The Electrical Fundamentals of Communication

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ELECTRICAL FUNDAMENTALS
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PREFACE

The title of this book, "The Electrical Fundamentals of Communication," defines its scope. It presents the electrical fundamentals upon which communication, including the three divisions telegraphy, telephony, and radio with its allied branches, is based.

The book is designed for the student of communication and the worker in the communication industry. It considers electrical phenomena, using explanations and illustrations taken from the communication industry itself. In this respect, it differs from other books on electrical fundamentals which include explanations and illustrations largely drawn from the power industry.

Care has been taken to keep the book well balanced. Illustrations and examples are included from all branches of communication. The theories and explanations given will withstand rigid examination. However, the terminology used and the method of presentation are simple and direct, adapting the book for beginning students and communications workers rather than for advanced students and engineers. Nevertheless, many of these will find the book useful in reviewing fundamentals.

Those examining the book may be surprised to find hydraulic and mechanical analogies missing. Here lies one great advantage of this book. The reader is presented the facts in electrical language and will learn them correctly the first time, instead of being obliged to study vague and inadequate analogies taken from other fields.

There is no substitute for sound training. If a student desires to become proficient in communication, or if a worker in the industry aspires to raise his knowledge to a higher plane, it can be done only by gaining a true knowledge of electrical fundamentals based on the explanations accepted as being technically correct. It is contended, however, that such explanations can be made readily understandable and interesting as well.

It is recognized that some readers will not want to go so far as others and that this book will be used by groups with wide individual interests. Thus the book has been written so that certain chapters may be omitted and so that the material may be studied in a sequence different from that given.

Industrial organizations have been of great assistance in providing illustrations and in other ways, and this service is gratefully acknowledged. Among those who have been helpful are General Electric Company, Westinghouse Electric and Manufacturing Company, American Telephone and Telegraph Company, Bell Telephone Laboratories, General Radio Company, RCA Manufacturing Company, Thomas A. Edison Company, Weston Electrical Instrument Corporation, National Carbon Company, Leeds and Northrup Company, International Resistance Company, and Electric Storage Battery Company.

Some of the illustrations used in this book have appeared in substantially the same form in other books by the author. He is indebted to John Wiley & Sons, Inc., and to The Macmillan Company for permission to use these illustrations.

The author is deeply indebted to his colleague Prof. H. B. Cockerline for making a detailed check of the manuscript and for suggesting many improvements. Appreciation is also expressed to R. H. Dearborn, dean of the School of Engineering, and to F. O. McMillan, head of the Department of Electrical Engineering, for approving this project and for their encouragement. The assistance of others who helped in numerous ways is acknowledged.

It is a distinct pleasure for the author to express his gratitude to his wife for her care in typing the manuscript and for the many helpful suggestions made.

ARTHUR L. ALBERT.

OREGON STATE COLLEGE,
August, 1942.

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ELECTRICAL FUNDAMENTALS OF COMMUNICATION

CHAPTER I

THE FUNDAMENTALS OF ELECTRONICS

In many fields of applied science it is easy to observe what is happening. It is possible *to see* the action taking place. Of course this is also true of certain electrical phenomena, such as an electric arc discharge in air or in a glass-enclosed gas-filled rectifier tube. But under most conditions, switchboard wiring, radio-set wiring, and other electric circuits *appear* the same whether or not they are carrying an electric current.

It is, however, possible to *visualize* what is happening in electric circuits even if the current and voltage cannot be seen. The practical electrical worker and the student of electricity alike have gone far in the mastery of electricity when a wire or a circuit carrying electric current *appears* (mentally) to them to be different from a circuit that is not energized. Thus, in beginning a study of electrical theory, the student must learn to form mental pictures of what is occurring. The word *theory* has a stigma attached to it in the minds of many, but theories are merely word expressions of mental pictures, at least in one sense.

Remember that "there is nothing as practical as a good sound theory," and let the mind form its mental pictures instead of trying to remember word laws and algebraic formulas. But, be sure that the pictures drawn are the correct ones, at least as electricity is known today. Also, if the following few pages should seem to be involved, keep in mind that the explanation of *modern communication is based on electronics.*

The Nature of Matter.—It is often erroneously thought that a battery or an electric generator actually *generates* or *makes* electricity. This is definitely not correct from the present viewpoint.

It is more correct to say that electricity makes up all matter. At least electric charges are found in all the matter comprising the physical objects of which this universe is composed. Before studying the electric current, it is, therefore, advisable to study the nature of matter.

Suppose that a piece of some solid object—a piece of copper wire for instance—is examined beneath a very powerful microscope. The image seen will be as shown in Fig. 1. The copper appears to be composed of grains held together in some mysterious manner.[1] These are **crystals** of copper, and under certain conditions a single crystal may be made quite large.

Fig. 1.—A photograph of copper, taken by microscopic means. This is called a photomicrograph and shows the grain or crystal structure of the copper (*Courtesy of Profs. S. H. Graf and C. E. Thomas, Oregon State College*).

Now as is well known, ordinary light will not pass through a metal, but a beam of X rays will penetrate thin sheets quite easily. Of course, X rays are fundamentally of the same nature as visible light. They are much shorter in wave length, however, contain much more energy, and thus have greater penetrating power. Hence if a beam of X rays is directed on one of these single

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[1] It would be very interesting at this point to study further the effect of mechanical treatment and heat-treatment on the grain size and other physical characteristics of a metal such as copper. Also, it is fascinating to reflect on the atomic forces that hold these particles together, for the grains or particles are not "glued" together in the usual sense of the word.

crystals of a metal, the X rays will pass through and emerge from the other side. By photographically studying the directions from which the rays emerge, it can be ascertained that the crystal of the metal is composed of rows upon rows of smaller particles arranged in the form of a lattice structure. An example of this is shown in Fig. 2. Each of these little submicroscopic particles is thought to be an **atom** of the metal.

To summarize: A metal such as copper can be examined with a microscope and shown to consist of *crystals* of copper. When these are further examined by X rays, the individual crystals are found to be composed of rows upon rows of particles or *atoms*. These atoms will now be further studied.

The Atom.—The reader is asked to study carefully the following statements regarding the structure of the atom. These paragraphs must be referred to time and again throughout the book.

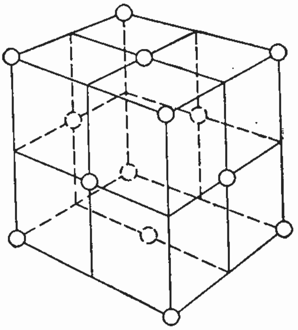

FIG. 2.—Most of the common metals consist of crystals composed of atoms arranged somewhat as shown. The small circles represent the atoms.

In the crystals of the solid matter (copper) considered in the preceding section, the atoms were considered to be held rigidly fixed in definite positions[1] within the copper crystal. In a gas, on the other hand, the atoms are not fixed in position but move about freely within the containing enclosure. But whether in a solid or a gas, the individual atoms themselves are made up in a definite way which will now be considered.[2]

An atom is pictured as being composed of *positive* charges of electricity called **protons** and *negative* charges of electricity called **electrons.** The electric charges on the proton and the electron, although differing in sign (one positive, the other negative), are of the same strength, or magnitude. Furthermore, the normal atom

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[1] The atoms are thought to vibrate slightly in these fixed positions. The greater the temperature of a solid, the larger will be these atomic vibrations.

[2] Of course modern physics has gone far beyond the simple picture given here of the internal atomic structure. The explanations here given are, however, sufficient for explaining most of the electrical phenomena encountered in the practical fields.

contains equal numbers of positive protons and negative electrons. As a result, a whole, or normal, atom is electrically neutral; that is, *the normal atom contains no resultant electric charge.*

The positive protons and negative electrons constituting the atom are not at all uniformly distributed within the space occupied by the atom. The atom consists of a central **nucleus** composed of all the protons and some of the electrons. The nucleus accordingly has a resultant *positive* charge. About this nucleus the remaining electrons (necessary to neutralize the excess positive charge of the nucleus and thus make the whole atom neutral) may be pictured as revolving in orbits.

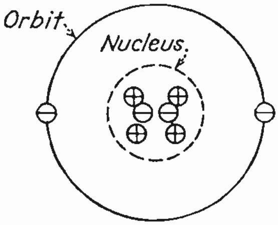

Fig. 3.—Bohr model of a helium atom. The central nucleus consists of four protons and two electrons. Two electrons are in orbits. The electrons are the small circles with the negative sign within and the protons are the circles enclosing the positive sign.

Although the protons and electrons have equal electric charges, the *proton is massive but the electron under ordinary conditions is not massive.* The nucleus contains all the protons and is, accordingly, dense and massive. The rest of the atom, containing the electrons moving in orbits, is relatively light, consisting largely of space. A typical gas atom is pictured in Fig. 3.

To summarize: An atom contains a nucleus considered for practical purposes to be composed of massive positive protons and light negative electrons. Because there are fewer electrons than protons in the nucleus, it has a resultant positive charge. The remaining electrons revolve in orbits about the nucleus, neutralizing the excess positive nuclear charge and thus making the atom as a whole neutral.

Laws of Electric Charges.—The reader will recall that he has heard much about electrons and very little about protons in his electrical studies to date. There is a very good reason for this. In general, protons are not detached from the atomic nucleus and they are accordingly less familiar, but the electrons in the outer orbits are readily separated from the rest of the atom, and their effects may be more easily observed. Although these electrons and protons are very minute electric charges, they obey the fundamental laws governing charges of electricity which will now be considered.

One of the fundamental laws of electricity is that like charges repel and that unlike charges attract. Thus if two bodies are suspended by strings as in Fig. 4 they will not be deflected as in Fig. 4*a* if there is no resultant charge on them. But they will swing apart as in Fig. 4*b* if the charges are alike (either both positive or both negative), and they will swing together as in Fig. 4*c* if the charges are unlike (that is, one positive and one negative). These are laws which have long been known and which can be readily demonstrated by almost anyone with such simple equipment as an ordinary pocket comb, bits of paper, and a silk cloth.

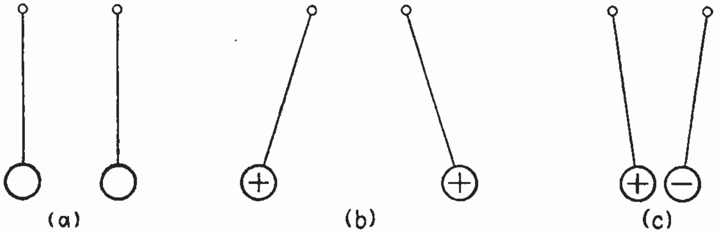

FIG. 4.—No forces will exist between two neutral bodies as in *a*. Bodies with like charges will be repelled as in *b*, and bodies with unlike charges will be attracted as in *c*.

Electrons are minute electric charges. They may be thought of as discrete negatively charged particles of matter. Since they are *charged* particles, they will follow the laws outlined in the preceding paragraph and illustrated by Fig. 4. This law, stated in other words and applying more specifically to electrons, is as follows: *Electrons themselves, being like charges, will repel each other; also, if an electron (negative charge) is free to move, it will move toward a place that is positively charged; that is, a place where there is a deficiency of electrons.*[1]

To summarize: Two neutral bodies exert no resultant force on each other and may be placed side by side as in Fig. 4*a* with no apparent effect. However, two like charges repel as in Fig. 4*b*, and two unlike charges attract as in Fig. 4*c*. From the usual viewpoint, *a body is positively charged if it has too few electrons and negatively charged if it has too many.* In general, explanations of electrical phenomena are based on electrons and their effects because (ac-

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[1] To say that a body has a deficiency of electrons is equivalent to saying that it has too many protons. But as previously mentioned, the proton is little used in explaining ordinary electrical effects.

cording to the accepted viewpoint) electrons are readily (as compared with the proton) separated from an atom and their actions are more readily observed.

The Electric Current.—Electrons are not only readily separated from an atom but as a matter of fact there is much evidence to show that *in a metal such as a copper conductor* **free electrons** *appear to exist.* That is, experiments show either that free electrons unattached to atoms must exist, or that there is such a free interchange of the orbital electrons between adjacent atoms that the result is the same; that is, at any instant the copper seems "full of free electrons." Thus, let it be assumed that there are electrons freely drifting about within a metal and that the circuit of Fig. 5 is to be considered.

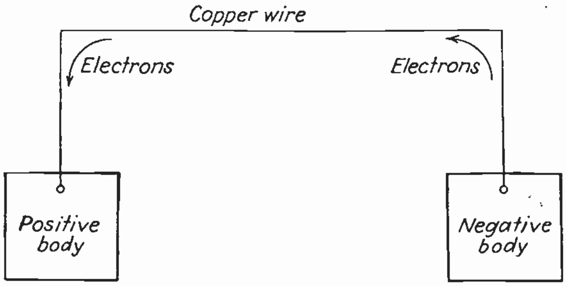

FIG. 5.—If a positive body (one containing too few electrons) is connected with a piece of copper wire to a negative body (one containing too many electrons), free electrons progressively flow from segment to segment of the copper wire until the electron deficiency of the positive body is supplied. At the end of the action both bodies are neutral and the electron current ceases to flow.

In this figure is shown a positive body (having too few negative electrons) and a negative body (having too many negative electrons). These two bodies of unlike charges are connected with a copper wire which was shown to contain free negative electrons.

At the instant the two bodies are connected, the positive body will attract the free negative electrons from the end of the copper and will pull some of them out. This will provide the positive body with the electrons it needs, and this body will become neutral (have equal positive and negative charges). Pulling free negative electrons from the end of the copper wire will however tend to leave the next segment of wire positive. This segment or portion of the copper wire will, therefore, pull free negative electrons from the next segment, and so on until the distant end is reached. Here the wire will take electrons from the negative body which has an excess of them. This action will, therefore, maintain the copper wire neutral and will also cause the negatively charged body to become neutral, that is, to contain equal numbers of positive and negative charges.

A review of the action accompanying Fig. 5 shows this: At the instant the copper wire was attached between the two unequally charged bodies (and for a brief period thereafter) *electrons moved within the copper wire.* The direction of their motion was *from the negative body to the positive body.* Also, it was seen that the motion of electrons or electronic flow soon ceased, because when the charges on the bodies were equalized, there was no attractive force left to cause the electrons to flow.[1] The *flow of electrons* just considered was an **electric current.** Thus, in quite simple terms, *a current of electricity may be defined as a progressive flow of electrons.*

To summarize: Electrons, constituting an electric current, flow for an instant when a conductor such as a copper wire is connected between a positively and a negatively charged body. This electric current soon becomes zero, however, because the flow of current neutralizes the differences in charges, and thus there is no attractive force left to cause electrons to move. The current flowing through the connecting wire consisted of the free electrons which are normally within a metal such as copper.

Potential Difference.—In the preceding section it was shown that an instantaneous electric current consisting of electrons flowed from one electrically charged body to another if their electric charges were different. This action was attributed to the attractive force the positively charged body had for the negative electrons. Another way of explaining the current flow would have been to say that the excess electrons on the negatively charged body repelled the free electrons within the copper wire. Or, perhaps it should be said that a combination of the two actions exist.

Any conflicting theories being neglected for the moment, one fact remains: *A current of electricity flowed along the wire for an*

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[1] Perhaps it should be added that at all times (that is, before as well as after neutralization) there is a *random* motion of electrons within the copper. There is, however, no *resultant* motion of negative electric charges (electrons) toward the positive terminal *after* neutralization of the two bodies is complete. It is the *resultant* motion of electrons which is considered to constitute a *flow* of current in a given direction.

instant, and then the current died out to zero. Thus, there was an *effect* (the current flow) so there must have been a *cause*. The cause was the difference in the charges on the two bodies.

Instead of always speaking of *differences in electric charges*, it is quite common to speak of **difference of potential.** Now the word *potential* has several meanings, and perhaps the best one to use here is *inherent ability*. Thus, when two bodies have unequal electric charges as just considered, they have a difference of potential; that is, they have the *inherent electrical ability* to cause a current to flow through the copper wire, because this was precisely what happened when the copper wire was connected between the two bodies. In simple terms, *a potential difference may be defined as the electrical condition, or force, that causes (or tends to cause) an electric current to flow.*

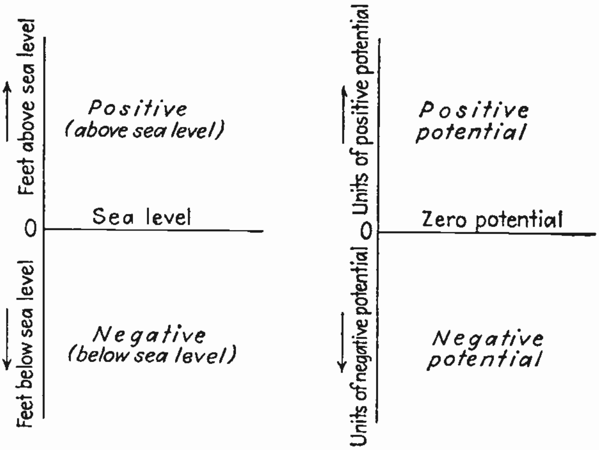

FIG. 6.—Showing the similarity between elevations above and below sea level and electrical potentials above and below some zero potential.

In measuring altitudes, sea level is used as a reference, and certain localities are referred to as being above or below sea level. Similarly, in electrical work, a body (such as the earth) may be taken as a reference and electrically charged bodies may be specified as so many units above or below this zero potential.[1] These relations are shown in Fig. 6.

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[1] Of course the potential of a body may be expressed with respect to any object, etc., but this does not at all falsify the above statement. The unit of measure is the volt, but since it has not been defined, it is not used here.

To summarize: When two bodies have unequal charges, a difference of electrical potential exists between them. This potential difference causes or tends to cause a current to flow. Thus, when the two bodies of Fig. 5 were connected, an instantaneous current flowed because of the existence of the potential difference caused by the unequal charges. The current flowed until the two charged bodies were neutralized and the potential difference disappeared.

Electromotive Force.—In the preceding section it was shown that an electric current would flow through a conductor such as a copper wire if the two ends of the wire were maintained at a difference of potential. In Fig. 5 and the accompanying discussion the potential difference was provided by two unequally charged bodies, but as soon as the unequal charges were neutralized, the current ceased to flow. Of course this was because after neutralization no difference of potential was present to force the free electrons along the wire.

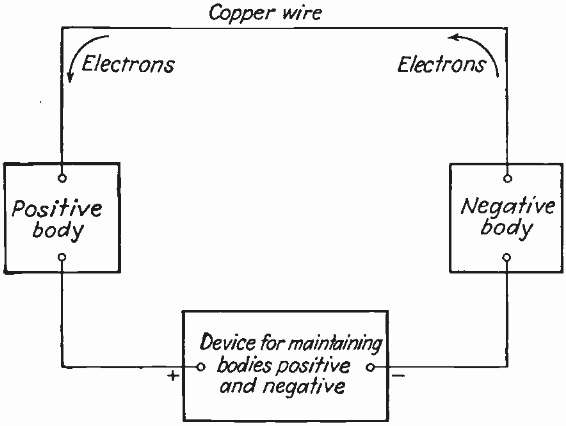

FIG. 7.—When a device such as indicated in this figure continuously maintains a difference of potential between two bodies (one positive and one negative) a continuous electric current will flow through the wire circuit connecting the two bodies.

It at once follows that if the circuit of Fig. 5 were extended as in Fig. 7 a continuous current would flow. Here a third element has been connected to the positive (+) and negative (−) bodies, and this new device *maintains* the bodies at unequal charges, and hence a *continuous* difference of potential exists between them. By an extension of the action previously explained, it is readily seen that this *continuous potential* will cause a *continuous current* to flow through the copper wire.

When a device such as shown in Fig. 7 maintains one body positive and another negative, it is good practice to state that the two bodies are maintained at a *difference of potential* because of the **electromotive force** that the device generates. These words electromotive force are often abbreviated **emf.**

To summarize: A current of electricity (composed of electrons) flows through a conductor when the two ends of the conductor are maintained at a difference of electrical potential, such as when a wire is connected between two unequally charged bodies. This current will tend to equalize the unequal charges, after which the current ceases to flow. If a device that continuously generates an *electromotive force* is connected between the two bodies, so that they are continuously maintained at a *difference of potential*, then a continuous current will flow.

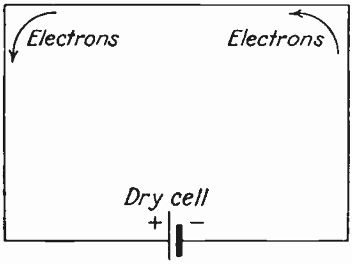

FIG. 8.—Here the positive plate or electrode of the dry cell represents the positive body of Fig. 7, and the negative electrode represents the negative body. The applied *electromotive force* which maintains the electrodes at a *difference of potential* and which causes the current flow is generated chemically within the cell.

Generation of Electromotive Force.—*An electromotive force may be defined as an electric force generated by a device (such as an electric dry cell) that causes a difference of potential to exist between the terminals of the device.* This established difference of potential will cause a current to flow through a wire or other electric circuit connected across the terminals. It is not necessary for the present to be concerned with the exact nature of the force (it is rather hard to "visualize" a force); instead, the student should concentrate on the statement in italics just given.

Further to explain these statements, suppose that Fig. 7 is developed into Fig. 8. Here the positive body of Fig. 7 is replaced by the positive plate or electrode marked +; also the negative body is replaced by the negative electrode marked −. These two electrodes are continuously maintained at a *difference of potential* by the *electromotive force* that is generated chemically within the dry cell. The long thin line and the short heavy line drawn parallel to each other as shown represent electric cells and batteries.

An electromotive force can be generated by other means, such as rotating machines called **electric generators.** In any event, however, the electromotive force maintains the terminals of the machine at a difference of potential, and by virtue of this a current flows through the connecting wire when the external circuit is closed.

To summarize: The electromotive force that is produced by electric generators and chemical cells is the electric force which in the final analysis forces an electric current through the externally connected circuit. Of course it does this by establishing a difference of potential between its terminals. However, the action is usually simplified by merely stating that the generated electromotive force[1] forces the current through the circuit.

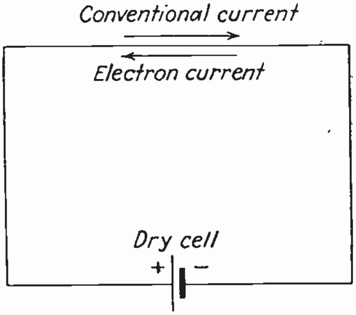

FIG. 9.—The purely fictitious "conventional current" is assumed to flow from positive to negative in the external circuit. Actually, a current of electricity consists of an electron flow in the opposite direction.

Direction of Current Flow.—In the preceding pages it has been repeatedly stated that an electric current consists of a flow of electrons. This is the present accepted view in electrical work. In Figs. 7 and 8 it is shown that the electrons flow in the connecting wire (representing the external circuit) from the *negative* to the *positive* terminal. This must be true if the current consists of *negative* electrons, because these negative particles of electricity would be attracted by the positive electrode (or terminal) and repelled by the negative electrode in accordance with the fundamental law given on page 5.

Now those students who have had some electrical experience may question these statements, because it is generally assumed that the electric current flows (in the external circuit) from the positive electrode (or terminal) of the electric cell or electric generator to the negative electrode.

Here the reader is confronted with one of the few inconsistent laws of electricity. The generally accepted viewpoints will be

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[1] Differences of potential resulting from an electromotive force are measured in volts. Hence in practical work it is ordinarily stated that the generated voltage forces the current through the circuit.

stated. (1) The *conventional electric current* is considered to flow (in the external circuit) from the positive to the negative terminal. (2) The *electron current* is thought to flow in the opposite direction. In this connection, it should be remembered that the *conventional* direction of current flow was assumed to be from positive to negative in the external circuit *before* the electron theory was known.

To summarize: The conventional direction of current flow is assumed to be from positive to negative in the external circuit. This current really consists of electrons flowing in the opposite direction. Thus in practice, direction of *current flow* means from positive to negative, but direction of *electron-current flow* means the opposite direction. In the preceding paragraphs the words "current flow in the *external* circuit" were extensively used. An examination of Fig. 8 will disclose that if the electron current flows from negative to positive in the circuit *external* to the cell, the electrons must move in the opposite direction *within* the cell.

The Law of Current Flow.—As the student masters electrical theory, he may be pleasantly surprised to find that the basic laws are not really new after all. They are merely the application to electric circuits of the fundamental laws of nature. Thus, it is no surprise to find that the familiar relation that *the result produced is directly proportional to the magnitude of the effort and inversely proportional to the magnitude of the opposing force* applies to electric circuits as well as to other fields.

This relation can be written in the form of equations as follows:

$$\text{Result} = \frac{\text{effort}}{\text{opposition}}, \tag{1}$$

or electrically speaking

$$\text{Current} = \frac{\text{electromotive force}}{\text{resistance}}. \tag{2}$$

These relations are known as Ohm's law, so named after an early investigator. They merely state what has been stressed in the preceding pages, that the applied electromotive force forces the current through the circuit. One new term is introduced, that of circuit **resistance.**

The Nature of Electric Resistance.—Referring back to Figs. 7 and 8 and the accompanying theory, it is recalled that the current flowing through the wire was shown to consist of *free electrons* that were already within the wire. But from this it should not be inferred that no opposition is offered to the flow of these electrons through the wire. Quite the opposite is true, because as the electrons flow through the wire, they encounter opposing internal atomic forces, and work must be done to push the electrons along their path. The fact that a wire carrying a current gets hot is ample evidence of the existence of internal forces or resistance opposing the flow of the current.

The electric resistance of a wire depends on many factors; certain of these are as follows:

1. Chemical composition of the wire.
2. Cross-sectional area and length of the wire.
3. Treatment during manufacture, such as heating and cooling, and method of drawing the metal into the wire.
4. Temperature of the wire.

These and other points will be considered elsewhere in this book.

To summarize: Although a current of electricity is thought to be a flow of free electrons along a wire under the influence of an externally applied electromotive force, this flow encounters internal opposing forces that cause the wire to offer an electric resistance to the current flow.

SUMMARY

A solid metal conductor such as copper appears in a microscope to be made of small grains or crystals.

Examination of such crystals by X rays shows them to be composed of a lattice structure of atoms.

Atoms consist of a central nucleus surrounded by revolving negative electrons. The central nucleus contains both negative electrons and positive protons.

Electrons are small negative charges. They are the basis of electrical explanations.

A body is positively charged if it contains too few electrons.

A body is negatively charged if it contains too many electrons.

A current of electricity is a progressive flow of electrons.

A current of electricity flows because a difference of potential forces the electrons through a conductor.

Differences of potential are caused by generated electromotive forces and IR drops.

An electron current flows from $-$ to $+$, but the conventional direction of current flow is from $+$ to $-$ in the external circuit.

There are three forms of the fundamental law governing the flow of direct current. This is known as Ohm's law, and the forms are $I = E/R$, $R = E/I$, and $E = IR$.

REVIEW QUESTIONS[1]

1. Why is it incorrect to say that a generator or battery actually generates an electric current?

2. What reasons are there for believing that a crystal of copper is composed of atoms, and how are they arranged?

3. Which is the more massive, the negative electron or the positive proton?

4. Are the electric charges on electrons and protons of the same magnitude? Of the same sign?

5. Enumerate the fundamental laws of electric charges.

6. What evidence is there to show that free electrons exist in a copper wire?

7. Do free electrons exist in metals other than copper?

8. Suppose you were told that in a wire there are *two* currents of electricity, one of protons moving in one direction and the other of electrons moving in the opposite direction. Would the statement be correct? Why?

9. Explain the relations among potential difference, electromotive force, and voltage.

10. What direction does the electron current have in a wire connected to a dry cell?

11. Referring to Question 10, discuss the relation of your answer to practical electrical work.

12. Is the theory of Ohm's law applicable only to electric circuits? Name an application in machines. Name one in hydraulics.

13. What external evidence is there for the fact that a wire offers resistance to current flow?

14. What factors determine the resistance of a conductor?

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[1] Obviously, there would be little gained by asking questions that could be answered merely by referring to the preceding summary. For this reason the questions asked throughout this book will be based on the main body of each chapter rather than on the summary.

CHAPTER II

DIRECT CURRENT

The previous chapter presented the basic facts regarding electrons and the various elementary electrical phenomena that are based on electronic action. It was explained that the commonly observed electrical effects, such as the flow of the electric current, are electronic in nature.

The discussion of the preceding chapter was of necessity somewhat theoretical. Now that the nature of the electric current is understood, however, it is possible to take up the more practical aspects of the electrical fundamentals of communication.

The Volt, Ohm, and Ampere.—Before discussing electric circuits further, and before the numerical solutions of simple problems can be given, it is necessary to introduce the units of measure. These are as follows:

1. The magnitude of the electric force or pressure is measured in **volts** and represented by E.
2. The magnitude of the opposition or opposing force a circuit offers is measured in **ohms** and represented by R.
3. The magnitude of the current which the voltage forces through the resistance is measured in **amperes** and represented by I.

Each of these terms honors early investigators in the field of electricity.

The numerical relations among these units is of much importance. Thus, letting each term of Eq. (2) be unity, it can be stated that a continuously applied voltage of *one volt* will force a current of *one ampere* through a resistance of *one ohm.*

Mathematically, there are three forms of Ohm's law as follows:

1. $I = E/R$, which states that the current in amperes which flows is directly proportional to the voltage in volts applied to the circuit and is inversely proportional to the resistance in ohms of the circuit.

2. $R = E/I$, which states that the resistance in ohms which a circuit offers is directly proportional to the applied voltage in volts and inversely proportional to the resulting current flow in amperes.

This form of the relation is obtained from (1) by multiplying each side of the equation by R/I and then canceling. Thus, $I = E/R$, or $RI/I = ER/RI$. Then canceling, $R = E/I$.

3. $E = IR$, which states that voltage in volts is the product of current in amperes and resistance in ohms.

This form of the relation is obtained from (2) by multiplying each side of the equation by I and then canceling. Thus, $R = E/I$, or $IR = IE/I$. Then canceling, $E = IR$.

To summarize: The units of volts, ohms, and amperes are used to measure voltage, resistance, and current in electric circuits. The magnitudes of these are so chosen that in the various forms of Ohm's law each unit is equal to unity (1.0) in the practical system of electric units[1] used most widely.

Other Units—Voltage, Resistance, and Current.—As previously stated, voltage is measured in *volts*, resistance is measured in *ohms*, and current is measured in *amperes*. These are the basic units, but multiples and subdivisions of these units are commonly used. These are best illustrated by the following tabulation:

Voltage units
  Kilovolt = 1000 volts
  Millivolt = $\frac{1}{1000}$ volt
  Microvolt = 1/1,000,000 volt
Resistance units
  Megohm = 1,000,000 ohms
  Kilohm = 1000 ohms
Current units
  Milliampere = $\frac{1}{1000}$ ampere
  Microampere = 1/1,000,000 ampere

The reader may be surprised to find that no distinguishing symbols such as ma for milliampere or mohm for kilohm are listed. This is because no such symbols have been universally

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[1] For the sake of standardization, exact definitions of the volt, ohm, and ampere are specified. These exact definitions are of such a nature as to be largely of interest to the physicist and the highly theoretical electrical worker. As a matter of curiosity it may interest the student to know that a current of 1 ampere consists of a procession of some 6,290,000,000,000,000,000 electrons passing a given point per second.

accepted, and it is, therefore, advisable to use the entire word rather than some more or less arbitrary abbreviation.

Simple Series Circuits.—A simple series circuit is shown in Fig. 10, consisting of a dry cell producing by chemical means an electromotive force of 1.5 volts and having an internal resistance[1] of 0.1 ohm connected in series with a resistor having a resistance of 9.9 ohms. The symbol shown under the word *resistor* of Fig. 10 is the proper method of representing a resistor.

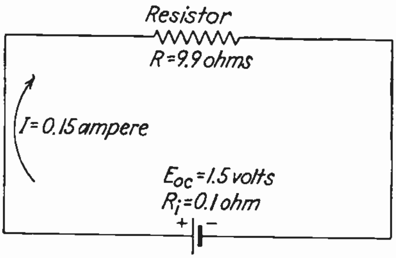

FIG. 10.—A simple series circuit. The internal resistance R_i of the dry cell also acts to limit the current flow; that is, the dry cell electromotive force must force the current through the cell itself as well as through the external circuit.

To solve such a circuit for the current that will flow, the following steps are taken:

Step 1. Add all the resistances to obtain the total resistance R_t of the circuit. $R_t = 0.1 + 9.9 = 10$ ohms.

Step 2. Use Ohm's law to find the current.

$$I = \frac{E_{oc}}{R_t} = \frac{1.5 \text{ volts}}{10 \text{ ohms}} = 0.15 \text{ ampere.}$$

Thus it is found that the cell voltage of 1.5 volts will cause a current of 0.15 ampere to flow; this current will be in the direction indicated, from the positive (+) terminal of the dry cell around through the resistor to the negative (−) terminal and then on through the cell. Note in particular that the same current flows through all parts of the series circuit even through the battery itself. This is the first of the fundamental laws of the *series* circuit: *The current in all parts of a series circuit is the same.*

Mention is again made of the fact that the current flows from the positive to the negative terminal in the external circuit. This is, of

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[1] The instructor may wonder why the internal resistance of the dry cell is not neglected as is so often the case. It is believed that it is a grave mistake to do so. In the general case, the internal resistance (impedance) of the source cannot be neglected. Only in the case of constant-voltage systems (such as are closely simulated by power systems) is it correct to neglect internal losses. Any generator or cell must have an internal resistance (impedance), and therefore, it is advisable to begin considering it at once.

course, the *conventional* current. The electrons themselves are flowing in the opposite direction.

The second fundamental law of the series circuit will now be considered. For this purpose, Fig. 10 has been redrawn, and in Fig. 11 an ammeter A and voltmeters V have been added to the circuit. The manner in which these operate need not be considered at present. Suffice it to say that the ammeter measures the current flow in amperes, the voltmeters measure voltage (or potential difference) in volts, and that these instruments may be connected into the circuit as shown without disturbing the circuits in any way. Also, a switch S has been added.

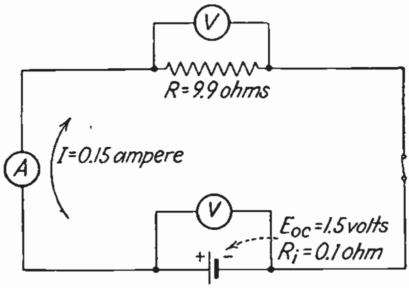

FIG. 11.—This represents the circuit of Fig. 10 redrawn to include an ammeter A and voltmeters V.

The method of solving Fig. 11 is the same as for Fig. 10, because it is assumed that the presence of the ammeter and the voltmeters does not disturb the circuit; that is, Figs. 10 and 11 are electrically identical.

The ammeter has been placed in *series* with the rest of the circuit, because this instrument measures the current in amperes flowing *in* the circuit. Only one ammeter is necessary because of the fact that the current in all parts of a series circuit is the same.

The current relations having been disposed of, let attention now be directed to the voltage relations. *Before* the switch S is closed, only one of the three instruments will show a reading; this one is the voltmeter across the dry cell. It will read 1.5 volts, which is the **open-circuit voltage**[1] of the cell, usually designated by E_{oc}.

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[1] So-called, because the switch S is open, and the cell is not forcing a current through the circuit. Cell open-circuit voltage is related to potential difference and electromotive force as follows: The chemically generated electromotive force within the cell causes the two plates and hence the two cell terminals to be at a difference of potential as indicated by the plus (+) and minus (−) signs. Of course the voltmeter actually measures this potential difference, but it is good practice merely to say that the voltmeter measures the cell voltage.

Now suppose that the switch S of Fig. 11 is closed so that the cell forces a current through the circuit. The reading of the voltmeter connected across the cell will *drop* slightly the instant the switch is closed for the following reason: Voltage is the electrical pressure that forces current through the resistance of a circuit. A certain amount of the cell voltage is required at each resistance. On page 16, the third statement is that $E = IR$; that is, voltage in volts is the product of current in amperes and resistance in ohms. Thus, a **voltage drop** of $0.15 \times 0.1 = 0.015$ volt will occur within the cell itself. Of course this voltage value is *opposite* to the open-circuit cell voltage of 1.5 volts (because if it were not opposite, the voltmeter reading would increase, giving more voltage to force more current, etc.). Hence, at the instant the switch is closed and current begins to flow, the reading of the voltmeter becomes $E_{cc} = 1.5 - 0.015 = 1.485$ volts. This is the **closed-circuit voltage** E_{cc} as distinguished from the open-circuit voltage E_{oc}.

Let the reading of the voltmeter connected across the resistor be now investigated. After the switch is closed, this voltmeter will read $E_{cc} = IR = 0.15 \times 9.9 = 1.485$ volts. The second law of the *series* circuit can now be stated as follows: *The sum of the voltage drops around a series circuit equals the total impressed voltage.* Thus, the sum of the voltage drops (or losses) across the resistor and across the internal resistance of the cell is $1.485 + 0.15 = 1.5$ volts, and this is the total voltage the cell impresses on the circuit.

To summarize: There are two fundamental laws that apply to the *series* circuit. (1) *The current in all parts of a series circuit is the same.* (2) *The sum of the voltage drops (or losses) around a series circuit equals the total impressed voltage.* For simplicity, only *one* source of voltage was considered, the more general case being treated on page 31.

Simple Parallel Circuits.—A simple parallel circuit is shown in Fig. 12, consisting of two resistors of 15 ohms and 25 ohms connected across a *constant* voltage[1] of 6 volts. A voltmeter and ammeters are included for measuring the impressed voltage and the currents in each circuit branch. As in the preceding section, it is considered that ammeters and voltmeters are ideal instruments and that their presence does not influence the circuit at all.

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[1] If a dry cell were used instead of a constant voltage, and if the internal resistance of the cell were not neglected (and it is often not negligible), then the problem would become one of series-parallel circuits instead of a simple parallel circuit which it is desired to study.

To solve such a circuit for the current that will flow from the 6-volt source, the following steps are taken:

Step 1. Find the current in each of the parallel branches by Ohm's law.

$$I_1 = \frac{E}{R_1} = \frac{6}{15} = 0.4 \text{ ampere}$$

$$I_2 = \frac{E}{R_2} = \frac{6}{25} = 0.24 \text{ ampere.}$$

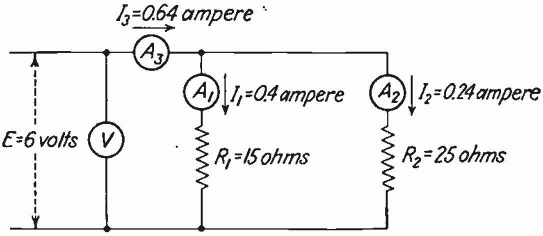

FIG. 12.—A simple parallel circuit composed of a 15-ohm resistor and a 25-ohm resistor in parallel.

Step 2. Add the branch currents to find the total line current flowing through the source.

$$I_t = I_1 + I_2 = 0.4 + 0.24 = 0.64 \text{ ampere.}$$

Embodied in these two steps are two fundamental laws of parallel circuits. The first of these is that *the voltage across each branch of a parallel circuit is the same.* This is evident from Fig. 12, because if the voltmeter reads 6 volts where it is shown, it will also read 6 volts if it is slid along to the right (remember that the ammeter theoretically has no influence). The use of this law is illustrated by Step 1.

In Step 2, the currents taken by each of the parallel branches were added to give the total line current from the source. This illustrates the second law, that *the sum of the currents flowing up to a point equals the sum of the currents flowing away.*

To summarize: In considering parallel circuits there are two simple laws applying: (1) *the voltage across each branch of a parallel circuit is the same, and* (2) *the sum of the currents flowing up to a point equals the sum of the currents flowing away.* The student should thoroughly master these simple laws of series and parallel circuits. Note that these circuits are in a sense opposites. *In* *series circuits the current is the same in all parts, and the separate voltage drops are added to give the total impressed voltage. In parallel circuits, however, the voltage is the same in all parts, and the separate currents are added to give the total line current.*

Division of Current in Parallel Circuits.—It is seen from the example just given that 0.4 ampere (400 milliamperes) of the total current of 0.64 ampere (640 milliamperes) flowed through the 15-ohm resistor. Thus, $^{400}/_{640}$, or $^5/_8$, of the total current flowed through the small resistor, and $^{240}/_{640}$, or $^3/_8$, of the total current flowed through the large resistor.

Now 15 ohms is $^{15}/_{40}$, or $^3/_8$, of the sum of the two resistances, and 25 ohms is $^{25}/_{40}$, or $^5/_8$, of the total parallel resistance. It can be stated, therefore, that *a current divides in a parallel circuit in inverse proportion to the ratio of the resistance of the path to the sum of the resistances of the parallel paths.* A simple proof of this is as follows: The voltage across two parallel circuits must be the same. Voltage equals IR; hence for two parallel paths R_1 and R_2

$$I_1R_1 = I_2R_2 \qquad \text{and} \qquad \frac{I_1}{I_2} = \frac{R_2}{R_1}.$$

Now

$$I_1 = \frac{I_2R_2}{R_1} \qquad \text{and} \qquad I_t = I_1 + I_2.$$

Hence

$$I_t = \frac{I_2R_2}{R_1} + I_2 = \frac{I_2(R_1 + R_2)}{R_1},$$

and

$$I_2 = \frac{I_tR_1}{R_1 + R_2}.$$

Equivalent Resistance.—In Step 2 of the solution accompanying the parallel circuit of Fig. 12 it is shown that the total current supplied by the 6-volt source is the sum of the branch currents, that is, $I_t = I_1 + I_2 = 0.4 + 0.24 = 0.64$ ampere. As was shown on page 16, voltage divided by current gives resistance. Suppose that the voltage in volts of Fig. 12 is divided by the total current in amperes supplied, what will be the significance of this resistance value?

Performing this calculation gives a value $R_e = 6/0.64 = 9.37$ ohms, and this is called the **equivalent resistance** of the 15-ohm resistor and the 25-ohm resistor in parallel. The subscript e has been used to designate "equivalent" in the calculation just made.

Thus it is shown that these two resistors in parallel take a total of 0.64 ampere from the 6-volt source. So does a single 9.37-ohm resistor. Hence, the 9.37-ohm resistor is *equivalent* to the other two in parallel. Note that *the equivalent resistance of two (or more) resistors in parallel is less than that of the smaller one alone.*

These relations can be expressed as follows:

$$I_t = I_1 + I_2 = \frac{E}{R_1} + \frac{E}{R_2},$$

and since the total current I_t equals the voltage divided by the equivalent resistance R_e as just explained,

$$I_t = \frac{E}{R_e} = \frac{E}{R_1} + \frac{E}{R_2}.$$

Dividing through by E gives

$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2}, \tag{3}$$

and if more than two resistors are in parallel, this equation becomes

$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \cdots. \tag{4}$$

By the use of simple algebra, Eq. (3) can be written

$$R_e = \frac{1}{(1/R_1) + (1/R_2)} \qquad \text{or} \qquad R_e = \frac{R_1 R_2}{R_1 + R_2}. \tag{5}$$

This last form is particularly useful and easily handled.

To summarize: When two or more resistors are connected in parallel across a source of voltage, they are *electrically equivalent* to a single resistor of *equivalent resistance* connected across that source. The equivalent resistance of a parallel combination is *less* than the resistance of any of the individual resistors.

Additional Series-circuit Solutions.—The solutions of series and parallel circuits are so very important that additional illustrations will be given. Thus in Fig. 13 is shown a series circuit composed of three resistors connected across a 24-volt lead-acid storage battery such as would be used as the common battery in a large telephone central office. Observe that no internal resistance is specified for this battery, although the internal resistance was considered for the dry cell of Figs. 10 and 11. In the case of the large central-office battery, the internal resistance is very low, and particularly when large resistance values are used such as in Fig. 13, the internal resistance is negligible and is therefore not considered in this problem.

Now with reference to Fig. 13, it was previously shown on page 17 that the *same current flows through each series element* and this gives the first fundamental law for the series circuit, that *the current in all parts of a series circuit is the same.* This current will be measured by the ammeter A connected in the circuit. The voltmeters V will measure the IR or voltage drop across each individual resistor.

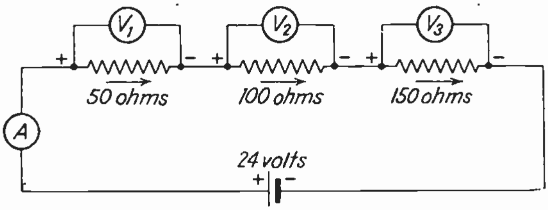

FIG. 13.—The current in all parts of a series circuit is the same. The voltmeters take negligible current, and read the IR or voltage drop across each resistor.

With reference to the typical solution given on page 17, the following steps are taken:

Step 1. Add all the resistances to obtain the total resistance R_t of the circuit.

$$R_t = 50 + 100 + 150 = 300 \text{ ohms.}$$

Step 2. Use Ohm's law to find the current.

$$I = \frac{E}{R_t} = \frac{24}{300} = 0.08 \text{ ampere, or } 80 \text{ milliamperes.}$$

The IR or voltage drop indicated by the various voltmeters connected across the resistors will be $E_1 = IR_1 = 0.08 \times 50 = 4$ volts; $E_2 = 0.08 \times 100 = 8$ volts; and $E_3 = 0.08 \times 150 = 12$ volts. Adding these gives $4 + 8 + 12 = 24$ volts, which is the voltage impressed by the battery.

This last summation again illustrates the second of the fundamental laws of the series circuit. As explained on page 19, this law states that *the sum of the voltage drops around a series circuit equals the total impressed voltage.* In other words, the electromotive force, or battery voltage, is all used up in forcing a current of electrons through the resistance of the circuit.

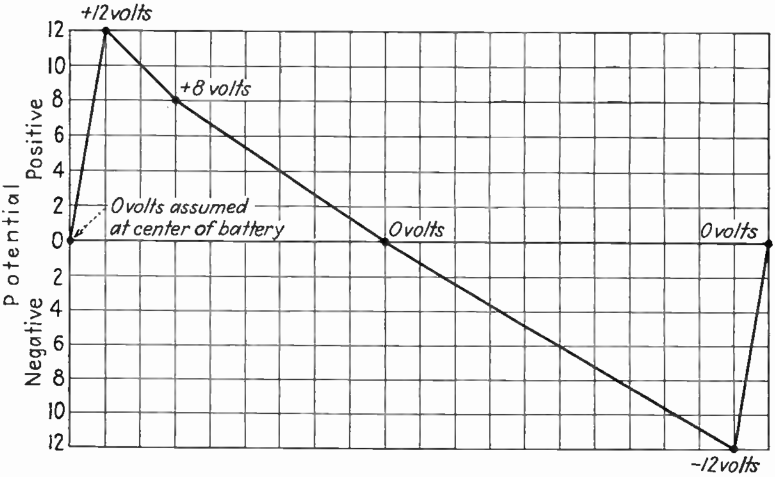

FIG. 14.—A graphical representation of the rise and fall of potential around the circuit of Fig. 13. The center of the battery has been taken as the point of reference. or zero potential. Either the positive terminal. or the negative terminal, or any other point in the circuit could have been taken as the point of reference. (No significance should be attached to the slopes of the lines; they are arbitrary in this figure.)

The portion of the total impressed voltage which is used in forcing the current through each resistor was shown to be equal to the IR drop across that resistor. As Fig. 13 shows, each of these voltage drops has a direction as indicated by the positive (+) and the negative (−) signs. If the impressed voltage of the battery were reversed, the current would of course flow in the opposite direction. This would cause the IR or voltage drops to be reversed, and the signs shown would be interchanged.

Rise and Fall of Potential.—On page 8 under Fig. 6 it was mentioned that if some reference point were taken, then potential rises and falls could be specified. Thus, a point in an electric circuit could be said to be at a potential of +10 volts or −10 volts just as a point on the surface of the earth could be specified as so many feet above or below sea level.

To illustrate this further, suppose that the potential at the middle of the battery of Fig. 13 is *specified* as *zero potential*; then, the positive terminal is +12 volts and the negative terminal −12 volts with respect to this center point. The potential difference or voltage relations in the series circuit of Fig. 13 could be shown as in Fig. 14.

This figure is explained as follows: By starting at the center point of the battery, the potential rises as shown in Fig. 14 to +12 volts. This increase is due to the chemically generated voltage within the first half of the battery cells. When a current flows through a resistor it must be accompanied by a potential drop. This is measured by the first voltmeter across the 50-ohm resistor of Fig. 13 and was shown in the previous section to be 4 volts. Thus at the end of the first resistor the voltage is $12 - 4 = 8$ volts.

Applying this same reasoning to the remaining resistors gives data for the rest of the graph of Fig. 14. The last portion, the rise from −12 to 0 volts, is due again to the chemically generated voltage of the last half of the battery cells.

To summarize: If the rises and falls in potential around a series circuit such as Fig. 13 are investigated, data for plotting a potential distribution graph such as Fig. 14 are obtained. Data for a similar graph could also be obtained by connecting one terminal of a center-zero 0- to 15-volt voltmeter to the center of the battery and observing the voltmeter readings as the free voltmeter terminal is successively touched to the various terminals of the resistors in the circuit.

Additional Parallel-circuit Solution.—As a further illustration of the theory of parallel circuits, the following solution is included. Here again it is good practice to neglect the internal resistance of the battery which is assumed to be of the large type used in telephone central offices; also, the resistors are large, and hence the current taken will be small. Even with a large battery having a low internal resistance, if low values of resistance are used (and as a result a large current will flow), it may be necessary to consider the effect of internal resistance.

The parallel circuit is shown in Fig. 15. As was shown on page 20 there are two fundamental laws applying: (1) *The voltage across each branch of a parallel circuit is the same;* and (2) *the sum of the currents in each branch equals the total line current.* These principles will be applied to solve the circuit as was done on page 20.

Step 1. Find the current in each of the parallel branches by Ohm's law.

$$I_1 = {}^{24}\!/\!_{50} = 0.48 \text{ ampere,}$$

$$I_2 = {}^{24}\!/\!_{100} = 0.24 \text{ ampere,}$$

$$I_3 = {}^{24}\!/\!_{150} = 0.16 \text{ ampere.}$$

Step 2. Add the branch currents to find the total line current flowing through the source.

$$I_t = I_1 + I_2 + I_3 = 0.48 + 0.24 + 0.16 = 0.88 \text{ ampere.}$$

As was shown on page 21, the *equivalent resistance* of the three resistors *in parallel* would be found by dividing the impressed voltage in volts by the total current in amperes taken by the parallel combination. Thus, $R_e = 24/0.88 = 27.3$ ohms. A value of 27.3 ohms if connected across a 24-volt source will take the same current as was taken by the parallel combination of Fig. 15.

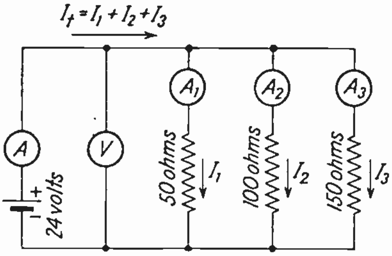

Fig. 15.—In a parallel circuit, the voltage across each branch is the same. The total current from the source is the sum of the branch currents.

According to the explanation given on page 22, the equivalent resistance of the parallel combination just considered could be calculated by Eq. (4). To check this theory,

$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{50} + \frac{1}{100} + \frac{1}{150}$$

$$= 0.02 + 0.01 + 0.0066 = 0.0366.$$

Then, since $1/R_e = 0.0366$, $R_e = 1/0.0366 = 27.3$ ohms, which is the value previously determined for the equivalent resistance of the parallel combination.

Conductance in Direct-current Circuits.—If only simple series and simple parallel circuits were encountered in practice, the equivalent resistance concept might not be so important. Likewise, this section might not be necessary. However, most circuits are not just series or parallel circuits but are combinations called series-parallel circuits. Although these are a little different from the circuits previously considered, they are not difficult to solve if the concept of equivalent resistance and the content of this section are clearly understood.

Now resistance has been defined as the *opposition* a circuit offers to the flow of current. Thus, it is stated that the resistance of a certain resistor is 10 ohms. If this resistor is connected across a constant source of 1.0 volt, the current will be, from Ohm's law, $I = 1.0/10 = 0.1$ ampere. Note that the *opposition* offered was *divided* into the voltage to obtain the current.

Suppose that instead of rating a resistor by the *opposition* it offers it is rated by the *ease* or *ability* it affords to current flow. This leads to a definition of the **conductance** of a circuit, *which is the measure of the assistance the circuit offers to the current flow.* Now assistance is the opposite of resistance, so conductance is the opposite of resistance. Thus, conductance is the *reciprocal* or opposite of resistance, and since resistance is measured in ohms, conductance is measured in **mhos** and is designated by the letter g. That is,

$$g = \frac{1}{R}. \tag{6}$$

Since from Ohm's law, $I = E/R$, it follows from Eq. (6) that $I = E\,1/R = Eg$. That is, *the current in amperes which flows in a circuit is equal to the product of the voltage in volts and the conductance of the circuit in mhos.*

Conductances, rather than resistances, are useful in parallel-circuit problems. On page 20 it was stated that in parallel circuits the separate branch currents are added to obtain the total line current. From this parallel-circuit theory and from the equations just given it follows that

$$I_t = I_1 + I_2 \quad \text{or} \quad \frac{E}{R_e} = \frac{E}{R_1} + \frac{E}{R_2} \quad \text{and that} \quad Eg_e = Eg_1 + Eg_2.$$

That is, the total current as given by Eg_e must equal the sum of Eg_1 and Eg_2, where E is the impressed voltage common to all and $g_e = 1/R_e$, $g_1 = 1/R_1$, and $g_2 = 1/R_2$.

Since the voltage is common to each of these terms (a parallel circuit is under consideration), it may be canceled from each term, leaving the expression

$$g_e = g_1 + g_2. \tag{7}$$

That is, *the equivalent conductance (which is the reciprocal of the equivalent resistance) of a parallel circuit is the sum of the conductances of each branch.* For a circuit consisting of more than two branches, additional conductance terms are added to Eq. (7).

To summarize: It is sometimes convenient to consider the *ease,* or *assistance,* a circuit offers to current flow instead of the resistance, or opposition. This ability to conduct current is defined as the circuit conductance and designated by g. Thus, current is equal to Eg as well as to E/R. Conductances are particularly useful in parallel circuits; in these the equivalent conductance is the sum of the conductance of each branch.

The Solution of Series-parallel Circuits.—As was previously mentioned, the circuits often encountered in practice are combinations of series and parallel types and are called **series-parallel circuits.**

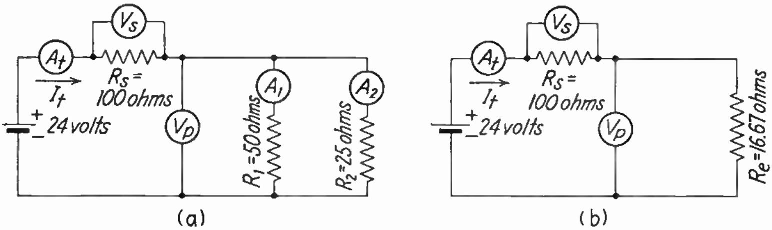

FIG. 16.—A series-parallel circuit is shown at a, and the equivalent circuit at b. The value of R_e is the equivalent resistance of R_1 and R_2 in parallel.

A circuit of this type is shown in Fig. 16. Suppose that the branch currents I_1 and I_2 as indicated by the ammeters A_1 and A_2, the total current I_t as indicated by the ammeter A_t, the voltage E_p across the parallel portion as indicated by V_p, and the voltage drop E_s indicated by the voltmeter V_s across the 100-ohm series resistor are all desired.

In analyzing this problem, it is at once evident that the currents taken by the parallel branches cannot be computed immediately, because the voltage E_p across this parallel portion is unknown. Of course, this voltage is the impressed voltage minus the IR drop E_s across the 100-ohm series resistor, but then the current I is unknown and cannot be found until E_p is known. Vague and difficult as this all seems, it is all made very simple by the use of either equivalent resistances or conductances as desired. In the following paragraphs both solutions will be explained.

Step 1. Calculate the equivalent resistance of the parallel portion of Fig. 16, using the equivalent resistance method of Eq. (4), page 22. $1/R_e = 1/R_1 + 1/R_2 = 1/50 + 1/25 = 0.02 + 0.04 = 0.06$, and $R_e = 1/0.06 = 16.67$ ohms. The circuit is now equivalent to that of Fig. 16*b*.

This may also be solved by the conductance method using Eq. (7). $g_e = g_1 + g_2 = 0.02 + 0.04 = 0.06$, and $R_e = 1/0.06 = 16.67$ ohms.

Step 2. Find the total equivalent circuit resistance, which is the sum of the series resistance, and the equivalent resistance of the parallel portion.

$$R_t = 100 + 16.67 = 116.67 \text{ ohms.}$$

Step 3. Calculate the current I_t which will flow.

$$I_t = \frac{E}{R_t} = \frac{24}{116.67} = 0.206 \text{ ampere.}$$

Step 4. Calculate the voltage E_p across the parallel portion of the circuit.

$$E_p = 24 - I_tR_s = 24 - 0.206 \times 100 = 24 - 20.6 = 3.4 \text{ volts.}$$

Step 5. The current through each branch can now be found. This may be calculated by either of the two following methods:

$$I_1 = \frac{E}{R_1} = \frac{3.4}{50} = 0.068 \text{ ampere}$$

and

$$I_2 = \frac{E}{R_2} = \frac{3.4}{25} = 0.136 \text{ ampere.}$$

$$I_1 = Eg_1 = 3.4 \times 0.02 = 0.068 \text{ ampere}$$

and

$$I_2 = Eg_2 = 3.4 \times 0.04 = 0.136 \text{ ampere.}$$

To summarize: In solving a series-parallel circuit, the first step is to find the equivalent resistance of the parallel portion. The equivalent resistance is then added to the series resistance to obtain the total series resistance. This value divided into the impressed voltage gives the total current flow. If the total current is known, the voltage drop across the series resistance can be computed. This drop, subtracted from the impressed voltage, gives the voltage across the parallel part of the circuit. With this voltage known, the separate branch currents can be found.

Solution of a Typical Telephone Circuit.—The simplified circuit of Fig. 17*a* is a portion of a common-battery telephone circuit, composed of a common-battery telephone at the left, the "subscriber's loop" of 22-gauge cable, and a part of the cord circuit and central-office battery. This circuit is of the series-parallel type.

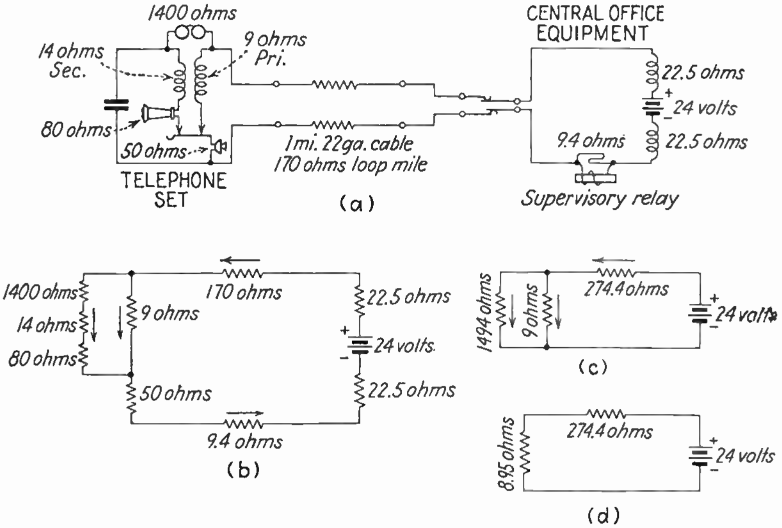

FIG. 17.—A circuit of the series-parallel type is the portion of a simplified common-battery telephone circuit shown at *a*. The steps in solving this circuit are given in *b*, *c*, and *d*.

The ringer, induction-coil secondary, and receiver are in parallel with the induction-coil primary, and this parallel combination is in series with the line, relay, and repeating coil.

Suppose that it is desired to find the current through the transmitter. The first step is to simplify the circuit as much as possible, which has been done in Fig. 17*b*. The condenser has been omitted since it does not pass direct current. The transmitter, cable, supervisory relay, and the two windings of the repeating coil are in series, giving a total resistance of 274.4 ohms. The ringer, induction-coil secondary, and receiver are in series, making a total of 1494 ohms across the 9-ohm primary as shown in Fig. 17*c*.

The equivalent resistance of the parallel circuit of the telephone set must be found. From the section on conductance, the equivalent conductance $g_e = 1/1494 + 1/9 = 0.00067 + 0.11111 = 0.11178$ mho. Since the equivalent resistance of the parallel combination is the reciprocal of the equivalent conductance, $R_e = 1/g_e = 1/0.11178 = 8.95$ ohms.

The current through the transmitter can now be found. By redrawing the circuit as in Fig. 17*d*, the total resistance offered to the battery is $R_t = 8.95 + 274.4 = 283.35$ ohms. The current flowing through the circuit and hence *the transmitter* will be $I = 24/283.35 = 0.0847$ ampere, or 84.7 milliamperes.

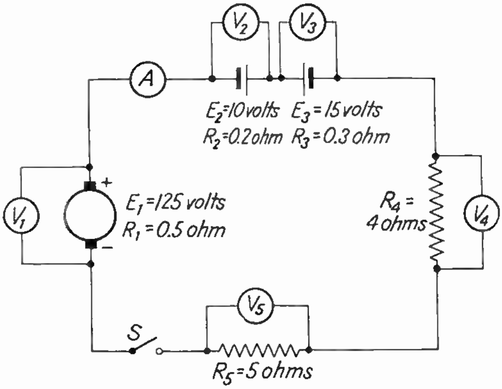

FIG. 18.—A series circuit containing more than one source of electromotive force. Each of these sources has internal resistance as represented by R_1, R_2, and R_3. Of course these internal resistances could be shown on the diagram, but this might be confusing, and it is common practice to omit them.

Circuits Containing Several Voltage Sources.—The circuits that have been considered thus far contained only one source of impressed voltage. In practice, however, circuits are often used which contain two or more sources of impressed voltage. Such a circuit is shown in Fig. 18, and its solution will now be considered.

The *open-circuit* voltages of the three sources are represented by E_1, E_2, and E_3, and these are the voltages that the voltmeters V_1, V_2, and V_3 would indicate when the switch S is *open*. The voltage E_1 is impressed by a direct-current generator, having a generated or open-circuit voltage of 125 volts and an internal resistance in the armature windings of 0.5 ohm. Note that the two batteries are reversed. It is desired to calculate the current that will flow as indicated by the ammeter A and also the voltage that will be across each element of the circuit when the switch S is closed.

Step 1. Calculate the total resistance. In series circuits this is the sum of the separate resistances. $R_t = 0.5 + 0.2 + 0.3 + 4.0 + 5.0 = 10$ ohms.

Step 2. Calculate the resultant impressed voltage acting around the circuit. The resultant voltage is $E_R = E_1 + E_2 - E_3 = 125 + 10 - 15 = 120$ volts. The impressed voltage E_3 is subtracted from the other two, because E_3 is connected in the *reverse* direction and its voltage *opposes* the effect of the other two. Voltage E_3 *tries* to cause a current flow in the *opposite* direction but is prevented from doing so by the voltages E_1 and E_2 which are larger and hence control the direction of the current flow.

Step 3. Calculate the current flow by Ohm's law. $I = E/R = 120/10 = 12$ amperes.

Step 4. Calculate the potential differences or voltages E_1, E_2, etc., that will be indicated by the various voltmeters V_1, V_2, etc. For the resistors, the voltages will be $E_4 = IR_4 = 12 \times 4 = 48$ volts and $E_5 = IR_5 = 12 \times 5 = 60$ volts. For the generator and the battery, the potential differences or terminal voltages will be the combined effect of the generated electromotive forces and the IR drops.

$$E_1 = \text{emf} - IR_1 = 120 - (12 \times 0.5) = 114 \text{ volts.}$$

$$E_2 = \text{emf} - IR_2 = 10 - (12 \times 0.2) = 7.6 \text{ volts.}$$

$$E_3 = \text{emf} + IR_3 = 15 + (12 \times 0.3) = 18.6 \text{ volts.}$$

It should be noted in particular that in computing E_1 and E_2 the internal IR voltage drops were *subtracted* from the generated electromotive force (emf) to find the terminal potential difference or voltage, but in calculating E_3 the internal IR drop was *added* to the generated emf to find the terminal voltage. The reason for this will be explained in the next section.

The Direction of *IR* Drops.—When a current flows through a resistor, it causes an IR drop in potential. The numerical magnitude or size of this potential drop depends only on the *magnitude* of the current and voltage, but *the direction of the IR drop depends on the direction of the current flow.*

Thus, in Fig. 19*a*, if a current is forced through the resistor as shown, the IR drop has the polarity indicated. If the current is reversed, the polarity is also reversed as shown in Fig. 19*b*. By referring back to Fig. 6 on page 8 and Fig. 14 on page 24, it is recalled that some points in a circuit may be at higher positive potentials than others. Conventional current (not electron current) flows from positive to negative, or from points of higher potential to points of lower potential. Therefore, in any circuit or device, *the direction of the current flow through resistance determines the direction of the IR voltage drop across the resistance.*

To apply this reasoning to sources of electromotive force containing internal resistance such as the generator and batteries of Fig. 18, consider Fig. 19*c*. The direction in which the current is flowing is as indicated by the arrow. This current will cause an IR_i drop as it flows through the internal resistance of the cell. This IR_i drop will be in the direction shown in Fig. 19*a* or from left to right. The *terminal* voltage as would be read by the voltmeter connected across this cell is now the *algebraic* [1] sum of the electromotive force and the internal IR_i drop. For Fig. 19*c* this is $E_{cc} = E_{oc} - IR_i$, explaining the first two calculations under Step 4 of the preceding illustration.

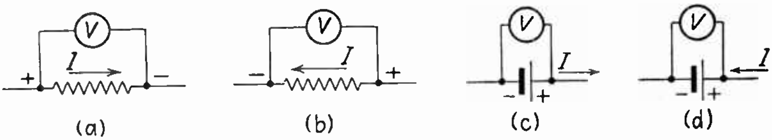

FIG. 19.—Illustrating the directions of *IR* voltage drops. The conventional direction of current flow through resistance is assumed to be from positive to negative (+ to −). Hence, changing the direction of the current will change the direction of the *IR* drop measured by the voltmeter *V* of *a* and *b*. In *c* and *d* the electromotive force or open-circuit voltage E_{oc} is represented by the battery plates. The IR_i drop within the battery reverses because the current is changed in direction. Thus, in *c* the closed-circuit voltage $E_{cc} = E_{oc} - IR_i$, and in *d* the relations are $E_{cc} = E_{oc} + IR_i$. In these equations R_i is the internal resistance of the battery.

Now suppose that the current through the cell is reversed as in Fig. 19*d*. As indicated in Fig. 19*b*, the direction of the IR_i drop is now reversed and is now in the direction to *add* to the electromotive force. That is, in passing through the circuit of Fig. 19*d*, a + to −, + to − sequence of the voltages exists, such as would be the connections used if maximum voltage were desired from two dry cells. Hence, the terminal voltage as would be read by the voltmeter of Fig. 19*d* is $E_{cc} = E_{oc} + IR_i$, explaining the last calculation under Step 4 of the preceding illustration.

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[1] This simply means that the sign (+ or −) of the voltages must be considered as well as their magnitudes.

Another way of looking at this matter is as follows: In Fig. 19*c* the direction of current flow is the direction in which the cell would normally force current through an external circuit. Since the cell is not a perfect device, its terminal voltage drops slightly when the current flows. (Note that an IR_i drop must occur within the cell because of its internal resistance, and surely this drop would not *increase* the voltage of the cell. If it did, to get more voltage, merely draw more current, etc.) Suppose that the cell under consideration is a storage cell. When it is being charged, as would occur in Fig. 19*d*, the external charging source must force the charging current through the cell in the reverse direction, that is, from positive to negative through the cell.

Now for charging, the IR_i drop inside the cell is reversed in direction, but the chemically generated voltage remains as before. Thus, as shown in the preceding paragraph, $E_{cc} = E_{oc} + IR_i$, explaining the last calculation Step 4, page 32. The charging current being forced through the cell by the charging source is opposed by both the chemically generated voltage of the cell and the voltage drop caused by the internal resistance of the cell.

Kirchhoff's Laws.—Much of the basic theory considered in this chapter can be summarized in the following two laws, bearing the name of an early electrical investigator.

First Law. The algebraic sum of the impressed electromotive forces around any closed electric circuit is equal to the algebraic sum of all the IR drops around that circuit. This is equivalent to saying that the algebraic sum of the electromotive forces minus the algebraic sum of the IR drops equals zero.

Second Law. The algebraic sum of the currents flowing to a point must equal the algebraic sum of the currents flowing away from that point. This is equivalent to saying that the algebraic sum of the currents flowing up to a point minus the algebraic sum of the currents flowing away equals zero. These two laws are of very great importance, and their meanings should be clearly understood. Their applications will now be considered.

Problem.—A 10-volt battery having an internal resistance of 0.1 ohm and a 20-volt battery having an internal resistance of 0.2 ohm are connected in parallel for charging with a generator having a generated electromotive force of 120 volts and an internal resistance of 0.2 ohm. The resistance of the wires from the generator to the two batteries is 0.8 ohm. The 10-volt battery has a resistor of 3.9 ohms, and the 20-volt battery has a resistor of 2.8 ohms in series for controlling the current. Calculate the charging current of each battery and the current from the generator.

Solution.—Step 1. Draw a clear diagram of the circuit, such as Fig. 20. In this diagram note that the positive terminals of the two batteries are connected toward the positive generator terminal. Current is assumed to flow out from the positive generator terminal and must flow into the positive battery terminals to charge them.

Step 2. On the diagram, write in all the known values of voltage, resistance, and current. Indicate the probable direction of current flow with arrows. Try to reason out the directions of current flow, but if these are not evident, they should be assumed. Use the letters i_1, i_2, i_3, etc., and e_1, e_2, e_3, etc., to indicate all unknown values, but keep the numbers of letters used to a minimum; for example, instead of marking the line current i_3, apply Kirchhoff's second law and designate it $i_1 + i_2$. Since the batteries are to be charged, i_1 and i_2 were taken as indicated.

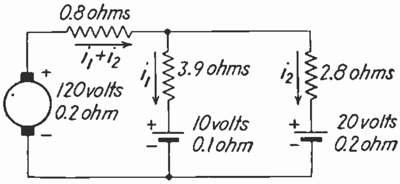


FIG. 20.—A circuit to be solved by Kirchhoff's laws.

Step 3. There are two unknown values in this problem. These are the currents i_1 and i_2. It is now necessary to use Kirchhoff's first law and write two equations in which these two unknown values occur. Starting at a point just below the negative terminal of the generator and going completely around the outside loop in a clockwise direction,

$$120 - 0.2(i_1 + i_2) - 0.8(i_1 + i_2) - 2.8i_2 - 0.2i_2 - 20 = 0.$$

(In writing this equation, imagine that you are actually walking around this circuit going up and down potential hills and making a graph such as Fig. 14.) Note that in going through a source of electromotive force such as a generator or a battery from − to + that the sign is positive because it represents an increase in potential, but in going through from + to − the sign is negative because this is a decrease in potential. Attention is also called to the fact that since a current always flows from a point of higher potential to a point of lower potential, when going around a loop in the same direction as the current, an *IR* drop is negative but in going around a circuit against the direction of current the drop is positive. Again starting at the point just below the generator and going around the inside loop in a clockwise direction,

$$120 - 0.2(i_1 + i_2) - 0.8(i_1 + i_2) - 3.9i_1 - 0.1i_1 - 10 = 0.$$

Step 4. Simplify the equations and solve them simultaneously. The first equation becomes $1.0i_1 + 4.0i_2 = 100$, and the second equation becomes $+5.0i_1 + 1.0i_2 = 110$. Multiplying the first equation through by -5.0 and solving,

$$\begin{aligned} -5.0i_1 - 20.0i_2 &= -500 \\ +5.0i_1 + \ 1.0i_2 &= \ \ 110 \\ \hline - 19.0i_2 &= -390 \\ i_2 &= 20.53 \text{ amperes.} \end{aligned}$$

Step 5. Calculate the remaining values of current. This may be done by substituting the values of i_2 in either equation. Substituting in the first,

$$\begin{aligned} 1.0i_1 + (4 \times 20.53) &= 100 \\ i_1 &= 100 - 82.12 \\ i_1 &= 17.88 \text{ amperes.} \end{aligned}$$

The generator current is the sum of the two branch currents, or

$$i_g = i_1 + i_2 = 20.53 + 17.88 = 38.41 \text{ amperes.}$$

Step 6. To check the results, write a voltage equation for a loop not previously considered, and substitute the computed values in this equation. For this, use the closed loop at the extreme right of Fig. 19. Starting just below the 10-volt battery and going around in a clockwise direction,

$$+0.1i_1 + 10 + 3.9i_1 - 2.8i_2 - 0.2i_2 - 20 = 0,$$

or

$$4.0i_1 - 3.0i_2 = 10.$$

Substituting the values of i_1 and i_2 in this expression,

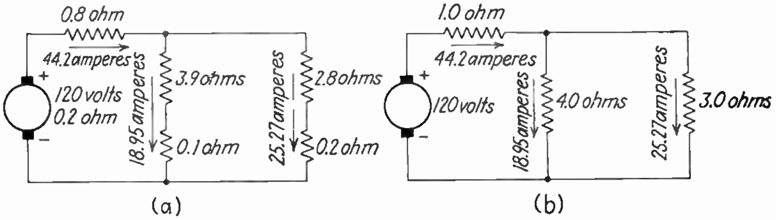
$$(4 \times 17.88) - (3 \times 20.53) = 10,$$

and

$$71.52 - 61.59 = 10 \text{ approximately.}$$

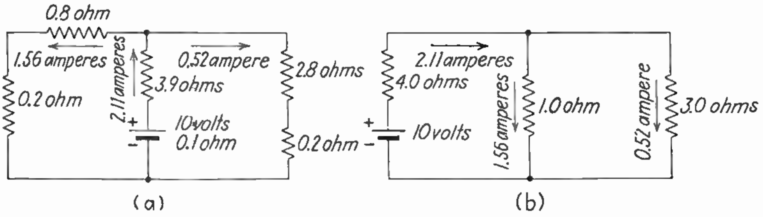
Solutions Using Principle of Superposition.—Some students may not be familiar with the method of solving equations simultaneously, and a method based on the **principle of superposition** is here offered as an alternative solution to the Kirchhoff-law method. This principle is in reality a theorem used in advanced communication engineering and may be stated for the purpose of this chapter as follows: *The current that flows at any point in a circuit, or the voltage between any two points in a circuit, due to a number of sources of electromotive force connected at various points in the circuit, is the algebraic sum of the separate currents or voltages at these points which would exist if each source of electromotive force were considered separately, each of the other sources being replaced at that time by a unit of equivalent internal resistance.* This principle will now be applied to the problem of Fig. 20.

Step 1. Redraw Fig. 20 as shown in Fig. 21*a*, replacing two of the battery sources of electromotive force with resistors of the same value as the internal resistance of the battery. Combining all the series resistances gives the simple series-parallel circuit of Fig. 21*b*. Solving this gives 1.72 ohms for the equivalent resistance of the parallel portion, 2.72 ohms for the total circuit resistance, and a generator current of 120/2.72 = 44.2 amperes directed as shown in Fig. 21*a*. Note that this is not the current actually flowing in the circuit of Fig. 20 but rather the current that would flow if the electromotive forces of the batteries were removed. The voltage across the parallel combination will be 120 − 44.2 × 1.0 = 75.8 volts. The branch currents will be 75.8/4 = 18.95 amperes and 75.8/3 = 25.27 amperes also as indicated on Fig. 21*a*.



(a) (b)

FIG. 21.—Partly illustrating the application of the principle of superposition to the solution of the circuit of Fig. 20. (See also Figs. 22 and 23.)

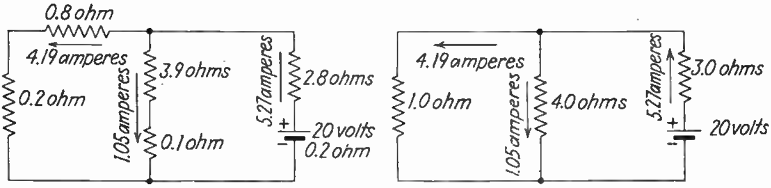


(a) (b)

FIG. 22.—Further illustration of the principle of superposition as applied to the solution of the circuit of Fig. 20. Note that the branches in *a* have been interchanged in *b* to give the circuit more the appearance of parallel circuits already familiar to the reader. This change should be noted in the final algebraic addition. That is, "superimpose" Figs. 21*a*, 22*a*, and 23*a* to find the resultant current (see discussion).

Step 2. Redraw Fig. 20 as shown in Fig. 22*a*, replacing the generator and one battery with their respective internal resistances. This circuit is now equivalent to that of Fig. 22*b*. The equivalent resistance of the parallel portion is 0.75 ohm, the total resistance is 4.75 ohms, and the current that would flow is 10/4.75 = 2.11 amperes. The voltage available across the parallel portion will be $10 - 2.11 \times 4 = 1.56$ volts. Hence, 1.56/1.0, or 1.56, amperes would flow to the left as indicated by the arrow of Fig. 22*a* and $1.56/3 = 0.52$ ampere would flow down through the right portion as shown by the arrow.

Step 3. It now remains to investigate what would be the results of replacing all sources of emf except the 20-volt battery with their internal resistances. This is done in Fig. 23*a*. The equivalent resistance of the parallel portion is 0.8 ohm, the total circuit resistance is 3.8 ohms, and the battery current would be $20/3.8 =$ 5.27 amperes directed as shown. The voltage across the parallel portion would be $20 - 5.27 \times 3 = 4.19$ volts. The currents in the other two branches would be 4.19 and 1.05 amperes, respectively.



(a) (b)

FIG. 23.—Completing the illustration of the application of the principle of superposition to the circuit of Fig. 20.

Step 4. Combine the currents algebraically to find the actual current flowing in each branch. For the generator, $i_g = 44.2 - 1.56 - 4.19 = 38.45$ amperes. For the 10-volt battery, $i_1 = 18.45 - 2.11 + 1.05 = 17.39$ amperes. For the 20-volt battery, $i_2 = 25.27 + 0.52 - 5.27 = 20.53$ amperes.

It is seen that these answers check within the usual limits of slide-rule accuracy.

The Rheostat.—The rheostat is an adjustable resistor so constructed that its resistance may be adjusted without opening the circuit in which it is connected.[1] Such a device is placed in series in the circuit as shown in Fig. 24, and by adjusting the amount of resistance, the current flow in the circuit may be controlled within

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[1] All definitions given in this book will be taken from the standards of the American Institute of Electrical Engineers or the standards of the Institute of Radio Engineers. The definition given above is from the Standards of the American Institute of Electrical Engineers.

wide limits, depending of course on the values of the circuit voltage and the resistance of the rheostat.

The value of resistance that a rheostat should have is calculated by Ohm's law. Thus suppose that some electrical device such as a relay having a direct-current resistance of 1225 ohms is to be connected to a 110-volt source. It is desired to be able to vary the current at will from 0.050 ampere (50 milliamperes), the current at which the relay closes, to 0.010 ampere (10 milliamperes), the current at which it opens. The question confronting the experimenter is this: What must be the value of the rheostat? The solution requires an application of Ohm's law.

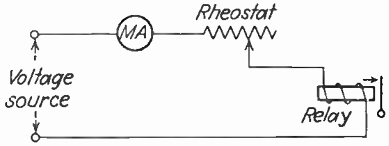

FIG. 24.—Showing a rheostat connected to control the current flowing in a relay. The rheostat is a variable resistance, the desired resistance being selected by moving the contact arm. See also Fig. 27.

Step 1. Find the maximum resistance that the rheostat must have. The minimum current that is to flow is 0.010 ampere. Thus, the circuit equation is

$$\text{Amperes} = \frac{\text{volts}}{\text{ohms}} \qquad \text{or} \qquad 0.01 = \frac{110}{1225 + R},$$

where R is the maximum resistance that the rheostat must have.

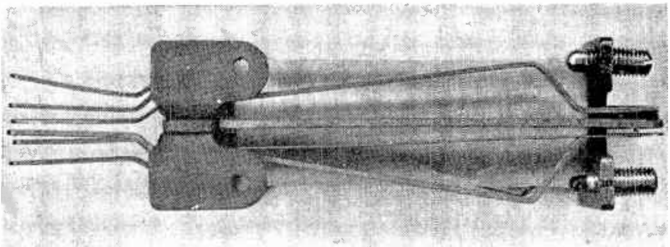

FIG. 25.—Photograph of a relay used in communication circuits.

Solving for R,

$$R = \frac{110}{0.01} - 1225 = 9775 \text{ ohms.}$$

Step 2. Find the minimum value that the rheostat must have. The maximum value of current is to be 0.050 ampere. Thus,

$$R = \frac{110}{0.05} - 1225 = 975 \text{ ohms.}$$

To limit the flow of current between the limits desired, a rheostat that was continuously variable from 0 to 10,000 ohms would be satisfactory.

The Voltage Divider.—A voltage divider[1] is a resistor provided with fixed or movable contacts and with two fixed terminal contacts; current is passed between the terminal contacts, and a desired voltage is obtained across a portion of the resistor. (The term potentiometer is often erroneously used for this device.) The symbol for a voltage divider is shown in Fig. 26, and a typical voltage divider of the variable-contact type is included in Fig. 27.

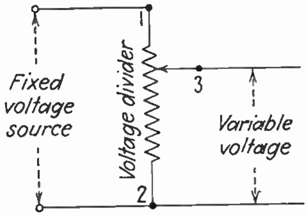

FIG. 26.—Showing a voltage divider connected to a fixed voltage source. The desired voltage is selected by varying the position of the contact arm. Terminals 1–2–3 are the terminals of the voltage divider.

It is not possible to set down in a few words simple rules applying to voltage dividers under all conditions of operation. This is not because the circuit arrangements are so difficult, but rather because they are so varied. In fact, the circuits are readily solved by the theory for series and parallel circuits discussed in this chap-

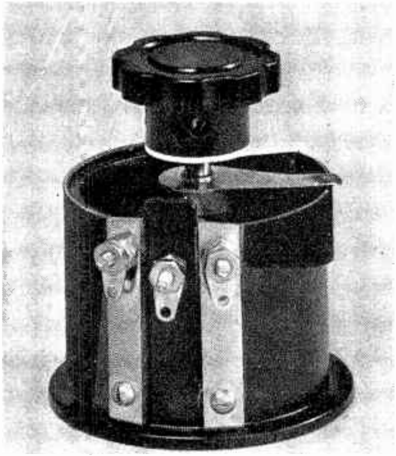

FIG. 27.—A commercial form of a wire-wound voltage divider very useful in communication circuits. (*Courtesy of General Radio Company.*)

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[1] This definition is from the Standards of the American Institute of Electrical Engineers.

ter. This will be more evident after studying the following illustration.

In Fig. 28, a 50,000-ohm voltage divider is connected across a 500-volt source. The purpose of this divider is to divide, or split up, the 500 volts into voltages that are available for doing specific tasks in the rest of the circuit.

By assuming that the various taps are open, a current of 500/50,000 = 0.010 ampere, or 10 milliamperes, will flow down through the circuit. The first tap includes 10,000 ohms. Thus the IR drop will be $0.01 \times 10{,}000 = 100$ volts across this section of the divider. Of course this means that the 100 volts will be between A and B and that point B is then 400 volts above ground which is taken as a reference point. The other voltages would be as indicated.

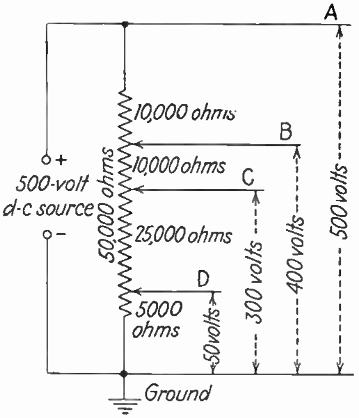

FIG. 28.—The 50,000-ohm voltage divider tapped as shown makes available the indicated voltages, provided but very little current is drawn.

It may be wondered why 50,000 ohms was taken as the value of the divider. There was no special reason, it simply would be a good value, taking only 10 milliamperes from the source. Why not, therefore, make the divider 100,000 ohms and take only 5 milliamperes, or better yet, why should it not be 500,000 ohms and take only 1 milliampere from the source?

The answer is merely that a value should be chosen which fits the circuit conditions. If the various taps are connected to devices or portions of the circuit that draw but little current, then, a voltage divider of very high resistance is suitable. If, however, the various taps must supply considerable current, then the divider must be of lower resistance, because *the current supplied must flow through a portion of the divider.* Of course this will add to the current that the divider itself takes, and will, therefore, increase the IR drop. This is one of the reasons why a voltage divider of lower resistance would probably be used to supply voltages to the plate circuits of vacuum tubes that take appreciable plate current.

SUMMARY

Three fundamental rules apply to series circuits.

1. The current in all parts of a series circuit is the same.

2. To find the total resistance of a series circuit, add the separate resistances.

3. The sum of the *IR* drops around a series circuit equals the algebraic sum of the impressed electromotive forces, or emfs.

Three fundamental rules apply to parallel circuits.

1. The voltage across each branch of a parallel circuit is the same.

2. The equivalent resistance of a parallel circuit is $1/R_e = 1/R_1 + 1/R_2$, etc. The equivalent conductance of a parallel circuit is $g_e = g_1 + g_2$, etc.

3. The sum of the currents flowing to a point equals the sum of the currents flowing away.

Kirchhoff's laws, stated as follows, are useful in solving certain circuits.

1. The algebraic sum of the impressed electromotive forces around any closed electric circuit is equal to the algebraic sum of all the *IR* drops around that circuit.

2. The algebraic sum of the currents flowing to a point must equal the algebraic sum of the currents flowing away from that point.

The principle of superposition (page 36) is useful in solving problems where Kirchhoff's laws are applicable but where their use is not desired.

REVIEW QUESTIONS

1. State Ohm's law in its three basic forms, discussing the units of measurement.

2. What multiples and subdivisions of the basic units of measure are commonly used?

3. State the laws applying to a series circuit.

4. Distinguish between the open-circuit and closed-circuit voltage of a cell.

5. State the laws applying to a parallel circuit.

6. Explain how the current divides in a parallel circuit having two branches.

7. When must the internal resistance of a storage battery be considered, and when may it be neglected?

8. Explain the relation between equivalent resistance and equivalent conductance.

9. Conductance is useful in solving circuits of what type?

10. Explain how to solve a simple series-parallel circuit.

11. How are voltages in series combined to obtain the resultant voltage?

12. What are the relations between an *IR* drop in a resistor and the magnitude and direction of the current?

13. A storage battery is being charged. How will the internal *IR* drop combine with the battery electromotive force?

14. State Kirchhoff's laws.

15. Two equations each having two unknowns are to be solved simultaneously. Explain how this is accomplished. (Can it be done for three equations each having three unknowns?)

16. What is the principle of superposition?
17. Explain how to solve circuits using the principle of superposition.
18. What is a rheostat?
19. What is a voltage divider?
20. State briefly the theory of a voltage divider.

PROBLEMS

1. The normal filament current of a type 1H4G vacuum tube is 0.060 ampere, and the normal voltage is 2.0 volts. Calculate the resistance of the filament.

2. Two dry cells of 1.48 volts *closed-circuit potential difference* are to be used to supply the current to the filament of Prob. 1. How should these dry cells be connected? How much resistance should be used with them? Draw a circuit diagram of the connections.

3. Three tubes of Prob. 1 are to be operated from the two dry cells. Draw a complete circuit diagram of the connections, and calculate the value of the current-limiting resistor that must be used. Assume that for these conditions the closed-circuit voltage falls to 1.45 volts per cell.

4. A dry cell shows an open-circuit voltage of 1.49 volts and a closed-circuit voltage of 1.47 volts when delivering 120 milliamperes. Calculate the internal resistance of the dry cell.

5. Calculate the internal resistance of the dry cells used in Probs. 2 and 3.

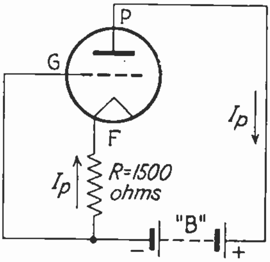

FIG. 29.—Circuit for Prob. 6.

6. A certain vacuum-tube circuit is arranged as in Fig. 29. The *electron* plate current equals 2.1 milliamperes and flows outside the tube as shown. Within the tube the *entire* current flows directly from the filament F to the plate P. A resistance of 1500 ohms is in the filament circuit. What is the magnitude and sign of the potential difference between the grid G and the filament F? If the B battery closed-circuit voltage is 89 volts, what is the magnitude and the sign of the potential difference between the grid G and plate P and between the plate P and filament F?

7. A telephone installer connects a simple portable handset consisting of a 50-ohm transmitter and a 275-ohm receiver across the subscriber's line in order to talk to the central office. The subscriber's residence is 1.5 miles from the central office, the circuit being 22-gauge cable having a resistance of 170 ohms per mile of cable pair. When the installer connects the handset, the line relay in the central office operates, signaling the operator, the circuit being as shown in Fig. 30. Compute the current through the transmitter, and determine the magnitude and direction of the voltage drop across each part of the circuit. The switchboard lamp takes 36 milliamperes. What is its resistance?

8. *Two* telephone sets such as shown in Fig. 17 are connected in *parallel* at the ends of the two line wires; that is, they are "bridged" across the line.

Calculate the equivalent resistance of the two sets so connected. Calculate the current flowing through the line, through each transmitter, and through each receiver.

9. Referring to the discussion on page 25 and to Fig. 14, plot a potential graph for the same circuit, using the *negative* terminal as the point of reference.

10. Referring to page 22, derive the two equations (5).

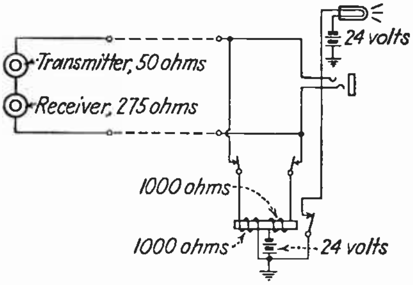

Fig. 30.—Circuit for Prob. 7.

11. Referring to the discussion on page 21, develop the laws applying to the current distribution for *three* resistors in parallel.

12. The current is 32.5 amperes when a certain storage battery is short-circuited. The current is 6.5 amperes when the same battery is connected to 1.44 ohms. Calculate the electromotive force of the battery and its internal resistance.

13. One battery has an electromotive force of 10 volts and an internal resistance of 0.5 ohm. Another battery has an electromotive force of 11.0 volts and an internal resistance of 0.6 ohm. When they are being charged in parallel, the current through the first battery is 2.2 amperes. What will be the current through the second?

14. Three batteries have electromotive forces of 10, 20, and 30 volts and internal resistances of 5, 3, and 2 ohms, respectively. Calculate the current through each battery if their positive terminals are connected to one point and their negative terminals are connected to another point.

15. Calculate the current in each part of the circuit if the common points of the batteries of Prob. 14 are connected to a 10-ohm resistor.

CHAPTER III

CONDUCTORS, RESISTORS, AND INSULATORS

In the preceding chapter the flow of electricity in direct-current circuits was considered. These circuits consist largely of *conductors*, such as connecting wires, *resistors* for limiting current flow, and *insulators* for confining the electric currents to the desired path.

In this chapter, these conductors, resistors, and insulators and their application to communication circuits will be discussed. The explanations will be limited to direct-current phenomena, alternating-current transmission being reserved for a later chapter.

It will be apparent to the reader that as a familiarity with electric circuits is gained there is no clear dividing line between conductors, resistors, and insulators. Of course, conductors would usually be found to have very low resistances, but what might be classed as a resistor in one circuit might be an insulator in another. Also, at room temperature a certain material may make a good insulator, but if heated to a high temperature it becomes a good conductor.

In vacuum-tube amplifiers, resistors of hundreds of thousand and even millions of ohms are used. This latter value would be considered as belonging to an insulator in most other circuits. Thus conductors, resistors, and insulators differ fundamentally largely in degree only. That is, the same basic laws of electric-current flow apply to each. It is thought that good conductors contain many free electrons, resistors contain less, and insulators contain but few. This will account, at least in an elementary way, for the low resistance of conductors, the higher resistance of resistors, and the very high resistance of insulators.

The Resistance of Solid Conductors.—As has been mentioned, the flow of electricity in a metallic conductor is a progressive motion of the free electrons already in the conductor. The battery or generator does not "generate" the current in the usual sense of the word. The electromotive force generated by the device is thought to "force," or "push," the free electrons around the circuit, the result being a current flow.

Since the current flow consists of the free electrons, and since there are a given number of free electrons in any given volume of a metallic conductor, the current-carrying ability of a conductor must depend on its dimensions. To explain these relations, consider the piece of copper bar of Fig. 31. This bar has a thickness t, a width w, and length l. The influence of these dimensions on the resistance R offered to the current I will now be explained.

For convenience, the solid bar shown in Fig. 31 is thought of as being made up of three thin pieces (shown dotted) placed close together. If each of these pieces were *longer*, the current I would have to flow farther, more opposition would exist.[1] It can be stated, therefore, that *the resistance of a conductor varies directly as the length l of the conductor.*

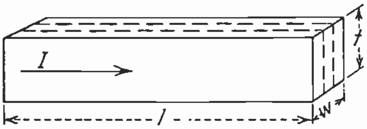

FIG. 31.—Copper bar showing dimensions which have influence on resistance.

Now suppose that a fourth thin piece of copper is placed alongside the three strips of Fig. 31, thereby *increasing the width w.* The current I will now have *four* paths in *parallel* instead of three (of course it is assumed that the ends are all connected). From the laws of the parallel circuit, the resistance of the *four* pieces would be *less* than the resistance of the three pieces. Similarly, it can be shown that increasing the thickness t has the effect of adding more paths in parallel and hence decreases the resistance. *The resistance of a conductor therefore varies inversely as the width w and the thickness t of the conductor.* There is of course, one other factor entering into the relations determining the actual resistance of a particular conductor such as the bar of Fig. 31. This factor is designated as ρ and is determined by the exact nature of the metal under consideration.

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[1] The term "more opposition would exist" may seem vague, because the nature of the opposition that a conductor offers to current flow has not been explained. The fact is, the exact nature of this opposition is vague. It is well known, however, and perfectly reasonable to assume that there is an opposition per unit length of wire. Hence the more unit lengths, that is, the greater the length l, the greater will be the total resistance R. That a wire offers opposition to current flow is readily proved by the fact that a conductor heats when a current flows through it. Thus, the foregoing statement is acceptable.

These relations can be expressed by the equation

$$R = \frac{\rho l}{wt} = \frac{\rho l}{a}, \tag{8}$$

where R is the resistance in ohms of a conductor, l is the length of the conductor in centimeters, a is the cross-sectional area of the conductor in square centimeters, and ρ is the resistivity, or resistance, per centimeter cube.

Note that in Eq. (8) the width and thickness multiply to give the cross-sectional area a. Observe also that all units are specified in centimeters. This is because the resistivity ρ is specified as the resistance per centimeter cube. A centimeter cube is a cube of the metal one centimeter on each side (it should *not* be confused with cubic centimeter, which is a measure of volume and not dimension). The resistivity ρ is usually specified in ohms per centimeter cube; however, it could be specified in ohms per inch cube, in which event all the dimensions above would be in inches.

To summarize: Since a current flow depends on the presence of free electrons, the resistance to current flow will depend on the number of free electrons present in the current path. The number of free electrons will depend on the dimensions of the path and the nature of the path; that is, whether it is copper, aluminum, etc. The resistance is found to vary directly as the length of the path, directly as the resistivity of the material, and inversely as the cross-sectional area.

Resistivity.—As previously defined, resistivity ρ is usually defined as the resistivity per centimeter cube. It was also stated that the resistivity of a specimen depended on the nature of that specimen. The discussion in this section being confined to common electric-conductor metals only, it must also be stated that the resistivity, and hence the final resistance of a conductor, depends also on the temperature.

For the common electric conductors such as copper and aluminum, *increasing the temperature increases the resistivity, and decreasing the temperature decreases the resistivity.* For this reason, where exact statements are made, the temperature at which a given resistivity holds should be specified. This same statement also applies to resistance. Measurements are usually assumed to be made at average room temperature, considered to be 20°C., or 68°F.

The resistivity of metals sometimes used as electric conductors is given in Table I.

TABLE I.—RESISTIVITY OF METALS USED AS ELECTRIC CONDUCTORS[1]

(Values in ohms per centimeter cube at 20°C.)

Aluminum (as used in wire)	0.000002828
Copper (annealed standard)	0.0000017241
Copper (hard-drawn No. 12 A.W.G.)	0.000001772
Iron (99.98% pure)	0.000010
Iron (wrought)	0.00001057
Lead	0.000022
Nickel	0.0000078
Silver (99.78% pure)	0.000001629

[1] The values here listed were largely taken from "Electrical Engineers' Handbook," 3d ed., Vol. V, Communication and Electronics, John Wiley & Sons, Inc., and from "Standard Handbook for Electrical Engineers," McGraw-Hill Book Company, Inc. Because of variations in individual specimens, values listed elsewhere may be slightly different from those in this table.

To summarize: The resistivity of a metal and the resulting resistance of a conductor vary with the temperature. Usually, the resistance is given at 20°C., or 68°F. (assumed to be average room temperature). For the common electric conductors, such as copper and aluminum, increasing the temperature increases the resistance, and vice versa.

Resistance Calculations.—Large copper bus bars are used in telephone central offices (for example) to extend from the power switchboard to the storage batteries. Suppose that it is desired to calculate the resistance of such a bus bar which is 27 feet long, 1.0 inch wide, and 6.0 inches thick. First, all these dimensions must be changed to centimeters, 1 inch equaling 2.54 centimeters. Then, $l = 27 \times 12 \times 2.54 = 822$ centimeters, $w = 2.54$ centimeters, and $t = 6 \times 2.54 = 15.24$ centimeters. The resistance of the bus bar is

$$R = \frac{\rho l}{wt} = \frac{0.0000017241 \times 822}{2.54 \times 15.24} = 0.0000367 \text{ ohm.}$$

As a second illustration, suppose that the resistance of an iron rod 10 feet long and ¼ inch in diameter is desired. The length in centimeters is $l = 10 \times 12 \times 2.54 = 305$ centimeters. The cross-sectional area is $a = \pi r^2 (r = \text{radius})$, or $a = 3.1416 \times (0.25 \times 2.54/2)^2 = 0.317$ square centimeter. In Table I, iron is listed with a resistivity of $\rho = 0.000010$ ohm per centimeter cube. This is for pure iron, but an iron rod would usually be of wrought iron. The resistivity of wrought iron is listed at 0.00001057 ohm per centimeter cube. Using this value,

$$R = \frac{\rho l}{a} = \frac{0.00001057 \times 305}{0.317} = 0.01016 \text{ ohm.}$$

For most electrical work a resistivity for copper of 0.00000172 ohm per centimeter cube is sufficiently accurate.

The Resistance of Wires.—It will be noted that the value of π (that is, $\pi = 3.1416$) entered into the calculation of the resistance of the iron rod of the preceding section. This was because the rod was circular in cross section and the value of the resistivity ρ was for a unit cube having a square cross section. It is quite clear that the resistance of a wire could be calculated in just the same manner as was done for the rod. However, each such calculation would involve the value $\pi = 3.1416$ and also squaring the radius.

There is nothing wrong with this, or even very involved, but it is bothersome, and the method of calculating the resistance of wires has been simplified. This has been done by using a *circular unit* in Eq. (8) instead of the centimeter cube which has a square cross section. That is, instead of using for ρ the resistance of a centimeter cube of the material, in wire calculations ρ' is used which is the resistance of a wire one mil (0.001 or one one-thousandth of an inch) in diameter and one foot long. *The resistance of this unit wire of circular cross section, one one-thousandth of an inch in diameter and one foot long, is known as the* **circular-mil-foot.**

The resistance at 20°C. of a high-grade annealed copper wire 1 mil in diameter and 1 foot long (that is, the resistance per circular-mil-foot) is 10.37 ohms. This value applies to a very pure form of copper arbitrarily defined to be of 100 per cent conductivity. It will be referred to as pure copper hereafter, meaning of course of a specified commercial grade of purity rather than as free from all impurities.

Annealed, or "soft," copper is used for magnet wire, for wiring parts, for cables, and for purposes where mechanical strength is not of prime importance. Hard-drawn copper wire is used for line conductors such as are used in constructing open-wire telephone lines, where strength is important. To form wires, the copper is successively passed through smaller and smaller dies while the temperature of the metal is relatively low. This process leaves mechanical stresses in the surface layers of the wire and otherwise affects the grain structure. This is a factor affecting the resistance. For hard-drawn wires the resistance per circular-mil-foot is about 4 per cent greater for small wires and 2 per cent greater for large wires (thus 3 per cent will be a satisfactory factor to apply to hard-drawn wires of common sizes such as number 12 or 10). The reason for this varying percentage is of course that the smaller the cross section of the wire, the greater will be the percentage of the material affected by the drawing process.

To summarize: A circular unit of measure, the circular-mil-foot, is used in calculating the resistance of wires. This avoids the use of the factor 3.1416. For pure annealed copper, the resistance per circular-mil-foot is 10.37 ohms at 20°C. For hard-drawn copper, the resistance is some 3 per cent higher.[1]

Wire-resistance Calculations.—As has previously been explained, a form of Eq. (8), $R = \rho' l/a$, is used to calculate the resistance of wires. It has been explained that ρ' is the resistance per mil-foot, and of course l would be in feet. It has been mentioned that a circular unit of measure is used for the area a, but this unit must be further investigated.

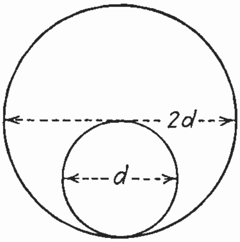

Fig. 32.—Illustrating the fact that the cross sectional areas of the two circles are proportional to their diameters squared.

The circular unit of measure used in wire calculations is the circular mil. *A circular mil is the area of a circle one mil (1/1000 inch) in diameter.* The use of the value $\pi = 3.1416$ is avoided, because this is a factor changing from circles to squares. However, *the cross-sectional area of a wire is proportional to the diameter squared.* This is illustrated by Fig. 32. Merely by inspection it is evident that the circle having a diameter of $2d$ has an area *four* times that of the circle having a diameter of d. Thus, the area in circular mils is proportional to the diameter squared. *To find the circular mil area, square the diameter in mils,* one mil equaling one one-thousandth of an inch.

One other factor must be introduced to complete the expression for the resistance of a wire. This is a conductivity term, because

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[1] In this book the resistance per circular-mil-foot is designated as ρ'. In most writings, ρ is used for both the centimeter-cube and the mil-foot values. The symbol ρ' is here used to avoid confusion.

all copper wire may not be so good a conductor as the pure annealed copper having a resistance per mil foot of $\rho' = 10.37$ ohms at 20°C. As was mentioned, hard-drawn copper has a resistance some 3 per cent higher; this is equivalent to saying that its conductivity is 97 per cent as great. Or, looked at from the standpoint of the circular-mil-foot, the resistance per circular-mil-foot of this hard-drawn copper would be $\rho' = 10.37/0.97 = 10.68$ ohms. An example of the calculation of the resistance of a wire will now be considered. In practice the circular-mil-foot is often shortened to **mil-foot.**

In the telephone industry a hard-drawn copper wire 0.165 inch in diameter has been used for constructing open-wire lines. Suppose it is desired to calculate the *direct-current* resistance per **loop mile.** A loop mile means a loop of wire one mile long. It takes two wires to make a loop, hence a *loop mile consists of two miles of wire.* The calculations are to be made at 20°C.

Step 1. Calculate the resistance per mil-foot for the hard-drawn wire, assuming the 3 per cent increase to apply.

$$\rho' = \frac{10.37}{0.97} = 10.68 \text{ ohms.}$$

Step. 2. Calculate the length in feet.

$$l = 5280 \times 2 = 10{,}560 \text{ feet.}$$

Step 3. Calculate the circular-mil area. This equals the diameter in mils squared.

$$d = 0.165 = 165 \text{ mils.} \qquad d^2 = (165)^2 = 27{,}200 \text{ circular mils.}$$

Step 4. Apply these values in the modified form of Eq. (8) to calculate the resistance of the wire.

$$R = \frac{\rho' l}{a} = \frac{10.68 \times 10{,}560}{27{,}200} = 4.16 \text{ ohms.}^1$$

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[1] If the reader happens to be a telephone worker with tables available he should not lose faith in this calculation just because it may not agree *exactly* with his tables. There are many reasons for this: This was specified to be the *direct-current* resistance; his tables may be for 1000 cycles, and the values may be slightly different. Also, the 3 per cent factor may not hold exactly, etc. Mention should also be made of the fact that in cables using twisted pairs (such as in telephone plant) the actual wire length of one conductor is about 5 per cent greater than that of the fabricated cable (that is, of the length of the cable measured along its sheath).

Wire Sizes.—It is common practice to specify the size of wires by corresponding gauge numbers. Unfortunately, there are several systems of gauges in common use. The most widely used system in this country is the American Wire Gauge (A.W.G.); this is also called the Brown and Sharpe (B. & S.) Gauge. All copper electric conductors used in power work follow this system. In communication, the copper wires used in telephone-central-office and outside-plant cables follow this system. Also, the conductors used in telephone-central-office and radio wiring are A.W.G. wires. These wire sizes and other useful data are given in Table II.

The data given in Table II require no accompanying explanation. There are certain "rules" by which the data applying to one size may be altered to give the characteristics of another. These data and additional information are given in various handbooks.[1] It is not believed advisable to memorize such relations unless special circumstances justify the effort.

Two other gauge systems are used, particularly in telephony. For iron and steel wires the U. S. Steel Wire Gauge (Stl.W.G.) is used; the Birmingham Wire Gauge (B.W.G.) is also used for iron wire. This same B.W.G. system is used for copper, however, as is the New British Standard Gauge. (N.B.S.G.) Handbooks should be consulted for data on these systems. As is evident, there is opportunity for much confusion. To avoid this, it has become good practice in telephone work to specify not the wire gauge but the wire diameter in mils, particularly when specifying open-wire conductors. Three types of hard-drawn copper conductors are often used for long-distance open-wire telephone lines. These are No. 8 B.W.G. (165 mil), No. 10 N.B.S.G. (128 mil), and No. 12 N.B.S.G. (104 mil). These wires have resistances of 4.02, 6.68, and 10.12 ohms, respectively, per loop mile at 20°C., according to reliable figures (see footnote, page 51). The reader is referred to handbooks for tables giving the safe carrying capacity of wires.

Effect of Temperature on Resistance.—As was briefly mentioned on page 3, temperature affects the atomic vibrations to a small extent. Since this is true, it is not surprising to find that temperature has a marked effect on the resistance of most conducting materials. In fact it can be stated that all pure metals of

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[1] For instance, "Electrical Engineers' Handbook," Vol. V, Communication and Electronics, John Wiley & Sons, Inc., or "Standard Handbook for Electrical Engineers," McGraw-Hill Book Company, Inc.

TABLE II.—WIRE TABLE FOR INTERNATIONAL STANDARD ANNEALED COPPER
American Wire Gauge (B. & S.)

Commonly used in the United States for copper, aluminum, and resistance wire. For hard-drawn copper wire, see page 49.

B. & S. gauge, No.	Diameter, mils, *d*	Area, circular mils, d^2	Ohms per 1000 ft. at 20°C., or 68°F.	Pounds per 1000 ft.
0000	460.00	211,600	0.04901	640.5
000	409.64	167,810	0.06180	508.0
00	364.80	133,080	0.07793	402.8
0	324.86	105,530	0.09827	319.5
1	289.30	83,694	0.1239	253.3
2	257.63	66,373	0.1563	200.9
3	229.42	52,634	0.1970	159.3
4	204.31	41,742	0.2485	126.4
5	181.94	33,102	0.3133	100.2
6	162.02	26,250	0.3951	79.46
7	144.28	20,816	0.4982	63.02
8	129.49	16,509	0.6282	49.98
9	114.43	13,094	0.7921	39.63
10	101.89	10,381	0.9989	31.43
11	90.742	8,234.0	1.260	24.93
12	80.808	6,529.9	1.588	19.77
13	71.961	5,178.4	2.003	15.68
14	64.084	4,106.8	2.525	12.43
15	57.068	3,256.7	3.184	9.858
16	50.820	2,582.9	4.016	7.818
17	45.257	2,048.2	5.064	6.200
18	40.303	1,624.3	6.385	4.917
19	35.890	1,288.1	8.051	3.899
20	31.961	1,021.5	10.15	3.092
21	28.462	810.10	12.80	2.452
22	25.347	642.40	16.14	1.945
23	22.571	509.45	20.36	1.542
24	20.100	404.01	25.67	1.223
25	17.900	320.40	32.37	0.9699
26	15.940	254.10	40.81	0.7692
27	14.195	201.50	51.47	0.6100
28	12.641	159.79	64.90	0.4837
29	11.257	126.72	81.83	0.3836
30	10.025	100.50	103.2	0.3042
31	8.928	79.70	130.1	0.2413
32	7.950	63.21	164.1	0.1913
33	7.080	50.13	206.9	0.1517
34	6.305	39.75	260.9	0.1203
35	5.615	31.52	329.0	0.0954
36	5.000	25.00	414.8	0.0757
37	4.453	19.82	523.1	0.0600
38	3.965	15.72	659.6	0.0476
39	3.531	12.47	831.8	0.0377
40	3.145	9.89	1049	0.0299

the types ordinarily used for electric conductors are affected in substantially the same way. Certain metallic alloys are available, however, having resistances that are affected very little.

In studying the effect of temperature on resistance, it should be remembered that the statements which are made apply, in general, *only over the usual operating range*, say from zero to several hundred degrees Fahrenheit. To study these relations, annealed copper will be used as an illustration because it is the most common electric conductor.

The relation between resistance and temperature for pure *annealed copper only* is shown in Fig. 33. This figure is interpreted as follows: If a specimen of copper has a resistance R_1 at temperature t, it will have a resistance R_2 at temperature t'. Over the usual operating range, a direct relation exists between temperature and resistance as indicated by the straight line. The slope of this line for copper *over the usual operating range* is such that if the line is extended back it will intersect the X axis at -234.5°C.

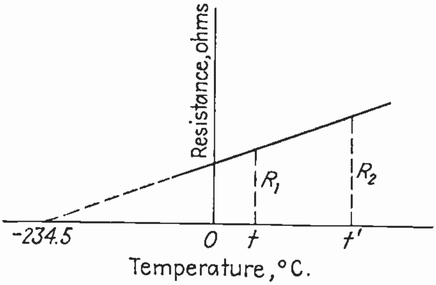

FIG. 33.—If the resistance of pure annealed copper is R_1 ohms at temperature t, it will be R_2 ohms at temperature t'. The resistance appears to be zero at a temperature of -234.5° C.

Now this would *indicate* that at -234.5°C. a copper conductor would have zero resistance, but this is not true. The line shown applies *only* over the *usual* operating range. In fact it is extended to zero arbitrarily as indicated by the fact that it is dotted.[1] How-

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[1] Theoretical and experimental evidence indicate that the temperature of zero resistance is -273°C. This is absolute zero, and a body can be no colder. Within a few degrees of absolute zero, the resistance of a conductor suddenly (almost) vanishes. Provided that it is once started, it is possible to have a large current flow in a conductor near absolute zero with no applied electromotive force. Such phenomena are discussed in books and articles on physics. For an example, see K. K. Darrow, Electricity in Solids, *Bell System Technical Journal*, Vol. III, No. 4.

ever, the value -234.5 is a very useful figure as will now be shown.

From Fig. 33, and the fundamental law of similar triangles (to be explained below), it follows that

$$\frac{R_1}{R_2} = \frac{234.5 + t}{234.5 + t'} \quad \text{and} \quad R_2 = \frac{R_1(234.5 + t')}{234.5 + t}. \qquad (9)$$

As an application of this, suppose that the resistance R_1 of a coil of wire at 20°C. is 6.4 ohms and that its resistance R_2 at 80°C. is desired. Then, from Eq. (9)

$$R_2 = \frac{6.4(234.5 + 80)}{234.5 + 20} = \frac{6.4 \times 314.5}{254.5} = 7.91 \text{ ohms.}$$

Eq. (9) and its application to Fig. 33 are based on the two triangles of Fig. 34. Here are shown two triangles *xyz* and *mno*. The angle θ of each triangle is the *same*. Now if these two triangles are drawn to scale and measured (as has been done in this figure) it follows that $y/n = x/m$. Thus, in Fig. 33 where $y = R_1$, $n = R_2$, $x = 234.5 + t$, and $m = 234.5 + t'$, the relations given by Eq. (9) are true.

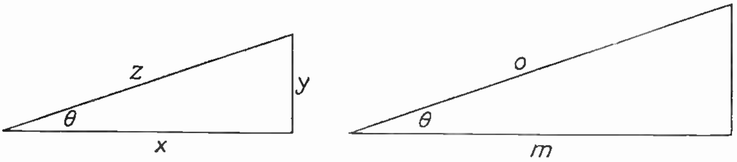

FIG. 34.—Illustrating the relations between similar triangles. If the angle θ is the same, then the lengths of the sides are proportional.

To summarize: For pure metals (as distinguished from alloys) *increasing* the temperature *increases* the resistance and *decreasing* the temperature *decreases* the resistance. Over the usual operating temperature range, a direct proportionality exists between these factors. This makes possible a simple method of calculation of resistance change as given by Eq. (9) based on Fig. 33.

Temperature Coefficient of Resistance.—As was emphasized before, the numerical relations of Fig. 33 and Eq. (9) hold only for pure annealed copper. Of course the general method will apply to any material over the temperature range where there is a straight line relation between temperature and resistance. But, *the value* 234.5 *applies only to annealed copper.* If similar values were available for other conductor materials, the method of Eq. (9) could be used. A different method is, however, used to calculate the change in resistance. This is based on a **temperature coefficient of resistance.**

By definition, the *temperature coefficient of resistance is the increase in resistance per ohm of original resistance at a temperature t per degree centigrade of temperature change above the initial temperature t.* Now that statement by itself seems vague, but its application is very simple and practical. This coefficient for pure annealed copper at a starting temperature of 20°C. is 0.00393 and is usually designated by α, the Greek letter alpha.

Now suppose that the problem of the preceding section is to be solved by this alternate method. From the definition of the temperature coefficient, the *increase* in temperature will be $6.4 \times 60 \times 0.00393 = 1.51$ ohms. The 6.4 is the "per ohm of original resistance," and the 60 is the "per degree temperature change" of the preceding definition. Of course, adding the increase in resistance to the original resistance gives the final resistance, or $R_2 = 1.51 + 6.4 = 7.91$ ohms, the same result as was previously obtained.

The more theoretically inclined readers may desire an equation to express the relations of the preceding paragraph. Such an equation may be derived as follows: The final resistance R_2 equals the original resistance R_1 plus the increase in resistance. From the definition of the temperature coefficient of resistance, the increase in resistance is the product of the original resistance, the change in temperature in degrees centigrade, and the temperature coefficient. Setting down these relations algebraically, the final resistance is

$$R_2 = R_1 + R_{\text{increase}},$$

$$R_{\text{increase}} = R_1(t_2 - t_1)\alpha,$$

and thus

$$R_2 = R_1 + R_1(t_2 - t_1)\alpha,$$

which becomes

$$R_2 = R_1[1 + \alpha(t_2 - t_1)]. \tag{10}$$

One word of warning regarding the use of Eq. (10) is necessary. If the original temperature t_1 is 20°C., then α_{20} the coefficient for a starting temperature of 20°C. must be used. If t_1 is for 0°C., then α_0 must be used. To generalize, if t is the original temperature, then α_t *a coefficient applying to that temperature* must be used. (This is an argument in favor of using the method on page 54 *for copper conductors.* Of course this method also could be used for other conductors if information such as contained in Fig. 33 were available for them.)

In practice, room temperature, assumed to be 20°C., is usually taken as the reference temperature. For this reason, temperature coefficients of resistance are usually specified, with 20°C. as a reference point. Coefficients for various metals used in electrical work are given in Table III.

Attention is called to two important facts in Table III. (1) The temperature coefficient of resistance with a starting temperature of 20°C. *is about* 0.004 *for many of the pure metals.* This figure is a handy one to remember. (2) For hard-drawn copper the *temperature coefficient is less* than for pure copper. (As shown in Table I, the *resistivity* is greater.)

TABLE III.—TEMPERATURE COEFFICIENT OF RESISTANCE FOR CERTAIN METALS

(Based on an initial temperature of 20°C.)

Aluminum (as used in wire)	0.0039
Copper (annealed standard)	0.00393
Copper (hard-drawn No. 12 A.W.G.)	0.00382
Iron (electrolytic melted *in vacuo*)	0.0055 approx.
Iron (wrought)	0.0055 approx.
Lead	0.00387
Nickel	0.0062
Silver (99.78% pure)	0.0040

Much has been said about the *increase* in resistance due to a *rise* in temperature, but little has been said about the *decrease* in resistance due to a *fall* in temperature. Such calculations are made by using Eqs. (9) and (10) in "reverse." No difficulty should be experienced in using Eq. (9); for Eq. (10), however, because the temperature coefficient is based on either 0 or 20°C., the following is recommended. First, change the + sign of the equation to − and calculate the resistance at either 0 or 20°C., depending on which coefficient is used. Then, recalculate the resistance at the desired temperature. In other words, if the resistance at 60 degrees is known and if it is desired at 25 degrees, calculate it at 20 degrees by changing the sign, and then recalculate at 25 degrees using Eq. (10) as it stands. Of course, short cuts are possible after experience has been gained.

To summarize: Although the method explained on page 54 is extremely useful for finding the increase in the resistance of copper conductors, data are not generally available so that this method can be applied to other metals. An alternate method, given by Eq. (10), is widely used. This involves the temperature coefficient of resistance which is defined as the increase in resistance *per ohm of original resistance at a temperature t* per degree of temperature rise above the initial temperature t. Since the statement in italics enters into determining the temperature coefficient, the starting temperature at which the coefficient applies must be given.

Temperature Measurements by Resistance Change.—The increase or decrease in resistance due to temperature change is made use of in the so-called **resistance thermometer.** A resistance thermometer can be devised merely by inserting a suitable wire in the region at which the *average* temperature is desired and then accurately measuring the variations of resistance with a Wheatstone bridge (page 243). This method is also very useful in finding the average temperature of transformer windings, for example. It should be noted that this does not give the temperature of the hottest spot at which charring and insulation failure would probably occur first.

As an illustration, suppose that the secondary windings of a small power output transformer have a direct-current resistance of 5.28 ohms at 20°C., but that after long operation a measurement shows that the resistance has risen to 6.22 ohms. Using Eq. (9),

$$\frac{6.22}{5.28} = \frac{234.5 + t}{234.5 + 20},$$

$$t = 65.3°\text{C}.$$

and the temperature rise = 65.3 − 20 = 45.3°C.

Resistors.—Resistors have been mentioned from time to time, but little has been said about them. It seems unnecessary to describe them in detail because the reader is no doubt very familiar with them. From this it should not be inferred that they are unimportant; in fact, they are *very* important as a look under a radio chassis will prove. Typical resistors are shown in Figs. 35 and 36. Additional information on resistors will appear at various points in this text (see pages 258 and 304).

Something should be said at this point regarding the wires used in wire-wound resistors. Such wires are usually alloys composed of such metals as nickel, chromium, iron, copper, and manganese. If the economic factors and the mechanical properties are disregarded, the electrical characteristics most desired are *high resistance* and *low change in resistance* with temperature variations. (Of course low inductance is also desirable, see page 305). Data regarding certain alloys extensively used in wire-wound resistors (and for other purposes) in communication are given in Table IV. It should be noted that all these coefficients are positive; that is, *increasing* the temperature *increases* the resistance. Alloys are available with negative coefficients.

FIG. 35.—Photograph of a resistor of the "composition" instead of wire-wound type. This resistor will only dissipate one watt. They are made in values of from a few ohms to millions of ohms.

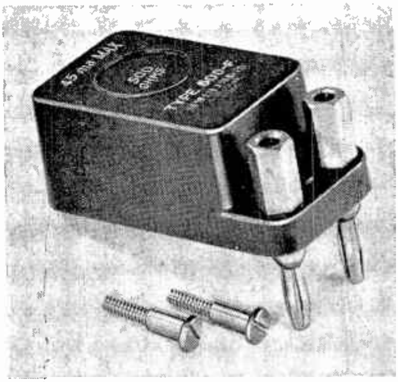

FIG. 36.—An accurate wire-wound noninductive resistor used in bridge measurements. (*Courtesy of General Radio Company.*)

TABLE IV.—CHARACTERISTICS OF RESISTANCE WIRE [1]

Name	Composition	Resistivity at 20°C., ohms per centimeter cube	Temperature coefficient based on 20°C.
Advance	Ni-Cu	0.000048	0.000018
Comet	Ni-Cr-Fe	0.000087	0.0007
German silver	Ni-Cu-Zn	0.0000338	0.00031
Manganin	Cu-Mn-Ni	0.0000478	0.000003
Nicrome II	Ni-Cr-Fe-Mn	0.0001095	0.000172

Cr = chromium   Cu = copper   Fe = iron
Ni = nickel   Mn = manganese   Zn = zinc

[1] These are merely typical of the alloys used and are not complete by far. Because of the various percentages of each metal used, the characteristics will vary. Handbooks should be consulted for detailed and exact data on these and other alloys.

Insulators.—It is usually stated that a current flows in a wire because the wire is a good conductor. Perhaps it would be just as exact to say that the current flows in the wire because the wire is surrounded by a good **insulator.** This insulation may be provided by an insulator such as a glass or porcelain support, or it may be the surrounding air itself. Insulation and insulators are very important, because they *actually touch* the energized wires.[1] Furthermore, they are important because they prevent injury to man and to equipment.

As was mentioned in previous pages, a material is a good conductor if it contains *many free electrons*. A material will be good for resistors if it contains *relatively few free electrons* (and has a low temperature coefficient of resistance, keeping its resistance quite constant as it heats). A material is a good insulator if it has *very few free electrons*, and therefore but an exceedingly small current can flow even if a fairly large voltage is applied.[2]

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[1] Also, as will be considered on p. 188, the insulators and the insulation are in the electric field produced by the voltage impressed on the conductors.

[2] Of course many other characteristics enter into determining the qualities of a material as an insulator. From an elementary electron viewpoint, all that is necessary however is that there are very few free electrons and that the application of a potential difference will not liberate, within the material, electrons which might then constitute a current flow.

Since only a very small current flows in a good insulator even upon the application of very high voltages, it follows that an insulating material has *very low conductivity* and *very high resistivity*. Suppose that it is desired to measure the resistivity of some insulating material such as Bakelite. As for a conductor, resistivity would be the resistance between the two opposite parallel faces of a *centimeter cube* of the material. But, there is a *great difference* involved in the path the measuring current will take in an insulator and in a conductor.

In measuring the resistance of a centimeter cube of a *metal* conductor, the metal offers such a *good* path that the current which flows over the surface is negligible. But in measuring the current which flows from one face to another parallel face of a centimeter cube of an insulating material such as Bakelite *the path over the surface as well as the path through the material itself must be considered.* In fact, if a centimeter cube of Bakelite were being tested, the flow of current over the surface could *even exceed* the flow of current through the body of the material. The flow of current or **leakage current** over the surface would be through paths offered by moisture and "impurities" on the surface. For example, salt spray on the surface of insulators near the ocean and damp alkali dust on insulators in desert areas conduct large leakage currents and reduce the insulation resistance of lines to very low values.

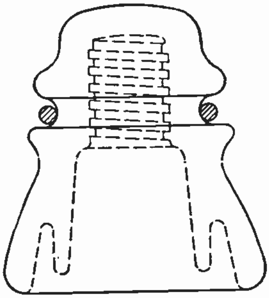

FIG. 37.—Cross section of a telephone insulator.

This principle is illustrated by Fig. 37 which represents the cross section of a telephone insulator. There are two paths for current flow from the conductor in the groove to the pin (often steel). One path is directly *through* the glass between the conductor (and the tie wire) and the pin. The other path is the leakage path *over the surface* of the insulator. As Fig. 37 indicates, this leakage path has been made quite long and the insulator so constructed that the inside portion of the path is kept as dry as practicable.

To summarize: An electric insulation is provided by a material having very few free electrons. In considering insulators, both the resistance to current flow through the body of the insulator and the resistance to the leakage current flow over the surface of the insulator must be considered.

Volume and Surface Resistivity.—Because of the phenomena just considered, *two* resistivities must be considered for insulating materials. These are the **volume resistivity,** or resistance per centimeter *cube* to current flow *through* the insulation, and the **surface resistivity,** or resistance per centimeter *square* to current flow *over the surface* of the insulation.

Since the surface leakage is very greatly affected by the moisture on the surface, the humidity at which the tests were made should be stated. Also all specimens should be cleaned in some accepted manner. Since the path over the surface and that through the material are *in parallel,* special test methods must be employed. Volume and surface resistivities for a few insulating materials are given in Table V.

TABLE V.—CHARACTERISTICS OF INSULATING MATERIALS[1]

Material	Volume resistivity, ohms per centimeter cube at 20–25°C.	Surface resistivity, ohms per centimeter square at 20–25°C.		
		Relative humidity		
		30%	50%	90%
Amber	5×10^{16}	......	6×10^{16}	3×10^{12}
Bakelite	2×10^{16}	4×10^{16}	8×10^{15}	8×10^{14}
Fiber (bone)	2×10^{10}	3×10^{10}	5×10^{9}	3×10^{7}
Glass (plate)	2×10^{13}	8×10^{13}	5×10^{10}	2×10^{6}
Isolantite	2.7×10^{14}	......	1×10^{15}	
Mica (muscovite)	2×10^{17}	1×10^{14}	2×10^{13}	8×10^{9}
Porcelain (wet process)	1×10^{14}	2×10^{13}	6×10^{11}	5×10^{6}
Rubber (hard)	1×10^{18}	6×10^{15}	3×10^{15}	2×10^{9}

[1] Values from "Electrical Engineers' Handbook," Vol. V, Communication and Electronics, John Wiley & Sons, Inc. A number like 2×10^{10} is 20,000,000,000, or 20 billion. Thus the volume resistivity of bone fiber is 20 billion ohms per centimeter cube.

To summarize: The surface and volume current paths are in parallel, and special means must be employed to measure these values. The surfaces of materials to be tested should be cleaned in an accepted manner, and the humidity should be specified (in addition to temperature) because the surface moisture affects the leakage to a marked degree.

Insulation-resistance Calculations.—It is seldom necessary in practical electrical work to calculate leakage currents; furthermore, such calculations are approximate, largely because of irregularly shaped insulators and the variable nature of the resistivity values. Nevertheless, it will be instructive to consider the means for calculating the volume and surface resistance of an insulating body such as in Fig. 38. This same reasoning will, of course, apply to a longer cylinder or rod.

The problem is to calculate the volume current *through* a glass cylinder and the leakage current *over the surface* of the glass cylinder, the dimensions and applied voltage being as shown in Fig. 38. The volume resistivity is 10^{15} ohms per centimeter cube, and the surface resistivity is 10^6 ohms for this particular specimen under the conditions involved. A circular metal plate in intimate contact with the glass covers each end of the cylinder.

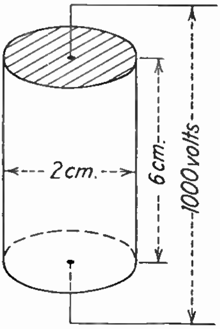

FIG. 38.—Glass cylinder used in illustrating volume and surface leakage.

Step 1. Calculate the current *through* the glass. From the theory on page 50, the resistance of the current path will be calculated in the same general manner as for a wire. Thus, $R = \rho l/a$. As just specified, for the volume of the glass, $\rho = 10^{15}$ ohms per centimeter cube, $l = 6$ centimeters, and a the cross-sectional area equals $\pi r^2 (r = \text{radius}) = 3.1416 \times 1 = 3.1416$ centimeters. Then,

$$R = \frac{10^{15} \times 6}{3.14} = 1.91 \times 10^{15} \text{ ohms.}$$

$$I = \frac{E}{R} = \frac{1000}{1.91 \times 10^{15}} = \frac{1000}{1{,}910{,}000{,}000{,}000{,}000}$$

$$= 0.000000000000524 \text{ ampere.}$$

Step 2. Calculate the current *over the surface* of the glass. To do this it must be imagined that the surface of the glass is "unrolled." If this were done, the surface would constitute a rectangle. The height of this rectangle would be the same as the height of the cylinder, or 6 centimeters. The width of the rectangle would be the distance around the glass cylinder. This is πd (d = diameter), or $2 \times 3.1416 = 6.2832$ centimeters. Now this unrolled surface constitutes a conductor. The resistance of this conductor will vary *directly* as the distance the current has to flow. This distance is the length of the rectangle which equals the height of the cylinder. Also, the resistance of the rectangle will vary *inversely* as the width, because the greater the width (that is, the greater the distance around the cylinder), the more current paths will be in parallel between the two ends of the cylinder. Applying these relations,

$$R = \frac{10^6 \times 6}{6.28} = 0.955 \times 10^6 \text{ ohms,}$$

and

$$I = \frac{E}{R} = \frac{1000}{0.955 \times 10^6} = 1.05 \times 10^{-3} = 0.00105 \text{ ampere.}$$

From these calculations it is apparent that the surface leakage over an insulator may be many times as great as the current passing through the body of the insulator. Thus the importance of clean dry insulators is apparent. Those readers who are familiar with insulation-resistance variations on long open-wire telephone lines will recall that on many lines the insulation resistance will rise quite high when the insulators have just dried following a good rain. At the other extreme, they will recall that the insulation resistance is often quite low when the insulators are dirty (as at the end of a dry period) and the line is enveloped in a damp fog. The insulation troubles due to salt spray and alkali dust were mentioned. In some desert regions, and in industrial areas, it has sometimes been necessary to *wash* the insulators with steam.[1]

SUMMARY

Conductors, resistors, and insulators are fundamentally the same, differing only in the *degree* in which they contain free electrons.

In fact the whole classification is purely relative. Resistors of say 10 megohms used in vacuum-tube circuits might be classed as insulators in other circuits. Furthermore an insulator may become a very good conductor if heated.

The resistance of a body varies directly as the length and inversely as the cross-sectional area of the body.

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[1] See for example, Howard, B. F., Controlling Insulation Difficulties in the Vicinity of Great Salt Lake, *Journal of the A.I.E.E.*, December, 1926.

The resistance per centimeter cube is known as the resistivity of a material.

For the common conductor materials, such as copper or aluminum, increasing the temperature increases the resistance, and vice versa.

To simplify the calculation involving wires, the measure *ohms per circular-mil-foot* instead of ohms per centimeter cube is used.

Many systems of wire gauges are used. The one most widely used in this country is the B. & S. (Brown and Sharpe) or A.W.G. (American Wire Gauge). Much confusion is prevented by specifying a wire by its diameter in mils (1/1000 inch).

Changes in resistance with temperature can be calculated for copper if it is remembered that the curves for copper indicate that at −234.5°C. the resistance of copper would be zero. (Actually the point is near absolute zero or −273°C.)

A temperature coefficient of resistance, defined on page 56, is also used to compute the changes in resistance with changes in temperature.

Alloys are available that have a very small change in resistance with temperature. Some alloys are even available that have a resistance decrease with temperature increase.

Alloys that have a high resistivity, but show small resistance variation, and that have good mechanical characteristics make good stable wire-wound resistors.

In the study of an insulator, the current flow over the surface of the insulator as well as through the insulator must be considered.

REVIEW QUESTIONS

1. Explain why the resistance of a conductor varies directly as the length and inversely as the cross-sectional area.

2. Does the same reasoning as given for Question 1 apply to an insulating rod?

3. Define resistivity.

4. Explain why the circular-mil-foot is used.

5. At what temperatures are resistivity values usually given?

6. From the standpoint of superior conductivity alone, rate the following: copper, aluminum, silver, and lead.

7. Is hard-drawn copper a better conductor than annealed copper? Rate its conductivity as compared with annealed copper.

8. What is meant when it is stated that a conductor is 128-mil copper?

9. What wire gauge is used for telephone cables? For radio push-back wire?

10. How many miles of wire are there in a loop mile? How many miles of wire are there in a loop inside a cable section 1 mile long?

11. Define the temperature coefficient of resistance.

12. What is meant by the statement that the temperature coefficient of resistance is negative?

13. What important precaution must be taken in using temperature coefficients?

14. What is an approximate temperature coefficient for many pure metals?

15. Name some of the electrical specifications you would set down for wire-wound resistors.

16. Explain how a simple resistance "thermometer" can be constructed and used.

17. In what basic manner does the calculation of insulation resistance differ from that of the calculation of conductor resistance? What procedures are essentially the same?

18. Explain the relation humidity bears to insulation resistance.

19. Do you regard insulation-resistance calculations such as on page 63 as highly accurate? How great an error might you expect?

20. Explain fully why the unit per square centimeter is used in specifying surface resistivity.

PROBLEMS

1. Copper is the most common material used for electric conductors. Assume that its *conductivity* is unity, and construct a table for the metals of Table I expressing their conductivity as a percentage of that of copper.

2. Aluminum is sometimes used for bus bars. Calculate the resistance at 20°C. of an aluminum bus bar 12 feet long and 6 inches by 1 inch in cross section.

3. Calculate the resistance at 20°C. of a copper tube having an inside diameter of 0.5 inch, a wall thickness of 0.075 inch, and a length of 16.2 feet.

4. Calculate the resistance at 20°C. of a lead strap which is 20 inches long, 2.1 inches wide, and 0.32 inch thick.

5. Calculate the resistance per foot at 20°C. of an annealed copper wire of 100 per cent conductivity. The wire is 0.229 inch in diameter. Repeat for a hard-drawn wire.

6. A small private-branch-exchange (p.b.x.) telephone switchboard must have a direct voltage of at least 18 volts impressed across it in order to operate satisfactorily. Under peak conditions the board draws 2.2 amperes. The voltage from the main central-office battery of 24 volts is supplied the p.b.x. board over three cable pairs connected in parallel; that is, three wires of three pairs are tied together in parallel for the positive supply and the three other wires of the pairs are tied together for the negative supply. The only pairs available are the three No. 22 A.W.G. Can the board be 1.18 miles (*cable miles*) from the central office and operate satisfactorily? What are your recommendations?

7. What must be the diameter in mils of an aluminum wire to have the same resistance as a hard-drawn No. 2 A.W.G. copper wire?

8. The resistance of an annealed copper wire is 10 ohms at 20°C. Calculate the resistance at 25°C. Calculate the final resistance if it is of hard-drawn copper.

9. Repeat Prob. 3 for 25°C.

10. Repeat Prob. 5 for the wires at 30°C.

11. The resistance of a copper relay winding is 184 ohms at 20°C. After carrying normal current for 20 minutes the resistance is found to be 189 ohms. Calculate the final temperature. Is this an excessive temperature rise?

12. The current through a certain relay winding is 0.144 ampere when 4 volts is impressed across it and the temperature is 22°C. What will the current be if the relay is 4°C. hotter?

13. A wire-wound 1000-ohm resistor is made of No. 36 A.W.G. Nicrome II wire. How many feet must be used at 20°C.? What will be the percentage change in resistance for a 10-degree temperature rise?

14. A radio-antenna strain insulator is made of porcelain, is 8 inches between the two metal end fittings, and is 1.5 inches in diameter. Calculate the direct-current leakage resistance of the insulator and the surface leakage resistance at 20°C. and 30 per cent relative humidity.

15. The direct-current insulation resistance between the two wires of a telephone line 50 miles long is 10,000,000 ohms. What is the insulation resistance per mile?

CHAPTER IV

DIRECT-CURRENT ELECTRIC POWER AND ENERGY

Any system of instantaneous communication is an energy transmission system. When a person speaks, energy in the form of sound waves travels to the listener's ear. The telephone transmitter starts electric impulses out on the line, and thus electric energy is conveyed to the distant telephone. The radio-broadcast station sends electric energy through space to the distant radio-receiving antenna.

The discussions of the preceding chapters have been largely confined to voltages and currents. Voltage by itself does not contain energy; and voltage by itself can do no work. The same is true of current alone. To transmit energy and to do work, voltage and current must act together.

Before considering this subject further, it is advisable to consider certain fundamentals upon which electric power and energy are based.

Force, Work, Energy, and Power. *Force.*—Suppose that the massive body M is placed on a rough surface as in Fig. 39 and it is desired to move the body along the surface *at a uniform rate.* A force F is applied to the body M to move it, but suppose the force is too small. Of course, under these conditions nothing at all happens. Now suppose that the force is increased until the body moves along the rough surface at a uniform rate. *Force may be defined as that which produces or tends to produce a change in the motion of a body.* It will be recalled that electromotive force was defined as that which produces or tends to produce current flow; in other words, voltage causes or tends to cause electrons to move.

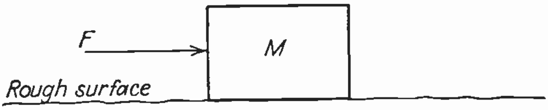

FIG. 39.—Work is done when the force F moves the mass M along the rough surface.

Work.—When the massive body of Fig. 39 slides along the rough surface, the *friction* between the body and the surface causes heat to be developed and the temperature to rise. Energy has been dissipated, and work has been done. *Work is defined as the production of motion against a resisting force.* In this instance, friction was the resisting force. In electric circuits, resistance offers the resisting force to current flow.

Energy.—Fuel oil is burned beneath a boiler so that the energy contained within the oil may be made available to operate a steam engine and do mechanical work by producing motion against a resisting force. *Energy may be defined as the ability to do work.* Work is something done, something accomplished; energy is the *ability* to do work. One of the most fundamental laws of nature is that of the **conservation of energy**: *Energy can be neither created nor destroyed but can be changed from one form to another.* Thus an electric dry cell changes chemical energy to electric energy, and an electric motor changes electric energy to mechanical energy. The efficiency of transformation from one form to another is less than 100 per cent. However, the energy is not lost in the usual sense of the word. The difference between the energy received and the energy transmitted can always be accounted for, usually in the form of heat dissipation.

Power.—A large "powerful" electric motor can do a job of work in a short time. A motor of low power rating will require a much longer time to do the same amount of work. *Power is the time rate of doing work.* If the two motors considered have the same voltage ratings, then the larger powerful motor which does the work in a short time will take a large current from the source. The current taken by the smaller motor will be low, but it will be taken for the longer time required to complete the task.

Electric Power.—Electric power is measured in **watts.** *Electric power in watts is numerically equal to the voltage in volts times the current in amperes.* That is, for direct-current values,

$$P = EI. \tag{11}$$

Thus, if a 6.3-volt storage battery is delivering 0.3 ampere directly to the heater of a type 77 radio vacuum tube, the power dissipated in the heater is $P = EI = 6.3 \times 0.3 = 1.89$ watts.

An illustration using an electric motor may be of interest. Suppose that six 6.0-volt storage batteries are connected in *series* to drive the motor of a motor-generator set used to supply electric energy to an amplifier in a large sound truck. When in operation, the motor takes 9.8 amperes. The power supplied the motor will then be $P = EI = 6 \times 6.0 \times 9.8 = 353$ watts. In the preceding paragraph the power delivered to the vacuum-tube heater was dissipated in the heater. For the motor just considered, much of the electric power delivered to the motor is converted into mechanical power for turning the generator connected to the common shaft.

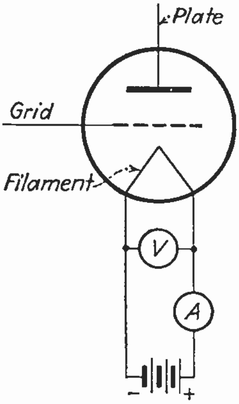

FIG. 40.—The ammeter *A* measures the current taken by the filament of the vacuum tube. The voltmeter *V* measures the voltage across the filament. From these values the current and power taken by the filament can be calculated.

Equation (11) can be written in at least two other very useful forms, explained by the use of Fig. 40, which represents a battery supplying electric power to a vacuum-tube filament. From Ohm's law, the current taken by the filament is $I = E/R$, the resistance of the filament is $R = E/I$, and the voltage drop across the filament is $E = IR$. Using these relations in Eq. (11), gives the following

$$P = EI = IRI = I^2R \tag{12}$$

and

$$P = EI = \frac{EE}{R} = \frac{E^2}{R}. \tag{13}$$

Assuming that, for this filament, $E = 6.3$ volts and $I = 0.3$ ampere, then $R = E/I = 6.3/0.3 = 21$ ohms. Then $P = EI = 6.3 \times 0.3 = 1.89$ watts, $P = I^2R = (0.3)^2 \times 21 = 1.89$ watts, or $P = E^2/R = (6.3)^2/21 = 1.89$ watts.

It is often convenient in electrical work to use units other than the watt for measuring power. Thus, one **kilowatt** = 1000 watts, one **milliwatt** = 1/1000 watt, and one **microwatt** = 1/1,000,000 watt. As illustrations, the output of a large broadcast station may be 50 kilowatts; the standard power input for testing long telephone lines is taken at 1.0 milliwatt; and the average speech power output of the human voice is about 10 microwatts. Another power unit often used is the **horsepower**; one horsepower equals 746 watts.

To summarize: *Power is the rate of doing work.* It tells how fast a device may be at accomplishing a job. Electric power is given by any one of three equations $P = EI = I^2R = E^2/R$. In applying these equations, be sure that E is the actual voltage *across the device*, that I is the actual current *through the device*, and that R is the resistance *of the device*. The kilowatt (1000 watts), milliwatt (1/1000 watt), and microwatt (1/1,000,000 watt) are extensively used for measuring power in electric circuits.

Measurement of Electric Power.—The student should remember that in this chapter power in *direct-current* circuits only is considered and that certain modifications are necessary to extend this theory to *alternating-current* circuits (see page 261).

The measurement of power in direct-current circuits is usually very simple and can ordinarily be done with only an ammeter and a voltmeter. There are certain precautions to be followed, and these will now be considered. For comparative purposes, Fig. 40 has been redrawn and extended in Fig. 41.

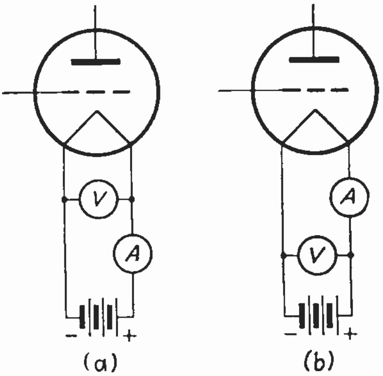

FIG. 41.—In *a* the ammeter *A* measures the sum of the current taken by the filament and that taken by the voltmeter *V*. In *b* the voltmeter *V* measures the *IR* voltage drop across the ammeter *A* plus the voltage across the filament.

It was stated on page 70 that the power dissipated in heating the filament of Figs. 40 (and 41*a*) would be $P = EI$, that is, the power dissipated in watts would be the product of the voltage across the filament in volts, and the current through the filament in amperes. This is true, but the ammeter A of Figs. 40 and 41*a* does not measure the current taken by the filament, it measures the sum of the current taken by the filament and the current taken by the voltmeter, because the voltmeter is a high-resistance electrical device in parallel with the filament.

It might be thought that all errors could be eliminated by connecting the circuit as in Fig. 41*b*, so that the voltmeter is below the ammeter and hence the ammeter reading does not include the current taken by the voltmeter. But in changing the connections from Fig. 41*a* to Fig. 41*b*, another error entered. In this second circuit the current is the correct value, but the voltage read by the voltmeter V is the sum of the voltage across the filament and the voltage drop across the ammeter A. A good ammeter is a very low-resistance device, but a poor ammeter may have sufficient resistance to cause an appreciable voltage drop across it.

Of course the logical question is this: Which way should the voltmeter and ammeter be connected so that the least error will result? The answer must be indefinite, it depends on the internal resistances of the ammeter and voltmeter and upon the resistance of the load.

It is not advisable to study ammeters and voltmeters at this time. However, it can be said that the internal resistance of an *ideal* voltmeter should be infinite, and that the internal resistance of the *ideal* ammeter should be zero. If these were the conditions, then either of the circuits of Fig. 41 would be satisfactory. From the practical standpoint, good high-resistance direct-current voltmeters, having a resistance of 1000 ohms per volt, will seldom cause much error even if connected as in Fig. 41*a*. Similarly, good low-resistance direct-current ammeters, having a resistance of about 0.01 ohm, will seldom cause much error even if connected as in Fig. 41*b*. High-resistance voltmeters and low-resistance ammeters should always be used; then, if the load resistance is low, use the connections of Fig. 41*a*, and if the load resistance is high, use the connections of Fig. 41*b*.

To summarize: Power in direct-current circuits is readily measured with a direct-current ammeter and voltmeter. A slight error may creep in, however. To avoid appreciable error, certain precautions outlined in this section should be followed.

Example of Power Calculations.—Calculations of direct-current power were treated on page 69. Additional examples will now be given. The circuit of Fig. 42 is the filter and "bleeder" of a power supply with the numerical values given on the diagram. Suppose it is desired to calculate the direct-current power loss in each choke coil and in the bleeder.

Step 1. Calculate the current flow. Since the condensers pass no direct current, the two chokes and the bleeder are in series with the direct-current voltage of 500 volts. The total resistance is $R_t = 200 + 200 + 30{,}000 = 30{,}400$ ohms. The current which will flow is $I = E/R_t = 500/30{,}400 = 0.01645$ ampere.

Step 2. Calculate the power loss in each choke coil. For each choke, $P = I^2R = (0.01645)^2 \times 200 = 0.0542$ watt. Or, this can be done by finding the voltage across each choke and then multiplying the voltage by the current as in Eq. (11). Thus, $E = IR = 0.01645 \times 200 = 3.29$ volts, and $P = EI = 3.29 \times 0.01645 = 0.0542$ watt.

Step 3. Calculate the power loss in the bleeder.

$$P = I^2R = (0.01645)^2 \times 30{,}000 = 8.12 \text{ watts.}$$

Or

$$E = IR = 0.01645 \times 30{,}000 = 493.5 \text{ volts.}$$

Then

$$P = EI = 493.5 \times 0.01645 = 8.12 \text{ watts.}$$

Step 4. The sum of the power dissipated in each element must equal the total power delivered by the source (from the conservation of energy principle), so this may be used as a check. The total power delivered is $P_t = 0.0542 + 0.0542 + 8.12 = 8.228$ watts. Also, $P_t = EI = 500 \times 0.01645 = 8.225$ which is in close agreement with the sum of the power taken by each element.

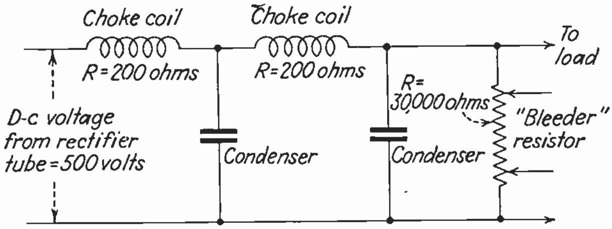

FIG. 42.—Circuit representing a filter for connection between a rectifier tube and a load such as vacuum-tube plates. The choke coils and condensers serve to keep alternating-current components from the rectifier tube from reaching the load. The "bleeder" is a voltage divider.

From the foregoing calculation it is evident that the bleeder resistor must be capable of continuously taking at least about 10 watts from the circuit. If current is taken at one or more of the various voltage taps shown in Fig. 42, then additional current will flow in portions of the bleeder, and the I^2R losses in those portions will be greater. For this reason, and to provide a factor of safety (that is, to assure that the bleeder will be of sufficient capacity under all reasonable conditions of operation), a 25-watt resistor would probably be selected.

Power-handling Capacity.—The preceding discussion leads directly to a consideration of the power-handling capacity of a device. It has been shown that when current flows through resistance, power in watts given by the relation $P = I^2R$ is dissipated.

This means that electric power is expended, and since from the law of conservation of energy power cannot be *lost* in the usual meaning of the word,[1] the power dissipated must be converted into heat.

Of course this generation of heat will raise the temperature of a resistor (or other electric device), and an excessive temperature rise may permanently damage the resistor. This is illustrated by Fig. 43 which includes four resistors, each of the same resistance but of different current-carrying ability and, hence, power-handling capacity.

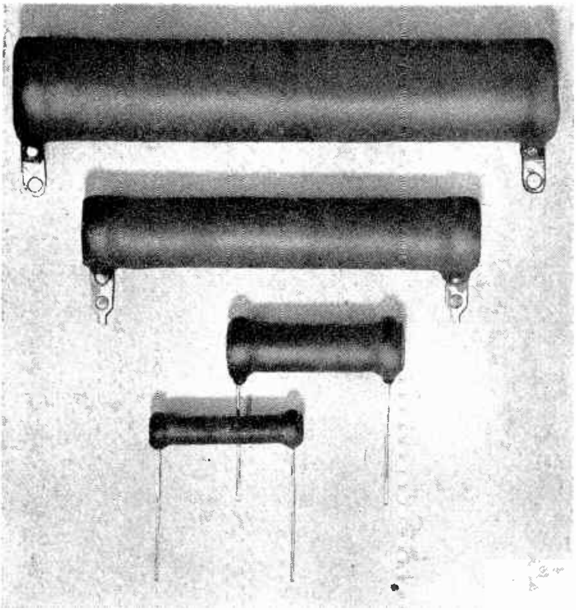

FIG. 43.—Four wire-wound resistors of the same resistance values. The different sizes make possible different current-carrying capacities. The upper resistor will dissipate 100 watts, the next lower one 50 watts, the third one 20 watts, and the lowest one 10 watts. (*Courtesy of International Resistance Company.*)

The question may be asked: Why are the resistors of higher current and power-handling capacity physically larger than those of lower power capacity? To answer this question the factors deter-

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[1] It is quite common in electrical practice to say that the *power loss* is a certain amount. Of course, what is meant is that *electric power* is lost, in that it is converted into heat or otherwise dissipated. (At high frequencies, for example, power is radiated into space.)

mining temperature rise must be considered. Most telephone, telegraph, and radio equipment is made to operate in air.[1] The rate at which heat is transferred from a resistor (or other device) is affected by the area of the resistor in contact with the air.

This means that a physically large resistor using large wire will be able to handle a larger current and dissipate more power than a physically small resistor wound with small wire. This also explains why cooling fins (such as on power tubes in radio-broadcast sets) are effective—they increase the radiating area in contact with the air. Of course moving cool air rapidly by these fins will also prevent an excessive temperature rise. An excessive temperature rise damages equipment in a variety of ways, but particularly in charring insulation and reducing its insulating properties and in warping and otherwise damaging metal parts.

To summarize: In the design and operation of electric equipment, care must be taken to select devices that have sufficient power-handling capacity. Thus, a few inches of a hairlike resistance wire may have 500 ohms resistance, but the current it can carry and its power-handling capacity are both extremely low. On the other hand, a few feet of a larger wire of the same material will have the same resistance and can carry a large current and dissipate much power without having a damaging temperature rise.

Electric Energy.—Man uses electricity to do his work for him: to carry his messages, to bring him entertainment, to illuminate his home, and to do his every bidding. As was shown, starting on page 68, work is the production of motion against an opposing force, power is the rate of doing work, and energy is the ability to do work. Thus, *electric energy* is provided by a dry cell or is purchased from the electric-utility company. Energy, or the ability to do work, is measured by the watt-hour meter at the electric service entrance; the purchaser may put this energy to use as he desires and may take the energy at any rate (within reason) that he may wish. That is, the power demand at a given instant depends on the needs of the user.

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[1] This is in contrast with electric-power equipment, where many of the devices, such as transformers, operate in oil. The oil more readily carries the heat to the surface of the tank where the large radiating area in contact with the air keeps the oil temperature low. Of course the oil also improves the insulation of the windings.

Electric power has been defined as the rate of doing work (electrically). Thus, power multiplied by the time over which the power is used gives work, or energy. That is,

$$\text{Work or energy} = Pt = \text{watts} \times \text{hours} = \text{watt-hours}. \quad (14)$$

Of course, work and energy [1] can be measured in other units such as watt-seconds and kilowatt-hours.

To summarize: Electric energy, which is the product of electric power and the time over which the power is used, is a unit of basic importance. Electric energy is usually measured in watt-hours and kilowatt-hours.

Examples of Energy Calculations. *Example* 1.—Suppose that in a large telephone central office a motor-generator set is being used to charge the central-office storage batteries. The generator delivers an average current of 460 amperes at 30.8 volts for 2.75 hours. The efficiency of the generator is 88 per cent, and the efficiency of the motor is 90.2 per cent. Calculate the cost of the energy to the telephone company if the rate is 1.0 cent per kilowatt-hour.

Step 1. Calculate the energy delivered by the generator.

$$W = EIt = \frac{30.8 \times 460 \times 2.75}{1000} = 38.9 \text{ kilowatt-hours.}$$

Step 2. Calculate the energy taken by the generator.

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} \quad \text{and} \quad \text{input} = \frac{\text{output}}{\text{efficiency}},$$

hence

$$\text{Energy input} = W_i = \frac{38.9}{0.88} = 44.2 \text{ kilowatt-hours.}$$

Step 3. Calculate the energy taken by the motor.

$$W_i = \frac{44.2}{0.902} = 49.1 \text{ kilowatt-hours.}$$

Step 4. Calculate the cost.

$$\text{Cost} = 49.1 \times 0.01 = \$0.491.$$

Example 2.—A large triode used in a radio transmitter requires 60 amperes at 20 volts for heating the filament. The station is on the air 16 hours a day

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[1] Remember, these two quantities are different but are measured in the same units. Energy is the ability to do work, and work is what is accomplished by energy. In other words, the efficiency of utilization enters. Thus, a person may be quite energetic, but without a reasonable degree of personal efficiency, little work is done.

and 30 days a month. Calculate the monthly cost of heating this filament if power costs 1.0 cent per kilowatt-hour.

Step 1. Calculate the power used.

$$P = EI = 20 \times 60 = 1200 \text{ watts} = 1.2 \text{ kilowatts.}$$

Step 2. Calculate the energy consumption per month.

$$W = Pt = 1.2 \times 16 \times 30 = 575 \text{ kilowatt-hours.}$$

Step 3. Calculate the cost per month.

$$575 \times 0.01 = \$5.75.$$

Example 3.—As in Fig. 44, four loud-speakers with 6-ohm field coils are located 246 feet from a 6.3-volt storage battery of negligible internal resistance. The coils are connected in parallel and are fed over a No. 12 A.W.G. annealed copper wire circuit. Calculate the current that each coil receives, the total power delivered to the coils, the total power supplied by the battery, the power loss in the line, and the efficiency of the transmitting circuit.

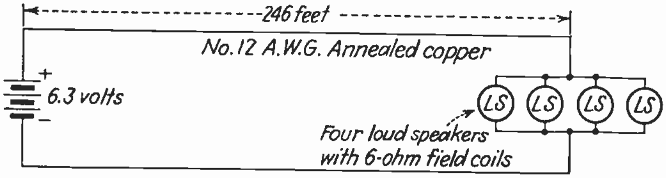

FIG. 44.—Showing a 6.3-volt storage battery supplying four loud-speaker fields in parallel.

Step 1. Calculate the line resistance. A circuit 246 feet long requires 492 feet of wire. The resistance per 1000 feet of wire this size is 1.588 ohms at 20°C. The resistance of the circuit will be $(492/1000) \times 1.588 = 0.78$ ohm.

Step 2. Calculate the equivalent resistance of the four coils in parallel. The equivalent resistance of four identical coils in parallel is one-fourth that of one coil alone. This follows from Eq. (4), page 22. Hence, $R_e = 6/4 = 1.5$ ohms.

Step 3. Calculate the total resistance of the circuit. This will be the sum of the equivalent resistance of 1.5 ohms and the wire resistance of 0.78 ohm or $R_t = 2.28$ ohms.

Step 4. Calculate the current that will flow. $I = E/R = 6.3/2.28 = 2.76$ amperes.

Step 5. Calculate the current each loud-speaker field coil will receive. Since there are four identical coils in parallel, each coil will receive one-fourth the current. Thus, each coil receives $2.76/4 = 0.69$ ampere.

Step 6. *This current is too low*, because the speaker coils are designed to operate directly across a 6-volt battery. A plentiful supply of No. 12 copper is on hand, so it is decided to run *two* wires in parallel for each side of the circuit. This will of course halve the resistance of the wire circuit making it $0.78/2 = 0.39$ ohm. The total resistance is now $1.5 + 0.39 = 1.89$ ohms. The current will be $6.3/1.89 = 3.33$ amperes, and this will give $3.33/4 = 0.83$ ampere for each field coil. This is deemed sufficient for the conditions of this installation.

Step 7. Calculate the power delivered to the coils. Power is equal to I^2R. The power delivered to the four speaker coils will be I^2R_e, where I is the total current delivered and R_e is the equivalent resistance of the coils in parallel. Thus, $P = (3.33)^2 \times 1.5 = 16.6$ watts.

Step 8. Calculate the total power supplied by the battery. This is easily calculated from $P = EI = 6.3 \times 3.33 = 21.0$ watts.

Step 9. Calculate the power loss in the line. This will be the difference between the power transmitted and the power received. Thus, $21.0 - 16.6 = 4.4$ watts. As a check, $P = I^2R$, where R is the line resistance. $P = (3.33)^2 \times 0.39 = 4.32$ watts, a fairly close check.

Step 10. Calculate the efficiency of the transmitting system. Efficiency equals power delivered to the load divided by the power supplied to the line by the battery. Efficiency $= 16.6/21.0 = 79$ per cent.

Power Transfer.—The efficiency of the circuit in the example just given is not high; however, only about 4.3 watts are lost, and such a small amount of power costs very little. In contrast, a transmission line in a commercial power system would probably be redesigned if calculations showed that its efficiency would be only 79 per cent. But in a power system, large amounts of power are to be transmitted, and a line of such low efficiency would prove costly to operate because it wasted so much power; in fact, 21 per cent of the power put into it.

In communication work, the magnitudes of the values of power involved are quite often very low. Thus a current of 0.1 ampere through a 60-ohm telephone transmitter is more than sufficient for its successful operation, and this represents a power of only $P = I^2R = (0.1)^2 \times 60 = 0.6$ watt. What difference does it make from a power or energy-cost standpoint if a few tenths of a watt are lost in a line? *In communication work, considerations other than efficiency often are the determining factors in circuit design.* One of the most important of these factors is **maximum power transfer.**

Before considering the theoretical relations involved in power transfer, several practical illustrations will be given. Suppose it is desired to design a battery-operated pack radio transmitter to be carried by a man for forest-fire patrol work or for military operations. The set is to be used only in emergencies, and then such things as efficiency lose most of their importance; portability and high signal output for a short period are the most desirable characteristics. In such a set, the lighter the batteries are and the greater the power drain from them (within reason, of course), the lighter and better the pack transmitter will be.

As a further illustration, consider the telephone transmitter previously discussed. In one sense, it is a generator or source of feeble electric impulses varying as the speech waves striking the transmitter diaphragm. Of course vacuum-tube amplifiers could be used at each telephone installation, but this would be reflected in the monthly rate because the capital investment and the maintenance would be greater for each telephone (and there would also be an energy charge involved). For such reasons as these, it is more practical not to use amplifiers but to design the telephone sets so that the maximum possible amount of electrical speech power is taken from the transmitter and delivered to the connected circuit and thence to the listening party.

In working out the rules for obtaining maximum power transfer from a source of electromotive force into a load, a dry cell will be considered, the cell having an open-circuit voltage E_{oc}, or electromotive force of 1.5 volts, and an internal resistance of 0.1 ohm. The circuit arrangement for studying these relations is shown in Fig. 45*a*. If a dry cell is operated continuously, internal chemical action causes the generated electromotive force to drop and the internal resistance to rise. For this reason, in making a test such as will be explained, *the switch S should be closed only for an instant.*

Condition 1. Adjust R_L equal to 0.05 ohm, which is one-half of R_i. Close the switch S, and quickly read the voltmeter V giving the voltage E_L across the load and the ammeter A giving the load current I_L. Calculate the power delivered to the load.[1] For the values given,

$$I_L = \frac{E_{oc}}{R_i + R_L} = \frac{1.5}{0.1 + 0.05} = \frac{1.5}{0.15} = 10 \text{ amperes.}$$

$$P_L = I_L^2 R_L = (10)^2 \times 0.05 = 5.0 \text{ watts.}$$

Condition 2. Adjust R_L equal to 0.075 ohm which is three-fourths of R_i, and repeat.

$$I_L = \frac{1.5}{0.175} = 8.57 \text{ amperes.} \quad P_L = (8.57)^2 \times 0.075 = 5.5 \text{ watts.}$$

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[1] In actually making the tests, the power delivered to the load will be $P_L = E_L I_L$. For the explanation here given, it will be necessary to calculate the current flowing and determine the power delivered from $P_L = I_L^2 R_L$.

Condition 3. Adjust R_L equal to 0.1 ohm, which equals R_i, and repeat.

$$I_L = \frac{1.5}{0.2} = 7.5 \text{ amperes.} \quad P_L = (7.5)^2 \times 0.1 = 5.62 \text{ watts.}$$

Condition 4. Adjust R_L equal to 0.125 ohm, which is five-fourths of R_i, and repeat.

$$I_L = \frac{1.5}{0.225} = 6.66 \text{ amperes.} \quad P_L = (6.66)^2 \times 0.125 = 5.55 \text{ watts.}$$

Condition 5. Adjust R_L equal to 0.15 ohm, which is six-fourths of R_i, and repeat.

$$I_L = \frac{1.5}{0.25} = 6.0 \text{ amperes.} \quad P_L = (6.0)^2 \times 0.15 = 5.4 \text{ watts.}$$

If the values of power delivered to the load are studied, it will be seen that these values *increase* as the load resistance is increased and made more nearly equal to the internal resistance. It will also be seen that the power delivered to the load resistance *is maximum* when the load resistance R_L equals the internal resistance R_i and that as greater values of R_L are used the power delivered *decreases*. These relations are plotted in Fig. 45*b*.

A somewhat startling result is revealed if a few additional computations are made. Since the efficiency of *power transfer* from the battery to the load equals the power delivered divided by the total power generated, and since the total power generated is $P_t = E_{cc}I_L$, the efficiencies for the five conditions considered are as follows:

Condition 1. $P_t = 1.5 \times 10 = 15.0$ watts.

$$\text{Eff} = \frac{5}{15} = 0.33 = 33\%.$$

Condition 2. $P_t = 1.5 \times 8.57 = 12.85$ watts.

$$\text{Eff} = \frac{5.5}{12.85} = 0.427 = 42.7\%.$$

Condition 3. $P_t = 1.5 \times 7.5 = 11.25$ watts.

$$\text{Eff} = \frac{5.62}{11.25} = 0.50 = 50\%.$$

Condition 4. $P_t = 1.5 \times 6.66 = 10.0$ watts.

$$\text{Eff} = \frac{5.55}{10} = 0.55 = 55\%.$$

Condition 5. $P_t = 1.5 \times 6.0 = 9.0$ watts.

$$\text{Eff} = \frac{5.4}{9} = 0.60 = 60\%.$$

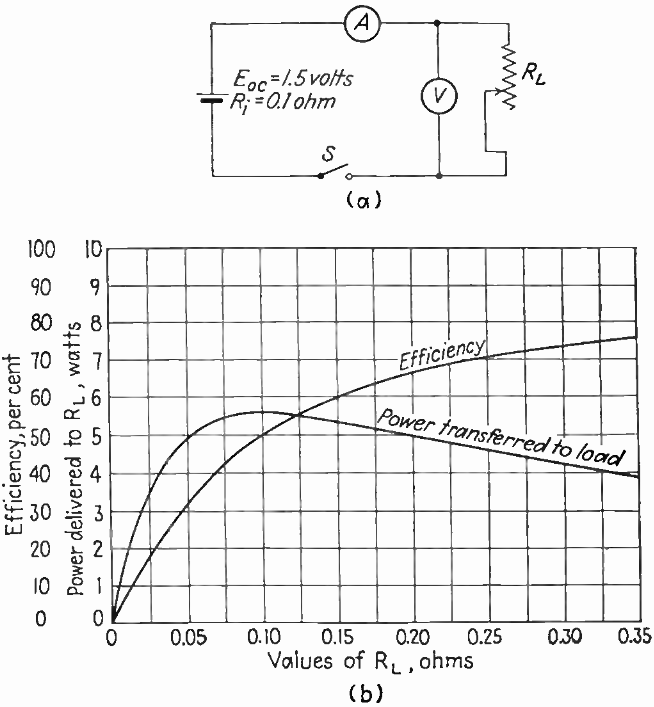

FIG. 45.—Circuit for studying power transfer; and results of the study. Maximum power is transferred from a source to a load when the resistance of the load equals the internal resistance of the source. At this point the efficiency is 50 per cent.

It is found that although the maximum power transfer occurs when the load resistance equals the internal resistance (that is, when $R_L = R_i$), under these conditions, *the efficiency is* 50 *per cent.*

To summarize: In much electrical work, high efficiency is very important, but in communication circuits, considerations other than high efficiency, such as maximum power transfer, are often the governing factors. *Maximum power transfer is obtained from a device such as a dry cell, battery, or generator when the load resistance equals the internal resistance, but under these conditions the efficiency is only* 50 *per cent.*

Transmission of Electric Energy.—Those engaged in communication are less often concerned with the transmission of electric energy in large amounts than are workers in the electric-power field. Therefore efficiency of transmission is often sacrificed in favor of such factors as maximum power transfer as considered in the preceding section. Nevertheless, instances do occur when efficiency of transmission is of great importance, and those in communication should be prepared to cope with such circumstances.

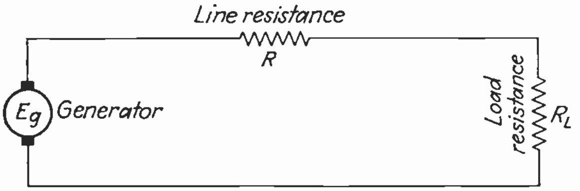

FIG. 46.—Because the power loss in the line is I^2R, much less loss will occur in transmitting power from the generator to the load if high voltage and low current is used.

Assuming that the line is reasonably short so that insulation leakage losses are negligible, and assuming that direct current only is to be transmitted, the following considerations apply: The power lost in the transmission line of Fig. 46 equals I^2R watts, where I is the current flowing in the line and R is the resistance of the line. Now the power entering the line at the generator is $P_g = E_gI$, and the power received by the load is $P_L = E_LI$, the current I being the same in all parts of the circuit because it is a simple series circuit, the leakage between wires being negligible.

Going back over these relations it is apparent that (1) a given amount of power may be transmitted at either high voltage or low current or low voltage and high current; (2) the power loss in the line is I^2R, and if high voltage and low currents are used, the line losses will be greatly decreased; and (3) the losses may be kept the same per cent of the power delivered, but much smaller wires may be used if high voltage and low currents are employed. This will of course reduce conductor costs and fixed charges (interest on investment, depreciation, etc.).

To summarize: If there is a choice in the matter, *transmitting electric energy at high voltage and low current is very desirable and effects material savings* up to a point where the high voltage becomes bothersome by increasing insulator costs and requiring special construction, etc. In practice, the selection of a wire size is often based on the costs involved.

Voltage Regulation.—Although the statement just made applies in many instances, **voltage regulation** is often a determining characteristic. This, again, is a factor of very great importance in power work but often of less importance in communication. The regulation of a line is defined as *the rise in the voltage at the distant end of the line when the load is changed from full load to no load, divided by the full load voltage.* Of course this definition can be reworded to apply to a dry cell, a storage battery, or an electric generator, because the same principles hold. Several examples of voltage regulation will be given.

Example 1.—The internal resistance of a direct-current generator is 0.46 ohm. The generator develops an electromotive force of 124 volts. Calculate the regulation of this generator when used with a 12.4-ohm load.

Step 1. Calculate the current that will flow.

$$I = \frac{E}{R_t} = \frac{124}{12.86} = 9.65 \text{ amperes.}$$

Step 2. Calculate the generator closed-circuit voltage, E_{cc}. This will equal the generated emf minus the IR drop within the generator.

$$E_{cc} = 124 - (9.65 \times 0.46) = 124 - 4.45 = 119.55 \text{ volts.}$$

Step 3. Calculate the regulation.

$$\text{Regulation} = \frac{E_{oc} - E_{cc}}{E_{cc}} = \frac{4.45}{119.55} = 0.037 = 3.7 \text{ per cent.}$$

Example 2.—The direct-current portion of a simple rectifier is as shown in Fig. 47. The rectified current (direct current) must flow through the circuit having the resistances shown. When the switch S is closed the load, assumed equal to 30,000 ohms, is connected. (In an actual circuit this load would probably be vacuum-tube plates.) The open-circuit voltage E_{oc} across points 1–2 is 365 volts. Calculate the regulation.

Step 1. As far as the 30,000-ohm load is concerned, the circuit to the left of points 1–2 is merely a generator of open-circuit E_{oc} voltage (switch S open) and a given internal resistance. The voltage existing across points 1–2 with the switch closed must be calculated. When this closed-circuit voltage E_{cc} is known, the regulation can be computed. The first step is to calculate the current that flows when the switch is open. The IR drop across the 50,000-ohm bleeder is 365 volts. Then the current will be $I = E/R = 365/50{,}000 = 0.0073$ ampere.

Step 2. Calculate the value of the total direct voltage impressed by the rectifier. This will equal the 365 volts (termed E_{oc}) plus the total IR drop in the rest of the circuit. The resistance of the remainder of the circuit is 1100 ohms. Then, $365 + (0.0073 \times 1100) = 373$ volts (approx.).

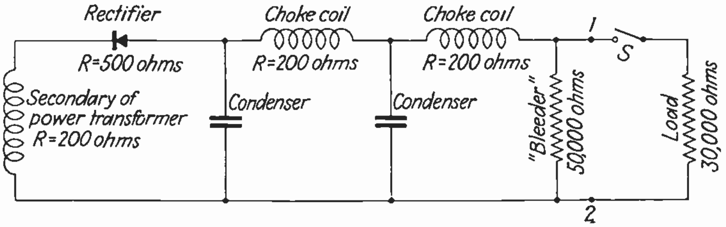

FIG. 47.—Circuit for calculating the voltage regulation of a rectifier and filter.

Step 3. It is now necessary to find the current that will flow through the rectifier when the load is connected. Before this can be done, the equivalent resistance of the bleeder and the load must be known. From page 21, this is $R_e = R_1R_2/(R_1 + R_2) = 50{,}000 \times 30{,}000/(50{,}000 + 30{,}000) = 18{,}750$ ohms. Then the total resistance offered to the rectified voltage is $R_t = 18{,}750 + 1100 = 19{,}850$ ohms. The current through the rectifier will equal the total rectified voltage divided by this load resistance, or $I_t = 373/19{,}850 = 0.0188$ ampere.

Step 4. Calculate the voltage E_{cc}, the voltage across the load with switch S closed. This is easily found from the product of the total current and the equivalent resistance. Thus, $E_{cc} = 0.0188 \times 18{,}750 = 352$ volts.

Step 5. Calculate the percentage regulation. This equals

$$\frac{E_{oc} - E_{cc}}{E_{cc}} = \frac{365 - 352}{352} = 0.037 = 3.7 \text{ per cent.}$$

Sources of Direct-current Energy.—The common sources of direct-current energy for communication circuits and equipment include rectifiers, direct-current generators, wet cells, dry cells, and storage batteries. Because vacuum tubes have not been thus far considered, rectifiers will not be treated until page 442; also, the principle of the electric generator (page 155) cannot be explained until the fundamentals of magnetism have been presented. Hence the following pages will consider only wet cells, dry cells, and storage batteries.

Wet Cells.—One of the earliest electrical phenomena known was that if two different metals were immersed in an electrolyte, such as dilute sulphuric acid, a difference of potential would exist between the two metals. Such an arrangement constitutes a simple **primary cell** of the **wet-cell** variety. The difference of potential existing between the two metals (and also between the two cell terminals connected to them) is caused by an electromotive force generated by the chemical action within the cell. Today wet cells are relatively unimportant as compared with dry cells and storage cells, but nevertheless there are many of them in use, and they will be briefly described.

The Gravity Cell.—This consists of a glass jar in which two different electrolytes are used. A copper sulphate solution and a solid copper electrode are placed in the bottom of the jar. A rubber-covered wire is connected to the copper electrode and leads to the external circuit. A zinc electrode, often cast in the form of a "crow's-foot" is suspended from the upper edge of the jar, and a zinc sulphate electrolyte is added on top of the copper sulphate electrolyte until the zinc electrode is completely covered. The two electrolytes have different densities or "weights" and are thus prevented from mixing. These cells were once very widely used and are still often found in isolated telegraph systems. They must be used on circuits that are in continuous operation or the cell is damaged internally by chemical action. The open-circuit electromotive force is a little over one volt.

Air Cell.—This cell is sometimes used with small magneto switchboards to supply current for the operator's telephone transmitter. It is quite widely used in battery-operated radio receiving sets to supply the energy for heating the tube filaments. For this purpose two cells are connected in series, giving the air-cell battery which has a voltage of about 2.5 volts when new. This drops but slightly during the normal life, finally reducing to about 2.0 volts at the end of its life. In this common form the air-cell battery will supply about 0.5 ampere for about 1200 hours.

The air cell consists essentially of a negative zinc plate and a porous carbon positive electrode immersed in an electrolyte of caustic soda. A cross-sectional diagram is shown in Fig. 48. The zinc is dissolved chemically during the operation. The complete chemical action is involved and will not be included. The name "air cell" is derived from the fact that air is absorbed in considerable quantities by the porous carbon electrode. The oxygen thus taken into the cell from the outside air combines with hydrogen liberated inside the cell at the positive porous carbon electrode to form water which enters the **electrolyte.** This reduces **polarization.**

Polarization is the name applied to the formation of a layer of gas on a cell electrode. Hydrogen is a gas often liberated at the positive electrode and clings to the electrode surface. If the positive electrode is covered with hydrogen, then the electrolyte cannot comein contact with the electrode material, the battery voltage is reduced, and the internal resistance of the cell is increased. Air-cell batteries are "primary" batteries and like dry batteries cannot be completely recharged as can storage batteries. For many purposes they are more economical than dry batteries.

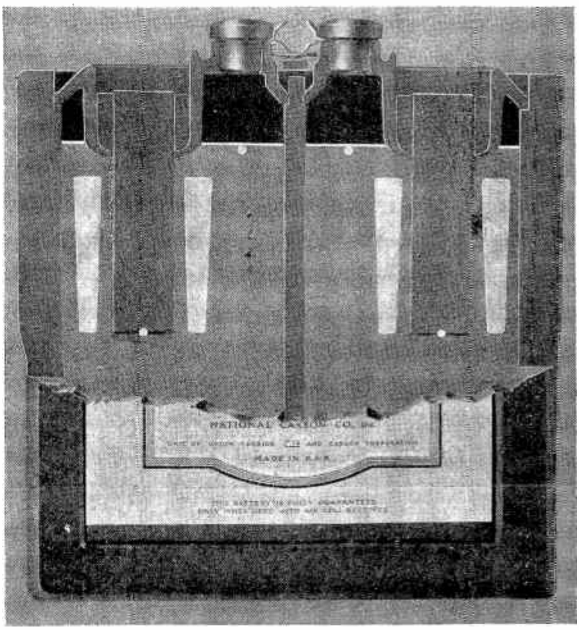

FIG. 48.—Cross section of an air-cell battery. The large carbon electrodes are shown at the center of each cell. The zinc electrodes are shown on each side of the carbon electrodes. (*Courtesy cf National Carbon Company.*)

Dry Cells.—Most readers will be quite familiar with dry cells. Of course they are not *dry;* if they were, they would be useless. A cross section of a commercial dry cell is shown in Fig. 49, and the materials used are indicated. Even in this familiar cell the complete chemical action is involved. Briefly, however, the action is as follows: The zinc is acted on chemically by the electrolyte held in the ground carbon mixture. The porous absorbent medium permits the electrolyte to attack the zinc but keeps the ground carbon mixture from short-circuiting the cell internally.

During action, hydrogen is given off at the positive carbon electrode, and this tends to cause polarization as previously considered. The manganese dioxide combines with the hydrogen to form water

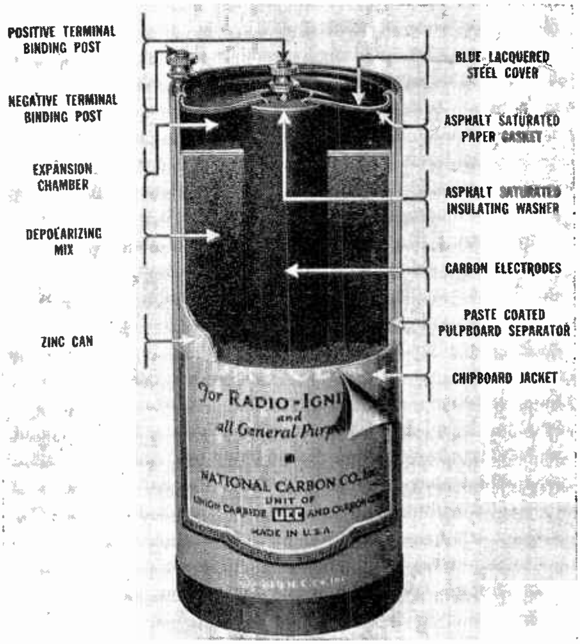

FIG. 49.—Cross section of the ordinary "dry" cell. (*Courtesy of National Carbon Company.*)

and is known as a **depolarizer.** The open-circuit voltage or emf of the cell is about 1.5 volts. It decreases a little with age, but cannot be used to indicate the internal condition of a cell. The best indication is whether it will maintain for a short time a current flow of about an ampere with a reasonable closed-circuit voltage. Actual test procedure for this will depend on the type of cell. The internal resistance is about 0.2 ohm, depending of course on the type of cell, its age, and the value of the test current.

Some dry cells are designed to supply heavier currents than others. Those used in telephone service are for low current use and a long life. Radio B batteries are composed of a large number of small dry cells connected in series. Excessive currents should never pass through these batteries. They should be tested with a voltmeter when delivering a reasonable current, for example, 20 milliamperes. If the ordinary 45-volt B battery falls below about 40 volts when such a current passes through it, the useful life has about expired.

The internal resistance of dry cells and B batteries may rise to very high values, even opening the circuit at times. For this reason, they should always be replaced when they have deteriorated. As is well known, internal chemical action often called **local action** causes such cells to deteriorate even when they are not in use. The advantages of new "fresh" batteries are apparent. Those who use dry cells in any considerable quantity should correspond with the manufacturer and operate the cells according to his recommendations.

Lead-acid Storage Cells.—The chemical actions are not reversible (at least to any great extent) in the primary cells considered in the preceding paragraph; that is, once they are discharged they are practically useless and must be discarded. The secondary cell, or storage cell, is similar in action to the primary cell, except that *the action is completely and readily reversible.* That is, once a storage cell has been discharged by allowing it to force a current through an external circuit, it can be charged and returned to its original chemical condition by forcing a current back through it in the direction opposite to that of discharge. Storage cells are generally connected in series and then referred to as **storage batteries.**

The essential parts of a storage cell are a positive and a negative plate immersed in an electrolyte. In a fully charged lead-acid cell the positive plate is lead peroxide (PbO_2) and the negative plate is spongy lead (Pb). The two plates are spaced with an electrical nonconducting **separator.** This structure is immersed in a dilute solution of sulphuric acid (H_2SO_4) held in a suitable hard-rubber or glass container, as shown in Fig. 50. In very large installations the cells are sometimes held in wooden lead-lined tanks.

In a fully charged cell, the positive plate is dark brown and the negative plate is gray. The fully charged specific gravity (weight of a given volume of the electrolyte as compared with the same volume of pure water) of the electrolyte is as high as about 1.300 (usually stated as 1300) for the automobile and radio-type storage cell, or as low as about 1.180 (or 1180) for large permanent installations. Generally speaking, a lower specific gravity gives a longer life; however, the specific gravity recommended by the manufacturer should always be used.

Theory of Lead-acid Storage Cells.—It must be mentioned that the complete theory of the action in a lead-acid storage cell is involved,[1] but from the practical standpoint, the following relations hold, the arrows indicating the direction of the reactions:

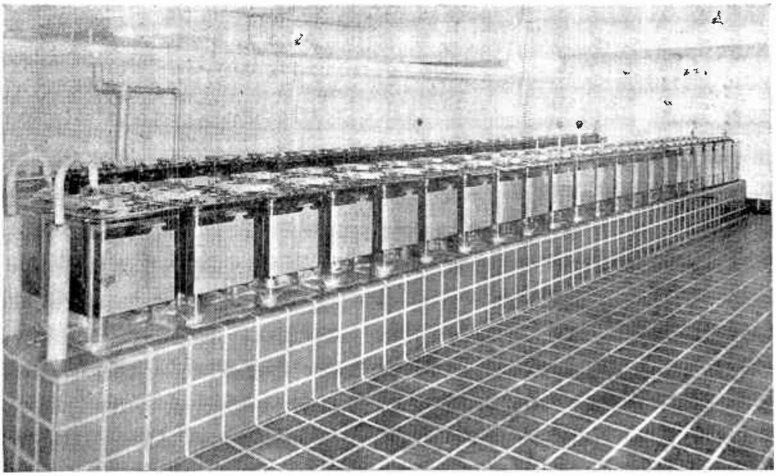

FIG. 50.—A group of high-capacity lead-acid storage cells installed in a large telephone central office. (*Courtesy of Electric Storage Battery Company.*)

$$PbO_2 + Pb + \underset{\xleftarrow{\text{Charge}}}{\overset{\xrightarrow{\text{Discharge}}}{2H_2SO_4}} = 2PbSO_4 + 2H_2O$$

This relation states that *when fully charged the lead-acid cell consists of a lead peroxide and a lead plate in dilute sulphuric acid and that when the cell is completely discharged each plate is converted to lead sulphate and the electrolyte becomes water.* On charging, the cell is returned to the original condition. The reader not familiar with chemistry should not worry about the numbers in front of the H_2SO_4, etc.; these are merely for "balancing" the equation.

Because the electrolyte gradually changes to water during the

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[1] See for instance, Vinal, G. W., "Storage Batteries," John Wiley & Sons, Inc.

discharge period, the specific gravity, as measured with a hydrometer, is used to indicate the condition of charge or discharge.

Characteristics of Lead-acid Batteries.—There are, generally speaking, two methods of charging batteries. (1) The constant-current method in which the battery is charged at a constant rate for about 90 per cent of the charging period, and then the rate is reduced to about 40 per cent of this rate for the rest of the charging period. (2) There is the constant-potential method, where the voltage is held constant and the battery is permitted to take whatever current it will. Because the electromotive force rises during charge, the battery itself gradually tapers the charging rate. There are also modifications of these two methods. In some instances (particularly in telephone installations) the energy source (such as the generator or rectifier) is of insufficient current capacity to supply the peak load. The battery is "floated" on the line, however, and supplies energy during peak loads and takes energy to charge during light loads.

Batteries are usually rated on an **ampere-hour basis**, but this neglects the closed-circuit voltage. Large batteries and in general those used in permanent installations are rated on an 8-hour basis. In automobile storage batteries, high starting currents are important, and shorter time ratings are specified. Unless otherwise stated, however, a 160-ampere-hour battery will deliver 20 amperes for 8 hours without its voltage falling below some specified value. Similarly, a 1000-ampere-hour battery will deliver 125 amperes for 8 hours without the end voltage falling below a specified value. The open-circuit voltage or electromotive force is about 2.2 volts per cell. Under full load the voltage drops to slightly under 2.0 volts per cell, and a cell is considered discharged when the closed-circuit voltage is about 1.75 volts per cell. Those values vary slightly with the type, age, specific gravity, and temperature.

The complete test characteristics of a lead-acid storage battery of the type used in a medium-sized telephone central office are given in Fig. 51. The closed-circuit voltage of the battery (terminal voltage when delivering current) is less than the open-circuit voltage (electromotive force) on discharge. This is largely due to the IR drop within the cells. On charge, the reverse is true, the closed-circuit voltage must be greater than the battery emf by an amount equal to the IR drop within the battery. The specific gravity falls gradually during discharge and rises during charge. Time is required for diffusion in the electrolyte, and changes in specific gravity lag slightly behind changes in the charge or discharge rate. The internal resistance varies as shown. This is due to changes in the plate material, to chemical changes in the electrolyte, and to the accumulation of gas in the electrolyte. A lead-acid battery gasses a little at all times but particularly when the charge is nearly completed. Excessive gassing may damage the plates, and it is common practice to reduce the charging rate when excessive gassing occurs. Hydrogen gas is given off in large quantities during the charging period, and *hydrogen gas is explosive.* A battery room should be well ventilated, and flames and sparks should be kept away from batteries. Sometimes, short-circuiting a battery may produce sparks that will cause the cells to explode.

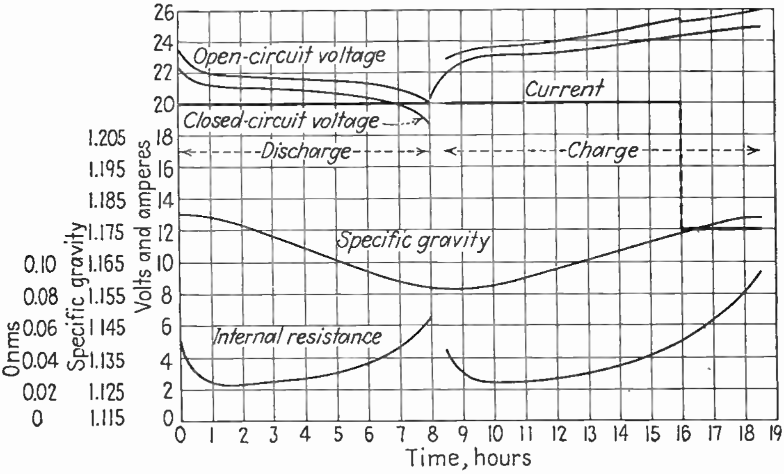

FIG. 51.—Characteristic curves for 11 lead-acid cells connected in series. Each cell rated at 160 ampere-hours. Normal current 20 amperes.

There are two methods of expressing and computing the efficiency of storage batteries. The ampere-hour output divided by the ampere-hour input on charging gives the **ampere-hour efficiency.** This is often of the order of 80 to 90 per cent and is somewhat misleading because it neglects the voltage variations. The **watt-hour efficiency** is a true measure of the efficiency of a battery as a source of energy. The ampere-hour output times the average closed-circuit voltage gives the watt-hours obtained from the cell.

The ampere-hour input times the average closed-circuit voltage during charging gives the energy input to a battery. The ratio of these two quantities gives the watt-hour efficiency.

Construction of Lead-acid Batteries.—Such details depend on the service the battery must give and on the manufacturer. In general, batteries for automobile and radio purposes are not designed for long life but rather for low first cost, reasonable life, and (for automobiles) high starting currents. Batteries for telephone central offices and similar purposes are higher in first cost, in current capacity, and are designed to last longer.

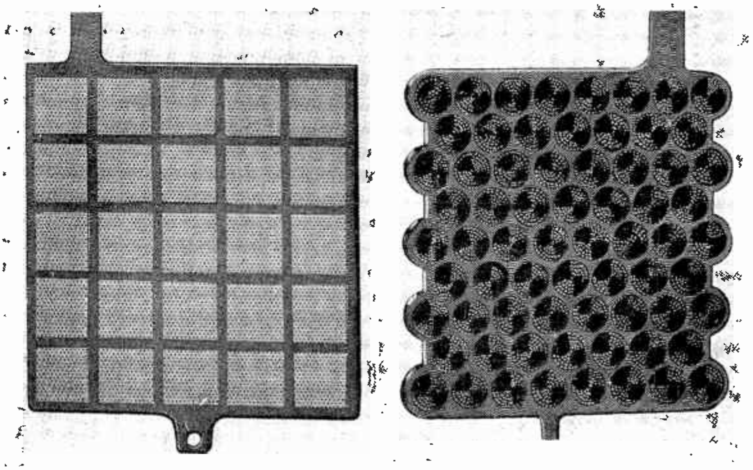

FIG. 52.—Types of plate used in the storage cells of Fig. 50. The negative plate shown at the left, and the positive plate at the right. (*Courtesy of Electric Storage Battery Company.*)

In all lead-acid batteries the active material of the plates is held in or on some supporting grid or other structure. This is to strengthen the plates. This supporting grid or frame is made of lead with a small amount of antimony added to strengthen the structure. The active material (that is, the lead peroxide for the positive plate and the spongy lead for the negative plate) is either pasted onto and into the grid or is held by other means. One common type of construction is shown in Fig. 52. In small batteries the separators are usually of wood, hard rubber, or combinations of these. Wood is also used in medium-sized batteries. Glass rods are sometimes used to separate the plates of large batteries.

In presenting the equations for charge and discharge on page 89, it was stated that when the storage battery was discharged the plates were converted to lead sulphate. If charging occurs within a reasonable length of time, this lead sulphate is all changed back to the original material. If a battery is allowed to stand in a discharged condition, however, it is difficult to get the sulphate to return to its original plate material. This is called **sulphation.** If the sulphation has not gone too far, charging the battery for a long time at a very low rate may gradually convert the sulphate back. Sulphation may cause the plates to buckle; so may discharging or charging at too high a rate.

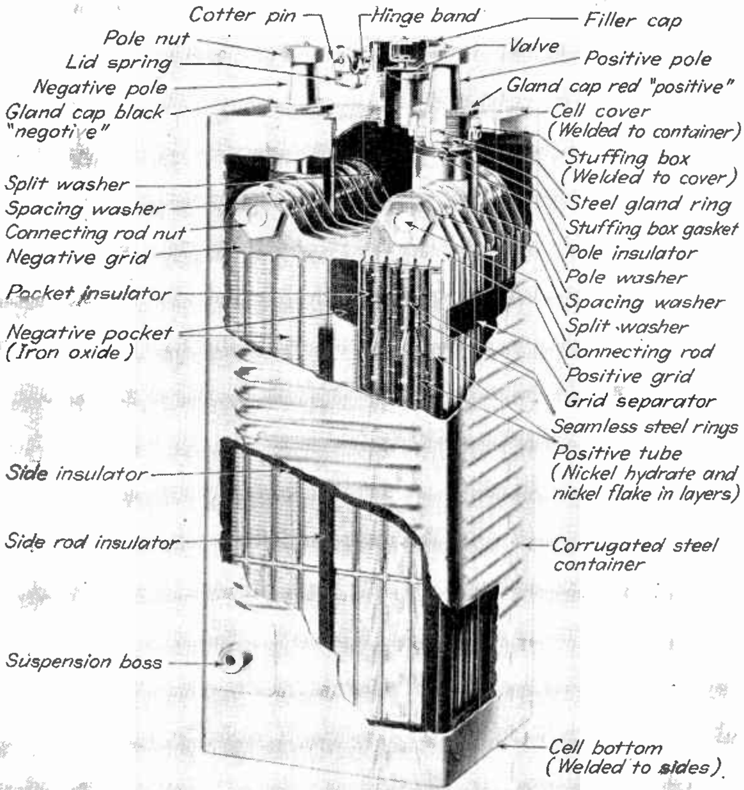

FIG. 53.—Rugged construction of the nickel-iron Edison storage cell. (*Courtesy of Thomas A. Edison, Inc.*)

Nickel-iron Storage Cells.—These are not so widely used for communication purposes as the lead-acid type, although they have certain characteristics such as extreme ruggedness, very long life, and other features well suiting them for certain installations. They are often called **Edison cells,** honoring their inventor.

The positive electrodes of this cell when charged consist of nickel peroxide held in steel tubes, and the negative plate consists of iron oxide held in steel pockets. The electrolyte is a potassium hydrate solution. The electrodes are spaced with rubber insulators, and the whole cell is mounted in a welded steel container.

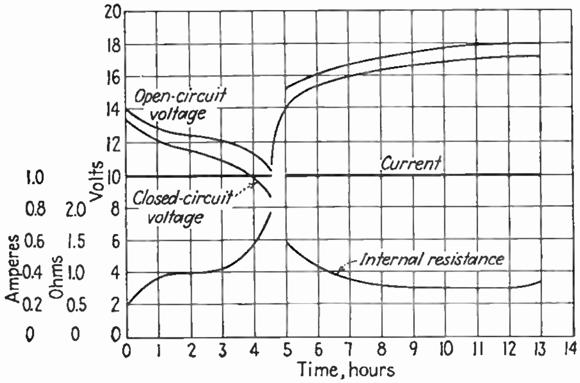

FIG. 54.—Characteristic curves for ten small nickel-iron Edison cells connected in series. Each cell rated at 4.5 ampere-hours. Normal current 1.0 ampere.

As Fig. 54 shows, the Edison cell has an electromotive force of about 1.4 volts per cell, and the voltage varies considerably on discharge. The specific gravity remains constant. It will stand much abuse, such as overcharge and undercharge, without apparent damage.

Electrolytic Conduction.—The communication worker must be acquainted with the essentials of electrolytic conduction, because of electrolysis to underground lead cable sheaths, if for no other reason.

Suppose that two perfectly clean electrodes of the same material (copper for instance) are placed in a glass container full of pure water and that the two electrodes are connected to an external source of potential differences as in Fig. 55. The source of voltage will be able to force almost no current at all through the circuit, because *pure water contains almost no carriers of electricity,* and hence there is almost no way for the charges to get across from one electrode to the other.[1] This brings out one important law of electrolytic conduction: *For current to flow through a liquid there must be carriers of electricity present in the liquid.*

If, however, a small amount of some acid or a pinch of common salt or soda is placed in the water, a current will immediately flow from one electrode to the other. This is because the acid (or the salt) produced **electrolytic ions** in the water. Thus, if sulphuric acid (H_2SO_4) were added, it would break up into some *positive* H_2 ions and some *negative* SO_4 ions. In chemistry, H stands for hydrogen, S for sulphur, and O for oxygen.

Under these conditions, the positive H_2 ions would be attracted to the *negative* plate where they would become neutralized (by receiving a negative charge), and they would then become ordinary hydrogen gas atoms and would cling to the electrode or bubble to the surface. The negative SO_4 ions would travel to the positive electrode where they would combine chemically with the copper, and copper sulphate ($CuSO_4$) would thus be formed. This would "eat away" the positive electrode.

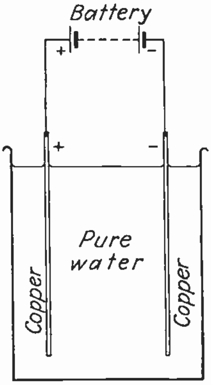

FIG. 55.—If the electrodes are clean and the water is pure, the battery can only force a very small current through the water, because, being pure, there are few carriers of electricity present.

To summarize: Ions flowing through a solution are, in a sense, actually carrying electric charges from one electrode to the other, and thus current could flow around through an external circuit. The energy required for the chemical process would be furnished by the electrical source applied outside the cell; it is the externally applied voltage which is maintaining the difference of potential at the electrodes which in turn causes the ions to migrate.

Laws of Electrolytic Action.—The two basic laws governing electrolytic action were early stated by Faraday. Because the reader may never have had elementary chemistry, and also because the quantitative relations involved are not necessary for a practical understanding of electrolysis, the complete theory of

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[1] Strictly speaking, under such ideal conditions as outlined above, it is presumed that absolutely no current would flow, but of course such conditions would be impossible to obtain by ordinary means.

electrolytic conduction need not be presented. The following abbreviated statement of Faraday's first law of electrolytic conduction is sufficient: *The mass of a substance deposited at an electrode is proportional to the quantity of electricity that has passed through the solution.* The quantity of electricity referred to is equal to the product of the current flowing and the time it flows. Unit quantity of electricity is the **coulomb**. One ampere flowing for one second equals one coulomb. That is,

$$\text{Coulombs} = \text{amperes} \times \text{seconds} \qquad \text{or} \qquad Q = It. \qquad (15)$$

In considering the way in which a current of electricity flowed through an electrolyte, it was shown that it could only flow when there were carriers of electricity such as charged ions in the electrolyte. Similarly, about the only way electricity can flow through the earth is by ion migration. Dry earth is a fairly good insulator, but earth contains various chemical salts and other sources of ions, and when earth is moist, it readily conducts current.

As was mentioned on page 95, when current flows it will chemically eat away at least one of the electrodes. Now suppose that a circuit consists of an underground cable sheath as one electrode, moist earth as the electrolyte, and the rail of a street railway as the other electrode. If current flows through this circuit in the correct direction, the cable sheath will be eaten away.

To summarize: Cable-sheath electrolysis is due to electrolytic action following the laws stated by Faraday. According to his first law, the corrosion will be proportional to the quantity of electricity flowing. Theoretically at least, a small current flowing a long time will cause the same corrosion as a large current flowing a short time.

Cable Electrolysis.—In Fig. 56 is shown a typical arrangement leading to cable electrolysis. In a typical direct-current railway system the trolley is positive and the rails are negative as indicated. If an underground telephone cable parallels the rails, the cable may pick up some of the railway current, may conduct it for a time, and then lose it back to the rails or to the substation ground. This tendency to pick up stray current is made greater if the bonds placed between the rails become loosened or broken. Where the current *enters* (conventional direction) the cable sheath, little if any harm is done; but where the current *leaves* the sheath, the lead sheath is gradually eaten away as in Fig. 57. This will, in time, permit the entrance of moisture which will ruin the cable.

Whether or not a cable sheath is collecting or losing current can be determined with a high-resistance (about 1000 ohms per volt) voltmeter. A 100–0–100 voltmeter (that is, one with a center zero and reading + or − 100 volts) is well suited for such measurements.[1] If the voltmeter is placed between the rails and the cable sheath at the right-hand portion of Fig. 56, the cable will be found to be negative with respect to earth and the rails; but when it is placed at the left the cable will be positive. Since conventional current flows from + to −, current will be leaving the cable at the left portion, and electrolysis will occur in this region. A **drainage bond** placed as indicated may largely prevent current flowing away from the cable through earth and, hence, may greatly reduce electrolysis in that area.

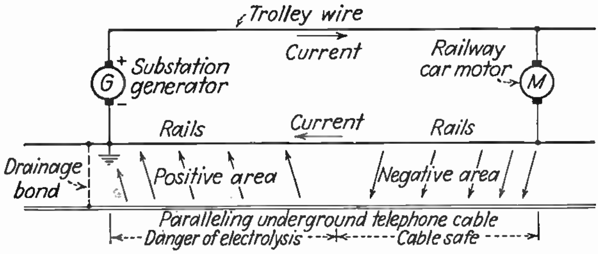

FIG. 56.—An underground telephone cable or other metallic body may gather current from a poorly designed or maintained street-railway system. Where the current leaves the cable and flows into the earth, electrolysis will probably occur. A drainage bond placed as shown will reduce the electrolysis at this point. Care must be used in installing bonds, however, because there is a limit to the current which may safely flow in a cable sheath. Also, placing a bond at one point may cause bad conditions at some other point.

Of course it is possible to have cable sheaths eaten away by causes other than street railways. Sometimes telephone companies send battery-charging current to remote installations over cable pairs as one side of the circuit and ground as the other. This may cause electrolysis. Also, local action may eat away a cable sheath. This term is used to include a variety of effects. Suppose that there

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[1] According to the theory for electrolytic cells (p. 85), a voltage will exist between any two dissimilar metals in an electrolyte. In making electrolysis studies it is convenient to use long prods to touch the voltmeter terminals to rails, earth, etc. For this reason it is advisable to tip the prods with lead cable-sheath material, because if an iron-tipped prod were stuck in the moist earth, a voltage due to iron and lead would be set up, and this would add or subtract from the electrolysis voltage.

are impurities in a cable sheath. Then minute primary cells would exist as formed by the impurities and the lead; and minute local currents would be set up which might cause damage. This is a source of much trouble in electric batteries. For instance, this is a factor limiting the "shelf life" of dry cells.

Certain forms of corrosion of lead cable sheaths may be confused with electrolysis. Cable splicers have been known to jam okum or dirty waste around cables where they enter manholes. This may cause local corrosion of the sheath. Also, it is now well established that creosoted-wood ducts and creosoted-wood boxes for splices (on armored cables which are buried directly in the ground) produce acetic acid which may badly corrode a cable sheath.

FIG. 57.—Damage done by electrolysis to a lead cable sheath. (*Courtesy of Bell Telephone Laboratories.*)

To summarize: Cable electrolysis occurs where underground cables lose current that they have collected from other areas. Drainage bonds often reduce electrolysis by metallically conducting the current away from the sheath. Self-corrosion and corrosion due to other sources may cause damage. Self-corrosion in electric cells is called local action.

Circuit Protection.—With the exception of the justifiable deviations to study briefly electrolytic conduction and electrolysis, this chapter has been devoted to electric power, energy, and sources of electric energy including primary cells and secondary cells, or storage batteries. Such sources of energy must be protected from overloads which might cause overheating and permanent damage to these devices. Also, electric devices such as tube filaments, relays, etc., must be protected from excess currents which might damage them. Of course a complete study of equipment protection would be quite extensive. Also, a complete study is impossible because the theory of magnetism, upon which circuit breakers (for example) depend, has not been considered. Thus the present discussion of circuit protection will consider only two protective devices, **fuses** and **heat coils.**

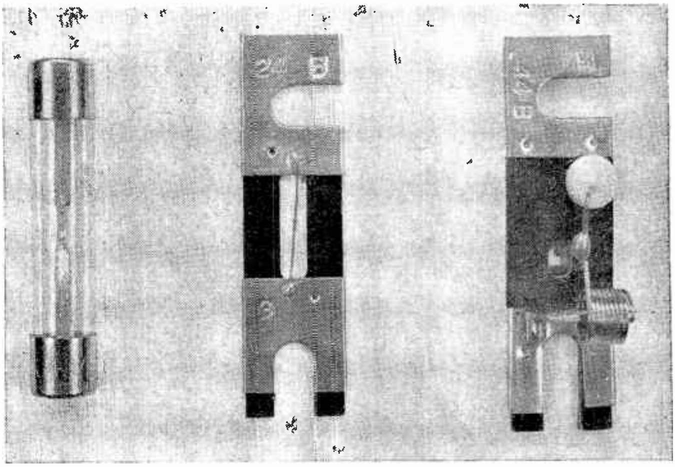

FIG. 58.—Fuses which are typical of those used in communication. The fuse wire of the fuse at the right is under tension and when it "blows" the spring raises the white glass bead, showing which fuse is open. This is important when hundreds of fuses are mounted in close proximity.

Fuses.—This is merely an electric conductor placed in series with the line wires to be protected against a high current flow. The fuse material through which the current flows is usually a link of a metallic alloy having a very low melting point. The temperature rise caused by the heat energy dissipated by the normal current is insufficient to melt the fuse. However, when an overload current flows, the I^2R loss in the resistor increases and the additional heat produced quickly raises the temperature of the fuse to the point where the fuse link melts and the fuse "blows." This opens the circuit and protects the equipment from further overload. In a telephone plant very small copper wires, such as a short length of No. 24 or 26 gauge cable, may be used to "fuse" the circuits, because these small wires will become overheated and melt on currents that the other circuits can safely withstand. Fuses typical of the communication industry are shown in Fig. 58. A fuse that would quickly blow if the safe current-carrying capacity of the line wires or equipment was exceeded [1] would be selected for a given installation.

Heat Coils.—A current might be only slightly above the normal current, but might persist for a long time. The accumulated heat over a period of several hours might be sufficient to char insulation, however. Now a circuit could be fused so that even the slightest overload would blow the fuses. However, the equipment could safely stand a slight overload without difficulty. Thus, circuits should not be fused too "closely," or fuse replacements and maintenance will be excessive. But, apparatus must be protected against slight overloads or "sneak currents" that persist for long periods. This has led to the development and use of heat coils in the telephone plant.

The typical heat coil consists essentially of a coil of fine wire wound on a copper tube into which a pin is inserted and soldered with an easily melting solder. A spring arrangement holds the pin under a force. If the heat generated by the current is sufficient, the solder melts and the spring forces the pin against a ground strip, grounding the circuit and protecting the central-office apparatus. A typical heat coil will carry 0.35 ampere for 3 hours but will blow in 210 seconds on 0.54 ampere.

SUMMARY

In general terms, force is that which produces or tends to produce a change in the motion of a body.

Work is defined as the production of motion against a resisting force.

Energy is the ability to do work. Energy can be neither created nor destroyed but can be changed from one form to another.

Power is the time rate of doing work. Electric power is measured in watts and equals volts times amperes. The expressions for power in direct-current circuits are $P = EI$, $P = I^2R$, and $P = E^2/R$. Power is also measured in kilowatts (1000 watts), milliwatts (1/1000 watt), and microwatts (1/1,000,000 watt).

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[1] In this connection, one must know his fuses. Some fuses that are rated at 10 amperes actually blow at about 30 amperes. Other manufacturers rate their fuses correctly. Fuses used for telephone circuits usually blow at just slightly above their marked rating.

The power-handling capacity of electric equipment is limited by the temperature rise, which if excessive may damage insulation.

Electric energy is numerically equal to the product of power and the time the power acts.

The maximum power is transferred from a device such as a dry cell or generator to a load such as a resistance when the resistance of the load equals the internal resistance of the cell or generator. Under these conditions the efficiency of power transfer is 50 per cent.

Voltage regulation is the rise in voltage when the load is charged from full load to no load divided by the full-load voltage.

Common sources of direct-current electric energy are primary cells and secondary cells. These include gravity cells, air cells, dry cells, lead-acid storage cells, and nickel-iron storage cells.

An electric current flows through a liquid electrolyte because of the presence of charged ions. Under certain conditions this action may cause cable corrosion.

Circuits are protected from overload by the use of fuses and heat coils.

REVIEW QUESTIONS

1. What is meant by the conservation of energy? Explain the application of this law to an amplifier driving a loud-speaker.

2. Explain why power bills, as they are called, are in reality bills for consumed electric energy.

3. How would you measure the electric power taken by a circuit? The electric energy?

4. Explain why short overloads of say 150 per cent may be tolerated by electric equipment and why such overloads cannot be continued for long periods of time.

5. A dry cell puts out electric energy. From where does this energy come?

6. Why is efficiency in handling power of less importance in communication circuits than in power circuits.

7. What are the conditions for maximum power transfer? What is the efficiency under these conditions?

8. Give several reasons why good voltage regulation is important in communication circuits. In power circuits. (Consider the incandescent electric light for instance.)

9. Why must the air cell absorb oxygen through the positive electrode? Where does the electric energy come from in this cell?

10. Why are the common electric cells called dry cells? What is the difference between a cell and a battery?

11. What is polarization? What is a depolarizer? What is local action?

12. Explain the fundamental difference between primary and secondary electric cells.

13. What is sulphation in lead-acid storage cells? How can it be prevented? How can it be treated?

14. Why are several positive and several negative plates used in cells of a storage battery? Are the plates connected in series or parallel?

15. Explain why hydrometer readings are useful in lead-acid battery operation.

16. Are hydrometer readings useful in nickel-iron battery operation? What is a useful reading?

17. Under what conditions will an electric current flow through a liquid?

18. Explain the meaning of the term electrolytic ion.

19. Explain how cable corrosion is caused by electrolysis. Are other underground objects such as water and gas mains similarly acted on?

20. Explain the fundamental difference between a fuse and a heat coil, and explain how each is used.

PROBLEMS

1. A large vacuum tube used in the power-output stages of a radio transmitter requires 50 amperes at 20 volts to keep the tungsten filament at operating temperature. Calculate the power taken in watts, the energy consumed during a 16-hour operating period, and the cost of this energy at 1.0 cent per kilowatt-hour. Calculate the resistance of the filament.

2. The filament of a small radio-receiver vacuum tube takes 50 milliamperes at 1.4 volts. Calculate the power taken in watts, the energy consumed during a 16-hour operating period, and the cost of this energy if taken from a 1.4-volt air-cell battery, which costs $2.75, and will supply 0.5 ampere for 1200 hours (page 85). Calculate the resistance of the filament.

3. Referring to Probs. 1 and 2, compare the cost of energy from a commercial power system with that from an air cell.

4. Repeat the example of power calculations given on page 72 if a 50,000-ohm bleeder is used.

5. Calculate the maximum values of the direct current which can be used with the following: (1) A 1000-ohm 1-watt resistor; (2) a 1000-ohm 10-watt resistor; and (3) a 1000-ohm 25-watt resistor. What maximum value of voltage may be impressed across each?

6. Repeat the illustrative example on page 77 if three wires in parallel are used for each side of the circuit.

7. From Example 1, page 83, calculate the regulation of the generator when connected to an 8.6-ohm load.

8. From Example 2, page 83, calculate the regulation if the bleeder is omitted. (The voltage across points 1–2 will no longer be 365 volts.)

9. Assume that the average resistance of a telephone transmitter is 60 ohms and that this transmitter, a 9-ohm induction coil, a 2000-ohm relay, and 10 miles of No. 22 gauge cable conductor are all connected in series and then across a central-office 24-volt storage battery. Calculate the power loss in each piece of apparatus.

10. Two resistors are in series with a 55-volt battery which has an internal resistance of 0.5 ohm. The current in the circuit is 20 amperes. The voltage drop across one resistor is twice that across the other. Calculate the power each resistor takes and the resistance of each resistor.

11. A four-cell storage battery has an electromotive force of 8.4 volts, and when short-circuited, the current is 20.8 amperes. What type of storage battery is it? What is the maximum power output?

12. A lead-acid storage cell consists of six positive and seven negative plates having a capacity of 25 amperes per positive plate. What would be the ampere-hour capacity of the cell? What average power and how much energy could be taken from the cell? (See curves, page 91.)

13. Two loud-speaker field coils in parallel take a total of 15 watts direct from a three-cell lead-acid storage battery. What is the resistance of each field coil? By allowing a safety factor of at least 2.0, what must be the minimum ampere-hour capacity of the battery if the speakers are to be used continuously for 12 hours?

14. Referring to page 100, calculate the relative amounts of energy dissipated in the heat coil under each condition mentioned. Assume that the resistance remains constant.

15. A small radio receiving set having five tubes is designed to operate from a 117-volt direct-current source. The filaments of the tubes are all in series. Two of these are 35-volt filaments, and three are 12.6-volt filaments. The current required is 0.15 ampere. Can they be operated directly across the 117-volt source? If not, what do you recommend?

CHAPTER V

ALTERNATING CURRENTS

The preceding pages were devoted to a study of direct-current phenomena. The various relations applying were considered in much detail. The entire presentation was based on the principle that a difference in electric pressure or voltage forces an electric current through the resistance of the circuit. These relations are expressed numerically by Ohm's law, which states that current = voltage/resistance, or $I = E/R$.

As was stressed when Ohm's law was presented, this law is in reality only the electrical expression for the universal principle that the result of anything is directly proportional to the magnitude of the effort and inversely proportional to the amount of opposition encountered.

In the preceding pages a direct voltage was always employed. This voltage was considered to act always in a given direction, that is, from plus to minus in the external circuit. Furthermore, it was of a constant value; that is, it did not change or vary with time. For example, if a storage battery had a generated electromotive force of 6.3 volts, this always stayed constant at substantially 6.3 volts.

Strictly speaking, direct currents and voltages merely have to act in *one direction, and they may vary considerably in magnitude.* Popular usage, however, applies the term direct currents and voltages to currents and voltages that are *essentially constant.*

In the following pages, currents and voltages that vary intentionally will be discussed. These alternating currents and voltages (in their simple form) vary uniformly with time.

In presenting any new subject, a firm foundation must be provided. The student who is to be successful in his work must realize the importance of this chapter. It is true that the material is of an abstract nature, but a mastery of it will ensure an understanding of the later chapters which contain the interesting applications to everyday electrical-communication work.

The Alternating Voltage.—Since voltage forces current through a circuit, the **alternating voltage** will be considered first. A source of alternating voltage familiar to the reader is the ordinary vacuum-tube oscillator. As the reader no doubt well knows, an oscillator will force an alternating current through a loud-speaker or a pair of headphones and cause them to give off a tone. Of course, the vacuum-tube oscillator, being a source of alternating voltage, will force a current through any other circuit as well.

A circuit composed of a vacuum-tube oscillator connected to a resistor is shown in Fig. 59. The wavy line at the center of the circle represents the oscillator for reasons that will be clear later (the wavy line represents a cycle, page 114).

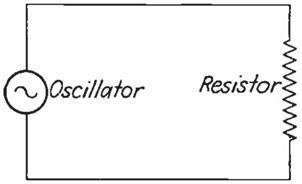


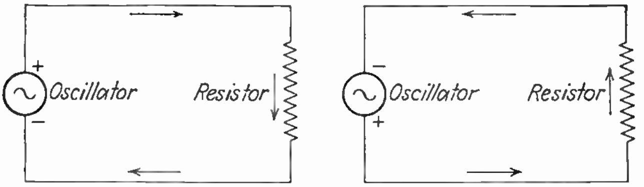
FIG. 59.—Circuit composed of a vacuum-tube oscillator connected to a resistor. The oscillator generates a sine-wave voltage which forces an alternating sine-wave current through the resistor.

As was previously stated, the vacuum-tube oscillator is a source of alternating voltage. To alternate, means to *follow in turn,* and this is precisely what the voltage of the oscillator is doing. No longer is the source of applied voltage constant as it was in direct-current circuits. No longer is one terminal positive all the time and the other terminal negative all the time. In the circuit of Fig. 59, the voltage applied by the oscillator is *alternating in nature.* First, the upper terminal of the oscillator is positive and the lower terminal is negative; then, the voltage *alternates,* and the upper terminal is negative and the lower terminal is positive. This process is repeated rapidly, many complete changes in polarity occurring per second.

To summarize: In alternating-current circuits the alternating voltage is directed first one way and then the opposite way. The vacuum-tube oscillator is a source of alternating voltage familiar to most communication workers.

The Alternating Current.—The viewpoint that potential difference or voltage forces current through a circuit has been repeatedly stressed. This is true for the circuit of Fig. 59 as well. If the upper terminal of the oscillator is positive at a given instant, then current will be forced to the *right* in the upper wire and to the *left* in the lower wire. When the rapidly alternating voltage changes, and the lower terminal becomes positive and the upper terminal becomes negative, these relations are reversed; that is, the current now flows to the *left* in the upper wire and to the *right* in the lower wire. These relations are pictured in Fig. 60.

In studying direct-current flow, it was explained that the current consisted of a flow of electrons. It was mentioned, however, on page 7 that an individual electron did not necessarily need to travel *completely* around the circuit; all that was needed was that a progressive flow of electrons occurred around the circuit. Of course some individual electrons may pass completely around the circuit.



(a)-Showing direction of current when upper oscillator terminal is positive

(b)-Showing direction of current when lower oscillator terminal is positive

FIG. 60.—As the alternating voltage of the oscillator changes in direction from + to −, the current which this voltage forces through the load resistor likewise changes.

In considering **alternating-current** flow, it is apparent that if the applied voltage changes *very slowly some* individual electrons may actually go completely around the circuit as in direct-current. It is also apparent, however, that if the alternating voltage is changing rapidly, then *very few if any electrons will have time* to pass completely around the circuit.

The flow of alternating current in a wire may be conveniently pictured as follows: The current flow consists of the free electrons just as in any circuit. These free electrons are, however, more or less confined to a given portion of the wire. First they are forced one way, then back the other way as the voltage changes. In other words, they *alternately* flow back and forth, constituting as they do an alternating current.

To summarize: An alternating voltage forces an alternating current through a circuit. This current consists of free electrons present within the conductor. When the alternating current is flowing, these free electrons may be pictured as rapidly moving back and forth within the conductor.

Alternating-voltage Variations with Time.—As has been mentioned, and as illustrated in Fig. 60, an *alternating voltage* is directed first one way and then the other, forcing an *alternating current* through the circuit. *If there is nothing in the circuit to cause a change in the relations, the alternating current must correspond exactly to the alternating voltage.* A resistor fulfills these requirements; thus, in Fig. 60 the current that flows will correspond exactly to the voltage impressed by the oscillator. It is important therefore to study alternating-voltage variations.

In describing alternating-voltage variations, it is convenient to speak of the **wave shape** of the voltage. To explain the meaning of wave shape, an illustration familiar to many communication engineers—that of a double-current telegraph system[1]—will be used.

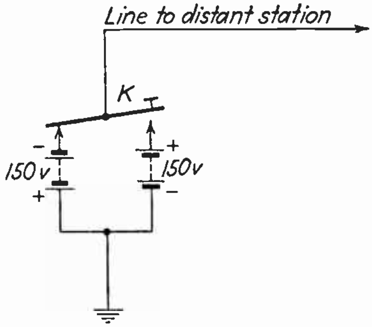


FIG. 61.—Diagram of sending-key circuit of a double-current telegraph system which sends out a square-wave signal such as Fig. 62. The opened position of the key *K* is as shown; depressing the key is assumed to close it.

The sending key of a double-current telegraph system is arranged in a circuit as shown in the simplified circuit of Fig. 61. When the key *K* is *up*, *negative* battery is applied between the line and ground. When the key is *depressed* to send the signals of a telegraph message, *positive* battery is applied between the line and ground.

Now assume that an experienced operator is rapidly sending out a uniform series of telegraphic dots with the key. Also, let it be assumed that the dots are all the same length, same intensity, have the same spaces between them, and that the lengths of these spaces equals the lengths of the dots. That is, the operator depresses the key for a certain length of time, then instantly releases it for the same length of time, then depresses it again, and so on. If this is done, then +150 volts will be impressed on the line for a given length of time, then −150 volts for the same length of time, and so on.

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[1] Double-current telegraph systems (as distinguished from the elementary single-current type) are very widely used in the United States for such purely practical reasons as more reliable operation.

These relations can be shown graphically, and for the elementary telegraph circuit of Fig. 61, they would be as in Fig. 62. At point 1 the key is closed, impressing a voltage of +150 voltages from line to ground, at point 2 the key is opened, impressing −150 volts from line to ground, etc. If the *time* required by the operator in manipulating the key is plotted on the X axis (horizontal) and the *magnitude* of the signal in volts is impressed on the Y axis (vertical), the **square-topped voltage wave**[1] of Fig. 62 results.

The reader may have much difficulty in finding any resemblance between the graphical plot of voltage versus time of Fig. 62 and any *wave* he ever saw. The next section should help to clarify this usage however.

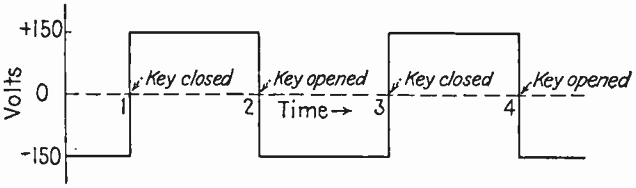

FIG. 62.—Square voltage wave impressed on a circuit by the operation of the key of Fig. 61.

To summarize: If a plot is made of *voltage versus time* as was done in this section for a double-current telegraph system, the *wave shape* of the *voltage wave, or the voltage impulse,* is graphically represented.

Sine-wave Voltages.—As is well known to most readers, a pure tone is often used in testing communication equipment. Thus, if a voltage from an oscillator that produces essentially a pure tone when connected *directly* to a loud-speaker is first passed through an amplifier and then to à loud-speaker, then if the loud-speaker does not give off a pure tone, it is a good indication that distortion exists in the amplifier. It requires little imagination to predict the nature of the sound given off by a loud-speaker if a voltage having the square shape of Fig. 62 were impressed on the loud-speaker. If the voltage were varying slowly, a series of dull thuds would be produced. If the voltage were varying rapidly, the result would

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[1] One experienced in telegraphy will recognize that this illustration makes several assumptions. For instance, it is assumed that the time required to change the key from one position to the other is negligible and that the circuit is such that the voltage *can* rise abruptly as shown.

be more like a series of clicks. *Thus, there must be considerable difference between the wave shape of the voltage of an oscillator producing a pure tone and the wave shape of Fig. 62, and such surely is the case.*

The voltage wave of the telegraph instrument sending out a series of uniform dots as shown in Fig. 62 would be called a **square wave.** The voltage output of a vacuum-tube oscillator that would produce a pure tone in a loud-speaker is called a **sine wave.** The difference between a telegraphic square wave and a sine wave is this: Instead of changing *immediately* from a positive maximum to a negative as in Fig. 62, a sine-wave voltage changes slowly and gradually from a positive maximum voltage to a negative maximum voltage, and vice versa.

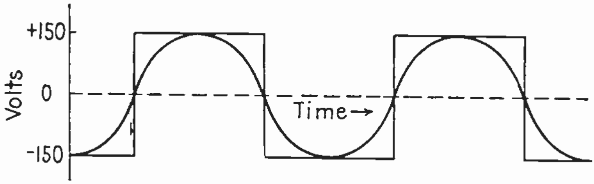

FIG. 63.—Square wave and pure sine wave of the same frequency.

A comparison of a square wave and a sine wave is shown in Fig. 63. These two waves are similar in some respects. For instance, the impulses exist for the same lengths of time, and the maximum intensities are the same. Actually, however, they are quite different. The reader may be surprised to find that the square-topped telegraph wave is the more complicated of the two.

To summarize: It has been shown that a pure sine wave of Fig. 63 is the simplest form that an alternating voltage can have. Although a square-topped voltage wave will cause a loud-speaker to give off a complex sound, *a pure sine wave will produce only a single pure tone or note.*

Sine-wave Currents.—Let the sine-wave voltage of Fig. 63 be plotted as in Fig. 64 with ordinates (vertical lines) drawn in at even time intervals. Now it is apparent that each point of intersection with the X axis is at a different instant as measured from the starting point or origin at the left. Thus, *each ordinate, or each dotted vertical line, represents the voltage at that instant or the* **instantaneous voltage** *of the pure sine wave.*

Thus, it is evident that the instantaneous-voltage value of a pure sine wave depends on the particular instant at which the voltage is measured. But, *voltage forces current through a circuit*, and thus if a pure sine-wave voltage is impressed on a circuit containing *only pure resistance*,[1] the current at each instant will be determined by the voltage at that instant. That is, if a pure sine-wave voltage is impressed on a circuit containing only pure resistance, a pure sine-wave current will flow through the circuit. These relations are shown in Fig. 65. Furthermore, the points at which the voltage wave and the current wave pass through zero will coincide for a circuit containing only resistance. When two waves coincide in this manner, they are said to be **in phase**.

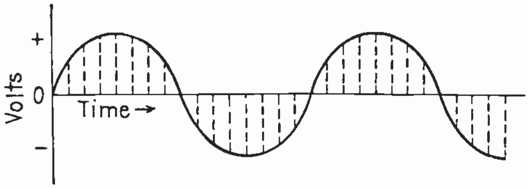

FIG. 64.—The height of each dotted line represents the instantaneous value of the voltage. Time is measured from the line at the left. Such a wave could also represent current as well.

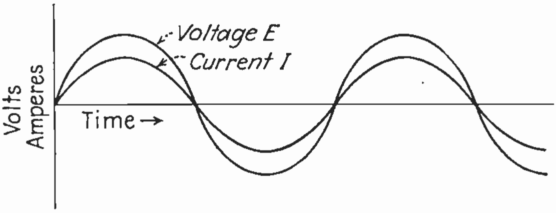

FIG. 65.—The sine-wave voltage E will force the current I through a circuit. In this figure I has been drawn smaller than E for no particular reason. Since volts and amperes are different quantities, they are plotted to different scales. These two waves are in phase.

To summarize: A pure sine-wave voltage will force a pure sine-wave current through a circuit containing only resistance. Furthermore, if only resistance is present, the various points such as zero and maximum values of the two waves will coincide and the waves will be *in phase*.

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[1] For the present, the discussion must be confined to circuits containing only pure resistance. Later, the discussions will become more generalized.

Characteristics of Sine Waves.—There are at least four very important and useful values associated with sine waves. Two of these, instantaneous and maximum values, need little additional discussion, at least for the present, but for the sake of completeness, they will be discussed again. This will be done with the aid of Fig. 66 which is a portion of Fig. 64 specially drawn for this purpose.

Instantaneous Values.—The **instantaneous values** of a pure sine wave are the values shown as dotted vertical lines between the zero line (or X axis) and the curve. Of course any number of such vertical lines can be drawn, but for convenience, only a few are usually laid off, and at regular intervals as shown. The instantaneous value of voltage or current depends on the *instant* at which it is desired. The letters i and e are used to represent instantaneous values.

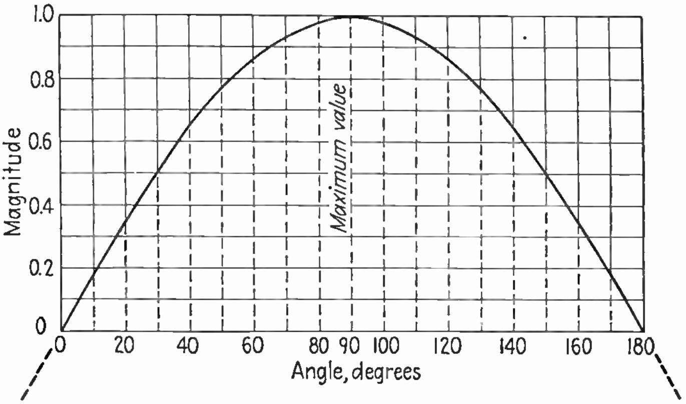

Fig. 66.—This represents one half cycle of a pure sine wave. The instantaneous values are shown by the dotted lines. This curve may represent either alternating voltage or current. It is plotted in angles instead of time as in previous figures. This may be done because the degrees represent the angle turned through in a given time.

Maximum Value.—This is, of course, the greatest or **maximum value** to which the voltage or current rises. It is marked on Fig. 66. The notations $I_{\max}$ and $E_{\max}$ are used to represent maximum values.

Average Value.—The **average value** of a sine wave is the average height of the curve of Fig. 66. Thus, if the height of the dotted ordinates is measured and the average taken, it will be found to be about 0.6. Actually, the average value found by taking a large number of ordinates is 0.637 of the maximum value. This is written

$$E_{av} = 0.637E_{max} \qquad \text{also} \qquad E_{max} = \frac{E_{av}}{0.637} = 1\ 57\ E_{av}.$$

$$I_{av} = 0.637I_{max} \qquad \text{also} \qquad I_{max} = \frac{I_{av}}{0.637} = 1.57\ I_{av}. \quad (16)$$

Effective Value.—The terms just considered have been fairly obvious. Now although there is nothing particularly difficult about effective value, it cannot be said to be obvious. The **effective value** of an alternating voltage or current (which it must be remembered is varying in magnitude at each instant) must be the same as a corresponding direct-current value. If this were not true, then 1 volt of alternating voltage would not produce the same *effect* on a resistor as would 1 volt of direct voltage. Also, 1 ampere of alternating current would not produce the same heating effect in a given resistor as would 1 ampere of direct current. That chaos would result from such a system is apparent.

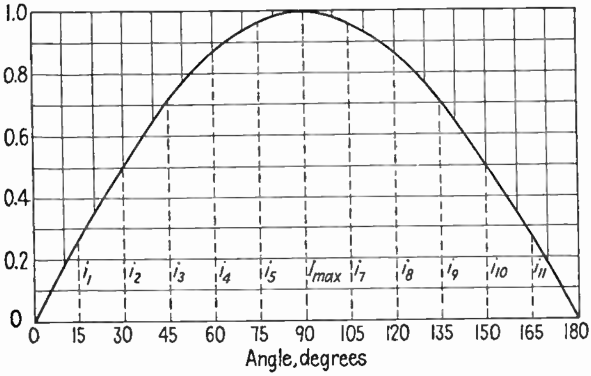

FIG. 67.—The effective value of a sine-wave value can be found by squaring the instantaneous current values from i_1 to i_{11} inclusive, adding these squared values together, averaging, and taking the square root. This value should be about 0.707 for a pure sine wave.

The effective value of a sine-wave voltage or current is defined as that value which will produce the same heating effect in a resistor[1]

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[1] This definition assumes that the resistor is of such type that its resistance and hence the power loss within it is the same with alternating as with direct current. The importance of this assumption will be made apparent on page 258

as will be produced by a given value of direct voltage or current. Referring to Fig. 67, this really means that the *average of the sum* of all the *instantaneous* i^2R losses in a resistor equals some I^2R loss, where I represents the *effective value* of the sine-wave current.

These relations can be stated as follows:

$$I^2R = \frac{i_1^2R + i_2^2R + i_3^2R + \cdots}{n},$$

where n is the number of instantaneous values taken. Then

$$I^2R = \frac{R(i_1^2 + i_2^2 + i_3^2 + \cdots)}{n}.$$

Canceling the R values gives

$$I^2 = \frac{i_1^2 + i_2^2 + i_3^2 + \cdots}{n},$$

and then

$$I = \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 + \cdots}{n}}. \qquad (17)$$

This equation can be interpreted as follows: *The effective value of a sine wave (current or voltage) is the square root of the average of the instantaneous values squared.*

Thus, with reference to either Fig. 66 or 67, the effective value of a sine-wave voltage or current in terms of the maximum value is found as follows: Find the numerical value of each ordinate (instantaneous value). These are plotted with respect to the maximum value as unity. Square each instantaneous value. Add all the instantaneous values. Divide by the number taken, and thus find the average or mean value. Then, take the square root of this number. The answer obtained will be about 0.7. The exact value is 0.707. Thus it can be stated that

$$E = 0.707E_{\max} \qquad \text{and} \qquad E_{\max} = \frac{E}{0.707} = 1.414E$$

$$I = 0.707I_{\max} \qquad \text{and} \qquad I_{\max} = \frac{I}{0.707} = 1.414E. \qquad (18)$$

Because of the way in which they are found, effective values are also known as **root-mean-square** values, or **r.m.s.** values.

To summarize: Instantaneous, maximum, average, and effective values of an alternating sine-wave voltage or current may be specified. Of these, the effective value is of very great importance, because it really ties the direct current and the alternating current units together so that 1 volt or 1 ampere produces the same heating effect in a *resistor* irrespective of whether it is alternating or direct current. The symbols E and I are used to specify these effective values. *In all problems and in all statements in alternating-current explanations, effective values of voltage and current are to be assumed, unless it is specifically stated that they are instantaneous, maximum, or average values.*

Frequency of a Sine Wave.—The reader is familiar with the term **frequency.** He knows that if he connects a telephone receiver or a loud-speaker to a source of low voltage (for example the secondary of a small filament transformer) from the 60-cycle power supply that a low-frequency (low-pitched) hum will be heard. He also knows that if a receiver or loud-speaker is connected to an oscillator the pitch of the sound is increased as the frequency or number of **cycles per second** is increased.

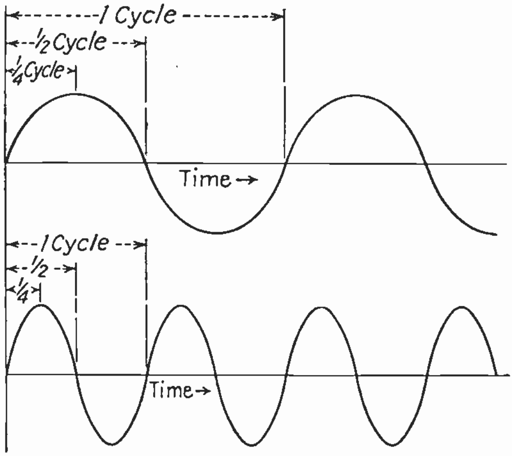
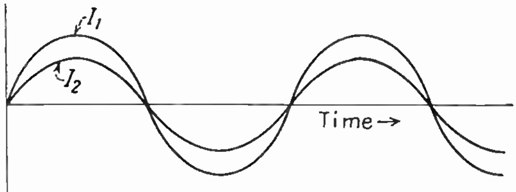


Fig. 68.—The sine wave below has *twice* the frequency of the one above. For this reason it takes only one-half the time for corresponding variations.

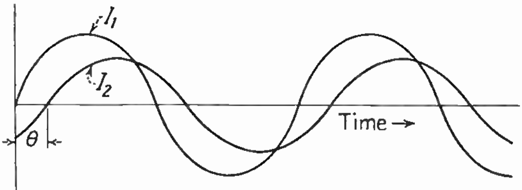
With reference to Fig. 68, *one cycle consists of a complete positive and negative impulse* as indicated. The *frequency* of a wave *is the number of cycles occurring per second.* The time required for one cycle to occur is $t = 1/f$. The time required for a half cycle is

$t = 1/2f$ and for one-fourth cycle it is $t = 1/4f$. That is, the time required for a sine-wave signal to build up from zero to maximum is $t = 1/4f$, where f is the frequency in cycles per second.

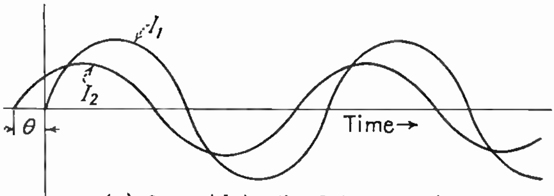
Phase Relations of Sine Waves.—Perhaps the reader has suspected, or has been told, that alternating currents are much more difficult to work with than are direct currents. The answer is that neither is difficult if well understood. But this is true: With alternating currents, more possibilities must be considered; in other words, the explanations must be generalized.



(a)-Two currents I_1 and I_2 in phase



(b)-Current I_2 lagging I_1 by an angle θ



(c)-Current I_2 leading I_1 by an angle θ

FIG. 69.—Showing currents in phase, lagging, and leading. The same figures could be used to represent voltages.

For example, take Kirchhoff's second law on page 34. This says that the algebraic sum (considering the + and − nature) of the currents flowing to a point must equal the algebraic sum of the currents flowing away from that point. In direct-current circuits, currents can flow only toward or away from a point. There are other possibilities to consider in alternating-current circuits, however. Two currents may be flowing together at a point, but *they may not be exactly in phase.*

Another example will be helpful. On page 69, it is stated that $P = EI$. Well, that is true for direct-current circuits, but as will be seen later, it may not be true for alternating-current circuits, because the current and voltage may not be in phase, that is, may not be acting together.

When two or more sine waves, whether they are currents or voltages, are in phase, they pass through corresponding values at the same instant. Two currents that are in phase are shown in Fig. 69*a*. The same two currents are shown out-of-phase in Fig. 69*b*. Here I_2 lags I_1. That is, time is being taken along the X axis, and "increasing positively" is taken as the *positive* direction of current flow. Thus, I_2 completes its variations after I_1, and hence I_2 lags in phase. Similarly in Fig. 69*c*, I_2 leads I_1. *Note that these statements are relative, depending on which wave is taken as the reference in stating the position of the other wave.*

To summarize: In alternating-current circuits the phase relations of the various waves must be considered. Two waves may be in phase (passing through corresponding variations at the same time), or they may be out of phase, with one wave leading or lagging the other.

Addition of Sine Waves of the Same Frequency.—When two sine-wave currents flow up to a point in a circuit, the current flowing away must equal their sum, added of course with proper regard to their phase relations. Two or more sine-wave voltages acting on a circuit are added similarly. To illustrate the addition of sine-wave values, three cases will be considered: (1) two currents in phase, (2) the second current lagging the first, and (3) the second current leading the first.

Addition of Sine Waves Which Are in Phase.—Two currents I_1 and I_2 are shown in phase in Fig. 70. The current I_1 is larger than I_2. The maximum value of I_1 is $I_{\max} = 4.0$ amperes, and for I_2, the maximum value is $I_{\max} = 2.0$ amperes. At each instant in the circuit, the *instantaneous* current components i_1 and i_2 are acting. Thus at each instant the resultant current must be $i_3 = i_1 + i_2$. If the reader will add the instantaneous values of i_1 and i_2 at a number of points, the resultant current I_3 will be produced.

The *maximum* value of this resultant current will be $I_{3\max} = I_{1\max} + I_{2\max} = 4 + 2 = 6$ amperes. The *effective value* of the resultant current will be $I_3 = 0.707 I_{3\max}$, which will of course equal the sum of the effective values of the separate currents. This reasoning shows that when two or more currents are in phase the maximum values may be added directly to give the maximum value of the total current and also that the effective values may be added directly to give the effective value of the total current. Of course this same reasoning applies to voltages. It should also be noted that the frequency of the total current is the same as the frequency of the separate currents.

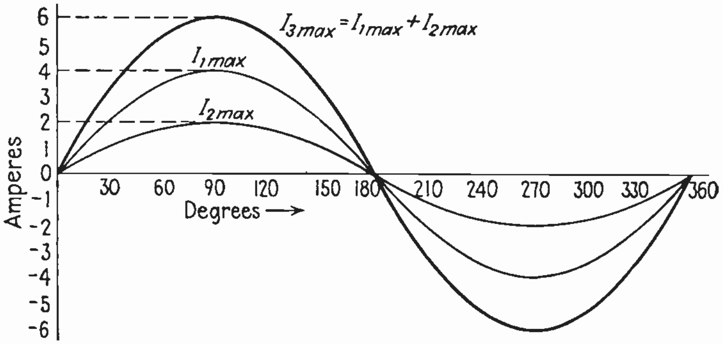


FIG. 70.—The two in-phase currents, $I_{1\max}$ and $I_{2\max}$, add to give the resultant current $I_{3\max}$.

Addition of Sine Waves out of Phase (Lagging).—The current I_2 of Fig. 71 is lagging I_1 by a certain time interval. That is, I_2 is shown increasing to its positive maximum value after I_1. The maximum values of I_1 and I_2 are the same as in Fig. 70. The total or resultant current I_3 must at each instant be the sum of the instantaneous values of I_1 and I_2. Adding the *corresponding instantaneous values algebraically*, because now sometimes positive and negative instantaneous values *act simultaneously*, gives the total current I_3. Note in particular that the maximum value of this current is not $I_{3\max} = I_{1\max} + I_{2\max} = 4 + 2 = 6$ amperes as it was when the currents were in phase. The maximum value $I_{3\max}$ of the resultant wave is less than this sum; so will be the effective value I_3 of the resultant wave, because $I_3 = 0.707 I_{3\max}$. Thus, when two sine-wave currents or voltages are out of phase, that is, *not acting exactly together at the same time*, the resultant voltage or current *will not* be the direct sum of the separate components. The frequency, it will be noted from Fig. 71, remains the same. The reader can readily prove to himself that if I_2 of Fig. 71 were of the same magnitude as I_1 and if I_2 were shifted along until it is one half cycle out of phase with I_2 then the value of the total current I_3 will be *zero* at every instant.

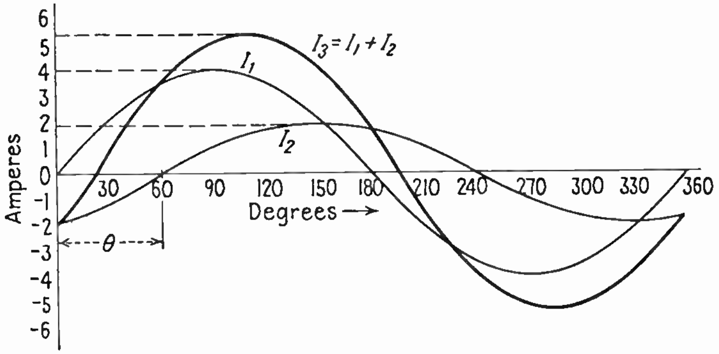


FIG. 71.—In this figure current I_2 lags I_1 by a certain time interval represented by the angle θ of 60 degrees. The sum of these two currents is I_3, obtained by adding instantaneous values.

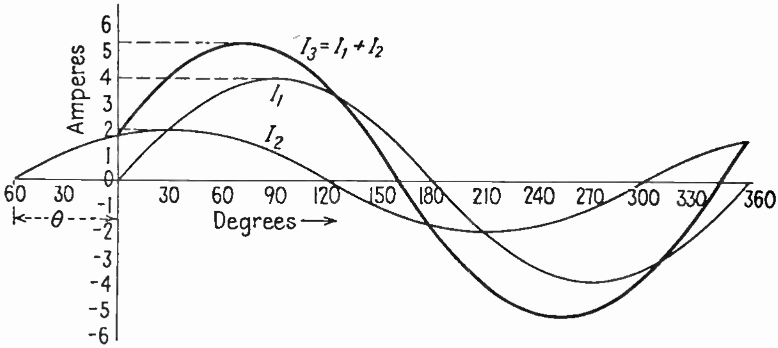


FIG. 72.—In this figure current I_2 leads I_1 by a time interval represented by the angle θ of 60 degrees. The sum of these two currents is I_3, obtained by adding instantaneous values.

Addition of Sine Waves out of Phase (Leading).—The sine wave I_2 of Fig. 72 is leading I_1, because I_2 has increased to a positive maximum before I_1. These two waves are combined to find the resultant value just as was done in the preceding case, that is, by adding algebraically the instantaneous values. This gives the resultant wave I_3. Further explanation seems unnecessary. The resultant wave is less than the direct sum of the two waves but of course has the same frequency.

Vector Representation of Sine Waves.—The methods of adding two or more sine-wave currents or voltages to find the resultant current or voltage was explained in the preceding section. Two or more sine waves are combined by adding (algebraically) the instantaneous values. But think how bothersome it would be always to stop and combine currents or voltages point by point in alternating-current work. Remember, that in alternating-current circuits currents cannot (in general) be added directly because they may be out of phase. The same applies to voltages.

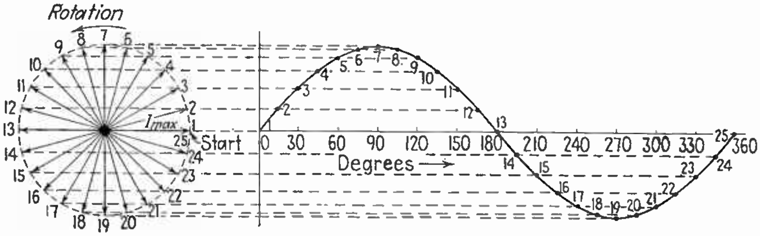


FIG. 73.—If the line I_{max} is rotated as indicated, and if at 15-degree (or other) intervals the line is stopped, and its vertical projection is carried over to the right and plotted at the corresponding angle, a sine wave will be created.

These difficulties have led to the adoption of **vectors** for combining currents and voltages in alternating-current circuits. The use of vectors greatly simplifies the solution of alternating-current problems. Now it is logical to assume that if vectors are useful in adding sine-wave currents and voltages then there must be a relation between vectors and sine waves. This is true, as will now be explained.

Suppose that the line I_{max} of Fig. 73 is rotating counterclockwise at a constant rate. This rate could be measured in *degrees per second*, there being 360 degrees in a complete circle or in one complete revolution of the line I_{max}. This rate could also be measured in *radians per second*, there being (by definition) 2π or 2×3.1416 radians in one complete revolution of the line I_{max}. Suppose that the angle through which the line I_{max} turns during time t from the assumed starting position is called ωt and is measured in radians. Then the angular velocity at which the line I_{max} revolves is $\omega t/t = \omega$ radians per second. In one revolution, 2π radians are covered, and this is done once for each cycle of the sine wave. Since there are f cycles per second, $\omega = 2\pi f$, where f is the frequency.

In Fig. 73, as the line $I_{\max}$ revolves counterclockwise it is assumed that at each point 1, 2, 3, 4, 5, etc. (representing 15-degree increments) the line is stopped instantaneously and the height of its vertical component is plotted as a Y-axis value against an X-axis value representing the angle through which the line has turned. This gives a sine-wave curve at the right. Therefore, from Fig. 73 it is evident that a sine-wave value (such as a sinusoidal alternating current) is created by a line such as $I_{\max}$ rotating as shown. In electrical terms, a sine-wave alternating current can be represented by a counterclockwise rotating line having a length $I_{\max}$ equal to the maximum value of the alternating current. Similarly, a sine-wave voltage can be represented by a counterclockwise rotating line having a length $E_{\max}$.

In Fig. 73 the line has a certain *length* $I_{\max}$. It also has an arrow head on one end. This indicates that the line also has *direction*. A quantity that has magnitude (such as length) and direction is often called a **vector quantity.**

To summarize: This section shows that a sine wave is generated by the vertical projection of a rotating vector. It has previously been shown that pure alternating currents of a single frequency could be represented by sine waves; therefore, an alternating current or an alternating voltage can be represented by a vector. *The advantage of this is that vectors can be added and subtracted quite readily as compared with sine waves.* This statement will be made clear later.

Vector Representation of Two or More Sine Waves.—The preceding section has shown that a vector can be used to represent a sine wave. It is logical, then, to assume that two vectors can be used to represent two sine waves, and this is true.

Representation of Waves in Phase.—With reference to Fig. 70, which shows, the addition of two sine waves that are in phase and to Fig. 73 which indicates how a vector represents a sine wave, it is apparent that the two vectors that would represent $I_{1\max}$ and $I_{2\max}$ of Fig. 70 and also the resultant vector $I_{3\max}$ (representing the sum of $I_{1\max}$ and $I_{2\max}$) must lie along the same straight line.

Representation of Waves out of Phase.—In Fig. 71 two sine waves with $I_{2\max}$ lagging $I_{1\max}$ were shown. These are reproduced in Fig. 74 together with the rotating vectors that may be thought to create them.[1] It is apparent that $I_{2\text{max}}$ must be placed behind $I_{1\text{max}}$ to conform to the fact that one sine wave is behind the other. *Remember that the vectors are assumed to be rotating counterclockwise.* Further examination of this figure discloses that just as the sum of the two sine waves can be represented by a single sine wave $I_{3\text{max}}$, so can the sum of the two vectors $I_{1\text{max}}$ and $I_{2\text{max}}$ be represented by a single vector $I_{3\text{max}}$. Although the case shown here was for a lagging current, it should not be difficult for the reader to draw out the relations for a leading current.

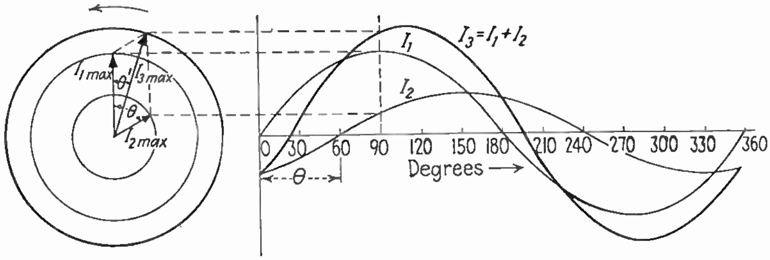


FIG. 74.—The vertical projections of the rotating vectors $I_{1\text{max}}$, $I_{2\text{max}}$, and $I_{3\text{max}}$ produce the sine waves shown. In this figure, I_2 lags I_1 by an angle θ of 60 degrees. The resultant current $I_{3\text{max}}$ lags $I_{1\text{max}}$ by the angle θ'. The subscript "max" is used in designating the vectors because they represent the maximum value of the waves.

To summarize: It has been shown that vectors may be used to represent either current or voltage sine waves. Also, because vectors are assumed to be rotating in a counterclockwise direction, they may be used to represent sine waves in phase, leading, or lagging. Just as one sine wave may be used to represent the sum of several other sine waves, so may one vector be used to represent the sum of several other vectors.

Addition of Vectors.—An examination of Fig. 74 should disclose that it is much simpler to add two vectors representing two currents

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[1] This wording may seem vague, but it is good in this sense: As a vector rotates, its vertical projection is different at each instant, as has been shown. Plotting the projections at the various points of rotation creates or generates a sine wave. The reader may imagine a parallel beam of light shining onto a rotating stick, the plane of rotation being parallel to the light rays. The shadow thrown by the stick on a screen behind the stick will be the vertical projection at each point of rotation. If the reader will imagine that the extreme end (only) of the projection or shadow makes a mark on the screen and that the screen is moved horizontally at a uniform speed, then a sine wave will be drawn out on the screen.

than it is to add two sine waves point by point. This fact certainly is true, and because of this the solution of most alternating-current problems includes vector addition (and also subtraction to be discussed in the following section). Such additions are possible by **graphic** and by **analytic** means. The graphic methods will be considered in this chapter.

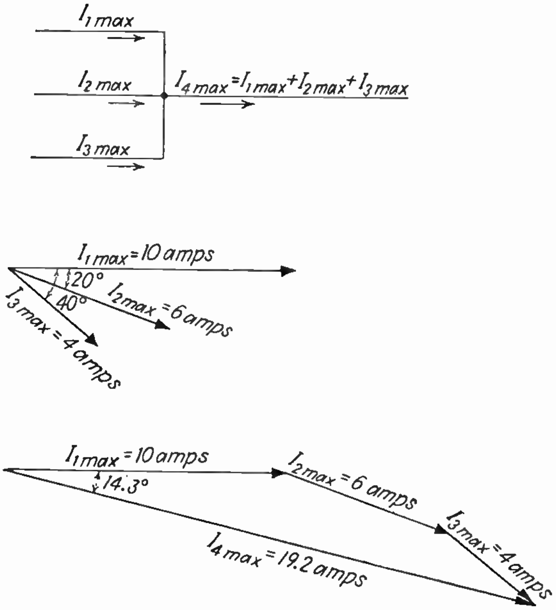


FIG. 75.—In the upper figure are shown three branch lines carrying currents $I_{1\max}$, $I_{2\max}$, and $I_{3\max}$ coming together to form $I_{4\max}$. The relative vector positions of these currents are shown in the center figure. At the bottom is shown the method of adding these separate currents to find the total current. The resultant value for $I_{4\max}$ is shown.

An ordinary unit such as gallons expresses only quantity, and thus gallons are added directly. *A vector, however, possesses both magnitude and direction,* and vectors must be added so that both these measures are considered. Figure 74 will show that if a line equal in length to $I_{2\max}$ is placed from the end of $I_{1\max}$ and that if this line extends in the same direction of $I_{2\max}$, then the end of this new line will coincide with the end of vector $I_{3\max}$.

Now vector $I_{3\max}$ is obtained by adding *sine waves* $I_{1\max}$ and $I_{2\max}$ as shown by the sine waves of Fig. 74. But it has just been shown that the length and direction of the resultant $I_{3\max}$ can also be found by adding one vector to the end of another, that is, by drawing a line identical to $I_{2\max}$ from the end of $I_{1\max}$. *Thus, adding currents and voltages vectorially is equivalent to adding them sinusoidally.*

The process involved in Fig. 74 has been extended in Fig. 75. Thus, suppose three branches each carrying sinusoidal alternating currents come together and one wire leads away. Suppose that $I_{1\max} = 10$ amperes, $I_{2\max} = 6$ amperes at an angle of 20 degrees behind $I_{1\max}$, and $I_{3\max} = 4$ amperes at an angle 40 degrees behind $I_{1\max}$, and that it is desired to find the resultant current $I_{4\max}$. This is found as Fig. 75 shows by successively laying off from the end of $I_{1\max}$ each vector at its proper angle and of its correct length. Then, a line $I_{4\max}$ connecting the end of the last vector with the origin of the first is the sum or **resultant** of the three values. Since all the vectors have been drawn to scale, $I_{4\max} = 19.2$ amperes, and it lags 14 degrees behind $I_{1\max}$. Of course the same method would be followed in adding voltages.

To summarize: To add vectors, successively add each vector to the end of the last vector, being certain that the vectors added are of the same length and at the same angle as the currents or voltages they represent; then, the resultant will be a line connecting the end of the last vector with the beginning of the first one.

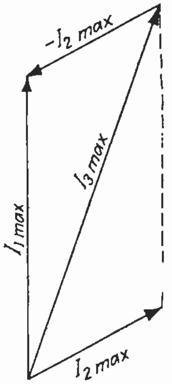


FIG. 76.—Showing how to subtract vectors. In this figure $I_{2\max}$ is changed in sign (making it $-I_{2\max}$) and added to $I_{3\max}$. This gives vector $I_{1\max}$.

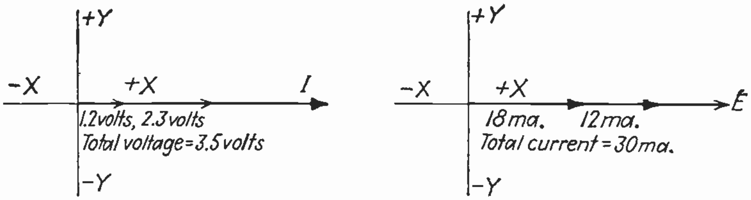
Subtraction of Vectors.—To illustrate this process, Fig. 74 will be used. Thus, suppose that $I_{3\max}$ and $I_{2\max}$ are known and it is desired to find $I_{1\max}$. *To subtract a vector, change its sign and add it* as shown in Fig. 76. Changing its sign means to place the arrowhead on the opposite end, in other words, reverse its direction. Thus when $I_{2\max}$ is reversed in direction and added from the end of $I_{3\max}$ and when a line connecting $I_{3\max}$ and the end of $I_{2\max}$ is drawn, this is vector $I_{1\max}$. Of course this same process can be used to subtract voltages or to subtract more than one vector from another.

Vectors Representing Effective Values.—On page 114 it was stressed that effective values were used instead of maximum values in most instances, yet in the preceding discussion of vectors, maximum values and not effective values were used. The reason this was done is as follows: It was a simple matter to show that two sine waves could be represented by a vector equal to the maximum value of the sine wave. It would not have been quite so easy to have shown the relation between a sine wave and an effective value.

However, the fact still remains that most values given for current or voltage are effective values. The factor 0.707 is the numerical relation between the effective and maximum value ($I = 0.707I_{\text{max}}$). Thus, if vectors of the maximum value of a sine wave can be used to represent sine waves, *vectors of the effective value of a sine wave can also be used to represent the sine waves.*

To summarize: In accordance with statements made earlier in this chapter, most specified alternating-current values are *effective* values. Therefore, in the following pages all values will be effective values and will be represented by I and E (not I_{max} and E_{max}) unless *otherwise* stated.

Examples of Vector Addition and Subtraction. *Vectors in Phase.*—The current through a resistor is *always in phase* with the voltage across the resistor. Two resistors are in series, and the current through them is 10 milliamperes. There is a drop of $E_1 = 1.2$ volts across one, and $E_2 = 2.3$ volts across the other. Calculate the voltage impressed across the combination. This is solved by the use of the vector diagram of Fig. 77*a*. The current is common, so it is taken as the base along the X axis. The 1.2 volts and the 2.3 volts are plotted to scale along this axis. The sum of these two voltages is the vector $E_1 + E_2 = 3.5$ volts. Here is an instance



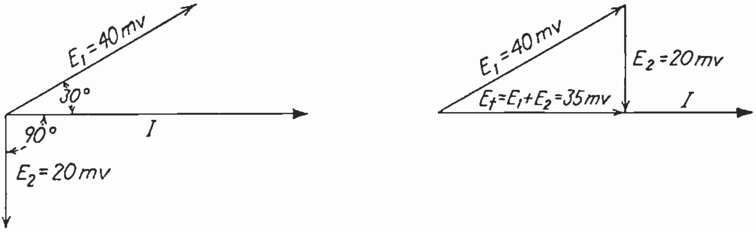
(a)-Adding two voltages 1.2 volts and 2.3volts which are in phase with each other and with the current

(b)-Adding two currents 18 milliamperes and 12 milliamperes which are in phase with each other and with the voltage

FIG. 77.—Illustrating the method of adding voltages and currents which are in phase. The X and Y designations refer to the axes used for representing vectors. Thus, the horizontal line is the X axis, the vertical line is the Y axis, and they are designated as + or − depending on their position with respect to the central origin.

where the vector solution and the numerical solution are the same, but *it is a special case true only because the two voltages are in phase.*

As a second illustration, suppose that two resistors are in parallel across an alternating voltage of 2.8 volts. One coil takes a current of $I_1 = 18$ milliamperes and the other $I_2 = 12$ milliamperes. Calculate the total line current. This is done graphically in Fig. 77*b*. In this instance the voltage is taken as the base along the X axis because the voltage is common in a parallel circuit. The currents are also laid off along the X axis because they are in phase with the voltage in pure resistance. The total current is $I_1 + I_2 = 30$ milliamperes. Here again the arithmetic sum and the vector sum are the same, *but only because the two currents are in phase.*



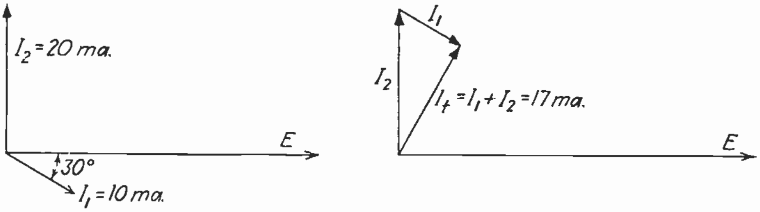
(a) - Vector diagram for series circuit discussed in text

(b) - Addition of vectors shown in (a)

FIG. 78.—Vector diagrams for a series circuit composed of 40 millivolts across a coil at an angle of 30 degrees with the current and 20 millivolts across a condenser at an angle of 90 degrees with the current. The diagram is drawn as in (*a*) because the current is common, and because the voltage across a coil leads the current, and the voltage across a condenser lags the current.

Vectors out of Phase.—The *current always lags the voltage* in a circuit containing a *coil.* (This is equivalent, of course, to saying that the voltage leads the current.) The *current always leads the voltage* in a circuit containing a *condenser.* (This is equivalent to saying that the voltage lags the current.) The reasons for these phase relations will appear later (page 247). Suppose that a *series* circuit consists of a coil and a condenser. The voltage across the coil is $E_1 = 40$ millivolts, and the phase angle is 30 degrees. The voltage across the condenser is $E_2 = 20$ millivolts, and the angle is 90 degrees. This problem is solved graphically by Fig. 78. The current is taken as the base because the current is the same (common) in all parts of a series circuit. The voltages are next laid off as indicated in accordance with the statements at the opening of this paragraph. Then the vectors are added, giving the total voltage $E_t = E_1 + E_2 = 35$ millivolts, and the angle measures to be 0 degrees.

Now suppose that a coil and a condenser are connected in *parallel* across a common source of voltage, that the coil takes a current of $I_1 = 10$ milliamperes with a phase angle of 30 degrees, and that the condenser takes a current of $I_2 = 20$ milliamperes at an angle of 90 degrees. The total line current will be the vector sum of the individual current. These currents are plotted and added as in Fig. 79. The voltage is taken as a base and the currents laid off in accordance with the statements made early in this section. When the vectors are added, it is found that the total current $I_t = I_1 + I_2 = 17$ milliamperes and the phase angle is 61 degrees leading the voltage.



(a) - Vector diagram for parallel circuit discussed in text

(b) - Addition of vectors shown in (a)

FIG. 79.—Vector diagram for a parallel circuit composed of a current of 10 milliamperes through a coil at an angle of 30 degrees with the voltage, and a current of 20 milliamperes through a condenser at an angle of 90 degrees with the voltage. Current through a coil lags the voltage, and through a condenser leads the voltage. These relations are shown in (*a*).

Solutions of Alternating-current Circuits.—In the preceding chapters, particularly Chap. II, the methods of solving series, parallel, and series-parallel circuits were considered in much detail. In explaining the methods employed, two important laws were illustrated. (1) In a series circuit the current in all parts is the same (common) and the sum of all the voltages around a closed circuit equaled the impressed voltages. (2) For a parallel circuit the voltage across each branch is the same (common) and the sum of the branch currents equals the total line current.

In the section just preceding this one the vector methods of adding alternating currents and of adding alternating voltages were considered. The fundamental laws governing the solution of alternating-current circuits are but generalizations of the laws governing direct-current circuits. These laws are as follows:

Series Alternating-current Circuits.—In the series circuit, the current is the same in all parts (that is, common to all parts). The *vector sum* of the voltages around a series circuit equals the impressed voltages.

Parallel Alternating-current Circuits.—In the parallel circuit, the voltage is the same across each branch (that is, common to each branch). The *vector sum* of the branch currents equals the total line current.

In other words, the direct-current circuit rules apply to alternating-current circuits if they are generalized by the addition of the word *vector*.

In the preceding section it was stated that the current through a coil lagged the voltage across the coil. Also, it was stated that the current through a condenser led the voltage across a condenser. These peculiarities are due to the **inductance** of the coil and the **capacitance** of the condenser. Inductance and capacitance are properties of alternating-current circuits which act with resistance to impede the flow of alternating current.

Before a meaningful discussion of inductance and capacitance can be given, the basic principles of magnetic and electric fields must be understood. For this reason, no attempt will be made at this time to consider further the solution of alternating-current circuits.

Nonsinusoidal Waves.—Pure sine waves were used in all the considerations in this chapter with one exception; this was the square-topped telegraph wave of Fig. 62. Waves that are not pure sine waves are called **nonsinusoidal waves.** The telegraph wave was a *uniform* nonsinusoidal wave, because it recurred exactly the same time after time.

It is the purpose of this section to show that *a steady recurring nonsinusoidal wave is composed of two or more steady pure sine waves.*

Nonsymmetrical Nonsinusoidal Wave.—A wave is nonsymmetrical if the positive and negative half cycles are *not* identical. At first thought wave I_3 of Fig. 80 *appears* to be identical, but it is not. Along the time axis the peak of the positive half cycle occurs first, and the peak of the negative half cycle occurs last. It is evident that I_3 is obtained by adding the instantaneous values of I_1 and I_2. The frequency of I_2 is twice that of I_1. If several such figures are drawn it will be found that adding waves of *even* multiples of frequency (that is, if f is the lowest frequency, then adding waves of $2f$, $4f$, etc.) always produces *nonsymmetrical nonsinusoidal waves.* Now I_3 is produced by adding the instantaneous values of I_1 and I_2. It follows, therefore, that any time a recurring wave such as I_3 is observed it can be broken down into two waves such as I_1 and I_2; in other words, the nonsymmetrical nonsinusoidal recurring wave I_3 of Fig. 80 is in reality composed of two pure sine waves of frequency f and $2f$.

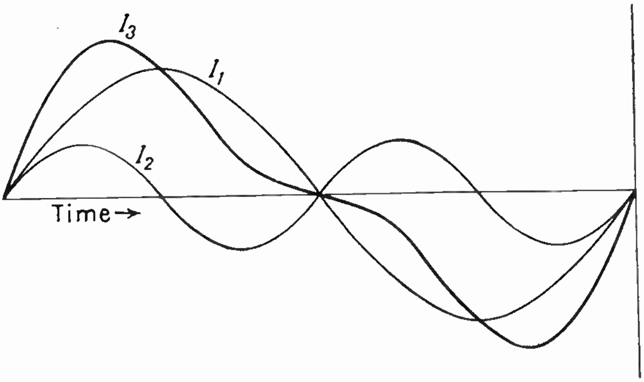


FIG. 80.—Addition of the two pure sine waves I_1, of frequency f, and I_2 of frequency $2f$ gives the nonsymmetrical nonsinusoidal wave I_3.

Symmetrical Nonsinusoidal Waves.—The resultant wave I_3 of Fig. 81 is a symmetrical wave because the positive and negative half cycles are identical. As is evident, it is formed by adding instantaneous values of the pure sine waves I_1 and I_2. An examination of I_2 will show it to have three times the frequency of I_1.

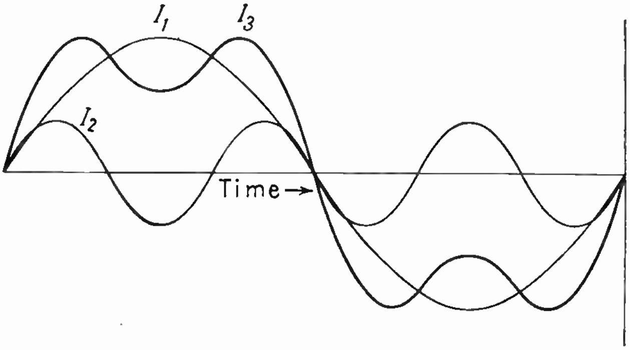


FIG. 81.—Addition of the two pure sine waves I_1 of frequency f, and I_2 of frequency $3f$ gives the symmetrical nonsinusoidal wave I_3.

because three cycles of I_2 are the same length as one cycle of I_1. If several such figures are drawn, it will be concluded that adding *odd* multiples of frequency (that is, f, $3f$, $5f$, etc.) always produces *symmetrical nonsinusoidal waves.*

To summarize: Waves are nonsinusoidal if they are not pure sine waves. They are nonsymmetrical if the positive and negative half cycles are not identical. Adding sine waves of *even* multiples (f, $2f$, $4f$, etc.) produces nonsymmetrical nonsinusoidal waves. Adding sine waves of *odd* multiples (f, $3f$, $5f$, etc.) produces symmetrical nonsinusoidal waves. *Any steady continuously recurring nonsinusoidal wave (whether symmetrical or nonsymmetrical) is composed of single-frequency pure sine waves.*

Harmonics.—In Fig. 80, the sine wave of lowest frequency f is called the **fundamental,** and the sine wave $2f$ of twice the frequency of the fundamental is designated as the **second harmonic.** Similarly, in Fig. 81 the sine wave of lowest frequency is again the fundamental, and the sine-wave frequency $3f$ of three times the fundamental is the **third harmonic.** That is, a nonsinusoidal wave is the summation, or resultant, of sine-wave components. The lowest frequency component is called the fundamental, a component of twice this lowest frequency is called the second harmonic, a component frequency three times the fundamental is the third harmonic, and so on.

Vector Representation of Nonsinusoidal Waves.—As was shown in Fig. 73, and the accompanying discussion, a vector may be used to represent a *sine wave.* A single vector cannot be used to represent a nonsinusoidal wave, a fact that will be apparent if Fig. 73 is studied.

It has also been shown by Figs. 80 and 81 that nonsinusoidal waves are composed of pure sine waves whose instantaneous values add to produce the resultant nonsinusoidal wave. This is the method of adding a wave of one frequency to a wave of another frequency. Vectors representing waves of the same frequency can be added or subtracted vectorially as was explained on page 121, but vectors representing waves of different frequency cannot be added or subtracted.

SUMMARY

An alternating voltage forces an alternating current through a circuit. An alternating voltage is directed first one way, then the opposite way.

An alternating current consists of free electrons which are present in the metallic conductors. These electrons may be pictured as alternately flowing back and forth.

The wave shape of a voltage or current may be shown graphically by plotting instantaneous values of voltage or current on the Y, or vertical, axis and values of time at which the values occur on the X, or horizontal, axis.

If this is done for a double-current telegraph system that is sending a uniform series of dots, a square-topped wave will be depicted.

The simplest form an alternating voltage or current can have is that of a pure sine wave. These waves have instantaneous, maximum, average, and effective values. The effective value is the quantity usually specified.

One complete cycle of a sine wave consists of a positive and a negative variation. The number of complete cycles occurring per second is called the frequency.

Two or more sine waves may be in-phase or out-of-phase, either leading or lagging.

Sine waves, regardless of phase relations, are added or subtracted by combining instantaneous values.

Sine-wave currents and voltages may be represented by vectors, which are quantities having direction and magnitude.

Vectors usually represent effective values and may be added or subtracted to find the resultant of sine-wave currents or voltages.

Only vectors representing the same frequencies may be combined.

Nonsinusoidal currents or voltages are composed of a fundamental and harmonics.

REVIEW QUESTIONS

1. Name at least three sources of alternating voltage in communication circuits. Are these pure sinusoidal voltages?

2. Explain the manner in which alternating current flows in a wire.

3. What relation exists between the motion of a clock pendulum and a sine wave?

4. Explain the relation between effective values of alternating current and direct current.

5. What is the phase relation between current and voltage in a circuit composed of

a. A resistor?

b. A condenser?

c. A coil?

6. Can you add effective values directly? Explain your answer.

7. Just why can a vector be used to represent a sine-wave quantity?

8. At what angular velocity does a vector rotate?

9. What is the relation between the frequency of a vector quantity and the angle turned through in a given time?

10. Explain how to add and subtract vectors.

11. How can you prove that a rotating vector generates a sine wave?

12. Can you add vectors of different frequency?

13. State the laws applying to alternating-current series circuits.

14. State the laws applying to alternating-current parallel circuits.

15. What is meant by the term nonsinusoidal wave?

16. Can a single vector represent a nonsinusoidal wave?

17. How are nonsinusoidal waves composed?

18. Can sine waves of different frequency be represented by vectors on the same vector diagram?

19. What are the reasons for stating that a square-topped double-current wave consists of a fundamental and a series of harmonics?

20. Would a rectified wave be rich in even or odd harmonics?

PROBLEMS

1. Calculate the numerical relation between average and effective values.

2. The following table gives the numerical relation between the instantaneous and maximum values for one-fourth of a sine wave. The rest of the wave is symmetrical. Plot the sine waves for two currents $I_{1\max} = 40$ milliamperes and $I_{2\max} = 25$ milliamperes, the currents being in phase. Plot a wave representing the sum of these two currents (refer to Fig. 70).

Angle, Degrees	Instantaneous Current
0	$i = 0.0I_{\max}$
15	$i = 0.259I_{\max}$
30	$i = 0.500I_{\max}$
45	$i = 0.707I_{\max}$
60	$i = 0.866I_{\max}$
75	$i = 0.966I_{\max}$
90	$i = 1.0I_{\max}$

3. Determine the effective values of each of the waves of Prob. 2.

4. Repeat Probs. 2 and 3 if $I_{2\max}$ lags $I_{1\max}$ by 30 degrees.

5. A vector is rotating at an angular velocity of 62,832 radians per second. What is the frequency of the current that it represents?

6. Referring to Fig. 75, if $I_{1\max} = 5$ milliamperes, $I_{2\max} = 3$ milliamperes at 30 degrees lagging $I_{1\max}$, and if $I_{3\max} = 5$ milliamperes at an angle of 40 degrees leading $I_{1\max}$, calculate the total current. Is this an effective, average, or maximum value?

7. A portion of a circuit consists of a main wire and two branches. The current in the main wire is 35 milliamperes, and the current in the other wire is 15 milliamperes, lagging that in the main wire by 36 degrees. Graphically determine the current in the second wire.

8. Draw a vector diagram showing the approximate relations of the various voltages in a series circuit composed of a resistor, a coil, and a condenser. Explain how to find the total impressed voltage.

9. Draw a vector diagram showing the approximate relations of the various currents in a parallel circuit composed of a resistor, a coil, and a condenser. Explain how to find the total line current.

10. A sinusoidal current having an effective current of 50 milliamperes and a frequency of 1000 cycles per second exists in the same circuit with a sinusoidal current of 20 milliamperes effective value and a frequency of 2000 cycles per second. The two components are in phase. Plot these two currents, and determine the wave shape of the resultant current.

CHAPTER VI

THE MAGNETIC FIELD AND INDUCTANCE

There are three fundamental parameters of electric circuits; these are resistance, inductance, and capacitance. In the preceding chapters, resistance has been treated in much detail, but largely from the direct-current standpoint. It will be considered again, as it acts in alternating-current circuits, on page 247.

The effect of a coil in causing the current through it to *lag* the voltage across it was mentioned in the preceding chapter. This peculiar (and useful) effect was due to the **inductance** of the coil. Likewise, it was stated that the current through a condenser *led* the voltage across it. This was due to the **capacitance** of the condenser (not capacity, see footnote, page 192). Inductance will be considered in this chapter and capacitance in the one following.

This property of inductance, which a coil of wire possesses, is very useful in many ways. Inductance and inductive effects are due to the **magnetic field** that a current produces. For this reason, it is necessary to study the magnetic field before considering inductance.

The Magnetic Field.—A **magnetic field** is a *region* in which **magnetic forces** act. Magnetic forces are somewhat familiar to all. The well-known permanent horseshoe magnet exerts an attractive force on a bit of iron and clasps the bit of iron to it. The magnetic field of the earth exerts a force on the magnetized compass needle, etc.

One of the easiest ways of studying a magnetic field is to place a stout piece of paper over a permanent bar magnet, dust iron filings on the paper, and tap the paper lightly. If this is done, the iron filings, being acted on by the magnetic forces of the permanent magnet, will arrange themselves in lines, somewhat as shown in Fig. 82. This is because magnetic forces are acting in a given direction. Thus it is said that the iron filings arrange themselves along **magnetic lines of force.** This concept of magnetic lines of force will be used extensively in the pages that follow. The student

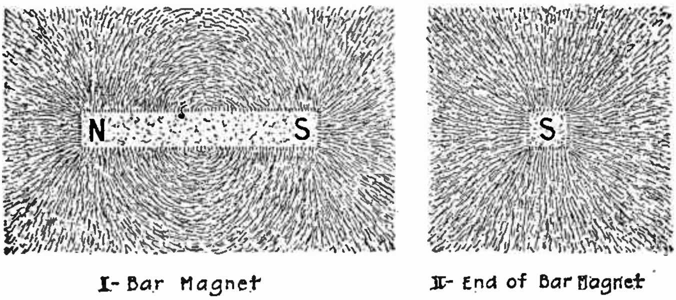


FIG. 82.—When iron filings are placed on a sheet of paper covering a permanent magnet, the iron filings arrange themselves along the lines of force issuing from the permanent magnet. (*From Croft.*)

must learn to visualize and think in terms of these magnetic lines of force. To assist in this, Figs. 83 and 84 have been included. Think of magnetic fields in terms of the way iron filings would act if they were placed on a paper in the field of influence of the magnet or coil. Or think of the field in terms of how a small compass needle would point if placed at various locations in the magnetic

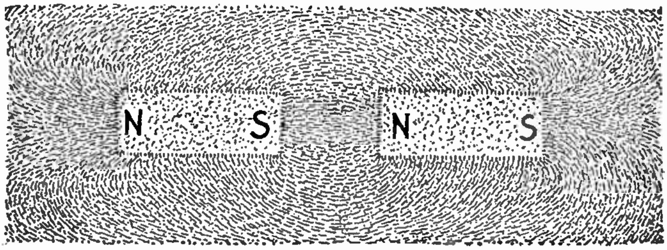


FIG. 83.—Showing how iron filings will arrange themselves if dusted on a piece of paper lying over two bar magnets with *opposite* magnetic poles adjacent. Magnetic lines of force are thought to act along the lines designated by the iron filings. (*From Croft.*)

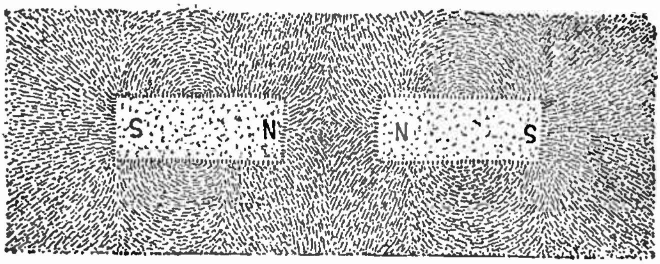


FIG. 84.—Showing how iron filings will arrange themselves if dusted on a piece of paper lying over two bar magnets with *like* magnetic poles adjacent. (*From Croft.*)

field. A compass needle always aligns itself with the lines of force, that is, it points in the direction in which they are acting.

To summarize: A permanent magnet is surrounded by a region in which magnetic forces act. This region is called the magnetic field and is considered as being composed of magnetic lines of force.

Laws of Magnetic Action.—As the reader has already surmised, many electrical laws (as well as others) are merely arbitrary assumptions. Once they are stated, however, they are rigidly adhered to, because they become the building blocks of electrical explanations.

Now as the reader well knows, a permanent bar magnet is assumed to have a north pole and a south pole, and the *lines of force are assumed to act or flow from the north pole to the south pole in the air and then back from the south pole to the north pole in the steel.*

It is generally well known that *unlike magnetic poles attract* each other. This is indicated by Fig. 83 which shows a north and a south pole together. It can be assumed that magnetic lines of force tend to shorten themselves like stretched rubber bands and that by this action the north and south poles would pull together.

But observation has shown that *two like poles repel* each other. This is indicated in Fig. 84. Here the lines of force are both directed out and repel each other, the magnets being thus forced apart.

To summarize: Several laws applying to magnetic lines of force have been enumerated.

1. Lines of force are continuous and tend to take the shortest path.
2. They are assumed to act or flow out of the north pole into air, through the air to the south pole, and back through the steel of the magnet to the north pole.
3. Lines of force established in the same direction repel each other. (Note spreading between the magnetic lines of Fig. 88, also note the forcing apart of the two magnets in Fig. 84.)
4. Conversely, lines of force in opposite directions attract each other.

Magnetic Field around a Wire.—As shown in Fig. 82, a magnetic field exists around a permanent magnet. This field may be photographed by placing iron filings on a paper placed over a magnet. If now a wire carrying a current of electricity is passed through a sheet of paper, and if iron filings are dusted on the paper, the filings will arrange themselves in concentric circles around the wire carrying the current. This action is illustrated by Fig. 85.

It follows, therefore, that a wire carrying a current is surrounded by a magnetic field, much as is a permanent magnet. In fact, there is no difference in the fundamental nature of the magnetic lines of force surrounding a magnet and the lines surrounding a wire carrying a current.

Although there is no difference in the nature of the lines of force themselves, there are several differences in the behavior of a magnetic field produced by a permanent magnet and that of a magnetic field *in air* around a wire. If the current in the wire is increased, the magnetic-field intensity is increased in direct proportion; that is, more lines of force are produced. Now if the current in the wire is decreased, the lines of force in air are decreased in the same ratio. In fact, if the current is decreased to zero, the magnetic field collapses to zero.

To summarize: A wire carrying a current is surrounded by magnetic lines of force constituting a magnetic field. *In air*, the strength of the field, that is, the number of lines of force, is directly proportional to the magnitude of the current flow.

Direction of Magnetic Field around a Wire.—The directions of the magnetic lines of force around a permanent magnet were previously considered. It was explained in the accompanying discussion that those directions were purely arbitrary. The arbitrarily assumed direction of the lines of force around a wire carrying a current is shown by the arrows on the lines of Fig. 85.

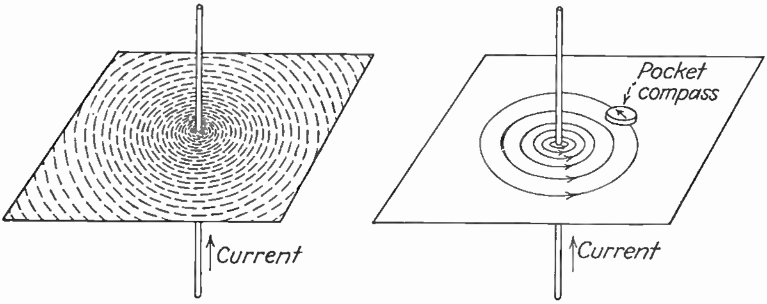


FIG. 85.—Iron filings placed on a piece of paper penetrated by a wire carrying a current will arrange themselves as shown in the first figure. The corresponding lines of magnetic force would be shown as in the figure to the right. For the direction of current shown, the pocket compass will point as indicated. If the current were reversed, the compass needle would also reverse.

Screw Rule for Field Direction.—The common wood screw or machine screw must be turned to the right (clockwise) to progress *in*. If the direction of the current in a wire is in the direction of the travel of a screw, the direction of the magnetic field will correspond to the direction the screw is being turned to force it in.

Right-hand Rule for Field Direction.—Another way of remembering the direction of the magnetic field around a wire carrying a current is as follows: Grasp the wire (or better, imagine you grasp it, because you might get a shock) with the right hand with the *thumb* pointing in the direction of current flow; then, the fingers will encircle the wire in the same direction as the lines of force.

Magnetic Field around Two Wires.—Of particular interest is the magnetic field around two wires carrying the same current, that is, two wires of a pair constituting a loop or a circuit. To make this clear, Fig. 86 has been drawn. It is assumed that the line is being studied at point x.

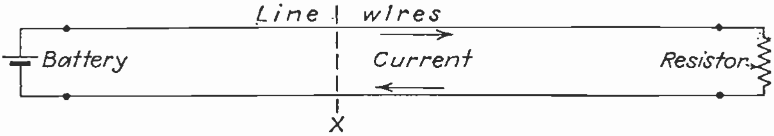


FIG. 86.—Two line wires constituting a circuit or loop.

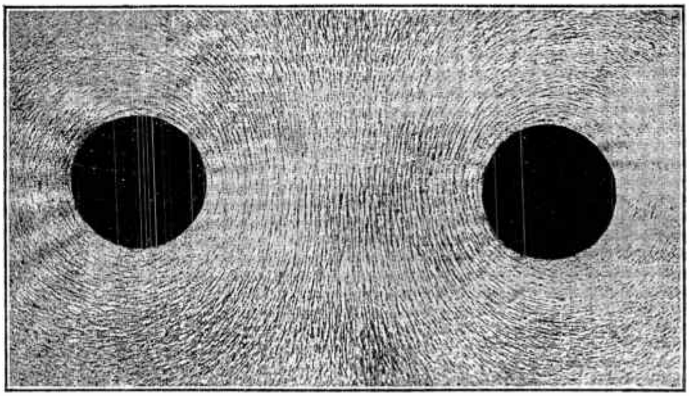


FIG. 87.—Illustrating the way in which iron filings arrange themselves when in the vicinity of a pair of wires carrying the same current. (*From Peek.*)

If the two wires are passed upward through a piece of paper which is again sprinkled with iron filings, these will arrange themselves as shown in Fig. 87.

By assuming that the currents in the two wires have the directions indicated, an application of one of the laws of the preceding section will show that the directions of the lines of force are as drawn in Fig. 88.

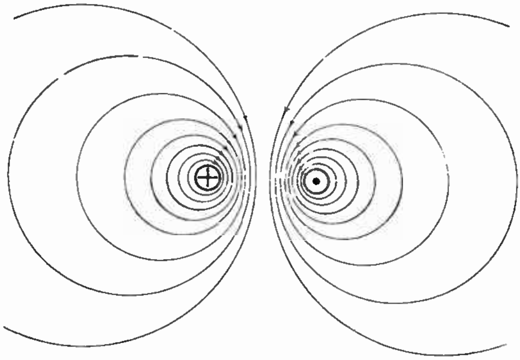


FIG. 88.—Showing the directions of the magnetic lines of force produced by two wires carrying currents in the directions indicated. On the cross section of one wire is placed a dot. This indicates that the current is flowing out. On the cross section of the other wire is placed a cross. This indicates that the current is in.

Thus it is seen that the magnetic field around two adjacent wires carrying the same current in the opposite direction is considerably distorted from the shape of the field around a single isolated wire.

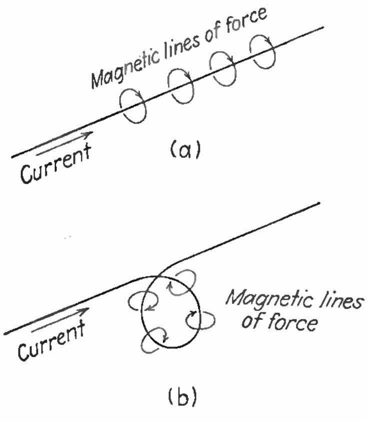


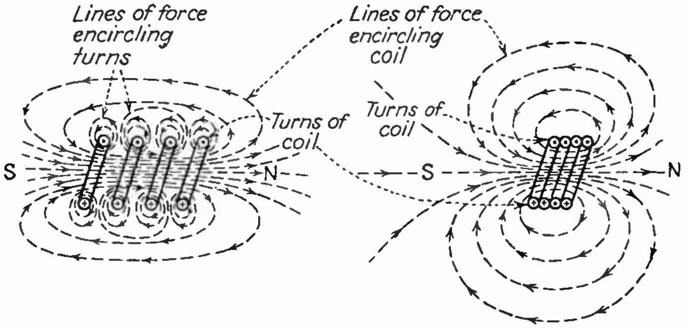
FIG. 89.—Magnetic lines of force link a straight line and a turn of wire as indicated.

Magnetic Field Produced by an Air-cored Coil.—The magnetic field around a single straight wire carrying a current is as shown in Fig. 89*a*. If now the wire is wound into a one-turn coil, the lines of force will thread this turn as indicated in Fig. 89*b*.

If several turns of wire are wound *loosely* together as shown by the cross-sectional view of Fig. 90, the resulting magnetic field will be as shown. Some of the lines will still encircle each individual wire, but in addition, some lines will pass through the center of the coil or **solenoid** and will pass completely around the turns, return-ing to the other end. Although no figure has been included, this action can be illustrated with iron filings.

Also in Fig. 90 the turns are shown closely pressed together. Although some lines of force will still encircle each individual turn, most of the magnetic field will now pass entirely through the coil as indicated.

It is apparent that the magnetic field produced by a coil of wire carrying a current has a shape similar to that of a permanent magnet. It is to be expected, therefore, that a coil of wire carrying a current should exhibit the same fundamental characteristics shown by permanent magnets, and of course this is true. Just like permanent magnets, coils carrying currents have north and south poles. Also, the other laws, like poles repelling and unlike poles attracting, also apply.



I-Turns expanded II-Turns closed

FIG. 90.—When a coil of wire or solenoid carries a current, magnetic lines of force are produced as indicated. When the turns of wire are close together, there is very little leakage flux; that is, flux which does not link all the turns. (*From Croft.*)

To summarize: A coil of wire or solenoid carrying a current establishes a magnetic field of the same general shape of that produced by a permanent magnet. This causes the coil of wire carrying a current to exhibit the same basic magnetic characteristics as does a permanent magnet.

Magnetic Field Produced by an Iron-cored Coil.—Permanent magnets are often made of steel, which of course is merely iron with a higher carbon content and with special heat-treatment; such permanent-magnet steel is very hard. Pure iron, on the contrary, makes but a poor permanent magnet and is by comparison soft.

Suppose, now, that a *core* of soft iron is inserted in the coil of Fig. 90 and the magnetic field is compared with that produced when the coil has an air core. It will be found that with an iron [1] core in the same coil and with the same current *the magnetic field will be greatly* increased.

The question immediately presenting itself is this: *Why are the magnetic lines of force increased*, giving a stronger magnetic field when the iron core is inserted? Theories of modern physics have an answer for this, but for the purpose of this book, the answer is: *The addition of the iron core causes the coil to have a stronger magnetic field because the iron core is a better conductor of magnetic lines of force than is air.*

Thus, the comparison of magnetic lines of force with an electric current becomes apparent. A given voltage will force a certain electric current through a circuit, but if the circuit is changed so that it is a *better conductor*, then a *larger* current will be caused to flow by the same voltage.

To summarize: The voltage of an electric circuit causes a current to flow. A coil of wire carrying a current produces a magnetic field. The path offered the lines of force is better in iron than air; thus, a given current in a given coil will produce a much stronger field if an iron core is inserted than if the core is merely air.

Closed Magnetic Circuits.—The coil of Fig. 90 with an iron core offers an **open magnetic path** for the lines of force. This path consists partly of iron and partly of air. The iron part of the path is fairly definite, but the air return path *is very indefinite.* According to the third law on page 135, lines of force in the same direction repel each other, and therefore they will spread out widely outside the coil. Of course these repel each other in the iron, but they stay in the iron because it is in the center of the coil, and because it offers a good path compared with air.

For convenience in studying magnetic effects, however, it is preferable to have a closed magnetic path as shown in Fig. 91*a*. It

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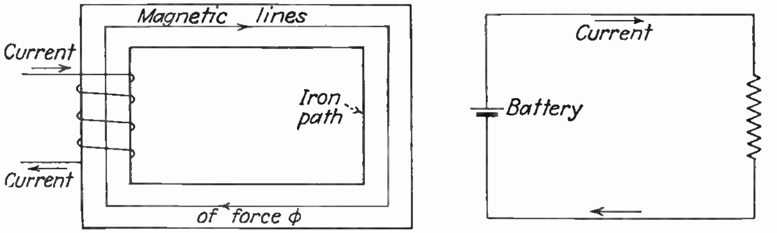
[1] The core is assumed to extend only the inside length of the coil. As no doubt the reader knows, many types of materials in addition to soft iron are used for magnetic cores. Among these are silicon-steel laminations and Permalloy. The words "iron core" are simple and expressive, but of course what is meant is a core of good magnetic properties, etc. Likewise, permanent magnets are seldom made of "pure" steel any more. Such materials as Alnico or cobalt-steel are usually employed.

is assumed that this path is so good compared with air that all the lines of force stay in the iron.

When a current of electricity is passed through the coil on the closed iron core, the magnetic effect of the individual turns unite to produce a magnetic field in the iron core. By applying the rule (page 137) it will be found that the lines of force that are established have the direction indicated.



(a)
The magnetomotive force produced by the coil of wire carrying a current forces the magnetic lines of force through the iron path

(b)
This is the electrical analogy to the magnetic circuit at (a). The electromotive force of the battery forces the current through the circuit

FIG. 91.—Magnetic and electric analogies.

Ohm's Law for the Magnetic Circuit.—The preceding sections have discussed the fact that a coil of wire carrying a current produces a magnetic field. A coil on a closed iron core was shown in Fig. 91*a*.

On page 12 it was stated that there was a fundamental law underlying all action: *The result produced is directly proportional to the magnitude of the effort and inversely proportional to the magnitude of the opposing force.* This basic law applied to electric circuits is termed Ohm's law. It was applied to the simple series circuit such as Fig. 91*b*. It was explained that an electromotive force forced the electric current through the resistance of the electric circuit.

The same reasoning is applied to the magnetic circuit of Fig. 91*a*. It is considered that the coil of wire carrying the current forces the magnetic lines of force through the magnetic circuit. It is easy to visualize it all in this way: The coil of wire carrying a current in effect generates a **magnetomotive force** (sometimes abbreviated mmf), and this *forces* the lines of force around the magnetic circuit offered by the iron core. After all, it is just as easy to form a picture of this magnetic action as of current flow; just think of the lines of force as actually flowing, if it helps.

Thus, what might be called Ohm's law for the magnetic circuit is as follows:

$$\text{Magnetic lines of force} = \frac{\text{magnetomotive force}}{\text{reluctance}}. \qquad (19)$$

Using the proper symbols, this law can be written

$$\phi = \frac{\mathfrak{F}}{\mathcal{R}}, \qquad \mathfrak{F} = \phi\mathcal{R}, \qquad \text{and} \qquad \mathcal{R} = \frac{\mathfrak{F}}{\phi}, \qquad (20)$$

where ϕ (phi) = total number of lines of force produced in maxwells,

$\mathfrak{F}$ = magnetomotive force in gilberts,

$\mathcal{R}$ = the opposition of the magnetic circuit.[1]

Magnetomotive Force.—It has been shown that magnetomotive force *forces* magnetic flux through the magnetic circuit. Magnetomotive force is analogous to electromotive force in this respect, as Fig. 91 illustrates.

It is found experimentally that if the current is increased, the magnetic lines of force will be increased in number. Also, it is found that if the current is held at the original value but if more turns are wound on the core the magnetic lines of force will be increased. From these facts, it is concluded that the magnetomotive force produced by a coil such as Fig. 91*a* is directly proportional to the product of the number of turns of the coil and the current the coil carries.

It is beyond the scope of this book to show the derivations of all the equations, particularly those which are rather theoretical. The equation for magnetomotive force is somewhat of this nature, so it will merely be stated that

$$\text{mmf} = \mathfrak{F} = 0.4\pi NI \text{ gilbert}. \qquad (21)$$

where N is the number of turns in the coil and I is the current in amperes it carries. In practice the constant 0.4π is sometimes dropped, and the relation

$$\text{Amperes} \times \text{turns} = NI \qquad (22)$$

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[1] The unit oersted was formerly applied to reluctance, but at present there is no name for this unit of measure.

is used to specify the magnetomotive force. Of course, when this is done, the unit is the ampere turn and *not* the gilbert.

To summarize: The magnetomotive force produced by a coil is proportional to the product of the number of turns in the coil and the current carried. When the current is in amperes, $0.4\pi NI$ gives the magnetomotive force in gilberts. A unit of magnetomotive force, NI = ampere turns, is sometimes used, particularly in the more practical phases of electrical work. Its use is probably decreasing and will not be stressed in this book.

Reluctance.—As Eq. (19) and the accompanying discussion state, *reluctance is the opposition offered by a magnetic circuit to the establishment of a magnetic field.* Thus, an iron core placed in a coil offers less reluctance than air, and a greater number of lines of force are established in the coil.

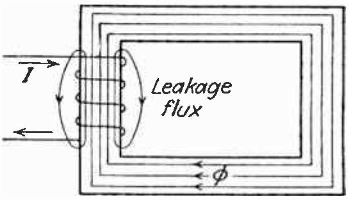

FIG. 92.—The magnetomotive force produced by the coil of wire carrying the current forces the magnetic flux ϕ through the iron core. Some leakage flux, however, does not follow entirely around the core.

For convenience of study, Fig. 91*a* is here reproduced as Fig. 92. The coil of N turns carrying a current of I amperes will produce a magnetomotive force of $0.4\pi NI$ gilbert. This magnetomotive force will establish a magnetic field in the iron core.[1]

Now since the magnetomotive force must *force* the magnetic lines through the iron, several factors will enter into determining the opposition to the lines of force offered by the iron. *The longer the iron path,* the greater the distance the lines must be forced, and hence *the greater will be the reluctance;* also, *the greater the cross-sectional area,* the more elementary paths will be in parallel, and *the less will be the reluctance.* A comparison of this statement with the resistance offered by a conductor as discussed on page 46 is helpful.

The final factor that must be considered in determining the reluctance of a magnetic circuit is a factor determined by the physical nature of the material constituting the magnetic circuit. In resistance calculations the resistivity in ohms per centimeter cube, a factor determined by the physical nature of the material, was in-

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[1] Some magnetic flux will also "leak" out of the core and will establish a weak magnetic field in the surrounding air. This so-called leakage flux will be treated on p. 152.

serted in the equation. It has already been shown that *iron is a better conductor of magnetic lines of force than is air.* Thus, if the magnetic-field-conducting ability of air is taken as unity,[1] then the magnetic-conducting abilities of all other materials, such as iron, can be expressed with respect to air.

The ratio of the magnetic-conducting ability of iron compared with air is called the **magnetic permeability,** or simply the **permeability,** and is designated by μ, the Greek letter mu. Therefore, if the reasoning of this paragraph and the one preceding is followed,

$$\text{Reluctance} = \mathcal{R} = \frac{l}{\mu A} \qquad \text{(no units)}, \tag{23}$$

where l is the length of the magnetic path in centimeters, A is the cross-sectional area of the path in square centimeters, and μ is as just defined, a factor expressing the magnetic-conducting ability of the path as compared with air. A comparison of this equation with Eq. (8), page 47, for resistance will show that the resistivity ρ for an electric conductor is placed above the dividing line, but that μ for a magnetic material is placed below the line. This is because ρ is a *resistance* or opposition term, but μ is a *conductance* term.

To summarize: The reluctance a magnetic path offers to the establishment of a magnetic field is directly proportional to the length of the field and inversely proportional to the permeability and the cross-sectional area.

Magnetic Flux and Flux Density.—It is convenient to speak of the magnetic flux instead of the longer term magnetic lines of force. Thus it is correct to say that magnetomotive force produces magnetic flux in a magnetic circuit. The term is a good one to use, because one definition of flux is a "a flowing," and it is often convenient to think of a magnetomotive force as establishing a flux or *a flowing* around the magnetic circuit.

It is also convenient to be able to express the density of the magnetic lines of force or flux established within the core. Density in the magnetic sense means this: *How many lines of force are there*

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[1] To be correct, vacuum is taken as unity, but for all practical purposes, air is the same.

passing through each square centimeter of cross-sectional area of the core, at right angles to the flux? Thus, the total flux ϕ produced, divided by the cross-sectional area in square centimeters, gives the flux density, or

$$\text{Flux density} = B = \frac{\phi}{A}\ \text{gausses,[1]} \qquad (24)$$

where ϕ is the total number of lines produced in the core and A is the cross-sectional area of the core in square centimeters.

Magnetizing Force.—With reference to Eqs. (21) and (23), it would seem that it should now be possible to proceed with the quantitative calculation of magnetic circuits, but unfortunately such is not the case. Equation (21) gives the magnetomotive force, but the difficulty is with Eq. (23). The *permeability μ is not a constant* (as is the resistivity ρ as used in resistance calculations). The permeability μ varies for each different flux density when an iron core is used. With an air core, the permeability is always unity, however.

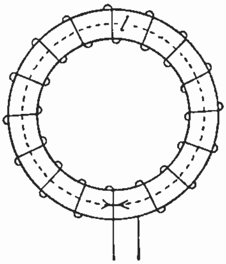

FIG. 93.—A uniform magnetic circuit uniformly wound with a coil of wire.

In devising a means of solving the difficulties caused by the permeability μ varying with the amount of flux in the core, a new term must be introduced. This is the **magnetizing force.** This term can easily be explained by reference to Fig. 93. Now the magnetomotive force applied by the coil to the iron core forces the magnetic flux around the iron path. Each unit length of the path has the same reluctance, and the flux passing through each unit path is the same. Since this is true, there is from Eq. (20) a magnetomotive force drop equal to $\phi\mathcal{R}$ per unit length of magnetic path. In other words, the applied magnetomotive force distributes itself around the magnetic circuit. *The total applied magnetomotive force divided by the length of the path gives the magnetizing force.* When

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[1] Lines of force in air are measured in maxwells (page 142), one maxwell being one line. Flux density or lines per square centimeter in any material other than air is measured in gausses. The reader can save himself confusion by just thinking of lines in either case and not worrying too much about the units.

the applied magnetomotive force is in gilberts ($0.4\pi NI$) and the length of the path l is in centimeters, the magnetizing force is

$$\text{Magnetizing force} = H = \frac{0.4\pi NI}{l} \text{ oersted.}^1 \qquad (25)$$

In fixing the meaning of magnetizing force in mind, attention should be focused on these facts: The value of permeability is a variable, but if the number of gilberts per centimeter (or oersteds) applied is known, curves can be used to find the permeability μ. That is, μ occurs in the reluctance equation (23); but μ is a variable (instead of being a constant like resistivity ρ). Hence *curves* for the specific magnetic material being used for the core must be consulted to find μ. Now to attempt to plot these curves in terms of total magnetomotive force would be of little use, but if the curves are plotted in terms of the magnetizing force H or the gilberts per centimeter it really means something, because this information can be used for cores of any dimensions.

There is a very interesting and useful relation between magnetizing force and lines of force per square centimeter in air or, more strictly speaking, in a vacuum. It can be proved theoretically that one gilbert applied to a centimeter cube of vacuum will produce one line of force through the cube. This means that *the gilberts per centimeter* (*oersteds*) *and the field intensity or lines* (*or maxwells*) *per square centimeter in air are numerically equal*. In other words a doughnut or ring-shaped core, such as Fig. 93, with a uniformly and closely wound coil (a form of winding often used in communication equipment) will have a magnetizing force of $H = 0.4\pi NI/l$ oersted, and this will be numerically equal to the lines of force per square centimeter of cross-sectional area if the core is air. This neglects **magnetic leakage.** The particular advantage of this will be more apparent after studying page 150. Whereas it would be difficult to measure the field intensity or lines per square centimeter in air, it is easy to calculate the magnetizing force ($H = 0.4\pi NI/l$), and the two are numerically equal.

To summarize: The total magnetomotive force applied to a core is not a very descriptive quantity. If the length is known, then the

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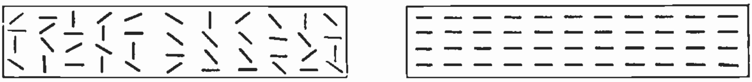
[1] This was formerly measured in gilberts per centimeter but is now measured in oersteds. The former term is so descriptive that it will be used often in this book to assist in clarifying the discussions.

magnetomotive force per unit length, or magnetizing force in gilberts per centimeter, or oersteds, is available. This term gives the actual magnetic stress applied to the iron and is useful in plotting curves. It works out theoretically that the magnetizing force and the lines per square centimeter in air are numerically equal.

Magnetization Curves.—From a practical point of view, iron or any other good magnetic flux conductor can be thought of as composed of a large number of very small permanent magnets. Each little magnet has its own magnetic field. When the iron is in an unmagnetized state the magnets point in random directions, and the net magnetic effect is zero.


(a)
In an unmagnetized piece of metal the "small permanent magnets" point in random directions

(b)
When a piece of metal is magnetized, the small "magnets" all align themselves, thus combining their magnetic effects

FIG. 94.—Illustrating a simple theory of magnetism.

Now suppose that a coil is placed over the iron core and a current is passed through the coil. The coil itself would produce some lines of force even with an air core. When it has an iron core, however, the magnetic field produced by the coil lines up the little permanent magnets, and they add their magnetic fields to the flux that would be produced by the coil alone. This action is illustrated by Fig. 94.

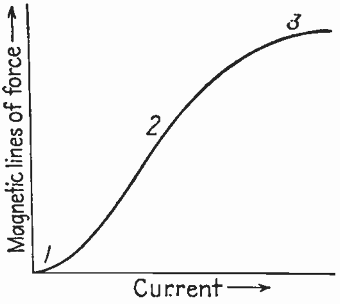


FIG. 95.—As the current in a coil wound on an iron core is increased, the flux in the core increases as shown. The three regions are marked 1, 2, and 3.

The relation between the magnetizing force and the resultant flux produced in an iron core is given in Fig. 95. Note that there are three regions to this curve: (1) where the current is very weak and the field produced by the coil is not sufficiently strong to overcome the "internal friction" of the small elementary magnets; (2) where the current has been increased to the point that the internal atomic friction is overcome and the magnets are all lining up; and (3) where the elementary magnets have all lined up and the iron is **saturated.**

This is of course but an approximate picture, because iron or other magnetic material certainly *is not* made up of small magnets. Nevertheless, iron is composed of atoms, and these contain revolving electrons. Now a current of electricity produces a magnetic field (page 136), and moving electrons constitute a current. Therefore, it is reasonable to assume that an atom may have a magnetic field and that it may, accordingly, be considered to be a small permanent magnet.

To summarize: There are three regions to the magnetization curve of a magnetic material such as iron. These are thought to be due to the action of the magnetizing force on the small atomic magnets. The atomic magnets probably represent the magnetic effects of the atoms themselves.

Permeability.—Magnetization curves for several materials used in electrical work are shown in Fig. 96. For convenience in solving problems, one set of curves is plotted on semilog paper. The small set of curves gives the characteristics plotted in the normal manner.

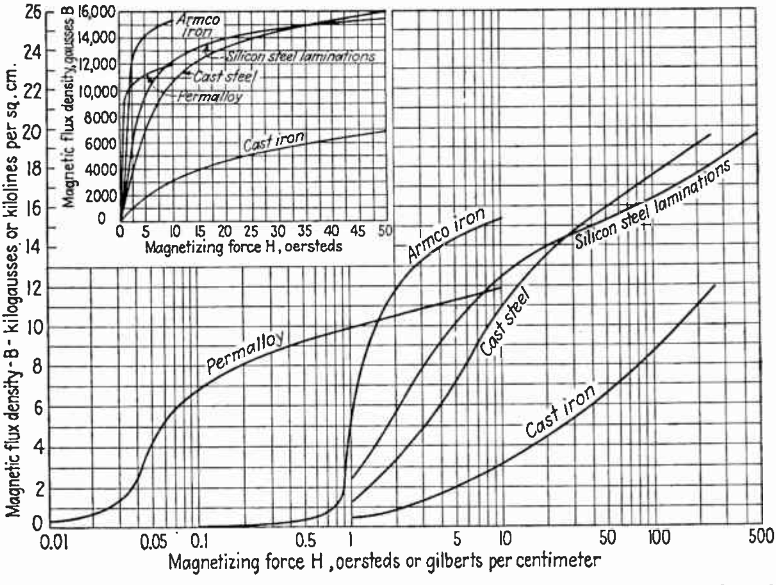


FIG. 96.—Magnetization curves for various metals. The lower curves are plotted with the magnetizing force H to a logarithmic scale for ease in obtaining values.

On page 144 it was stated that the ratio of the magnetic conducting ability of iron compared with air is called the magnetic permeability, or simply the permeability. The flux density B is a measure of the flux produced in an iron core. The magnetizing force H (page 145) is a measure of the flux produced in air. Therefore, the permeability becomes

$$\mu = \frac{B}{H}, \tag{26}$$

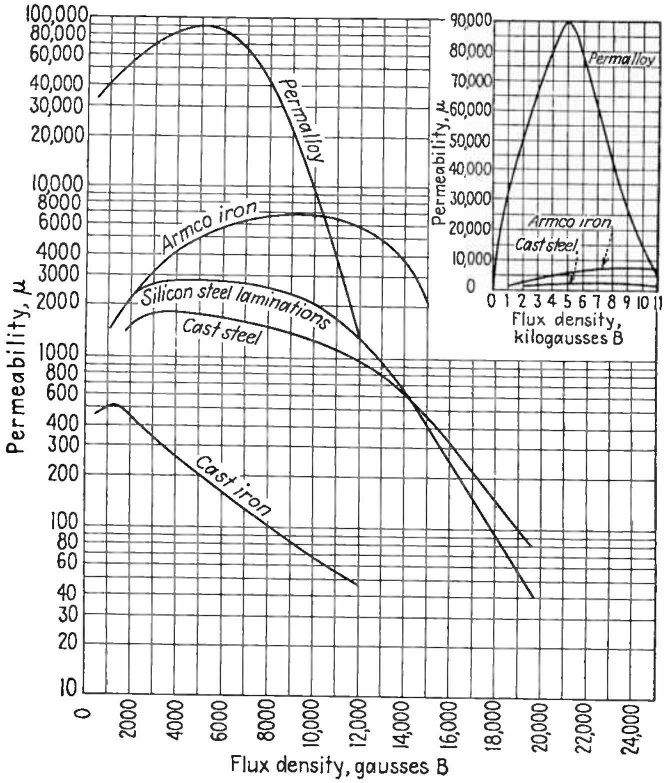


FIG. 97.—Permeability curves for the metals of Fig. 96.

where B is the flux density, or lines per square centimeter in the material being considered, and H is the field intensity, or lines per square centimeter in air measured in oersteds (or in gilberts per centimeter since this equals the field intensity, page 146).

The permeability curves of Fig. 97 are plotted from data obtained from Fig. 96. These curves are plotted on semilog paper for convenience in determining their numerical values and are also plotted in the usual manner for better comparing their characteristics relatively. Note that the curve marked "Permalloy" reaches a very high permeability of over 90,000, and even higher values have been obtained. This means that under comparable conditions over 90,000 times as many lines of force would be established in the Permalloy as would be set up in air. Also note that the Permalloys are good conductors of magnetic flux at very low values of H. This is advantageous because in communication circuits, the current, and hence H, is small.

The Permalloys are a group of magnetic alloys extensively used in communication equipment. They consist essentially of about 78.5 per cent nickel and 21.5 per cent iron. In some instances a small percentage of chromium or molybdenum is added, and the iron content is slightly reduced. Permalloys are very interesting and have made possible many of the remarkable developments of modern communication.[1]

To summarize: Permeability μ is the ratio of flux density B to field intensity H. Since the field intensity and the magnetizing force are equal (page 146), H can be calculated and B can be measured. The permeability of such alloys as Permalloy may be well over 90,000.

Magnetic-circuit Calculations. *Series Circuits.*—As the reader will observe in these calculations, there is a close similarity existing in the solving of magnetic problems and in the solving of electric-circuit problems.

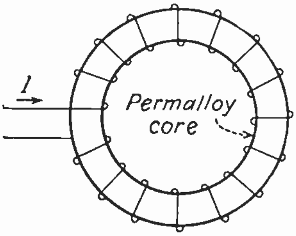


FIG. 98.—Magnetic circuit for Prob. 1.

Problem 1.—A Permalloy core is toroidal in shape and circular in cross section. The average length of the core is 18 centimeters, and the cross-sectional area is 1.4 square centimeters. A single layer of 206 turns of wire is placed uniformly around the coil and carries 625 milliamperes. Calculate the total flux produced. The circuit is shown in Fig. 98.

Solution.—Step 1. Calculate the magnetizing force H.

$$H = \frac{0.4\pi NI}{l} = 1.257 \times 206 \times \frac{0.625}{18} = 9.0 \text{ gilberts per centimeter, or oersteds.}$$

Step 2. From Fig. 96, find what flux density this magnetizing force will produce. This is found to be about 11,800 lines, or gausses.

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[1] See Elmen, G. W., Magnetic Alloys of Iron, Nickel, and Cobalt, *Electrical Engineering*, December, 1935.

Step 3. Calculate the total flux produced. This will be the product of the flux density and the area.

$$\phi = BA = 11{,}800 \times 1.4 = 16{,}500 \text{ lines, or maxwells.}$$

Problem 2.—An air gap of 0.05 centimeter is cut across the Permalloy core so that all the lines of force must cross this gap in addition to being established in the Permalloy. How many lines of force will the same current now establish?

Solution.—Step 1. The magnetic circuit now consists of two reluctances in series. The total reluctance will be

$$\mathcal{R}_t = \mathcal{R}_p + \mathcal{R}_a = \frac{l_p}{\mu_p A} + \frac{l_a}{\mu_a A},$$

where $\mathcal{R}_p$ is the reluctance of the Permalloy and $\mathcal{R}_a$ is the reluctance of the air. This problem cannot be solved directly because from Fig. 97 the value of μ depends on the value of the flux density B. A cut-and-try method must be used.

Step 2. The reluctance of the air gap is in series with the reluctance of the Permalloy. There is a total of $\mathcal{F} = 0.4\pi NI = 1.257 \times 206 \times 0.625 = 162$ gilberts magnetomotive force available. Since $\mathcal{F} = \phi\mathcal{R}$, this magnetomotive force of 162 gilberts must divide itself so that some will be across the air gap, forcing the flux through it, and the rest will be across the Permalloy. As the reader will find in solving such problems, a very large percentage of the total magnetomotive force will be needed across the air gap. Thus, as an *estimate* of the flux that will be produced, assume that *all* the available magnetomotive force is used across the air gap. Since the reluctance of the air gap is $\mathcal{R} = l/\mu A = 0.05/1 \times 1.4 = 0.0357$, the *assumed* flux will be $\phi = \mathcal{F}/\mathcal{R} = 162/0.0357 = 4500$ lines and $B = \phi A = 4500/1.4 = 3{,}210$ gausses.

Step 3. Let it now be assumed that the actual flux density is 3210 gausses. From Fig. 96, this will require a magnetizing force of 0.045 gilbert per centimeter, or oersted. Then, the total magnetomotive force required for the Permalloy will be $\mathcal{F} = Hl = 0.045 \times 18 = 0.81$ gilbert. Now only 0.81 gilbert is required for forcing the total flux of 4500 lines through the Permalloy, and this is so small that it may be neglected. Thus, in this particular problem, it is satisfactory to *assume that the final answer is* 4500 *lines*. If the magnetomotive force required to force the flux through the metallic part had not been negligible, it would have been necessary to assume a value of flux *smaller* than 4500 lines and then recalculate, and thus find the magnetomotive force required for the metallic path and the *new* value required for the air. The problem would be considered as solved when the sum of the two magnetomotive forces approximately equaled the total magnetomotive force available.

Parallel Circuit.—A very close similarity was shown to exist in the preceding problem between the solution of series magnetic and series electric circuits. Of course, the problem was made somewhat involved because of the fact that the permeability μ varies. In the following problem, a similarity will be found between the solution of parallel magnetic and parallel electric circuits.

Problem.—The core of a small 60-cycle power transformer is shown in Fig. 99. The material is silicon steel laminations. The cross-sectional area of each of the outside legs is 3.0 square centimeters and that of the central portion is 6.0 square centimeters. The average length of the flux path as indicated by the dotted lines is 16 centimeters. A coil of 500 turns is placed as indicated, and the current is 1.0 ampere. Calculate the flux in each portion of the magnetic circuit.

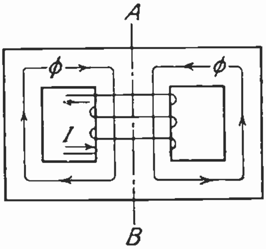

FIG. 99.—The core of a simple 60-cycle power transformer is often made as indicated.

Solution.—Step 1. This problem can be solved easily by considering the core to be composed of two separate cores as divided by the line AB. The flux in the center portion will be the sum of the flux in the two outer parts.

Step 2. Since the magnetomotive force applied to each part is the same, just as the voltage applied to two resistors connected in parallel to a battery is the same, the magnetizing force applied to each outside portion will be

$$H = \frac{0.4\pi NI}{l} = 1.257 \times 500 \times \frac{1}{16} = 39.3 \text{ gilberts per centimeter, or oersted.}$$

Step 3. From the magnetization curve of Fig. 96 for silicon steel, a value of $B = 15{,}000$ gausses corresponds to this magnetizing force.

Step 4. The total flux will be the product of this flux density and the cross-sectional area.

$$\phi = BA = 15{,}000 \times 3 = 45{,}000 \text{ lines.}$$

Step 5. Calculate the total flux. Since the flux from both sides passes through the middle portion, the flux passing through the coil will be the sum of the flux in each of the outside portions, or 90,000 lines.

Magnetic Leakage.—As is evident, magnetic calculations are approximate as compared with other computations. Many approximations are made, such as estimating the length of the flux path. Magnetic leakage is also a factor making the computations approximate.

Considering electric-current flow, for the moment, it is usually thought that the current stays in the wire because the wire is surrounded by insulation. An alternate point of view is that the current stays in the wire because the wire is such a good conductor compared with the insulation. In fact, the conductivity of the copper is millions of times that of the surrounding insulation. Little wonder, then, that the current chooses the conductor in which to flow and that the leakage current is small.

In considering the flow of magnetic flux around a magnetic circuit, the same fundamental principles are found to exist. The magnetic flux prefers to follow the iron path only because it is a better path than offered by air. But, contrasted with the electric circuit considered in the preceding paragraph, the permeability of iron is at best only some 5000 times that of air, and even the permeability of Permalloy is only about 100,000 times that of air. Therefore, it should be expected that the leakage of magnetic lines of force or **leakage flux** is high in magnetic circuits, and such is the case.

Several illustrations of leakage flux will now be considered. In Fig. 90 it is shown that certain lines of force do not encircle the entire coil, but only individual turns, or at best a few turns. These constitute leakage flux. In Fig. 91 a closed magnetic path was shown, and to avoid confusion it was inferred that all the lines of force would be confined to the iron. Actually, however, there will be some leakage flux even with a *closed* core as indicated in Fig. 92, page 143. If the iron path has high reluctance, or if an air gap is cut in the path (page 151), then the increased reluctance will "encourage" the establishment of leakage flux, because the magnetic path offered by the core is now not so superior to that offered by air. Distributing the windings uniformly around the core such as in Fig. 93 also distributes the applied magnetomotive force and cuts down the tendency for leakage.

Magnetic Shielding.—As shown in the preceding section, considerable magnetic leakage may exist around electrical apparatus. Also, the earth's magnetic field immerses all objects. In certain instances these stray magnetic fields may be very bothersome, causing instrument errors, deflecting cathode-ray tubes (particularly those operated at low anode voltages and with correspondingly low electron speeds), and producing other unwanted effects.

Where such constant magnetic fields are bothersome, the device being influenced is entirely enclosed in a box or case of good magnetic-conducting ability, such as iron. As shown in Fig. 100, a metallic box of good flux-conducting ability will distort the normal magnetic field and conduct the lines around the device or circuit to be shielded. This, of course, prevents the constant field from influencing the device or circuit. If one shield is insufficient, a second may be used.[1]

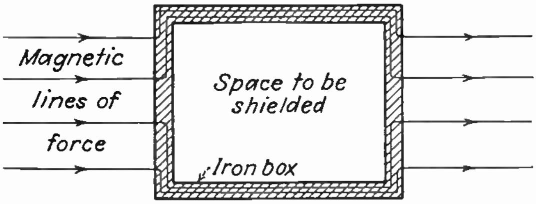

FIG. 100.—An iron box conducts the magnetic lines of force around the space or object to be shielded.

Motor Action.—As the reader probably knows, when a wire immersed in a magnetic field carries a current, the wire experiences a force tending to force it out of the field. This is, of course, the principle on which the electric motor operates.

The reason the wire moves can be explained by Fig. 101. The reaction between the magnetic field and the lines of force produced by the wire is such that the field is strengthened on one side of the wire and weakened on the other, and the wire is accordingly forced in the direction of the weaker field.

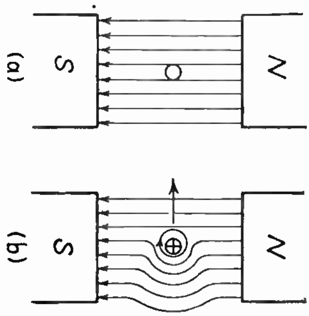

FIG. 101.—In (*a*) the wire is in a magnetic field but carries no current. In (*b*) the wire is carrying a current in, and the magnetic field will be distorted as shown. This will force the wire to the left. The field will be strengthened on the right side of the wire where the lines are in the same direction, and weakened on the left side where the lines are in the opposite direction.

The force acting on the wire is proportional to the strength of the current flow, to the length of the wire in the field, and to the field strength. The reasons for this follow directly from Fig. 101.

There is a rule, known as the "left-hand rule" for figuring out the direction of motion when a wire carrying a current is placed in a magnetic field. It is sincerely believed that it is far more difficult to attempt to remember such

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[1] If the stray field is alternating, then the shield should be laminated (page 169).

a rule than it is to figure the direction out with a simple sketch such as Fig. 101.

Generator Action.—If the reader will refer back to Fig. 92, page 143, he will observe that the electric circuit and the magnetic circuit are *linked together* like the links of a chain. This is also evident from other illustrations which have been included.

The loop of wire in Fig. 102*a* is linked with the lines of force shown by dots coming out of the paper, the dots representing the ends of arrowheads. It may not be evident at once that a linkage exists in this instance. But, referring to the first law on page 135, it is stated that lines of force are continuous.

The electric generator, which is used to generate most of the electric energy used today, is based on the principle of *changing* flux linkages. This principle is known as Lenz's law, and may be stated as follows: *Whenever there is a change made in the flux linking a circuit, there is induced in the circuit an electromotive force (or voltage) that tends to oppose the change.* This law will now be illustrated.

When the loop of Fig. 102*a* is moved to position *b*, *no* voltage is induced in the wire, because in position *b* the same number of lines of force are linked as in position *a*. When the loop has moved to position *c*, however, an electromotive force is induced, because the flux linkages are now changed. This electromotive force will cause a potential difference to exist between the ends of the loop, and a current of electricity will flow in an external circuit if such is attached to the ends of the loop.

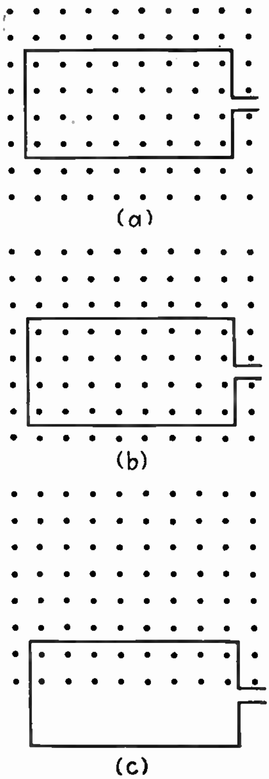

Fig. 102.—Diagrams illustrating generator action. The dots represent magnetic lines of force.

According to Lenz's law an electromotive force will be induced *which tends to oppose the change.* If the loop of Fig. 102 is open, the electromotive force will be generated as stated, but no *electrical* force will exist opposing the motion. If the loop is closed, however.

a current will flow, and this current, by motor action, will oppose the motion. It is obvious, for example, that an electric generator must develop an opposing force so that the steam turbine must do work in turning the generator. Also, it is well known that a hand-cranked telephone magneto generator turns easily when it is open-circuited, but turns very hard when it is short-circuited.

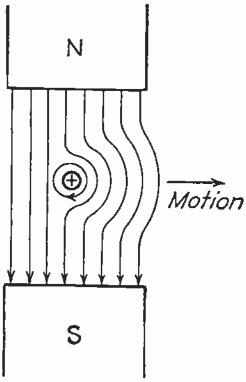

Fig. 103.—If the magnetic field is directed downward, and if the motion of the wire is to the right, then the voltage induced in the wire will be directed as shown. This direction will be such that if it is allowed to force a current through the wire, the magnetic field produced by the current will be as indicated, in such a direction as to cause magnetic effects opposing the motion of the wire.

This principle makes it possible to determine readily the direction of the electromotive force which is induced in a conductor. To illustrate this principle, Fig. 103 has been added. Here is shown a magnetic field directed downward and a wire that is being moved to the right. The question is this: What will be the direction of the induced electromotive force? The answer is that from Lenz's law the electromotive force must be in such a direction that if the external circuit (see Fig. 102) is closed and a current is allowed to flow then this current will be in such a direction as to oppose, by motor action, the motion generating it.

Since lines of force in the same direction add and lines in the opposite direction subtract, and since for the motion to the right to be opposed, the field must be *intense* at the right of the wire (see Fig. 101), then the lines of force must encircle the wire as shown. In order that lines of force may encircle the wire as shown, the current must be directed *in* (page 137), and this is indicated by the cross at the center of the wire, the cross representing the tail of an arrow. *This means that the electromotive force will be directed in.*

A right-hand rule has been developed from which the direction of the induced electromotive force can be deduced. But it is felt that the student will be further ahead if the time available is spent studying the method explained above rather than trying to remember a rule of thumb.

With reference to Fig. 102, it has been stated that an electromotive force or voltage is induced when the flux linkages are changed.

The magnitude of this voltage depends on the *rate of change of the linkages*, that is, on the flux linkages changed per second.[1]

To summarize: An electromotive force or voltage is induced in an electric circuit whenever a change is made in the flux linking that circuit. The voltage will be in such a direction that if a current is permitted to flow it will, by motor action, oppose the mechanical force producing the motion.

Self-inductance.—As was explained in the preceding section, a voltage was induced in a loop of wire when the loop was moved so that the flux linkages were changed. Now the same result is obtained when the loop remains fixed in position but the magnetic field is changed. All that is necessary for a voltage to be induced is to have a change in flux linkages, irrespective of how the linkages are changed. This action can be explained by Fig. 104.

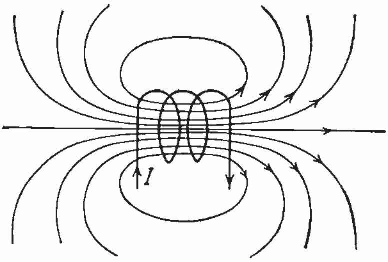

FIG. 104.—Illustrating the way magnetic lines of force link an air-cored coil carrying a current of I amperes.

Here is shown a simple coil of wire carrying a current which will establish a magnetic field as indicated. The number of lines of force established will depend on the strength of the current flowing. As long as the current remains fixed in magnitude and direction, nothing will happen except of course the wire will heat a little. If the current is *increased*, however, more lines of force will be established. This will change the flux linking the coil, and a voltage will be induced in the coil. If the current is *decreased*, then the flux linking the coil will be decreased, and a voltage will again be induced.

It is apparent that this action depends on some peculiar property that a coil has, because if the wire is stretched out straight this action will (largely) disappear. No such property was found in a

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[1] In discussing induced voltages, a *wire* loop has been considered. Theoretically, a voltage would be induced in *any* object, even a loop of glass tubing if the flux linking the glass loop is changed.

resistor, for example. *The property of an electric circuit by virtue of which a voltage is induced in the circuit when the current flowing in the circuit is changed is called* **self-inductance.**

To summarize: Note in particular the fact that self-inductance is a *property* of a circuit, just as resistance is a property or a characteristic of a circuit. Self-inductance is not an induced voltage but is the property that causes the voltage to be induced when the current is changed.[1]

The Nature of Self-inductance.—From Lenz's law, the direction of the induced voltage will be such as to oppose the change. Now the *change* is a change in the magnitude of the current. Thus if the current is being *increased*, the voltage will be induced in such a direction as to *oppose* the increase; if the current is being *decreased*, the voltage will be induced in such a direction as to tend to keep it from decreasing. Because of this opposing relationship, the voltage is often called a **back voltage.**

Self-inductance is, therefore, similar to inertia in physical bodies. Thus a body at rest tends to remain at rest, and a body in motion tends to remain in motion. Similarly, self-inductance, or the *inertia effect* of electric circuits, tends to oppose changes in the current flowing, irrespective of whether they are increases or decreases in current.

The Unit of Self-inductance.—Electrical units are often named after the early experimenters who did much to further knowledge in a particular field. The unit of self-inductance is named after Henry.

A circuit has a self-inductance of one henry when a rate of change of one ampere per second causes an induced voltage of one volt. This is the way in which the unit of self-inductance is correctly defined, but the use of this particular definition requires the application of higher mathematics which is probably unfamiliar to the reader. Thus, for the purpose of this book the unit of self-inductance will be defined as follows: *A circuit has a self-inductance of one henry when an average rate of change of one ampere per second causes an induced average voltage of one volt.* (Note insertion twice of the word *average.*)

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[1] There are three fundamental properties or characteristics of electric circuits; these are resistance, inductance, and capacitance. This last property will be considered in the following chapter. Resistance, inductance, and capacitance are also called circuit parameters (or measures) as at the opening of this chapter.

According to the fundamental relations defining the various electrical units, *an average voltage of one volt is induced in a circuit when* 10^8 *flux linkages are changed in one second.* Now flux linkages are the product of flux and turns of wire or ϕN. Therefore, for *any circuit*, the average voltage induced will be $E_{av} = N\phi/10^8t$. But from the definition of self-inductance, $E_{av} = LI/t$. Therefore,

$$\frac{LI}{t} = \frac{N\phi}{10^8t} \quad \text{and} \quad L = \frac{N\phi}{10^8I}. \tag{27}$$

To summarize: Inductance is a *property* of an electric circuit by virtue of which a back voltage is induced when the current in the circuit is changed. In order for a voltage to be induced in a circuit, flux linkages must be changed. Flux linkages are the product of lines of force ϕ and the turns N with which they are linked. Considering all factors, and as Eq. (27) shows, a circuit has a self-inductance of one henry when a current of one ampere causes a flux linkage of 10^8 lines. This linkage can be due to many turns and little flux, or vice versa.

Calculations of Self-inductance.—The reader may be disappointed to find that equations are not included in this book whereby the inductance of various coils can be calculated. This book is devoted to the electrical fundamentals. The fundamental principles of self-inductance have been given, and Eq. (27) is the fundamental expression for the self-inductance of *any* circuit.

Now it may be easy or it may be very difficult to calculate the self-inductance of a circuit; it all depends on how easy or how difficult it is to calculate the flux ϕ linking the turns N. As an example, consider Fig. 105. The current is 1.2 amperes, the number of turns is 480, the cross section of the core is 4.36 square centimeters, the average length of the flux path in the core is 12.8 centimeters, and the material is silicon steel laminations. Calculate the inductance.

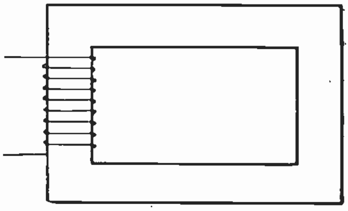

FIG. 105.—Magnetic circuit for problem on self-inductance.

Solution.—Step 1. Equation (27) will be used to find the inductance. The leakage flux will be neglected. This is a uniform magnetic circuit. The first step is to calculate the magnetizing force. $H = 0.4\pi NI/l = 1.257 \times 480 \times 1.2/12.8 = 56.4$ gilberts per centimeter, or oersteds.

Step 2. From the curve of Fig. 96, find out how much flux density will be produced by this magnetizing force. From the silicon steel curve this is $B = 15{,}500$. Then $\phi = BA = 15{,}500 \times 4.36 = 67{,}500$ lines.

Step 3. Use this value in Eq. (27), and calculate the inductance.

$$L = \frac{N\phi}{10^8 I} = \frac{480 \times 67{,}500}{10^8 \times 1.2} = \frac{32{,}400{,}000}{120{,}000{,}000} = 0.27 \text{ henry.}$$

Note in particular that the current enters into the equation for inductance and that the characteristic curve for the metal must be used. This means that *a coil with a core of magnetic material such as iron has a value of inductance depending on the current used.* To prove this statement, assume that the current value is 0.8 ampere. Then, $H = 1.257 \times 480 \times 0.8/12.8 = 37.7$ gilberts per centimeter, or oersteds. The corresponding value from the curve is $B = 15{,}000$. Then $\phi = 15{,}000 \times 4.36 = 65{,}300$ lines. Using this value, $L = (480 \times 65{,}300)/(10^8 \times 0.8) = 0.392$ henry. This shows that the inductance is *higher* with a smaller current. No generalization should be made from this, however; it all depends on which side of the maximum permeability point the magnetizing force lies. Thus with reference to Fig. 97, where $B = 15{,}500$, $\mu = 300$, but when $B = 15{,}000$, $\mu = 400$; but, if the flux densities were below the value giving the maximum permeability, then *decreasing* the current as was done in these problems would *decrease* and not increase the inductance.

To summarize: It is very easy to calculate inductance from the fundamental relations given by Eq. (27) *if* it is possible to calculate ϕ. This can be done only when the magnetic circuit is known. Where a closed iron path exists, ϕ can be found quite accurately. Where a coil has an air core, ϕ cannot be readily calculated. For this reason, the inductance of *air-cored coils* having vague magnetic paths is best determined by the use of empirical equations based on experimental observations. Such equations are given in handbooks and textbooks on communication.

Illustrative Problems on Inductance.—According to the definition, the inductance of a circuit is one henry when an average rate of change of one ampere per second induces an average voltage of one volt. That is, $E_{av} = LI/t$. A coil has an inductance of 0.068 henry and is carrying a 1000-cycle current of 72 milliamperes. The resistance of the coil is negligible. Calculate the average voltage across the coil.

Solution.—Step 1. The voltage across the coil will be the back voltage due to the self-inductance. Before this can be found, the average rate of change of current must be computed. From Fig. 68, page 114, a 1000-cycle current will rise from zero to maximum in 1/4000 second. Then the average rate of change will be $I/t = 0.072 \times 1.414 \times 4000 = 407$ amperes per second. (The value 1.414 was introduced because the problem stated that the current was 72 milliamperes and did not state whether it was the average, effective, or maximum value. When no statement is given, it is assumed to be the effective value.)

Step 2. Knowing the average rate of change of current, calculate the induced voltage. $E_{av} = LI/t = 0.068 \times 405 = 27.7$ volts, average voltage. If the effective value is wanted, it will equal $(27.7/0.637) \times 0.707 = 30.7$ volts.

Current Rise and Decay in Inductive Circuits.—In Fig. 106 is shown a coil and a resistance in series and so arranged that if the switch is thrown *down* the battery will force a current through the circuit. The manner in which the current rises in this circuit will now be explained.

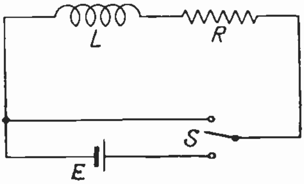

FIG. 106.—When the switch S is thrown to the down position the battery forces a current through the coil L and the resistor R.

First, assume that the inductance is *not* in the circuit. If this is true and the switch is thrown down, the current will *immediately* rise to the value $I = E/R$. Now consider that *both* the inductance and resistance *are* in the circuit. With inductance in the circuit, the current will *not* immediately rise to the final value of $I = E/R$. It will *slowly* rise to this final value for the following reason.

Before the switch is thrown down there is no magnetic flux linking the turns of the coil. When the switch is thrown down, current flows through the coil and this current establishes a magnetic field. As this field is built up around the coil, the flux linkages are changed, and this induces a back voltage in the coil. From Lenz's law, this voltage will be directed so as to oppose the change (increase) in current. The maximum rate of change of flux linkage is when the current first starts to flow. The back voltage will therefore be maximum at this instant and will slowly decrease, gradually allowing the current to get larger until it reaches the final value of $I = E/R$.

In Fig. 107 is shown the manner in which the current rises with respect to time. The *ratio* of L/R determines how fast the current increases. With large L and small R, it will rise very slowly, but if

L is small and R is large, it will rise rapidly. The ratio L/R is called the **time constant** of the coil. It is the time in seconds required for a current starting from zero to reach 63 per cent of its final value.

Suppose that after the switch of Fig. 106 has been thrown to the down position and the current has reached a constant maximum value the switch is suddenly thrown to the *up* position. The battery source of voltage has now been removed, and the current through the coil starts to decrease. This causes the lines of force to collapse, and from Lenz's law this will induce a voltage in the coil tending to keep the current flowing. Thus the current will decay according to the curve of Fig. 107. This curve will be of the same shape as the current rise curve, but upside down. The rate at which the current dies out is determined by the ratio L/R. With large L and small R, the current will die out slowly, but if L is small and R is large, it will decay rapidly. As before, the ratio of L/R is the *time constant* and, for current decay, gives the time in seconds required for the current to decrease to 37 per cent of its original value.

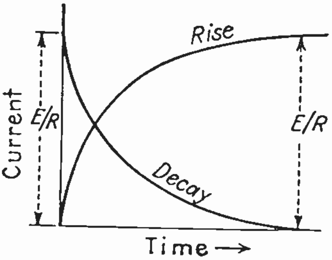

FIG. 107.—Illustrating the way in which the current increases and decreases in an inductive circuit.

To summarize: Inductance in electrical circuits is similar to the common inertia effect. Inductance tends to prevent current changes. It tends to keep the current from increasing and tends to keep it from decreasing. The laws governing the rise and decay of current in an inductive circuit are the same.

Energy Stored in a Magnetic Field.—Suppose that a current is flowing in a coil of high inductance and low resistance and the circuit is *suddenly* interrupted by opening the line switch. When this is done, a spark and perhaps even an arc occurs at the switch blade. Now the establishment of this arc will require a reasonably high voltage, and the presence of the arc denotes energy being dissipated. The question is, where does this high voltage and the required energy come from? The answer is that they come from the magnetic field.

As explained in the preceding section, as the current builds up, the back voltage opposes it. On page 75 it was explained that energy was the ability to do work and that work was the production of motion against a resisting force. Thus, when the current is being built up against the back voltage, work is done, and this stores energy in the magnetic field around the coil. An analogy is this: When you accelerate or increase the speed of an automobile, the inertia of the automobile (corresponding to the inductance of the coil) opposes the increase in speed (just as inductance opposes an increase in the current). Much energy is stored in the automobile as is evident when you attempt to stop suddenly. Similarly, much energy is stored in the magnetic field as evidenced by the spark or arc when the switch is suddenly opened.

For a magnetic field *in air*, the energy in joules or watt-seconds is

$$W = \frac{LI^2}{2}, \qquad (28)$$

where L is the inductance of the *air-cored coil* in henrys and I is the direct current or maximum value of alternating current in amperes. When dealing with power equipment that may be highly inductive and may be carrying large currents, care must be exercised in opening switches because of instrument or even personal injury. In such circuits the current should first be reduced to a low value with a series rheostat or by other means.

Mutual Inductance.—When two circuits, *not in electrical contact*, are so situated that the magnetic flux produced by one circuit links with the other, a voltage will be induced in the second circuit when a change is made in the current flowing in the first. Of course this is because changing the current in the first circuit changes the

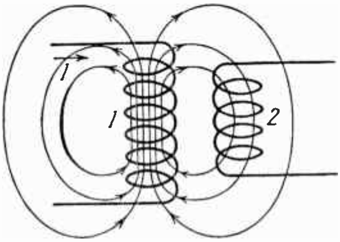

FIG. 108.—The lines of magnetic force produced by coil (1) link with coil (2), and will induce a voltage in coil (2) if the flux linkages are changed. With these two coils the coupling is in air.

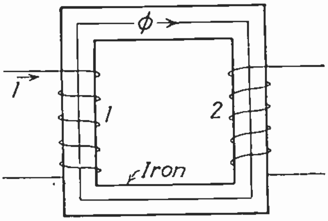

FIG. 109.—The lines of magnetic force produced by coil (1) will induce a voltage in coil (2) if the flux linkages are changed. The iron magnetic circuit provides the coupling medium for these coils.

flux linking the second circuit. Such circuits are shown in Figs. 108 and 109.

The property of two electric circuits, not in electrical contact, by virtue of which a voltage is induced in one circuit when the current is changed in the other is known as mutual inductance. Note that mutual inductance is also defined as a *property* just as for self-inductance. *Two circuits have a mutual inductance of one henry when an average rate of change of one ampere per second in one coil induces an average voltage of one volt in the other.*

Following the same general reasoning as for self-inductance, it can be shown that for mutual inductance

$$M = \frac{N_s\phi}{10^8 I_p}, \tag{29}$$

where M is the mutual inductance in henrys, N_s is the number of secondary turns linked by the flux ϕ, and I_p is the primary current in amperes. Of course the transformer and other electrical equipment operate because of mutual inductance.

Calculations of Mutual Inductance.—From Eq. (29), it follows that to calculate mutual inductance the method used for calculating self-inductance must be followed. Thus, suppose that in Fig. 105 a secondary coil of 240 turns of wire is placed on the same magnetic core. For the same data given on page 159, the flux linking the secondary will be 67,500 lines. Then, the mutual inductance will be, from Eq. (29), $M = (240 \times 67{,}500)/(10^8 \times 1.2) = 0.135$ henry.

Thus it is seen that when two coils are on closed magnetic cores, or even on good cores with small air gaps, it is easy to calculate the mutual inductance. But, when the coils are surrounded by air, or when there is an excessive amount of leakage, calculations must be made by the use of empirical equations such as given in handbooks.

Hysteresis.—The shape of a typical magnetization curve was shown in Fig. 95. As was mentioned in discussing this curve, the changes in the magnetic field lag behind the changes in the applied magnetomotive force. This is because of the so-called "molecular friction" within the iron. Typical magnetization curves for various materials were also shown in Fig. 96. It is important to note that these curves were all taken with direct current.

In much electrical apparatus such as transformers, for example, the magnetic-core material is subjected to an alternating magnetic field instead of a constant magnetic field as just considered. It is therefore necessary to study magnetic materials under alternating field conditions, such as would occur in the core of Fig. 110.

In this circuit the alternating-voltage source at the left forces an alternating current through the circuit, one cycle of which is shown in Fig. 111. Suppose that the magnetic core is silicon steel and that it is initially in an unmagnetized condition. As the alternating current rises from 0 to $+I_{\max}$, a corresponding magnetizing force ($H = 0.4\pi NI/l$) is impressed on the core. The magnetic flux in the core will build up as shown by the *broken* curve 0 to B of Fig. 112. This curve has the same shape as that of Fig. 95, and is explained in the same manner (see page 147). Thus from 0 to B the "atomic magnets" are gradually lined up against the internal "atomic friction" and the field is established. Work is done in establishing the field, and energy is stored in this magnetic field as explained in a preceding section.

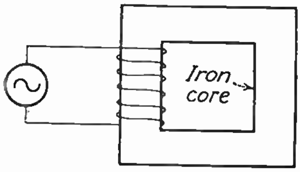

FIG. 110.—Magnetic cores, such as those in transformers and chokes, are subjected to alternating magnetic fields. The term "iron core" as used above merely indicates any of the various materials used, such as silicon-steel laminations, or powdered and compressed Permalloy.

Now when the current decreases from $I_{\max}$ to 0 of Fig. 111, the magnetic flux also decreases, but not in direct proportion. When the current is zero, there is an amount of residual flux in the core as represented by the line 0 to C of Fig. 112. This tendency of a material to retain magnetism is called **hysteresis**. As the flux decreases, the linkages are changed, a voltage is induced in the coil, and some, but not all, of the stored energy is returned to the circuit. The flux does not fall back to zero because of the internal atomic re-

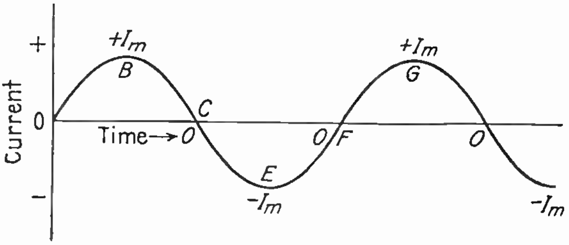

FIG. 111.—In the circuit of Fig. 110 the alternating current here shown is assumed to flow. (Actually, if a pure sine-wave voltage is impressed, the current will not be exactly a pure sine wave but will contain harmonics, page 129.)

sistance of the miniature atomic magnets. *The difference between the energy put into the circuit and the energy returned to the circuit represents a heat loss due to hysteresis.*

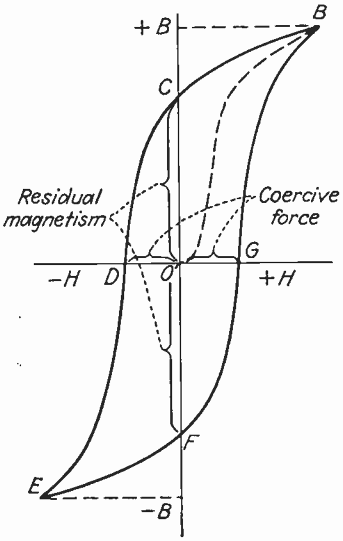

FIG. 112.—When a pure sine-wave current such as in Fig. 111 flows through a coil on a magnetic core as in Fig. 110, the magnetic flux produced changes according to the hysteresis loop B–C–D–E–F–G–B. The values $-H$ and $+H$ are the magnetizing forces, varying directly with the current through the coil.

The alternating current has now changed from a positive maximum to zero, and the flux has fallen only to C. As the current increases to $-I_{\max}$, the flux decreases to a zero value at D and then reaches at point E a negative flux value of $-B$. When the alternating current returns to zero, the flux follows the line E to F, a negative value of residual flux equal to 0 to F being left in the iron. When the current of Fig. 111 again increases from 0 to $+I_{\max}$, the flux will follow the curve FGB.

During current changes from 0 to $+I_{\max}$ and 0 to $-I_{\max}$, energy is put into the magnetic field, and during the current decreases, energy is returned by the magnetic field. Energy is lost in "shifting the atomic magnets," and this heats the core. *The area of the hysteresis loop of Fig.* 112 *represents the power loss for each cycle*[1] *of the alternating current.* The power loss per cycle is given by an equation based on the work of Steinmetz as follows:

$$P = \frac{KfVB^{1.6}}{10^7 \text{ watts.}} \cdot \tag{30}$$

According to this equation, P is the power loss in watts when the current has a frequency of f cycles per second, when V is the volume of the iron in cubic centimeters and B is the value of the maximum flux density in gausses, or lines per square centimeter. Typical

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[1] In explaining the hysteresis effect, it was assumed that the core was initially unmagnetized. This required a little over one cycle to trace the hysteresis loop. Once established, each cycle traces once around the loop.

values of K are for hard tungsten steel, 0.058; for pure iron, 0.003; and for silicon steel sheet, 0.0006.

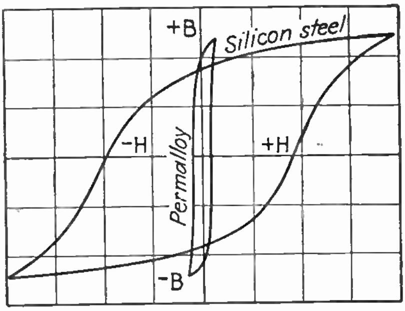

FIG. 113.—Hysteresis loops for silicon steel and for Permalloy. The area of the curve is directly proportional to the hysteresis power loss each cycle.

To summarize: When an alternating magnetic field exists in a magnetic material such as iron, energy is required to reverse repeatedly the miniature atomic magnets of which it is convenient to consider the iron to be composed. The power loss for each cycle is represented by the area of the hysteresis curve. A comparison between the losses caused by silicon steel and Permalloy is shown in Fig. 113.

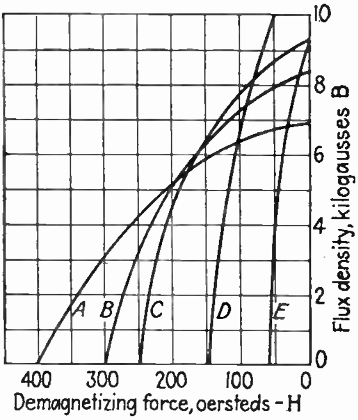

FIG. 114.—Permanent magnetic characteristics of magnetic alloys. This will be recognized as a portion of the hysteresis curve. A large $-H$ value indicates a large coercive force, or demagnetizing force required to reduce the residual flux $+B$ to zero. (A) Nickel-aluminum-iron. (B) Cobalt-molybdenum-iron. (C) Cobalt-steel. (D) Cobalt-tungsten-iron. (E) Chromium-steel. (*Data from Williams, C. S., Permanent Magnet Materials, Electrical Engineering, January,* 1936.)

Permanent Magnets.—Permanent magnets were treated early in this chapter and should now be discussed again in view of the theory that has just been presented. It was noted in Fig. 112 that there was a certain **residual magnetism.** If a material has the ability to retain a large amount of residual magnetism and has other desirable characteristics, it will be suitable for permanent magnets.

The characteristics of several permanent-magnet materials are shown in Fig. 114. Not only is a large value of permanent magnetism desirable, but the **coercive force,** that is, the force required to reduce the magnetism to zero, is also a measure of good permanent-magnet characteristics. The more coercive force required to force the flux to zero, the greater will be the internal atomic friction tending to keep the atomic magnets lined up.

To summarize: When certain metals are subjected to a strong magnetizing force (by placing them within a coil carrying a current, or better yet, by placing them across the air gap of a strong **electromagnet**[1]) much of the magnetism is retained when the magnetizing force is removed. This residual magnetism makes the metal a permanent magnet.

Eddy Currents.—As has been stressed throughout this chapter, whenever the flux linking a circuit is changed, a voltage is induced in that circuit. Now the core of magnetic material shown in Fig. 110 carries magnetic flux. Furthermore, each individual portion of the core is linked by flux from other parts of the core. Thus as an alternating flux rises to maximum, falls to zero, rises to maximum in the opposite direction, etc., the flux linking various parts of the core is being changed continually. This change will induce voltages in the core, and as a result **eddy currents** will flow. These eddy currents will produce I^2R losses, causing a heating of the core.

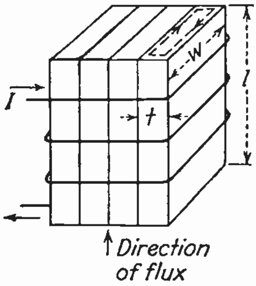

Fig. 115.—Magnetic cores are usually made of thin sheets or laminations to reduce eddy-current losses. The paths of the eddy currents in one of the laminations are as indicated by the broken line.

A portion of a magnetic core is shown in Fig. 115, and the path of the eddy-current flow is as indicated by the broken lines. Of course, these currents alternate in direction in accordance with the flux variations. If the resistances of the eddy-current paths are low, as they would be for a large volume of metal, the eddy currents become very large, and the heating of the core is excessive.

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[1] The word electromagnet is often used to designate a device composed of a magnetic core of low flux-retaining characteristics wound with many turns of wire. When a current flows through the wire, a very strong magnetic field may be produced, but when the current ceases to flow, the flux dies out to a very small value. The word electromagnet therefore distinguishes the device from a permanent magnet.

To reduce this effect, magnetic cores are often **laminated**, that is, made up of thin sheets as shown in Fig. 115. The sheets are insulated from each other electrically by the natural oxide which forms on the surface, or by insulating varnishes. The power loss W in *watts per cubic centimeter* for a laminated core is given by

$$W = \frac{1.64 t^2 f^2 B^2}{10^{16} \rho}, \tag{31}$$

where t is the thickness of laminations in centimeters, f is the frequency in cycles per second, B is the flux density in gausses, and ρ is the resistivity of the material in ohms per centimeter cube. The resistivity of a commercial grade of silicon steel may be about 60×10^{-6} ohm. Silicon is added to increase the resistivity which from Eq. (31) will reduce the power loss. The desirability of thin laminations is apparent. Note that the loss varies as the *square* of the frequency.

High-grade iron was the standard material for the cores of communication transformers and coils for many years. Because the frequencies employed were higher (and the losses were greater than in power circuits where laminations suffice to hold down the loss), the iron was first made in thin sheets electrolytically, then powdered, then mixed with some insulating compound such as shellac, and finally compressed into cores. Each individual particle of the magnetic core was thus insulated from each other particle, the losses being thus kept to a reasonable value. A similar method is now used in making Permalloy cores.

Eddy currents are caused to flow in any metallic body that is subjected to an alternating magnetic field. Thus the metallic shields around coils have eddy currents flowing in them.

To summarize: Eddy currents are caused to flow in cores of transformers, coils, or in any metallic object that is affected by an alternating field. Where the frequency is high, such as in communication circuits, these losses may become excessive, because the loss varies as the frequency squared. Special cores made of iron or Permalloy "dust" keep the losses within reason. In general all metallic objects exposed to alternating fields should be laminated or other precautions should be taken to prevent excessive eddy-current losses.

The Relay.—A photograph of a relay used in telephone-dial switching systems is shown in Fig. 116 and a schematic electrical

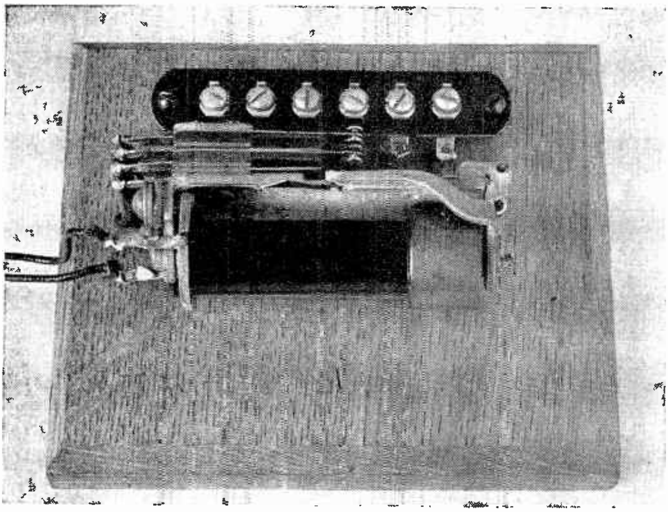

FIG. 116.—A communication relay with a copper collar or "slug" on the magnetic circuit as indicated at the lower right. The current which is induced in the collar when the electric circuit is opened and the magnetic lines of force start to die out tends to keep the magnetic flux in the core at a constant value, and this makes the relay slow-acting. This particular relay is mounted for laboratory tests on a block of wood.

diagram in Fig. 117. The magnetic circuit consists of the core of the coil, the holding frame, and the armature. When the relay is energized by passing a current through the coil, the magnetic field created pulls up the armature, operating the switching contacts. When the current is interrupted, the armature falls back because of gravity or a spring. Because of the fact that flux variations do not exactly follow variations in magnetizing force, and for other reasons, a typical relay may close on, for example, 65 milliamperes but may not open until the current is reduced to 38 milliamperes.

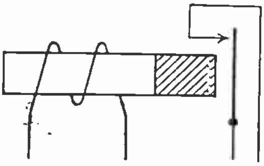

FIG. 117.—Symbol for a slow-operating relay. The cross-lined portion represents the copper collar which gives the relay its slow-operating characteristics. Sometimes the cross-lines are omitted and the letters *SO* are written in to designate the slow-operating features.

Both the photograph and the diagram of the relay here included show a massive copper "slug" on the magnetic circuit. This makes a relay "slow acting" in the following manner. When the current in the windings is varied, changing the magnetic flux in the core, very large eddy currents will flow in the copper slug. These will, according to Lenz's law (page 155), produce magnetic effects tending to keep the flux from being changed. This will make the relay slow acting.

The Circuit Breaker.—The circuit breaker is an electromagnetic device used to protect circuits from overload because of excessive currents. In its common form, the circuit breaker consists of a coil of wire on an iron core, part of which is pivoted to move. Usually the current-carrying contacts are closed by hand or other means against a spring force, the contacts themselves being tightly held to prevent chattering.

When the current becomes excessive, the armature is pulled up which trips the spring mechanism, and the contacts thus quickly open. Because of inductance, the rapid opening of a circuit breaker may induce a high voltage in a circuit. This may cause the circuit breaker to arc across as it is opened. To prevent excessive burning, air circuit breakers are often arranged so that the arcs occur between carbon electrodes. Large circuit breakers are often immersed in oil to absorb the energy of the arc.

SUMMARY

A magnetic field is a region in which magnetic forces act. A magnetic field is considered to be composed of magnetic lines of force. A magnetic line of force represents the direction of the magnetic force at a particular point. The density of the lines represents the intensity of the field.

Magnetic lines of force are continuous and tend to take the shortest path. Lines of force are assumed to flow from the north to the south pole in air and then back through the magnet. Lines of force in the same direction repel each other, and lines in the opposite direction attract each other.

A wire carrying a current is surrounded by a magnetic field. The strength of the field in air is proportional to the strength of the current. A magnetic field also exists around two wires of a pair.

A coil of wire carrying a current produces a magnetic field. If an iron core is placed in the coil, the magnetic field is greatly increased.

Magnetomotive force is considered to force magnetic lines of force through the magnetic circuit. Magnetic lines of force ϕ = magnetomotive force $\mathcal{F}$/reluctance.

Magnetomotive force is measured in gilberts ($0.4\pi NI$) or in ampere turns (NI).

The reluctance of a magnetic path is $\mathcal{R} = l/\mu A$ (no unit).

Magnetic flux density equals $B = \phi/A$ gausses.

Magnetizing force is measured in gilberts per centimeter, or oersteds. Magnetizing force is the magnetomotive force gradient. Magnetizing force and field intensity in lines per square centimeter are numerically the same in air.

Permeability = B/H and is the ratio of the lines of force per square centimeter established in a material to the lines of force that would be established in air.

The shape of the magnetization curve is explained on the basis of miniature atomic magnets.

Because the permeability is a variable and because leakage flux makes the exact path difficult to determine, many magnetic calculations are only approximate.

Devices that are to be shielded from stray constant magnetic fields should be enclosed in a box or shield of good flux-conducting ability such as iron.

A wire carrying a current in a magnetic field will experience a force tending to push it from the field.

One of the most important of all electrical principles is Lenz's law. This law states that a change in flux linkage is accompanied by an opposing induced electromotive force.

Self-inductance is a property of a circuit which causes a back voltage to be induced when the current is changed. Self-inductance is measured in henrys.

Mutual inductance is the property of two circuits which causes a voltage to be induced in one when the current in the other is changed.

Self-inductance causes a current in the circuit to rise and decay slowly.

Energy equaling $\frac{1}{2}LI^2$ is stored in a magnetic field in air.

Magnetic hysteresis is a name applied to the phenomenon of the magnetic flux in a core of magnetic material lagging behind the magnetizing force. Hysteresis causes a power loss and a heating of the iron core.

Eddy currents are currents caused to flow in magnetic cores and metallic objects when flux linkages are changed.

REVIEW QUESTIONS

1. Name the three fundamental properties of electric circuits.

2. What is a magnetic field? How can you prove one exists?

3. What are the important properties of magnetic lines of force? Can they cross each other?

4. Explain how to determine the direction of the magnetic field around a wire.

5. How does a coil of wire establish a magnetic field?

6. Why does a coil with an iron core produce a stronger field than the same coil in air?

7. State the three forms of Ohm's law for magnetic circuits.

8. Define magnetomotive force, and give the unit of measure.

9. Define magnetizing force, and give the unit of measure.

10. What is the relation between magnetizing force and field intensity?

11. In what important ways does reluctance differ from resistance?

12. Define permeability.

13. Explain why the magnetization curve has its peculiar shape.

14. What is the shape of the magnetization curve for air alone?

15. Compare the shape of the magnetization curve for an iron core with an air gap in with the shape of the curve for iron alone.

16. Why is Permalloy so well suited for telephone equipment?

17. Give the cause and remedy for magnetic leakage.

18. State Lenz's law.

19. Explain how Lenz's law applies to motor action.

20. Explain how Lenz's law applies to generator action.

21. Can a loop of wire be moved in a magnetic field without inducing a voltage in it?

22. Can a long straight rod be moved in a magnetic field without inducing a voltage in it?

23. Define self-inductance, and explain its likeness to inertia. Define the unit of measure.

24. Define mutual inductance, and define its unit of measure.

25. Why are silicon-steel laminations used for transformer cores?

26. What is the significance of a hysteresis loop?

27. What is the relation between frequency and hysteresis loss?

28. What is the relation between frequency and eddy-current loss?

29. Does the magnitude of the current affect hysteresis loss?

30. Does the magnitude of the current in the coil affect eddy-current loss?

PROBLEMS

1. Two permanent magnets are 0.5 inch wide and 3.5 inches long. They are placed parallel 1 inch apart with their north poles at the same end. Draw these to scale, and sketch in the magnetic field, showing with arrowheads the directions of the lines of force.

2. A flux density of 10,500 gausses exists in a Permalloy core. What is the magnetomotive force if the core is 10 centimeters long? What is the reluctance?

3. A total flux of 5200 lines exists in a Permalloy core of 0.82 centimeter cross-sectional area. The core is 12 centimeters long and uniformly wound with 316 turns of wire. What is the current in the wire? What is the permeability?

4. A magnetic circuit consists of 12 centimeters of silicon steel and 4 centimeters of cast steel. The cross-sectional area of the cast steel is 2.2 square centimeters and that of the silicon steel is 1.8 square centimeters. The total flux is 22,000 lines. First assume that the two metals are in intimate contact, and calculate the magnetomotive force; then assume two machined surface contacts between the metals, and estimate the additional magnetomotive force required.

5. Solve Probs. 1 and 2 starting on page 150 if the current is 5.50 milliamperes.

6. Solve the magnetic circuit of Fig. 99, page 152, if the current is 1.5 amperes.

7. A magnetic field has an intensity of 8000 lines per square centimeter. A wire 42 centimeters long is moved through the field at the average rate of 120 feet per second. Calculate the average voltage induced in the wire.

8. Calculate the self-inductance of the coil of Prob. 1, page 150.

9. Calculate the self-inductance of the coil of Prob. 2, page 151.

10. Calculate the self-inductance of the coil of Fig. 99.

11. Calculate the self-inductance of the problem on page 159 if the current is 3.6 amperes.

12. Calculate the mutual inductance if a secondary of 260 turns is wound on the core of Fig. 105. Use primary currents of 1.2 amperes and 3.6 amperes.

13. A coil of wire with an air core has a self-inductance of 0.056 henry. Calculate the energy stored in the coil if it carries a current of 12.6 amperes.

14. Referring to Fig. 113, page 167, determine the approximate ratio of the hysteresis loss in silicon steel to that in Permalloy.

CHAPTER VII

THE ELECTRIC FIELD AND CAPACITANCE

As was mentioned at various points in the last chapter, there are three fundamental properties of electric circuits; these were shown to be resistance, inductance, and capacitance. Resistance and inductance have now been discussed, and this chapter will be devoted to capacitance.

The preceding chapter discussed magnetic fields, particularly stressing the field produced by a coil carrying a current. Because magnets and compasses are common objects, the basic magnetic phenomena are rather widely understood. The study of a wire carrying a current led to a consideration of the inductance of a circuit. As was mentioned in Chap. V, the current through a coil *lags* the voltage across the coil. This is caused by the inductance of the coil.

In this chapter **electric fields** will be discussed, particularly the electric field produced by a voltage impressed between the two parallel plates of a **condenser.**[1] Some readers may be surprised to find that *a voltage establishes an electric field,* just as a current produces a magnetic field. This phenomena is just as easy to understand and just as real as the magnetic field; the difference is that high voltage is not a particularly convenient or safe plaything, and experimentation with it is not commonplace.

The establishment of an electric field in a condenser or **capacitor** [2] is the result of the property of a circuit called **capacitance.** This

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[1] There is no connection between the word *condenser* and such ideas as compressing or condensing. The term was early applied to electric-circuit elements such as two plates insulated from each other, and the usage has continued. As will be shown later in this chapter, electricity *can be stored* in a condenser, but not compressed or condensed in the usual sense of the word.

[2] The words condenser and capacitor have substantially the same meaning. The word condenser is more widely used in communication, and capacitor is more widely used in power. They will be used interchangeably in this text to familiarize the reader with both usages. The same usage applies to resistance and resistor and to inductance and inductor.

property will be found to be very useful in circuit design. It is this property that causes the current through a condenser or capacitor to lead the voltage across it as was mentioned in Chap. V.

The Electric Field.—As was mentioned in the preceding paragraphs, the magnetic field is fairly familiar to most people at all interested in, or associated with, electricity. The magnetic field is proved to exist by placing iron filings near a wire carrying a current or near a permanent magnet and by observing how the iron filings line up along the magnetic lines of force. The iron filings are caused to line up because they are better conductors of the magnetic lines of force than is the air. The lines of magnetic force desire to flow through each small bit of iron as far as possible, and hence the lines of force twist each bit of iron around until it is pointing in the same direction as the force at that point.

Now the *electric field* will be found to be just as easy to understand; in fact, as will be shown on page 178, there are reasons why the electric field is even *easier* to calculate than is the magnetic field. Thus, suppose that the plates of a parallel-plate *condenser* are arranged as in Fig. 118*a* and a voltage is impressed between them. Next, a piece of paper is placed over the plates, and hard rubber dust is sprinkled on the paper. When this is done, and the paper is tapped slightly, the bits of hard rubber will align themselves as shown.[1]

The reason for this reaction is that the bits of hard rubber are better conductors of the electric lines of force than is air. Thus, the electric lines of force desire to follow the hard-rubber particles as far as possible, and so they exert a force on them and line them up as explained. The direction in which the bits of hard rubber are pointing at a given location is the direction of the lines of force at that location. Although hard rubber was mentioned here, this same effect could be indicated by small particles of Bakelite, mica, colored glass, etc.

The shape of the electric field as determined by the electric lines of force between the positive and negative plates of the condenser of Fig. 118*a* is represented by the lines of force of Fig. 118*b*. It should be noted that these electric lines of force are not continuous as were magnetic lines of force. Electric lines merely go from the *positive to the negative* plate. Electric lines of force in the same

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[1] It is probably inadvisable for the reader to attempt such an experiment because of the high-voltage technique involved.

direction repel each other as indicated at the edge of the condenser.

To summarize: It is found experimentally that if a voltage is applied between two parallel metallic plates, such as those of a condenser, an electric field will be established between the two

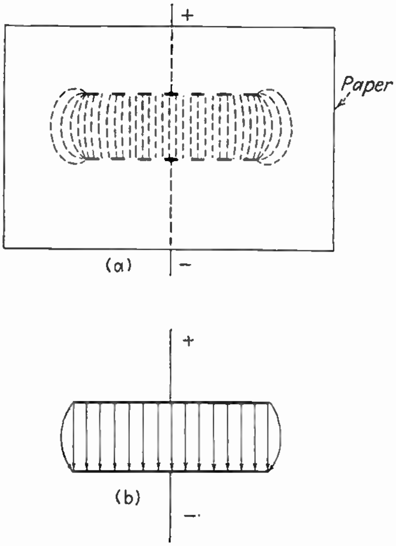

Fig. 118.—When a piece of paper that has been "dusted" with hard-rubber filings is placed over a high-voltage condenser and the paper is tapped, the bits of hard rubber will align themselves as shown in (*a*). This leads to the conclusion that electric lines of force or an electric stress exists between the condenser plates. The electric lines of force prefer to follow the hard-rubber particles because they offer a better path than air. Thus, the lines of force align the particles so that the lines may flow through the particles for their entire length.

plates. Such a field is considered as composed of electric lines of force passing from the positive to the negative plate.

The Dielectric.—The *material or substance* between the plates of the condenser of Fig. 118 is said to be a **dielectric.** In the condenser just considered the dielectric was air. The electric lines of force were established in this air dielectric. The dielectric used between the plates must be a good *insulator*, or the plates will be electrically short-circuited. Now suppose that some insulating material other than air, for instance hard rubber, is placed entirely within the space between the plates. An investigation will show that now *a very much stronger or more dense field exists.*

The reader will probably suspect the reason why the field is more dense or stronger with the hard rubber between the plates than with air. The field is stronger because *the hard rubber is a better conductor of electric lines of force than is the air.* So are all good insulating materials such as Bakelite, mica, glass, or wood.

For comparable conditions, the *ratio* of the lines produced in a dielectric (such as Bakelite, glass, etc.) to the lines that would be produced in air (more specifically, vacuum) is called the **dielectric constant** of the dielectric material. The dielectric constant is usually represented by K. This is also called **relative permitivity** and **specific inductive capacity.** Typical values of the dielectric constant for common materials are given in Table VI.

The dielectric constants given will vary in different books because of individual differences in specimens and perhaps because of variations in test conditions as well. In addition to the values given above, gases under ordinary conditions have a dielectric constant of about 1.0; transil oil, about 2.5; ethyl alcohol, about 27, and *pure* water, 81. The dielectric constant of a material is truly *constant* from a practical standpoint under usual conditions. This is in contrast with the permeability of magnetic materials which varies widely. This fact makes cut-and-try solutions unnecessary in electric-field calculations, and to this extent such calculations are more simple than are magnetic-field calculations where the variable nature of the permeability must be considered.

To summarize: The dielectric is the material between the plates of a condenser. The dielectric constant is the ratio of the electric-flux-conducting ability of the material compared with that of air, or more correctly vacuous space.

Ohm's Law for the Dielectric Circuit.[1]—In the production of an electric field in a **dielectric circuit** the same fundamental law considered elsewhere (page 12) applies: The result is directly proportional to the magnitude of the effort and inversely proportional to the magnitude of the opposing force.

As was shown in Fig. 118, the application of a voltage to the plates of the condenser established an electric field. This was true because *the voltage forced the electric lines of force through the dielectric.* In accordance with the basic principle again presented in the pre-

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[1] As the term is to be used in the following pages, the dielectric circuit will be the dielectric material in which the electric field is produced. This is also sometimes called a dielectric field.

ceding paragraph, the amount of flux produced will be directly proportional to the magnitude of the applied voltage and inversely proportional to the magnitude of the opposing force.

TABLE VI.—DIELECTRIC CONSTANTS FOR COMMON MATERIALS [1]

Material	Dielectric constant, K	Frequency range,[2] kc.
Bakelite	4.5	100–2000
Cloth (varnished)	4.5	Less than 2
Cloth (varnished)	2.5	100–2000
Fiber	2.5	Less than 2
Fiber	5.0	100–2000
Glass (special electrical)	10.0	Less than 2
Glass (common plate)	7.6	100–2000
Glass (Pyrex "radio")	4.4	100–2000
Mica (muscovite)	4.5	Less than 2
Mica (U.S.A. clear)	8.7	100–2000
Paper (dry Kraft)	3.5	Less than 2
Porcelain (wet process)	7.0	100–200
Pressboard (untreated)	2.9	Less than 2
Pressboard (oiled)	4.5	Less than 2
Rubber (hard)	2.8	Less than 2
Rubber (hard)	3.0	100–2000
Wood (paraffined maple)	4.1	Less than 2

[1] Largely assembled from "Electrical Engineer's Handbook," Vol. V, Communication and Electronics, John Wiley & Sons, Inc., and from "Standard Handbook for Electrical Engineers," McGraw-Hill Book Company, Inc. The student should consult such handbooks for additional data, and for further information regarding the materials listed here.

[2] Range at which dielectric constants hold.

Thus, what might be called Ohm's law for the electric circuit is as follows:

$$\text{Electric lines of force} = \frac{\text{voltage}}{\text{elastance}}. \qquad (32)$$

Using the conventional symbols, this law can be written

$$\psi' = \frac{E}{S}, \qquad E = \psi' S, \qquad \text{and} \qquad S = \frac{E}{\psi'}, \tag{33}$$

where ψ' (psi) is the total electric flux produced in **coulombs**,[1] E is the applied voltage (or potential difference) in volts, and S is the **elastance** or opposition offered by the dielectric to the establishment of the field and is measured in **darafs.**

To summarize: The voltage applied to the plates of a parallel-plate condenser forces electric lines of force through the dielectric material between the plates of the condenser. In the practical system, one volt applied to a dielectric having an elastance of one daraf will produce one coulomb of electric flux.

The Elastance of a Dielectric.—A sincere effort has been made to present the direct-current circuit, the magnetic circuit, and the dielectric circuit all on the same basis of a force causing a flow through an opposition. This makes all the presentations follow logically one after the other. It does, in the case of the dielectric circuit, require certain out of the ordinary treatments, all of which are, however, perfectly legitimate.

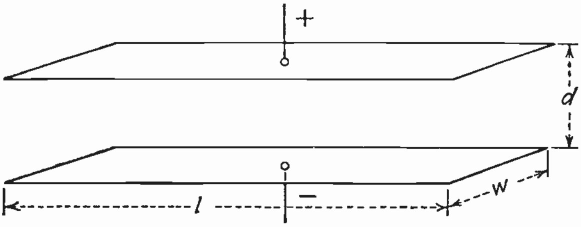

FIG. 119.—Representing the condenser plates of Fig. 118, drawn to indicate the dimensions more clearly.

The plates of the condenser of Fig. 118 are redrawn in Fig. 119 the better to show their dimensions. Let it be supposed that the space within this condenser is filled with air, and further let the area (area = width × length = $w \times l$) be large and the distance d between the plates be small so that the **fringing effect**[2] is negligible.

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[1] In accordance with the practical system of units. The prime is used to prevent confusion where ψ represents the electric flux in lines not coulombs.

[2] By fringing effect is meant the spreading of the lines of force near the edge of the condenser; this is plainly evident in Fig. 118. If the distance between the plates is small and the area of the plates is large, the fringing flux is but a small percentage of the total and is negligible.

From the relations of Eq. (32), an applied voltage will produce a given electric field in the air. If, now, the plates are spread apart, so that d is greater, *less* lines will be established, because the impressed voltage now has to force the lines a greater distance. If, however, the area is increased by making the plates larger, then the electric flux will be *increased*, because there are now more dielectric paths in parallel through which the flux can flow, and hence the elastance S of Eq. (33) is less. And last, if a dielectric other than air—for example, glass—is placed between the plates, the electric flux will be increased because glass is a better conductor of electric flux than is air.

Considering all these factors, it can be written that the elastance S varies as d/KA, and that the elastance

$$S = \frac{1.131 \times 10^{13} d}{KA}. \tag{34}$$

Where d is the thickness of the dielectric in centimeters, A is the area of the dielectric (or of *one* plate in Fig. 119), and K is the dielectric constant of the dielectric (the ratio of the flux-conducting ability of the dielectric compared to that of air), S as given by Eq. (34) will give the elastance in darafs. The significance of the numerical constant in Eq. (34) will be treated on page 190.

To summarize: The elastance, or flux-opposing property of a dielectric, varies directly as the thickness and inversely as the area and the relative flux-conducting ability of the dielectric as compared with air.

Dielectric Strength.—Readers *interested in radio are urged to read every word carefully.* High voltages are encountered in radio circuits, and high voltages often cause equipment failures. In this section certain phenomena causing such failures will be considered. These phenomena are simple, but unfortunately they are seldom clearly understood by most communication workers.

It has been shown (Fig. 118 and accompanying discussion) that a voltage impressed on a dielectric causes electric forces or stresses within the dielectric. If the voltage on a particular specimen is not too great, then when the voltage is removed the stress will disappear, and the dielectric material is in its original condition. If, however, the voltage is excessive, then the stress will be more than the material can stand, the dielectric material will actually be broken down, and a current of electricity will flow through it, heating it and damaging it further until its properties as an insulator are ultimately destroyed.

Now a given thickness of dielectric might safely withstand a certain impressed voltage, but a *thinner* piece of the same material might break down. In other words, it is not definite to say, for example, "a piece of glass will stand 60,000 volts without puncturing," what must be said is that a piece of glass will stand a certain voltage *per unit thickness* without puncturing. This is a specific statement, and if the volts or kilovolts per unit thickness which various dielectric materials can stand without puncturing is known, then the relative properties of the various materials can be compared and equipment can be designed for high-voltage service.

The volts per unit thickness which a dielectric can stand without rupture is known as the **dielectric strength** *of that material.* This value can be expressed in volts per inch, volts per mil (0.001 inch), volts per centimeter, or volts per millimeter (0.001 meter). A reasonable measure is *volts per centimeter,* which will be used in this book. The dielectric strength for certain materials is shown in Table VII. Of course kilovolts may also be used.

TABLE VII.—DIELECTRIC STRENGTH FOR COMMON MATERIALS

(Values in kilovolts per centimeter)

Material	Dielectric Strength
Bakelite	100–280
Cloth (varnished)	169
Fiber	69
Glass (electrical)	800–3300
Glass (common plate)	60–120
Mica (U.S.A. clear)	2250
Paper (dry Kraft)	300–400
Porcelain (wet process)	57
Pressboard (untreated)	50–120
Pressboard (oiled)	292
Rubber (hard)	173
Wood (paraffined maple)	45

The values given in Table VII were, in general, assembled from the same sources as those of Table VI. If several sources of such data are compared, large variations in the values given may be found. This again is due to individual variations in test specimens and perhaps to differences in test procedure as well; it is difficult to standardize such tests completely.

Gases such as air are, of course, used as dielectrics. As mentioned on page 178, their dielectric constants are approximately unity even over wide pressure ranges. The dielectric strength for air is about 31 kilovolts per centimeter under normal conditions. The dielectric strength for a gas at high pressures may be very much greater than under ordinary conditions. This makes possible the construction of high-pressure gas condensers with small spacings between plates.

Liquids such as oil are also used as dielectrics. As was shown on page 178, the dielectric constant of a liquid may be very high, but for the oils such as mineral oil ($K = 2.2$ approx.) and castor oil ($K = 4.7$) the dielectric constant is relatively low. The dielectric strength of most insulating oils is 30 to 40 kilovolts per 0.1 inch. Certain oils such as Pyranol ($K = 4.5$) are explosion-proof. Castor oil is widely used in oil-filled communication condensers. Gaseous and liquid dielectrics are advantageous because they are not permanently disrupted by the passage of an arc; in other words, they are self-sealing.

To summarize: Solids, gases, and oils are used for dielectrics in communication condensers. The thickness of the dielectric must be sufficiently great that the condenser will not break down. The volts per unit thickness which a condenser can stand without puncturing is the dielectric strength.

Voltage Gradient.—Suppose that Bakelite of various thicknesses is available. A piece 1 centimeter thick is placed between the electrodes of Fig. 120 and the voltage gradually increased until the Bakelite breaks down. From Table VII, Bakelite will stand at

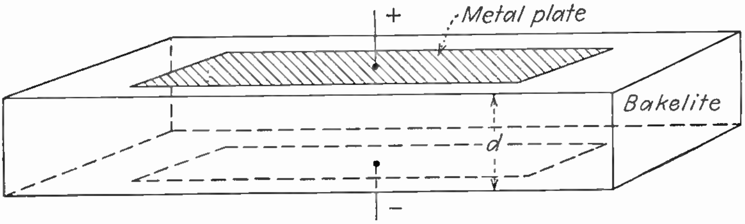

FIG. 120.—Illustrating the principle of voltage gradient. The voltage applied between the two metal plates divided by the thickness d gives the voltage gradient. In practice special plates (electrodes) with edges rolled back would be used to prevent flux concentrations at the edges (see Fig. 118).

least 100 kilovolts, or 100,000 volts per centimeter. Thus (assuming the lower value applies), *if the spacing is* 1 *centimeter,* 100,000 *volts must be applied before puncturing occurs.*

Now suppose that a piece of Bakelite 0.5 centimeter thick is placed between the electrodes of Fig. 120 and that they are firmly pressed against it. If the voltage on the electrodes is gradually increased, it will be found that the 0.5 centimeter thickness of Bakelite punctures at a voltage of approximately 50,000 volts. Similarly, a piece 0.25 centimeter thick will puncture at about 25,000 volts. Also, a piece 0.1 centimeter thick will puncture at about 10,000 volts.

Computing the *volts per centimeter* in each instance gives the following figures:

$$E = \frac{100,000}{1.0} = 100,000 \text{ volts per centimeter.}$$

$$E = \frac{50,000}{0.5} = 100,000 \text{ volts per centimeter.}$$

$$E = \frac{25,000}{0.25} = 100,000 \text{ volts per centimeter.}$$

$$E = \frac{10,000}{0.10} = 100,000 \text{ volts per centimeter.}$$

Thus, the volts *per unit thickness* is the same in each of the instances considered. *Volts per unit thickness is defined as the* **voltage gradient.** As is apparent from the preceding computations, voltage gradient is merely volts/distance.

To summarize: A very high gradient, that is, many *volts per unit thickness* can be obtained by applying relatively few volts to a very thin sheet of material.

Voltage Distribution in Dielectrics.—Before taking up this particular subject, it is desirable to review certain fundamentals of electric circuits. Therefore consider Fig. 121. With 6 volts applied to the circuit, of 3 ohms total resistance, a current of $I = 6/3 = 2$ amperes will flow. The voltage across the upper resistor will be $E_1 = IR_1 = 2 \times 2 = 4$ volts; and the voltage across the lower resistor will be $E_2 = IR_2 = 2 \times 1 = 2$ volts. Note that the 6 volts is divided directly proportional to the resistances.

Now let this same reasoning be applied to dielectrics. Suppose that an experimental condenser made as in Fig. 122 has a voltage E across it. This total E will force a flux of ψ lines through the dielectric as indicated. Since these lines must extend from the positive to the negative plate, the lines in all part of the dielectric are the same.[1]

As shown in Fig. 122, the space between the plates is actually filled with *two* dielectrics. These two dielectrics are *in series* to the electric lines of force; that is, just as the same current passes through each of the series resistors of Fig. 121, so must the same electric flux pass through each of the two series dielectrics.

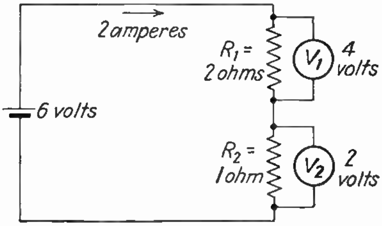

FIG. 121.—Illustrating the way in which voltages are distributed across resistors in series.

According to Eqs. (32) and (33), the electric lines of force $\psi' = E/S$, where E is the *total* voltage impressed across the two electrodes and S is the *total* elastance of the two dielectrics. It also follows from Eq. (33) that the voltage E_1 across dielectric 1 will be $E_1 = \psi' S_1$, and that across dielectric 2 will be $E_2 = \psi' S_2$.

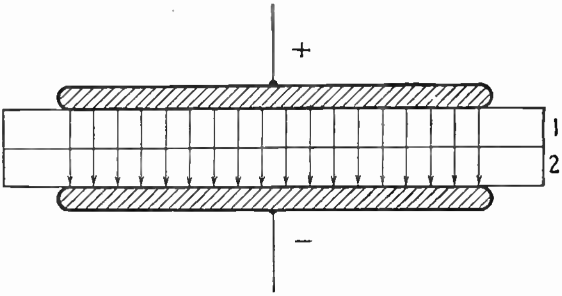

FIG. 122.—The two dielectrics, 1 and 2, are in series and therefore the same electric field passes through each. The metal plates have rounded edges to prevent flux concentration at sharp edges. This diagram shows a uniform field, but in practice there would be a fringing effect or a nonuniform field at the edges. If the area of the plates is large this effect is negligible.

To summarize: Just as in the circuit of Fig. 121, where the total

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[1] Of course this is with reference to the position of the dielectric under the plates and also assumes that the distance between the plates is reasonably low and that the plates are large, so that fringing is negligible.

impressed voltage was divided across the resistors in direct proportion to their resistances, so in Fig. 122 the total voltage will be divided across the dielectric in direct proportion to the elastance of each dielectric.

Calculations of Voltage Distribution.—The principle discussed in the preceding section is of the greatest importance in electrical work wherever high voltages are employed, and high voltages *are* employed in radio. To further illustrate the principle, the following illustrations will be given.

Example 1.—An experimental condenser such as Fig. 122 has a dielectric consisting of 0.1 centimeter of Bakelite and 0.3 centimeter of hard rubber. It is desired to impress 10,000 volts at radio frequencies across this condenser, and the voltage distribution across each dielectric is desired.

Solution.—Step 1. The dielectric flux ψ is the same in each dielectric, as explained in the preceding section. It is not necessary to calculate this flux. Neither is it necessary to calculate the elastances, but their *relative magnitudes* must be known. To find these, the values of the dielectric constant K must be known. For Bakelite $K = 4.5$, and for hard rubber $K = 3.0$ (Table VI).

Step 2. Determine the relative magnitudes of the elastances. In general, elastance varies as d/KA. The area of each dielectric is the same, so A may be neglected; then, the elastances are proportional to d/K.

Step 3. Find the ratios of the elastances. For the Bakelite the elastance is proportional to $0.1/4.5 = 0.0222$, and for the hard rubber, the elastance is proportional to $0.3/3 = 0.1$. The total elastance will be proportional to the sum of these, or to $0.0222 + 0.1 = 0.1222$. Then, $0.1/0.1222 = 0.82$, and $0.0222/0.122 = 0.18$. This means that of the total elastance between the two parallel plates 82 per cent is due to the hard rubber and 18 per cent is due to the Bakelite.

Step 4. Calculate the voltage distribution. The hard rubber will have $0.82 \times 10{,}000 = 8200$ volts across it, and the Bakelite will have $0.18 \times 10{,}000 = 1800$ volts across it, because the total impressed voltage will divide in direct proportion to the elastances.

Step 5. Calculate the voltage gradient. The hard rubber was 0.3 centimeter thick and has 8200 volts across it. This is a voltage gradient of $8200/0.3 = 27{,}400$ volts per centimeter. From Table VII hard rubber will readily stand this gradient. The gradient for the Bakelite will be $1800/0.1 = 18{,}000$ which also is far within the safe limit of operation.

Example 2.—The spacing between the plates of an air condenser is 1.0 centimeter, and it is desired to put 25,000 volts on the condenser. Now this is close to the safe operating limit of air, which is about 31,000 volts per centimeter. It is feared that the condenser will break down due to a voltage surge. Some large sheets of common window glass 0.25 centimeter thick are available. It is decided to place two of these solid sheets between the plates to prevent arcing across. This is done, and the voltage is impressed. When the voltage is applied, the condenser suddenly fails after a moment of operation. What happened?

Solution.—When the 0.5 centimeter of glass (two sheets) was placed in the space between the condenser plates, the condenser then had a dielectric of 0.5 centimeter of glass in series with 0.5 centimeter of air.

Air has a dielectric constant of about 1.0, but the common window glass used had a constant of 6.0. The thicknesses and areas of the two dielectrics are the same, and thus from Eq. (34) the elastances will vary inversely as the dielectric constant. This means that the elastance of the air will be 6.0 times that of the glass. Then $6/7 = 0.857$ or 85.7 per cent of the total voltage will appear across the air, and $1/7 = 0.143$ or 14.3 per cent will appear across the glass.

This means that $0.857 \times 25{,}000 = 21{,}400$ volts will *initially* be impressed across the 0.5 centimeter of air, and 3600 volts across the 0.5 centimeter of glass. This means that the voltage gradient on the air will be $21{,}400/0.5 = 42{,}800$ volts per centimeter. But air will only stand about 31,000 volts per centimeter and will break down. When the air breaks down, it will become a conductor, and this will throw the entire 25,000 volts on the 0.5 centimeter of glass. The window glass will then have a voltage gradient of $25{,}000/0.5 = 50{,}000$ volts per centimeter impressed on it. This it would safely stand if it were not for the heating of the air next to it. This sudden heating might lower the dielectric strength sufficiently to cause the glass to break down, permitting the plates to arc across and ruin the condenser and perhaps other equipment in the circuit as well.

Now if this particular experiment were tried, under favorable conditions the glass might hold up. Nevertheless, this example serves to illustrate how the addition of insulating material might actually cause a breakdown by shifting the voltage distribution. Such matters are of very great importance wherever high voltages are involved.

In the design of high-voltage insulators, bushings, and other equipment, care must be taken not to overstress the air surrounding the object. As an example, consider the radio interference caused by high-voltage insulators such as Fig. 123. The head of this insulator is covered with a conducting material as is plainly evident and for the following reason. The first insulators had no conducting coating. Since a high voltage existed between the conductor and tie wire as one electrode of a condenser and the crossarm pin as the other, a strong electric field would be set up in the insulator porcelain as a dielectric.

Now the area of the conductor and tie wire in particular was very small, and thus the electric field would be very intense where the lines converged upon these wires. In addition to putting a high-voltage gradient on portions of the porcelain, a very high gradient would be impressed on the very thin layer of air between the porcelain and the wires. (Remember that $E = \psi/S$ and that the elastance of air is high compared with that of porcelain; also, that voltage gradient equals volts divided by thickness and the air layer is quite thin). This very high gradient might cause the air to break down intermittently, the result being a series of small sparks that produce radio noise. Covering the head with a conducting layer gives a large area which reduces flux concentrations, and the conducting layer also eliminates the air layer which has high elastance.

FIG. 123. A high-voltage power-line insulator with a metal cap to reduce radio interference.

• To summarize: Wherever an electric field passes through a dielectric of high elastance in series with one of low elastance, the voltage will be distributed so that the greater voltage will exist across the larger elastance. Wherever there are *flux concentrations,* such as when voltages are impressed between *sharp points* instead of between plates or spheres, the voltage gradient will be great, and the air or other dielectric may be broken down near these points, although it remains normal elsewhere in between the points. Air pockets and layers in dielectrics or in insulation (such as in high-voltage cables or high-voltage bushings) should be avoided, because since the elastance of air is high, the air may be overstressed, break down, cause local heating, and eventually cause failure of the dielectric or insulation.

Dielectric Hysteresis.—Although this statement is open to criticism, all magnetic materials are quite alike—at least they are all metallic in nature. Not so for dielectrics; they may be and commonly are gases, liquids, solids, or even vacuous space, which might be defined as an ideal dielectric in some respects. Thus it is seen from the varied natures of dielectrics that they act similarly in some respects but quite differently in others.

In studying magnetic materials, it was found that a certain amount of magnetic hysteresis loss occurred each cycle. This was caused by the energy required to reverse the small "atomic magnets" as they were considered to be.

If an alternating voltage is impressed on a dielectric, it will be found that (with the possible exception of vacuous space) energy is required to reverse the electric flux in the dielectric. That is, there is a heat loss per cycle within the dielectric which can be accounted for only in simple terms by saying that it represents the energy required to reverse the electric field. This loss is often termed **dielectric hysteresis.** Such losses are very important at radio frequencies because of the rapidity with which the voltage, and thus the electric field, changes.

Because of the varied nature of dielectrics as explained in the first paragraph of this section, no attempt will be made to give a physical explanation of dielectric hysteresis other than as follows: In gases, in liquids, and in dielectrics usually called solids, there is moisture and there are also **ions,** or charged particles. The electric lines of force will act on these, causing ionic motions and hence energy losses. Also, it is conceivable that electric lines of force may reach into the atoms themselves (which consist largely of space and orbital electrons) and may distort the atoms slightly during each cycle, causing an energy loss.

To summarize: When an alternating electric voltage is impressed on a dielectric producing an alternating electric field within the dielectric, there are energy losses caused by the alternating electric field, and these are often called dielectric hysteresis losses.

Origin of Electric Lines of Force.—In introducing the concept of electric lines of force, it was stated that the lines extended from the positive plate or electrode through the dielectric and to the negative plate. Also, it was stated that the voltage forced the lines through the dielectric. These statements are in accordance with good practice. However, the point has been reached in this discussion where it is advisable to examine further the source of the electric lines of force.

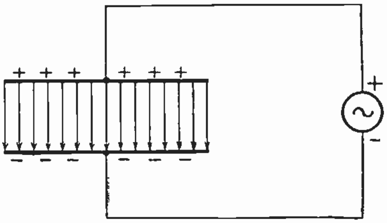

FIG. 124.—Showing a condenser connected to a source of alternating voltage.

Completing the circuit of Fig. 118 gives Fig. 124. With reference to page 6, it follows that the positive generator terminal will pull in electrons from the wire, finally leaving the upper condenser plate positive. Also, the negative generator terminal will force electrons away from it, and thus make the lower condenser plate negative. This means that the upper plate can be considered to have *positive electric charges* on it and the lower plate to have *negative electric charges* on it.

Lines of electric force are considered to extend from positive to negative charges. It is these lines of force which cause the forces between charged bodies (page 5).

Each electric charge is a small portion or quantity of electricity. From each unit[1] charge there are 4π lines of force. There are 3×10^9 of these elementary unit charges in *one coulomb*, which is the practical unit of electrical measure. Thus, if there is 1 coulomb of electricity on the plates of the condenser, then there are $4\pi \times 3 \times 10^9 = 37.7 \times 10^9$ lines of electric force extending through the condenser.

To summarize: The actual origin of the electric lines of force is the electric charges on which these lines terminate. The lines of force extend from the positive to the negative charges.

Capacitance.—In considering the flow of electric current, it was shown that $I = E/R$ and also that $I = EG$, where G was defined as the *conductance* of the circuit and was measured in *mhos*. In considering the establishment of an electric field, it was shown that $\psi' = E/S$, where S was the elastance of the dielectric, corresponding to the resistance of the current path. It follows, then, that

$$\psi' = CE, \tag{35}$$

where C is the **capacitance** of the circuit in **farads.** Thus the term capacitance is similar to the term conductance and is a measure of the *ease* with which lines of electric force are conducted by the dielectric. Capacitance is, therefore, the *reciprocal* of the elastance.

Capacitance has an added meaning, however, because it was shown that charges must exist on the plates of a condenser in order that an electric field may be established, because the electric lines of force actually extend from positive to negative charges. Thus, capacitance may be used to determine the electric lines of force [Eq. (35)], or *capacitance may be used to find the charge or quantity*

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[1] This refers to a unit of measure and not to an electron. It is convenient to refer to positive charges on a condenser. Thus, a deficiency of electrons leaves a condenser plate positively charged.

of electricity stored on the plates, because for each 4π lines there must be a small electric charge present.

The quantity of electricity stored in a condenser (on the plates) is given by the expression

$$Q = CE, \tag{36}$$

where Q is in **coulombs,** C is the capacitance of the circuit in *farads,* and E is the voltage impressed across the condenser in volts. *One coulomb is the amount of electricity carried past a point when one ampere flows for one second.*

To summarize: There is a relation between the number of electric lines of force and quantity of electricity, because according to the system established, 4π lines of electric force originate on each unit positive charge and terminate on each negative charge.

Definitions of Capacitance.—Condensers are very important circuit elements, possessing as they do one of the fundamental electrical properties, that of *capacitance.* There are two ways of defining capacitance; one way is based on the phenomenon that occurs when a *direct* voltage is connected across a condenser, and the other method is based on the phenomenon when an *alternating* voltage is impressed on a condenser. The phenomenon occurring with a direct voltage will be treated first.

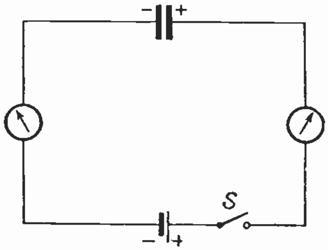

FIG. 125.—Immediately after closing the switch S connecting the battery to the condenser, electrons will be attracted by the battery electrodes from one condenser plate and forced to the other plate charging the condenser + and − as indicated. Both milliammeters will deflect slightly as indicated, and in opposite directions.

In Fig. 125 is shown a condenser connected to a battery. It is assumed that initially there is no resultant charge on the plates of the condenser (which is equivalent to saying that there are equal numbers of positive and negative charges on the plates). When the switch S is closed, the positive terminal of the battery *attracts electrons* from the condenser plate to the right, and this leaves the right plate *positive.* At the same instant, the negative battery terminal *repels electrons* in the wire to the left condenser terminal; this forces it to go *negative.* Both milliammeters would show slight instantaneous deflections of the *same* magnitude. This would be expected because *the current in all parts of a series circuit must be the same* (page 17).

It is thus seen that a condenser becomes charged electrically when a direct voltage is impressed on it. From this viewpoint, therefore, *capacitance*[1] *may be defined as the property of a condenser which causes it to take a charge when a voltage is connected across it.* According to Eq. (36), *the farad is the amount of capacitance that will cause a condenser or circuit to take a charge of one coulomb when a voltage of one volt is connected across it.* The units microfarad (1/1,000,000 farad) and micromicrofarad (1/1,000,000,000,000 farad) are widely used.

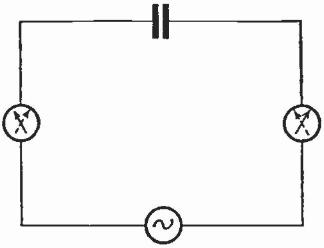

FIG. 126.—When an alternating voltage is connected to a condenser, the voltage alternately forces the plates positive and negative. This will cause the instruments in series to deflect slightly first in one direction and then the other (if they are direct-current instruments). This indicates that an alternating current is flowing in the line wires, *and the condenser is in series in these line wires.* It is always assumed that the current in all parts of a series circuit is the same. Thus, the alternating line current is assumed to flow "through" a condenser, although the condenser plates are insulated from each other.

Capacitance will now be considered from the alternating-voltage standpoint. In many respects, this treatment is far more important in communication than the other, because of the widespread use of alternating voltages in telegraphy, telephony, and radio.[2] This viewpoint will be explained with the aid of Fig. 126.

When the *right* terminal of the alternating-voltage source of Fig. 126 is *positive*, the condenser will be charged as in Fig. 125, and each milliammeter will experience a current flow in a certain direction. This will correspond to the *positive* half cycle of the alternating

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[1] This property was formerly called electrostatic capacity, or capacity. It is still called capacity, even in legitimate circles, and by good electrical authorities. This is, queerly enough, particularly true in communication. Nevertheless, the correct word is, according to the best standards, *capacitance* and not capacity. Capacity is used to measure, for example, the power-handling *capacity* of a machine, the capacity of an alternator, etc. The term permittance is used for capacitance, particularly in the high-voltage power field.

[2] The reader may challenge this statement, but it is correct from the viewpoint that any signal which continually recurs in the same magnitude and at the same rate can be considered to be composed of a fundamental and har monics, each of which is an alternating value (page 127).

voltage of the source. During the next, or *negative*, half cycle of the alternating voltage, the polarity of the voltage source will be reversed, and the condenser will be charged opposite to that shown in Fig. 125. This means that the charges have flowed in the opposite directions in the wires and that each milliammeter will experience a current flow in the opposite direction. Thus, *when an alternating voltage is connected to a condenser, an alternating current will flow in the connecting wires.*

In considering the foregoing definitions, it should be noted that current flows to the condenser only when the voltage is changed. This statement applies to a direct voltage, because only when the voltage was *changed* from zero to the direct-voltage value by closing switch S of Fig. 125 did the current flow. Thus, *capacitance may be defined as the property of a condenser (or circuit) which allows (or permits) a current to flow when the voltage across it is changed.* The *unit of capacitance* may also be defined on this basis. *A condenser has a capacitance of one farad when an average rate of change in impressed voltage of one volt per second causes an average current flow of one ampere.* It is particularly important to become thoroughly familiar with these two definitions.

To summarize: Capacitance is one of the three basic circuit properties (resistance and inductance are the other two). Resistance is a property of a circuit, inductance is a property of a circuit, and likewise capacitance is a property of a circuit. Capacitance may be defined either on a direct- or on an alternating-voltage basis. The latter is more important in communication.

Capacitance of a Parallel-plate Condenser.—Condensers are often made of parallel plates separated by air. This type is particularly common in radio circuits where small variable capacitances are needed. Also, air condensers are employed because the dielectric losses of some of the dielectrics at the high radio frequencies would be excessive. Mica condensers are also widely used in radio circuits, particularly for small fixed condensers. At low frequencies, such as in audio amplifiers and telephone circuits, impregnated paper is often used for the dielectric. A good grade of paper impregnated with paraffin, wax, or oil makes an excellent dielectric and one that is quite inexpensive.

The capacitance of a condenser is easily calculated if the dimensions and the dielectric constant of the dielectric are known. With reference to Eq. (34), page 181, it was shown that the elastance

$S = 1.131 \times 10^{13}\ d/KA$ farads. In the preceding section it was shown that the capacitance was the reciprocal of the resistance, or that $C = 1/S$. From this it follows that the capacitance of a parallel-plate condenser is

$$C = \frac{1}{1.131 \times 10^{13} \dfrac{d}{KA}} = \frac{8.842KA}{10^{14}d} \text{ farads.} \tag{37}$$

In this expression K is the dielectric constant, A is the area of the dielectric (see following paragraph) in square centimeters, and d is the thickness of the dielectric in centimeters.

With reference to Fig. 119 and page 180, the manner in which the elastance (and hence the capacitance) varied was shown. It was the *area of the dielectric* which was one factor determining the elastance. Thus, in Fig. 127, which represents a multiplate condenser, it is the *total area of the dielectric* which is a factor determining the elastance and hence the capacitance. Of course, condensers are often built with the plate area substantially equal to the area of the dielectric. Thus in a parallel-plate condenser consisting of *two* plates, A [of Eq. (37)] would often be specified as the area of *one* plate. If the parallel-plate condenser consisted of three plates, A would be taken as equal to *twice* the area of one plate, and as Fig. 127 makes clear, A would be taken as *four* times the area of one plate for a five-plate condenser, and so on.

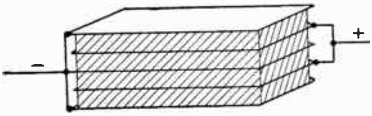

FIG. 127.—A multiplate condenser. It is the total cross-sectional area of the dielectric which determines the capacitance.

A word for the more theoretically inclined reader will now be given regarding the numerical constant of Eq. (37). The manner in which this follows from the expression for elastance should be evident, but the question is, how does this numerical value originally get into the expression for elastance? This question will not be answered in detail, because little would be gained by so doing. Suffice it to say that it comes from the systems of units used. There are 4π lines of electric force from each statcoulomb of electricity; there are 3×10^9 statcoulombs in 1 coulomb; and there are 300 volts in 1 statvolt. Multiplying these gives $4\pi \times 3 \times 10^9 \times 300 = 1.131 \times 10^{13}$. The relations studied in this chapter were originally called "electrostatics," and the units of measure were set up in the **electrostatic system.** Practical electrical workers use the **practical system** (ohm, volt, ampere, coulomb, etc.), and to obtain the correct answer the conversion factor just developed must be used.

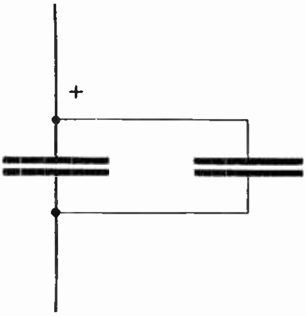

FIG. 128.—When a second condenser is connected in series with another, it is equivalent to increasing the area of the first condenser. Thus, the equivalent capacitance of two condensers in parallel is the sum of their separate capacitances.

To summarize: The capacitance of a condenser varies directly as the area A of the dielectric (or as the number of plates minus one), directly as the dielectric constant K of the material between the plates, and inversely as the distance d between the plates.

Condensers in Parallel and in Series.—It has been shown that the capacitance of a condenser varies directly as the area A of the plate [Eq. (37)]. Suppose that two condensers are connected in parallel as shown in Fig. 128. Clearly, this would be equal to increasing the area of the plates (or more correctly the dielectric as explained in the preceding section) of C_1 by the area of the plates of C_2. Then, *the capacitance of two condensers in parallel is equal to the sum of the capacitances of the separate condensers.* That is,

$$C_p = C_1 + C_2 + C_3 + \cdots. \qquad (38)$$

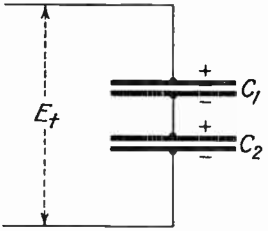

FIG. 129.—When two condensers are in series, the charge on each condenser and the electric flux in each condenser must be the same.

It is interesting to note that capacitances combine as do direct-current conductances in parallel direct-current circuits. Just as conductances represent the *ease* with which current flows, so does capacitance represent the *ease* with which electric flux is established.

In considering the equivalent capacitance of several condensers in series, refer to Fig. 129. The *electric flux* through each condenser *must be the same,* because of these facts: When the upper plate of C_1 is made positive, the lower plate of C_1 must be equally negative.

Also, the positive charge on the upper plate of C_2 will equal the negative charge on the lower plate of C_1, etc. Remember that the lower plate of C_1 and the upper plate of C_2 are tied together but *insulated* from all other objects, and hence the foregoing statement must hold. If one plate goes negative, the other must go equally positive. *If the charges are the same, the flux must be the same in each condenser, because* 4π *lines originate on each charge.*

Now according to Fig. 129, the total voltage E_t will be distributed across the two condensers, because from Eq. (35), $\psi' = CE$, and $E = \psi'/C$. Now $E_1 = \psi'/C_1$, $E_2 = \psi'/C_2$, and $E_t = \psi'/C_e$, where C_e is the equivalent capacitance of the series combination. Since $E_t = E_1 + E_2$ and ψ' are common, it can be written that

$$\frac{\psi'}{C_e} = \frac{\psi'}{C_1} + \frac{\psi'}{C_2}, \qquad \text{and} \qquad \frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots. \tag{39}$$

Referring back to page 22, an equation of this form can be written

$$C_e = \frac{C_1C_2}{C_1 + C_2}. \tag{40}$$

It is often necessary to know how the voltage divides when two or more condensers are placed in series. This can be explained on the basis, previously shown, that the electric flux must be the same in each condenser. Then, the voltage drop across each condenser must equal the total impressed voltage across the two in series. That is, since $E = \psi'S = \psi'/C$,

$$E_t = E_1 + E_2 = \frac{\psi'}{C_1} + \frac{\psi'}{C_2}. \tag{41}$$

This equation shows that *when two or more condensers are connected in series across a source of voltage the voltage is distributed across the condensers inversely proportional to their capacitance.* That is, the condenser of low capacitance will have the greater voltage, and vice versa.

To summarize: Capacitances correspond, in a way, to conductances and are combined accordingly. The equivalent capacitance of two or more condensers *in parallel* is the sum of the separate capacitances. The reciprocal of the equivalent capacitance of two or more is equal to the sum of the reciprocals of the individual capacitances.

Problems Involving Capacitance.—This section will explain the methods used in solving problems involving capacitance.

Example 1.—A condenser having a capacitance of 1.2 microfarads and a condenser having a capacitance of 2.16 microfarads are connected in *parallel* across a direct voltage of 48.2 volts. Calculate the equivalent capacitance, the charge each condenser will take, and the total charge.

Solution.—Step 1. The two condensers are in parallel, and according to Eq. (38) their capacitances will add directly. Thus, $C_e = C_1 + C_2 = 1.2 + 2.16 = 3.36$ microfarads.

Step 2. The charge each condenser will take is, from Eq. (36), $Q_1 = C_1E = 1.2 \times 10^{-6} \times 48 = 57.6 \times 10^{-6}$ coulomb, or 57.6 microcoulombs; $Q_2 = C_2E = 2.16 \times 10^{-6} \times 48 = 103.7 \times 10^{-6}$ coulomb, or 103.7 microcoulombs.

Step 3. The total charge will be $57.6 + 103.7 = 161.3$ microcoulombs.

Example 2.—Repeat Example 1 if the two condensers are connected in series. Also calculate the voltage distribution.

Solution.—Step 1. The equivalent capacitance will be found by Eq. (39). Thus, $1/C_2 = 1/C_1 + 1/C_2 = 1/1.2 + 1/2.16 = 0.834 + 0.463 = 1.297$, and $C_e = 1/1.297 = 0.772$ microfarad. The equivalent capacitance can also be found from Eq. (40). Thus, $C_e = C_1C_2/(C_1 + C_2) = 1.2 \times 2.16/(1.2 + 2.16) = 2.59/3.36 = 0.772$ microfarad. Note that the equivalent capacitance is less than the smaller capacitance.

Step 2. Before calculating the charge each condenser will take, it is advisable to calculate the voltage across each condenser. From the preceding section the voltage will divide across the condensers inversely as their capacitances. Then $(1.2/3.36) \times 48.2 = 17.2$ volts will be across condenser C_2 having a capacitance of 2.16 microfarads, and $(2.16/3.36) \times 48.2 = 31.0$ volts will be across condenser C_1 having a capacitance of 1.2 microfarads. (As a check, $17.2 + 31.0 = 48.2$ volts, the total impressed voltage.)

Step 3. The charge taken by the first condenser will be $Q_1 = C_1E_1 = 1.2 \times 10^{-6} \times 31.0 = 37.2 \times 10^{-6}$ coulomb, or 37.2 microcoulombs. The charge taken by the second condenser will be $Q_2 = C_2E_2 = 2.16 \times 10^{-6} \times 17.2 = 37.2 \times 10^{-6}$ coulomb, or 37.2 microcoulombs. *The total charge will be the same as each separate charge.*[1] As a check, the charge taken by the equivalent capacitance will be $Q_e = C_eE = 0.772 \times 10^{-6} \times 48.2 = 37.2 \times 10^{-6}$ coulomb, or 37.2 coulombs.

Example 3.—A paper condenser consists of a thin paper dielectric between layers of metal foil. Sometimes the paper is paraffined, sometimes waxed, and sometimes saturated in oil. In a certain paraffined-paper condenser the paper is 0.0015 centimeter thick, and the dielectric constant is $K = 3.5$. Calculate the area this condenser must have to give a capacitance of 1.0 microfarad.

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[1] The reason for this is left to the student (see Question 17, page 207).

Solution.—This problem can be solved by Eq. (37), page 194. Thus, $1.0 \times 10^{-6} = (8.842 \times 3.5 \times A)/(10^{14} \times 0.0015)$. Solving for the area, $A = 4.86 \times 10^3 = 4850$ square centimeters. If the paper is 10 centimeters wide, this means that a strip 485 centimeters, or $485/(2.54 \times 12) = 15.9$ feet long, must be rolled between metal foil into a condenser.

Example 4.—Calculate the average value of the alternating current that will flow through a 2.18-microfarad condenser when it is connected across a voltage of 30.7 volts at 1000 cycles.

Solution.—Step 1. The basis for solving this problem is the definition for capacitance on page 193, and the average rate of change of voltage must be known. Since it is not otherwise stated, the value of 30.7 volts given is the effective value. The maximum value is $E_{\max} = 30.7 \times 1.414 = 43.4$ volts. For a sinusoidal alternating voltage, which this is assumed to be, the voltage will increase from 0 to a maximum value of 43.4 volts in 1/4 cycle, or in 1/4000 second. The average rate of change of voltage is $E_{\max}/t = 43.4/(1/4000) = 43.4 \times 4000 = 173{,}500$ volts per second.

Step 2. According to the definition on page 193, a condenser has a capacitance of one farad when an average voltage change of one volt per second produces an average current flow of one ampere. That is, $I_{av} = C \times (E/t)$. For this problem, $I_{av} = 2.18 \times 10^{-6} \times 173{,}500 = 0.378$ ampere.

Condenser Charge and Discharge Current.—In the definitions of capacitance starting on page 191 and in Figs. 125 and 126 accompanying the discussion, it was shown that capacitance was a property of a condenser or a circuit which permitted a *current* to flow when the applied *voltage was changed.* Now this is in marked contrast with inductance (page 158) which caused a *back voltage* to exist when the *current was changed.* This inductive effect was likened to inertia in mechanical systems. When the *voltage is changed* on a circuit containing capacitance, a *current flows.* This capacitive effect is often likened to **compliance**[1] in mechanical systems.

The current that flows in a circuit containing only capacitance is known as the condenser **charging current.** If the voltage impressed on the circuit is a pure sine-wave alternating voltage, the charging current will be a pure sine-wave alternating current (Example 4). As was mentioned on page 176, this current will, however, lead the voltage by 90 degrees, but a further treatment of this phenomenon will be reserved for Chap. IX. The shape of the curve for the current that flows when a *direct* voltage is impressed

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[1] To comply means "to yield." A spring complies when subjected to a mechanical force. A circuit containing capacitance complies and passes an alternating current when an alternating voltage is impressed on it.

on a condenser is of much importance in electrical work, and this will now be considered.

A circuit for studying condenser charging current is shown in Fig. 130. Assume that the condenser initially is in an uncharged condition; then, the voltage across it must be zero. When the switch S is thrown *down*, the battery is connected to the condenser, and a charging current flows to the condenser. The characteristics of this charging current will now be considered.

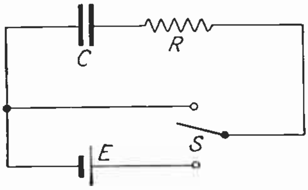

FIG. 130.—Circuit for charging and discharging the condenser C through a resistor R.

As previously mentioned, there is initially no charge in the condenser, and no voltage across it. Therefore, *at the instant the switch is closed, the condenser appears to be a short circuit,* and the resistance R is the only opposition offered to the current flow. Thus as indicated in Fig. 131, the current immediately jumps up to the value $I = E/R$. As the current continues to flow, however, charges are carried to the plates of the condenser, and a voltage is gradually built up across it. This voltage will oppose the impressed battery voltage and will assist the resistance in opposing the current flow. When the voltage across the condenser builds up equal to the battery voltage, the current equals zero, and the condenser is charged.

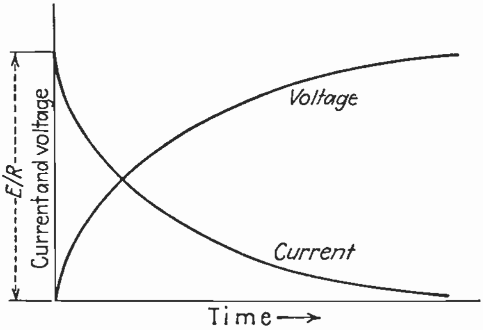

FIG. 131.—When an uncharged condenser is connected through the resistor R to a source of voltage E as in Fig. 130, the current instantly rises to the value E/R and then decreases as electric charges reach the plates and build up an opposing voltage across the condenser as indicated by the voltage curve.

If now the switch is thrown to the *up* position when the condenser is fully charged with a voltage E across it, the current immediately rises to the value $I = E/R$. As the condenser discharges, however, its voltage gradually drops, and since it is forcing the current through the circuit, the current gradually drops also. The shape of the current-decay curve is the same as the current-rise curve shown in Fig. 131.

Since the series resistance R is the factor limiting the maximum value of the condenser charge or discharge current, the series resistance determines the time required to charge or discharge a given condenser. In general it can be written that the product of the resistance and capacitance is the **time constant** of a capacitive circuit. When $t = RC$, that is, when the time in seconds equals the product of the resistance in ohms and the capacitance in farads, then the current will have fallen to about 37 per cent of its original value. The time constant of a capacitive circuit is, therefore, the time required for the current to fall to 37 per cent of its original value.

To summarize: When an uncharged condenser is connected to a source of voltage, the current immediately rises to a value $I = E/R$, and then gradually decreases as a charge builds up in the condenser. Also, when a fully charged condenser is discharged through a resistor, the current immediately jumps up to the value $I = E/R$ and then gradually decreases as the condenser discharges. For a given condenser, the series resistance regulates the time of charge and discharge.

Energy Stored in a Condenser.—If the reader has ever come in contact with the terminals of a highly charged condenser, he is quite certain to have been made aware of the fact that *energy is stored in a charged condenser.* In fact, the discharge of a condenser through the human body may result in serious injury and may even be fatal.

The amount of energy stored in a condenser is given by the relation

$$W = \frac{CE^2}{2}, \qquad (42)$$

where W is the energy in joules (power of one watt acting for one second), C is the capacitance in farads, and E is the maximum value of the voltage in volts. The energy is stored in the electric field in the dielectric.

Dielectric Absorption.—No material is a perfect insulator, and there is, therefore, a small **leakage current** when a voltage is applied to a condenser (see page 61). Such a leakage current is *very* small in a good air or mica condenser and quite small in a good paper condenser, because the resistance of a condenser is very high, usually many millions of ohms.

When a constant voltage is applied to a condenser, the charging current flows first (page 198). Then for a time a current will flow that may be larger than the purely leakage current. This is because many dielectric materials, particularly a nonuniform dielectric such as paraffined paper, seem to *absorb* charges; this current is accordingly called an **absorption current,** and the phenomenon is termed **dielectric absorption.**

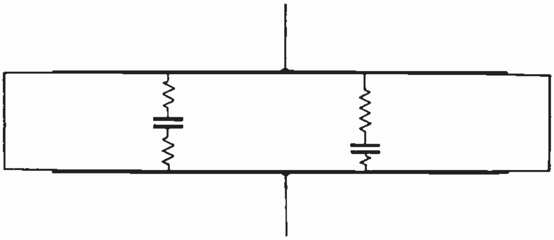

FIG. 132.—A nonuniform dielectric acts as if it were composed of a large number of very small condensers in series with very high values of resistance. Two such circuits are shown. The effect of these is to cause the dielectric to absorb charges.

Thus if a condenser is charged by connecting it to a source of voltage for only an instant, it will not take so great a charge as if connected for a longer period. A condenser with a nonuniform dielectric may be thought of as made up as shown in Fig. 132. That is, discontinuities (fibers in paper, for example) in the dielectric may be considered as being minute condensers connected to the plates by very high resistors (the other portions of the dielectric). Time is therefore required for these minute condensers to charge slowly through these high resistors.

If a condenser has been charged for a period of, say, several minutes (until most of the internal minute condensers have become charged) and is then discharged for an instant, these internally absorbed charges will not have time to all leak back through the high internal resistances to the plates. Thus, if the condenser is discharged a second time, or even a third time (or more), small additional electric discharges will be obtained. From this reasoning it would appear that some condensers (with nonuniform dielectrics) should show a slightly lower capacitance at high frequencies, and such is the case.

Force between Condenser Plates.—As has been explained before (page 5), two bodies having opposite charges attract each other. Opposite charges exist on the plates of a condenser, and therefore there is an attractive force between these plates. This force can be shown to be

$$F = \frac{CE^2 \times 10^7}{2d}. \tag{43}$$

The force F will be in dynes when C is the capacitance in farads, E is the applied voltage in volts, and d is the distance in centimeters between the plates.

Electrolytic Condensers.—This book is designed to consider the electrical fundamentals of communication equipment instead of describing the equipment in detail. A brief discussion of condensers was given on page 193. Before closing this chapter, a few words should be included about the electrolytic condenser.

It has been known for many years that suitable electrodes in an electrolytic solution would allow current to flow readily in one direction but would allow but little current to flow in the opposite direction. Such a device constitutes an **electrolytic rectifier,** because if an alternating voltage is impressed on it, it will carry appreciable current only in one direction. This rectifying action is attributed by some to the formation of a gaseous layer on one electrode and by others to the formation of an oxide layer; this latter view is accepted here.

Considering the electrolytic rectifier *in the high-resistance direction,* it may be classed as an **electrolytic condenser.** It consists of two electrodes or "plates" (the metallic electrode and the electrolyte) separated by a dielectric composed of the oxide layer.[1] Electrolytic condensers in their simple form can be used only on circuits containing both direct- and alternating-current components. If the direct-current component in the form of a "polarizing voltage" is not present, or if the electrolytic condenser is not

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[1] In his book "Electrolytic Capacitors," P. M. Deeley of the Cornell-Dubilier Co. states that the dielectric is the layer of oxide. This view will be accepted here. Deeley's book is recommended to those desiring detailed information on electrolytic condensers.

connected into the circuit correctly, as indicated by the positive and negative markings on the container, then the oxide layer will not be built up, or will be destroyed, and the electrolytic condenser may be damaged.

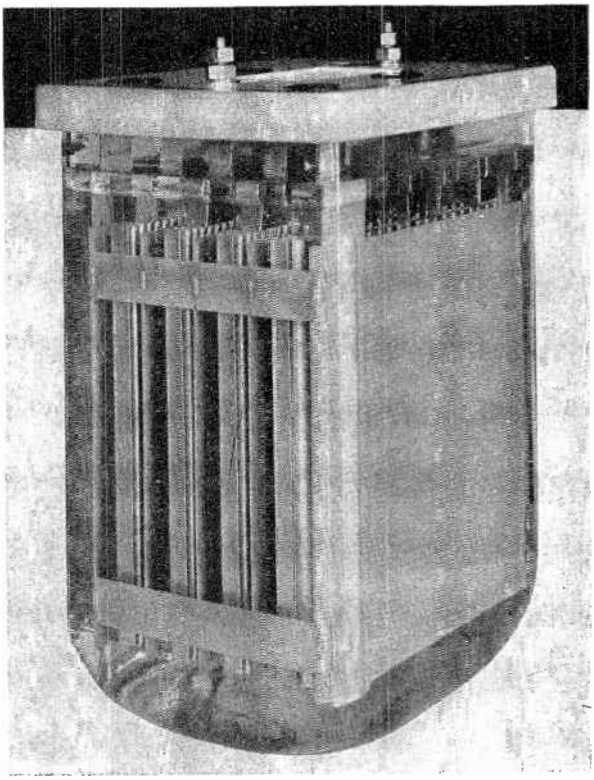

FIG. 133.—An aluminum electrolytic condenser of the type used in telephone central-office battery filter systems. (*Courtesy of American Telephone and Telegraph Company.*)

Electrolytic condensers may be of either the *wet* or *dry* types. These will be briefly considered by the following paragraphs.

Wet Electrolytic Condensers.—A form of the **wet electrolytic condenser** widely used in battery filter circuits in telephone plant is shown in Fig. 133. The positive anode is of corrugated aluminum. The oxide film is formed on this plate. The negative electrode is a flat aluminum plate; its only purpose is to make contact with the electrolyte. This is necessary because glass jars are used for the container. (In the common type of wet electrolytic condenser to be considered in the next paragraph, the metal container serves as the electrode for making contact with the electrolyte.) The electrolyte used in this condenser is a mixture of ammonia, boric acid, and water.[1]

The form of the wet electrolytic condenser used in audio and radio circuits is no doubt familiar to the reader. The containing cylindrical "can" is usually of aluminum, although copper containers have been used. The electrolyte within the container is a water solution of boric acid and ammonium or sodium borate. Special antifreeze solutions are sometimes used. The positive or anode structures are of various forms. The aluminum constituting this anode may be coiled, wound, or pleated to give larger area. It is stated (see footnote, page 202) that the oxide film covering the anode is about 0.00001 centimeter thick and that, because of this *extremely* thin dielectric, an anode surface area of 100 square centimeters will give a condenser capacitance of 8.85 microfarads.

Dry Electrolytic Condensers.—Of course these are not dry in the strict sense, but the electrolyte is held in some absorbing material. The structure of the dry electrolytic condenser does not resemble that of the wet type.

In a common form of the *dry* electrolytic condenser both the positive anode and the negative cathode consist of strips of annealed aluminum. These are insulated from each other by a strip of special paper, although cloth was used in the earlier types. This paper is saturated with the electrolyte, and the combination of aluminum foils and paper separator is rolled into a compact roll which is inserted in the container. The paper used in a 600-volt condenser is only about 0.008 inch thick, but remember it is not the paper but the very thin anode oxide layer which is the dielectric. The electrolytes used in dry electrolytic condensers are quite complicated and will not be further described except to say that they range from slightly liquid fluids to fudgelike substances. In some dry electrolytic condensers, where very high capacitance and small physical dimensions are desired, a cotton gauze coated with high purity aluminum threads is used for the anode. This gives a large effective area.

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[1] For a complete treatment see H. O. Siegmund, The Aluminum Electrolytic Condenser, *Bell System Technical Journal*, January, 1929

As was previously mentioned, both the wet and dry electrolytic condensers must be operated on a pulsating direct current; that is, in circuits containing a direct polarizing voltage in addition to the alternating-voltage component. If this is not done, and if the condensers are not correctly connected, they will be ruined. *Special* dry electrolytic condensers for use on alternating voltages have been developed, however. These consist essentially of two of the ordinary dry electrolytic elements, which are connected reversed so that the combination always has a high internal resistance and does not permit the passage of a large current when the polarity of the alternating voltage is such that the condenser conducts (see opening paragraphs of this section).

SUMMARY

An electric field is established in a condenser (or capacitor) when a voltage is impressed across it. This field consists of electric lines of force or an electric stress in the dielectric or material between the plates of the condenser. This field can be readily detected by the alignment of small bits of electric flux-conducting material, such as hard rubber or Bakelite dust placed in the vicinity of charged conductors.

For a given voltage impressed across a condenser, more electric lines of force will be established in a dielectric material such as mica than will be established in air. The ratio of the flux established in a material to that established in vacuum (space) is the dielectric constant.

The magnitude of the electric field produced in a dielectric is directly proportional to the impressed voltage and inversely proportional to the elastance. The voltage may be considered as forcing the electric lines of force through the elastance of the dielectric. An alternating electric field in a dielectric causes a dielectric hysteresis heat loss.

If the electric field or stress within the dielectric is too great, the material may be overstressed and may puncture. The dielectric strength is the volts per unit thickness which a dielectric can stand without puncturing.

The volts per unit thickness, or voltage gradient, in a dielectric is an indication of the stress produced. A high voltage gradient may be produced by a small voltage impressed on a very thin specimen. The voltage across a dielectric is proportional to the product of density of the electric field and the elastance.

When two dielectrics are in series so that the flux passing through them is the same, the impressed voltage will distribute itself across the dielectrics in direct proportion to the individual elastances. Since capacitance is inversely proportional to elastance, the voltage distribution will be inversely proportional to the capacitances.

The electric lines of force in the dielectric originate on positive and negative charges on the plates of the condenser. The charge in coulombs taken by a condenser equals the product of the voltage across the condenser and the capacitance of the condenser.

The capacitance of a condenser and the unit of measure may be defined from either the direct- or alternating-current standpoint; the latter method is probably the more useful. Capacitance is the property that allows or permits a current to flow when the voltage is changed. A circuit has a capacitance of one farad when an average rate of change of one volt per second causes an average current flow of one ampere. For a parallel-plate condenser, the capacitance equals

$$C = \frac{8.842KA}{10^{14}d}.$$

The capacitance will be in farads when the dimensions are in centimeters and K is as listed on page 179.

When two or more condensers are in parallel, the equivalent capacitance is the sum of the separate capacitances. When two or more condensers are in series, the reciprocal of the equivalent capacitance is the sum of the reciprocals of the separate capacitances. That is, $1/C_e = 1/C_1 + 1/C_2 + \cdots$.

When a condenser is charged, the current immediately rises to a value of $I = E/R$ and then gradually decreases as a charge is built up on the condenser plates. When a condenser is discharged, the current immediately rises to the value $I = E/R$ and then gradually decreases as the charges leave the plates. Energy is stored in a condenser equal to $W = CE^2/2$, where W is in joules when C is in farads and E is in volts.

Condensers with nonuniform dielectrics appear to absorb charges into the dielectric. This is known as dielectric absorption and affects the charge and discharge characteristics of a condenser.

REVIEW QUESTIONS

1. Why are electric fields less generally understood than magnetic fields?

2. Give one important reason why electric-field calculations are easier than magnetic calculations.

3. For comparable conditions, would more electric lines of force be produced in glass or hard rubber?

4. What is meant by the dielectric circuit?

5. Explain Ohm's law for the dielectric circuit.

6. Define elastance, explain how dimensions affect it, and give the unit of measure.

7. Discuss dielectric strength and voltage gradient, and explain the relation between them.

8. What is meant by fringing?

9. Explain how the impressed voltage divides across dielectrics in series.

10. Explain the importance of elastance in the design of high-voltage bushings.

11. How does dielectric hysteresis vary with frequency?

12. What reasons are there to assume that lines of force emanate from electric charges (see also page 5)?

13. Define capacitance and the unit of measure.

14. To what does capacitance compare in the electric circuit?

15. What is the equivalent capacitance of condensers in series? In parallel?

16. How does the impressed voltage divide across condensers in series?

17. Referring to Example 2, Step 3, page 197, why is the total charge the same as the separate charges?

18. What is meant by the time constant of a condenser and a resistor in series?

19. Explain the relations between the dimensions of a parallel-plate condenser and its capacitance.

20. Why do electrolytic condensers have such high capacitance?

21. What is the magnitude of the initial value of the current when a condenser is charged or discharged? What is dielectric absorption?

22. Explain how energy is stored in a condenser.

23. What is meant by positive charges on the plates of a condenser?

24. Why should air pockets be avoided in porcelain insulators?

25. Why are rounded edges instead of sharp edges used in high-voltage condensers?

PROBLEMS

1. Referring to Fig. 119, page 180, l = 15 centimeters, w = 10 centimeters, d = 1.0 centimeter, and the dielectric is Bakelite having a strength of 100 kilovolts per centimeter and a constant of 4.5. Calculate the elastance of the condenser.

2. If 10,000 volts is impressed across the condenser, calculate the electric flux that will be established.

3. Calculate the capacitance of the condenser of Prob. 1.

4. Calculate the charge the condenser will take under the condition of Prob. 2.

5. Calculate the energy that will be stored in the dielectric.

6. A piece of "electrical" glass is 0.1 inch thick. How many volts may be safely impressed on it?

7. The plates of an air condenser are 0.05 inch apart. May 5000 volts be impressed across it safely? If not, what voltage may be?

8. A parallel-plate condenser consists of seven plates of metal each having an area of 1.8 square inch between which are placed sheets of "U.S. clear" mica 0.03 inch thick. Calculate the capacitance of the condenser

9. What voltage may safely be placed across the condenser of Prob. 8?

10. A condenser having a capacitance of 1.2 microfarads and a maximum voltage rating of 600 volts is connected in series with a condenser having a capacitance of 1.8 microfarads and a maximum voltage rating of 400 volts. What is the equivalent capacitance?

11. Will it be safe to connect this series combination across 980 volts?

12. An experimental condenser is constructed with a layer of glass 0.5 centimeter thick and a layer of oiled pressboard 1.0 centimeter thick. Using the lowest values listed on page 182 for dielectric strength, calculate the voltage distribution in the condenser when 10,000 volts is impressed across it. Is this a safe working voltage?

13. Allowing a safety factor of 2.0, what is the maximum voltage that may be safely impressed across the condenser of Prob. 12?

14. Calculate the average value of the current through a 2.2-microfarad condenser if an 800-cycle sine-wave voltage of 26.4 volts is impressed across it.

15. A 20-cycle sine-wave ringing voltage of 84 volts peak value is impressed across a 2.0-microfarad condenser. What will be the effective value of the current that will flow?

CHAPTER VIII

ELECTRIC MEASURING INSTRUMENTS[1]

> I often say that when you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of science, whatever the matter may be.
>
> —LORD KELVIN, 1883.

No more appropriate introductory words for the chapter on electric measuring instruments can be found. The rapid progress in electricity has been due in a good measure to the ability to measure accurately, and usually quite easily, all the quantities desired. There is no branch of applied science that is provided with more accurate, flexible, and abundant instruments than the electrical field.

Instruments and Meters.—The reader, particularly if he is actively engaged in industry, may be surprised to find that what he has always called **meters** are defined as **instruments.** An *instrument* is defined[2] as "a device for measuring the present value of the quantity under observation." A *meter* is defined[2] as "a device that measures and registers the integral of an electrical quantity with respect to time."

Thus, milliammeters, ammeters, millivoltmeters, voltmeters, wattmeters, and similar devices measuring the current, voltage,

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[1] Many of the illustrations used in this chapter were obtained from the instrument manufacturers. Some of these were line drawings of simple form, and others were elaborate "phantom" views. In order to prevent confusing these illustrations with the actual product of each manufacturer, by a special agreement individual acknowledgments under each illustration furnished are not included. Those who courteously supplied illustrations and other services in connection with this chapter are General Radio Co., Weston Electrical Instrument Corp., General Electric Co., and Westinghouse Electric and Manufacturing Co.

[2] Standards of the A.I.E.E.

or power *at a given instant* are properly called instruments. Devices such as the familiar watt-hour meter which measures the quantity of electric energy taken over a period of time are properly called meters. Although these distinctions may seem trivial, the correct usage of terms prevents confusion.

The D'Arsonval Instrument.—This instrument is very important because most direct-current measurements are made with it, and also because it is used in conjunction with thermocouples, copper oxide rectifier units, and vacuum tubes to make many alternating-current measurements.

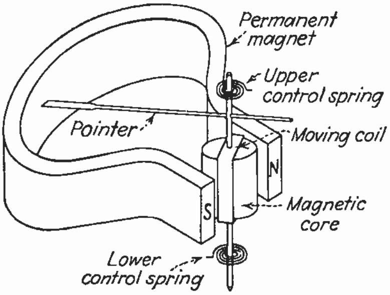

FIG. 134.—Essential parts of a D'Arsonval instrument. This forms the basis of most direct-current instruments.

A working sketch of the D'Arsonval movement is shown in Fig. 134. A permanent magnet of an alloy such as tungsten steel, cobalt steel, or aluminum-nickel-cobalt (Alnico) produces a strong magnetic field. A fine coil of wire is pivoted and is free to move within the field. The current to be measured (or a portion of it) is passed through the moving coil which is situated in the constant field of the permanent magnet. The motor action (page 154) resulting from the magnetic reactions of the field produced by the moving coil and the field produced by the magnet causes the coil and the pointer attached to it to turn; the strength of the current is thus indicated.

The Permanent-magnet Moving-coil Mechanism.—The simple principle just explained is the basis for a large majority of electrical measuring instruments. Of course, the different manufacturing details and the variations for different applications cause instruments of different manufacturers to vary slightly. The "phantom" view of Fig. 135 will show the details of a typical permanent-magnet moving-coil, or D'Arsonval, instrument.

Pole pieces are attached to the ends of the permanent magnet. These are usually of soft iron and serve at least two purposes. (1) They intensify the field at the ends of the permanent magnet and concentrate the magnetic lines of force within the region desired. In both Figs. 134 and 135 a cylindrical magnetic core is visible between the ends of the magnets. This core is also usually of soft iron, and it serves to help in providing a good magnetic path between magnet poles. (2) The soft-iron pole pieces acting with the soft-iron cylindrical magnetic core produce a uniform air gap between the ends of the permanent magnet.

This uniform air gap is visible in Fig. 135. The moving coil extends down through this air gap and is free to turn in it. To be certain that the arrangement is clear, Fig. 136 has been included. The pole pieces and the magnetic core or cylinder thus provide a uniform air gap in which *the magnetic flux is uniformly distributed.*

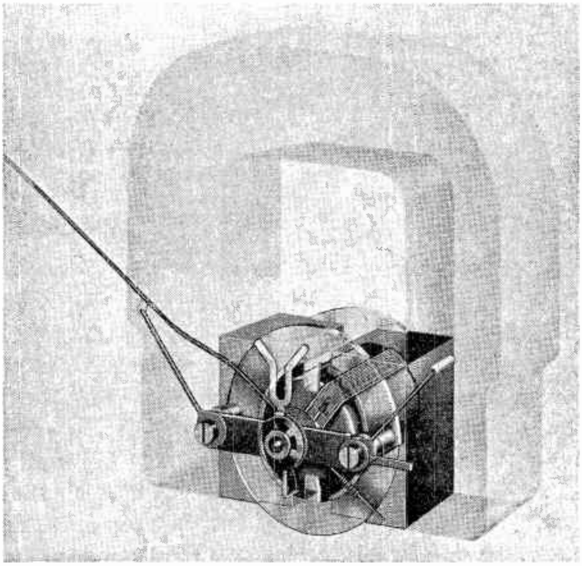

FIG. 135.—A "phantom" view showing the details of construction of a typical permanent-magnet moving-coil direct-current instrument.

The torque or turning effort exerted on the moving coil and the attached pointer is proportional to the product of the current through the coil and the magnetic field in which the coil rests. If the magnetic field is uniform across all parts of the air gap, then the deflection of the pointer will at all times be the same for a given current increase; in other words, the scale under the pointer to indicate the current magnitude will be uniform.

Current is conducted to and away from the moving coil by passing it through the spiral springs or by passing it through an auxiliary metallic spiral. The spiral springs provide the opposing force against which the moving coil acts, and also return the pointer to zero.

An electric instrument must be **damped**; if not, the pointer will not go direct to a given value of current, but will swing to and fro, only finally coming to rest after a considerable period. To provide damping, the moving coil is wound on a light aluminum frame. The edges of this frame are clearly visible in Fig. 135. When this frame moves in the magnetic field, from Lenz's law (page 155) a voltage will be induced in it, and currents will flow in the frame in such a direction as to oppose the motion. Because of this action, the pointer will swing somewhat more slowly to a given position, but once it arrives, there will be little in the way of oscillating motion about that position.

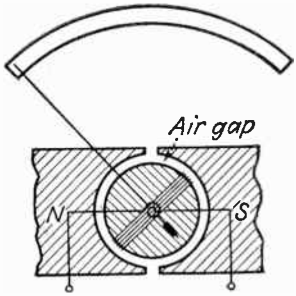

FIG. 136.—Showing how the soft-iron cylinder and pole pieces produce a uniform air gap within which the moving coil turns. A shunt (page 214) is connected across this "movement" when it is used as a large milliammeter or as an ammeter, and resistance is connected in series when it is used as a voltmeter.

To summarize: The permanent-magnet moving-coil mechanism consists essentially of a permanent magnet which together with soft-iron pole pieces and a core produce a uniform magnetic field in an air gap. A moving coil wound on an aluminum frame with an attached pointer is placed in this air gap. The motor action between the current in the coil and the magnetic field in the air gap causes the coil to turn, the pointer being thus deflected. The motion of the pointer is opposed by currents that flow in the aluminum frame, and this action damps the instrument.

Ideal and Actual Instruments.—As has been brought out repeatedly in this text, and as no doubt the reader well knows, voltmeters are placed directly across the voltage to be measured, and ammeters are connected in series with the circuit in which the current is to be measured.

The *ideal voltmeter* for direct-current measurements (and the present discussions are confined to direct current) should have an infinite resistance. Then, its presence across the line would not in any way disturb the circuit conditions. If its resistance is fairly low, however, it will take current from the source of voltage, and if the internal resistance of the source is high, the internal voltage drop will reduce the external voltage that reaches the load. In other words, a good voltmeter should not change the circuit conditions.

The *ideal ammeter* for direct-current measurements should have zero internal resistance. If it has appreciable resistance, then the current passing through the load will be less than it was before the ammeter was connected.

In actual practice, the voltmeters used for direct-current measurements have very high internal resistance. Voltmeters with an internal resistance of 1000 ohms per volt of range are common in communication work, and they are available with much higher internal resistances, but at an increased cost. Thus, a 0- to 100-volt voltmeter will have an internal resistance of 100,000 ohms.

A typical 0- to 5-ampere ammeter has an internal resistance of 0.03588 ohm. When measuring the full current of 5 amperes, the drop across the ammeter will be $E = 5.0 \times 0.03588 = 0.1794$ volt. This small resistance and low voltage drop will be negligible in most circuits in which the ammeter might be used.

Direct-current Milliammeters.—These are of great importance in communication because in this field the currents to be measured are often very small. This is particularly true in vacuum-tube circuits.

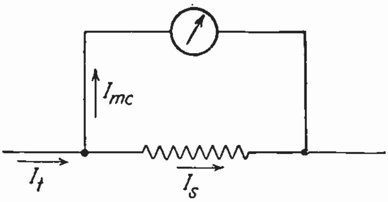

FIG. 137.—In a milliammeter for very small currents the current to be measured passes directly through the moving coil. In larger milliammeters a shunt is used as shown above. Then, but a small value of the total current passes through the moving coil.

The ordinary direct-current milliammeter consists essentially of a permanent-magnet moving-coil element such as shown in Figs. 134 and 135. In a typical series of milliammeters marketed by one manufacturer, the *entire* current to be measured passes *directly* through the moving coil in all instruments from the 1.0-milliampere (maximum reading) to the 30.0-milliampere instrument. The internal resistances of certain of these milliammeters are as follows: 1.0 milliampere, 105 ohms; 2.0 milliampere, 27 ohms; 5.0 milliampere, 12 ohms; 10 milliampere, 9.3 ohms; 25 milliampere, 1.2 ohms.

In this particular series of instruments, the entire current is not passed through the moving coil in sizes above 30 milliamperes. In these instruments, a **shunt** is placed in *parallel* with the moving coil, and only a portion of the current to be measured passes through the moving coil, the remainder passing through the shunt. This shunt is internally mounted in the instrument case.

A shunt is merely a conductor of low resistance placed in parallel with the moving coil as shown in Fig. 137. The design of shunts for use with milliammeters will now be considered.

Problem.—A radio-service man with a small business finds he cannot afford a large array of instruments of various sizes and decides to investigate the possibilities of purchasing one 0- to 1-milliampere instrument and of using several shunts with it so that measurements of a wide range of currents are possible. The milliammeter has an internal resistance of 105 ohms. He decides to design shunts for measuring 5.0 and 25.0 milliamperes, respectively.

Solution.—With reference to Fig. 137, when the total current I_t is 5.0 milliamperes, the current I_{mc} through the moving coil must be 1.0 milliampere to produce a full-scale deflection. Under these conditions the drop across the moving coil will be $E_{mc} = I_{mc}R_{mc} = 0.001 \times 105 = 0.105$ volt. Since the shunt is in parallel with the moving coil, the voltage across each must be the same; therefore, $E_s = 0.105$ volt. Now 5.0 milliamperes is flowing in the main circuit, and the resistance of the shunt must be such that 1.0 milliampere flows through the moving coil and 4.0 milliamperes, or 0.004 ampere, flows through the shunt. The drop across the shunt was found to equal 0.105 volt. The resistance of the shunt must be $R_s = E_s/I_s = 0.105/0.004 = 26.25$ ohms.

For measuring 25.0 milliamperes maximum, the current through the moving coil and the voltage drop across the coil must be the same as in the preceding example. The current through the shunt will be 24 milliamperes, or 0.024 amperes, for a full-scale deflection. The resistance of the shunt must be $R_s = E_s/I_s = 0.105/0.024 = 4.375$ ohms.

To summarize: One milliammeter and a series of shunts can be used for measurements over a wide range. The milliammeter and shunts can be mounted in a box, if desired, and a multiple rotary tap switch can be arranged to select the various ranges. It is assumed in all these discussions that the resistance of all connecting wires is negligible.

Direct-current Ammeters.—The ordinary direct-current ammeter consists essentially of a *shunted* permanent-magnet moving-coil element such as shown in Figs. 134 and 135. Basically, the direct-current ammeter and the larger sizes of direct-current milliammeters are essentially the same. Of course, minor variations exist among instruments manufactured by different firms, but fundamentally they are the same, consisting of a moving-coil element and a shunt.

A typical 0- to 5-ampere ammeter has an internal resistance of 0.03588 ohm. This is the resistance that is offered by the internal circuit between the binding posts and is composed of the moving-coil element in parallel with the proper shunt. When the instrument deflects to full-scale value, the voltage drop across the ammeter will be $E_a = I_a R_a = 5.0 \times 0.03588 = 0.1794$ volt.

It is sometimes desirable to provide *external* shunts to use with an ammeter such as just considered. A well-assorted group of shunts will make possible the measurement of a wide range of currents with a single instrument. Thus, if it were desired to use the 0- to 5-ampere ammeter just considered to measure a maximum value of 50 amperes, the resistance of an external shunt would be $R_s = E_s/I_s = 0.1794/45 = 0.003987$ ohm. A voltage of 0.1794 volt was taken, because this must be the drop across the ammeter when it is carrying the full-scale value of 5.0 amperes. This must also be the drop across the parallel shunt. The current through the shunt is 45 amperes, because 50 amperes is the value of the current in the external circuit and 5 amperes of this must flow through the ammeter; therefore 45 amperes flow in the external shunt.

Direct-current Millivoltmeters.—In discussing milliammeters, it was stated that the internal resistance of a typical 0- to 1-milliammeter was 105 ohms. For a full-scale deflection, the voltage drop across this instrument would be $E = IR = 0.001 \times 105 = 0.105$ volt. Suppose its internal resistance were only 100 ohms. Then the voltage drop across it for full-scale deflection would be $E = IR = 0.001 \times 100 = 0.100$ volt, or 100 millivolts. Thus, the scale of a milliammeter could be divided into millivolts instead of milliamperes, and the milliammeter would then become a millivoltmeter. Mechanically and electrically there is no basic difference between a direct-current millivoltmeter and milliammeter.

From the preceding sections it was shown that an ammeter consists essentially of a moving-coil element and a shunt. It follows, therefore, that a shunted millivoltmeter can be used to measure current. Thus, one millivoltmeter and a series of relatively inexpensive shunts can be used for a wide range of measurements.

Problem.—Suppose a 0- to 200-millivolt millivoltmeter having an internal resistance of 8.00 ohms is available and it is desired to design a series of shunts to use with it, so that currents with maximum values of 1.0, 5.0, 10.0, and 50 amperes will produce full-scale deflections.

Solution.—For a full-scale deflection, the drop across the instruments is the full-scale deflection of 200 millivolts, or 0.2 volt. With this voltage drop the current through the millivoltmeter must be $I = E/R = 0.2/8.00 = 0.025$ ampere. The current through each shunt will be the total current it is desired to measure minus the current through the millivoltmeter. The drop across the shunt must equal that across the millivoltmeter because they are in parallel. Thus, the values of the shunts will be as follows:

1.0-ampere shunt: $R_s = \dfrac{0.2}{0.975} = 0.2053$ ohm.

5.0-ampere shunt: $R_s = \dfrac{0.2}{4.975} = 0.04022$ ohm.

10.0-ampere shunt: $R_s = \dfrac{0.2}{9.975} = 0.02005$ ohm.

50.0-ampere shunt: $R_s = \dfrac{0.2}{49.975} = 0.004005$ ohm.

It is of course important that the temperature coefficient (page 55) of the material of which the shunt is constructed is low so that the resistance of the shunt will not vary with the current flow.

Direct-current Voltmeters.—This instrument is merely the basic permanent-magnet moving-coil mechanism with a large amount of resistance in series with the moving coil. As previously mentioned, inexpensive voltmeters for communication measurements (where the amount of power from batteries, rectifiers, etc., is low) commonly have an internal resistance of 1000 ohms per volt of range.

Thus a 0- to 100-volt voltmeter would have an internal resistance of 100,000 ohms. Such an instrument will take $I = E/R = 100/100{,}000 = 0.001$ ampere, or 1.0 milliampere, from the circuit in which it is connected to produce a full-scale deflection. This is in contrast with many direct-current voltmeters designed for use in the power industry; in such instruments it is not uncommon to find that they would have resistances of only 100 ohms per volt.

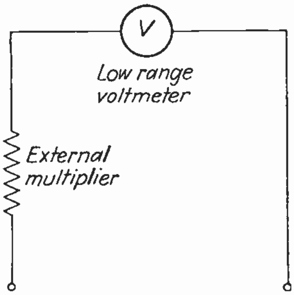

Fig. 138.—A high resistance connected in series with a low-range voltmeter makes possible the reading of larger voltages.

A single low-range voltmeter can be used with a series of **multipliers**; readings over a wide range are thus made possible. Such multipliers are merely series resistances as will now be explained and are connected as in Fig. 138. Multipliers are readily designed as follows:

Problem.—A radio-service man has a 0- to 10-volt 1000-ohm per volt voltmeter. He wishes to construct two external multipliers, one so that the range will be 0 to 100 volts, and the other so that the range will be 0 to 500 volts.

Solution.—The resistance of the 0- to 10-volt instrument is 10,000 ohms, and the current required for a full-scale deflection is $I = E/R = 10/10{,}000 = 0.001$ ampere. To make the instrument read 100 volts maximum, $100 - 10 = 90$ volts must be absorbed in the external multiplier resistance. Since the current through the circuit for a full-scale deflection is 0.001 ampere, the resistance of the external resistance will be $R = E/I = 90/0.001 = 90{,}000$ ohms. The multiplier required to make possible the reading of 500 volts maximum must have a resistance of $R = 490/0.001 = 490{,}000$ ohms. These multipliers should also be constructed of material having a low temperature coefficient.

It is of course possible to use series resistance with a milliammeter and measure voltage with it. It will be recalled that a milliammeter is merely a basic permanent-magnet moving-coil instrument. An ammeter can be made from a milliammeter merely by placing a suitable shunt across it; likewise, a voltmeter can be made from a milliammeter by placing a resistance in series with it as will now be shown.

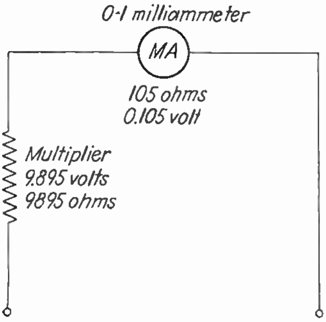

FIG. 139.

Problem.—The radio-service man mentioned on page 214 desires to use the same 0- to 1-milliampere milliammeter for measuring voltages of 0 to 10, 0 to 100, and 0 to 500 volts. How can this be done?

Solution.—The internal resistance of the milliammeter is 105 ohms, and a current of 0.001 ampere produces a full-scale deflection. For this deflection the voltage drop across the instrument is $E = IR = 0.001 \times 105 = 0.105$ volt. Thus, if the final instrument of Fig. 139 is to indicate a full-scale deflection when connected across 10 volts, then the voltage drop across the multiplier must be $E_m = 10 - 0.105 = 9.895$ volts. The multiplier resistance that will cause this drop is $R_m = E_m/I = 9.895/0.001 = 9895$ ohms. The multiplier resistance required for measuring 100 volts maximum would be $R_m = (100 - 0.105)/0.001 = 99{,}895$ ohms. The multiplier resistance for measuring 500 volts maximum would be $R_m = (500 - 0.105)/0.001 = 499{,}895$ ohms.

Combination Ammeter and Voltmeter.—As has been shown in the preceding pages, a simple milliammeter can be used with appropriate shunts or with appropriate multipliers to measure either currents or voltages over wide ranges. It is, therefore, possible to mount a single milliammeter in a cabinet with appropriate resistors and switches and have a combination ammeter and voltmeter that will be versatile and relatively inexpensive.

A convenient circuit for such an instrument is shown in Fig. 140. The various shunts can easily be made from resistance wire. The multipliers are often made from composition resistors which are purchased slightly oversized and then ground or filed away to the exact value desired.

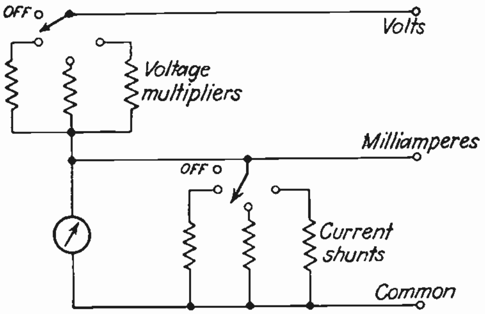

FIG. 140.—A combination milliammeter (or ammeter) and voltmeter may be made with a single permanent-magnet moving-coil instrument and appropriate resistors.

The Ohmmeter.—It is very convenient to have available a device for quickly measuring, with a fair degree of accuracy, the resistance of a resistor or of a circuit. Such an instrument is popularly called an *ohmmeter* and is often combined into the circuit of the combination ammeter and voltmeter just considered; of course it uses the same moving-coil element.

A simple ohmmeter can be considered as a series circuit composed of say a 20-volt voltmeter in series with a 22.5-volt battery, an adjustable rheostat, and two binding posts or say test leads. The instrument is designed to read directly on its scale the values of the resistance in ohms connected between the test leads.

First, suppose that the test leads are connected *directly* together and the variable rheostat adjusted until the instrument reads 20 volts. This is the full-scale value and corresponds to *zero* ohms. Now suppose the battery voltage and the rheostat remain fixed and a 50-ohm resistor is connected between the test leads. The voltage reaching the voltmeter is now less than formerly, because of the added *IR* drop in the externally connected 50-ohm resistor. This particular reading on the scale can be marked as "50 ohms." Similarly, other values can be marked off on the scale, and an ohmmeter is then available.

Measurements of Insulation Resistance with a Voltmeter.—A high-resistance voltmeter (of the order of 100,000 ohms) is very convenient for measuring high resistances, such as the **insulation resistance** of an open-wire telephone or telegraph line or a cable circuit. The values obtained are somewhat approximate for very high resistances (say over 10,000,000 ohms) but they are sufficiently reliable for most purposes and serve as an excellent check of the operating conditions of the line. Furthermore, the measurements are easily and quickly made, the result being that this method is well adapted for daily routine checks.

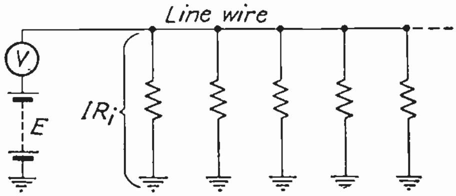

FIG. 141.—Illustrating the principles involved in measuring the insulation resistance of a telegraph, telephone, or power line with a voltmeter. The resistances between the wire and ground represent "leaks" over faulty insulators and through trees, brush, etc. This same method of measuring insulation can be applied to any device or circuit, such as measuring the insulation resistance of a motor, or between the coils and the core of a transformer.

In Fig. 141 is shown an arrangement for using a voltmeter to measure the insulation resistance of a line. When connected as shown, the voltmeter is in series with the battery and the *equivalent* resistance of all the leakage path to ground. This equivalent resistance is, of course, the insulation resistance R_i of the line.

With reference to Fig. 141, suppose that the voltmeter V is a 0- to 100-volt 1,000-ohm per volt voltmeter and that E is two 45-volt radio B batteries. When connected as indicated, the voltmeter and the equivalent or insulation resistance of the line are in series. The current through the voltmeter will be the same as that through the equivalent or insulation resistance of the line.

There will be an $E_v = IR_v$ drop across the voltmeter, and *it will indicate this voltage value on the scale.* There will be an $E_i = IR_i$ drop across the insulation resistance of the line. These two drops must equal the impressed voltage from the battery, or $E_B = IR_v + IR_i$, and $IR_i = E_B - IR_v$. Now I can easily be found, because

$I = E_v/R_v$. Making this substitution $E_vR_i/R_v = E_B - E_vR_v/R_v$, and from this

$$R_i = \frac{(E_B - E_v)R_v}{E_v}. \tag{44}$$

To summarize: The insulation resistance of a line or device may be measured by connecting a high-resistance voltmeter and a battery to the line as shown in Fig. 141. Then, from Eq. (44), the insulation resistance of the line in ohms can easily be found. This entire apparatus may be mounted in a small portable box for convenience. Insulation resistance is usually expressed in **megohms,** one megohm equaling 1,000,000 ohms. Of course, this method is suited in general for any high-resistance measurements, such as measuring the insulation resistance of a motor.

Other Methods of Measuring Direct-current Values.—The preceding pages have been devoted to the measurement of direct voltages and currents. The only instrument that was considered for these measurements was the permanent-magnet moving-coil type.

Now the reader may know quite well that other types of instruments will measure direct voltages and currents and he may wonder why they have not been considered. The answer is this: Other instruments will measure direct voltages and currents satisfactorily. They are, however, really alternating-current instruments. *The really important instruments for direct voltage and current measurements are of the permanent-magnet moving-coil type* which has been treated in considerable detail.

The following pages of this chapter will largely be devoted to alternating-current instruments. These may, for convenience, be divided into two classifications: (1) those alternating-current instruments which are designed for use in low-frequency (50- or 60-cycle) power circuits; and (2) those alternating-current instruments which are designed for use in communication circuits. Of course, there are several classifications for communication instruments, and these will be considered at the appropriate point (page 228).

Electrodynamic Instrument.—The basic difference between this instrument and the permanent-magnet moving-coil instrument is that no permanent magnet is used; the magnetic field for reacting with the moving coil is produced by a field coil instead. As will be evident after studying the electrodynamic instrument, it can be used to measure direct as well as alternating values.

The operating features of the **electrodynamic instrument** (also called electrodynamometer) are shown in Fig. 142. As indicated, there are two fixed field coils and two movable coils[1] mounted shown. The manner in which these are connected in ammeters, voltmeters, and wattmeters will now be considered.

The Electrodynamic Ammeter.—If the movable coils and the fixed field coils are connected *in series* as in Fig. 143, the device can be used for a low-range alternating-current ammeter and milliammeter. The current will pass through each coil. The field coils will produce a magnetic field, and the movable coils will produce another magnetic field. The pointer will deflect in the same direction for *alternating* currents because the current in both the fixed and movable coils alternates at the same instant. This is in contrast with the permanent-magnet moving-coil type considered earlier in this chapter. In these instruments the field produced by the permanent magnet is always directed the same way; therefore, if alternating current were passed through the movable coil, the

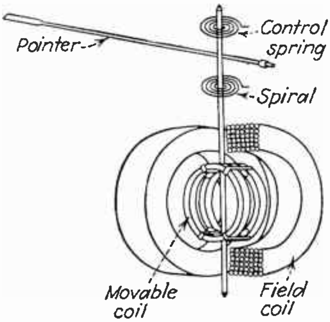

FIG. 142.—Showing the operating features of an electrodynamic instrument used for measuring alternating currents and voltages.

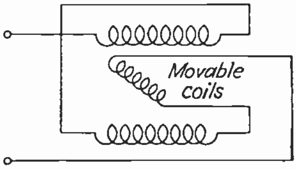

FIG. 143.—In the electrodynamic ammeter for measuring small currents the fixed and movable coils are connected in series.

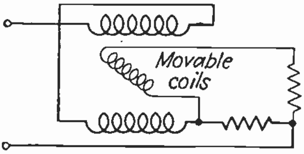

FIG. 144.—In the electrodynamic ammeter for measuring large currents, a small portion only of the total current is passed through the movable coil. The movable coil is carried on pivots in jeweled bearings and must be made of rather small wire as compared with the fixed field coils.

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[1] Of course one movable coil symmetrically placed on the shaft will work just as well.

pointer would tend to move first in one direction and then in the other. For measuring larger values of current, the total current is passed through the fixed field coils, but only a portion is shunted through the movable coils as Fig. 144 indicates.

The Electrodynamic Voltmeter.—For measuring voltages, the fixed field coils and the movable coils are connected *in series*, and a high value of resistance is added in series with this combination as shown in Fig. 145. The fixed field coils produce one magnetic field, and the movable coils produce another magnetic field. The reaction between these two fields turns the movable coil and the attached pointer.

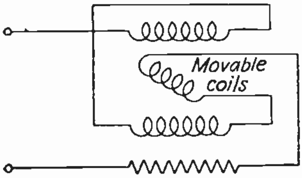

FIG. 145.—In the electrodynamic voltmeter the fixed and movable coils are in series and a high resistance is placed in series with the coils to limit the current flow when the voltmeter is connected across a source of voltage.

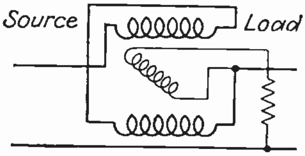

FIG. 146.—In the electrodynamic wattmeter the current taken by the load passes through the fixed coils. The voltage across the load is impressed on the movable coil in series with a fixed resistor.

In both the electrodynamic ammeter and voltmeter, the magnetic field produced by the fixed coil is proportional to the current flowing. Also, in each of these instruments the field produced by the movable coil is proportional to the current flowing. The force on the movable coil and the attached pointer is caused by the interaction of these two fields and is proportional to their product. Thus, since the current flowing is in a sense used *twice*, the deflection of the ammeter is *proportional to the current squared.* Likewise, in the electrodynamic voltmeter, the deflection is *proportional to the voltage squared* (because the voltage forces the current through the coils). For this reason, the scale is crowded together at the lower end and is open at the upper end. Of course, because of the various positions occupied by the movable coil, this variation of the deflection with the current squared does not hold exactly.

The Electrodynamic Wattmeter.—A circuit of this instrument is shown in Fig. 146. The current taken by the load passes through the fixed field coils and produces a magnetic field. The voltage across the load forces a current through the movable coils and establishes a magnetic field. Thus, two magnetic fields are produced, one proportional to the current the load takes and the other proportional to the voltage across the load. The force on the movable coil and hence its deflection are determined by the instantaneous strengths of these two fields.

If the currents and voltages are not in phase (page 120), the force at any instant will be proportional to the product of their in-phase components, that is, to $EI \cos \theta$, where E is the voltage on the load, I is the current through the load, and θ is the angle between the current and voltage. Now the power taken in alternating-current circuits is $P = EI \cos \theta$ (page 261). Thus, the electrodynamic wattmeter of Fig. 146 measures the power taken in an alternating-current circuit. Of course, it will also measure the power taken by a direct-current circuit.

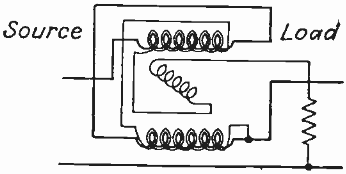

FIG. 147.—Connections for a compensated electrodynamic wattmeter. See text for details.

By referring to Fig. 146, it is evident that the current flowing through the fixed field coils is greater than it should be by the amount of current flowing through the movable coils. This will cause the deflection to be greater than it should be. If the movable coils are attached to the "source" side, then the voltage across the movable coils will not be that across the load, but will be the load voltage plus the voltage drop in the fixed coils. Both these connections will cause small errors. This effect has been discussed previously (page 71) in considering connections for ammeters and voltmeters. To remedy these errors, electrodynamic wattmeters of the better grade are **compensated** as shown in Fig. 147. As will be noted, additional **compensating coils** are wound on the fixed field coils. The current taken by the movable coils is passed back through these compensating coils so that a demagnetizing effect is produced. In other words, in Fig. 146 the current carried by the fixed field coils and the magnetic field produced by them is too great by the amount of current taken by the movable coils. Therefore, compensating coils are wound on the field coils, and the current taken by the movable coils is passed back through these compensating coils so as to produce a demagnetizing effect offsetting the excess field.

A phantom view of an electrodynamic instrument is shown in Fig. 148. It will be noted that the interior of the instrument is placed within a laminated-iron shield to prevent errors due to stray fields. If instruments are not carefully shielded, and many of the cheaper ones are not, care must be taken to prevent errors due to stray fields.

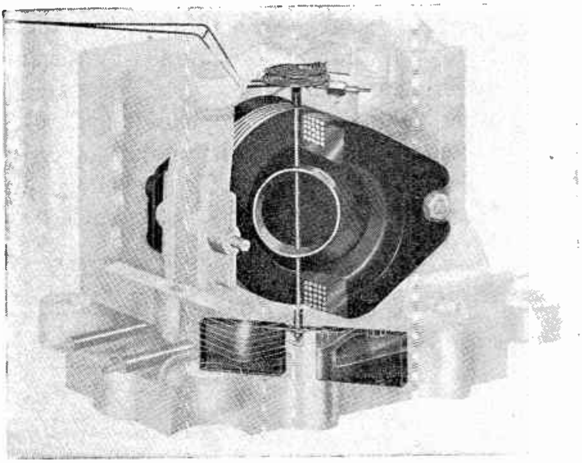

FIG. 148.—Working principle of an electrodynamic instrument.

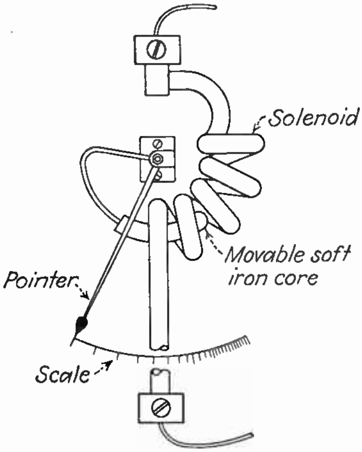

FIG. 149.—Early magnetic-vane mechanism of the suction type.

Moving-iron Instruments.—A large number of such instruments are possible, but only three need be treated here, they being illustrative of the principles involved.

Suction Type.—This principle is illustrated by Fig. 149, and little need be said regarding its operation. It is apparent that the current passing through the coil of wire or solenoid will produce a magnetic field which will "suck" in the movable iron core and cause a deflection of the attached pointer. This instrument will operate on either alternating or direct current.

Magnetic-vane Type.—This instrument is illustrated in Fig. 150. One or more soft-iron vanes are so placed in an inclined coil that when no current flows through the coil the iron vanes are almost at right angles to the axis of the coil. When a current flows through the coil, a magnetic field is produced along the axis of the coil and the soft-iron vanes tend to line up with this field so that they can carry the lines of force.[1] This instrument will operate on either direct or alternating current.

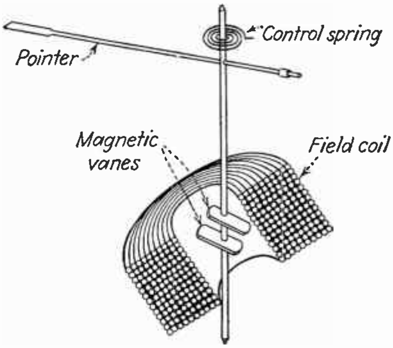

FIG. 150.—Working principle of a magnetic-vane type instrument.

Concentric-vane Type.—This principle is illustrated by Fig. 151. For the ammeter a small number of turns of large wire is used for the coil, and in the voltmeter a large number of small turns (plus some series resistance) is used. Fastened to the inside of the coil is a fixed cylindrical piece of soft iron. Attached to the shaft down the center of the coil is another piece of soft iron. The two pieces of iron are held but a small distance apart. When a current flows through the coil, the pieces of iron become magnetized and repel

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[1] This discussion is only for an ammeter. Of course the same principle will work for a voltmeter if the field can be made sufficiently strong without taking an excessive current. In one series of "inclined-coil" instruments, the voltmeter has a coil of fine wire at the center of the fixed inclined coil, and the same current that passes through the fixed inclined coil passes through the movable coil. In the inclined-coil wattmeter, a central movable coil carries the current proportional to the voltage, and the fixed inclined coil carries the load current. Strictly speaking, these two instruments are electrodynamic instruments. The American Institute of Electrical Engineers defines an electrodynamic instrument as an "instrument which depends for its operation on the reaction between the current in one or more moving coils and the current in one or more fixed coils."

each other. The piece of iron that is fixed to the inside of the coil is tongue-shaped or otherwise made nonsymmetrical so that the inner movable soft-iron pole piece deflects (because of the mutual repelling force) in a given direction. The final shape of the scale can be modified by the shape of the fixed pole piece. The concentric-vane instrument is not used for a wattmeter, but the instrument can be used either on alternating or direct current.

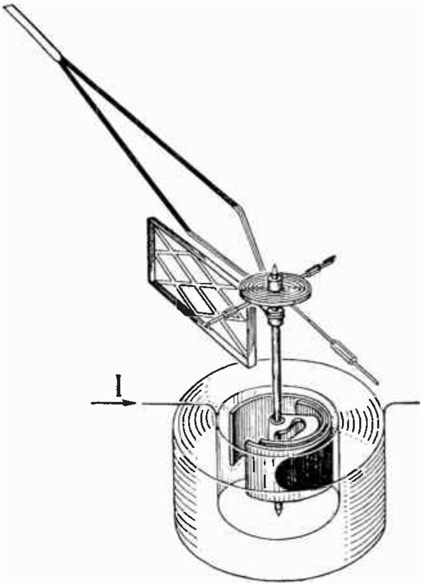

Fig. 151.—Principles of construction of the concentric-vane type instrument.

Damping Alternating-current Instruments.—In modern electric instruments the motion of the movable parts is opposed by one or more spiral control springs. Nevertheless, they must be damped to prevent oscillations about a value before the pointer finally comes to rest. The method of damping a direct-current instrument was considered on page 212. The methods of damping alternating-current instruments will now be considered.

Air Vanes.—Aluminum air vanes are often attached to the movable assembly (see Fig. 148 and Fig. 151). If the air vanes move more or less in open air, they are not very effective. If, however, the air vane is confined to a tight-fitting enclosure, and if it has a small hole in it for a "dash-pot" effect, the damping is very effective.

Eddy Currents.—These were used for damping direct-current instruments, and a modification of this principle is also used for damping alternating-current instruments. This is done as follows: Attached somewhere to the movable system is a thin disk of aluminum; it may be attached to the opposite end of the pointer assembly where it also acts as a counterweight, or it may be attached to the shaft in some convenient position. This aluminum disk turns (when the pointer moves) between the poles of one or more strong permanent magnets. When it does this, eddy currents will flow in the disk, and from Lenz's law (page 155), these currents will be so directed that the motion will be opposed. In this way the instrument is effectively damped.

Increasing Instrument Ranges.—It is often necessary to use a low-range instrument for measuring large current or voltage values. Examples of this were given on pages 214 and 216, where it was shown how to calculate the values of shunts and multipliers for increasing the ranges of ammeters and voltmeters. These were direct-current and voltage instruments, and shunts and multipliers are widely used with instruments of this type.

Shunts and multipliers are also sometimes used with certain types of alternating-current and voltage instruments. In so doing however, care must be taken to prevent errors; because inductance, effective resistance, and frequency are involved, shunts designed on simple direct-current theory may not be correct.

In alternating-current practice it is common to use a specially designed current transformer of various ratios to increase the range of an ammeter. Also, it is common practice to use a specially designed voltage transformer of various ratios to increase the range of a voltmeter. Thus, one ammeter and current transformer and one voltmeter and voltage transformer can be used to make a wide range of measurements.

Electrostatic Instruments.—The voltmeters that have been considered thus far really functioned because of currents and magnetic fields; but, since the current and field could be made proportional to the line voltage, the scale of the instrument could be calibrated in volts.

The electrostatic instrument is a true voltage-operated device. When a voltage is applied to two plates of a condenser, charges flow to these plates, an electric field is produced in the dielectric, and a repelling force is produced between the two plates. If one of the plates is arranged to move and has a pointer attached, then this principle can be used to measure voltage, either alternating or direct. Electrostatic ammeters and wattmeters are also possible but are perhaps only suited to experimental work.

The electrostatic voltmeter has many advantages over the electromagnetic type. Its internal resistance is very high, and when used for measuring direct voltages the current taken is insignificant. Because of its internal capacitance, it takes a small amount of leading current (page 176) from the line when used in alternating-current circuits. The electrostatic voltmeter is comparatively inexpensive, is fairly rugged, and although not extensively used in communication, is well suited for many measurements in this field because it takes so little power for its operation.

Communication Instruments.—Of course the permanent-magnet moving-coil direct-current instruments discussed in the first part of this chapter are widely used in communication practice for measuring direct currents and voltages. Also, they are used with copper oxide rectifiers (page 229), with thermocouples (page 231), and with vacuum-tube instruments to measure alternating-currents and voltages. Relatively speaking, however, the types of alternating-current instruments previously discussed are seldom used, except for measurements in the 50- or 60-cycle power-supply circuits.

Before further discussing communication instruments, it is well to consider the alternating-current values encountered in communication circuits.

Frequency.—Direct-current (which is in a sense alternating-current of zero frequency) is very extensively used. Very low frequencies of a few cycles per second are encountered in television and other circuits. The audio frequencies cover a band from about 50 to 12,000 cycles. Then there are power-line and telephone-carrier systems occupying various frequencies from about 10,000 to 150,000 cycles. Above this and extending up to billions of cycles per second come the many radio-communication uses. Thus, measurements must be made of alternating currents and voltages ranging in frequency from a few cycles per second to billions of cycles per second.

Current.—Very small currents are adequate for carrying communication messages. Instruments must be available for measuring alternating currents of a few microamperes to many amperes such as would be present in the antenna circuit of a large radio transmitter.

Voltage.—What was said regarding communication currents applies to voltages. In communication circuits, voltages of a few microvolts to thousands of volts must be measured.

Power.—The power limitation in communication circuits is one of the important factors seriously limiting measurements. The amount of power in a communication circuit may be only a few microwatts (the output of a microphone, for instance), or it may be only a few milliwatts (as in a telephone conversation). Of course, the power level may also be high, such as the input to the antenna in a large radio station.

Now unless either the frequency or the voltage is too high, large values of current, voltage, and power are much easier to measure than are small values. But consider for the moment the measurement in a circuit in which the voltage is only a few volts, the current is only a few milliamperes, and the power is only a few milliwatts. The power required to operate an alternating-current instrument of the types considered on pages 220 to 226 is greater than the useful communication power in the circuit being used to transmit the message or program. It is not uncommon to find that 10 watts is required to operate an alternating-current voltmeter, for example, such as is used in 60-cycle power circuits.

For these and other reasons, special instruments are used in alternating-current and voltage measurements in communication. Two of these, the copper oxide instruments and the thermocouple instruments, will be considered in this chapter.

The Copper Oxide Rectifier Instrument.—In about 1926 it was found that if a thin layer of cuprous oxide (usually referred to as copper oxide) was formed on a layer of copper a current flowed (conventional direction) quite readily from the oxide to the copper, but that the path was a very high resistance in the opposite direction. Thus, the copper oxide element is a rectifier, and is used in communication measuring instruments as will now be explained (see also page 435).

The circuit is arranged as shown in Fig. 152. Each of the rectifier elements in the arms of the bridge is a very small copper oxide rectifier disk or disks. The four are usually combined into a single unit with the proper leads brought out. Across one portion of the bridge is connected a sensitive permanent-magnet moving-coil direct-current instrument. The alternating current to be measured is passed in at the other pair of bridge terminals.

Assume that, for the moment, the *right* terminal is positive and the left is negative. Then, current will flow in at the right, up through the rectifier element, down through the direct-current instrument, and out to the left through the lower-left rectifier element. When the next half of the alternating current is impressed on the bridge, the *left* terminal will be positive and the right terminal will be negative. Then, the current will flow in at the left terminal, up through the element, down through the direct-current instrument, and out through the element at the lower right. Note that the alternating current is rectified so that it can be measured by a direct-current instrument.

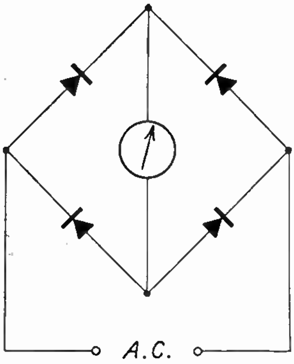

Fig. 152.—Connections of a typical copper oxide rectifier-type instrument.

The instrument of Fig. 152 is a **copper oxide alternating-current milliammeter.** For many purposes, it is very satisfactory, but for some communication purposes it is not. (1) It should not be used to measure direct current because it is not correctly calibrated to

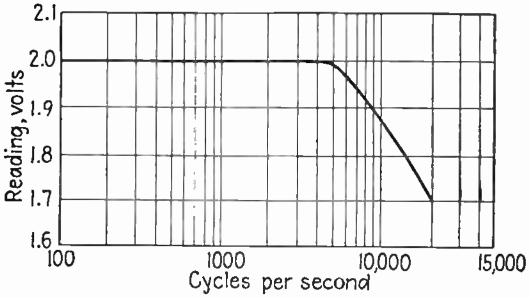

Fig. 153.—Calibration curve for a typical copper oxide rectifier voltmeter with 2.0 volts held across its terminals and the frequency varied.

read accurately on direct current. This instrument should be calibrated on alternating current. (2) Although it will indicate, it does not read accurately on very low frequencies or on the higher audio frequencies. The most accurate range of most copper oxide instruments is from about 100 to 5000 cycles per second. There is capacitance between the copper oxide layer and the copper, and when the frequency becomes sufficiently high the effect of this capacitance seems to be to shunt out current from the instrument, causing the reading to drop. At 10,000 cycles, a typical copper oxide instrument reads about 7.5 per cent low. (3) A milliammeter should have very low internal resistance, but the internal resistance of a copper oxide milliammeter is fairly high (a typical one having a resistance of almost 1000 ohms). Furthermore, this resistance is found to vary widely, depending on the amount of current being measured.

FIG. 154.—A typical copper oxide rectifier voltmeter.

The copper oxide milliammeter is recommended only for special purposes where the characteristics enumerated are of no disadvantage. The copper oxide voltmeter, however, is a splendid general-utility instrument for audio-communication purposes.

If a high value of resistance, say about 20,000 ohms (for as low as a 0- to 2-volt voltmeter), is connected in series with the bridge circuit of Fig. 152, the device becomes a copper oxide alternating-current voltmeter. Because it contains a high value of series resistance, its internal impedance is quite constant and is quite high as it should be for a voltmeter. The copper oxide voltmeter has a frequency error as explained for the ammeter and as shown in Fig. 153. This limits its use for most purposes to measurements below about 10,000 cycles. A typical copper oxide voltmeter is shown in Fig. 154.

Thermocouples.—Measurements of alternating currents and voltages in communication circuits are extensively made with **thermocouples.** Several different types of thermocouples are possible; three of the most common are shown in Fig. 155. Each of these thermocouples consists essentially of two wires (A and B) of *unlike* metals fastened together at the **thermocouple junction.** When this junction is heated, a direct voltage exists between the two ends of the unlike wires. If these two ends are connected to a sensitive permanent-magnet moving-coil instrument, a current will be forced through the instrument and its pointer will deflect.

Three methods are shown in Fig. 155 for heating the thermocouple junction. In the **mutual type** of thermocouple, the alternating current to be measured is passed through the thermocouple itself, rather than through a separate heater. The alternating current to be measured divides, part passing through the instrument and part through the thermocouple. The instrument will have a small amount of inductance, and thus the current division will depend on the frequency of the current to be measured. This introduces a **shunting error.** Also, there is a bad **reversal error** with the mutual type; that is, the deflection of the instrument is not the same for the same value of current in different directions through the thermocouple.

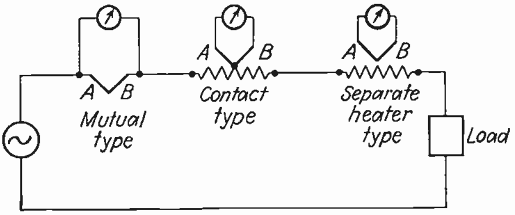

Fig. 155.—The various types of thermocouples connected for measuring currents.

The **contact type** of thermocouple shown in Fig. 155 is a widely used type and is very satisfactory. In this, the current to be measured is passed through a **heater** which makes metallic contact with the thermocouple junction and readily transmits heat to it. With this thermocouple no shunting error exists, and the reversal error is usually negligible.

The **separate-heater type** of thermocouple, also shown in Fig. 155, is particularly well suited for measurements in radio-frequency circuits. It has been specially designed to reduce the errors that thermocouples introduce into radio-frequency circuits. Because of the importance of this in communication measurements, it will be discussed in the following separate section.

Thermocouples at Radio Frequencies.—Below frequencies of a few thousand cycles per second, measurements are usually relatively easy to make. Above about 10,000 cycles per second, more attention must be given to the grounding and shielding of circuits. When measurements are made at radio frequencies, say from 100,000 cycles on up to billions of cycles, an inexperienced person can get almost any answer to his measurements if his instruments or technique is faulty.

Now the contact-type thermocouple generally used for audio and the lower radio frequencies is a rather simple device. It consists essentially of a heater which may have only a few ohms resistance if it is to measure large currents or may be as high as 1000 ohms if it is to measure small currents. In contact with this heater is the thermocouple junction. When the alternating current to be measured is passed through the heater, power equal to I^2R is dissipated in the heater, and this raises the temperature of the thermocouple junction. This causes a direct voltage to exist between the cold terminals of the thermocouple, and this voltage will force a direct current through the sensitive moving-coil instrument. Although special wires are used in high-grade thermocouples, a voltage will be produced between any two unlike wires in contact, for instance, between copper and iron. In sensitive thermocouples these wires are very delicate, and they are mounted, as indicated in Fig. 156, in an evacuated glass bulb. This protects them from mechanical injury, prevents air from conducting heat away from the heater, and makes the thermocouple more sensitive.

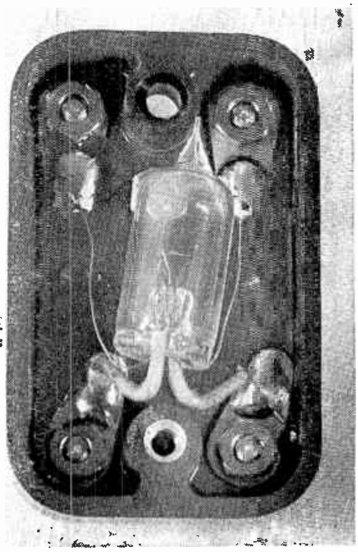

FIG. 156.—The wires of a sensitive thermocouple are very fine and are mounted in an evacuated glass bulb which is placed in a suitable plastic holder.

If the thermocouple is casually examined, its electrical characteristics will appear to be those of a simple pure resistance, because only the heater is inserted in the circuit in which measurements are desired. But, if a critical examination is made, the equivalent circuit of a thermocouple is found to appear as shown in Fig. 157. The various elements of this drawing are described below it.

Because the reactances of these small inductances and capacitances will vary with frequency, it is evident that a calibration made at one frequency will not hold at another frequency. Nevertheless, by using extreme care in the design of a thermocouple for radio frequencies, these stray effects can be kept to a minimum.[1] The result of this careful design is that thermocouples having the excellent characteristics of Fig. 158 are available. Note that the error is only about 1 per cent at 200,000,000 cycles.

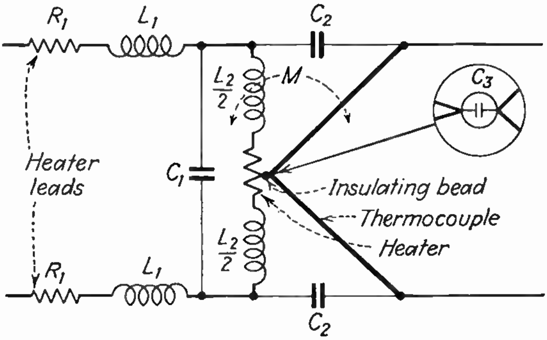

FIG. 157.—The equivalent circuit of a thermocouple at very high frequencies. L_1 and L_2 are the series inductances of the leads and heater wire, respectively. R_1 represents the resistance of the leads. C_1 is the stray capacitance between the leads. C_2 represents the capacitance between the heater and thermocouple, and M is the mutual inductance between these two. C_3 is the capacitance between the thermojunction and the heater.

In Fig. 157, it will be noted that an insulating bead is placed between the heater and the thermocouple junction. This bead *electrically* insulates the thermocouple junction from the heater wire. However, the material chosen for the bead has good heat-conducting properties, so that the heat from the heater is readily conducted to the junction. By the careful choice and construction of this bead, the separate-heater-type thermocouple has been made almost as sensitive as the contact type in which the heater and junction are in metallic contact.

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[1] For a good summary of this subject, see the catalogue of the General Radio Co., from which Figs. 157 and 158 were taken.

As the enlarged view of the bead in Fig. 157 indicates, there is but very little capacitance C_3 between the heater and the thermocouple. This capacitance C_3 plus the small capacitances C_2 and the mutual inductive effect of M form the only electric circuit between the heater and the measuring circuit attached to the thermocouple junction.

To explain completely the advantages of the separate-heater type of thermocouple requires a knowledge of balanced and unbalanced circuits which will be considered on page 325. For the present, suffice it to say that in radio-frequency measurements the less the circuit is disturbed the better. Inserting a *contact-type thermocouple* directly connects the thermocouple, the leads, and the measuring instrument to the heater. Thus, a large amount of stray capacitance is connected to one side of the circuit, and this may disturb the original circuit conditions. Inserting a *separate-heater-type thermocouple* does not disturb the circuit to nearly so great an extent, because the thermocouple, the leads, and the measuring instrument are not in direct metallic contact with the heater but are connected to it only by the small stray capacitances C_2 and C_3 and by the mutual inductance M of Fig. 157.

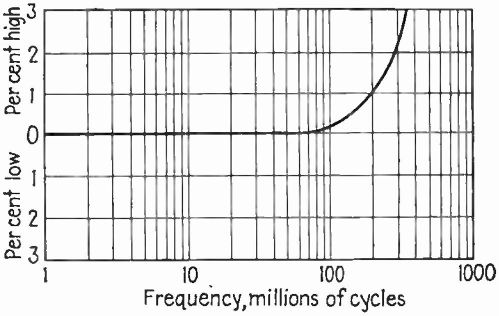

Fig. 158.—Percentage error for a good high-frequency thermocouple.

Of course, inserting the heater in *any* circuit, irrespective of the frequency, disturbs the circuit. This must be tolerated, however, if a thermocouple is to be used. However, the effect may be entirely negligible. For instance, if an oscillator having an internal impedance of several hundred ohms is connected to a load of several hundred ohms, inserting a thermocouple with a heater of a few ohms will not cause appreciable error. About the only rule that can be stated is to examine the circuit before the thermocouple is inserted and estimate the error that will result. Usually a mental Ohm's law analysis will suffice.

There may be certain carefully balanced radio-frequency circuits (again, this will be clearer after the statements on page 325 have been studied) in which if a thermocouple is inserted in one wire a small resistance of the same value should be inserted in the other wire.

To summarize: It must be emphasized that care must be taken in the operation of thermocouples if reliable measurements are to be made. However, once the proper technique is developed, thermocouples will be found to be invaluable for measurements in communication circuits.

Voltage Measurements with Thermocouples.—In the preceding discussions the use of thermocouples in current measurements has largely been considered. For making current measurements, the thermocouple heater is inserted in series with the circuit in which the current is to be measured.

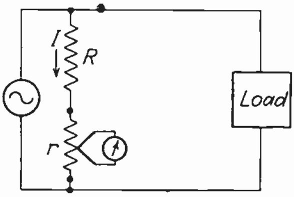

FIG. 159.—Method of measuring voltage with a thermocouple.

For measuring an unknown voltage, the thermocouple and a protective series resistance are connected as shown in Fig. 159. The resistance of the series protective resistance plus that of the heater must be sufficiently large that the current through the thermocouple heater will not exceed the safe value. The voltage between the line wires is $E = I(R + r)$, where R is the resistance of the protective resistor, r is the heater resistance, and I is the current through the heater, determined from the deflection of the instrument and its calibration curve.

The thermocouple heater and the series protective resistance of Fig. 159 take power from the circuit, and there may be instances where such a circuit cannot be used. With reference to page 83, if the regulation is poor, care should be taken to remove the thermocouple from the circuit before the load is removed. If this is not done, the rise in voltage may cause an excessive current through the heater and may burn out the heater. In fact, a sensitive thermocouple has *extremely* small and delicate wires in it and is easily burned out, after which it is useless, because ordinarily thermocouples cannot be repaired.

Calibration of Thermocouples.—If a given thermocouple is always used with the same moving-coil type measuring instrument, then the measuring instrument can be calibrated to read directly in milliamperes. Often the thermocouple is installed within the instrument case, and in this instance, the instrument would probably be calibrated to read directly in milliamperes.

It is common practice, however, to have several thermocouples of various ranges to use with one moving-coil instrument. Thus, a wide range of measurements is possible without an excessive investment. A calibration curve must be available, however, for each thermocouple and instrument combination. Such a calibration curve can be made with data obtained from a circuit such as shown in Fig. 160.

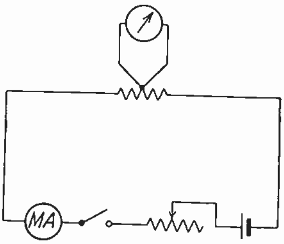

FIG. 160.—Circuit for calibrating a thermocouple.

It will be recalled (page 112) that a given value of direct current and the *effective* or root-mean-square (r.m.s.) value of alternating current produce the same heating effect in a resistor. For this reason, the simple direct-current circuit of Fig. 160 may be used to calibrate a thermocouple. In using this circuit, it will be found that the thermal lag of the thermocouple makes it laborious to set on exact values of current as indicated by the moving-coil instrument. For this reason it saves time to adjust the current to values read on the direct-current milliammeter being used as a standard.

This same method of calibration (Fig. 160) can be used for instruments with self-contained thermocouples provided that the thermocouple within the instrument is of the contact or separate-heater type. Instruments are sometimes built that do not use thermocouples of this type and that will not deflect on direct current. Such instruments must be calibrated on alternating current.

In the use of thermocouples, it should be ascertained if the heater resistances given in catalogues and often marked on the thermocouples are correct. This is very important in using thermocouples to measure voltages. Some manufacturers accurately measure the heater resistance and stamp it on the thermocouple housing or otherwise mark it.

If there is any question about the heater resistance, it can be readily measured by placing the heater as the unknown arm of a bridge such as shown in Fig. 237, page 359. With this bridge, measurements can be made at several frequencies to determine if the resistance varies with frequency (for a good thermocouple the resistance is constant over a wide range). Also, by varying the input voltage to the bridge and by observing the deflection of the thermocouple instrument, measurements can be made to determine if the resistance of the thermocouple heater varies with the current through it (for a good thermocouple, this also is quite constant). Of course a simple Wheatstone bridge may also be used to measure the resistance of the heater to direct current. In any such measurements it is advisable to have the instrument connected to the thermocouple and observe the deflection carefully so that the thermocouple will not be burned out.

SUMMARY

Measurements form the basis on which progress in communication is made.

Electric instruments are used to measure present values. Meters are used to measure quantities of electricity or electric energy.

Instruments are called meters by many persons, but this usage is incorrect.

The D'Arsonval or permanent-magnet moving-coil mechanism is the basic device for making almost all direct-current instruments.

This mechanism consists essentially of a strong permanent magnet, suitably shaped pole pieces and cylindrical core, and a moving coil. The current to be measured, or a portion of it, is passed through the moving coil, and the reaction of the field produced by this current and the field of the permanent magnet causes a deflection proportional to the current being measured.

The ideal ammeter or milliammeter should have zero resistance (or impedance), and the ideal voltmeter or millivoltmeter should have infinite resistance (or impedance).

The permanent-magnet moving-coil mechanism by itself is either a sensitive milliammeter or millivoltmeter, depending on how it is calibrated.

The usual direct-current voltmeter merely consists of a permanent-magnet moving-coil mechanism with a large protective resistance in series.

The usual direct-current ammeter is merely a shunted permanent-magnet moving-coil mechanism.

A single moving-coil mechanism may be mounted in a case and provided with both shunts and multipliers which may be switched into the circuit, the result being a combination ammeter and voltmeter.

A voltmeter may be used as an ohmmeter or may be used to measure high resistances, such as the insulation resistance of a line.

The basic electrodynamic unit consists of two fixed coils which produce a magnetic field and one or two movable coils which produce another magnetic field. The reaction of these two fields causes a deflection of the movable coil and attached pointer.

In the electrodynamic ammeter, the fixed and movable coils may be connected in series and the entire current passed through them for a low-reading instrument. For a large ammeter, only a portion of the current is passed through the movable coil.

In the electrodynamic voltmeter the fixed and movable coils are usually connected in series, and a protective resistance is also connected in series to limit the current flow when measuring voltages.

In the electrodynamic wattmeter, the current taken by the load is passed through the fixed coils, and the voltage across the load is impressed across the movable coil and a series protective resistor.

Moving-iron-type instruments are widely used in making measurements in alternating-current circuits. There are several types of these instruments.

Electric instruments must be damped. In the direct-current permanent-magnet moving-coil instrument, damping is accomplished by the currents induced in the moving coil when it turns in the permanent magnetic field. In alternating-current instruments, damping is accomplished by air vanes or by currents induced in a disk or vane that moves between the poles of a permanent magnet.

Electrostatic instruments, having many uses in communication, also are available.

Because of the high frequencies and the low power levels in communication circuits, instruments for communication purposes often must have special characteristics.

The copper oxide rectifier instrument is very useful for approximate measurements at the lower communication frequencies.

Thermocouples are very widely used in making alternating-current measurements in communication circuits.

Thermocouples may be used up to frequencies of several hundred million cycles per second if they are properly designed.

Thermocouples can be used to measure either currents or voltages. They are easily calibrated on direct current, except for certain special types.

REVIEW QUESTIONS

1. Explain the difference between an instrument and a meter.

2. Why are pole pieces and the fixed cylindrical soft-iron core used in the direct-current permanent-magnet moving-coil instrument?

3. What is the relation between the current through the moving coil and the deflection of the pointer? Why is it so?

4. How is the direct-current instrument damped? Is this quite effective?

5. Fundamentally, how does a sensitive direct-current milliammeter differ from a sensitive millivoltmeter?

6. What is the fundamental difference between a direct-current ammeter and voltmeter?

7. How would you calculate the power consumed by an ammeter? By a voltmeter?

8. Why are voltmeters of very high internal resistance most useful in communication?

9. What is a shunt? A multiplier?

10. Explain the operation of an ohmmeter.

11. Explain the theory of the electrodynamic ammeter, voltmeter, and wattmeter.

12. Discuss the relative effect of stray magnetic fields on unshielded electrodynamic instruments as compared with unshielded permanent-magnet moving-coil instruments.

13. How should electrodynamic instruments be shielded?

14. Enumerate the common types of moving-iron instruments, and explain the operation of each type.

15. How are alternating-current instruments shielded?

16. Explain the operation of the electrostatic instrument.

17. Why are the measurement problems in communication different from those in the power industry?

18. What are the advantages and limitations of copper oxide instruments?

19. Explain the principle of operation of the thermocouple.

20. What are the advantages of the separate-heater type thermocouple over the contact-type thermocouple at radio frequencies?

PROBLEMS

1. A direct voltage of 10 volts is impressed across an ammeter and a load in series. A 0- to 10-volt voltmeter of 62 ohms per volt is connected *directly* across the source. The ammeter has an internal resistance of 0.01 ohm, and the resistance of the load is 2.94 ohms. Calculate the percentage error in the reading of the voltmeter compared with the true voltage across the load.

2. Referring to Prob. 1, calculate the error in the current reading if the voltmeter is connected on the load side of the ammeter.

3. A 0- to 100-volt voltmeter has an internal resistance of 62 ohms per volt. How much power will be consumed by it when it is measuring a full-scale value? Make the same calculation for a voltmeter having an internal resistance of 1000 ohms per volt.

4. Calculate the power consumed by a milliammeter having a range of 0 to 10 milliamperes and an internal resistance of 9.3 ohms when reading a full-scale value.

5. Referring to the problem on page 214, design a shunt to use with the milliammeter so it can measure a full-scale current of 100 milliamperes, also of 1.0 ampere.

6. Referring to Fig. 140, page 218, suppose that the instrument has an internal resistance of 105 ohms and a full-scale deflection of 1.0 milliampere. Design shunts so that the instrument will measure maximum values of 25 milliamperes, 500 milliamperes, and 5 amperes. Also, design multipliers so that the instrument will measure maximum values of 10, 100, and 500 volts.

7. The moving coil of an ammeter has a resistance of 10.8 ohms, and 0.008 ampere is required to produce a full-scale deflection. What should be the value of a shunt to make possible the measurements of 10 amperes maximum.

8. It is desired to measure an unknown voltage. A 0- to 150-volt voltmeter having an internal resistance of 14,750 ohms is connected across the voltage but it goes off scale. A 0- to 100-volt voltmeter having a resistance of 12,570 ohms is connected in series with the first instrument, and the two are then connected across the voltage. The smaller instrument reads 90 volts. What is the total voltage?

9. It is desired to find the resistance of a voltmeter. An adjustable calibrated rheostat is placed in series with the voltmeter, and the resistance inserted by the rheostat is varied. How will you know when the resistance inserted by the rheostat equals that of the voltmeter?

10. A 0- to 150-volt voltmeter having an internal resistance of 27,560 ohms is connected in series with two 45-volt B batteries and an unknown resistor. The voltmeter reads 62.7 volts. What is the value of the resistor?

11. A voltmeter having an internal resistance of 100,000 ohms is connected between a telephone line and ground as shown in Fig. 141, page 219. Two 45-volt radio B batteries are connected in series. The voltmeter reads 3.9 volts. Calculate the insulation resistance of the line.

12. If the line of Prob. 11 is 36.8 miles long, what is the insulation resistance per mile?

13. The resistance of a thermocouple heater is 1000 ohms. Calculate the power loss in it when 0.5 milliampere is flowing through it.

14. Suppose that the thermocouple of Prob. 13 is to be used with a series resistor to measure 10 volts. The moving-coil instrument used with the thermocouple produces a full-scale deflection when the heater current is 1.0 milliampere. How much series resistance should be used with the thermocouple, and where should it be placed?

15. An oscillator has an internal impedance of 600 ohms pure resistance and produces an open-circuit voltage of 40.2 volts. It is connected to a load of 750 ohms pure resistance. A thermocouple that will carry 2.0 milliamperes safely, that has a heater resistance of 1000 ohms, and that has 24,000 ohms in series with it is connected across the resistor to measure the voltage. Will the thermocouple be burned out if the load is disconnected from the circuit?

CHAPTER IX

ALTERNATING-CURRENT CIRCUITS

Certain of the theoretical aspects of alternating voltages and currents were considered in Chap. V. The discussion was quite generalized, being largely devoted to a description of sinusoidal waves and to the method of handling vector quantities representing the sine-wave values.

In Chap. V it was mentioned that in alternating-current circuits the current could lead, lag, or be in phase with the voltage. It was stated that in a capacitive circuit (such as a condenser) the current *led* the voltage, that in an inductive circuit (such as a coil) the current *lagged* the voltage, and that in a purely resistive circuit the current was *in phase* with the voltage. The reasons for these statements were not given, however.

In this chapter the discussion of alternating voltages and currents will be continued. The treatment will be more specific, however, because the general features of sinusoidal waves and of vectors were previously described. Also, in this chapter extensive numerical examples will be given. The reader is urged to refer to Chap. V whenever necessary to refresh his memory regarding the important basic principles discussed there.

Generation of a Sine-wave Voltage.—It is customary in introducing this subject to use an alternating-current power **generator** or **alternator** to illustrate the principles involved. In accordance with the purpose of this book, however, the hand-driven magneto used to generate the bell-ringing current in a local-battery or magneto telephone set will be chosen.

A typical magneto generator is shown in Fig. 161, and a working diagram is shown in Fig. 162. As is evident, this magneto generator consists electrically of a coil of wire which is rapidly rotated in a magnetic field produced by several permanent horseshoe magnets. Mechanically, there is a train of gears to increase the rotation of the coil and a mechanism attached to the shaft so that when the handle is turned an electric contact is made with the external circuit (the telephone line), but when the handle is not being turned, the switch is opened and the magneto ringing generator is not connected to the line. The advantage of this is as follows: Many telephone sets are often connected at various points along the line; however, the only time a magneto is connected is when a person rings on the line. Of course, this method prevents a loss of both ringing power and speech power, which would result if the magnetos at the various points were permanently connected to the line.

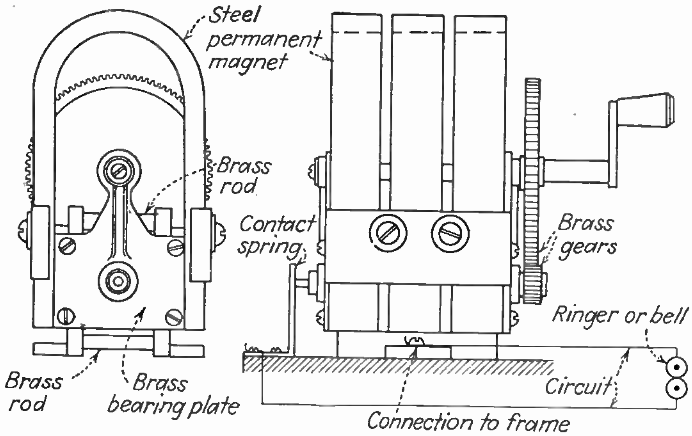

FIG. 161.—Working parts of a telephone magneto-ringing generator used with local-battery telephone sets.

A third simplified view of the rotating portion of the generator is shown in Fig. 163. The poles of the horseshoe magnet have been omitted, and only the magnetic lines of force and a single turn of the rotating coil have been shown. From Lenz's law (page 155) an electromotive force is induced in a coil of wire whenever the magnetic flux linkages are changed. Thus it follows that in Fig. 163 an electromotive force will be induced in the single-turn coil of

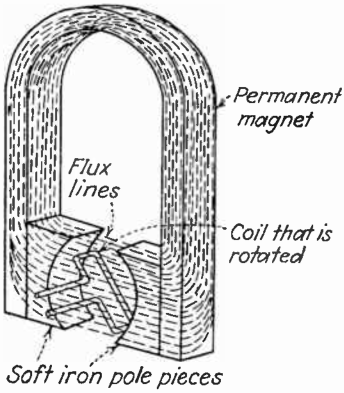

FIG. 162.—Elements of a telephone magneto-ringing generator.

wire (representing the rotor of the hand magneto generator) when this coil is rotated in the magnetic field.

A study of Fig. 163 will disclose that at the *instant* the plane of the coil is at right angles to the magnetic lines of force the coil is not changing flux linkages but is merely "sliding along" the lines of force; hence, at this instant the voltage induced is *zero* in accordance with Lenz's law. Similarly, a study will show that at the *instant* the plane of the coil is parallel to the magnetic field the rate of change of flux linkages is greatest and the induced voltage will be maximum.[1]

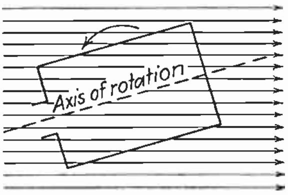

FIG. 163.—Simplified drawing of a coil rotating in a magnetic field, illustrating generator action.

As the reader no doubt suspects, if the coil of wire of Fig. 163 is rotated at a *uniform rate in a uniform magnetic field*, a pure sine-wave voltage will be induced in the coil. Of course, this is true. Two of the instantaneous sine-wave values have been considered, that is, when the wave is zero and when it is maximum. A study of this figure will also disclose that as the coil makes one complete revolution in the magnetic field the flux linkages are changed first in one direction and then in the other; this means that *positive* and *negative* half cycles of electromotive force constituting *one complete cycle* are generated in the magneto windings per revolution. Of course, these positive and negative values are purely relative with respect, for example, to one wire (say the upper wire of Fig. 163) of the coil. That is, one half is merely designated as positive and the other half as negative.

In between the positive and negative values, the *instantaneous values* of the induced electromotive force are as given by the relation

$$e = E_{\max} \sin \omega t, \tag{45}$$

where e is the instantaneous value of the voltage in volts when $E_{\max}$ is the maximum value of the sine-wave voltage in volts, ω is the

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[1] Such principles are readily expressed by appropriate mathematical equations, but these are avoided in this book. To assist in clarifying the above statements, let the following facts be remembered: From Lenz's law the voltage induced *at a given instant* depends on the rate of change of magnetic flux linkages *at that instant*. When the coil is at right angles to the magnetic field, the flux *linkages* are maximum, and during a small time interval the *rate of change* of linkages is very small. On the other hand, when the coil is parallel to the magnetic field the flux *linkage* is zero, but the rate of change during a very short period of time is maximum.

angular velocity of rotation of the coil in degrees per second ($\omega = 2\pi f$, page 120), and t is the time in seconds from the position of maximum flux linkages and zero induced voltage.

A word must now be said about the portion "sin" of Eq. (45). Equation (45) could be stated as follows: *The instantaneous value* e *of the voltage equals the maximum value* $E_{\max}$ *multiplied by the sine of the angle* (the angle being measured from the zero voltage point as a reference). This statement is based on a branch of mathematics known as **trigonometry** and may be unfamiliar to the reader. This need not be bothersome, however, because the principle involved is quite simple. Values of the sines of certain angles are given in Table VIII. More detailed tables are given on page 523. As is evident, the sine values repeat themselves and thus give the positive and negative half cycle.

TABLE VIII.—SINES OF CERTAIN ANGLES

Angle, degrees	Value	Angle, degrees	Value
0	0.000	195	−0.259
15	0.259	210	−0.500
30	0.500	225	−0.707
45	0.707	240	−0.866
60	0.866	255	−0.966
75	0.966	270	−1.000
90	1.000	285	−0.966
105	0.966	200	−0.866
120	0.866	315	−0.707
135	0.707	330	−0.500
150	0.500	345	−0.259
165	0.259	360	−0.000
180	0.000		

Magnitude of Generated Voltage.—In the preceding section it was shown that a sine-wave voltage would be generated by a coil of wire rotating at a uniform rate in a uniform magnetic field.[1] The magnitude of the voltage induced at any instant was shown to de-

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[1] The hand-driven magneto generator actually used in telephone sets does not generate a pure sine wave because a pure sine wave is not needed. No attempt is made to construct the pole pieces of the magnets or the coil so that the voltage generated is exactly a pure sine wave.

pend on the magnitude of the maximum value and the position of the coil at that instant. The method of calculating this maximum value will now be considered, but first the equation for the average voltage must be reviewed.

According to page 159, the *average* voltage is given by the relation $E_{av} = N\phi/10^8t$, where E_{av} is the *average* voltage in volts induced in a coil of N turns when the flux ϕ is changed from maximum to zero in time t seconds. Now for a coil of wire such as shown in Fig. 163, the total flux linkage is completely changed *four times per cycle,* and each cycle occurs in $t = 1/f$ seconds (page 114); hence, the average rate of change of flux linkages $N\phi/(1/4f) = 4f\phi N$ linkages per second, where ϕ is the total flux passing through the coil of N turns when its plane is at right angles to the lines of force. Then, the voltage induced in a coil of N turns which is revolving at a uniform rate in a uniform magnetic field will be

$$E_{av} = \frac{4fN\phi}{10^8}. \tag{46}$$

From page 112, $E_{av} = 0.637E_{max}$. Therefore, Eq. (46) may be written

$$0.637E_{max} = \frac{4fN\phi}{10^8}, \qquad \text{and} \qquad E_{max} = 1.57 \times \frac{4fN\phi}{10^8}.$$

The value of $1.57 \times 4 = 6.28 = 2\pi$, where $\pi = 3.1416$. Thus, the equation for the *maximum value* becomes

$$E_{max} = \frac{2\pi fN\phi}{10^8} \qquad \text{or} \qquad \frac{\omega N\phi}{10^8}, \tag{47}$$

where $\omega = 2\pi f$ (see page 120). Referring to Eq. (45), page 244, the equation for the instantaneous voltage induced in the coil of Fig. 162, or 163 representing the rotor of the hand-driven magneto generator of Fig. 161, is

$$e = \frac{2\pi fN\phi}{10^8} \sin \omega t = \frac{\omega N\phi}{10^8} \sin \omega t. \tag{48}$$

Numerical Example of Generator Action. *Problem.*—The rotor of a simple hand-operated magneto generator is composed of 2500 turns of fine wire closely bound together so that as the rotor is turned the voltage induced in each turn is the same at each instant. The maximum value of the generated voltage wave is 76.4 volts when the handle is turned at a speed of 110 revolutions per minute. The gear ratio is 1 to 5. Calculate the lines of force which the permanent magnets produce across the air gap and which link with the coil.

Solution.—Step 1. The frequency must be calculated. As was previously shown (page 244) when one complete revolution is made with a two-pole machine such as Fig. 161, one complete cycle is generated. For each turn of the handle, the rotor will turn five times; hence, the rotor will turn at the rate of $110 \times 5 = 550$ revolutions per minute, or $550/60 = 9.17$ cycles per second.

Step 2. Calculate the flux, using Eq. (47). Solving for the flux in this equation gives $\phi = 10^8 E_{\max}/\omega N$. Substituting the known values in this expression, $\phi = 10^8 \times 76.4/2 \times 3.1416 \times 9.17 \times 2500 = 53{,}000$ lines.

Current and Voltage Relations in Resistance Only.—When an alternating voltage is impressed on a circuit containing *resistance only*, the current at each instant is equal in magnitude to the instantaneous value of the voltage divided by the resistance. Therefore, the current rises and falls in exact accordance with the voltage producing it. These relations are shown in Fig. 164*b*. Since the current and voltage pass through corresponding values at exactly the same instant, they are *in phase* in circuits containing *resistances only*. This is shown by the vectors in Fig. 164*c*.

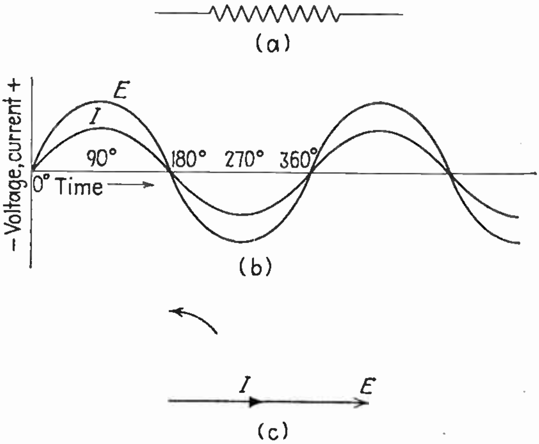

FIG. 164.—When a voltage is impressed across a circuit containing only resistance, the current which flows is in phase with the voltage as shown by the sine waves and the vectors.

Current and Voltage Relations in Inductance Only.—To explain the relation between current and voltage in a circuit containing inductance only, it is necessary to refer to the fundamental definition of inductance given on page 158. Inductance is defined as the *property* of a circuit (a coil, for instance) by virtue of which a back voltage is induced whenever the current in the circuit is changed. Now when a voltage is impressed on an inductive circuit, this *induced back voltage keeps the current from immediately rising to the final maximum value* as was explained on page 161 and shown by Fig. 107.

Assume that an alternating voltage is impressed on the inductive circuit of Fig. 165*a* and that this impressed voltage is rising from the zero to the maximum positive value. A current also attempts to rise because this impressed voltage tries to force a current through the inductance. This current change (rise) induces a back voltage, and this opposes the impressed voltage, allowing the current to rise but slowly. *The sine-wave current that flows when a sine-wave alternating voltage is impressed on a circuit containing inductance only lags 90 electrical degrees behind the voltage producing it.* This is easily verified experimentally or proved mathematically.

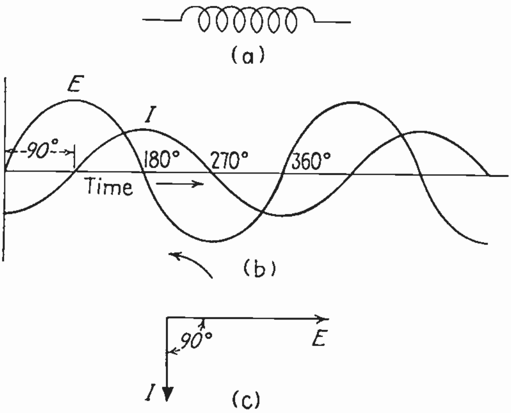

FIG. 165.—When an alternating voltage is impressed on a circuit containing inductance only, the current which flows lags the voltage by 90 degrees as shown by the sine waves and the vectors.

These phase relations are shown in Fig. 165*b*. As is indicated, the impressed voltage passes through its zero and maximum values (and all others as well) *ahead* of the current, which of course means that the current lags the voltage by 90 electrical degrees, there being 360 electrical degrees in 1 cycle. The angle θ is the angle of lag, it being 90 degrees as shown for a circuit containing inductance only. The corresponding vectors are shown in Fig. 165*c*. These are for *effective values* (page 113). Thus, whereas Fig. 165*b* illustrates that the instantaneous values lag by an angle θ of 90 degrees, Fig. 165*c* shows that the *effective values* lag by this same angle.

In these figures, the voltage is taken *as the base*, and the position of the current is expressed *with respect to it.* Thus, in Fig. 165*b* note that the voltage wave starts at zero, and in Fig. 165*c* note that the voltage vector starts at zero which is represented by the fact that it is drawn as a horizontal line extending to the right.

To summarize: In a circuit containing only inductance, the current lags the voltage by 90 degrees. This may be shown by sine waves or by vectors.

Current and Voltage Relations in Capacitance Only.—On page 191, capacitance was defined as the *property* of a condenser (or circuit) which allows a current to flow when the voltage across it is changed. When a steady voltage is impressed on a *perfect* condenser, an initial charging current flows, which finally reaches zero (page 198), and then nothing further happens. If the voltage is changed, however, then a current again flows because of the property of capacitance.

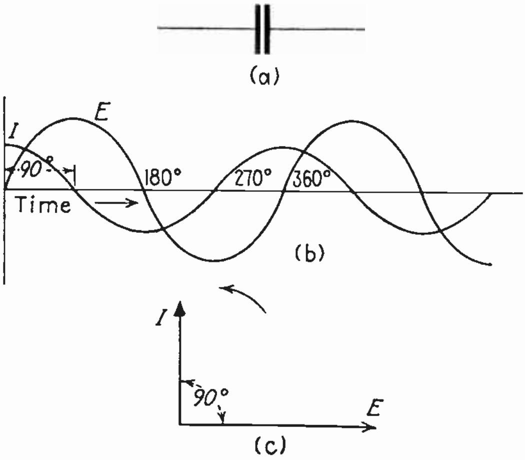

FIG. 166.—When an alternating voltage is impressed on a circuit containing only capacitance, the current which flows leads the voltage by 90 degrees, as the sine waves and vectors indicate.

Assume that an alternating voltage is impressed on the circuit of Fig. 166, consisting of a *perfect* condenser. This alternating voltage wave is shown in Fig. 166*b*. At the instant the impressed voltage wave is maximum, no current will flow through the condenser, because *at the instant the voltage wave is maximum, the voltage is not changing.* At the instant the impressed alternating voltage wave is passing through zero, it is actually changing at the maximum rate. In fact, it is not only changing in magnitude but is also changing in direction. Thus, *at the instant the impressed voltage is passing through zero, the current is maximum,* because the current through a condenser is caused by a voltage change, and the voltage change is maximum at this instant. Furthermore, *the current through a circuit containing only capacitance leads the voltage impressed across the circuit by* 90 *electrical degrees.* This statement is easily verified experimentally or proved mathematically.

These phase relations are shown in Figs. 166*b* and *c*. Note that in each of these figures the voltage is taken as the base and the current wave and the current vector are shown 90 degrees ahead of the voltage. Remember that a vector diagram is assumed to be rotating in a counterclockwise direction.

To summarize: In a circuit containing only capacitance, the current leads the voltage by 90 degrees. This can be shown by either sine waves or vectors.

Inductive Reactance.—According to the theory on page 159, the average voltage that will exist across a coil when the current through the coil is changing is $E_{\text{av}} = LI/t$, where L is the inductance in henrys and I/t is the average rate of current change in amperes per second. By simple algebra, this equation may also be written

$$I = \frac{E_{\text{av}}t}{L}. \qquad (49)$$

Let this equation now be interpreted in alternating-current terms. The current I of this equation was the *maximum value* (page 161), which the current reached, and becomes I_{max} of an alternating-current equation. In alternating-current circuits, the current desired would be the effective value, or $I = I_{\text{max}}/1.414$. Also, the time t in Eq. (49) required for the current to change from zero to maximum in the alternating-current instance becomes $t = 1/4f$ (see page 114). Also, $E_{\text{av}} \times 0.707/0.637$ or $1.11\ E_{\text{av}} = E$ (page 112). Making these substitutions, necessary to convert Eq. (49) to apply to alternating current and voltages,

$$I = \frac{E}{1.414 \times 1.11 \times 4f \times L} = \frac{E}{6.2832fL} = \frac{E}{2\pi fL} = \frac{E}{X_L}. \qquad (50)$$

This equation is interpreted as follows: When an alternating voltage of E volts effective value is impressed on a circuit containing *only* inductance, the effective value of the current I in amperes which flows will equal the voltage E in volts divided by the product $2\pi fL$ in ohms, where f is the frequency of the voltage in cycles per second and L is the inductance of the circuit in henrys. The quantity $2\pi fL$ is the **inductive reactance** of an inductive circuit and is represented by X_L and measured in **ohms.**

From the form of Eq. (50), it can be stated that *the inductive reactance of a circuit is the opposition the inductance of the circuit offers to the flow of current.* Because of this opposing effect, inductive reactance is measured in ohms. It should also be noted that the magnitude of the inductive reactance depends on two things, *frequency* and *inductance.* Furthermore, the current that does flow will lag the voltage by 90 degrees (page 248).

To summarize: A circuit containing inductance offers an inductive reactance of $X_L = 2\pi fL$ ohms to the flow of alternating current. This inductive reactance is an expression for the effect of the back voltage induced, according to Lenz's law, when the flux linkages are changed.

Calculations of Inductive Reactance. *Example* 1.—A coil has an inductance of 0.125 henry and negligible resistance. What current will the coil take when connected across 34.2 volts, 1000 cycles?

Solution.—Step 1. Calculate the inductive reactance.

$$X_L = 2\pi fL = 6.2832 \times 1000 \times 0.125 = 785 \text{ ohms.}$$

Step 2. Calculate the current flowing. Since the resistance is negligible, the reactance is the only opposition to the current flow. From Eq. (50),

$$I = \frac{E}{X_L} = \frac{34.2}{785} = 0.0436 \text{ ampere.}$$

Example 2.—A radio-frequency choke coil having an inductance of 5.5 millihenrys is to be used at a frequency of 1200 kilocycles. Calculate the inductive reactance.

$$X_L = 2\pi fL = 6.2832 \times 1{,}200{,}000 \times 0.0055 = 41{,}500 \text{ ohms.}$$

Capacitive Reactance.—The expression for capacitive reactance is found by the same general method as in the preceding section. Thus, on page 193 it is stated that $I_{\text{av}} = CE_{\text{max}}/t$, where E is the *maximum direct voltage* which changes in the time t. For the alternating-current equation, effective values are desired, because it is these values which are always used except in unusual instances. The effective current is $I = I_{av} \times 1.11$, the effective voltage is $E = E_{max}/1.414$ and $t = 1/4f$. Making these substitutions,

$$I = CE \times 1.11 \times 1.414 \times 4f = 2\pi fCE = \frac{E}{X_C}, \qquad (51)$$

where I is in amperes when E is in volts. The term $X_C = 1/2\pi fC$, and is the **capacitive reactance** of a condenser or circuit and is measured in **ohms,** when f is the frequency of the impressed voltage in cycles per second and C is the capacitance of the circuit in farads. The current as given by Eq. (51) will lead the voltage by 90 degrees.

Thus, in a circuit containing only capacitance, a certain amount of capacitive reactance equal to $X_C = 1/2\pi fC$ ohms exists. This reactance is the *reciprocal* of a term containing the frequency and the capacitance.[1] The capacitive reactance of a circuit *opposes* current flow.

To summarize: A circuit containing capacitance offers a capacitive reactance of $X_C = 1/2\pi fC$ ohms to the flow of alternating current. This capacitive reactance is the quantitative expression for the *inverse* of the property of a condenser in permitting current flow when the voltage is changed.

Calculations of Capacitive Reactance. *Example* 1.—A condenser has a capacitance of 0.5 microfarad and is connected across 34.2 volts at 1000 cycles. What current will flow?

Solution.—Step 1. Since no statement to the contrary is made, it is assumed that the condenser is perfect, consisting only of capacitance. Calculate the capacitive reactance.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{6.2832} \times 1000 \times 0.0000005 = \frac{1}{0.00314} = 318 \text{ ohms.}$$

Step 2. Calculate the current that will flow. From Eq. (51),

$$I = \frac{E}{X_C} = \frac{34.2}{318} = 0.1075 \text{ ampere.}$$

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[1] Past explanations have defined capacitance as the ability of a dielectric to conduct electric flux or as the property of permitting current flow when the voltage is changed. It should be noted that capacitive reactance is the inverse effect; it is the opposition offered by a condenser to alternating-current flow.

Example 2.—Calculate the reactance of a 15-micromicrofarad condenser at 1500 kilocycles.

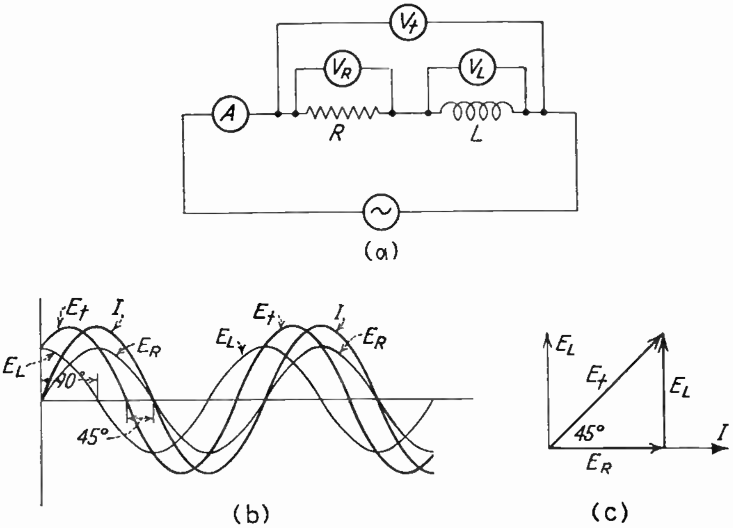
$$X_C = \frac{1}{2\pi fC} = \frac{1}{6.2832 \times 1{,}500{,}000 \times 0.000000000015}$$

$$= \frac{1}{6.2832 \times 1.5 \times 10^6 \times 1.5 \times 10^{-11}}$$

$$= \frac{1}{6.2832 \times 2.25 \times 10^{-5}} = \frac{1 \times 10^4}{1.414} = 0.707 \times 10^4$$

$$= 7070 \text{ ohms.}$$

Resistance and Inductance in Series.—In Fig. 167 is shown a circuit composed of resistance and inductance in series. An ammeter A measures the current that flows through the circuit. The voltmeter V_t measures the total voltage impressed on the circuit, and voltmeters V_R and V_L measure the voltage drops across the resistor R and the inductive coil L.



(a)

(b)

(c)

FIG. 167.—For drawing these figures the current is taken as the base because it is common to all elements of a series circuit. The voltage drop E_R across the resistor is in phase with the current, and the voltage drop E_L across the coil is 90 degrees ahead of the current. These two voltages add to give the total voltage E_t. In the vector diagram E_L is shown twice. This is because it has been moved over and added from the end of E_R to find E_t.

Now this is a simple series circuit and follows the fundamental laws of a series circuit, generalized, of course, for alternating-current applications. One fundamental law for the series circuit is that *the current in all parts of a series circuit is the same.* The other law (modified for the alternating-current circuit) is that *the vector sum of the voltage drops around a circuit must equal the impressed voltage.* Note the presence of the word *vector.*

If I represents the current in the circuit, there will be a voltage drop of $E_R = IR$ across the resistor R. This voltage drop will be measured by the voltmeter V_R and will be in phase with the current (page 247).

Equation (50), page 250, may be written in the form $E = IX_L$. Thus, when a current flows through a coil, there is a voltage drop across the coil equal to $E_L = IX_L$, and this voltage drop will be read by voltmeter V_L. From page 248, this voltage drop V_L will *lead* the current through the coil by 90 degrees.[1]

If it is assumed that the circuit is so adjusted that $E_R = E_L$, then the sine-wave relations will be as shown in Fig. 167*b*. Here is shown the current as the base (starting at zero). The voltage drop E_R across the resistor is shown *in phase* with the current, and the voltage drop E_L across the coil is shown 90 degrees ahead of the current. With reference to page 118, it follows that the total impressed sine-wave voltage existing across the voltmeter E_t will be the sum of the two sine waves E_R and E_L.

As was previously explained (page 121) it is usually more convenient to add vector values than to add sine-wave values. The vector value of the voltage E_t is the vector sum of the voltages E_R and E_L. These are shown in their proper phase relations in Fig. 167*c*, and they are there added to give E_t. Note that the voltage E_t makes an angle with the current of 45 degrees leading. This is only because E_R and E_L were assumed the same. Altering the values of R, L, or f would change this 45-degree relationship.

To summarize: When resistance and inductance are in series, the same current must flow through each. There is an $E_R = IR$ voltage drop across the resistance and an $E_L = IX_L$ voltage drop across the inductance. Because the voltage drop across a resistor is in phase with the current and the voltage drop across an inductance is 90 degrees ahead of the current, these two voltage drops must be added vectorially at right angles to give the total impressed voltage.

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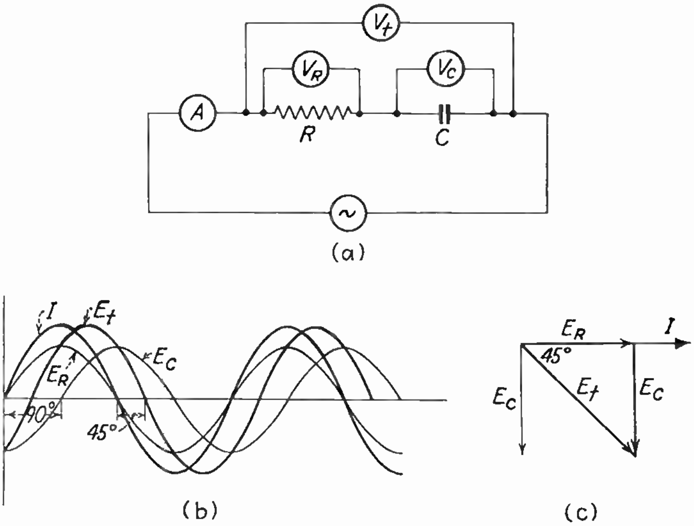
[1] Voltage drop as here used means that portion of the impressed voltage which is used in forcing alternating current through a resistor, coil, or condenser. Voltage drop as here used does not mean the opposing voltage, like the back voltage induced in a coil, for example.

Resistance and Capacitance in Series.—The circuit of Fig. 168*a* shows resistance and capacitance in series, with an ammeter to measure the current flow and voltmeters V_R, V_C, and V_t to measure each of the voltages. As in any series circuit, the current is the same in each part, and the vector sum of the voltage E_R across the resistor and the voltage E_C across the condenser equals the total impressed voltage E_t.



(a)

(b)

(c)

FIG. 168.—For drawing these figures the current is taken as the base because it is common to all elements of a series circuit. The voltage drop E_R across the resistor is in phase with the current, and the voltage drop E_C across the condenser is 90 degrees behind the current. These two vectors add to give the total voltage E_t. In the vector diagram E_C is shown twice. This is because it has been moved over and added to the end of E_R to find E_t.

Of course the voltage drop across the resistor will be $E_R = IR$. Likewise, the voltage drop across the condenser will be $E_C = IX_C$. The voltage drop across the resistor will be in phase with the current, and the voltage drop across the condenser will *lag* the current by 90 degrees, because the current leads the voltage or the voltage lags the current by 90 degrees in pure capacitance. The relations between these sine waves are shown in Fig. 168*b*.

The vector sum of the two voltages are obtained graphically as shown in Fig. 168*c*. For drawing the vector diagram, the current is taken as the base because it is common to both portions of the circuit. The voltage drop E_R across the resistor is then laid off along the current, because current and voltage are in phase in a resistor. The voltage drop E_C across the condenser is then laid off 90 degrees behind the current because in pure capacitance the voltage lags the current by 90 degrees. As is evident, the total impressed voltage E_t is the vector sum of these two individual voltages. It should be noted that the total voltage E_t makes an angle of 45 degrees *lagging* with the current. Again, this is only because E_R and E_C were the same. Altering the values of R, L, or f would change this 45-degree relationship.

To summarize: When resistance and capacitance are in series, the same current must flow through each. There is an $E_R = IR$ voltage drop across the resistance and an $E_C = IX_C$ voltage drop across the capacitance. Because the voltage drop across a resistor is in phase with the current and the voltage drop across a condenser is 90 degrees behind the current, these two voltage drops must be added vectorially at right angles to give the total impressed voltage.

Impedance.—Referring to Fig. 167*c*, the vector diagram for the voltages in a series circuit composed of resistance and inductance is a triangle composed of IR as a base, IX_L as the side at right angles, and E_t as the closing side, or hypotenuse, of the triangle. Likewise, in the vector diagram of Fig. 168*c*, IR is the base, IX_C the side at right angles (only in this case it points in the opposite direction), and E_t is the hypotenuse.

From geometry, the length of the hypotenuse is equal to the square root of the sum of the squares of the length of each of the sides. That is, for the E_R and E_X values,

$$E_t = \sqrt{E_R^2 + E_X^2} = \sqrt{(IR)^2 + (IX^2)} = I\sqrt{R^2 + X^2} = IZ, \quad (52)$$

where Z is the impedance of the circuit and $Z = \sqrt{R^2 + X^2}$. Impedance is measured in ohms, just as are resistance and reactance.

By referring again to the vector diagrams of Figs. 167 and 168, which for convenience have been reproduced in Fig. 169, it is seen that the current is common to each side and thus may be dropped, giving the impedance triangles R, X, Z. Note that R, X, and Z have *no arrowheads*. They are merely quantities and must be multiplied by current to become vectors.

Thus it is seen that impedance is composed of resistance R and reactance X combined at right angles. From Eq. (52) it follows that

$$E_t = IZ, \qquad I = \frac{E_t}{Z}, \qquad \text{and} \qquad Z = \frac{E_t}{I}. \tag{53}$$

These relations are interpreted as follows: In the series circuits of Figs. 167 and 168, both the resistance R and the reactance X are in series to oppose the current flow when the voltage E_t is impressed across the circuits. The current that does flow is given by the relation $I = E_t/Z$, where $Z = \sqrt{R^2 + X^2}$. *Impedance is the total opposition a circuit offers to the flow of alternating current.*

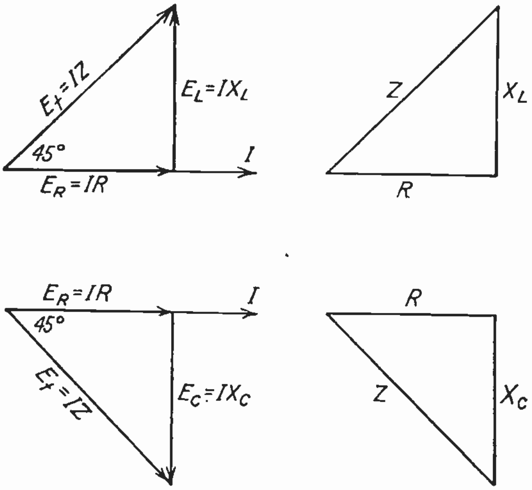


FIG. 169.—Here have been drawn the vector diagrams from Figs. 167 and 168. Dividing the voltage values by the currents gives the impedance diagrams at the right. The upper impedance diagram is for an inductive circuit, and the lower one for a capacitive circuit. It is not necessary to plot a capacitive impedance diagram downward as here done. Usually, it is drawn upward as is the inductive impedance diagram. Note that the arrowheads are omitted from the impedance diagram. Resistance, reactance, and impedance are *not* vectors revolving with respect to time as are current and voltage.

To summarize: In a series circuit the resistance and reactance add at right angles to give the impedance which is, therefore, the total opposition a circuit offers to alternating-current flow.

Effective Resistance.—The reader is encouraged to give special attention to this section because of its particular importance in communication practice.

When a direct current of I amperes flows into a circuit of R ohms, the power dissipated is I^2R watts, but if an alternating current of the *same effective value* is passed through the same circuit, the power dissipated will (generally) be greater. This increase in power dissipation with alternating current over the power dissipation with the same value of direct current can be explained only on the basis of an *increase in* the **effective resistance,** because alternating and direct currents are defined on the basis of the same heating effect (see page 112).

The following illustration should be helpful in explaining the meaning of the term effective resistance. Suppose that an iron-cored coil is connected through a wattmeter and an ammeter to a source of *direct* voltage of E volts as in Fig. 170. The ammeter will indicate a value of I_{dc} amperes and the wattmeter will indicate a value of P_{dc} watts. Since $P = I^2R$, $R = P/I^2$. Then, the resistance offered to the direct current is $R_{dc} = P_{dc}/I_{dc}^2$. This value of resistance is usually called the **direct-current resistance,** or the **ohmic resistance.** Now suppose that an *alternating* voltage is impressed on the coil; both the ammeter and wattmeter will read different values. The *resistance* offered to the alternating current flow will be $R_{ac} = P_{ac}/I_{ac}^2$. This value of resistance is usually called the **alternating-current resistance,** or the **effective resistance.** The alternating-current or effective resistance of any circuit (such as Fig. 170) *is greater* (if but only a slight amount) than the direct-current or ohmic resistance, because *the heating effect in any circuit is greater (if but slightly) with alternating than with direct current.*

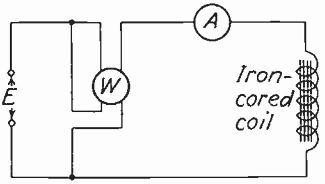


FIG. 170.—The effective resistance of a device such as the iron-cored coil shown equals the power measured by the wattmeter divided by the current squared. The lines through the coil represent the laminated iron core.

The increased heat loss with alternating values is largely due to the following causes: (1) Hysteresis losses (page 166) in the conductors (if of magnetic material such as iron wire) and in the magnetic core or surrounding magnetic materials cause the losses to be greater and, therefore, increase the effective resistance. (2) Eddy currents (page 168) in the current conductors, in the magnetic core, or in surrounding metallic objects (shields, for example) cause additional losses and thus increase the effective resistance. (3) Magnetic skin effect decreases the effective conducting area of the wire and increases the effective resistance. (4) Dielectric hysteresis losses (page 189) in dielectrics, insulation of wires, and in surrounding objects exposed to electric fields also increase the effective resistance. In alternating-current practice, when resistance is specified, it is, unless otherwise stated, the effective resistance.[1] This increases with frequency, and if the circuit has a magnetic core, the effective resistance will vary with the magnitude of the measuring current.

Skin Effect.—As mentioned, skin effect decreases the conducting area of a wire and this increases the effective resistance. The action is as follows: When a wire carries a current, the magnetic lines of force encircling the wire originate at the center of the wire, and fall back to the center when the current stops flowing. If the current and the field are alternating, the rising and falling magnetic field cuts the wire, and induces a back voltage in it. More lines cut the center portion than the outer layers, and a greater back voltage is induced near the center of the wire than near the surface. In effect, this increases the impedance at the center of the wire, and the current is crowded to the surface layers, thus decreasing the conducting area, and increasing the effective resistance. The higher the frequency, the greater is skin effect (see references page 52).

Alternating-current Power.—At any instant, the power in an alternating-current circuit is given by the expression

$$p = ei, \tag{54}$$

where p is the *instantaneous* power in watts, when e and i are the instantaneous voltage and current in volts and amperes. Instantaneous values of the power p are shown in Fig. 171 for pure resistance, pure inductance, and pure capacitance. Remember that if *either* e or i is negative p is negative, and that if *both* e and i are negative p is positive. (From algebra minus × minus = plus, and plus × minus = minus.)

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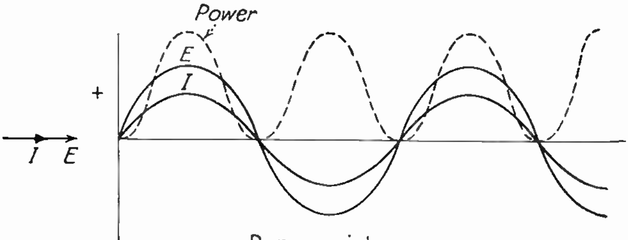
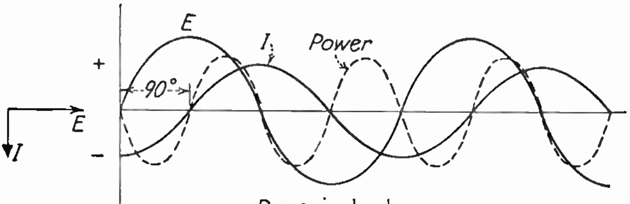
[1] The student is cautioned against confusing impedance and effective resistance. Impedance is the total opposition to current flow. It is given by the relation $Z = \sqrt{R^2 + X^2}$. The value of R in this equation is the *effective resistance.* Effective resistance is a part of impedance.

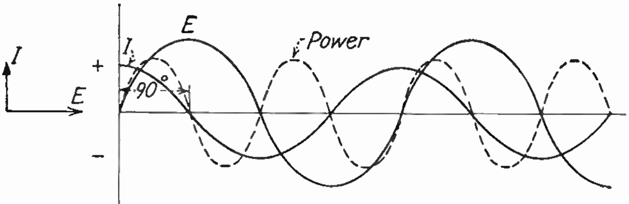
Note that for pure resistance the power lies always on the positive side of the axis. This means that power *always* flows into the circuit. For pure inductance, note that the positive and negative power pulses are of equal size. This means that during one-half of the cycle power is taken from the source and is stored in the magnetic field (page 162) and that during the other half cycle power is returned to the source by the collapsing of the magnetic field. The same reasoning applies to a circuit of pure capacitance; power is stored in the condenser during one half cycle and returned by the condenser during the next half cycle.



Pure resistance



Pure inductance



Pure capacitance

FIG. 171.—Showing the vector and instantaneous current, voltage, and power relations in circuits composed of pure resistance, inductance, and capacitance.

To summarize: A pure resistance takes power from the source during the entire cycle. For pure inductance or capacitance, however, power is taken from the source during one portion of the cycle and is stored in the magnetic or electric field. During another part of the cycle the power is returned to the source.

Apparent Power, Power Factor, and Reactive Power.—Suppose that a circuit contains some resistance and some inductance and that the current is, therefore, not lagging the voltage by 90 degrees but by some angle θ. These conditions are shown in Fig. 172; also, the power wave has been plotted. It will be noted that for a circuit of *resistance and inductance, where the phase angle is less than* 90 *degrees, a net power flows into the circuit as indicated by the fact that the negative portion of the power curve is much smaller than the positive portion.* Of course, the same reasoning would apply to resistance and capacitance.

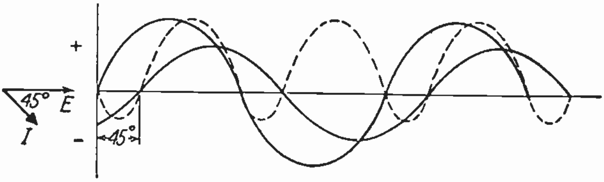


FIG. 172.—When the current is not exactly 90 degrees out of phase, then there is a net power input as shown by the fact that the + power is greater than the − power. In this figure the angle θ by which the current lags the voltage is 45 degrees.

It is apparent that the *angle* between the current and voltage determines the amount of power taken for given magnitudes of current and voltage. When these are in phase, it was seen that the *maximum amount of power* was taken; when they were 90 degrees out of phase, *no net power* was taken; and when they were out of phase by some angle of less than 90 degrees, *some net power* was taken. Equation (54) gave the instantaneous power. By using effective values as would be measured by electric instruments,

$$P = EI \cos \theta. \tag{55}$$

The E and I of Eq. (55) need little explanation other than that they are the effective voltage in volts and the effective current in amperes. The angle θ is the angle in degrees between the current and voltage and $\cos \theta$ is a value obtained from the table on page 523. For the meaning of the term $\cos \theta$, see page 288.

Here is encountered one of the interesting characteristics of alternating-current circuits. When a voltage E is impressed on a circuit and a current I flows into the circuit, it *appears* as if the value of the power taken should be $E \times I$, but this is the **apparent power** *and not the true power.* The true power, or net power actually dissipated in the circuit, is given by Eq. (55). True power is measured in watts, but apparent power is not watts but volts times amperes and is measured in **volt-amperes.**

Thus, the *apparent power* is EI, and the *true power* is $EI \cos \theta$. The ratio of the true power in a circuit to the apparent power is defined as the **power factor.** That is,

$$\text{Power factor} = \frac{EI \cos \theta}{EI} = \cos \theta. \tag{56}$$

The power factor of a circuit is numerically equal to the cosine of the angle between the current and voltage. The power factor of a circuit is an indication of whether or not the circuit is reactive, and to what extent.

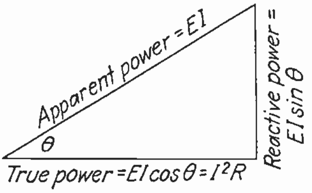


FIG. 173.—True power, reactive power, and apparent power are related as shown.

These values are all related as the sides of a triangle as shown in Fig. 173. The third side is known as the **reactive power** and is also measured in volt-amperes.

To summarize: When an alternating voltage is impressed on a circuit composed of resistance and inductance or resistance and capacitance, a net power flows into the circuit. This power can be calculated by Eq. (55). Apparently, the power flowing equals EI, and so this is called the apparent power. Actually, the true power is $EI \cos \theta$, where the value $\cos \theta$ is numerically equal to the power factor. This latter term is widely used in the power applications of electricity but is not so extensively used in communication.

Calculations of Resistance and Inductance in Series.—A circuit is arranged as in Fig. 174*a*. Make a complete analysis of the circuit, and plot impedance and vector diagrams to scale.

Solution.—Step 1. Calculate the inductive reactance.

$$X_L = 2\pi f L = 6.2832 \times 60 \times 0.17 = 64.1 \text{ ohms.}$$

Step 2. Calculate the impedance. From Eq. (52) this will be $Z = \sqrt{R^2 + X^2} = \sqrt{(50)^2 + (64.1)^2} = 81.2$ ohms. This should also be found graphically by laying off the 50 ohms as a base, the 64.2 ohms at right angles, and then measuring the length of the hypotenuse. This hypotenuse is the impedance, as shown in Fig. 174*b*.

Step 3. Calculate the current.

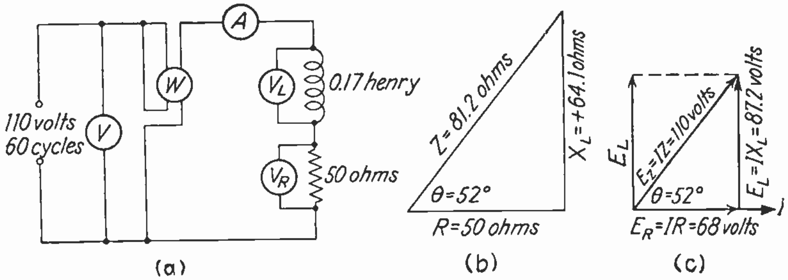
$$I = \frac{E}{Z} = \frac{110}{81.2} = 1.36 \text{ amperes.}$$

Step 4. Calculate the voltage drop across the coil and that across the resistor.

$$E_L = IX_L = 1.36 \times 64.1 = 87.2 \text{ volts.}$$

$$E_R = IR = 1.36 \times 50 = 68 \text{ volts.}$$

Step 5. Plot a vector diagram. This is done in Fig. 174*c*. The current is taken as the base, because it is common to both the coil and the resistor. The drop across the resistor is plotted along this base, because the voltage across a resistor and the current through a resistor are in phase. The drop across the coil is next plotted 90 degrees ahead of the current, because the voltage is 90 degrees ahead of the current for pure inductance. Next, the total line voltage is calculated by adding vector E_L to the head of vector E_R (page 121). The vector sum of these two is vector $E_Z = IZ$, which is the voltage drop across the total impedance of the circuit equal, of course, to the impressed voltage. The angle θ is the angle by which the voltage leads the current (or current lags the voltage as is usually stated for an inductive circuit). This angle can be measured with a protractor.



(a) (b) (c)

Fig. 174.—Impedance and vector diagrams for resistance and inductance in series. The voltmeters V_L and V_R measure the voltage drops $E_L = IX_L$ and $E_R = IR$. In the vector diagram E_L is moved over to the right to add it to the end of E_R.

Step 6. Calculate the apparent power, true power, and power factor. Apparent power $= EI = 110 \times 1.36 = 149.5$ volt-amperes. *The true power taken by any circuit always equals* I^2R *where* I *is the current in the circuit and* R *is the effective resistance of that circuit.* $P = (1.36)^2 \times 50 = 92.4$ watts. Power factor = true power/apparent power $= 92.4/149.5 = 0.618$. The wattmeter of Fig. 174*a* should indicate 92.4 watts.

Step 7. Find the angle between the current and voltage. From Eq. (56), the power factor as computed in Step 6 is numerically equal to the cosine of the angle. Referring to page 543, the corresponding angle will be approximately 51 degrees 50 minutes. This should agree with the angle found graphically in Step 5.

Calculations of Resistance and Capacitance in Series.—A condenser and a resistor are connected in series as indicated in Fig.

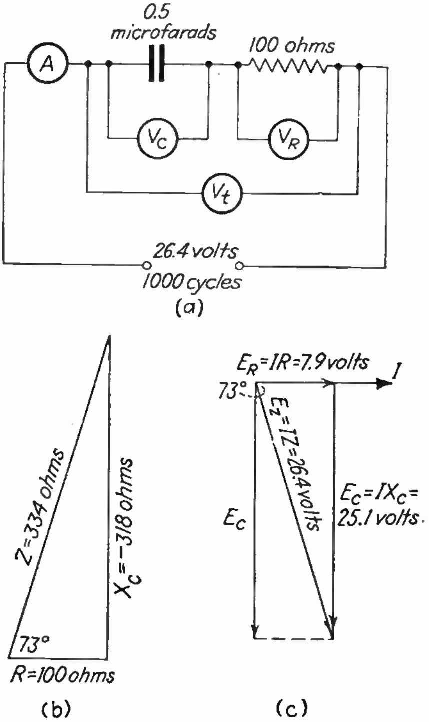


FIG. 175.—Impedance and vector diagrams for resistance and capacitance in series. The voltmeters V_C and V_R measure the voltage drops $E_C = IX_C$, and $E_R = IX_R$. In the vector diagram E_C is moved over to the right to add it to the end of E_R. $R = 100$ ohms; $C = 0.5$ microfarad.

175*a*. Make a complete analysis of the circuit, and plot impedance and vector diagrams to scale.

Solution.—Step 1. Calculate the reactance of the condenser.

$$X_c = \frac{1}{2\pi fC} = \frac{1}{6.2832 \times 1000 \times 0.0000005} = 318 \text{ ohms.}$$

Step 2. Calculate the impedance. $Z = \sqrt{R^2 + X^2} = \sqrt{(100)^2 + (318)^2} = 334$ ohms. The impedance diagram of Fig. 175*b* is constructed by laying off 100 ohms as the horizontal base and then 318 ohms vertically upward [1] from it. The hypotenuse is then drawn in, and its length is the

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[1] The reader may wonder why this is not laid off vertically downward instead of upward. It could but does not have to be, because reactance is not

impedance and should equal 334 ohms. The measured angle is 73 degrees.

Step 3. Calculate the current.

$$I = \frac{E}{Z} = \frac{26.4}{334} = 0.079 \text{ ampere.}$$

Step 4. Calculate the voltage drop across the condenser and across the resistor.

$$E_c = IX_c = 0.079 \times 318 = 25.1 \text{ volts.}$$

$$E_R = IR = 0.079 \times 100 = 7.9 \text{ volts.}$$

Step 5. Plot a vector diagram. The current is taken as the base because it is common to both the condenser and the resistor. The IR drop of 7.9 volts is laid off to scale along this base. The IX_C voltage drop of 25.1 volts is laid off 90 degrees behind the current, because the voltage lags the current in a condenser. This voltage is then added to the end of the voltage across the resistor to find the total line voltage E_t. This measures to be 26.4 volts, and the angle by which the line voltage lags the current (or the current leads the voltage) is 73 degrees as measured before.

Step 6. Calculate the apparent power, true power, and power factor. The apparent power $= EI = 26.4 \times 0.079 = 2.08$ volt-amperes. The true power *is always* I^2R, so $P = (0.079)^2 \times 100 = 0.624$ watt. The power factor is the ratio of these two values, or P.F. $= 0.624/2.08 = 0.30$, and this equals $\cos \theta$.

Step 7. Find the angle between the current and voltage. From the tables of cosine values (on page 532), the angle corresponding to a cosine of 0.30 is about 72 degrees and 33 minutes, or for all practical purposes 73 degrees as found graphically.

Calculations of Resistance and Inductance in Parallel.—A circuit composed of resistance and inductance in parallel is shown in Fig. 176*a*. A complete analysis of the circuit is desired. Note that the resistance of the coil is negligible.

Solution.—Step 1. Calculate the reactance of the coil.

$$X_L = 2\pi fL = 6.2832 \times 100{,}000 \times 0.002 = 1257 \text{ ohms.}$$

Step 2. Calculate the current taken by the coil.

$$I_L = \frac{E}{Z_L} = \frac{0.86}{1257} = 0.000685 \text{ ampere, or } 0.685 \text{ milliampere.}$$

Step 3. Calculate the current taken by the resistor.

$$I_R = \frac{E}{Z_R} = \frac{0.86}{750} = 0.00115 \text{ ampere, or } 1.15 \text{ milliamperes.}$$

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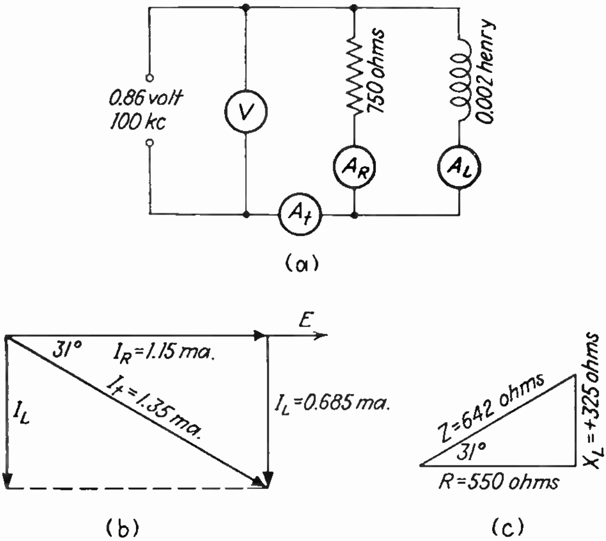
a vector value, although, as will be seen later (page 290), reactances are either positive or negative, and this fact would give grounds for plotting it downward. Ordinary practice is to plot impedance diagrams as has been done here

Step 4. Plot the vector diagram, and find the total current. This is done in Fig. 176*b*. The voltage is taken as the base, because it is common to both of the parallel units. The current through the resistor I_R is laid off along the voltage because it is in phase with it; and the current I_L through the inductance is placed 90 degrees behind the voltage, because the current lags the voltage by 90 degrees in pure inductance. Adding I_L downward from the end of I_R gives the total current of $I_t = 1.35$ milliamperes. A measurement with a protractor shows the angle θ by which the total current lags the total voltage to be about 31 degrees.



(a)

(b) (c)

FIG. 176.—Vector diagram for resistance and inductance in parallel. The diagram for the equivalent impedance is also shown. Ammeter A_L measures the current I_L through the coil, ammeter A_R measures the current I_R through the resistance, and ammeter A_t measures I_t, the total current.

Step 5. Calculate the total current. Since the current I_R taken by the resistor is 90 degrees out of phase (at a right angle) to the current I_L taken by the coil, they form two sides of a triangle, the third side of which represents the total current, then, $I_t = \sqrt{I_R^2 + I_L^2} = \sqrt{(0.000685)^2 + (0.00115)^2} = 0.00134$ ampere, or 1.34 milliamperes, which is in close agreement with the value found graphically.

Step 6. Calculate the power taken by the circuit. No power will be taken by the inductance, because it is considered to be a pure inductance and the current through it lags the voltage across it by 90 degrees. The power taken by the resistor is $P = I_R^2R = (0.00115)^2 \times 750 = 0.00000132 \times 750 = 0.00099$ watt, or 0.99 milliwatt.

Step 7. Calculate the equivalent impedance of the two units in parallel

$$Z_t = \frac{E}{I_t} = \frac{0.86}{0.00134} = 642 \text{ ohms.}$$

Step 8. Draw the triangle representing the impedance of a series circuit which is equivalent to the parallel circuit just considered. This equivalent impedance is 642 ohms, and it must be broken down into resistive and reactive components if the equivalent series circuit is to be found. The angle between the resistance and impedance of the impedance diagram will be the same as the angle between the voltage and total current of the vector diagram. Thus, it is possible to draw the impedance diagram of Fig. 176*c* as follows: First draw a horizontal line. From the left end of this line, draw a line 642 units long at the angle of 31 degrees found in Step 4. From the end of this line, draw a vertical line. This forms a right triangle, the base of which represents the resistance and the height of which represents the reactance of *a series circuit that is equivalent to the parallel circuit* of Fig. 176*a*.

Calculations of Resistance and Capacitance in Parallel.—A circuit composed of resistance and capacitance in parallel is shown in

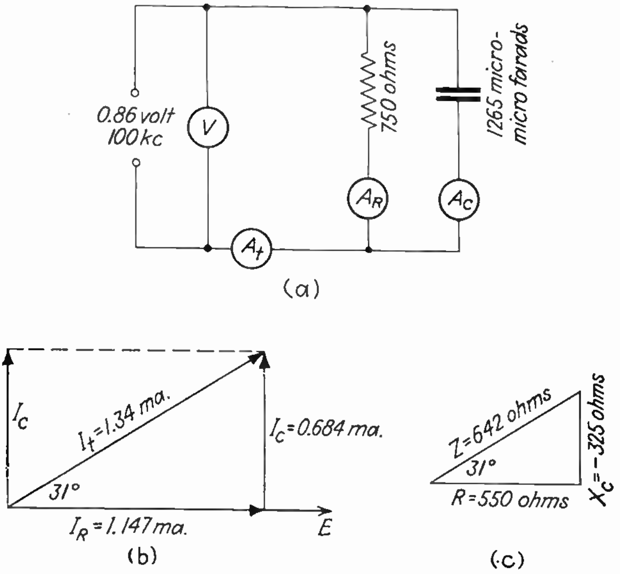


FIG. 177.—Vector diagram for resistance and capacitance in parallel. The diagram for the equivalent impedance is also shown. Ammeter A_C measures the current I_C through the condenser, ammeter A_R measures the current I_R through the resistance, and ammeter A_t measures I_t the total current. (Note: The values calculated for the circuits of Figs. 176 and 177 should be identical from a practical standpoint. The differences are largely due to sliderule approximations.)

Fig. 177*a*. A complete analysis of the circuit is desired. It will be seen that the general method of attack is the same as for the preceding section.

Solution.—Step 1. Calculate the reactance of the condenser.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{6.2832 \times 100{,}000 \times 1265 \times 10^{-12}} = 1259 \text{ ohms.}$$

Step 2. Calculate the current taken by the condenser.

$$I_C = \frac{E}{X_C} = \frac{0.86}{1257} = 0.000684 \text{ ampere, or } 0.684 \text{ milliampere.}$$

Step 3. Calculate the current taken by the resistor.

$$I_R = \frac{E}{R} = \frac{0.86}{750} = 0.001147 \text{ ampere, or } 1.147 \text{ milliamperes.}$$

Step 4. Plot the vector diagram, and find the total current. This is done as in Step 4 of the preceding section and as indicated in Fig. 177*b*. The current I_C through the condenser is of course placed 90 degrees ahead of the voltage. The total line current I_t is found to be 1.35 milliamperes, and the angle $\theta = 31$ degrees.

Step 5. The total current is the same as found in the preceding section.

Step 6. The power taken is the same as in the preceding section.

Step 7. The impedance is found as in the preceding section.

Step 8. The impedance diagram has the same numerical values and is found in the same way as in the preceding section. This diagram is reproduced in Fig. 177*c*.

Resistance, Inductance, and Capacitance in Series.—This combination is of great importance in communication, and the principles involved should be thoroughly studied. In the discussion that follows, it will be assumed that the units shown in Fig. 178*a* consist of *pure* resistance, inductance, and capacitance.

If the frequency of the source is f cycles per second, then the reactance of the coil will be $X_L = 2\pi fL$ and the reactance of the condenser will be $X_C = 1/2\pi fC$. The generator of terminal voltage E_g will force a current I through the circuit. This current will be *the same* in all parts of the series circuit.

This current will encounter opposition to its flow in each unit. There will, therefore, be a voltage drop across the resistance of $E_R = IR$ volts, a drop across the inductance of $E_L = IX_L$ volts, and a drop across the capacitance of $E_C = IX_C$ volts. For a series circuit the sum of the voltage drops must equal the total impressed voltage E_t.

The three separate voltages E_R, E_L, and E_C are shown in their correct positions in the vector diagram of Fig. 178*b*. The current I is taken as the base because it is common to each unit. The voltage drop $E_R = IR$ across the resistor is *in phase* with the current as shown. The voltage drop $E_L = IX_L$ is drawn 90 *degrees ahead* of the current, because this is the phase relation for pure inductance. Similarly, the voltage drop $E_C = IX_C$ is drawn 90 *degrees behind* the current. In this diagram, it is assumed that the capacitive reactance $X_C = 1/2\pi fC$ is greater than the inductive reactance $X_L = 2\pi fL$. *Since frequency enters oppositely into these two values, it is apparent that for a given value of inductance and capacitance X_C can be made equal to X_L merely by varying the frequency.*

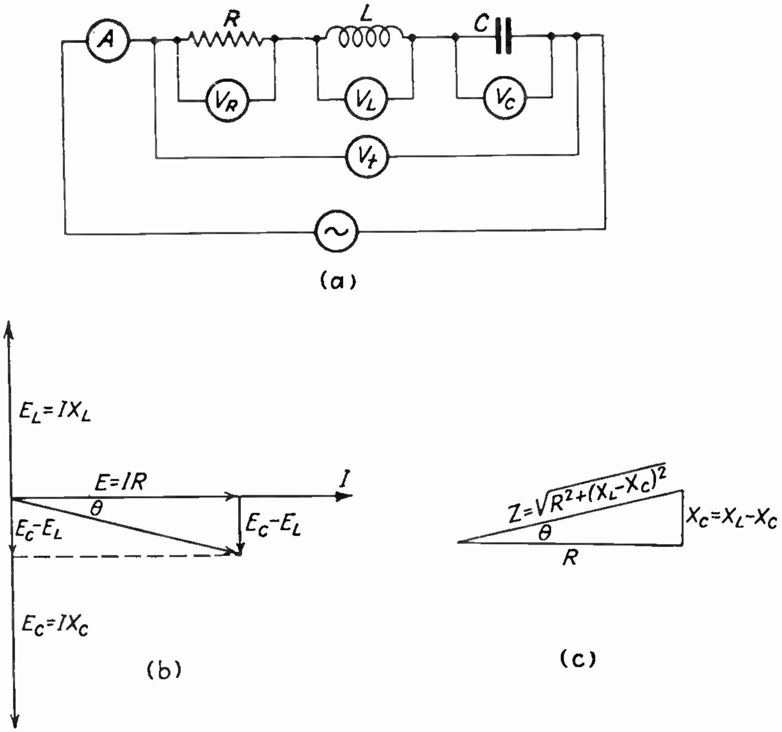


FIG. 178.—Circuit, vector diagram, and impedance diagram for resistance, inductance, and capacitance in series.

Referring to Fig. 178*b*, it is apparent that E_L and E_C are exactly opposed. Numerically, therefore, the voltage across the portion of the circuit composed of L and C would be $E_C - E_L$. This value is shown on Fig. 178*b*. The total voltage E_t will equal this voltage added vectorially to the voltage E_R as indicated. *It is seen that E_C and E_L act oppositely in series circuits. Now $E_C = IX_C$, and $E_L =$*

IX_L. Since I is the same in the condenser and the coil, it follows that in reality it is the reactances $X_C = 1/2\pi fC$ and $X_L = 2\pi fL$ that are different.

Again referring to Fig. 178*b*, it is seen that E_C is negative and E_L is positive because E_C lags I and E_L leads I as previously explained on pages 248 and 250. From the reasoning of the preceding paragraph it follows that it must be X_C which is *negative* and X_L which is *positive*, and such is the case. *From this it follows that the equivalent reactance of two or more reactances in series is the algebraic sum of the separate reactances.* Thus for the series circuit of Figs. 178*a*, $X_e = X_L - X_C$, and the equivalent impedance diagram for the series circuit of Fig. 178*a* is as shown in Fig. 178*c*.

As mentioned previously in this section, the capacitive reactance X_C can be made equal to the inductive reactance X_L merely *by varying the frequency.* If this is true, then $X_e = X_L - X_C$ would equal zero *at that frequency.* Thus, in a circuit composed of resistance, inductance, and capacitance in series, *there is one frequency at which the equivalent reactance is zero and at which frequency the resistance is the only opposition left to the current flow.* This frequency is the one at which the inductive and capacitive reactances are equal, that is, at which $2\pi fL = 1/2\pi fC$. Solving this equation for frequency gives

$$f_r = \frac{1}{2\pi\sqrt{LC}}. \tag{57}$$

When L is the inductance of the circuit in *henrys* and C is the capacitance in *farads*, f_r is the resonant frequency in cycles per second.

To summarize: When a circuit consists of resistance, inductance, and capacitance in series, the impedance of the circuit is found by constructing a right triangle with the resistance as the base and with the difference between the reactances $(X_L - X_C)$ as the vertical part of the right triangle. The impedance is the length of the line required for the hypotenuse of the triangle. The current is then equal to the applied voltage divided by this impedance. For one frequency, $X_L = X_C$ and $IX_L = IX_C$. This is known as the resonant frequency of the circuit.

Calculations of *R*, *L*, and *C* in Series. *Example* 1.—Solve the series circuit of Fig. 179 for the equivalent impedance, the current, and the voltage across each unit if the frequency is 9000 cycles.

Solution.—Step 1. Calculate the equivalent reactance. $X_e = X_L - X_C = 2\pi fL - 1/2\pi fC = (6.2832 \times 9000 \times 0.05) - [1/(6.2832 \times 9000 \times 0.005 \times 10^{-6})] = 2827 - 3536 = -709$ ohms. The negative sign indicates that *capacitive* reactance predominates and that the frequency is *off resonance.*

Step 2. Calculate the impedance.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(100)^2 + (709)^2} = 714 \text{ ohms.}$$

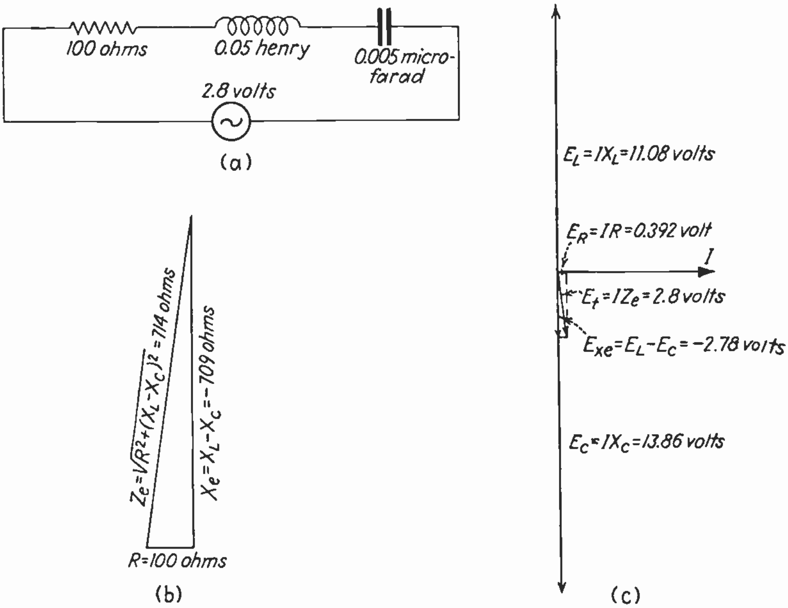


FIG. 179.—Diagrams for analyzing a series circuit at a frequency near resonance.

Step 3. Calculate the current. $I = E/Z = 2.8/714 = 0.00392$ ampere, or 3.92 milliamperes.

Step 4. Calculate the voltage drops. $E_R = IR = 100 \times 0.00392 = 0.392$ volt. $E_L = IX_L = 0.00392 \times 2827 = 11.08$ volts. $E_C = IX_C = 0.00392 \times 3536 = 13.86$ volts. Note that these voltages *do not add numerically to equal the total impressed line voltage, but they do add vectorially to equal the total impressed line voltage as the reader can readily demonstrate.*

Example 2.—Solve the circuit of Fig. 179 at a frequency of 11,000 cycles per second.

Solution.—Step 1. Calculate the equivalent reactance. $X_e = X_L - X_C = 2\pi fL - 1/2\pi fC = (6.2832 \times 11{,}000 \times 0.05) - [1/(6.2832 \times 11{,}000 \times 0.005 \times 10^{-6})] = 3460 - 2900 = 560$ ohms. Note that this value is positive indicating the *inductive* reactance predominates, and that the frequency is again *off resonance.*

Step 2. Calculate the impedance.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(100)^2 + (560)^2} = 570 \text{ ohms.}$$

Step 3. Calculate the current.

$$I = \frac{E}{Z} = \frac{2.8}{750} = 0.00492 \text{ ampere, or } 4.92 \text{ milliamperes.}$$

Step 4. Calculate the voltage drops.

$$E_R = IR = 100 \times 0.00492 = 0.492 \text{ volt.}$$

$$E_L = IX_L = 0.00492 \times 3460 = 17.0 \text{ volts.}$$

$$E_C = IX_C = 0.00492 \times 2900 = 14.24 \text{ volts.}$$

Note that these voltages *do not add numerically to equal the total impressed line voltage, but they do add vectorially to equal the total impressed line voltage* as the reader also can readily demonstrate.

Example 3.—Solve the circuit of Fig. 179 at the frequency for which the circuit will be in resonance.

Solution.—Step 1. Calculate the resonant frequency by Eq. (57).

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{6.2832\sqrt{0.05 \times 0.005 \times 10^{-6}}} = \frac{1}{0.0000994}$$

$$= 10{,}060 \text{ cycles.}$$

Step 2. Calculate the equivalent reactance. $X_e = X_L - X_C = 2\pi fL - 1/2\pi fC = (6.2832 \times 10{,}060 \times 0.05) - [1/(6.2832 \times 10{,}060 \times 0.005 \times 10^{-6})] = 3160 - 3160 = 0$ ohm (approximately).

Step 3. Calculate the impedance. Since the circuit is in resonance and the resultant reactance is zero, the impedance becomes the resistance of 100 ohms in the circuit. This is all that is effective in holding back the current flow.

Step 4. Calculate the current.

$$I = \frac{E}{Z} = \frac{2.8}{100} = 0.028 \text{ ampere, or } 28 \text{ milliamperes.}$$

Step 5. Calculate the voltage drops.

$$E_R = IR = 0.028 \times 100 = 2.8 \text{ volts.}$$

$$E_L = IX_L = 0.028 \times 3160 = 88.5 \text{ volts.}$$

$$E_C = IX_C = 0.028 \times 3160 = 88.5 \text{ volts.}$$

Note that since the circuit is in resonance, the inductive and capacitive reactances neutralize eath other. The resistance only is effective in controlling the current flow. Also note that *with only* 2.8 *volts applied* 88.5 *volts exist across the coil and across the condenser.* If R is reduced, this voltage drop across the coil and condenser *will be increased to even a greater value. This effect is very important.*

Practical Example of Coil and Condenser in Series.—In the problems just considered it was assumed that the coil was without resistance and that all the loss in the circuit was represented by the resistor. Actually, in the practical circuit there would be some loss in the coil, and hence all the resistance in the circuit would not be concentrated in the single resistor. The loss in a good paper or mica condenser is ordinarily negligible except at very high frequencies and in special circuits. In the following example an actual circuit such as might be encountered in practice is considered.

Example.—An air-cored coil has a resistance of 48.2 ohms and an inductance of 0.056 henry. It is in *series* with a condenser of 0.58 microfarad capacitance and negligible effective resistance as shown in Fig. 180. The source will impress a voltage of 12.8 volts at 1000 cycles on the circuit. Make an analysis of the circuit.

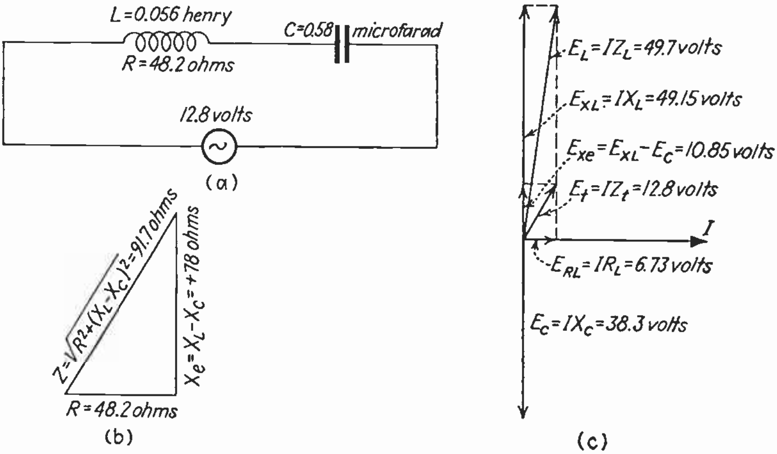


FIG. 180.—Observe that this time in adding the various vectors, they were not moved over to the end of the other vector to which they were to be added. Actually drawing a vector the second time sometimes complicates a drawing, and so it is common practice to just "dot in" the values as here shown. This was also done in Fig. 179.

Solution.—Step 1. Calculate the equivalent reactance. $X_e = X_L - X_C = 2\pi fL - 1/2\pi fC = (6.2832 \times 1000 \times 0.056) - [1/(6.2832 \times 1000 \times 0.58 \times 10^{-6})] = 352 - 274 = 78$ ohms inductive reactance.

Step 2. Calculate the total circuit impedance. This will be composed of the resistance of the coil and the equivalent reactance.

$$Z_t = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(48.2)^2 + (78)^2} = 91.7 \text{ ohms.}$$

Step 3. Calculate the current.

$$I = \frac{E}{Z_t} = \frac{12.8}{91.7} = 0.1396 \text{ ampere, or } 139.6 \text{ milliamperes.}$$

Step 4. Calculate the drop in voltage across the condenser and across the coil. The drop in voltage across the condenser will be $E_C = IX_C = 0.1396 \times 274 = 38.3$ volts. The drop across the coil will be the vector sum of the voltage drop E_{RL} (L being used to represent the inductance coil) caused by the resistance of the coil and the voltage drop caused by the reactance of the coil. The first will be $E_{RL} = IR_L = 0.1396 \times 48.2 = 6.73$ volts, and the second will be $E_{XL} = IX_L = 0.1396 \times 352 = 49.15$ volts. The impedance *of the coil* is $Z_L = \sqrt{R_L^2 + X_L^2} = \sqrt{(48.2)^2 + (352)^2} = 356$ ohms. The total voltage drop across the coil will be $E_L = IZ_L = 0.1396 \times 356 = 49.7$ volts. The vector diagram is shown in Fig. 180*c* and was drawn as follows: This is a series circuit, so I is taken as a base, it being common. The value of $E_{RL} = 6.73$ volts is drawn along this base. The value of E_{XL} is drawn at right angles leading. The vector sum of E_{RL} and $E_{XL} = E_{ZL}$, the IZ_L drop across the coil. This line is then drawn. The voltage across the condenser $E_C = IX_C$ is next drawn lagging the current by 90 degrees. This voltage E_C is then subtracted from E_{XL} the reactive voltage drop across the coil. The total line voltage will be the vector sum of this *resultant* reactive voltage drop and the resistive voltage drop. This is represented by $E_t = IZ_t$. This equals the impressed voltage of 12.8 volts.

Resistance, Inductance, and Capacitance in Parallel.—The preceding pages were devoted to resistance, inductance, and capacitance *in series*. This circuit combination is of great importance in communication. Likewise, circuits of resistance, inductance, and capacitance *in parallel* are also of great importance, and such a circuit will now be considered.

The circuit shown in Fig. 181*a* is assumed to be composed of *pure* resistance, inductance, and capacitance in parallel. As for any parallel circuit, *the impressed voltage is common to each branch, and the total current I_t as read by the ammeter A_t is the vector sum of the branch currents.*

The current taken by the resistor is $I_R = E/R$. The current taken by the coil is $I_L = E/X_L = E/2\pi fL$, and the current taken by the condenser is $I_C = E/X_C = E/(1/2\pi fC) = E2\pi fC$. Their reactances are considered for the coil and condenser only, because they are assumed to be pure and their effective resistances are, therefore, negligible.

In drawing the vector diagram of Fig. 181*b*, the voltage E is taken as the base because it is common to each branch. The current I_R through the resistor is plotted in phase with the voltage. The current I_C is plotted 90 degrees ahead of the voltage, and the current I_L through the coil is plotted 90 degrees behind the voltage, because these are their proper phase relations (pages 248 and 249).

Since I_L and I_C are 180 degrees out of phase and are therefore directly opposing, they are combined to find the resultant reactive current. The numerical value of this reactive current will be $I_C - I_L$. In the circuit shown, it is assumed that I_L exceeds I_C and that the net reactive current *lags* the voltage. The total current I_t is then the vector sum of the net reactive current $I_C - I_L$ and the resistive current I_R.

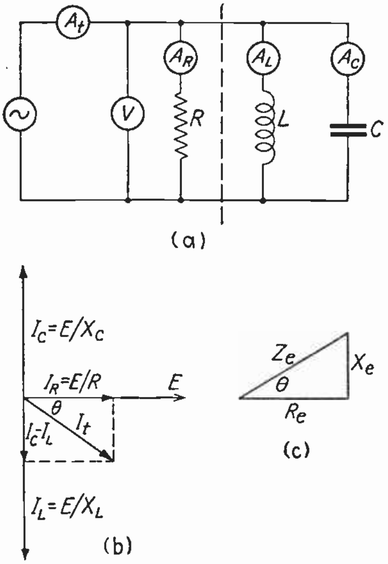


FIG. 181.—Illustrating the steps in the solution of a parallel circuit. The parallel circuit takes a resultant or total current I_t when a voltage E is impressed across it. Thus, the parallel circuit has an equivalent series circuit represented by (c).

The source of voltage forces I_t into the circuit. This current lags the voltage by an angle θ. This combined parallel circuit R, L, and C is, therefore, equivalent to some resistor and coil in series, because *the proper series circuit can be made to draw the same lagging current as the parallel circuit.* Thus, the equivalent impedance of the parallel circuit is $Z_e = E/I_t$. The equivalent resistance and reactance composing this equivalent impedance are found as follows: As in Fig. 181*c*, draw a horizontal line. Lay off the equivalent impedance Z_e *to scale*, making the angle θ (by which the current lags the voltage) with the horizontal. From the end of Z_e, drop a vertical line. Then, the right triangle is complete, and the values of the equivalent resistance and reactance can be measured. From the value of reactance, the inductance of the coil ($L = X_L/2\pi f$) can be found if desired.

For a parallel circuit such as Fig. 181*a*, varying the frequency or varying the inductance or capacitance will change conditions such that I_C can be made equal to I_L. *If this is done, then the net, or re-*

sultant, current taken by L and C (only) in parallel is zero. To the reader, this may seem to be an absurd statement; nevertheless, with a perfect coil and a perfect condenser, the statement is true. Of course, in practice, a coil with no resistance is a practical impossibility. Thus, the statement is true in theory only, but a circuit can easily be constructed in which I_L and I_C are very large and yet their total current is extremely small.

In a parallel circuit, when I_L and I_C are 180 degrees out of phase and equal in magnitude, *the portion of the circuit to the right of the broken line* of Fig. 181*a* may as well be disconnected, since it draws no current and its input impedance is therefore infinite. Under these conditions, the only current that flows from the source of voltage is the current taken by the resistor. This current will be in phase with the voltage, and the circuit is in a condition of antiresonance.[1]

To summarize: In a parallel circuit the voltage is the same across each branch. The total current is the vector sum of the currents in each branch. When perfect coils and condensers are in parallel, the currents they take at some frequency are equal in magnitude and 180 degrees out of phase. At this frequency the reactive currents cancel and the total current taken is in phase with the voltage.

Calculations of *R*, *L*, and *C* in Parallel. *Example* 1.—Make a complete analysis of the circuit of Fig. 182 at a frequency of 9000 cycles.

Solution.—Step 1. Calculate the current taken by each branch.

$$I_R = \frac{E}{R} = \frac{2.8}{1000} = 0.0028 \text{ ampere, or 2.8 milliamperes.}$$

$$I_L = \frac{E}{X_L} = \frac{2.8}{6.2832 \times 9000 \times 0.05} = \frac{2.8}{2830}$$

$$= 0.00099 \text{ ampere, or 0.99 milliampere.}$$

$$I_C = \frac{E}{X_C} = 2.8 \times 6.2832 \times 9000 \times 0.005 \times 10^{-6}$$

$$= 0.000792 \text{ ampere, or 0.792 milliampere.}$$

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[1] This subject will be treated further on page 315. The terms resonance, parallel resonance, and current resonance are also used to describe this condition. The term antiresonance as here used is popular in communication, especially in telephony. When the resistance of the coil is negligible, Eq. (57) gives the antiresonant frequency. In many circuits this equation is satisfactory but it is not strictly correct as will be shown on page 317.

Step 2. Calculate the total current. The net reactive or out-of-phase current will be $I_C - I_L = 0.000792 - 0.00099 = -0.000198$ ampere. The negative sign shows that the net current is lagging. The total current will be $I_t = \sqrt{(0.000198)^2 + (0.0028)^2} = 0.00281$ ampere, or 2.81 milliamperes (approximately). The frequency is slightly below antiresonance.

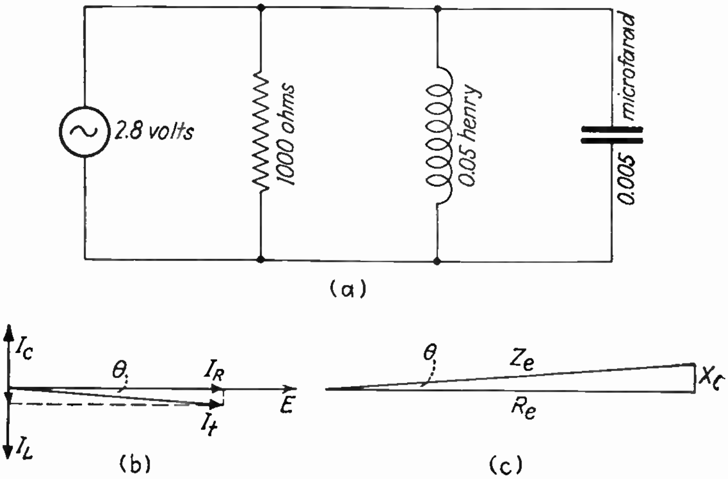


FIG. 182.—Parallel circuit almost in resonance and vector and impedance diagrams to scale. Numerical values given in text.

Step 3. Calculate the equivalent impedance. $Z_e = E/I_t = 2.8/0.00281 =$ 997 ohms (approximately).

Example 2.—Solve the circuit of Fig. 182 at a frequency of 11,000 cycles.

Solution.—Step 1. Calculate the current taken by each branch.

$$I_R = \frac{E}{R} = \frac{2.8}{1000} = 0.0028 \text{ ampere, or 2.8 milliamperes.}$$

$$I_L = \frac{E}{X_L} = \frac{2.8}{6.2832 \times 11{,}000 \times 0.05} = \frac{2.8}{3455}$$

$$= 0.00081 \text{ ampere, or 0.81 milliampere.}$$

$$I_C = \frac{E}{X_C} = 2.8 \times 6.2832 \times 11{,}000 \times 0.005 \times 10^{-6}$$

$$= 0.000965 \text{ ampere, or 0.965 milliampere.}$$

Step 2. Calculate the total current. The net reactive or out-of-phase current will be $I_C - I_L = 0.000965 - 0.00081 = 0.000155$ ampere, or 0.155 milliampere. This net current will lead the voltage, because the current through the condenser is greater. The total current will be $I_t =$

$\sqrt{(0.000155)^2 + (0.0028)^2} = 0.00281$ ampere, or 2.81 milliamperes (approximately). The frequency is slightly *above* antiresonance.

Step 3. Calculate the equivalent impedance. $Z_e = E/I_t = 2.8/0.00281 = 997$ ohms (approximately).

Example 3.—Solve the circuit of Fig. 182 at the frequency at which the circuit is antiresonant.

Solution.—Step 1. Since it is assumed that neither the coil nor the condenser contains resistance, the frequency of antiresonance will be as given by Eq. (57). Thus, $f_r = 1/(2\pi\sqrt{LC}) = 1/(6.2832\sqrt{0.05 \times 0.005 \times 10^{-6}}) = 10{,}060$ cycles.

Step 2. Calculate the current taken by each branch.

$$I_R = \frac{E}{R} = \frac{2.8}{1000} = 0.0028 \text{ ampere, or 2.8 milliamperes.}$$

$$I_L = \frac{E}{X_L} = \frac{2.8}{6.2832 \times 10{,}060 \times 0.05} = \frac{2.8}{3160}$$
$$= 0.000886 \text{ ampere, or 0.886 milliampere.}$$

$$I_C = \frac{E}{X_C} = 2.8 \times 6.2832 \times 10{,}060 \times 0.005 \times 10^{-6}$$
$$= 0.000886 \text{ ampere, or 0.886 milliampere (approximately).}$$

Step 3. Calculate the total current. The net reactive or out-of-phase current will be $I_L - I_C = 0.000886 - 0.000886 = 0$ ampere. The total current taken from the source by the parallel circuit will be that taken by the resistor, or 0.0028 ampere.

Step 4. Calculate the equivalent impedance. The equivalent impedance of a circuit is $Z_e = E/I_t = 2.8/0.0028 = 1000$ ohms, which is the resistance of the parallel resistor.

Practical Problem of Coil and Condenser in Parallel.—In the problems just given it was assumed that the coil was without resistance. All coils have resistance, of course, and it must often be considered. Except under special conditions, the loss in a good paper or mica condenser is negligible. Such a circuit will now be considered.

Example.—An air-cored coil has a resistance of 48.2 ohms and an inductance of 0.056 henry. It is in *parallel* with a condenser having a capacitance of 0.58 microfarad, as indicated in Fig. 183*a*. The source will impress a voltage of 12.8 volts at 1000 cycles on the circuit. Make an analysis of the circuit.

Solution.—Step 1. Calculate the reactance of the condenser. $X_C = 1/2\pi fC = 1/(6.2832 \times 1000 \times 0.58 \times 10^{-6}) = 274$ ohms.

Step 2. Calculate the impedance of the coil. $X_L = 2\pi fL = 6.2832 \times 1000 \times 0.056 = 352$ ohms. The impedance will be $Z_L = \sqrt{R_L^2 + X_L^2} = \sqrt{(48.2)^2 + (352)^2} = 355$ ohms.

Step 3. Calculate the current in each branch. $I_C = E/X_C = 12.8/274 = 0.0467$ ampere. $I_L = E/Z_L = 12.8/355 = 0.036$ ampere.

Step 4. Calculate the total line current. This will be done vectorially as in Fig. 183*b*. Take the voltage as the base, because this is a parallel circuit and the voltage is common to all branches. Next, plot the current I_C to scale 90 degrees ahead of the voltage. The current I_L should next be plotted. It will lag the voltage E, but the angle is not known. It can be found from the impedance triangle, however, because these impedance relations determine the current relations. By plotting the

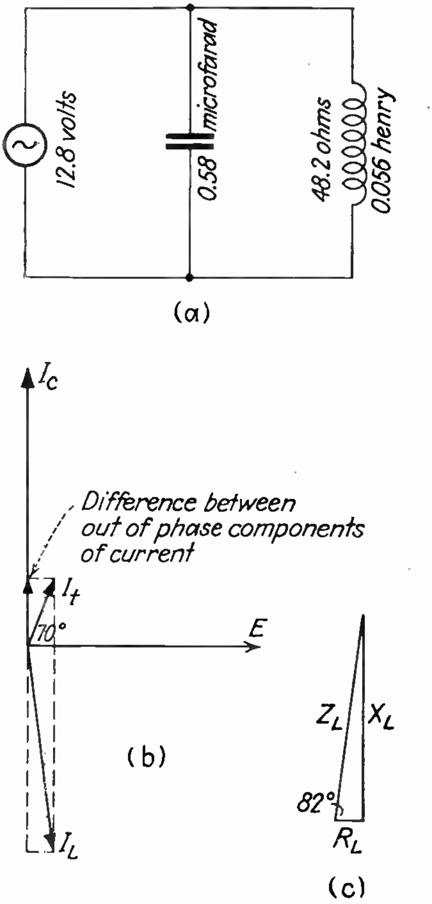


FIG. 183.—Illustrating the method of finding the current taken by a condenser and a coil having resistance, when these two are connected in parallel. The impedance diagram is for the coil. The equivalent impedance of the parallel circuit can be drawn as explained in previous illustrations.

impedance triangle for this problem as in Fig. 183*c*, it is found that the angle between Z_L and R_L is 82 degrees. Plotting the value of $I_L = 0.036$ ampere 82 degrees behind the voltage E locates the current through the coil on the vector diagram of Fig. 183*b*. The total current can be found by adding I_C and I_L vectorially. This gives $I_t = 0.012$ ampere, and with a protractor it is found to *lead* the voltage by about 70 degrees.

Step 5. Calculate the equivalent impedance of the coil and condenser in parallel. $Z_e = E/I_t = 12.8/0.012 = 1065$ ohms. If it is desired to calculate the resistive and reactive components of this impedance, it can be done by drawing an impedance diagram to scale.

SUMMARY

A sine-wave alternating voltage is induced in a coil of wire when it is rotated in a magnetic field. The equation for the *instantaneous voltage* is $e = E_{\max} \sin \omega t$ volts.

The *average voltage* induced in a coil of wire revolving in a magnetic field is given by the equation $E_{\text{av}} = N\phi/10^8 t$ volts. The *maximum voltage* generated is $E_{\max} = 2\pi f N\phi/10^8$ volts.

In pure resistance, the current through the resistor is exactly in phase with the voltage.

In pure inductance, the current through the coil lags the voltage by 90 degrees; or conversely, the voltage leads the current by 90 degrees.

In pure capacitance, the current through the condenser leads the voltage by 90 degrees; or conversely, the voltage lags the current by 90 degrees.

Inductance is the property of a circuit which causes a back voltage to be induced when the current is changed. Inductance, accordingly, opposes the flow of alternating current which is continually changing. This opposition is due to the inductive reactance and equals $X_L = 2\pi f L$ ohms.

Capacitance is the property of a circuit which permits a current to flow when the voltage is changed. The reciprocal of this "permitting" effect is an *opposing* effect called capacitive reactance. Numerically, this equals $X_L = 1/2\pi f C$ ohms.

In an alternating-current circuit containing only inductance, the current equals the voltage divided by the inductive reactance, or $I = E/X_L$.

In an alternating-current circuit containing only capacitance, the current equals the voltage divided by the capacitive reactance, or $I = E/X_C$.

When a circuit contains only resistance and inductance in series, the total opposition offered to the current is the impedance $Z_t = \sqrt{R^2 + X_L^2}$. The current flowing is $I = E/Z_t$. There will be a voltage drop $E_R = IR$ across the resistor and a voltage drop $E_L = IX_L$ across the coil. The total voltage will be the vector sum of these, because E_R is in phase with the current and E_L is 90 degrees ahead of the current; then, $E_t = \sqrt{E_R^2 + E_X^2}$.

When a circuit contains only resistance and capacitance in series, the statements of the preceding paragraph hold, it being remembered, of course, that the voltage across a coil leads the current by 90 degrees but the voltage across a perfect condenser lags the current by 90 degrees.

Impedance is the total opposition a circuit offers to the flow of alternating current. Impedance Z is composed of resistance R and reactance X. Numerically, $Z = \sqrt{R^2 + X^2}$.

The effective resistance of an alternating-current circuit is a fictitious value of resistance which represents all the power losses in the circuit. Effective resistance in ohms equals the power loss in watts divided by the current in amperes squared; that is, $R_{ac} = P_{ac}/I_{ac}^2$.

In alternating-current circuits, the power is not equal to the product of current and voltage but is equal to the product of the current and voltage multiplied by the cosine of the angle between them; that is, $P = EI \cos \theta$.

The value $\cos \theta$ is numerically equal to the power factor of the circuit. The power factor is defined as the ratio of the true power P to the EI product, or apparent power. P.F. $= (EI \cos \theta)/EI = \cos \theta$.

When a circuit contains only resistance and inductance in parallel, the voltage is the same across each branch, $I_R = E/R$ and $I_X = E/X_L$. The total current is the vector sum of these branch currents, and since the current in the resistor is in phase with the voltage and the current in the coil lags the voltage by 90 degrees, this total current is $I_t = \sqrt{I_R^2 + I_X^2}$. The equivalent impedance of the circuit is $Z_e = E/I_t$. This equivalent impedance can be separated into two components of R_e and X_e by graphic methods.

When a circuit contains only resistance and capacitance in parallel, the statements of the preceding paragraph hold, it being remembered, of course, that the current through a coil lags the voltage by 90 degrees but the current through a condenser leads the voltage by 90 degrees.

In a circuit composed of pure resistance, inductance, and capacitance in series, the current is common to all parts. This current equals the voltage divided by the impedance of the combination. This impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$. There is an IR drop across the resistor, an IX_L drop across the inductor, and an IX_C drop across the condenser. The total impressed voltage is the vector sum of these individual voltages. If the inductive reactance $X_L = 2\pi fL$ equals the capacitive reactance $X_C = 1/2\pi fC$, then the voltage drops across the coil and condenser neutralize each other because they are equal in magnitude and 180 degrees out of phase. Then the resistance of the circuit is the only opposition to the current flow. This condition is called resonance and occurs at the frequency $f_r = 1/2\pi\sqrt{LC}$.

In a circuit composed of pure resistance, inductance, and capacitance in parallel, the voltage is common to each branch. The current through each branch equals the voltage divided by the resistance or reactance of that branch. The total current I_t equals the vector sum of the individual currents. The equivalent impedance $Z_e = E/I_t$. This impedance can be separated into two components R_e and X_e by graphical methods. When the current through pure inductance equals the current through pure capacitance, they cancel because they are 180 degrees out of phase. The only current then taken from the source is that taken by the resistor. The circuit is in antiresonance under these conditions. For the conditions of pure inductance and capacitance, the frequency of antiresonance is the same as that for resonance, $f_r = 1/2\pi\sqrt{LC}$.

REVIEW QUESTIONS

1. State the law governing the operation of the telephone magneto generator.

2. For one rotation of the coil, how many cycles are produced in the generator of Fig. 162?

3. Why is no voltage induced in the coil of Fig. 163 when it is at right angles to the field?

4. What is the word meaning of the equation $e = E_{\text{max}} \sin \omega t$?

5. Explain why the current lags the voltage in an inductive circuit.

6. Explain why the current leads the voltage in a capacitive circuit.

7. In drawing vector diagrams, what is meant by the base?

8. In drawing vector diagrams, when should the current be taken as the base, and when should the voltage be taken as the base?

9. What is meant by inductive and capacitive reactances? How do they act in circuits?

10. Enumerate the steps to follow in solving a series circuit. A parallel circuit.

11. What is meant by resonance? By antiresonance?

12. Define impedance. Define effective resistance. Are they related?

13. Define power in alternating-current circuits, and explain how to calculate it.

14. Define apparent power, power factor, and reactive power.

15. Referring to page 272, Step 5, discuss any possible hazards to personnel or equipment when a circuit is in resonance.

16. What is meant by the equivalent reactance of a circuit?

17. Are resistance, reactance, and impedance vectors? Why?

18. Are IR, IX, and IZ vectors? Why?

19. Knowing the resistance and reactance, how can the impedance be found by other than graphic means?

20. Knowing the impedance and resistance, how can the impedance be found by other than graphic means?

21. Knowing the reactance, how can the capacitance of a condenser be computed?

22. How do hysteresis and eddy currents affect the value of the impedance measured between the terminals of a coil?

23. How does the magnitude of inductive reactance vary with frequency?

24. How does the magnitude of capacitive reactance vary with frequency?

25. When are the frequencies of resonance and antiresonance given by the same equation?

PROBLEMS

1. The rotor of a simple hand-operated magneto generator is composed of 2000 turns of wire closely bound together. The handle is turned at the rate of 100 revolutions per minute, and the gear ratio is 1 to 4.5. In the position of maximum linkage, the flux through the coil is 100,000 lines. Calculate the frequency and the maximum value of the induced voltage.

2. Derive an equation giving the frequency in terms of the revolutions per second and the number of pairs of poles.

3. Calculate the inductive reactance of a 0.058-henry coil at frequencies of 0, 500, 1000, 1500, 2000, and 2500 cycles per second. Plot a curve of these with frequency on the X axis and reactance in ohms on the Y axis. Inductive reactance is *positive*, so plot values *up* from the X axis.

4. Calculate the capacitive reactance of a 0.62-microfarad condenser at the frequencies of Prob. 3. Capacitive reactance values are *negative*. On the same set of axes as for Prob. 3, plot the capacitive reactance values *down* from the X axis. If the coil and condenser of Probs. 3 and 4 were connected in series, mark the resonant frequency on your reactance curves.

5. A coil has an inductance of 0.08 henry. It is in series with a resistance of 72 ohms. The voltage is 62.5 volts at 100 cycles. Calculate the current and the voltage across each unit. Plot sine waves with proper phase relations for the voltages. Add these sine waves to find the wave of the total impressed voltage. Determine the angle between the total voltage and the current. Add the voltages vectorially, and find the angle.

6. A certain load when connected to 110 volts, 60 cycles, takes 50 watts, and the current is 0.5 ampere. Calculate the impedance, the effective resistance, the apparent power, the power factor, and the angle between the current and voltage.

7. The impedance of a coil at 120 cycles is 73 ohms, and its resistance is 39 ohms. What is its inductance?

8. A 0.11-henry coil having a resistance of 20 ohms is connected across 110 volts 60 cycles. Make a complete analysis of the circuit.

9. A condenser having a capacitance of 0.4 microfarad is in series with a 200-ohm resistor, and this series combination is across 32.8 volts at 1000 cycles. Make a complete analysis of the circuit.

10. Make an analysis of the circuit of Fig. 176*a*, page 266, at an impressed voltage of 2.86 volts, 100 kilocycles.

11. Make an analysis of the circuit of Fig. 177*a*, page 267, at an impressed voltage of 2.86 volts, 100 kilocycles.

12. Make an analysis of the circuit of Fig. 179*a*, page 271, at an impressed voltage of 10.8 volts, and a frequency of 8500 cycles.

13. Referring to Prob. 12, if the circuit is in resonance, what will be the voltage across the coil, and what will be the voltage across the condenser? What will be the voltage across each of these if the resistance is reduced to 10 ohms?

14. Make an analysis of the circuit of Fig. 182*a*, page 277, at an impressed voltage of 10.8 volts and a frequency of 8500 cycles.

15. *a*. What condenser will produce resonance with a 0.03-henry coil at 1000 cycles?

b. A coil tunes to resonance at 560 kilocycles with a 340-micromicrofarad condenser. What is the inductance of the coil?

c. A 100-microhenry coil and a 0.0002-microfarad condenser are resonant at what frequency?

CHAPTER X

ALGEBRAIC REPRESENTATION OF ALTERNATING-CURRENT QUANTITIES

In the early chapters of this book, direct-current circuits were considered. In these circuits, amperes were added to amperes, volts to volts, and ohms to ohms. No phase relations needed to be considered, the solutions always being simple additions and subtractions.

When alternating-currents were considered, it was found that because the currents were not always in phase and because the voltages were not always in phase vector solutions were necessary. Also, it was found that resistances and reactances must be combined at right angles to find the impedance of a circuit. It was shown how to make these calculations merely by plotting the values to scale and graphically combining them.

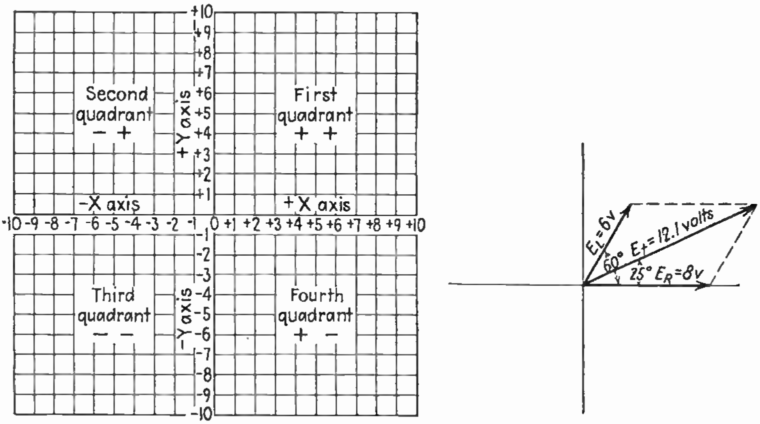
Now the graphic solution is a good one and is extensively used in practice. In many problems it is the most simple and direct way to proceed, but particularly for advanced work it is less flexible than other methods.

In this chapter certain phases of these more advanced methods of handling alternating-current problems will be discussed. In doing this, it is not intended to abandon the graphic solution. In fact, the graphic solution will be continued throughout this book, whenever it is applicable.

It was at first intended to place the material in this chapter in an Appendix. This has not been done because it was feared that in an Appendix it would be less effective. It has been decided to place it where the student is sure to find it and where it will be useful in studying the chapters that follow.

And now for a final word to the reader: If his high-school mathematics has long since become hazy, he should not despair, because there is nothing at all difficult in this chapter. If he does not care to follow this chapter through, it will in no way prevent his understanding the rest of the book. It is hoped that the majority of the readers will master this chapter, however, because it will be a key to unlocking a vast storehouse of technical information.

Methods of Representing Vectors.—Since alternating currents and voltages are treated as vectors, and since resistances and reactances must be added at right angles to give impedances, much of the solution of alternating-current circuits has to do with handling these peculiar quantities. The various ways of representing vectors will now be considered.



(a) (b)

FIG. 184.—Illustrating how vectors are plotted on a system of rectangular coordinates.

Vectors Expressed Graphically.—Vectors, such as alternating currents and voltages, are completely described when their *magnitudes* and their *directions* are given. Before they can be drawn graphically, they must have a *background* on which their magnitudes and directions can be indicated. Such a background is the system of **rectangular coordinates** shown in Fig. 184*a*. Thus suppose that in a simple series circuit composed of a resistor and an inductance coil there is a voltage drop of 8 volts across the resistor and a drop of 6 volts across the coil and that the voltage across the coil leads the current through the coil by 60 degrees. These values would be plotted as shown in Fig. 184*b*, and as is seen, they fall in the first quadrant of the system of Fig. 184*a*. The total voltage is also shown found by the method of graphical addition discussed on page 125. This total voltage is about 12.1 volts, and the angle it makes with the current is about 25 degrees.

Vectors Expressed Analytically.—Referring to Fig. 184*b*, there is a *horizontal* projection [1] on the X axis of the voltage across the coil; likewise, there is a vertical projection on the Y axis of the voltage across the coil. In Fig. 184*b*, the horizontal projection of the voltage across the coil is $E_{LH} = 3.0$ volts; also, the vertical projection is $E_{LV} = 5\ 2$ volts. The subscript L has been used to designate the inductance coil. Thus, the voltage across the coil is completely specified when its horizontal and vertical values are given. Similarly, from Fig. 184*b* the horizontal projection of the total voltage is $E_{tH} = 11.0$ volts, and the vertical projection is $E_{tV} = 5.2$ volts. These projections are called **components** and are referred to as the **horizontal component** and the **vertical component.** A current or voltage value is accordingly completely specified when its horizontal and vertical components are given.

Vectors Expressed Algebraically.—Merely by definition let it be said that *whenever a component is preceded by the letter j the component is a vertical component.* This simple statement forms the basis of a method of expressing values algebraically.[2] The advantage of this is that values thus expressed can be added, subtracted, multiplied, divided, and otherwise handled algebraically without drawing them out graphically. Furthermore, by the graphic method, vectors can only be added and subtracted. Expressed algebraically, the voltage across the resistor previously considered would be $E_R = 8 + j0$ volts; that is, the voltage across the resistor consists of 8 volts in phase with the current and zero volts at right angles (vertical) with the current, because the voltage across a resistor is in phase with the current through it. The voltage across the coil would be written $E_L = 3 + j5.2$ volts; that is, the voltage across

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[1] Imagine that parallel rays of light are shining vertically on the vector representing the voltage across the coil. The shadow it throws on the X axis is the horizontal projection. Also, imagine that parallel rays of light are shining horizontally on this same vector. The shadow it throws on the Y axis is the vertical projection.

[2] An expression in which the letter j is used is called a complex number. Thus, $10 + j20$ is a complex number, and the method of handling such expressions is called complex algebra. Because of the mental barrier created by the word complex, it has not been stressed. Nothing is complex if you are interested in it and understand it, and there is nothing complex about the use of the letter j to distinguish between in-phase and out-of-phase components.

the coil consists of 3.0 volts in phase with the current and 5.2 volts at right angles ahead of the current. Had the voltage across the coil lagged the current, the expression for the voltage would have been $E'_L = 3 - j5.2$ volts. That is, $+j$ indicates a leading component which puts the vector in the first quandrant, but $-j$ indicates a lagging component which puts the vector in the fourth quadrant. The total voltage is $E_t = E_R + E_L = (8 + j0) + (3 + j5.2) = 11 + j5.2$ volts.

Vectors Expressed in Polar Coordinates.—The algebraic method just described employs the system of rectangular coordinates of Fig. 184a as its basis. This system consists of horizontal and vertical measurements. The polar system uses only length of the vector and its angle for locating a vector. This is similar to the graphic method. Thus, in the polar system, the voltage across the resistor previously considered would be $E_R = 8\underline{/0^\circ}$ volts; that is, it consists of a voltage of 8 volts at an angle of zero degrees with the current. The voltage across the inductance coil previously considered would be $E_L = 6\underline{/60^\circ}$ volts; that is, it consists of a voltage of 6 volts at an angle of 60 degrees *leading* the current. Similarly, the total voltage across the circuit is $E_t = 12.1\underline{/25^\circ}$ volts. Now if the voltage had *lagged* the current, it would have been written $E_t = 12.1\overline{\backslash 25^\circ}$ (see also page 298).

To summarize: There are several ways of expressing and combining vectors. Each of these systems has certain advantages, and each has certain limitations. The graphical method has been treated in much detail, particularly in Chap. IX, and little need be added in explanation of it. The following pages will consider the other methods in much detail.

Trigonometric Functions.—Before proceeding with the methods of representing vector values standing for alternating currents and voltages, a few words must be included regarding the fundamentals of trigonometry.

Trigonometry applies in its simple form to right triangles like the ones that have been used in the vector solutions given at various points in this book. There are six important trigometric functions, but only three of these need be studied for solving electric circuits. The uses to which trigonometry will be put are largely these: (1) If the magnitude of a vector and its angle are known, the in-phase or resistive component (horizontal) and the out-of-phase or reactive component (vertical) can be found *without* plotting the vector graphically and measuring the values with a scale. (2) If the in-phase and reactive components of a vector are known, its magnitude and angle may be found *without* plotting the vector graphically.

To explain these simple relations, the right triangle of Fig. 185 is presented. The horizontal, or in-phase, component is marked b, the vertical, or out-of-phase, component is marked a, and the hypotenuse is marked c. For any such right triangle, the following relations hold *by definition*.

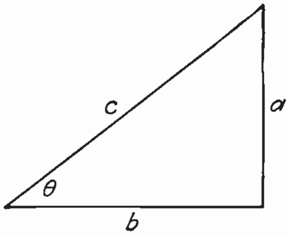

FIG. 185.—In the right triangle shown here, the horizontal side is $b = c \cos \theta$, and the vertical side is $a = c \sin \theta$.

$$\sin \theta = \frac{a}{c}, \qquad a = c \sin \theta, \qquad \text{and} \qquad c = \frac{a}{\sin \theta}. \qquad (58)$$

$$\cos \theta = \frac{b}{c}, \qquad b = c \cos \theta, \qquad \text{and} \qquad c = \frac{b}{\cos \theta}. \qquad (59)$$

$$\tan \theta = \frac{a}{b}, \qquad a = b \tan \theta, \qquad \text{and} \qquad b = \frac{a}{\tan \theta}. \qquad (60)$$

When spoken or written separately, sin becomes **sine,** cos becomes **cosine,** and tan becomes **tangent.** Tables of sines, cosines, and tangents sufficient for most purposes are included on page 523. More complete tables, where necessary, can be found in most handbooks and in books on mathematics. To illustrate the application of trigonometry to electrical problems, several examples will now be given. These will be drawn out so that the trigonometric calculations may be verified graphically.

Example 1.—A voltage consists of 10 volts at an angle of 50 degrees leading the current. Calculate the in-phase, or resistive, component (horizontal) and the out-of-phase, or reactive, component (vertical).

Solution.—These relations are shown in Fig. 186. The 10 volts correspond to side c of Fig. 185. It is desired to find side a (the out-of-phase, or reactive, component) and side b (the in-phase, or resistive, component). Referring to Eq. (58), side $a = c \sin \theta$. The value of sine 50° is given in the tables on page 543 as sin 50° = 0.7660. Then, side a, the out-of-phase, or reactive, component E_X of the 10 volts is $E_X = 10 \times 0.7660 = 7.66$ volts. Referring to Eq. (59), the in-phase, or resistive, component is $b = c \cos \theta$. The value of cos 50° is given in the tables on page 543 as cos 50° = 0.6428. Then, side b, the in-phase, or resistive, component E_R of the 10 volts is $E_R = 10 \times 0.6428 = 6.428$ volts. Thus, 10 volts at an angle of 50 degrees is composed of two components, the out-of-phase, or reactive, component (vertical) of $E_X = 7.66$ volts, and the in-phase, or resistive, component (horizontal) of $E_R = 6.428$ volts. These values can be verified graphically in Fig. 186.

Example 2.—A voltage consists of 10 volts at an angle of 37 degrees lagging the current. Calculate the in-phase, or resistive, and the out-of-phase, or reactive, components.

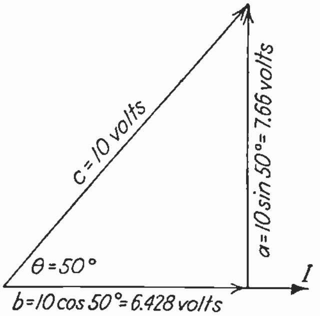

FIG. 186.—In this figure the voltage of 10 volts leads the current by an angle of 50 degrees. This voltage is composed of a horizontal and a vertical component. In this instance the vertical component is positive, that is, directed upward.

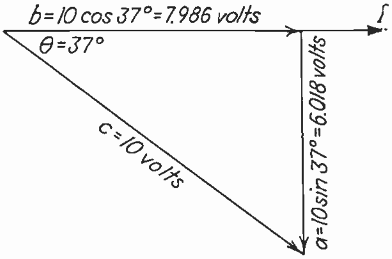

FIG. 187.—In this figure the voltage of 10 volts lags the current by 37 degrees. This voltage vector is composed of a horizontal and a vertical component. In this instance the vertical component is negative, that is, directed downward.

Solution.—These relations are shown in Fig. 187. Note that the vector is now in the fourth quadrant, and thus the out-of-phase, or reactive, component must be negative. This reactive component will be $E_X = 10 \sin 37° = 10 \times 0.6018 = -6.018$ volts. The resistive component will be $E_R = 10 \cos 37° = 10 \times 0.7986 = 7.986$ volts. Thus, 10 volts at an angle of 37 degrees lagging is composed of two components, a reactive component of $E_X = -6.018$ volts and a resistive component of $E_R = 7.986$ volts. These values can be verified graphically in Fig. 187.

Example 3.—A current lags a voltage. This current consists of an out-of-phase, or reactive, component of $I_X = 7.9$ milliamperes and an in-phase, or resistive, component of $I_R = 6.2$ milliamperes. What is the value of the magnitude of the current, and what angle does it make with the voltage?

Solution.—Step 1. For solving this problem, the tangent relationships of Eq. (60) will first be used. Thus, $\tan \theta = a/b$. Now in this problem (refer to Fig. 185), $a = I_X = 7.9$ milliamperes and $b = I_R = 6.2$ milliamperes. Thus, $a/b = 7.9/6.2 = 1.275$. This value is the tangent of the angle, and from the tables on page 524, the angle corresponding to this tangent is slightly less than 52 degrees. Thus, the total value of the current will lag the voltage by an angle of about 52 degrees.

Step 2. The two components of the current and its angle are known. Calculate the total or real value of the current. This can be done by either Eq. (58) or (59). Suppose Eq. (59) is selected. Then, the real value of the current is equivalent to side c of Fig. 185, and this is $c = b/\cos\theta$. Thus, $I = 6.2/\cos 52° = 6.2/0.6156 = 10.06$ milliamperes. This can also be found from the relation $I = \sqrt{(6.2)^2 + (7.9)^2} = 10.04$ milliamperes (which checks reasonably well).

Vectors Expressed Analytically.—As was explained on page 286, in expressing vectors analytically their in-phase, or resistive (horizontal), and their out-of-phase, or reactive (vertical), components are stated. In adding and subtracting vectors, the in-phase, or resistive, components are added or subtracted, and the out-of-phase, or reactive, components are added or subtracted. Several illustrations of these principles will now be given.

Example 1.—In Fig. 188 is shown a coil of 0.087 henry inductance and 49.4 ohms resistance in series with a 0.68-microfarad condenser. A current of 87 milliamperes at 1000 cycles flows through the coil. Calculate the voltage, stating both the magnitude and the angle.

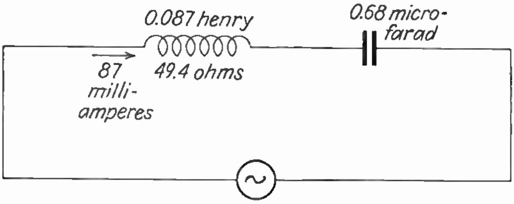

FIG. 188.—A series circuit composed of a coil and a condenser in series and carrying a current of 87 milliamperes.

Solution.—Step 1. Calculate the reactance of the coil and of the condenser.

$$X_L = 2\pi fL = 6.2832 \times 1000 \times 0.087 = 547 \text{ ohms.}$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{6.2832 \times 1000 \times 0.68 \times 10^{-6}} = -234 \text{ ohms.}$$

Note that the sign of the capacitive reactance is marked negative as it is always assumed to be.

Step 2. Calculate the in-phase and reactive components of the voltage across the coil and the condenser.

Voltage across coil. $E_R = IR = 0.087 \times 49.4 = 4.3$ volts

$E_X = IX_L = 0.087 \times 547 = 47.6$ volts.

Voltage across condenser. $E_X = IX_C = 0.087 \times -234 = -20.4$ volt.

There is no in-phase component.

Step 3. To find the total impressed voltage, add all in-phase or resistive components and add all out-of-phase or reactive components. Resistive component of total voltage = 4.3. (The only resistive component is that due to the coil.) Reactive component of total voltage = 47.6 − 20.4 = 27.2 volts. Thus, the impressed voltage consists of an in-phase, or resistive (horizontal), component of 4.3 volts, and an out-of-phase, or reactive (vertical), component of 27.2 volts.

Step 4. Find the magnitude of the total voltage.

$$E_t = \sqrt{(4.3)^2 + (27.2)^2} = 27.5 \text{ volts approximately.}$$

Step 5. Find the angle. Because the net reactive voltage drop is positive, the voltage will *lead* the current and will lie in the first quadrant of Fig. 184*a*. From Eq. (56), $\tan \theta = a/b = 27.2/4.3 = 6.32$. The angle corresponding to this tangent is, from the tables on page 528, about 81 degrees. That is, the voltage lags the current by about 81 degrees. If it had been desired, Step 4 could have been omitted and the magnitude of the voltage could have been found by Eq. (54) or (55). Using Eq. (54), $c = a/\sin \theta = 27.2/\sin 81° = 27.2/0.9876 = 27.5$ volts, approximately. Of course, all these solutions could have been made graphically, but the purpose of this section is to show how to make vector solutions without resorting to vector diagrams.

Example 2.—A circuit consists of two coils and a condenser in parallel. The current in coil 1 is 62.0 microamperes and lags the voltage by 47 degrees. The current in coil 2 is 53.7 microamperes and lags the voltage by 59 degrees. The current through the condenser is 109.2 microamperes and leads the voltage by 90 degrees. Calculate the total current flowing.

Solution.—Step 1. Find the resistive and reactive components of each current. This can be done by Eqs. (58) and (59). The resistive components will each be $b = c \cos \theta$, and the reactive components will each be $a = c \sin \theta$ (Fig. 185).

$$I_{R1} = 62.0 \cos 47° = 62.0 \times 0.6819 = 42.2 \text{ microamperes.}$$

$$I_{X1} = 62.0 \sin 47° = 62.0 \times 0.7313 = -45.3 \text{ microamperes.}$$

The minus sign indicates that the current lags the voltage as is true for a coil.

$$I_{R2} = 57.3 \cos 59° = 57.3 \times 0.5150 = 29.5 \text{ microamperes.}$$

$$I_{X2} = 57.3 \cos 59° = 57.3 \times 0.8571 = -49.2 \text{ microamperes.}$$

The minus sign indicates that the current lags the voltage as is true for a coil.

The entire condenser current of 109.2 is reactive and is positive, because the current through a condenser leads the voltage across the condenser.

Step 2. Calculate the resistive component and the reactive component of the total line current.

$$\text{Resistive component of total current} = 42.2 + 29.5$$
$$= 71.7 \text{ microamperes.}$$

$$\text{Reactive component of total current} = -45.3 - 49.2 + 109.2$$
$$= 14.7 \text{ microamperes.}$$

Because the net reactive current is positive, the total current will lead the voltage, indicating that the effect of the condenser predominates.

Step 3. Determine the angle by which the current leads the voltage. From Fig. 185 and Eq. (60), $\tan \theta = a/b = 14.7/71.7 = 0.205$. From the tables on page 529, the angle corresponding to this tangent is about 11 degrees 35 minutes.

Step 4. Calculate the real value of the current flowing. This will correspond to side c of Fig. 185. From Eq. (56), $c = a/\sin \theta$, and thus $I_t = 14.7/\sin 11° 35' = 14.7/0.2008 = 73.2$ microamperes. As a check, $I_t = \sqrt{(71.7)^2 + (14.7)^2} = 73.2$ microamperes. Considering the approximations made, these two values are in satisfactory agreement.

Vectors Expressed Algebraically.—This system is very useful for handling vector quantities. Furthermore, like everything else, it is *not* difficult (despite the fact that some people avoid it) if a certain few principles are mastered and if the rudiments of algebra are known.

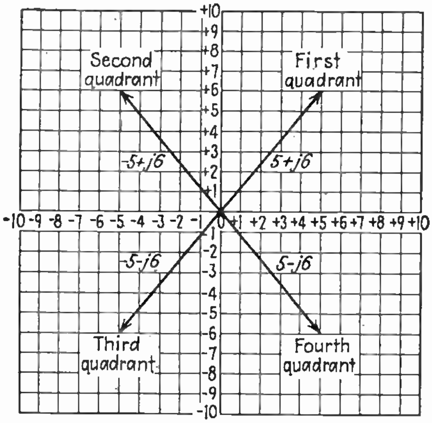

FIG. 189.—Indicating how the sign preceding each component of a vector fixes it in one of four quadrants.

On page 286 it was stated that when a value was preceded by the letter j it indicated that it was at right angles to a value that was not preceded by j. Referring to Fig. 184*a* and Fig. 189, a vector $I_1 = 5 + j6$ would be located as indicated in the *first quadrant.* Now if the vector were $I_2 = -5 + j6$, it would be located in the *second quadrant.* Similarly, if the vector were $I_3 = -5 - j6$, it would be located in the *third quadrant,* and if it were $I_4 = 5 - j6$, it would be located in the *fourth quadrant.*

The proof of this statement is beyond the scope of this book, but the term $j = \sqrt{-1}$. In mathematics this is called i, but j is used in electrical work to prevent confusion with current i. Now here is an important rule: *Whenever j^2 occurs in an equation, it can be replaced by -1, because $j^2 = j \times j = \sqrt{-1} \times \sqrt{-1} = -1$.*

The algebraic method of handling vectors is commonly used to perform four types of operation: addition, subtraction, multiplication, and division. These will now be considered separately.

Algebraic Vector Addition.—In this the same fundamental law is followed: *Add in-phase, or resistive, components together, and add out-of-phase, or reactive, components together.* This is best explained by an illustration.

Example.—A circuit consists of a coil and a condenser in series. The voltage across the coil is 2.68 volts, and it leads the current by 41 degrees. The voltage across the condenser is 1.97 volts, and it lags the current by 90 degrees. Calculate the total current.

Solution.—Step 1. Find the resistive and reactive components of the voltage across the coil. From Fig. 185 and Eq. (59), the in-phase, or resistive, component will be $E_R = 2.68 \cos 41° = 2.68 \times 0.7547 = 2.02$ volts. The out-of-phase, or reactive, component will be from Eq. (58), $E_X = 268 \sin 41° = 2.68 \times 0.6560 = 1.76$ volts.

Step 2. Write the algebraic expression for the voltage across the coil and the voltage across the condenser.

$$\text{Voltage across coil} = E_L = 2.02 + j1.76.$$

$$\text{Voltage across condenser} = E_C = 0 - j1.97.$$

The $+j$ value in the voltage across the coil indicates that the value which it precedes is at right angles *ahead* of the first term in the expression. The $-j$ value in the voltage across the coil indicates that the value which it precedes is at right angles *behind* the first term of the expression (which happens to be zero in this instance).

Step 3. Add the two algebraic expressions to find the total voltage across the coil and condenser in series.

$$\begin{aligned} E_L &= 2.02 + j1.76 \\ E_C &= 0.0 \;\; - j1.97 \\ \hline E_L + E_C &= 2.02 - j0.21 \end{aligned}$$

Thus the total voltage is a vector value composed of 2.02 volts in phase with the current and 0.21 volts 90 degrees behind the current.

Algebraic Vector Subtraction.—The rule learned in algebra for subtracting quantities was to change the sign and add. This will be made clear by the following example.

Example.—A circuit is composed of two impedances in parallel. The algebraic expression for the total current taken by the two impedances is $I_t = 12.8 - j6.4$ milliamperes. The current taken by the first impedance is $I_1 = 4.9 - j12.4$ milliamperes. What is the current taken by the second impedance? What is the nature of each impedance?

Solution.—Step 1. The total current is the sum of the branch currents, or $I_t = I_1 + I_2$; therefore $I_2 = I_t - I_1$. Remembering the rule previously given,

$$\begin{aligned} I_t &= 12.8 - j6.4 \\ -I_1 &= -4.9 + j12.4 \\ \hline I_2 = I_1 + (-I_1) &= 7.9 + j6.0 \\ \text{Thus, } I_2 &= 7.9 + j6.0 \text{ milliamperes.} \end{aligned}$$

Step 2. The nature of the first impedance is that of resistance and inductance because the reactive portion of the current is negative as shown by the $-j$ portion, indicating that the current lags the voltage. The nature of the second impedance is that of resistance and capacitance because the reactive component of the current leads the voltage as indicated by the $+j$.

Multiplication of Algebraic Values.—As was mentioned on page 256, resistance, reactance, and impedance are not *vector* values. However, resistance and reactance are added at right angles to give impedance. As will be recalled, inductive reactance is positive, and capacitive reactance is negative. Thus the expression for the impedance of a circuit containing *resistance* and *inductive* reactance is

$$Z = R + jX, \tag{61}$$

and the expression for the impedance of a circuit containing *resistance* and *capacitive reactance* is

$$Z = R - jX. \tag{62}$$

As is well known, the voltage across any circuit is the product of the current through that circuit and the impedance of that circuit. Two examples will serve to illustrate the principles involved.

Example 1.—A coil has a resistance of 10 ohms and an inductance of 0.0005 henry. It is carrying 1.7 milliamperes at 50,000 cycles. Take the current as the base, and find the expression for voltage across the coil.

Solution.—Step 1. Calculate the reactance of the coil.

$$X_L = 2\pi fL = 6.2832 \times 50{,}000 \times 0.0005 = 157 \text{ ohms.}$$

Step 2. Write the algebraic expression for the impedance of the coil and for the current through the coil. For a coil, $Z = R + jX = 10 + j157$ ohms. Since the current is taken as the base, it will lie entirely along the $+X$ axis and will have no reactive component; that is, $I = 1.7 + j0$ milliamperes.

Step 3. Multiply the expression for the current by that for the impedance to find the voltage across the coil. This will now be done following the simple rules of algebra.

$$\begin{array}{l} 10 \;\; + j157 \\ 1.7 + j0 \\ \hline 17 \;\; + j267 \end{array}$$

The expression for the voltage across the coil is $17 + j267$ millivolts. This is reasonable, because the voltage across a coil should lead the current through a coil, and the $+j267$ component indicates that these relations are fulfilled.

Example 2.—The impedance of a circuit containing both resistance and reactance in series is $Z = 21 - j267$ ohms. A current having a value of $I = 6 - j9$ milliamperes passes through this circuit. What is the expression for the voltage across this circuit? (The fact that the current has a reactive component instead of no reactive component as in the preceding example merely means that in the example now being considered the current is not taken as the base.)

Solution.—It is merely necessary to multiply these expressions, because $E = IZ$. This should be done algebraically as follows:

$$\begin{array}{rrr} 21 & - j267 & \\ 6 & - j9 & \\ \hline 126 & - j1602 & \\ & - j189 & + j^2 2403 \\ \hline 126 & - j1791 & + j^2 2403 \end{array}$$

But, as was shown on page 293, the value j^2 can always be replaced by -1, then, the value $j^2 2403 = -2403$. Now all values without a j in front can be combined. Thus the answer to the multiplication becomes $E = IZ = 126 - j1791 - 2403 = -2277 - j1791$ millivolts, and this is the expression for the voltage across the circuit containing the resistance and capacitance and carrying the current specified. Just to be sure that this is a reasonable answer, the vector diagram of Fig. 190 has been plotted. First is shown the current $I = 6 - j9$. This is plotted in the fourth quadrant as its reactive value specifies. It is laid off by taking 6 units along the $+X$ axis and 9 units down along the $-Y$ axis. Next, the value for the voltage is located. This is 2277 millivolts along the $-X$ axis and 1791 millivolts along the $-Y$ axis. (Current and voltage are not the same quantities and need not be plotted to scale, but in drawing Fig. 190, the angles have been made correct.) The final angle between the current through the circuit and the voltage across the circuit is about 85 degrees 30 minutes. By remembering that vectors are assumed to be rotating in a counterclockwise direction (page 121), it is seen that the vectors are in their correct position, because the circuit contains resistance and reactance, and in such a circuit the voltage should lag the current.

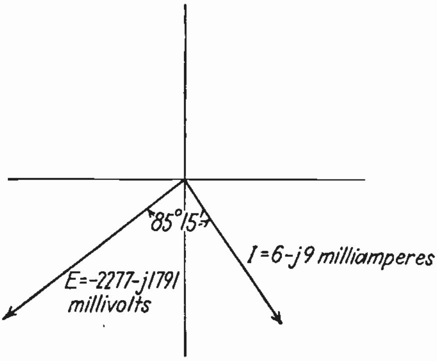

FIG. 190.—The current $I = 6 - j9$ milliamperes flowing through an impedance of $Z = 21 - j267$ ohms causes a voltage drop of $-2277 - j1791$ millivolts.

Division of Algebraic Values.—This will be illustrated by two examples.

Example 1.—A coil having a resistance of 18 ohms and an inductance of 0.8 millihenry is connected across a source of voltage of 1.8 volts at 50,000 cycles. Take the voltage as the base, and calculate the current through the circuit.

Solution.—Step 1. Calculate the reactance of the coil.

$$X_L = 2\pi fL = 6.2832 \times 50{,}000 \times 0.0008 = 251 \text{ ohms.}$$

Step 2. Write the expressions for the voltage and for the impedance of the coil. Remember that the voltage is to be taken as the base.

$$\text{Voltage across coil} \quad E = 1.8 + j0.$$

$$\text{Impedance of coil} \quad Z = 18 + j251.$$

Step 3. Divide the expression for the voltage by the expression for the impedance to obtain the expression for the current. Thus,

$$I = \frac{E}{Z} = \frac{1.8 + j0}{18 + j251}.$$

Now here is a rather complex fraction to solve, and to do it, a process of multiplication will be resorted to. Multiplying both the numerator and the denominator of a fraction by the same factor does not change the value of the fraction. Thus, the numerator and denominator will each be multiplied by an expression having the same *numerical* values but with the sign of the reactance *opposite* to that of the impedance which is being divided into the voltage to find the current. Then,

$$I = \frac{E}{Z} = \frac{1.8 + j0}{18 + j251} \times \frac{18 - j251}{18 - j251} = \frac{32.4 - j452}{63{,}324}$$

$$= 0.000512 - j0.00714 \text{ ampere.}$$

$$\begin{array}{l} 18 \quad - j251 \\ \;1.8 + j0 \\ \hline 32.4 - j452 \end{array} \qquad \begin{array}{lll} 18 + j251 & & \\ 18 - j251 & & \\ \hline 324 + j4520 & & \\ \quad\;\; - j4520 & - j^2 63{,}000 & \\ \hline 324 & - j^2 63{,}000 & = 324 + 63{,}000 = 63{,}324 \end{array}$$

(If it is remembered to replace j^2 by -1.)

The reactive value of the current is negative, and this is correct, because the current through the coil should lag the voltage.

Example 2.—The equation for the voltage across a circuit is $E = 8 + j12$ volts, and the impedance of the circuit is $Z = 20 - j64$ ohms. Find the expression for the current through the circuit.

Solution.—Divide the voltage by the impedance to obtain the current.

$$I = \frac{E}{Z} = \frac{8 + j12}{20 - j64} = \frac{8 + j12}{20 - j64} \times \frac{20 + j64}{20 + j64} = \frac{-608 + j752}{4496}$$

$$= -0.135 + j0.167 \text{ ampere.}$$

$$\begin{array}{ll} 8 + j12 & \\ 20 + j64 & \\ \hline 160 + j240 & \\ \quad + j512 + j^2 768 & \\ \hline 160 + j752 + j^2 768 & = -608 + j752 \end{array} \qquad \begin{array}{lll} 20 - j64 & & \\ 20 + j64 & & \\ \hline 400 - j1280 & & \\ \qquad\; j1280 & - j^2 4096 & \\ \hline 400 & - j^2 4096 & = 4496 \end{array}$$

(If it is remembered to replace j^2 by -1.)

In this problem a capacitive impedance is divided into a voltage which lies in the first quadrant. The current should be ahead of the voltage as in any capacitive circuit. The equation for the current places it in the second quadrant as should be expected.

Vectors Expressed in Polar Coordinates.—The algebraic method of representing vectors in polar coordinates which was considered in the preceding pages is very useful. Four operations were demonstrated; these were addition, subtraction, multiplication, and division.

In the pages that follow, the method of expressing vectors in polar coordinates will be considered. This method was briefly considered on page 287. In this method a system of polar coordinates or concentric circles and lines representing angles is used to express vectors as shown in Fig. 191. On this polar system several vectors will be plotted to illustrate the principles involved. As explained on page 287, an angle $\underline{/\theta}$ is leading, and $\overline{/\theta}$ is lagging; sometimes, $\underline{/\theta}$ denotes leading, and $\underline{/-\theta}$ denotes lagging. The details of the method of handling polar values will now be explained.

Example.—A circuit consists of a coil having a resistance of 28 ohms and an inductance of 0.038 henry in parallel with a condenser having no equivalent resistance and a capacitance of 0.48 microfarad, and this parallel combination is connected across a voltage of 56.4 volts at 1200 cycles. Calculate the current through each branch of the circuit, the total current, and, using the voltage as a base, plot all vectors as polar values.

Solution.—Step 1. Calculate the reactance of the coil and of the condenser.

$$X_L = 2\pi fL = 6.2832 \times 1200 \times 0.038 = 286 \text{ ohms.}$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{6.2832 \times 1200 \times 0.48 \times 10^{-6}} = 276 \text{ ohms.}$$

Step 2. Calculate the impedance of the coil and the condenser.

$$Z_L = \sqrt{(28)^2 + (286)^2} = 288 \text{ ohms.}$$

$$Z_C = \sqrt{(0)^2 + (276)^2} = 276 \text{ ohms.}$$

Step 3. Calculate the current that will flow through the coil and through the condenser.

$$I_L = \frac{E}{X_L} = \frac{56.4}{288} = 0.196 \text{ ampere.}$$

$$I_C = \frac{E}{X_C} = \frac{56.4}{276} = 0.204 \text{ ampere.}$$

Step 4. Express these currents as polar values, using the voltage as the base. To do this it is necessary to find the angles by which the current leads or lags the voltage. For the coil, the current will lag the voltage. The angle by which it lags is determined by the resistance, reactance, and impedance. The angle between the impedance and the resistance is found from the relation $\cos \theta = R/Z = 28/288 = 0.0972$, and $\theta = 84$ degrees 30 minutes, approximately. The current through the condenser will lead the voltage across the condenser by 90 degrees, because there is no resistive component to the impedance of the condenser.

$$I_L = 0.196\overline{/84^\circ\ 30'} \text{ ampere.}$$

$$I_C = 0.204\underline{/90^\circ} \text{ ampere.}$$

Step 5. Plot the currents as indicated in Fig. 191. First, the impressed voltage is drawn in at the angle 0, because it is the reference. It can be merely a line and need not be to scale, because it is the only voltage involved. Next, the current through the coil is drawn in. Its length is 0.196 unit, and it is placed 84 degrees 30 minutes behind the voltage. Then the current through the coil is drawn to a length of 0.204 unit and 90 degrees ahead of the voltage.

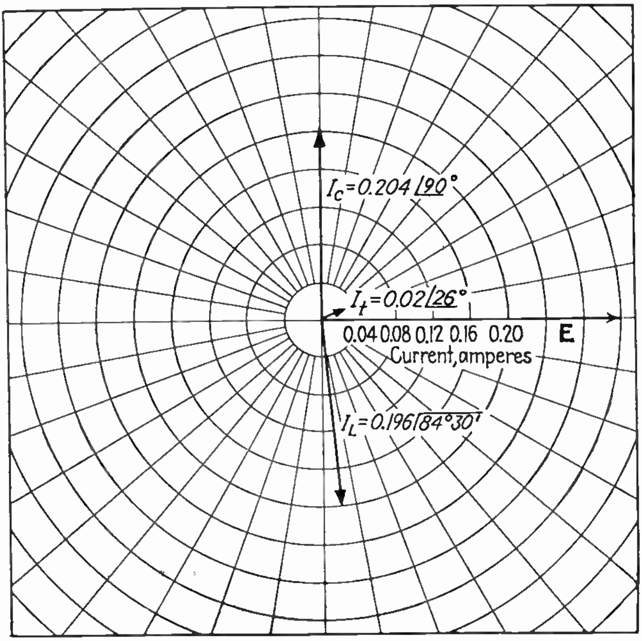

Fig. 191.—Showing the method of representing vectors on polar coordinates.

Step 6. Calculate the total current. The polar values of the two currents as given under Step 4 cannot be added directly to obtain the total resultant current, *because they do not lie at the same angle.* To add vectors, their in-phase and out-of-phase components must be added as explained in previous pages. But, the polar vector can be added graphically on Fig. 191. When this is done it is found that the total current is about 0.02 ampere and the angle is about 26 degrees ahead of the voltage. The polar expression for this current is $I = 0.02\,\underline{/26^\circ}$ ampere.

To summarize: Rectangular coordinates *must* be used for adding and subtracting vectors and *can* be used for multiplying and dividing vectors. As will be shown in this section, polar coordinates *cannot* be used for addition and neither can they be used for subtraction. As will be shown in the following section, polar coordinates *can* be used for multiplication, division, and for squaring vector values and *must* be used for taking the roots of vectors. In other words, only in the case of multiplication, division, and squaring numbers is there a choice available between the two systems.

Rules for Solving Polar Values.—These are so simple that in most instances a mere statement of the rule to be followed and a single example will be sufficient. Before proceeding, however, two terms must be introduced. For a polar value such as $I = 10\underline{/30^\circ}$, the value 10 represents the *length* of the vector and is called the **modulus.** The value $\underline{/30^\circ}$ is the *angle* the vector makes with the reference point and is called the **argument.**

Multiplication.—To multiply two polar vectors, *multiply* the two moduli and *add* the angles algebraically. Thus if a current of $I = 0.8\underline{/30^\circ}$ flows through an impedance of $Z = 20\overline{/60^\circ}$, the voltage across the impedance will be the product of these two, or $E = IZ = (0.8\underline{/30^\circ})\ (20\overline{/60^\circ}) = 16\overline{/30^\circ}$ volts. It will be recalled that impedance is not really a vector, but it does contain in-phase and out-of-phase components and can be expressed as a polar value. (In calculating the angle, $\underline{/30^\circ}$ designates a *positive* angle of +30 degrees, and $\overline{/60^\circ}$ designates a *negative* angle of -60 degrees. Therefore, $(+30^\circ) + (-60^\circ) = -30^\circ = \overline{/30^\circ}$.)

Division.—To divide one polar vector by another, *divide* one modulus by the other and *subtract* the angles algebraically. Thus if a voltage of $E = 20\underline{/60^\circ}$ exists across an impedance of $276\overline{/88^\circ}$, the current through the impedance is $0.0725\underline{/148^\circ}$ ampere.

Raising to Powers.—To raise a polar vector to a power, raise the modulus to that power, and multiply the argument angle by that power. Thus, if $I = 2\underline{/30^\circ}$, $I^2 = 4\underline{/60^\circ}$.

Taking a Root.—To take the root of a polar vector, take the root of the modulus, and divide the argument or angle by the root. Thus, if $I^2 = 49\underline{/30^\circ}$, then $I = 7\underline{/15^\circ}$.

A few words of warning must be given regarding the sign of the angle which an impedance must have in a polar expression. Suppose that a coil has an impedance of 10 ohms and an angle of 60 degrees. And suppose that a voltage of 100 volts is impressed across this coil and that the voltage is to be taken as a base. Now as is well known, the current must lag the voltage in the coil. Thus, to solve this problem and to have a negative angle in the answer requires that the impedance of the coil be expressed

$Z = 10\underline{/60^\circ}$. Then, $I = E/Z = 100\underline{/0^\circ} \,/\, 10\underline{/60^\circ} = 10\overline{/60^\circ}$. In calculating the final angle, the angles of the voltage and current must be subtracted. Then, $\underline{/0} - \underline{/60^\circ} = 0 - (+60^\circ) = -60^\circ = \overline{/60^\circ}$. Similarly, it can be shown that the impedance of a condenser must be expressed as a negative angle. Thus, the polar expression for *the impedance of an inductive circuit is* $Z\underline{/\theta}$, and the polar expression for *the impedance of a capacitive circuit is* $Z\overline{/\theta}$. The way to remember this is that inductive reactance (and the angle) is positive and that capacitive reactance (and the angle) is negative. These relations can also be proved by studying polar multiplication.

SUMMARY

Vector values representing alternating currents and voltages may be expressed graphically, analytically, by algebraic expressions based on rectangular coordinates, or by the polar coordinate system.

In the analytic method of studying vectors, they are broken down into vertical and horizontal components.

Separating a vector into its components is easily done by elementary trigonometry. For the purpose of this book, three trigonometric functions, $\sin\theta$, $\cos\theta$, and $\tan\theta$ were introduced.

In expressing vectors algebraically, the letter j, having a value of $\sqrt{-1}$, is placed before the reactive, or out-of-phase (vertical), component.

When expressed algebraically in rectangular coordinates, vectors may be added, subtracted, multiplied, and divided by following the usual rules of algebra and remembering to replace j^2 by -1 whenever it occurs.

Vectors in the polar form cannot be added or subtracted (except by doing it graphically), but such operations as multiplication, division, squaring, and taking roots are very simple in this form.

The impedance of a coil or of a condenser can also be expressed in either the rectangular or polar form even if, strictly speaking, impedance is not a vector.

For rectangular coordinates

The impedance of a circuit containing inductance is $Z_L = R + jX$.

The impedance of a circuit containing capacitance is $Z_C = R - jX$.

For polar coordinates

The impedance of a circuit containing inductance is $Z_L = Z\underline{/\theta}$

The impedance of a circuit containing capacitance is $Z_C = Z\overline{/\theta}$.

REVIEW QUESTIONS

1. In studying alternating-current circuits, why cannot currents or voltages be added directly?
2. How are currents or voltages added in alternating-current circuits?
3. May impedances be added directly? How are they combined?

4. What is the sign of capacitive reactance, of inductive reactance, of resistance, and of impedance?

5. Give the signs preceding the components of vectors lying in each quadrant.

6. If the X axis is the reference, which components of a vector are plotted vertically and which are plotted horizontally?

7. If the sine of an angle is known, how is the angle found?

8. If the hypotenuse of a right triangle and the angle are known, how can the other two sides be found?

9. What is meant by the letter j and by $-j$ in an algebraic equation for a vector?

10. If the algebraic expression for a vector is known, how is the length of the vector found?

11. Is an impedance a vector quantity? May it be written algebraically?

12. In what quadrants would the following vectors be placed: $I_1 = -6 + j2$; $I_2 = 6 - j12$; $I_3 = 6 + j12$; $I_4 = -6 - j12$?

13. What is the rule for subtracting algebraic vector values?

14. How do you divide algebraic vector values?

15. If a vector is $E = 60 + j80$, how would it be expressed in polar form?

16. Why cannot polar expressions be added except by graphic means?

17. What is the rule for multiplying, dividing, and taking the root of polar values?

18. What is the difference between $\underline{/\theta}$ and $\overline{/\theta}$?

19. Is the angle of a polar term representing the impedance of a coil positive or negative?

20. How must the impedance of one element be added to that of another to give the total impedance?

PROBLEMS

1. Solve Example 1, page 290, graphically.

2. Solve Example 2, page 291, graphically.

3. Solve the example given on page 293 graphically.

4. Solve the example given on page 294 graphically.

5. On page 296 it is stated that the angle between the current through the circuit and the voltage across the circuit is approximately 85 degrees. Prove this to be true graphically.

6. Graphically add the vectors in Fig. 191, and check the answer given on page 299.

7. Multiply the expressions $I = 5 - j8$ and $Z = 20 - j60$. Find the real value of the voltage.

8. Divide the expression $E = 20 + j40$ by $Z = 5 + j6$. Find the real value of the current.

9. Change the data of Prob. 7 into the polar form, and perform the multiplication.

10. Change the data of Prob. 8 into polar form, and perform the division.

CHAPTER XI

ELECTRIC NETWORKS

Resistors, inductors, and capacitors[1] are combined in many ways to form electric circuits or **networks**. In their usual form these networks have two input and two output terminals. Thus, from a generalized point of view, all the common electric circuits are networks; these include such familiar devices as amplifiers, filters, attenuators, and telephone lines.

In the power industry, the power networks operate at a single frequency, usually 60 cycles per second. In communication, the networks usually must operate over a band of frequencies and often this band is very wide, for example, in television video amplifiers, where frequencies of a few cycles to several million cycles must be passed.

Networks are sometimes quite complicated, and when a network diagram is examined, the whole is very confusing. A complicated network must be *analyzed* into its component parts. This means that to be able to understand complicated networks a complete mastery of the elements comprising networks must be attained.

For this reason, before attempting to discuss even the simplest of networks, the elements from which they are made will be considered. Certain aspects of this will be a review of explanations that have been given, but this material should be studied carefully. Progress in electricity, or in any other field, depends more often on having a complete knowledge of a few basic fundamentals and the enthusiasm and ability to apply these fundamentals than it does on having a superficial knowledge of many facts.

This chapter will, therefore, start with simple resistors, inductors, and capacitors. It will then consider the basic combinations of these, and at the end of the chapter will discuss certain fundamental networks composed of these basic combinations.

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[1] In this chapter the terms *resistor*, *inductor*, and *capacitor* will be used instead of resistance, inductance or coil, and condenser. This is done not only because it is good usage but also to familiarize the student with these terms.

Resistors.—These have been discussed elsewhere, particularly on pages 58 and 74, and this material should be reviewed.

Resistors should, first of all, have a very low temperature coefficient of resistance (page 55), so that their resistance will not change materially with temperature. Resistors should also be constructed so that their effective resistance (page 258) does not change appreciably with frequency. Also, resistors should be adequate in size and heat-radiating ability (page 74) to dissipate the power desired. Resistors should be noninductive; that is, they should produce but a very small magnetic field when they carry a current. Also, resistors should have very low distributed capacitance.

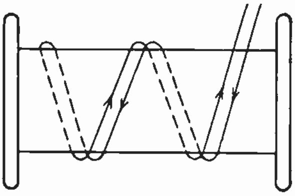

FIG. 192.—Illustrating how a double winding of resistance wire is placed on a coil to reduce the inductance of a resistor.

Resistors that are made of metallicized strips or of semiconducting masses are essentially noninductive. Wire-wound resistors must be especially constructed so that their inductance is negligible. One method of winding noninductive resistors is to wind a double wrapping of wire on the core as indicated in Fig. 192. The same current will then pass through adjacent wires in the *opposite* directions. In this way the tendency of one wire to produce a magnetic field is neutralized by the tendency of the other wire to produce a magnetic field. Very little field is produced because of this "bucking," or neutralizing action, so very little inductance can exist between the terminals of the resistor.

Resistors may also be made noninductive by winding the wire on a thin flat card. The flux produced by unit current is a measure of the inductance (page 159). If the reluctance of the magnetic circuit is high, the flux produced will be low. Reluctance is inversely proportional to the area of the core. If a resistor is wound on a thin card, the cross section of the core is very small, the reluctance is quite large, the flux produced will be low, and the inductance will be negligible. Such resistors (Fig. 193) are widely used in telephone switchboards where they have the additional advantage of occupying but little space. Inductance in resistors acts *in series* with the resistance.

Capacitance in resistors acts *in parallel* with the resistance. The capacitance between turns can be kept low by using wire of high resistance so but few turns need be used. Also, keeping the turns spread apart lowers the capacitance (but may increase the inductance). Usually, what little stray capacitance exists between the turns of a wire-wound resistor can be tolerated in any circuits where such resistors are used. Stray capacitance in metallicized-strip resistors and in composition resistors is also usually tolerable. Of course, these statements apply at audio and at the usual radio frequencies. At ultra-high frequencies and in work with microrays, stray effects are extremely bothersome. The equivalent circuit for a resistor is shown in Fig. 194.

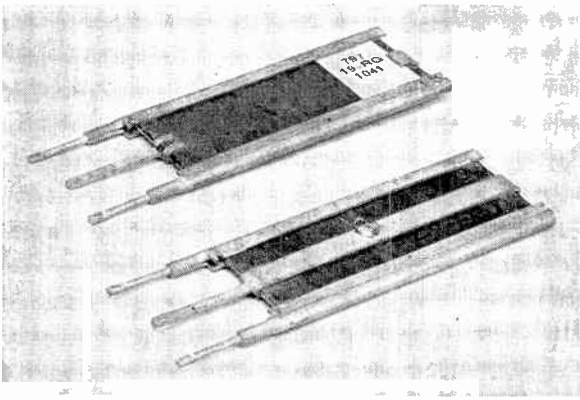

FIG. 193.—A type of wire-wound resistor used in communication. The wire is wound on a thin card, reducing the cross-sectional area A, and hence the inductance as explained in the text. (*Courtesy of Bell Telephone Laboratories.*)

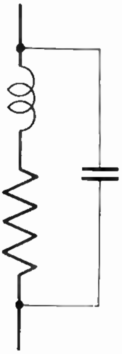

FIG. 194.—A resistor contains inductance and stray capacitance which may be of importance at very high frequencies.

Inductors.—It is more difficult to make good inductors than good resistors or capacitors. Inductors are bothered with effective resistance (page 257), which acts in series with the turns of the coil, and with capacitance, which acts in parallel with the turns of the coil.

Resistance can be kept low by using wires of low resistance if the coil has an air core, and by keeping the field produced by the coil from linking with, and inducing eddy currents in, metallic objects such as coil shields. If the coil has a core of iron or other magnetic material, then reducing any of the factors that cause effective resistance (page 258) will improve the inductor. Coils are rated as to their **figure of merit** or **Q** which is designated as *the ratio of the inductive reactance to the resistance.* Thus,

$$Q = \frac{\omega L}{R}. \tag{63}$$

Of course, the R in this expression is the effective resistance which may vary widely with frequency, a fact that is often overlooked. Also, the frequency at which a given value of Q applies should be stated.

The way in which distributed capacitance acts between the turns of a coil is illustrated by Fig. 195. At very high frequencies more current actually may pass between the terminals of some coils through the distributed capacitance than is flowing through the windings themselves. Such a coil would, at these frequencies, *exhibit the characteristics of a capacitor.* Thus it is important to use coils for the purpose for which they are designed; an audio-frequency choke coil might be useless in radio-frequency circuits. Many methods have been devised for reducing the distributed capacitance between turns. Fine wire, with turns spaced a small distance apart and crisscrossed rather than laid parallel, reduces the distributed capacitance in radio-frequency chokes. Also, it may be better to obtain the desired inductance by adding several small inductors in series than to use one large closely spaced winding. This construction is shown in Fig. 196. At audio

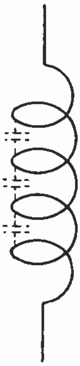

FIG. 195.—Distributed capacitance exists between the turns of a coil. Some resistance is also introduced by the wire composing the coil.

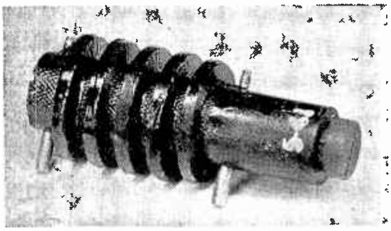

FIG. 196.—Showing the construction of a radio-frequency choke coil. The windings are sectionalized and otherwise arranged to reduce the stray or distributed capacitance. (*Courtesy of Bell Telephone Laboratories.*)

frequencies the distributed capacitance of coils is seldom bothersome (for exceptions see pages 362 and 458).

Capacitors.—As was shown on page 198 the area of the metal foil used in a paper condenser is quite large. This means that some of the current flowing in and out of the condenser has a path of some length to traverse. There will, therefore, be resistance and inductance in this path. (Remember that even a straight wire has inductance.) If there are appreciable losses in the dielectric, power will be dissipated and the condenser will have effective resistance (page 258). The equivalent series circuit for a condenser is as shown in Fig. 197. In the air condensers and mica condensers used in radio circuits, the losses are quite low and the inductance is negligible in most instances if short leads are used.

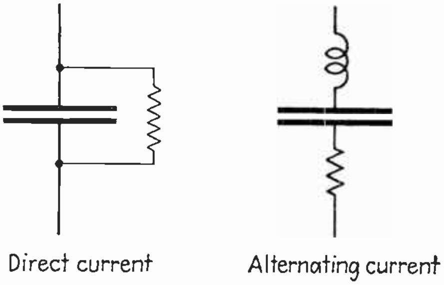

FIG. 197.—Equivalent circuits for a condenser, showing the direct-current and the alternating-current series equivalent circuits. Parallel equivalent circuits may also be drawn for the alternating-current case, but the series equivalent circuit is usually more useful. Often the inductance shown is negligible, and for condensers with no losses, the equivalent series resistance approaches zero.

To summarize the preceding three sections: "Stray" resistance, inductance, and capacitance effects exist in resistors, inductors, and capacitors. At audio and the lower radio frequencies these effects are usually not very bothersome, but at high frequencies they are very important. Units are designed to minimize these effects. In the pages that follow, it will be assumed that perfect resistors, inductors, and capacitors are available.

Resistance and Inductance in Series.—This was considered on page 262. By assuming that the resistance R of the resistor and the inductance L of the inductor are constant with frequency and that the coil has no losses or stray effects, the impedance Z of the resistor and inductor in series is, at any frequency,

$$Z = \sqrt{R^2 + (2\pi fL)^2} \text{ ohms.} \tag{64}$$

The relations are shown in Fig. 198. As is evident, the impedance of this series circuit *increases* with frequency.

If an alternating current of *constant* effective value but of varying frequency is passed through this circuit, the voltage across the circuit will be $E = IZ$. Since Z *increases* with frequency, the voltage across the series circuit composed of a resistor and an inductor will increase as the frequency is increased.

If an alternating voltage of *constant* effective value but of varying frequency is impressed on this series combination, then the current will decrease as the frequency is increased.

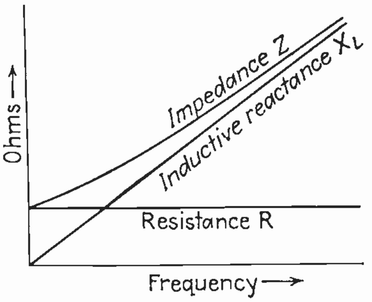

FIG. 198.—Resistance, reactance, and impedance curves for resistance and inductance in series.

At very low frequencies, the circuit will be largely *resistive* because the inductive reactance is low; hence, the current and voltage will be closely in phase, and the power factor will approach unity. As the frequency is increased, however, the circuit becomes more and more reactive, and the current and voltage become increasingly out of phase.

Resistance and Capacitance in Series.—This was considered on page 264. By assuming that the resistance R of the resistor and the capacitance C of the capacitor are constant with frequency and that the capacitor has no losses or stray effects, the impedance Z of the resistor and capacitor is, at any frequency,

$$Z = \sqrt{R^2 + (1/2\pi fC)^2} \text{ ohms.} \tag{65}$$

These relations are shown in Fig. 199. As is evident, the impedance of this series circuit *decreases* with frequency.

If an alternating current of *constant* effective value but of varying frequency is passed through this circuit, the voltage across the circuit will be $E = IZ$. Since Z *decreases* with frequency, the voltage across the series circuit composed of a resistor and a capacitor will decrease as the frequency is increased.

If an alternating voltage of *constant* effective value but of varying frequency is impressed on this series combination, then the current will increase as the frequency is increased.

At very low frequencies the circuit will be largely *reactive* because the capacitive reactance is so high; hence, the current and voltage will be greatly out of phase, and the power factor will approach zero. As the frequency is increased, however, the circuit becomes more and more resistive, and the current and voltage become more and more in phase. Note the inverse relationship existing between resistance and inductance in series and resistance and capacitance in series as the frequency is increased.

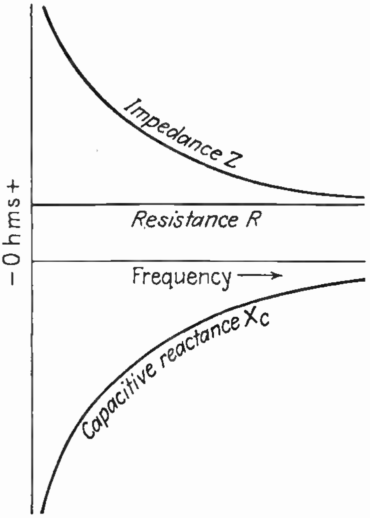

FIG. 199.—Resistance, reactance, and impedance curves for resistance and capacitance in series. Capacitive reactance is negative.

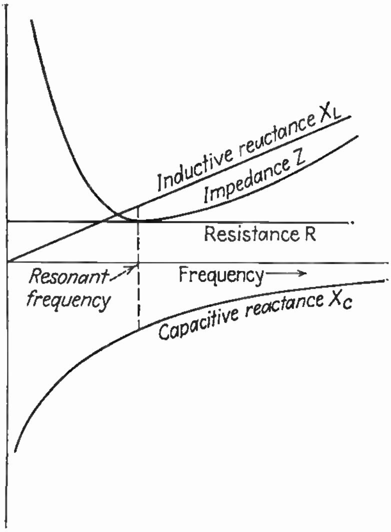

FIG. 200.—Showing how the reactances and impedance vary with frequency in a circuit composed of resistance, inductance, and capacitance in series. At the resonant frequency the inductive and capacitive reactances are equal in magnitude and neutralize each other. Then, the impedance of the circuit equals the resistance.

Resistance, Inductance, and Capacitance in Series.—This was considered on page 268. The characteristics of this circuit must be the combined characteristics of Figs. 198 and 199. The impedance of the circuit at any frequency will be

$$Z = \sqrt{R^2 + [2\pi fL - (1/2\pi fC)]^2} \text{ ohms.} \qquad (66)$$

These relations are shown in Fig. 200.

At low frequencies the inductive reactance $X_L = 2\pi fL$ is low, but the capacitive reactance $X_C = 1/2\pi fC$ is high. The equivalent impedance of the circuit is accordingly a very high capacitive reactance. At high frequencies the inductive reactance is high, but the capacitive reactance is low. The equivalent reactance is accordingly a very large value of inductive reactance. At some intermediate frequency, *the inductive reactance equals the capacitive reactance, and at this point the circuit is in resonance.* From Eq. (57), page 270, the frequency of resonance was shown to be $f = 1/2\pi\sqrt{LC}$. The equivalent impedance of the circuit is high at low frequencies, drops until the resonance point is reached when the impedance of the circuit equals the resistance [because, as Eq. (66) shows, at some frequency the inductive and capacitive reactances neutralize each other], and then the equivalent impedance rises again until it becomes a very high value at high frequencies.

If an alternating current of *constant* effective value but of varying frequency is passed through the circuit, the voltage across the circuit will be $E = IZ$. Because Z varies as indicated in Fig. 200, the voltage across the circuit will be high at low frequencies, drop to a small value at resonance, and again rise to high values at high frequencies.

If an alternating voltage of *constant* effective value but of varying frequency is impressed on this series circuit, the current will be low at low frequencies, high at the resonance point, and will again drop to a low value.

At low frequencies the current will be greatly out of phase with the voltage, and the power factor will be low. At the resonance frequency the equivalent impedance of the circuit as viewed[1] at the input terminals will be pure resistance, and the power factor will be unity. At high frequencies the current will again be out of phase with the voltage, and the power factor will be low.

To summarize: The important characteristics of a series resonant circuit are low impedance equal to pure resistance and large in-phase current flow.

Susceptance, Conductance, and Admittance.—The preceding pages have been devoted to *series* circuits, and those which follow will consider *parallel* circuits. On page 26, where parallel direct-current circuits were discussed, it was shown that, in some in-

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[1] This way of regarding a circuit is good, because, in a sense, the voltage source looks into the circuit and then forces in a current of the proper phase relation and magnitude to satisfy circuit conditions.

stances, it was more convenient to treat parallel circuits on a conductance basis than from the resistance standpoint. This is particularly true in series-parallel circuits which can be conveniently solved only if the conductance method is used.

In this chapter, alternating-current parallel circuits are being considered. Such parallel circuits can be solved by the impedance method outlined on page 274. However, the only way in which series-parallel circuits can be solved conveniently is by a method similar to the conductance method of page 26.

In direct-current *series* circuits, resistances are added to obtain the total resistance of a circuit. In direct-current parallel circuits, conductances (reciprocals of resistances) are added to obtain the total conductance of a circuit.

In alternating-current *series* circuits, impedances are added (by adding their separate components, of course) to obtain the total impedance of a circuit. In alternating-current parallel circuits, **admittances** are added (again, their separate components) to find the total admittance of a circuit.

In *direct-current* circuits, the conductance is equal to the reciprocal of the resistance. In alternating-current circuits, *the admittance is equal to the reciprocal of the impedance.*

Now impedance is given by the general formula $Z = R + jX$. Taking

$$Y = \frac{1}{Z} = \frac{1}{R + jX} = \frac{1}{R + jX}\,\frac{R - jX}{R - jX}$$

$$= \frac{R - jX}{R^2 + X^2} = \frac{R}{Z^2} - j\frac{X}{Z^2} = G - jB. \quad (67)$$

Therefore, $Y = G - jB$, where Y is the **admittance,** $G = R/Z^2$ is the **conductance,** and $B = X/Z^2$ is the **susceptance,** all measured in **mhos.** Note that alternating-current conductance is equal to R/Z^2, whereas direct-current conductance is merely $1/R$. Observe also that in the general form of the equation for admittance the term $-j$ instead of $+j$ occurs.

The practical advantage of all this will become more evident as parallel circuits are studied. For the moment, let it merely be stated that

1. The voltage across a circuit multiplied by the admittance of the circuit gives the current flowing in that circuit.

2. The voltage across a circuit multiplied by the conductance of the circuit gives the *in-phase component* of current in that circuit.

3. The voltage across a circuit multiplied by the susceptance of the circuit gives the 90-degree out-of-phase or **quadrature component** of current in that circuit.

Resistance and Inductance in Parallel.—This was discussed on page 265. It was there shown that in parallel circuits the current in each branch was computed, and then the separate currents were added vectorially to find the total current. It often happens that seemingly difficult things are easy to apply, and this is indeed true for admittance. The manner in which it is used to solve a parallel circuit will now be explained.

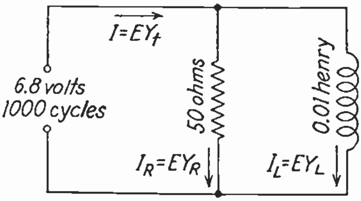


FIG. 201.—Circuit composed of resistance and inductance in parallel.

Example.—In Fig. 201 is shown a parallel circuit. It is desired to find the total current and its phase position, the voltage being taken as the base because it is common to both branches.

Solution.—Step 1. Calculate the impedance of the inductor. $X_L = 2\pi fL = 6.2832 \times 1000 \times 0.01 = 62.8$ ohms. $Z_L = R + jX = 0 + j62.8$ ohms, because the resistance of the inductor is negligible.

Step 2. Write the equation for the impedance of the resistor. $Z_R = 50 + j0$ ohms.

Step 3. Calculate the admittance of the inductor and of the resistor.

$$Y_L = \frac{1}{Z_L} = \frac{1}{0 + j62.8} = \frac{1}{0 + j62.8}\,\frac{0 - j62.8}{0 - j62.8} = \frac{0 - j62.8}{3950} = 0 - j0.0159 \text{ mho.}$$

$$Y_R = \frac{1}{Z_R} = \frac{1}{50 + j0} = \frac{1}{50 + j0}\,\frac{50 - j0}{50 - j0} = \frac{50 - j0}{2500} = 0.02 - j0 \text{ mho.}$$

Step 4. Calculate the total admittance of the circuit. This will be the sum of the separate admittance. $Y_t = Y_R + Y_L = (0.02 - j0) + (0 - j0.0159) = 0.02 - j0.0159$ mho.

Step 5. Calculate the current. The current through a circuit equals the voltage across a circuit multiplied by the admittance of the circuit. Since it was stated that the voltage was to be the base, it will be $E = 6.8 + j0$. $I_t = EY_t = (6.8 + j0)(0.02 - j0.0159) = 0.136 - j0.108$ ampere. This is the vector expression for the current through the circuit composed of the resistor and inductor in parallel. Note that the current will lag the voltage as it should because the circuit is inductive.

The equivalent impedance (see page 275) of the circuit of Fig. 201 is equal to the reciprocal of the total admittance. Therefore,

$$Z_e = \frac{1}{Y_t} = \frac{1}{0.02 - j0.0159} = \frac{1}{0.02 - j0.0159}\,\frac{0.02 + j0.0159}{0.02 + j0.0159}$$

$$= \frac{0.02 + j0.0159}{0.000653} = 30.6 + j24.3 \text{ ohms.}$$

Now this value is for 1000 cycles per second. It is of interest to investigate the variations over a wide frequency range in the equivalent impedance offered between the terminals of the circuit of Fig. 201.

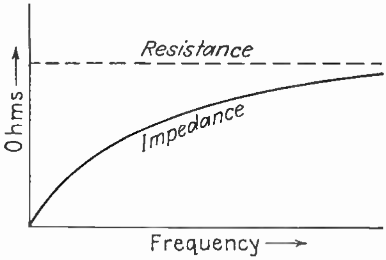


FIG. 202.—The equivalent impedance of the circuit of Fig. 201 is very low at low frequencies, and increases as the frequency is made higher.

When the frequency is very low, the reactance $X_L = 2\pi fL$ offered by the inductor is very low, and a large current will be taken by the inductor. This current will lag the voltage by almost 90 degrees. At this frequency, the equivalent impedance approaches a value of zero inductive reactance. Of course current is taken by the resistor, but this is insignificant compared with that taken by the inductor. As the frequency is increased and becomes very high, the reactance of the inductor becomes great, and the current it takes is negligible. At very high frequencies, therefore, substantially the only current taken from the source is that through the resistor. At high frequencies the equivalent impedance becomes approximately the resistance of the resistor, and of course the current taken is almost in phase with the voltage. These relations are shown in Fig. 202.

Comparing these calculations with the graphic solution for a similar problem indicates that the graphic solution is the simpler, and so it is, *for simple problems.* But as mentioned before, there are circuits that cannot be solved conveniently by means other than the admittance method explained here.

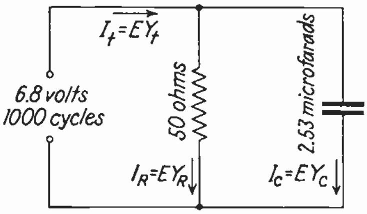


FIG. 203.—Circuit composed of resistance and capacitance in parallel.

Resistance and Capacitance in Parallel.—This was discussed on page 267. The method of attack is the same as in the preceding section.

Example.—With reference to the parallel circuit of Fig. 203, it is desired to calculate the total current and its phase position with the voltage taken as the base because it is common.

Solution.—Step 1. Write the equation for the impedance of the capacitor and of the resistor. $X_C = 1/2\pi fC = 1/(6.2832 \times 1000 \times 2.53 \times 10^{-6}) =$ 62.9 ohms. $Z_C = 0 - j62.9$ ohms. Note that the general form for impedance is $Z = R + jX$. Since capacitive reactance is negative, $Z_C = R - jX_C$. The impedance of the resistor is $Z_R = 50 + j0$ ohms.

Step 2. Calculate the admittance of the capacitor and of the resistor.

$$Y_C = \frac{1}{Z_C} = \frac{1}{0 - j62.9} = \frac{1}{0 - j62.9}\,\frac{0 + j62.9}{0 + j62.9} = \frac{0 + j62.9}{3950} = 0 + j0.0159 \text{ mho.}$$

$Y_R = 0.02 - j0$ (as in the preceding example).

Step 3. Calculate the total admittance of the circuit.

$Y_t = Y_R + Y_C = (0.02 - j0) + (0 + j0.0159) = 0.02 + j0.0159$ mho.

Step 4. Calculate the total current. $I_t = EY_t = (6.8 + j0)(0.02 + j0.0159) = 0.136 + j0.108$ ampere. This is the vector expression for the current taken by the resistor and capacitor in parallel. Note that the reactive portion of this current is *positive* as it should be for a circuit containing capacitance where the current leads the voltage.

If a calculation is made similar to that of the preceding example, it will be found that the equivalent impedance of the circuit of Fig. 203 is $Z_e = 30.6 - j24.3$. This is at 1000 cycles per second. When the frequency is very low, the reactance of the capacitor will be very great, and the current taken by it will be negligible. At a low frequency, therefore, the equivalent impedance of the circuit will be the approximate resistance of the resistor, and the current will be in phase with the voltage. At high frequencies, the reactance of the capacitor becomes very low, and the current taken by it becomes very large. At high frequencies, therefore, the equivalent impedance of the circuit becomes substantially a low value of capacitive reactance, and the current leads almost 90 degrees ahead of the voltage. These relations are shown in Fig. 204. Note the inverse relationship existing between resistance and inductance in parallel and resistance and capacitance in parallel.

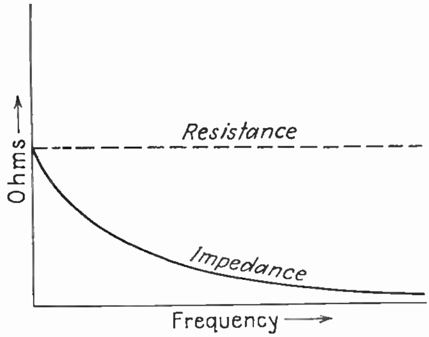


FIG. 204.—The equivalent impedance of the circuit of Fig. 203 at low frequencies is equal to the resistance and decreases as the frequency is made higher.

Resistance, Inductance, and Capacitance in Parallel.—This was considered on page 274. There it was found that at the antiresonant frequency the lagging current taken by the inductor neutralized the leading current taken by the capacitor and that the only resultant current taken from the source (at this antiresonant frequency) was that taken by the resistor. At this frequency, the equivalent impedance was the resistance of the resistor. These relations will be studied again.

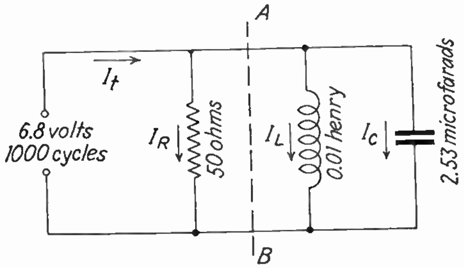


FIG. 205.—Circuit composed of resistance, inductance, and capacitance in parallel.

Example.—A parallel circuit is composed of the elements of Figs. 201 and 203 and is shown in Fig. 205. Make a complete analysis of this circuit.

Solution.—Step 1. Write the expression for the admittance of each branch These were determined in the two preceding examples.

$$Y_R = 0.02 - j0, \ Y_L = 0 - j0.0159, \ Y_C = 0 + j0.0159.$$

Step 2. Write the expression for the total admittance.

$$Y_t = Y_R + Y_L + Y_C = 0.02 - j0 \text{ mho.}$$

Step 3. Calculate the total current that will flow. This is a parallel circuit, so the voltage will be taken as the base.

$$I_t = EY_t = (6.8 + j0)(0.02 - j0) = 0.136 - j0 \text{ ampere.}$$

It is evident that this circuit is in antiresonance (often called resonance, parallel resonance, or current resonance) at 1000 cycles. There is a current through the capacitor equal to $I_C = EY_C = (6.8 + j0)(0 + j0.0159) = 0 + j0.108$ ampere. As is evident from the $+j$, this current leads the voltage by 90 degrees. Also, there is a current through the inductor equal to $I_L = EY_L = (6.8 + j0)(0 - j0.0159) = 0 - j0.108$ ampere; this current lags the voltage by 90 degrees. As previously stated, these two currents neutralize each other, and for this reason no current flows from the source to the right of the line AB. Of course in this statement a pure inductor and capacitor are assumed.

The condition will now be discussed in which the inductor is not perfect but contains a small amount of resistance. This condition was treated on page 278. If the inductor contains a small amount of resistance, then the current through the inductor will lag the voltage by an angle less than 90 degrees. Nevertheless, a condition of antiresonance can still exist, because, if the out-of-phase component of the current through the inductor equals the current through the capacitor, no reactive component will be supplied by the generator, and the equivalent impedance of the parallel circuit will be pure resistance.

According to the third statement on page 312, the voltage across a circuit multiplied by the susceptance of the circuit gives the out-of-phase component of the current through the circuit. As has just been shown, when the out-of-phase component of a capacitor is equal to the out-of-phase component of a parallel inductor, a condition of antiresonance exists. From Eq. (67), the susceptance of a capacitor is $B_C = X_C/Z_C^2$ and the susceptance of an inductor is $B_L = X_L/Z_L^2$. Now when a capacitor and an inductor are in parallel, the resistance of the capacitor is negligible, but the resistance of an inductor often is not negligible. The susceptance of the capacitor is $B_C = X_C/Z_C^2 = X_C/(R_C^2 + X_C^2)$, but for a good condenser $R_C = 0$, so $B_C = X_C/X_C^2 = 1/X_C = 2\pi fC$, where $X_C = 1/2\pi fC$. The susceptance of an inductor having resistance is $B_L = X_L/Z_L^2 =$

$2\pi fL/[R_L^2 + (2\pi fL)^2]$. Equating these two susceptances to find the frequency of antiresonance, here designated as f_{ar}

$$2\pi fC = \frac{2\pi fL}{R_L^2 + (2\pi fL)^2} \qquad \text{and} \qquad f_{ar} = \frac{1}{2\pi}\sqrt{\frac{L - CR_L^2}{CL^2}}. \quad (68)$$

Note that the resistance of the inductor enters into the equation for antiresonance, and hence *in a parallel circuit the resistance of the inductor helps determine the antiresonant frequency.* Also note that the resistance does not enter into determining the resonant point of a series circuit (page 270). In a parallel circuit, if the resistance of *both* the capacitor and inductor is *negligible,* then Eq. (68) becomes $f_{ar} = \dfrac{1}{2\pi\sqrt{LC}}$, and this equation is the same as that for the series circuit.

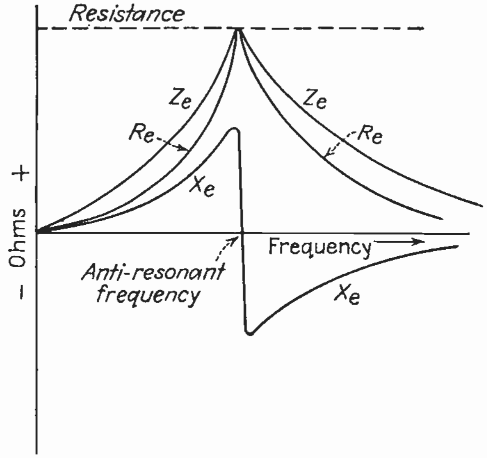


FIG. 206.—Showing the way in which the equivalent resistance, reactance, and impedance vary for the parallel circuit of Fig. 205. It will be noted that the impedance equals the resistance at antiresonance. Circuits which have little resistance in them will have a "sharp" impedance curve, and will be "sharply" tuned. Such circuits have a high Q, or ratio of $\omega L/R$.

Usually, when either a series or a parallel circuit is used, it is to obtain certain impedance variations *between its terminals.* Thus, the manner in which the equivalent or input impedance of a parallel circuit varies with frequency is a very important consideration. The variations for the circuit of Fig. 205 are shown in Fig. 206. At very *low* frequencies the reactance of the inductor is very small, and it takes a very large current which lags the voltage by about 90 degrees. Thus, the equivalent impedance will be very low and highly inductive. At very *high* frequencies the reactance of the capacitor is very low, and hence it takes a very large current which leads the voltage by about 90 degrees. At high frequencies, the impedance will be very low and will be highly reactive. At some intermediate frequency, the leading current taken by the capacitor will equal and neutralize the lagging current taken by the inductor. Then, as previously mentioned, no resultant current will flow to the right of line AB of Fig. 205, and at this antiresonant frequency the only current taken will be that by the resistor. The equivalent or input impedance at this frequency will, therefore, equal the resistance of the resistor. If the inductor has a small amount of resistance, then a small current will flow to the inductor at antiresonance, and the equivalent input impedance of the parallel circuit will be slightly less than the resistance of the resistor.

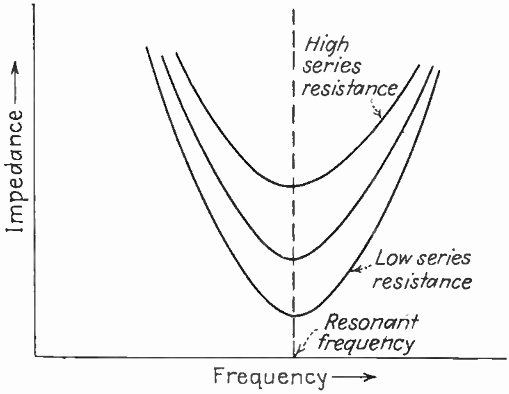
With reference to Fig. 206, it has been explained why the equivalent impedance Z_e varies as indicated. Also, it has been shown that the equivalent reactance of the circuit is inductive below antiresonances, zero at antiresonance (because the equivalent impedance equals the resistance), and capacitive above antiresonance. These relations are shown. Since in general $Z_e = \sqrt{R_e^2 + X_e^2}$, the value of the equivalent resistance R_e must fall below the impedance curve, somewhat as shown.

If an alternating current of *constant* effective value but of varying frequency is passed through this circuit, the voltage across the circuit will be $E = IZ_e$. Because the equivalent impedance Z_e varies as indicated in Fig. 206, the voltage across the circuit will be low at both low and high frequencies but will be maximum at the frequency of antiresonance.

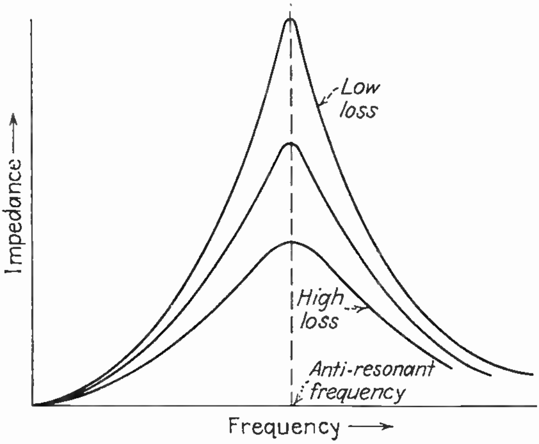
If an alternating voltage of *constant* effective value but of varying frequency is impressed on this circuit, a large current which is lagging by almost 90 degrees will flow at low frequencies, and a large current which is leading by almost 90 degrees will flow at high frequencies. At the frequency of antiresonance, a small current will flow, and this current will be in phase with the voltage.

To summarize: Series and parallel circuits are opposites. The series circuit offers a low impedance at resonance, and the parallel circuit offers a high equivalent impedance at antiresonance. These facts are of the *greatest importance* in communication.

Sharpness of Resonance.—A series circuit composed of *pure* resistance, inductance, and capacitance was discussed on page 309.



(a)-Series circuit



(b)-Parallel circuit

FIG. 207.—The sharpness of resonance is greatly affected by the loss in the circuit. (*a*) effect of the series resistance on the characteristics of a series circuit, and (*b*) effect of the parallel resistance (or losses in a coil) on the characteristics of a parallel circuit. A low resistance gives a low loss, and in radio this is termed a high Q (page 306). Conversely, high resistance gives high loss, and the circuit has a low Q.

It was shown that the impedance was high at frequencies both below and above the resonant frequency. At the resonant frequency, the impedance of the circuit became pure resistance and dropped to a low value equal to the resistance of the resistor. If the series circuit consisted only of an inductor and a capacitor, then at the resonant frequency the impedance would drop to a value of resistance equal to the effective resistance of the coil. Circuits in which the resistance is low and in which the impedance accordingly drops to a very low value at resonance are said to be **sharply tuned.** This is illustrated by Fig. 207*a*.

In discussing the parallel circuit composed of *pure* resistance, inductance, and capacitance, it was shown on page 310 that the equivalent impedance was low at both low frequencies and high frequencies. At the antiresonant frequency, the equivalent impedance of the parallel circuit increased and became pure resistance equal to the value of the parallel resistor. If the circuit consists only of an inductor and a capacitor in parallel, then the impedance will rise to a very high value determined by the effective resistance of the inductor. The value of the equivalent resistance at antiresonance will equal the reciprocal of the conductance $G_L = R_L/(R_L^2 + X_L^2)$. In a parallel circuit composed of a good condenser and a coil having a low effective resistance, the impedance will rise to a very high value and, again, the circuit will be said to be *sharply tuned.* This action is illustrated by Fig. 207*b*.

Series-parallel Circuits.—As was previously mentioned, a series-parallel circuit offers certain characteristics that make necessary the use of the admittance method for its convenient solution. A large variety of series-parallel circuits are encountered in practice. For the purpose of illustrating the principles involved, the circuit shown in Fig. 208 has been chosen. To save laborious calculations and better to illustrate the principles involved, a simple circuit and convenient values have been chosen.

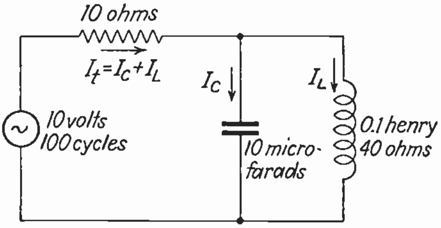


FIG. 208.—A series-parallel circuit.

Example.—Solve the circuit of Fig. 208 for the current through the inductor.

Solution.—Of course the current through the inductor will equal the voltage across the inductor divided by its impedance. But, the voltage across the inductor is not known; it equals the total voltage minus the voltage drop in the resistor, and this voltage drop involves the current through the inductor. Thus, the following solution is required. Certain of the calculations have not been shown in detail because the methods have been fully explained previously.

Step 1. Calculate the equivalent impedance of the inductor and capacitor in parallel.

$$Z_L = 40 + j(2\pi f L) = 40 + j62.8 \text{ ohms.}$$

$$Z_C = 0 - j(1/2\pi f C) = 0 - j159 \text{ ohms.}$$

$$Y_L = \frac{1}{Z_L} = \frac{1}{40 + j62.8} = 0.0072 - j0.0113 \text{ mho.}$$

$$Y_C = \frac{1}{Z_C} = \frac{1}{0 - j159} = 0 + j0.0063 \text{ mho.}$$

$$Y_t = Y_L + Y_C = 0.0072 - j0.005 \text{ mho.}$$

$$Z_e = \frac{1}{Y_t} = \frac{1}{0.0072 - j0.005} = 93.7 + j65.2 \text{ ohms.}$$

Step 2. Find the total impedance connected to the generator.

$$Z_t = Z_e + Z_R = (93.7 + j65.2) + (10.0 + j0) = 103.7 + j65.2 \text{ ohms.}$$

Step 3. Take the voltage as the base, and find the total current I_t.

$$I_t = \frac{E_t}{Z_t} = \frac{10 + j0}{103.7 + j65.2} = 0.0692 - j0.0434 \text{ ampere.}$$

Step 4. Calculate the voltage drop across the series resistor.

$$E_R = I_R R = (0.0692 - j0.0434)(10 + j0) = 0.692 - j0.434 \text{ volt.}$$

Step 5. Calculate the voltage across the parallel portion.

$$E_p = E_t - E_R = (10 + j0) - (0.692 - j0.434) = 9.308 + j0.434 \text{ volts.}$$

Step 6. Calculate the current through the inductor.

$$I_L = E_p Y_L = (9.308 + j0.434)(0.0072 - j0.0113) = 0.072 - j0.102 \text{ ampere.}$$

Coupled Circuits.—Mutual inductance was discussed on page 163. Two circuits not in metallic contact are often coupled electrically through the **mutual inductance** between them. It is on this principle that the transformer and other familiar devices operate.

Two coils having a mutual inductance of M henrys are shown in Fig. 209. The alternating current I_p in the primary produces an alternating magnetic field, and this magnetic field links with the turns of the secondary. This alternating magnetic field induces a voltage in the secondary. This induced voltage is $E_s = -j\omega M I_p$, where E_s is the effective value of the voltage induced in the secondary, when M is the mutual inductance in henrys, I_p is the effective value of the primary current, and $\omega = 2\pi f$. The value $-j$ indicates that the voltage E_s induced in the secondary lags 90 degrees behind the current I_p inducing it.

Now if the switch S in the secondary circuit is closed so that a current flows, this secondary will attempt to establish a magnetic field. Two magnetomotive forces now exist; one caused by the current in the primary coil, and the other caused by the current in the secondary coil. These two magnetomotive forces will oppose each other, and the result is that a smaller resultant magnetic field will exist than if the switch S is opened. The current that flows in the primary of a coil having no losses is opposed by the back electromotive force induced in the primary. If a current flows in the secondary, there will be less flux linking the primary, and hence the current relations are altered. In other words, *the effect of a current flow in the secondary is felt in the primary.*

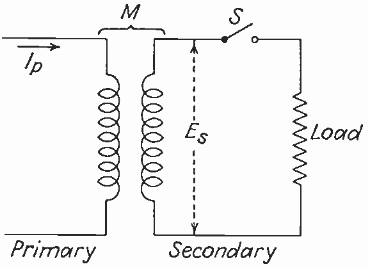


FIG. 209.—The effect of the secondary on the primary when switch S is closed is to couple an impedance of $(\omega M)^2/Z_s$ into the primary, where Z_s is the *total* secondary impedance including that of the load plus that of the secondary of the coil.

Since the current flow in the primary is altered by a current flow in the secondary, the *impedance of the primary circuit is changed by the current in the secondary.* The effect of the secondary circuit on the primary is to add an impedance $(\omega M)^2/Z_s$ in the primary. This will add to the total primary impedance Z_p in determining the current flow in the primary.

If two coils are **closely coupled,** then substantially all the flux produced by the primary links with the secondary. Thus, a primary and a secondary properly placed on a core of excellent flux-conducting properties are closely coupled and the **coefficient of coupling** approaches 1.00. This condition is represented by a good iron- or Permalloy-cored transformer. Some leakage flux exists in all devices, however, and the coupling is less than 100 per cent (or less than 1.00). The coefficient of coupling is given by the relation

$$k = \frac{M}{\sqrt{L_p L_s}}, \qquad (69)$$

where M is the mutual inductance, L_p is the primary inductance, and L_s is the secondary inductance, all in henrys.

Coupled-circuit Solutions.—As stated in the preceding section, the effect of a secondary circuit is to couple an impedance of $(\omega M)^2/Z_s$ into the primary. An illustration will now be given.

Example.—In the circuit of Fig. 210, the frequency is 1000 cycles per second. Calculate the effect of the secondary on the primary.

Solution.—Step 1. Calculate Z_s. Of course, all solutions must be in vector algebra. The impedance Z_s is the *total* secondary impedance. $Z_s = (R_s + R) + j2\pi f L_s = (3.88 + 5) + j(6.2832 \times 1000 \times 0.0058) = 8.88 + j36.5$ ohms.

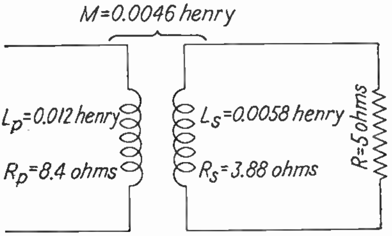


FIG. 210.—A coupled circuit.

Step 2. Calculate the effect of this reflected into the primary.

$$Z_{\text{reflect}} = \frac{(2\pi f M)^2}{Z_s} = \frac{(6283 \times 0.0046)^2}{8.88 + j36.5} = \frac{834}{8.88 + j36.5}$$

$$= \frac{834}{8.88 + j36.5} \cdot \frac{8.88 - j36.5}{8.88 - j36.5} = \frac{7420 - j30450}{1410} = 5.26 - j21.8.$$

Step 3. Calculate the total primary impedance. The total primary impedance is $Z_{tp} = Z_p + Z_{\text{reflect}} = [8.4 + j(6.2832 \times 1000 \times 0.012)] + (3.59 - j22.3) = (8.4 + j75.4) + (5.26 - j21.8) = 13.66 + j53.6$ ohms.

Note that the effect of coupling an inductive secondary into the primary has the effect of adding a *capacitive reactance* into the primary, because the sign preceding the j of the reflected impedance is opposite to the secondary impedance. An investigation will show that coupling a capacitive secondary will be equivalent to adding *inductive reactance* into the primary.

To summarize: By using the simple relations just outlined, coupled circuits may be broken down into equivalent noncoupled circuits. In the example just considered, with the equivalent primary impedance known, the primary current can be computed if a given voltage is impressed across it. With this primary current known, the induced secondary voltage can be found, and this, divided by the entire secondary impedance, will give the secondary current. Of course, vector algebra should be used throughout. If the reader desires the derivation of the equation for this reflected impedance, it can be found in most books devoted to advanced communication theory.

The Transformer.—As has been previously mentioned, the transformer consists of a primary and a secondary. In radio-frequency circuits it is common practice to use transformers with air cores, although transformers with finely powdered and compressed iron cores (or with other similarly constructed cores to reduce eddy currents) are sometimes used. In general, in radio-frequency transformers the coefficient of coupling is considerably less than unity, and the coupled-circuit theory such as previously explained must be used.

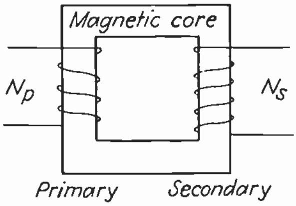


FIG. 211.—Transformer with N_p primary turns and N_s secondary turns.

If the coupling is very good, then approximations may be used which simplify calculations. These hold very closely for 60-cycle power transformers and approximately for audio-frequency transformers which always have cores of magnetic materials.

Considering the transformer of Fig. 211, and assuming that the coupling is 100 per cent, then *all* the magnetic flux produced by the primary links the secondary. Then, from the theory on page 159, the effective voltage induced in the primary (which in an ideal transformer must equal the impressed voltage) will be $E_p = 1.11\ N_{p\phi}/10^8t$, and the voltage induced in the secondary will be $E_s = 1.11\ N_{s\phi}/10^8t$. Dividing the last equation by the first gives

$$\frac{E_s}{E_p} = \frac{N_s}{N_p} \qquad \text{or} \qquad E_s = \frac{N_sE_p}{N_p}. \tag{70}$$

This equation states that the voltage induced in the secondary of a closely coupled transformer equals the primary voltage multiplied by the ratio of secondary to primary turns.

Again on the assumption that the transformer is an ideal device with close coupling and no losses, the volt-amperes in the primary and the volt-amperes in the secondary must be the same. This is based on the fact that power $= EI \cos \theta$, that an *ideal* transformer must deliver as much power as it receives, and that it cannot change the *EI product* or the phase angle θ. Thus, $E_sI_s = E_pI_p$, and $E_s/E_p = I_p/I_s$; but, it has been shown that $E_s/E_p = N_s/N_p$, and therefore,

$$\frac{I_p}{I_s} = \frac{N_s}{N_p} \qquad \text{and} \qquad I_s = \frac{I_pN_p}{N_s}. \tag{71}$$

Thus it is seen that the current ratio in the primary and secondary of a closely coupled transformer is opposite to the voltage ratio. In other words, if a transformer *steps up* voltage, it *steps down* current, and vice versa.

In 60-cycle power work, transformers are known as "voltage changers," but in communication practice they are often regarded as "impedance changers." If the primary of a transformer is connected to a source, the primary impedance is $Z_p = E_p/I_p$, and if the secondary is connected to a load, the secondary impedance is $Z_s = E_s/I_s$. But, $E_s = E_pN_s/N_p$, and $I_s = I_pN_p/N_s$, thus

$$Z_s = \frac{E_pN_s}{N_p} \times \frac{N_s}{I_pN_p} = \frac{N_s^2E_p}{N_p^2I_p} = \left(\frac{N_s}{N_p}\right)^2 Z_p$$

and

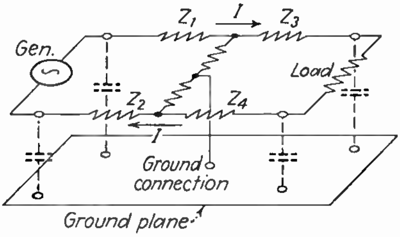
$$Z_p = \left(\frac{N_p}{N_s}\right)^2 Z_s. \tag{71a}$$

That is, the impedance Z_p measured across the primary winding of an *ideal* transformer when the secondary is connected to a load Z_s is equal to Z_s times the turns ratio squared. These equations apply closely to good audio-frequency transformers used in communication.

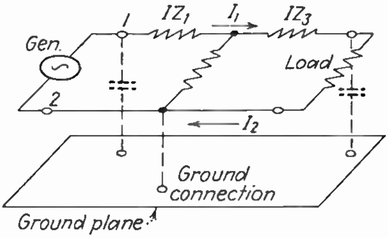
Balanced and Unbalanced Networks.—Communication networks (a general term including *all* circuits) are of two general types: (1) those which are **balanced** with respect to ground, and (2) those which are **unbalanced** with respect to ground. The term "ground" may be the earth itself or some other object, such as a large conducting plane.

A balanced network is shown in Fig. 212*a*. If the impedances in each side are the same, and if the stray capacitances to ground are the same, then the current in each side of the network will be the same and the circuit is balanced. If the circuit is to be grounded, this should be done by connecting the center point of the shunt impedance.

An unbalanced network is shown in Fig. 212*b*. In this circuit the impedance of one side of the circuit is different from that of the other. Although the stray capacitances to ground may still be about the same, the series IZ drops in the two sides will be different, and different currents will flow through the stray capacitances to ground. In other words, because of the IZ drops in wire 1, current will flow down through and up through the stray capacitances indicated; no such action will exist for wire 2. Thus, I_1 will be different from I_2, and the circuit will be unbalanced.



(a)-A balanced network having equal impedances Z in each line wire



(b)-An unbalanced network having unequal impedances in each line wire

FIG. 212.—Circuits which are balanced and unbalanced with respect to ground.

Whether a circuit is to be balanced or unbalanced depends on the performance expected. Push-pull vacuum-tube circuits are balanced. Single-tube circuits are unbalanced. Long-distance telephone circuits are balanced. Many remote rural telephone circuits, operated with one wire and an earth return, are unbalanced. In general, this can be stated: If one side of a circuit is grounded, the circuit is unbalanced; if the center point is grounded, it is balanced. The fundamental rule to follow is this: Never connect unbalanced and balanced circuits directly together; connect them through an isolating transformer, and preferably one with an internal shield which may be grounded. If this is not done the circuits may be noisy, have crosstalk, or may oscillate.

Symmetrical and Nonsymmetrical Networks.—With reference to Fig. 212*a*, if $Z_1 = Z_2 = Z_3 = Z_4$, then the network or circuit is **symmetrical**; that is, it is identical as viewed *in* at each end. If $Z_1 = Z_2$, but these do not equal $Z_3 = Z_4$, then the network is nonsymmetrical. This means that the impedance as viewed *in* from the generator end will not be the same as the impedance viewed *in* at the receiving end.

The same reasoning applies to the network of Fig. 212*b*. If Z_1 does not equal Z_3, then the impedances as viewed from either end will not be the same. Networks may, therefore, be classified as follows:

1. Balanced symmetrical.
2. Balanced nonsymmetrical.
3. Unbalanced symmetrical.
4. Unbalanced nonsymmetrical.

Also, any of these networks may be either grounded or ungrounded.

Transpositions.—Circuits are often carefully shielded to prevent the interchange of energy with other circuits or with other parts of the same circuit. Such an interchange may cause oscillations, may cause crosstalk, or may result in noisy circuits.

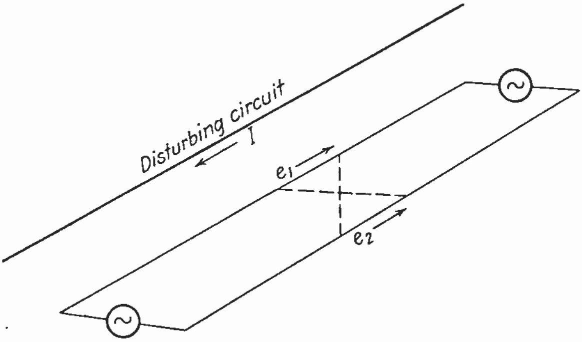


FIG. 213.—Illustrating the effect of transpositions.

Some circuits depend on balance to prevent noise and crosstalk as shown by Fig. 213. If the telephone circuit is **untransposed,** then the circuit is unbalanced with respect to the disturbing circuit and the magnetic field produced by the current in the disturbing circuit will induce a voltage into the paralleling telephone circuit.

Wire 2 is farther away from the power line than wire 1; therefore, e_2 will be less than e_1, and there will be a resultant voltage left to force a bothersome current around through the telephone circuit, causing noise or crosstalk. If the two wires are transposed as indicated by the dotted lines, then the over-all voltage induced in each wire will be the same, and there will be no resultant voltage to force a current around the circuit. Although this is but one aspect of the effect of transpositions, it illustrates their action and the way in which a balanced circuit resists external induction. In a sense, paired and twisted wire is transposed. For a complete consideration of transpositions, the reader should consult a modern book on electric communication.

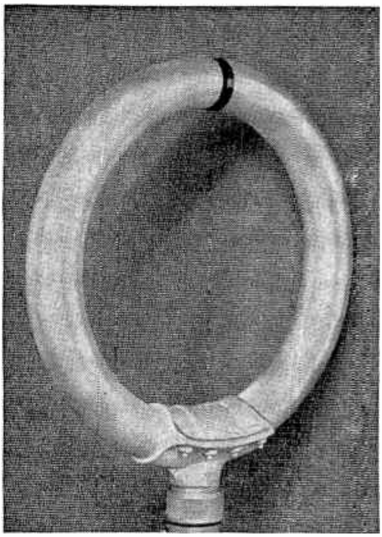


FIG. 214.—A shielded loop antenna such as used in an aircraft radio compass. The metallic shield is broken at the top as shown to prevent the shield from acting as a short-circuited turn. (*Courtesy of Bell Telephone Laboratories.*)

Shielding.—It is often necessary to shield circuit elements such as resistors, inductors, and capacitors from the effects of stray alternating electric and magnetic fields. Thus if a microphone input transformer is not carefully shielded, it may pick up hum from the 60-cycle power transformer. Electric instruments must also be shielded from stray fields (page 224). Shielding against direct or low-frequency magnetic fields is accomplished by enclosing the circuit or device within an iron box which conducts the magnetic flux *around* the space within (page 154).

The examples just given are of shielding against the effects of stray *magnetic fields.* It is often necessary to shield equipment and circuits against the effect of stray *electric fields.* This can be done by enclosing the object or circuit in a "can" of good conducting material. If this is done the lines of force of the stray electric field will terminate on the shield and will not penetrate to the equipment within.

A complete discussion of shielding is beyond the scope of this text. Again, the reader must refer to modern books on electric communication for more complete discussions and for references to the literature. But, it is desired to point out that shielding may affect circuits adversely in several ways.

1. If a coil with an air core is placed in a metallic shield, the magnetic field produced by the coil will cause circulating currents to flow in the shield. These circulating currents will take power from the coil, and this will affect the input impedance of the coil and cause it to have less inductance and greater effective resistance. If the shield can be broken so that circulating currents are reduced or even made zero, then electric shielding can be achieved without affecting the coil appreciably. The shielded loop of Fig. 214 is an example of this.

2. The shield may increase stray capacitive effects. Thus, the impedance of Fig. 215*a* has distributed capacitance to ground as shown. If this impedance is used at the higher audio frequencies, and particularly at radio frequencies, the impedance that it offers between its terminals will depend on its position with respect to ground and also will vary each time it is moved or an observer approaches it, etc. If now the impedance is enclosed in a metallic shield as indicated, it still has a stray capacitance to the shield, and this may even be *greater* than the stray capacitance existing previously, but *it is fixed in value* and independent of position, observers, etc. The capacitance of the shield to ground is variable, but this causes no variation in the impedance of the device. It may cause trouble with other portions of the circuit, in which event the system of shielding must be extended.

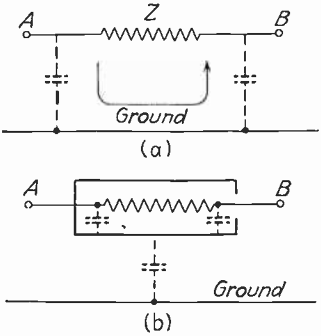


FIG. 215.—In the upper illustration the impedance is unshielded and its terminal impedance as measured between points A–B varies with respect to ground. The lower impedance is shielded, and its terminal impedance value is independent of position with respect to ground.

If the frequency is above about 2000 cycles per second, and particularly if it is quite high, merely enclosing a device in a shield of low-current conductivity will be effective against both magnetic and electric fields. For magnetic fields, the circulating currents that are produced in the shield by the stray field must oppose the stray magnetic field causing them. In a sense the magnetic fields produced by these circulating currents form a barrier through which the stray magnetic field causing them cannot penetrate. This action shields the device within the shield of low current conductivity, and is in accordance with Lenz's law.

Theorems.—Certain electrical principles have become so well established that they are often stated as **theorems.** These theorems are capable of being proved theoretically and verified experimentally. Particularly in communication, however, it is sometimes convenient to refer to them without giving the proof. Several of the theorems useful in communication will now be stated and briefly discussed. For the purpose and scope of this book, they will be given in modified form. In general the statements that follow apply to **linear networks** which are composed of units the impedances of which do not vary with the magnitude of either the current or voltage. The statements also apply to **steady-state conditions** as distinct from **transient** conditions, which obtain, for instance, immediately after the switch is closed energizing a circuit.

The Principle of Superposition.—The current that flows at any point or the voltage that exists between any two points in a network, owing to the simultaneous action of a number of sources of electromotive force at various points throughout a circuit, is the sum of the currents or voltages at these points which would exist if each source of electromotive force were considered separately, each of the other sources being replaced at that time by a unit of equivalent internal impedance.

This theorem was considered on page 36 and was explained in detail for direct-current circuits. The method of applying it to alternating-current circuits is similar, except that the various steps must be performed vectorially, preferably using vector algebra.

The Reciprocity Theorem.—If any source of electromotive force E located at one point in a circuit produces a current I at any other point in the circuit, the same source of electromotive force E acting at the second point will produce the same current I at the first point.

In addition to applying to steady-state conditions in linear networks, it is assumed in the reciprocity theorem that the units in the network are **bilateral** (that is, having the same impedance in each direction) and not **unilateral,** like a rectifier unit which carries current well in one direction only.

Thévenin's Theorem.—If an impedance Z is connected between any two points of a circuit, the resulting current I through this impedance is the ratio of the potential difference E between the points (prior to the connection) divided by the sum of the connected impedance Z and an impedance Z′, which is the impedance of the circuit measured between the two points before connecting impedance Z.

Two illustrations will serve to make this clear. In Fig. 216 is shown a portion of a network, chosen at random. Suppose that it is desired to connect an impedance Z between points $X - Y$ and that it is desired to know what current will flow through this impedance. From Thévenin's theorem, the current that will flow through the impedance Z will be $I = E/(Z + Z')$, where E is the potential difference between points $X - Y$ prior to the connection and Z' is the impedance that would be measured between points $X - Y$ (perhaps with a bridge, page 343) back into the network.

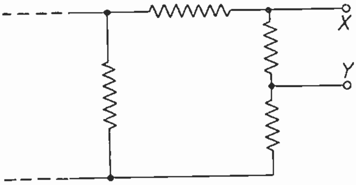


FIG. 216.—Circuit for illustrating Thévenin's theorem.

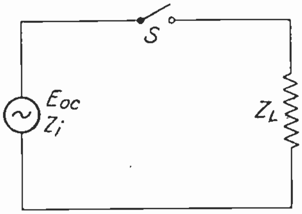


FIG. 217.—Circuit for illustrating the application of Thévenin's theorem.

As a second illustration, consider Fig. 217 consisting of a generator of internal impedance Z_i and open-circuit voltage E_{oc}. According to Thévenin's theorem, the current that will flow through the load impedance Z_L when the switch S is closed is $I = E_{oc}/(Z_i + Z_L)$. This theorem has, in this form, been used throughout this book. It is very important because it illustrates the fact that in communication all calculations must consider the internal impedance of the source. Contrast this with a power system where voltage regulators keep the voltage very constant, irrespective of whether or not the load is connected. Thus, if it is desired to calculate the current that flows through a 10-ohm resistor when connected to a 110-volt 60-cycle outlet, $I = E/Z = 110/10 = 11$ amperes. But, if it is desired to calculate the current that will flow when a load is connected to a communication oscillator, the equation is $I = E_{oc}/(Z_i + Z_L)$ as previously explained. In other words, *in communication circuits that are not voltage regulated the internal impedance must be considered.*

Impedance Matching.—There are at least two reasons for matching the impedance of one circuit to that of another circuit connected to it. (1) To prevent wave reflection, as will be explained on page 378. (2) To secure maximum transfer of power from one circuit to another. This section will deal entirely with this last phenomenon.

The proper relation for securing maximum power transfer in direct-current circuits was explained on page 78, where it was shown that when the resistance of the load equaled the internal resistance of the source the power transfer was maximum and the efficiency was 50 per cent.

From alternating-current relations it follows that if the internal impedance of the generator or other source of power contains *inductive* reactance, and if the impedance of the load contains an equal amount of *capacitive* reactance, no resultant reactance will exist in the circuit. And of course, if the inverse relations of a source having capacitive reactance and a load having inductive reactance exist, then again no resultant reactance exists in the circuit.

If no resultant reactance exists in a circuit, and if the resistance of the load equals that of the source, then the power transfer from the source to the load is maximum. This is sometimes known as the power-transfer theorem. Impedances having equal resistances, and equal and opposite reactances, are known as **conjugate impedances.**

Now under practical conditions it is not always that these conjugate impedance conditions can be obtained. For example, suppose that a vacuum-tube amplifier is to drive a loud-speaker. The amplifier output is usually slightly inductive and so is the loud-speaker input. They are simply made that way because they work on inductive principles (the amplifier contains an output transformer, and the loud-speaker contains a voice coil). In considering the conditions for maximum power transfer when the reactances are *not* opposite as previously considered, it can be shown that *if the magnitude of a load impedance but not the angle is varied, the maximum power will be taken from the source by the load when the magnitude of the load impedance is equal to the magnitude of the impedance of the source.* This also is sometimes stated as a theorem. For the proof of this the reader again is referred to more advanced books on communication.

As an example of this last theorem, suppose that the magnitude of a load impedance is 50 ohms and that it is desired to connect it to an oscillator having an internal impedance of 500 ohms. From Eq. (71*a*) a transformer should be used which has a ratio of $Z_p/Z_s = (N_p/N_s)^2 = 500/50 = 10$, and $N_p/N_s = \sqrt{10} = 3.17$. Therefore, a transformer having a ratio of 3.17 primary turns to 1.0 secondary turn should be inserted between the amplifier and the load.

Impedance-transforming Circuits.—As has been shown in the preceding section, circuits should be matched if the maximum power is to be transferred from one circuit to another. At low frequencies such as those in audio circuits, closely coupled transformers with magnetic cores are usually employed for matching impedances. At radio-frequencies, however, different circuits instead of transformers are most useful for matching impedances. These are called **impedance-transforming circuits.**[1]

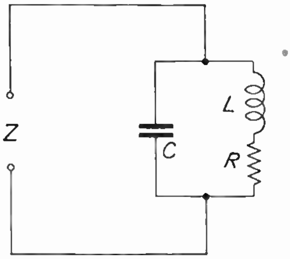


FIG. 218.—An impedance-transforming circuit. At the frequency of antiresonance, the input impedance is pure resistance, and is a value greater than R.

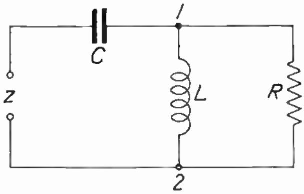


FIG. 219.—An impedance-transforming circuit. At some frequency the input impedance at Z will be pure resistance and will be a value less than R.

The simple antiresonant circuit of Fig. 218 may be used as an impedance-transforming circuit. A calculation based on the principles explained in this chapter will show that at antiresonance the input impedance Z measured between the points indicated will be pure resistance of a value *higher* than the resistance of R. Thus, the value of R has been *increased*, and this principle may be used in impedance matching. Furthermore, the value of R may be directly inserted in series with L or may be reflected into L by coupled-circuit theory.

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[1] Space limitations and the scope of this text permit of but a very brief discussion of impedance-transforming circuits. For complete information the reader is referred to W. L. Everitt, "Communication Engineering," McGraw-Hill Book Company. Inc.

Another simple impedance-transforming circuit is shown in Fig. 219. The impedance measured between the points 1–2 will contain a resistance term less than the resistance of R. This statement can be verified experimentally or by parallel-circuit theory. Then, if the correct capacitor is added in series as at C, the reactance of the impedance existing between the points 1–2 will be neutralized. Then, the input impedance Z measured between the points indicated will be pure resistance but will be a value *lower* than that of the resistor R. Such a circuit can be used to *decrease* a resistance. The circuits here considered can be used to match vacuum tube–power output circuits to antennas in radio transmitters.

The Decibel.—This unit is extensively used to express ratios in communication, in sound, and in other fields. Probably the first use of the decibel was to measure *power ratios* in telephone circuits. As was mentioned on page 79, the amount of power in such circuits is very low, and a practicable wattmeter is not available. For this and other reasons, power is measured as a ratio in decibels, rather than in milliwatts. A *bel* equals ten decibels, but is seldom used.

The ratio of the power W_1 in a termination or load when a device or a circuit is *removed* to the power W_2 in the termination or load when the device is *inserted* is a measure of the loss (or gain if an amplifier) that the circuit introduces. When the ratio is $10^{0.1}$ the **insertion loss** is 1 decibel. If the loss is n decibels, the expression is

$$\frac{W_1}{W_2} = 10^{0.1 \times n} \qquad \text{and} \qquad n = 10 \log_{10} \frac{W_1}{W_2}. \tag{72}$$

Since a wattmeter is not available for communication use, it became the practice to measure power losses and gains in telephone circuits indirectly by measuring the currents and voltages in the circuit. Power is always equal to I^2R. Therefore, Eq. (72) may be written

$$n = 10 \log_{10} \frac{W_1}{W_2} = 10 \log_{10} \frac{I_1^2 R_1}{I_1^2 R_2} = 20 \log_{10} \frac{I_1}{I_2} + 10 \log_{10} \frac{R_1}{R_2}. \tag{73}$$

Power is also equal to $EI \cos \theta$. Thus,

$$n = 10 \log_{10} \frac{W_1}{W_2} = 10 \log_{10} \frac{E_1 I_1 \cos \theta_1}{E_2 I_2 \cos \theta_2} = 10 \log \frac{E_1^2 Z_2 \cos \theta_1}{E_2^2 Z_1 \cos \theta_2}$$

$$= 20 \log_{10} \frac{E_1}{E_2} + 10 \log_{10} \frac{Z_2}{Z_1} + 10 \log_{10} \frac{\cos \theta_1}{\cos \theta_2}. \tag{74}$$

The various steps here involved require a knowledge of logarithms. If the reader is without this knowledge, it is suggested that he consult some textbook on mathematics or accept the derivations as stated.

For explaining Eqs. (72) and (73), Fig. 220 has been included. Now W_1 is represented by the power that reaches the load with nothing inserted and is either I^2R or $EI \cos \theta$. Similarly, the power W_2 is the power that reaches the same load with the loss inserted. For the same load $R_1 = R_2$. Thus, in Eq. (73), $R_1/R_2 = 1.0$, and $10 \log 1.0 =$ zero. Then, the loss in decibels is $n = 20 \log_{10} I_1/I_2$. Similarly, in Eq. (74), Z_2/Z_1 and $\cos \theta_1/\cos \theta_2$ both equal 1.0 because the same load is being considered. Therefore $10 \log Z_2/Z_1 + 10 \log_{10} \cos \theta_1/\cos \theta_2 = 0$, and hence the loss in decibels is $n = 20 \log_{10} E_1/E_2$.

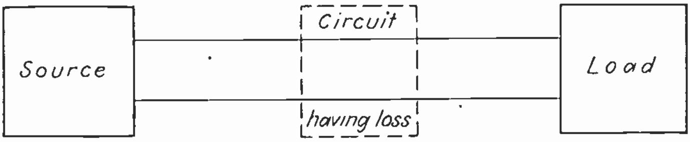


FIG. 220.—Circuit for studying the theory of the decibel.

These statements show that in circuits where the impedances are the same (that is, circuits that are, say, matched throughout for maximum power transfer) either current ratios or voltage ratios can be used to determine the *power loss* in decibels. But, unless $R_1 = R_2$, $Z_1 = Z_2$, and $\cos \theta_1 = \cos \theta_2$, the true power loss will not be given unless all of Eqs. (73) or (74) are considered, not merely the first portions. Figure 221 can be used for approximate results.

Now there is absolutely nothing incorrect about using the expression $n = 20 \log_{10} E_1/E_2$ for giving a *voltage ratio* in decibels; neither is it incorrect to use $n = 20 \log_{10} I_1/I_2$ to give a *current ratio* in decibels. The point is, however, that only under matched circuit conditions as are usually found in telephone circuits do these ratios also give the power ratio. In other words, in using the decibels it should be explicitly stated whether it refers to power ratio, current ratio, voltage ratio, or some of the other uses of the decibel, one of which will now be discussed.

If a **zero power level** is chosen, the decibel can be used to express power as so many decibels above or below that level. For example, if zero power level is arbitrarily defined at 0.001 watt, or 1.0 milliwatt, then when the power in a circuit was n decibels above zero level, the actual power in milliwatts could be computed from Eq. (72). Also, a voltage-measuring device could be used to give the power level, in a circuit of known impedance, in decibels above or below the zero reference level. No universally accepted level has ever been designated for this purpose. In fact, the power at some part of the circuit, say in input, may be arbitrarily taken as zero level, and the power at other points may be expressed with respect to it.

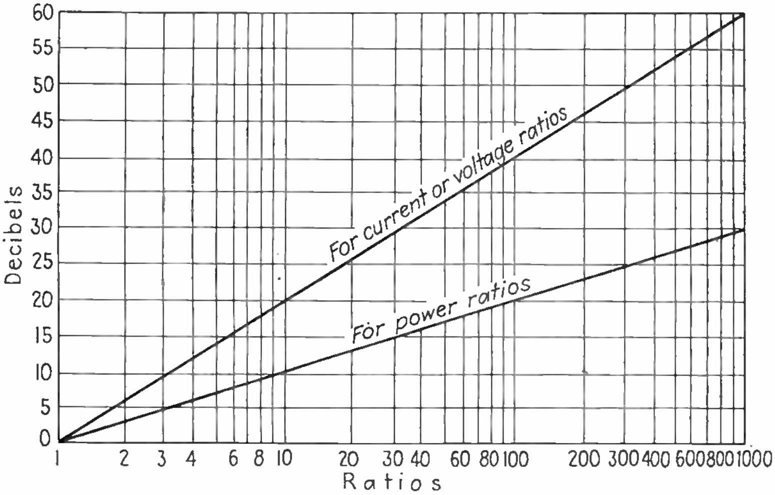


FIG. 221.—Curves for gain or loss in decibels, from current, voltage, or power ratios.

For measuring program levels on broadcast circuits transmitting speech and music, where the currents, voltage, and power are varying rapidly, the characteristics of the instrument affect the reading. For example, if the instrument is highly damped (page 212) it will not be able to follow rapid changes in the program. To standardize the matter, industry has defined and accepted a **volume-level indicator** (which is merely a voltmeter with specified characteristics) and a **volume unit** or VU. This volume unit is based on a zero level of 1.0 milliwatt in a circuit of 600 ohms impedance.

Electric Filters.—In communication circuits it is often desirable to insert a network that will freely pass currents of one band of frequencies but that will greatly attenuate (decrease) currents of frequencies outside this band. Such selective networks are called **filters,** or **wave filters.**

Filters are composed of inductors and capacitors having losses as low as commercially obtainable. In elementary filter design it is assumed, therefore, that the inductors and capacitors contain no internal effective resistance. The design of filters is a subject for advanced communication studies. There are, however, certain characteristics of filters that should be presented at this time.

The terminal or input impedance of a filter will first be considered. Suppose that a filter is connected between a generator and a load. If the generator is going to send power to the load, the filter must take power from the generator so that this power can be passed on to the load. Thus, even though a filter is composed of pure inductances and capacitances, it must offer a *resistance* load to the generator at all frequencies that are to be passed so that it will take power from the generator at these frequencies. Since the inductors and capacitors are without loss, all this power taken is passed on to the load. At all frequencies that are not to be passed, the input impedance must be a pure *reactance,* so that no power will be taken at these frequencies.

The various combinations of the series and parallel inductors and capacitors give the filter these characteristics. It may seem strange at first that a network made of reactances only can look like a resistance at certain frequencies, but a review of resonant and antiresonant circuits will show this to be possible. Although filters can be considered in an elementary way from the standpoint of resonance, this method of approach is not ordinarily used. In fact, no attempt will be made to explain filter action other than as has been stated, because such an attempt would probably only confuse the reader should accepted filter theory ever be studied.

The Low-pass Filter.—A single section of a **low-pass filter** is shown in Fig. 222*a*. Several sections of such a filter are often combined into one structure. With the load connected, the ideal filter should have attenuation characteristics somewhat as shown in Fig. 222*b*. The low-pass filter passes all frequencies up to a **cutoff frequency** and then attenuates all frequencies above this cutoff frequency. From a practical viewpoint, the reactances of the inductors are low at low frequencies, and they readily pass such frequencies. Also, the reactance of the capacitor is low to high frequencies, and it tends to shunt high frequencies. But of course, this is an elementary explanation and should not be taken too seriously.

A simple low-pass filter such as shown in Fig. 222*a* can be calculated by the following equations:

$$L = \frac{Z_k}{2\pi f_c} \qquad \text{and} \qquad C = \frac{1}{\pi f_c Z_k}, \tag{75}$$

where f_c is the cutoff frequency desired and Z_k is the iterative impedance in ohms the filter is to have, L and C will be in henrys and farads, respectively. Iterative impedance in a filter means the input impedance a filter offers the source. It is defined as the input impedance of an infinite series of sections such as Fig. 222*a* connected one to the other. The meaning of this term will be made evident on page 377. Suffice it to say at this time that the iterative impedance usually selected should be such that the circuits are matched. This concept will also be made clear on page 378.

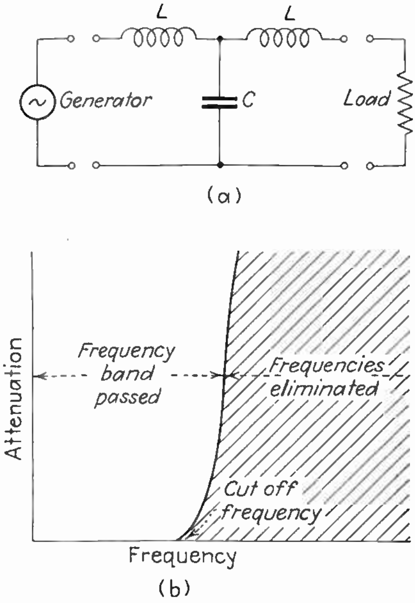
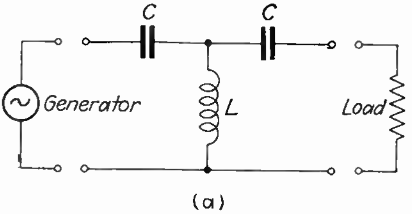


FIG. 222.—Circuit and characteristic curves for a simple low-pass filter.

The High-pass Filter.—A single section of a **high-pass filter** is shown in Fig. 223*a*, and its characteristics with a connected load are somewhat as shown in Fig. 223*b*. The high-pass filter passes all frequencies from very high values down to the cutoff value and attenuates all below this frequency. Again from the practical point of view, the series capacitor readily passes the high frequencies and the shunt-connected inductor by-passes the low frequencies so that they do not reach the load.



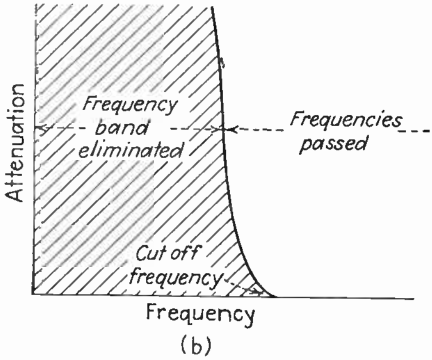


FIG. 223.—Circuit and characteristic curves for a simple high-pass filter.

A simple high-pass filter such as shown in Fig. 223*a* can be readily calculated by the following equations:

$$L = \frac{Z_k}{4\pi f_c} \qquad \text{and} \qquad C = \frac{1}{2\pi f_c Z_k}, \tag{76}$$

where f_c is the cutoff frequency desired and Z_k is the iterative impedance in ohms the filter is to have, L and C will be in henrys and farads, respectively.

SUMMARY

Electric networks are composed of resistors, inductors, and capacitors, and the characteristics of these circuit elements such as their effective resistance and stray inductance and capacitance must be known.

The figure of merit, or Q, of an inductor is the ratio $\omega L/R$. An inductor with a high Q has low losses, that is, a low effective resistance.

A circuit composed of a resistor and an inductor in series has an impedance equal to the resistance at a very low frequency and has a higher impedance at higher frequencies because of the increased inductive reactance of the inductor.

A circuit composed of a resistor and a capacitor in series has a very high impedance at low frequencies, and the impedance decreases and approaches the resistance of the resistor at very high frequencies.

A circuit composed of a resistor, inductor, and capacitor in series is in resonance at the frequency at which the inductive and capacitive reactances are equal. At the resonant frequency, the impedance of the circuit is a value of pure resistance equal to the resistance present in the circuit.

The admittance Y of a circuit is the reciprocal of the impedance. The total current flowing in a circuit equals the admittance multiplied by the impressed voltage.

The conductance G of a circuit is the in-phase component of the admittance. The in-phase component of the total current equals the voltage multiplied by the conductance.

The susceptance B of a circuit is the out-of-phase component of the admittance. The out-of-phase component of the total current equals the voltage multiplied by the susceptance.

In parallel circuits the admittance of each branch is added to obtain the total admittance. The reciprocal of this admittance equals the equivalent impedance of the parallel combination.

A circuit composed of a resistor, inductor, and capacitor in parallel is in antiresonance at the frequency at which the inductive and capacitive susceptance are equal. At the antiresonant frequency the equivalent impedance of the circuit is a value of pure resistance equal to the parallel resistor (if the inductor contains no resistance).

The resistance present in either a series or a parallel circuit determines the sharpness of resonance.

Circuits are coupled through mutual inductance. The effect of the secondary is to reflect an impedance of $(\omega M)^2/Z_s$ into the primary.

A transformer with a core of good magnetic material is a closely coupled circuit. Such a transformer may be regarded as a voltage changer, as a current changer, or as an impedance changer.

Networks may be balanced or unbalanced, symmetrical or nonsymmetrical, and may be composed of linear or nonlinear elements which may be either bilateral or unilateral.

Transpositions and shielding reduce coupling with other circuits and minimize crosstalk and noise.

Certain theorems such as the principle of superposition, the reciprocity theorem, Thévenin's theorem, and impedance-matching theorems are very useful in communication work.

For maximum transfer of power from a generator to a load, the resistances should be equal and the reactances equal and opposite. If these reactance conditions cannot be met, then the magnitudes of the impedances should be the same.

Transformers and impedance-transforming networks are useful in matching impedances.

The decibel is used to specify power, voltage, or current ratios. Care should be exercised to prevent confusing these usages.

Electric-wave filters are widely used in communication to select currents on the basis of frequency.

REVIEW QUESTIONS

1. Under what conditions is it possible for the measured impedance of a coil to have a capacitive reactance component?

2. What is meant by the figure of merit, or Q, of a coil?

3. In a parallel circuit composed of a capacitor and an inductor, what will be the effect of the Q of the inductor in the impedance at antiresonance? On the frequency of antiresonance?

4. What determines the frequency at which a series circuit is resonant?

5. What determines the impedance of a resonant series circuit?

6. Define admittance, conductance, susceptance, and explain their significance.

7. How much current is theoretically taken by a parallel circuit composed of pure inductance and capacitance at antiresonance? What is its impedance?

8. Explain why the resistance in a parallel antiresonant circuit varies as it does.

9. When should series resonant circuits be used, and when should parallel antiresonant circuits be used?

10. How can a series resonant circuit be used to increase or "step up" voltage?

11. How can a parallel antiresonant circuit be used to increase or "step up" current?

12. Briefly, what procedure is followed in solving series-parallel circuits?

13. What is the effect on the primary of a load coupled into the secondary?

14. What determines the coefficient of coupling between two coils?

15. How are resistance, inductance, and capacitance each reflected into a primary?

16. How can a transformer be used as an impedance matcher?

17. Distinguish between balanced and unbalanced and symmetrical and nonsymmetrical networks.

18. What is meant by the term linear bilateral impedance?

19. How should an object be shielded against a low-frequency alternating magnetic field?

20. How should an object be shielded against a high-frequency alternating magnetic field?

21. State Thévenin's theorem, and explain its application.

22. What impedance relations should exist for maximum power transfer?

23. Under what conditions may current and voltage ratios be used to compute power ratios in decibels?

24. What is meant by zero level, and how is it useful?

25. What is meant by saying that the program level is +6 VU (volume units)?

PROBLEMS

1. Calculate the figure of merit, or Q, at 1000 cycles of an inductor that has an inductance of 0.048 henry and a resistance of 4.8 ohms.

2. A radio-frequency plate choke coil has an inductance of 30 microhenrys and a resistance of 2.2 ohms. Calculate its reactance and impedance at 10,000, 100,000, 500,000, and 1,000,000 cycles. Plot curves showing the variations of X and Z for these frequencies.

3. A capacitor of 0.0005 microfarad is in series with a 1000-ohm resistor. Calculate the reactance and impedance at 10,000, 100,000, 500,000, and 1,000,000 cycles. Plot curves showing the variations of X and Z for these frequencies.

4. Solve the parallel circuit of Fig. 201 at a frequency of 1200 cycles.

5. Solve the parallel circuit of Fig. 203 at a frequency of 1200 cycles.

6. A circuit is composed of a 100-ohm resistor, a 0.01-henry inductor, and a 0.1-microfarad capacitor in series. The inductor has a resistance of 5.5 ohms. Calculate the resonant frequency, the impedance at resonance, and the impedance 1000 cycles each side of resonance. At resonance, what will be the voltage across each unit if 3.8 volts is applied across the combination?

7. The units of Prob. 6 are connected in parallel. Neglect the resistance of the inductor, and calculate the frequency of antiresonance. Consider the resistance of the inductor, and calculate the frequency of antiresonance.

8. Calculate the equivalent impedance of the circuit of Prob. 7 when the resistance of the inductor is considered, and when it is not.

9. Neglect the resistance of the inductor of Prob. 7, and calculate the equivalent circuit impedance 1000 cycles each side of the antiresonant frequency.

10. An inductor and a mica capacitor are in antiresonance at a frequency of 500,000 cycles. The inductance of the inductor is 0.0005 henry and its resistance is 6.1 ohms. Calculate the equivalent resistance of the circuit at the antiresonant frequency.

11. Solve the circuit of Fig. 208 at a frequency of 60 cycles.

12. Solve the circuit of Fig. 210 at 1200 cycles per second.

13. If the circuit of Fig. 210 is connected to a 1000-cycle oscillator of 542 ohms internal resistance and 32.4 volts open-circuit voltage, use the voltage as the base, and calculate the expression for the primary current, the expression for the open-circuit secondary voltage, and the expression for the secondary current.

14. Using a table of logarithms, calculate the voltage ratio in decibels for voltage ratios of 2, 5, 10, 25, 50, 100, 500, and 1000.

15. Calculate the values of the inductors and capacitors for a high-pass filter and a low-pass filter each with a cutoff frequency of 1000 cycles, and to work in 600-ohm telephone circuits.

CHAPTER XII

BRIDGE CIRCUITS

Bridges of various types are used in many ways for measurements in communication circuits. In fact, it may be said that the bridge is as old as the communication industry itself. The subject of bridges is quite large; however, most measurements are made with a few simple types.

Resistance, inductance, and capacitance can be accurately measured with bridges. Measurements of either the direct-current ohmic resistance or the alternating-current effective resistance can be made. Measurements of the effective resistance and inductance of a coil can be made simultaneously. This may be done with or without direct current in the windings. Bridges can be devised for measuring the effective resistance and capacitance of a condenser. For electrolytic condensers a polarizing voltage may be provided. Bridge circuits have also been devised for locating faults, such as short circuits or grounds on telephone lines. Bridges are of two general types, direct current and alternating current. Direct-current bridges will now be considered.

The Wheatstone Bridge.—Most bridge circuits used in practice can be traced back to the basic Wheatstone bridge of Fig. 224. The values of the resistors R_A, R_B, R_S, and R_X are varied as necessary so that with switch S closed no deflection of the **galvanometer** G occurs. The galvanometer is a very sensitive uncalibrated moving-coil permanent-magnet instrument.

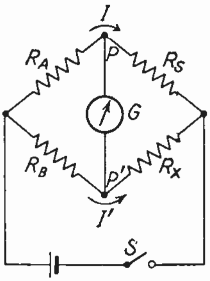


FIG. 224.—The Wheatstone bridge.

Now if the galvanometer does not deflect, then no difference of potential (that is, no voltage) exists between points P-P' where the galvanometer G is connected. Also, if the galvanometer does not deflect, then no current is flowing through the galvanometer, and the current in R_A equals that in R_S, and the current in R_B

equals that in R_X. If points P-P' are at the same potential, the voltage across R_A must equal that across R_B; furthermore, the voltage across R_S must equal that across R_X. Each of these voltages is an IR drop. Therefore it can be written that $IR_A = I'R_B$ and that $IR_S = I'R_X$.

Writing these relations in terms of current ratios gives the following:

$$\frac{I}{I'} = \frac{R_B}{R_A} \qquad \text{and} \qquad \frac{I}{I'} = \frac{R_X}{R_S}. \tag{77}$$

These two different resistance ratios are equal to the same thing and may, accordingly, be equated. Then,

$$\frac{R_B}{R_A} = \frac{R_X}{R_S} \cdot \qquad \text{and} \qquad R_X = \frac{R_BR_S}{R_A}. \tag{78}$$

These are the fundamental bridge equations and will be extensively used.

In balancing this bridge, the current through the galvanometer was made to equal zero with a voltage applied as shown. According to the reciprocity theorem, when a bridge is balanced, the positions of the battery and galvanometer may be interchanged and the bridge will still be in balance. Diagrams will often be found in which the battery and galvanometer are interchanged from the positions of Fig. 224.

Wheatstone-bridge Measurements.—In an actual Wheatstone bridge the resistors R_A, R_B, and R_S are usually accurately calibrated variable resistors often called **arms** of the bridge. The element R_X of Fig. 224 is the unknown resistor under test. Note that in Eq. (78), $R_X = (R_B/R_A)R_S$. In words, the *ratio* of R_B to R_A multiplied by the value of R_S equals the resistance of the unknown resistor R_X. Arms R_B and R_A are accordingly known as **ratio arms.** The resistor R_S is the standard resistor against which the unknown is compared.

In making Wheatstone-bridge measurements, a ratio (R_B/R_A) is selected so that with a convenient setting of R_S a galvanometer **null reading** or balance is obtained. Thus, suppose that $R_X =$ 1000 ohms and $R_S = 100$ ohms; then, at balance R_B/R_A must equal 10. Now since R_B and R_A are often each variable from 1 ohm up to perhaps 10,000 ohms, this ratio 10 could be obtained by many different combinations, for example, $R_B = 10$ ohms and $R_A = 1$ ohm.

In general, however, it will be better always to use larger resistances to obtain the desired ratios. One reason for this is that any contact error in the arms will be a larger percentage of 1 ohm than it is of, say, 100 ohms. Thus, in general, it would be better to use $R_B = 1000$ ohms and $R_A = 100$ ohms to obtain a ratio of $R_B/R_A = 10$. If the unknown R_X were 0.1 ohm, for instance, and if R_S were still set on 100, then the ratio of R_B/R_A must equal 0.001 to satisfy Eq. (78). Such a ratio would usually have to be obtained by using a ratio $R_B = 10$ and $R_A = 10{,}000$, or in some bridges $R_B = 1$ and $R_A = 1000$. The use of such a small value for R_B could be avoided by reducing R_S from 100 to 10 ohms.

The Wheatstone bridge of Fig. 224 is often used to measure the direct-current or ohmic resistance of circuits that are inductive. If this is to be done, the bridge will be badly out of balance the *instant* after the switch is closed or opened. This is, of course, due to the fact that as the current builds up and dies out back voltages are produced because of the inductance. This inductive action may cause the galvanometer to experience violent "kicks" when the battery switch is closed and opened. To avoid this, it is common practice to arrange two button-type switches side by side, one connected in the battery supply and the other in the galvanometer circuit. When making measurements in inductive circuits, the battery switch is first pressed, and then the galvanometer switch is pressed; also, the galvanometer switch is first released and then the battery switch. In this way the galvanometer is not in the circuit when the inductive surges occur.

To summarize: The Wheatstone bridge is very useful and can be used for a wide range of measurements if R_A, R_B, and R_S of Fig. 224 are each variable, say from 1 to 10,000 ohms.

The Murray Loop.—Open-wire telephone and telegraph lines sometimes become crossed, short-circuited, or grounded, particularly during bad storms. On cable circuits, such **faults** occur, but less frequently. By using modifications of the Wheatstone bridge, it is possible to locate the distances to such faults. One bridge arrangement for this is the **Murray loop** shown in Fig. 225.

In the circuit here shown, a **ground** exists on line wire 2. It is desired to find the distance X from the test point to the ground, so that a lineman may be sent out to clear the trouble. A good wire is connected or "patched" at the distant station to the faulty wire, and the bridge is balanced.

To take the general case first, suppose that L is the length of the entire loop in feet, that r_1 is the resistance per foot of wire 1, and that r_2 is the resistance per foot of wire 2. Such a condition might be encountered in practice if a clear wire *the same size* as the faulty wire was not available for the tests.

Under these conditions, $R_A = R_A$ and $R_B = R_B$ of Fig. 224. The length of the clear wire is $0.5L$, and this length multiplied by its resistance per foot r_1 is $0.5Lr_1$ which is the resistance of the clear wire. The length $0.5L - X$ is the distance from the fault to the distant office, and this length multiplied by its resistance per foot r_2 is $(0.5L - X)r_2$ the resistance from the fault to the distant office.

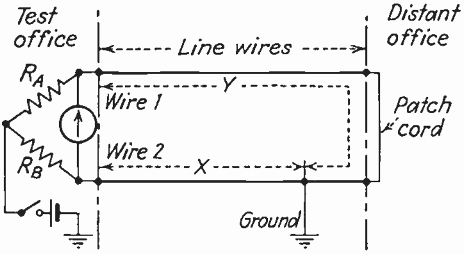


FIG. 225.—The Murray loop.

The resistance R_S of Fig. 224 corresponds to the total resistance from the test point to the distant office via the clear wire and back on the faulty wire to the ground; that is, $R_S = 0.5Lr_1 + (0.5L - X)r_2$. The resistance R_X of Fig. 224 corresponds to the total resistance from the test point to the ground via the faulty wire and is $R_X = Xr_2$. Substituting these values in Eq. (78) for the condition when the bridge is balanced gives

$$R_X = \frac{R_B R_S}{R_A} \qquad \text{and} \qquad Xr_2 = \frac{R_B[0.5Lr_1 + (0.5L - X)r_2]}{R_A}.$$

From this,

$$X = \frac{0.5LR_B(r_1 + r_2)}{r_2(R_A + R_B)}. \tag{79}$$

As previously designated, X will be the distance in feet from the test point to the ground by the Murray loop method when L is the total loop length in feet and r_1 and r_2 are the resistances per foot of the clear and faulty wires, respectively.

If the clear wire and the faulty wire have the same resistance per foot, then it can easily be shown that when the Murray loop circuit is balanced the distance X to the fault is

$$X = \frac{R_B L}{R_A + R_B}. \qquad (80)$$

In this equation X will be in feet when L is the total length of the loop in feet. Of course the distances may be expressed in miles if it will be more convenient.

The Varley Loop.—Another method of finding faults on lines and cables is the **Varley-loop** circuit of Fig. 226. This arrangement

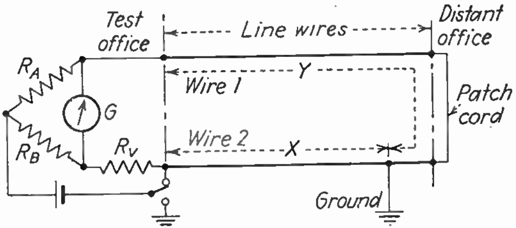


FIG. 226.—The Varley loop.

offers the additional possibility of throwing the switch up and measuring the resistance of the loop or of any resistor by the Wheatstone-bridge method.

To take the general case first, suppose that L is the length of the entire loop in feet, that r_1 is the resistance per foot of wire 1, and that wire r_2 is the resistance per foot of wire 2.

Under these conditions, $R_A = R_A$ and $R_B = R_B$ of Fig. 224. The length of the clear wire is $0.5L$, and this multiplied by its resistance per foot r_1 is $0.5Lr_1$ which is the resistance of the clear wire. The length $0.5L - X$ is the distance from the fault to the distant office, and this length multiplied by its resistance per foot r_2 is $(0.5L - X)r_2$ the resistance from the fault to the distant office.

The resistance R_S of Fig. 224 corresponds to the total resistance from the test point to the office via the clear wire and back on the faulty wire to the ground; that is, $R_S = 0.5Lr_1 + (0.5L - X)r_2$. The resistance R_X of Fig. 224 corresponds *in the Varley loop* to the total resistance from the test point to the ground via the faulty wire *plus* the Varley-loop resistor R_V; that is, $R_X = Xr_2 + R_V$. Substituting these values in Eq. (78), for the condition of balance,

$$R_X = \frac{R_B R_S}{R_A} \qquad \text{and} \qquad Xr_2 + R_V = \frac{R_B[0.5Lr_1 + (0.5L - X)r_2]}{R_A}.$$

From this,

$$X = \frac{0.5LR_B(r_1 + r_2) - R_A R_V}{r_2(R_A + R_B)}. \tag{81}$$

As previously designated, X will be the distance in feet from the test point to the ground by the Varley-loop method when L is the total loop length in feet and r_1 and r_2 are the resistance per foot of the clear and faulty wires, respectively.

If the clear wire and the faulty wire have the same resistance per foot, then it can easily be shown that when the Varley loop is balanced the distance X to the fault is

$$X = \frac{R_B L - R_A R_U}{R_A + R_B}. \tag{82}$$

In this equation X will be in feet when L is the total length of the loop in feet, or the distances may be expressed in miles.

Alternating-current Bridges.—Inductance, capacitance, and effective resistance are conveniently measured in communication practice by alternating-current bridges. Although the simple Wheatstone bridge is satisfactory for making most direct-current measurements, a number of different alternating-current bridges are possible and are used. These are based on the Wheatstone-bridge principle.

In the alternating-current bridge a source of alternating voltage must be substituted for the battery, and an alternating-current detector must be substituted for the galvanometer. A vacuum-tube oscillator makes a splendid source of alternating voltage, but tuning-fork oscillators, microphone hummers, and buzzers may also be used in some instances. For a detector of the null point or balance of the bridge, a good grade of double-receiver headphones is usually quite satisfactory.

Sometimes it is very desirable to place a vacuum-tube voltage amplifier between the bridge and the headset to increase the intensity of the receiver output and make a more precise bridge balance possible. Also, it is sometimes well to use a tuned detector such as a wave analyzer to detect the condition of balance. This is particularly desirable if harmonics of the fundamental frequency are bothersome in adjusting the bridge. Most telephone receivers are highly resonant to the frequencies around 1000 cycles per second. Thus, if a bridge were being balanced at some low frequency to which the receiver was insensitive, a harmonic, although below the fundamental in amplitude, might be very loud.

Equal-ratio-arm Bridge.—By this term is meant that the ratio arms R_A and R_B of Fig. 227 are of the same value, perhaps 1000 ohms each, giving a 1 to 1 ratio. Several different combinations are possible as will now be explained.

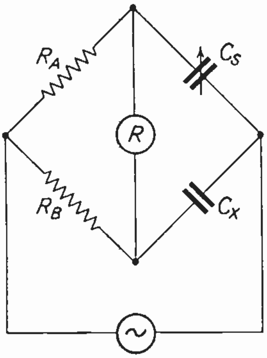


FIG. 227.—Bridge circuit for measuring the capacitance C_X of an unknown capacitor by balancing the bridge with a standard capacitor C_S.

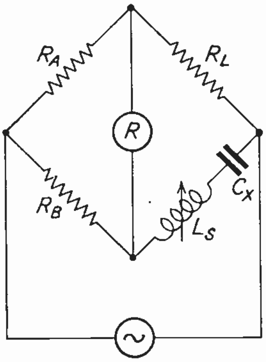


FIG. 228.—Bridge circuit for measuring the capacitance C_X of an unknown capacitor by balancing the bridge with a standard inductor L_S.

Capacitance Measurements with Standard Capacitor.—With arms R_A and R_B equal, the variable standard condenser C_S must be adjusted to equal the unknown condenser C_X for a null point of minimum tone in the receiver R. Very large and very small values of capacitance can be measured with the bridge of Fig. 227, depending on the capacitance of the standard condenser C_S. For measuring small values, the wires to the condensers should be kept short and should *not* be twisted, but rather should be spread apart.

Capacitance Measurements with Standard Inductor.—A bridge circuit for measuring the capacitance of an unknown capacitor by comparing it with a variable standard inductor is shown in Fig. 228. As previously considered, R_A equals R_B. When the inductor is varied until at the test frequency the inductive reactance equals the capacitive reactance, then series resonance occurs in the L_S–C_X arm. Therefore,

$$2\pi f L_S = \frac{1}{2\pi f C_X} \qquad \text{and} \qquad C_X = \frac{1}{(2\pi f)^2 L_S}. \tag{83}$$

The value of C_X will be in farads when L_S is in henrys and f is the frequency in cycles per second. The resistor R_L is equal to the *effective resistance* of the standard inductor L_S; then, R_L balances out this resistance, and a more complete null adjustment is possible.

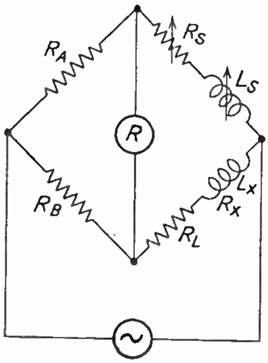


FIG. 229.—Bridge circuit for measuring the effective resistance R_X and the inductance L_X of an unknown inductor by balancing the bridge with a standard inductor L_S and standard resistor R_S.

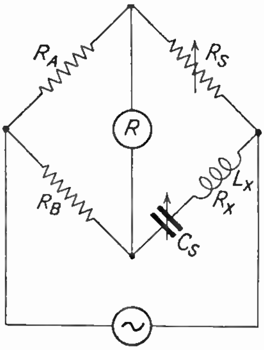


FIG. 230.—Bridge circuit for measuring the effective resistance R_X and the inductance L_X of an unknown inductor by balancing the bridge with a standard capacitor C_S and standard resistor R_S.

Inductance Measurements with Standard Inductor.—In the bridge of Fig. 229, R_A and R_B are again equal. The unknown inductor has inductance L_X and *effective resistance* R_X. These two values must be balanced by varying L_S and R_S in the opposite bridge arms. The value of R_L in series with the unknown inductor offsets or balances the effective resistance of the standard inductor. If this is done, then the setting of R_S will be the true effective resistance of the unknown inductor. If R_L is not added as shown, then R_S will be less than R_X by the effective resistance of L_S.

This is the first bridge in which two adjustments, that of R_S and L_S, must be made. A good procedure is as follows: Vary L_S over a wide range, and note any decrease in the signal; then, leave L_S set on what appears to be the lowest value. Next, vary R_S over a wide range, and this should "sharpen" the tuning; that is, the null point should grow more evident. Leave R_S set where the tone is least, and again adjust L_S. Proceed in this manner until the slightest adjustment of R_S or L_S increases the tone quite distinctly. If this is not accomplished, the bridge usually is not satisfactorily balanced. If considerable harmonics exist in the oscillator output, do not attempt to eliminate all noise from the receiver, but instead concentrate on removing the fundamental which is, of course, the lowest tone present. It is remarkable how the ear can be trained in bridge balancing.

Inductance Measurements with Standard Capacitor.—This circuit is shown in Fig. 230, where R_A equals R_B as before. This circuit is similar to Fig. 230. The value of C_S and R_S are varied until a null point is reached; then, the inductive and capacitive reactances are equal,

$$\frac{1}{2\pi f C_S} = 2\pi f L_X \qquad \text{and} \qquad L_X = \frac{1}{(2\pi f)^2 C_S}. \tag{84}$$

The value of R_S at the null setting will equal the effective resistance R_X of the inductor under test.

Unequal-ratio-arm Bridge.—The alternating-current bridge circuits that have been considered have been special types in that they had equal ratio arms ($R_A = R_B$) and that each bridge was limited as to what it could do. The next bridge to be considered will be generalized, and the basic alternating-current bridge equations will be derived. In the bridge shown in Fig. 231, R_A *does not equal* R_B, and *impedances* are shown at Z_S and Z_X. If desired, impedances could also have been placed at R_A and R_B, but in the usual bridge these are variable ratio arms of pure resistance.

By applying the same reasoning as for the Wheatstone bridge on page 343, at balance no current flows through the receiver, and the current through R_A and Z_S is the same. Likewise, the current through R_B and Z_X is the same. The voltage drop IR_A must equal $I'R_B$, because the points across which the receiver is connected must be at the same potential for no current through it and no tone from it. Also, the voltage drop IZ_S must equal $I'Z_X$ in both magnitude and phase. Then,

$$\frac{I}{I'} = \frac{R_B}{R_A}, \quad \frac{I}{I'} = \frac{Z_X}{Z_S} \qquad \text{and} \qquad \frac{R_B}{R_A} = \frac{Z_X}{Z_S}. \tag{85}$$

Now the general expression for an impedance is $Z = R + jX$. Writing this in for the impedances of Eq. (85) gives $R_B/R_A = (R_X + jX_X)/(R_S + jX_S)$. Because of the fact that resistances and reactances act at right angles (page 257) in the bridge of Fig. 231, the resistances must be equated separately and the reactances must also be equated separately. For this reason the balance equations become

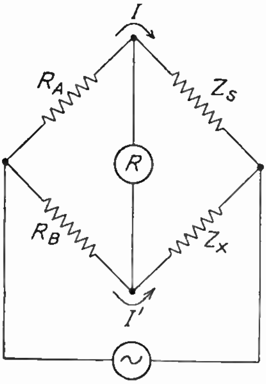


FIG. 231.—Circuit for deriving generalized bridge equations.

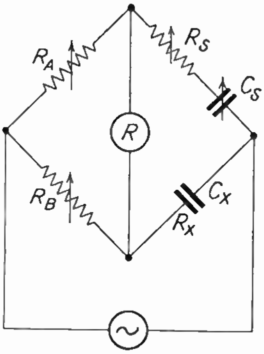


FIG. 232.—Unequal-ratio-arm bridge for measuring the capacitance C_X and effective resistance R_X of an unknown capacitor with a standard resistance R_S and a standard capacitor C_S.

$$\frac{R_B}{R_A} = \frac{R_X}{R_S}, \quad \frac{R_B}{R_A} = \frac{X_X}{X_S}, \quad R_X = \frac{R_B R_S}{R_A}, \quad \text{and} \quad X_X = \frac{R_B X_S}{R_A}. \tag{86}$$

The application of these principles will now be shown.

Capacitance Measurements with Standard Capacitor.—This bridge is essentially the same as shown in Fig. 227, page 349, with the exception that now R_A and R_B are not equal but variable to give different ratios for Eq. (86). For convenience this bridge has been reproduced in Fig. 232 with a resistor R_S added to measure the effective resistance R_X of the unknown capacitor C_X under test. (Of course this refinement could also be added to Fig. 227 if advisable.)

Expanding Eq. (86) to apply to capacitive measurements,

$$\frac{1}{2\pi f C_X} = \frac{R_B(1/2\pi f C_S)}{R_A} \quad \text{and} \quad C_X = \frac{C_S R_A}{R_B}. \tag{87}$$

Note that for measuring capacitors the ratio of the ratio arms R_A and R_B *appear in the inverse form* because of the reciprocal nature of the equation for capacitive reactance. The effective *series* resistance of the unknown capacitor will be as given by Eq. (86), $R_X = R_BR_S/R_A$. With such a bridge, one standard capacitor can be used for a very wide range of measurements.

Capacitance Measurements with Standard Inductor.—Such an arrangement would ordinarily be used only with an *equal-ratio-arm* bridge, and was treated on page 349.

Inductance Measurements with Standard Inductor.—This was considered for the *equal-ratio-arm* bridge on page 350. The circuit of Fig. 229 is used with the *unequal-ratio-arm* bridge now being considered with one modification: It is no longer advisable to use the balancing resistor R_L to offset the effective resistance of the standard inductor. This resistor was used in the *equal-ratio-arm* bridge so that R_S of Fig. 229 would read direct. Since a calculation must be made anyway with the *unequal-ratio-arm* bridge, the resistor R_L is omitted, but the effective resistance R_L of the standard inductor L_S must be known, and in the computations, R_L must be added to whatever resistance R'_S it takes to balance the bridge to give the true value of R_S for the calculations that follow.

Expanding Eq. (86) to apply to inductive measurements,

$$2\pi f L_X = \frac{2\pi f L_S R_B}{R_A} \quad \text{and} \quad L_X = \frac{L_S R_B}{R_A}. \tag{88}$$

Note that the ratio-arm values are not inverted as for Eq. (87). The effective resistance R_X of the inductor is found from Eq. (86) by the relation $R_X = R_BR_S/R_A$, but again attention is directed to the fact that R_S is the sum of the resistance setting R'_S of the variable resistor in series with L_S and the effective resistance R_L of the standard inductor. With this bridge, one standard inductor can be used for making measurements over a wide range of values.

As was brought out on page 258, the effective resistance of a coil with an iron core depends in part on the hysteresis loss. This loss varies directly with the frequency and depends on the magnitude of the current (page 166). Eddy-current losses in the iron core also contribute to the effective resistance loss, and eddy-current losses also vary greatly with both the frequency and the magnitude of the current. Also, the magnitude of the current affects the inductance. Thus, to obtain reliable values for the inductance and effective resistance of a coil with a core of magnetic material, this current used in making the measurements should be approximately of the same frequency and magnitude as the current with which the coil will be used in practice.

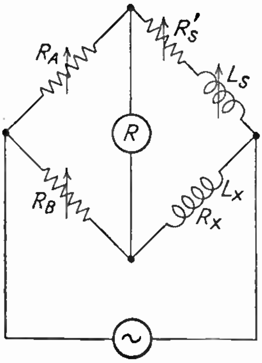


FIG. 233.—Unequal-ratio-arm bridge for measuring the inductance L_X and effective resistance R_X of an inductor with a standard resistance R'_S and a standard inductor L_S.

Thus, for very precise work, Fig. 233 should be modified as follows: Place a voltage divider across the oscillator, and feed the bridge from the variable arm of this divider. Place an appropriate thermocouple in the arm with the coil under test. By varying the voltage fed to the bridge, the test current through the coil can be held at the desired value. Of course the thermocouple heater resistance must be subtracted from the value calculated by the relation $R_X = R_B R_S / R_A$. It is advisable to check the resistance of the heater to ascertain if its resistance is substantially constant at various current values and if it agrees with its rated resistance value. The bridge can be used to do this by removing the inductive elements. The standard inductor L_S must have an effective resistance R'_L that is constant, or

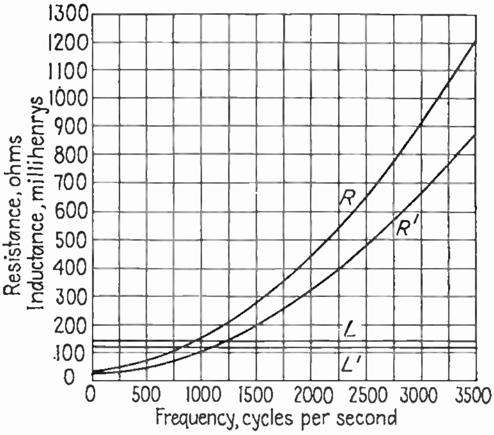


FIG. 234.—Illustrating the relations between self-inductance, effective resistance, and frequency of a coil with a closed magnetic core of powdered and compressed iron. Values of L and R taken with 1.5 milliamperes through the windings, and L' and R' taken with 1.0 milliampere through the windings.

a further correction will be required. The effective resistance will usually be sufficiently constant at audio frequencies if the standard inductor has an air core. Typical curves taken at various audio frequencies on an iron-cored coil by the method here outlined are shown in Fig. 234.

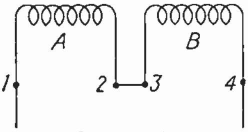
Inductance Measurements with Standard Capacitor.—Such an arrangement would ordinarily be used only with an *equal-ratio-arm* bridge, and was treated on page 350.

Precautions in Bridge Measurements.—In general it is true that bridge measurements are easily and quickly made and with a high degree of accuracy. Reasonable precautions must be followed, however, for reliable results. It is difficult in a few words to enumerate the precautions to be taken, but the following points may be helpful.

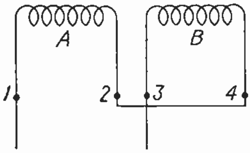
Remember that if inductance is to be measured the magnitude of the current through the coil will affect both the inductance and effective resistance if the coil has a core of magnetic material. The inductance itself should not change appreciably with frequency over the audio range. Be certain that the bridge is balanced sharply. It is usually difficult to balance bridges at frequencies below 300 cycles. If frequencies above about 5000 cycles are employed, the bridge must usually be carefully shielded, and this does not mean merely enclosing the bridge in a metal container, but the individual resistors, etc., must be shielded. In other words, only specially constructed high-frequency bridges should be used at frequencies above about 5000 cycles. Special radio-frequency bridges are available commercially which will make accurate measurements throughout the broadcast band and above. These are particularly useful in making impedance measurements on broadcast antennas.

Stray capacitances are particularly troublesome in causing errors in bridge measurements. For example, the observer may notice that the balance is changed when the headphones are touched or when the bridge dials are touched but not varied. Much of this trouble can be minimized by using an isolating transformer between the bridge output terminals and the headphone leads. An input transformer between the oscillator and the bridge may also be helpful. In general, these transformers should be carefully constructed with a shield between the primary and secondary which may be grounded. These may be 1 to 1 ratio transformers, or ones with other convenient ratios. Usually a bridge should be grounded, but there are instances where this is neither necessary nor advisable. If the reader desires a detailed discussion on bridges, particularly practical information on their assembly, a series of articles, "Impedance Bridges Assembled from Laboratory Parts," starting in the July, 1941, issue of the *General Radio Experimenter*, is recommended.

The Measurement of Mutual Inductance.—Mutual inductance can be measured by the bridge arrangements previously considered for measuring self-inductance. The way to do this is as follows: Connect the primary and secondary in series; this will use two of the leads. Take the other two (one primary and one secondary), connect them to the impedance bridge, and measure the inductance between them. Then, reverse the connections of the primary and secondary, and again measure the inductance of the two coils. The mutual inductance will equal the difference between the two readings divided by 4. The reason for this will now be explained.



(a)-Coils aiding



(b)-Coils opposing

FIG. 235.—Connections for determining mutual inductance.

When two coils are coupled by mutual inductance, the following action occurs: The current in coil A induces a back voltage in coil A because of the self-inductance L_1; also, the current in coil A induces a voltage in coil B because of the *mutual inductance* M. Furthermore, the current in coil B induces a back voltage in coil B because of the self-inductance L_2, and the current in coil B induces a voltage in coil A because of the mutual inductance M. If the two coils are connected *aiding* as in Fig. 235*a*, then the induced voltages all *add*. If the coils are connected *opposing* as in Fig. 235*b*, then the voltages due to the *self*-inductance add, but the voltages due to the *mutual* inductance oppose the voltages of *self*-inductance and subtract from them.

Now inductance is made evident in a circuit because a voltage is induced when the current is changed, for instance, when an alternating current flows through it. Thus, if all induced voltages add as in Fig. 235*a*, it appears that a *large* value of inductance exists between the terminals 1–4. If the induced voltages oppose as in Fig. 235*b*, it appears that a *small* value of inductance exists between terminals 1–3. Since the inductance between the terminals is proportional to the voltages induced, and since these voltages are determined by the self- and mutual inductances, it can be written that

$$\text{Aiding} \quad L_{1-4} = L_1 + 2M + L_2.$$

$$\text{Opposing} \quad L_{1-3} = L_1 - 2M + L_2.$$

Subtracting the last equation from the first gives

$$L_{1-4} - L_{1-3} = 4M \qquad \text{and} \qquad M = \frac{L_{1-4} - L_{1-3}}{4}. \tag{89}$$

Thus, the mutual inductance equals the difference between the measured inductance with the coils aiding and the measured inductance with the coils opposing divided by 4.

The principles discussed in the preceding paragraphs make possible a variable inductance standard, often called an **inductometer** or **variometer** *which has a substantially constant effective resistance and which has a continuously variable inductance.* With reference to certain of the bridge circuits (for instance, Fig. 229), it was explained how a small resistor R_L was used in the opposite arm of the bridge to balance the effective resistance of the standard variable inductor L_S. This could be accomplished by simple means only if the effective resistance of the standard inductor is constant. Thus, an inductance standard that gave different inductances by *varying taps* would have a varying resistance and would

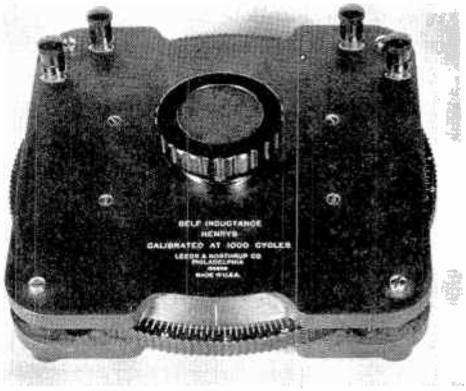


FIG. 236.—A commercial form of a standard variable inductor or Inductometer which provides a variable inductance with substantially constant effective resistance between its terminals. (*Courtesy of Leeds and Northrup Company.*)

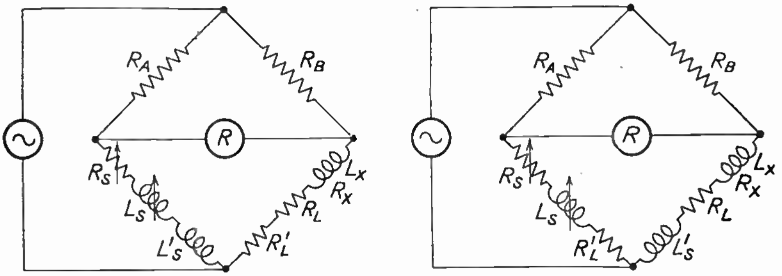
not have a continuously variable inductance. But, if a standard inductor is made as in Fig. 235*a*, and if the mutual coupling or mutual inductance M can be changed by varying the position of one coil with respect to the other, then a continuously variable inductance standard with constant resistance is possible. A commercial form of such an inductor is shown in Fig. 236.

The Impedance Bridge.—Many of the bridge principles of the preceding pages are incorporated into a single bridge circuit rather extensively used in telephone systems and called an **impedance bridge.** Just as for the bridges previously discussed, however, it measures resistance, inductance, and capacitance. With this impedance bridge there are four possible circuit arrangements as shown in Fig. 237. Each of these circuits is set by two key switches, and no other circuit changes are necessary. The resistors R_A and R_B are the same, giving a 1 to 1 ratio.

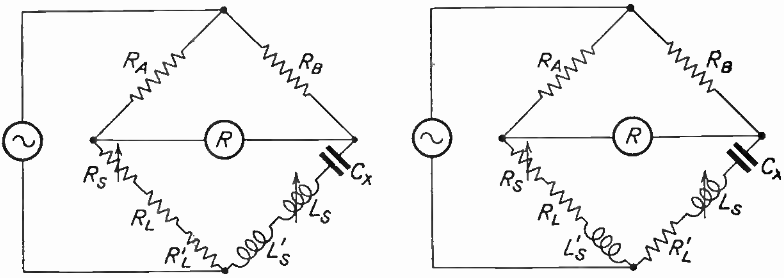
Measurements of High Inductance.—The key switches are set so that the circuit is as shown in Fig. 237*a*. This circuit is for measuring a large value of inductance which is connected as indicated. The inductance L_X of the unknown is balanced by and equal to the sum of the *variable* standard inductor (or inductometer) L_S and the fixed standard inductor L'_S. The effective resistance R_X of the unknown is balanced by and equal to the resistor R_S. The effective resistances of the inductometer and the fixed inductance are balanced out by resistors R_L and R'_L placed in series with the unknown as indicated. This action was discussed on page 350.

Measurements of Low Inductance.—Because of the self-inductance that is present in an inductometer, it is not possible to reduce the inductometer setting to a zero value. Hence, for making measurements on inductors of low inductance the key switches are thrown so that the bridge is arranged as in Fig. 237*b*. In this circuit, the fixed inductor L'_S, which usually has an inductance of 0.1 henry, is placed *in series* with the unknown inductance; note, also, that the resistor R'_L which balances the effective resistance of the fixed inductor is shifted to the opposite arm. With these arrangements, the inductance of the unknown inductor is L_S-L'_S. Since the inductance of the fixed inductor is 0.1 henry, and since the lowest setting of the inductometer is about 0.1 henry, with the bridge arranged as in Fig. 237*b* very low values of inductance can be measured. The setting of R_S will equal the effective resistance of the unknown.

Measurements of Low Capacitance.—On the assumption that the frequency is the same, a capacitor of low capacitance has a high reactance, and a capacitor of high capacitance has low reactance, because $X_C = 1/2\pi fC$. The circuit of Fig. 237*c* is arranged by setting the key switches in the proper positions. This places both the variable inductometer L_S and the fixed inductor L'_S in series with the unknown capacitor. Note that the resistors R_L and R'_L balancing the inductometer effective resistance and the fixed inductor effective resistance are placed in the opposite arms. With these circuit arrangements, series resonance is produced by making the inductive reactance equal the capacitive reactance, and



(a)- Measurement of high inductance (b)-Measurement of low inductance



(c)-Measurement of low capacitance (d)-Measurement of high capacitance

FIG. 237.—Various circuit combinations of an impedance bridge in a form widely used in telephone practice.

$$X_L = X_C \quad \text{or} \quad 2\pi fL = \frac{1}{2\pi fC_X} \quad \text{and} \quad C_X = \frac{1}{(2\pi f)^2 L}, \tag{90}$$

where L equals the *sum* of L_S and L'_S in henrys and f is the frequency in cycles per second, C_X will be in farads. Ordinarily, the capacitor under test will have no losses (which means zero effective resistance), so R_S will be zero.

Measurements of High Capacitance.—As previously mentioned, high capacitance gives low reactance. Thus, but a small amount of inductance will be necessary in series with a large capacitor to produce resonance. Thus, for measuring the capacitance of a large capacitor, the key switches are thrown to the position for Fig. 237*d*. Now *only part* of the inductometer value L_S is effective in producing resonance because L'_S is in the opposite arm and offsets part of L_S. The capacitance of the unknown capacitor is found by Eq. (90), but now L in this equation is the difference between L_S and L'_S.

Incremental Inductance.—One of the fundamental equations for the self-inductance of a coil is $L = N\phi/10^8I$. From this equation, the magnetic flux ϕ produced by current I is a measure of the inductance L. If the reader will refer to page 148, it will be found that the permeability μ of an iron-cored coil varies widely for different values of magnetizing current. Now the permeability is one of the factors determining the flux produced by a current. Therefore, the inductance L must vary with the current I.

A very important example of this is in the large iron-cored choke coils used in rectifier-filter circuits. These coils carry the direct current from the rectifier tube to the load, and because of their inductance, they largely prevent the alternating-current components, which would cause hum, from flowing through the load. Accordingly, the amount of direct current through the coils will be a factor in determining the inductance offered to the alternating current flow as will now be explained.

The saturation curve for an iron-cored coil is as shown in Fig. 238. The direct-current component I through such a coil will determine the point of operation on this curve such as shown by the dotted line A, and this will establish a magnetic flux ϕ. Now if an alternating-current component is also flowing through the coil, as it does in rectifier-filter chokes, then the resultant current will be pulsating in nature and will vary above and below I and point A as shown by the dotted lines I_1 and I_2. This alternating-current variation will produce a flux change ϕ_1 to ϕ_2, and this will induce a back voltage in the coil. Since the back voltage is a measure of the self-inductance, the self-inductance in this instance will be *high*, because ϕ_1 to ϕ_2 is a large change.

Now suppose that the direct current through the coil is changed to I' and that the *same* value of alternating current flows through the coil as before as represented by I'_1 to I'_2. When these lines are projected over to the Y axis, it is seen that but a very small flux change occurs, and therefore for the *same* alternating current change a much lower back voltage would be induced. The coil, accordingly, has less inductance with direct current I' through it than it has with current I through it. *The inductance offered to small current changes is called the* **incremental inductance.**

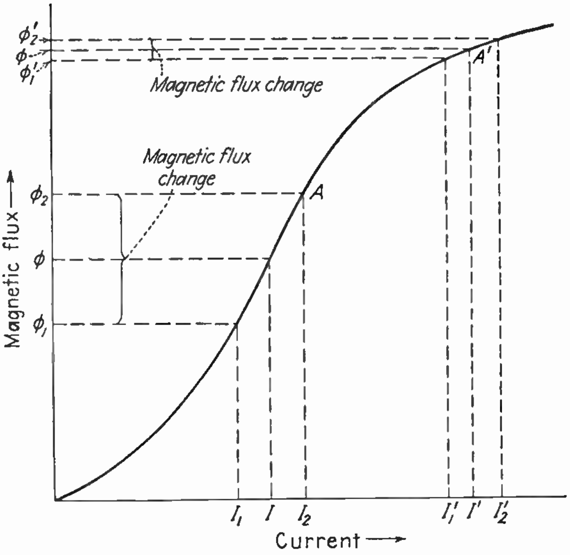


FIG. 238.—The flux change produced in an iron-cored coil depends on the saturation current I and I'.

Measurements of Incremental Inductance.—The incremental inductance of an iron-cored coil is not particularly easy to measure. Furthermore, unless it is measured in a very careful manner, almost any result can be obtained. There have been circuits and methods developed which *appear* to be entirely satisfactory, but if they are carefully studied it will be found that the results they give are not reliable. The so-called modified Hay-bridge circuit which will now be discussed will give reliable results if the method to be outlined is carefully followed.[1]

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[1] Another method which appears to be very good will be found in an article by E. H. Meier and D. L. Waidelich, The Measurement of Iron-cored Choke Inductance, *Communications*, November, 1941.

The modified Hay bridge is arranged as in Fig. 239. Starting at the right, the oscillator and voltage divider furnishes a variable voltage of the test frequency. This may be a low-voltage 60-cycle source if no low-frequency oscillator is available. *The frequency at which these tests are made must be low,* because if it is not, then the distributed capacitance between the turns of the coil will offer a shunt path to the test current and will materially affect the measured values. A frequency of 100 cycles is often used; this test frequency should never exceed, say, about 300 to 400 cycles, depending of course on the coil.

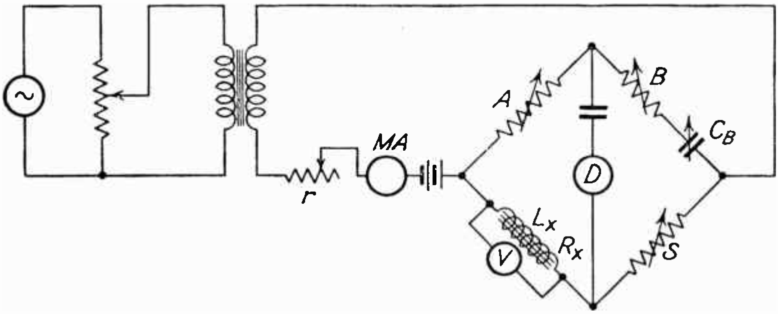

FIG. 239.—Circuit for measuring the incremental inductance of an iron-cored coil. This circuit shows a very high impedance voltmeter connected across the unknown coil L_XR_X so that the alternating voltage can be held constant. An alternate and perhaps a better method is to place a small resistance in series with the coil and measure the drop across the resistance, thus being able to hold the alternating current constant. In either instance, the voltmeter must have a very high impedance, and must not pass direct current.

The transformer shown merely prevents direct-current from flowing through the voltage divider and oscillator. It may be omitted, but it is best to have it, otherwise each adjustment of the voltage divider will change the direct current in the coil under test. The direct current is caused to flow by the battery, is adjusted by the series rheostat, and measured by the milliammeter. A check of the bridge circuit will show that, because of the various condensers, direct current flows only through the two lower arms of the bridge, one of which is the coil under test. It is important to note that resistor *S* must have sufficient heat-dissipating capacity to carry the direct current safely. Also, since the current flows through arm *S*, it is best to leave this arm set during bridge balancing to keep from varying the direct current.

Because of the low frequency used, headphones are not very satisfactory, although they can be used for approximate results. The best null indicator is a vacuum-tube amplifier detector with a tuned input circuit; a wave analyzer works very well. It is best to have some type of a circuit that can be sharply tuned to the test frequency.

Before the tests are made, the core of the coil should be thoroughly demagnetized. This may easily be done with 60-cycle current. Then if measurements are to be made at various direct-current values, it is well to start in with the lowest value of direct current to be used and balance the bridge, then take the next larger value of direct current and balance the bridge, etc. *Do not proceed in the reverse order*, and if in adjusting for a given direct-current value it is exceeded, demagnetize the core before proceeding.

It will be found that the value of the alternating-current component through the coil causes slight variations in the incremental inductance. To be able to measure this alternating current, a small resistor is placed in series with the coil, and the alternating-voltage drop across this resistor is maintained constant by varying the voltage divider across the oscillator. A vacuum-tube voltmeter can be used to measure this voltage drop. The resistance of this small resistor must be subtracted from the final calculated value (Eq. 91). As an alternate method, it has been found satisfactory to hold the alternating voltage across the coil constant, but theoretically the other method is better when a series of measurements are made at various direct-current values because the impedance of the coil varies.

The derivations of the equations giving the inductance and the effective resistance of the coil are involved and will not be included. At balance,

$$L_X = ASC_B, \qquad R_X = ABS\omega^2C_B, \qquad \text{and} \qquad Q = \frac{1}{B\omega C_B}. \qquad (91)$$

In these equations L_X will be in henrys and R_X will be in ohms, when A, B, and S are in ohms, C_B is in farads, and $\omega = 2\pi f$. These equations are approximate but are satisfactory for most work. In Fig. 240 are shown curves taken by the Hay-bridge method.

To summarize: The incremental inductance and the effective resistance of an iron-cored inductor can be measured by the modified Hay bridge. A number of precautions are advisable if reliable results are to be obtained. Because the incremental inductance and effective resistance of an iron-cored coil vary greatly with the amount of direct current, when the inductance of such coils is stated the value of direct current at which this inductance holds should also be given; for example, 12 henrys at 20 milliamperes.

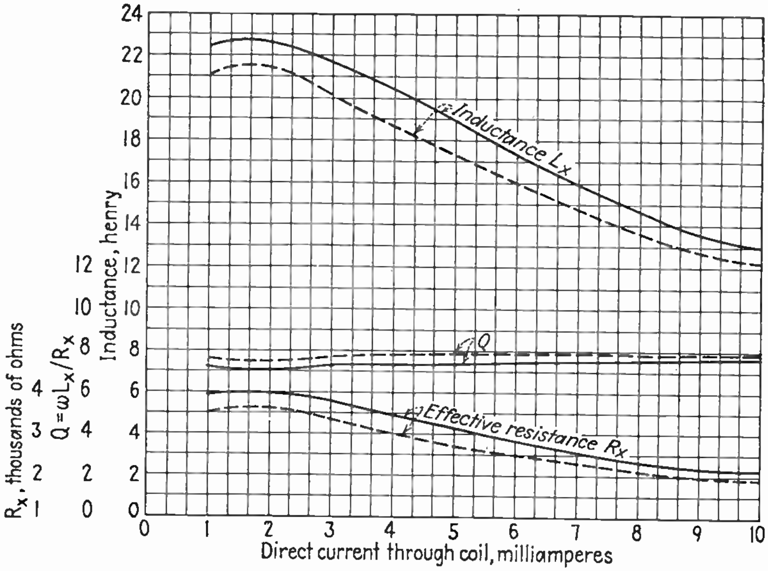

FIG. 240.—Characteristics of a power supply filter choke coil. Inductance L_X, effective resistance R_X, and $Q = \omega L/R_X$ measured at 200 cycles and with various values of direct current using the bridge of Fig. 239. For the solid curve the alternating voltage across the coil was held at 1.5 volts, and for the dotted curve it was held at 1.0 volt.

Only recently is this method of rating coils being followed in practice.

Capacitance Bridge with Polarizing Voltage.—As was discussed on page 202, an electrolytic condenser must be used on circuits in which pulsating voltages exist so that the direct-voltage component will keep the condenser polarized. When the capacitance of an electrolytic condenser is to be measured on a bridge, the bridge must provide a direct polarizing voltage for the condenser.

Such a bridge circuit is shown in Fig. 241. This bridge is but a modification of Fig. 232, and the same equations hold. A condenser is added in series with the oscillator to prevent direct current from flowing through it, and a large inductor is connected in series with the direct-voltage supply to prevent this circuit from affecting the bridge balance.[1]

When measuring the capacitance and effective resistance of an electrolytic condenser, the normal direct working voltage should be impressed on the condenser. The losses in electrolytic condensers are greater than in other types of condensers, and therefore the effective resistance as given by Eq. (86) is larger. The resistance R_X and the capacitive reactance produced by the capacitance C_X of the condenser act at right angles (page 257) to give the impedance of the condenser. From these values, the power factor can be calculated, because they determine the phase angle between the voltage and current. Either the power factor or this phase angle can be used to specify the merits of *any capacitor.*

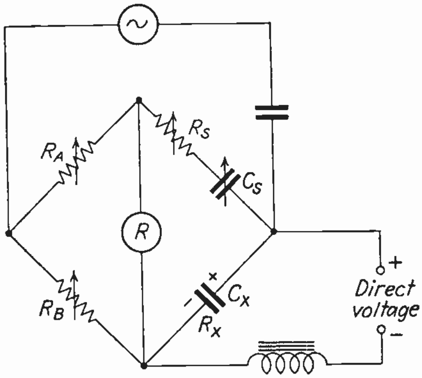

FIG. 241.—Circuit for determining the characteristics of electrolytic condensers. A polarizing voltage is supplied.

The Bridge Transformer.—This interesting device is used in many telephone circuits to permit two-way conversation over one pair of wires. It is also used in bridge measuring circuits. The bridge transformer can be explained by Fig. 242. From the discussions that have been given in this chapter, it is evident that the bridge circuit of Fig. 242*a* will be balanced if $Z_E = Z_W$ in both magnitude and sign, and if the two coils are identical. Then, by applying the reciprocity theorem (page 330), it is evident that the circuit of Fig. 242*b* also will be balanced. Without giving a

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[1] P. M. Deeley, in his book "Electrolytic Capacitors," recommends that this inductor be as large as 500 henrys. His book is suggested for further data on testing electrolytic capacitors. Published by the Cornell-Dubilier Electric Corp., South Plainfield, N. J.

detailed proof, Fig. 242*c* will also be balanced if the two transformers are identical.

This last circuit can be extended to Fig. 243. By analogy with Fig. 242*c*, no voltage will exist across Z_i when the transformer windings are identical, and $Z_E = Z_W$. Therefore, Z_i may be the *input* circuit to a vacuum-tube amplifier in a **telephone repeater,**[1] and the generator may be the *output of the amplifier which will not feed back into the input.* If it did, the amplifier would "sing" or "howl" (oscillate) as it is called in telephone parlance.

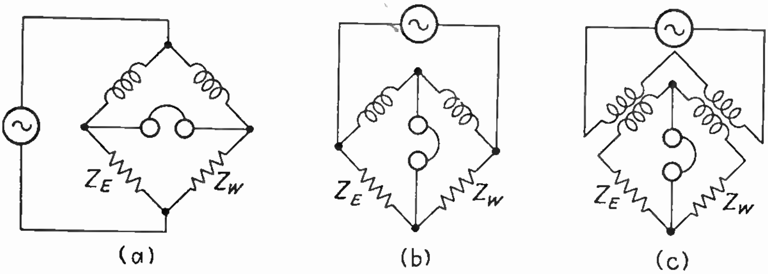

Fig. 242.—Development of the theory of the hybrid coil or bridge transformer.

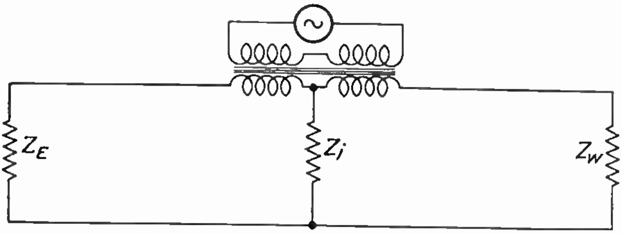

Fig. 243.—Circuit of the bridge transformer.

Figure 243 also provides a very useful bridge circuit for making measurements. If Z_W is a resistor, then Z_E must be a resistor of equal resistance in order that no tone may be produced by a receiver connected at Z_i; similarly, inductors or capacitors can be measured.

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[1] A term applied to the entire circuit of an amplifying device inserted in telephone lines to amplify the speech waves that have been weakened by the line losses.

SUMMARY

Resistance, inductance (both self- and mutual), and capacitance can be accurately measured with bridges.

The Wheatstone bridge is the basic type from which other bridges are derived.

According to the reciprocity theorem, when a bridge is balanced the source and the null indicator may be interchanged and it will still be balanced.

The Murray and Varley loops are circuit modifications of the Wheatstone bridge and are useful in locating faults on lines and cables.

Alternating-current bridges are also but modifications of the direct-current types.

Alternating-current bridges are driven by an alternating-voltage source such as an oscillator and often use a telephone receiver as a null indicator.

Both equal and unequal ratio-arm alternating-current bridges are used.

For accurate and reliable results, certain precautions must be observed in making bridge measurements. This is particularly true at high frequencies where specially shielded and balanced bridges must be employed.

Mutual inductance can easily be determined by taking one-fourth of the difference between the inductances with the coils connected aiding and connected opposing.

Incremental inductance is defined as the inductance offered to small current changes. One factor determining it is the point on the magnetic saturation curve of an iron-cored coil at which the incremental change occurs. This is determined by the strength of the direct-current component flowing in the coil. Incremental inductance can be measured with a special bridge circuit.

Electrolytic capacitors must have a polarizing voltage across them, and a special bridge circuit supplying this voltage is possible.

A bridge transformer is used to make two-way amplification possible in a two-wire circuit. It is also used in bridge measuring circuits.

REVIEW QUESTIONS

1. Name the two general types of bridges.

2. What is the application of the reciprocity theorem to bridge circuits?

3. Why should large values of resistance be used to obtain a given ratio?

4. What precautions must be used in the direct-current Wheatstone bridge when measuring the resistance of a coil of wire?

5. What is the advantage of the Varley loop over the Murray loop?

6. Why is a tuned detector sometimes desirable as a null indicator?

7. Under what conditions is an alternating-current bridge balanced?

8. Why can a standard inductance be used to measure capacitance?

9. Why does the effective resistance of an air-cored coil vary but slightly with frequency?

10. Why does the effective resistance of an iron-cored coil vary greatly with frequency?

11. Explain how stray capacitance may affect the bridge balance (see Fig. 215, page 329).

12. How is mutual inductance measured?

13. Explain the principle of the inductometer.

14. Explain what is meant by incremental inductance.

15. Why does incremental inductance vary with the direct current through a coil?

16. Enumerate the precautions to be used in measuring incremental inductance.

17. Why should the effective resistance of an iron-cored coil vary with the amount of direct current through the coil?

18. Referring to Eq. (91), what is meant by Q?

19. Referring to Fig. 241, what value of direct voltage should be used on the bridge?

20. How would you proceed to demagnetize an iron-cored coil with a 60-cycle supply?

PROBLEMS

1. Referring to the Murray loop on page 346, suppose that the loop length is not known and that it is not convenient to determine it, but the loop resistance and the resistance per foot of each wire are known. Derive equations giving the distance to the fault when the two wires are different, and when they are the same.

2. Repeat Prob. 1 for the Varley loop.

3. Referring to the statements on page 353, explain why capacitance measurements with a standard inductor ordinarily would be made only with an equal-ratio-arm bridge.

4. Referring to Fig. 234, explain why the curves are as shown.

5. Referring to page 363, draw a circuit and explain how to demagnetize an iron-cored coil with 60-cycle alternating current, and also draw a circuit and explain how to demagnetize a coil with direct current.

6. Referring to Eq. (91), prove that the equation for Q is correct.

7. If a capacitor has an effective resistance of R_X and a capacitance of C_X, write the equation for the power factor. How could the phase angle be found?

8. Explain the shapes of the curves in Fig. 240.

CHAPTER XIII

THE TRANSMISSION OF ELECTROMAGNETIC WAVES

For a person to talk by telephone over the connecting wire circuit between two distant cities, electric energy, controlled by the speaker's voice, must travel along the wires. Also, for a broadcast program or a radio message to be transmitted from the radio station to the receiving set, electric energy must travel through space.

In transmitting direct-current energy and in transmitting low-frequency 60-cycle energy, it is usually entirely satisfactory merely to consider that the currents in the wires and the voltages between the wires carry the electric energy from one point to another. When working with higher frequencies such as in telephony and in radio, and also when long distances are involved as in these two fields, a new concept of the transmission of electric energy must be had in order to understand clearly the problems involved.

It is the purpose of this chapter to develop this new concept and to explain the basic principles involved. The transmission of electric energy over long wire circuits and through space by "wireless" means will both be considered. Transmission over wire lines will be considered first.

Electromagnetic Waves.—In dealing with direct currents and low-frequency alternating currents over circuits that were of no great physical length the method of attack used was as follows: The voltage impressed on the circuit forced a current through the circuit, and the power reaching the load was equal to EI for direct currents and $EI \cos \theta$ for alternating currents. The product of the power thus calculated and the time involved gave the energy transmitted to the load.

Now an impressed voltage always produces an *electric field*, and this field is proportional to the strength of the voltage. Also, a flow of current always produces a *magnetic field*, and the strength of the field is proportional to the strength of the current, at least when the field is in air. Here is a new way of regarding the matter, however: From a fundamental point of view, perhaps after all what is called voltage is there only because the electric field manifests itself as a "voltage," and perhaps what is called current is there only because the magnetic field causes a phenomenon that has been termed "current." In other words, it is just as fundamentally correct to say that the energy transmitted equals $t\psi\phi \cos \theta$ as it is to say the energy equals $tEI \cos \theta$; that is, the energy transmitted between two points equals the product of the time t, the electric field strength ψ, the magnetic field strength ϕ, and the cosine of the angle between the fields. Electric fields and magnetic fields are out of phase if the voltages and currents associated with them are out of phase.

Thus, when a voltage from an alternating source is impressed on two long parallel wires (such as a telephone line) electric fields and magnetic fields will exist along the line. These fields will travel from the source to the distant load. Energy is stored in electric fields (page 200) and in magnetic fields (page 162), and these fields *must* travel from the source to the load to carry energy from one point to another. Strictly speaking, electric energy is not carried by voltages and currents (even if it does *equal* $tEI \cos \theta$), the energy is carried by the electric and magnetic fields in which it is stored. Electric energy traveling along a wire line (or through space) in the form of electric and magnetic field variations is termed an **electromagnetic wave.**

Production of Electromagnetic Waves.—As was explained on page 119, a sine-wave voltage is produced by a rotating voltage vector. When the voltage vector lies along the *horizontal*, the *instantaneous* value of the impressed voltage is *zero*. When the voltage vector is *vertical upward*, the instantaneous value of the impressed voltage is a *positive maximum*. When the voltage vector is *vertical downward*, the instantaneous value of the impressed voltage is a *negative* maximum. When a voltage is impressed on two long parallel wires, this voltage produces an electric field *between* the wires. Also, a current flows in the wires and magnetic lines of force encircle the wires. The electric and magnetic fields constitute an electromagnetic wave, and this wave travels along the wire line with a velocity approaching that of the velocity of light, or 186,300 miles per second. Actually, for a typical telephone line the velocity is, for practical purposes, 180,000 miles per second. No wonder instantaneous electrical communication is possible.

To explain the production of an electromagnetic wave, Fig. 244 is included. Here is shown a source of alternating voltage connected to an open-wire line of infinite length. Assume that this source is an oscillator impressing a pure sine-wave voltage on the line. This sine-wave voltage is not shown, but the rotating voltage vector that generates the sine-wave voltage (page 119) is shown at the left. The voltage vector is shown rotating counterclockwise, and it is assumed that it has turned through 360 degrees, from A to B to C and back to A, thereby impressing *one cycle* of alternating voltage on the line.

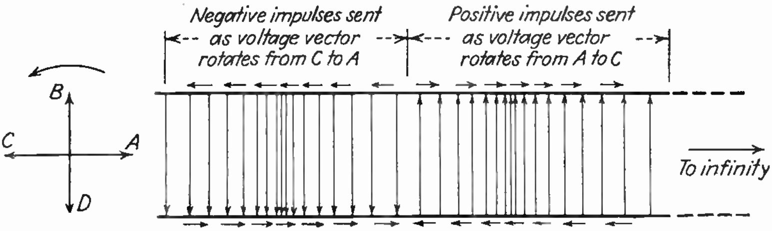

FIG. 244.—The voltage applied to the end of a line establishes an electromagnetic wave along the line.

As the voltage vector starts out from A, its *instantaneous* value is at first very small, but it keeps increasing and becomes a positive maximum at B. Although it is very small at first, it nevertheless starts a weak electric field down the line; also, a small current flows into the line, and this starts a weak magnetic field down the line. The voltage and current continue to get stronger at each instant from A to B, and the electric and magnetic fields flowing into the line also tend to get stronger at each instant. From B to C the instantaneous value of the impressed voltage and the current is getting smaller, and likewise the electric and magnetic fields entering the line at each instant become smaller. *Once the electric and magnetic impulses enter the (infinite) line, however, they never return but continue on down the line.*

Now consider what happens as the voltage vector continues on around from C to D. The voltage is now increasing negatively, and the current now flows into the line in the opposite direction; also, the electric and magnetic fields started down the line will be in the opposite direction. This action is the same from point D back to A with the exception that the instantaneous action is decreasing.

It must be remembered that at each instant *the fields produced at each preceding instant are traveling down the line with a velocity of* 180,000 *miles per second.* Thus, with reference to Fig. 244, traveling *positive* electric and magnetic lines of force were produced when the voltage vector rotated from A to C, and these traveled to the position shown while the voltage vector rotated from C on around back to A.

Because of the difficulty in drawing the magnetic lines of force encircling the wires, they have not been shown in Fig. 244, but instead, arrows representing the current in each wire have been drawn. The length of these current arrows indicate the intensity of the current and the strength of the magnetic field. The arrows between the wires represent the electric field; where these arrows are close together, the field is strong. The direction of these electric lines of force represent the direction of the voltage between the wires, and the closeness or intensity represents the strength of the voltage. Note that in this wire line *the voltage is in different directions at different sections; and the currents are also in different directions at different points.* This is exactly what is happening at different points along a line that is transmitting electric energy in the form of electromagnetic waves. The reason why the method of explaining the action of long lines is different from that of considering ordinary circuits is now evident.

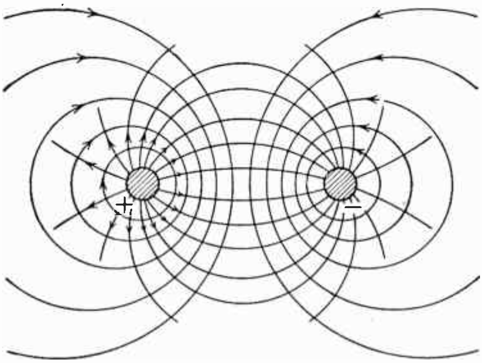

FIG. 245.—Electric field between a pair of wires and the magnetic field around the wires when one wire is positive and the other is negative. These two fields progressing together down the pair of wires constitute an electromagnetic wave.

The magnetic lines of force around the wires were not shown in Fig. 244, because of difficulty in representing them. Their distribution and their relation with the electric lines of force are shown in the "end-on" view of the line in Fig. 245.

Electrically Long Lines.—As was shown by Fig. 244 and the accompanying discussion, when a sinusoidal voltage is impressed on a line an electromagnetic wave is propagated down the line. This wave was shown to consist of electric and magnetic lines of force. For convenience in drawing, in Fig. 244 only the electric lines of force were shown. The magnetic component was depicted by current arrows. The electric lines of force were directed one way for positive values and the opposite way for negative values. In regions where the field was intense, many lines were shown. Now this is a good way to picture an electromagnetic wave, but it is somewhat laborious.

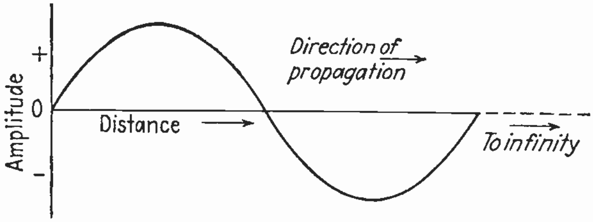

FIG. 246.—An electromagnetic wave along a wire line may be represented by a curve as shown. This illustration is for a line with no loss.

An electromagnetic wave may also be represented by a curve such as in Fig. 246. If the input impedance of the line is pure resistance, then the input current will be in phase with the voltage. Thus, this curve could then represent both the current and voltage components or both the electric and magnetic components of the electromagnetic wave. If the input impedance of the line contained a small amount of capacitive reactance as is usually the case, then the current curve would be slightly ahead of the voltage curve. Note that although the curve of Fig. 246 is a sinusoidal wave, it represents voltage variations (or current variations, or electromagnetic-field variations) at a given instant *along the line.* In Fig. 246, *distance* is taken along the X axis. The sine waves that were previously considered in this book were not space waves as is Fig. 246; they merely represented variations *with respect to time.* In Fig. 246 are shown *instantaneous values at a given instant along a line.* In other figures, current and voltages *were represented* by sine waves, but the electromagnetic impulses that flow down a line when a sinusoidal voltage is impressed *are actually waves.*

In Fig. 246 is shown one complete **wave length.** One wave length is equivalent to 360 electrical degrees because it is due to one complete revolution of the voltage vector of Fig. 244 or to one complete cycle of the impressed voltage. At a frequency of f cycles per second, f complete wavelengths are established in the line each second, and these *can travel only* a distance of V miles (or other convenient units) per second, where V is the wave velocity in the line. Therefore, the length of the wave will be

$$\lambda = \frac{V}{f}, \tag{92}$$

where λ is the length of the wave in the same units as used to express the wave velocity V, where f is the frequency of the impressed signal voltage in cycles per second.

Suppose two parallel wires 300 miles long are available and that a 60-cycle voltage is impressed across them as if they were a power line. One wave length would be $\lambda = 180{,}000/60 = 3000$ miles, and thus only 300/3000, or 0.1, wave length could exist on the line. Now 300 miles is very long for a power-transmission line. Since they are usually so much shorter in practice, at a 60-cycle power frequency, waves are not distributed along them as shown in Fig. 246. Wave-propagation theory does not usually need to be applied in power-line design.

Now suppose that a voltage having a frequency of 1000 cycles per second is impressed on the line. This frequency is not high and is even somewhat below the average of the important voice frequencies. For 1000 cycles, $\lambda = 180{,}000/1000 = 180$ miles, and $300/180 = 1.66$ wave lengths will exist on the line.

Now in the first instance an **electrically short line** is being considered because the 60-cycle frequency is so low that the line has only a small fraction of a wave length on it, but in the second case an **electrically long line** is under consideration because it has a considerable portion of a wave length on it; in fact, it has 1.66λ. If the frequency is very high, say a few million cycles per second, then the wave length will be very short, and many waves will exist on a relatively short section of line. Waves but a few centimeters long may be produced electrically.

To summarize: Whether a line should be designated as electrically long or electrically short depends both upon the frequency and the physical length of the line.

The Transmission Line.—For the purposes of this book and for most communication installations, a **transmission line** consists of two parallel conductors, often of hard-drawn copper wire. When an alternating voltage is impressed between the sending end of the line, a current flows into the line. For very high radio frequencies, a few feet of parallel conductors becomes a transmission line. This line can be thought of as conducting the current and voltage to the distant end, but for communication purposes, especially at radio frequencies, a better point of view is that *the transmission line guides the electromagnetic wave* from the sending end to the receiving end.

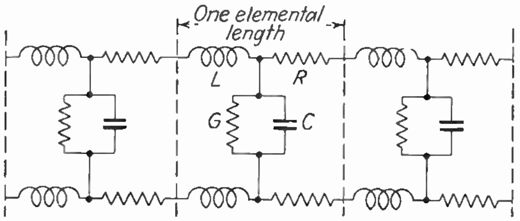

FIG. 247.—Each elemental length of a transmission line consists of inductance *L*, capacitance *C*, resistance *R*, and leakage *G*. For an elemental length these are, of course, very small values.

Now the two-wire transmission line is far more than just two wires. Each elemental length of line has capacitance between the wires, each elemental length of line has leakage between the wires, each elemental length of line has inductance, and lastly, each elemental length has resistance. Several elemental lengths of a transmission line are as shown in Fig. 247. This diagram applies to two short parallel rods used in very high-frequency radio as well as to the two long parallel wires of a transmission line.

The Propagation of an Electromagnetic Wave.—The actual electric circuit of a transmission line was shown in Fig. 247. If an alternating voltage is impressed on this line, an alternating current will flow into the line. The voltage will establish an electric field, the current will produce a magnetic field, and these two fields acting together will produce the electromagnetic wave that travels along the line carrying the electric energy to the distant receiving end. This action was shown in Figs. 244, 245, and 246.

Not all the electromagnetic energy that starts out reaches the distant end, however; in an actual line such as Fig. 247 some of the energy, and often much of the energy, is lost in the line. Power losses are equal to E^2/R or to I^2R (page 70), and thus there are losses in the line produced by the voltage across each elemental length of line, and there are also current losses due to the currents flowing. These losses at any point in the line must take energy from the passing electromagnetic wave. Because of this, the electromagnetic wave and the accompanying voltages and currents are decreased in amplitude. This decrease in amplitude as the wave progresses along the line is called the **line attenuation.** This action is indicated in Fig. 248.

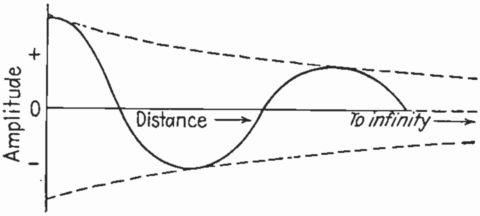

FIG. 248.—If a line has attenuation, the amplitude of the wave decreases with distance along the line. The dotted lines show this decrease. It has the shape shown for the following reason: Suppose that the first unit length of line has a transmission efficiency of 80 per cent; then, at the end of it 80 per cent of the original wave exists. At the end of the second section, there will be left 80 per cent of 80 per cent or 64 per cent of the original wave, etc.

Another factor influencing the transmission of an electromagnetic wave along a line is the velocity of the wave. It was stated on page 370 that the velocity was almost 180,000 miles per second for a typical telephone circuit. This wave velocity at a given frequency is determined by the same factors that determine the line attenuation; that is, the capacitance, leakage, inductance, and resistance of the line.

The beginning student and the practical worker would benefit little by including at this point the equations by which the attenuation and velocity are calculated. Instead, the equations and typical calculations will be treated later in the chapter (page 389) so that a greater knowledge of the subject will exist. For the telephone worker such data are available in tables.

To summarize: As an electromagnetic wave travels along an actual transmission line, the line losses in each elemental section extract energy from the wave and cause the wave and the accompanying current and voltage to be decreased in amplitude or attenuated.

Characteristic Impedance.—Each transmission line *of a given type* has the same characteristics within close limits. Thus, all telephone circuits made of hard-drawn copper wire of a given size and spaced 1 foot apart on the same type insulation, etc., will offer the same attenuation under similar weather conditions. If the wires are spaced closer together, then the inductance will be *decreased* and the capacitance will be *increased*, but still all similar lines will have the same characteristics.

Now suppose that the network of Fig. 247 is considered. Here three elemental lengths are shown. In a very long line a very great number of these elemental lengths would be included. Suppose that an oscillator is connected to a very long section of transmission line; let it be supposed that the transmission line is *infinite* in length, and hence it will contain an infinite number of these sections.

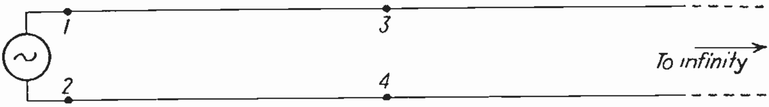

FIG. 249.—Line for studying characteristic impedance.

Such an *infinite* line is shown in Fig. 249. For simplicity the elemental resistances, inductances, etc., of Fig. 247 have not been drawn, but they are present nevertheless. Now suppose that a generator is connected to the input terminals of the line; the impressed voltage will cause a current flow into the line. The **input impedance** of the *infinite* line equals $Z_O = E/I$, and for a given frequency the input impedance of all identical lines is closely the same. *The input impedance of an infinite line is defined as the* **characteristic impedance** *of that line.* Characteristic impedance varies slightly with frequency but not with line length (see next section).

Line Termination.—As has been repeatedly stressed in this textbook, there is at best but little energy in most communication circuits, and the *maximum power output* from a generator or line to a load must be obtained. The question now to be settled is this: What impedance should terminate a transmission line so that the maximum amount of power will be transferred by the line to the load?

This question is readily answered by reference to Fig. 249. When an electromagnetic wave is started out along an *infinite* transmission line the wave will go on forever. Now when an electromagnetic wave comes to points 3-4 of Fig. 249, it goes right on, because the impedance of the circuit ahead of it is just the same as what it has passed through. Now if the length of line between points 1-2 and points 3-4 is removed from an *infinite* line and impedance measurements made from points 3-4 on to infinity, the measured impedance at points 3-4 will be the same as at points 1-2; that is, it will be Z_0 the characteristic impedance. This holds true because subtracting a *finite* length of line from an *infinite* line does not change the characteristics of the line.

But in practice the lines used are *finite* sections like the section 1-2 to 3-4. How should this finite line be terminated for maximum power transfer? The answer is to terminate the finite section 1-2 to 3-4 in an impedance load that is the same as the characteristic impedance. Then, when the electromagnetic wave strikes the load it will see an impedance the same as the line, will enter the load just as it would enter the line on to infinity, and the energy in the wave will be used in the load as desired. This gives another definition of characteristic impedance: *The characteristic impedance of a line equals the input impedance of a line terminated in its characteristic impedance.* Thus, as mentioned in the preceding section, line length does not influence the characteristic impedance.

To summarize: The practical application of the theory of this section is as follows: A telephone transmission line, or a radio transmission line, must be connected to a telephone set, or to a radio antenna, or must be terminated in some load. For the best results the impedance of the termination should equal the characteristic impedance of the line.

Wave Reflection.—The discussion in the preceding section gave consideration to the fact that the load impedance and the line impedance should be the same for the best energy transfer. If only electrically short networks were being considered instead of electrically long transmission lines, then the power or energy transfer would ordinarily be all that must be considered. But in the case of an electrically long transmission line, an additional factor must be considered. This factor is **wave reflection.**

Wave reflection on a long line causes an effect similar to an echo in acoustics (page 501). When an electromagnetic wave that is traveling down a line strikes *any* discontinuity in the line, some of the wave energy is reflected back to the sending end. Theoretically, a poor line splice or a broken insulator can cause reflection, but actually it must be a larger discontinuity such as going from a circuit of one type to a circuit of another type to cause noticeable reflection. Now when an electromagnetic wave traveling down a wire strikes a big discontinuity like an open circuit or a short circuit, then *complete* reflection occurs because the wave cannot go on.

If a person speaks over a telephone connected to a line on which there is reflection, his voice comes back to him after an interval; this may be bothersome. If the circuit is short, and if the wave velocity is high, the time interval may be so small that the echo is not even apparent. On the other hand, if the circuit is long, and if the wave velocity is low, the time interval between the time of speaking and the return of the echo will be large, and a distinct and bothersome echo will be heard. On *electrically short* circuits it is ordinarily only necessary to match circuits to obtain maximum power transfer or to obtain the proper load on vacuum tubes, for instance. On electrically long circuits, however, the proper impedance match also prevents wave reflection. If a sine-wave alternating voltage is impressed on the line instead of speech as has just been considered, then **standing waves** will be caused along the line if it is open- or short-circuited.

The Formation of Standing Waves of Voltage.—There is much misconception regarding the nature and effect of standing waves. As a matter of fact, it takes much imagination to picture a "standing" wave. Treated properly, they are not at all difficult to understand. In explaining their nature, however, it is best to assume a transmission line having very low loss. It would be possible to have standing waves on a power line, a telephone line, or on a radio-frequency transmission line. Power and telephone lines are ordinarily operated with loads terminating them and are seldom operated open- or short-circuited and, hence, under conditions of operation do not have standing waves on them. But, radio-frequency transmission lines are often operated either open- or short-circuited to *utilize* the effect of standing waves. Thus, in the following discussion it will be considered that the transmission line is two parallel wires some few feet long and that the source is a very high radio-frequency oscillator. The attenuation of the line will be considered negligible.

The formation of a standing voltage wave[1] can be explained by reference to Fig. 250. This figure represents the *instantaneous voltage* conditions existing along an *open-circuited* line that has no attenuation. An alternating-voltage source such as a vacuum-tube

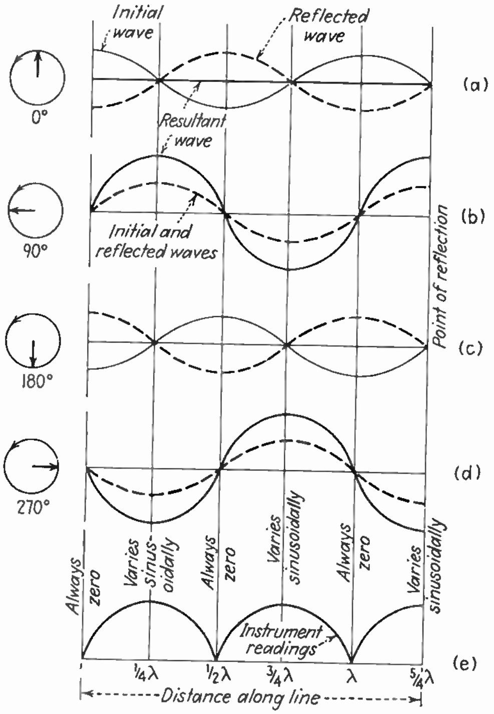

FIG. 250.—Curves of voltage relations on an open-circuited line or current relations on a short-circuited line five-fourths wavelengths long, and having no attenuation. At certain points the voltage or current is always zero; at all other points they vary sinusoidally, points of maximum change being indicated. The voltmeters or milliammeters will read 0.707 of the maximum changes, giving the lower curve called a standing wave. Note that these instruments do not follow the instantaneous sinusoidal variations.

oscillator impresses a vector voltage $E_{\max}$ on the line. Conditions on the line are shown for four different vector voltage positions. The line is $\frac{5}{4}\lambda$ in length. In this discussion the resultant wave (which may be either current or voltage) is made up of an initial and a reflected component. Figure 250*a* represents *voltage* condi-

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[1] This discussion and Fig. 250 are based on the excellent explanations of reflection given by W. L. Everitt in Chap. V of his book "Communication Engineering," McGraw-Hill Book Company, Inc.

tions at the various points along the line *at the instant* the impressed voltage is maximum and positive (page 119). At this instant the *initial voltage wave* at a point ¼λ from the sending end is zero, because ¼λ = 90 degrees, and the voltage wave now at point ¼λ was sent out by the impressed or sending-end voltage value 90 degrees previously, and *then the impressed voltage was zero.* Also, at point ½λ, at the instant under consideration the initial voltage wave will be a negative maximum value, because the voltage wave *at this point at this instant* left the oscillator 180 degrees before when the oscillator voltage was a *negative maximum.* Following this same reasoning, the open end of the line is 5/4λ or 450 degrees from the oscillator, and the initial voltage wave there at the instant under consideration is what left the oscillator 450 degrees before. At the *open end* of the line the initial voltage wave is reflected just as it would have gone on, and the various positions of the reflected voltage wave are as shown. Now it is only possible to have one resultant voltage wave along a line, but this resultant may be composed of two or more separate components. Thus, in Fig. 250*a* the resultant voltage wave made up of the initial and reflected voltage waves is zero all along the line *at this instant;* in other words, *at this instant the voltage is zero at all points* along this line having no losses and 5/4λ in length.

In Fig. 250*b* are shown the conditions all along the line at the instant that the impressed voltage is in the position shown. The initial voltage wave at point ¼λ left the sending end 90 degrees earlier and, hence, is a positive maximum. The initial voltage wave at ½λ left 180 degrees earlier and is, accordingly, zero. The initial voltage wave at other points also can be predicted. The initial voltage wave is reflected from the distant end just as it would have gone on, and the reflected voltage wave is displaced along the line as shown. The initial and reflected voltage waves combine to give the resultant voltage wave as indicated.

The instantaneous distribution of the initial, reflected, and resultant voltage waves along the line for two other positions of the impressed voltage are shown in Figs. 250*c* and *d*. For Fig. 250*c*, note that at the instant depicted the resultant voltage wave is *zero* at all points along the line. Figure 250*d* is similar to Fig. 250*b*, except that all values are reversed.

And now how does this all lead to a standing wave? To explain this, suppose that an observer is at point ¼λ, that one is at point

½λ, that observers are also at points ¾λ, λ, and 5/4λ, and that each of these observers connects a voltmeter across the line. Now as the vector voltage rotates because it is being impressed by a sine-wave source, the *instantaneous* voltage value at each point changes, but from Fig. 250, the observers at ½λ and λ will read zero voltage, because the resultant voltage wave is always zero at this point. The observers at points ¼λ, ¾λ, and 5/4λ will read certain effective voltage values, which are the largest of any values along the line. At these last three points, and at all points except the zero points, an oscilloscope connected to the line will show a sinusoidal voltage to exist. If the *effective* values of the voltage along the line are measured with a voltmeter and plotted, the so-called standing voltage wave of Fig. 250*e* is produced. Note that this is not a wave in the usual sense of the word but merely a plot of effective voltage values.

The Formation of a Standing Current Wave.—The same line as in Fig. 250 will be considered, the line being open-circuited, without attenuation, and the frequency being such that the line is 5/4λ long. Four vector positions of the *current entering* the line are considered in Fig. 251 and the initial, reflected, and resultant current waves are shown. In studying this figure, note that when the current reaches the open end of the line it *does not* reflect just as it would have traveled on, but reflects with a change in sign of 180 degrees. This is because when the electric charges traveling in a given direction strike the open end of the line they are reflected back in the *opposite* direction which changes their direction 180 degrees. (For the voltage wave previously considered, the direction of the lines of force between the two wires, and hence the direction of the voltage between the wires, was the same before and after reflection.)

In Fig. 251*e* are shown the effective values of currents which various observers would measure with suitable milliammeters at different points along the line. Note that the so-called standing wave of current (Fig. 251*e*) is displaced ¼λ from the standing wave of voltage (Fig. 250*e*).

Standing Waves on Lines Open- or Short-circuited.—The standing-wave relations between voltage and current on an open-circuited line 5/4λ in length and having no attenuation were considered in the preceding sections. The relations on the same line when it is *short-circuited* will now be considered.

With reference to Figs. 250 and 251, for the open-circuited line it was found that at the distant end of the line the voltage was maximum and the current was zero. Now for the *short-circuited* line just the *opposite* is true. Drawing individual diagrams as was previously done will show that for the short-circuited line the voltage at the distant end is always zero and the current is maximum.

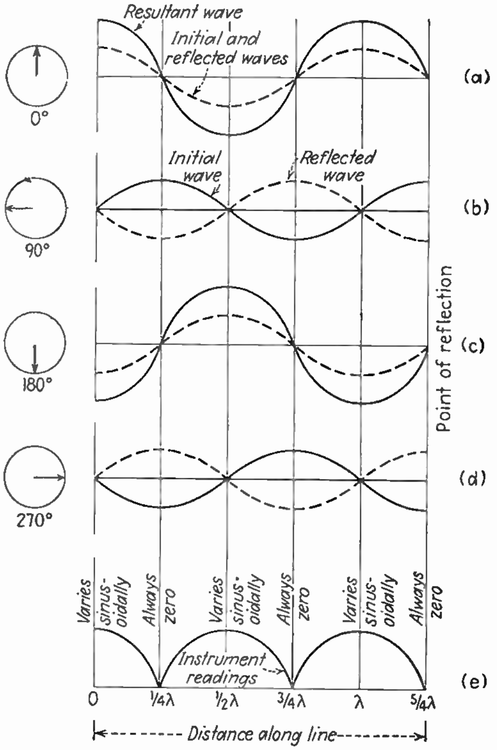

FIG. 251.—Curves of current relations on an open-circuited line or voltage relations on a short-circuited line five-fourths wavelengths long, and having no attenuation. At certain points the current or voltage is always zero, and at other points they vary sinusoidally.

Without the detailed drawing of figures similar to Figs. 250 and 251, it follows that the so-called standing waves in open-circuited and short-circuited lines are as shown in Fig. 252.

Now the question arises, suppose that the line is not $\frac{5}{4}\lambda$ in length, then what are the current and voltage relations? The

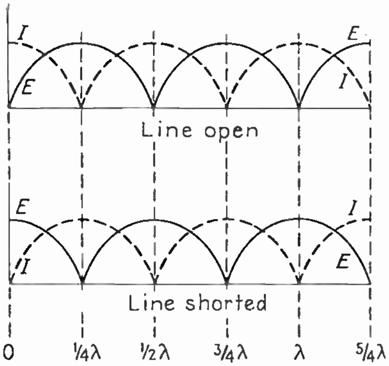

FIG. 252.—Current and voltage relations on open and shorted lines which have no attenuation.

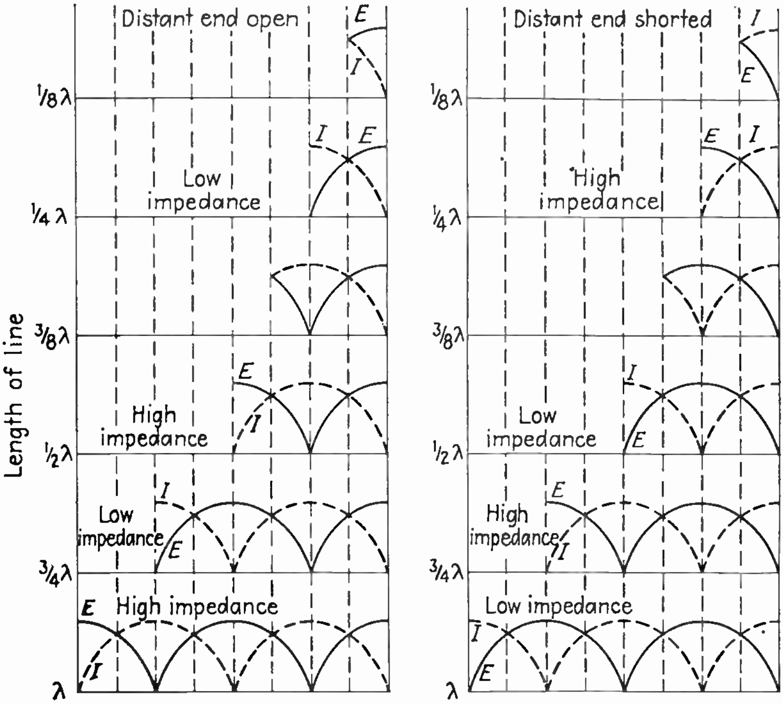

FIG. 253.—The current and voltage relations on open and shorted lines are as indicated for lines of various lengths. Relations at the distant end fix the conditions at the sending end; that is, at the distant end of an open line the current is always zero, and at the distant end of a shorted line the voltage is always zero, etc. The relations in this figure are for a line with negligible attenuation.

answer is simple: Fig. 252 is the basic drawing to refer to. It is not necessary to draw detailed sketches in each instance such as Figs. 250 and 251. The relations of Fig. 252 are sufficient. The distant end fixes the condition. Merely select the point along the line measured from the receiving end equal to the length of line in question, and the relations will be depicted. To illustrate, Fig. 253 has been drawn showing conditions on lines of various length. Note that these are merely sections of Fig. 252 measured from the *receiving end.*

The condition of reflected waves on a transmission line having considerable attenuation will not be considered as it is relatively unimportant. Suffice it to say that because of attenuation the reflection phenomena are less pronounced.

Input Impedance of Lines Having Reflection.—Consider for a moment the input impedance of an open-circuited line $\frac{1}{4}\lambda$ in length. At the *sending* end the voltage is very low and the current is high. For the theoretical line having no attenuation, the input voltage approaches zero, and hence the *input impedance approaches zero.* The voltage at the receiving end is a large value, and hence the $\frac{1}{4}\lambda$ open-circuited line acts like a voltage *step-up* transformer of *low* input impedance. The current is stepped *down.*

The short-circuited line $\frac{1}{4}\lambda$ in length acts like a *high* input impedance transformer which steps voltage *down* and steps the current *up.* Similarly, the other lengths have the characteristics shown. Note that where the lines are not exact multiples of a quarter wave length, such as $\frac{1}{8}\lambda$ or $\frac{3}{8}\lambda$, input impedances between the two extreme values exist. It should also be noted that these relations hold for lengths of line which are $\frac{1}{4}\lambda$, etc., at a *given frequency.* Thus, the circuits are frequency selective.

To summarize: Lines of various lengths exhibit very interesting relations at a given frequency. Input impedances can be varied at will by changing either the line length or the impressed frequency. Such **resonant lines,** as they are often called, have many very important applications, particularly in radio where they may be used as transformers, filters, and for other purposes.[1]

Input Impedance Relations in Resonant Lines.—This is completely considered[2] by Everitt to which the reader is referred for

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[1] See Terman, F. E., Resonant Lines in Radio Circuits, *Electrical Engineering,* July, 1934.

[2] See Everitt, W. L., "Communication Engineering," McGraw-Hill Book Company, Inc.

details. To explain how the input impedance varies for lines of different lengths, the vector diagram of Fig. 254 is included.

Open-circuited Line Less than $\frac{1}{4}\lambda$ *in Length.*—The reference for this diagram is the X axis. This line is considered to be just slightly less than $\frac{1}{4}\lambda$ or 90 degrees in length. Since the distant end of the line fixes input conditions, the voltage E_0 at the distant open end will be taken as the reference on the X axis. This places E_i the initial voltage slightly less than 90 degrees ahead of E_0 say by the angle α. On the assumption that the characteristic impedance of the line is largely resistive, as is true for open-wire lines and coaxial cables at radio frequencies, the current I_i of the initial wave will be in phase with the voltage as shown. Likewise, the current I_0 *arriving* at the open end is *in phase* with the voltage E_0, but when the current is reflected from the open end it is reversed in direction or turned through 180 degrees as is shown for $-I_0$. But the *reflected* component of voltage E_R *now* at the input of the line is α degrees *behind* E_0, because it is an impulse that left the distant end α degrees ago. Also, the reflected component of current

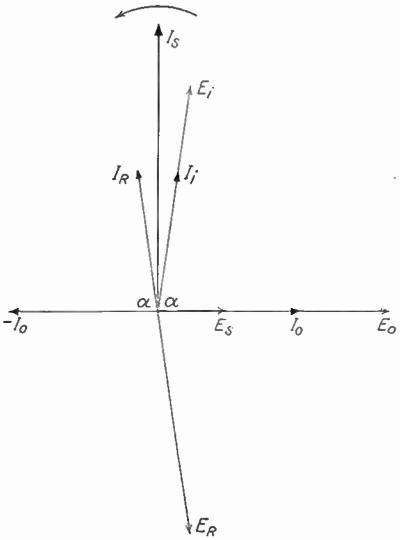

FIG. 254.—Vector diagram for an open-circuited line having no attenuation and less than $\frac{1}{4}\lambda$ in length. For such a line the input impedance is pure capacitive reactance as proved by the fact that the current I_S at the sending end leads the sending-end voltage E_S by 90 degrees.

I_R *now* at the input of the line is α degrees behind $-I_0$. The resultant of the initial current I_i and reflected current I_R gives the sending-end input current I_S; also, the resultant of the initial voltage E_i and the reflected voltage E_R gives the sending-end voltage E_S. Note that in Fig. 254 when the length of the line is less than ¼λ, the angle between the current and voltage is 90 degrees leading. Thus for an open-circuited line having no attenuation and less than ¼λ in length, the input impedance is a *pure capacitive reactance.*

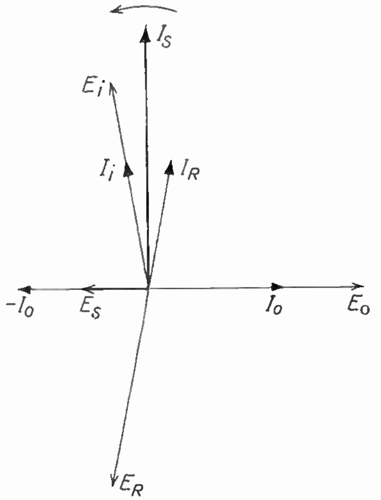

FIG. 255.—Vector diagram for an open-circuited line having no attenuation and greater than ¼λ in length. For such a line the input impedance is *pure inductive reactance* as proved by the fact that the current I_S at the sending end lags the sending-end voltage E_S by 90 degrees.

Open-circuited Line Greater than ¼λ in Length.—In Fig. 255 the length of the line is slightly greater than ¼λ, and the angle α will be greater than 90 degrees. This will give the conditions shown, and for an open-circuited line having no attenuation and more than ¼λ in length the input impedance is a *pure inductive reactance.*

Short-circuited Line Less than ¼λ in Length.—A vector diagram similar to Figs. 254 and 255 will show that when a line having no loss is shorted and less than ¼λ in length the input impedance is *pure inductive reactance.* In drawing the diagram, the voltage at the receiving end is turned through 180 degrees instead of the current as in previous figures.

Short-circuited Line Greater Than $\frac{1}{4}\lambda$ in Length.—A vector diagram similar to Figs. 254 and 255 will show that when a line having no loss is shorted and more than $\frac{1}{4}\lambda$ in length the input impedance is *pure capacitive reactance.* Again, the voltage at the receiving end is turned through 180 degrees and not the current for a short-circuited line.

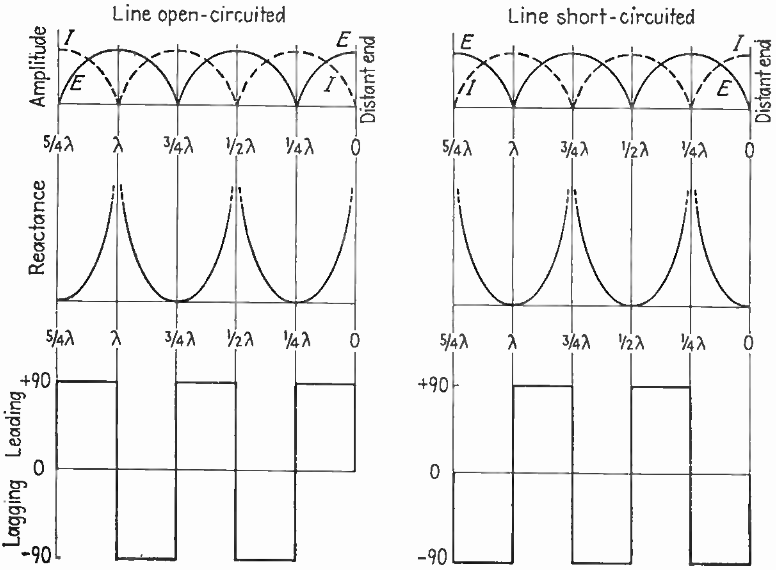

FIG. 256.—Input impedance characteristics of open-circuited and short-circuited lines having no attenuation, but of various lengths, measured from right to left. Considering the open-circuited line, and starting at the right, if the length of line is less than $\frac{1}{4}\lambda$, the voltage and current will be as shown, the input impedance will be pure capacitive reactance, as indicated by the lower curve. If the length of line is greater than $\frac{1}{4}\lambda$, but less than $\frac{1}{2}\lambda$, the input impedance will be a pure inductive reactance. For the short-circuited line the results are reversed.

Input Impedance of Any Resonant Line.—On the basis of the preceding discussion, it is possible to draw figures from which the input impedance of a line of any length can be determined. To find the nature of the input impedance of any line either open- or short-circuited, measure off from the right-hand end of either of the diagrams of Fig. 256 the length of the line in question; then, the curves give the input impedance characteristics of the line. For example, the input-impedance of an open-circuited line $\frac{1}{8}\lambda$ in length is a reasonably low value of pure capacitive reactance.

At ¼λ, it is a very *low* value of pure resistance, and at ½λ, it is a very *high* value of pure resistance.

Transmission-line Calculations.—In the preceding pages the electrical principles underlying the transmission of electromagnetic waves along two parallel wires have been discussed. It was shown that a line had a certain characteristic impedance, that the wave traveled down the line with a velocity approaching that of light, and that the wave (current and voltage) was attenuated as it passed down the line. The applications of these principles to a transmission line will now be considered. The equations will not be derived; for this, reference is made to more advanced books on communication.

Characteristic Impedance.—For a parallel-wire transmission line, the characteristic impedance Z_O is

$$Z_O = \sqrt{\frac{R + j2\pi fL}{G + j2\pi fC}} \text{ ohms.} \qquad (93)$$

For a typical hard-drawn copper telephone-line side circuit with wires 0.104 inch in diameter and spaced 12 inches apart under standard dry-weather conditions at 1000 cycles, $R = 10.15$ ohms, $L = 0.00366$ henry, $C = 0.00837$ microfarad, and $G = 0.29$ micromho, all constants being for 1 mile of line composed of *two* wires. Neglecting the effect of the line leakage G, which may be done for dry-weather conditions, and substituting these values in Eq. (93) gives

$$Z_O = \sqrt{\frac{10.15 + j6283 \times 0.00366}{+j6283 \times 0.00837 \times 10^{-6}}} = \sqrt{\frac{10.15 + j23.0}{+j52.6 \times 10^{-6}}}$$
$$= 692\underline{/12^\circ} = 680 - j144 \text{ ohms.}$$

The characteristic impedance of a line may be calculated from the relation $Z_O = \sqrt{Z_{OC}Z_{SC}}$, where Z_{OC} is the input impedance measured with the distant end open-circuited and Z_{SC} is the input impedance measured with the distant end short-circuited. Typical measurements for a line are $Z_{OC} = 669 - j207$, and $Z_{SC} = 692 - j81.5$. Using the relation previously given,

$$Z_O = \sqrt{(669 - j207)(692 - j81.5)} = 696\underline{/12^\circ} = 680 - j145.$$

These values were taken on an artificial line in the laboratory, but the values previously calculated were from the constants of an actual open-wire line.

Propagation Constant.—The **propagation constant** is composed of two parts; the first is the **attenuation constant** which determines how the electromagnetic wave is attenuated as it travels down the line, and the second is the **phase constant** which determines the wave velocity. The propagation constant

$$\gamma = \sqrt{(R + j2\pi fL)(G + j2\pi fC)}. \tag{94}$$

Using the values for the line previously specified and again neglecting the leakage G, $\gamma = \sqrt{(10.15 + j23.0)(+j52.6 \times 10^{-6})} = 0.0364\underline{/78} = 0.00756 + j0.0356$.

The velocity with which an electromagnetic wave travels along a wire is given by the relation $V = 2\pi f/\beta$. The value of β is the second term just calculated. For a frequency of 1000 cycles, $V = 6283/0.0356 = 176{,}500$ miles per second. This is below the theoretical value of 186,300 but is about correct for an open-wire line of this type.

The first term of the propagation constant γ is the attenuation constant α. As calculated, it is in **nepers**. The attenuation constant in decibels equals the attenuation constant in nepers multiplied by 8.686. For the line being considered, the loss $= 8.686 \times 0.00756 = 0.0657$ decibels per mile of line.

Calculations of Voltage or Current along a Line.—As an electromagnetic wave flows along a line it is attenuated; that is, its amplitude is decreased. This means that the effective values of the current and voltage decrease along the line. The attenuation constant and the loss in decibels are the factors that determine how rapidly the current or voltage decreases along the line. These values may be calculated as will now be shown.

Example.—A voltage of 1.2 volts at 1000 cycles is impressed on the circuit that has been considered previously. Calculate the current that will enter the line and the current, voltage, and power 150 miles from the sending end.

Solution.—Step 1. The current that enters the line will be the impressed voltage divided by the characteristic impedance (the assumption being that the line is terminated in its characteristic impedance).

$$I = \frac{1.2}{692} = 1.73 \text{ milliamperes.}$$

Step 2. Calculate the loss of the line. Loss $= 0.0657 \times 150 = 9.85$ decibels.

Step 3. Calculate the voltage at the end of the line. With reference to page 336, from the curve there shown, 9.85 decibels gives a voltage ratio of about 3.0. The received voltage will be 1.2/3.0 = 0.40 volt approximately.

Step 4. Calculate the current at the end of the line. This will be in the same ratio as the voltage. The received current will be 1.73/3.0 = 0.58 milliampere approximately.

Step 5. Calculate the power. The current of 0.58 milliampere reaches the distant end of the line which is terminated in its characteristic impedance. The resistance component of this is 680 ohms. The current will flow through this. Power = I^2R = $(0.58 \times 10^{-3})^2 \times 680$ = 0.229 milliwatt.

Distortion.—Communication workers are largely concerned with the transmission of speech, music, and telegraph signals between points separated some distance apart. The requirements that must be met are as follows:

1. *Volume.*—The intensity of the received signals must be sufficient for the operation of communication equipment, for example, the listening party's telephone.

2. *Noise.*—The noise induced must not be excessive. If amplifiers are used the volume may drop to a low level, provided that the **signal-to-noise** ratio remains high. If it does not, then amplifying the signal also amplifies the noise, and the output of the amplifier is too noisy for intelligibility.

3. *Distortion.*[1]—In transmitting the signal from one point to another, it must pass through much apparatus and over various lines. These must not distort the signal. *Distortion may be defined as a change in wave form.* That is, if a signal starts out with a given wave shape, and if the equipment through which it passes changes its wave shape, the signal has been distorted. There are three types of distortion which will now be treated separately.

Frequency Distortion.—When a change is made in the relative magnitudes of the different frequency components of a signal, it is known as **frequency distortion** (also called amplitude distortion). With reference to Eq. (94), it is seen that frequency enters into the equation for the propagation constant which also gives the attenuation constant. Thus, the attenuation constant of a transmission line will vary with frequency, and when a complex signal composed of many frequencies is impressed on the sending end, the various components will be attenuated differently and the wave will arrive at the receiving end slightly distorted. In practice this can be corrected by an equalizer if it is serious. Another example is

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[1] More exact definitions of the terms which follow will be found in "American Standard Definitions of Electrical Terms," A.I.E.E. (1941 Edition).

an amplifier that does not amplify all frequencies equally; such an amplifier would distort a complex signal by accentuating certain frequency components.

Delay Distortion.—If the velocity of transmission is different for the various frequency components of a complex wave, **delay distortion** results. This is illustrated by the calculations on page 390. The velocity of transmission of a line will vary with frequency, and thus certain components will arrive at the receiving end with phase relations that are altered with respect to the other components. A phase shift occurs in many amplifiers. If the phase shift of a signal in passing through an amplifier is different for the various components of a complex wave, **phase distortion** results.

Nonlinear Distortion.—If frequency components are present in the output of a device which were not in the input of the device, it is called **nonlinear distortion.** Nonlinear distortion is caused by the hysteresis effect in coils and transformers with iron cores. As shown on page 166, there the flux changes do not *exactly* follow the current changes because of hysteresis. Now flux changes induce voltage; therefore, if a sine-wave current flows through a coil, the voltage across the coil (or the voltage induced in the secondary of the coil) will not be a pure sine wave if the coil has a highly saturated magnetic core. This means that new frequencies or *harmonics* are produced. Also, if a pure sine-wave voltage is impressed on a coil with a saturated magnetic core, the current will be distorted and will contain harmonic. Vacuum-tube amplifiers will also generate harmonics if they are not properly designed and operated.

Coaxial Cables.—These have been used for transoceanic telegraph circuits for many years and are also being developed for land telephone purposes. They are extensively used for connecting radio transmitters to their antennas, and the **coaxial cable** in this form will be considered here. In its usual form such a cable consists of a solid copper conductor held at the center of an outer copper tube by insulating spacers of hard rubber or some suitable ceramic material (see Fig. 257). The outer tube acts as one side of the transmitting circuit and is also a very effective shield (page 329). Because of skin effect the high-frequency signals being transmitted travel along the inside of the sheath, and the unwanted currents induced by stray fields travel along the outside of the copper tube and do not penetrate within.

The characteristic impedance of a coaxial cable at high frequencies is almost *pure resistance* and equals

$$Z_O = \sqrt{\frac{L}{C}} \text{ ohms,} \tag{95}$$

where L is the inductance in henrys and C is the capacitance in farads for any convenient length. For a typical coaxial cable the characteristic impedance is about 70 ohms. For further numerical data on coaxial cables a communication handbook or advanced books on communication are recommended.

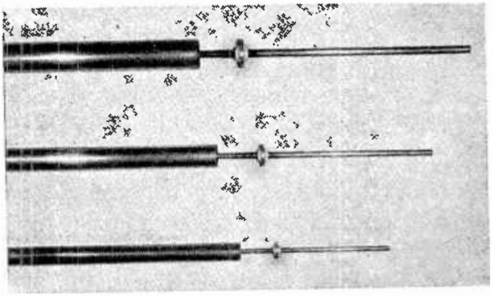

FIG. 257.—Coaxial cables of various sizes. The insulating spacers hold the small solid conductor at the center of the copper tube constituting the outer conductor. (*Courtesy of Isolantite Company.*)

An electromagnetic wave travels along a coaxial cable in the same general way as for a transmission line, with one exception. For an open-wire transmission line the wave, composed of the electric and magnetic components, exists *about* the wires, but for the coaxial cable the electromagnetic wave is (theoretically) all confined to the region within the tube. This means that interference between coaxial circuits is negligible. Otherwise, coaxial cables exhibit all the characteristics of open-wire lines, including wave attenuation and reflection phenomena. The velocity of wave travel along a line is assumed to be that of light.

Wave Guides.—As has been mentioned, in wire transmission the wires may be thought of as being merely *guides* to *direct* the electromagnetic wave being transmitted to the desired location. The wave containing the energy being transmitted may be thought of as flowing through the medium (often space) surrounding the wires.

Now this process can be reversed. An electromagnetic wave may be transmitted down a **wave guide** without any wires or other conductors at all. A glass or Bakelite rod, or a tube of water, will act as a wave guide because it has a higher dielectric constant and is a better conductor of electromagnetic waves than is air. A wave guide may be a hollow metal tube filled with air. An electromagnetic wave will travel down this tube because the induced currents would prevent its escaping from the metal tube. Wave guides are used with electromagnetic waves but a few centimeters in length.

The Radio Antenna.—*A system of conductors for radiating or receiving electromagnetic waves is an* **antenna.** The reciprocity theorem (page 330) can be extended to antennas. If a voltage E impressed on a local transmitting antenna causes a current I in a distant receiving antenna, then the same voltage E impressed on the distant antenna will cause the same current I in the local antenna. This relation holds only when conditions are comparable and means, in simple words, that what is said about an antenna in transmitting also applies to the same antenna in receiving. For example, if an antenna transmits equally well in all directions, it will receive equally well from all directions; also, if it is directional in transmitting, it will be directional in receiving. Thus, if transmitting antennas are studied, the same principles apply to similar antennas in receiving.

Before considering antennas themselves, certain basic principles will be reviewed. Thus, consider the two long parallel wires of Fig. 258*a*. Energy in the form of an electromagnetic wave flows into the parallel-wire circuit and is transmitted along the line to the distant open end where substantially all the energy is reflected back toward the sending end. Of course it is probable that some energy escapes from the open end of the line into space, but very little. This is because the wires are so close together that they really affect the space near the end of the wires but very little. If the parallel wires of Fig. 258*a* have negligible attenuation, then all the energy that starts out will be reflected and the net power input to the line must be zero. Now the generator voltage forces a current into the line, but since the power taken is zero, in the power equation $P = EI \cos \theta$, the value $\cos \theta$ must be zero which means that the current and voltage are 90 degrees apart *with respect to the time* with which they pass through corresponding values. The input impedance of the circuit of Fig. 258*a* must be pure reactance (see page 385).

Now suppose that the ends of the wires are separated as shown in Fig. 258*b*. It will be found that the generator now sends power into the line and that the input impedance is *no longer* pure reactance but contains a resistance component. Also, the expression $P = EI \cos \theta$ is no longer zero, so the current and voltage are no longer 90 degrees out of phase *with respect to time.* Since it may be assumed that the wires themselves are without attenuation, *where does the energy go?* The answer is simply this: The magnetic and electric components of the electromagnetic wave are no longer largely confined to a small region, but they now excite a large volume of space and are beginning to radiate electromagnetic waves into space quite effectively.[1]

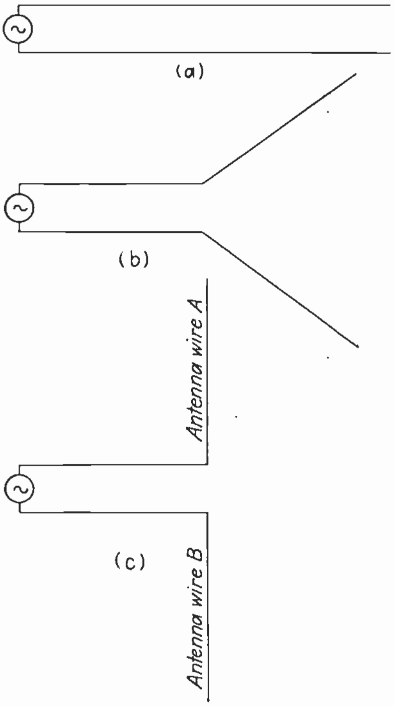

FIG. 258.—How a pair of parallel wires can be developed into an antenna.

If the wires are further separated as in Fig. 258*c*, the system now becomes a transmission line feeding an antenna. Under the proper conditions, this antenna now excites a large amount of space and *very* effectively radiates energy in the form of electromagnetic waves into space. If the antenna properly terminates the transmission line in the characteristic impedance of the line, then there will be no wave reflection, the

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[1] An analogy would be this: Suppose that you were standing 50 feet from the end of a 2-inch pipe into which someone was speaking at the distant end. Very little sound would be heard because the pipe did not excite many air particles, only the small number directly in front of the pipe opening. Now suppose a large horn is screwed onto the end of the pipe; the sound radiated will then be intense, because the large volume of air within the horn is excited.

input impedance to the line at radio frequencies will be largely pure resistance, and the input current at the generator will be in phase with the voltage *with respect to time*. The input power will be $P = EI \cos \theta$, and $\cos \theta$ will be unity. This power will be radiated by the antenna, the circuit losses being assumed zero.

The Radiation of Electromagnetic Waves.—Suppose that the radio-frequency generator (which may be now regarded as a radio transmitting set) is connected at the junction of the transmission line and antenna of Fig. 258*c* and that the transmission line is removed. This gives the vertical radiator of Fig. 259. The generator or transmitting set connected at the center of the antenna is impressing a pure sine-wave voltage on the antenna.

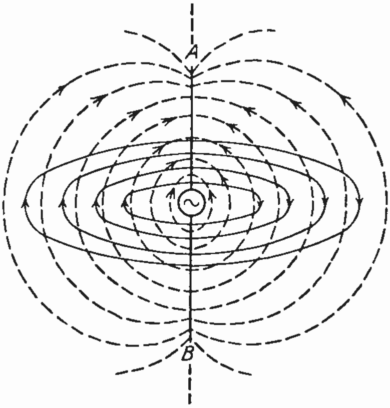

FIG. 259.—Electric field (broken lines), and magnetic field around a vertical wire antenna *AB*.

At that part of the cycle in which the upper generator terminal is *negative*, electrons in the antenna will be repelled and will flow up wire *A*. At this same instant, the lower generator terminal will be positive and will attract electrons up out of wire *B*. A *conventional* current will accordingly flow *down* the antenna, and lines of *magnetic* force will accordingly encircle it somewhat as shown by the solid circles. When the generator polarity reverses during the next half cycle, the direction of motion of the charges reverses. As a consequence the conventional direction of current flow changes, and the magnetic lines of force will encircle the antenna in the opposite direction. Also, when the upper generator terminal is negative and the lower one is positive, *electric* lines of force will be established as indicated between the two portions of the antenna. When the generator voltage is reversed, the electric lines of force will be reversed.

Now at the very low frequencies which were previously considered (page 162), it was assumed that the energy stored in a magnetic field was all returned to the source, and this *is* quite true at low frequencies. Also, in studying electric fields, such as a condenser, it was assumed that the energy stored in the condenser was all returned to the circuit when the condenser was discharged, and this *is* quite true for a condenser.

The phenomenon shown in Fig. 259 is quite different, however; now a *large amount* of space has been excited, and at a very high frequency; the antenna has acted on the space and set up an *electromagnetic wave motion* in it, and there is no particular reason why space should return this energy to the antenna. Therefore, electromagnetic waves travel out from the antenna as a source as the high-frequency generator voltage varies. This constitutes the **radio wave** which contains the electric energy transmitting the signal to the distant receiving antenna. Not much energy would be radiated from an antenna at low frequencies, but it is readily radiated at the very high radio frequencies.

Reflection of Electromagnetic Waves.—Suppose that only one-half the antenna of Fig. 259 is used and is connected to one generator (radio transmitter) terminal and the other terminal is grounded. This arrangement is given in Fig. 260, somewhat exaggerated of course. Such a grounded system will produce an electromagnetic field equal to one-half that of Fig. 259.

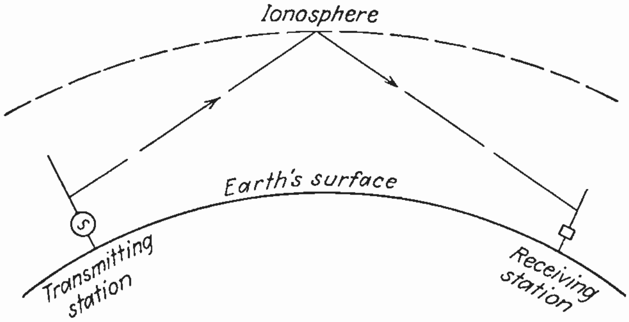

FIG. 260.—How the ionosphere reflects radio waves back to the earth.

Now it might be assumed that this would radiate a ray of electromagnetic waves in the direction shown and that these would never reach a radio-receiving set at point *X* for instance. That this is not true is due to the **ionosphere,** or **Kennelly-Heaviside** layer as it is often called.

The ionosphere is a layer of ionized gas (page 426) at some 50 to 100 miles above the surface of the earth. One factor contributing to its ionization is the sun's rays, so the apparent layer height will vary with the time of day and season of the year. From a practical standpoint the ionosphere may be regarded as a conducting shell surrounding the earth, and this shell reflects the electromagnetic waves back to the surface of the earth. In this way, stations a great distance away can be heard.

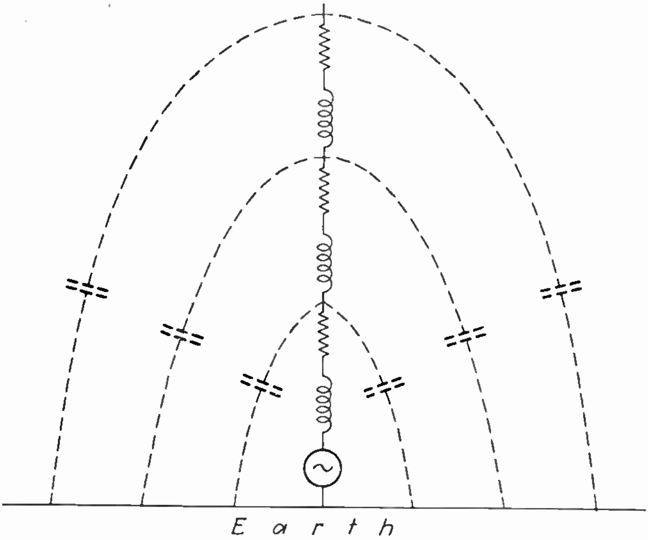

FIG. 261.—An antenna is in reality a complex network consisting of series resistance and inductance and shunt capacitance.

Antenna Input Impedance.—As was explained in the discussion accompanying Fig. 258*c*, the circuit consists of a transmission line feeding an antenna. The antenna is, accordingly, the terminating load on the transmission line, and to match this load properly to the line the input characteristics of an antenna must be known.

By now it should be clear to the reader that at high frequencies there is more to a circuit than just what is seen by the eye. Thus it was previously explained that the two parallel wires of a transmission line actually comprised a complex network such as shown in Fig. 247, page 375. By referring to Figs. 258*a*, *b*, and *c*, it follows that the antenna is not simply a straight wire but really a complex network, each clemental length having resistance and inductance, and each elemental length of antenna wire A of Figs. 258 and 259 having capacitance to each elemental length of antenna wire B. This has been shown in Fig. 261. Now in any circuit a generator can only force a current through a *completed* circuit, and the distributed capacitance between the antenna and ground in Fig. 261 completes the path for current flow.

A circuit composed of distributed inductance and capacitance elements will be resonant at certain frequencies, and this is precisely what happens for an antenna such as shown in Fig. 261. *At certain frequencies the input impedance of the antenna at the point where the generator is located will be a value of pure resistance.*

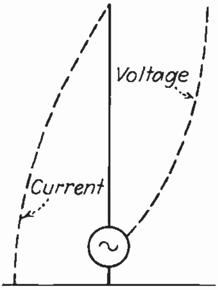

FIG. 262.—Magnitude of the current and voltage at various points along an antenna when the length of the antenna equals $\lambda/4$, where λ is the wave length at the frequency f of the transmitter driving the antenna. This is a space distribution and not a time distribution. From this diagram it should not be inferred that the current entering the antenna is 90 degrees out-of-phase with the voltage across the antenna. If this relation existed, no power could enter the antenna.

This equivalent resistance may be low, or it may be quite high, depending on the manner in which the antenna is constructed, etc. For example, if the antenna were composed of resistance wire, then much of the power entering the antenna would be lost in I^2R losses in the wire. If the antenna is composed of conductors of low loss, and if it is isolated in space so that circulating currents are not induced in surrounding conducting objects and also so that dielectric losses do not occur, the antenna will radiate the energy put into it, but it still takes power and will have an input resistance. If an antenna is driven at the frequency at which its input impedance is largely pure resistance, *then the current entering the antenna will be in phase with the voltage; that is, they will pass through corresponding values at the same time.*

With reference to Fig. 261, the current at the *extreme* end of the antenna is zero, because there is negligible capacitance between this *extreme* end and ground. Also, the current flowing in the antenna is *greatest* at the point where the antenna is connected to the generator. Thus for a straight vertical wire antenna the *current distribution* is as shown in Fig. 262 when the frequency f is such that the length of the wire is equal to $\frac{1}{4}\lambda$, where $\lambda = V/f = 186{,}300/f$

miles. Also for Fig. 262, the *voltage distribution* will be as shown, because in a sense the antenna is a transmission line, and the voltage at the distant end of a line may approach zero.[1]

Note in particular that for the current-distribution curve and the voltage-distribution curve these curves represent the *magnitude* of the current or voltage at each point in the antenna. These curves represent a *space* distribution because they are plotted against the *length* of the antenna. They do not represent a *time* relation. This point is stressed because many have the opinion that Fig. 262 indicates that the current entering an antenna and voltage across an antenna are *out of phase* by 90 degrees. The time relation between current and voltage is determined by the input impedance of the antenna, which may be a pure resistance at certain frequencies. If the input current and the voltage impressed across an antenna were 90 degrees out of phase, then the power input $P = EI \cos \theta$ would be zero. Then, no power would enter the antenna and none would be radiated.

Antenna Coupling Circuits.—It has been shown that an antenna is an electric network having an input impedance. The final tube in the radio-transmitting set must work into this impedance. If the antenna does not offer the correct load impedance to the radio transmitting set, then an impedance-transforming network called a **coupling circuit** can be inserted between the transmitting set and the antenna (see page 333).

It may be that the transmitter house must be located some distance from the antenna. If so, it is usually connected to the antenna with an open-wire transmission line or a coaxial cable. It is possible (page 385) to use the transmission line itself as an impedance-matching circuit, or it may be operated as a line terminated in its characteristic impedance. These principles have been discussed earlier in this chapter. The details of these methods will be found in books on radio engineering.

Receiving Antennas.—It has been shown that a transmitting antenna sets up an electric and a magnetic field about it and that it radiates an electromagnetic wave into space. In *free space* the electric and magnetic components of an electromagnetic wave must be of equal strength. This is in accordance with the principle of elec-

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[1] This may also be regarded as caused by the fact that the distributed capacitance is low and the IX_C drop will be high, even though the I may be very small.

tromagnetic induction that any relative motion between a magnetic field and a conductor will induce a voltage in the conductor. Although this law is usually stated for conductors, a voltage would also be induced in a glass wire, but negligible current would flow because of the high resistance. Space, like glass, is a dielectric, and if a magnetic field sweeps through space, a voltage will be induced and an electric field produced.

Now the current and voltage of a *transmitting* antenna will send out an electromagnetic wave, and when this electromagnetic wave sweeps past a *receiving* antenna it induces a voltage in it, and this voltage will cause a current flow. If the antenna is arranged as in Fig. 263, the generator shown represents the resultant induced voltage. For simplicity the distributed capacitance between each elemental length of the antenna and ground has been lumped into one capacitor shown dotted.

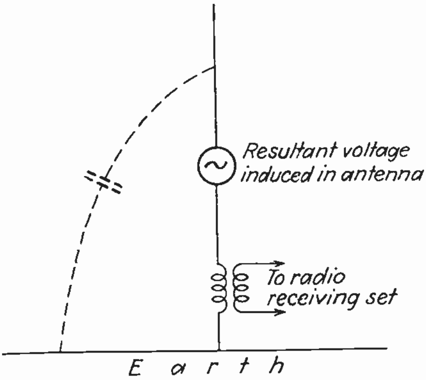

FIG. 263.—Action of a receiving antenna.

This resultant generated voltage will force a current to ground and through the antenna coil of the radio set. This current will in turn induce a voltage in the secondary of the antenna coil which will be impressed on the vacuum-tube circuits for amplification and detection or demodulation. Condensers may be added in series with the antenna coil or in parallel with it to tune it to a particular band.

SUMMARY

When a voltage is impressed on the two parallel wires of a transmission line a current flows into the circuit. The impressed voltage produces an electric field between the wires, and the current produces a magnetic field around the wires. These two fields constitute an electromagnetic wave which travels down the line.

The velocity with which an electromagnetic wave travels down a line approaches the velocity of light, or 186,300 miles per second. In a typical open-wire telephone line, a wave having a frequency of 1000 cycles per second travels with a velocity of about 180,000 miles per second.

A line in which several wave lengths exist is an electrically long line. Several wave lengths may exist in a line if the line is long, if the frequency is high, or if the wave velocity is low. An electrically short line is one in which only a fraction of a wave length exists. The wave length $\lambda = V/f$.

A transmission line has a characteristic impedance. This equals the input impedance of an infinite line or of a finite line terminated in its characteristic impedance.

When an electromagnetic wave strikes any discontinuity in a line, wave reflection occurs. Therefore a wave is reflected from an open-circuited line, from a short-circuited line, or in fact from any termination that does not equal the characteristic impedance of the line.

The combining and interfering of the initial and reflected waves at various points along a line cause maximum and minimum voltages and currents along an open or shorted line. These are called standing waves.

For open-circuited or short-circuited lines having no attenuation, the input impedance is either pure resistance or pure reactance, depending on the impressed frequency.

The characteristic impedance of a transmission line may be calculated from the line constants of resistance, inductance, capacitance, and leakage. Characteristic impedance is independent of line length but varies with frequency. The characteristic impedance of a radio-frequency line is largely pure resistance.

The attenuation of a transmission line may also be calculated from the line constants. Attenuation varies with the length of the line and with the frequency.

Distortion may be defined as a change in wave form.

Frequency distortion is caused by a line that offers different attenuation to different frequencies or by an amplifier that amplifies the various frequencies differently.

Delay distortion results when a line transmits waves of different frequencies at different velocities.

Nonlinear distortion causes new frequency components to be created.

Coaxial cables composed of a center conductor and an outer sheath are extensively used in radio practice.

The radio-transmitting antenna is an electric circuit that serves to radiate electric energy in the form of electromagnetic waves into space.

Electric energy is effectively radiated into space at the high radio frequencies but not at low frequencies.

Electromagnetic waves are reflected back to the earth by the ionosphere.

Like any electric circuit, the radio antenna has a definite input impedance, depending on its dimensions and the test frequency. The input impedance must contain a large resistance component if power is to flow into the antenna.

The electromagnetic wave sweeping past the receiving antenna induces a signal voltage in it. This causes a current flow through the primary of the antenna-coupling coil of the radio receiving set, and this in turn induces a signal voltage in the secondary that is connected to the vacuum tubes.

REVIEW QUESTIONS

1. Explain why the electric power being transmitted in a line is proportional to the product of the electric field and the magnetic field strength and the cosine of the angle between them.

2. What is an electromagnetic wave?

3. How is an electromagnetic wave set up in a transmission line?

4. What reason is there for saying that communication by electrical means is instantaneous?

5. What is an electrically long line? An electrically short line?

6. Why is the characteristic impedance of a line independent of length?

7. How does line attenuation cause signal distortion?

8. What happens to the electric component of an electromagnetic wave when it strikes the open end of a transmission line? What happens to the magnetic component?

9. What happens to the electric component of an electromagnetic wave when it strikes the shorted end of a transmission line? What happens to the magnetic component?

10. Briefly explain how a standing-voltage wave is found on an open-circuited line.

11. Referring to Fig. 253, for open-circuited lines, what lengths of line offer high input impedances? What lengths offer low input impedances?

12. Repeat Question 11 for a short-circuited line.

13. Why may an open-circuited or a short-circuited transmission line be regarded as a transformer?

14. Explain why the input impedance of an open-circuited line having no attenuation must be either a high value of resistance, a low value of resistance, or pure reactance.

15. What is meant by signal-to-noise ratio, and what is its importance?

16. Name and briefly explain the three types of distortion.

17. Can standing waves exist on a coaxial cable?

18. The losses in a transmitting antenna are assumed to be zero. How can the antenna input impedance contain a resistance component?

19. What is the ionosphere, and why is it useful in long-distance radio communication?

20. What is the reciprocity theorem as applied to radio antennas?

PROBLEMS

1. The velocity of an electromagnetic wave in a given transmission line 100 miles long is 180,000 miles per second. Calculate the frequency that will produce one full wave length in the line. Also calculate the frequency that will produce one-half wave length and one-fourth wave length.

2. Assuming that the wave travels with the velocity of light, what is the frequency that will establish a 5-centimeter wave on a line? A 0.5-centimeter wave?

3. Figure 250 represents the instantaneous voltage conditions existing along an open-circuited line. Draw a similar figure showing the relations on a short-circuited line.

4. A bridge such as shown in Fig. 237, page 359, was used to measure the open-circuited impedance of a line. The measured resistance was 672 ohms and the inductance was -0.372 henry, using a test frequency of 1000 cycles. Write the expressions for the open-circuited impedance in both the rectangular and polar systems.

5. When the short-circuited impedance of the line of Prob. 4 was measured, the resistance was found to be 695 ohms and the inductance -0.012 henry. Write the expressions for the short-circuited impedance in both the rectangular and polar systems.

6. Using the values in Probs. 4 and 5, calculate the characteristic impedance of the line.

7. The constants at 1000 cycles of a certain telephone line per mile are $R = 4.11$ ohms, $L = 0.00337$ henry, and $C = 0.00915$ microfarad. Neglect the effect of the leakage, and calculate the characteristic impedance.

8. Using the values of Prob. 7, calculate the propagation constant.

9. Calculate the wave velocity and the loss per mile in decibels for the line of Prob. 7.

10. If the line is terminated in its characteristic impedance and is 400 miles long, calculate the input current and the current, voltage, and power at the receiving end when a voltage of 1.5 volts at 1000 cycles is impressed on the sending end.

11. The equivalent circuit of a certain broadcast antenna is a 7.7-ohm resistor in series with a 0.001-microfarad capacitor. One kilowatt is being put into the antenna at 550 kilocycles. Calculate the equivalent input impedance of the antenna, and the series inductance required to produce resonance.

12. Referring to Prob. 11, calculate the current that flows into the antenna, the voltage across the antenna, and the angle between the voltage and current.

13. A shunt-excited or grounded vertical broadcast antenna tower is being fed by a sloping wire attached some distance up from the base. The input impedance between ground and the end of the sloping wire extending up the tower is $70 + j500$ ohms at 675 kilocycles. The antenna is to be fed from the transmitting set by a coaxial cable having a characteristic impedance of $Z_o = 70 + j0$ ohms. The sheath of the cable is grounded at the broadcast tower. The sloping wire is to be attached to the central coaxial cable conductor. How should this be done for maximum power transfer at the junction?

14. If 5.0 kilowatts are put into the coaxial cable at the transmitter, what will be the input current and the impressed voltage?

15. If a current of 5.97 amperes flows into the coaxial cable of Prob. 13, what will be the impressed voltage and the power entering the cable?

CHAPTER XIV

FUNDAMENTAL PRINCIPLES OF VACUUM TUBES

Vacuum tubes are largely responsible for making modern communication systems possible. These tubes may be regarded in two ways: (1) from the electronic viewpoint, considering what happens within a vacuum tube; and (2) how a vacuum tube functions in a circuit, considering its characteristics as a circuit element.

In this chapter the vacuum tube will be considered from the electronic viewpoint, and in the following chapter it will be treated as a circuit element. The term *vacuum tube* is used to include tubes that are evacuated to a high degree and also gas-filled rectifier tubes.

The fundamentals of electronics were considered in Chap. I. There it was shown that all metals contain free electrons and that ordinary electrical effects, such as current flow, are due to the motion of these free electrons. It is suggested that these principles be reviewed.

Now in vacuum tubes, and in other electronic devices such as photocells, it is necessary to extract the electrons from metals so that they may be acted on by the various electrodes within the tube. The ways in which electrons can be readily separated from metals and made useful within vacuum tubes are as follows:

1. *Thermionic Emission.*—When a metal is sufficiently heated, electrons are given off.
2. *Secondary Emission.*—If a beam of rapidly moving electrons strikes a metallic object, **secondary electrons** are liberated from the metal.
3. *Cold-cathode Emission.*—If even a cold metal is subjected to a *very* strong electric field, electrons are pulled out of the metal.
4. *Photoelectric Emission.*—When a beam of suitable light is directed on certain metals, electrons are liberated.

The Thermionic Cathode.—The vacuum tube, so widely used in radio transmitting and receiving sets and in telephone equipment, contains a heated **thermionic cathode** which emits electrons into the space immediately surrounding this cathode. These emitted or ejected electrons are then acted upon by the various electrodes within the tube, the tube is thus given its ability to amplify, etc.

The emission of electrons from the heated thermionic cathode is somewhat like the evaporation of a liquid. The heat energy supplied to a liquid causes particles of the liquid to be thrown from the surface. The heat energy supplied to a thermionic cathode causes free electrons to be thrown from the surface of the metal.[1]

The thermionic cathodes used in ordinary vacuum tubes are of two general types. (1) The **filament type** in which the heating current passes directly through a metallic wire or ribbon, and this filament emits the electrons. (2) The **separate-heater,** or **indirectly-heated, type,** which is a metallic cylinder (coated as will be explained later) within which is a separate wire for heating. The metallic cylinder emits the electrons; the wire within is insulated from the cylindrical cathode and merely supplies heat to it.

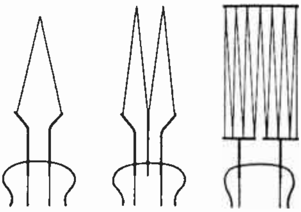

FIG. 264.—Directly heated cathodes of the filament type.

Filament-type Thermionic Cathodes.—Filaments usually are shaped as in Fig. 264. The wires may be made of a pure metal, they may be of thoriated tungsten, or they may be oxide coated.

Pure Metal Filaments.—Because of its very great mechanical strength at high temperatures and because of certain other desirable features, tungsten is widely used for the filaments of the large power-handling tubes used in the output stages of radio transmitters.

Thoriated-tungsten Filaments.—During the manufacture of the wire, a small amount of thorium oxide is added to the tungsten. Then, after the filament is mounted within the tube the thoriated-tungsten filament is activated. This is done by heating the fila-

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[1] Distinctions between evaporation and thermionic emission are as follows: In the evaporation of a liquid, actual particles of the liquid are thrown off and are borne away so that the pan of liquid "dries up." In thermionic emission, electrons are thrown off, but electrons are merely electric charges and are not particles of metal as are the atoms (p. 3). Also, the external connections bring the electrons back to cathode. Thus it follows that a thermionic cathode will last for many years if it is properly operated. The thermionic cathode merely acts as a source of electrons, or negative electricity.

ment to a high temperature which "boils" some of the thorium to the surface. This very thin layer of thorium gives the thoriated-tungsten filament a much higher emission under comparable conditions than for pure tungsten alone; in fact, it is about one thousand times as great.

Oxide-coated Filaments.—This cathode consists of a ribbon of nickel or some suitable alloy on which a coating of barium and strontium is placed. Before the tube is put into service, the coating must be activated somewhat as for the thoriated-tungsten filament just described. This activating process "boils" a layer of barium to the surface, and this layer gives the oxide-coated filament extraordinary emitting properties. The strontium is used to strengthen the oxide coating mechanically. The oxide coating increases the emission millions of times over what it would be for a pure metal at the same temperature.

Pure tungsten filaments operate at a very high temperature of about 2500°K. (degrees Kelvin).[1] Thoriated-tungsten filaments operate at an intermediate temperature of about 1500°K., and oxide-coated filaments operate at a low temperature of about 1000°K. The tungsten filament is the most rugged, the thoriated-tungsten filament is less rugged, and the oxide-coated filament is the weakest of the three. Pure tungsten filaments are used in the large power tubes, thoriated-tungsten in those of intermediate size, and oxide-coated filaments in the small radio-receiving-type tubes and also in gas-filled tubes. In the case of both the thoriated-tungsten type filament and the oxide-coated type, it is the very thin layer of thorium or barium on the surface of the filament which causes the excellent emission. These layers seem to accomplish this, not by emitting the electrons themselves, but rather by making it easier for the electrons to come out from beneath the thin layers.

Separate-heater Type Cathodes.—From the circuit-design standpoint there are many advantages in having a separate heater rather than a filament. Also, in radio receivers it is desired to heat the cathode with alternating current. In radio receivers the received signal strength is very low in the first tubes. The alternating-current hum caused in these tubes may be comparable to the signal strength if filament-type tubes are used.

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[1] Zero in the Kelvin system is −273°C.

There are two types of separate-heater cathodes. One is shown in Fig. 265*a*. It consists of an oxide-coated cylinder of nickel or other suitable metal inside of which is a hairpin-shaped heater wire insulated from the metal cylinder with a tubular ceramic insulator. This construction makes the cathode slow in heating, because of the time required to raise the structure to the operating temperature.

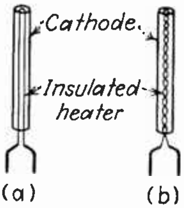


FIG. 265.—Separate-heater type cathodes. The cathode consists of a metal cylinder covered with an oxide coating. The cathode which emits the electrons is indirectly heated by the wires within.

The second type of indirectly heated cathode is shown in Fig. 265*b*. In this the cathode is not insulated throughout its length with a ceramic insulator; instead, it is insulated from the metal cylinder at each end and is spiraled within so that it is held in the center of the cylinder by its own spring tension. This cathode heats much more rapidly than the other type because the tubular ceramic insulator is not present and thus there is less material to bring up to temperature when the tube is turned on. For this reason it is more widely used.

The Negative Space Charge.—When the filament of a thermionic vacuum tube is heated, electrons are given off by the filament into the region immediately surrounding the filament. This is shown by Fig. 266. When the negative electrons are given off, they take negative electricity away from the filament, and this action leaves the filament positive. The filament, accordingly, attracts the electrons back to it.

Thus, in a tube such as Fig. 266 containing only a filament, an equilibrium condition is soon reached in which negative electrons are being thrown off from the filament and the filament is pulling an equal number back to it. Since the space surrounding the filament is filled with negative electric charges because of the presence of the electrons, it is said to be a **negative space charge**.

Actually, the space charge extends into other portions of the tube than the region immediately surrounding the filament. Practically, however, the intensity is so much greater adjacent to the filament that in ordinary radio-receiving-type tubes it is usually considered that the space charge is a sheath surrounding the filament. The way in which the space charge appears around a separate-heater cathode is shown in Fig. 267.

The Diode.—A diode is a two-electrode vacuum tube containing a heated cathode and an **anode, or plate.** The conventional way of drawing a diode is as shown in Fig. 268. Of course, this is not the way a tube is actually constructed, the details of construction of a typical thermionic vacuum tube being shown in Fig. 269.

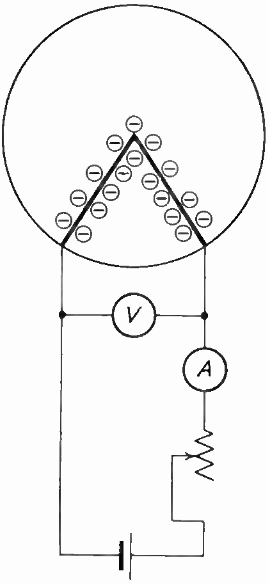


FIG. 266.—When a filament is heated, negative electrons are given off. These form as a sheath or layer around the filament, although they also exist in other parts of the tube as well.

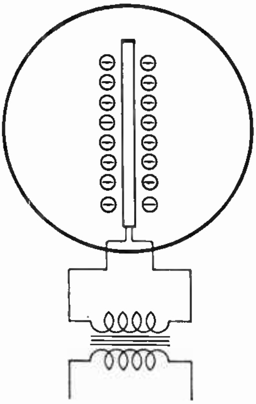


FIG. 267.—When an indirectly heated cathode reaches operating temperature, negative electrons are given off. These may be thought of as forming a sheath about the cathode, although they also exist in other parts of the tube.

Referring to Fig. 268, if the anode or plate is made positive, electrons will be attracted to it from the intense space charge region. These electrons will flow on around the circuit to the cathode. When they first leave the cathode they make way, so to speak, for other electrons from the cathode. Thus, when the plate is positive an electron current flows around the circuit, and this current will be measured by the milliammeter.

As is shown in Fig. 268, the plate voltage is variable. The relation between the plate voltage and the electron current through the tube at a given cathode temperature is shown by Fig. 270. For obtaining curve A the cathode is held at a given temperature and the plate voltage is varied. As the plate voltage is increased the plate current, consisting of electrons from the space-charge region, keeps increasing until the electrons are taken from the cathode as fast as they are given off. After this plate-voltage value has been reached, a further increase in plate voltage does not cause a corresponding increase in plate current. When this condition has been reached, it is sometimes said that the tube is **voltage saturated.** Increasing the cathode temperature will provide more electrons, and the plate current will now increase along curve B.

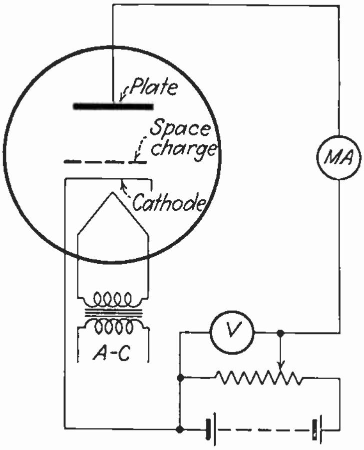


FIG. 268.—Conventional way of drawing a diode using a separately heated cathode. The space charge has here been shown as a layer of negative electricity. Actually, it consists of a random distribution of negative electrons.

The relation between cathode temperature and plate current for a given plate voltage is shown in Fig. 271. The cathode must be raised to a temperature sufficient to emit electrons before plate current can flow. The plate current then increases as the temperature is raised, until as the cathode becomes quite hot the electron emission builds up such a large negative space charge that the plate voltage cannot overcome it and draws a current through it from the filament. This is indicated by the flattening off of curve A of Fig. 271 and is known as **temperature saturation.** Applying a larger voltage to the plate will overcome the space charge and permit a larger current, curve B, to flow.

To summarize: In a diode (as well as in other thermionic vacuum tubes) the negative space charge is a controlling factor in limiting the plate current that flows through the tube. The positive plate voltage must, in effect, reach down through the negative space charge and overcome it sufficiently to pull a current through the tube from the cathode to the plate. Thermionic vacuum tubes are often called **valves**; this is quite an appropriate term, because they act like an electron valve, in that they allow more or less current to flow through themselves and the connected external circuit.

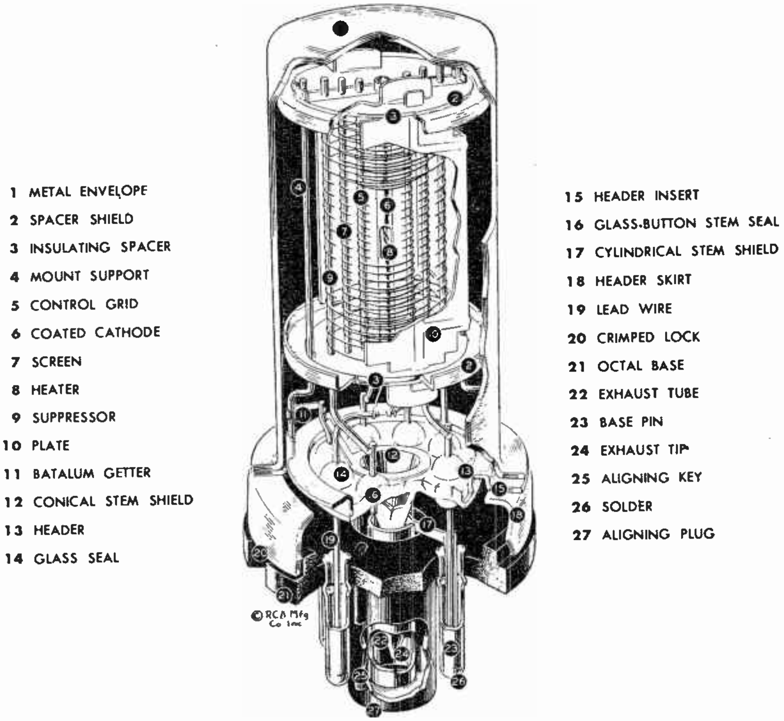


FIG. 269.—Details of construction of an "all metal" radio vacuum tube. (*Courtesy of RCA Manufacturing Company, Inc.*)

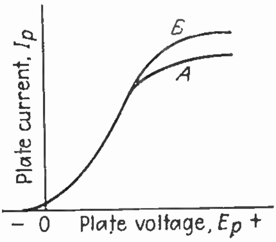


FIG. 270.—Variations in diode plate current with plate voltage at two cathode temperatures. The flattening is due to voltage saturation. As will be noted from this curve, some current will flow even if the plate is slightly negative because of the initial velocities which some of the electrons have when they are emitted.

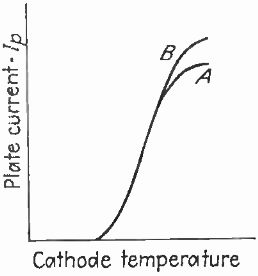


FIG. 271.—Variations in diode plate current with cathode temperature for two different plate voltages E_p and E_p'. The flattening is due to temperature saturation. The cathode temperature is varied by changing the heater current.

The Triode.—As shown in the preceding section, the negative space charge composed of electrons emitted by the cathode is a powerful factor in controlling the plate current through the tube. Now suppose that a mesh or coil of wire called a **grid** is placed near this space-charge region. If the grid is made positive, it will neutralize some of the negative space charge, and this action will allow the positive plate to pull a larger current through the tube. If the grid is made negative, it will assist the negative space charge in holding back the electrons from the filament and will reduce the plate current through the tube. By this action, the grid will be able to *control* the current through the tube, the assumption being that the filament temperature and the plate voltage are held constant.

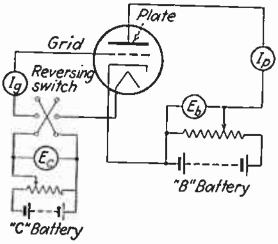


FIG. 272.—Schematic diagram for a triode and its test circuit. The "C" battery and its voltage divider makes available a variable voltage E_c for the grid. This voltage may be reversed so that a positive value may be applied. The "B" battery and its associated voltage divider provides a variable voltage E_b for the plate.

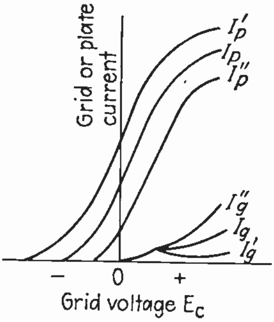


FIG. 273.—Plate and grid currents at plate voltages E_b, E_b' and E_b'' as the grid voltage E_c is varied. (Plate and grid currents plotted to different scales.)

This action is explained with the aid of Fig. 272. The filament is held constant at the correct operating temperature. The grid is made negative by throwing the switch down and positive by throwing it up. The grid voltage is varied by the voltage divider across the C battery, and the plate voltage is varied by the voltage divider across the B battery.

With the plate voltage at E_b, the grid voltage E_c is made highly negative and then slowly varied to positive values. When the grid is highly negative it will repel the electrons, and no plate current will flow. As the grid is made less negative, a current I_p flows to the plate as Fig. 273 shows. When the grid becomes positive, a small grid current I_g flows to the grid. If a higher plate voltage E'_b is used, then it will take a larger negative grid bias to repel the electrons and completely stop the plate-current flow. With a larger plate voltage, plate current I'_p and grid current I'_g results. If a low plate voltage E''_b is used, then curves I''_p and I''_g are produced.

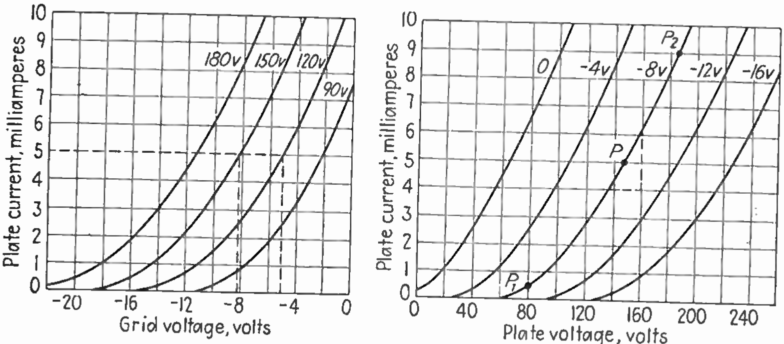
The peculiar shape of the grid current curve of Fig. 273 is caused by secondary emission (page 418). At about 10 volts positive, the grid begins to draw an appreciable current and the electrons striking the grid may knock out secondary electrons. Now many of these secondary electrons will be pulled over by the highly positive plate. Thus, electrons are flowing *to* the grid from the cathode and *from* the grid itself to the plate. A check of the current directions in Fig. 272 will show that secondary emission results in a drop in grid current. However, as the grid is made more positive, it begins to "reclaim" these secondary electrons and pulls them back to it.

To summarize: The grid in the triode gives the vacuum tube a control element, and it is this feature which makes the tube so useful. A feeble signal voltage impressed between the grid and cathode can control a large current flow.

Vacuum-tube Coefficients.—In calculating the gain, power output, etc., for vacuum tubes, several coefficients (sometimes called constants) are used. These will now be explained.

Amplification Factor.—The grid is in a very strategic position; it is close to the space charge region where the electrons are "milling about" and are accordingly easily influenced. To reach the plate, the electrons must pass through the grid and, accordingly, must do its bidding. A few volts on the grid may completely nullify several hundred volts on the plate. The **amplification factor** is a measure of the effectiveness of the grid with respect to the plate. The ratio of the plate-voltage change to the opposing grid-voltage change to prevent any variation in the plate current flowing gives the amplification factor. Thus, if the plate voltage is increased positively 40 volts the plate current would tend to rise, but if the grid voltage is decreased 5 volts negative and the plate current returns to its former value, then the +40-volt plate-voltage change is offset by a −5-volt grid-voltage change. Then, the amplification factor, designated by the Greek letter mu, is $\mu = 40/5 = 8$. The best way to find the amplification factor graphically is indicated beneath Fig. 274*a*. The amplification factor is a ratio having no unit of measure.

Mutual Conductance.—At several places (page 26) conductance has been defined. Conductance is a factor which if multiplied by voltage gives the current in a circuit. **Grid-plate transconductance** is a more correct term than mutual conductance. The "trans" implies that the effect is carried over from one circuit to another; thus, grid-plate transconductance means the effect of a grid-voltage change in producing a plate-current change. The mutual conductance of a tube is, therefore, a factor measured in mhos which if multiplied by grid signal voltage gives the plate signal current. In



(a)-Grid voltage-plate current curves at several values of plate voltage

(b)-Plate voltage-plate current curves at several values of grid voltage

FIG. 274.—Characteristic curves for a triode. The amplification factor is a measure of the relative effectiveness of the grid with respect to the plate. In (*a*) if the plate voltage is increased from 120 volts to 150 volts, and if at the same time the grid voltage is decreased from −5 volts to −8.3 volts, the plate current remains constant at 5 milliamperes. Thus, 3.3 volts change in grid voltage is as effective as 30 volts change in plate voltage and the amplification factor is $\mu = 30/3.3 = 9.1$.

The alternating-current plate resistance, which is the opposition the tube offers to the flow of alternating current, is found from (*b*). Assume that the tube is being operated at point *P* with −8 volts on the grid, 147 volts on the plate, and with a plate current of 5 milliamperes. An alternating current would vary about point *P*, going above *P* and below *P*. The plate resistance offered to alternating current will be the voltage at point *P* divided by the current, because the tube is being operated at these values. The length of the base of the triangle drawn at point *P* is 25 volts, and the height of the triangle is 2.2 milliamperes or 0.0022 ampere. The alternating-current plate resistance will be $r_p = E/I = 25/0.0022 = 11{,}350$ ohms. The plate resistance depends on the point of operation, whether at *P*, P_1, or P_2. The direct-current plate resistance, as distinguished from the alternating-current plate resistance, at any point *P* is merely the value of plate voltage divided by the corresponding value of plate current.

The grid-plate transconductance can be found from (*a*) by erecting a small triangle at the operating point as in (*b*). Or, the grid-plate transconductance can be found from the relation $g_m = \mu/r_p$. For this tube $g_m = 9.1/11{,}350 = 0.0008$ mho = 800 micromhos.

other words, in amplification, the alternating signal voltage is impressed on the grid. Multiplying this value by the mutual conductance gives the alternating-current component of the plate current. The graphical method of determining the mutual conductance of a triode is explained in Fig. 274*a*.

Plate Resistance.—As will be made quite clear in the next chapter, a vacuum tube as an amplifier is similar to a generator. The vacuum tube (because of its amplification factor) generates in the plate circuit a signal voltage μ times greater than the alternating signal voltage impressed on the grid. This generated voltage forces a current through the load connected in the plate circuit. But, just as for any generator, the source of voltage must also force the alternating current through the internal resistance of the generator as well as through the external resistance (or impedance) of the load. The **plate resistance** of a vacuum tube is the opposition measured in ohms offered by the tube to an *alternating-current* flow through its plate circuit. Resistance causes heat loss, and so does plate resistance cause a heat loss *within* the tube, largely due to the effect of the opposing space charge. The plate resistance of a tube, it should be carefully noted, applies to alternating-current and *not* direct-current flow. The graphic method of finding the plate resistance of a tube is given in Fig. 274*b*. Variations in the

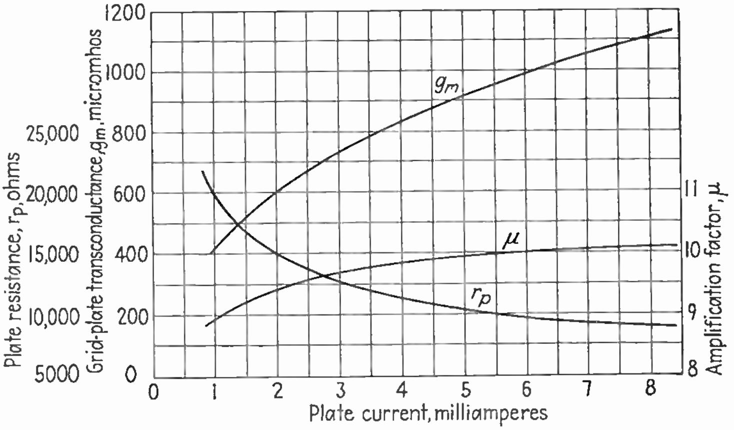


Fig. 275.—Variations in grid-plate transconductance, amplification factor, and alternating-current plate resistance with plate current. Grid voltage held constant at −8 volts, plate voltage varied. The curves are for a typical triode similar to that used for Fig. 274. The relations among these values are as follows: $r_p = \mu/g_m$, $g_m = \mu/r_p$, and $\mu = g_m r_p$.

amplification factor, mutual conductance, and plate resistance are shown in Fig. 275.

To summarize: The important vacuum-tube coefficients are the amplification factor μ, the mutual conductance g_m, and the alternating-current plate resistance r_p. These are related as follows: $\mu = r_p g_m$, $g_m = \mu/r_p$, and $r_p = \mu/g_m$.

The Tetrode.—As is well known to many readers, the tetrode, or four-electrode thermionic vacuum tube, has a higher plate resistance and a higher amplification factor than a triode, or three-electrode tube. This is because a **screen grid** has been added to the tube. The screen grid also reduces, or prevents entirely, the tendency of the tube to **oscillate** by shielding the control grid from the plate. The reason for this will be explained on page 475. Note that it is now necessary to designate which grid is being considered. What is called the control grid corresponds to the grid in a triode.

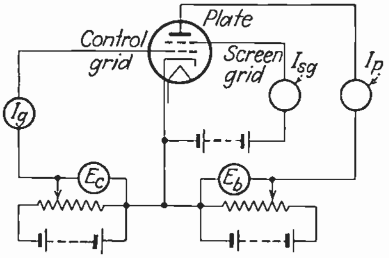
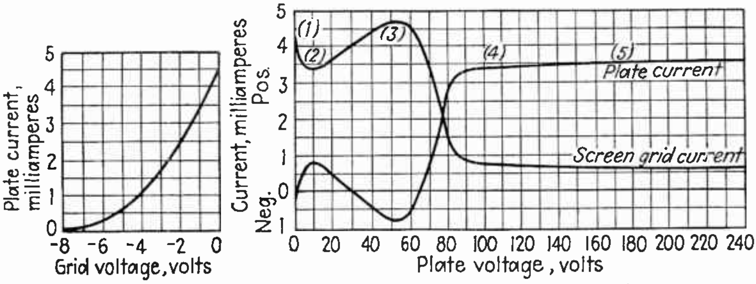


FIG. 276.—Schematic diagram for a tetrode or screen-grid tube and its test circuit. The screen-grid voltage is held at a constant positive voltage as indicated. The plate voltage E_b and the grid voltage E_c may be varied.

The characteristics of the screen-grid tube can be explained by Fig. 276. Note that the screen grid is placed between the control grid and the plate. In this position, it can largely prevent the voltage on the plate from affecting the important negative space-charge region close to the cathode. It will be recalled that in this region the electrons emitted by the hot cathode are at low velocities. To illustrate the action, it can be said that in this region the negative electrons are "milling about" undecided as to whether they should go on, or go back to the cathode which they left positively charged by their departure.

Now the control grid is close to this space charge and can readily control the electrons. The plate, in addition to being farther away as it was in the triode, is shielded by the screen grid. Because of this, the plate has about lost even what little control it formerly had over the electron stream or plate current through the tube. In studying the triode, it was explained that the amplification factor was a measure of the effectiveness of the grid, as compared with the plate, in controlling the electron stream through the tube. In the screen-grid tube, the plate has, as stated, lost almost all control over the electron stream, and the amplification factor is, accordingly, quite high, being 400 for a typical tube. Also, because the plate voltage has little control and it is hard for the plate potential to draw current through the tube, the plate resistance of the tube is high, being 400,000 ohms for a typical screen-grid tube.

Characteristics of a Tetrode.—The relations between plate current and grid voltage are shown in Fig. 277*a* and are obtained with a circuit arranged as in Fig. 276. Note that the screen grid is held at a fixed positive potential, often about +90 volts. This potential must be positive to pull the electrons through the control grid.



(a)-Grid voltage-plate current curves

(b)-Plate voltage-plate current curves

FIG. 277.—Characteristic curves for a screen-grid tube or tetrode. Because the plate is so fully shielded by the screen grid, the plate voltage has little effect on the plate current. For this reason, the graphical solutions of Fig. 274 cannot be applied with accuracy. For the meaning of points (1), (2), etc., see the accompanying discussion. The operating region from about 10 to 50 volts can be regarded as having negative resistance characteristics because an *increase* in plate voltage causes a *decrease* in plate current.

Now once the electrons are highly accelerated, as they are by the positive screen grid, they tend to travel in straight lines, and some of them pass between the wires of the screen grid; once they are through, they are drawn on to the positive plate. Thus, the plate eventually receives a current if the control grid, which has first control, permits the electrons to pass.

Figure 277*a* was obtained by holding the plate at a fixed positive voltage and by varying the control-grid voltage. As previously mentioned, the screen-grid voltage is held constant at a positive value. Plate currents at various plate voltages result in curves that fall almost on top of each other, proving that the plate has little control over the plate current. No attempt has been made to reproduce such curves. Applying the method discussed in Fig. 274 indicates that the tetrode would have a very high amplification factor. This method is not satisfactory because it is not accurate. Special circuits are used for finding the coefficients of tetrodes and pentodes (see Standards of the Institute of Radio Engineers).

The plate voltage–plate current curves of Fig. 277*b* are particularly interesting and can be explained as follows: As previously mentioned, the screen grid is held at a positive voltage of, say, +90 volts. Now at point 1 the plate is at zero volts; therefore, the plate will take no current, but the screen grid will take a large current. At point 2, the plate now has a positive voltage on it and starts to draw a few electrons to it. There are, then, fewer electrons available for the screen grid, and its current drops. But now, secondary emission again comes into play (pages 405 and 413), and its effect will now be explained.

Although the plate is not highly positive at point 2, its potential is, nevertheless, sufficient to add a considerable velocity to the electrons that pass through the screen grid. When these rapidly moving electrons strike the plate, they knock electrons out of it. These secondary electrons have two choices; they may return to the plate where they originated, or they may go over to the screen grid. Slightly beyond point 2 of Fig. 277*b*, the plate potential is only about +20 volts but the screen-grid potential is +90 volts, and so many of these secondary electrons go over to the grid. As the plate is made more positive, the electrons are given higher velocities, a greater secondary emission occurs, and the screen grid attracts a larger group of these electrons.

The increase in the screen-grid current and the decrease in the plate current at point 3 can be explained by Fig. 277*b*. The screen grid is now drawing electrons from both the cathode and the plate, and these two currents unite in passing down through the screen-grid milliammeter and cause the screen-grid current to increase. The secondary electrons must return to the plate through the milliammeter in the plate circuit, and in the figure just referred to, they move *up*. The normal electron plate current from the cathode flows *down* through the milliammeter in the plate circuit, and thus the *net* current, which is the current that the milliammeter measures, decreases as shown at point 3 of Fig. 277*b*. In fact, as the plate voltage is increased so that greater secondary emission occurs, the net plate current may even go negative, *indicating that the number of secondary electrons liberated from the plate and going over to the screen grid exceeds in number the electrons from the cathode to the plate.* All operating conditions will not show this negative plate current, but it can easily be demonstrated.

As the plate is made more positive than the screen grid, such as at point 4 of Fig. 277*b*, the plate is now able to draw all the secondary electrons back to it, and the plate voltage–plate current curve flattens out as at point 5. A screen-grid tube is normally operated over the region centered about point 5. The reader may be surprised to find that secondary emission occurs even in triodes. In these, however, there is no positive screen grid to pull the secondary electrons away from the vicinity of the plate, and thus the peculiar effects of Fig. 277*b* do not show up. Because the plate voltage–plate current curve is quite flat in the vicinity of point 5 where the tube is usually operated, the screen-grid tube has a high alternating plate-circuit resistance.

To summarize: In the screen-grid tube, or tetrode, a second grid called the screen grid is placed between the plate and the control grid. This screen grid is held at a constant positive potential. Because of the presence of this screen grid, the voltage on the plate has little influence in determining the plate-current flow, but the control grid is very effective. This gives the screen-grid tube, or tetrode, a high amplification factor and a high resistance. The presence of the positive screen grid shows up the effect of secondary emission which produces peculiar variations in the plate-current curve at low plate-voltage values.

The Pentode.—Although the screen-grid tube, or tetrode, offers many advantages over the triode for certain uses, the plate-current variations at low plate voltages are not desirable. The **pentode,** or five-electrode thermionic vacuum tube, is provided with a means of *suppressing* this secondary emission effect. Now at the start, note carefully that in the pentode the secondary emission is not prevented, but *the effect of it is suppressed.* This is accomplished by the introduction of a third grid, called a **suppressor grid,** between the plate and the screen grid. This suppressing action will now be explained.

The characteristics of a suppressor-grid pentode can be found with the circuit shown in Fig. 276. The suppressor grid is connected directly to the cathode, and hence no additional circuit features are required. The arrangement of the electrodes and their typical operating potentials are as shown in Fig. 278. In this tube the control grid operates as before and largely determines the current flow through the tube. The screen grid gives the electrons their initial acceleration, and their velocity carries many of them through the screen grid, through the suppressor grid, and on toward the positive plate.

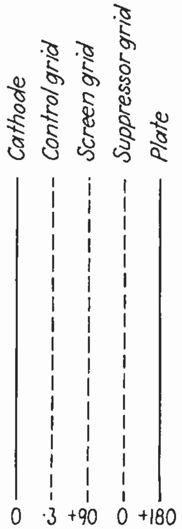


FIG. 278.—Order of the electrodes in a suppressor-grid pentode tube. Also, operating potentials are shown. As indicated, the cathode and the suppressor grid are both at zero potential because they are tied together and grounded. In some suppressor-grid pentodes the screen grid may operate at the same potential as the plate.

The entire positive voltage of the plate is impressed between the plate and the suppressor grid, because this grid and the cathode are tied together. Secondary electrons are emitted from the plate as in any vacuum tube, but because of the presence of the suppressor grid which is at zero (cathode) voltage, these secondary electrons cannot go to the positive screen grid as they did in the tetrode when the plate voltage is low.

As a result of the action of the suppressor grid, the undesired *effects* of secondary emission are eliminated. This gives the suppressor-grid pentode a very smooth plate voltage–plate current curve as shown in Fig. 279. Now the suppressor grid does another thing: It further prevents the potential of the plate from reaching down into the space-charge region in which the control grid so effectively acts. As a result, the plate voltage has lost substantially all control over the electron stream. The result is that a typical suppressor-grid pentode has a very high amplification factor, as great as 1200 or over, and a very high plate resistance of over 1,000,000 ohms.

The fact that the plate voltage has so little effect on the plate current would cause the grid voltage–plate current curves at various plate voltages to fall almost on top of each other. Thus, in Fig. 279, no attempt has been made to show the various curves. The flat plate voltage–plate current curve of this same figure indicates a very high plate resistance as explained under Figs. 274 and 277.

To summarize: The introduction of a third grid, placed between the plate and screen grid, suppressed the effect of secondary emission, although electrons are knocked out of the plate as before. The suppressor grid prevents the secondary electrons from flowing to the screen grid. The suppressor grid further shields the plate from the space-charge region, however, and thus makes both the amplification factor and plate resistance very high.

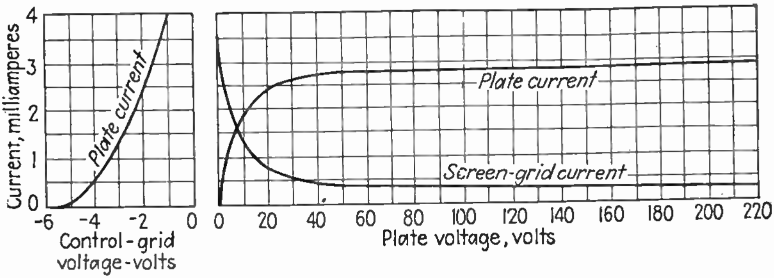


FIG. 279.—Typical characteristic curves for a suppressor-grid pentode comparable to the screen-grid tetrode of Fig. 277. The suppressor grid eliminates the undesired secondary emission effect, giving the smooth curves shown.

Remote Cutoff Tubes.—If the control grid voltage–plate current curves of Figs. 274, 277, and 279 are examined, it will be seen that the plate-current curves come quite gradually down to zero and then abruptly cut off. In other words, the slopes of these curves are about the same down to almost the cutoff value where the plate current ceases to flow.

Certain tubes called **remote cutoff tubes** (or variable μ or supercontrol tubes) are constructed so that they do not cut off sharply but gradually as shown in Fig. 280. Instead of the curve reducing to zero as in the figures referred to in the preceding paragraph, they reduce gradually to zero and cut off at a grid voltage of perhaps -50 volts. This special feature is obtained by using a control grid which is nonuniformly wound or which is unsymmetrical in some way. The application of these tubes will be considered on page 488.

Multigrid Tubes.—A vast array of tubes are on the market today. It is somewhat bewildering to start studying them. To help clarify this situation, the following paragraphs are added.

Multigrid, or multielectrode, tubes may be classified as tubes containing more electrodes than a triode. There are two classes of multigrid tubes. The *first* class includes tubes designed to perform some function that cannot be performed readily by a triode. The *second* class includes tubes having additional electrodes to perform, simultaneously, more than one function.

As an example of the first class, consider the screen-grid tube and the pentode. As will be shown later (page 475) these tubes make excellent radio-frequency amplifiers, but special neutralizing circuits must be employed when a triode is used at radio frequencies. As an example of the second class, consider a "duplex-diode high-mu triode." This tube contains two diode structures and a high-amplification triode structure in the same evacuated enclosure.

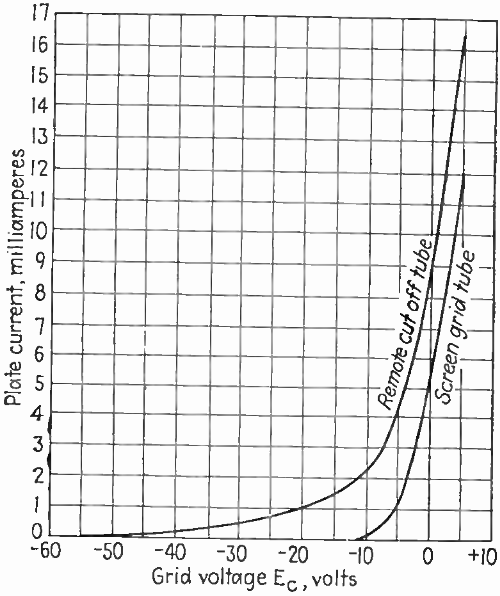


FIG. 280.—A comparison of the cutoff characteristics of a remote cutoff or super-control tube and an ordinary screen-grid tube. This effect is achieved by having a nonuniform control grid, several methods being applicable. These are often called variable-mu tubes, because the amplification factor will depend on the control grid bias voltage used (see Prob. 3 page 441).

To summarize: It can be said that the basic tube structures are diodes, triodes, tetrodes, and pentodes. All other ordinary tubes are but combinations of these. When examining a new tube, the classification into which it falls should be ascertained, and then if the four basic tube types are understood, the operation of the new tube can readily be explained.

The Beam-power Tube.—In the preceding sections no distinction has been made between tubes which handle small amounts of power and those which handle large amounts of power. These distinctions will be given some consideration in the next chapter. In general, however, it can be said that whether a given triode, tetrode, or pentode will handle much power or not depends on its size, etc. One obvious error in this statement is as follows: The more grids there are within a tube, the more the plate current is limited. As the reader may know, triodes are very popular as voltage amplifiers, and for handling very large amounts of power they are always used.

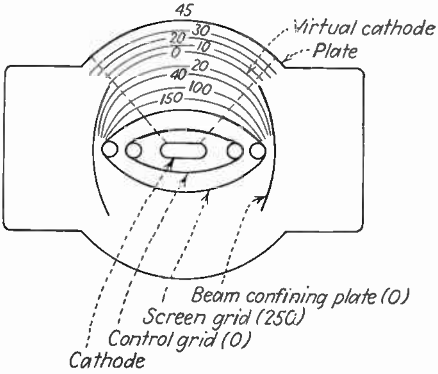


FIG. 281.—Top view of electrodes in the so-called beam power tube showing potentials in the beam when the plate is at a low potential.

Power pentodes are well suited for handling a few watts of power, but the presence of the suppressor grid places some limitations on their operation. Thus, if a tube could be designed which had no suppressor grid, but in which the effects of secondary emission were controlled by other convenient means, a tube would be provided which for some purposes would prove superior.

In the so-called **beam-power tube** shown schematically in Fig. 281, the effects of secondary emission are controlled and the erratic variations of Fig. 277 prevented without a suppressor grid. The beam-power tube contains a cathode, a control grid, a screen grid, and a plate which operates about as has been previously considered, and in addition it has a beam-confining plate which is at ground or cathode potential. A complete analysis of the action of this tube involves a detailed study of the electric fields produced within the tube by each electrode. Briefly, the action is as follows: The numerical values within the schematic tube structure of Fig. 281 indicate the voltage at each point with respect to the cathode. Note that opposite the ends of the beam-confining plate a zero line marked "virtual cathode" exists. In effect, the zero potential of this electrode extends on across the tube. With such a voltage distribution within the tube the secondary electrons will return to the plate just as in the suppressor-grid pentode where the suppressor grid at cathode potential prevents the secondary electrons from going to the positive screen grid. The beam-power tube has about the same characteristics, therefore, as a suppressor-grid power pentode.

To summarize: In the beam-power tube the advantages of suppressing secondary emission are obtained with a beam-confining plate rather than with a suppressor grid. One advantage of this is a more efficient tube of smaller size than a comparable power pentode.

The Gas Diode.—Two-electrode vacuum tubes or diodes are extensively used as rectifiers to obtain direct current from an alternating-current power supply. The rectified direct-current component of the plate current flows through the tube. If the internal resistance of the tube is high, there will be a large voltage drop within the tube. This tends to cause poor regulation and low efficiency (page 83). Thus, if the internal resistance of a diode can be reduced, the tube will be a better rectifier from the standpoint of efficiency and regulation.

Several factors, such as the shapes and spacings of the cathode and plate (or anode), contribute to determining the internal resistance of a diode. For a given tube structure, the negative space charge is largely responsible for the voltage drop within the tube. This is because, as previously explained on page 410, the negative space charge opposes the flow of negative electrons making up the plate current as they flow from the cathode to the plate. From this it follows that if the negative space charge within a tube can be neutralized the electrons constituting the plate current can pass through the tube with less opposition and the tube will be a better rectifier.

The negative electrons making up the space charge must be there, because they are in effect a reservoir from which the positive plate draws a current. However, if the negative space charge can be neutralized, the electrons will still be there, but the opposing space charge effect will be largely gone.[1] To accomplish this neutralizing effect, gas at a low pressure is introduced into a rectifier tube, or, more commonly, mercury is placed in an evacuated tube, and at the low pressure existing the mercury vaporizes and fills the tube with a mercury vapor or gas. The functioning of the mercury vapor in neutralizing the space charge and reducing the voltage drop within the tube will now be explained.

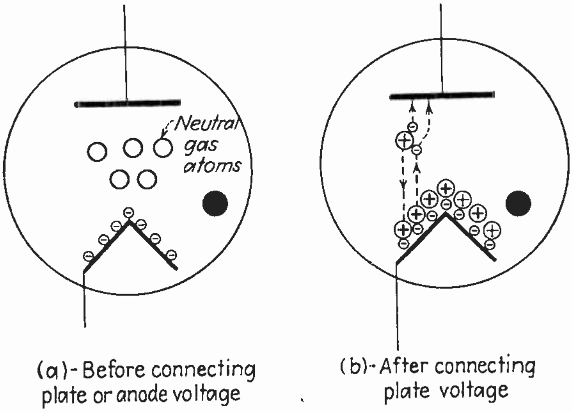


FIG. 282.—Action of ionization by collision in a gas diode, and the resulting neutralization of the space charge. The large black dot is the symbol for indicating that the tube is filled with gas.

An enlarged view of a gas diode is shown in Fig. 282*a*. For the condition of this figure it is assumed that the filament is up to temperature but that the plate voltage has *not* been applied. A negative space charge will exist around the filament, and neutral gas atoms will exist throughout the tube. Several of these gas atoms are shown; since they are neutral, no + or − sign is shown on them.

Now consider what happens at the instant a positive plate voltage is applied. This positive voltage will not affect the neutral atoms but will draw negative electrons toward it and give them a

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[1] Perhaps it is well to explain that although the positive plate does draw its current from the negative space charge, this current flows around through the external circuit to the cathode and then on to the space-charge region to replace the electrons previously withdrawn. Thus the plate current has a complete path through the tube and in effect flows through the negative space charge which causes a voltage drop just as in any electric circuit.

high velocity. As previously mentioned, the tube will be filled with gas atoms, and when the high-velocity electrons are traveling to the plate, many collisions between high-velocity electrons and gas atoms will occur.

An atom of gas is thought to be composed of a positive heavy central nucleus about which negative electrons are revolving. There are enough revolving electrons to equal the positive charge on the nucleus, and thus the gas atom as a whole is neutral. But when a high-velocity electron on its way from cathode to plate encounters a gas atom in its path, it may **ionize** the atom by tearing one or more of its revolving electrons from it.

When an otherwise neutral gas atom is ionized, it is broken up into positive and negative charged particles called **ions.** The negative ions will be the electrons torn away from the atom; although they will have the same charge as the positive ions, the electrons are small, have little mass, and are readily acted on by a voltage and quickly move about within the tube. The positive ions, on the contrary, are heavy (because they are really massive gas atoms minus a "light" electron), and they move very slowly as compared with the negative ion. As soon as a gas atom is ionized, the positive plate quickly draws over the negative ions, and the negative space-charge region and the negative cathode start to draw over the positive ions. These are massive, however, and do not readily move. The result is that after ionization the electrons are quickly withdrawn leaving the massive positive ions in the vicinity of the space-charge region. This accumulation of massive "sluggish" positive ions largely overcomes the negative space charge. This action is illustrated by the lower curve of Fig. 283.

This question immediately rises: If the negative space charge is largely overcome by the accumulation of positive ions, is there any voltage drop at all across a gas diode? Yes, the voltage across a gas diode must be high enough to give the ions velocities sufficient to cause ionization by collision as the phenomenon is called. For a mercury-vapor tube the theoretical ionizing potential is about 10.4 volts. From the characteristic curves of Fig. 283 it is seen that the voltage drop for a mercury-vapor tube is about this value and that it is independent of the magnitude of the current flow. It should be added, however, that such a relation cannot go on indefinitely and that if the rated current of a gas tube is exceeded the tube will be permanently damaged, and may even arc across. In the operation of gas tubes, the filament should always be at the correct operating temperature before plate voltage is applied. This is to ensure that there is a sheath of negative electrons around the filament. These act somewhat as a "buffer" for the positive electrons traveling to the filament. Although these move slowly, they are massive, and if they are not "cushioned" by the negative space charge, they may have sufficient energy actually to ruin the oxide-coated filament when they strike it.

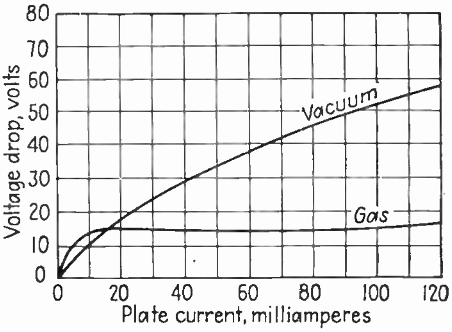


FIG. 283.—Internal voltage drops between cathode and anode (or plate) for comparable gas and vacuum rectifier tubes.

To summarize: In a gas-filled diode, ionization by collision between rapidly moving electrons and neutral gas atoms occurs. This causes massive positive ions to be formed within the tube. These positive ions slowly move toward the filament or cathode where they largely neutralize the negative space charge. Since the negative space charge is the factor limiting current flow and causing a voltage drop within a high-vacuum tube, when this space charge is neutralized in the gas tube, current readily flows through it with but a low voltage drop. This results in the tube being an efficient rectifier.

The Gas Triode.—If a grid is placed in a gas-filled thermionic diode, it becomes a **gas triode,** which goes by the trade name Thyratron. This is also called a grid-controlled gas-filled tube, but when its action is studied, it appears that it might well be called a grid *lose control* rather than a grid-controlled tube, because the tube functions in exactly that way. This action is explained as follows:

The gas triode is pictured in Fig. 284 and consists of an oxide-coated thermionic cathode, a grid, and a plate or anode. Gas or mercury vapor at a low pressure fills the tube. Suppose that the filament is at operating temperature, that the grid is made, say, 15 volts *negative*, and that the plate is 50 volts *positive* for a typical gas triode such as used in cathode-ray tube sweep circuits. Now the grid is so highly negative that it repels all electrons back to the negative space-charge region, and no current flows to the plate.

If the grid is slowly made less negative, however, a grid voltage will be reached at which the positive plate can finally pull through an electron current. Thus far, the tube has functioned as a high-vacuum tube would function, because the gas atoms were neutral, nothing having happened to disturb them.

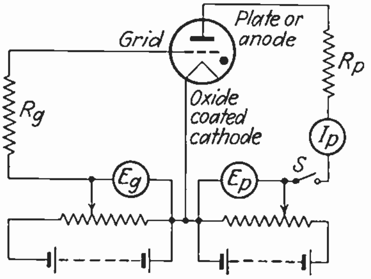


FIG. 284.—Schematic diagram for a grid-controlled gas-filled tube (often called a thyratron) and a test circuit.

The instant an electron current flows to the plate, however, all this changes. The electrons reach a high velocity, and many of the gas atoms are ionized, or broken down into positive and negative charges, as was explained for the gas diode. Also as in the gas diode, as soon as ionization occurs the slow-moving positive ions leisurely travel over to the negative filament or cathode. Here they largely neutralize the negative space charge, and only a low voltage of about 14 volts is required between the cathode and plate to pull a very large electron current through the tube (see Fig. 283). For this reason, the resistor R_p is placed in the plate circuit. If it is not there to limit the current flow, then the tube will arc across and will be ruined.

Now let the action at the grid be considered. As soon as ionization occurs within the tube, the negative grid pulls positive ions toward it and repels negative electrons away from it. The massive positive ions slowly move toward the grid, but the negative electrons dart away quickly. The result is that a sheath of positive ions surround the grid, and its negative voltage is neutralized. For this reason, the grid *loses control* as was previously mentioned.

The characteristic curves for a gas triode are shown in Fig. 285 and are obtained in the following manner. The filament is at operating temperature, the plate voltage is made zero, and the grid voltage is adjusted to some negative value, say -15 volts. Then, the plate voltage is increased until the tube conducts, as noted by the milliammeter in the plate circuit. The plate-voltage value at which this occurs is observed and plotted as on Fig. 285 with the corresponding grid-voltage value. Then, the plate voltage is reduced to zero so that the tube stops conducting. As soon as this happens, the positive and negative ions all recombine, **deionization** occurs, and the gas returns to a neutral state. Next, the grid potential is adjusted to −14 volts, and then the plate is slowly made positive until ionization again occurs. This value of plate voltage is observed and plotted. Additional points are taken in this manner until the curve is complete. Because the grid draws current after it loses control, a current-limiting resistor R_g must be placed in the grid circuit as Fig. 284 indicates.

To summarize: In the gas-filled triode, the negative grid can control the tube action (to a certain extent) by preventing current flow until the positive plate draws an electron current from the negative space-charge region at the cathode. Then ionization occurs, and the grid loses control. Certain uses of the gas triode will be considered on page 495.

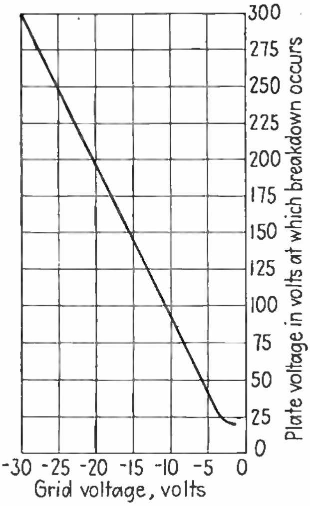


FIG. 285.—Characteristics for a typical grid-controlled gas-filled tube. This tube contained argon gas.

Cold-cathode Two-electrode Gas Tubes.—These tubes consist of two electrodes in a glass bulb or tube filled with neon, argon, or similar gases. No thermionic or heated cathode is used; the cathode is relatively cold. Such devices are often called **neon tubes.** The type used in communication circuits will be treated in this section rather than the variety used for neon signs.

A convenient circuit for testing a cold-cathode gas tube is shown in Fig. 286. Voltage divider R_2 is first cut in to serve as a protective resistance to limit the current flow after the tube starts to conduct. Next, the voltage divider R_1 is varied until the tube suddenly breaks down. This breakdown occurs at some voltage as at point 1, Fig. 287, and then the voltage across the tube immediately drops to point 2. The reason for this sudden drop in voltage will now be explained.

In any body of gas, such as within the tube, there are a very few free ions, even before ionization by collision (as explained in the two previous sections) occurs. These few free ions are produced by natural causes such as by light rays. As soon as the voltage is applied to the tube, these few free ions are attracted to the electrodes, the positive ions moving toward the negative electrode, and the negative ions moving toward the positive electrode.

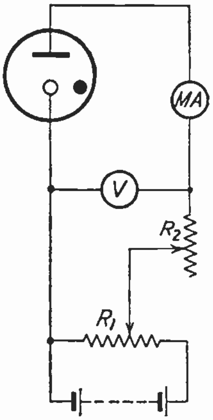


FIG. 286.—Circuit for studying the characteristics of a cold-cathode two-electrode gas tube.

As these ions move toward their respective electrodes, they gather momentum and may reach velocities sufficient to cause ionization by collision. When this occurs at point 1 on Fig. 287, the ions produced by collision now join the current flow, and the voltage quickly drops to point 2, which is the voltage required to sustain the flow.

In general it can be said that the action occurring within a gas tube is more complicated than that within a high-vacuum type. This is because of the presence of both positive and negative ions and because of other somewhat vague ionic phenomena. But this can be said: Electrons must flow from the cold cathode into the tube, because electrons are leaving at the plate and flowing on around the wire circuit as indicated by the deflection of the milliammeter in the plate or anode circuit.

In the thermionic tube, the heat supplied the cathode from the supply lines gives the electrons the energies they need to escape from the cathode, but in the tube now being considered, the cathode is relatively cold. Although the exact action is somewhat obscure, it is possible that the slow-moving positive ions approaching the cold cathode accumulate in a layer or sheath very close to the cathode. This would cause a potential to exist between the sheath and the cathode, and because of the small distance between the sheath and the cathode, the voltage gradient (page 183) would be very great. If this is sufficient, and there are some reasons for believing it is, then the very strong electric field (page 187) may actually pull the electrons out of the relatively cold cathode. This is the third type of emission mentioned on page 405. Whatever the nature of the action, electrons must either come from the cathode or must be produced by ionization by collision between positive ions and gas atoms in the immediate vicinity of the cathode, or the gas would soon have all the ions withdrawn and no current would flow.

If the electrodes within the tube are two parallel disks close together, a small area of the negative electrode will be observed to glow as soon as the current flows at point 2 of Fig. 287. If now resistor R_2 of Fig. 285 is decreased so that more current flows, a greater portion of the negative electrode or cathode glows, the area depending on the current flow. From point 2 to 3 the voltage drop across the tube is almost independent of the current magnitude. It is this property of the tube which makes the tube so useful in certain circuit applications. Since the voltage drop across the tube does not increase much with an increase in current, the internal resistance of the tube *decreases* with an increase in current. At about point 3, the cathode is completely covered with a glow, and after this the voltage drop across the tube increases. The usual operating region is between points 2 and 3 of Fig. 287.

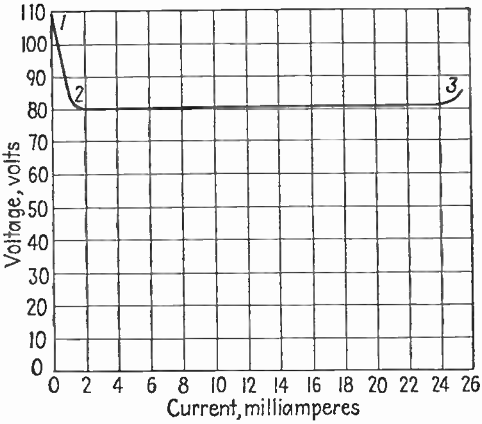


FIG. 287.—For a cold-cathode two-electrode gas-filled tube the voltage across the tube remains substantially constant for a wide range of currents. It is this principle which is used in a voltage regulator.

To summarize: In the cold-cathode two-electrode gas tube after ionization occurs there is almost no increase in voltage across the tube with an increased current flow over a wide operating range. This is one of the very useful properties of this tube.

Cold-cathode Three-electrode Gas Tubes.—These have been called **ionic tubes** in the technical literature. The principles involved will be explained in this section, the circuit applications being left for the next chapter.

In this cold-cathode three-electrode gas tube there are two gaps across which action occurs. One is a small gap between the **control anode** and the cathode, and the other is a larger gap between the **main anode** and the cathode. The characteristics are obtained by using a circuit such as shown in Fig. 286 and testing first one gap and then the other. In doing this, care should be taken to follow the directions for electrode connections as specified by the manufacturer.

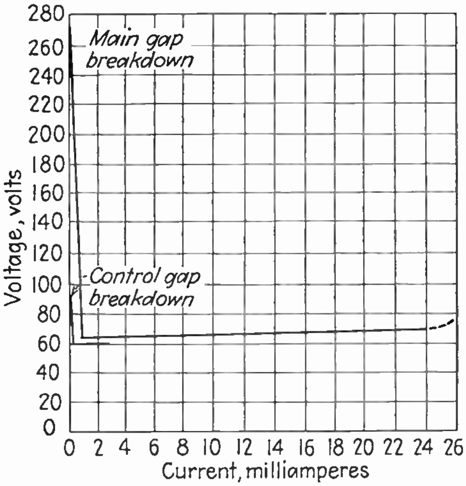


FIG. 288.—Characteristics of the control gap and main gap for a cold-cathode three-electrode gas tube.

These characteristics are shown in Fig 288. It will be observed that the control gap breaks down at a much lower voltage than the main gap and can conduct only a small current flow. This is made use of in certain circuit applications somewhat as follows: The control gap is held at a voltage just below breakdown. Then, the feeble impulse to control the circuit is also impressed on the control gap. This raises the control gap above its breakdown value of +90 volts as shown in Fig. 288. The control anode then attracts the few free electrons within the tube (page 430), and these cause ionization by collision. This fills the tube with ions and permits the main gap to break down at a low voltage and a relatively large current to flow.

Phototubes.—Photoelectric emission was the last method of liberating electrons from metals as mentioned on page 405. The phototube consists of a light-sensitive cathode and a positive current-collecting anode. These are contained in a glass bulb which may be either highly evacuated or may contain a small amount of some gas such as argon. In most of the common phototubes in ordinary use, the cathode is a composite cesium-oxygen-silver layer.

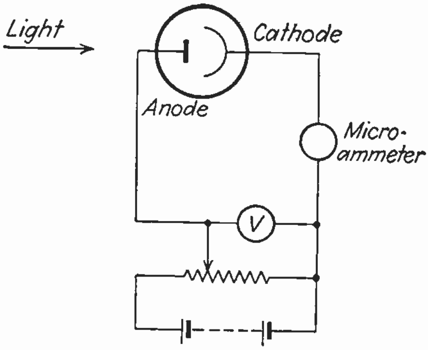


Fig. 289.—Circuit for testing a phototube. The conventional symbol is here shown. From this it would appear that the anode shields the cathode from the illumination. Actually, this is not true because the anode is merely a small wire. For a gas phototube a current limiting resistor should be placed in series with the anode or cathode to protect the tube from current overload which might result in an arc across the tube.

The High-vacuum Phototube.—The light falling on the light-sensitive cathode causes electrons to be emitted from the active material. *The number of electrons emitted is directly proportional to the light intensity.* This is a very important law, and if it were not true, many of the applications of the phototube would not be possible. Some cathode surfaces are sensitive to one type of light, and other cathodes respond to different types of light; for instance certain cathodes are good emitters for red light, but other types may largely respond to ultraviolet. Nevertheless, in all instances, the emission, when it does occur, is proportional to the light intensity.

A circuit for testing a phototube is shown in Fig. 289. The light intensity at the phototube can be measured with a commercial light meter and can be varied by moving the light source with respect to the phototube. The anode or plate voltage can be varied by adjusting the voltage divider.

By referring to the curves in Fig. 290*a*, it is seen that at a given illumination, say 0.5 lumen, an anode or plate current flows just as soon as this electrode is made positive. However, an anode potential is soon reached at which the anode is taking the electrons just as fast as they are liberated, and the curve flattens out. The characteristics of the tube at an illumination of 0.1 lumen are also shown. If the voltage is held constant, but the illumination is varied, Fig. 290*b* results. This curve demonstrates the linear relation between illumination and current output. A typical phototube is shown in Fig. 291.

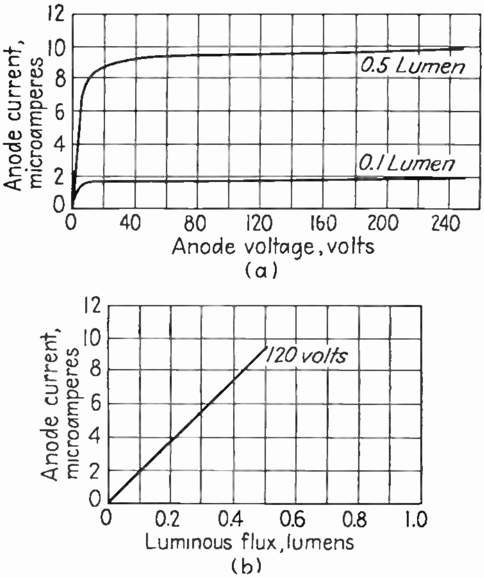


FIG. 290.—Curves showing the characteristics of a high-vacuum phototube. The upper curves show variations with anode voltage. The lower curve proves that the output of a phototube varies directly with the light striking the cathode. The intensity of illumination at the phototube cathode is measured in lumens, the lumens being given by the relation $F = AC/d^2$, where A is the effective projected area of the cathode, C is the intensity of the light source in candlepower, and d is the distance between the source and the cathode.

The Gas Phototube.—In the gas phototube, electrons are emitted from the cathode by the light striking its surface. These electrons are drawn over to the positive plate. The curve flattens out as for the high-vacuum tube, but at higher values of plate voltage the electrons achieve velocities sufficiently high to cause ionization by collision. These characteristics are shown in Fig. 292*a* and are readily obtained with a circuit such as shown in Fig. 289.

In the gas phototube the ionization by collision provides an additional source of electrons, and the current through the tube is greater under comparable conditions than in the high-vacuum type. Also, it is probable that the presence of the positive ions affects the operation much as in a thermionic gas tube. As shown in Fig. 292*b*, there is essentially a linear relation between illumination and current flow, a condition essential for most uses of the phototube.

Because the gas phototube is more sensitive than the high-vacuum tube, the gas phototube has largely supplanted the high-vacuum type which was the first one used. In using and testing the gas phototube, care must be taken in using a circuit such as Fig. 289 to ensure that the current through the tube is not excessive. If the current through the tube and the voltage drop across it are permitted to become excessive, the tube may arc across and become permanently damaged. Thus, in working with gas phototubes it is usually advisable to insert a current-limiting resistor of at least a megohm in the plate or anode circuit.

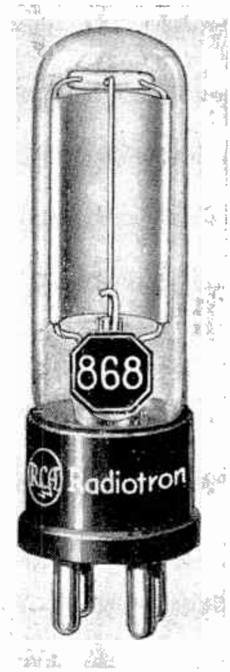
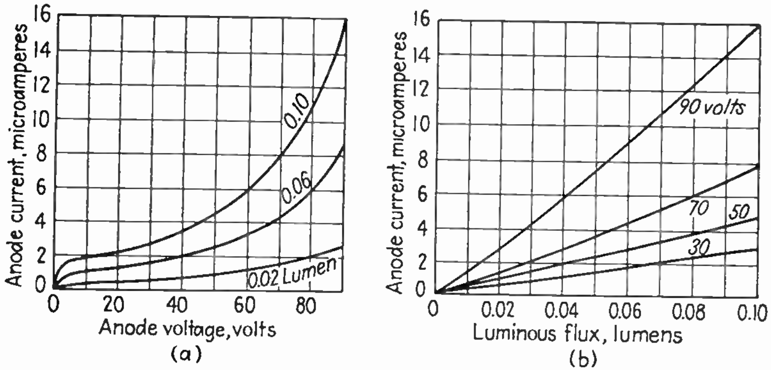


FIG. 291.—A phototube. (*Courtesy of RCA Manufacturing Company, Inc.*)

The Copper Oxide Rectifier.—This device was discussed on page 229, where its use in alternating-current instruments was considered. There it was shown that the copper oxide rectifier was composed of disks of copper on which a layer of cuprous oxide had been formed. A stack of these disks make up a rectifier element. Current passes quite readily from the oxide to the copper, but very little current will flow in the opposite direction. This statement is for conventional current which is in the direction opposite to electron-current flow.

The statements just made are equivalent to saying that the resistance of a copper oxide rectifier is low if the oxide is made positive and the copper negative, but the resistance is quite high if the opposite polarities are applied. These relations are shown in Fig. 293. An examination of this figure reveals that a wide range of resistance values is obtainable depending on the voltage value impressed on the device. Since the copper oxide rectifier is, in effect, a variable resistor, it is often called a copper oxide **Varistor.** The reader may be surprised to find that the copper oxide Varistor is replacing vacuum tubes for many purposes in telephone applications.



(a) (b)

FIG. 292.—Curves showing the characteristics of a gas phototube. Compare with curves of Fig. 291.

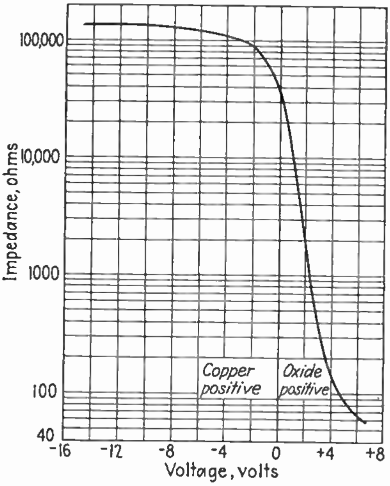


FIG. 293.—Characteristics of a typical copper oxide Varistor such as used for modulation and demodulation in the new carrier-telephone systems. Those used for rectifying power are larger and have lower resistance than the unit here shown.

The Photocell.—A review of the theory of phototubes will disclose that they merely act like variable high resistances or as electric valves. Thus, a plate or anode voltage is impressed upon them, and they merely let more or less current flow through the tube, depending on the intensity of illumination at the cathode. There are, however, **photocells** (as distinguished from phototubes) which generate an electromotive force when light shines on the active material. The resulting voltage between the cell terminals forces current through a load connected to the photocell. No batteries or other sources of energy are required. A typical photocell is shown in Fig. 294.

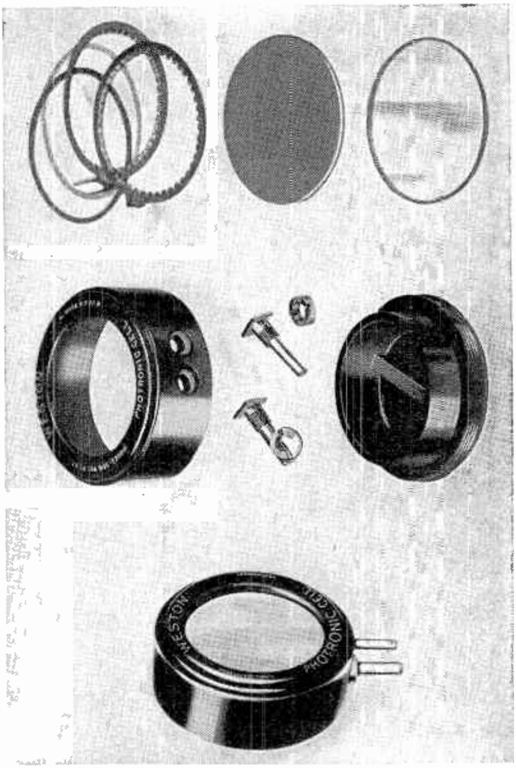


FIG. 294.—A photocell and its component parts. (*Courtesy of Weston Electrical Instrument Corporation.*)

The photocell consists essentially of a metal disk on which a layer of light-sensitive material has been formed or placed. A *very* thin layer of conducting metal is deposited on top of the light-sensitive layer. The conducting layer must be so thin that light can readily pass through it and penetrate the light-sensitive layer beneath. The thin metallic layer forms one electrode, and the metal disk is the other. Two types of these cells have been on the market. One is the copper oxide type called a **Photox** cell, and the other a selenium on iron type called a **Photronic** cell.

As Fig. 293 shows, conventional current readily passes from the oxide to the copper. This means that electrons pass but poorly in this direction. But when light shines into the oxide layer, it imparts energy to the electrons, and the light *forces* the electrons into the copper and makes it negative. This action will leave the oxide and the thin metallic layer electrode positive. An electromotive force is generated within the cell and a voltage exists between its terminals. Thus, if the external circuit is made complete, a conventional current will flow through the external load from the positive oxide to the negative copper. A similar action occurs in the selenium-iron type.

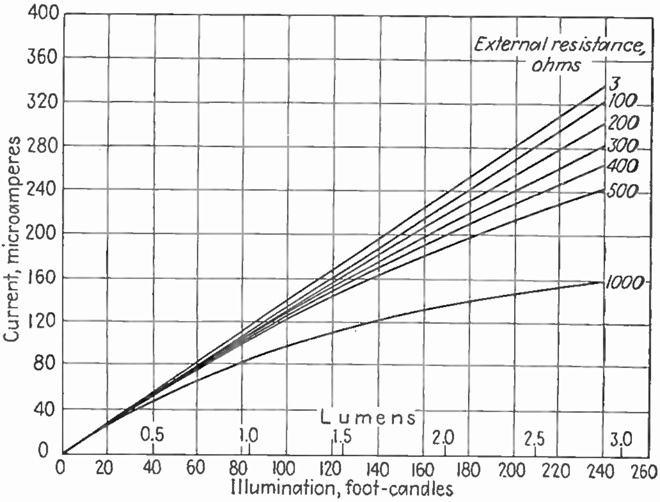


FIG. 295.—Effect of external resistance on current output of the Photronic cell.

Now as has been explained, conventional current readily flows from oxide to metal, and electrons readily flow from the metal to the copper oxide (or selenium). Thus, when the light forces electrons into the copper (or iron), they tend to flow back *within the cell* quite easily. If the external circuit is of high resistance, this tendency will be very pronounced. This action gives the photocell a *low* internal resistance. As Fig. 295 shows, a linear or straight-line relationship only exists in photocells when the external or load resistance is low. In effect, therefore, the photocell is a small generator producing a terminal open-circuit voltage proportional to the illumination of the cathode. For maximum power transfer, the external or load resistance should equal the internal resistance of the source. Since the photocell has low internal resistance, a load of low resistance must be used with it for maximum power transfer.

SUMMARY

Electrons are liberated from metals by thermionic emission, by secondary emission, by cold-cathode emission, and by photoelectric emission.

In the thermionic cathode, the heat energy supplied the electrons enables them to escape from the metal. Thermionic cathodes are pure tungsten filaments, thoriated-tungsten filaments, and oxide-coated filaments or separately heated cathodes.

In a diode a negative space charge exists as a sheath around the cathode and consists of electrons thrown off by the heated cathode. This space charge opposes the flow of electrons from cathode to plate and causes the tube to have an internal-resistance effect.

A control grid is placed between the cathode and plate of a triode. This grid is close to the space-charge region and exerts much control on the current through the tube.

Tube coefficients or constants that are of much use in circuit design are the amplification factor μ, the alternating-current plate resistance r_p, and the mutual conductance g_m. Three relations, $\mu = r_p g_m$, $g_m = \mu/r_p$, and $r_p = \mu/g_m$, are helpful.

The screen grid placed in the tetrode shields the plate so completely that the amplification factor and plate resistance are quite high. Secondary emission from the plate causes a peculiarly shaped plate voltage–plate current curve.

A suppressor grid is placed in the pentode to suppress the effects of secondary emission. This grid further shields the plate, the result being that the amplification factor and plate resistance are very high.

In the beam-power tube, the objectionable effects of secondary emission are prevented by special construction and the use of a beam-confining plate which causes a region of zero potential between the positive plate and the other electrodes. This prevents the secondary electrons from leaving the region of the positive plate.

In the gas diode, the negative electrons flowing to the positive plate or anode cause ionization by collision with the neutral gas atoms. The massive positive ions so produced slowly move toward the negative cathode where they neutralize the negative space charge and make it possible for an electron current to flow from cathode to plate with less voltage drop than in a high-vacuum tube.

In the gas triode, a grid is placed between the plate or anode and the cathode. The grid is made negative and can prevent current flow through the tube until a critical positive plate voltage is impressed. When this voltage is reached, an electron current flows through the tube and ionization by collision results. The negative grid then becomes surrounded by positive ions, entirely loses control of the plate current until the plate voltage is removed, and deionization occurs.

Ionization by collision also occurs in the cold-cathode two-electrode gas tube. Over a large part of the operating curve of the tube, increasing the current does not increase the voltage drop across the tube.

Two anodes and two gaps exist in the cold-cathode three-electrode gas tube. One gap more easily breaks down than the other and is used as a control gap to "fire" the main gap.

The phototube acts like an electronic valve controlled by the light striking a light-sensitive cathode which emits electrons. These flow to the positive plate or anode. A fundamental principle is that the number of electrons emitted is directly proportional to the illumination intensity.

Both high-vacuum and gas phototubes are used. In the latter, ionization by collision occurs, and a larger current flows giving a more sensitive tube.

A copper oxide rectifier unit consists of a copper disk on which is formed a layer of cuprous oxide. Conventional current flows readily from the oxide to the copper, and the electrical resistance is low in this direction. The resistance is very high to current flow in the opposite direction.

If light falls on the oxide layer, electrons are forced into the copper, and if suitable connections are made a current will flow in an external circuit. For circuits of low resistance, the current is substantially proportional to the intensity of illumination. No battery or other source of voltage is needed; the copper oxide photocell generates its own voltage and the energy extracted from the light is sufficient to supply the losses in the circuit.

REVIEW QUESTIONS

1. Name the ways in which electrons can be removed from a metal, and give an example of an electronic device operating on each principle.

2. What materials are used for thermionic cathodes?

3. Explain the principle of operation of the separate-heater type cathode.

4. What causes a negative space charge in a thermionic vacuum tube?

5. What is the effect of a negative space charge on the plate or anode current in a diode?

6. How does the grid control the plate current in a triode?

7. Referring to Fig. 274*a*, why is the grid current smaller for larger plate voltages?

8. What causes the dip in the grid current curves of Fig. 274*a*?

9. Explain what is meant by amplification factor, mutual conductance, and alternating-current plate resistance.

10. What is the effect of adding the screen grid to a tube?

11. Why does not secondary emission occur from the negative control grid?

12. Explain how the suppressor grid functions in a pentode.

13. Why do the screen-grid tube and the suppressor-grid pentode have an amplification factor so much greater than a triode?

14. How may multigrid tubes be classified?

15. How are the bothersome effects of secondary emission prevented in a beam-power tube?

16. Why is gas used in diodes?

17. Why is not the voltage drop across a gas diode zero?

18. Explain why a positive ion sheath forms around the negative grid in a gas triode.

19. What is meant by deionization?

20. Why do the free ions produced by natural causes play such an important role in a cold-cathode gas tube?

21. What property of a two-electrode gas tube makes it so important as a circuit element?

22. What is a very important law of photoemission?

23. What is accomplished by placing gas in a phototube?

24. Discuss the theory of the copper oxide rectifier.

25. How does a photocell differ from a phototube?

PROBLEMS

1. Three filaments of different materials are the same size. To obtain an emission current of 20 milliamperes from each filament, 0.6 watt is required to heat the oxide-coated filament, 9.0 watts is required to heat the thoriated-tungsten filament, and 45.0 watts is required to heat the tungsten filament. Assume that the oxide-coated filament is perfect, and express the emission of the others in percentage with respect to it.

2. From a tube manual, or experimentally, obtain the required data on a triode and graphically determine μ, r_p, and g_m, and the direct-current plate resistance.

3. The grid voltage–plate current curve for a remote cutoff tube is shown in Fig. 280. This curve is at a given value of plate voltage, and the curve for a different value of plate voltage would be displaced a uniform distance from the one shown. Following the method explained accompanying Fig. 274, write a discussion proving that the tube actually has a variable amplification factor.

4. Referring to Fig. 283, compare the plate-circuit efficiencies of a gas tube and a similar high-vacuum tube at 100 milliamperes output.

5. Referring to Fig. 287, calculate the resistance at five current values between points 2 and 3, and plot a curve showing the relation between current and resistance.

6. Referring to Fig. 290*b*, calculate values and plot curves showing the relations between luminous flux and phototube resistance.

7. Referring to the 90-volt curve of Fig. 292*b*, calculate values, and plot curves showing the relations between luminous flux and phototube resistance.

8. Referring to Figs. 290*a* and 292*a*, calculate the increase in sensitivity due to the presence of the gas. The tubes are identical except that one is a high-vacuum tube and the other contains gas.

9. Referring to Fig. 293, calculate the current at +6 volts and at −6 volts and the rectification ratio.

10. Referring to Fig. 295, calculate the power that the photocell will put out into a 500-ohm resistor at 20, 40, and 60 foot-candles, and calculate the energy output during a 12-hour period.

CHAPTER XV

VACUUM TUBES AS CIRCUIT ELEMENTS

The fundamental electronic principles of vacuum tubes were considered in the preceding chapter. Very little was said, however, about the uses of vacuum tubes or their performance in circuits. Vacuum tubes as circuit elements will be considered in this chapter; that is, the circuit performance of vacuum tubes instead of their electronic principles will be discussed.

Before continuing, it should again be mentioned that this book is directed toward explaining the fundamentals of communication, rather than being a treatment of modern communication itself. For this reason, only the basic circuits will be discussed; usually, if these are clearly understood, modern communication systems are readily understood.

Rectifier Theory.—A simple bridge rectifier circuit using a copper oxide rectifier was considered on page 230. This basic circuit is used in instruments, as was explained, and is also used in modulator and demodulator circuits (page 489).

A **half-wave rectifier** supplying a load is shown in Fig. 296. The tube shown may be either a gas or a high-vacuum tube. When the plate is positive, current flows through the tube and the load. When the plate is negative, no current flows, or rather, it should be said that during the negative half cycle the *net current flow is zero*. In any event, the net current that flows through the load resistor *appears* to be merely a pulsating direct current.

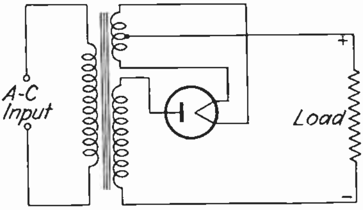


Fig. 296.—A half-wave rectifier connected to a load.

If the reader is experienced in communication, he will find the following treatment of rectifiers different from that usually given. The common explanation is somewhat as follows: A filter composed of choke coils and condensers is placed between the rectifier tube and the load (which is often the vacuum tubes of a radio set) to "smooth out" the current wave. This statement is entirely insufficient to explain filtering action. A far more meaningful explanation will now be given, and the explanation is one that is easily verified.

In Fig. 296 is shown a half-wave rectifier tube connected direct to a pure resistance load. Now suppose that an oscilloscope *having no condenser in the input circuit* is connected across the load resistance. If this is done, the trace of the oscilloscope will appear as in Fig. 297. Since this is the voltage across the resistor, it must be an IR drop, and because the resistance is constant, the shape of the net current wave must be the same as this voltage wave, that is, it consists of pulses of current flowing during the positive half cycle.

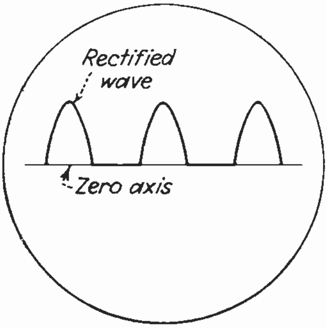


FIG. 297.—The appearance of a rectified half wave on the end of a cathode-ray oscilloscope tube if the oscilloscope circuit contains no input condenser.

Thus far, the explanation has been as usual, but now suppose that a direct-current voltmeter is connected across the resistor. If this is done, the voltmeter will read a value of 0.318 $E_{\max}$, where $E_{\max}$ is the maximum or peak value of the voltage across the load resistor. Now the voltmeter reads only the direct-current component of the complex voltage wave of Fig. 297 which the oscilloscope shows to be across the load. It follows, therefore, that if there is a direct voltage across the load there must be a direct current through the load, and this fact can be demonstrated by placing a direct-current ammeter in series with the load.

Now suppose that a pair of headphones is connected across the load or a part of it if the voltage is too high. When this is done, a complex tone, corresponding to the alternating-current "power hum," will be heard. This is because there are alternating-voltage components across the load resistor, and there can be only alternating-voltage components across a resistor when there are alternating currents through the resistor. As a matter of fact, if some type of tunable receiver is used, it will be found that the complex tone, usually called **alternating-current hum,** is composed of definite frequencies.

A wave analyzer is in a sense a tunable receiver giving the frequency and magnitude of the separate components of a complex voltage connected across it. If a wave analyzer is used to study the complex alternating voltage causing the alternating-current hum in the load of Fig. 296, it will be found that each of the separate components making up the complex rectified wave of Fig. 297 is as shown in Table IX. This is in reality a partial analysis of the rectified wave of Fig. 297; higher harmonics also exist but they are usually negligible. In this table, $E_{\max}$ is the maximum value of the rectified voltage wave but may be $I_{\max}$ if current components are desired.

TABLE IX.—COMPONENTS EXISTING IN THE OUTPUT OF A HALF-WAVE RECTIFIER

(For 60-cycle input)

Component	Frequency, Cycles	Magnitude
First component—direct current.	0	$0.318\ E_{\max}$
Second component—fundamental.	60	$0.500\ E_{\max}$
Third component—second harmonic.	120	$0.212\ E_{\max}$
Fourth component—fourth harmonic. . . .	240	$0.042\ E_{\max}$

Now the reader may be skeptical of all this, but a simple test will demonstrate its truthfulness. In a preceding paragraph it was stated that if an oscilloscope *having no condenser in its input circuit* is connected across the load resistor, then the voltage across the load due to the rectifier tube would be as indicated in Fig. 297. Note that in this figure the voltage trace lies *all* above the center line. But, the usual cathode-ray oscilloscope *does* have a condenser in its input circuit, and if an ordinary oscilloscope is used, then the trace of the voltage wave will be as shown in Fig. 298. Note that the trace is now partly below the center line. The question that immediately comes up is why this is true.

The somewhat startling answer is this: When the ordinary cathode-ray oscilloscope with a condenser in the input is connected across the load resistor of a half-wave rectifier, the trace is due to the alternating-current components (causing alternating-current hum) in the rectified voltage wave. What is seen is *thought to be* a rectified pulsating direct voltage wave, but it really is only the alternating-voltage components, because no direct-current components can pass through the condenser in the input circuit of the oscilloscope.

To summarize: It may be concluded that a rectifier is a **distorter** and that it takes the pure sine-wave voltage and distorts it by changing its wave form to that of Fig. 297. Such a wave contains a direct-current component and various alternating-current components as indicated in Table IX.

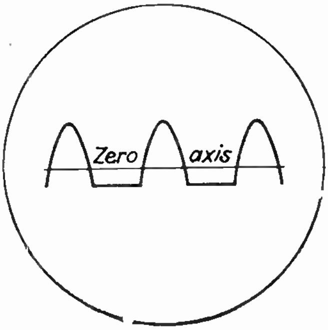


FIG. 298. The appearance of a rectified half wave on the end of a cathode-ray oscilloscope tube *if* the oscilloscope input circuit contains a condenser, as is usually the case. The wave drops down below the zero axis by an amount equal to the direct-current component, because only the alternating components can pass through the input condenser.

The Filter.—Circuits can easily be built which will pass currents of certain frequencies but which will not pass currents of other frequencies. Thus the low-pass filter (page 337) will pass all frequencies from direct current up to the cutoff point.

In Fig. 299 a low-pass filter is connected between the rectifier and the load. The rectifier tube may be considered as a rather complex generator impressing on the filter input a complex voltage composed of direct and alternating components as in Fig. 297. The purpose of the filter is to pass the direct-current component to the load and to prevent any alternating currents from flowing through the load. This it will do if the cutoff frequency is sufficiently low.

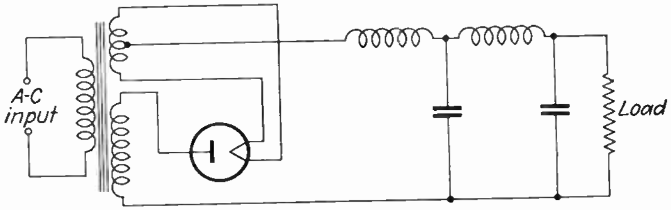


FIG. 299.—A half-wave rectifier connected to a load through a choke-input filter.

In analyzing and designing filters, an alternate and a very helpful explanation is as follows: Consider Fig. 300, which has a single choke coil of negligible resistance and high inductance in series with the load, and a condenser of 16 microfarads across the load. If the inductance of the choke coil is 12 henrys, then the reactance it offers to the first alternating term given in Table IX will be $X_L = 2\pi fL = 6.28 \times 60 \times 12 = 4520$ ohms. This would allow but a small amount of current of the lowest frequency to flow to the condenser and load in parallel (even if they had zero impedance). Even less current would flow for the higher harmonics of Table IX, because their magnitude is less, their frequency is greater, and the reactance of the choke coil will be higher. The condenser assists in the following manner: The reactance of a 16-microfarad condenser to the first component will be $X_C = 1/2\pi fC = 1/(6.28\times60\times0.000016) = 166$ ohms. Now the load resistance is quite high, usually several thousand ohms, and thus what little alternating current does pass through the choke coil will largely be shunted through the condenser. Thus the action of the choke coil and condenser is to keep alternating current from flowing through the load but to freely permit direct current to do so. If more complete suppression of the alternating current is necessary, then an additional coil and condenser can be used as in Fig. 299.

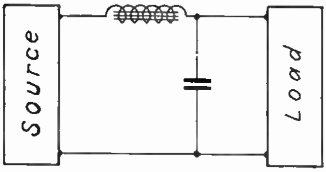


FIG. 300.—Circuit for making an approximate analysis of filter action.

Rectifiers.—A complete form of a half-wave rectifier was shown in Fig. 299. **Full-wave rectifiers** are more commonly used, and the rectifier of Fig. 301 is of this full-wave type. The shape of the voltage wave that such a rectifier impresses on the filter and load is as shown in Fig. 302, and an analysis of this complex wave is given in Table X. Higher harmonics also exist, but they are usually negligible. Note that in the full wave, the magnitude of the direct-

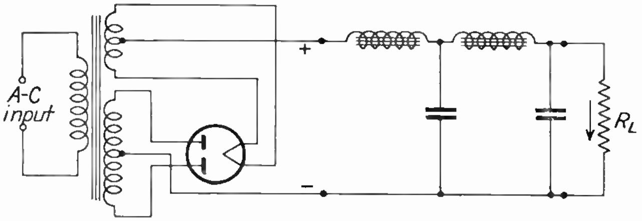


FIG. 301.—A typical full-wave rectifier unit with the inductance in the positive lead. The filtering is less effective if the inductance is placed in the negative lead.

TABLE X.—COMPONENTS EXISTING IN THE OUTPUT OF A FULL-WAVE RECTIFIER

(For 60-cycle input)

Component	Frequency, Cycles	Magnitude
First component—direct current	0	$0.637\ E_{\max}$
Second component—second harmonic	120	$0.425\ E_{\max}$
Third component—fourth harmonic	240	$0.085\ E_{\max}$
Fourth component—sixth harmonic	360	$0.036\ E_{\max}$

current value is twice that for the half wave for the same peak value of $E_{\max}$. This peak value depends on the transformer secondary voltage. Also, note that the first alternating current is of higher frequency and lower magnitude than for the half wave. This means that under comparable conditions a greater direct voltage exists for the full-wave rectifier, and because the alternating components are of higher frequency, filtering is more complete. These are among the reasons that full-wave rectifiers are widely used. This same table may be used to analyze a current $I_{\max}$ into its components.

Either high-vacuum or gas tubes may be used in rectifiers. When gas tubes are used, they should be used only with "choke-input" filters such as shown in Figs. 299 and 301. If gas tubes are used with "condenser-input" filters (having a condenser directly across the tube), an excessive peak current flows which may damage a gas tube.

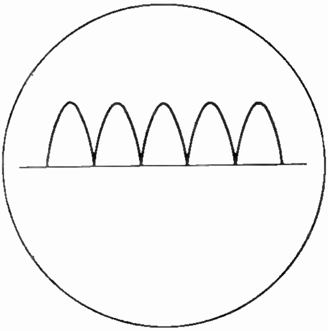


FIG. 302.—The appearance of the output of a full-wave rectifier. This is the way it would appear on an oscilloscope with no condenser in the oscilloscope input circuit. With a condenser in the input, the wave would shift below the zero axis by an amount equal to the direct-current component.

As the reader may know, filters with condenser input are widely used with high-vacuum tubes because a greater direct-voltage output is obtained from such a combination. The reason for it is this: With a condenser input the voltage impressed across the filter is not exactly like the half waves and full waves that have been considered. The voltage wave impressed across a filter with condenser input is so changed in shape that it contains a *larger* direct-voltage component. This means that for Tables IX and X the first term is larger. Thus, a greater direct-voltage output is obtained from rectifiers with condenser-input filters; also the harmonics are smaller.[1]

Rectifier Calculations.—As a practical example of rectifier design, consider the following:

Example.—A full-wave gas-rectifier tube is connected to a transformer with a 750-volt center-tapped secondary. The filter is of the choke-input type using two 12-henry chokes each of 200 ohms direct-current resistance, two 8-microfarad condensers, and a load resistance of 5000 ohms. Calculate the percentage ripple at the load. The circuit is shown in Fig. 303. An approximate solution satisfactory for most purposes is as follows:

Solution.—Step 1. This is a full-wave rectifier, and the *lowest* frequency existing in the voltage impressed on the filter will be, from Table X, 0.425 E_{max} and this will be a 120-cycle component. The reactance offered by a 12-henry choke to a 120-cycle component will be $X_L = 2\pi f L = 6.28 \times 120 \times 12 = 9040$ ohms. The reactance offered by an 8-microfarad condenser to a 120-cycle component will be $X_C = 1/2\pi f C = 1/(6.28 \times 120 \times 8 \times 10^{-6}) = 166$ ohms. For a center-tapped 750-volt secondary, the maximum, or peak, value of the rectified wave will be $E_{\text{max}} = (750/2)\,1.414 = 530$ volts. Assuming no drop in the tube or transformer, the magnitude of the first alternating term or 120-cycle component will be $530 \times 0.425 = 225$ volts peak value.

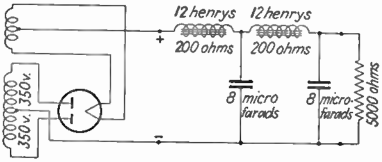


FIG. 303.—Full-wave rectifier circuit showing typical values.

Step 2. Calculate the alternating-current 120-cycle component that will flow through the load. This can be done by ordinary series-parallel circuit theory, but an *approximate* solution is as follows: Assume that the 225 volts, 120 cycles is all impressed directly on the first choke, and assume that the reactance of the first condenser is zero. Then, the effective value of current that will flow down through the condenser will be $I = 225 \times 0.707/9040 = 0.0176$ ampere. Now the reactance of the first condenser is not zero but 166 ohms, and this current flowing through it will cause a voltage drop of $E = 0.0176 \times 166 = 2.92$ volts. Next

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[1] A more complete analysis of this becomes quite involved. For additional information, the reader is referred to M. B. Stout, Analysis of Rectifier Filter Circuits, *Electrical Engineering*, September, 1935.

assume that the second condenser has zero reactance, and *approximately* calculate the current $I_1 = 2.92/9040 = 0.000323$ ampere. The drop across the condenser will be $E = 0.000323 \times 166 = 0.0536$ volt. This will force an alternating current of $I_2 = 0.0536/5000 = 0.0000107$ ampere through the load.

Step 3. Calculate the direct current through the load. The direct component of the voltage is (Table X, page 447) $E = 0.637\ E_{\text{max}} = 0.637 \times 530 = 338$ volts. By neglecting the drop in the transformer and the tube, the direct current flowing through the load will be $I = 338/5400 = 0.0625$ ampere.

Step 4. Calculate the percentage ripple. This will be the alternating current flowing through the load divided by the direct current flowing through the load times 100. Percentage ripple $= 0.0000107 \times 100/0.0625 = 0.017$ per cent.

This solution applies only to the lowest frequency, but the magnitudes of the higher harmonics are less because the reactance offered by the chokes is greater and by the condensers is less, so usually only the first alternating-current component need be considered.

Regulated Power Supplies.—In the approximate calculations made in the preceding sections, the voltage drops due to the direct current passing through the tube and the choke coils were neglected. However, such direct voltage losses do occur because of the voltage required across the tube (Fig. 283) and because a typical filter choke coil has a direct-current resistance of about 200 ohms. This means that the voltage drops and the voltage across the load will change if the current being drawn is varied (see page 83). Also, even if the load is held constant, line-voltage variations of the power circuit will cause the voltage output of the rectifier to fluctuate, and this will vary the voltage across the load.

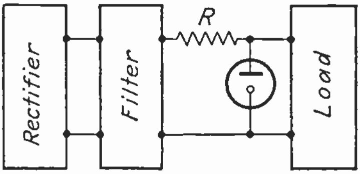

FIG. 304.—How a cold-cathode tube can be used as a voltage regulator.

Such variations in the direct voltage across the load can be prevented or at least minimized by the proper use of a cold-cathode gas diode (page 429) as a regulator. As shown in Fig. 287, page 431, the voltage drop across such a tube is constant over a wide current range. This relation is made use of as in Fig. 304. The regulator tube and a resistor are connected between the filter and the load. If the direct voltage across the tube tends to rise for any reason, the tube draws a larger current. This will increase the voltage drop across R and absorb the voltage rise.

Another way of looking at the operation of the circuit of Fig. 304 is this: So long as the current limits of Fig. 287 are not exceeded, then the voltage shown in this figure cannot be exceeded. Thus, the tube holds the voltage across the load constant within the limits indicated. For regulating higher voltages, two or more tubes may be connected in series.

Amplifiers.—Before amplifiers are discussed, it should be made clear that many types exist. Classified as to the results achieved, there are two basic types of amplifiers, **voltage amplifiers** and **power amplifiers.** It is absolutely necessary to distinguish clearly between these two types because the design methods are entirely different.

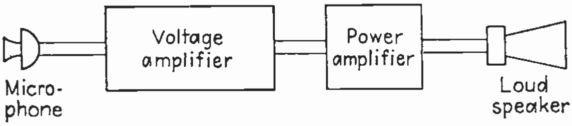

FIG. 305.—The essential parts of a sound-amplifying or public-address system.

To explain the difference between these two basic types of amplifiers, consider Fig. 305 which represents the essential parts of a sound-amplifying or public-address system. The microphone picks up the sound waves and puts out a very feeble voltage. This is impressed on the voltage amplifier where the *voltage* is increased in the successive stages until it is quite large. This amplified voltage is then impressed on the power amplifier, and this portion of the amplifier delivers the *power* that operates the loud-speaker and radiates acoustic power into the auditorium. In the design of amplifiers, it must clearly be kept in mind whether a voltage amplifier or a power amplifier is desired.

Before further considering amplifiers, it should be mentioned again that in this book the electrical fundamentals instead of the applications are considered. Extensive construction and design data are found in vacuum-tube manuals, handbooks, and technical journals, and it is not the purpose of this book to duplicate these sources of information.

Vacuum Tubes as Linear Amplifiers.—The purpose of a vacuum tube as an amplifier is to take a weak alternating signal and strengthen it. Before this can be explained, however, it is necessary to show how the alternating signal progresses through the tube. This can be done with the aid of Fig. 306.

Here is shown the dynamic[1] grid voltage–plate current curve of a triode. The voltage E_C is the direct voltage applied to the grid to bias the tube to the proper operating point, which for a distortionless amplifier is about the center of dynamic curve. Then, the alternating signal voltage to be amplified is impressed on the grid, and it causes the grid voltage to vary around the bias voltage as shown. When the grid voltage goes more positive, the plate cur-

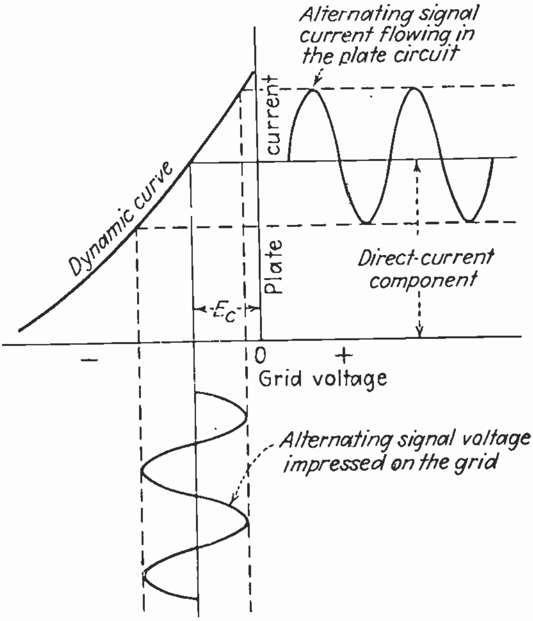

FIG. 306.—The alternating signal voltage on the grid causes an alternating-current flow in the plate circuit.

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[1] The curves of Fig. 274, p. 414, are called static curves and show the relation between grid voltage and plate current when there is no resistance in the plate circuit. But as will be shown in this section a tube must be operated with a load resistor in the plate circuit to obtain a signal-voltage drop to impress on the grid of the next tube. Tubes operated with load resistance operate on their dynamic curves. Further consideration of the dynamic curves will be deferred until a later section because they are most important when studying power amplifiers.

rent increases above the value it has when only the bias voltage is applied. Also, when the grid goes more negative, the plate current decreases below the value with no signal applied.

From this it is seen that two voltages, a direct biasing voltage and the alternating signal voltage to be amplified, are impressed on the grid. Likewise, in the plate circuit two currents flow, a direct-current component and an alternating-current component.

Speech and music are composed of alternating voltage and current components, and thus vacuum-tube amplifiers are designed to pass these alternating components. From the alternating-current standpoint a vacuum tube as a linear or nondistorting amplifier is equivalent to a generator as shown in Fig. 307.

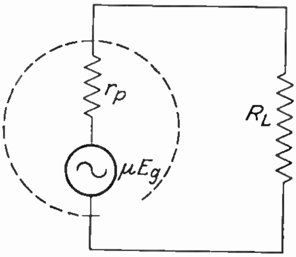

FIG. 307.—The vacuum tube as a linear or nondistorting amplifier acts as if it were a generator of voltage μE_g and internal resistance r_p, where μ is the amplification factor of the tube, E_g is the alternating signal voltage impressed on the grid, r_p is the plate resistance and R_L is the load resistance connected in the plate circuit. The term E_s is often used to designate the alternating signal voltage impressed on the grid. A minus sign is sometimes placed before the value μE_g to designate that the plate-voltage variations are 180 degrees out-of-phase with the grid signal voltage.

The grid of the tube is μ times as effective in controlling plate-current flow as is the plate. Then, an alternating signal voltage of E_g volts when impressed on the grid acts as a voltage of μE_g volts existing in the plate circuit and in series with the alternating plate resistance r_p. This principle is of very great importance.

With the circuit of Fig. 307, the current flow through the load resistor R_L is

$$I = \frac{\mu E_g}{r_p + R_L}. \tag{96}$$

The voltage existing across the load resistor is

$$E_{R_L} = IR_L = \frac{\mu E_g R_L}{r_p + R_L}. \tag{97}$$

The voltage amplification $A_V = E_{R_L}/E_g$ represents the voltage increase or gain per stage of amplification and is

$$A_V = \frac{E_{R_L}}{E_g} = \frac{\mu R_L}{r_p + R_L}. \tag{98}$$

The power delivered to the load resistor R_L is

$$P = I^2R_L = \frac{(\mu E_g)^2R_L}{(r_p + R_L)^2}. \tag{99}$$

These basic equations are useful in the design of most amplifiers.

Resistance-coupled Voltage Amplifiers Using Triodes.—As was shown in the preceding section a vacuum tube acts like a generator which takes the alternating signal voltage E_g impressed on the grid and increases it to a voltage μE_g in the plate circuit. This voltage acts to force an alternating current through the plate circuit composed of the plate resistance of the tube and the load resistance.

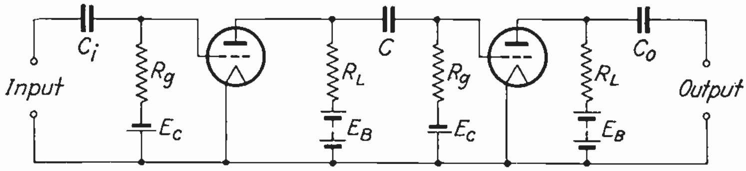

FIG. 308.—Two-stage resistance-coupled voltage amplifier using triodes.

The alternating signal voltage drop across the plate resistance is the voltage available to impress on the grid of the next tube for further amplification. This circuit forms the nucleus of the resistance-coupled amplifier of Fig. 308.

By starting at the input end, the functioning of the various components are as follows: The condenser C_i prevents any direct voltages which might be in a source connected to the input circuit from reaching the grid of the first tube. Such a direct voltage might incorrectly bias the grid of the tube. The condenser C_i allows the alternating signal voltage to force an alternating signal current through the grid resistor R_g, down through the battery, and back to the input terminal. The alternating signal voltage drop across R_g is impressed for amplification on the grid of the tube. The value R_g should have depends on the internal impedance of the source and usually should be very much larger than the source so that it will not draw excessive current from the source. The grid is essentially an open circuit for all but high frequencies and passes negligible direct current. The battery E_C biases the tube through resistor R_g to the proper operating point. Negligible direct current flows through R_g.

Between the first and second tubes is a network composed of R_L, C, and R_g. The resistor R_L is the load resistor discussed in the preceding section. The condenser C is necessary to isolate the plate-supply battery from the grid of the second tube. It is absolutely necessary that this condenser have a very high insulation resistance. The grid resistor R_g functions essentially as explained in the preceding paragraph.

Now it is apparent that the *total* load circuit connected to the first tube of Fig. 308 is *not* merely the resistor R_L but is the network R_L, C, and R_g. Thus the equivalent circuit of the tube is no longer that of Fig. 307 but becomes that of Fig. 309. Also, a capacitance C_g has been added. This represents the input capacitance[1] of the second tube plus the capacitance of connecting wires.

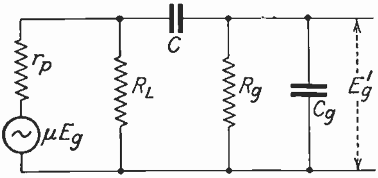

FIG. 309.—Equivalent circuit for one stage of Fig. 308. The voltage E_g would be impressed on the grid of the second tube for further amplification.

Design of Resistance-coupled Audio Voltage Amplifiers Using Triodes.—The design procedure for the resistance-coupled audio amplifier such as Fig. 308 is as follows: At an intermediate frequency of 1000 cycles, the circuits of Fig. 308 and 309 are equivalent to that of Fig. 310*a*. The frequency is so low that negligible current flows through the capacitance C_g, and its effects are negligible so it has been omitted. Also, the capacitance of condenser C is so large that the voltage drop across it is negligible, and it has been omitted. For a triode resistance-coupled audio voltage amplifier using tubes having a μ of about 10 and a plate resistance of about 10,000 ohms, R_L is usually about 30,000 to 50,000 ohms. The grid resistor is usually several hundred thousand ohms. For an approximate solution its effect may be neglected, or a better method is to compute the equivalent resistance of R_L and R_g in parallel and

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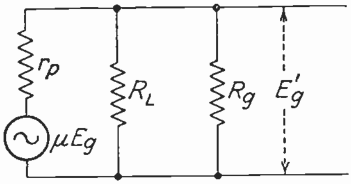
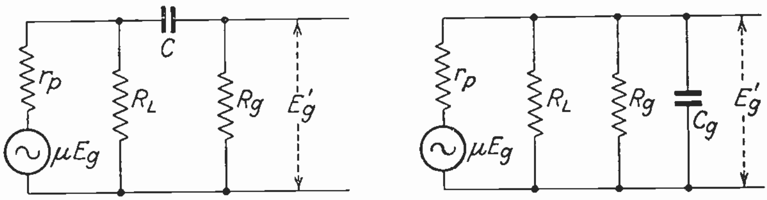
[1] The grid-input capacitance of a triode operating as an amplifier is greater than the interelectrode capacitance as given in tube manuals. This grid-input capacitance is given by the relation $C_g = C_{gf} + C_{gp}\,(1 + A_V)$, where C_{gf} is the interelectrode capacitance between grid and filament, C_{gp} is the capacitance between grid and plate, and A_V is the voltage amplification given by Eq. (98). The derivation of this equation can be found in more advanced books. For a practical case, this value of grid-input capacitance may be about 70 micromicrofarads for a small triode.

use this equivalent resistance for R_L in Eq. (98) to compute the voltage gain. This can be converted to decibels (page 334) and will be substantially the maximum voltage gain the amplifier can have at any frequency.

At the very low audio frequencies such as 50 cycles per second, the drop in voltage across condenser C becomes appreciable, and the equivalent circuit becomes Fig. 310*b*. Now it is the IR_g voltage drop across the grid resistor which is impressed across the grid of the second tube. This same current I must flow through the condenser C, and there will be a drop across it as well. These two voltages are 90 degrees out of phase, however. At the frequency at which $X_C = 1/2\pi fC = R_g$, approximately 70 per cent of the maximum voltage output as calculated in the preceding paragraph will be impressed across the grid of the second tube. Drawing a simple vector diagram will prove this to be true.



(a)-Intermediate frequency



(b)-Low frequency

(c)-High frequency

FIG. 310.—Equivalent circuits at various frequencies for a resistance-coupled voltage amplifier.

At the high audio frequencies such as 10,000 cycles per second, the drop in voltage across C becomes negligible, but the current down through condenser C_g is appreciable. The equivalent circuit is now Fig. 310*c*. It is possible to write design equations giving the voltage output at high frequencies. This is done by writing the expression for the current through the condenser C_g and then multiplying this current by the reactance of C_g. For ordinary values of R_L, C, R_g, and C_g, the resistance-coupled amplifier works up to 10,000 or 15,000 cycles without much drop in voltage amplification. For the practical design of such amplifiers, the data given in tube manuals should be followed. Such data give both amplification and response.

In the design information previously considered, the effects of the circuit on the direct current flowing in the plate circuit and the direct voltage reaching the plate are neglected. The direct current flowing through resistor R_L causes a direct voltage drop of I_pR_L. If the plate current is 2.5 milliamperes and R_L is 40,000 ohms, then the direct voltage drop will be $E = I_pR_L = 0.0025 \times 40{,}000 = 100$ volts. If 90 volts is desired on the plate, then the total voltage applied by the battery or rectifier must be 190 volts. Although of little importance in amplifiers operated with rectifiers, it is of great importance in amplifiers operated with batteries that this resistor R_L be kept low. Anyway, there is little additional voltage amplification to be gained by making R_L greater than about four or five times r_p as can be easily proved.

To summarize: The theory of resistance-coupled amplifiers has been considered in much detail because of its great importance. If it is clearly understood, then the other amplifiers can be easily explained. In the design of a resistance-coupled audio voltage amplifier, a reliable estimate of the gain and frequency response can be obtained by considering the circuit at intermediate, low, and high frequencies.

Resistance-coupled Audio Voltage Amplifiers Using Pentodes.—Resistance-coupled amplifiers using pentodes or screen-grid tubes are similar. Pentodes are often employed because of the greater amplification obtainable. A suppressor-grid pentode is shown connected in the circuit of Fig. 311.

The general features of this circuit are the same as those of the resistance-coupled amplifier circuit using triodes. Of course a source of positive voltage for the screen grid must be provided. The grid-input capacitance discussed in the footnote on page 454 is greatly reduced because of the shielding action of the suppressor grid and the screen grid.

For a typical suppressor-grid pentode, $\mu = 1200$ and $r_p = 1{,}000{,}000$ ohms. Various values of R_L and R_g may be used in an amplifier employing this tube; among these are $R_L = 250{,}000$

ohms, R_g = 500,000 ohms, and C = 0.008 microfarad. It will be noted that these resistances are very much greater than used with a triode. The reason that such high resistances must be used is this: The plate resistance of a typical suppressor-grid voltage-amplifying pentode is very high, often 1,000,000 ohms or over. If a high value of plate load resistance is not used, then most of the signal voltage generated (μE_g) will be lost inside the tube and very little will reach the external circuit for amplification in the next tube. The question immediately presents itself: Why not use a load resistor of four or five times the plate resistance? The answer is that the direct current for the plate would have to flow through this resistance, and the high voltage drop would be objectionable.

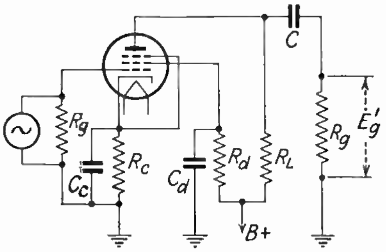


FIG. 311.—A suppressor-grid pentode connected in a resistance-coupled amplifier circuit. For a typical circuit, $B+$ is 90 volts, R_L is 250,000 ohms, R_g is 500,000 ohms, R_d is 1,180,000 ohms, R_C is 2600 ohms, C_d = 0.03 microfarad, C_C = 3.2 microfarads, C = 0.005 microfarad, and the voltage gain is approximately 85.

A typical circuit such as used in practice is shown in Fig. 311, and the values of the resistors and condensers used are shown below this figure. The important electrical principles involved will now be discussed. First, it is noted that no C battery is employed. The control grid is made negative with respect to the cathode by the direct voltage drop $E = IR_C$ in resistor R_C caused by the combined plate current and screen-grid current which must return to the cathode through resistor R_C. But, there is also an alternating-current component flowing back to the cathode, and if this flows through R_C, a voltage drop will occur which will impress a signal back on the control grid and this will interfere with the signal impressed on the grid to be amplified. Hence the resistor R_C is by-passed with a large capacitor which offers a path of extremely low impedance even to the lowest frequencies to be amplified. Since the impedance approaches zero, the voltage drop is negligible and no interference results. The condenser C_d across the resistor in the screen-grid circuit serves as a similar by-pass and prevents the screen-grid voltage from fluctuating which would interfere with operation. The direct voltage that reaches the screen grid is the voltage +B (plate supply voltage) minus the drop in resistor R_d

caused by the screen-grid current. Also, the direct voltage on the plate is $+B - I_pR_L$. The gain and frequency response can be determined much as for a triode. The gain of a pentode amplifier such as Fig. 311 can be calculated accurately enough for most results by a very simple formula derived as follows: Referring to Eq. (98), if R_L is small compared with r_p, then the equation becomes $A_V = \mu R_L/r_p = g_m R_L$, because $g_m = \mu/r_p$. But note, this holds *only* when R_L is small compared with r_p, and the incorrect use of this equation may cause very bad errors.

To summarize: Because of the very high internal resistance of the suppressor-grid pentode, high values of coupling resistance must be used. Although for purely practical reasons the circuit cannot be designed to utilize the high amplification factor of the tube effectively, voltage amplifications can be obtained with a pentode which exceed those obtained with other commercially available tubes.

Transformer-coupled Audio Voltage Amplifiers Using Triodes.—Interstage transformers were formerly quite extensively used to couple together vacuum tube audio voltage amplifiers using triodes. They are not used with screen-grid tubes or suppressor-grid voltage-amplifying pentodes because the plate resistances of such tubes are very high, and it is very difficult to build good audio transformers to work satisfactorily in circuits having high impedance; for one reason, the transformer would need very high inductances which would require many turns of wire, and this would result in excessive distributed capacitance.

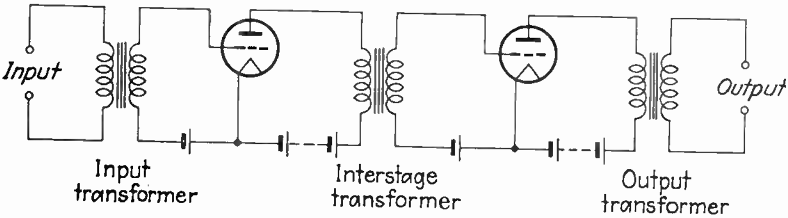


FIG. 312.—A simple two-stage transformer-coupled class *A* audio-frequency voltage amplifier.

A transformer-coupled audio-frequency voltage amplifier using triodes is shown in Fig. 312. A complete analysis of this circuit is quite difficult, because to study the performance at all audio frequencies many factors such as primary and secondary inductance, stray capacitance, and mutual inductances must be considered at each frequency band. However, an approximate solution can be readily made as follows: The alternating-signal voltage impressed on the grid appears as an amplified voltage μE_g in the plate circuit. The primary impedance of the transformer is high at intermediate frequencies and substantially all this amplified voltage will appear across the primary. Such interstage transformers usually have a step-up ratio from primary to secondary of about 1 to 3, and this ratio will be called N. Then, the voltage appearing on the secondary of the transformer will be $E_s = \mu E_g N$, and the voltage gain per stage will be

$$A_V = \frac{E_s}{E_g} = \mu N. \tag{100}$$

From a purely practical viewpoint there is little involved in the practical design of such an amplifier except to compute the gain from the equation just given, and then buy transformers that will have good frequency response and cause little nonlinear distortion in the circuits in which they are to be used. Remember that a "skimpy" transformer may show a good frequency response and little nonlinear distortion if tested with a very small signal but may be quite poor at the signal level at which it is to be used.

Transformer-coupled Radio-frequency Voltage Amplifiers Using Pentodes.—Such a circuit is shown in Fig. 313. Because of excessive hysteresis and eddy-current losses (page 164), it is common practice to use air coupling instead of magnetic cores for these transformers. Special magnetic cores are sometimes used, however, and with good results. In either instance the magnetic coupling be-

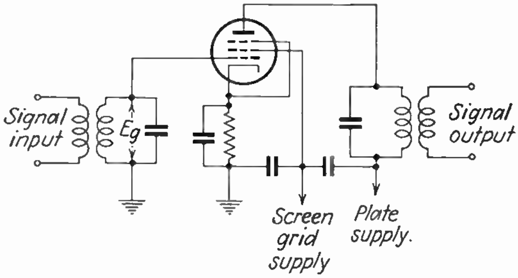


FIG. 313.—Simplified circuit of a pentode in a voltage-amplifying transformer-coupled radio-frequency circuit.

tween the primary and secondary is very low, and coupled-circuit theory (page 321) must be used to calculate the gain per stage rather than the simple method of the preceding section.

As shown in Fig. 313, the input and output circuits of the amplifier are both tuned to antiresonance for the same frequency. Then, the load in the plate circuit is a pure resistance for this frequency, because at antiresonance the impedance of a tuned parallel circuit is a value of pure resistance. The radio-frequency signal voltage E_g impressed on the grid appears as an amplified voltage μE_g in the plate circuit, and this voltage must force a radio-frequency signal around through a circuit composed of the plate resistance r_p and the equivalent resistance offered by the antiresonant primary with the coupled secondary. Because the design equations are somewhat involved and not easy to apply, design data on this circuit will not be presented. It usually turns out that the load resistance offered is greatly below the plate resistance of the pentode and a voltage gain per stage of only a few per cent of the amplification factor of the tube is obtained.

Screen-grid tubes and pentodes are used in radio-frequency amplifiers not only because of their higher voltage amplification, but also because of their stability. The screen grid and also the suppressor grid shield the control grid from the plate, and the tendency for feedback is greatly lessened if not entirely eliminated. This will be further treated on page 475.

Power Amplifiers.—The preceding discussions have been confined to voltage amplifiers as distinguished from power amplifiers. Voltage amplifiers are usually employed to increase the weak signal voltage up to the point where the voltage is sufficiently strong to drive a power tube. Then, the power tube or power-output tube furnishes the power to the loud-speaker or antenna.

In selecting a tube for a power amplifier, there is no object in using a large tube and driving it to only part of its full output capacity; rather, it is better to select as small a power tube as can be used without excessive distortion. No longer is the objective that of obtaining maximum voltage gain. In designing a power amplifier, it is necessary to consider the dynamic curve of Fig. 314.

The static curves shown in Fig. 274*a*, page 414, and Fig. 314 can be used to predict the operation of a triode *with no resistance* in the plate circuit, but tubes are operated *with* resistance in the plate circuit, and hence dynamic and not static curves must be used. In voltage amplifiers the tubes usually are not driven to the extreme degree that they are in power amplifiers. It is usually safe to assume that negligible distortion occurs in voltage amplifiers, and the

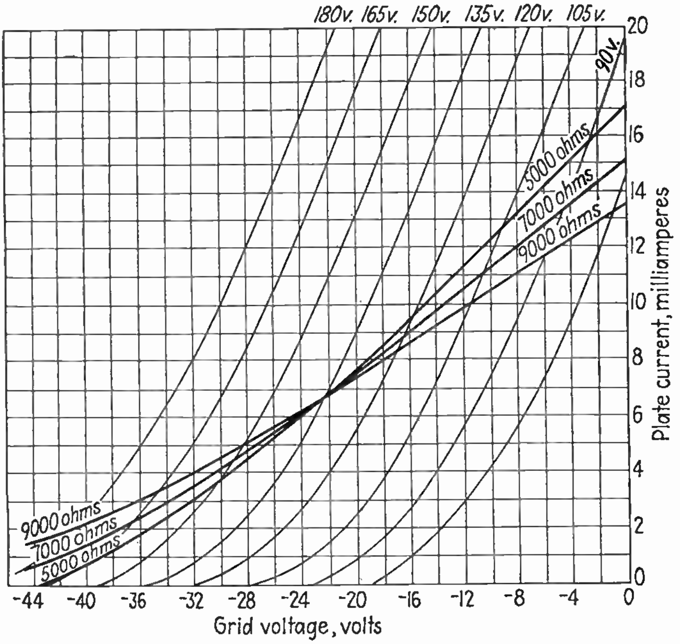


FIG. 314.—If a tube is operated without resistance in the plate circuit, then the output plate current flows in accordance with the static curves. If, however, there is a load resistance in the plate circuit, then the tube operates not along a static curve, but instead it operates along a dynamic curve. When a load is in the plate circuit, then when the current increases the voltage on the plate becomes less, and when the current decreases the voltage on the plate becomes greater than under static conditions. In other words, the plate-current variations follow the dynamic curves of this figure. Three curves are shown, corresponding to three values of plate-load resistance for a small power triode. The curve at 7000 ohms is the proper curve for maximum undistorted power output. To find these dynamic curves proceed as follows: The point of operation assumed is -22.5 volts on the grid, and $+135$ volts on the plate. To find where the 5000-ohm curve crosses the 105-volt curve, $135 - 105 = 30$ volts, $30/5000 = 0.006$ ampere. Add this value to the current at the point of operation; thus, $6 + 6.7 = 12.7$ milliamperes. Other points are found in a similar manner. The dynamic curves can also be taken experimentally.

dynamic curves need not be considered. The power-amplifier tubes are usually driven as hard as possible, and thus the dynamic curves must be considered. From the dynamic curve the distortion can be predicted.

The dynamic curve shows the relation between grid voltage and plate current with a load resistor in the plate circuit. The relations of Fig. 314 have been redrawn in Fig. 315, omitting the static curves. The dynamic curve is found to be slightly curved upward. Thus, if a signal voltage is impressed on the grid as indicated, the *positive* half cycles of the plate current will be larger than they should be; and the *negative* half cycles will be smaller than they should be. If the dynamic curve were a straight line as shown by the dotted line of Fig. 315, then there would be negligible current distortion as shown by the dotted sine wave. The difference between the actual wave and the pure sine wave is largely caused by the second harmonic created in the tube because the dynamic curve is not straight.

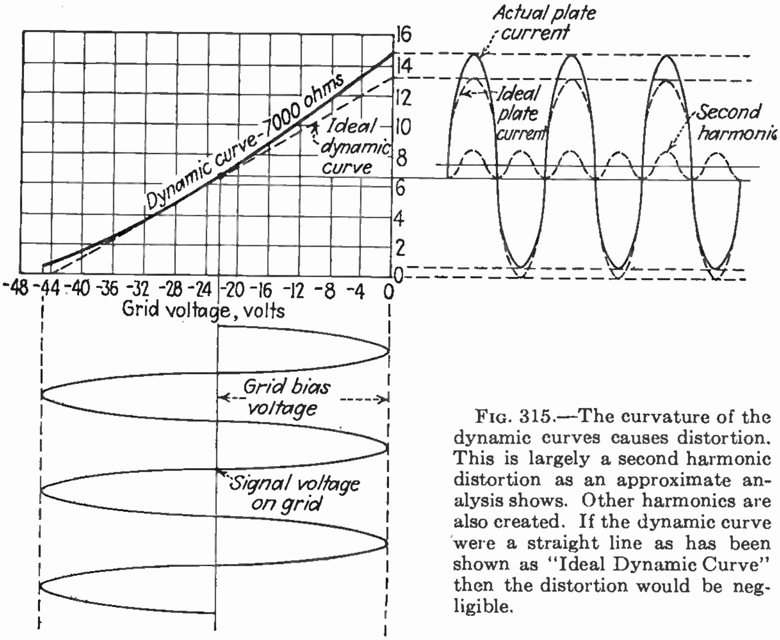


FIG. 315.—The curvature of the dynamic curves causes distortion. This is largely a second harmonic distortion as an approximate analysis shows. Other harmonics are also created. If the dynamic curve were a straight line as has been shown as "Ideal Dynamic Curve" then the distortion would be negligible.

Now there are two conflicting principles involved in the design of a power tube. (1) A power tube is, like any vacuum tube, equivalent to a generator of internal impedance μE_g and internal plate resistance r_p. From page 78 it is seen that the maximum power is obtained from a source into a load when the internal resistance of the source equals the resistance of the load. Thus, *from this stand-*

point alone, a load resistance *equal* to the plate resistance of the tube would be selected. (2) From Fig. 314 it is seen that the greater the load resistance, the straighter will be the dynamic curve, and from Fig. 315 the less will be the distortion. It follows, therefore, that in designing a power amplifier these two opposing effects must both be considered and the best practical solution found.

Of course, some distortion can be tolerated, and it has been found in practice that about 5 per cent distortion will be unnoticed. That is, if the peak value of the second harmonic of Fig. 315 is less than 5 per cent of the fundamental, the signal quality will be good. It works out that if the resistance of the load is about twice that of the plate resistance of the tube, then the distortion will be less than 5 per cent. From Fig. 45, page 81, it is seen that this gives about the same output as for the matched condition of internal resistance equal to load resistance. Thus, in the design of power amplifiers the load resistance is made about equal to twice the plate resistance, and this fact is a good one to keep in mind.

Any experimenter or designer of vacuum-tube circuits will be certain to have available tube manuals published by the tube manufacturers, and in these will be found described graphical methods and equations by which vacuum-tube circuits can be calculated. These usually will give a load resistance equal to a little less than twice the plate resistance, but this latter relation is close enough for most purposes.

To summarize: The design of a power-amplifier stage involves a study of the dynamic curves of a vacuum tube. The problem is to obtain the maximum amount of power from the tube without excessive distortion. This dictates that the load resistance should be about twice the alternating-current plate resistance of the tube.

The Output Transformer.—Referring to Fig. 315, it will be observed that the peak value of the alternating signal voltage impressed on the grid just equals the grid-bias voltage; in other words, the grid is driven to zero and to twice the bias value but is *not* driven positive. Since the grid is not driven positive, negligible current flows in the grid circuit, and negligible input power is taken by the grid circuit. The driving voltage is impressed on the grid from a preceding *voltage* amplifier which may be either resistance or transformer coupled into the grid of the power tube.

It is usually desired that the power tube drive some device such as a loud-speaker. Now if an impedance bridge such as Fig. 237, page 359, is used to measure the input impedance to a loud-speaker of the dynamic type, it will be found that the input impedance of the voice coil equals about 10 ohms resistance and is slightly inductive. Obviously, a power tube cannot work directly into such a device, because the plate resistance of a small triode power tube is, say, 2000 ohms, but the resistance of the loud-speaker voice coil to which it must supply signal power is only about 10 ohms. Also, remember the relations for maximum undistorted power output of the preceding section—the load resistance should be about twice the plate resistance, or about 4000 ohms.

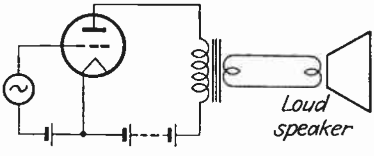


FIG. 316.—By the use of an inequality transformer a 10-ohm voice coil may be made to appear as a high-resistance load in the plate circuit of a vacuum tube.

Now it was shown on page 325 that a transformer could be regarded as an *impedance* transformer and that it would change impedance in proportion to the square root of the turns ratio. Thus, if a transformer having a turns ratio of $N_p/N_s = \sqrt{4000/10} = 20$ is placed between the tube and the loud-speaker voice coil, the input impedance at the primary of the transformer will *appear* to be 4000 ohms, and the power tube will therefore be working into 4000 ohms which is the correct value for maximum undistorted power output. These relations are shown in Fig. 316.

Before concluding this section, a word should be said about power pentodes which are often used in radio sets as the power-output tube to drive the loud-speaker. The same basic principles apply but with this exception: Because of the peculiar shapes of the static curves, the dynamic curves for high load resistances are far from being straight lines, and if a load resistance of twice the plate resistance is used, excessive distortion will result. For this reason a load resistance very much lower than the plate resistance is used. For a typical suppressor-grid *power* pentode having a plate resistance of about 60,000 ohms, a load resistance as low as about 7000 ohms must be used. Of course with this large mismatch much of the available signal power is lost within the tube, but even so, the power pentode has a high amplification factor (about 150 for the tube being considered which is about 40 times that of a comparable triode) and therefore a large amount of output power can be obtained with a low grid signal voltage.

Push-pull Power Amplifiers.—A push-pull circuit is shown in Fig. 317. As is evident, with no signal applied at the input, the two tubes have the same grid voltage E_c and the same plate voltage E_B. If each tube is biased as in Fig. 315, then each tube with no signal applied will pass the same amount of direct current. But, these two direct currents will flow through each half of the primary of the output transformer in *opposite* directions, and hence the magnetomotive forces they produce will oppose, and no magnetic flux will be produced in the transformer core for the no-signal condition. This permits economies in transformer design because there are negligible magnetic saturation effects (page 361) due to the direct plate currents, and the core may be smaller.

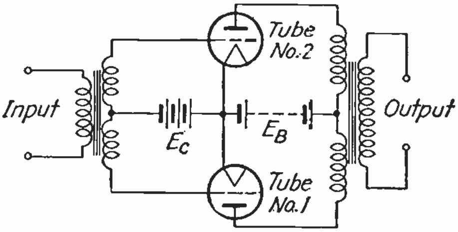


FIG. 317.—Circuit for two output triodes in push-pull.

When an alternating signal voltage is applied to the input circuit, one grid is driven positive and the other grid is driven an equal amount negative. One tube will accordingly take *more* plate current, and the other tube will take *less* current. On the next half cycle of the applied signal voltage the action will be reversed, and the first tube will draw less current and the second tube will draw more current. As a result of this action, each tube passes an alternating current of opposite sign in opposite directions in each half of the primary. The result of these two "opposites" is a positive action, and the magnetic effects produced by each tube in each primary do not cancel as for the direct-current components but add and pass signal power to the output. These relations are shown in Fig. 318.

A complete analysis [1] of this method shows that the proper load resistance for the two tubes is less than that for one tube alone and that the second harmonic component, which causes most of the dis-

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[1] Thompson, B. J., Graphical Determination of Performance of Push-pull Audio Amplifiers, *Proceedings of the Institute of Radio Engineers*, April, 1933

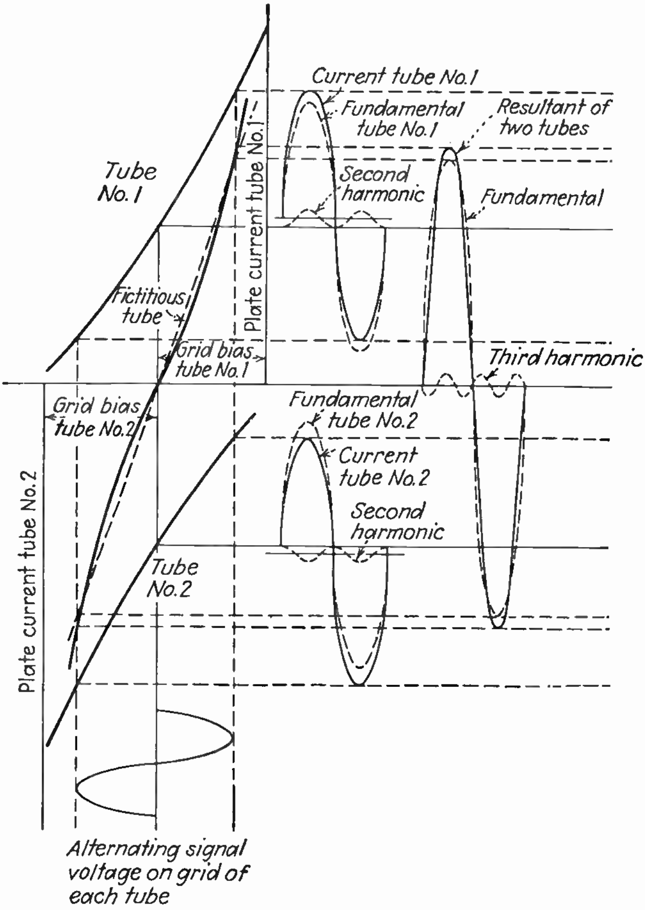

FIG. 318.—Diagram for analyzing the output of two triodes in push-pull. The actual plate current of each tube is the same and is as shown. Because of the fact that the second harmonics are oppositely related as shown, they cancel out in the primary of the output transformer. The fundamental components do not cancel out, however, but their effects add as shown by the resultant current of the two tubes. This is equivalent to assuming that the resultant current comes from one fictitious tube having the characteristics shown. Note that odd harmonics such as the third do not cancel, and hence appear in the output of the push-pull amplifier.

tortion in single-tube-triode power amplifiers, is *balanced out* [1] in the power-output transformer. In the design of the triode power amplifier, a load resistance equal to about twice the plate resistance was selected (at some sacrifice of power) to reduce the second-harmonic distortion. With tubes in push-pull, however, this distortion is canceled out in the transformer. Thus, the load can be matched to the tubes and more than twice the output of a single tube can be obtained by two tubes in push-pull.

Class A, B, and C Power Amplifiers.—The power-output tubes that have been considered up to the present were biased to about one-half the cutoff value and plate current flowed through the tubes at all times. When an alternating voltage was impressed on such a tube, more and less plate current flowed, an alternating-current component being thus given in the plate circuit. Such tubes, biased to about half the cutoff value, are said to be operated in **class A.**

For certain purposes to be explained later, tubes are operated with their grids biased to cutoff, or special tubes have been designed which almost cutoff with no bias. Such tubes pass plate current only on the *positive* half cycle of the signal voltage applied to the grid, because the negative half cycle of the applied signal voltage merely drives the grid more negative, and it is already at the plate cutoff value. Such tubes are said to be operated in **class B.**

Power tubes in radio transmitters are sometimes operated biased far beyond plate-current cutoff, and a large alternating signal voltage is impressed. Then, plate current flows only during the peak of the positive half cycle, and such tubes are said to be operated in **class C.**

From this discussion, and from the explanations that will follow, it will be seen that the plate current in class B and class C power amplifiers flows in pulses, similar to those shown in Fig. 297 for the half-wave rectifier. Now it was shown on page 444 that the current in a half-wave rectifier contained a direct-current component, a fundamental alternating-current component, and various alternating-current harmonic components. An analysis of the plate current flowing in a class B or a class C amplifier will definitely show that this "pulse" current is made up in this same way.

If the reader is a practical radio worker, he may have been taught

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[1] This can be shown graphically, and the analysis will be found explained under Fig. 318.

to regard the plate current in class B and class C amplifiers differently, just as was the case for a rectifier. But, only by accepting new viewpoints and new ideas can mental progress be made, and it is sincerely hoped that the reader will accept this analytical way of looking at current pulses as a step leading to a better understanding of the equipment with which he daily works.

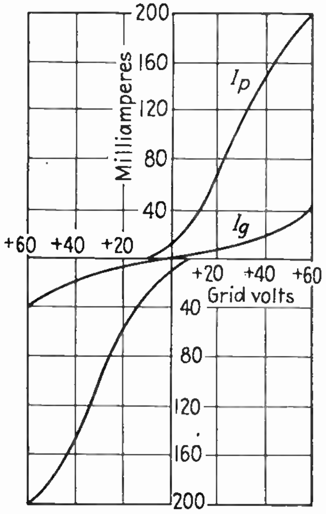

FIG. 319.—Dynamic curves for two tubes operated as class *B* triodes. Plate current I_p and grid current I_g for plate-to-plate loads per tube of 1450 ohms.

The Class B Power Amplifier.—These amplifiers are used for two purposes, as audio power amplifiers and as radio power amplifiers. They must, therefore, be considered separately.

Class B Audio Power Amplifiers.—These amplifiers use a circuit similar to Fig. 317. The tubes are usually of the special type which operate essentially at cutoff with no grid bias voltage. Their dynamic characteristics are shown in Fig. 319. Their operation differs from the class A push-pull power amplifier as follows: In the class A push-pull circuit, plate current flows at all times, and when the grids are driven alternately positive and negative by the applied signal voltage, the tubes pass more or less plate current and cause alternating-current components to flow in the primary of the output transformer. Now in the class B push-pull circuit, an alternating signal voltage is again impressed on the grid, but because of the peculiar characteristics each tube passes plate current *only* when its grid is driven positive.

A complete analysis of the class B tubes can be made much as in Fig. 318 for class A push-pull. Because each tube passes only positive half cycles, each tube acts as a half-wave rectifier tube (page 443). When studying rectifiers it was shown that the output from a half-wave rectifier contained a fundamental and even harmonics. The even harmonics will be *canceled out* in the primary of the transformer, but the fundamental components *combine* in the primary passing the alternating signal through the tube.

As is shown in Fig. 319, the grids of the tubes will draw current when they are driven positive. This means that the stage preceding the class B push-pull stage must supply power as it drives the grids of the push-pull tubes and, hence, must be designed to have sufficient power capacity. Transformers for push-pull class B power amplifiers must be specially designed and used with the proper tubes. Because class B tubes operate essentially at cutoff, but little plate current flows unless they have a signal voltage applied, although in class A tubes plate current flows at all times. Thus the class B power amplifier is more efficient than a comparable class A power amplifier but, on a comparable basis, the output quality of the class A tube is better. Well-designed class B amplifiers have good characteristics, however.

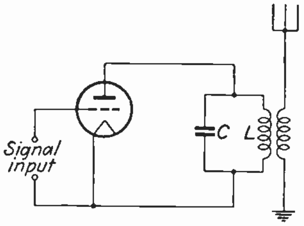

FIG. 320.—In the class *B* radio-frequency power amplifier the antiresonant tuned parallel circuit *L–C* suppresses the harmonics. Substantially, the only alternating voltage existing across this tuned circuit will be voltages of frequencies close to the antiresonant frequency.

Class B Radio Power Amplifiers.—As was previously shown, the class B tube is a rectifier, and although a component having the same frequency of the alternating signal voltage exists in the output, even harmonics also exist. In the class B audio amplifier, two tubes are used in push-pull, so that the harmonics will cancel out in the primary of the output transformer.

Although two class B tubes are often operated in push-pull in radio transmitters, this need not be done. The harmonics can be suppressed by a parallel antiresonant tuned circuit placed in the load circuit of a single class B tube as in Fig. 320. Since the parallel circuit plays such an important role, the explanations on page 316 should be reviewed.

If a parallel circuit is in antiresonance or tuned to the signal frequency it is desired to pass, then the input impedance of this circuit will be a large value of pure resistance to this frequency but will be a low, largely reactive, value of impedance to all other frequencies. This means that if a tuned parallel circuit is placed in the plate circuit of the class B radio-frequency power amplifier, the load offered the tube will be resistive at the signal frequency to which the circuit is tuned. The tube will then supply power to this load into which is reflected the resistance component of the device to which power is to be supplied, for example, to a transmitting antenna (page 394). To other frequencies, the load is reactive and is low in value; therefore, *for these frequencies*, little power will flow into the circuit and but little voltage will exist across the parallel circuit.

A class B radio-frequency power amplifier can be used to amplify a modulated radio-frequency signal (page 484). The carrier and the two side bands lie sufficiently close together that the action explained in the preceding paragraph holds for these three components.

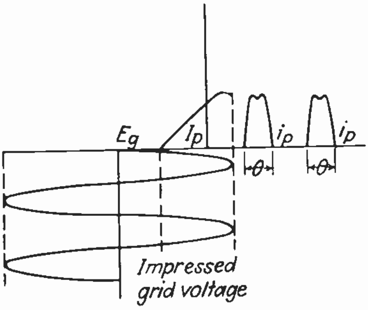

FIG. 321.—Grid voltage and plate current in class *C* amplification. The plate current may show "dips" if the tube is driven with large grid voltage as indicated. This is because the grid current which flows may become great enough actually to reduce the plate current as shown. The angle θ is called the angle of flow. It is less than 180 degrees for class *C* operation.

Class C Radio Power Amplifiers.—These amplifiers operate in a manner similar to the class B amplifier just considered, but the grid is biased further beyond cutoff. Plate current flows for only a small part of the positive half cycle as Fig. 321 shows. Since the angle of flow θ is so small, the distortion is very great. This amplifier cannot be used for voice frequencies or for a voice-modulated carrier (page 484). The class C amplifier is used for amplifying single frequency waves (unmodulated carrier, page 484) and for radiotelegraph amplification. This wave is also similar to the rectified half wave of Fig. 297, and it also contains a direct-current component, a fundamental, and various harmonics. Because the direct-current component is very low, the amplifier is quite efficient. Just as for the class B amplifier, the tuned circuit in the plate lead offers a high resistance to the fundamental and takes power at this frequency. If it is well designed it takes negligible power for the harmonics. Thus, in Fig. 320 only power at the fundamental would be radiated. Because grid current flows in the class C amplifier, power must be supplied to the grid circuit.

The "Tank" Circuit.—The radio literature contains extensive references to "tank" circuits, "circulating" currents, "flywheel" action, and "wave traps." From the standpoint of accepted electrical fundamentals, such terms are very confusing and lead to a vague understanding of the performance of radio circuits.[1]

Now the so-called tank circuit is the tuned parallel antiresonant load circuit used in class B and class C radio-frequency amplifiers. The performance of antiresonant circuits has been considered at several points in this book, in particular on page 316. Often this antiresonant circuit is coupled directly to a radio-transmitting antenna as shown in Fig. 320. Then, the antiresonant circuit follows the impedance-transformation theory considered on page 333 and couples the antenna to the tube so that the tube supplies power to the antenna.

It is often stated in discussing tank circuits that power flows into the tank circuit in "spurts" corresponding to the shape of the current pulses of Fig. 321 and that the "circulating currents" in the tank circuit "fill in the missing half cycle," etc. Such statements are meaningless. When an experienced chemist looks at a test tube, he does not merely see the milky liquid therein; instead, he "sees" various amounts of several elements all blended to give the milky liquid. When one experienced in communication looks with an oscilloscope at pulses of current flowing in a circuit, he sees not only the resultant image produced on the end of the tube, but he mentally visualizes the direct-current and various alternating-current components within.

When such a wave, composed of many components, flows through a load, the only components that can deliver power to the load are those components to which the load offers pure resistance. Power cannot be delivered to a pure reactance. Thus, in the class B or class C amplifier which is tuned to the fundamental frequency component, appreciable power only flows at this frequency. An oscilloscope connected across the tuned circuit will show a sine-wave voltage to exist. This is because the tuned circuit offers a high impedance only to the fundamental frequency to which it is tuned, and thus the voltage across it is due almost

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[1] Unfortunately, in the early days of radio it was very difficult to make accurate measurements, and this led to many erroneous explanations of radio phenomena. In fact, the opinion became prevalent that "the frequency was so high that radio circuits did not follow the usual electrical laws." Of course, this is incorrect; it may be necessary to consider such things as stray capacitance which is negligible at low frequencies, and it may be necessary to generalize certain electrical laws, but radio circuits do follow the same basic electrical principles as circuits at other frequencies.

entirely to the fundamental component. Thus if the signal voltage impressed on the grid is a pure sine wave (the fundamental), then the voltage across the tuned tank circuit can be only a pure sine wave of this same frequency. In **frequency multipliers** the tuned plate circuit load (tank) may be tuned to some frequency other than the fundamental; then, the frequency of the voltage across it will be due to the harmonic of the complex current of Fig. 321 to which it is tuned.

Oscillators.—In discussing the various types of amplifiers it has been shown that a small voltage applied to the grid of the tube will be increased to a large voltage in the output circuit. Also, it has been shown that with either a negligible amount of power input to the grid circuit or a small amount of input power a large amount of power will be put into the output circuit.

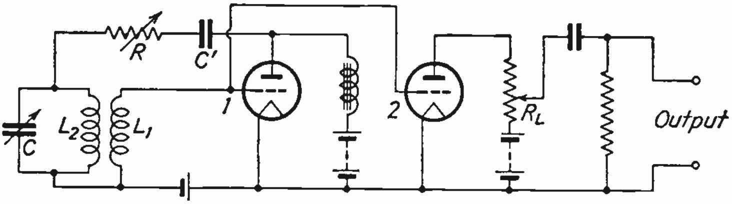

Fig. 322.—Circuit of an audio-frequency oscillator. The output terminals may be connected to additional voltage and power-amplifying stages. Or, the output circuit may be removed and a transformer substituted for R_L.

Since more power is put out than put into an amplifier, and since more power is available in the output circuit than is needed in the input circuit, a signal can be coupled back from the output to the input circuit, and the tube will oscillate and can be used as a source of alternating-current power.

Audio Oscillators.—Audio oscillators are usually operated as class A amplifiers with their grids driven slightly positive, as will be later explained. A typical audio oscillator is shown in Fig. 322. Tube 1 is the oscillator tube and tube 2 is a power amplifier, also operated essentially in class A for audio purposes. Note that the two grids are tied together and, therefore, their voltages must vary in unison. The plate of the oscillator tube (tube 1) is connected back to the cathode through a circuit composed of R, C', and the antiresonant circuit C, L_2. Condenser C' serves to isolate the plate and grid potentials. The variable resistor R controls the current that flows to the tuned circuit L_2, C.

As has been shown for the class B and C amplifiers, the only appreciable voltage existing across a tuned parallel circuit such as C, L_2 will be at the frequency to which this circuit is in antiresonance and at which a high impedance is offered to the current flow. By coupled-circuit action some of this voltage will appear across L_2. Thus, the tuned circuit determines the frequency of the signal introduced on the grid of the oscillating tube (and also the amplifying tube) and in this way controls the frequency of oscillation. After the circuit is first energized, the oscillations gradually build up until the peak of the positive half cycle induced on the grid drives the grid slightly positive. Then the grid draws a small amount of power which means that the grid input becomes resistive. This resistance by coupled-circuit theory will be introduced into the tuned circuit C, L_2 and this will make the tuning less "sharp" (page 317), it will offer a lower impedance, and the voltage across it will not increase. In this way, the magnitude of the oscillations become stabilized.

The second tube is usually a class A amplifier which may have a resistance load in its plate circuit or may be more efficiently coupled to its external load through an output transformer. This tube also serves as a "buffer" to isolate the external load from tube 1. If this is not done, the external load may affect the frequency of oscillation. For audio amplifiers a 1 to 1 ratio iron-cored transformer works well at L_1, L_2; C' may be about 0.1 microfarad and should have negligible leakage. The variable resistor R may be variable from 0 to 100,000 ohms. The value of C should be such as to give antiresonance at the audio frequency desired. This is approximately $f = 1/(2\pi\sqrt{LC})$ and may be more correctly calculated in accordance with parallel-circuit theory. For oscillations to build up, the signal impressed on the grid circuit must have the proper phase relation. Thus, if the circuit does not oscillate, the first thing to do is to reverse the connections of coil L_1.

Radio Oscillators.—As has been explained, audio oscillators are amplifiers in which part of the output signal is fed back from the plate to the grid circuit. Likewise, radio-frequency oscillators are usually class C amplifiers in which part of the output signal is fed back to the grid circuit. A typical circuit is shown in Fig. 323.

The principles of operation of this circuit are as have been explained. Output power can be obtained by inductively coupling a coil with L_2, and from coupled-circuit theory this will be equivalent to inserting a resistance R_2 in series with coil L_2. This helps fix the frequency at which L_2, C offers the maximum impedance and at which the circuit will oscillate.

To obtain high efficiency, the grid is driven with a large alternating signal voltage induced in L_1. It is usually desired that the oscillator operate as a class C amplifier to obtain high efficiency. But, a class C amplifier is biased beyond cutoff, and if a fixed bias of this value is used, no plate current would flow and the tube would never start to oscillate. For this reason the tube is self-biased by a resistor R_g and a condenser C_g placed in the grid circuit. Then, when the circuit is energized, oscillations will build up and the grid will be driven positive. Of course an oscilloscope would show that the grid current would flow in spurts at the peaks of the positive half cycles. On the basis of previous explanations, it is now understood that a direct-current component would exist in such a current, and this direct-current component flowing through the resistor R_g will bias the tube. Of course the alternating voltage fed back by L_1 must reach the grid of the tube. The condenser C_g offers a low-impedance path to the flow of alternating-current components.

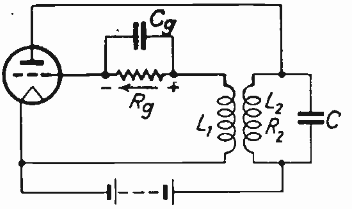

FIG. 323.—A class C oscillator, self-biased by the grid resistor–grid condenser combination.

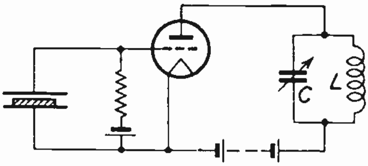

FIG. 324.—A quartz crystal oscillator.

Quartz crystals are used to control the frequency of oscillation of radio-frequency oscillators. These quartz crystals are placed in the grid circuit as shown in Fig. 324. Quartz has this property: If a voltage is impressed across it, its dimensions will change, and conversely, if its dimensions are changed, a voltage will exist between opposite faces. The quartz crystal of the proper dimensions is placed in a holder as indicated. Voltage is fed back through the interelectrode capacitance (to be explained in the next section), and the quartz crystal actually vibrates mechanically. This vibration generates a voltage which is impressed on the grid of the tube where the signal is amplified, and voltage is again fed back.

Neutralization.—As mentioned in the preceding paragraph, a signal voltage may be fed back from the plate to the grid circuit through the interelectrode capacitance in the tube. The capacitance causing this is the grid-plate capacitance C_{gp} of Fig. 325. Although it exists, of course, within the tube it is drawn, for convenience, connected as shown by the dotted lines.

Now this capacitance between the grid and plate will feed back a signal much as in Fig. 322, and the circuit will oscillate. To be specific, alternating current will flow from the plate through C_{gp} and down through the equivalent impedance of the antiresonant circuit L_1C_1 to the cathode. The voltage drop across L_1C_1 due to this current flow is impressed between the grid and cathode where it is amplified, and this action will cause the oscillations to be sustained.

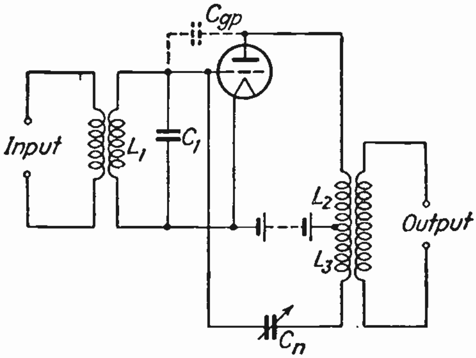

FIG. 325.—Circuit for preventing oscillations due to feedback through the grid-plate capacitance. Coils L_2 and L_3 are closely coupled.

This tendency to oscillate can be prevented in a simple manner as follows: Referring to Fig. 325, coils L_2 and L_3 are inductively coupled, and during the part of the cycle when the upper end of L_2 is positive (this is also the potential of the plate) the lower end of L_3 will be negative. In other words, the voltages across L_2 and L_3 are 180 degrees out of phase. Then, the interelectrode capacitance C_{gp} will feed back a current, and this will cause a voltage drop across circuit L_1C_1; but the condenser C_n will feed back a current of the opposite instantaneous polarity, and it will cause a voltage drop across circuit L_1C_1 which is 180 degrees out of phase. These two voltages will then neutralize each other, and there will be no net signal voltage between grid and cathode to cause oscillations.

Of course the desired input signal will be amplified. Other neutralizing circuits are possible, one in particular being of fundamental interest. If a coil with a condenser in series is placed directly between the plate and grid, these two may be adjusted until they are in antiresonance with C_{gp} at the frequency at which the circuit is to amplify. Then, the impedance between plate and grid will be very high, and negligible current will flow. If the current from plate to tuned circuit L_1C_1 is low, then little voltage due to feedback will exist across them, and oscillations will not occur.

Ordinarily, neutralization is only necessary with triodes. In fact, this feedback between plate and control grid was probably the factor that led to the development of the screen-grid tube. In these, the screen grid shields the control grid from the plate and the capacitance is made so low that the feedback is ordinarily not sufficient to cause oscillations. Because of the added presence of the suppressor grid in the pentode, shielding is even better and the tendency to oscillate less. Thus screen-grid tubes and pentodes are usually used in radio circuits whenever it is possible. For radio transmitters, however, triodes must be used where the power level is high because the power-handling capacity of screen-grid tubes and pentodes is limited.

Negative Feedback.—In considering audio oscillators on page 472, it was explained that the oscillations were sustained by the feedback of a signal from the plate to the grid. In discussing crystal oscillators on page 474, it was mentioned that feedback through the grid-plate capacitance kept the crystal oscillating. Also, in the preceding section it was shown that feedback through the grid-plate capacitance would cause oscillations. It will be noted that in each of these instances the signal fed back caused the output to *increase*, and this is called **positive feedback.**

Although it would seem foolish ever to reduce the gain of an amplifier by reversing the feedback and *decreasing* the signal applied, this is done in **negative feedback amplifiers,** a schematic design of which is shown in Fig. 326. The alternating signal input voltage E_g is impressed on an amplifier of total gain A_V. Without feedback, the output voltage would of course be A_VE_g, but with feedback the output voltage is reduced to a value E_0. The variable negative feedback circuit impresses a certain amount βE_0 of this output voltage back on the input so that the net signal input is *reduced* and is $(E_g - \beta E_0)$. Then, the magnitude of the output voltage is $E_0 = (E_g - \beta E_0)A_V$. Now output voltage E_0 divided by input voltage E_g equals the voltage gain of any amplifier, and for the negative feedback amplifier this ratio can be determined from the equation just written for the output voltage as follows: $E_0 = (E_g - \beta E_0)A_V$, $E_0 = E_gA_V - \beta E_0A_V$, $E_0 + \beta E_0A_V = E_gA_V$, $E_0(1 + \beta A_V) = E_gA_V$, and

$$\text{voltage gain with negative feedback} = \frac{E_0}{E_g} = \frac{A_V}{1 + \beta A_V}. \quad (101)$$

With large amounts of negative feedback, the term βA_V becomes large and the gain is reduced, approaching the value $1/\beta$. Then, the gain of the amplifier is largely independent of A_V. According to Eq. (98), the voltage gain of an amplifier *without* negative feedback is $A_V = \mu R_L/(r_p + R_L)$. Both μ and r_p depend on operating conditions (Fig. 275, page 415) such as the plate-supply voltage. But, the gain of the negative feedback amplifier is independent of these factors and is very stable, because its gain will not vary with plate-supply variations. The negative-feedback amplifier also has low distortion because the portion of the output signal fed back into the grid circuit counteracts any tendency the amplifier has to cause distortion. In a sense, it oppositely predistorts the input signal so it comes out undistorted.

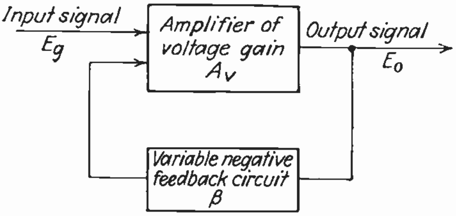

FIG. 326.—Schematic circuit of the controlled negative feedback amplifier.

The Principle of Modulation.—As it pertains to communication, to modulate means *to regulate, vary, or inflect.* By the process of **modulation,** a basic carrier wave is caused to vary in accordance with a modulating wave such as the voice that it is desired to transmit.

The preceding is essentially the popular version of modulation, but the process of modulation is far more interesting than this indicates. In fact, modulation can be effected and the carrier component in the output can be entirely suppressed. But before continuing with modulation, let the necessity for modulation be considered.

The process of modulation is one of the basic steps involved in radio communication and in carrier-current telephony over wire lines. *Modulation is merely a method of frequency-band translation.* Modulation is a system by which the audio frequencies of the voice can be raised to a band of higher frequencies. **Demodulation** is a process of the same fundamental nature as modulation; but this term is applied to the process of lowering the band of frequencies back to the audible range.

Modulation is necessary in radio communication for at least two reasons: (1) it is difficult to radiate very low frequencies into space, and (2) even if it were possible to do so, then all radio stations would be on the same frequency and no selectivity would be possible. Modulation is necessary in carrier telephony to obtain a large number of talking channels over one pair of wires. At the input end, the various voice channels are raised to different high-frequency bands and they are then transmitted to the distant end, selected by band-pass filters, and then demodulated and returned to their original audio frequencies.

In discussing modulation it is possible to describe the process by drawing diagrams, etc. These diagrams are not all that is needed, however. Again, merely looking at a milky solution does not tell the chemist much; he must analyze the solution into its component parts. Merely looking at diagrams and wave shapes does not tell the communication man much about the fundamental process of modulation. To understand modulation, the process and the results of the process must be analyzed.

As mentioned elsewhere in this book, a theorem is a principle so well established that it is accepted as a law. Ohm's law is a theorem. It is believed that the process of modulation is so fundamental, so universal, and so readily demonstrated that it should be stated and accepted as a theorem. Although modulation can be accomplished by other means, the one most widely used is modulation by **nonlinear impedances.** A nonlinear impedance is an impedance which varies with the magnitude, sign, or frequency of an alternating current or voltage impressed across it. The **basic modulation theorem** is as follows: *Whenever signal voltages of two different frequencies are simultaneously impressed on a circuit having a nonlinear impedance, sum and difference fre-*

quencies will be created. This is the true meaning of modulation, and a careful consideration of it will promote a better understanding of radio and carrier telephony. On this basis, Fig. 327 is presented as giving a clear picture of a radio system.

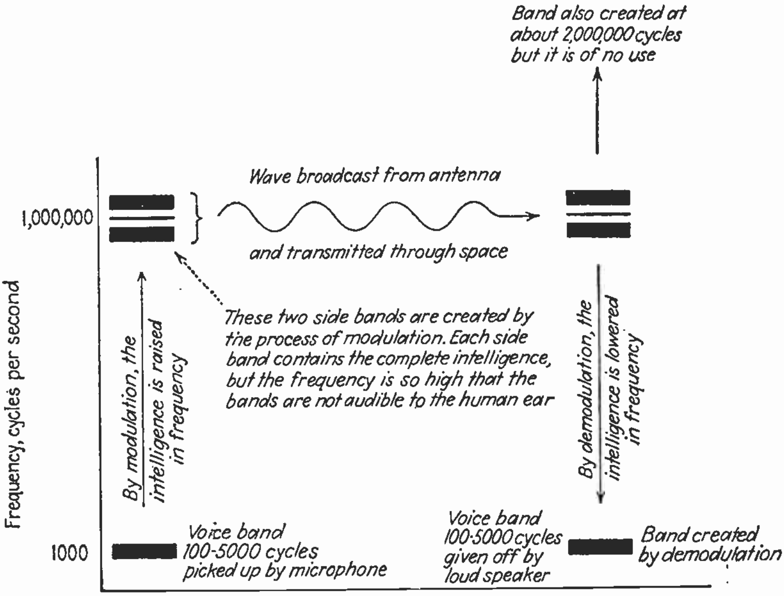

FIG. 327.—The essentials of a typical radio system. The voice band must be raised to a high frequency for efficient radiation and for selectivity. The voice band of 100–5000 cycles and a carrier frequency of 1,000,000 cycles are simultaneously impressed on a nonlinear device such as a vacuum-tube circuit which is biased for operation on the curved portion of the E_g-I_p curve. As stated in the modulation theorem, this will result in the creation of two side bands, the upper side band covering a range of from 1,000,100 to 1,005.000 cycles, and the lower side band covering a range of from 999,900 to 995,000 cycles. In ordinary radio broadcast these two side bands and the carrier are then radiated, the signal therefore being composed of three components. After reception by the radio receiver, these three components are again demodulated or distorted in a nonlinear impedance. Again, sum and difference frequencies are created, and the difference frequency is the desired audible signal.

Systems of Modulation.—Many systems of modulation are possible, but in a book of this type the systems considered must be limited. Fundamentally, modulation can occur in a vacuum tube in either the grid circuit (because of the nonlinear relation between E_g and I_g, page 412) or in the plate circuit (because of the nonlinear relation between E_g and I_p, page 414).

Plate-circuit Modulation by Grid Injection.—A circuit for accomplishing this is shown in Fig. 328. The tube is biased as indicated in Fig. 329 so that distortion results as indicated. According to Fig. 328, the audio signal, represented by a single frequency A (instead of a band, see Fig. 327), and the carrier signal, as shown by B, are simultaneously injected (or impressed) into the grid circuit. The instantaneous grid voltage will vary around the bias value as indicated by curve C. This will cause a distorted plate current to flow as shown by D.

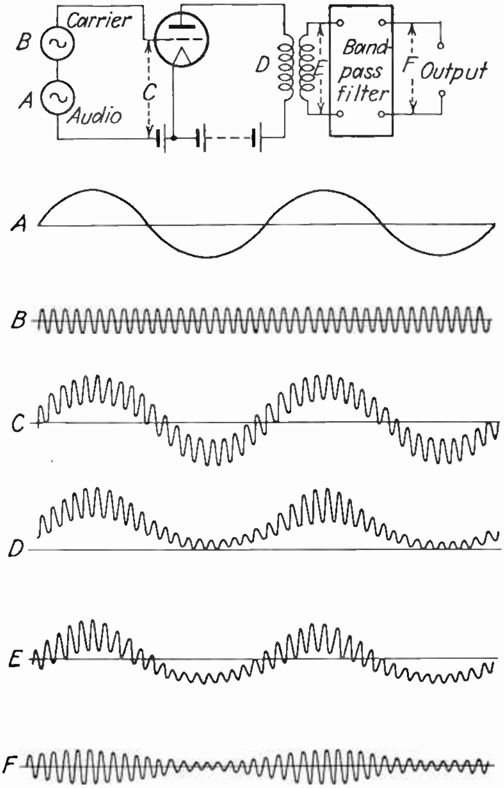

Fig. 328.—Circuit for studying plate-circuit square-law modulation by grid injection, and analysis of currents and voltages in various parts of the circuit.

In previous discussions of modulation and in considering Fig. 327, it was stated that in the process of modulation two side bands were created. This is true, but in the output of a modulator other frequencies and a direct-current component also exist; among these is the voice frequency. In radio transmitters this component is removed because it will not pass through radio amplifiers or out over the antenna. In carrier-current telephone systems of modulation (represented by Fig. 328), the direct-current component is removed by the transformer giving *E*, and the audio-frequency component is removed by the filter; thus the familiar modulated wave *F* is produced. This wave contains the carrier and the two side bands. Although this discussion was for carrier telephony, the principles apply to radio as well, as will now be explained.

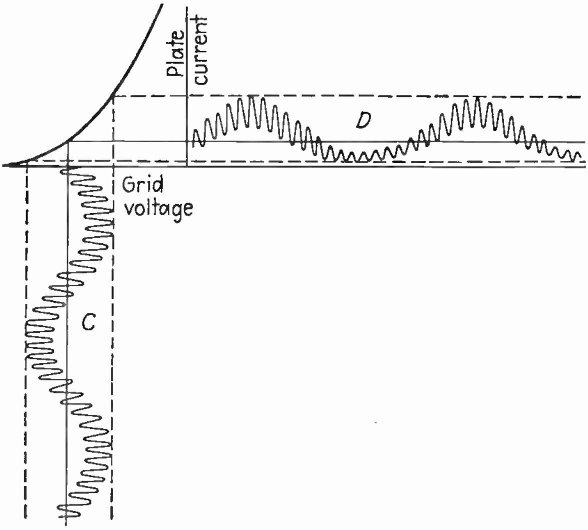

FIG. 329.—Circuit for illustrating how distortion is produced. These correspond to curves *C* and *D* of Fig. 328.

For radio modulation by grid injection, the tube of Fig. 330 is biased to operate as a class C amplifier. Thus, with only the carrier voltage impressed but no voice impressed, as indicated by the first part of the curve, the tube is merely a class C amplifier. When the voice is impressed as indicated by the presence of E_a, the signal voltage on the grid is now the combination of the two waves. This will cause the distorted output current shown at the right. This current wave is distorted, because it does not have the same wave shape as the impressed voltage signal. This current will contain, along with other components, the carrier and the two side bands. The antiresonant plate circuit will offer a high impedance to the carrier and the side bands but not to the other frequencies. This means that the only appreciable voltage existing across

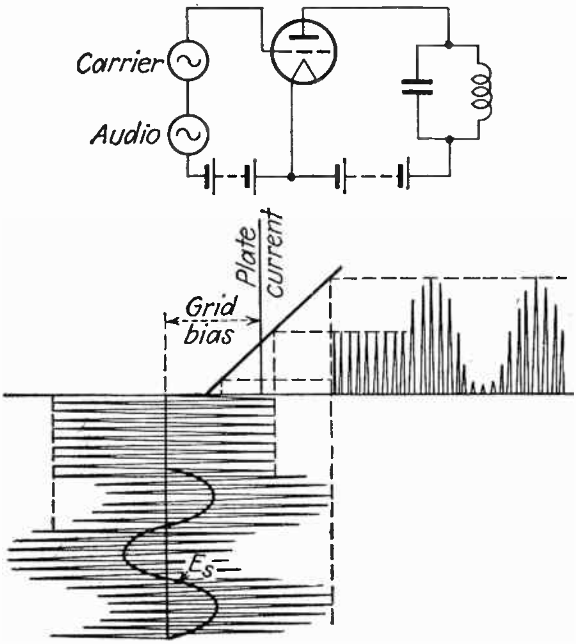

FIG. 330.—System of plate-circuit modulation by grid injection as used in radio circuits. The tube is biased to operate as a class C amplifier. The action here shown is both before and after the speech signal E_s is impressed.

the tuned circuit will be due to the carrier and the side bands, and this signal will be further amplified or impressed on the antenna.

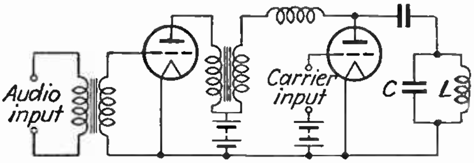

FIG. 331.—The essential parts of a radio transmitter, showing plate-circuit modulation by plate injection.

This system is sometimes called grid modulation, but this is incorrect. The modulation occurs in the plate circuit because of the nonlinear relation between the total resultant grid voltage E_g and the plate current I_p.

Plate-circuit Modulation by Plate Injection.—This system is shown in Fig. 331 and is the essential part of a radio transmitter.

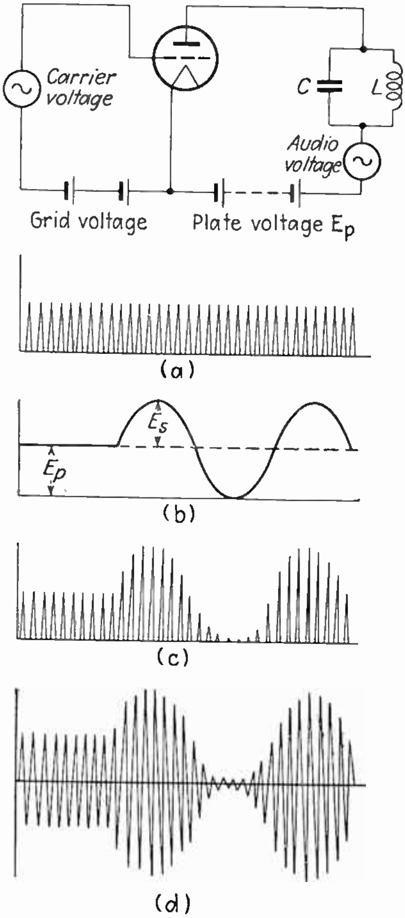

FIG. 332.—Equivalent circuit for a modulated class C amplifier, and curves for analyzing its operation. (*a*) Plate current with only carrier impressed. (*b*) Direct voltage E_p and speech signal voltage E_s. (Often called E_g.) (*c*) Resulting plate current when carrier voltage and signal voltage are simultaneously impressed. (*d*) Voltage drop across the tuned circuit composed of the inductance and the condenser. This so-called modulated wave contains the carrier and the side bands.

The audio input is impressed on an audio amplifier (often called a modulator, but incorrectly because this leads to believing that it produces the distortion, when it merely amplifies the audio signal), and the amplified voice signal is impressed in the plate circuit of a tube biased to operate as a class C amplifier. The carrier frequency from a crystal oscillator is amplified and impressed on the grid of the class C tube.

The circuit of Fig. 331 is equivalent to that of Fig. 332 which also shows the shapes of the various components and is sufficient to explain the action. This comes under the classification of impressing two signals on a nonlinear impedance in this way: The instantaneous plate voltage varies at an audio rate and this varies the plate resistance of the tube in accordance with the audio signal.

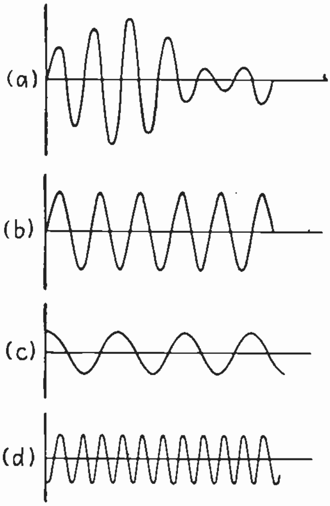

FIG. 333.—The so-called amplitude modulated wave of (*a*) can be analyzed into a carrier wave (*b*) and lower (*c*) and upper (*d*) side bands. Conversely, (*b*), (*c*), and (*d*) can be shown graphically to combine and give the modulated wave (*a*). The above illustrations are not exactly to scale. Note in particular that the amplitude of the carrier component (*b*) does not vary for a modulated wave.

Nature of a Modulated Carrier.—It has been mentioned that the modulated carrier, contrary to the usual viewpoint, is not just a wave of single frequency varying in amplitude, but is really a wave composed of three components, the carrier, the upper side band, and the lower side band. The intelligence is in the side bands, and they alone vary in accordance with the speech or music to be transmitted. *The carrier frequency component is constant in amplitude both before and after modulation.* If instead of the voice a single frequency such as a constant 1000-cycle note is used to modulate the transmitter, then the side bands will be constant-amplitude single-frequency terms as shown in Fig. 333. If a carrier of 1,000,000 cycles is modulated by a frequency of 1000 cycles, the upper side band will be a single frequency of 1,001,000 cycles, and the lower side band will be a single frequency of 999,000 cycles.

If the carrier is completely modulated, the minimum values of the resulting amplitude-modulated wave will just strike the zero axis. This is called 100 per cent modulation and is shown graphically in Fig. 334. For 100 per cent modulation, the amplitude of each side band is one-half that of the carrier, and each side band contains $16\frac{2}{3}$ per cent of the total power in the completely modulated wave. This fact leads to the correct but startling conclusion that since no intelligence is carried by carrier-frequency component, $66\frac{2}{3}$ per cent of the power output of a broadcast station is wasted, and this is true, with reservations, because the carrier is necessary at the radio receiver to cause demodulation (Fig. 327). In fact, in most systems of commercial radio and carrier telephony (as distinguished from broadcasting), only one side band is transmitted, because each side band has the complete intelligence within itself.

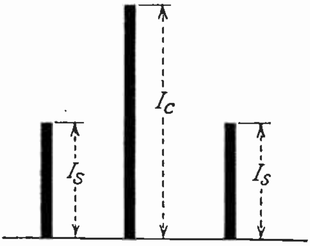

FIG. 334.—Relative heights of the carrier component I_c and the two side bands I_s for 100 per cent modulation.

Demodulation.—As was explained on page 478, demodulation is the same fundamental process as modulation. Two (or more) frequencies are simultaneously impressed on a nonlinear circuit, and sum and difference frequencies are created. The process of demodulation merely lowers the intelligence from high radio frequencies to a low audible band. This is also called **detection,** particularly in radio parlance.

Plate-circuit Demodulation.—The basic principle of this system of demodulation or detection is the same as that considered on page 480, and for this reason it will be considered first. This system is extensively used in carrier-current telephony, and the basic circuit is shown in Fig. 335. The tube is biased to work on the nonlinear or curved portion of the E_g-I_p (grid voltage–plate current) curve.

This system illustrates demodulation where only the lower side band has been transmitted and is shown and introduced at A. The original carrier-frequency component was suppressed at the sending end and has been generated in a local oscillator at the receiving end and is shown and introduced at B. The two waves acting together cause the grid voltage to vary as at C which is distorted because of the nonlinear E_g-I_p relations. This distortion creates sum and difference frequencies. The difference frequency is the voice.[1] The transformer removes the direct-current component, and the filter removes all undesired high-frequency components; thus only the desired audio-frequency signal is left.

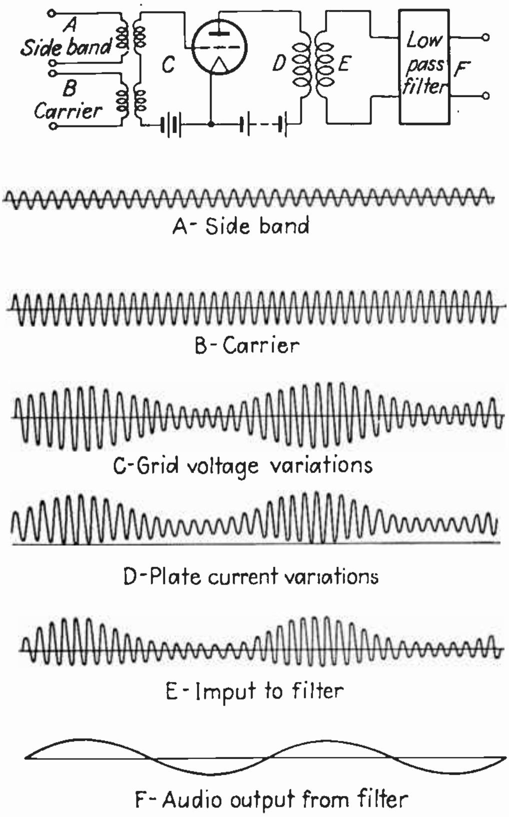

Fig. 335.—Details of plate-circuit square law or nonlinear demodulation or detection. The tube is operated as in Fig. 329, and this method is fundamentally the same as Fig. 328.

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[1] This is explained as follows: At the transmitting end, a carrier of frequency C and a voice of frequency V are modulated. In the output of the system the carrier component C, the upper side band $C + V$, and the lower side band $C - V$ exist. In the system here considered, only $C - V$ is transmitted. At the demodulator the locally generated carrier C and this lower side band $C - V$ are again distorted, and again sum and difference frequencies are created, the sum will be $C + (C - V)$ in frequency, but the lower side band will be $C - (C - V)$ which leaves the voice frequency V.

Diode Detection.—Diode detection as it is called is the most widely used system of demodulation in radio receiving sets. A diode will pass a plate current only when the plate is positive. These relations are shown in Fig. 336. Therefore, when a modulated signal composed of a carrier and the two side bands is impressed on a diode having the characteristics of Fig. 336, a badly distorted current will flow in the plate circuit. This distorted current will contain sum and difference frequencies, and the difference frequency will be the audible speech or music program which is desired.

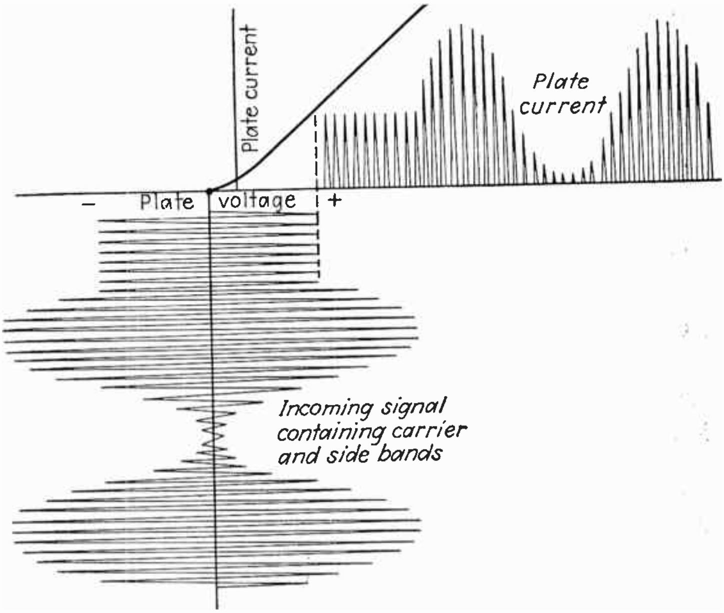


FIG. 336.—Diode detection. The distorted plate current contains the desired audio-frequency component.

A complete diode-detection circuit such as used in a radio receiving set is shown in Fig. 337. The tube is a diode-triode combination. The carrier and two side bands are impressed between the diode plates and the cathode. Because the diode is a nonlinear impedance (Fig. 336), the current which flows in the diode element is distorted and the desired audio-frequency component flows. This audio current must provide an audio voltage for further amplification; thus, the audio current flows through R_1 and causes an audio-signal-voltage drop across it. A part of this audio voltage is impressed on the grid of the triode portion by the C_2, R_2 combination, R_2 being a manual volume control. The resistor R_1 is shunted with a capacitor C_1 which offers low impedance to the carrier-frequency components, so that negligible voltage for these components exists across R_1.

The circuit of Fig. 337 also provides an automatic volume-control voltage in this way: Assume for a moment that the transmitting station is not being modulated, so that the carrier frequency only is being received. The path in space between the transmitting and receiving sets is not constant, so that the received signal varies from time to time or **fades.** Now the diode will rectify the carrier-frequency signal, and a direct-current component proportional to the strength of the received signal will be created, and this direct current will flow through R_1 and will cause a direct voltage drop across it.

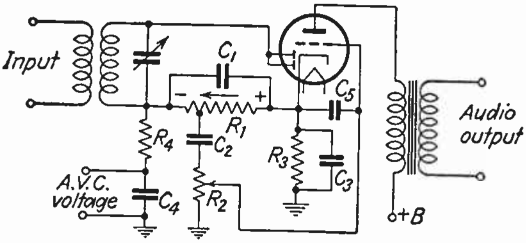
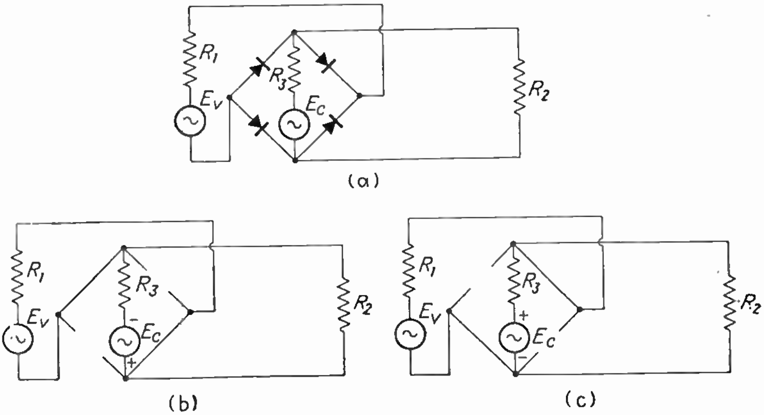


FIG. 337.—Method of obtaining an automatic volume-control voltage and an audio-signal voltage from the same tube. Amplification of the audio signal is also produced in the triode portion of the tube.

When the received signal is strong, the direct voltage will be large, but when the received signal is weak, the direct voltage will be small. This direct-voltage drop across R_1 is allowed to vary the charge on condenser C_4 through resistor R_4. This means that the voltage across condenser C_4 will slowly rise and fall whenever the rectified carrier component rises and falls. This voltage is then used to bias a variable-mu or remote cutoff tube (page 421). Thus, when the received carrier signal is strong, a large direct voltage will exist across C_4 and will highly bias a tube and give it low amplification. When the received carrier signal is weak, a small direct voltage will exist across C_4 and put but a small bias on the tube and give a large amplification to make up for the weak received signal.

Since as previously explained the carrier-frequency component does *not* vary when a signal is modulated, this automatic volume-control circuit operates as explained even when a modulated signal is being received.

Copper Oxide Varistors as Modulators and Demodulators.—The copper oxide rectifier, or Varistor (page 436), is used in a bridge circuit in modern carrier telephone systems instead of vacuum tubes to produce modulation. Now the curves of Fig. 293, page 436, show that the Varistor is a nonlinear device much like a diode. The Varistor has very low resistance for current in one direction and very high resistance for current in the other. If two frequencies are simultaneously impressed on such a device, sum and difference frequencies, or side bands, will be created as previously explained for nonlinear devices.



(a)

(b)

(c)

FIG. 338.—Circuits for studying the copper oxide Varistor as a modulator. The generator E_c represents the carrier voltage, and E_v the voice voltage. The resistor R_2 represents the outgoing line impedance.

The circuit actually used is shown in Fig. 338. For this circuit it is assumed that the resistance is either zero or infinite (open). The carrier frequency E_c is made larger than the audio or voice voltage E_v, and thus the carrier voltage varies the impedance offered the voice voltage in accordance with the relations of Fig. 293, page 436, and as shown in Fig. 338*b* and *c*. The voice voltage and carrier voltage acting together badly distort the signals, and sum and difference frequencies, or side bands, will flow in the load resistor R_2. Thus, modulation has resulted.

As has been stressed in these pages, demodulation is the same fundamental process. Thus, if a carrier and side bands are impressed on the circuit of Fig. 338*a*, the desired audio or voice component will be created and demodulation has resulted. Copper oxide Varistors cannot be used for modulation and demodulation at radio frequencies largely because of their internal capacitance.

Frequency Modulation.—The systems that have been previously considered are known as **amplitude-modulation** systems because

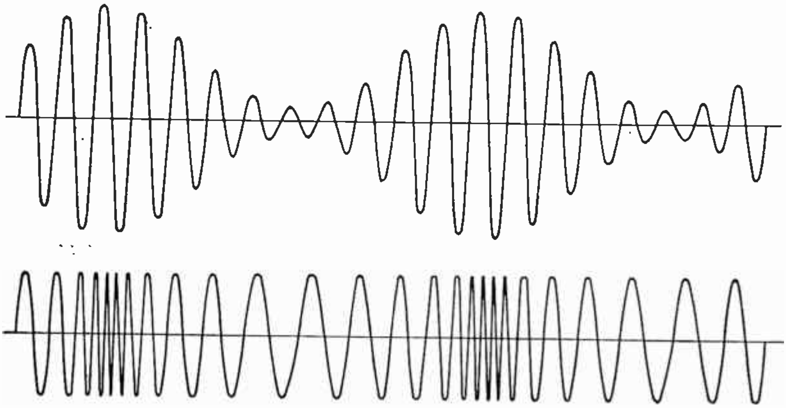


FIG. 339.—Showing an amplitude-modulated wave (above) and a frequency-modulated wave (below).

when the signal is viewed with an oscilloscope it *appears* that the carrier is merely varying in amplitude in accordance with the audible signal, although it has been made clear that the carrier is constant and that the fluctuations that appear are really caused by the added presence of the created side bands.

A system of **frequency modulation** has been developed, and if this signal is photographed (which perhaps is the best way to observe it), it will appear somewhat as in Fig. 339. Now here again it is found that many erroneous views exist regarding this signal. It *appears* that it is merely a carrier which varies in frequency; however, a mathematical treatment or an analysis with a wave analyzer shows that this is definitely *not true.* In fact, in this system the carrier-frequency component is constant in frequency and varies in amplitude, and, further, the carrier-frequency component can actually be *zero*, and the output of a frequency modulation system will look just like Fig. 339. The signal has this peculiar shape because in the process of distortion **side frequencies** (as distinguished from side bands in amplitude modulation) are created. Also, although only two side bands are created in amplitude modulation, a large number of side frequencies may be created in frequency modulation, the number of these and their relative strengths or amplitudes varying with several factors such as the frequency of the modulating signal.

Circuit for Frequency Modulation.—There are several systems of frequency modulation, the most typical being depicted in Fig. 340, the circuit being, of course, simplified to the essential parts. The frequency of oscillation of an oscillator is usually fixed by a tuned circuit such as *L-C*. If a reactance which varies as the signal to be transmitted is connected across *L-C*, then the oscillator output will be frequency modulated. The reactance tube is so operated that it presents such a variable reactance. This is accomplished as will now be explained.

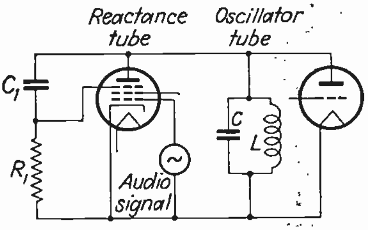


FIG. 340.—Illustrating the reactance-tube method of producing frequency modulation.

There will be a carrier-frequency voltage across the tuned *L-C* circuit because this tube generates the carrier frequency. This will force a current to the left and down through C_1-R_1. Condenser C_1 is made small and offers a very high reactance to this current; then, the voltage drop across C_1 will be large compared with that across R_1; also, these voltages will be almost 90 degrees out of phase. The voltage across R_1 will be impressed on the grid and will cause an alternating-current flow in the output which has the same frequency as that of the voltage across the tuned circuit *L-C* but will be 90 degrees out of phase with this voltage. Thus, the circuit to the left of the tuned circuit *L-C appears to be reactive.* Now a tube is used in which the amplified output will vary with the magnitude of the audio signal on the grid. Then, the signal on the grid controls the *amount* of reactive current flowing from the plate of the reactance tube to the tuned circuit *L-C*. In this way, a reactance that varies with the audio signal is connected across the tuned circuit of the oscillator, and the output *frequency* of the oscillator is frequency modulated in accordance with the audio signal.

Reception of a Frequency-modulated Signal.—The basic principles rather than the actual circuits will be considered. First, after reception and amplification of the frequency-modulated signal, it is passed through a **limiter** circuit. This circuit passes a signal of only a given amplitude and assures that no variations in the amplitude of the frequency-modulated signal exist. Next, the frequency-modulated signal is passed through a **discriminator,** and it is the purpose of this circuit to change the frequency-modulated signal into an amplitude-modulated signal which is then demodulated and thus gives the audio-frequency signal.

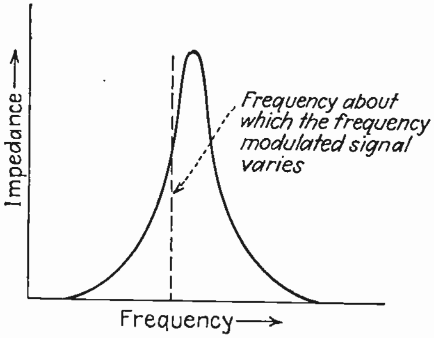


FIG. 341.—The principle of receiving a frequency-modulated signal when a frequency-modulated current of constant amplitude is impressed on the circuit as shown. Because a different impedance is offered by the circuit to each frequency, the voltage across the circuit will vary in accordance with the variations in frequency. In this way a frequency-modulated signal is changed to an amplitude-modulated signal.

In an actual frequency-modulation radio receiver, several of these functions are combined into a single circuit. Rather than describe this circuit, the principle will be explained by the use of Fig. 341. Suppose that a signal which is varying in frequency is impressed as indicated on a circuit which is tuned above the basic carrier frequency of the frequency-modulated signals. When the frequency rises, a greater impedance will be offered, and the voltage across the tuned circuit will rise. When the frequency drops, a smaller impedance is offered, and the voltage across the tuned circuit will be less. In this way, the frequency-modulated signal is converted to an amplitude-modulated signal which can then be demodulated into an audio signal. The discriminator circuit actually used is similar to an automatic frequency-control circuit.

Frequency-modulation systems are characterized by freedom from station interference and noise.

The Cathode-ray Oscilloscope.—A circuit for a cathode-ray oscilloscope tube is shown in Fig. 342. It consists of an indirectly heated cathode which supplies a source of electrons. To reach the distant end of the tube where the image is produced, these electrons must pass through the negative grid G_1. This grid largely controls the *number* of electrons flowing down the tube, and it is used to control the intensity of the spot on the screen. The electron beam then flows through anodes or plates A_1 and A_2, and the relative potentials of these two anodes focuses the electron beam on the end of the tube. The deflecting plates D_1, D_2, and D_3, D_4 serve to deflect the electron beam. The beam then impinges on the end of the tube, which is coated with a material that will fluoresce (give off visible light) when the electron beam strikes it.

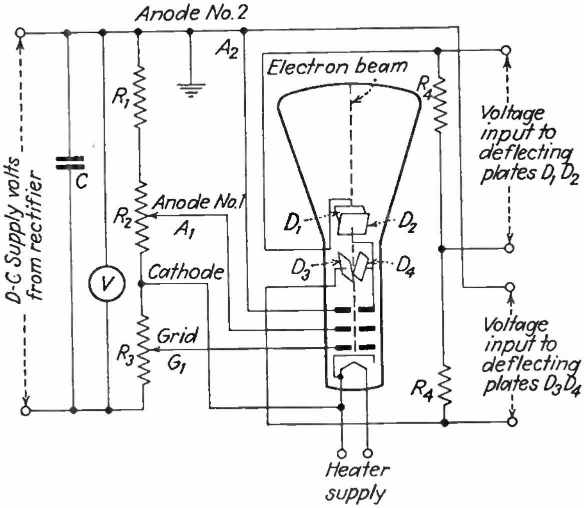
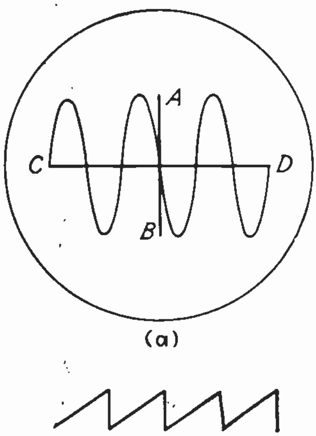


FIG. 342.—Simplified connections of a typical cathode-ray oscilloscope.

Suppose that Fig. 343 represents the end of a cathode-ray oscilloscope tube. If an alternating-signal voltage is placed on the pair of plates that causes vertical deflection, a straight line AB will result; that is, the electron beam will merely be moved up and down. Now suppose those plates are not energized but that a special "saw-tooth" voltage such as Fig. 343*b* is impressed on the other set of deflecting plates. These will cause a horizontal deflection *C-D*, but this deflection will have peculiar characteristics. The electron beam will slowly move from *C* to *D* and will return almost instantaneously because of the saw-tooth voltage applied. Thus if a sine-wave alternating voltage is impressed on the vertical deflecting plates, and a saw-tooth voltage is impressed on the horizontal deflecting plates, the alternating signal will be drawn out from *C* to *D* and will trace a sine wave, but it will snap back so quickly it will leave no visible trace. This is shown by the horizontal line *C-D* of Fig. 343*a*.



(a)

(b)

FIG. 343.—If an alternating signal voltage is applied alone to the vertical deflecting plates of a cathode-ray tube, the alternating voltage will merely cause the electron beam to move up and down producing the trace *A-B*. If a "saw-tooth" wave such as *b* is applied alone to the horizontal deflecting plates of a cathode-ray tube, the gradual building up of the wave will deflect the beam slowly across the end of the tube, and will quickly let the beam go back. This will cause the horizontal trace *C-D*. When the alternating sinusoidal signal voltage and the saw-tooth waves are applied simultaneously, the saw-tooth wave stretches out the sinusoidal voltage so that the sine wave trace or wave form is visible.

The saw-tooth wave is obtained from a circuit such as shown in Fig. 344, employing a grid-controlled gas-filled triode, or Thyratron, page 427. The grid is biased negatively a certain amount. The anode-supply voltage will *slowly* charge condenser *C* through the resistor *R* (page 198). When *C* reaches a sufficient potential, the grid loses control and the condenser discharges *very rapidly* through the tube, which has low resistance; thus the voltage across the condenser is reduced to a low value and the grid regains control. If a portion of the voltage across the condenser is used, it will be essentially a saw-tooth wave.

Phototube Circuits.—The principle of operation of the phototube was discussed on page 433. A simple light-operated phototube relay circuit will now be discussed. Such circuits are convenient for operating a counter when packages pass on a conveyer belt and for similar purposes. With the circuit arranged as indicated in Fig. 345, the light beam will fall on the cathode of the phototube, and an electron current will flow from cathode to anode, which is equivalent to having a conventional current flow in the direction shown by the arrow. The direct-voltage drop across resistor R is impressed on the grid of the tube. If the phototube current is sufficient, and if R is at least about 1,000,000 ohms, then the plate voltage on the tube can be such that the thermionic vacuum tube is about at cutoff, and thus but little plate current is drawn.

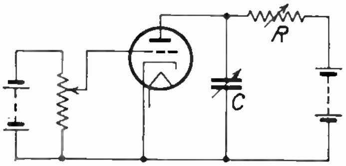


FIG. 344.—Circuit for generating a saw-tooth wave.

When the beam of light is interrupted by an object on the conveyer belt, then the phototube current falls to a very low value. This results in little bias voltage on the thermionic vacuum tube, and a large plate current flows. This plate current operates the counter mechanism.

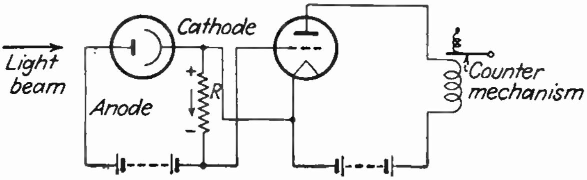


FIG. 345.—Simple circuit for a photoelectric counter.

Vacuum-tube Relay Circuits.—The electronic principles of the cold-cathode gas-filled tube were discussed on page 429. This tube is used in practice as a rectifier, as a voltage regulator, and as a relay. Its action as a rectifier is similar to that of any rectifier (page 442), and its action as a voltage regulator is similar to that explained on page 449. As a relay, however, it offers possibilities beyond those of other devices.

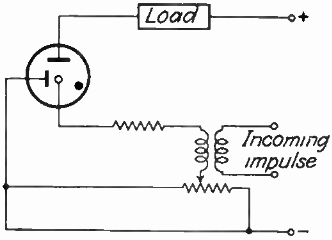


FIG. 346.—Simple relay circuit using a cold-cathode tube.

Now the principle of a relay is that a feeble signal may be able to control a large amount of power. Thus the cold-cathode tube will act as follows: Suppose that the circuit is arranged as in Fig. 346. The control anode is made positive with respect to the cathode, but not quite sufficiently positive to break down. Then, an incoming voltage impulse will "trip off" the control anode, and the control gap will break down. This action will fill the tube with ions which will permit the main gap between the cathode and main anode to break down, and a current will flow through the load. Relay tubes such as this may be arranged in a variety of ways to perform remote operations when acted upon by feeble impulses.

SUMMARY

It is usually assumed that a rectifier produces a pulsating direct-current flow and that the choke coils and condensers in the filter "smooth" it out. A complete analysis, however, shows that a rectifier distorts the impressed voltage, an output composed of direct-current and various alternating-current components being thus produced. The filter allows the direct current to flow, but largely prevents the alternating-current components from flowing.

The direct voltage output of a rectifier and filter will vary with the impressed alternating voltage and also with the magnitude of the current drawn by the load. A cold-cathode gas tube can be used as a voltage regulator for such power supplies.

Amplifiers are of two types, voltage amplifiers and power amplifiers. The voltage amplifier is usually placed first to increase the feeble signal until it is sufficiently strong to drive a power tube.

The voltage amplification of a resistance-coupled stage is easily found by the expression $A_V = \mu R_L/(r_p + R_L)$.

The power delivered to a load resistor R_L is $P = (\mu E_g)^2 R_L/(r_p + R_L)^2$.

In the design of a resistance-coupled audio amplifier, the gain per stage at an intermediate frequency of about 1000 cycles is calculated by the equation previously given, and the low- and high-frequency response is often merely estimated.

For voltage amplifiers using triodes, the load resistance should be three to five times the plate resistance of the tube.

In power amplifiers, for maximum undistorted (not to exceed 5 per cent) power output, the load resistance should be about twice the plate resistance.

These relations do not hold for either voltage or power amplifiers using pentodes. It is recommended that data given in tube manuals be consulted for design information.

Transformer-coupled audio amplifiers are easily designed by the relations $A_V = \mu N$, where N is the turns ratio. Such simple relations do not apply to radio transformer-coupled voltage amplifiers.

The power-output transformer is an impedance matcher which reflects the proper load impedance into the plate circuit of the output tube.

Push-pull power amplifiers offer certain advantages over single tubes. More than twice the power output of a single tube can be obtained, and even harmonic distortion is canceled out.

Voltage amplifiers are operated in class A; audio power amplifiers are operated in class A or class B. and radio-frequency power amplifiers are operated in class B or class C.

The so-called "tank" circuit is just a parallel circuit following the usual electrical laws. No such explanations as "flywheel" action are necessary to explain its operation.

Vacuum-tube oscillators are merely amplifiers with tuned circuits in their plates or grids which feed back a signal voltage from the plate to the grid circuits.

Quartz crystals are often used to stabilize radio-frequency oscillators.

Because of the capacitance between the plate and the grid in a triode, some of the output signal may be fed back as a voltage on the grid. This action may cause the triode to oscillate, in which event a voltage of the opposite phase is intentionally fed back and the triode is neutralized.

Controlled negative feedback is used in oscillators to make them independent of the supply voltage variations and to reduce their distortion.

Amplitude modulation consists of simultaneously impressing two signals on a nonlinear or distorting circuit, by which action two side bands are created.

Demodulation is fundamentally the same as modulation.

Both modulation and demodulation can be produced in either the grid or plate circuit of a vacuum tube.

Copper oxide Varistors are used as modulators and demodulators in low-frequency circuits such as carrier telephone systems.

In amplitude modulation, the amplitude of the carrier appears to vary. This is due to the presence of the created side bands. In frequency modulation, the frequency of the signal appears to vary. This is due to the presence of the created side frequencies.

REVIEW QUESTIONS

1. Discuss the various components comprising a rectified half wave and a full wave.

2. What is the purpose of the filter in a rectifier circuit?

3. For the same impressed alternating voltage, is a greater direct-current output obtained from a half-wave rectifier or a full-wave rectifier?

4. Which is the easier to filter, a half wave or a full wave? Why?

5. What type of tube is used for a voltage regulator, and why does it regulate?

6. What is meant by a linear amplifier, and how should a tube be operated as such?

7. What is the dynamic curve of a tube, and when must it be considered?

8. Referring to a resistance-coupled voltage amplifier, what causes the response at low frequencies, and at high frequencies, to drop off?

9. On page 454 it is stated that condenser *C* of Fig. 308 must have very high insulation resistance. Why is this true?

10. In resistance-coupled voltage amplifiers using pentodes, why is such a low value of load resistance used?

11. Briefly discuss the operation of the power-output transformer from the impedance-matching viewpoint.

12. Enumerate the advantages of the push-pull circuit.

13. Explain the difference between classes A, B, and C power amplifiers.

14. The output of a tube in class B or C is badly distorted. Explain how the undesired frequency components are eliminated.

15. Discuss the operation of the "tank" circuit from the load-impedance standpoint as contrasted with the "flywheel-action" explanation.

16. Why are tubes in class A used in audio oscillators and tubes in class C used in radio oscillators?

17. Explain why the grid-biasing polarities indicated on Fig. 323 are as shown.

18. How can feedback in a triode be neutralized by connecting a condenser and a coil from plate to grid?

19. Why is neutralization ordinarily not necessary with screen-grid tubes and pentodes?

20. Why is negative feedback used in amplifiers?

21. What components constitute the amplitude-modulated wave radiated in commercial broadcasting?

22. State the theorem applying to modulation by nonlinear impedance.

23. What reasons are there for regarding modulation and demodulation or detection as the same process?

24. Explain how an audio signal is obtained in the circuit of Fig. 337. How is the automatic volume-control voltage obtained?

25. Compare a frequency-modulated signal with an amplitude-modulated signal.

26. Does the amplitude of the carrier frequency vary in an amplitude-modulated signal? Does it vary in a frequency-modulated signal?

27. What is the purpose of the reactance tube in a frequency-modulated system?

28. Why is a "saw-tooth" oscillator used in a cathode-ray oscilloscope?

29. What is the purpose of the grid in a cathode-ray oscilloscope?

30. How is the beam in a cathode-ray tube focused?

PROBLEMS

1. Referring to Fig. 300, assume that the choke coil has a direct-current resistance of 200 ohms and an inductance of 12 henrys, that the capacitance of the condenser is 16 microfarads, that the load resistance is 5000 ohms, and that the peak of the half wave is 380 volts. Neglect the voltage drops in the transformer and the tube, and calculate the values of the direct current and the lowest-frequency alternating current which will flow through the load. Calculate the percentage ripple.

2. Repeat Prob. 1 for a full wave of the same peak value.

3. A triode has an amplification factor of 9.3 and a plate resistance of 10,300 ohms. Calculate the voltage output of a practical resistance-coupled voltage amplifier using this tube with 2 volts signal impressed on the grid.

4. A triode has an amplification factor of 3.5 and a plate resistance of 2000 ohms. Calculate the power output of a practical transformer-coupled power amplifier using this tube driven to maximum undistorted output with 25 volts bias on the grid.

5. The plate resistance of a power triode is 2000 ohms. Draw a curve showing the power output plotted on the *Y* axis for various values of load resistance plotted on the *X* axis. Also plot a curve showing the efficiency at various load resistances. What conclusions do you draw?

6. Two stages of transformer-coupled amplification are used in an amplifier. The input, interstage, and output transformers each has a turns ratio of 1 to 3, and the triodes that are used have amplification factors of 12.7. Calculate the voltage gain, the decibels gain, and the voltage output if 0.01 volt is impressed on the input.

7. Referring to Fig. 316, a 10-ohm resistor is connected across the output of a stepdown 20 to 1 turn (ideal) push-pull transformer. What will be the impedance measured across the entire primary, and across one-half of the primary?

8. A carrier telephone system is arranged to pass a speech band from about 100 to 4000 cycles and modulates a 30,000-cycle wave. What will be the frequencies of the side bands?

9. Referring to page 485, it is stated that 16⅔ per cent of the total power output of a broadcast station is contained in each side band and 66⅔ per cent is in the carrier. Prove this to be true.

10. The phototube of Fig. 345 will allow 13 microamperes to flow through a 2,000,000-ohm resistor when the light beam shines on its cathode and 3 microamperes when the beam is interrupted and only stray light strikes its cathode. What voltage will exist across the resistor for each condition? If a thermionic tube having the characteristics of Fig. 314, page 461. is used, what current change will be available for operating the counter?

CHAPTER XVI

ELECTROACOUSTICS

Systems of voice communication are much concerned with acoustics. Both telephone and radio systems are actuated by sounds from the air, using electroacoustic devices to change the sound waves to electric impulses. Also, after transmission to the distant location, electroacoustic devices are once again used but this time to change from electric impulses to sound waves to actuate the listener's ear.

Those engaged in sound amplification and radio are deeply concerned with the acoustical characteristics of studios and auditoriums. The important role played by acoustics is seldom fully appreciated and certainly poorly understood, except by the acoustical expert. A sound system may sound unsatisfactory in an acoustically incorrect auditorium but will prove excellent in an auditorium that is acoustically corrected.

Because of their great importance to those in communication, sound, acoustics, speech, electroacoustic devices, and noise will be considered in this chapter.

Sound.—There are two closely related but distinct meanings of the word *sound*. It may be considered *objectively* as a wave motion in air, or it may be considered *subjectively* as a sensation produced in the organs of hearing.

From the objective, or physical, viewpoint, sound is a wave motion consisting of condensations and rarefactions in the air. To produce sound waves, some object such as a loud-speaker diaphragm must vibrate or a column of air as in a horn must be set in motion.

Sound waves serve to transmit energy from the vibrating body to the listener's ear. Sound waves must, therefore, contain energy, although it is very small. Sound waves can be represented by sine waves as are electric impulses. Sound waves add, interfere, and have other characteristics in common with electromagnetic waves. But, whereas electromagnetic waves are transmitted through vacuous space, sound waves must have air particles (or gas, or some other physical medium) for their propagation.

Sound waves have a definite frequency and amplitude. They pass through zero and maximum values just as any other waves. Sound travels very slowly, compared with electromagnetic waves, the velocity at sea level and at 20°C., or 68°F., being about 1125 feet per second.

Sound Transmission, Absorption, and Reflection.—This section is *very* important and should be carefully studied.

When sound waves traveling from a source strike a *rigid* wall, several important phenomena occur as shown in Fig. 347. Reflection of the sound waves occurs; in fact if the walls are covered with some material such as painted wood or plaster, about 96 per cent of the sound energy is **reflected.** Some of the sound energy is **absorbed** at the surface; this will be fully treated later. What sound energy is left after reflection and *surface* absorption is **transmitted** into the wall. To pass through this rigid wall, the particles of the material comprising the wall must be set in vibration. Some of the energy starting through the wall is absorbed. Because the velocity of sound in a solid material is greater than in air, the sound ray is bent, or refracted. Because of the surface reflection and absorption, and the absorption within the wall itself, the sound transmitted through a rigid wall is very weak.

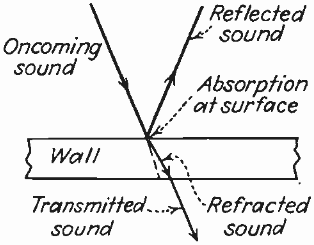


Fig. 347.—Phenomena occurring when a sound wave strikes a rigid wall.

If, however, a wall is solid but contains cracks or other small openings, then much sound energy will leak through. Also, if the wall is thin and not rigid, then it will readily vibrate and, like any vibrating body, will radiate much sound energy on the opposite side of the wall. When a wall or other object vibrates, because of the internal frictional losses it extracts energy from the sound waves causing it to vibrate.

Echoes and Beats.—When sound strikes a rigid wall or other object, sound energy is reflected. This may be a large percentage of the total sound energy. If a speaker is in the open but near a large reflecting object such as a wall, he may speak but may hear no reflected sound. This is because the distance is so small that the time interval between the original and reflected sound is not sufficient to distinguish between these two sounds. If the speaker steps back so that the time interval is about one-seventeenth of a second, he will hear a distinct **echo.**

Now it is generally considered that when two sounds of different frequencies are simultaneously produced in a room they "beat together" and produce a "beat note." This statement is very misleading, however. It is strictly true that two waves of different frequencies do combine as shown in Fig. 348, but no beat note is produced in the air, *the beat note is produced by the ear.* This is related to modulation, a beat note being similar to the lower side band. In modulation, the two frequencies must be impressed on a *nonlinear impedance* to create the side band. In acoustics, the ear is the nonlinear impedance.[1]

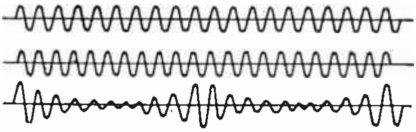


FIG. 348.—Two pure tones of the same amplitude but of different frequencies combine as shown. The beat note which the ear "hears" is, however, created by the hearing mechanism itself.

That acoustic beats are produced in the ear and not the air can be demonstrated as follows: Connect two good loud-speakers to separate oscillators of good wave form. Set one oscillator on 1500 cycles and the other on 2000 cycles. You will then *hear* a 500-cycle beat note. Put a good microphone in the sound field, amplify the signal, and analyze the wave. A wave analyzer will show that the resultant wave contains only a 1500- and a 2000-cycle component, although an oscilloscope will picture the resultant wave as in Fig. 348. Nevertheless, if a 500-cycle wave is not picked up by the microphone and shown to be present by the wave analyzer, it just is not present, and if heard, it must be a sensation created by the ear. It should be noted, however, that the air itself is not exactly a linear transmitting medium and that with *very* intense sounds

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[1] A study of the ear and the mechanism of hearing is a fascinating subject, but beyond the scope of this text. For a summary of this subject and for additional information on the material covered in this chapter, the writer takes the liberty of suggesting his book "Electrical Communication," John Wiley & Sons, Inc.

some noticeable distortion will occur and an appreciable beat note will be created; but, this is not the usual phenomenon that is so often demonstrated with two tuning forks, etc.

Reverberation.—In a preceding section it was shown that if a speaker were in the open near a large reflecting object a distinct echo could be heard. But, if a speaker is in a room, or especially an auditorium, he is surrounded by reflecting surfaces, and when he speaks no distinct echo will (usually) be heard, but rather his words will come back to him as a "jumble" of sounds. This prolonging of sounds after the source has ceased to emit is termed **reverberation.** In auditoriums, an excessive **reverberation time** is the most common of all acoustic faults.

If the sounds in a room or auditorium are audible for too long a period, they may prevent the listener from understanding the next words spoken. The cause of this prolonging of sounds is as follows: If the room or auditorium contains little furnishing such as draperies, carpets, or upholstery; if there are but few people present; and if the walls and ceiling are covered with ordinary painted wood or plaster; then, there is nothing present to absorb the energy in the sound waves. A sound contains energy, and if the surfaces in the room are smooth and hard, then about 96 per cent of the energy will be reflected as shown in Fig. 347, and the sound waves must encounter many reflections before they are reduced to inaudibility. Thus, sounds are prolonged.

If a room has an excessive reverberation time, there is only one thing to do, and that is to introduce materials into the room that will absorb the sound waves. Materials such as draperies, rugs, and special acoustic wall coverings offer porous surfaces. It is generally assumed that the air particles in the small openings or pockets of a porous surface move back and forth under the influence of the sound waves and dissipate the energy in the sound waves by friction on the sides of the minute pockets of the porous material. If in addition to this, the material is caused to vibrate, frictional losses due to flexing the material will also damp out the sound waves.

As mentioned, an excessive reverberation time is the most common of acoustic faults. Almost invariably, church, school, and similar auditoriums have excessive reverberation time. This can be corrected only by adding sound-absorbing material to the room. In many instances carpets, rugs, draperies, etc., are impractical or would not give sufficient absorption, and sound-absorbing material must be placed on the ceiling and walls. Obviously, if this is done when the auditorium is being constructed instead of after it is built, money can be saved. In most instances the so-called heat-insulating boards will also provide sufficient sound absorption if they are placed on the inner surfaces of an auditorium. Then, over a period of a few years they will pay for themselves in fuel savings in addition to improving the acoustics. Such materials should be painted with special acoustic paint instead of oil paint if the acoustic properties are to be unimpaired.

For years the prevailing opinion has been that if fine wires are strung near the ceiling of an auditorium the presence of the wires will improve the acoustics. The presence of wires does not affect the acoustics of an auditorium to any measurable extent. What happens is this: Let us assume that a high-school auditorium has faulty acoustics. Perhaps as a community project the folks gather and string hundreds or perhaps thousands of feet of fine wires back and forth near the ceiling. Then, a special event is planned to celebrate the installation of the wires. The auditorium is filled to capacity, and the acoustics are found to be greatly improved, but *this is because of the presence of the audience* and not because of the wires. As will be shown in the next section, a person's clothing absorbs much sound energy, and the acoustics of almost any auditorium are at least fair when it is filled with people.

Calculation of Reverberation Time.—The desirable reverberation time depends on the size of the room. Typical values are given by Table XI. The reverberation time can be approximately measured by loudly blowing a horn in the room and measuring the time that the sound remains audible after the blowing is suddenly ceased. A frequency of 512 cycles is commonly used in approxi-

TABLE XI.—ACCEPTABLE REVERBERATION TIME

Size of Room, Cubic Feet	Time, Seconds
1,000	0.8
5,000	1.0
10,000	1.1
50,000	1.4
100,000	1.6
500,000	1.9
1,000,000	2.0

mate reverberation measurements. The reverberation time can be computed by the formula developed by Sabine,

$$T = \frac{0.05V}{a}, \tag{102}$$

where T is the **reverberation time** in seconds, when V is the room volume in cubic feet and a is the total sound-absorbing power of the material exposed to the sound waves. This equation does not hold exactly for very highly damped rooms such as radio-broadcast studios. For such rooms, a value $0.027V/S$ should be subtracted from Eq. (102), where V is the total room volume in cubic feet and S is the total room surface in square feet.

Referring to Eq. (102), it was stated that a was the total sound-absorbing power of the material in the room. Each type of surface has a different **sound-absorbing coefficient,** typical values being shown in Table XII.

TABLE XII.—APPROXIMATE SOUND-ABSORBING COEFFICIENTS

(512 Cycles)

Material	Absorption Units
Brick wall	0.03
Glass	0.03
Linoleum	0.03
Plaster	0.03
Varnished or painted wood	0.03
Open window	1.00
Air vents	0.75
Carpet, thin	0.10
Carpet, thick	0.40
Insulating boards	0.25
Acoustic coverings (use manufacturer's data. Coefficients vary from about 0.40 to 0.90):	
Adult person	4.0
Plain wood seats	0.15
Seat cushions	1.0

Calculation of Reverberation Time.—A small high-school auditorium is 80 feet long, 50 feet wide, and 25 feet high. The plans show that the ceiling and walls are to be plastered. Of the wall area, 20 per cent is to be windows. There are to be 500 seats upholstered with cloth (not imitation leather) each having an absorption of 1.5 units. The plans are submitted to a consultant who is requested to determine if the reverberation time of the auditorium will be satisfactory with one-half audience and with full audience.

Solution.—The sound-absorption coefficients given in Table XII are only approximate but are sufficiently accurate for most purposes. It will be noted that the coefficients for the first five common materials are given as the same value, 0.03. Actually, they are not exactly the same, but they are so nearly so that little error is introduced if this assumption is made. Assuming that the values are the same simplifies the calculations.

Step 1. Calculate the total absorbing power of the auditorium when empty. Assume that for the floor, walls, and ceiling, the coefficient is the same.

Ceiling area $= 80 \times 50 = 4000$ square feet

Floor area $= 80 \times 50 = 4000$ square feet

Wall area $= (2 \times 25 \times 50) + (2 \times 25 \times 80) = 2500 + 4000 = 6500$ square feet.

Total area $= 4000 + 4000 + 6500 = 14{,}500$ square feet

Absorption of walls, etc. $= 14{,}500 \times 0.03 = 435$ units

Absorption of seats $= 500 \times 1.5 = 750$ units

Total absorption $= 1185$ units

Step 2. Calculate the total absorbing power of the walls, etc., and the seats when the audience is one-half capacity. As shown in Table XII, an adult person has an absorption of 4.0 units. It also will be considered that an occupied seat offers an absorption of 4.0 units.

Absorption of audience $250 \times 4.0 = 1000$ units

Absorption of vacant seats $250 \times 1.5 = 325$ units

Absorption of walls, etc. $= 435$ units

Total absorbing power $= 1760$ units.

Step 3. Calculate the total absorbing power of the walls, etc., and the seats and with a full audience.

Absorption of audience $500 \times 4.0 = 2000$ units

Absorption of walls, etc. $= 435$ units

Total absorbing power $= 2435$ units.

Step 4. Calculate the reverberation time of the auditorium when empty, when it has half-capacity audience, and when it is filled. This will be done using Eq. (102), $t = 0.05\ V/a$.

Empty $t = 0.05 \times 100{,}000/1185 = 4.2$ seconds

Half audience $t = 0.05 \times 100{,}000/1760 = 2.8$ seconds

Full audience $t = 0.05 \times 100{,}000/2435 = 2.1$ seconds.

Comparing these figures with the values given in Table XI discloses that the reverberation characteristics would be satisfactory with full audience and would be fair with half audience. Thus, no acoustic material need be added to reduce the reverberation time. The upholstered seats do much to reduce the reverberation time, because without them the time would be $t = 0.05 \times 100{,}000/435 = 11.5$ seconds. It is probable that without the upholstered seats (for example, if wooden chairs were used) it would be necessary to add sound-absorbing material. There is much to be said for cloth upholstered seats and carpets in an auditorium, and part of their cost should be charged against the acoustical improvement.

As previously mentioned, it was assumed that the coefficient for each type of surface was the same value of 0.03. Actually, some of the common materials such as wood and glass are higher, and some are lower, and the value 0.03 gives good results. If it is desired, the area of each separate surface material may be computed, and this area multiplied by its exact coefficient (consult handbooks on communication) gives the absorption of that surface. Then, the total absorption will be the sum of absorption of each individual surface.

Speech Sounds.—The sounds of speech are produced by the vocal cords and resonating air cavities of the head, and are also produced by passing air through small openings of the mouth and over the teeth. Speech sounds combine in a very irregular manner and when speech sounds are picked up by a microphone the electric currents which flow produce a very erratic wave form as Fig. 349 indicates.

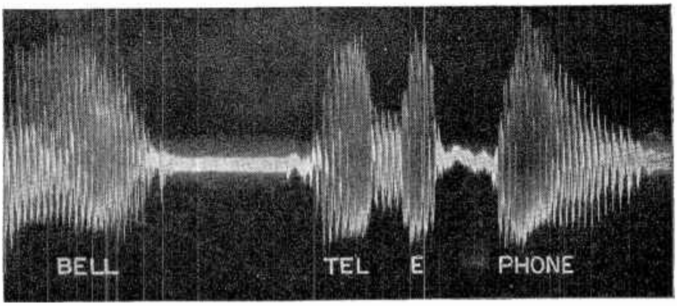

FIG. 349.—Wave form of speech sounds. (*Courtesy of the Bell Telephone System.*)

If speech sounds are carefully analyzed,[1] they will be found to contain frequencies from as low as 100 cycles to as high as 10,000 cycles. For *high-quality* speech, the electrical circuits should pass a band of from about 100 to at least 5000 cycles. For *commercial telephony*, where intelligence is of great importance but high quality is not, a band of from about 250 to 3000 cycles will give good service.

Compared with many other physical phenomena, speech is very weak. The *average* speech power is about 10 microwatts. Thus, the communication engineer has very little power to start with, because a microphone will pick up but a small amount of this power and its efficiency in converting from acoustic to electric power is low.

Musical Sounds.—Musical sounds, when converted by a microphone into electric impulses, also produce a signal of complex wave form. Extensive studies have been made to determine the characteristics of music. As a result of these studies, it has been found that the various musical instruments contain frequencies as low as 50 cycles and as high as 15,000 cycles.

The communication worker is vitally concerned with the frequency range that must be transmitted to give musical reproduction of excellent quality. As a result of exhaustive tests,[2] in which experts listened to music with various frequency components removed, it was proved that little of the quality was lost if the electric system would pass without distortion a band from 50 to 8000 cycles. In fact, a band of 100 to 5000 cycles will give good reproduction. The tone control on radio-broadcast receivers has proved that the general public is not too greatly interested in having high frequencies. The volume range in music is very great, being as great as 100,000 to 1, but it was also found that a range of 10,000 to 1 gave good reproduction.

Hearing.—The ear and the process of hearing are as complex as they are interesting. The normal ear can hear sounds from about 20 to 20,000 cycles, although many ears can hear sounds of higher frequency. The ear does not hear equally well over this range of frequencies, however. Also, the frequency-response characteristics of the ear depend upon the intensity of the sounds.

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[1] Fletcher, Harvey, "Speech and Hearing," D. Van Nostrand Company.

[2] Snow, W. B., Audible Frequency Ranges of Music, Speech, and Noise, *Journal of the Acoustical Society of America*, July, 1931.

For very loud sounds, the frequency characteristics of the ear are substantially flat; that is, if the sounds are loud the ear hears low frequencies and high frequencies equally well. For average sounds, such as the voice in conversation, the ear is most sensitive over the band from about 500 to 5000 cycles. For very weak sounds, such as the sound level reproduced by a telephone receiver, the ear is most sensitive from about 1000 to 5000 cycles.

In the design and operation of communication systems, it is of great importance to know what happens if the low frequencies are eliminated and what happens if the high frequencies are eliminated. It has been found that if only frequencies *below* 1000 cycles are transmitted, 60 per cent of the intelligibility is lost, but only 15 per cent of the energy. Also, these tests showed that if only frequencies *above* 1000 cycles are transmitted, only about 15 per cent of the intelligibility is lost, but that 85 per cent of the energy is removed. From these tests it follows that the energy (boominess) is in the low frequencies but that the intelligibility and the energy giving musical instruments their individual quality is in the high frequencies.

Noise Measurements.—Noise may be defined as any unwanted sound. Thus, radio music may be noise to you if you are concentrating on a good story. But, in the popular sense, noise is that disagreeable jumble of sounds of miscellaneous origins. Some years ago noise was tolerated as a necessary evil, but not so today. Noise may be studied, analyzed, and reduced by appropriate means.

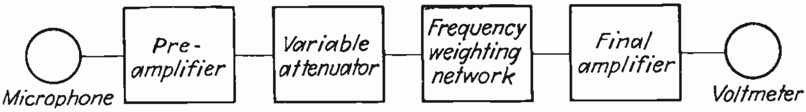

FIG. 350.—Schematic diagram of a sound level or noise meter.

The **sound-level meter** is used to measure noise. The intensity of the noise is measured in decibels above a zero level (page 335). The zero level is approximately the intensity of a sound which is just audible to the human ear. The component parts of a sound-level meter are shown in Fig. 350. The variable attenuator is to provide different ranges on the measuring instrument.

The frequency-weighting network is inserted to give the sound-level meter characteristics similar to those of the human ear for the following reason. Suppose that measurements are being made of a noise of moderate intensity but which contains many low frequencies. At moderate intensities the ear is not quite so sensitive to low frequencies. If a sound-level meter that was equally sensitive to all frequencies were used to measure this noise, it would give a certain reading. But, to the ear this noise (which contains many low frequencies) would not seem particularly loud. To make the meter judge a noise as does the ear, frequency-weighting networks are provided which give the meter essentially the characteristics of the ear.

When studying a noise with a sound-level meter, the electrical output may be analyzed into the various frequency bands. This gives an indication of the physical nature of the noise, and such knowledge may lead to a clue to its reduction, particularly if a study is being made of the noise from some machine.

Noise may be reduced by isolating vibrating parts on rubber mountings so that the machine structure and the floor are not set in vibration. Or, perhaps the machine may be insulated by enclosing it in a heavy box lined with sound-absorbing material. A noisy room may often be quieted by placing sound-absorbing material on the walls and ceilings. Such reduction is due to the sound-absorbing properties of materials rather than to sound-insulating properties as popularly stated.

Transmitters and Microphones.—In a system of communication it is necessary to have some device to convert the speech and music sound waves into electric impulses. In telephony this device is usually called a **transmitter,** and in radio it is called a **microphone.** In this section these devices will be considered from the standpoint of the basic electrical principles involved.

Modifier-type Transmitters and Microphones.—These modify or cause variations in either the voltage across them or the current flowing through them. The sound waves merely act to *control* an external source of power.

In the **condenser transmitter,** or microphone, a very thin diaphragm is supported a small distance from a metal plate and is insulated from it. A voltage is impressed between the diaphragm and plate which form, in reality, the plates of a condenser. When sound waves strike the diaphragm it moves in and out, and this changes the capacitance of the microphone and causes a charging current to flow through a resistor. The voltage drop across this resistor will vary closely in accordance with the sound waves, and this voltage is then amplified. The condenser microphone was very popular in the early days of sound motion pictures and radio and is still used for some purposes today.

In the **single-button carbon-granule transmitter,** a small cup is partly filled with carbon granules as indicated in Fig. 351. The sound waves striking the diaphragm cause a small carbon plunger closing the front of the cup to move in and out. As this occurs, the carbon granules are compressed when the plunger moves in, and this *decreases* the resistance of the current path from the carbon plunger, through the granules, and to the carbon disk at the back. As the plunger moves out, the resistance of the granules is *increased.* A battery is connected in series with the carbon-granule path. When no sound strikes the diaphragm, a comparatively steady direct current flows. When sound waves strike the diaphragm, the current varies in accordance with the sound waves. These current impulses flowing through a distant receiver will reproduce the sound. If the transmitter of Fig. 351 is held horizontally the granules will drop away from the carbon disk, and current path actually may become opened. To provide a nonpositional transmitter, an arrangement such as shown in Fig. 352 is used. With this arrangement the carbon-granule path does not open with the transmitter in a horizontal position.

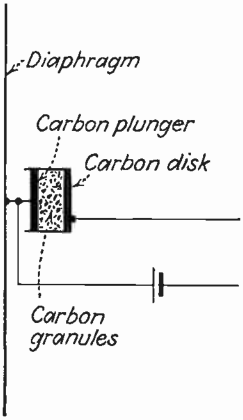

FIG. 351.—The single-button carbon-granule transmitter or microphone consists of a thin diaphragm which causes a movable carbon plunger to move back and forth. This alternately compresses and expands a mass of carbon granules which are held in a brass cup, but insulated from the cup. In this way the resistance of the granules is caused to vary in accordance with the sound waves striking the diaphragm. This in turn causes the electric current to follow the voice variations.

In the **double-button carbon-granule transmitter,** two buttons are provided, and the circuit of Fig. 353 is used. This device was extensively used in early radio and sound-amplifying systems. As will be noted, when the diaphragm moves in, the resistance of the granules in one cup increases and that in the other cup decreases. Because of this action, the current in one half of the transformer decreases and that in the other half increases, causing the magnetic effects to add. Second harmonic distortion occurs in a single-button transmitter, and this cancels out in the transformer as in the push-pull amplifier (page 465). In the double-button transmitter the diaphragm is stretched tightly, giving a natural period outside the usual audio range. The quality of a good double-button transformer is excellent, but objectionable carbon hiss exists.

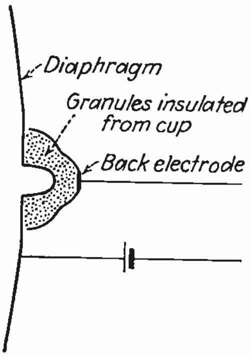

FIG. 352.—In nonpositional transmitters or microphones such as used in handsets, it is not possible for the granules to fall away from the diaphragm which also serves as the front electrode. Of course a seal is placed between the cup and diaphragm.

Generator-type Transmitters and Microphones.—In these no external voltage is applied as in the modifier type. They are essentially small electrical generators driven by energy from the sound waves. Since only about 10 microwatts of power is available in average conversational speech, and since these microphones (as they are usually called) are not very efficient in converting from sound waves to electric impulses, the output of the generator-type microphone is very low. This is in contrast with the modifier type, which merely acts to control an external source of power and which has a large output in comparison.

In the so-called **dynamic microphone** a small thin coil is attached to a diaphragm in such a way that the coil is free to move in a very strong magnetic field when sound waves strike the diaphragm. This action induces an electromotive force in the coil, and this electromotive force is then amplified with vacuum tubes. There are many varieties of these dynamic microphones on the market, and they have proved very satisfactory. The basic principles are illustrated by Fig. 354.

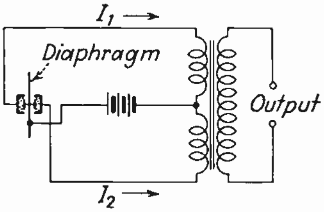

FIG. 353.—Circuit for a double-button carbon-granule transmitter or microphone.

The **ribbon, or velocity, microphone** is of the generator type but differs from other microphones in that it does not have a diaphragm. Instead, a very thin corrugated metallic ribbon is suspended in a very strong magnetic field. When the ribbon moves under the influence of sound waves, an electromotive force is induced in the ribbon, and this is amplified with vacuum tubes. The acoustic action is different from that for the other types for this reason: If a diaphragm is used, the back of it is to some extent sealed off (it must be open to barometric pressure changes); therefore, if a diaphragm is used, the pressure of the sound waves moves the diaphragm. In the usual ribbon microphone, both the front and the back of the microphone are open. The air particles in motion due to the sound waves move back and forth through the narrow slits between the ribbon and the pole pieces. This narrow slit has an acoustical "impedance," and because of the velocity of the air particles, a difference of acoustical pressure or a pressure gradient will exist between the two sides of the ribbon and cause it to move. The ribbon, or velocity, microphone is sensitive to sound from the front or the back but not from the side as indicated in Fig. 355. This gives the ribbon microphone a "figure-of-eight" directivity or response pattern. Microphones using dia-

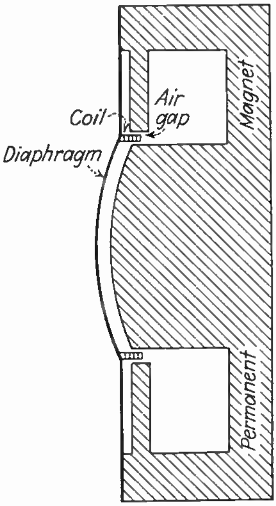

FIG. 354.—Elements of a dynamic microphone. It is essentially nondirectional. Sounds from any direction cause an increase and decrease of pressure at the front of the diaphragm, and it moves in and out.

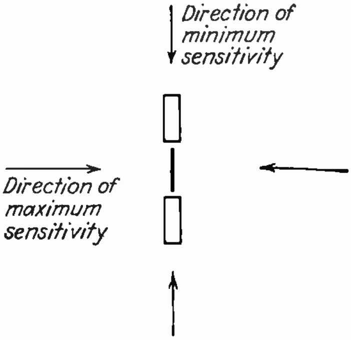

FIG. 355.—Top view of a ribbon microphone. The ends of the pole pieces are shown and the ribbon hangs down between them. For sounds coming at right angles to the ribbon the response is maximum. Sounds coming parallel to the ribbon merely flow by it, causing a negligible motion.

phragms are almost equally sensitive to sounds from all directions (unless they are specially constructed); that is, they are nondirectional and their directivity or response pattern is essentially a circle.

In the **crystal microphone** the generating element is a crystal of Rochelle salt which generates a voltage, corresponding to the sound waves, by virtue of its piezoelectric effect, explained as follows: Certain crystals, such as the type now being considered, produce an electromotive force if they are mechanically deformed in any way. Thus if a Rochelle salt crystal is provided with two metallic electrodes on opposite faces, and if sound waves are allowed to strike the crystal, a voltage, varying in accordance with the sound waves, will exist between the crystal electrodes. This voltage can then be amplified. In some crystal microphones, several crystal elements may be arranged in a series-parallel grouping to give a higher voltage output and a lower internal impedance. Also, some crystal microphones are equipped with a diaphragm that transmits its motion through a suitable mechanical connection to the crystal. Such a microphone is more sensitive but has a slightly poorer frequency response than those in which the sounds impinge directly on the crystal.

In certain combination microphones, the advantages of the pressure-operated microphone and the velocity, or pressure-gradient operated, microphone are combined to advantage. A pressure-operated microphone, such as the dynamic type, is essentially nondirectional; that is, it has a circular response pattern. As stated on page 513, a velocity, or pressure-gradient-operated microphone receives from the front and rear but not from the sides; that is, it has a figure-of-eight response pattern. If microphones of these two types are combined into one housing and are properly connected, then the circular response pattern of the dynamic elements combines with the figure-of-eight pattern of the ribbon element to give a highly directional combination, sensitive to sounds only from the front. In addition, the dynamic element or the ribbon element may be used alone, a choice of response patterns being thus given.

To summarize: There are two main types of transmitters or microphones. These are the modifier type and the generator type. In the former, the microphone merely controls the power output from a source such as a battery. In the latter, the operating power comes from the acoustic power in the sound waves. The generator type is in all respects a generator with a generated electromotive force causing an open-circuit voltage and having an internal impedance. The current-modifying carbon-granule type has the greatest output, but this is obtained at the sacrifice of quality. It is generally considered that radio microphones are very "sensitive." Telephone transmitters have a far greater output, however. It is the connected amplifier that makes the radio microphone so sensitive.

Receivers and Loud-speakers.—In these there are two parts to consider. (1) There is the **motor element,** or the portion that takes the electric power and converts it into mechanical power. (2) There is the **acoustic radiator** which takes the mechanical power and converts it into acoustic power or sound waves. The motor elements and acoustic radiators of importance in modern communication equipment will now be considered.

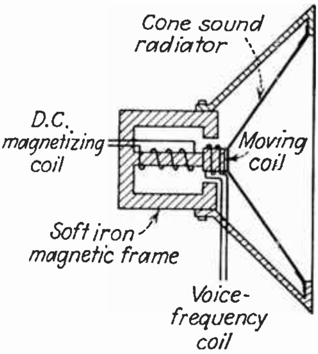

FIG. 356.—Essential parts of a moving-coil or dynamic loud-speaker.

Motor Elements.—In the **moving-coil, or electrodynamic, motor element,** a voice coil is free to move in an air gap in which is concentrated a very strong magnetic field. This magnetic field may be produced by a coil of wire carrying a current (Fig. 356) or by a strong permanent magnet. The amplified alternating signal currents pass through the voice coil and cause it to move back and forth in accordance with the signal variations. If a suitable acoustic radiator is attached to the voice coil, sound will be radiated into the air. The load impedance that a voice coil offers is quite low, being in the vicinity of 10 ohms *resistance* when being driven to about one-half capacity. The reader may justly wonder why the input impedance to a *coil* would be almost *pure resistance.* The answer is this: If a loud-speaker is radiating power, it must take power from the source. If it takes power, then it must offer a resistance load. Thus, when a good moving-coil dynamic speaker is being driven at a considerable output, it offers almost a pure resistance load. Of course, it would not do this at very low loads or if the field were suddenly removed (which should never be done).

In the **moving-armature,** or **magnetic, motor element** some portion of the magnetic circuit vibrates. The ordinary **telephone receiver** (Fig. 357) is of this classification. In this a permanent magnet with soft-iron pole pieces produces a strong pull on a thin iron diaphragm. Around the soft-iron pole pieces are wrapped coils of fine wire which carry the voice currents. These voice currents produce magnetic fields which add to and subtract from the magnetic pull of the permanent magnets. This causes the diaphragm to move in and out in accordance with the voice currents, and the diaphragm radiates sound waves. In the ordinary telephone receiver the iron diaphragm is part of the motor element and also the acoustic radiator. The many types of magnetic loud-speakers also are of the **moving-armature type.** One of these of the balanced-armature type is shown in Fig. 358. The alternating signal currents pass through the voice coil on the balanced iron armature and cause it to move about the pivot in accordance with the signal current. The motion of this armature is transmitted to an acoustic radiator. The telephone receiver and the magnetic loud-speaker both must offer a large resistive component of the input impedance when radiating sound. Ordinarily, their input impedance is high and inductive, however, because they have many turns of fine wire for a voice coil.

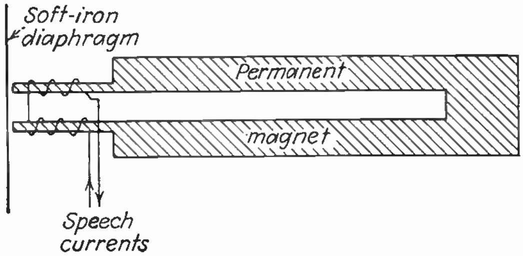

FIG. 357.—In the ordinary permanent-magnet telephone receiver the speech currents passing through the coils on the pole tips cause the soft-iron diaphragm to move in accordance with the current variations, thus reproducing the speech. The pole tips are soft iron welded on the permanent magnets.

The **crystal motor element** is used in receivers and in loud-speakers for reproduction at low power levels. The operation is the reverse of the crystal microphone. An alternating signal voltage is impressed on the crystal, and this signal voltage causes the dimensions of the crystal to change in accordance. These changes in dimensions will directly radiate sounds into the air, or they may be transmitted by a mechanical linkage to an acoustic radiator.

Acoustic Radiators.—The diaphragm of a telephone receiver is the acoustic radiator in addition to being a part of the motor element. In most other systems, the acoustic radiator and the motor element are separate.

The cone used with the so-called **dynamic loud-speaker** (Fig. 356) is a common type of acoustic radiator. The cone should be stiff so that it will move without flexing, and it should be free at the edges. The purpose of the cone is, of course, to impart energy to the air, creating a sound wave. The cone should be as light as possible in keeping with rigidity so that its inertia will be but a small factor. The inertia will affect the input impedance to the voice coil.

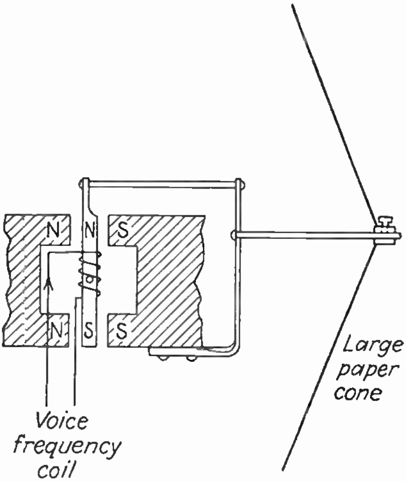

FIG. 358.—Cross-section of a magnetic balanced-armature driving unit coupled to a cone loud-speaker.

A baffle is used with the cone of a dynamic speaker, and the baffle is part of the acoustic radiator. The purpose of the baffle is this: When the cone moves out, a compression of the air is caused on the front side, and a reduction of pressure is caused on the back side. When the cone moves back, a compression of the air occurs on the back side, and a reduction of pressure occurs on the front side. It is in this way that a sound wave is created. Now if the cone is not large, then there is nothing to prevent the air from the compression side from flowing over the edge of the cone and equalizing the pressure on the back side. If this is done, the sound wave will be neutralized and little sound will be radiated. If a large baffle, say about 30 inches square, is used, then the length of the path is increased, and considerable sound is radiated before any cancellation can occur. Theoretically, the dimensions of the baffle should be such that the shortest air path between the front and the back of the cone is at least one-fourth the wave length of the lowest note to be reproduced. In a sense, the cabinet provides a baffle in the ordinary radio set.

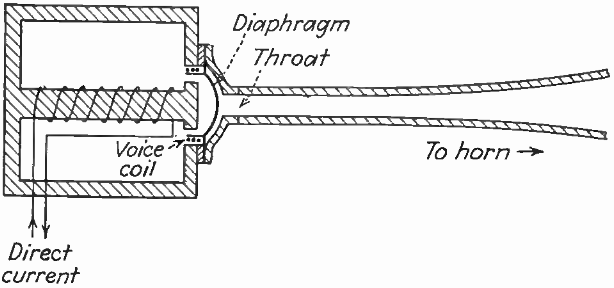

FIG. 359.—Details of a dynamic driving unit attached to a long horn. A small coil of wire is attached to the arched diaphragm. This coil is represented by dots, and is situated in a strong magnetic field which is produced either by an electromagnet carrying direct current (as shown) or by a permanent magnet. The voice currents pass through the small coil attached to the diaphragm, and cause the diaphragm to move in accordance with the voice currents.

The **loud-speaker horn** is a portion of the acoustic radiator in the **horn-type loud-speaker.** The horn is attached to a driving unit or motor element as shown in Fig. 359. The motor element is of the dynamic type, but instead of having a large diaphragm, it has a *small* diaphragm. The diaphragm is in a small air chamber containing little air, so that there is insufficient air present to act as a cushion on the diaphragm. Thus, as the diaphragm is driven out it forces air out of the throat of the horn, and when it moves in, it sucks air in through the throat of the horn. This action prevents violent movements of the diaphragm, and it is "damped" by the column of air in the horn to which it imparts energy and which radiates sound. The horn does not merely direct sound. It provides a volume of air which is affected by the diaphragm. Without a horn the diaphragm would only radiate a feeble sound, because it would have no acoustic load on it. The horn must have a logarith-
]

mic taper, with the cross-sectional area doubling with equal distances along its length. This is to prevent "impedance discontinuities" which will cause acoustic reflections and hence nonuniform transmission to the mouth of the horn. The mouth must be large if low frequencies are to be effectively radiated, the throat must be small to put a good load on the diaphragm, and the rate of taper must not be too great for good frequency response. All this means that a good horn must be quite long.

The so-called **directional baffles** or **short horns** are a compromise between the flat baffle and the horn. Thus, it is common practice to mount rather small dynamic speakers with cone diaphragms in horns that are fairly short. These horns improve the acoustic load on the diaphragm and make it directional and more efficient.

To summarize: Receivers and loud-speakers are composed of two parts, the motor element and the acoustic radiator. As was previously mentioned, the microphone that converts from acoustic power to electric power is not efficient. Likewise the power-conversion efficiency of loud-speakers that change from electric impulses back to sounds is very low. In approximate figures, the efficiency of a telephone receiver is probably less than 1 per cent, the efficiency of a dynamic speaker in a baffle is about 2 to 5 per cent, the efficiency of a dynamic speaker in a directional baffle or short horn is about 5 to 10 per cent, and the efficiency of a well-designed driving unit on a good long horn is as high as 50 per cent.

SUMMARY

Sound may be considered objectively as a wave motion or subjectively as a sensation.

Sound waves occur as pressure waves in the air and travel at about 1125 feet per second.

When sound waves strike a rigid wall, some of the sound energy is transmitted through the wall, some is absorbed as heat within the wall and at the surface of the wall, and some energy is absorbed.

An echo is a sound reflection that is distinct. The phenomenon of two waves beating together is a phenomenon of hearing.

Reverberation is caused by wave reflection, but the reflections are a prolonging or "jumble" of sound. Reverberation is in a sense indistinct echoes. Reverberation is the most common acoustical fault of auditoriums.

In the construction of an auditorium, its acoustical characteristics need not be left to chance. Its reverberation time can be calculated, and additional sound-absorbing material can be added to give the correct reverberation time.

Speech sounds are very weak compared with ordinary amounts of power, only about 10 microwatts being present in the ordinary conversational tone. The frequencies present in speech cover a band from at least 100 to 10,000 cycles, but a band from 250 to 3000 cycles gives good intelligible telephone conversations.

Musical sounds cover a frequency range from at least 50 cycles to 15,000 cycles, but a band from 50 to 8000 cycles will give excellent reproduction.

The average ear will respond to frequencies from about 20 to 20,000 cycles but is most sensitive at several thousand cycles.

Noise measurements are made in decibels above an arbitrary zero level which is approximately the threshold of hearing.

Transmitters and microphones are used to change from acoustic to electric impulses. They are classified as modifier types or as generator types.

Receivers and loud-speakers are composed of two basic parts, the motor element and the acoustic radiator.

QUESTIONS

1. Give several reasons why a study of acoustics and related phenomena are important in communication.

2. Explain what occurs when a sound wave strikes a solid wall, and what happens when a wave strikes a nonrigid wall.

3. Explain the acoustic phenomenon called beating.

4. What is meant by reverberation and by reverberation time?

5. An auditorium is highly reverberant, and sound-absorbing material is added to the ceiling. When this is done, an echo is apparent. What is the probable cause, and what is a possible remedy?

6. What types of materials make good sound absorbers? What practical considerations are there in selecting a sound-absorbing material for an auditorium?

7. Discuss the characteristics of the human ear, and explain how these affect the design of a sound-level meter for measuring noise.

8. Enumerate the basic types of microphones, and give an example of each type.

9. Why must the power output of a generator-type microphone be very low as compared with that of the modifier type?

10. Why does the ribbon microphone have a figure-of-eight response pattern?

11. What two basic parts are there to a loud-speaker?

12. What would happen if during operation the field excitation were suddenly removed from a dynamic speaker?

13. Why must the input impedance of a loud-speaker be largely resistive when it is radiating sound?

14. What is the function of a baffle on a loud-speaker?

15. What is the function of the loud-speaker horn?

PROBLEMS

1. Referring to the discussion on page 502, how far must a speaker be from a reflecting object before an echo is noticeable? If a speaker is 500 feet from a large reflecting surface, what will be the "echo time"?

2. Select some large meeting hall or auditorium in your neighborhood, calculate its reverberation time, and determine if it is correct when the audience is one-half capacity, and when it is full. Make recommendations for its acoustic correction if such is advisable.

3. On page 508 the volume range of an orchestra is given as 100,000 to 1. What is this range expressed in decibels? The range for acceptable reproduction is given as 10,000 to 1. Express this in decibels.

4. The output impedance of an amplifier is 15 ohms, and it is to operate four horn driving units the voice coils of which are 15 ohms each. How should the voice coils be connected for best results? No transformer is to be used. If 25 watts electric power is put into the system, what will be the approximate acoustic power output?

5. A dynamic loud-speaker having a 10-ohm voice coil is to be connected to a vacuum tube having an optimum load impedance of 2000 ohms resistance. How should these connections be made?

APPENDIX

NATURAL TRIGONOMETRIC FUNCTIONS

Of angles for each minute from 0° to 90°, correct
to five significant figures

0°

′	sin	tan	cot	cos	
0	.00000	.00000	∞	1.0000	60
1	029	029	3437.7	000	59
2	058	058	1718.9	000	58
3	087	087	1145.9	000	57
4	116	116	859.44	000	56
5	.00145	.00145	687.55̄	1.0000	**55**
6	175̄	175̄	572.96	000	54
7	204	204	491.11	000	53
8	233	233	429.72	000	52
9	262	262	381.97	000	51
10	.00291	.00291	343.77	1.0000	**50**
11	320	320	312.52	.99999	49
12	349	349	286.48	999	48
13	378	378	264.44	999	47
14	407	407	245.55	999	46
15	.00436	.00436	229.18	.99999	**45**
16	465	465	214.86	999	44
17	495̄	495̄	202.22	999	43
18	524	524	190.98	999	42
19	553	553	180.93	998	41
20	.00582	.00582	171.89	.99998	**40**
21	611	611	163.70	998	39
22	640	640	156.26	998	38
23	669	669	149.47	998	37
24	698	698	143.24	998	36
25	.00727	.00727	137.51	.99997	**35**
26	756	756	132.22	997	34
27	785	785	127.32	997	33
28	814	815̄	122.77	997	32
29	844	844	118.54	996	31
30	.00873	.00873	114.59	.99996	**30**
31	902	902	110.89	996	29
32	931	931	107.43	996	28
33	960	960	104.17	995	27
34	.00989	.00989	101.11	995	26
35	.01018	.01018	98.218	.99995̄	**25**
36	047	047	95.489	995̄	24
37	076	076	92.908	994	23
38	105	105	90.463	994	22
39	134	135̄	88.144	994	21
40	.01164	.01164	85.940	.99993	**20**
41	193	193	83.844	993	19
42	222	222	81.847	993	18
43	251	251	79.943	992	17
44	280	280	78.126	992	16
45	.01309	.01309	76.390	.99991	**15**
46	338	338	74.729	991	14
47	367	367	73.139	991	13
48	396	396	71.615	990	12
49	425	425	70.153	990	11
50	.01454	.01455̄	68.750	.99989	**10**
51	483	484	67.402	989	9
52	513	513	66.105	989	8
53	542	542	64.858	988	7
54	571	571	63.657	988	6
55	.01600	.01600	62.499	.99987	**5**
56	629	629	61.383	987	4
57	658	658	60.306	986	3
58	687	687	59.266	986	2
59	716	716	58.261	985	1
60	.01745	.01746	57.290	.99985̄	0
	cos	cot	tan	sin	′

89°

1°

′	sin	tan	cot	cos	
0	.01745	.01746	57.290	.99985̄	60
1	774	775̄	56.351	984	59
2	803	804	55.442	984	58
3	832	833	54.561	983	57
4	862	862	53.709	983	56
5	.01891	.01891	52.882	.99982	**55**
6	920	920	52.081	982	54
7	949	949	51.303	981	53
8	.01978	.01978	50.549	980	52
9	.02007	.02007	49.816	980	51
10	.02036	.02036	49.104	.99979	**50**
11	065	066	48.412	979	49
12	094	095̄	47.740	978	48
13	123	124	47.085	977	47
14	152	153	46.449	977	46
15	.02181	.02182	45.829	.99976	**45**
16	211	211	45.226	976	44
17	240	240	44.639	975̄	43
18	269	269	44.066	974	42
19	298	298	43.508	974	41
20	.02327	.02328	42.964	.99973	**40**
21	356	357	42.433	972	39
22	385	386	41.916	972	38
23	414	415̄	41.411	971	37
24	443	444	40.917	970	36
25	.02472	.02473	40.436	.99969	**35**
26	501	502	39.965	969	34
27	530	531	39.506	968	33
28	560	560	39.057	967	32
29	589	589	38.618	966	31
30	.02618	.02619	38.188	.99966	**30**
31	647	648	37.769	965̄	29
32	676	677	37.358	964	28
33	705̄	706	36.956	963	27
34	734	735	36.563	963	26
35	.02763	.02764	36.178	.99962	**25**
36	792	793	35.801	961	24
37	821	822	35.431	960	23
38	850	851	35.070	959	22
39	879	881	34.715	959	21
40	.02908	.02910	34.368	.99958	**20**
41	938	939	34.027	957	19
42	967	968	33.694	956	18
43	.02996	.02997	33.366	955	17
44	.03025̄	.03026	33.045	954	16
45	.03054	.03055	32.730	.99953	**15**
46	083	084	32.421	952	14
47	112	114	32.118	952	13
48	141	143	31.821	951	12
49	170	172	31.528	95̄0	11
50	.03199	.03201	31.242	.99949	**10**
51	228	230	30.960	948	9
52	257	259	30.683	947	8
53	286	288	30.412	946	7
54	316	317	30.145̄	945	6
55	.03345̄	.03346	29.882	.99944	**5**
56	374	376	29.624	943	4
57	403	405̄	29.371	942	3
58	432	434	29.122	941	2
59	461	463	28.877	940	1
60	.03490	.03492	28.636	.99939	0
	cos	cot	tan	sin	′

88°

2°

′	sin	tan	cot	cos	
0	.03490	.03492	28.636	.99939	60
1	519	521	.399	938	59
2	548	550	28.166	937	58
3	577	579	27.937	936	57
4	606	609	.712	935	56
5	.03635	.03638	27.490	.99934	55
6	664	667	.271	933	54
7	693	696	27.057	932	53
8	723	725	26.845	931	52
9	752	754	.637	930	51
10	.03781	.03783	26.432	.99929	50
11	810	812	.230	927	49
12	839	842	26.031	926	48
13	868	871	25.835	925	47
14	897	900	.642	924	46
15	.03926	.03929	25.452	.99923	45
16	955	958	.264	922	44
17	.03984	.03987	25.080	921	43
18	.04013	.04016	24.898	919	42
19	042	046	.719	918	41
20	.04071	.04075	24.542	.99917	40
21	100	104	.368	916	39
22	129	133	.196	915	38
23	159	162	24.026	913	37
24	188	191	23.859	912	36
25	.04217	.04220	23.695	.99911	35
26	246	250	.532	910	34
27	275	279	.372	909	33
28	304	308	.214	907	32
29	333	337	23.058	906	31
30	.04362	.04366	22.904	.99905	30
31	391	395	.752	904	29
32	420	424	.602	902	28
33	449	454	.454	901	27
34	478	483	.308	900	26
35	.04507	.04512	22.164	.99898	25
36	536	541	22.022	897	24
37	565	570	21.881	896	23
38	594	599	.743	894	22
39	623	628	.606	893	21
40	.04653	.04658	21.470	.99892	20
41	682	687	.337	890	19
42	711	716	.205	889	18
43	740	745	21.075	888	17
44	769	774	20.946	886	16
45	.04798	.04803	20.819	.99885	15
46	827	833	.693	883	14
47	856	862	.569	882	13
48	885	891	.446	881	12
49	914	920	.325	879	11
50	.04943	.04949	20.206	.99878	10
51	.04972	.04978	20.087	876	9
52	.05001	.05007	19.970	875	8
53	030	037	.855	873	7
54	059	066	.740	872	6
55	.05088	.05095	19.627	.99870	5
56	117	124	.516	869	4
57	146	153	.405	867	3
58	175	182	.296	866	2
59	205	212	.188	864	1
60	.05234	.05241	19.081	.99863	0
	cos	cot	tan	sin	′

87°

3°

′	sin	tan	cot	cos	
0	.05234	.05241	19.081	.99863	60
1	263	270	18.976	861	59
2	292	299	.871	860	58
3	321	328	.768	858	57
4	350	357	.666	857	56
5	.05379	.05387	18.564	.99855	55
6	408	416	.464	854	54
7	437	445	.366	852	53
8	466	474	.268	851	52
9	495	503	.171	849	51
10	.05524	.05533	18.075	.99847	50
11	553	562	17.980	846	49
12	582	591	.886	844	48
13	611	620	.793	842	47
14	640	649	.702	841	46
15	.05669	.05678	17.611	.99839	45
16	698	708	.521	838	44
17	727	737	.431	836	43
18	756	766	.343	834	42
19	785	795	.256	833	41
20	.05814	.05824	17.169	.99831	40
21	844	854	17.084	829	39
22	873	883	16.999	827	38
23	902	912	.915	826	37
24	931	941	.832	824	36
25	.05960	.05970	16.750	.99822	35
26	.05989	.05999	.668	821	34
27	.06018	.06029	.587	819	33
28	047	058	.507	817	32
29	076	087	.428	815	31
30	.06105	.06116	16.350	.99813	30
31	134	145	.272	812	29
32	163	175	.195	810	28
33	192	204	.119	808	27
34	221	233	16.043	806	26
35	.06250	.06262	15.969	.99804	25
36	279	291	.895	803	24
37	308	321	.821	801	23
38	337	350	.748	799	22
39	366	379	.676	797	21
40	.06395	.06408	15.605	.99795	20
41	424	438	.534	793	19
42	453	467	.464	792	18
43	482	496	.394	790	17
44	511	525	.325	788	16
45	.06540	.06554	15.257	.99786	15
46	569	584	.189	784	14
47	598	613	.122	782	13
48	627	642	15.056	780	12
49	656	671	14.990	778	11
50	.06685	.06700	14.924	.99776	10
51	714	730	.860	774	9
52	743	759	.795	772	8
53	773	788	.732	770	7
54	802	817	.669	768	6
55	.06831	.06847	14.606	.99766	5
56	860	876	.544	764	4
57	889	905	.482	762	3
58	918	934	.421	760	2
59	947	963	.361	758	1
60	.06976	.06993	14.301	.99756	0
	cos	cot	tan	sin	′

86°

4°

′	sin	tan	cot	cos	
0	.06976	.06993	14.301	.99756	**60**
1	.07005̄	.07022	.241	754	59
2	034	051	.182	752	58
3	063	080	.124	750	57
4	092	110	.065	748	56
5	.07121	.07139	14.008	.99746	**55**
6	15̄0	168	13.951	744	54
7	179	197	.894	742	53
8	208	227	.838	740	52
9	237	256	.782	738	51
10	.07266	.07285	13.727	.99736	**50**
11	295̄	314	.672	734	49
12	324	344	.617	731	48
13	353	373	.563	729	47
14	382	402	.510	727	46
15	.07411	.07431	13.457	.99725	**45**
16	440	461	.404	723	44
17	469	490	.352	721	43
18	498	519	.300	719	42
19	527	548	.248	716	41
20	.07556	.07578	13.197	.99714	**40**
21	585̄	607	.146	712	39
22	614	636	.096	710	38
23	643	665	13.046	708	37
24	672	695̄	12.996	705	36
25	.07701	.07724	12.947	.99703	**35**
26	730	753	.898	701	34
27	759	782	.85̄0	699	33
28	788	812	.801	696	32
29	817	841	.754	694	31
30	.07846	.07870	12.706	.99692	**30**
31	875̄	899	.659	689	29
32	904	929	.612	687	28
33	933	958	.566	685̄	27
34	962	.07987	.520	683	26
35	.07991	.08017	12.474	.99680	**25**
36	.08020	046	.429	678	24
37	049	075	.384	676	23
38	078	104	.339	673	22
39	107	134	.295̄	671	21
40	.08136	.08163	12.251	.99668	**20**
41	165̄	192	.207	666	19
42	194	221	.163	664	18
43	223	251	.120	661	17
44	252	280	.077	659	16
45	.08281	.08309	12.035̄	.99657	**15**
46	310	339	11.992	654	14
47	339	368	.950	652	13
48	368	397	.909	649	12
49	397	427	.867	647	11
50	.08426	.08456	11.826	.99644	**10**
51	455̄	485	.785	642	9
52	484	514	.745̄	639	8
53	513	544	.705̄	637	7
54	542	573	.664	635̄	6
55	.08571	.08602	11.625̄	.99632	**5**
56	600	632	.585	630	4
57	629	661	.546	627	3
58	658	690	.507	625̄	2
59	687	720	.468	622	1
60	.08716	.08749	11.430	.99619	**0**
	cos	**cot**	**tan**	**sin**	′

85°

5°

′	sin	tan	cot	cos	
0	.08716	.08749	11.430	.99619	**60**
1	745̄	778	.392	617	59
2	774	807	.354	614	58
3	803	837	.316	612	57
4	831	866	.279	609	56
5	.08860	.08895	11.242	.99607	**55**
6	889	925̄	.205̄	604	54
7	918	954	.168	602	53
8	947	.08983	.132	599	52
9	.08976	.09013	.095	596	51
10	.09005	.09042	11.059	.99594	**50**
11	034	071	11.024	591	49
12	063	101	10.988	588	48
13	092	130	.953	586	47
14	121	159	.918	583	46
15	.09150	.09189	10.883	.99580	**45**
16	179	218	.848	578	44
17	208	247	.814	575	43
18	237	277	.780	572	42
19	266	306	.746	570	41
20	.09295̄	.09335	10.712	.99567	**40**
21	324	365̄	.678	564	39
22	353	394	.645̄	562	38
23	382	423	.612	559	37
24	411	453	.579	556	36
25	.09440	.09482	10.546	.99553	**35**
26	469	511	.514	551	34
27	498	541	.481	548	33
28	527	570	.449	545	32
29	556	600	.417	542	31
30	.09585̄	.09629	10.385	.99540	**30**
31	614	658	.354	537	29
32	642	688	.322	534	28
33	671	717	.291	531	27
34	700	746	.260	528	26
35	.09729	.09776	10.229	.99526	**25**
36	758	805	.199	523	24
37	787	834	.168	520	23
38	816	864	.138	517	22
39	845	893	.108	514	21
40	.09874	.09923	10.078	.99511	**20**
41	903	952	.048	508	19
42	932	.09981	10.019	506	18
43	961	.10011	9.9893	503	17
44	.09990	040	.9601	5̄00	16
45	.10019	.10069	9.9310	.99497	**15**
46	048	099	.9021	494	14
47	077	128	.8734	491	13
48	106	158	.8448	488	12
49	135̄	187	.8164	485	11
50	.10164	.10216	9.7882	.99482	**10**
51	192	246	.7601	479	9
52	221	275	.7322	476	8
53	250	305̄	.7044	473	7
54	279	334	.6768	470	6
55	.10308	.10363	9.6493	.99467	**5**
56	337	393	.6220	464	4
57	366	422	.5949	461	3
58	395̄	452	.5679	458	2
59	424	481	.5411	455	1
60	.10453	.10510	9.5144	.99452	**0**
	cos	**cot**	**tan**	**sin**	′

84°

6°

′	sin	tan	cot	cos	
0	.10453	.10510	9.5144	.99452	60
1	482	540	.4878	449	59
2	511	569	.4614	446	58
3	540	599	.4352	443	57
4	569	628	.4090	440	56
5	.10597	.10657	9.3831	.99437	**55**
6	626	687	.3572	434	54
7	655	716	.3315	431	53
8	684	746	.3060	428	52
9	713	775	.2806	424	51
10	.10742	.10805̄	9.2553	.99421	**50**
11	771	834	.2302	418	49
12	800	863	.2052	415	48
13	829	893	.1803	412	47
14	858	922	.1555	409	46
15	.10887	.10952	9.1309	.99406	**45**
16	916	.10981	.1065̄	402	44
17	945̄	.11011	.0821	399	43
18	.10973	040	.0579	396	42
19	.11002	070	.0338	393	41
20	.11031	.11099	9.0098	.99390	**40**
21	060	128	8.9860	386	39
22	089	158	.9623	383	38
23	118	187	.9387	380	37
24	147	217	.9152	377	36
25	.11176	.11246	8.8919	.99374	**35**
26	205̄	276	.8686	370	34
27	234	305	.8455	367	33
28	263	335̄	.8225	364	32
29	291	364	.7996	360	31
30	.11320	.11394	8.7769	.99357	**30**
31	349	423	.7542	354	29
32	378	452	.7317	351	28
33	407	482	.7093	347	27
34	436	511	.6870	344	26
35	.11465̄	.11541	8.6648	.99341	**25**
36	494	570	.6427	337	24
37	523	600	.6208	334	23
38	552	629	.5989	331	22
39	580	659	.5772	327	21
40	.11609	.11688	8.5555	.99324	**20**
41	638	718	.5340	320	19
42	667	747	.5126	317	18
43	696	777	.4913	314	17
44	725̄	806	.4701	310	16
45	.11754	.11836	8.4490	.99307	**15**
46	783	865	.4280	303	14
47	812	895̄	.4071	300	13
48	840	924	.3863	297	12
49	869	954	.3656	293	11
50	.11898	.11983	8.345̄0	.99290	**10**
51	927	.12013	.3245̄	286	9
52	956	042	.3041	283	8
53	.11985̄	072	.2838	279	7
54	.12014	101	.2636	276	6
55	.12043	.12131	8.2434	.99272	**5**
56	071	160	.2234	269	4
57	100	190	.2035	265	3
58	129	219	.1837	262	2
59	158	249	.1640	258	1
60	.12187	.12278	8.1443	.99255̄	**0**
	cos	cot	tan	sin	′

83°

7°

′	sin	tan	cot	cos	
0	.12187	.12278	8.1443	.99255̄	60
1	216	308	.1248	251	59
2	245̄	338	.1054	248	58
3	274	367	.0860	244	57
4	302	397	.0667	240	56
5	.12331	.12426	8.0476	.99237	**55**
6	360	456	.0285̄	233	54
7	389	485	8.0095̄	230	53
8	418	515̄	7.9906	226	52
9	447	544	.9718	222	51
10	.12476	.12574	7.9530	.99219	**50**
11	504	603	.9344	215	49
12	533	633	.9158	211	48
13	562	662	.8973	208	47
14	591	692	.8789	204	46
15	.12620	.12722	7.8606	.99200	**45**
16	649	751	8424	197	44
17	678	781	.8243	193	43
18	706	810	.8062	189	42
19	735	840	.7882	186	41
20	.12764	.12869	7.7704	.99182	**40**
21	793	899	.7525	178	39
22	822	929	.7348	175̄	38
23	851	958	.7171	171	37
24	880	.12988	.6996	167	36
25	.12908	.13017	7.6821	.99163	**35**
26	937	047	.6647	160	34
27	966	076	.6473	156	33
28	.12995̄	106	.6301	152	32
29	.13024	136	.6129	148	31
30	.13053	.13165	7.5958	.99144	**30**
31	081	195̄	.5787	141	29
32	110	224	.5618	137	28
33	139	254	.5449	133	27
34	168	284	.5281	129	26
35	.13197	.13313	7.5113	.99125	**25**
36	226	343	.4947	122	24
37	254	372	.4781	118	23
38	283	402	.4615	114	22
39	312	432	.4451	110	21
40	.13341	.13461	7.4287	.99106	**20**
41	370	491	.4124	102	19
42	399	521	.3962	098	18
43	427	550	.3800	094	17
44	456	580	.3639	091	16
45	.13485	.13609	7.3479	.99087	**15**
46	514	639	.3319	083	14
47	543	669	.3160	079	13
48	572	698	.3002	075̄	12
49	600	728	.2844	071	11
50	.13629	.13758	7.2687	.99067	**10**
51	658	787	.2531	063	9
52	687	817	.2375	059	8
53	716	846	.2220	055̄	7
54	744	876	.2066	051	6
55	.13773	.13906	7.1912	.99047	**5**
56	802	935	.1759	043	4
57	831	965	.1607	039	3
58	860	.13995̄	.1455	035̄	2
59	889	.14024	.1304	031	1
60	.13917	.14054	7.1154	.99027	**0**
	cos	cot	tan	sin	′

82°

8°

′	sin	tan	cot	cos	
0	.13917	.14054	7.1154	.99027	60
1	946	084	.1004	023	59
2	.139755̄	113	.0855̄	019	58
3	.14004	143	.0706	015̄	57
4	033	173	.0558	011	56
5	.14061	.14202	7.0410	.99006	55
6	090	232	.0264	.99002	54
7	119	262	7.0117	.98998	53
8	148	291	6.9972	994	52
9	177	321	.9827	990	51
10	.14205	.14351	6.9682	.98986	50
11	234	381	.9538	982	49
12	263	410	.9395	978	48
13	292	440	.9252	973	47
14	320	470	.9110	969	46
15	.14349	.14499	6.8969	.98965	45
16	378	529	.8828	961	44
17	407	559	.8687	957	43
18	436	588	.8548	953	42
19	464	618	.8408	948	41
20	.14493	.14648	6.8269	.98944	40
21	522	678	.8131	940	39
22	551	707	.7994	936	38
23	580	737	.7856	931	37
24	608	767	.7720	927	36
25	.14637	.14796	6.7584	.98923	35
26	666	826	.7448	919	34
27	695̄	856	.7313	914	33
28	723	886	.7179	910	32
29	752	915	.7045̄	906	31
30	.14781	.14945	6.6912	.98902	30
31	810	.14975̄	.6779	897	29
32	838	.15005̄	.6646	893	28
33	867	034	.6514	889	27
34	896	064	.6383	884	26
35	.14925̄	.15094	6.6252	.98880	25
36	954	124	.6122	876	24
37	.14982	153	.5992	871	23
38	.15011	183	.5863	867	22
39	040	213	.5734	863	21
40	.15069	.15243	6.5606	.98858	20
41	097	272	.5478	854	19
42	126	302	.5350	849	18
43	155̄	332	.5223	845̄	17
44	184	362	.5097	841	16
45	.15212	.15391	6.4971	.98836	15
46	241	421	.4846	832	14
47	270	451	.4721	827	13
48	299	481	.4596	823	12
49	327	511	.4472	818	11
50	.15356	.15540	6.4348	.98814	10
51	385̄	570	.4225	809	9
52	414	600	.4103	805̄	8
53	442	630	.3980	800	7
54	471	660	.3859	796	6
55	.155̄00	.15689	6.3737	.98791	5
56	529	719	.3617	787	4
57	557	749	.3496	782	3
58	586	779	.3376	778	2
59	615̄	809	.3257	773	1
60	.15643	.15838	6.3138	.98769	0
	cos	cot	tan	sin	′

81°

9°

′	sin	tan	cot	cos	
0	.15643	.15838	6.3138	.98769	60
1	672	868	.3019	764	59
2	701	898	.2901	760	58
3	730	928	.2783	755	57
4	758	958	.2666	751	56
5	.15787	.15988	6.2549	.98746	55
6	816	.16017	.2432	741	54
7	845̄	047	.2316	737	53
8	873	077	.2200	732	52
9	902	107	.2085	728	51
10	.15931	.16137	6.1970	.98723	50
11	959	167	.1856	718	49
12	.15988	196	.1742	714	48
13	.16017	226	.1628	709	47
14	046	256	.1515	704	46
15	.16074	.16286	6.1402	.98700	45
16	103	316	.1290	695̄	44
17	132	346	.1178	690	43
18	160	376	.1066	686	42
19	189	405	.0955	681	41
20	.16218	.16435	6.0844	.98676	40
21	246	465	.0734	671	39
22	275	495	.0624	667	38
23	304	525	.0514	662	37
24	333	555̄	.0405	657	36
25	.16361	.16585̄	6.0296	.98652	35
26	390	615̄	.0188	648	34
27	419	645̄	6.0080	643	33
28	447	674	5.9972	638	32
29	476	704	.9865̄	633	31
30	.16505̄	.16734	5.9758	.98629	30
31	533	764	.9651	624	29
32	562	794	.9545̄	619	28
33	591	824	.9439	614	27
34	620	854	.9333	609	26
35	.16648	.16884	5.9228	.98604	25
36	677	914	.9124	600	24
37	706	944	.9019	595̄	23
38	734	.16974	8915	590	22
39	763	.17004	.8811	585	21
40	.16792	.17033	5.8708	.98580	20
41	820	063	.8605	575	19
42	849	093	.8502	570	18
43	878	123	.8400	565	17
44	906	153	.8298	561	16
45	.16935̄	.17183	5.8197	.98556	15
46	964	213	.8095	551	14
47	16992	243	.7994	546	13
48	.17021	273	.7894	541	12
49	05̄0	303	.7794	536	11
50	.17078	.17333	5.7694	.98531	10
51	107	363	.7594	526	9
52	136	393	.7495̄	521	8
53	164	423	.7396	516	7
54	193	453	.7297	511	6
55	.17222	.17483	5.7199	.98506	5
56	250	513	.7101	501	4
57	279	543	.7004	496	3
58	308	573	.6906	491	2
59	336	603	.6809	486	1
60	.17365̄	.17633	5.6713	.98481	0
	cos	cot	tan	sin	′

80°

10°

′	sin	tan	cot	cos	
0	.17365̄	.17633	5.6713	.98481	60
1	393	663	.6617	476	59
2	422	693	.6521	471	58
3	451	723	.6425̄	466	57
4	479	753	.6329	461	56
5	.17508	.17783	5.6234	.98455	55
6	537	813	.6140	450	54
7	565	843	.6045	445	53
8	594	873	.5951	440	52
9	623	903	.5857	435̄	51
10	.17651	.17933	5.5764	.98430	50
11	680	963	.5671	425̄	49
12	708	.17993	.5578	420	48
13	737	.18023	.5485	414	47
14	766	053	.5393	409	46
15	.17794	.18083	5.5301	.98404	45
16	823	113	.5209	399	44
17	852	143	.5118	394	43
18	880	173	.5026	389	42
19	909	203	.4936	383	41
20	.17937	.18233	5.4845	.98378	40
21	966	263	.4755̄	373	39
22	.17995̄	293	.4665̄	368	38
23	.18023	323	.4575	362	37
24	052	353	.4486	357	36
25	.18081	.18384	5.4397	.98352	35
26	109	414	.4308	347	34
27	138	444	.4219	341	33
28	166	474	.4131	336	32
29	195̄	504	.4043	331	31
30	.18224	.18534	5.3955	.98325	30
31	252	564	.3868	320	29
32	281	594	.3781	315̄	28
33	309	624	.3694	310	27
34	338	654	.3607	304	26
35	.18367	.18684	5.3521	.98299	25
36	395	714	.3435̄	294	24
37	424	745̄	.3349	288	23
38	452	775̄	.3263	283	22
39	481	805̄	.3178	277	21
40	.18509	.18835̄	5.3093	.98272	20
41	538	865	.3008	267	19
42	567	895	.2924	261	18
43	595	925	.2839	256	17
44	624	955	.2755	250	16
45	.18652	.18986	5.2672	.98245	15
46	681	.19016	.2588	240	14
47	710	046	.2505̄	234	13
48	738	076	.2422	229	12
49	767	106	.2339	223	11
50	.18795	.19136	5.2257	.98218	10
51	824	166	.2174	212	9
52	852	197	.2092	207	8
53	881	227	.2011	201	7
54	910	257	.1929	196	6
55	.18938	.19287	5.1848	.98190	5
56	967	317	.1767	185̄	4
57	.18995	347	.1686	179	3
58	.19024	378	.1606	174	2
59	052	408	.1526	168	1
60	.19081	.19438	5.1446	.98163	0
	cos	cot	tan	sin	′

79°

11°

′	sin	tan	cot	cos	
0	.19081	.19438	5.1446	.98163	60
1	109	468	.1366	157	59
2	138	498	.1286	152	58
3	167	529	.1207	146	57
4	195	559	.1128	140	56
5	.19224	.19589	5.1049	.98135̄	55
6	252	619	.0970	129	54
7	281	649	.0892	124	53
8	309	680	.0814	118	52
9	338	710	.0736	112	51
10	.19366	.19740	5.0658	.98107	50
11	395̄	770	.0581	101	49
12	423	801	.0504	096	48
13	452	831	.0427	090	47
14	481	861	.035̄0	084	46
15	.19509	.19891	5.0273	.98079	45
16	538	921	.0197	073	44
17	566	952	.0121	067	43
18	595̄	.19982	5.0045	061	42
19	623	.20012	4.9969	056	41
20	.19652	.20042	4.9894	.98050	40
21	680	073	.9819	044	39
22	709	103	.9744	039	38
23	737	133	.9669	033	37
24	766	164	.9594	027	36
25	.19794	.20194	4.9520	.98021	35
26	823	224	.9446	016	34
27	851	254	.9372	010	33
28	880	285̄	.9298	.98004	32
29	908	315̄	.9225̄	.97998	31
30	.19937	.20345	4.9152	.97992	30
31	965	376	.9078	987	29
32	.19994	406	.9006	981	28
33	.20022	436	.8933	975	27
34	051	466	.8860	969	26
35	.20079	.20497	4.8788	.97963	25
36	108	527	.8716	958	24
37	136	557	.8644	952	23
38	165̄	588	.8573	946	22
39	193	618	.8501	940	21
40	.20222	.20648	4.8430	.97934	20
41	250	679	.8359	928	19
42	279	709	.8288	922	18
43	307	739	.8218	916	17
44	336	770	.8147	910	16
45	.20364	.20800	4.8077	.97905̄	15
46	393	830	.8007	899	14
47	421	861	.7937	893	13
48	450	891	.7867	887	12
49	478	921	.7798	881	11
50	.20507	.20952	4.7729	.97875̄	10
51	535	.20982	.7659	869	9
52	563	.21013	.7591	863	8
53	592	043	.7522	857	7
54	620	073	.7453	851	6
55	.20649	.21104	4.7385	.97845̄	5
56	677	134	.7317	839	4
57	706	164	.7249	833	3
58	734	195̄	.7181	827	2
59	763	225	.7114	821	1
60	.20791	.21256	4.7046	.97815̄	0
	cos	cot	tan	sin	′

78°

12°

′	sin	tan	cot	cos	
0	.20791	.21256	4.7046	.97815̅	60
1	820	286	.6979	809	59
2	848	316	.6912	803	58
3	877̅	347	.6845	797	57
4	905̅	377	.6779	791	56
5	.20933	.21408	4.6712	.97784	55
6	962	438	.6646	778	54
7	.20990	469	.6580	772	53
8	.21019	499	.6514	766	52
9	047	529	.6448	760	51
10	.21076	.21560	4.6382	.97754	50
11	104	590	.6317	748	49
12	132	621	.6252	742	48
13	161	651	.6187	735	47
14	189	682	.6122	729	46
15	.21218	.21712	4.6057	.97723	45
16	246	743	.5993	717	44
17	275̅	773	.5928	711	43
18	303	804	.5864	705̅	42
19	331	834	.5800	698	41
20	.21360	.21864	4.5736	.97692	40
21	388	895̅	.5673	686	39
22	417	925	.5609	680	38
23	445	956	.5546	673	37
24	474	.21986	.5483	667	36
25	.21502	.22017	4.5420	.97661	35
26	530	047	.5357	655̅	34
27	559	078	.5294	648	33
28	587	108	.5232	642	32
29	616	139	.5169	636	31
30	.21644	.22169	4.5107	.97630	30
31	672	200	.5045	623	29
32	701	231	.4983	617	28
33	729	261	.4922	611	27
34	758	292	.4860	604	26
35	.21786	.22322	4.4799	.97598	25
36	814	353	.4737	592	24
37	843	383	.4676	585	23
38	871	414	.4615	579	22
39	899	444	.4555̅	573	21
40	.21928	.22475̅	4.4494	.97566	20
41	956	505	.4434	560	19
42	.21985̅	536	.4373	553	18
43	.22013	567	.4313	547	17
44	041	597	.4253	541	16
45	.22070	.22628	4.4194	.97534	15
46	098	658	.4134	528	14
47	126	689	.4075̅	521	13
48	155̅	719	.4015	515̅	12
49	183	750	.3956	508	11
50	.22212	.22781	4.3897	.97502	10
51	240	811	.3838	496	9
52	268	842	.3779	489	8
53	297	872	.3721	483	7
54	325	903	.3662	476	6
55	.22353	.22934	4.3604	.97470	5
56	382	964	.3546	463	4
57	410	.22995̅	.3488	457	3
58	438	.23026	.3430	450	2
59	467	056	.3372	444	1
60	.22495	.23087	4.3315̅	.97437	0
	cos	cot	tan	sin	′

77°

13°

′	sin	tan	cot	cos	
0	.22495	.23087	4.3315̅	.97437	60
1	523	117	.3257	430	59
2	552	148	.3200	424	58
3	580	179	.3143	417	57
4	608	209	.3086	411	56
5	.22637	.23240	4.3029	.97404	55
6	665	271	.2972	398	54
7	693	301	.2916	391	53
8	722	332	.2859	384	52
9	750	363	.2803	378	51
10	.22778	.23393	4.2747	.97371	50
11	807	424	.2691	365̅	49
12	835	455̅	.2635	358	48
13	863	485	.2580	351	47
14	892	516	.2524	345̅	46
15	.22920	.23547	4.2468	.97338	45
16	948	578	.2413	331	44
17	.22977	608	.2358	325̅	43
18	.23005̅	639	.2303	318	42
19	033	670	.2248	311	41
20	.23062	.23700	4.2193	.97304	40
21	090	731	.2139	298	39
22	118	762	.2084	291	38
23	146	793	.2030	284	37
24	175̅	823	.1976	278	36
25	.23203	.23854	4.1922	.97271	35
26	231	885̅	.1868	264	34
27	260	916	.1814	257	33
28	288	946	.1760	251	32
29	316	.23977	.1706	244	31
30	.23345̅	.24008	4.1653	.97237	30
31	373	039	.1600	230	29
32	401	069	.1547	223	28
33	429	100	.1493	217	27
34	458	131	.1441	210	26
35	.23486	.24162	4.1388	.97203	25
36	514	193	.1335	196	24
37	542	223	.1282	189	23
38	571	254	.1230	182	22
39	599	285̅	.1178	176	21
40	.23627	.24316	4.1126	.97169	20
41	656	347	.1074	162	19
42	684	377	.1022	155̅	18
43	712	408	.0970	148	17
44	740	439	.0918	141	16
45	.23769	.24470	4.0867	.97134	15
46	797	501	.0815	127	14
47	825	532	.0764	120	13
48	853	562	.0713	113	12
49	882	593	.0662	106	11
50	.23910	.24624	4.0611	.97100	10
51	938	655̅	.0560	093	9
52	966	686	.0509	086	8
53	.23995̅	717	.0459	079	7
54	.24023	747	.0408	072	6
55	.24051	.24778	4.0358	.97065̅	5
56	079	809	.0308	058	4
57	108	840	.0257	051	3
58	136	871	.0207	044	2
59	164	902	.0158	037	1
60	.24192	.24933	4.0108	.97030	0
	cos	cot	tan	sin	′

76°

14°

′	sin	tan	cot	cos	
0	.24192	.24933	4.0108	.97030	60
1	220	964	.0058	023	59
2	249	.24995̄	4.0009	015	58
3	277	.25026	3.9959	008	57
4	305	056	.9910	.97001	56
5	.24333	.25087	3.9861	.96994	55
6	362	118	.9812	987	54
7	390	149	.9763	980	53
8	418	180	.9714	973	52
9	446	211	.9665	966	51
10	.24474	.25242	3.9617	.96959	50
11	503	273	.9568	952	49
12	531	304	.9520	945̄	48
13	559	335̄	.9471	937	47
14	587	366	.9423	930	46
15	.24615	.25397	3.9375	.96923	45
16	644	428	.9327	916	44
17	672	459	.9279	909	43
18	700	490	.9232	902	42
19	728	521	.9184	894	41
20	.24756	.25552	3.9136	.96887	40
21	784	583	.9089	880	39
22	813	614	.9042	873	38
23	841	645̄	.8995̄	866	37
24	869	676	.8947	858	36
25	.24897	.25707	3.8900	.96851	35
26	925	738	.8854	844	34
27	954	769	.8807	837	33
28	.24982	800	.8760	829	32
29	.25010	831	.8714	822	31
30	.25038	.25862	3.8667	.96815̄	30
31	066	893	.8621	807	29
32	094	924	.8575̄	800	28
33	122	955̄	.8528	793	27
34	151	.25986	.8482	786	26
35	.25179	.26017	3.8436	.96778	25
36	207	048	.8391	771	24
37	235	079	.8345̄	764	23
38	263	110	.8299	756	22
39	291	141	.8254	749	21
40	.25320	.26172	3.8208	.96742	20
41	348	203	.8163	734	19
42	376	235̄	.8118	727	18
43	404	266	.8073	719	17
44	432	297	.8028	712	16
45	.25460	.26328	3.7983	.96705̄	15
46	488	359	.7938	697	14
47	516	390	.7893	690	13
48	545̄	421	.7848	682	12
49	573	452	.7804	675̄	11
50	.25601	.26483	3.7760	.96667	10
51	629	515̄	.7715	660	9
52	657	546	.7671	653	8
53	685	577	.7627	645	7
54	713	608	.7583	638	6
55	.25741	.26639	3.7539	.96630	5
56	769	670	.7495̄	623	4
57	798	701	.7451	615	3
58	826	733	.7408	608	2
59	854	764	.7364	600	1
60	.25882	.26795̄	3.7321	.96593	0
	cos	cot	tan	sin	′

75°

15°

′	sin	tan	cot	cos	
0	.25882	.26795̄	3.7321	.96593	60
1	910	826	.7277	585	59
2	938	857	.7234	578	58
3	966	888	.7191	570	57
4	.25994	920	.7148	562	56
5	.26022	.26951	3.7105̄	.96555̄	55
6	050	.26982	.7062	547	54
7	079	.27013	.7019	540	53
8	107	044	.6976	532	52
9	135̄	076	.6933	524	51
10	.26163	.27107	3.6891	.96517	50
11	191	138	.6848	509	49
12	219	169	.6806	502	48
13	247	201	.6764	494	47
14	275	232	.6722	486	46
15	.26303	.27263	3.6680	.96479	45
16	331	294	.6638	471	44
17	359	326	.6596	463	43
18	387	357	.6554	456	42
19	415	388	.6512	448	41
20	.26443	.27419	3.6470	.96440	40
21	471	451	.6429	433	39
22	500	482	.6387	425̄	38
23	528	513	.6346	417	37
24	556	545̄	.6305̄	410	36
25	.26584	.27576	3.6264	.96402	35
26	612	607	.6222	394	34
27	640	638	.6181	386	33
28	668	670	.6140	379	32
29	696	701	.6100	371	31
30	.26724	.27732	3.6059	.96363	30
31	752	764	.6018	355	29
32	780	795	.5978	347	28
33	808	826	.5937	340	27
34	836	858	.5897	332	26
35	.26864	.27889	3.5856	.96324	25
36	892	921	.5816	316	24
37	920	952	.5776	308	23
38	948	.27983	.5736	301	22
39	.26976	.28015̄	.5696	293	21
40	.27004	.28046	3.5656	.96285̄	20
41	032	077	.5616	277	19
42	060	109	.5576	269	18
43	088	140	.5536	261	17
44	116	172	.5497	253	16
45	.27144	.28203	3.5457	.96246	15
46	172	234	.5418	238	14
47	200	266	.5379	230	13
48	228	297	.5339	222	12
49	256	329	.5300	214	11
50	.27284	.28360	3.5261	.96206	10
51	312	391	.5222	198	9
52	340	423	.5183	190	8
53	368	454	.5144	182	7
54	396	486	.5105	174	6
55	.27424	.28517	3.5067	.96166	5
56	452	549	.5028	158	4
57	480	580	.4989	150	3
58	508	612	.4951	142	2
59	536	643	.4912	134	1
60	.27564	.28675̄	3.4874	.96126	0
	cos	cot	tan	sin	′

74°

16°

′	sin	tan	cot	cos	
0	.27564	.286755̄	3.4874	.96126	60
1	592	706	.4836	118	59
2	620	738	.4798	110	58
3	648	769	.4760	102	57
4	676	801	.4722	094	56
5	.27704	.28832	3.4684	.96086	55
6	731	864	.4646	078	54
7	759	895	.4608	070	53
8	787	927	.4570	062	52
9	815	958	.4533	054	51
10	.27843	.28990	3.4495	.96046	50
11	871	.29021	.4458	037	49
12	899	053	.4420	029	48
13	927	084	.4383	021	47
14	955̄	116	.4346	013	46
15	.27983	.29147	3.4308	.960055̄	45
16	.28011	179	.4271	.95997	44
17	039	210	.4234	989	43
18	067	242	.4197	981	42
19	095̄	274	.4160	972	41
20	.28123	.29305	3.4124	.95964	40
21	150	337	.4087	956	39
22	178	368	.4050	948	38
23	206	400	.4014	940	37
24	234	432	.3977	931	36
25	.28262	.29463	3.3941	.95923	35
26	290	495̄	.3904	915̄	34
27	318	526	.3868	907	33
28	346	558	.3832	898	32
29	374	590	.3796	890	31
30	.28402	.29621	3.3759	.95882	30
31	429	653	.3723	874	29
32	457	685̄	.3687	865	28
33	485	716	.3652	857	27
34	513	748	.3616	849	26
35	.28541	.29780	3.3580	.95841	25
36	569	811	.3544	832	24
37	597	843	.3509	824	23
38	625̄	875̄	.3473	816	22
39	652	906	.3438	807	21
40	.28680	.29938	3.3402	.95799	20
41	708	.29970	.3367	791	19
42	736	.30001	.3332	782	18
43	764	033	.3297	774	17
44	792	065̄	.3261	766	16
45	.28820	.30097	3.3226	.95757	15
46	847	128	.3191	749	14
47	875	160	.3156	740	13
48	903	192	.3122	732	12
49	931	224	.3087	724	11
50	.28959	.30255	3.3052	.95715	10
51	.28987	287	.3017	707	9
52	.29015̄	319	.2983	698	8
53	042	351	.2948	690	7
54	070	382	.2914	681	6
55	.29098	.30414	3.2879	.95673	5
56	126	446	.2845	664	4
57	154	478	.2811	656	3
58	182	509	.2777	647	2
59	209	541	.2743	639	1
60	.29237	.30573	3.2709	.95630	0
	cos	cot	tan	sin	′

73°

17°

′	sin	tan	cot	cos	
0	.29237	.30573	3.2709	.95630	60
1	265̄	605̄	.2675̄	622	59
2	293	637	.2641	613	58
3	321	669	.2607	605̄	57
4	348	700	.2573	596	56
5	.29376	.30732	3.2539	.95588	55
6	404	764	.2506	579	54
7	432	796	.2472	571	53
8	460	828	.2438	562	52
9	487	860	.2405̄	554	51
10	.29515	.30891	3.2371	.95545	50
11	543	923	.2338	536	49
12	571	955	.2305̄	528	48
13	599	.30987	.2272	519	47
14	626	.31019	.2238	511	46
15	.29654	.31051	3.2205	.95502	45
16	682	083	.2172	493	44
17	710	115̄	.2139	485̄	43
18	737	147	.2106	476	42
19	765	178	.2073	467	41
20	.29793	.31210	3.2041	.95459	40
21	821	242	.2008	450	39
22	849	274	.1975	441	38
23	876	306	.1943	433	37
24	904	338	.1910	424	36
25	.29932	.31370	3.1878	.95415	35
26	960	402	.1845	407	34
27	.29987	434	.1813	398	33
28	.30015	466	.1780	389	32
29	043	498	.1748	380	31
30	.30071	.31530	3.1716	.95372	30
31	098	562	.1684	363	29
32	126	594	.1652	354	28
33	154	626	.1620	345	27
34	182	658	.1588	337	26
35	.30209	.31690	3.1556	.95328	25
36	237	722	.1524	319	24
37	265̄	754	.1492	310	23
38	292	786	.1460	301	22
39	320	818	.1429	293	21
40	.30348	.3185̄0	3.1397	.95284	20
41	376	882	.1366	275̄	19
42	403	914	.1334	260	18
43	431	946	.1303	257	17
44	459	.31978	.1271	248	16
45	.30486	.32010	3.1240	.95240	15
46	514	042	.1209	231	14
47	542	074	.1178	222	13
48	570	106	.1146	213	12
49	597	139	.1115	204	11
50	.30625̄	.32171	3.1084	.95195	10
51	653	203	.1053	186	9
52	680	235̄	.1022	177	8
53	708	267	.0991	168	7
54	736	299	.0961	159	6
55	.30763	.32331	3.0930	.95150	5
56	791	363	.0899	142	4
57	819	396	.0868	133	3
58	846	428	.0838	124	2
59	874	460	.0807	115̄	1
60	.30902	.32492	3.0777	.95106	0
	cos	cot	tan	sin	′

72°

18°

′	sin	tan	cot	cos	
0	.30902	.32492	3.0777	.95106	60
1	929	524	.0746	097	59
2	957	556	.0716	088	58
3	.30985̄	588	.0686	079	57
4	.31012	621	.0655	070	56
5	.31040	.32653	3.0625	.95061	55
6	068	685	.0595	052	54
7	095	717	.0565̄	043	53
8	123	749	.0535̄	033	52
9	151	782	.0505̄	024	51
10	.31178	.32814	3.0475̄	.95015	50
11	206	846	.0445	.95006	49
12	233	878	.0415	.94997	48
13	261	911	.0385	988	47
14	289	943	.0356	979	46
15	.31316	.32975	3.0326	.94970	45
16	344	.33007	.0296	961	44
17	372	040	.0267	952	43
18	399	072	.0237	943	42
19	427	104	.0208	933	41
20	.31454	.33136	3.0178	.94924	40
21	482	169	.0149	915	39
22	510	201	.0120	906	38
23	537	233	.0090	897	37
24	565̄	266	.0061	888	36
25	.31593	.33298	3.0032	.94878	35
26	620	330	3.0003	869	34
27	648	363	2.9974	860	33
28	675	395̄	.9945̄	851	32
29	703	427	.9916	842	31
30	.31730	.33460	2.9887	.94832	30
31	758	492	.9858	823	29
32	786	524	.9829	814	28
33	813	557	.9800	805̄	27
34	841	589	.9772	795	26
35	.31868	.33621	2.9743	.94786	25
36	896	654	.9714	777	24
37	923	686	.9686	768	23
38	951	718	.9657	758	22
39	.31979	751	.9629	749	21
40	.32006	33783	2.9600	.94740	20
41	034	816	.9572	730	19
42	061	848	.9544	721	18
43	089	881	.9515	712	17
44	116	913	.9487	702	16
45	.32144	.33945	2.9459	.94693	15
46	171	.33978	.9431	684	14
47	199	.34010	.9403	674	13
48	227	043	.9375̄	665̄	12
49	254	075	.9347	656	11
50	.32282	.34108	2.9319	.94646	10
51	309	140	.9291	637	9
52	337	173	.9263	627	8
53	364	205	.9235	618	7
54	392	238	.9208	609	6
55	.32419	.34270	2.9180	.94599	5
56	447	303	.9152	590	4
57	474	335	.9125̄	580	3
58	502	368	.9097	571	2
59	529	400	.9070	561	1
60	.32557	.34433	2.9042	.94552	0
	cos	cot	tan	sin	′

71°

19°

′	sin	tan	cot	cos	
0	.32557	.34433	2.9042	.94552	60
1	584	465	.9015̄	542	59
2	612	498	.8987	533	58
3	639	530	.8960	523	57
4	667	563	.8933	514	56
5	.32694	.34596	2.8905	.94504	55
6	722	628	.8878	495̄	54
7	749	661	.8851	485	53
8	777	693	.8824	476	52
9	804	726	.8797	466	51
10	.32832	.34758	2.8770	.94457	50
11	859	791	.8743	447	49
12	887	824	.8716	438	48
13	914	856	.8689	428	47
14	942	889	.8662	418	46
15	.32969	.34922	2.8636	.94409	45
16	.32997	954	.8609	399	44
17	.33024	.34987	.8582	390	43
18	051	.35020	.8556	380	42
19	079	052	.8529	370	41
20	.33106	.35085̄	2.8502	.94361	40
21	134	118	.8476	351	39
22	161	150	.8449	342	38
23	189	183	.8423	332	37
24	216	216	.8397	322	36
25	.33244	.35248	2.8370	.94313	35
26	271	281	.8344	303	34
27	298	314	.8318	293	33
28	326	346	.8291	284	32
29	353	379	.8265	274	31
30	.33381	.35412	2.8239	.94264	30
31	408	445̄	.8213	254	29
32	436	477	.8187	245̄	28
33	463	510	.8161	235̄	27
34	490	543	.8135	225	26
35	.33518	.35576	2.8109	.94215	25
36	545	608	.8083	206	24
37	573	641	.8057	196	23
38	600	674	.8032	186	22
39	627	707	.8006	176	21
40	.33655̄	.35740	2.7980	.94167	20
41	682	772	.7955̄	157	19
42	710	805	.7929	147	18
43	737	838	.7903	137	17
44	764	871	.7878	127	16
45	.33792	.35904	2.7852	.94118	15
46	819	937	.7827	108	14
47	846	.35969	.7801	098	13
48	874	.36002	.7776	088	12
49	901	035	.7751	078	11
50	.33929	.36068	2.7725	.94068	10
51	956	101	.7700	058	9
52	.33983	134	.7675̄	049	8
53	.34011	167	.765̄0	039	7
54	038	199	.7625̄	029	6
55	.34065	.36232	2.7600	.94019	5
56	093	265	.7575̄	.94009	4
57	120	298	.755̄0	.93999	3
58	147	331	.7525̄	989	2
59	175̄	364	.75̄00	979	1
60	.34202	.36397	2.7475̄	.93969	0
	cos	cot	tan	sin	′

70°

20°

′	sin	tan	cot	cos	
0	.34202	.36397	2.747$\bar{5}$	.93969	60
1	229	430	.74$\bar{5}$0	959	59
2	257	463	.7425	949	58
3	284	496	.7400	939	57
4	311	529	.7376	929	56
5	.34339	.36562	2.7351	.93919	**55**
6	366	59$\bar{5}$	.7326	909	54
7	393	628	.7302	899	53
8	421	661	.7277	889	52
9	448	694	.7253	879	51
10	.34475	.36727	2.7228	.93869	**50**
11	503	760	.7204	859	49
12	530	793	.7179	849	48
13	557	826	.715$\bar{5}$	839	47
14	584	859	.7130	829	46
15	.34612	.36892	2.7106	.93819	**45**
16	639	92$\bar{5}$	.7082	809	44
17	666	958	.7058	799	43
18	694	.36991	.7034	789	42
19	721	.37024	.7009	779	41
20	.34748	.37057	2.6985	.93769	**40**
21	775	090	.6961	759	39
22	803	123	.6937	748	38
23	830	157	.6913	738	37
24	857	190	.6889	728	36
25	.34884	.37223	2.6865	.93718	**35**
26	912	256	.6841	708	34
27	939	289	.6818	698	33
28	966	322	.6794	688	32
29	.34993	355	.6770	677	31
30	.35021	.37388	2.6746	.93667	**30**
31	048	422	.6723	657	29
32	075	45$\bar{5}$	.6699	647	28
33	102	488	.6675	637	27
34	130	521	.6652	626	26
35	.35157	.37554	2.6628	.93616	**25**
36	184	588	.660$\bar{5}$	606	24
37	211	621	.6581	596	23
38	239	654	.6558	585	22
39	266	687	.6534	575	21
40	.35293	.37720	2.6511	.9356$\bar{5}$	**20**
41	320	754	.6488	55$\bar{5}$	19
42	347	787	.6464	544	18
43	37$\bar{5}$	820	.6441	534	17
44	402	853	.6418	524	16
45	.35429	.37887	2.639$\bar{5}$	.93514	**15**
46	456	920	.6371	503	14
47	484	953	.6348	493	13
48	511	.37986	.6325	483	12
49	538	.38020	.6302	472	11
50	.35565	.38053	2.6279	.93462	**10**
51	592	086	.6256	452	9
52	619	120	.6233	441	8
53	647	153	.6210	431	7
54	674	186	.6187	420	6
55	.35701	.38220	2.616$\bar{5}$	.93410	**5**
56	728	253	.6142	400	4
57	755	286	.6119	389	3
58	782	320	.6096	379	2
59	810	353	.6074	368	1
60	.35837	.38386	2.6051	.93358	**0**
	cos	cot	tan	sin	′

69°

21°

′	sin	tan	cot	cos	
0	.35837	.38386	2.6051	.93358	60
1	864	420	.6028	348	59
2	891	453	.6006	337	58
3	918	487	.5983	327	57
4	945	520	.5961	316	56
5	.35973	.38553	2.5938	.93306	**55**
6	.36000	587	.5916	295	54
7	027	620	.5893	28$\bar{5}$	53
8	054	654	.5871	274	52
9	081	687	.5848	264	51
10	.36108	.38721	2.5826	.93253	**50**
11	135	754	.5804	243	49
12	162	787	.5782	232	48
13	190	821	.5759	222	47
14	217	854	.5737	211	46
15	.36244	.38888	2.571$\bar{5}$	.93201	**45**
16	271	921	.5693	190	44
17	298	95$\bar{5}$	.5671	180	43
18	325	.38988	.5649	169	42
19	352	.39022	.5627	159	41
20	.36379	.39055	2.560$\bar{5}$	.93148	**40**
21	406	089	.5583	137	39
22	434	122	.5561	127	38
23	461	156	.5539	116	37
24	488	190	.5517	106	36
25	.3651$\bar{5}$	.39223	2.5495	.9309$\bar{5}$	**35**
26	542	257	.5473	084	34
27	569	290	.5452	074	33
28	596	324	.5430	063	32
29	623	357	.5408	052	31
30	.36650	.39391	2.5386	.93042	**30**
31	677	42$\bar{5}$	.536$\bar{5}$	031	29
32	704	458	.5343	020	28
33	731	492	.5322	.93010	27
34	758	526	.5300	.92999	26
35	.36785	.39559	2.5279	.92988	**25**
36	812	593	.5257	978	24
37	839	626	.5236	967	23
38	867	660	.5214	956	22
39	894	694	.5193	945	21
40	.36921	.39727	2.5172	.9293$\bar{5}$	**20**
41	948	761	.5150	924	19
42	.3697$\bar{5}$	79$\bar{5}$	.5129	913	18
43	.37002	829	.5108	902	17
44	029	862	.5086	892	16
45	.37056	.39896	2.5065	.92881	**15**
46	083	930	.5044	870	14
47	110	963	.5023	859	13
48	137	.39997	.5002	849	12
49	164	.40031	.4981	838	11
50	.37191	.4006$\bar{5}$	2.4960	.92827	**10**
51	218	098	.4939	816	9
52	24$\bar{5}$	132	.4918	805	8
53	272	166	.4897	794	7
54	299	200	.4876	784	6
55	.37326	.40234	2.485$\bar{5}$	.92773	**5**
56	353	267	.4834	762	4
57	380	301	.4813	751	3
58	407	33$\bar{5}$	.4792	740	2
59	434	369	.4772	729	1
60	.37461	.40403	2.4751	.92718	**0**
	cos	cot	tan	sin	′

68°

22°

′	sin	tan	cot	cos	
0	.37461	.40403	2.4751	.92718	60
1	488	436	.4730	707	59
2	515̄	470	.4709	697	58
3	542	504	.4689	686	57
4	569	538	.4668	675̄	56
5	.37595	.40572	2.4648	.92664	55
6	622	606	.4627	653	54
7	649	640	.4606	642	53
8	676	674	.4586	631	52
9	703	707	.4566	620	51
10	.37730	.40741	2.4545	.92609	50
11	757	775	.4525̄	598	49
12	784	809	.4504	587	48
13	811	843	.4484	576	47
14	838	877	.4464	565	46
15	.37865̄	.40911	2.4443	.92554	45
16	892	945	.4423	543	44
17	919	.40979	.4403	532	43
18	946	.41013	.4383	521	42
19	973	047	.4362	510	41
20	.37999	.41081	2.4342	.92499	40
21	.38026	115̄	.4322	488	39
22	053	149	.4302	477	38
23	080	183	.4282	466	37
24	107	217	.4262	455̄	36
25	.38134	.41251	2.4242	.92444	35
26	161	285	.4222	432	34
27	188	319	.4202	421	33
28	215̄	353	.4182	410	32
29	241	387	.4162	399	31
30	.38268	.41421	2.4142	.92388	30
31	295	455	.4122	377	29
32	322	490	.4102	366	28
33	349	524	.4083	355̄	27
34	376	558	.4063	343	26
35	.38403	.41592	2.4043	.92332	25
36	430	626	.4023	321	24
37	456	660	.4004	310	23
38	483	694	.3984	299	22
39	510	728	.3964	287	21
40	.38537	.41763	2.3945̄	.92276	20
41	564	797	.3925	265	19
42	591	831	.3906	254	18
43	617	865	.3886	243	17
44	644	899	.3867	231	16
45	.38671	.41933	2.3847	.92220	15
46	698	.41968	.3828	209	14
47	725̄	.42002	.3808	198	13
48	752	036	.3789	186	12
49	778	070	.3770	175	11
50	.38805	.42105̄	2.3750	.92164	10
51	832	139	.3731	152	9
52	859	173	.3712	141	8
53	886	207	.3693	130	7
54	912	242	.3673	119	6
55	.38939	.42276	2.3654	.92107	5
56	966	310	.3635̄	096	4
57	.38993	345̄	.3616	085̄	3
58	.39020	379	.3597	073	2
59	046	413	.3578	062	1
60	.39073	.42447	2.3559	.92050	0
	cos	cot	tan	sin	′

67°

23°

′	sin	tan	cot	cos	
0	.39073	.42447	2.3559	.92050	60
1	100	482	.3539	039	59
2	127	516	.3520	028	58
3	153	551	.3501	016	57
4	180	585̄	.3483	.92005̄	56
5	.39207	.42619	2.3464	.91994	55
6	234	654	.3445̄	982	54
7	260	688	.3426	971	53
8	287	722	.3407	959	52
9	314	757	.3388	948	51
10	.39341	.42791	2.3369	.91936	50
11	367	826	.3351	925̄	49
12	394	860	.3332	914	48
13	421	894	.3313	902	47
14	448	929	.3294	891	46
15	.39474	.42963	2.3276	.91879	45
16	501	.42998	.3257	868	44
17	528	.43032	.3238	856	43
18	555̄	067	.3220	845̄	42
19	581	101	.3201	833	41
20	.39608	.43136	2.3183	.91822	40
21	635̄	170	.3164	810	39
22	661	205̄	.3146	799	38
23	688	239	.3127	787	37
24	715̄	274	.3109	775	36
25	.39741	.43308	2.3090	.91764	35
26	768	343	.3072	752	34
27	795̄	378	.3053	741	33
28	822	412	.3035	729	32
29	848	447	.3017	718	31
30	.39875̄	.43481	2.2998	.91706	30
31	902	516	.2980	694	29
32	928	550	.2962	683	28
33	955̄	585	.2944	671	27
34	.39982	620	.2925	660	26
35	.40008	.43654	2.2907	.91648	25
36	035̄	689	.2889	636	24
37	062	724	.2871	625̄	23
38	088	758	.2853	613	22
39	115̄	793	.2835̄	601	21
40	.40141	.43828	2.2817	.91590	20
41	168	862	.2799	578	19
42	195̄	897	.2781	566	18
43	221	932	.2763	555̄	17
44	248	.43966	.2745̄	543	16
45	.40275̄	.44001	2.2727	.91531	15
46	301	036	.2709	519	14
47	328	071	.2691	508	13
48	355̄	105	.2673	496	12
49	381	140	.2655	484	11
50	.40408	.44175̄	2.2637	.91472	10
51	434	210	.2620	461	9
52	461	244	.2602	449	8
53	488	279	.2584	437	7
54	514	314	.2566	425	6
55	.40541	.44349	2.2549	.91414	5
56	567	384	.2531	402	4
57	594	418	.2513	390	3
58	621	453	.2496	378	2
59	647	488	.2478	366	1
60	.40674	.44523	2.2460	.91355̄	0
	cos	cot	tan	sin	′

66°

24°

′	sin	tan	cot	cos	
0	.40674	.44523	2.2460	.91355	60
1	700	558	.2443	343	59
2	727	593	.2425	331	58
3	753	627	.2408	319	57
4	780	662	.2390	307	56
5	.40806	.44697	2.2373	.91295	55
6	833	732	.2355	283	54
7	860	767	.2338	272	53
8	886	802	.2320	260	52
9	913	837	.2303	248	51
10	.40939	.44872	2.2286	.91236	50
11	966	907	.2268	224	49
12	.40992	942	.2251	212	48
13	.41019	.44977	.2234	200	47
14	045	.45012	.2216	188	46
15	.41072	.45047	2.2199	.91176	45
16	098	082	.2182	164	44
17	125	117	.2165	152	43
18	151	152	.2148	140	42
19	178	187	.2130	128	41
20	.41204	.45222	2.2113	.91116	40
21	231	257	.2096	104	39
22	257	292	.2079	092	38
23	284	327	.2062	080	37
24	310	362	.2045	068	36
25	.41337	.45397	2.2028	.91056	35
26	363	432	.2011	044	34
27	390	467	.1994	032	33
28	416	502	.1977	020	32
29	443	538	.1960	.91008	31
30	.41469	.45573	2.1943	.90996	30
31	496	608	.1926	984	29
32	522	643	.1909	972	28
33	549	678	.1892	960	27
34	575	713	.1876	948	26
35	.41602	.45748	2.1859	.90936	25
36	628	784	.1842	924	24
37	655	819	.1825	911	23
38	681	854	.1808	899	22
39	707	889	.1792	887	21
40	.41734	.45924	2.1775	.90875	20
41	760	960	.1758	863	19
42	787	.45995	.1742	851	18
43	813	.46030	.1725	839	17
44	840	065	.1708	826	16
45	.41866	.46101	2.1692	.90814	15
46	892	136	.1675	802	14
47	919	171	.1659	790	13
48	945	206	.1642	778	12
49	972	242	.1625	766	11
50	.41998	.46277	2.1609	.90753	10
51	.42024	312	.1592	741	9
52	051	348	.1576	729	8
53	077	383	.1560	717	7
54	104	418	.1543	704	6
55	.42130	.46454	2.1527	.90692	5
56	156	489	.1510	680	4
57	183	525	.1494	668	3
58	209	560	.1478	655	2
59	235	595	.1461	643	1
60	.42262	.46631	2.1445	.90631	0
	cos	cot	tan	sin	′

65°

25°

′	sin	tan	cot	cos	
0	.42262	.46631	2.1445	.90631	60
1	288	666	.1429	618	59
2	315	702	.1413	606	58
3	341	737	.1396	594	57
4	367	772	.1380	582	56
5	.42394	.46808	2.1364	.90569	55
6	420	843	.1348	557	54
7	446	879	.1332	545	53
8	473	914	.1315	532	52
9	499	950	.1299	520	51
10	.42525	.46985	2.1283	.90507	50
11	552	.47021	.1267	495	49
12	578	056	.1251	483	48
13	604	092	.1235	470	47
14	631	128	.1219	458	46
15	.42657	.47163	2.1203	.90446	45
16	683	199	.1187	433	44
17	709	234	.1171	421	43
18	736	270	.1155	408	42
19	762	305	.1139	396	41
20	.42788	.47341	2.1123	.90383	40
21	815	377	.1107	371	39
22	841	412	.1092	358	38
23	867	448	.1076	346	37
24	894	483	.1060	334	36
25	.42920	.47519	2.1044	.90321	35
26	946	555	.1028	309	34
27	972	590	.1013	296	33
28	.42999	626	0997	284	32
29	.43025	662	.0981	271	31
30	.43051	.47698	2.0965	.90259	30
31	077	733	.0950	246	29
32	104	769	.0934	233	28
33	130	805	.0918	221	27
34	156	840	.0903	208	26
35	.43182	.47876	2.0887	.90196	25
36	209	912	.0872	183	24
37	235	948	.0856	171	23
38	261	.47984	.0840	158	22
39	287	.48019	.0825	146	21
40	.43313	.48055	2.0809	.90133	20
41	340	091	.0794	120	19
42	366	127	.0778	108	18
43	392	163	.0763	095	17
44	418	198	.0748	082	16
45	.43445	.48234	2.0732	.90070	15
46	471	270	.0717	057	14
47	497	306	.0701	045	13
48	523	342	.0686	032	12
49	549	378	.0671	019	11
50	.43575	.48414	2.0655	.90007	10
51	602	450	.0640	.89994	9
52	628	486	.0625	981	8
53	654	521	.0609	968	7
54	680	557	.0594	956	6
55	.43706	.48593	2.0579	.89943	5
56	733	629	.0564	930	4
57	759	665	.0549	918	3
58	785	701	.0533	905	2
59	811	737	.0518	892	1
60	.43837	.48773	2.0503	.89879	0
	cos	cot	tan	sin	′

64°

26°

′	sin	tan	cot	cos	
0	.43837	.48773	2.0503	.89879	60
1	863	809	.0488	867	59
2	889	845	.0473	854	58
3	916	881	.0458	841	57
4	942	917	.0443	828	56
5	.43968	.48953	2.0428	.89816	55
6	.43994	.48989	.0413	803	54
7	.44020	.49026	.0398	790	53
8	046	062	.0383	777	52
9	072	098	.0368	764	51
10	.44098	.49134	2.0353	.89752	50
11	124	170	.0338	739	49
12	151	206	.0323	726	48
13	177	242	.0308	713	47
14	203	278	.0293	700	46
15	.44229	.49315̄	2.0278	.89687	45
16	255̄	351	.0263	674	44
17	281	387	.0248	662	43
18	307	423	.0233	649	42
19	333	459	.0219	636	41
20	.44359	.49495	2.0204	.89623	40
21	385	532	.0189	610	39
22	411	568	.0174	597	38
23	437	604	.0160	584	37
24	464	640	.0145̄	571	36
25	.44490	.49677	2.0130	.89558	35
26	516	713	.0115	545	34
27	542	749	.0101	532	33
28	568	786	.0086	519	32
29	594	822	.0072	506	31
30	.44620	.49858	2.0057	.89493	30
31	646	894	.0042	480	29
32	672	931	.0028	467	28
33	698	.49967	2.0013	454	27
34	724	.50004	1.9999	441	26
35	.4475̄0	.50040	1.9984	.89428	25
36	776	076	.9970	415	24
37	802	113	.9955	402	23
38	828	149	.9941	389	22
39	854	185	.9926	376	21
40	.44880	.50222	1.9912	.89363	20
41	906	258	.9897	350	19
42	932	295̄	.9883	337	18
43	958	331	.9868	324	17
44	.44984	368	.9854	311	16
45	.45010	.50404	1.9840	.89298	15
46	036	441	.9825	285̄	14
47	062	477	.9811	272	13
48	088	514	.9797	259	12
49	114	550	.9782	245	11
50	.45140	.50587	1.9768	.89232	10
51	166	623	.9754	219	9
52	192	660	.9740	206	8
53	218	696	9725	193	7
54	243	733	.9711	180	6
55	.45269	.50769	1.9697	.89167	5
56	295	806	.9683	153	4
57	321	843	.9669	140	3
58	347	879	.9654	127	2
59	373	916	.9640	114	1
60	.45399	.50953	1.9626	.89101	0
	cos	cot	tan	sin	′

63°

27°

′	sin	tan	cot	cos	
0	.45399	.50953	1.9626	.89101	60
1	425̄	.50989	.9612	087	59
2	451	.51026	.9598	074	58
3	477	063	.9584	061	57
4	503	.099	.9570	048	56
5	.45529	.51136	1.9556	.89035̄	55
6	554	173	.9542	021	54
7	580	209	.9528	.89008	53
8	606	246	.9514	.88995̄	52
9	632	283	.95̄00	981	51
10	.45658	.51319	1.9486	.88968	50
11	684	356	.9472	955̄	49
12	710	393	.9458	942	48
13	736	430	.9444	928	47
14	762	467	.9430	915	46
15	.45787	.51503	1.9416	.88902	45
16	813	540	.9402	888	44
17	839	577	.9388	875	43
18	865̄	614	.9375̄	862	42
19	891	651	.9361	848	41
20	.45917	.51688	1.9347	.88835	40
21	942	724	.9333	822	39
22	968	761	.9319	808	38
23	.45994	798	.9306	795̄	37
24	.46020	835	.9292	782	36
25	.46046	.51872	1.9278	.88768	35
26	072	909	.9265̄	755̄	34
27	097	946	.9251	741	33
28	123	.51983	.9237	728	32
29	149	.52020	.9223	715̄	31
30	.46175̄	.52057	1.9210	.88701	30
31	201	094	.9196	688	29
32	226	131	.9183	674	28
33	252	168	.9169	661	27
34	278	205̄	.9155	647	26
35	.46304	.52242	1.9142	.88634	25
36	330	279	.9128	620	24
37	355	316	.9115̄	607	23
38	381	353	.9101	593	22
39	407	390	.9088	580	21
40	.46433	.52427	1.9074	.88566	20
41	458	464	.9061	553	19
42	484	501	.9047	539	18
43	510	538	.9034	526	17
44	536	575	.9020	512	16
45	.46561	.52613	1.9007	.88499	15
46	587	65̄0	.8993	485	14
47	613	687	.8980	472	13
48	639	724	.8967	458	12
49	664	761	.8953	445̄	11
50	.46690	.52798	1.8940	.88431	10
51	716	836	.8927	417	9
52	742	873	.8913	404	8
53	767	910	.8900	390	7
54	793	947	.8887	377	6
55	.46819	.52985̄	1.8873	.88363	5
56	844	.53022	.8860	349	4
57	870	059	.8847	336	3
58	896	096	.8834	322	2
59	921	134	.8820	308	1
60	.46947	.53171	1.8807	.88295̄	0
	cos	cot	tan	sin	′

62°

28°

′	sin	tan	cot	cos	
0	.46947	.53171	1.8807	.882955̄	60
1	973	208	.8794	281	59
2	.46999	246	.8781	267	58
3	.47024	283	.8768	254	57
4	05̄0	320	.875̄5	240	56
5	.47076	.53358	1.8741	.88226	**55**
6	101	395	.8728	213	54
7	127	432	.8715	199	53
8	153	470	.8702	185	52
9	178	507	.8689	172	51
10	.47204	.53545̄	1.8676	.88158	**50**
11	229	582	.8663	144	49
12	255	620	.865̄0	130	48
13	281	657	.8637	117	47
14	306	694	.8624	103	46
15	.47332	.53732	1.8611	.88089	**45**
16	358	769	.8598	075	44
17	383	807	.8585̄	062	43
18	409	844	.8572	048	42
19	434	882	.8559	034	41
20	.47460	.53920	1.8546	.88020	**40**
21	486	957	.8533	.88006	39
22	511	.53995̄	.8520	.87993	38
23	537	.54032	.8507	979	37
24	562	070	.8495̄	965̄	36
25	.47588	.54107	1.8482	.87951	**35**
26	614	145	.8469	937	34
27	639	183	.8456	923	33
28	665̄	220	.8443	909	32
29	690	258	.8430	896	31
30	.47716	.54296	1.8418	.87882	**30**
31	741	333	.8405̄	868	29
32	767	371	.8392	854	28
33	793	409	.8379	840	27
34	818	446	.8367	826	26
35	.47844	.54484	1.8354	.87812	**25**
36	869	522	.8341	798	24
37	895̄	560	.8329	784	23
38	920	597	.8316	770	22
39	946	635	.8303	756	21
40	.47971	.54673	1.8291	.87743	**20**
41	.47997	711	.8278	729	19
42	.48022	748	.8265	715̄	18
43	048	786	.8253	701	17
44	073	824	.8240	687	16
45	.48099	.54862	1.8228	.87673	**15**
46	124	900	.8215	659	14
47	15̄0	938	.8202	645̄	13
48	175	.54975	.8190	631	12
49	201	.55013	.8177	617	11
50	.48226	.55051	1.8165̄	.87603	**10**
51	252	089	.8152	589	9
52	277	127	.8140	575̄	8
53	303	165	.8127	561	7
54	328	203	.8115̄	546	6
55	.48354	.55241	1.8103	.87532	**5**
56	379	279	.8090	518	4
57	405̄	317	.8078	504	3
58	430	355̄	.8065	490	2
59	456	393	.8053	476	1
60	.48481	.55431	1.8040	.87462	0
	cos	cot	tan	sin	′

61°

29°

′	sin	tan	cot	cos	
0	.48481	.55431	1.8040	.87462	60
1	506	469	.8028	448	59
2	532	507	.8016	434	58
3	557	545	.8003	420	57
4	583	583	.7991	406	56
5	.48608	.55621	1.7979	.87391	**55**
6	634	659	.7966	377	54
7	659	697	.7954	363	53
8	684	736	.7942	349	52
9	710	774	.7930	335̄	51
10	.48735	.55812	1.7917	.87321	**50**
11	761	85̄0	.7905	306	49
12	786	888	.7893	292	48
13	811	926	.7881	278	47
14	837	.55964	.7868	264	46
15	.48862	.56003	1.7856	.8725̄0	**45**
16	888	041	.7844	235	44
17	913	079	.7832	221	43
18	938	117	.7820	207	42
19	964	156	.7808	193	41
20	.48989	.56194	1.7796	.87178	**40**
21	.49014	232	.7783	164	39
22	040	270	.7771	15̄0	38
23	065	309	.7759	136	37
24	090	347	.7747	121	36
25	.49116	.56385	1.7735	.87107	**35**
26	141	424	.7723	093	34
27	166	462	.7711	079	33
28	192	501	.7699	064	32
29	217	539	.7687	05̄0	31
30	.49242	.56577	1.7675̄	.87036	**30**
31	268	616	.7663	021	29
32	293	654	.7651	.87007	28
33	318	693	.7639	.86993	27
34	344	731	7627	978	26
35	.49369	.56769	1.7615	.86964	**25**
36	394	808	.7603	949	24
37	419	846	.7591	935	23
38	445̄	885̄	.7579	921	22
39	470	923	.7567	906	21
40	.49495	.56962	1.7556	.86892	**20**
41	521	.57000	.7544	878	19
42	546	039	.7532	863	18
43	571	078	.7520	849	17
44	596	116	.7508	834	16
45	.49622	.57155̄	1.7496	.86820	**15**
46	647	193	.7485̄	805	14
47	672	232	.7473	791	13
48	697	271	.7461	777	12
49	723	309	.7449	762	11
50	.49748	.57348	1.7437	.86748	**10**
51	773	386	.7426	733	9
52	798	425	.7414	719	8
53	824	464	.7402	704	7
54	849	503	.7391	690	6
55	.49874	.57541	1.7379	.86675	**5**
56	899	580	.7367	661	4
57	924	619	.7355	646	3
58	95̄0	657	.7344	632	2
59	.49975̄	696	.7332	617	1
60	.50000	.57735	1.7321	.86603	0
	cos	cot	tan	sin	′

60°

30°

	sin	tan	cot	cos	
0	.50000	.57735	1.7321	.86603	**60**
1	025	774	.7309	588	59
2	050	813	.7297	573	58
3	076	851	.7286	559	57
4	101	890	.7274	544	56
5	.50126	.57929	1.7262	.86530	**55**
6	151	.57968	.7251	515	54
7	176	.58007	.7239	501	53
8	201	046	.7228	486	52
9	227	085̅	.7216	471	51
10	.50252	.58124	1.7205̅	.86457	**50**
11	277	162	.7193	442	49
12	302	201	.7182	427	48
13	327	240	.7170	413	47
14	352	279	.7159	398	46
15	.50377	.58318	1.7147	.86384	**45**
16	403	357	.7136	369	44
17	428	396	.7124	354	43
18	453	435	.7113	340	42
19	478	474	.7102	325̅	41
20	.50503	.58513	1.7090	.86310	**40**
21	528	552	.7079	295	39
22	553	591	.7067	281	38
23	578	631	.7056	266	37
24	603	670	.7045̅	251	36
25	.50628	.58709	1.7033	.86237	**35**
26	654	748	.7022	222	34
27	679	787	.7011	207	33
28	704	826	.6999	192	32
29	729	865	.6988	178	31
30	.50754	.58905̅	1.6977	.86163	**30**
31	779	944	.6965	148	29
32	804	.58983	.6954	133	28
33	829	.59022	.6943	119	27
34	854	061	.6932	104	26
35	.50879	.59101	1.6920	.86089	**25**
36	904	140	.6909	074	24
37	929	179	.6898	059	23
38	954	218	.6887	045̅	22
39	.50979	258	.6875	030	21
40	.51004	.59297	1.6864	.86015̅	**20**
41	029	336	.6853	.86000	19
42	054	376	.6842	.85985	18
43	079	415	.6831	970	17
44	104	454	.6820	956	16
45	.51129	.59494	1.6808	.85941	**15**
46	154	533	.6797	926	14
47	179	573	.6786	911	13
48	204	612	.6775	896	12
49	229	651	.6764	881	11
50	.51254	.59691	1.6753	.85866	**10**
51	279	730	.6742	851	9
52	304	770	.6731	836	8
53	329	809	.6720	821	7
54	354	849	.6709	806	6
55	.51379	.59888	1.6698	.85792	**5**
56	404	928	.6687	777	4
57	429	.59967	.6676	762	3
58	454	.60007	.6665̅	747	2
59	479	046	.6654	732	1
60	.51504	.60086	1.6643	.85717	**0**
′	cos	cot	tan	sin	

59°

31°

′	sin	tan	cot	cos	′
0	.51504	.60086	1.6643	.85717	**60**
1	529	126	.6632	702	59
2	554	165	.6621	687	58
3	579	205̅	.6610	672	57
4	604	245̅	.6599	657	56
5	.51628	.60284	1.6588	.85642	**55**
6	653	324	.6577	627	54
7	678	364	.6566	612	53
8	703	403	.6555	597	52
9	728	443	.6545̅	582	51
10	.51753	.60483	1.6534	.85567	**50**
11	778	522	.6523	551	49
12	803	562	.6512	536	48
13	828	602	.6501	521	47
14	852	642	.6490	506	46
15	.51877	.60681	1.6479	.85491	**45**
16	902	721	.6469	476	44
17	927	761	.6458	461	43
18	952	801	.6447	446	42
19	.51977	841	.6436	431	41
20	.52002	.60881	1.6426	.85416	**40**
21	026	921	.6415̅	401	39
22	051	.60960	.6404	385	38
23	076	.61000	.6393	370	37
24	101	040	.6383	355	36
25	.52126	.61080	1.6372	.85340	**35**
26	151	120	.6361	325̅	34
27	175	160	.6351	310	33
28	200	200	.6340	294	32
29	225	240	.6329	279	31
30	.5225̅0	.61280	1.6319	.85264	**30**
31	275̅	320	.6308	249	29
32	299	360	.6297	234	28
33	324	400	.6287	218	27
34	349	440	.6276	203	26
35	.52374	.61480	1.6265	.85188	**25**
36	399	520	.6255̅	173	24
37	423	561	.6244	157	23
38	448	601	.6234	142	22
39	473	641	.6223	127	21
40	.52498	.61681	1.6212	.85112	**20**
41	522	721	.6202	096	19
42	547	761	.6191	081	18
43	572	801	.6181	066	17
44	597	842	.6170	051	16
45	.52621	.61882	1.6160	.85035	**15**
46	646	922	.6149	020	14
47	671	.61962	.6139	.85005̅	13
48	696	.62003	.6128	.84989	12
49	720	043	.6118	974	11
50	.52745	.62083	1.6107	.84959	**10**
51	770	124	.6097	943	9
52	794	164	.6087	928	8
53	819	204	.6076	913	7
54	844	245̅	.6066	897	6
55	.52869	.62285̅	1.6055	.84882	**5**
56	893	325	.6045̅	866	4
57	918	366	.6034	851	3
58	943	406	.6024	836	2
59	967	446	.6014	820	1
60	.52992	.62487	1.6003	.84805̅	**0**
′	cos	cot	tan	sin	

58°

32°

′	sin	tan	cot	cos	
0	.52992	.62487	1.6003	.84805̄	60
1	.53017	527	.5993	789	59
2	041	568	.5983	774	58
3	066	608	.5972	759	57
4	091	649	.5962	743	56
5	.53115	.62689	1.5952	.84728	55
6	140	730	.5941	712	54
7	164	770	.5931	697	53
8	189	811	.5921	681	52
9	214	852	.5911	666	51
10	.53238	.62892	1.5900	.84650	50
11	263	933	.5890	635̄	49
12	288	.62973	.5880	619	48
13	312	.63014	.5869	604	47
14	337	055̄	.5859	588	46
15	.53361	.63095	1.5849	.84573	45
16	386	136	.5839	557	44
17	411	177	.5829	542	43
18	435	217	.5818	526	42
19	460	258	.5808	511	41
20	.53484	.63299	1.5798	.84495	40
21	509	340	.5788	480	39
22	534	380	.5778	464	38
23	558	421	.5768	448	37
24	583	462	.5757	433	36
25	.53607	.63503	1.5747	.84417	35
26	632	544	.5737	402	34
27	656	584	.5727	386	33
28	681	625	.5717	370	32
29	705	666	.5707	355̄	31
30	.53730	.63707	1.5697	.84339	30
31	754	748	.5687	324	29
32	779	789	.5677	308	28
33	804	830	.5667	292	27
34	828	871	.5657	277	26
35	.53853	.63912	1.5647	.84261	25
36	877	953	.5637	245	24
37	902	.63994	.5627	230	23
38	926	.64035̄	.5617	214	22
39	951	076	.5607	198	21
40	.53975	.64117	1.5597	.84182	20
41	.54000	158	.5587	167	19
42	024	199	.5577	151	18
43	049	240	.5567	135	17
44	073	281	.5557	120	16
45	.54097	.64322	1.5547	.84104	15
46	122	363	.5537	088	14
47	146	404	.5527	072	13
48	171	446	.5517	057	12
49	195	487	.5507	041	11
50	.54220	.64528	1.5497	.84025	10
51	244	569	.5487	.84009	9
52	269	610	.5477	.83994	8
53	293	652	.5468	978	7
54	317	693	.5458	962	6
55	.54342	.64734	1.5448	.83946	5
56	366	775	.5438	930	4
57	391	817	.5428	915̄	3
58	415	858	.5418	899	2
59	440	899	.5408	883	1
60	.54464	.64941	1.5399	.83867	0
	cos	cot	tan	sin	′

57°

33°

′	sin	tan	cot	cos	
0	.54464	.64941	1.5399	.83867	60
1	488	.64982	.5389	851	59
2	513	.65024	.5379	835	58
3	537	065̄	.5369	819	57
4	561	106	.5359	804	56
5	.54586	.65148	1.535̄0	.83788	55
6	610	189	.5340	772	54
7	635̄	231	.5330	756	53
8	659	272	.5320	740	52
9	683	314	.5311	724	51
10	.54708	.65355	1.5301	.83708	50
11	732	397	.5291	692	49
12	756	438	.5282	676	48
13	781	480	.5272	660	47
14	805̄	521	.5262	645̄	46
15	.54829	.65563	1.5253	.83629	45
16	854	604	.5243	613	44
17	878	646	.5233	597	43
18	902	688	.5224	581	42
19	927	729	.5214	565̄	41
20	.54951	.65771	1.5204	.83549	40
21	975	813	.5195̄	533	39
22	.54999	854	.5185	517	38
23	.55024	896	.5175	501	37
24	048	938	.5166	485̄	36
25	.55072	.65980	1.5156	.83469	35
26	097	.66021	.5147	453	34
27	121	063	.5137	437	33
28	145	105̄	.5127	421	32
29	169	147	.5118	405̄	31
30	.55194	.66189	1.5108	.83389	30
31	218	230	.5099	373	29
32	242	272	.5089	356	28
33	266	314	.5080	340	27
34	291	356	.5070	324	26
35	.55315̄	.66398	1.5061	.83308	25
36	339	440	.5051	292	24
37	363	482	.5042	276	23
38	388	524	.5032	260	22
39	412	566	.5023	244	21
40	.55436	.66608	1.5013	.83228	20
41	460	65̄0	.5004	212	19
42	484	692	.4994	195	18
43	509	734	.4985̄	179	17
44	533	776	.4975	163	16
45	.55557	.66818	1.4966	.83147	15
46	581	860	.4957	131	14
47	605	902	.4947	115̄	13
48	630	944	.4938	098	12
49	654	.66986	.4928	082	11
50	.55678	.67028	1.4919	.83066	10
51	702	071	.4910	05̄0	9
52	726	113	.4900	034	8
53	750	155̄	.4891	017	7
54	775̄	197	.4882	.83001	6
55	.55799	.67239	1.4872	.82985	5
56	823	282	.4863	969	4
57	847	324	.4854	953	3
58	871	366	.4844	936	2
59	895	409	.4835̄	920	1
60	.55919	.67451	1.4826	.82904	0
	cos	cot	tan	sin	′

56°

34°

′	sin	tan	cot	cos	
0	.55919	.67451	1.4826	.82904	**60**
1	943	493	.4816	887	59
2	968	536	.4807	871	58
3	.55992	578	.4798	855̅	57
4	.56016	620	.4788	839	56
5	.56040	.67663	1.4779	.82822	**55**
6	064	705	.4770	806	54
7	088	748	.4761	790	53
8	112	790	.4751	773	52
9	136	832	.4742	757	51
10	.56160	.67875̅	1.4733	.82741	**50**
11	184	917	.4724	724	49
12	208	.67960	.4715̅	708	48
13	232	.68002	.4705	692	47
14	256	045	.4696	675	46
15	.56280	.68088	1.4687	.82659	**45**
16	305̅	130	.4678	643	44
17	329	173	.4669	626	43
18	353	215	.4659	610	42
19	377	258	.4650	593	41
20	.56401	.68301	1.4641	.82577	**40**
21	425̅	343	.4632	561	39
22	449	386	.4623	544	38
23	473	429	.4614	528	37
24	497	471	.4605̅	511	36
25	.56521	.68514	1.4596	.82495̅	**35**
26	545̅	557	.4586	478	34
27	569	600	.4577	462	33
28	593	642	.4568	446	32
29	617	685	.4559	429	31
30	.56641	.68728	1.4550	.82413	**30**
31	665̅	771	.4541	396	29
32	689	814	.4532	380	28
33	713	857	.4523	363	27
34	736	900	.4514	347	26
35	.56760	.68942	1.4505̅	.82330	**25**
36	784	.68985	.4496	314	24
37	808	.69028	.4487	297	23
38	832	071	.4478	281	22
39	856	114	.4469	264	21
40	.56880	.69157	1.4460	.82248	**20**
41	904	200	.4451	231	19
42	928	243	.4442	214	18
43	952	286	.4433	198	17
44	.56976	329	.4424	181	16
45	.57000	.69372	1.4415̅	.82165̅	**15**
46	024	416	.4406	148	14
47	047	459	.4397	132	13
48	071	502	.4388	115̅	12
49	095	545̅	.4379	098	11
50	.57119	.69588	1.4370	.82082	**10**
51	143	631	.4361	065	9
52	167	675̅	.4352	048	8
53	191	718	.4344	032	7
54	215̅	761	.4335̅	.82015	6
55	.57238	.69804	1.4326	.81999	**5**
56	262	847	.4317	982	4
57	286	891	.4308	965	3
58	310	934	.4299	949	2
59	334	.69977	.4290	932	1
60	.57358	.70021	1.4281	.81915	**0**
	cos	cot	tan	sin	′

55°

35°

′	sin	tan	cot	cos	
0	.57358	.70021	1.4281	.81915	**60**
1	381	064	.4273	899	59
2	405	107	.4264	882	58
3	429	151	.4255̅	865	57
4	453	194	.4246	848	56
5	.57477	.70238	1.4237	.81832	**55**
6	501	281	.4229	815̅	54
7	524	325̅	.4220	798	53
8	548	368	.4211	782	52
9	572	412	.4202	765̅	51
10	.57596	.70455	1.4193	.81748	**50**
11	619	499	.4185̅	731	49
12	643	542	.4176	714	48
13	667	586	.4167	698	47
14	691	629	.4158	681	46
15	.57715̅	.70673	1.415̅0	.81664	**45**
16	738	717	.4141	647	44
17	762	760	.4132	631	43
18	786	804	.4124	614	42
19	810	848	.4115̅	597	41
20	.57833	.70891	1.4106	.81580	**40**
21	857	935	.4097	563	39
22	881	.70979	.4089	546	38
23	904	.71023	.4080	530	37
24	928	066	.4071	513	36
25	.57952	.71110	1.4063	.81496	**35**
26	976	154	.4054	479	34
27	.57999	198	.4045	462	33
28	.58023	242	.4037	445	32
29	047	285	.4028	428	31
30	.58070	.71329	1.4019	.81412	**30**
31	094	373	.4011	395̅	29
32	118	417	.4002	378	28
33	141	461	.3994	361	27
34	165̅	505	.3985	344	26
35	.58189	.71549	1.3976	.81327	**25**
36	212	593	.3968	310	24
37	236	637	.3959	293	23
38	260	681	.3951	276	22
39	283	725	.3942	259	21
40	.58307	.71769	1.3934	.81242	**20**
41	330	813	.3925	225	19
42	354	857	.3916	208	18
43	378	901	.3908	191	17
44	401	946	.3899	174	16
45	.58425̅	.71990	1.3891	.81157	**15**
46	449	.72034	.3882	140	14
47	472	078	.3874	123	13
48	496	122	.3865	106	12
49	519	167	.3857	089	11
50	.58543	.72211	1.3848	.81072	**10**
51	567	255	.3840	055	9
52	590	299	.3831	038	8
53	614	344	.3823	021	7
54	637	388	.3814	.81004	6
55	.58661	.72432	1.3806	80987	**5**
56	684	477	.3798	970	4
57	708	521	.3789	953	3
58	731	565	.3781	936	2
59	755̅	610	.3772	919	1
60	.58779	.72654	1.3764	.80902	**0**
	cos	cot	tan	sin	′

54°

36°

′	sin	tan	cot	cos	
0	.58779	.72654	1.3764	.80902	60
1	802	699	.3755	885̄	59
2	826	743	.3747	867	58
3	849	788	.3739	850	57
4	873	832	.3730	833	56
5	.58896	.72877	1.3722	.80816	55
6	920	921	.3713	799	54
7	943	.72966	.3705	782	53
8	967	.73010	.3697	765̄	52
9	.58990	055	.3688	748	51
10	.59014	.73100	1.3680	.80730	50
11	037	144	.3672	713	49
12	061	189	.3663	696	48
13	084	234	.3655̄	679	47
14	108	278	.3647	662	46
15	.59131	.73323	1.3638	.80644	45
16	154	368	.3630	627	44
17	178	413	.3622	610	43
18	201	457	.3613	593	42
19	225̄	502	.3605	576	41
20	.59248	.73547	1.3597	.80558	40
21	272	592	.3588	541	39
22	295	637	.3580	524	38
23	318	681	.3572	507	37
24	342	726	.3564	489	36
25	.59365	.73771	1.3555	.80472	35
26	389	816	.3547	455̄	34
27	412	861	.3539	438	33
28	436	906	.3531	420	32
29	459	951	.3522	403	31
30	.59482	.73996	1.3514	.80386	30
31	506	74041	.3506	368	29
32	529	086	.3498	351	28
33	552	131	.3490	334	27
34	576	176	.3481	316	26
35	.59599	.74221	1.3473	.80299	25
36	622	267	.3465	282	24
37	646	312	.3457	264	23
38	669	357	.3449	247	22
39	693	402	.3440	230	21
40	.59716	.74447	1.3432	.80212	20
41	739	492	.3424	195̄	19
42	763	538	.3416	178	18
43	786	583	.3408	160	17
44	809	628	.3400	143	16
45	.59832	.74674	1.3392	.80125	15
46	856	719	.3384	108	14
47	879	764	.3375	091	13
48	902	810	.3367	073	12
49	926	855̄	.3359	056	11
50	59949	.74900	1.3351	.80038	10
51	972	946	.3343	021	9
52	.59995	.74991	.3335̄	.80003	8
53	.60019	.75037	.3327	.79986	7
54	042	082	.3319	968	6
55	.60065	.75128	1.3311	.79951	5
56	089	173	.3303	934	4
57	112	219	.3295̄	916	3
58	135	264	.3287	899	2
59	158	310	.3278	881	1
60	.60182	.75355	1.3270	.79864	0
	cos	cot	tan	sin	′

53°

37°

′	sin	tan	cot	cos	
0	.60182	.75355	1.3270	.79864	60
1	205̄	401	.3262	846	59
2	228	447	.3254	829	58
3	251	492	.3246	811	57
4	274	538	.3238	793	56
5	.60298	.75584	1.3230	.79776	55
6	321	629	.3222	758	54
7	344	675	.3214	741	53
8	367	721	.3206	723	52
9	390	767	.3198	706	51
10	.60414	.75812	1.3190	.79688	50
11	437	858	.3182	671	49
12	460	904	.3175	653	48
13	483	95̄0	.3167	635	47
14	506	.75996	.3159	618	46
15	.60529	.76042	1.3151	.79600	45
16	553	088	.3143	583	44
17	576	134	.3135	565̄	43
18	599	180	.3127	547	42
19	622	226	.3119	530	41
20	.60645	.76272	1.3111	.79512	40
21	668	318	.3103	494	39
22	691	364	.3095	477	38
23	714	410	.3087	459	37
24	738	456	.3079	441	36
25	.60761	.76502	1.3072	.79424	35
26	784	548	.3064	406	34
27	807	594	.3056	388	33
28	830	640	.3048	371	32
29	853	686	.3040	353	31
30	.60876	.76733	1.3032	.79335	30
31	899	779	.3024	318	29
32	922	825	.3017	300	28
33	945	871	.3009	282	27
34	968	918	.3001	264	26
35	.60991	.76964	1.2993	.79247	25
36	.61015̄	.77010	.2985	229	24
37	038	057	.2977	211	23
38	061	103	.2970	193	22
39	084	149	.2962	176	21
40	.61107	.77196	1.2954	.79158	20
41	130	242	.2946	140	19
42	153	289	.2938	122	18
43	176	335	.2931	105̄	17
44	199	382	.2923	087	16
45	.61222	.77428	1.2915	.79069	15
46	245̄	475̄	.2907	051	14
47	268	521	.2900	033	13
48	291	568	.2892	.79016	12
49	314	615̄	.2884	.78998	11
50	.61337	.77661	1.2876	.78980	10
51	360	708	.2869	962	9
52	383	754	.2861	944	8
53	406	801	.2853	926	7
54	429	848	.2846	908	6
55	.61451	.77895̄	1.2838	.78891	5
56	474	941	.2830	873	4
57	497	.77988	.2822	855̄	3
58	520	.78035̄	.2815	837	2
59	543	082	.2807	819	1
60	.61566	.78129	1.2799	.78801	0
	cos	cot	tan	sin	′

52°

38°

′	sin	tan	cot	cos	
0	.61566	.78129	1.2799	.78801	60
1	589	175	.2792	783	59
2	612	222	.2784	765	58
3	635̄	269	.2776	747	57
4	658	316	.2769	729	56
5	.61681	.78363	1.2761	.78711	**55**
6	704	410	.2753	694	54
7	726	457	.2746	676	53
8	749	504	.2738	658	52
9	772	551	.2731	640	51
10	.61795	.78598	1.2723	.78622	**50**
11	818	645	.2715	604	49
12	841	692	.2708	586	48
13	864	739	.2700	568	47
14	887	786	.2693	55̄0	46
15	.61909	.78834	1.2685̄	.78532	**45**
16	932	881	.2677	514	44
17	955	928	.2670	496	43
18	.61978	.78975	.2662	478	42
19	.62001	.79022	.2655̄	460	41
20	.62024	.79070	1.2647	.78442	**40**
21	046	117	.2640	424	39
22	069	164	.2632	405	38
23	092	212	.2624	387	37
24	115̄	259	.2617	369	36
25	.62138	.79306	1.2609	.78351	**35**
26	160	354	.2602	333	34
27	183	401	.2594	315	33
28	206	449	.2587	297	32
29	229	496	.2579	279	31
30	.62251	.79544	1.2572	.78261	**30**
31	274	591	.2564	243	29
32	297	639	.2557	225̄	28
33	320	686	.2549	206	27
34	342	734	.2542	188	26
35	.62365	.79781	1.2534	.78170	**25**
36	388	829	.2527	152	24
37	411	877	.2519	134	23
38	433	924	.2512	116	22
39	456	.79972	.2504	098	21
40	.62479	.80020	1.2497	.78079	**20**
41	502	067	.2489	061	19
42	524	115	.2482	043	18
43	547	163	.2475̄	025̄	17
44	570	211	.2467	.78007	16
45	.62592	.80258	1.2460	.77988	**15**
46	615	306	.2452	970	14
47	638	354	.2445̄	952	13
48	660	402	.2437	934	12
49	683	45̄0	.2430	916	11
50	.62706	.80498	1.2423	.77897	**10**
51	728	546	.2415	879	9
52	751	594	.2408	861	8
53	774	642	.2401	843	7
54	796	690	.2393	824	6
55	.62819	.80738	1.2386	.77806	**5**
56	842	786	.2378	788	4
57	864	834	.2371	769	3
58	887	882	.2364	751	2
59	909	930	.2356	733	1
60	.62932	.80978	1.2349	.77715̄	**0**
	cos	cot	tan	sin	′

51°

39°

′	sin	tan	cot	cos	
0	.62932	.80978	1.2349	.77715̄	**60**
1	955̄	.81027	.2342	696	59
2	.62977	075̄	.2334	678	58
3	.63000	123	.2327	660	57
4	022	171	.2320	641	56
5	.63045	.81220	1.2312	.77623	**55**
6	068	268	.2305̄	605̄	54
7	090	316	.2298	586	53
8	113	364	.2290	568	52
9	135	413	.2283	55̄0	51
10	.63158	.81461	1.2276	.77531	**50**
11	180	510	.2268	513	49
12	203	558	.2261	494	48
13	225	606	.2254	476	47
14	248	655̄	.2247	458	46
15	.63271	.81703	1.2239	.77439	**45**
16	293	752	.2232	421	44
17	316	800	.2225̄	402	43
18	338	849	.2218	384	42
19	361	898	.2210	366	41
20	.63383	.81946	1.2203	.77347	**40**
21	406	.81995̄	.2196	329	39
22	428	.82044	.2189	310	38
23	451	092	.2181	292	37
24	473	141	.2174	273	36
25	.63496	.82190	1.2167	.77255̄	**35**
26	518	238	.2160	236	34
27	540	287	.2153	218	33
28	563	336	.2145	199	32
29	585	385̄	.2138	181	31
30	.63608	.82434	1.2131	.77162	**30**
31	630	483	.2124	144	29
32	653	531	.2117	125	28
33	675	580	.2109	107	27
34	698	629	.2102	088	26
35	.63720	.82678	1.2095	.77070	**25**
36	742	727	.2088	051	24
37	765̄	776	.2081	033	23
38	787	825	.2074	.77014	22
39	810	874	.2066	.76996	21
40	.63832	.82923	1.2059	.76977	**20**
41	854	.82972	.2052	959	19
42	877	.83022	.2045	940	18
43	899	071	.2038	921	17
44	922	120	.2031	903	16
45	.63944	.83169	1.2024	.76884	**15**
46	966	218	.2017	866	14
47	.63989	268	.2009	847	13
48	.64011	317	.2002	828	12
49	033	366	.1995	810	11
50	.64056	.83415	1.1988	.76791	**10**
51	078	465̄	.1981	772	9
52	100	514	.1974	754	8
53	123	564	.1967	735	7
54	145̄	613	.1960	717	6
55	.64167	.83662	1.1953	.76698	**5**
56	190	712	.1946	679	4
57	212	761	.1939	661	3
58	234	811	.1932	642	2
59	256	860	.1925̄	623	1
60	.64279	.83910	1.1918	.76604	**0**
	cos	cot	tan	sin	′

50°

40°

′	sin	tan	cot	cos	
0	.64279	.83910	1.1918	.76604	60
1	301	.83960	.1910	586	59
2	323	.84009	.1903	567	58
3	346	059	.1896	548	57
4	368	108	.1889	530	56
5	.64390	.84158	1.1882	.76511	55
6	412	208	.1875	492	54
7	435̄	258	.1868	473	53
8	457	307	.1861	455̄	52
9	479	357	.1854	436	51
10	.64501	.84407	1.1847	.76417	50
11	524	457	.1840	398	49
12	546	507	.1833	380	48
13	568	556	.1826	361	47
14	590	606	.1819	342	46
15	.64612	.84656	1.1812	.76323	45
16	635̄	706	.1806	304	44
17	657	756	.1799	286	43
18	679	806	.1792	267	42
19	701	856	.1785̄	248	41
20	.64723	.84906	1.1778	.76229	40
21	746	.84956	.1771	210	39
22	768	.85006	.1764	192	38
23	790	057	.1757	173	37
24	812	107	.175̄0	154	36
25	.64834	.85157	1.1743	.76135̄	35
26	856	207	.1736	116	34
27	878	257	.1729	097	33
28	901	308	.1722	078	32
29	923	358	.1715	059	31
30	.64945̄	.85408	1.1708	.76041	30
31	967	458	.1702	022	29
32	.64989	509	.1695̄	.76003	28
33	.65011	559	.1688	.75984	27
34	033	609	.1681	965̄	26
35	.65055	.85660	1.1674	.75946	25
36	077	710	.1667	927	24
37	100	761	.1660	908	23
38	122	811	.1653	889	22
39	144	862	.1647	870	21
40	.65166	.85912	1.1640	.75851	20
41	188	.85963	.1633	832	19
42	210	.86014	.1626	813	18
43	232	064	.1619	794	17
44	254	115̄	.1612	775	16
45	.65276	.86166	1.1606	.75756	15
46	298	216	.1599	738	14
47	320	267	.1592	719	13
48	342	318	.1585	700	12
49	364	368	.1578	680	11
50	.65386	.86419	1.1571	.75661	10
51	408	470	.1565̄	642	9
52	430	521	.1558	623	8
53	452	572	.1551	604	7
54	474	623	.1544	585	6
55	.65496	.86674	1.1538	.75566	5
56	518	725̄	.1531	547	4
57	540	776	.1524	528	3
58	562	827	.1517	509	2
59	584	878	.1510	490	1
60	.65606	.86929	1.1504	.75471	0
	cos	cot	tan	sin	′

49°

41°

′	sin	tan	cot	cos	
0	.65606	.86929	1.1504	.75471	60
1	628	.86980	.1497	452	59
2	65̄0	.87031	.1490	433	58
3	672	082	.1483	414	57
4	694	133	.1477	395̄	56
5	.65716	.87184	1.1470	.75375	55
6	738	236	.1463	356	54
7	759	287	.1456	337	53
8	781	338	.145̄0	318	52
9	803	389	.1443	299	51
10	.65825	.87441	1.1436	.75280	50
11	847	492	.1430	261	49
12	869	543	.1423	241	48
13	891	595̄	.1416	222	47
14	913	646	.1410	203	46
15	.65935̄	.87698	1.1403	.75184	45
16	956	749	.1396	165̄	44
17	.65978	801	.1389	146	43
18	.66000	852	.1383	126	42
19	022	904	.1376	107	41
20	.66044	.87955	1.1369	.75088	40
21	066	.88007	.1363	069	39
22	088	059	.1356	05̄0	38
23	109	110	.1349	030	37
24	131	162	.1343	.75011	36
25	.66153	.88214	1.1336	.74992	35
26	175̄	265	.1329	973	34
27	197	317	.1323	953	33
28	218	369	.1316	934	32
29	240	421	.1310	915̄	31
30	.66262	.88473	1.1303	.74896	30
31	284	524	.1296	876	29
32	306	576	.1290	857	28
33	327	628	.1283	838	27
34	349	680	.1276	818	26
35	.66371	.88732	1.1270	.74799	25
36	393	784	.1263	780	24
37	414	836	.1257	760	23
38	436	888	.1250	741	22
39	458	940	.1243	722	21
40	.66480	.88992	1.1237	.74703	20
41	501	.89045̄	.1230	683	19
42	523	097	.1224	664	18
43	545̄	149	.1217	644	17
44	566	201	.1211	625	16
45	.66588	.89253	1.1204	.74606	15
46	610	306	.1197	586	14
47	632	358	.1191	567	13
48	653	410	.1184	548	12
49	675̄	463	.1178	528	11
50	.66697	.89515	1.1171	.74509	10
51	718	567	.1165̄	489	9
52	740	620	.1158	470	8
53	762	672	.1152	451	7
54	783	725̄	.1145	431	6
55	.66805̄	.89777	1.1139	.74412	5
56	827	830	.1132	392	4
57	848	883	.1126	373	3
58	870	935	.1119	353	2
59	891	.89988	.1113	334	1
60	.66913	.90040	1.1106	.74314	0
	cos	cot	tan	sin	′

48°

42°

′	sin	tan	cot	cos	
0	.66913	.90040	1.1106	.74314	60
1	93$\bar{5}$	093	.1100	295	59
2	956	146	.1093	276	58
3	978	199	.1087	256	57
4	.66999	251	.1080	237	56
5	.67021	.90304	1.1074	.74217	55
6	043	357	.1067	198	54
7	064	410	.1061	178	53
8	086	463	.1054	159	52
9	107	516	.1048	139	51
10	.67129	.90569	1.1041	.74120	50
11	151	621	.103$\bar{5}$	100	49
12	172	674	.1028	080	48
13	194	727	.1022	061	47
14	215	781	.1016	041	46
15	.67237	.90834	1.1009	.74022	45
16	258	887	.1003	.74002	44
17	280	940	.0996	.73983	43
18	301	.90993	.0990	963	42
19	323	.91046	.0983	944	41
20	.67344	.91099	1.0977	.73924	40
21	366	153	.0971	904	39
22	387	206	.0964	88$\bar{5}$	38
23	409	259	.0958	865	37
24	430	313	.0951	846	36
25	.67452	.91366	1.0945	.73826	35
26	473	419	.0939	806	34
27	49$\bar{5}$	473	.0932	787	33
28	516	526	.0926	767	32
29	538	580	.0919	747	31
30	.67559	.91633	1.0913	.73728	30
31	580	687	.0907	708	29
32	602	740	.0900	688	28
33	623	794	.0894	669	27
34	64$\bar{5}$	847	.0888	649	26
35	.67666	.91901	1.0881	.73629	25
36	688	.9195$\bar{5}$	.0875	610	24
37	709	.92008	.0869	590	23
38	730	062	.0862	570	22
39	752	116	.0856	551	21
40	.67773	.92170	1.08$\bar{5}$0	.73531	20
41	79$\bar{5}$	224	.0843	511	19
42	816	277	.0837	491	18
43	837	331	.0831	472	17
44	859	385	.0824	452	16
45	.67880	.92439	1.0818	.73432	15
46	901	493	.0812	413	14
47	923	547	.0805	393	13
48	944	601	.0799	373	12
49	965	655	.0793	353	11
50	.67987	.92709	1.0786	.73333	10
51	.68008	763	.0780	314	9
52	029	817	.0774	294	8
53	051	872	.0768	274	7
54	072	926	.0761	254	6
55	.68093	.92980	1.0755	.73234	5
56	11$\bar{5}$	.93034	.0749	21$\bar{5}$	4
57	136	088	.0742	19$\bar{5}$	3
58	157	143	.0736	175	2
59	179	197	.0730	155	1
60	.68200	.93252	1.0724	.73135	0
	cos	cot	tan	sin	′

47°

43°

′	sin	tan	cot	cos	
0	.68200	.93252	1.0724	.73135	60
1	221	306	.0717	116	59
2	242	360	.0711	096	58
3	264	41$\bar{5}$	.070$\bar{5}$	076	57
4	28$\bar{5}$	469	.0699	056	56
5	.68306	.93524	1.0692	.73036	55
6	327	578	.0686	.73016	54
7	349	633	.0680	.72996	53
8	370	688	.0674	976	52
9	391	742	.0668	957	51
10	.68412	.93797	1.0661	.72937	50
11	434	852	.0655	917	49
12	45$\bar{5}$	906	.0649	897	48
13	476	.93961	.0643	877	47
14	497	.94016	.0637	857	46
15	.68518	.94071	1.0630	.72837	45
16	539	125	.0624	817	44
17	561	180	.0618	797	43
18	582	235	.0612	777	42
19	603	290	.0606	757	41
20	.68624	.94345	1.0599	.72737	40
21	645	400	.0593	717	39
22	666	455	.0587	697	38
23	688	510	.0581	677	37
24	709	565	.057$\bar{5}$	657	36
25	.68730	.94620	1.0569	.72637	35
26	751	676	.0562	617	34
27	772	731	.0556	597	33
28	793	786	.0550	577	32
29	814	841	.0544	557	31
30	.68835	.94896	1.0538	.72537	30
31	857	.94952	.0532	517	29
32	878	.95007	.0526	497	28
33	899	062	.0519	477	27
34	920	118	.0513	457	26
35	.68941	.95173	1.0507	.72437	25
36	962	229	.0501	417	24
37	.68983	284	.049$\bar{5}$	397	23
38	.69004	340	.0489	377	22
39	025	395	.0483	357	21
40	.69046	.95451	1.0477	.72337	20
41	067	506	.0470	317	19
42	088	562	.0464	297	18
43	109	618	.0458	277	17
44	130	673	.0452	257	16
45	.69151	.95729	1.0446	.72236	15
46	172	78$\bar{5}$	.0440	216	14
47	193	841	.0434	196	13
48	214	897	.0428	176	12
49	235	.95952	.0422	156	11
50	.69256	.96008	1.0416	.72136	10
51	277	064	.0410	116	9
52	298	120	.0404	095	8
53	319	176	.0398	075	7
54	340	232	.0392	055	6
55	.69361	.96288	1.0385	.7203$\bar{5}$	5
56	382	344	.0379	.7201$\bar{5}$	4
57	403	400	.0373	.7199$\bar{5}$	3
58	424	457	.0367	974	2
59	44$\bar{5}$	513	.0361	954	1
60	.69466	.96569	1.0355	.71934	0
	cos	cot	tan	sin	′

46°

44°

′	sin	tan	cot	cos	
0	.69466	.96569	1.0355	.71934	**60**
1	487	625	.0349	914	59
2	508	681	.0343	894	58
3	529	738	.0337	873	57
4	549	794	.0331	853	56
5	.69570	.96850	1.0325	.71833	**55**
6	591	907	.0319	813	54
7	612	.96963	.0313	792	53
8	633	.97020	.0307	772	52
9	654	076	.0301	752	51
10	.69675̄	.97133	1.0295	.71732	**50**
11	696	189	.0289	711	49
12	717	246	.0283	691	48
13	737	302	.0277	671	47
14	758	359	.0271	650	46
15	.69779	.97416	1.0265	.71630	**45**
16	800	472	.0259	610	44
17	821	529	.0253	590	43
18	842	586	.0247	569	42
19	862	643	.0241	549	41
20	.69883	.97700	1.0235	.71529	**40**
21	904	756	.0230	508	39
22	925̄	813	.0224	488	38
23	946	870	.0218	468	37
24	966	927	.0212	447	36
25	.69987	.97984	1.0206	.71427	**35**
26	.70008	.98041	.0200	407	34
27	029	098	.0194	386	33
28	049	155	.0188	366	32
29	070	213	.0182	345	31
30	.70091	.98270	1.0176	.71325	**30**
31	112	327	.0170	305̄	29
32	132	384	.0164	284	28
33	153	441	.0158	264	27
34	174	499	.0152	243	26
35	.70195̄	.98556	1.0147	.71223	**25**
36	215	613	.0141	203	24
37	236	671	.0135̄	182	23
38	257	728	.0129	162	22
39	277	786	.0123	141	21
40	.70298	.98843	1.0117	.71121	**20**
41	319	901	.0111	100	19
42	339	.98958	.0105	080	18
43	360	.99016	.0099	059	17
44	381	073	.0094	039	16
45	.70401	.99131	1.0088	.71019	**15**
46	422	189	.0082	.70998	14
47	443	247	.0076	978	13
48	463	304	.0070	957	12
49	484	362	.0064	937	11
50	.70505̄	.99420	1.0058	.70916	**10**
51	525	478	.0052	896	9
52	546	536	.0047	875	8
53	567	594	.0041	855̄	7
54	587	652	.0035̄	834	6
55	.70608	.99710	1.0029	.70813	**5**
56	628	768	.0023	793	4
57	649	826	.0017	772	3
58	670	884	.0012	752	2
59	690	.99942	.0006	731	1
60	.70711	1.0000	1.0000	.70711	**0**
	cos	cot	tan	sin	

45°

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