅝ inch internal diameter, is fitted into an appropriate case, the mica diaphragm being attached by means of a small screw to the main diaphragm of the

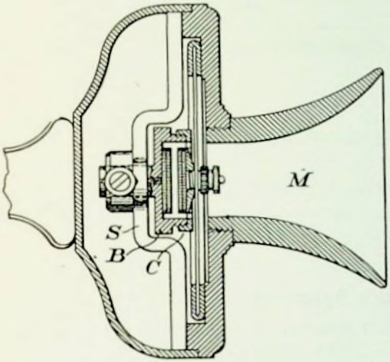

Fig. 270.—White's Solid Back Microphone.

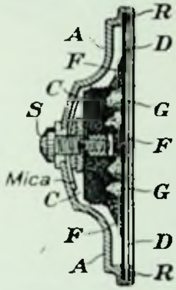

Fig. 271.—Inset Transmitter.

transmitter, which is 2½ inches diameter and 0·022 inch thick. The construction is detailed in Fig. 270. The resistance is 30 ohms for local circuits, and 55 ohms for C.B. circuits.

The second form, the inset transmitter, is a complete unit in itself, which is simply slipped into an appropriate case (Fig. 271). It comprises a carbon block screwed to the back of a shallow case. A carbon diaphragm is employed as shown, and the carbon granules are kept in place in the intervening space by means of a double turn of flannel bound round the outside edge of the carbon block.

Receivers.—The standard Post Office receiver consists of two long bar magnets yoked together at one end. The other ends are fitted with L-shaped pole pieces which carry the coils of the instrument.

The whole is assembled in a brass case, the rim of which carries the diaphragm, as shown in Fig. 272. The passage of varying currents through the coils produces vibrations of the diaphragm.

The magnets are of tungsten steel, the diaphragm being of stalloy, 10 mils thick. The coils are wound with 605 turns of 40 S.W.G. single silk-covered tinned copper wire, the total resistance being of the order of 60 ohms.

A good modern receiver will respond to a current of 3×10^{-9} amperes, the power required being from 1 to 2 microwatts at about 800 cycles.

Even so, the energy efficiency is estimated to be as low as 0·1 per cent. The actual movement of the diaphragm is of the order of 3×10^{-8} inches.

The permanent magnet is employed to increase the sensitivity. The magnetic pull is proportional to $\frac{B^2}{8\pi}$. The vibration due to a speech current

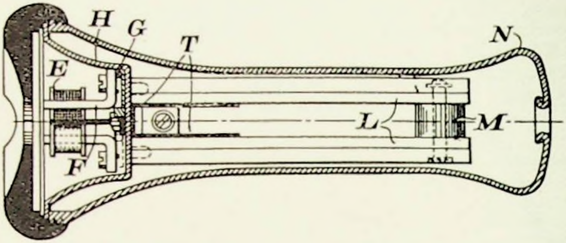

FIG. 272.—STANDARD RECEIVER.

s thus proportional to $(B+\delta B)^2-(B-\delta B)^2=4B\delta B$. Hence the response depends not only on δB, but on B. Without a polarising field the telephone is not a practical proposition.

Subscriber's Apparatus.

In outlying districts the magneto system is employed. The skeleton connections are shown in Fig. 273. The generator G is a simple hand-driven magneto machine for sending an alternating current along the line. This operates a ringer at the other end, which attracts the attention of the called subscriber. The magneto is normally short-circuited, owing to

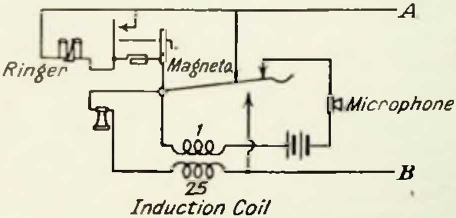

FIG. 273.—DIAGRAM OF SUBSCRIBER'S APPARATUS (MAGNETO SYSTEM).

its high impedance. When the handle is turned, however, this short circuit is automatically removed, and the ringer is short-circuited instead, thus permitting full voltage to be applied to the line. The removal of the receiver from the switch hook connects up the microphone battery, and also connects the receiver directly across the line.

The microphone is not placed directly in the line circuit owing to the high resistance of the circuit comprising the line and associated apparatus. The current variations in the microphone are thus transferred to the line through a small transformer or induction coil. The primary, which has a resistance of 1 ohm, is wound with single silk-covered wire 23 S.W.G., the secondary being wound with 32 S.W.G. to a resistance of 25 ohms. The core is $\frac{5}{32}$ inch in diameter, being composed of soft iron 24 S.W.G.

The magneto delivers 50 to 75 volts at frequencies of 30 to 40 per second, according to the speed of turning.

For towns, however, where the length of line from the exchange is comparatively short, the central battery system is employed. Here a 24-volt battery is stationed at the exchange, and the microphone current is supplied

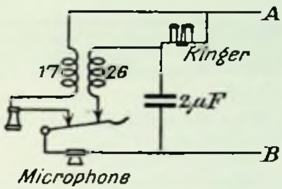

FIG. 274.—SUBSCRIBER'S APPARATUS (C.B. SYSTEM).

from this battery via the line. The circuit is shown in Fig. 274. The magneto is dispensed with, since the lifting of the switch hook completes the line circuit, and operates a line relay at the exchange.

The microphone is connected in the line circuit directly in series with the receiver and the secondary of an induction coil. A high resistance microphone (55 ohms) is employed, but even so, the quality of the speech would be poor if it were not for the induction coil. The primary of the coil is connected in series with a condenser across the microphone. The current variations then produce surges in this condenser circuit, which induce E.M.F.'s in the secondary of the coil in the same direction as the E.M.F.'s already existing therein. The quality and strength of the speech is much improved by this device.

A 2 μF condenser is employed, and a special type of induction coil having a primary winding of 1,400 turns wound to a resistance of 26 ohms, the secondary having 1,700 turns, the resistance being 17 ohms only.

Switchboard Apparatus.

Two subscribers are connected to one another by means of a switchboard in the telephone exchange. The subscriber attracts the attention of the operator, who connects his or her head telephone set to the line, ascertains the number required, completes the requisite connections, and withdraws her set from the line.

The connections are made by means of plugs and spring " jacks." These plugs may be two or three way, the construction of the more usual three-way plug being shown in Fig. 275.

When inserted into a jack the three insulated sections of the plug make contact with three springs, so completing the connections. The plug pushes the springs aside slightly, and this motion may be utilised to make or break auxiliary circuits if desired. Such jacks are called break jacks, in distinction to the simple contact type which are termed branching jacks.

The connecting cords are apt to be troublesome in practice. Three connections have to be carried in the single cord, and the constant bending is apt to cause failures by breaks or short circuits. The connections are usually made of thin stranded tinsel wires, and a partial fracture gives an intermittent contact which sets up microphonic noises. To overcome this, cords have been constructed with concentric insulated spirals of thin steel

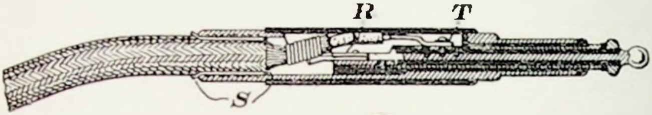

FIG. 275.—SECTION OF C.B. PLUG.

tape for the conductor. These have a higher resistance, but are much more reliable. A new form of tinsel cord, however, has been introduced recently, and is superseding the steel cords.

To attract the operator's attention, electromagnetic relays are employed, which either produce a mechanical indication, or make a local circuit for a visual indication. The former type may be self-restoring, or may be restored by the insertion of the jack in answering the call.

The second type of relay carries a light pivoted armature, which is attracted to the core on the passage of current. The motion of the armature presses

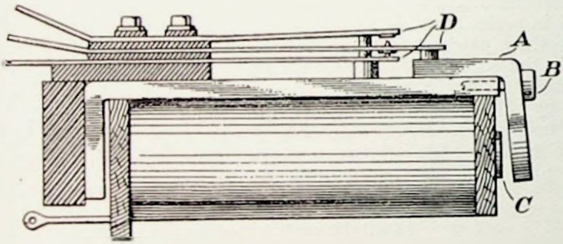

FIG. 276.—TELEPHONE RELAY.

two springs into contact. Several sets of springs may be operated by one relay. Fig. 276 shows an instrument which makes one contact, and breaks one.

There are many different types of relay, and for information concerning windings, etc., the reader is referred to Poole's "Practical Telephone Handbook."

Multiple Switchboards.—In an exchange handling a large number of lines, more than one operator has to be employed. The system then adopted is as follows:

1. Each operator handles the incoming calls of a number of subscribers (this number being anything up to 150 or 200), the connections from which, together with the indicating lamps, etc., are grouped in front of the operator. The operator has a number of cords to enable several calls to be in operation simultaneously.

2. All the lines in the exchange are terminated in jacks, which are stacked in banks arranged in numerical order. The banks are so arranged as to be contained completely in a lateral width of about 10 feet, after which the numbers start again. Thus, each number is " multipled " throughout the exchange. An operator receiving a call first plugs one end of a cord into the subscriber's jack, and then plugs the other end into the number required, which, owing to the multipling, is always within reach. She then rings the called subscriber by connecting the line to a motor driven magneto generator.

3. If a subscriber requires connection to a subscriber on another exchange, the operator plugs the second end of the cord into a junction jack, which transfers the connection to the distant exchange, where the call is treated as an incoming call, and the connection completed.

There are many details which cannot be considered here. Reference may, however, be made to the engaged test. If a line or junction is engaged, the sleeve of the jack is connected to a special generator giving a busy back tone. The operator, before completing the connection, touches the jack sleeve with the plug tip to see whether the line is free or not.

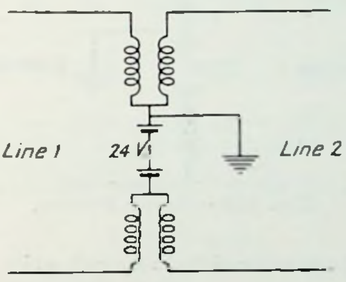

FIG. 277.—HAYES SYSTEM.

The connection of the several lines to the common battery may be accomplished in several ways, two of the principal methods being shown in Figs. 277 and 279.

In the Hayes system the line currents are fed through repeating coils. Any variations of the currents in one line will be transformed into the other line through this agency.

The four repeating coils are all wound on one core in a toroidal form, one such repeater being required for each cord. The windings are about 23 ohms resistance.

Fig. 278 shows the arrangement of such a coil.

The Stone system replaces the repeating coil with a single choke coil. The current then splits in the two lines in inverse proportion to their resist-

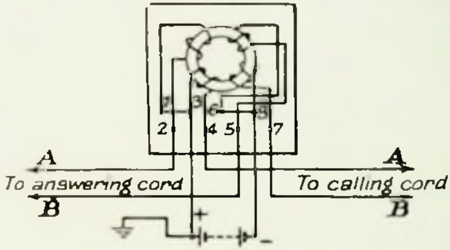

FIG. 278.—REPEATING COIL.

ances. Any variation of current in one line is automatically transferred to the other, because the total current supplied by the battery must remain

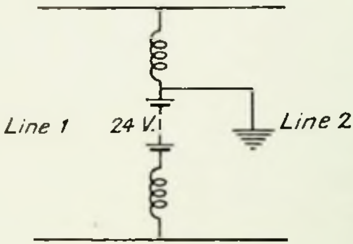

FIG. 279.—STONE SYSTEM.

constant owing to the presence of the choke coil, which will not permit any variation.

This system is only used where the lengths of the lines are not too unequal.

Aerial Line Construction.

Hard-drawn bronze wire is now almost universally employed for the connecting lines, although copper wire is used on trunk routes.

Table XXXIII. (p. 245) gives some particulars of wires in use in this country.

The ohm mile constant is the product of the resistance per mile of a given wire and its weight in pounds. This is constant for a given material.

Since $\frac{\text{ohm mile constant}}{\text{weight in pound / mile}}$ = resistance per mile, the practice of classifying conductors in terms of their weight per mile has become general.

TABLE XXXIII.

WIRES USED FOR AERIAL LINES.

Material.	*Nearest S.W.G.*	*Diameter in Mils (Minimum).*	*Weight in Pounds per Mile.*	*Resistance per Mile at 60° F. (Standard Weight).*	*Ohm Mile Constant.*	*Minimum Breaking Loads in Pounds (Standard Weight).*	*For what Purpose Used.*
			W	R	R × W	S	
H.D. copper	$4\frac{1}{2}$	221	800	1·09	878	2,400	Long trunk lines.
	6	191	600	1·46	878	1,800	Long trunk lines.
	8	156	400	2·19	878	1,250	Long trunk lines.
	$9\frac{1}{2}$	135	300	2·92	878	950	Long trunk lines.
	$11\frac{1}{2}$	110	200	4·39	878	650	Shorter trunk lines.
	13	95	150	5·85	878	490	Shorter trunk lines.
	14	78	100	8·72	878	330	Short trunk and junction lines.
Bronze wire	13	95	150	12·14	1,820	715	Long local lines.
	16	65	70	26·00	1,820	345	Long local lines.
	18	50	40	45·50	1,820	200	Local line distribution.

(Poole's "Telephone Handbook.")

Sag in Wires.—The sag in a wire depends on the tension. The temperature has considerable effect also, causing expansion of the wire and consequent increase of sag with reduction of tension.

The curve of the wire is a catenary. If the two ends are at the same height, then—

$$t = \frac{w \times s^2}{8d} \text{ nearly,}$$

where t = tension pounds,
s = span,
d = sag,
w = weight of wire in pounds/foot run,
= 0·0189 for 100 pounds copper,
= 0·00757 for 40 pounds bronze.

The following empirical data extracted from Poole's "Telephone Handbook" will probably be useful.

The safe sag is given in the tables below for various spans. The tension with this safe sag is also given for 100 pound copper and 70 pound bronze. The tension with another conductor is $T \times \frac{W}{100}$ and $T \times \frac{W}{70}$ respectively, where W is the weight of the conductor in pounds/mile.

TABLE XXXIV.

Sag and Tension for Hard-Drawn Copper (Factor of Safety 4 at 22° F.).

Temperature (Degrees F.).	*Span (Yards).* 30	40	50	55	60	70	80	90	100	*Tension for 100 Pound Copper (Pounds).*
	Sag in Inches.									
20 to 30	3·0	5·5	8·5	10·5	12·25	16·75	22·0	27·75	34·0	75
30 ,, 40	3·25	5·75	9·25	11·25	13·25	18·0	23·5	29·75	36·75	70
40 ,, 50	3·5	6·25	10·0	12·0	14·25	19·5	25·25	32·0	39·5	65
50 ,, 60	4·0	6·75	10·75	13·0	15·5	21·0	27·5	34·5	42·75	60
60 ,, 70	4·25	7·5	11·75	14·25	17·0	23·0	30·0	34·75	46·75	55
70 ,, 80	4·75	8·25	13·0	15·75	18·5	25·0	33·0	41·5	51·25	50
80 ,, 90	5·25	9·25	14·25	17·25	20·5	28·0	36·5	46·25	57·0	45
90 ,, 100	6·0	10·25	16·0	19·0	23·0	31·5	41·0	52·0	64·0	40

TABLE XXXV.

Sag and Tension for Bronze Wire (Factor of Safety 3 at 22° F.).

Temperature (Degrees F.).	*Span (Yards).* 55	60	65	70	75	*Tension for 70 Pound Bronze (Pounds).*
	Sag in Inches.					
17 to 27	5·0	5·88	6·88	7·88	9·13	110
27 ,, 36	5·25	6·13	7·25	8·38	9·5	105
36 ,, 46	5·5	6·38	7·5	8·75	10·0	100
46 ,, 56	5·75	6·75	8·0	9·25	10·5	95
56 ,, 66	6·0	7·13	8·5	9·75	11·25	90
66 .. 76	6·38	7·5	9·0	10·25	11·75	85
76 ,, 86	6·75	8·0	9·5	10·88	12·5	80
86 ,, 96	7·25	8·5	10·13	11·63	13·5	75
96 ,, 106	7·75	9·13	10·75	12·5	14·5	70

If the allowable stress at the lowest temperature has been decided upon, the temperature required to produce a given tension is given by—

$$t = \frac{l^2 w^2}{24c}\left(\frac{1}{T_1^2} - \frac{1}{T^2}\right) + \frac{1}{Ewc}(T_1 - T).$$

where t = temperature in degrees F. *above minimum*,
T = stress allowed at minimum temperature t_{min}.
T_1 = stress at temperature $(t + t_{min})$,
E = modulus of elasticity,
c = coefficient of linear expansion,
l = span in feet,
w = weight per foot run.

For hard-drawn copper:

$T = 135$ for 100 pound copper,
$E = 17{\cdot}8 \times 10^6$,
$c = 8{\cdot}5 \times 10^{-6}$.

For bronze:

$T = 80$ for 40 pound bronze,
$E = 17{\cdot}8 \times 10^6$,
$c = 8{\cdot}87 \times 10^{-6}$.

Poles.—Poles are usually of Norwegian or Swedish fir, and are rendered weather-proof by forcing creosote through the pores. If unpainted, about 12 pounds/cubic feet of creosote should be absorbed. For painted poles only 4 pounds/cubic feet is absorbed, a special process known as "rupining" being carried out. The creosote does not then ooze out, and painting is possible.

The weight of such poles is about $(44 + K)$ pounds per cubic foot, where—

$K = 12$ for creosoted poles,
4 for rupinised poles.

The dimensions of some of the principal poles employed in telegraph construction are attached.

TABLE XXXVI.

DIAMETERS OF POLES (INCHES).

	Light.		*Medium.*		*Stout.*	
Length (Feet).	*Diameter (Top).*	*Diameter (Bottom).*	*Diameter (Top).*	*Diameter (Bottom).*	*Diameter (Top).*	*Diameter (Bottom)*
20	5	6	—	—	—	—
24	5	6½	5½	8	—	—
28	5	7	5¾	8½	—	—
34	5	7½	6	9¼	7½	11¼
40	5	8	6	9¾	7½	12
50	5¼	9½	6½	11½	7¾	13¾
60	5½	11	7	13¼	8	15½
70	6½	13	7	14¾	8	17
80	—	—	7	16¼	8	18¾

(Poole.)

The poles are planted to a depth varying from 4 feet for 24 feet poles to 8 feet for 80 feet poles, but this figure depends to some extent on the nature of the ground.

In erecting poles, a stepped hole is first cut; the pole is then lifted by means of ladders, which are moved downwards a little at a time, as indicated in Fig. 280. For very heavy poles a derrick may be employed (Fig. 281).

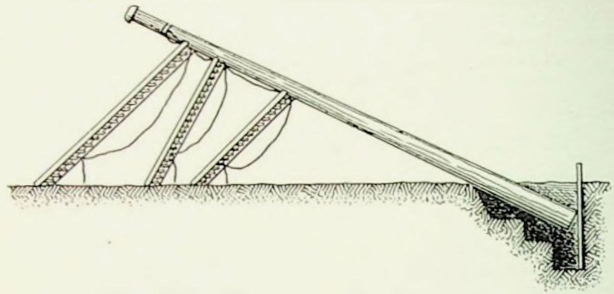

Fig. 280.—Method of Raising Pole.

Ropes.—The safe load on the lifting ropes is important. This may be taken as $W = 120\ G^2$, where G is the girth in inches, the breaking load being given by $W = 605\ G^2$.

These figures apply to manilla ropes. For hemp ropes the figure is about 20 per cent. less.

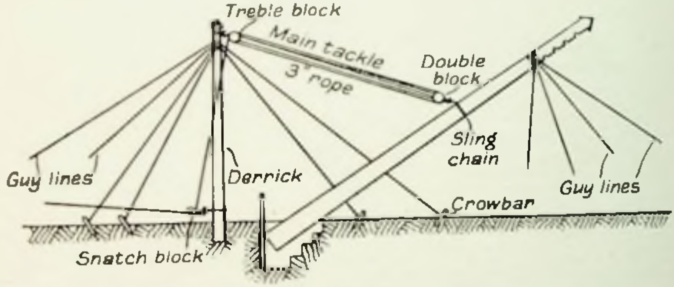

Fig. 281.—Raising Heavy Pole.

Stays.—Where there is an unbalanced force due to a bend in the route, a termination, or a wind load, stays are attached at the point of resultant pull on the pole. They are anchored in the ground to stay blocks in undercut holes, as shown in Fig. 282. If it is impossible to stay the pole owing to lack of space, a trussed pole may be employed (Fig. 283). In this case the ground resistance of the pole should be increased with stay blocks at A and B.

Stresses in Poles.—The stresses in the poles and stays may be determined by the usual laws, and need not be considered here.

Joints.—Dry joints are usually found satisfactory for telephone work. There are several types of joint. The McIntyre joint consists in slipping a copper tube over the two wires, as in Fig. 284. The whole is then given five or six twists. The tube protects the joint from dust and damp.

For heavier conductors a Britannia joint is employed. The two ends of the wire are overlapped and bound with tinned wire, the whole being sweated up afterwards.

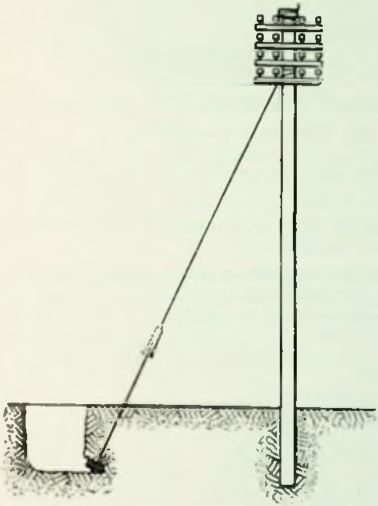

FIG. 282.—STAYED POLE.

FIG. 283.—TRUSSED POLE.

For binding wires to insulators special binding tapes are employed, as the hard copper or bronze is liable, by continual chafing, to destroy the glaze on the insulator. Hence binders of soft copper are employed. Fig. 285 (*a*)

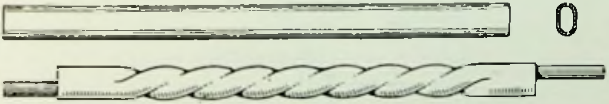

FIG. 284.—COPPER SLEEVE JOINT.

shows a simple bind in. For making off, the wire is first wrapped with a copper tape, and then made off with binding wire, as shown in Fig. 285 (*b*).

High Frequency Joints.—If the wires are carrying high frequency current the joint should always be sweated, since the current only flows in the surface

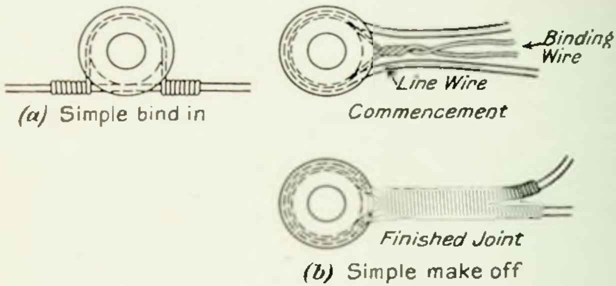

Fig. 285.—Method of Binding-In.

of the wire; it is thus essential to make contact all round the wire, and not at one point only.

Transposition.—Cross talk may occur between lines due to mutual action. At telephone frequencies this action is chiefly electrostatic, and may be reduced and often eliminated by transposition of the various wires.

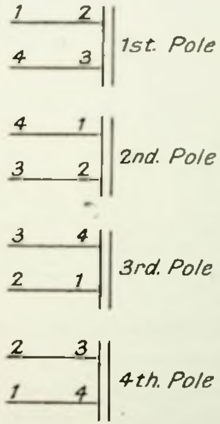

Fig. 286.—Twist System.

The simplest system is the twist system, whereby four wires, forming two circuits, are continuously rotated. The capacity effects between each circuit and any other wire is then the same over a long distance, and hence mutual effects are avoided. Instead of a twist, the wires may run parallel for a short distance, and then change places. This produces the same effect, and has certain advantages from the erecting point of view. Figs. 286 and 287 illustrate the two systems.

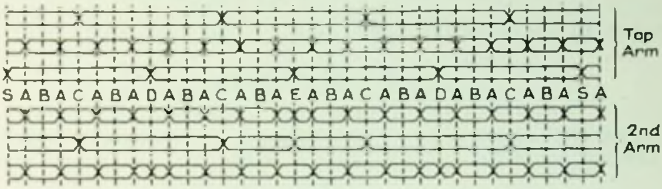

Fig. 287.—Transposition Diagram for Twelve-Wire Route.

Mutual inductive effects are also eliminated if the wires are suitably arranged—*e.g.*, in Fig. 286 the circuits must be 1 3 and 2 4 respectively, so forming two loops which are always mutually at right angles.

Underground Cables.

Aerial lines are being largely superseded by underground cables. For telegraph work it is essential to keep the capacity between any two wires of a pair as low as possible. To this effect "air-space" cables are employed. The wires are insulated with a loose wrapping of paper tape wound round the wire during the manufacture of the cable. The individual pairs are

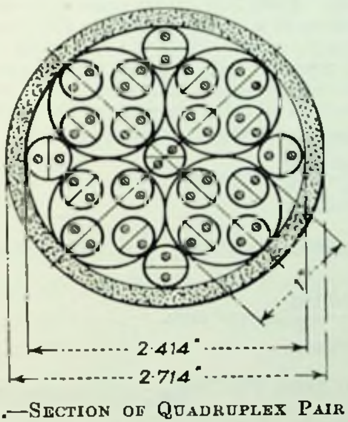

Fig. 288.—Section of Quadruplex Pair Cable.

twisted, and the whole cable is also twisted to avoid mutual effects, the spacing between all pairs being kept uniform as far as possible. Finally, the whole is encased in a lead sheath.

Fig. 288 gives an idea of a four-core cable. It will be seen that there are sixteen main pairs, and five "worming" pairs to fill in the gaps.

The thickness of the sheath should be not less than—

$$t = 0{\cdot}051 + 0{\cdot}038\sqrt{\frac{S}{0{\cdot}7584}},$$

S being the area of the unsheathed cable.

TABLE XXXVII.

Resistance and Capacity of Underground Cable.

Weight of Conductor (Pounds per Mile).	*Resistance (Ohms per Mile).*	*Capacity Wire to Wire (μF per Mile).*
6·5	135·1	·08
10	87·8	·08
12·5	70·3	·075
20	43·9	·06
40	21·9	·056
70	12·5	·065
100	8·8	·065
150	5·8	—
200	4·4	—

Cables are made up in four forms, according to the type of core.

(*a*) Twin core, the wires being simply twisted together. A number of such twin cores may be twisted to form an n pair cable.

(*b*) Multiple twin core, in which two twin pairs are twisted to form a core. Two such cores may again be twisted to form a four-pair core, and so on.

(*c*) Quadruple twin. Here four single pairs are twisted to form a core, and the cable is made up of n such cores. Fig. 288 is on this system.

(*d*) Screened conductor cable. This is a telegraph cable in which single wires are employed, wrapped with three layers of paper, and covered with soft copper tape 0·3 inch wide and less than 3 mils thick. This screen is connected (by contact) to all other screens and the lead sheath.

Inductance and Capacity of Lines.—Single wire:

$$C = \frac{0{\cdot}0388}{\log_{10}\frac{2h}{r}}\ \mu F \text{ per mile},$$

$$L = 0{\cdot}0805 + 0{\cdot}742 \log_{10}\frac{2h}{r} \text{ millihenries per mile.}$$

Loop line:

$$C = \frac{0{\cdot}0194}{\log_{10}\frac{d}{r}}\ \mu F \text{ per mile.}$$

$$L = 0{\cdot}1609 + 1{\cdot}482 \log_{10}\frac{d}{r} \text{ millihenries per mile.}$$

where h = height above ground, d = distance apart of wires, r = radius of wire, in any similar units.

These values are for isolated wires, and will be increased by 5 to 20 per cent. by the presence of other wires.

Transmission Formulæ.

The currents and voltages at any point of a telephone transmission line can be obtained from certain fairly simple formulæ.

The problem is somewhat complicated by the fact that the constants of the line are uniformly distributed throughout the length thereof. There are two effects to be considered:

1. Drop in potential, due to resistance and inductance.
2. Drop in current, due to capacity and leakage.

Let R = resistance of line per unit length,
G = leakage of line per unit length = 1/insulation resistance.
L = inductance of line per unit length,
C = capacity of line per unit length.

Then at a distance x from the receiving end, the drop of potential over an element dx is—

$$dv = i\mathrm{R}dx + \mathrm{L}dx \cdot \frac{di}{dt},$$

therefore
$$\frac{dv}{dx} = i\mathrm{R} + \mathrm{L}\frac{di}{dt}.$$

Similarly,
$$\frac{di}{dx} = v\mathrm{G} + \mathrm{C}\frac{dv}{dt}.$$

Let
$$i = \mathrm{I}\sin\omega t.$$

Then
$$\frac{di}{dt} = \mathrm{I}\,\omega\cos\omega t = j\omega i.$$

Similarly,
$$\frac{dv}{dt} = j\omega v.$$

Hence
$$\frac{dv}{dx} = i(\mathrm{R} + j\omega\mathrm{L})$$
$$\frac{di}{dx} = v(\mathrm{G} + j\omega\mathrm{C}).$$

Then
$$\frac{\partial^2 v}{\partial x^2} = \frac{\partial i}{\partial x}(\mathrm{R} + j\omega\mathrm{L}) = v(\mathrm{G} + j\omega\mathrm{C})(\mathrm{R} + j\omega\mathrm{L}).$$

Similarly,
$$\frac{\partial^2 i}{\partial x^2} = i(\mathrm{R} + j\omega\mathrm{L})(\mathrm{G} + j\omega\mathrm{C}).$$

If $\mathrm{Z} = \mathrm{R} + j\omega\mathrm{L}$ and $\mathrm{Y} = \mathrm{G} + j\omega\mathrm{C}$ and $\sqrt{\mathrm{ZY}} = a$,

then
$$\frac{\partial^2 v}{\partial x^2} = (\mathrm{ZY})v. \qquad \frac{\partial^2 i}{\partial x^2} = (\mathrm{ZY})i$$
$$= a^2 v. \qquad\qquad = a^2 i.$$

Solving these differential equations—

$$i = \mathrm{M}\varepsilon^{ax} + \mathrm{N}\varepsilon^{-ax}$$
$$= \mathrm{P}\cosh ax + \mathrm{Q}\sinh ax,$$
$$v = \frac{1}{\mathrm{Y}}\frac{di}{dx} = \sqrt{\frac{\mathrm{Z}}{\mathrm{Y}}}\left\{\mathrm{P}\sinh ax + \mathrm{Q}\cosh ax\right\}$$

At the receiving end $i = i_r$, $x = 0$, $v_r = i_r Z_r$, whence the values of P and Q may be solved, giving—

$$i_x = i_r\left(\cosh ax + Z_r\sqrt{\frac{Y}{Z}}\sinh ax\right).$$

$$v_x = i_r\sqrt{\frac{Z}{Y}}\left(\sinh ax + Z_r\sqrt{\frac{Y}{Z}}\cosh ax\right).$$

Characteristic Impedance.—The apparent impedance at the sending end:

$$Z_s = \frac{v_s}{i_s} = \sqrt{\frac{Z}{Y}}\,\frac{\tanh al + Z_r\sqrt{\frac{Y}{Z}}}{1 + Z_r\sqrt{\frac{Y}{Z}}\tanh al}.$$

In a very long line $\varepsilon^{-al} << \varepsilon^{al}$ and $\tanh al = 1$.

Then $$Z_s = \sqrt{\frac{Z}{Y}} = Z_o.$$

This quantity Z_o is called the "characteristic" or "surge" impedance of the line. If—

$$Z_r = 0 \text{ (far end short-circuited)},\ Z_s = \sqrt{\frac{Z}{Y}}\tanh al,$$

$$Z_r = \infty \text{ (far end open)},\ Z_s = \sqrt{\frac{Z}{Y}}\coth al,$$

$$Z_o = \sqrt{\left(\sqrt{\frac{Z}{Y}}\,.\,\tanh al \times \sqrt{\frac{Z}{Y}}\coth al\right)} = \sqrt{\frac{Z}{Y}},$$

i.e., Z_o is the geometric mean between the apparent sending end impedances with $Z_r = 0$ and $Z_r = \infty$.

Attenuation and Distortion.—It has been shown that—

$$v = N\varepsilon^{-ax} + M\varepsilon^{ax}.$$

If x is measured from the sending end, ε^{-ax} decreases rapidly, and ε^{ax} increases. The first term represents the forward wave, and the second the reflected wave, which, owing to damping, is usually negligible except at the far end, where it is, of course, important.

The state of affairs is as represented in Fig. 289.

Hence, neglecting reflection,

$$v = N\varepsilon^{-ax} \text{ and } N = v_s,$$

therefore $$i = \frac{v_s}{Z_o}\varepsilon^{-ax}.$$

Now
$$\begin{aligned} a = \sqrt{ZY} &= \sqrt{(R + j\omega L)(G + j\omega C)} \\ &= \sqrt{(RG - \omega^2 LC) + j(GL\omega + RC\omega)} \\ &= p + jq. \end{aligned}$$

Hence
$$\begin{aligned} v_x = v_s\,\varepsilon^{-ax} &= v_s\,\varepsilon^{-(p+jq)x}, \\ &= v_s\varepsilon^{-px}\,\varepsilon^{-jqx}, \\ &= v_s\varepsilon^{-px}\left\{\cos qx - j\sin qx\right\}. \end{aligned}$$

In physical language this means that the voltage at a distance x from the sending end has been attenuated by a factor ϵ^{-px}, and is, moreover, leading on the sending voltage by an angle $\theta = \tan^{-1}qx$.

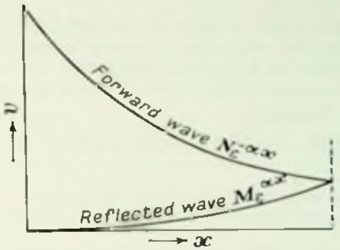

FIG. 289.—ILLUSTRATING MEANING OF TRANSMISSION FORMULÆ.

The factor p is thus termed the "attenuation constant" of the line, and it may easily be shown that—

$$p = \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}} \text{ nearly if } L\omega >> R.$$

In order to reduce the attenuation, therefore, p must be kept small. This may be done in several ways.

1. Reduce R. This is expensive.
2. Reduce G. This is always kept as low as possible, and the first term $\frac{R}{2}\sqrt{\frac{C}{L}}$ is in consequence the more important.
3. Reduce C. This is also expensive. In overhead lines C is naturally small, while in underground cables air-space cables are used, as has already been seen, to keep the capacity low.
4. Increase L. This is a practicable solution. A limit occurs when $\frac{G}{2}\sqrt{\frac{L}{C}}$ becomes appreciable, but up to this limit L may be, and is increased with advantage.

Loaded Lines.—The process of inserting inductance in the lines is known as loading. All long-distance telephone lines are loaded. The loading may be continuous, obtained by wrapping iron wire round the cable, a method suggested and employed by Krarup.

The more general method, however, is to insert loading coils at definite points. This method is due to Pupin, and the loading coils are sometimes called Pupin coils. The coils should be spaced according to either of the following rules, which are different ways of expressing the same thing:

(a) The wave must pass through 6,500 to 7,500 coils in one second (G.P.O.).
(b) $D^2CL = 21$, where—

D = distance between coils in miles.
C = capacity per mile (μF).
L = inductance per mile (μH) (Western Electric Company).

It is found in practice that speech is satisfactory if $pl < 2{\cdot}5$, and is good if $pl = 1{\cdot}5$, or less.

The insertion of loading coils on overhead wires tends to introduce troublesome effects due to high voltage surges, and lightning arresters have to be inserted.

Power Transmitted.—The power transmitted is proportional to v_s^2. Hence, if the received power is to be the same in each case, the transmitted power for a line n times as long as a given line will be ε^{2pln}. If $pl = 3{\cdot}5$, the ratio of powers required if the length of line is doubled is over 1,000.

Distortion. Wave Velocity:

$$q = \sqrt{\frac{\{(RG - \omega^2LC)^2 + \omega^2(RC + GL)\}^{\frac{1}{2}} - RG + \omega CL^2}{2}}.$$

It will be seen that this depends on the frequency. Hence different frequencies will arrive with different phase angles and distortion will occur.

Moreover, the wave length $\lambda = \frac{2\pi}{q}$ and the wave velocity $= \lambda \times f = \frac{\omega}{q}$. The velocity is thus different for different frequencies, which is a further source of distortion.

Heaviside has shown that if $\frac{R}{G} = \frac{L}{C}$, p and the wave velocity are independent of the frequency. This condition, however, is not economically practicable. The best conditions obtain when L is large, so that the insertion of loading coils is, to a certain extent, a remedy for distortion as well as attenuation.

The above expression for q is rather involved. Since the approximate value of p, the attenuation constant, is fairly easily obtained $\left(= \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}\right)$, q may more readily be deduced from the expression—

$$q = \sqrt{p^2 - (RG - \omega^2LC)}.$$

The wave velocity is then obtainable from the formula $v = \frac{\omega}{q}$.

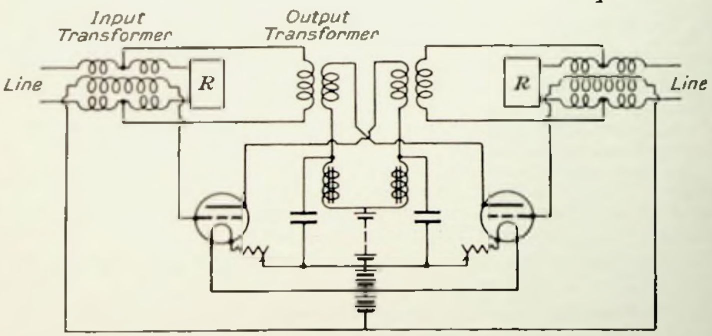

Fig. 290.—Telephone Repeater.

Repeaters.—The introduction of the thermionic valve has rendered possible the use of telephone repeaters. Details of some of the experiments made by the Post Office on repeaters may be obtained from a paper by Sir William Noble on "The Long-Distance Telephone System of the United Kingdom" (*Journ. I.E.E.*, vol. lix., p. 389). Fig. 290 gives particulars of the pattern eventually adopted. It will be observed that the output from the valves is introduced across the middle points of a special transformer, one side of which goes to line and the other to a balancing network. This

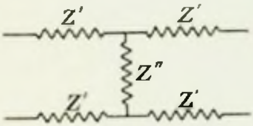

FIG. 291.—REPEATER NETWORK T CONNECTION.

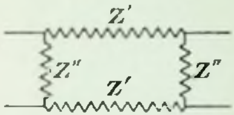

FIG. 292.—REPEATER NETWORK RECTANGULAR CONNECTION.

prevents any feeding back of the energy to the transmitting end, so eliminating howling.

The successful operation depends to a large extent on the balancing network. Two types of network may be employed. For the T circuit shown in Fig. 291 the impedances Z′ and Z″ should be—

$$Z' = Z_0 \tanh \frac{l}{2} p,$$

$$Z'' = \frac{Z_0}{\sinh lp}.$$

where Z_0 = characteristic impedance of actual line
p = attenuation constant of actual line.
l = length of actual line.

For the arrangement shown in Fig. 292 the expressions are—

$$Z' = Z_0 \sinh lp.$$

$$Z'' = \frac{Z_0}{\tanh \frac{l}{2} p}.$$

One or other of these types will match any given line. If substitution in one set of formulæ gives an impossible condition (*e.g.*, a negative resistance), the other type of network should be used.

The values of cosh x, sinh x, ε^x, etc., may be obtained from Fig. 293 overleaf. More accurate values are given in Appendix A.

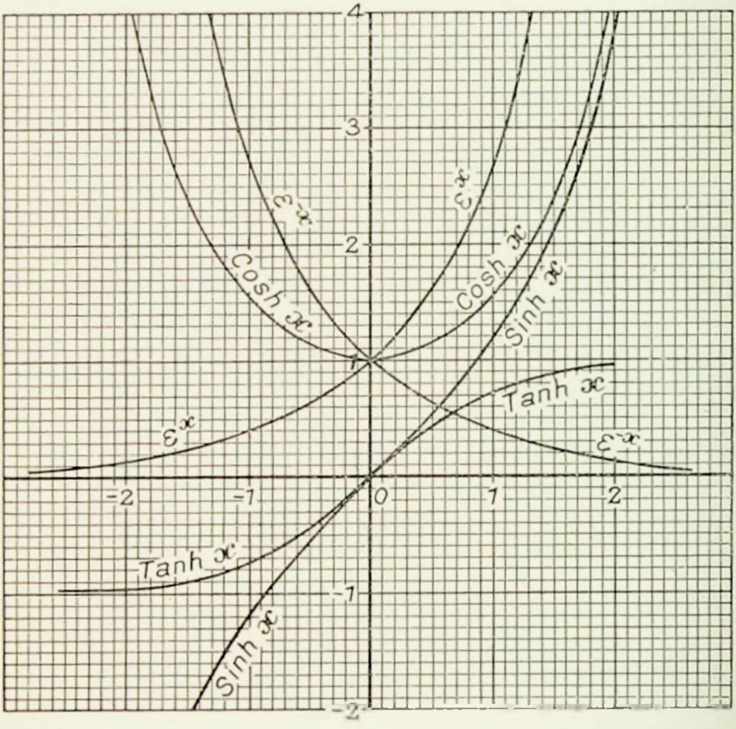

Fig. 293.—Graph of the Exponential and Hyperbolic Functions.

For more exact values see Appendix A.

APPENDICES

APPENDIX A

APPENDIX A.

British Weights and Measures.

Length.

12 inches	= 1 foot.
3 feet	= 1 yard.
6 feet	= 1 fathom.
5½ yards	= 1 rod, pole or perch.
22 yards	= 1 chain.
220 yards	= 1 furlong.
8 furlongs	= 1 mile = 1,760 yards = 5,280 feet.
6,080 feet	= 1 nautical mile = 1·1515 mile.

Surface.

144 square inches	= 1 square foot.
9 square feet	= 1 square yard.
30¼ square yards	= 1 square pole.
40 square poles	= 1 rood.
4 roods	= 1 acre = 4,840 square yards.
640 acres	= 1 square mile.

Weight (Avoirdupois)

27·344 grains	= 1 dram.
16 drams	= 1 ounce = 437½ grains.
16 ounces	= 1 lb. = 7,000 grains.
14lbs.	= 1 stone.
28lbs.	= 1 quarter.
4 quarters	= 1 cwt.
20 cwt.	= 1 ton.

Weight (Troy).

4 grains	= 1 carat.
24 grains	= 1 pennyweight (dwt.)
20 dwt.	= 1 ounce = 480 grains.
12 ounces	= 1 lb. = 5,760 grains.

Note.—The grain Avoirdupois and the grain Troy are the same.

Mensuration of Surfaces.

Area of triangle .. .. = Base × ½ perpendicular.

" circle .. .. = Diameter^2 × ·7854.

" sector of circle .. = Length of arc × ½ radius.

" " " .. = $\dfrac{\text{Number of degrees in arc} \times \text{area of the circle}}{360}$

Area of parabola .. .. = Base × $\frac{2}{3}$ height.

Frustum of a parabola .. = $\frac{2}{3}$ height $\frac{\text{base}^3 - \text{top}^3}{\text{base}^2 - \text{top}^2}$

Area of ellipse .. .. = Long axis × ·7854 short axis.
,, cycloid .. .. = Area of generating circle × 3.
Surface of cylinder .. = Area of both ends + length × circumference.
,, cone .. .. = Area of base + circumference of base × $\frac{1}{2}$ slant height.
,, sphere .. .. = Diameter^2 × 3·14159.
,, frustum .. .. = Sum of girt at both ends × $\frac{1}{2}$ slant height + area of both ends.
,, zone on sphere between parallel planes = $2 \pi r$ × distance between planes.

Mensuration of Solids.

Cylinder = Area of one end × length.
Sphere = Diameter^3 × 0·5236.
Segment of sphere = 0·5236 H ($H^2 + 3 R^2$), where H = height of segment and R = radius of the base of the segment.
Cone or pyramid = Area of base × $\frac{1}{3}$ perpendicular height.
Frustum of cone or pyramid = $\frac{1}{3}$ H ($A + a + \sqrt{A \times a}$). When A and a = Areas of the ends, H = Perpendicular height.
Frustum of cone = 0·2618 H ($D^2 + d^2 + D.\ d$). When D and d = the diameters of each end, H = Perpendicular height.
Wedge = Area of base × $\frac{1}{2}$ perpendicular height.
Frustum of wedge = $\frac{1}{2}$ H ($A + a$), when A and a = Area at each end, H = Perpendicular height.
Parallopiped or prism = area of one face × distance from opposite face.

TABLE I.

Conversion Table.

To convert.	*Multiply by*	*Reciprocal.*
Lengths.		
Mils to millimetres	0·0254	39·37
Mils to inches	0·001	1000·0
Inches to millimetres	25·4	0·03937
Inches to centimetres	2·54	0·3937
Inches to metres	0·0254	39·3701
Feet to centimetres	30·48	0·032808
Feet to metres	0·3048	3·2808
Feet to kilometres	0·0003048	3280·8
Feet to nautical miles	0·0001644	6080·0
Feet to miles	0·0001894	5280·0
Yards to metres	0·91449	1·0936
Yards to kilometres	0·0009144	1093·6
Yards to miles	0·0005682	1760·0
Miles to metres	1609·34	0·00062137
Miles to kilometres	1·60934	0·62137
Nautical miles to kilometres	1·853	0·5396

TABLE I.—*continued.*

To convert	Multiply by	Reciprocal.
Areas.		
Circular mils to square mils	0·78539	1·2732
Circular mils to square millimetres	0·0005067	1974
Circular mils to square inches	0·0000007854	1273240
Square inches to square millimetres	645·16	0·00155
Square inches to square centimetres	6·4516	0·155
Square feet to square metres	0·0929	10·764
Square yards to square metres	0·836126	1·196
Square yards to square miles	0·0000003228	3097600
Square miles to square kilometres	2·59	0·3861
Square miles to acres	640·0	0·00156
Volumes.		
Circular mil feet to cubic inches	0·000009425	106100
Cubic inches to cubic centimetres	16·387	0·06104
Cubic feet to cubic metres	0·028317	35·3148
Cubic yards to cubic metres	0·76455	1·307
Cubic inches to pints	0·02886	34·67
Cubic inches to litres	0·016387	61·024
Cubic inches to gallons (Imperial)	0·00361	277·11
Cubic feet to litres	28·317	0·035315
Cubic feet to gallons (Imperial)	6·2358	0·160365
Cubic yards to litres	764·553	0·001308
Pints to litres	0·5682	1·760
Gallons (Imp.) to litres	4·5410	0·220216
Gallons (Imp.) to cubic centimetres	4541	0·0002202
Gallons (Imp.) to U.S.A. gallons	1·1997	0·8335
Weights.		
Dynes to milligrammes	1·01919	0·98117
Dynes to grammes	0·00101919	981·17
Dynes to pounds (av.)	0·000002198	445000
Grains to milligrammes	64·8	0·0154323
Grains to grammes	0·0648	15·4323
Grains to ounces (av.)	0·00229	437·5
Grains to ounces (Troy)	0·00208	480
Grains to pounds (av.)	0·00014286	7000
Ounces to grammes	28·3495	0·03527396
Ounces to kilogrammes	0·0283495	35·27396
Pounds (av.) to grammes	453·5924	0·0022046
Pounds (av.) to kilogrammes	0·4535924	2·204622
Cwts. to kilogrammes	50·80235	0·019684
Tons to kilogrammes	1016·047	0·0009842
Tons to U.S.A. tons (2,000 lbs.)	1·12	0·89286
Tons to tonneaux (1,000 kilogrammes)	1·016047	0·9842

TABLE I.—*continued.*

To convert	*Multiply by*	*Reciprocal.*
Pressures.		
Pounds per sq. in. to gm. per sq. cm. ..	70·307	0·014223
Pounds per sq. in. to head of water in ft. at 62° F.	2·30925	0·4330
Pounds per sq. in. to head of water in metres at 62° F.	0·70386	1·4207
Pounds per sq. in. to kg. per sq. cm. ..	0·070307	14·223
Pounds per sq. in. to tons per sq. ft. ..	0·064286	15·556
Pounds per sq. in. to atmospheres ..	0·068	14·7
Tons per sq. ft. to head of water in ft. at 62° F.	35·92	0·02784
Tons per sq. ft. to head of water in metres at 62° F.	10·949	0·09133
Tons per sq. ft. to kg. per sq. cm. ..	1·0937	0·9143
Tons per sq. ft. to atmospheres ..	1·058	0·945
Atmospheres to head of water in inches at 62° F.	407·4	0·002455
Atmospheres to head of water in feet at 62° F.	33·95	0·02945
Atmospheres to head of water in metres at 62° F.	10·35	0·0966
Atmospheres to head of mercury in inches	29·92	0·03342
Atmospheres to head of mercury in millimetres	760	0·001316
Atmospheres (760 mm. Hg. at 0°C.) to bars (dynes per sq. cm.)	$1{\cdot}0132.10^6$	$0{\cdot}9871.10^{-6}$
Bars to millibars	0·001	1000
Velocities.		
Feet per sec. to metres per sec. ..	0·3048	3·2808
Feet per sec. to miles per hour ..	0·6816	1·4667
Feet per sec. to kilometres per hour ..	1·0973	0·91133
Feet per min. to metres per sec. ..	0·00508	196·854
Feet per min. to miles per hour ..	0·01136	88
Miles per hour to metres per min. ..	26·82	0·03729
Miles per hour to kilometres per hour ..	1·6093	0·62138
Knots to feet per hour	6080	0·0001644
Knots to metres per hour	1853·13	0·0005396
Knots to miles per hour	1·1515	0·8684
Knots to nautical miles per hour ..	1	1
Weights per Unit Length.		
Pounds per ft. to kilogrammes per metre ..	1·48858	0·67178
Pounds per mile to kilogrammes per kilometre	0·28186	3·54786

TABLE I.—*continued.*

To convert	*Multiply by*	*Reciprocal.*
Weights per Unit Volume.		
Pounds per cub. ft. to kg. per cub. metre	16·02	0·06243
Pounds per cub. yd. to kg. per cub. metre	0·5933	1·686
Power.		
Watts to foot pounds per minute ..	44·2567	0·0226
Watts to H.P.	0·001341	746
Watts to kilogrammetres per second ..	0·102	9·81
H.P. to foot pounds per second ..	550	0·00182
H.P. to foot pounds per minute ..	33,000	0·0000303
H.P. to kilogrammetres per second ..	76·07	0·01314
H.P. to metric horse-power	1·013	0·987
Work and Thermal Units.		
Joules to ergs	10^7	10^{-7}
Joules to foot pounds	0·7373	1·3565
Joules to calories	0·2386	4·186
Joules to Kilogrammetres	0·102	9·81
Joules to B.Th.U.	0·0009474	1055
Joules to watt hours (1 joule = 1 watt-second)	0·0002778	3600
Kilogrammetres to foot pounds ..	7·233	0·1383
Kilogrammetres to calories	2·341	0·427
Kilogrammetres to B.Th.U.	0·009295	107·56
Foot pounds to B.Th.U.	0·001285	778
Foot pounds to watt hours	0·0003766	2656
Calories to B.Th.U.	0·003968	251·9
Miscellaneous.		
Pounds of water to litres	0·4536	2·2046
Pounds of water to gallons	0·1	10
Pounds of water to cubic feet	0·016037	62·43
Amperes per sq. in. to amperes per sq. mm.	0·00155	645·16
Common to Neperian logarithm ..	2·3026	0·43429
Microhms per in.^{3} to microhms per cm.^{3}	2·54	0·394
Temp. Coeff. per ° F. to temp. coeff. per ° C.	1·8	0·556
Lines per sq. cm. to kilolines per sq. in.	0·00645	155
C.G.S. units to amp. turns per inch ..	2·02	0·495
C.G.S. units to amp. turns per cm. ..	0·7956	1·257

TABLE II.

Areas and Circumferences of Circles.

Dia.	Circum.	Area.	Dia.	Circum.	Area.	Dia.	Circum.	Area.
1/32	·0981	·00077	1 5/8	5·1051	2·0739	4 3/16	13·155	13·772
1/16	·1963	·00307	1 11/16	5·3014	2·2365	4 1/4	13·351	14·186
3/32	·2945	·0069	1 3/4	5·4978	2·4052	4 5/16	13·547	14·606
1/8	·3927	·01227	1 13/16	5·6941	2·58	4 3/8	13·744	15·033
5/32	·4908	·0192	1 7/8	5·8905	2·7611	4 7/16	13·94	15·465
3/16	·589	·02761	1 15/16	6·0868	2·9483	4 1/2	14·137	15·904
7/32	·6872	·0376	2	6·2832	3·1416	4 9/16	14·333	16·394
1/4	·7854	·04909	2 1/16	6·4795	3·3410	4 5/8	14·5 9	16·8
9/32	·8835	·0621	2 1/8	6·6759	3·5465	4 11/16	14·725	17·257
5/16	·9817	·0767	2 3/16	6·8722	3·7584	4 3/4	14·922	17·72
11/32	1·0799	·0928	2 1/4	7·0686	3·976	4 13/16	15·119	18·19
3/8	1·1781	·1104	2 5/16	7·2649	4·2	4 7/8	15·315	18·665
13/32	1·2762	·1296	2 3/8	7·4613	4·4302	4 15/16	15·511	19·147
7/16	1·3744	·1503	2 7/16	7·6576	4·6664	5	15·708	19·635
15/32	1·4726	·1725	2 1/2	7·854	4·9087	5 1/16	15·904	20·129
1/2	1·5708	·1963	2 9/16	8·0503	5·1573	5 1/8	16·1	20·629
17/32	1·6689	·2216	2 5/8	8·2467	5·4119	5 3/16	16·296	21·135
9/16	1·7771	·2485	2 11/16	8·443	5·6723	5 1/4	16·493	21·647
19/32	1·8653	·2768	2 3/4	8·6394	5·9395	5 5/16	16·689	22·166
5/8	1·9635	·3068	2 13/16	8·8357	6·2126	5 3/8	16·886	22·69
21/32	2·0616	·3382	2 7/8	9·0321	6·4918	5 7/16	17·082	23·221
11/16	2·1598	·3712	2 15/16	9·2284	6·7772	5 1/2	17·278	23·758
23/32	2·258	·4057	3	9·4248	7·0686	5 9/16	17·474	24·301
3/4	2·3562	·4417	3 1/16	9·6211	7·3662	5 5/8	17·671	24·85
25/32	2·4543	·4793	3 1/8	9·8175	7·6699	5 11/16	17·867	25·406
13/16	2·5525	·5185	3 3/16	10·014	7·9798	5 3/4	18·064	25·967
27/32	2·6507	·5591	3 1/4	10·21	8·2957	5 13/16	18·261	26·535
7/8	2·7489	·6013	3 5/16	10·406	8·618	5 7/8	18·457	27·108
29/32	2·847	645	3 3/8	10·602	8·9462	5 15/16	18·653	27·688
15/16	2·9452	·6903	3 7/16	10·799	9·2807	6	18·85	28·27
31/32	3·0434	·737	3 1/2	10·995	9·6211	6 1/8	19·24	29·46
1	3·1416	·7854	3 9/16	11·191	9·968	6 1/4	19·63	30·67
1 1/16	3·3379	·8866	3 5/8	11·388	10·32	6 3/8	20·02	31·91
1 1/8	3·5343	·994	3 11/16	11·584	10·679	6 1/2	20·42	33·18
1 3/16	3·7306	1·1075	3 3/4	11·781	11·044	6 5/8	20·81	34·47
1 1/4	3·927	1·2271	3 13/16	11·977	11·416	6 3/4	21·20	35·78
1 5/16	4·1233	1·353	3 7/8	12·173	11·793	6 7/8	21·60	37·12
1 3/8	4·3197	1·4848	3 15/16	12·3 9	12·177	7	21·99	38·48
1 7/16	4·516	1·6229	4	12·566	12·566	7 1/8	22·38	39·87
1 1/2	4·7124	1·7671	4 1/16	12·762	12·962	7 1/4	22·77	41·28
1 9/16	4·9087	1·9175	4 1/8	12·959	13·364	7 3/8	23·17	42·71

TABLE II.—(*contd.*).

Dia.	Circum.	Area.	Dia.	Circum.	Area.	Dia.	Circum.	Area.
7½	23·56	44·17	12⅝	39·66	125·18	17¾	55·76	247·45
7⅝	23·95	45·66	12¾	40·05	127·67	17⅞	56·15	250·94
7¾	24·34	47·17	12⅞	40·44	130·19	18	56·54	254·47
7⅞	24·74	48·70	13	40·84	132·73	18⅛	56·94	258·01
8	25·13	50·26	13⅛	41·23	135·29	18¼	57·33	261·58
8⅛	25·52	51·84	13¼	41·62	137·88	18⅜	57·72	265·18
8¼	25·91	53·45	13⅜	42·01	140·50	18½	58·12	268·80
8⅜	26·31	55·08	13½	42·41	143·13	18⅝	58·51	272·44
8½	26·70	56·74	13⅝	42·80	145·80	18¾	58·90	276·11
8⅝	27·09	58·42	13¾	43·19	148·48	18⅞	59·29	279·81
8¾	27·49	60·13	13⅞	43·58	151·20	19	59·69	283·53
8⅞	27·88	61·86	14	43·98	153·93	19⅛	60·08	287·27
9	28·27	63·62	14⅛	44·37	156·70	19¼	60·47	291·04
9⅛	28·66	65·39	14¼	44·76	159·48	19⅜	60·86	294·83
9¼	29·06	67·20	14⅜	45·16	162·30	19½	61·26	298·64
9⅜	29·45	69·02	14½	45·55	165·13	19⅝	61·65	302·49
9½	29·84	70·88	14⅝	45·94	167·98	19¾	62·04	306·35
9⅝	30·23	72·75	14¾	46·34	170·87	19⅞	62·44	310·24
9¾	30·63	74·66	14⅞	46·73	173·78	20	62·83	314·16
9⅞	31·02	76·58	15	47·12	176·71	20⅛	63·22	318·09
10	31·41	78·54	15⅛	47·51	179·67	20¼	63·61	322·06
10⅛	31·80	80·51	15¼	47·91	182·65	20⅜	64·01	326·05
10¼	32·20	82·51	15⅜	48·30	185·66	20½	64·40	330·06
10⅜	32·59	84·54	15½	48·69	188·69	20⅝	64·79	334·10
10½	32·98	86·59	15⅝	49·08	191·74	20¾	65·18	338·16
10⅝	33·38	88·66	15¾	49·48	194·82	20⅞	65·58	342·25
10¾	33·77	90·76	15⅞	49·87	197·93	21	65·97	346·36
10⅞	34·16	92·88	16	50·26	201·06	21⅛	66·36	350·49
11	34·56	95·08	16⅛	50·65	204·21	21¼	66·76	354·65
11⅛	34·95	97·20	16¼	51·05	207·40	21⅜	67·15	358·84
11¼	35·34	99·40	16⅜	51·44	210·59	21½	67·54	363·05
11⅜	35·73	101·62	16½	51·83	213·82	21⅝	67·93	367·28
11½	36·13	103·87	16⅝	52·23	217·07	21¾	68·33	371·54
11⅝	36·52	106·14	16¾	52·62	220·35	21⅞	68·72	375·82
11¾	36·91	108·43	16⅞	53·01	223·65	22	69·11	380·13
11⅞	37·30	110·75	17	53·40	226·98	22⅛	69·50	384·46
12	37·69	113·09	17⅛	53·79	230·33	22¼	69·90	388·82
12⅛	38·09	115·46	17¼	54·19	233·70	22⅜	70·29	393·20
12¼	38·48	117·86	17⅜	54·58	237·10	22½	70·68	397·61
12⅜	38·87	120·27	17½	54·97	240·52	22⅝	71·07	402·04
12½	39·27	122·71	17⅝	55·37	243·97	22¾	71·47	406·49

TABLE II.—(*contd.*).

Dia.	Circum.	Area.	Dia.	Circum.	Area.	Dia.	Circum.	Area.
22⅞	71·86	410·97	28	87·96	615·75	33⅛	104·06	861·79
23	72·25	415·47	28⅛	88·35	621·26	33¼	104·46	868·30
23⅛	72·64	420·00	28¼	88·75	626·79	33⅜	104·85	874·84
23¼	73·04	424·55	28⅜	89·14	632·35	33½	105·24	881·41
23⅜	73·43	429·13	28½	89·53	637·94	33⅝	105·63	888·00
23½	73·82	433·73	28⅝	89·93	643·59	33¾	106·03	894·62
23⅝	74·22	438·36	28¾	90·32	649·18	33⅞	106·42	901·25
23¾	74·61	443·01	28⅞	90·71	654·84	34	106·81	907·92
23⅞	75·00	447·69	29	91·10	660·52	34⅛	107·20	914·61
24	75·39	452·39	29⅛	91·49	666·22	34¼	107·59	921·32
24⅛	75·79	457·11	29¼	91·89	671·95	34⅜	107·99	928·06
24¼	76·18	461·86	29⅜	92·28	677·71	34½	108·38	934·82
24⅜	76·57	466·63	29½	92·67	683·49	34⅝	108·77	941·60
24½	76·97	471·43	29⅝	93·06	689·29	34¾	109·17	948·42
24⅝	77·36	476·26	29¾	93·46	695·12	34⅞	109·56	955·25
24¾	77·75	481·10	29⅞	93·85	700·98	35	109·95	962·11
24⅞	78·14	485·97	30	94·24	706·86	35⅛	110·34	968·99
25	78·54	490·87	30⅛	94·64	712·76	35¼	110·74	975·90
25⅛	78·93	495·79	30¼	95·03	718·69	35⅜	111·13	982·84
25¼	79·32	500·74	30⅜	95·42	724·64	35½	111·52	989·80
25⅜	79·71	505·71	30½	95·82	730·62	35⅝	111·92	996·78
25½	80·11	510·70	30⅝	96·21	736·62	35¾	112·31	1003·79
25⅝	80·50	515·72	30¾	96·60	742·64	35⅞	112·70	1010·82
25¾	80·89	520·76	30⅞	96·99	748·69	36	113·09	1017·87
25⅞	81·29	525·83	31	97·39	754·76	36⅛	113·49	1024·96
26	81·68	530·93	31⅛	97·78	760·87	36¼	113·88	1032·06
26⅛	82·07	536·04	31¼	98·17	766·99	36⅜	114·27	1039·19
26¼	82·46	541·19	31⅜	98·56	773·14	36½	114·66	1046·34
26⅜	82·86	546·35	31½	98·96	779·31	36⅝	115·06	1053·52
26½	83·25	551·54	31⅝	99·35	785·51	36¾	115·45	1060·73
26⅝	83·64	556·76	31¾	99·74	791·73	36⅞	115·84	1067·96
26¾	84·03	562·00	31⅞	100·14	797·98	37	116·24	1075·21
26⅞	84·43	567·26	32	100·53	804·25	37⅛	116·63	1082·49
27	84·82	572·55	32⅛	100·92	810·54	37¼	117·02	1089·79
27⅛	85·21	577·87	32¼	101·31	816·86	37⅜	117·41	1097·11
27¼	85·60	583·20	32⅜	101·71	823·21	37½	117·81	1104·46
27⅜	86·00	588·57	32½	102·10	829·57	37⅝	118·20	1111·84
27½	86·39	593·96	32⅝	102·49	835·97	37¾	118·59	1119·24
27⅝	86·78	599·37	32¾	102·88	842·39	37⅞	118·99	1126·66
27¾	87·17	604·80	32⅞	103·28	848·83	38	119·38	1134·11
27⅞	87·57	610·26	33	103·67	855·30	38⅛	119·77	1141·59

TABLE II.—(*contd.*).

Dia.	Circum.	Area	Dia.	Circum.	Area.	Dia.	Circum.	Area.
38¼	120·16	1149·08	42¼	132·73	1401·99	46¼	145·29	1680·01
38⅜	120·56	1156·61	42⅜	133·12	1410·29	46⅜	145·69	1689·11
38½	120·95	1164·15	42½	133·51	1418·62	46½	146·08	1698·23
38⅝	121·34	1171·73	42⅝	133·91	1426·99	46⅝	146·47	1707·37
38¾	121·73	1179·32	42¾	134·30	1435·36	46¾	146·87	1716·54
38⅞	122·12	1186·94	42⅞	134·69	1443·77	46⅞	147·26	1725·73
39	122·52	1194·59	43	135·08	1452·20	47	147·65	1734·94
39⅛	122·91	1202·26	43⅛	135·48	1460·66	47⅛	148·04	1744·19
39¼	123·30	1209·95	43¼	135·87	1469·14	47¼	148·44	1753·45
39⅜	123·70	1217·67	43⅜	136·26	1477·63	47⅜	148·83	1762·74
39½	124·09	1225·42	43½	136·66	1486·17	47½	149·22	1772·05
39⅝	124·48	1233·18	43⅝	137·05	1494·73	47⅝	149·62	1781·39
39¾	124·87	240·98	43¾	137·44	1503·30	47¾	150·01	1790·76
39⅞	125·27	1248·79	43⅞	137·83	1511·91	47⅞	150·40	1800·14
40	125·66	1256·64	44	138·23	1520·53	48	150·79	1809·56
40⅛	126·05	1264·50	44⅛	138·62	1529·19	48⅛	151·19	1818·99
40¼	126·45	1272·40	44¼	139·01	1537·86	48¼	151·58	828·46
40⅜	126·84	1280·31	44⅜	139·40	1546·55	48⅜	151·97	1837·95
40½	127·23	1288·25	44½	139·80	1555·29	48½	152·36	1847·45
40⅝	127·62	1296·22	44⅝	140·19	1564·03	48⅝	152·76	1856·99
40¾	128·02	304·20	44¾	140·58	1572·81	48¾	153·15	1866·55
40⅞	128·41	1312·21	44⅞	140·98	1581·61	48⅞	153·54	1876·14
41	128·80	1320·26	45	141·37	1590·43	49	153·94	885·74
41⅛	129·19	1328·32	45⅛	141·76	1599·28	49⅛	154·33	1895·37
41¼	129·59	1336·41	45¼	142·15	1608·16	49¼	154·72	1905·04
41⅜	129·98	1344·52	45⅜	142·55	1617·04	49⅜	155·11	1914·72
41½	130·37	1352·66	45½	142·94	1625·97	49½	155·60	1924·42
41⅝	130·77	1360·82	45⅝	143·33	1634·92	49⅝	155·90	1934·15
41¾	131·16	1369·00	45¾	143·72	1643·89	49¾	156·29	1943·91
41⅞	131·55	1377·21	45⅞	144·12	1652·89	49⅞	156·68	1953·69
42	131·94	1385·44	46	144·51	1661·90	50	157·08	1963·50
42⅛	132·33	1393·70	46⅛	144·90	1670·95			

TABLE III.

Powers, Roots and Reciprocals.

n	n^2	n^3	$\sqrt{n}$	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\frac{1}{n}$	n
1·10	1·2100	1·33100	1·04881	3·31662	1·03228	2·22398	·909091	1·10
1·20	1·4400	1·72800	1·09545	3·46410	1·06266	2·28943	·833333	1·20
1·30	1·6900	2·19700	1·14018	3·6055	1·09139	2·35134	·769231	1·30
1·40	1·9600	2·74400	1·18322	3·74166	1·11869	2·41014	·714286	1·40
1·50	2·2500	3·37500	1·22474	3·87298	1·14471	2·46621	·666667	1·50
1·60	2·5600	4·09600	1·26491	4·00000	1·16961	2·51984	·625000	1·60
1·70	2·8900	4·91300	1·30384	4·12311	1·19348	2·57128	·588235	1·70
1·80	3·2400	5·83200	1·34164	4·24264	1·21644	2·62074	·555556	1·80
1·90	3·6100	6·85900	1·37840	4·35890	1·23856	2·66840	·526316	1·90
2·00	4·0000	8·00000	1·41421	4·47214	1·25992	2·71442	·500000	2·00
2·10	4·4100	9·26100	1·44914	4·58258	1·28058	2·75893	·476191	2·10
2·20	4·8400	10·6480	1·48324	4·69042	1·30059	2·80204	·454546	2·20
2·30	5·2900	12·1670	1·51658	4·79583	1·32001	2·84387	·434783	2·30
2·40	5·7600	13·8240	1·54919	4·89898	1·33887	2·88450	·416667	2·40
2·50	6·2500	15·6250	1·58114	5·00000	1·35731	2·92402	·400000	2·50
2·60	6·7600	17·5760	1·61245	5·09902	1·35707	2·96250	·384615	2·60
2·70	7·2900	19·6830	1·64317	5·19615	1·39248	3·00000	·370370	2·70
2·80	7·8400	21·9520	1·67332	5·29150	1·40946	3·03659	·357142	2·80
2·90	8·4100	24·3890	1·70294	5·38516	1·42604	3·07232	·344828	2·90
3·00	9·0000	27·0000	1·73205	5·47723	1·44225	3·10723	·333333	3·00
3·10	9·6100	29·7910	1·76068	5·56776	1·45810	3·14138	·322581	3·10
3·20	10·2400	32·7680	1·78885	5·65685	1·47361	3·17480	·312500	3·20
3·30	10·8900	35·9370	1·81659	5·74456	1·48881	3·20753	·303030	3·30
3·40	11·5600	39·3040	1·84391	5·83095	1·50369	3·23961	·294118	3·40
3·50	12·2500	42·8750	1·87083	5·91608	1·51829	3·27107	·285714	3·50
3·60	12·9600	46·6560	1·89737	6·00000	1·53262	3·30193	·277778	3·60
3·70	13·6900	50·6530	1·92354	6·08276	1·54668	3·33222	·270270	3·70
3·80	14·4400	54·8720	1·94936	6·16441	1·56049	3·36198	·263158	3·80
3·90	15·2100	59·3190	1·97484	6·24500	1·57406	3·39121	·256410	3·90
4·00	16·0000	64·0000	2·00000	6·32456	1·58740	3·41995	·250000	4·00
4·10	16·8100	68·9210	2·02485	6·40312	1·60052	3·44822	·243902	4·10
4·20	17·6400	74·0880	2·04939	6·48074	1·61343	3·47603	·238095	4·20
4·30	18·4900	79·5070	2·07364	6·55744	1·62613	3·50340	·232558	4·30
4·40	19·3600	85·1840	2·09762	6·63325	1·63864	3·53035	·227273	4·40
4·50	20·2500	91·1250	2·12132	6·70820	1·65096	3·55689	·222222	4·50
4·60	21·1600	97·3360	2·14476	6·78233	1·66310	3·58305	·217391	4·60
4·70	22·0900	103·823	2·16795	6·85565	1·67507	3·60883	·212766	4·70
4·80	23·0400	110·592	2·19089	6·92820	1·68687	3·63424	·208333	4·80
4·90	24·0100	117·649	2·21359	7·00000	1·69850	3·65931	·204082	4·90
5·00	25·0000	125·000	2·23607	7·07107	1·70998	3·68403	·200000	5·00
5·10	26·0100	132·651	2·25832	7·14143	1·72130	3·70843	·196078	5·10
5·20	27·0400	140·608	2·28035	7·21110	1·73248	3·73251	·192308	5·20
5·30	28·0900	148·877	2·30217	7·28011	1·74351	3·75629	·188679	5·30
5·40	29·1600	157·464	2·32379	7·34847	1·75441	3·77976	·185185	5·40
5·50	30·2500	166·375	2·34521	7·41620	1·76517	3·80295	·181818	5·50

TABLE III.—Powers, Roots and Reciprocals—(*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\frac{1}{n}$	n
5·60	31·3600	175·616	2·36643	7·48331	1·77581	3·82586	·178571	5·60
5·70	32·4900	185·193	2·38747	7·54983	1·78632	3·84850	·175439	5·70
5·80	33·6400	195·112	2·40832	7·61577	1·79670	3·87088	·172414	5·80
5·90	34·8100	205·379	2·42899	7·68115	1·80679	3·89300	·169492	5·90
6·00	36·0000	216·000	2·44949	7·74597	1·81712	3·91487	·166667	**6·00**
6·10	37·2100	226·981	2·46982	7·81025	1·82716	3·93650	·163934	6·10
6·20	38·4400	238·328	2·48998	7·87401	1·83709	3·95789	·161290	6·20
6·30	39·6900	250·047	2·50998	7·93725	1·84691	3·97906	·158730	6·30
6·40	40·9600	262·144	2·52982	8·00000	1·85664	4·00000	·156250	6·40
6·50	42·2500	274·625	2·54951	8·06226	1·86626	4·02073	·153846	**6·50**
6·60	43·5600	287·496	2·56905	8·12404	1·87578	4·04124	·151515	6·60
6·70	44·8900	300·763	2·58844	8·18535	1·88520	4·06155	·149254	6·70
6·80	46·2400	314·432	2·60768	8·24621	1·89454	4·08166	·147059	6·80
6·90	47·6100	328·509	2·62679	8·30662	1·90378	4·10157	·144928	6·90
7·00	49·0000	343·000	2·64575	8·36660	1·91293	4·12129	·142857	**7·00**
7·10	50·4100	357·911	2·66458	8·42615	1·92200	4·14082	·140845	7·10
7·20	51·8400	373·248	2·68328	8·48528	1·93098	4·16017	·138889	7·20
7·30	53·2900	389·017	2·70185	8·54400	1·93988	4·17934	·136986	7·30
7·40	54·7600	405·224	2·72029	8·60233	1·94870	4·19834	·135135	7·40
7·50	56·2500	421·875	2·73861	8·66025	1·95743	4·21716	·133333	**7·50**
7·60	57·7600	438·976	2·75681	8·71780	1·96610	4·23582	·131579	7·60
7·70	59·2900	456·533	2·77489	8·77496	1·97468	4·25432	·129870	7·70
7·80	60·8400	474·552	2·79285	8·83176	1·98319	4·27266	·128205	7·80
7·90	62·4100	493·039	2·81069	8·88819	1·99163	4·29084	·126582	7·90
8·00	64·0000	512·000	2·82843	8·94427	2·00000	4·30887	·125000	**8·00**
8·10	65·6100	531·441	2·84605	9·00000	2·00830	4·32675	·123457	8·10
8·20	67·2400	551·368	2·86356	9·05539	2·01653	4·34448	·121951	8·20
8·30	68·8900	571·787	2·88097	9·11043	2·02469	4·36207	·120482	8·30
8·40	70·5600	592·704	2·89828	9·16515	2·03279	4·37952	·119048	8·40
8·50	72·2500	614·125	2·91548	9·21954	2·04083	4·39683	·117647	**8·50**
8·60	73·9600	636·056	2·93258	9·27362	2·04880	4·41400	·116279	8·60
8·70	75·6900	658·503	2·94958	9·32738	2·05671	4·43105	·114943	8·70
8·80	77·4400	681·472	2·96648	9·38083	2·06456	4·44796	·113636	8·80
8·90	79·2100	704·969	2·98329	9·43398	2·07235	4·46474	·112360	8·90
9·00	81·0000	729·000	3·00000	9·48683	2·08008	4·48140	·111111	**9·00**
9·10	82·8100	753·571	3·01662	9·53939	2·08776	4·49794	·109890	9·10
9·20	84·6400	778·688	3·03315	9·59166	2·09538	4·51436	·108696	9·20
9·30	86·4900	804·357	3·04959	9·64365	2·10294	4·53065	·107527	9·30
9·40	88·3600	830·584	3·06594	9·69536	2·11045	4·54684	·106383	9·40
9·50	90·2500	857·375	3·08221	9·74679	2·11791	4·56290	·105263	**9·50**
9·60	92·1600	884·736	3·09839	9·79796	2·12532	4·57886	·104167	9·60
9·70	94·0900	912·673	3·11448	9·84886	2·13267	4·59470	·103093	9·70
9·80	96·0400	941·192	3·13050	9·89949	2·13997	4·61044	·102041	9·80
9·90	98·0100	970·299	3·14643	9·94987	2·14723	4·62607	·101010	9·90
10·00	100·000	1000·00	3·16228	10·0000	2·15443	4·64159	·100000	**10·00**

TABLE IV.—Inches to Decimals of a Foot.

In.	0	1/32	1/16	3/32	1/8	5/32	3/16	7/32	1/4	9/32	5/16	11/32	3/8	13/32	7/16	15/32
0	·0000	·0026	·0052	·0078	·0104	·0130	·0156	·0182	·0208	·0234	·0260	·0286	·0312	·0339	·0365	·0391
1	·0833	·0859	·0885	·0911	·0937	·0964	·0990	·1016	·1042	·1068	·1094	·1120	·1146	·1172	·1198	·1224
2	·1667	·1693	·1719	·1745	·1771	·1797	·1823	·1849	·1875	·1901	·1927	·1953	·1979	·2005	·2031	·2057
3	·2500	·2526	·2552	·2578	·2604	·2630	·2656	·2682	·2708	·2734	·2760	·2786	·2812	·2839	·2865	·2891
4	·3333	·3359	·3385	·3411	·3437	·3464	·3490	·3516	·3542	·3568	·3594	·3620	·3646	·3672	·3698	·3724
5	·4167	·4193	·4219	·4245	·4271	·4297	·4323	·4349	·4375	·4401	·4427	·4453	·4479	·4505	·4531	·4557
6	·5000	·5026	·5052	·5078	·5104	·5130	·5156	·5182	·5208	·5234	·5260	·5286	·5312	·5339	·5365	·5391
7	·5833	·5859	·5885	·5911	·5937	·5964	·5990	·6016	·6042	·6068	·6094	·6120	·6146	·6172	·6198	·6224
8	·6667	·6693	·6719	·6745	·6771	·6797	·6823	·6849	·6875	·6901	·6927	·6953	·6979	·7005	·7031	·7057
9	·7500	·7526	·7552	·7578	·7604	·7630	·7656	·7682	·7708	·7734	·7760	·7786	·7812	·7839	·7865	·7891
10	·8333	·8359	·8385	·8411	·8437	·8464	·8490	·8516	·8542	·8568	·8594	·8620	·8646	·8672	·8698	·8724
11	·9167	·9193	·9219	·9245	·9271	·9297	·9323	·9349	·9375	·9401	·9427	·9453	·9479	·9505	·9531	·9557

In.	1/2	17/32	9/16	19/32	5/8	21/32	11/16	23/32	3/4	25/32	13/16	27/32	7/8	29/32	15/16	31/32
0	·0417	·0443	·0469	·0495	·0521	·0547	·0573	·0599	·0625	·0651	·0677	·0703	·0729	·0755	·0781	·0807
1	·1250	·1276	·1302	·1328	·1354	·1380	·1406	·1432	·1458	·1484	·1510	·1536	·1562	·1589	·1615	·1641
2	·2083	·2109	·2135	·2161	·2188	·2214	·2240	·2266	·2292	·2318	·2344	·2370	·2396	·2422	·2448	·2474
3	·2917	·2943	·2969	·2995	·3021	·3047	·3073	·3099	·3125	·3151	·3177	·3203	·3229	·3255	·3281	·3307
4	·3750	·3776	·3802	·3828	·3854	·3880	·3906	·3932	·3958	·3984	·4010	·4036	·4062	·4089	·4115	·4141
5	·4583	·4609	·4635	·4661	·4688	·4714	·4740	·4766	·4792	·4818	·4844	·4870	·4896	·4922	·4948	·4974
6	·5417	·5443	·5469	·5495	·5521	·5547	·5573	·5599	·5625	·5651	·5677	·5703	·5729	·5755	·5781	·5807
7	·6250	·6276	·6302	·6328	·6354	·6380	·6406	·6432	·6458	·6484	·6510	·6536	·6562	·6589	·6615	·6641
8	·7083	·7109	·7135	·7161	·7188	·7214	·7240	·7266	·7292	·7318	·7344	·7370	·7396	·7422	·7448	·7474
9	·7917	·7943	·7969	·7995	·8021	·8047	·8073	·8099	·8125	·8151	·8177	·8203	·8229	·8255	·8281	·8307
10	·8750	·8776	·8802	·8828	·8854	·8880	·8906	·8932	·8958	·8984	·9010	·9036	·9062	·9089	·9115	·9141
11	·9583	·9609	·9635	·9661	·9688	·9714	·9740	·9766	·9792	·9818	·9844	·9870	·9896	·9922	·9948	·9974

TABLE V.

Decimal and Millimetre Equivalents of Fractions of an Inch.

Fractions of an inch.	Decimals.	Millimetres.	Fractions of an inch.	Decimals.	Millimetres.
1/64	·015625	0·395	33/64	·515625	13·0968
1/32	·031250	0·793	17/32	·531250	13·4937
3/64	·046875	1·185	35/64	·546875	13·8096
1/16	·062500	1·5875	9/16	·562500	14·287
5/64	·078125	1·985	37/64	·578125	14·6843
3/32	·093750	2·381	19/32	·593750	15·08125
7/64	·109375	2·765	39/64	·609375	15·4781
1/8	·125000	3·1749	5/8	·625000	15·875
9/64	·140625	3·565	41/64	·640625	16·2718
5/32	·156250	3·968	21/32	·656250	16·66875
11/64	·171875	4·345	43/64	·671875	17·0656
3/16	·187500	4·7624	11/16	·687500	17·462
13/64	·203125	5·165	45/64	·703125	17·8593
7/32	·218750	5·556	23/32	·718750	18·25625
15/64	·234375	5·855	47/64	·734375	18·653125
1/4	·250000	6·3499	3/4	·750000	19·050
17/64	·265625	6·7468	49/64	·765625	19·4468
9/32	·281250	7·143	25/32	·781250	19·84375
19/64	·296875	7·5406	51/64	·796875	20·2406
5/16	·312500	7·9374	13/16	·812500	20·637
21/64	·328125	8·3343	53/64	·828125	21·0343
11/32	·343750	8·731	27/32	·843750	21·43125
23/64	·359375	9·1281	55/64	·859375	21·828125
3/8	·375000	9·5248	7/8	·875000	22·225
25/64	·390625	9·8218	57/64	·890625	22·6218125
13/32	·406250	10·300	29/32	·906250	23·01875
27/64	·421875	10·7156	59/64	·921875	23·4156
7/16	·437500	11·1120	15/16	·937500	23·812
29/64	·453125	11·5093	61/64	·953125	24·209375
15/32	·468750	11·91	31/32	·968750	24·60625
31/64	·484375	12·3031	63/64	·984375	25·003125
1/2	·500000	12·700	1	1·000000	25·400

TABLE VI.—Logarithms.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

Note.—To convert Common Logarithms to Hyperbolic Logarithms multiply by 2·30258509.

TABLE VI.—(*contd.*).

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

Note.—**To convert Common Logarithms to Hyperbolic Logarithms multiply by 2·30258509.**

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1054	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3000	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3408	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5072	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7601	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

TABLE VIII.

Neperian or Hyperbolic Logarithms.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1·0	0·0000	0100	0198	0296	0392	0488	0583	0677	0770	0862	10	19	29	38	48	57	67	76	86
1·1	0·0953	1044	1133	1222	1310	1398	1484	1570	1655	1740	9	17	26	35	44	52	61	70	78
1·2	0·1823	1906	1989	2070	2151	2231	2311	2390	2469	2546	8	16	24	32	40	48	56	64	72
1·3	0·2624	2700	2776	2852	2927	3001	3075	3148	3221	3293	7	15	22	30	37	44	52	59	67
1·4	0·3365	3436	3507	3577	3646	3716	3784	3853	3920	3988	7	14	21	28	35	41	48	55	62
1·5	0·4055	4121	4187	4253	4318	4383	4447	4511	4574	4637	6	13	19	26	32	39	45	52	58
1·6	0·4700	4762	4824	4886	4947	5008	5068	5128	5188	5247	6	12	18	24	30	36	42	48	55
1·7	0·5306	5365	5423	5481	5539	5596	5653	5710	5766	5822	6	11	17	23	29	34	40	46	51
1·8	0·5878	5933	5988	6043	6098	6152	6206	6259	6313	6366	5	11	16	22	27	32	38	43	49
1·9	0·6419	6471	6523	6575	6627	6678	6729	6780	6831	6881	5	10	15	20	26	31	36	41	46
2·0	0·6931	6981	7031	7080	7129	7178	7227	7275	7324	7372	5	10	15	20	24	29	34	39	44
2·1	0·7419	7467	7514	7561	7608	7655	7701	7747	7793	7839	5	9	14	19	23	28	33	37	42
2·2	0·7885	7930	7975	8020	8065	8109	8154	8198	8242	8286	4	9	13	18	22	27	31	36	40
2·3	0·8329	8372	8416	8459	8502	8544	8587	8629	8671	8713	4	9	13	17	21	26	30	34	38
2·4	0·8755	8796	8838	8879	8920	8961	9002	9042	9083	9123	4	8	12	16	20	24	29	33	37
2·5	0·9163	9203	9243	9282	9322	9361	9400	9439	9478	9517	4	8	12	16	20	24	27	31	35
2·6	0·9555	9594	9632	9670	9708	9746	9783	9821	9858	9895	4	8	11	15	19	23	26	30	34
2·7	0·9933	9969	0006	0043	0080	0116	0152	0188	0225	0260	4	7	11	15	18	22	25	29	33
2·8	1·0296	0332	0367	0403	0438	0473	0508	0543	0578	0613	4	7	11	14	18	21	25	28	32
2·9	1·0647	0682	0716	0750	0784	0818	0852	0886	0919	0953	3	7	10	14	17	20	24	27	31
3·0	1·0986	1019	1053	1086	1119	1151	1184	1217	1249	1282	3	7	10	13	16	20	23	26	30
3·1	1·1314	1346	1378	1410	1442	1474	1506	1537	1569	1600	3	6	10	13	16	19	22	25	29
3·2	1·1632	1663	1694	1725	1756	1787	1817	1848	1878	1909	3	6	9	12	15	18	21	25	28
3·3	1·1939	1969	2000	2030	2060	2090	2119	2149	2179	2208	3	6	9	12	15	18	21	24	27
3·4	1·2238	2267	2296	2326	2355	2384	2413	2442	2470	2499	3	6	9	12	15	17	20	23	26
3·5	1·2528	2556	2585	2613	2641	2669	2698	2726	2754	2782	3	6	8	11	14	17	20	22	25
3·6	1·2809	2837	2865	2892	2920	2947	2975	3002	3029	3056	3	5	8	11	14	16	19	22	25
3·7	1·3083	3110	3137	3164	3191	3218	3244	3271	3297	3324	3	5	8	11	13	16	19	21	24
3·8	1·3350	3376	3403	3429	3455	3481	3507	3533	3558	3584	3	5	8	10	13	16	18	21	23
3·9	1·3610	3635	3661	3686	3712	3737	3762	3788	3813	3838	3	5	8	10	13	15	18	20	23
4·0	1·3863	3888	3913	3938	3962	3987	4012	4036	4061	4085	2	5	7	10	12	15	17	20	22
4·1	1·4110	4134	4159	4183	4207	4231	4255	4279	4303	4327	2	5	7	10	12	14	17	19	22
4·2	1·4351	4375	4398	4422	4446	4469	4493	4516	4540	4563	2	5	7	9	12	14	16	19	21
4·3	1·4586	4609	4633	4656	4679	4702	4725	4748	4770	4793	2	5	7	9	12	14	16	18	21
4·4	1·4816	4839	4861	4884	4907	4929	4951	4974	4996	5019	2	5	7	9	11	14	16	18	20
4·5	1·5041	5063	5085	5107	5129	5151	5173	5195	5217	5239	2	4	7	9	11	13	15	18	20
4·6	1·5261	5282	5304	5326	5347	5369	5390	5412	5433	5454	2	4	6	9	11	13	15	17	19
4·7	1·5476	5497	5518	5539	5560	5581	5602	5623	5644	5665	2	4	6	8	11	13	15	17	19
4·8	1·5686	5707	5728	5748	5769	5790	5810	5831	5851	5872	2	4	6	8	10	12	14	16	19
4·9	1·5892	5913	5933	5953	5974	5994	6014	6034	6054	6074	2	4	6	8	10	12	14	16	18
5·0	1·6094	6114	6134	6154	6174	6194	6214	6233	6253	6273	2	4	6	8	10	12	14	16	18
5·1	1·6292	6312	6332	6351	6371	6390	6409	6429	6448	6467	2	4	6	8	10	12	14	16	18
5·2	1·6487	6506	6525	6544	6563	6582	6601	6620	6639	6658	2	4	6	8	10	11	13	15	17
5·3	1·6677	6696	6715	6734	6752	6771	6790	6808	6827	6845	2	4	6	7	9	11	13	15	17
5·4	1·6864	6882	6901	6919	6938	6956	6974	6993	7011	7029	2	4	5	7	9	11	13	15	16

Note.—To convert Hyperbolic Logarithms to Common Logarithms multiply by 0·43429448.

TABLE VIII.—(*continued*).

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
5·5	1·7047	7066	7084	7102	7120	7138	7156	7174	7192	7210	2	4	5	7	9	11	13	14	16
5·6	1·7228	7246	7263	7281	7299	7317	7334	7352	7370	7387	2	4	5	7	9	11	12	14	16
5·7	1·7405	7422	7440	7457	7475	7492	7509	7527	7544	7561	2	3	5	7	9	10	12	14	16
5·8	1·7579	7596	7613	7630	7647	7664	7681	7699	7716	7733	2	3	5	7	9	10	12	14	15
5·9	1·7750	7766	7783	7800	7817	7834	7851	7867	7884	7901	2	3	5	7	8	10	12	13	15
6·0	1·7918	7934	7951	7967	7984	8001	8017	8034	8050	8066	2	3	5	7	8	10	12	13	15
6·1	1·8083	8099	8116	8132	8148	8165	8181	8197	8213	8229	2	3	5	6	8	10	11	13	15
6·2	1·8245	8262	8278	8294	8310	8326	8342	8358	8374	8390	2	3	5	6	8	10	11	13	14
6·3	1·8405	8421	8437	8453	8469	8485	8500	8516	8532	8547	2	3	5	6	8	9	11	13	14
6·4	1·8563	8579	8594	8610	8625	8641	8656	8672	8687	8703	2	3	5	6	8	9	11	12	14
6·5	1·8718	8733	8749	8764	8779	8795	8810	8825	8840	8856	2	3	5	6	8	9	11	12	14
6·6	1·8871	8886	8901	8916	8931	8946	8961	8976	8991	9006	2	3	5	6	8	9	11	12	14
6·7	1·9021	9036	9051	9066	9081	9095	9110	9125	9140	9155	1	3	4	6	7	9	10	12	13
6·8	1·9169	9184	9199	9213	9228	9242	9257	9272	9286	9301	1	3	4	6	7	9	10	12	13
6·9	1·9315	9330	9344	9359	9373	9387	9402	9416	9430	9445	1	3	4	6	7	9	10	12	13
7·0	1·9459	9473	9488	9502	9516	9530	9544	9559	9573	9587	1	3	4	6	7	9	10	11	13
7·1	1·9601	9615	9629	9643	9657	9671	9685	9699	9713	9727	1	3	4	6	7	8	10	11	13
7·2	1·9741	9755	9769	9782	9796	9810	9824	9838	9851	9865	1	3	4	6	7	8	10	11	12
7·3	1·9879	9892	9906	9920	9933	9947	9961	9974	9988	0001	1	3	4	5	7	8	10	11	12
7·4	2·0015	0028	0042	0055	0069	0082	0096	0109	0122	0136	1	3	4	5	7	8	9	11	12
7·5	2·0149	0162	0176	0189	0202	0215	0229	0242	0255	0268	1	3	4	5	7	8	9	11	12
7·6	2·0281	0295	0308	0321	0334	0347	0360	0373	0386	0399	1	3	4	5	7	8	9	10	12
7·7	2·0412	0425	0438	0451	0464	0477	0490	0503	0516	0528	1	3	4	5	6	8	9	10	12
7·8	2·0541	0554	0567	0580	0592	0605	0618	0631	0643	0656	1	3	4	5	6	8	9	10	12
7·9	2·0669	0681	0694	0707	0719	0732	0744	0757	0769	0782	1	3	4	5	6	8	9	10	11
8·0	2·0794	0807	0819	0832	0844	0857	0869	0882	0894	0906	1	3	4	5	6	8	9	10	11
8·1	2·0919	0931	0943	0956	0968	0980	0992	1005	1017	1029	1	2	4	5	6	7	9	10	11
8·2	2·1041	1054	1066	1078	1090	1102	1114	1126	1138	1150	1	2	4	5	6	7	9	10	11
8·3	2·1163	1175	1187	1199	1211	1223	1235	1247	1258	1270	1	2	4	5	6	7	8	10	11
8·4	2·1282	1294	1306	1318	1330	1342	1353	1365	1377	1389	1	2	4	5	6	7	8	10	11
8·5	2·1401	1412	1424	1436	1448	1459	1471	1483	1494	1506	1	2	4	5	6	7	8	9	11
8·6	2·1518	1529	1541	1552	1564	1576	1587	1599	1610	1622	1	2	3	5	6	7	8	9	10
8·7	2·1633	1645	1656	1668	1679	1691	1702	1713	1725	1736	1	2	3	5	6	7	8	9	10
8·8	2·1748	1759	1770	1782	1793	1804	1815	1827	1838	1849	1	2	3	5	6	7	8	9	10
8·9	2·1861	1872	1883	1894	1905	1917	1928	1939	1950	1961	1	2	3	4	6	7	8	9	10
9·0	2·1972	1983	1994	2006	2017	2028	2039	2050	2061	2072	1	2	3	4	6	7	8	9	10
9·1	2·2083	2094	2105	2116	2127	2138	2148	2159	2170	2181	1	2	3	4	5	7	8	9	10
9·2	2·2192	2203	2214	2225	2235	2246	2257	2268	2279	2289	1	2	3	4	5	6	8	9	10
9·3	2·2300	2311	2322	2332	2343	2354	2364	2375	2386	2396	1	2	3	4	5	6	7	9	10
9·4	2·2407	2418	2428	2439	2450	2460	2471	2481	2492	2502	1	2	3	4	5	6	7	8	10
9·5	2·2513	2523	2534	2544	2555	2565	2576	2586	2597	2607	1	2	3	4	5	6	7	8	9
9·6	2·2618	2628	2638	2649	2659	2670	2680	2690	2701	2711	1	2	3	4	5	6	7	8	9
9·7	2·2721	2732	2742	2752	2762	2773	2783	2793	2803	2814	1	2	3	4	5	6	7	8	9
9·8	2·2824	2834	2844	2854	2865	2875	2885	2895	2905	2915	1	2	3	4	5	6	7	8	9
9·9	2·2925	2935	2946	2956	2966	2976	2986	2996	3006	3016	1	2	3	4	5	6	7	8	9

THE FOLLOWING ARE THE VALUES OF POWERS OF ϵ:—

ϵ	2·7183	ϵ^{-1}	0·3679	$\epsilon^{\frac{1}{2}}$	1·6487	$\epsilon^{-\frac{1}{2}}$	0·6065	$\epsilon^{\frac{1}{2}}$	1·6457	$\epsilon^{-\frac{1}{2}}$	0·6065
ϵ^{2}	7·3891	ϵ^{-2}	0·1353	$\epsilon^{\frac{3}{2}}$	4·4816	$\epsilon^{-\frac{3}{2}}$	0·2231	$\epsilon^{\frac{1}{3}}$	1·3956	$\epsilon^{-\frac{1}{3}}$	0·7165
ϵ^{3}	20·086	ϵ^{-3}	0·04979	$\epsilon^{\frac{5}{2}}$	12·182	$\epsilon^{-\frac{5}{2}}$	0·0821	$\epsilon^{\frac{1}{4}}$	1·2840	$\epsilon^{-\frac{1}{4}}$	0·7788
ϵ^{4}	54·598	ϵ^{-4}	0·01832	$\epsilon^{\frac{7}{2}}$	33·114	$\epsilon^{-\frac{7}{2}}$	0·0302	$\epsilon^{\frac{1}{5}}$	1·2214	$\epsilon^{-\frac{1}{5}}$	0·8187
ϵ^{5}	148·41	ϵ^{-5}	0·006738	$\epsilon^{\frac{1}{4}}$	1·2840	$\epsilon^{-\frac{1}{4}}$	0·7788	$\epsilon^{\frac{1}{6}}$	1·1814	$\epsilon^{-\frac{1}{6}}$	0·8465
ϵ^{6}	403·43	ϵ^{-6}	0·002479	$\epsilon^{\frac{1}{8}}$	1·1331	$\epsilon^{-\frac{1}{8}}$	0·8825	$\epsilon^{\frac{1}{7}}$	1·1536	$\epsilon^{-\frac{1}{7}}$	0·8669
ϵ^{7}	1096·6	ϵ^{-7}	0·0009119	$\epsilon^{\frac{1}{16}}$	1·0645	$\epsilon^{-\frac{1}{16}}$	0·9394	$\epsilon^{\frac{1}{8}}$	1·1331	$\epsilon^{-\frac{1}{8}}$	0·8825
ϵ^{8}	2981·0	ϵ^{-8}	0·0003355	$\epsilon^{\frac{1}{32}}$	1·0317	$\epsilon^{-\frac{1}{32}}$	0·9692	$\epsilon^{\frac{1}{9}}$	1·1175	$\epsilon^{-\frac{1}{9}}$	0·8948
ϵ^{9}	8103·1	ϵ^{-9}	0·0001234					$\epsilon^{\frac{1}{10}}$	1·1052	$\epsilon^{-\frac{1}{10}}$	0·9048
ϵ^{10}	22026	ϵ^{-10}	0·0000454								

ϵ^{π}	23·1407	$\epsilon^{-\pi}$	43214 × 10⁻⁶	$\epsilon^{\frac{\pi}{2}}$	4·8105	$\epsilon^{-\frac{\pi}{2}}$	20788 × 10⁻⁵	$\epsilon^{\frac{\pi}{4}}$	2·1933	$\epsilon^{-\frac{\pi}{4}}$	4559 × 10⁻⁴
$\epsilon^{2\pi}$	535·491	$\epsilon^{-2\pi}$	1867 × ,,	$\epsilon^{\frac{3\pi}{2}}$	111·32	$\epsilon^{-\frac{3\pi}{2}}$	898·33 × ,,	$\epsilon^{\frac{3\pi}{4}}$	10·557	$\epsilon^{-\frac{3\pi}{4}}$	947·8 × ,,
$\epsilon^{3\pi}$	12391·7	$\epsilon^{-3\pi}$	80·699 × ,,	$\epsilon^{\frac{5\pi}{2}}$	2576·0	$\epsilon^{-\frac{5\pi}{2}}$	38·820 × ,,	$\epsilon^{\frac{5\pi}{4}}$	50·754	$\epsilon^{-\frac{5\pi}{4}}$	197·0 × ,,
$\epsilon^{4\pi}$	286752	$\epsilon^{-4\pi}$	3·487 × ,,	$\epsilon^{\frac{7\pi}{2}}$	59609·6	$\epsilon^{-\frac{7\pi}{2}}$	1·6776 × ,,	$\epsilon^{\frac{7\pi}{4}}$	244·15	$\epsilon^{-\frac{7\pi}{4}}$	40·96 × ,,
$\epsilon^{5\pi}$	6635627	$\epsilon^{-5\pi}$	·1507 × ,,	$\epsilon^{\frac{9\pi}{2}}$	1379406	$\epsilon^{-\frac{9\pi}{2}}$	·0725 × ,,	$\epsilon^{\frac{9\pi}{4}}$	1174·48	$\epsilon^{-\frac{9\pi}{4}}$	8·514 × ,,

TABLE IX.

Natural Sines.

	0′	6′	12′	18	24′	30′	36′	42′	48′	54′	1	2	3	4	5
0°	0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	15
5	0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10	1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	12	14
11	1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	2079	2096	2113	2130	2147	2164	2181	2198	2215	2232	3	6	9	11	14
13	2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20	3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
23	3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30	5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	10	12
36	5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12
37	6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11
40	6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10

Note.—For values above 45° use Cosine Table—e.g., sin 60° = cos (90—60°) = cos 30°.

TABLE X.

Natural Cosines.

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5
0°	1·000	1·000 nearly.	1·000 nearly.	1·000 nearly.	1·000 nearly.	9999	9999	9999	9999	9999	0	0	0	0	0
1	9998	9998	9998	9997	9997	9997	9996	9996	9995	9995	0	0	0	0	0
2	9994	9993	9993	9992	9991	9990	9990	9989	9988	9987	0	0	0	1	1
3	9986	9985	9984	9983	9982	9981	9980	9979	9978	9977	0	0	1	1	1
4	9976	9974	9973	9972	9971	9969	9968	9966	9965	9963	0	0	1	1	1
5	9962	9960	9959	9957	9956	9954	9952	9951	9949	9947	0	1	1	1	2
6	9945	9943	9942	9940	9938	9936	9934	9932	9930	9928	0	1	1	1	2
7	9925	9923	9921	9919	9917	9914	9912	9910	9907	9905	0	1	1	2	2
8	9903	9900	9898	9895	9893	9890	9888	9885	9882	9880	0	1	1	2	2
9	9877	9874	9871	9869	9866	9863	9860	9857	9854	9851	0	1	1	2	2
10	9848	9845	9842	9839	9836	9833	9829	9826	9823	9820	1	1	2	2	3
11	9816	9813	9810	9806	9803	9799	9796	9792	9789	9785	1	1	2	2	3
12	9781	9778	9774	9770	9767	9763	9759	9755	9751	9748	1	1	2	3	3
13	9744	9740	9736	9732	9728	9724	9720	9715	9711	9707	1	1	2	3	3
14	9703	9699	9694	9690	9686	9681	9677	9673	9668	9664	1	1	2	3	4
15	9659	9655	9650	9646	9641	9636	9632	9627	9622	9617	1	2	2	3	4
16	9613	9608	9603	9598	9593	9588	9583	9578	9573	9568	1	2	2	3	4
17	9563	9558	9553	9548	9542	9537	9532	9527	9521	9516	1	2	3	4	4
18	9511	9505	9500	9494	9489	9483	9478	9472	9466	9461	1	2	3	4	5
19	9455	9449	9444	9438	9432	9426	9421	9415	9409	9403	1	2	3	4	5
20	9397	9391	9385	9379	9373	9367	9361	9354	9348	9342	1	2	3	4	5
21	9336	9330	9323	9317	9311	9304	9298	9291	9285	9278	1	2	3	4	5
22	9272	9265	9259	9252	9245	9239	9232	9225	9219	9212	1	2	3	4	6
23	9205	9198	9191	9184	9178	9171	9164	9157	9150	9143	1	2	3	5	6
24	9135	9128	9121	9114	9107	9100	9092	9085	9078	9070	1	2	4	5	[illegible]
25	9063	9056	9048	9041	9033	9026	9018	9011	9003	8996	1	3	4	5	6
26	8988	8980	8973	8965	8957	8949	8942	8934	8926	8918	1	3	4	5	6
27	8910	8902	8894	8886	8878	8870	8862	8854	8846	8838	1	3	4	5	7
28	8829	8821	8813	8805	8796	8788	8780	8771	8763	8755	1	3	4	6	7
29	8746	8738	8729	8721	8712	8704	8695	8686	8678	8669	1	3	4	6	7
30	8660	8652	8643	8634	8625	8616	8607	8599	8590	8581	1	3	4	6	7
31	8572	8563	8554	8545	8536	8526	8517	8508	8499	8490	2	3	5	6	[illegible]
32	8480	8471	8462	8453	8443	8434	8425	8415	8406	8396	2	3	5	6	8
33	8387	8377	8368	8358	8348	8339	8329	8320	8310	8300	2	3	5	6	8
34	8290	8281	8271	8261	8251	8241	8231	8221	8211	8202	2	3	5	7	8
35	8192	8181	8171	8161	8151	8141	8131	8121	8111	8100	2	3	5	7	8
36	8090	8080	8070	8059	8049	8039	8028	8018	8007	7997	2	3	5	7	9
37	7986	7976	7965	7955	7944	7934	7923	7912	7902	7891	2	4	5	7	9
38	7880	7869	7859	7848	7837	7826	7815	7804	7793	7782	2	4	5	7	9
39	7771	7760	7749	7738	7727	7716	7705	7694	7683	7672	2	4	6	7	9
40	7660	7649	7638	7627	7615	7604	7593	7581	7570	7559	2	4	6	8	9
41	7547	7536	7524	7513	7501	7490	7478	7466	7455	7443	2	4	6	8	10
42	7431	7420	7408	7396	7385	7373	7361	7349	7337	7325	2	4	6	8	10
43	7314	7302	7290	7278	7266	7254	7242	7230	7218	7206	2	4	6	8	10
44	7193	7181	7169	7157	7145	7133	7120	7108	7096	7083	2	4	6	8	10

N.B.—Numbers in difference columns to be subtracted, not added.
For values above 45° use Sine Table.

TABLE XI.

Natural Tangents.

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5
0°	0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	14
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	16
15	2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	16
18	3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	6	10	13	17
20	3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	10	14	18
25	4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	10	14	19	24
39	8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

Note.—For values above 45° use Cotangent Table.

TABLE XII.

Natural Cotangents.

Difference-columns not useful here, owing to the rapidity with which the value of the cotangent changes. (Applies to rows 0°–9°.)

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5
0°	Inf.	573·0	286·5	191·0	143·2	114·6	95·49	81·85	71·62	63·66					
1	57·29	52·08	47·74	44·07	40·92	38·19	35·80	33·69	31·82	30·14					
2	28·64	27·27	26·03	24·90	23·86	22·90	22·02	21·20	20·45	19·74					
3	19·08	18·46	17·89	17·34	16·83	16·35	15·89	15·46	15·06	14·67					
4	14·30	13·95	13·62	13·30	13·00	12·71	12·43	12·16	11·91	11·66					
5	11·43	11·20	10·99	10·78	10·58	10·39	10·20	10·02	9·845	9·677					
6	9·5144	3572	2052	0579	9152	7769	6427	5126	3863	2636					
7	8·1443	0285	9158	8062	6996	5958	4947	3962	3002	2066					
8	7·1154	0264	9395	8548	7720	6912	6122	5350	4596	3859					
9	6·3138	2432	1742	1066	0405	9758	9124	8502	7894	7297					
10	5·6713	6140	5578	5026	4486	3955	3435	2924	2422	1929	1	2	3	4	5
11	5·1446	0970	0504	0045	9594	9152	8716	8288	7867	7453	74	148	222	296	370
12	4·7046	6646	6252	5864	5483	5107	4737	4374	4015	3662	63	125	188	252	314
13	4·3315	2972	2635	2303	1976	1653	1335	1022	0713	0408	53	107	160	214	267
14	4·0108	9812	9520	9232	8947	8667	8391	8118	7848	7583	46	93	139	186	232
15	3·7321	7062	6806	6554	6305	6059	5816	5576	5339	5105	41	82	122	163	204
16	3·4874	4646	4420	4197	3977	3759	3544	3332	3122	2914	36	72	108	144	180
17	3·2709	2506	2305	2106	1910	1716	1524	1334	1146	0961	32	64	96	129	161
18	3·0777	0595	0415	0237	0061	9887	9714	9544	9375	9208	29	58	87	115	144
19	2·9042	8878	8716	8556	8397	8239	8083	7929	7776	7625	26	52	78	104	130
20	2·7475	7326	7179	7034	6889	6746	6605	6464	6325	6187	24	47	71	95	118
21	2·6051	5916	5782	5649	5517	5386	5257	5129	5002	4876	22	43	65	87	108
22	2·4751	4627	4504	4383	4262	4142	4023	3906	3789	3673	20	40	60	79	99
23	2·3559	3445	3332	3220	3109	2998	2889	2781	2673	2566	18	37	55	74	92
24	2·2460	2355	2251	2148	2045	1943	1842	1742	1642	1543	17	34	51	68	85
25	2·1445	1348	1251	1155	1060	0965	0872	0778	0686	0594	16	31	47	63	78
26	2·0503	0413	0323	0233	0145	0057	9970	9883	9797	9711	15	29	44	58	73
27	1·9626	9542	9458	9375	9292	9210	9128	9047	8967	8887	14	27	41	55	68
28	1·8807	8728	8650	8572	8495	8418	8341	8265	8190	8115	13	26	38	51	64
29	1·8040	7966	7893	7820	7747	7675	7603	7532	7461	7391	12	24	36	48	60
30	1·7321	7251	7182	7113	7045	6977	6909	6842	6775	6709	11	23	34	45	56
31	1·6643	6577	6512	6447	6383	6319	6255	6191	6128	6066	11	21	32	43	53
32	1·6003	5941	5880	5818	5757	5697	5637	5577	5517	5458	10	20	30	40	50
33	1·5399	5340	5282	5224	5166	5108	5051	4994	4938	4882	10	19	29	38	48
34	1·4826	4770	4715	4659	4605	4550	4496	4442	4388	4335	9	18	27	36	45
35	1·4281	4229	4176	4124	4071	4019	3968	3916	3865	3814	9	17	26	34	43
36	1·3764	3713	3663	3613	3564	3514	3465	3416	3367	3319	8	16	25	33	41
37	1·3270	3222	3175	3127	3079	3032	2985	2938	2892	2846	8	16	23	31	39
38	1·2799	2753	2708	2662	2617	2572	2527	2482	2437	2393	8	15	23	30	38
39	1·2349	2305	2261	2218	2174	2131	2088	2045	2002	1960	7	14	22	29	36
40	1·1918	1875	1833	1792	1750	1708	1667	1626	1585	1544	7	14	21	28	34
41	1·1504	1463	1423	1383	1343	1303	1263	1224	1184	1145	7	13	20	26	33
42	1·1106	1067	1028	0990	0951	0913	0875	0837	0799	0761	6	13	19	25	32
43	1·0724	0686	0649	0612	0575	0538	0501	0464	0428	0392	6	12	18	25	31
44	1·0355	0319	0283	0247	0212	0176	0141	0105	0070	0035	6	12	18	24	30

N.B.—Numbers in difference columns to be subtracted, not added.

For values above 45° use Tangent Table.

Hyperbolic Functions.

$$\sinh x = \frac{E^x - E^{-x}}{2} \qquad \coth x = \frac{\cosh x}{\sinh x}$$

$$\cosh x = \frac{E^x + E^{-x}}{2} \qquad \operatorname{sech} x = \frac{1}{\cosh x}$$

$$\tanh x = \frac{\sinh x}{\cosh x} \qquad \operatorname{csch} x = \frac{1}{\sinh x}$$

Relations between hyperbolic and trigonometric functions
$j = \sqrt{-1}$

$$\cos j\,x = \cosh x$$
$$\sin j\,x = j \sinh x$$

Also

$$\sin\,(a + j\,b) = \sin a \cos j\,b + \cos a \sin j\,b$$
$$= \sin a \cosh b + j \cos a \sinh b$$

giving the typical complex form $x + j\,y$

TABLE XIII.

Hyperbolic Functions.

		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	sinh	0.0000	0.0100	0.0200	0.0300	0.0400	0.0500	0.0600	0.0701	0.0801	0.0901
	cosh	1.0000	1.0001	1.0002	1.0005	1.0008	1.0013	1.0018	1.0025	1.0032	1.0041
	tanh	0.0000	0.0100	0.0200	0.0300	0.0400	0.0500	0.0599	0.0699	0.0708	0.0898
0.1	sinh	0.1002	0.1102	0.1203	0.1304	0.1405	0.1506	0.1607	0.1708	0.1810	0.1911
	cosh	1.0050	1.0061	1.0072	1.0085	1.0098	1.0113	1.0128	1.0145	1.0162	1.0181
	tanh	0.0997	0.1096	0.1194	0.1293	0.1391	0.1489	0.1587	0.1684	0.1781	0.1878
0.2	sinh	0.2013	0.2115	0.2218	0.2320	0.2423	0.2526	0.2629	0.2733	0.2837	0.2941
	cosh	1.0201	1.0221	1.0243	1.0266	1.0289	1.0314	1.0340	1.0367	1.0395	1.0423
	tanh	0.1974	0.2070	0.2165	0.2260	0.2355	0.2449	0.2543	0.2636	0.2720	0.2821
0.3	sinh	0.3045	0.3150	0.3255	0.3360	0.3460	0.3572	0.3678	0.3785	0.3892	0.4001
	cosh	1.0453	1.0484	1.0516	1.0549	1.0584	1.0619	1.0655	1.0692	1.0731	1.0770
	tanh	0.2913	0.3004	0.3095	0.3185	0.3275	0.3364	0.3452	0.3540	0.3627	0.3714
0.4	sinh	0.4108	0.4216	0.4325	0.4434	0.4543	0.4653	0.4764	0.4875	0.4986	0.5098
	cosh	1.0811	1.0852	1.0895	1.0939	1.0984	1.1030	1.1077	1.1125	1.1174	1.1225
	tanh	0.3800	0.3885	0.3969	0.4053	0.4136	0.4219	0.4301	0.4382	0.4462	0.4542

TABLE XIII.—(*contd.*).

∠		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
	sinh	0.5211	0.5324	0.5438	0.5552	0.5666	0.5782	0.5897	0.6014	0.6131	0.6248
0.5	cosh	1.1276	1.1329	1.1383	1.1438	1.1494	1.1551	1.1609	1.1669	1.1730	1.1792
	tanh	0.4621	0.4700	0.4777	0.4854	0.4930	0.5005	0.5080	0.5154	0.5227	0.5299
	sinh	0.6367	0.6485	0.6605	0.6725	0.6846	0.6967	0.7090	0.7213	0.7336	0.7461
0.6	cosh	1.1855	1.1919	1.1984	1.2051	1.2119	1.2188	1.2258	1.2330	1.2402	1.2476
	tanh	0.5370	0.5441	0.5511	0.5581	0.5649	0.5717	0.5784	0.5850	0.5915	0.5980
	sinh	0.7586	0.7712	0.7838	0.7966	0.8094	0.8223	0.8353	0.8484	0.8615	0.8748
0.7	cosh	1.2552	1.2628	1.2706	1.2785	1.2865	1.2947	1.3030	1.3114	1.3199	1.3286
	tanh	0.6044	0.6107	0.6169	0.6231	0.6292	0.6352	0.6411	0.6469	0.6527	0.6584
	sinh	0.8881	0.9015	0.9150	0.9286	0.9423	0.9561	0.9700	0.9840	0.9981	1.0122
0.8	cosh	1.3374	1.3464	1.3555	1.3647	1.3740	1.3835	1.3932	1.4029	1.4128	1.4229
	tanh	0.6640	0.6696	0.6751	0.6805	0.6858	0.6911	0.6963	0.7014	0.7064	0.7114
	sinh	1.0265	1.0409	1.0554	1.0700	1.0847	1.0995	1.1144	1.1294	1.1446	1.1598
0.9	cosh	1.4331	1.4434	1.4539	1.4645	1.4753	1.4862	1.4973	1.5085	1.5199	1.5314
	tanh	0.7163	0.7211	0.7259	0.7306	0.7352	0.7398	0.7443	0.7487	0.7531	0.7574
	sinh	1.1752	1.1907	1.2063	1.2220	1.2379	1.2539	1.2700	1.2862	1.3025	1.3190
1.0	cosh	1.5431	1.5549	1.5669	1.5790	1.5913	1.6038	1.6164	1.6292	1.6421	1.6552
	tanh	0.7616	0.7658	0.7699	0.7739	0.7779	0.7818	0.7857	0.7895	0.7932	0.7969
	sinh	1.3356	1.3524	1.3693	1.3863	1.4035	1.4208	1.4382	1.4558	1.4735	1.4914
1.1	cosh	1.6685	1.6820	1.6956	1.7093	1.7233	1.7374	1.7517	1.7662	1.7808	1.7956
	tanh	0.8005	0.8041	0.8076	0.8110	0.8144	0.8178	0.8210	0.8243	0.8275	0.8306
	sinh	1.5095	1.5276	1.5460	1.5645	1.5831	1.6019	1.6209	1.6400	1.6593	1.6788
1.2	cosh	1.8107	1.8258	1.8412	1.8568	1.8725	1.8884	1.9045	1.9208	1.9373	1.9540
	tanh	0.8337	0.8367	0.8397	0.8426	0.8455	0.8483	0.8511	0.8538	0.8565	0.8591
	sinh	1.6984	1.7182	1.7381	1.7583	1.7786	1.7991	1.8198	1.8406	1.8617	1.8829
1.3	cosh	1.9709	1.9880	2.0053	2.0228	2.0404	2.0583	2.0764	2.0947	2.1132	2.1320
	tanh	0.8617	0.8643	0.8668	0.8693	0.8717	0.8741	0.8764	0.8787	0.8810	0.8832
	sinh	1.9043	1.9259	1.9477	1.9697	1.9919	2.0143	2.0369	2.0597	2.0827	2.1059
1.4	cosh	2.1509	2.1700	2.1894	2.2090	2.2288	2.2488	2.2691	2.2896	2.3103	2.3312
	tanh	0.8854	0.8875	0.8896	0.8917	0.8937	0.8957	0.8977	0.8996	0.9015	0.9033
	sinh	2.1293	2.1529	2.1768	2.2008	2.2251	2.2496	2.2743	2.2993	2.3245	2.3499
1.5	cosh	2.3524	2.3738	2.3955	2.4174	2.4395	2.4619	2.4845	2.5074	2.5305	2.5538
	tanh	0.9052	0.9069	0.9087	0.9104	0.9121	0.9138	0.9154	0.9170	0.9186	0.9202
	sinh	2.3756	2.4015	2.4276	2.4540	2.4806	2.5075	2.5346	2.5620	2.5896	2.6175
1.6	cosh	2.5775	2.6013	2.6255	2.6499	2.6746	2.6995	2.7245	2.7502	2.7760	2.8020
	tanh	0.9217	0.9232	0.9246	0.9261	0.9275	0.9289	0.9302	0.9316	0.9329	0.9342
	sinh	2.6456	2.6740	2.7027	2.7317	2.7609	2.7904	2.8202	2.8503	2.8806	2.9112
1.7	cosh	2.8283	2.8549	2.8818	2.9090	2.9364	2.9642	2.9922	3.0206	3.0493	3.0782
	tanh	0.9354	0.9367	0.9379	0.9391	0.9402	0.9414	0.9425	0.9436	0.9447	0.9458
	sinh	2.9422	2.9734	3.0049	3.0367	3.0689	3.1013	3.1340	3.1671	3.2005	3.2341
1.8	cosh	3.1075	3.1371	3.1669	3.1972	3.2277	3.2585	3.2897	3.3212	3.3530	3.3852
	tanh	0.9468	0.9478	0.9488	0.9498	0.9508	0.9518	0.9527	0.9536	0.9545	0.9554
	sinh	3.2682	3.3025	3.3372	3.3722	3.4075	3.4432	3.4792	3.5156	3.5523	3.5894
1.9	cosh	3.4177	3.4506	3.4838	3.5173	3.5512	3.5855	3.6201	3.6551	3.6904	3.7261
	tanh	0.9562	0.9571	0.9579	0.9587	0.9595	0.9603	0.9611	0.9619	0.9626	0.9633
	sinh	3.6269	3.6647	3.7028	3.7414	3.7803	3.8196	3.8593	3.8993	3.9398	3.9806
2.0	cosh	3.7622	3.7987	3.8355	3.8727	3.9103	3.9483	3.9867	4.0255	4.0647	4.1043
	tanh	0.9640	0.9647	0.9654	0.9661	0.9668	0.9674	0.9680	0.9686	0.9693	0.9699
	sinh	4.0219	4.0635	4.1056	4.1480	4.1909	4.2342	4.2779	4.3221	4.3666	4.4117
2.1	cosh	4.1443	4.1847	4.2256	4.2668	4.3085	4.3507	4.3932	4.4362	4.4797	4.5236
	tanh	0.9705	0.9710	0.9716	0.9722	0.9727	0.9732	0.9738	0.9743	0.9748	0.9752
	sinh	4.4571	4.5030	4.5494	4.5962	4.6434	4.6912	4.7394	4.7880	4.8372	4.8868
2.2	cosh	4.5679	4.6127	4.6580	4.7037	4.7499	4.7966	4.8437	4.8914	4.9395	4.9881
	tanh	0.9757	0.9762	0.9767	0.9771	0.9776	0.9780	0.9785	0.9789	0.9793	0.9797
	sinh	4.9370	4.9876	5.0387	5.0903	5.1425	5.1951	5.2483	5.3020	5.3562	5.4109
2.3	cosh	5.0372	5.0868	5.1370	5.1876	5.2388	5.2905	5.3427	5.3954	5.4487	5.5026
	tanh	0.9801	0.9805	0.9809	0.9812	0.9816	0.9820	0.9823	0.9827	0.9830	0.9834

TABLE XIII.—(*contd.*).

∠		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.4	sinh	5.4662	5.5221	5.5785	5.6354	5.6929	5.7510	5.8097	5.8689	5.9288	5.9892
	cosh	5.5569	5.6119	5.6674	5.7235	5.7801	5.8373	5.8951	5.9535	6.0125	6.0721
	tanh	0.9837	0.9840	0.9843	0.9846	0.9849	0.9852	0.9855	0.9858	0.9861	0.9864
2.5	sinh	6.0502	6.1118	6.1741	6.2369	6.3004	6.3645	6.4293	6.4946	6.5607	6.6274
	cosh	6.1323	6.1931	6.2545	6.3166	6.3793	6.4426	6.5066	6.5712	6.6365	6.7024
	tanh	0.9866	0.9869	0.9871	0.9874	0.9876	0.9879	0.9881	0.9884	0.9886	0.9888
2.6	sinh	6.6947	6.7628	6.8315	6.9009	6.9709	7.0417	7.1132	7.1854	7.2583	7.3319
	cosh	6.7690	6.8363	6.9043	6.9729	7.0423	7.1123	7.1831	7.2546	7.3268	7.3998
	tanh	0.9890	0.9892	0.9895	0.9897	0.9899	0.9901	0.9903	0.9905	0.9906	0.9908
2.7	sinh	7.4063	7.4814	7.5572	7.6338	7.7112	7.7894	7.8683	7.9480	8.0285	8.1098
	cosh	7.4735	7.5479	7.6231	7.6991	7.7758	7.8533	7.9316	8.0106	8.0905	8.1712
	tanh	0.9910	0.9912	0.9914	0.9916	0.9917	0.9919	0.9920	0.9922	0.9923	0.9925
2.8	sinh	8.1919	8.2749	8.3586	8.4432	8.5287	8.6150	8.7021	8.7902	8.8791	8.9689
	cosh	8.2527	8.3351	8.4182	8.5022	8.5871	8.6728	8.7594	8.8469	8.9352	9.0244
	tanh	0.9926	0.9928	0.9929	0.9931	0.9932	0.9933	0.9935	0.9936	0.9937	0.9938
2.9	sinh	9.0596	9.1512	9.2437	9.3371	9.4315	9.5268	9.6231	9.7203	9.8185	9.9177
	cosh	9.1146	9.2056	9.2976	9.3905	9.4844	9.5792	9.6749	9.7716	9.8693	9.9680
	tanh	0.9940	0.9941	0.9942	0.9943	0.9944	0.9945	0.9946	0.9948	0.9949	0.9950
3.0	sinh	10.018	10.119	10.221	10.324	10.429	10.534	10.640	10.748	10.856	10.966
	cosh	10.068	10.168	10.270	10.373	10.476	10.581	10.687	10.794	10.902	11.011
	tanh	0.9951	0.9952	0.9953	0.9953	0.9954	0.9955	0.9956	0.9957	0.9958	0.9959
3.1	sinh	11.076	11.188	11.301	11.415	11.530	11.647	11.764	11.883	12.003	12.124
	cosh	11.121	11.233	11.345	11.459	11.574	11.690	11.806	11.925	12.044	12.165
	tanh	0.9960	0.9960	0.9961	0.9962	0.9963	0.9963	0.9964	0.9965	0.9966	0.9966
3.2	sinh	12.246	12.369	12.494	12.620	12.747	12.876	13.006	13.137	13.269	13.403
	cosh	12.287	12.410	12.534	12.660	12.796	12.915	13.044	13.175	13.307	13.440
	tanh	0.9967	0.9968	0.9968	0.9969	0.9969	0.9970	0.9971	0.9971	0.9972	0.9972
3.3	sinh	13.538	13.674	13.812	13.951	14.092	14.234	14.377	14.522	14.668	14.816
	cosh	13.575	13.711	13.848	13.987	14.127	14.269	14.412	14.556	14.702	14.850
	tanh	0.9973	0.9973	0.9974	0.9974	0.9975	0.9975	0.9976	0.9976	0.9977	0.9977
3.4	sinh	14.965	15.116	15.268	15.422	15.577	15.734	15.893	16.053	16.215	16.378
	cosh	14.999	15.149	15.301	15.455	15.610	15.766	15.924	16.084	16.245	16.408
	tanh	0.9978	0.9978	0.9979	0.9979	0.9979	0.9980	0.9980	0.9981	0.9981	0.9981
3.5	sinh	16.543	16.709	16.877	17.047	17.219	17.392	17.567	17.744	17.923	18.103
	cosh	16.573	16.739	16.907	17.077	17.248	17.421	17.596	17.772	17.951	18.131
	tanh	0.9982	0.9982	0.9983	0.9983	0.9983	0.9984	0.9984	0.9984	0.9985	0.9985
3.6	sinh	18.285	18.470	18.655	18.843	19.033	19.244	19.418	19.613	19.811	20.010
	cosh	18.313	18.497	18.682	18.870	19.059	19.250	19.444	19.639	19.836	20.035
	tanh	0.9985	0.9985	0.9986	0.9986	0.9986	0.9987	0.9987	0.9987	0.9987	0.9988
3.7	sinh	20.211	20.415	20.620	20.828	21.037	21.249	21.463	21.679	21.897	22.117
	cosh	20.236	20.439	20.644	20.852	21.061	21.272	21.486	21.702	21.919	22.139
	tanh	0.9988	0.9988	0.9988	0.9989	0.9989	0.9989	0.9989	0.9989	0.9990	0.9990
3.8	sinh	22.339	22.564	22.791	23.020	23.252	23.486	23.722	23.961	24.202	24.445
	cosh	22.362	22.586	22.813	23.042	23.273	23.507	23.743	23.982	24.222	24.466
	tanh	0.9990	0.9990	0.9990	0.9991	0.9991	0.9991	0.9991	0.9991	0.9992	0.9992
3.9	sinh	24.691	24.939	25.190	25.444	25.700	25.958	26.219	26.483	26.749	27.018
	cosh	24.711	24.959	25.210	25.463	25.719	25.977	26.238	26.502	26.768	27.037
	tanh	0.9992	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993	0.9993	0.9993	0.9993
4.0	sinh	27.290	27.564	27.842	28.122	28.404	28.690	28.979	29.270	29.564	29.862
	cosh	27.308	27.583	27.860	28.139	28.422	28.707	28.996	29.287	29.581	29.878
	tanh	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994

TABLE XIII.—(*contd.*).

∠		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
4.1	sinh	30.162	30.465	30.772	31.081	31.393	31.709	32.028	32.350	32.675	33.004
	cosh	30.178	30.482	30.788	31.097	31.409	31.725	32.044	32.365	32.691	33.019
	tanh	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995
4.2	sinh	33.336	33.671	34.009	34.351	34.697	35.046	35.398	35.754	36.113	36.475
	cosh	33.351	33.686	34.024	34.366	34.711	35.060	35.412	35.768	36.127	36.490
	tanh	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996
4.3	sinh	36.843	37.214	37.588	37.965	38.347	38.733	39.122	39.515	39.913	40.314
	cosh	36.857	37.227	37.601	37.979	38.360	38.746	39.135	39.528	39.925	40.326
	tanh	0.9996	0.9996	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997
4.4	sinh	40.719	41.129	41.542	41.960	42.382	42.808	43.238	43.673	44.112	44.555
	cosh	40.732	41.141	41.554	41.972	42.393	42.819	43.250	43.684	44.123	44.566
	tanh	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
4.5	sinh	45.003	45.455	45.912	46.374	46.840	47.311	47.787	48.267	48.752	49.242
	cosh	45.014	45.466	45.923	46.385	46.851	47.321	47.797	48.277	48.762	49.252
	tanh	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
4.6	sinh	49.737	50.237	50.742	51.252	51.767	52.288	52.813	53.344	53.880	54.422
	cosh	49.747	50.247	50.752	51.262	51.777	52.297	52.823	53.354	53.890	54.431
	tanh	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
4.7	sinh	54.969	55.522	56.080	56.643	57.213	57.788	58.369	58.955	59.548	60.147
	cosh	54.978	55.531	56.089	56.652	57.221	57.796	58.377	58.964	59.556	60.155
	tanh	0.9998	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.8	sinh	60.751	61.362	61.979	62.601	63.231	63.866	64.508	65.157	65.812	66.473
	cosh	60.759	61.370	61.987	62.609	63.239	63.874	64.516	65.164	65.819	66.481
	tanh	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4.9	sinh	67.141	67.816	68.498	69.186	69.882	70.584	71.293	72.010	72.734	73.465
	cosh	67.149	67.823	68.505	69.193	69.889	70.591	71.300	72.017	72.741	73.472
	tanh	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
5.0	sinh	74.203	74.949	75.702	76.463	77.232	78.008	78.792	79.584	80.384	81.192
	cosh	74.210	74.956	75.709	76.470	77.238	78.014	78.798	79.590	80.390	81.198
	tanh	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
5.1	sinh	82.008	82.832	83.665	84.506	85.355	86.213	87.079	87.955	88.839	89.732
	cosh	82.014	82.838	83.671	84.512	85.361	86.219	87.085	87.960	88.844	89.737
	tanh	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
5.2	sinh	90.633	91.544	92.464	93.394	94.332	95.281	96.238	97.205	98.182	99.169
	cosh	90.639	91.550	92.470	93.399	94.338	95.286	96.243	97.211	98.188	99.174
	tanh	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000
5.3	sinh	100.17	101.17	102.19	103.22	104.25	105.30	106.36	107.43	108.51	109.60
	cosh	100.17	101.18	102.19	103.22	104.26	105.31	106.37	107.43	108.51	109.60
	tanh	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5.4	sinh	110.70	111.81	112.94	114.07	115.22	116.38	117.55	118.73	119.92	121.13
	cosh	110.71	111.82	112.94	114.08	115.22	116.38	117.55	118.73	119.93	121.13
	tanh	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5.5	sinh	122.34	123.57	124.82	126.07	127.34	128.62	129.91	131.22	132.53	133.87
	cosh	122.35	123.58	124.82	126.07	127.34	128.62	129.91	131.22	132.54	133.87
	tanh	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5.6	sinh	135.21	136.57	137.94	139.33	140.73	142.14	143.57	145.02	146.47	147.95
	cosh	135.22	136.57	137.95	139.33	140.73	142.15	143.58	145.02	146.48	147.95
	tanh	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5.7	sinh	149.43	150.93	152.45	153.98	155.53	157.09	158.67	160.27	161.88	163.51
	cosh	149.44	150.94	152.45	153.99	155.53	157.10	158.68	160.27	161.88	163.51
	tanh	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5.8	sinh	165.15	166.81	168.48	170.18	171.89	173.62	175.36	177.12	178.90	180.70
	cosh	165.15	166.81	168.49	170.18	171.89	173.62	175.36	177.13	178.91	180.70
	tanh	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5.9	sinh	182.52	184.35	186.20	188.08	189.97	191.88	193.80	195.75	197.72	199.71
	cosh	182.52	184.35	186.21	188.08	189.97	191.88	193.81	195.75	197.72	199.71
	tanh	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

If $y = x^n$, $\frac{dy}{dx} = nx^{n-1}$.

,, $y = \sin\theta$, $\frac{dy}{dx} = \cos\theta$.

,, $y = \cos\theta$, $\frac{dy}{dx} = -\sin\theta$.

,, $y = \tan\theta$, $\frac{dy}{dx} = \sec^2\theta$.

,, $y = \cot\theta$, $\frac{dy}{dx} = -\operatorname{cosec}^2\theta$

,, $y = \sec\theta$, $\frac{dy}{dx} = \tan\theta\ \sec\theta = \frac{\sin\theta}{\cos^2\theta}$

,, $y = \operatorname{cosec}\theta$, $\frac{dy}{dx} = -\cot\theta\ \operatorname{cosec}\theta = -\frac{\cos\theta}{\sin^2\theta}$

,, $y = \sin^{-1}\frac{x}{a}$, $\frac{dy}{dx} = \frac{1}{\sqrt{a^2-x^2}}$

,, $y = \cos^{-1}\frac{x}{a}$, $\frac{dy}{dx} = -\frac{1}{\sqrt{a^2-x^2}}$

,, $y = \tan^{-1}\frac{x}{a}$, $\frac{dy}{dx} = \frac{a}{a^2+x^2}$

,, $y = \cot^{-1}\frac{x}{a}$, $\frac{dy}{dx} = -\frac{a}{a^2+x^2}$

,, $y = \sec^{-1}\frac{x}{a}$, $\frac{dy}{dx} = \frac{a}{x\sqrt{x^2-a^2}}$

,, $y = \operatorname{cosec}^{-1}\frac{x}{a}$. $\frac{dy}{dx} = -\frac{a}{x\sqrt{x^2-a^2}}$

,, $y = e^x$, $\frac{dy}{dx} = e^x$

,, $y = e^{ax}$, $\frac{dy}{dx} = ae^{ax}$

,, $y = a^x$, $\frac{dy}{dx} = a^x \log_e a$

,, $y = \log x$, $\frac{dy}{dx} = \frac{1}{x}$

List of Integrals.

(Logarithms are to the base e unless otherwise stated).

Rational Algebraic Integrals

1 $\int x^m dx = \frac{x^{m+1}}{m+1}$ when $m \neq -1$

2 $\int \frac{dx}{x} = \log x$

3 $\int (ax+b)^m dx = \frac{(ax+b)^{m+1}}{a(m+1)}$ when $m \neq -1$

4 $\int \frac{dx}{ax+b} = \frac{1}{a}\log(ax+b)$

5 $\int \frac{x\,dx}{ax+b} = \frac{1}{a^2}\left(ax+b-b\log(ax+b)\right)$

6 $\int \frac{x\,dx}{(ax+b)^2} = \frac{1}{a^2}\left(\frac{b}{ax+b}+\log(ax+b)\right)$

7 $\int \frac{x^2dx}{ax+b} = \frac{1}{a^3}\left(\frac{(ax+b)^2}{2} - 2b(ax+b) + b^2\log(ax+b)\right)$

8 $$\int \frac{x^2dx}{(ax+b)^2} = \frac{1}{a^3}\left(ax + b - \frac{b^2}{ax+b} - 2b\log(ax+b)\right)$$

9 $$\int \frac{dx}{x(ax+b)} = \frac{1}{b}\log\frac{x}{ax+b}$$

10 $$\int \frac{dx}{x(ax+b)^2} = \frac{1}{b(ax+b)} + \frac{1}{b^2}\log\frac{x}{ax+b}$$

11 $$\int \frac{dx}{x^2(ax+b)} = -\frac{1}{bx} + \frac{a}{b^2}\log\frac{ax+b}{x}$$

12 $$\int \frac{dx}{x^2(ax+b)^2} = -\frac{2ax+b}{b^2x(ax+b)} + \frac{2a}{b^3}\log\frac{ax+b}{x}$$

13 $$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

14 $$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\frac{x-a}{x+a} = -\frac{1}{a}\tanh^{-1}\frac{a}{x}$$

$\int \frac{dx}{ax^2+b}$ reduces to 16 or 17 by taking the factor $\frac{1}{a}$ outside the integral sign.

15 $$\int \frac{dx}{(ax^2+b)^m} = \frac{x}{2(m-1)\,b\,(ax^2+b)^{m-1}} + \frac{2m-3}{2(m-1)b}\int\frac{dx}{(ax^2+b)^{m-1}} \text{ when } m \neq 1$$

16 $$\int \frac{xdx}{(ax^2+b)^m} = -\frac{1}{2(m-1)\,a\,(ax^2+b)^{m-1}} \text{ when } m \neq 1$$

17 $$\int \frac{xdx}{ax^2+b} = \frac{1}{2a}\log(ax^2+b)$$

18 $$\int \frac{x^2dx}{ax^2+b} = \frac{x}{a} - \frac{b}{a}\int\frac{dx}{ax^2+b}$$

19 $$\int \frac{x^2dx}{(ax^2+b)^m} = -\frac{x}{2(m-1)\,a\,(ax^2+b)^{m-1}} + \frac{1}{2(m-1)\,a}\int\frac{dx}{(ax^2+b)^{m-1}} \text{ when } m \neq 1$$

20 $$\int \frac{dx}{ax^3+b} = \frac{k}{3b}\left(\sqrt{3}\tan^{-1}\frac{2x-k}{k\sqrt{3}} + \log\frac{k+x}{\sqrt{k^2-kx+x^2}}\right)$$

21 $$\int \frac{xdx}{ax^3+b} = \frac{1}{3ak}\left(\sqrt{3}\tan^{-1}\frac{2x-k}{k\sqrt{3}} - \log\frac{k+x}{\sqrt{k^2-kx+x^2}}\right)$$

where $k = \sqrt[3]{\frac{b}{a}}$ (for 20 and 21)

22 $$\int \frac{dx}{x(ax^n+b)} = \frac{1}{bn}\log\frac{x^n}{ax^n+b}$$

Let $X = ax^2 + bx + c$ and $q = b^2 - 4ac$

23 $$\int \frac{dx}{X} = \frac{1}{\sqrt{q}}\log\frac{2ax+b-\sqrt{q}}{2ax+b+\sqrt{q}} \text{ when } q > 0$$

24 $$\int \frac{dx}{X} = \frac{2}{\sqrt{-q}}\tan^{-1}\frac{2ax+b}{\sqrt{-q}} \text{ when } q < 0$$

For the case $q = 0$, use formula 9 with $m = -2$

25 $$\int \frac{dx}{X^n} = -\frac{2ax+b}{(n-1)qX^{n-1}} - \frac{2(2n-3)a}{q(n-1)}\int \frac{dx}{X^{n-1}} \quad \text{when } n \neq 1$$

26 $$\int \frac{x\,dx}{X} = \frac{1}{2a}\log X - \frac{b}{2a}\int \frac{dx}{X}$$

27 $$\int \frac{(mx+n)\,dx}{X} = \frac{m}{2a}\log X + \frac{2an-bm}{2a}\int \frac{dx}{X}$$

28 $$\int \frac{x^2\,dx}{X} = \frac{x}{a} - \frac{b}{2a^2}\log X + \frac{b^2-2ac}{2a^2}\int \frac{dx}{X}$$

Integrals Involving $\sqrt{ax+b}$

$$\int \sqrt{ax+b}\,dx$$

$$\int \frac{dx}{\sqrt{ax+b}}$$

$$\int (ax+b)^n \sqrt{ax+b}\,dx$$

$$\int \frac{dx}{(ax+b)^n\sqrt{ax+b}}$$

These may all be integrated by formula 9

29 $$\int x\sqrt{ax+b}\,dx = \frac{2(3ax-2b)\sqrt{(ax+b)^3}}{15a^2}$$

30 $$\int x^2\sqrt{ax+b}\,dx = \frac{2(15a^2x^2-12abx+8b^2)\sqrt{(ax+b)^3}}{105a^3}$$

31 $$\int x^m\sqrt{ax+b}\,dx = \frac{2}{a(2m+3)}\left(x^m\sqrt{(ax+b)^3} - mb\int x^{m-1}\sqrt{ax+b}\,dx\right)$$

32 $$\int \frac{\sqrt{ax+b}\,dx}{x} = 2\sqrt{ax+b} + \sqrt{b}\log\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}} \quad \text{when } b>0$$

33 $$\int \frac{\sqrt{ax+b}\,dx}{x} = 2\sqrt{ax+b} - 2\sqrt{-b}\tan^{-1}\sqrt{\frac{ax+b}{-b}} \quad \text{when } b<0$$

For the case $b = 0$, use formula 4

34 $$\int \frac{\sqrt{ax+b}\,dx}{x^m} = -\frac{1}{(m-1)b}\left(\frac{\sqrt{(ax+b)^3}}{x^{m-1}} + \frac{(2m-5)a}{2}\int \frac{\sqrt{ax+b}\,dx}{x^{m-1}}\right) \quad \text{when } m \neq 1$$

35 $$\int \frac{x\,dx}{\sqrt{ax+b}} = \frac{2(ax-2b)}{3a^2}\sqrt{ax+b}$$

36 $$\int \frac{x^2\,dx}{\sqrt{ax+b}} = \frac{2(3a^2x^2-4abx+8b^2)}{15a^3}\sqrt{ax+b}$$

37 $$\int \frac{x^m\,dx}{\sqrt{ax+b}} = \frac{2}{a(2m+1)}\left(x^m\sqrt{ax+b} - mb\int \frac{x^{m-1}dx}{\sqrt{ax+b}}\right) \quad \text{when } m \neq \tfrac{1}{2}$$

38 $$\int \frac{dx}{x\sqrt{ax+b}} = \frac{1}{\sqrt{b}} \log \frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}} \text{ when } b > 0$$

39 $$\int \frac{dx}{x\sqrt{ax+b}} = \frac{2}{\sqrt{-b}} \tan^{-1} \sqrt{\frac{ax+b}{-b}} \text{ when } b < 0$$

For the case $b = 0$, use formula 4

40 $$\int \frac{dx}{x^m\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{(m-1)bx^{m-1}} - \frac{(2m-3)a}{(2m-2)b} \int \frac{dx}{x^{m-1}\sqrt{ax+b}} \text{ when } m \neq 1$$

Integrals Involving $\sqrt{x^2 \pm a^2}$ and $\sqrt{a^2 - x^2}$

(These are special cases of the more general integrals given in the next section.)

41 $$\int \sqrt{x^2 \pm a^2}\,dx = \tfrac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \log (x + \sqrt{x^2 \pm a^2})\right]^*$$

42 $$\int \sqrt{a^2 - x^2}\,dx = \tfrac{1}{2}\left(x\sqrt{a^2 - x^2} + a^2 \sin^{-1} \frac{x}{a}\right)$$

43 $$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log (x + \sqrt{x^2 \pm a^2})^*$$

44 $$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a}$$

45 $$\int x\sqrt{x^2 \pm a^2}\,dx = \tfrac{1}{3} \sqrt{(x^2 \pm a^2)^3}$$

46 $$\int x^2\sqrt{x^2 \pm a^2}\,dx = \frac{x}{4} \sqrt{(x^2 \pm a^2)^3} \mp \frac{a^2}{8} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \log (x + \sqrt{x^2 \pm a^2})\right]^*$$

47 $$\int x\sqrt{a^2 - x^2}\,dx = -\tfrac{1}{3} \sqrt{(a^2 - x^2)^3}$$

48 $$\int x^2\sqrt{a^2 - x^2}\,dx = -\frac{x}{4} \sqrt{(a^2 - x^2)^3} + \frac{a^2}{8} \left(x\sqrt{a^2 - x^2} + a^2 \sin^{-1} \frac{x}{a}\right)$$

49 $$\int \frac{\sqrt{a^2 \pm x^2}\,dx}{x} = \sqrt{a^2 \pm x^2} - a \log \frac{a + \sqrt{a^2 \pm x^2}}{x}^*$$

50 $$\int \frac{\sqrt{x^2 - a^2}\,dx}{x} = \sqrt{x^2 - a^2} - a \cos^{-1} \frac{a}{x}$$

51 $$\int \frac{\sqrt{x^2 \pm a^2}\,dx}{x^2} = -\frac{\sqrt{x^2 \pm a^2}}{x} + \log (x + \sqrt{x^2 \pm a^2})^*$$

*In these formulas we may replace

$$\log (x + \sqrt{x^2 + a^2}) \text{ by } \sinh^{-1} \frac{x}{a}$$

$$\log (x + \sqrt{x^2 - a^2}) \text{ by } \cosh^{-1} \frac{x}{a}$$

$$\log \frac{a + \sqrt{a^2 + x^2}}{x} \text{ by } \sinh^{-1} \frac{a}{x}$$

$$\log \frac{a + \sqrt{a^2 - x^2}}{x} \text{ by } \cosh^{-1} \frac{a}{x}$$

52 $$\int \frac{\sqrt{a^2 - x^2}}{x^2}\,dx = -\frac{\sqrt{a^2 - x^2}}{x} - \sin^{-1}\frac{x}{a}$$

53 $$\int \frac{x\,dx}{\sqrt{a^2 - x^2}} = -\sqrt{a^2 - x^2}$$

54 $$\int \frac{x\,dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$$

55 $$\int \frac{x^2\,dx}{\sqrt{x^2 \pm a^2}} = \frac{x}{2}\sqrt{x^2 \pm a^2} \mp \frac{a^2}{2}\log\left(x + \sqrt{x^2 \pm a^2}\right)^*$$

56 $$\int \frac{x^2\,dx}{\sqrt{a^2 - x^2}} = -\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$$

57 $$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\cos^{-1}\frac{a}{x}$$

58 $$\int \frac{dx}{x\sqrt{a^2 \pm x^2}} = -\frac{1}{a}\log\left(\frac{a + \sqrt{a^2 \pm x^2}}{x}\right)^*$$

59 $$\int \frac{dx}{x^2\sqrt{x^2 \pm a^2}} = \pm\frac{\sqrt{x^2 \pm a^2}}{a^2 x}$$

60 $$\int \frac{dx}{x^2\sqrt{a^2 - x^2}} = -\frac{\sqrt{a^2 - x^2}}{a^2 x}$$

61 $$\int \sqrt{(x^2 \pm a^2)^3}\,dx = \tfrac{1}{4}\left[x\sqrt{(x^2 \pm a^2)^3} \pm \frac{3a^2x}{2}\sqrt{x^2 \pm a^2} + \frac{3a^4}{2}\log\left(x + \sqrt{x^2 \pm a^2}\right)\right]^*$$

62 $$\int \sqrt{(a^2 - x^2)^3}\,dx = \tfrac{1}{4}\left[x\sqrt{(a^2 - x^2)^3} + \frac{3a^2x}{2}\sqrt{a^2 - x^2} + \frac{3a^4}{2}\sin^{-1}\frac{x}{a}\right]$$

63 $$\int \frac{dx}{\sqrt{(x^2 \pm a^2)^3}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}}$$

64 $$\int \frac{dx}{\sqrt{(a^2 - x^2)^3}} = \frac{x}{a^2\sqrt{a^2 - x^2}}$$

Integrals involving $\sqrt{ax^2 + bx + c}$

Let $X = ax^2 + bx + c$ and $q = b^2 - 4ac$

65 $$\int \frac{dx}{\sqrt{X}} = \frac{1}{\sqrt{a}}\log\left(\sqrt{X} + \frac{2ax + b}{2\sqrt{a}}\right) \text{ when } a > 0$$

66 $$\int \frac{dx}{\sqrt{X}} = \frac{1}{\sqrt{-a}}\sin^{-1}\frac{(-2ax - b)}{\sqrt{q}} \text{ when } a < 0$$

67 $$\int \frac{x\,dx}{\sqrt{X}} = \frac{\sqrt{X}}{a} - \frac{b}{2a}\int \frac{dx}{\sqrt{X}}$$

68 $$\int \frac{(mx + n)\,dx}{\sqrt{X}} = \frac{m\sqrt{X}}{a} + \frac{2an - bm}{2a}\int \frac{dx}{\sqrt{X}}$$

69 $$\int \frac{x^2\,dx}{\sqrt{X}} = \frac{(2ax - 3b)\sqrt{X}}{4a^2} + \frac{3b^2 - 4ac}{8a^2}\int \frac{dx}{\sqrt{X}}$$

70 $$\int \frac{dx}{x\sqrt{X}} = -\frac{1}{\sqrt{c}}\log\left(\frac{\sqrt{X} + \sqrt{c}}{x} + \frac{b}{2\sqrt{c}}\right) \text{ when } c > 0$$

* See note on previous page.

71 $\int \frac{dx}{X\sqrt{x}} = \frac{1}{\sqrt{-c}} \sin^{-1} \frac{bx + 2c}{x\sqrt{q}}$ when $c < 0$

72 $\int \frac{dx}{x\sqrt{X}} = -\frac{2\sqrt{X}}{bx}$ when $c = 0$

73 $\int \frac{dx}{(mx+n)\sqrt{X}} = \frac{1}{\sqrt{k}} \log \left[\frac{\sqrt{k} - m\sqrt{X}}{mx + n} + \frac{bm - 2an}{2\sqrt{k}} \right]$ when $k > 0$

74 $\int \frac{dx}{(mx+n)\sqrt{X}} = \frac{1}{\sqrt{-k}} \sin^{-1} \left[\frac{(bm - 2an)(mx + n) + 2k}{m(mx + n)\sqrt{q}} \right]$ when $k < 0$

75 $\int \frac{dx}{(mx+n)\sqrt{X}} = -\frac{2m\sqrt{X}}{(bm - 2an)(mx + n)}$ when $k = 0$

where $k = an^2 - bmn + cm^2$

76 $\int \frac{dx}{x^2\sqrt{X}} = -\frac{\sqrt{X}}{cx} - \frac{b}{2c} \int \frac{dx}{x\sqrt{X}}$

77 $\int \sqrt{X}\, dx = \frac{(2ax + b)\sqrt{X}}{4a} - \frac{q}{8a} \int \frac{dx}{\sqrt{X}}$

78 $\int x\sqrt{X}\, dx = \frac{X\sqrt{X}}{3a} - \frac{b(2ax + b)\sqrt{X}}{8a^2} + \frac{bq}{16a^2} \int \frac{dx}{\sqrt{X}}$

79 $\int x^2\sqrt{X}\, dx = \frac{(6ax - 5b)X\sqrt{X}}{24a^2} + \frac{(5b^2 - 4ac)(2ax + b)\sqrt{X}}{64a^3} - \frac{(5b^2 - 4ac)q}{128a^3} \int \frac{dx}{\sqrt{X}}$

80 $\int \frac{\sqrt{X}\, dx}{x} = \sqrt{X} + \frac{b}{2} \int \frac{dx}{\sqrt{X}} + c \int \frac{dx}{x\sqrt{X}}$

81 $\int \frac{\sqrt{X}\, dx}{mx + n} = \frac{\sqrt{X}}{m} + \frac{bm - 2an}{2m^2} \int \frac{dx}{\sqrt{X}} + \frac{an^2 - bmn + cm^2}{m^2} \int \frac{dx}{(mx + n)\sqrt{X}}$

82 $\int \frac{\sqrt{X}\, dx}{x^2} = -\frac{\sqrt{X}}{x} + \frac{b}{2} \int \frac{dx}{x\sqrt{X}} + a \int \frac{dx}{\sqrt{X}}$

83 $\int \frac{dx}{X\sqrt{X}} = -\frac{2(ax + b)}{q\sqrt{X}}$

84 $\int X\sqrt{X}\, dx = \frac{2(2ax + b)X\sqrt{X}}{8a} - \frac{3q(2ax + b)\sqrt{X}}{64a^2} + \frac{3q^2}{128a^2} \int \frac{dx}{\sqrt{X}}$

Miscellaneous Irrational Integrals

85 $\int \sqrt{2ax - x^2}\, dx = \frac{x - a}{2} \sqrt{2ax - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x - a}{a}$

86 $\int \frac{dx}{\sqrt{2ax - x^2}} = \cos^{-1} \frac{a - x}{a}$

87 $$\int \sqrt{\frac{mx+n}{ax+b}}\,dx = \int \frac{(mx+n)dx}{\sqrt{amx^2+(bm+an)x+bn}}$$

then use formula 68

Logarithmic Integrals

88 $$\int \log_a x\,dx = x \log_a \frac{x}{a}$$

89 $$\int \log x\,dx = x(\log x - 1)$$

90 $$\int x^m \log_a x\,dx = x^{m+1}\left(\frac{\log_a x}{m+1} - \frac{\log_a e}{(m+1)^2}\right)$$

91 $$\int x^m \log x\,dx = x^{m+1}\left(\frac{\log x}{m+1} - \frac{1}{(m+1)^2}\right)$$

Exponential Integral

92 $$\int a^x dx = \frac{a^x}{\log a}$$

93 $$\int e^x dx = e^x$$

94 $$\int xe^x dx = e^x(x-1)$$

95 $$\int x^m e^x dx = x^m e^x - m\int x^{m-1}e^x dx$$

Trigonometric Integrals

N.B.—In these formulas m and n are *positive integers* unless otherwise indicated.

96 $$\int \sin x\,dx = -\cos x$$

97 $$\int \sin^2 x\,dx = \tfrac{1}{2}(x - \sin x \cos x)$$

98 $$\int \sin^n x\,dx = \int (1-\cos^2 x)^{\frac{n-1}{2}} \sin x\,dx, \text{ when } n \text{ is odd}$$

Then expand $(1-\cos^2 x)^{\frac{n-1}{2}}$ and use formula 109

99 $$\int \sin^n x\,dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$

when n is even

100 $$\int \frac{dx}{\sin^n x} = -\frac{\cos x}{(n-1)\sin^{n-1} x} + \frac{n-2}{n-1}\int \frac{dx}{\sin^{n-2} x}$$

when n is odd, $\neq 1$

101 $$\int \frac{dx}{\sin^n x} = \int \csc^n x\,dx \text{ when } n \text{ is even.}$$

Then use formula 128

102 $$\int \cos x\,dx = \sin x$$

103 $$\int \cos^2 x\,dx = \tfrac{1}{2}(x + \sin x \cos x)$$

104 $\int \cos^n x\,dx = \int (1 - \sin^2 x)^{\frac{n-1}{2}} \cos x\,dx$, when n is odd.

Then expand $(1 - \sin^2 x)^{\frac{n-1}{2}}$ and use formula 109

105 $\int \cos^n x\,dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x\,dx$ when n is even

106 $\int \frac{dx}{\cos^n x} = \frac{\sin x}{(n-1)\cos^{n-1} x} + \frac{n-2}{n-1} \int \frac{dx}{\cos^{n-2} x}$ when n is odd, $\neq 1$

107 $\int \frac{dx}{\cos^n x} = \int \sec^n x\,dx$ when n is even. Then use formula 124

108 $\int \sin^n x \cos x\,dx = \frac{\sin^{n+1} x}{n+1}$

109 $\int \cos^n x \sin x\,dx = -\frac{\cos^{n+1} x}{n+1}$

(108, 109) n is any constant -1

110 $\int \sin^2 x \cos^2 x\,dx = \frac{4x - \sin 4x}{32}$

111 $\int \frac{dx}{\sin x \cos x} = \log \tan x$

$\int \sin^m x \cos^n x\,dx$, $\int \frac{\sin^m x\,dx}{\cos^n x}$, $\int \frac{\cos^n x\,dx}{\sin^m x}$ and $\int \frac{dx}{\sin^m x \cos^n x}$ may be reduced to integrals given above by the use of the following reduction formulas, in which r and s are any integers *positive* or *negative*.

112 $\int \sin^r x \cos^s x\,dx = \frac{\cos^{s-1} x \sin^{r+1} x}{r+s} + \frac{s-1}{r+s} \int \sin^r x \cos^{s-2} x\,dx$ when $r + s \neq 0$

113 $\int \sin^r x \cos^s x\,dx = -\frac{\sin^{r-1} x \cos^{s+1} x}{r+s} + \frac{r-1}{r+s} \int \sin^{r-2} x \cos^s x\,dx$ when $r + s \neq 0$

114 $\int \sin^r x \cos^s x\,dx = \frac{\sin^{r+1} x \cos^{s+1} x}{r+1} + \frac{s+r+2}{r+1} \int \sin^{r+2} x \cos^s x\,dx$ when $r \neq -1$

115 $\int \sin^r x \cos^s x\,dx = -\frac{\sin^{r+1} x \cos^{s+1} x}{s+1} + \frac{s+r+2}{s+1} \int \sin^r x \cos^{s+2} x\,dx$ when $s \neq -1$

116 $\int \tan x\,dx = -\log \cos x$

117 $\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \frac{\tan^{n-3} x}{n-3} + \frac{\tan^{n-5} x}{n-5}$ .... $\pm \tan x \mp x$ when n is even

118 $\int \tan^n x\,dx = \int (\sec^2 x - 1)^{\frac{n-1}{2}} \tan x\,dx$ when n is odd.

Then expand $(\sec x^2 - 1)^{\frac{n-1}{2}}$ and use formula

119 $\int \cot x\,dx = \log \sin x$

120 $\int \cot^n x\,dx = -\frac{\cot^{n-1} x}{n-1} + \frac{\cot^{n-3} x}{n-3} - \frac{\cot^{n-5} x}{n-5}$

$\ldots\ldots\ldots \pm \cot x \pm x$ when n is even

121 $\int \cot^n x\,dx = \int (\csc^2 x - 1)^{\frac{n-1}{2}} \cot x\,dx$ when n is odd.

Then expand $(\csc x^2 - 1)^{\frac{n-1}{2}}$ and use formula 131

122 $\int \sec x\,dx = \log(\sec x + \tan x)$

123 $\int \sec^2 x\,dx = \tan x$

124 $\int \sec^n x\,dx = \int (\tan^2 x + 1)^{\frac{n-2}{2}} \sec^2 x\,dx$ when n is even

Then expand $(\tan^2 x + 1)^{\frac{n-2}{2}}$ and use formula 132

125 $\int \sec^n x\,dx = \int \frac{dx}{\cos^n x}$ when n is odd, and use formula 106

126 $\int \operatorname{cosec}^2 x\,dx = -\cot x$

127 $\int \operatorname{cosec} x\,dx = \log(\operatorname{cosec} x - \cot x)$

128 $\int \operatorname{cosec}^n x\,dx = \int (\cot^2 x + 1)^{\frac{n-2}{2}} \operatorname{cosec}^2 x\,dx$ when n is even.

Then expand $(\cot^2 x + 1)^{\frac{n-2}{2}}$ and use formula 130

129 $\int \operatorname{cosec}^n x\,dx = \int \frac{dx}{\sin^n x}$ when n is odd, and use formula 100

130 $\int \sec^n x \tan x\,dx = \frac{\sec^n x}{n}$

131 $\int \operatorname{cosec}^n x \cot x\,dx = -\frac{\operatorname{cosec}^n x}{n}$

(130–131) where n is *any constant* $\neq 0$

132 $\int \tan^n x \sec^2 x\,dx = \frac{\tan^{n+1} x}{n+1}$

133 $\int \cot^n x \operatorname{cosec}^2 x\,dx = -\frac{\cot^{n+1} x}{n+1}$

(132–133) where n is *any constant* $\neq -1$

134 $\int \frac{dx}{a + b \sin x} = \frac{-1}{\sqrt{a^2 - b^2}} \sin^{-1} \frac{b + a \sin x}{a + b \sin x}$ when $a^2 > b^2$

135 $\int \frac{dx}{a + b \sin x} = \frac{+1}{\sqrt{b^2 - a^2}} \log \frac{b + a \sin x - \sqrt{b^2 - a^2}\,(\cos x)}{a + b \sin x}$

136 $$\int \frac{dx}{a+b\cos x} = -\frac{1}{\sqrt{a^2-b^2}}\sin^{-1}\left[\frac{b+a\cos x}{a+b\cos x}\right] \text{ when } a>b>0$$

$$\text{or } \frac{1}{\sqrt{a^2-b^2}}\sin^{-1}\left[\frac{\sqrt{a^2-b^2}\,\sin x}{a+b\cos x}\right] \quad a>b>0.$$

$$\text{or } \frac{1}{\sqrt{a^2-b^2}}\tan^{-1}\left[\frac{\sqrt{a^2-b^2}\,\sin x}{b+a\cos x}\right] \quad a>b>0.$$

137 $$\int \frac{dx}{a+b\cos x} = \frac{1}{\sqrt{b^2-a^2}}\log\left[\frac{b+a\cos x+\sqrt{b^2-a^2}\,\sin x}{a+b\cos x}\right] \text{ when } b^2>a^2,\ a<0$$

138 $$\int \sqrt{1-\cos x}\,dx = -2\sqrt{2}\cos\frac{x}{2}$$

139 $$\int \sqrt{(1-\cos x)^3}\,dx = \frac{4\sqrt{2}}{3}\left(\cos^3\frac{x}{2} - 3\cos\frac{x}{2}\right)$$

140 $$\int x\sin x\,dx = \sin x - x\cos x$$

141 $$\int x^2\sin x\,dx = 2x\sin x + (2-x^2)\cos x$$

142 $$\int x\cos x\,dx = \cos x + x\sin x$$

143 $$\int x^2\cos x\,dx = 2x\cos x + (x^2-2)\sin x$$

Inverse Trigonometric Integrals

144 $$\int \sin^{-1}x\,dx = x\sin^{-1}x + \sqrt{1-x^2}$$

145 $$\int \cos^{-1}x\,dx = x\cos^{-1}x - \sqrt{1-x^2}$$

146 $$\int \tan^{-1}x\,dx = x\tan^{-1}x - \log\sqrt{1+x^2}$$

147 $$\int \cot^{-1}x\,dx = x\cot^{-1}x + \log\sqrt{1+x^2}$$

148 $$\int \sec^{-1}x\,dx = x\sec^{-1}x - \log\left(x+\sqrt{x^2-1}\right)$$

$$= x\sec^{-1}x - \cosh^{-1}x$$

149 $$\int \operatorname{cosec}^{-1}x\,dx = x\operatorname{cosec}^{-1}x + \log\left(x+\sqrt{x^2-1}\right)$$

$$= x\operatorname{cosec}^{-1}x + \cosh^{-1}x$$

Hyperbolic Integrals

150 $$\int \sinh x\,dx = \cosh x$$

151 $$\int \cosh x\,dx = \sinh x$$

152. $\int \operatorname{sech}^2 x\, dx = \tanh x.$

153. $\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1}\frac{x}{a} = \log\left\{x + \sqrt{x^2 + a^2}\right\}$

154. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\frac{x}{a} = \log\left\{x + \sqrt{x^2 - a^2}\right\}$

155. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \frac{1}{a}\tanh^{-1}\frac{x}{a} = \frac{1}{2a}\log\frac{a+x}{a-x}$

Quadratic Equation.

If $ax^2 + bx + c = o$

Then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

To find the sum of any number of terms in an arithmetical progression.

Let a be the first term, n the number of terms, l the last term, S the sum, d the common difference.

$$S = \frac{n}{2}(a + l) = \frac{n}{2}\left[2\, a + (n - 1)\, d\right]$$

To find the sum of any number of terms in a geometrial progression.

Let r be the common ratio.

$$S = a\,\frac{r^n - 1}{r - 1}$$

Combinations and Permutations.

The combinations of C of n things r at a time $= {}_nC_r$

$${}_nC_r = \frac{\underline{|n}}{\underline{|r}\;\underline{|n-r}} = {}_nC_{n-r}$$

The permutations P of n things r at a time $= {}_nP_r$

$${}_nP_n = n\,(n-1)\,(n-2) - - - 3\,.\,2\,.\,1 = \underline{|n}$$

$${}_nP_r = n\,(n-1)\,(n-2) - - - (n - r + 1) = n_r$$

$${}_nP_r = {}_nC_r \times \underline{|r}$$

Binomial Theorem.

$$(1 \pm x)^n = 1 \pm nx + \frac{n\,(n-1)}{1\,.\,2}x^2 \pm \frac{n\,(n-1)\,(n-2)}{1\,.\,2\,.\,3}x^3 + - - -$$

Maclaurin's Theorem.

$$f\,(x) = f\,(0) + xf'\,(0) + \frac{x^2}{1\,.\,2}f''\,(0) + - - -$$

Taylor's Theorem.

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{1 \cdot 2} f''(x) + \cdots$$

Values of π.

$\pi = 3{\cdot}14159265.$

$\frac{\pi}{4} = 0{\cdot}78539816.$

$\frac{\pi}{180} = 0{\cdot}01745329.$

$\pi^2 = 9{\cdot}86960440.$

$\pi^3 = 31{\cdot}00627668.$

$\frac{1}{\pi} = 0{\cdot}31830989.$

$\frac{1}{\pi^2} = 0{\cdot}10132118.$

$\frac{1}{\pi^3} = 0{\cdot}03225153.$

$\sqrt{\pi} = 1{\cdot}77245385.$

$\sqrt[3]{\pi} = 1{\cdot}46459189.$

$\frac{1}{\sqrt{\pi}} = 0{\cdot}56418958.$

$\frac{1}{\sqrt[3]{\pi}} = 0{\cdot}68278406.$

$\log \pi = 0{\cdot}49714987.$

TABLE XIV.

VALUES OF g AT SEA LEVEL AND GIVEN LATITUDES.

System.	L = 0°	L = 10°	L = 20°	L = 30°	L = 40°	L = 50°	L = 60°
British ..	32·088	32·095	32·108	32·130	32·159	32·197	32·215
C.G.S. ..	978·03	978·24	978·63	979·32	980·18	981·06	981·91

Subtract 0·01 from g for approximately every 33 metres of height above sea level, and add if below sea level.

g at London = 981·17 cm. per sec. per sec.
= 32·1912 feet per sec. per sec.

Conversion of Thermometric Scales.

Temperature Fahrenheit = $\frac{9}{5}$ (temp. Cent.) + 32.
" Centigrade = $\frac{5}{9}$ (temp. Fahr.— 32)

Alternately.

To convert temp. Fahr. to Cent.,— Add 40, multiply by $\frac{5}{9}$, and subtract 40.

" " " Cent. to Fahr.,— Add 40, multiply by $\frac{9}{5}$, and subtract 40.

Absolute zero = — 273·1 deg. C. = — 491·6 deg. F.

T_{abs} = 273·1 + deg. C.

T_{abs} = 459·6 + deg. F.

TABLE XV.

Specific Gravity corresponding to the Degrees of the Baume Hydrometer, at 15° C.

(For specific gravities less than 1.)

Degrees Baumé.	*Sp. Gr.*	*Degrees Baumé.*	*Sp. Gr.*	*Degrees Baumé.*	*Sp. Gr.*
10	1·000	28	0·889	46	0·800
11	0·993	29	0·883	47	0·796
12	0·986	30	0·878	48	0·791
13	0·979	31	0·873	49	0·787
14	0·973	32	0·867	50	0·783
15	0·967	33	0·862	51	0·778
16	0·960	34	0·857	52	0·774
17	0·953	35	0·852	53	0·770
18	0·947	36	0·847	54	0·766
19	0·941	37	0·842	55	0·762
20	0·935	38	0·837	56	0·758
21	0·929	39	0·832	57	0·754
22	0·923	40	0·827	58	0·750
23	0·917	41	0·823	59	0·746
24	0·911	42	0·818	60	0·742
25	0·905	43	0·814	61	0·738
26	0·900	44	0·809	62	0·735
27	0·894	45	0·804		

Note.—The following formula enables degrees Baumé to be converted into specific gravities:—

$$\text{Specific gravity} = \frac{140}{\text{Degrees Baumé} + 130}$$

Conversely, the specific gravity may be converted into degrees Baumé by the equation:—

$$\text{Degrees Baumé} = \frac{140}{\text{sp. gr.}} - 130$$

For liquids heavier than water.

$$\text{Specific gravity} = \frac{145}{145 - \text{Degrees Baumé.}}$$

A hydrometer may be tested by placing it in water at 4 deg. C. If it registers 1, it is correct.

TABLE XV (A).

Specific Gravity corresponding to the Degrees of the Baume Hydrometer.

(For specific gravities greater than 1.)

Degrees Baumé.	*Sp. Gr.*	*Degrees Baumé.*	*Sp. Gr.*	*Degrees Baumé.*	*Sp. Gr.*	*Degrees Baumé.*	*Sp. Gr.*
0	1·000	19	1·147	37	1·337	55	1·596
1	1·007	20	1·157	38	1·349	56	1·615
2	1·014	21	1·166	39	1·361	57	1·634
3	1·020	22	1·176	40	1·375	58	1·653
4	1·028	23	1·185	41	1·388	59	1·671
5	1·034	24	1·195	42	1·401	60	1·690
6	1·041	25	1·205	43	1·414	61	1·709
7	1·049	26	1·215	44	1·428	62	1·729
8	1·057	27	1·225	45	1·442	63	1·750
9	1·064	28	1·235	46	1·456	64	1·771
10	1·072	29	1·245	47	1·470	65	1·793
11	1·080	30	1·256	48	1·485	66	1·815
12	1·088	31	1·267	49	1·500	67	1·839
13	1·096	32	1·278	50	1·515	68	1·864
14	1·104	33	1·289	51	1·531	69	1·885
15	1·113	34	1·300	52	1·546	70	1·909
16	1·121	35	1·312	53	1·562	71	1·935
17	1·130	36	1·324	54	1·578	72	1·960
18	1·138						

TABLE XVI.

Specific Gravity corresponding to Degrees on the Twaddell Hydrometer.

Deg. Twadd.	Spec. Grav.	Deg. Twadd.	Spec. Grav.	Deg. Twadd.	Spec. Grav.	Deg. Twadd.	Spec. Grav.	Deg. Twadd.	Spec. Grav.
1	1·005	24	1·120	47	1·235	80	1·40	126	1·63
2	1·010	25	1·125	48	1·240	82	1·41	128	1·64
3	1·015	26	1·130	49	1·245	84	1·42	130	1·65
4	1·020	27	1·135	50	1·250	86	1·43	132	1·66
5	1·025	28	1·140	51	1·255	88	1·44	134	1·67
6	1·030	29	1·145	52	1·260	90	1·45	136	1·68
7	1·035	30	1·150	53	1·265	92	1·46	138	1·69
8	1·040	31	1·155	54	1·270	94	1·47	140	1·70
9	1·045	32	1·160	55	1·275	96	1·48	142	1·71
10	1·050	33	1·165	56	1·280	98	1·49	144	1·72
11	1·055	34	1·170	57	1·285	100	1·50	146	1·73
12	1·060	35	1·175	58	1·290	102	1·51	148	1·74
13	1·065	36	1·180	59	1·295	104	1·52	150	1·75
14	1·070	37	1·185	60	1·300	106	1·53	152	1·76
15	1·075	38	1·190	62	1·31	108	1·54	154	1·77
16	1·080	39	1·195	64	1·32	110	1·55	156	1·78
17	1·085	40	1·200	66	1·33	112	1·56	158	1·79
18	1·090	41	1·205	68	1·34	114	1·57	160	1·80
19	1·095	42	1·210	70	1·35	116	1·58	162	1·81
20	1·100	43	1·215	72	1·36	118	1·59	164	1·82
21	1·105	44	1·220	74	1·37	120	1·60	166	1·83
22	1·110	45	1·225	76	1·38	122	1·61	168	1·84
23	1·115	46	1·230	78	1·39	124	1·62		

TABLE XVII.

Electric Equivalents of Heat Units, &c.

Unit.	Equivalent value in other units.	Unit.	Equivalent value in other units.
1 k.w. hour =	1,000 watt hours. 1·34 h.p. hours. 2,656,400 ft. lb. 3,600,000 joules. 3,440 heat units. 366,848 k.g.m. 0·229 lb. coal oxidised with perfect efficiency. 3 lb. water evaporated at 212° F. 22·9 lb. of water raised from 62° to 212° F.	1 h.p. =	746 watts. 0·746 k.w. 33,000 ft. lb. per minute. 550 ft. lb. per sec. 2,580 heat units per hour. 43 heat units per minute. 0·71 heat units per second. ·172 lb. coal oxidised per hour. 2·25 lb. water evaporated per hour at 212° F.
1 h.p. hour =	0·746 k.w. hour. 1,980,000 ft. lb. 2,580 heat units. 273,740 k.g.m. 0·172 lb. coal oxidised with perfect efficiency. 2·25 lb. water evaporated at 212°F. 17·2 lb. of water raised from 62° to 212° F.	1 joule =	1 watt second. 0·000000278k.w.hr. 0·102 k.g.m. 0·00094 heat unit. ·73 ft. lb.
1 k.w. =	1,000 watts. 1·34 h.p. 2,656,400 ft. lb. per hour. 44,240 ft. lb. per minute. 737 ft. lb. per second. 3,440 heat units per hour. 57·3 heat units per minute. ·955 heat units per second. 0·229 lb. coal oxidised per hour. 3 lb. water evaporated per hour at 212° F.	1 ft. lb. =	1·36 joules. 0·1383 k.g.m. 0·000000377k.w.hr. 0·000129 heat unit. 0·0000005 h.p. hr.
		1 watt =	1 joule per second. 0·00134 h.p. 0·001 k.w. 3·44 heat units per hour. 0·73 ft. lb. per sec. 0·003 lb. of water evaporated per hour. 44·24 ft. lb. per minute.

TABLE XVII.—*continued*

Unit.	*Equivalent value in other units.*	*Unit.*	*Equivalent value in other units.*
1 watt per sq. in =	8·2 thermal units per sq. ft. per minute. 120° F. above surrounding air (Japanned cast iron surface). 66° C. above surrounding air (Japanned cast iron surface).	1 kilogram-metre =	7·23 ft. lb. 0·00000366 h.p. hr. 0·00000272 k.w. hr. 0·0092 heat units.
1 heat B.T. unit =	1,048 watt seconds. 772 ft. lb. 0·252 kg.-deg. centigrade. 108 k.g.m. 0·000291 k.w. hour. 0·000388 h.p. hour. 0·0000667 lb. coal oxidised. 0·00087 lb. water evaporated at 212° F.	1 lb. bituminous coal oxidised with perfect efficiency=	15,000 heat units. 0·98 lb. anthracite coal oxidised. 2·1 lb. dry wood oxidised. 15 cubic ft. illuminating gas. 4·37 k.w. hours (theoretical value). 5·81 h.p. hours (theoretical value). 11,590,000 ft. lb. (theoretical value). 13·1 lb. of water evaporated at 212° F.
1 heat unit (per sq. ft.) per min. =	0·121 watts per sq. in. 0·01745 k.w. 0·00232 h.p.	1 lb. water evaporated 212° F.=	0·33 k.w. hour. 0·44 h.p. hour. 1·180 heat units. 124·200 k.g.m. 1,219,000 joules. 887,800 ft. lb. 0·076 lb. of coal oxidised.

TABLE XIX.

Boiling Points of Water at Different Vacua.

Vacuum Millimetres of Mercury.	*Vacuum Inches of Mercury.*	*Temperature Centigrade.*	*Temperature Fahrenheit.*
756	29·74	0·0	32
754	29·67	4·4	40
751	29·56	10	50
747	29·40	15·6	60
741	29·18	21·1	70
734	29·89	26·7	80
724	28·50	32·2	90
712	28·00	37·8	100
708	27·88	38·7	101·83
657	25·85	52·3	126·15
605	23·81	60·8	141·52
553	21·78	67·2	153·01
502	19·74	72·5	162·28
450	17·70	76·8	170·06
398	15·67	80·5	176·85
346	13·63	83·7	182·86
295	11·60	86·75	188·27
243	9·56	89·5	193·22
191	7·52	91·9	197·75
139	5·49	94·3	201·96
88	3·45	96·8	205·87
36	1·42	98·5	209·55
0	0·00	100	212

TABLE XX.

Boiling Points of Various Liquids at Different Vacua.

	Constant	*Atmospheric pressure at 14·7lbs per sq. in.* °F.	*20·7" vacuum or 625 mm.* °F.	*24·06" vacuum or 611 mm.* °F.	*27·9" vacuum or 710 mm.* °F.	*29·5" vacuum or 750 mm.* °F.
Alcohol	0·904	173	124	118	75	28
Ether	1·0	94	40	22	13	7
Acetic Acid	1·164	247	184	164	121	59
Benzine	1·125	176	115	95	54	4
Turpentine (oil of) ..	1·329	318	246	222	176	85
Butyric Acid ..	1·228	322	256	232	189	124
Glycerin	1·25	554	486	464	419	351
Mercury	2·0	662	560	531	459	351
B. Naphthol ..	2·0	554	446	410	338	230
Carbolic Acid ..	1·2	352	287	266	219	158
Cresol	1·2	374	309	293	244	179

TABLE XVIII.—The Properties of Dry Saturated and Supersaturated Steam.

Absolute pressure in lbs. per sq. in., = gauge pressure + 14·69.	Temperature °F.	Heat in B.Th.U. to raise temp. of 1 lb. of water from 32° to t°F.	Total Heat of formation in B.Th.U. from water at 32°F. to steam at t°F. = h + L.	Volume of 1 lb. of steam in cubic feet.	Entropy of 1 lb. of dry saturated steam, from 32°F.	Absolute pressure in lbs. per sq. in., = gauge pressure + 14·69.	Temperature °F.	Heat in B.Th.U. to raise temp. of 1 lb. of water from 32° to t°F.	Total Heat of formation in B.Th.U. from water at 32°F. to steam at t°F. = h + L.	Volume of 1 lb. of steam in cubic feet.	Entropy of 1 lb. of dry saturated steam from 32°F.
P	t*†	h	H*‡	V*	ϕ*	P	t*†	h	H*‡	V*	ϕ*
1 (27·96")§	101·7	70·04	1102·4	333·0	1·9719	36	260·9	230·00	1169·4	11·59	1·6874
2 (25·91")	126·1	94·37	1113·6	173·5	1·9159	38	264·1	233·26	1170·6	11·02	1·6831
3 (23·87")	141·5	109·76	1120·6	118·6	1·8833	40	267·2	236·39	1171·7	10·50	1·6792
4 (21·83")	153·0	121·27	1125·7	90·54	1·8630	42	270·2	239·39	1172·8	10·03	1·6754
5 (19·79")	162·3	130·56	1129·8	73·44	1·8422	44	273·0	242·28	1173·7	9·603	1·6719
6 (17·75")	170·1	138·40	1133·2	61·91	1·8277	46	275·7	245·06	1174·7	9·212	1·6685
7 (15·70")	176·9	145·21	1136·1	53·59	1·8156	48	278·4	247·75	1175·6	8·853	1·6651
8 (13·66")	182·9	151·25	1138·6	47·28	1·8049	50	280·9	250·36	1176·5	8·520	1·6620
9 (11·62")	188·3	156·70	1140·9	42·34	1·7956	52	283·4	252·87	1177·4	8·213	1·6589
10 (9·58")	193·2	161·66	1143·0	38·39	1·7874	54	285·8	255·32	1178·2	7·928	1·6561
11 (7·54")	197·8	166·23	1144·9	35·11	1·7799	56	288·2	257·69	1178·9	7·663	1·6533
12 (5·49")	202·0	170·46	1146·6	32·37	1·7731	58	290·4	260·00	1179·7	7·415	1·6506
13 (3·45")	205·9	174·40	1148·2	30·03	1·7669	60	292·6	262·25	1180·4	7·184	1·6479
14 (1·41")	209·6	178·11	1149·8	28·02	1·7611	62	294·8	264·43	1181·1	6·966	1·6453
14·69 (0")	212·0	180·0	1150·7	26·79	1·7573	64	296·8	266·57	1181·8	6·761	1·6429
15	213·0	181·61	1151·2	26·27	1·7557	66	298·9	268·64	1182·4	6·571	1·6405
16	216·3	184·92	1152·5	24·73	1·7506	68	300·9	270·67	1183·1	6·388	1·6382
17	219·5	188·06	1153·7	23·37	1·7458	70	302·8	272·66	1183·7	6·218	1·6359
18	222·4	191·06	1154·9	22·16	1·7414	72	304·7	274·60	1184·3	6·056	1·6337
19	225·2	193·92	1156·0	21·06	1·7373	74	306·6	276·49	1184·9	5·902	1·6315
20	228·0	196·66	1157·1	20·08	1·7333	76	308·4	278·35	1185·5	5·757	1·6294
22	233·1	201·82	1159·1	18·37	1·7258	78	310·2	280·17	1186·0	5·618	1·6275
24	237·8	206·61	1160·9	16·93	1·7189	80	311·9	281·95	1186·6	5·487	1·6256
26	242·2	211·09	1162·5	15·71	1·7126	82	313·6	283·70	1187·1	5·362	1·6237
28	246·4	215·29	1164·1	14·66	1·7069	84	315·3	285·41	1187·6	5·241	1·6218
30	250·3	219·26	1165·6	13·74	1·7016	86	317·0	287·10	1188·1	5·127	1·6200
32	254·0	223·02	1166·9	12·94	1·6966	88	318·6	288·75	1188·6	5·018	1·6183
34	257·6	226·59	1168·2	12·22	1·6919	90	320·2	290·37	1189·1	4·913	1·6165

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92	321·7	291·97	1189·5	4·813	1·6148
94	323·3	293·54	1190·0	4·717	1·6131
96	324·8	295·08	1190·4	4·624	1·6115
98	326·3	296·60	1190·9	4·535	1·6098
100	327·7	298·09	1191·3	4·451	1·6081
105	331·2	301·73	1192·3	4·251	1·6044
110	334·7	305·24	1193·3	4·070	1·6007
115	338·0	308·62	1194·2	3·903	1·5972
120	341·1	311·89	1195·1	3·751	1·5938
125	344·2	315·05	1195·9	3·609	1·5906
130	347·2	318·12	1196·7	3·479	1·5875
135	350·1	321·10	1197·5	3·358	1·5846
140	353·0	324·00	1198·2	3·245	1·5818
145	355·7	326·82	1199·0	3·140	1·5791
150	358·4	329·57	1199·7	3·041	1·5765
155	361·0	—	1200·4	2·949	1·5740
160	363·5	334·85	1201·0	2·862	1·5715
165	366·0	—	1201·6	2·781	1·5691
170	368·4	339·89	1202·2	2·703	1·5666
175	370·7	—	1202·8	2·631	1·5643
180	373·0	344·71	1203·4	2·562	1·5620
185	375·3	—	1203·9	2·496	1·5598
190	377·5	349·33	1204·4	2·435	1·5577
195	379·7	—	1204·9	2·376	1·5557
200	381·8	353·7	1205·4	2·320	1·5538
210	386·0	358·04	1206·4	2·216	1·5502
220	390·0	362·17	1207·3	2·120	1·5465
230	393·8	366·15	1208·1	2·034	1·5429
240	397·6	370·01	1209·0	1·954	1·5395
250	401·2	373·75	1209·7	1·880	1·5362
260	404·7	377·38	1210·5	1·811	1·5332
270	408·1	380·91	1211·2	1·748	1·5303

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280	411·4	384·34	1211·8	1·689	1·5274
290	414·6	387·68	1212·5	1·634	1·5246
300	417·8	390·93	1213·1	1·583	1·5219
320	423·8	—	1214·3	1·489	1·5167
340	429·6	—	1215·4	1·406	1·5119
360	435·1	—	1216·4	1·333	1·5074
380	440·4	—	1217·4	1·266	1·5032
400	445·5	—	1218·3	1·206	1·4991
420	450·5	—	1219·1	1·152	1·4952
440	455·2	—	1220·0	1·102	1·4915
460	459·8	—	1220·7	1·057	1·4880
480	464·3	—	1221·4	1·016	1·4846
500	468·6	—	1222·2	0·977	1·4814
550	478·9	—	1223·7	0·893	1·4739
600	488·3	—	1225·0	0·822	1·4667
650	497·2	—	1226·2	0·762	1·4601
700	505·5	—	1227·2	0·710	1·4541
750	513·4	—	1228·1	0·665	1·4486
800	520·9	—	1229·0	0·626	1·4434
850	528·0	—	1229·6	0·591	1·4381
900	534·7	—	1230·1	0·559	1·4333
950	541·1	—	1230·5	0·531	1·4287
1000	547·3	—	1230·9	0·506	1·4244
1100	558·9	—	1231·7	0·462	1·4164
1200	569·7	—	1232·2	0·425	1·4086
1300	579·8	—	1232·6	0·393	1·4017
1400	589·3	—	1232·8	0·366	1·3951
1500	598·2	—	1232·8	0·343	1·3891
1600	606·7	—	1232·8	0·322	1·3834
1700	614·8	—	1232·8	0·304	1·3783
1800	622·4	—	1232·7	0·288	1·3731
1900	629·8	—	1232·6	0·273	1·3683
2000	636·8	—	1232·3	0·260	1·3635

* These values are taken from The Enlarged Callendar Steam Tables by permission of the Author and the Publishers Messrs. Arnold & Co.

† Absolute temperature $(T) = t + 459{\cdot}6$ F.

‡ For calculating $H - B$, $B = 835{\cdot}2$.

§ The vacuum is referred to a barometric pressure of 30″ of mercury at 62° F.

TABLE XXI.

Standard Dimensions of B.S. Whitworth and B.S. Fine Screw Threads.

Nominal dia. of Screw. *inches.*	*inches.*	*Number of Threads per inch.* *B.S.W.*	*B.S.F.*	*Core Diameter. inches.* *B.S.W.*	*B.S.F.*	*Cross-Sectional Area at bottom of thread of bolt, sq. in.* *B.S.W.*	*B.S.F.*
$\frac{7}{32}$	·2188	—	28	—	·1731	—	·0235
$\frac{1}{4}$	·2500	20	26	·1860	·2007	·0272	·0316
$\frac{9}{32}$	·2813	—	26	—	·2320	—	·0423
$\frac{5}{16}$	·3125	18	22	·2411	·2543	·0458	·0508
$\frac{3}{8}$	·3750	16	20	·2950	·3110	·0683	·0760
$\frac{7}{16}$	·4375	14	18	·3460	·3664	·0940	·1054
$\frac{1}{2}$	·5000	12	16	·3933	·4200	·1215	·1385
$\frac{9}{16}$	·5625	12	16	·4558	·4825	·1632	·1828
$\frac{5}{8}$	·6250	11	14	·5086	·5335	·2032	·2235
*$\frac{11}{16}$	·6875	11	14	·5711	·5960	·2562	·2790
$\frac{3}{4}$	·7500	10	12	·6219	·6433	·3038	·3250
*$\frac{13}{16}$	·8125	10	12	·6844	·7058	·3679	·3913
$\frac{7}{8}$	·8750	9	11	·7327	·7586	·4216	·4520
*$\frac{15}{16}$	·9375	9	—	·7952	—	·4966	—
1	1·0000	8	10	·8399	·8719	·5540	·5971
$1\frac{1}{8}$	1·1250	7	9	·9420	·9827	·6969	·7585
$1\frac{1}{4}$	1·2500	7	9	1·0670	1·1077	·8942	·9637
*$1\frac{3}{8}$	1·3750	6	8	1·1616	1·2149	1·0597	1·1592
$1\frac{1}{2}$	1·5000	6	8	1·2866	1·3399	1·3001	1·4100
*$1\frac{5}{8}$	1·6250	5	8	1·3689	1·4649	1·4718	1·6854
$1\frac{3}{4}$	1·7500	5	7	1·4939	1·5670	1·7528	1·9285
*$1\frac{7}{8}$	1·8750	4·5	—	1·5904	—	1·9866	—
2	2·0000	4·5	7	1·7154	1·8170	2·3111	2·5930
*$2\frac{1}{8}$	2·1250	4·5	—	1·8404	—	2·6602	—
$2\frac{1}{4}$	2·2500	4	6	1·9298	2·0366	2·9249	3·2577
*$2\frac{3}{8}$	2·3750	4	—	2·0548	—	3·3161	—
$2\frac{1}{2}$	2·5000	4	6	2·1798	2·2866	3·7318	4·1065
*$2\frac{5}{8}$	2·6250	4	—	2·3048	—	4·1721	—
$2\frac{3}{4}$	2·7500	3·5	6	2·3841	2·5356	4·4641	5·0535
*$2\frac{7}{8}$	2·8750	3·5	—	2·5091	—	4·9445	—
3	3·0000	3·5	5	2·6341	2·7439	5·4496	5·9133

* The British Engineering Standards Association recommends that for general use these sizes of *B.S.W.* screw threads be dispensed with.

TABLE XXII.

DIMENSIONS OF BRITISH ASSOCIATION (B.A.) SCREW THREADS.

Designating Number.	*Full Diameter. Bolt and Nut.*		*Approx. No. of Threads per inch.*	*Core Diameter Bolt and Nut.*	*Approx. Cross-Sec. Area at Bottom of Thread.*
	mm.	*inches.*		*mm.*	*sq. mm.*
0	6·0	·236	25·4	4·80	18·10
1	5·3	·209	28·2	4·22	13·99
2	4·7	·185	31·3	3·73	10·93
3	4·1	·161	34·8	3·22	8·14
4	3·6	·142	38·5	2·81	6·20
5	3·2	·126	43·1	2·49	4·87
6	2·8	·110	47·9	2·16	3·66
7	2·5	·098	52·9	1·92	2·89
8	2·2	·087	59·1	1·68	2·22
9	1·9	·075	65·1	1·43	1·61
10	1·7	·067	72·6	1·28	1·29
11	1·5	·059	81·9	1·13	1·00
12	1·3	·051	90·7	0·96	0·72
13	1·2	·047	102·0	0·90	0·64
14	1·0	·039	110·0	0·72	0·41
15	0·90	·035	121·0	0·65	0·33
16	0·79	·031	—	0·56	0·25
17	0·70	·028	—	0·50	0·20
18	0·62	·024	—	0·44	0·15
19	0·54	·021	—	0·37	0·11
20	0·48	·019	—	0·34	0·09

TABLE XXIII.

Whitworth's Standard Threads for Pipes.

Dimensions in inches.			*Number of threads per inch.*
Nominal bore of pipe.	*Diameter of pipe.*	*Diameter at bottom of thread.*	
⅛	·382	·336	28
¼	·518	·451	19
⅜	·656	·589	19
½	·826	·734	14
⅝	·902	·811	14
¾	1·04	·949	14
⅞	1·189	1·097	14
1	1·309	1·192	11
1⅛	1·492	1·375	11
1¼	1·65	1·533	11
1⅜	1·745	1·628	11
1½	1·882	1·765	11
1⅝	2·022	1·965	11
1¾	2·16	2·042	11
1⅞	2·245	2·128	11
2	2·347	2·23	11
2⅛	2·467	2·351	11
2¼	2·587	2·47	11
2⅜	2·794	2·678	11
2½	3	2·882	11
2⅝	3·124	3·009	11
2¾	3·247	3·13	11
2⅞	3·367	3·251	11
3	3·485	3·368	11
3¼	3·698	3·581	11
3½	3·912	3·795	11
3¾	4·125	4·008	11
4	4·340	4·223	11

TABLE XXIV

Sizes and Weights of Metal Sheets.

Size			*Thickness*		*Weight: lbs. per Sq. Foot*			
S.W.G.	*Birmingham Sheet Gauge*		*Inches*	*Mm.*	*Aluminium*	*Brass*	*Copper*	*Steel*
	Old Style	*Legalized 1914*						
			·375	9·53	5·26	16·4	17·3	15·2
3/0			·372	9·45	5·22	16·3	17·2	15·0
		1	·353	8·96	4·96	15·5	16·3	14·3
2/0			·348	8·84	4·89	15·2	16·1	14·1
1/0			·324	8·23	4·55	14·2	14·9	13·1
		2	·315	8·00	4·42	13·8	14·5	12·7
			·312	7·93	4·38	13·7	14·4	12·6
1	40		·300	7·62	4·21	13·1	13·8	12·1
	39		·289	7·34	4·06	12·7	13·3	11·7
		3	·280	7·11	3·93	12·3	12·9	11·3
	38		·278	7·06	3·90	12·2	12·8	11·2
2			·276	7·01	3·87	12·1	12·7	11·1
	37		·270	6·86	3·79	11·8	12·5	10·9
3			·252	6·40	3·54	11·0	11·6	10·2
	36	4	·250	6·35	3·51	10·9	11·5	10·1
	35		·238	6·05	3·34	10·4	11·0	9·62
4			·232	5·89	3·26	10·2	10·7	9·38
		5	·223	5·66	3·13	9·76	10·3	9·02
	34		·216	5·49	3·03	9·46	9·96	8·74
5			·212	5·38	2·98	9·92	9·78	8·59
	33		·200	5·08	2·81	8·76	9·23	8·10
		6	·198	5·03	2·78	8·67	9·13	8·01
6			·192	4·88	2·70	8·41	8·85	7·76
			·187	4·75	2·62	8·19	8·62	7·56
	32		·182	4·62	2·56	7·97	8·40	7·36
7		7	·176	4·47	2·47	7·71	8·12	7·11
	31		·166	4·22	2·33	7·27	7·65	6·71
8			·160	4·06	2·25	7·01	7·38	6·48
		8	·157	3·99	2·20	6·88	7·24	6·35
	30		·150	3·81	2·11	6·58	6·92	6·08
9			·144	3·66	2·02	6·31	6·64	5·82
		9	·140	3·56	1·97	6·13	6·45	5·66
	29		·136	3·45	1·91	5·96	6·28	5·50
10			·128	3·25	1·80	5·61	5·90	5·18
		10	·125	3·18	1·76	5·48	5·76	5·06
	28		·124	3·15	1·74	5·43	5·72	5·02
11			·116	2·95	1·53	5·08	5·35	4·69
	27		·112	2·85	1·57	4·91	5·17	4·54
		11	·111	2·82	1·56	4·86	5·12	4·49
12			·104	2·64	1·46	4·56	4·80	4·21
	26		·100	2·54	1·41	4·38	4·61	4·05
		12	·099	2·51	1·39	4·34	4·57	4·00
13			·092	2·34	1·29	4·03	4·24	3·72
	25		·090	2·29	1·26	3·94	4·15	3·64
		13	·088	2·24	1·24	3·85	4·06	3·56
	24		·082	2·08	1·15	3·59	3·78	3·32
14			·080	2·03	1·12	3·50	3·69	3·24

The above weights are subject to a variation of 2 per cent.

TABLE XXIV—*continued.*

Size			*Thickness*		*Weight: lbs. per Sq. Foot*			
S.W.G.	*Birmingham Sheet Gauge* *Old Style*	*Birmingham Sheet Gauge* *Legalized 1914*	*Inches*	*Mm.*	*Aluminium*	*Brass*	*Copper*	*Steel*
		14	·078	1·98	1·10	3·42	3·60	3·16
	23		·077	1·96	1·08	3·37	3·55	3·12
15	22		·072	1·83	1·01	3·15	3·32	2·91
		15	·070	1·78	0·982	3·06	3·23	2·83
	21		·068	1·73	0·955	2·98	3·14	2·75
	20		·065	1·65	9·914	2·85	3·00	2·63
16			·064	1·63	0·900	2·80	2·95	2·59
	19		·063	1·60	0·885	2·76	2·91	2·55
		16	·062	1·58	0·871	2·72	2·86	2·51
	18		·060	1·53	0·843	2·63	2·77	2·43
17		17	·056	1·42	0·786	2·45	2·58	2·27
	17		·055	1·40	0·773	2·41	2·54	2·22
	16		·051	1·30	0·716	2·23	2·35	2·06
		18	·050	1·27	0·702	2·19	2·31	2·02
18			·048	1·22	0·675	2·10	2·22	1·94
	15		·047	1·19	0·660	2·06	2·17	1·90
		19	·044	1·12	0·618	1·93	2·03	1·78
	14		·042	1·07	0·590	1·84	1·94	1·70
19			·040	1·02	0·562	1·75	1·85	1·62
		20	·039	0·991	0·548	1·71	1·80	1·58
	13		·038	0·965	0·534	1·66	1·75	1·54
20			·036	0·914	0·506	1·58	1·66	1·46
	12	21	·035	0·899	0·492	1·53	1·62	1·42
21			·032	0·813	0·449	1·40	1·48	1·29
	11	22	·031	0·793	0·435	1·36	1·43	1·25
22		23	·028	0·711	0·393	1·23	1·29	1·13
	10		·027	0·686	0·379	1·18	1·25	1·09
		24	·025	0·635	0·351	1·09	1·15	1·01
23			·024	0·610	0·337	1·05	1·11	0·970
	9		·023	0·584	0·323	1·01	1·06	0·930
24		25	·022	0·559	0·309	0·964	1·01	0·890
	8		·021	0·533	0·295	0·920	0·969	0·850
25		26	·020	0·508	0·281	0·876	0·922	0·809
	7		·019	0·483	0·267	0·832	0·876	0·769
26			·018	0·457	0·253	0·791	0·083	0·728
		27	·017	0·432	0·239	0·745	0·784	0·688
27			·0164	0·417	0·230	0·718	0·756	0·664
	6		·0160	0·406	0·225	0·701	0·738	0·647
		28	·0156	0·397	0·219	0·683	0·720	0·631
28			·0148	0·376	0·208	0·648	0·683	0·599
	5		·0140	0·356	0·196	0·613	0·645	0·566
		29	·0139	0·353	0·195	0·609	0·641	0·562
29			·0136	0·346	0·191	0·596	0·627	0·550
30			·0124	0·315	0·174	0·543	0·572	0·502
		30	·0123	0·312	0·173	0·539	0·567	0·498
	4		·0120	0·305	0·169	0·525	0·553	0·486
31			·0116	0·294	0·163	0·508	0·535	0·469

The above weights are subject to a variation of 2 per cent

TABLE XXIV—*continued.*

Size			*Thickness*		*Weight: lbs. per Sq. Foot*			
S.W.G.	*Birmingham Sheet Gauge* *Old Style*	*Birmingham Sheet Gauge* *Legal-ized 1914*	*Inches*	*Mm.*	*Alu-minium*	*Brass*	*Copper*	*Steel*
		31	·0110	0·279	0·155	0·482	0·507	0·445
32			·0108	0·274	0·152	0·473	0·498	0·437
33	3		·0105	0·267	0·147	0·460	0·485	0·425
		32	·0098	0·249	0·138	0·429	0·452	0·396
34			·0092	0·233	0·129	0·403	0·424	0·372
	2		·0090	0·228	0·126	0·394	0·415	0·364
		33	·0087	0·221	0·122	0·381	0·402	0·352
35			·0084	0·213	0·118	0·368	0·388	0·340
	1		·0080	0·203	0·112	0·350	0·369	0·324
		34	·0077	0·195	0·108	0·337	0·355	0·312
36			·0076	0·193	0·107	0·333	0·351	0·308
		35	·0069	0·175	0·097	0·302	0·318	0·279
37			·0068	0·172	0·0935	0·298	0·314	0·275
		36	·0061	0·155	0·0857	0·267	0·282	0·247
38			·0060	0·152	0·0843	0·263	0·277	0·243
		37	·0054	0·137	0·0759	0·236	0·249	0·218
39			·0052	0·132	0·0731	0·228	0·240	0·210
40		38	·0048	0·122	0·0675	0·210	0·222	0·194
41			·0044	0·112	0·0618	0·193	0·203	0·178
		39	·0043	0·109	0·0604	0·188	0·198	0·174
42			·0040	0·102	0·0562	0·175	0·185	0·162
		40	·0039	0·0991	0·0548	0·171	0·180	0·158
43			·0036	0·0915	0·0506	0·158	0·166	0·146
		41	·0034	0·0864	0·0478	0·149	0·157	0·138
44			·0032	0·0813	0·0450	0·140	0·148	0·129
		42	·0031	0·0788	0·0436	0·136	0·143	0·125
45			·0028	0·0711	0·0394	0·123	0·129	0·113
		43	·0027	0·0686	0·0379	0·118	0·125	0·109
46		44	·0024	0·0610	0·0337	0·105	0·111	0·097
		45	·0022	0·0559	0·0309	0·096	0·101	0·089
47			·0020	0·0508	0·0281	0·088	0·092	0·081
		46	·0019	0·0483	0·0267	0·083	0·088	0·077
		47	·0017	0·0432	0·0239	0·075	0·078	0·069
48			·0016	0·0406	0·0225	0·070	0·074	0·065
		48	·0015	0·0381	0·0211	0·066	0·069	0·061
		49	·0013	0·0330	0·0183	0·057	0·060	0·053
49		50	·0012	0·0305	0·0169	0·053	0·055	0·049
		51	·0011	0·0280	0·0155	0·048	0·051	0·045
50			·0010	0·0254	0·0140	0·044	0·046	0·040
SPECIFIC GRAVITY	..	..	..		2·71	8·45	8·89	7·8
RATIO OF WEIGHTS	..	..	..		1·00	3·12	3·28	2·88

The above weights are subject to a variation of 2 per cent.

TABLE XXV.

Horse-Power that Steel Shafting will Transmit.

Revolutions per minute.	*Diameter of Shaft in inches.*								
	1½	1¾	2	2¼	2½	3	4	6	8
	Horse-Power Transmitted.								
50	3·3	5·3	8·0	10·9	15·6	27	64	216	512
60	4·0	6·4	9·6	13·1	18·8	32	77	259	614
70	4·7	7·5	11·2	15·2	21·9	38	89	302	717
80	5·4	8·5	12·8	17·4	25·0	43	102	346	819
90	6·0	9·6	14·4	19·6	28·1	49	115	389	922
100	6·7	10·7	16·0	21·8	31·2	54	128	432	1024
110	7·4	11·8	17·6	23·9	34·4	59	141	475	1126
120	8·1	12·9	19·2	26·1	37·5	65	154	518	1229
130	8·7	13·9	20·8	28·3	40·6	70	166	562	1331
140	9·4	15·0	22·4	30·5	48·8	76	179	605	1434
150	10·1	16·1	24·0	32·6	46·9	81	192	648	1536
160	10·8	17·1	25·6	34·8	50·0	86	205	691	1638
170	11·5	18·2	27·2	37·0	53·1	92	218	734	1741
180	12·2	19·3	28·8	39·2	56·3	97	230	778	1843
190	12·8	20·4	30·4	41·3	59·4	103	243	821	1945
200	13·5	21·4	32·0	43·5	62·5	108	256	864	2048
225	15·2	24·1	36·0	49·0	70·3	122	288	972	2304
250	16·9	26·8	40·0	54·4	78·1	135	320	1080	2560
275	18·6	29·5	44·0	59·9	85·9	149	352	1188	2816
300	20·3	32·2	48·0	65·3	93·7	162	384	1296	3072
325	21·9	34·8	52·0	70·7	101·6	176	416	1404	3328
350	23·6	37·5	56·0	76·2	109·4	189	448	1512	3584
375	25·3	40·2	60·0	81·6	117·2	203	480	1620	3840
400	27·0	42·9	64·0	87·0	125·0	216	512	1728	4096
425	28·7	45·6	68·0	92·5	132·8	230	544	1836	4352
450	30·4	48·2	72·0	97·9	140·6	243	576	1944	4608
475	32·1	50·9	76·0	103·4	148·4	257	693	2052	4864
500	33·7	53·6	80·0	108·8	156·2	270	640	2160	5120

TABLE XXVI.

Horse-Power that Leather Belts will transmit, per inch of Width.

Velocity of Belt in Feet per Minute.	*Best Oak-tanned Belts.*			*Best Link or Chain Belts.*					
	Single Belts.	*Light Double Belts.*	*Heavy Double Belts.*	*in.* 3/8	*in.* 1/2	*in.* 5/8	*in.* 3/4	*in.* 7/8	*in.* 1
100	0·15	0·21	0·27	0·13	0·15	0·17	0·20	0·24	0·27
200	0·30	0·42	0·55	0·25	0·29	0·35	0·40	0·47	0·55
300	0·45	0·64	0·82	0·38	0·44	0·52	0·60	0·71	0·82
400	0·61	0·85	1·09	0·51	0·58	0·69	0·80	0·95	1·09
500	0·76	1·06	1·36	0·64	0·73	0·86	1·00	1·18	1·36
600	0·91	1·27	1·64	0·76	0·87	1·04	1·20	1·42	1·64
700	1·06	1·49	1·91	0·89	1·02	1·21	1·40	1·65	1·91
800	1·21	1·70	2·18	0·92	1·16	1·38	1·60	1·89	2·18
900	1·36	1·91	2·45	1·05	1·31	1·55	1·80	2·13	2·45
1000	1·51	2·12	2·73	1·27	1·45	1·73	2·00	2·36	2·73
1100	1·67	2·33	3·00	1·40	1·60	1·90	2·20	2·60	3·00
1200	1·82	2·55	3·27	1·53	1·75	2·07	2·40	2·84	3·27
1300	1·97	2·76	3·55	1·65	1·89	2·25	2·60	3·07	3·55
1400	2·12	2·97	3·82	1·78	2·04	2·42	2·80	3·31	3·82
1500	2·27	3·18	4·09	1·91	2·18	2·59	3·00	3·55	4·09
1600	2·42	3·39	4·36	2·04	2·33	2·76	3·20	3·78	4·36
1700	2·58	3·61	4·64	2·16	2·47	2·94	3·40	4·02	4·64
1800	2·73	3·82	4·91	2·29	2·62	3·11	3·60	4·25	4·91
1900	2·88	4·03	5·18	2·42	2·76	3·28	3·80	4·49	5·18
2000	3·03	4·24	5·45	2·55	2·91	3·45	4·00	4·73	5·45
2100	3·18	4·45	5·73	2·67	3·05	3·63	4·20	4·96	5·73
2200	3·33	4·67	6·00	2·80	3·20	3·80	4·40	5·20	6·00
2300	3·49	4·88	6·27	2·93	3·35	3·97	4·60	5·44	6·27
2400	3·64	5·09	6·55	3·05	3·49	4·15	4·80	5·67	6·55
2500	3·79	5·30	6·82	3·18	3·64	4·32	5·00	5·91	5·82
2600	3·94	5·52	7·09	3·24	3·78	4·49	5·20	6·15	7·09
2700	4·09	5·73	7·36	3·28	3·85	4·66	5·40	6·38	7·36
2800	4·24	5·94	7·64	3·31	3·86	4·73	5·60	6·62	7·64
2900	4·39	6·15	7·91	3·32	3·87	4·78	5·80	6·85	7·91
3000	4·50	6·36	8·18	3·31	3·86	4·75	5·97	7·09	8·18
3100	4·60	6·58	8·45	3·30	3·85	4·73	5·96	7·33	8·45
3200	4·69	6·79	8·70	3·28	3·82	4·71	5·94	7·37	8·73
3300	4·77	7·00	8·86	3·24	3·77	4·70	5·92	7·35	8·88
3400	4·84	7·21	8·96	3·19	3·71	4·64	5·87	7·32	8·86
3500	4·90	7·31	9·06	3·13	3·61	4·50	5·78	7·26	8·80
3600	4·95	7·40	9·16	3·05	3·50	4·37	5·67	7·16	8·73
3700	4·99	7·48	9·24	2·96	3·39	4·26	5·55	7·01	8·58
3800	5·03	7·54	9·29	2·84	3·28	4·15	5·41	6·87	8·41
3900	5·06	7·60	9·34	2·72	3·13	4·02	5·20	6·70	8·27
4000	5·08	7·64	9·37	2·58	2·95	3·84	5·01	6·48	8·04
4200	5·10	7·70	9·38	2·27	2·55	3·37	4·52	5·98	7·51
4500	5·07	7·69	9·27	1·64	1·77	2·45	3·68	5·05	6·55
5000	4·82	7·42	8·75	0·42	0·55	0·31	1·56	2·78	4·32

TABLE XXVII.

Friction Torque and Stored Energy for Various Classes of Machinery.*

Description of work.	Friction or load torque at end of accelerating period.	Stored energy at full speed.
	Per cent.	ft.-lb./b.h.p.
Toothed gears	—	15/18
Loose pulley, as for rotor only	2½	
Shop shafting	20/80 (aver. 45)	800/1 000
Hoisting full load on cranes	80/100	6/7
Hoisting full load on colliery winders	100	500/1 000
Small rolls and bar-bending machines	10	700/1 000
Medium punching machines	10/15	2 000
Medium shearing machines	10/15	3 000/3 500
Large shearing machines	10/15	14 000/18 000
Large rolling mills	5/10/15	500/1 000
	5/10	2 000/3 000
	10/20	5 000/8 000
	15/20	18 000/65 000
Planing machines	20/40	7 000/10 000
Disc saws for wood	20/50	1 900/3 800
Band saws for wood	7½	2 500/10 000 (generally 4 000/10 000)
High-speed disc saws for steel	7½ at start, rising to 20 at full speed.	14 000/17 000
Travelling motion of a crane with full load	75/100	400
Centrifugal hydro-extractor	35/50 at start, rising to 75/100 at full speed.	60 000/80 000
Centrifugal pump, valve closed	5/7½ at start, rising to 30/50 at full speed.	
Centrifugal pump, valve open	5/7½ at start, rising to 100 at full speed.	
Ram pump delivery against vertical head or accumulator ..	100	100/200
Air compressor starting on by-pass	6	100/200
Haulage, clutches out	7½/12½	300/900
Haulage, clutches in	100	1 000/2 000
Conveyor	100	850/1 000
Rubber pulping mill	2½	50/60
Corn grinding mill	100	7 500
Lathes and drills	20/25	300

* J. Anderson, M.I.E.E. *Journal I.E.E.* Vol. 60, No. 310.

Physical Properties of the Elements.

TABLE XXVIII.

ATOMIC WEIGHT, SPECIFIC GRAVITY, SPECIFIC HEAT, MELTING POINT, BOILING POINT.

Name.	*Symbol.*	*Atomic Weight.*	*Specific Gravity.*	*Specific Heat, 0°C. to 100°C.*	*Melting Point, °C.*	*Boiling Point, °C.*
Aluminium	Al	27·1	2·6	0·212	658	1800
Antimony	Sb	120·2	6·69	0·0495	632	1500*
Argon ..	A	39·9	1·40	—	—188	—186·9
Arsenic ..	As	74·96	5·73	0·083	850*	360*
Barium ..	Ba	137·37	3·75	0·068	850*	—
Bismuth ..	Bi	208·0	9·78	0·030	268	1430*
Boron ..	B	11·0	2·55	0·307	2350*	—
Bromine ..	Br	79·92	3·15	0·107	—7·3	63
Cadmium ..	Cd	112·4	8·67	0·055	320·7	762
Calcium ..	Ca	40·07	1·52	0·180	810	—
Carbon ..	C	12·00	3·51	0·310	> 3600	volatilises
Cerium ..	Ce	140·25	7·02	0·045	650*	—
Chlorine ..	Cl	35·46	1·51	0·226	—101	— 34
Chromium	Cr	52·00	6·92	0·0998	<1775	2200
Cobalt ..	Co	58·97	8·71	0·106	1500	—
Copper ..	Cu	63·57	8·89	0·0933	1083	2300
Fluorine ..	F	19·00	1·14	—	—223	—187
Gold ..	Au	197·2	19·25	0·031	1050	2240
Helium ..	He	3·99	—	—	<—271	—267
Hydrogen	H	1·008	0·070	—	—259	—252
Iodine ..	I	126·92	4·8	0·054	113·5	250
Iridium ..	Ir	193·1	22·42	0·032	1950*	2600
Iron ..	Fe	55·8	7·7	0·115	1530	2450
Lead ..	Pb	207·1	11·36	0·031	327·4	1500
Lithium ..	Li	6·94	0·534	0·85	180	1400
Magnesium	Mg	24·32	1·72	0·246	651	1100
Manganese	Mn	54·93	7·4	0·122	1260	1900
Mercury ..	Hg	200·6	13·55	0·033	— 38·85	360

* Uncertain.

Physical Properties of the Elements—Table XXVIII. *(Continued).*

Name.	*Symbol.*	*Atomic Weight.*	*Specific Gravity.*	*Specific Heat. 0°C. to 100°C.*	*Melting Point, °C.*	*Boiling Point, °C.*
Molybdenum	Mo	96·0	8·5	0·066	2500*	3620
Nickel ..	Ni	58·68	8·9	0·109	1452	2330
Nitrogen ..	N	14·0	0·83	—	—214	—194·4
Osmium ..	Os	190·9	22·5	0·031	2500	—
Oxygen ..	O	16·00	1·14	—	—218	—181·4
Palladium	Pd	106·7	11·4	0·071	1550	2550
Phosphorus	P	31·04	2·34	0·19	44	287
Platinum	Pt	195·2	21·52	0·032	1775	2450
Potassium .	K	39·1	0·87	0·170	62·1	667
Rhodium	Rh	102·9	12·4	0·058	2000	—
Rubidium	Rb	85·45	1·53	—	38·5	696
Ruthenium	Ru	101·7	12·3	0·061	1800	—
Selenium ..	Se	79·2	4·55	0·084	217	665
Silicon ..	Si	28·3	2·10	0·183	1420	—
Silver ..	Ag	107·88	10·6	0·056	968	1955
Sodium ..	Na	23·00	0·97	0·283	97·6	742
Strontium	Sr	87·63	2·54	—	about 900*	—
Sulphur ..	S	32·07	2·05	0·163	114·5	450
Tantalum ..	Ta	181·5	16·6	0·036	2850*	—
Tellurium ..	Te	127·5	6·25	0·048	525ϕ	1400
Thallium ..	Tl	204·0	11·85	0·0335	294	1280
Thorium ..	Th	232·4	11·0	0·0276	‡	—
Tin ..	Sn	119·0	7·40	0·056	232	2270
Titanium	Ti	48·1	3·5	0·112	1800*	—
Tungsten ..	W	184·0	18·85	0·035	<3000§	3700†
Uranium ..	U	238·5	18·7	0·028	1700*	—
Vanadium ..	V	51·0	5·5	0·115	1700*	—
Zinc ..	Zn	65·37	7·20	0·093	419	930
Zirconium	Zr	90·6	4·14	0·066	1700	—

* Uncertain

‡ Almost infusible. Bureau of Standards gives — 1700° C.

ϕ The U.S. Bureau of Standards gives 452° C.

§ Uncertain. Bureau of Standards gives 3400° C.

† Uncertain. Langmuir, Phys. Rev. 1913, gives 5830.

Physical Properties of Various Metals.

TABLE XXIX.

SPECIFIC RESISTANCE, RELATIVE CONDUCTIVITY, TEMPERATURE COEFFICIENT OF RESISTANCE AND ELECTROCHEMICAL EQUIVALENT.

Metal.	*Specific Resistance at 18°C. Microhms per cm³*	*Relative Conductivity Copper = 100.*	*Temperature Coefficient. 0° to 100°C.*	*Electro-chemical equivalent. Grammes per ampere-hour.*
Aluminium ..	2·94	60·0	·00380	0·337 (3) ‖
Antimony	40·50	4·1	·00389	—
Bismuth	119·0	1·4	·00420	2·586 (3)
Cadmium	7·54	22·3	·00419	2·096 (2)
Copper, drawn ..	1·78	100·0	·00428	1·186 (2)
„ annealed ..	1·69*			2·372 (1)
Gold	2·42	76·8	·00393¶	2·451 (3)
Iridium	5·30	32·6	—	2·399 (3)
Iron, wrought ..	12·00	22·1§	·00625	0·695 (3) / 1·042 (2)
Lead	20·80	8·5	·00411	3·860 (2)
Magnesium	4·35	38·8	·00381	0·454 (2)
Mercury	94·30	1·75	·00720	3·73 (2) / 7·46 (1)
Nickel	11·8	23·8	·00622	1·094 (2)
Platinum Wire (pure)	11·0†	15·4	·00350	1·816 (4) / 3·632 (2)
„ annealed	8·98‡	—	·00247	—
Silver	1·66	103·6	·00377	4·025 (1)
Tantalum	14·5	10·2	·00330	1·350 (5)
Thallium	17·6	9·6	·00400	2·54 (3)
Tin	11·3	14·0	·00440	1·109 (4) / 2·219 (2)
Tungsten	5·0	33·6	·00510	3·431 (2)
Zinc	6·10	31·4	·00370	1·219 (2)

* At 15·5° C.
† Dewar & Fleming.
‡ Matthiessen.
§ Pure. Cast-iron, hard = 1·7. Wrought-iron, hard = 5·2.
„ soft = 2·3. „ soft = 7·3.
‖ The figures in brackets denote the valency of the metal at which the electro-chemical equivalents are given.
¶ At 20° C.

TABLE XXX.

Tensile Strength, Modulus of Elasticity, and Thermal Conductivity.

Metal.	*Tensile Strength, lbs. per sq. in.*	*Modulus of Elasticity, lbs. per sq. in.*	*Thermal Conductivity, cal. per cm³ per °C. per sec.*
Aluminium, H.D. wire ..	24,000 to 33,000	10,000,000	0·51
Antimony	—	—	0·044
Bismuth	—	—	0·019
Cadmium	—	—	0·22
Copper, H.D. wire ..	55,000 to 63,000	18,000,000	0·89
Gold, wrought	20,000 to 30,000	8,000,000	0·70
Iron, cast	19,500	16,000,000	0·095
,, wrought	50,000 to 80,000	29,000,000	·167 to ·207
Lead	1,800 to 2,500	720,000	0·084
Magnesium	20,000 to 30,000	—	0·37
Mercury	—	—	0·015
Nickel	40,000 to 85,000	25,000,000	0·14
Platinum, wrought ..	30,000 to 50,000	24,000,000	0·16
Silver, wrought	40,000 to 45,000	9,800,000	1·10
Steel, wire	50,000 to 300,000	30,000,000	0·108
Tantalum	130,000	—	—
Tin, cast	5,000	3,500,000	0·15
Tungsten, wire	600,000	—	—
Zinc, sheets	16,000	13,400,000	0·26

TABLE XXXI.

COMPOSITION, MELTING POINTS, SPECIFIC GRAVITIES, WEIGHTS PER CUBIC FOOT, AND CUBIC INCH OF VARIOUS ALLOYS.

Alloy.	Approximate Melting Point.		Specific Gravity.	Weight per cubic-foot. lbs.	Weight per cubic inch. lbs.	Remarks.
	Fahr.	Cent.				
Brass (Cu 66, Zn 34) ..	1652	900	8·39	522·6	0·3024	English Standard Brass.
„ („ 60, „ 40) ..	1641	894	8·40	523·2	0·3048	Muntz Metal.
„ („ 70, „ 30) ..	1454	790	8·44	525·6	0·3042	Tubes and Wire.
„ („ 50, „ 50) ..	1598	870	8·29	516·4	0·2989	Brazing Solder.
Aluminium Brass (Cu 59, Zn 39, Fe 1, Al 1)	1652	900	8·41	523·8	0·3031	Resists Corrosion. Strong and tough.
Brazing Metal (Cu 94, Zn 6)	1832	1000	8·84	550·5	0·3186	
Bronze (Cu 80, Sn 20) ..	1634	890	8·85	551·1	0·3189	Bell Metal.
„ („ 75, „ 25) ..	1472	800	8·80	548·1	0·3175	Bell Metal.
„ („ 91, „ 9) ..	1886	1030	8·79	547·5	0·3168	For Castings.
„ („ 70, „ 30) ..	1382	750	8·93	556·2	0·3219	Speculum Metal.
„ („ 88, „ 10 Zn 2)	1880	972	8·50	529·4	0·3064	Admiralty Gunmetal. Machine Parts.
Manganese Bronze (Cu 59, Zn 39, Fe 0·5, Mn 1·5)	1650	900	8·60	535·6	0·3100	Resists Corrosion. Strong and tough.
Phosphor Bronze (Cu 94, Sn 6, traces of Phosphorus)	1922	1050	8·94	556·8	0·3222	Cold Rolling. Sheets and Tubes.
Phosphor Bronze (Cu 92, Sn 8, P up to 0·3%)	1832	1000	8·68	540·6	0·3129	Castings.

TABLE XXXII.

Specific Gravities and Weights of Various Materials.

Specific Gravity × 62·28 = weight of cubic foot in lbs.
,, ,, × ·03604 = ,, ,, ,, inch ,, ,,

Substance.	Specific Gravity.	Weight of a cubic foot. lbs.
Acid, acetic . .	1·06	66
,, oxalic . .	1·64	102
,, fluoric	1·50	93
,, hydrochloric	1·20	75
,, nitric . .	1·22	76
,, sulphuric	1.85	115
Air	·001293	·08072
Alcohol, pure	·79	49
,, proof 50% alcohol	·93	58
Amber . .	1·08	67
Asbestos, starry . .	3·07	191
Ash, European	·52–·95	32½–59
Asphaltum . .	·91	57
,, . .	1·65	103
Beeswax . .	·97	60
Benzine . .	·83	51½
Boxwood . .	1·28	80
Carbon, retort	3·50	219
Chalk . .	2·50	156
Cork . .	·24	15
Ebonite . .	1·15	71½
Ebony . .	1·21	75½
Emery . .	4·00	249
Fir, Spruce . .	·35–·60	21¾–37½
Glass, Crown	2·49	155
,, flint . .	3·078	192
,, optical	3·45	215
,, window	2·64	164
Gutta percha	·98	61
Gypsum . .	2·17	135
Horn . .	1·69	105
Ice	·92	57
India rubber, pure	·93	58
Ivory . .	1·83	114
Lignum vitæ	1·33	83
Lime, quick	·843	53
Mahogany, Spanish	·86	53½
Marble . .	2·72	169
Mica . .	2·80	174
Naphtha . .	·85	53
Oak, English	·70–1·04	43–64½
Oil, linseed	·94	59
,, olive . .	·915	57
Petroleum . .	·89	55½
Pine, yellow	·50	32
Plaster of Paris	1·18	73
Plumbago . .	2·10	131
Porcelain . .	2·30	143
Quartz . .	2·66	166
Red-lead . .	8·94	557
Resin . .	1·09	68
Sand . .	1·90	118
Slate . .	2·8	174
Spermaceti . .	·94	59
Spirit, rectified 95% alcohol	·82	51
Talc . .	2·7	168
Tallow . .	·94	59
Tar	1·02	64
Teak . .	·62	38½
Turpentine . .	·87	54
Water . .	1·00	62
,, sea . .	1·03	64
Walnut, English . .	·63–·71	39–44½

TABLE XXXIII.

Mean Coefficients of Linear Expansion of Solids.*

($l = l_0(1 + a\,t)$, where a = the coeff. of linear expansion. The coefficient of cubical expansion = $3a$).

Substance.	*Coeff. of exp.* (a) *per* $°C. \times 10^6$	*Substance.*	*Coeff. of exp.* (a) *per* $°C. \times 10^6$
Aluminium	25·5	Phosphor Bronze, 96·6	
Antimony	12·0	Cu. 2 Sn. 0·2 P. ..	16·8
Bismuth	15·7	Platinum-Iridium, 10 Ir.	8·7
Carbon (graphite) ..	7·9	Platinum-Silver, 33 Pt.	15·0
Cadmium	28·8	Solder, 2 Pb. 1 Sn., 50°C.	25·0
Cobalt	12·3		
Copper	16·7	*Miscellaneous.*	
Gold	13·9	Cement and Concrete	10 to 14
Iridium	6·5	Ebonite	64 to 77
Iron (cast)	10·2	Glass	5·7 to 9·7
,, (wrought)	11·9	Gutta-Percha	198
Steel	10·5 to 11·6	Ice, 10° to 0°	50·7
Lead	27·6	Marble, white Carrara,	
Nickel	12·8	15° C.	1·4 to 3·5
Palladium	11·7	Paraffin Wax, 0°–40°C.	c. 110
Platinum	8·9	Porcelain	2·5 to 3·4
Selenium, 40° C. ..	36·8	Quartz (crystal), ‖ axis	7·5
Silver	18·8	,, ,, ⊥ axis	13·7
Tin	21·4	Silica (fused), 0°–30°C.	0·42
Tungsten, 27° C. ..	4·4	,, ,, 0°–100°C.	0·50
,, 2,027° C. ..	7·3	,, ,, 0°–1000°C.	0·54
Zinc	26·3	Slate	
Alloys.		*Woods.*	
Brass, 66 Cu. 34 Zn. ..	18·9	Beech (along grain) ..	c. 3
Bronze, 32 Cu. 2 Zn. 5 Sn.	17·7	,, (across grain) ..	c. 60
Constantan	17·0	Oak (along grain) ..	c. 5
Duralumin	22·6	Mahogany (along grain)	c. 3
German Silver, 60 Cu.		,, (across grain)	c. 40
15 Ni. 25 Zn., (50° C.)	18·4	Pine (along grain) ..	c. 5
Gunmetal (Admiralty) ..	18·1	,, (across grain) ..	c. 34
Nickel Steel, 10% Ni. ..	13·0	Ash (along grain) ..	c. 9
,, ,, 20% Ni. ..	19·5	Maple and Walnut	
,, ,, 30% Ni. ..	12·0	(along grain)	c. 6
,, ,, 36 Ni. (Invar)†	0·9	Do. (across grain)	c. 48
,, ,, 40% Ni. ..	6·0		

* Based on Kaye and Laby, "Physical and Chemical Constants."

† Invar is obtainable in three qualities, with a range of coefficients of (— 0·3 to + 2·5) × 10^{-6} at ordinary temperatures.

Fusible Alloys.

Fusible alloys, having a melting point below 100°C. are made by melting together lead, tin, bismuth, and cadmium. The melting points of these metals are :—

Lead	327°C.
Tin	232°C.
Bismuth	268°C.
Cadmium ..	320°C.

The following table shows the composition of some fusible alloys and their melting points :—

TABLE XXXIV.

FUSIBLE ALLOYS.

Alloy.	*Lead* %	*Tin* %	*Bismuth* %	*Cadmium* %	*Melting Point* °C.
Wood's ..	25	12·5	50	12·5	65
Lipowitz's ..	27	13	50	10	65
FusibleAlloy	27·5	10	27·5	34·5	75
" "	25	50	—	25	86
Lichtenberg's	30	20	50	—	91
Arcet's ..	25	25	50	—	94
Rose's ..	28	24	48	—	95
Newton's ..	31·2	18·8	50	—	94
Eutectic of	32	16	52	—	96

TABLE XXXV.

Thermal Resistivity of Various Metals, Mineral Substances, Insulating Materials, Liquids, Gases, etc.

Material.	*Degrees C per Watt per cm.*^{3}
Metals.	
Pure Copper 18° C.	0·261
Commercial Copper 18° C.	0·286
Pure Iron 18° C.	1·5
Steel, 1% Carbon 18° C.	2·22
Cast Iron 100° C.	2·5
Transformer Steel (4% Si) 20°-250° C.	3·12
Brass 20°-250° C.	0·77
German Silver (30% Ni) 20°-250° C.	3·57
Constantan (60 Cu, 40 Ni) 18° C.	3·45
Calorite (65 Ni, 15 Fe, 13 Cr, 7 Mn) 20°-250° C.	6·25
Mineral Substances.	
Graphite	0·281
Marble	33·4
Slate	50·0
Glass	91·0
Quartz	66·7
Porcelain	50 to 100
Insulating Materials.	
Asbestos Paper	400—650
Bakelite and Linen Tape	370
Cambric, varnished	400
Cloth, varnished. Transverse	450
,, ,, Longitudinal	230
Cotton Tape, varnished	370
Ebonite	550
Fullerboard, varnished T.	620
,, ,, L.	150
,, untreated T.	420
,, ,, L.	160
Guttapercha	420
Linen Tape, varnished and baked	670
Mica, pure	180 to 250
Insulating Materials (contd.).	
Mica Paper	625
Micanite (19% shellac)	1000
,, (11% shellac)	830
Micafolium T.	430
,, L.	90
Paper	770
Paraffin Wax	385
Presspalm (untreated)	590
Rope P. untreated	830
Rope Paper and Oil	715
Rope Paper, treated with varnish	590
Rubber, Para.	625
Rubber Tape	230
Shellac	400
Vulcanized Fibre	350
Wool, pure, loose	2100
,, ,, tightly packed	4300
Miscellaneous Materials.	
Cement	345
Cork	1840
Oak (⊥ to grain)	540
(∥ to grain)	260
Maple (⊥ to grain)	560
,, (∥ to grain)	230
Pine, soft white (⊥ to grain)	625
Liquids.	
Glycerine 25° C.	417
Paraffin Oil	715
Petroleum 23° C.	625
Water 25° C.	175
Gases.	
Air, at 20° C.	4020
Air, at 100° C.	3330

Note;—

1 watt = 0·2390 g-cal. per sec.
1 B.Th. U = 252 g-cal.
1 g-cal. = 0·00397 B.Th. U.
1 B. Th. U. = 1055 watt-sec.
1 g-cal. = 4·184 watt-sec.
T = transverse.
L = longitudinal.

TABLE XXXVI.—Physical Properties of Resistance Materials.

ohms per in.^{3} = 0·394 ohms per cm.^{3}
ohms per circular mil foot = 6·02 × microhms per cm.^{3}
ohms per square mil foot = 4·73 × microhms per cm.^{3}

Name of Material	Approximate Composition	Sp. Resistance: Microhms per cm³	Sp. Resistance: at °C.	Temp. Coefficient per °C.	Coeff. of Expansion per °C.	Tensile Strength tons per sq. in.	Melting point °C.	Specific Heat	Specific Gravity	Maximum Working Temperature °C.
*Brightray ..	80 Ni ; 20 Cr. ..	100	16	·00019	·000015	59	1375	0·106	8·35	1100
†Cromaloy IV	,, ,, ..	100	20	·000083 (average)	—	—	1380	—	8·35	1100
*Redray ..	85 Ni ; 15 Cr. ..	93	16	·000253	·000015	55	1380	0·108	8·28	1000
†Cromaloy III	,, ,, ..	93	20	·00012 (average)	—	—	1380	—	8·25	1050
‡Kromore ..	Ni-Cu ..	90	20	·000186	—	—	—	—	8·5	1100
*Glowray ..	80 Ni ; 15 Cr ; 5 Fe	100	16	·00042	·0000164	47	1400	0·112	8·27	800
†Cromaloy II	,, ,, ..	110	20	·00012 (average)	—	—	—	—	8·15	1000
‡Nichrome ..	Ni-Cr-Fe ..	110	24	·00017	—	—	1480	—	8·15	1000
‖Nickel Chrome	,, ,, ..	100	15	·00042	—	47	1550	—	8·15	1000
*Dulray ..	Ni-Fe-Cr ..	87	16	·00071	·0000071	43	1490	0·113	8·13	500
‡No. 193 Alloy	,, ,, ..	87	24	·00072	—	—	—	—	8·137	650
*Ferrozoid ..	30 Ni ; 70 Fe ..	84	16	·00076	·0000071	42	1490	0·113	8·13	400
*Ferry ..	40 Ni ; 60 Cu ..	48	16	·000022	·0000146	39	1250	0·098	8·9	300
‡Advance ..	,, ,, ..	49	24	·000018	—	—	—	—	8·9	550
‖Eureka Constantan	,, ,,	48	15	·000022	·0000144	40	1250	0·10	8·9	300
*Corronil ..	70 Ni ; 26 Cu ; 4 Mn ..	50	16	·00056	·000014	40·7	1400	0·106	8·8	500

•Cupro	20 Ni ; 80 Cu	26	16	·000285	—	23	1175	—	8·97	300–400
Manganin	84 Cu ; 12 Mn ; 4 Ni	47	15	·000015	—	—	—	—	—	—
•Tarnac	Cu-Mn-Ni	39	16	·000017	·0000177	29	1150	0·097	8·39	—
•B.B. (Nickel Silvers)	30 Ni ; 55 Cu ; 15 Zn	40	16	·00021	·0000157	34·8	—	—	8·8	300
•Zodiac (Nickel Silvers)	25 Ni ; 58 Cu ; 17 Zn	36	16	·00023	·0000159	37·8	—	—	8·93	300
•Grade 1 (Nickel Silvers)	20 Ni ; 63 Cu ; 17 Zn	31	16	·00027	·0000161	30	—	—	8·78	300
•Grade 4 (Nickel Silvers)	9 Ni ; 58 Cu ; 33 Zn	21	16	·00047	·0000173	29·5	—	—	8·6	300
•Nickel	Pure nickel	10·2	16	·005	·0000127	36	1452	0·107	8·9	—
•Mangonic	Ni-Mn	14·95	16	·0035	·000014	37·1	1450	0·110	8·67	—
Platinoid	Nickel Silver + 1 or 2% Tungsten	41·7	0	·0003	—	—	—	—	9·0	—
Platinum-Silver	1 Pt ; 2 Ag.	31·5	0	·00024	—	—	—	—	—	—
Platinum-Iridium	80 Pt ; 20 Ir.	31	0	·00082	—	—	—	—	—	—
Copper-Nickel-Aluminium	87 Cu ; 6·7 Ni ; 6·5 Al.	15	0	·00065	—	—	—	—	—	—
§No-Mag.		140	20	·0009	—	—	—	—	—	—
Manganese-Steel	12 Mn ; 80 Fe	67	0	·00127	—	—	—	—	—	—
Silicon Steel	3½ Si.	55	20	—	—	35	—	—	7·5	—
Nickel Steel	4½ Ni.	29·5	0	·0020	—	39	—	—	—	—
Cast iron	Ordinary	70–100	20	·0020	—	—	—	—	—	—
,, ,,	Malleable	25	20	—	—	—	—	—	—	—

• Henry Wiggin & Co., Ltd. † A. C. Scott & Co., Ltd. ‡ British Driver-Harris Co., Ltd.
|| London Elec. Wire Co. & Smiths, Ltd. § Ferranti, Ltd.

TABLE XXXVII.

Comparison of Wire Gauges.

No.	American or Brown & Sharpe's	Old English or London	Birmingham or Stubs	W. & M. and Roebling	New British Standard	U.S. Standard
0000	.460	.454	.454	.393	.400	.406
000	.40964	.425	.425	.362	.372	.375
00	.36480	.380	.380	.331	.348	.344
0	.32495	.340	.340	.307	.324	.313
1	.28930	.300	.300	.283	.300	.281
2	.25763	.284	.284	.263	.276	.266
3	.22942	.259	.259	.244	.252	.250
4	.20431	.238	.236	.225	.232	.234
5	.18194	.220	.220	.207	.212	.219
6	.16202	.203	.203	.192	.192	.203
7	.14428	.180	.180	.177	.176	.188
8	.12849	.165	.165	.162	.160	.172
9	.11443	.148	.148	.148	.144	.156
10	.10189	.134	.134	.135	.128	.141
11	.09074	.120	.120	.120	.116	.125
12	.08081	.109	.109	.105	.104	.109
13	.07199	.095	.095	.092	.092	.0938
14	.06408	.083	.083	.080	.080	.0781
15	.05706	.072	.072	.072	.072	.0703
16	.05082	.065	.065	.063	.064	.0625
17	.04525	.058	.058	.054	.056	.0563
18	.04030	.049	.049	.047	.048	.0500
19	.03589	.040	.042	.041	.040	.0438
20	.03196	.035	.035	.035	.036	.0375
21	.02846	.0315	.032	.032	.032	.0344
22	.025347	.0295	.028	.028	.028	.0313
23	.022571	.027	.025	.025	.024	.0281
24	.0201	.025	.022	.023	.022	.0250
25	.0179	.023	.020	.020	.020	.0219
26	.01594	.0205	.013	.018	.018	.0188
27	.014195	.01875	.016	.017	.0164	.0172
28	.012641	.0165	.014	.016	.0148	.0156
29	.011257	.0155	.013	.015	.0136	.0141
30	.010025	.01375	.012	.014	.0124	.0125
31	.008928	.01225	.010	.0135	.0116	.0109
32	.00795	.01125	.009	.013	.0108	.0102
33	.00708	.01025	.008	.011	.010	.0094
34	.0063	.0095	.007	.010	.0092	.0086
35	.00561	.009	.005	.0095	.0084	.0078
36	.005	.0075	.004	.009	.0076	.0070
37	.00445	.0065	......	.0085	.0068	.0066
38	.003965	.00575	......	.008	.006	.0063
39	.003531	.005	......	.0075	.0052	......
40	.003144	.0045	......	.007	.0048	......

TABLE XXXIX.

WEIGHTS IN GRAINS PER YARD OF STANDARD ANNEALED COPPER WIRES.

Diameter.	Weight.	Diameter.	Weight.
Inch.	Grains per yard.	Inch.	Grains per yard.
0·0010	0·06358	0·0072	3·296
0·0012	0·09156	0·0076	3·672
0·0014	0·1246	0·0080	4·069
0·0016	0·1628	0·0084	4·486
0·0018	0·2060	0·0088	4·924
0·0020	0·2543	0·0092	5·381
0·0022	0·3077	0·0096	5·860
0·0024	0·3662	0·0100	6·358
0·0026	0·4298	0·0104	6·877
0·0028	0·4985	0·0108	7·416
0·0030	0·5722	0·0112	7·976
0·0032	0·6511	0·0116	8·555
0·0036	0·8240	0·0120	9·136
0·0040	1·017	0·0124	9·776
0·0044	1·231	0·0130	10·74
0·0048	1·465	0·0136	11·76
0·0052	1·719	0·0142	12·82
0·0056	1·994	0·0148	13·93
0·0060	2·289	0·0156	15·47
0·0064	2·604	0·0164	17·10
0·0068	2·940	0·0172	18·81
		0·0180	20·60

Bare Annealed Copper Wire.

The following has been extracted from British Standard Specification No. 128 (1922):—

Note.—See also B.S.S. No. 7 for Insulated Annealed Copper Conductors for Power and Light, Vol. I.

(*a*) International Standards of Resistance for Copper.

The following Standards have been fixed by the International Electrotechnical Commission :—

(i) At a temperature of 20° C. the resistance of a wire of standard annealed copper one metre in length and of a uniform section of one square millimetre is 1/58 ohm (0·017241 . . . ohm).

(ii) At a temperature of 20° C. the density of standard annealed copper is 8·89 grammes per cubic centimetre.

(iii) At a temperature of 20° C. the "constant-mass" temperature coefficient of resistance of standard annealed copper, measured between two potential points rigidly fixed to the wire, is 0·00393 = 1/254·45 . . . per degree C. (and at 60° F. is 0·0022221 = 1/450·025 . . . per degree F.)

(iv) As a consequence, it follows from (i) and (ii) that at a temperature of 20° C. the resistance of a wire of standard annealed copper of uniform section one metre in length and weighing one gramme is (1/58) × 8·89 = 0·15328 . . . ohm.

(*b*) Coefficient of Linear Expansion of Standard Annealed Copper.

The coefficient of linear expansion of standard annealed copper, between 60° F. (15·6° C.) and 68° F. (20° C.) is 0·00000944 per °F. (0·0000170 per 1° C.)

(*c*) Density of Standard Annealed Copper at 60° F.

The density of standard annealed copper at a temperature of 60° F. is 8·892015, and the weight of one cubic foot of copper 555·1108 lbs.

(*d*) Resistance of a Conductor at 60° F.

For the purpose of calculating Table XXXVIII the resistance of a conductor of standard annealed copper at 60° F., 1,000 yards in length, and of a uniform cross-sectional area of one square inch, has been taken as 0·0240079 ohm.

(*e*) Constant for ascertaining the Sectional Area of a Wire from the Weight per Yard.

The constant for ascertaining the sectional area, expressed as a part of a square inch, from the weight in grains per yard of a plain wire can be taken as 0·000012353.

(*f*) Constants for converting Values from British to Metric Measure.

The following constants, being the Board of Trade legal values, have been adopted in the preparation of tables below :—

1 inch = 2·54 centimetres.
7,000 grains = 453·592 grammes.
1 grain = 0·0648 grammes.
15·432 grains = 1 gramme.

The following constants are based on these :—

1 ohm per 1,000 yards = 1·09361 ohms per kilometre.
1 lb. per 1,000 yards = 0·496055 kgs. per kilometre.

(g) Definition of Annealed Wire.

The term "annealed" shall imply a wire which will satisfy the following requirements :—

A sample wire of 10 inches long shall be slowly and steadily stretched, and shall show without fracture a minimum elongation as follows :—

0·0076 inch diameter and under	15 per cent.
Above 0·0076 to 0·020 inch diameter ..	20 per cent.
Above 0·020 inch diameter	30 per cent.

(h) Formulæ for calculating Resistance of Wires.

The resistance of a plain standard annealed copper wire at a temperature of 60° F. is $\frac{1943 \cdot 53}{W.}$ ohms per 1,000 yards. The resistance of a tinned wire at a temperature of 60° F. from 0·0076 inch diameter up to and including 0·036 inch diameter is $\frac{1982 \cdot 40}{W.}$ ohms per 1,000 yards; and the resistance of a tinned wire 0·038 inch diameter and above is $\frac{1962 \cdot 97}{W.}$ ohms per 1,000 yards.

The value of the denominator (W) in each of the above vulgar fractions is the weight per yard, in grains, of the wire under test.

The weights per yard in grains of wires from 0·0010 to 0·0180 inch diameter are given in Table XXXIX.

TABLE XXXVIII.

British Standard Sizes of Annealed Copper Wires.

Superseded S.W.G. Size.	Standard Diameter.		Calculated Sectional Area.		Weight per 1,000 yards.	Standard Resistance at 6° F.		Current rating Amperes @	
	Inch.	*mm.*	*Square Inch.*	*Square mm.*	*Pounds.*	*Per 1000 yds. Ohms.*	*Per lb. Ohms.*	*1,000 per Sq. Inch*	*I.E.E.*
50	·0010	·0254	·0000007854	·0005067	·009083	30568	3365000	·0007	
49	·0012	·0305	·0000011310	·0007297	·013079	21230	1623000	·0011	
	·0014	*·0356*	*·0000015394*	*·0009932*	*·017803*	*15596*	*876000*	*·0015*	
48	·0016	·0406	·000002011	·0012972	·02325	11941	513500	·0020	
	·0018	*·0457*	*·000002545*	*·0016417*	*·02943*	*9435*	*320600*	*·0025*	
47	·0020	·0508	·000003142	·002027	·03633	7642	210300	·0031	
	·0022	*·0559*	*·000003801*	*·002452*	*·04396*	*6316*	*143660*	*·0038*	
46	·0024	·0610	·000004524	·002919	·05232	5307	101440	·0045	
	·0026	*·0660*	*·000005309*	*·003425*	*·06140*	*4522*	*73650*	*·0053*	
45	·0028	·0711	·000006158	·003973	·07121	3899	54750	·0061	
	·0030	*·0762*	*·000007069*	*·004560*	*·08175*	*3396*	*41550*	*·0070*	
44	·0032	·0813	·000008042	·005189	·09301	2985	32090	·0080	
43	·0036	·0914	·000010179	·006567	·11772	2359	20040	·0101	
42	·0040	·1016	·000012566	·008107	·14533	1910·5	13146	·0125	
41	·0044	·1118	·000015205	·009810	·17585	1578·9	8979	·0152	
40	·0048	·1219	·000018096	·011675	·2093	1326·7	6430	·0180	
39	·0052	·1321	·00002124	·013701	·2456	1130·5	4603	·0212	
	·0056	*·1422*	*·00002463*	*·015890*	*·2848*	*974·7*	*3422*	*·0246*	
38	·0060	·1524	·00002827	·018241	·3270	849·1	2597	·0282	
	·0064	*·1626*	*·00003217*	*·02075*	*·3720*	*746·3*	*2006*	*·0321*	

37	·0068	·1727	·00003632	·02343	·4200	661·1	1574·0	·0363
	·0072	*·1829*	*·00004072*	*·02627*	*·4700*	*589·7*	*1252·3*	*·0407*
36	**·0076**	**·1930**	**·00004536**	**·02927**	**·5246**	**529·2**	**1008·7**	**·0453**
	·0080	*·2032*	*·00005027*	*·03243*	*·5813*	*477·6*	*821·6*	*·0502*
35	·0084	·2134	·00005542	·03575	·6409	433·2	676·0	·0564
	·0088	*·2235*	*·00006082*	*·03924*	*·7034*	*394·7*	*561·2*	*·0608*
34	**·0092**	**·2337**	**·00006648**	**·04289**	**·7688**	**361·2**	**469·8**	**·0664**
	·0096	*·2438*	*·00007238*	*·04670*	*·8371*	*331·7*	*396·2*	*·0723*
33	·0100	·2540	·00007854	·05067	·9083	305·7	336·5	·0785
	·0104	*·2642*	*·00008495*	*·05481*	*·9824*	*282·6*	*287·7*	*·0849*
32	**·0108**	**·2743**	**·00009161**	**·05910**	**1·0594**	**262·1**	**274·4**	**·0916**
	·0112	*·2845*	*·00009852*	*·06356*	*1·1394*	*243·7*	*213·9*	*·0985*
31	·0116	·2946	·00010568	·06818	1·2222	227·2	185·87	·1056
	·0120	*·3048*	*·00011310*	*·07297*	*1·3079*	*212·3*	*162·30*	*·1131*
30	**·0124**	**·3150**	**·00012076**	**·07791**	**1·3966**	**198·80**	**142·35**	**·1207**
	·0130	*·3302*	*·00013273*	*·08563*	*1·5350*	*180·87*	*117·83*	*·1327*
29	·0136	·3454	·00014527	·09372	1·6800	165·27	98·37	·1452
	·0142	*·3607*	*·00015837*	*·10217*	*1·8315*	*151·60*	*82·77*	*·1583*
28	**·0148**	**·3759**	**·00017203**	**·11099**	**1·9895**	**139·55**	**70·14**	**·1720**
	·0156	*·3962*	*·00019113*	*·12331*	*2·210*	*125·61*	*56·82*	*·1911*
27	·0164	·4166	·0002112	·13628	2·443	113·65	46·52	·2112
	·0172	*·4369*	*·0002324*	*·14990*	*2·687*	*103·33*	*38·45*	*·2324*
26	**·018**	**·4572**	**·0002545**	**·16417**	**2·943**	**94·35**	**32·06**	**·2545**
	·019	*·4826*	*·0002835*	*·18292*	*3·279*	*84·68*	*25·82*	*·2835*

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The sizes printed in *italics* are *not* recommended for general use

(Continued on next page.)

TABLE XXXVIII.—British Standard Sizes of Annealed Copper Wires– (*continued*).

Superseded S.W.G. Size.	Standard Diameter.		Calculated Sectional Area.		Weight per 1,000 yards.	Standard Resistance at 60° F.		Current rating Amperes @	
	Inch.	mm.	Square Inch.	Square mm.	Pounds.	Per 1000 yds. Ohms.	Per lb. Ohms.	1,000 per Sq. Inch.	I.E.E.
25	·020	·5080	·0003142	·2027	3·633	76·42	21·03	·3142	
	·021	·5334	·0003464	·2235	4·006	69·31	17·305	·3464	
24	·022	·5588	·0003801	·2453	4·396	63·16	14·366	·3801	
	·023	·5842	·0004155	·2680	4·805	57·78	12·026	·4155	
23	·024	·6096	·0004524	·2919	5·232	53·07	10·144	·4524	
	·025	·6350	·0004909	·3167	5·677	48·91	8·615	·4909	
	·026	·6604	·0005309	·3425	6·140	45·22	7·365	·5309	
	·027	·6858	·0005726	·3964	6·621	41·93	6·333	·5726	
22	·028	·7112	·0006158	·3973	7·121	38·99	5·475	·6158	2·5
	·029	·7366	·0006605	·4261	7·639	36·35	4·758	·6605	
	·030	·7620	·0007069	·4560	8·175	33·96	4·155	·7069	
21	·032	·8128	·0008042	·5189	9·301	29·85	3·209	·8042	3·3
	·034	·8636	·0009079	·5858	10·500	26·44	2·518	·9079	
20	·036	·9144	·0010179	·6567	11·772	23·59	2·004	1·0179	4·0
	·038	·9652	·0011341	·7313	13·116	21·17	1·6140	1·1341	
19	·040	1·0160	·0012566	·8107	14·533	19·105	1·3146	1·2566	5·3
	·042	1·0668	·0013854	·9938	16·022	17·329	1·0815	1·3854	
	·044	1·1176	·0015205	·9810	17·585	15·789	·8978	1·5205	
	·046	1·1684	·0016619	1·0722	19·220	14·446	·7516	1·6619	
18	·048	1·2192	·0018096	1·1675	20·93	13·267	·6340	1·8096	7·2
	·050	1·2700	·0019635	1·2668	22·71	12·227	·5385	1·9635	
	·052	1·3208	·002124	1·3701	24·56	11·305	·4603	2·124	

	·054	*1·3716*	*·002290*	*1·4776*	*26·49*	*10·483*	*·3958*	*2·290*	
17	·056	1·4224	·002463	1·5890	28·48	9·747	·3422	2·463	9·8
	·058	*1·4732*	*·002642*	*1·7046*	*30·56*	*9·087*	*·2974*	*2·642*	
	·060	*1·5240*	*·002827*	*1·8241*	*32·70*	*8·491*	*·2597*	*2·827*	
16	**·064**	**1·6256**	**·003217**	**2·0755**	**37·20**	**7·463**	**·2006**	**3·217**	**12·9**
	·068	*1·7272*	*·003632*	*2·3430*	*42·00*	*6·611*	*·15740*	*3·632*	
15	·072	1·8288	·004072	2·6268	47·09	5·897	·12523	4·072	16·3
	·076	*1·9304*	*·004536*	*2·9267*	*52·46*	*5·292*	*·10087*	*4·536*	
14	**·080**	**2·0320**	**·005027**	**3·2429**	**58·13**	**4·776**	**·08216**	**5·027**	**19**
	·084	*2·1336*	*·005542*	*3·5753*	*64·09*	*4·332*	*·06760*	*5·542*	
	·088	*2·2352*	*·006082*	*3·9239*	*70·34*	*3·947*	*·05612*	*6·082*	
13	·092	2·3368	·006648	4·2888	76·88	3·612	·04698	6·648	23
	·096	*2·4384*	*·007238*	*4·6698*	*83·71*	*3·317*	*·03962*	*7·238*	
	·100	*2·5400*	*·007854*	*5·0671*	*90·83*	*3·057*	*·03365*	*7·854*	
12	**·104**	**2·6416**	**·008495**	**5·4805**	**98·24**	**2·826**	**·02877**	**8·495**	**28**
	·108	*2·7432*	*·009161*	*5·9102*	*105·94*	*2·621*	*·02474*	*9·161*	
	·112	*2·8448*	*·009852*	*6·3561*	*113·94*	*2·347*	*·02139*	*9·852*	
11	·116	2·9464	·010568	6·8183	122·22	2·272	·018587	10·568	32
	·120	*3·0480*	*·011310*	*7·2966*	*130·79*	*2·123*	*·016230*	*11·310*	
	·124	*3·1496*	*·012076*	*7·7911*	*139·66*	*1·9880*	*·014235*	*12·076*	
10	**·128**	**3·2512**	**·012868**	**8·3019**	**148·82**	**1·8657**	**·012537**	**12·868**	**35**
	·132	*3·3528*	*·013685*	*8·8289*	*158·26*	*1·7544*	*·011085*	*13·685*	
	·136	*3·4544*	*·014527*	*9·3721*	*168·00*	*1·6527*	*·009837*	*14·527*	
	·140	*3·5560*	*·015394*	*9·9315*	*178·03*	*1·5596*	*·008760*	*15·394*	
9	·144	3·6576	·016286	10·5071	188·34	1·4741	·007827	16·286	38

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(Continued on next page.)

TABLE XXXVIII.—British Standard Sizes of Annealed Copper Wires.—(*continued*).

Superseded S.W.G. Size.	Standard Diameter.		Calculated Sectional Area.		Weight per 1,000 yards.	Standard Resistance at 60° F.		Current rating Amperes @	
	Inch.	mm.	Square Inch.	Square mm.	Pounds.	Per 1000 yds. Ohms.	Per lb. Ohms.	1,000 per Sq. Inch.	I.E.E.
	·148	*3·7592*	*·017203*	*11·0989*	*198·95*	*1·3955*	*·007014*	*17·203*	
	·152	*3·8605*	*·018146*	*11·7070*	*209·9*	*1·3231*	*·006305*	*18·146*	
8	**·160**	**4·0640**	**·02011**	**12·9717**	**232·5**	**1·1941**	**·005135**	**20·11**	**44**
	·168	*4·2672*	*·02217*	*14·3013*	*256·4*	*1·0830*	*·004225*	*22·17*	
7	·176	4·4704	·02433	15·6958	281·4	·9868	·003507	24·33	48
	·184	*4·6736*	*·02659*	*17·1551*	*307·5*	*·9029*	*·002936*	*26·59*	
6	**·192**	**4·8768**	**·02895**	**18·6792**	**334·8**	**·8292**	**·002476**	**28·95**	**53**
5	·212	5·3848	·03530	22·7734	408·2	·6801	·0016661	35·30	60
4	**·232**	**5·8928**	**·04227**	**27·2730**	**488·9**	**·5679**	**·0011617**	**42·27**	**65**
3	·252	6·4008	·04988	32·1780	576·8	·4814	·0008345	49·88	74
2	**·276**	**7·0104**	**·05983**	**38·5990**	**691·9**	**·4013**	**·0005800**	**59·83**	**83**
1	·300	7·6200	·07069	45·6037	817·5	·3396	·0004155	70·69	92
1/0	**·324**	**8·2296**	**·08245**	**53·1921**	**953·5**	**·2912**	**·0003054**	**82·45**	**102**
2/0	·348	8·8392	·09511	61·3643	1100·0	·2524	·0002295	95·11	114
3/0	**·372**	**9·4488**	**·10869**	**70·1202**	**1256·9**	**·2209**	**·00017574**	**108·69**	**123**
4/0	·400	10·1600	·12566	81·0732	1453·3	·19105	·00013146	125·66	135
5/0	**·432**	**10·9728**	**·14657**	**94·5638**	**1695·1**	**·16379**	**·00009663**	**146·57**	**150**
6/0	·464	11·7856	·16909	109·0921	1955·5	·14198	·00007260	169·09	165
7/0	**·500**	**12·7000**	**·19635**	**126·6769**	**2271·0**	**·12227**	**·00005385**	**196·35**	**178**

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TABLE XL.

American Wire Tables. Details of Standard American Bare Annealed Copper Strands.*

B. & S. Gauge.	Circular Mills (nominal).	Number and Diameter (in inches) of Wires in Strand.	Diameter of Strand. Inches.	Weight in lbs. Per 1,000 feet.	Weight in lbs. Per mile.	Resistance (ohms) at 60° Fah. Per 1,000 feet.	Resistance (ohms) at 60° Fah. Per mile.
—	2000000	127/·125	1·625	6117	32300	0·005250	0·02772
—	1750000	91/·139	1·529	5417	28600	0·005924	0·03128
—	1500000	91/·128	1·408	4602	24300	0·006987	0·03689
—	1250000	91/·117	1·287	3839	20270	0·008367	0·04412
—	1000000	91/·105	1·155	3097	16350	0·01038	0·05480
—	950000	91/·102	1·122	2917	15400	0·01100	0·05817
—	900000	91/·100	1·100	2807	14820	0·01145	0·06044
—	850000	61/·118	1·062	2619	13830	0·01227	0·06477
—	800000	61/·114	1·026	2447	12920	0·01314	0·06940
—	750000	61/·111	·999	2320	12250	0·01398	0·07380
—	700000	61/·107	·963	2155	11380	0·01492	0·07876
—	650000	61/·103	·927	1994	10530	0·01610	0·08500
—	600000	61/·099	·891	1845	9740	0·01743	0·09204
—	550000	61/·095	·855	1699	8970	0·01893	0·09994
—	500000	37/·116	·812	1527	8060	0·02074	0·1095
—	450000	37/·110	·770	1371	7240	0·02292	0·1210
—	400000	37/·104	·728	1227	6480	0·02581	0·1363
—	350000	37/·097	·679	1066	5630	0·02966	0·1566
—	300000	37/·090	·630	918·6	4850	0·03443	0·1818
—	250000	37/·082	·574	762·1	4024	0·04152	0·2192
0000	211600	19/·105	·525	642·0	3390	0·05174	0·2732
000	167772	19/·094	·470	513·3	2710	0·06149	0·3247
00	133079	19/·084	·420	410·0	2165	0·07703	0·4067
0	105625	19/·075	·375	327·6	1730	0·09661	0·5101
1	83694	19/·066	·330	253·8	1340	0·1247	0·6586
2	66358	7/·097	·291	201·5	1064	0·1566	0·8272
3	52624	7/·087	·261	162·1	856	0·1947	1·028
4	41738	7/·077	·231	127·3	672	0·2487	1·313
5	33088	7/·069	·207	101·9	538	0·3096	1·635
6	26244	7/·061	·183	79·74	421	0·3960	2·091
8	16512	7/·048	·144	49·33	260·5	0·6398	3·378

Hard-drawn strands will have a resistance 2% higher than above.
Weights may vary 2% up or down.
Resistance may vary 2% up.

Note.—A Circular Mil is a unit of area equal to the area of a circle 1 mil in diameter. Such a circle has an area of 0·7854 $\left(\frac{\pi}{4}\right)$ square mil. Hence 1 circular mil equals 0·7854 sq. mil. A wire 10 mils in diameter has a cross-sectional area of 100 circular mils, or 78·54 square mils.

* Manufactured by the British Insulated & Helsby Cables, Ltd.

TABLE XLI.

American Wire Tables.

Dimensions, Woights, etc., of Pure Soft Copper Wire.

B. & S. Gauge.	Diameter Mils (*d*)	Area.		Weight.		Resistance at 60° F	
		Circular Mils. (d^2) 1 Mil. = ·001 in.	Sq inches (diam. in inches)² × ·7854.	Lbs. per 1,000 ft.	Lbs. per Mile.	Ohms per 1,000 ft.	Ohms per Mile.
000000	580·0	336400	0·264210	1017	5370	·03034	·1602
00000	516·5	266772	0·209520	806·6	4259	·03825	·2020
0000	460·0	211600	0·166190	639·8	3378	·04823	·2547
000	409·6	167772	0·131770	507·3	2678	·06082	·3212
00	364·8	133079	0·104520	402·4	2124	·07669	·4050
0	325·0	105625	0·082958	319·4	1686	·09661	·5102
1	289·3	83694	0·065733	253·0	1336	·1219	·6440
2	257·6	66358	0·052117	200·6	1059	·1538	·8122
3	229·4	52624	0·041331	159·1	840·1	·1939	1·024
4	204·3	41738	0·032781	126·2	666·3	·2445	1·291
5	181·9	33088	0·025987	100·0	528·2	·3085	1·629
6	162·0	26244	0·020612	79·35	419·0	·3890	2·054
7	144·3	20822	0·016354	62·96	332·4	·4901	2·588
8	128·5	16512	0·012969	49·92	263·6	·6181	3·264
9	114·4	13087	0·010279	39·57	208·9	·7798	4·118
10	101·9	10384	0·0081553	31·39	165·8	·9828	5·190
11	90·7	8226·5	0·0064611	24·87	131·3	1·241	6·551
12	80·8	6528·6	0·0051276	19·74	104·2	1·563	8·255
13	72·0	5184·0	0·0040715	15·69	82·78	1·969	10·40
14	64·1	4108·8	0·0032271	12·42	65·59	2·485	13·12
15	57·1	3260·4	0·0025607	9·858	52·05	3·130	16·53
16	50·8	2580·6	0·0020268	7·802	41·20	3·956	20·89
17	45·3	2052·1	0·0016117	6·204	32·76	4·973	26·26
18	40·3	1624·1	0·0012756	4·910	25·93	6·283	33·18
19	35·9	1288·8	0·0010122	3·897	20·57	7·917	41·81
20	32·0	1024·0	0·00080425	3·096	16·35	9·966	52·63
21	28·5	812·25	0·00063794	2·456	12·97	12·56	66·36
22	25·3	640·09	0·00050273	1·935	10·22	15·95	84·20
23	22·6	510·76	0·00040115	1·544	8·154	19·98	105·5
24	20·1	404·01	0·00031731	1·222	6·450	25·26	133·4
25	17·9	320·41	0·00025165	0·9688	5·115	31·85	168·2
26	15·9	252·81	0·00019856	0·7644	4·036	40·37	213·2
27	14·2	201·64	0·00015837	0·6097	3·219	50·62	267·3
28	12·6	158·76	0·00012469	0·4800	2·534	64·29	339·5
29	11·3	127·69	0·00010029	0·3861	2·038	79·74	422·1
30	10·0	100·00	0·000078540	0·3023	1·596	102·1	539·0
31	8·93	79·74	0·000062631	0·2411	1·273	128·0	675·9
32	7·95	63·20	0·000049639	0·1911	1·009	161·5	852·8
33	7·08	50·13	0·000039369	0·1516	0·8002	203·6	1075
34	6·30	39·69	0·000031173	0·1200	0·6336	257·1	1358
35	5·61	31·47	0·000024718	0·09515	0·5024	324·4	1713
36	5·00	25·00	0·000019635	0·07559	0·3991	408·3	2156
37	4·45	19·80	0·000015553	0·05987	0·3161	515·5	2722
38	3·96	15·68	0·000012316	0·04741	0·2503	650·8	3437
39	3·53	12·46	0·0000097868	0·03768	0·1989	818·8	4324
40	3·14	9·86	0·0000077437	0·02981	0·1574	1035·0	5466

TABLE XLII

Aluminium Wire.

S.W.G. Sizes.

Size of Wire	*Diam.*	*Sectional Area*	*Resistance at 20° C. (hard drawn)*		*Weight*		*Current for 50° F. rise*
			Ohms per 1,000 ft.	*Ohms per mile*	*Lbs. per 1,000 ft.*	*Lbs. per mile*	
S.W.G.	*Ins.*	*Sq. ins.*					*Amps*
7/0	·500	·196	0·069	0·366	231	1215	212
6/0	·464	·169	0·080	0·425	199	1050	193
5/0	·432	·147	0·092	0·487	173	909	176
4/0	·400	·126	0·108	0·570	148·	780	158
3/0	·372	·109	0·125	0·658	128	674	142
2/0	·348	·0951	0·143	0·755	112	590	129
0	·324	·0825	0·165	0·870	97·0	511	117
1	·300	·0707	0·192	1·015	83·0	438	106
2	·276	·0598	0·227	1·20	70·8	371	94
3	·252	·0499	0·272	1·44	59·7	309	83
4	·232	·0423	0·321	1·70	50·0	262	74
5	·212	·0353	0·385	2·03	41·0	219	65
6	·192	·0289	0·470	2·48	34·0	179	57
7	·176	·0243	0·560	2·96	28·6	151	50
8	·160	·0201	0·673	3·57	23·6	125	44
9	·144	·0163	0·830	4·40	19·1	101	38
10	·128	·0129	1·05	5·56	15·1	79·8	32
11	·116	·0106	1·28	6·77	12·4	65·6	28
12	·104	·00849	1·27	8·46	10·9	52·7	24
13	·092	·00665	2·04	10·8	7·8	41·2	20
14	·080	·00503	2·70	14·3	5·9	31·2	17
15	·072	·00407	3·33	17·6	4·8	25·3	14
16	·064	·00322	4·23	22·3	3·8	20·0	12
17	·056	·00246	5·50	29·2	2·89	15·3	10
18	·048	·00181	7·50	39·6	2·13	11·2	8·1
19	·040	·00126	10·8	57·0	1·48	7·80	6·3
20	·036	·00102	13·3	70·4	1·20	6·31	5·4
21	·032	·000804	16·9	89·3	0·94	4·99	4·9
22	·028	·000616	22·0	116·5	0·72	3·82	3·8
23	·024	·000452	30·0	159·0	0·53	2·80	3·1
24	·022	·000380	35·7	189·0	0·45	2·36	2·7
25	·020	·000314	43·4	229·0	0·37	1·95	2·4

The above resistances and weights are subject to a variation of 2 per cent.

TABLE XLIII.

Physical Properties of Aluminium and Copper.

		Aluminium.	Copper.
Chemical Symbol		Al.	Cu
Atomic Weight (oxygen = 16)		27·1	63·57
Position in Electro-chemical series		14	33
Melting Point ° Cent.		658	1083
" " ° Fahr.		1216	1981
Specific Heat (water = 1) at 20° C. cals. .		0·214	0·095
Specific Thermal Conductivity cals. per °C. per sq. cm. per cm. per sec.		0·504	0·895
Approximate Relative Heat Conductivity (silver = 100%)		50	90
Coefficient of Linear Expansion, per ° C. ..		0·000024	0·0000167
" " " " ° F. ..		0·0000133	0·00000945
Specific Gravity, rolled or drawn		2·71	8·89
Tensile Strength, Hard Drawn wire (No. 10 s.w.g.), lb./sq. in. ..		26,000	50,000
" " Annealed wire (No. 10 s.w.g.), lb./sq. in. ..		14,000	29,000
Modulus of Elasticity, lb. per sq. in.		10×10^6	$17·5 \times 10^6$
Elastic Limit as percentage of tensile strength ..		70 per cent.	75 per cent.
Specific Resistance in microhms per cm. cube at 20° C. (68° F.) ..	Annealed ..	2·8159	1·7241
	Hard Drawn	2·8735	1·7585
Specific Resistance in microhms per in. cube at 20° C. (68° F.) ..	Annealed ..	1·1086	0·6788
	Hard Drawn	1·1313	0·6924
Ohms per metre-gramme at 20° C. (68° F.)	Annealed ..	0·0762	0·1533
	Hard Drawn	0·0777	0·1564
Resistance of solid Conductor, 1,000 yds. long by 1 sq. in. cross section ohms	Annealed ..	0·0399	0·0244
	Hard Drawn	0·0407	0·0249
Coefficient of increase of Resistance with Temperature ..	Per ° C. ..	0·00390	0·00393
	Per ° F. ..	0·00218	0·0022221
Weight per 1,000 yds. by 1 sq. in. cross section lb.		3,510	11,520
For H.D. Conductors of equal resistance—			
Ratio of diameters		1·28	1·0
Ratio of sectional areas		1·64	1·0
Ratio of weight		0·50	1·0
For H.D. Conductors of equal temperature rise—			
Ratio of diameters		1·18	1·0
Ratio of sectional areas		1·39	1·0
Ratio of weight		0·424	1·0

Current Carrying Capacity of Aluminium Conductors.

Flat bars—$C = kA^{0.45}\ S^{0.5}$

where C is the current in amperes.

A is the sectional area in sq. ins.

S the perimeter in ins.

$k = 258$ for a 20°C. temperature rise, and 329 for a 30°C. temp. rise.

Round rod—$C = kD^{1.4}$

where D is the diameter in ins., and $k = 442$ for 20° C. rise and 563 for 30° C. rise.

Bare Cable—$C = kA^{0.45}\ D^{0.5}$

where A is the effective section of the aluminium in sq. ins. (neglecting the steel section in a steel-cored cable), D is the overall diameter in ins., and $k = 493$ for a 20° C. rise and 626 for a 30° C. rise.

Specific Inductive Capacity.

The Specific Inductive Capacity or Permittivity of a substance is the ratio of the capacity of a condenser when its plates are separated by this substance, to the capacity of a similar condenser when its plates are in a vacuum. The S.I.C. is taken as unity for a vacuum, but that of air at atmospheric pressure is very nearly unity, and may be taken as unity for all practical purposes.

The Specific Inductive Capacity is also called the Dielectric Constant or Dielectric Coefficient.

The dielectric constant of most dielectrics remains fairly constant up to frequencies of 10,000, after which it begins to drop, but in the case of good dielectrics such as ebonite, mica, etc., it varies very little over the whole range up to 1,000,000 cycles. In the case of gutta percha, however, the dielectric constant falls off rapidly at low frequencies, then becoming more constant.

It is essential, in measuring the dielectric constant, to use mercury electrodes so as to eliminate air films.

See Tables XLIV. and XLV.

Electric Strength.

Investigations made by the Electrical Research Association have shown that the methods of test, which have been employed in the past for the determination of electric strength, do not permit of comparison between different classes of dielectrics, and in consequence no reliable data are at present available. Figures for electric strength would be misleading, and for this reason are not given here.

For comparative purposes it is necessary that tests on the various materials should be carried out in a standardized manner, and the following conditions must be specified :—

Temperature, relative humidity, thickness of material, time and manner of application of the potential, size and shape of electrodes.

The Electrical Research Association has such a specification under consideration.

For further data, see page 1, Vol. II.

Power Losses in Insulating Materials.

From researches carried out by the American Telephone and Telegraph Co. and the Western Electric Co. Inc.,* it has been shown that a satisfactory measure of power loss in a dielectric is the product of phase angle and dielectric constant. In the case of ordinary insulation, where the object is to provide a mechanical support or separator, this product is a true measure of the energy loss per unit volume. In the case of a condenser, where the object is to obtain a given capacity, the phase difference alone determines the power loss, since in this case the effect of the smaller volume of dielectric is exactly balanced by the smaller volume of dielectric required. The data in Table XLIV were obtained by the resistance variation method as given in the Bureau of Standards Circular No. 74, mercury electrodes being used.

TABLE XLIV.

DIELECTRIC CONSTANT, PHASE DIFFERENCE AND THEIR PRODUCT, AT VARIOUS FREQUENCIES.

Material.	*Frequency Cycles per sec.*	*Dielectric Constant.*	*Phase Difference. Degrees.*	*Product.*
Phenol Fibre A ..	295,000	5·9	2·9	17·1
	500,000	5·8	2·9	16·8
	670,000	5·7	2·9	16·5
	1,040,000	5·6	3·3	18·5
Phenol Fibre B ..	190,000	5·8	2·2	12·7
	500,000	5·6	2·5	14·0
	675,000	5·6	2·6	14·6
	975,000	5·6	2·8	15·7
Phenol Fibre C ..	200,000	5·4	2·1	11·3
	395,000	5·4	2·2	11·8
	685,000	5·3	2·3	12·2
	975,000	5·2	2·4	12·5
Phenol Fibre D ..	194,000	5·4	4·2	22·7
	500,000	5·2	3·9	20·3
	695,000	5·2	3·9	20·3
	1,000,000	5.1	3.8	19.4
Wood (Oak)	300,000	3·2	2·1	6·7
	425,000	3·3	2·0	6·6
	635,000	3·3	2·2	7·3
	1,060,000	3·3	2·4	7·9
Wood (Maple) ..	500,000	4·4	1·9	8·4
Wood (Birch) ..	500,000	5·2	3·7	19·2
Hard Rubber ..	210,000	3·0	0·5	1·5
	440,000	3·0	0·5	1·5
	710,000	3·0	0·5	1·5
	1,126,000	3·0	0·6	1·8
Flint Glass	500,000	7·0	0·24	1·68
	720,000	7·0	0·24	1·68
	890,000	7·0	0·23	1·61
Plate Glass	500,000	6·8	0·4	2·7
Cobalt Glass	500,000	7·3	0·4	2·9
Pyrex Glass	500,000	4·9	0·24	1·18

* The Bell System Technical Journal, Vol. I, No. 2, Nov. 1922.

TABLE XLV.

Specific Inductive Capacity.

Substance	*S.I.C.*	*Substance*	*S.I.C.*
SOLIDS.		*LIQUIDS.*	
Asphalt	2·5	Alcohol, methyl	35·4
Bakelite	4 to 6	,, ethyl, 14·7°	26·8
Calcite	7·5	,, amyl 20°	16·0
Carbon	5·5	Benzine 20° C.	2·28
Cambric, Varnished	4·5—5·5	,, 74° C.	2·18
Celluloid	4 to 16	Benzol	2·29
Ebonite	2—3·5	Bisulphide of Carbon, 11° C.	1·9—2·2
Fibre (horn) dry	2·5	Bromine	3·2
,, ,, oiled	4·5—5·5	Carbon Tetrachloride, 18° ..	2·25
Fluorite	6·8	Chloroform	5·2
Glass, common	3—3·25	Ethyl Acetate	6·7
,, flint	6—7	,, Chloride	10·9
,, plate	5—8	,, Ether, α = ·005 ..	4·4
Guttapercha	4·2	Glycerine	39
,, high frequency	2·46	Nitrobenzine, 17°	34
Gypsum	6·3	Oil, Castor	4·8
Ice (— 2° C.)	93·9	,, Linseed	3·5
Indiarubber pure	2·12	,, Olive	3·2
,, vulcanized ..	2·6—2·9	,, Paraffin	0—2·6
,, hard	3	,, Sperm	3
Ivory	6·9	,, Transformer	2—2·25
Marble..	8·3	Petroleum Spirit	2·1
Mica (Muscovite)	7	Toluene, α = ·001	2·3
,, (amber normal) ..	6	Turpentine, commercial ..	2·23
,, (,, silver)	5	,, oil of	1·95
Paper, dry	2—2·8	Water, at 14° C.	83·8
,, paraffined	3·7	,, at 25° C.	75·7
,, oiled	4	Xylene	2·4
Paraffin wax	2—2·3		
Pitch	1·8		
Porcelain	4·4—6·8	*GASES.*	
Pressboard, dry	2·5—4		
,, oiled	4·7—7	Air at about 760 mm. pressure	1·000580
Quartz..	3·5—4·5	Vacuum	1·00000
Resin	2·5	Carbon dioxide (760 mm.) ..	1·000989
Rock Salt	5·6	Chlorine (liquid)	1·97
Selenium (16°)	6·3	Helium (760 mm.)	1·000074
Shellac, h.f.	2·75	Hydrogen (760 mm.) ..	1·000282
,, ...	2·75—3·73	,, (liquid)	1·21
Silk (density 1·51)	4·6—4·9	Nitrogen (760 mm.) ..	1·000606
Silica, fused	3·5—3·8	,, (liquid)	1·42
Slate	6·5—7·5	Nitrous oxide (760 mm.) ..	1·001129
Sulphur	3—4	Oxygen (760 mm.)	1·000547
Sylvin	4·9	,, (liquid)	1·46
Vaseline	2·2		
Varnished cloth, yellow ..	5·5		
,, ,, black ..	4·5		
Wood (treated)	3—3·5		

TABLE XLVI.

Volume and Surface Resistivity of Solid Dielectrics.*

Material.	*Volume Resistivity at 22° C. Ohms/cm.**	*Surface Resistivity at 75% humidity.*
Amberite	5×10^{14}	2×10^{14}
Bakelite (¹)	2×10^{11}	2×10^{9}
Bakelite (²)	2×10^{7}	8×10^{10}
Bakelite (³)	2×10^{16}	3×10^{11}
Bakelite Micarta	5×10^{10}	2×10^{8}
Beeswax, yellow	2×10^{15}	7×10^{14}
Celluloid	2×10^{10}	2×10^{10}
Condensite	4×10^{10}	2×10^{11}
Dielectrite	5×10^{13}	1×10^{8}
Duranoid	3×10^{16}	9×10^{10}
Fibre, hard	2×10^{10}	9×10^{8}
,, red	5×10^{9}	2×10^{9}
Galalith, black	2×10^{10}	4×10^{10}
,, white	1×10^{10}	8×10^{8}
Glass, German	5×10^{13}	5×10^{9}
,, plate	2×10^{13}	6×10^{9}
Gummon	3×10^{12}	2×10^{10}
Hard Rubber	1×10^{18}	2×10^{11}
Ice (at 0° C.)	7×10^{7}	—
Ivory	2×10^{8}	7×10^{8}
Marble, Italian	1×10^{11}	1×10^{8}
Mica, clear	2×10^{17}	1×10^{11}
,, Black Spotted African	4×10^{15}	3×10^{10}
,, Brown African, clear	2×10^{15}	7×10^{10}
,, Indian ruby, stained	5×10^{13}	6×10^{8}
,, ,, ,, slightly stained ..	5×10^{14}	—
Moulded Mica	1×10^{16}	1×10^{11}
Paraffin wax	1×10^{14}	9×10^{13}
Porcelain, unglazed	3×10^{14}	4×10^{9}
,, glazed	—	3×10^{9}
Quartz	2×10^{14}	4×10^{13}
,, fused	over 5×10^{18}	4×10^{13}
Redmanite	2×10^{14}	5×10^{13}
Rosin	5×10^{16}	4×10^{14}
Sealing wax	8×10^{15}	6×10^{13}
Shellac	1×10^{16}	8×10^{11}
Slate	1×10^{8}	4×10^{7}
Stabalite	3×10^{18}	8×10^{9}
Sulphur	1×10^{17}	4×10^{15}
Tetrachlornaphthalene	5×10^{16}	1×10^{14}
Wood, paraffined		
,, Mahogany	4×10^{13}	4×10^{11}
,, Maple	3×10^{10}	8×10^{10}
,, Poplar	5×10^{11}	1×10^{11}
,, Walnut	5×10^{8}	—
Water (distilled)	7×10^{10}	—

* U.S. Bureau of Standards. Scientific Paper, No. 234, by H. L. Curtis.

Note.—(¹) Filling material, paper; condensing agent, Ammonia.
(²) Filling material, vegetable fibre; condensing agent, Caustic Soda.
(³) Filling material, vegetable fibre; condensing agent, Ammonia.

The catalytic agent has a marked influence on the final product, and the binding material influences the surface resistivity.

TABLE XLVII.

Electro-Chemical Equivalents.

(The electro-chemical equivalent of an element is proportional to its chemical equivalent or combining weight, and the values in Column 6 of the Table following are based on the atomic weights given in Column 3, and on the determination of the electro-chemical equivalent of silver.

The last four columns have been calculated from Column 6, in order to facilitate calculations in practical electro-metallurgical work).

Element	*Symbol.*	*Atomic weight.*	*Valency.*	*Chemical equivalent.*	*Electro-chem equivalent (gm. per coulomb)*	*Lbs. per 1,000 ampere-hours.*	*Ampere hours per lb.*	*Kg. per 1,000 ampere-hours.*	*Ampere hours per kg.*
1	2	3	4	5	6	7	8	9	10
ELECTRO-POSITIVE—									
Copper (cuprous)	Cu	63·0	1	63·0	0·000 6542	5·192	192·6	2·355	424·6
Hydrogen	H	1	1	1	0·000 01038	0·08242	12130	0·03738	2·675
Mercury (mercurous) ..	Hg	199·8	1	199·8	0·002 075	16·47	60·73	7·469	133·9
Potassium	K	39·04	1	39·04	0·000 4054	3·218	310·8	1·459	685·2
Silver	Ag	107·66	1	107·66	0·001 118	8·873	112·7	4·025	248·5
Sodium	Na	22·99	1	22·99	0·000 2387	1·895	527·8	0·8595	1164
Copper (cupric)	Cu	63·0	2	31·5	0·000 3271	2·596	385·2	1·178	849·2
Iron (ferrous)	Fe	55·9	2	27·95	0·000 2902	2·304	434·1	1·045	957·0
Lead	Pb	206·4	2	103·2	0·001 072	8·506	117·6	3·858	259·2
Magnesium	Mg	23·94	2	11·97	0·000 1243	0·9866	1014	0·4475	2235
Mercury (mercuric) ..	Hg	199·8	2	99·9	0·001 037	8·234	121·5	3·735	267·8
Nickel	Ni	58·6	2	29·3	0·000 3043	2·415	414·1	1·085	912·9
Tin (stannous)	Sn	117·8	2	58·9	0·000 6116	4·854	206·0	2·202	454·2
Zinc	Zn	64·9	2	32·45	0·000 3370	2·674	373·9	1·213	824·3
Aluminium	Al	27·3	3	9·1	0·000 09450	0·7500	1333	0·3402	2940
Gold	Au	196·2	3	65·4	0·000 6791	5·391	185·5	2·445	409·0
Iron (ferric)	Fe	55·9	3	18·63	0·000 1935	1·536	651·2	0·6966	1436
Tin (stannic)	Sn	117·8	4	29·45	0·000 3058	2·427	412·0	1·101	908·3
ELECTRO-NEGATIVE—									
Bromine	Br	79·75	1	79·75	0·000 8282	6·573	152·1	2·981	335·4
Chlorine	Cl	35·37	1	35·37	0·000 3673	2·915	343·0	1·322	756·3
Iodine	I	126·53	1	126·53	0·001 314	10·43	95·87	4·730	211·4
Oxygen	O	15·96	2	7·98	0·000 08287	0·6577	1520	0·2983	3352
Nitrogen	N	14·01	3	4·67	0·000 04850	0·3849	2958	0·1746	5728

Electro-Chemical Series of Metals.

1.—Potassium +	7—Cadmium	13—Copper
2—Sodium	8—Nickel	14—Mercury
3—Magnesium	9—Cobalt	15—Silver
4—Manganese	10—Lead	16—Platinum
5—Zinc	11—Tin	17—Gold
6—Iron	12—Bismuth	18—Antimony —

Primary Cells.

The principal characteristics of primary cells are given in Table XLVIII.

Two-fluid Chromic Acid Cell.

In the single-fluid type of bichromate cell there is local action between the zinc and the chromic acid, the zinc being consumed even when amalgamated. In the Fuller cell, used in the U.S.A., the amalgamated zinc is immersed in a sulphuric acid solution contained in a porous pot, the chromic acid solution and carbons being in the outer chamber, whereby the zinc is not appreciably consumed.

Standard Cells.

The Clark Cell.—This cell is a B.O.T. standard. There are a large number of modifications in existence, but in principle they do not vary. Its principal defect is its large temperature coefficient (0·0011). Professor Cathcart's form of Clark cell has a temperature coefficient of only 0·00038 per °C. The Hibbert cell consists of chlorides instead of sulphates of zinc and ,yrucrem and has a temperature coefficient of 0·0011 per °C.

The formula for calculating the E.M.F. of a standard Clark cell at various temperatures is :—

$E_t = 1{\cdot}4326 - 11{\cdot}9\ (t - 15)\ 10^{-4} - 0{\cdot}07\ (t - 15)^2\ 10^{-4}$, where 1·4326 is the voltage at 15° C.

TABLE XLVIII.

Characteristics of Primary Cells.

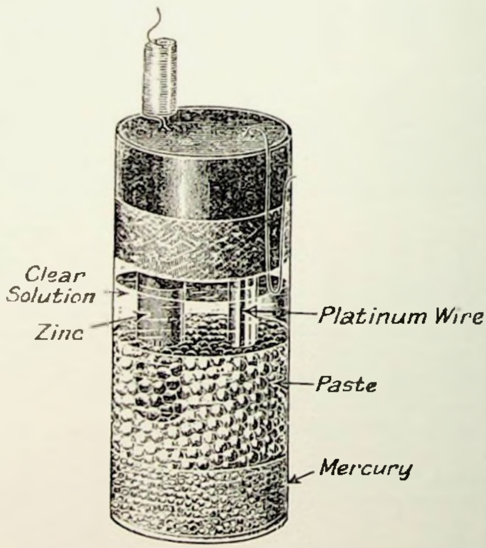
Name of Cell.	E.M.F.	Anode.	Cathode.	Electrolyte.	Depolariser.	Internal Resistance.	Chemical Reaction.
Clark	1·433 at 15° C.	Zinc	Mercury	Zinc Sulphate, or Chloride	Mercurous Sulphate, or Chloride	·3—·5	$Zn + Hg_2SO_4 = ZnSO_4 + 2Hg$
Weston ..	1·083 at 20° C.	Cadmium	Mercury	Cadmium Sulphate	Mercurous Sulphate	900	$Cd + CdSO_4 = CdSO_4 + 2Cd$
Bunsen ..	1·9	Zinc	Carbon	Dilute H_2SO_4	Strong Nitric Acid	·1—·2	$Zn + H_2SO_4 + 2HNO_3 = ZnSO_4 + 2H_2O + 2NO_2$
Grove ..	1·9	Zinc	Platinum	Dilute H_2SO_4	Strong Nitric Acid	·1—·2	Same as for Bunsen
Leclanché ..	1·47	Zinc	Carbon	Salammoniac	Manganese Dioxide	1—3	$Zn + 2NH_4Cl + 2MnO_2 = ZnCl_2 + 2NH_3 + H_2O + Mn_2O_3$
Bichromate ..	2·0	Zinc	Carbon	Dilute H_2SO_4	Bichromate of Potash	·5—1·5	$3Zn + 2CrO_3 + 6H_2SO_4 = Cr_2(SO_4)_3 + 3ZnSO_4 + 6H_2O$
Daniell ..	1·1	Zinc	Copper	Zinc Sulphate	Copper Sulphate	2—5	$Zn + CuSO_4 = ZnSO_4 + Cu$
Dry	1·5	Zinc	Carbon	Salammoniac	Manganese Dioxide	·1—·5	Same as for Leclanché
Edison-Lelande	0·95	Zinc	Copper-Oxide	Sodium or Potassium Hydroxide	Cuprous Oxide	·2—1·0	$Zn + 2NaOH + CuO = Na_2ZnO_2 + H_2O + Cu$

Fig. 1 shows a standard Clark cell, and Table XLIX gives the E.M.F. at various temperatures.

TABLE XLIX.

E.M.F. OF CLARK AND WESTON CELLS AT VARIOUS TEMPERATURES.

Temp. ° C.	*Clark.* *volts.*	*Weston.* *volts.*	*Temp.* ° C.	*Clark.* *volts.*	*Weston.* *volts.*	*Temp.* ° C.	*Clark.* *volts.*	*Weston.* *volts.*
5	1·4436	1·0187	12	1·4359	1·0186	19	1·4282	1·0183
6	1·4425	1·0187	13	1·4348	1·0185	20	1·4271	1·0183
7	1·4414	1·0187	14	1·4337	1·0185	21	1·4260	1·0183
8	1·4403	1·0187	15	1·4326	1·0184	22	1·4249	1·0182
9	1·4392	1·0186	16	1·4315	1·0184	23	1·4238	1·0182
10	1·4381	1·0186	17	1·4304	1·0184	24	1·4227	1·0181
11	1·4370	1·0186	18	1·4293	1·0184	25	1·4216	1·0181



[*H. Tinsley & Co.*

Fig. 1.—Clark Standard Cell, 3″ × 1″.

The Weston Cell.

The Weston or Cadmium cell is also a B.O.T. standard and is used in preference to the Clark cell. It has a temperature coefficient of only about 0·00004 per °C., and is depolarized much more quickly than the Clark cell. It has a large temperature-time lag.

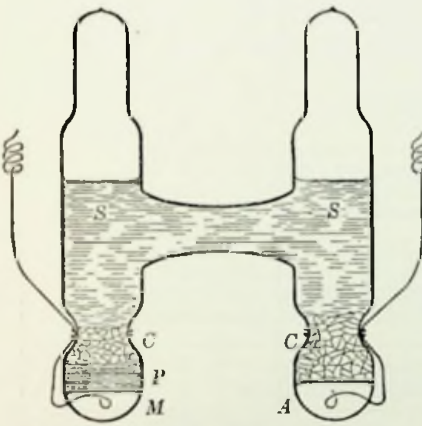
The E.M.F. at various temperatures is calculated from the formula:—

$$E_t = 1{\cdot}0183 - 40{\cdot}6\,(t - 20)\,10^{-6} - 0{\cdot}95\,(t - 20)^2\,10^{-6} + 0{\cdot}01\,(t - 10)^3\,10^{-6}$$

where 1·0183 = the voltage at 20° C.

Table XLIX gives the E.M.F. at various temperatures.

Fig. 2 shows a Standard Weston cell.



M = Mercury.

P = Mercurous Sulphate Paste.

C = Crystals of Cadmium Sulphate.

S = Saturated Solution of Cadmium Sulphate.

A = Cadmium and Mercury Amalgam.

Fig. 2.—Weston Cell.

TABLE L.

SPECIFIC GRAVITY, SPECIFIC RESISTANCE AND TEMPERATURE COEFFICIENT OF ELECTROLYTES.

ρ = *resistivity in ohms per cm.³ at 18° C.*

%Strength	Sulphuric Acid. H_2SO_4			Nitric Acid. HNO_3			Hydrochloric Acid. HCl			Silver Nitrate. $AgNO_3$			Potassium Hydrate. KOH		
by weight.	ρ	Temp. Coeff.* %per °C.	Sp. Gr.	ρ	Temp. Coeff.* %per °C.	Sp. Gr.	ρ	Temp. Coeff.* %per °C.	Sp. Gr.	ρ	Temp. Coeff.* %per °C.	Sp. Gr.	ρ	Temp. Coeff.* %per °C.	Sp. Gr.
5	4·82	1·2	1·033	3·90	1·5	1·029	2·55	1·6	1·0242	39·3	2·2	1·043	5·84	1·9	1·045
10	2·57	1·3	1·069	2·18	1·4	1·058	1·59	1·6	1·0490	21·4	2·2	1·090	3·19	1·9	1·092
15	1·65	1·4	1·105	1·64	1·4	1·089	1·35	1·6	1·0744	14·7	2·2	1·141	2·36	1·9	1·141
20	1·54	1·5	1·143	1·41	1·4	1·121	1·32	1·5	1·1001	11·6	2·1	1·197	2·01	2·0	1·191
25	1·40	1·5	1·182	1·31	1·4	1·154	1·39	1·5	1·1262	9·50	2·1	1·257	1·86	2·1	1·244
30	1·36	1·6	1·223	1·28	1·4	1·187	1·52	1·5	1·1524	8·11	2·1	1·323	1·85	2·3	1·295
35	1·39	1·7	1·264	1·31	1·4	1·220	1·70	1·5	1·1775	7·18	2·1	1·396	1·97	2·4	1·349
40	1·48	1·8	1·307	1·37	1·5	1·253	1·95	1·5	1·2007	6·44	2·1	1·479	2·23	2·7	1·406
50	1·87	1·9	1·399	1·59	1·6	1·320	—	—	—	5·44	2·1	1·677	—	—	1·528
60	2·70	2·1	1·503	1·96	1·6	1·377	—	—	—	4·80	2·1	1·919	—	—	—
70	4·66	2·6	1·616	2·54	1·5	1·424	—	—	—	—	—	—	—	—	—
80	9·13	3·5	1·733	3·76	1·3	1·461	—	—	—	—	—	—	—	—	—

%Strength	Zinc Sulphate. $ZnSO_4$			Magnesium Sulphate. $MgSO_4$			Copper Sulphate. $CuSO_4$			Sodium Chloride. $NaCl$			Sal-ammoniac. NH_4Cl		
by weight.	ρ	Temp. Coeff.* %per °C.	Sp. Gr.	ρ	Temp. Coeff.* %per °C.	Sp. Gr.	ρ	Temp. Coeff.* %per °C.	Sp. Gr.	ρ	Temp. Coeff.* %per °C.	Sp. Gr.	ρ	Temp. Coeff.* %per °C.	Sp. Gr.
5	52·3	2·2	1·052	39·3	2·3	—	52·3	2·2	1·050	14·9	2·2	1·0354	10·9	2·0	—
10	31·4	2·3	1·108	24·1	2·4	—	31·4	2·3	1·103	8·33	2·1	1·0724	5·67	1·8	—
15	24·1	2·3	1·168	20·9	2·5	—	24·1	2·3	1·161	6·15	2·1	1·1105	3·89	1·7	—
20	21·9	2·4	1·236	20·9	2·7	—	—	—	—	5·14	2·2	1·1501	2·98	1·6	—
25	21·4	2·6	1·307	24·1	2·9	—	—	—	—	4·70	2·3	1·1913	2·50	1·5	—
30	22·9	3·0	1·382	—	—	—	—	—	—	—	—	—	—	—	—
35	[illegible]	[illegible]	—	—	—	—	—	—	—	—	—	—	—	—	—

* The temperature coefficient is negative.

Note.—The resistivity of a 5% solution of Alum $KAl(SO_4)_2$ at 18° C. is 39·3 ohms per c.c., and the temp. coeff. is 2% per ° C.

I.E.C. Temperature Limits for Electrical Machinery.

Table LI gives the temperature limits and temperature rises for electrical machinery as specified by the International Electrotechnical Commission.*

TABLE LI.

I.E.C. Temperature Limits.

Nature of Insulation of Winding, or name of part.	*Highest permissible observable temperature.* ° C.	*Highest permissible temperature rise.* ° C.
Cotton, paper or silk, non-impregnated[1] ..	80	40
„ „ „ impregnated[1] ..	95	55
„ „ „ immersed in oil ..	95	55
Enamelled wire[1]	95	55
Mica, asbestos, glass, porcelain, micanite and similar compositions	115	75
Insulated windings permanently short-circuited	100	60
Non-insulated windings permanently short-circuited	110	70
Commutator slip rings	90	50
Bearings	80	40
Iron core immersed in oil	95	55
„ „ in contact with windings.. ..	Same as the windings.	
„ „ not in contact with windings nor immersed in oil, shall not exceed that of the windings and in no case shall it exceed	110	70

[1] *Note.*—Single-layer windings. An increase of 5°C. is permitted in the case of coils, revolving or stationary, with single layer windings when not immersed in oil.

Miscellaneous Constants.†

Elementary electrical charge, charge on electron	$e = 4{\cdot}774 \times 10^{-10}$ esu (M)
½ charge on α particle	$= 1{\cdot}591 \times 10^{-20}$ emu
Mass of an electron	$m = 9{\cdot}01 \times 10^{-28}\ g$
Radius of an electron	about 2×10^{-13} cm.
Ratio e/m, small velocities	$e/m = 1{\cdot}766 \times 10^{7}$ emu g^{-1}
Number of molecules per gram molecular weight (Avogadro constant)	$N = 6{\cdot}062 \times 10^{23}$ (M)

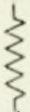
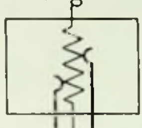
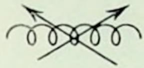
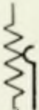
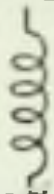
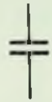
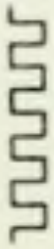
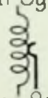
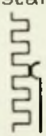
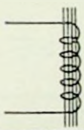
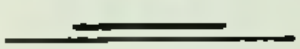
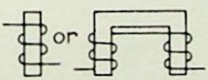
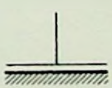
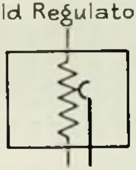
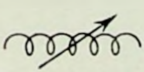
* Copies of the I.E.C. publications may be obtained from 28 Victoria Street, S.W.1.

† Smithsonian Physical Tables, 1920.

Number of gas molecules per cm^3, 76 cm. 0° C. (Loschmidt's number)	$n = 2{\cdot}705 \times 10^{19}$ (M)
Number of gas molecules per cm^3, 0° C. at 1×10^6 bars..	$= 2{\cdot}670 \times 10^{19}$
Kinetic energy of translation of a molecule at 0° C.	$E_o = 5{\cdot}621 \times 10^{-14}$ erg (M)
Constant of molecular energy E_o/T = change of translational energy per °C.	$\epsilon = 2{\cdot}058 \times 10^{-16}$ erg/°C (M)
Mass of hydrogen atom	$= 1{\cdot}662 \times 10^{-24}$ *g* (M)
Radius of hydrogen molecule, about ..	10^{-8} cm.
Mean free path, ditto, 76 cm., 0° C., about	$L = 1{\cdot}6 \times 10^{5}$ cm/sec.
Sq. rt. mean sq. velocity, ditto, 76 cm., 0° C.	$G = 1{\cdot}84 \times 10^{5}$ cm/sec.
Arithmetical average velocity, ditto, 76 cm., 0° C.	$\Omega = 1{\cdot}70 \times 10^{5}$ cm/sec.
Average distance apart of molecules, 76 cm., 0° C.	$= 3 \times 10^{-7}$ cm.
Boltzmann gas constant = constant of entropy equation = $R/N = p_oV_o/TN = (\frac{2}{3})\,\epsilon$	$k = 1{\cdot}372 \times 10^{-16}$ erg/°C.
Volume per mol (*e*) or gram-molecular weight of ideal gas, 76 cm., 0° C., ($1{\cdot}01323 \times 10^6$ bars)	= 22·412 litres.
Ditto, 1×10^6 bars, 0° C. (75 cm. H*g*) ..	= 22·708 litres.
Gas constant: $PV_m = RT$, V_m = vol. molec. wt, in g when P in g/cm.^{2}, V_m in cm^3	R = 84·780 *g*—cm./°C.
when P in atmospheres, V_m in litres ..	R = 0·08204 *l*—atm/°C.
when P in dynes, V_m in cm^3	$R = 8{\cdot}315 \times 10^7$ ergs/°C.
Absolute zero = 0° Kelvin	= — 273·13 °C.
1 bar = 10^6 dynes/cm.^{2} = 1·013 kg/cm^2	= 0·987 atmosphere.
Mechanical equivalent of heat, 1g (20°C.) cal.	$= 4{\cdot}184 \times 10^7$ ergs. = 4·184 Joules.
Faraday constant	F = 96494 coulombs.
Velocity of light *in vacuo*	$C = 2{\cdot}99860 \times 10^{10}$ cm/sec.
Planck's element of action	$h = 6{\cdot}547 \times 10^{27}$ erg. cm. (M)
Rydberg's fundamental frequency ..	$V_o = 3{\cdot}28880 \times 10^{15}$ sec.$^{-1}$.
Rydberg's constant, V_o/C	N = 109678·7
Wien's constant of spectral radiation ..	$C_2 = 1{\cdot}4312$ for λ in cm. (M)
Stefan-Boltzmann constant of total radiation	$\sigma = 5{\cdot}72 \times 10^{-12}$ watt/cm.^{2} (M)
Grating space in calcite	*d* = 3·030 A
Grating space in rock-salt (Uhler, Cooksey)	$= 2{\cdot}814 \times 10^{-8}$ cm.
P.D. in volts for X-rays of wave length λ in cm = V λ = *hc*/*e*	$= 1{\cdot}241 \times 10^{-4}$ volt cm.

Note.—(M) = Millikan, *Phil. Mag.* 34, I, 1917.)

British Standard Graphical Symbols for resistances, rheostats, inductance and capacity.

Fixed Resistance,  (practically non-inductive.)	Field Regulator,  Potentiometer Type.	Inductive Regulator, (up and down.) 
Variable Resistance.  (practically non-inductive.)	Inductance, (General Symbol.)  also, Fixed Air Core Inductance.	Condenser. 
Fixed Non-inductive Resistance. 	Variable Inductance, (General Symbol.)  also, Variable Air Core Inductance.	Variable Condenser. 
Variable Non-inductive Resistance. 	Iron Core Inductance, (Choke Coil.) 	Distributed Capacity. 
Liquid Resistance. 	Electro-Magnet. 	Earth Capacity. 
Field Regulator. 	Inductive Regulator. 	

INTERNATIONAL SYMBOLS.

TABLE LII. Quantities.

Name of quantity.	*Symbol.*	*Symbols recommended for case in which principal symbol is not suitable.*	*Name of quantity.*	*Symbol.*	*Symbols recommended for case in which principal symbol is not suitable.*
1. Length	l	In dimensional equations the capital letters L, M, T, are to be employed.	20. Resistivity	ρ	
2. Mass	m		21. Conductance	G	
3. Time	t		22. Quantity of electricity ..	Q	
4. Angles	a, β, γ		23. Flux-density, electrostatic	D	
5. Acceleration of gravity	g		24. Capacity	C	
6. Work	A	W	25. Dielectric constant ..	ϵ	
7. Energy	W	U†	26. Self-inductance ..	L	
8. Power	P		27. Mutual inductance ..	M	
9. Efficiency	η		28. Reactance	X	
10. No. of turns in unit of time	n		29. Impedance	Z	Script, heavy-faced or special type.
11. Temp. Centigrade ..	t	θ, δ	30. Reluctance	S	
12. Temperature absolute ..	T	Θ	31. Magnetic flux	Φ	
13. Period	T		32. Flux-density, magnetic	B	
14. $2\pi/T$	ω		33. Magnetic field	H	
15. Frequency	f	ν	34. Intensity of magnetisation	J	
16. Phase displacement ..	ϕ		35. Permeability	μ	
17. Electromotive force ..	E		36. Susceptibility	κ	
18. Current	I		37. Difference of potential, electric	V	U
19. Resistance	R		38. Magnetomotive force ..	*	

* Symbol to be proposed by the National Committees.

† This symbol is used in thermo-dynamics.

Radiation Wave-length Limits.

Hertzian waves, longest	1,000,000·0 cm.
,, ,, shortest	0·2 cm.
Infra-red, longest, restrahlung, focal isolation	0·03 cm.
Infra-red, spectroscopically studied	0·002 cm.
Visible, longest	0·00008 cm.
,, shortest	0·00004 cm.
Ultra-violet, Lyman, shortest[1]	0·000006 cm.
X-rays, longest	0·00000012 cm.
,, shortest	0·000000001 cm.
γ rays longest	0·000000013 cm.
,, shortest	0·0000000007 cm.

TABLE LIII.

Units.—Signs for Names of Units.

Signs for names of electrical units to be employed only after numerical values :—

Name of Unit.	*Sign.*
1. Ampere	A
2. Volt	V
3. Ohm	*
4. Coulomb	C
5. Joule	J
6. Watt	W
7. Farad	F
8. Henry	H
9. Volt-coulomb	VC
10. Watt-hour	Wh
11. Volt-ampere	VA
12. Ampere-hour	Ah
13. Milliampere	mA
14. Kilowatt	kW
15. Kilovolt-ampere	kVA
16. Kilowatt-hour	kWh

* As a sign for the ohm, one of the two letters O or Ω is provisionally recommended. The letter Ω should no longer be used for megohm.

[1] 0·0000032 cm. (*Millikan-Sawyer*, 1919).

TABLE LIV.

Mathematical Symbols and Rules.

Name.	*Symbol.*	*Symbols recommended for case in which principal symbol is not suitable.*
Total differential	d	*d*
Partial differential	δ	
Base of Napierian logarithms	e	ε
Imaginary $\sqrt{-1}$	i	ȷ
Ratio of circumference to diameter ..	π	
Summation	Σ	
Summation, integral	∫	

m sign for milli- μ sign for micro- or micr-
k sign for kilo- M sign for mega- or meg-

1. Ordinary numerals as exponentials shall exclusively be used to represent powers. (In consequence, it is desirable that the expression $\sin^{-1} x$, $\tan^{-1} x$, employed in certain countries be expressed by arc sin x, arc tan x).

2. The comma and the full-stop shall be employed for separating the decimals according to the custom of the country, but the separation between any three digits constituting a whole number shall be indicated by a space and not a full-stop or a comma (1 000 000).

3. For the multiplication of numbers and geometric quantities, indicated by two letters, it is recommended to use the sign ×, and the full-stop only when there is no possible ambiguity.

4. To indicate division in a formula, it is recommended that the horizontal bar of the colon be employed. Nevertheless, the oblique line may be used when there is no possibility of ambiguity ; when necessary, ordinary brackets ( ), square brackets [ ], and braces { } may be employed to obtain clearness.

Abbreviations for Weights and Measures.

Length :—m ; km ; dm ; cm ; mm ; $\mu = 0{\cdot}001$ mm.
Surface :—a ; ha ; m^2 ; km^2 ; dm^2 ; cm^2 ; mm^2.
Volume :—l ; hl ; dl ; cl ; ml ; m^3 ; km^3 ; dm^3 ; cm^3 ; mm^3.
Mass :—g ; t ; kg ; dg ; cg ; mg.

TABLE LV.

The Greek Alphabet.

Name.	*Large.*	*Small.*	*Commonly used to designate.*
alpha ..	A	α	angles, coefficients.
beta ..	B	β	angles, coefficients.
gamma ..	Γ	γ	specific gravity.
delta ..	Δ	δ	density, variation, partial differential.
epsilon ..	E	ϵ	base of hyperbolic logarithms, dielectric constant.
zeta ..	Z	ζ	co-ordinates, coefficients.
eta	H	η	hysteresis (Steinmetz) coefficient, efficiency
theta ..	Θ	θ	angular phase displacement, time constant.
iota ..	I	ι	
kappa ..	K	κ	susceptibility, constant.
lambda ..	Λ	λ	conductivity, wave length.
mu	M	μ	permeability.
nu	N	ν	reluctivity, frequency.
xi	Ξ	ξ	output coefficient.
omicron ..	O	o	
pi	Π	π	circumference ÷ diameter.
rho	P	ρ	resistivity.
sigma ..	Σ	σ	(cap.), summation ; leakage coefficient.
tau	T	τ	time-phase displacement, time constant.
upsilon ..	Y	υ	
phi	Φ	ϕ	flux, phase displacement.
chi	X	χ	
psi	Ψ	ψ	angular velocity in time.
omega ..	Ω	ω	(small), angular velocity in space.

TABLE LVI.

Electrical Units.

Quantity.		*Name of Unit.*	*C.G.S. Electro-Magnetic Units.**	*Dimensions of Unit.*
Current	..	Ampere	10^{-1}	$L^{\frac{1}{2}} M^{\frac{1}{2}} T^{-1}$
Potential	..	Volt	10^{8}	$L^{\frac{3}{2}} M^{\frac{1}{2}} T^{-2}$
Resistance	..	Ohm	10^{9}	LT^{-1}
Quantity	..	Coulomb	10^{-1}	$L^{\frac{1}{2}} M^{\frac{1}{2}}$
Energy	..	Joule	10^{7}	$L^{2} MT^{-2}$
Power	..	Watt	10^{7}	$L^{2} MT^{-3}$
Capacity	..	Farad	10^{-9}	$L^{-1} T^{2}$
Induction	..	Henry	10^{9}	L

International and B.O.T. Units.

The International Ohm is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14·4521 grammes in mass, of a constant cross-sectional area, and of a length of 106·300 cm. It is equivalent to 1·00052 C.G.S. Units.

The International Ampere is the unvarying electric current, which, when passed through a solution of nitrate of silver in water, deposits silver at the rate of 0·00111800 gramme per second.

The International Volt is the electrical pressure, which, when steadily applied to a conductor the resistance of which is one International ohm, will produce a current of one International ampere.

The E.M.F. of a Weston Normal Cell at 20° C. is 1·0183 International Volts, and that of a Clark Cell at 15° C. is 1·4326 International Volts.

Miscellaneous Units.

A dyne is the force which, acting upon a gramme for 1 second, gives it a velocity of 1 cm. per second.

An erg is the work done by a dyne working through a distance of 1 cm.

---

* The ratio of electrostatic to electro-magnetic units is denoted by v, the value of which is 3×10^{10} cm. per second (the velocity of light). The relation between the two sets of units is as follows:—

v electro-static units of current = 1 electro-magnetic unit.
v ,, quantity = 1 ,, ,,
v^2 ,, capacity = 1 ,, ,,
1 ,, unit of E.M.F. = v ,, ,,
1 ,, resistance = v^2 ,, ,,

For example—

1 microfarad = 10^{-15} electromagnetic units.
= $10^{-15} v^2$ electrostatic units.
= $10^{-15} \times 9 \times 10^{20}$ = 900,000 electrostatic units.

The Magnetic Circuit.

Unit Magnetic Pole is one of such strength that, when placed at a distance in air of 1 cm. from a similar pole of unit strength, will repel it with a force of 1 dyne. The number of lines of force which pass through a unit magnetic pole is 4π (= 12·57).

The *Force* exerted between two magnetic poles is proportional to the product of their strength, and inversely proportional to the square of the distance between them.

The *Magnetic Moment* (M) is the product of the distance between the poles and the pole strength of a magnet. $M = mL$.

The *Intensity of Magnetisation* (J) is the magnetic moment per unit volume. $J = \frac{M}{V}$

The *Strength of a Magnetic Field* at any point is measured by the force with which it acts upon a unit magnetic pole. If a pole of strength m is placed in a field of strength H, the force is mH dynes. If it is then placed at a distance of d centimetres from a second pole of strength n, then the force will be

$$\frac{m \times n}{d^2} \text{ dynes.}$$

The *Magnetomotive Force* (F) is the product of the current and the number of turns.

$$F = \frac{4\pi}{10} NI = 1{\cdot}257\ NI, \text{ where } NI = \text{the ampere-turns.}$$

The *Magnetic Field Intensity* or *Magnetising Force* (H) is defined as the *m.m.f.* per unit length of path.

$$H = \frac{F}{l} = \frac{4\pi}{10\,l} NI = \frac{1{\cdot}257\ NI}{l}$$

$$= 1{\cdot}257 \text{ ampere-turns per cm.}$$

$$= 0{\cdot}495 \text{ ampere-turns per in.}$$

Magnetic Flux (Φ) may be defined as a tube of lines of force which is complete and closed on itself, the number of lines of force being constant throughout. If a section be taken anywhere across the tube the constant number of lines of force will be a defined amount of magnetic flux.

$$\Phi = \mu HA \text{ (where A = the cross-section).}$$

$$= \mu A \frac{4\pi\ NI}{10\,l} = \frac{\mu A}{l} \cdot \frac{4\pi\ NI}{10} = \frac{F}{R} = \frac{m.m.f.}{\text{reluctance}}$$

where R, the *Reluctance*, $= \frac{l}{\mu A}$

The *Flux Density* (B) is defined as the flux (Φ) per unit area, perpendicular to the direction of the lines of force.

$$B = \frac{\Phi}{A}$$

If the flux is measured in maxwells, and the area (A) in sq. cm., B is expressed in maxwells per sq. cm. One maxwell per sq. cm. is called a Gauss. B is also expressed in kilolines per sq. cm. or per sq. inch (see Table LX).

C

Figs. 3 and 4 give typical normal induction curves for the usual magnetic materials.*

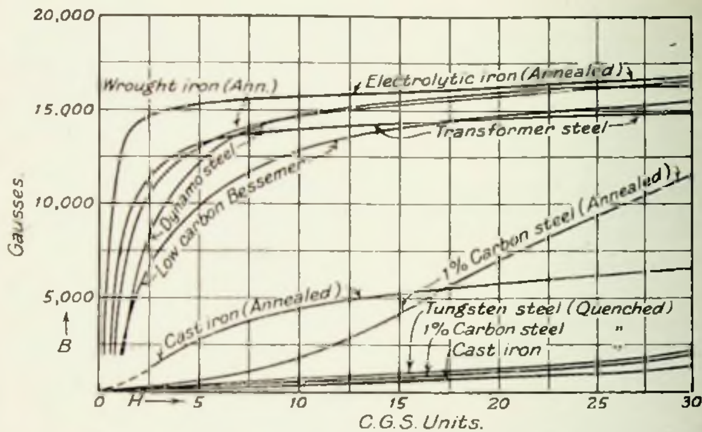


Fig. 3.—Showing typical normal induction curves for the usual commercial magnetic materials for magnetizing forces less than 30 C.G.S. units.

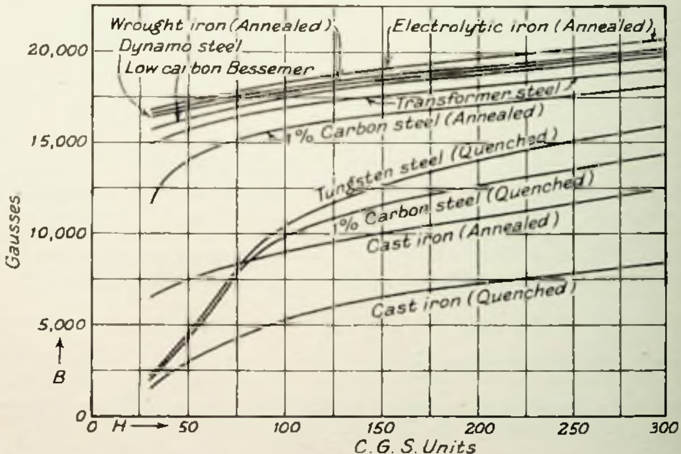


Fig. 4.—Showing typical normal induction curves for the usual commercial magnetic materials for magnetizing forces between 30 and 300 C.G.S. units.

* Bureau of Standards.

The *Permeability* (μ) of a magnetic material is the ratio of the flux density B induced in it to the magnetising field H. B represents the total number of lines of force induced per sq. cm., and H the actual magnetising field operating within the material.

$$\therefore \mu = \frac{B}{H}.$$

The *Magnetic Susceptibility* (k) represents the magnetisability of a material, in which the ratio of intensity J to the magnetising field H is the measure, so that

$$J = kH.$$

When k is positive the material is paramagnetic, and when negative the material is diamagnetic.

Air is regarded as having unit permeability. Iron, nickel, cobalt, chromium, manganese and other metals are more permeable than air and are called paramagnetic. Antimony, arsenic, bismuth and phosphorus are less permeable than air and are called diamagnetic.

At ordinary temperatures nickel is magnetic to about two-thirds the value of iron. It continues to be magnetic up to about 350° C., but beyond this it becomes entirely non-magnetic.

Hysteresis (h). If a magnetic body is carried round a cycle from one state of magnetisation to another, the magnetisation will lag behind the magnetic force, so that the curve connecting B and H will be quite different according to the direction of change of field.

If a complete curve is drawn connecting B with H for a whole cycle from one value of H to any other and then back to the original value of H, the curve connecting B and H will form a closed loop (Fig. 5), the area of which

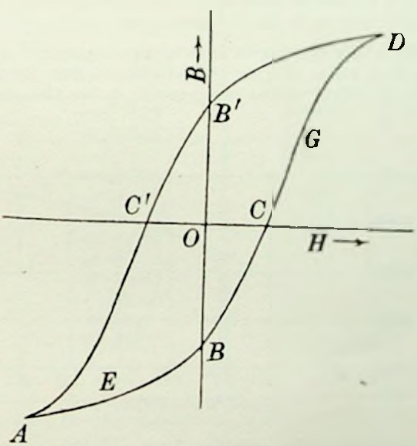


Fig. 5.

is a measure of the energy absorbed in carrying the magnetisation through the cycle. This energy is known as the hysteresis loss, and is usually expressed in ergs per cm^3 per cycle.

The slope of the curve $\left(\frac{B}{H}\right)$ indicates the permeability at any point.

Remanence (B_{rem}) is the magnetisation which exists in the specimen when, during a cycle of magnetisation, the magnetising field is reduced to zero from some maximum value of H. In Fig. 5, OB′ is the residual magnetism.

Reversal of the magnetising force (H) reduces and finally reverses the flux in the specimen, which is shown by the curve B′C′A.

The *Coercive Force* (H_c) is the demagnetising force necessary to reduce the induction B to zero from any specified value. In Fig. 5 H_c is given by OC′.

Coercivity is the demagnetising force necessary to reduce B to zero from its saturation value.

Steinmetz Coefficient (η).

Dr. Steinmetz established the relation between the maximum flux density and the energy (hysteresis) loss in the relation

$$W_h = \eta B_{max}^{1 \cdot 6} f, \text{ ergs per cm.}^3 \; (\times \; 10^{-7} = \text{watts per cm.}^3)$$

where W_h is the hysteresis loss per cm^3., f the frequency and η a constant depending on the nature of the magnetic material. The value of the exponent is an average. It has been shown by various authorities that at very low flux densities the energy loss is more nearly represented by B^2 than $B^{1 \cdot 6}$. The curve in Fig. 6, due to Dr. Lloyd, for various transformer irons, indicates the general trend with change of flux density.

The co-efficient η varies widely with various materials, from 0·0007 for good quality silicon-iron to 0·025 for hard cast steel (see Table LVII).

Table LVIII gives the hysteresis loss in watts per in.^{3} and per cm.^{3} per cycle per second for a value of $\eta =$ ·001 at various flux densities. To obtain the hysteresis loss at other values of η, multiply by the appropriate value.

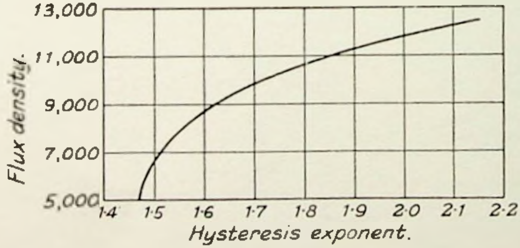


Fig. 6.—Curve showing the relation between the exponent and the flux density in the formula $W_h = \eta \; B^n$.

TABLE LVII.

Hysteresis Coefficient for Various Materials.

Material.	η	*Material.*	η
British Silicon Steel, 3·1 Si . .	0·00095 to	Barrett's Aluminium iron . .	0·00068
(Transformer iron) . .	0·001	Good sheet iron	0·002 to
American Silicon Steel, 3·5 Si	0·00086		0·004
German Silicon Steel, 3·8 Si	0·00080	Ordinary sheet iron . .	0·0045
American Transformer Steel	0·0031 to	Soft annealed cast steel . .	0·008
(unannealed)	0·0049	Cast steel	0·012
American Transformer Steel	0·00105 to	Cast iron	0·0162
(annealed)	0·00174	Hardened cast steel . .	0·025
British Transformer Steel	0·00122 to	Hensler Alloy I	0·012
(annealed)	0·00129	Hensler Alloy II	0·0024
Average Sheet Steel	0·002	Electrolytic Iron	0·009
Sankey's Lohys iron . .	0·0012		

TABLE LVIII.

Hysteresis Loss per Cycle per Second for $\eta = \cdot 001$.

B per sq. in.	*Watts lost per in.*^{3} *per cycle per sec. for* $\eta = \cdot 001$.	*B per sq. cm.*	*Watts lost per cm.*^{3} *per cycle per sec. for* $\eta = \cdot 001$.
35,000	·0018643	6,000	·00011093
40 000	·0023083	7,000	·00014196
45 000	·0027870	8,000	·00017577
50,000	·0032988	9,000	·00021220
55,000	·0038422	10,000	·00025125
60,000	·0044162	11,000	·00029255
70,000	·0056514	12,000	·00033625
80,000	·0069975	13,000	·0003822
90,000	·0084487	14,000	·0004303
100,000	·0100000	15,000	·0004805
110,000	·011647	16,000	·0005328
120,000	·013387	17,000	·0005871
		18,000	·0006433

N.B.—To obtain the watts lost at other values of η, multiply by the appropriate value.

TABLE LIX. Magnetic Properties of Iron and Steel.*

Material.	% Carbon	% Alloying Constituent	State of Hardness	B_{max}	B_{rem}	I_{rem}	H_c
Swedish Wrought Iron	Trace	—	Very soft	17,400	6,900	550	0·8
Softest Selected Iron	"	—	"	17,430	10,400	804	0·44
Piano Steel Wire	0·95 ?	—	Annealed, soft	14,500	10,400	824	22·0
" "	0·95 ?	—	Glass, hard	12,600	9,600	760	40·0
Low Carbon Steel	0·06	—	Quenched 1,000° C. soft	19,800	7,812	625	3·4
High " "	1·2	—	Quenched 800° C. hard	15,080	8,060	645	58·0
Haarlem Magnet Steel	—	—	Hard	16,900	10,048	800	56·0
Allevard Steel	0·59	5·5 Wo	Not quenched	18,700	11,250	900	26·0
" "	0·59	5·5 Wo	Quenched 770° C.	17,500	10,500	800	73·0
Böhlers Styrian Steel	—	—	Soft	17,850	9,950	790	34·0
" " "	—	—	Hard	14,000	7,570	600	75·0
Remy Tungsten Steel	—	—	Hard	15,145	10,175	808	63·0
" " "	—	—	Very hard	16,070	10,040	800	77·0
Medium Tungsten Steel	0·89	3·08 Wo	Quenched 760° C. hard	11,000	7,330	572	58·8
Whitworth " "	0·51	4·01 Wo	Quenched 900° C. hard	—	—	610	37·0
Molybdenum Steel	1·25	3·36 Mo	Hard	10,000 ?	4,651	370	85·0
Chilled Cast Steel	—	—	Chilled at 1,000° C.	9,000 ?	1,800	220	52·0
Lodestone	—	—	—	—	—	350	50·0
High Carbon Steel	1·2	—	Quenched 905° C.	—	—	264	48·0
Cast Iron	—	—	—	—	—	312	3·8
Manganese Steel	—	—	Annealed	—	—	43	21·5
Grey Cast Iron	—	—	—	—	—	250	13·67
Chrome Steel	—	—	—	—	—	690	52·5
Tungsten Steel	—	—	—	—	—	806	71·5
Chrome Steel	—	16 Cr	Tempered	—	—	1,030	22·0
Chrome Steel	—	—	—	—	—	286	56·0
Alloy Steel	—	5 Cr. 3 Mn. 4 Mo	Forged	—	—	905	55·8
Nickel Molybdenum Steel	—	10 Ni. 5 Mo	—	—	—	810	36·7
Molybdenum Chromium Steel	0·5	10 Mo. 4 Cr.	Quenched	—	—	835	78·0
Nickel Tungsten Steel	—	10 Ni. 5 Wo	Quenched	—	—	820	35·2
Molybdenum Manganese Steel	0·5	1 Mn. 2 Mo	Quenched	—	—	835	39·6
" " "	—	10 Mo. 1 Mn.	Quenched	—	—	855	58·4
Tungsten Steel	0·43	8·72 Wo. 0·24 Mn.	—	—	—	474	37·5
Cobalt Steel	—	35 Co.	—	—	—	827	24·6

* Drysdale & Jolly, "Electrical Measuring Instruments."

Eddy Current Losses.

If the cycle of magnetisation is performed many times per second, the rapid cutting of the lines of induction through the metal gives rise to induced eddy currents (called Foucault currents), which waste energy; the total losses are therefore greater than the hysteresis losses. The eddy current losses are proportional to the square of the induced currents, and are therefore proportional to f^2, B^2 and g^2, where f is the frequency, and g the form factor of the secondary induced voltage

$$\left(= \frac{V_{R.M.S.}}{V_{mean}}, \text{ which for sine wave} = \frac{\pi}{2\sqrt{2}} = 1{\cdot}1107\right).$$

$\therefore$ Eddy current losses $= kf^2\, g^2\, B^2$ (k being a constant for a particular specimen)

and the Total losses $W = \eta\, f\, B^{1\cdot6} + k\, f^2\, g^2\, B^2$. ergs per cm^3.

Fig. 7 shows typical curves of total losses for high and low resistance

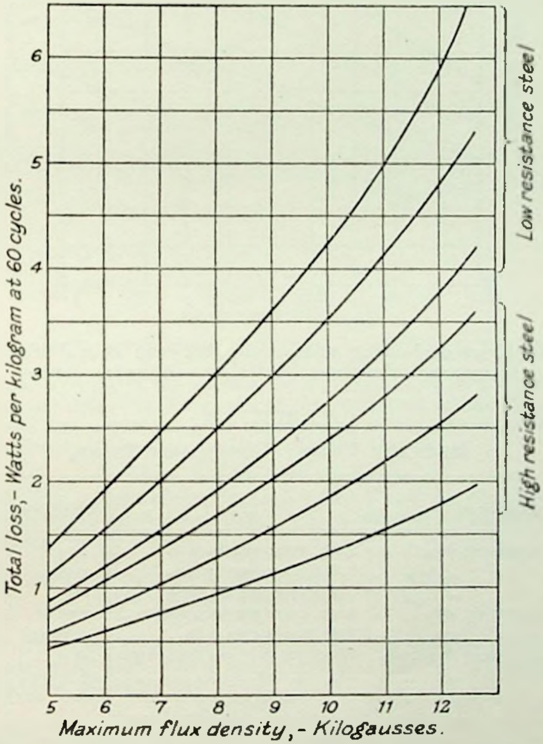


Fig. 7.—Showing typical curves of total core losses for the usual materials used in transformer and armature construction.

steels as used in transformer and armature construction. High resistance steels contain about 3% silicon and have low specific gravity (about 7·5) and low core loss. Low resistance steels are relatively pure iron and have a sp. gr. of about 7·7 and a core loss approximately double that of the high resistance materials. Fig 7*a* shows the relationship between induction density and iron loss for transformer sheet steel at various frequencies.

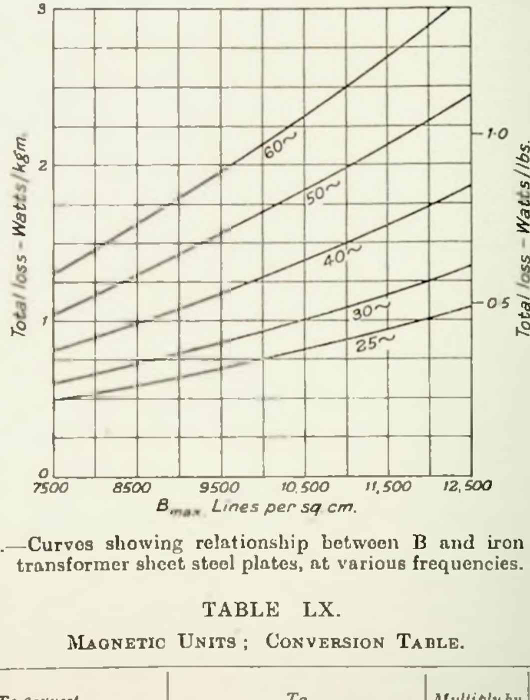


Fig. 7a.—Curves showing relationship between B and iron loss for transformer sheet steel plates, at various frequencies.

TABLE LX.

Magnetic Units ; Conversion Table.

To Convert	*To*	*Multiply by*	*Reciprocal*
Amp. turns per cm. length ..	Amp. turns per inch, *l*. ..	2·54	0·3938
,, ,, ,, ,, ,,	C.G.S. lines per sq. cm. ..	1·257	0·7956
,, ,, ,, ,, ,, ..	C.G.S. lines per sq. inch ..	8·1	0·1234
C.G.S. lines per sq. cm. ..	Amp. turns per inch length ..	2·02	0·495
,, ,, ,, ,, ..	Kilo lines per sq. inch	0·00645	155·0
Ergs per cm.^{3} per cycle per sec.	Watts per cm.^{3} per cyc per sec.	10^{-7}	107
,, ,, ,, ,, ,,	,, ,, kg. ,, ,, ,,	13×10^{-6}	77×10^3
,, ,, ,, ,, ,,	,, ,, lb. ,, ,, ,,	$5·9 \times 10^{-6}$	169×10^3
,, ,, ,, ,, ,,	,, ,, lb. per 50 cycles per sec.	$2·95 \times 10^{-6}$	339×10^3

Note.—A gauss is one C.G.S. line per sq. cm. A maxwell is one C.G.S. line.

Effect of Electric Current.

(1) The field due to a straight wire of infinite length at a distance r cm. from the axis of the conductor is

$$H = \frac{2\,i}{10\,r} \text{ lines per cm.}$$

where i is the current in C.G.S. Units = amps $\times$ 10^{-1}.

(2) The field within a long straight solenoid, where the length is large compared with its diameter, is

$$H = \frac{4\,\pi\,N\,i}{10} \text{ lines per cm.}$$

(its direction being determined by the right-hand screw rule), where N is the number of turns. At the ends of the solenoid the strength is half that at the middle.

(3) The field due to a closed circular conductor is

$$H = \frac{2\,\pi\,r^2\,i}{10\,(\sqrt{r^2 + l^2})^3} = \frac{0{\cdot}2\,\pi\,r^2\,i}{(\sqrt{r^2 + l^2})^3} \text{ lines per cm.}$$

where i is the current carried by the conductor bent in the form of a ring of radius r cm., and H is the magnetising force at a point along the axis.

$$\text{When } l = o,\ H = \frac{0{\cdot}2\,\pi\,i}{r},$$

and when l is very great in comparison with r

$$H = \frac{0{\cdot}2\,\pi\,r^2\,i}{l^3}$$

(4) The force on a conductor carrying a current in a magnetic field is

$$F = 10{\cdot}2\,i\,Bl\,10^{-8}\,(Kg.),$$

where i is the current in the conductor, B (lines per sq. cm.) is the density of the magnetic field, l is the length of the conductor in cm. The direction of the axis of the conductor is at right angles to the direction of the field. If the direction of i and B form an angle θ, the force is

$$F = 10{\cdot}2\,iBl\,10^{-8} \sin\theta\ (Kg.)$$

The force F is perpendicular to both i and B and its direction is determined by the right-hand screw rule. The conductor tends to move away from the denser field.

If B is in lines per sq. in., l in inches, and I in amps.,

$$F = 8{\cdot}85\ IBl\ 10^{-8} \sin\theta\ (\text{lb.})$$

(5) If a conductor moving in a magnetic field cuts across the lines of flux or a magnetic field moves across a conductor, the instantaneous induced E.M.F. in the conductor is

$$e = kBlv,$$

where B is the flux density, l the length of conductor, v the relative velocity between the flux and the conductor, and k a coefficient depending on the units employed. When e is in volts, B in lines per sq. cm., l in cm., and v in cm. per second, $k = 10^{-8}$.

(6) (*a*) The relative direction of flux, E.M.F. and motion in a generator may be determined by Fleming's rule, as shown in Fig. 8.

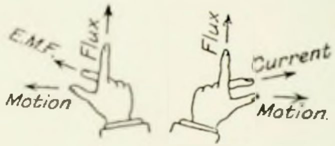


Fig. 8. Fleming's Rule.

The thumb, forefinger and middle finger of the right hand being held at right angles to each other, the hand is placed so that the forefinger is in the direction of the flux, and the thumb in the direction of motion. The middle finger will then indicate the direction of the generated E.M.F.

In the case of a motor, using the left hand, and pointing the forefinger along the direction of the magnetic field, and the middle finger along the direction of the current, the thumb will indicate the direction of the force and therefore the resulting motion.

(*b*) In the right-hand screw rule, if the current flows in the direction of rotation of a right-hand screw, the flux is in the direction of the progressive movement of the screw. If the current in a straight conductor is in the direction of the progressive motion of a right-hand screw, then the flux encircles this conductor in the direction in which the screw must be rotated in order to produce this motion.

Resistance.

Ohm's Law.

If I = the current in amps., E = the E.M.F. in volts, and R = the resistance in ohms,

$$I = \frac{E}{R} \text{ amps.}\ ; \quad E = I\,R \text{ volts}\ ; \text{ and } R = \frac{E}{I} \text{ ohms.}$$

$$\text{Watts} = E\,I = \frac{E^2}{R} = I^2 R \; (= \cdot 24\, I^2 R \text{ calories}).$$

Resistances in Series and Parallel.

When several resistances are in series, the total resistance is the sum of the separate resistances :

$$R = r_1 + r_2 + r_3 + - - - - -$$

In the case of resistances in parallel, the combined resistance is the reciprocal of the sum of the reciprocals of each resistance :

$$R = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + - - - -} = \frac{r_1\, r_2\, r_3}{r_1\, r_2 + r_1\, r_3 + r_2\, r_3}$$

The reciprocal of the resistance $\left(\frac{1}{R}\right)$ is the Conductivity.

Resistivity.

Resistivity or Specific Resistance is the resistance between the opposite faces of a substance, 1 cm. in length and 1 sq. cm. in cross-sectional area, and is expressed in ohms or microhms per cm.^{3}.

The resistance of a substance neglecting temperature is :

$$R = \frac{\rho\, l}{a} \text{ ohms.}$$

where ρ = the specific resistance, l = the length and a = the cross-sectional area.

Resistivity may be measured by means of a moving-coil voltmeter of resistance r. About 500 volts are applied to the resistance R in series with the voltmeter. If V is the applied voltage and v the reading of the voltmeter then :

$$\frac{R}{r} = \frac{V - v}{v}$$

$$\therefore R = \frac{r\,(V - v)}{v} \text{ ohms.}$$

Temperature Coefficient.

The resistance of a material varies with temperature, that of pure metals increasing with rise of temperature, and that of insulators decreasing with increase of temperature. The resistance of carbon and electrolytes decreases with rise of temperature. Alloys change less than pure metals, some having a negative temperature coefficient.

If R is the resistance at any temperature t, R_o the resistance at 0°C., and a the temperature coefficient per °C. :

$$R = R_o\ (1 + a\, t) \text{ ohms.}$$

If R_1 is the resistance at an initial temperature t_1, R the resistance at any other temperature t, and a the temperature coefficient at the initial temperature, then :

$$R = R_1 \left\{ 1 + a\,(t - t_1) \right\}$$

$$\text{or } t - t_1 = \frac{R - R_1}{R_1\, a}$$

For copper, which has a temp. coeff. of about 0·004* per °C. :

$$t - t_1 = \frac{250\,(R - R_1)}{R_1}$$

(The temperature coefficient per degree F. = $\frac{5}{9}$ that per degree C.).

Table LXI. gives the temperature factor for high conductivity copper from 0°C to 200°C. For use of Table see note at end of Table.

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* The figure is actually 0·00393 at 20° C, and 0·004265 at 0° C.

TABLE LXI.

TEMPERATURE FACTOR BY WHICH THE OBSERVED VALUE OF THE RESISTANCE OF HIGH CONDUCTIVITY COPPER AT ANY TEMPERATURE MUST BE MULTIPLIED IN ORDER TO ASCERTAIN THE STANDARD VALUE AT 20° C.

Temp. C°	*Factor.*	*Temp. C°*	*Factor.*	*Temp. C°*	*Factor.*	*Temp. C°*	*Factor.*	*Temp. C°*	*Factor.*
0	1·0853	20	1·0000	40	·9271	60	·864	80	·809
1	1·0807	21	·9961	41	·9238	61	·861	81	·806
2	1·0761	22	·9922	42	·9204	62	·858	82	·804
3	1·0716	23	·9883	43	·9171	63	·855	83	·801
4	1·0671	24	·9845	44	·9138	64	·852	84	·799
5	1·0626	25	·9807	45	·9105	65	·849	85	·796
6	1·0582	26	·9770	46	·9073	66	·847	86	·794
7	1·0538	27	·9732	47	·9041	67	·844	87	·791
8	1·0495	28	·9695	48	·9009	68	·841	88	·789
9	1·0452	29	·9658	49	·8977	69	·838	89	·786
10	·10409	30	·9622	50	·8945	70	·835	90	·784
11	1·0367	31	·9586	51	·891	71	·833	91	·781
12	1·0325	32	·9550	52	·888	72	·830	92	·779
13	1·0283	33	·9514	53	·885	73	·827	93	·777
14	1·0241	34	·9478	54	·882	74	·825	94	·774
15	1·0200	35	·9443	55	·879	75	·822	95	·772
16	1·0160	36	·9408	56	·876	76	·819	96	·770
17	1·0119	37	·9374	57	·873	77	·817	97	·767
18	1·0079	38	·9339	58	·870	78	·814	98	·765
19	1·0039	39	·9305	59	·867	79	·811	99	·763
20	1·0000	40	·9271	60	·864	80	·809	100	·760
100	·760	120	·717	140	·679	160	·644	180	·613
102	·756	122	·713	142	·675	162	·641	182	·610
104	·751	124	·709	144	·672	164	·638	184	·607
106	·747	126	·705	146	·668	166	·635	186	·604
108	·742	128	·701	148	·665	168	·632	188	·602
110	·738	130	·698	150	·661	170	·628	190	·599
112	·734	132	·694	152	·658	172	·625	192	·596
114	·730	134	·690	154	·654	174	·622	194	·593
116	·725	136	·686	156	·651	176	·619	196	·590
118	·721	138	·683	158	·648	178	·616	198	·588
120	·717	140	·679	160	·644	180	·613	200	·585

Note.—The value for the temperature coefficient of annealed high conductivity copper is ·00426, referred to 0° C. This is equivalent to ·00393, referred to 20° C.; or approximately 0·4 per cent. of the resistance at 20° C., a figure which is easily remembered and is sufficiently accurate for ordinary work. The Table has been calculated from the exact figure.

Examples of the Use of the Table.—(1) The resistance of a coil of copper strip is found to be ·144 ohms when its temperature as indicated by a centigrade thermometer is 27°: What is the standard value—that is to say, the resistance at 20° C.?

The factor for 27°C is ·973, hence at 20° C. the resistance will be ·144 × ·973 = ·140 ohms.

(2) On another day the same coil is measured at a temperature of 12° C. and found to be ·136 ohms.

The factor for 12° is 1·033, hence the standard value will be

·136 × 1·033 = ·140 ohms as before.

(3) After continued use the coil is found to have a resistance of ·142 ohms at 36° C.

The factor for 36° is ·941, hence the resistance at 20° C. will be ·142 × ·941 = ·134 ohms, showing a considerable fall below the original value of ·140 ohms.

Such a decrease indicates either a partial short circuit in the windings or an excessive leakage between the terminals.

(4) A motor armature, of which the standard value is ·045 ohms, is found to measure ·057 ohms after running at full load for several hours: What is the temperature attained by the windings?

The factor is $\frac{\text{standard value}}{\text{hot value}} = \frac{\cdot 045}{\cdot 057} = \cdot 79$; reference to the Table shows that the temperature is 87° C.

Kirchoff's Laws.

The following laws relate to continuous currents in divided circuits; they cannot be applied to alternating currents:—

(*a*) The algebraic sum of the currents in all wires which meet at a point = 0. Currents flowing to a junction are positive, and those from the junction are negative.

(*b*) The algebraic sum of the products of current and resistance in each branch of a network equals the algebraic sum of the electromotive forces of the network.

Wheatstone Bridge.

The Wheatstone Bridge consists essentially of four resistances as shown in Fig. 9; a and b are the ratio arms of known resistance, and may be either equal, or as 1,000 to 100, or as 10 to 1 or 1 to 10 or 100 to 1,000. The arm r is an adjustable resistance, and x is the unknown resistance to be found.

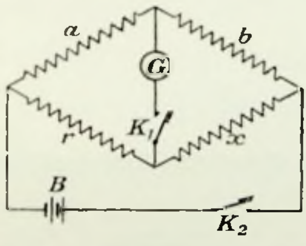


Fig. 9. Wheatstone Bridge.

The arm r is adjusted until $\frac{r}{x} = \frac{a}{b}$. When this balance is obtained the current from the battery B will divide between the two parallel circuits and there will be no deflection of the galvanometer G. The best value for the ratio arms is that which makes them most nearly equal to the resistance under test. The keys k_1 and k_2 are placed one below the other so that on depressing the handle the battery circuit is closed first and opened last.

The unknown resistance $x = \frac{r\,b}{a}$.

A sliding contact may be used to alter the adjustable resistance r, or adjustment may be effected as in the Post Office Bridge, where a number of resistance coils for the arms a, b and r are mounted in a box across gaps in a heavy brass bar, the gaps being short-circuited by plugs. A resistance is inserted by removing the corresponding plug.

The principle of the Wheatstone Bridge is used in many forms. For A.C. measurements the resistances must be non-inductive, and balance is obtained by the use of a telephone or a vibration galvanometer. In the case of a telephone, balance is indicated when there is no sound in the telephone.

Galvanometer and Shunt.

Resistance of Galvanometer and Shunt.

If G and s are the resistances of galvanometer and shunt respectively their joint resistance R is :

$$R = \frac{1}{\frac{1}{G} + \frac{1}{s}} = \frac{G\,s}{G + s}$$

Currents in Galvanometer and Shunt.

If I, I_G, and I_s are the currents through the battery, galvanometer and shunt respectively, and E is the potential difference :

$$I_G = \frac{E}{G}, \text{ and } I_s = \frac{E}{s}$$

$$\therefore \frac{I_G}{I_s} = \frac{s}{G}, \text{ or } I_G = \frac{s}{G} I_s \text{ and } I_s = \frac{G}{s} I_G$$

$$\text{But } I = I_s + I_G = I_s + \frac{s}{G} I_s = I_s \left(1 + \frac{s}{G}\right) = I_s \left(\frac{G + s}{G}\right)$$

$$\therefore I_s = I \left(\frac{G}{G + s}\right), \text{ and } I_G = I \left(\frac{s}{G + s}\right) \text{ or } I = I_G \left(\frac{G + s}{s}\right)$$

where $\frac{G + s}{s}$ is the "Multiplying Power" of the shunt.

Resistance of Shunt to give required Multiplying Power.

The multiplying power (n) is :

$$n = \frac{G + s}{s} = \frac{G}{s} + 1$$

$\therefore$ the resistance of shunt to give any required multiplying power is

$$s = \frac{G}{n - 1}.$$

$$\text{and } \frac{s}{G} = \frac{1}{n - 1}$$

In shunt boxes the multiplying powers are 10, 100 and 1,000, and they are marked $\frac{1}{9}$, $\frac{1}{99}$ and $\frac{1}{999}$, being the ratios of the currents passed through G and s; that is, the ratio of the resistances $\frac{s}{G}$ are as $\frac{1}{9}$, $\frac{1}{99}$ or $\frac{1}{999}$.

Compensating Resistance for Galvanometer and Shunt.

In order to make the joint resistances of the galvanometer and shunt equal to that of the unshunted galvanometer, a compensating resistance (R_c) must be added in the main circuit, so that :

$$G = \frac{G\,s}{G+s} + R_c, \text{ or } R_c = G - \frac{G\,s}{G+s} = G - \frac{G}{n} = G\left(1 - \frac{1}{n}\right)$$

$$= G\left(\frac{n-1}{n}\right) \text{ ohms.}$$

The Ayrton Universal Shunt.

This shunt is so arranged that it can be used with any galvanometer (Fig. 10).

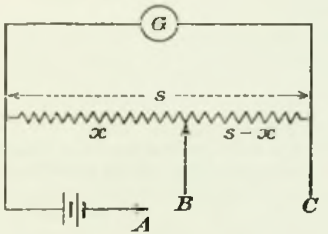


Fig. 10. Universal Shunt.

With A connected to C the current through the galvanometer will be $I_G = I\,\frac{s}{s+G}$, where I is the battery current.

If A is connected to B instead of C.

$$I'_G = I\,\frac{x}{s+G}$$

Therefore $I'_G = I_G\,\frac{x}{s}$

Hence the shunt ratio at any position of B along s is $\frac{x}{s}$ and is independent of the galvanometer resistance.

Inductance.

Inductance (L) is the ratio of the magnetic flux linked with and due to a conductor. to the current strength carried.

The self-induced voltage is proportional to the rate of change of the current in the circuit,

$$E = -L\,\frac{dI}{dt}$$

where L is the coefficient of self-induction.

The magnetic energy stored in a circuit is proportional to the square of the current and is

$$W = \tfrac{1}{2}\,LI^2 \text{ (L being in henries and W in joules).}$$

Helmholtz's Law.

Inductance in a circuit retards the increase of current, and the strength of the current may be calculated at any time after the circuit is closed from the formula

$$I = \frac{E}{R}\left(I - e^{-\frac{Rt}{L}}\right)$$

where I is the current after t seconds, E the voltage, R the resistance, L the coefficient of self-induction in henries, and e is the base of the Napierian logarithm (= 2·71828).

The ratio of t to $\frac{L}{R}$ is called the "Time Constant" of the circuit, and the percentage which the instantaneous value of the current bears to the full value depends upon this ratio.

Inductance of Solenoids.

For a straight coil uniformly wound with n turns of wire per cm. length, provided that the length of the coil is large compared with its diameter, and that the winding consists of one layer of thin wire, the inductance is

$$L = 1{\cdot}257\, n^2\, l\, A 10^{-8} \text{ henries,}$$

where n is the number of turns per cm. length, A is the cross-section of the flux in sq. cm., and l is the average length of the flux in cm.

The inductance of a long straight coil of several layers of wire and with an iron core of inside radius, a,

$$L = 4n^4\, l d^2\, r^2 \left[1 + (\mu - 1)\frac{a^2}{r^2} + \frac{d}{r} + \frac{d^2}{3r^2}\right] 10^{-9} \text{ henries.}$$

where r is the inside radius of the winding, d is its radial thickness, and μ is the relative permeability of the core with respect to the air. If there is no iron core $a = 0$.

Inductance of Transmission Line.

The self-inductance of a single-phase transmission line is

$$0{\cdot}0805 + 0{\cdot}741 \log\left(\frac{d - r}{r}\right) \text{ millihenries per mile,}$$

where d is the distance between centres of conductors, and r is the radius of the conductor.

For overhead lines $(d - r)$ may be taken as d, since r is relatively small. For further data see "Overhead Transmission Lines," page 221, Vol. I.

Mutual Inductance.

When two independent circuits (a) and (b) are near to each other, their electro-magnetic energy consists of three parts: (i) the part due to the linkages of the flux produced by circuit (a) with the current in (a); (ii) that due to the flux produced by the circuit (b) with the current in (b); and (iii) that due to the current in each circuit linking with the flux produced by the other circuit.

The total energy of the system is

$$W = \tfrac{1}{2} L_a I_a^2 + \tfrac{1}{2} L_b I_b^2 + I_a I_b L_m$$

where L_m is the coefficient of mutual inductance (henries) of the two circuits.

$$\text{Also } e_a = -L_m \frac{dI_b}{dt} \text{ and } e_b = -L_m \frac{dI_a}{dt}$$

Choking Coil.

The back E.M.F. set up by a choking coil by reason of its self-induction is

$$E_b = 4{\cdot}44\ NfBA \times 10^{-8} \text{ volts,}$$

where E_b = the back E.M.F.
N = number of turns.
f = frequency.
B = flux density (lines per cm².).
A = area of core.

Capacity.

Capacity is the ratio of an electric charge on a conductor to the electric potential difference producing the charge,

$$Q = C \times E \times 10^{-6}$$

where Q = the electric charge in coulombs, C = the electrostatic capacity in microfarads, E = the potential difference in volts.

If I is the current in amperes passed into a condenser during t seconds

$$Q = It.$$

The energy stored in a condenser of capacity C farads when charged to a potential difference of E volts is

$$W = \tfrac{1}{2}CE_{.}^{2} \text{ joules.}$$

(1 microfarad = 10^{-15} electromagnetic units = 900,000 electrostatic units.)

Condensers in Series and in Parallel.

When condensers are placed in parallel the resultant capacity is equal to the sum of the capacities of the condensers.

$$C = C_1 + C_2 + C_3 + \text{---}$$

When condensers are connected in series, the resultant capacity is the reciprocal of the sum of the reciprocals of the capacity of each condenser.

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \text{----}} = \frac{C_1\,C_2\,C_3}{C_1\,C_2 + C_1\,C_3 + C_2\,C_3}.$$

Specific Inductive Capacity.

The Specific Inductive capacity, or dielectric constant, is the ratio of the capacity of a condenser, the plates of which are separated by a given substance, to the capacity of a similar condenser, the plates of which are separated by a vacuum (see Table XLV).

Capacity of Plate Condensers.

When a condenser consists of two parallel plates, the capacity is

$$C = \frac{\epsilon A}{4\pi t} \text{ electrostatic units,}$$

where ϵ is the specific inductive capacity of the dielectric (= 1 for air), A is the total effective area of the plates in sq. cm., t is the thickness of the dielectric in cm.

The capacity in microfarads is

$$C = \frac{\epsilon A}{4\pi t\,(9 \times 10^5)} = 0{\cdot}08842 \times 10^{-6}\,\frac{\epsilon A}{t}\ \mu F.$$

If the dimensions are in sq. in. and inches,

$$C = 0{\cdot}2246 \times 10^{-6}\,\frac{\epsilon A}{t}\ \mu F.$$

If, instead of a single pair of metal plates, there are N similar plates with dielectric between, alternate plates being connected in parallel

$$C = 0{\cdot}08842 \times 10^{-6}\,\frac{(N-1)\,\epsilon A}{t}\ \mu F \text{ (dimensions in cm.).}$$

Capacity of Variable Condenser with Semicircular Plates.

If N = the total number of parallel plates,
r_1 = the outside radius of the plates in cm.
r_2 = the inner radius of plates in cm.
t = the thickness of dielectric in cm.
ϵ = dielectric constant.

Then for the position of maximum capacity (movable plates between the fixed plates),

$$C = 0{\cdot}1390\,\epsilon\,\frac{(N-1)\,(r_1^2 - r_2^2)}{t} \times 10^{-6}\,\mu\,F.$$

Capacity of two Coaxial Cylinders.

If r_1 = radius of outer cylinder in cm.
r_2 = ,, ,, inner ,, ,, ,,
l = length of each cylinder in cm.
ϵ = dielectric constant.

$$\text{then } C = \frac{0{\cdot}2416\,\epsilon\,l}{\log \frac{r_1}{r_2}} \times 10^{-6}\,\mu\,F.$$

The same formula gives the capacity of a single conductor cable with grounded metal sheath (see below).

Capacity of two Concentric Spheres.

If r_1 = the inner radius of the outside sphere in cm.
r_2 = the radius of the inside sphere.
ϵ = the dielectric constant,

$$\text{then } C = 1{\cdot}112\,\epsilon\,\frac{r_1 r_2}{r_1 - r_2} \times 10^{-6}\,\mu\,F.$$

Capacity of Overhead Lines.

The capacity between two parallel overhead conductors is

$$\frac{0{\cdot}0194}{\log \frac{d}{r}} \text{ microfarads per mile}$$

where d is the distance between centres of conductors, and r the radius of the conductor (dimensions in inches or cm.).

In 3-phase circuits the capacity between wire and neutral is twice the above value, and will also be twice the above value for the Y-capacity of the line.

The capacity of a single overhead conductor with ground return is

$$C = \frac{0{\cdot}03882}{\log \frac{2h}{r}}\,\mu\,F \text{ per mile}, = \frac{0{\cdot}02413}{\log \frac{2h}{r}}\,\mu\,F \text{ per km.}$$

where h is the height of the conductor above the ground.

For further data see "Overhead Transmission Lines," page 225, Vol. I.

Capacity of Cables.

The capacity of a concentric cable, or of a single conductor cable with grounded metal sheath is

$$\frac{0{\cdot}03882\,\epsilon}{\log \frac{D}{d}}\,\mu\,F \text{ per mile,}$$

where ϵ is the specific inductive capacity of the dielectric, D the diameter over the insulation surrounding the inner conductor, and d the diameter of the inner conductor. This is the same formula as for two concentric cylinders (see above).

The mutual capacity of a twin conductor cable is

$$\frac{0 \cdot 0194\epsilon}{\log \frac{2a\,(R^2 - a^2)}{r\,(R^2 + a^2)}}\ \mu\text{ F per mile,}$$

where ϵ is the dielectric constant, a is the distance from the centre of the cable to the centre of the conductors, R is the inside radius of the lead sheath and r is the radius of the conductors. The capacity of a single conductor is twice that given by the above formula.

The mutual capacity of a three-phase three-conductor cable is

$$\frac{0 \cdot 03882\epsilon}{\log \frac{3a^2\,(R^2 - a^2)^3}{r^2\,(R^6 - a^6)}}\ \mu\text{ F per mile,}$$

the dimensions being the same as above. The capacity of a single conductor is twice that given by this formula.

Charging Current.

The capacity or charging current (I_c) of a single phase circuit is

$$I_c = 2\pi f\,E\,C\,10^{-6}\text{ amps.,}$$

where E is the pressure between wires at the generator end and C the capacity of the wires in μ F. Where $f = 50$ cycles

$$I_c = 0 \cdot 000314\ E\,C\text{ amps.}$$

The charging current of a three phase circuit is $\frac{2}{\sqrt{3}}$ $(= 1 \cdot 155)$ times that of a single phase circuit, with equal spacing between phases.

Alternating Current Circuits.

Inductive Reactance.

In a circuit containing inductance only the current lags 90 deg. behind the applied E.M.F.

$$E = 2\pi f\,L\,I\text{ volts}$$

where E is the R.M.S. voltage, I the current, L the inductance in henries and f the frequency in cycles per second.

$$\therefore\ 2\pi f\,L = \frac{E}{I}\text{ ohms}$$

where $2\pi f\,L$ $(= \omega\,L)$ is the inductive reactance.

Impedance.

In a circuit containing resistance and inductance in series, the impedance is

$$Z = \sqrt{r^2 + (\omega\,L)^2}$$

$$\text{so that}\quad I = \frac{E}{\sqrt{r^2 + (\omega\,L)^2}}$$

where r is the energy component of the impedance and ωL the wattless component of the impedance.

Admittance is the reciprocal of impedance $= \frac{1}{Z}$.

$$\frac{1}{Z} = \sqrt{g^2 + b^2}$$

where g is the *Conductance* and b the *Susceptance*.

Capacity Reactance.

In a circuit containing capacity and resistance in series, the current leads the applied E.M.F. by 90 deg., and the impedance is

$$Z = \sqrt{r^2 + \left(\frac{1}{2\pi f C}\right)^2} \text{ ohms.}$$

where C is in farads, and $\frac{1}{2\pi f C}\left(=\frac{1}{\omega C}\right)$ is the capacity reactance, or condensance.

Inductive and Capacity Reactance.

In a circuit containing inductance, capacity and resistance in series, the impedance is

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The condition for the capacity to annul the inductance is when

$$\omega L = \frac{1}{\omega C} \text{ or } \omega = \frac{1}{\sqrt{LC}}$$

That is, when $f = \frac{1}{2\pi\sqrt{LC}}$.

This condition is called *Resonance.*

Phase Angle.

In Fig. 11 ϕ is the angle of lag, and $\tan\phi = \frac{2\pi f L}{r}$

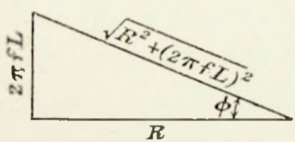


Fig. 11.

That is, the tangent of the angle of lag is equal to the ratio of the inductive reactance to the resistance. ϕ is called the phase angle.

Similarly where there is capacity in the circuit, $\tan\phi = \frac{2\pi f C}{r}$.

Form Factor.

The form factor is the ratio of the R.M.S. value of the voltage to the mean value.

$$E_{R.M.S.} = \frac{E_{max}}{\sqrt{2}} = 0{\cdot}7071\ E_{max}$$

$$E_{mean} = \frac{E_{max}}{\frac{\pi}{2}} = \frac{2\,E_{max}}{\pi} = 0{\cdot}6366\ E_{max}.$$

∴ the form factor for a sine wave is

$$\frac{E_{R.M.S.}}{E_{mean}} = \frac{0{\cdot}7071}{0{\cdot}6366} = 1{\cdot}11$$

The *Amplitude Factor* $= \frac{E_{max}}{E_{R.M.S.}}$

Power Factor.

In Fig. 12 KW is the true load, KVA the total or apparent load, and the remaining side of the triangle the wattless load, all in watts.

The ratio between the true watts and the apparent watts (volt amperes) is the power factor, and is the cosine of the angle of lag ($\cos \phi$).

$$\frac{\text{KW}}{\text{KVA}} = \cos \phi = \text{power factor.}$$

$$\frac{\text{Wattless load}}{\text{KVA}} = \sin \phi = \text{wattless factor.}$$

$$\frac{\text{Wattless load}}{\text{KW}} = \tan \phi.$$

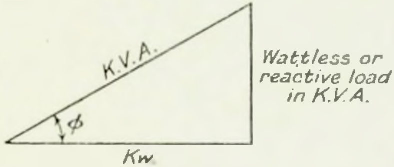


Fig. 12.

Oscillatory Discharge.

If the resistance is greater than $\sqrt{\frac{4L}{C}}$, the circuit will be non-oscillatory, and the discharge will be unidirectional. If r is less than $\sqrt{\frac{4L}{C}}$, it will be oscillatory, and the frequency of oscillation will be $\frac{1}{2\pi\sqrt{LC}}$.

The expression $\sqrt{\frac{L}{C}}$ is called the *Surge Impedance.*

Formulæ for Wave Length, etc.

Wave length in metres is

$\lambda_m = \frac{v}{f}$, where v is the velocity of light and f the frequency.

$$\therefore \lambda_m = \frac{299 \cdot 8 \times 10^6}{f} \text{ metres.}$$

But $\frac{1}{f} = \frac{2\pi}{\omega}$, where $\omega = 2\pi f$.

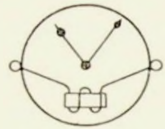
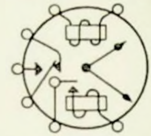
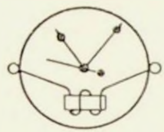
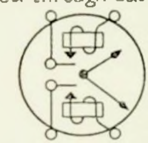
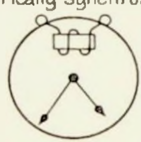
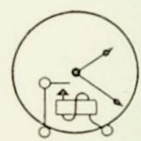
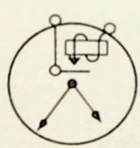
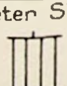
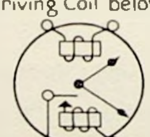
$\therefore \lambda_m = \frac{299 \cdot 8 \times 10^6 \times 2\pi}{\omega} = \frac{1884 \times 10^6}{\omega} = 1884 \times 10^6 \sqrt{LC}$, where $\omega = \frac{1}{\sqrt{LC}}$, L being in henries and C in farads.

$\therefore \lambda = 1884 \sqrt{LC}$ where L is in μH and C in μF

and $f = \frac{10^6}{2\pi\sqrt{LC}} = \frac{159{,}200}{\sqrt{LC}}$ where L is in μH and C in μF.

Measuring Instrumenis.

British Standard Graphical Symbols.

Ammeter. (A) Milliammeter. (mA)	Electric Clock, electrically driven from Master Clock. 	Synchronised Clock, as above, with Auto-Switch. 
Microammeter. (μA) Voltmeter. (V)	Electric Clock as above, and with Seconds Hand. 	Synchronised Clock, as above, but synchronised through Cut-out. 
Wattmeter. (W) Frequency Meter. (F)	Mechanical Clock, Weight or Spring driven, electrically synchronised. 	Self-winding Clock. 
Power Factor Meter. (PF) Galvanometer. (Gal.)	Mechanical Clock, as above, with Cut-out. 	Overload Relay, Constant Time Limit. OR/CT Reverse Power Relay. R/PR
Differential Galvanometer. (D.Gal.) Ammeter Shunt. 	Synchronised Clock, Synchronising Coil above, Driving Coil below. 	Overload Relay, Inverse Time Limit. OR/IT Reverse Current Relay. R/CR

Measurement of Power with Wattmeters.

The diagrams given in Figs. 13 to 19 are self-explanatory, and illustrate the use of single-phase wattmeters under various conditions. W is the power to be measured, and W_1, W_2 and W_3 are the wattmeter readings.

In the case of four-wire unbalanced load circuits, the connections are similar to those shown in Fig. 16, except that the pressure windings are joined to a fourth or neutral wire. Three wattmeters are essential in this case. Three complete elements can be obtained in one instrument.

With a balanced three-phase system in which the neutral point is not accessible, either two wattmeters can be used, connected as shown in Fig. 17, or the connections can be made as shown in Fig. 19. One wattmeter is sufficient in this case, but, in order to create an artificial neutral point, a " star resistance " is necessary. This star resistance may be self-contained in the instrument, or may consist of an external box. The total power is three times the wattmeter reading, or the scale of the instrument can be arranged to indicate the total power direct.

Power Factor Measurement with a Wattmeter.

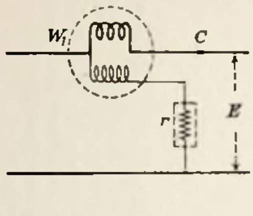
$$\text{Power factor} = \frac{\text{Volts} \times \text{amperes} \times \cos\phi}{\text{Volts} \times \text{ampere}} = \frac{\text{True power}}{\text{Apparent power}} = \cos\phi.$$

From the above it is clear that $\cos\phi$ is equal to the watts divided by the product of the amperes and the volts, consequently the power factor can always be determined by means of an ammeter, a voltmeter, and a wattmeter. In the case of a single-phase system one of each is required, the wattmeter being connected up as shown in Fig. 13. A two-phase system may be dealt with as two single-phase systems. With a balanced three-phase load the connections would be as shown in Figs. 18 or 19. In either case the ammeter is connected in one of the lines and the voltmeter to the neutral point—actual or artificial as the case may be. In the latter case, unless special precautions are taken, a common star resistance cannot be used for both wattmeter and voltmeter since the arm in parallel, with which the voltmeter is connected, would then have a lower resistance than the other two. In the case of an unbalanced three-phase load the wattmeters are connected as shown in Figs. 16 or 17, and three ammeters are connected, one in each line. As a rule the voltages are sufficiently well-balanced to warrant the use of a single voltmeter.

It is useful to note that, with the arrangement shown in Fig. 17, the angle of lag or lead can, in the case of a balanced three-phase load, be deduced from the ratio of the two wattmeter readings, since :—

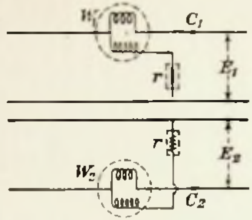
$$\frac{W_2}{W_1} = \frac{\sqrt{3}\ E\ C_2 \cos(30 - \phi)}{\sqrt{3}\ E\ C_1 \cos(30 + \phi)} = \frac{\cos(30 - \phi)}{\cos(30 + \phi)}$$

where W_1 and W_2 are the respective wattmeter readings and ϕ the angle of lag of the system. The load being balanced $C_1 = C_2$. At a power factor of unity ($\phi = 0$) the two wattmeters indicate the same (*i.e.*, $W_1 = W_2$). At a power factor of 0·5 ($\phi = 60$) one instrument indicates zero, and the other all the load. At lower power factors than this the former wattmeter reverses, and at zero power factor ($\phi = 90$) W_1 again equals W_2, but in this case the two instruments read in opposite directions.



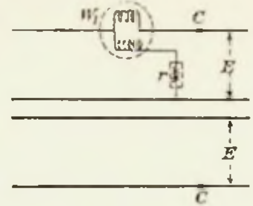
$W = W_1$

Fig. 13.
Single Phase.



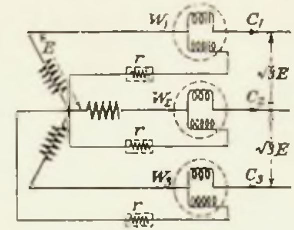
$W = W_1 + W_2$

Fig. 14.
Two-Phase (unbalanced).
Independent or Common return.



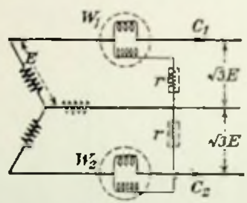
$W = 2W_1$

Fig. 15.
Two-Phase (balanced).
Independent or Common return.



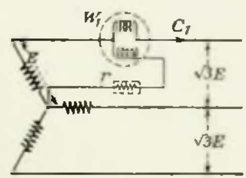
$W = W_1 + W_2 + W_3$

Fig. 16.
Three-Phase (unbalanced).
Using three instruments.



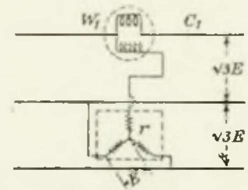
$W = W_1 + W_2$

Fig. 17.
Three-Phase (unbalanced).
Using two instruments.



$W = 3W_1$

Fig. 18.
Three-Phase (balanced).
Neutral point available.



$W = 3W_1$

Fig. 19.
Three-Phase (balanced).
Neutral point not available.

Measurement of Power by Three Voltmeter and Three Ammeter Methods.

Three Voltmeter Method.

For this measurement three voltmeters, an ammeter and a non-inductive resistance are required.

The non-inductive resistance R is placed in series with the apparatus X, which is absorbing the power to be measured, R being about the same resistance as X. The voltmeters V_1, V_2 and V_3, and the ammeter A are connected as shown in Fig. 20. The currents taken by the voltmeters must be small as compared with the main current.

The power taken by X is :—

$$W = \frac{A}{2V_1} ({V^2}_3 - {V^2}_1 - {V^2}_2) \text{ watts.}$$

The power factor is equal to :—

$$({V^2}_3 - {V^2}_1 - {V^2}_2)\ (2\ V_1\ V_2)$$

A disadvantage of this method is that the result varies as the difference of the squares of the readings.

Three Ammeter Method.

If three ammeters and a voltmeter are used as in Fig. 21 with the non-inductive resistance R in parallel with X, the power taken by X is measured by :—

$$W = \frac{V}{A_2} ({A^2}_3 - {A^2}_2 - {A^2}_1)$$

The same disadvantage holds for this as for the three voltmeter method.

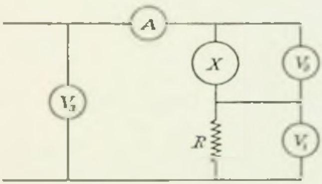


Fig. 20.
Three Voltmeter Method.

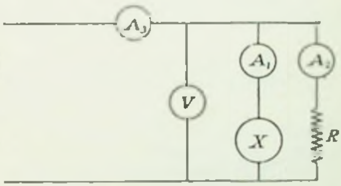


Fig. 21.
Three Ammeter Method.

APPENDIX B

WIRING RULES, REGULATIONS, ETC.

REGULATIONS FOR THE ELECTRICAL EQUIPMENT OF BUILDINGS.*
(FORMERLY I.E.E. WIRING RULES).

Definitions.

(*Note*—Definitions 1 to 40 have been omitted here: Definitions 44 to 54 will be found under B.S. Specification No. 168. See page 456, Vol. I.).

41. *Systems of Wiring.*

A. *Two-wire.*—A two wire system of wiring is one comprising two conductors between which the load may be connected, the wiring being effected by either of the following methods:

(*a*) *Two-conductor, insulated.*—Conductors insulated throughout are provided for all connections to both poles of the supply, the conductors being separate, twin, or concentric.

(*b*) *Two-conductor, earthed.*—Conductors are provided throughout for all connections to both poles of the supply, those connected to one pole being insulated thoughout, and those connected to the other being uninsulated throughout and efficiently earthed. The uninsulated conductor, known as the "external" conductor, completely surrounds the whole length of the other, known as the "internal" conductor.

> *Note.*—Except with the consent of the Electricity Commissioners no conductor directly connected to the public supply system may be earthed.

B. *Three-wire.*—A three-wire system of wiring is one comprising three conductors, one of which known as the "neutral" or "middle," is maintained at a potential midway between the potentials of the other two, referred to as the "outer" conductors. Part of the load may be connected directly between the outer conductors, and the remainder divided as evenly as possible into two parts connected respectively between the middle and each outer conductor.

C. *Two-phase Three-wire.*—A two-phase three-wire system of wiring is one comprising three conductors between one of which, known as the "common return," and the other two are maintained respectively alternating differences of potential displaced in phase by one-quarter of a period.

D. *Three-phase Three-wire.*—A three-phase three-wire system of wiring is one comprising three conductors between successive pairs of which are maintained alternating differences of potential successively displaced in phase by one-third of a period.

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E. *Two-phase Four-wire.*—A two-phase four-wire system of wiring is one comprising four conductors divided into two pairs which have maintained between their conductors alternating differences of potential displaced in phase by one-quarter of a period.

F. *Three-phase Four-wire.*—A three-phase four-wire system of wiring is one comprising four conductors, three of which are connected as in a three-phase three-wire system, the fourth being connected to the neutral point of the supply.

42. *Balanced.*—A three-wire system of generation or supply is said to be "balanced" when :—

A. In a case of direct-current or single-phase alternating-current systems of generation or supply the loads connected between the middle and each of the outer conductors are equal.

B. In the case of a three-phase system of generation or supply the load carried by any combination of two conductors is equal to the load carried by any other combination of two conductors.

> *Note.*—In the case of a three-phase four-wire system of generation or supply, in addition to condition (*b*) above, the loads connected between the middle and each of the outer or "phase" conductors are also equal.

Generating Plant.

55. *Dynamos and Alternators.*—Where an electric supply is generated upon the consumer's or other private premises, the generators, except in the case of extra-low pressure plant having a capacity not exceeding 5 kilowatts, shall conform in all respects to the British Standard Specification applicable to such machines.

56. *Situation of Generating Plant.*—

A. Generators, other than those defined in Regulations 49, 50, 51 and 53, shall be placed in well-ventilated rooms where inflammable or explosive dust or gases cannot accumulate. In situations where inflammable materials are stored or handled, generators may be placed only if suitably and adequately protected.

B. Generators shall be placed in positions in which they are not exposed to risk of mechanical injury or to damage from water, steam or oil.

C. No unprotected woodwork or other combustible material shall be within a distance of 12 inches (30 cm.) measured horizontally from or within 4 feet (120 cm.) measured vertically above the generators.

57. *Earthing of Generating Plant.*—When the supply is at medium pressure the generating plant shall have its bedplate and frame earthed (see Regulations 96 to 103).

58. *General Requirements for Secondary Batteries.*—When apparatus is supplied from secondary batteries the same general regulations shall be observed as apply to similar apparatus fed from generators developing the same difference of potential.

59. *Arrangement of Secondary Batteries*—

A. Every battery shall be so arranged that a potential difference exceeding 50 volts does not exist between adjacent cells without adequate protection, and that each cell shall be readily accessible from the top and from at least one side.

B. In a lead-sulphuric acid battery having more than 33 cells, and in a nickel-iron alkaline battery having more than 53 cells, the cells shall be supported on glass or vitreous porcelain insulators. In addition thereto, the stands shall be insulated where a battery comprises more than 56 lead-sulphuric acid cells or 88 nickel-iron alkaline cells.

C. When acid is used as an electrolyte for the cells, the battery connecting-bolts, unless of the non-corrosive type, shall be kept covered with petroleum jelly.

D. Cells having containers not sealed or provided with screw-down covers shall be fitted with spray arresters.

E. Celluloid shall not be employed in the construction of non-portable batteries; and where it is used for portable batteries the charging arrangements shall be such that if the cases become ignited the risk of a fire spreading shall be minimized.

60. *Ventilation of Secondary Battery Room.*—The room in which batteries are placed shall be thoroughly well ventilated.

61. *Control of Secondary Batteries.*—Suitable means shall be provided for controlling the current with which a battery is being worked. As a minimum this shall comprise an automatic cut-in and cut-out switch and fusible cut-out, or alternatively a circuit breaker with overload and reverse-current trips.

Switchboards.

62. *Situation of Switchboards.*—Switchboards shall be placed only in dry situations and in well-ventilated rooms where inflammable or explosive dust or gases cannot accumulate, and they shall be so arranged as to prevent access of acid fumes from batteries to the boards.

63. *General Construction of Switchboards.*—

A. Switchboards shall be constructed wholly of durable non-ignitable non-absorbent materials, and all insulation shall be of permanently high electric strength and insulation resistance.

B. If semi-insulating materials such as marble or slate be used, all conducting parts shall be insulated from the slate or marble slab with mica or other non-hygroscopic insulating material, except in the case of extra-low-pressure plant having a capacity not exceeding 5 kilowatts.

C. Where the frames of switchboards have to be earthed (see Regulations 96 to 103), suitable terminals shall be provided to which the earth connection can be made.

D. The various live parts shall be so arranged, by suitable spacing or shielding with non-ignitable insulating materials, that an arc cannot be maintained between any such parts or between such parts and earth.

E. The arrangement of all parts shall be such that the connections to all instruments and apparatus can be readily traced.

F. All parts, including connections, shall be readily accessible; and no fuse shall be fixed on the back of the board.

G. All nuts fixed at the back of switchboards shall be effectively locked so that they cannot become loose.

H. All omnibus bars and connections on switchboards shall be in accordance with British Standard Specification No. 159.

J. All circuits, instruments and important apparatus shall be clearly and indelibly labelled for identification. If detachable name-plates be employed they shall be non-ignitable and, if of metal, shall be so disposed as not to involve risk of causing short-circuits or earths.

K. The connecting wires to every voltmeter and its pivot lamp (if any) and to every earth lamp shall be protected by a fuse on each insulated pole.

L. Switches shall be so arranged that their blades or moving parts are disconnected from the supply in the "off" position.

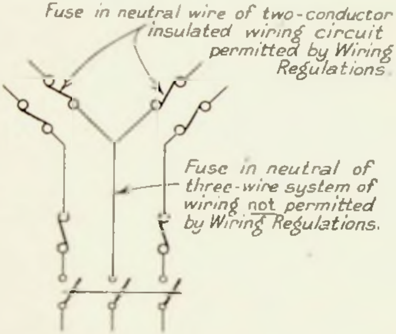
M. In every case in which switches and fuses are fitted on the same pole, these switches shall be so arranged that the fuses are disconnected from the supply when their respective switches are in the " off " position.

N. Where a scheme of colouring is employed to distinguish switchboard omnibus bars and connections to individual poles or phases, such scheme of colouring shall be in accordance with British Standard Specification No. 158.

Note.—The standard colours adopted for the identification of cables (see Regulation 85) are different from those specified in Regulation 63 N for switchboard connections.

O. The arrangement of omnibus bars carrying alternating currents shall be in accordance with British Standard Specification No. 158.

64. *Main Switchgear.*—Every main switchboard shall be fitted with the following switchgear as a minimum :—



(*Note.*—It is important to distinguish between a fuse in the neutral conductor of a three-wire system of wiring and a fuse in that conductor of a two-conductor insulated wiring circuit which is connected to the neutral of a three-wire system of wiring. See diagram).

(1) *Two-wire Systems of Wiring.*

A. When only one generator is installed, or when the supply is derived from a single two-wire service from an external source :

(*a*) For the generator or service main :—

(i.) If a two-conductor insulated system of wiring, either a double-pole overload circuit breaker or a double-pole linked switch with a fuse on each pole.

(ii) If a two-conductor earth system of wiring, either a single-pole overload circuit breaker or a single-pole switch with a single-pole fuse on the insulated pole.

(*b*) For each outgoing circuit from the switchboard or main fuses :—

(i) If a two-conductor insulated system of wiring, either a double-pole overload circuit breaker or a double-pole linked switch with a fuse on each pole.

(ii) If a two-conductor earthed system of wiring, either a single-pole overload circuit breaker or a single-pole switch with a single-pole fuse on the insulated pole.

B. When more than one generator is installed, the generators not being arranged to run in parallel, or when the supply is derived from a duplicate two-wire service from an external source :

(*a*) For each generator or service main :—

(i) If a two-conductor insulated system of wiring, either a double-pole overload circuit breaker, or a double-pole linked switch with a fuse on each pole.

(ii) If a two-conductor earthed system or wiring, either a single-pole overload circuit breaker, or a single-pole switch with a single-pole fuse on the insulated pole.

(*b*) For each outgoing circuit from the switchboard or main fuses :

(i) If a two-conductor insulated system of wiring, a double-pole change-over switch with either a double-pole overload circuit breaker or a fuse on each pole.

(ii) If a two-conductor earthed system of wiring, a single-pole change-over switch with either a single-pole overload circuit breaker or a fuse on the insulated pole.

C. When more than one generator is installed, the generators being arranged to run in parallel :

(*a*) For each generator, if shunt wound, a circuit breaker with overload and reverse-current trips. This circuit breaker shall be :—

(i) If a two-conductor insulated system of wiring, double-pole.

(ii) If a two-conductor earthed system of wiring, single-pole.

(*b*) For each generator, if compound wound, a circuit breaker with overload and reverse-current trips, and a single-pole equalizer switch so interlocked with the circuit breaker that this equalizer switch must be closed before the circuit breaker and cannot be opened until the main circuit is broken. This circuit breaker shall be :—

(i) If a two-conductor insulated system of wiring, double-pole.

(ii) If a two-conductor earthed system of wiring, single-pole.

(*c*) For each outgoing circuit from the switchboard or main fuses :—

(i) If a two-conductor insulated system of wiring, either a double-pole overload circuit breaker or a double-pole linked switch with a fuse on each pole.

(ii) If a two-conductor earthed system of wiring, either a single-pole overload circuit breaker or a single-pole switch with a single-pole fuse on the insulated pole.

(2) *Three-wire Systems of Wiring.*

A. When only two generators each wound for half pressure and acting as balancers are installed, or when the supply is derived from a three wire service from an external source :—

For each generator, or for each side of the service, either a double-pole overload circuit breaker or a double-pole linked switch with a fuse on each pole.

B. When more than one generator is installed, the generators being arranged to run in parallel, whether the machines be wound for the full pressure or for half pressure and act as balancers :—

(*a*) For each generator, if shunt wound, a double-pole circuit breaker with overload and reverse-current trips.

(*b*) For each generator, if compound wound, a double-pole circuit breaker with overload and reverse-current trips, and a single-pole equalizer switch so interlocked with the circuit breaker that this equalizer switch must be closed before the circuit breaker and cannot be opened until the main circuit is broken.

C. In the case of A or B above, for each outgoing circuit from the switchboard or main fuses :—

(*a*) If a three-conductor circuit, either a double-pole overload circuit breaker controlling the outer conductors, or a double-pole linked switch controlling the outer conductors with a fuse on each outer conductor. A fuse or unlinked switch shall not be included in the neutral conductor, but this requirement does not preclude the provision of an isolating link therein for testing purposes.

(*b*) If a two-conductor insulated circuit taken from the neutral and one outer, either a double-pole overload circuit breaker, or a double-pole linked switch with a fuse on each pole.

(*c*) If a two-conductor earthed circuit taken from the neutral and one outer, either a single-pole overload circuit breaker on that pole which is connected to the outer conductor, or a single-pole switch and single-pole fuse on that pole which is connected to the outer conductor. Such circuits are only permissible (see Regulation 88) if the neutral conductor of the supply be known to be earthed at the source of supply without a circuit breaker or added resistance.

(3) *Two-phase Three-wire Systems of Wiring.*

A. For each generator, or service main when the supply is derived from an external source, either a triple-pole circuit breaker with overload trips on each phase, or a triple-pole linked switch with a fuse on each conductor except the neutral.

B. For each outgoing three-conductor circuit from the switchboard or main fuses, either a triple-pole circuit breaker with overload trips on each phase, or a triple-pole linked switch with a fuse on each conductor except the neutral.

C. For each outgoing two-wire circuit from the switchboard or main fuses, either a double-pole circuit breaker with overload trip on at least one conductor, or a double-pole linked switch with a fuse on each conductor.

(4) *Three-phase Three-wire Systems of Wiring.*

A. For each generator, or service main when the supply is derived from an external source, either a triple-pole circuit breaker with overload trips on at least two phases, or a triple-pole linked switch with a fuse on each conductor.

B. For each outgoing three-conductor circuit from the switchboard or main fuses, either a triple-pole circuit-breaker with overload trips on at least two phases, or a triple-pole linked switch with a fuse on each conductor.

C. For each outgoing two-wire circuit from the switchboard or main fuses to which a supply is given from any two of the three conductors, either a double-pole overload circuit breaker or a double-pole linked switch with a fuse on each conductor.

(5) *Two-phase Four-wire Systems of Wiring.*

A. For each generator, or service main when the supply is derived from an external source, either a four-pole circuit breaker with overload trips on each phase, or a four-pole linked switch with a fuse on each conductor.

B. For each outgoing four-conductor circuit from the switchboard or main fuses, either a four-pole circuit breaker with overload trip on at least one conductor of each phase, or a four-pole linked switch with a fuse on each conductor.

C. For each outgoing two-wire circuit from the switchboard or main fuses, either a double-pole circuit breaker with overload trip on at least one conductor, or a double-pole linked switch with a fuse on each conductor.

(6) *Three-phase Four-wire Systems of Wiring.*

A. For each generator, or service main when the supply is derived from an external source, a triple-pole circuit breaker with an overload trip on each phase, or a triple-pole linked switch with a fuse on each phase.

B. For each outgoing three-conductor circuit from the switchboard or main fuses to which a supply is given from the three phases, either a triple-pole circuit breaker with overload trips on at least two phases, or a triple-pole linked switch with a fuse on each phase.

C. For each outgoing two-wire circuit from the switchboard or main fuses to which a supply is given from one of the three phases and the neutral when the neutral is known to be earthed at the source of supply without a circuit breaker or added resistance, either a single-pole overload circuit breaker, or a single-pole switch and single-pole fuse on that side of the circuit which is connected to one of the three phases. When the neutral is not earthed at the source of supply without a circuit breaker or added resistance, for each two-wire circuit either a double-pole overload circuit breaker, or a double-pole linked switch with a fuse on each conductor.

D. For each outgoing four-wire circuit used to supply a distribution board from which three-wire or two-wire circuits radiate, either a triple pole circuit breaker with an overload trip on each phase, or a triple-pole linked switch with a fuse on each phase. A fuse or unlinked switch shall not be included in the neutral conductor, but this requirement does not preclude the provision of an isolating link for testing purposes.

65. *Instruments.*—Every main switchboard when the supply is not derived from an external source, shall be provided with the following instruments as a minimum :—

A. *Two-wire Systems of Wiring.*

(*a*) When only one generator is installed, one ammeter and one voltmeter.

(*b*) When more than one generator is installed, the generators not being arranged to run in parallel, an ammeter for each generator and one voltmeter for use on any generator ; the voltmeter shall be fitted with a linked double-pole multiple-way switch or plug.

(*c*) When more than one generator is installed, the generators being arranged to run in parallel, an ammeter for each generator, and two voltmeters ; also a synchronizing device for paralleling purposes if the current be alternating.

For compound machines the ammeter shall be connected on the pole opposite to that to which the equalizer connection is made.

One of the voltmeters shall be fitted with a linked double-pole multiple way switch or plug enabling it to be connected to any one generator before the machine is put in circuit ; the other voltmeter shall be permanently connected to the omnibus bars.

B. *Three-wire Systems of Wiring.*

In addition to the instruments required for two-wire systems, a voltmeter shall be connected between the neutral and each outer omnibus bar ; also a central-zero ammeter if direct current, or central-zero wattmeter if alternating, in the main neutral conductor.

C. *Two-phase and Three-phase Systems of Wiring.*

(*a*) When only one generator is installed, an ammeter on each phase and one voltmeter.

(*b*) When more than one generator is installed, the generators being arranged to run in parallel, an ammeter on each phase for each generator, two voltmeters and a synchronizing device for paralleling purposes. One of these voltmeters shall be fitted with a linked double-pole multiple way switch or plug enabling it to be connected to one phase of any one generator before the machine is put in circuit. The other voltmeter shall be permanently connected to one phase of the omnibus bars. All these voltmeter connections shall be made to the same phase in each case.

Note—All instruments should preferably conform to British Standard Specification No. 89.

66. *Earth Testing.*—Where private generating plant is installed, main switchboards shall be provided with suitable means for indicating the state of the insulation of the system, either by lamps or voltmeters or otherwise, except in the case of extra-low pressure plant having a capacity not exceeding 5 kilowatts.

67. *Switches and Circuit Breakers.*—Every switch, fuse-switch and other circuit-breaker shall comply with the following requirements:—

A. All parts shall be so proportioned that when the normal working current for which they are designed flows through them continuously their temperature shall not rise above that of the surrounding air more than 36 degrees F. (20 degrees C.) in the case of switches rated below 100 amperes, and 54 degrees F. (30 degrees C.) in the case of switches rated at 100 amperes or above.

B. Each fuse-switch when opening the circuit as a switch, and each switch, shall break the circuit without permitting an arc to be maintained when a current 50 per cent greater than that for which it is rated is flowing under a pressure 50 per cent in excess of the pressure of supply. Each fuse switch when opening the circuit as a fuse, and each circuit breaker, shall comply with Regulation 68 for fusible cut-outs.

Note.—The construction and dimensions of switches and circuit breakers conforming to the requirements of this Regulation are embodied in British Standard Specifications Nos. 109 and 110, which further provide for complete interchangeability of parts.

C. Every circuit-opening device shall be so constructed and arranged that when placed in the off position it cannot accidentally move sufficiently to close the circuit.

D. If the current to be interrupted be sufficiently large to cause damage to the main contacts, suitable arrangements shall be made for the easy renewal of the parts on which the arc is formed.

E. The handles and their attachments shall be mechanically strong, and the exposed surface shall consist either entirely of insulating material or of metal. They shall be so designed and arranged that the hand of the operator cannot accidentally touch live metal or be injured through an arc arising from the switch or the blowing of an adjacent fuse. They shall not operate through unprotected slots.

F. The bases shall be of durable, non-ignitable, non-absorbent insulating material and shall comply with the following conditions:—

(*a*) Semi-hygroscopic materials such as slate or marble, if used, shall be free from metallic veins, cracks or other defects.

(*b*) The slabs shall be planed all over and, if of slate, treated, after drying, with a damp-proof medium, all holes being similarly treated.

(*c*) Bolts for securing marble or slate slabs to a metallic framework or case shall be insulated from the slabs, and the latter from the framework or case, by non-hygroscopic insulating bushes and washers.

G. When switches are not fixed on a switchboard, the live parts shall be enclosed by covers of rigid metal or non-conducting, non-ignitable material. In positions in which they are liable to mechanical injury, the covers, unless of rigid metal, shall be protected by a suitable guard. Metal cases shall be well clear of live parts and, if necessary, protection shall be provided to prevent arcing to the case.

H. All switches fixed in positions exposed to the weather, to drip, or to an excessively moist atmosphere, shall be contained in weather-proof cases which shall be provided with cable glands or bushings, or be adapted to receive screwed conduit, according to the way in which the cables entering the fittings are run.

Note.—Suitable glands are embodied in British Standard Specifiation No. 94, which further provides for complete interchangeability of parts.

J. Every electromagnetic circuit breaker shall be provided with suitable means of adjustment for determining the current at which it shall open, and shall be so arranged that it cannot be held in against this current.

K. Circuit breakers shall be so arranged and placed that no combustible material is endangered by their coming into action.

68. *Fusible Cut-outs.*

A. For *extra low pressure*, every cut-out shall comply with the following requirements:—

(*a*) All parts other than the fusible metal shall be so proportioned that their temperature shall not rise more than 54 degrees F. (30 degrees C.) above that of the surrounding air when the normal working current for which they are designed flows through them continuously.

(*b*) The fusing current, when the time taken for the fuse to blow is one minute (two minutes in the case of a lead-tin alloy fuse), shall be double the carrying capacity of the smallest cable which the fuse controls, provided that no fuse smaller than one rated to blow at 8 amperes need be inserted in any final sub-circuit.

Note.—The current-carrying capacities of rubber and paper-insulated cables are given in Tables II and III respectively (pages clxii and clxiv), and for the purpose of Regulation 68 the carrying capacity of a flexible cable or cord shall be considered to be that of a rubber-insulated cable of equal cross-section.

Tables XI and XII (pages clxxii and clxxiii) give the approximate fusing currents for wires of copper and lead-tin alloy respectively.

(*c*) The base shall be of durable, non-ignitable, non-absorbent insulating material.

(*d*) The circuit contacts and their terminals shall be so spaced or shielded that an arc cannot be maintained when the fuse blows.

(*e*) The fuse shall be of such construction, or be so guarded or placed, as to prevent danger from overheating, arcing and the scattering of hot metal or other substances when it blows.

(*f*) Fuses shall not be placed in ceiling roses, in switches other than fuse switches or those of the metal-covered type which comply with the Home Office Regulations, or in wall plugs or sockets. A fuse rated at not more than 5 amperes may, however, be placed in an intermediate adapter designed for insertion into a wall socket and for receiving the pins of a smaller plug connected to a consuming device taking 5 amperes or less, provided that in all cases the wall socket or sockets shall be protected by sub-circuit fuses mounted in accordance with Regulation 68 (*g*) below. Where such an adapter is used it shall not be sunk below the surface of the wall and its base shall comply with Regulation 68 (*c*) above.

(*g*) When cut-outs are not fixed on a switchboard they shall be grouped on distribution boards or, unless completely enclosed, shall be contained within cases conforming in all respects to the requirements specified in Regulation 69 below.

(*h*) Except as provided in Regulation 68 A (*f*) above, cut-outs in which the fuses are without removable carriers and are protected by close-fitting covers shall be used only on extra-low-pressure circuits and provided that :—

(i) They are ventilated in such a manner that fused metal cannot be ejected.

(ii) The maximum power in the building does not exceed 1,000 watts.

B. For *low pressure* every cut-out shall comply with the following requirements, in addition to the above requirements A (*a*) to (*h*):—

(*a*) It shall be provided with a suitable incombustible and insulating carrier for the fuse, of such form as to protect a person handling it from shock and burns ; and contacts shall be provided on the carrier to which the ends of the fuse can be readily attached.

(*b*) The base shall be provided with fixed circuit contacts of such form as to retain the carrier in position in the presence of vibration.

(*c*) The bus-bars, fixed contacts, removable contacts and fuses shall be so shielded as to protect a person against contact with live metal when the fuse carrier is being inserted or removed.

(*d*) Cut-outs for use with low pressure shall be of two grades designated respectively—" Ordinary Duty " and " Heavy Duty." The term " Ordinary Duty " is applied to a cut-out or a fuse carrier when the maximum short-circuit current in the circuit cannot exceed the following values :—

Maximum Working Current Amperes.	*Maximum Short-Circuit Current for " Ordinary Duty " Cut-out Amperes.*
10	1,000
30	2,000
60	4,000
100	6,500

" Heavy Duty " cut-outs shall be used on circuits where " Ordinary Duty " cut-outs are inadmissible. Both types of fuse shall break the circuit, under such conditions as are above specified, without damage to the cut-out or its surroundings.

C. For *medium pressure* every cut-out shall comply with the requirements set out in A (*a*) to (*h*) and B (*a*), (*b*) and (*c*) above.

Note.—British Standard Specification No. 88, for Low-Pressure Electric Cut-Outs, provides for complete interchangeability of parts.

69. *Section and Distribution Boards.*

A. The general design and construction of section and distribution boards shall conform to the requirements of Regulations 63 and 67 so far as they are applicable. The fuses fitted in section and distribution boards shall conform to the requirements of Regulation 68.

B. Every section or distribution board shall be contained within a protecting case.

C. In earthed concentric wiring systems, section and distribution boards shall, in addition to complying with the foregoing conditions, be contained in cases in which provision is made for the following :—

(*a*) If of metal, the attachment to the case of all external conductors of the concentric cables entering it.

(*b*) If of wood, a sheet of incorrodible metal, of the same area and shape as the base of the case, interposed between it and the wall or other support to which it is attached. This sheet of metal shall be not less than $\frac{1}{16}$ inch in thickness and be electrically and mechanically connected to all the external conductors of the concentric cables entering the case, by means of a metal bar, or rod, or bare wire conductor, of which the resistance shall not be greater than that of the inner conductor of the cable feeding the board.

D. The design, construction and arrangements of the cut-outs and metal box shall be such that an arc cannot be set up to the case or between poles when the fuse is melted by a short-circuit current in accordance with Regulation 68 B (*d*). For the purpose of this test, the metal box and its cover shall be efficiently earthed.

E. All soft-wood cases shall be lined with non-ignitable insulating material which shall be clear of all live parts by not less than 1 inch (2·5 cm.) Cases of hard-wood, such as teak, need not be so lined.

F. Boxes not provided with backs forming an integral portion thereof shall have a non-ignitable insulating shield between their contents and any structure to which they are fixed.

G. If glass fronts be provided they shall clear all live parts by not less than 1 inch (2·5 cm.), and such fronts may be regarded as insulating shields.

H. All cases fixed in positions exposed to the weather, to drip, or to an abnormally moist atmosphere, shall be provided with cable glands or bushings (see Note to Regulation 67 H), or be adapted to receive screwed conduit, according to the way in which the cables entering the cases are run.

J. The cases of section and distribution boards shall be of sufficiently ample dimensions to allow of easy leading in and access for the attachment of cables, and for conduits and fixing nuts.

K. Where cut-outs are grouped on a board they shall be fixed vertically, and a shield of non-ignitable insulating material shall be inserted between cut-outs of opposite polarity when placed one above the other.

Conductors of Cables.

70. *Material of Conductors.*

A. All conductors, other than the outer conductors of earthed concentric systems, shall be of annealed copper and shall conform to British Standard Specification No. 7.

B. When the insulating covering of the conductor may contain sulphur, each wire shall be efficiently and uniformly coated with tin free from all impurities.

71. *Standard Sizes of Conductors.*

A. The sizes of conductors as given in British Standard Specification No. 7 are recognized as standard.

B. The standard sizes and resistances of conductors for flexible cables and flexible cords are set out in Tables I and V (pages clxi and clxvi) respectively.

72. *Minimum Size of Conductor.*—No cable having a conductor of nominal sectional area less than 0·0015 square inch (1/·044in.) shall be used except for wiring fittings, for which a conductor having a nominal sectional area not less than 0·001 square inch (1/·036in.) may be employed. Where the design of a fitting renders it impossible to use a conductor of this size, a flexible cord having a conductor of nominal sectional area not less than 0·0006 square inch (14/·0076in.) may be used.

73. *Maximum Size of Single Wire.*—All conductors having a nominal sectional area exceeding 0·003 square inch (1/·064in.) shall be stranded.

74. *Current-carrying Capacity of Conductors.*

A. The size of conductors shall be so selected that :—

(*a*) For lighting, the fall in pressure from the consumer's terminals in the case of a public supply, or from the omnibus bars of the main switchboard controlling the various circuits in the case of private generating plant, to any and every point on the installation does not exceed 1 volt plus 3 per cent. of the pressure at the consumer's terminals, or omnibus bars as the case may be, when the conducters are carrying the maximum demand under the practical conditions of service.

(*b*) In no case, whether for lighting, heating or power, does the current exceed that given in Tables II and III (pages clxii and clxiv) for each size of conductor when the maximum current referred to in (*a*) above is being carried.

(*c*) Regulation 72 is complied with.

Note.—Tables II and III also show the total length of conductor in circuit that will give a drop of pressure of 1 volt when the respective maximum currents are being carried.

B. In the case of flexible cables and flexible cords, it being advisable to restrict the maximum current, the values shown in Table IV (page clxv) and column 9 of Table VI (page clxvii) respectively shall be adopted.

Note.—Two ratings are given for flexible cables (Table IV), the lower rating applying to flexible cables supplying portable domestic appliances.

Insulation and Protective Covering of Conductors.

75. *Insulation of Conductors.*—Except as provided in Regulation 88 (earthed concentric wiring) all conductors shall be insulated either by being carried on insulators (see Regulation 86) or by the use of insulated cables.

76. *Types of Cables.*

A. Cables shall be single, twin, three-core, four-core, concentric or triple concentric in accordance with the dimensions and other requirements of British Standard Specification No. 7, and only approved types shall be employed. The following types have been approved :—

Vulcanized rubber-insulated cables.

(*a*) Taped as in Regulation 77 (A) and compounded.

(*b*) Taped as in Regulation 77 (A) and braided.

(*c*) Sheathed with a closely-fitting seamless covering of commercially pure lead having a smooth exterior surface and of uniform radial thickness in accordance with British Standard Specification No. 7.

(*d*) Lead covered as in (*c*) above, and bedded and armoured, with or without serving or braiding over the armour; the bedding, armouring and serving or braiding (if any) to be in accordance with British Standard Specification No. 7.

(*e*) Bedded and armoured, with or without serving or braiding over the armour; the bedding, armouring and serving or braiding (if any) to be in accordance with British Standard Specification No. 7.

(*f*) Encased in a closely fitting covering of brass, copper, or equally hard incorrodible metal in accordance with Regulation 82.

(*g*) Sheathed with a closely-fitting seamless covering containing not less than 95 per cent. of commercially pure lead (the remainder consisting of rarer metals) and in other respects complying with (*c*) above.

(*h*) Covered with tough rubber compound (see Regulation 83).

Paper-insulated Cables.

(*j*) Sheathed with a closely-fitted seamless covering of commercially pure lead having a smooth exterior surface and of uniform radial thickness in accordance with British Standard Specification No. 7.

(*k*) Lead covered as in (*j*) above, and bedded and armoured, with or without serving or braiding over the armour; the bedding, armouring and serving or braiding (if any) to be in accordance with British Standard Specification No. 7.

(*l*) Encased in a closely-fitting covering of brass, copper or equally hard incorrodible metal in accordance with Regulation 82.

(*m*) Sheathed with a closely-fitting seamless covering containing not less than 99 per cent. of commercially pure lead (the remainder consisting of rarer metals), and in other respects complying with (*j*) above.

B. If it be desired to use types of cables insulated otherwise than specified in Regulation 76 (A), sample lengths shall be submitted, together with a report from a recognized testing authority (such as the National Physical Laboratory) as to the behaviour, properties and life of the insulating materials employed, for consideration by the Institution of Electrical Engineers, with a view to their use being permitted, provisionally or otherwise, if found satisfactory.

> *Note.*—Evidence of insulation resistance alone will not be considered to be sufficient for the purpose of Regulation 76 (B).
>
> The requirements in regard to any additional types of cables that may be permitted will be issued later as a supplement to these Regulations.

C. Of the types of cables specified in (A) above, the following shall not be used for alternating current except in connection with an earthed concentric system in which the sheathing forms one conductor:—

(*a*) Single cables armoured with steel wire or tape or encased in a ferrous sheath.

(*b*) Single cables encased in brass, copper or equally hard incorrodible metal.

(*c*) Single, unarmoured, lead-covered cables having a conductor of nominal sectional area greater than 0·25 square inch.

(*d*) Single unarmoured cables sheathed with a covering containing not less than 95 per cent. of commercially pure lead (the remainder consisting of rarer metals) and having a conductor of nominal sectional area greater than 0·25 square inch.

Note.—Where cables of the types referred to in Regulation 76 C (*c*) and (*d*) having a nominal sectional area of 0·25 square inch or less are used for alternating current, it is desirable that the lead and return should be placed as near as possible to each other.

77. *Vulcanized Rubber-insulated Cables, except Flexible Cords.*

A. Vulcanized rubber-insulated cables shall be insulated with a label of pure rubber next to the conductor, an intermediate layer of vulcanizing rubber, and an outer jacket of vulcanizing rubber. These three layers shall together constitute the dielectric, and its radial thickness shall be in accordance with British Standard Specification No. 7. The dielectric shall be surrounded by a layer of tape, and the whole shall be vulcanized together. In the case of cables having an outer protective covering of "tough rubber" (see Regulation 83) the tape may be omitted.

B. Braided cables shall have an exterior braiding of hemp, cotton or jute, thoroughly impregnated with a protective compound of such a nature as not to have any deleterious action on the rubber or armouring, as the case may be. The finish of the braiding shall be smooth and uniform.

78. *Paper-insulated Cables.*—Paper-insulated cables shall be insulated with a covering of paper impregnated with a chemically neutral insulating compound. The radial thickness of dielectric shall be in accordance with British Standard Specification No. 7.

79. *Types of Flexible Cords.*

Two kinds of insulation for flexible cords are recognized as standard, viz. :—

(1) High Insulation.

(2) Medium Insulation.

High Insulation flexible cords shall be insulated in one of the following two ways, and the radial thickness of rubber insulation shall not be less than that specified in column 2 or column 3 of Table VI (page clxvii) according to the insulating material used :—

(*a*) Each conductor, which shall be composed of plain copper wires, shall be lapped with cotton and shall have two layers of pure rubber overlapped with cotton.

(*b*) Each conductor, which shall be composed of copper wires efficiently and uniformly coated with tin free from all impurities, shall have one layer of pure rubber and two layers of vulcanizing rubber.

Medium Insulation flexible cords shall be insulated in one of the following two ways, and the radial thickness of rubber insulation shall not be less than that specified in column 4 or column 5 of Table VI according to the insulating material used :—

(*c*) Each conductor, which shall be composed of plain copper wires, shall be lapped with cotton and shall have two layers of pure rubber overlapped with cotton.

(*d*) Each conductor, which shall be composed of copper wires efficiently and uniformly coated with tin free from all impurities, shall have two layers of vulcanizing rubber.

80. *Protective Covering of Flexible Cables and Cords.*—Flexible cables and cords shall be provided with one of the following protective coverings, but of these (*a*) and (*b*) shall not be used where the cable or cord is liable to the risk of mechanical damage :—

(*a*) Natural or non-ignitable artificial silk braiding ;

(*b*) Glacé cotton braiding ;

(*c*) Hemp, cotton or jute braiding, thoroughly compounded ;

(*d*) Wire armouring comprising a flexible braiding or spiral of galvanized steel or phosphor bronze wire in addition to the covering specified in (*c*) ;

(*e*) Hard cord braiding in addition to the covering specified in (*c*) ;

(*f*) Tough rubber compound (see Regulation 83) applied directly to the insulated core or to two or more such cores laid up together.

81. *Twin Flexible Cords.*

A. Twin twisted flexible cords shall be used only for fixed wiring, fixed fittings, and portable lamp standards ; in all other positions, and for all other purposes, flexible cords made up to a circular or oval section shall be employed.

Note.—The use of flexible cords made up to a circular or oval section is recommended for all portable fittings.

B. Medium Insulation (kind 2) flexible cords shall be used only between ceiling roses and pendant fittings and for the internal wiring of fittings.

C. The maximum weight carried by a twin twisted flexible cord shall be as follows :—

Number and diameter of wires comprising conductor.	Maximum permissible weight.
14/·0076 in.	3 lb.
23/·0076 in.	5 lb.
40/·0076 in.	10 lb.

Note.—Where a weight greater than 10 lb. has to be supported Regulation 104 D. shall apply.

82. *Hard Metal Sheathing of Cables.*—Cables sheathed as specified in Regulation 76 A (*f*) and (*l*) shall comply with the following requirements :—

A. The dielectric shall be surrounded with a closely fitting sheathing of brass, copper or other equally hard and incorrodible metal, the method of application of the sheathing and the nature of the dielectric being such that the sheathing can be readily removed without damage to the dielectric. The sheathing shall be watertight and wholly metallic, any joints in it being made by soldering or brazing.

B. The sheathing shall be of sufficient rigidity to ensure that the cable will not sag when supported in a horizontal position by clips at 2ft. (60 cm.) intervals.

C. The sheathing shall be sufficiently flexible to allow of the cable being bent to a radius equal to six times its diameter without cracking of the sheathing or damage to the dielectric, and the cable must reasonably retain its original shape.

D. The sheathing shall have a minimum thickness of 0·01 inch for all sizes of conductor up to and including 0·003 square inch (1/·064 in.), and of 0·015 inch for all sizes of conductor above 0·003 square inch (1/·064 in.) and up to and including 0·0225 square inch (7/·064 in.)

83. *Tough Rubber Compound.*—This covering, when used as a protection to vulcanized rubber-insulated cables, shall form a closely fitting sheath filling the external irregularities of the laid-up cores in the case of multicore

cables, and concentric with the conductor when single core, and shall be capable of offering a high degree of resistance to abrasion, acids, oils and alkalis. The radial thickness of this sheath shall not be less than that specified in British Standard Specification No. 7.

84. *Tests of Dielectric of Cables.*

A. The dielectric of cables, except flexible cords (see Clause D below), insulated with vulcanized rubber or impregnated paper shall withstand the pressure test and other tests specified in British Standard Specification No. 7. Subsequent to such pressure test and whilst the cable is still immersed in water the insulation resistance at a temperature of 60° F. (15·6° C.), after one minute's electrification at a pressure of at least 500 volts, shall not be less than that given in Table VII (page clxviii).

B. The insulation resistance of each insulated conductor of a multicore cable, except flexible cords, shall not be less than that given in Table VII for single conductors of the same sectional area.

C. The insulation resistance of the dielectric separating the two conductors of a concentric cable shall not be less than that given in Table VII for single conductors having the same diameter as the inner conductor.

D. The dielectric of multicore flexible cords, except in the case of High Insulation (see Regulation 79) cords with vulcanized rubber insulation, shall withstand for 15 minutes the alternating pressure and frequency set out in column 3 of Table VIII (page clxix) for the respective kinds of insulation indicated therein, the flexible cord being in a dry state at the time of test and the test being made between conductors. In the case of High Insulation cords with vulcanized rubber insulation the flexible cord shall have previously been immersed in water for 24 hours and shall be still so immersed at the time of test, and the test shall be taken between each conductor and earth, the conductor or conductors not under pressure being earthed. Subsequent to such pressure test, the insulation resistance of flexible cords with vulcanized rubber insulation at a temperature of 60° F. (15·6° C.), after one minute's electrification at a pressure of at least 500 volts, shall not be less than that given in Table IX (page clxix), this test in the case of the High Insulation kind being made whilst the flexible cord is still immersed in water.

> *Note.*—The above tests are intended to be carried out at the Cable Manufacturer's works, the pressure being derived from a source having a rated output of not less than 5 kilowatts. Cables and flexible cords which have to be tested when immersed in water can be tested before the protective coverings are applied, but, if desired, the pressure test can be made on the finished cable, the protective coverings of such cables, however, being thereby damaged.

85. *Identification of Cables by Colour.*—Where colours are used to distinguish the conductors the following shall be employed (where polarities are indicated they refer to the polarity up to the lamp or other point when the switch is closed) :—

A. For *direct-current* systems of generation or supply :

(*a*) Two-wire circuits connected to a two-wire system of wiring—
Red for positive or switch wire.
Black (or blue) for negative.

(*b*) Two-wire circuits connected to the neutral and one side only of a three-wire system of wiring—

Red for outers.
Black (or blue) for neutral.

(c) Two-wire or three-wire circuits connected to a three-wire system of wiring except as in (b) above.

Red for positive or switch wire.
Black (or blue) for neutral.
White (or yellow) for negative or switch wire.

B. For *alternating-current* systems of generation or supply :

(a) Single-phase, two-wire system of wiring—
Red for one conductor or switch wire.
Black (or blue) for other conductor.

(b) Single phase, three-wire system of wiring—
Red for one conductor or switch wire.
Black (or blue) for neutral.
White (or yellow) for other conductor or switch wire.

(c) Two-phase, three-wire system of wiring—
Red for one phase.
Black (or blue) for neutral.
White (or yellow) for other phase.

(d) Two-phase, four-wire system of wiring—
Red for one phase.
White (or yellow) for other phase.

(e) Three-phase, three-wire system of wiring—
Each conductor red, white (or yellow) and green respectively.

(f) Three-phase, four-wire system of wiring—
Red, white (or yellow) and green for the three phases.
Black (or blue) for neutral.

Note.—The standard colours adopted for the identification of switchboard connections (see Regulation 63 N) are different from those specified in Regulation 85 for cables.

Installing and Fixing of Conductors and Cables.

86. *Bare Conductors.*

A. Bare conductors may be used as collector or trolley wires for travelling cranes and similar appliances, and for battery connections.

B. They shall be supported upon insulators and so spaced that risk of accidental contact between the conductors themselves or between conductors and walls, structural or other metal work, is reduced to a minimum.

C. At each straining point, i.e., at the ends of each conductor, efficient straining gear fitted with double insulation shall be provided.

D. The circuit supplying current to such bare conductors shall, except in the case of the regulating cells of batteries, be protected by either a circuit breaker or a switch and fuse, as is specified for similar circuits with continuously insulated conductors.

E. Bare conductors extended to positions liable to lightning discharge shall be fitted with lightning arresters on each pole or phase.

F. Wall rosettes or brackets used as supports for span wires shall not be fixed within 12 inches (30 cm.) of any gas pipe.

G. Except as above specified, bare conductors shall only be used in positions not ordinarily accessible to unauthorized persons and under such circumstances as may be sanctioned by the Fire Office insuring the risk.

87. *Cables.*

Class L. Taped and Braided Cables.

Taped and braided cables such as are specified in Regulation 76 A (*b*) may be used without the further protection of conduit or casing provided that :—

(1) They are open to view throughout their length and, in particular, are not installed under floors or within partitions or buried in plaster.

(2) They are kept away from all structural metal work.

(3) In any position in which they would be liable to mechanical damage, and wherever they are within 6 feet above the floor, they are adequately protected.

(4) They are kept away from gas and water pipes.

(5) They are secured by porcelain cleats, or by clips, saddles or clamps which are so spaced as to prevent the cables coming into contact, and which have smooth or rounded edges that will not indent or damage the braiding.

(6) In damp situations they are spaced not less than ½ inch away from walls, ceilings and floors by means of saddles or cleats which, together with their fixings, are of non-absorbent, non-rusting material.

(7) When passing through floors, walls, partitions or ceilings they are protected by being enclosed in metal, porcelain or other non-absorbent non-ignitable conduits, the ends of which are bushed or so arranged as to prevent abrasion.

(8) When passing through party walls or fire-resisting floors, the conduits referred to in (7) above are close fitting, and the holes through which they pass are plugged with fire-clay or similar non-ignitable material, no space through which fire might spread being left around or inside the conduits.

Class M1. Metal-sheathed Cables.

Metal-sheathed cables such as are specified in Regulation 76 A (*c*), (*g*), (*j*) and (*m*), may be used without the further protection of conduit or casing provided that they are installed in accordance with the above condition (4) and, in addition, provided that :—

(9) They are secured, at intervals sufficiently short to prevent appreciable sagging of the cable, by clips, saddles or clamps constructed of such material as will not be liable to set up electrolytic action with the sheathing and having smooth or rounded edges which will not indent or damage the sheathing.

(10) When vertical, cables are fixed by the same means, with supports at the same intervals, as when horizontal, unless they be inaccessible, when a length not exceeding 10ft. may be allowed between the supports, provided that the upper support firmly grips the cable or wire and that where there is a change of direction from horizontal to vertical they are brought over a rounded support of a radius not less than six times the external diameter of the sheathing.

(11) Effectual means are taken to ensure that all metallic envelopes of cables are efficiently earthed and made electrically continuous throughout their length by means of soldered joints or, alternatively, by bonding clamps specially designed for the purpose or forming part of joint boxes and similar fittings in which the cables terminate.

(12) The electrical resistance of the metallic envelope of cables in a complete installation, measured between such envelope at a point near the main switch and any other point of the installation, does not exceed 2 ohms.

(13) If liable to mechanical damage they are adequately protected, having regard to the nature of their sheathing or casing.

(14) In damp situations and where exposed to the weather the saddles and fixtures referred to in (9) above are of non-rusting material.

(15) When passing through steel or iron structural work, the holes through which they pass are bushed to prevent abrasion.

(16) When passing through party walls or fire-resisting floors, the holes through which they pass are plugged with fire-clay or similar non-ignitable material.

(17) When run under floors or behind partitions all connections are made in boxes of ample capacity and of non-absorbent, non-ignitable material (*see* Regulation 93 C.)

Class M2. Hard Metal-Sheathed Cables.

Hard metal-sheathed cables such as are specified in Regulation 76 A (*f*) and (*l*) may be used without the further protection of conduit or casing provided that they are installed in accordance with the above conditions (4), and (9) to (17) inclusive.

Class R. Armoured Cables.

Armoured cables such as are specified in Regulation 76 A (*d*), (*e*) and (*k*), may be used without the further protection of conduit or casing provided that they are installed in accordance with the above conditions (4), (9), (10), (13), (14), and (17). In addition, when lead sheathed such sheathing shall comply with conditions (11) and (12), and when not lead sheathed the armouring shall so comply.

Class S. Cables covered with Tough Rubber Compound.

Cables protected in accordance with Regulation 76 A (*h*) may be used without the further protection of conduit or casing provided that they are installed in accordance with the above conditions (4), (9), (10), and (13) to (17) inclusive.

Class T1. Conduits screwed.

All classes of cable specified in Regulation 76 A may be enclosed in steel conduits provided that the conduits are installed in accordance with the above conditions (4), (13), (14), (16), and (17), and, in addition, provided that :—

(18) The conduits, together with their fittings, are made in accordance with British Standard Specification No. 31 and if used for circuits of medium pressure are of heavy gauge.

(19) The conduits are mechanically and electrically continuous across all joints therein and are earthed in accordance with Regulations 96 to 103.

(20) The electrical resistance of the conduit in a complete installation, measured between the conduit at a point near the main switch and any other point of the installation does not exceed 2 ohms.

(21) Ventilating outlets are provided at the highest and lowest points of each circuit to allow circulation of air through the conduit.

(22) The conduits of each circuit are erected complete before the cables are drawn in.

Note.—Table X shows the capacity of conduits for the simultaneous drawing-in of conductors.

(23) Provision is made at the ends of all conduits to prevent abrasion of the covering of cables emerging therefrom.

(24) The ends of conduits where terminating at accessories and fittings are screwed thereto or provided with lock-nuts, or are led into separate blocks, preferably of non-ignitable material.

(25) No elbows or tees, unless of the inspection type, are used, except at the ends of conduits immediately behind fittings or accessories; and no bend has a radius smaller than 2½ times the outside diameter of the conduit.

(26) In damp situations, and where exposed to the weather, the conduits are welded, brazed or solid drawn.

Class T2. Metal Conduits not screwed.

All classes of cable specified in Regulation 76 A may be enclosed in conduits of steel or other metal provided that such conduits are installed in accordance with the above conditions (4), (13), (14), (16), (17), (19), (20), (21), (22), (23), (25), (26), and in addition provided that:—

(27) The ends of the conduits where terminating at accessories and fittings are adequately clamped thereto, or are led into separate blocks, preferably of non-ignitable material.

Note.—Plain slip sockets do not comply with the above conditions, some form of screwed or grip joint which will give ample and permanent electrical conductivity and mechanical rigidity throughout being necessary.

Class T3. Non-Metallic Conduits.

All classes of cable specified in Regulation 76 A may be enclosed in conduits of non-ignitable, non-absorbent, damp-proof material, provided that such conduits are mechanically continuous and strong and are installed in accordance with the above conditions (4), (13), (14), (16), (17), (21), (22), (23), (25) and (27).

Class W. Wood Casing.

All classes of cable specified in Regulation 76 A may be enclosed in wood casing provided that:

(28) Wood casing is used only in dry situations.

(29) It is not buried in plaster or cement, nor fixed in contact with gas pipes or water pipes or immediately below the latter.

(30) The capping is secured by screws.

(31) If the casing forms part of ornamental woodwork, ready access is provided to the cables contained therein.

Class Z. Flexible Cords.

High Insulation (kind 1) twin or multicore flexible cords such as are specified in Regulation 79 may, in addition to being used for pendant and portable appliances, be installed provided that conditions (3) and (4) above are complied with, and also provided that:—

(32) They are used only for low-pressure sub-circuits carrying currents not exceeding 6 amperes from distribution boards.

(33) If insulated with pure rubber they are not used in damp situations.

(34) They are supported on porcelain or other equally effective insulating cleats fixed at intervals not exceeding 3 ft., such cleats being so designed and placed that the cords are securely and permanently spaced away from walls, ceilings and structural metal work.

(35) They are open to view throughout their length, except where protected in accordance with condition (3) above and conditions (38) and (39) below.

(36) The premises do not come within the provisions of the Factory and Workshop Acts and the Coal Mines Regulations Acts.

(37) They are not used in shops, warehouses, or places of public resort.

(38) Where passing directly through floors or division walls they are protected by non-ignitable damp-proof conduits.

(39) Where issuing from fittings and unavoidably passing into ceilings they are enclosed in non-ignitable tubes terminating in non-ignitable junction boxes.

88. *Earthed Concentric Wiring.*

A. Earthed concentric wiring shall only be used when connected to systems of supply :—

(*a*) So as to derive the supply from the secondary side of transformers or converters so arranged that the public supply system is electrically insulated therefrom ; or

(*b*) In which earthed concentric wiring has been approved by the Electricity Commissioners ; or,

(*c*) Consisting of an independent generating plant.

B. Every earthed concentric installation shall be so arranged that the internal conductors are protected by a single-pole switch and fuse placed in a position easily accessible to the consumer and situated as near as possible to :—

(*a*) The point or points of entry of the service main or to the secondary of the transformer, in the case of a public supply ; or,

(*b*) The generator, in the case of a private plant.

C. Regulation 94 B shall apply to earthed concentric installations.

D. When the supply is direct current the external conductor shall always be the one nearest to earth potential and shall, where possible, be negative to the inner conductor ; also the difference of potential between any two points in the external conductors shall not exceed :—

(*a*) Seven volts, if the internal conductors be connected to the positive pole of the system ; or

(*b*) One and a half volts, if the internal conductors be connected to the negative pole of the system.

Note.—As Regulation 88 D is framed with a view to minimizing the risk of electrolytic action, alternating-current installations are exempt from its provisions.

E. From the position or positions at which the installation is earthed, concentric wiring shall be employed throughout up to all fixed positions for fittings or accessories. At the positions at which the external conductor ceases to surround the internal conductor the latter shall be separated from the surface upon which the fitting or accessory is mounted by an incorrodible metal plate or terminal box to which the external conductor is electrically connected. This requirement does not preclude the interposition of a wooden block between the metal plate and the fitting or accessory mounted thereon, provided that this metal plate covers the principal recess in the wooden block.

F. Where the metal sheathing of a cable is used as one conductor, the resistance of the sheathing shall not be greater than that of the inner conductor when measured at a temperature of 60° F.

G. Joints, however made, in the external conductor shall be of such a nature that the conductivity of the conductor is not reduced.

H. All circuits, lamps and appliances shall be controlled and protected by single-pole circuit breakers, or switches and fuses which shall be inserted in the internal conductor of the circuit. No circuit breaker, switch or fuse shall be included in the external conductor.

J. Ordinary accessories may be used, but if lampholders having central contacts be employed, such central contacts shall be connected to the internal conductor.

K. Lamp fittings may be wired with two separate wires, one being insulated and connected to the internal conductor and the other to the metal work of the fitting.

L. Twin flexible cords may be used between fixed points and portable or pendant fittings. If such flexible cords terminate in plug and socket connections these connections shall be of either the concentric or two-pin type, and if two-pin plugs be used, either they shall have pins of unequal diameter or other means shall be provided to prevent the connections being reversed.

89. *Selection of Cable Runs.*

A. Cables shall be fixed as far as possible in accessible positions, so chosen that they are not exposed to drip or accumulation of water or oil, or to high temperature from boilers, steam pipes, or other hot objects, or to risk of mechanical damage.

B. The runs shall be selected to accord with the following requirements :—

(*a*) No rubber-insulated cable shall be bent to a radius shorter than twice its overall diameter if unarmoured, or to a radius shorter than three times its overall diameter if lead-covered and /or armoured.

(*b*) No paper-insulated cable (whether armoured or not) shall be bent to a radius shorter than six times its overall diameter.

Note.—These represent the permanent bends ; the arrangement of the runs must be such that the cable is not subjected to bends of this kind during drawing-in or erection.

90. *Bunching of Cables.*

A. When installed in wooden casing, cables carrying direct or alternating current may, if desired, be bunched whatever their polarity, provided that :—

(*a*) The number of cables bunched is not more than 10 if the sectional area of each cable does not exceed 0·007 square inch (7/·036in.) ;

6 if the sectional area of any cable exceeds 0·007 square inch (7/·036in.) but does not exceed 0·0225 square inch (7/·064in.) ;

4 if the sectional area of any cable exceeds 0·0225 square inch (7/·064in.) but does not exceed 0·1 square inch (19/·083in.) ;

3 if the sectional area of any cable exceeds 0·1 square inch (19/·083in.).

(*b*) The size of the casing does not exceed that necessary to accommodate the maximum number of cables permissible.

B. When installed in metal conduits, cables carrying direct current may, if desired, be bunched whatever their polarity, but if carrying alternating current they shall always be bunched.

Note.—Where large cables are bunched in casing or conduit the current rating given in Tables II, III and IV should be reduced in order to ensure that the cables are not overheated.

91. *Bell and Telephone Circuits.*—Cables which are used in connection with the electric bells, telephone and signalling apparatus, etc., in a building shall be kept away from, and not be installed in the same casing or conduit as, the cables used for the distribution of the electricity supply throughout the building.

92. *Cable Sockets and other Connections.*

A. The ends of all conductors having a sectional area above 0·04 square inch (19/·052in.) shall be provided with a soldering socket (preferably made in accordance with British Standard Specification No. 91) of such a size that all the strands of the conductor can enter the socket simultaneously.

> *Note.*—It is advisable that stranded conductors of smaller size than the above should be fitted with such sockets.

B. Where a cable socket or terminal is used the cable shall be so supported that there is no appreciable stress on the socket or terminal.

C. Soldering fluids containing acid or other corrosive substances shall not be used.

D. When soldering or securing the ends of conductors to sockets or terminals, the dielectric shall not be removed farther than is necessary to allow the conductor to enter and completely fill the socket or terminal and be properly soldered. Dielectric damaged by the application of heat during the process of soldering shall be cut away.

E. The braid, lead, or other covering over the dielectric, including the tape in contact therewith, shall be cut back at least half an inch from the end of the dielectric.

F. In the case of paper-insulated cables, the exposed conductor and dielectric shall be protected from moisture by being suitably sealed with insulating compound.

G. The ends of stranded conductors unprovided with cable sockets shall be made solid by soldering in the case of all conductors insulated with paper, and, in the case of those insulated with rubber, when the cables are fixed in damp situations.

93. *Connections between Cables.*

A. Connections between cables, other than flexible cords, shall be made either by soldered joints or by mechanical connectors.

(*a*) Every joint shall be easily accessible and mechanically and electrically sound. The conductors shall be soldered together, a flux free from acid or other corrosive substance being employed, and the resistance of the soldered joint shall not be greater than that of an equivalent length of the largest conductor included in the joint. In the case of rubber-insulated cables the joint shall be lapped with rubber, to a thickness not less than that of the dielectric, and with waterproofed protecting tapes so as to render it moisture-proof, and if the cable be sheathed with tough rubber compound the joint shall be enclosed in a joint box complying with 93 C below, the protective covering or the cable being maintained up to a point within such box. In the case of paper-insulated cables, the joint after being insulated with suitably impregnated tapes shall be enclosed either in a joint box complying with 93 C below, or in a lead sleeve wiped on to the cable sheathings ; the box or sleeve, as the case may be, being filled with an insulating compound impervious to moisture.

(*b*) Every mechanical connector shall be easily accessible, and shall have a resistance not greater than that of an equivalent length of the largest conductor to which the connector is fixed. It shall be effectively enclosed in a non-ignitable box complying with 93 C below, the protective covering of the cable being maintained up to a point within such box and the latter being filled in the case of paper-insulated cables with an insulating compound impervious to moisture.

B. Connections between cables and flexible cords, or between lengths of flexible cords, shall in every case be made by means of mechanical connectors shrouded in non-ignitable insulating material contained within suitable receptacles which, in the case of lamp fittings, may form part of such fittings and which, if not on the surface of the wall or ceiling, shall be non-ignitable.

C. Joint boxes shall be constructed wholly of durable, non-ignitable, non-absorbent materials, and all insulation shall be of permanently high electric strength and insulation resistance. The live parts shall be so arranged, by suitable spacing and shielding with non-ignitable insulating materials, that conductors of opposite polarity or different phase cannot be readily short-circuited. If used in damp situations, joint boxes shall be weather- and moisture-resisting.

Main Distribution.

94. *Control of Main Supply.*

A. Every installation shall be adequately protected by suitable controlling apparatus (see Regulation 64) easily accessible to the consumer and situated as near as possible to :—

(*a*) The point or points of entry of the service main, in the case of a public supply, or

(*b*) The generator, in the case of a private plant.

B. If the supply undertaking provide and install the controlling apparatus laid down in 94 A (*a*), such apparatus if under the control of the consumer need not be duplicated by the latter unless so required by the supply undertaking ; but in the case of a private plant, where the house containing the generator referred to in 94 A (*b*) is isolated from the building in which the installation is supplied by this generator, duplicate controlling apparatus shall be installed at the point at which the main cables enter the building.

C. In installations in which the normal working current in any circuit or circuits connected to the main distribution board exceeds 100 amperes there shall be provided, in addition to a fuse or circuit breaker with overload trip, an automatic device whereby the installation or faulty circuit, as the case may be, shall be disconnected in the event of a leakage to earth. This device may be set to operate with any prescribed value of the leakage current, provided that such value does not exceed 100 amperes.

D. Three-wire circuits supplied from a three-wire system of generation or supply and two-wire circuits whether supplied from a two-wire system, from a three-wire system, from a two- or three-phase four-wire system or from a three-phase three-wire system of generation or supply, shall be controlled either by a circuit breaker or by a switch and fuse on each insulated conductor. When more than one pole is provided with switchgear the circuit breakers or switches shall be linked.

E. No fuse or unlinked single-pole switch shall be inserted on a main circuit in that conductor of an installation which is connected to earth at the source of supply without a circuit breaker or added resistance, or derives its polarity from a supply conductor so connected. (See diagram in connection with Regulation 64.)

> *Note.*—Any system of supply having a neutral conductor not publicly declared or known to be earthed without circuit breaker or added resistance shall be considered for the purpose of this Regulation to be a system having an insulated neutral.

F. The neutral conductor of a three-wire or of a three-phase four-wire system of generation or supply, whether earthed or not, shall never be interrupted while the outer or phase conductors, as the case may be, are closed; consequently, no fuse or unlinked switch may be included in the neutral conductor. A triple-pole linked switch controlling any circuit shall not connect the outer conductors to the supply before connecting the neutral, or open the neutral before the outer conductors have been opened.

G. Every circuit supplied from all the conductors of a two-phase system of generation or supply shall be controlled by a circuit breaker with overload trip on each conductor; or by a switch and fuse on each live conductor and a switch only on the neutral conductor. The circuit breakers or switches shall be linked.

H. Every circuit supplied from all the conductors of a three-phase system of generation or supply shall be controlled either by a triple-pole circuit breaker with overload trips on at least two phases, unless the neutral conductor is earthed, when trips shall be fitted on all three phases, or by a triple-pole linked switch with a fuse on each of the three conductors. No fuse shall be fixed on the neutral conductor if earthed.

J. In every case in which single-pole switches are required by these Regulations they shall be fitted on the same pole throughout the installation.

> *Note.*—When a supply is given from an outside source through more than one pair of terminals, it is desirable that the pressure between every pair of terminals, and also the extreme pressure, shall be indicated by clear and indelible labels.

95. *Sub-division of Circuits.*

A. The maximum number of points that may be connected in parallel to a final sub-circuit shall be as follows:—

Where the total rating of the points supplied from the sub-circuit does not exceed

6 amperes	10 points
8 ,,	6 ,,
10 ,,	4 ,,
20 ,,	2 ,,

Final sub-circuits supplying one lamp or appliance are not limited as to current-carrying capacity.

> *Note.*—In applying this Regulation it should be observed that when the fusing current of the fuse controlling a final sub-circuit exceeds 8 amperes the smallest cable or flexible cord which is used for any purpose on such circuit must be capable of carrying continuously a current not less than one-half of such fusing current (*see* Regulation 68 A (*b*) and Tables II. III, IV and VI).

B. Every final sub-circuit shall be connected to a sub-distribution board.

C. Every sub-distribution board shall be connected to a separate way on a main distribution board. Each main distribution board shall in turn be connected either to a separate way on the main switchboard, or to one way of a distribution board for larger currents, which in turn shall be connected to the main switchboard, or to one way of a distribution board for still larger currents.

The number of such distribution boards intervening between the final sub-circuit distribution boards and the main switchboard will depend upon the size and disposition of the installation.

Note.—For the purposes of Regulation 95, several distribution boards (whether main or sub) may be regarded as a single board, provided that their omnibus bars are connected together by cables of uniform size throughout and the bars are connected to the cables through fuses or disconnecting links.

Earthing.

96. *Conditions where Earthing is necessary.*

A. For all pressures of supply, earthing of metal objects other than the conductors shall be effected in the following cases :—

(*a*) In bathrooms, lift shafts, the immediate neighbourhood of running machinery, and all places where even a slight shock might lead to serious accident, all exposed metal liable to become alive should the insulation become defective shall be earthed.

(*b*) Where any metal liable to become alive should the insulation become defective is so situated that there is risk of accidental contact with earthed metal, it shall either be protected against such accidental contact or be earthed.

(*c*) The metal sheathings of cables installed in accordance with Regulation 87 (Class M1 and Class M2) shall be earthed.

B. Where the pressure of supply exceeds the limits of extra low pressure (see Regulation 10)—

(*a*) If the conditions are such that a person touching any metal liable to become alive should the insulation become defective is likely to be simultaneously making contact with earth, such metal shall be earthed.

(*b*) All metal conduits installed in accordance with Regulation 87 (Class T1 and Class T2) shall be earthed as near as conveniently possible to the point of entry of the supply, but isolated lengths of metal conduit need not be earthed except in the conditions specified in A (a) and (*b*) above.

Note.—A person is likely to be making contact with earth if standing on a floor which is conducting through damp or otherwise, also if in the immediate neighbourhood of masses of metal connected to earth, e.g., structural steelwork, gas stoves, water taps, etc.

Where the metal cases of switches, distribution boards or other apparatus have to be earthed, special precautions should be taken to guard against the risk of shock or burning to anyone when working on live conductors in or adjacent to such apparatus.

97. *Damp Plaster and Concrete.*—Damp plaster or damp concrete in contact with metal liable to become alive shall be considered, for the purpose of Regulation 96, to form part of such metal.

98. *Joints in Metallic Conduits and Sheathings.*—When the metallic conduits or sheathings of cables have to be earthed, or are themselves used as earthing connections, every joint in such conduit or sheathing shall be so made that the current-carrying capacity of the joint shall not be less than that of the conduit or sheathing itself.

99. *Steel Constructional Work and Machinery.*

A. (*a*) In buildings of steel frame construction, metal which has to be earthed may, where convenient, be connected to the structural steelwork, provided that such steelwork is itself earthed.

(*b*) Where the structural steelwork is not earthed, metal which is not earthed and which is liable to become alive should the insulation become defective shall be protected from contact with the structure.

B. Where electrical apparatus is mounted on machinery, e.g., cranes and lifts, the metal covers and frames of such apparatus, and the metal conduits or sheathing of the conductors, shall be connected to the machinery, which shall itself be earthed.

Note.—It is desirable that all steel structures should be earthed.

100. *Position of Metal in Bathrooms.*—In bathrooms all exposed metal liable to become alive shall, in addition to being earthed, be placed out of reach of a person standing in the bath. Lampholders shall have their exposed metal parts efficiently earthed, or, alternatively, all parts liable to be handled when replacing a lamp shall be constructed of insulating material.

101. *Precautions in Earthing.*

A. Great care shall be taken to secure as far as possible that the earthing systems shall be such that the combined resistance of the earthing lead and of the earthing system itself is low enough to permit the passage of the current necessary to operate the fuse or the earth leakage trip of the circuit breaker protecting the circuit.

B. Water pipes used as an earthing system shall have metal to metal joints throughout.

C. Pipes conveying gas or an inflammable liquid shall not be used as an earthing system.

Note.—The armouring of cables cannot in all cases be relied upon for the purpose of earthing.

102. *Earthing Leads.*

A. Every conductor used as an earthing lead shall be of high conductivity copper, protected against corrosion by tinning or otherwise and against mechanical injury, and its sectional area shall not be less than one-half that of the largest of the conductors to be protected, provided that no conductor of less sectional area than 0·0045 square inch (7/·029in.) shall be used as an earthing lead.

B. All connections of the earthing lead to the installation and to the earthing system itself shall be easily accessible.

C. If more than one plate or tubular earth be employed for one earthing system they shall be efficiently and permanently connected together.

D. Wherever an earthing lead is connected to a pipe, conduit, cable sheath, armour or other cylindrical earth a substantial metal clip shall be used, and the contact surfaces shall be clean. For armoured cables such clips shall be so designed as to grip firmly the whole of the wires of the armouring without damage to the insulation. For lead-sheathed armoured cables the principal contact shall be with the lead, but the clip shall be so designed as to grip the armouring firmly without damage to the lead.

E The ends of all earthing leads having a sectional area of 0·007 square inch (7/·036in.) and above shall be provided with a soldering socket, preferably made in accordance with British Standard Specification No. 91, of such a size that all the strands of the conductor can enter the socket simultaneously.

103. *Provision of Earthing Terminals.*—All current-consuming devices, whether portable or fixed, designed for pressures exceeding 100 volts direct current or 30 volts alternating current, shall be provided with a terminal or other suitable means for earthing all exposed metal liable to be charged should the insulation become defective.

Fittings.

104. *Construction of Fittings.*

A. Fittings shall be so designed and constructed that the passages for the insulated conductors are of ample size and are free from rough projections and sharp angles or bends. All outlets shall have well-rounded edges or be bushed.

B. Fittings shall be so designed, and the insulated conductors so installed, that no stress can be applied by the conductors to any terminal to which they may be connected.

C. Fittings shall be so designed and fixed that neither dust nor moisture can readily accumulate on live parts.

D. Where a fitting exceeds 10 pounds in weight it shall be supported by several flexible cords so that the maximum weight to which any cord is subjected shall not be greater than that specified in Regulation 81 C, or other means of support such as a metal chain shall be provided.

E. Open-type fittings shall not be furnished with inflammable shades unless such shades be kept free from contact with the lamps by suitable guards or supports. Celluloid shall not be used for shades or candle-tubes or in any situation near a lamp.

F. Enclosed-type fittings shall be provided with a removable glass receptacle arranged to enclose the lamp completely and of such size or construction as to prevent undue heating of the lamp; and if the position of the fitting be such that the glass receptacle is liable to mechanical damage, the glass shall be protected by a suitable wire guard.

G. Fittings, whether fixed or portable, wherever exposed to drip or externally condensed moisture shall be of the weatherproof type.

105. *Enclosed Fittings.*—Enclosed fittings shall be used :—

A. In spaces where inflammable or explosive dust or gas is liable to be present or where inflammable goods are stored. In such situations the fittings shall be of strong construction, having airtight external globes of thick glass provided with substantial guards.

B. In positions in which the lamp is either near to, or can swing into contact with, readily combustible materials.

106. *Portable Fittings.*

A. In all cases where portable metal fittings have to be earthed in accordance with Regulations 96 to 103 all metal parts shall be connected to earth by means of a third conductor in the flexible cord. If this flexible cord have a metal armouring, this armouring shall, in addition to the aforementioned conductor, be efficiently connected electrically at one end to the metal frame of the fitting, and at the other end to the earthed metal of the plug-and-socket connection. Such fittings shall have strong metal guards in metallic contact with the rest of the uninsulated metal of those fittings.

B. Portable fittings with non-metallic frames, or the only metal parts of which are the guards surrounding the lamp or lamps, need not be earthed as described in 106 A provided that these guards are not, and cannot come, in metallic contact with the lampholder or lampholders. Such fittings shall be made of treated hard wood, or of some suitable non-ignitable composite material capable of withstanding rough usage in service.

C. Where portable fittings, appliances or accessories are likely to be used, the pressure between any two points in one room or compartment shall not exceed 250 volts unless the fittings, appliances or accessories between which there may be a higher pressure are so situated that they cannot be brought within 6 feet of each other.

107. *Ceiling Roses.*

A. Ceiling Roses shall not be used for pressures exceeding 250 volts.

B. Not more than two pairs of flexible cords shall be attached to one ceiling rose unless it be specially designed for multiple pendants.

Note.—Suitable ceiling roses are embodied in British Standard Specification No. 67, which provides for complete interchangeability of parts.

108. *Base Blocks.*—Where a fitting or accessory has to be mounted upon any surface liable to become damp it shall, unless in contact with the sheathing of an earthed system, be mounted upon a block of treated hard wood or upon some non-hygroscopic material in addition to its own base.

Accessories.

109. *Fixing of Accessories.*—Regulation 108 shall apply to accessories.

110. *Accessories Liable to Arc Prohibited in Certain Situations.*—Accessories liable to arc when operated shall not be installed in position in which inflammable or explosive dust or gas is liable to be present.

111. *Lampholders.*

A. Lampholders shall comply with the appropriate British Standard Specification, and where there is no such Specification the following conditions shall apply :—

(*a*) Lampholders shall be wholly of non-ignitable material, and all metal parts shall be of robust proportions.

(*b*) Lampholders in weatherproof portable fittings shall have their uninsulated metal parts in metallic contact with the frames of such fittings.

B. Lampholders in open-type portable fittings shall be insulated from the fitting by means of insulating material which is of adequate mechanical strength and which will not soften at 302° F. (150° C.), and they shall be so shielded by means of similar insulating material that they cannot be touched accidentally by a person using the fitting or replacing a lamp. This shield shall be so extended as to prevent the lamp cap being touched when the lamp is in the holder.

Note.—Standard bayonet and Goliath lampholders are embodied in British Standard Specifications Nos. 52 and 98 respectively.

C. Switch lampholders shall be provided with further means of control in the same room.

112. *Plugs and Sockets.*

A. Sockets shall be constructed so that they cannot be accidentally short-circuited whether the plug be in or out, and so that a pin of the plug cannot be made to earth either pole of the socket.

B. Plugs connected to apparatus which has a rated capacity exceeding 1 ampere, but not exceeding 10 amperes, shall be constructed with :—

(*a*) a hand shield, the entry for the flexible cable being at the side of the shield remote from the hand, and, if desired, a switch being fixed adjacent to the socket ; or

(*b*) the plug so interlocked with a switch on the socket that it is impossible to insert or withdraw the plug when the switch is in the "on" position ;

provided that every socket, or group of sockets, carrying more than 300 watts shall have a switch in an accessible position.

C. Plugs having a rated capacity exceeding 10 amperes shall be constructed with a hand shield, the entry for the flexible cable being at the side of the shield remote from the hand, and shall be provided with a switch immediately adjacent to the socket and preferably interlocked as in Regulation 112 B (*b*).

D. The clearances of plugs and sockets not interlocked with switches shall be such that an arc cannot be maintained if the plug be pulled out while a current 50 per cent greater than that for which the plug and socket are rated is flowing under a pressure 50 per cent. in excess of the pressure of supply.

E. Where the socket is attached to a floor the contacts shall be below the floor level so that there can be no risk of contact between live metal and the floor covering. Sockets so fitted shall be provided with a strong incombustible cover, either hinged, or screwed and secured by a chain.

F. The bases of sockets shall be of tough, non-ignitable, non-conducting, non-hygroscopic material, and all live parts shall have such dimensions and clearances as are laid down in British Standard Specification No. 73.

G. The covers of sockets and plugs shall be either of insulating material or of rigid metal well clear of all live parts or provided with an insulating lining.

Note.—Bases and covers made of hard wood conform to the requirements specified in Regulation 112 F and G.

H. Weatherproof plugs and sockets shall be used wherever exposed to rain, drip or externally condensed moisture. Such accessories shall be of specially robust construction and be provided with efficient means to maintain the sockets weatherproof when the plug is removed therefrom. When a loose cover is employed for this purpose, it shall be anchored to the socket by means of a chain. When the plug is inserted in its socket the combined fitting and its interlocking switch, if any, shall also be weatherproof.

J. If concentric sockets be used on an earthed concentric system of wiring, the centre contact of the socket shall be connected to the insulated conductor.

K. Where tough-rubber-sheathed flexible cord is used, a suitable clamp shall be provided to grip the protecting covering of the flexible cord.

L. When sockets and plugs have provision for earthing, the current-carrying capacity of the earthing contact shall comply with the requirements in regard to earthing (see Regulations 96 to 103).

M. In places where petrol-driven conveyances are stored or repaired, plugs and sockets shall be placed not less than 6 feet above the floor level.

N. Lampholder adapters shall not be used as plug connections for appliances taking more than 2 amperes each.

Note.—Suitable two-pin plugs and sockets (non-interlocked) are embodied in British Standard Specification No. 73, which provides for complete interchangeability of parts.

113. *Incandescent Lamps.*

A. Incandescent lamps shall be provided with caps of a pattern as follows :—

Up to and including 100 watts ..	Standard bayonet (B.C.)
Above 100 watts and not exceeding 200 watts	Edison screw (E.S.)
Above 200 watts	Goliath (G.E.S.)

Note.—Suitable Goliath lamp caps are embodied in British Standard Specification No. 98.

B. Lamp caps shall, in all cases, be fixed to the lamps with non-hygroscopic cement, and the apertures in the contact blocks shall be completely filled with solder.

C. The insulation resistance measured between the contact blocks and the ring of the cap of an incandescent lamp shall not be less than 50 megohms.

D. Fittings for lamps shall be so designed as to provide for adequate dissipation of heat from such lamps.

114. *Mercury Vapour Lamps.*

A. The connections to the terminals of the lamp tube of mercury vapour lamps shall be so constructed that loosening of the contact or overheating cannot occur.

B. The resistance and solenoid shall be completely enclosed in a metal case, and any apertures in the case for purposes of ventilation shall be made on the sides only, and be covered with fine wire gauze.

C. In positions where inflammable or explosive dust or gases are liable to be present, mercury vapour lamps shall not be used.

115. *Arc Lamps.*

A. Arc lamps shall have the whole of their live parts insulated from the frame or case.

B. Except as provided by 115 C below, where the floor immediately underneath an arc lamp is formed of combustible material, or where heated particles of carbon might fall and constitute a danger to persons walking underneath, every such arc lamp shall be provided with a globe or lantern arranged to intercept such falling particles of carbon. Globes of 12 inches (30 cm.) diameter and over shall be contained within wire netting of not greater than 3 inch (7·5 cm.) mesh so arranged as to prevent pieces of broken glass falling therefrom.

C. In situations in which an open arc is essential, as in photographic work, the floor immediately underneath the lamp shall, if necessary, be protected from falling particles of carbon by an incombustible covering.

D. If fitted in situations where combustible material is present, open inverted arc lamps shall have metal reflectors rigidly attached beneath the arc, which at all times shall be below the level of the upper edge of the reflector. This reflector shall project radially at least 15 inches (38 cm.), and in hazardous risks 21 inches (53 cm), measured horizontally beyond the arc.

E. Arc lamps shall not be fitted in positions in which inflammable gases may accumulate.

F. Every arc lamp circuit shall be controlled by a fuse and switch on each insulated pole. When more than one pole is insulated the switches shall be linked.

116. *Electric Signs.*—Every electric sign shall comply with the following requirements :—

A. It shall be readily accessible for inspection and attention.

B. Its frame shall be earthed in accordance with Regulation 96.

C. If Regulation 95 is not complied with, the sign shall be connected to the main supply by independent wiring and its circuit shall be controlled by a separate double-pole switch and a fuse on each pole.

D. Where elaborate switching and flashing apparatus is installed a special non-ignitable enclosure shall be provided.

E. If fixed in the open air—

(*a*) It shall be weatherproof ;

(*b*) Only non-ignitable materials shall be used in its construction except for letters and designs, for which hard wood (teak or oak) is permissible ;

(*c*) All external wiring shall be of Class R, Class S, Class T1 or Class T3 (see Regulation 87), and in the case of Class T1 the conduits shall be galvanized.

Motors.

117. *Types.*

A. Motors may be of any of the types enumerated in British Standard Specification No. 168, or of the immersible type, and all motors rated at more than one brake horse-power shall conform in all respects to that Specification.

B. The frame of every motor shall be provided with a suitable terminal to which the earthing lead may be connected.

118. *Position.*

A. Motors shall, wherever possible, be placed in well-ventilated spaces in which inflammable gases cannot accumulate. Where these conditions cannot be complied with, the motors shall be of the flame-proof or pipe-ventilated type with inlet and outlet connected to the outer air.

B. Motors fixed in situations in which the surrounding air exceeds the limit of temperature permitted for the cooling air in the appropriate British Standard Specification shall be of special construction, or alternatively of the pipe-ventilated, forced-draught or induced-draught type, connected by ventilating ducts to a source of cool air supply.

C. Motors shall, as far as possible, be placed in positions in which they are not exposed to risk of mechanical injury or to damage from water, steam or oil. Motors necessarily exposed to such conditions shall have suitable types of enclosing frames selected from the standard " types of enclosure " specified in British Standard Specification No. 168 and here defined in Regulations 44 to 54.*

D. Pipe-ventilated, forced-draught and induced-draught motors shall be supplied with air as cool as possible, and the air intakes shall be guarded against the admission of dirt and/or moisture.

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* These have been omitted from these regulations, but will be found in B.S.S. No. 168 on page 456.

E. No unprotected woodwork or other combustible material shall be within a distance of 12 inches (30 cm.) measured horizontally from, or within 4 feet (120 cm.) measured vertically above, any motor, unless such motor be of the totally enclosed, flame-proof or pipe-ventilated type with inlet and outlet connected to the outer air. A metal plate or tray extending 12 inches (30 cm.) beyond the base of the machine shall be placed under every open-type machine which is mounted on a floor consisting of wood or other combustible material.

119. *Control of Motors.*

A. Every motor shall be protected by efficient means suitably placed and so connected that the motor and all apparatus in connection therewith may be isolated from the supply; provided, however, that when one point of the system of generation or supply is connected with earth, it shall not be necessary to disconnect on that side of the system which is connected to earth.

> *Note.*—In the case of motors not exceeding one brake horse-power, a plug and socket will be considered to be an efficient method of isolation from the supply.

B. Every motor shall be provided with an efficient switch or switches for starting and stopping, so placed as to be easily operated by the person controlling the motor; and every motor having a rating exceeding one-half horse-power shall in addition be provided with —

(*a*) Means for automatically opening the circuit if the supply pressure falls sufficiently to cause the motor to stop;

(*b*) In the case of direct-current motors a starter or switch for limiting the current taken when starting and accelerating;

(*c*) In the case of alternating current motors, such starter or switch for limiting the current taken, when starting and accelerating to the value (if any) required by the supply undertaking.

C. In every place in which a machine is being driven by a motor there shall be means at hand for either switching off the motor or stopping the machine if necessary to prevent danger.

> *Note.*—Suitable motor starters are embodied in the following British Standard Specification:—No. 82, No. 117, No. 140, No. 141, No. 147, No. 155, and No. 167.

Resistances and Machine Control Gear.

> *Note.*—Regulations 120 and 121 do not apply to apparatus having a capacity less than 60 watts.

120. *General Construction.*

A. The general construction of all resistances and machine control gear shall be in accordance with the appropriate British Standard Specification.

B. All live parts shall be so guarded as to prevent accidental contact with them.

C. The frame of every resistance and control gear shall be provided with a suitable terminal to which the earthing lead can be connected.

D. Resistances shall be so proportioned and placed that they do not rise to such a temperature as to impair their durability, and they shall be so disposed within their cases that no accessible part of such cases shall rise to a temperature higher than 176° F. (80° C.)

E. Internal connections shall not be soldered, and all such connections, unless self-supporting or rigidly fixed in position, shall be continuously insulated with non-ignitable material or beads.

F. Suitable terminals with cable sockets (preferably made in accordance with British Standard Specification No. 91) shall be provided for the attachment of external leads, and shall be so situated that such leads enter the case below the resistances and are not exposed at any point to a high temperature.

121. *Position.**

A. All resistances and control gear shall, as far as possible, be placed :—

(*a*) In positions in which they will not be exposed to risk of mechanical injury or to damage from water, steam, or oil ;

(*b*) In well-ventilated spaces in which inflammable gases or dust cannot accumulate.

Where necessarily exposed to such conditions, control gear shall in the case of (*a*) be completely enclosed, and in the case of (*b*) be flame-proof ; and resistances shall in the case both of (*a*) and (*b*) be completely enclosed.

B. All woodwork or other combustible material which is within a distance of 24 inches (60 cm.) measured vertically above, or 12 inches (30 cm.) measured vertically below, or 6 inches (15 cm.) measured in any other direction from, the frames or cases containing resistances shall be protected with non-ignitable material.

122. *Electric Lifts.*—Every lift or hoist operated electrically shall comply with the following requirements :—

A. It shall be operated from a circuit which is independent of the lighting installation.

B. The multicore trailing cable shall comprise the requisite number of conductors to keep the motor wiring and the control and safety devices entirely separate.

C. All cables in the lift or hoist shaft, except trailing cables, shall be enclosed in metal conduits complying with Regulation 87 (Class T1 or Class T2), the control and motor leads being in separate conduits.

Heating and Cooking Appliances.

123. *General Construction of Heating and Cooking Appliances.*

A. All appliances shall be so constructed and mounted that their supports and those parts which have necessarily to be handled in their operation cannot become heated to a temperature exceeding 130° F. (54° C.). The heating elements shall be of materials durable at the highest temperature to which they attain, and be so arranged that they can be readily replaced.

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* See Note above Regulation 120.

B. Precautions shall be taken with regard to their surroundings as in the case of non-electrical heating appliances.

C. The support and frame of every appliance shall be provided with a suitable terminal to which the earthing lead may be connected.

D. The connections between heating elements shall be effected either by parts of the elements themselves or by material having heat-resisting properties similar to those of the elements. The junction between the elements and switches or external connecting leads shall be effected without solder by suitable connections, which shall be so placed that the temperature of no part of the switch or terminal connections can rise above 176° F. (80° C.).

E. All connections between elements or between elements and main terminals shall, unless self-supporting, or rigidly fixed in position, be continuously insulated with suitable non-ignitable material.

F. All live parts of cooking appliances shall be so guarded that the cooking utensils cannot be brought into contact with them.

124. *Control of Heating and Cooking Appliances.*

A. Appliances shall be protected by a fuse on each insulated pole.

B. Appliances whether portable or fixed shall be controlled as a whole by a switch, which in the case of portable appliances shall be fitted on a wall.

C. Where a switch or switches are fixed on the frame of a portable luminous heating appliance, at least one section of the heating element shall not be controlled by such switches, so that the luminous heating element is permanently connected to the wall-plug or similar device in order to indicate that the circuit is not broken and that current is still flowing.

125. *Portable Heating and Cooking Appliances.*—Portable appliances shall be of such shape or be so weighted that they cannot easily be overturned.

126. *Protection of Combustible Materials.*—Heating and cooking appliances shall not be fixed near combustible materials unless the latter are suitably protected.

Testing of Completed Installation.

> *Note.*—The following tests are intended to ensure that the installation is in a satisfactory state at the time of completion. The value of systematically inspecting and testing apparatus and circuits cannot be too strongly urged, and such periodical tests are essential if the installation is to be maintained in a sound condition and undue deterioration thereof detected. All defects thus discovered should be made good without loss of time. The attention of consumers should be drawn to the importance of maintaining all apparatus and fittings in a clean and dry condition.

127. *Tests to be carried out.*—Before an installation is permanently put into service the following tests shall be carried out :—

Insulation Resistance.—

A. The insulation resistance shall be measured by applying a direct-current pressure of not less than twice the working pressure, but under no circumstances exceeding 500 volts, between earth and the whole system of conductors or any section thereof, with all lamps and fuses in place, and all switches on. Where main switchboards are installed they may be excluded from this insulation test, provided that they have satisfactorily withstood a pressure test.

B. The insulation resistance of the lighting circuits measured as in A above shall not be less in megohms than 25 divided by the number of points on those circuits, except that the insulation resistance of any final lighting sub-circuit need not exceed 1 megohm.

C. The insulation resistance between the case or framework and every live part of each individual dynamo, motor, heater, arc lamp or other appliance complete with its switch and control gear, regulating resistance and similar accessories, shall not be less than half a megohm when measured as in A above.

Note.—In addition to the foregoing tests, it is advisable wherever practicable to take an insulation test between all the conductors connected to one pole or phase and all the conductors connected to the other pole or phase of a system, except in the case of concentric systems having an earthed outer conductor.

Continuity of Metal Sheathing.

The metal conduits or metallic envelopes of cables in all cases where such methods are used for the mechanical protection of electrical conductors shall be tested for electrical continuity, and the electrical resistance of such conduits or sheathing, measured between a point near the main switch and any other point of the completed installation, shall not exceed 2 ohms.

Earthing.

(Investigations are being made with a view to specifying the conditions required for the satisfactory earthing of an installation, and it is proposed to include such a specification in a later edition of these Regulations.)

128. *Care Required in Making Additions to Installations.*—Before an addition is made to an installation care shall be taken to ascertain whether the existing conductors, switches, etc., affected by the addition are of sufficient capacity for the augmented current which they may have to carry.

Note.—Alternative plug positions are often provided for electric heating appliances, and in such cases it should be ascertained whether the existing conductors are of sufficient size to allow of the simultaneous use of apparatus connected to more than one plug.

TABLE I. Flexible Cables: Dimensions and Resistance of Conductors.

(See Regulation 71 B.)

Nominal Area	Number and Diameter of Wires comprising Conductor				Resistance per 1 000 Yards at 60° F. (15·6° C.)		
	Diameter 0·010 inch	Diameter 0·012 inch	Diameter 0·018 inch	Diameter 0·029 inch	Standard	Maximum allowable for Plain Wires	Maximum allowable for Tinned Wires
1	2	3	4	5	6	7	8
sq. in.					ohms	ohms	ohms
0·1	140/·010	97/·012*	—	—	1·29	2·34	2·39
0·0145	195/·010	—	60/·018*	—	1·64	1·68	1·72
0·0225	296/·010	—	91/·018*	—	1·08	1·11	1·13
0·03	—	266/·012	117/·018*	—	0·847	0·864	0·881
0·04	—	368/·012	163/·018*	—	0·606	0·618	0·631
0·06	—	557/·012	248/·018*	—	0·400	0·408	0·416
0·075	—	705/·012	313/·018	121/·029*	0·316	0·323	0·329
0·1	—	—	416/·018	160/·029*	0·238	0·243	0·247
0·12	—	—	482/·018	186/·029*	0·206	0·210	0·214
0·15	—	—	610/·018	235/·029*	0·163	0·166	0·169
0·2	—	—	810/·018	312/·029*	0·122	0·125	0·127
0·25	—	—	1017/·018	392/·029*	0·0974	0·0993	0·101
0·3	—	—	—	481/·029	0·0794	0·0810	0·0826
0·4	—	—	—	646/·029	0·0591	0·0603	0·0614
0·5	—	—	—	792/·029	0·0482	0·0491	0·0501

* For trailing cables and similar purposes.

TABLE 11.

VULCANIZED RUBBER CABLES: CURRENT-CARRYING CAPACITY (SUBJECT TO VOLTAGE DROP) AND CORRESPONDING FALL IN PRESSURE.

(See Regulation 68 A (b) Note and 74 A (b)).

Nominal Area of Conductor 1	*Number and Diameter (in.) of Wires comprising Conductor* 2	*Single Cables run in Pairs* 3	*Concentric or Twin Cable* 4	*Three-core Cable* 5	*Approximate Total Length in Circuit (Lead and Return) for 1-volt Drop with Maximum Permissible Current (Col. 3)* 6
sq. in.		*amps.*	*amps.*	*amps.*	*feet*
0·001	1/·036	4·1	3·5	—	30
0·0015	1/·044	6·1	5·2	—	30
0·002	3/·029	7·8	6·7	—	30
0·003	3/·036	12·0	10·3	—	29
0·003	1/·064	12·9	11·1	—	29
0·0045	7/·029	18·2	15·7	13·6	28
0·007	7/·036	24·0	20·6	18·0	33
0·01	7/·044	31·0	26·6	23·2	39
0·0145	7/·052	37·0	32·0	27·8	45
0·0225	7/·064	46·0	39·0	34·0	55
0·03	19/·044	53·0	46·0	40·0	61
0·04	19/·052	64·0	55·0	47·0	71
0·06	19/·064	83·0	71·0	59·0	83
0·075	19/·072	97·0	83·0	69·0	90
0·1	19/·083	118·0	100·0	83·0	98
0·12	37/·064	130·0	118·0	90·0	103
0·15	37/·072	152·0	126·0	105·0	112
0·2	37/·083	184·0	149·0	126·0	123
0·25	37/·093	214·0	170·0	146·0	132
0·3	37/·103	240·0	188·0	—	145
0·4	61/·093	288·0	220·0	—	162
0·5	61/·103	332·0	249·0	—	172
0·6	91/·093	384·0	—	—	181
0·75	91/·103	461·0	—	—	185
0·85	127/·093	512·0	—	—	190
1·0	127/·103	595·0	—	—	200

Note to Table II.—The figures given in Table II apply to single cables run in pairs in iron conduits or in wood casing, and to single cables sheathed with tough rubber compound, and to concentric, twin, and three-core cables of any finish run singly.

The maximum permissible currents (subject to voltage drop) for the various sizes of conductors up to 1 square inch in cross-sectional area are shown in columns 3, 4 and 5 of the Table, which allow for a rise in temperature of 20 degrees F. (11·1 degrees C.) for rubber-insulated cables. For sizes below 0·007 square inch the Table is based on a current density of 4,000 amperes per square inch.

The Table refers to situations where the temperature of the air does not exceed 80° F. (26·6° C.), and thus the normal maximum running temperature is 100° F. (37·7° C.). Rubber-insulated cables should not be allowed to attain a temperature higher than 120° F. (48·8° C.) for long periods, or for a short period 130° F. (54·4° C.). The figures, therefore, in the latter case allow of a margin of 30 degrees F. (16·7 degrees C.).

Where the temperature of the air exceeds 80° F. (26·6° C.), the permissible current should be reduced so that the maximum temperature of the rubber-insulated cables does not exceed the figures given above.

The further limitation of the size of conductor by the permissible drop in voltage is dealt with in Regulation 74A (*a*).

Note to Table III.—The figures given in Table III apply to single cables run in pairs and to concentric, twin and three-core cables run singly.

The maximum permissible currents (subject to voltage drop) for the various sizes of conductors up to 1 square inch in cross-sectional area are shown in columns 3, 4 and 5 of the Table, which allows for a rise in temperature of 50 degrees F. (27·7 degrees C.) for impregnated-paper cables. For sizes below 0·017 square inch the Table is based on a current density of 4,000 amperes per square inch.

The Table refers to situations where the temperature of the air does not exceed 80° F. (26·6° C.) and thus the normal maximum running temperature is 130° F. (54·4° C.). Impregnated-paper lead-covered cables for pressures not exceeding 660 volts should not be allowed to attain a permanent temperature higher than 176° F. (80° C.) and the figures therefore allow of a margin of 46 degrees F. (25·6 degrees C.).

Where the temperature of the air exceeds 80° F. (26·6° C.), the permissible current should be reduced so that the maximum temperature of the impregnated-paper lead-covered cables does not exceed the figures given above.

The further limitation of the size of conductor by the permissible drop in voltage is dealt with in Regulation 74A (*a*).

TABLE III.

Impregnated-paper and Lead-covered Cables: Current-carrying Capacity (subject to Voltage Drop) and Corresponding Fall in Pressure.

(See Regulations 68A (*b*) Note, and 74A (*b*)).

Nominal Area of Conductor 1	*Number and Diameter (in.) of Wires comprising Conductor* 2	*Single Cables run in Pairs* 3	*Concentric or Twin Cable* 4	*Three-core cable* 5	*Approximate Total Length in Circuit (Lead and Return) for 1-volt Drop** 6
sq. in.		*amps.*	*amps.*	*amps.*	*feet*
0·001	1/·036	4·1	3·5	—	30
0·0015	1/·044	6·1	5·2	—	30
0·002	3/·029	7·8	6·7	—	30
0·003	3/·036	12·0	10·3	—	29
0.003	1/·064	12·9	11·1	—	29
0·0045	7/·029	18·2	15·7	13·6	28
0·007	7/·036	28·0	4·0	21·0	27
0·01	7/·044	42·0	36·0	31·0	27
0·0145	7/·052	57·0	49·0	43·0	28
0·0225	7/·064	75·0	65·0	56·0	32
0·03	19/·044	87·0	76·0	66·0	35
0·04	19/·052	104·0	89·0	76·0	41
0·06	19/·064	135·0	116·0	97·0	48
0·075	19/·072	157·0	135·0	111·0	52
0·1	19/·083	191·0	162·0	134·0	57
0·12	37/·064	210·0	177·0	146·0	60
0.15	37/·072	246·0	204·0	170·0	65
0·2	37/·083	296·0	240·0	203·0	72
0·25	37/·093	343·0	265·0	233·0	78
0·3	37/·103	385·0	302·0	258·0	85
0·4	61/·093	464·0	354·0	—	95
0·5	61/·103	540·0	405·0	—	100
0·6	91/·093	624·0	—	—	105
0·75	91/·103	738·0	—	—	109
0·85	127/·093	815·0	—	—	116
1·0	127/·103	932·0	—	—	121

* With maximum permissible current (Col. 3.).

TABLE IV.

RUBBER-INSULATED FLEXIBLE CABLES: CURRENT-CARRYING CAPACITY.

(See Regulation 74B.)

Nominal Area of Conductor	*Maximum Current Permissible (subject to Voltage Drop)*	
	For Portable Domestic Appliances	*For other Purposes than those specified in Col. 2*
1	2	3
sq. in.	*amps.*	*amps.*
0·00	16	24
0·0145	19	28
0·0225	23	35
0·03	27	41
0·04	33	49
0·06	—	63
0·075	—	74
0·1	—	90
0·12	—	100
0·15	—	117
1·2	—	140
0·25	—	163
0·3	—	184
0·4	—	220
0·5	—	254

Note.—The current values given above are based on calculated areas, allowing 2 per cent. for the laying-up of the wires comprising the stranded conductor.

TABLE V.

Flexible Cords: Dimensions and Resistance of Conductors.

(See Regulation 71 B.)

Nominal Area	Ordinary Flexible Cords: Number and Diameter (in.) of Wires comprising Conductor	Ordinary Flexible Cords: Resistance per 1 000 yards at 60° F. (15·6° C.): Standard	Ordinary Flexible Cords: Resistance per 1 000 yards at 60° F. (15·6° C.): Maximum allowable for Plain Wires	Ordinary Flexible Cords: Resistance per 1 000 yards at 60° F. (15·6° C.): Maximum allowable for Tinned Wires	Flexible Cords with Tough Rubber Sheathing: Number and Diameter (in.) of Wires comprising Conductor	Flexible Cords with Tough Rubber Sheathing: Resistance per 1 000 yards at 60° F. (15·6° C.): Standard	Flexible Cords with Tough Rubber Sheathing: Resistance per 1 000 yards at 60° F. (15·6° C.): Maximum allowable
1	2	3	4	5	6	7	8
sq. in.		*ohms*	*ohms*	*ohms*		*ohms*	*ohms*
0·0006	14/·0076	39·7	40·5	41·3	7/·012*	40·5	41·3
0·001	23/·0076	24·2	24·6	25·2	11/·012†	24·6	25·1
0·0017	40/·0076	13·9	14·2	14·4	16/·012‡	14·2	14·4
0·003	70/·0076	7·94	8·10	8·26	28/·012‡	8·10	8·26
0·0048	110/·0076	5·05	5·15	5·25	44/·012‡	5·15	5·25
0·007	162/·0076	3·43	3·50	3·57	65/·012‡	3·50	3·57

* 5 tinned copper; 2 tinned steel. † 9 tinned copper; 2 tinned steel. ‡ All tinned copper.

TABLE VI.

Flexible Cords: Thickness of Insulation and Current-carrying Capacity.

(See Regulations 74 B and 79.)

	Minimum Thickness of Dielectric (Ordinary Flexible Cords)				*Flexible Cords with Tough Rubber Sheathing*			
Nominal Area of Conductor	*High Insulation (Kind 1)*		*Medium Insulation (Kind 2)*		*Thickness of Tough Rubber*	*Overall Diameter*		*Maximum Current permissible (subject to Voltage Drop)*
	Pure Rubber	*Pure and Vulcanizing Rubber*	*Pure Rubber*	*Vulcanizing Rubber*		*250 volts*	*600 volts*	
1	2	3	4	5	6	7	8	9
sq. in.	*in.*	*in.*	*in.*	*in.*	*in.*	*in.*	*in.*	*amps.*
0·0006	0·020	0·033	0·015	0·028	0·050	0·200	0·244	1·2
0·001	0·020	0·034	0·015	0·029	0·050	0·214	0·255	2·0
0·0017	0·020	0·035	0·015	0·030	0·050	0·227	0·267	3·6
0·003	0·020	0·036	0·015	0·031	0·050	0·248	0·290	6·2
0·0048	0·020	0·038	0·015	0·032	0·050	0·268	0·308	10·0
0·007	0·020	0·039	—	—	0·050	0·288	0·328	12·0

TABLE VII.

Insulation Resistance of Cables.

(See Regulation 84 A, B and C.)

Conductor		Minimum Insulation Resistance, Megohms for a Mile Length at 60° F. (15·6° C.)			
		Rubber-insulated Cables			Paper-insulated Cables
Nominal Area of Cable	Number and Diameter (in.) of Wires	600 megohm Grade*	2 500-megohm Grade*	660-volt Grade†	
1	2	3	4	5	6
sq. in.		megohms	megohms	megohms	megohms
0·001	1/·036	2 000	5 000	5 000	140
0·0015	1/·044	2 000	5 000	5 000	140
0·002	3/·029	1 250	4 500	4 500	140
0·003	3/·036	1 250	4 500	4 500	140
0·003	1/·064	2 000	5 000	5 000	140
0·0045	7/·029	1 250	4 500	4 500	140
0·007	7/·036	900	4 000	4 000	140
0·01	7/·044	900	4 000	4 000	140
0·0145	7/·052	900	4 000	4 000	140
0·0225	7/·064	900	3 500	3 500	130
0·03	19/·044	750	3 500	3 500	125
0·04	19/·052	750	3 000	3 000	115
0·06	19/·064	750	3 000	3 000	100
0·075	19/·072	600	3 000	3 000	85
0·1	19/·083	600	3 000	3 000	80
0·12	37/·064	600	3 000	3 000	75
0·15	37/·072	600	3 000	3 000	60
0·2	37/·083	600	2 500	2 500	55
0·25	37/·093	600	2 500	2 500	50
0·3	37/·103	600	2 500	2 500	50
0·4	61/·093	600	2 500	2 500	50
0·5	61/·103	600	2 500	2 500	45
0·6	91/·093	600	2 500	2 500	40
0·75	91/·103	600	2 500	2 500	40
0·85	127/·093	600	2 500	2 500	35
1·0	127/·103	600	2 500	2 500	35

* For (*a*) Direct-current systems for pressures not varying from earth potential by more than 250 volts.

(*b*) Three-phase systems, with centre point earthed, for pressures not more than 500 volts between phases.

† For pressures not varying from earth potential by more than 660 volts.

TABLE VIII.

Test Pressures for Flexible Cords.

(See Regulation 84 d.)

Kind	*Insulating Material*	*Test Pressure and Frequency*	*Nature of Test*
1	2	3	4
		volts	
High Insulation (1)	Pure Rubber	1 500 at 25–100 ~	Between Conductors, in dry state
Medium Insulation (2)	Pure Rubber	1 000 at 25–100 ~	
Medium Insulation (2)	Vulcanizing Rubber	1 500 at 25–100 ~	
High Insulation (1)	Pure and Vulcanizing Rubber	1 010 at 25–100 ~	In Water, after 24 hours' immersion

TABLE IX.

Insulation Resistance of Flexible Cords having Vulcanized Rubber Insulation.

(See Regulation 84 d.)

Nominal Area of Conductor	*Number and Diameter (in.) of Wires comprising Conductor*	*Minimum Insulation Resistance, Megohms for a Mile Length at 60° F. (15·6° C.)*	
		High Insulation	*Medium Insulation*
1	2	3	4
sq. in.			
0·0006	14/·0076	1 250	300
0·001	23/·0076	1 250	300
0·0017	40/·0076	1 250	300
0·003	70/·0076	1 250	300
0·0048	110/·0076	1 250	300
0·007	162/·0076	900	300

TABLE X.

Capacity of Conduits (Type B, Screwed, B.S.S. No. 31) for the Drawing-in of Conductors.

(See Regulation 87, Class T.1.)

Size of Conduit			½ *in.*	⅝ *in.*	¾ *in.*	1 *in.*	1¼ *in.*	1½ *in.*	2 *in.*	2½ *in.*
Internal Diameter (Approximate)			0·388 *in.*	0·498 *in.*	0·606 *in.*	0·856 *in.*	1·106 *in.*	1·34 *in.*	1·816 *in.*	2·316 *in.*
Nominal Area of Conductor	*Number and Diameter (in.) of Wires comprising Conductor*	*Approximate Overall Diameter*	*Maximum Number of Conductors*							
1	2	3	4	5	6	7	8	9	10	11
sq. in.		*in.*								
0·0015	1/·044	0·173	2	3	5	8	—	—	—	—
0·002	3/·029	0·195	—	3	4	8	—	—	—	—
0·003	3/·036	0·215	—	2	4	8	—	—	—	—
0·003	1/·064	0·197	—	2	4	8	—	—	—	—
0·0045	7/·029	0·226	—	—	3	5	7	—	—	—

0·007	7/·036	0·259	—	—	2	5	7	—	—	—
0·01	7/·044	0·287	—	—	2	4	7	—	—	—
0·0145	7/·052	0·317	—	—	—	2	5	7	—	—
0·0225	7/·064	0·359	—	—	—	2	4	7	—	—
0·03	19/·044	0·393	—	—	—	—	3	5	7	8
0·04	19/·052	0·441	—	—	—	—	2	4	7	8
0·06	19/·064	0·513	—	—	—	—	—	3	5	6
0·075	19/·072	0·596	—	—	—	—	—	—	4	4
0·1	19/·083	0·663	—	—	—	—	—	—	3	4
0·12	37/·064	0·702	—	—	—	—	—	—	2	3
0·15	37/·072	0·768	—	—	—	—	—	—	2	2

Note.—Table X shows the capacity of conduits for the simultaneous drawing-in of conductors, but disregard of this Table will not be deemed to be non-compliance with the Wiring Regulations. The Table applies to 250-volt, vulcanized rubber, braided cables in accordance with British Standard Specification No. 7, and to conduits, type (B) screwed, which comply with British Standard Specification No. 31.

The grouping in one tube of more than two of the larger cables is not recommended, and where it is done the current rating given in Tables II, III, and IV should be reduced to ensure that the cables are not overheated.

TABLE XI.

Approximate Fusing Currents of Copper Wires in Free Air*

(See Regulation 68 A (*b*) Note.)

Diameter of Wire	*Equivalent S.W.G. Size*	*Fusing Current*	*Maximum Safe Working Current (see Note)*
1	2	3	4
inch		*amps.*	*amps.*
0·0092	34	8·6	4·3
0·010	33	9·8	4·9
0·0108	32	11·0	5·5
0·0120	—	12·8	6·4
0·0124	30	13·5	6·8
0·0148	28	17	8·6
0·018	26	22	11
0·022	24	30	15
0·028	22	41	21
0·029	—	43	22
0·036	20	62	31
0·040	19	73	37
0·044	—	86	43
0·048	18	98	49
0·052	—	111	56
0·056	17	125	63
0·064	16	156	78
0·072	15	191	96
0·080	14	229	115

*See Note under Table XII.

TABLE XII.

APPROXIMATE FUSING CURRENTS OF LEAD-TIN ALLOY (LEAD 75 PER CENT., TIN 25 PER CENT.) WIRES IN FREE AIR.

(See Regulation 68 A (*b*) Note.)

Diameter of Wire	*Equivalent S.W.G. Size*	*Fusing Current*	*Maximum Safe Working Current (See Note)*
1	2	3	4
inch.		*amps.*	*amps.*
0·020	25	3	2·0
0·022	24	3·5	2·3
0·024	23	4	2·6
0·028	22	5	3·3
0·032	21	6	4·1
0·036	20	7	4·8
0·048	18	10	7·0
0·064	16	16	11·0

Note.—Tables XI and XII refer to wires in free air and of the following lengths :—*Copper*—2½ to 3½ inches for wires up to 0·018 inch diameter; and not less than 4 inches for larger wires. *Lead-Tin Alloy*—2½ to 3½ inches.

The values given in the Tables may be taken to be correct where the fuse wire passes through an asbestos tube and does not closely touch the tube, but they do not apply where a substantial length of the wire is in contact with a porcelain holder. The tendency of the latter design is to increase the working capacity of the fuse, i.e., more current is required to melt the fuse, and if great accuracy is required the fusing current should be determined for the fuse holder in question.

For copper wires, the values of the currents given in Table XI are those necessary to fuse the wire in one minute, and are not appreciably different for other periods (the current required to fuse the wire in two hours being, in general, over 90 per cent. of that required to melt the wire in one minute.)

For the lead-tin alloy the currents given in Table XII are those necessary to fuse the wire in two minutes.

In every case the relation between the fusing current and the maximum safe-running current is based on values which will not produce an excessive temperature under normal running conditions. The actual temperature-rise at the hottest part of the fuse wire will be from 100 to 150 degrees C. for copper and 50 to 75 degrees C. for the lead-tin alloy.

TABLE XIII.

Standard Dimensions of Steel Conduits.

CONDUIT (Outside Diameter), Size of .. *ins.*	½	⅝	¾	1	1¼	1½	2	2½
Threads per inch	18	18	16	16	16	14	14	14
Depth of Thread .. .. .. ins.	·0356	·0356	·0400	·0400	·0400	·0457	·0457	·0457
Maximum Length of Thread on Ends ins.	7/16	½	9/16	11/16	¾	13/16	15/16	1 1/16
Minimum Length of Thread on Ends ins.	⅜	7/16	½	⅝	11/16	¾	⅞	1
Nominal Thickness—Class A (Plain) ins.	·040	·040	·048	·048	·056	·064	·064	·072
Minimum Thickness—Class A (Plain) ins.	·036	·036	·044	·044	·052	·060	·060	·068
Nominal Thickness—Class B (Screwed) ins.	·056	·064	·072	·072	·072	·080	·092	·092
Minimum Thickness—Class B (Screwed) ins.	·052	·060	·068	·068	·068	·076	·088	·088
Calculated weight per 100 ft., in lb., unenamelled and not including couplers — Class A (Plain) ..	20	26	37	50	73	100	135	191
Calculated weight per 100 ft., in lb., unenamelled and not including couplers — Class B (Screwed)	27	39	53	73	93	124	192	242

TABLE XIV.

Standard Dimensions of Fittings.

CONDUIT (*Outside Diameter*), *Size of* .. ins.	½	⅝	¾	1	1¼	1½	2	2½
Plain Coupler, Minimum Length of ins.	1⅛	1¼	1⅜	1½	1⅝	1⅞	2⅛	2⅜
Screwed Coupler, Minimum Length of ins.	1	1⅛	1¼	1½	1⅝	1¾	2	2¼
Fittings, Cast (Machined Part), Minimum Thickness of .. .. .. ins.	1/16	1/16	1/16	1/16	3/32	3/32	3/32	⅛
Fittings, Tube (Machined Part), Minimum Thickness of—Class A .. .. ins.	·036	·036	·044	·044	·052	·060	·060	·068
Fittings, Tube (Machined Part), Minimum Thickness of—Class B .. .. ins.	·052	·060	·068	·068	·068	·076	·088	·088
Split Elbows, Number of Lugs and Size of Screws for	3 No. 2 B.A.	3 No. 2 B.A.	4 No. 2 B.A.	4 No. 2 B.A.	4 ¼" Whit.	5 ¼" Whit.	5 ¼" Whit.	5 ¼" Whit.
Split Normal Bends, Number of Lugs and Size of Screws for	5 No. 2 B.A.	5 No. 2 B.A.	5 No. 2 B.A.	6 No. 2 B.A.	6 ¼" Whit.	6 ¼" Whit.	7 ¼" Whit.	7 ¼" Whit.
Split Tees, Number of Lugs and Size of Screws for	6 No. 2 B.A.	6 No. 2 B.A.	6 No. 2 B.A.	6 No. 2 B.A.	6 ¼" Whit.	8 ¼" Whit.	8 ¼" Whit.	8 ¼" Whit.
Running Couplers, Length of .. ins.	6	6	6	6	9	9	9	9
Recessed Portion in Class A or Threaded Portion in Class B for all Fittings, Length of .. — Minimum, ins.	7/16	½	9/16	11/16	¾	13/16	15/16	1 1/16
Recessed Portion in Class A or Threaded Portion in Class B for all Fittings, Length of .. — Maximum (Class B only) .. ins.	½	9/16	⅝	¾	13/16	⅞	1	1⅛
Lock-nuts, Thickness .. .. ins.	3/16	3/16	¼	¼	¼	5/16	⅜	⅜
" Width across Flats .. ins.	·815	·915	1·092	1·472	1·662	2·040	2·570	3·150
" Whitworth Spanner .. ins.	7/16	½	⅝	¾	1	1¼	1⅝	2

B.S. Specification for Steel Conduits and Fittings for Electrical Wiring, No. 31—1923.

The following are extracts from B.S. Specification No. 31 :—

1. *Conduit.* (*a*) Two classes of Steel Conduit for Electrical Wiring are recognized as Standard :—

Class A.—Plain.
Class B.—Screwed.

Class A consists of light gauge Conduit of the thicknesses and dimensions given in Table XIII. Class A Conduit is either close-joint, brazed, welded or solid drawn. The Coupler joining the lengths of tubing is a sleeve, and neither the ends of the Conduit nor the Coupler are screwed.

In the close-joint tubes, the edges of the steel strip although brought tightly together in the process of manufacture, are not metallically joined in any way.

Class B consists of heavy gauge Conduit of the thicknesses and dimensions given in Table XIII. Class B Conduit is either brazed, welded or solid drawn. Both ends of the Conduit are screwed.

Manufacture.

2. *Material and Method of Manufacture.*—Steel Conduits shall be either close-joint, brazed, welded or solid drawn as ordered, and shall be of mild steel and free from burrs and internal roughness. The Conduit Fittings (Table XIV) shall be made of steel or malleable iron. All steel shall have a tensile strength of not less than 18 tons nor more than 24 tons per square inch of section, and an elongation of not less than 15 per cent. in a length of 8 inches. Malleable castings shall be well annealed and free from internal projections.

For the capacity of conduits, see Table X of this Appendix.

TABLE XV.

Size of Cable for use with Heating and Cooking Appliances.*

Capacity of Appliance. watts	*Size of Cable*		
	100 *volts*	220 *volts*	260 *volts*
500	1/·044″	1/·036″	1/·036″
1,000	3/·036″	1/·044″	1/·036″
1,500	7/·029″	3/·029″	1/·044″
2,000	7/·029″	3/·029″	3/·029″
2,500	7/·036″	3/·036″	3/·029″
3,000	7/·044″	3/·064″	3/·036″
4,000	7/·052″	7/·029″	7/·029″
5,000	7/·064″	7/·036″	7/·029″
6,000	19/·052″	7/·044″	7/·036″
7,000	19/·052″	7/·052″	7/·044″
8,000	19/·064″	7/·052″	7/·052″
9,000	19/·072″	7/·064″	7/·052″
10,000	19/·072″	7/·064″	7/·064″

* This Table does not belong to the above B.S. Specification.

I.E.E. Regulations for the Electrical Equipment of Ships (1919).

The following is an abstract of the chief regulations :—

Generating Plant.

Particulars are given of standard steam engines and standard internal combustion engines. The rules in this part have reference to rating regulations, field regulators, general construction, etc.

Switchboards.

Materials are dealt with generally, on ordinary lines. If semi-insulating materials, such as marble or slate, are used, all conducting parts are to be insulated from the slate or marble with mica or micanite, and the slab is to be similarly insulated as a whole. Omnibus bars and ordinary connecting conductors are to be so proportioned that their average temperature will not rise more than 54° F. above that of the surrounding air and no part is to rise more than 9 deg. above the average. Schemes of colouring for conductors are specified. In regard to main switchgear, the requirements are set out in a general way according to the various systems that may be employed. Means are to be provided for indicating the state of insulation of the system to earth. Fuses are divided into two grades, namely, for ordinary duty and heavy duty. Ordinary duty fuses must protect the circuit without damage when a current 33 times the fusing current flowing under the normal pressure of the circuit is suddenly thrown on. Heavy duty fuses must protect a circuit when a current 330 times the fusing current is thrown on.

Conductors.

With regard to conductors, not more than two joints are to be made during manufacture in any one of the individual wires forming 1,000 yards complete cable, and no joint is to be within 12 in. of any other joint in the same ring and all joints made or manufactured are to be brazed or electrically welded. No cable is to have a conductor of less sectional area than 0·0015 sq. in., except for wiring of metal fittings. The sectional area of the conductor of the cable for such fittings must not be less than 0·001 sq. in. All conductors having an effective sectional area exceeding 0·0033 sq. in. must be stranded. The drop in voltage allowed is 2 volts plus 3 per cent. in the case of lighting and 2 volts plus 5 per cent. in the case of power.

The cable employed may be either vulcanized rubber or paper insulated. The V.I.R. cables may be (1) braided, (2) lead covered, (3) lead covered and armoured and (4) braided and armoured. The paper insulated cables may be (1) lead covered and (2) lead covered and armoured. Rules are given in regard to armouring and tests of dielectric. A system of colours is specified for the identification of cables.

Flexible cords are to be insulated with vulcanized rubber and provided with a protective covering of (*a*) silk braiding, (*b*) glace cotton braiding, (*c*) hemp, cotton or jute braiding, thoroughly compounded, (*d*) wire armouring comprising flexible braiding or spiral of galvanized steel or phosphor bronze wire in addition to the covering specified in (*c*), (*e*) hard cord braiding in addition to the covering specified in (*c*), or (*f*) tough rubber sheathing applied directly to the laid-up insulated cores.

The ends of all cables having a sectional area of 0.007 sq. in. and above are to be provided with a soldering socket. Cables may be (*a*) run in wood casing (any class of cable) in dry situations or (*b*) may be armoured cables and lead covered cables when not run in wood casing, in which case they are to be secured by metal clips having smooth or rounded edges ; the electrical resistance between any two points of the metallic envelopes of such cables must then not exceed 2 ohms. Such clips are to be spaced in accordance with the table given in the Rules. Armoured cables having a sectional area of 0·25 sq. in. and upwards may be carried on metal hangers. If cables are exposed to mechanical damage they are to be protected by sheet iron plating or by heavy gauge screwed conduit. No elbows are to be used, and no bend is allowed having a smaller radius than 2½ times the outside diameter of the conduit. Cables in machinery spaces or where inevitably exposed to the weather or to the action of sea water are to be lead covered and may be armoured in addition, with or without braiding over the armour. Braided cables of the same polarity in wood casing may be bunched. Joints during erection are to be made by means of joint boxes. All cables passing through ducts or watertight bulkheads are to be provided with deck tubes or watertight glands as the case may be.

Earthing.

Various rules are given of the usual type in regard to sub-division and control of circuits. Earthing connections are to be of copper having the same sectional area as the working conductor, but not less than 0·003 sq. in. for all sizes up to 0·007 sq. in. Above this size a conductor of not less than 0·007 sq. in. must be provided for every 50 amperes of working current or part thereof. Rules are given for the prevention of interference with magnetic compasses. Certain rules are also given in regard to secondary batteries.

Fittings.

In regard to fittings, rules are given providing for ample passages, freedom from stress on the conductors, avoidance of dust, moisture, etc. All weatherproof fittings, whether fixed or portable, are to be so protected as to withstand immersion under a 3 ft. head of water before fixing. After fixing they must be capable of withstanding the application of a stream of water ejected from the open end of an ordinary wash deck hose under a head of 15 ft. for a period of a quarter of a minute, the outlet of the hose being 6 ft. away from the fitting. Open type fittings are not to be used in spaces where inflammable or explosive dust or gases may be present, or near combustible materials. Weatherproof portable fittings must have all metal parts connected to earth by means of a third conductor in the flexible cord. If this cord is of metal armouring, the armouring may form the earthing conductor, provided it is sufficiently connected at either end. Such fittings are to have strong metal guards in metallic contact with the rest of the uninsulated metal in these fittings. Open type portable fittings with metal frames must have all their metal parts, other than the lamp holder or lamp holders, connected to earth. Open type portable fittings with non-metal frames need not be earthed, provided that the guards are not in, and cannot come into, metallic contact with the lamp-holder or lamp holders. Such fittings are to be made of treated hard wood, or of some suitable composition capable of standing rough usage in service. Lamp-holders are to be of the Goliath type for lamps consuming more than 300 watts. Lamp-holders in open-type portable fittings are to be insulated from the fittings by means of insulating material. All plugs and sockets having a rated capacity exceeding 5 amperes are to be provided with an interlock switch.

Motors and other Appliances.

Motors above ½ B.H.P. must conform in all respects with British standardization rules No. 168 of 1923. Motors fitted in compartments in which the temperature is liable to exceed 120° F. must be either of the pipe-ventilated type connected by ventilating ducts to the spaces in which the temperature is not liable to exceed 120° F. or of the various forced draught type connected to fans supplying air at or below that temperature.

Switch parts and protective devices of control gears must be so proportioned that their temperature shall not rise more than 54° F. above that of the surrounding air when working with normal current. All live parts must be enclosed by metal covers which are to be clear of such live parts by not less than ¾ in. Those portions of the cover in proximity to working contacts must be lined with incombustible insulating material. Resistances must be so proportioned that no accessible part shall rise to a higher temperature than 130° F.

Those parts of heating and cooking appliances which have to be handled in their operation must not become heated to a temperature exceeding 130° F. All connections between elements and main terminals, unless self-supporting or rigidly fixed, must be continuously insulated with porcelain beads. All appliances for pressures exceeding 150 volts, whether portable or fixed, must be provided with a terminal or other suitable means for earthing all uninsulated parts. All appliances are to be protected by a fuse on each insulated pole. They must be controlled as a whole by a switch, which in the case of portable appliances must be fitted on a bulkhead. Every portable appliance must have at least one section of the element controlled only on the live side of the wall plug or other similar connection between the flexible conductor and the fixed portion of the circuit. Non-luminous appliances must be provided with a visual indication to show that current is passing through. No appliance taking more than 1,000 watts is to be portable.

All apparatus for internal communications must be constructed to take the full pressure of the source of supply, without the interposition of any external resistance; and if the pressure of supply exceeds 25 volts, all circuits and accessories must be designed in all respects in accordance with the rules for lighting and power circuits.

Some rules are given in regard to lightning conductors and also in regard to wiring for oil ships.

The insulation resistance before putting an installation into service must not be less in megohms than 10 divided by the number of lamps on the circuits, except that the insulation resistance of any final lighting sub-circuit need not exceed 1 megohm. The insulation resistance between the case or framework in every live part of each individual dynamo, heater or other appliance complete must not be less than half a megohm.

Home Office Regulations on the Use of Electricity in Factories and Workshops.*

Exemptions.—Nothing in Regulations 2, 3, 4, 7, 9, 10, 11, 15, 16, 17, 21, 22, 23, 24, 25, 26, 28, 29, 30 and 31 shall apply, unless, on account of special circumstances the Secretary of State shall give notice to the occupier that this exemption does not apply (*a*) To any system in which the pressure does not exceed low pressure direct or 125 volts alternating; (*b*) in any public supply generating station, to any system in which the pressure between it and earth does not exceed low pressure; (*c*) in any above-ground sub-station for public supply, to any system not exceeding low pressure.

* This includes generating and transforming stations, and distribution.

2. Nothing in these Regulations shall apply to any service lines or apparatus on the supply side of the consumer's terminals or to any chamber containing such service lines or apparatus where the supply is given from outside under Board of Trade regulations; provided always that no live metal is exposed so that it may be touched.

3. If the occupier can show, with regard to any requirement of these Regulations, that the special conditions in his premises are such as adequately to prevent danger, that requirement shall be deemed to be satisfied; and the Secretary of State may by Order direct that any class of special conditions defined in the Order shall be deemed for the purposes of all or any of the requirements of these Regulations adequately to prevent danger, and may revoke such Order.

4. Nothing in these Regulations shall apply to any process or apparatus used exclusively for electro-chemical or electro-thermal or testing or research purposes; provided such process be so worked and such apparatus so constructed and protected and such special precautions taken as may be necessary to prevent danger.

5. The Secretary of State, may by Order, exempt from the operation of all or any of these Regulations any premises to which any special rules or regulations under any other Act as to the generation, transformation, distribution or use of electrical energy apply; and may revoke such Order.

6. The Secretary of State may, if satisfied that safety is otherwise practically secured, or that exemption is necessary on the ground of emergency or special circumstances, grant such exemption by Order, subject to any conditions that may be prescribed therein; and may revoke such Order.

7. Nothing in these Regulations shall apply to domestic factories or domestic workshops.

Regulations.

1. All apparatus and conductors shall be sufficient in size and power for the work they are called upon to do, and so constructed, installed, protected, worked and maintained as to prevent danger so far as is reasonably practicable.

2. All conductors shall either be covered with insulating material, and further efficiently protected where necessary to prevent danger, or they shall be so placed and safeguarded as to prevent danger so far as is reasonably practicable.

3. Every switch, switch fuse, circuit-breaker and isolating link shall be: (*a*) so constructed, placed or protected as to prevent danger; (*b*) so constructed and adjusted as accurately to make and to maintain good contact; (*c*) provided with an efficient handle or other means of working insulated from the system, and so arranged that the hand cannot inadvertently touch live metal; (*d*) so constructed or arranged that it cannot accidentally fall or move into contact when left out of contact.

4. Every switch intended to be used for breaking a circuit and every circuit-breaker shall be so constructed that it cannot with proper care be left in partial contact. This applies to each pole of double-pole or multipole switches or circuit-breakers. Every switch intended to be used for breaking a circuit and every circuit-breaker shall be so constructed that an arc cannot accidentally be maintained.

5. Every fuse, and every automatic circuit-breaker used instead thereof, shall be so constructed and arranged as effectively to interrupt the current before it so exceeds the working rate as to involve danger. It shall be of

such construction or be so guarded or placed as to prevent danger from overheating, or from arcing or the scattering of hot metal or other substance when it comes into operation. Every fuse shall be either of such construction or so protected by a switch that the fusible metal may be readily renewed without danger.

6. Every electrical joint and connection shall be of proper construction as regards conductivity, insulation, mechanical strength and protection.

7. Efficient means, suitably located, shall be provided for cutting off all pressure from every part of a system, as may be necessary to prevent danger.

8. Efficient means suitably located shall be provided for protecting from excess of current every part of a system, as may be necessary to prevent danger.

9. Where one of the conductors of a system is connected to earth, no single pole switch, other than a link for testing purposes or a switch for use in controlling a generator, shall be placed in such conductor or any branch thereof. A switch, or automatic or other cut-out may, however, be placed in the connection between the conductor and earth at the generating station, for use in testing and emergencies only.

10. Where one of the main conductors of a system is bare and uninsulated, such as a bare return of a concentric system, no switch, fuse or circuit-breaker shall be placed in that conductor, or in any conductor connected thereto, and the said conductor shall be earthed. Nevertheless, switches, fuses or circuit-breakers may be used to break the connection with the generators or transformers supplying the power; provided that in no case of bare conductor the connection of the conductor with earth is thereby broken.

11. Every motor, converter and transformer shall be protected by efficient means suitably placed, and so connected that all pressure may thereby by cut off from the motor, converter or transformer as the case may be, and from all apparatus in connection therewith; provided, however, that where one point of the system is connected to earth, there shall be no obligation to disconnect on that side of the system which is connected to earth.

12. Every electrical motor shall be controlled by an efficient switch or switches for starting and stopping, so placed as to be easily worked by the person in charge of the motor. In every place in which machines are being driven by any electric motor there shall be means at hand for either switching off the motor or stopping the machines if necessary to prevent danger.

13. Every flexible wire for portable apparatus, for alternating currents or for pressures above 150 volts direct current, shall be connected to the system either by efficient permanent joints or connections or by a properly constructed connector. In all cases where the person handling portable apparatus or pendant lamps with switches, for alternating current or pressures above 150 volts direct current, would be liable to get a shock through a conducting floor or conducting work or otherwise, if the metal-work of the portable apparatus became charged, the metal-work must be efficiently earthed; and any flexible metallic covering of the conductors shall be itself efficiently earthed and shall not itself be the only earth connection for the metal of the apparatus. And a lampholder shall not be in metallic connection with the guard or other metal-work of a portable lamp. In such places, and in any place where the pressure exceeds low pressure, the portable apparatus and its flexible wire shall be controlled by efficient means suitably located, and capable of cutting off the pressure,

and the metal-work shall be efficiently earthed independently of any flexible metallic cover of the conductors, and any such flexible covering shall itself be independently earthed.

14. The general arrangement of switchboards shall, so far as reasonably practicable, be such that (*a*) all parts which may have to be adjusted or handled are readily accessible; (*b*) the course of every conductor may where necessary be readily traced; (*c*) conductors not arranged for connection to the same system are kept well apart, and can where necessary be readily distinguished; (*d*) all bare conductors are so placed or protected as to prevent danger from accidental short-circuit.

15. Every switchboard having bare conductors normally so exposed that they may be touched, shall, if not located in an area or areas set apart for the purposes thereof, where necessary be suitably fenced or enclosed. No person except an authorized person, or a person acting under his immediate supervision, shall for the purpose of carrying out his duties have access to any part of an area so set apart.

16. All apparatus appertaining to a switchboard and requiring handling shall, so far as practicable, be so placed or arranged as to be operated from the working platform of the switchboard, and all measuring instruments and indicators connected therewith shall, so far as practicable, be so placed as to be observed from the working platform. If such apparatus be worked or observed from any other place, adequate precautions shall be taken to prevent danger.

17. At the working platform of every switchboard and in every switchboard passage-way if there be bare conductors exposed or arranged to be exposed when live so that they may be touched, there shall be a clear and unobstructed passage of ample width and height, with a firm and even floor. Adequate means of access, free from danger, shall be provided for every switchboard passage-way.

The following provisions shall apply to all such switchboard working platforms and passage-ways constructed after January 1st, 1909, unless the bare conductors, whether overhead or at the sides of the passage-ways, are otherwise adequately protected against danger by divisions or screens or other suitable means:—

(*a*) Those constructed for low-pressure and medium-pressure switchboards shall have a clear height of not less than 7 ft. and a clear width measured from bare conductor of not less than 3 ft.

(*b*) Those constructed for high-pressure and extra high-pressure switchboards, other than operating desks or panels working solely at low pressure, shall have a clear height of not less than 8 ft. and a clear width measured from bare conductor of not less than 3ft. 6in.

(*c*) Bare conductors shall not be exposed on both sides of the switchboard passage-way unless either (i) the clear width of the passage is, in the case of low pressure and medium pressure, not less than 4 ft. 6 in., and in the case of high pressure and extra high pressure, not less than 8 ft., in each case measured between bare conductors, or (ii) the conductors on one side are so guarded that they cannot be accidentally touched.

18. In every switchboard for high pressure or extra high pressure:—

(*a*) Every high-pressure and extra high-pressure conductor within reach from the working platform or in any switchboard passage-way shall be so placed or protected as adequately to prevent danger.

(*b*) The metal cases of all instruments working at high pressure or extra high pressure shall be either earthed or completely enclosed with insulating covers.

(*c*) All metal handles of high-pressure and extra high-pressure switches and, where necessary to prevent danger, all metal gear for working the switches shall be earthed.

(*d*) When work has to be done on any switchboard, then, unless the switchboard be otherwise so arranged as to secure that the work may be carried out without danger, either (i) the switchboard shall be made dead, or (ii) if the said switchboard be so arranged that the conductors thereof can be made dead in sections, and so separated by permanent or removable divisions or screens from all adjoining sections of which the conductors are live, that work on any section may be carried out without danger, that section on which work has to be done shall be made dead.

19. All parts of generators, motors, transformers or other similar apparatus at high pressure or extra high-pressure, and within reach from any position in which any person employed may require to be, shall be, so far as reasonably practicable, so protected as to prevent danger.

20. Where a high-pressure or extra high-pressure supply is transformed for use at a lower pressure, or energy is transformed up to above low-pressure, suitable provision shall be made to guard against danger by reason of the power-pressure system becoming accidentally charged above its normal pressure by leakage or contact from the higher-pressure system.

21. Where necessary to prevent danger, adequate precautions shall be taken either by earthing or by other suitable means to prevent any metal other than the conductor from becoming electrically charged.

22. Adequate precautions shall be taken to prevent any conductor or apparatus from being accidentally or inadvertently electrically charged when persons are working thereon.

23. Where necessary adequately to prevent danger, insulating stands or screens shall be provided and kept permanently in position, and shall be maintained in sound condition.

24. Portable insulating stands, screens, boots, gloves or other suitable means shall be provided and used when necessary adequately to prevent danger, and shall be periodically examined by an authorized person.

25. Adequate working space and means of access, free from danger, shall be provided for all apparatus that has to be worked or attended to by any person.

26. All those parts of premises in which apparatus is placed shall be adequately lighted to prevent danger.

27. All conductors and apparatus exposed to the weather, wet corrosion, inflammable surroundings or explosive atmosphere, or used in any process or for any special purpose, other than for lighting or power, shall be so constructed or protected, and such special precautions shall be taken as may be necessary adequately to prevent danger in view of such exposure or use.

28. No person except an authorized person or a competent person acting under his immediate supervision shall undertake any work where technical knowledge or experience is required in order adequately to avoid danger; and no person shall work alone in any case in which the Secretary of State directs that he shall not. No person except an authorized person, or a competent person over 21 years of age acting under his immediate supervision, shall undertake any repair, alteration, extension, cleaning or such work where technical knowledge or experience is required in order to avoid

danger, and no one shall do such work unaccompanied. Where a contractor is employed, and the danger to be avoided is under his control, the contractor shall appoint the authorized person, but if the danger to be avoided is under the control of the occupier, the occupier shall appoint the authorized person.

29. Instructions as to the treatment of persons suffering from electric shock shall be affixed in all premises where electrical energy is generated, transformed or used above low-pressure, and in such premises, or classes of premises, in which electrical energy is generated, transformed or used at low-pressure, as the Secretary of State may direct.

30. Every sub-station shall be substantially constructed, and shall be so arranged that no person other than an authorized person can obtain access thereto otherwise than by the proper entrance, or can interfere with the apparatus or conductors therein from outside; and shall be provided with efficient means of ventilation and be kept dry.

31. Every sub-station shall be under the control of an authorized person, and none but an authorized person or a person acting under his immediate supervision shall enter any part thereof where there may be danger.

32. Every underground sub-station not otherwise easily and safely accessible shall be provided with adequate means of access by a door or trap-door, with a staircase or ladder securely fixed and so placed that no live part of any switchboard or any bare conductor shall be within reach of a person thereon; provided, however, that the means of access to such sub-station shall be by a doorway and staircase (*a*) if any person is regularly employed therein, otherwise than for inspection or cleaning, or (*b*) if the sub-station is not of ample dimensions and there is therein either moving machinery other than ventilating fans, or extra high-pressure.

Memorandum on Home Office Regulations, dealing with Low-Pressure and Medium-Pressure Switchboards, with special reference to distribution boards and motor starting panels.

It is sometimes overlooked that the term "switchboard" includes all classes of switchboards, from large station switchboards down to distribution fuse boards for lighting circuits and motor starting panels. Mistakes sometimes leading to serious accidents or involving somewhat extensive alterations, have been made by occupiers and others in not considering the conditions under which a switchboard is to be placed, particularly as to whether the conditions permit of the use of (*a*) an open type switchboard, i.e., one having the conductors exposed, or (*b*) one enclosed in a cabinet which has to be opened for use, thereby exposing the live metal; or whether one having all live metal permanently protected is necessary.

(*a*) *Switchboards for use on systems to which Exemption* 1 *does not apply, i.e., in which the pressure exceeds* 125 *volts alternating or* 250 *volts direct.*

Fuse holders which have to be handled whilst live, must be in all cases in accordance with the requirements of Regulations 3 (*c*) and 5 (3rd part). The construction should be such that, in putting the fuse holder in or out of the board, (*a*) it is impossible for the hand inadvertently to touch any live part either of the fuse holder itself or of any adjacent live metal on the board, e.g., the fixed contacts, and (*b*) the hand is screened from the flash should the fuse blow at the moment of being inserted.

All the regulations affecting switchboards are applicable. Thus, a switchboard, whether large or small, if it has bare conductors normally exposed so that they may be touched, must either be in an area set apart for the purpose and to which only authorized and competent persons may have access, or it must be suitably fenced or enclosed (Regulation 15).

The alternative method of enclosing the switchboard in a cabinet is very commonly adopted, but even so, the conditions must be carefully considered. Even if enclosed in a cabinet, the door of which has to be opened for use, thereby exposing bare conductors so that they may be touched, there must be a clear and unobstructed working platform 3 feet wide (Regulation 17). Whether in a cabinet or not, it must be so placed that all apparatus which requires handling is within reach from the working platform (Regulation 16), and there must be an insulating stand (Regulation 23). Large switchboards are generally placed where all the above conditions can be complied with without difficulty. Smaller switchboards such as distribution fuse boards or motor panels, cannot, however, always be placed so as to comply with these requirements. Thus, distribution boards may have to be placed out of reach from the floor, so that there is no working platform, or they may have to be placed over iron floor plates or damp ground where an insulating stand is impracticable, or they may be on a works where they may have to be used by persons devoid of any technical knowledge.

Where the requirements referred to cannot be complied with, or where for any reason it is not convenient to comply with them, other means of safety must be adopted. All that is necessary to avoid having to comply with the requirements of these regulations is to provide a switchboard having no conductors exposed or arranged to be exposed when live so that they may be touched. Such complete protection is provided in the case of certain distribution fuse boards. In others the protection provided is not entirely complete, but is such that, under the terms of Exemption 3, it may be regarded as "adequate to prevent danger." Thus, all live parts are so protected that they cannot be *inadvertently* touched by a person handling the fuse holders, although it may be quite possible for him to touch them intentionally. As by the terms of the Exemption the responsibility for the adequacy of the protection provided rests with the occupier, he must take care in adopting such alternative means of safety that it is adequate. Switchboards constructed on such lines are now obtainable from a number of manufacturers, both for small distribution boards suitable for lighting circuits or larger ones for power circuits, but not all of those purporting to be in compliance with the requirements are adequately protected. Such construction may also effect compliance with Regulation 16 (last sentence) in regard to small distribution boards placed out of reach from the ground, provided that adequate means of access, e.g., a stepladder or ladder which can be securely placed, is available when required. For larger boards placed high up a platform is, however, generally necessary for safe access and working in any case.

Similar protection of all live parts within the cabinet may also be necessary in the case of metal cabinets, even when placed within reach from an insulating floor of proper width. Thus, if the metal cabinet has a door hinged at the top, the operator has to hold the door open with one hand whilst handling the apparatus with the other and he is therefore in connection with earth, despite the insulating stand. With a side-hinged door which can be opened 180° or thereabouts, there may be no need for this protection under these conditions although it is often desirable, or the cabinet may be made of hard wood or other suitable insulating material.

Distribution boards enclosed in cabinets, but not otherwise protected (i.e., having live metal exposed when the door is open) are therefore permissible only if placed in accordance with the requirements of Regulations 16, 17 and 23; i.e., they must be within reach of the ground or working platform, there must be a three-feet space in front and there must be an insulating stand. The cabinet should be preferably made of insulating material, but if of metal, the door must not be hinged at the top. For use under any other conditions there should be adequate protection of the live metal within the cabinet.

The question of accessibility of distribution boards, from the point of view of convenience of operation, is often overlooked. It is not of great consequence in regard to boards containing fuses only and which require handling only occasionally, as when a fuse requires renewal or a circuit has to be made dead. If, however, they contain switches for controlling the branch circuits, it is obviously more convenient that they should be readily accessible. Whether containing switches or not, they should not be placed unnecessarily high up or otherwise inconveniently. To place them in positions involving danger to the attendant, as over running machinery or close to shafting, as is sometimes done, is contrary to the requirements of Regulations 1 and 25.

Similarly, as regards convenience of operation, switches for controlling the branch circuits are better placed outside the cabinet, so that it is not necessary to open the door every time a switch has to be turned on or off. If, however, they are within the cabinet, convenience of operation is sometimes secured by the switch handles being extended through the base of the cabinet.

There are other types of completely protected switchgear, suitable for use under practically any conditions. Several firms make ironclad switchgear conveniently arranged, so that new panels can be added as required. With such switchgear, the requirements as to the fuses (Regulations 5 and 3 (c)) must be properly arranged for, and no live metal must be exposed in the fuse chambers so that it may be touched inadvertently, when the fuse chamber is opened for renewal of a fuse. This may be arranged by the protection of all live metal within the fuse chamber and the use of properly protected switch-fuses; or by the fuses and therefore all the conductors within the fuse chamber being on the dead side of the switch controlling the circuit, in which case safety is further ensured if the fuse chamber door is interlocked with the switch so that it cannot be opened until the switch is "off"; or by the use of properly constructed "fuse-switches." Where oil switches are used, there must be means of making the oil switch dead for examination or repairs (Regulation 7). In some this is provided for by isolating switches, arranged to be operated from outside the casing, and in others by the oil switches being arranged on slides so that they can be withdrawn from contact with the bus-bars.

Totally enclosed ironclad switchgear for motor panels is now quite common. Here again attention should be paid particularly to the requirements as to renewal of the fuses (*see* also under Regulation 5).

(*b*) *Switchboards for use on systems to which Exemption* 1 *applies, i.e. in which the pressure does not exceed* 125 *volts alternating or* 250 *volts direct.*

A number of the Regulations do not apply in the case of such systems, and there is consequently greater freedom in the use of switchboards having conductors exposed or arranged to be exposed when live. Thus, Regulations 15, 16, 17 and 23 do not apply. At the same time there is no exemption from the requirements of Regulations 1, 5 and 14. Regulation 1 very definitely, although in general terms, requires that safety shall be provided for. Thus, switchboards having live metal exposed, although not required to be in an area set apart for the purpose, should obviously not be placed where persons are liable to run into them or touch live metal in passing or where engaged in their employment. There are no specified requirements as to working platforms and passage-ways. Nevertheless, these should be of reasonable width and where there would otherwise be danger of shock to earth, insulating flooring should be provided. Subject, however, to reasonable precautions being taken, open-type switchboards, motor starting panels etc., are permitted without the full restrictions required in the case of those for higher pressures. The protection to be afforded will depend very much upon the particular circumstances of each case, and apart from the question of safety of employees, enclosed type switchgear may be necessary for the protection of the apparatus against damage. Enclosed type motor control panels are advisable in all cases where there is not plenty of space, and particularly where women are employed. Where machines, driven by motors, have starting panels attached or otherwise so placed that the workers are liable to touch them when in contact with earth, e.g., laundry or printing machinery or machine tools, the switchgear should be of totally enclosed type.

Similarly, distribution fuse boards should in general be enclosed in cabinets. Regulation 5 must be complied with. Fuses must be of such a type or so enclosed as to prevent scattering and must be so constructed that they may be readily renewed without danger. They must be so constructed that the hand is shielded from the arc or hot metal should a fuse blow on being replaced on the board on a short circuit. Unless the distribution board is placed where the person renewing a fuse is insulated from earth, either as regards the floor or by reason of the fuses being in a metal cabinet with which he may be in contact, the fuse holders must also be so constructed that there is no risk of touching live metal when handling them. The busbars and other live parts within the cabinet need not, however, be further protected. Similarly, if there are switches in the cabinet, so long as they have proper insulated handles no further enclosure of the parts is necessary. Again, distribution fuse boards need not be placed within reach of the ground if this is not convenient, but if they contain switches in addition it is much better that they should be placed within convenient reach. To place them in positions involving danger to the attendant, as over-running machinery or close to the shafting, is contrary to the requirements of Regulation 1.

Regulations as to the Use of Electricity in Mines under the Coal Mines Act.

I.—Below Ground.

117. *Duties.*—It shall be the duty of the mine owner, agent, and manager to comply with and enforce the following regulations, and it shall be the duty of all workmen and persons employed to conduct their work in accordance with the regulations.

118. *Definitions.*—"Pressure" means the difference of electrical potential between any two conductors, or between a conductor and earth as read by a hot wire or electrostatic voltmeter.

"Low Pressure" means a pressure in a system normally not exceeding 250 volts where the electrical energy is used.

"Medium Pressure" means a pressure in a system normally above 250 volts, but not exceeding 650 volts, where the electrical energy is used.

"High Pressure" means a pressure in a system normally above 650 volts, but not exceeding 3,000 volts, where the electrical energy is used or supplied.

"Extra-high Pressure" means a pressure in a system normally exceeding 3,000 volts, where the electrical energy is used or supplied.

"System" means an electrical system in which all the conductors and apparatus are electrically connected to a common source of electromotive force.

"Concentric System" means a system in which the circuit in a conductor or conductors, called the inner conductor, is completed through one or more conductors, called the outer conductor, arranged so that the inner conductor is insulated and the outer conductor is disposed over the insulation of, and more or less completely around, the inner conductor.

"Conductor" means an electrical conductor arranged to be electrically connected to a system.

"Apparatus" means electrical apparatus, and includes all apparatus, machines, and fittings in which conductors are used, or of which they form a part.

"Circuit" means an electrical circuit forming a system or branch of a system.

"Covered with insulating material" means adequately covered with insulating material of such quality and thickness that there is no danger.

"Metallic covering" means iron or steel armouring, with or without a lead or other metallic sheath as the conditions of the case may require, or an iron or steel pipe surrounding two or more conductors.

"Bare" means not covered with insulating material.

"Live" means electrically charged.

"Dead" means at, or about, zero potential, and disconnected from any live system.

"Open Sparking" means sparking which, owing to the lack of adequate provision for preventing the ignition of inflammable gas external to apparatus, would ignite such inflammable gas.

"Earthed" means connected to the general mass of earth in such manner as will ensure at all times an immediate discharge of electrical energy without danger.

"Earthing system" means an electrical system in which all the conductors are earthed.

"Switchgear" means switches or fuses, conductors, and other apparatus in connection therewith, used for the purpose of controlling the current or pressure in any system or part of a system.

"Authorized person" means a person appointed in writing by the manager of the mine to carry out certain duties incidental to the generation, transformation, distribution, or use of electrical energy in the mine, such person being a person who is competent for the purposes of the rule in which the term is used.

"Electrician" means a person appointed in writing by the manager of the mine to supervise the apparatus in the mine and the working thereof, such person being a person who is over 21 years of age, and is competent for the purposes of the rule in which the term is used.

"Danger" means danger to health or danger to life or limb from shock, burn, or other injury to persons employed, or from fire or explosion attendant upon the generation, transformation, distribution or use of electrical energy.

"Use" of electricity means the conversion of electricity into mechanical energy, heat, or light for the purpose of providing mechanical energy, heat, or light.

119. *Notices for H.M. Inspector.*—Notices shall be sent to the Inspector of the division, on the forms prescribed by the Secretary of State, as

follows :—namely, (i) Notice of the intention to introduce apparatus into any mine, or into any ventilating district in any mine. (ii) Notice of the intention to introduce or re-introduce electricity into any mine where the use of electricity has previously been prohibited by Sec. 60 (1) of the 1911 Act. (iii) On or before the 21st of January in every year an annual return, giving the size and type of apparatus and any particulars which may be required by the Secretary of State as the circumstances of its use.

If the Inspector of the division does not object in writing, within one calendar month from the receipt by him of the notice, to the carrying out of either of the intentions specified in the first or second notices, the owner shall be entitled to carry out such intention or intentions. Provided that this regulation shall not apply to telephones and signalling apparatus.

120. *Plan.*—A proper plan on the same scale as that kept at the mine in fulfilment of the requirements of the Act shall be kept in the office at the mine showing the position of all fixed apparatus in the mine, other than cables, telephones and signalling apparatus. The said plan shall be corrected as often as may be necessary to keep it reasonably up to date, and it shall be produced to an inspector of mines at any time on his request.

121. *Notices to Workmen.*—The following notices, constructed of durable material, shall be exhibited where necessary : (i) A notice prohibiting any person other than an authorized person from handling or interfering with apparatus. (ii) A notice containing directions as to procedure in case of fire. This notice shall be exhibited in every place containing apparatus, other than cables, telephones and signalling apparatus. (iii) A notice containing directions as to the restoration of persons suffering from the effects of electric shock. (iv) A notice containing instructions how to communicate with the person appointed under Regulation 128 (*a*). This notice shall be exhibited at the shaft bottom.

122. *Lighting, Telephones, and Fire Buckets.*—(*a*) In all places lighted by electricity where a failure of the electric light would be likely to cause danger, one or more safety lamps or other proper lights shall be kept continuously burning. (*b*) Efficient telephonic or other equivalent means of communication shall be provided for communicating between the place in which the switchgear provided under Regulation 128 (*a*) is erected and the shaft bottom or main distributing centre in the pit. (*c*) Fire buckets of suitable capacity, filled with clean dry sand ready for immediate use in extinguishing fires, shall be kept in every place containing apparatus, other than cables, telephones and signalling apparatus.

123. *Housing of apparatus and Working Space.*—(*a*) Where necessary to prevent danger or mechanical damage, transformers and switchgear shall be placed in a separate room, compartment or box. (*b*) Unless the apparatus is so constructed, protected and worked as to obviate the risk of fire, no inflammable material shall be used in the construction of any room, compartment or box containing apparatus, or in the construction of any of the fittings therein. Each such room, compartment or box shall be substantially constructed and shall be kept dry. (*c*) Adequate working space and means of access clear of obstruction and free from danger shall be provided for all apparatus that has to be worked or attended to by any person, and all handles intended to be operated shall be conveniently placed for that purpose.

124. *The Construction of Apparatus and the Insulation of a System.*—(*a*) All apparatus and conductors shall be sufficient in size and power for the work they may be called upon to do, and so constructed, installed, protected, worked and maintained as to prevent danger so far as is reasonably practicable. (*b*) All insulating material shall be chosen with special regard to the circumstances of its proposed use. It shall be of mechanical strength sufficient for its purpose, and, so far as is practicable, it shall be of such a character or so protected as fully to maintain its insulating properties under working conditions of temperature and moisture. (*c*) Every part of a system shall be kept efficiently insulated from earth, except that (i) the neutral point of a polyphase system may be earthed at one point only ; (ii) the mid-voltage point of any system, other than a concentric system, may be earthed at one point only ; and (iii) the outer conductor of a concentric system shall be earthed. Where any point of a system is earthed it shall be earthed by connection to an earthing system at the surface of the mine. (*d*) Efficient means shall be provided for indicating any defect in the insulation of a system.

125. *Earthing.*—(*a*) All metallic sheaths, coverings, handles, joint boxes, switchgear frames, instrument covers, switch and fuse covers and boxes, and all lamp holders, unless efficiently protected by an earthed or insulating covering made of fire-resisting material, and the frames and bedplates of generators, transformers and motors (including portable motors) shall be earthed by connection to an earthing system at the surface of the mine. (*b*) Where the cables are provided with a metallic covering constructed and installed in accordance with Regulation 129 (*e*), such metallic covering may be used as a means of connection to the earthing system. All the conductors of an earthing system shall have a conductivity at all parts and at all joints at least equal to 50 per cent. of that of the largest conductor used solely to supply the apparatus a part of which it is desired to earth. Provided that no conductor of an earthing system shall have a cross-sectional area of less than 0·022 of a square inch. (*c*) All joints in earth conductors and all joints to the metallic covering of the cables shall be properly soldered or otherwise efficiently made, and every earth conductor shall be soldered into a lug of each for its terminal connections. No switch, fuse or circuit-breaker shall be placed in any earth conductor.

This rule shall not apply (except in the case of portable apparatus) to any system in which the pressure does not exceed low-pressure direct current or 125 volts alternating current.

126. *Use of High (or Extra-High) Pressure Current.*—(*a*) Where electricity is distributed at a pressure higher than medium pressure (i) it shall not be used without transformation to medium or low-pressure, except in fixed machines in which the high or extra-high pressure parts are stationary ; and (ii) motors under 20 h.p. shall be supplied with current through a transformer stepping down to medium or low pressure. (*b*) Where energy is transformed, suitable provision shall be made to guard against danger by reason of the lower-pressure apparatus becoming accidentally charged above its normal pressure by leakage from or contact with the higher-pressure apparatus.

127. *Switchgear, etc.**—Switchgear and all terminals, cable ends, cable joints and connections of apparatus shall be constructed and installed so that : (i) All parts shall be of mechanical strength sufficient to resist rough usage. (ii) All conductors and contact areas shall be of ample current-carrying capacity, and all joints in conductors shall be properly soldered or otherwise efficiently made. (iii) The lodgment of any matter likely to

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* A memorandum states that Switchgear should comply with the British Standard Specifications (see pages 263 et seq.).

diminish the insulation and of coal dust on or close to live parts shall be prevented. (iv) All live parts shall be so protected or enclosed as to prevent accidental contact by persons and danger from arcs or short-circuits, fire or water. (v) Where there may be risk of igniting gas, coal dust or other inflammable material, all parts shall be so protected as to prevent open sparking.

128. *Control of the Supply of Current.*—(*a*) Properly constructed switchgear for cutting off the supply of current to the mine shall be provided at the surface of the mine, and during the time any cable is live a person authorized to operate the said switchgear shall be available within easy reach thereof. Lightning arresters, properly adjusted and maintained, shall be provided where necessary to prevent danger. (*b*) Efficient means, suitably placed, shall be provided for cutting off all pressure from every part of a system as may be necessary to prevent danger. (*c*) Such efficient means shall be provided for cutting off all pressure automatically from the part or parts of the system affected in the event of a fault, as may be necessary to prevent danger. (*d*) Every motor shall be controlled by switchgear for starting and stopping, so arranged as to cut off all pressure from the motor and from all apparatus in connection therewith, and so placed as to be easily worked by the person appointed to work the motor. (*e*) If a concentric system is used, no switch, fuse or circuit-breaker shall be placed in the outer conductor, or in any conductor connected thereto, except that, if required, a reversing switch may be inserted in the outer conductor at the place where the current is being used. Nevertheless, switches, fuses or circuit-breakers may be used to break the connection with the generators or transformers supplying the electricity ; provided that the connection of the outer conductor with the earthing system shall not thereby be broken.

129. *Cables.*—All cables, other than flexible cables for portable apparatus and signalling wires, shall comply with the following requirements : (*a*) They shall be covered with insulating material (except that the outer conductor of a concentric system may be bare). The lead sheath of lead-sheathed cables and the iron or steel armouring of armoured cables shall be of not less thickness respectively than is recommended by the Engineering Standards Committee. (*b*) They shall be efficiently protected from mechanical damage and supported at sufficiently frequent intervals and in such a manner as adequately to prevent danger and damage to the cables. (*c*) Concentric cables, or two-core or multi-core cables protected by a metallic covering, or single core cables protected by a metallic covering, which shall contain all the conductors of the circuit, shall be used (i) where the pressure exceeds low pressure ; (ii) where the roadway conveying the cables is also used for mechanical haulage, and (iii) where there may be risk of igniting gas, coal dust or other inflammable material. Provided that if the medium pressure d.c. system is used (i) two single-core cables protected by metallic coverings may be used for any circuit if the said metallic coverings are bonded together by earth conductors so placed that the distance between any two consecutive bonds is not greater than 100 ft. measured along either cable, and (ii) two single-core cables covered with insulating material efficiently protected otherwise than by a metallic covering may be used in gate roads (except in gate roads which are also used for mechanical haulage or where there may be risk of igniting gas, coal dust or other inflammable material) for the purpose of supplying portable apparatus. (*d*) Cables unprotected by a metallic covering shall be properly secured by some non-conducting and readily breakable material to efficient insulators. (*e*) The metallic covering of every cable shall be (i) electrically continuous throughout ; (ii) earthed, if it is required by Regulation 125 (*a*) to be earthed, by a connection to the earthing system of not less conductivity than the same length of the said metallic covering; (iii) efficiently protected against corrosion where necessary: (iv) of a conductivity at all parts and at all joints at least equal to 50 per cent. of the conductivity of the largest conductor enclosed by the said metallic covering and (v) where there may be risk of igniting gas, coal dust, or other inflammable material so constructed as to prevent as far as is practicable any fault in leakage of current from the live conductors from causing open sparking. Provided that where two single-core cables protected by metallic coverings bonded together in accordance with paragraph (*c*) of this Regulation are used for a circuit, the conductivity of each of the said metallic coverings at all parts and at all joints shall be at least equal to 25 per cent. of the conductivity of the conductor enclosed thereby. (*f*) Cables and conductors where joined up to motors, transformers, switchgear and other apparatus shall be installed so that (i) they are mechanically protected by securely attaching the metallic covering (if any) to the apparatus, and (ii) the insulating material at each cable end is efficiently sealed so as to prevent the diminution of its insulating properties. Where necessary to prevent abrasion, or to secure gas-tightness, there shall be properly constructed bushes.

130. *Portable Apparatus.*—(*a*) Flexible cables for portable apparatus shall be two-core or multi-core and covered with insulating material, which shall be efficiently protected from mechanical damage. If a flexible metallic covering be used either as the outer conductor of a concentric system or as a means of protection from mechanical damage, the same shall not alone be used to form an earth conductor for the portable apparatus. (*b*) Every flexible cable for portable apparatus shall be connected to the system and to the portable apparatus itself by a properly constructed connector. (*c*) At every point where flexible cables are joined to main cables a switch capable of entirely cutting off the pressure from the flexible cables shall be provided. (*d*) No lamp-holder shall be in metallic connection with the guard or other metal-work of a portable lamp.

131.—*Supervision and Working of Apparatus.*—(*a*) Every person appointed to work, supervise, examine or adjust any apparatus shall be competent for the work that he is set to do. No person, except an electrician or a competent person acting under his supervision, shall undertake any work where technical knowledge or experience is required in order adequately to avoid danger. (*b*) An electrician shall be appointed in writing by the manager to supervise the apparatus. If necessary for the proper fulfilment of the duties detailed in the succeeding sections of this rule, the manager shall also appoint in writing an assistant or assistants to the electrician. (*c*) The electrician shall be in daily attendance at the mine. He shall be responsible for the fulfilment of the following duties, which shall be carried out by him or by an assistant or assistants duly appointed under subsection (*b*): (i) the thorough examination of all apparatus (including the testing of earth conductors and metallic coverings for continuity) as often as may be necessary to prevent danger and (ii) the examination and testing of all new apparatus and of all apparatus re-erected in a new position in the mine before it is put into service in the new position; provided that in the absence of the electrician for more than one day the manager shall appoint in writing an efficient substitute. (*d*) The electrician shall keep at the mine a log-book made up of daily log-sheets kept in the form prescribed by the Secretary of State. The said log-book shall be produced at any time to an inspector of mines on his request. (*e*) Should there be a fault in any circuit the part affected shall be made dead without delay, and shall remain so until the fault has been remedied. (*f*) All apparatus shall be kept clear of obstruction and free from dust, dirt and moisture as may be necessary to prevent danger. Inflammable or explosive material shall not be stored in any room, compartment or box containing apparatus, or in the vicinity of apparatus. (*g*) Adequate precautions shall be taken by earthing or other suitable means to discharge electrically any conductor or apparatus, or any adjacent apparatus if there is danger therefrom, before it is handled, and to prevent any conductor or apparatus from being accidentally or inadvertently electrically charged when persons are working thereon. While lamps are being changed the pressure shall be cut off. Provided that this paragraph shall not apply to the cleaning of commutators and slip-rings working at low or medium pressures. (*h*) The person authorized to work an electrically-driven coal-cutter or other portable machine shall not leave the machine while it is working, and shall, before leaving the working place, ensure that the pressure is cut off from the flexible trailing cable which supplies such coal-cutter or other portable machine. Trailing cables shall not be dragged along by the machine when working. (*i*) Every flexible cable shall be examined periodically (if used with a portable machine, at least once in each shift by the person authorized to work the machine), and if found damaged or defective it shall forthwith be replaced by a spare cable in good and substantial repair. Such damaged or defective cable shall not be further used underground until after it has been sent to the surface and there properly repaired.

132. *The use of Electricity where Inflammable Gas is likely to be present.*—In any part of a mine in which inflammable gas, although not normally present, is likely to occur in quantity sufficient to be indicative of danger, the following additional requirements shall be observed: (i) All cables, apparatus, signalling wires and signalling instruments shall be constructed, installed, protected and worked and maintained so that in the normal working thereof there shall be no risk of open sparking. (ii) All motors shall be constructed so that when any part is live all rubbing contacts (such as commutators and slip-rings) are so arranged or enclosed as to prevent open sparking. (iii) The pressure shall be switched off apparatus forthwith if open sparking occurs, and during the whole time that examination or adjustment disclosing parts liable to open sparking is being made. The pressure shall not be switched on again until the apparatus has been examined by the electrician or one of his duly appointed assistants and the defect (if any) has been remedied or the adjustment made. (iv) Every electric lamp shall be enclosed in an air-tight fitting, and the lamp globe itself shall be hermetically sealed. (v) A safety lamp shall be provided and used with each motor when working, and should any indication of fire-damp appear from such safety lamp, the person appointed to work the motor shall forthwith cut off the pressure therefrom and report the matter to a fireman, examiner or deputy or other official.

133. *Shot Firing.*—Current from lighting or power circuits shall not be used for firing shots. (*b*) Shot-firing cables shall be covered and protected as provided by Regulation 130 (*a*) for flexible cables. Adequate precautions shall be taken to prevent them from touching other cables and apparatus.

134. *Signalling.*—(*a*) Where electricity is used for signalling the pressure in any one circuit shall not exceed 25 volts. (*b*) Contact-makers shall be so constructed as to prevent the accidental closing of the circuit. (c) Adequate precautions shall be taken to prevent signal and telephone wires from touching cables and other apparatus.

135. *Electric Re-lighting of Safety Lamps.*—(*a*) All re-lighting apparatus shall be so constructed, worked and maintained as to preclude the accumulation of explosive gas within it. (*b*) Re-lighting apparatus shall not be used in any part of a mine to which Regulation 132 applies. (*c*) All safety lamps when re-lighted shall be examined before being issued.

136. *Locomotives.*—(*a*) Haulage by electric locomotives on the overhead trolley-wire system is prohibited in any mine in which coal is worked. (*b*) Haulage by electric locomotives on the overhead trolley-wire system may be used in mines other than coal mines, and haulage by storage battery locomotives may be used in any mine, with the consent in writing first obtained of the Secretary of State in all cases, and subject to such conditions affecting safety as may be prescribed by him.

137. *Exemptions.*—(*a*) Any of the requirements of this part of these Regulations shall not apply in any case in which exemption is obtained from the Secretary of State on the ground either of emergency or special circumstances, on such conditions as the Secretary of State may prescribe. (*b*) The requirements of this part of these Regulations which relate to the construction of cables and other apparatus shall not before January 1st, 1920, apply to any apparatus which was in use before June 1st, 1911, and which had been constructed or had before June 1st, 1911, been adapted so as to comply with the requirements relating to the construction of electrical apparatus in mines in force before that date, unless the inspector of the division, by written notice served on the owner, agent, or manager as regards either all or any of the said requirements of the foregoing rules, so directs. If the owner, agent, or manager within 14 days after the receipt of such notice objects to comply with the requirements specified in the notice, the matter shall be settled in manner provided by the Act for settling disputes.

II.—Above Ground.

The Regulations set out above are applicable to electrical apparatus used above ground, subject to the following amendments :—

1. In Regulation 118 and section (*c*) of Regulation 131, the words "at the surface of the mine" shall be substituted in every case for the words "in the mine."

2. Regulations 119, 120, 121 (iv), 122 (*b*), 123 (*b*), 128 (*a*), 132, 133, 134 (*a*), 135, 136 and 137 (*b*) shall not apply.

3. For Regulation 123 (*a*) the following regulation shall be substituted: "Where necessary to prevent danger or mechanical damage, apparatus shall be placed in a separate room, compartment or box or fireproof construction. Inflammable or explosive material shall not be stored in any such room, compartment or box."

4. Regulation 125 (*b*) shall be amended by adding to the end : "Except that in the case of a portable lamp or other apparatus of small current capacity, connected to the system by means of a flexible cable complying with Regulation 130, the cross sectional area of the earthing conductor in the flexible cable shall not be required to be greater than the cross-sectional area of either of the live conductors in the same flexible cable."

5. The word "regulation" shall be substituted for "rule" in the second paragraph of Regulation 125 (*c*).

6. For Regulation 129 the following regulation shall be substituted: "Unless so placed or otherwise safeguarded as to prevent danger, all cables other than flexible cables for portable apparatus and signal and telephone wires shall comply with the following requirements : (*a*) They shall be

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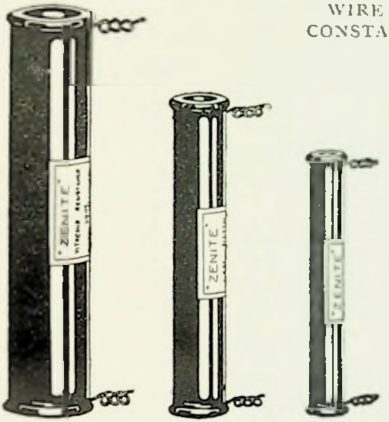

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covered with insulating material (except that the outer conductor of a concentric system may be bare). The lead sheath of lead-sheathed cables and the iron or steel armouring of armoured cables shall be of not less thickness respectively than is recommended by the British Engineering Standards Association. (*b*) They shall be efficiently protected from mechanical damage and supported at sufficiently frequent intervals and in such a manner as adequately to prevent danger and damage to the cables. (*c*) Concentric cables, or two-core or multi-core cables protected by a metallic covering or single-core cables protected by metallic covering which shall contain all the conductors of the circuit, shall be used (i) where the pressure exceeds low pressure, and (ii) where there may be risk of igniting coal dust or other inflammable material. Provided that if the medium pressure d.c. system is used two single core cables protected by metallic coverings may be used for any circuit. (*d*) Cables unprotected by a metallic covering shall be properly secured to efficient insulators. (*e*) The metallic covering of every cable shall be (i) electrically continuous throughout; (ii) earthed, if it is required by Regulation 125 (*a*) to be earthed, by a connection to the earthing system of not less conductivity than the same length of the said metallic covering; (iii) efficiently protected against corrosion where necessary; (iv) of a conductivity at all parts and at all joints at least equal to 50 per cent. of the conductivity of the largest conductor enclosed by the said metallic covering; and (v) where there may be risk of igniting coal dust or other inflammable material so constructed as to prevent, as far as is practicable, any fault or leakage of current from the live conductors from causing open sparking. (*f*) Cables and conductors, where joined up to motors, transformers, switchgear and other apparatus, shall be installed so that (i) they are mechanically protected by securely attaching the metallic covering (if any) to the apparatus; and (ii) the insulating material at each cable end is efficiently sealed so as to prevent the diminution of its insulating properties. Where necessary to prevent abrasion there shall be properly constructed bushes."

7. Regulation 130 (*c*) shall be amended to read:—" At every point where flexible cables for portable apparatus are joined to main cables a switch capable of entirely cutting off the pressure from the flexible cables shall be provided."

8. The second paragraph of Regulation 131 (*f*) shall not apply.

9. For Section (*h*) of Regulation 131 the following shall be substituted: " The person authorized to work an electrically driven portable machine shall not leave the machine while it is working, and shall, before leaving the machine, ensure that the pressure is cut off from the flexible trailing cable which supplies such portable machine. Trailing cables shall not be dragged along by the machine when working."

10. In Section (i) of Regulation 131 the words "repaired or" shall be inserted before " replaced." The last sentence shall not apply.

11. In Regulation 134 (*b*) the words " used for signalling " shall be inserted after " contact makers."

For the **Ministry of Transport's Regulations as to Electrical Power on Tramways**, see page 408, Vol I.

For the **Electricity Commissioners' Regulations for Overhead Lines** see page 174, Vol I.

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