THERMIONIC VALVE CIRCUITS — WILLIAMS

PITMAN

THERMIONIC VALVE CIRCUITS

BY

EMRYS WILLIAMS

PH.D., B.ENG., M.I.E.E., M.Brit.I.R.E.

Professor of Electrical Engineering in the University College of South Wales and Monmouthshire

FOURTH EDITION

LONDON

SIR ISAAC PITMAN & SONS, LTD.

First edition 1942
Second edition 1944
Reprinted 1945
Reprinted 1947
Third edition 1952
Revised and reprinted 1955
Reprinted 1956
Reprinted 1958
Fourth edition 1961

SIR ISAAC PITMAN & SONS, LTD.
PITMAN HOUSE, PARKER STREET, KINGSWAY, LONDON, W.C.2
THE PITMAN PRESS, BATH
PITMAN HOUSE, BOUVERIE STREET, CARLTON, MELBOURNE
22–25 BECKETT'S BUILDINGS, PRESIDENT STREET, JOHANNESBURG

ASSOCIATED COMPANIES
PITMAN MEDICAL PUBLISHING COMPANY, LTD.
39 PARKER STREET, LONDON, W.C.2
PITMAN PUBLISHING CORPORATION
2 WEST 45TH STREET, NEW YORK
SIR ISAAC PITMAN & SONS (CANADA), LTD.
(INCORPORATING THE COMMERCIAL TEXT BOOK COMPANY)
PITMAN HOUSE, 381–383 CHURCH STREET, TORONTO

MADE IN GREAT BRITAIN AT THE PITMAN PRESS, BATH
F1—(T.205)

PREFACE

TO THE FOURTH EDITION

This book is not about valves but about valve-circuits. Many more engineers and converted physicists are engaged on valve-circuits (in telephony, radio communication, television, telemetry, automatic control, radar, computation, instrumentation, etc.) than are engaged in valve design and development. There is a broad measure of disagreement, however, on the question of how much a valve-circuit engineer should know about the inside of a valve. In one well-known public examination, the examination paper on valve-circuits includes questions on a wide coverage of solid-state physics and the physics of electronic emission.

This book takes the opposite point of view, namely that there is so much of real interest to be said about valve-circuits that a single, very elementary chapter must suffice for valves themselves. Even eighteen years ago (when the first edition was a slim volume which fitted the pocket both physically and figuratively) it would have been a masterpiece of understatement to say that enough was known about valve-circuits to fill a book. Today enough is known to fill a library. This book has therefore two main purposes: firstly, to serve as a reliable guide to this wide field of knowledge and, secondly, to *teach* the subject. To achieve these ends, the scope of the book must be strictly limited to valve circuits. To extend the scope, either by including the physics of the valve, or by including cathode-ray tubes and all the other electron tubes which have become important, would mean forgoing that prodigality of words which distinguishes teaching from the skilled précis writing to be found in the paper-starved Proceedings of professional institutions.

Electronics has become the handmaiden of almost all other sciences. As a result many physicists and other scientists find themselves in need of a more thorough and detailed understanding of valve-circuitry than could be provided by a degree-course in pure science. It is hoped that this book will meet their need. It is hoped, too, that it will help to dispel the idea that the valve-circuit engineer needs a kind of intuition, a "feel" for circuits, which cannot be explained or communicated. This myth persists as a result of the habit of *ad hoc* explanation of apparently unrelated circuits and the failure to point out their common features of circuit-technique. For this reason, it has been thought worth while in this book to place more than usual emphasis on various quite simple features (the a.c./d.c. separating circuit, earthing and screening, etc.) and to stress and re-stress the importance of the incremental equivalent circuit.

It is impossible to exaggerate the importance of equivalent circuits as a means of facilitating both qualitative thinking and quantitative analysis. In this connexion it is not only the equivalent circuit of the valve itself which is useful, but also similar circuits (each consisting merely of an e.m.f. in series with an impedance) which can serve as equivalents for a valve plus its anode load impedance, or for valve plus anode load impedance plus cathode lead impedance.

The chief innovation in this new edition is the inclusion of a set of problems with answers. Since the book is not directed towards coaching students for examinations, the problems are not taken from recognized public examination papers but are specially prepared for use with this text. Many of them are very easy problems, designed to provide the reader with a quick check on whether he has understood the text. Among these are some trick questions which tempt the unwary to give an answer when, in fact, insufficient data is provided. These questions are intended as a dire warning against various common pitfalls, arising from loose thinking. The trick questions would lose their effectiveness if the reader were notified of them in advance, but—on the assumption that reviewers read prefaces and students do not—we may direct attention to problems 8 and 16 as typical cases. There are a hundred problems on amplifiers alone. Some of the problems are exercises in circuit design, and some are exercises in the techniques of analysis, including graphical methods of analysis. In the "Answers" section, short (usually numerical) answers are given for the easy or straightforward problems, but the more difficult problems are discussed at greater length. In the preface to an earlier edition of this book it was stated that one of the author's objects was to cover all the main types of valve circuit in such a way that the student may be in a position to understand, or even foresee, further developments. It has therefore been thought legitimate to include a few problems which, in addition to serving as exercises, introduce the reader to known circuits and techniques for which space could not be afforded in the body of the book. For example, it is only among the problems (and in the index) that any mention is made of the constant-current equivalent circuit, of valve-bridges for measuring amplification factor and mutual conductance, of the phase-shifting amplifier and of various methods of calculation, applicable to cathode-coupled stages.

If an author does not know what is wrong with his book after eighteen years, it can only be because his best friends are afraid to tell him. The writer's friends have been commendably free from restraint and his less inhibited students have quite clearly indicated those parts of the book which needed re-writing. A substantial part of the revision which has gone into this new edition is directed towards easier understanding and increased clarity. A number of well-known oscillator circuits, described separately in earlier editions, have been grouped together under the heading "Closed-circuit Oscillators" and a common simple theory is now presented for oscillators of this class. Similarly, a new heading, "Shunt-connected Feedback," serves to correlate a group of circuits previously treated separately (viz. adding circuits, the see-saw circuit, differentiating and integrating circuits using feedback).

For any author who tries to provide in one book a reliable guide to a very large and growing field of knowledge, the main problem is that of deciding what to leave out. The decision has been guided by two rules: first, that space must be found for fundamental principles, and that a class of circuits therefore deserves mention if it introduces any new principle. The second rule is the obvious one, that widespread use of any technique justifies devoting space to explaining it. For both of these reasons, space has been found in this edition for balanced modulators, the cascode amplifier, the Foster-Seeley discriminator, the ratio-detector and (belatedly) for the subject of rectifiers with choke-input filters. The rapid increase in the application of digital computer circuit-techniques faces the author with a difficulty, since even a modest review of the subject would occupy many pages. Rather than omit the subject altogether, a few appetite-whetting sections have been included in the chapter on pulses and pulsed circuits, viz. sections on pulse-counting, binary numbers, gate-circuits for digital computers, and a binary adding circuit. A short paragraph on decibels has been added to the introductory chapter at the beginning of the book and a short list of suggestions for further reading is given at the end of all chapters except the first one.

An apology is perhaps needed for the author's non-conformity in two respects. The first is the use of the Greek letter ρ for the anode incremental-impedance of a valve, instead of the more usual r_a. The second is his dalliance with such outmoded words as "coil" and "condenser" (instead of "inductor" and "capacitor"). He will not complain if his dalliance in this matter earns him the appellation of "dallior."

A kind friend has suggested that this book should now be scrapped and re-written in terms of transistors. The publishers, with an ear to the ground and an eye on the market, feel that the time for such ruthlessness has not yet come. The suggestion, however, is not an irresponsible one. The day is not far off when it will be *more* important for an electronic engineer to have a knowledge of transistor circuitry than of valve circuitry. In the meantime it is comforting (for teacher and student alike) to reflect that so far as circuitry is concerned, valves and transistors have more in common than in contrast, and that a study of either serves as a good basis for an understanding of the other.

EMRYS WILLIAMS

UNIVERSITY COLLEGE OF SOUTH WALES AND MONMOUTHSHIRE
CARDIFF

PREFACE

TO THE FIRST EDITION

THIS book is based on a lecture course given by the author to third-year students for the degree of B.Sc. in Electrical Engineering in the University of Durham. It represents an attempt to include in one small volume the theory of the operation and design of thermionic valve circuits. This theory is to be found included in the larger books on communications engineering and radio engineering, and also, in a very qualitative form, in the semi-popular books on wireless. But there appears to be no convenient textbook dealing exclusively with this subject and suitable for universities, technical colleges, and electrical engineers trained in the days before the development of the valve.

The book assumes a knowledge of alternating current theory and of mathematics up to the standard of a university student at the end of his second year. For the benefit of those whose university days are no more than a pleasant (if irrelevant) memory, the alternating current theory used throughout the book is summarized in the first chapter.

It will be seen that extensive references to the literature of the subject are not given. For further reading and for the study of specialized applications of the theory, the reader is referred to the following—

1. *Radio Engineering*, F. E. Terman (many references to the literature).

2. *A Critical Review of Literature on Amplifiers for Radio Reception*, H. A. Thomas. Radio Research Board Special Report No. 9 (H.M. Stationery Office).

3. The references to the literature published monthly in the *Wireless Engineer*.

EMRYS WILLIAMS

KING'S COLLEGE
NEWCASTLE UPON TYNE

CONTENTS

CHAPTER VIII

CHAPTER I

SUMMARY OF A.C. THEORY AND MATHEMATICS REQUIRED

1. A SINUSOIDAL alternating current of peak value, I, and frequency, f, has an instantaneous value given by

$$i = I \sin \omega t$$

where $\omega = 2\pi f$.
An alternating current whose instantaneous value is given by the equation

$$i = I \sin (\omega t \pm \phi)$$

is said to lead/lag by an angle ϕ with respect to the alternating current $I \sin \omega t$. The quantity ϕ is the angle of phase difference between the two alternating currents.

2. As an alternative to specifying an alternating current by an equation, as above, or by the graph of that equation, we may completely specify an alternating current of known frequency by means of a vector. The length of the vector will equal the peak value (or "amplitude") of the alternating current, and the angle between the vector and some arbitrarily chosen direction (usually the horizontal) will equal the angle of phase difference between the alternating current and some arbitrarily chosen alternating quantity.

The instantaneous value of the alternating current is then the length of the projection of the vector on the vertical, the vector being assumed to rotate anti-clockwise with angular velocity $2\pi f$ radians per second. Hence the graph of the alternating current may be plotted if the vector representing it is given. Clearly an alternating *voltage* may also be represented by a vector in a similar manner.

3. It is convenient to express a vector in the form

$$a + jb$$

where a is the horizontal component of the vector, and b its vertical component (just as a force may be resolved into two components in directions at right angles to each other). The symbol j is used to prefix the vertical component.

The length of the vector, whose horizontal and vertical components are respectively a and b, is clearly $\sqrt{a^2 + b^2}$. This statement is written

$$|a + jb| = \sqrt{a^2 + b^2}$$

The length of a vector is variously known as its "Modulus," "Numerical Value," or "Amplitude." The angle which a vector so expressed makes with the horizontal will be $\tan^{-1} b/a$, or alternatively

$$\cos^{-1} \frac{a}{\sqrt{a^2 + b^2}}$$

4. Addition, subtraction, multiplication, and division of vector quantities expressed in the form $a + jb$, may be carried out by assuming that j equals $\sqrt{-1}$, e.g.

$$(a + jb)(c + jd) = (ac - bd) + j(ad + bc)$$

5. For sinusoidal alternating currents and voltages Kirchhoff's second law may be amended to read: For any closed circuit, the sum of the vectors for the external e.m.f.'s, taken in a given direction round the circuit, equals the sum of the products of current vectors and impedance vectors taken in the same direction. The vectors for the impedance of a resistance, R, an inductance, L, and a capacitance, C, are respectively, R, $j\omega L$, and $1/(j\omega C)$, where ω denotes $2\pi f$.

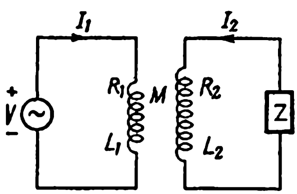


FIG. 1

Kirchhoff's first law may be amended to read: The sum of the vectors for the currents flowing *to* any point of a circuit is equal to zero.

Where there is mutual inductance, M, between two circuits, 1 and 2, there will be an additional e.m.f. vector, $j\omega MI_2$, in circuit 1, and an additional e.m.f. vector, $j\omega MI_1$, in circuit 2. For instance, in Fig. 1 (in which the coils are assumed wound in the same sense on the same former) the vector equations are

$$V = (R_1 + j\omega L_1)I_1 + j\omega MI_2$$
$$O = (R_2 + j\omega L_2 + Z)I_2 + j\omega MI_1$$

If the coils are wound in the opposite sense (i.e. one wound clockwise and the other wound anti-clockwise) the sign of M would become negative in both equations. In either case the equations give

$$I_1 = \frac{V}{R_1 + j\omega L_1 + \dfrac{\omega^2 M^2}{R_2 + j\omega L_2 + Z}}$$

Also $$I_2 = j\omega MI_1/(R_2 + j\omega L_2 + Z)$$

It is usual to express these equations in words, thus formulating the following rules*—

(*a*) The primary current is the quotient of the primary voltage and the effective primary impedance, the latter being the sum of the true primary impedance and a term which is the quotient of $\omega^2 M^2$ and the true secondary impedance.

* Applicable only when voltages, currents, and impedances are expressed as vectors (i.e. in the j-notation).

(*b*) The secondary current is the quotient of the secondary e.m.f. (viz. $j\omega MI_1$) and the true secondary impedance.

6. The rules for forming the *impedance vectors* (expressed in the j-notation) for series and parallel impedances are the same as for series and parallel resistances in direct-current circuits.

7. The procedure for determining the current in any branch of a network when all the applied e.m.f.'s are known is as follows: Assume vectors, I_1, I_2, I_3, etc., for the currents in the various branches. By applying Kirchhoff's laws, derive a number of simultaneous equations. Solve these algebraically for the required unknown, remembering that j may be taken as $\sqrt{-1}$ (see Vector Algebra, below). Express the resultant vector in the form $a + jb$. The peak value of the current it represents is then $\sqrt{a^2 + b^2}$.

In the case where there is only one applied e.m.f., the expression for the required current vector will be expressible in the form $V/(a + jb)$. In this case the peak value of the current is $V/\sqrt{a^2 + b^2}$, and the current lags behind the voltage by an angle $\tan^{-1} b/a$.

8. The power dissipated in any circuit element, the sine-wave voltage across which has a peak value, $\hat{V}$, and the current through which has a peak value, $\hat{I}$, is given by

$$W = \frac{\hat{V}}{\sqrt{2}} \cdot \frac{\hat{I}}{\sqrt{2}} \cdot \cos\phi$$

where ϕ is the angle of phase difference between current and voltage. The quantities $\hat{V}/\sqrt{2}$ and $\hat{I}/\sqrt{2}$ are known as the Root Mean Square values of the voltage and current respectively. The quantity $\cos\phi$ is known as the Power Factor.

Algebra of A.C. Vectors

If
$$a + jb = c + jd$$
then
$$a = c$$
and
$$b = d$$

$$|p + jq| = \sqrt{p^2 + q^2}$$

$$\left|\frac{p + jq}{r + js}\right| = \frac{|p + jq|}{|r + js|}$$

$$|(p + jq)(r + js)| = |(p + jq)| \times |(r + js)|$$

To express $\dfrac{p + jq}{r + js}$ in the form $a + jb$, proceed as follows—

$$\frac{p + jq}{r + js} = \frac{(p + jq)\,(r - js)}{(r + js)\,(r - js)}$$
$$= \frac{(pr + qs) + j\,(qr - ps)}{r^2 + s^2}$$

Notation and Symbols

Small letters are used to denote instantaneous values of alternating quantities. Vector quantities are denoted by capital letters, e.g. V, I. The peak value (or "numerical value," or "modulus") of a vector V is denoted by $|V|$, where it is thought necessary to distinguish it from the vector quantity. Otherwise it is written simply as V. Where it is thought necessary to distinguish between peak values and r.m.s. values of sinusoidal alternating quantities, symbols such as $\hat{V}$ and V_{rms} are used. Many relations between peak values of voltages and currents are applicable also to r.m.s. values, since V_{rms} equals $\hat{V}/\sqrt{2}$, and in such cases the capital letters are used without either suffix or circumflex accent.

Some of the symbols used are given below in order to facilitate reference to the equations—

ρ = Anode impedance of a valve.
μ = Amplification factor.
g_m = Mutual conductance.
g_c = Conversion conductance.
(VA) = Voltage amplification ratio.

Suffixes a, g, and c are used to denote "anode," "grid," and "cathode."

C_{ag}, C_{gc}, C_{ga} = Inter-electrode capacitances.

The symbols V_1 and V_2 are used to denote the input and output voltage vectors, respectively, of an amplifier or other valve circuit. V_g and V_a denote the vectors for the alternating components of the grid and anode voltages. When, as is most usually the case, the input voltage of an amplifier is the only alternating voltage in the grid circuit, V_1 and V_g are identical; but it is necessary to distinguish between them in the case of feed-back amplifiers, etc. (See page 104.)

Thevenin's Theorem

This theorem states that any network of linear* impedances, having two output terminals, may be replaced by, and is equivalent to, a single source of e.m.f. in series with a single impedance. Fig. 2 illustrates this for a particular a.c. network, but the theorem applies to networks having steady e.m.f.'s or e.m.f.'s varying with time in any manner whatsoever. The theorem goes on to say that the e.m.f. in the equivalent circuit is equal to that p.d. which would exist between the output terminals of the actual circuit when these terminals are open (i.e. when no external circuit is connected to them) and the impedance in the equivalent circuit is equal to that

* This excludes such non-linear elements as resistors whose resistance is a function of the current, so that their v–i graphs are non-linear.

impedance which would be measurable between the actual output terminals when all internal e.m.f.'s are reduced to zero. Using these rules we can determine the equivalent circuit for any actual circuit. For the circuit of Fig. 2 for instance, we have

$$E_0 = V_1Z_2/(Z_1 + Z_2) \text{ and } Z_0 = Z_1Z_2/(Z_1 + Z_2)$$

We know, therefore, that if any load impedance Z be connected to the actual circuit, the current which will flow through Z will be the

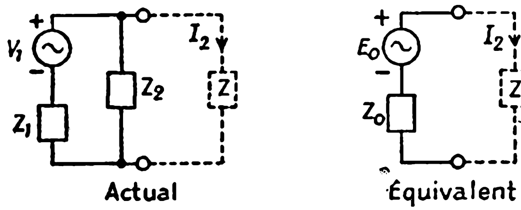


FIG. 2

same as if it were connected not to the actual circuit, but to the equivalent circuit, i.e. we know that

$$\left.\begin{aligned} I_Z &= E_0/(Z_0 + Z) \\ \text{and} \qquad V_Z &= E_0Z/(Z_0 + Z) \end{aligned}\right\} \quad . \quad . \quad . \quad (I.1)$$

The quantities E_0 and Z_0 are known respectively as the *Output E.M.F.* and the *Output Impedance* of the network. The output impedance must not be confused with the load impedance, which latter is the impedance connected across the output terminals, external to the network (Z in Fig. 2). The output impedance is "the internal impedance of the equivalent simple source."

Thevenin's theorem may be used to simplify the analysis of networks (a.c., d.c., or pulsed e.m.f.'s). If, for example, one wishes to determine the current through a given branch Z of the network, one may regard that branch as a load impedance fed from the remainder of the network, and proceed to determine the Thevenin equivalent of this remainder. When E_0 and Z_0 have been calculated, the current through the branch Z is given by equation (I.1).

Output impedance is an important factor where problems of matching are concerned, and it is often necessary to determine the output impedance of an amplifier. In such cases the above method of determining Z_0 (by "looking into" the output terminals, with all internal e.m.f.'s reduced to zero) may present difficulties, and the following is an easier method. Derive an expression for the *output voltage* of the network when supplying a load Z, and re-arrange this expression until it is of the form

$$V_{output} = \frac{aZ}{b + Z} \quad . \quad . \quad . \quad . \quad (I.2)$$

in which a and b must each be independent of Z (since E_0 and Z_0, being each a property of the network *in the absence of Z*, are necessarily independent of the load Z). The output impedance is then given by b and the output e.m.f. by a.

The Charging of a Condenser through Resistances

If, in the circuit of Fig. 3 (a), the condenser is initially uncharged and the switch is closed at time $t = 0$ (i.e. if time be measured from

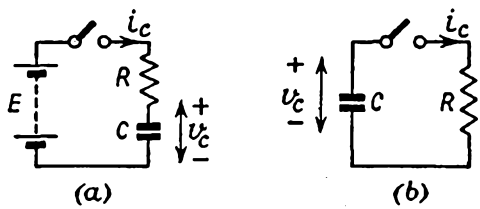


FIG. 3

the instant at which the switch is closed) then the values of the current and of the condenser voltage at any subsequent time t are given by

$$i_c = \frac{E}{R} e^{-t/T}$$

$$v_c = E\,(1 - e^{-t/T})$$

where T denotes the product RC, known as the Time Constant.

If, in the circuit of Fig. 3 (b), the condenser is initially charged so that v_c equals E (v_c being measured in the direction shown) and if the switch be closed at time $t = 0$, then at any subsequent time t

$$i_c = \frac{E}{R} e^{-t/T}$$

$$v_c = E\, e^{-t/T}$$

Both of the above cases, and also the case in which the condenser of Fig. 3 (a) is initially charged to some given p.d., are covered by the following general formula

$$v_c = v_2 + (v_1 - v_2)e^{-t/T} \quad . \quad . \quad . \quad (I.3)$$

where v_1 and v_2 are respectively the initial and final values of v_c. Also

$$i_c = i_1 e^{-t/T} \quad . \quad . \quad . \quad . \quad (I.4)$$

where i_1 is the initial value of the condenser current.

Any network of resistances and e.m.f.'s can be treated by means of equations (I.3) and (I.4) provided that it includes only one capacitance. The capacitance branch may be regarded as the "load" and then the remainder of the network can be simplified, by means of Thevenin's theorem, into the form of Fig. 3. The initial value of i_c may usually be calculated as the current which would flow if the condenser were replaced by a constant e.m.f. equal in magnitude to the initial value of v_c. In the case of Fig. 3 (*a*), of course, with v_c initially zero, this corresponds to substituting a short-circuit for the condenser.

Distortion

The amplification of a voltage or current implies increasing its amplitude without altering its waveform. Distortion of the waveform can take place in the following ways—

1. By the introduction of harmonics. This is known as *Harmonic Distortion.*

2. If the voltage or current to be amplified is not initially sinusoidal, but consists of fundamental and harmonics, or if it consists of a number of components of different frequency, then all the components must receive the same amplification if the waveform is to be unchanged, i.e. the amplification must be independent of frequency. If the amplification is not independent of frequency there is said to be *Attenuation Distortion,* sometimes also called *Frequency Distortion.* (*N.B.* This does *not* imply that the frequency is altered.)

3. In the case of a composite voltage, as above, the relative phase of the components must be the same before and after amplification. The distortion resulting when this condition is not satisfied is known as *Phase Distortion.* Phase distortion is unimportant in sound reproduction.

The above three types of distortion have been defined with reference to the amplification of a current or voltage. Clearly, when any alternating quantity is to be communicated and reproduced, the same considerations apply.

Amplitude-modulated Alternating Quantities

The instantaneous value of an alternating voltage of amplitude, $\hat{V}$, and frequency, f, is given by

$$v = \hat{V} \sin wt$$

where ω equals $2\pi f$. "Modulation" of the amplitude simply means changing the amplitude. Periodic modulation of the amplitude (which is what "Modulation" has come to mean) is the regular variation of the amplitude, to and fro, in the manner of an alternating current. Fig. 133, p. 225 shows an alternating voltage with its amplitude modulated in this way.

If the amplitude is to vary periodically between limits $\hat{V} + \delta\hat{V}$, and $\hat{V} - \delta\hat{V}$, the simplest possible expression for the amplitude is $\hat{V} + \delta\hat{V} \sin pt$. In this case the amplitude is sinusoidally modulated at a frequency $p/2\pi$. This frequency is, of course, lower than the original frequency, $\omega/2\pi$. The expression for the instantaneous value is now

$$v = (\hat{V} + \delta\hat{V} \sin pt) \sin \omega t$$

If $\delta\hat{V}$ equals $\hat{V}$, the amplitude varies between zero and $2\hat{V}$. In general we write $\delta\hat{V}$ equal to a fraction, m, of $\hat{V}$, this fraction being known as the *Depth of Modulation.* This gives

$$v = \hat{V}\,(1 + m \sin pt) \sin \omega t$$

Modulated alternating voltages and currents are the basis of radio telephony. The frequencies $p/2\pi$ and $\omega/2\pi$ are known respectively as the *Modulation Frequency* and the *Carrier Frequency.*

Frequency-modulated Alternating Quantities

Such alternating quantities have a frequency which varies from instant to instant. In the simplest case the frequency varies periodically about a mean value (known as the *Carrier Frequency*) the periodic variations taking place at a lower frequency (known as the *Modulation Frequency*). For instance, if the frequency is to vary periodically between limits $f_c + \delta f$ and $f_c - \delta f$, a simple expression for the frequency at time t would be

$$f = f_c + \delta f \cos pt$$

where $p/2\pi$ is the modulation frequency. The quantity δf is known as the *Frequency Deviation,* and corresponds to the modulation depth in amplitude modulation.

The rotating vector (see page 1) which denotes an alternating quantity rotates with an angular velocity equal to 2π times the frequency. Thus the vector denoting a frequency modulated alternating voltage rotates with a velocity given by

$$\omega = 2\pi[f_c + \delta f \cos pt]$$

The angular position reached by such a rotating vector after time t is clearly given by

$$\theta = \int \omega\, dt$$

$$= 2\pi f_c t + \frac{2\pi\, \delta f}{p} \sin pt$$

The instantaneous value of the alternating voltage, being the projection of the vector upon the vertical, is thus given by $\hat{V} \sin \theta$, i.e.

$$v = \hat{V} \sin \left[2\pi f_c t + \frac{2\pi\, \delta f}{p} \sin pt \right]$$

Decibels

It is sometimes more convenient to give the logarithms of numbers instead of referring to the numbers themselves, either because one is dealing with numbers which range from small up to very large values (the logarithm of a million, to base ten, is only six) or because some logarithmic law is involved, e.g. the law relating the physical intensity of a sound to its "loudness" as adjudged by a human being. The voltage-gain of an amplifier (i.e. the ratio of its output voltage, V_2, to its input voltage, V_1) is a quantity which is often expressed logarithmically, for both of the above reasons. The phrase used is that "the amplifier has a voltage-gain of so many Bels," the number of Bels being calculated as $2 \log_{10}(V_2/V_1)$. The factor, 2, arises from an earlier usage in which this particular mode of expression was reserved exclusively for the logarithms of *power* ratios or *energy* ratios.

The decibel is a unit which is one-tenth the size of a Bel. Thus to calculate the voltage-gain of an amplifier in decibels one merely takes $20 \log_{10}(V_2/V_1)$. Clearly, voltage-gains of 10, 100, 1,000, correspond respectively to 20 dB, 40 dB and 60 dB. A doubling of voltage, i.e. a voltage-gain of 2, corresponds almost exactly to 6 decibels. Frequent reference is made to "three decibels," which corresponds to a voltage ratio of $\sqrt{2}$, because of the convention which associates a ratio of $\sqrt{2}$ with "cut-off" (see page 44).

CHAPTER II

THE THERMIONIC VALVE

A THERMIONIC valve, or thermionic vacuum tube, consists of two or more electrodes in an evacuated space. One of the electrodes (that known as the cathode) is maintained at a high temperature. Valves with two, three, four, five, six, etc., electrodes are known respectively as Diodes, Triodes, Tetrodes, Pentodes, Hexodes, etc.

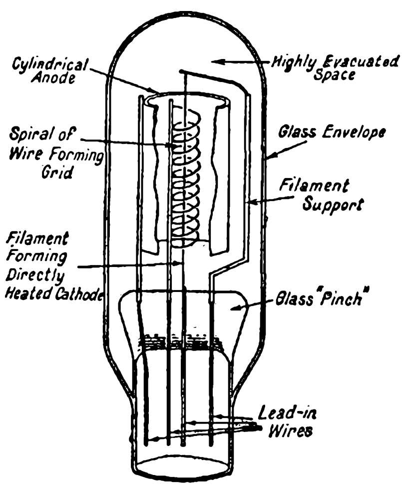


FIG. 4. DIRECTLY HEATED TRIODE

Hot bodies emit electrons, which are particles of negative electricity. In the thermionic valve the function of the hot cathode is to emit electrons, and there is another electrode known as the Anode whose function it is to collect the electrons. Thus positive electricity flows through the valve from anode to cathode, and returns to the anode by way of the external circuit connecting anode and cathode, setting up various useful voltages in this "anode circuit." In general, thermionic valve circuit theory may be said to be the determination of the currents and voltages in the anode circuit from a consideration of the properties of the valve in conjunction with well-known alternating current circuit theory.

A diode has only a cathode and an anode. A triode has in addition an electrode known as a Grid, usually in the form of a mesh of wire or metal gauze interposed between cathode and anode; through this grid the electrons must pass. The anode is usually maintained at a positive potential with respect to the cathode, in order to attract the particles of negative electricity. The potential of the grid with respect to the cathode will clearly affect the flow of electrons. Another external circuit, known as the Grid Circuit, is connected between grid and cathode, its function being to control the flow of electrons, and thus the current in the anode circuit, by controlling the potential of the grid with respect to the cathode. A tetrode has two grids, a pentode has three, and so on.

The cathode of a thermionic valve may be heated directly, by constructing it in the form of a wire or filament and passing direct current through it; or it may be indirectly heated by means of a very small electric heater in contact with it inside the evacuated space. Fig. 4 is a drawing of a rather old fashioned triode. Modern triodes use improved methods of supporting the electrode structures. Fig. 5 shows the symbols used to represent various valves in

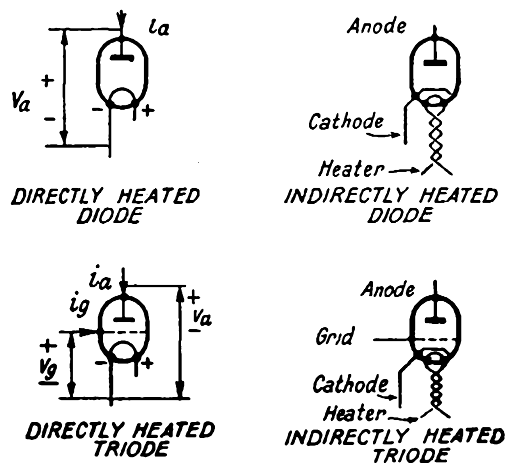


FIG. 5

Note that in the case of the directly heated valves the grid and anode voltages are measured from the negative end of the cathode

drawing circuit diagrams. Fig. 5 also shows the conventional direction of measurement of anode-voltage, v_a, anode-current, i_a, grid-voltage, v_g, and grid-current, i_g.

Thermionic Emission

The rate of emission of electrons from a hot body depends on the temperature and on the nature of the body. It can be shown that the rate of emission, n, is given by

$$n = AT^2e^{-b/T} \quad . \quad . \quad . \quad . \quad \text{(II.1)}$$

where A and b are constants and T is the absolute temperature. The cathode of a thermionic valve does not lose at this rate, however, unless the electrons are removed (by being drawn off to the anode), for the electrostatic field of the electrons in the space around the cathode will repel electrons back into the cathode as fast as they are emitted. Thus the rate of departure of electrons from the cathode is usually less than n, being equal to the rate at which electrons are drawn off by the anode. The electrons in the space around the cathode are spoken of as a Space Charge.

Diode-characteristic

The curve showing how the anode-current of a diode varies with variation of the anode-voltage is known as the Characteristic of the diode, and may be determined experimentally using the circuit of Fig. 6. Fig. 7 shows a typical characteristic. Note that—

(*a*) When the anode-voltage is negative no anode-current flows, for the anode then repels the electrons.

(*b*) The anode-current increases with the anode-voltage, slowly at first and more rapidly for greater values of anode-voltage. This

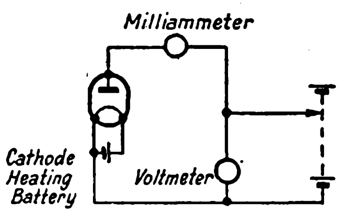


FIG. 6

Note. The cathode heating battery is often omitted from circuit diagrams

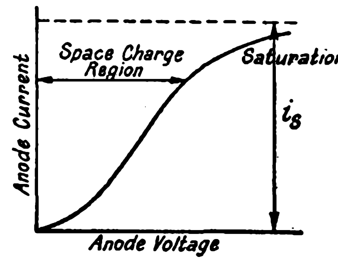


FIG. 7. CHARACTERISTICS OF DIODE

part of the curve is often referred to as the space-charge-limitation region. The relation between anode-voltage and anode-current may be deduced from the consideration that the anode-current resulting from the application of a given anode-voltage will be such that the resultant electrostatic field at the cathode due to the anode and the space charge (i.e. the negative charges in transit from cathode to anode) will be zero. A greater current (i.e. more electrons in transit) would give a negative field at the cathode, which would decrease the total number of electrons leaving the cathode per second, i.e. it would reduce the anode-current to a value at which the resultant field at the cathode was zero. Similarly a smaller anode-current would increase to this equilibrium value. The relationship has been deduced mathematically for simple geometrical systems of anode and cathode, and shown to be

$$i_a \propto v_a^{3/2} \qquad . \quad . \quad . \quad . \quad (II.2)$$

(*c*) As the anode-voltage is increased still further the current becomes asymptotic to $i_a = i_s$, where i_s has the value n electrons per second as given by equation (II.1). This indicates that the anode-voltage is sufficiently great to draw off electrons as fast as the cathode can emit them. This state of affairs is known as Saturation, and i_s is known as the Saturation Current.

Diode Connected to A.C. Generator

Fig. 8 shows the result of connecting a generator of sinusoidal A.C. across a diode. Note that—

(*a*) Current flows only during the positive half-cycle of v_a, so that the resulting current is unidirectional but pulsating. The production

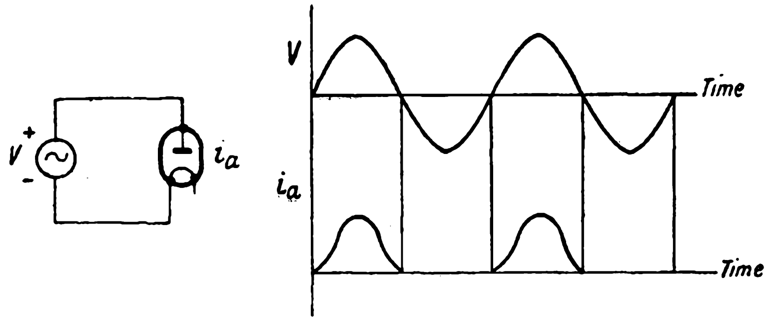


FIG. 8

of unidirectional current from alternating current is known as Rectification.

(*b*) The waveform of the current pulses is not the same as the positive half of a sine wave. Had the diode-characteristic been a straight line passing through the origin, this would have been so. Thus, curvature of the diode-characteristic causes distortion of the waveform. This is the first example, of many which we shall meet, of distortion caused by curvature of the characteristics of a valve.

Triode-characteristics

The anode-current of a triode depends on both the anode-voltage and the grid-voltage, and the set of curves showing how the

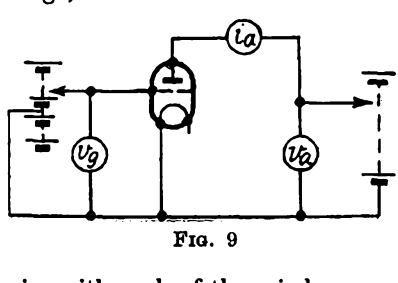


FIG. 9

anode-current varies with each of these is known as the Characteristics of the triode. Fig. 9 is the circuit used for determining the characteristics experimentally, and will be seen to consist of a battery and potentiometer for applying a variable positive voltage to the anode, a battery and potentiometer for applying a variable voltage, either positive or negative, to the grid, together with instruments for measuring v_a, v_g, and i_a. The procedure would be to keep v_a constant at a chosen value, and to observe the variation of i_a with v_g. This would then be repeated for other values of v_a and a set of curves of i_a against v_g would be drawn. Fig. 10 shows typical characteristics. An alternative procedure would clearly be to keep v_g constant at a chosen value, and to observe the variation of i_a with v_a, repeating for other values of v_g. In this case the characteristics could conveniently be plotted as in Fig. 11. The two sets of curves (Figs. 10 and 11) give exactly the same information, plotted in a different way, and, given one set, the other set may easily be drawn. For some purposes it is convenient to express the characteristics in one form (i_a/v_g for various fixed values of v_a) and for other purposes in the other form (i_a/v_a for various fixed values of v_g). The two forms are sometimes referred to as the "Anode-characteristics" (Fig. 11) and the "Mutual-characteristics" (Fig. 10). See also page 358.

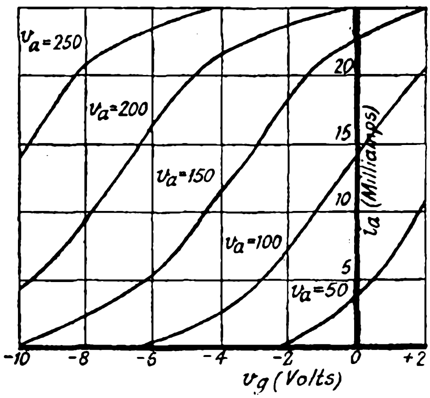


Fig. 10

The characteristics indicate that, as for the diode, no anode-current will flow if the anode-voltage is negative, and also that the anode-current increases smoothly with increasing anode-voltage. The anode-current is seen also to increase with increasing grid-voltage. This is because when the grid is positive it produces an electrostatic field at the cathode which adds to that of the anode.

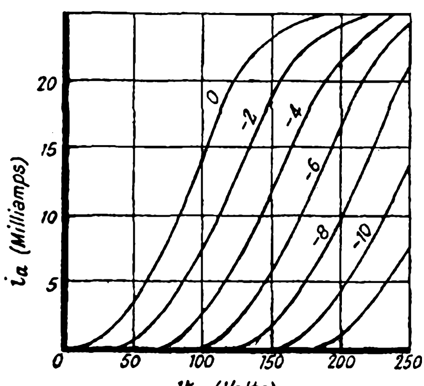


Fig. 11

The figure attached to each curve indicates the value of v_g for which it applies

Thus, for a given anode-voltage, the force attracting electrons from cathode towards grid and anode may be increased by increasing the positive grid-voltage. This increases the number of electrons per second setting out from the cathode. When these electrons reach the grid they have a velocity sufficient to carry most of them through the spaces between the grid-wires. Nearly all these electrons thus proceed to the anode; the grid collects hardly any. Similarly, a negative voltage on the grid will reduce the field at the cathode, thus reducing the anode-current. If the negative grid-voltage be made sufficiently large the anode-current will be seen to be completely cut off. The magnitude of the cut-off value of grid-voltage is much smaller than the anode-voltage; this is because the grid is much nearer the cathode than is the anode, and thus its effect on the electrostatic field at the cathode is greater. The ratio of the anode-voltage to that negative grid-voltage which just suffices to cut off anode current gives a measure of the relative effectiveness of grid-voltage and anode-voltage in controlling the anode-current. (See "Amplification Factor," page 19.)

As in the case of the diode there is a maximum number of electrons per second which the cathode can emit, so that there is a Saturation effect. (The maximum number of electrons which the cathode can emit per second depends, as we have seen, on the temperature of the cathode, but in the operation of thermionic valves the temperature of the cathode is kept constant.)

There is another characteristic of the triode which is sometimes of interest, namely, the curve indicating the variation of the grid-current, i_g, with variation of grid-voltage. In shape this is similar to Fig. 7, being very nearly zero for v_g negative. The values of i_g will be very much smaller than those of i_a since most of the electrons pass *through* the grid.

A.C. Generator Connected in Grid Circuit of Triode

In the most common applications of the triode the anode-current is made to vary periodically by the connexion of an alternating voltage in the grid circuit, as in Fig. 12. Thus the grid circuit is often referred to as the Input Circuit and the anode circuit referred to as the Output Circuit.

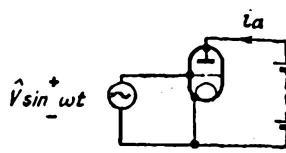


FIG. 12

If the voltage of the a.c. generator be sinusoidal, as in Fig. 13, then we may plot the variation of i_a with time by reference to the characteristics of the valve in question. If these are as shown in Fig. 10, we have simply to select the curve corresponding to the anode-voltage in Fig. 12, and from it determine the value of i_a corresponding to any given value of v_g. The lower dotted curve of Fig. 13 shows the anode-current for a small alternating voltage, and the full curve for a large alternating voltage. Note that—

(*a*) The application of the alternating voltage adds an a.c. component to the existing d.c. component of i_a.

(*b*) If the applied alternating grid-voltage is small enough to involve only the more nearly linear (i.e. straight) portions of the characteristics, the resulting a.c. component of i_a is nearly sinusoidal (as dotted curve). If the amplitude of the applied A.C. is large, however, the curvature of the characteristics causes waveform distortion (as full curve).

There is another possible source of waveform distortion. During the positive half-cycles of the applied voltage, grid-current will flow. Its waveform will be very much as shown in Fig. 8. Unless the a.c. generator has zero internal impedance, this grid-current, in flowing

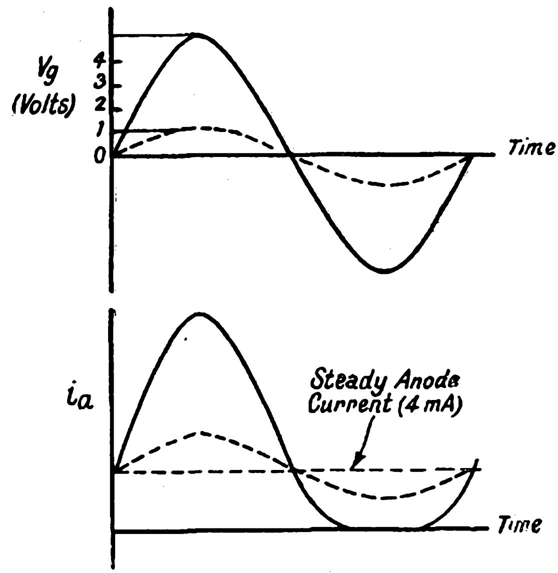


FIG. 13

through the generator, will set up distorted voltages across the internal impedance. Since the grid-voltage will be the sum of the generator voltage and these distorted voltages, there will be waveform distortion of the output current quite independently of the distortion mentioned in (a) above. This could be prevented by connecting in series with the generator in the grid circuit a steady negative voltage (e.g. a battery). If this "bias voltage" were made greater than the peak voltage of the generator, then v_g would never become positive and no grid-current would flow. Fig. 14 shows the circuit, and the resulting curves of v_g and i_a.

The Triode Amplifier (Choice of Operating Voltages)

The triode amplifier will be discussed at length at a later stage, but it may be said here that the basic principle of the valve amplifier is the inclusion of a high resistance or impedance in the anode circuit, so that when the alternating voltage, which it is desired to amplify, is applied in the grid circuit the resulting alternating current in the anode circuit shall produce an alternating voltage across this resistance or impedance which is larger than the alternating voltage applied in the grid circuit.

The anode battery and grid battery voltages (E_a and E_g, Fig. 14) are chosen so as to avoid waveform distortion, i.e. so that—

(*a*) The grid-voltage shall never become positive.

(*b*) Only the more nearly linear portions of the characteristics shall be traversed.

Condition (*a*) means that the minimum negative E_g which may be used is equal to the peak voltage of the A.C. to be amplified. Condition (*b*) will be considered in more detail later.

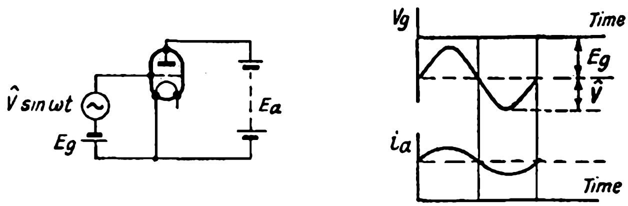


FIG. 14

Supply Voltages: Nomenclature

Where batteries are used, E_a and E_g may be referred to as the anode-battery and the grid-battery voltages, but they are more generally spoken of as the High-tension voltage and the Grid-bias voltage—abbreviated to h.t. and g.b.

Equation of Linear Portions of Triode-characteristics

To formulate an analytical theory of the operation of the triode (i.e. to express its performance in terms of algebraic equations, instead of making all calculations graphically) we must derive an equation for the relation between v_a, v_g, and i_a, hitherto expressed only in the form of a series of curves. Since only the more nearly linear parts of these curves are normally called into play in the operation of the valve, we shall assume that the characteristics are a set of straight lines having the same slopes and being displaced horizontally by the same distances as the more nearly linear parts of the curves. We observe that the slopes (e.g. in Fig. 10) are about the same, i.e. that the curves are parallel. Moreover, we see that for a set of curves as shown in Fig. 10 (i.e. with values of v_a equally spaced) the horizontal distances between the curves are equal. Thus, we may re-draw the characteristics as in Fig. 15.

The equation of any one of the lines may be written

$$i_a = av_g + b \quad . \quad . \quad . \quad . \quad . \quad (II.3)$$

where b, the intercept on the axis of i_a, has a different value for each straight line of the set, i.e. b depends upon the anode-voltage v_a. Since the lines are equally spaced for equal increments of v_a, b will be a linear function of v_a, and may be written

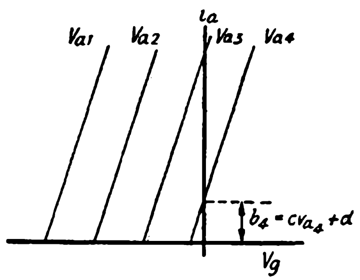


FIG. 15

$$b = cv_a + d$$

Substituting this in equation (II.3)

$$i_a = c\left(v_a + \frac{a}{c}v_g\right) + d$$

If v_a and v_g be changed simultaneously by amounts δv_a and δv_g (not necessarily small), then so long as only the more nearly linear parts are used, we may find the change in i_a by substitution of the values $v_a + \delta v_a$ and $v_g + \delta v_g$ in the above equation. This gives

$$\delta i_a = c\left(\delta v_a + \frac{a}{c}\,.\,\delta v_g\right) \quad . \quad . \quad . \quad \text{(II.4)}$$

Now δv_g and δv_a represent the changes of the grid- and anode-voltages over and above their steady values. In an amplifier, where both grid- and anode-voltages consist of the sum of a steady component and an alternating component, δv_g and δv_a may be used to represent the alternating components of grid- and anode-voltages, so that the vector equation relating the corresponding alternating currents and voltages will be

$$I_a = c\left(V_a + \frac{a}{c}V_g\right) \quad . \quad . \quad . \quad \text{(II.5)}$$

In this equation a and c are constants for the valve. The equation is usually written

$$I_a = \frac{1}{\rho}\left(V_a + \mu V_g\right) \quad . \quad . \quad . \quad \text{(II.6)}$$

Parameters of the Triode

The constants μ and ρ in the above equation, and their ratio μ/ρ (written g_m), are known as the Parameters of the triode. Clearly only two of these three constants are necessary to define the characteristics, but each of the three will be seen to have a special significance. They are named as follows—

MUTUAL CONDUCTANCE, g_m

This quantity will be seen to be the rate of change of i_a with v_g, if v_a (upon which i_a also depends) remains constant. Thus, the mutual conductance is the slope of the more nearly linear parts of the curves of i_a against v_g (i.e. curves as in Fig. 10). Typical values of g_m for small valves are from 1 milliamp per volt to 10 milliamps per volt, sometimes expressed as from 1 to 10 millimhos.

VALVE IMPEDANCE, ρ

The rate of change of i_a with v_a, if v_g remains constant, will be seen to be $1/\rho$. Thus the impedance of the triode is the reciprocal of the slope of the curves of i_a against v_a (i.e. curves as in Fig. 11). Clearly, if an a.c. generator were connected in the anode circuit and the grid-voltage were kept constant, the impedance presented to the generator would have this value, ρ.

Typical values of impedance for triode valves range from 2,000 to 100,000 ohms. Another name for Valve Impedance is "Anode Incremental Resistance."

AMPLIFICATION FACTOR, μ

An increase of anode-current may be caused either by an increase of anode-voltage or by an increase of grid-voltage. The amplification factor may be considered to be the ratio of the increments of anode-voltage and of grid-voltage which separately cause the same increase of anode-current. Perhaps it is better to consider the anode-voltage decreased by an amount δv_a, and the grid-voltage simultaneously increased by just such an amount as will keep the anode-current constant (say, δv_g). Then μ is the numerical value of the ratio $\delta v_a/\delta v_g$. Mathematically it is *minus* the ratio of the simultaneous small *increases* in v_a and v_g, the anode current remaining constant. We may summarize the above as follows—

$$\left.\begin{aligned} g_m &= \frac{\partial i_a}{\partial v_g} \\ \rho &= \frac{\partial v_a}{\partial i_a} \\ \mu &= -\frac{\partial v_a}{\partial v_g} \end{aligned}\right\} \quad . \quad . \quad . \quad (\text{II.7})$$

$$g_m = \mu/\rho \quad . \quad . \quad . \quad . \quad (\text{II.8})$$

Typical values of μ for triode valves range from 5 to 100.

Measurement of the Parameters

There are special circuits for direct measurement of the parameters of a valve, but the most obvious method of determining them is to plot the characteristics and estimate the parameters therefrom.

The mutual conductance may be estimated as the slope of the curves of i_a against v_g. In practice these curves will not be linear, nor will the curves for different values of v_a be quite parallel. For small valves, manufacturers usually state the value of the slope for an anode-voltage of 100 volts and a grid-voltage of zero. That is,

g_m is taken as the slope at v_g = zero of the curve for which $v_a = 100$ volts.

Similarly, the impedance may be estimated as the reciprocal of the slope of the curve of i_a against v_a for a grid-voltage of zero, the slope being measured where $v_a = 100$ volts. The amplification factor may then be found by multiplying the values obtained for g_m and ρ.

The Screen-grid Tetrode

The diode may be thought of as a two-terminal circuit element which may be connected in series with other circuit elements. The triode may be thought of as a four-terminal device with input and output terminals, the input terminals being the grid and cathode, and the output terminals being the anode and cathode. The screen-grid valve has two grids; the grid nearer the cathode plays exactly the same part in the operation of this valve as the grid plays in the operation of the triode, and is therefore known as the Control Grid. The other grid, known as the Screen-grid, is not a part either of the input circuit or of the output circuit, but acts merely as a sort of auxiliary electrode by means of which, as we shall see, the shape of the characteristics can be adjusted. Thus, no a.c. circuits are connected to the screen-grid.

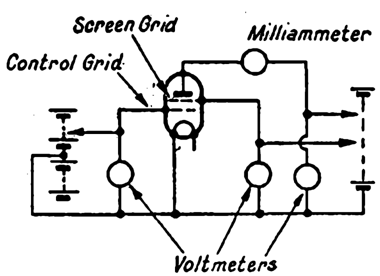


FIG. 16. CIRCUIT FOR MEASURING CHARACTERISTICS OF SCREEN-GRID VALVE

The tetrode is usually operated with the screen-grid maintained at a potential somewhat lower than that of anode, e.g. by connecting this grid to a tapping on the h.t. battery (as in Fig. 16). The internal resistance of this battery is small, so that so far as alternating voltages are concerned the screen-grid is short-circuited to the cathode. (Where other methods of applying the steady positive potential are used, it is usual to connect a fairly large condenser between screen-grid and cathode, this condenser being of so low a reactance at the frequency of the A.C. that it is virtually a short-circuit. See p. 74.)

The characteristics of a screen-grid valve may be measured in just the same way as for a triode (circuit as Fig. 16), the screen-grid voltage being kept constant at the value at which the valve is intended to operate. Fig. 17 shows typical characteristics. Comparing these with Fig. 11, which shows the corresponding characteristics for a triode, the following differences will be noticed—

(*a*) The curves rise steeply from the origin instead of rising slowly at first and then curving upwards.

(*b*) Over a certain range of anode-voltage $\frac{\partial i_a}{\partial v_a}$ is negative, and thus its reciprocal, the impedance of the valve, is negative, an increase of v_a causing a decrease of i_a.

(*c*) For higher anode-voltages the curves rise again and flatten out. That this is not a saturation effect follows from the fact that the curves do not tend to a common maximum value (as would be the case if the anode were collecting electrons as fast as the cathode could emit them).

Secondary Emission

Before proceeding to discuss the reasons for (*a*), (*b*), and (*c*) above, it must be pointed out that thermionic emission (i.e. the emission of electrons from a hot body) is not the only source of electrons in a thermionic valve. The positively-charged anode attracts electrons to it, and the impact of each of these electrons with the anode may result in the emission of several electrons from the material of the anode. This is known as Secondary Emission. It occurs in triode valves, but there the secondary electrons all return to the anode. In the screen-grid valve, however, there is a positively charged screen-grid near the anode, to which the secondary electrons may be attracted—depending on whether the anode or the screen-grid is the more positive.

The part of the characteristics for which, as mentioned in (*b*) above, the slope is negative, has values of anode-voltage less than the value of screen-grid voltage. Thus, most of the secondary electrons will be collected by the screen-grid and will not return to the anode. The effect of an increase of anode-voltage in these circumstances is to increase two things: first, it will increase the rate at which electrons reach the anode from the cathode; and, secondly, it will increase the rate of secondary emission, i.e. the rate at which electrons leave the anode for the screen-grid. Since each primary electron may cause the emission of several secondary electrons from the anode, the rate at which electrons leave the anode will increase more rapidly than the rate at which they arrive at the anode, and the anode-current will thus decrease as a result of an increase of anode-voltage. If the secondary emission is sufficiently profuse the anode current may even become negative for a short range of anode-voltage values.

For values of anode-voltage higher than the screen-grid voltage the secondary electrons will return to the anode. For low values of anode-voltage there is little secondary emission. In both of these cases an increase of anode-voltage will cause an increase of anode-current.

The reason for the steeply rising initial part of the curves is that even when the anode-voltage is zero there is a positive electrostatic field at the cathode due to the screen-grid. This will bring electrons to the vicinity of the anode and, as the anode-voltage increases, an increasing number of these electrons will be drawn off by the anode instead of being collected by the screen-grid. The flattening out of the curves at higher values of anode-voltage is due to the fact that the screen-grid shields the cathode from the effect of the anode, so that changes of anode-voltage cause comparatively small changes in the number of electrons leaving the cathode per second. That this flattening of the curves is not due to saturation is obvious from the fact that the different curves do not flatten out to the same value of anode-current.

We are, however, more interested in the effect of these characteristics on the performance of the valve than in the reasons for their

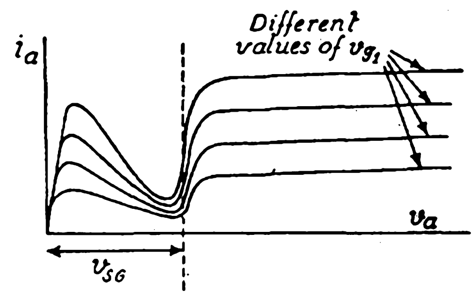


FIG. 17. CHARACTERISTICS OF SCREEN-GRID TETRODE

being shaped as they are. Let us compare the values of corresponding parameters in the triode and the screen-grid valve.

Parameters of the Screen-grid Valve

In view of the irregular shape of the curves of i_a against v_a it would seem, at first sight, impossible to quote a single value for the impedance of the valve (i.e. the reciprocal of the slope of these curves). But, since, in the more common uses of the screen-grid valve, only the portions of the curves to the right of the dotted line in Fig. 17 are traversed, we may consider only these parts of the curves.

Since the slope of the curves in this region is very small the impedance of the screen-grid valve is very large. Valve-impedances greater than a megohm are quite common. The control-grid of the screen-grid valve, however, controls the anode-current just as in a triode, and the value of the mutual conductance of a screen-grid valve is similar to that of a triode. Hence, the amplification factor, being the product of the impedance and the mutual conductance, is much larger than that of a triode. Values of μ greater than 1,000 are quite common.

There are certain other applications of the screen-grid valve which make use of the downward sloping part of the characteristics; these are mentioned in Chapter V under the heading, "Dynatron Oscillator."

It is interesting to note that the screen-grid was originally introduced not with the object of modifying the shape of the characteristics, but in order to provide electrostatic screening between the anode and control-grid. This matter is discussed in Chapter IV.

The Pentode

If a third grid be added to the screen-grid valve, being placed between the anode and the screen-grid, and being connected directly to the cathode, the valve becomes the well-known Pentode. Since the potential of this grid is negative with respect to the anode, the secondary electrons emitted by the anode will be repelled by it and will return to the anode without ever reaching the screen-grid. Thus, the downward sloping part of the characteristics is eliminated by the introduction of this Suppressor Grid, as it is called. This means that the alternating anode-voltage may be greater for a pentode than for a screen-grid valve, without encroaching on the less linear parts of the characteristics. Otherwise the pentode is similar to the screen-grid valve, having high values of amplification factor and impedance. The external connexions also may be similar to those of the screen-grid valve, since the suppressor grid is often connected to the cathode within the valve. Typical characteristics of a pentode are shown in Fig. 18. See also Fig. P–3, page 360.

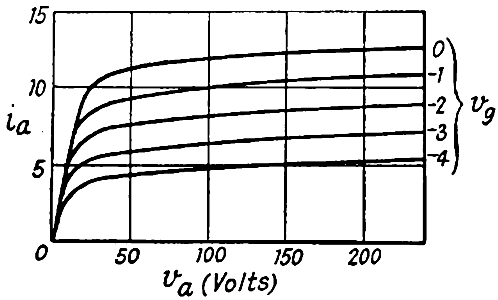


FIG. 18. CHARACTERISTICS OF PENTODE

The suppressor grid is made of fairly open mesh, for otherwise (being at cathode potential) it might prevent primary electrons from ever reaching the anode.

Gas-filled Valves

When the space between the anode and cathode of a diode contains gas, then the electrons emitted by the cathode and travelling towards the positive anode are liable to collide with molecules of the gas. If at the time of the collision an electron has sufficient velocity, and therefore sufficient kinetic energy, it may ionize the molecule with which it collides, i.e. the kinetic energy of the electron will be used to remove an electron from the gas molecule, thus splitting the molecule into two parts—a further electron and a molecule lacking an electron. The latter is known as a positive ion. The amount of kinetic energy required to split off an electron from a gas molecule depends only on the nature of the gas. *Thus for a given gas there is a given electron velocity which is necessary before ionization by collision can occur.*

Electrons travelling from cathode to anode within a diode derive their acceleration from the positive anode-voltage. As they proceed towards the anode their velocity increases, and they strike the anode (unless they have previously collided with a gas molecule) with their maximum velocity. For small anode-voltages, even this maximum velocity may be less than that required to produce ionization by collision, but as the anode-voltage is increased a point will be reached when electrons (not having previously collided) are reaching the neighbourhood of the anode with a velocity sufficient to cause ionization of any gas molecule with which they may collide. *For lower anode-voltages than this value, the gas-filled diode behaves very much as an ordinary hard* (*i.e. high vacuum*) *diode, but when this voltage is reached ionization of the gas occurs and the behaviour changes abruptly.*

The effect of ionization is twofold. First, the positive ions travel towards the cathode—slowly, because their mass is so much greater than that of an electron. *Their presence in the space between anode and cathode cancels the effect of the space charge, and the anode-current rises to a value equal to the whole electron emission of the cathode.* Second, the movement of the additional electrons and of the positive ions, under the influence of the anode voltage, further contributes to the anode-current. If there were a really profuse production of further electrons and positive ions (e.g. if the anode-voltage were so high that an electron travelling from cathode to anode was very greatly accelerated and also if the gas pressure were sufficient for a greatly increased number of collisions to take place) the rise of anode-current might be sufficient to cause an arc, with resulting damage to the valve.

At atmospheric pressure the molecules of a gas are so crowded together that an electron has not far to go before colliding with a gas molecule—and in this short distance it does not acquire sufficient kinetic energy to produce ionization by collision unless exceedingly large anode-voltages are used (and then the ionization is so profuse that damage to the valve is certain to take place). Ionization by collision may be brought about with normal anode-voltages *by using very much lower gas pressures* so that the electron has further to move before colliding with a gas molecule, and can acquire sufficient kinetic energy in this time. The gas pressures used in practice are of the order of a few millionths of an atmosphere, and at such pressures only a few parts in 10^{12} of the space are actually occupied by gas molecules (though there are millions of them in this small space) so that most of the electrons from the cathode reach the anode without a collision. Near the end of their path, however, the electrons have sufficient kinetic energy to cause ionization if they do collide with a gas molecule, and those that do so produce positive ions and further electrons, with the resulting cancellation of the space charge and sudden rise of anode-current.

Now the potential difference between two points is defined as the work done on unit charge moving between those two points. The work done on an electron in moving from cathode to anode is thus $v_a e$, the product of the anode-voltage and the charge on an electron. All this work becomes kinetic energy of the electron. Thus, if the kinetic energy of an electron near the end of its path is required to be sufficient to cause ionization by collision, and since the energy required to ionize a gas molecule is a constant for the gas, it follows that *the anode-voltage must be at least a certain value and that this value is a constant for the particular gas and is independent of the dimensions of the valve.* This voltage is known as the *Ionizing Potential* of the gas. For mercury vapour it is 10·4 volts, and for neon 21·5 volts.

The same argument applies for a gas-filled *triode* (often known by the trade name, "Thyratron")—a minimum anode-voltage is required to cause the sudden rise of anode-current. It might appear that, as above, this minimum anode-voltage is a constant for the gas and independent of everything else, including the grid-voltage. But it must be remembered that the grid-voltage, if sufficiently negative, may altogether prevent the passage of electrons to the anode (see page 15) and if there are no electrons there can be no collisions of the type discussed. For any given negative grid-voltage, therefore, the anode-voltage must not only be at least equal to the ionizing potential for the gas, but must be sufficiently large to overcome the repulsion by the grid of electrons as they leave the cathode. Thus the critical value of the anode-voltage does depend upon the grid-voltage. The characteristics of a gas-filled triode are usually measured by observing, for several values of negative grid-voltage, the critical anode-voltage necessary to produce the sudden rise of anode-current which denotes that ionization by collision is taking place, and plotting critical anode-voltage against negative grid-voltage. The slope of the resulting graph is known as the Control Factor of the valve, and is approximately equal to the amplification factor of the same valve if without gas.

Just as the positive ions resulting from the ionization neutralize the space charge, so also they neutralize the effect of the negative grid-voltage while ionization continues. The positive ions around the grid sufficiently screen it from the cathode for the grid-voltage to be completely unable to affect the anode-current. *Thus once ionization has taken place it cannot be made to cease by increasing* *the negative grid-voltage*, but only by decreasing the anode-voltage below the ionizing potential of the gas.

Some of the applications of gas-filled triodes are discussed in Chapter V.

Suggestions for Further Reading

This book is about valve circuits and not primarily about valves. This introductory chapter is intended to provide a minimum of background information about valves themselves. Readers wishing to know more of the physics background or of the construction of valves are referred to *Electronics* by P. Parker (Edward Arnold).

CHAPTER III

AMPLIFIERS

THE principle of the valve amplifier has already been stated. The anode circuit is completed through an impedance (referred to as the Load) and, of course, through the h.t. supply. The alternating voltage to be amplified is connected in the grid circuit, the effect of this being to introduce an alternating current in the anode circuit, which, in flowing through the load impedance, produces a voltage greater than the original voltage.

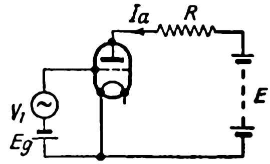


FIG. 19. BASIC AMPLIFIER CIRCUIT

The load may take various forms. Fig. 19 shows a circuit with the usual resistive load, R. Clearly the potential difference across R will be the sum of a direct and an alternating voltage. The methods by which the alternating voltage alone is tapped off for use or for further amplification will be considered later in this chapter.

Graphical Determination of Steady Anode-current

To determine the steady anode-current in the circuit of Fig. 19 we observe that the anode-current, i_a, is a function of v_g and v_a, but that this last quantity, v_a, is now no longer an independent variable, but is related to the anode-current by the equation

$$v_a = E_a - Ri_a$$

where E_a is the h.t. voltage. This equation, together with the relationship of i_a with v_a and v_g expressed in the characteristics, enables us to determine the value of i_a for any value of v_g. Since the relation between i_a, v_a, and v_g is in graphical form, the calculation is, of course, a graphical one.

First, the characteristics are plotted in the form of a series of curves of i_a against v_a, each curve being for a single value of v_g. The graph of the equation

$$v_a = E_a - Ri_a$$

is then superimposed upon the curves (AE, Fig. 20). This graph will be a straight line, its intercepts on the horizontal and vertical axes being E_a and E_a/R respectively, and its slope being $-1/R$. The ordinate of the point of intersection of this line with the curve which corresponds to a given value of v_g will give the value of i_a resulting from that particular value of v_g. In particular, the intersection of the straight line with the curve corresponding to the value of the grid-bias voltage will give the value of the steady anode-current, or "quiescent anode current," as it is sometimes called.

Graphical Determination of Voltage Amplification

It will be clear that the above graphical method may be used to determine the value of i_a corresponding to *any* value of v_g, i.e. we may plot the variation of i_a throughout the alternating current cycle. Figs. 20 and 21 illustrate this for a sinusoidal applied

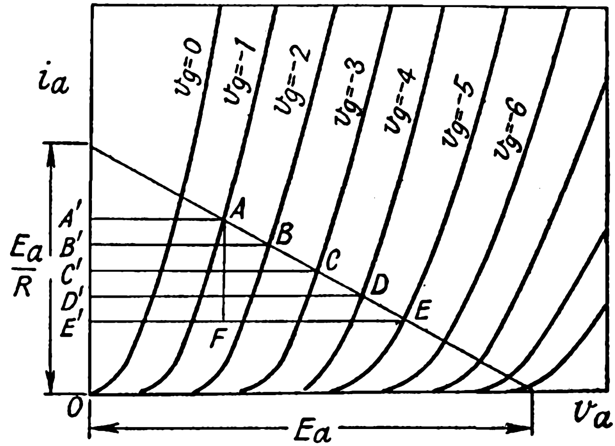


FIG. 20. LOAD-LINE DIAGRAM

voltage of maximum value 2 volts, and a grid-bias voltage of − 3 volts. (As stated in Chapter II, the negative grid-bias voltage must exceed the peak value of the alternating voltage input if waveform distortion is to be avoided. Throughout the remainder of the discussion it will be assumed that this condition is satisfied, unless the contrary is specifically stated.) Fig. 21 shows the graph of the alternating input voltage against time. At times corresponding to the points *O*, *S*, and *W*, the instantaneous value is zero, and v_g is simply equal to the grid bias, − 3 volts, and at these times the value of i_a will be equal to the value of the steady anode-current. At times corresponding to the points *P* and *R* in Fig. 21 the instantaneous value of the alternating input voltage is + 1 volt, and the resultant value of v_g is − 2 volts. Hence at these instants the value of i_a is given by *OB'* in Fig. 20. In this way the value of i_a may be plotted throughout the a.c. cycle. If the curve is to be drawn in considerable detail it will, of course, be necessary to have available characteristic curves for small increments of grid-voltage.

This calculation is similar to that illustrated in Fig. 13; there, however, the anode-voltage, v_a, was constant and equal to E_a since there was no load. Such a circuit is of no use in practice.

The Voltage Amplification is the ratio of the output alternating voltage to the input alternating voltage; we shall denote it by the symbol (VA). For the circuit of Fig. 19 the voltage amplification is the ratio of the product, $R\hat{I}_a$ to $\hat{V}_1$, where $\hat{I}_a$ and $\hat{V}_1$ are the peak values of the alternating anode-current and the input voltage respectively. In the case illustrated in Fig. 21 the value of $\hat{V}_1$ was 2 volts, and the peak value of the anode-current may be found by

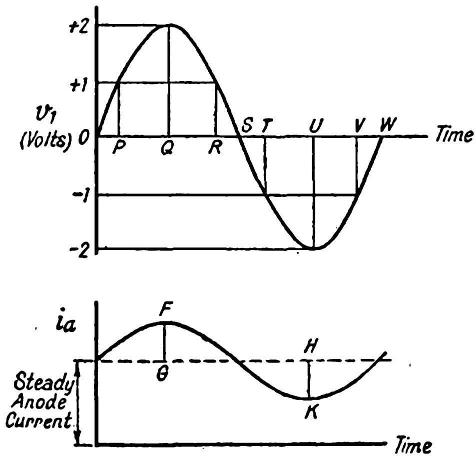


FIG. 21

taking the mean of the positive peak, FG, and the negative peak, HK, in Fig. 21,

i.e. $$\hat{I}_a = \frac{FG + HK}{2}$$

$$= \frac{C'A' + C'E'}{2} \text{ in Fig. 20} \quad . \quad . \text{(III.1)}$$

FG and HK (Fig. 21) will be unequal only if there is waveform distortion; in a well-designed amplifier circuit they will be very nearly equal. In calculating the voltage amplification of a given circuit it is of course unnecessary to draw out the waveform of the anode-current as in Fig. 21 for the peak output voltage is given by $R\hat{I}_a$, i.e. $\frac{1}{2}R(A'E')$. This is equal to $\frac{1}{2}EF$ in Fig. 20 (since the slope of AE is equal to $1/R$), and may thus be read off from the graph. Note that $\frac{1}{2}EF$ also gives the peak value of the alternating component of the *anode-voltage*. For this reason the output voltage of an amplifier is often defined as the alternating component of its anode-voltage.

The construction of Fig. 20 is sometimes spoken of as the Load Line construction, and the line *ABCDE* as the Load Line.

Graphical Determination of Harmonic Distortion

If *FG* and *HK* in Fig. 21 are unequal there is waveform distortion, and the extent of the distortion may be expressed by making a Fourier analysis of the resulting waveform of i_a, and stating the amplitudes of the various harmonics, usually expressing them as percentages of the amplitude of the fundamental. The way in

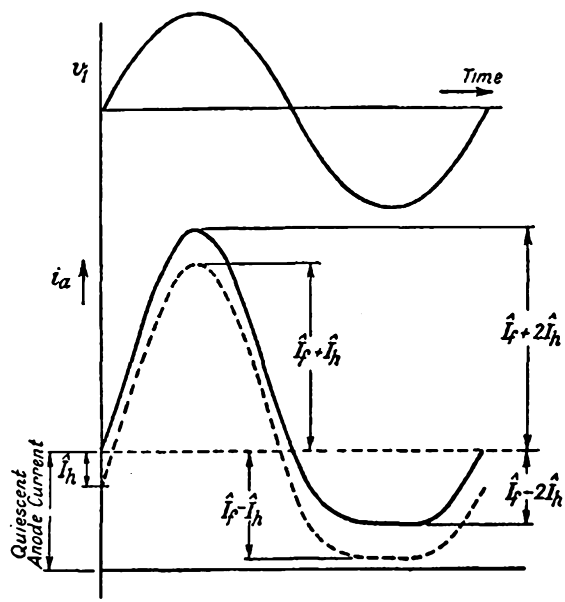


FIG. 22 (*a*)

which the waveform is usually distorted may be seen by examining the characteristics of Fig. 20. The intersections with the load line are evenly spaced except at the bottom right-hand end of the line and thus the distortion will take the form of a reduction of amplitude of the negative peak, the graph of the anode-current being as shown in Fig. 22 (*a*) (full curve). Let us try to synthesize this graph by adding to the steady anode-current an alternating current consisting of fundamental and second harmonic. Fig. 22 (*b*) shows how the relative phases of fundamental and harmonic must be chosen to give the desired type of waveform, viz. a waveform having a positive peak greater than the negative peak. The positive and negative peak values of this two-component alternating current are respectively $\hat{I}_f + \hat{I}_h$ and $\hat{I}_f - \hat{I}_h$, where the suffixes *f* and *h* denote fundamental and second harmonic respectively. The addition of this two-component alternating current alone would not, however, give us the desired graph, but would give instead the graph shown dotted in Fig. 22 (*a*). This graph must be raised by an amount equal to $\hat{I}_h$, the peak value of the second harmonic current, in order to

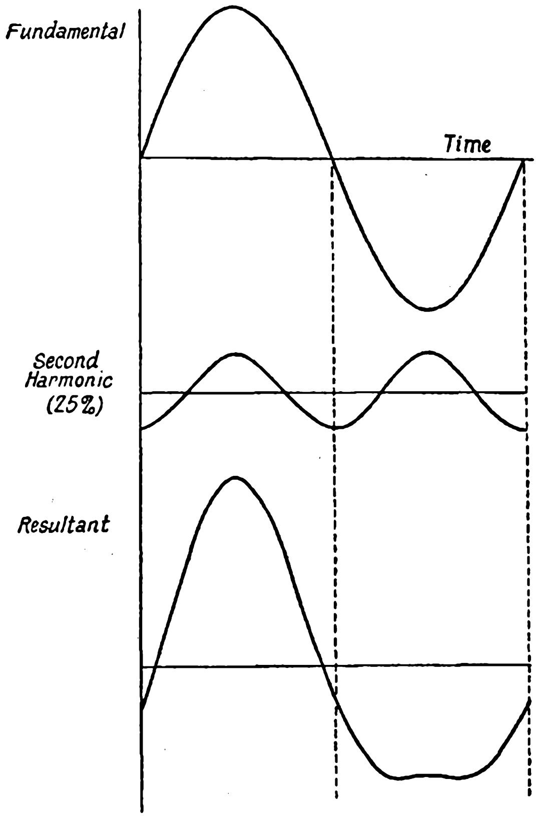


FIG. 22 (*b*)

give the desired graph, i.e. a direct current equal in magnitude to $\hat{I}_h$ must be added. Thus, we see that the desired graph may be built up by adding three components to the steady anode-current—

1. A current of the fundamental frequency.
2. A second harmonic current (phase as shown).
3. A direct current. (The production of a direct current from an alternating voltage constitutes Rectification.)

The amplitude of the positive peak of i_a, measured from the value of the steady, or quiescent, anode-current corresponds to the length $A'C'$ on the load line diagram of Fig. 20; the amplitude of the negative peak so measured corresponds to the length $C'E'$ on the load line diagram. We see from Fig. 22 (*a*) that the amplitude of the positive peak measured from the value of the quiescent anode-current is equal to $\hat{I}_f + 2\hat{I}_h$, and that of the negative peak is equal to $\hat{I}_f - 2\hat{I}_h$

i.e.
$$A'C' = \hat{I}_f + 2\hat{I}_h$$
$$C'E' = \hat{I}_f - 2\hat{I}_h$$

giving
$$\hat{I}_f = \frac{A'C' + C'E'}{2}$$
$$\hat{I}_h = \frac{A'C' - C'E'}{4}$$

Thus the Percentage Second Harmonic

$$= \frac{1}{2} \cdot \frac{A'C' - C'E'}{A'C' + C'E'} \times 100 \text{ per cent} \quad . \quad . \text{(III.2)}$$

This gives us a method of estimating the percentage second harmonic distortion from the load-line diagram. It is to be noted that the method assumes that no harmonics other than the second are present; to the extent that this is untrue the method is inaccurate, and the percentage given by equation (III.2) is better called the Percentage of *Equivalent* Second Harmonic. The matter is treated more fully in Chapter VI under the heading "Anode-circuit Detection."

An ideal amplifier would, of course, cause no distortion at all, but since no valve has absolutely straight and evenly-spaced characteristics there will always be some distortion in practice. Since the curvature of the characteristics becomes of less importance when only a small length of the curve is traversed, the percentage harmonic distortion will be smaller when the values of the input voltage are smaller. The design of an amplifier consists of choosing the operating voltages and the valve in such a way as to reduce the distortion to a minimum, whilst at the same time securing sufficient amplification, handling the required input voltages, and economizing in direct-current supplies. The greater the distortion that can be tolerated, the greater the voltage which can be handled by any given valve. A figure of five per cent is often quoted for the permissible second harmonic distortion in the amplification of speech voltages, but this figure clearly depends upon the quality of reproduction desired.

Graphical Determination of A.C. Power in Load

If the load consists of a resistance, R, and the peak value of the alternating anode-current is $\hat{I}_a$, the a.c. power dissipated in the load will be $\frac{1}{2}R\hat{I}^2_a$. The construction of Fig. 20 may be used to determine $\hat{I}_a$, and then, using the notation of that figure, the power is given by

$$\tfrac{1}{2}R(A'E'/2)^2$$

i.e.
$$\frac{R \,.\, A'E'^2}{8} \quad . \quad . \quad . \quad . \quad (III.3)$$

or alternatively,

$$\frac{AF \,.\, FE}{8} \quad . \quad . \quad . \quad . \quad (III.4)$$

which is one quarter of the area of the triangle AEF. *The problem of securing maximum output power is thus a question of fitting in the largest possible triangle* AEF.

In the above we have considered only resistive loads. When the load has reactance as well as resistance the equation of the load line is no longer

$$v_a = E - R \,.\, i_a$$

but will be given by

$$v_a = E - R \,.\, i_a - L \,.\, di_a/dt$$

in the case of an inductive reactance. When the distortion is small, i.e. the alternating anode-current is approximately sinusoidal, this equation is that of an ellipse lying along the line

$$v_a = E - R \,.\, i_a$$

and shrinking to this straight line as the reactance tends to zero. The length of the ellipse, measured along this straight line, will depend upon the amplitude of the input voltage, just as the length of the line traversed depends upon the amplitude of the input voltage in the case of a resistive load. The case of the reactive load will not be further considered here.

Circuits with A.C. Load Resistance Differing from D.C. Load Resistance

Fig. 23 shows a circuit of the type we are now to consider. Circuits of this type are very common in practice, as will be seen in the section on Multi-stage Amplifiers. Without discussing at this stage the particular purpose of this circuit, we shall consider the application to it of the graphical construction given in the preceding paragraphs.

So far as direct current is concerned we have the relation

$$v_a = E - R_1 i_a$$

and thus the value of the steady anode-current may be found by superimposing upon the characteristics a load line of slope $-1/R_1$, in the manner already described. So far as alternating currents are concerned, however, the resistance R_2 and the condenser C are connected in parallel with R_1. In circuits of this type the values of R_2 and C are such that the reactance of the condenser is very small with respect to R_2 for all frequencies which the amplifier will be called upon to handle. Thus for alternating currents the condenser may be regarded as a short circuit and R_2 is virtually connected directly in parallel with R_1. The effective load for alternating currents is thus a resistance $\frac{R_1R_2}{R_1 + R_2}$ so that for alternating currents the load line has a slope equal to the reciprocal of this effective load resistance, i.e. a slope numerically greater than that of the direct-current load line. The point whose co-ordinates give the steady values of anode-current and anode-voltage (namely, the point of intersection of the direct-current load line with the characteristic for a value of grid-voltage equal to the grid-bias voltage) must be a point on the alternating-current load line. The alternating-current load line is, therefore, constructed by drawing through the steady working point a line of slope $\frac{-(R_1 + R_2)}{R_1R_2}$ as shown in Fig. 24. The determination of voltage amplification, distortion, and power then proceeds as already described, using this new load line.

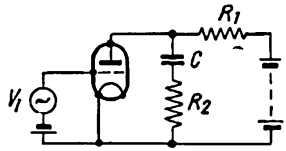


FIG. 23

By far the most commonly occurring case of a.c. load lines differing from d.c. load lines is the case of a transformer-coupled resistance load as shown in Fig. 35, page 48. It is shown on page 48 that to connect the load R into the circuit in this way, instead of connecting it directly into the anode-circuit, is equivalent, *so far as A.C. is concerned*, to connecting a resistance R/n^2 directly into the anode-circuit, where n is the ratio of the number of turns on the load side of the transformer to the number of turns on the anode-circuit side of the transformer. So far as D.C. is concerned, however, the only resistance in the anode-circuit is R_1, the resistance of the primary winding of the transformer. The d.c. load line, drawn through the E_a point on the horizontal axis, has therefore a slope equal to $-1/R_1$. In practice R_1 is usually so small that this d.c. load line can barely be distinguished from a vertical line, and the usual approximate method is to draw the d.c. load line vertical. The a.c. load line has a slope which is numerically the reciprocal of $R_1 + R/n^2$. This is approximately n^2/R. This case differs from the case illustrated in Figs. 23 and 24 in that the slope of the a.c. load line is now numerically less than that of the d.c. load line. In all cases, the two load lines intersect at the steady working point.

One conclusion, perhaps rather surprising at first, which can be made from the load-line diagram for a circuit with a transformer-coupled load resistance, as in Fig. 35 (page 48), is that, in such circuits, the anode-voltage attains values higher than the h.t. voltage, E_a.

The Dynamic Characteristic

The characteristics of a valve show how the anode-current varies with the two independent variables, grid-voltage and anode-voltage. In an amplifier circuit these two variables are no longer independent,

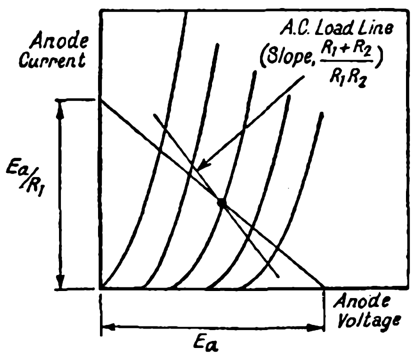


FIG. 24

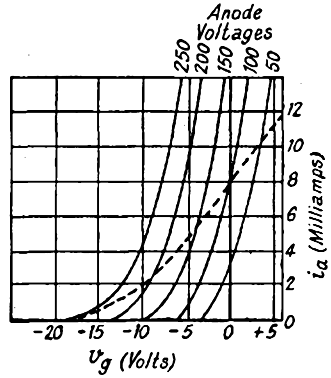


FIG. 25. DYNAMIC CHARACTERISTIC (DOTTED) FOR—
E_a = 250 volts
R = 20,000 ohms

since the anode-voltage and anode-current are related by the equation

$$v_a = E - R \cdot i_a$$

In such a circuit, therefore, there is a single relationship between grid-voltage and anode-current, and we have shown how this relationship may be conveniently expressed by drawing a load line on the characteristics. It is sometimes convenient to express the relationship as a graph of anode-current against grid-voltage and such a graph is known as the Dynamic Characteristic.

The dynamic characteristic may conveniently be plotted from the ordinary or static characteristics if the latter are given in the form of a set of curves of anode-current against grid-voltage, each curve being for a given value of anode-voltage (e.g. as in Fig. 10). Since each curve is for a given value of anode-voltage, and since the anode-voltage is related to the anode-current by the above equation, each curve must refer to a given value of anode-current, and this value can be very easily calculated. Marking off a point on each curve at the appropriate value of anode-current, we shall have a series of points on the dynamic characteristic. This construction is shown in Fig. 25, and it will be seen that the dynamic characteristic has roughly the same shape as the static characteristics, but a smaller slope.

The drawing of the dynamic characteristic is rather more laborious than the drawing of the load line, although both give exactly the same information. It is perhaps easier to see under what conditions distortion occurs by considering the dynamic characteristic. If the grid-voltage is such that only the straighter part of the dynamic characteristic is traversed, there will be a linear relation between grid-voltage and anode-current, and the distortion will be very small. This corresponds to the statement that if only that part of the load line is traversed where the characteristics are evenly spaced, the distortion will be very small.

Equivalent Anode Circuit of a Triode

When an accurate determination of voltage amplification or output power is required, the above graphical methods must be used. A very valuable general theory of the operation of valve circuits may be developed, however, from equation (II.6), which is the equation for the linear parts of the characteristics of any valve, the coefficients ρ and μ (known as the Impedance and Amplification Factor respectively) varying from valve to valve.

Consider an amplifier circuit with a load of impedance Z. The characteristics give us

$$I_a = \frac{1}{\rho}\left(V_a + \mu V_g\right)$$

using capital letters to denote alternating current vectors In addition we have (dealing with alternating quantities only)

$$V_a = -Z \,.\, I_a$$

Combining these equations we have the important result

$$I_a = \frac{\mu V_g}{Z + \rho} \quad . \quad . \quad . \quad . \text{(III.5)}$$

This shows that the alternating anode-current in the load, Z, is the same as that which would flow if the load were connected not to the valve but to a generator of e.m.f. μV_g having an internal resistance ρ. This gives us a simple equivalent of the triode valve, valid so long as only the more nearly linear parts of the characteristics are traversed.

An analysis of the performance of any triode circuit in which only the more linear parts of the characteristics are traversed may now be made by drawing an equivalent anode circuit, in which the valve is replaced by a generator of e.m.f. μV_g* and internal

* Usually the vector V_g is simply equal to the input voltage vector V_1. But sometimes there are other a.c. voltages in the grid circuit.

impedance ρ. The basic circuit of Fig. 19 thus has an equivalent anode circuit as shown in Fig. 26. The direction of the e.m.f. μV_g is anti-clockwise round the circuit since the anode-current, I_a, measured in the conventional direction, is in phase with the grid-voltage V_g. This follows from the fact that an increase of grid-voltage causes an increase of anode-current.

Any analysis which is based upon the equivalent circuit of Fig. 26 inherently assumes that the valve characteristics are linear since the derivation of Fig. 26 was based on this assumption. If the operating point traverses only a small portion of the valve characteristics in the course of the a.c. cycle (i.e. if the alternating components of i_a, v_a and v_g are small), or if the movement of the operating point is confined to the more linear parts of the characteristics, the analysis based on the equivalent circuit will yield reasonably accurate results. When large portions of the valve characteristics are traversed, the only accurate method of analysis is the graphical construction already described (the load-line diagram). A warning should perhaps be given that Fig. 26 is an equivalent circuit only for the incremental (i.e. alternating) components of currents and voltages, since its derivation is based on the incremental equation (II.4).

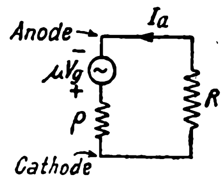


FIG. 26. EQUIVALENT ANODE CIRCUIT

Conditions for Maximum Voltage Amplification and Maximum Power

The voltage amplification in the basic circuit is given by

$$(VA) = \frac{R \,.\, I_a}{V_1}$$

$$= \frac{\mu R}{R + \rho} \qquad \qquad . \quad . \quad . \quad . \ (\text{III}.6)$$

This expression clearly increases continuously with R, and tends to a maximum value, μ, as R approaches infinity. Thus, in the design of such a circuit R should be made as large as possible. Since the steady anode-current flows through R, however, there will be a steady RI drop in this resistance. Thus, as R is increased, an increasingly large high-tension voltage will be required if the steady anode-voltage and anode-current are to be kept at the same values. The value of R which may be used in practice is therefore limited by the high-tension voltage available.

The a.c. power in the resistance R is given by

$$\text{Power} = R \,.\, I_a^2$$

$$= \frac{(\mu V_1)^2}{(R + \rho)^2} \,.\, R$$

This expression is zero for R equal to zero, and zero for R equal to infinity, and reaches a maximum value at some intermediate value of R. This value of R will be given by

$$\frac{d}{dR}\left[\mu^2 V_1^2 R/(R+\rho)^2\right] = 0$$

giving

$$R = \rho$$

Thus the output power is greatest when the resistance load presented to the valve is equal to the valve impedance. The maximum power will then be given by $\mu^2 V_1^2/4\rho$. These conclusions could also have been reached by considering the load-line diagram. (See also, "Output Stages," page 56.)

The Variable-mu Valve

This is the name given to a valve designed to have characteristics which, instead of being as straight as possible for a considerable portion of their length, have a constant curvature. This means that the slope of the $i_a - v_g$ characteristics increases continuously with grid-voltage. The usefulness of an amplifier using such a valve lies in the fact that the values of the parameters may be varied by varying the grid-bias voltage. For example, the voltage amplification may be increased or decreased respectively by decreasing or increasing the (negative) grid-bias voltage. With large input voltages such an amplifier will clearly produce considerable distortion as a result of the curvature of the characteristics; for this reason such amplifiers usually handle only small input voltages. The most important application of variable-mu valves is to Automatic Gain Control (Chapter VI).

Curvature of the characteristics can be secured by making the grid of the valve of fine mesh at one end and of open mesh at the other end, the mesh being graded from one end to the other. A grid of fine mesh exerts a greater control over the electron stream than does a grid of open mesh. The finer the mesh, the higher the mutual conductance and the smaller the value of negative grid-voltage required to reduce the anode-current to zero. The characteristics of a valve with a grid of graded mesh may be considered as the sum of the characteristics of a number of valves differing only in the fineness of the mesh of the grid. For large negative grid-voltages, anode-current will flow only in the valves with grids of very open mesh; the mutual conductance of these valves is small. For small negative grid-voltages or for positive grid-voltages, anode-current will flow in all the valves, and the mutual conductance will be high, since it will be largely determined by the mutual conductance of the valves having grids of very fine mesh. Thus, the mutual conductance (i.e. the slope of the $i_a - v_g$ curves) will increase continuously with grid-voltage, over a wide range of grid-voltage.

Multi-stage Amplifiers

A multi-stage amplifier is one in which the voltage to be amplified is connected in the grid circuit of the first valve, the output voltage of this first valve is in some way connected in the grid circuit of the second valve for further amplification, the output voltage of the second valve is connected in the grid circuit of the third valve, and so on, the amplification being accomplished in a number of stages. If the purpose of the amplifier was to give as great a voltage output as possible, all stages would be designed to give the maximum possible voltage amplification. Usually, however, the purpose of the amplifier is to give as great a power as possible in a given load; in this case all stages except the last are designed to give maximum voltage amplification, and the last stage—known as the Output Stage—is designed to give maximum power output.

The general requirements in multi-stage amplifiers may be stated as follows—

(i) The output voltage (i.e. alternating eomponent only) of one stage shall be connected into the input circuit of the next stage for further amplification.

(ii) All stages shall work from a common h.t. supply.

(iii) All stages shall work from a common heater supply.

(iv) There shall be no paths between the input and output terminals of the various stages except those necessitated by (i). (This is to prevent the unfortunate consequences of unwanted "feed-back," and does not apply to amplifiers which employ feed-back deliberately.)

In the remainder of this chapter we shall consider methods of interstage coupling (i.e. methods of connecting the output voltage of one valve in the grid circuit of the next valve), difficulties arising from the use of the same high-tension supply for all stages, the design of output stages, and the push-pull circuit.

A.C./D.C. Separating Circuits

A.c./d.c. separating circuits are used very extensively in all kinds of electronic circuits, and are of particular importance in interstage coupling. The function of these circuits is to separate the alternating component from the direct component of a voltage which is the sum of two such components. There are many circuits which can effect this separation but the simplest, and by far the most used, is shown in Fig. 27. The resistance R and the capacitance C are connected in series across the

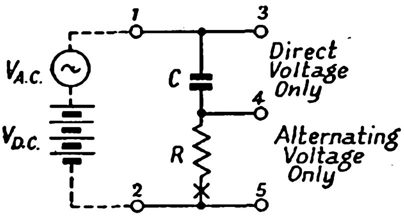


FIG. 27. A.C./D.C. SEPARATING CIRCUIT

composite voltage, and may be thought of as forming a voltage divider. Considering first the alternating component, we see that the voltage across C is given by

$$V_c = \frac{1/j\omega C}{R + 1/j\omega C} \cdot V_{AC}$$

and this may be made negligibly small by choosing R and C so that R is large with respect to the condenser reactance at all relevant frequencies. The alternating voltage across R is given by

$$V_R = \frac{R}{R + 1/j\omega C} \cdot V_{AC}$$

and this may be made approximately equal to V_{AC} by the same method. Considering the steady (or direct) component, we see that if the condenser has no leakage resistance there is no current through R and therefore no direct voltage across R. It follows that the whole direct voltage, V_{DC} appears across C. The design rules for the circuit are thus that *R shall be made very large with respect to the condenser reactance at the lowest frequency to be used, and that the condenser shall have negligible leakage resistance.* Throughout the above we have considered the device to be "on open-circuit," i.e. the required component of voltage when separated out is connected to a circuit having infinite impedance. In most applications this is in fact the case, and the above analysis is valid. Even if the device is not on open-circuit, its performance is approximately as described above provided that the impedance to which its output is connected is large with respect to R and gives no d.c. component across C.

Three points of technique are worth mentioning in connexion with the use of these circuits—

(*a*) *Use in circuits with a Common Negative.* As explained later, it is usual for one side of all input and output voltages to be connected to a common point (usually the negative end of the h.t. voltage, and often referred to as Earth). Thus R or C respectively must be connected to this common negative, according as the alternating or the direct component is required as the output voltage. See Fig. 28 (*a*) and (*b*).

(*b*) *Cases where the output voltage is required to be the alternating component of the input, plus a fixed direct voltage.* In such a case the circuit may be broken at X (Fig. 27) and the required direct voltage inserted. The circuit becomes as shown in Fig. 28 (*d*) and it is easy to show that v_{DE} is the sum of the alternating component of v_{AE} plus the inserted direct component v_{BE}.

(*c*) *Parallel Addition of two components.* The problem of Fig. 28 (*c*) often arises, in which one has an alternating voltage v_{AE} and a direct voltage v_{BE}, each with one side earthed; and one is required to produce between D and E a p.d. equal to the sum of these two components, viz. $v_{AE} + v_{BE}$. The circuit used is that of Fig. 28 (*d*). It may seem paradoxical that what we have called a Separating Circuit should be used for *combining* in this way, but the method

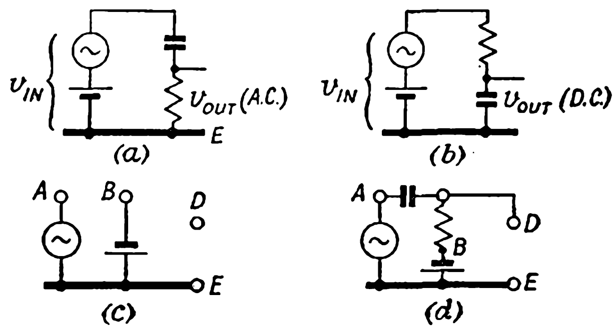


FIG. 28

used is essentially that of paragraph (*b*) above, and the design rule is $R \gg 1/\omega C$ as enunciated for the a.c./d.c. separating circuit. This is an important method of applying grid-bias voltage, whether fixed or derived from an A.G.C. circuit.

Resistance-capacitance Coupling

As a result of the requirement that all stages of a multi-stage amplifier shall use a common h.t. supply, the two stages which are to be coupled together must first be connected as in Fig. 29 (*a*). We

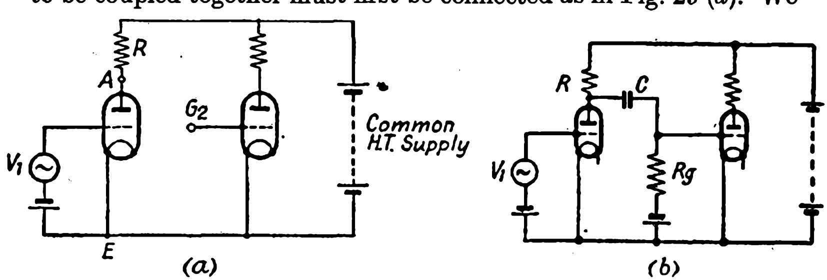


FIG. 29. RESISTANCE-CAPACITANCE COUPLING

must now connect the output voltage of the left-hand stage into the input circuit of the right-hand stage. It has been shown (page 29) that the output voltage is the alternating component of the anode-voltage, i.e. in Fig. 29 (*a*) the alternating component of v_{AE}. We wish to produce, between G_2 and E, a voltage which is the sum of the alternating component of v_{AE} and a direct voltage, the latter being suitably chosen to provide grid-bias for the second valve. This is exactly the same problem as that discussed in paragraph (*b*) on page 40, and the method used is to connect a suitable a.c./d.c. separating circuit across AE, including the required grid-bias voltage as indicated in Fig. 28 (*d*). This results in the circuit of Fig. 29 (*b*).

The voltage amplification of the first stage of a resistance-capacitance coupled amplifier is the ratio of the input voltage of the second valve to the input voltage of the first valve, i.e. the ratio V_2/V_1 where V_2 is the alternating voltage across R_g.

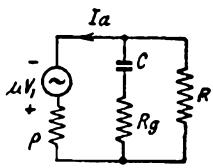


FIG. 30

The voltage amplification will not be merely that of the basic circuit, viz.

$$\frac{\mu R}{\rho + R}$$

since it will be affected by the components C and R_g.

Fig. 30 shows the equivalent anode circuit of the first stage of a resistance-capacitance coupled amplifier. If the a.c./d.c. separating circuit is correctly designed (i.e. $R_g \gg 1/\omega C$), then R and R_g are effectively in parallel. The effective load resistance is thus less than R, being $RR_g/(R + R_g)$.

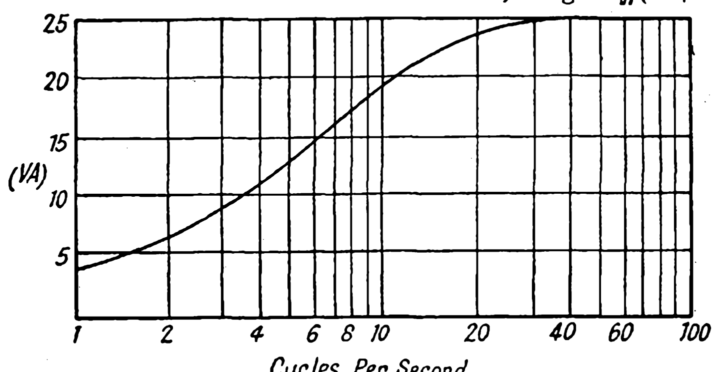


FIG. 31. FREQUENCY DISTORTION IN RESISTANCE-CAPACITANCE COUPLED AMPLIFIER STAGE HAVING—

ρ = 10,000 ohms μ = 30
R = 50,000 ohms C = 0·01 microfarad
R_g = 2 megohms

This means that the voltage amplification (which increases with the value of the load resistance, see page 37) has been reduced by the addition of the coupling circuit. The reduction can be made negligible, however, by making R_g large with respect to R, in which case, the voltage amplification of the stage will be very nearly equal to that of the basic circuit. It is to be noted, however, that at very low frequencies the reactance, $1/\omega C$, may be of the same order of magnitude as R_g. When this occurs the resistance, R_g, and the condenser, C, must be regarded as a voltage divider connected across R, and the voltage V_2 will then be only a fraction

$$\frac{R_g}{R_g + 1/j\omega C}$$

of the total voltage developed across the load, R. The voltage amplification thus falls off at low frequencies and we have frequency distortion. The *magnitude* of the above fraction is given by

$$\frac{1}{\sqrt{1 + \frac{1}{\omega^2 C^2 R_g^2}}}$$

and the magnitude of the voltage amplification will be

$$|(VA)| = \frac{\mu R}{\rho + R} \cdot \frac{1}{\sqrt{1 + \frac{1}{\omega^2 C^2 R_g^2}}} \quad . \quad . \quad . \quad \text{(III.7)}$$

It should be noted that this expression is accurate only if R_g is large with respect to R. Plotting equation (III.7) against frequency we shall see the extent of the frequency distortion. Fig. 31 shows such a curve for a valve of impedance 10,000 ohms and amplification factor 30, with values of R, R_g, and C respectively of 50,000 ohms, 2 megohms, and 0·01 microfarad. It will be seen that frequency distortion may be avoided down to very low frequencies by using values of capacitance readily obtainable in practice—provided R_g is of the order of a megohm. It may be remarked here that the use of much larger values of R_g than this is impracticable for reasons connected with the flow of undesirable negative grid-current (i.e. current *out of* the valve at its grid, resulting from the collection of positive ions by the grid, or the emission of electrons by the grid). Negative grid-current, in flowing through R_g, produces a positive bias voltage. A type of runaway effect is possible in high-μ valves if R_g is made too large.

A general expression for the voltage amplification, making no assumptions as to the values of the circuit components, may easily be derived from the equivalent circuit. The total impedance presented to the generator may be written

$$Z = \rho + \frac{R(R_g + 1/j\omega C)}{R + R_g + 1/j\omega C}$$

so that the whole alternating anode-current will be $\mu V_1/Z$. The current flowing through R_g will thus be

$$\frac{R}{R + R_g + 1/j\omega C} \cdot \frac{\mu V_1}{Z}$$

and the voltage amplification will be given by

$$(VA) = \frac{RR_g}{R + R_g + 1/j\omega C} \cdot \frac{\mu}{Z}$$

which equals

$$\frac{\mu RR_g}{\rho R + (\rho + R)R_g - \dfrac{j(\rho + R)}{\omega C}}$$

its magnitude being

$$|(VA)| = \frac{\mu RR_g}{\sqrt{\{\rho R + (\rho + R)R_g\}^2 + (\rho + R)^2/\omega^2C^2}} \quad \text{.(III.8)}$$

If $R_g \gg R$ this reduces to the form given in equation (III.7).

The angle of phase difference between V_2 and V_1 will be

$$\tan^{-1} \frac{(\rho + R)/\omega C}{\rho R + (\rho + R)R_g}$$

or, if $R_g \gg R$, $\tan^{-1} \dfrac{1}{\omega CR_g}$

Since this is a non-linear function of frequency there will be phase distortion; the phase relation between the fundamental and harmonics of a complex wave may not be quite the same after amplification. As has been pointed out in the introductory chapter, phase distortion is not harmful in sound reproduction.

The rules for the design of a resistance-capacitance coupled stage may be summarized as follows—

$$R \gg \rho$$
$$R_g \gg R$$

C to be large enough to make $\dfrac{1}{\omega C} \ll R_g$ for all frequencies which are to be amplified.

When informed that $\dfrac{1}{\omega C}$ must be small with respect to R_g the reader may legitimately ask, "How small?"

To answer this question we must look again at Fig. 31 which shows that the amplification falls off very gradually as the frequency is decreased. This particular graph is asymptotic to an amplification value of 25, and appears to be a horizontal line for frequencies above about 30 c/s. Equation (III.7) shows, however, that at any frequency which one might mention, even at 1,000 c/s, the amplification is less than the asymptotic value, 25, by a finite amount. It is therefore impossible to state at what frequency the amplification begins to be cut-off—except by an arbitrary definition of "cut-off." The established convention is to say that "cut-off" shall mean a reduction of voltage-amplification to $1/\sqrt{2}$ of the asymptotic value. This is the same as a reduction of 3 decibels (see page 9). We define the "cut-off frequency" as the frequency at which the gain is reduced 3 decibels below the asymptotic value of gain.

It is fairly easy to see, from equation (III.7), that the gain will be $1/\sqrt{2}$ of the asymptotic value, $\mu R/(\rho + R)$, when $\omega C R_g = 1$, i.e. at a frequency given by $1/(2\pi C R_g)$. This is therefore the cut-off frequency, for any given values of C and R_g. If a value of R_g has been decided upon, and if the lowest frequency to be amplified is known, we can then choose C so that $1/(2\pi R_g C)$ becomes equal to the value of this lowest frequency. We shall thus have ensured that the amplification of the stage will be constant to within 3 decibels, for all frequencies which are to be handled. (The reduction of amplification which occurs at higher frequencies is discussed in Chapter IV.)

Choke-capacitance Coupling

This is similar to resistance-capacitance coupling except that a choke coil of high inductance and low resistance takes the place of the load resistance, R. The circuit is thus more a modification of the basic circuit than a new type of coupling.

Such a modification of the basic circuit (i.e. the substitution of a choke coil for the resistance R) will give us an equivalent circuit consisting of a generator of voltage μV_1 and internal resistance ρ, in series with an impedance $R + j\omega L$, where R and L respectively are the resistance and inductance of the choke coil. Thus

$$(VA) = (R + j\omega L)I_a/V_1$$

$$= \frac{\mu(R + j\omega L)}{\rho + R + j\omega L}$$

so that the numerical value, or magnitude, of the voltage amplification will be

$$|(VA)| = \mu\sqrt{\frac{R^2 + \omega^2 L^2}{(\rho + R)^2 + \omega^2 L^2}} \quad . \quad . \text{(III.9)}$$

The usual practice is to design the choke coil to have a large inductance and a small resistance, in which case R may be neglected in the above expression, giving

$$|(VA)| = \frac{\mu\omega L}{\sqrt{\rho^2 + \omega^2 L^2}} \quad . \quad . \quad . \text{(III.10)}$$

If $\omega L \ll \rho$, the voltage amplification becomes simply $\mu\omega L/\rho$, i.e. directly proportional to frequency, giving severe frequency distortion. Such a circuit is sometimes used for special purposes, where it is desirable to have an amplification proportional to frequency, but in the orthodox design of a distortionless amplifier ωL is made large with respect to ρ, thus giving a voltage amplification which is approximately equal to μ at all frequencies. At very low frequencies, of course, the value of the reactance, ωL, will become small, thereby reducing the amplification and making it a function of frequency. Using the expression of equation (III.10) the voltage amplification may be plotted against frequency; Fig. 32 shows such curves for a valve of impedance 10,000 ohms and amplification factor 30, used

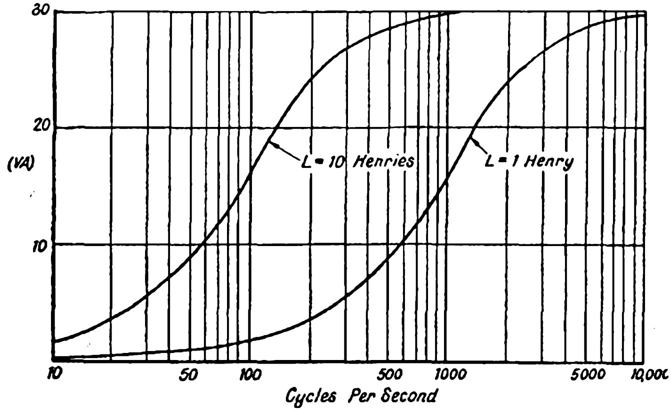


FIG. 32. FREQUENCY DISTORTION IN AMPLIFIER STAGE WITH CHOKE LOAD

$\mu = 30$
$\rho = 10{,}000$ ohms

with choke coils of inductance 10 henrys and 1 henry. From equation (III.10) it is easy to show that the cut-off frequency, defined as the frequency at which the gain is 3 decibels below the asymptotic value, is given by $f_c = \rho/2\pi L$.

The important advantage of using a choke as a load is that owing to its small resistance to direct current and large impedance to

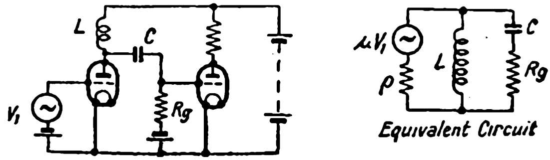


FIG. 33 CHOKE-CAPACITANCE COUPLING

alternating current, large amplification is possible without having a large steady *RI* drop in the load. Thus, a much smaller high-tension voltage is required for the same working anode-voltage. The disadvantage is that a really large inductance is required if the amplification is to remain constant down to the lowest audio-frequencies. In audio-frequency amplifiers of this type iron-cored coils are necessary to give the requisite inductance, but in high-frequency amplifiers a smaller inductance is required and air-cored chokes are often used.

In Fig. 33 is shown a choke-capacitance coupled circuit and its equivalent circuit. Since the arm formed of R_g and C is in parallel with the choke coil, L, the effective load impedance will be less than either ωL or R_g. At frequencies for which $\omega L \gg R_g$ the effective load impedance will be R_g; thus R_g should be made large with respect to ρ. As in the case of resistance-capacitance coupling the voltage across the load is divided between R_g and C; thus here again $1/\omega C$ should be small with respect to R_g.

The rules for the design of a choke-capacitance coupled stage may therefore be summarized as follows—

L to be large enough to make $\omega L \gg \rho$ for all frequencies which are to be amplified.

$$R_g \gg \rho$$

C to be large enough to make $1/\omega C \ll R_g$ for all frequencies which are to be amplified.

A general expression for the magnitude of the voltage amplification, from which the overall frequency distortion (resulting from the unduly small reactance of L and the unduly large reactance of C at low frequencies) may be estimated, may be derived in the same way as was equation (III.8). This will be found to give

$$|(VA)| = \frac{\mu\omega L}{\sqrt{\left\{\omega L + \frac{\rho}{R_g}\left(\omega L - \frac{1}{\omega C}\right)\right\}^2 + \left(\rho + \frac{L}{R_g C}\right)^2}} \quad \text{(III.11)}$$

Transformer Coupling

In this circuit (illustrated in Fig. 34) the primary of a step-up transformer is connected in the anode circuit of the first valve, and the secondary of the transformer is connected directly in the grid circuit of the second valve. Apart from the capacitance of the condenser formed of the grid and cathode structures, the secondary is open-circuited, thus the impedance looking into the primary of the transformer will be very large, the effective load impedance will be large, and there will be a large voltage amplification. The direct-current load, however, is equal to the resistance of the primary, which may be made small; so that this circuit, like choke-capacitance coupling, requires a smaller high-tension voltage than the basic circuit.

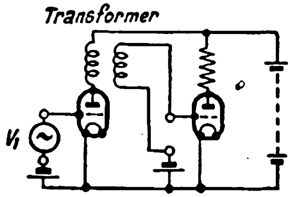


FIG. 34. INTERSTAGE TRANSFORMER COUPLING

Before deriving an expression for the voltage amplification of the circuit of Fig. 34, we shall consider the circuit of Fig. 35 which shows another modification of the basic circuit, namely, the connexion of the load R into the anode circuit through a transformer instead of directly. We shall first make the assumption that the transformer is an ideal transformer, i.e. we shall neglect the resistance and leakage-inductance of its windings, and assume that the secondary voltage is always n times as great as the primary voltage. The secondary current must then be $1/n$ times as great as the primary current, in order to have the input power equal to the output power, and hence the ratio of voltage to current on the primary side will be $1/n^2$ times as great as the ratio of voltage to current on the secondary side. Since the ratio of voltage to current on the secondary side is equal to R, this ratio must equal R/n^2 on the primary side. *Thus the connexion into the anode circuit of a step-up transformer of ratio, n, its secondary being closed by a resistance, R, is equivalent to connecting a resistance, R/n^2, directly in the anode circuit.*

The alternating anode-current will be given by

$$I_a = \frac{\mu V_1}{\rho + R/n^2}$$

and the primary voltage will be the product of this with R/n^2, the

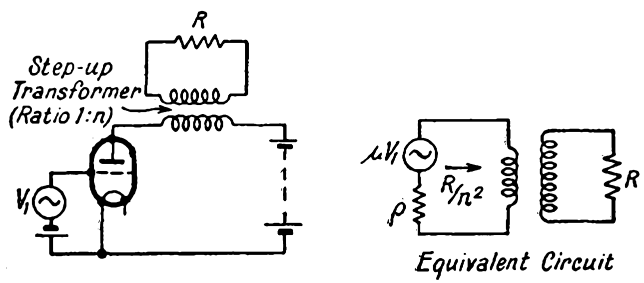


FIG. 35

secondary voltage being n times as great. The voltage amplification is the ratio of the secondary voltage to V_1, and will be given by

$$(VA) = \frac{\mu R/n^2}{\rho + R/n^2} \times n = \frac{\mu R}{\rho n + R/n}$$

This will be zero for n equal to zero or infinity, and will reach a maximum at some intermediate value of n, given by

$$\frac{d}{dn}\left(\rho n + R/n\right) = 0$$

i.e. $$\rho - R/n^2 = 0$$

i.e. $$n = \sqrt{R/\rho} \quad . \quad . \quad . \text{(III.12)}$$

This is the value of n which makes R/n^2, the resistance of the effective load, equal to the valve impedance, ρ. This is a particular case of the general rule that *the voltage amplification of such a circuit is a maximum when the transformer ratio is so chosen as to make the impedance presented to the valve numerically equal to the valve impedance.* This adjustment of the transformer ratio is spoken of as "matching" the load to the valve impedance.

The proof of this rule in the case when the secondary is closed by an impedance $(R + j\omega L)$ is as follows—

$$(VA) = \frac{\mu(R + j\omega L)/n^2}{\rho + (R + j\omega L)/n^2} \times n = \frac{\mu(R + j\omega L)}{\rho n + (R + j\omega L)/n}$$

$$\therefore \quad |(VA)| = \frac{\mu\sqrt{R^2 + \omega^2 L^2}}{\sqrt{(\rho n + R/n)^2 + \omega^2 L^2/n^2}} \quad . \quad . \quad . \quad . \text{ (III.13)}$$

The value of n which gives the maximum voltage amplification will therefore, be given by

$$\frac{d}{dn}\left[\left(\rho n + R/n\right)^2 + \omega^2 L^2/n^2\right] = 0$$

which gives

$$n = \sqrt{\frac{\sqrt{R^2 + \omega^2 L^2}}{\rho}} \quad . \quad . \quad . \text{ (III.14)}$$

Thus, with a reactive load, matching is possible only at a single frequency, whereas in the case of a purely resistive load the transformer ratio for maximum amplification is $\sqrt{R/\rho}$ which is independent of frequency. With this ratio the voltage amplification becomes

$$(VA)_{max} = \frac{n \,.\, \mu \,.\, R/n^2}{\rho + R/n^2} = \tfrac{1}{2}\,\mu n \quad . \quad . \text{ (III.15)}$$

If the same resistance were connected directly in the anode circuit the voltage amplification would be

$$\frac{\mu R}{\rho + R}$$

and since this expression is less than $\frac{1}{2}\mu n$ (except when R equals ρ, in which case the two expressions are equal), it follows that the voltage amplification is increased by the introduction of a transformer of suitable ratio.

In the above we have considered the conditions for maximum voltage amplification in the case where the actual load impedance (that closing the secondary) is fixed, and the transformer ratio may be varied. We shall now consider the case in which we are free to choose both R and n, the aim being to secure the maximum voltage amplification from the transformer coupled stage. It has been shown that the voltage amplification is given by

$$(VA) = \frac{\mu R}{\rho n + R/n}$$

Hence for any given transformer, i.e. for a given value of n, the amplification increases continuously with R, and is greatest when R equals infinity, i.e. when the transformer secondary is open-circuited as it effectively is in Fig. 34. In this case the value of the above expression becomes μn, and it would appear that the amplification may be made as great as desired by making n sufficiently great. There are two reasons why this is not so; the first is that the assumption of an ideal transformer breaks down when we consider the transformer secondary open-circuited, since the impedance looking into the primary of an open-circuited transformer is not infinite, as in the ideal case, but is equal to $(R_1 + j\omega L_1)$ where R_1 and L_1 are the resistance and inductance of the primary winding. The second reason is that the transformer ratio is limited, as we shall see, by considerations of frequency distortion.

Considering a stage using a transformer of primary resistance, R_1, primary inductance, L_1, mutual inductance, M, and having its secondary open-circuited, we see that the load is effectively a choke coil formed of the primary winding of the transformer. The voltage across the primary will therefore be

$$\frac{\mu V_1(R_1 + j\omega L_1)}{\rho + R_1 + j\omega L_1}$$

and since R_1 is usually small with respect to both ρ and ωL_1, this may be written

$$\frac{j\omega L_1 . \mu V_1}{\rho + j\omega L_1}$$

The voltage induced in the secondary will be $j\omega M . I_a$ where

$$I_a = \frac{\mu V_1}{\rho + j\omega L_1}$$

$$\therefore \qquad V_2 = \frac{j\omega M . \mu V_1}{\rho + j\omega L_1}$$

Thus, the secondary voltage is M/L_1 times as great as the primary voltage, and we shall write M/L_1 equal to n, the step-up ratio of the transformer. Thus

$$(VA) = \frac{j\omega L_1 . \mu n}{\rho + j\omega L_1}$$

and $$|(VA)| = \frac{\mu n \,.\, \omega L_1}{\sqrt{\rho^2 + \omega^2 L_1^2}} \quad . \quad . \quad . \text{(III.16)}$$

This is a function of frequency except when ωL_1, the primary reactance of the transformer, is large with respect to ρ. Thus, for distortionless amplification L_1 must be large enough to make $\omega L_1 \gg \rho$ at all frequencies which are to be amplified; otherwise there will be frequency distortion of the type illustrated in Fig. 32.

Equation (III.16) shows that the voltage amplification is proportional to n, the step-up ratio of the transformer. This ratio may be increased either by decreasing the number of turns on the primary or by increasing the number of turns on the secondary.

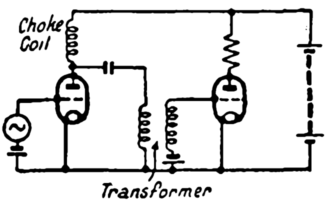


FIG. 36. PARALLEL-FED TRANSFORMER COUPLING

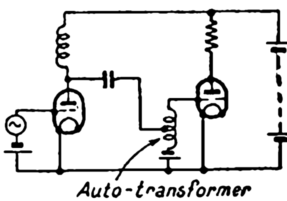


FIG. 37

The former procedure is limited by the consideration that the primary inductance must not be allowed to become too small; the latter procedure, if taken too far, causes the secondary to have too great a self-capacitance. This has two effects: first, it results in the secondary being virtually closed by a low capacitive reactance at the higher frequencies instead of being open-circuited, thereby reducing the amplification; secondly, it causes a considerable increase in amplification at the high frequency at which the inductance and self-capacitance of the secondary resonate. (This matter is discussed further in Chapter IV, under the heading "Tuned Transformer-coupled Amplifier.") Thus, the extent to which the ratio may be increased is limited, and the greatest ratio used in practice is about 4 : 1.

There are various useful modifications of the simple transformer-coupled circuit shown in Fig. 34. Fig. 36 shows a circuit designed to avoid the passage of the steady anode-current through the primary of the transformer, for this goes some way toward saturating the iron core of the transformer and thus reduces the primary inductance. The circuit of Fig. 36 is sometimes spoken of as Parallel-fed Transformer coupling. In Fig. 37 the same transformer is shown connected as an auto-transformer.

We may sum up the discussion of transformer coupling as follows: transformer coupling gives a large amplification, but causes greater distortion than resistance-capacitance coupling. Not only is there the dwindling amplification at both high and low frequencies as already described, but with iron-cored transformers there is harmonic distortion resulting from the non-linear characteristics of the iron.

Direct-coupled Amplifiers

All the amplifiers we have considered hitherto have had as their object the amplification of alternating voltages. Were a steady (or d.c.) voltage substituted for the alternating input voltage in any of these amplifiers (thus constituting simply an additional grid-bias voltage), the anode-current and anode-voltage of the first stage would be dependent upon the value of the d.c. input voltage, but the anode-currents and voltages of the subsequent stages would be quite unaffected by the input voltage. This is because

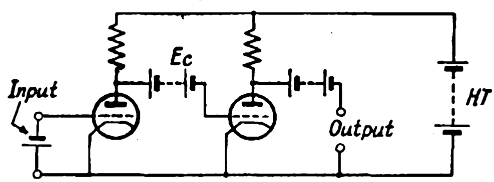


FIG. 38. D.C. AMPLIFIER

the coupling circuits so far considered do not pass on from stage 1 to stage 2 the d.c. component of the load voltage of stage 1. They are, in fact, deliberately designed to avoid so doing, in order that the steady grid voltage of stage 2 may be determined only by the grid-bias supply of that stage.

The phrase "D.C. Amplifier" is used to denote an amplifier having a d.c. output voltage whose value is dependent on the value of the d.c. input voltage; small permanent changes of the d.c. input voltage are required to cause proportionately larger permanent changes of the output voltage. D.C. amplifiers are almost always required, however, to amplify alternating voltages as well as d.c. voltages; in short, the interstage coupling circuits must be capable of passing on the d.c. component as well as the a.c. component. Apart from this, the usual requirements of interstage coupling (see page 39) still apply. Fig. 38 shows a simple circuit for d.c. coupling using a common h.t. supply. The anode of stage 1 is connected to the grid of stage 2 through a battery E_c. The voltage of this battery is so chosen as to avoid positive grid-voltage on stage 2 (with consequent distortion and excessive anode-current). Clearly the grid-potential of stage 2 is less, by a constant voltage E_c, than the anode-potential of stage 1, and hence all changes of anode-potential (whether permanent changes of the d.c. component, or alternating changes) are communicated to the grid of the next stage. The voltage E_c of the coupling battery will have to be slightly greater than the anode-voltage of valve 1, i.e. of the same order as the h.t. supply voltage. In an amplifier with several stages, using this method of interstage coupling, we should require several such d.c. supply voltages, and since these voltages have no common point, they would either have to be supplied by separate rectifiers or by separate batteries. For this reason, the method of Fig. 38 is seldom used.

Fig. 39 shows a method which is sometimes known as the "third rail" method, since in addition to the h.t. positive "rail" and the h.t. negative "rail" a "third rail" is provided whose potential is considerably negative with respect to the negative side of the h.t. supply. Consider the interstage coupling between the first and second valves in Fig. 39. The series circuit $R_1R_2E_2$ is connected directly across the first valve, from anode to cathode. Provided that R_1

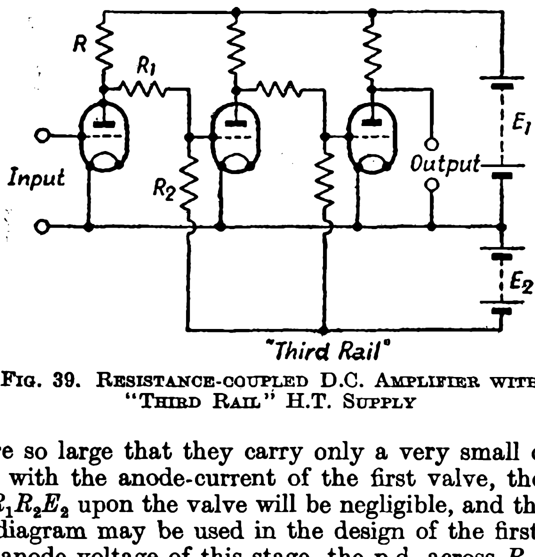


FIG. 39. RESISTANCE-COUPLED D.C. AMPLIFIER WITH "THIRD RAIL" H.T. SUPPLY

and R_2 are so large that they carry only a very small current, as compared with the anode-current of the first valve, the shunting effect of $R_1R_2E_2$ upon the valve will be negligible, and the ordinary load-line diagram may be used in the design of the first stage. If v_a be the anode-voltage of this stage, the p.d. across R_1 and R_2 in series will be $v_a + E_2$. By making R_1/R_2 equal to v_a/E_2, the potential of the grid of the second valve could be brought to cathode-potential. In practice R_1/R_2 is made slightly greater than v_a/E_2, so that the quiescent grid-voltage of the second stage (i.e. the effective bias-voltage) shall be slightly negative.

An analysis of a d.c. amplifier may legitimately be made by means of the valve equivalent circuit since, although we have derived the equivalent circuit in terms of a.c. vectors, it is in fact based on the incremental equation (II.4). Such an analysis (based on the equivalent circuit) can yield no information about the whole d.c. component of either the anode-current or anode-voltage, but relates only to the increments produced in these quantities by the application of the input voltage. Drawing the equivalent circuit for

Fig. 39, it is seen that R_1 and R_2 form a voltage-divider across the output voltage of the first stage, the grid-circuit of stage 2 receiving only a fraction $R_2/(R_1 + R_2)$ of the output voltage of stage 1. This is a disadvantage of the method, but it may be minimized by making R_1/R_2 as small as conveniently possible. Since R_1/R_2 is approximately equal to v_a/E_2, it follows that the circuit should be designed to use as small a value of anode-voltage as is consistent with operating on the linear parts of the characteristics, and also that a large value of E_2 should be used. In practice E_2 is usually about half as great as E_1. It should be noted that E_2 is not required to supply much current: it may be thought of as a common grid-bias supply voltage (see also "Cathode Coupling," page 79 and Problem 34, page 367).

In any valve circuit the values of the various anode-currents are subject to slow fluctuations and drifts (e.g. due to slight variations of h.t. and heater supply voltages, to ageing of valves, etc.). These drifts are usually too slight to have any marked effect on the gain of an amplifier. Such slow fluctuations are not passed by the interstage couplings of an a.c. amplifier, but in a d.c. amplifier the drifts in the first stage behave as spurious signals and are amplified by the succeeding stages, sometimes giving an output voltage larger than the desired output voltage. Drift becomes a more and more serious obstacle the smaller the input voltage, and many ingenious circuits have been designed in an attempt to overcome it.

Earthing and Screening

In a multistage amplifier (or in any apparatus using a number of valve stages, e.g. radio receivers and transmitters) it is usual to have a common h.t. supply voltage for all stages. It follows that there is one point in the circuit (viz. the h.t. negative terminal) which is common to the anode circuits of all stages, and also to all the grid circuits. For example, in Figures 29 (*b*), 33, 34, 36 and 37, the horizontal line at the base of each figure is common to both anode circuits and both grid circuits. Moreover, in cases where the h.t. voltage is produced by rectification of an alternating voltage derived from a.c. mains, one side of this alternating voltage will usually be connected to the h.t. negative and thus also to this common point (see Figs. 140, 141, 145, 146). Since only one side of the h.t. voltage and only one side of the alternating voltage derived from the a.c. mains is connected to the various grid circuits, we do not expect that these voltages will send currents through the impedances in the grid circuits. But if the other side of either of these voltages is connected, via unwanted leakage resistance paths and stray capacitance paths, to other points in the various grid circuits, then these voltages will cause unwanted currents to flow in the grid circuits, thus setting up spurious input voltages, whose magnitude may be quite comparable with the wanted input voltages.

Fig. 40 (*a*) illustrates this. Z represents the impedance in one of the grid circuits (e.g. the resistance R_g in Fig. 29 (*b*)). It is required that neither the h.t. voltage, E, nor the alternating voltage, $\hat{V}$, shall send current through the impedance Z. Yet the use of a common h.t. supply has made it unavoidable that one side of E and one side of $\hat{V}$ shall be connected to one side of Z (and to one side of every impedance such as Z in the whole multistage apparatus). Now there will inevitably be leakage paths of finite resistance, R_1, from the point G to the nearest large mass of conducting material (which is usually the metal chassis, and is denoted by the letter C in Fig. 40). There will inevitably be similar paths, R_2, from the points A and B to the chassis, and R_1 and R_2 in series constitute a path by which $\hat{V}$ can send unwanted currents through Z. R_1 and R_2 may be of the order of hundreds of megohms or more,

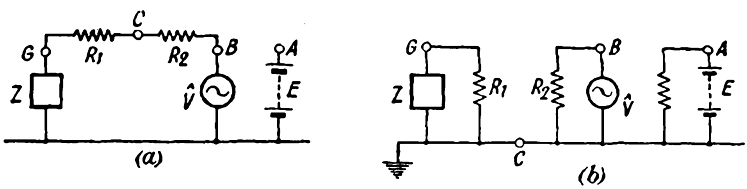


FIG. 40

and thus the leakage currents will be small, but the magnitude of the voltages which such currents will produce across Z depends on the relative magnitudes of Z and the leakage resistances. In audio-frequency amplifiers the magnitude of Z may be of the order of megohms, and hence appreciable unwanted voltages may be set up.

The fact that the chief leakage paths are by way of the nearest large mass of conducting material makes possible a simple remedy, viz. to connect this nearest mass of conducting material (the chassis) to the common point, as shown in Fig. 40 (*b*). The effect of this is clearly to prevent the existence of a path by which $\hat{V}$ or E can send current through Z, since R_1 and R_2 now become shunted across Z and $\hat{V}$ respectively. The nearest mass of conducting material is often referred to as "Earth," and the above remedy is spoken of as "Earthing the common negative."

$\hat{V}$ may denote any alternating voltage one side of which is connected to the common negative; this includes not only the mains-frequency voltage of the rectifier (if any) but also the input and output voltage of every stage which uses the common h.t. supply, and may therefore include h.f. voltages. Now there will normally be a stray capacitance between the points G and B (Fig. 40 (*a*)), formed of the condenser whose plates are the wiring and actual conducting parts of Z and of the circuit in which $\hat{V}$ exists. This capacitance may be very small, but if $\hat{V}$ is a high frequency voltage, the reactance of the capacitance may be fairly small at that high frequency. Again, this stray capacitance path permits $\hat{V}$ to send current through Z, and to set up unwanted voltages across Z. The remedy is to divide the capacitance into two parts, by interposing a conducting screen (of sheet metal or metal gauze) geometrically between G and B. The circuit then becomes similar to that of Fig. 40 (a), but with capacitances instead of resistances, the point C now denoting the metal screen. The connexion of the metal screen to the common negative then suffices to cause the two capacitances to appear simply as shunts across $\hat{V}$ and Z, with no capacitance path by which $\hat{V}$ can send current through Z. This procedure is known as "Electrostatic Screening," and it should be noted that

(i) the screen need not completely surround either Z or $\hat{V}$, provided it is sufficient to split the capacitance between them into two separate capacitances in series;

(ii) the screen must be "earthed."

The screen-grid is included in a tetrode or pentode valve for the purpose of providing electrostatic screening in order to reduce the capacitive coupling between anode and control-grid. The deleterious effects of such coupling are discussed in Chapter V.

Spurious voltages may also be produced by unwanted mutual inductance between various parts of the circuit (e.g. between leads, coils, etc.). The remedy in this case is Electromagnetic Screening, viz. the *complete* surrounding of the parts in question by a metal screening can of good conductivity. It is not essential that this screen should be earthed, but it is usual to earth it in order that it may provide electrostatic screening at the same time.

Output Stages

The last stage, or output stage, of a multi-stage amplifier is usually required to feed as much power as possible into a predetermined load, such as a loudspeaker, a low impedance line, or a measuring instrument. The first consideration is thus the design of a circuit to feed maximum power to a given load *for a given input voltage*.

It was shown on page 37 that this condition for maximum output power is that the impedance presented to the valve shall equal the valve impedance. Unless there is available a valve of the desired impedance the load must be coupled to the anode circuit of the output valve by means of a transformer—usually referred to as an Output Transformer to distinguish it from an Interstage Transformer.

In the case of a pure resistance load the transformer ratio which will give maximum power for a given input voltage is that which matches the load to the valve impedance, viz.

$$n = \sqrt{R/\rho}$$

In the case of a load which has both resistance and reactance the ratio is (from equation III.14)

$$n = \sqrt{\frac{\sqrt{R^2 + \omega^2 L^2}}{\rho}},$$

but matching can clearly be accomplished only at a single frequency. Thus an output stage feeding a load which has reactance is bound to cause frequency distortion. The case of a loudspeaker load is interesting; the impedance of most loudspeakers is a very complicated function of frequency, and undergoes large changes both of magnitude and phase when the frequency is changed by quite small amounts. This is caused by resonances of the mechanical system, which also cause the sensitivity of the loudspeaker to vary greatly with frequency. In such a case the ratio of the output transformer is chosen to give least overall frequency distortion, consistent with a reasonable output power, the choice being made empirically.

The second consideration in the design of an output stage is the choice of a valve "large" enough to give the required power without undue harmonic distortion. In amplifiers used for the reproduction of sound "undue harmonic distortion" is usually taken to mean 5 per cent second harmonic distortion or over. We must, therefore, consider the estimation of *the maximum power which can be delivered by a given valve subject to the second harmonic distortion not exceeding 5 per cent.*

First let us consider an output valve of impedance, ρ, coupled to a load, R, by means of a transformer of ratio $\sqrt{R/\rho}$. The output power and second harmonic distortion resulting from the application of a given input voltage in the grid circuit may be estimated by the load-line method described earlier in this chapter. It must be noted that this is a circuit having an alternating-current load resistance different from the direct-current load resistance. The latter is simply equal to the resistance of the transformer primary, whereas the former is R/n^2, and since n equals $\sqrt{R/\rho}$, this equals ρ and the load is matched to the valve impedance. The direct-current load line of slope $1/R_1$ tells us the value of the steady anode-current and the steady anode-voltage. The alternating-current load line of slope $1/\rho$ and drawn through the point whose co-ordinates are the values of the steady anode-current and voltage, enables us to calculate the power delivered to the load (neglecting losses in the transformer, which are usually small) and the harmonic distortion.

Both the power and the distortion increase as the input voltage is increased. When finally the second harmonic distortion has increased to 5 per cent, the output power and the input voltage have reached their maximum permissible values with this particular circuit. It is almost always found, however, that if the calculation be repeated for an alternating-current load line corresponding to a slightly greater load resistance,* a greater input voltage may be applied before the second harmonic distortion reaches 5 per cent—and a greater output power may be secured. The alternating-current load resistance which allows maximum power to be delivered by the valve, subject to a limitation of 5 per cent second harmonic distortion, may be found by trying load lines of various slopes, and is referred to as the "Optimum Load for Maximum Undistorted Output Power." Its value for triodes is often about twice that of the valve impedance, and is usually stated for the various valves described in the catalogues of valve manufacturers. (See also Problem III.43, page 369.)

In the orthodox design of output stages, therefore, the transformer ratio is so chosen that the resistance presented to the valve is equal to this optimum load resistance and not equal to ρ. It is to be noted that this does not normally cause an appreciable reduction of the output power resulting *from a given input voltage.* For an input voltage, V_1, the output power, W, is given by

$$W = I_a{}^2 \,.\, R/n^2$$

$$= \left(\frac{\mu V_1}{\rho + R/n^2}\right)^2 . \, R/n^2 \qquad . \qquad . \text{(III.17)}$$

from which it may be seen that the impedance, R/n^2, presented to the valve may vary from $\frac{1}{2}\rho$ to 2ρ without W being reduced by more than about 11 per cent.

The design of an output stage may thus be summarized as follows—

$$n = \sqrt{\frac{R}{\text{Optimum load resistance}}}$$

where the optimum load resistance is the reciprocal of the slope of that load line which gives maximum power output subject to a limit of 5 per cent harmonic distortion.

Valves in Parallel

If sufficient power cannot be obtained from a given valve, two or more similar valves may be connected in parallel by joining all the cathodes, all the grids and all the anodes, and treating the combination as a single valve. The power delivered by a single valve is given in equation (III.17) and, if the load is matched to the valve impedance, this becomes

$$W = \frac{\mu^2 V^2{}_1}{4\rho}$$

When an input voltage, V_1, is applied to two similar valves in

* The above refers to an output stage using a triode. See also "Pentode Amplifier Circuits," page 72.

parallel, we have in the equivalent anode circuit two generators in parallel, each of voltage μV_1 and internal resistance ρ. This is equivalent to a single generator of voltage μV_1 and internal resistance $\frac{1}{2}\rho$. For maximum power the load presented to the combination of valves should, therefore, be $\frac{1}{2}\rho$, i.e. the transformer ratio should be

$$n = \sqrt{\frac{R}{\frac{1}{2}\rho}}$$

The power delivered to the load is then given by

$$W = \left(\frac{\mu V_1}{\frac{1}{2}\rho + R/n^2}\right)^2 . R/n^2$$

$$= \frac{\mu^2 V_1^2}{2\rho}$$

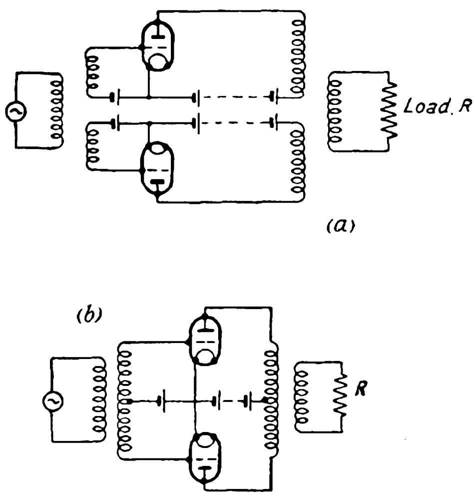


FIG. 41. PUSH-PULL AMPLIFIER

which is twice that delivered by a single valve as might have been expected.

Since the two valves in parallel are equivalent to a single valve of the same amplification factor and one-half the impedance, the characteristics of the equivalent single valve will be exactly the same as those of each of the two separate valves but with the scale of anode-current multiplied by two. It follows that the optimum load for maximum undistorted output power for the two valves in parallel will be one-half of that for each of the valves separately.

Push-pull Circuits

The push-pull circuit is another method of combining two similar valves so that the combination is equivalent to a single valve which will deliver a greater power. This circuit has important advantages over the parallel circuit and the push-pull principle is widely applied not only to amplifier circuits but also to modulators, detectors, and oscillators. Push-pull circuits are especially important in dealing with high frequencies.

When two valves are used in parallel the input voltages of the two valves are obviously in phase, and the two anode-currents are in phase and are added in passing through the common load. The principle of the push-pull circuit is the connexion of two valves so that the two input voltages are in antiphase; the two anode-currents will then also be in antiphase so that their difference (i.e. the result of reversing the phase of one of them and then adding) will be a large useful anode-current similar to that in the parallel circuit. Fig. 41 (*a*) shows a single-stage transformer-coupled amplifier using two valves in push-pull. The input transformer has two secondary windings arranged so that the input voltages to the two valves are in antiphase, i.e. so arranged that when the grid of the upper valve is positive with respect to its cathode, the grid of the lower valve is negative with respect to its cathode. The output transformer has two primary windings so arranged that the magnetization of the core shall be the difference of that produced by each winding separately, i.e. if the anode-currents of the two valves measured in the conventional direction are equal there is no resultant magnetization of the core. The circuit may be simplified considerably, as shown in Fig. 41 (*b*), by dispensing with two separate secondary windings on the input transformer and two separate primary windings on the output transformer, and substituting in each case a single winding with a centre-tap, i.e. a connexion to the middle point of the winding. This is simply equivalent to joining the two cathodes. We may then use common high-tension, heater, and grid-bias supplies. Of course, if it were necessary that the two valves should have different operating voltages this simplification would not be possible, but since the importance of the push-pull principle arises from the use of similar valves operating in an exactly similar condition the circuit of Fig. 41 (*a*) is never seen in practice.

The first advantage of the push-pull circuit is that despite the curvature of the valve-characteristics the circuit cannot cause the introduction of even-order harmonics, provided that the valves are exactly similar and are used in a similar condition. This may be shown as follows

Let the input voltage of the first valve be $V_1 \sin \omega t$; the input voltage of the second valve will then be $-V_1 \sin \omega t$. Let us suppose that each valve used separately would produce a second harmonic in addition to the amplified fundamental, and let the amplitude of the harmonic be k times that of the fundamental. Then the anode-current of the first valve may be written

$$i_{a1} = I_a \sin \omega t - kI_a \cos 2\omega t$$

the function $-\cos 2\omega t$ being chosen to give the proper phase relation as shown in Fig. 22 (*b*), page 31, for harmonic and fundamental, though we shall see later that this is not essential to the proof. It might be expected at first sight that since for alternating voltages v_{g1} equals $-v_{g2}$, the alternating part of the anode-current of the second valve would be simply $-i_{a1}$, but this is not so. Fig. 42 shows the graphs of the two anode-currents, the fundamental and second harmonic being shown separately.

In drawing these graphs, the criterion for deciding the phase of the second harmonic relative to the fundamental was that the graph

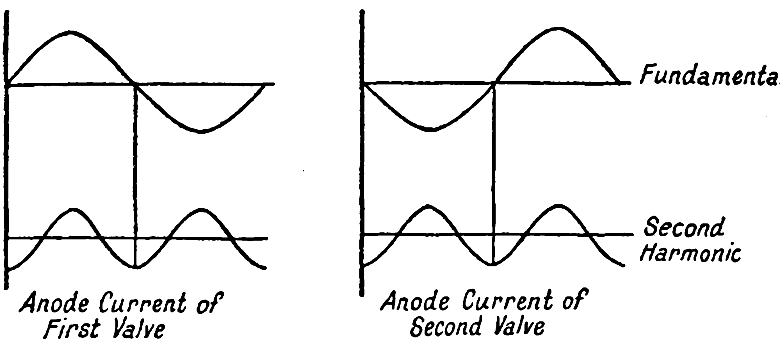


FIG. 42

of the resultant anode-current (viz. the sum of fundamental and second harmonic) should have a larger positive excursion than negative excursion. This criterion applies equally to the two valves, since the effect of curved characteristics upon the waveform will be the same for both valves. The anode-current of the second valve is, therefore, given by

$$i_{a2} = -I_a \sin \omega t - kI_a \cos 2\omega t$$

whence $$(i_{a1} - i_{a2}) = 2I_a \sin \omega t.$$

Thus there is no second harmonic in the output of the push-pull circuit.

The general proof that no even-order harmonics are produced in such a circuit, making no assumptions as to the phase of the harmonics produced by a single valve, is as follows. Consider the positive half-cycle of anode-current in the first valve; this will be taking place during a negative half-cycle of anode-current in the second valve. Now the graph of each even-order harmonic repeats itself exactly in each half-cycle of the fundamental, since there are an exact number of waves of each even-order harmonic in each half-cycle of the fundamental. Thus the graph of a particular even-order harmonic is the same in a positive half-cycle as in a negative half-cycle, and is therefore the same in the two valves although the first valve is experiencing a positive half-cycle of anode-current when the second is experiencing a negative half-cycle. In taking the difference between the two anode-currents the even-order harmonics will therefore cancel out.

For odd-order harmonics, however, there is an odd number of half-waves in each half-cycle of the fundamental, so that the graph of a particular odd-order harmonic in the negative half-cycle of the fundamental is that in the positive half-cycle with the sign reversed. Thus the graph of a particular odd-order harmonic in the anode-current of the second valve is that in the first valve with the sign reversed. In taking the difference between the two anode-currents the odd-order harmonics will thus not cancel out, but will add, as do the fundamentals. There will thus be odd-order harmonic distortion, but no even-order.

It is interesting to note that the current which passes through the high-tension supply is the sum of the steady anode-currents plus the even-order harmonics produced by the valve. It will be seen later in this chapter that it is advantageous to reduce in this manner the alternating current passing through the high-tension supply. (See sections on "Decoupling Circuits" and "Automatic Bias Circuits.")

The reduction of distortion by the use of a push-pull circuit means that a larger power may be delivered by two valves in push-pull than by two valves in parallel, subject to the harmonic distortion not becoming unduly large.

The second major advantage of the push-pull circuit—and the most important advantage in high-frequency work—is that if the cathodes be connected to earth the circuit is electrically symmetrical with respect to earth. By disposing the mechanical parts of the circuit symmetrically with respect to the earthed surroundings the stray capacitances to earth may also be made symmetrical with respect to earth, so that the electrical symmetry is preserved.

Another advantage of the push-pull circuit is that the steady anode-currents of the two valves pass through the two halves of the primary winding of the transformer in opposite directions, thereby causing no steady magnetization. (Cf. parallel-fed transformer coupling.)

Composite Characteristics for Valves in Push-pull

The push-pull circuit is perhaps most widely used in output stages, though it may of course be used for any stage of an amplifier. The determination of the optimum load resistance for maximum undistorted power output, and hence the determination of the best transformer ratio, may not be made simply from a knowledge of the optimum load for a single valve of the type used, as in the case of valves in parallel, but involves the drawing of Composite Characteristics. The drawing of these characteristics constitutes a graphical construction for calculating the difference between the alternating anode-currents of two valves having input voltages equal in amplitude but opposite in phase.

The load line for a single valve is the locus of points whose co-ordinates are simultaneous values of anode-voltage and anode-current; the variation of anode-current is investigated by allowing a point to oscillate along this line, moving from curve to curve in a manner governed by the variation of grid-voltage, since each curve corresponds to a single value of grid-voltage. The centre of oscillation is the point (known as the "Quiescent Point") whose co-ordinates are the values of the steady anode-voltage and anode-current. In the push-pull case we wish to investigate the variation of a quantity which is the difference between two anode-currents, by observing the co-ordinates of a single oscillating point. We must thus superimpose the characteristics of two equal valves in such a way that an increase of the anode-voltage of one valve means a corresponding decrease of the anode-voltage of the other. Moreover, we must note that the anode-voltages of the two valves pass through their steady values at the same instant, so that the scales of anode-voltage for the two sets of characteristics must read the steady value together. Fig. 43 shows the construction. The set of curves for one of the valves is reversed both horizontally and vertically and placed below the set of curves for the other valve, the horizontal displacement of the origin being such as to allow the scales to coincide at the value of the steady anode-voltage, the quiescent points, P_1 and P_2, being in the same vertical straight line. We next proceed to combine the two curves which correspond to a value of grid-voltage equal to the steady grid-bias supply voltage, giving a single curve showing the variation of the difference of the anode-currents as the anode-voltage of one valve increases and the other decreases, the grid-voltage remaining constant. This is done simply by adding curves A and B, to give the dotted curve, C; since curve B is upside down all its ordinates are negative, and the *addition* of curves A and B gives the *difference* between the anode-currents. Finally, we combine other pairs of curves in the same way, forming pairs by taking from one set of curves the curve corresponding to a grid-voltage exceeding the steady value by a given amount, and taking from the other set the curve corresponding to a grid-voltage which is less than the steady value by the same amount. This choice, of course, arises from the fact that the input voltages to the two valves are in antiphase, one increasing as the other decreases.

We have now drawn the composite characteristics, showing the variation of $(i_{a1} - i_{a2})$ with the anode-voltage of either valve, each curve being for a single value of grid-voltage, and taking account of the fact that the input voltages are in antiphase. It remains to draw a composite load line.

In Fig. 43 the straight lines $X_1P_1Y_1$ and $X_2P_2Y_2$ are the individual load lines for the two valves, the slope of each being the reciprocal of the impedance presented to each valve, i.e. the impedance looking

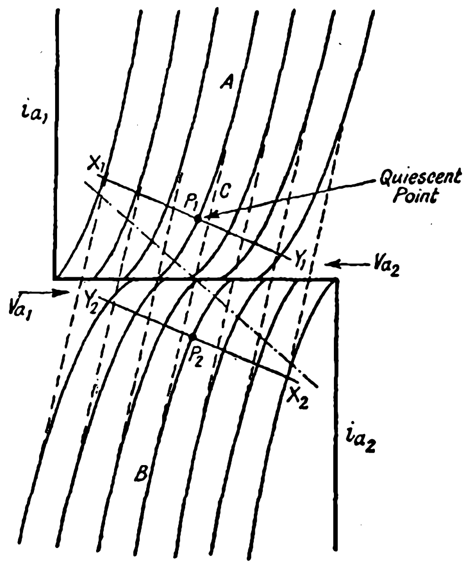


FIG. 43. COMPOSITE CHARACTERISTICS FOR PUSH-PULL CIRCUIT

into each primary winding of the output transformer of Fig. 41 (*a*) or Fig. 41 (*b*). Writing

$$n = \frac{\text{Number of secondary turns}}{\textit{Total}\text{ number of primary turns}}$$

the impedance looking into the whole of the primary winding will be R/n^2. The impedance looking into one-half of the primary winding with the other half disconnected would be $R/(2n)^2$, but this will not be the impedance presented to each valve since in the circuit we are considering there is a generator connected to each half, the two generators sending current through the primary winding in the same direction. In this case the impedance presented to each generator will be $R/2n^2$; this may be seen as follows.

Let the voltage across each half of the primary be $\hat{V}_p$ and the voltage across the secondary be $\hat{V}_s$. The ratio $\hat{V}_s/\hat{V}_p$ will be equal to the turns ratio, $2n$, since every turn is experiencing the same rate of change of flux. Since, however, the total input power must equal the output power,

$$2\hat{V}_p\hat{I}_p = \hat{V}_s\hat{I}_s$$

where $\hat{I}_p$ is the current in each primary and $\hat{I}_s$ is the secondary current. Hence

$$\hat{I}_p = \frac{\hat{V}_s\hat{I}_s}{2\hat{V}_p} = \frac{\hat{I}_s}{2} \cdot 2n = n\hat{I}_s$$

Thus the impedance looking into each primary will be

$$\frac{\hat{V}_p}{\hat{I}_p} = \frac{\hat{V}_s/(2n)}{n\hat{I}_s} = \frac{\hat{V}_s}{\hat{I}_s} \cdot \frac{1}{2n^2} = \frac{R}{2n^2}$$

$\therefore$ Slope of individual load line $= 2n^2/R$

The new composite load line, to be used in conjunction with the composite characteristics, must be formed by adding the lines $X_1P_1Y_1$ and $X_2P_2Y_2$, since it must relate the anode-voltage to the difference of the anode-currents. Its slope will thus be $4n^2/R$. Since this is the reciprocal of one-quarter of R/n^2, it is sometimes stated that the equivalent load for the composite characteristics is one quarter of the anode-to-anode load, R/n^2.

The composite characteristics are the characteristics of the equivalent single valve, the difference between the anode-currents of the individual valves being analogous to the anode-current of the equivalent single valve. Calculations of power output and waveform distortion are made by drawing the composite load line for the load under consideration and proceeding exactly as for a single valve. Since there are no even-order harmonics if the valves are exactly similar, the third harmonic is usually the largest, and it may be evaluated by plotting the waveform of the equivalent anode-current and calculating in the usual way one-third of the sum of three ordinates equally spaced throughout the cycle of the fundamental. For convenience the values of the three ordinates may be determined without plotting the whole waveform, by observing the values of the equivalent anode-current at values of the input voltage one-third and two-thirds of the way through the cycle, the value of the steady anode-current constituting the third ordinate. It is wise to check such calculations experimentally. The optimum load resistance and maximum "undistorted" power output subject to a stated limit of third harmonic distortion may be found by trying various load lines, as in the case of a single valve. The required transformer ratio will be given by

$$n = \sqrt{\frac{R}{\text{Anode-to-anode load resistance}}}$$

where the anode-to-anode load resistance is four times the reciprocal of the slope of the desired load line.

For maximum power output with a given input voltage (as distinct from maximum "undistorted" power output) the impedance presented to each valve should be equal to the valve impedance, ρ, i.e. $R/2n^2$ should be made equal to ρ. The slope of the composite load line will then be $2/\rho$, which is numerically equal to the slope of the straight parts of the composite characteristics, since the slope of the straight parts of the individual characteristics is $1/\rho$.

There are two errors which must be guarded against in using the composite characteristics: first, the grid-voltages stated on the curves refer to the input voltage of one valve only. The total grid-to-grid input voltage will be twice the input voltage to each valve.

Secondly, the composite characteristics are valid for push-pull circuits in which the output is transformer-coupled, but not for circuits in which two resistances take the place of the two halves of the primary winding of the transformer of Fig. 41 (*b*). In this latter case the performance of the circuit must be analysed by considering the valves separately, drawing load lines for the individual loads, calculating the individual output voltages and adding them in the correct phase. It might appear at first sight that this would give the same result as the method of composite characteristics, but this is not so. The drawing of composite characteristics, involving as it does the superposition of the scales of anode-voltage, inherently assumes that at any instant the anode-voltage of one valve exceeds the steady value by the same amount that the anode-voltage of the other valve falls short of the steady value. In a push-pull circuit with resistance loads this is not necessarily so, for the form of the negative half-wave of the anode-voltage may differ from the form of the positive half-wave, as a result of the curvature of the characteristics. In the transformer-coupled case, however, the two halves of the primary winding are equal, and are linked by the same flux. The voltages across the two halves are, therefore, equal, and the anode-voltages of the two valves must thus be of the same amplitude and waveform, but 180° out of phase.

It is interesting to note that, since the alternating anode-voltages of the two valves in a transformer-coupled push-pull amplifier circuit must be equal and opposite, whereas this would not be indicated by the drawing of individual load lines, each valve, considered separately, cannot be functioning in the manner corresponding to the individual load line. It is fairly easy to show that the individual load lines which would give the same result as the composite method are curves instead of straight lines.

Decoupling Circuits

A difficulty arises when all the valves of a multi-stage amplifier have a common high-tension supply or a common grid-bias supply. Since the alternating anode-currents of all stages pass through the source of high tension, and since the latter will not have zero resistance, there will be alternating voltages set up across it. Thus, the high-tension supply to, say, the first valve is not simply a steady voltage but is the sum of a steady voltage and the alternating voltages due to the passage through the source of high tension of the anode-currents of all the other valves. Thus we are, in effect, feeding back some of the amplified voltage to the first stage of the amplifier for further amplification. It might be thought that this would simply increase the overall amplification, which would be advantageous, but it can be shown (see "Regeneration and Oscillation," Chapter V) that if the voltage fed back exceeds a certain value the circuit will generate voltages of an entirely different

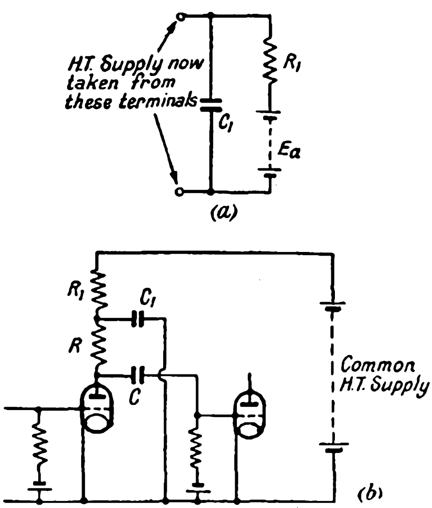


FIG. 44. ANODE-CIRCUIT DECOUPLING

frequency from the input voltage, this frequency depending only upon the circuit constants. Under these conditions such a circuit will generate alternating voltages of this particular frequency even though there is no input voltage. It therefore becomes an oscillator, i.e. an apparatus which generates alternating voltages, although power is fed to it from direct-current supplies only. In an amplifier, of course, this "self-oscillation" is to be avoided, and when it occurs it is referred to as "Parasitic Oscillation."

The object of decoupling circuits is to avoid parasitic oscillation by preventing the alternating anode-current from passing through the source of high tension. This is done by shunting the high tension with a condenser of large capacitance, with the object of providing a path of lower impedance for the alternating current. The impedance of the source of high tension (which will be either a battery or a mains-rectifier circuit) is usually so low, however, that impossibly large condensers would be required; to avoid this the impedance is artificially increased by the addition of a resistance in series with the high-tension supply. Fig. 44 (*a*) shows a decoupling circuit, and Fig. 44 (*b*) shows a stage of a resistance-capacitance coupled amplifier using such a circuit.

The choice of the values of R_1, the decoupling resistance, and C_1, the decoupling capacitance, is made as follows. R_1 must not be made too large, on account of the reduction of the steady anode-voltage by the RI drop across it. With valves having anode-currents of not more than a few milliamps, a value of 10,000 ohms is convenient. C_1 must then be chosen so that it effectively shunts R_1 at all frequencies to be amplified, i.e. we must make $1/\omega C_1$ small with respect to R_1. If parasitic oscillation occurs at a lower frequency, C_1 must be increased so that $1/\omega C_1 \ll R_1$ at this frequency.

In addition to by-passing alternating current from the high-tension supply, the decoupling circuit will be seen to prevent any alternating voltage which has become included in the high-tension supply from appearing in the various anode circuits. Fig. 44 (*a*) shows that R_1 and C_1 form a voltage-divider across the high-tension supply, and if $1/\omega C_1$ is small with respect to R_1 scarcely any of the alternating voltage will appear across the condenser, C_1, from which the high-tension supply is now taken.

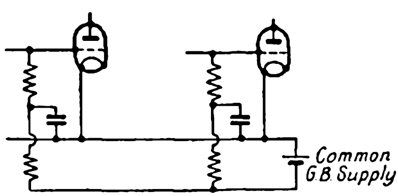


FIG. 45. GRID-BIAS DECOUPLING

Decoupling circuits may also be used with the grid-bias supply. The principle is exactly the same: the impedance of the grid-bias supply is increased by the connexion of a resistance in series with it, and the two are shunted by a condenser. Fig. 45 shows a grid decoupling circuit. Such circuits are of limited usefulness, since each stage may be made to provide its own grid-bias voltage automatically as described in the next paragraph.

Automatic Bias Circuits

The principle of the automatic bias circuit is illustrated in Fig. 46 (*a*). It will be seen that a resistance, R_1, known as the "bias resistance," is included in the anode circuit between the cathode (or, in the case of a directly heated triode as is shown in Fig. 5, the negative end of the filament) and the high-tension supply. The passage of the steady anode-current through this resistance results in a difference of potential between its ends, the lower end being negative with respect to the upper end. This difference of potential appears in the grid circuit, making the grid negative with respect to the cathode, and taking the place of the grid-bias supply. For a valve operating with a steady grid-voltage of $-E_g$ and a steady anode-current of i_{a0} the required value of the bias-resistance will be given by

$$R_1 = E_g/i_{a0}$$

The steady anode-voltage will be reduced by an amount, E_g, but this is usually negligible with respect to the anode-voltage.

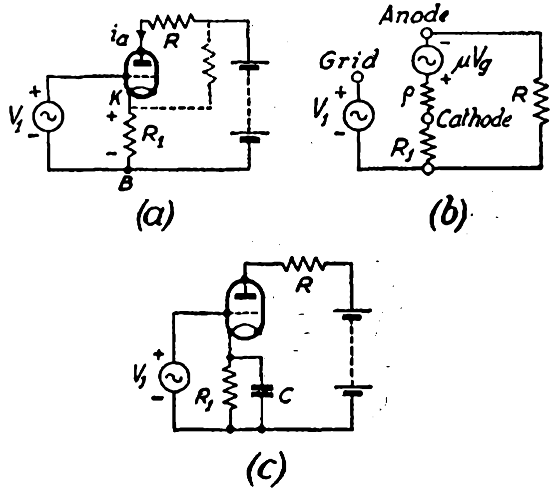


FIG. 46

For some purposes it is necessary to have a grid-bias voltage large enough to cut-off the flow of anode-current. Clearly, this cannot be achieved with this circuit since the grid-bias voltage is produced by the flow of anode-current. Cut-off bias can, however, be provided by connecting an additional resistance (shown dotted) between K and h.t. positive.

In the simple circuit of Fig. 46 (a) the alternating part of the anode-current would also pass through R_1. We shall see that this is undesirable. Fig. 46 (b) shows the equivalent circuit. (Note the directions of the voltages V_1 and μV_g. These must be such that the anode-current, I_a, measured in the conventional direction, shall be in phase with V_1.) The total alternating grid-voltage is no longer simply the input voltage V_1, but is given by

$$V_g = V_1 - R_1 I_a$$

and from Fig. 46 (b)
$$I_a = \frac{\mu V_g}{\rho + R + R_1}$$

giving
$$I_a = \frac{\mu V_1}{\rho + R + R_1(1 + \mu)}$$

$$(VA) = \frac{RI_a}{V_1} = \frac{\mu R}{\rho + R + R_1(1 + \mu)} \quad . \quad . \quad \text{(III.18)}$$

Now if the resistance R_1 were not included in the grid circuit, i.e. if the lower end of the input voltage were connected to K instead of B (Fig. 46 (a)), the voltage across R would be $\dfrac{\mu R \,.\, V_1}{\rho + R + R_1}$, so that the inclusion of R_1 in the grid circuit has reduced the amplification in the ratio $\dfrac{1}{1 + \dfrac{\mu R_1}{\rho + R + R_1}}$.

This will sometimes be an appreciable reduction. It may be avoided by connecting a large condenser across the resistance R_1 (see Fig. 46 (c)). Denoting the impedance vector of R_1 and this condenser in parallel by Z_1, we see that Z_1 must now be substituted for R_1 in equation (III.18).

Since Z_1 tends to zero with increasing frequency (the condenser acting as an a.c. short-circuit across the bias-resistance, R_1) the amplification tends to the value $\mu R/(\rho + R)$ with increasing frequency. On the other hand, as the frequency approaches zero, Z_1 tends to R_1, and the amplification tends to the lower value given by (III.18). The response curve of the stage, which shows amplification plotted against frequency, thus rises from the lower value (equation (III.18)) and asymptotically approaches the value $\mu R/(\rho + R)$ as the frequency is increased. In order to decide what value of bias-capacitance, C, to use in a given case, it is necessary to evaluate the cut-off frequency, defined on a 3 decibel basis, i.e. the frequency at which the voltage amplification is $1/\sqrt{2}$ of the asymptotic value. It should be noted that it is possible for the reduced amplification given in equation (III.18) not to be as small as $1/\sqrt{2}$ times the asymptotic amplification, $\mu R/(\rho + R)$. In this case there is no cut-off frequency, for the amplification does not fall by as much as 3 decibels even at zero frequency. In such a case it may not be thought worth while to include a bias condenser. We shall restrict attention to cases in which there is a substantial loss of amplification unless a bias condenser be used, i.e. cases in which, as seen from equation (III.18), $R_1(1 + \mu) \gg \rho + R$.

The impedance vector Z_1, for R_1 and C in parallel, may be written as $R_1/(1 + j\omega CR_1)$. Using this expression to replace R_1 in equation (III.18) and re-arranging, we have

$$(VA) = \frac{\mu R}{\rho + R} \times \frac{1 + j\omega CR_1}{1 + \dfrac{R_1(1 + \mu)}{\rho + R} + j\omega CR_1}$$

The cut-off frequency is the frequency at which the magnitude of the right-hand quotient becomes equal to $1/\sqrt{2}$. Since we are restricting attention to cases in which $R_1(1 + \mu)$ is appreciably larger than $(\rho + R)$ (the other cases being those in which there is, in any case, only a small reduction of amplification), we can write

$$\frac{1}{\sqrt{2}} = \sqrt{\frac{1 + \omega^2C^2R_1^{\,2}}{\dfrac{R_1^{\,2}(1 + \mu)^2}{(\rho + R)^2} + \omega^2C^2R_1^{\,2}}}$$

which gives, after squaring both sides,

$$\omega^2C^2R_1^{\,2} = \frac{R_1^{\,2}(1 + \mu)^2}{(\rho + R)^2} - 2 \approx \frac{R_1^{\,2}(1 + \mu)^2}{(\rho + R)^2}$$

whence $$\omega \approx \frac{1 + \mu}{C(\rho + R)} \approx \frac{g_m}{C(1 + R/\rho)}$$

This gives $g_m/2\pi C(1 + R/\rho)$ as an approximate value for the cut-off frequency. In the case of an amplifier stage using a pentode, this becomes approximately $g_m/2\pi C$, since R is usually small with respect to ρ for a pentode. It is of interest to note that these expressions for f_c are independent of the bias-resistance R_1, provided only that R_1 is sufficiently large to cause an appreciable reduction of amplification in the absence of a bias condenser. In such cases, they enable us to calculate the required bias-capacitance if the cut-off frequency is specified. For example, if the gain is required to be constant to within 3 decibels down to a cut-off frequency of 20 c/s, in a stage using a pentode whose g_m is 5 mA/v, the minimum permissible value of C is calculated as

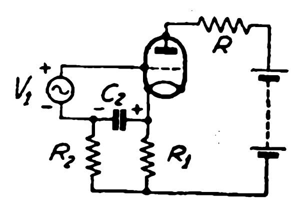


FIG. 47

$$g_m/2\pi f_c = 40 \text{ microfarads.}$$

To provide such large values of capacitance, cheaply and in a reasonably small space, electrolytic condensers must be used. The problem is eased by the fact that the voltage rating of these condensers can be quite small. In some cases, bias condensers can be omitted without causing a very great reduction of amplification. The smaller the anode load resistance, and the larger the mutual conductance, the greater is the need for a bias condenser.

Fig. 47 shows an automatic bias circuit of a different kind. R_2 and C_2 form an A.C./D.C. separating circuit across the bias resistor R_1. The grid-to-cathode voltage is clearly the algebraic sum of the input voltage V_1 and the p.d. across C_2. If the values of R_2 and C_2 are correctly chosen (i.e. if C_2 be made so large that $1/\omega C_2 \ll R_2$ at all relevant frequencies) then the p.d. across C_2 will be pure D.C. and equal to the d.c. component of the p.d. across R_1. Thus the grid-to-cathode voltage is simply the combination of the input voltage and the required d.c. bias-voltage. Note that this circuit cannot be used when there is a requirement that one side of the input voltage shall be earthed (i.e. connected to h.t. negative.) This precludes the use of this bias circuit with *RC* coupled stages. It is useful, however, with transformer coupled stages, the transformer secondary winding taking the place of V_1 in Fig. 47. To make $1/\omega C_2$ small with respect to R_2 it is not necessary to make C_2 very large since R_2 may be made very large.

The design of such an automatic bias circuit may, therefore, be summarized as follows—

$$R_1 = E_g/i_{a0}$$

$$R_2 \gg R_1$$

C_2 to be made sufficiently large for $1/\omega C_2 \ll R_2$ for all frequencies which are to be amplified.

Pentode Amplifier Circuits

Since the control-grid and anode of a pentode serve exactly the same purpose as the control-grid and anode of a triode, the connexions to these two electrodes in a pentode amplifier stage are the same as in a triode amplifier stage. The alternating voltage to be amplified is connected between control-grid and cathode (the "input circuit") together with the necessary grid-bias voltage, while the load impedance is connected in the anode-to-cathode circuit (the "output circuit") in series with the required h.t. voltage. Inter-stage coupling circuits, automatic bias circuits, and anode-decoupling circuits may be used with a pentode just as they are used with a triode.

The screen-grid and suppressor-grid may be regarded as auxiliary electrodes: they play no part in the a.c. circuits of the valve and they do not appear in the a.c. equivalent circuit, which is the same (page 36) as the a.c. equivalent circuit of a triode. The requirement which governs the connexions made to the suppressor-grid is that this grid must be at a sufficiently negative potential with respect to the anode, to turn back secondary electrons which might otherwise go from the anode to the screen-grid. Obviously the most convenient course is to connect the suppressor-grid directly to the cathode. Such a connexion is often provided inside the valve, by the valve manufacturers. The requirements which govern the connexions made to the screen-grid are (*a*) that it shall be at cathode-potential so far as A.C. is concerned (this arises from considerations of screening, page 54), and (*b*) that its steady potential, with respect to cathode, shall be positive and large enough to attract electrons from the cathode so that they may be collected by the anode. This steady potential may be the same as the steady potential of the anode or it may be somewhat lower. The two most common ways of achieving this are (i) to connect a voltage divider, formed of two suitable resistances in series, across the h.t. supply and to connect the screen-grid to the point of junction of the two resistors, or (ii) merely to connect the screen-grid to h.t. positive through a suitably chosen dropping-resistance, R_s (see Fig. 48). The screen-grid current, i_s (which is usually of the order of a quarter of the anode-current, and arises from the fact that the screen collects some of the electrons, though most of them proceed to the anode) flows through this resistance, entering the valve at the screen-grid. The screen-potential is thus made lower than h.t. positive potential by an amount $R_s i_s$.

All these connexions are shown together in the apparently complicated circuit diagram of Fig. 48. The student of electronics will

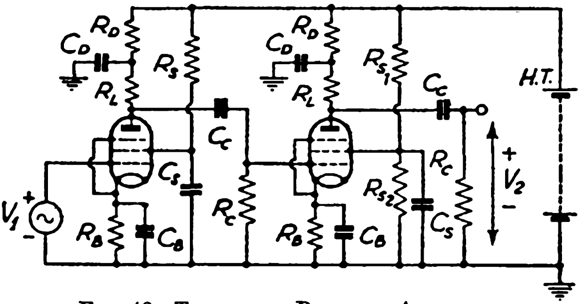


FIG. 48. TWO-STAGE PENTODE AMPLIFIER

be called upon to face many more complicated circuit diagrams than this and it is perhaps not out of place to say a word here about the technique of understanding elaborate circuit diagrams. This technique consists chiefly of the rapid *recognition* of standard circuit elements, e.g. anode-load resistances, interstage couplings, decoupling circuits, automatic bias-circuits, etc., all of which feature in Fig. 48. (Fig. 48 also includes one item which has not yet been explained, but which is dealt with in the next paragraph.)

The alternating voltage in the control-grid circuit will introduce an alternating component of screen-grid current, in just the same way as it introduces an alternating component of anode-current. The flow of this alternating component of screen-grid current through R_s, or through the resistors forming the voltage divider if method (i) be used, will cause the screen-grid voltage, like the anode-voltage, to have an alternating component, since v_s is equal to $E_{H.T.} - R_s i_s$. The minus sign in this expression makes it clear that the alternating component of screen-grid voltage is 180 degrees out of phase with i_s and therefore with the alternating voltage on the control grid. Thus the anode-current is subject to the control of two grids, and the voltages on these grids are in antiphase. The screen-grid, being further away from the cathode than is the control-grid, is less able to influence the anode-current, but nevertheless does influence it, and the result is a reduction of the amplification. The remedy is to connect, between screen-grid and cathode (or between screen-grid and h.t. negative) a capacitance large enough to have a very low reactance at all relevant frequencies, so that the screen-grid is virtually short-circuited to cathode, as far as A.C. is concerned. (See Fig. 48.) An alternative outlook on this remedy is to regard the screen-to-cathode condenser as a low impedance by-pass, which avoids the flow of A.C. through R_s. For this reason, such condensers are called "Screen by-pass condensers."

A reasonably good estimate of the reduction of amplification, when no by-pass condenser is used, may be made by writing the alternating component of screen-grid voltage as

$$-\mu_s R_s V_1/(\rho_s + R_s)$$

This expression is derived by considering the cathode, control-grid and screen-grid as constituting a triode whose parameters are μ_s and ρ_s. (The value of ρ_s will be only approximately equal to $\partial v_s/\partial i_s$, but the approximation will be a fairly close one if the load in the anode circuit is small compared to the valve-impedance, ρ, as is usually the case with a pentode.) The parameter μ_s expresses the relative ability of the control-grid and the screen-grid to influence the screen-current, and therefore also their relative ability to influence the anode-current. We may therefore divide the alternating component of screen-grid voltage by μ_s, and say of the resulting voltage, $-V_1R_s/(\rho_s + R_s)$, that it gives the effective reduction of control-grid alternating voltage. The total effective alternating voltage between control-grid and cathode is thus reduced from V_1 to $V_1 - V_1R_s/(\rho_s + R_s)$, i.e. from V_1 to $V_1/(1 + R_s/\rho_s)$. The voltage-amplification or "gain," of the pentode stage will thus be reduced by a factor $1 + R_s/\rho_s$. In some cases this is an appreciable reduction.

A by-pass capacitance, C, connected between screen-grid and cathode, appears in parallel with R_s, so far as A.C. is concerned. Thus, in the expression for the gain reduction factor, $1 + R_s/\rho_s$, we must replace R_s by $R_s/(1 + j\omega CR_s)$, giving

$$\text{gain reduction factor} = \frac{1 + R_s/\rho_s + j\omega CR_s}{1 + j\omega CR_s}$$

If we restrict attention to cases where, in the absence of a by-pass condenser, there would be a substantial reduction of gain, i.e. to cases where $R_s/\rho_s \gg 1$, we may write

$$\text{gain reduction factor} = \sqrt{\frac{(R_s/\rho_s)^2 + \omega^2C^2R_s^2}{1 + \omega^2C^2R_s^2}}$$

Defining the cut-off frequency as the frequency at which the gain reduction factor is $\sqrt{2}$ (corresponding to 3 decibels), we may equate the above expression to $\sqrt{2}$, giving

$$\omega^2 C^2 R_s^2 = (R_s/\rho_s)^2 - 2 \approx (R_s/\rho_s)^2$$

whence the cut-off frequency, $\omega/2\pi$, is given by $1/(2\pi C_s \rho_s)$. This approximate relationship enables us to decide upon a suitable value of by-pass capacitance, in terms of the desired cut-off frequency and the value of ρ_s, in cases where the gain would be substantially reduced if no by-pass condenser were used. An approximate value of ρ_s is given by $(n\rho_T)$, where n is the ratio of d.c. anode-current to d.c. screen-grid current, and ρ_T is the impedance of the same valve when used as a triode by joining the anode to the screen-grid.

The behaviour of an amplifier circuit using a pentode is determined by the same equations as for a triode circuit, but it must be remembered that the values of ρ and μ are very much higher for the pentode, being commonly as high as 500,000 ohms and 1,000 respectively, while the mutual conductance, g_m, is of the same order as for a triode. Thus the load impedance, Z, is usually small with respect to ρ, and equation (III.5) becomes approximately

$$I_a = \mu V_g/\rho = g_m V_g$$

As this is independent of Z, the pentode is equivalent simply to a generator of a constant A.C., $g_m V_g$, and the voltage amplification is $g_m Z$, where Z is the impedance in the anode circuit. The mutual conductance is the significant parameter in the case of pentode valves, and valve manufacturers often do not quote values of the other parameters.

In view of the very high amplification factor, pentodes are much more commonly used than triodes in voltage amplifier stages, and are also much used in output stages. The estimation of the optimum load impedance for maximum undistorted output power may be carried out graphically just as for a triode, but the values of optimum load impedance are greater in the pentode case, and thus different output transformer ratios must be used. Owing to the difference in the shape of the pentode and triode-characteristics, the largest triangle AEF (Fig. 20, p. 28) is secured by using a fairly steeply sloping load-line in the case of a pentode, and hence the optimum load impedance for maximum output power is much smaller than the valve impedance. Also the harmonic distortion in the two cases differs in the relative proportions of the various order harmonics, and in general the pentode is considered to produce greater harmonic distortion. It will be seen in the next chapter that pentodes have very considerable advantages over triodes in the matter of amplifying radio- and higher audio-frequencies. As the pentode has all the advantages of the screen-grid valve without its obvious disadvantage (the irregular nature of the characteristic curves), screen-grid valves are comparatively little used. Screen-grid valves have been manufactured, however, having pentode-like characteristics, the valve being designed in such a way that a concentration of electrons near the anode acts as does the suppressor-grid of a pentode. These valves are known as "Beam Tetrodes."

One important precaution must be mentioned in connexion with transformer-coupled pentode amplifier stages. If the transformer secondary becomes open-circuited (either as a result of faulty connexions or in the process of switching) while the full input voltage, V_1, is being applied, the load impedance in the anode circuit will become very large, being simply the primary reactance of the transformer. Equation (III.6), or alternatively the load-line diagram, shows that under these conditions the voltage amplification will become very large, and the resulting high voltages set up across the valve will be sufficient to damage it.

The reader may ask why, if the above be true, the pentode is not normally used with a very large load impedance in order to secure large amplification. The answer is that for small input voltages the pentode is so used, and as such is extremely valuable; for large inputs, however, the curvature of the characteristics clearly limits the variation of anode-current during the positive half-cycle of the input voltage, and the use of a very large load impedance would introduce severe distortion. This may be seen by drawing load lines for various impedances.

Push-pull Input Circuits

We have seen that a push-pull stage needs two input voltages, equal in magnitude and differing in phase by 180°. The only method shown hitherto for deriving these two input voltages from a single input voltage is the transformer with centre-tapped secondary shown in Fig. 41 (*b*). It is sometimes important to avoid the use of transformers for this purpose, either because they would introduce excessive distortion or because the required frequency range is such as could not be handled by a transformer. Various other circuits, known as Push-pull Input Circuits, are then used to derive the two necessary input voltages.

At first sight it would seem that this can be done simply by connecting a resistance (AB) across the single voltage to be amplified with a tapping (C) at the mid-point of the resistance. The two voltages, AC and BC, would then provide the two required input voltages, the tapped resistance taking the place of the tapped secondary in Fig. 41 (*b*). This simple method is usually not permissible, however, since one side of the voltage to be amplified is usually connected to the common negative (see "Earthing and Screening," page 54). This would certainly be the case if the voltage to be amplified were derived from a previous stage of the same multi-stage amplifier of which the push-pull stage forms part. The problem is thus to derive two voltages "balanced with respect to earth" from a single voltage which is "unbalanced with respect to earth."

Two circuits for doing this are shown in Fig. 49 (see also "The Seesaw Circuit," page 123). Fig. 49 (*a*) is simply a normal two-stage *R*–*C* coupled amplifier, the requisite output voltages being the alternating components of the two anode-voltages. If the a.c./d.c. separating circuit, R_2C_2, is properly designed (viz. if R_2 is large with

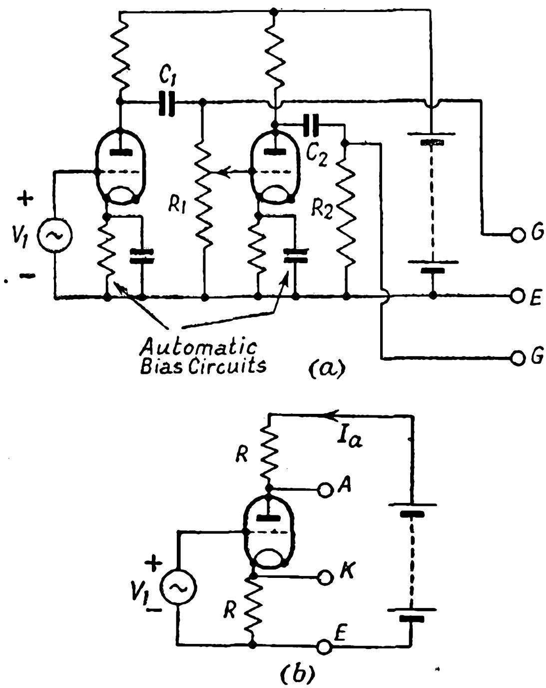


FIG. 49. PHASE INVERTERS
(Push-pull input circuits)

respect to $1/\omega C_2$ for all relevant values of ω), the phase change in the second stage is almost exactly 180°, so that $V_{G'E}$ and $V_{G''E}$ are 180° out of phase as required. These two voltages are also required to be equal in magnitude. We see that $V_{G''E}$ is equal to $n(VA)_2V_{G'E}$, where n (< 1) is the ratio of the voltage divider R_1 and $(VA)_2$ is the gain of the second stage. It follows that the tapping point on R_1 must be so chosen as to make $n(VA)_2$ equal to unity, i.e. the gain of the second stage is to be exactly cancelled out by the voltage-division in R_1. The second stage is, in fact, simply used as a phase inverter.

Fig. 49 (*b*) is drawn to show the principle of another push-pull input circuit, the complete circuit being shown in Fig. 50. The two balanced voltages are taken as the alternating components of v_{AE} and v_{KE} (Fig. 49 (*b*)), and it is clear that they are given by $-RI_a$ and $+RI_a$ respectively, being thus equal in magnitude and 180° out of phase. The equivalent circuit and the expression for the voltage amplification are as in Fig. 46 (*b*) and equation (III.18)

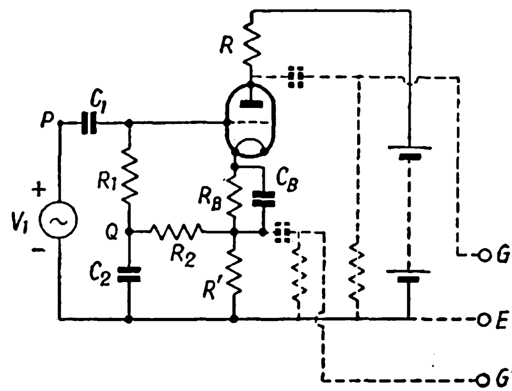


FIG. 50

respectively, but with R substituted for R_1. The gain may be written as follows

$$(VA) = 1\bigg/\left(1 + \frac{2}{\mu} + \frac{1}{g_m R}\right)$$

from which it is seen that the gain is less than unity, but may be made to approach unity by using a valve having a μ large with respect to 2, and by making R large with respect to $1/g_m$. This means that the load resistance R can be made considerably smaller than in the case of an ordinary amplifier stage; and so also can the h.t. supply voltage.

The p.d. across the cathode-resistance (v_{KE}, Fig. 49 (*b*)) is of the correct polarity to provide the grid-bias voltage, but will usually be found to be much too large. Accordingly two separate resistances, R_B and R' (Fig. 50) are connected in series in the cathode lead and an ingenious arrangement of a.c./d.c. separating circuits may be used, so that only the d.c. component of v_{RB} and only the a.c. component of $V_{R'}$ are included between grid and cathode. The value of R_B is chosen so that $R_B i_a$ gives the required grid-bias. R_B is shunted by a capacitance C_B sufficiently large to constitute a short circuit to A.C., so that the only p.d. across R_B is the direct voltage required for grid-bias. R_2 and C_2 constitute an a.c./d.c. separating circuit connected across R', and hence the p.d. across C_2 is the direct component of the p.d. across R'. Finally, an a.c./d.c. separating circuit R_1C_1 is connected between the points P and Q. The p.d. between these two points has a direct component (that across C_2) and an alternating component (viz. V_1). Thus the p.d. across R_1 is simply equal to V_1. We are now in a position to see what are the direct and alternating components of v_g, the grid-to-cathode voltage, for we have evaluated v_{R_B}, v_{R_2}, and v_{R_1}, and the grid-to-cathode voltage is the sum of these three. In this way we see that the direct component of v_g is simply the grid-bias voltage developed across R_B, while the alternating component of v_g is $V_1 - R'I_a$, just as for the simple circuit of Fig. 49 (*b*).

Observe that the above argument proceeds by determining the behaviour of part of the circuit in the absence of the rest, then adding a further part and determining its behaviour, and then yet another part, and so on, until the complete circuit is explained. A warning must be given against "cascaded arguments" of this kind. The conclusions reached by such an argument remain valid only if it can be shown that the behaviour of the part first considered remains the same after the addition of the subsequent parts as it was in isolation. This is ensured if the impedance of each added part is very large with respect to that of the parts to which it is added. For the circuit here considered this means that we must make $R_1 \gg R_2 \gg R'$. To avoid having to use an unduly large value of R_1, one may make $R_1 \gg R_2$, but make R_2 of the same order as R'. The effect of this can be seen by drawing the equivalent circuit, all condensers being therein considered to be short-circuits. It will be found that R_2 then appears in parallel with R'. Thus, to ensure that the two output voltages are equal in magnitude, we must make R equal to the parallel combination of R' and R_2. The design rules for this circuit may thus be summarized as follows—

$$R'R_2/(R' + R_2) = R \gg 1/g_m$$

$$1/\omega C_1 \ll R_1 ;\ R_1 \gg R_2 ;\ 1/\omega C_2 \ll R_2$$

$$R_B = \text{(Desired g.b. voltage)} \div \text{(mean } i_a)$$

The two R–C coupling circuits shown dotted in Fig. 49 should be identical. An alternative biasing circuit is shown in Fig. 83 (*b*).

Cathode Coupling

Fig. 51 (*a*) shows two valve-amplifier stages using a common bias resistance, R, but with no other connexion between the two stages (apart from the use of a common h.t. supply voltage). At first sight it may appear that this circuit makes no provision for taking an output voltage from stage 1 and feeding it to stage 2 for further amplification. Since, however, no bias condenser is used across R_1 it follows that the a.c. component of the anode-current of stage 1 flows through R_1 and that therefore the p.d. across R_1 will have an alternating component. Because R_1 is connected directly between grid and cathode of valve 2, the alternating component of this p.d. appears in the grid circuit of valve 2. The circuit does, therefore, constitute a form of interstage coupling and it will be noted that this form of coupling is suitable for d.c. amplifiers. Positive grid-bias voltages (not shown in Fig. 51) would normally be used in both grid circuits to counteract part of the large negative bias voltage developed across R_1 (see also Fig. 52).

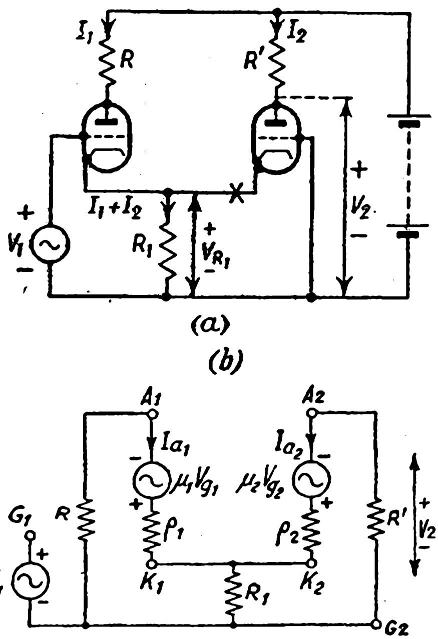


FIG. 51. CATHODE COUPLING

To investigate the properties of this circuit let us first suppose the circuit to be broken at the point X. The first stage is now similar to Fig. 46 (a), p. 69; the alternating component of the potential of the cathode with respect to h.t. negative is given by

$$V_{R_1} = \frac{\mu_1 R_1}{\rho_1 + R + R_1(1 + \mu_1)} \cdot V_1 \quad . \quad . \quad \text{(III.19)}$$

where ρ_1 and μ_1 are the parameters of the first valve. This p.d. (which we regard as the output voltage of stage 1) is in phase with V_1 and is smaller than V_1. The first stage thus gives no gain.

By restoring the connexion at the point X (Fig. 51 (a)) we now feed the output voltage of stage 1 to the grid circuit of stage 2. Until this connexion is restored, stage 2 is passive in the sense that it incorporates no alternating e.m.f.'s and passes no alternating current. The connexion of such a passive network across R_1 will effectively connect an impedance in shunt across R_1, thereby reducing V_{R_1}, but cannot otherwise affect the performance of stage 1. In particular, it cannot alter the phase of V_{R_1}. The grid-cathode voltage of stage 2 is equal to $- V_{R_1}$ (V_{R_1} being defined as in Fig. 51 (a)) and is thus 180° out of phase with V_1 and I_{a1}. It follows that I_{a2} is also 180° out of phase with I_{a1} and that *the overall* *phase-shift of the two-stage cathode-coupled amplifier is zero.* It also follows that, whatever the values of the circuit components, I_{a2} must be of smaller amplitude than I_{a1}, for the current through R_1 is the sum of these two antiphase currents and yet we have seen that the voltage across R_1 is in phase with I_{a_1}.

To derive an expression for the overall gain of the circuit we draw the equivalent a.c. circuit, as in Fig. 51 (*b*). In the analysis of this and other similar equivalent circuits, it is necessary to write down an expression for the grid-to-cathode voltages (in this case V_{g_1} and V_{g_2}) and this is facilitated by labelling the valve electrodes (A_1, G_1, K_1; A_2, G_2, K_2) on the equivalent circuit diagram. Thus, for the equivalent circuit of Fig. 51 (*b*) we have

$$V_{g_1} = V_{G_1K_1} = V_1 - R_1(I_{a_1} + I_{a_2})$$
$$V_{g_2} = V_{G_2K_2} = - R_1(I_{a_1} + I_{a_2})$$

The application of Kirchhoff's second law to the two closed meshes gives

$$\mu_1 V_{g_1} = I_{a_1}(\rho_1 + R_1 + R) + I_{a_2}R_1$$
$$\mu_2 V_{g_2} = I_{a_1}R_1 + I_{a_2}(\rho_2 + R_1 + R')$$

Substituting the above values for V_{g_1} and V_{g_2} into these last two equations, we have

$$\mu_1 V_1 = I_{a_1}[\rho_1 + R + R_1(1 + \mu_1)] + I_{a_2}R_1(1 + \mu_1)$$
$$0 = I_{a_1}R_1(1 + \mu_2) + I_{a_2}[\rho_2 + R' + R_1(1 + \mu_2)]$$

These equations can now be solved for I_{a_2}, for the purpose of evaluating the output-voltage V_2, which is equal to $-R'I_{a_2}$. This gives

$$(VA) = \frac{V_2}{V_1}$$
$$= \frac{\mu_1 R_1(1 + \mu_2)R'}{R_1(1 + \mu_2)(\rho_1 + R) + (\rho_2 + R')\{\rho_1 + R + R_1(1 + \mu_1)\}} \quad . \quad . \quad . \text{(III.20)}$$

This may be written as

$$(VA) = \frac{A_1A_2}{1 + kA_1} \quad . \quad . \quad . \quad . \quad . \text{(III.21)}$$

where

$$\left.\begin{aligned} A_1 &= \frac{\mu_1 R_1}{\rho_1 + R + R_1(1 + \mu_1)} \\ A_2 &= \frac{(1 + \mu_2)R'}{\rho_2 + R'} \\ k &= \frac{\rho_1 + R}{\rho_2 + R'} \cdot \frac{\mu_2 + 1}{\mu_1} \end{aligned}\right\} \quad . \quad . \quad . \text{(III.22)}$$

It will be seen that A_1 is less than unity, but that with large values of μ_1 it will approach unity provided that $R_1(1 + \mu_1)$ is made large with respect to $\rho_1 + R$. If pentodes are used, ρ_1 and μ_1 will be so large that this condition becomes $R_1 \gg \rho_1/\mu_1$, i.e. $R_1 \gg 1/g_{m_1}$. The value of A_1 is increased (only very slightly if pentodes are used) by making R equal to zero, but particular applications of this circuit retain this resistance for special purposes. The expression A_2 is similar to the expression for the gain of a basic amplifier, and R' should therefore be made as large as possible subject to operating on the straight parts of the characteristics. Usually $k < 1$.

The circuit of Fig. 51 (*a*), which is sometimes named the "Long-tailed Pair," may be modified for use as a *Difference-amplifier*, i.e. an

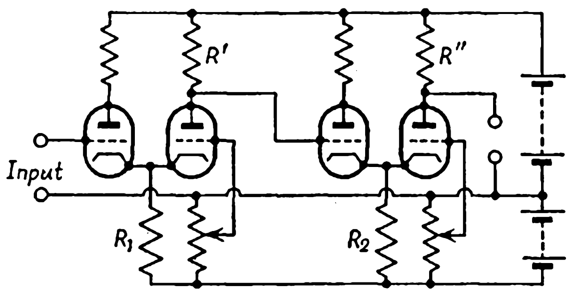


FIG. 52. LONG-TAILED PAIRS IN A D.C. AMPLIFIER

amplifier having two independent input voltages and whose output voltage is required to be an amplified version of the difference between these two input voltages. For this purpose, the connexion from the right-hand grid to h.t. negative is broken and the second input voltage, V_1', is inserted at the break. The output voltage, V_2, is taken as in Fig. 51 (*a*). If we assume that the circuit is acting in a linear manner (i.e. without distortion) we may say that the total output voltage is the sum of those which would be produced by V_1 acting alone and by V_1' acting alone. We have seen that the former is in phase with V_1 and it is easy to see that the latter is in antiphase with V_1'. Thus the two outputs subtract as required, and V_2 is equal to $(VA).V_1 - (VA)'V_1'$, where (VA) and $(VA)'$ denote respectively the magnitudes of the gain with V_1 acting alone and with V_1' acting alone.

For the circuit to be a difference-amplifier, we require that (VA) and $(VA)'$ shall be equal, in order that V_2 may be given by $(VA)\ (V_1 - V_1')$. In the circuit as it stands, (VA) and $(VA)'$ are not equal. They may be made equal by using identical valves, reducing R to zero, and making R_1 large with respect to $1/g_m$ (so that $R_1(1 + \mu) \gg \rho$). Equations (III.21) and (III.22) then show that (VA) has the value $\mu R'/(2\rho + R')$, and it may be shown that $(VA)'$ has the same value.

Apart from their use as comparators, difference-amplifiers have the great merit of being able to amplify the potential difference between two points (say, A and B) *neither of which is at earth potential.* In using the long-tailed pair for this purpose, one would connect A to the grid of one valve and B to the grid of the other. The output voltage would then be $(VA)(V_{AE} - V_{BE})$ which is the same as $(VA).V_{AB}$.

Fig. 52 shows how two cathode-coupled pairs may be cascaded in a multistage d.c. amplifier. The values of the resistances in the second pair (valves 3 and 4) are chosen so as to make the potential of the cathodes slightly positive with respect to the anode of valve 2, so permitting a direct connexion between this anode and the grid of valve 3.

Design of Amplifiers

In general the design of amplifiers starts from a specification of the output power (or in some cases, output voltage) required from a given input voltage, with a given limit of permissible distortion. The output stage is designed first, the major part of the design procedure being the choice of a valve which will give a sufficient output with the stated limit of distortion. This is carried out as follows—

1. Select a valve.
2. Using the load-line construction, and trying various values of load and grid-bias voltage, determine the maximum output power subject to the given limit of distortion. The maximum permissible value of steady anode-voltage will in general be stated by the valve manufacturers.

Having found a suitable valve, and determined the values of grid-bias voltage, input voltage, and load, the circuit of the output stage may be designed. The design of the stage before the output stage may now be carried out in a similar manner, assuming an output voltage equal to the input voltage required for the output stage. We then proceed to the design of the preceding stage—and so on.

Suggestions for further reading

1. F. Langford Smith (Editor), *Radio Designer's Handbook* (Iliffe).
2. Valley and Wallman, "Vacuum Tube Amplifiers," *Radiation Laboratory Series*, vol. 19.
3. F. E. Terman, *Electronic and Radio Engineering* (McGraw-Hill).

CHAPTER IV

AMPLIFIERS (*continued*)

AMPLIFIERS which are required to handle only a single frequency, or a narrow band of frequencies, are of considerable importance in Communications Engineering. The parallel *L–C* circuit (i.e. a coil of inductance, *L*, in parallel with a condenser of capacitance, *C*) is used extensively in such amplifiers, since at the resonant frequency it has a very high impedance, suitable for the load of a voltage-amplifying valve. Thus, in such amplifiers the *L–C* circuit is designed to resonate at the frequency to be amplified. By making the capacitance variable (by the use of a variable condenser) the *L–C* circuit may be "tuned" to any given frequency, i.e. its resonance frequency may be varied until it is equal to the particular frequency to be amplified.

Tuned Amplifiers

The single frequency to be amplified is usually a radio-frequency. This means that the product, *LC*, is small, and a suitable circuit is easily designed using quite small condensers and coils. Not only does the *L–C* circuit provide a very convenient form of high impedance load at radio-frequencies, but it provides almost the only available form of high impedance load, since a simple high resistance, as used in audio-frequency amplifiers, is shunted by its own self-capacitance (the small capacitance between its ends). This means that at high frequencies the resistance is shunted by a low capacitive reactance, and its resultant impedance is much lower than at audio-frequencies.

Single frequency amplifiers are usually required not only to amplify voltages of the given frequency, but also to avoid amplifying voltages of neighbouring frequencies. An amplifier which gives a finite voltage amplification at a single frequency, but zero amplification at all other frequencies, would be said to have infinite "Selectivity," whereas an amplifier whose amplification is the same at all frequencies would be said to have zero selectivity. Before defining quantitatively the selectivity of an amplifier, we must consider the properties of the parallel *L–C* circuit.

The Parallel L–C Circuit

The impedance of a circuit consisting of a capacitance, *C*, in parallel with a coil of resistance, *R*, and inductance, *L*, is given by

$$Z = \frac{(R + j\omega L)\dfrac{1}{j\omega C}}{R + j\omega L + 1/j\omega C}$$

Let us restrict attention to circuits in which the coil resistance, *R*, though not zero, is very small. As we shall see later, this is a restriction to which practical circuits, used in frequency-selective amplifiers, must conform. At one particular value of frequency ωL will be equal to $1/\omega C$. We shall denote this frequency by f_0 and shall use ω_0 to denote the corresponding value of ω, viz. $2\pi f_0$. At this frequency the term $(\omega L - 1/\omega C)$ in the denominator of the above expression will be zero, leaving the denominator equal only to R, which is very small. At this frequency, therefore, the impedance Z is very large. This frequency, f_0, is known as the resonance frequency, and is given by

$$\omega_0^2 = 1/LC \quad \text{or} \quad f_0 = 1/2\pi\sqrt{LC} \quad . \quad . \quad \text{(IV.1)}$$

There is a convenient and well-known approximation which we can adopt if we further restrict attention to a small range of frequencies centred on the resonance frequency. Within such a range of frequencies, and for low resistance coils such as we are considering, R will be very small compared to the coil reactance, ωL. We may therefore neglect R in the numerator of the above expression for Z, *but we may not neglect it in the denominator where it is added to a quantity which vanishes at one particular frequency.* This gives the well-known result

$$Z = \frac{L/C}{R + j(\omega L - 1/\omega C)} \quad . \quad . \quad . \quad \text{(IV.2)}$$

At the resonance frequency, for which ωL is equal to $1/\omega C$, we see that $Z = L/CR$, and the absence of j from this expression for Z denotes that Z is a pure resistance at this frequency. This quantity, L/CR, is known as the Dynamic Resistance of the tuned circuit. It is the maximum value attained by Z at any frequency.

The numerical value of the impedance Z, at any frequency within the range considered, is given by

$$|Z| = (L/C)/\{R^2 + (\omega L - 1/\omega C)^2\}^{\frac{1}{2}} \quad . \quad . \quad \text{(IV.3)}$$

Fig. 53 shows this quantity plotted against frequency for two typical cases. The selectivity of the circuit is the quantity which expresses its ability to present a high impedance at the resonance frequency and a low impedance at all other frequencies. Thus the greater the sharpness of the curve, the greater will be the selectivity. We therefore require some quantitative way of expressing the sharpness of the curve. A convenient measure of the sharpness could be given by quoting the width (AB, Fig. 53) at a stated distance down the curve, expressing the width as a fraction of the resonance frequency. The greater this width, the less sharp the curve, and the smaller the selectivity; thus the selectivity would be better defined as the *reciprocal* of this width, i.e. the reciprocal of the band width at a stated distance down the curve, the band width being expressed as a fraction of the resonant frequency. With reference to the stated distance down the curve, it is found convenient to measure the bandwidth at an impedance $1/\sqrt{2}$ of the maximum impedance. Thus

$$\text{Selectivity, } Q = \frac{f_0}{2\delta f} = \frac{\omega_0}{2\delta\omega} \quad . \quad . \quad \text{(IV.4)}$$

where $2\delta f$ is the bandwidth, i.e. $\delta\omega$ is the increase or decrease of ω required to reduce the impedance from Z_{max} to $Z_{max}/\sqrt{2}$. In other words, we are defining $\delta\omega$ so that the value of ω at point A of Fig. 53 is $\omega_0 - \delta\omega$ and the value of ω at point B is $\omega_0 + \delta\omega$.

This is the selectivity of the parallel L–C circuit as such. The selectivity of a complete amplifier circuit is defined as the ratio

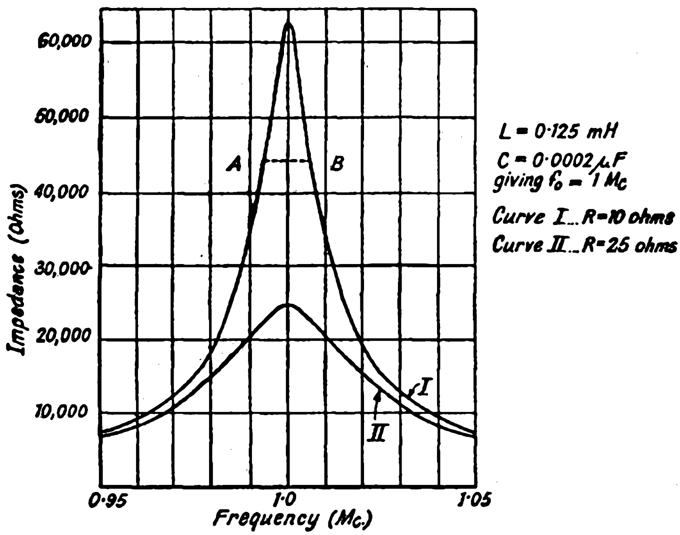


FIG. 53

$\omega_0/2\delta\omega$ where $\delta\omega$ is the change of ω required to reduce the amplification from $|(VA)|_{max}$ to $|(VA)|_{max}/\sqrt{2}$. In view of the use of the factor $1/\sqrt{2}$, the range of frequency corresponding to $2\delta\omega$ is often called "the 3 decibel bandwidth" of the amplifier, and we may write—

$$\text{3 dB bandwidth} = \frac{f_0}{Q\text{-value of amplifier}} \quad . \quad \text{(IV.5)}$$

The selectivity of the parallel L–C circuit is usually denoted by the letter Q, and may be expressed simply in terms of L, C, and R. For this purpose, it is necessary to evaluate $\delta\omega$ in terms of L, C, and R in order to substitute for $\delta\omega$ in equation (IV.4). In other words, we must first evaluate the frequencies at which $|Z|$ is equal to $Z_{max}/\sqrt{2}$, this being the value which $|Z|$ has at points A and B in Fig. 53. Referring to equation (IV.3) we see that, at a frequency which makes $(\omega L - 1/\omega C)$ equal to $\pm R$, the numerical value of the impedance will become $(L/C)/\sqrt{2R^2}$, which is $Z_{max}/\sqrt{2}$. There will be one frequency for which $(\omega L - 1/\omega C) = - R$ and another frequency for which $(\omega L - 1/\omega C) = + R$, corresponding to points A and B in Fig. 53. Writing $\omega_0 + \delta\omega$ in place of ω we thus have

$$(\omega_0 + \delta\omega)L - \frac{1}{(\omega_0 + \delta\omega)C} = \pm R$$

$$\omega_0 L(1 + \delta\omega/\omega_0) - (1/\omega_0 C)(1 + \delta\omega/\omega_0)^{-1} = \pm R$$

Since we have restricted our attention to a narrow band of frequencies centred on the resonance frequency, $\delta\omega/\omega_0$ will be small compared to 1, and we may approximate by writing $(1 - \delta\omega/\omega_0)$ in place of $(1 + \delta\omega/\omega_0)^{-1}$. Hence

$$\omega_0 L(1 + \delta\omega/\omega_0) - (1/\omega_0 C)(1 - \delta\omega/\omega_0) = \pm R$$

Now ω_0 is the value of ω at the resonance frequency, i.e. the value of ω for which $\omega L = 1/\omega C$. Thus we may write $\omega_0 L$ in place of $1/\omega_0 C$ in the above equation. Then, solving for $\delta\omega$, we have

$$\delta\omega = \pm R/2L \quad . \quad . \quad . \quad (IV.6)$$

This means that, at point A in Fig. 53, ω has the value $(\omega_0 - R/2L)$ and at point B, ω has the value $(\omega_0 + R/2L)$. The range of ω from A to B, which we have denoted by $2\delta\omega$, is thus equal to R/L. Substitution of R/L for $2\delta\omega$ in equation (IV.4) gives

$$\text{Selectivity, } Q = L\omega_0/R \quad . \quad . \quad . \quad (IV.7)$$

Since $L\omega_0$ is equal to $1/\omega_0 C$, an alternative expression for Q is $1/\omega_0 CR$. Since ω_0 is equal to $1/\sqrt{LC}$, a further alternative expression for Q is $\sqrt{L/(CR^2)}$. Remembering that Z_{max} is L/CR, it is easy to show that

$$R : L\omega_0 : Z_{max} :: 1 : Q : Q^2 \quad . \quad . \quad (IV.8)$$

Q-values of 100 are often attained in practice.

Tuned-anode Amplifier

This is the name given to the circuit shown in Fig. 54, in which the load of the valve is a parallel L–C circuit tuned to the frequency to be amplified. The voltage amplification is given by

$$(VA) = \frac{\mu Z}{\rho + Z} \quad . \quad . \quad . \quad . \quad (IV.9)$$

and at the frequency to be amplified this will give

$$(VA)_{max} = \frac{\mu \frac{L}{RC}}{\rho + \frac{L}{RC}} \quad . \quad . \quad . \quad . \quad (IV.10)$$

L and C must be so chosen that if f is the frequency to be amplified

$$f = \frac{1}{2\pi\sqrt{LC}}$$

It is usual to provide a small variable condenser, known as a Trimmer, in parallel with C, so that an accurate adjustment may be made experimentally.

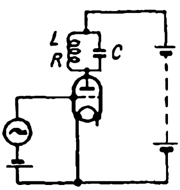


FIG. 54

Clearly for large amplification R should be made small; also with a given value of R, the amplification will be greater the greater the ratio L/C. We must also consider the effect of these quantities on the selectivity.

The selectivity of the amplifier is given by

$$Q_1 = \frac{\omega_0}{2\delta\omega}$$

where $\delta\omega$ is the increase or decrease of ω required to reduce the numerical value of the voltage amplication to $1/\sqrt{2}$ of its maximum value.

The general expression for the voltage amplification at any frequency is, from equations (IV.2) and (IV.9)

$$(VA) = \frac{\mu L/C\rho}{R + L/C\rho + j(\omega L - 1/\omega C)}$$

whence

$$|(VA)| = \frac{\mu L/C\rho}{\sqrt{(R + L/C\rho)^2 + (\omega L - 1/\omega C)^2}}$$

This expression has its maximum value at the resonance frequency, and in this case the second of the two added terms under the square root sign disappears. If now ω be increased to $(\omega_0 + \delta\omega)$, $\delta\omega$ being as defined above, the voltage amplification will have been reduced to $1/\sqrt{2}$ of its maximum value, i.e. the second of the two added terms under the square root sign will no longer be zero but will have increased until it is equal to the first term

i.e. $$R + L/C\rho = (\omega_0 + \delta\omega)L - (\omega_0 + \delta\omega)C$$

Proceeding exactly as on page 87, the right-hand side of this equation can be shown to be simply $2L\delta\omega$.

Thus
$$Q_1 = \frac{\omega_0}{2\delta\omega} = \frac{\omega_0}{R/L + 1/C\rho}$$
$$= \frac{Q}{1 + Z_{max}/\rho} \quad . \quad . \quad . \text{(IV.11)}$$

where Q is the selectivity of the L–C circuit alone. This shows that for a given tuned circuit the selectivity of the amplifier increases with ρ, tending to equal the selectivity of the tuned circuit as ρ becomes large with respect to the dynamic resistance of the latter. Thus for high selectivity we require ρ large *w.r.t.* Z_{max}, whereas equation (IV.10) shows that for high amplification we require Z_{max} large *w.r.t.* ρ. Where both high selectivity and high amplification are required it is usual to use a pentode, so that considerable amplification can be secured even though ρ is larger than Z_{max}.

Amplifiers employing tuned circuits very frequently have an input consisting of a number of voltages of different frequency, of which only one is to be amplified. In such cases the condenser of the L–C circuit is made variable, and is adjusted until the resonance frequency of the L–C circuit is equal to the frequency of the voltage to be amplified. The voltage amplification is given by equation (IV.10) and since this is a function of the capacitance, C, the other quantities remaining constant, it follows that the voltage amplification will vary with the frequency to be amplified, being greater the higher the frequency. Circuits are designed to correct for this.

Tuned Transformer-coupled Amplifiers

The output of a tuned-anode amplifier stage, as described above, may be fed into the grid circuit of the next stage by using resistance-capacitance coupling, the circuit then being similar to that shown in Fig. 29 (*b*), but with the load resistance replaced by a parallel L–C circuit. The self-capacitance of the resistor in the grid circuit then merely shunts the tuned circuit. This alters the tuning, but does not reduce the load impedance. The only disadvantage of this shunt capacitance is that it increases the minimum attainable tuning capacitance, and thus lowers the maximum frequency to which the L–C circuit may be tuned. At higher frequencies other methods must be used. Of these, tuned transformer-coupling, as illustrated in Fig. 55, is the most common. The valve shown in Fig. 55 is a pentode; its screen-grid is connected to a tapping on the high-tension battery, and also through a large capacitance to earth. The parallel L–C circuit is here formed of the secondary winding of the transformer, L_2, R_2, and the condenser C, and is arranged to resonate at the frequency to be amplified, i.e. L_2 and C are chosen so that—

$$f = \frac{1}{2\pi\sqrt{L_2C}}$$

To calculate the voltage amplification we must first determine the impedance of the load effectively presented to the valve. The transformer primary has impedance $R_1 + j\omega L_1$, but owing to the presence of the secondary this will be increased by $\frac{\omega^2 M^2}{Z_2}$, where Z_2 is the impedance of the secondary circuit. Since the secondary

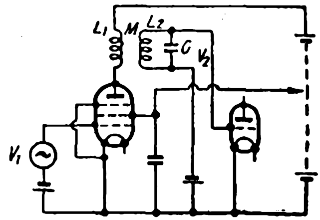


FIG. 55

circuit is arranged to resonate, its impedance will be simply R_2, and the impedance presented to the valve will be

$$R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2}$$

Thus $$I_a = \frac{\mu V_1}{\rho + R_1 + j\omega L_1 + \dfrac{\omega^2 M^2}{R_2}} = \text{approx. } \frac{\mu V_1}{\rho + \dfrac{\omega^2 M^2}{R_2}}$$

The e.m.f. induced into the secondary winding will be $j\omega M \,.\, I_a$ and thus the secondary current will be $j\omega M \,.\, I_a/R_2$

$$\therefore \qquad V_2 = \frac{1}{j\omega C} \cdot \frac{j\omega M \,.\, I_a}{R_2} = \frac{\mu V_1 M}{\rho R_2 C + \omega^2 M^2 C}$$

Writing ω^2 equal to $1/L_2C$, and re-arranging we have

$$(VA) = V_2/V_1 = \mu \cdot \frac{M/L_2}{\rho\Big/\left(\dfrac{L_2}{R_2C}\right) + M^2/L_2^2} \qquad \text{(IV.12)}$$

Now M/L_2 is the effective transformer ratio* from the secondary L_2 to the primary L_1 (being equal to the coefficient of coupling times $\sqrt{L_1/L_2}$), so that, writing n for M/L_2

$$(VA) = \frac{\mu n}{\left(\rho \div \dfrac{L_2}{R_2C}\right) + n^2}$$

The value of n which gives a maximum value of (VA) may be found by writing

$$\frac{d}{dn}\left[\frac{\mu n}{\left(\rho \div \dfrac{L_2}{R_2C}\right) + n^2}\right] = 0$$

* Cf. page 50, line 30.

which gives $$n^2 = \rho \div \frac{L_2}{R_2C}$$

or $$M = \sqrt{\rho R_2 L_2 C}$$

That is, for maximum amplification the transformer ratio should be such that the impedance presented to the valve is equal to ρ. Under these conditions the amplification becomes equal to $\mu/2n$, n having the value given by $n^2 = \rho R_2 C/L_2$. Thus the maximum amplification may be written $\frac{1}{2}(\mu g_m Z')^{\frac{1}{2}}$, where Z' denotes L_2/R_2C. It follows that the valve which will give the highest amplification is the valve which has the highest value of the product μg_m.

The selectivity of this amplifier may be calculated as was the selectivity of the tuned-anode amplifier; it is given by

$$Q_1 = \frac{Q_0}{1 + \dfrac{M^2\omega_0^2}{\rho R_2}}$$

where Q_0 is the selectivity of the L–C circuit, i.e. $\omega_0 L_2/R_2$. Thus, we see that for a given L–C circuit the selectivity increases as M is reduced. In order to improve the selectivity such circuits are frequently operated with a lower value of M than that required to give maximum amplification.

An important application of the above theory is the tapped tuned-anode amplifier shown in Fig. 56. Here the transformer is replaced by an auto-transformer, and the auto-transformer ratio can be varied during operation in such a way as to secure the best compromise between amplification and selectivity.

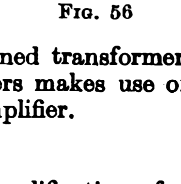


FIG. 56

As stated in Chapter III, the effect of the self-capacitance of the secondary winding of audio-frequency transformers is to cause an increased amplification at the frequency at which this self-capacitance resonates with the inductance of the winding. In short, the circuit is behaving as a tuned transformer amplifier. The design of audio-frequency amplifiers makes use of this effect to extend the frequency range of the amplifier.

Band-pass Coupling

In Communications Engineering the uniform amplification of a narrow band of frequencies is a more common requirement than the amplification of a single frequency. In view of their selectivity the circuits already described in this chapter cannot amplify uniformly all the frequencies within a band. Circuits are therefore designed to incorporate Band-pass Filters. The subject of filter design is extensive, and lies outside the scope of this book. It so happens, however, that the most commonly used band-pass filter consists merely of two similar L–C circuits inductively coupled together, as shown in

Fig. 57, and that an indication of its working can be given without recourse to general theory.

The voltage amplification of such a circuit as that shown in Fig. 57 is shown plotted as a function of frequency in Fig. 58. The three cases illustrated are for the same circuit but with three different values of the mutual inductance, M. It will be seen that the shape of the curve depends on the value of M. In practice the value of M is adjusted so that the form of the curve is similar to that shown in curve B of Fig. 58. The circuit then amplifies, more or less uniformly, voltages having frequencies within a narrow band and discriminates against voltages of frequencies outside this band.

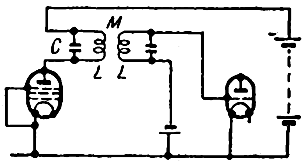


FIG. 57

Let us examine the impedance of the load presented to the valve, assuming for simplicity that the circuits are free from resistance.

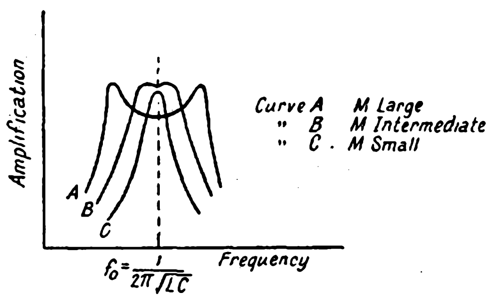


FIG. 58

This impedance is formed of the capacitance, C, in parallel with the primary of the transformer. The impedance of the primary of the transformer is the sum of its self-impedance, $j\omega L$, and the added impedance, $\dfrac{\omega^2 M^2}{j\omega L + \dfrac{1}{j\omega C}}$, due to the presence of the secondary. Proceeding round the primary closed circuit we therefore have the following impedances—

$$\frac{1}{j\omega C}, \quad j\omega L, \quad \frac{\omega^2 M^2}{j\omega L + \dfrac{1}{j\omega C}}$$

If at any frequency the sum of these impedances becomes zero, the primary L–C circuit will be in resonance and the impedance presented to the valve will be infinite. Adding them, equating to zero and solving for ω^2 we have

$$\omega = \frac{1}{\sqrt{(L \pm M)C}}$$

i.e.
$$\omega = \frac{\omega_0}{\sqrt{1 \pm \frac{M}{L}}}$$

where
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

We thus see that the impedance presented to the valve becomes infinite (or in the practical case, where there is resistance, very large) at two frequencies; these frequencies are just above and just below the natural frequency of each of the two separate L–C circuits, and their separation (the width of the band) increases as M is increased. The voltage across the primary will thus be a maximum at these two frequencies. If we were to proceed to calculate the primary current, the voltage induced into the secondary, and finally the voltage across the condenser in the secondary circuit, we should find that this voltage varied with frequency in the manner shown in Fig. 58.

Effect of Inter-electrode Capacitances on the Input Impedance of a Triode Amplifier Circuit

The inter-electrode capacitances of a triode valve are three: the grid-cathode capacitance, C_{gc}; the grid-anode capacitance, C_{ga}; and the anode-cathode capacitance, C_{ac}. Each is the geometrical capacitance between the pair of structures quoted in its title.

The Input Impedance of an amplifier stage is simply the impedance which the stage presents to the input voltage, i.e. the impedance measured between grid and cathode while the valve is operating. In this section we shall evaluate this impedance for various conditions and circuits. The effect of the input impedance on the performance of the amplifier will be considered in the next section.

The discussion will be limited to amplifiers in which no grid-current is allowed to flow. In these circumstances it might appear at first sight that the input impedance of the triode amplifier circuit would be equal to the reactance of the grid-cathode capacitance, and thus negligible except at very high frequencies. The whole of the anode circuit is, however, linked to the grid circuit through the grid-anode capacitance, and the equivalent circuit of the amplifier becomes as shown in Fig. 59 (*a*). The relative directions of the e.m.f.s. shown in this figure simply indicate that the grid-voltage and anode e.m.f. measured in the conventional directions, are, as we have already seen, 180° out of phase.

In analysing the performance of this equivalent circuit we shall not take account of C_{ac} or C_{gc}; the first of these can be taken into account by modifying the impedance of the load, Z (in the case where Z is a parallel L–C circuit C_{ac} is necessarily a part of the tuning capacitance), and the second may be considered to be in parallel with the value of input impedance which we shall determine. Thus, for the purpose of analysis, Fig. 59 (*b*) becomes our circuit. Since the input impedance is equal to V_1/I_g we must proceed to

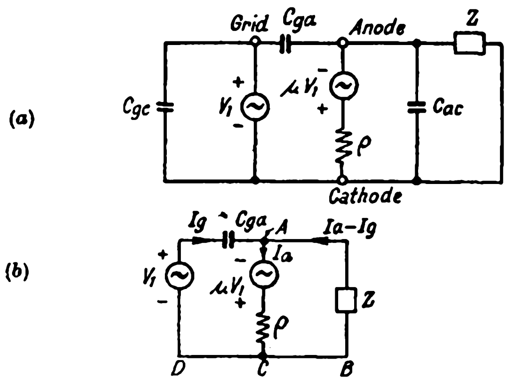


FIG. 59. EQUIVALENT CIRCUIT OF AMPLIFIER SHOWING INTER-ELECTRODE CAPACITANCES

determine I_g. The application of Kirchhoff's laws to the two closed circuits, ACD, ACB, gives

$$\left.\begin{aligned} V_1(1+\mu) &= I_g \cdot \frac{1}{j\omega C_{ga}} + \rho I_a \\ \mu V_1 &= I_a(\rho + Z) - I_g Z \end{aligned}\right\} \quad . \quad . \quad \text{(IV.13)}$$

Solving for I_g we have

$$I_g = V_1 \cdot \frac{j\omega C_{ga}(\rho + Z + \mu Z)}{\rho + Z + j\omega C_{ga}\rho Z}$$

and after re-arrangement the input impedance may be written

$$\text{Input Impedance} = \frac{V_1}{I_g} = \frac{1}{j\omega C_{ga}} \cdot \frac{1 + \dfrac{Z}{\rho + Z} \cdot j\omega C_{ga}\rho}{1 + \dfrac{\mu Z}{\rho + Z}} \quad . \quad \text{(IV.14)}$$

We shall now consider the significance of this result with reference to a few special cases.

(*a*) AUDIO-FREQUENCIES

Taking 5 $\mu\mu$F. as an average value of C_{ga} and ρ as 10,000 ohms, we find that the value of the product, $\omega C_{ga}\rho$, which appears in the numerator of equation (IV.14), is 0·003 at a frequency of 10,000 c.p.s., so that for audio-frequencies the numerator of equation (IV.14) is almost exactly unity. To simplify we shall evaluate the input *admittance*, i.e. the reciprocal of the input impedance.

$$\text{Input Admittance} = j\omega C_{ga}\left(1 + \frac{\mu Z}{\rho + Z}\right)$$
$$= j\omega C_{ga}\left\{1 + \left|\frac{\mu Z}{\rho + Z}\right| \cos\theta + j\left|\frac{\mu Z}{\rho + Z}\right| \sin\theta\right\}$$

where θ is the phase angle of the vector, $\frac{Z}{\rho + Z}$. For example, if Z consists of a resistance, R, the phase angle is zero; if Z is a reactance, jX, the above vector becomes

$$\frac{jX}{\rho + jX}$$

If X is positive (i.e. inductive) the phase angle of the numerator is 90°, and of the denominator $\tan^{-1}\frac{X}{\rho}$, so that the phase angle of the whole vector is $\left(90° - \tan^{-1}\frac{X}{\rho}\right)$. If X is negative (i.e. capacitive) the phase angle of the whole vector lines between zero and $-$ 90°.

The imaginary part of the input admittance is

$$j\omega C_{ga}\left(1 + \left|\frac{\mu Z}{\rho + Z}\right| \cos\theta\right)$$

which will be seen to be the admittance of a condenser of capacitance

$$C_{ga}\left(1 + \left|\frac{\mu Z}{\rho + Z}\right| \cos\theta\right)$$

The real part of the input admittance is

$$-\omega C_{ga}\left|\frac{\mu Z}{\rho + Z}\right| \sin\theta$$

which will be seen to be the admittance of a resistance, negative when Z is inductive, and numerically equal to the reciprocal of the above expression. Since the whole input admittance is the sum of the real and imaginary parts given above, it is equivalent to the admittance of the condenser and the negative resistance *in parallel*.

The most important case is that of an audio-frequency amplifier with resistive load. Z then is a resistance, R, which in a well-designed amplifier using triodes will be large with respect to ρ. Also $\sin\theta$ becomes zero, and $\cos\theta$ unity. Thus in this case the input impedance is simply that of a condenser of capacitance

$$C_{input} = C_{ga}(1 + \mu) \quad . \quad . \quad . \quad \text{(IV.15)}$$

(*b*) Higher Frequencies

If the load impedance is large with respect to ρ, equation (IV.14) may be written

$$Z_{input} = \frac{1}{j\omega C_{ga}(1+\mu)} + \frac{\rho}{1+\mu}$$

which is the impedance of a capacitance, $C_{ga}(1+\mu)$, in series with a resistance $\rho/(1+\mu)$.

If the load consists of an inductive reactance, $j\omega L$, small with respect to ρ, equation (IV.14) becomes

$$Z_{input} = \frac{1}{j\omega C_{ga}} \cdot \frac{1-\omega^2 LC_{ga}}{1+\dfrac{j\omega L\mu}{\rho}}$$

so that

$$\text{Input Admittance} = \frac{j\omega C_{ga}}{1-\omega^2 LC_{ga}} - \frac{\omega^2 LC_{ga}\mu}{\rho(1-\omega^2 LC_{ga})}$$

i.e. the input impedance consists of a capacitance in parallel with a resistance which is negative for frequencies below the very high frequency at which the inductance resonates with the grid-anode capacitance.

Substitution in equation (IV.14) will show that for a capacitive load the input impedance consists of a capacitance in parallel with a resistance which is always positive.

Input Impedance and Amplifier Performance

Consider the resistance-capacitance coupled circuit of Fig. 29 (*b*) and its equivalent circuit, Fig. 30. The input impedance of the second valve, which we neglected when these figures were under discussion, should appear in parallel with R_g in the equivalent circuit. It is thus effectively in parallel with the load of the first valve.

Assume that the second valve has μ equal to 20, and C_{ga} equal to 5 $\mu\mu$F., and has a load impedance large with respect to the valve impedance. Then by equation (IV.15) its input capacitance is 105 $\mu\mu$F. To this must be added the grid-cathode capacitance, which we will suppose is also equal to 5 $\mu\mu$F. This makes a total of 110 $\mu\mu$F., the reactance of which at 10,000 c.p.s. is 145,000 ohms, and at a megacycle only 1,450 ohms. It is thus apparent that at radio-frequencies the load impedance of the first valve is seriously lowered by having this low capacitance connected in parallel with it, and the voltage amplification is reduced to a very small figure. At the higher audio-frequencies, too, the effect is serious.*

* This reduction of amplification at the higher audio-frequencies is known as the "Miller Effect."

It is here that the importance of the pentode lies, for the screen-grid, being at earth potential so far as alternating current is concerned, constitutes a screen between grid and anode, and almost eliminates C_{ga}. This capacitance is commonly as low as 0·005 $\mu\mu$F. in pentodes designed for radio-frequency amplification, but pentodes used in audio-frequency amplifiers can successfully eliminate this form of frequency distortion without the necessity for such careful screening. To reduce C_{ga} it is important to have the anode and grid leads well separated both outside the valve and inside. This is facilitated by bringing out either the grid or the anode lead to a terminal at the top of the valve, instead of bringing it through the valve base together with the other leads.

In addition to an input capacitance there is an input resistance, which, in various circumstances, can be positive or negative. A positive resistance in parallel with the input capacitance accentuates the reduction of amplification at radio-frequencies mentioned above; moreover, it reduces the selectivity in the case of tuned amplifiers. A negative resistance has even more disastrous effects. The significance of a negative resistance is that the voltage and current in it are 180° out of phase, so that instead of absorbing power it is a source of power. In our case this power is fed back from the anode circuit. The danger of self-oscillation as a result of such feed-back has already been mentioned in the section on "Decoupling," and we may simply state here that parasitic oscillations may occur when conditions are such that the input resistance of any stage of an amplifier is negative. Tuned amplifiers using triodes are quite impracticable for this reason, unless special circuits are employed. (See section on "Neutralizing".)

It is interesting to consider the tuned-anode amplifier from this point of view. At the resonant frequency of the parallel L–C circuit the load is a high resistance; at lower frequencies it is inductive, and at higher frequencies capacitive. Now we have shown that with capacitive loads the input resistance is positive, whereas with inductive loads it may be negative. Thus, we see that the input resistance of the tuned-anode amplifier passes from negative to positive as the frequency is increased. Even if parasitic oscillation does not occur the amplification is likely to be increased at frequencies below resonance and decreased at frequencies above resonance, thus distorting the curve of amplification against frequency.

The choke-coupled amplifier, as described in the previous chapter, may be used for the amplification of high frequencies. As it is not a tuned amplifier its selectivity is zero, and it may be used for the amplification of a wide band of frequencies. Although it is not a tuned amplifier, there is in parallel with the inductance of the choke its own self-capacitance and also C_{ac}, and the circuit will resonate at a certain frequency. It is usual to design the choke so that its resonant frequency lies just below the band of frequencies to be amplified, thus ensuring that within this band the load impedance is slightly capacitive and thus the input resistance is positive.

Effect of Inter-electrode Capacitances on the Voltage Amplification of a Single-stage Amplifier

Even apart from the reduction of amplification resulting from the shunting effect of the input impedance, we cannot expect that the voltage amplification for the circuit of Fig. 59 will be given by the simple expression of equation (III.6), page 37, which was derived for the basic circuit in the absence of inter-electrode capacitances.

The voltage amplification will be given by $Z(I_a - I_g)/V_1$ (using the notation of Fig. 59), where I_a and I_g must be found from equations (IV. 13). Solving these equations for I_a, we have

$$I_a = V_1 \cdot \frac{\mu + j\omega C_{ga} Z(1 + \mu)}{\rho + Z + j\omega C_{ga} Z\rho}$$

whence, after re-arrangement, the voltage amplification is found to be

$$(VA) = \frac{\mu Z}{\rho + Z} \cdot \frac{1 - \dfrac{j\omega C_{ga}\rho}{\mu}}{1 + \dfrac{Z}{\rho + Z} \cdot j\omega C_{ga}\rho}$$

Thus, if Z is a pure resistance R, the numerical value of the voltage amplification is reduced by the factor

$$\sqrt{\frac{1 + \dfrac{\omega^2 C_{ga}^2 \rho^2}{\mu^2}}{1 + \left(\dfrac{R}{\rho + R}\right)^2 \omega^2 C_{ga}^2 \rho^2}}$$

This factor is approximately unity except at frequencies so high that $\omega C_{ga}\rho$ is no longer small with respect to unity. We thus conclude that (VA) remains very nearly equal to $\mu Z/(\rho + Z)$, the only reduction in the voltage amplification being that considered in the previous section, viz. the reduction arising from the lowering of the value of Z as a result of inter-electrode capacitance in the succeeding stage.

Neutralizing

Neutralizing is the modification of the circuit of an amplifier in such a way as to counteract the effect of the grid-anode capacitance. The best-known neutralizing circuit is that known as the Neutrodyne and is shown in Fig. 60 (*a*). The effect of this circuit is to eliminate the negative input resistance and to reduce greatly the input capacitance. Thus, the feed-back from anode circuit to grid circuit will be prevented, and the amplifier will not oscillate. It is fairly easy to show, without a complete analysis, that the circuit of Fig. 60 (*a*) eliminates feed-back. The equivalent circuit is shown in Fig. 60 (*b*) and may be redrawn in the form shown in Fig. 60(*c*) in which the generator, μV_1, of internal resistance, ρ, is omitted. To complete this equivalent circuit this generator should be connected across the points *AE*. Note, however, that owing to the transformer action between the two sections, L_1 and L_2, of the coil, the voltage between the points *BE* is approximately a constant times the voltage between the points *EA*. Thus, instead of considering the anode generator connected across the points *EA*, we may consider a generator of a rather greater voltage connected across the points *AB*. The resulting circuit is that of an alternating-current bridge,

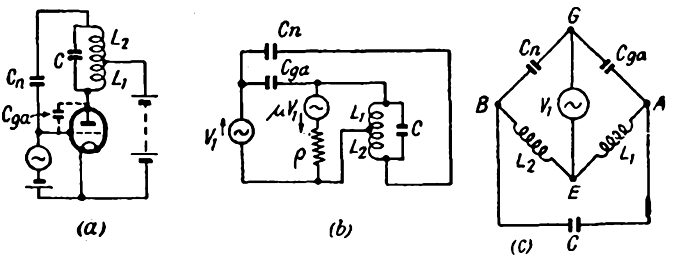


FIG. 60

which will be balanced if C_n/C_{ga} equals L_1/L_2. When this condition obtains the anode generator can send no current through the grid circuit (denoted in our bridge by the generator V_1). Thus the feed-back from anode circuit to grid circuit is prevented.

A simpler physical picture of the operation of this circuit is perhaps obtained as follows. With C_n open-circuited in Fig. 60 (*a*), feed-back occurs from anode circuit to grid circuit because the alternating anode-voltage sends current through the grid circuit *via* C_{ga}. The remedy is to send an equal and opposite current through the grid circuit from some other point in the anode circuit. It is therefore necessary to find a point in the anode circuit whose voltage (with respect to cathode) is equal and opposite to that of the anode, and to connect this point also to the grid through a condenser equal to C_{ga}. Now the tapping point on coil L_1L_2 is at cathode potential so far as a.c. is concerned, so that it is possible, by suitably choosing the tapping point, to make the potential of the upper end of L_2 as much above cathode-potential at every instant as the potential of the anode is below. Neutralizing is therefore effected by connecting the upper end of L_2 to the grid through the condenser C_n. In practice C_n is made variable, and its capacitance adjusted by trial and error until feed-back is eliminated (the criterion being that, with the h.t. switched off, the input voltage shall not succeed in setting up a voltage across the load in the anode circuit). Adjustment of the tapping point on L_1L_2 would be equally effective but less convenient.

The Cascode Stage

We have stated (on page 97) that triodes may not be used in tuned amplifiers unless neutralizing is employed (because of the

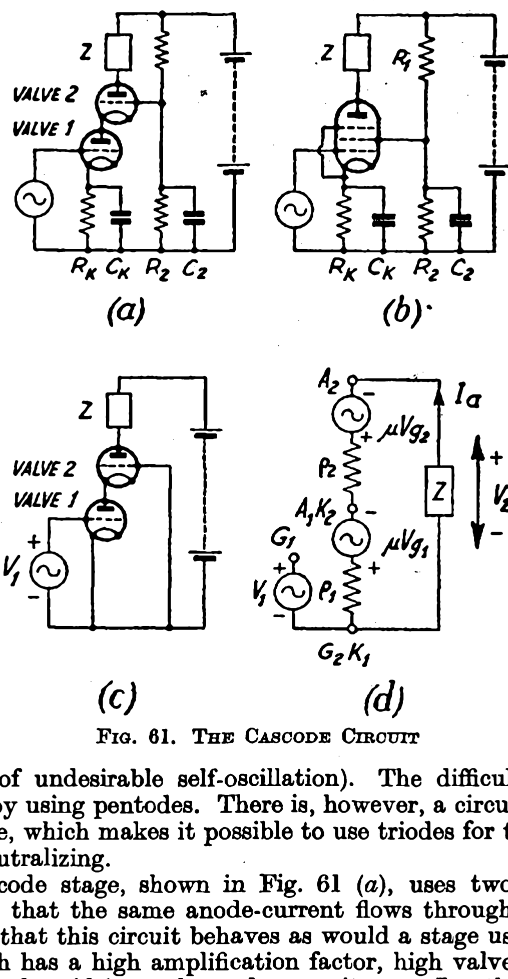


FIG. 61. THE CASCODE CIRCUIT

likelihood of undesirable self-oscillation). The difficulty may be overcome by using pentodes. There is, however, a circuit known as the Cascode, which makes it possible to use triodes for this purpose without neutralizing.

The Cascode stage, shown in Fig. 61 (*a*), uses two triodes in cascade, so that the same anode-current flows through both. We shall show that this circuit behaves as would a stage using a single valve which has a high amplification factor, high valve-impedance and low anode-grid inter-electrode capacitance. In other words, it behaves very much as would a pentode, while avoiding any disadvantages inherent in pentodes (e.g. partition noise). Comparison of Figs. 61 (*a*) and (*b*) shows that the circuit is somewhat similar in appearance to that of a pentode stage, though this must be regarded as fortuitous since the two circuits operate in quite different ways to produce the same result.

Fig. 61 (*c*) shows the circuit stripped of all non-essentials, and it will be seen that the input-voltage V_1 is applied in the grid-circuit of the lower valve and may therefore be expected to produce corresponding variations (i.e. to produce a signal component) of the anode-voltage of this lower valve. The upper valve has its grid connected to h.t. negative. The only way in which the upper valve manages to secure an input voltage, in its grid-to-cathode circuit, is by virtue of the inclusion of the lower valve's anode-voltage in the cathode lead of the upper valve. The output voltage is taken between the upper anode and h.t. negative. It is, therefore, in the usual way, equal to the signal component of the p.d. across the load, Z.

Before turning to a quantitative analysis, let us look at the so-called "non essentials" which distinguish the practical circuit of Fig. 61 (*a*) from the simple circuit of Fig. 61 (*c*). The parallel combination of R_K and C_K is merely an automatic-bias circuit (see page 68). The purpose of the components R_1, R_2, and C_2 is to ensure that the grid of the upper valve is connected to h.t. negative, so far as A.C. (or the "signal component") is concerned while at the same time the d.c. potential of this grid is raised to within a few volts of the potential of the lower anode, i.e. the potential of the upper cathode. This is necessary to ensure that the upper valve operates with a suitable value of grid-bias voltage, which would not be the case in the simplified circuit of Fig. 61 (*c*). When the upper valve is operated with a suitable grid-bias voltage, no grid-current flows and thus no signal-frequency current flows in R_1 or R_2. Thus, to a first approximation, C_2 is unnecessary, but it is retained as a precaution against stray feed-back currents.

Fig. 61 (*d*) shows the incremental equivalent circuit (or "a.c. equivalent circuit"). The showing, and labelling, of the electrodes A_1 G_1 K_1 and A_2 G_2 K_2 on such a diagram is most strongly recommended as a standard procedure. It enables one to see at a glance what algebraic expressions to substitute for V_{g1} and V_{g2}, symbols which, by established convention, denote *grid-to-cathode* voltages. V_{g1} is the amount by which the potential of G_1 exceeds that of K_1 and is here merely equal to V_1. V_{g2} is the amount by which the potential of G_2 exceeds that of K_2 and is given by

$$\begin{aligned} V_{g_2} &= \mu_1 V_{g_1} - \rho_1 I_a \\ &= \mu_1 V_1 - \rho_1 I_a \end{aligned}$$

Substituting for V_{g_1} and V_{g_2} in the equation

$$\mu_1 V_{g_1} + \mu_2 V_{g_2} = I_a(\rho_1 + \rho_2 + Z)$$

and solving for I_a, we have

$$I_a = V_1 \frac{\mu_1\mu_2 + \mu_1}{\rho_1(1 + \mu_2) + \rho_2 + Z} \quad . \quad . \quad . \text{(IV.16)}$$

This is the same anode-current as would be sent through the load Z by a valve having an amplification factor

$$\mu_{effective} = \mu_1\mu_2 + \mu_1 \approx \mu_1\mu_2$$

and having a valve-impedance given by

$$\rho_{effective} = \rho_1(1 + \mu_2) + \rho_2 \approx \mu_2\rho_1$$

The mutual conductance of such an equivalent valve would be given by

$$g_{m_{effective}} = \frac{\mu_{effective}}{\rho_{effective}} = \frac{\mu_1\mu_2}{\mu_2\rho_1} = g_{m_1}$$

In other words, the mutual conductance of the equivalent valve is, to a first approximation, independent of the parameters of the upper triode.

These effective parameters will have numerical values similar to those of a pentode—viz. large values of μ and ρ. The voltage-gain will be given by

$$\frac{V_2}{V_1} = -\frac{ZI_a}{V_1} \approx \frac{-\mu_1\mu_2 Z}{\mu_2\rho_1 + Z} \quad . \quad . \quad . \text{(IV.17)}$$

The product $\mu_2\rho_1$ will usually be of the order of a megohm. Since Z is not likely to be much larger than this (and will usually be smaller) voltage gains will usually be only a fraction of $\mu_1\mu_2$. For the more usual values of $|Z|$, $|Z|$ will be small with respect to $\mu_2\rho_1$ and the voltage-gain will be approximately $-g_{m_1}Z$. Thus the stage gain is approximately independent of the parameters of the upper valve.

Before we attempt to calculate the effective grid-anode capacitance of a cascode stage, it will be convenient to calculate the input-impedance of such a stage, i.e. the ratio V_1/I_1 where V_1 is the input voltage and I_1 is any current which V_1 may be called upon to supply. We shall, of course, assume that the lower valve has a grid-bias voltage adequate to prevent the flow of grid-current. Also we shall neglect the component of I_1 which flows through the grid-cathode capacitance of the lower valve, since this may be taken into account later (if required) merely by adding a capacitive shunt reactance to the value of Z_{in} which we shall calculate. Thus the only remaining path for I_1 is through C_{ga_1}, the grid-anode capacitance of the lower valve. I_1 will be given by $j\omega C_{ga_1}V_{ga_1}$, where V_{ga_1} is the p.d. between grid and anode of the lower valve and is equal to $V_1 - V_{a_1}$. Hence

$$I_1 = j\omega C_{ga_1}(V_1 - V_{a_1})$$

and $$Z_{in} = \frac{V_1}{I_1} = \frac{1}{j\omega C_{ga_1}(1 - V_{a_1}/V_1)}$$

To evaluate V^{a_1} we use equation (IV.16) to substitute for I_a in the equation

$$V_{a_1} = -\mu_1 V_1 + \rho_1 I_a$$

This will be found to give

$$\frac{V_{a_1}}{V_1} \approx \frac{-\mu_1 Z}{\mu_2\rho_1 + Z}$$

whence $$Z_{in} \approx \frac{1}{j\omega C_{ga_1}\left(1 + \dfrac{\mu_1 Z}{\mu_2\rho_1 + Z}\right)} \quad . \quad . \quad \text{(IV.18)}$$

Now we have shown that the cascode stage is equivalent to an orthodox stage using a single valve whose parameters are

$$\mu_{eff} \approx \mu_1\mu_2 \quad \text{and} \quad \rho_{eff} \approx \mu_2\rho_1$$

To evaluate the effective grid-anode capacitance, C_{eff}, we now ask ourselves what value of grid-anode capacitance this equivalent valve would require to have, to give the same input impedance as the cascode stage, when using the same load, Z. The input impedance of the stage using the equivalent valve would be (cf. p. 95)

$$Z_{in} = \frac{1}{j\omega C_{eff}\left(1 + \dfrac{\mu_{eff} Z}{\rho_{eff} + Z}\right)}$$

$$= \frac{1}{j\omega C_{eff}\left(1 + \dfrac{\mu_1\mu_2 Z}{\mu_2\rho_1 + Z}\right)}$$

Equating this to the input impedance of the cascode stage, as given in equation (IV.18), and solving for C_{eff}, we have

$$C_{eff} = C_{ga_1} \cdot \frac{\mu_2\rho_1 + Z(1 + \mu_1)}{\mu_2\rho_1 + Z(1 + \mu_1\mu_2)}$$

If $|Z|$ is larger than or of the same order as the triode impedance ρ_1, this gives an effective grid-anode capacitance, C_{eff}, equal to C_{ga_1}/μ_2 approximately. Low values of $|Z|$ in the cascode will not suffice to secure the advantage of a low effective grid-anode capacitance.

Negative Feed-back Amplifiers

The principle of negative feed-back is the returning of a portion of the amplified voltage in the anode circuit to the grid circuit, the connexions being made so that the voltage returned, or fed back, is 180° out of phase with the external voltage, V_1, which is to be amplified. The total alternating grid-voltage is thus less than V_1 and the amplification is reduced. This, of course, is a disadvantage, but we shall see that it is more than offset by the advantages of this circuit.

Perhaps the simplest example of a circuit employing negative feed-back is that of Fig. 46(a), page 69, already discussed in the section on "Automatic Bias Circuits." It was shown in that section that the voltage amplification was reduced in the ratio

$$\frac{1}{1 + \dfrac{\mu R_1}{\rho + R + R_1}}$$

Now the amplification without feed-back was shown to be

$$(VA)_0 = \frac{\mu R}{\rho + R + R_1}$$

Moreover, the fraction of the output voltage fed back to the grid or input circuit is R_1/R, since the output voltage is RI_a and the voltage fed back is $R_1 I_a$. Thus, the ratio in which the amplification is reduced may be written

$$\frac{1}{1 + n(VA)_0} \quad . \quad . \quad . \quad . \quad . \quad (IV.19)$$

where n is the fraction of the output voltage fed back to the input and $(VA)_0$ is the amplification that there would be in the absence of negative feed-back.

We shall now show without reference to any particular circuit that the amplification is reduced in the ratio given in equation (IV.19) for any amplifier using negative feed-back. Fig. 62 shows schematically an amplifier (denoted by the rectangle) the magnitude of whose amplification is $(VA)_0$ with an input voltage V_1. By feeding back a fraction, n, of the output voltage as shown, it has been converted to a negative feed-back amplifier. Despite the fact that the voltage to be amplified is still V_1, the total effective input voltage is now less than this by an amount equal to the voltage fed back. We shall denote the total effective input voltage by V. The output voltage must therefore be $(VA)_0 V$ and, the fraction fed back being n, the total effective input

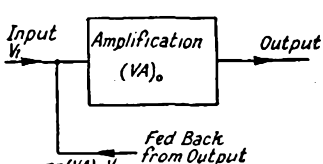


FIG. 62

voltage which we have denoted by V must be equal to $V_1 - n(VA)_0V$

$$V = V_1 - n(VA)_0V$$

whence
$$V = \frac{V_1}{1 + n(VA)_0}$$

But the output voltage is $(VA)_0V$, so that the voltage amplification must be given by

$$(VA) = \frac{(VA)_0V}{V_1}$$

$$= \frac{(VA)_0}{1 + n(VA)_0} \quad . \quad . \quad . \text{(IV.20)}$$

the amplification thus being reduced in the ratio given in equation (IV.19).

The first important deduction to be made from this relation is that if a sufficient fraction of the voltage is fed back for the number $n(VA)_0$ to be large with respect to unity, the voltage amplification of the negative feed-back amplifier becomes simply $1/n$; i.e. *the amplification is independent of the amplification of the original amplifier.* Valves may be changed or may deteriorate, h.t. and l.t. voltages may fluctuate, but the amplification of the negative feed-back amplifier is unaffected. As an example, consider an amplifier having $(VA)_0$ equal to 1,000, converted into a negative feed-back amplifier by the feeding back of one-tenth of the output voltage. Here n is 1/10 and from equation (IV.20) we calculate that the amplification is 9·9. If now the amplification of the original amplifier changes to 500, the amplification of the feed-back amplifier changes only to 9·8.

Next observe that since the amplification of the feed-back amplifier is approximately $1/n$, it is free from frequency distortion except in so far as n is a function of frequency. If n is independent of frequency, then the feed-back amplifier is free from frequency distortion even if the original amplifier is not so. Moreover, to design a feed-back amplifier having its amplification a given function of frequency, it is simply necessary to make n a similar function of frequency by including a suitable network in the feed-back circuit.

Let us next compare the harmonic content of the output voltage (arising from curvature of the characteristics) with and without feed-back. We shall assume that the original amplifier, when giving an output voltage of $(VA)_0V$, generates a second harmonic of $h_0(VA)_0V$, the harmonic content thus being h_0; and we shall determine the second harmonic content, h, of the feed-back amplifier when giving the same output voltage. The second harmonic voltage generated by the feed-back amplifier when giving an output voltage, $(VA)_0V$, will be $h(VA)_0V$. Thus a second harmonic voltage, $-nh(VA)_0V$, will be fed back and amplified. The total generated second harmonic voltage which we have denoted by $h(VA)_0V$ will be made up of the second harmonic production of the original amplifier, $h_0(VA)_0V$, and the amplified fed-back second harmonic, $-nh(VA)_0^2V$, i.e.

$$h(VA)_0V = h_0(VA)_0V - nh(VA)_0^2V$$

whence

$$h = \frac{h_0}{1 + n(VA)_0}$$

That is, the harmonic content is reduced in the same ratio as is the amplification. For example, if an amplifier with $(VA)_0$ equal to 1,000 has 5 per cent second harmonic distortion at a given output voltage, its conversion to a negative feed-back amplifier with n equal to one-tenth will reduce the second harmonic distortion *at the same output voltage* to 0·0495 per cent.

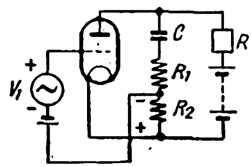


Fig. 63

In the same way it can be shown that the noise voltage generated by the valve is reduced in the same ratio as the amplification, by the application of negative feed-back.

Feed-back Amplifier Circuits

The obvious method of returning a fraction of the output voltage of an amplifier to its input circuit is to apply the output voltage to the input terminals of a voltage-divider. The output of this voltage-

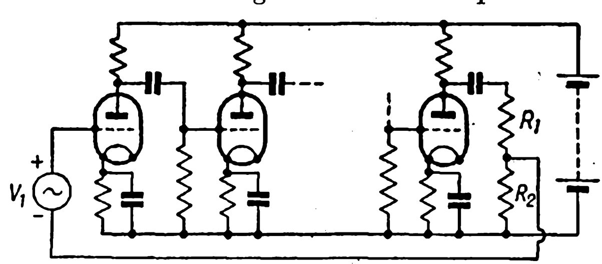


Fig. 64. Negative Feed-back Amplifier with an Odd Number of Stages

divider is then taken to a break in the input circuit of the amplifier. A practical circuit for accomplishing this is shown in Fig. 63, in which R_1R_2 is the voltage-divider in series with a blocking condenser C (of large capacitance and therefore small reactance) whose only function is to prevent the h.t. supply from discharging continuously through RR_1R_2. If the reactance of C is negligible with respect to R_1, then a fraction $R_2/(R_1 + R_2)$ of the output voltage is fed back to the input circuit, V_g being $(V_1 - nV_2)$ where n equals $R_2/(R_1 + R_2)$. It is clear that the feed-back is negative (i.e. that V_g is not equal to $V_1 + nV_2$) since the output voltage of a single-stage amplifier with a resistive load is 180° out of phase with its input voltage. During the positive half cycle of V_1 therefore the p.d. across R_2 will be a "grid-depressing" p.d. as shown in Fig. 63.

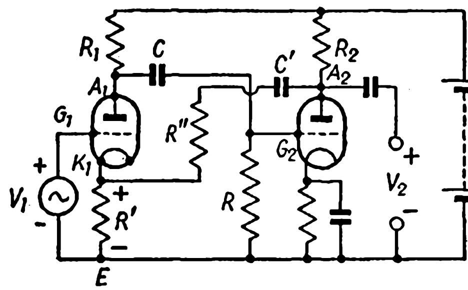


FIG. 65. NEGATIVE FEED-BACK AMPLIFIER WITH EVEN NUMBER OF STAGES

Providing that $(R_1 + R_2)$ is large compared to the load R, $(VA)_0$ will be given by

$$(VA)_0 = \frac{\mu R}{\rho + R}$$

and the overall amplification of this negative feed-back amplifier may be found by substituting these expressions for n and $(VA)_0$ in equation (IV.20).

This method of feed-back may be applied to any amplifier whose output voltage is 180° out of phase with its input voltage, i.e. to any resistance-loaded amplifier with an odd number of stages as shown in Fig. 64.

For an amplifier with an *even* number of resistance-loaded stages, the output voltage is in phase with the input voltage. The method of Fig. 64 cannot therefore be used, as the voltage fed back would be in phase with the input voltage V_1, thus giving positive feed-back instead of negative feed-back. Fig. 65 shows a circuit which may be used to apply negative feed-back to a two-stage amplifier. The feed-back path is by way of R'', and if this path be broken (say, between R'' and C') the circuit becomes an ordinary R–C coupled two-stage amplifier, with the exception that the cathode resistor, R', of the first stage is not shunted by the usual bias condenser. There is, therefore, some negative feed-back in the first stage of the amplifier even with the main feed-back path broken; but this is merely incidental. To check that the overall feed-back is in fact negative (and not positive) observe that V_{a_2}, the anode-voltage of the second stage, will be 180° out of phase with the anode-voltage of the first stage, and so in phase with V_1. Connecting V_{a_2} across the voltage-divider $R'R''$ may therefore be expected to produce a voltage V_{K1E} in phase with V_1 as indicated by the $\pm$ signs on Fig. 65. During the positive half cycle of V_1 this is a "grid-negative-making" voltage and therefore subtracts from V_1, giving negative feed-back.

Since it is not easy to deduce the values of (VA), and n by inspection, let us make a detailed analysis of this circuit. We shall assume

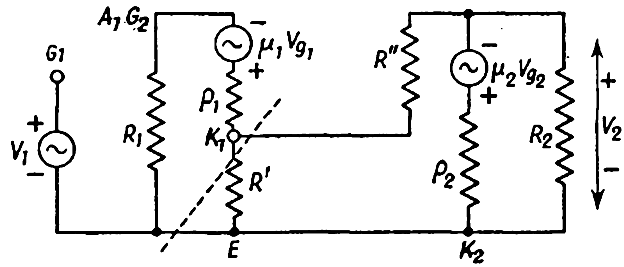


FIG. 66

that the usual design rules for R–C couplings are followed, i.e. $1/\omega C \ll R \gg R_1$ and $1/\omega C' \ll R'' \gg R_2$. This means that in drawing the equivalent circuit, C and C' may be regarded as short-circuits, while R may be omitted as having negligible shunting effect upon R_1. The analysis of the equivalent circuit (Fig. 66) may be greatly simplified by replacing the whole of the network to the right of the dotted line by a simple circuit consisting of an e.m.f. E_0 in series with a resistance R_0 (as prescribed by Thevenin's Theorem, see page 4). Since we are assuming $R'' \gg R_2$, E_0 and R_0 are given by

$$E_0 = \frac{R'}{R' + R''} \cdot V_2 \text{ and } R_0 = R'R''/(R' + R'')$$

The simplified equivalent circuit is as shown in Fig. 67 from which we have

$$I_{a_1} = \frac{\mu_1 V_{g_1} - E_0}{\rho_1 + R_1 + R_0}$$

and

$$V_{g_1} = V_1 - E_0 - R_0 I_{a_1}$$

Substituting the second of these equations in the first, and solving for I_{a_1} we have

$$V_{g_2} = V_{a_1} = -R_1 I_{a_1} = -R_1 \cdot \frac{\mu_1 V_1 - (1 + \mu_1)E_0}{\rho_1 + R_1 + R_0(1 + \mu_1)}$$

For the second stage, remembering that $R'' \gg R_2$, we may write

$$V_2 = \frac{-\mu_2 R_2 V_{g_2}}{\rho_2 + R_2}$$
$$= \frac{\mu_2 R_2 R_1}{\rho_2 + R_2} \cdot \frac{\mu_1 V_1 - (1 + \mu_1)E_0}{\rho_1 + R_1 + R_0(1 + \mu_1)}$$

Finally we substitute into this the expressions for E_0 and R_0 given above, and solve for V_2. This gives

$$(VA) = \frac{V_2}{V_1}$$
$$= \frac{\dfrac{\mu_1 \mu_2 R_1 R_2}{\rho_2 + R_2}}{\rho_1 + R_1 + \dfrac{R'R''}{R' + R''}(1 + \mu_1) + \dfrac{R'}{R' + R''} \cdot \dfrac{(1 + \mu_1)\mu_2 R_1 R_2}{\rho_2 + R_2}}$$

i.e. $$(VA) = \frac{(VA)_1 (VA)_2}{1 + n(VA)_1(VA)_2} \quad . \quad . \quad . \quad . \quad (IV.21)$$

where
$$(VA)_1 = \frac{-\mu_1 R_1}{\rho_1 + R_1 + (1 + \mu_1)R'R''/(R' + R'')} \quad . \quad (IV.22)$$
$$(VA)_2 = \frac{-\mu_2 R_2}{\rho_2 + R_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (IV.23)$$
$$n = (1 + 1/\mu_1) \cdot \frac{R'}{R' + R''} \approx \frac{R'}{R' + R''} \quad . \quad (IV.24)$$

The popularity of this circuit arises from its being able to give a reasonably large stabilized amplification, (VA), since $n(VA_1)(VA)_2$

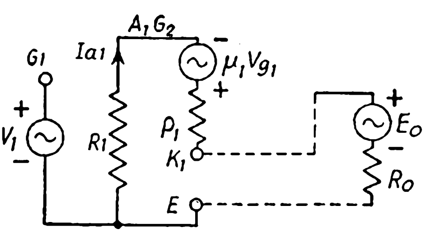


FIG. 67

may be made large compared to unity with quite small values of n. It has the further advantage over the circuits of Fig. 64 that one side of the input voltage, V_1, may be earthed to the common negative.

A three-stage negative feed-back amplifier in which one side of the input voltage may be earthed to the common negative is shown in Fig. 68. The usual rules for R–C coupling are observed, i.e. $1/\omega C_1' \ll R_1' \gg R_1$ and $1/\omega C_2' \ll R_2' \gg R_2$. Feed-back is secured by the use of a common cathode resistor, R, for the first and last stages. The second stage merely amplifies the output voltage of the first and passes it on to the third stage with a phase change of

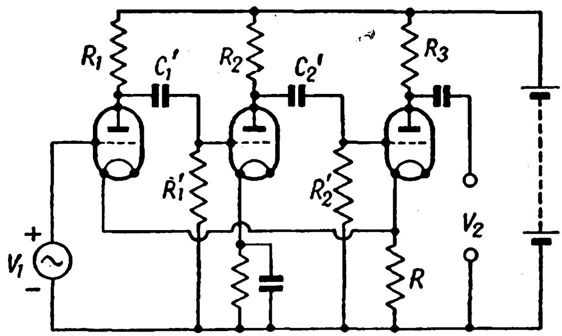


FIG. 68 NEGATIVE FEED-BACK AMPLIFIER WITH AN ODD NUMBER OF STAGES
(Permitting earthing of one input terminal)

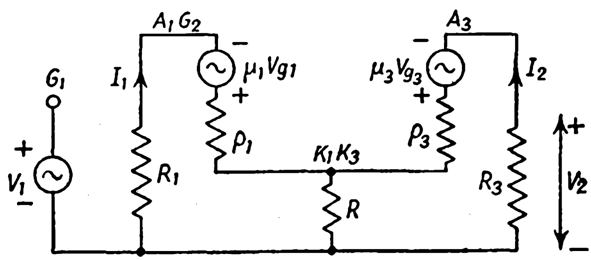


FIG. 69

180°. Thus we may draw the equivalent circuit as in Fig. 69, omitting the second stage, but taking the alternating input voltage to the third stage as $\left(-\frac{\mu_2 R_2}{\rho_2 + R_2}\right) \times V_{a_1}$ or $-(VA)_2 V_{a_1}$. Since V_a is equal to $-R_1 I_1$ this gives $V_{g_3} = (VA)_2 R_1 I_1 - R(I_1 + I_2)$

Also $\quad V_{g_1} = V_1 - R(I_1 + I_2)$

The application of Kirchhoff's Law to the two closed circuits gives

$$\mu_1 V_{g_1} = (\rho_1 + R_1 + R)I_1 + RI_2$$
$$\mu_3 V_{g_3} = (\rho_3 + R_3 + R)I_2 + RI_1$$

Substituting for V_{g_1} and V_{g_3} in these equations we have

$$\mu_1 V_1 = \{\rho_1 + R_1 + R(1 + \mu_1)\} I_1 + R(1 + \mu_1)I_2$$
$$0 = \{\rho_3 + R_3 + R(1 + \mu_3)\} I_2 + [R(1 + \mu_3) + \mu_3(VA)_2R_1]I_1$$

These two equations may now be solved for I_2, whence the output voltage V_2 is given by R_3I_2. We then have

$$(VA) = \frac{V_2}{V_1} = \frac{(VA)_1(VA)_2(VA)_3K}{1 - n(VA)_1(VA)_2(VA)_3K} \quad . \quad . \quad (\text{IV}.25)$$

$$\left.\begin{aligned} \text{where } (VA)_1 &= \frac{\mu_1R_1}{\rho_1 + R_1 + R(1+\mu_1)} \\ (VA)_2 &= \frac{\mu_2R_2}{\rho_2 + R_2} \\ (VA)_3 &= \frac{\mu_3R_3}{\rho_3 + R_3 + R(1+\mu_3)} \\ n &= \left(\frac{R}{R_3}\right)(1 + 1/\mu_1) = \text{approx. } R/R_3 \\ \text{and} \quad K &= 1 - (1 + 1/\mu_3)\frac{R}{(VA)_2R_1} \\ &= \text{approx. } 1. \end{aligned}\right\} \quad . \quad (\text{IV}.26)$$

Thus, provided that the product $n(VA)_1(VA)_2(VA)_3$ is made large with respect to unity, this circuit will give a stabilized amplification of approximately $1/n$, i.e. R_3/R.

Many other negative feed-back circuits have been devised. In general, if the circuit includes a transformer it is immaterial whether the number of stages is odd or even, for the phase of the feed-back may be reversed simply by interchanging the leads to one of the transformer windings. Some particular circuit applications of negative feed-back are treated separately later (see "The Cathode-follower Stage," "The Seesaw Circuit," "The Miller Integrator").

Error-actuated Automatic-control Systems (Servos)

There is a very close analogy between negative feed-back amplifiers and servos—so close that a book on "Network Analysis and Feed-back Amplifier Design" (H. W. Bode, D. Van Nostrand Co., Inc.) has become the vade-mecum of servo engineers. A very brief introduction to the subject of servos is thus not out of place here, partly because the student of feed-back amplifiers is in a position to learn something of servos with a minimum of effort, and partly because it is sometimes instructive to view negative feed-back amplifiers from the servo point of view.

The principle of "actuation by error" can be applied to the control of almost any quantity. To illustrate the principle, let us consider a large rotatable mechanism whose angular position is to be precisely controlled from a distance (Remote Position Control, abbreviated to R.P.C.). It is required that when the control knob (or "input shaft") is turned to any given angular position the distant rotatable mechanism (or "output shaft") shall turn to exactly the same angular position, i.e. it shall reproduce the movement of the input shaft as precisely as possible. Let an electric motor be used to drive the output shaft for this purpose (this motor and the output shaft being required to remain stationary, of course, whenever the input shaft is stationary). To apply the error-actuation principle we may now use two devices one of which will give a

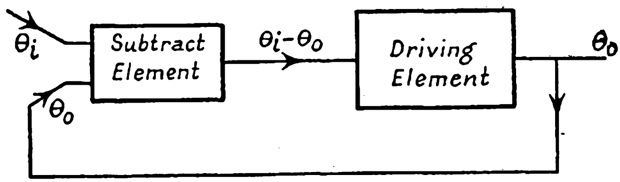


FIG. 70

voltage directly proportional to the angular deviation of the output shaft from some datum position, and the other of which will give a similar voltage indicating the position of the input shaft. These two voltages will be equal when the output shaft has taken up the same angular position as the input shaft, i.e. the difference between the two voltages will then be zero. But when the output shaft position is *not* the same as the input shaft position (in which case we require

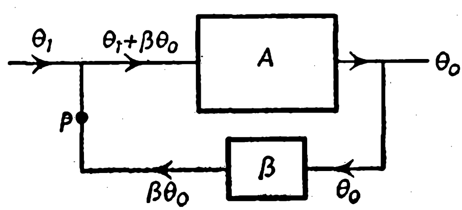


FIG. 71

that the motor shall turn the output shaft to restore coincidence of the two positions) the difference between the two voltages will not be zero. This difference voltage (known as the Error) is therefore used as the input to the driving motor, usually after amplification. Fig. 70 shows a generalized block diagram of an error-actuated automatic-control system, in which θ_i represents the input (or controlling) quantity and θ_0 represents the output (or controlled) quantity. The analogy with feed-back amplifiers is obvious, the feed-back ratio (n) being 100 per cent. The quantity fed back is not usually the whole output quantity θ_0, however. By feeding back in addition a quantity proportional to $d\theta_0/dt$, the system can be given an ability to "anticipate," since (in the above example) the *required motor torque* clearly depends not only on the existing error, but also on whether this error is already rapidly decreasing. In this manner "overshoots" or "hunting" may be prevented. A more general schematic diagram is therefore that shown in Fig. 71 in which the fed-back quantity is denoted by $\beta\theta_0$. In this figure the upper diagram represents the driving element, and A is the ratio of its output and input quantities. The lower rectangle represents the feed-back path and β represents the ratio of its output and input quantities expressed in operational form, e.g. $\beta\theta_0$ might equal $K_1\theta_0 + K_2(d\theta_0/dt)$. This, in operational form, gives $\beta\theta_0$ equal to $(K_1 + K_2p)\theta_0$ where p denotes d/dt. If only sinusoidal variations were considered, we should have the vector β equal to $K_1 + j\omega K_2$.

For this system we see that

$$A\ (\theta_i + \beta\theta_0) = \theta_0$$

$$\theta_0/\theta_i = \frac{A}{1 - A\beta} \quad . \quad . \quad . \text{(IV.27)}$$

If β is equal to -1 (i.e. 100 per cent negative feed-back) and A tends to infinity, θ_0 is maintained almost exactly equal to θ_i. If β is equal to $K_1 + K_2p$, and K_1 equals -1, θ_0 will be equal to θ_1 whenever $d\theta_0/dt$ is zero, i.e. after the system has come to rest. It should perhaps also be pointed out that, as a result of mechanical inertia, etc., the quantity A is in general complex.

The feed-back amplifier circuit of Fig. 64 may be regarded as an electronic servo system, if the voltage across R_2 be thought of as the output quantity. The "error" is thus the difference between V_1 and the voltage across R_2. Since the object of a servo system is to maintain the error as small as possible it follows that, if the amplification is large enough, the voltage across R_2 will be maintained equal to V_1. The true output voltage of this feed-back amplifier is, however, $(R_1 + R_2)/R_2$ times the voltage across R_2; it can thus be seen that the gain of the feed-back amplifier is stabilized at a value $(R_1 + R_2)/R_2$. This suggests a useful approximate method of estimating the stabilized amplification of any given feed-back amplifier circuit, viz. first identify the voltage which is subtracted from V_1 to give the "error"; next determine, by inspection of the circuit, the ratio of the true output voltage to this subtracted voltage; this ratio gives the stabilized amplification of the amplifier.

General Equation for Feed-back of any Phase

A comparison of equation (IV.20) and equation (IV.27) shows the difference between the two notations we have used. The servo quantities θ_i and θ_0, correspond of course to the input and output voltages, V_1 and V_2 respectively, of the feed-back amplifier case. The quantity A in the servo case, corresponds to the vector for $(VA)_0$ in the case of a feed-back amplifier. The quantity β is the vector gain (or, as it is sometimes called, the Transfer Function) of the feed-back path. The symbol n in equation (IV.20) thus corresponds to $|\beta|$. Equation (IV.20), which applies only to *negative* feed-back, is merely a particular case of the more general equation which applies to feed-back of any phase, viz.

$$(VA) = \frac{V_2}{V_1} = \frac{A}{1 - A\beta} \quad . \quad . \quad . \quad (IV.28)$$

where A is the vector ratio V_2/V_1 for the forward path alone (i.e. the original amplifier before the application of feed-back) and β is the vector ratio V_2/V_1 for the feed-back path alone. The definition of A and β demands a convention for the positive directions of V_1 and V_2. Since one of the input terminals and one of the output terminals is normally connected to earth (i.e. to the common negative), the accepted convention is to take V_1 as the potential of the other input terminal with respect to earth, and similarly for V_2. With this convention the basic amplifier of Figs. 19 and 26 would have A equal to $-\mu R/(\rho + R)$.

If the product $A\beta$ is real and negative we have negative feed-back; if it is real and positive we have positive feed-back. Thus, for negative feed-back, $A\beta$ is equal to $-|A\beta|$ and equation (IV.28) becomes

$$\frac{V_2}{V_1} = \frac{A}{1 + |A| . |\beta|}$$

which, with the appropriate change of notation, is the same as equation (IV.20).

If the magnitude of $(1 - A\beta)$ exceeds unity, the application of the feed-back has reduced the amplification and the feed-back is said to be "degenerative." If the magnitude of $(1 - A\beta)$ is less than unity, the feed-back has brought about an increase in amplification and is said to be "regenerative." If $(1 - A\beta)$ is zero the gain is infinite, which implies that the circuit can give a finite output voltage with zero input voltage, i.e. the circuit is self-oscillatory.

Nyquist's Criterion of Stability

The word "stable" has a variety of meanings, even within the field of telecommunications. The gain of an amplifier and the voltage of a supply are said to be stabilized if they are maintained constant. An oscillator is said to be stable if its frequency is maintained constant. An amplifier is said to be stable if it does not oscillate, and Nyquist's criterion for the stability of feed-back amplifiers is the general condition that they shall not oscillate. (Readers who are new to the subject of Oscillation are advised to read the first six pages of Chapter V before proceeding with this section.)

In general, the amplification of an amplifier is a function of frequency, and so also is the phase difference between the input and output voltages. Thus a feed-back amplifier which is designed to have the product $A\beta$ real and negative over a certain range of frequency (i.e. to have a phase shift of 180° "round the loop"), so giving true negative feed-back, may have a vector $A\beta$ whose angle is very different from 180° at some other frequency. If, at any particular frequency, $A\beta$ becomes equal to $+1$, the amplifier will act as an oscillator, giving an output voltage of that particular frequency even when it has no external input voltage. This is possible because the amplifier can supply its own input voltage by way of the feed-back path, the gain "round the loop" being equal to unity and the phase shift being zero. A rough and ready remedy for this state of affairs is to include in the circuit such components as will greatly reduce the gain at the frequency at which oscillation would otherwise occur, but which will not cause appreciable reduction of gain within the desired frequency range. For instance, if the frequency of the unwanted oscillation were much higher than the desired frequency, one could try the effect of shunting small condensers across one or more of the load resistances of the amplifier. Such trial and error methods are superseded by the general method given by Nyquist* who first showed that the criterion for stability was that the locus of the tip of the $A\beta$ vector should not enclose the point whose co-ordinates are (1, 0).

Let us suppose that Fig. 71 represents a feed-back amplifier. If the circuit is broken at the point P, we can determine the vector $A\beta$ by connecting a sinusoidal input voltage, V_1, at the point marked θ_i and measuring the voltage V_2 at the point P, on the β-side of the break. Clearly $A\beta = V_2/V_1$. If measurements of both magnitude and phase be made, at a large number of frequencies, we shall have data for plotting the vector $A\beta$ at a large number of frequencies and may thus draw the locus which the tip of this vector follows as the frequency is varied. Consider the three loci shown in Fig. 72. Locus (*b*) passes through the point $(+1, 0)$, so that it is clear that an amplifier having this characteristic would oscillate at the particular frequency corresponding to this point on the locus. (A suitable scale of frequencies can be marked along the locus for convenience, in any given case.) Locus (*c*) shows that the amplifier to which it applies has pure positive feed-back at some frequency, since the vector $A\beta$ is real and positive at the point Q. But at this point the length of the vector, OQ, is less than unity, i.e. the feed-back is too small for the amplifier to supply its own input voltage, and oscillation does not occur.

The amplifier to which locus (*a*) applies also has pure positive feed-back, at the frequency corresponding to the point R. The

* H. Nyquist, "Regeneration Theory," *Bell System Technical Journal.* Vol. 11, p. 126 (Jan. 1932).

length of the vector $A\beta$ at this frequency is greater than unity, i.e. the feed-back is more than great enough for the amplifier to supply its own input voltage. It is shown in Chapter V (page 154) that under these conditions the "amplifier" will generate oscillations whose amplitude increases with time, the amplitude eventually reaching equilibrium at a value which makes the valve parameters such that the magnitude of $A\beta$ is reduced exactly to unity.

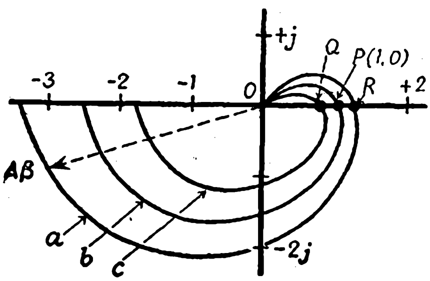


FIG. 72. POLAR PLOT OF THE LOOP GAIN

The advantage of this method of analysis is that it facilitates the design of networks which can be cascaded with the amplifier to restore stability. Multiplying $A\beta$ by the V_2/V_1 vector for the network chosen, frequency by frequency, we obtain the Nyquist locus for the modified amplifier. The added network must clearly be such as will prevent the locus from enclosing the point (1, 0), and Bode (page 111) has developed procedures for the design of such networks.

Voltage Feed-back and Current Feed-back

It has already been shown (page 105) that the application of negative feed-back stabilizes the gain of an amplifier, i.e. the gain becomes relatively independent of $(VA)_0$, the gain of the original amplifier before feed-back was applied. If the product $n(VA)_0$ is made large compared to unity, the gain becomes approximately equal to $1/n$. Now Fig. 46 (*a*) is the circuit of a negative feed-back amplifier but care must be used in applying these conclusions to this particular circuit. If, for instance, the gain of the "original amplifier" be changed by altering the value of the load R, we might expect at first sight, that the gain of the overall amplifier would be relatively unaffected. But this is not true, because changing the value of R changes also the feed-back ratio n, which is equal to R_1/R. Moreover if the load of this amplifier is not simply a pure resistance, but has reactance, it is obvious that the feed-back ratio, R_1/Z, becomes both complex and frequency-dependent; in such a case we are no longer feeding back a constant fraction of the output voltage, nor is the fed-back voltage exactly 180° out of phase with V_1. What we are doing is to feed-back a voltage which is at every instant directly proportional to the *output current.*

Let us look at this simple circuit from the point of view of Error Actuation. The grid voltage or "error" is the difference of the externally applied input voltage V_1 and the p.d. R_1I_a. Thus, if the gain be sufficient, this error will be kept small, i.e. the circuit tends to maintain R_1I_a equal to V_1. This it will do, or tend to do, irrespective of variations in the valve parameters or the load impedance, even if the latter be made reactive—provided only that the gain remains high. What is stabilized is therefore *not* the output voltage (i.e. the alternating p.d. across the load) but the alternating p.d. across R_1. In terms of load quantities, we may say that the alternating *current* through the load, I_a, tends to be maintained equal to V_1/R_1. It is thus not the gain (V_2/V_1) which is stabilized, but the *transfer-admittance*, (I_a/V_1).

When the fed-back voltage is directly proportional to the output *voltage* we say that the feed-back is Voltage Feed-back; when it is directly proportional to the output *current*, the feed-back is said to be Current Feed-back. Voltage feed-back stabilizes the gain (V_2/V_1) of the overall amplifier, making it approximately equal to the reciprocal of the feed-back ratio. Current feed-back stabilizes the transfer-admittance (I_2/V_1) of the overall amplifier, making it approximately equal to the admittance of the element across which the feed-back voltage is derived.

The distinction between voltage feed-back and current feed-back is in one sense an artificial distinction, depending on the definition of the output voltage. The circuit of Fig. 46 (*a*) for instance, may be said to have voltage feed-back or current feed-back respectively, according as the output voltage is taken to be the p.d. across R_1 or the p.d. across R, the circuit remaining exactly the same. The circuits of Figs. 64 and 68 have voltage feed-back and current feed-back respectively, if in each case the output voltage be taken as the p.d. across the load of the last valve.

Effect of Feed-back on Input and Output Impedances

The quantity known as the Output Impedance has been defined in Chapter I (page 5). It may be thought of as the internal impedance of the equivalent simple source. The output impedance of an amplifier which is free from feed-back is simply equal to the valve impedance, ρ, of its output stage. The application of negative feed-back changes the output impedance, however, and we shall show that voltage feed-back decreases the output impedance whereas current feed-back increases it.

Let Fig. 73 (*a*) be the equivalent circuit of an amplifier without feed-back, but to which we are about to apply feed-back. (For instance, Fig. 73 (*a*) might represent the amplifier of Fig. 64, but with V_1 disconnected from the position there shown, and connected instead between the grid of the first valve and h.t. negative.) The output impedance is Z_0, and the output e.m.f., E_0 is equal to AV_1 where the vector A is the product of the gain of all the previous stages and $-\mu$ for the output valve. Both Z_0 and E_0 are thus independent of the load impedance Z, as they are required to be by Thevenin's Theorem (see pages 5 and 6).

Now let a voltage βV_2 be fed back to the input circuit as shown in Fig. 73 (*b*). E_0 in this figure now becomes $A(V_1 + \beta V_2)$ which is no

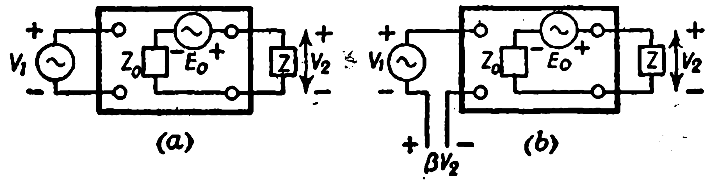


FIG. 73

longer independent of the load impedance Z, since V_2 is a function of Z, being given by—

$$V_2 = \frac{Z}{Z + Z_0} \cdot E_0$$
$$= \frac{Z}{Z + Z_0} \cdot A(V_1 + \beta V_2)$$

Thus the circuit as drawn in Fig. 73 (*b*) is not the Thevenin equivalent circuit of the feed-back amplifier, and Z_0 (the output impedance of the amplifier before feed-back was applied) can no longer be taken as the output impedance. To determine the output impedance of the amplifier when feed-back is applied, we may solve for V_2 from the above equation, giving

$$V_2 = \frac{AZV_1}{Z_0 + Z(1 - A\beta)} \quad . \quad . \quad . \quad \text{(IV.29)}$$

First let us consider the case of voltage feed-back, and determine the output impedance by writing V_2 in the form of equation (I.2) in which a and b must be independent of Z. For voltage feed-back, as distinct from current feed-back, the quantity β is independent of Z and we may write

$$V_2 = \frac{\dfrac{A}{1 - A\beta} \cdot V_1 Z}{\dfrac{Z_0}{1 - A\beta} + Z}$$

showing that the output e.m.f. is $V_1A/(1 - A\beta)$ and the output impedance is $Z_0/(1 - A\beta)$. For *negative* feed-back, $A\beta$ is real and negative, and thus we see that the output impedance is reduced by negative voltage feed-back.

In the case of current feed-back, the fed-back voltage, βV_2, is directly proportional to the output current I_2. Let this voltage be derived by the passage of I_2 through an impedance Z_{FB},* so that

$$\beta V_2 = \pm Z_{FB} I_2$$

the positive or negative sign being taken according to the particular method of connexion used (which depends upon the number of stages).

Thus
$$\beta = \pm Z_{FB} I_2 / V_2$$
$$= \pm Z_{FB}/Z$$

Substituting this into equation (IV.29) and re-arranging the resulting expression so that it is in the form of equation (I.2) we have—

$$V_2 = \frac{A V_1 Z}{Z_0 \pm A Z_{FB} + Z} \quad . \quad . \quad . \text{(IV.30)}$$

showing that the output e.m.f. is $A V_1$ and the output impedance is $(Z_0 \pm A Z_{FB})$. For *negative* feed-back $A\beta$ must be real and negative, i.e. the denominator in equation (IV.29) or equation (IV.30) must be greater than $Z_0 + Z$, and hence $\pm A Z_{FB}$ in equation (IV.30) must be positive. Thus the output impedance is increased by negative current feed-back.

As examples of these effects upon the output impedance, we may consider the voltage feed-back circuit of Fig. 63 and the current feed-back circuit of Fig. 46 (*a*). For the latter, it is clear from equation (III.18), and the equation above it, that the output impedance is increased to $\{\rho + R_1(1 + \mu)\}$. For the former (Fig. 63) we have

$$(VA) = \frac{V_2}{V_1} = \frac{\mu R}{\rho + R + \mu R R_2/(R_1 + R_2)}$$
$$= \frac{\mu R}{\rho + R + \mu n R}$$

where n denotes $R_2/(R_1 + R_2)$.

Writing this in the form of equation (I.2)

$$V_2 = \frac{\dfrac{\mu}{1 + \mu n} \cdot V_1 R}{\dfrac{\rho}{1 + \mu n} + R}$$

whence it is seen that the output impedance has been reduced to $\rho/(1 + \mu n)$.

By using a combination of voltage feed-back and current feed-back, known as Bridge Feed-back, the output impedance may be maintained the same as in an amplifier without feed-back. Fig. 74 shows the equivalent circuit of an amplifier with bridge feed-back, the "bridge" being the closed circuit $\rho\ R_1\ R_2\ R_3$, with the output voltage taken across one diagonal and the feed-back voltage across

* This impedance, Z_{FB}, is of course part of Z_0.

the other. If, for simplicity, we assume that $(R_1 + R_2) \gg Z$, it is easy to show that

$$V_2 = \frac{\dfrac{\mu V_1 Z}{1 + \mu n}}{\dfrac{\rho + R_3(1 + \mu)}{1 + \mu n} + Z}$$

where n again denotes $R_2/(R_1 + R_2)$. The output impedance is thus given by

$$\frac{\rho + R_3(1 + \mu)}{1 + \mu n}$$

If R_1R_3 is made equal to ρR_2, this expression for the output impedance reduces to $\rho + R_3$, and thus the output impedance is unaffected by the application of feed-back. The gain, however, is reduced and the normal advantages of negative feed-back remain.

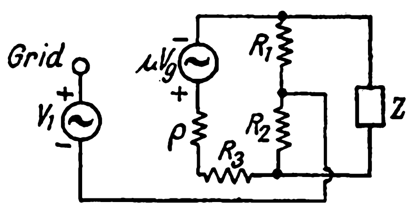


FIG. 74. EQUIVALENT CIRCUIT OF THE BRIDGE FEED-BACK

It should be noted that if a feed-back amplifier (other than the bridge type) is used in conjunction with other apparatus in such a way that impedance matching is required, it is necessary to match the load to the output impedance *as modified by feed-back.*

The application of feed-back affects also the value of the input impedance of an amplifier. The input impedance is increased or decreased according as the input voltage and fed-back voltage are combined in series (as in all the circuits considered hitherto), or in shunt. An example of the increase in input impedance caused by series-connected feed-back is given in the section headed "The Cathode-follower stage." An example of the decrease caused by shunt-connected feed-back is given in the next section.

Shunt-connected Feed-back

All the feed-back circuits considered up to this point have used series-connected feed-back. The fed-back voltage has been inserted into a break in one of the input leads, as in Fig. 73 (*b*), so that it has appeared in series with the input-voltage source.

Fig. 75 (*a*) illustrates shunt-connected feed-back. The output terminals of the amplifier are connected back to the input terminals through an impedance Z_2. In Fig. 75 (*a*) only one of the output terminals needs to be connected in this way; the other is already connected to one input terminal by way of the common h.t. negative line, or "earth" line. Using our terms rather loosely, we may say that the output is shunted across the input. The generalized shunt-connected feed-back circuit of Fig. 75 (*a*) is the basis of Differentiating Circuits, Integrating Circuits (page 329), Adding circuits in analogue computers (page 125), and the See-saw circuit (page 123).

If the input source had zero internal impedance and there were no series impedance Z_1, then the making of the Z_2 connexion, shunting the output across the input, would leave the input voltage unchanged. The voltage-amplification, or gain, would be unchanged.

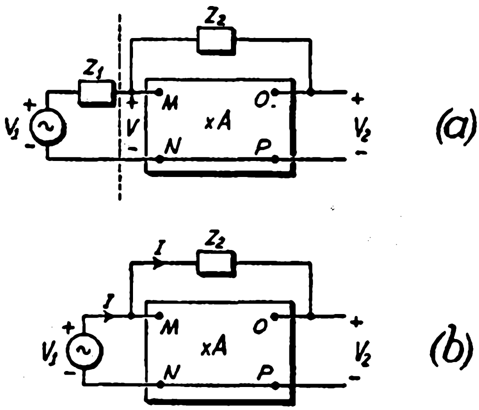


FIG. 75. SHUNT-CONNECTED FEEDBACK

The situation would be as illustrated in Fig. 75 (*b*). As a step in analysing the circuit of Fig. 75 (*a*) we shall consider Fig. 75 (*b*) in detail.

We shall assume that the amplifier represented by the rectangle has infinite input impedance and that it has the property of giving a voltage, V_2, across its output terminals, OP, which is always A times the voltage V_1 which is applied across its input terminals, MN (A may be positive or negative or complex, according as the phase-shift of the amplifier is zero, 180 degrees or some other value). Since the source, V_1 (which has no impedance), is connected directly across the input terminals, the voltage across these terminals will be V_1 and the output voltage, V_2, will be AV_1. Since the amplifier represented by the rectangle has infinite input impedance, the current I sent by V_1 must all flow through Z_2, as indicated in Fig. 75 (*b*). This current must be equal to $(V_1 - V_2)/Z_2$ which is the same as $V_1(1 - A)/Z_2$. The ratio V_1/I gives the effective input impedance, Z_{in}, presented to the source V_1.

$$\therefore \qquad Z_{in} = Z_2/(1 - A) \qquad . \quad . \quad . \text{(IV.31)}$$

It is now a simple matter to deduce the properties of the shunt-connected feed-back amplifier of Fig. 75 (*a*). The part of the circuit to the right of the dotted line is the same as the circuit of Fig. 75 (*b*). Thus the input impedance, looking to the right from the dotted line, must be the same as that given in equation (IV.31). This input impedance and Z_1, taken together, form a voltage-divider connected across V_1. The voltage at the input terminals, *MN*, is thus given by $V_1 Z_{in}/(Z_1 + Z_{in})$ and the output voltage will be A times this. The overall gain of the circuit is thus

$$(VA) = V_2/V_1 = AZ_{in}/(Z_1 + Z_{in})$$

Substituting for Z_{in} from equation (IV.31) and re-arranging

$$(VA) = \frac{A}{1 + (Z_1/Z_2)(1 - A)} \quad . \quad . \quad \text{(IV.32)}$$

The feed-back may be made *negative* feed-back by making A real and negative (e.g. as in a single-stage amplifier with a resistance load) and using pure resistances, R_1 and R_2, for Z_1 and Z_2. This particular case is illustrated in Fig. 76, in which C represents a d.c. blocking condenser, having so small a reactance that it may be considered a short-circuit to A.C. Since A is here real and negative, we may write it equal to $-|A|$, where $|A|$ is the *numerical value* of the voltage-amplification of the amplifier represented by the rectangle in Fig. 75 (*a*). Substituting in equation (IV.32), we see that the overall amplification of this negative feed-back amplifier is given by

$$(VA) = \frac{-|A|}{1 + (R_1/R_2)(1 + |A|)} \quad . \quad . \quad \text{(IV.33)}$$

If $|A|$ is large enough, this expression for the amplification reduces to $-R_2/R_1$, i.e. the overall amplification becomes independent of the amplification of the amplifier A.

A direct analysis of the particular circuit of Fig. 76, based upon its equivalent circuit, gives*

$$(VA) = V_2/V_1 = \frac{r - R_2|A|}{r + R_2 + R_1(1 + |A|)}$$

in which r denotes $\rho R/(\rho + R)$ and $|A|$ denotes $\mu R/(\rho + R)$. This reduces to equation (IV.33) if R_1 and R_2 are large with respect to ρ.

The input impedance of the circuit of Fig. 76 is seen from equation (IV.31) to be $R_1 + R_2/(1 + |A|)$.

A simple method of evaluating the output impedance is to suppose the input voltage turned down to zero and a test-e.m.f., V, substituted for the load, R, in the anode circuit of Fig. 76. The output impedance will then be given by V/I, where I is the current supplied by the test-e.m.f. If the resistances R_1 and R_2 are large with

* It is of interest to note that the gain becomes zero if R_2 is given the value $1/g_m$.

respect to ρ, the current I will be given by $(V + \mu V_g)/\rho$. Since V_g will be merely $VR_1/(R_1 + R_2)$, it follows that the output impedance must be

$$Z_0 = \frac{\rho}{1 + \mu R_1/(R_1 + R_2)}. \quad . \quad . \quad (IV.34)$$

Except in cases where R_1 is much smaller than R_2, this will be a fairly low output impedance. If $\mu \gg 1$ and R_1 is of the same order

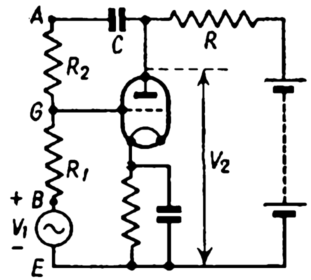


FIG. 76. SINGLE STAGE WITH SHUNT-CONNECTED NEGATIVE FEEDBACK

of magnitude as R_2, equation (IV.34) may be written in a form which relates the output impedance to the overall gain of the amplifier, viz.

$$Z_0 \asymp \text{approximately } \frac{1 + |A|}{g_m} \quad . \quad . \quad (IV.35)$$

The Seesaw Circuit

The Seesaw circuit makes use of the circuit of Fig. 76, with R_1 made equal to R_2. Hence from equation (IV.33)

$$(VA) = \frac{-|A|}{2 + |A|}$$

which is approximately equal to -1, *irrespective of the value of* A, *if* A *is large enough.* The usefulness of the circuit is as a phase-inverter whose output voltage magnitude is automatically held equal to its input voltage magnitude, even though A may alter (i.e. even though l.t. and h.t. supply voltages may fluctuate or valves be replaced). The chief application of this stabilized phase-inverter is as a push-pull input stage. As such it has advantages over the two push-pull input circuits shown in Fig. 49, page 77. For instance, the balance of the two output voltages in Fig. 49 (*a*) will be disturbed if the gain of the second stage varies, while the balance in Fig. 49 (*b*) will be disturbed if unequal impedances are connected across the two output voltages. The balance of the two output voltages of the seesaw circuit (V_1 and V_2, Fig. 76) is undisturbed both by change of gain and change of load impedance.

A more precise balance may be obtained by making R_2 slightly greater than R_1. To determine the required ratio, we may write (VA) equal to -1 in equation (IV.33) and solve for R_1/R_2. This gives

$$\frac{R_2}{R_1} = \frac{|A| + 1}{|A| - 1}$$

i.e.

$$\frac{R_2}{R_1} = \frac{(\mu + 1)R + \rho}{(\mu - 1)R - \rho}$$

The name "Seesaw Circuit" is derived from the "Tilting-line" diagram of Fig. 77, which is sometimes used to explain the operation of this circuit. This diagram is simply the graph of the potential distribution along the total resistance $R_1 + R_2$, the ordinates being potentials with respect to h.t. negative and AB being a scale of resistance. The letters $ABEG$ correspond to the lettering on Fig. 76. Since the potentials of points A and B are the instantaneous values of v_1 and v_2, and these voltages are 180° out of phase, we see that there must be some point E which is permanently at zero potential. When the balance is exact (viz. $\hat{V}_2 = \hat{V}_1$) this "fulcrum" will be at the mid-point of AB. It follows that an exact balance cannot be secured by tapping the grid G on to the precise mid-point of $(R_1 + R_2)$, for if that were so there would be zero alternating grid-voltage.

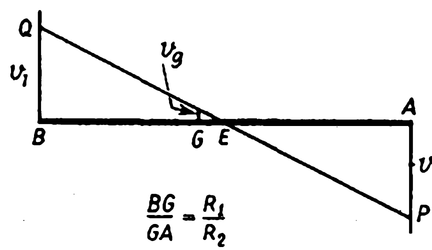


FIG. 77

Fig. 78 shows how the output from a previous stage may be coupled to the input of the seesaw circuit, the two output voltages being taken from AE and BE. This circuit is derived from Fig. 76 as follows. The source V_1 is removed from Fig. 76 and the leads BE are connected instead, across the output voltage of a previous stage. The output terminals of the previous stage are taken to be its anode and h.t. negative. To avoid feeding a steady voltage component to the grid circuit of the seesaw valve, the direct connexion between this grid and the point G (Fig. 76) is replaced by an R–C coupling circuit $R'C'$ (Fig. 78). This makes the blocking condenser (C, Fig. 76) unnecessary. The other feature of note in Fig. 78 is that the two valves are not provided with separate automatic-bias circuits, but both derive their grid-bias from a common bias resistor R_B. The current through this resistor is the sum of the two anode-currents. For the alternating components of these anode-currents we have

$$I_{a_1} = - V_1/R_L$$

and $$I_{a_2} = - V_2/R$$

where V_1 and V_2 have the same meaning as in Fig. 76. Since V_1 and V_2 are equal in magnitude and 180° out of phase, we can make this total alternating component zero by making R_L equal to R. The condenser C_B may then be dispensed with.

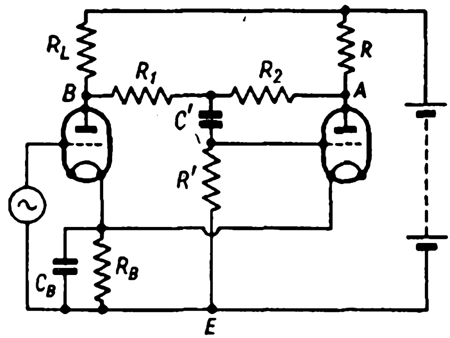


Fig. 78. The Seesaw Circuit

It is not sufficient to make R_1 equal to R_2 in Fig. 78, since the resistance in Fig. 78, which really corresponds to R_1 in Fig. 76, is the parallel combination of R' with $(R_1 + \rho_1)$ where ρ_1 is the valve impedance of the first valve. In general R_1 will be large with respect to ρ_1. But it is not usually possible to make R' large with respect to R_1, since R_1 must itself be large with respect to R_L in order that the second stage shall not have too low an input impedance. A single resistance with a variable tapping may be used for $(R_1 + R_2)$ and the tapping point may then be adjusted by experiment. It is perhaps worth noting that the two output voltages of this phase-inverter are associated with two different output impedances.

The seesaw circuit is also known as the Anode Follower, since the anode-potential follows the potential of the input terminal (albeit 180° out of phase). The name Anode Follower has also been used (or misused?) to describe circuits based upon Fig. 76 in which R_1 is not even approximately equal to R_2.

Adding Circuits

The requirement often arises, particularly in analogue computers, that the output voltages of a number of amplifiers shall be added. Almost always, these output voltages have each one side "earthed" (i.e. one of the two output terminals of each amplifier is connected to a common negative line). The simple method of addition, by connecting all the output voltages in series, is thus precluded.

Fig. 79 (a) shows a method of producing an output voltage, v_0, which is proportional to the sum of the four voltages v_1, v_2, v_3, and v_4, each of which has one side "earthed." The calculation of v_0 in terms of v_1, v_2, v_3, v_4 and R is most easily accomplished by using a modification of Thevenin's Theorem to replace the whole of the circuit to the left of the dotted line by a single e.m.f., e_0, in series with a single resistance, R_0. R_0 will be $R/4$, being the impedance seen looking to the left from the dotted line, with all e.m.f.s turned down to zero. The modification of Thevenin's Theorem (sometimes known as Norton's Theorem) which enables us to derive e_0, consists of the rule that e_0 is given by the product $R_0 i_{sc}$, where i_{sc} is the current that would flow through a short-circuiting link if connected across the load R_L. Clearly i_{sc} would be given by $(v_1/R + v_2/R$

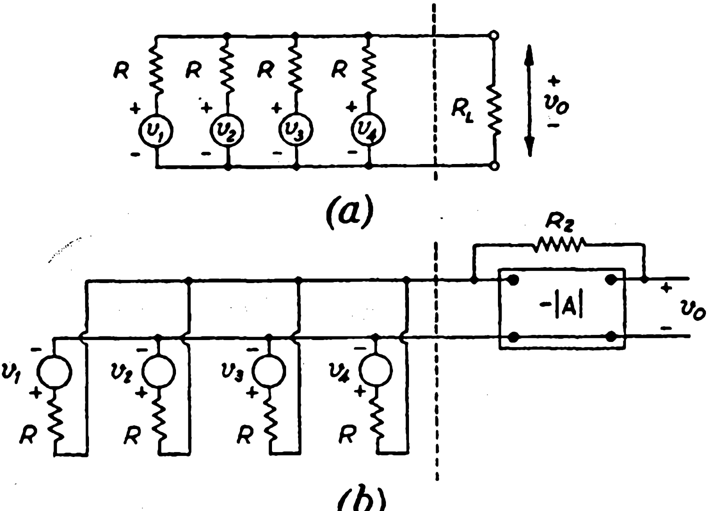


FIG. 79. ANALOGUE ADDING CIRCUIT

$+ v_3/R + v_4/R)$. It follows that e_0 must be $(v_1 + v_2 + v_3 + v_4)/4$, and that

$$v_0 = \tfrac{1}{4}(v_1 + v_2 + v_3 + v_4) \cdot \frac{R_L}{R_L + R/4}$$

In general, with n voltages to be added,

$$v_0 = \frac{\frac{1}{n} R_L \Sigma v_1}{R_L + R/n} = \frac{\Sigma v_1}{n + R/R_L} \quad . \quad . \quad \text{(IV.36)}$$

In an analogue computer, the number n of voltages which a particular adding circuit is required to add on any one occasion will vary according to the particular computation being undertaken. It is thus a requirement that the constant of proportionality between v_0 and v_1 shall be independent of n. Equation (IV.36) shows that approximate independence of n can be achieved by making R/R_L large compared to the largest number of voltages ever to be added. This involves making the constant of proportionality very small, which in turn means that additional amplification must be provided.

This is where shunt-connected feed-back can be used with advantage, as shown in the adding circuit of Fig. 79 (*b*). The amplifier denoted by the rectangle is designed to have a phase-shift of 180 degrees and its vector gain A may therefore be written as $-|A|$. The set of sources to the left of the dotted line is equivalent to a single source having an e.m.f. $(1/n)\Sigma v_1$ and an internal resistance R/n. From equation (IV.31), page 121, the input impedance looking to the right at the dotted line is $R_2/(1+|A|)$. Thus the input voltage of the amplifier denoted by the rectangle is

$$(1/n)\Sigma v_1 \cdot \frac{R_2/(1+|A|)}{R/n + R_2/(1+|A|)}$$

The output voltage, v_0, being $-|A|$ times this, is given by

$$v_0 = \frac{-|A|\Sigma v_1}{n + (R/R_2)(1+|A|)}. \qquad \text{(IV.37)}$$

The constant of proportionality between v_0 and v_1 may thus be made more or less independent of n, for any desired values of R and R_2, merely by making $|A|$ large enough. For example, the requirement might be that the constant of proportionality with n equal to 2 shall not differ by more than one per cent from the constant of proportionality with n equal to 10. The necessary value of $|A|$ to ensure this could then be found by solving for $|A|$ in the equation.

$$2 + (R/R_2)(1+|A|) = \tfrac{99}{100}\{10 + (R/R_2)(1+|A|)\}$$

and would be given by $790R_2/R$.

Note that, as $|A|$ approaches infinity, the constant of proportionality approaches $-R_2/R$. Thus the adding circuit of Fig. 79 (*b*) enjoys the distortion-reducing properties of negative feed-back. In general, for use in an analogue computer, the amplifier will need to be a d.c. amplifier.

Wide-band Amplifiers

When an amplifier is required to handle a wide range of frequencies (e.g. from below the lowest audio-frequency to two or three megacycles per second, as in the case of video-frequency amplifiers for television) the difficulty of securing uniform amplification for all frequencies becomes considerable. The chief difficulty is that the input impedance of each stage is effectively connected across the load of the previous stage, and that this input impedance is a function of frequency, becoming small at the higher frequencies. By using pentodes (see page 97) we may avoid the severe reduction of input impedance which would result from the grid-anode capacitance of a triode. But there remains the grid-cathode capacitance and the effect of the residual grid-anode capacitance, so that at frequencies of more than about a megacycle per second it is impossible to produce input impedances of more than a few thousand ohms. In single-frequency amplifiers, using tuned circuits, this input impedance (which is largely a capacitive reactance) merely forms part of a tuned circuit and so does not lower the load impedance of the previous stage—but tuned circuits cannot, of course, be used in wide-band amplifiers.

The extent to which the amplification is reduced at a given high frequency is easily determined. Let us assume that pentodes are used and that C_{ga} is thus so small as to have negligible effect. The anode-cathode capacitance of a given stage appears directly across its load resistance R and the grid-cathode capacitance of the next stage also appears in parallel with this. Let the total effective capacitance which shunts the load be C, so that the total effective load impedance, Z, is the parallel combination of R and $1/j\omega C$. We then have

$$(VA) = \mu Z/(\rho + Z) = \mu/(1 + \rho/R + j\omega C\rho)$$

The magnitude of this is

$$(VA) = \frac{\mu}{\sqrt{(1 + \rho/R)^2 + (\omega C\rho)^2}} \quad . \quad . \quad (IV.38)$$

The response curve (i.e. the graph of amplification against frequency) thus falls off at high frequencies. At low frequencies the gain is simply $\mu R/(\rho + R)$, but it is reduced to $1/\sqrt{2}$ times this value, i.e. reduced by 3 decibels when the frequency has a value given by

$$1 + \rho/R = \omega C\rho$$

i.e.

$$f_c = \frac{1/\rho + 1/R}{2\pi C} \quad . \quad . \quad . \quad . \quad . \quad (IV.39)$$

$$= \text{approx. } 1/(2\pi RC) \text{ for pentodes}$$

where f_c is the frequency at which the response curve has dropped to $1/\sqrt{2}$ of its maximum height (sometimes spoken of as the Cut-off Frequency or Half-power Frequency). The value of this frequency may be increased (i.e. the flat portion of the response curve may be prolonged to a region of higher frequency) by decreasing the value of R, although this has the disadvantage of lowering the height, viz. $\mu R/(\rho + R)$, of the flat portion.

This may be expressed by saying that one may barter gain for bandwidth. Neglecting any low-frequency cut-off, the bandwidth is from 0 c/s to the value of f_c given in (IV.39). The product of this bandwidth and the gain, $\mu R/(\rho + R)$, will be seen to be

$$\text{Gain} \times \text{Bandwidth} = g_m/2\pi C$$

The value of C can be reduced by a layout which minimizes stray wiring capacitances but it cannot be reduced below an absolute minimum value which is the sum of C_{ac} for the valve concerned and C_{gc} for the following valve. It is interesting to note that this "gain-bandwidth product" is a function only of the valve.

The overall gain of a multi-stage amplifier is the product of the gains of the individual stages. Assuming n identical stages, the overall gain will be the right-hand side of (IV.38) raised to the nth power. Abbreviating by writing A for $\mu R/(\rho + R)$ and r for $\rho R/(\rho + R)$, we may write this overall gain of an n-stage amplifier as

$$(VA)_n = A^n(1 + \omega^2 r^2 C^2)^{-n/2} = A^n(1 + \omega^2/\omega_c^2)^{-n/2}$$

where ω_c denotes the cut-off value of ω for a single stage of the amplifier, viz. $1/(rC)$.

The cut-off frequency of the overall n-stage amplifier is defined in the same way as for a single-stage amplifier. It is the frequency at which the overall gain is reduced to $A^n/\sqrt{2}$. We may therefore evaluate the cut-off frequency of the n-stage amplifier by solving for ω in the following equation

$$A^n/\sqrt{2} = A^n(1 + \omega^2/\omega_c^2)^{-n/2}$$

whence

$$\omega = \omega_c\sqrt{2^{1/n} - 1} \quad . \quad . \quad . \quad \text{(IV.40)}$$

i.e. the upper cut-off frequency for the n-stage amplifier is $\sqrt{2^{1/n} - 1}$ times the upper cut-off frequency for a single stage.

Various devices have been used for securing a further prolongation of the flat portion of the response curve, including the addition of inductance in series with the load R, forming, in conjunction with the coupling capacitance, a heavily-damped tuned circuit. There are also a number of circuits for reducing the frequency distortion at the low-frequency end of the response curve. In particular, the reduction of gain due to the automatic-bias circuit may be offset by the increase of gain (as the frequency decreases) arising from the decoupling circuit.

The Cathode Follower Stage

We have seen above that a multi-stage amplifier may be made to cater for a wide range of frequencies by reducing the values of the load resistances, the necessary reduction depending upon the magnitudes of the inter-electrode (and stray) capacitances and the highest frequency which is to be amplified. This entails a loss of amplification or, if a stated amplification is required, an increase in the number of stages. In practice, conditions are often such that the number of stages required would be more than doubled. In these circumstances it is more economical to use a "buffer stage" between successive amplifier stages.

The cathode follower is such a buffer stage and is connected between two stages of such an amplifier for the purpose of avoiding

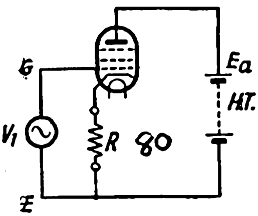


FIG. 80. CATHODE FOLLOWER

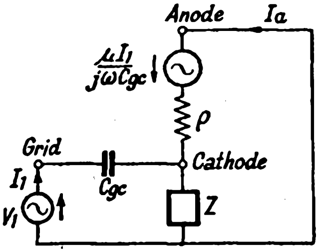


FIG. 81

the connexion of the low input impedance of the second stage across the high load impedance of the first. It is clearly necessary that

(i) The cathode follower stage shall not itself have a low input impedance;

(ii) The output voltage of the cathode follower shall not be unduly reduced by the connexion to it of the low input impedance of the second stage. In other words, the cathode follower shall have a low output impedance. (See definition of output impedance, page 5.)

As we shall see, the cathode follower satisfies these requirements, but it gives no amplification. The circuit is shown in Fig. 80, in which V_1 represents the input voltage (from stage 1 of the amplifier) and the voltage across the high resistance R is the output voltage (connected to the input circuit of stage 2 of the amplifier). The total effective load impedance of the cathode-follower will be the parallel combination of the resistance R and the input impedance of the next stage, and we shall denote this by Z. From the equivalent circuit of this simple stage (which will be similar to Fig. 46 (b), but with the right-hand resistance short-circuited and Z written for R_1) we see that

$$I_a = \mu V_g/(\rho + Z)$$

and $$V_g = V_1 - ZI_a$$

giving $$V_2 = ZI_a = \frac{\mu V_1 Z}{\rho + Z(1+\mu)}$$

This expression for the output voltage may be written as

$$\frac{V_1 \dfrac{\mu Z}{1+\mu}}{\dfrac{\rho}{1+\mu} + Z}$$

showing that this same voltage would be produced across Z were the latter connected not to the cathode follower but to an a.c. generator of internal impedance $\rho/(1 + \mu)$ and having an e.m.f. equal to $\mu V_1/(1 + \mu)$. The output impedance of the cathode-follower is therefore $\rho/(1 + \mu)$, or approximately $1/g_m$. It follows that, so long as the load impedance Z is large with respect to $1/g_m$, the voltage across the load of the cathode follower will be

$$\mu V_1/(1 + \mu),$$

or approximately equal to the input voltage.

Since the value of $1/g_m$ will be only a few hundred ohms for most valves, the output voltage of the cathode follower (for a constant alternating input voltage) will remain constant even though the input impedance of the succeeding stage falls as low as a few thousand ohms. Condition (ii) above is thus satisfied, and we see that the cathode follower gives, of itself, no amplification, its output voltage being approximately equal to its input voltage. Moreover, since the voltage across R is approximately equal to the input voltage V_1, and these two voltages round the grid circuit are 180° out of phase, it follows that the alternating voltage between grid and cathode is very small, The cathode potential may be said to "follow" the grid potential.

In order that the input impedance of the cathode follower shall be high, it is essential that no grid-current shall flow. The requisite grid-bias voltage to ensure this is automatically provided by the d.c. RI drop across R. The chief remaining factor operating against high input impedance is the grid-cathode capacitance, C_{gc}. The alternating voltage across this capacitance is equal to the alternating grid-voltage, and we have seen that this is very small. It follows that the current through C_{gc} will be much smaller than if the whole input voltage were connected across it, and thus that the input impedance is much larger than the impedance of C_{gc} alone. Fig. 81 shows the equivalent circuit taking account of C_{gc}. Applying Kirchhoff's laws to the two closed circuits and solving for I_1 we find that the input impedance vector is given by

$$Z_{in} = \frac{V_1}{I_1} = \frac{1}{j\omega C_{gc}} + \frac{Z}{\rho + Z} \cdot \left(\rho + \frac{\mu}{j\omega C_{gc}}\right)$$

For a pentode, ρ will be much larger than Z, so that Z_{in} reduces to

$$Z_{in} = Z + \frac{1 + g_m Z}{j\omega C_{gc}}$$

Hence, provided $g_m Z$ is large with respect to unity, the input impedance will be large with respect to the reactance of C_{gc}. Both from considerations of output impedance and of input impedance therefore, the design rule for the cathode follower is that Z shall be large with respect to $1/g_m$ at all relevant frequencies. Since the screen-grid of a pentode is effectively connected to the cathode so far as A.C. is concerned, the inter-electrode capacitance existing between screen-grid and control grid, C_{gs}, will be directly in parallel with C_{gc}, so that the above input impedance will become the sum of the load Z and the impedance of a capacitance equal to

$$(C_{gc} + C_{gs})/(1 + g_m Z)$$

In a cathode follower using a triode valve, the anode will be effectively connected to h.t. negative (i.e. earth) so far as A.C. is concerned, and thus the grid-anode capacitance will appear directly across the input voltage, in parallel with the value of Z_{in} given in the above equation.

The cathode follower may be regarded as a negative feed-back amplifier using voltage feed-back, and with a feed-back ratio of 100 per cent. As shown in the section on "Negative Feed-back Amplifiers," such an amplifier would have a low output impedance and a voltage amplification of approximately unity. More important, the cathode-follower enjoys the other advantages of negative feed-back, viz. independence of reasonably large variation in supply voltages and valve parameters, and freedom from distortion and noise. The use of the cathode follower as a buffer amplifier is, of course, not confined to wide-band amplifiers. It may be used wherever a low impedance load is to be connected to a high impedance source.

Dynamic Characteristics of the Cathode Follower

A warning has already been given that any analysis based on the valve equivalent circuit is accurate only if the alternating components of v_a, v_g and i_a are small. This condition is satisfied in many applications of the cathode follower, since V_g is $1/(1 + \mu)$ of the input voltage V_1, as a result of the inherent negative feed-back. The values of ρ and μ which are to be used in equations derived by the use of the equivalent circuit are, however, the values corresponding to the d.c. operating point, which is defined by the values of v_g and v_a in the quiescent condition. In this section we shall deal with the calculation of the quiescent condition for a given cathode-follower circuit, using a graphical method which can be extended to determine the voltage amplification and distortion when a large alternating input voltage is applied.

Consider the circuit of Fig. 80 and assume that the valve characteristics are given and that the values of E_a and R are specified. The following three relations must be satisfied—

(i) $v_g = V_{GE} - Ri_a$

(ii) $v_a = E_a - Ri_a$

(iii) The relation between i_a, v_a and v_g which is expressed graphically by the static characteristics of the valve.

By subtracting the first two of these equations, we obtain

$$v_a - v_g = E_a - v_{GE}$$

or (iv) $$v_{GE} = E_a - v_a + v_g$$

This is in a form more suitable for our present purpose than equation (i). The method of calculation is simply the graphical solution of the three simultaneous relations (ii), (iii) and (iv) above. Relations (ii) and (iii) can be combined by the conventional load-line method giving a diagram similar to Fig. 20. The co-ordinates of the points of intersection tell us how i_a (or v_a) varies with v_g, but until equation (iv) is used we do not know how i_a varies with v_{GE}. By using equation (iv), however, we can assign a value of v_{GE} to each point of intersection, since the values of v_a and v_g can be read from the graph and E_a is specified. Plotting i_a against v_{GE} we then have the dynamic characteristic of the cathode follower. This will be of the form shown in Fig. 83, from which we can determine the quiescent anode-current as the intercept OP on the vertical axis.

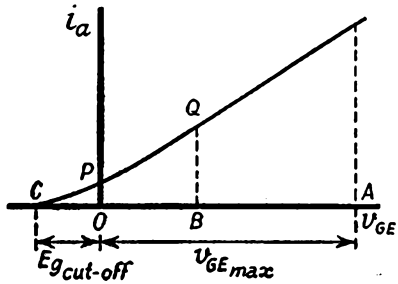


FIG. 82

The values of the valve parameters corresponding to this quiescent condition can then be determined from the static characteristics. Since the quiescent point for the circuit of Fig. 80 is usually found to be low down in the curved parts of the characteristics, the values of the parameters will usually be degraded as compared with those stated by the valve manufacturer for the straight parts. For this reason, and for a further reason explained below, it is usually advisable to use a positive grid-bias voltage with the cathode follower, as shown in Fig. 83 (*a*) and (*b*).

To avoid severe distortion the grid-voltage, v_g, must be kept within certain limits. Just as in the case of a basic amplifier, v_g must never become positive lest grid-current should flow; nor must it become so negative that the anode-current is cut off. The points on the load line corresponding to these two limiting conditions are (*a*) the intersection of the load line with the graph labelled $v_g = 0$, and (*b*) the intersection of the load line with the axis of v_a. Using equation (iv) we can assign a value of v_{GE} to each of these points. The value of v_{GE} corresponding to (*a*) (where $v_g = 0$) may be seen to be equal to the horizontal projection of that portion of the load line intercepted between the axis of v_a and the graph labelled $v_g = 0$. We shall denote this value by v_{GEmax}; it is the maximum permissible p.d. of grid above earth. The value of v_{GE} corresponding to (*b*) is simply $- Eg_{cut\text{-}off}$, as in a basic amplifier.

Thus v_{GE} may range between $-Eg_{cut\text{-}off}$ and the large positive value v_{GEmax} (see Fig. 82). The largest peak-alternating input voltage which can be accommodated is equal to one half of this range, viz. $\frac{1}{2}(v_{GEmax} + Eg_{cut\text{-}off})$. To accommodate this maximum input voltage the quiescent point must be placed in the middle of the range, i.e. a positive bias OB must be used in the grid circuit, the point B

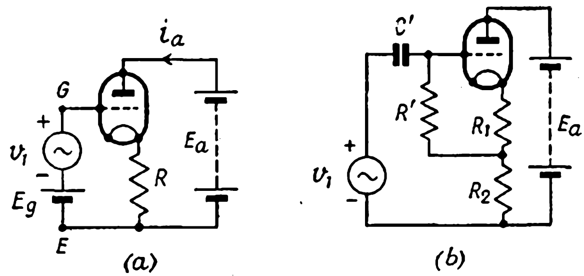


FIG. 83. BIASING CIRCUITS FOR CATHODE FOLLOWER

being midway between the points A and C. The required positive bias for maximum input is given by

$$OB = OA - BA = OA - \tfrac{1}{2}(AC)$$
$$= \tfrac{1}{2}(v_{GEmax} - Eg_{cut\text{-}off})$$

The positive grid-bias voltage may be applied either as in Fig. 83 (*a*) or using the automatic grid-bias circuit of Fig. 83 (*b*). In Fig. 83 (*b*), $R'C'$ is an a.c./d.c. separating circuit (see page 39) connected across v_1 and R_2 in series. Thus the p.d. across R' has no direct component, but is given by

$$v_{R'} = v_1 - R_2[i_a]_{AC}$$

The total grid-to-cathode p.d., v_g, is made up of $v_{R'}$ and v_{R_1} and is thus given by

$$v_g = v_1 - R_2[i_a]_{AC} - R_1 i_a$$

which may be written

$$v_g = v_1 - (R_1 + R_2)i_a + R_2[i_a]_{DC}$$

Comparing this with the grid-voltage in Fig. 83 (*a*), viz. $v_1 - Ri_a + E_g$ we see that Fig. 83 (*a*) and Fig. 83 (*b*) are exactly equivalent provided that

$$R = R_1 + R_2$$

and
$$E_g = R_2[i_a]_{DC}$$

The analysis of the circuit of Fig. 83 (*b*) for a given valve and given values of R_1 R_2 and E_a, may thus be made by applying the above graphical method to its equivalent circuit, Fig. 83 (*a*), and we can plot the dynamic characteristic (Fig. 82) in the manner already described, using a load line whose slope is $1/(R_1 + R_2)$. But E_g is not explicitly known, i.e. we have as yet no means of locating the quiescent point on the dynamic characteristic. The value of E_g depends upon E_a, R_1, R_2 and the valve characteristics. It may be determined as follows. In the quiescent condition, i.e. with no alternating input voltage, the following d.c. equations hold for the circuit of Fig. 83 (*b*)

$$v_a = E_a - (R_1 + R_2)i_a$$
$$v_g = - R_1 i_a$$

The first of these equations simply tells us that the quiescent point must lie on the load line. The second equation shows that this point will be the intersection of the load line with the locus of points for which v_g equals $- R_1 i_a$. This locus may be drawn on the load-line diagram by marking on each static characteristic curve a point whose ordinate is $1/R_1$ times the numerical value of v_g for the curve in question. The curve joining all these points is the required locus. The ordinate of the point of intersection of the locus with the load line, gives $[i_a]_{DC}$ and the product $R_2[i_a]_{DC}$ gives us the equivalent positive-bias voltage, E_g, appropriate to Fig. 83 (*b*).

We may *design* R_1 to give any desired value of quiescent anode-current $[i_a]_{DC}$ simply by marking a point on the load line having its ordinate equal to the desired quiescent anode-current, and reading from the characteristics the value of grid-voltage which corresponds to this marked point (say v_{g_0}). Then

$$R_1 = v_{g_0}/[i_a]_{DC}$$

The Cathode Follower as a Voltage Stabilizer

The cathode follower is so called because the potential of the cathode follows fairly closely the potential of the grid, i.e. in Fig. 80 the p.d. across R follows fairly closely the grid-to-earth p.d. V_1. We have seen that this is true so far as the alternating components of the two potentials are concerned. We shall show also, that if the circuit is suitably designed, the same is true of the direct components, and that if a direct voltage be substituted for V_1 in Fig. 80, the p.d. across R will remain very nearly equal to this voltage despite considerable variations in the h.t. supply voltage or in the value of R itself.

A voltage stabilizer is a device which is interposed between a supply voltage and its load for the purpose of maintaining constant the load voltage. In the absence of the voltage stabilizer (i.e. with the load connected directly across the supply) the load voltage may vary for two reasons—(*a*) because the supply voltage is varying, (*b*) because the load resistance has been changed. The second of these two effects arises from an unduly high "source impedance" (i.e. the output impedance) in the supply circuit, as a result of which, there is an appreciable internal voltage drop in the supply circuit when current is flowing to the load. Change of the load resistance alters the load current, this alters the internal voltage drop in the supply circuit, and so the load voltage changes. Such a supply circuit is said to have poor "regulation."

Perhaps the simplest of all voltage stabilizers is that shown in Fig. 84, in which a resistance R_s and a cold-cathode gas-filled diode G are connected in series across the supply voltage. Even if the

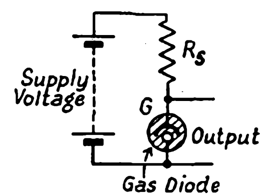


FIG. 84. SIMPLE VOLTAGE STABILIZER

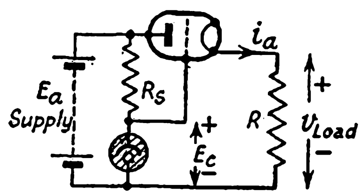


FIG. 85. CATHODE FOLLOWER VOLTAGE STABILIZER

supply voltage varies greatly, the voltage across G is scarcely altered, the current through G changing in such a way that the variation of supply voltage is taken up across R_s. This circuit has the disadvantage that it cannot supply any appreciable current to the load.

For the cathode follower circuit of Fig. 80 to be regarded as a voltage stabilizer, the h.t. voltage must be taken to represent the supply voltage, R must be taken to represent the load, and V_1 be taken to represent a very stable d.c. control voltage. It may seem a little unreasonable to include "a very stable voltage" among the requirements for producing a "stabilizer"; but the source of this very stable voltage need not be capable of producing any current, whereas the stabilizer of which it forms a part *is* capable of so doing. In fact, the circuit of Fig. 84 may be used as the source of our very stable control voltage, in which case the cathode follower stabilizer circuit becomes as shown in Fig. 85.

Using the notation of Fig. 85 we have—

$$v_a = E_a - Ri_a$$
$$v_g = E_c - Ri_a$$

Provided that the operating point of the valve is in the linear region of the characteristics, we may adopt the following equation for the valve (see page 18).

$$i_a = \frac{1}{\rho}(v_a + \mu v_g) + d$$

Solving these three simultaneous equations for i_a we have

$$v_L = Ri_a = \frac{E_a + \mu E_c + \rho d}{\rho/R + 1 + \mu} \quad . \quad . \text{(IV.41)}$$

Now we require that this voltage v_L shall be reasonably independent of the values of both E_a and R. By making $(\mu E_c + \rho d) \gg E_a$ we can make the numerator of the expression for v_L sensibly independent of E_a; and by making $(1 + \mu) \gg \rho/R$, or $g_m \gg 1/R$, we can make the denominator sensibly independent of R.

Let us consider these two requirements separately. The numerator requirement clearly demands a high value of μ. This suggests the use of a pentode. A further argument in favour of a pentode is that the quantity d (which is the ordinate of the point where the straight part of the $v_g = 0$ characteristic cuts the i_a axis, if this straight part is produced to the left) is positive for a pentode but negative for a triode. Triodes are often used, however, since pentodes have the disadvantage of requiring a stable screen-voltage.

The denominator requirement limits the applicability of this type of stabilizer to loads whose resistance is large with respect to the available values of $1/g_m$. By connecting two or more valves in parallel we can increase the effective value of g_m and so permit the application of this stabilizer to smaller loads, but this sometimes causes difficulty as a result of unequal sharing of the current between the valves.

If both of the above conditions are satisfied, and if in addition μE_c is large compared to ρd (as is often the case) and μ is large compared to unity, then equation (IV.41) shows that v_L is simply equal to the control voltage, E_c.

The cathode-follower voltage stabilizer may be thought of as the series connexion of an unstabilized supply voltage, a voltage-dropping valve, and the load. The voltage drop across the valve depends on its effective d.c. resistance, and this depends upon its grid-voltage. The circuit is so arranged that, if the load voltage falls, the grid-voltage is made to rise, so decreasing the d.c. resistance of the voltage-dropping valve and tending to reinstate the load voltage to its original value. This is but one of a large number of stabilizer circuits; for further information the reader is referred to F. V. Hunt and R. W. Hickman, "Electronic Voltage Stabilizers," *Review of Scientific Instruments*, Vol. 10, pp. 6–21, (Jan. 1939).

Energy Interchange and Anode Efficiency

Consider an output stage operating with steady anode-voltage, v_0, steady anode-current, i_0, and with alternating anode-voltage and current of peak values, $\hat{V}$ and $\hat{I}$, respectively. For resistive loads the alternating anode-voltage and current will be 180° out of phase, since an increase of anode-current means an increase of voltage across the load and therefore a decrease of anode-voltage. Thus

$$i_a = i_0 + \hat{I} \sin \omega t$$
$$v_a = v_0 - \hat{V} \sin \omega t$$

The instantaneous value of power dissipation in the valve will be

$$v_a i_a = v_0 i_0 - \hat{V}\hat{I} \sin^2 \omega t + v_0 \hat{I} \sin \omega t - \hat{V} i_0 \sin \omega t$$

The mean value of this gives the power dissipation in the valve, viz.

$$W = v_0 i_0 - \tfrac{1}{2}\hat{V}\hat{I}$$
$$= v_0 i_0 - V_{rms} I_{rms} \quad . \quad . \quad . \text{(IV.42)}$$

Now the only source of power is the high-tension supply, and the total power supplied by it *to the valve* is $v_0 i_0$. Of this an amount $V_{rms} I_{rms}$ is converted into output power (i.e. a.c.) in the load, and the remainder, as shown above, constitutes the power dissipation in the valve.

On switching off the input voltage, the power dissipation in the valve increases by an amount equal to the useful power which was being fed into the load. Energy dissipated in a valve is converted into heat at the anode, and it is not uncommon for the anodes of large valves to be run glowing red. Still larger valves are water-cooled, a stream of water flowing through a hollow anode. Under these conditions the switching off of the input voltage will cause an appreciable rise of temperature, and the valve may be destroyed. It is usual, therefore, to switch off the h.t. supply before switching off the input voltage.

The Anode Efficiency of a valve is defined as the ratio of its output power to the total d.c. power supplied to it by way of the anode circuit. In the case considered above

$$\text{Efficiency} = \frac{V_{rms} I_{rms}}{v_0 i_0} \quad . \quad . \quad . \text{(IV.43)}$$

(We are seldom interested in the overall efficiency, which takes account also of the power from the l.t. supply.)

Let us consider the maximum value of anode efficiency which can be attained. If distortion is to be avoided, only the straighter parts of the characteristic curves must be traversed, and the anode-current must certainly not be allowed to fall to zero and remain zero for part of the cycle (as in Fig. 13 for example). Let us make the optimistic assumption that the peak anode-current, $\hat{I}$, may be made as great as the steady anode-current, i_0, so that the anode-current is reduced to zero at one point of the cycle but does not remain zero for part of the cycle. Similarly, the peak anode-voltage, $\hat{V}$, may not exceed v_0, but we shall assume that it may be made as great as v_0. In these circumstances the expression for the efficiency becomes equal to 50 per cent. This is thus the maximum attainable efficiency for an approximately sinusoidal anode-current.

Class B and Class C Amplifiers

We have seen that with an approximately sinusoidal anode-current the anode efficiency cannot exceed 50 per cent. If we allow the waveform of the anode-current to be distorted so that it is not even approximately sinusoidal, much higher efficiencies may be attained.

There are two very important types of amplifier, known as Class B and Class C, which work with non-sinusoidal anode-currents, and yet contrive to give a sinusoidal output voltage. The Class C amplifier is a single-frequency amplifier using a parallel L–C circuit as load, the circuit being tuned to the frequency to be amplified, as described in the earlier part of this chapter. To determine the voltage across the load we may consider the non-sinusoidal anode-current to consist of fundamental and harmonics; each of these components, in passing through the load, will set up a voltage across it, but as the load impedance is large at the fundamental frequency and small at other frequencies all these voltages will be small except that of the fundamental frequency. The voltage across the tuned circuit is thus almost sinusoidal.

The Class B amplifier uses a push-pull circuit, the negative grid-bias voltage of each of the two valves being adjusted until it is just great enough to reduce the anode-current to zero, this value of grid-bias voltage being spoken of as the Cut-off Value. Since, during the negative half-wave of the input voltage, the resultant negative grid-voltage of the valve is greater than the cut-off value, no anode-current will flow during this half-cycle. During the positive half-wave of the input voltage, however, current will flow, and the waveform of the anode-current will consist of a series of half-waves, as shown in Fig. 86. Since the input voltages of the two valves are 180° out of phase, anode-current will flow alternately in the two valves. The effective primary current in the output transformer is the difference of the two anode-currents and is, therefore, sinusoidal.

The great advantage of the Class B amplifier is that no anode-current flows when the input voltage is zero, so that there is no drain on the high-tension supply when the amplifier is idling.

Let us estimate the maximum anode efficiency attainable with Class B operation. Fig. 87 shows curves of anode-current and voltage for one of the two valves of such an amplifier. Denoting the peak values (as shown on the curves) by $\hat{I}$ and $\hat{V}$, the output power of the valve is $\frac{1}{2}(\frac{1}{2}\hat{V}\hat{I})$ since current is flowing for only half of the time. The total power supplied from the h.t. is the product of v_0, the steady anode-voltage, and i_0, the current which would be registered by a d.c. milliammeter in the anode circuit. The value of i_0 will be $\hat{I}/\pi$ since this is the mean value of the curve of anode-current against time. Thus the anode efficiency is $\frac{\frac{1}{2}\hat{V}\hat{I}}{(v_0\hat{I}/\pi)}$. $\hat{V}$ cannot exceed v_0, and

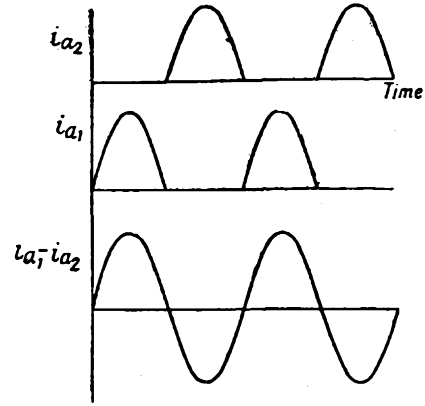


FIG. 86. CLASS B OPERATION—RELATION BETWEEN THE ANODE-CURRENTS OF THE TWO VALVES

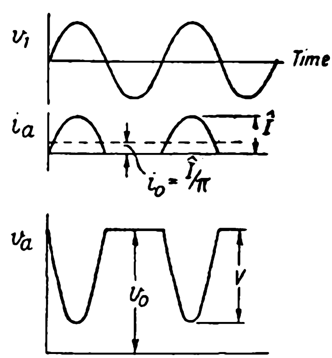


FIG. 87. CLASS B OPERATION—VOLTAGE AND CURRENT RELATIONS IN ONE OF THE VALVES

assuming that it can reach this value, the maximum efficiency becomes $\pi/4$, or 78·5 per cent.

In Class B amplifiers anode-current flows only during the positive half-cycle of the input voltage, i.e. only during 180° of the rotation of the input voltage vector. We say that the Angle of Flow is 180°. The Class C amplifier does not rely upon the device of adding half-waves in order to provide a sinusoidal output, so that its angle of flow need not be exactly 180°. If the negative grid-bias voltage be made larger than the cut-off value, anode-current will flow during only a part of the positive half-cycle of the input voltage, and the angle of flow will be less than 180°. The mean anode-current, i_0, will now be less than $\hat{I}/\pi$ (where $\hat{I}$ is as before the peak value of the anode-current); hence for the same peak anode-current the total power taken from the h.t. supply (i.e. the denominator in the expression for the efficiency) is less than in the Class B case. The useful a.c. power (the numerator in this expression) is also less than in the Class B case, for although the peak anode-current be the same, this current now flows for less than one half-cycle. It may be shown,* however, that as the angle of flow is decreased, the numerator decreases less quickly than the denominator, so that decreasing the angle of flow increases the efficiency. For this reason the Class C amplifier uses a negative grid-bias voltage larger than the cut-off value.

We may, therefore, say that the classification of amplifiers is made as follows—

Class A: Angle of flow 360°, i.e. negative grid-bias voltage less than cut-off value.

Class B: Angle of flow 180°, i.e. negative grid-bias voltage equal to cut-off value.

Class C: Angle of flow less than 180°, i.e. negative grid-bias voltage greater than cut-off value.

Amplifiers considered in the earlier sections of this book come into the first class. Amplifiers in which the anode-current is cut off for less than half of each cycle (i.e. the angle of flow is between 180° and 360°) are said to belong to Class AB.

Design of Class B Audio-frequency Amplifier

The Class B amplifier is widely used for audio-frequency amplification where economy of h.t. current and high-efficiency working are important. Unless careful attention is paid to design and operation, however, it suffers from the disadvantage of producing considerably more distortion than the Class A amplifier.

The performance of the Class B amplifier may be investigated, and the choice of optimum load impedance may be made, by drawing the composite characteristics for the push-pull stage as already

* The peak anode-current occurs when the grid-voltage is at its maximum value, and therefore when the anode-voltage is at its minimum value, which latter is not far from zero. The decreasing of the angle of flow restricts the duration of anode-current more and more, until eventually current would flow only at the instant of minimum anode-voltage. Since the product of anode-voltage and anode-current at a given instant is the power dissipation at that instant, it will be seen that decreasing the angle of flow greatly decreases the dissipation.

described. Static characteristics for the first valve are turned upside down and placed below the static characteristics for the second valve with the axes of anode-voltage coinciding, the points on these axes corresponding to the steady anode-voltage, v_0, being superposed (point *B*, Fig. 88). Next the load line is drawn for each valve, *AB*, *BC*. In the Class B case we see that these lines pass through the same point, *B*, for the load lines must intersect the axis of anode-voltage at a voltage equal to the h.t. supply voltage, and in the Class B case this voltage (corresponding as it does to zero

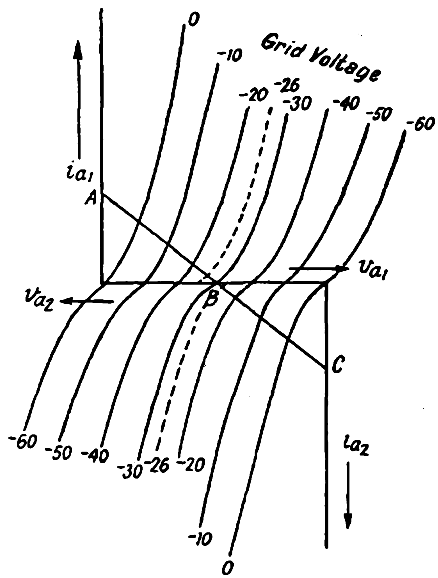


FIG. 88. COMPOSITE CHARACTERISTICS FOR CLASS B STAGE
(Incorrect Bias-voltage)

anode-current) is equal to the steady anode-voltage. We now proceed to add the two curves, one for each valve, corresponding to a grid-voltage equal to the value of the grid-bias voltage (— 30 volts in Fig. 88). Now the grid-bias voltage is such as to reduce the anode-current to zero with an anode-voltage of v_0. Thus, the curves concerned are the two curves running up and down from the point *B*; since the upper one has zero ordinate for all points to the left of *B* and the lower one has zero ordinate for all points to the right of *B*, the sum of the two curves is simply the continuous curve formed of the two separate curves. Finally we combine other pairs of curves, forming a pair by taking from the upper set the curve corresponding to a grid-voltage exceeding the grid-bias voltage by a given amount, and from the lower set the curve corresponding to a grid-voltage which is less than the grid-bias voltage by the same amount. As in the case of the first pair, the resulting curve in each case is simply the continuous curve formed of the separate curves of the pair. The resultant composite characteristics are thus not linear, having a kink at zero anode-current. The result of this will be distortion, a sinusoidal input voltage giving an anode-current of the form shown in Fig. 89 (*a*).

This may be avoided by a slight reduction of the negative grid-bias voltage (from − 30 to − 26 volts in the case shown).

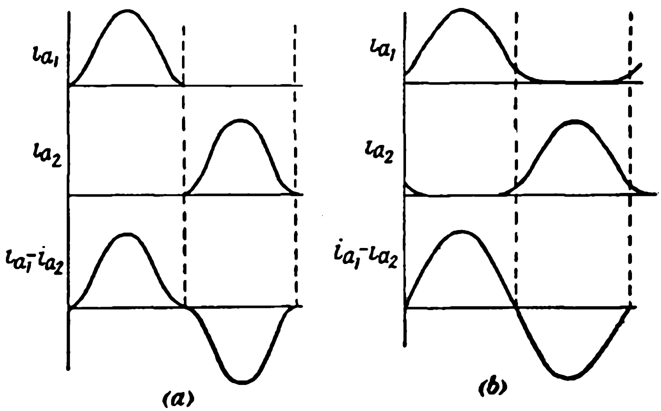


FIG. 89

Let us consider how this modifies the composite characteristics. The important change is that the two curves which we first proceed to add are no longer continuous, since they correspond to a slightly lower grid-bias voltage than that which makes the anode-current zero for an anode-voltage equal to v_0. A careful choice of grid-bias voltage will enable us to make the composite characteristics linear, as shown in Fig. 90, thus avoiding distortion of the type shown in Fig. 89 (*a*). Another way of regarding the slight reduction of negative grid-bias voltage from the cut-off value, is that it causes a slight increase of the angle of flow. The waveform of the anode-current pulse of each valve will still be as in Fig. 89 (*a*) but the waveform of the effective current, $i_{a1} - i_{a2}$, will be sinusoidal as shown in Fig. 89 (*b*), since the two pulses now overlap slightly.

An important point to notice in connexion with the use of the composite characteristic construction is that the composite load line has not, as in the Class A case, double the slope of the individual load line. The composite load line is the sum of the two load lines, *AB*, *BC*, neither of which continues beyond the point *B* (for this would imply that the anode-current of a valve may be negative). Thus, the composite load line is simply *CA*, and its slope is the reciprocal of the resistance presented to each valve. In the case of the Class A push-pull amplifier we found that the resistance presented to each valve was $R/2n^2$, where R is the resistance closing the transformer secondary, and n is the ratio

$$\frac{\text{Number of secondary turns}}{\text{Total number of primary turns}}$$

This was deduced by considering a transformer with a generator connected to each of its two primary windings and a resistance, R, across the secondary, and estimating the ratio of the voltage and current of each generator. In the case of the Class B amplifier, however, each generator is inoperative while the other is passing current, so that the resistance presented to each valve is the same as it would be if the primary winding in the circuit of the other valve were not present, viz. $R/(2n)^2$. The slope of the composite load line is the reciprocal of this. Thus, we have, after all, the same rule as in the Class A case, viz. *the slope of the composite load line is the reciprocal of one-quarter of the anode-to-anode load.*

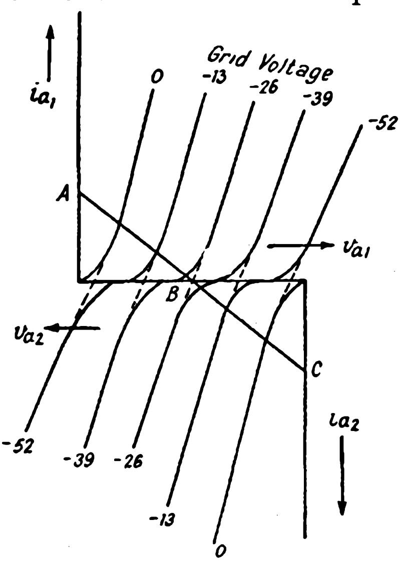


FIG. 90. COMPOSITE CHARACTERISTICS FOR CLASS B STAGE
(Correct Bias Voltage)

Using the composite load-line diagram, the optimum load for maximum "undistorted" output power may be estimated in the same way as for a Class A amplifier, and the distortion may be estimated for any given input voltage. The correct adjustment of grid-bias voltage is somewhat critical. Moreover, the correct value depends upon the value of the h.t. voltage, so that if either the h.t. voltage or the grid-bias voltage varies during operation there is liable to be considerable distortion. Further, the two valves must be accurately matched to each other. In general it may be said that the Class B amplifier can never give as distortionless amplification as the Class A amplifier, but it has the advantage of higher efficiency and of taking no current from the h.t. supply when there is no input voltage.

As we stated in Chapter II, if the grid is allowed to become positive during part of the cycle, grid-current will flow. Its waveform will be far from sinusoidal, and the distorted voltages which this grid-current sets up in flowing through the impedance (usually a high resistance) connected in the grid circuit, will be amplified along with the input voltage. The discussion of amplifiers up to this point has, therefore, been confined to amplifiers in which the grid-voltage is not allowed to become positive. Distortionless amplification may take place, however, even if the grid-voltage is allowed to become positive, provided that we make the impedance in the grid circuit sufficiently small for the voltage set up in it by the grid current to be negligibly small. By doing this we are effectively connecting a low impedance across the load of the previous valve. It will, therefore, be necessary to match this low impedance to the impedance of the previous valve; in short, the previous stage must be designed as an output stage, and the low resistance in the grid circuit of the valve whose grid is being driven positive must be considered as its load. The previous stage is known as a Driver Stage.

Allowing the grid-voltage to become positive means that a greater output power may be secured from a given valve, since the operating point may swing farther to the left along the load line. Output stages frequently work with the grid driven positive; in particular, almost all Class B and Class C stages are so designed. A suffix, 2, is often added to the class letters to indicate that the grid is driven positive, and a suffix 1 to indicate that the grid is not driven positive, (e.g. Class AB_1, Class AB_2).

The static characteristics for positive grid-voltages are fairly linear and evenly spaced up to a point. When the positive grid-voltage exceeds the anode-voltage, however, electrons begin to be attracted to the grid rather than to the anode, and the grid-current increases rapidly at the expense of the anode-current. If distortion is to be avoided in amplifiers in which the grid is driven positive, the limitation must be imposed that the grid-voltage must never exceed the anode-voltage. Since the maximum value, $v_{g\,max}$, of the grid-voltage, and the minimum value, $v_{a\,min}$, of the anode-voltage, occur at the same point of the cycle, this condition becomes

$$v_{g\,max} \leqslant v_{a\,min}$$

In calculating the maximum attainable anode efficiency for Class B operation, we made the assumption that the peak anode-voltage could equal the steady anode-voltage. In an amplifier in which the grid is driven positive this assumption no longer holds, the maximum value which the peak anode-voltage can attain being $v_0 - v_{a\,min}$. The anode efficiency is therefore given by

$$\text{Anode Efficiency} = \frac{\frac{1}{2}\hat{V}\hat{I}}{v_0\hat{I}/\pi} = \frac{\frac{1}{2}(v_0 - v_{a\,min})\hat{I}}{v_0\hat{I}/\pi}$$

$$= \frac{\pi}{4}(1 - v_{a\,min}/v_0) \qquad . \qquad . \text{(IV.44)}$$

Graphical Determination of the Performance of the Class C Amplifier

The load line method of calculation fails in the case of the Class C amplifier (and in all cases where the load is not a constant resistance at all frequencies) for the load line is the graphical representation of the relation

$$v_a = E - Ri_a$$

and, if the coefficient of i_a in this relation is a function of frequency, different load lines would have to be used for the fundamental and the various harmonics of the anode-current. Moreover, we have seen that the waveform of the anode-current in the Class C amplifier is far from sinusoidal, whereas the anode-voltage is sinusoidal. The load-line construction would not give this result.

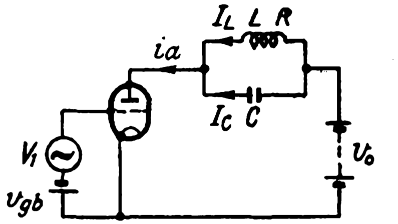


FIG. 91

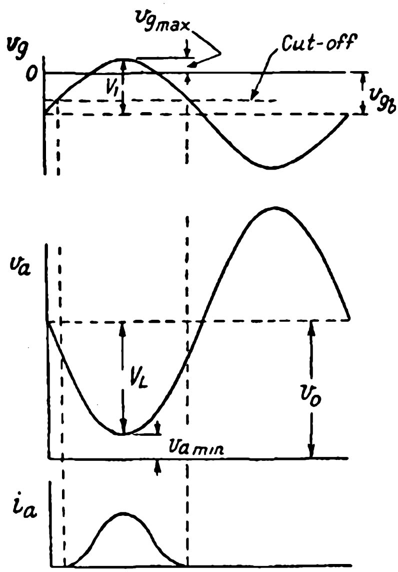


FIG. 92. OPERATION DIAGRAM FOR CLASS C AMPLIFIER

Let us consider the circuit of Fig. 91 operated as a Class C amplifier, i.e. with a negative grid-bias voltage greater than the cut-off value. How may the output voltage, V_L, be predicted for given circuit constants, valve characteristics, operating voltages and input voltage? First we plot the grid-voltage, v_g, against time, as shown in Fig. 92. The *form* of the anode-voltage will be as shown, but the amplitude, V_L, remains to be calculated. The anode-voltage must be sinusoidal and 180° out of phase with the input voltage. V_L must now be found by a process of trial and error.

Assume a value for V_L, and draw the curve of v_a against time as shown in Fig. 92. Knowing thence the value of v_a and v_g at any instant throughout the cycle, we may now find the anode-current, i_a, at any instant, and plot the curve of i_a. Except when v_g exceeds the cut-off value, of course, i_a will be zero. From the curve of i_a against time, the mean anode-current, i_0, such as would be registered by a d.c. milliammeter in the anode circuit, may be calculated. It will be given by

$$i_0 = \frac{\text{Area under the curve of } i_a}{\text{Periodic time}}$$

The product, $v_0 i_0$, will then give the total power supplied from the h.t. supply, which is, of course, the sum of the output power developed in the load and the power dissipation of the valve. The latter may be determined by plotting the product, $v_a i_a$, which is the instantaneous value of the anode power dissipation, and taking the mean value,

$$\text{Power dissipation in valve} = \frac{\text{Area under curve of } (v_a i_a)}{\text{Periodic time}}$$

Subtracting this from the total power, $v_0 i_0$, we shall have the output power.

But the output power is also given by

$$\text{Output power} = R I_{L\,rms}^2$$

$$= \frac{R V_{L\,rms}^2}{R^2 + \omega^2 L^2} = (\text{approx.}) \frac{R V_{L\,rms}^2}{\omega^2 L^2}$$

whence,

$$V_{L\,rms} = \sqrt{(\text{Output power}) \cdot \frac{\omega^2 L^2}{R}}$$

If the value of V_L thus calculated is the same as that originally assumed, the value is correct; otherwise a further value must be assumed and the process repeated.

Having determined V_L, the anode efficiency may be determined; it is the ratio of the output power to the total power taken from the h.t. supply.

Design of Class C Amplifier

As we saw in the last section of the previous chapter, the major part of the design of an output stage consists of the choice of a valve which can give a sufficient output power. Let us consider the conditions for maximum power output from a Class C stage. The method of estimating the output power under given conditions was indicated in the previous section, and from what was said there it will be seen that the output power increases with increasing peak anode-current, peak load voltage, V_L, and angle of flow. To secure the maximum output from a valve we must therefore make these three quantities as large as possible.

The first of them, the peak anode-current, we may make equal to the maximum safe emission from the cathode as given by the valve manufacturers. Thus, the estimation of the maximum output power from a given valve begins by fixing the peak anode-current.

This gives us one point on the lowest curve of Fig. 92. Next we must make V_L as large as possible; a factor which limits the increase of V_L is as follows. The greater V_L, the lower is the minimum anode-voltage, $v_{a\,min}$, and this, of course, is the value of the anode-voltage at the instant when the peak anode-current is flowing. Now the peak anode-current is fixed, so that we can determine, from the characteristics of the valve, suitable pairs of values of $v_{a\,min}$ and $v_{g\,max}$, these being the values of anode- and grid-voltage at the instant when peak anode-current is flowing. The smaller we make $v_{a\,min}$ the larger must we make $v_{g\,max}$ to secure the correct value of peak anode-current. If $v_{a\,min}$ and $v_{g\,max}$ are of the same order, or if $v_{a\,min}$ is less than $v_{g\,max}$, excessive grid-current will flow, and the power required from the previous stage will be unnecessarily large. We must use some such rule as that $v_{g\,max}$ shall not exceed 80 per cent of $v_{a\,min}$. The next step in estimation of the maximum power output is therefore to determine from the characteristics the values of $v_{a\,min}$ and $v_{g\,max}$ which will give the correct peak anode-current, $v_{g\,max}$ being equal to 0·8 $v_{a\,min}$. This gives us a point on each of the two upper curves of Fig. 92, and we require to know only the grid-bias voltage and the steady anode-voltage to complete the figure. The maximum safe anode-voltage will usually be known, and hence the steady anode-voltage may be found and the middle curve completed. Some value of grid-bias voltage greater than the cut-off value is next chosen and the other two curves drawn. From this figure we may now estimate the output power and the power dissipation of the valve. If the latter exceeds the maximum safe dissipation, greater values of negative grid-bias voltage must be tried, until the grid-bias voltage and the output power corresponding to the maximum permissible dissipation are found.

Having selected a valve capable of the output power required and determined the operating voltages as above, it remains to choose suitable values of L, R, and C. In the circuit of Fig. 91 the output power is developed in the resistance, R, and this resistance is thus the actual load. (To this extent our use of the phrase, "Load Impedance," to denote the impedance of the parallel L–C circuit has been a misnomer.) In practice the load, R, is not included directly in the parallel circuit, but is coupled to this circuit either inductively or in the manner of resistance-capacitance coupling. In either case, Fig. 91 is an equivalent circuit provided that the resistance, R, shown in that Figure is the sum of the resistance of the coil and the resistance effectively in series with the coil as a result of the coupling of the actual load to the parallel L–C circuit. We have seen that the operation of the Class C amplifier depends entirely upon the use of a parallel L–C circuit with high selectivity. If the selectivity is not sufficiently high, the non-sinusoidal anode-current will set up across the L–C circuit harmonic voltages which are not negligible with respect to the fundamental voltage, and the output will not be sinusoidal. The parallel L–C circuit must, therefore, be designed to have as little resistance as possible, and in addition the resistance, R', effectively in series with the coil as a result of the coupling of the load to the parallel L–C circuit, must not be so great as to reduce the resultant selectivity below about ten. The resultant selectivity, Q, is equal to $L\omega_0/R$, or approximately $L\omega_0/R'$, since R is very little greater than R'. Now the output power, W, is given by

$$W = RI_{L\,rms}^2 = \text{approx.}\ \frac{RV_{L\,rms}^2}{\omega_0^2 L^2}$$

$$= \frac{V_{L\,rms}^2}{Q\omega_0 L}$$

whence

$$L = \frac{V_{L\,rms}^2}{Q\omega_0 W} \quad . \quad . \quad . \quad . \quad . \quad (\text{IV.45})$$

so that if a value of Q be assumed, L may be calculated, and hence also R. C is then calculated from the relation

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

We have outlined above the design of a Class C output stage, starting from a specification of power output and frequency. It may be summarized as follows—

1. Select a valve.
2. Build up the operation diagram (Fig. 92), trying various values of grid-bias voltage to find the maximum output. Note that, having chosen a valve, we have fixed the characteristics, and the values of $i_{a\,max}$, $v_{a\,max}$, and the maximum power dissipation of the valve; also that some such rule as making $v_{g\,max}$ equal to 80 per cent of $v_{a\,min}$ must be used.
3. If the output thus calculated is sufficient, note the required operating voltages. If not, try other valves.
4. Assuming a minimum permissible selectivity of the parallel L–C circuit with its coupled load, calculate L, R, and C.

The above is a theme on which many variations may be played. For example, if the resistance, R', effectively in series with the coil, is specified, in addition to the required output power, then V_L is fixed in advance, since

$$W = \frac{V_{L\,rms}^2 R}{\omega_0^2 L^2} = \frac{V_{L\,rms}^2}{RQ^2}$$

In this case, after determining $v_{a\,min}$ and $v_{g\,max}$ we must examine whether the maximum anode-voltage ($v_{a\,min} + 2V_L$) is a permissible value.

Self-bias arrangements for Class C amplifiers and oscillators will be considered in Chapter V.

Suggestions for Further Reading

1. F. Langford Smith (Editor), *Radio Designer's Handbook* (Iliffe).
2. Valley and Wallman, "Vacuum Tube Amplifiers," *Radiation Laboratory Series*, vol. 19.
3. F. E. Terman, *Electronic and Radio Engineering* (McGraw-Hill).
4. E. K. Sandeman, *Radio Engineering* (Chapman & Hall).
5. K. R. Sturley, *Radio Receiver Design* (Chapman & Hall).
6. J. C. West, *Servomechanisms*.
7. F. A. Benson, *Voltage Stabilizers* (Electronic Engineering Monograph).
8. E. E. Zepler, *The Technique of Radio Design* (Chapman & Hall).

CHAPTER V

REGENERATION AND OSCILLATION

WHAT is variously known as Regeneration or Retroaction is simply positive feed-back, i.e. the feeding back of a fraction of the output voltage to the input circuit of an amplifier, the voltage fed back being in phase with the input voltage. The total alternating grid-voltage of the first stage of the amplifier is thus the sum of the voltage fed back and the external voltage to be amplified, and the amplification is increased. In Chapter IV (page 114) it was shown that, for an amplifier with feed-back

$$(VA) = \frac{A}{1 - A\beta}$$

where A is the vector gain of the amplifier and β is the vector gain of the feed-back path. For the fed-back voltage to be in phase with the external input voltage, the product $A\beta$ must be a real positive number—as it would be, for example, if there were a phase-shift of 180° in the amplifier and a further phase-shift of 180° in the feed-back path. Thus, for positive feed-back, we may write

$$(VA) = \frac{(VA)_0}{1 - n(VA)_0} \quad . \quad . \quad . \quad . \quad (V.1)$$

where $(VA)_0$ is the magnitude of A, and the feed-back ratio, n, is the magnitude of β.

As the feed-back ratio is increased, the amplification increases, and the equation seems to indicate that when n is increased to a value which makes the product $n(VA)_0$ equal to unity, the amplification becomes infinite. This may be interpreted as meaning that when the feed-back ratio attains this value a finite output voltage may be produced with zero input voltage. In such circumstances the apparatus will be functioning not as an amplifier but as an oscillator, i.e. it will generate alternating current, drawing its energy entirely from the h.t. supply.

We may thus regard an oscillator as a regenerative amplifier having a feed-back ratio sufficiently large for the amplifier to provide its own input voltage. This point of view sometimes enables us to make a simple approximate analysis of an oscillator circuit. In the pages which follow we shall make a detailed analysis of various oscillator circuits, but for the purpose of illustration let us consider here the oscillator circuit shown in Fig. 93 (a). This is simply a regenerative single-frequency amplifier, the feed-back being accomplished by mutual inductive coupling between the output and input circuits. We shall derive the condition for oscillation by considering how large the mutual inductance,

M, must be made for the amplifier to be able to provide its own input voltage.

The equivalent circuit is shown in Fig. 93 (b), V denoting the alternating grid-voltage. The amplifier is a single-frequency amplifier, the frequency of amplification being $1/2\pi\sqrt{LC}$, and oscillation

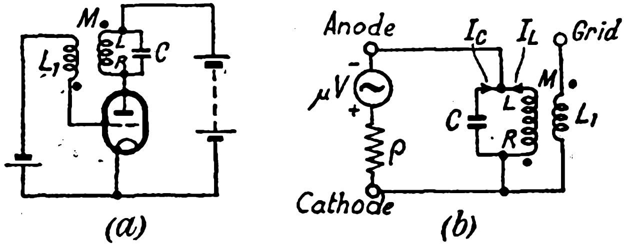


FIG. 93. TUNED-ANODE OSCILLATOR

will therefore occur at this frequency. At this frequency the load impedance is L/RC, so that the voltage across the coil L is

$$V_L = \frac{\mu V(L/RC)}{\rho + L/RC}$$

Dividing this by $(R + j\omega L)$ we have the coil current I_L. The e.m.f. which this current induces into the grid circuit is

$$V_{FB} = j\omega M \, . \, I_L = \frac{j\omega M \, . \, V_L}{R + j\omega L}$$

The resistance, R, is usually small with respect to ωL, so that this becomes

$$V_{FB} = \left(\frac{M}{L}\right) V_L = \frac{M}{L} \cdot \frac{\mu V L/RC}{\rho + L/RC}$$

To maintain oscillation this fed-back voltage V_{FB} must be such that it exactly provides the input voltage V. Equating V_{FB} and V, we have

$$M = \frac{L + \rho RC}{\mu} \quad . \quad . \quad . \quad . \quad (V.2)$$

This is the condition that the positive feed-back shall be great enough for the "amplifier" to supply its own input voltage. It is known as the Maintenance Equation, and its significance is further considered on page 155.

Tuned-anode Amplifier with Regeneration by Mutual Inductive Coupling

The circuit of Fig. 93 (a) may be used as a regenerative amplifier, the input being included in series with the grid circuit. The voltage of the generator in the equivalent circuit then becomes

$$\mu(V_1 + j\omega M \, . \, I_L)$$

and we have, at the resonance frequency,

$$I_L = V_L/(j\omega L) = \frac{\mu(V_1 + j\omega M \,.\, I_L)\,(L/RC)}{\rho + L/RC}\frac{1}{j\omega L}$$

After solving this for I_L, we have

$$(VA) = j\omega L \,.\, I_L/V_1 = \frac{\mu(L/RC)}{\rho + L/RC - \mu M/RC} \quad . \quad \text{(V.3)}$$

This should be compared with equation (IV.10), page 88, which gives the voltage-amplification in the absence of regeneration. Equation (V.3) may be written in the form given in equation (V.1) since the voltage fed back is $j\omega M \,.\, I_L$, which may be written nV_L, where

$$n = j\omega M/j\omega L = M/L$$

It is fairly easy to show that regeneration increases the selectivity of the amplifier. Equation (V.3) gives the voltage amplification at the resonance frequency; an expression for the amplification at any frequency may be derived by the application of Kirchhoff's laws to the equivalent circuit, and may be written in the same form as equation (V.1). With this equation in mind, let us consider the selectivity of (VA) as compared with that of $(VA)_0$. If the frequency be displaced from the resonance value $(VA)_0$ will decrease, while the quantity $(1 - n(VA)_0)$ will increase. Thus (VA), as given by equation (V.1), will decrease more than will $(VA)_0$. Thus the curve of (VA) against frequency will be sharper than that of $(VA)_0$, showing that regeneration has increased the selectivity.

Tuned-anode Oscillator with Mutual Inductive Coupling

This is shown in Fig. 93 (*a*) and we have already derived the condition of oscillation, equation (V.2). This equation might equally well have been derived by writing the condition that the amplification given in equation (V.3) should become infinite, i.e. simply equating the denominator of equation (V.3) to zero.

If the mutual inductance, M, has a smaller value than that given in equation (V.2) oscillation will not take place. Note also that the sign of M may be changed from positive to negative or vice versa by interchanging the connexions to the coil in the grid circuit. Unless the connexions are so arranged that the sign of M is correct, i.e. so that the voltage fed back is of the correct phase to provide the input voltage which produces it, oscillation will not take place.

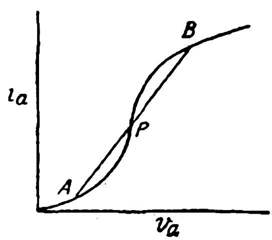


FIG. 94

The above analysis gives us no indication of what takes place when M exceeds the value given in equation (V.2), nor of the amplitude of the oscillation. These matters are bound up with the curvature of the characteristics, whereas the treatment which adopts an equivalent circuit inherently assumes linear characteristics. Since the characteristics are in reality not linear, the values of the parameters are not constants for the valve, but depend also upon the operating voltages. For instance, the impedance, ρ, is the reciprocal of the slope of the $v_a - i_a$ characteristics, and since these are curved it is clearly a function of v_a. What is more important is that the effective value of a parameter depends upon the amplitude of the oscillations. Consider for instance the impedance of a valve for various amplitudes of anode-current. If the point P (Fig. 94) represents the steady operating conditions, then for very small amplitudes the impedance is the reciprocal of the slope of the tangent to the curve at the point P. For a larger amplitude, corresponding to the traversing of a length, APB, of the characteristic, the impedance is increased to the reciprocal of the slope of AB. Thus in an oscillator the values of the parameters depend upon the amplitude of oscillation, and in general these values are degraded (i.e. ρ is increased and g_m and μ are decreased) as the amplitude increases. *If the value of M is greater than the critical value required to produce oscillation, then the amplitude will increase to such a point that the parameters satisfy equation* (V.2) *with the particular value of M in use.*

Since the amplitude is limited only by the curvature of the characteristics, the distortion increases with the amplitude, i.e. with M. For low distortion, the steady anode- and grid-voltages should be such as to bring the operating point to the middle of the straight portion of the characteristic. As in the Class C amplifier, the distortion is reduced to small proportions if the selectivity of the parallel L–C circuit is high. In circuits using iron-cored coils, especially if no air gap is used, the matter is further complicated by the fact that the inductance of the coil changes with the amplitude of the current through it. Thus an increase of output is accompanied by a decrease of frequency. Moreover, the non-linear relation between current and magnetic flux is a further source of distortion, this distortion increasing with amplitude.

We concluded above that the frequency of oscillation of the tuned-anode mutual-coupled oscillator was equal to the resonance frequency of its parallel L–C circuit. A detailed analysis of the circuit will show us that the oscillation frequency is slightly higher than the resonance frequency of the parallel L–C circuit. Such an analysis may be made either by equating to infinity the general expression for the amplification at any frequency, the circuit being considered as a regenerative amplifier, or by the method given below.

This method applies Kirchhoff's laws to the equivalent circuit, but instead of using vector notation (and thereby *assuming* that the oscillation will be sinusoidal) it is concerned with instantaneous values of voltage and current. Apart from the assumption of linear characteristics inherent in the adoption of an equivalent circuit, this may be considered a rigorous method of analysis. Moreover, the circuit is not considered as a regenerative amplifier, i.e. no input voltage need be assumed. Writing i_1 and i_2 respectively for the instantaneous values of the currents I_L and I_C in Fig. 93 (b), we see that the voltage induced into the grid circuit is $M(di_1/dt)$. The instantaneous value of the e.m.f. μV in Fig. 93 (b) thus becomes $\mu M(di_1/dt)$, and the application of Kirchhoff's laws gives

$$-L\frac{di_1}{dt} + \frac{1}{C}\int i_2 dt = Ri_1$$

and

$$\mu M\frac{di_1}{dt} - L\frac{di_1}{dt} = \rho(i_1 + i_2) + Ri_1$$

Solving for i_1, we have

$$\rho LC\frac{d^2i_1}{dt^2} + (\rho RC + L - \mu M)\frac{di_1}{dt} + (\rho + R)i_1 = 0$$

This is a second order linear differential equation with constant coefficients, and its solution is thus

$$i_1 = Ae^{-\alpha t}\sin(\omega t + B)$$

where A and B are arbitrary constants and

$$\alpha = \frac{\rho RC + L - \mu M}{2\rho LC}$$

$$\omega = \sqrt{(1 + R/\rho)/LC - \alpha^2} \quad . \quad . \quad (V.4)$$

The form of this solution indicates that i_1 may oscillate sinusoidally with an amplitude $Ae^{-\alpha t}$, i.e. an amplitude which decreases exponentially with time, the rate of decrease depending upon the constant α. For example, when the h.t. supply is switched on, an oscillation will be set up; its frequency will be given by equation (V.4) and its amplitude will decrease (i.e. the oscillation will die out) at a rate depending on the value of α. If α is zero, the rate of decay will be zero, and the oscillation will be sustained. Thus the condition for sustained oscillation is given by equating α to zero, and the result will be found to agree with equation (V.2). When α is zero, equation (V.4) becomes

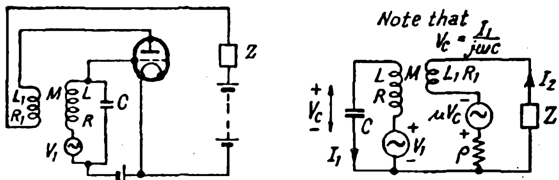
$$\omega = \sqrt{(1 + R/\rho)/LC}$$
$$= \omega_0\sqrt{1 + R/\rho} \quad . \quad . \quad . \quad (V.5)$$

where ω_0 is the resonant frequency of the parallel L–C circuit.

The Significance of the Maintenance Equation

The above analysis confirms what we have already said about the effect of increasing M beyond the value specified by the maintenance equation. If M be increased beyond the value which makes α equal to zero, then α becomes negative and the amplitude, $Ae^{-\alpha t}$, increases with time. Since the values of the valve parameters depend upon this amplitude (being, in general, degraded as the amplitude increases), the increase in amplitude does not proceed indefinitely. The amplitude reaches an equilibrium value when it has increased to such a magnitude that the changed values of the parameters satisfy the maintenance equation, (i.e. make α equal to zero) for the greater value of M.

The maintenance condition may be regarded either as an equation which must be satisfied whenever oscillation is taking place, or it may be regarded as an inequality (i.e. M must be greater than, or equal to, a certain critical value). If the values assigned to μ and



(a) (b)

FIG. 95. TUNED-GRID REGENERATIVE AMPLIFIER

ρ are those values corresponding to the tangent to the characteristics at the operating point, then the maintenance condition is to be regarded as an inequality. If, however, μ and ρ denote the effective values of the parameters at the equilibrium value of oscillation amplitude, then the maintenance condition is to be regarded as an equality.

Tuned-grid Regenerative Amplifier with Mutual Inductive Coupling

This important circuit is shown in Fig. 95 (*a*). The input voltage, V_1, which we have shown connected in series with the coil of the parallel L–C circuit, is in practice a voltage induced in this coil by mutual inductive coupling to the previous stage. A small coil, L_1R_1, mutually coupled to the parallel L–C circuit, is connected in series with the load, Z. Let us determine the voltage amplification, at the resonance frequency, using the notation of the equivalent circuit, Fig. 95 (*b*). For the L–C circuit, at the resonance frequency, we have

$$V_1 + j\omega MI_2 = RI_1$$

and since the alternating grid-voltage equals the voltage across C, viz., $I_1/j\omega C$, we have the following equation for the anode circuit.

$$j\omega MI_1 + \frac{\mu I_1}{j\omega C} = I_2(\rho + R_1 + j\omega L_1 + Z)$$

R_1 and $L_1\omega$ will normally be very small with respect to $(\rho + Z)$ so that the right-hand side of the second equation becomes simply $I_2(\rho + Z)$. Solving these equations and writing $1/LC$ for ω^2, we have

$$I_1 = \frac{V_1}{R - \frac{\mu M}{C(\rho + Z)} \cdot \left(1 - \frac{M}{\mu L}\right)} \quad . \quad . \quad (\text{V.6})$$

$$I_2 = \frac{\frac{\mu V_1}{j\omega CR}\left(1 - \frac{M}{\mu L}\right)}{\rho + Z - \frac{\mu M}{RC}\left(1 - \frac{M}{\mu L}\right)}$$

Thus $$(VA) = ZI_2/V_1 = \frac{\frac{\mu Z}{j\omega CR}\left(1 - \frac{M}{\mu L}\right)}{\rho + Z - \frac{\mu M}{RC}\left(1 - \frac{M}{\mu L}\right)}$$

This equation is not of the simple form shown in equation (V.1) but has a factor $(1 - M/\mu L)$ in the numerator. The reason for this is as follows. The circulating current in the parallel L–C circuit induces a voltage in the anode circuit quite apart from the action of the valve. If the valve were removed the voltage induced in the anode circuit would be $j\omega M . V_1/R$. But

$$j\omega M \;.\; V_1/R = j\omega M \;.\; j\omega C \;.\; (V_1/R)\,(1/j\omega C) = -\,\omega^2 MC \;.\; V_C$$
$$= (-\,M/L)V_C$$

This is a fraction, $M/\mu L$, of the voltage of the generator in the equivalent circuit, and is 180° out of phase with the voltage of this generator. In practice this fraction will usually be small with respect to unity, and the factor, $(1 - M/\mu L)$, may be omitted, giving

$$I_1 = \frac{V_1}{R - \frac{\mu M}{C(\rho + Z)}}$$

$$(VA) = \frac{\mu Z/j\omega CR}{\rho + Z - \frac{\mu M}{RC}}$$

The first of these equations shows that if Z be a resistance, R', then regeneration reduces the effective resistance of the parallel L–C circuit by an amount, $\frac{\mu M}{C(\rho + R')}$, thereby increasing the selectivity. The second equation shows the effect of regeneration in increasing the amplification.

If M be increased, a point will be reached at which self-oscillation takes place. The critical value of M may be found by writing the amplification equal to infinity. This gives

$$M = \frac{CR(\rho + R')}{\mu} \quad . \quad . \quad . \quad \text{(V.7)}$$

It will be seen that this is also the value of M which makes the selectivity of the parallel L–C circuit infinite.

Tuned-grid Oscillator with Mutual Inductive Coupling

It follows that the tuned-grid circuit with inductive coupling may also be used as an oscillator, in which case Z (and, of course, V_1) may be omitted. The frequency of oscillation will be very nearly equal to that of the parallel L–C circuit, and the condition for oscillation will be given by equation (V.7), but with R' equal to zero.

Although it is possible to deduce the behaviour of this oscillator from the above analysis of the regenerative tuned-grid amplifier, the results may be obtained more simply from a straightforward analysis of the oscillator circuit. The equivalent circuit is as in Fig. 95 (*b*), but with the source, V_1, and the load, Z, omitted. The application of Kirchhoff's laws gives, for any frequency,

$$j\omega M I_2 = I_1(R + j\omega L + 1/j\omega C)$$

$$j\omega M I_1 + \mu I_1/j\omega C = (\rho + j\omega L_1)I_2$$

providing, as is usually the case, R_1 is negligible compared to ρ. Eliminating I_1 and I_2 between these equations, we shall have a complex equation involving only the frequency and the circuit constants. By treating the real and imaginary parts of this equation separately we may form two equations, one of which will give us the oscillation frequency, and the other the condition for sustained oscillation or Maintenance Equation.

Proceeding thus, we first write the second of the above equations as

$$I_2(\rho + j\omega L_1) = I_1(j\omega M + \mu/j\omega C)$$

Dividing this into the first of the above equations and cross multiplying, we eliminate I_1 and I_2, giving

$$(\rho + j\omega L_1)\,(R + j\omega L + 1/j\omega C) = j\omega M\,(j\omega M + \mu/j\omega C)$$

Considering only the imaginary terms we have

$$\omega L_1 R + \omega L \rho - \rho/\omega C = O$$

whence the frequency of oscillation is given by

$$\omega = \frac{1}{\sqrt{LC\left(1 + \frac{L_1 R}{L\rho}\right)}}$$

which is very nearly equal to $1/\sqrt{LC}$.

Considering the real terms only we have

$$\rho R - \omega^2 L L_1 + L_1/C = \frac{\mu M}{C} - \omega^2 M^2$$

and if in this equation we write $1/LC$ for ω^2 we have

$$\rho R = (1 - M/\mu L)\,(\mu M/C)$$

The term $M/\mu L$ is small compared to unity. This may be seen by writing M equal to $k\sqrt{LL_1}$, where k is the coupling coefficient. $M/\mu L$ then becomes $(k/\mu)\sqrt{L_1/L}$. Since k is less than unity and L_1 is small compared to L, we see that the term $M/\mu L$ may be neglected in comparison with unity. With this approximation, the Maintenance Equation becomes

$$M = \frac{\rho RC}{\mu}$$

which is in agreement with equation (V.7).

Variable Regeneration

The degree of regeneration which may be used in amplifiers is limited by the fact that sustained oscillation will occur if the regeneration is increased beyond a certain point. The condition for self-oscillation has been derived for the two regenerative amplifiers hitherto considered, and will be seen to involve the valve parameters and the circuit constants. Now in practice slight variations of the supply voltages, and hence of the valve parameters, are unavoidable. Moreover, variations of temperature will affect the physical dimensions of coils and condensers, and hence slight variations of the circuit constants are also unavoidable. Thus, in an amplifier with fixed regeneration, the degree of regeneration may not be made quite so large as is indicated by the condition for sustained oscillation, lest these small variations should cause the valve parameters and circuit constants to take up values which would cause oscillation. The closeness with which the degree of regeneration may approach the value given by the condition for sustained oscillation depends upon the magnitude of the small variations. Further, in a single-frequency amplifier, tunable to various frequencies, the degree of regeneration in general depends upon the particular frequency being amplified. (For instance, in the tuned-anode regenerative amplifier already considered, the value of M may not exceed that given by equation (V.2). Since this is a function of C, and tuning is accomplished by variation of C, it follows that the maximum permissible value of M depends upon the frequency to which the amplifier is tuned.) For these reasons, variable regeneration is necessary where it is desired to secure the maximum regeneration for any frequency and whatever the temperature and the supply voltages. Variable regeneration is also necessary in an oscillator in which it is required that the amplitude of oscillation shall be as small as possible, with the object of eliminating distortion; it may also be used as a means of controlling the amplitude of oscillation.

The simplest method of varying the regeneration in the two regenerative amplifiers so far considered is by variation of M, this being accomplished by actually moving one coil relative to the other. This method is cumbersome and also varies the stray capacitance between the coils. Another method is indicated in Fig. 96, which differs from Fig. 95 (a) in that the coil, L_1, is no longer in series with the h.t. supply, but is parallel-fed by means of the choke, L', and the variable condenser, C'. If the reactance of L' is large the equivalent circuit approximates to that of Fig. 95 (b), provided that $1/j\omega C'$ be substituted for Z. Making this substitution in the equation for I_1 already deduced for this equivalent circuit, we see that, in this method of varying the regeneration, variation of C' slightly affects the tuning of the grid L–C circuit. In a practical circuit of this kind the positions of L_1 and C' would be reversed, in order to have the moving plates of the condenser at earth potential and thus avoid the effects of hand-capacitance.

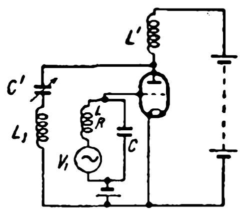


FIG. 96. VARIABLE REGENERATION

Variable regeneration may also be secured by connecting a variable resistance across the coil, L_1, in Fig. 93 (a) or Fig. 95 (a), or in series with the anode coil of a tuned-grid oscillator, though this last arrangement also results in variation of the steady anode-voltage.

Closed-circuit Oscillators

Oscillators of this type have a circuit which simply consists of a single closed circuit (i.e. a number of two-terminal impedances

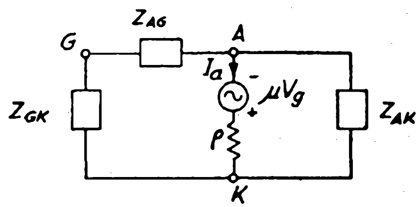


FIG. 97. PRINCIPLE OF CLOSED-CIRCUIT OSCILLATORS

connected in series to form a closed loop), with the anode, cathode and grid of a valve tapped on at three different points of the closed circuit, as in Fig. 97. Denoting the impedance between the tapping points by Z_{AK}, Z_{GK} and Z_{AG}, as shown in Fig. 97, we shall deduce a general condition for sustained oscillation in such circuits. The resulting equation will enable us to "invent" a number of oscillator circuits of this type.

The total load impedance Z' presented to the valve is $(Z_{GK} + Z_{AG})$ in parallel with Z_{AK}. The anode-current I_a will be given by substituting this value for Z' into the expression $\mu V_g/(\rho + Z')$. The fraction of the anode-current which flows through Z_{GK} is $Z_{AK}/(Z_{AK} + Z_{GK} + Z_{GA})$. Since this current flows upwards through Z_{GK} in Fig. 97, the p.d. V_{GK} is $- Z_{GK}Z_{AK}I_a/(Z_{AK} + Z_{GK} + Z_{GA})$. This gives

$$V_{GK} = \frac{-Z_{GK}Z_{AK}}{Z_{AK} + Z_{GK} + Z_{AG}} \cdot \frac{\mu V_g}{\rho + \dfrac{Z_{AK}(Z_{GK} + Z_{AG})}{Z_{AK} + Z_{GK} + Z_{AG}}}$$

$$= \frac{-Z_{GK}Z_{AK}\mu V_g}{\rho(Z_{AK} + Z_{GK} + Z_{AG}) + Z_{AK}(Z_{GK} + Z_{AG})}$$

The condition for sustained oscillation is simply the condition that the "amplifier" shall provide its own input voltage, i.e. the condition that V_{GK} shall suffice exactly to provide V_g, where V_g is the e.m.f. of the generator in the equivalent circuit. The condition is thus obtained by writing V_g equal to the above expression for V_{GK}. If V_g were zero, the circuit would not be oscillating. If V_g is not zero, we may divide throughout by V_g and re-arrange the equation to give the following general condition for sustained oscillation—

$$\rho(Z_{GK} + Z_{AK} + Z_{AG}) = -Z_{AK}(Z_{GK} + \mu Z_{GK} + Z_{AG}) \quad \text{(V.8)}$$

In trying to devise circuits which will satisfy this equation, let us confine our attention to circuits formed entirely of pure reactances (formed, in practice, of condensers and high-Q coils). This would mean that

$$Z_{GK} = jX_{GK};\ Z_{AK} = jX_{AK};\ Z_{AG} = jX_{AG}$$

The left-hand side of equation (V.8) would then be purely imaginary and the right-hand side would be real. The two sides could be equal only if both were zero, i.e. if

$$X_{GK} + X_{AK} + X_{AG} = 0 \quad \text{(V.9)}$$

and
$$X_{GK} + \mu X_{GK} + X_{AG} = 0$$

whence
$$X_{GK} = X_{AK}/\mu \quad \text{(V.10)}$$

Equation (V.9) shows that oscillation will take place at a frequency which makes the sum of the three reactances zero, i.e. at the resonance frequency of the "closed circuit" mentioned in the title. It follows that two of the reactances must be of like sign (i.e. both must be inductances or both must be capacitance) and the third must be of the opposite sign to these two, for if all three were of like sign their sum could not be zero. Equation (V.10), however, tells us that X_{GK} and X_{AK} must be reactances of like sign. There are thus two and only two basic possible circuits. These are shown in Fig. 98 (*a*) and (*b*). In Fig. 98 (*a*), X_{GK} and X_{AK} are inductances and X_{AG} is a capacitance. In Fig. 98 (*b*), X_{GK} and X_{AK} are capacitances and X_{AG} is an inductance. Fig. 98 (*a*) gives us the valve-circuit shown in Fig. 99, which is sometimes called the Armstrong oscillator. Fig. 98 (*b*) leads us to the Colpitts oscillator, which is treated on page 168.

In these two, basic, closed-circuit oscillators, each of the impedances Z_{AK}, Z_{GK} and Z_{AG} is formed of a single coil or condenser.

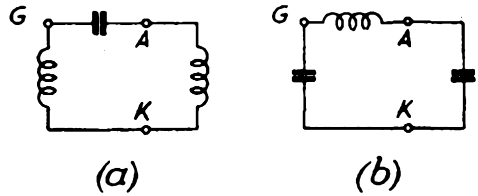


FIG. 98. CLOSED-CIRCUIT OSCILLATORS: THE TWO BASIC POSSIBILITIES

If each is permitted to be a little more complicated than this, many other closed-circuit oscillators can be devised. The tuned-anode tuned-grid oscillator, page 170, and the crystal-controlled oscillator shown in Fig. 107, page 183, are examples of closed-circuit oscillators.

In all cases, the oscillation frequency is the frequency at which the closed-circuit resonates (i.e. the frequency at which $X_{AK} + X_{GK} + X_{AG}$ is zero) and the maintenance condition is that the grid-circuit reactance, X_{GK}, shall be at least $1/\mu$ times as large as the anode-circuit reactance, X_{AK}. These conclusions are of course modified by the presence of resistance in the closed-circuit (see, for example, equation (V.12)).

The Derivation of Practical Circuits from Given Equivalent Circuits

The argument of the previous section enables us to deduce the a.c. equivalent circuits of a number of different oscillators. What is the procedure, thereafter, for drawing a practical circuit, suitable for wiring up? In some cases the procedure is very simple; for example, to transform the equivalent circuit of Fig. 98 (*a*) to the practical circuit of Fig. 99, the procedure consists simply of the following two steps: (i) draw the symbol for the valve and connect its anode to the point marked *A*, its cathode to the point marked *K* and its grid to the point marked *G*, and (ii) insert the h.t. voltage source in a break in the anode-to-cathode circuit, positioning it so that the h.t. negative terminal is connected to cathode. The requirement that the cathode and the h.t. negative terminal shall be as nearly as possible at the same potential arises from the consideration that leakage conductance in the h.t. supply source will usually bring the valve heater to approximately the potential of h.t. negative, coupled with the fact that in small valves the insulation from heater to cathode is not able to withstand any large difference of potential.

There are a number of reasons why the procedure is not always as simple as the above. Sometimes, in the given equivalent circuit, there is no d.c. path from cathode to anode. In such a case it will obviously not suffice to insert the h.t. supply source in a break in the anode-to-cathode circuit. The second step must then be changed to the following: (ii) connect an additional path from anode to cathode, for the purpose of supplying d.c. anode-current to the valve, including in this d.c. path a choke or other high a.c. impedance. Since this added, d.c., anode-circuit is *shunted* across the a.c. anode-circuit which was present in the given equivalent circuit, the resulting practical circuit is known as a "Shunt-fed Circuit," as distinct from the "Series-fed Circuit" which results from the first-mentioned procedure above.

The process is still not complete until one has assured oneself that there is a d.c. path from cathode to grid, for the purpose of establishing the grid-bias voltage. Even if the latter is to be zero, such a path must be provided. If there is no such path, a high resistance should be connected between grid and cathode.

Finally, if there is a d.c. path from anode to grid (which two electrodes are intended to be at quite different d.c. potentials) this d.c. path must be broken by interposing a "d.c. blocking condenser," i.e. a condenser whose capacitance is sufficiently large for its reactance at the operating frequency to be negligibly small in comparison with the other circuit elements.

All these points of procedure are well illustrated by the circuit of Fig. 102 which is the practical circuit derived from Fig. 98 (*b*). Two further practical points are perhaps worth mentioning (though the experimenter will in any case learn them from bitter experience). The first is that long leads radiate power into space and the resulting losses represent the introduction of effective resistances into the circuit. This may prevent a low-power h.f. oscillator circuit from oscillating. The second is the advisability of anode-circuit decoupling both to avoid losses in the h.t. supply source and, in a laboratory, to avoid sharing the a.c. output of the oscillator with ungrateful colleagues. For simplicity, most of the oscillator circuits in this chapter are, however, shown without decoupling.

The Armstrong Oscillator

As explained on page 162, the Armstrong oscillator is one of the two basic closed-circuit oscillators. The circuit is shown in Fig. 99. If the resistance of the coils be neglected, the general theory of closed-circuit oscillators shows that the frequency of oscillation is given by $\dfrac{1}{2\pi\sqrt{(L_1 + L_2)C}}$ and that the maintenance condition is $L_1 \geqslant L_2/\mu$.

No longer neglecting the resistance of the coils, we have from the equivalent circuit

$$I_1[\rho + (R_1 + j\omega L_1)(1 + \mu) + 1/(j\omega C)] = I_2\rho$$

$$I_1(\rho + \mu R_1 + \mu j\omega L_1) = I_2(\rho + R_2 + j\omega L_2)$$

Dividing these equations, cross-multiplying, and equating imaginary parts, we have the frequency of oscillation

$$\omega^2 = \frac{1 + R_2/\rho}{C\left[L_1 + L_2 + \dfrac{(R_1L_2 + R_2L_1)(1 + \mu)}{\rho}\right]} \quad . \quad \text{(V.11)}$$

Equating real parts we have the condition of oscillation

$$\rho(R_1 + R_2) + R_1R_2(1 + \mu) + L_2/C - \omega^2L_1L_2(1 + \mu) = 0$$

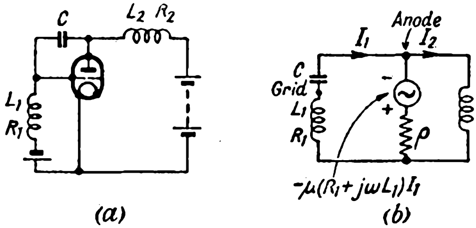


FIG. 99

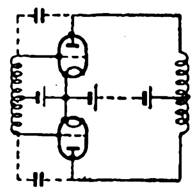


FIG. 100

Substituting $1/(L_1 + L_2)C$ as an approximate value for ω^2, this becomes

$$L_1 = L_2/\mu + \frac{[\rho(R_1 + R_2) + R_1R_2(1 + \mu)]\,(L_1 + L_2)C}{\mu L_2} \quad . \quad \text{(V.12)}$$

We may look upon L_1/L_2 as indicating the feed-back ratio. Oscillation can take place only if this ratio is as great as the value given by equation (V.12), i.e. if the left-hand side of equation (V.12) is equal to or greater than the right-hand side.

This oscillator is a useful high-frequency oscillator, the capacitance, C, being provided entirely by the grid-anode capacitance of the valve. It may be used in a push-pull form (Fig. 100), the symmetry of which proves useful at high frequencies.

The Hartley Oscillator

The Hartley oscillator may be considered to be a development of the Armstrong oscillator. If, in Fig. 99 (a), the positions of L_1 and the bias-supply were interchanged, and at the same time the positions of L_2 and the h.t. supply were interchanged, the equivalent circuit would be unaffected, but we should now have the two coils L_1 and L_2 connected together, with the cathode-lead tapped-on at their junction. The advantage of making this apparently insignificant alteration is that a single coil of bare wire may now be used and the cathode-lead may be clipped on to this coil at any desired point, so making it an easy matter to vary the ratio L_2/L_1 which controls the amplitude of oscillation.

Altering the positions of the two coils in the way described above does not alter the equivalent circuit. Using a single, tapped coil, however, in place of the two coils, introduces mutual inductance into the circuit, and thus *does* alter the equivalent circuit. In fact, it converts the circuit from the Armstrong oscillator to the Hartley oscillator.

This much-used oscillator is shown in its simplest form in Fig. 101 (*a*), and Fig. 101 (*b*) shows an alternative form in which the parallel *L–C* circuit, or Tank Circuit, is parallel-fed by means of the choke, L', of large reactance at the oscillation frequency, and the condenser, C', of small reactance at the oscillation frequency. These two forms are the Series-fed and Shunt-fed forms; in the former, alternating current flows through the h.t. supply, and it is usual to shunt a large capacitance across this lest its impedance should be appreciable. The equivalent circuit for both arrangements is shown in Fig. 101 (*c*), and it will be seen that the circuit consists of a parallel *L–C* circuit with part of the coil included in the anode circuit and the remainder in the grid circuit.

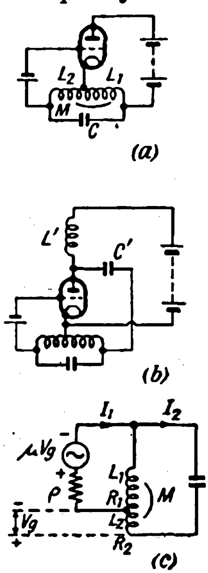


FIG. 101. HARTLEY OSCILLATOR

A preliminary approximate analysis of the circuit may be made by considering the coil free from resistance, i.e. $R_1 = R_2 = 0$. First let us show that, if this be so, the impedance presented by the tank circuit to the generator, μV_g, becomes infinite at a frequency given by

$$\omega = \frac{1}{\sqrt{C(L_1 + L_2 + 2M)}} \qquad . \quad . \quad (V.13)$$

The impedance of the tank circuit, considered as a closed loop, is simply the reactance

$$j\omega L_1 + j\omega L_2 + 2j\omega M + 1/j\omega C$$

and this becomes zero at a frequency given by equation (V.13). Now looking into the tank circuit from the generator, μV_g, we see that it consists of two parallel branches. At the frequency given by equation (V.13) the sum of the impedances of these two branches is zero, i.e. their impedances are equal and opposite. The currents sent through them by the generator, μV_g, must therefore also be equal and opposite. I_1 will thus be zero, and the impedance presented to the generator will be infinite. It follows that the voltage across section 1 of the coil will be μV_g. The oscillator, however, provides its own input voltage, V_g, by the inclusion of section 2 of the coil in the grid circuit. Thus, for sustained oscillation, the ratio of the voltages across sections 1 and 2 of the coil must be μ. But the ratio of these voltages is equal to

$$\frac{j\omega L_1 I_2 + j\omega M I_2}{j\omega L_2 I_2 + j\omega M I_2} \quad \text{i.e.} \quad \frac{L_1 + M}{L_2 + M}$$

Thus

$$\mu = \frac{L_1 + M}{L_2 + M} \quad . \quad . \quad . \quad . \quad \text{(V.14)}$$

This is thus the condition for oscillation, the oscillation frequency being given by equation (V.13).

A complete analysis, taking account of the resistance of the coil, may be made by applying Kirchhoff's laws to the two closed circuits of Fig. 99 (*c*), having careful regard to the directions of all induced e.m.f.'s, and writing V_g equal to the voltage across the section 2 of the coil, i.e.

$$V_g = (R_2 + j\omega L_2)I_2 + j\omega M(I_2 - I_1)$$

This procedure gives the following two equations—

$$I_1(\rho + R_1 + j\omega L_1 - \mu j\omega M)$$
$$= I_2[R_1 - \mu R_2 + j\omega(L_1 + M) - j\omega(L_2 + M)\mu]$$
$$I_1(R_1 + j\omega L_1 + j\omega M)$$
$$= I_2[R_1 + R_2 + j\omega L_1 + j\omega L_2 + 2j\omega M + 1/j\omega C]$$

Dividing these equations, we have a single equation involving only frequency and the circuit constants. This equation is the condition that the voltage fed back is equal in magnitude and phase to the input voltage required to produce it. Cross-multiplying, and equating the imaginary parts, we have

$$(\rho + R_1)\left(L_1 + L_2 + 2M - \frac{1}{\omega^2 C}\right) + (R_1 + R_2)(L_1 - \mu M)$$
$$= R_1(L_1 + M - \mu L_2 - \mu M) + (L_1 + M)(R_1 - \mu R_2)$$

which gives

$$\omega^2 = \frac{1 + R_1/\rho}{C\left[(L_1 + L_2 + 2M) + \dfrac{(R_1 L_2 + R_2 L_1)(1 + \mu)}{\rho}\right]} \quad . \quad . \quad \text{(V.15)}$$

As R_1 and R_2 are normally small, the oscillation frequency given by this expression is not very different from that of equation (V.13).

Cross-multiplying and equating real parts, we have

$$(\rho + R_1)(R_1 + R_2) - \omega^2(L_1 - \mu M)(L_1 + L_2 + 2M) + \frac{L_1 - \mu M}{C}$$
$$= R_1(R_1 - \mu R_2) - \omega^2(L_1 + M)(L_1 + M - \mu L_2 - \mu M)$$

in which the second and third terms on the left-hand side cancel each other if we assume that ω is sufficiently accurately given by equation (V.13). The equation then becomes

$$\omega^2(L_1 + M)^2$$
$$= \omega^2\mu(L_1 + M)(L_2 + M) - \rho(R_1 + R_2) - R_1R_2(1 + \mu) \quad \text{(V.16)}$$

We shall show that this maintenance equation implies that there are *two* positions of the cathode tapping point (see Fig. 101 (*a*)) at which sustained oscillation can commence, and that oscillation can be sustained when the tapping point is placed between these two positions, but not when it is outside them. This, of course, is to be expected, for when the tapping point is at the grid end of the coil the alternating grid-voltage must be zero, and when it is at the anode end, the alternating anode-voltage must be zero.

First note that the term $R_1R_2(1 + \mu)$ in equation (V.16) is usually small with respect to $\rho(R_1 + R_2)$, the ratio of the terms being approximately $g_mR_1R_2/(R_1 + R_2)$. We shall write x for $(L_1 + M)$, so that $(L_2 + M)$ becomes $(L - x)$, where L is the whole inductance of the coil, viz. $L_1 + L_2 + 2M$. Equation (V.16) then becomes

$$\omega^2x^2 = \mu\omega^2x\,(L - x) - \rho(R_1 + R_2)$$

Now different values of x (viz. $L_1 + M$) correspond to different positions of the tapping point, (e.g. $x = 0$ at the anode end of the coil, and $x = L$ at the grid end). Thus the fact that our maintenance condition is a quadratic in x means that there are two possible tapping points at which oscillation can be sustained. Solving for x, and writing $1/(LC)$ for ω^2, we have

$$x = \tfrac{1}{2}L\,\frac{\mu}{1 + \mu}\left\{1 \pm \sqrt{1 - \frac{4}{g_mR_D}\cdot\frac{1 + \mu}{\mu}}\right\}$$

where R_D is the Dynamic Resistance (L/RC) of the whole tank circuit. Since $4/(g_mR_D)$ will normally be small compared to unity, we may further approximate as follows

$$x = \tfrac{1}{2}L\,\frac{\mu}{1 + \mu}\left\{1 \pm \left(1 - \frac{2}{g_mR_D}\cdot\frac{1 + \mu}{\mu}\right)\right\}$$
$$= \text{either } (a) \quad L/(g_mR_D)$$
$$\text{or } (b) \quad L\left(\frac{\mu}{1 + \mu} - \frac{1}{g_mR_D}\right)$$

The first of these, (a), is the smaller value of x, and corresponds to a position of the tapping point near the anode end of the coil. The second value, (b) corresponds to a position near the grid end. For the idealized case of resistanceless coils ($R_D = \infty$), position (a) is at the anode end, but position (b) is not quite at the grid end of the coil.

For a coil of given resistance (R_D finite) the two positions (a) and (b), move towards each other as the valve parameters are degraded, i.e. as g_m and μ decrease. Now the valve parameters depend upon the oscillation amplitude, being in general degraded as the amplitude increases. Hence, if the tapping point be placed at some position between (a) and (b), the amplitude of oscillation will increase until the values of the parameters are such that one of the expressions, (a) or (b), becomes equal to the value of x for that particular position of the tapping point.

The Colpitts Oscillator

The Colpitts oscillator is derived from the circuit of Fig. 98 (b). Fig. 102 (a) shows the circuit, and its equivalent circuit is shown in Fig. 102 (b). As explained on page 163, no series-fed version can exist for this oscillator. Moreover, since the a.c. equivalent circuit provides no d.c. path between cathode and grid, a shunt path must be included (R' in Fig. 102 (a)) to provide for the application of the grid-bias voltage. The low-reactance blocking condenser C' is inserted to avoid an undesirable d.c. path between anode and grid. If L' is of large inductance and R' of large resistance, then these two elements constitute high impedance shunts across the alter-

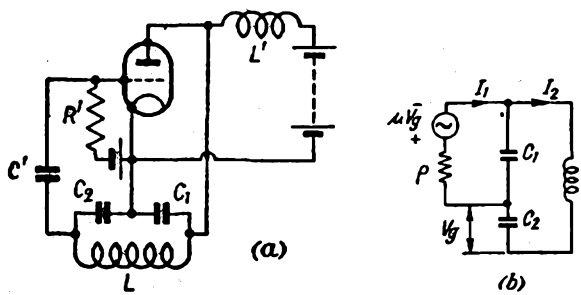


FIG. 102. COLPITTS OSCILLATOR

nating-current circuits, and need not appear in the equivalent circuit. In cases where the impedance of the h.t. supply is not negligibly small it is usual to shunt it by means of a large capacitance, as mentioned in connexion with the Hartley circuit.

From the general theory of closed-circuit oscillators (page 161) we see that oscillation occurs at a frequency which makes $X_{AK} + X_{GK} + X_{AG}$ equal to zero. These three reactances are respectively $-1/\omega C_1$, $-1/\omega C_2$, and ωL. It follows that the oscillation frequency is given by

$$\omega^2 = \frac{1}{L}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) \quad . \quad . \quad . \quad \text{(V.17)}$$

and that the condition of oscillation is

$$C_2 = \mu C_1 \quad . \quad . \quad . \quad . \quad \text{(V.18)}$$

An analysis taking account of the resistance of the coil may be made by applying Kirchhoff's laws to the equivalent circuit, and writing V_g equal to the voltage across the condenser included in the grid circuit, i.e. equal to $I_2/j\omega C_2$. We then have

$$I_1/(j\omega C_1) = I_2\left(R + j\omega L + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}\right)$$

and $$I_1\left(\rho + \frac{1}{j\omega C_1}\right) = I_2\left(\frac{1}{j\omega C_1} - \frac{\mu}{j\omega C_2}\right)$$

Dividing these equations, cross-multiplying, and equating imaginary parts, we have

$$\omega^2 = \frac{1}{L}\left[\frac{1}{C_2} + \frac{1}{C_1}(1 + R/\rho)\right] \quad . \quad . \quad \text{(V.19)}$$

giving the oscillation frequency. Equating real parts we have

$$\rho R + L/C_1 = \frac{(1+\mu)}{\omega^2 C_1 C_2}$$

and substituting for ω the value given by equation (V.17), which is very nearly equal to that given by equation (V.19), the condition for oscillation becomes

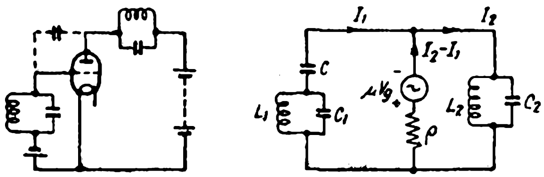
$$C_2/C_1 = \mu - \frac{\rho R(C_1 + C_2)}{L} \quad . \quad . \quad . \quad \text{(V.20)}$$

We may look upon C_1/C_2 as indicating the feed-back ratio, so that if C_1/C_2 is less than the value given by equation (V.20), oscillation cannot take place, the values taken for μ and ρ being those corresponding to the steady operating voltages.

For use at higher frequencies, the Colpitts oscillator may be modified by omitting the condensers C_1 and C_2, their places being taken by the inter-electrode capacitances C_{ac} and C_{gc}. If a differential condenser be substituted for the two condensers C_1 and C_2, for the purpose of varying the oscillation amplitude without altering the frequency, it will be found that there are two threshold settings of the differential condenser. This corresponds to the two threshold tapping-points in a Hartley Oscillator.

The Tuned-anode Tuned-grid Oscillator

At high frequencies the self-capacitances of the coils of an Armstrong oscillator become important, and instead of having simply inductances in the grid and anode circuits we find we have parallel L–C circuits. The oscillator then becomes as shown in Fig. 103 (*a*), the equivalent circuit being shown in Fig. 103 (*b*). This, the tuned-anode tuned-grid oscillator with inter-electrode capacitance coupling, is a circuit of importance, not only because of the extent of its use in practice but also because the equivalent circuit of many comparatively complicated valve oscillating systems reduces to that of this oscillator. For example, when parasitic oscillation occurs (i.e. undesirable sustained oscillation in an amplifier or oscillator) it may be found that at the frequency of the parasitic oscillation various circuit components are effectively short circuits, whereas the reactance of leads assumes appreciable proportions, and the circuit reduces to a tuned-anode tuned-grid oscillator.



(*a*) (*b*)

FIG. 103. TUNED-ANODE TUNED-GRID OSCILLATOR

Parasitic oscillation may often be eliminated by including a resistance in the grid circuit; in view of the importance of stray capacitances and lead inductances at high frequencies, this resistance should be connected as near the grid terminal of the valve as possible, in order to ensure that it *is* connected in series with the grid circuit. The effect of such a resistance may be seen from equation (V.12): as R_1 is increased, the right-hand side of this equation increases. For given values of the other circuit constants the equation can only remain satisfied (and oscillation can only be maintained) with a valve having a larger value of μ.

The tuned-anode tuned-grid oscillator is of the closed-circuit type and it has already been shown (page 161) that, in such oscillators, the reactances X_{AK} and X_{GK} must each be of opposite sign to the reactance X_{AG}. In the tuned-anode tuned-grid oscillator, which we are considering, X_{AG} is capacitive. It therefore follows that both X_{AK} and X_{GK} must be inductive. Now a parallel L–C circuit is inductive only at frequencies below its resonance frequency. Hence it follows that the frequency of oscillation is lower than the resonance frequency of either of the tuned circuits, L_1C_1 and L_2C_2 (Fig. 103). The circuit is often built with variable condensers for C_1 and C_2. Variation of either of these alone will vary both the frequency and the amplitude of oscillation. The desired amplitude of oscillation at the desired frequency can be secured by a process of successive adjustment first of one condenser, then of the other.

A push-pull form of this oscillator is often used. For the push-pull circuit only two variable condensers are required, the circuit being similar to the push-pull Armstrong oscillator shown in Fig. 100, but with one variable condenser connected between the two grids and the other variable condenser connected between the two anodes. It is not permissible, of course, to use variable condensers of such a construction that one side has to be earthed.

At higher frequencies, a form of tuned-anode tuned-grid oscillator may be used in which short lengths of transmission-line (parallel rod, or co-axial cable) take the place of the tuned circuits. This is possible because of the properties of a transmission-line at frequencies which make its physical length approximately equal to one quarter of a wavelength. If one end of the line be closed by a short-circuit, then at such frequencies the input impedance looking into the other end has the same form as the impedance of a parallel coil-condenser circuit. The input impedance rises to a high, resistive value (corresponding to the dynamic resistance of a parallel coil-condenser circuit) at the frequency which makes the length of the line exactly equal to a quarter of a wavelength, and the impedance falls rapidly as the frequency is increased or decreased from this value. The reasoning which showed that the oscillation frequency of a tuned-anode tuned-grid oscillator is lower than the resonance frequency of either of its tuned circuits can also be used to show that the oscillation frequency of the oscillator using transmission lines is lower than the frequency at which the longer of the two lines has a length of a quarter of a wavelength. The two lines are usually provided with sliding short-circuits, movement of which can be used to control the oscillation frequency and the oscillation amplitude in the same way as these quantities are controlled by the variable condensers when *L–C* circuits are used.

The Dynatron Oscillator

The types of oscillator hitherto considered all depend for their operation on the feeding back to the grid circuit of part of the output voltage. The Dynatron oscillator does not depend on this, but on the fact that over a certain range of anode and screen voltages the screen-grid valve can behave as a negative resistance, i.e. when the anode-voltage, measured in the conventional direction, is increased, the anode-current, measured in the same direction, decreases (see Chapter II).

As explained in Chapter II, the effect is due to secondary emission from the anode, and takes place when the anode-voltage is less than the screen-grid voltage. Under these conditions the slope of the $i_a - v_a$ characteristics will be negative, as shown in Fig. 17, and since the valve impedance, ρ, is defined as the reciprocal of this slope, ρ will be negative. If the downward sloping parts of the characteristics are fairly linear, we may assume that this negative value of ρ is a constant within this range of anode-voltage. As shown in Fig. 17, however, the value of ρ depends upon the grid-voltage, since the negative slope is different for the various curves, the slope increasing with grid-voltage. The set of curves shown in Fig. 17 is for a given value of screen-grid voltage; for other values of screen-grid voltage there will be other sets of curves.

The dynatron oscillator operates with a fixed grid-voltage and a fixed screen-grid voltage, so that we are concerned with one particular curve of one particular set. The circuit arrangements to secure fixed grid and screen-grid voltages are shown in Fig. 104 (*a*). The grid circuit includes a steady voltage (the grid-bias voltage) but no alternating voltage; the h.t. voltage is connected between the screen-grid and cathode, and a condenser, C', is also connected between screen-grid and cathode. The capacitance of C' is made large enough for its reactance at the frequency of oscillation to be very small, and thus, so far as alternating current is concerned, the screen-grid is effectively short-circuited to the cathode. The anode circuit includes the parallel L–C circuit and a lower h.t. voltage than that used for the screen-grid circuit.

Since there is no alternating grid-voltage and the impedance, ρ, is negative, the equivalent circuit consists simply of a negative resistance, ρ, connected across the L–C circuit, as shown in Fig. 104 (*b*).

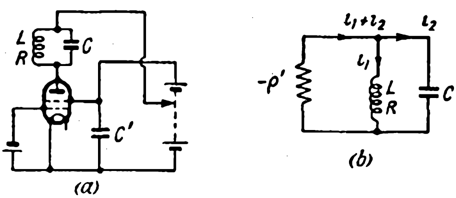


FIG. 104. DYNATRON OSCILLATOR

With the notation of this figure and writing $-\rho'$ for ρ, so that ρ' is the numerical value of the reciprocal of the slope of the i_a-v_a characteristic,

$$-\rho'(i_1 + i_2) + L\frac{di_1}{dt} + Ri_1 = 0$$

$$L\frac{di_1}{dt} + Ri_1 - \frac{1}{C}\int i_2 dt = 0$$

By differentiating the second of these equations with respect to time we may express i_2 in terms of i_1. Substituting this result into the first equation we have

$$\rho' LC\frac{d^2 i_1}{dt^2} + (\rho' RC - L)\frac{di_1}{dt} + (\rho' - R)\, i_1 = 0$$

This means that there will be sustained sinusoidal oscillations if the coefficient of $\frac{di_1}{dt}$ is zero, i.e. if

$$\rho' = \frac{L}{CR} \quad . \quad . \quad . \quad . \quad (V.21)$$

The frequency of the oscillation will be given by

$$\omega = \sqrt{\frac{\rho' - R}{\rho' LC}} = \sqrt{\frac{1}{LC}} \cdot \sqrt{1 - R/\rho'} \quad . \quad . \quad (V.22)$$

If ρ' is greater than L/RC, any oscillation which may be set up in the anode circuit (e.g. by the switching on of the h.t. supply) will decay exponentially, and will not be sustained. If ρ' is less than L/RC, the oscillation will increase in amplitude until the value of ρ' corresponding to the amplitude finally reached satisfies equation (V.21). Substitution of equation (V.21) into equation (V.22) shows that the oscillation frequency is given by $\omega_0\sqrt{1 - 1/Q^2}$ where ω_0 is $1/2\pi\sqrt{LC}$ and Q is the selectivity of the L–C circuit. Since this expression for the oscillation frequency is independent of the valve parameters, the frequency-stability (see page 178) of this oscillator is inherently greater than that of other oscillators mentioned hitherto.

The assumption of linear characteristics in the downward sloping region leads to the result that the oscillation will be sinusoidal, but we have seen that the amplitude of the oscillation is limited solely by the curvature. Thus, it follows, as in the case of other oscillators already considered, that the distortion will increase as the amplitude is increased.

It is possible to use a triode in a dynatron circuit, the grid taking the place of the screen-grid, and being at a positive potential with respect to the anode. Under these conditions a large anode-current and a large grid-current will flow, usually resulting in damage to the valve. The anode-current may be reduced by reducing the anode and grid-voltages proportionately, but this increases ρ'.

Power Oscillators : Class C

So far we have discussed only Class A oscillators, i.e. oscillators in which anode-current flows throughout the whole of the cycle, the grid-voltage never falling so low as to cut off the anode-current. Class A oscillators have a limited usage where small output only is required, but in cases where considerable power is required the Class C oscillator is always used.

We have pointed out that an oscillator may be considered as an amplifier which supplies its own input voltage, and it follows that most of what has been said about amplifiers applies also to oscillators. In particular, Class C operation, with an angle of flow of less than 180°, gives high efficiency. Moreover, the output power may be increased by allowing the alternating grid-voltage to swing the grid positive with respect to cathode for part of the cycle. In this case, power is dissipated in the grid circuit. In the case of an amplifier, power dissipated in the grid circuit is supplied by the previous stage, and the extent to which the grid is allowed to become positive is limited by the excessive demands upon the previous stage. In an oscillator, however, the power dissipation in the grid circuit is supplied from the h.t. supply, as is the anode dissipation of the valve and the useful output power. Thus, the extent to which the grid of an oscillator is allowed to swing positive is limited by considerations of efficiency, the efficiency being given by

$$\text{Efficiency} = \frac{\text{Useful output power}}{\text{Total power supplied by h.t.}}$$

$$= \frac{\left\{\begin{array}{l}\text{Total power supplied by h.t.}\\ -\text{Anode dissipation}\\ -\text{Grid dissipation}\end{array}\right\}}{\text{Total power supplied by h.t.}}$$

Consider the oscillator of Fig. 93 (*a*) operating as a Class A oscillator. Let M be increased so that the alternating anode- and grid-voltages become very large. To increase the output still further, let the steady anode-voltage be increased, the steady anode-current increasing with it, and thus also the anode dissipation. If we now increase the grid-bias voltage (the steady negative grid-voltage) so that for more than half of the cycle the total grid-voltage is sufficiently negative to cut off the anode-current, we shall have Class C operation. The anode dissipation will be reduced, the efficiency will be increased, and the waveform of the anode-current will be far from sinusoidal, consisting of a peak of current lasting for less than half of the cycle. Nevertheless, as a result of the selective L–C circuit, the anode-voltage will be approximately sinusoidal, as explained in the section on Class C amplifiers.

Since in Class C operation the grid-bias voltage is sufficiently large for anode-current to flow for less than half of the cycle, it follows that the anode-current of a Class C amplifier will be zero when there is no input voltage. This means that when the h.t. voltage is first switched on to a Class C oscillator no anode-current will flow; the operating point is in a region where the characteristics simply lie along the zero axis. By definition, the impedance of the valve under these quiescent conditions is infinite. Thus, oscillation cannot take place. The oscillation must be started by using initially a smaller grid-bias voltage, and increasing this, after oscillation has commenced, to a value greater than the cut-off value. This procedure may be avoided by the use of a self-bias circuit.

Self-bias for Class C Oscillator (Rectification Bias)

It is shown in Chapter VI that, if an alternating voltage be connected across a diode in series with a circuit consisting of a resistance in parallel with a capacitance, rectification takes place, and a steady voltage proportional to the amplitude of the alternating voltage appears across the resistance. If such a circuit (a resistance in parallel with a capacitance) be connected in series with the input voltage in the grid circuit of an oscillator or amplifier, the grid and cathode of the valve will function as a diode, and a steady voltage, roughly proportional to the alternating grid-voltage, will appear across the resistance, and will serve as a grid-bias supply. (See Fig. 134, p. 227, and lower part of Fig. 156, p. 267.)

This self-bias device has three important properties. First, an oscillator with self-bias is self-starting, for when the h.t. supply voltage is first switched on there is zero grid-bias voltage, instead of a grid-bias voltage greater than the cut-off value, as in the case of an oscillator with fixed bias. Anode-current thus flows and oscillation commences. As the amplitude of oscillation increases, so the grid-bias voltage increases.

Secondly, the oscillator may operate as a Class C oscillator. We have seen that oscillation commences when the grid-bias voltage is zero, and that as the amplitude increases the grid-bias voltage increases with it. Now, in an oscillator with fixed bias, the amplitude of oscillation depends upon the bias voltage in addition to the values of the circuit constants. So also in an oscillator with self-bias. Thus the amplitude will continue increasing until the point is reached where further increase of the grid-bias voltage would cause a reduction of amplitude. If the grid-bias voltage were to increase further, the amplitude would decrease, and as the bias voltage is secured by rectification from the alternating grid-voltage, the bias voltage would be forced to decrease. The grid-bias voltage finally attained may be made such as to cause Class C operation, i.e. greater than the cut-off value.

Finally, the peak value of the alternating grid-voltage in such a circuit always *exceeds* the grid-bias voltage, so that the grid becomes positive with respect to cathode for a small part of the cycle. This follows from the theory of the rectifier (see Chapter VI). Thus, we see that the use of self-bias tends to produce the conditions required for maximum output.

The value of the self-bias resistance is usually of the order of 10,000 ohms, and the capacitance of the condenser should be such that its reactance at the frequency of oscillation is small with respect to the self-bias resistance. As shown in Chapter VI, if the capacitance of the condenser is too great the steady voltage across the resistance is sluggish in following changes of amplitude of the alternating voltage. In the case of an oscillator with self-bias this means that the bias voltage will be sluggish in following the change of amplitude of the oscillations. The result of this may be to change entirely the sequence of events, described above, by which the amplitude of oscillation increases to a value at which it remains stable. If the bias voltage increases beyond a certain value, the amplitude will decrease, as we have seen, but if the bias voltage is sluggish in following this decrease of amplitude, the decrease may proceed to the point at which oscillation ceases. Then, slowly, the bias voltage will decrease until it reaches a value which permits oscillation to recommence. The whole cycle of events will then be repeated, with the result that the oscillation is continually interrupted. This effect has been called "Squegging"; it is prevented by reducing the capacitance of the condenser.

Design of Class C Oscillator

The procedure in the design of a Class C oscillator follows that given in Chapter IV for the Class C amplifier, but with the following additions.

Having determined L, C, and R for the tank circuit, further circuit constants remain to be determined. For example, in the tuned-anode oscillator with mutual inductive coupling M must be determined. From the operation diagram (Fig. 92) which has been built up in the earlier stages of the design, the ratio of the alternating anode- and grid-voltages is known. Now the alternating grid-voltage is the product of ωM and the coil current, and the coil current is very nearly equal to the quotient of the coil voltage and ωL, so that

$$\frac{\text{Alternating grid-voltage}}{\text{Alternating anode-voltage}} = M/L$$

The value of M is found from this relation. Knowing the coefficient of coupling (i.e. $M/\sqrt{LL_1}$) for the type of coil used, the value of the grid circuit inductance may be determined. For other oscillators than the tuned-anode type, a relation similar to the above may be derived, giving the ratio of the alternating grid- and anode-voltages in terms of the circuit constants, and thus providing a means of determining the remaining circuit constants.

Next, from the operation diagram and the grid-current characteristics of the valve, the grid-current at any instant of the cycle may be determined, and a curve of grid-current against time plotted. The mean grid-current, i.e. the current which would be read by a d.c. milliammeter in the grid circuit, may be estimated from this curve, since

$$\text{Mean grid-current} = \frac{\text{Area under grid-current curve}}{\text{Periodic time}}$$

Dividing this mean grid-current into the required grid-bias voltage, we have the value of the self-bias resistance.

Effect of Grid-current on Frequency

The method of determining the oscillation frequency for the various oscillator circuits considered earlier in this chapter involved the assumption that the internal impedance between the grid and cathode terminals of the valve was infinite, i.e. that the grid circuit was open-circuited at the valve. Clearly, if grid-current flows, this does not represent the facts, and an impedance of some sort must be included in the equivalent circuit to close the grid circuit. If the grid-cathode inter-electrode capacitance may be neglected, this impedance is taken as a resistance, its value being such that the product of the resistance and the square of the r.m.s. value of the grid-current gives the grid power dissipation, i.e. the rate of dissipation of energy inside the valve as a result of the flow of grid-current.* Let us examine the result of adding such a resistance, R_g, in the equivalent circuit of the tuned-anode oscillator shown in Fig. 93.

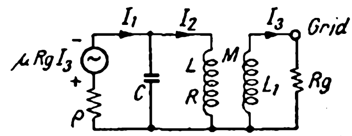


FIG. 105

The equivalent circuit becomes as shown in Fig. 105, and applying Kirchhoff's laws we have, with the notation of this figure,

$$\left.\begin{aligned} I_1\left(\rho + \frac{1}{j\omega C}\right) - \frac{I_2}{j\omega C} + \mu R_g I_3 &= 0 \\ \frac{I_1}{j\omega C} - I_2\left(R + j\omega L + \frac{1}{j\omega C}\right) + j\omega M I_3 &= 0 \\ I_2 j\omega M = I_3(R_g + j\omega L_1) \end{aligned}\right\} \quad . \text{(V.23)}$$

Eliminating I_1 between the first and second equations we have

$$I_2\left(\rho R + \frac{L}{C} + j\omega L\rho + \frac{\rho + R}{j\omega C}\right) = I_3\left(\frac{M}{C} + j\omega M\rho - \frac{\mu R_g}{j\omega C}\right)$$

Dividing this by the third equation, cross-multiplying and equating imaginary parts, we have

$$\frac{j\omega M^2}{C} = j\omega L\rho R_g + \frac{(\rho + R)R_g}{j\omega C} + j\omega L_1\rho R + \frac{j\omega L L_1}{C}$$

giving

$$\omega^2 = \frac{1 + R/\rho}{LC + \dfrac{L_1 CR}{R_g} + \dfrac{L_1 L - M^2}{\rho R_g}}$$

This expression for the oscillation frequency reduces to equation (V.5) when R_g is infinite, i.e. when the circuit operates without grid-current. The expression shows that, if grid-current flows, the oscillation frequency is not exactly equal to the resonance frequency

* The grid power dissipation in a given case may be found by plotting against time the product of grid-current and grid-voltage, and determining the mean value.

of the tank circuit ***even in the ideal case of a tank circuit free from resistance.*** In this case the oscillation frequency becomes

$$\omega = \frac{\omega_0}{\sqrt{1 + \frac{LL_1 - M^2}{\rho R_g LC}}} \quad . \quad . \quad . \quad (V.24)$$

where ω_0 is the resonance frequency of the tank circuit.

Similar expressions showing the effect of grid-current on the oscillation frequency of other types of oscillator may be derived along the same lines as the above.

Frequency Stability of Oscillators

The frequency stability of an oscillator expresses the degree of constancy of the oscillation frequency. In oscillators employed for radio transmission and measurement a very high degree of constancy is necessary, and in most cases the permissible frequency variation is less than one part in ten thousand.

We have seen that the oscillation frequency is a function of the circuit constants and of the valve constants. The chief cause of variation of frequency is variation of the circuit constants with temperature, e.g. due to variation in the dimensions of coils and condensers. To eliminate these variations, special coils and condensers, designed so that their electrical constants vary very little with temperature, may be used, and these may be kept at constant temperature. The chief cause of variation of the valve constants is variation of the supply voltages, and this may be limited by voltage stabilizing devices. Where the supply voltage is derived by rectification from alternating-current mains, however, there is usually a small alternating voltage (known as "Ripple") superimposed upon the steady voltage, its frequency being that of the mains voltage or else a multiple of this value, depending upon the type of rectifier used. This will cause a periodic variation of the valve parameters, and thus a periodic variation of oscillation frequency. In general this Frequency Modulation must be reduced to very small proportions if the oscillator is to be of any practical value. Variation of oscillation frequency with variation of the valve parameters, and in particular this undesirable frequency modulation, may be reduced by the choice of suitable circuit and valve constants, and also by the use of special circuits. The following are some points to consider.

First, as the resistance of the tank circuit decreases and the valve impedance increases, the oscillation frequency tends to become independent of the valve constants. This may be seen from equations (V.5), (V.11), (V.15), (V.19), for the particular oscillators to which these equations apply. In order to increase frequency stability, a resistance is sometimes connected in series with the valve in the anode circuit, thereby artificially increasing the valve impedance. Next, we see from equations (V.11), (V.15), and (V.24) that, for the circuits to which these equations refer, the oscillation frequency tends to become less dependent on the valve constants as the inductance of the tank circuit is decreased, i.e. as the L/C ratio is reduced. Further, in view of the effect of grid-current upon oscillation frequency, as discussed in the previous section, grid-current should be avoided, or reduced, where high stability is required.

For oscillators in which grid-current flows, the oscillation frequency is not exactly equal to the resonant frequency of the tank circuit even in the ideal case of a resistanceless tank circuit, but is a function of ρ and R_g. By the inclusion of a suitable reactance in the anode or grid circuit of such oscillators, the oscillation frequency in the ideal case of a resistanceless tank circuit may be made equal to the resonance frequency of the tank circuit, i.e. independent of the valve parameters. The value of reactance required proves to be independent of the valve parameters. Consider the tuned-anode oscillator using grid-current, for which the oscillation frequency with a resistanceless tank circuit is given in equation (V.24). If a capacitance, C_1, be included in the anode circuit in series with the anode, equations (V.23) become as follows, the tank circuit resistance being neglected.

$$I_1\left(\rho + \frac{1}{j\omega C} + \frac{1}{j\omega C_1}\right) - \frac{I_2}{j\omega C} + \mu R_g I_3 = 0$$

$$\frac{I_1}{j\omega C} - I_2\left(j\omega L + \frac{1}{j\omega C}\right) + j\omega M I_3 = 0$$

$$I_2 j\omega M = I_3(R_g + j\omega L_1)$$

Proceeding in the same manner as in the previous section, we have

$$\omega = \frac{1}{\sqrt{LC}} \cdot \sqrt{\frac{1 + \dfrac{L_1}{\rho R_g C_1}}{1 + \dfrac{(LL_1 - M^2)(1 + C/C_1)}{\rho R_g LC}}}$$

If C_1 be chosen so that

$$\frac{C_1}{C} = \frac{k^2}{1 - k^2}$$

where k is the coefficient of coupling, i.e. $M/\sqrt{LL_1}$, then this expression for the oscillation frequency becomes $1/\sqrt{LC}$, which is the resonant frequency of the tank circuit. Clearly we cannot simply connect a capacitance of this value in series with the anode, for there would then be no direct-current path. In practice, therefore, the capacitance is connected in series with the anode and a choke of high reactance at the oscillation frequency is connected across it to provide a path for the direct current. A better method is to "parallel feed" the tank circuit by means of this choke and the capacitance, C_1. In practice the value of C_1 would be adjusted by experiment.

Similar reactances may be added to the other oscillator circuits which we have described. For example, it may be shown that the oscillation frequency of the Colpitts oscillator when grid-current is permitted is given by

$$\omega = \sqrt{\frac{1}{L}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) + \frac{1}{\rho R_g C_1 C_2}}$$

but that the addition of an inductance, LC_2/C_1, in series with the anode, makes the oscillation frequency equal to the resonant frequency of the resistanceless tank circuit. The proof of this is left as an exercise for the reader.

When an oscillator circuit is coupled to its load circuit, variation of the temperature of the load circuit will cause variation of the oscillation frequency. Moreover, oscillators generating large powers are themselves sources of heat. Where high stability is required, therefore, it is usual to use a small power oscillator not coupled to the load circuit, but feeding an amplifier which is coupled to the load circuit. The small power oscillator, which is spoken of as the Master Oscillator, should be designed for the maximum frequency stability. The amplifier stage is sometimes loosely described as a Driven Oscillator.

Several special circuits have been used as high stability master oscillators, but oscillator circuits employing piezo-electric crystals are now almost exclusively used for this purpose.

Crystal-controlled Oscillators

Crystal-controlled oscillators are oscillators in whose circuit is included a slab cut from a crystal of piezo-electric material such as quartz or Rochelle Salt. A slab cut in a particular direction from a piezo-electric crystal has the two following properties. First, if two opposite faces of the slab are coated with conducting material to form a condenser, then a voltage applied to the plates of this condenser produces a mechanical stress in the slab. For example, the application of a voltage, V, as shown in Fig. 106, would produce a stress tending to increase the length, x. The magnitude of this mechanical stress would be the product of the electrical stress, V/t, and the piezo-electric constant, ε, for the material. The resulting elongation will depend upon the value of Young's modulus for the material, since

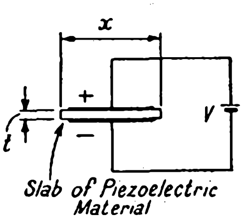


FIG. 106

$$\text{Stress/Strain} = \text{Young's Modulus, } Y$$

Thus the strain, $\delta x/x$, will be given by

$$\delta x/x = \text{Stress}/Y = \frac{\varepsilon V}{tY}$$

$$\text{i.e. } \delta x = \frac{\varepsilon V x}{tY}$$

Second, if a mechanical strain be produced in the slab, by mechanical means *or as above*, electrical charges will be released in the material, and will charge up the condenser. For example, if the slab shown in Fig. 106 be elongated to a length $x + \delta x$, so that the strain is $\delta x/x$, the charge per unit area produced on the electrodes will be the product of the strain and the piezo-electric constant, ε. A positive charge will be produced on the upper plate of Fig. 106, so that if the strain were initially produced by the connexion of a voltage, V, as shown in the figure, the total charge on the condenser will be the sum of the charge liberated as a result of the strain and the charge normally to be expected as a result of charging up the condenser. The latter will be the product of V and the capacitance $kxy/4\pi t$, where k is the dielectric constant of the material of the slab and y is the length of the slab perpendicular to the plane of the page. (For readers who prefer rationalized units, the 4π should be deleted here and throughout what follows.) The former will be the product of the strain, the piezo-electric constant and the area, i.e. $(\delta x/x)\varepsilon xy$. Thus

$$\text{Total charge} = \frac{kxyV}{4\pi t} + \left(\frac{\delta x}{x}\right)\varepsilon xy$$

If the strain is initially produced by the application of the voltage, V, then we may substitute $\dfrac{V\varepsilon}{tY}$ for the strain, $\delta x/x$, giving

$$\text{Total charge} = \frac{kxyV}{4\pi t} + \frac{xy\varepsilon^2 V}{tY}$$

This is the charge which would be produced on a simple condenser of capacitance

$$\frac{kxy}{4\pi t} + \frac{xy\varepsilon^2}{tY}$$

when charged to a voltage, V. Thus we may say that the equivalent circuit of the slab of piezo-electric material *for steady voltages* is a capacitance equal to that of the simple condenser formed by the slab and electrodes, in parallel with a capacitance

$$C = \frac{xy\varepsilon^2}{tY} \quad . \quad . \quad . \quad . \quad . \quad (V.25)$$

When the applied voltage is alternating, a slightly more complicated equivalent circuit must be adopted. The mechanical stress will be an alternating one, and the crystal will execute longitudinal forced vibrations. For a given alternating stress the amplitude of these vibrations will be a maximum when the frequency of the applied voltage is equal to the mechanical resonance frequency of the crystal slab. The greater the amplitude of the vibrations, the greater the amplitude of the alternating current set up as a result of the alternating strain. This alternating current will augment the current, sometimes described as the charging current, normally flowing through the capacitance, $kxy/4\pi t$, just as the charge due to the steady strain augments that due to the charging of the condenser. With a fixed amplitude of alternating voltage, the total current will become very great at the resonance frequency of the crystal slab; in fact, if the mechanical resistance (i.e. friction, air resistance, etc.) were zero, the amplitude of vibration, and with it the current, would become infinite. Clearly the crystal is behaving in very much the same way as a series L–C circuit* having the same resonance frequency as the natural frequency of mechanical vibrations, and it can be shown that the equivalent circuit of the crystal is, in fact, a series combination of inductance, capacitance, and resistance. The value of the resistance will depend on the mechanical resistance, and since this is normally extremely small, so also will be the resistance of the equivalent circuit. In parallel with this series L–C circuit we must consider the normal capacitance of the slab-and-electrodes to be connected. The complete equivalent circuit for the slab of crystal is thus as shown at the left-hand side of Fig. 107 (*c*), namely $LRCC_1$. It is to be noted that the ratio of the capacitance C_1 to the capacitance C, is $kY/4\pi\varepsilon^2$, the value of which for quartz is about 180, so that by far the greater part of the capacitive reactance is in the series L–C circuit.

The most commonly used circuit arrangement for crystal-controlled oscillators is shown in Fig. 107 (*a*). Since the crystal itself provides no path for D.C., the high resistance, R', is included in parallel with it for the purpose of applying the grid-bias voltage. Alternatively, the grid-bias battery may be omitted, in which case the arrangement becomes a self-bias circuit, the capacitance necessary being provided by the crystal. The equivalent circuit is shown in Fig. 107 (*b*), the part LCC_1 being the equivalent circuit of the crystal. The shunting effect of R' upon the crystal is negligible if R' is sufficiently large, and R' is therefore not shown in the equivalent circuit. The circuit is very similar to that of Fig. 103,

* Contrast this with the behaviour of an electro-mechanical system such as a moving-coil loudspeaker. If we consider such a system as having a single resonant frequency, then its equivalent electrical circuit is a *parallel* L–C circuit. Instead of producing a current proportional to the amplitude of the mechanical vibrations, such a system produces a back e.m.f. proportional to the amplitude of the vibrations. This back e.m.f. is greatest at the frequency of mechanical resonance, so that the amplitude of the current falls to a minimum at this frequency. The analogy in this case is thus the *parallel* resonant circuit.

and it is clear that this is a closed-circuit type of oscillator (see page 160). It follows therefore that the circuit of Fig. 107 can oscillate only if both the anode L–C circuit and the crystal equivalent circuit are inductive. Now the crystal equivalent circuit is

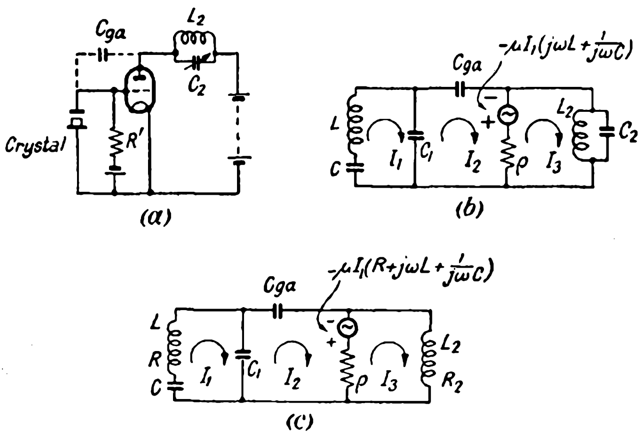


FIG. 107. PIERCE-MILLER CRYSTAL OSCILLATOR AND EQUIVALENT CIRCUITS

inductive, i.e. its total reactance is positive, only between the frequencies

$$f_1 = \frac{1}{2\pi\sqrt{LC}}$$

$$f_2 = \frac{1}{2\pi\sqrt{\dfrac{LCC_1}{C+C_1}}} = f_1\sqrt{1 + C/C_1}$$

This follows from the fact that the total reactance may be written as

$$\frac{(\omega L - 1/\omega C)(-1/\omega C_1)}{\omega L - 1/\omega C - 1/\omega C_1}$$

The frequencies f_1 and f_2 are very nearly equal since, in the equivalent circuit of the crystal, the capacitance C is always small compared to C_1. Thus, the frequency of oscillation must lie in this narrow band; it is therefore very nearly equal to f_1, the mechanical resonance frequency of the crystal. Further, the anode L–C circuit must be inductive at the oscillation frequency; thus the resonance frequency of the anode L–C circuit must be higher than the mechanical resonance frequency of the crystal.

Let us make a preliminary analysis of the circuit, neglecting the resistances in the anode L–C circuit and in the equivalent circuit of the crystal. Applying Kirchhoff's laws to the circuit of Fig. 107 (*b*), and writing the impedance of the anode L–C circuit as

we have $$\frac{j\omega L_2}{1-\omega^2 L_2 C_2}$$

$$I_1\left(j\omega L + \frac{1}{j\omega C} + \frac{1}{j\omega C_1}\right) = \frac{I_2}{j\omega C_1}$$

$$I_1\left(j\omega L\mu + \frac{\mu}{j\omega C} - \frac{1}{j\omega C_1}\right) + I_2\left(\frac{1}{j\omega C_1} + \frac{1}{j\omega C_{ga}} + \rho\right) - I_3\rho = 0$$

$$I_1\left(j\omega L\mu + \frac{\mu}{j\omega C}\right) + I_2\rho - I_3\left(\rho + \frac{j\omega L_2}{1-\omega^2 L_2 C_2}\right) = 0$$

Eliminating I_3 from the last two equations, we have

$$I_1\left[\frac{\mu L_2/C - L_2/C_1 - \mu\omega^2 L L_2}{1-\omega^2 L_2 C_2} - \frac{\rho}{j\omega C_1}\right] =$$
$$-I_2\left[\frac{L_2/C_1 + L_2/C_{ga}}{1-\omega^2 L_2 C_2} + \rho\left(\frac{1}{j\omega C_1} + \frac{1}{j\omega C_{ga}} + \frac{j\omega L_2}{1-\omega^2 L_2 C_2}\right)\right]$$

Dividing this by the first of the three equations above, cross-multiplying and equating imaginary parts

$$\frac{1}{j\omega C_1}\left(\frac{\mu L_2/C - L_2/C_1 - \mu\omega^2 L L_2}{1-\omega^2 L_2 C_2}\right) =$$
$$-\left(j\omega L + \frac{1}{j\omega C} + \frac{1}{j\omega C_1}\right)\left(\frac{L_2/C_1 + L_2/C_{ga}}{1-\omega^2 L_2 C_2}\right)$$

giving

$$\omega^2 = \frac{1}{LC}\cdot\frac{1 + \dfrac{1 + C_1/C_{ga} + C/C_{ga}}{\mu}}{1 + \dfrac{1 + C_1/C_{ga}}{\mu}} \quad . \quad . \quad (V.26)$$

This shows that the oscillation frequency is primarily determined by the crystal constants, and is influenced only slightly by the other circuit constants. In particular, the change of frequency due to thermal expansion and contraction of the anode coil and condenser and of the inter-electrode dimensions is very small indeed. The variation with temperature of the crystal constants is comparatively small; where extreme frequency stability is required the crystal may be maintained at a constant temperature.

To derive the condition for maintained oscillations, we must equate real parts after cross-multiplying the two equations above. This gives

$$-\frac{\rho}{\omega^2 C_1^2} = \rho\left(j\omega L + \frac{1}{j\omega C} + \frac{1}{j\omega C_1}\right)\left(\frac{j\omega L_2}{1-\omega^2 L_2 C_2} + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_{ga}}\right)$$

From equation (V.26) we see that the oscillation frequency is closely approximate to f_1. Thus, substituting $1/LC$ for ω^2, the condition for oscillation becomes

$$L_2C_2 = LC - L_2C_{ga} \quad . \quad . \quad . \quad (V.27)$$

Thus, L_2C_2 must be smaller than LC, and it follows that the resonant frequency of the anode L–C circuit must be higher than the oscillation frequency, as we deduced earlier. This circuit is thus inductive at the oscillation frequency and could be replaced by a single coil of suitable inductance. The sole purpose of the condenser, C_2, is to effect an accurate adjustment of the effective inductance of the anode circuit in order to produce oscillation.

A complete analysis of the circuit must take account of the resistance of the anode coil and the resistance in the crystal equivalent circuit. To simplify this analysis we shall assume that the anode circuit is simply a coil of inductance, L_2, and resistance, R_2, in place of the parallel L–C circuit. The equivalent circuit then is as shown in Fig. 107 (c), which gives

$$I_1\left(R + j\omega L + \frac{1}{j\omega C} + \frac{1}{j\omega C_1}\right) = \frac{I_2}{j\omega C_1}$$

$$I_1\left[\mu\left(R + j\omega L + \frac{1}{j\omega C}\right) - \frac{1}{j\omega C_1}\right] + I_2\left(\rho + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_{ga}}\right) - I_3\rho = 0$$

$$I_1\mu\left(R + j\omega L + \frac{1}{j\omega C}\right) + I_2\rho - I_3(\rho + R_2 + j\omega L_2) = 0$$

Eliminating I_3 between the last two equations we have

$$I_1\left[(\mu RR_2 - L_2/C_1 - \omega^2 LL_2\mu + \mu L_2/C) + \left(j\omega LR_2\mu + j\omega L_2R\mu + \frac{\mu R_2}{j\omega C} - \frac{\rho + R_2}{j\omega C_1}\right)\right]$$

$$= -I_2\left[\rho R_2 + L_2(1/C_1 + 1/C_{ga}) + (\rho + R_2)\left(\frac{1}{j\omega C_1} + \frac{1}{j\omega C_{ga}}\right) + j\omega L_2\rho\right]$$

To derive the maintenance condition, divide this by the first of the three equations, cross-multiply and equate real parts. This gives

$$-\frac{1}{j\omega C_1}\left(j\omega LR_2\mu + j\omega L_2R\mu + \frac{\mu R_2}{j\omega C} - \frac{\rho + R_2}{j\omega C_1}\right)$$

$$= \rho RR_2 + RL_2(1/C_1 + 1/C_{ga}) + \left(j\omega L + \frac{1}{j\omega C} + \frac{1}{j\omega C_1}\right) \times \left[(\rho + R_2)\left(\frac{1}{j\omega C_1} + \frac{1}{j\omega C_{ga}}\right) + j\omega L_2\rho\right]$$

Writing $1/LC$ for ω^2 and neglecting the second-order term ρRR_2. this becomes

$$L_2 = L\left(\frac{C}{C_{ga}}\right) \cdot \frac{1 + R_2/\rho}{1 + \dfrac{R(1 + \mu + C_1/C_{ga})}{\rho}}$$

This is of the form

$$L_2 = \kappa L \cdot \frac{1 + a/\rho}{1 + b/\rho} \quad . \quad . \quad . \quad \text{(V.28)}$$

Let us examine the variation of the amplitude of oscillation as the value of L_2 is varied. This will correspond to the effect of varying C_2 in the circuit of Fig. 107 (*a*). It has already been pointed out that, owing to the curvature of the characteristics, the effective value of ρ is a function of the amplitude of oscillation. For very small amplitude the value of ρ is simply $(\partial v_a/\partial i_a)$ evaluated at values of v_a and v_g equal to the steady operating voltages, or, in other words, the reciprocal of the slope of the $v_a - i_a$ characteristic at the quiescent point. For larger amplitudes, the value of ρ becomes greater. The relation given in equation (V.28) between L_2 and ρ, clearly gives us also a relation between L_2 and amplitude of oscillation, it being remembered that a range of ρ from the quiescent value to infinity corresponds to the whole range of amplitude from zero to infinity. Equation (V.28) shows that the variation of L_2 with ρ is a smooth change from a value $\kappa L \,.\, a/b$ when ρ is zero, to a value κL as ρ tends to infinity. The value of L_2 corresponding to the quiescent value, ρ_0, is

$$\kappa L \cdot \frac{1 + a/\rho_0}{1 + b/\rho_0}$$

and the values of L_2 which will permit oscillation must, therefore, lie between this value and the value κL. Since a and b are small with respect to ρ_0, we see that the value of L_2 corresponding to the quiescent value, ρ_0 (i.e. the value of L_2 which will just cause oscillation), is not very different from the value κL corresponding to an infinite value of ρ. The range of values of L_2 which will permit oscillation is thus very small. Within this range, as L_2 is changed from the value corresponding to ρ_0, to the value κL, the amplitude will increase to a large value, and then as L_2 is further increased oscillation will cease abruptly.

In the circuit of Fig. 107 (*a*) the inductance of the anode circuit is varied by means of the variable capacitance, C_2. Oscillation can take place only within a small range of values of C_2, and as the value of C_2 is decreased through this range the amplitude of oscillation increases, oscillation ceasing abruptly at the end of the range. If the oscillator uses self-bias, the value of the grid-bias voltage will always be slightly less than the peak value of the alternating grid-voltage, and will thus be a measure of the amplitude of oscillation. If, therefore, a direct-current milliammeter be included in series with the self-bias resistor, the reading of this milliammeter will indicate the amplitude of oscillation. More conveniently, we may connect a direct-current milliammeter in the anode circuit; when no oscillation is taking place this meter will simply register the quiescent value of the anode-current, but when oscillation occurs, and the grid-bias voltage rises almost to the peak alternating grid-voltage, the meter reading will fall. As the value of C_2 is decreased, the meter reading will remain steady until C_2 enters the range of values which will permit oscillation. As C_2 is further decreased, the meter reading will be continuously reduced, until, just as the value of C_2 is approaching the end of the range, it rises very rapidly to the quiescent value, and oscillation ceases.

The circuit of Fig. 107 (*a*) is sometimes used for the precision measurement of small capacitances, or small changes of capacitance. A standard variable condenser is used for C_2, and its capacitance is first adjusted to bring the milliammeter reading to some convenient point on the steep portion, *BC*, of Fig. 108, so that the circuit is in the condition where the milliammeter reading is most sensitive to changes of capacitance. The capacitance to be measured is then connected in parallel with the standard condenser, and the latter adjusted until the total capacitance is the same, i.e. until the milliammeter reading is again brought to the same point. The difference between the two values of the standard capacitance gives the capacitance to be measured.

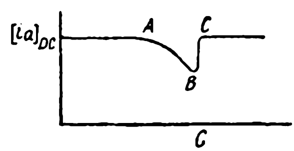


FIG. 108

It is to be noted that while the oscillation frequency is primarily determined by the crystal constants, it is affected to a very small extent by the other circuit constants. In particular, the oscillation frequency is slightly affected by the value of C_2 and, as C_2 is increased, the resulting decrease in amplitude is accompanied by a slight reduction of frequency.

A General Theorem

The reader will have observed that three distinct methods have been used in this chapter for making an analysis of the behaviour of an oscillator.

(i) The oscillator has been regarded as an amplifier which, by circuit coupling between the grid and anode circuits, supplies its own input voltage. Except in the treatment of the Dynatron oscillator this method has been used throughout the chapter.

(ii) The oscillator may be regarded as a regenerative amplifier, and the oscillation frequency and maintenance condition may be derived by writing the voltage amplification equal to infinity. An indication of the method was given on page 158 in connexion with the tuned-grid regenerative amplifier.

(iii) The valve (anode to cathode) may be regarded as an impedance having a resistance component which is negative. This was the method adopted in the analysis of the Dynatron oscillator, in which case the valve impedance was regarded simply as a negative resistance.

The third of the above methods may be extended to cover all valve oscillators (except such special types as positive-grid oscillators, magnetrons, etc., which are not dealt with in this book). The method is based on the theorem that for all valve oscillators

$$Z + \frac{\rho}{1 + \mu N} = 0 \quad . \quad . \quad . \quad \text{(V.29)}$$

where Z is the vector impedance of the whole external circuit connected between anode and cathode, and N is the complex ratio, V_g/V_a, of the grid and anode voltage vectors. This may be proved as follows.

Fig. 109 (*a*) shows the equivalent a.c. circuit of the valve, and from this we see that

$$V_a = -\mu V_g + \rho I_a$$

If the circuit coupling between grid and anode circuits is such as to maintain V_g equal to NV_a, where N may be complex, then

$$V_a = -\mu N V_a + \rho I_a$$

whence $$V_a/I_a = \frac{\rho}{1 + \mu N}$$

The valve thus behaves as an impedance of vector $\rho/(1 + \mu N)$. For instance, if the circuit is such as to maintain the grid-voltage greater than a fraction $1/\mu$ of the anode-voltage and 180° out of phase with the anode-voltage, the valve will behave as a negative resistance with no reactance. In general, as we shall see, the valve of an oscillator behaves as a series combination of negative resistance and reactance, the latter not being zero.

The oscillator circuit may now be re-drawn as in Fig. 109 (*b*) which simply shows two impedances in series—the valve effective impedance, $\rho/(1 + \mu N)$, and the impedance Z of the whole external circuit connected between anode and cathode. The application of Kirchhoff's law to this circuit gives

$$I_a\left(Z + \frac{\rho}{1 + \mu N}\right) = 0$$

whence either I_a is zero (i.e. there is no sustained oscillation) or else the condition given in equation (V.29) is satisfied.

As an example of the use of this theorem consider the tuned-anode oscillator of Fig. 93, page 152. The impedance vector Z is given by

$$\frac{(R + j\omega L)\,(1/j\omega C)}{R + j\omega L + 1/j\omega C}$$

and $$V_g = j\omega M I_L = -j\omega M V_a/(R + j\omega L)$$

whence $$N = -j\omega M/(R + j\omega L)$$

After substituting these expressions into equation (V.29) and resolving into real and imaginary parts, we have the oscillation frequency and maintenance condition as already given in equations (V.2) and (V.5).

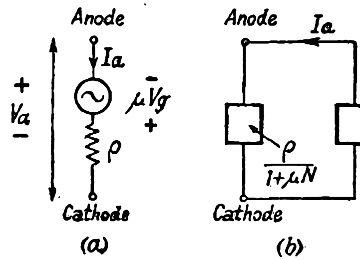


FIG. 109

The Transitron Oscillator

The Transitron oscillator is of interest as illustrating an entirely different way of using a pentode valve.

In the great majority of its applications, the pentode is used simply as a triode but with two auxiliary grids (the screen and suppressor) which are maintained simply at d.c. potentials with respect to the cathode and play no part in the a.c. circuits. In certain applications of the pentode, the suppressor is used as an additional control-grid, i.e. the suppressor is permitted to have an a.c. component. In the transitron oscillator, and in circuits derived from it, *both the suppressor and screen voltages are permitted to have a.c. components.*

In the transitron circuit neither the anode nor the first grid (that normally referred to as the Control Grid) plays a part in the a.c. circuits; the anode-voltage and first grid-voltage are pure d.c., these electrodes functioning simply as auxiliary electrodes. *Instead of using the first grid-voltage to control the anode-current, as in a triode, we have here the novel device of using the suppressor voltage to control the screen current.* Thus in a transitron-connected pentode-amplifier the load is connected in the screen circuit, and the input voltage is connected in the suppressor circuit, as illustrated in Fig. 110 (*a*).

Let us see how it is possible for the suppressor voltage to control the screen current. In Chapter II we saw that variations of the anode-voltage of a pentode have very little effect on the total space current (i.e. on the total number of electrons leaving the cathode per second, and setting off towards the screen and anode). This is because the screen grid effectively screens the cathode region from the influence of the anode-voltage. For a fixed voltage on the first grid the total space current is thus decided by the screen voltage, but the proportions in which it divides between the anode and the screen are decided by the anode-voltage and suppressor voltage. We saw in Chapter II that with the suppressor connected to the cathode in the orthodox way, the proportion of the space current going to the anode increases with anode-voltage until finally nearly

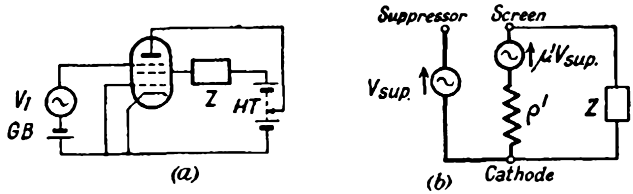


FIG. 110. PRINCIPLE OF TRANSITRON-CONNEXION

the whole space current becomes anode-current. If now the suppressor voltage be made increasingly negative, the repulsion of electrons by this electrode increases, and the proportion of the space current which reaches the anode is reduced. The proportion going to the screen is thus increased and it follows that making the suppressor voltage negative *increases* the screen current. Contrast this with the behaviour of a triode, where making the control-grid voltage negative *decreases* the controlled current (i.e. in the case of a triode, the anode-current). Herein lies the key to the special properties of the transitron-connected pentode, for if a basic amplifier be constructed using such a pentode and with a resistive load, the current in the output circuit (i.e. screen circuit) is 180° out of phase with the input voltage. Hence the a.c. components of the input voltage and the screen voltage are in phase, and *positive feed-back may be applied simply by R–C coupling the output to the input.*

It would be convenient to make an analysis of circuits using the transitron-connected pentode by the adoption of an equivalent circuit for the valve. Assuming linear characteristics (screen current plotted against screen voltage and suppressor voltage), and by analogy with the case of the triode, it may be shown that the equivalent circuit is as shown in Fig. 110 (*b*), in which the direction of the e.m.f. $\mu' Vsup.$ is opposite to the direction normally adopted for the e.m.f. μV_g in the equivalent circuit of a triode. This expresses the fact that the a.c. component of the current entering the valve at the screen is 180° out of phase with the a.c. component of the suppressor voltage which produces it.

The circuit of the transitron oscillator is shown in Fig. 111 (*a*), and it will be seen to be a tuned amplifier using a parallel *L–C* circuit as load and supplying its own input voltage by *R–C* coupling the whole output voltage to the input circuit. The equivalent circuit is shown in Fig. 111 (*b*) and an analysis may be made by writing down the condition that the "amplifier" supplies its own input voltage. The *R–C* coupling circuit is designed to feed as much as possible of the a.c. component of the screen voltage to the suppressor, and for this purpose R' is made large with respect to $1/(\omega C)$ at the frequency used. Let us assume therefore that the effect of the circuit $R'C'$ in shunting the load may be neglected

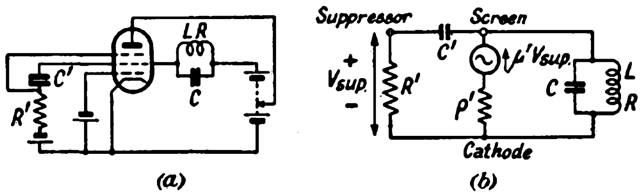


FIG. 111. TRANSITRON OSCILLATOR

and that the whole of the a.c. screen voltage is fed to the suppressor. Then, denoting the impedance vector of the parallel *L–C* circuit by Z, we see from the equivalent circuit that

$$V_{screen} = \frac{\mu' V_{sup}.Z}{\rho' + Z}$$

The condition that the voltage fed back (in this case the whole of V_{screen} as given above) shall suffice to provide the input voltage, is given by writing V_{screen} equal to V_{sup}. This gives

$$\mu' Z = \rho' + Z$$

which may be written

$$Z + \frac{\rho'}{1 - \mu'} = 0$$

(This same result could have been obtained by the use of equation (V.29), remembering that the sign of μ is changed and that N in this case is unity.) The result indicates that the parallel *L–C* circuit is effectively connected in series with a negative resistance of magnitude $\rho'/(\mu' - 1)$. The analogy with the Dynatron oscillator is obvious, and shows that such a circuit will oscillate at a frequency very nearly equal to the resonance frequency of the *L–C* circuit, provided that the dynamic impedance of the latter is at least as great as the negative resistance furnished by the valve.

Resistance-capacitance Oscillators

A number of oscillator circuits have been devised in which no inductance is included, the circuit consisting only of the valve (or valves) and a network of resistances and capacitances. One of these circuits is similar to the transitron oscillator of Fig. 111 (*a*) except that a pure resistance R is substituted for the coil LR, and a smaller capacitance is used for the condenser C'.

The equivalent circuit is as shown in Fig. 111 (*b*) but with L equal to zero. The total load Z presented to the valve is the parallel combination of R, $1/j\omega C$ and $(R' + 1/j\omega C')$

i.e.
$$\frac{1}{Z} = \frac{1}{R} + j\omega C + \frac{1}{R' + 1/j\omega C'}$$

The resulting suppressor voltage, V_{sup}, is thus a fraction

$$R'/(R' + 1/j\omega C')$$

of the screen-voltage, and is therefore given by

$$V_{sup} = \frac{R'}{R' + 1/j\omega C'} \cdot \frac{\mu' Z}{\rho' + Z} \cdot V_{sup}$$

This gives

$$\mu' R' = (R' + 1/j\omega C')(1 + \rho'/Z)$$

Substituting the expression for $1/Z$ into this equation and equating imaginary parts, we have the frequency of oscillation given by

$$\omega^2 = \frac{1}{CC'R'\left(\dfrac{\rho' R}{\rho' + R}\right)}$$

To determine the maintenance condition, we equate real parts of the above equation, giving

$$\mu' = 1 + \rho'/R + (\rho'/R')(1 + C/C')$$

If both R and R' be made large this can be satisfied with quite small values of transitron amplification factor, μ'. If, in addition, C/C' is made small, or if C is removed from the circuit altogether, then the value of μ' is liable to be much larger than is necessary to satisfy the maintenance equation, and the circuit ceases to generate sinusoidal oscillations, becoming instead the Transitron relaxation oscillator (see page 213).

A further resistance-capacitance oscillator may be produced by substituting a pure resistance R for the L–C circuit in Fig. 111 (*a*), adding resistance R_1 in series with C', and adding capacitance C_1 in parallel with R'. This may be thought of as a zero phase-shift amplifier with feed-back applied via the network $R_1C'R'C_1$. Now this network consists of two impedances in series, viz.

$$Z_1 = R_1 + 1/j\omega C'$$

and
$$Z_2 = \frac{R'/j\omega C_1}{R' + 1/j\omega C_1}$$

and the feed-back ratio vector is thus given by

$$\beta = Z_2/(Z_1 + Z_2)$$

It is not difficult to show that β is real at one and only one frequency, this being the frequency at which Z_1 and Z_2 have the same phase angle, and being given by

$$\omega^2 = \frac{1}{C_1C'R_1R'}$$

At this frequency therefore we have positive feed-back and, provided that the gain is adequate, the circuit will oscillate.

R–C oscillators using this principle need not necessarily use a transitron stage. The feed-back network consisting of Z_1 and Z_2 may be applied to any zero phase-shift amplifier, e.g. to a two-stage resistance-capacitance coupled amplifier with resistance loads. If the phase-shift of the amplifier is not exactly zero (i.e. if its complex gain, A, is not purely real), oscillation will still occur but at a slightly different frequency—namely the frequency at which the product $A\beta$ is real. The frequency of oscillation will thus depend upon the amplifier, and not only upon $R_1R'C_1$ and C'. To reduce this dependence, and thereby increase the frequency stability of the oscillator, heavy negative feed-back may be applied to the amplifier before the network Z_1Z_2 is connected to it. A suitable circuit is that of the negative feed-back amplifier shown in Fig. 65; to convert this into an *R–C* oscillator, it is simply necessary to connect Z_1Z_2 in series between A_2 and E, and to join G_1 to the junction of Z_1 and Z_2 (the input source V_1 being, of course, omitted).

Yet a further type of *R–C* oscillator is that known as the Phase-shift Oscillator. It consists of a triode with a resistance load, its anode-voltage being fed back to the grid circuit through three (or more) cascaded a.c./d.c. separating circuits, i.e. terminals 1 and 2 of a network such as that shown in Fig. 27, are connected to anode and cathode respectively, terminals 4 and 5 being connected to the input terminals (1 and 2) of a second similar network, which is followed in the same way by a third similar network. Terminal 4 of the third network is then connected to the grid of the valve. It can be shown that at one particular frequency, depending upon the values of resistance and capacitance, a cascaded network of this kind produces a phase-shift of 180° between its input and output voltages. Since a phase-shift of about 180° is also produced by the valve, this is what is required to enable the circuit to "supply its own input voltage," and so act as an oscillator.

The Multivibrator: Relaxation Oscillators

We have said that an oscillator is an amplifier which supplies its own input voltage by feeding back to the input circuit a part or the whole of the output voltage. All the oscillators which we have so far considered have been *single frequency* amplifiers with regeneration, i.e. they have used some form of selective circuit. Consider the two-stage *R–C* coupled amplifier shown in Fig. 112. This is an aperiodic amplifier, i.e. it amplifies equally over a wide range of frequency. Apart from the small phase change due to the coupling-capacitance, C, the output voltage of such an amplifier is in phase with the input voltage; thus we might expect that the circuit would function as an oscillator if the output voltage were connected into the input circuit by making the connexion shown dotted in Fig. 112. Analysis of the circuit in the manner employed for other oscillators

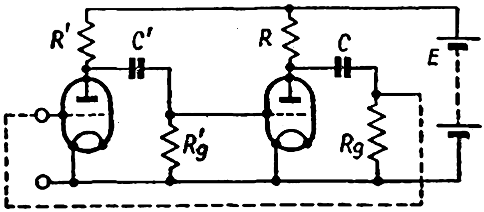


FIG. 112. FREE-RUNNING MULTIVIBRATOR

in this chapter gives as the condition for oscillation the requirement that the overall amplification of the circuit shall be greater than or equal to unity, but the equation giving the condition that the voltage fed back shall be of the correct phase to sustain oscillation (i.e. the equation which normally defines the frequency of oscillation) simply becomes, as we might expect,

$$1/\omega C = 1/\omega C' = 0$$

which cannot be satisfied for any finite frequency. We therefore conclude that the circuit is not capable of sinusoidal oscillation. Experiment shows, however, that the circuit can oscillate, the waveform of the oscillations being as shown in Fig. 114. This oscillator is known as the Multivibrator.

In the section "Pulsed Amplifiers" (page 320) it is shown that if the grid-voltage of the first valve in a two-stage *R–C* coupled amplifier (e.g. as in Fig. 112 but without the dotted connexion) be suddenly changed to a new value, the output terminal (i.e. the top of the right-hand resistance R_g in Fig. 112) will have its potential changed in the same direction. This change in potential of the output terminal will not be a permanent change, since *R–C* coupling circuits will not transmit D.C. Instead, the potential of the output terminal, after an initial sudden change, will return exponentially to its original value, i.e. zero. For the present it is sufficient to observe that a sudden change of input voltage will cause a larger sudden change of output voltage, and that the two will be of the same polarity.

We can now see the effect of making the connexion shown dotted in Fig. 112, and thus connecting the output voltage into the input circuit. The resulting circuit is in a condition of unstable equilibrium, for any slight random change of input voltage (and such slight changes are continually occurring as a result of thermal agitation, etc.) will produce a larger change of output voltage, which is thereupon fed back to the input circuit where it adds to the original change; the larger input voltage is then amplified, fed back, re-amplified, fed back, and so on. The whole process is almost instantaneous and results in a "landslide of grid-voltage" in the direction of the original slight change.

This precipitate unidirectional change of grid-voltage does not proceed to infinity, nor does it destroy the amplifier! Let us see what limits it. Throughout the whole of the "landslide" the p.d. of neither coupling condenser changes; for the process is almost instantaneous—and condenser voltages cannot change instantaneously but only at a rate determined by the time-constant of their associated circuits. Thus the sudden change in potential of each anode is communicated to the grid of the other valve, and it follows that while the grid-voltage of one valve is suddenly increasing, that of the other is suddenly decreasing, i.e. becoming negative. When it becomes sufficiently negative, the anode-current of that valve will be cut off and the valve will no longer amplify. The landslide can then proceed no farther.

The result of making the dotted connexion is thus that the anode-current of one valve is suddenly cut off. The anode-voltage of that valve therefore rises suddenly to the potential of h.t. positive. Since the coupling condenser voltages cannot change suddenly, this means that the grid of the other valve is suddenly driven positive with respect to cathode, with a consequent sudden rise in the anode-current of this other valve and a sudden fall in its anode potential. This fall is communicated to the grid of the valve which is cut off, and drives the grid still farther negative, i.e. well beyond the cut-off point.

The landslide is now completed and we have a situation as illustrated in Fig. 113, in which the cut-off valve has been omitted because it is not carrying any current. Since the whole process has been instantaneous the two coupling condensers still have their original voltages. The coupling condenser C' will now proceed to charge exponentially from its original voltage to the full h.t. voltage, E, the charging current flowing in the direction shown by the arrow, and thus producing a positive grid-voltage for the right-hand valve. Hence grid-current will flow in that valve. The valve (from cathode to grid) will behave as a fairly low resistance r_g shunting R_g'. The result of this shunting of R_g' is twofold—

(*a*) The time-constant of the charging process is reduced from $C'(R' + R_g')$ to a value which is little greater than $C'R'$.

(*b*) The positive grid-voltage produced by the flow of charging current through the parallel combination of R_g' and r_g will not be more than a few volts. As the condenser C' charges, so this small grid-voltage will decay exponentially to zero.

Just as a charging process is occurring in the circuit of the coupling condenser C', so a discharging process is occurring in the circuit of C. This condenser C is connected, in series with the resistor R_g, directly between A_2 and h.t. negative. As a result of the landslide an increased current is now flowing through R and hence the p.d. between A_2 and h.t. negative has been reduced. The condenser C must therefore discharge, the discharge current flowing in the direction of the arrow and producing an RI drop across R_g which provides the grid-voltage (more negative than the cut-off value) of the other valve. As C discharges, the discharge current through R_g decays exponentially* towards zero. It follows that the negative grid-voltage of the cut-off valve also decays exponentially towards zero. A time will therefore come when the valve is no longer cut off; anode-current will again flow, the valve will be able to amplify, and a further landslide will occur. This landslide is precipitated by the rising grid-voltage of the valve which was previously cut off, and the direction of the landslide will now be such as to drive this grid positive with respect to cathode, and to cut off anode-current in the other valve.

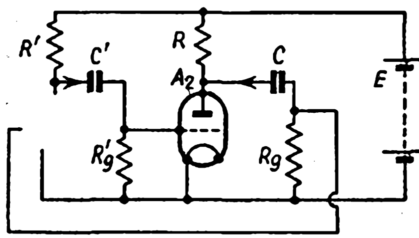


FIG. 113

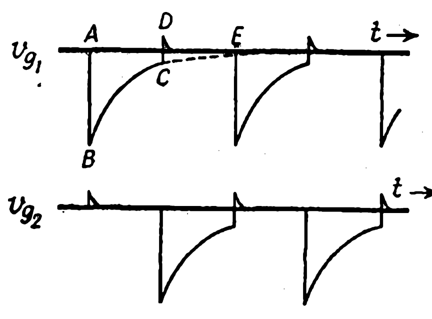


FIG. 114. MULTIVIBRATOR WAVEFORMS

Hence we see that the oscillation which takes place in a multivibrator consists of a sudden cut-off of the anode-current first in one valve, then in the other, then in the first again, and so on. The interval between successive landslides is the time required for the grid-voltage of the valve which is cut off to decay from its initial value (more negative than cut-off) to the cut-off value. The

* The decay is not quite exponential since the positive grid-voltage of the valve is changing slightly while C is discharging.

waveform of the grid-voltages will be as shown in Fig. 114. The portions AB and CD represent the landslides, while BC and DE show the exponential decrease of grid-voltage as the coupling condensers charge and discharge. If the circuit uses two identical stages (i.e. $R = R'$; $R_g = R_g'$; $C = C'$; and similar valves) the duration of BC will be the same as that of DE, and the multivibrator is said to be symmetrical.

The frequency of oscillation is given by $1/(\tau + \tau')$ where τ is the duration of BC and τ' is the duration of DE. To determine τ and τ' we require to know the time constant of each discharge, and the ratio of the voltages at the beginning and end of the discharges. During the period BC, conditions are as shown in Fig. 113

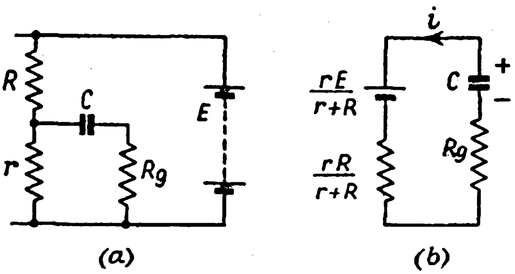


FIG. 115

and C is discharging. Since the grid-voltage of valve 2 is only slightly positive during this period, we shall make the simplifying assumption that it remains at zero throughout the period. The valve (from anode to cathode) thus behaves as a resistor whose current-voltage graph is simply the static characteristic for $v_g = 0$. Again we make a simplifying assumption, viz. that this graph is a straight line passing through the origin, so that the valve behaves (during this period) simply as a constant resistance r. Thus, to determine τ, we may redraw the relevant parts of the circuit as in Fig. 115 (a) and derive the simpler equivalent circuit of Fig. 115 (b) by making use of Thevenin's Theorem (see page 4). From this equivalent circuit we see at once that the time constant of the discharge is given by

$$T = C\{R_g + rR/(r + R)\} \qquad . \quad . \quad (V.30)$$

Now the condenser C, which during the period we are considering is discharging, was the *charging* condenser in the previous period (when valve 2 was cut off instead of valve 1). During that previous period it became charged to the full h.t. voltage, E. During the present period, therefore, it commences its discharge from the same voltage, E, and Fig. 115 (b) shows that it discharges exponentially towards a voltage $rE/(r + R)$. At the commencement of the discharge the condenser current is given by

$$i = \frac{E - \dfrac{rE}{r + R}}{R_g + \dfrac{rR}{r + R}} = \frac{ER}{rR_g + RR_g + rR}$$

and the voltage across R_g (which provides the negative grid-voltage to cut-off valve 1) is simply R_g times this current. Thus the negative grid-voltage of valve 1 decays according to the equation

$$v = \frac{ERR_g}{rR_g + RR_g + rR} \cdot e^{-t/T}$$

the time, t, being measured from the commencement of the discharge. When t has reached a value τ, v has reached the cut-off value of negative grid-voltage, $v_{cut\text{-}off}$, so that we may write

$$v_{cut\text{-}off} = \frac{ERR_g}{rR_g + RR_g + rR} \cdot e^{-\tau/T}$$

This gives

$$\tau = T \log_e \frac{(E/v_{cut\text{-}off})}{1 + r/R + r/R_g}$$

The ratio $(E/v_{cut\text{-}off})$ is approximately equal to the amplification factor, μ, of the valve concerned (valve 1, during period BC). Thus—

$$\tau = T \log_e \frac{\mu_1}{1 + r/R + r/R_g} \quad . \quad . \quad (V.31)$$

where $\quad T = C\{R_g + rR/(r + R)\}$

The expression for τ' (the discharge time during period DE) is of exactly the same form, but with C', R', R_g', r', and μ_2 taking place of C, R, R_g, r, and μ_1. Finally we have

$$f = 1/(\tau + \tau') \quad . \quad . \quad . \quad . \quad (V.32)$$

(For a discussion of the accuracy of these results, the reader is referred to E. R. Schenk, "The Multivibrator—Applied Theory and Design," *Electronics*, Jan. 1944.) See also Problem V.17, page 379.

During the period BC, the condenser C is discharging, and v_g is decreasing exponentially from a fairly large negative value (AB, Fig. 114) towards zero. This negative value AB, is large with respect to $v_{cut\text{-}off}$, so that the discharge has proceeded almost to completion before it is interrupted by the landslide at the point C. As a result of this, the graph of v_{g_1} against time (Fig. 114) is approaching the horizontal by the time the landslide occurs, and therefore slight variations in the value of $v_{cut\text{-}off}$ (e.g. due to slight fluctuations of supply voltage, or to spurious voltages induced in the circuit) can cause inordinately large variations in the time of occurrence of the landslide, and also in the frequency of oscillation. The effect is known as Jitter and can be reduced by taking the lower ends of the resistors R_g and R_g' not to h.t. negative, as in Fig. 112, but to h.t. positive. This means that, during the discharge of the condenser C, the voltage v_{g_1} is changing exponentially from a fairly large negative value towards an even larger positive value, viz. the h.t. voltage E. The discharge is thus very far from completion when it is interrupted by the landslide, and the graph of v_{g_1} against time has a steep slope when it reaches the small negative value $v_{cut\text{-}off}$. Thus slight variations in the value of $v_{cut\text{-}off}$ cause very much smaller variations of frequency.

At first sight, it might appear that in connecting the two grid resistors to h.t. positive we are permitting the grid-voltages to reach dangerously large positive values during the periods when the valves are not cut off. If, however, these two resistors have resistance of the order of a megohm or more, the flow of grid-current through them will produce so large an RI drop that the grid-potential can rise only a volt or two above cathode-potential. Expressing the same thing in different words, we may say that the grid-cathode resistance of the valve (i.e. the input resistance) is of the order of a few thousand ohms when grid-current is flowing; this grid-cathode resistance and the much larger resistance of R_g together form a voltage-divider across the h.t. voltage, with the result that only a very small voltage appears between grid and cathode.

The multivibrator may be used as a pulse-generator. It is also a convenient generator of *alternating voltage with a far from sinusoidal waveform*, i.e. with a large number of harmonic components of reasonably large amplitude. As such, it is a valuable asset in frequency measurement, since if the fundamental frequency be set at a known value, then the frequencies of the harmonic components are known to be exact multiples of this frequency.

The fundamental frequency may be automatically set equal to that of a standard frequency oscillator by including in the anode circuit of one valve of the multivibrator a voltage of this standard frequency, e.g. by including in the anode circuit of one valve a resistance through which current of the standard frequency flows. A general discussion of the mechanism by which the frequency of an oscillator is modified by coupling to the oscillator a second oscillator having a slightly different frequency is outside the scope of this book, but the case of the multivibrator provides an interesting case of such synchronization. We have seen that at a certain point in the cycle of operations the grid-voltage of valve 1 is more negative than cut-off, and that this grid-voltage must fall to the cut-off value before a voltage landslide can again take place, since the overall amplification is clearly zero so long as the anode-current of the valve remains zero. Decrease of the negative grid-voltage of valve 1 is not, however, the only method of re-starting the anode-current, an alternative method being to increase the anode-voltage. If, as the grid-voltage of valve 1 is recovering, the anode-voltage is suddenly increased by some external influence, anode-current will flow, and as already explained, the anode-current of the next valve, will suddenly fall to zero. Thus, cut-off of the anode-current in valve 2 may be synchronized with the external voltage. If this external voltage is periodic and of frequency slightly greater than the multivibrator, the multivibrator will be pulled into step with the external voltage. If the frequency of the external voltage is roughly equal to a multiple of the multivibrator frequency, synchronization will again take place; for instance, if the external frequency is about five times the fundamental frequency of the multivibrator then every fifth peak of the external voltage will serve to re-start the anode-current of valve 1, and the multivibrator frequency will become exactly one-fifth of the external frequency. This is sometimes described as synchronizing the fifth harmonic of the multivibrator with an external voltage. When a multivibrator is used for frequency division in this way the effect of jitter is to produce some uncertainty as to which of the synchronizing pulses will precipitate the voltage landslides. For this reason it is best to return the grid resistors to h.t. positive (as described above) instead of to h.t. negative.

Each half-cycle of the multivibrator oscillation may be divided into two quite distinct parts, *AB* and *BC*, Fig. 114. During the first part the circuit is acting as an amplifier; during the second part it is acting simply as a combination of condensers and resistances, the former being charged by way of the latter. The first type of action is periodically relaxed in favour of the second, and vice versa. This type of oscillation is known as Relaxation Oscillation. The simplest relaxation oscillator is perhaps the series connexion of a d.c. supply voltage, a resistance and a condenser, a gas diode being shunted across the condenser. The condenser charges up through the resistance, but as soon as its voltage reaches the striking voltage of the gas diode the latter becomes a conductor. The condenser then proceeds to discharge through the diode until its voltage falls to the extinguishing voltage of the diode, when the diode ceases to conduct and the process is repeated (compare Fig. 120, page 207).

The Bi-stable Multivibrator and the Flip-flop (or Mono-stable Multivibrator)

The multivibrator circuit of Fig. 112 is sometimes described as the Free-running Multivibrator, to distinguish it from two closely related circuits, viz. the bi-stable multivibrator and the Flip-flop (or mono-stable multivibrator). Each of the three consists of a two-stage untuned amplifier with the whole of the output voltage fed back to the input circuit. They differ only in the method of interstage coupling, but this difference brings about very important differences in their behaviour. The simplest circuits of a bi-stable multivibrator and Flip-flop are shown in Figs. 116 and 118 respectively, and we may summarize the methods of interstage coupling as follows—

Circuit	First Coupling	Second Coupling
Free-running Multivibrator . .	*R–C* coupling	*R–C* coupling
Flip-flop . . . (mono-stable)	*R–C* coupling	*D.C.* coupling
Bi-stable Multivibrator . .	*D.C.* coupling	*D.C.* coupling

Since each of the three devices is an untuned amplifier with 100 per cent positive feed-back it follows that each is capable of a voltage landslide, limited only by cut-off of one or other of the

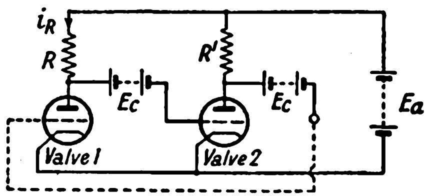


Fig. 116. Simple Bi-stable Multivibrator

two valves (as described on page 195). Having attained this limiting condition, the three circuits behave differently. In the free-running multivibrator, as we have seen, the condenser voltages then change until the circuit is again able to amplify, at which point a voltage landslide again occurs, this time with the other valve cutting off. The free-running multivibrator thus cannot remain in the cut-off condition indefinitely, and we may say that the two limiting conditions reached are "unstable conditions."

Consider, however, the circuit of Fig. 116. It might be thought that the two valves, whose circuits are identical, will have equal anode currents. As soon as the connexion shown dotted is made, however, the slightest random voltage impulse can initiate a voltage landslide, and the anode-current of one valve will therefore suddenly rise to a high value, while that of the other will suddenly cease. The circuit of Fig. 116, unlike the free-running multivibrator, can remain in this condition indefinitely, since the grid-voltage of the valve which is cut off (say valve 2) is maintained by a circuit in which there are no condensers (whose voltage could change by charging or discharging). On the contrary, the grid-voltage of valve 2 is given by

$$\begin{aligned} v_{g2} &= v_{a1} - E_c \\ &= E_a - Ri_R - E_c \end{aligned}$$

and i_R is determined in turn by v_{g1} which has one, and only one, possible value when valve 2 is cut off, viz.

$$v_{g1} = E_a - E_c$$

(This same condition could have been brought about simply by open-circuiting the anode of valve 2, in which case v_{g1} would automatically be given by the above equations. It follows that, in designing the circuit, E_c must be made sufficiently large for v_{g2} to be negative and greater than the cut-off value with v_{g1} having the value given above.)

From the symmetry of Fig. 116 it can be seen that there are *two stable conditions*, viz. with valve 1 cut off, or with valve 2 cut off. With the circuit in either of these conditions it is possible to change to the other condition only by injecting an external voltage large enough, and of the correct polarity, to cause the valve which is cut off to conduct again. Such a voltage may take the form of a voltage pulse of very short duration, but the circuit will nevertheless remain "switched-over" after the pulse has ceased. The external voltage pulse necessary to "trigger" the circuit from one stable condition to the other may be injected into either grid circuit or either anode circuit. The requisite polarity will depend upon the point of injection, the criterion being that it shall cause anode-current to flow in the valve which was cut off.

One way to trigger the circuit is to inject a negative voltage pulse into the anode-circuit of the valve which is cut off. The negative pulse will be communicated to the grid of the other valve, and will cause this other valve suddenly to pass less anode-current, thereby producing a sudden rise of its anode-voltage. This sudden rise of voltage will be communicated to the grid of the cut-off valve and will trigger the circuit to its other stable condition. Observe that the requirement is that the externally applied negative triggering pulse shall be fed to the anode of the cut-off valve, i.e. the valve which, at the time, has the higher anode-voltage. An ingenious device is often used, whereby the negative triggering pulse is fed to *both* anodes, but in each case through a diode valve connected as shown in Fig. 226, page 347. A diode valve can conduct if its anode is at a higher potential than its cathode. In this case the cathodes of the two diodes are connected together and form the trigger-input terminal. The anode of one diode (being connected to the anode of the cut-off triode) is at h.t. positive potential. The anode of the other diode (being connected to the anode of the conducting triode) is at a much lower potential. The trigger input terminal (constituted by the diode cathode) is normally maintained at h.t. positive potential so that neither diode conducts. When it is desired to trigger the circuit, the potential of the trigger input terminal and therefore of the diode cathodes is made to fall suddenly. This fall will suffice to make only one of the diodes conduct, namely the diode whose anode is at the higher potential. This is the diode which is connected to the anode of the cut-off triode. This device therefore serves to route the negative trigger pulse to the very place where it is required. It does not matter which valve is cut off

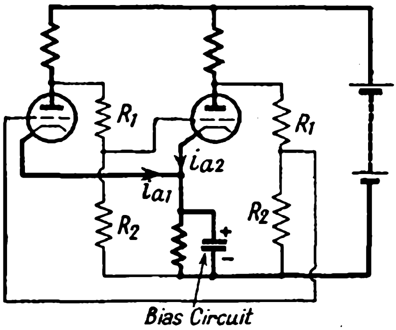


FIG. 117. RESISTANCE-COUPLED KIPP RELAY

at the time; the trigger pulse will be directed by the diodes to the anode of the cut-off valve, and so will succeed in triggering the circuit.

Bi-stable multivibrators, triggered in this way, are often used in pulse-counting circuits (see Chapter VIII). The circuit is sometimes called a "Scale of Two," because it requires two successive input pulses to take it through one cycle of operation.

D.C. interstage coupling using batteries, as shown in Fig. 116, is of little practical value. Fig. 117 shows a bi-stable circuit using a more practical form of coupling. This is similar to the interstage coupling shown in the d.c. amplifier of Fig. 39 (page 53), but the use of a "third-rail" supply is avoided by incorporating an automatic-bias circuit in Fig. 117. Such automatic bias circuits cause a reduction of gain if used in d.c. amplifiers, as explained on page 70. In this particular circuit, however, the total anode-current of the two valves remains constant, for when one is cut off, the current in the other immediately rises. Hence, by allowing this total current to flow through a common bias resistor, a constant direct voltage is provided which serves as grid-bias, or "third-rail" voltage, for both valves.

Both interstage couplings in a bi-stable multivibrator must be d.c. couplings. It is not necessary, however, that they should be identical, nor even that they should be of a similar type. There is a well-known bi-stable multivibrator circuit, known as the Schmitt Trigger Circuit, in which one of the interstage couplings is cathode-coupling (see page 79) and the other is resistance-coupling. The Schmitt circuit is similar to Fig. 119, page 205, but with a resistor substituted for C and the upper end of R_g connected to h.t. negative instead of h.t. positive.

FLIP-FLOP CIRCUITS. The Flip-flop (Fig. 118) is a hybrid of the bi-stable and free-running multivibrators. Like them, it is capable of a voltage landslide limited only by cut-off of one valve or the

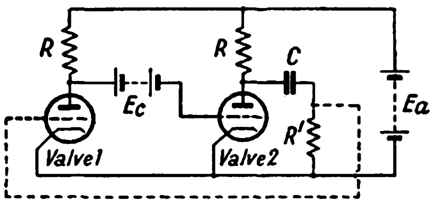


FIG. 118. FLIP-FLOP CIRCUIT

other. Of the two limiting conditions thus reached, *one is stable and the other unstable*. For instance, when valve 1 (Fig. 118) is the valve whose anode-current is cut off, the cut-off is effected by v_{g1} which, being equal to the RI drop across R', is directly proportional to the discharging current of the condenser C (flowing upwards through R'). This discharging current is initially large, producing a value of v_{g1} sufficient to cut off i_{a1}; but the discharging current falls exponentially, and so v_{g1} will automatically rise towards zero and i_{a1} will again be permitted to flow. The limiting condition with valve 1 cut off (Fig. 118) is therefore an unstable condition and cannot be maintained indefinitely.

As soon as v_{g1} has risen sufficiently to allow the amplifier again to amplify, a voltage landslide will again take place, and being initiated by the rising v_{g1}, will be limited by cut-off of valve 2, not valve 1, the anode-current of valve 1 rising suddenly to a high value. Thus the other limiting condition is now attained, and this limiting condition is stable, i.e. it can be maintained indefinitely. This is so because the grid-voltage of the valve which is now cut off (valve 2 in Fig. 118) can be maintained without the agency of any condenser. This grid-voltage is given by

$$v_{g2} = E_a - Ri_{a1} - E_c$$

and i_{a1} is controlled by the value of v_{g1}. If the condition with i_{a2} cut off is to be stable (i.e. capable of being maintained permanently, even after the charging current of C has ceased), the above expression for v_{g2} must be more negative than the cut-off value when i_{a1} has the value corresponding to zero grid-voltage on valve 1. This gives us a design rule for the magnitude of E_c.

The properties of the Flip-flop are thus as follows. Before the completion of the connexion shown dotted, the current through R' is zero, as in the stable limiting condition. The completion of the dotted connexion, therefore, leaves the circuit in its stable condition. It can be "triggered" to the other limiting condition only by the injection of an external voltage of sufficient magnitude and of correct polarity to make valve 2 conduct again. As in the case of bi-stable circuits, the correct polarity will depend

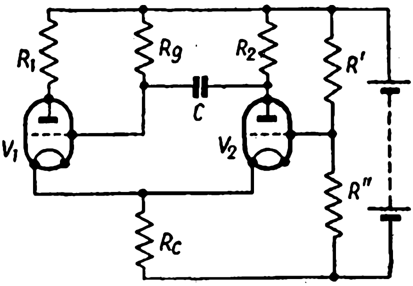


FIG. 119. CATHODE-COUPLED FLIP-FLOP

on the point of injection. Also the external voltage may be an impulsive voltage of very short duration. As soon as valve 2 is again made to conduct in this way, a voltage landslide will occur, being limited by cut-off of valve 1. The circuit is now in its unstable condition, and will automatically return to the stable condition (without the necessity of an external voltage) after a period of time determined by the rate of discharge of the condenser C. By suitable choice of values of C, R and R' this period may be varied from the order of a microsecond to several seconds. By the inclusion of d.c. milliammeters in the two anode circuits, and with the time delay arranged to be several seconds, the properties of the Flip-flop can be easily demonstrated; the requisite external voltage impulse for switching-over from the stable to the unstable condition may be injected simply by connecting the grid of valve 2 to high tension positive through the resistance of the body, i.e. by touching both at once with the fingers.

To avoid the use of a coupling battery, a d.c. interstage-coupling circuit of the type shown in Fig. 39 (page 53) may be used together with a "third-rail" voltage supply. (See Fig. 197, page 314.) An even better method is to use cathode coupling (as described on page 80). The circuit then becomes as shown in Fig. 119, in which the output of valve 2 is fed to valve 1 by resistance-capacitance coupling (R_g, C) the coupling resistor being returned to h.t. positive as described on page 199. The output of valve 1 is fed to valve 2 by cathode coupling (R_c). A positive-bias voltage is provided for the grid of valve 2 by means of the voltage-divider $R'R''$, but the total grid-bias voltage of this valve in the stable condition remains more negative than the cut-off value, since there is a large negative bias derived from R_c. The "condenser-fed" valve, valve 1, cannot be maintained cut off indefinitely, and therefore this is the valve which carries current in the quiescent or stable condition, valve 2 being cut off. It follows that, to trigger the circuit to its unstable condition, positive pulses may be applied to the grid of valve 2, or negative pulses to the grid of valve 1, since either could cause valve 2 to conduct and so precipitate a voltage landslide. The output voltage of this Flip-flop circuit may be taken either as the voltage between anode 1 and h.t. negative, or the voltage between anode 2 and h.t. negative, according to whether a positive or a negative output pulse is required. It is perhaps worth noting that the free-running multivibrator circuit of Fig. 112 can be made to behave as a Flip-flop by including a suitable negative grid-bias voltage in series with one of the grid resistors. If this bias-voltage is made large enough, a stable condition is produced in which the valve so biased remains cut-off.

The Flip-flop has a great variety of uses, e.g. for producing a pulse of long duration from a pulse of short duration, or for the relaying of pulses, or for the introduction of a time delay, or to put an amplifier out of action (or into action) for a brief period following the injection of the voltage pulse. Another interesting application is as a trigger circuit for a hard valve time-base. (See page 210.) In this latter case, the circuit has been used with a tetrode in place of a triode for one of the valves, the "screen-grid" being used as the control grid for the purpose of Fig. 118, and the grid nearest the cathode being used as an extra control grid solely for the injection of the external voltage.

The author has not been able to discover which of that happy company of radar pioneers was responsible for the inspired title "Flip-flop." "Flip" suggests the triggering process and "flop" certainly suggests the precipitate descent from an unstable state. Attempts to popularize the (highly logical) names "Flip-flip" and "Flop-flop" for bi-stable and free-running circuits respectively have met with little success. It seems that there is a level of flippancy which must not be exceeded.

The Thyratron Time-base

A Time-base is an oscillator which is required to have a particular form of non-sinusoidal waveform. The need for such a device arises in the use of cathode-ray oscilloscopes, where it is necessary to impart to a beam of electrons a deflexion which shall be a specified function of time. The most common requirement is that the beam of electrons in the cathode-ray tube shall be deflected sideways with a uniform velocity until a certain deflexion is reached, and then the beam shall be returned as rapidly as possible to its starting point, the whole process being repeated *ad lib.* This means that the graph of deflexion against time is of the form shown in Fig. 120 (*a*), which is usually described as a Saw-tooth waveform.

The commonest way of deflecting the beam of electrons is by making the beam pass through an electrostatic field which varies with time in the same manner as Fig. 120 (*a*), and for this purpose it is necessary that a voltage having the same waveform shall be connected across a pair of plates (known as the Deflecting Plates) between which the beam passes. The apparatus for generating this voltage is the Time-base, and the remainder of this chapter is concerned with the operation of such circuits.

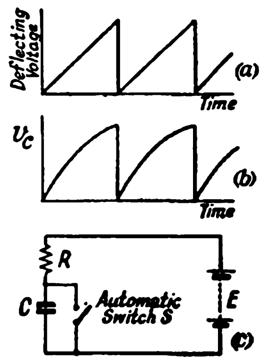


FIG. 120

In general, time-base circuits are relaxation oscillators, and Fig. 120 (*c*) illustrates the principle upon which most of them work. A condenser C is charged through a resistance R by a d.c. supply voltage E. The condenser voltage will rise exponentially with time from the moment the circuit is completed. The voltage across this condenser is taken as the output voltage, and can be made to have a waveform as in Fig. 120 (*b*) if the condenser be suddenly discharged at regular intervals. For this purpose an automatic switch (S in Fig. 120 (*c*)) is required, which shall close for a very short time as soon as the condenser voltage has reached the maximum value which it is required to have. While this switch is closed, the condenser will discharge through it; the discharge will not, of course, be instantaneous, but will be very rapid if the resistance of the discharge path through S is small. The three time-bases discussed in this chapter differ only in the nature of the automatic switch used.

Fig. 120 (*a*) differs from Fig. 120 (*b*) in that the rise of voltage in the latter is not uniform. It may be made uniform by substituting for the constant resistance R some device which passes a constant current, whatever the voltage across it, for then the charging current will be constant, and thus the rate of rise of charge on the condenser will be constant. Since the voltage across a condenser is proportional to the charge upon it, it follows that the condenser voltage will rise at a constant rate, and the graph showing the condenser voltage against time will be a straight line as required. A pentode valve (connexions to anode and cathode) provides just such a device, provided that the voltages on the three grids remain constant at values similar to those used in the normal operation of a pentode. Fig. 18 shows that over a wide range of anode-voltage the anode-current remains almost constant, at a value depending on the first grid-voltage and the screen voltage. But for low anode-voltages, the anode-current varies considerably with anode-voltage. When the pentode is connected in place of the resistance R in Fig. 120 (c), however, the voltage across it from anode to cathode is the difference between the d.c. supply voltage E and the condenser voltage. It can easily be arranged that the switch S operates before the condenser voltage has risen to any large fraction of E, so that there always remains a substantial fraction of E across from anode to cathode of the pentode. It is convenient to provide

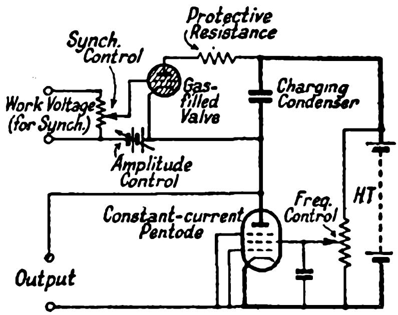


FIG. 121. THYRATRON TIME-BASE

the screen voltage of the pentode from the same h.t. supply as is used for the charging of the condenser (see Fig. 121); this makes it necessary to interchange the positions of R (now replaced by the pentode) and C in Fig. 120 (c), connecting the pentode on the negative side of the condenser.

Many types of automatic switch may be used across the condenser. The time-base which is the subject of this section uses a gas-filled triode (often known by the trade. name, "Thyratron"). The relevant properties of such a valve (see page 25) are as follows—

(a) As the anode-voltage is increased from a low value, the anode-current is initially very small, but when a critical anode-voltage is reached the anode-current rises suddenly to the full saturation current of the cathode. The valve is said to "strike."

(b) The value of this critical anode-voltage depends upon the negative grid-bias voltage, being approximately μ times the negative grid-voltage where μ is the amplification factor of the same valve if evacuated.

(c) Once the valve has struck, the grid-voltage exercises no further control over the anode-current, and it is not possible to cause the anode-current to revert to zero by increasing the negative grid-voltage.

(d) The valve may, however, be made to cease conducting if the anode-voltage be sufficiently decreased. It is not sufficient to reduce the anode-voltage below the critical value which was necessary for the valve to strike; it is necessary to reduce it approximately to the ionization potential of the gas. For instance, a mercury vapour valve with a control factor of 20 and a negative grid-voltage of 6 would require its anode-voltage to be raised to 20 times the negative grid-voltage (i.e. to 120 volts) before it would strike, but thereafter it would be necessary to reduce the anode-voltage to about 15 volts in order to cause the anode-current suddenly to cease.

Such a valve can therefore act as an automatic switch if its anode and cathode are connected across the charging condenser as in Fig. 121. The anode-voltage of the gas-filled valve is then equal to the condenser voltage, and as the condenser charges the anode-voltage rises. As soon as the condenser voltage has risen to the striking voltage of the valve (the time taken will depend upon the values of the steady screen and first-grid voltages of the pentode) the valve will strike and become a good conductor through which the condenser can discharge rapidly. To prevent the condenser-discharging current being sufficiently large to damage the gas-filled valve (through which it flows) a small protective resistance is included in series with the valve; this resistance will still be sufficiently small to permit a rapid discharge. The condenser will discharge until the anode-voltage of the gas-filled valve has fallen to the low value at which the valve ceases to conduct. Then charging of the condenser through the pentode will begin again, and the process will be repeated *ad lib.*

It is usually required that the frequency and amplitude of the output voltage of the time-base shall be adjustable. For a given striking voltage of the gas-filled valve the frequency depends only on the rate of rise of condenser voltage during charging. This may be varied either by varying the capacitance of the charging condenser or by varying the value of the constant charging-current passed by the pentode. The value of the pentode current may be adjusted either by adjusting its first-grid voltage or its screen voltage, and provision is usually made for adjusting one of these voltages in order to give a fine control of frequency. A coarse adjustment of frequency is usually made by switching-in condensers of different capacitance. The *amplitude* of the output voltage of the time-base depends only on how far the condenser voltage is allowed to rise before the gas-filled valve strikes (i.e. on the value of the striking voltage), and may therefore be controlled by adjusting the negative grid-voltage of this valve.

A further requirement is that of "synchronizing." When the time-base is used in conjunction with a cathode-ray oscilloscope to show the waveform of some alternating voltage (the alternating voltage under examination is often spoken of as the "work voltage") it is necessary that the frequency of the time-base shall be an exact sub-multiple of that of the work voltage, for otherwise the picture seen on the fluorescent screen of the cathode-ray tube is either moving or else an unwanted complicated pattern. The adjustment of time-base frequency may be made manually (as described above), but since the adjustment is required to be exact, manual adjustment is not always easy. There is a very simple device which causes the time-base to synchronize accurately with the chosen sub-multiple of the frequency of the work voltage, provided that an approximate manual adjustment has been made. The device consists of connecting in the grid circuit of the gas-filled triode a fraction of the work voltage. Since the voltage required to cause striking depends on the grid-voltage, this voltage-requirement will now fluctuate at the frequency of the work voltage, i.e. the valve is predisposed to strike at the positive peaks of the work voltage. If the frequency of striking be manually adjusted until it is nearly equal to the required submultiple, then the lowering of the voltage-requirement which occurs whenever there is a positive peak of the work voltage will ensure that striking always takes place at one of these peaks. Thus the time-base frequency will become an exact submultiple of the frequency of the work voltage.

Finally, it will be noted from Fig. 121 that the output voltage is not taken as the voltage across the condenser, but the voltage across the charging pentode. Since, however, this voltage is the difference between the constant h.t. voltage and the saw-tooth voltage across the condenser, it may be regarded as the required voltage with an additional d.c. component.

The Puckle Time-base*

The Puckle time-base also operates on the principle of charging a condenser through a constant current pentode, as shown in Fig. 120 (*c*), but uses a Flip-flop as the automatic switch.

It will be remembered (see page 204) that the Flip-flop circuit is that of a two-stage amplifier with the whole of the output voltage coupled back to the input circuit. Thus there are two inter-stage couplings—one of them is d.c. coupling and the other is Resistance-Capacitance coupling. The properties of the Flip-flop have been shown to be as follows—

(*a*) In its stable condition, anode-current is completely cut off in that valve which immediately follows the d.c. coupling (hereafter referred to, for convenience, as the d.c.-fed valve), while a fairly large anode-current flows through the other valve (the condenser-fed valve).

(*b*) Certain types of stimulus can cause the anode-current of the condenser-fed valve to fall suddenly to zero, while the anode-current of the d.c.-fed valve simultaneously rises to a fairly large value.

(*c*) This condition is an unstable one (as the negative grid voltage

* See O. S. Puckle, *Time Bases.* (Chapman and Hall, 1943.)

of the condenser-fed valve is produced by the flow of a condenser-charging current through the resistance in its grid circuit, and this charging-current decays with time). After a period of time decided by the circuit constants, the currents will revert to their values in the stable condition, viz. zero anode-current in the d.c.-fed valve.

(*d*) The criterion that the stimulus mentioned in (*b*) above shall succeed in "triggering" the circuit from the stable to the unstable condition, is that the stimulus shall be such as to cause anode-current to flow in the d.c.-fed valve.

There is thus one valve (the d.c.-fed valve) which normally is a non-conductor, and which can be caused to conduct for a short period by a suitable stimulus. The connexion of this valve across the charging-condenser of a time-base circuit will thus provide a suitable automatic switch for periodically discharging the condenser and causing the charging process to begin all over again. The stimulus *available* is the rising voltage across the condenser, and the circuit must therefore be so arranged that this particular stimulus suffices to trigger it, i.e. so that when the condenser voltage reaches a certain value the first valve shall conduct.

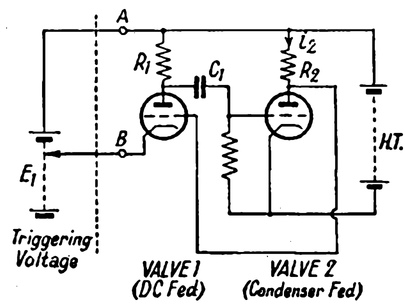


FIG. 122

For this purpose the circuit of the Flip-flop is re-arranged as shown in Fig. 122. It will be noted that the stimulus is applied by variation of the cathode-potential of valve 1. The stimulus is shown in Fig. 122 as simply a variable d.c. voltage: actually the charging-condenser of the time-base is connected across the points AB, and as this condenser charges up, the tapping-point B may be considered to slide down the battery E_1 of Fig. 122. Let us deduce the effect of this.

With zero anode-current in valve 1—the stable condition of the Flip-flop—the potential of the anode of this valve is the same as that of the positive side of the h.t. supply (right-hand side of Fig. 122), whereas the grid of valve 1 is at a potential lower than this by the voltage drop, R_2i_2, in the load of valve 2. Thus when the tapping-point B is at the positive end of E_1, the anode-to-cathode voltage of valve 1 is zero and the grid is considerably negative with respect to the cathode. As the point B slides down E_1, the anode and grid of valve 1 remain at these same potentials, while the cathode-potential is lowered. Thus, at the same time, the anode-voltage of the valve is increased and the negative grid-voltage is decreased. A point will be reached at which anode-current will commence to flow in valve 1, and this suffices to trigger the Flip-flop circuit, i.e. to cause anode-current in valve 1 to rise suddenly to a large value and in valve 2 to fall suddenly to zero. (The voltage landslide process by which this takes place is described in the sections on the Multivibrator and the Flip-flop.)

Since, in the Puckle time-base, the voltage E_1 of Fig. 122 is provided by the charging-condenser voltage, this condenser being connected across the points AB, it follows that when valve 1 suddenly becomes a good conductor the condenser will discharge rapidly through this valve in series with R_1, the value of the latter being small. This means that the tapping point B is returned

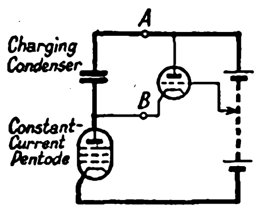


FIG. 123

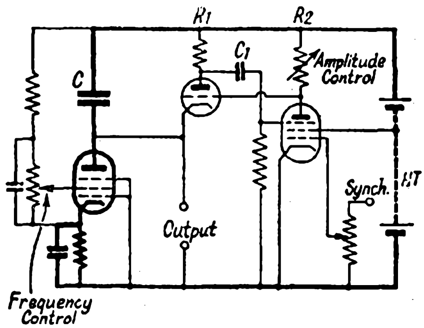


FIG. 124. PUCKLE TIME-BASE

rapidly towards the positive end of E_1. This causes the anode-current of valve 1 to be cut off, which in turn precipitates a return to the stable condition with valve 2 conducting. The whole process of sliding the tapping point down the battery E_1 then recommences.

It is interesting to note that a time-base circuit could be formed by using only valve 1 of Fig. 122 as the automatic switch, as shown in Fig. 123. Anode-current would commence to flow in this valve as soon as the charging-condenser voltage reached a sufficient value, and would increase rapidly as the condenser voltage increased. The increase, however, would not be sufficiently rapid, and the addition of valve 2 to form a Flip-flop circuit is called for. The three valves in the Puckle time-base are often referred to as the Charging Pentode, Discharging Valve, and Accelerating Valve, respectively.

Fig. 123 shows clearly that the potential to which the condenser can charge before the discharge valve begins to conduct depends upon the grid-voltage of this valve, since the cathode-potential must fall to the region of the grid-potential. Control of the grid-potential of this valve therefore serves to control the amplitude of the time-base output voltage. In Fig. 122, the grid-potential of valve 1 is the same as that of the anode of valve 2, which is an amount $R_2 i_2$ lower than the potential of h.t. positive. In this case, therefore, adjustment of R_2 controls the potential of the grid of valve 1 and serves to adjust the time-base amplitude.

Fig. 124 shows the complete circuit of the Puckle time-base. It will be noted that a pentode is used as the condenser-fed valve of the Flip-flop, the suppressor playing the part of the control grid. The first-grid of this pentode also functions as a control grid for the purpose of synchronization. A fraction of the work voltage is connected in the first-grid circuit, so reducing i_2 at each negative peak of the work voltage. This predisposes the Flip-flop to trigger at one of these peaks. The d.c. voltages for the various grids of the constant-current pentode are derived from the voltage-divider formed of the three resistances at the left-hand side of the diagram, the shunting condensers being to preclude the possibility of a.c. voltages appearing on these grids. As in the Thyratron time-base, a fine control of frequency is provided by an adjustment of the screen voltage of the constant-current pentode.

The Transitron Time-base (or Fleming-Williams Time-base)*

This time-base, like those described above, works on the principle illustrated in Fig. 120 (*c*), viz. the charging of a condenser through a resistance, followed by its rapid discharge through an automatic switch, this switch being triggered by the rising voltage of the condenser. The automatic switch used in this time-base is a Transitron Relaxation Oscillator.

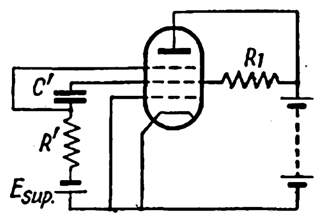


FIG. 125. TRANSITRON RELAXATION OSCILLATOR

The circuit of the Transitron Relaxation Oscillator is shown in Fig. 125, and will be seen to be identical with that of the ordinary transitron oscillator (Fig. 111 (*a*)) except that a resistance load R_1 is substituted for the parallel L–C circuit. In the section on the transitron it was shown that the screen voltage in a transitron-connected pentode is in phase with the suppressor voltage which produces it, so that positive feed-back can be achieved simply by resistance-capacitance coupling the output electrode (i.e. the screen) to the control electrode (i.e. the suppressor). When using a parallel L–C circuit as load, as in the ordinary transitron oscillator, adequate positive feed-back results in sinusoidal oscillation. With a resistance load it means that any sudden change of suppressor voltage is amplified, fed-back, re-amplified, fed back, and so on—so that the initial sudden change becomes a precipitate change in the same direction, limited only by the eventual displacement of the electrode

* B. C. Fleming-Williams, *Wireless Engineer*, 1940, Vol. 17, p. 161.

voltages to regions of the characteristics where the suppressor voltage exerts insufficient control over the screen current.

While the electrode voltages are such that the suppressor adequately controls the screen current, any random impulse, however minute, can initiate this voltage landslide. It follows that the circuit cannot remain in this sensitive condition, but will automatically take up one of its two limiting conditions. In one of these limiting conditions the suppressor has been forced positive, so that the anode-current is large and the screen current small: in the other, the suppressor has been forced negative, so that the anode-current is small and the screen current large.

Neither of these limiting conditions is stable. The attainment of a limiting condition is extremely rapid—so rapid that the voltage across the coupling condenser C' has not time to change. After the limiting condition is attained, however, the voltage across this condenser slowly changes, and the suppressor voltage slowly changes towards the value E_{sup}—which it would eventually reach were the charging of condenser C' allowed to become complete. Long before this happens, however, the changing suppressor voltage has reached the range of values in which it can exercise control of the screen current, and the other limiting condition is therefore suddenly precipitated, the sudden change being initiated by the movement of the suppressor voltage away from its value in the first limiting condition. Thus the circuit behaves as a relaxation oscillator, alternating between one limiting condition and the other.

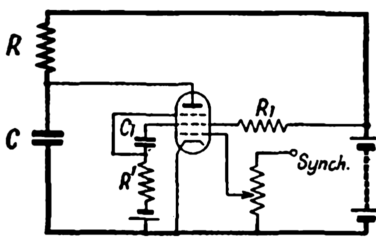


FIG. 126. TRANSITRON TIME-BASE

Fig. 126 shows the complete circuit of the Transitron time-base. The heavy lines show the orthodox condenser-charging circuit. It will be seen that the d.c. supply voltages for anode and screen are not derived (as they were in Fig. 125) from a common h.t. supply. The screen supply voltage E is here the same as that used for the charging of the condenser C, but the anode is supplied simply from the voltage across this condenser. It follows that in the early stages of the charging of this condenser the anode-voltage slowly rises from zero or a low value. Until it reaches a certain value the relaxation oscillator is necessarily inoperative—for the control of the screen current exercised by the suppressor voltage is dependent on the ability of the anode to collect electrons. Under these conditions, therefore, nearly the whole of the space current goes to the screen. As the voltage across the charging-condenser C rises, the anode-current slowly rises, the screen current slowly falls and the screen voltage rises. As soon as the anode-voltage has reached such a value that the suppressor can exert control over the screen current, a voltage landslide occurs, the screen current suddenly falls and the anode-current suddenly rises to a fairly large value. The valve, from anode to cathode, has now become a fairly good conductor, and the condenser C discharges rapidly through it. The discharge proceeds until the other limiting condition (i.e. small anode-current) is suddenly precipitated, when the whole process begins all over again.

Linear, instead of exponential, rise of condenser voltage may be secured by substituting a constant current pentode for the resistance R. Alternatively, the rise of voltage may be made approximately linear by making the charging voltage supply, E, large with respect to the highest voltage attained by the condenser.

Finally, Fig. 126 shows that the first-grid voltage is not kept constant, but that a fraction of the work voltage is connected in this grid circuit for the purpose of synchronizing.

Other circuits which can generate a linear sweep of voltage (e.g. the Miller integrator, the Miller transitron, and the Bootstrap circuit) are described in Chapter VIII.

Suggestions for Further Reading

1. H. A. Thomas, *Theory and Design of Valve Oscillators* (Chapman & Hall).

2. O. S. Puckle, *Time Bases* (Chapman & Hall).

3. D. A. Levell, *Pulse and Time-base Generators* (Pitman).

4. J. F. Reintjes and G. T. Coate (M.I.T. Radar School), *Principles of Radar* (McGraw-Hill).

5. F. E. Terman, *Electronic and Radio Engineering* (McGraw-Hill).

CHAPTER VI

DETECTORS AND RECTIFIERS

RECTIFICATION is the process of producing direct current from alternating current, or a direct voltage from an alternating voltage. When this is done for the purpose of providing a d.c. power supply, the apparatus used is known as a Rectifier. When, however, the production of a direct voltage from an alternating voltage is a certain part of the process of communicating information by means of electrical signals, the apparatus used is known as a Detector. This part of the process of communication is called Detection or Demodulation. The input voltage to a power rectifier is sine-wave A.C. of constant amplitude and constant frequency. The alternating input voltage to a Detector, on the other hand, has an amplitude which is varying ("modulated")—often in accordance with the output voltage of a distant microphone. The term "carrier-frequency" is used to denote the frequency of this alternating voltage and the term "modulation-frequency" is used to denote the frequency at which the amplitude of the alternating voltage is varying. The carrier-frequency is always much larger than the modulation-frequency—usually more than a hundred times as large.

It is usually required that the d.c. output voltage of a detector shall vary in direct proportion to the amplitude of its alternating input voltage. This is known as Linear Detection (see also page 264). If the input voltage is the sinusoidally modulated voltage

$$v_1 = V_1 (1 + m \sin pt) \sin \omega t \quad . \quad . \quad (\text{VI.1})$$

where ω is 2π times the carrier frequency, p is 2π times the modulation frequency, and m is the modulation depth, then to be directly proportional to the amplitude of this voltage the output voltage must be

$$\begin{aligned} v_2 &= kV_1 (1 + m \sin pt) \\ &= kV_1 + mkV_1 \sin pt \quad . \quad . \quad . \quad (\text{VI.2}) \end{aligned}$$

The first term in this equation represents a direct voltage proportional to the amplitude of the unmodulated carrier voltage; the second term represents an alternating voltage of the modulation frequency, and is the useful output of the detector.

The Diode Detector

Consider a circuit consisting of a diode, a condenser and a source of A.C., all connected in series. The diode permits current to flow in one direction only. This current will charge the condenser, and since there can be no current in the opposite direction the condenser cannot discharge. The condenser voltage will be in such a direction as to oppose the passage of current through the diode. As long as the condenser voltage is lower than the peak value of the source of applied voltage, there will be at certain parts of the cycle a resultant voltage tending to send current through the diode, and thus to charge the condenser to a still higher voltage. The condenser will thus charge to a voltage equal to the peak value of the applied voltage, and will remain so charged.

This circuit as it stands succeeds in producing a direct voltage directly proportional to (in fact, equal in magnitude to) the amplitude of the applied alternating voltage. It is not a detector circuit, however, since it cannot respond to changes in amplitude of the applied voltage. For instance, if the amplitude of the applied voltage were to decrease, or even if the applied voltage were to be switched off, the condenser voltage (which we are considering as the output voltage of the detector) would remain the same, since there is no path by which the condenser could discharge. The connexion

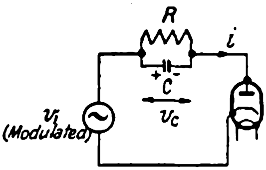


Fig. 127. Diode Detector

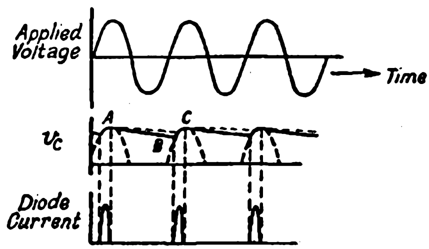


Fig. 128

of a high resistance across the condenser, making the circuit as shown in Fig. 127, provides such a path, and the circuit is then that of a diode detector.

A good way of deducing the behaviour of such circuits is to regard the diode as a switch, operated automatically by the rest of the circuit. Since a diode cannot conduct when its anode-voltage is negative, we see that in such a case the "switch" must be considered to be open (thereby blocking the passage of current). In the circuit of Fig. 127, the anode-voltage is the sum of two components, the condenser voltage and the applied voltage, v_1. The condenser voltage is a negative contribution to the anode-voltage. The applied voltage v_1, is alternately a positive and a negative contribution. It follows that the "switch" will be open throughout the negative half cycles of v_1 and throughout those portions of the positive half-cycle when the instantaneous value of v_1 is less than the condenser voltage.

We thus have a criterion for deciding whether the switch is open or closed at any given instant. Consider next what happens when the switch is closed, and what happens when it is open. With the switch open, there is no path by which v_1 can send current; the only current which can flow is the current in the RC loop in the upper part of the circuit diagram, namely the current due to the discharge of the condenser through the resistance. With the switch closed, the condenser is connected directly across v_1 and its p.d. will be equal to v_1. (We have made an approximation in regarding the diode as a switch, for in practice the condenser voltage will be slightly less than v_1 as a result of the small p.d. across the diode.)

With the above in mind, it is easy to deduce the behaviour of the circuit. Let the condenser be initially uncharged when the applied voltage v_1 is first switched on. On the arrival of the first positive half-cycle of v_1, the "switch" will close and v_1 will be connected directly across the condenser. Thus as the instantaneous value of v_1 increases, the condenser voltage will increase, right up to the peak value. (A more accurate picture would show the condenser voltage lagging somewhat behind v_1 as the result of the resistance of the diode.) As soon as the positive peak has been passed, the instantaneous value of v_1 begins to decrease. For the condenser voltage to decrease, the condenser would have to discharge. Its only discharge path, however, is by way of R, which is a very high resistance. Thus the condenser can discharge only very slowly. Almost at once therefore we have the situation in which v_1 is smaller than the condenser voltage—a situation which demands that the "switch" must open (Point A on Fig. 128).

With the switch open, the condenser discharges through R, its voltage falling exponentially towards zero, along the full line AB in Fig. 128. The time-constant of this discharge is the product RC. If RC has a very large value, this discharge will proceed more slowly, as shown by the upper dotted line in Fig. 128.

While the condenser is discharging, the a.c. cycle is proceeding. v_1 decreases, passes through zero, and executes its negative half-cycle. Throughout all this, the anode-voltage of the diode is negative and so the "switch" remains open. The applied voltage now begins its next positive half-cycle. At the instant marked B in Fig. 128, the rising graph of v_1 (shown dotted) intersects the falling graph of the condenser voltage. The slightest further increase of v_1 gives us a positive anode-voltage for the diode. The "switch" therefore closes, the condenser is again connected to v_1, and the cycle which we have described begins to repeat itself.

Five points should be noted: (i) that the condenser voltage is D.C. with a ripple on it, (ii) that the magnitude of this D.C. is approximately equal to the peak value of the applied alternating voltage, (iii) that the ripple can be made smaller by increasing the time-constant RC, (iv) that diode-current flows only during the short intervals such as BC, and (v) that these short intervals become shorter still as RC is increased.

In the above paragraphs, we have considered what happens in each cycle of the carrier-frequency. At the same time, a much slower process is taking place; the amplitude of the alternating voltage is varying at the modulation frequency, alternately increasing and decreasing. During a period when the amplitude is decreasing, successive positive peaks are smaller. The height of the point C in Fig. 128 would then not be so great as the height of the point A. If the time-constant RC is too large, or if the height of the positive peak at C is too small, the line AB may not intersect the (dotted) rising curve of v_1, but may pass over the top of the half sine-wave. Instead of following the amplitude of the alternating input-voltage, the condenser-voltage would then continue decaying exponentially until next the amplitude began to rise. During this part of the modulation cycle, there would be no relation between the input and output voltages of the detector. This constitutes distortion.

Let us calculate the maximum permissible value of the product RC which will enable the condenser voltage to decrease rapidly enough to give distortionless detection of the modulated voltage expressed in equation (VI.1). The voltage at time t of a condenser initially charged to a voltage V_0 and discharging through a resistance R, is given by

$$v_c = V_0 e^{-t/RC}$$

so that the rate of discharge is

$$-dv_c/dt = \frac{V_0}{RC} e^{-t/RC} = v_c/RC \quad . \quad . \quad \text{(VI.3)}$$

That is, the rate of discharge at the instant when the voltage is v_c is v_c/RC.

Now for distortionless detection the condenser voltage must be given by equation (VI.2), viz.,

$$v_c = kV_1(1 + m \sin pt)$$

and its rate of decrease is therefore required to be

$$-dv_c/dt = -kV_1 mp \cos pt$$

We know, however, that its rate of decrease will be v_c/RC, i.e.

$$\frac{kV_1}{RC}(1 + m \sin pt)$$

Thus, the rate of discharge will be sufficiently great if

$$\frac{kV_1}{RC}(1 + m \sin pt) > -kV_1 mp \cos pt$$

i.e. if

$$RC < \frac{1 + m \sin pt}{-mp \cos pt}$$

This must be so throughout the discharge, i.e. for all values of t.

By differentiation with respect to t we may show that the right-hand side of the above inequality has a minimum value when t is such that

$$\sin pt = -m$$

and the condition therefore becomes

$$RC < \frac{\sqrt{1-m^2}}{mp} \quad . \quad . \quad . \quad . \quad (VI.4)$$

Clearly this condition cannot be satisfied for m equal to unity (100 per cent modulation). This is to be expected, since theoretically an infinite time is required for the voltage of a discharging condenser to fall to zero, whereas the amplitude of a fully modulated voltage periodically falls to zero.

The relation given in equation (VI.4) may be so written as to give the ratio of the reactance of C at the modulation frequency to the resistance R, i.e.

$$\frac{(1/pC)}{R} > \frac{m}{\sqrt{1-m^2}}$$

The result of making the product RC larger than indicated above is that the condenser voltage cannot follow the modulation, so that part of the waveform of the condenser voltage will have the shape of the discharge curve instead of the shape of the modulation envelope. This, therefore, causes harmonic distortion in the (modulation frequency) output voltage of the detector.

There is a further complication arising from the connexion of a resistance across the condenser. We have seen that in the absence of such a resistance the condenser voltage will ultimately rise to a value equal to the peak value of the applied voltage. Now when current is flowing through the diode there is a voltage across it, the relation between voltage and current being as shown in Fig. 7. If this graph were a straight line we could say that whenever current was passing through the diode it behaved as a fixed resistance, of value equal to the reciprocal of the slope of the graph. Since the graph is a curve, however, we may only say that the diode behaves as a resistance whose value depends upon the current flowing through it. Now consider the diode in Fig. 127 replaced by a resistance, and the applied alternating voltage replaced by a steady voltage in such a direction as to cause current to flow through the diode. In such a circuit, when the condenser has finally charged, current will continue to flow through the two resistances in series, and if we denote the diode resistance by r_d the voltage across the condenser will be the same as the voltage across R, viz. a fraction $\frac{R}{R+r_d}$ of the applied voltage. Similarly in the case of an alternating applied voltage the condenser will charge only to a fraction of the peak value of the applied alternating voltage. This fraction can be made to approximate to unity if R be made large with respect to r_d. The ratio of the condenser voltage to the peak value of the applied alternating voltage is known as the Detector Efficiency and is the coefficient k in equation (VI.2).

We have seen that the value of r_d is not a constant but depends on the current flowing through the diode. For small currents the slope of the graph (Fig. 7) is small, and r_d will be large. For large currents r_d will be smaller, and it will be easier to make R large with respect to r_d, and thus to secure a high detector efficiency. It follows that the detector efficiency will be higher for large values of applied voltage, since these will involve large values of diode current.

Since the detector efficiency depends upon the magnitude of the input voltage, it follows that there will not be, during modulation, a truly linear relation between the amplitude of the applied voltage and the condenser, or output, voltage. This again will cause harmonic distortion of the output voltage. This distortion may be made fairly small, and it may be estimated by drawing dynamic characteristics of the diode as explained in the next section. It is important to note that if R be made large with respect to the largest value which r_d assumes in the detection of a given modulated alternating voltage, the detector efficiency will approximate to unity; then, although the detector efficiency will vary throughout the modulation cycle, its value will never be very far removed from unity, and the variation (and hence the distortion) will be small.

Another important factor is the inter-electrode capacitance, C_{ac}, of the diode. Let us consider the values of the impedance of the various parts of the circuit at the carrier frequency. Since the product RC is large with respect to the periodic time of the carrier alternations, it follows that R is large with respect to $1/\omega C$, so that at the carrier frequency we have, in effect, simply the capacitance C in series with the non-linear impedance of the diode, which is itself shunted by C_{ac}. Now if C_{ac} is larger than C, its reactance will be smaller than that of C, and the larger part of the carrier frequency voltage which should appear across the diode will appear across C. To avoid this C should be made several times as large as C_{ac}.

We may summarize what we have said of the diode detector as follows—

1. The product RC should be large in order to avoid the presence of carrier frequency components in the output.

2. The limit to the value of RC is given by equation (VI.4). If this limit is exceeded there will be harmonic distortion of the modulation frequency output voltage.

3. The resistance R should be large to avoid low detector efficiency, and to reduce the distortion which results from the detector efficiency being a function of the amplitude of the input voltage. This distortion will also be smaller for large input voltages, which are therefore to be recommended.

4. The capacitance C should be several times as large as the inter-electrode capacitance, C_{ac}.

The choice of values for R and C is based on the above considerations. Since R is to be large and RC may not be larger than indicated by equation (VI.4) it follows that C should be made no greater than necessary. A value of from five to ten times C_{ac} is usually adopted. The largest value of R consistent with equation (VI.4) is then used.

Dynamic-characteristics of Diode Detector

The dynamic-characteristics, shown in Fig. 129, are a means of expressing the behaviour of a diode valve to which are applied a direct voltage and a sinusoidal alternating voltage in series. This is, in effect, what occurs when an unmodulated voltage is applied to the circuit of Fig. 127, provided that we may consider the capacitance C a short-circuit at the carrier frequency. In Fig. 129 of course the value of the direct voltage is a function of the alternating voltage.

The dynamic-characteristics give the value of the mean diode current, i_{mean} (such as would be read by a d.c. milliammeter), for simultaneously applied d.c. and a.c. voltages, V_{DC}, and V_{rms}, respectively. The curves are usually obtained experimentally by the circuit of Fig. 130, though they could be determined graphically from the static-characteristic (Fig. 7). First the a.c. voltage-divider,

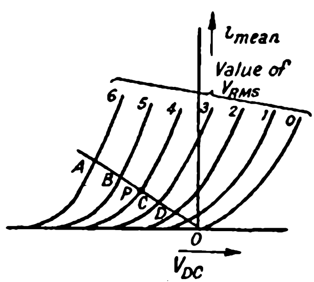


FIG. 129. DYNAMIC-CHARACTERISTICS OF DIODE

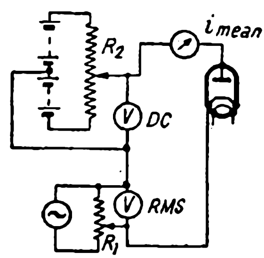


FIG. 130. CIRCUIT FOR MEASURING DYNAMIC-CHARACTERISTICS

R_1, is set to give zero V_{rms}, and the curve of i_{mean} against V_{DC} is taken by varying the setting of the voltage-divider, R_2. This, of course, simply gives the static-characteristic. V_{rms} is next given a small value, and a similar curve taken. Current will now flow even for negative values of V_{DC}. The process is repeated for equal increments of V_{rms}. Since a negative d.c. voltage equal in magnitude to $\sqrt{2}\,V_{rms}$ is sufficient to prevent the flow of current, each curve will reach the horizontal axis at a value of V_{DC} equal to $\sqrt{2}\,V_{rms}$.

In these curves i_{mean} is the dependent variable and V_{DC} and V_{rms}

are independent variables. In the circuit of Fig. 127, however, V_{DC} and V_{rms} are not independent quantities; in fact, V_{DC} and i_{mean} are related by the equation

$$V_{DC} = -Ri_{mean}$$

since the d.c. output voltage is produced by the passage of the mean diode current through R. If we draw on Fig. 129 a straight line having this equation, the abscissa of the point of intersection of this line with the curve for a given value of V_{rms} will give the value of the steady output voltage resulting from the application of this given value of V_{rms} in the circuit of Fig. 127. As explained in the previous section, this voltage will be less than the peak value of the applied voltage, owing to the resistance r_d of the diode.

Using this construction we may investigate the effect of a modulated alternating input voltage in Fig. 127. Let us assume that the r.m.s value of the unmodulated carrier voltage is 4 volts and that m is 50 per cent, so that the r.m.s. value varies between 2 volts and 6 volts. The abscissae of the points A, B, P, C, and D will give the values of V_{DC} corresponding to values of V_{rms} equal to 6, 5, 4, 3, and 2 volts, so that we may proceed to plot the waveform of the modulation-frequency output voltage corresponding to a sinusoidal modulation. The harmonic distortion may be estimated in a similar manner to that used in dealing with load-line diagrams for amplifier valves. It will be seen that the points of intersection are more crowded at the lower end of the line AB, indicating that greater harmonic distortion will occur with a fully modulated input voltage, or with a small input voltage.

Input and Output Circuits of Diode Detector

The input circuit of a detector usually consists of a parallel L–C circuit (across which the modulated voltage appears) forming part of the aerial circuit or part of the output circuit of a radio-frequency or intermediate-frequency amplifier. The output circuit is the means of coupling the modulation frequency component of the detector output voltage into the grid circuit of an audio-frequency amplifier. Fig. 131 shows such a circuit; it will be seen that the detector R–C circuit is connected on the cathode side of the diode in order to make it possible to have the cathodes of both valves at the same potential. We have seen that the voltage across R resulting from an unmodulated input voltage is a steady d.c. voltage, but that when the input voltage is modulated there is added to this an alternating voltage of the modulation frequency. It is the latter

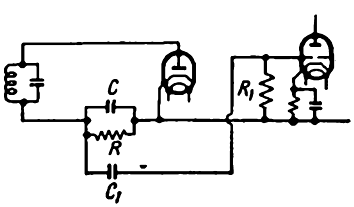


FIG. 131

alone which we wish to connect into the grid circuit of the amplifier stage. (The former would constitute a grid-bias voltage which would be a function of the carrier amplitude; this voltage is, however, the basis of automatic volume control methods.) The condenser, C_1, and resistance R_1, connected in series across R, form an a.c./d.c. separating circuit, and if the reactance of C_1 at the modulation frequency be made small with respect to R_1, the whole of the d.c. component of the detector output voltage will appear across C_1, and the modulation frequency component across R_1.

The dynamic-characteristic method of examining the performance of a diode detector is based upon the fact that the output voltage is produced by the passage of the mean diode current through the resistance R, so that

$$V_{DC} = -Ri_{mean}$$

The mean diode current is a steady direct current when the input voltage is unmodulated, but varies with the modulation when the input is modulated. By the connexion of R_1 and C_1 across R in Fig. 131 we have provided a parallel path for the varying component of the mean diode current, but not for the steady component. Since the reactance of C_1 at the modulation frequency is small with respect to R_1, the impedance of the two paths in parallel is

$$\frac{RR_1}{R + R_1}$$

We must now treat the two components of the output voltage separately. Using V_{DC} now to represent the steady component and V_{mod} to represent the modulation frequency component, we have

$$V_{DC} = -R(i_{mean})_{DC}$$

$$V_{mod} = -\frac{RR_1}{R + R_1}(i_{mean})_{mod}$$

Fig. 132 shows the required modification of the dynamic-characteristic diagram. A straight line OP of slope $1/R$ is first drawn; the point of intersection of this line with the curve for V_{rms} equal to the r.m.s. value of the unmodulated carrier (i.e. $V_1/\sqrt{2}$, with the notation of equation (VI.1)) will give the magnitude of V_{DC}, the d.c. component of the output voltage. Through this point, P, a straight line $ABCD$ must be drawn, having a slope $(R + R_1)/RR_1$, i.e. a greater slope than $1/R$. The intersections of this steeper line with the various curves must now be used in plotting the waveform of the mean diode current, and in estimating the harmonic distortion.

It is apparent from Fig. 132 that the detector cannot handle a fully modulated input voltage (m equal to 100 per cent) without severe distortion. Consider for example a fully modulated input voltage whose r.m.s. value varies in the course of modulation from zero to 4 volts, i.e. over the whole range of values shown in Fig. 132. The ordinates of the points A, B, P, and C will give the values of i_{mean} corresponding to r.m.s. values of the input voltage of 4, 3, 2, and 1 volts, and the waveform of i_{mean} (and thus of the output voltage) may be plotted as shown in Fig. 133. For all r.m.s. values of the input voltage less than about $\frac{3}{4}$ of a volt, i_{mean} is zero, with the result that the bottom of the negative half-wave is sliced off. This effect is less marked the less the difference between the slopes of the straight lines OP and APD. This difference may be made small, and the distortion reduced, by making R_1 large with respect to R. This "slicing-off of the negative half-wave" will, of course, not occur

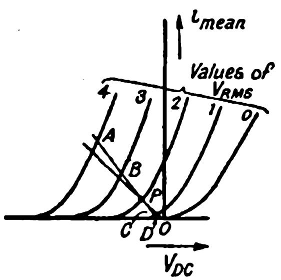


FIG. 132

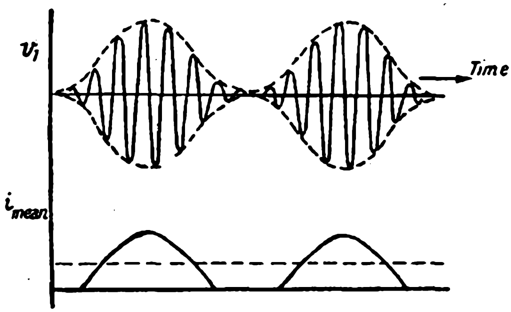


FIG. 133

for modulation depths less than a certain value, this value becoming smaller as the difference between the slopes of the two lines increases.

Let us next consider the effect of the detector on the preceding circuit. Both the selectivity and the amplification of the preceding circuit would be seriously reduced by the connexion of a resistance across the parallel L–C circuit, the reduction being greater the lower the value of the resistance. The impedance presented by the diode detector is not simply that of a resistance, but we may define the Effective Input Resistance of the detector circuit as that resistance which would draw the same power from the source, or in this case from the parallel L–C circuit. The lower the effective input resistance, the greater the reduction of amplification and selectivity of the preceding circuit, so that we must design the detector to have as high an effective input resistance as possible.

In a diode detector designed in accordance with the considerations already given in the previous section, the diode current consists of pulses of current, of short duration, occurring when the input voltage is passing through its peak value (see Fig. 128). The power drawn from the source is thus approximately equal to the product of the peak input voltage and the mean diode current. The mean diode current is the current through R, and since the voltage across R is kV_1, where k is the detector efficiency and V_1 is the peak value of the input voltage, this current must be kV_1/R, and the power must be kV_1^2/R. The value of the resistance, R_{input}, which would draw the same power from the source is therefore given by

$$\frac{(V_1/\sqrt{2})^2}{R_{input}} = kV_1^2/R$$

giving

$$R_{input} = R/(2k) \quad . \quad . \quad . \quad . \quad \text{(VI.5)}$$

In a diode detector not designed so that the efficiency is high the duration of the current pulses would be longer, and equation (VI.5) would give too high a value for the effective input resistance. Equation (VI.5) shows that for a high effective input resistance R should be made large, a condition already imposed by other considerations.

With a detector output circuit as shown in Fig. 131 the effective input resistance will differ for modulated and unmodulated input voltages, since for current of the modulation frequency the resistance effectively in parallel with C is not R but $RR_1/(R + R_1)$, whereas for direct current it is simply R. The modulation of the carrier voltage is equivalent to the addition to it of sidebands (see page 259). The effective input resistance to the carrier is $R/2k$; when sidebands are added, the effective input resistance to the sidebands is

$$\frac{RR_1/(R + R_1)}{2k}$$

Since the effective input resistance to the sidebands is lower than the effective input resistance to the carrier, the sideband amplification of the preceding stage will be less than the amplification of the carrier, i.e. the ratio of sideband amplitude to carrier amplitude is reduced by connexion to the detector circuit. In other words, the detector itself reduces the modulation depth of the voltage applied to it. This makes it of less consequence that, as a result of the output circuit, the detector cannot handle fully modulated voltages without severe harmonic distortion.

The Triode Grid-circuit Detector (or Cumulative-Grid Detector)

This type of detector, shown in Fig. 134, treats the grid and cathode of a triode valve as constituting a small diode within the triode valve, the grid of the triode being the anode of this diode. Using this diode, a diode detector circuit of the form just described is built up, and the output voltage of this detector, which is of course the voltage across the resistance R, is thus automatically connected in the grid circuit of the triode. This voltage is amplified by the triode, the amplified voltage appearing across the load impedance connected in the anode circuit of the triode. The chief advantage of this circuit over the diode detector circuit is that it combines in one valve a detector and an amplifier. Its disadvantages arise from the fact that, in addition to connecting into the grid circuit of the amplifier valve the useful modulation-frequency component of the detector output, we have also connected into this circuit the d.c. component of the detector output and the whole of the detector input voltage.

The four points summarized in the section on the "Diode Detector" apply also to this detector. The paragraphs dealing with the effect of the detector output circuit do not apply, since in this case there is no such output circuit. This means that the triode grid-circuit detector is better able than the diode detector to give distortionless detection of deeply modulated voltages. In general, however, the values of the effective resistance, r_d, of the "diode" formed by the grid and cathode are larger than the values for a diode valve, making it more difficult to make R large with respect to r_d. Thus it is more difficult to secure a high detector efficiency and to avoid the distortion resulting from variation of the detector efficiency during the modulation cycle.

The voltage passed on from a diode detector to the next stage consists only of the modulation frequency component. In the triode grid-circuit detector, however, the grid-voltage of the amplifier is the sum of the modulated radio-frequency input voltage, the modulation-frequency component of the detector output, and the d.c. component of the detector output. The effect of this is seriously to limit the amplitude of the useful input voltage which the amplifier can handle without overloading (i.e. without producing severe distortion). Consider, for example, an ordinary Class A amplifier valve; to estimate roughly the maximum permissible input voltage we draw on the characteristics a load line corresponding to the particular values of the load impedance and h.t. voltage to be used (as in Fig. 20). The grid-voltage must never become so negative that the operating point moves into the region where the points of intersection of the load line and the curves are crowded together; nor must the grid-voltage ever become positive. In the triode grid-circuit detector this latter consideration is not valid since the operation of the circuit depends on the flow of grid-current (i.e. of "diode" current). The former rule, however, holds good. Now, with the notation of equation (VI.2), the total grid-voltage may be written

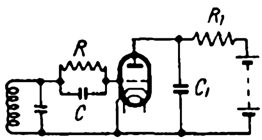


FIG. 134. CUMULATIVE-GRID DETECTOR

$$v_g = V_1(1 + m \sin pt) \sin \omega t - kV_1(1 + m \sin pt)$$

k being the detector efficiency. The graph of this quantity is shown in Fig. 135, and it will be seen that it varies between a small positive value and the value

$$-V_1(1 + m)(1 + k)$$

The maximum permissible value of V_1 is given by equating the above expression to the maximum permissible negative grid-voltage.

The amplitude of the *useful* input voltage is only mkV_1, and if this were the only input voltage we could make $2mkV_1$ equal to the maximum permissible swing of input voltage without overloading. Thus in this detector circuit we are using only a fraction

$$\frac{2mk}{(1+m)(1+k)}$$

of the available grid-voltage range for the useful input voltage. For a detector efficiency of 80 per cent and a modulation depth of

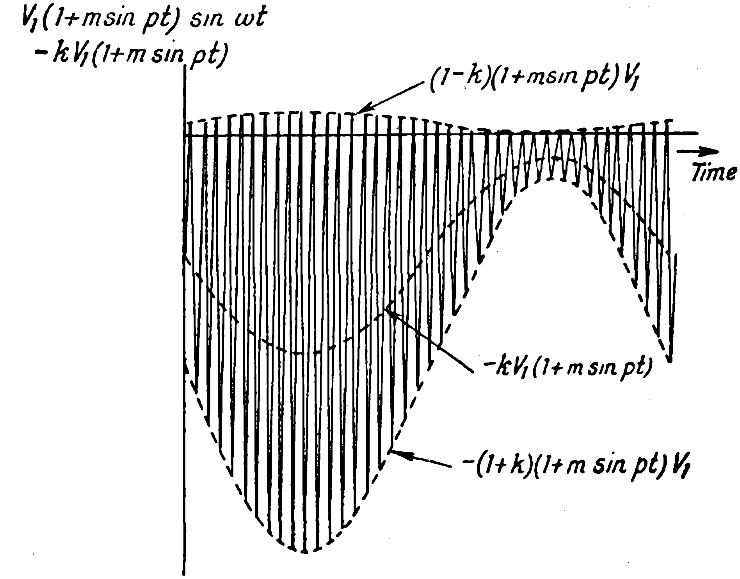


FIG. 135. RESULTANT GRID-VOLTAGE OF CUMULATIVE-GRID DETECTOR WITH MODULATED INPUT

the same value this ratio is 0·34. The chief disadvantage of this type of detector, therefore, is that only a small input voltage is permissible if distortion is to be avoided.

Another disadvantage arises from the fact that the input admittance of the valve is high at the radio-frequency; this results from the effect of the grid-anode capacitance, as discussed in Chapter IV. This increases the damping effect on the parallel L–C circuit, and also reduces the amplitude of the voltage reaching the "diode." In order to reduce the input admittance of the valve at the radio-frequency, a condenser C_1, of small reactance at this frequency, is shunted across the load in the anode circuit. This also helps to prevent the production of a radio-frequency voltage across the load, and the consequent danger of overloading the next stage. A better method is to couple the valve to load through a low-pass filter of the type having a shunt condenser input.

It is to be noted that when an alternating voltage is applied to a detector of this type, the steady anode-current decreases. A d.c. milliammeter may be used in the anode circuit to give an indication of the amplitude of the applied alternating voltage. (See section on "Valve Voltmeters.") Although this section has been entitled "The Triode Grid-circuit Detector," pentodes are frequently used for Grid-circuit Detection of this type.

Anode-circuit Detection

When considering distortion in an amplifier stage (Chapter III) we saw that the curvature of the valve characteristics resulted in harmonic distortion, and that the application of a sinusoidal input voltage produced, in addition to a sinusoidal anode-current of the same frequency, a second harmonic anode-current *and an increment of steady anode-current*. This latter constitutes detection. Detection of this type depends upon the curvature of the valve characteristics and is known as Anode-bend Detection.

The circuit of an anode-bend detector is thus the same as that of an amplifier, but the operating voltages are not so chosen that the straight parts of the characteristics are used, but so that the most sharply curved parts are used.

Consider an input voltage $V_1 \sin \omega t$ applied to the simple amplifier circuit shown in Fig. 19, and let the grid-bias voltage be of magnitude E_g. The total grid-voltage will then be

$$v_g = -E_g + V_1 \sin \omega t$$

The dynamic-characteristic (e.g. Fig. 25) expresses the relation between anode-current, i_a, and grid-voltage, v_g, for a given resistive load, R, and a given h.t. voltage. Let its equation be

$$i_a = f(v_g)$$

so that the quiescent anode-current is simply

$$i_{a0} = f(-E_g)$$

and the value of i_a when the input voltage is applied will be

$$i_a = f(-E_g + V_1 \sin \omega t)$$

which may be expanded by Taylor's Theorem to

$$i_a = f(-E_g) + \frac{V_1 \sin \omega t}{\underline{|1}} . f'(-E_g) + \frac{V_1^2 \sin^2 \omega t}{\underline{|2}} . f''(-E_g) + \dots$$

For small values of V_1 we may neglect terms of higher order than the second, and then, denoting the above values of the first and second differential coefficients by D_1 and D_2, we have

$$i_a - i_{a0} = D_1 V_1 \sin \omega t + \tfrac{1}{4} D_2 V_1^2 - \tfrac{1}{4} D_2 V_1^2 \cos 2\omega t \quad . \quad \text{(VI.6)}$$

Thus we have an expression for the change of anode-current caused by the application of the input voltage. The first term represents the alternating current, the second term the rectified current which constitutes the useful output of the detector, and the third term the second harmonic component.* Since the rectified component is proportional not to V_1 but to V_1^2, we see that we have here square-law detection and not linear detection. Although square-law detection is of use in the special case of heterodyne detection (q.v.), this type of detection of a modulated voltage would result in harmonic distortion of the modulation frequency output.

Anode-circuit detection of this particular kind is thus very little used. Anode-circuit detection may be carried out, however, using the same circuit, by making use not of the curvature of the characteristics but of the fact that the characteristics are discontinuous at zero anode-current, i.e. that for negative grid-voltages greater than the cut-off value no anode-current will flow. In this form of detector the grid-bias voltage is made about equal to the cut-off value. The application of an unmodulated input voltage will then cause current to flow during the positive half-cycles but not during the negative half-cycles, in the same way as occurs in each valve of a Class B amplifier. The resulting pulsating anode-current may be resolved into a d.c. component, having a value equal to the mean value of the pulses, together with a carrier-frequency component and various harmonics. It is this d.c. component which constitutes the useful output of the detector, for when a modulated input voltage is applied to this detector the value of the d.c. component varies in accordance with the *amplitude* of the input voltage, i.e. in accordance with the modulation. Linear detection is possible in so far as the range of amplitude covered by the modulation lies on the straighter part of the dynamic-characteristic. For example, let the input consist of a carrier of peak value 2 volts modulated to a depth of 50 per cent. The range of amplitude covered by the modulation is then from 1 to 3 volts. Let the grid-bias voltage be — 5; then, provided the dynamic-characteristic be reasonably linear over the range — 4 to — 2 volts, there will be linear detection. It is to be noted that with this type of detection the application of the input voltage causes an *increase* of anode-current, whereas in grid-circuit detection a *decrease* of anode-current is the result.

While this simple treatment illustrates the principle of the detector, however, it is not valid in practice, since in practical circuits a condenser is usually connected across the load. The value of the load

* This same result might have been obtained more simply, but less rigorously, by assuming the equation of the dynamic-characteristic to be simply

$$i_a = a + bv_g + cv_g^2$$

Substituting for v_g, we may obtain an expression, similar to that obtained above, for the change of anode-current caused by the application of the input voltage.

impedance is thus very different at carrier and modulation frequencies, and any treatment based upon the dynamic-characteristic is invalid. The purpose of this condenser is to by-pass high-frequency currents from the load. This device (or the alternative method of using a low-pass filter) ensures that the output voltage of the detector is free from carrier-frequency components, which might overload the succeeding stage. The reactance of this condenser at the carrier frequency must clearly be small with respect to the load impedance, but at the highest modulation frequency the reactance must be large with respect to the load impedance; where the two frequencies are not widely different a compromise is of course necessary.

The design and analysis of the performance of a given circuit are best based on the Rectification Diagram, a set of curves similar to the so-called Dynamic-characteristics of a diode. We are interested in the variation of the mean value, i_{mean}, of the anode-current with amplitude of the input voltage. Now i_{mean} is a function both of the peak value, $\hat{V}_1$, of the input voltage, and of the anode-voltage, v_a. Since the condenser already mentioned by-passes the carrier-frequency component of the anode-current, only i_{mean} flows through the load. Thus, if the load is a resistance R, v_a is related to i_{mean} by the equation

$$v_a = E_a - Ri_{mean} \quad . \quad . \quad . \quad (\text{VI.7})$$

The rectification diagram thus consists of a set of curves giving the variation of i_{mean} with v_a for various values of $\hat{V}_1$, upon which is superimposed a straight line having the above equation. The curves may be obtained experimentally using the same circuit as is used to obtain the ordinary static characteristics but with the addition of a measured variable alternating voltage in the grid circuit. The value of the grid-bias voltage used throughout is conveniently made equal to the value to be used in the detector circuit. Typical curves are shown in Fig. 136, each curve being for a different value of $\hat{V}_1$. The right-hand curve, corresponding to $\hat{V}_1$ equal to zero (i.e. no alternating grid-voltage), is clearly the same as the static characteristic for the value of grid-bias voltage chosen. If the grid-bias voltage chosen is the cut-off value corresponding to an h.t. voltage, E_a, then the straight line whose equation is (VI.7) will intersect the horizontal axis at the same point as does the right-hand curve. If the chosen value of grid-bias voltage is smaller than the cut-off value corresponding to E_a, the line will cross the horizontal axis at

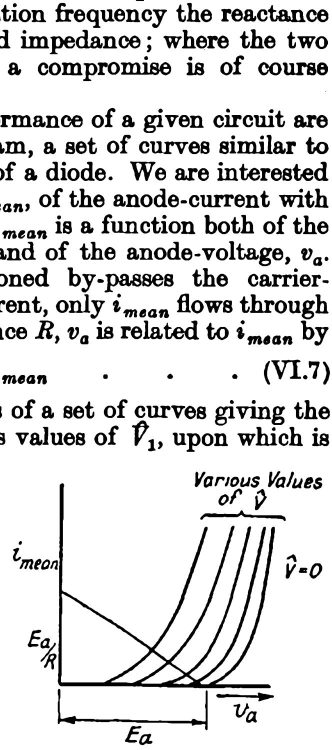


FIG. 136. RECTIFICATION CHARACTERISTICS OF ANODE-BEND DETECTOR

a point to the right of the right-hand curve, the ordinate of the point of intersection of the straight line and the right-hand curve giving the steady anode-current which will flow in the absence of any alternating input voltage.

The diagram is very similar to the load-line diagram for an amplifier stage, the differences being that ordinates in the rectification diagram are the mean values of the anode-current instead of the instantaneous values of the whole anode-current, and that each curve is drawn for a given *peak* value of the input voltage instead of for a given *instantaneous* value of it. The use of the two diagrams is similar. The load-line diagram tells us the instantaneous value of anode-current corresponding to a given instantaneous value of grid-voltage, whence knowing the waveform (i.e. the sequence of instantaneous values) of the grid-voltage, we may plot the waveform of the anode-current: the rectification diagram tells us the value of the mean anode-current corresponding to a given amplitude of input voltage, whence, knowing the variation of amplitude of the input voltage (i.e. the modulation envelope), we may plot the waveform of the mean anode-current. This is, of course, the useful modulation-frequency output of the detector. From this construction, therefore, we can determine the waveform and amplitude of the output voltage resulting from any given modulated input voltage.

With a carrier amplitude of $\hat{V}_c$, and a modulation depth m, $\hat{V}_1$ will vary between $(1 + m)\hat{V}_c$ and $(1 - m)\hat{V}_c$. If the points of intersection of the straight line with the curves are evenly spaced within this range, there will be distortionless detection. It will be seen that at the lower end of the line the points of intersection are crowded together; thus for small input voltages or for fully modulated input voltages there cannot be distortionless detection. The smaller the slope of the straight line the greater will be the output voltage for a given input, but the greater also will be the distortion, since the line will then intersect the less evenly spaced parts of the curves. The slope of the line is the reciprocal of the load resistance, so that it follows that the choice of load resistance must be a compromise between high output voltage and high distortion. Careful choice of load and operating voltages can reduce the distortion to a comparatively small percentage except for small input voltages.

The maximum permissible input voltage is decided by the consideration that grid-current shall not flow, i.e. that the total grid-voltage shall not be allowed to become positive at any part of the cycle. This consideration is imposed both in order to avoid distortion (as in the case of an amplifier) and also because the flow of grid-current would reduce the high input impedance which is a valuable feature of the anode-circuit detector. In the case of an input voltage having a carrier amplitude of $\hat{V}_c$ and modulated to a depth of 100 per cent, the maximum positive value of the input voltage is $2\hat{V}_c$, and this must not be sufficient to make the grid positive, i.e. the grid-bias voltage must equal $2\hat{V}_c$. The maximum permissible carrier amplitude, assuming 100 per cent modulation depth, is thus one-half of the maximum allowable grid-bias voltage, i.e. one-half of the cut-off grid-bias voltage corresponding to the maximum allowable h.t. voltage.

Comparing the rectification diagram with the load-line diagram, we see the possibility of forming an equivalent anode circuit for this type of detector. This may be shown to consist of a generator of internal resistance

$$\partial v_a/\partial i_{mean}$$

and having an e.m.f. equal to

$$m\hat{V}_c\frac{\partial v_a}{\partial \hat{V}_1}$$

We may summarize the properties of the anode-circuit detector by saying that it can handle an input voltage greater than that of the grid-circuit detector but not so great as that of the diode detector; it has a high input impedance; but it distorts with small input voltages.

Anode-circuit detection may take place in an overloaded grid-circuit detector, for anode-circuit detection is nothing more than a way of finding a use for the d.c. component of an amplifier's distortion. We have observed that in grid-circuit detection there is a decrease of anode-current, whereas in anode-circuit detection there is an increase. Thus the occurrence of anode-circuit detection in a grid-circuit detector disturbs the relationship between the amplitude of the input voltage and the mean anode-current. Instead of the mean anode-current decreasing continuously with the amplitude of the input voltage, a point will be reached at which further increase of amplitude will cause an increase of mean anode-current. This means that the overloading of a grid-circuit detector results in severe distortion.

Valve Voltmeters

Valve voltmeters for measuring alternating voltages are essentially detectors followed by d.c. measuring instruments. For example, the circuit of Fig. 127 (with the voltage to be measured taking the place of v_1) gives a voltage across R which is a measure of (and approximately equal to the peak value of) the input voltage. Hence the direct current through R is proportional to the input voltage, and by including a d.c. micro-ammeter in series with R, this circuit may be used as a valve voltmeter. Similarly the circuit of Fig. 134, with the voltage to be measured taking the place of the parallel L–C circuit shown, may be used as a valve voltmeter by including a d.c. meter to measure the anode-current. An anode-circuit detector may be similarly used.

In telecommunications practice it is commonly necessary to measure the potential difference across a circuit of high impedance. For this purpose a voltmeter of even higher impedance is necessary, for otherwise the application of the voltmeter to the circuit modifies the circuit by shunting the voltmeter impedance across the circuit impedance. Ideally, therefore, the voltmeter should itself have infinite impedance. Valve voltmeters provide the only practicable "infinite impedance" voltmeters (electrostatic voltmeters being unsuitable for low voltages or for high frequencies). The input impedance of the diode (Fig. 127) and cumulative-grid (Fig. 134) types of valve voltmeter is limited by the fact that their operation depends on the passage of current around the input circuit. Moreover, the *range* of the cumulative-grid type is seriously limited (see page 228). As a result, the most commonly used type of valve voltmeter is that using an anode-circuit detector (sometimes referred to as the anode-bend type).

The input impedance of the anode-circuit detector, if grid-current be avoided, is limited only by the grid-cathode capacitance and leakage conductance, and may be made very large by careful design. The necessary condition for the avoidance of grid-current is that the peak input voltage shall not exceed the grid-bias voltage. If the latter be adjusted to the cut-off value, then the range of measurable input voltages is limited to peak values less than or equal to the value of the cut-off grid-bias. Clearly the range may be extended by increasing the high tension voltage, since this increases the value of the cut-off grid-bias voltage. Alternatively, the range may be extended by using a grid-bias voltage greater than the cut-off value, but this has the disadvantage that the circuit will not respond to input voltages whose positive peaks are not sufficient to raise the grid potential above the cut-off value. A commonly used method of increasing the range is to use an automatic grid-bias circuit (R and C in parallel in the cathode lead) with suitably chosen bias resistance. This means that grid-bias voltage will be less than the cut-off value for small input voltages, but will increase as the rectified component of the anode-current increases, attaining values larger than the cut-off value for large input voltages. As the input voltage increases, the grid-bias voltage automatically increases. Different values of bias-resistance may be used to provide different voltage ranges.

Since the anode-current corresponding to zero input voltage will not be zero with such an arrangement, it is convenient to include a "backing-off circuit"—i.e. a circuit consisting of a small d.c. voltage in series with a variable resistance, which is connected directly across the milliammeter for the purpose of passing just sufficient current through the milliammeter in the opposite direction. This series resistance is adjusted until the meter reads zero for zero input voltage; this adjustment usually differs for each range-setting.

The most common requirement is the measurement of sinusoidal alternating voltages, and for this reason it is usual to calibrate valve voltmeters by using known sinusoidal alternating voltages. When the waveform of the voltage to be measured is non-sinusoidal this calibration cannot be relied upon. But it is interesting to note that, since the diode valve voltmeter mentioned above produces a current through the micro-ammeter which is proportional to the positive peak of the voltage being measured, it can be relied upon to measure the positive peak accurately (using the r.m.s. sinusoidal calibration and multiplying by $\sqrt{2}$) whatever the waveform of the input voltage. By reversing the voltmeter connexions the other peak may be measured in a similar way.

Further, the anode-circuit detector type of valve voltmeter, when using a grid-bias of the cut-off value, produces an anode-current which is roughly proportional to the *average* value of the positive half-wave of the voltage being measured, and can therefore be used to measure this quantity approximately, using the r.m.s. sinusoidal calibration and multiplying by $(2\sqrt{2})/\pi$, whatever the waveform of the input voltage. Similarly any valve voltmeter giving a d.c. output proportional to the mean of the square of the input voltage can be used to measure r.m.s. values, using the sinusoidal calibration, whatever the waveform of the measured voltage. (See Square-law Detection, page 230.)

Automatic Gain Control

If the voltage amplification of an amplifier be continuously regulated so that it is inversely proportional to the amplitude of the alternating input voltage, the amplitude of the output voltage will be constant. For instance, if

$$(VA) = \kappa/V_1$$

then the output voltage, V_2, will be given by

$$V_2 = V_1(VA) = \kappa$$

Such amplifiers have important applications, the most important being perhaps in radio reception, where it is convenient to have a constant output voltage from the radio-frequency amplifier (and thus also a constant input voltage to the detector stage) in order to reduce variation of the output of the receiver with varying signal-strength caused by fading. The control of the amplification may be made automatic, in which case, however, the device usually does not succeed in giving an absolutely constant output voltage, but considerably reduces the rate of change of output voltage with input voltage.

Automatic gain control circuits operate by applying the output voltage of the amplifier to a detector, and using the d.c. component of the detector output voltage (i.e. the first term in equation (VI.2)) as grid-bias voltage for a variable-mu valve (see Chapter III) which constitutes one stage of the amplifier. The larger the output of the amplifier, the larger the grid-bias voltage of the variable-mu valve, and thus the smaller the amplification.

If, as in a radio receiver, the output of the amplifier is normally fed to a detector, the d.c. component of the output voltage of this

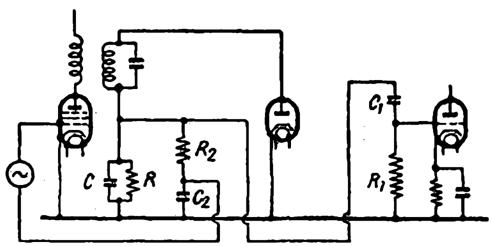


FIG. 137. AUTOMATIC GAIN CONTROL. SINGLE DIODE FOR DETECTION AND A.G.C.

same detector may be used to furnish the grid-bias voltage of the variable-mu valve. To illustrate this, Fig. 137 shows a single-stage amplifier, using a variable-mu pentode coupled to a diode detector. (Connexions irrelevant to the argument are not shown.) The diode

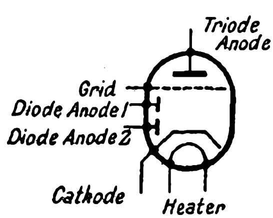


FIG. 138. DOUBLE-DIODE TRIODE

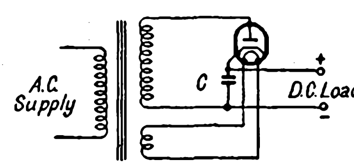


FIG. 139. HALF-WAVE RECTIFIER

detector now has two output circuits: one, R_1C_1, is as already described and serves to feed the modulation-frequency output voltage to an amplifier valve; the other, R_2C_2, forms an a.c./d.c. separator across the detector R–C circuit, the d.c. component of the detector output voltage appearing across C_2, which is included in the grid circuit of the variable-mu valve.

A separate detector is often used for automatic gain control purposes—even where the amplifier output would normally be fed to a detector. In the double-diode-triode valve, Fig. 137, two diodes having a common cathode are included in the same evacuated glass bulb as a triode (which also uses the same cathode). One of these diodes may be used for automatic gain control, and the other used normally for detection. The triode part of the valve may then be used to amplify the modulation frequency output voltage of the second diode, by connecting this output voltage into the grid circuit of the triode.

Diode Rectifier Systems

By a rectifier we mean an apparatus fed from an a.c. supply and designed to provide a d.c. supply. The simplest possible rectifier system consists merely of the series connexion of an a.c. supply, a diode (or other one-way conductor) and the load. Such a system would be satisfactory only for a limited number of simple applications, such as battery charging. The output current through the load would be unidirectional, as required, but it would be very far from constant. It would in fact consist of alternate half-sine waves. The output current may be made more nearly constant by the following successive steps—

(*a*) Either by connecting a condenser, known as a Reservoir Condenser, across the load or by connecting a choke-coil in series with the load.

(*b*) By making a push-pull version of the circuit, known as a Full-wave rectifier.

(*c*) By adding a low-pass filter network, known as a Smoothing Circuit.

First let us consider the addition of a reservoir condenser as in (*a*) above.

Reservoir-Condenser Rectifiers

In principle the circuit is exactly the same as the diode detector circuit of Fig. 127, page 217, but the practical circuit of Fig. 139 shows also how a transformer may be used, both in order to supply the heater of the diode and in order to step-up or step-down the voltage of the a.c. mains, so that any desired value of d.c. output voltage may be secured. If a directly heated diode be used (i.e. one in which the heater itself functions as cathode) the cathode connexion may be taken from a centre-tap on the heater winding of the mains transformer.

The load constitutes a resistance R connected directly across the reservoir condenser, C. The behaviour of the circuit is thus as explained for the diode detector of Fig. 127 and the graphs of condenser voltage and diode current are again as in Fig. 128, page 217.

Since the d.c. load is connected across the condenser C, the load voltage is the same as the condenser voltage, and its mean value will be slightly less than the peak value of the transformer secondary voltage. We see from Fig. 128 that this voltage, though unidirectional, is not steady, but has a ripple superimposed upon its d.c. component. Since the object of the rectifier is to provide a d.c.

supply, this is undesirable. We know from our previous consideration of the diode detector that the ripple may be made small by making the product RC large. The value of R is fixed by the nature of the load; it therefore seems that the capacitance C should be made large. An upper limit to this capacitance is set, however, by the fact that as C is increased, other things remaining equal, the value of the peak current through the diode increases, and this peak current must not be allowed to exceed the maximum safe value specified by the manufacturers of the valve. The relationship between C and the peak diode current may be seen as follows. The diode current (lowest graph, Fig. 128) consists of a number of pulses. This current may be resolved into a d.c. component of value equal to the mean of the pulses, together with a number of harmonically related a.c. components. Now the whole of this current must pass through the circuit consisting of R and C in parallel. The entire d.c. component will pass through R, since d.c. cannot flow continuously through a condenser. This d.c. component constitutes the load current. As C is increased two things will happen: the mean condenser voltage will increase slightly, and since this is the load voltage, the d.c. load current will increase slightly; secondly, the duration of the pulses of diode current will decrease. Thus increase of C means that the duration of the pulses decreases, whereas the mean value of the pulses increases slightly. It therefore follows that the peak value of the pulses will increase, and to reduce the ripple voltage to a small percentage of the d.c. component by this means would involve peak currents some twenty or thirty times as great as the load current.

The ripple voltage is usually reduced to a sufficiently small value by interposing a low-pass filter between the reservoir condenser C and the load. A single-stage filter of a type commonly used is shown in Fig. 140. The cut-off frequency of the filter is, of course, made lower than the fundamental frequency of the ripple voltage. This usually involves an inductance of several henrys and a capacitance of several microfarads. The resistance of the choke should be low in order to avoid poor "Regulation." The filtering-out of the ripple is known as "Smoothing."

The rectifier shown in Fig. 139 makes no use of the negative half-waves of the applied alternating voltage, and is known as a Half-wave Rectifier. During the negative half-wave the two voltages in series with the diode (the applied voltage and the voltage of the condenser) are both in such a direction as to prevent the flow of diode current. If the condenser voltage is approximately equal to the peak value of the applied voltage (as it will be when the load current is switched off) then the maximum reverse voltage applied to the diode is equal to twice the peak value of the applied alternating voltage. This must be borne in mind in selecting a valve for a given rectifier.

Full-wave Rectifiers

The half-wave rectifier is little used, since the full-wave rectifier (Fig. 141) gives a smaller ripple voltage and a higher ripple frequency (thus making filtering easier) and also eliminates d.c. magnetization of the mains transformer. The full-wave rectifier is a push-pull version of the half-wave rectifier, i.e. it consists of a half-wave rectifier to which has been added a second a.c. supply voltage (by the device of centre-tapping the secondary winding of the mains transformer) equal to, but 180° out of phase with, the first, and connected to the *RC* circuit through a diode which can conduct during the half-cycle when the first diode is non-conducting. During one half-cycle charging current flows to the condenser *C*, round the circuit *ABCD*, and during the next half-cycle round the circuit *EBCD*, the two currents charging the condenser in the same direction. Since the condenser cannot now discharge for longer than half a cycle, the ripple frequency is *twice* the mains frequency; also the condenser cannot discharge so far, and thus the ripple voltage is smaller. The whole of the current

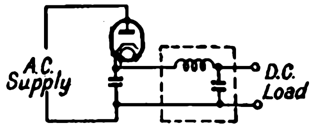


FIG. 140. SMOOTHING CIRCUIT (WITHIN THE RECTANGLE)

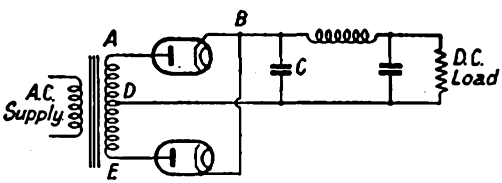


FIG. 141. FULL-WAVE RECTIFIER

through one of the diodes flows through one half of the transformer secondary, and the whole of the current through the other diode flows in the opposite direction through the other half of the secondary. Thus, there can be no d.c. magnetization of the core. In Fig. 141 a smoothing filter is shown connecting the reservoir condenser to the load.

Rectifiers using a reservoir condenser are much used for providing an h.t. supply for valve circuits. Their two chief disadvantages are that they have poor Regulation (i.e. that their output voltage falls if more d.c. output current is demanded) and that they require diodes whose current-rating is several times larger than the d.c. output of the rectifier. As indicated in paragraph (*a*) on page 237, the alternative to a Reservoir Condenser Rectifier is a Series Choke Rectifier. In the next section we shall see how the mere connexion of a choke-coil in series with the load can change the current from a series of half-sine waves to a nearly constant D.C.

Series-Choke Rectifiers

If a unidirectional but pulsating e.m.f. be connected across a resistance load, the current through the load will, of course, have the same waveform as the applied e.m.f. It may be considered to be the sum of a d.c. component and an a.c. component. By the inclusion of a choke-coil (an iron-cored coil with high self-inductance and very low resistance) in series with the load, we greatly increase the impedance of the circuit to A.C. without appreciably altering the resistance of the circuit to D.C. *We conclude that the connexion of a large enough inductance in series with a resistance load and a specified pulsating e.m.f. will give us a current which is nearly constant D.C.*

Care is required, however, in applying this conclusion to diode-rectifier circuits. To illustrate the pitfalls let us consider the half-wave rectifier formed by connecting in series a sine-wave a.c. supply, a diode, a load resistance and a choke. Can we conclude that, if the inductance of the choke is great enough, a nearly constant D.C. will flow? We certainly cannot! If this were so, we would be faced with the paradox that the diode would be conducting throughout both half-cycles. This would mean that the applied a.c. e.m.f. would be connected to the load and the choke throughout both half-cycles and that the current flowing through the load would be A.C. The conclusion that a nearly constant D.C. would flow is therefore false.

The reason for this false conclusion is that the general principle, stated in italics above, is applicable only where there is a *specified pulsating e.m.f.* The combination of an a.c. supply and a diode cannot be regarded as a *specified* e.m.f., for the e.m.f. which it provides is a function of the circuit to which it is connected. The question of whether or not the diode is conducting at any given instant of the cycle is decided by the polarity of the anode-voltage at that instant—and the anode-voltage depends not only on the a.c. supply but also on the self-induced e.m.f. in the choke-coil.

Although the half-wave series-choke rectifier cannot be considered to provide "a specified e.m.f.," the *full-wave* series-choke rectifier (Fig. 142) *can* be so considered. It is clear from the symmetry of the circuit of Fig. 142 that the two diodes will conduct for equal times, for what one diode does in a particular half-cycle will be repeated by the other diode in the next half-cycle. It is also clear that the two diodes cannot both be conducting at any one instant, for this would give a short-circuit across the transformer secondary and would lead to the conclusion that one diode was conducting in the prohibited direction, viz. from cathode to anode. Finally, it is clear that there can be no finite period of the cycle when both diodes are non-conducting, for during such a period there would be no current through the choke and no self-induced e.m.f., whence it would follow that one of the diodes would have a positive anode-voltage and would conduct. We thus conclude that each diode conducts for exactly half the total time. This means that each diode "switches-on" at the beginning of the half-cycle which makes its anode positive and "switches-off" at the end of

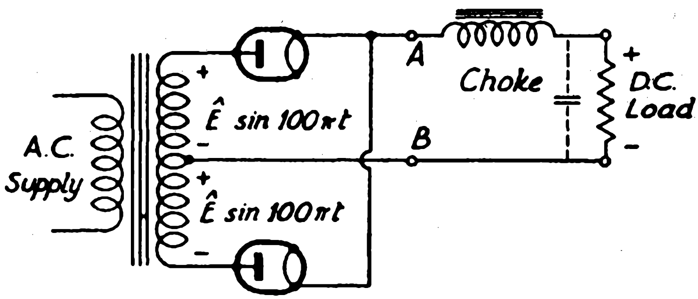


FIG. 142. SERIES-CHOKE FULL-WAVE RECTIFIER

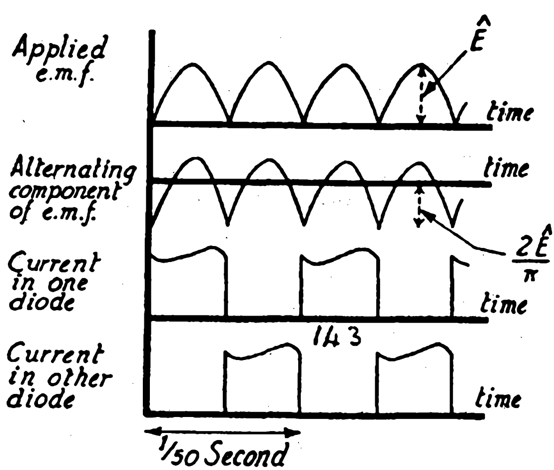


FIG. 143

this half-cycle. The p.d. between the points A and B in the circuit of Fig. 142 must therefore be as shown in Fig. 143 and the circuit may be considered as supplying this specified pulsating e.m.f.

The principle stated in italics on page 240 may therefore be applied to the full-wave series-choke rectifier. The applied e.m.f. (Fig. 143) is the sum of a d.c. component and an a.c. component. The magnitude of the d.c. component, being the mean height of the graph, is given by $2\hat{E}/\pi$ where $\hat{E}$ is the peak value of the alternating voltage across each half of the transformer secondary. The d.c. component of the current through the load will be

$$(2\hat{E}/\pi)/(R_{load} + R_{choke} + R_{transformer})$$

and the d.c. output voltage of the rectifier will be R_{load} times this current. This will be approximately $2\hat{E}/\pi$ since the resistance of the choke and transformer will usually be much smaller than the load resistance. Note that a reservoir-condenser rectifier, using the same transformer, would give a higher d.c. output voltage.

The a.c. component of the applied e.m.f. will be as shown in Fig. 143. Its frequency is twice the frequency of the a.c. mains (100 c/s for a mains frequency of 50 c/s). Its waveform is non-sinusoidal and it can therefore be resolved into fundamental and harmonic sine-wave components. It can be shown by Fourier analysis that the amplitudes of the fundamental and harmonics are as follows—

Order of harmonic .	Fund'l	2nd	3rd	4th	*n*th
Frequency . . .	100	200	300	400	$100n$
Amplitude . . .	$\frac{4\hat{E}}{3\pi}$	$\frac{4\hat{E}}{15\pi}$	$\frac{4\hat{E}}{35\pi}$	$\frac{4\hat{E}}{63\pi}$	$\frac{4\hat{E}}{(4n^2-1)\pi}$
Choke reactance . .	$200\pi L$	$400\pi L$	$600\pi L$	$800\pi L$	$200n\pi L$

Since the amplitude of the largest harmonic (i.e. the second) is only 20 per cent of the amplitude of the fundamental, we can make an approximate estimate of the resulting *current* amplitude by considering the fundamental only. (The error in this estimate will be much less than 20 per cent, both because the reactance of the choke at the second harmonic frequency is twice the reactance presented to the fundamental and because there will be a phase difference between the fundamental and second harmonic components.)

Neglecting the small resistance of the choke-coil and the small leakage-reactance of the transformer secondary winding, we may write the peak value of the fundamental (100 c/s) component of the ripple current as

$$\hat{I}_{ripple} = \frac{4\hat{E}/3\pi}{\sqrt{R^2_{load} + (200\pi L)^2}}$$

and the direct current as

$$2\hat{E}/(\pi R_{load})$$

The ratio of these two is therefore

$$\frac{\hat{I}_{ripple}}{I_{DC}} = \frac{(\frac{2}{3})\,R_{load}}{\sqrt{R_{load}^2 + (200\pi L)^2}} = \frac{\frac{2}{3}}{\sqrt{1 + \left(\frac{200\pi L}{R_{load}}\right)^2}}$$

This same expression gives the peak ripple voltage as a fraction of the d.c. voltage. If the peak value of the ripple is required to be a very small fraction, $1/n$, of the D.C., then clearly $200\pi L/R_{load}$ must be large with respect to unity and we have the following approximate relation—

$$\frac{1}{n} = \frac{\frac{2}{3}}{200\pi L/R_{load}}$$

Hence the value of choke self-inductance, L, necessary to reduce the peak value of the ripple to a small fraction $1/n$ of the D.C., is given approximately by

$$L = nR_{load}/300\pi \quad . \quad . \quad . \quad \text{(VI.8)}$$

A much smaller value of inductance will suffice to reduce the ripple voltage to the same small amplitude if a suitable capacitance, C, be added in parallel with the load resistance, i.e. directly across the d.c. output terminals of the rectifier as shown dotted in Fig. 142. Without such a capacitance, we may say that the applied e.m.f. of Fig. 143 is connected across a voltage-divider whose two elements are the choke and the load resistance. The ripple e.m.f. (second graph in Fig. 143) is shared by these two circuit elements, the element with the larger impedance getting the larger share of ripple voltage. By connecting a capacitance across the load resistance we can greatly lower the impedance of this element of the voltage-divider and so greatly reduce its share of the ripple voltage.

If this capacitance is made large enough for its reactance at the fundamental frequency of the ripple (100 c/s) to be much smaller than the load resistance, then the impedance of the load and capacitance in parallel may be taken as that of the capacitance alone. The peak value of the fundamental component of the ripple voltage across the load is therefore a fraction $(1/\omega C)/(\omega L - 1/\omega C)$ of the fundamental component, $4\hat{E}/3\pi$, of the applied e.m.f. We have already assumed, however, that the reactance $1/\omega C$ is much smaller than R_{load}, and we know that the choke reactance ωL is larger than R_{load}—so we may reasonably write $(\omega L - 1/\omega C)$ as merely ωL. This gives

$$\hat{V}_{ripple} = \frac{4\hat{E}/3\pi}{\omega^2 LC}, \quad \therefore \frac{\hat{V}_{ripple}}{V_{DC}} = \frac{2}{3\omega^2 LC}$$

in which ω is 2π times the ripple frequency, i.e. 2π times *twice* the mains frequency. For 50 c/s mains,

$$\frac{\hat{V}_{ripple}}{V_{DC}} = \frac{1{\cdot}7}{\text{(Inductance in henrys) (Capacitance in microfarads)}} \quad . \quad . \quad . \quad \text{(VI.9)}$$

The discerning reader may have observed that there is a serious gap in the above argument. We have carefully justified the application of the principle printed in italics on page 240 to the full-wave series-choke rectifier. We then proceeded to add a shunt capacitance without checking that the principle would thereafter still remain applicable. Let us belatedly remedy this omission. The argument that the two diodes must conduct for equal periods remains valid. The argument that both diodes cannot conduct at the same time remains valid. But, in view of the p.d. across the capacitance, it is no longer possible to argue that there can be no period during the cycle when both diodes are non-conducting. With the capacitance in circuit, therefore, we may merely conclude that each diode conducts for half the total time *or for less than half*. Experiment shows that for values of load resistance greater than a certain critical value, each diode conducts for less than half of each cycle, whereas for values of load resistance less than this critical value, each diode conducts for exactly half of the cycle. In this latter regime, therefore, we may continue to apply the above method of analysis. Fortunately it is always in this latter regime that one chooses to use the rectifier, because the Regulation is poor for values of load resistance greater than the critical value, i.e. for load currents smaller than a certain value.

We may determine this critical value by examining the impedance presented to the fundamental component. Assuming, as before, that the reactance of the choke is much larger than the reactance of the condenser, and than the choke resistance, and than the transformer leakage reactance, we see that the impedance presented by the whole circuit to the ripple frequency component of the applied e.m.f. is merely ωL, and that the d.c. resistance presented by the whole circuit to this e.m.f. is merely R_{load}. Thus the d.c. and fundamental a.c. components of the current through the choke are as follows—

$$I_{DC} = \frac{2\hat{E}/\pi}{R_{load}} \quad \text{and} \quad \hat{I}_{ripple} = \frac{4\hat{E}/3\pi}{\omega L}$$

It is interesting to note that the addition of the condenser causes the ripple current *through the choke* to increase, though it reduces the ripple voltage across the load. Clearly by increasing the value of R_{load} sufficiently, it would be possible to reduce I_{DC} until it was no greater than $\hat{I}_{ripple}$. The sum of the d.c. component and the a.c. component of the choke current would then fall to zero at one instant of the cycle. What would happen if the load resistance were still further increased, and the d.c. output current were thereby still further decreased? The answer given by an analysis of the kind we have used will be false, for that answer would be that the a.c. component of current would now exceed the d.c. component and the current through the choke would, for a short part of the cycle, flow in the opposite direction. Since neither of the diodes feeding the choke can carry reverse current, this is impossible. What would happen in fact is that the diodes would cease to conduct for a short part of the cycle. Since our method of analysis is based on the assumption of an applied e.m.f. as shown in Fig. 143, it is not surprising that it cannot be applied to this regime.

The critical value of load resistance is that value which makes I_{DC} equal to $\hat{I}_{ripple}$. This value is thus given by

$$\frac{4\hat{E}/3\pi}{\omega L} = \frac{2\hat{E}/\pi}{R_{load}}$$

in which ω is 2π times the fundamental frequency of the ripple. For a 50 c/s supply, ω is 200π and we have

$$R_{critical} = 1{\cdot}5\omega L = 943L \qquad . \quad . \text{ (VI.10)}$$

If the load-resistance were disconnected, or open-circuited, there would be no d.c. path and, since the diodes can conduct only in one direction, there could be no steady-state alternating current. What would happen in this case is that the condenser would charge to a p.d. $\hat{E}$, the peak value of the alternating voltage across one-half of the transformer secondary, and that thereafter no current would flow. In other words, the open-circuit output voltage of the rectifier is $\hat{E}$, the same value as for a reservoir-condenser rectifier. If now a high resistance load were connected to the output terminals, the circuit would behave very much in the same way as does a reservoir-condenser rectifier.

The diodes would conduct in short pulses and, in the intervals between these pulses, the load current would be supplied by the condenser. Since this constitutes a periodic discharge of the condenser, the d.c. output voltage would fall. If more current be drawn from the rectifier (by the connexion of a lower load-resistance to the output terminals) the d.c. output voltage will fall further and the width of the current pulses will increase. When the load-resistance is reduced to the critical value given in equation (VI.9) the d.c. output voltage will have fallen to $2\hat{E}/\pi$ and the width of the current pulses will have increased to half a cycle. Further reduction of the load resistance (i.e. further increase of d.c. output current) will not cause a further widening of the diode current pulses nor will it lower the output voltage (except by a small amount depending on the resistance of the choke and the transformer winding and the small effective resistance of the conducting diodes). For this value and higher values of output current, the regulation of this rectifier is good and it is in this regime that the series-choke rectifier with smoothing condenser is used. In order to ensure that the current shall never fall below the critical value, a resistance having the value given in (VI.10) (or a slightly lower value) is permanently connected across the output terminals, as part of the rectifier. This is known as a "bleeder" resistance.

Each diode of a full-wave series-choke rectifier conducts for half a cycle during each cycle of mains voltage. If there were no ripple current, the current-waveform for each diode would consist of a rectangular pulse, the height of whose flat top would be equal to the d.c. load current. The addition of a ripple component changes the shape of the top of the pulse, somewhat as shown in the lower graphs of Fig. 143, but the maximum height is still not very different from the d.c. load current. It follows that for the same load current, a series-choke rectifier can use valves with a smaller current-rating than could a reservoir-condenser rectifier.

Bridge Rectifiers

Fig. 144 shows another full-wave rectifier circuit, known as the Bridge-connected rectifier. (Arrowheads represent the diodes.) During one half-cycle of the a.c. supply, current flows through the load along the zig-zag path *PBAQ*, and during the other half-cycle along the path *QBAP*, flowing through the load in the same direction in both half-cycles. As in Fig. 141, the ripple voltage is smaller than in a corresponding half-wave rectifier, and the ripple frequency is twice the supply frequency. There is no d.c. magnetization of the mains transformer, a centre-tapped secondary winding is not required, and (for the same overall secondary voltage of the transformer) this circuit gives almost twice as great a d.c. load voltage

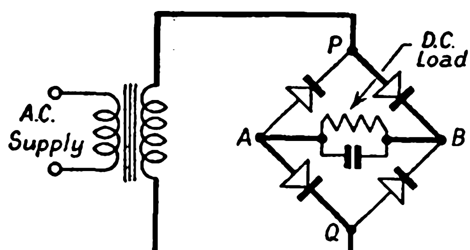


FIG. 144. BRIDGE RECTIFIER

as does Fig. 141. The obvious disadvantage is the added expense of four diodes, each with its separate heater supply, but the circuit is often used with metal rectifiers substituted for these diodes. A.C. measuring instruments of the "rectifier type" often use this circuit, in which case a d.c. instrument is connected in place of the load, and *P* and *Q* become the terminals of the a.c. instrument.

Voltage-multiplying Rectifier Systems

Fig. 145 shows a voltage-doubling rectifier system. It consists of two parallel circuits, each consisting of a condenser in series with a diode, connected across the a.c. supply voltage (or, in this case, the transformer secondary). The diodes are in opposite directions, so that one conducts during one half-cycle and the other during the other half-cycle, and the condensers are charged in opposite directions (as shown in Fig. 145). The load is then connected across the two condensers in series, the load voltage being twice the condenser voltage. Since the condensers are charged alternately during successive half-cycles it follows that the ripple frequency is again twice the mains frequency.

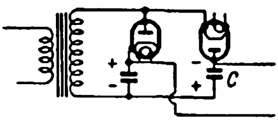


FIG. 145. VOLTAGE-DOUBLING RECTIFIER

The voltage-multiplying rectifier system shown in Fig. 146 has been used to provide a d.c. voltage of half a million for the operation of an X-ray tube (in which case smoothing is not important), the peak value of the transformer secondary voltage being 125 KV. The operation of apparently complicated rectifier systems such as this may be understood by considering each closed circuit separately, beginning with the simplest closed circuit which includes the applied alternating voltage, and using *the rule that in a closed circuit consisting of a diode, condenser, and varying applied voltage, the condenser will charge to a voltage equal to the maximum value of the voltage applied in the direction in which the diode can conduct.* This very useful rule (which of course breaks down if the diodes do not conduct at all, or if the time-constants are too small) is sometimes extended to diode circuits in which no condensers are present, by expressing it as follows: "*The anode-voltage of each diode reaches the value zero but never becomes positive.*"

For example, in the circuit of Fig. 146 consider first the closed circuit TC_1V_1. The diode V_1 can conduct in an anti-clockwise direction round this circuit, and the maximum applied voltage in such a direction is simply the peak value, $\hat{V}$, of the transformer secondary voltage. Thus C_1 will charge to a voltage $\hat{V}$, the left-hand plate being positive as shown. Now that we know the voltage of C_1 we can regard T and C_1 together as an applied voltage, and can apply our rule to the closed circuit $C_1TC_2V_2$. The diode V_2 can conduct in a clockwise direction round this circuit, and the maximum applied voltage in such a direction is the peak value, $\hat{V}$, plus the voltage of the condenser C_1, i.e. the maximum applied voltage in the conducting direction is $2\hat{V}$. The condenser C_2 will thus charge to a voltage $2\hat{V}$, the left-hand plate being positive. We can now consider C_1, T, and C_2 together as our source of voltage, and treat the closed circuit $C_2TC_1C_3V_3$. The conducting direction is anti-clockwise, and the maximum voltage applied in this direction is

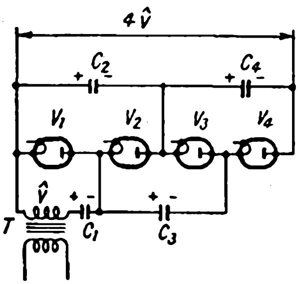


FIG. 146. VOLTAGE-QUADRUPLING RECTIFIER

$$2\hat{V} + \hat{V} - \hat{V}$$

being the sum of the voltages on C_2, T, and C_1 respectively. Thus C_2 will charge to a voltage $2\hat{V}$, the left-hand plate being positive. Finally consider the combination $C_2TC_1C_3$ as the source of voltage in the closed circuit $C_2TC_1C_3V_4C_4$. The conducting direction is clockwise, and the maximum voltage in this direction is

$$-2\hat{V} + \hat{V} + \hat{V} + 2\hat{V}$$

(being the voltages on C_2, T, C_1, and C_3 respectively). Thus C_4 will charge to a voltage $2\hat{V}$, the left-hand plate being positive. The load is connected across C_2 and C_4 in series, giving a voltage of $4\hat{V}$.

Phase-Sensitive Rectifiers

All the rectifiers considered hitherto have been such as will give a d.c. output whose polarity (i.e. direction of current flow) is quite independent of the phase of the a.c. input. If the phase of the input were reversed, the d.c. output voltage of these rectifiers would be entirely unchanged. This is, of course, a necessary requirement in rectifiers which are intended to provide an unvarying d.c. power supply. In servo-mechanisms and a.c. data-transmission systems, however, it is often necessary to use Phase-sensitive Rectifiers, i.e. rectifiers in which a phase reversal of the input voltage must cause a reversal of the polarity of the d.c. output voltage.

This requirement arises from the use of an *alternating* voltage to indicate the angular position of an output element (see page 112, line 9). As the position of the output element approaches the zero or reference position, this alternating voltage decreases in amplitude. As the output element passes through the zero position, the amplitude of the alternating voltage passes through zero, and its phase is reversed. Without this phase reversal there would be nothing to indicate the sign (or "sense") of the displacement of the output element (i.e. whether it is displaced to the left or the right). To convert this a.c. indicating voltage to a d.c. indicating voltage, a phase-sensitive rectifier (better called a "detector") is required, since an ordinary (phase-insensitive) rectifier would give the same polarity of output whatever the phase of the input, and would thus fail to give any indication of the sign of the displacement.

The indicating voltage which forms the input to a phase-sensitive rectifier has usually a low frequency, 1,000 c/s or less. The phase-sensitive rectifier incorporates a second alternating voltage of the same frequency (known as the Reference Voltage), and the input voltage is always either in phase with, or 180° out of phase with this reference voltage. The function of the phase-sensitive rectifier is, first, to rectify (i.e. to produce a d.c. output voltage whose instantaneous value shall be proportional to the amplitude of the alternating input voltage) and, second, to cause the output current to flow in one direction through the load when the input-voltage is in phase with the reference voltage, but to cause this current to flow in the opposite direction through the load when the input and reference voltages are 180° out of phase. To see how this requirement can be met, let us reconsider the simple rectifier circuit of Fig. 127, p. 217, with the condenser omitted for simplicity.

This circuit rectifies because the diode behaves as a synchronous switch, making the circuit conduct during alternate half cycles. But in this circuit there is no option as to *which* half-cycles shall be the conducting ones. The diode can conduct only during the positive half-cycles of v_1: it is, in short, a *directional* synchronous switch. What is required in a phase-sensitive rectifier is a non-directional

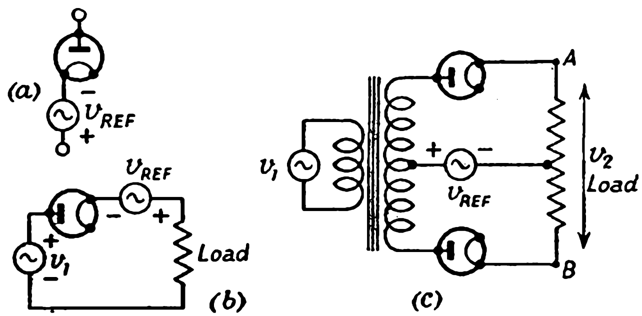


FIG. 147

synchronous switch, operated by the reference voltage, i.e. a switch which when closed can conduct in either direction, and which is closed always during the positive half-cycles of the *reference* voltage. If the positive half-cycles of the input voltage coincide with the positive half-cycles of the reference voltage, then the input voltage will send current through this switch in the positive direction; whereas if the negative half-cycles of the input coincide with the positive half-cycles of the reference voltage, the input voltage will send current through the switch in the opposite direction. In both cases rectification is achieved, since conduction occurs only during alternate half-cycles; but the direction of this conduction depends on whether the input and reference voltages are in phase or 180° out of phase.

The simplest non-directional synchronous switch, operated by the reference voltage, is simply the series connexion of the reference voltage and a diode (Fig. 147 (*a*)). To form a phase-sensitive rectifier we simply substitute this circuit for the diode of an ordinary (phase insensitive) rectifier the result being as shown in Fig. 147 (*b*). Provided that the amplitude of the reference voltage is made greater than the largest expected amplitude of input voltage, the diode will conduct only during the positive half-cycles of the reference voltage, and the input voltage will send a component of current through the load either clockwise or anticlockwise (in Fig. 147 (*b*)) according as it is in phase or in antiphase with the reference voltage. This simple circuit, however, has the serious disadvantage that the reference voltage itself also sends current through the load, in a direction which is independent of the phase of the input voltage.

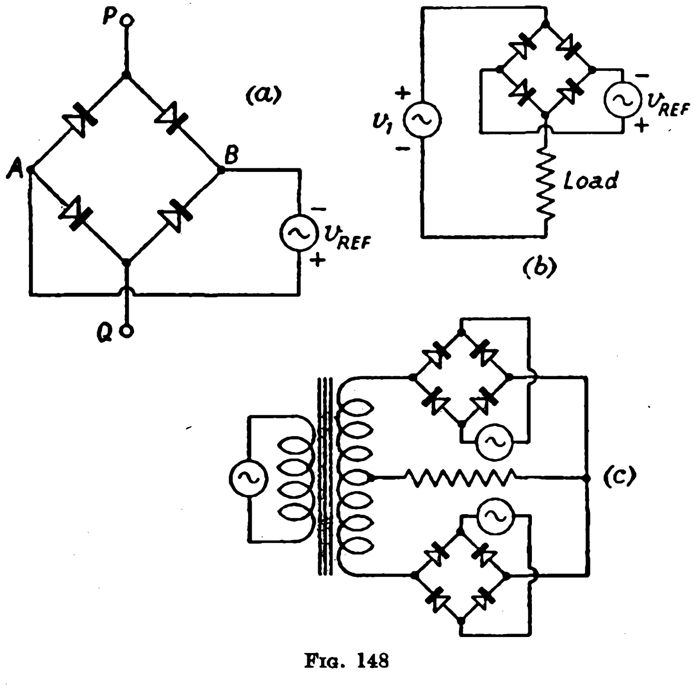


FIG. 148

This means that the total rectified current through the load will not be zero when the input voltage is zero. The difficulty can be overcome by building this type of non-directional synchronous switch into a rectifier with a centre-tapped load as shown in Fig. 147 (c). The single reference voltage in this circuit (provided that it is made large enough) prescribes the conducting periods of both diodes, independently of the phase of the input voltage. Thus the diodes conduct simultaneously but unequally, the phase of the input voltage determining which diode carries the greater current. This in turn determines the polarity of the total load voltage v_2. If the input voltage falls to zero, the diodes conduct equally and the average value of v_2 becomes zero.

Fig. 148 (*a*) shows a non-directional synchronous switch which is free from the disadvantages of the circuit shown in Fig. 147. (The diodes are again denoted by arrowhead symbols in this figure. These symbols may also be taken to represent metal rectifiers.) During the positive half-cycles of v_{ref} all four rectifier elements conduct, so that there is a conducting path between the terminals PQ, each arm conducting in the direction of the arrowhead. An external voltage applied across PQ, will now be able to send current in either direction. For instance, if the external voltage is such as to make P positive with respect to Q, it will produce a current which divides equally between the paths PAQ and PBQ, and the current due to v_{ref} will be augmented in branches AQ and PB, and diminished in branches PA and BQ. (It is important to note that the reference voltage must be made large enough to prevent the diminished currents from falling to zero, since the branches concerned would then cease to conduct, and v_{ref} would cease to be the sole arbiter of whether the network conducts or not.) As a result of the balanced bridge structure, v_{ref} produces no component of current through the external circuit connected across PQ, except in so far as the balance is disturbed by curvature of the characteristics of the rectifying elements.

We are now in a position to take as a prototype any ordinary (phase insensitive) rectifier circuit and derive from it the corresponding phase-sensitive rectifier circuit simply by substituting the circuit shown in Fig. 148 (*a*) for each diode (or rectifier element) in the prototype. Two examples of this are shown in Fig. 148 (*b*) and (*c*). The prototypes of these circuits are the ordinary half-wave and full-wave rectifiers of Fig. 140 and Fig. 141 respectively. A phase-sensitive rectifier may also be derived from the bridge-circuit rectifier of Fig. 144 (the result being a bridge each arm of which is itself a bridge). It should be noted that if the prototype incorporates more than one diode, then the derived circuit will incorporate more than one non-directional synchronous switch and thus more than one reference voltage. In such cases the phases of the various reference voltages must be made to correspond to the conduction direction of the diodes in the prototype circuit, so that any two paths which conduct simultaneously in the prototype will also conduct simultaneously in the derived circuits. Also it is clear that reservoir condensers and smoothing circuits may in general be used with phase-sensitive rectifiers just as with ordinary phase-insensitive rectifiers.

Fig. 149 shows a phase-sensitive rectifier (or detector) using triodes. It will be seen that the input circuits are arranged just as in a push-pull amplifier, and the alternating reference voltage takes the place of the h.t. supply voltage. Thus conduction occurs only during the positive half cycles of the reference voltage. The two valves conduct simultaneously but unequally. As in the circuit of Fig. 147 (*c*), the question of which valve passes the greater current is decided by the relative phase of the input and reference voltages which thus decides the polarity of the output voltage.

Frequency-modulation: Discriminators

The Discriminator plays the same role in the reception of frequency-modulated signals as the Detector plays in the reception of amplitude-modulated signals. An a.m. signal is a high-frequency alternating voltage whose *amplitude* has been made to vary in

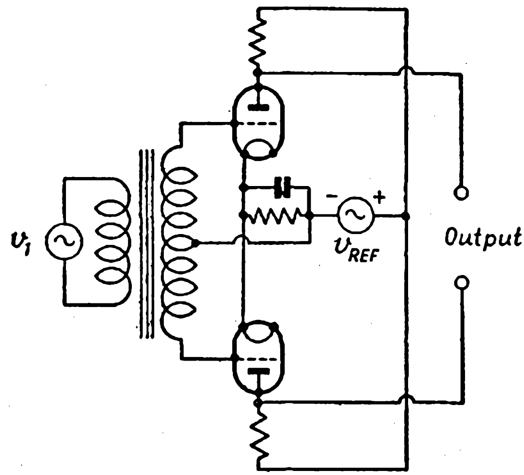


Fig. 149. Phase-sensitive Detector

accordance with the (lower frequency) information voltage. The task of the detector is thus to give an output-voltage whose instantaneous value shall vary in direct proportion with the *amplitude* of the h.f. alternating voltage. An f.m. signal is a high-frequency alternating voltage whose *frequency* has been made to vary in accordance with the (lower frequency) information voltage. The task of the discriminator is thus to give an output-voltage whose instantaneous value shall vary in direct proportion with the *frequency* of the h.f. alternating voltage.

This is usually accomplished by first feeding the f.m. signal-voltage to the input terminals of a circuit which has a response curve (i.e. a graph of gain against frequency) which is a sloping straight line. The f.m. input voltage has a constant amplitude but its frequency is varying. Since the gain of the chosen circuit is a linear function of frequency, the *amplitude* of the output voltage will now vary in precisely the same way as the frequency is varying. We thus have an amplitude-modulated h.f. output voltage (it is frequency-modulated as well, but this does not affect the argument) and can proceed to use an ordinary detector circuit to extract the information-voltage.

The response curve of a tuned-anode amplifier stage has the form of a resonance-curve similar to Fig. 53, page 86. Part of one side of this resonance curve could be regarded, very approximately, as a straight line. By tuning the circuit so that the centre-frequency (i.e. the carrier frequency) of the f.m. signal coincided with the middle of this "straight" response curve, the reception of f.m. signals would be possible. This method would be unsatisfactory, however, not only because the "straight" response curve is really far from straight, but because of the difficulty of accommodating the necessary range of frequency deviation on the side of a steep resonance curve.

Several circuits have been used to provide a more suitable sloping response-curve. Fig. 150 illustrates the principle of a circuit which is used both in the Foster-Seeley discriminator and in the discriminator which has come to be known as the Ratio Detector. If a current I_1 flows through the primary winding of a loosely coupled h.f. transformer (Fig. 150 (*a*)) then the primary voltage is given approximately by $V_1 = j\omega L_1 I_1$, and the voltage across the tuned secondary is given by

$$V_2 = (1/j\omega C)I_2$$

$$= \frac{1}{j\omega C} \cdot \frac{j\omega M I_1}{R_2 + j\omega L_2 + 1/j\omega C}$$

$$= \frac{1}{j\omega C} \cdot \frac{j\omega L_1 I_1 (M/L_1)}{R_2 + j\omega L_2 + 1/j\omega C}$$

$$= V_1 \cdot \frac{(M/L_1)}{j\omega C(R_2 + j\omega L_2 + 1/j\omega C)}$$

The angle of the numerator vector is zero and the angle of the denominator vector is

$$\phi = 90° + \tan^{-1} (\omega L_2 - 1/\omega C)/R_2$$

Thus V_2 lags V_1 by this angle ϕ. The value of ϕ will depend upon the frequency of the current, being 90° at the resonance frequency (at which $\omega L_2 = 1/\omega C$), and being greater or less than 90° according as the frequency is higher or lower than the resonance frequency.

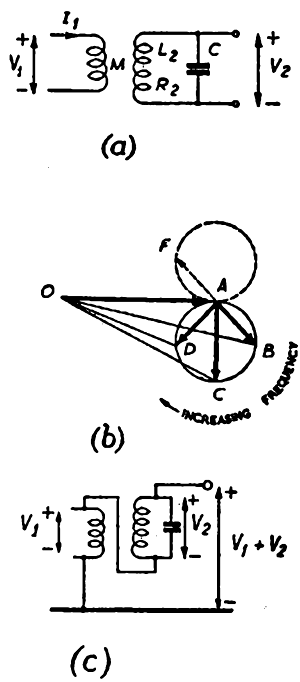


FIG. 150

Fig. 150 (*b*) is a vector diagram in which OA represents the primary voltage, V_1, and AB, AC, AD represent the secondary voltage V_2 at three possible frequencies. It is fairly easy to show, from the above equation, that, as the frequency varies, the locus of the tip of the secondary voltage vector will be approximately a circle. In Fig. 150 (*c*) the primary and secondary voltages are shown connected in series for the purpose of adding the vectors V_1 and V_2. The total output voltage vector, in Fig. 150 (*b*), will thus be OB at a certain frequency below resonance, OC at resonance, and OD at a certain frequency above resonance. Clearly, the length of this vector is very dependent on frequency. If a (constant amplitude) frequency-modulated input-voltage were used, the

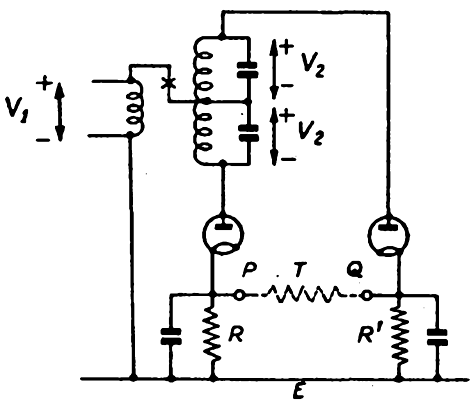


FIG. 151. PRINCIPLE OF THE FOSTER-SEELEY DISCRIMINATOR

output voltage would not be of constant amplitude, but its amplitude would vary throughout the modulation cycle, being greater when the frequency was below the value $1/2\pi\sqrt{L_2C}$ and smaller when the frequency was above this value. The values of L_2 and C would of course be chosen to make the resonance frequency equal to the carrier-frequency of the particular f.m. input voltage. The resulting amplitude-modulated output-voltage could now be used as the input voltage to any conventional detector circuit, and the output of the detector circuit would then give the required, low frequency, signal-voltage.

The linearity of the conversion from f.m. to a.m. can be improved, however, and at the same time the output voltage can be increased, by using two identical tuned secondary circuits and connecting them as shown in Fig. 151. This is the Foster-Seeley method. Two diode detectors are used. The input voltage to the right-hand detector is the h.f. voltage $V_1 + V_2$ as described above. The input voltage to the left-hand detector is seen to be the h.f. voltage $V_1 - V_2$. Instead of adding a V_2 vector such as AB (Fig. 150 (*b*)) to the V_1 vector, OA, we must now subtract the vector AB from the vector OA. This is the same as adding to OA a vector $- V_2$, i.e. the vector AF. The vector for $V_1 - V_2$, the h.f. voltage applied to the right-hand detector, is therefore OF at the time when the vector for the voltage applied to the other detector is OB. As the frequency varies, in the course of the modulation cycle, we can imagine the line, BF, oscillating clockwise and anticlockwise about the centre-point, A, being vertical when the frequency is passing through the resonance value. For frequencies below resonance, the amplitude of the h.f. voltage applied to the right-hand detector will be greater than the mean amplitude, OC and the amplitude of the h.f. voltage

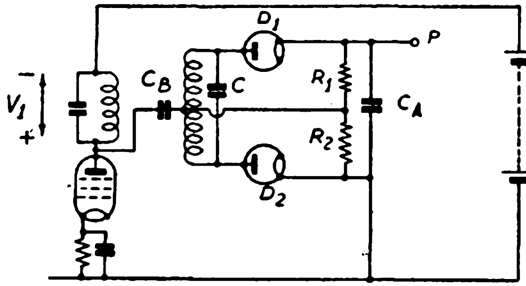


FIG. 152. FOSTER-SEELEY DISCRIMINATOR

applied to the left-hand detector will be less than OC—and vice versa above resonance.

The output voltage of the complete discriminator is taken as the difference between the output voltages of the two detectors. As the frequency of the h.f. input voltages passes through the carrier-frequency, the output voltage of the discriminator will pass through zero and change its polarity, which is exactly what is required.

The circuit shown in Fig. 151 gives a balanced output suitable for use as the input voltage to a push-pull amplifier. More often than not, however, an unbalanced output is required, i.e. an output with one side earthed. A second reason why the circuit of Fig. 151 is not generally used, is that the primary of the h.f. transformer is usually part of the anode circuit of a previous valve stage—which means that a d.c. blocking condenser would have to be included at the point marked X in Fig. 151. Since, however, the sum of the anode-currents of the two diodes (which is D.C.) flows along the lead marked X, it is not permissible to block the d.c. path. The difficulty is overcome in the practical circuit of Fig. 152. In this circuit, the output-voltage of the discriminator is the p.d. between the point P and h.t. negative. We shall show that the blocking condenser, C_B, is acting also as a reservoir condenser.

For ease of explanation the circuit is re-drawn in Fig. 153. (This is not an "equivalent" circuit but the actual circuit re-drawn, the only omission being the pentode of the previous stage. Also, the single tuning-condenser C, in Fig. 152, has been shown in Fig. 153 as two tuning condensers, one for each half of the secondary coil. The lettering in Fig. 153 corresponds to that in Fig. 152.)

The behaviour of circuits such as this (consisting of diodes, e.m.f.s, reservoir-condensers and leak-resistances) can be understood by referring to the principle established earlier in this chapter (page 247) that in each closed circuit the condenser will charge to a p.d. equal to the maximum value of the voltage applied in the

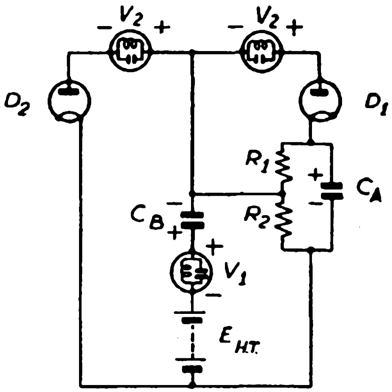


Fig. 153

conducting direction of the diode. Applying this rule to the left-hand closed circuit (formed of D_2, V_2, C_B, V_1 and E_{HT}) we see that

$$v_{C_B} = E_{HT} + |V_1 - V_2|$$

Considering the closed circuit formed of V_2, D_1, C_A, E_{HT}, V_1 and C_B, we see that

$$v_{C_A} + v_{C_B} = E_{HT} + |V_1 + V_2|$$

Hence the output voltage of the discriminator, being equal to the p.d. across C_A, is given by

$$v_{C_A} = |V_1 + V_2| - |V_1 - V_2| \quad . \quad . \text{(VI.11)}$$

exactly as in the simpler circuit of Fig. 151.

To avoid interference and to reduce noise, it is desirable that an f.m. radio receiver should not also respond to a.m. transmissions. For this purpose, a limiter (see page 309) may be included in the receiver. If, however, a discriminator could be designed which would inherently not respond to a.m. signal voltages, then a separate limiter stage could be dispensed with. The Foster-Seeley discriminator, described above, unfortunately has not this desirable property. If the amplitude of its h.f. input-voltage fluctuates, then the amplitudes of the primary and secondary voltages of the tuned transformer of Fig. 151 will also fluctuate, as will the output-voltage given by equation (VI. 11).

Clearly, what is required is some circuit which, instead of giving an output-voltage proportional to the *difference* between $|V_1 + V_2|$ and $|V_1 - V_2|$, will give an output-voltage proportional to the *ratio* $|V_1 + V_2|/|V_1 - V_2|$. Such a discriminator would respond to f.m. input-voltages but not to a.m. input-voltages.

By modifying the Foster-Seeley discriminator in a manner which we shall describe below, it is possible to reduce very considerably its response to a.m. input-voltages without unduly reducing its response to f.m. input-voltages. There is no part of the resulting circuit which can really be thought of as designed to respond to the ratio of the voltages $|V_1 + V_2|$ and $|V_1 - V_2|$. Yet, because the modified circuit gives an output voltage which depends upon which of these two voltages is the greater, without depending very much upon the magnitude of either, it has come to be known as the "Ratio Detector."

Let us look again at the Foster-Seeley circuit shown in Fig. 151. The p.d.s across R and R' are equal when the input frequency is equal to the carrier frequency. An increase of input frequency above the carrier value (in the course of the modulation cycle) increases one of these p.d.s and decreases the other by the same amount. A decrease of input frequency has the opposite effect. It follows that v_{PE} and v_{QE} (Fig. 151) each have the same d.c. component, V_{DC}, and have alternating components, at the modulation frequency, which are equal in amplitude but 180° out of phase. Assuming sine-wave modulation, at a modulation frequency $p/2\pi$, we may write

$$v_{PE} = V_{DC} + \hat{V}_P \sin pt$$
$$v_{QE} = V_{DC} - \hat{V}_P \sin pt$$

Thus the output-voltage, v_{PQ}, is given by

$$\begin{aligned} v_{PQ} &= v_{PE} - v_{QE} \\ &= 2\hat{V}_P \sin pt \end{aligned}$$

Note that both V_{DC} and $\hat{V}_P$ are proportional to the amplitude of the h.f. input-voltage to the discriminator.

The modification which converts the Foster-Seeley circuit into the Ratio-Detector consists, first, in the reversal of the polarity of one of the diodes shown in Fig. 151. Suppose that the two connexions to the right-hand diode are interchanged. The sign of v_{QE} is thereby reversed and we have

$$v_{PE} = V_{DC} + \hat{V}_P \sin pt$$
$$v_{QE} = -V_{DC} + \hat{V}_P \sin pt$$

The p.d. between the terminals P and Q is now $2V_{DC}$. It can no longer serve as the output-voltage of the discriminator since its magnitude is unaffected by the frequency of the input signal and depends only on the amplitude of that signal. If, however, a high resistance be connected between the terminals P and Q, we can obtain a suitable output-voltage by taking the p.d. between earth (E) and a centre-tap (T) on this resistance. This p.d. is given by

$$\begin{aligned} v_{TE} &= v_{TP} + v_{PE} \\ &= -\tfrac{1}{2}v_{PQ} + v_{PE} \\ &= -\tfrac{1}{2}(2V_{DC}) + V_{DC} + \hat{V}_P \sin pt \\ &= \hat{V}_P \sin pt \end{aligned}$$

By using this method of connexion we have halved the useful output-voltage of the discriminator. But, in the course of producing this reduced output-voltage, we have produced between P and Q a d.c. voltage which is proportional to the input carrier amplitude—and which will fluctuate if the carrier amplitude fluctuates. It is found that, by connecting a large enough capacitance between P and Q, the carrier amplitude at the discriminator input can be prevented from fluctuating (except very slowly) and the Ratio Detector circuit can in this way be made unresponsive to amplitude modulation. This is usually explained in terms of the variable damping of the tuned h.f. transformer by the effective input-resistance of the diode circuits.

Suggestions for Further Reading

1. P. Parker, *Electronics* (Edward Arnold).
2. E.E. Staff M.I.T., *Applied Electronics* (Wiley).
3. F. E. Terman, *Electronic and Radio Engineering* (McGraw-Hill).
4. H. Rissik, *Mercury-arc Current Converters* (Pitman).
5. F. Langford Smith (Editor), *Radio Designer's Handbook* (Iliffe).

CHAPTER VII

FREQUENCY CHANGERS AND MODULATORS

MODULATION is the production of a modulated alternating voltage, e.g. of the form expressed in equation (VI.1). The chief use of a modulated alternating voltage is the communication of information which is in the form of low-frequency (usually audio-frequency) voltages, *by the transmission of high-frequency voltages*. The transmission of high-frequency voltages has great advantages over the transmission of the original low-frequency "information voltages" (e.g. in Radio Communication and Carrier Telephony). Although the waveforms of the voltages produced by the falling of everyday sounds upon a microphone are very complex, we shall assume, for purposes of analysis, the simplest possible "information voltage," viz. the sinusoidal voltage

$$v_1 = V_p \sin pt$$

Amplitude modulation is the operation of producing a voltage

$$v = V(1 + m \sin pt) \sin \omega t \quad . \quad . \quad \text{(VII.1)}$$

from the information voltage, as above, and the high-frequency voltage

$$v_2 = V_\omega \sin \omega t$$

Since equation (VII.1) may be expanded in the form

$$v = V \sin \omega t + \tfrac{1}{2}mV \cos (\omega - p)t - \tfrac{1}{2}mV \cos (\omega + p)t \quad \text{(VII.2)}$$

we see that the input voltage of a modulator consists of two voltages, one of frequency $p/2\pi$, the modulation frequency, and the other of frequency $\omega/2\pi$, the carrier frequency: and that its output voltage consists of three components: one of the carrier frequency, one of frequency the sum of the input frequencies, and the other of frequency the difference of the input frequencies. These sum and difference frequencies are the "Sideband Frequencies," the components represented by the second and third terms of equation (VII.2) being known as the Sideband Voltages. It is important to note that in general the carrier frequency is large with respect to the modulation frequency, so that the sideband frequencies differ relatively little from the carrier frequency, the three output voltages forming a narrow band of high frequencies.

Frequency changing is the operation of altering the frequency of a given alternating voltage by the addition or subtraction of a given number of cycles per second. Its chief use is in the Superheterodyne method of radio reception, which is described later in this chapter. The input of a frequency changer consists of the voltage whose frequency is to be changed

$$v_1 = V_1 \sin \omega_1 t$$

together with a voltage having a frequency equal to the number of cycles per second by which it is required to change the frequency of v_1, e.g.

$$v_2 = V_2 \sin \omega_2 t$$

The output of the frequency changer is then required to be a voltage

$$v = kV_1 \sin (\omega_1 \pm \omega_2)t$$

the positive or negative sign being taken according as the change of frequency required is an increase or a decrease. The phase relation between input voltages and output voltage is unimportant, so that, to be quite general, the quantity $(\omega_1 \pm \omega_2)t$ should be increased by an arbitrary constant.* In view of the fact that two voltages of different frequencies are mixed in a frequency changer, this piece of apparatus is often called a "Mixer," and also a "Converter." In the Superheterodyne frequency changer the change of frequency required is a decrease; one of the two input voltages is received by transmission from a distance and is known as the Signal Voltage, while the other input voltage is generated especially for the purpose of frequency changing by an oscillator situated at the receiver, and is known as the Oscillator Voltage. It will be convenient to refer to the two voltages by these names. The signal frequency and oscillator frequency are normally both high frequencies.

There are the following points of similarity between a Modulator and a Frequency Changer—

(i) Both have two input voltages of different frequency.

(ii) Both produce sum and/or difference frequencies.

The points of difference are—

(i) In the Modulator the input frequencies are very different, the carrier frequency being large with respect to the modulation frequency; as a result the sum and difference frequencies are not very different from the carrier frequency. In the Frequency Changer the two input frequencies are of the same order, and as a result the difference frequency is not even approximately equal to the signal frequency.

(ii) Modulators are commonly required to give large power output, whereas Frequency Changers are not so required.

A great variety of circuits has been produced, both for modulation

* When two or more voltages are having their frequencies changed simultaneously, however, the phase relations in the output are required to bear a definite relation to those in the input. Failure in this respect constitutes Phase Distortion; in many of the applications of the frequency changer, phase distortion is unimportant.

and for frequency changing. Some modulator circuits may be used for frequency changing, and vice versa. Before discussing individual circuits we shall show how, when two voltages of different frequency are applied to an ordinary amplifier, curvature of the characteristics can cause, in addition to harmonic distortion, the production of components in the output voltage having sum and difference frequencies. This is known as Inter-modulation and can be a serious form of distortion in an amplifier.

Inter-modulation

Let the two voltages, $v_1 = V_1 \sin \omega_1 t$ and $v_2 = V_2 \sin \omega_2 t$, be connected in the grid circuit of a simple amplifier stage, in series with each other and with a grid-bias voltage $-E_g$. Let the load be a resistance, R, and let the dynamic-characteristic for this load be not a straight line but the quadratic

$$i_a = a + bv_g + cv_g^2$$

Now $v_g = -E_g + V_1 \sin \omega_1 t + V_2 \sin \omega_2 t$, and substituting this into the equation of the dynamic-characteristic we have

$$\begin{aligned} i_a = a - bE_g + cE_g^2 \\ + (b - 2E_g c)(V_1 \sin \omega_1 t + V_2 \sin \omega_2 t) \\ + \tfrac{1}{2}cV_1^2 + \tfrac{1}{2}cV_2^2 \\ - \tfrac{1}{2}cV_1^2 \cos 2\omega_1 t - \tfrac{1}{2}cV_2^2 \cos 2\omega_2 t \\ + 2cV_1V_2 \sin \omega_1 t \sin \omega_2 t \end{aligned} \quad . \quad . \quad . \quad \text{(VII.3)}$$

It is the last term in which we are interested at the moment. The terms in the first line give the quiescent anode-current, i.e. the anode-current in the absence of v_1 and v_2. The terms in the second line give the properly amplified components. The terms in the third give the rectified components. The terms in the fourth line give the second harmonic distortion. The fifth line may be written

$$cV_1V_2 \cos (\omega_1 - \omega_2)t - cV_1V_2 \cos (\omega_1 + \omega_2)t$$

and thus we have components of anode-current having frequencies equal to the sum and difference of the two input frequencies. These components are known as Inter-modulation components. In an amplifier they constitute distortion, and such distortion always accompanies the harmonic distortion resulting from curvature of the characteristics. In a modulator or a frequency changer these components constitute the useful output. For example, in a modulator $\omega_1/2\pi$ and $\omega_2/2\pi$ would be respectively the modulation frequency and the carrier frequency, and the two inter-modulation terms taken in conjunction with the term

$$(b - 2E_g c)V_2 \sin \omega_2 t$$

would give a suitably modulated alternating current.

In a frequency changer $\omega_1/2\pi$ would be the signal frequency, and

$\omega_2/2\pi$ the oscillator frequency. The term $cV_1V_2\cos(\omega_1 - \omega_2)t$ is then of the desired form, being of the difference frequency and having an amplitude proportional to that of v_1.

In the above we have seen one method by which frequency changing might be accomplished. The principle of the method is simply the addition of the signal voltage and the oscillator voltage (by connecting them in series), and the application of the summed voltage to the grid circuit of an amplifier arranged to produce "curvature distortion." Variable-mu valves may be used for this purpose. Such an amplifier is simply a square-law detector. In

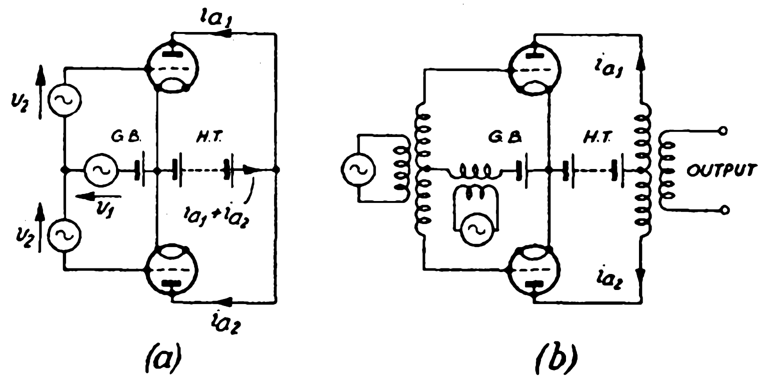


FIG. 154. BALANCED MODULATOR

view of the fact that in a Superheterodyne circuit there is a normal detector in addition to the frequency-changing stage, the detector used for frequency changing is spoken of as the First Detector and the normal detector as the Second Detector.

The useful output voltage of a frequency changer stage is always accompanied by components of other frequencies. The useful output voltage is segregated from the unwanted components by making the load of the valve a parallel *L–C* circuit tuned to the desired difference frequency.

The Balanced Modulator

We have shown in the previous section that modulation can be accomplished by connecting the carrier voltage and the low-frequency modulating voltage in series in the grid-cathode circuit of an amplifier which has non-linear characteristics. A rather surprising result can be produced by using a push-pull amplifier for this purpose. To explain this, let us first consider the push-pull circuit of Fig. 154 (*a*). Observé that this circuit has two input voltages, v_1 and v_2, having different frequencies. The input-voltage v_2 is connected in the usual push-pull manner (cf. Fig. 41, page 59).

The input-voltage v_1, however, is connected in the common lead and so affects both valves in the same way. Thus the grid-voltages of the two valves, v_{g1} and v_{g2}, are given by

$$v_{g1} = -E_g + v_1 + v_2 = -E_g + V_1 \sin \omega_1 t + V_2 \sin \omega_2 t$$
$$v_{g2} = -E_g + v_1 - v_2 = -E_g + V_1 \sin \omega_1 t - V_2 \sin \omega_2 t$$

We again assume as in the previous section, that the valve characteristics are curved so that

$$i_{a_1} = a + bv_{g1} + cv_{g1}^2$$
$$i_{a_2} = a + bv_{g2} + cv_{g2}^2$$

Substituting for v_{g1} in the first of these equations, we shall obtain an expression for i_{a1} which is identical with equation (VII.3). Substituting for v_{g2} in the second equation, we shall obtain for i_{a2} another similar expression, differing from i_{a1} only in that $-V_2$ must everywhere be substituted for V_2. The first, third and fourth lines of the expression, as set out in equation (VII.3), page 261, will be unaffected by this substitution of $-V_2$ for V_2, and will be the same in the expression for i_{a2} as in the expression for i_{a1}.

We wish to derive an expression for $i_{a1} - i_{a2}$, because the output voltage derived from a push-pull stage (in the manner shown in Fig. 154 (*b*)) is proportional to the difference between the anode-currents.

It is easy to see that

$$i_{a_1} - i_{a_2} = 2(b - 2E_g c)V_2 \sin \omega_2 t + 4cV_1 V_2 \sin \omega_1 t \sin \omega_2 t$$

Thus if the input-voltage v_2 be made the carrier voltage, and the input voltage v_1 be made the modulating voltage, we shall have an output which consists of carrier and sidebands, unencumbered by any unwanted components.

On the other hand, if we reverse the rôles of v_1 and v_2, making v_1 the carrier and v_2 the modulating voltage, we shall have an output voltage consisting only of the sideband-components (without the carrier) and a low modulating-frequency component. The latter can easily be filtered out by the connexion of a tuning-condenser across the output coil, the capacitance being chosen to resonate with the output coil at the carrier frequency. (Note that the sideband frequencies are very near the carrier frequency.)

The balanced modulator may therefore be used in two ways: either for straightforward amplitude-modulation, or to give an output consisting of the sideband components only. This latter application is important in suppressed carrier systems of communication and also in the Armstrong method of frequency modulating.

The circuit of the balanced modulator may take many different forms. In general, one may take any modulator circuit, build it into a push-pull form, using the normal push-pull input connexion for one of the input voltages. The other input voltage must then be so connected that it affects both halves of the push-pull circuit in the same way and therefore produces no output component of its own frequency.

Frequency Changing by Linear Detection (Envelope Detection)

In this method of frequency changing, the signal voltage and oscillator voltage are added (by connecting them in series) and the sum of the two voltages is applied to a linear detector—usually an anode-circuit detector of the type which is biased to cut-off (see "Detectors"). Instead of the usual resistive load, the load is a parallel L–C circuit tuned to the required difference frequency.

Since a linear detector gives an output voltage whose instantaneous value is proportional to the amplitude of the input voltage, we may class it as an "Envelope Detector," i.e. whose output voltage is the envelope of its input voltage. Thus, to understand the working of this circuit, we must visualize the graph of the input voltage, i.e. the graph of the sum of the signal voltage and oscillator voltage.

Let us first consider the simple case in which the oscillator voltage is equal in amplitude to the signal voltage. Although not a practical case this will serve to illustrate what is meant by the envelope of the graph of the sum of the two voltages. This sum is given by

$$\begin{aligned} v &= V(\sin \omega_1 t + \sin \omega_2 t) \\ &= 2V \cos \tfrac{1}{2}(\omega_1 - \omega_2)t \,.\, \sin \tfrac{1}{2}(\omega_1 + \omega_2)t \end{aligned}$$

Now the required difference frequency is often of the order of 10 per cent of the signal frequency, and sometimes less. In other words the frequency $\frac{1}{2}(\omega_1 - \omega_2)/2\pi$ is much smaller than the frequency $\frac{1}{2}(\omega_1 + \omega_2)/2\pi$. Thus in the above expression the function $\cos \frac{1}{2}(\omega_1 - \omega_2)t$ is a slowly varying function compared to $\sin \frac{1}{2}(\omega_1 + \omega_2)t$, and we may consider the sum voltage as a voltage of frequency $\frac{1}{2}(\omega_1 + \omega_2)/2\pi$ whose amplitude is the relatively slowly varying quantity $2V \cos \frac{1}{2}(\omega_1 - \omega_2)t$. The graph of the sum voltage will thus be as shown in Fig. 155 (*a*) in which the graph of $2V \cos \frac{1}{2}(\omega_1 - \omega_2)t$ is shown dotted. [Note that when this quantity becomes negative, the "amplitude" of the sum voltage is just the same as if it were positive; in other words, note that the graph of the sum voltage is similar in the positive and negative half-cycles of the quantity $2V \cos \frac{1}{2}(\omega_1 - \omega_2)t$.] Fig. 155 (*b*) shows the voltage which will be produced by the application of the sum voltage to a linear detector. This detector output voltage will be seen to be the sum of a steady d.c. voltage and an alternating voltage of the required difference frequency, viz. $(\omega_1 - \omega_2)/2\pi$. Its waveform, however, is far from sinusoidal and the process is not a satisfactory method of frequency changing.

It is easy to see without mathematical reasoning that the shape of the graph of the sum voltage will be as shown in Fig. 155 (*a*). The two input voltages may be considered of nearly equal frequency, so that their relative phase slowly changes. Initially they are in phase and the peak value of the sum voltage is $2V$. When they have become 180° out of phase, the peak value of the sum voltage is zero. Thus, as the phase change progresses the peak value rises and falls from zero to $2V$.

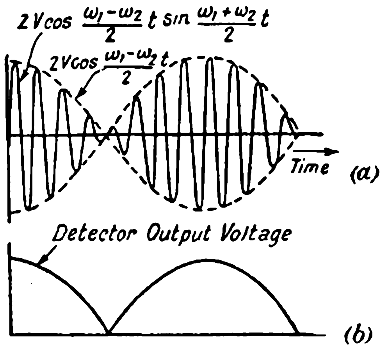


FIG. 155

Let us next consider the case in which, as in practice, the oscillator voltage is much greater than the signal voltage. Here the same non-mathematical reasoning tells us that, as the phase change progresses, the peak value of the sum voltage will rise and fall between the values $(V_2 - V_1)$ and $(V_2 + V_1)$, where V_1 and V_2 are respectively the peak values of the signal voltage and oscillator voltage. To determine the exact shape of the envelope we must revert to mathematical treatment and proceed to find the equation of the envelope, i.e. the equation of the curve passing through the maxima and minima of the sum curve,

$$v = V_1 \sin \omega_1 t + V_2 \sin \omega_2 t$$

To simplify the notation we shall write the difference frequency $(\omega_1 - \omega_2)/2\pi$ as a fraction, n, of the signal frequency $\omega_1/2\pi$, so that

$$\omega_2 = \omega_1(1 - n)$$

and
$$v = V_1 \sin \omega_1 t + V_2 \sin (1 - n)\omega_1 t$$

The values of t at which the sum curve has maxima and minima are given by dv/dt equal to zero, i.e. by

$$O = \omega_1 V_1 \cos \omega_1 t + \omega_1(1 - n) V_2 \cos (1 - n)\omega_1 t$$

Expanding $\cos (1 - n)\omega_1 t$ and abbreviating $\cos n\omega_1 t$ to x, this gives

$$\tan \omega_1 t = -\frac{\dfrac{V_1}{(1-n)V_2} + x}{\sqrt{1 - x^2}} \quad . \quad . \quad . \quad \text{(VII.4)}$$

Now the first equation above (the equation of the sum voltage) may be written

$$v = (\sin \omega_1 t)\,(V_1 + V_2 x - V_2\sqrt{1 - x^2} \,.\, \cot \omega_1 t) \quad . \ \text{(VII.5)}$$

Eliminating ω_1 between equations (VII.4) and (VII.5) we shall have the equation of the curve passing through the maxima and minima of the sum curve. Let us therefore substitute in (VII.5) the values of $\cos \omega_1 t$ and $\sin \omega_1 t$ derived from equation (VII.4), remembering that

$$\sin \omega_1 t = \frac{\tan \omega_1 t}{\sqrt{1 + \tan^2 \omega_1 t}}$$

This will be found to give

$$v = \frac{V_1^2 + V_2^2(1 - n) + V_1 V_2 x(2 - n)}{\sqrt{V_1^2 + V_2^2 (1 - n)^2 + 2 V_1 V_2 x(1 - n)}}$$

If n is small with respect to unity, then we have, approximately,

$$v = \sqrt{V_1^2 + V_2^2 + 2 V_1 V_2 x}$$

or, substituting for x, the equation of the envelope is

$$v = \sqrt{V_1^2 + V_2^2 + 2 V_1 V_2 \cos (\omega_1 - \omega_2)t} \qquad . \text{ (VII.6)}$$

We have not yet used the fact that the signal voltage, V_1, is small with respect to the oscillator voltage, V_2, so that equation (VII.6) is a general expression for the equation of the envelope, subject only to the assumption that the difference frequency is much smaller than the signal frequency. Equation (VII.6) shows that whatever the relative magnitudes of V_1 and V_2, square law detection will give a sinusoidal voltage of the desired difference frequency. This was to be expected from what we have said of inter-modulation. If V_1 and V_2 are equal, the right-hand side of equation (VII.6) reduces to

$$2 V_1 \cos \tfrac{1}{2}(\omega_1 - \omega_2)t$$

which is the result already obtained for the amplitude of the sum curve in the case of equal signal and oscillator voltages.

If now V_1 is small with respect to V_2, equation (VII.6) may be written

$$v \simeq V_2 + V_1 \cos (\omega_1 - \omega_2)t$$

and this, being the equation of the envelope, is also (with the inclusion of a constant of proportionality) the equation of the output voltage of the linear detector. Thus, we can successfully produce a sinusoidal voltage of the difference frequency by linear detection *provided that the oscillator voltage is large with respect to the signal voltage.* Actually the production of harmonics of the difference frequency is not so serious a matter as might at first be supposed, since the load of the frequency changer is a parallel *L–C* circuit tuned to the difference frequency, and thus only voltages of this frequency can appear in the output of the frequency changer.

Fig. 156 shows a frequency changer stage of this type. The upper valve is the linear detector and the lower one the oscillator. It will be seen that the detector valve is a pentode, thereby reducing capacitive coupling (via the inter-electrode capacitance of the valve) between the parallel *L–C* circuits on the input and output sides of this valve. Such coupling prevents completely independent tuning of the two L–C circuits.

The phenomenon resulting from the addition of two sinusoidal quantities of nearly equal frequency is well known in acoustics and is known as the phenomenon of Beats. The word Heterodyne is now widely used to describe the phenomenon of the addition of two

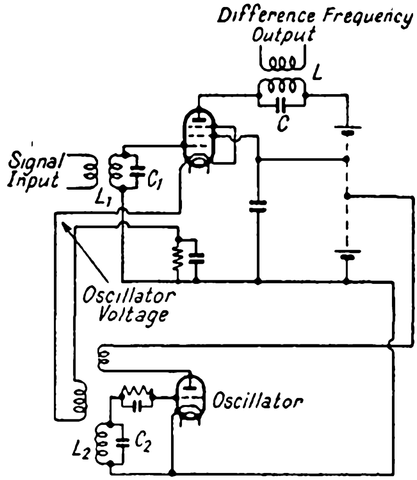


FIG. 156

alternating electrical quantities of nearly equal frequency—the envelope of the sum curve having the difference frequency.

Modulation by Linear Rectification

Modulation and frequency changing are in essence the same operation, the chief difference being in the relative magnitudes of the input frequencies (see page 260). In the method of frequency changing using a linear detector, the two input voltages were added and the sum voltage applied to a linear detector. The graph of the sum voltage, as we have seen, was of the heterodyne form, and the waveform of the detector output was proportional to the heterodyne envelope, and was thus of the difference frequency.

In the method of modulation by linear rectification the two input voltages (viz. the low-frequency modulating voltage and the high-frequency carrier voltage) are added, and the sum voltage applied to a linear rectifier. In the modulator case, however, the two input frequencies are not of the same order, and thus the sum graph is not of the heterodyne form, but is as shown in Fig. 157. The connexion of this voltage directly across a linear rectifier (e.g. a diode with as straight a characteristic as possible) will produce a current as shown in the lowest graph of Fig. 157. Such a current may be resolved approximately into a modulated alternating current of non-sinusoidal waveform together with a current of the modulation frequency and a d.c. component. The first of these three is the useful output, and may be further resolved into a modulated sinusoidal current together with unwanted harmonics of carrier and sidebands. By including in series with the sum voltage and the diode a parallel L–C circuit tuned to the carrier frequency, the desired modulated voltage (being a narrow band of frequencies whose mean value is the carrier frequency) may be separated from the unwanted components, for the parallel L–C circuit has a large impedance at frequencies near the carrier frequency, and a small impedance at other frequencies.

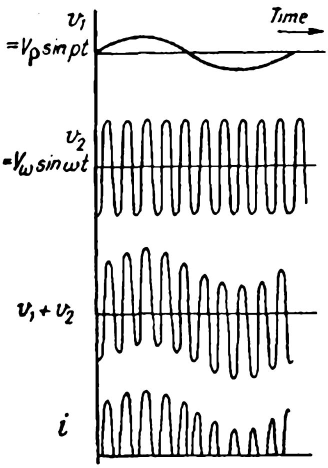


FIG. 157

Frequency Changers and Modulators Incorporating Rectifier Bridges

In the section on Phase-sensitive Rectifiers, in Chapter V, it was shown how a circuit consisting of four rectifiers, connected in the form of a bridge, could act as a switch, opened and closed by a reference voltage applied across one diagonal (Fig. 148 (*a*), page 250). There are two ways in which this "switch" may be used for frequency changing. In both of them, the carrier voltage takes the place of the reference voltage—the voltage which opens and closes the "switch."

In the first, or Series method, the lower-frequency modulating voltage is connected through this "switch" to the load. The circuit is similar to that of Fig. 148 (*b*), but with v_1 taken to be the modulating voltage and v_{REF} taken to be the carrier voltage. (Cf. also Fig. 201, page 317.) In the second, or Shunt method, the modulating voltage is connected directly across the load, and the "switch" is shunted across the load. In both circuits, the "switch" is opened and closed, by the carrier voltage, a large number of times in each cycle of the modulating voltage. It is as though the modulating voltage were being sampled or inspected, rapidly and at regular intervals. In the series circuit, the modulating voltage is communicated to the load whenever the "switch" is closed, and blocked when the switch is open. In the shunt circuit, the modulating voltage is communicated to the load whenever the switch is open, but is short-circuited when the "switch" is closed.

Provided that the carrier voltage is sufficiently large, it is legitimate to regard the bridge circuit as a switch. In that case, the output voltage will be proportional to the product of the modulating voltage and a regular square-wave of the carrier-frequency, f_c. Such a square-wave can be resolved into fundamental and odd-order harmonic components (see page 288).

If we assume a sine-wave modulating voltage of frequency f_M, then the product of the modulating voltage with the fundamental component of the square-wave gives us, in the output voltage, two components whose frequencies are $f_c + f_M$ and $f_c - f_M$. Other output components will have frequencies $3f_c \pm f_M$, $5f_c \pm f_M$, etc., but the amplitudes of these will be 1/3, 1/5, etc., of the amplitude of the two first named components. Finally it will be seen that, as a result of the product of the modulating voltage with the d.c. component of the square-wave, there is an output component of the modulating frequency.

The two first named components above, of frequencies $f_c \pm f_M$, would, if combined with a component of the carrier frequency, give an amplitude-modulated output voltage whose envelope is proportional (as required) to the waveform of the modulating voltage. These two circuits may therefore be regarded as suppressed-carrier modulators.

There is a further circuit, well known to telephone engineers (and used more with metal rectifiers or crystal rectifiers than with diodes) in which a bridge structure of four rectifiers acts as a reversing switch. It is known as a Ring Modulator. Its bridge-structure differs from that of the two circuits described above in that all four rectifiers have their conducting-direction "the same way round the ring" (Fig. 158 (*a*)). In Fig. 158 (*b*), which shows the complete circuit, the "ring" (drawn in heavy lines) appears as a crossed lattice, which is a well-known alternative way of drawing a bridge circuit. The carrier voltage is represented by v_c and the modulating voltage by v_M. Once again, the carrier voltage acts as a reference voltage, which maintains two of the rectifiers open during one carrier half-cycle (Fig. 158 (*c*)) and the other two open during the other carrier half-cycle (Fig. 158 (*d*)).

The modulating voltage, v_M, thus always has a path by which it may send current (*via* the transformers) through the load, but the route is changed, and the current thereby reversed, on the advent of every new half-cycle of the carrier voltage.

The deduction of the frequencies of the components present in the output-voltage follows exactly the same lines as for the series and shunt bridge-circuits considered earlier in this section, except that the square "switching-waveform" now has no d.c. component. As a result of this, the output of the ring-modulator contains no component of the modulation frequency. The other components have the same frequencies as in the series and shunt bridge-circuits and the ring-modulator can thus be used either for frequency-changing or for suppressed carrier modulation.

Valves with Two Control Grids

All the valves which we have considered hitherto (with the exception of the diode and valves such as the double-diode-triode, which latter can be considered simply as two or more valves within a single glass envelope) have had an anode-current which is a function of only two variables—viz. the anode-voltage and the grid-voltage. The extra electrodes present in the pentode and screen-grid valves have had constant potentials applied to them throughout

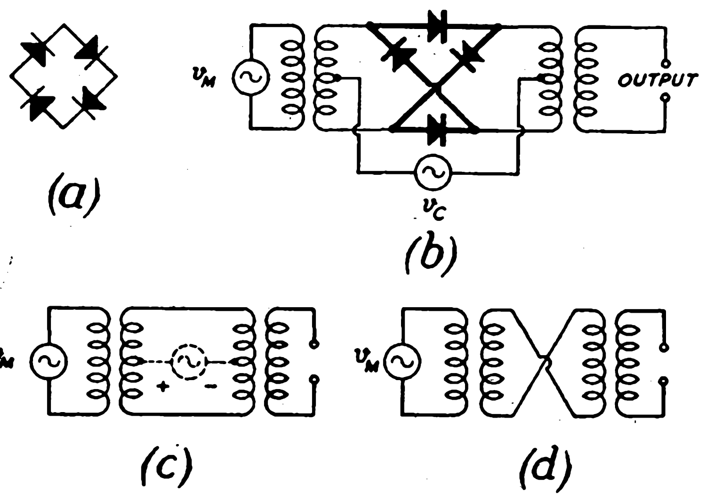


FIG. 158. THE RING MODULATOR

the operation of the valve, and so, while influencing the shape of the valve characteristics, have not added to the number of variables in the equation of the characteristics. Such valves may, as we have seen, be treated as triodes with unusually large values of impedance and amplification factor.

Let us suppose that an extra grid be added to a triode valve, being placed between the anode and the first control grid. Let the two grid-voltages, v_{g1} and v_{g2}, be variables. The anode-current, i_a, is then a function of three variables—

$$i_a = f(v_a, v_{g1}, v_{g2})$$

By deriving the equation of the characteristics, we shall see that it is possible to effect frequency changing with such a valve, the signal voltage being connected in one grid circuit and the oscillator voltage in the other.

If we keep the voltage on the second grid constant (and at not too large a value), the static-characteristics showing the variation of i_a with

v_a and v_{g1} are similar to the static-characteristics of a simple triode, i.e. they have straightish portions parallel to each other, and are equally spaced for equal increments of voltage between curves. Considering in particular the plot of the characteristics in the form of curves of i_a against v_{g1}, each curve being for a particular value of v_a, let us note the effect of varying v_{g2}. Fig. 159 shows this effect. The full-line graph is that of i_a against v_{g1} for fixed values of v_a and v_{g2}. If v_{g2} be increased (i.e. its negative value decreased) to v_{g2}', the characteristic becomes as shown by the broken line. If v_{g2} be further increased to v_{g2}'' the characteristic becomes as shown dotted.

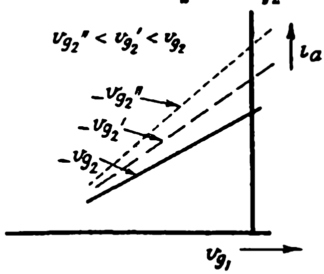


FIG. 159. IDEALIZED CHARACTERISTICS OF A BI-GRID VALVE

The equation of the individual characteristic is clearly of the form

$$i_a = av_{g1} + b$$

where both a and b are functions of v_{g2}. Over a certain range of voltage they may be considered linear functions of v_{g2}, and we shall write

$$a = cv_{g2} + d$$
$$b = ev_{g2} + f$$

giving

$$i_a = dv_{g1} + ev_{g2} + cv_{g1}v_{g2} + f$$

Now the coefficient, f, is a function of v_a. (None of the other coefficients can be a function of v_a, for this would imply that the valve impedance, $\partial v_a/\partial i_a$, was a function of v_{g1} or v_{g2}.) We shall write

$$f = gv_a + h$$

Substituting this into the previous equation we have the complete equation of the static characteristics, viz.

$$i_a = gv_a + dv_{g1} + ev_{g2} + cv_{g1}v_{g2} + h$$

which we may write

$$i_a = \frac{1}{\rho}(v_a + \mu_1 v_{g1} + \mu_2 v_{g2} + \mu_c v_{g1}v_{g2}) + k \quad . \text{(VII.7)}$$

In the use of such a valve for frequency changing, v_{g1} would be the signal voltage in series with a suitable grid-bias voltage, i.e.

$$v_{g1} = -E_{g1} + V_1 \sin \omega_1 t$$

while v_{g2} would be the oscillator voltage, again in series with a suitable grid-bias voltage, i.e.

$$v_{g2} = -E_{g2} + V_2 \sin \omega_2 t$$

Further, if the load were a resistance, R, we should have

$$v_a = E_a - Ri_a$$

Substituting these three equations into equation (VII.7) we have

$$i_a = \frac{1}{\rho + R}\left[\begin{array}{l} \quad\mu_1 V_1 \sin \omega_1 t + \mu_2 V_2 \sin \omega_2 t \\ + \frac{1}{2}\mu_c V_1 V_2 \cos (\omega_1 - \omega_2)t - \frac{1}{2}\mu_c V_1 V_2 \cos (\omega_1 + \omega_2)t \\ + \text{constant terms} \end{array}\right]$$

The useful output is the component of the difference frequency, i.e.

$$[i_a]_{\omega_1 - \omega_2} = \frac{\frac{1}{2}\mu_c V_1 V_2 \cos (\omega_1 - \omega_2)t}{\rho + R} \quad . \quad . \quad \text{(VII.8)}$$

Conversion Conductance of Frequency Changers

Equation (VII.8) shows that for a bi-grid valve with load R, the amplitude, $[I]_{\omega_1 - \omega_2}$, of the useful component of the anode-current is related to the amplitude of the signal voltage by the equation

$$[I]_{\omega_1 - \omega_2} = \frac{m_c V_1}{\rho + R} \quad . \quad . \quad . \quad \text{(VII.9)}$$

where m_c denotes $\frac{1}{2}\mu_c V_2$. The useful component of the anode-current of any type of frequency changer may be expressed in the form of equation (VII.9). This expression enables us to formulate an equivalent circuit, for the current $m_c V_1/(\rho + R)$ is equal to the current which would flow through R if it were connected across a generator having the following particulars:

$$\text{e.m.f.} = m_c V_1$$
$$\text{Frequency} = (\omega_1 - \omega_2)/2\pi$$
$$\text{Internal Impedance} = \rho$$

Equation (VII.9) refers only to one component of the anode-current —the useful component—and so this equivalent circuit is, of course, valid only for this particular component.

Valves with two control grids often have in addition a screen-grid and a suppressor grid, so that the characteristics may be of the pentode type. In such a case it is likely that the valve impedance, ρ, will be larger than R. If ρ is large with respect to R, equation (VII.9) reduces to

$$[I]_{\omega_1 - \omega_2} = m_c V_1/\rho = g_c V_1$$

The analogy between the quantities ρ, m_c and g_c in the equivalent circuit of the frequency changer, and the corresponding quantities, ρ, μ and g_m, in the equivalent circuit of an amplifier, is obvious. In particular, g_c corresponds to the mutual conductance, and is known as the Conversion Conductance.

The mutual conductance may be defined as the alternating anode-current per unit alternating grid-voltage *in a circuit with zero load impedance*. The Conversion Conductance may be defined as the difference-frequency alternating current per unit alternating signal-voltage *in a circuit with zero load impedance.*

The above equivalent circuit theorem gives the following useful relation: the difference-frequency voltage developed across a load, Z, will be

$$[V]_{\omega_1 - \omega_2} = \frac{\rho g_c V_1 Z}{\rho + Z} \quad . \quad . \quad . \quad \text{(VII.10)}$$

or, if the load, Z, is small with respect to the valve impedance, ρ, as in the case of a pentode frequency changer, the output voltage becomes $g_c V_1 Z$.

In the case of the bi-grid valve considered above, g_c is equal to $\mu_c V_2/2\rho$. Hence, as might be expected, the value of the conversion conductance depends on the oscillator voltage, V_2. The value of conversion conductance quoted by valve manufacturers is the value corresponding to the maximum permissible oscillator voltage.

The Hexode Mixer Valve

The hexode mixer valve is a screen-grid valve to which has been added an extra control grid, and also the screen-grid extended to surround the new control grid. The valve is shown diagrammatically in Fig. 160. As in the ordinary pentode valve, the screen-grid is maintained at a constant voltage above cathode. The suppressor grid (if any) is connected to cathode. The signal voltage is connected in the first control-grid circuit and the oscillator voltage in the second. The valve is thus simply a screen-grid valve with two control grids screened from each other. The object of this screening is to prevent capacitive coupling between the two parallel L–C circuits connected in the two control grid circuits, as this would render their tuning not independent.

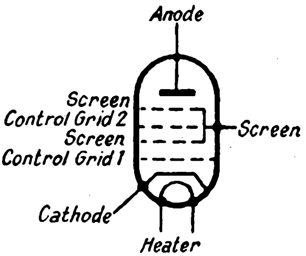


FIG. 160. HEXODE VALVE

The screen-grid characteristics make the impedance, ρ, very large, so that, in general, equation (VII.10) becomes

$$[V]_{\omega_1 - \omega_2} = g_c Z V_1$$

The anode-circuit load is not normally a resistance, but is a parallel L–C circuit tuned to the difference frequency, thus isolating the useful component of the output.

The Pentagrid (Heptode) and Triode-Hexode Valves

These two frequency-changer valves are both valves having two control grids and having in addition a triode oscillator valve in the same glass envelope.

Fig. 161 shows the pentagrid and its circuit. It will be seen to be a screen-grid valve with the addition of the electrodes *OA* and *OG*, and with the screen completely surrounding the signal control-grid. All the electrodes are directly in the electron stream from cathode to anode. The operation of the valve is as follows: *OA*, *OG* and the cathode operate as the anode, grid and cathode respectively of a triode oscillator. The oscillator anode, *OA*, is a very sparse structure, which consists usually simply of two straight wires. While it is adequate as an anode of the triode oscillator, it is a negligible barrier to the electrons. Nor does its potential greatly affect the main anode-current. The main anode-current is a function almost entirely of the potentials of the oscillator grid, *OG*, the signal control-grid and the main anode; and the valve therefore operates as a screen-grid valve with two control-grids. The oscillator voltage is the voltage of the first control-grid, while the signal voltage is connected in the second control-grid circuit. The circuits connected to *OA* and *OG* may be those of any triode oscillator which can give an adequate alternating grid-voltage—the obvious choice being the tuned-grid oscillator with mutual inductive coupling. The screen-grid is maintained at a constant positive potential in the normal way. The mesh of the signal control-grid is usually graded, to give a variable-mu characteristic. A modification of the pentagrid valve having an added suppressor grid to secure pentode characteristics has been called the Octode.

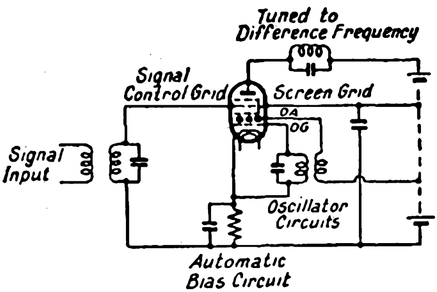


FIG. 161. THE PENTAGRID FREQUENCY CHANGER

The triode-hexode is no more than a hexode mixer valve and a triode oscillator valve enclosed in the same evacuated envelope. The two have a common cathode, but separate electron streams. The grid of the triode is electrically connected to one of the control-grids of the hexode within the valve.

Modulated Amplifiers and Oscillators

We have seen that modulation may be accomplished by using an amplifier with curved characteristics, the carrier voltage and the modulating voltage being connected in series in the grid circuit, and the anode-circuit load being a parallel *L–C* circuit tuned to the carrier frequency. We may regard such a modulator as a carrier frequency amplifier employing a variable-mu valve, the amplification of the amplifier thus depending upon the value of the grid-bias voltage. We may further regard the grid-bias voltage as consisting of a steady voltage (the true grid-bias voltage) in series with the modulating voltage, the latter being a relatively slowly varying quantity compared to the carrier voltage. Thus the amplification of the carrier voltage varies in accordance with the low-frequency modulating voltage, and the result is a modulated output voltage. Such a modulator is known as a Modulated Amplifier.

Since an oscillator is simply an amplifier which, by the use of coupling between the anode and grid circuits, provides its own input voltage, the same principle may be applied to an oscillator. By connecting the modulating voltage in the grid circuit of the oscillator which initially produces the carrier voltage, a modulated voltage may be produced across the load. Such a circuit is known as a Modulated Oscillator—like the modulated amplifier its operation depends upon the curvature of the characteristics. The oscillation frequency (see Chapter V) depends not only on the circuit constants but, to a small extent, also on the values of the valve parameters. Since the characteristics are not linear, the values of the parameters depend on the value of the grid-bias voltage, which in our case includes the modulating voltage. It follows that, while the grid bias is being made to vary at the modulation frequency (by the inclusion of the modulating voltage in the grid circuit) there will be a corresponding variation of oscillation frequency in addition to the desirable variation of amplitude. This variation of frequency is known as Frequency Modulation. While frequency modulation is the basis of a method of communication which has become of great importance, it is nevertheless undesirable in a circuit of the type under consideration, and must here be considered a form of distortion.

Modulated amplifiers may, as an alternative to including the modulating voltage in the grid circuit, include this voltage in the anode circuit. This method is in fact more commonly used. Consider a carrier-frequency amplifier whose input voltage is $V_\omega \sin \omega t$. If the valve characteristics are sensibly linear we may draw an equivalent anode circuit consisting simply of a generator of e.m.f. $\mu V_\omega \sin \omega t$ and of internal resistance, ρ, in series with the load. Clearly the inclusion of a voltage, $V_p \sin pt$, in the anode circuit will not produce modulation, but will simply add an anode-current

$$(V_p \sin pt)/(\rho + Z)$$

where Z is the load impedance. The operation of the circuit thus again depends upon the curvature of the characteristics. The simplest conception of such a modulator is that of a carrier-frequency amplifier whose amplification depends, as a result of the curvature of the characteristics, on the h.t. supply voltage. The h.t. voltage is made to vary according to the modulation—simply by the inclusion of the modulation voltage in series with the fixed h.t. supply voltage—and thus the amplification varies according to the modulation, and the result is a modulated output voltage. Class C amplifiers may be modulated in this way, for the angle of flow, provided it is small, increases with the h.t. voltage, and thus the output voltage also increases. There are several methods of coupling the modulating voltage (which is usually the output of an audio-frequency amplifier) into the anode circuit of the modulated amplifier. Fig. 162 shows one method. The valve on the left-hand side is the last stage of the audio-frequency amplifier; that on the right is a choke-capacitance coupled carrier-frequency amplifier. Instead of connecting the upper end of the high-frequency choke, L_2, directly to the h.t. supply voltage, it is connected to the h.t.

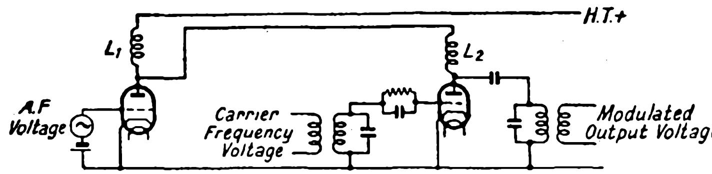


FIG. 162. CHOKE-COUPLED MODULATOR

by way of the choke L_1. The voltage across this choke, which is the output voltage of the audio-frequency amplifier, is thus connected in the anode circuit of the carrier-frequency amplifier. It should be noted that the load of the audio-frequency amplifier is the high impedance of the choke L_1, in parallel with the lower impedance presented by the right-hand valve (the impedance of the high-frequency choke L_2, at audio-frequencies will be negligible). The production across this comparatively low impedance load of the requisite audio-frequency voltage to effect modulation usually means a considerable power output from the audio-frequency amplifier. It may be shown that the ratio of the power required from the carrier-frequency amplifier to that required from the audio-frequency amplifier is approximately equal to the ratio of the carrier-frequency power in the load to the sideband frequency power in the load. In other words, the extra power in the load as a result of the addition of sidebands must be supplied by the audio-frequency amplifier. The r.m.s. value of the voltage expressed in equation (VII.2) is the square root of the sum of the squares of the r.m.s. values of the individual components, viz.,

$$V_{rms} = \sqrt{\left(\frac{V}{\sqrt{2}}\right)^2 + \left(\frac{\frac{1}{2}mV}{\sqrt{2}}\right)^2 + \left(\frac{\frac{1}{2}mV}{\sqrt{2}}\right)^2}$$

$$= \frac{V}{\sqrt{2}}\sqrt{1 + m^2/2} \quad . \quad . \quad . \quad . \quad \text{(VII.11)}$$

It follows that the ratio of the sideband power to the carrier power is equal to $m^2/2$.

Frequency-modulation: The Reactance Valve

Amplitude modulation is the controlling of the amplitude of a high-frequency voltage in accordance with the instantaneous value of an audio-frequency voltage (see page 7). Frequency-modulation is the controlling of the *frequency* of a high-frequency voltage in accordance with the instantaneous value of an a.f. voltage or low-frequency voltage. The resulting frequency-modulated voltage has a constant amplitude and a frequency which varies periodically about a mean value. Both systems are in use for radio telephony, frequency-modulation being the more recent and having certain advantages over the older system.

Since the frequency of an oscillator is approximately equal to the resonance frequency of an L–C circuit, the process of frequency-modulation can take place by periodic variation of either L or C in the L–C circuit. The Reactance-valve Circuit, which is the subject of this section, is a valve circuit having an input impedance which is a pure reactance (usually equivalent to a simple inductance) whose value depends on the mutual conductance of the valve. Using a variable-mu valve it is possible to vary the mutual conductance simply by varying the grid-bias voltage, and hence the reactance provided by this circuit may be varied periodically by periodic variation of the grid-bias voltage. The reactance valve circuit may thus be connected across the L–C circuit of the oscillator, and the oscillation frequency be varied periodically. The audio-frequency voltage in accordance with which it is desired to vary the oscillation frequency is used to effect the variation of the grid-bias voltage of the reactance valve.

The principle of the reactance valve circuit is shown in Fig. 163. In essence it is simply a two-terminal network, the two terminals (AA in Fig. 163) being the anode and cathode terminals of a valve. In this diagram a hypothetical a.c. generator V is shown connected across the terminals for the purpose of determining the input impedance. The input impedance vector will be given by V/I.

Now the current I is dependent upon both the anode-voltage and the grid-voltage. The former is simply V. The latter is derived from a phase-splitting circuit (not shown in Fig. 163) connected across V. The phase-splitting circuit is designed to give a grid-voltage V_g differing in phase from V by 90°, either lagging or leading. Let us consider the case in which V_g lags behind V by 90° and has a magnitude which is a fraction n of that of V, i.e.

$$V_g = -jnV$$

Fig. 164 shows the equivalent circuit, and gives the input impedance as

$$Z_{in} = V/I = \rho/(1 - j\mu n)$$

By designing the circuit so that μn is large with respect to unity we may produce an input impedance given by

$$Z_{in} = \rho/(-j\mu n) = j\left(\frac{1}{ng_m}\right) \quad . \quad . \quad \text{(VII.12)}$$

This means that the input impedance is equivalent to that of a pure inductance whose value is inversely proportional to the

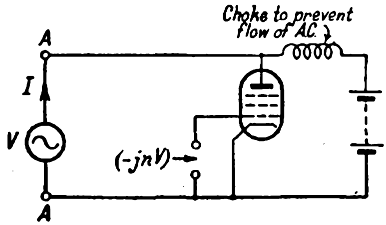


FIG. 163. PRINCIPLE OF REACTANCE VALVE

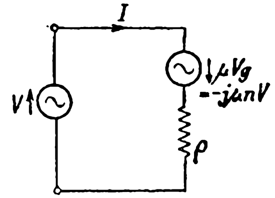


FIG. 164

mutual conductance, g_m. The effect of making V_g lead V by 90°, instead of lagging by 90° as above, may be seen by writing $-j$ for j in the above result; it is seen that in this case the input

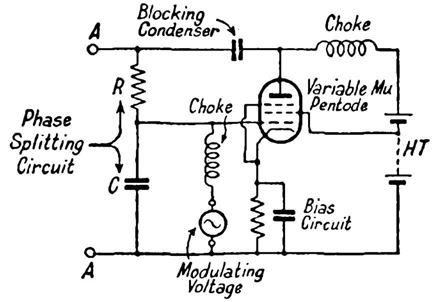


FIG. 165. REACTANCE-VALVE CIRCUIT

impedance is equivalent to that of a capacitance whose value is directly proportional to the mutual conductance of the valve.

Fig. 165 shows the complete circuit of a Reactance Valve using a phase-splitting circuit consisting of a resistance R in series with a capacitance C. With such a circuit V_g will be given by

$$V_g = \frac{V(1/j\omega C)}{R + 1/(j\omega C)}$$

The angle of phase difference between V_g and V may thus be made approximately 90° by making R large with respect to $1/(\omega C)$, in which case

$$V_g = V/(j\omega CR)$$

i.e. the fraction n is $1/(\omega CR)$ and V_g lags behind V by 90°. Thus for this particular phase-splitting circuit

$$Z_{in} = j\omega CR/g_m \quad . \quad . \quad . \quad \text{(VII.13)}$$

which is equivalent to a pure inductance of magnitude CR/g_m.

We have seen that the circuit must be designed so that, for the range of frequencies used,

$$1/(\omega C) \ll R$$

and

$$\mu n \gg 1$$

For the phase-splitting circuit shown, the latter condition becomes that $\mu/(\omega CR)$ shall be large with respect to unity, which may be written

$$1/(\omega C) \gg R/\mu$$

If the quantity $1/(\omega C)$ is to be both small with respect to R and large with respect to R/μ, it is clearly necessary that μ shall be large. For this reason, and also to avoid the effect of grid-anode capacitance (which would interfere with the operation of the phase-splitting circuit), it is necessary to use a pentode valve. It should also be noted that the impedance of the phase-splitting circuit is itself connected across the terminals of the reactance valve, so that the whole input impedance is the parallel combination of the phase-splitting circuit (approximately equal to R) and the input impedance as determined above.

It is interesting to note that with any impedance Z taking the place of the condenser C in the phase-splitting circuit, the input impedance is given by R/g_mZ, which is the inverse impedance of Z with a constant of inversion equal to R/g_m. As before, this is subject to the conditions that $|Z|$ shall be small with respect to R and large with respect to R/μ, and also there is the impedance of the phase-splitting circuit in parallel with the impedance R/g_mZ.

Heterodyne Reception of C.W. Signals

Communication by the Continuous Wave method (usually abbreviated to "C.W.") does not use modulated voltages. The information is conveyed by the starting and stopping of an unmodulated voltage according to a pre-arranged code. All that is needed, therefore, for the reception of such signals is a method of causing the received high-frequency voltage to produce an audio-frequency voltage, the latter then being applied to a telephone receiver. The required operation is clearly frequency changing. Frequency changing in this case is most conveniently accomplished by adding to the received signal voltage an oscillator voltage whose frequency differs from the signal frequency by the requisite audio frequency, and applying the sum voltage to a detector.

Fig. 166 shows a suitable circuit. In this circuit a single triode valve is made to function both as oscillator and detector. The detector is of the triode grid-circuit type. Although the parallel *L–C* circuit is tuned not to the signal frequency but to the oscillator frequency, these two frequencies differ only by an audio frequency, and thus signal voltages can appear in the grid circuit.

The same circuit may be used for the reception of radio telephony (amplitude-modulated h.f. voltages). In this case it functions as a combination of regenerative h.f. amplifier and cumulative-grid detector, and is known as the Autodyne circuit. The oscillation-control condenser then becomes a regeneration control (see Fig. 96,

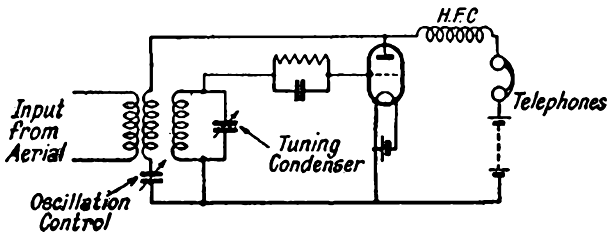


FIG. 166. THE AUTODYNE CIRCUIT

page 160) and is adjusted to have as large a capacitance as is possible without self-oscillation. The Autodyne circuit has the merits of sensitivity, selectivity, and simplicity, and has been widely used for radio reception. Its chief disadvantage is that the degree of regeneration is affected by changes of the tuning capacitance (see page 159) and also that tuning of the *L–C* circuit is affected by changes in the regeneration control. The operation of the circuit thus calls for patience.

The Superheterodyne Method of Radio Reception

We have said that the phenomenon resulting from the addition of two alternating voltages of nearly equal frequency is known as a Heterodyne. When the envelope frequency, which is equal to the difference frequency, is higher than the audio-frequency range, the phenomenon is known as a Supersonic Heterodyne. This name, abbreviated to Superheterodyne, has come to be used to describe a method of radio reception in which, nowadays, the heterodyne principle is sometimes not involved at all.

The principle of the method is to reduce to a lower value the frequency of the signal voltage derived from the aerial circuit, and to follow this by the amplification of this lower frequency voltage and its detection in the usual way. Consider, for example, the reception by this method of a signal whose frequency is 5 Mc/s and whose amplitude is modulated at a frequency of 1,000 c/s. The voltage derived from the aerial circuit consists of three components—the carrier voltage, of frequency 5 Mc/s, and the sideband voltages, of frequencies 5·001 Mc/s and 4·999 Mc/s—the whole constituting a band of frequencies centred round 5 Mc/s. The first operation—that of frequency changing—is designed to reduce this voltage to one having a band of frequencies centred round some lower frequency, say 500 kc/s. This is accomplished by the use of a frequency changer having an oscillator voltage of frequency 4·5 Mc/s. Subtracting 4·5 Mc/s from the three signal frequencies, we see that the output of the frequency changer consists of three components having frequencies 501 kc/s, 500 kc/s, and 4·99 kc/s. Together these three components constitute a voltage of frequency 500 kc/s with its amplitude modulated in the same way as was the aerial voltage. This voltage is then amplified and detected in the usual way.

Radio receivers are not normally designed to receive signals of only one carrier frequency, but are fitted with controls by means of which the receiver is made capable of receiving signals of any frequency within a wide range. These controls usually vary the capacitance of the condensers in the various parallel L–C circuits, thus varying the resonance frequency of these circuits. The adjustment is known as Tuning. In the superheterodyne method of reception the frequency of the oscillator in the frequency-changer stage is varied according to the frequency of the signal which it is desired to receive, in such a way as to make the difference frequency always the same. In other words, whatever the frequency of the signal voltage, the frequency of the output voltage of the frequency changer remains constant. This constant frequency is known as the Intermediate Frequency, and the subsequent amplifier as the Intermediate Frequency Amplifier. In a receiver with an intermediate frequency of 500 kc/s tuned to receive a signal of frequency f kc/s, the oscillator frequency, in kilocycles per second, would be

$$f_{oscillator} = f - 500$$

A difference frequency of the same value, 500 kc., could be produced by making the oscillator frequency $(f + 500)$ kc.; the use of an oscillator frequency higher than the signal frequency is, in fact, becoming common practice.

The first advantage of the superheterodyne method is that, by substituting intermediate-frequency amplification for radio-frequency amplification it reduces the complexity and cost of the required amplifier. Particularly is this the case in the reception of ultra short wave (i.e. very high frequency) signals. The second advantage is that, in the superheterodyne method it is necessary when tuning to vary only the oscillator frequency, whereas, in the simple receiver employing only radio-frequency amplification, it is necessary to vary the resonance frequency of all the parallel L–C circuits in the radio-frequency amplifier. In both cases, of course, tuning also involves a variation of one, or where band-pass tuning is used, two capacitances in the aerial circuit. It is usual to arrange mechanical coupling by which all necessary condensers may be suitably varied simultaneously by the operation of a single control. This is known as Ganging.

The third advantage of the superheterodyne method is that it gives a great increase of selectivity. In Chapter IV it was explained that the selectivity of an amplifier is its ability to amplify voltages of one particular frequency—or of a particular narrow band of frequencies—while not amplifying voltages of other frequencies. In radio-communication the signals of the various transmitters are differentiated by their carrier frequency, so that selectivity implies the ability to receive the signal from one particular transmitter without receiving at the same time signals from other transmitters. Consider a superheterodyne type of receiver tuned to receive a signal of frequency, f, and let there be present in the aerial circuit in addition an unwanted signal of frequency differing by n per cent from f. The frequency changer reduces the frequency of both these signals by the same amount. If f_{IF} be the intermediate frequency, the two carrier frequencies present in the output of the frequency changer will be f_{IF} and a frequency differing from this by $nf/100$. The percentage difference in frequency is now nf/f_{IF}, and f/f_{IF} is usually fairly large. If f/f_{IF} is equal to 10, then the intermediate-frequency amplifier, instead of having to differentiate between two frequencies with a difference of n per cent, has only to differentiate between two frequencies with a difference of $10n$ per cent, and the selectivity is ten times that of the I.F. amplifier alone.

With the receiver tuned to receive a frequency, f, the oscillator frequency will be $(f - f_{IF})$, and the signal of frequency, f, then produces a difference frequency, f_{IF}, as desired. An unwanted signal of frequency $(f - 2f_{IF})$ would, however, also produce a voltage of the requisite difference frequency, and this voltage would then be amplified by the intermediate-frequency amplifier together with the desired voltage. A spurious addition to the desired signal is known as Interference, and this particular kind of interference is known as Second Channel Interference. To avoid it, the unwanted signal of frequency, $(f - 2f_{IF})$, must not be allowed to reach the frequency changer. This can be accomplished by using a selective aerial circuit, or a stage of tuned radio-frequency amplification, with the tuning controls ganged to the oscillation frequency control in such a way that the aerial circuit is always tuned to a frequency differing from the oscillation frequency by an amount f_{IF}.

The Heterodyne Oscillator

For many purposes in Communications Engineering it is necessary to have an oscillator whose frequency may be varied continuously over the whole audio-frequency range by the operation of a single control, the output voltage remaining constant. The heterodyne oscillator provides such a piece of apparatus.

A simple oscillator, say of the type shown in Fig. 93 (*a*), would not meet the requirements. Its frequency could be varied smoothly and continuously by the use of a variable condenser in the parallel *L–C* circuit, but since the oscillation frequency is inversely proportional to the square root of the capacitance it follows that the ratio of the maximum available capacitance to the minimum available capacitance would have to equal the square of the ratio of the maximum frequency required to the minimum frequency required. Thus if the frequency range required were 10 c/s to 10,000 c/s, the range of capacitance available in the single variable condenser

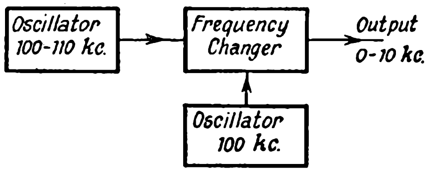


FIG. 167. HETERODYNE OSCILLATOR

would have to equal a million to one. The minimum available capacitance is the residual capacitance of the condenser when its electrodes are as far separate as the mechanical arrangement will allow, and to this must be added the anode-cathode inter-electrode capacitance of the valve, since this is in parallel with the parallel *L–C* circuit. Such considerations show that it is not possible to secure a sufficient range of capacitance in a single variable condenser. (Further condensers may be switched in parallel, of course, but this gives a step-by-step variation of frequency instead of the continuous variation required.)

Another objection to the use of the simple oscillator of Fig. 93 (*a*) is that with a wide variation of capacitance there will be a wide variation of oscillation amplitude, for the capacitance *C* appears in equation (V.2), the equation giving the condition for sustained oscillation, and it follows that the oscillation amplitude will depend upon *C*.

The solution of the problem is to generate a high frequency, say ten times as great as the highest frequency required, and then reduce this frequency, by means of frequency changing, to the value required. For example, the simple oscillator of Fig. 93 (*a*) may be used to generate a voltage whose frequency may be varied from 100,000 c/s to 110,000 c/s. This voltage may then be applied to a frequency changer whose oscillator frequency is fixed at 100,000 c/s. The frequency of the output voltage of the frequency changer will then range from zero to 10,000 c/s as required. A schematic diagram of the arrangement is shown in Fig. 167.

In the above example, only a 10 per cent variation of frequency is required of the simple oscillator, and this needs only a 21 per cent variation of capacitance, which is easily secured. The waveform of the voltage applied to the detector is of the heterodyne type, and thus square-law detection is necessary if the output voltage is to be sinusoidal. Alternatively, the fixed frequency voltage must be made small with respect to the variable frequency voltage.

The Heterodyne Analyser

Another application of the heterodyne principle is seen in the measuring instrument known as the Heterodyne Analyser. The function of an analyser is to measure separately the amplitudes of the various components of a voltage formed of the sum of a number of voltages of different frequency. Perhaps its most important use

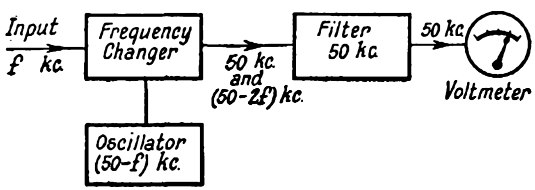


FIG. 168. HETERODYNE WAVE-ANALYSER

is the measurement of the amplitudes of fundamental and harmonics in a non-sinusoidal alternating voltage.

Clearly the measurement could be carried out by the use of a single-frequency amplifier followed by an a.c. voltmeter, but the use of a simple tuned amplifier for this purpose gives greatly insufficient selectivity. The selectivity could be increased by the use of a narrow-band-pass filter with very sharp cut-off, but the pass frequency would have to be made continuously variable. The problem is again solved by frequency changing. Fig. 168 shows a schematic diagram of the circuit. The frequency of the voltage to be measured is increased to a fixed high frequency, e.g. 50 kc/s. For this purpose the oscillator frequency must be $(50{,}000 - f)$, where f is the frequency of the voltage being measured. The sum frequency is then 50 kc/s. The frequency changer is followed by a very sharply tuned narrow-band-pass filter, preferably a crystal filter, which accepts only a narrow band of frequencies centred about 50 kc/s. Thus, when the oscillator frequency is made $(50{,}000 - f)$ the only component of the input voltage which can affect the voltmeter is the component of frequency f. The arrangement is similar to the superheterodyne method in that it has a fixed intermediate frequency, thus making a variable-frequency filter unnecessary.

The tuning control varies the oscillator frequency, viz. $(50{,}000 - f)$, but is graduated to read f, the frequency of the component which is affecting the voltmeter.

Suggestions for Further Reading

1. D. G. Tucker, *Modulators and Frequency Changers* (Macdonald).
2. K. R. Sturley, *Radio Receiver Design* (Chapman & Hall).
3. F. Langford Smith (Editor), *Radio Designer's Handbook* (Iliffe).
4. F. E. Terman, *Electronic and Radio Engineering* (McGraw-Hill).

CHAPTER VIII

PULSES AND PULSED CIRCUITS

A CURRENT (or voltage) pulse is said to occur when a unidirectional current flows (or a unidirectional voltage appears) between two points for a short period only, the current (or voltage) being zero for a much longer period before and afterwards. The simplest case is that of a current or voltage which is simply switched on and then off, remaining at a constant value throughout the small interval between the switching operations. The graph of voltage or current against time in such a case is rectangular, and a pulse of this kind is known as a Rectangular pulse. A rectangular pulse is completely defined by stating its peak value, its duration (sometimes spoken of as the pulse width or pulse length) and the time of its occurrence. Voltage and current pulses, often having a duration of the order of a microsecond (i.e. 10^{-6} sec.) are used extensively and for a variety of purposes. These include telecommunication by the various systems of Pulse Modulation, the location of distant objects by radar methods, the synchronizing of television receivers and transmitters, the representation of numbers in digital computers, and timing operations in general.

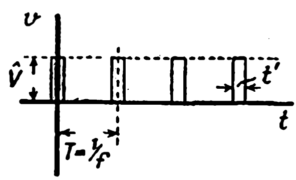


FIG. 169. RECTANGULAR PULSE WAVEFORM

In many of these applications the pulse is repeated at regular intervals. The number of pulses occuring per second is then known as the pulse repetition frequency, and its reciprocal gives the periodic time, i.e. the time interval between the leading edges of successive pulses. Fig. 169 shows such a system of repeated pulses. It can be resolved by Fourier analysis into the sum of a d.c. component and a series of alternating components in the form of fundamental and harmonics. With the notation of Fig. 169, the voltage v at time t may be written

$$v = a_0 + a_1 \cos \omega t + a_2 \cos 2\omega t + a_3 \cos 3\omega t + \ldots + a_n \cos n\omega t + \ldots \quad \text{(VIII.1)}$$

in which a_0 is the d.c. component, a_1 is the amplitude of the fundamental a.c. component, a_2 is the amplitude of the second harmonic, and so on. These coefficients a_0, a_1, a_2, etc. may be shown to be as follows—

$$a_0 = \hat{V}\,(t'/T)$$

$$a_n = \frac{2}{\pi}\cdot\frac{\hat{V}}{n}\sin\,(n\pi t'/T) \quad \text{(VIII.2)}$$

The ratio of pulse duration to periodic time, t'/T, which occurs in the above expressions, is often called the Duty Ratio, s. It may be thought of as the time for which the voltage is "switched on" divided by the total time. Note that $s = t'/T = ft'$.

It follows that

$$a_n = 2s\hat{V}\frac{\sin n\pi s}{n\pi s} \quad . \quad . \quad . \text{(VIII.3)}$$

This enables us to see how the amplitudes of the various harmonics compare with the amplitude of the fundamental. Fig. 170 shows the graph of a_n against n (i.e. the graph of the amplitude of a harmonic against its frequency, the frequency being expressed as a multiple

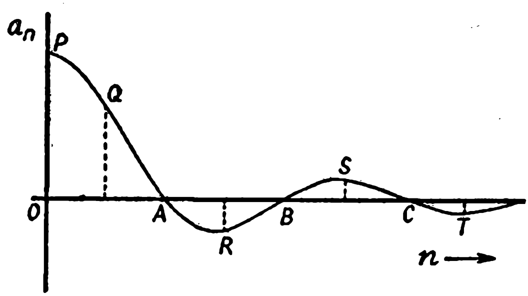


FIG. 170

of the fundamental frequency). The graph is the same (but to a different scale) as the well-known graph of $(\sin\theta)/\theta$ plotted against θ. If θ be expressed in radians, this latter graph has an ordinate of 1 when θ is zero, and the graph passes through zero when θ has the values π, 2π, 3π, etc., corresponding to the points A, B, C, etc. in Fig. 170. Clearly, not all points on this graph have a physical significance, but only those points corresponding to an integral value of n. For example, if the duty ratio, s, were equal to 1/1,000, the values of $(n\pi s)$ for the fundamental, second harmonic and third harmonic would be respectively $\pi/1{,}000$, $2\pi/1{,}000$, $3\pi/1{,}000$. At the point A, Fig. 170, where $\sin(n\pi s)$ is zero, $(n\pi s)$ is equal to π. Thus, at this point n equals $1/s$, i.e. $n = 1{,}000$. In this case there are 1,000 significant points on the graph, each representing an harmonic, between the points O and A. The point representing the fundamental $(n = 1)$ is not at the point P but always to the right of that point.

Going to the other extreme, let us consider the case in which the value of s is $\frac{1}{2}$, i.e. the pulse duration is one half of the periodic time, giving so-called square waves. The values of $(n\pi s)$ for the fundamental, second harmonic and third harmonic are respectively $\pi/2$, π, and $3\pi/2$. Thus the point A now corresponds to the second harmonic, the point B to the fourth harmonic, and it is seen that far fewer points on the graph of Fig. 170 have physical significance. Moreover, in this particular case of square waves the even-order harmonics all have zero amplitude. The amplitudes of the fundamental, third harmonic, fifth harmonic, etc. are given respectively by the magnitudes of the ordinates of the points Q, R, S, etc. At the points, Q, R, S, the values of n are 1, 3, and 5 respectively, so that the corresponding amplitudes (given by $2\hat{V}/n\pi$) are $\frac{2\hat{V}}{\pi}$, $\frac{2\hat{V}}{3\pi}$, and $\frac{2\hat{V}}{5\pi}$.

From the above it is seen that, for very small values of duty ratio (i.e. for pulses of very short duration compared to the periodic time), the amplitudes of the harmonics are very small compared to $\hat{V}$, but there are a great many of them—all of comparable magnitude. The amplitude does not decrease rapidly as one considers harmonics of increasing order. For instance, if the duty ratio be 1/1,000, the amplitude of the 1,500th harmonic is a fraction $2/3\pi$ of the amplitude of the fundamental, being given by the ordinate of the point R, Fig. 170. Amplifiers which are required to handle voltage pulses of short duration must therefore have a response curve which is level, up to quite high frequencies. If it be assumed (somewhat arbitrarily) that, to avoid distortion of the pulse waveform, it will suffice to amplify all harmonics whose amplitude is greater than a certain fraction of the fundamental amplitude, then we may calculate the highest frequency which we require the amplifier to handle. Consider the frequency corresponding to the point T in Fig. 170. For this point $(n\pi s)$ is equal to $7\pi/2$, and thus n is equal to $7/(2s)$. The frequency of this harmonic is $7/(2s)$ times the fundamental frequency, i.e. $7/(2sT)$. Since s equals t'/T, this gives

$$f_{max} = 7/(2t')$$

The amplitude of this particular harmonic is $2/7\pi$ (or 9·09%) of the amplitude of the fundamental. Thus, to ensure the amplification of all harmonics whose amplitude is greater than 9·09 per cent of the fundamental, one must have a response curve which is level up to a frequency given by $f_{max} = 7/2t'$. Note that this is independent of the repetition frequency and depends only upon the pulse duration.

The subject-matter of this chapter falls roughly into two categories, namely, the discussion of circuits (such as pulse-generators and pulse-shaping circuits) which are concerned *only* with pulses, and the discussion of the behaviour of circuits described in earlier chapters when such circuits are handling pulses.

There are two methods of calculating the response of a circuit to regularly repeated pulses. The first method is to calculate the response to each of the separate components of the pulses, (viz. d.c., fundamental, and harmonics) and to combine the individual responses to give the total response. This method is usually too laborious;

except in cases where the circuit is such that the response to most of the separate components is either zero or of a very simple form. The second method is to calculate the response of the circuit to a single pulse (i.e. to a single switching-on process followed, after an interval t', by a single switching-off process). If the voltage and current changes caused by the single pulse have substantially decayed to zero before the next pulse is due to occur, then this next pulse will produce the same changes of voltage and current as did the first pulse, and the response of the circuit to the repeated pulses may be taken to be simply a regular repetition of the response to a single pulse.

As an illustration, consider the application of a single e.m.f. pulse to a circuit consisting of a resistance R in series with a capacitance C. Initially the e.m.f. is zero (this does not mean that the input terminals are open-circuited, but that they are maintained at the same potential, i.e. effectively short-circuited) and the condenser voltage is zero. When the e.m.f. is switched on, the condenser voltage rises exponentially and with a time-constant RC towards the peak voltage value of the pulse. After an interval t' the input terminals are again effectively short-circuited, and the condenser voltage begins to fall towards zero from whatever value it had reached, falling exponentially and with a time-constant RC. If this time-constant is small with respect to the periodic time of the repeated pulses, then the condenser voltage will have decayed substantially to zero before the next pulse occurs, and the response to this next pulse will be simply a repetition of the response to the first. If, on the other hand, the time-constant is large with respect to the repetition period, the first method of analysis may be used. If $RC \gg T$ we may write $RC \gg 2\pi/\omega$, where ω is 2π times the fundamental frequency (viz. the repetition frequency). Thus $R \gg \frac{2\pi}{\omega C}$, which shows that the resistance is large with respect to the condenser reactance at the fundamental frequency, and therefore still larger with respect to the reactance at the harmonic frequencies. It follows that none of the alternating components of the e.m.f. produce any appreciable voltage across the condenser. The resultant condenser voltage is thus that which would be produced by the d.c. component of the e.m.f. alone, i.e. the resultant condenser voltage is $s\hat{V}$ and the circuit is functioning as an a.c./d.c. separating circuit.

If the time-constant is neither much smaller than the periodic time nor much larger than it, but instead the two are of the same order of magnitude, a modification of the second method may sometimes be used. Writing v for the unknown value to which the condenser voltage has fallen at the instant when any one pulse commences, we may derive an expression for the value to which the condenser voltage rises during this pulse. This will be the value from which the condenser voltage next begins to fall, and an expression may be derived for the value to which it will have fallen at the onset of the succeeding pulse. This must clearly be equal to v since, in the steady state, the changes of condenser voltage will be regularly repeated. Equating the two, we may solve for v, the resulting expression being

$$v = \hat{V}\,\frac{e^{t'/RC} - 1}{e^{T/RC} - 1} \qquad . \quad . \quad . \quad \text{(VIII.4)}$$

The condenser voltage will thus rise and fall between two values, namely, v and $ve^{(T-t')/RC}$. This is illustrated in Fig. 177 for a duty ratio of 1/3, in which case the condenser voltage, v_2, rises and falls between 0·23 $\hat{V}$ and 0·45 $\hat{V}$.

Pulse Modulation

Since a sine-wave alternating current, or voltage, is completely defined by stating its amplitude, frequency and phase, there are only three possible ways of modulating it to make it carry a lower frequency signal. These are, first, amplitude-modulation (in which the amplitude is varied in accordance with the instantaneous value of the low-frequency signal); second frequency-modulation (in which the frequency is varied in the same way); and third, phase-modulation (in which the phase is varied in the same way).

A whole set of further methods of modulation becomes available, however, if the high frequency sine-wave is first amplitude-modulated by a system of recurrent pulses, i.e. if the h.f. voltage, or current, is switched on and off so that its envelope becomes as shown in Fig. 169. For example, the width t' of the pulses may now be varied in accordance with the low-frequency signal, giving *pulse-width modulation*; or the time of occurrence of each pulse may be modulated (i.e. varied), being advanced or retarded by a time which is proportional to the instantaneous value of the low-frequency signal, so giving *pulse-position modulation*. One could also vary the pulse repetition frequency in accordance with the signal. It will be seen that in such systems there are *two successive modulation processes*: in the first the pulse-voltage (whose waveform is as in Fig. 169) is modulated by the low-frequency signal; and in the second, the high-frequency sine wave is amplitude-modulated by the pulse-voltage.

The first of these processes is sometimes spoken of as a Sampling process, since the waveform of the low-frequency signal is, so to speak, inspected at a number of instants, (viz. the instants at which the pulses occur) and some attribute of the pulse (e.g. its width) is adjusted in accordance with the results of the inspection. Those portions of the signal waveform which occur *between* the inspections contribute nothing to the final modulated voltage. In fact the complete signal waveform is not transmitted but merely those portions which occur when the h.f. pulse is "switched on"; the signal is transmitted only by transmitting "a number of points on the graph," and the receiver may be thought of as drawing a smooth graph through these points. For there to be sufficient points on the graph, the pulse-repetition frequency must of course be higher than the highest signal-frequency which is to be transmitted. The frequency of the h.f. sine-wave carrier must also be large with respect to the pulse-repetition frequency.

Some advantages of communication by pulse-modulation methods are as follows—

(i) Economy of power and reduction in the rating of components, since the transmitter is switched on for only a portion of the time.

(ii) The possibility of multi-channel communication with a single carrier-frequency, by sampling a number of different signal-waveforms in a regular repeated sequence.

(iii) The ability to modulate oscillators which are not easily susceptible to amplitude-variation but which can be easily switched on and off by the pulse-voltages.

It would not be appropriate to attempt a fuller discussion of pulse-modulation systems here, but some features of a hypothetical pulse-modulation system will now be described for the purpose of illustrating some of the operations which require to be carried out upon pulses. The manner in which these operations are carried out will then be explained in the later sections of this chapter. Superscript reference numbers refer to the list of operations on page 295.

Consider a four-channel width-modulated system, the envelope of the transmitted h.f. sine wave being as shown in Fig. 171. If this waveform were studied over a longer period of time than is shown in the figure, it would be seen that the width of the pulses marked *A A A* . . . , is not constant. This width, in fact, varies in accordance with the instantaneous value of the signal in channel *A*. The width of the pulses marked *B B B* . . . varies quite independently, since it is controlled by quite a different signal, viz. the signal in channel *B*. Similarly for channels *C* and *D*. The pulses marked *S*, and known as synchronizing pulses, are included for the purpose of identifying the individual channel pulses: at the receiver, the *A* channel pulses are identifiable as those pulses which occur first after the synchronizing pulses.

The modulating apparatus includes a sinusoidal oscillator whose frequency is $1/T$, where T is as shown in Fig. 171. Next comes a circuit[1*] which, when fed with the output voltage of this oscillator, produces the rectangular pulses *S*. The first step in producing the channel pulses, *ABCD*, is the generation of a triangular[2] or sawtooth waveform which is of periodic time T and is initiated by the synchronizing pulses *S* (see upper graph in Fig. 172). This sawtooth

* The reference numbers refer to the list of operations on page 295.

voltage is fed in parallel to the input circuits of four valves, each of which has a bias-voltage more negative than the cut-off value. These valves would normally be non-conducting in the absence of the sawtooth input-voltage, but each conducts when this input voltage sufficiently overcomes the bias-voltage. By making the

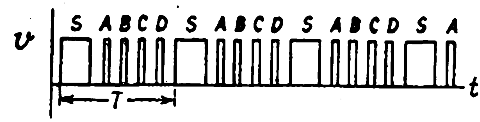


FIG. 171. FOUR-CHANNEL PULSE MODULATION

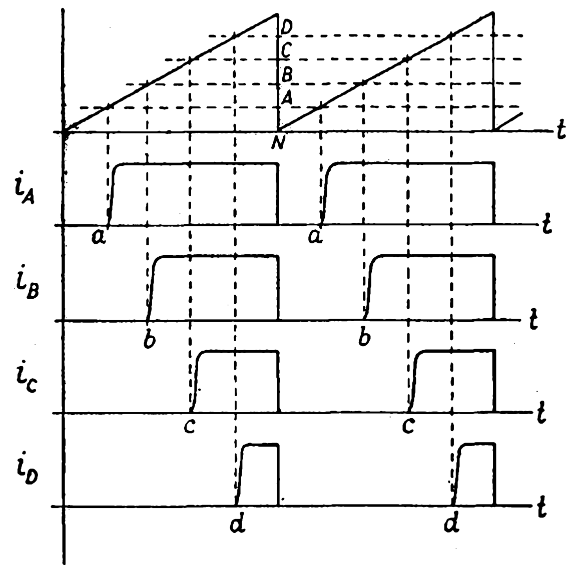


FIG. 172. AMPLITUDE COMPARISON

bias-voltages different in the four cases,[3] it can be arranged that the valve A conducts first, then valve B, then valve C, and finally valve D. This is illustrated in Fig. 172, in which AN, BN, CN, and DN represent the positive values of input voltage required to make the respective valves conduct. Each of these four stages is followed by a Differentiating Circuit,[4] i.e. a circuit which gives an output voltage proportional to the rate of change of the input voltage. The output voltages of these four differentiating circuits will thus consist of positive pulses commencing at the instants a, b, c, and d respectively, together with negative pulses corresponding to the steep portions of the original sawtooth. The positive pulses are the required channel pulses; the negative pulses are not required and can be removed[5] from the waveform.

The next step is the process of width-modulating the channel pulses in accordance with the four low-frequency signals. We need consider only one channel. First, the channel pulse is converted into a positive sawtooth[6] voltage of short duration. This sawtooth voltage is now fed to the input circuit of a valve which uses the

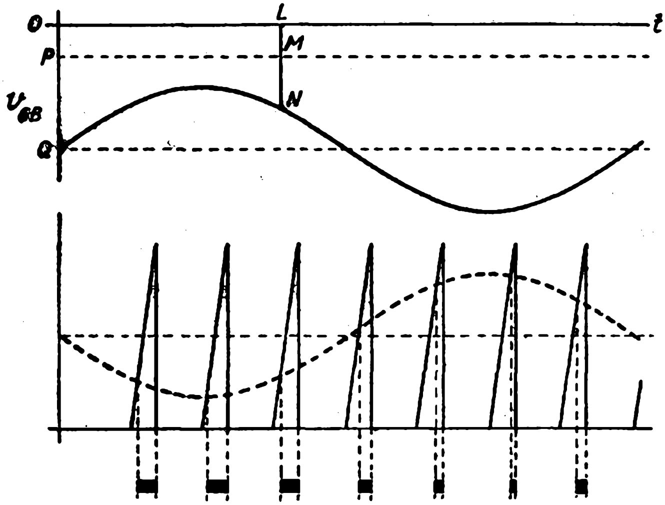


FIG. 173. PULSE-WIDTH MODULATION. ADDITION AND AMPLITUDE SELECTION

low-frequency signal as its negative-bias voltage. When the positive-sawtooth voltage overcomes the negative-bias voltage, the valve will conduct.[7] The greater the negative bias, the smaller will be the time for which the valve conducts. Thus the *duration* of the flow of anode-current is controlled by the low-frequency signal as illustrated in Fig. 173. The top graph in this figure shows the grid-bias voltage plotted against time. This grid-bias voltage is the sum of a steady voltage *OQ* and the low-frequency signal voltage. The cut-off value of grid-voltage is represented by *OP*, so that the required positive voltage for anode-current to flow is given at any instant by the vertical distance between the line *PM* and the curve (e.g. at the instant *L*, a positive voltage *MN* would be required). The lower graph shows, superposed, the sawtooth-input voltage and this required positive voltage. The short horizontal lines at the bottom of the figure represent the time-duration of the resulting current flow, and it will be seen that width modulation has been achieved. It remains to shape[8] the resulting pulses so that they are rectangular and of constant amplitude, and to combine the four sets of width-modulated pulses and the synchronizing pulses. The resulting voltage waveform will be as in Fig. 171, and may now be used to modulate the sinusoidal h.f. carrier.

The receiving apparatus is similar to an orthodox radio receiver, up to and including the detector stage, the output voltage of which has a waveform similar to Fig. 171. It is now necessary to separate the four channels, and for this purpose the synchronizing pulses are first extracted. This is accomplished by an Integrating Circuit,[9] i.e. a circuit whose output voltage at a given instant is proportional to the area under the graph of the input voltage up to the instant. If the input voltage be the alternating component only of Fig. 171, the output voltage will be a series of triangular pulses, that corresponding to the synchronizing pulse being of considerably greater amplitude than those corresponding to the channel pulses. The smaller pulses may now be removed from the waveform[10] (e.g. by the use of a valve stage with so great a bias-voltage that only the larger pulses cause anode-current to flow).

After extraction in this way, the synchronizing pulses are used to produce a duplicate set of channel pulses, in the same way that the synchronizing pulses were first used in the transmitter (references 2, 3, 4, and 5 above). These duplicate channel pulses are, of course, unmodulated, and exist in four separate circuits A, B, C, and D. They are arranged to be slightly wider (i.e. of longer duration) than the maximum width of the incoming modulated pulses. They are to be used for separating the incoming channel pulses into four separate circuits, one for each channel.

This process of separation is carried out in four Gating Circuits[11] (or "gate circuits"), each consisting of a pentode in which grid 1 and grid 3 are both used as control grids. Let us consider the gating circuit which isolates the pulses of channel A. The whole output voltage of the detector, (viz. the whole incoming signal as in Fig. 171) is fed to grid 1. Anode current is prevented from flowing, however, by the use of a sufficient negative-bias voltage in the circuit of grid 3.

Grid 3 may, in fact, be considered as a switch which renders the pentode inoperative except when some positive voltage appears in the circuit of grid 3 and overcomes the negative bias in that circuit. We clearly wish the valve to become operative only during the times when the modulated A-channel pulses are occurring. Accordingly we feed the duplicate A-channel pulses to the circuit of grid 3. The output voltage of this gating valve then consists of the modulated A-channel pulses only. The pulses of the other three channels are isolated in a similar way by the other three gating valves.

It remains to carry out the inverse process of width-modulation, i.e. to recover the low-frequency modulating voltage from these width-modulated pulses. This can be done simply by means of a low-pass filter, but the following alternative method illustrates certain possibilities. The width modulated pulses (assumed to be positive pulses for the purpose of argument) are fed to a differentiating circuit.[12] The output voltage of the differentiating circuit will consist of a series of very narrow positive pulses corresponding to the leading edges of the width-modulated pulses, and a series of very narrow negative pulses corresponding to the trailing edges. Now the trailing edges are equally and regularly spaced along the time axis but the leading edges are position-modulated (see Fig. 173). By removing[13] the narrow negative pulses from the waveform, we are left with a series of position-modulated narrow, positive pulses. These are now used to trigger a sawtooth generator,[14] for instance, they may be used to operate the "automatic switch" in the basic time-base circuit of Fig. 120 (page 207). Between the pulses, the condenser voltage of Fig. 120 will rise in a sawtooth fashion, while on the occurrence of a pulse the condenser voltage will fall to zero and immediately begin to rise again. The "teeth of the saw" will be irregular because the triggering pulses are position-modulated. Some of the teeth will be higher than others because the interval between the triggering pulses is longer for those teeth than for others. In short, the resulting sawtooth voltage is *amplitude modulated.* An orthodox a.m. detector may now be used to recover the low-frequency modulating voltage.

Operations upon Pulses and Associated Waveforms

The above description of a multi-channel pulse-modulation system (though necessarily condensed) may serve to introduce the reader to many of the usual operations upon pulses and associated waveforms. Superscript numbers, 1 to 14, have been used throughout the description for ease of reference to the more important operations, which may now be listed as follows—

1. The production of pulses from sine-waves.

2. The generation of triangular waveforms, to be initiated by pulses.

3. The production of a pulse at the instant when a given waveform reaches a certain prescribed amplitude. This process is known as *amplitude comparison.*

4. Differentiation with respect to time.

5. The removal of that part of a waveform which is above or below a prescribed amplitude, or between certain limits of amplitude. (Alternatively, the selection of only that part of a waveform which is above or below a prescribed amplitude, or between certain limits of amplitude.) This process is known as *amplitude selection.*

6. As item 2 above.

7. *Modulated amplitude selection.* Similar to item 5 above, except that the "prescribed amplitude" is a variable.

8. The shaping of pulses.

9. Integration with respect to time.
10. As item 5 above.
11. The process of "gating," or selecting those parts of a waveform which occur between certain prescribed times. This process has the general name of *time selection*.
12. As item 4 above.
13. As item 5 above.
14. As item 2 above.

To this list must be added such obvious processes as the generation of pulses, their amplification, addition and subtraction, multiplication and division. There is the process of "d.c. restoration" (page 322) and the more refined processes of pulse counting and pulse delaying. The list is, then, far from being complete, but many of the more complex processes can justly be omitted from such a list on the grounds that they are simply combinations of these basic processes. For further information the reader is referred to item 5 of the bibliography at the end of this chapter.

Differentiating and Integrating Circuits

If the graph shown in Fig. 174 (*a*) be differentiated with respect to time (i.e. if we plot the slope of the graph against time) the result is as shown in Fig. 174 (*b*), and consists of a series of pulses, each of infinite amplitude and of infinitesimally small duration, the pulses being alternately positive and negative. If the sides of the original pulses in Fig. 174 (*a*) were not quite vertical, then the pulses derived by differentiation would have a finite amplitude and duration. A differentiating circuit is a circuit which gives an output voltage whose graph against time is the graph obtained by differentiating the input-voltage graph with respect to time. Such circuits may be used to derive narrow voltage pulses from any input voltage whose graph shows a steeply sloping portion. Similarly, an integrating circuit is a circuit which gives an output voltage whose graph against time is that graph which would be obtained by integrating the graph of the input voltage with respect to time.

If the graph shown in Fig. 174 (*a*) be integrated with respect to time (i.e. if we plot against time the area under the graph, measured up to the instant in question) the result is as shown in Fig. 174 (*c*). This graph is not periodic, the reason for this being that the graph of Fig. 174 (*a*) contains a d.c. component a_0, the integration of which gives a term a_0t. If the input voltage of an integrating circuit be only the alternating component of a train of voltage pulses, as in Fig. 174 (*d*), then the integrated waveform will be as shown in Fig. 174 (*e*).

The actual circuits used as differentiating circuits and integrating circuits do not give an output voltage which is accurately equal to the required differentiated or integrated voltage, but the result approximates to what is required. The simplest differentiating circuit is that shown in Fig. 175 (*a*). Considering instantaneous values of voltage and current for this circuit we have

$$i = C\frac{dv_c}{dt}$$

Also
$$v_2 = Ri = RC\frac{dv_c}{dt}$$

Now the input voltage, v_1, is equal to $v_c + v_2$, and if the resistance R is made smaller and smaller v_c tends to equality with v_1; and

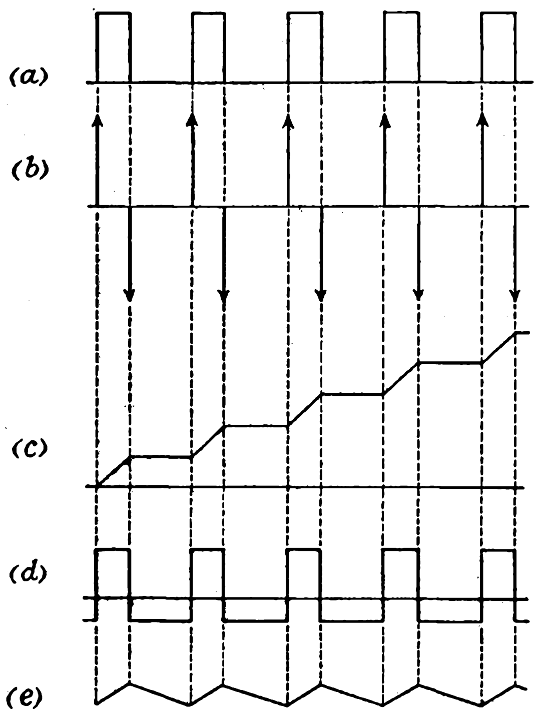


FIG. 174. DIFFERENTIATION AND INTEGRATION OF WAVEFORMS

v_2 tends to zero. In these circumstances the output voltage v_2, although very small, may be written

$$v_2 = RC\frac{dv_c}{dt} \simeq RC\frac{dv_1}{dt} \quad . \quad . \quad . \quad \text{(VIII.5)}$$

If the input voltage applied to this circuit be a train of recurrent rectangular pulses, and if the product RC be small with respect to the pulse duration, the waveform of the output voltage may be calculated by considering separately the effect of each switching-on

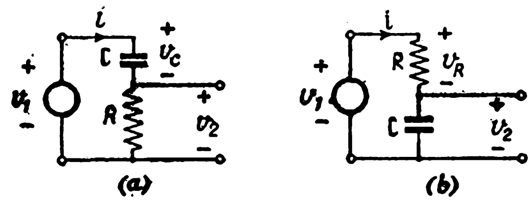


FIG. 175

(a) Differentiating circuit
(b) Integrating circuit

process and each switching-off process. The resulting waveform of Fig. 176 is at least a first approximation to the true differentiated waveform of Fig. 174 (b). The width of the output pulses may be reduced (thus improving the approximation) by decreasing the time-constant RC. In theory this should cause no reduction in the magnitude of the output pulses, which should remain equal to $\hat{V}$ as shown in Fig. 176. In practice, however, the best "rectangular" input pulses have sides whose slope is finite (not infinite), and equation (VIII.5) shows that in this case a reduction of the time-constant, RC, will reduce the magnitude of the output pulses.

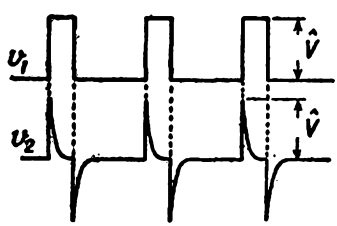


FIG. 176. QUASI-DIFFERENTIATION WITH THE CIRCUIT OF FIG. 175 (a)

The simplest integrating circuit is that shown in Fig. 175 (b). Considering instantaneous values of voltage and current for this circuit we have

$$v_2 = \frac{1}{C}\int i \, dt = \frac{1}{C}\int \frac{v_R}{R} \, dt$$

Now the input voltage v_1, is equal to $v_R + v_2$, and if the resistance R is made larger and larger v_R tends to equality with v_1, and v_2 tends to zero. In these circumstances the output voltage v_2, although very small, may be written

$$v_2 = \frac{1}{RC}\int v_R \, dt \simeq \frac{1}{RC}\int v_1 dt \quad . \quad . \quad \text{(VIII.6)}$$

If the input voltage applied to this circuit be a train of recurrent rectangular pulses, and if the product RC be large with respect to the pulse duration it is fairly simple to show that the graph of the output voltage is as shown in the second line of Fig. 177. This is a first approximation to the waveform of Fig. 174 (e). The circuit has therefore not integrated the input waveform, but has integrated the alternating component of the input waveform. Moreover, the output waveform has a d.c. component in addition to the desired integrated waveform, this d.c. component being equal to the d.c. component of the input voltage. Nevertheless, the circuit is known as an integrating (or quasi-integrating) circuit, and serves to produce output voltages which have a linearly increasing or decreasing portion

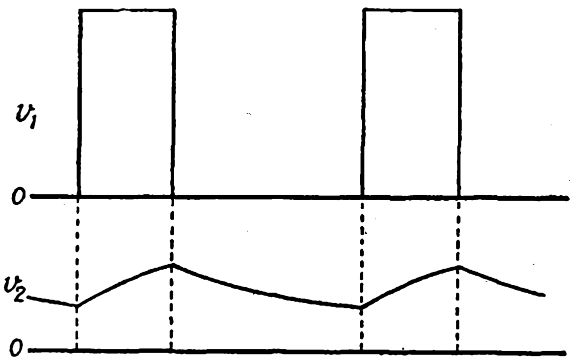


FIG. 177 QUASI-INTEGRATION WITH THE CIRCUIT OF FIG. 175 (b)

when fed with input voltages having a waveform which includes a flat horizontal portion.

It is instructive to compare this simple integrating circuit with the a.c./d.c. separating circuit of Fig. 27. The two circuit configurations are identical and the design rule for the a.c./d.c. separating circuit, (viz. $R \gg 1/\omega C$) amounts to the same thing as the design rule for the integrating circuit, (viz. $RC \gg T$). The output terminals are those from which, in an a.c./d.c. separating circuit, one would expect to derive the d.c. component. This d.c. component will not be completely free from alternating components since R cannot be *infinitely* large with respect to $1/\omega C$. We see that the waveform of the residual alternating component across the d.c. output terminals is an approximation to the integral of the alternating component of the input voltage.

The Generation of Pulses

Voltages having pulse waveform may be derived from many of the circuits described in the earlier chapters of this book. Pulses of anode-current occur in a Class C amplifier or oscillator (Fig. 92) and in a diode detector (Fig. 128). Pulses of grid-current occur in a cumulative-grid detector (Fig. 135) and in a self-bias (rectification-bias) circuit. The output voltage of an asymmetrical multivibrator (e.g. Fig. 112 with unequal time constants) consists of narrow pulses of one polarity alternating with wider pulses of the opposite polarity. In time-bases developed from the basic circuit of Fig. 120,* pulses of current flow in the leads of the condenser, C, and voltage pulses may be derived from a resistance inserted in these leads. The fact that the pulses generated by these circuits are not rectangular is not a serious disadvantage, since shaping circuits (see page 307) may be used to make them very nearly so.

Fairly narrow pulses may be produced by differentiating any waveform which has sudden changes of slope, e.g. a square or

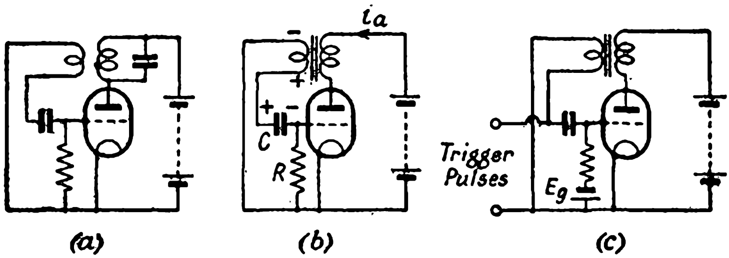


FIG. 178

(a) Orthodox tuned-anode mutual-coupled oscillator
(b) Free-running blocking oscillator
(c) Triggered blocking oscillator

sawtooth waveform or the output voltage waveform of a multivibrator, or of many relaxation oscillators. When used for this purpose, a differentiating circuit is often referred to as a *Peaking Circuit.* Still narrower pulses may be generated by the use of a monostable multivibrator (page 204), the pulse duration being the time taken for the circuit to revert to its stable state after it has been triggered to its unstable state. Triggering pulses are needed to operate such a monostable circuit, but these need not be particularly narrow pulses. Monostable circuits, such as the monostable multivibrator, may also be used as Delay Circuits, in which case the incoming pulse is used for triggering, and the required delayed pulse is derived by differentiation of the output waveform. The delayed pulse is the result of differentiating the steep part of the output waveform which corresponds to the "landslide" by which the circuit returns to its stable state.

A much-used pulse generator is the *Blocking Oscillator.* There

* This circuit (Fig. 120) is in fact the basic circuit of many so-called "driven pulse generators," the automatic switch S being a hard valve which is normally maintained non-conducting by the use of an adequate negative grid-bias voltage, but which is rendered conducting for a very short interval by the application of a positive "driver pulse" in its grid-circuit. Such "driven pulse generators" are in reality pulse power amplifiers and we shall not consider them further.

are several forms of blocking oscillator but all consist of an orthodox feed-back type oscillator using a grid-circuit similar to a self-bias (or "rectification-bias") circuit and using a transformer to provide the feed-back. The oscillator-tuning condenser is usually omitted. Fig. 178 (*b*) shows a blocking-oscillator circuit derived from the sinusoidal oscillator of Fig. 178 (*a*).

It is convenient to think of a blocking oscillator as the limiting case of a squegging oscillator (page 176). The difference between the behaviour of a squegging oscillator and that of a blocking oscillator is that in the former several positive half-cycles of grid voltage occur before the condenser *C* acquires a voltage sufficient to prevent further oscillation, whereas in the blocking oscillator this occurs in the first positive half-cycle. The waveform of a blocking oscillator thus consists of a large positive pulse of grid-voltage (accompanied by a large negative pulse of anode-voltage), after which the charge on the condenser *C* leaks away through *R* until anode-current is no longer cut off, the pulse then being repeated, and so on. This difference in performance is secured by increasing the feed-back coupling, probably to such an extent that the circuit equation has so large a negative damping term as to be non-oscillatory. Instead of sinusoidal oscillation, therefore, we have a "landslide" similar to that which occurs in the multivibrator, and it is not really permissible to speak of "the first half-cycle." The mechanism of the landslide is readily to be understood: if for any reason the anode-current i_a (Fig. 178 (*b*)) is increasing, an e.m.f. of the polarity shown will be induced in the grid winding of the transformer, and this will further increase the anode-current by raising the grid-voltage. Similarly any decrease in the anode-current induces a grid-voltage which enhances the decrease. During the landslide in which the anode-current is increasing (*AB* Fig. 179) the grid-voltage is driven extremely positive, and the resulting grid-current begins charging the condenser *C* with a grid-making-negative polarity. The subsequent landslide *CD* drives the grid negative and the charge on *C* holds the grid-voltage below the cut-off value. Anode-current therefore ceases while the condenser *C* discharges through *R*, the grid-voltage approaching zero exponentially. When the point *E* is reached, anode-current recommences and the cycle is repeated. Traces of the shock-excited natural oscillation of the transformer windings are usually visible on an oscillogram; the *Q*-value of the transformer is usually made low to minimize these oscillations.

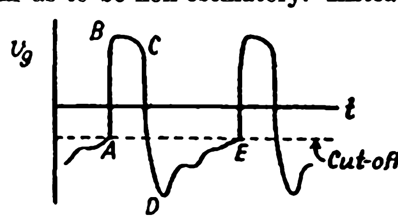


FIG. 179. GRID-VOLTAGE WAVEFORM OF BLOCKING OSCILLATOR

The blocking oscillator may be made monostable, as indicated in Fig. 178 (c), by the inclusion of a bias-voltage E_g greater than the cut-off value. The portion *DE* of the waveform is then no longer asymptotic to zero but to a value $-E_g$, and pulses do not occur except in response to triggering. When the pulse repetition frequency is required to be held at a precise value, it is advisable to use a monostable blocking oscillator in preference to a free-running version, the trigger pulses being derived from a master

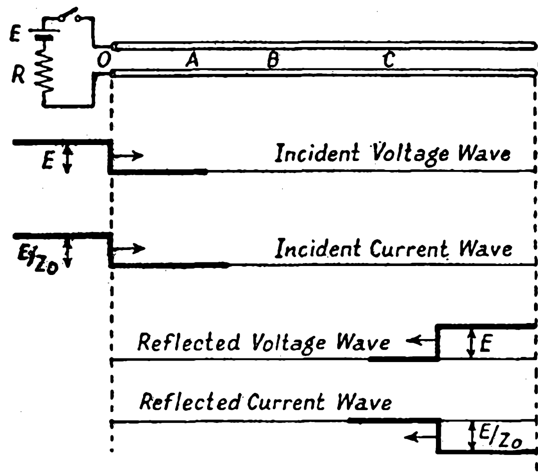


FIG. 180

oscillator. Large amplitude pulses of duration less than one micro-second can be successfully generated by a blocking oscillator provided that care is taken in designing the transformer.

Where narrow pulses with a precisely-known duration are required, *Delay-line Pulse Generators* may be used. There are many forms of delay-line pulse generators but all depend upon the fact that any variations of voltage which are applied at the input end of a line do not immediately appear at the other end. For instance, if a steady (i.e. d.c.) voltage be suddenly connected between the conductors at the end *O* of the two-wire line shown in Fig. 180, no voltage will appear between the conductors at the point *A* until a short time has elapsed. The "switch-on" will occur at point *B* even later than at the point *A*, and later still at points *C* and *D*. If the effects of resistance be neglected, it can be shown that the voltage between the conductors at any point of the line and at any instant is given by the ordinate (at that point and at that instant) of a travelling graph. This travelling graph is a graph of voltage against distance; its shape is deduced from the consideration that in travelling past the input end of the line it must produce the required time variation of the ordinate at the point O (corresponding to the known time-waveform of the input voltage). Its speed of travel is constant and given by $1/\sqrt{LC}$, where L and C are respectively the inductance per unit length of line and the capacitance per unit length of line. For the switching-on of a d.c. voltage, the travelling graph will clearly have the "step" shape shown in Fig. 180. Further relevant properties of lines may be summarized as follows—

(i) The above travelling graph of voltage is accompanied by a travelling graph of current, the two graphs being of exactly the same shape and their ordinates being related by

$$v/i = \sqrt{L/C}$$

where the quantity $\sqrt{L/C}$ is a pure resistance, known as the Characteristic Impedance of the line, and denoted by Z_0.

(ii) When travelling graphs reach the far end of a line they are usually reflected; but there is no reflection if the far end of the line is closed by a resistance equal in magnitude to Z_0.

(iii) When more than one travelling graph occurs at the same time (e.g. as a result of reflection) the total voltage between the conductors at any point and at any instant is the sum of the ordinates of all the travelling graphs (at that point and at that instant).

(iv) At an open-circuited end, a travelling graph of voltage is reflected without change of sign, and a travelling graph of current is reflected with* change of sign as shown in Fig. 180. At a short-circuited end, a travelling graph of voltage is reflected with* change of sign, and a travelling graph of current is reflected without change of sign.

(v) It follows from (i) that when only one travelling graph exists (e.g. when the far end of the line is closed by a resistance equal to the characteristic impedance, so that no reflection occurs), the ratio of the voltage and current at all points of the line and at all instants is equal to Z_0. In particular, the input impedance of a line on which only one travelling graph exists at the input end is equal to Z_0. This means that, whatever the conditions at the far end of the line, the input impedance of the line is equal to Z_0 until such time as the first reflected graph arrives back at the input end.

Fig. 181 shows how a short-circuited line may be used to produce narrow pulses. The input end of the line is fed from a square-wave generator whose internal impedance is arranged to be equal to Z_0, the characteristic impedance of the line. This ensures that waves returning to the input end, after reflection at the short-circuit, are not further reflected but are absorbed. The behaviour of the system is illustrated in Fig. 181 by showing the travelling graph in successive positions in lines 1 to 14 of the diagram. The left-hand dotted vertical and the right-hand dotted vertical both correspond to the input

* Clearly this must be so, since the sum of the incident and reflected graphs at the end of the line is required to be zero in these two cases.

end of the line, and the diagram should be thought of as being folded along the middle vertical line (thus superposing the incident and reflected graphs) for the reflected graphs to travel in the correct direction. The waveform of the travelling graph will be the same as the time-waveform of the applied e.m.f. e, viz. alternately zero

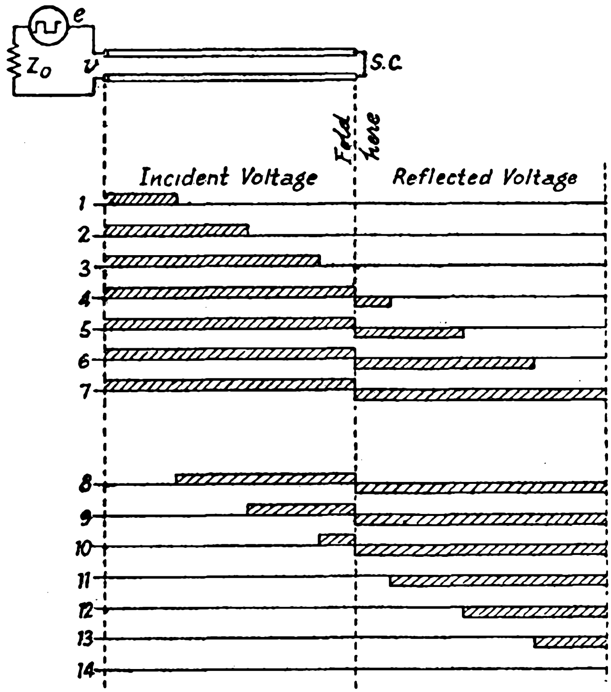


FIG. 181. SHORT-CIRCUITED DELAY LINE AS PULSE GENERATOR

and a constant positive value, with vertical "steps" from one to the other. In line 1 of the diagram the first of these steps is shown advancing along the line. When it reaches the short-circuited end, the graph is reflected with change of sign (lines 4 to 7). The total voltage, v, at the input end of the line is the sum of the ordinates at the left-hand and right-hand verticals. In lines 1 to 6, this total voltage is positive. In line 7, it is zero, since the inverted reflected graph has reached the input end of the delay line. This occurs at a time $2l/c$ after the occurrence of the positive step of applied e.m.f., where l is the length of the line and c the velocity of propagation along the line. The conditions illustrated in line 7 persist until the occurrence of the negative step of applied e.m.f. Since the periodic time of the applied e.m.f. is very much greater than $2l/c$, it follows that a much greater time elapses between lines 7 and 8 of the diagram than between lines 6 and 7. Also, the total voltage, v, at the input end of the line remains zero for a very much longer period than that for which it was positive. In fact only a very short positive pulse of input voltage, v, occurs (duration $2l/c$) immediately following the positive step of applied e.m.f., e. Similarly, by tracing the progress of the negative step, it can be seen that there is a short negative pulse of input voltage immediately following the negative step of applied e.m.f. The amplitude of these pulses is one half of the peak amplitude of the applied e.m.f., since during the occurrence of the pulses the input impedance of the line is equal to the internal impedance of the source.

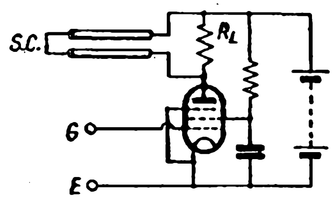


FIG. 182. CIRCUIT OF A DELAY-LINE PULSE GENERATOR

Fig. 182 shows the circuit of a pulse generator using this principle. The grid-circuit is supplied with a square-wave voltage sufficient to switch the anode-current regularly on and off. The equivalent circuit of the network connected to the input of the delay line is seen (by Thevenin's Theorem) to be a square-wave e.m.f. of peak value $R_L i_{a\,max}$ in series with a resistance equal to the parallel combination of R_L and the valve impedance. This resistance should be made equal to Z_0. Since the valve impedance of a pentode is so high, this means that R_L should be made equal to Z_0. The equivalent circuit then corresponds exactly to Fig. 181, and the voltage across R_L (being the input voltage of the delay line) consists of narrow pulses as shown in Fig. 183. Since the pulse amplitude is $\frac{1}{2}R_L i_{a\,max}$ it follows that R_L, and therefore Z_0 should be as large as possible. This particular circuit is therefore used with a high-impedance delay line, usually an artificial line formed of several lumped series inductances and shunt capacitances.

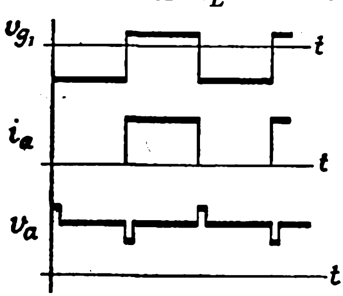


FIG. 183. DELAY-LINE PULSE GENERATOR WAVEFORMS

Instead of using a resistor to provide the resistance R_L in Fig. 182, we may use a second delay line, having the same characteristic impedance as the first, with its isolated end terminated by a resistance equal to this characteristic impedance. The input impedance of such a delay line is equal to Z_0 and can take the place of R_L. The generated pulses then appear across the input terminals of this second delay line, are propagated along it, and appear across its terminating resistance after a time delay determined by its length. Thus we have a pulse generator which produces two pulses with an adjustable time delay between them.

Delay lines may be incorporated into other pulse generators (e.g. the monostable multivibrator or the blocking oscillator) to control the pulse duration accurately. For example, a short-circuited delay line may be connected between grid and cathode of the normally-off valve in a monostable multivibrator. When the circuit is triggered, and the grid of this normally-off valve is driven positive by the resulting landslide, a large positive step of voltage travels along the delay line. After a time $2l/c$ the inverted reflected step returns to the input end of the line, i.e. a negative step of voltage is applied between grid and cathode. This serves to precipitate the return landslide, cutting off the "normally-off" valve and leaving the circuit once more in its stable condition. The width (i.e. time duration) of the resulting pulse is thus reduced to the value $2l/c$. An alternative method is to use an open-circuited delay-line in place of a coupling condenser.

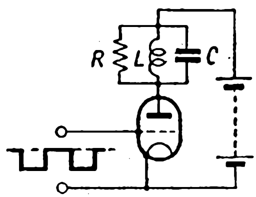


FIG. 184. RINGING CIRCUIT

A circuit which is sometimes used for the generation of fairly narrow pulses is the *Ringing Circuit* of Fig. 184. This is simply a triode-amplifier stage with a tuned load, its grid circuit being fed with negative-going square waves whose amplitude is sufficient to switch the anode-current regularly off and on. The sudden switching-off of anode-current, shock-excites natural oscillations in the tuned circuit in much the same way as a bell may be rung by striking it with a hammer. The shunt resistance R is included to ensure that these natural oscillations decay rapidly. The sudden *switching-on* of anode-current does not produce natural oscillation since, when anode-current is flowing, the valve-impedance is effectively connected across the tuned circuit, with the result that the circuit is so heavily damped as to be non-oscillatory. The resulting waveform of anode-voltage is shown in Fig. 185. The width t' at the base of the positive pulses is approximately $\pi\sqrt{LC}$. The waveform of the input voltage need not be square, but must include a steep negative-going step.

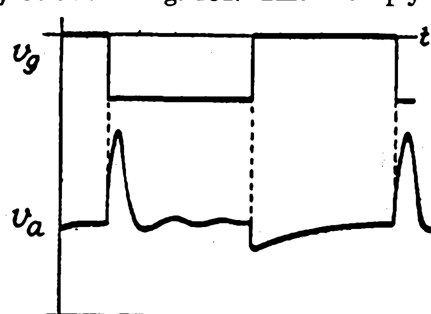


FIG. 185. WAVEFORMS IN RINGING CIRCUIT

The Shaping and Sharpening of Pulses by Amplitude Selection

The name "Shaping Circuit" often connotes circuits which convert a waveform to any desired shape, but we shall use it in the restricted sense of circuits which, when fed with input voltage pulses of non-rectangular waveform, give output voltage pulses whose waveform is rectangular.

Fig. 186 illustrates the most used principle of shaping. We require a circuit whose voltage-current graph (or whose input-voltage/output-voltage graph) is of the form shown by the thick line in the top left part of the figure. The application to such a circuit of the non-rectangular pulse voltage shown in the lower part of the figure will then give a rectangular output pulse as shown.

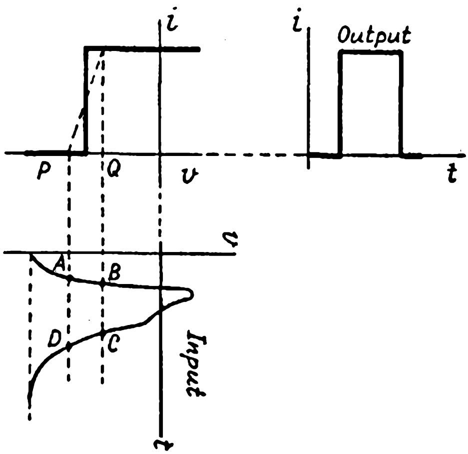


FIG. 186. PRINCIPLE OF SHAPING

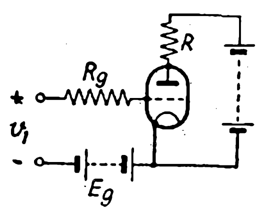


FIG. 187. SIMPLE SHAPING CIRCUIT

The amplitude of the output pulse is independent of the input voltage, being determined entirely by the vertical height of the "step" in the circuit characteristic graph. If the step is not in the form of a vertical line, but is sloping as shown by the broken line, the sides of the output pulse will not be vertical but will depend upon the slope of those portions of the input waveform (*AB* and *CD*, Fig. 186) which lie between the same voltage limits as the break in the characteristic. It is therefore desirable that the slope of these portions shall be large. It may be made so by amplification of the input waveform before applying it to the shaping circuit, so making its total amplitude as large as conveniently possible with respect to the voltage-interval, *PQ*, of the circuit characteristic.

A simple shaping circuit is shown in Fig. 187. When the input voltage v_1 is large and negative, anode-current is cut off. When v_1 is large and positive, the grid-voltage is positive and grid-current flows through R_g and the grid-cathode space of the valve. The effective resistance of this grid-cathode space (for positive grid-voltages) is of the order of 1,000 ohms. If R_g be made large with respect to this (e.g. 100,000 ohms or more) the grid-cathode voltage will be, by voltage-divider action, of the order of 1 per cent or less of the applied positive voltage, $(v_1 - E_g)$. This means that the grid-cathode voltage v_g will be equal to $(v_1 - E_g)$ when this quantity is negative, but will be neglibibly small when $(v_1 - E_g)$ is positive. We may say that, as $(v_1 - E_g)$ rises above zero, v_g remains effectively "clamped" at zero. The graph of anode-current against input voltage is thus of the required form for shaping.

Fig. 188 shows how this circuit may be used to produce pulses when using a large amplitude sine-wave input together with a value of E_g much greater than the cut-off value of v_g. By adjusting E_g to equal the cut-off value, the circuit may be made to shape sine waves into square waves. The pulses of Fig. 188 may be made more nearly rectangular, i.e. their sides may be further steepened, by amplification and further shaping of the same kind. The pulses so produced may be "sharpened," or made of shorter duration by differentiation, amplification and further shaping in a circuit similar to Fig. 187.

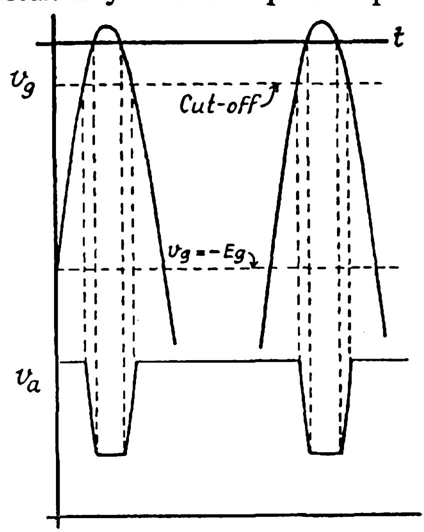


FIG. 188. PRODUCTION OF PULSES BY SHAPING OF SINE WAVE

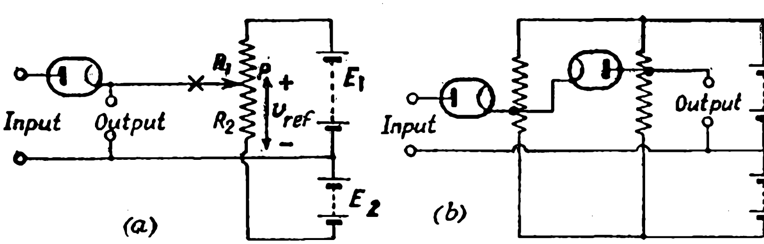


FIG. 189. AMPLITUDE SELECTION WITH SERIES DIODES

The shaping process effected by the circuit of Fig. 187 is really the operation of *amplitude selection* (see page 295) and many circuits have been devised to perform this operation. For example, the output voltage of the circuit of Fig. 187 (*a*) consists of only those portions of the voltage waveform which are above the reference-voltage level $(E_1R_2 - E_2R_1)/(R_1 + R_2)$. If the diode in this circuit were reversed, the output voltage would consist of only those portions of the input voltage waveform which were *below* this same voltage level. The diode acts as a series switch, and only when it is conducting is the input voltage communicated to the output terminals. When it is not conducting (i.e. when its anode is at a lower potential than its cathode) the output voltage is "clamped" at the reference level. When it is conducting, the potential of the point P is no longer at the reference level, for the input voltage is causing additional currents to flow in the voltage-divider resistances, with resulting RI drops. The load presented to the input voltage source is (if the impedance of the diode be neglected) the parallel combination of the resistances R_1 and R_2; this should be made large with respect to the output impedance of that source. Alternatively, a resistance may be inserted at the point X. By cascading two such circuits, as in Fig. 189 (*b*), we can arrange that the output voltage shall consist of only those portions of the input waveform which lie between certain reference levels.

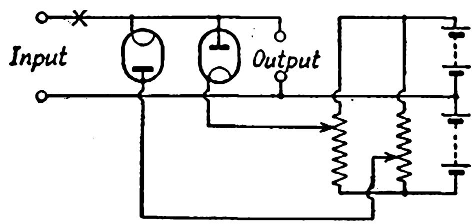


FIG. 190. AMPLITUDE SELECTION WITH SHUNT DIODES

The same results may be achieved by using diodes as *shunt switches* as shown in Fig. 190, which simply consists of two diodes, each in series with a biasing-off voltage, connected directly across the input voltage. When either of the diodes conducts, it may be thought of as short-circuiting the input voltage—or more accurately, as "clamping" the output terminal at a reference level equal to the biasing-off voltage. For the short-circuiting action to be fully effective the resistance values in this circuit are required to be small with respect to the internal impedance of the source; alternatively, the latter may be artificially increased by inserting a resistance at the point X. The action of such circuits is variously described as *clamping* or *limiting*. Diodes performing this function are often built into other circuits, in addition to their use in stages which perform this sole function. For example, a shunt diode may be used to prevent the potential of any point in a circuit from rising above a certain reference level (in which case the *anode* of the diode is connected to the point in question and the *cathode* to a point at the reference level) or from falling below a certain reference level (in which case the *cathode* of the diode is connected to the point in question and the *anode* to a point at the reference level). The clamping of the grid-voltage at the value zero in Fig. 187, is really accomplished by a shunt diode, viz. the diode formed of the grid and cathode of the valve. Instead of making use of the phenomenon of cut-off in that circuit, we could add a further shunt diode with its cathode connected to the grid of the triode and its anode connected to a point at the required negative reference level.

We have assumed, hitherto, that the anode-current of a diode is zero for negative anode-voltages and finite for positive anode-voltages, i.e. that anode-current commences to flow as the anode-voltage increases through the value zero. Both measurement and theory show, however, that this is not accurately true; instead, anode-current commences to flow as the increasing anode-voltage reaches a small negative value. This means that the biasing voltage must be made slightly different from the desired "slicing" level, but this in itself is not a serious disadvantage. A more serious fault is that the value of anode-voltage at which current starts to flow is

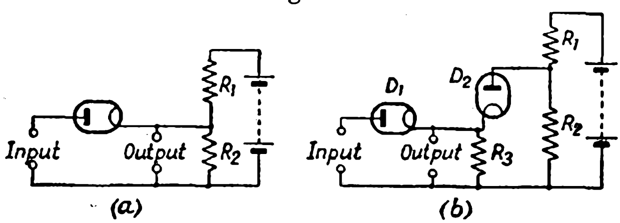


FIG. 191. STABILIZATION OF REFERENCE LEVEL AGAINST VARIATION OF CATHODE TEMPERATURE

not a constant, but depends on the cathode-temperature of the diode, so that the "slicing" level is liable to vary with small variations in heater voltage. A figure which is quoted for this variation is one tenth of a volt change in level for a 10 per cent change in heater voltage. The level can be partly stabilized against such variations as shown in Fig. 191. The unstabilized circuit of Fig. 191 (*a*) is similar to Fig. 189 (*a*), except that it caters only for a positive "slicing" level. If the temperature of the cathode in this circuit were to increase, anode-current would be able to flow for slightly larger negative values of anode-voltage, i.e. smaller positive values of input voltage would suffice to "close the series switch." The "slicing" level is thus lowered. To compensate for this we wish to raise the biasing voltage by a corresponding amount. This is done in Fig. 191 (*b*), by connecting a voltage-divider across R_2, this voltage-divider being formed of the resistance R_3 and a second diode D_2, which should be similar to D_1 and subject to the same variations of cathode temperature (D_1 and D_2 are preferably the two halves of a double diode valve with common cathode). As the cathode temperature increases, the effective d.c. resistance of this second diode will decrease and the biasing voltage (i.e. the d.c. voltage across R_3) will increase as required.

The triode shaping circuit of Fig. 187 has a characteristic roughly of the form shown in Fig. 186, the bottom bend being the result of cutting off the anode-voltage by negative grid-voltage and the top bend resulting from the clamping of grid-voltage at approximately zero by the flow of grid-current. An important alternative method of producing this top bend in the characteristic is available if a pentode be used. The process is known as Bottoming, and requires that the pentode shall operate with a load resistance and an h.t. supply voltage such as will give a load line which passes below the "knee" of the valve characteristics, as shown in Fig. 192. In this region the characteristics for different grid-voltages are seen to coalesce. As the grid-voltage is increased towards zero from a large negative value, the operating point moves up the load line from curve to curve, but its motion ceases when it reaches the point where the curves coalesce (point P in Fig. 192). The anode-voltage then "bottoms" and can be reduced no further by increase of grid-voltage.

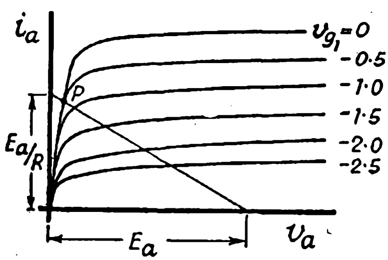


FIG. 192. BOTTOMING IN A PENTODE CIRCUIT

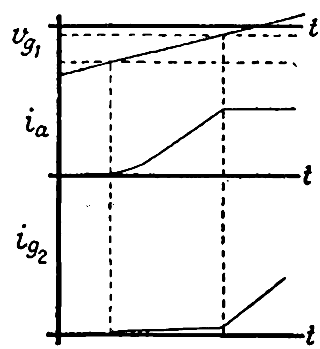


FIG. 193. VARIATION OF ANODE-CURRENT AND SCREEN CURRENT DURING BOTTOMING

Nevertheless, further increase of grid-voltage increases the total space current, and since the anode-current does not increase, it follows that there must be an increase of screen current. Fig. 193 shows the changes of anode-current and screen current which result from a uniform increase of control-grid voltage in a circuit having a load line as shown in Fig. 192. A shaping circuit using bottoming is similar in appearance to Fig. 187, except that the valve is a pentode and the resistor R_g may be omitted. This resistor is sometimes retained, however, for the purpose of avoiding excessive screen current and control-grid current.

A cathode-follower type of circuit which achieves amplitude-selection is shown in Fig. 194. In this circuit

$$v_g = v_1 - Ri_a + E_1$$

The value of input voltage v_1 which makes v_g equal to the cut-off value (at which, of course, $i_a = 0$) is thus given by

$$v_{1\,cut\text{-}off} = v_{g\,cut\text{-}off} - E_1$$

For all values of v_1 more negative than this, the anode-current will be cut off, there will be no RI drop across R, and the output voltage will be clamped at the value $-E_1$. The level at which the input voltage is sliced is therefore given by $v_{g\,cut\text{-}off} - E_1$; this level is controlled by E_1 but not equal to it. The part of the input waveform above this level will be reproduced at the output terminals above the voltage level $-E_1$, since the gain of a cathode follower is approximately unity. E_1 may be either of the polarity shown in Fig. 194, or of the opposite polarity. This circuit has the merit of low-output impedance and high-input impedance. It may be modified to eliminate also the portions of the waveform above a second, higher-reference level either by the insertion of a high resistance in the grid circuit (as in Fig. 187), or by the use of a bottoming pentode.

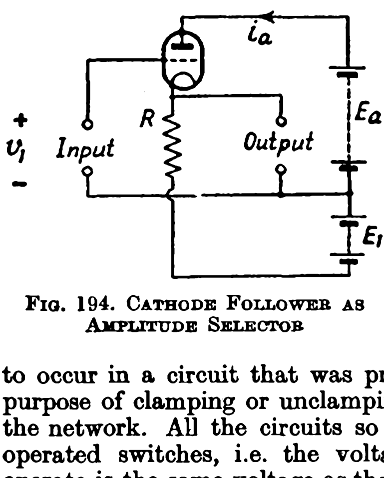


FIG. 194. CATHODE FOLLOWER AS AMPLITUDE SELECTOR

Electronic Switches and Time Selection

The amplitude-selection circuits described in the previous section may be thought of as incorporating electronic switches, for when the input voltage of such circuits reaches a prescribed level a switch is closed (in the sense that conduction is made to occur in a circuit that was previously non-conducting) for the purpose of clamping or unclamping the potential of some point in the network. All the circuits so far described are, however, self-operated switches, i.e. the voltage which causes the switch to operate is the same voltage as that upon which the switch operates. The electronic switches now to be discussed are operated by an independent external voltage. (This distinction between the two types is the same as the distinction between the ordinary rectifier and the phase-sensitive rectifier on page 248.)

Perhaps the best known electronic switch circuit is that shown in Fig. 195. This circuit is simply an amplifier stage which can be switched off and on by depressing or restoring the suppressor voltage, v_{g3}. If the suppressor-bias voltage E_{g3} be made sufficient to cut off anode-current, and if the control voltage be in the form of positive pulses (so that anode-current is restored for the duration of each pulse) then the only parts of the input waveform which will be reproduced in the output are those parts which occur at the same time as a pulse of control voltage. For the remainder of the time the anode-voltage will simply be equal to E_a. This is the process of time-selection (page 296) and may be used, among other things, for separating the individual channel pulses in a multi-channel pulse-modulation system (page 294). Unless v_{g1} happens to be at or below the cut-off value when the sudden suppressor-switching processes occur, there will be sudden changes of anode-current at these instants, i.e. the waveform of the control voltage pulses will also appear in the output voltage and in an oscillogram the desired output voltage will appear on a "pedestal." This is sometimes a disadvantage, though the pedestal can be eliminated subsequently. The circuit is also known as a *Coincidence Circuit* or *Gate Circuit.*

In the above circuit the switching process is accomplished by making use of one of the electrodes (g_3) of a valve which is already being used for a different purpose, viz. as an amplifier. Most switch circuits involve the use of one or more additional valves. There are many such circuits, varying in complexity according to the duty which is required of them. Some of the simpler ones are shown in Fig. 196. These circuits are switches in the sense that the impedance looking into the switch terminals (*S.T.*) is either very low (i.e. "switch closed") or very high (i.e. "switch open") according to the value of the voltage connected to the *O.V.* terminals. In the circuits of Fig. 196 (*a*), (*b*), and (*f*), the switch terminals are open whenever the anode-voltage of the diode is negative. In the circuits of Fig. 196 (*c*), (*d*), and (*e*), the switch terminals are open whenever the grid-voltage of the triode is more negative than the cut-off value. A *pulse voltage* is usually applied to the *O.V.* terminals, so that the operating voltage has alternately the value zero (in which case the switch is closed) or the necessary positive or negative value for holding the switch open.

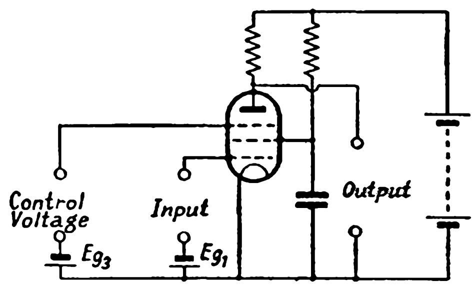


FIG. 195. TIME SELECTION ("COINCIDENCE CIRCUIT")

The switch terminals of Fig. 196 (*a*) will "short-circuit" two points of a network, if the impedance looking into the network at those two points is large with respect to the impedance of the diode and the source of operating voltage, provided also that the network is such as will maintain one of these points (viz. that to which the diode cathode is to be connected) always negative with respect to the other. The operating voltage required to make the switch open must be of a polarity which will oppose conduction (i.e. with the upper *O.V.* terminal negative) and must be at least as large as the largest voltage applied between the switch-terminals. Similarly for Fig. 196 (*b*), but with opposite polarities of both voltages. These provisions seem so stringent that the reader may be forgiven for supposing (quite wrongly) that this switch circuit will find few

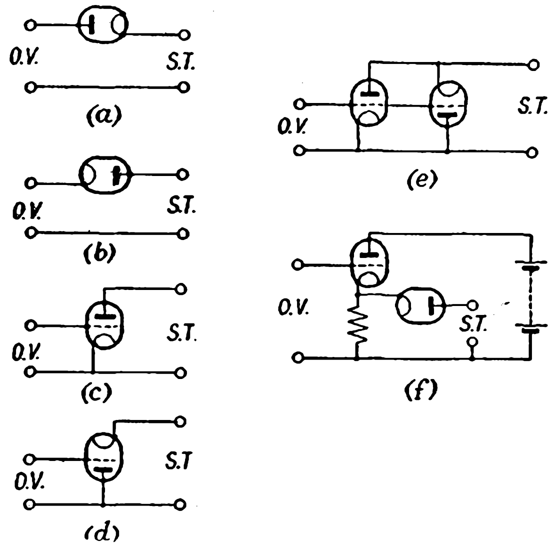


FIG. 196. ELECTRONIC SWITCH CIRCUITS
(All are unidirectional switches except (*e*) which is bi-directional)
O.V. = Operating Voltage *S.T.* = Switch Terminals

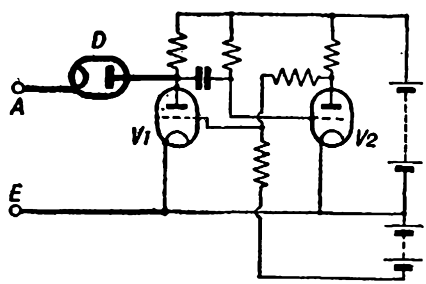


FIG. 197. DIODE SWITCH FOR TRIGGERING MONOSTABLE MULTIVIBRATOR

applications! Fig. 197 illustrates the application of Fig. 196 (*b*) for triggering a monostable multivibrator. Valve 1 is cut off in the stable condition and its anode-voltage is then high; the application of a negative pulse to this anode will initiate the landslide to the unstable condition. Accordingly terminal *A* is normally maintained at h.t. positive potential, but its potential is made to fall suddenly (and then return to h.t. positive) when the landslide is required to occur. The advantage of using this switch-circuit for the application of the trigger voltage is that the switch is open, and thus the trigger source is isolated from the Flip-flop circuit, except during the very short period of triggering. In practice the negative trigger pulses would be *RC* coupled to terminal *A*, with the coupling resistance returned to h.t. positive.

The switch-circuits of Fig. 196 (*c*) and (*d*), like those shown at (*a*) and (*b*), require that the switch terminals shall encounter voltages of a single polarity in the network to which they are applied

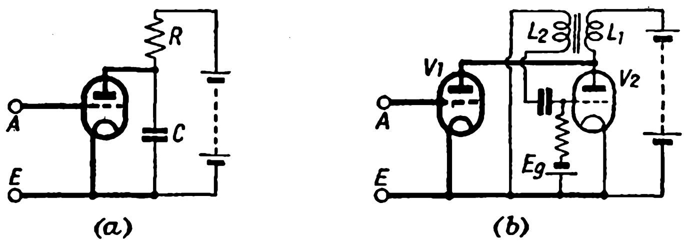


FIG. 198

(*a*) Production of triangular pulses
(*b*) Triggered blocking-oscillator

(viz. upper switch terminal, positive in (*c*) and negative in (*d*)). Such circuits are called One-way Clamps or Unipolar Switches, in contrast with circuits such as that shown at (*e*), which, being a combination of circuits (*c*) and (*d*), can function with its switch terminals connected to a voltage of either polarity. Fig. 198 (*a*) shows the switch circuit of Fig. 196 (*c*) used in a circuit for generating triangular pulses while Fig. 198 (*b*) shows its application to the triggering of a blocking oscillator. In Fig. 198 (*a*) the output voltage is taken across *C*. The terminal *A* is normally held at the potential of terminal *E*, thus maintaining the switch closed. It will be seen that this does not mean that the condenser *C* becomes completely discharged; its p.d. does, however, fall to a low value. The application of short pulses to *A* succeeds in cutting off the valve for short periods, and during each of these periods the condenser charges through *R*, being discharged again when the switch is re-closed at the end of the pulse. In Fig. 198 (*b*) the switch is normally open, terminal *A* being held sufficiently negative with respect to *E* for the anode-current of valve 1 to be cut off. The anode-current of valve 2 (the blocking oscillator) is also normally cut off by the bias voltage E_g. A sudden positive excursion of the potential of *A* causes an increasing current through L_1, and this induces a grid-making-positive e.m.f. in L_2 and precipitates the action of the blocking oscillator. A further example of the use of this switch-circuit is illustrated in Fig. 223 (*c*), (page 342), in which case a pentode takes the place of the triode. The switch circuit of Fig. 196 (*f*) is simply that of Fig. 196 (*b*), but with the operating voltage applied through a cathode follower.

Fig. 199 shows an ingenious application of the diode switch circuit of Fig. 196 (*b*) for the purpose of time selection. Consider first the circuit of Fig. 199 (*a*) in which the switch circuit D_1 is shown by heavy lines, and the letters *O.V.* and *S.T.* denote its "operating voltage" and "switch terminals" respectively. When the switch terminals are open, i.e. when D_1 is non-conducting, the signal voltage (positive pulses) is connected directly across R, D_2 and R_2 in series and thus produces a corresponding voltage across the output terminals. When the switch terminals are closed, the

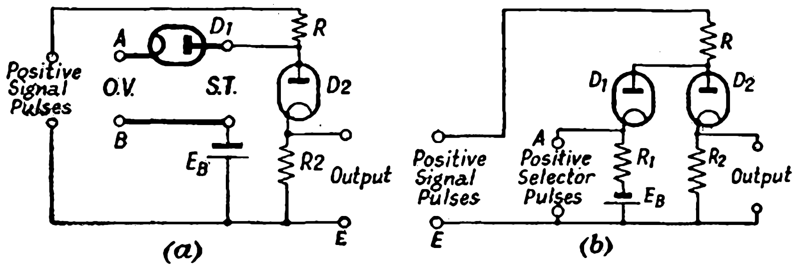


FIG. 199. TWO-DIODE TIME-SELECTION CIRCUIT

anode of D_2 is connected by the switch to a point whose potential is negative with respect to its cathode. D_2 therefore ceases to conduct, the output terminals are isolated, and the output voltage is zero. The required operating voltage, v_{AB}, consists of positive pulses applied to the *O.V.* terminals and the amplitude of these pulses must be greater than the sum of E_B and the peak output voltage. For the duration of each of these positive selector pulses, D_1 is held non-conducting and the signal pulses therefore succeed in producing pulses of output voltage. In the absence of selector pulses, there will still be a path between A and B and the positive signal voltages will cause D_1 to conduct. The switch is thus closed, D_2 ceases to conduct, and the output voltage is zero. In the practical circuit of Fig. 199 (*b*) a resistance R_1 is connected across the *O.V.* terminals (A and B) and the selector pulses (which constitute the operating voltage) are applied between A and earth instead of between A and B.

The above two-diode circuit can be used only with an input voltage of one polarity. Fig. 200 shows a two-diode switch circuit, suitable for time selection, which can be used with an input voltage of either polarity or both polarities (e.g. alternate positive and negative pulses). The circuit requires push-pull selector pulses, the negative-selector pulses being applied at the terminals *NE* and the simultaneous positive-selector pulses at the terminals *PE*. For the duration of the selector pulse, both diodes are non-conducting and the part of the circuit which is drawn in heavy lines can be considered in isolation; the output voltage is thus equal to the input voltage for the duration of each selector pulse. In the absence of selector pulses, current flows round the closed circuit $E_1R_1D_1D_2R_2E_2$. If the two diodes are accurately matched, and R_1 is equal to R_2 and E_1 is equal to E_2, the output terminal Q will be at earth potential in the absence of an input voltage. The network which maintains Q at earth potential is a low-impedance network, and the application of input voltages of either polarity, via the fairly-high resistance R, thus fails to produce any appreciable change of potential at Q.

Fig. 201 shows a further switch circuit, suitable for time selection. The selector pulses are transformer coupled into the vertical diagonal of the bridge and are of such polarity as will cause current to flow in all diodes, thus providing a conducting path between input and output terminals. In the absence of the selector pulses there is no such path available. This is exactly the same circuit as that used for phase-sensitive rectification and shown in Fig. 148 (*b*), page 250. As a time-selector circuit it has the advantage that, owing to the bridge type of circuit structure, there is no tendency for the high frequency components of the selector pulses to be communicated to the output terminals by way of the inter-electrode capacitances of the diodes. When the circuit is used with recurrent pulses, a rectified voltage of the polarity shown is built up on the condenser; this serves to hold the anode-voltage of all diodes negative except during the selector pulses.

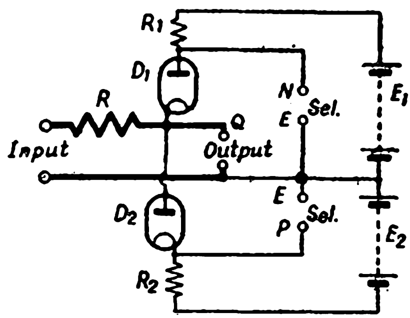


FIG. 200. TIME SELECTION BY BALANCED DIODES

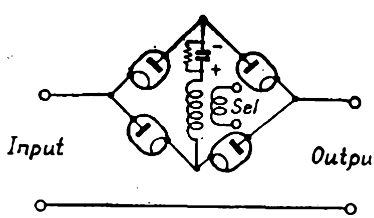


FIG. 201. BRIDGE-TYPE SWITCH CIRCUIT

Pulsed-amplifier Circuits

If an amplifier stage could exist in the simple form shown in Fig. 19 then the application of rectangular voltage-pulses in its grid-circuit would produce undistorted rectangular pulses in the anode-circuit. In practice, however, a circuit which is built to conform with the simple diagram of Fig. 19 has inter-electrode capacitances and stray capacitances which modify its performance at the higher frequencies. It was shown in the earlier part of this chapter that a wide frequency spectrum is associated with recurrent pulse voltages, the width of the frequency band increasing as the pulse width decreases. The dependence of the amplifier's gain and phase-shift upon frequency therefore distorts the pulse waveform. The gain and phase-shift of inter-stage coupling circuits (even the simple *RC* coupling circuit) are also dependent upon frequency, so that waveform distortion occurs here also.

It is the purpose of this section to examine the waveform distortion produced in amplifiers and their coupling circuits when handling rectangular pulses. The approach will not be by considering the frequency-dependence of the gain and phase-shift, in conjunction with a knowledge of the pulse spectrum, but by the simpler method of considering the effect of individual switching processes (step-functions) upon circuits composed of the requisite resistances and condensers (see page 289). To illustrate the method, let us first consider the effect of anode-cathode capacitance alone, as in the circuit of Fig. 202 (*a*). Let the input voltage pulses be such as will

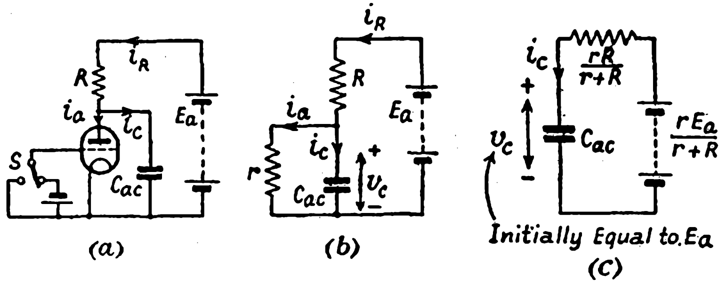


FIG. 202

switch the grid-voltage between zero and some negative value greater than the cut-off value; such an input voltage could be produced by movement of the switch *S* from side to side. Consider the switch first in the right-hand position (i.e. with anode-current cut off) and assume that it has been in that position sufficiently long for any transient currents and voltages to have decayed to zero. The anode-voltage, and therefore the condenser voltage, will be equal to E_a. Let the switch now be moved instantaneously to the left-hand position. After sufficient time has elapsed, a steady state will be reached in which the anode-voltage and the condenser voltage have fallen to a value lower than E_a, but this state is not reached immediately, because the condenser voltage cannot change instantaneously but only at a rate determined by the time-constant of the associated circuit. During the change, the equivalent circuit will be as in Fig. 202 (*b*), in which r represents the resistance of the valve. As a first approximation we shall take r as the d.c. resistance of the valve at the *end* of the period considered, i.e. in the resulting steady state. The d.c. resistance is simply the ratio of v_a to i_a and could be determined in a given case from the load-line diagram. See also Problems VIII, 19 and 20, page 389.

For *small* changes of anode-current it would be more correct to draw an equivalent circuit of the orthodox form, i.e. as in Fig. 26, page 37. Such a circuit would serve to determine the *increments* of the various currents and voltages as a function of time, and the valve impedance ρ would take the place of our resistance r. The circuit of Fig. 202 (*b*) is *not* an *incremental* equivalent circuit, but deals with the whole magnitudes of the various currents and voltages. Fig. 202(*b*) may be simplified to Fig. 202 (*c*) by the application of Thevenin's Theorem. This shows that the time-constant T of the resulting exponential changes is given by $rRC_{ac}/(r + R)$. Also the initial value of v_c is equal to E_a, and its final value, which we shall denote by v_{cf}, is equal to $rE_a/(r + R)$. Thus by equation (I.3)

$$v_c = v_{cf} + (E_a - v_{cf})\, e^{-t/T}$$

$$= (r + R \,.\, e^{-t/T})\, E_a/(r + R) \qquad . \qquad . \text{ (VIII.7)}$$

and

$$i_c = \left\{\left(\frac{rE_a}{r+R} - E_a\right)\bigg/\frac{rR}{r+R}\right\} e^{-t/T}$$

$$= -\,(E_a/r)e^{-t/T} \qquad . \qquad . \qquad . \qquad . \text{ (VIII.8)}$$

To determine the anode-current i_a, n ote that (from Fig. 202 (*b*) ri_a is always equal to v_c. Thus

$$i_a = v_c/R$$

The current i_R may now be determined since it is the sum of i_a and i_c.

From these equations we can sketch the graphs of v_a, i_a, i_c, and i_R against time, as in the left-hand portion of Fig. 203. The form of these graphs could have been deduced without recourse to mathematics. In particular we could have foreseen that the anode-current would immediately rise to a value higher than the final steady value, by observing that immediately after the switching process v_a is still equal to E_a, whereas ultimately it falls to a lower value, the grid-voltage meantime remaining constant at zero. It also follows that, immediately after the switching process, both ends of the resistance R are at a potential E_a above earth; thus i_R is zero, i_c is equal to $-\,i_a$, and the whole of the *initial* anode-current is supplied by the discharging capacitance.

When the switch S is moved back to the right-hand side, anode-current is immediately cut off and the equivalent circuit is as in

Fig. 202 (*b*) but with r equal to infinity (i.e. open-circuited). The condenser voltage therefore rises exponentially from $E_a r/(r + R)$ to E_a, with a time constant RC. The currents i_R and i_c are equal; both decay to zero from an initial value $E_a/(r + R)$, which is the value at which i_a had settled down after the previous switch-on. We can now see the whole distorted output waveform corresponding to one pulse (graph of v_a against t in Fig. 203). *For the distortion to be small we require that the time-constant RC shall be small with respect to the pulse duration.* Fig. 204 (*a*, *b* and *c*) shows some of the output waveforms which may be obtained with various values of time-constant. This circuit is sometimes used, with an artificially increased C_{ac}, for the production of quasi-triangular waveforms such as Fig. 204 (*c*).

Fig. 204 (*d*) shows additional distortion due to the *grid-anode* capacitance. The initial small sudden rise is due to the fact that C_{ga} and C_{ac} are effectively in series across the input voltage, in a circuit of zero resistance (and therefore of zero time-constant). When the sudden rise of input voltage occurs,

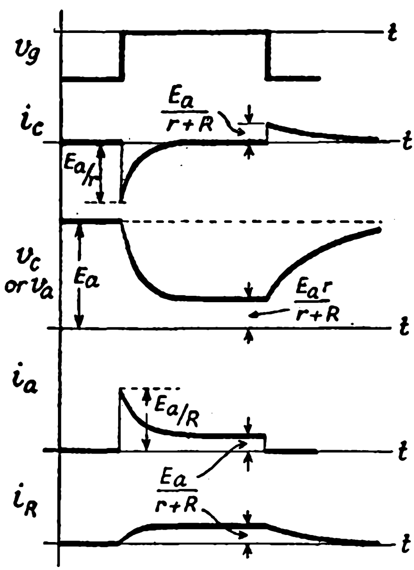


FIG. 203. WAVEFORMS OCCURRING IN CIRCUIT OF FIG. 202

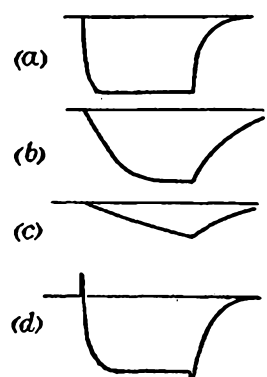


FIG. 204. PULSE-WAVEFORM DISTORTION CAUSED BY INTER-ELECTRODE CAPACITANCE

(*a*), (*b*) and (*c*) Anode-cathode capacitance. (*d*) Anode-grid and anode-cathode capacitance

it therefore results in an immediate charging of these inter-electrode capacitances, the input voltage being "shared" between C_{ga} and C_{ac} in inverse proportion to the capacitances. Thereafter, the slower process shown in Fig. 203 continues as before. The effect shown in Fig. 204 (*d*) is not observable in practice if the sides of the input pulse are insufficiently steep.

Let us next consider the case in which the valve stage feeds an *RC* coupling circuit (R_1, C_1, Fig. 205). We shall begin by neglecting C_{ac} and shall show later how its effect may be taken into account. The equivalent circuit is shown in Fig. 205 (*b*), and may be converted to the simpler form of Fig. 205 (*c*) by the application of Thevenin's theorem. From this circuit we see that the time-constant is

$$T = C_1\left\{R_1 + \frac{rR}{r+R}\right\} \approx C_1R_1. \qquad \text{(VIII.9)}$$

Since C_1 is initially charged to a potential difference of E_a, the current i_c will be given by

$$i_c = \left\{\frac{\dfrac{rE_a}{r+R} - E_a}{R_1 + \dfrac{rR}{r+R}}\right\}e^{-t/T}$$

$$= \frac{-RE_a\, e^{-t/T}}{rR_1 + rR + RR_1} \qquad \text{(VIII.10)}$$

The output voltage is given by

$$v_2 = R_1 i_c$$

and is shown plotted against time in the left-hand part of Fig. 206.

When the switch S is moved back to the right-hand position, the

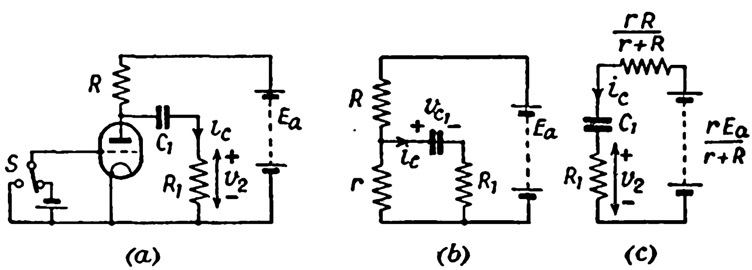


FIG. 205

anode-current is immediately cut off, and the equivalent circuit is as in Fig. 205 (*b*) but with r equal to infinity. The time-constant for this part of the waveform is therefore $C_1(R + R_1)$. Current now flows downwards through R_1, recharging the condenser. The output voltage, $R_1 i_c$, therefore reverses in polarity as shown in Fig. 206, and decays exponentially towards zero. The initial value of the current through R_1 is $(E_a - V_c)/(R + R_1)$, where V_c denotes the p.d. of the coupling condenser at the instant of occurrence of the second switching process.

We can now see the whole distorted waveform corresponding to one pulse and we observe that, whereas the effect of anode-cathode capacitance was to reduce the steepness of the *sides* of the pulse waveform, the effect of the *RC* coupling circuit is to distort the *top and bottom* of the waveform. *For the distortion to be small we require that the time-constant,* R_1C_1, *of the coupling circuit shall be large with respect to the pulse duration.* In a reasonably well-designed circuit, the combined effect of anode-cathode capacitance and *RC* coupling will be to distort the pulse as shown in Fig. 207, since the transients produced by the former are of short duration and may be considered to have died out before the slowly changing p.d. of the coupling condenser has had time to alter.

It is interesting to note that harmonic distortion, due to the curvature of the valve characteristics, plays no part in distorting the waveform of *rectangular* pulses, since the grid-voltage of the amplifier changes instantaneously from one value to another and back again. In practice, however, it is not possible to produce absolutely rectangular pulses with sides of infinite slope, and curvature of the characteristics will thus cause the shape of the output pulse-sides to differ from the shape of the input-pulse sides. Amplifiers in which the anode-current is cut off by the negative excursion of the input voltage, and in which also the negative excursion of anode-voltage is limited (either by bottoming or by the use of a grid-resistor) are known as Overdriven Amplifiers. They are really shaping circuits rather than amplifiers.

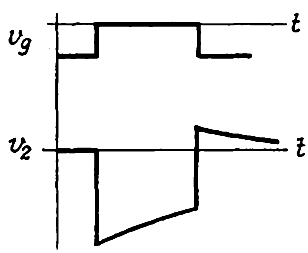


FIG. 206. DISTORTION OF PULSE WAVEFORM BY *R–C* COUPLING

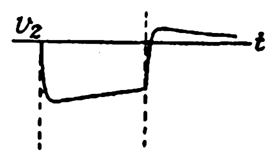


FIG. 207. COMBINED EFFECT OF ANODE-CATHODE CAPACITANCE AND *R–C* COUPLING

D.C. Restoration (or Level-setting)

A voltage which consists of recurrent pulses, as shown in the graph of Fig. 169, has a d.c. component. It might be thought, therefore, that a d.c. amplifier would be required when amplifying pulse-voltages. In view of the added complexity of d.c. amplifiers, however, ordinary a.c. amplifiers are normally used; these pass on the alternating components of the waveform but suppress the d.c. component. It is then necessary to restore the d.c. component after the process of amplification has been accomplished. The same applies after pulse voltages have passed through any network (e.g. a transformer) which removes the d.c. component.

It is not sufficient, however, to add a *fixed* d.c. component. The d.c. component of a rectangular pulse waveform is the product of the pulse amplitude and the duty ratio. In cases where the d.c. component is allowed to be suppressed, and is to be restored at a later stage, the d.c. voltage which must be added at that later stage depends upon both the pulse amplitude and the duty ratio. If either of these alter, then the d.c. restoration circuit is required to make an appropriate change in the added d.c. component. An alternative (but equivalent) viewpoint is that the d.c. restoration

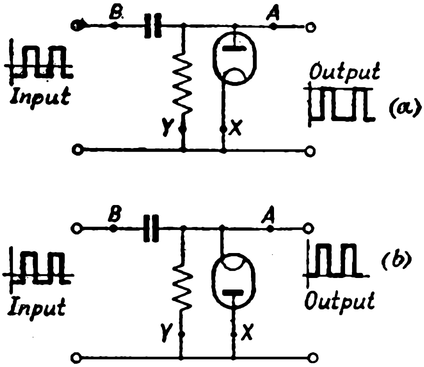


FIG. 208. D.C. RESTORATION CIRCUITS

circuit shall equalize the voltage level at the base (or at the top) of all pulses, and this is in fact what such circuits do.

We thus see that the d.c. component cannot be added simply in the way that a constant grid-bias voltage is added, but rather in the way that a variable grid-bias voltage is added by a self-bias circuit of the rectification-bias type. In such circuits (see page 174) the bias voltage is automatically adjusted to equality with the peak value of the input voltage. The simplest d.c. restoration circuit is in fact identical with the rectification-bias circuit, except that an actual diode takes the place of the "grid-cathode diode." The circuit of Fig. 208 (*a*) introduces the requisite d.c. component to set the *top* of the output waveform at zero level, while the circuit of Fig. 208 (*b*) sets the *base* of the output waveform at zero level.

The behaviour of these circuits is perhaps most easily understood by first considering the resistor to be removed, so that the circuit simply consists of a single closed loop formed of the input source, the condenser and the diode, all in series. Since the diode can conduct in only one direction, the condenser will be charged progressively, having no opportunity to discharge. The condenser p.d. will be of such a polarity as to oppose conduction. It follows that the condenser p.d. will rise to such a value as finally just prevents any further conduction through the diode, i.e. to such a value as will permit the anode voltage of the diode to reach, in the course of each cycle, the threshold value for conduction (approximately zero) but not to exceed this value. (Cf. the rule stated in italics on page 247.) The condenser p.d. will then remain constant. In Fig. 208 (*a*), the output voltage is simply equal to the anode-voltage, v_{AX}, and is thus the sum of the input voltage and such a d.c. component as will set the voltage level of the top of the waveform at the value zero.

Similarly the anode-voltage of the diode in the other circuit, Fig. 208 (*b*), is just able to reach the value zero but not to become positive. The output voltage, v_{AX}, is *minus* the anode-voltage, and is thus the sum of the input voltage and such a d.c. component (viz. the condenser p.d.) as will set the voltage level of the *bottom* of the waveform at the value zero.

The addition of the resistance is necessary to permit the condenser voltage to follow changes (in particular, decreases) of pulse amplitude. In the absence of the resistance, the condenser voltage would be unable to decrease at all, since no discharge path is available. Just as for the diode detector circuit, the time-constant *RC* should be made larger than the pulse repetition period in order to avoid undue ripple of the condenser p.d.; but this time-constant must not be so large that it prevents the condenser p.d. decreasing as quickly as may be required by a decrease of pulse amplitude.

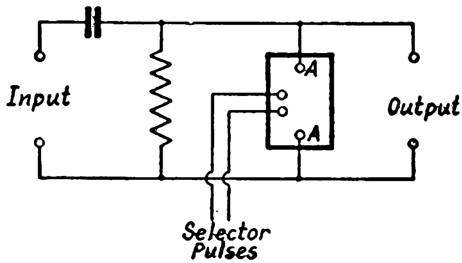


FIG. 209. LEVEL-SETTING CIRCUIT

The level of the top (or bottom) of the output waveform can be set at a level differing from zero by the addition of a suitable constant voltage after d.c. restoration. The insertion of a d.c. voltage at point *X* (Fig. 208) is equivalent to inserting the same d.c. voltage in series with the output, viz. at point *A*.

Fig. 209 shows an ingenious development of the above d.c. restoration circuit. Its purpose is to set some specified point on the waveform, other than the top or bottom, at zero level. (It is not normally used with rectangular pulse input voltages.) The rectangle denotes an electronic switch, actuated by selector pulses of the same frequency as the input voltage and of small duty ratio. For the duration of each selector pulse, the switch-terminals *AA* are effectively short-circuited; for the remainder of the cycle they are open-circuited. The behaviour of the circuit is very similar to that of Fig. 208 (*a*) except that, instead of having a diode which conducts for only a short period at the positive peak of the input waveform, we have an electronic switch which conducts for only a short period at some other point of the waveform, the position of this point being determined by the time of occurrence of the selector pulses. The condenser p.d. is thus nearly constant D.C. of a magnitude equal to that of the input voltage at the time the selector pulses occur.

Pulsed Decoupling Circuits

The form and function of decoupling circuits has been explained in Chapter III (see Fig. 44). In a properly designed anode-decoupling circuit, the alternating components of anode-current flow through the decoupling condenser, while only the d.c. component flows through the decoupling resistor and the h.t. supply circuit. Consider the problem of determining the steady operating conditions (e.g. the d.c. component of i_a) and the output voltage in such a circuit, the valve characteristics and circuit values being known. In the normal usage of the circuit, with an alternating input voltage and reasonably small distortion, the problem is straightforward. A line is drawn upon the i_a–v_a characteristics, its intercept upon the v_a axis being equal to the h.t. supply voltage, and its downward slope being the reciprocal of the sum of the load and decoupling resistances. The d.c. operating point is then located as the intersection of this load line with the particular characteristic curve

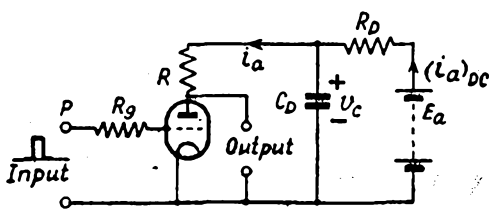


FIG. 210

corresponding to the grid-bias voltage used. An a.c. load line (or dynamic load line) is then drawn through this point, its downward slope being the reciprocal of the load resistance. The output voltage may then be determined if the alternating input voltage is known.

Consider, however, the overdriven amplifier circuit shown in Fig. 210, in which the input voltage consists of rectangular pulses of sufficient amplitude to make the potential of the point P alternate between a large positive value and a negative value sufficient to cut off anode-current. The grid-voltage will not follow the positive excursion, but will be unable to rise above zero as a result of the flow of grid-current through R_g (see page 199). Thus the grid-voltage is alternately zero and a negative value larger than the cut-off value; and the anode-current alternates between a value i_{max} and zero. How may i_{max} be determined?

From the circuit we see that

$$v_a = E_a - Ri_a - R_D[i_a]_{DC}$$

where $[i_a]_{d.c.}$ denotes the d.c. component, or mean value, of i_a. This is equal to si_{max}, where s is the duty ratio of the input pulses, viz. the ratio of pulse width to repetition period. During each positive pulse of input voltage we thus have

$$v_{a\,min} = E_a - Ri_{max} - sR_D i_{max}$$

This is the equation of a load line having an intercept E_a upon the v_a axis, and of downward slope—

$$\frac{1}{R + sR_D}$$

The intersection of this load line with the characteristic curve for zero v_g gives the required value of i_{max}.

We see from this that if the duty ratio is increased, the magnitude of the output pulses will decrease. The effect is small, however, if R_D is small compared to R, and also if the duty ratio never exceeds a very small value (i.e. for very narrow pulses). The effect may be thought of as arising from the d.c. component of the input-voltage pulses, which varies as the duty ratio. Thus as the duty ratio increases, and this positive d.c. component of the input voltage increases, the "effective grid-bias" voltage is decreased, the mean anode-current is increased, the RI drop across R_D is increased, and there is therefore a smaller effective h.t. supply voltage.

It may be thought that the method of calculation (given above) which uses both an a.c. and a d.c. load line could be applied to the overdriven circuit simply by resolving the grid-voltage pulses into a d.c. and an a.c. component, this d.c. component of the grid-voltage being then regarded as part of the effective grid-bias voltage. The mean anode-current determined in this way would, however, be incorrect since, owing to the non-linearity of the valve characteristics, a measure of rectification of the a.c. component takes place, and this also contributes to the mean anode-current. Expressing this differently, a grid-voltage of $(1 - s)v_{g\,cut\text{-}off}$ will not give an anode-current of exactly si_{max}, and the method is therefore unsound.

Throughout the above we have assumed the decoupling circuit to be successful in diverting even the slightest fraction of the a.c. component of i_a from the h.t. supply circuit, so that the current through R_D was pure d.c. equal to $[i_a]_{DC}$. This would require that the capacitance C_D be such as to make $1/(\omega C_D)$ infinitely small with respect to R_D. In such an ideal situation, the condenser voltage would be pure d.c. It is a simple matter to determine how large the decoupling capacitance must be if its p.d. is to be permitted to fluctuate by a specified small amount δv_c. Provided δv_c is small, the values of i_{max} and $[i_a]_{DC}$ determined above (on the assumption that δv_c was zero) may be taken as sufficiently accurate. Thus we see that when i_a is cut off (viz. for a period of $(1 - s)T$ in each cycle), a current $[i_a]_{DC}$ is being supplied by the h.t. circuit to charge the decoupling condenser; while for the duration of the pulse (viz. for a period sT in each cycle) the decoupling condenser is discharging and is supplying a current $(i_{max} - [i_a]_{DC})$ to the valve. The total change of voltage, on charge or discharge, is thus given by

$$\delta v_c = \frac{\delta q}{C_D} = \frac{(1 - s)T\,[i_a]_{DC}}{C_D}$$

Thus if the periodic time T, the mean anode-current (or the maximum anode-current) and the duty ratio are known, the required decoupling capacitance can be determined for a given fluctuation of v_c.

Pulsed Automatic Grid-bias Circuits (Cathode Bias)

Automatic grid-bias circuits of the type shown in Fig. 211 are intended to produce a p.d. across the parallel combination of R_B

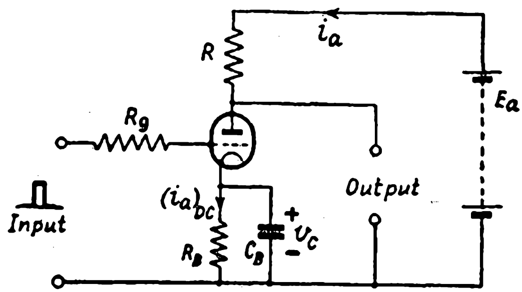


FIG. 211

and C_B which shall be, as far as possible, a pure d.c. so that it may serve as a constant grid-bias voltage (see Chapter III, page 68). For this purpose the value of C_B is made sufficiently large for its reactance to be small compared to R_B at all input frequencies. Let us consider the determination of the mean anode-current and of the output voltage in a pulsed, overdriven amplifier stage using this circuit, assuming that the valve characteristics, circuit components and input voltage are known.

Let the input voltage be of rectangular-pulse waveform, and of sufficient magnitude to make the grid-cathode voltage alternate between zero (as a result of the presence of R_g) and a value more negative than cut-off. The anode-current will be successively switched between zero and a value which we shall denote by i_{max}. Assuming that C_B is large enough for the p.d. across it to be effectively d.c., the current through R_B will be at all times simply the d.c. component of i_a, viz. si_{max}, where s is the duty ratio. While the valve is conducting we have

$$v_g = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{VIII.11})$$

$$v_a = E_a - Ri_{max} - R_B(si_{max}) \quad . \quad . \quad (\text{VIII.12})$$

Thus the value of i_{max} may be determined from the point of intersection, P, of the characteristic for zero v_g with a load line drawn for an h.t. supply voltage of E_a and an effective load resistance of $(R + sR_B)$. As s is varied, this load line pivots about its lower end (say, Q) attaining its maximum steepness as s approaches zero. If a horizontal line be drawn through P, to meet the load line for $s = 0$ in the point T, it is easy to show that the bias-voltage, sR_Bi_{max} is given by PT, and that the output pulse magnitude, Ri_{max}, is given by the horizontal projection of TQ. This graphical construction shows that, if s be increased, the bias-voltage will increase and the output voltage pulse will decrease. If R_B is not too small, it is possible by increasing s to make the bias-voltage exceed the cut-off value corresponding to E_a. This means that the anode-current can be regularly switched on and off with an input voltage consisting of positive pulses without any negative excursion at all (i.e. with an input voltage similar to that shown in Fig. 169).

The necessary positive excursion of input voltage to ensure that v_g is driven to zero is given by sR_Bi_{max}; this increases as s is increased. It is therefore possible that a circuit which is satisfactorily overdriven with narrow pulses would fail to reach zero grid-voltage if the duty ratio were greatly increased, even though the amplitude of the input pulses were unchanged.

If the positive excursion of input voltage is insufficient to cause grid-current to flow, or if the resistor R_g is omitted from the circuit of Fig. 211, the determination of i_{max} becomes more difficult. Equation (VIII.12) still applies, but for equation (VIII.11) we must now substitute—

$$v_{g\,max} = v_{1\,max} - sR_Bi_{max} \quad . \quad . \quad (\text{VIII.13})$$

Equations (VIII.12), (VIII.13) and the relation between i_a, v_a and v_g given by the characteristics, suffice to determine the three unknowns, viz. $v_{g\,max}$, $v_{a\,min}$ and i_{max}. Probably the easiest method is to plot the dynamic-characteristic (i_a plotted against v_g) corresponding to an h.t. voltage of E_a and an effective load resistance of $(R + sR_B)$, and to superpose upon it the graph of equation (VIII.13). The latter is a straight line with an intercept $v_{1\,max}$ on the v_g axis and a slope of $-1/(sR_B)$.

The Pulsed Cathode Follower

When the load of a pulsed cathode follower is shunted by a capacitance (e.g. stray capacitance, or a cable capacitance) care must be taken that the anode-current does not become cut off by the negative step in the input-voltage waveform. So long as the valve is conducting, the output impedance of a cat hode follower is small, and thus the time-constant of the shunt capacitance taken with this output impedance is sufficiently small to allow rapid charging and discharging of the capacitance. The output-voltage waveform is able very nearly to follow the input-voltage waveform. The charging of the shunt capacitance cannot, however, be instantaneous. Thus the negative step of a rectangular input-voltage pulse may, if the input amplitude is large enough, cut off the anode-current before the p.d. of the shunt capacitance has had time to change. Once this has happened, the capacitance can discharge only at a rate determined by the larger time-constant which is the

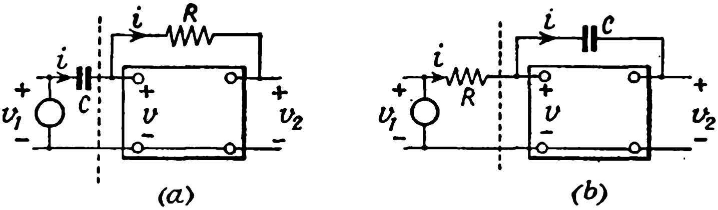


FIG. 212

(a) Differentiating circuit (b) Integrating circuit

product of the shunt capacitance and the cathode-circuit resistance. The output voltage then has an exponential waveform and is unrelated to the input waveform until the valve again conducts.

By deliberately introducing shunt capacitance across the cathode resistor, in order to produce distortion of this kind, the cathode follower can be made to function as a detector of amplitude-modulated pulse voltages. Provided that the time-constant of the capacitance taken with the output impedance of the cathode follower is small compared to the pulse duration, the cathode-to-earth voltage will rise by approximately the same amount as the positive step of input voltage. Provided that the time-constant of the shunt capacitance taken with the cathode-circuit resistance is large with respect to the repetition period, there will be comparatively little discharge of the condenser between pulses. The output waveform will then be similar to that of a pulsed diode-detector. This circuit is sometimes known as the Infinite-impedance Detector.

Differentiating and Integrating Circuits using Feed-back

Differentiating circuits and integrating circuits using feed-back amplifiers can give a better approximation to the desired output waveform than can the simple circuits of Fig. 175. Block diagrams of such circuits are shown in Fig. 212. Both circuits incorporate an amplifier (denoted by the rectangle) designed to give an output voltage v_2 which is $-A$ times its input voltage v, i.e. an amplifier which, if used with a sinusoidal input-voltage, would give a sinusoidal output-voltage 180° out of phase with the input voltage, as would an amplifier with an odd number of resistance-loaded stages. The two circuits differ only in the positions and magnitudes of R and C. If the input impedance of the amplifier is assumed infinite, the same current will flow through R as flows through C, in both circuits.

In the differentiating circuit of Fig. 212 (a), the input voltage v_1 is effectively connected through the capacitance C to a pure resistance R_{in}. This may be proved, and R_{in} may be evaluated, by isolating that part of the circuit to the right of the broken line, then considering a voltage v to be applied to the input terminals, and determining the resultant current i. The circuit considered would thus be similar to Fig. 212 (a), but with a "straight-through" connexion in place of C. For such a circuit we should have

$$i = (v - v_2)/R$$

Also $$v_2 = - Av$$

therefore $$i = v(1 + A)/R$$

$$R_{in} = v/i = R/(1 + A) \quad . \quad . \text{(VIII.14)}$$

For the purpose of evaluating the current i in the differentiating circuit of Fig. 212 (a) we may therefore replace everything to the right of the broken line by a single resistance equal to $R/(1 + A)$, and the circuit is then of the same form as the simple differentiating circuit of Fig. 175 (a). Thus the application of a step-function input voltage (e.g. the sudden switching-on of a constant voltage $\hat{V}$ as illustrated in Fig. 213) will produce a voltage across the hypothetical resistance $R/(1 + A)$ given by

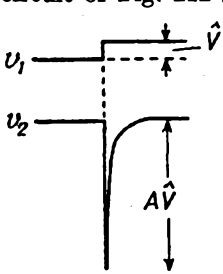


FIG. 213. INPUT AND OUTPUT VOLTAGES OF FIG. 212 (a)

$$v = \hat{V} e^{-t/T}$$

where $$T = RC/(1 + A)$$

This is the voltage marked v, across the input terminals of the amplifier in Fig. 212 (a) and the output voltage will therefore be $- A$ times this, i.e.

$$v_2 = - Av = - A\hat{V} e^{-t/T}$$

We may summarize by saying that this feed-back circuit greatly decreases the time-constant (changing it from RC to $RC/(1 + A)$) and compensates for the resulting decrease of output-pulse magnitude (see page 298) by amplifying the output voltage by a factor A. It also reverses the polarity of the derived pulses.

The same effect could be produced merely by using a simple differentiating circuit (of the form shown in Fig. 175 (*a*)) with a very much reduced value of R, and following this with an ordinary (non-feed-back) amplifier. The circuit of Fig. 212 (*a*) is indeed equivalent to such a circuit, its equivalent circuit being shown in Fig. 214. But it is important to observe that in a differentiating circuit wired-up as in Fig. 214 (i.e. with an actual fixed resistor where $R/(1 + A)$ is shown in Fig. 214) any stray fluctuation in the value of the amplification A will directly affect the magnitude of the output voltage. The output voltage magnitude will in fact be directly proportional to A. This is not the case when using the feed-back differentiating circuit of Fig. 212 (*a*). The equivalent circuit (Fig. 214) shows that if A varies, the effective resistance in the RC circuit varies. For instance if A decreases slightly, $R/(1 + A)$ will increase slightly. This will cause a slight deterioration in the shape of the output pulses (a slight widening), but equation (VIII.5) shows that their magnitude will be increased, thus compensating for the decrease in A. By making A large with respect to unity we can secure output pulses whose magnitude is relatively independent of the amplification A.

The feed-back differentiating circuit has a further advantage over the combination of a simple RC differentiating circuit followed by an amplifier, namely that the output waveform is not so seriously affected by frequency distortion and phase distortion in the amplifier. This may be seen as follows. For each of the Fourier components of the input voltage, we may evaluate a complex ratio V_2/V_1 for the circuit of Fig. 212 (*a*). Starting with the vector equations

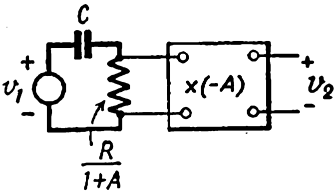


FIG. 214. EQUIVALENT CIRCUIT OF FIG. 212 (*a*)

$$V_1 - V_2 = (R + 1/j\omega C)I$$

and

$$V_2 = -AV = -A(V_1 - I/j\omega C)$$

we eliminate I, and re-arrange the resulting equation to give

$$\frac{V_2}{V_1} = \frac{-j\omega CRA}{1 + A + j\omega CR} \quad . \quad . \quad \text{(VIII.15)}$$

Over a certain range of frequencies (from zero up to some high frequency) $R/(1 + A)$ will be small with respect to $1/\omega C$, i.e. $\omega CR \ll 1 + A$. This range of frequencies will increase as A is increased. Within this range, we may approximate as follows

$$V_2 \simeq -j\omega V_1 \cdot \frac{CRA}{1 + A} \quad . \quad . \quad \text{(VIII.16)}$$

Since multiplying by $j\omega$ constitutes differentiation of the rotating vector with respect to time, we see that all components within this frequency range are accurately differentiated. The departure of the whole output voltage from the true differentiated waveform arises from the phase-distortion and frequency-distortion to which the harmonics of higher frequency are subjected, together with variation of the factor $A/(1 + A)$ from component to component. Equation (VIII.16) shows that, provided that A is large compared to unity, the amplification, A, may vary with frequency without affecting the ratio V_2/V_1 for components within a certain frequency range. Thus if this frequency range has been made wide enough to give an adequate approximation to true differentiation, the introduction of frequency distortion in the amplifier will not make this approximation any less adequate.

In view of the low-input impedance of this feed-back differentiating circuit (viz. $1/(j\omega C) + R/(1 + A)$ to a component of frequency $\omega/2\pi$) it is usual to feed the input voltage through a cathode-follower stage. The amplifier itself may also be given a cathode-follower output stage, in which case the output electrode becomes the cathode of this last stage.

Fig. 212 (b) shows a block diagram of an integrating circuit using feed-back. As in the case of the differentiating circuit the amplifier is made such that the output voltage is equal to $-A$ times the input voltage, A being positive. Assuming that the input impedance of the amplifier is infinite, and using the notation of Fig. 212 (b) to refer to instantaneous values of voltage and current, we see that

$$v_1 - v_2 = Ri + \frac{1}{C}\int idt$$

and

$$v_2 = -Av = -A\,(v_1 - Ri)$$

Eliminating v_2 between these two equations we have

$$v_1 = Ri + \frac{1}{(1 + A)C}\int idt \qquad . \qquad . \text{(VIII.17)}$$

The simple integrating circuit of Fig. 175 (b) gives an equation of exactly similar form

$$v_1 = Ri + \frac{1}{C}\int idt$$

By comparing these two equations we see that, in the feed-back integrating circuit, v_1 finds itself effectively connected across a resistance R and a capacitance $C\ (1 + A)$ in series. It follows that that part of the circuit of Fig. 212 (b) to the right of the broken line has an input impedance which is simply the impedance of a capacitance $C(1 + \text{A})$.

If the input voltage consists of a step-function (e.g. the sudden switching-on of a constant voltage $\hat{V}_1$ as illustrated in Fig. 215) the resultant voltage across the hypothetical capacitance $C(1 + A)$, and thus also the resultant voltage across the input terminals of the amplifier, will be given by

$$v = \hat{V}_1(1 - e^{-t/T})$$

where
$$T = RC(1 + A)$$

The output voltage v_2 will be $-A$ times this, namely—

$$v_2 = -A\hat{V}_1(1 - e^{-t/T}) \qquad . \quad . \text{(VIII.18)}$$

the graph of which is shown in Fig. 215.

We may summarize by saying that this feed-back circuit greatly increases the time-constant (changing it from RC to $RC(1 + A)$) and compensates for the resultant reduction of output-voltage magnitude by amplifying the output voltage by a factor A. It also reverses the polarity of the output voltage.

The same effect could be produced merely by using a simple integrating circuit (of the form shown in Fig. 175 (b)) with a very much increased value of C, and following this with an ordinary (non-feed-back) amplifier. The feed-back integrator, however, has advantages over the non-feed-back circuit, in much the same way as the feed-back differentiator was shown to have advantages over the non-feed-back circuit. To show these advantages we may evaluate the complex ratio V_2/V_1 for each of the Fourier components of the input voltage of the feed-back integrator circuit of Fig. 212 (b). In view of the similarity between the integrator and differentiator circuits of Fig. 212, the resulting equation is similar to equation (VIII.15) but with $1/j\omega C$ written in place of R, and vice versa, viz.

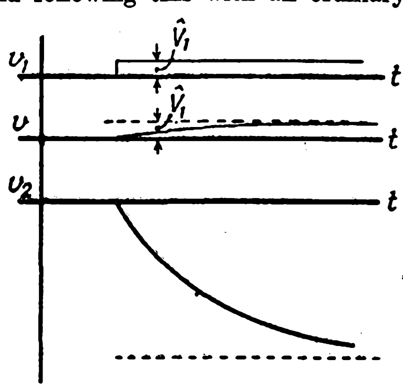


FIG. 215. INPUT AND OUTPUT VOLTAGES OF FIG. 212 (b)

$$\frac{V_2}{V_1} = \frac{-A\dfrac{1}{j\omega CR}}{1 + A + \dfrac{1}{j\omega CR}} \qquad . \quad . \text{(VIII.19)}$$

If the circuit be so designed that $1/\omega CR$ is negligibly small with respect to $(1 + A)$ at the repetition frequency (and therefore also at all higher frequencies) we may approximate by writing

$$V_2 = \frac{-A}{1+A} \cdot \frac{1}{RC} \cdot \frac{V_1}{j\omega} \quad . \quad . \quad . \quad \text{(VIII.20)}$$

Since division of a rotating vector by $j\omega$ constitutes integration with respect to time, we see that (subject to the above approximation) the output voltage is the sum of a series of components each one of which is the integrated version of the corresponding component in the input voltage. If the factor $A/(1 + A)$ were the same for all components (i.e. for all frequencies) we could conclude that the whole output voltage would be the integrated version of the input voltage. This can be secured by making the magnitude of A large compared to unity, for then the factor $A/(1 + A)$ is almost exactly equal to unity, and thus relatively independent of frequency *even if A is a function of frequency.* A further advantage is that, since $A/(1 + A)$ is relatively independent of A, the magnitude of the output voltage will be almost unaffected by changes in the gain of the amplifier.

The Miller Integrator

Feed-back integrator circuits of the type discussed in the previous section, and illustrated by the block diagram of Fig. 212 (*b*) are often referred to as Miller Integrators. The simplest circuit of this type is probably that shown in Fig. 216 in which terminals 1 and E are the input terminals, and the output voltage is taken between A and E. R and C have the same function as in the block diagram of Fig. 212 (*b*).

The components within the dotted rectangle are intended to represent a source of step-function input voltage: the input voltage will rise abruptly by an amount δv_1 when the switch is moved from right to left. The response of the circuit to a step-function of input voltage can easily be deduced without reference to the more general theory given in the previous section. Since C is connected between anode and grid we may refer to the Miller Effect, viz. the effect of grid-anode capacitance on the input impedance of a resistance-loaded amplifier (see equation (IV.15), page 95). The input impedance looking to the right at points 2 and E, and neglecting stray capacitance and inter-electrode capacitances, will be that of a capacitance $C(1 + A)$, where A denotes $\mu R'/(\rho + R')$. The input voltage is thus virtually connected across the series combination of a resistance R and an effective capacitance $C(1 + A)$. The increment of grid-voltage resulting from the sudden increase δv_1 in input voltage will therefore be the voltage change across this effective capacitance, viz.

$$\delta v_g = \delta v_1(1 - e^{-t/T})$$

where

$$T = RC(1 + A)$$

Assuming that the condenser-discharge current which flows through the load R' is so small that it causes negligible changes of anode-voltage* we may write

$$\delta v_a = -A\delta v_g$$
$$= -A\delta v_1(1 - e^{-t/T}) \qquad . \quad . \quad \text{(VIII.21)}$$

Thus, on the occurrence of a sudden rise, δv_1, of input voltage, the anode-voltage falls exponentially through a voltage $A\delta v_1$ with a time-constant $RC(1 + A)$ where A is given by $\mu R'/(\rho + R')$.

While the above statement truly describes the behaviour of the circuit for a small step of input voltage, the statement must be modified if the input voltage step is large. Consider for instance the effect of stepping the input voltage from a small negative value to a large positive value. The step δv_1 will then be sufficiently large for $A\delta v_1$ to be larger than the initial anode-voltage v_{a0}. The anode-voltage cannot therefore fall by this amount, for its final value would then be negative. Nevertheless it will commence falling, as if towards the negative value $v_{a0} - A\delta v_1$, with a time-constant $RC(1 + A)$. To determine the final value to which v_a will in fact fall, we note that when steady conditions are eventually attained there will be no further current in the condenser leads, so that the condenser could then be removed from the circuit without altering the anode-voltage. It is clear that in a circuit such as Fig. 216, with the input voltage large and positive, and with no condenser present, grid-current would be flowing through the large resistance R, and the grid would thereby be maintained at cathode-potential or very slightly higher. This is a clamping effect, and we may say that the exponential fall of anode-voltage proceeds, as if towards a negative voltage $v_{a0} - A\delta v_1$, until the accompanying rise of grid-voltage (see Fig. 217) brings the grid to the potential of the cathode. The grid-potential is then clamped at this value, and the change of anode-voltage ceases. The resulting

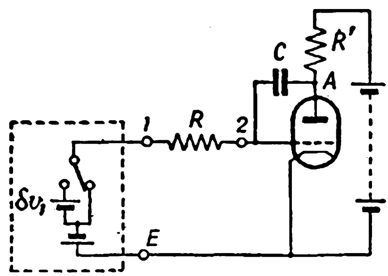


FIG. 216. SIMPLE MILLER INTEGRATOR CIRCUIT WITH STEP-FUNCTION INPUT

* An accurate analysis, not neglecting the voltage drop due to the flow of condenser discharge current upwards through R', shows that there is a sudden small rise in anode-voltage at the instant when the sudden rise of input voltage occurs. The magnitude of this sudden rise of anode-voltage is

$$\frac{\delta v_1}{1 + (1 + A)R/\rho'}$$

where ρ' represents the parallel combination of the load resistance, R', and the valve impedance, ρ.

value of anode-voltage may be found, if required, from the intersection of the load line with the static-characteristic for v_g equal to zero.

It is important to note that, if δv_1 is large, the anode-voltage may have fallen by only a small fraction of the ultimate unrealizable fall, $A\delta v_1$, when the fall is cut short by the clamping of the grid-

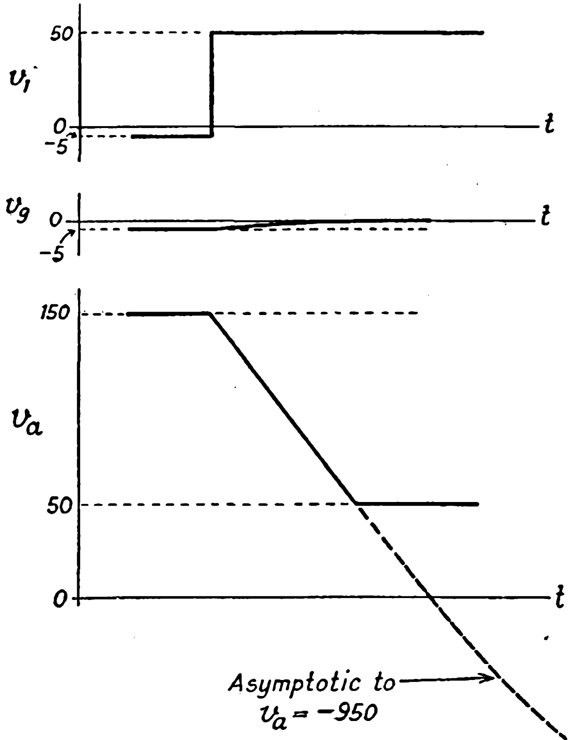


FIG. 217. WAVEFORMS OCCURRING IN FIG. 216

potential. In this case the fall of anode-voltage will be almost exactly linear with time as shown in Fig. 217 (in which A has been given the value 20). The Miller integrator circuit is extensively used in this way for the generation of linear sweeps.

When so used, the circuit is not truly an integrator of an externally applied input voltage: the function of the input voltage is merely to initiate the linear sweep at an accurately defined instant, i.e. to operate the switch in Fig. 216. Thus, when considering the circuit as a linear-sweep generator, we no longer regard the part of Fig. 216 within the dotted rectangle as representing an input voltage;

the components within the rectangle are now part of the circuit of the linear-sweep generator. To secure a large value of δv_1 (so that the actual fall of v_a shall be only a small fraction of $(1 + A)\delta v_1$) the left-hand end of the resistor R may be switched from a small negative voltage to the h.t. positive terminal, as illustrated in Fig. 218 (*a*). In this circuit the linear run-down of anode-voltage is initiated by moving the switch S_1 from the lower to the upper position. A slightly different method of initiating the run-down is shown in

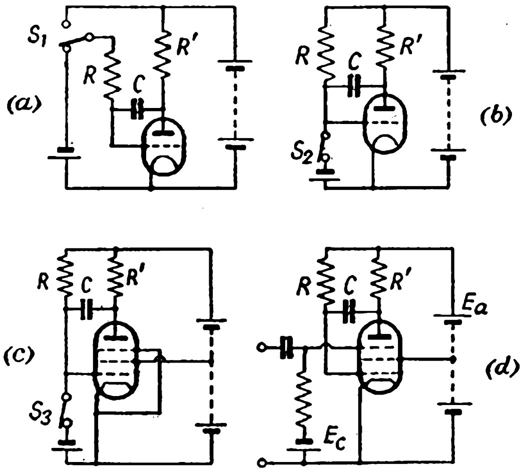


FIG. 218. INITIATING THE RUN-DOWN IN A MILLER SWEEP GENERATOR
(Each circuit is shown in its quiescent condition)

Fig. 218 (*b*). This circuit, with the switch S_2 open, is identical with Fig. 218 (*a*) with its switch S_1 in the upper position. With the switches in their other positions, the circuits are not identical but both give the same quiescent values of grid-voltage, anode-current, anode-voltage, and (therefore) condenser voltage. The advantage of the circuit shown at (*b*) over that shown at (*a*) is that it permits a more rapid return to the quiescent condition when the switch is returned to the position shown in the diagram.

Fig. 218 (*c*) is the same as Fig. 218 (*b*) except that a pentode is used in place of a triode. The behaviour of the circuit is changed in one respect by the use of a pentode—namely, in that the end of the run-down may now be brought about not by the flow of grid-current, but by bottoming (see page 311). If the load line for the particular values of E_a and R' cuts the pentode characteristics below the "knee," then the negative grid-voltage, rising towards zero during the run-down, will eventually reach a value where any further rise causes no change of anode-current. When this happens, there is no longer any feed-back, and the condenser can discharge very much faster than during the run-down. The grid-voltage therefore rises rapidly to zero. As it does so the total space current of the valve increases. Since the anode-current remains constant, this increase constitutes an increase of screen current.

In the circuit of Fig. 218 (*d*) a biasing voltage E_c is connected in the suppressor-to-cathode circuit and this voltage is made such that the suppressor grid is sufficiently negative to cut off anode-current. The run-down is initiated by feeding a positive-voltage pulse to the input terminals, thus raising the suppressor voltage and permitting anode-current to flow for the duration of the pulse. During the period for which the suppressor voltage is maintained at the positive value (i.e. during the pulse), the usual relation between i_a, v_a and v_{g1} will obtain, as expressed in the static-characteristics, and there will thus be an exponential run-down of v_a as already deduced. During the sudden rise of suppressor voltage, however, at the onset of the initiating pulse, there will be a sudden rise of anode-current and therefore a sudden fall of anode-voltage at the commencement of the run-down, as shown by the line NP, Fig. 219. This fall of anode-voltage is communicated, by way of the condenser C, to the control-grid and its magnitude may be determined by the following considerations.

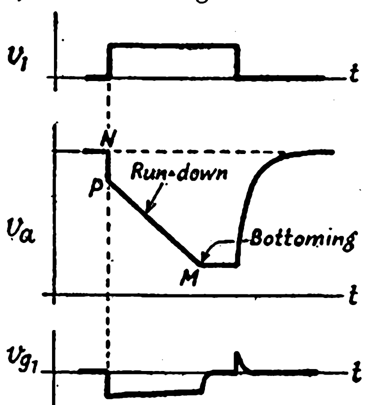


FIG. 219. WAVEFORMS OCCURRING IN MILLER SWEEP CIRCUIT OF FIG. 218 (*d*)

In the quiescent condition, i.e. with no voltage applied across the input terminals, the suppressor voltage is negative, the anode-current is zero, and the anode-voltage is equal to the full h.t. voltage E_a. The condenser voltage is given by $v_a - v_{g1}$, and since v_{g1} is held at zero (or a very small positive value) by the flow of grid-current through R, it follows that the condenser voltage in the quiescent condition is equal to E_a (or very slightly less). When the suppressor-voltage suddenly rises, at the onset of the positive pulse of input-voltage, the condenser voltage cannot immediately change, so that at the start of the run-down, we shall still have v_a and v_{g1} related by the equation.

$$v_a - v_{g1} = E_a$$

The rising suppressor voltage will thus cause the anode-current to increase suddenly to the value at which this equation and the relation expressed by the valve characteristics are simultaneously satisfied. In the load-line diagram of Fig. 220, *ABCD* is the locus of points for which $v_a - v_{g1}$ is equal to E_a. (This locus may be drawn simply by marking off on each curve the point whose abscissa, v_a, is equal to $E_a + v_{g1}$, so that the abscissae of *A*, *B*, *C*, and *D* respectively are equal to E_a, $E_a - 1$, $E_a - 2$, and $E_a - 3$.) The values of v_a, i_a and v_{g1} at the start of the run-down will therefore be given by the co-ordinates of the point *P*, viz. the point of intersection of this locus with the load line for the appropriate values of E_a and R'.

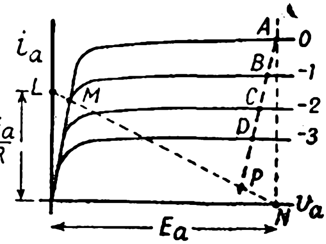


FIG. 220. LOAD-LINE DIAGRAM FOR MILLER SWEEP GENERATOR

The letters *M*, *N*, and *P* correspond to the points marked by the same letters in Fig. 219

Fig. 219, in which the lettering corresponds to that in Fig. 220, shows that the run-down is terminated by bottoming at the point *M*, the control-grid voltage then rising rapidly until clamped by the flow of grid-current at a value just above zero, with an accompanying rise of screen current. Conditions will then remain unchanged until the end of the input pulse, when the suppressor voltage will suddenly fall and the anode-current will be cut off. The only current flowing through R' thereafter will be the condenser charging current, whose path will be the closed circuit of E_a, R', C and the grid-cathode space of the valve. Since the control-grid voltage is positive, the resistance of this last-named section of the path is comparatively small, and the time-constant of the charging process is approximately equal to $R'C$. As the charging current through R' decays exponentially to zero, the anode-voltage rises exponentially towards the value E_a, and the initial conditions are finally reinstated.

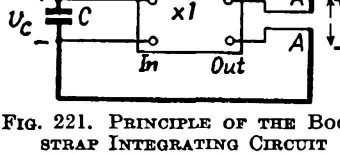


FIG. 221. PRINCIPLE OF THE BOOTSTRAP INTEGRATING CIRCUIT

The Bootstrap Circuit

The so-called "Bootstrap Circuit" is a linear-sweep generator, closely related to the Miller Integrator. The principle of its operation is elegant and simple. Fig. 221 shows a block diagram of the circuit. It may be thought of as a modification of the simple integrating circuit in which (Fig. 175 (*b*)) a resistance and condenser in series are connected directly across the input voltage, and the output voltage is taken as the voltage across the condenser. The modification consists of breaking the circuit (at points *AA* in Fig. 221) for the purpose of injecting an e.m.f. which is derived from, and maintained equal to, the voltage across the condenser. Such an e.m.f. may be obtained from the output terminals of an amplifier with gain equal to unity, its input terminals being connected across the condenser. If the input impedance of the amplifier may be taken as infinite, the application of Kirchhoff's second law gives the following equation for the circuit—

$$v_1 + v_c = Ri + v_c \qquad \text{(VIII.22)}$$

whence

$$v_1 = Ri$$

and the output voltage, v_2, being simply the condenser voltage, v_c, is given by

$$v_2 = \frac{1}{C}\int idt = \frac{1}{RC}\int v_1 dt$$

It appears, therefore, that here we have an accurate integrating circuit, and that the response to a step-function of input voltage would be a linear rise of output voltage with time, limited only by overloading of the amplifier. Practical versions of the circuit, however, do not accurately implement the block diagram, and on the whole it may be said that the bootstrap circuit and the Miller integrator give comparable accuracy if the amplifiers in the two circuits are of comparable complexity and cost.

The easiest realization of the block diagram uses a cathode follower as the unity-gain amplifier, and the loss of accuracy arises from the fact that the gain of a cathode follower is always slightly less than unity. This means that the circuit equation, corresponding to (VIII.22) above, becomes

$$v_1 + kv_c = Ri + v_c$$

where k is slightly less than unity. This equation may be written as follows

$$\frac{v_1}{1-k} = \frac{Ri}{1-k} + \frac{1}{C}\int idt$$

This shows that, for the purpose of evaluating the current and the condenser voltage, we may regard the condenser as being effectively connected across a resistance $R/(1-k)$ in series with an e.m.f. $v_1/(1-k)$. Thus if the input voltage, v_1, be a step-function, of magnitude, $\hat{V}$ the condenser voltage will be given by

$$v_c = \frac{\hat{V}}{1-k}(1 - e^{-t/T}) \qquad \text{(VIII.23)}$$

where

$$T = RC/(1-k) \gg RC$$

The output voltage is usually taken from the output of the cathode follower in order to avoid connecting a load impedance across the condenser.

Fig. 222 (*a*) shows a version of the bootstrap circuit using a cathode-follower. The circuit is drawn in such a way as to emphasize its derivation from the block diagram of Fig. 221. It will be seen that the connexion AB is redundant, and that the circuit may be redrawn as in Fig. 222 (*b*). It is, of course, a serious disadvantage that neither side of the input voltage can be earthed to h.t. negative; as a result of this, the circuit is seldom used as an integrator, but rather as a linear-sweep generator.

In the circuit of Fig. 222 (*b*) the condenser-charging current flows through R', but if R is sufficiently large this current will be small, and we may neglect the voltage drop which it produces across R'. The behaviour of the cathode-follower is then unaffected by the connexion of the charging circuit to its load resistance and we may write

$$v_2 = \frac{\mu R'}{\rho + R'(1+\mu)} \cdot \delta v_c = k\delta v_c$$

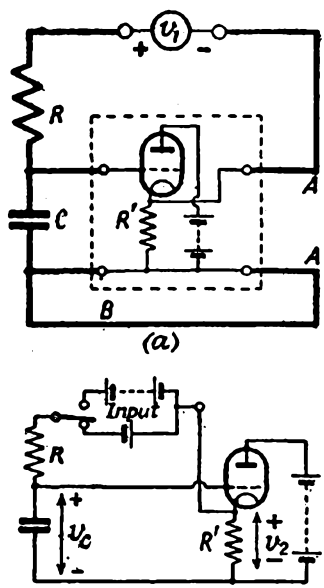
Equation (VIII.23) then shows that, on the occurrence of the sudden rise of input voltage, δv_1, the change of condenser voltage will be given by

$$\delta v_c = \frac{\delta v_1}{1-k}\,(1 - e^{-t/T})$$

and the output change, δv_2, will be k times this. Substituting for k we then have

$$\delta v_2 = \frac{\mu R' \delta v_1}{\rho + R'}(1 - e^{-t/T}) \quad . \text{ (VIII.24)}$$

where $T = \left[1 + \dfrac{\mu R'}{\rho + R'}\right] RC$



(b)

FIG. 222. BOOTSTRAP INTEGRATING CIRCUIT

(*a*) Drawn in such a way as to show the derivation from Fig. 221
(*b*) Practical circuit

The grid-cathode voltage is given by $v_c - v_2$ and must therefore rise by an amount δv_1, the rise being exponential and the time-constant having the same value T.

To use the circuit as a linear sweep generator, δv_1 is made large and positive, so that the rise δv_2 is never completed. Instead, the exponential change is interrupted by the flow of grid-current, which (since it produces a low resistance path between grid and cathode) clamps the grid-cathode voltage at a value near zero, and prevents further change of anode-current. A substantially-linear rise of output voltage is secured by making δv_1 so large that only a small fraction of the ultimate rise (as given by equation (VIII.24)) has been achieved when grid-current limits the process.

Just as in the case of the Miller sweep generator, a variety of methods may be used to initiate the sweep. Instead of switching the upper end of R as in Fig. 222 (b), we may connect the large positive value of v_1 permanently in circuit (as in Fig. 223 (a)) but with the grid held negative by means of a switch, S. The sweep is then initiated by opening the switch S. Fig. 223 (b) shows a method of avoiding the use of the floating battery (or unearthed d.c. supply) E_1. The large capacitance C_D here takes the place of

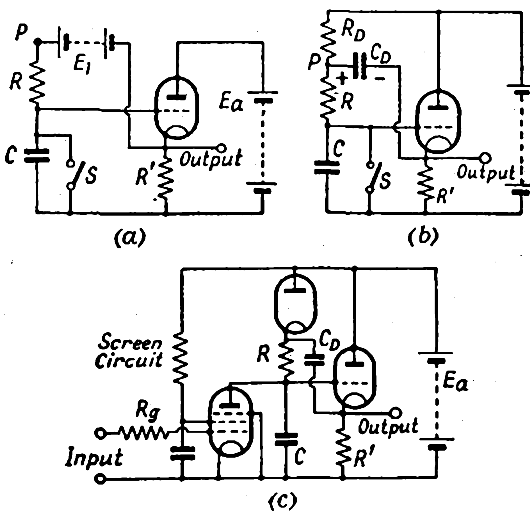


FIG. 223. VARIATIONS OF THE BOOTSTRAP SWEEP GENERATOR

the battery E_1 in Fig. 223 (a). If this capacitance be made so large that it is an effective short-circuit to A.C. of the repetition frequency (i.e. the frequency with which the switch S is opened and closed), then there can be no alternating p.d. between the cathode and the point P. The p.d. across C_D is thus effectively d.c. and can take the place of E_1. In practice C_D will supply a charging current through R to C during the periods for which the switch is open, and this will cause its voltage to fall slightly. But during the periods when the switch is closed, C_D will be recharged through R_D.

The potential of the point P (Fig. 223 (a) and (b)) will rise and fall with the cathode (i.e. output) potential. If R_D (Fig. 223 (b)) be replaced by a diode, with its cathode connected to point P, the anode-voltage of the diode will fall and rise as the potential at P rises and falls. In the quiescent condition, with switch S closed and the cathode of the triode at its lowest potential, the diode will conduct, since it is connected in a series closed circuit formed by E_a, R and the closed switch S. The condenser C_D will therefore charge to a voltage equal to E_a minus the quiescent value of the output voltage (since the left-hand side of C_D is connected to E_a through the diode, and the right-hand side is connected to the output terminal). When the switch S is opened, to initiate the linear rise of output voltage, the potential at P will rise, the diode will cease to conduct, and it will not conduct again until, on the closure of switch S, the potential of the point P falls.

Fig. 223 (*c*) is similar to Fig. 223 (*b*) but with two modifications. The first is the use of a diode as just described. The second is the replacement of the switch S by the pentode switching valve. If the input voltage to the pentode consist of square waves, then the positive half-cycles will drive the control-grid positive (subject to the limiting action of R_g) and during this period anode-current will flow in the pentode by way of E_a, the diode and R. Since R is large, the relevant load line for the pentode will have very small slope, and its anode-voltage will be very low indeed. This means that the condenser C is effectively short-circuited (just as by the switch S in the previous circuits), or at least discharged to a very low voltage. During the negative half-cycles of the square-wave input, the anode-current of the pentode is cut off, i.e. the switch is effectively opened, and the linear sweep can proceed.

The Screen-coupled Phantastron

The circuit which goes by this extraordinary name is a combination of the Transitron relaxation oscillator and the Miller sweep generator. As such it is a free-running relaxation oscillator whose output waveform includes a linear sweep, i.e. a substantial part of the waveform consists of a sloping, straight line. It may be made monostable (as distinct from free-running) in which case it remains in a quiescent state until triggered by an external voltage.

The transitron relaxation oscillator (page 213) is a circuit in which the suppressor and screen (i.e. the transitron control electrode and the transitron output electrode) are RC coupled, thus giving positive feed-back. As a result voltage landslides occur, in one of which the suppressor voltage suddenly goes negative (with the result that the screen current increases at the expense of the anode-current) while in the other the suppressor voltage suddenly goes positive (with the result that the anode-current is reinstated, at the expense of the screen current). The anode-current is thus alternately cut-off and reinstated. Consider the transitron relaxation oscillator of Fig. 224 (*a*), in which grid 1 is clamped at about cathode-potential by the flow of grid-current through R_1. (The suffixed numbers denote the grids to which particular circuit components are connected.) Let us suppose that, during one of the periods when the anode-current is cut-off (and the suppressor voltage is returning exponentially towards zero from a negative value) the circuit is opened at the point X, and a resistance R_a is inserted. Since no anode-current is flowing, this will not immediately affect the currents and voltages in the circuit; nor will the further addition of a capacitance C_{1A}, connected between anode and the first grid, provided that the capacitance is already charged to the p.d. between the points to which it is to be connected. The circuit is now as shown in Fig. 224 (*b*) and the suppressor voltage, v_{g3}, is rising towards the region of the characteristics where it will be able to control the sharing of

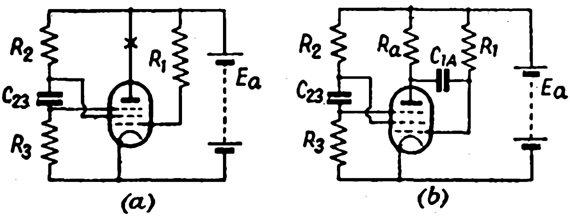


FIG. 224
(*a*) Transitron relaxation oscillator
(*b*) Free-running screen-coupled Phantastron derived from (*a*)

the electron stream between anode and screen grid. When it reaches this region, it precipitates a voltage landslide, for the rise of suppressor voltage reduces the screen current, so raising the voltage of the screen, which is coupled back to the suppressor through C_{23}. In the absence of the added components, R_a and C_{1A}, the anode-current would immediately rise considerably and the screen-current would fall considerably.

As a result of the addition of R_a, however, any sudden rise of anode-current will produce a sudden fall of anode-voltage. Also, as a result of the addition of C_{1A}, this sudden fall will be communicated to the first grid (G_1), thus tending to offset the rise of anode-current by reducing the total space current. The net result of this is that the rise of anode-current will be greatly curtailed. Moreover, the sudden fall of v_{g1} will cause a sudden lowering of the screen current, which is in any case a reduced fraction of the total space current now that the suppressor has become positive. The resulting rise of screen voltage is, of course, communicated to the suppressor grid, so that the rise of suppressor voltage will be appreciable.

The graphs of Fig. 225 illustrate these changes; the broken vertical line marks the instant at which the components R_a and C_{1A} were supposedly connected into the circuit, and the point P marks the occurrence of the first subsequent landslide. The Miller run-down of v_a then commences, and the positive suppressor voltage decays exponentially to zero (PQ), as the charge on C_{23} re-adjusts itself. The time-constant of the run-down is much greater than the time-constant of the decaying suppressor voltage and the latter returns to zero in the early stages of the run-down. But no landslide occurs when the suppressor voltage reaches the sensitive region. The reason for this is that such a landslide would involve a sudden rise in anode-voltage, which would be communicated to the first grid by way of C_{1A}, thus tending to increase the anode-current and so to offset the rise of anode-voltage. Since the G_1-to-anode mutual conductance is greater than the G_3-to-G_2 mutual

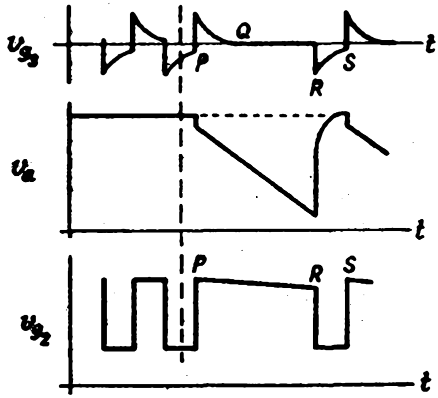


FIG. 225. WAVEFORMS IN SCREEN-COUPLED PHANTASTRON OF FIG. 224 (*b*)

conductance, this effectively prevents the cumulative action which would result in a landslide.

Instead, the linear run-down of v_a proceeds until it is ended by bottoming at point *R* (Fig. 225). When bottoming occurs a further landslide is precipitated, for the anode-voltage is held constant at a low value, and further rise of v_{g1} causes a rise in screen current, and consequently a fall in screen voltage and a fall in suppressor voltage. The anode-current (in the valve) is cut off as a result of this landslide, but the anode-voltage does not rise at once to the value E_a since there is a voltage-drop in R_a as a result of the downward passage through this resistor of the current which is recharging C_{1A}. As this current decays, the anode-voltage rises to E_a and the cycle (*PQRS*) is completed.

This circuit may be used as a free-running time-base generator. Alternatively it may be made monostable by substituting a d.c. coupling (e.g. of the "third-rail" type) for the *RC* coupling between screen and suppressor, the steady suppressor-bias being sufficient to hold the anode-current cut-off indefinitely in the stable condition. The circuit may then be triggered by a positive pulse in the suppressor-cathode circuit. Another name for this circuit is the Miller Transitron.

There are several other combinations of the Miller sweep circuit with relaxation oscillators and flip-flops, among which are the Cathode-coupled Phantastron and the Sanatron. The latter is a combination of a Miller sweep circuit and a monostable multivibrator.

Circuits for Counting Pulses

Several types of circuit have been developed for the counting of electrical pulses, some of them involving special valves. We shall restrict attention here to pulse-counting circuits which are based on the bi-stable multivibrator, described in Chapter V, pages 200–204.

The bi-stable multivibrator is a two-stage direct-coupled amplifier with the whole of its output voltage direct-coupled back to its input terminals. It has two stable conditions. In one of these, the anode-current of one valve is cut-off; in the other, the anode-current of the other valve is cut off. The circuit can be triggered from one of these stable conditions to the other by a negative pulse of voltage applied to the anode of the particular valve which, at the time, happens to be cut off. As explained on page 202, if an incoming negative pulse be fed to both anodes, and if a diode be included in the path to each anode, then the two diodes will serve to route the pulse to the anode of the valve which is cut off and not to the other valve.

Two successive triggering pulses will therefore suffice to take the bi-stable multivibrator circuit through one complete cycle of operation. The first pulse changes it from one of its stable states to the other; the second pulse restores the original stable state.

Consider the valve which, before the arrival of the pulses, was carrying anode-current. On the arrival of the first pulse, the anode-voltage of this valve will suddenly rise. On the arrival of the second pulse, the anode-voltage of this same valve will suddenly fall. A negative pulse can be derived from this sudden fall of anode-voltage and we shall regard this negative pulse as the output voltage of the circuit. There is thus one negative output pulse for each two negative input pulses.

The output pulses from one bi-stable multivibrator can be used as the input pulses for a second bi-stable multivibrator, and the output pulses from this second bi-stable multivibrator can be used as input pulses for a third, and so on. The purpose of cascading bi-stable circuits in this way, as we shall see below, is to provide an apparatus for counting pulses at a high speed. The most obvious method of cascading bi-stable circuits for this purpose will be, first, to select one valve of each bi-stable pair as the output valve, and to connect its anode through a condenser and series resistance to h.t. positive. The resulting *RC* circuit, if made to have a sufficiently small time-constant, would differentiate (see page 298) the step-shaped anode-voltage and would give an output-voltage across the resistance. This voltage pulse could then be fed via diodes, to the two anodes of the next bi-stable pair in the manner described on page 202; the two diode cathodes would be connected together and also connected to the end of the differentiating resistance remote from h.t. positive.

An alternative method of cascading is shown in Fig. 226. Here the above-mentioned differentiating circuit is omitted and a fraction of the step-shaped output voltage of the triggering bi-stable pair is communicated (again via diodes) to the next, or triggered bi-stable pair. The differentiation which converts this step-voltage into a pulse takes place in the circuit of the triggered bi-stable pair, as a

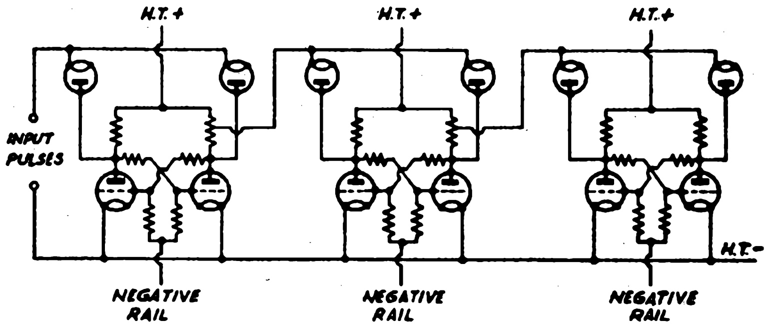


FIG. 226. THREE CASCADED BI-STABLE MULTIVIBRATORS USED AS A BINARY COUNTING CIRCUIT

result of the addition of a condenser across each anode-grid resistance. In a circuit used for fast counting, these grid-anode condensers would, in any case, have to be included for a different but related reason (viz. to avoid the integrating effect of the capacitances which shunt the grid-to-negative rail resistances) and the circuit of Fig. 226 is therefore to be preferred. To take the third stage through its cycle of operation, two output pulses would be required from the second stage, and this would demand four output pulses from the first stage. Consequently eight pulses would be required at the input terminals of the first stage. With n such bi-stable circuits connected in cascade 2^n input pulses would be required to take the whole circuit through one cycle of operation.

To show how an indication is given of the number of pulses counted (i.e. to show how one reads-off the answer), let us suppose that we have four such cascaded bi-stable multivibrators. Each of them has two valves. Before any input pulses are fed in, let us connect a d.c. milliammeter into the anode circuit of each valve which at the time is cut-off. There will thus be one milliammeter in each of the four multivibrator circuits. All four of these milliammeters will of course read zero until we begin to feed pulses into the circuit. When the first pulse arrives, the milliammeter in the first stage will pass current and give a reading, but the other three milliammeters will continue to read zero. The arrival of a second pulse will restore the reading of the first milliammeter to zero but, since the first of the cascaded multivibrators has now received two input pulses, a pulse will be sent from the first multivibrator to the

Number of Pulses	Fourth Milliammeter	Third Milliammeter	Second Milliammeter	First Milliammeter
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

second multivibrator. The second milliammeter will therefore pass current and give a reading. The table shows which milliammeters are reading after any given number of pulses have arrived at the input terminals. (A zero indicates that the milliammeter in question is not reading; a figure 1 indicates that it is reading.)

It will be seen that no two lines of this table are the same. Thus we have a code in which any number from 1 to 15 may be expressed, and the reading of our pulse counter is given directly in this code.

Instead of using d.c. milliammeters as described above, it is cheaper and simpler merely to connect a low-consumption neon glow-lamp across the load resistance of the appropriate valves. When the valve is passing current the lamp will glow; when it is cut off, the lamp will be dark. The uninitiated reader may well comment that it will often be necessary to count numbers very much greater than 15, and that the memorizing of the "code" may then be rather difficult. We must therefore reveal the fact that this so-called "code" happens to be the well-known system of "Binary numbers."

Binary Numbers

When man first learnt to count, he used his fingers. (The word "digit" means also "finger.") Having ten fingers, he produced a system of counting in tens, in tens of tens (hundreds), in tens of tens of tens (thousands), etc. It is possible that space-travellers of the future will discover intelligent beings, with eight fingers, who count in eights, in eights of eights, etc., instead of in our decade system. The basis of all their counting would be the number eight. Their figures would include no figure 8 or figure 9, as these would be written 10 and 11 respectively.

The system of counting which uses the number 2 as its basis is known as the Binary system. This system includes no figures other than 0 and 1. The ordinary decimal numbers 2, 4, 8, 16, etc. would be written in the binary system as 10, 100, 1,000, 10,000, etc. Clearly the right-hand digit gives us the number of units, the next digit to the left gives us the number of two's, the next digit gives us the number of $(2)^2$'s, and so on. The decimal numbers 0 to 15 are listed in binary notation, in the above table.

We have seen in the previous section how a series of lamps (on the front panel of an electronic counter) can indicate a binary number. Each lamp indicates one digit. If the lamp is dark, that digit is "zero"; if the lamp is lit, that digit is "one." The complete number (all the digits) is indicated at one and the same instant. An alternative method is to use only a single flashing lamp to indicate all the digits one after another in a time sequence. This is known as the *serial* method, since a *series* of current pulses would flow through the lamp one after the other.

Our mention of a flashing lamp, in defining the serial method of representing binary numbers, was for the purpose of illustration only. No lamp could flash in and out rapidly enough to follow the pulses of current in a modern digital computer. The essence of the serial method is that the binary number is indicated by a train of current (or voltage) pulses, successive digits of the number being indicated in successive time intervals. If a pulse occurs in a particular time interval, the particular digit assigned to that interval is "one." If no pulse occurs, the digit is "zero."

A ten-digit binary number might be indicated by a train of pulses lasting ten microseconds, each successive digit (usually beginning with the "units" digit—i.e. reading the number "backwards"), being indicated in successive 1 microsecond periods. The chosen pulse-width might be ½ microsecond. A *regular* repetition of pulses (with no pulses missing) would indicate the number 1111111111. If, say, the second, third, fourth, sixth and seventh pulses were missing the number indicated would be 1 1 1 0 0 1 0 0 0 1. (Remember that the number is read backwards, in the same way, and for the same reason, that we begin at the right-hand end when adding two long numbers.)

Gate Circuits for Electronic Digital Computers

Consider the addition of two binary numbers as set out below—

1 0 1	i.e. Decimal number five.
0 1 1	i.e. Decimal number three.
1 0 0 0	i.e. Decimal number eight.

What sort of circuits would be required to carry out this addition process electronically, i.e. what sort of circuits are required in a Binary Adder?

Such an adder would require two pairs of input terminals. To one pair of input terminals would come a train of pulses representing the binary number 101 (viz. a pulse, no pulse, then another pulse). Simultaneously, a train of pulses representing the number 011 would come to the other input terminals (viz. a pulse, another pulse, no pulse). The device would be required to produce at its output terminals a train of pulses corresponding to the binary number 1000 (viz. no pulse, no pulse, no pulse, a pulse).

Consider first the addition of the "units" column (the right-hand column). The requirements are as follows—

(i) If there is no pulse in either input channel during this period, then there shall be no output pulse $(0 + 0 = 0)$.

(ii) If there is a pulse in only one of the two input channels during this period then there shall be an output pulse $(0 + 1 = 1$ and $1 + 0 = 1)$.

(iii) If there is a pulse in both input channels during this period, then there shall be no output pulse, but an additional pulse shall be fed in during the later period corresponding to the second column of figures. This corresponds to saying: Carry "one" forward to the next column. $(1 + 1 = 10)$.

To meet requirement (ii) above, we need a circuit which will give an output pulse when an input pulse appears in *either one or the other* of the input channels. Such a circuit is called an "OR-Gate."

To meet requirement (iii) above, we need a circuit which will give an output pulse only if an input pulse appears in one input channel *and at the same time* an input pulse appears in the other channel. Such a circuit is called an "AND-Gate." Its output pulse would not be fed to the output terminals of the adder but, being a "carry" pulse, would be fed through a delay circuit to the input of other circuits.

The carry-pulse, after the appropriate delay, would therefore appear at the input terminals of these other circuits at the same time as the pulses corresponding to the figures in the second column from the right. This means that the circuits dealing with the second column pulses must be circuits with *three* input channels—one channel for the pulses of one of the numbers which are to be added, one channel for the pulses of the other number and one channel for the carry-pulses. In all columns except the "units" column, we are thus required to add *three* binary digits. We conclude that there is a need for "AND-Gate" circuits and for "OR-Gate" circuits each having three input channels.

"AND"-GATE CIRCUITS. In Fig. 227, the gate circuit is the part of the diagram to the right of the dotted line. To the left of the

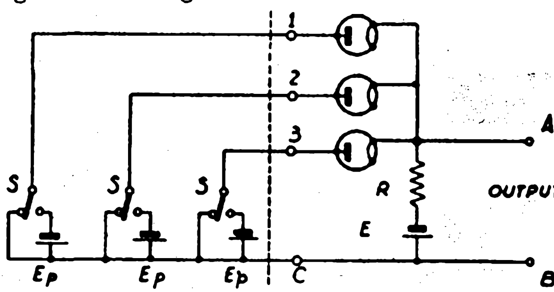


FIG. 227. "AND"-GATE

dotted line are shown, schematically, three negative pulse-generators connected to the three input circuits (terminals 1, 2 and 3 and a common input terminal, C). The pulse generators are presumed to operate by the instantaneous movement of the switch, S, from left to right, followed a little later by the instantaneous return of the switch from right to left. In a period when no input pulses are forthcoming in any of the three input channels, the three switches are all in the left-hand position. It is clear that, during such a period, the three diodes are in parallel and behave as a single diode. Moreover, the d.c. e.m.f. E causes this single diode to have a positive anode-voltage and it therefore conducts. With all the switches S in the left-hand position, this single diode will be seen to be connected directly across the output terminals AB. The resistance of the diodes when conducting is very small compared with the resistance R, and we may therefore regard the single diode as short-circuiting the output terminals AB. The output voltage, v_{AB}, is thus zero.

If one of the switches, S, moves from left to right (to represent the arrival of a negative pulse in one channel) then the anode-voltage of the diode concerned will be given by $-E_P - v_{AB}$. This particular diode will thus cease to conduct. The two remaining diodes will still conduct and they will continue to short-circuit the output terminals AB. If two of the switches, S, move from left to right, the two diodes concerned will cease to conduct, but the output terminals AB will still be short-circuited by the remaining diode. Only if all diodes are prevented from conducting will there be an output voltage. The output voltage (Fig. 227) will then be $-E$.

Thus, as required, the condition for an output pulse is the simultaneous arrival of negative input pulses in all input channels. The output pulse will be a negative pulse and its magnitude will be E.

"OR"-GATE CIRCUITS. In Fig. 228, the gate circuit is the part of the diagram to the right of the dotted line. As in the previous section, the part of the diagram to the left of the dotted line represents three negative pulse-generators, assumed to operate by the movement of the switches, S, from left to right and back again. Comparing Fig. 228 with Fig. 227 we note that the only difference

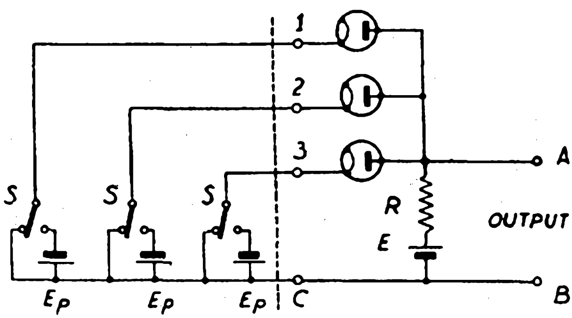


FIG. 228. "OR"-GATE

is that the polarities of all diodes have been reversed, and also the polarity of the d.c. e.m.f. E. The polarity of the input pulses is again considered to be negative.

In the quiescent condition, with no incoming pulses, all switches S are considered to be in the left-hand position. The three diodes are thus in parallel. The d.c. e.m.f. E is of the correct polarity to make the diodes conduct. The conducting diodes are seen to be directly across the output terminals AB. Since the resistance of each diode when conducting is very much smaller than the resistance R, the diodes may be considered to short-circuit the output terminals. Thus, in the quiescent condition, the output voltage, v_{AB}, is zero.

Now let one of the switches S in Fig. 228 move to the right, to represent the arrival of a negative pulse in one input channel only. In view of the symmetry of the circuit, it does not matter which of the three switches we consider to be moved. Let it be the left-hand switch, associated with diode No. 1. The contribution of E_P to the anode-voltage of this diode is a positive contribution. (Cf. the "and"-gate of the previous section, where E_P was a negative contribution to the anode-voltage.) Diode No. 1 will therefore continue to conduct. The whole of its anode-current flows through the high resistance R so that the voltage across the diode itself will be very small, and it may be considered a short circuit. If a short-circuit be substituted for diode No. 1 in Fig. 228, it can be seen at once—

(*a*) that the output voltage, v_{AB}, is $-E_P$, since the left-hand pulse-generator is now connected directly across the output terminals, and

(*b*) that diodes Nos. 2 and 3 are connected in parallel, directly across the output terminals. The anode-voltage of both these diodes thus becomes negative and they both cease to conduct.

Thus, as required, the condition for the production of an output pulse is that a negative pulse shall arrive in any one input channel. The output pulse will be negative and its magnitude will be E_P.

It is to be observed that this "OR"-gate will still give an output pulse if input pulses arrive simultaneously in *more than one* input channel.

INHIBITING GATE CIRCUITS. The purpose of an inhibiting gate is to block the passage of pulses through a circuit in response to a pulse (known as an inhibiting pulse). As an example of the need for such circuits, let us reconsider the "OR"-gate of the previous section.

An "OR"-gate will give an output pulse if there is an input pulse in any one of its input channels. It will also give an output pulse if pulses arrive simultaneously in *more than one* input-channel. This "OR"-gate would therefore not serve our purpose if the requirement were that there should be an output pulse when there is an input pulse in one *and only one* of the input channels.

This requirement can be met, however, if we feed the output pulses of the "OR"-gate through an inhibiting-gate, and arrange that the inhibiting gate shall block the passage of these output pulses whenever simultaneous pulses occur in two or more input channels. The necessary inhibiting pulse could be derived by connecting the input channels, two at a time, to "AND"-gates and using the output pulses of the three "AND"-gates as the input pulses for a second "OR"-gate. This second "OR"-gate would provide an output pulse only if there were output pulses from one or more of the "AND"-gates. The output pulse of this "OR"-gate would therefore serve as the inhibiting pulse for our purpose. (Cf. shaded parts of Fig. 230.) Fig. 229 shows a simple inhibiting-gate circuit. The gate-circuit proper is the part of the diagram to the right of the dotted line. To the left of the dotted line are shown two elementary pulse generators, presumed to operate by the movement of switches S_1 and S_2 from left to right, and then back from right to left. It will be seen that this circuit calls for a positive inhibiting pulse, though the input pulse is, as usual, a negative pulse.

The circuit differs from a two-input "AND"-gate (cf. Fig. 227) only in the inclusion of the d.c. e.m.f. E_B. In the quiescent condition, with no input pulse and no inhibiting pulse, we can consider both switches, S_1 and S_2 (Fig. 229) to be in the left-hand position. The upper diode is conducting and the output voltage, v_{AB}, is zero, as for the "AND"-gate. The e.m.f. E_B prevents the lower diode from conducting.

Movement of S_1 to the right, corresponding to the arrival of an input pulse, prevents the upper diode from conducting, since E_P is

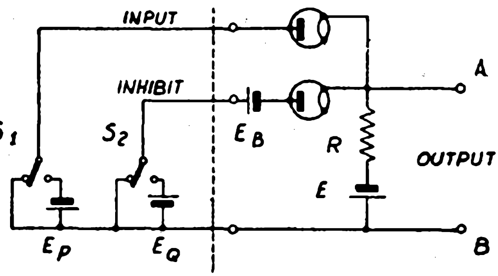


FIG. 229. INHIBITION GATE

a negative contribution to its anode-voltage. Neither diode is now conducting and the output voltage, v_{AB}, falls to the value $-E$. Thus, in the absence of an inhibiting pulse, the arrival of an input pulse produces an output pulse.

If now switch S_2 be considered to be moved to the right, representing the arrival of a positive inhibiting pulse, the inhibiting e.m.f. E_Q will cancel the effect of the e.m.f. E_B, the lower diode will conduct, and the output voltage, v_{AB}, will be zero whether the switch S_1 is in the right or left-hand position.

The fact that a positive inhibiting pulse is required for the operation of this gate-circuit means that, in a system where negative pulses are used throughout, we must include a polarity-inverting circuit to provide the required inhibiting pulse.

As an example of the application of these three gate circuits, Fig. 230 shows the circuit of a complete binary adder. This is a schematic diagram in which a single line denotes a pair of leads. Arrows are used to make it clear which is the input end of each gate circuit. The word "UNLESS" denotes an inhibiting gate.

The combination of the six shaded elements constitutes a gate which gives an output pulse when an input pulse appears *in one and only one* of the three input channels, as described above (in the first two paragraphs of the section on Inhibiting Gate Circuits). There will be an output pulse from the Inhibiting Gate if there is an input pulse in either input channel or in the delay channel, *unless* there are input pulses in two or more of these three channels. This is the same thing as saying that there will be an output pulse from the Inhibiting Gate if there is an input pulse in one and only one of these three input channels.

There will be an output pulse from the whole network either if there are simultaneous pulses in the two input channels and the "carry" channel $(1 + 1 + 1 = 11)$ or if there is a pulse in only one of these three channels $(1 + 0 + 0 = 1)$. There will be a

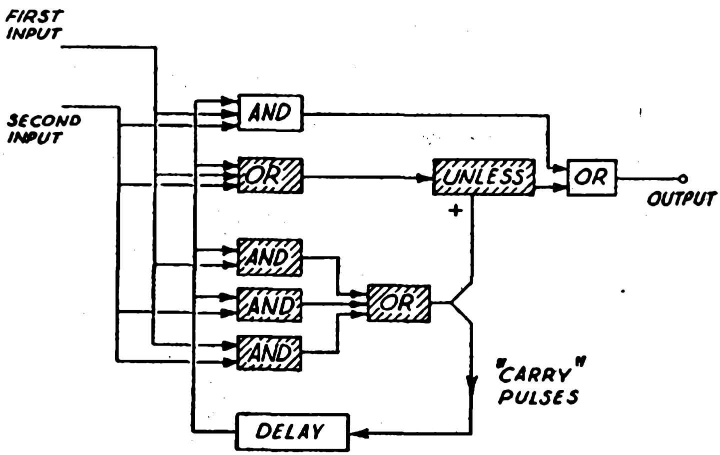


Fig. 230. Binary Adding Circuit

"carry" pulse if there are simultaneous pulses in any two or more of these three channels.

Thus a train of pulses denoting one binary number can be fed along one input channel, and simultaneously another train of pulses denoting another binary number can be fed along the second input channel, and the output of the circuit of Fig. 230 will then automatically be the required train of pulses to denote the sum of the two binary numbers.

There are gate-circuits using pentodes. The coincidence circuit of Fig. 195, page 313, is really a two-input "AND"-gate. The diode circuits just described are closely related to the diode circuits discussed in the section on "Electronic Switches and Time Selection," page 312. There are decade counting circuits (see Problem No. 30, page 391) as well as binary counting circuits. There are circuits which convert decade numbers into binary numbers and vice versa. The few circuits which we have been able to describe in these last pages merely serve as an introduction to the new and growing subject of computer circuitry.

Suggestions for Further Reading

1. W. B. Lewis, *Electrical Counting* (Cambridge University Press).
2. M. V. Wilkes, *Automatic Digital Computers* (Methuen).
3. Glasoe and Lebacqz, "Pulse Generators," *Radiation Laboratory Series*, Vol. 5.
4. Valley and Wallman, "Vacuum Tube Amplifiers," *Radiation Laboratory Series*, Vol. 18.
5. Chance and others, "Waveforms," *Radiation Laboratory Series*, Vol. 19.
6. Chance and others, "Electronic Time Measurements," *Radiation Laboratory Series*, Vol. 20.
7. Solley, Starr and Valley, "Cathode Ray Tube Displays," *Radiation Laboratory Series*, Vol. 22.
8 M.I.T. Radar School Staff, *Principles of Radar* (McGraw-Hill).
9. D. A. Levell, *Pulse and Time-base Generators* (Pitman).
10. A. C. D. Haley and W. E. Scott, *Analogue and Digital Computers* (Newnes).

PROBLEMS

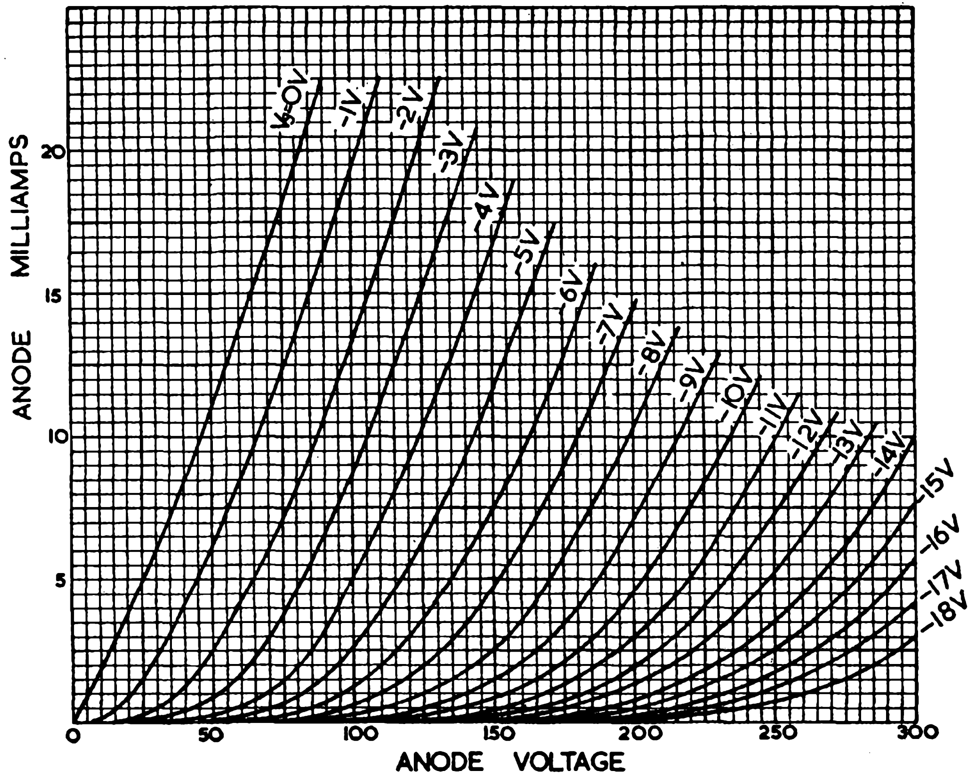
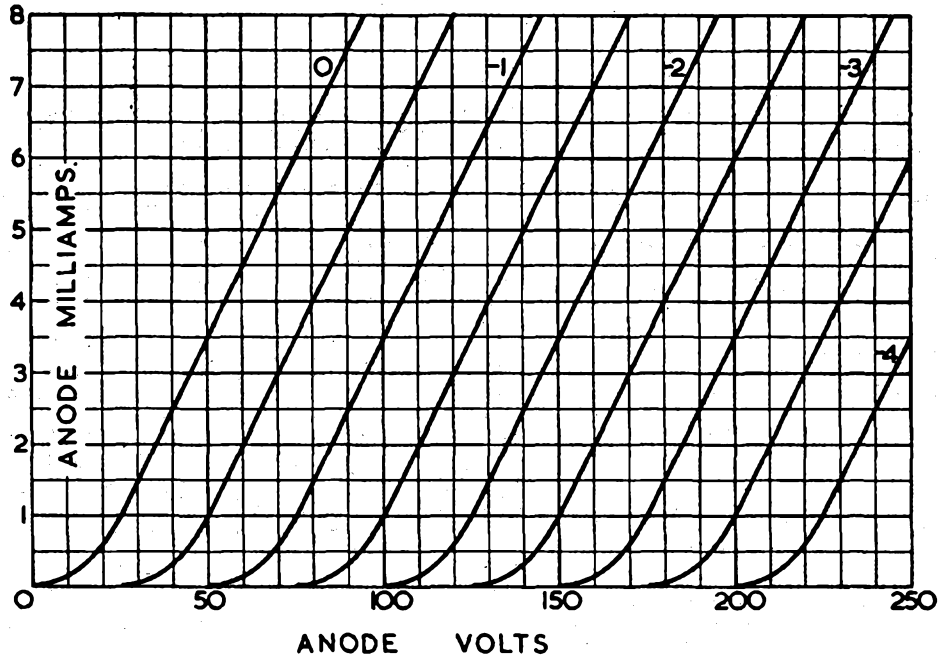

ANODE MILLIAMPS
20
15
10
5
0
50
100
150
200
250
300
ANODE VOLTAGE
Vg=0V
-1V
-2V
-3V
-4V
-5V
-6V
-7V
-8V
-9V
-10V
-11V
-12V
-13V
-14V
-15V
-16V
-17V
-18V

Fig. P-1



ANODE MILLIAMPS.
ANODE VOLTS
0
50
100
150
200
250
1
2
3
4
5
6
7
8
0
-1
-2
-3
-4

Fig. P-2

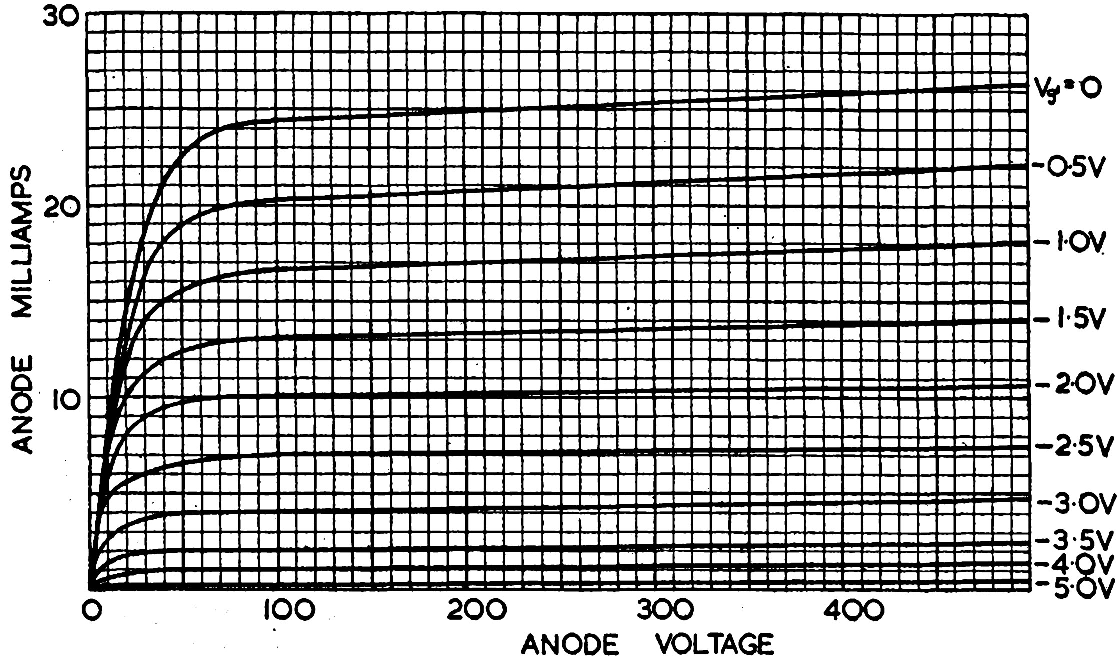


FIG. P-3. PENTODE CHARACTERISTICS, DRAWN FOR A SCREEN-GRID VOLTAGE OF 250 V

PROBLEMS

Answers and solutions to these problems are given on pages 393 to 419. These problems include some trick questions which cannot be solved with the data provided. The explanation of why a particular problem cannot be solved is given under "Answers." An asterisk is used to mark problems which introduce the reader to useful information or techniques not included in the body of the book.

Chapter II

II.1. How many wires need to be brought out through the glass envelope of each of the following valves? (Assume that no two electrodes are connected together inside the valve.)

(*a*) Indirectly heated diode
(*b*) directly heated triode
(c) indirectly heated tetrode
(*d*) indirectly heated pentode
(*e*) directly heated double-diode with common cathode
(*f*) indirectly heated double-triode with separate cathodes and separate heaters
(*g*) indirectly heated double-diode-triode with common cathode.

II.2. Evaluate g_m, μ and ρ for the triode whose characteristics are shown in Fig. P–1 (page 358), at an anode-voltage of 100 volts and a grid-voltage of − 2 volts.

II.3. A certain triode valve has its grid-voltage maintained at zero and it is found that, when the anode-voltage is 80, the anode-current is 1·6 milliamp. What is the valve-impedance, ρ?

II.4. What will be the horizontal spacing, in volts, between adjacent *anode*-characteristics of a valve which has $g_m = 5$ mA/v and $\rho = 12{,}000$ ohm, if the characteristics are drawn for grid-voltages of 0, − 1, − 2, − 3, etc.?

II.5. What will be the vertical spacing, in milliamps, between adjacent characteristics as in problem 2?

II.6. Is the valve mentioned in problem 4 a triode, a screen-grid tetrode or a pentode?

II.7. Write down the equation of the linear parts of the characteristics of the valve specified in problem 4 (using volts and amps as units) given that the anode-current is 15 mA when the anode-voltage is 240 and the grid is directly connected to the cathode.

II.8. If the valve mentioned in problems 4 to 7 above is operated with an anode-voltage of 120, what value of negative grid-voltage would be required to give an anode-current of 1 microamp (i.e. 10^{-6} amp)?

II.9. How could the value of the constant "intercept-current," *d* (page 18) be found from the characteristic curves? If one is told that, for a certain valve, $d = +\ 2$ mA, what can one conclude about the valve? Evaluate the intercept-current, *d*, for the valves whose characteristics are shown in Fig. P–2 and Fig. P–3.

II.10. Devise a method of evaluating the amplification factor by measuring a single length,

(*a*) on the *anode* characteristics of a valve (see problem 2)
(*b*) on the *mutual* characteristics of a valve.

II.11. Could the methods of measuring amplification factor, implied in problem 10, be used in practice for a pentode?

II.12. The valve specified in problem 4 has its grid-voltage maintained constant at − 2 volts. The anode-voltage is given by 250 + 3·6 sin 1,000*t*. What will be the peak value of the alternating component of anode-current?

II.13. The valve specified in problem 4 has its anode-voltage maintained constant at 300 volts. The grid-voltage is given by $-1 + 0{\cdot}5 \sin 2{,}000t$. What will be the peak value of the alternating component of anode-current?

II.14. A certain screen-grid tetrode, operated with a screen-grid voltage V_s and a control-grid voltage of zero, has an anode-current of 1 mA when the anode-voltage is 120 volts, and an anode-current of 0·9 mA when the anode-voltage is raised to 130 volts. What information does this give us about the value of V_s?

II.15. For a certain triode, it is found that when $v_g = -1$ volt and $v_a = 240$ volt, the anode-current is 3 mA. Can you evaluate ρ and g_m?

II.16. A certain triode valve has $v_g = 0$ (i.e. the grid is connected to the cathode). An h.t. voltage of 250 v in series with a resistance of 10,000 ohms is connected between anode and cathode, with the negative side connected to the cathode. The valve-impedance, ρ, is 40,000 ohms. What anode-current will flow?

II.17. Assume that, for the valve in the circuit of Fig. 12, page 15, the grid-current is zero whenever the grid-voltage is negative, whereas for positive grid-voltage

$$(i_g \text{ in milliamps}) = \tfrac{1}{2}\,(v_g \text{ in volts})$$

Plot roughly the waveform of v_g which would result from including 1 megohm in series with the source $V \sin \omega t$, where V is 4 volts. Between what limits will v_g vary?

II.18. Draw a rough graph to show how the anode-current of a pentode depends on the screen-grid voltage, the voltages of G3 and anode being held constant at zero and a positive value respectively.

***II.19.** From the anode characteristics of a pentode, as given in Fig. P–3, page 360, deduce and plot the mutual characteristics (i_a plotted against v_{g_1}) for the following values of anode-voltage: 400, 100, 50, 20, 10.

II.20. A triode-valve whose characteristics are as given in Fig. P–1, page 358, has an anode-voltage of 150 and an anode-current of 9 mA. From the characteristic curves, determine the value of the grid-voltage.

II.21. A triode valve whose characteristics are as given in Fig. P–1, page 358, has a battery connected directly between anode and grid, the positive side being connected to the anode. The internal resistance of the battery is negligibly small. A 1,000-ohm resistance is connected between cathode and grid. The current through this resistance is 10 milliamps. What is the battery voltage?

II.22. In the circuit of Fig. P–4, $E_a = 200$ v, $R = 500$ ohms and each valve has characteristics as given in Fig. P–1, page 358. The current through R is 20 mA. Determine—

(*a*) the anode-voltage of each valve
(*b*) the grid-voltage for valve 1
(*c*) the anode-current of each valve
(*d*) the grid-voltage for valve 2
(*e*) the battery-tapping voltage, E_g.

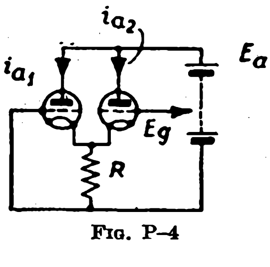


FIG. P–4

***II.23.** Plot graphs showing roughly how each anode-current in Fig. P–4 changes as E_g is increased from -20 v to $+20$ v. Take $E_a = 200$ v, $R = 1{,}000$ ohms and the characteristics as in Fig. P–1, page 358.

II.24. Deduce the shape of the graphs mentioned in the previous problem by assuming a linear algebraic equation for the valve characteristics and using the following relations—

$$v_{a_1} = v_{a_2} = E_a - R_k(i_{a_1} + i_{a_2})$$
$$v_{g_1} = -R_k(i_{a_1} + i_{a_2})$$
$$v_g = E_g - R_k(i_{a_1} + i_{a_2})$$

In particular, derive expressions for

(a) the value of E_g required just to cut off i_{a_1}
(b) the value of E_g required just to cut off i_{a_2}
(c) the value of i_{a_2} when i_{a_1} is just cut off
(d) the value of i_{a_1} when i_{a_2} is just cut off.

***II.25.** *Discharge of a condenser through a triode.* A condenser, of capacitance C and charged to a p.d. E, is connected across a triode valve, from anode to cathode, the positive side being connected to the anode. The grid-voltage is maintained constant. By assuming a linear equation for the valve, prove that the condenser voltage falls exponentially, with a time constant ρC, towards a voltage which is given by the intercept of the linearized characteristic on the anode-voltage-axis. (Hint: derive a differential equation for the anode-current. See also problem II.26.)

***II.26.** *Whole-current equivalent circuit for a triode valve.* A box with two terminals contains an unknown electric circuit. It is known that the circuit does not include a triode valve but it may include a diode. Any measurements which are made from outside the box give exactly the same results as if the box contained a triode valve with its anode connected to one terminal, with its cathode connected to the other and with a small battery maintaining the grid negative with respect to the cathode.

(a) Suggest what the box may contain.

(b) Hence devise an equivalent anode circuit for a triode with assumed linear characteristics.

(c) Use this equivalent circuit to provide an easier solution for problem II.25, above.

Chapter III

III.1. A simple amplifier circuit (Fig. 19, page 27) uses the triode valve whose characteristics are shown in Fig. P–1, page 358. The h.t. voltage is 300, the load is a resistance of 20,000 ohms and the grid-bias voltage (or "quiescent negative grid-voltage") is 6. What will be the steady (or "quiescent") values of anode-current and anode-voltage?

III.2. If the input voltage of the above amplifier is a sine wave whose peak value is 2 volts, between what limits will the anode-voltage vary in the course of each a.c. cycle? What will be the voltage amplification?

III.3. If the peak-value of the sine-wave input voltage to the above amplifier be now increased to 6 volts, evaluate (a) the voltage-amplification, (b) the percentage of equivalent second harmonic distortion, and (c) the a.c. power in the load.

III.4. For a given basic amplifier circuit, working on the linear parts of its characteristics, it was found that doubling the load resistance increased the voltage gain in the ratio 6 : 5. In what ratio would further doubling cause the gain to increase?

III.5. A single-stage amplifier, working on the linear parts of the valve-characteristics, has its voltage gain, G, measured for each of a series of different values of load resistance, R. A graph is plotted showing how $1/G$ (ordinate) depends upon $1/R$ (abscissa). What will be the shape of the graph and how can the valve parameters be calculated from it?

***III.6.** Show that a triode valve which has a resistance R shunted across it, from anode to cathode, behaves as a triode with a reduced amplification factor and a reduced valve impedance, the reduction factor in both cases being $1 + \rho/R$, where ρ is the value of the original valve impedance.

III.7. A simple voltage-amplifier stage uses a pentode whose characteristics are similar to those shown in Fig. P–3, page 360, except that the currents are only one-tenth of those shown. A load resistance of one megohm is directly connected in the anode circuit. The input voltage has a peak value of 10 millivolts and the negative grid-bias voltage is 2. Approximately how large need the h.t. voltage be made? If an h.t. voltage of 1,200 were used what would be approximately the steady anode-voltage and the voltage amplification?

III.8. A simple amplifier circuit (Fig. 19, page 27) uses a triode valve whose characteristics (Fig. P–2, page 359) are more or less linear for anode-currents exceeding ½ mA, but very curved for anode-currents smaller than this value. To avoid distortion, therefore, the rough rule is adopted that the anode-current shall not be allowed to fall below 1 mA in the course of the a.c. cycle. The load is a resistance of 40,000 ohms. What is the minimum permissible value of h.t. voltage in each of the following conditions?

(*a*) Peak sine-wave input, 1 volt; grid-bias, 1 volt

(*b*) Peak sine-wave input, 2 volts; grid-bias, 2 volts

(*c* Peak sine-wave input, 0·5 volt; grid-bias, 3·5 volts

III.9. If, with the conditions of problem III.8, the h.t. voltage were fixed at 250, and if one were free to choose the value of the load resistance, what would be the largest permissible load resistance in each of the following circumstances?

(*a*) Peak sine-wave input, 1·5 volts; grid bias, 2 volts

(*b*) Peak sine-wave input, 2 volts; grid bias, 2 volts

III.10. A simple amplifier circuit (Fig. 19, page 27) uses a valve whose characteristics (Fig. P–2, page 359) are more or less linear for anode-currents exceeding ½ mA but very curved for anode-currents smaller than this value. To avoid distortion, the rough rule is adopted that the anode-current shall not be allowed to fall below 1 mA in the course of the a.c. cycle. There are the further restrictions that the anode-current must at no time exceed 8 mA and that the anode-voltage must at no time exceed 225 v. A sine-wave input voltage of any desired magnitude is available. (*a*) What value of load resistance must be used to secure maximum a.c. output power? (*b*) What values of h.t. and grid-bias voltage must be used in conjunction with this optimum load resistance? (*c*) What will be the required peak-value of the input voltage? (*d*) What will be the value of the a.c. output power?

III.11. An amplifier stage, using the valve whose characteristics are shown in Fig. P–1, page 358, has a 20,000-ohm load resistance connected into its anode-circuit through a 1 : 1 transformer. The transformer may be assumed perfect. The h.t. voltage is 150, and the grid-voltage is the sum of a negative bias of 6 volts and a sine-wave A.C. whose peak value is 6 volts. Draw the A.C. load line superposed on the anode characteristics and hence determine the limiting values between which the anode-current and the anode-voltage will swing in the course of the a.c. cycle.

III.12. A transformer-coupled output stage (Fig. 35, page 48) uses the valve mentioned in problem III.10 and is subject to the same rough rule and the same restrictions upon anode-current and anode-voltage. The load is a resistance of 10 ohms. The transformer may be assumed perfect.

(*a*) What must be the transformer ratio to secure maximum a.c. output power?

(*b*) What h.t. voltage must be used?

III.13. A transformer-coupled output stage has an input voltage of constant amplitude and well within the range which it can handle without overloading. The valve impedance, ρ, is 4,000 ohms and the load is a 10-ohm resistor. What transformer ratio will give maximum output power? By what percentage would the output power be decreased if the transformer ratio were (*a*) one half, (*b*) twice the optimum value?

III.14. Derive the equation of the dynamic characteristic of a basic amplifier stage (Fig. 19, page 27) assuming a linear equation for the static characteristics of the valve. What is the slope of the dynamic characteristic in the case of a valve using a load resistance equal to the valve impedance and having a mutual conductance of 4 mA/v.

***III.15.** Why is the dynamic characteristic of a pentode amplifier stage, with a resistive load, almost indistinguishable from the mutual static characteristics?

III.16. The equation of the dynamic characteristic of a certain triode amplifier stage is given by

$$i_a = a + bv_g + cv_g^2$$

If the grid-voltage is made up of a bias-voltage, $-E$, in series with a sine-wave alternating voltage, $V \sin \omega t$, show that the anode-current is the sum of the following components and evaluate each of them: D.C., fundamental, second harmonic.

III.17. A simple amplifier stage using a triode operates on the linear parts of the valve characteristics, for which $\rho = 20{,}000$ ohms and $g_m = 3$ mA/v. The load is a resistance of 40,000 ohms.

(*a*) Calculate the voltage gain.

(*b*) Express this gain in decibels.

(*c*) If the quiescent anode-current is 2·5 mA and the peak input voltage is 1·5, between what limits will the anode-current alternate in the course of the a.c. cycle?

(*d*) To what value would the gain be increased by doubling the load resistance?

III.18. The amplifier stage of problem III.17 (*c*) operates with a grid-bias of 2·5 volts, produced by the use of a cathode-resistance. What must be the value of the cathode-resistance, R_k? To what value will the voltage gain be reduced if this resistor is not shunted by a condenser?

III.19. An amplifier stage using a pentode has a quiescent anode-current of 5 mA. Grid-bias is secured by means of a 600-ohm resistance (shunted by a bias-condenser) in the cathode-lead. The input is an alternating voltage of peak value 3 volts. Between what limits does the control-grid voltage vary in the course of the a.c. cycle?

III.20. Is it possible by using a cathode-bias resistor in a pentode amplifier stage (and *without* the use of a bleeder-resistance as shown dotted in Fig. 46 (*a*)) to produce a negative control-grid voltage large enough to cut off anode-current?

III.21. An output stage using the pentode whose characteristics are given in Fig. P–3, page 360, is to be operated subject to the following restrictions—

(*a*) the control-grid voltage shall never become positive

(*b*) the anode-voltage shall never exceed 400 volts

(*c*) the distortion arising from non-linearity of the characteristics shall not be allowed to become so bad that the anode-current is cut-off for part of the a.c. cycle, though the anode-current may be permitted to fall to zero at one point of the cycle.

Determine the approximate value of the optimum anode-load resistance for maximum output power and calculate this maximum value of output power.

III.22. An output stage using the pentode whose characteristics are given in Fig. P–3, page 360, has an h.t. voltage of 250 v and a negative grid-bias voltage of 1·5 v. The input voltage is a sine wave of peak value 1·5 volts. The load is a resistance of 150 ohms, coupled into the anode circuit by means of a transformer (assumed perfect). The transformer has a 10 : 1 turns ratio from its anode circuit side to its load side.

(*a*) Between what limits does the anode-voltage swing?

(*b*) Between what limits does the anode-current swing?

(*c*) What is the a.c. output power?

III.23. If the overall voltage-amplification of the two-stage, *R–C* coupled amplifier of Fig. 29 (*b*), page 41, be plotted against the frequency of the input voltage, it will be seen that, for frequencies higher than a certain value, the voltage amplification is more or less constant, say at a value A. In terms of R_g and C, derive an expression for the frequency at which the voltage amplification is $A/\sqrt{2}$. (Assume $R_g \gg R$.)

III.24. Amplifiers using *R–C* coupling are said to "cut off" at low frequencies, though the "cut-off" is in fact merely a gradual reduction of voltage amplification as the frequency becomes lower and lower. It is usual to define the "cut-off frequency" as the frequency at which the voltage amplification has fallen by 3 decibels. Adopting this definition, derive an expression for the cut-off frequency of the amplifier circuit shown in Fig. 29 (*b*), page 41.

III.25. Derive an expression for the cut-off frequency (defined as in problem III.24) of a simple single-stage amplifier, using a valve whose parameters are ρ, g_m and μ, and having as its load a coil whose resistance is negligible and whose inductance is L.

III.26. An amplifier stage with a choke load uses a valve whose impedance, ρ, is 10,000 ohms. Assuming that the resistance of the choke is negligibly small compared to 10,000 ohms, calculate the least permissible value of choke-inductance if the voltage amplification at 50 c/s is to be within 20 per cent of the (asymptotic) amplification at higher frequencies.

III.27. Derive an expression for the 3 dB cut-off frequency of a single-stage amplifier using a triode (ρ and μ) with a choke load (inductance L, resistance negligible) and an un-bypassed cathode-bias resistor, R_B.

III.28. A voltage amplifier stage uses a resistance load which is four times the valve impedance, ρ, and a cathode-bias resistance which is one-tenth as great as the valve impedance. The amplification factor of the valve is 40. What will be the voltage amplification

(*a*) if no bias-condenser be used?

(*b*) at a frequency such that the reactance of the bias-condenser is equal to the bias-resistance?

III.29. A certain *R–C* interstage-coupling (e.g. R_g and C in Fig. 29 (*b*), page 41) uses a one-megohm resistor and is required to have a response curve which is level, to within 3 dB, down to a frequency of 10 c/s. How large need the coupling capacitance be made?

A four-stage, *R–C* coupled amplifier, using three *R–C* couplings as described above, has each coupling capacitance equal to the minimum value as calculated above for a single coupling. By how many decibels is the overall gain less than the "level value" at a frequency of 10 c/s?

***III.30.** A multi-stage, *R–C* coupled amplifier has n identical interstage couplings, each having a coupling capacitance C and a coupling resistance R. Show that the overall voltage amplification is a constant divided by $(1 + x^2)^{n/2}$, where x denotes $1/2\pi fCR$. Hence show that, at a frequency f, the response curve has fallen $10n \log_{10}(1 + x^2)$ decibels below the "level value."

III.31. Calculate the cut-off frequency, defined on a 3 dB basis, for a four-stage, *R–C* coupled amplifier with three identical interstage couplings, if each coupling resistance is one megohm and each coupling capacitance is a tenth of a microfarad.

III.32. Choose a minimum permissible battery-voltage, E_c, for the d.c. amplifier circuit of Fig. 38, page 52, given that the first valve has characteristics as shown in Fig. P–1, page 358, that its load resistance is 30,000 ohms, that the h.t. voltage is 300 and that the instantaneous value of the input voltage will vary between $-$ 0·9 and $-$ 1.

III.33. A resistance-coupled d.c. amplifier, similar to Fig. 39, page 53, uses three identical valves, each having characteristics as shown in Fig. P–1, page 358. The resistances R_1 and R_2 are each one megohm and the second interstage coupling also consists of two one-megohm resistances. The h.t. voltage is 300 and each load resistance is 30,000 ohms. It is known that the instantaneous value of the input voltage will vary between $-$ 0·9 and $-$ 1. Determine—

(*a*) the limits between which the anode-voltage of the first stage will vary

(*b*) the minimum permissible value of the third rail voltage, E_3, if the grid of the second valve is not to be driven positive

(c) the voltage gain, up to the grid of the second valve. (N.B.: the gain of the first stage may be calculated using linear equations and taking the following values of parameters—

$$\rho = 4{,}000 \text{ ohms}, \mu = 20)$$

(*d*) the limits between which the anode-voltage of the second stage will vary, assuming that the value of the third-rail voltage, E_2, is as calculated in (b) above

(e) whether the same value of E_2 is suitable for the second interstage coupling. If it is not suitable, suggest a way in which the circuit might be modified.

*III.34. Fig. P–5 shows an alternative method of resistance-resistance direct-coupling without the use of a third rail voltage. The cathode-resistance

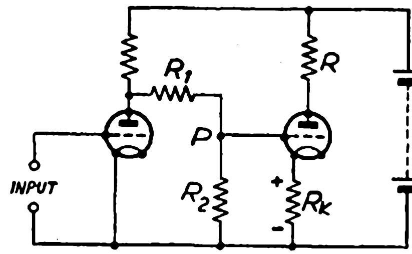


FIG. P–5

R_k is made sufficiently large to raise the potential of the cathode above earth (i.e. above h.t. negative) by about the same voltage as the point P is above earth, so avoiding making the grid of the second valve positive with respect to its cathode.

In a particular case, it is required that the quiescent condition of the second valve shall be $v_g = -5$ v, $i_a = 5$ mA, and it is known that the quiescent anode-voltage of the first valve is 110 volts. Given that $R_1 = R_2$, that the h.t. voltage is 360, and that the characteristics of the second valve are as shown in Fig. P–1, page 358, determine—

(a) the required value of R_k

(b) the required value of R.

III.35. Draw an equivalent a.c. circuit for the push-pull valve amplifier shown in Fig. 41, page 59. Given the following data, and assuming that only the linear parts of the characteristics are involved, calculate the output voltage across the load resistance R.

For each valve, $\rho = 5{,}000$ ohms, $\mu = 20$

Load resistance, $R = 400$ ohms

Input voltage = 10 volts, r.m.s.

Input transformer: total number of turns on the right-hand winding is four times the number on the left-hand winding.

Output transformer: total number of turns on the left-hand winding is ten times the number on the right-hand winding.

III.36. A transformer-coupled push-pull output-stage uses identical triode valves, each having characteristics as shown in Fig. P–1, page 358. The transformers may be assumed perfect. Plot the composite characteristics (the use of tracing paper is recommended) for each of the following conditions—

(a) h.t. voltage = 150, g.b. voltage = 6

(b) h.t. voltage = 150, g.b. voltage = 10.

III.37. A voltage-amplifier stage using automatic bias, as in Fig. 46 (a), page 69, has a load resistance of 60,000 ohms and a bias-resistance of 500 ohms. The valve parameters are: $\mu = 15$, $\rho = 4{,}000$ ohms. What value of capacitance must be shunted across the bias-resistance to ensure that the response curve remains level, to within 3 decibels, down to a frequency of 2 c/s?

*III.38. Fig. P–6 shows a circuit which is used for the direct measurement of the amplification factor μ, of a valve. V represents an audio-frequency alternating e.m.f. and R and r represent calibrated variable resistances. The procedure is to adjust r until no sound is heard in the headphones (which indicates that no A.C. is flowing in the anode circuit).

Draw the equivalent a.c. circuit for this circuit: deduce the relationship which exists between r, R and μ when no sound is heard in the headphones.

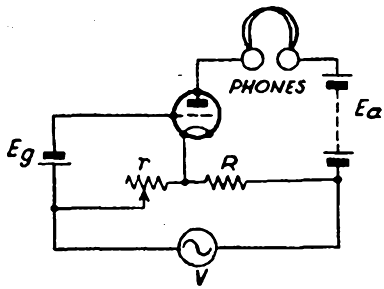


Fig. P–6

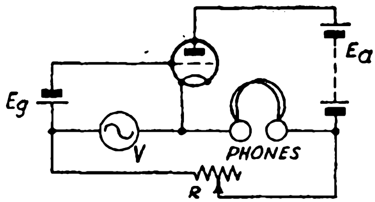


Fig. P–7

***III.39.** Fig. P–7 shows a circuit which is used for the direct measurement of the mutual conductance, g_m, of a valve. V represents an audio-frequency e.m.f. and R a calibrated variable resistance. The procedure is to adjust R until no sound is heard in the high-resistance headphones (which indicates that the alternating component of the anode-voltage is zero).

Draw the equivalent a.c. circuit for this circuit: deduce the relationship which exists between R and g_m when no sound is heard in the headphones.

III.40. Draw a dynamic characteristic (plotting instantaneous values of i_a against instantaneous values of v_1) for the phase-inverter circuit of Fig. 49 (b), page 77, given that $R = 5{,}000$ ohms, $E_{HT}\cdot = 200$ volts and that the valve characteristics are as shown in Fig. P–1, page 358. State the quiescent value of the anode-current and its value at an instant when $v_1 = +\ 50$ volts.

III.41. A triode valve has its anode connected through a resistance R to the positive terminal of a battery whose voltage is E. The cathode is connected through a resistance R_k to the negative terminal of the battery. The grid is tapped on to the battery at a potential E_g above its negative terminal. Show that

$$v_a = E - (R + R_k)i_a \qquad \text{(i)}$$
$$v_g = E_g - R_k i_a \qquad \text{(ii)}$$

Show how graphs having the equations (i) and (ii) could be plotted on the static characteristics and hence devise a graphical method of determining the anode-current when the valve characteristics are given and the values of E, E_g, R and R_k are known.

If the characteristics are as in Fig. P–1, page 358, and $E = 300$ v, $E_g = 75$ v, $R = 20{,}000$ ohms, $R_k = 10{,}000$ ohms, what will be the value of the anode current?

(N.B. Compare with problem III.42.)

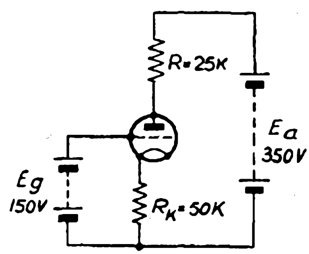


Fig. P–8

***III.42.** (a) Show how to deduce quickly a reasonably accurate approximate value for the anode current in the circuit of Fig. P–8, without a knowledge of the valve characteristics, knowing only that the valve has a high μ, and that it is operating on the linear parts of its characteristics. (N.B. 50 K in this figure denotes 50,000 ohms.)

(b) Hence calculate also a reasonably good approximate value of v_a, the anode-cathode voltage.

(c) Assuming in addition that the valve characteristics are as in Fig. P–2, page 359, deduce a good approximate value for the grid-cathode voltage, v_g, based on the answers to parts (a) and (b) of this problem.

(d) To what value must R be changed to make the grid-cathode voltage zero?

*III.43. There is a proof that, subject to certain assumptions, the optimum value of load resistance for a triode output stage (to give maximum power output) is equal to 2ρ. Can you devise such a proof? The assumptions are as follows—

(a) linear valve characteristics, $i_a = \frac{1}{\rho}(v_a + \mu v_g)$

(b) transformer coupled load, using a perfect transformer

(c) prescribed h.t. voltage, E

(d) an unlimited sine-wave input signal voltage is available

(e) the distortion may not be so great that the anode-current is cut-off for any part of the cycle, but the anode-current may be allowed just to fall to zero at one point of the cycle.

(Hints: Draw a load line on the assumed-linear anode-characteristics, the lower end of the load line being on the v_a axis, and the upper end being on the anode-characteristic for $v_g = 0$. The a.c. power output is one-quarter of the triangular area under the load line. The abscissa of the *mid-point* of the load line is the prescribed h.t. value, E.)

*III.44. An alternative equivalent a.c. circuit for a valve, known as the "constant-current" equivalent circuit, is sometimes used instead of the "constant-voltage" equivalent circuit shown in Fig. 26, page 37. Fig. P–9, shows this alternative equivalent circuit. The two intersecting circles denote

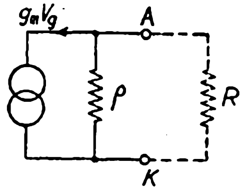


FIG. P–9

a constant-current a.c. generator, i.e. a source whose internal impedance is so very much higher than any impedance which could be used as a load that the value of the current supplied remains the same whatever load impedance is connected to the generator. (It follows that a constant-current generator may be said to have infinite internal impedance and also an infinite e.m.f., but that the ratio e.m.f./impedance is finite, being equal to the current supplied.)

Prove that the connexion of any impedance (such as R) to the circuit of Fig. P–9 will cause the same current to flow through R, as if R were connected instead to the circuit of Fig. 26.

Suggest a simplification of the constant-current equivalent circuit which will give approximately correct results in cases where the valve is a pentode.

*III.45. A "long-tailed pair" cathode-coupled amplifier has a circuit similar to that shown in Fig. 51 (a), page 80, except that $R = 0$ and that two identical pentodes are used instead of the two triodes. Draw an a.c. equivalent circuit, using the constant-current equivalent circuit mentioned in problem III.44 above. Making the approximation that the valve impedance, ρ, is infinite, derive an expression for the voltage-gain of the amplifier.

*III.46. Calculate the voltage-gain of the "long-tailed pair" mentioned in the previous question, if the input voltage, V_1, were transferred from valve 1 to valve 2, the grid of valve 1 being merely connected to h.t. negative ("earth"). The output voltage, as before, is to be taken as the alternating component of the p.d. between the anode of valve 2 and earth.

*III.47. Each valve of the long-tailed pair circuit shown in Fig. P–10 has characteristics as given in Fig. P–1, page 358. $E_a = 200$ v, $E_k = 100$ v, $R = 20$ K, $R_k = 10$ K.

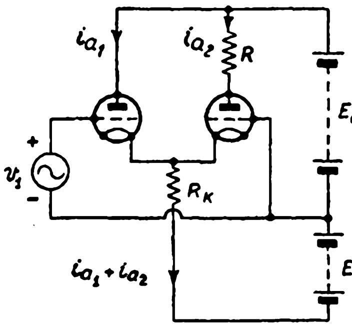


Fig. P–10

Find the approximate quiescent value of each anode-current.

*III.48. With the conditions of problem III.47, determine approximately what positive excursion of v_1 is required to cut off anode-current in valve 2.

*III.49. In the circuit of Fig. P–10, take $E_a = 200$ v, $E_k = 100$ v, $R_k = 10$ K, and the valve characteristics as in Fig. P–1, page 358. What is the largest value of R which may be used if grid-current is not to flow in the second valve when v_1 is such as to cut off anode-current in the first valve? With this value of R, what value of v_1 is required to cut off anode-current in the first valve?

*III.50. Fig. P–11 shows the circuit of a phase-shifting amplifier. Note that the anode-load resistance is equal to the resistance in the cathode-lead, each being R. The resistance R_p is large with respect to R.

Derive an expression for the gain-vector of this stage and show that, if the capacitance C be varied, the phase-shift of the stage varies without any change of magnitude of the gain.

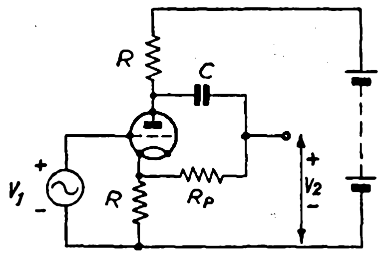


Fig. P–11

*III.51. *Grounded-grid amplifier.* A simple amplifier circuit is formed by connecting an h.t. supply-voltage from anode to cathode of a triode, connecting the grid to h.t. negative, and then inserting the alternating input-voltage in a break in the cathode-lead and inserting a load resistance, R_L, in a break in the anode lead. The output voltage is taken as the a.c. component of the p.d. between anode and h.t. negative.

(*a*) What is the phase-relationship between the output-voltage (potential of anode above h.t. negative) and the input-voltage (potential of cathode above h.t. negative)?

(*b*) Derive an expression for the voltage-gain.

(*c*) Derive an expression for the input impedance.

(*d*) To what do the expressions derived in (*b*) and (*c*) above approximate if a pentode is used instead of a triode? (Assume $\rho \gg R_L$ and $\mu \gg 1$.)

Chapter IV

IV.1. A parallel coil-condenser circuit is resonant at 500 Kc/s. The condenser has capacitance 500 $\mu\mu$F (i.e. 500 pF) and the Q-value of the circuit is 70. Calculate the inductance and resistance of the coil and also the value of the dynamic resistance (i.e. impedance at resonance).

IV.2. A parallel coil-condenser circuit has $Q = 50$ and its impedance at resonance is R_D. The resonance frequency is 1 Mc/s. Over what range of frequency is the impedance of the circuit greater than $R_D/\sqrt{2}$?

IV.3. A single-stage tuned-anode amplifier, using a triode, has the above coil-condenser circuit as its anode load. Given that $R_D = 30{,}000$ ohms, $\rho = 20{,}000$ ohms and $\mu = 40$, and neglecting the effects of stray capacitance, calculate

(*a*) the voltage gain at the resonance frequency

(*b*) the effective Q-value of the amplifier

(*c*) the frequency-bandwidth over which the voltage gain is within 3 decibels of the maximum gain.

IV.4. A single-stage tuned-anode amplifier using a pentode ($g_m = 5$ mA/v, $\rho = 1$ megohm) has the same tuned circuit as in problem IV.3 above for its anode load (i.e. $f = 1$ Mc/s, $Q = 50$, $R_D = 30{,}000$). How will the answers to parts (*a*), (*b*) and (*c*) of problem IV.3 be changed?

***IV.5.** A high-Q, parallel coil-condenser circuit (coil inductance L, coil resistance R, shunt capacitance C) is shunted by a grid-resistance R', known to be very large compared with R. Show that the admittance vector of this circuit may be written as follows

$$Y = \frac{1}{r + j\omega L} + j\omega C$$

where r denotes $(R + L/CR')$. Hence show that the circuit still behaves as a parallel coil-condenser circuit but with a Q-value reduced to

$$Q_{effective} = Q_{original}/(1 + R_D/R')$$

where R_D is the dynamic resistance, L/RC.

IV.6. A single-stage tuned-anode amplifier uses a pentode ($g_m = 2{\cdot}5$ mA/v, $\rho = 240{,}000$ ohms) and a tuned-circuit load whose Q-value is 60 and whose dynamic resistance is 100,000 ohms. As a result of the input resistance of the next stage of the amplifier, together with the grid-resistance ("coupling resistance"), the tuned-circuit is effectively shunted by a resistance of 400,000 ohms. Calculate the gain at resonance and the effective Q-value of the stage.

IV.7. A single-stage tuned-anode amplifier uses a pentode whose impedance, ρ, is 1 megohm and whose mutual conductance is 3 mA/v. The L–C circuit consists of a coil, of inductance 20 microhenry, tuned by a variable condenser whose maximum and minimum capacitances are 100 pF and 20 pF (these values include all parallel stray capacitance). What frequency range and what band of wavelengths can the amplifier be tuned through? Assuming that the Q-value of the L–C circuit is 50 at the high-frequency end of the tuning range, calculate the gain and the selectivity of the amplifier at that frequency.

IV.8. A band-pass filter of the coupled tuned-circuit type shown in Fig. 57, page 92, uses two coils, each having inductance 500 microhenrys. The pass-band is required to be from 500 to 520 Kc/s. Determine the value of each tuning capacitance and the approximate value of the coefficient of coupling between the coils.

IV.9. A single-stage triode amplifier ($\mu = 28{\cdot}5$, $\rho = 10$ K) has a load resistance of 20 K, and a grid-anode capacitance of 7·5 pF. The grid-cathode capacitance is 9 pF. The stage is fed from a source with an internal resistance of 100 K. Calculate the upper cut-off frequency, on a 3-decibel basis. (Neglect anode-cathode capacitance.)

IV.10. A two-stage, audio-frequency amplifier uses triode valves each having $\rho = 10{,}000$ ohms, $\mu = 30$, $C_{ga} = 3$ pF, $C_{gk} = 5$ pF. The amplifier uses R–C coupling between the stages, the grid-resistances being 2 megohms each. Each load resistance is 50,000 ohms.

(*a*) What is the input impedance of the second stage at a frequency of 100 Kc/s?

(*b*) What is the effective load impedance (in ohms) of the first stage at a frequency of 100 Kc/s?

(*c*) What will be the voltage amplification of the first stage at a frequency of 100 Kc/s?

IV.11. A voltage amplifier stage uses a valve for which $\rho = 20{,}000$ ohms and $\mu = 70$. The load is a resistance of 50,000 ohms. The sum of the anode-cathode inter-electrode capacitance and stray capacitance across the load is $100/\pi$ pF. Calculate the upper cut-off frequency evaluated on a 3-decibel basis. To what resistance must the load be reduced if cut-off is not to occur below 1·4 Mc/s? What will then be the gain of the stage at low frequencies? (The input voltage source is assumed to have low internal impedance.)

IV.12. A four-stage R–C coupled amplifier (*four* couplings, if the output voltage is also R–C coupled) has an overall response curve with cut-off frequencies (defined on a 3-decibel basis) 40 c/s and 10 Kc/s. If one makes the assumption that the four stages are identical, what would be the upper and lower cut-off frequencies of any one stage?

IV.13. Approximately what fraction of the output voltage of a high-gain amplifier must be fed back in order to produce a negative feed-back amplifier with an overall gain of 100?

IV.14. An amplifier having a voltage gain of 500 : 1 is converted to a negative feed-back amplifier by the feeding back of 2 per cent of the output voltage to the input circuit. What will be the gain of the resulting negative feed-back amplifier? If the gain of the original amplifier were to fall by 25 per cent (i.e. from 500 : 1 to 375 : 1) by what percentage would the overall gain be reduced?

IV.15. A three-stage negative feed-back amplifier using the circuit of Fig. 64, page 106, has $R_1 = 1$ megohm and $R_2 = 50{,}000$ ohms. With R_2 short-circuited, the gain is found to be 2,100. What will be the gain of the amplifier with the short-circuit removed?

IV.16. A two-stage negative feed-back amplifier using the circuit of Fig. 65, page 107, has $R' = 2{,}000$ ohms and $R'' = 100{,}000$ ohms. With R'' open-circuited, the gain is found to be 1,530. What will be the gain of the amplifier with R'' reinstated?

IV.17. A three-stage negative feed-back amplifier using the circuit of Fig. 68, page 110, has $R = 500$ ohms and $R_3 = 20{,}000$ ohms. With a condenser of large capacitance, connected across R, the gain is found to be 1,160. What will be the gain of the amplifier with this condenser removed?

***IV.18.** A degenerative feed-back amplifier has a forward amplifier whose gain vector is $|A|\underline{/\theta}$. The gain vector, β, of the feed-back path is real and negative. Derive an expression for the phase-shift of the overall amplifier and show that, if the loop gain be large enough, this phase-shift tends to be independent of θ.

IV.19. A negative feed-back amplifier uses the circuit shown in Fig. 63, page 106, with the exception that the load, R, instead of being connected directly in the anode circuit is transformer-coupled to the anode circuit (cf. Fig. 35, page 48). Given the following data and assuming linear operation determine the optimum transformer ratio, to give maximum power output for a given input voltage.

Valve impedance, $\rho = 8{,}000$ ohms
Amplification factor, $\mu = 40$
$R_1 = 0{\cdot}9$ megohm, $R_2 = 0{\cdot}1$ megohm
Load resistance $= 16$ ohms.

***IV.20.** Prove that, for a feed-back amplifier with a sine-wave input-voltage

$$\frac{1}{\text{Gain Vector}} = \frac{1}{A} - \beta$$

where A and β are the gain vectors of the forward amplifier and the feed-back path respectively. Show also that, if the loop gain $A\beta$ be real and negative, the magnitude of the overall gain G is given by

$$\frac{1}{|G|} = \frac{1}{|A|} + |\beta|$$

By differentiating this equation with respect to $|A|$, consider the effect of *small changes* in $|A|$ and prove that

$$\frac{d|G|/|G|}{d|A|/|A|} = \frac{|G|}{|A|}$$

Express this result in words.

IV.21. The loop gain of a certain feed-back amplifier is 49. If the forward gain (sometimes called "the internal gain") were to decrease by 1 per cent, *approximately* what would be the percentage decrease of overall gain?

IV.22. If the output stage of the amplifier of the previous question generates a 10 per cent second harmonic, what will be the second harmonic content of the overall amplifier?

***IV.23.** From problem IV.20, we see that the magnitudes of A, β and G (the internal gain, the feed-back ratio and the overall gain) in a negative feed-back are related as follows—

$$\frac{1}{|G|} = \frac{1}{|A|} + |\beta|$$

Let the internal gain be reduced by a per cent; let β be unaltered. The result of this will be that the *overall* gain is reduced. Let it be reduced by g per cent. Then

$$\frac{1}{(1 - g/100) \,.\, |G|} = \frac{1}{(1 - a/100) \,.\, |A|} + |\beta|$$

By solving these two simultaneous equations, show that

$$|A| = |G| \,.\, \frac{100/g - 1}{100/a - 1}$$

(See next problem for an application of this result.)

IV.24. A negative feed-back amplifier is to be designed with an overall gain of 50. A further requirement is that the overall gain shall not fall by more than 1 per cent when the internal gain falls by 25 per cent. By using the result of the previous problem (or by any preferred method) calculate the internal gain required for this amplifier.

IV.25. A three-stage amplifier is similar to that shown in Fig. 64, page 106, except that the input voltage (from a low-impedance source) is connected directly between h.t. negative and the grid of the first valve, and the tapping point between R_1 and R_2 is connected directly to the grid of the first valve. How does feed-back of this kind affect the gain of the amplifier?

IV.26. Explain, without detailed analysis, what would be the effect of connecting a high resistance between the anodes of a two-stage, *R–C* coupled, triode amplifier.

IV.27. A certain amplifier has an infinite input impedance and a voltage gain of 999, with 180 degrees phase-shift. What will be the input impedance and the gain of the amplifier if it is modified by connecting a 1-megohm resistance between the unearthed output terminal and the unearthed input terminal?

What will be the output voltage produced by an alternating input e.m.f. of peak value 10 millivolts if the input source has an internal resistance of 4,000 ohms?

IV.28. An adding circuit, using an amplifier with shunt-connected negative feed-back, is to be so designed that it can be used to add any number of input voltages up to sixteen, with the requirement that its sensitivity (or gain) shall not change by more than ½ per cent, when the number of contributing sources is changed. If the contributing sources all have the same value of output resistance (R) and if the feed-back resistance in the shunt-connected feed-back amplifier is five times as great ($5R$), what is the minimum permissible internal gain of the amplifier?

***IV.29.** The two important criteria of performance for a d.c. voltage stabilizer are the output-resistance ($-\,\partial v_L/\partial i_a$ with E_a constant) and the stabilization ratio ($\partial E_a/\partial v_L$ with R constant, using the notation of Fig. 85, page 136). The value of the output resistance tells us how effective is the stabilization against changes of load. The value of the stabilization ratio tells how effective is the stabilization against variations of supply voltage. Either by using the equations on page 136, or by an independent examination of incremental equivalent circuit, derive expressions for these two criteria, applicable to the stabilizer of Fig. 85.

***IV.30.** Fig. P–12 shows the circuit of a d.c. voltage stabilizer.

(*a*) Explain how it works.

(*b*) If this stabilizer is to be supplied from an unstabilized 350-volt d.c. supply and is to provide a stabilized 250-volt, 10-mA supply, calculate the

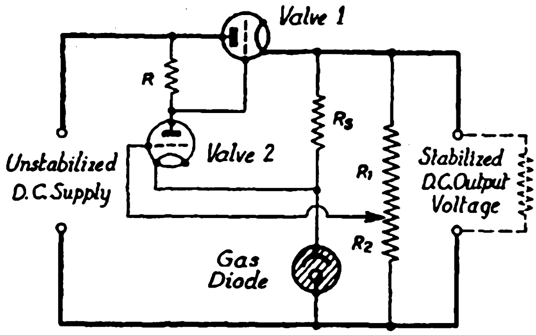


FIG. P–12

required resistance R. Valves 1 and 2 respectively have characteristics as shown in Fig. P–1 and Fig. P–2, and the voltage of the gas diode (or stabilizer diode) is 150 volts.

(*c*) At what point must the tapping-point be set on the potentiometer ($R_1 + R_2$)?

IV.31. A cathode follower uses a valve for which $\rho = 20{,}000$ ohms and $\mu = 80$. Choose a suitable value for the cathode resistor, R_k, and give approximate values of the resulting output-impedance and voltage gain.

IV.32. Prove that the Thevenin equivalent circuit of a cathode follower, using a valve with assumed-linear characteristics, consists of an e.m.f. $\mu/(\mu + 1)$ times the input voltage, in series with an output impedance $\rho/(1 + \mu)$.

IV.33. Is the output impedance of a cathode follower at all dependent on the grid-cathode inter-electrode capacitance? Prove that the output admittance of a cathode follower is $1/Z_{gc} + (1 + \mu)/\rho$ where Z_{gc} is the impedance between grid and cathode.

Is the output impedance of a cathode follower at all dependent on the internal impedance of the source which feeds its input-terminals?

IV.34. (*a*) A certain cathode follower has a load which is a resistance equal to the valve-impedance, ρ. What is the slope of the dynamic characteristic?

(*b*) A certain cathode follower has a load resistance of 20,000 ohms. Give an approximate numerical value for the slope of the dynamic characteristic.

IV.35. A simple cathode follower (e.g. Fig. 80, page 130) uses an h.t. supply voltage of 200 volts. The resistance R is 20 K ohms. The valve characteristics are as in Fig. P–1, page 358. To what potential above h.t. negative may the first grid be raised before the grid-cathode voltage becomes positive?

IV.36. The valve whose characteristics are shown in Fig. P–1, page 358, is used in a cathode-follower circuit. The cathode-resistance is 20 K ohms. If the peak-to-peak swing of the alternating input-voltage is 250 volts, and if the grid is never to become positive with respect to the cathode, nor is anode-current to be cut off, what is the smallest permissible h.t. voltage?

How would the answer to this question be changed if the cathode-resistance were altered to 40 K ohms?

***IV.37.** The valve whose characteristics are shown in Fig. P–1, page 358, has its anode connected to the positive end of a 200-volt h.t. supply and its cathode connected through a 2 K ohm resistance in series with an 18 K ohm resistance to the negative end of the supply. The grid is connected, through a high resistance, to the junction of the 2 K and 18 K ohm resistances. There are no other connexions. What will be the p.d. between grid and cathode?

IV.38. (*a*) The valve in the circuit of Fig. 83 (*b*), page 134, has characteristics as in Fig. P–1, page 358. The h.t. voltage, E_a, is 300 volts and the sum of R_1 and R_2 is 20 K ohms. What value must be used for R_1 if the quiescent anode-current is to be 7·5 mA?

(*b*) Repeat this calculation assuming that the valve is a pentode with characteristics as shown in Fig. P–3, page 360. The pentode operates with a screen-grid voltage of 250, this being the value for which the characteristics in Fig. P–3 are drawn.

IV.39. A cathode-follower circuit as in Fig. 83 (*b*), page 134 (characteristics as in Fig. P–1, $E_a = 200$ volts, $(R_1 + R_2) = 20$ K ohms), is used with a sine-wave alternating input voltage. What value must be used for R_1 in order to accommodate the largest possible input voltage amplitude without either grid-current flowing or cut-off occurring?

IV.40. A certain cathode-follower stage has an output impedance R_o. What would its output impedance become if a resistance R were connected (*a*) in the anode lead. (*b*) from anode to cathode (with a suitable d.c.-blocking condenser). (*c*) From anode to grid (again with a blocking condenser)?

IV.41. By considering the equivalent circuit of the cathode follower shown in Fig. 83 (*b*), page 134, show that the alternating component of anode-current is related to the alternating input-current by the equation

$$I_a/I_1 = \frac{\mu R' - R_2}{\rho + R_1(1 + \mu) + R_2}$$

This suggests that there will be zero alternating anode-current if R' be made equal to R_2/μ. Give a physical explanation of this.

IV.42. An e.m.f. consisting of regularly repeated rectangular, positive pulses is fed to an a.c./d.c. separating circuit similar to that of Fig. 27, page 39. The source impedance is negligibly small and R may be taken as very large compared to the condenser reactance at all relevant frequencies. The input pulse amplitude is 10 volts and the repetition frequency is 1 Kc/s. Calculate the positive peak value and the negative peak value of the p.d. across R (i.e. the maximum positive and negative excursions of output voltage) when the pulse duration (or "on" time) is (*a*) 10 microseconds, (*b*) 200 microseconds.

***IV.43.** Design a cathode follower, using the valve whose characteristics are shown in Fig. P–1, page 358, and adopting the circuit shown in Fig. 83 (*b*), page 134, for each of the following input voltages (v_1 in Fig. 83 (*b*)).

(*a*) Square wave, 200 v peak-to-peak, frequency 1,000 c/s.

(*b*) Negative pulses of 200 volts, repeated at 1,000 c/s, with a pulse duration of 200 microseconds (see problem IV.42).

IV.44. A transmitting valve has a rated maximum anode-dissipation of 250 watts. What is the theoretical maximum a.c. output power which can be derived from this valve (*a*) when class A operation is used, (*b*) when class B operation is used.

IV.45. A Class A output stage, operating without distortion, and with a sinusoidal a.c. input voltage, has an anode-voltage which swings between 50 and 450, and an anode-current which swings between 4 and 30 milliamps. Calculate the a.c. output power, the anode dissipation and the anode efficiency.

IV.46. A Class B amplifier in which the grid is permitted to swing 40 volts positive has a quiescent anode-voltage of 250.

(*a*) What is the maximum permissible swing of anode-voltage?

(*b*) What anode-efficiency is obtained using this anode-voltage swing?

(*c*) What anode-efficiency would be obtained with an anode-voltage swing of half this maximum permitted value?

IV.47. A Class B push-pull output stage has a load of 30 ohms coupled to the anode circuits by means of a transformer which has, in each half of the primary winding, 10 times as many turns as in the secondary winding.

(*a*) What is the effective load resistance presented to each valve?

Each valve has $\mu = 10$ and $\rho = 2{,}000$ ohms. The peak sine-wave input voltage per valve is 200 volts, and the h.t. supply voltage is 1,500 volts. Assuming that the grid-bias is set exactly to the cut-off value and that the valve characteristics are linear right down to zero current, calculate

(*b*) the peak anode-current of each valve

(*c*) the d.c. component of each anode-current

(*d*) the limits between which the anode-voltage swings during each a.c. cycle

(*e*) the total a.c. output power

(*f*) the power drawn from the h.t. supply

(*g*) the anode-efficiency.

IV.48. A certain Class B push-pull output stage uses valves whose maximum safe anode-current is 300 mA, and maximum safe anode-voltage 1,900 volts. The valve characteristics in the positive grid region show that an anode-current of 300 mA can be attained with any of the following pairs of voltages.

Anode-voltage . .	200	140	100	90
Grid-voltage . .	0	50	100	150

Calculate—

(*a*) the required h.t. voltage (i.e. quiescent anode-voltage)

(*b*) the maximum peak anode-voltage

(*c*) the optimum load resistance to be presented to each valve

(*d*) the total power required from the h.t. supply for the push-pull stage

(*e*) the resulting anode efficiency.

IV.49. Prove that, in a Class B amplifier,

$$\text{Anode dissipation} = \frac{2v_{H.T.}\,\hat{I}_a}{\pi}(1 - \text{Efficiency}).$$

A certain Class B amplifier uses valves subject to two limiting conditions, i.e. that the anode dissipation must not exceed 100 watts and that the anode-current must not exceed 600 mA. It is designed to work at 60 per cent efficiency and with an h.t. voltage of 2,000. If the alternating input voltage were gradually increased in amplitude, from zero, which of the above limiting conditions would be attained first?

What h.t. voltage will ensure that both limiting conditions are attained simultaneously?

IV.50. On the assumption that the valve characteristics are of the form

$$i_a = A(v_a + \mu v_g)^B$$

where A, B and μ are constants for the valve, derive an expression for the angle of flow in a tuned-anode Class C output stage in terms of the a.c. and the d.c. components of the grid- and anode-voltages. Hence show that, for an angle of flow of 120 degrees

$$\mu E_g - v_{HT} = \tfrac{1}{2}(\mu \hat{V}_1 - \hat{V}_a)$$

IV.51. A Class C output stage uses a valve whose characteristics are as shown in Fig. 11, page 14, but with all the numbers multiplied by ten (i.e. the axes show 0 to 200 mA and 0 to 2,500 volts, and the grid-voltages range from 0 to − 100 volts). The maximum safe values of anode-current and anode-voltage are respectively ¼ amp and 2,500 volts. The amplifier is to be designed so that these voltages are just attained but so that the grid never swings positive.

(a) What value of steady anode-voltage must be used?

(b) What will be the peak value of the alternating component of the voltage across the tuned load?

(c) If the angle of flow is 120 degrees, what is the value of anode-voltage at the instant of cut-off, and what is the steady grid-bias voltage?

IV.52. In the design of a Class C output stage to the specification given below, it is decided that the a.c. voltage across the anode-tuned circuit shall be 2,000 volts r.m.s. Determine suitable values of tuning inductance and capacitance.

Specification—A.c. output power = 700 watts.

Frequency = 1·24 Mc/s.

To avoid undue harmonic production, the effective Q-value of the tank circuit and its coupled load may be reduced to 10 but not lower.

IV.53. A Class C output stage has an h.t. voltage of 2,000 and an a.c. voltage of 1,600 volts, peak, across the tuned load. The valve has characteristics showing that the negative grid-voltage required to cut off anode-current is always one-twentieth of the anode-voltage at the instant concerned. The angle of flow is 120 degrees. The grid is allowed to swing positive up to the value given by 50 per cent of the minimum anode-voltage. What is the peak value of the alternating component of the grid-voltage? What is the grid-bias voltage?

***IV.54.** An amplifier stage using a suppressor-controlled (or "Transitron-connected") pentode as in Fig. 110 (a), page 190, has a bias-resistance connected in the cathode lead. The first grid, $G1$, is connected directly to the cathode. The bias-resistance is not shunted by a capacitance. Will there be negative feed-back and a reduction of gain, in the same way as there would be in a stage using a triode valve?

***IV.55.** Is it possible, by the use of a cathode-bias resistance in a suppressor-controlled pentode stage (as in the previous problem) to make the suppressor grid so negative that anode-current is cut off?

Chapter V

V.1. Two tuned-anode, mutual-coupled oscillators use identical L–C circuits, each of which has a dynamic resistance of 50,000 ohms. One oscillator uses a triode ($\rho = 10{,}000$ ohm, $\mu = 30$) and the other uses a pentode ($\rho = 500{,}000$ ohms, $\mu = 1{,}500$). What will be the ratio of the mutual inductances needed to sustain oscillation in the two circuits?

V.2. A tuned-grid mutual-coupled oscillator uses a variable condenser as a tuning capacitance. Its maximum capacitance is 500 pF and its minimum capacitance, including stray capacitance in parallel, is 40 pF. What is the ratio of the values of mutual inductance required to sustain oscillation at the two ends of the tuning range? (Assume that the coil-resistance is independent of frequency.)

What would this ratio be for a tuned-anode mutual-coupled oscillator using the same variable condenser in conjunction with a coil of inductance 200 microhenrys and resistance 20 ohms, the valve-impedance being 25,000 ohms?

V.3. A tuned-anode mutual-coupled oscillator uses a coil whose resistance at the oscillation frequency is 40 ohms. As a result of variations in supply voltage, the valve-impedance is liable to fluctuate between the limits $5{,}000 \pm 200$ ohms. What percentage fluctuations of oscillation frequency will result?

V.4. A certain tuned-anode tuned-grid oscillator uses identical tuned circuits in the anode circuit and in the grid circuit, the tuning capacitance in each case being eight times the grid-anode inter-electrode capacitance of the valve. The resonance frequency of each is 15 Mc/s. What will be the oscillation frequency? (Neglect the resistance of the coils.)

***V.5.** Prove that if the valve impedance be artificially increased by the connexion of a pure inductance L' in series in the anode lead of a tuned-anode mutual-coupled oscillator, the oscillation frequency will be given by the equation

$$\omega^2 = \frac{1}{LC} \cdot \frac{1 + R/\rho}{1 + RL'/\rho L}$$

What is the new maintenance condition? Suggest a useful value to assign to L', giving reasons.

V.6. Plot against frequency the reactance of the equivalent circuit of a piezo-electric crystal, neglecting resistance, and remembering that C_1 (Fig. 107, page 183) is much larger than C.

V.7. A certain amplifier has infinite input-impedance, output impedance equal to a pure resistance r, and output e.m.f. equal to m times the input voltage and in phase with the input voltage (i.e. the output circuit is equivalent to an a.c. generator of e.m.f. mV_1 and having an internal resistance r).

This amplifier is converted to an oscillator by having a parallel coil-condenser circuit connected across the output terminals, and the whole of the output-voltage fed back to the input terminals. Derive the maintenance equation of this oscillator and determine the value of ω at which oscillation will take place (in terms of m, r and the constants L, C, R of the coil-condenser circuit).

V.8. If, in the previous question, the output voltage is fed back not directly to the input terminals but through a small capacitance C', how will the answers to the question be altered?

V.9. Three identical amplifiers are connected in cascade and two leads are taken, from the output terminals of the third, back to the input terminals of the first. Each amplifier has an output resistance r and an output e.m.f. which is m times its input voltage but 180 degrees out of phase with the input voltage. A capacitance C is used as load impedance for each amplifier (i.e. connected directly across the output terminals).

Under what conditions will this network generate sustained sinusoidal oscillations and what will be the oscillation frequency?

V.10. A modified Dynatron oscillator has the usual coil-condenser circuit replaced by the primary winding of an h.f. transformer, the secondary winding of which has a capacitance C shunted across it. If the valve impedance is $-r$ and the two coils of the transformer are identical (each having inductance L and resistance R), determine the oscillation frequency and the maintenance condition.

V.11. The tuning capacitance of a certain Dynatron oscillator is made up of a calibrated variable condenser in parallel with a fixed capacitance (strays, etc.) whose magnitude is unknown. At a setting C_1 of the variable condenser, the oscillation frequency is found to be f_1. At a second setting C_2, the oscillation frequency is f_2. Assuming that the Q-value of the coil is very high, derive expressions for

(*a*) the inductance of the coil

(*b*) the total unknown capacitance shunting the variable condenser.

V.12. The solution of the second order differential equation

$$a\frac{d^2i}{dt^2} + b\frac{di}{dt} + ci = 0$$

is oscillatory only if $b^2 < 4ac$. What is the form of the solution if b is negative and $b^2 > 4ac$? Show that this condition could occur in the tuned-anode mutual-coupled oscillator if μM were greater than

$$L + \rho RC + 2\sqrt{\rho LC(\rho + R)}$$

Discuss the physical significance of this, if any.

V.13. A two-stage R–C coupled amplifier using identical pentodes (mutual conductance $= g_m$, valve impedance assumed infinite) has an unshunted resistance R in each cathode lead. The control-grid of the first valve is connected to h.t. negative. The load resistance of the first stage is R_L and that of the second stage is zero. The R–C coupling may be assumed perfect (i.e. the potential of the control-grid of valve 2 may be assumed to follow exactly the potential of the anode of valve 1). Draw an a.c. equivalent circuit using the constant-current equivalent circuit for each valve (see problem III.44). Hence show that the input impedance presented to an e.m.f. connected between the two cathodes is a resistance which is negative if R_L is greater than

$$\frac{2}{g_m}\left(1 + \frac{1}{g_m R}\right)$$

Draw the circuit diagram of an oscillator which makes use of this property.

***V.14.** In the theory of closed-circuit oscillators, it is shown that there are only two basic possibilities, namely the Armstrong oscillator and the Colpitts oscillator. Show that, if the valve used is a suppressor-controlled pentode, there are four basic possible oscillator circuits.

V.15. A multivibrator with the circuit of Fig. 112, page 194, uses valves whose characteristics are as shown in Fig. P–1, page 358. Each anode-load resistance is 20 K, the h.t. voltage is 200 and it may be assumed that the positive excursions of grid-voltage are negligibly small. Between what limits does the anode-voltage of each valve vary in the course of the multivibrator cycle?

***V.16.** It is found experimentally that the anode-voltage waveform of a multivibrator is as shown in Fig. P–13. Explain the reasons for the rounding-off at A and for the overshoot at B.

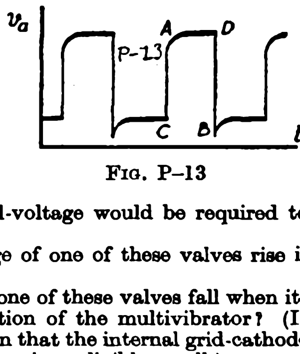


FIG. P–13

***V.17.** A symmetrical free-running multivibrator, formed of a two-stage R–C coupled amplifier with the anode of the second valve R–C coupled back to the grid of the first valve, uses valves whose characteristics are as shown in Fig. P–2, page 359. Both coupling capacitances are 0·01 microfarad, both grid-resistances are 1 megohm and both anode load-resistances are 40 K. The h.t. voltage is 200.

(i) What minimum value of negative grid-voltage would be required to cut off anode-current in one of the valves?

(ii) To what value would the anode-voltage of one of these valves rise if its anode-current were suddenly cut off?

(iii) To what value does the grid-voltage of one of these valves fall when its anode-current is cut off in the normal operation of the multivibrator? (It may be assumed for the purpose of this question that the internal grid-cathode resistance of a valve with a positive grid-voltage is negligibly small.)

(iv) What will be the time constant of the exponential rise of grid-voltage?

(v) How long does the grid-voltage take to change from the value calculated in (iii) above to the value calculated in (i) above?

(vi) What will be the frequency of this multivibrator?

V.18. A certain symmetrical, free-running multivibrator, operating with zero grid-bias and an h.t. voltage of 250 gives an output of frequency 1 Kc/s. The grid-voltage waveform shows a sudden fall from zero to − 50 volts and a sudden rise from − 5 volts to approximately zero. To what value will the frequency rise if the grid-resistor be returned to h.t. positive instead of to h.t. negative?

***V.19.** The multivibrator of problem V.18 is to be synchronized to a frequency of 1·5 Kc/s by the injection of narrow rectangular, positive voltage pulses into both grid circuits. What is the smallest value of pulse-amplitude which may be used?

V.20. A bi-stable multivibrator with floating-battery interstage coupling (as in Fig. 116, page 201, but with a 1-megohm resistance added in each grid-lead) uses valves whose characteristics are as shown in Fig. P–2, page 359.

The two anode-load resistances are each 25 K, and the h.t. voltage is 200. What is the minimum permissible coupling-battery voltage? If coupling batteries each of 100 volts are used, what is the minimum value of positive pulse, injected into the grid circuit of the cut-off valve, which will suffice to trigger the circuit to its other stable state?

V.21. In the circuit of the previous problem, used with 100 volt coupling batteries, what is the minimum value of negative pulse, injected into the grid circuit of the conducting valve, which will suffice to trigger the circuit?

V.22. A certain symmetrical bi-stable multivibrator, using the circuit of Fig. 117, has $R_1 = R_2 = 1$ megohm, anode-load resistances of 20 K and a common cathode-bias resistance of 20 K. The h.t. voltage is 300. The valves used have characteristics as in Fig. P–2, page 359. Determine—

(*a*) the anode-current of the valve which is conducting

(*b*) the grid-voltage of the valve which is not conducting

(*c*) the amplitude of the "step" of anode-voltage which occurs when the circuit is triggered

(*d*) the minimum permissible amplitude of triggering pulse to be applied to the grid of one of the valves.

V.23. Re-draw the circuit of Fig. 117 with the load resistance of the right-hand valve removed and the anode of this valve connected directly to h.t. positive. Break the connexion between the anode of the right-hand valve and R_1 and, instead, connect the upper end of this right-hand R_1 to h.t. positive. Finally remove the condenser. Explain how the resulting circuit can act as a bi-stable multivibrator.

V.24. The cathode-coupled flip-flop shown in Fig. 119, page 205 uses valves whose characteristics are given by Fig. P–2, page 359. $R_1 = 0$, $R_2 = 20$ K, $R_c = 20$ K, $R_g = 1$ megohm. The left-hand valve, when conducting, is required to have an anode-current of 5 mA.

(*a*) Determine the necessary h.t. voltage.

(*b*) Given that $R' = 0{\cdot}5$ megohm, determine the minimum permissible value of R'' to ensure that the right-hand valve is cut off in the quiescent condition.

(*c*) Given that $R' = 0{\cdot}5$ megohm, what is the minimum value of R'' which will ensure that the grid-voltage of the right-hand valve reaches the value zero when anode-current is cut off in the left-hand valve.

(*d*) Determine the condenser p.d. in the quiescent condition.

(*e*) Given that R'' is made larger than either of the values determined in (*b*) and (c) above, determine the value to which the grid-cathode potential of the left-hand valve falls when the circuit is first triggered.

(*f*) Given that $C = 100$ pF, calculate the time-interval between triggering and automatic return to the quiescent condition (i.e. the time-delay between "flip" and "flop").

V.25. A certain Thyratron time-base uses a constant-current pentode which has characteristics as shown in Fig. P–3, page 360. The circuit operates with a screen voltage of 250 (viz. the value for which the graphs of Fig. P–3 are drawn), and a control-grid voltage of -3. The capacitance of the timing condenser is $0{\cdot}01\ \mu$F.

(*a*) What will be the rate of rise of voltage across this condenser?

(*b*) Taking the extinction potential of the Thyratron as 15 volts, its control factor as 15 and its grid-voltage as -10, what will be the amplitude of the resulting saw-tooth voltage?

(c) What will be the frequency of the time base? (Assume that the fly-back time is negligibly small.)

***V.26.** Assuming that the cut-off value of grid-voltage for a triode is given by v_a/μ, derive an expression for the value of v_{AB} in the Puckle time-base circuit of Fig. 122, page 211, which will trigger the circuit and cause the left-hand valve to conduct.

V.27. A Puckle time-base circuit uses a 20 K load resistance in the anode-circuit of the accelerating valve and the current through this is alternately zero and 10 mA. The h.t. supply voltage is 300 and the amplification factor of the discharging triode is 30. What will be the approximate frequency of the time-base if the charging pentode is set to pass a constant-current of 5 mA and the charging capacitance is 0·05 μF. (Neglect fly-back time.)

***V.28.** Prove that the frequency of a Puckle time-base is given approximately by the formula

$$f = \frac{i_1/i_2}{CR}$$

where i_1 is the anode-current of the constant-current pentode, i_2 is the anode-current of the accelerating valve, C is the capacitance of the timing condenser and R is the anode-load resistance of the accelerating valve.

Chapter VI

VI.1. (*a*) Between what limits does the anode-voltage of the diode in Fig. 127 (page 217) vary, if v_1 is an unmodulated, constant amplitude alternating voltage of peak value $\hat{V}$?

(*b*) If a d.c. e.m.f., E, be added to the circuit in series with the constant amplitude a.c. e.m.f., between what limits will the anode-voltage of the diode vary?

VI.2. An e.m.f. $\hat{V} \sin \omega t$, a capacitance C and a diode are connected in series to form a single closed loop. A further circuit, consisting of a d.c. e.m.f. E in series with a resistance R, is shunted across the diode. The time constant RC is large with respect to the periodic time of the A.C.

(*a*) Between what limits does the anode-voltage of the diode vary?

(*b*) What is the mean value of the current through R?

VI.3. A condenser is connected in series with the alternating input-voltage of the diode detector shown in Fig. 127 (page 217), its capacitance being equal to that of the reservoir condenser (C, Fig. 127). How will this affect the waveform and the mean value of the voltage across the reservoir condenser?

VI.4. (*a*) What would be the effect of connecting a fixed d.c. e.m.f., E, in series with the unmodulated a.c. e.m.f. shown in the diode detector circuit of Fig. 127, page 217?

In such a circuit the condenser voltage is found to be 10 v D.C. or 18 v D.C. according to the polarity of the d.c. e.m.f. The detection efficiency in both cases may be taken as 80 per cent.

(*b*) What is the magnitude of the d.c. e.m.f.?

(*c*) What is the peak value of the alternating e.m.f.?

VI.5. A diode detector having a detection efficiency of 0·9 has a steady radio frequency input voltage whose carrier amplitude is 10 volts peak, modulated to a depth of 50 per cent at an audio frequency of 1,000 c/s. Between what limits does the condenser-voltage vary? Between what limits does the anode-voltage of the diode vary?

VI.6. What is the maximum permissible time constant, RC, for a diode detector designed to receive without distortion a telephony signal whose modulation depth may vary between 0 and 80 per cent, and whose modulation frequency may vary between 40 c/s and 8,000 c/s?

VI.7. Design a diode detector to the following specification—

Maximum modulation depth	70%
Maximum modulation frequency	50 Kc/s
Inter-electrode capacitance of diode (including strays)	10 pF.

VI.8. Fig. P–14 represents the dynamic (or detection) characteristics of a certain diode.

(*a*) What must be the labelling of the various curves, from right to left?

(*b*) What would be the amplitude of the audio-frequency output voltage from a detector using the given diode with a leak resistance of 80,000 ohms, and with an input voltage whose carrier amplitude was 3 volts r.m.s., modulated to a depth of 66·6 per cent?

(c) What would be the detection efficiency in (b) above?

(d) Between what limits would the diode voltage vary?

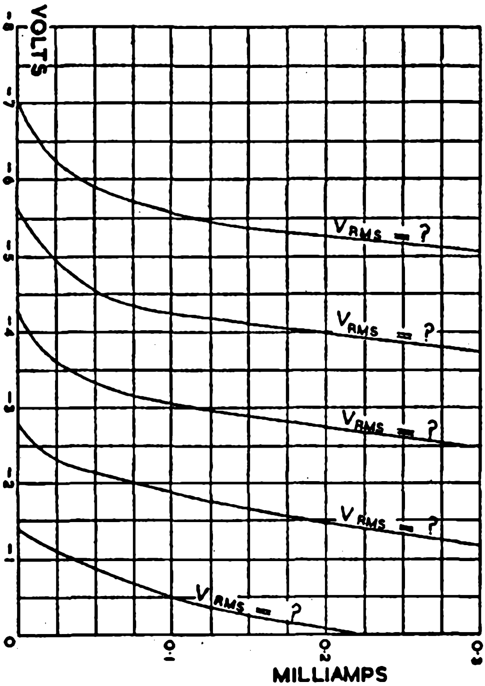


Fig. P–14

VI.9. If the diode detector described in part (b) of the previous question has, shunted across its leak-resistance, a circuit formed of a 20,000-ohm resistance in series with a large capacitance, determine—

(a) the magnitude of the d.c. voltage across the leak-resistance resulting from the application of an unmodulated carrier of amplitude 3 volts r.m.s.

(b) the magnitude of the positive and negative peaks of the audio output voltage if this carrier were modulated 66 per cent

(c) the detection efficiency.

VI.10. A valve oscillator using a self-bias circuit (e.g. as in Fig. 156, page 267) has a grid-voltage whose instantaneous value is found to vary between + 1 and − 13 volts. What is the detection efficiency of the grid circuit and what is the a.c. component of the grid-voltage?

VI.11. A cumulative-grid detector uses a triode valve for which $\mu = 30$ and $\rho = 4{,}000$ ohms. It has an amplitude-modulated input voltage whose carrier amplitude is 1 volt peak, the modulation depth being 40 per cent. The detection efficiency of the grid circuit may be taken as 75 per cent.

(a) Between what limits does the voltage across the R–C circuit vary?

(b) Between what limits does the grid-voltage vary?

(c) If the dynamic characteristic of the anode circuit of the triode (see page 230) cuts the horizontal axis at $v_g = -2$ volts, will there necessarily be distortion?

(d) Assuming that the load of the triode is a 50,000 ohms resistance and that there is no distortion, what will be the amplitude of the modulation-frequency output-voltage?

VI.12. An anode-circuit detector of the cut-off type uses a valve whose cut-off value of grid-voltage is given by $0{\cdot}05v_a$. If the detector is to handle input-voltages modulated up to a depth of 80 per cent and of carrier amplitude up to 4 volts peak, what is the minimum h.t. voltage which can be used without producing grid-current?

VI.13. If the detector of the previous question be used as a valve voltmeter, using an h.t. voltage of 250 and fixed grid-bias voltage,

(*a*) what will be the voltage-range of the valve-voltmeter if the grid-bias be set exactly to the cut-off value?

(*b*) what grid-bias voltage would be required to provide a larger range of measurable voltage (with suppressed zero), the lowest indication on this range being 80 per cent of the maximum of the first range? What would be the maximum indication on this larger range?

(*c*) if the average value of the mutual conductance of the valve, for the relevant parts of the characteristics, is 2,000 micromhos, approximately what range of d.c. ammeter (or microammeter) will be required in the anode circuit of this valve voltmeter? (Assume that the resistance of the ammeter is low enough for the dynamic characteristic not to differ from the static characteristic.)

(*d*) Will the indication (i.e. the ammeter deflexion) be the same for the maximum rated input voltages of the two ranges?

VI.14. Valve-voltmeters of types (*a*), (*b*) and (*c*) below are calibrated using sinusoidal alternating voltages, the scales being engraved to indicate r.m.s. values. A square wave of amplitude $\pm$ 10 volts is applied to each. What will the readings be?

(*a*) diode voltmeter

(*b*) square-law voltmeter

(*c*) anode cut-off type of voltmeter with fixed grid-bias voltage.

VI.15. An alternative form of the diode-detector of Fig. 127 has the resistance R connected not across the condenser but across the diode. By an argument similar to that on page 226, show that the effective input-resistance of this modified diode detector is given by $R/(1 + 2k)$.

VI.16. For each of the rectifiers listed below, state (i) the ripple frequency, (ii) the maximum reverse voltage across the valves, (iii) the transformer ratio (assuming negligible losses in the valves and transformer). Each rectifier is fed from 250 volt 50 c/s a.c. mains and gives an output of 1,500 v D.C.

(*a*) Half-wave rectifier with reservoir condenser

(*b*) Full-wave rectifier with reservoir condenser

(*c*) Full-wave rectifier with series choke

(*d*) Bridge rectifier with reservoir condenser.

VI.17. A full-wave rectifier using a reservoir condenser supplies a full-load direct current of 200 mA. Assuming (for the purpose of this exercise only!) that anode-current flows through the diodes in rectangular pulses each of duration one-tenth of a cycle, calculate the peak-current rating of the diodes.

VI.18. A full-wave series-choke rectifier, fed from 50 c/s a.c. mains and supplying a direct current of 250 mA at 500 volts, uses a choke whose inductance is 20 henrys. What will be the peak-amplitude of the fundamental component of the ripple voltage? The ripple is to be reduced to a peak value of 5 volts by the addition of a condenser across the output terminals. How large must the capacitance of this condenser be made?

VI.19. What value of bleeder resistance should be used in the rectifier of the previous question

(*a*) with the condenser used,

(*b*) without a condenser?

***VI.20.** (*a*) What is the peak value of the largest permissible sine-wave secondary current of a single-phase mains transformer rated at 250/10,000 volts, 1 KVA?

(*b*) This transformer has its primary fed from 250 v a.c. mains. Its secondary is centre-tapped and used to feed a full-wave series-choke rectifier. What will be the d.c. output voltage of the rectifier?

(c) To what value can the direct current supplied by the rectifier be increased without making the RI^2 loss in the transformer-secondary exceed its rated value?

(d) What will be the d.c. power output of the rectifier when the load-current has the value calculated in (b) above?

***VI.21.** A full-wave series-choke rectifier is required to supply a d.c. load of 1,500 watts. What is the minimum permissible KVA rating of the transformer? (See previous problem.)

VI.22. In the phase-sensitive rectifier of Fig. 147, page 249, the peak value of the reference voltage is 50 volts and the transformer ratio (total number of secondary turns divided by numbers of primary turns) is 4. The total load resistance from A to B is 5,000 ohms. Calculate the d.c. component of the output voltage between A and B when the peak value of the input voltage, v_1, is 5 volts. Assume that the losses in the transformer and in the diodes are negligible.

VI.23. Each diode in a phase-sensitive rectifier circuit of the bridge type (cf. Fig. 148 (b), page 250) has a forward resistance of 500 ohms and an infinite reverse resistance. The largest anticipated peak-value of the input voltage, v_1, is 55 volts and the load resistance is 5,000 ohms. What is the minimum permissible peak-value of the reference voltage?

Chapter VII

VII.1. What band of frequencies is occupied by an amplitude-modulated alternating voltage (or current) whose carrier frequency is 5 Mc/s and for which the modulating frequency may lie anywhere between zero and 10 Kc/s?

***VII.2.** An alternating voltage whose carrier frequency is 10 Mc/s is phase-modulated, i.e. its phase is periodically advanced and retarded, 1,000 times per second. The variation of phase is sinusoidal and the peak phase-shift is 50 radians. Prove that the same resultant voltage could be produced by periodically increasing and reducing the frequency, 1,000 times per second. What would be the peak-deviation of the frequency from the carrier value?

VII.3. An amplitude-modulated alternating voltage (carrier-frequency 2 Mc/s, modulating frequency 20 Kc/s) and an unmodulated alternating voltage of frequency 5 Mc/s are connected in series in the grid circuit of an amplifier which has a dynamic characteristic given by

$$i_a = a + bv_g + cv_g^2$$

Give a list of the frequencies of all the sine-wave components of the resulting anode-current. Show that these are grouped in a number of clearly defined frequency bands and comment on each.

VII.4. An unmodulated alternating voltage of frequency 1 Mc/s is switched on and off at a frequency of 10 Kc/s, the "on"-time being equal to the "off"-time. Give a list of the frequencies of the sine-wave components of the resulting "square-wave modulated" voltage and indicate the relative amplitudes of these components.

***VII.5.** A pentode is used in a frequency-changer circuit. The signal voltage is connected in the first grid circuit. The oscillator voltage is connected in the suppressor-grid circuit and is so large that it can be considered merely as switching the anode-current on and off at the oscillator frequency.

(a) By considering the change of anode-current as the product of a sine wave and a rectangular on-off pulse (see previous problem) show that the anode-current will contain a component whose frequency is the difference of the oscillator frequency and the signal frequency.

(b) Assuming that the anode-load impedance is small with respect to the valve impedance, derive an expression for the amplitude of the desired component of anode-current (in terms of the mutual conductance of the valve and the peak-value of the signal input-voltage).

(c) Derive an expression for the conversion conductance.

VII.6. Using the same approach as in the above two problems, deduce the relative amplitudes of fundamental, second harmonic, third harmonic and fourth harmonic components of a waveform which is a sine wave with all negative half-cycles suppressed.

VII.7. (*a*) A simple amplifier has a dynamic characteristic which gives i_a as a quadratic in v_g. Its quiescent anode-current is 3 mA. When a sine-wave input-voltage is applied, the fundamental component of the anode-current is found to have a peak value of 2 mA, and the d.c. component of the anode-current rises to 3·25 mA. What is the percentage of second harmonic distortion?

(*b*) The same amplifier is found to produce a 20 per cent second harmonic when operating with an h.f. sine-wave input voltage of peak value 2 volts. If a lower frequency voltage, of peak value 0·6 volts, be injected into the grid circuit, in series with the h.f. input voltage, what will be the percentage modulation-depth of the resulting modulated output-voltage?

***VII.8.** (*a*) Derive an expression for the slope of the $i_a - v_{g_1}$ characteristics of a hexode (assumed linear).

(*b*) A certain hexode frequency-changer uses a sine-wave oscillator voltage of peak value $\hat{V}_3$, in series with a d.c. bias E, in the oscillator-grid circuit. Between what extreme values does the slope of the $i_a - v_{g_1}$ characteristics vary in the course of one cycle of the oscillator voltage?

(c) Prove that the conversion-conductance is given by one-quarter of the difference between these extreme values of slope.

VII.9. A certain hexode valve has $i_a - v_{g_1}$ characteristics in the form of a set of upward-sloping straight lines intersecting at the point ($i_a = 0$, $v_{g_1} = -6$ v). The particular straight line for $v_{g_3} = 0$ intersects the anode-current axis at 9 mA and that for $v_{g_3} = 8$ v intersects the anode-current axis at 1 mA. When $v_{g_3} = -9$ v, anode-current is cut off.

(*a*) What grid-bias voltage should be used in the oscillator grid circuit?

(*b*) What peak value of sine-wave oscillator voltage will then just swing v_{g_3} to zero at the positive peak?

(*c*) What will be the conversion conductance when using this oscillator voltage?

(*d*) To what value could the conversion conductance be increased by using an oscillator voltage which made v_{g_3} zero throughout the positive half-cycle and more negative than the cut-off value throughout the negative half-cycle?

VII.10. (*a*) What is the root-mean-square value of a sinusoidally amplitude-modulated voltage whose peak value varies between 20 v and 180 v in the course of the modulation cycle?

(*b*) If this voltage be connected across a resistance, what will be the ratio of sideband power to carrier power?

(*c*) What is the root-mean-square value of a sinusoidally frequency-modulated voltage, whose peak amplitude is 100 v, and whose frequency varies between 10·1 Mc/s and 9·9 Mc/s in the course of the modulation cycle?

***VII.11.** A certain class C radio-frequency amplifier has an h.t. supply voltage of 1,000 and, with its prescribed a.c. input voltage, draws a direct current of ½ amp from this h.t. supply. It is found that, if the h.t. voltage be varied, the d.c. drawn from it varies in direct proportion and so does the radio frequency output voltage.

For the purpose of amplitude modulation, an audio-frequency source, of peak value 600 volts, is connected in series with the h.t. supply.

(*a*) What direct current will now be drawn from the h.t. supply?

(*b*) What is the peak value of the a.f. current drawn from the h.t. supply?

(*c*) What modulation depth is produced?

(*d*) What is the total power supplied by h.t. source?

(*e*) What is the total power supplied by the a.f. source?

(*f*) Show that the ratio of the powers supplied by the a.f. source and the h.t. source is the same as the ratio of the sideband power to the carrier power.

VII.12. Draw the circuit of a reactance valve using a phase-splitting circuit formed of a resistance and a choke. Derive an expression for the input impedance and its phase angle. (Assume that the phase-splitting choke is free from resistance and that the phase-splitting resistance is very large.)

VII.13. A reactance valve circuit of the type shown in Fig. 165 is to operate at a carrier frequency of 0·5 Mc/s. The phase-splitting resistance is 600,000 ohms. Select a suitable value for the phase-splitting capacitance, given that the variable-mu pentode has an amplification factor of 900.

What will be the value of the effective inductance furnished by the circuit when the grid-bias voltage is such as to give a mutual conductance of 1 mA/v?

VII.14. The inductive reactance provided by a certain reactance valve furnishes 10 per cent of the total tuning inductance of a frequency-modulated oscillator. If, in the course of the modulation cycle, the mutual conductance of the valve varies between 1 and 2 mA/v, and the carrier frequency is 50 Mc/s, what is the resulting frequency deviation?

VII.15. The resonance frequency of a high-Q parallel coil-condenser circuit is f_0. To what will the resonance-frequency change if a reactance-valve (as in Fig. 165) be connected across the coil? The self-inductance of the coil is L.

VII.16. (*a*) Show that a variable reactance can still be produced by the reactance-valve circuit of Fig. 165 if the positions of the phase-splitting elements, R and C, be interchanged.

(*b*) What conditions would govern the relative magnitudes of R and $1/\omega C$ in this alternative circuit?

(*c*) If these conditions are satisfied, which would give the larger input reactance, Fig. 165 or the modified circuit with R and C interchanged?

VII.17. (*a*) Without making the usual approximations, derive an expression for the input impedance at terminals AA of the reactance-valve circuit of Fig. 165 (p. 278).

(*b*) Derive an expression for the phase angle of this input impedance in the form $(90° - \theta)$, on the assumption that $1/\omega C$ is made equal to $R\sqrt{\mu}$ (see problem VII.13).

***VII.18.** What danger arises from slight, unwanted variations of h.t. or heater voltages in the circuit of a reactance-valve which is being used as a frequency modulator in a radio transmitter?

***VII.19.** How could a hexode be used as a reactance valve?

VII.20. What is the effect of increasing the intermediate frequency of a superhet receiver

(*a*) on the overall selectivity?

(*b*) on the required pre-mixer selectivity to avoid second-channel interference?

What is the advantage of making the oscillator frequency greater than the signal frequency?

***VII.21.** In ganging the aerial tuning control and the oscillator frequency control in a superhet receiver, is it sufficient to use identical ganged variable condensers for these two purposes and simply to use a different value of inductance for aerial signal tuning and oscillator tuning?

VII.22. On what does the amplitude of the a.f. voltage of a heterodyne oscillator depend?

Where constancy of output voltage amplitude is required, it is usual to have unequal input voltages to the frequency changer, the input derived from the fixed-frequency oscillator being the smaller. What is the advantage of this?

Chapter VIII

VIII.1. The sum of a 12 v positive d.c. e.m.f. and a regularly repeated negative pulse voltage of duty ratio 20 per cent and peak value − 12 v can be regarded as a regularly repeated positive pulse voltage. What will be the duty ratio of the positive pulses?

VIII.2. Express the amplitude of the 15th harmonic as a percentage of the amplitude of the fundamental for each of the following—

(*a*) rectangular, positive voltage pulses of width 5 microseconds, and repetition frequency 1 Kc/s

(*b*) a square wave alternating voltage

(*c*) a voltage consisting of regularly repeated half sine-waves as in Fig. 143 (page 241).

VIII.3. For the pulse voltage mentioned in VIII.2 (*a*) above, what frequency band would be required to include all harmonics whose amplitudes are equal to or greater than $2/11\pi$ (i.e. 5·8 per cent) of the amplitude of the fundamental?

VIII.4. A rectangular pulse voltage, with a duty ratio of 10 per cent, a repetition frequency of 1 Kc/s and a peak value of 10 v is applied across a circuit consisting of a 0·1 megohm resistance in series with a capacitance of 3,000 pF. Between what limits will the condenser voltage vary?

VIII.5. To what does equation (VIII.4) tend as the time constant, RC, tends to infinity? What is the physical significance of this result?

VIII.6. If the delay-line shown in Fig. 181, page 304, were open-circuited at its right-hand end (instead of short-circuited as shown) and if the input e.m.f. consisted of 10-volt rectangular positive pulses, of duty ratio 50 per cent (i.e. square waves) and of frequency 1 Kc/s, and if the delay time (i.e. $2l/c$) were 10 microseconds, what would be the waveform of the p.d., v, at the input-end of the line?

VIII.7. A delay-line pulse-generator (as in Fig. 182) uses a line whose characteristic impedance, Z_0, is 1,000 ohms. The length of the line is such that it would constitute one-quarter of a wavelength at a frequency of 100 Kc/s. The pentode has characteristics as shown in Fig. P–3 (page 360), and the input voltage, v_{GE}, is a square wave of frequency 1 Kc/s and of sufficient amplitude to make the control-grid voltage alternate between zero and a negative value greater than the cut-off value. Calculate (*a*) the duty-ratio, and (*b*) the amplitude of the resulting negative pulses of output voltage.

VIII.8. A shaping circuit similar to Fig. 187, page 307, uses the triode valve whose characteristics are shown in Fig. P–2, page 359. The anode-load resistance R is 50 K and the h.t. voltage is 200. The resistance R_g is sufficiently large to prevent the grid-potential rising by any measurable amount above cathode-potential.

(*a*) If the input voltage, v_1, is a large alternating voltage, between what limits will the anode-voltage vary?

(*b*) If the input voltage consists of 50-volt positive pulses, whose sides are steepest in the region of + 25 v, suggest a suitable value for the bias-voltage E_g.

(*c*) If the input voltage consists of 50-volt negative pulses, whose sides are steepest in the region of − 25 v, suggest a suitable value for the bias-voltage.

(*d*) Suggest circuits by which the required bias-voltage in (*b*) and (c) above could be attained without the use of a battery.

VIII.9. (*a*) Plot a graph of the output voltage, v_{AB}, for each of the four circuits shown in Fig. P–15. In each case the input voltage is a sine wave of

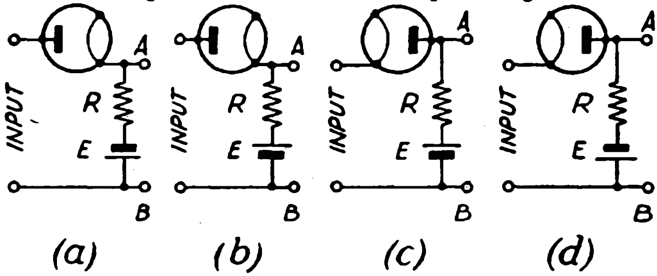


FIG. P–15

peak amplitude 100 v, E is 60 v, and R is very large compared to the internal impedance of the a.c. source supplying the input terminals. The diode may be considered to have negligible resistance when conducting, and to be non-conducting for negative values of anode-voltage.

(b) Mark on your graph the values of the upper and lower limits of v_{AB}.

VIII.10. How would the answers to the previous problem be changed if the output resistance of the a.c. source were 1,000 ohms, the diode resistance when conducting were 1,000 ohms, and R = 18 K ohms?

VIII.11. Plot a graph of the output voltage, v_{AB}, for each of the four circuits shown in Fig. P–16. In each case, the input voltage is a sine wave of peak amplitude 100 v, E is 60 v, and R is very large compared to the effective resistance of the diode when conducting. The diode may be considered to be non-conducting for all negative values of anode-voltage. Mark on your graph the values of the upper and lower limits of v_{AB}.

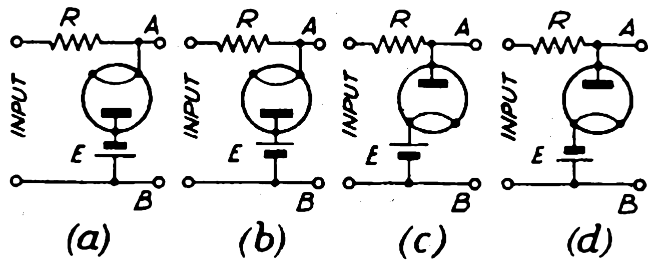


FIG. P–16

VIII.12. How would the answers to the previous question be changed if R = 9 K, the output resistance of the source were 10 K, and the resistance of the diode when conducting were 1 K?

VIII.13. A shaping circuit, similar to Fig. 187 (page 307) but using a pentode (characteristics as in Fig. P–3, page 360) has an h.t. voltage of 300. The input is a large alternating voltage. Between what limits does the anode-voltage vary if the load resistance is (a) 15 K, (b) 20 K, (c) 30 K?

VIII.14. Plot a graph of output voltage against input voltage for the cathode-follower amplitude-selector circuit of Fig. 194, page 312, taking the valve characteristics as in Fig. P–2, page 359, and using the following values—

$$E_a = 150 \text{ v}, E_1 = 50 \text{ v}, R = 40 \text{ K}.$$

Between what values will the output voltage range if the input voltage is a sine wave of peak amplitude 250 v and if there is a 2-megohm resistance in the lead to the grid?

VIII.15. Assume that for the diode switch circuits shown in Fig. 196 (a) and (b) the O.V. terminals are fed from a low impedance source whose e.m.f. consists of 10 Kc/s square waves of peak value ± 20 volts, and that the right-hand terminals are fed from a high-impedance source whose e.m.f. is a 1-Kc/s sine wave of peak value 10 v. For each circuit, plot the waveform of the p.d. across the right-hand terminals.

VIII.16. Assume that for the three switch circuits shown in Fig. 196 (c), (d) and (e), the O.V. terminals are fed from a source with an internal resistance of 1 megohm and an e.m.f. consisting of 10-Kc/s square waves of peak value ± 80 v, and that the right-hand terminals are fed from a high impedance source whose e.m.f. is a 1-Kc/s sine wave of peak amplitude 50 v. For each circuit plot the waveform of the p.d. across the right-hand terminals.

VIII.17. In the cathode-follower switching-circuit of Fig. 196 (f), page 314, assume that the switch terminals, S, T, are fed from a high-impedance source whose e.m.f. is a 10 Kc/s sine wave of peak-amplitude 50 v. The O.V. terminals are supplied from a source whose internal resistance is 1 megohm. Plot the waveform of the p.d. across the switch terminals, S, T, if the e.m.f. of the source supplying the O.V. terminals is

(*a*) 100-v positive, rectangular pulses with a repetition frequency of 100 Kc/s and a duty ratio of 50 per cent

(*b*) as (*a*) but *negative* pulses

(*c*) a 100-Kc/s square-wave alternating e.m.f. of peak value $\pm$ 100 v. Assume that the p.d. across the resistance would be 90 v, if the grid were connected directly to the cathode, and would be 4 v if the grid were connected directly to earth, and that a negative grid-voltage of 5 would be required to cut off anode-current.

VIII.18. A triode valve (characteristics as in Fig. P–2, page 359) has an h.t. voltage of 200 and an anode-load resistance of 100 K. A 0·001 microfarad condenser is connected between anode and cathode. What will be the condenser p.d.

(*a*) if the grid-voltage is maintained equal to zero?

(*b*) if the grid-voltage is maintained at a negative value sufficient to cut off anode-current?

(*c*) if the grid-voltage consists of 10 v rectangular, negative, voltage-pulses of frequency 50 c/s and duty ratio 0·5 per cent?

(*d*) if the grid-voltage consists of 10 v rectangular, negative, voltage pulses of frequency 50 c/s and duty ratio 99·5 per cent?

***VIII.19.** If the "whole-current equivalent anode-circuit" described in the answer to problem II.26 be substituted for the resistance r in the equivalent circuit of Fig. 202 (*b*), page 318, what change will need to be made in Fig. 202 (*c*)?

VIII.20. A triode whose characteristics are as shown in Fig. P–2, page 359, operates with an h.t. voltage of 200 and an anode-load resistance of 15,000 ohms. A 1-microfarad condenser is connected between anode and cathode. The grid-voltage is first maintained at a negative value sufficient to cut off anode-current, and is then suddenly switched to zero.

(*a*) To what value will the anode-voltage have fallen 3 milliseconds later?

(*b*) If, after anode-current has been flowing for a considerable time, the grid-voltage is suddenly made negative and sufficiently large to cut off anode-current, to what value will the anode-voltage have risen 7·5 milliseconds after the cut-off?

VIII.21. One valve of an R–C coupled amplifier suddenly has its grid-voltage made large and negative so that anode-current is cut off. (Assume that the next valve is removed from its holder, so that there is no complication due to its passing grid-current.) Given the particulars below, calculate

(*a*) to what value the anode-voltage immediately rises

(*b*) to what value the anode-voltage will have risen 15 milliseconds after the cut-off.

Working anode-voltage	=	129 v
Load resistance, R_L	=	50 K
Coupling resistance, R_g	=	250 K
Coupling capacitance, C	=	0·1 μF
H.t. voltage	=	200 v

***VIII.22.** One valve of an R–C coupled amplifier, with the same particulars as in the previous question, has its grid-voltage maintained for a substantial time at the cut-off value. The grid-voltage is then suddenly returned to the normal operating value, viz. $-$ 2 v. Given the further particulars below, calculate

(*a*) the time constant of the condenser discharge

(*b*) the p.d. across the coupling resistor, R_g, 12·9 milliseconds after the restoration of anode-current.

Valve impedance, ρ = 10 K ohm. Amplification factor, μ = 50. Intercept-current of linearized characteristics, d = $-$ 1·5 mA.

VIII.23. An amplifier stage with a resistance load and using anode-circuit decoupling has a grid-bias voltage sufficient to cut off anode-current, and an input voltage in the form of 50 c/s, rectangular, positive pulses of duty ratio 1 per cent. The mean, or d.c., anode-current is 5 mA. How large must the decoupling capacitance be made if the p.d. across it is to be d.c. with no more than 1-volt fluctuation?

VIII.24. An amplifier stage with a 20-K resistance load and a 20-K decoupling resistance (and a large decoupling capacitance) uses the valve whose characteristics are shown in Fig. P–2, page 359. The input voltage consists of rectangular positive pulses which switch the grid-voltage between zero and a negative value great enough to cut off anode-current. The h.t. voltage is 200. Between what limits does the anode-voltage vary when the duty-ratio of the positive input-pulses is (*a*) 0·1 per cent, (*b*) 50 per cent?

VIII.25. A triode valve whose characteristics are as shown in Fig. P–2, page 359, uses an h.t. voltage of 200 and an anode-load resistance R_L of 30 K ohm. Between cathode and h.t. negative is connected a resistance R_K of 20 K ohm. A large capacitance is connected across R_K, so that the p.d. across it is d.c. with a negligible fluctuation. Rectangular input pulses of duty ratio 50 per cent are fed in, between grid and h.t. negative. The magnitude of these pulses is just sufficient to swing the grid-cathode voltage between zero and the cut-off value. Determine—

(*a*) the peak anode-current

(*b*) the bias-voltage across R_K

(*c*) the limits of variation of the output voltage (i.e. the p.d. between anode and h.t. negative)

(*d*) whether the input pulses are positive pulses or negative pulses, and their amplitude.

VIII.26. A triode used in a cathode follower, with an h.t. voltage E and a cathode-load resistance R, has a condenser connected across R. The input voltage, connected between grid and h.t. negative, consists of repeated rectangular positive pulses, of duty ratio s and peak value $\hat{V}$. Adopting a linear equation for the valve characteristics, derive expressions for the limits between which the output voltage, v_R, varies,

(*a*) if the pulse repetition frequency is very low

(*b*) if the pulse repetition frequency is very high.

Also, (*c*) find what is the smallest amplitude, $\hat{V}$, of the high-frequency positive input pulses which will ensure that anode-current is cut off for part of the cycle.

VIII.27. A simple Miller-integrator circuit uses a valve whose characteristics are as shown in Fig. P–2, page 359. The anode-load resistance is 40 K ohms, the h.t. voltage is 200 and the capacitance between grid and anode is 0·5 microfarad. The resistance in the grid-lead is 2 megohms and the end of this resistance remote from the grid is suddenly switched from a negative potential, sufficient to cut off anode-current, to the full h.t. positive potential.

(*a*) To what value will the anode-voltage fall?

(*b*) How long will it take to fall to this value?

(*c*) What is the initial rate of fall of anode-voltage?

(*d*) How long *would* the anode-voltage take to fall to this same value if the initial rate of fall were maintained, i.e. if the "run-down" were linear?

VIII.28. (*a*) An amplifier with one earthed input terminal and one earthed output terminal has an infinite input-impedance and always gives an output voltage which is $-A$ times its input voltage, where A is real and positive. The amplifier is modified by having a resistance R_1 and a capacitance C connected in series from the non-earthed input terminal to the non-earthed output terminal. Derive an equivalent circuit for the input impedance of the modified amplifier.

(*b*) If a positive step-function e.m.f., $+E$, be connected through a high resistance R to the input terminals of the modified amplifier, what will be the form of the output-voltage?

***VIII.29.** (*a*) Show that Fig. P–17 is an equivalent circuit for the Miller-integrator circuit of Fig. 216, page 335. In this equivalent circuit, A denotes $\mu R'/(\rho + R')$ and r denotes $\rho R'/(\rho + R')$.

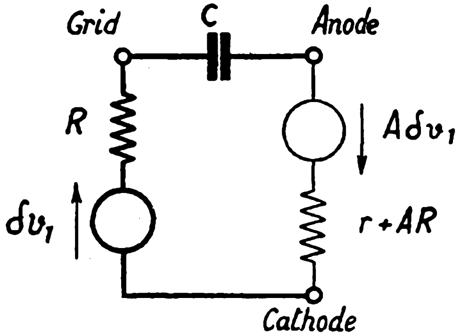


FIG. P–17

(*b*) Hence show that the inclusion of a resistance, equal in value to $1/g_m$, in series with C, will eliminate the unwanted sudden small rise of voltage which occurs in the ordinary Miller integrator (see footnote on page 335) when the positive step-function of voltage is applied in its input circuit.

VIII.30. The triggering-sequence table below shows the condition of each valve in a binary counter formed of four cascaded bi-stable multivibrators (see Fig. 226, page 347). In this table, a zero means that no anode-current is flowing and a 1 means that anode-current is flowing. Triggering pulses can be derived from a particular valve only when it changes from 0 to 1. Triggering pulses can trigger a bi-stable circuit only when applied to a valve in condition 0. In the cascaded circuit, triggering pulses are normally *derived* only from right-hand valves, but are normally *applied* to both left- and right-hand valves.

(*a*) Complete the table (up to 16 input pulses).

(*b*) What do the arrows indicate?

Number of input pulses	1st bi-stable circuit		2nd bi-stable circuit		3rd bi-stable circuit		4th bi-stable circuit	
	L	*R*	*L*	*R*	*L*	*R*	*L*	*R*
0	0	1	0	1	0	1	0	1
1	1	0	0	1	0	1	0	1
2	0	1	1	0	0	1	0	1
3	1	0	1	0	0	1	0	1
4	0	1	0	1	1	0	0	1

State how the triggering-sequence table would be altered in each of the following separate circumstances—

(*c*) if the diode feeding the left-hand valve of the 3rd bi-stable circuit were to become inoperative (*d*) if the diode feeding the right-hand valve of the 3rd bi-stable circuit were to become inoperative

(*e*) if an additional diode were connected from a tapping on the anode load of the right-hand valve of bi-stable circuit 1 to the anode of the right-hand valve of bi-stable circuit 4 for the purpose of feeding triggering pulses forward.

N.B. It is to be assumed that a small time-delay is associated with the triggering process. On an occasion when an input pulse triggers the first bi-stable pair, and as a result the first pair triggers the second pair, and as a further result the second pair triggers the third pair, there will be three such time-delays. On such an occasion, the feed-forward connexion from the output of the first pair to the fourth pair will therefore enable a forward-fed triggering pulse to reach the fourth pair before the arrival of the triggering pulse from the third pair.

(*f*) Prove that a decade counter can be formed by making the feed-forward connexion described in (*e*) above, and also providing a feed-back connexion, through a diode, from a tapping on the load resistance of the left-hand valve of the fourth bi-stable pair back to the left-hand anode of the second bi-stable pair.

ANSWERS AND COMMENTS ON THE PROBLEMS

II.1. (*a*) 4, (*b*) 4, (*c*) 6, (*d*) 7, (*e*) 4, (*f*) 10, (*g*) 7.

II.2. $\rho = 4{,}000$ ohms, $\mu = 19$, $g_m = 4{\cdot}75$ mA/v.

II.3. This problem cannot be solved with the data provided. The quotient (80 v/1·6 mA) is not ρ, but the d.c. resistance of the valve under these particular operating conditions.

II.4. 60 v.

II.5. 5 mA.

II.6. The low value of ρ shows that the valve is a triode.

II.7. $i_a = \dfrac{1}{12{,}000} \cdot (v_a + 60v_g) - 0{\cdot}005.$

II.8. This problem cannot be solved with the data provided, because a current so small as 1 microamp is not on the linear parts of the characteristics.

II.9. The value of d is given by the intercept, on the vertical axis, of the straight line obtained by producing (i.e. extending) the linear part of the $v_g = 0$ characteristic.

Only screen-grid valves and pentodes have positive values of d. For a triode, d is negative. For Fig. P–2, $d = -1{\cdot}5$ mA. For Fig. P–3, $d = 24$ mA.

II.10. (*a*) If anode characteristics are plotted for $v_g = 0, -1$ v, -2 v, etc. (i.e. 1 v increments of v_g), μ will be equal to the horizontal spacing between adjacent graphs.

(*b*) If mutual characteristics are plotted with an increment δv_a of anode-voltage between adjacent graphs, then μ is the quotient of δv_a and the horizontal spacing between adjacent graphs.

II.11. Not unless anode characteristics are drawn for *very small increments* of grid-voltage. For a pentode, μ is of the order of 1,000.

II.12. 0·3 mA.

II.13. 2·5 mA.

II.14. Only that v_s is greater than 130 volts.

II.15. No. Both ρ and g_m are the ratios of incremental quantities. g_m is the *slope* of the mutual characteristics and $1/\rho$ is the *slope* of the anode characteristics, whereas the data provided merely enable us to plot a single point on the characteristics.

II.16. This problem cannot be solved with the data provided. These data consist merely of a statement of the *slopes* of the anode characteristics and the mutual characteristics. If the quantity d were also given, and if it could be assumed that the characteristics were linear, the problem could be solved.

II.17. When the grid-voltage is positive, the valve (from grid to cathode) presents a resistance of 2,000 ohms. The voltage across this 2,000-ohm resistance, which is v_g, will be $2{,}000/(10^6 + 2{,}000)$ of the generator e.m.f. during the positive half-cycles. Thus the peak value during the positive half-cycles will be approximately 8 mV. The negative peak value is 4 v. Both half-cycles are half sine-waves, though the positive half-cycle is almost indistinguishable from a horizontal line.

II.18. The graph will be similar to the anode characteristics of a triode.

II.19. See Fig. P–18.

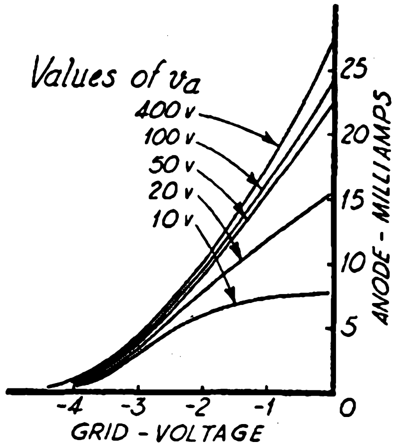


FIG. P–18

II.20. $-5{\cdot}6$ v.
II.21. $v_a = 233$ v. Thus, battery voltage $= 243$ v.
II.22. (*a*) $v_{a_1} = v_{a_2} = 190$ v, since p.d. across $R = 10$ v
(*b*) $v_{g_1} = -10$ v
(*c*) $i_{a_1} = 3{\cdot}5$ mA, from characteristics
$i_{a_2} = 16{\cdot}5$ mA, by subtraction of i_{a1} from 20 mA
(*d*) $v_{g_2} = -6{\cdot}1$ v, from characteristics
(*e*) $E_g = 3{\cdot}9$ v, since $v_{g_2} = E_g - V_R$
II.23. Method: assume a value for $(i_{a_1} + i_{a_2})$. Using the method of problem 22, calculate i_{a_1}, i_{a_2}, and E_g. Repeat for other assumed values of $(i_{a_1} + i_{a_2})$. The value of E_g which makes i_{a_1} zero is found as follows. The cut-off value of v_{g_1} when v_{a_1} is about 200, is about -18 v. This must be produced entirely by the p.d. across R. Thus $i_{a_2} = 18$ mA and $v_{a_2} = 200 - 18 = 182$ v, whence from the characteristics, $v_{g_2} = -5{\cdot}5$ v, giving $E_g = 12{\cdot}5$ v.

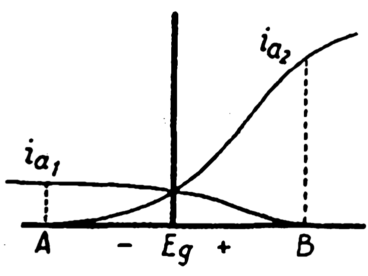


FIG. P–19

The negative value of E_g required to cut off i_{a_2} is found by first determining i_{a_1}, assuming that valve 2 is removed. Chapter III gives a load-line method, but at this stage a trial and error method may be used. Try any value of i_{a_1}, calculate v_{a_1} and v_{g_1} and test by reference to the characteristics, whether these values of v_{a_1} and v_{g_1} would in fact give the value of i_{a_1} which you have assumed. The correct value is $4{\cdot}5$ mA.

Having found i_{a_1}, calculate Ri_a (viz. $4{\cdot}5$ v). For cut-off v_{g_2} must be -18 v, thus $E_g = -13{\cdot}5$ v.

The form of the graphs is shown in Fig. P–19.

II.24. Substituting these three equations into the equation of the characteristics, assumed linear, viz.

$$i_a = \frac{1}{\rho}(v_a + \mu v_g) + d$$

we have

$$i_{a_1} = \frac{E_a - g_m R E_g(1 + \mu) + \rho d}{\rho + 2R(1 + \mu)}$$

$$i_{a_2} = \frac{E_a + g_m E_g(R + \mu R + \rho) + \rho d}{\rho + 2R(1 + \mu)}$$

It must be remembered that the equation for the linear characteristics is valid only if both anode-currents are positive. The above two equations for i_{a_1} and i_{a_2}, therefore, are applicable only to the part AB of Fig. P–19.

(a) $E_g = \dfrac{E_a + \rho d}{g_m R(1 + \mu)}$

(b) $E_g = -\dfrac{E_a + \rho d}{g_m[\rho + R(1 + \mu)]}$

(c) $i_{a_2} = \dfrac{E_a + \rho d}{R(1 + \mu)}$ for i_{a_1} just cut off

(d) $i_{a_1} = \dfrac{E_a + \rho d}{\rho + R(1 + \mu)}$ for i_{a_2} cut off.

II.25. Assume the following equation for the valve characteristics—$i_a = \frac{1}{\rho}(v_a + \mu v_g) + d$. From this equation, $dv_a/dt = \rho di_a/dt$. Also the condenser current is given by $i_a = -C dv_a/dt$, in which we may write $\rho di_a/dt$ in place of dv_a/dt.

$$\therefore di_a/dt = -i_a/\rho C$$

The solution of this differential equation is $i_a = Ae^{-t/\rho C}$ where A is the value of i_a at $t = 0$, i.e. A is the initial current, $\frac{1}{\rho}(E + \mu v_g) + d$.

It was shown above that $dv_a/dt = \rho di_a/dt$ and it follows from this that $v_a = \rho i_a +$ a constant

$$\therefore v_a = (E + \mu v_g + \rho d)e^{-t/\rho C} + \text{a constant}$$

By using the information that $v_a = E$ at $t = 0$, we find that this constant must be $-(\mu v_g + \rho d)$

$$\therefore v_a = -(\mu v_g + \rho d) + (E + \mu v_g + \rho d)e^{-t/\rho C}$$

The intercept of each characteristic on the voltage axis is given by $-(\mu v_g + \rho d)$. Writing this as v_0, we have

$$v_a = v_0 + (E - v_0)e^{-t/\rho C}$$

II.26. (a) The box could contain a diode valve with its anode connected to one of the terminals and with its cathode connected to the positive side of a battery v_0, and with the negative side of the battery connected to the other terminal.

(b) The above circuit, (a), is an equivalent circuit for a triode provided (i) that the diode, when it is conducting, be assumed to act as a constant resistance equal to ρ, and (ii) that the d.c. e.m.f. of the battery, v_0, be assigned the value $-(\mu v_g + \rho d)$ where d has the same meaning as on page 18. Note that, for a triode, d is negative.

(c) The equivalent circuit corresponding to problem II.25 is a single, closed loop formed of the charged condenser, the diode (whose resistance when conducting is ρ) and the d.c. e.m.f. $-(\mu v_g + \rho d)$, or v_0. It is clear that the condenser p.d. will fall exponentially towards a value v_0, with a time constant equal to ρC.

III.1. $i_a = 7{\cdot}5$ mA. $v_a = 150$ v.

III.2. v_a swings between 120 v and 178 v
Voltage amplification = 14·5.

III.3. (*a*) 14

(*b*) $\frac{1}{2} \cdot \frac{95 - 73}{95 + 73} = 6{\cdot}56$ per cent

(*c*) 180 milliwatts

III.4. 10 : 9.

III.5. The graph will be a straight line. Its slope will be $1/g_m$ and the intercept on the vertical axis will be $1/\mu$.

III.6. Hint: apply Thevenin's theorem to the equivalent circuit of the valve.

III.7. From the characteristics, the lowest anode-voltage which can be used is about 70 v, giving an anode-current of 1 mA. This would produce a p.d. of 1,000 v across the 1-megohm load. Hence required h.t. supply voltage = 1,070 v.

With $E_a = 1{,}200$, v_a will be about 190 v (from load-line diagram). Voltage amplification = 500 approx.

III.8. (*a*) 165 v (*b*) 265 v (*c*) 265 v

III.9. (*a*) 50,000 ohms (*b*) 25,000 ohms

III.10. (*a*) 18,600 ohms (*b*) $E_a = 244$ v, $E_g = 2$ v
(*c*) 2 v (*d*) 114 milliwatts

III.11. i_a swings between 14·3 mA and 3·8 mA.
v_a swings between 55 v and 223 v.

III.12. (*a*) 43·1 : 1 (*b*) 160 v

III.13. 20 : 1
(*a*) and (*b*). Decreased by 36 per cent in both cases.

III.14. $i_a = \dfrac{E_a + \rho d + \mu v_g}{\rho + R}$; Slope = 2 mA/v

III.15. Because the mutual characteristics for different anode-voltages are almost indistinguishable from each other (cf. problem II. 19).

III.16. D.c. component $= a - bE + cE^2 + \frac{1}{2}cV^2$
Fundamental component $= V(b - 2cE)\sin \omega t$
Second Harmonic component $= -\frac{1}{2}cV^2 \cos 2\omega t$

III.17. (*a*) 40 (*b*) 32·04 dB
(*c*) Between one and four milliamps (*d*) 48

III.18. Gain reduced to 20.

III.19. The current through the cathode-resistance is the sum of the anode-current and the screen-grid current. As we do not know the latter, we have not enough data to solve this problem.

III.20. No. When cut-off is brought about by the control-grid, both the anode-current and the screen-grid current are reduced to zero. (But see also problem IV.54.)

III.21. 13,000 ohms. 1 watt.

III.22. (*a*) 85 v to 385 v (*b*) 24·3 mA to 4·5 mA (*c*) 0·75 watts.

III.23., **III.24.** $f = \dfrac{1}{2\pi CR}$

III.25. $f = \rho/2\pi L$

III.26. 42·4 henrys.

III.27. $f = \dfrac{\rho + R_B(1 + \mu)}{2\pi L}$

III.28. (*a*) 17·6 (*b*) $Z_B = R_B/(1 + j)$. Gain = 21·8.
Note that the frequency mentioned in (*b*) is not the cut-off frequency.

III.29. $C = 0{\cdot}0159\ \mu$F, 9 dB

III.31. $f = 1/\{2\pi R_g C\sqrt{2^{\frac{1}{3}} - 1}\} = 3{\cdot}12$ c/s

III.32. The grid of the second valve must not be positive with respect to its cathode when the potential of the first anode is at its highest, viz. when the input voltage is making the first grid 1 volt negative. Intersection of the load line (E_a = 300 v, R = 30 K ohms) with the characteristic for $v_g = -1$ v, gives v_a = 58 v. Thus E_c = 58 v.

III.33. (a) v_{a1} varies between 58 v and 56·2 v

(b) E_2 = 58 v

(c) $\frac{1}{2}\mu R/(\rho + R) = 8{\cdot}82$

(d) v_{a2} varies between 41 v and 56 v. (Since v_{g2} varies between zero and − 0·88 v.)

(e) With the same value of E_2, v_{a3} would swing between − 8·5 v and − 1 v which is satisfactory. Had it been unsatisfactory, a voltage divider could be used across E_2 somewhat as shown in Fig. 52, p. 82.

III.34. (a) The second grid (point P) is 55 v above earth (i.e. above h.t. negative). The second cathode must therefore be 60 v above earth, giving R_k = 12,000 ohms.

(b) From the characteristics, the anode-voltage of the second valve is 117 v. Thus the p.d. across R is (360 − 117 − 60) v. Dividing this by 5 mA, R = 36,600 ohms.

III.35. A.c. component of each grid-voltage = 20 v. The resistance, R, presented to each valve, in its anode circuit, is 20,000 ohms. A.c. voltage across each half of the primary of the output transformer = 20 $\mu R/(\rho + R)$ = 320 v. A.c. voltage across 400 ohm load = 64 v.

III.36. See Figs. P–20 and P–21.

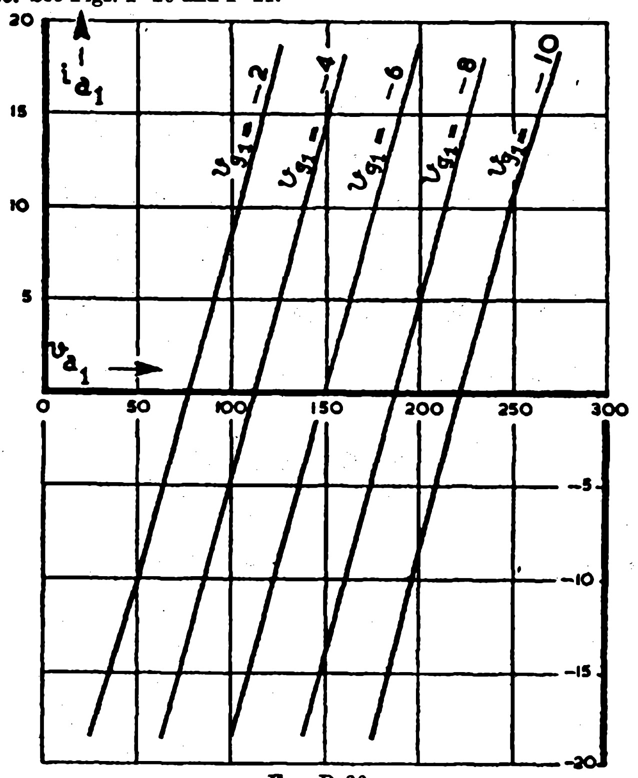


FIG. P–20

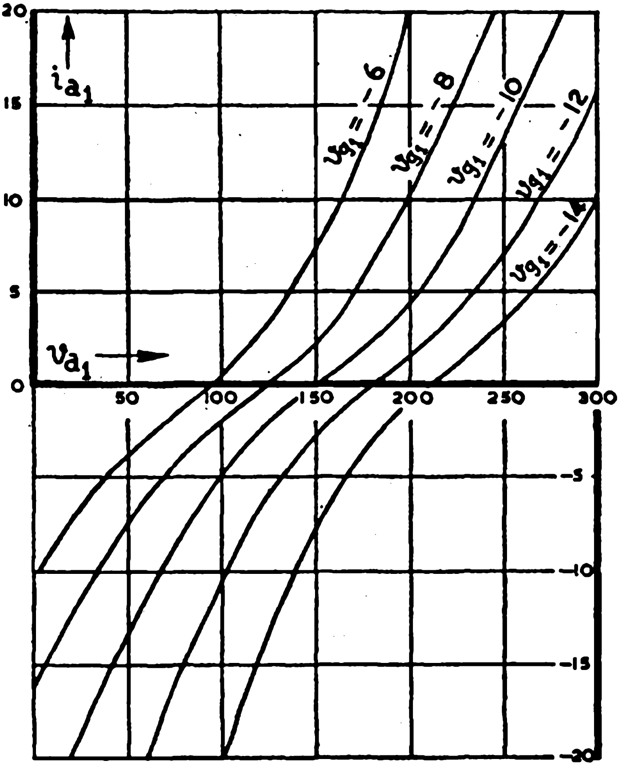


FIG. P–21

III.37. Without using *any* shunt capacitance, the gain does not fall by as much as 3 dB, even as the frequency tends to zero.

III.38. $\mu = R/r$

III.39. $g_m = 1/R$

III.40. Draw a load line of slope $-1/2R$. For each point of intersection, read off i_a and v_g, and calculate v_1 as $(v_g + Ri_a)$. Plot i_a against v_1.

For $v_1 = 50$ v, $i_a = 10{\cdot}45$ mA.

III.41. Equation (i) is represented by a load line of slope $-1/(R + R_k)$ cutting the horizontal axis at $v_a = E_a$. To plot equation (ii), select any one of the static characteristics, note its value of v_g, substitute this value of v_g into equation (ii) and hence calculate a value of i_a. Mark a point on the selected curve, having this calculated value of i_a. Repeat for other curves. The line joining all the marked points is the required graph. The intersection of this graph with the load line gives the actual value of i_a. With the values given in the problem, $i_a = 7{\cdot}7$ mA.

III.42. (*a*) The valve must be passing anode-current (for otherwise its grid-voltage would be + 150 v) and the value of v_g must therefore be within the "grid-base." Since the valve has a high μ, this means that the grid-voltage is within a few volts of zero. The p.d. across R_k must therefore be *approximately* 150 v/50,000 ohms, i.e. 3 mA.

(*b*) It follows that an approximate value for v_a is given by 350 v − (75,000 ohms × 3 mA), i.e. 125 v.

(*c*) From the characteristics, the value of v_g for $v_a = 125$ v and $i_a = 3$ mA is − 1·6 v, which gives us a better approximate value for the p.d. across R_k viz. 151·6 v. This gives 3·03 mA as a better approximate value for i_a, and 122·6 v as a better approximate value for v_a.

(*d*) With $v_g = 0$, the p.d. across R_k is exactly 150 v and the anode-current is exactly 3 mA. From the static characteristics, $V_a = 45$ v. Thus the drop across R (with 3 mA flowing through it) is 350 − 150 − 45, i.e. 155 v. Hence $R = 51{,}667$ ohms (approx. 52 K).

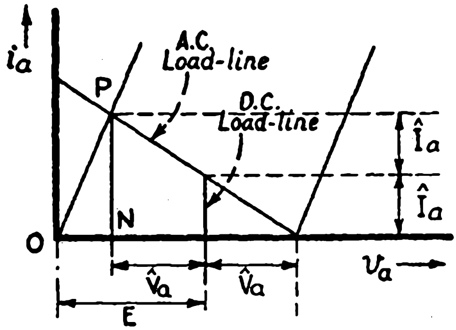


FIG. P–22

III.43. See Fig. P–22.

A.C. Power $= \frac{1}{2}\hat{V}_a\hat{I}_a$

But $\rho = ON/PN = (E - \hat{V}_a)/2\hat{I}_a$

whence $\hat{V}_a = E - 2\rho\hat{I}_a$

$\therefore$ Power $= \frac{1}{2}(E - 2\rho\hat{I}_a)\hat{I}_a$

To find maximum power, differentiate with respect to $\hat{I}_a$ and equate to zero, giving $\hat{I}_a = E/4\rho$ whence $\hat{V}_a = E/2$. The quotient $\hat{V}_a/\hat{I}_a$ then gives $R_{optimum}$ as 2ρ.

III.44. Both Fig. P–9 and the more conventional equivalent circuit (Fig. 26, p. 37) give a current through R equal to $\mu V_g/(\rho + R)$. (N.B. $g_m\rho = \mu$.) The suggested simplification, applicable only where $R \ll \rho$, is to omit ρ from the circuit of Fig. P–9, so that the equivalent circuit becomes merely a constant-current source, whose current is $g_m V_g$.

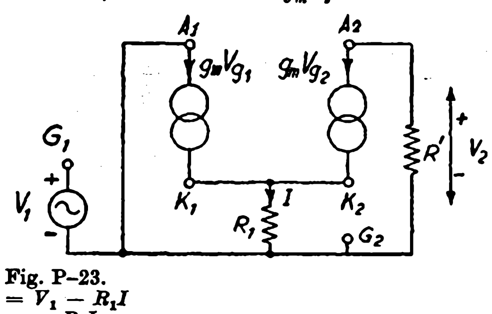


FIG. P–23

III.45. See Fig. P–23.

$V_{g_1} = V_1 - R_1 I$

$V_{g_2} = -R_1 I$

$I = g_m V_{g_1} + g_m V_{g_2}$

$V_2 = -R' g_m V_{g_2}$

Eliminating V_{g_1}, V_{g_2} and I from these four equations, we have $V_2/V_1 = g_m^2 R_1 R'/(1 + 2g_m R_1)$.

III.46. $V_2/V_1 = \{-g_m{}^2R_1R'/(1 + 2g_mR_1)\} \cdot \left(1 + \frac{1}{g_mR_1}\right).$

Note that, if $g_mR_1 \gg 1$, the answers to problems III.45 and III.46 would become $\frac{1}{2}g_mR'$ and $-\frac{1}{2}g_mR'$.

III.47. Compare with problem III.42. As a first approximation, take the p.d. across R_k as E_k, giving $i_{a_1} + i_{a_2} = 10$ mA. This also means that v_{a_1} is approximately 200 v, and that a load line may be drawn for valve 2 using an h.t. voltage of 200 and with a vertical intercept of 200 v/20 K, viz. 10 mA. For any value of v_g, i_{a_1} and i_{a_2} may be read off from the static characteristics and from this load line respectively. (N.B. v_g is the same for both valves in the quiescent condition.) Hence calculate $i_{a_1} + i_{a_2}$ for each of a number of values of v_g (two values actually suffice, viz. $v_g = -8$ v and $v_g = -10$ v) and mark a point on each static characteristic at the value of $(i_{a_1} + i_{a_2})$ calculated. The intersection of this line with the horizontal line for $i_a = 10$ mA gives a second approximation to the value of v_g (viz. $-8{\cdot}6$ v). Finally read off the values of i_{a_1} and i_{a_2} for this value of v_g.

Answers: $i_{a_1} = 2{\cdot}3$ mA, $i_{a_2} = 8{\cdot}5$ mA.

III.48. As a first approximation take the p.d. across R_k as E_k (cf. problem III.42). This means that v_{a_2} is approximately 200 v and the required value of v_g to cut off i_a is seen from the characteristics to be approximately -18 v. This, however, must be equal to $(E_k - R_ki_{a_1})$ and hence $R_ki_{a_1} = 118$ v. This gives a second approximation to v_{a_2} as 172 v with which value we can arrive at a second approximation to v_{g_2}, viz. -16 v. Thus $R_ki_{a_1}$ is 116 v and $i_{a_1} = 11{\cdot}6$ mA. v_{a_1} is 184 v. From the static characteristics, therefore, $v_{g_1} = -6{\cdot}8$ v. This, however, must be equal to $v_1 + E_k - R_ki_{a_1}$, i.e. $v_1 = 9{\cdot}2$ v.

III.49. v_{g_2} is required to be zero when i_{a_1} is zero. This means that $R_ki_{a_2} = E_k = 100$ v. Thus i_{a_2} is exactly 10 mA. From the characteristics, the value of v_{a_2} corresponding to 10 mA and zero grid-voltage is 47 v. Thus the p.d. across R, with 10 mA flowing through it, is $300 - 147$, i.e. 153 v. Hence $R = 15{,}300$ ohms.

$v_{g_1} = v_{g_2} + v_1$ and $v_{g_2} = 0$. Thus $v_{g_1} = v_1$. The characteristics show that, with $v_{a_1} = 200$, the value of v_{g_1} required to cut off i_{a_1} is -18 v. Thus the required value of v_1 is -18 v.

III.50. $$\frac{V_2}{V_1} = \frac{\mu R}{\rho + R(2 + \mu)} \cdot \frac{1 - j\omega CR_p}{1 + j\omega CR_p}$$

The magnitude (or "modulus") of V_2/V_1 is $\mu R/(\rho + 2R + \mu R)$. The angle of phase-shift is $-2\tan^{-1}(\omega CR_p)$. This quantity ranges from zero to -180 degrees as C ranges from 0 to ∞.

III.51. (*a*) The output voltage will be in phase with the input voltage.
(*b*) Gain $= (1 + \mu)R_L/(\rho + R_L)$
(*c*) $Z_{IN} = (\rho + R_L)/(1 + \mu)$
(*d*) Gain $\approx g_mR_L$ $\quad Z_{IN} \approx 1/g_m$

IV.1. $R : (\omega_0L \text{ or } 1/\omega_0C) : R_D :: 1 : Q : Q^2$

$1/\omega_0C = 2{,}000/\pi$. Thus $R_D = 140{,}000/\pi$ and $R = \frac{200}{7\pi}$

$L = 1/\omega_0{}^2C = 1/500\pi^2 = 200$ microhenrys.

IV.2. From 0·99 Mc/s to 1·01 Mc/s

IV.3. (*a*) 24 (*b*) 20 (*c*) 975 Kc/s to 1,025 Kc/s

IV.4. (*a*) 146 (*b*) 48·5 (*c*) 989·7 Kc/s to 1,010·3 Kc/s

IV.6. The shunt resistance of 400,000 ohms reduces the effective value of R_D to 80,000 ohms and reduces the effective value of Q to 48 (see problem IV.5). Using these values the problem now becomes similar to problem IV.4. Gain = 150. Effective value of Q for the amplifier stage = 36.

IV.7. 7·96 Mc/s to 3·56 Mc/s.
37·7 metres to 84·3 metres.
Gain = 143; Q_{eff} = 47·6.

IV.8. The coefficient of coupling is M/L or, more generally $M/\sqrt{L_1L_2}$. It is denoted by k. The peak frequencies are given by $f_0/\sqrt{1 \pm k}$ where f_0 is the geometric mean frequency. For small values of k, these peak frequencies are approximately $f_0(1 \mp k/2)$ and the bandwidth between them is therefore kf_0. Here kf_0 = 20 Kc/s and f_0 is approximately 510 Kc/s. Thus k = 0·039. C = 195 pF.

IV.9. $\mu Z/(\rho + Z)$ is real and equal to 19. Thus the total input capacitance is 9 pF + (20 × 7·5 pF), i.e. 159 pF. The cut-off frequency is $\frac{1}{2\pi RC}$ where R = 100 K and C = 159 pF. This gives a frequency of 10 Kc/s.

IV.10. (*a*) A capacitive reactance of 19·2 K ohms (83 pF at 100 Kc/s).
(*b*) An impedance of 18 K ohms (formed of the parallel combination of a 2-megohm resistance, a 50-K ohm resistance and the above capacitive reactance).
(c) 22·9. Cf. equation (IV.38).

IV.11. 350 Kc/s. 4·35 K ohms. 12·5.

IV.12. n = 4 and $\sqrt{2^{1/n} - 1}$ = 0·435. f_1 = 40 × 0·435 c/s = 17·4 c/s. f_2 = (10/0·435) Kc/s = 23 Kc/s.

IV.13. 1%.

IV.14. 45·5. 2·94%.

IV.15. 20·8.

IV.16. 49·3.

IV.17. Insufficient data provided. See equations (IV.26).

IV.18.
$$\text{Phase-shift} = \theta - \tan^{-1}\frac{|A| \,.\, |\beta| \sin\theta}{1 + |A| \,.\, |\beta| \cos\theta}$$

If $|A| \,.\, |\beta| \gg 1$ the gain vector reduces to $1/|\beta|$ and thus the phase-shift tends to zero.

IV.19. $\rho/(1 + \mu n)$ = 1,600. Transformer ratio = $\sqrt{1{,}600/16}$ = 10.

IV.20. "For small changes, the percentage change of overall gain bears the same ratio to the percentage change of internal gain as the overall gain bears to the internal gain" (i.e. the ratio of the percentage-changes is the same as the ratio of the gains themselves).

IV.21. One fiftieth of one per cent.

IV.22. 0·2%.

IV.24. A = 1,650.

IV.25. The gain of the amplifier will not be affected.

IV.27. Input impedance = 1,000 ohms. The gain remains − 999. Input voltage = input e.m.f. × 1,000/(1,000 + 4,000) = 2 mV. Output = 1·998 v.

IV.28. 13,919.

IV.29. Assuming E_a constant and considering the variation of v_L with i_a (i_a being varied by varying the load resistance, R), equation (IV.41) gives $v_L(\rho/R + 1 + \mu)$ = constant, i.e. $\rho i_a + (1 + \mu)v_L$ = constant. Differentiation with respect to i_a gives the output resistance, $-\partial v_L/\partial i_a$ as $\rho/(1 + \mu)$. Assuming R constant, and considering the variation of v_L due to varying E_a, differentiation of equation (IV.41) with respect to v_L gives the stabilization ratio, $\partial E_a/\partial v_L$, as $1 + \mu + \rho/R$.

IV.30. (*a*) The principle of this stabilizer is the same as the principle illustrated in Fig. 85, page 136, except that a further triode (valve 2) has been added to amplify the "error-voltage." This error-voltage is obtained by comparing the constant p.d. across the gas-diode ("the reference p.d.") with the p.d. across R_2, which is a fraction $R_2/(R_1 + R_2)$ of the output voltage.

The *difference* between these two p.d.s is applied between grid and cathode of valve 2. If, for any reason, the output voltage rises, the grid-potential of valve 2 will rise, its anode-current will increase, and the p.d. across its load resistance, R, will increase. Since R is connected between anode and grid of the "dropping valve" (valve 1), the grid-potential of that valve will be lowered and the current supplied to the load, through valve 1, will be reduced. This offsets the rise of output voltage which was the starting point of our argument.

$(R_1 + R_2)$ is a high-resistance potentiometer. The position of the tapping point on this potentiometer controls the value of the output voltage.

(b) For valve 1, $i_a = 10$ mA and $v_a = (350 - 250)$ v. From the characteristics (Fig. P–1) we can thus find v_g ($-$ 2·7 v by interpolation). The p.d. between grid and anode of valve 1 is then 102·7 v and this will be also the p.d. across R. The anode-voltage of valve 2 is thus $(350 - 150 - 102 \cdot 7) = 97 \cdot 3$ v. We want valve 2 to be working somewhere in the middle of its control-range (or "grid base"), say with a grid-voltage of $-$ 1 v (see characteristics Fig. P–2). This would make the anode-current of valve 2 equal to 3·2 mA. Hence $R = 102 \cdot 7$ v/3·2 mA = 32 K ohms. The value of R is not critical, since the output voltage can be adjusted by sliding the tapping point on the voltage divider, $(R_1 + R_2)$.

(c) The p.d. across R_2 is the sum of the gas-diode p.d. (150 v) and the grid-voltage of valve 2 ($-$ 1 v). The p.d. across $(R_1 + R_2)$ is 250 v.

$$\therefore \; R_2/(R_1 + R_2) = 149/250$$

IV.31. R_K could be made equal to $10/g_m$, i.e. 2,500 ohms. Output impedance = approximately $1/g_m = 250$ ohms. The voltage gain is approximately 1.

IV.33. Yes. To evaluate the output impedance we may consider V_1 in Fig. 80 made equal to zero, R removed, and a test e.m.f. V connected across the output terminals. If the current sent by V is I, then the output impedance is given by V/I. Obviously, any grid-cathode impedance, Z_{gc}, directly shunts the test e.m.f., V, and appears in parallel with the output impedance as calculated in the absence of Z_{gc}.

The input impedance of the source V_1 which feeds the input terminals of the cathode follower must be added to Z_{gc} in this expression for the output-impedance. When Z_{gc} tends to infinity, the effect of the source impedance on the output impedance is eliminated.

IV.34. (a) Slope $= g_m/(2 + \mu)$

(b) The slope of the dynamic characteristic is $\mu/(\rho + R + \mu R)$. If $\mu \gg 1$ and $\mu R \gg \rho$, this reduces to $1/R$. In this case (for normal commercial valves) these conditions are satisfied and the slope is approximately 1/(20,000 ohms).

IV.35. The load line intersects the characteristic for $v_g = 0$ at an anode-voltage of 38. $v_1 = E_a - v_a + v_g$. The value of v_1 for this point of intersection is therefore $(200 - 38 + 0) = 162$ volts.

IV.36. As in the previous question, the maximum positive excursion of the grid above earth potential is given by the projection upon the v_a axis of that part of the load line intercepted between the v_a axis and the characteristic for $v_g = 0$. The maximum negative excursion of the grid below cathode potential is $v_{g\,cut\,off}$, and is much smaller than the maximum positive excursion. It is a safe rule therefore to say that the whole peak-to-peak swing of the input voltage must be accommodated in the available positive excursion, i.e. that the projected length of the load-line intercept must be 250 volts. Thus the load line intersects the $v_g = 0$ curve at an anode-current of (250/20), i.e. 12·5 mA. A load line of slope $-$ 1/20,000 mho, drawn through this point intersects the v_a axis at the h.t. voltage of 306.

For a cathode-resistance of 40 K ohms, the same procedure gives an h.t. voltage of 278.

IV.37. Denoting the two resistances by R and $9R$, we have $v_a = E_a - 10Ri_a$ and $v_g = -Ri_a$. Thus $E_a - v_a + 10v_g = 0$. Draw the load line ($E_a = 200$ v, $R_{total} = 20$ K) and for each point of intersection of load line and characteristic curves evaluate $E_a - v_a + 10v_g$. From left to right the values are 162, 109, 60, 14, $-$ 30 (corresponding to $v_g = 0, -2, -4, -6, -8$ v). By interpolation, we find the point where $(E_a - v_a + 10v_g)$ is zero, viz. at $v_g = 6 \cdot 7$ v.

IV.38. (*a*) Where the load line reaches an ordinate of 7·5 mA, $v_g = -6$. $\therefore R_1 = 6$ v/7·5 mA = 800 ohms.

(*b*) Both the anode-current and the screen-grid current flow through R_1 and R_2. No information is given about the screen-current. Thus the problem cannot be solved.

IV.39. For $E_a = 200$ v, $v_{g_{cut-off}}$ = approx. -18 v $\therefore v_g$ may range between 0 and -18 v. Let quiescent value be -9 v. Load line cuts characteristic for $v_g = -9$ v at $i_a = 2$ mA $\therefore R_1 = 9$ v/2 mA = 4,500 ohms. For a more accurate method, see problem IV.43.

IV.40. R_0 denotes $\rho/(1 + \mu)$.

(*a*) New output impedance $= (\rho + R)/(1 + \mu) = R_0 + R/(1 + \mu)$.

(*b*) Cf. problem III.6. Since the effective value of g_m is unchanged, $1/g_m$ is unchanged and, to a first approximation, the output impedance is unchanged by the addition of R. More accurately, R_0 changes to $R_0(1 + 1/g_m R)$.

(*c*) R is a direct shunt across the input voltage. R_0 will be unaffected. (But cf. problem IV.33.)

IV.41. (Hint: note that $\mu V_g = \mu R' I_1 - \mu R_1 I_a$. The reactance of C' is assumed very small.) Physical explanation: compare the circuit with Fig. P–6, problem III.38. Even if the valve were withdrawn from its socket, V_1 would produce a p.d. across R_2. This constitutes an a.c. p.d. in the anode circuit, which tends to produce an a.c. anode-current in antiphase to that produced by the influence of the grid.

IV.42. (*a*) $+ 9 \cdot 9$ v, $- 0 \cdot 1$ v (*b*) $+ 8$ v, $- 2$ v

IV.43. (*a*) First, the total cathode-resistance and E_{HT} are determined. The total cathode-resistance $(R_1 + R_2)$ must be large with respect to $1/g_m$ and also large enough to limit the peak anode-current to a reasonable value. Take $(R_1 + R_2) = 20$ K. The h.t. voltage must be large enough to accommodate the input-voltage swing (-100 v to $+100$ v). Using the method of problem IV.36, we decide to make $E_{HT} = 250$ v. Then, from the load-line diagram, we can calculate the value of v_{GE} (i.e. $E_a - v_a + v_g$) for each point of intersection. (In practice, the calculation need be made for only one or

v_g	0	-2	-4	-6	-8	-10	-12	-14	-16	-18
v_{GE}	202	167	136	106	77	53	34	16	4	-6

two points of intersection.) This shows that $-6\frac{1}{2}$ v would be a suitable quiescent value of v_g. The corresponding quiescent anode-current is 5·2 mA. Hence $R_1 = 6\frac{1}{2}$ v/5·2 mA, i.e. 1,250 ohms. (Since the value of $R_1 + R_2$ is not at all critical, the design can be adjusted to make possible the use of preferred values of resistance.)

(*b*) The only essential difference between parts (*a*) and (*b*) of this question lies in the fact that, in case (*b*), the input voltage has a d.c. component (-40 v). It can be seen that Figs. 83 (*a*) and (*b*) are no longer equivalent when v_1 has a d.c. component, for the d.c. component will affect the quiescent anode-current in Fig. 83 (*a*) but not in Fig. 83 (*b*). To maintain the equivalence, we must delete v_1 from Fig. 83 (*a*) and substitute for it $[v_1]_{AC}$, the a.c. component only of the input voltage v_1.

We begin, therefore, by subtracting the d.c. component from v_1. The remaining a.c. component ranges from $+40$ v to -160 v. To convert this to a range of $+200$ v to zero (the available range of v_{GE} in the above table) a positive bias (Fig. 83 (*a*)) of 160 v is required. The quiescent value of v_{GE} in Fig. 83 (*a*) would then be 160 v which, by interpolation, corresponds to $v_g = -2 \cdot 5$ v with $i_a = 8$ mA. This gives $R_1 = 2 \cdot 5$ v/8 mA = 312 ohms.

IV.44. (*a*) 250 watts (*b*) 915 watts

IV.45. $P = 1 \cdot 3$ watts, $D = 2 \cdot 95$ watts, $\eta = 30 \cdot 6\%$.

IV.46. (*a*) Using the empirical rule that $v_{g\,max}$ shall not exceed $0{\cdot}8v_{a\,min}$, we have $v_{a\,min} = 50$ v $\therefore\ \hat{V}_a = 200$ v.

$$(b)\ \frac{\pi}{4}\left(1 - \frac{50}{250}\right) = 62{\cdot}8\% \qquad (c)\ \frac{\pi}{4}\left(1 - \frac{150}{250}\right) = 31{\cdot}4\%$$

IV.47. (*a*) 3,000 ohms (*b*) 400 mA (c) $400/\pi$ mA
(*d*) 2,700 v to 300 v (*e*) 240 watts (*f*) 382 watts (*g*) 62·8%

IV.48. (*a*) and (*b*). Using the design rule that $v_{g\,max}$ must never exceed $v_{a\,min}$, we see that $v_{a\,min}$ must be 100 v $\therefore\ v_{HT} = 1{,}000$ v, $\hat{V}_a = 900$ v. (If the alternative design rule be used, viz. that $v_{g\,max}$ must not exceed $0{\cdot}8v_{a\,min}$, then we must use the tabulated figures to plot v_a/v_g, and from the graph read off the required value of $v_{a\,min}$. The figure would be 110 v.) (*c*) 3,000 ohms (*d*) $600/\pi$ watts (*e*) 70·7%.

IV.49. The dissipation would exceed the permitted value before the peak anode-current reached its permitted value.

Required h.t. = 654 v.

IV.50. Cut-off will occur when $v_a/v_g = -\mu$. But $v_a = v_{HT} - \hat{V}_a \sin \omega t$ and $v_g = -E_g + \hat{V}_1 \sin \omega t$. After substituting and solving for ωt, we have

$$\text{angle of flow} = 2\left\{90° - \sin^{-1}\frac{v_{HT} - \mu E_g}{\hat{V}_a - \mu \hat{V}_1}\right\}$$

IV.51. (*a*) From the characteristics we can find the value of v_a when $v_g = 0$ and $i_a = 200$ mA, viz. 1,200 v $\therefore\ 2\hat{V}_a = (2{,}500 - 1{,}200)$ v and $\hat{V}_a = 650$ v $\therefore\ v_{HT} = (2{,}500 - 650)$ v $= 1{,}850$ v.
(*b*) 650 v (c) 1,525 v. 200 v bias.

IV.52. $L = 73{\cdot}5$ microhenrys, $C = 225$ pF.

IV.53. $\hat{V}_g = 520$ v, $E_g = 320$ v.

IV.54. All that G3 does is to control the sharing of the total space-current between i_a and i_{g_2}. If either of these increases as a result of suppressor control, it can only be at the expense of the other (see page 190). Thus, if both i_a and i_{g_2} flow through the cathode-bias resistance, there will be no feedback. If only i_a flows through this resistance, there will be negative feedback. If only i_{g_2} flows through the cathode-bias resistance, there will be positive feedback. This can be arranged by the use of anode-circuit decoupling, the decoupling condenser being returned to cathode instead of to h.t. negative.

IV.55. It is possible. Try it in your laboratory with a 6F 33 valve. G1 must be connected directly to cathode, and G3 must be connected to h.t. negative.

V.1. 5·45 : 1

V.2. 12·5 : 1 for tuned-grid oscillator.
2·05 : 1 for tuned-anode oscillator.

V.3. 0·03% or 3 parts in ten thousand.

V.4. 13·41 Mc/s

V.5. The new maintenance condition is almost exactly the same as the maintenance condition of the conventional tuned-anode oscillator.

If L' be made equal to L, ω^2 becomes exactly $1/LC$. The frequency of oscillation is then theoretically independent of the valve parameters.

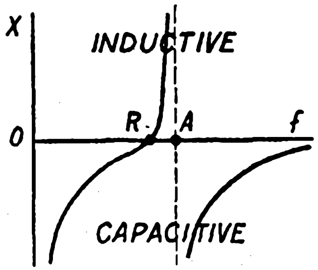


Fig. P–24

V.6. See Fig. P–24. The resonance frequency, corresponding to point R, is given by $\omega^2 = 1/LC$. The "anti-resonance" frequency, corresponding to point A, is given by $\omega^2 = (1 + C/C_1)/LC$. C is less than 1 per cent of C_1, which means that R and A are very close together.

V.7. The fed-back voltage is $mV_1Z/(r + Z)$ where Z denotes

$$(R + j\omega L)(1/j\omega C)/\{R + j\omega L + 1/j\omega C\}$$

For sustained oscillation, this fed-back voltage must be correct in magnitude and phase to supply V_1. Equating it to V_1 and separating real and imaginary parts, we have

$$m = 1 + rRC/L \quad . \quad . \quad . \quad . \text{ Maintenance equation}$$
$$\omega^2 = \{1 - (m - 1)R/r\}/LC = (1 - Q^{-2})/LC$$

V.8. Not at all, since the amplifier has infinite input-impedance. (In practice there would be at least a finite input-capacitance, which would mean that the minimum amplifier-gain for sustained oscillation would have to be greater in the ratio $1 + C_{input}/C'$.)

V.9. Gain of each stage $= -m/(1 + j\omega Cr)$. Overall gain of three stages $= -m^3/(1 + j\omega Cr)^3$. This must be equal to 1, for sustained oscillation. Considering the phase angle of this overall gain-vector, $180° - 3\tan^{-1}\omega Cr = 0 \quad \therefore \tan^{-1}\omega Cr = 60°$ and $\omega = \sqrt{3}/Cr$. This gives the oscillation frequency.

Considering the magnitude of the overall gain vector $m^3/(1 + \omega^2C^2r^2)^{3/2} = 1$. Substituting $\sqrt{3}$ for ωCr, this gives $m > 2$, as the condition for sustained oscillation.

V.10. $r - R - j\omega L = \omega^2M^2/(R + j\omega L + 1/j\omega C)$. Cross-multiplying and separating real and imaginary parts, we have the oscillation frequency given by $\omega^2 = (1/LC)(r - R)/(r - 2R)$ and the following condition for sustained oscillation—

$$\frac{L}{RC} = \frac{(r - R)(r - 2R)}{k^2(r - R) - R} \approx \frac{r}{k^2}$$

Thus the approximate condition is $k^2 > r/R_D$.

V.11. (a) $L = (1/f_1^2 - 1/f_2^2)/4\pi^2(C_1 - C_2)$

(b) $C_0 = (C_1f_1^2 - C_2f_2^2)/(f_2^2 - f_1^2)$

V.12. This is the case where there is so much negative resistance that the circuit is non-oscillatory. The general solution is

$$i = Ae^{K_1t} + Be^{K_2t}$$

where both K_1 and K_2 are positive, being given by

$$-\frac{b}{2a}\{1 \pm \sqrt{1 - 4ac/b^2}\}$$

The situation is therefore unstable, the current rising at an ever-increasing rate. This is what occurs in a relaxation oscillator.

V.13. Fig. P–25 shows the incremental equivalent-circuit, from which it is seen that

$$V = R(g_mV_{g_2} + I) - R(g_mV_{g_1} - I)$$
$$V_{g_1} = -R(g_mV_{g_1} - I)$$
$$V_{g_2} = -R(g_mV_{g_2} + I) - R_Lg_mV_{g_1}$$

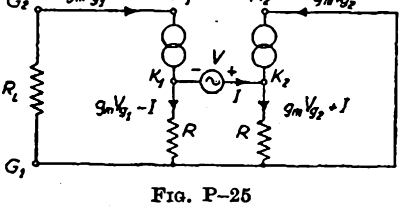


FIG. P–25

Elimination of V_{g_1} and V_{g_2} gives V/I as

$$\frac{R}{1 + g_mR}\left\{2 - \frac{g_m^2R\,R_L}{1 + g_mR}\right\}$$

Fig. P–26 shows a possible oscillator circuit.

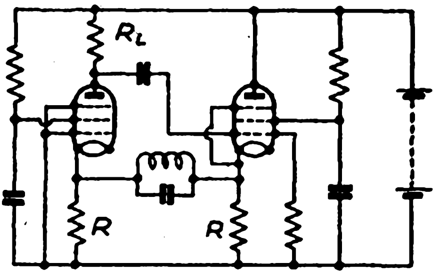


Fig. P–26

V.14. As explained on page 190, we may adopt an incremental equivalent circuit for a suppressor-controlled pentode. Apart from the fact that $G2$ and $G3$ take the place of anode and first grid, the circuit is very similar to the incremental equivalent circuit of a triode. The complete equivalent circuit of a closed-circuit oscillator using a suppressor-controlled pentode will thus be very similar to Fig. 97. But note that the e.m.f. $\mu' V_{sup}$ (see Fig. 110) will be directed *upwards*, in contrast to the e.m.f. μV_g in Fig. 97. The sign of μ will thus be changed in the resulting equations, as compared with those derived for triodes (page 161). In particular

$$X_{G2K} = -\mu' X_{G3K}$$

It follows that the reactances in the circuits of $G2$ and $G3$ must be of *opposite* sign. This gives two basic possibilities, viz. L and C, or C and L. In each of these cases we have a free choice as to whether the third reactance in the closed circuit (X_{G2G3}) shall be positive or negative. There are therefore *four* possible basic oscillator circuits when suppressor-controlled pentodes are used.

V.15. 200 v to 38 v.

V.16. It is explained on page 196 that the sudden rise of grid-voltage carries the grid slightly positive with respect to cathode but that thereafter the grid voltage decays rapidly (CDE, Fig. 114) to zero. The fall of v_a from D to B (Fig. P–13) corresponds to this rise of grid-voltage. To the right of point B, the slight rise of v_a corresponds to the rapid decay of v_g to zero.

The rise of anode-voltage from C to A corresponds to the fall of grid-voltage to a negative value greater than the cut-off value. The *anode-current* ceases immediately, but there remains some flow of current through the anode load resistance (say R, in Fig. 112) as a result of the charging current flowing to the condenser C. As this charging current decays to zero, the anode-voltage rises to the full h.t. voltage. This explains the "rounding off" of the waveform at A.

V.17. (i) 4 v

(ii) 200 v

(iii) The valve whose grid-voltage rises suddenly to zero, experiences a sudden fall of anode-voltage from 200 v to 52 v (value found for intersection of load line with the characteristic for $v_g = 0$). The grid-voltage of the other valve therefore falls to $-$ 148 v.

(iv) 0·01 second approx.

(v) 0·036 second

(vi) 13·8 c/s

V.18. The new half-period is given in microseconds by

$$500 \left(\log_e \frac{300}{255}\right) \Big/ \left(\log_e \frac{50}{5}\right)$$

Hence the new frequency is 14·18 Kc/s.

V.19. If v is the required pulse-amplitude, then $\left(\log_e \frac{50}{v+5}\right) \Big/ \left(\log_e \frac{50}{5}\right) = \frac{333}{500}$. This gives $v = 5{\cdot}8$ volts.

V.20. $E_c = 72$ v. Minimum effective pulse voltage = 28 v.

V.21. 0·784 v

V.22. (*a*) 5·7 mA (*b*) − 21 v (*c*) 114 v (*d*) 17·3 v

V.23. The circuit becomes the Schmitt trigger circuit (see page 204).

V.24. (*a*) When the left-hand valve conducts, it has $v_g = 0$, $i_a = 5$ mA. From the characteristics, $v_a = 65$ v; $\therefore E_{HT} = 65 + (5\text{ mA} \times 20\text{ K}) = 165$ v.

(*b*) The p.d. across R_c is 100 v. If the anode-current of the right-hand valve is cut off, its anode-voltage will be (165 − 100) v. From the characteristics, the cut-off value of v_g corresponding to this anode voltage is − 1·3 v. Thus the potential of the grid of the right-hand valve must be no more than 98·7 v above h.t. negative. $\therefore 165R''/(R' + R'')$ must be 98·7, i.e. $R'' = 746$ K.

(*c*) With anode-current cut off in the left-hand valve, the anode-voltage of the right-hand valve is $E_{HT} - (R_2 + R_c)i_a$. Thus we draw a load line for $E_{HT} = 165$ v and $(R_2 + R_c) = 40$ K. The intersection of this load line with the characteristic for $v_g = 0$ gives $i_a = 3$ mA. Thus the cathode is 60 v above h.t. negative and R'' must be chosen so that the grid is not less than 60 v above h.t. negative. $165R''/(R' + R'') = 60$, giving $R'' = 286$ K.

(*d*) In the quiescent condition, the grid of the left-hand valve, like its cathode, is 100 v above h.t. negative. This is thus the potential of the left-hand side of C. The right-hand side of C is at h.t. positive potential (i.e. 165 volts above h.t. negative). $\therefore v_c = 65$ v.

(*e*) Immediately after triggering, the potential of the cathodes is 60 v above h.t. negative. The left-hand grid is 65 v below the potential of the right-hand anode (since the condenser p.d. has not had time to change), i.e. 40 v above h.t. negative. Thus the grid-cathode voltage of the left-hand valve is − 20.

(*f*) The condenser p.d. is changing exponentially from 65 v (with the right-hand side positive) towards 60 v (with the left-hand side positive). v_g is therefore changing from − 20 v towards + 105, being given by

$$v_g = -20 + 125e^{-t/T}$$

where

$$T = R_gC = 10^{-4} \text{ sec.}$$

Throughout this period the anode-voltage of the left-hand valve remains at 105 v. From the characteristics, the cut-off grid-voltage corresponding to this anode-voltage, is − 2·1 v. The delay time, t, is therefore given by

$$-2{\cdot}1 = -20 + 125e^{-t/T}$$

which gives $t = 194$ microseconds.

V.25. (*a*) The characteristics show that, with this particular valve, the "constant-current" varies from 4 to 4·7 mA depending upon which part of the anode-voltage range is used. We shall assume that the current is constant at 4 mA. Since $i_a = C\,dv/dt$, $dv/dt = 4 \,.\, 10^5$ volt/sec.

(*b*) From 15 v to 150 v, viz. 135 v

(*c*) Sweep-time = $135/(4 \,.\, 10^5)$ = 337·5 microseconds

$\therefore$ frequency = 2·96 Kc/s

V.26. $v_{g\ cut\text{-}off} = -v_a/\mu = -v_{AB}/\mu$. Also $v_g = v_{AB} - R_2i_2$

$\therefore$ Required $v_{AB} = (R_2i_2)/(1 + 1/\mu)$

V.27. 500 c/s

V.28. We assume that, each time the condenser is discharged, it is discharged completely. The sweep-time is thus the time required to charge the condenser from zero to a p.d. given by $R_2i_2/(1 + 1/\mu)$. (See problem V.26.) This p.d. is approximately R_2i_2 if $\mu \gg 1$.

The charge on the condenser, corresponding to a p.d. R_2i_2, is CR_2i_2. Since the charging current is constant at a value i_1, the time taken to build up this charge is CR_2i_2/i_1. The time-base frequency (neglecting fly-back time) is therefore i_1/CR_2i_2.

VI.1. (*a*) Approximately between zero and $-2\hat{V}$.

(*b*) If E is in a direction such as to oppose conduction and is larger than $\hat{V}$, no current will flow at all; the limits of v_a will then be $-E+\hat{V}$ and $-E-\hat{V}$. If E is less than $\hat{V}$ or is in the conducting direction of the diode, the limits of v_a will be 0 and $-2\hat{V}$, as in part (*a*).

VI.2. (*a*) Note the similarity with Fig. 28 (*d*), page 41. By applying Thevenin's theorem to the circuit which feeds the diode one can reduce the problem to that of VI.1 (*b*) above. Except in cases where no anode-current flows at all, the limits of v_a will again be 0 and $-2\hat{V}$.

(*b*) The p.d. across R is v_a+E (or v_a-E, if the d.c. e.m.f. is in the conducting direction). But $v_a=-\hat{V}+\hat{V}\sin\omega t$. Hence $v_R=E-\hat{V}+\hat{V}\sin\omega t$. The mean current through R is therefore $(\hat{V}-E)/R$. N.B. If $E>\hat{V}$ and is in a direction to oppose conduction, no D.C. flows at all.

VI.3. The circuit can no longer function as a detector, since there is no path by which the added condenser can discharge. If a constant amplitude alternating e.m.f. were used as input voltage, v_1, then after a few cycles, all current would cease. The added condenser would then be charged up to a voltage equal to $\hat{V}_1$ and the original reservoir condenser would be uncharged. ($v_e=0$.)

VI.4. (*a*) Depending on the polarity of E, the condenser p.d. will be either $k(E+\hat{V}_1)$ or $k(-E+\hat{V}_1)$.

(*b*) Subtracting the two results from (*a*), above, gives $2kE$ as 8 v, whence $E=5$ v.

(*c*) Adding the same two results gives $2k\hat{V}_1$ as 28 v, whence $\hat{V}_1=17{\cdot}5$ v.

VI.5. v_c varies between 13·5 v and 4·5 v. v_a varies between + 0·5 v and − 28·5 v.

VI.6. 15 microseconds.

VI.7. $C=50$ pF, $R=65$ K.

VI.8. (*a*) The right-hand curve reaches the horizontal axis at a voltage of − 1·4. This means that a negative d.c. voltage of 1·4 v is just sufficient to prevent the flow of diode-current. The peak value of the applied alternating voltage must therefore be 1·4 v. This right-hand curve thus carries the label, "$V_{RMS}=1$ volt." Similarly the other curves are labelled, "$V_{RMS}=2$ v," "$V_{RMS}=3$ v," "$V_{RMS}=4$ v," etc.

(*b*) Draw a load line through the origin, having a slope of − (1/80,000) mho. The h.f. amplitude varies between 5 v and 1 v. The intersections of the load line with the curves labelled 5 v and 1 v have abscissae of 5·7 v and 1·2 v. Thus the peak amplitude of the a.f. output voltage of the detector is

$$\tfrac{1}{2}(5{\cdot}7-1{\cdot}2)=2{\cdot}25\text{ v}$$

(*c*) With the notation of equations (VI.1) and (VI.2), mkV_1 has been shown, in (*b*) above, to be 2·25 v. Since $m=0{\cdot}66$ and $V_1=3\sqrt{2}$, it follows that $k=2{\cdot}25/(2\sqrt{2})=0{\cdot}795$.

(*d*) The extreme limits of v_a are $-V(1+m)(1+k)$ and $V(1+m)(1-k)$, i.e. 12·7 v and 1·45 v.

VI.9. (*a*) 3·4 v (*b*) (5·3 − 3·4) = 1·9 v; (3·4 − 2·7) = 0·7 v
(*c*) $(5{\cdot}3-2{\cdot}7)/(4\sqrt{2})=46\%$

VI.10. The grid-bias voltage lies half-way between + 1 v and − 13 v, viz. − 6 v. The peak value of the a.c. component of grid-voltage is 7 v. $\therefore\ k=86\%$.

VI.11. (*a*) 1·05 v and 0·45 v
(*b*) + 0·35 v and − 2·45 v
(*c*) Yes (*d*) 8·3 v

VI.12. 144 v

VI.13. (*a*) $(12{\cdot}5/\sqrt{2})$ v r.m.s. (*b*) 22·5 v; $(22{\cdot}5/\sqrt{2})$ v r.m.s.
(*c*) $(25/\pi)$ mA
(*d*) No. There will be a smaller deflection on the larger range.

VI.14. (*a*) The diode voltmeter reads $1/\sqrt{2}$ of the peak value of the positive half-cycle, i.e. $10/\sqrt{2}$ v.

(*b*) The square-law voltmeter reads the r.m.s. value, i.e. 10 v.

(*c*) The anode-cut-off type of voltmeter reads $\pi/\sqrt{2}$ times the mean value (over a whole cycle) of the positive half-cycle, i.e. $5\pi/\sqrt{2}$ v.

VI.15. If, as is usual, $R \gg 1/\omega C$, then the Thevenin equivalent of the circuit which feeds the diode is identical with Fig. 127. It follows that the anode-voltage of the diode is the same in this alternative circuit as in the circuit of Fig. 127, namely $(-k\hat{V}_1 + \hat{V}_1 \sin \omega t)$. This is also the p.d. across R and the current through R is therefore $-k\hat{V}_1/R + (\hat{V}_1 \sin \omega t)/R$. The total current supplied by v_1 is the sum of this and i_a. Also $i_a = [i_a]_{DC} + [i_a]_{AC} = k\hat{V}_1/R + [i_a]_{AC}$. Thus the total current supplied by v_1 is $(\hat{V}_1 \sin \omega t)/R + [i_a]_{AC}$. In Fig. 127, the total current supplied by v_1 was $[i_a]_{DC} + [i_a]_{AC}$, of which the d.c. term contributed nothing to the power. Thus, in this alternative circuit, the power drain on v_1 is greater by an amount $\hat{V}_1^2/2R$ than in the circuit of Fig. 127. With the notation of page 226, therefore,

$$\hat{V}_1^2/2R_{input} = k\hat{V}_1^2/R + \hat{V}_1^2/2R$$

which gives R_{input} as $R/(1 + 2k)$.

VI.16. (*a*) (i) 50 c/s (ii) 3,000 v (iii) $6/\sqrt{2}$, step-up ratio
(*b*) (i) 100 c/s (ii) 3,000 v (iii) $12/\sqrt{2}$, secondary centre-tapped
(*c*) (i) 100 c/s (ii) $1{,}500\pi$ v (iii) $6\pi/\sqrt{2}$, secondary centre-tapped
(*d*) (i) 100 c/s (ii) 1,500 v (iii) $6/\sqrt{2}$.

VI.17. 1 amp

VI.18. About 50 volts. $C = 8{\cdot}5\ \mu$F.

VI.19. (*a*) 19,000 ohms. (*b*) No bleeder resistance is necessary.

VI.20. (*a*) $(\sqrt{2}/10)$ amp.

(*b*) $10{,}000\sqrt{2}/\pi$ volts.

(*c*) If the total resistance of the secondary winding be R_2, then the RI^2 loss with sine-wave current will be $\frac{1}{2}R_2\hat{I}_2^2$. In the rectifier, each half of the winding carries a current equal to the load-current, I_{DC}, for one-half of the time, and thus the RI^2 loss in each half of the secondary winding is $\frac{1}{2}(\frac{1}{2}R_2)I^2_{DC}$. For the same total RI^2 loss on A.C. and D.C. we have

$$2(\tfrac{1}{2})(\tfrac{1}{2}R_2)I_{DC}^2 = \tfrac{1}{2}R_2\hat{I}_2^2 \quad \therefore\ I_{DC} = \hat{I}_2 = (\sqrt{2}/10) \text{ amp}$$

(*d*) $(10{,}000\sqrt{2}/\pi)(\sqrt{2}/10) = 637$ watts.

VI.21. (1,500 watts)/(0·637) = 2·35 KVA

VI.22. $20/\pi$ volts

VI.23. 55 volts

VII.1. From 5·01 Mc/s to 4·99 Mc/s

VII.2. See equation on page 8. Peak phase-shift, $\hat{\theta}$, is $2\pi\,\delta f/p$. Hence δf is the product of $\hat{\theta}$ and the modulating frequency. $\delta f = 50$ Kc/s.

VII.3. (All figures in megacycles.)

(i) 0·02 and 0·04. The modulating frequency and second harmonic of same (square law detection).

(ii) 1·98, 2 and 2·02. Amplified version of the original, modulated voltage in the input. (Amplification.)

(iii) 2·98, 3 and 3·02. A 3-Mc/s difference-frequency component modulated at 20 Kc/s. (Frequency changing.)

(iv) 3·96, 4 and 4·04. Second harmonic distortion of the original modulated voltage. (Frequency multiplication.)

3·98, 4 and 4·02. Second harmonic of the carrier of the original modulated voltage, modulated at the original modulating frequency.

(v) 6·98, 7 and 7·02. A 7-Mc/s sum-frequency component modulated at 20 Kc/s. (Frequency changing.)

VII.4. (For harmonic analysis of square waves, refer to page 288.) The resulting voltage is the product of $\hat{V} \sin \omega t$ and a square pulse wave form.

This square pulse waveform has an ordinate + 1 from $t = 0$ to $t = 50$ microseconds, then zero ordinate for the next 50 microseconds, then ordinate + 1 again for the next 50 microseconds, and so on. The square pulse waveform can be resolved into a d.c. component, of value + 0·5, a fundamental 10-Kc/s sine-wave component of peak amplitude $2/\pi$ and a series of odd order harmonic components, having frequencies 30 Kc/s, 50 Kc/s, 70 Kc/s, etc., whose peak amplitudes are $2/3\pi$, $2/5\pi$, $2/7\pi$, etc. The frequencies and amplitudes of the sine-wave components *of the product of* $\hat{V}$ sin ωt *and this square pulse waveform* are thus as follows.

Kc/s . .	1,000	1,000 ± 10	1,000 ± 30	1,000 ± 50	etc.
Peak amplitude .	$\hat{V}/2$	$\hat{V}/\pi$	$\hat{V}/3\pi$	$\hat{V}/5\pi$	etc.

Warning: remember the *halving* which arises because

$$\sin \omega_1 t \sin \omega_2 t = \tfrac{1}{2} \cos (\omega_1 - \omega_2)t - \tfrac{1}{2} \cos (\omega_1 + \omega_2)t)$$

VII.5. (*a*) We need consider only the fundamental component of the on-off pulse waveform. Its frequency is f_{osc}. The product of this fundamental component with the signal sine-wave voltage will give components whose frequencies are $f_{osc} \pm f_{signal}$.

(*b*) The on-off waveform varies between the values 0 and + 1. The peak amplitude of its fundamental component is $2/\pi$. It is the product term $\hat{V}_1 \sin (\omega_{signal}\, t) \,.\, \frac{2}{\pi} \sin (\omega_{osc}\, t)$ which gives us the required difference frequency component of anode-current. Its peak amplitude will be $g_m \hat{V}_1/\pi$.

(*c*) $g_c = g_m/\pi$.

VII.6. The waveform in question is the product of sin ωt, i.e. a sine wave of frequency f, and an on-off waveform of the same frequency. The latter has a d.c. component of 0·5, a fundamental component whose peak amplitude is $2/\pi$, and odd-order harmonic components whose peak-amplitudes are $2/n\pi$ where n is the order of the harmonic. The product of sin ωt with the fundamental component gives $2f$ and zero as the sum and difference frequencies, the amplitude in each case being $1/\pi$. The product of sin ωt with the whole on-off waveform gives

$$(\sin \omega t)\left(\frac{1}{2} + \frac{2}{\pi}\sin \omega t + \frac{2}{3\pi}\sin 3\omega t + \frac{2}{5\pi}\sin 5\omega t + \ldots\right)$$

$$= \frac{1}{2}\sin \omega t + \left(\frac{1}{\pi} - \frac{1}{\pi}\cos 2\omega t\right) + \left(\frac{1}{3\pi}\cos 2\omega t - \frac{1}{3\pi}\cos 4\omega t\right)$$

$$+ \left(\frac{1}{5\pi}\cos 4\omega t - \frac{1}{5\pi}\cos 6\omega t\right) + \ldots$$

$$= \frac{1}{\pi} + \frac{1}{2}\sin \omega t - \frac{2}{3\pi}\cos 2\omega t - \frac{2}{15\pi}\cos 4\omega t - \frac{2}{35\pi}\cos 6\omega t - \ldots$$

Thus the relative amplitudes of the fundamental and harmonics are: Fundamental, 1; 2nd harmonic, $4/3\pi$; 3rd harmonic, zero; 4th harmonic, $4/15\pi$; 5th harmonic, zero; etc.

(Cf. the tabulated values on page 242.)

VII.7. (*a*) If $i_a = a + bv_g + cv_g^2$ and $v_g = V_1 \sin \omega_1 t$, then

$$i_a = a + \tfrac{1}{2}cV_1^2 + bV_1 \sin \omega t - \tfrac{1}{2}cV_1^2 \cos 2\omega t$$

From the data provided, $\frac{1}{2}cV_1^2$ is 0·25 mA. Thus the percentage of 2nd harmonic is 100(0·25 mA)/(2 mA), i.e. 12·5%.

(*b*) $v_g = V_1 \sin \omega t + V_p \sin pt$ where $V_1 = 2$ v and $V_p = 0{\cdot}6$ v. The carrier frequency component of i_a is $bV_1 \sin \omega t$. The sideband components are $cV_1V_p \cos (\omega - p)t - cV_1V_p \cos (\omega + p)t$. By comparison with equation (VII.2), we see that the modulation depth, m, is $2cV_p/b$. But $\frac{1}{2}cV_1/b$ is 20%.

$\therefore\ m = 0{\cdot}8V_p/V_1 = 24\%$.

VII.8. (*a*) From equation (VII.7), $\partial i_a/\partial v_{g_1} = (\mu_1 + \mu_c v_{g_2})/\rho$.

(*b*) The extreme values of v_{g_2} are $(-E + \hat{V}_2)$ and $(-E - \hat{V}_2)$. The extreme values of the slope are $(\mu_1 - \mu_c E + \mu_c V_2)/\rho$ and $(\mu_1 - \mu_c E - \mu_c V_2)/\rho$.

(*c*) The difference between these extreme values of slope is $2\mu_c V_2/\rho$. On page 273, it is shown that the conversion conductance, g_c, is $\mu_c V_2/2\rho$ which is therefore one-quarter of the difference between the extreme values of slope.

VII.9. (*a*) 4·5 v (*b*) 4·5 v (*c*) $\frac{3}{4}$ mA per volt
(*d*) $(1{\cdot}5/\pi)$ mA per volt. (See problem VII.5.)

VII.10. (*a*) The modulation depth is 0·8. $V_{r.m.s.} = \dfrac{100}{\sqrt{2}}\sqrt{1 + 0{\cdot}32} = 81{\cdot}2$ v

(*b*) $m^2/2 = 32\%$.

(*c*) The r.m.s. value of an f.m. voltage is the same as if it were not modulated, viz. in this case $100/\sqrt{2}$ v.

VII.11. (*a*) The same as before, viz. $\frac{1}{4}$ amp
(*b*) 150 mA (*c*) 60%
(*d*) 250 watts (*e*) 45 watts
(*f*) Each ratio is 18%

VII.12. The coil L takes the place of the phase-splitting condenser C in Fig. 165. A blocking condenser will be needed in the lead from the junction of L and R. If the values are so chosen that, at the carrier frequency, $R \gg \omega L \gg R/\mu$, then the impedance looking into terminals AA will be approximately $R/j\omega L g_m$ (i.e. phase angle $= -90°$ as for a capacitance).

VII.13. To satisfy the requirements that $R \gg 1/\omega C \gg R/\mu$, make $1/\omega C = R/\sqrt{\mu}$, i.e. 20,000 ohms. This gives $C = 15{\cdot}9$ pF. The resulting inductive reactance is $\omega CR/g_m = \sqrt{\mu}/g_m = 30{,}000$ ohms.

VII.14. The 10 per cent provided by the reactance valve changes to 20 per cent when g_m is halved, and the oscillator frequency therefore changes in the ratio $\sqrt{1{\cdot}1/1{\cdot}2}$, i.e. 0·957. This gives a total frequency swing from 1 Mc/s to 957 Kc/s. (The question could also be construed as meaning that the added reactance varies from 10 per cent to 5 per cent, in which case the oscillator frequency would swing between 1 Mc/s and 1·026 Mc/s.)

VII.15. $f_o\sqrt{1 + g_m L/CR} \approx f_0(1 + g_m L/2CR)$, if $g_m L/CR \ll 1$.

VII.16. (*a*) $Z_{in} \approx 1/j\omega C R g_m$ if the conditions in (*b*) below are satisfied.

(*b*) $R \ll 1/\omega C \ll \mu R$.

(*c*) They will give the same magnitude of input reactance, viz. $\sqrt{\mu}/g_m$.

VII.17. (*a*) $Z_{AA} = \rho(1 + j\omega CR)/(1 + \mu + j\omega CR)$.
(*b*) Phase angle of Z_{AA} is $90° - \tan^{-1} 2/\sqrt{\mu}$.

VII.18. Such variations will cause the mutual-conductance g_m of the reactance-valve to vary, and will thus introduce unwanted variations of frequency of the transmitter. All f.m. transmitters using a reactance valve employ a special correcting circuit to overcome this trouble. The special circuit has the function of comparing the transmitter frequency with that of a built-in crystal oscillator, using a discriminator for this purpose (see page 252). Any error in the transmitter frequency gives a proportionate d.c. output voltage from the discriminator and this is fed to the bias circuit of the reactance valve in order to correct the error.

VII.19. By connecting the modulating voltage in the circuit of G_3, and using an R–C phase-splitting circuit, connected from anode to cathode, to provide the a.c. component of v_{g_1}.

VII.20. (*a*) Increasing the intermediate frequency reduces the overall selectivity.

(*b*) Increasing the intermediate frequency reduces the necessary pre-mixer selectivity required to avoid second-channel interference.

The advantage of making the oscillator frequency greater than the signal frequency is that, in tuning the receiver through a given range of frequency, a smaller percentage variation of tuning capacitance is required. This means that the variation of oscillator *amplitude*, from one end of the tuning range to the other, will be reduced.

VII.21. The suggested method would *not* be satisfactory. Instead of giving a constant *difference* between the signal-tuning frequency and the oscillator frequency, it would give a constant *ratio* of these two frequencies. To secure a constant difference-frequency, additional fixed condensers must be added to the two circuits. The resulting "tracking" of the two frequencies is not exact, but is sufficiently accurate for practical purposes.

VII.22. If the amplitude of one h.f. oscillator is small compared to the amplitude of the other, the difference-frequency output of the frequency-changer will be independent of the larger amplitude. Since the amplitude of the variable frequency h.f. oscillations varies with tuning, constancy of output can be achieved by making this amplitude larger than that of the fixed-frequency oscillations.

VIII.1. 80%

VIII.2. (*a*) 99% (*b*) 6·67% (*c*) 0·3%

VIII.3. 1·1 Mc/s

VIII.4. See equation (VIII.4). 0·15 v and 2·96 v

VIII.5. As $RC \to \infty$, $e^{t'/RC} \to (1 + t'/RC)$. Thus the whole expression for v tends to $\hat{V}t'/T$. The condenser voltage rises and falls between this value and a value which is $e^{(T - t')/RC}$ times as great. But when $RC \to \infty$, $e^{(T - t')/RC} \to 1$. Thus the two limiting values of voltage coincide and the condenser voltage merely remains constant at the value $\hat{V}t'/T$. The circuit is then acting as a perfect a.c./d.c. separating circuit, since $\hat{V}t'/T$ is in fact the d.c. component of the pulse input voltage.

VIII.6. From $t = 0$ to $t = 10$ microseconds, $v = 5$ volts. For the next 490 microseconds, $v = 10$ volts. For the next 10 microseconds, $v = 5$ volts. For the next 490 microseconds, $v = 0$. The cycle then repeats itself.

VIII.7. (*a*) The statement that, at a frequency of 100 Kc/s, the length of the line is one-quarter of a wavelength means that in 10 microseconds (i.e. the periodic time corresponding to a frequency of 100 Kc/s) a wave could travel four times the length of the line. The delay time, $2\,l/c$, is the time taken by a wave in travelling *twice* the length of the line. The delay time is thus 5 microseconds and this is also the pulse duration. Duty ratio = (pulse duration)/(repetition period) = 5/1,000.

(*b*) The value of the h.t. voltage is not stated, but the characteristics show that, whatever the h.t. voltage, the anode-current will be about 25 mA when $v_g = 0$. Thus the output pulse amplitude is $\frac{1}{2}(1{,}000)(0{\cdot}025)$ v = 12·5 v.

VIII.8. (*a*) Between 48 v and 200 v

(*b*) 27 v

(*c*) 27 v, but reversed polarity.

(*d*) See Fig. P–27.

VIII.9. Consider circuit (*a*) in Fig. P–15. If the diode is conducting, it may be replaced by a short-circuit. The circuit then shows that the input voltage is connected directly across the output terminals, AB. Hence, whenever the diode is conducting, $v_{AB} = 100 \sin \omega t$. When the diode is not conducting, $v_{AB} = -60$ v.

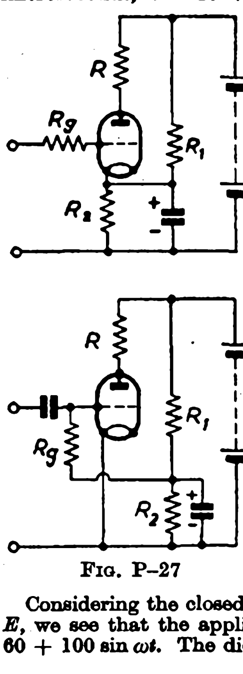


Fig. P–27

Considering the closed circuit formed of the input source, the diode, R and E, we see that the applied e.m.f. in the conducting direction of the diode is $60 + 100 \sin \omega t$. The diode will conduct only when this quantity is positive, i.e. whenever $100 \sin \omega t > -60$. The v_{AB} waveform for circuit (*a*) is thus as shown in Fig. P–28 (*a*). Corresponding waveforms for circuits (*b*) to (*d*) are shown in Fig. P–28 (*b*), (*c*) and (*d*).

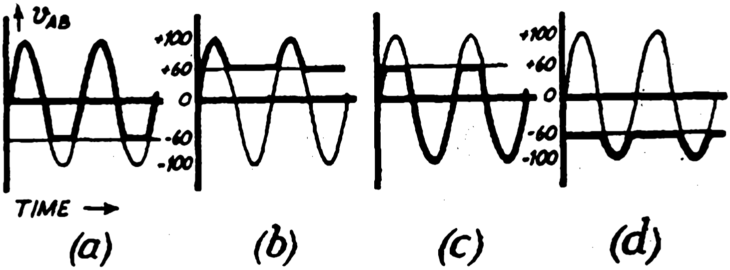


FIG. P–28

A useful rule for such circuits is that, if terminal A is connected to the *anode* of the diode, the potential of A will be prevented from rising above a limiting value given by E, whereas if terminal A is connected to the *cathode* of the diode, the potential of A will be prevented from falling below a limiting value given by E.

VIII.10. There will be no change in the times for which the various diodes conduct. There will be no change in value of v_{AB} during the periods when the diodes are not conducting. During the conducting periods, however, v_{AB} will be changed. Consider circuit (*a*) with the diode conducting. The anode-current during such a period is given by $(60 + 100 \sin \omega t)/(20{,}000)$. Hence v_{AB} is $(Ri_a - 60)$, i.e. $-60 + (18/20)(60 + 100 \sin \omega t)$ which is $(-6 + 90 \sin \omega t)$. The maximum positive excursion of v_{AB} will be reduced to 84 v.

(*b*) v_{AB} varies between 96 v and 60 v.
(*c*) v_{AB} varies between 60 v and − 84 v.
(*d*) v_{AB} varies between − 60 v and − 96 v.

VIII.11. These four circuits differ from the four circuits of problem VIII.9 in that the diodes, by conducting, "short circuit" the output voltage, whereas in the circuits of problem VIII.9 the diodes, by conducting, communicate the input voltage to the output terminals.

Consider circuit (*a*) in Fig. P–16. When the diode conducts, $v_{AB} = -60$ v. When the diode does not conduct, $v_{AB} = 100 \sin \omega t$. By considering the closed loop formed of the input source, R, the diode and E, we see that the e.m.f. applied in the conducting direction is $60 + 100 \sin \omega t$. Thus the diode will conduct (and v_{AB} will be − 60 v) whenever $100 \sin \omega t$ is greater than − 60 v. The waveforms of v_{AB} for these four circuits are given by the same graphs as in Fig. P–28.

VIII.12. The parts of the four waveforms which correspond to the non-conducting periods are unchanged. The flat portions of the waveform become slightly curved. The extreme values of v_{AB} are as follows—

(*a*) + 100 v, − 62 v   (*b*) + 100 v, + 52 v
(*c*) + 62 v, − 100 v   (*d*) − 52 v, − 100 v

VIII.13. The large alternating input voltage serves to swing the grid-voltage from a negative value, greater than the cut-off value, to a very small positive value (approximately zero, if R_g is large enough). The lower limit of anode-voltage is, however, brought about by bottoming (see page 311). The limits of anode-voltage are (*a*) 28 to 300, (*b*) 17 to 300, (*c*) 12 to 300.

VIII.14. (*a*) Draw a load line for an h.t. voltage of 200 and a resistance of 40K. For each intersection of the load line with the characteristic curve read off v_a and calculate v_{OUT} as $(E_a - v_a)$. Calculate v_{IN} as $(E_a - v_a + v_g)$ Plot v_{OUT} against v_{IN}.

v_{IN}	98	57	16	− 25	− 54	More negative still.
v_{OUT}	98	58	18	− 22	− 50	− 50

(*b*) The 2-megohm resistance in the grid-lead serves to prevent the grid-potential from rising more than a fraction of a volt above zero. The left-hand entry in the above table corresponds to $v_g = 0$. The output voltage will therefore range from − 50 v to + 98 v.

VIII.15. See Fig. P–29 (*a*) and (*b*). These waveforms are deduced by noting that, when the diode conducts the p.d. across the "switch terminals" will be almost exactly equal to the e.m.f. of the low-impedance source. The diode will conduct when the total circulatory e.m.f. in the conducting direction is positive, i.e. when the O.V. terminal e.m.f. exceeds the S.T. e.m.f. in circuit (*a*), and *vice versa* in circuit (*b*).

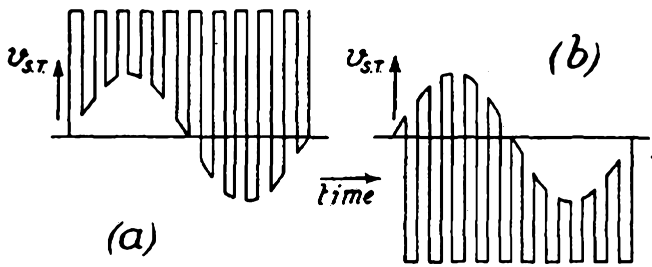


FIG. P–29

VIII.16. It is clear that, for circuit (*c*) the grid-cathode voltage is alternately zero and more negative than the cut-off value. The same can be seen to be true for circuit (*d*). (The grid-cathode e.m.f. is $v_{GA} + v_{AK} = v_{O.V.} - v_{S.T.}$, and is thus the difference between an 80-volt square wave and a 50-volt sine wave. During the positive half-cycle of the square wave, the e.m.f. applied between *G* and *K* lies somewhere in the range 80 ± 50 v. This applied e.m.f. is positive, but the resistance in the grid-lead keeps v_g at approximately zero. During the negative half-cycle of the square wave, the e.m.f. applied between *G* and *K* lies somewhere in the range − 80 ± 50 v and anode-current is cut-off.)

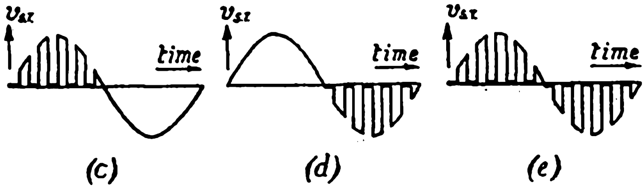


FIG. P–30

When anode-current is cut off, the p.d. between the "switch terminals" is equal to the sine-wave e.m.f. applied across these terminals. In circuits (c) and (*d*) the whole of one half-cycle of this sine wave is reproduced at the "switch terminals," because during this period the *anode*-voltage of the triode is negative. Fig. P–30 shows the waveform of the p.d. across the switch-terminals.

In circuit (*e*), one of the two anode-voltages is always positive and anode-current can be cut-off only by v_g being further negative than the cut-off value. This is the case during each negative half-cycle of the square wave. (See Fig. P–30 (*e*).)

In all three circuits the sine-wave source has a high internal impedance and, if a load line were drawn, its slope would be very small. It follows that, when anode-current flows, the anode-voltage is nearly zero.

VIII.17. (*a*) Triode anode-current is never cut off. The p.d. of the diode-cathode above h.t. negative alternates between + 4 v and + 90 v at a frequency of 100 Kc/s. When it is 90 v the diode is non-conducting and the p.d. across the switch terminals is the e.m.f. of the 10 Kc/s sine wave. When it is + 4 v, the p.d. across the switch terminals is either + 4 v or the sine-wave e.m.f. whichever is less at the instant in question. It follows that the whole of the negative half cycle of the sine wave is reproduced. See Fig. P–31 (*a*).

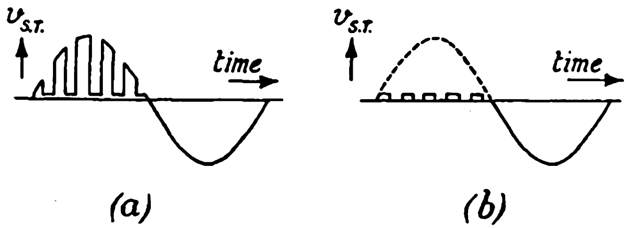


FIG. P–31

(*b*) With negative pulses applied to the O.V. terminals the potential of the diode-cathode above h.t. negative will alternate between + 4 v and zero. The diode will conduct for almost the whole of the positive half-cycle of the sine-wave voltage, but will be non-conducting throughout the negative half-cycle of the sine wave. (The output waveform, Fig. P–31 (*b*) is of only academic interest!)

(*c*) Similar to (*a*), except that the potential of the diode-cathode now alternates between + 90 v and zero.

VIII.18. (*a*) 32 v (*b*) 200 v

(*c*) Anode-current is cut-off for 100 microseconds every 20,000 microseconds. The time constant with anode-current cut off is 100 microseconds. During the cut-off period, the condenser p.d. rises exponentially from 32 v to 94 v. When anode-current re-commences, the time-constant is less than 10 microseconds (see problem No. II.25) and the condenser p.d. rapidly falls again to 32 v and remains at this value for nearly 20 milliseconds.

(*d*) Anode-current flows for only 100 microseconds every 20,000 microseconds. The condenser p.d. falls rapidly from 200 v to 32 v when anode-current commences. When anode-current is cut off, the condenser p.d. rises towards 200 v with a time constant of 100 microseconds. In ½ millisecond it will have reached about 199 v. For the remaining 19½ milliseconds of the cycle it is substantially at 200 v.

VIII.19. The resistance $rR/(r + R)$ at the top of Fig. 202 (c) must be changed to $\rho R/(\rho + R)$ and the d.c. e.m.f. at the right-hand side of the diagram becomes $(E_a - dR)\rho/(\rho + R)$.

VIII.20. (*a*) The equivalent circuit is as in Fig. 202 (*c*), page 318, with the modifications suggested in problem VIII.19 above. For this valve, $\rho = 10$ K, $\mu = 50$, $d = -1{\cdot}5$ mA. The effective e.m.f. in this equivalent, single-loop circuit is $(200 + 15 \times 1{\cdot}5)(10/25) = 89$ v. The effective series resistance is the parallel combination of 10 K and 15 K, i.e. 6 K. The time constant is thus 6 milliseconds.

$$\therefore \; v_c = 89 + (200 - 89)e^{-\frac{1}{2}} = 156{\cdot}3 \text{ v}$$

(b) For rising anode-voltage, the time constant is 15 K × 1 μF, i.e. 15 milliseconds. The anode-voltage at the instant of cut-off is 89 v.

$$\therefore \; v_a = 200 + (89 - 200)e^{-\frac{1}{2}} = 132{\cdot}7 \text{ v}$$

VIII.21. (a) Before cut-off, $v_a = 129$ v (from load line). Thus the p.d. across C is 129 v. Immediately after cut-off, the charging current flowing through the anode load resistance is (200 − 129)/(250 K + 50 K) and the drop across the 50-K load resistance is thus (50/300)(71), i.e. 11·8 v. Hence the immediate sudden rise of v_a is from 129 v to 188·2 v.

(b) The time constant of the charging process is (300 K × 0·1 μF), i.e. 30 milliseconds.

$$\therefore \; v_a = 200 + (188{\cdot}2 - 200)e^{-\frac{1}{2}} = 193 \text{ v}$$

VIII.22. (a) Time constant, $T = C\left(R_g + \dfrac{\rho R_L}{\rho + R_L}\right)$

$$= (0{\cdot}1\ \mu\text{F})\left(250\text{ K} + \frac{50 \times 10}{60}\text{ K}\right) = 25{\cdot}8 \text{ milliseconds.}$$

(b) The charging e.m.f. (cf. problem VIII.19 and Fig. 205 (c)) is $(E_a\rho + v_oR_L)/(\rho + R_L)$, i.e. 7,750/60 = 129·2 v. The condenser voltage at the instant of restoration of anode-current is the full h.t. voltage, 200 v. Thus the p.d. across the coupling resistance is $(200 - 129)(250/258)e^{-t/T}$. Taking T as 25·8 milliseconds and t as 12·9 milliseconds this gives $v_2 = 41{\cdot}6$ v.

VIII.23. 99 microfarads (see page 327).

VIII.24. (a) Draw a load line with intercepts 200 v and 10 mA. The intersection with the characteristic for $v_g = 0$ gives $i_{max} = 6{\cdot}2$ mA. The mean anode-current, si_{max}, is thus only 6·2 microamp, and the voltage drop across the decoupling resistance is negligible (only 124 millivolt). With i_a cut-off, $v_a = 200$ v. With $i_a = 6{\cdot}2$ mA, $v_a = 200 - (20\text{ K})(6{\cdot}2\text{ mA}) = 76$ v.

(b) Draw a load line with intercepts 200 v and 6·7 mA. The intersection with the characteristic for $v_g = 0$ gives $i_{max} = 4{\cdot}6$ mA. The mean anode-current is thus 2·3 mA and the d.c. p.d. across the decoupling resistance is (20 K)(2·3 mA) = 46 v. When i_a is cut off, $v_a = 200 - 46 = 154$ v. When i_{max} is flowing, the drop across the 20-K load is 92 v and thus the anode-voltage is (154 − 92), i.e. 62 v. Note that this value cannot be read from the load line as the peak current of 4·6 mA does not flow through the 20-K decoupling resistance.

VIII.25. (a) 3·7 mA (b) 37 v (c) 89 v and 200 v
(d) Positive input pulses are required, of magnitude at least 37 v.

VIII.26. (a) At very low frequencies, the condenser has negligible effect, except to produce a slight rounding of the corners of the waveform. $i_a = \dfrac{1}{\rho}(v_a + \mu v_g) + d$ where $v_a = E - Ri_a$ and $v_g = v_{IN} - Ri_a$ whence $i_a = (E + \mu v_{IN} + \rho d)/(\rho + R + \mu R)$. The limiting values of Ri_a correspond to $v_{IN} = 0$ and $v_{IN} = \hat{V}$ and are therefore given by

$$\frac{R(E + \rho d)}{\rho + R(1 + \mu)} \quad \text{and} \quad \frac{R(E + \mu\hat{V} + \rho d)}{\rho + R(1 + \mu)}$$

(b) At very high frequencies, the current through R is almost pure D.C. and thus the condenser p.d. does not vary between two limits but is constant. To determine its value, note that

$$v_a = E - v_c$$
$$v_g = v_{IN} - v_c$$

and

$$v_c = Ri_{mean} = R[i_{min} + s(i_{max} - i_{min})]$$
$$= R[si_{max} + (1 - s)i_{min}]$$

Using the equation of the characteristics to derive i_{max} and i_{min} in terms of v_c, and solving, we have

$$v_c = \frac{R(E + s\mu\hat{V} + \rho d)}{\rho + R(1 + \mu)}$$

(c) The condition that $i_{min} = 0$ is

$$\hat{V} > \frac{E + \rho d}{s g_m R(1 + \mu)}$$

VIII.27. (a) 52 volts (from load-line diagram).

(b) $A = \mu R'/(\rho + R') = 40$. (This could also be determined from the load-line diagram.) The time constant is 41 RC, i.e. 41 seconds. The "aiming value" of the exponential fall of v_a is $(200 - 40\delta v_1)$ and v_1 has changed from the cut-off value ($-$ 4 v) to + 200 v. The "aiming voltage" is therefore (200 $-$ 8,160). The time, t, for v_a to fall from 200 v to 52 v is thus given by

$$(200 - 52) = 8{,}160(1 - e^{-t/T})$$

whence $t = 755$ milliseconds.

(c) Since $v_a = 200 - 8{,}160(1 - e^{-t/T})$, the rate of fall of v_a is given by $-dv_a/dt = (8{,}160/T)e^{-t/T}$ and its value at $t = 0$ is $8{,}160/T$, i.e. 199 v/second.

(d) (200 $-$ 52)/199 = 744 milliseconds. Note that a comparison of the answers to parts (b) and (d) of this question gives a measure of the linearity of the run-down of anode-voltage.

VIII.28. (a) The series combination of a resistance $R/(1 + A)$ and a capacitance $C(1 + A)$.

(b) A sudden fall of voltage, of magnitude $AER_1/\{R_1 + R(1 + A)\}$, followed by an exponential fall, of magnitude $AER(1 + A)/\{R_1 + R(1 + A)\}$. The total fall is AE. The time constant of the exponential fall is

$$C\{R_1 + R(1 + A)\}.$$

VIII.29. (a) First draw the orthodox equivalent circuit ($\mu\delta v_g$, ρ). Then combine the elements $\mu\delta v_g$, ρ and R' by Thevenin's theorem, giving a single-loop circuit formed of δv_1, R, C, $A\delta v_g$ and r. If i is the current in this circuit then $\delta v_g = \delta v_1 - Ri$. Substitution of this for δv_g leads to a circuit equation identical with that of the given equivalent circuit.

(b) The section from cathode to grid of the given equivalent circuit, and the section from grid to anode, are unchanged sections of the original circuit of Fig. 216. It is therefore legitimate to show the new resistance (say R_1) in series with C, between grid and anode. The circuit equation then becomes

$$\delta v_1(1 + A) = \{r + R_1 + R(1 + A) + 1/pC\}i$$

in which p denotes d/dt and $1/p$ denotes integration with respect to time. The output voltage is the p.d. between anode and cathode and is given by

$$\delta v_2 = \delta v_1 - (R + R_1 + 1/pC)i$$

Substituting for i in this, from the equation above, we have

$$\delta v_2 = (-A\delta v_1)\frac{R_1 - r/A + 1/pC}{r + R_1 + R(1 + A) + 1/pC}$$

By making R_1 equal to r/A (which is equal to $1/g_m$) we simplify δv_2 to the following expression, the form of which shows that, when δv_1 is a step-function, there will be no sudden jump of anode-voltage

$$\delta v = \frac{-A\delta v_1}{1 + pC\{r + R_1 + R(1 + A)\}}$$

VIII.30. (*a*) The completed table is as shown.

No. of input pulses	First bi-stable pair		Second bi-stable pair		Third bi-stable pair		Fourth bi-stable pair	
	L	*R*	*L*	*R*	*L*	*R*	*L*	*R*
0	0	1	0	1	0	1	0	1
1	1	0	0	1	0	1	0	1
2	0	1	1	0	0	1	0	1
3	1	0	1	0	0	1	0	1
4	0	1	0	1	1	0	0	1
5	1	0	0	1	1	0	0	1
6	0	1	1	0	1	0	0	1
7	1	0	1	0	1	0	0	1
8	0	1	0	1	0	1	1	0
9	1	0	0	1	0	1	1	0
10	0	1	1	0	0	1	1	0
11	1	0	1	0	0	1	1	0
12	0	1	0	1	1	0	1	0
13	1	0	0	1	1	0	1	0
14	0	1	1	0	1	0	1	0
15	1	0	1	0	1	0	1	0
16	0	1	0	1	0	1	0	1

(*b*) The arrows show where the *effective* triggering pulses originate and to which valve they are fed. Every arrow must therefore start from a 1 and go to a 0.

(*c*) Lines 0, 1, 2 and 3 would be unaltered. Line 4 would read (0, 1) (0, 1) (0, 1) (0, 1) which is the same as line 0. Four pulses would thus take the circuit through one complete cycle. The last four valves would be wasted.

(*d*) Lines 0–7 inclusive would be unaltered. Line 8 would read the same as line 0. The last two valves would be wasted.

(*e*) The second, fourth, and sixth pulses all cause a sudden restoration of current in the right-hand valve of the first bi-stable pair, and thus communicate (feed forward) a negative voltage-step to the right-hand triode-anode of the fourth pair. Since this valve is carrying current, however, the fed-forward pulses are ineffective and the fourth bi-stable pair is not triggered by them. The same is true of the events immediately following the eighth input pulse, since the fed-forward voltage-step arrives at the fourth pair *before* this pair is triggered by the third pair. The table is thus unaltered right up to the arrival of the tenth pulse. This tenth pulse produces the situation (0, 1) (1, 0) (0, 1) (0, 1) which is the same situation as was produced by the second pulse (see the third line of the table). The eleventh input pulse thus produces the same situation as did the third pulse, and so on. The eighteenth input pulse produces the same situation as did the tenth and the second. Clearly eight input pulses are sufficient to take the circuit through one complete cycle of operation.

(*f*) As explained in (*e*) above, the triggering-sequence table remains unchanged until the arrival of the eighth input pulse. This pulse causes all four bi-stable pairs to be triggered in succession. The pulse fed forward from the first pair to the fourth pair is ineffective because it arrives at the right-hand anode before anode-current is cut off. The triggering of the fourth pair by the third pair, following the arrival of this eighth input pulse, causes a negative voltage-step to be fed back to the left-hand anode of the second pair. The left-hand anode-current has already been cut off when the fed-back voltage-step arrives and the second pair is therefore triggered for a second time. The final situation after eight input pulses, is (0, 1) (1, 0) (0, 1) (1, 0). After nine input pulses we have (1, 0) (1, 0) (0, 1) (1, 0). The tenth input pulse triggers the first pair, which in turn triggers both the second and fourth pairs, giving the situation (0, 1) (0, 1) (0, 1) (0, 1). This is the same as the starting condition in the first line of the table. Thus ten input pulses suffice to take the whole circuit through one cycle of operation, and to give one final output pulse.

INDEX

THERMIONIC VALVE CIRCUITS - WILLIAMS

PITMAN