MANUAL OF

MATHEMATICS

RULES LAWS CHARTS

CALCULATIONS
DEFINITIONS
FORMULAE
ON ARITHMETIC
TRIGONOMETRY
GEOMETRY
ALGEBRA
CALCULUS.

1/-

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ALGEBRA

POWERS AND ROOTS

(1) $a^0 = 1$. (2) $(\sqrt[3]{a})^3 = a$. (3) $\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$.

(4) $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$. (5) $\sqrt[3]{a} \times \sqrt[3]{a} \times \sqrt[3]{a} = a$. (6) $\sqrt[3]{a^2} = a^{\frac{2}{3}}$.

(7) $a^m b^m = (ab)^m$. (8) $\sqrt[3]{\frac{1}{a}} = a^{\frac{1}{3}}$. (9) $\sqrt[n]{a} \div \sqrt[n]{b} = \sqrt[n]{\frac{a}{b}}$.

(10) $a^m \times a^n = a^{m+n}$. (11) $a^m \div b^m = \left(\frac{a}{b}\right)^m$. (12) $a^{\frac{1}{n}} = \sqrt[n]{a}$.

(13) $a^{\frac{m}{n}} = \sqrt[n]{a^m}$. (14) $\frac{1}{a^{-n}} = a^n$. (15) $a^{-n} = \frac{1}{a^n}$.

(16) $a^m = \sqrt[\frac{1}{m}]{a}$. (17) $a^m \div a^n = a^{m-n}$. (18) $\sqrt[n]{a} \times \sqrt[m]{b} = \sqrt[nm]{a^m b^n}$.

(19) $a\sqrt[n]{b} = \sqrt[n]{a^n b}$

FACTORS

$(x+a)(x+b) = x^2 + x(a+b) + ab$.

$(x+a)(x-b) = x^2 + x(a-b) - ab$.

$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$.

$(a \pm b)^2 = a^2 \pm 2ab + b^2$.

$(a^3 \pm b^3) \div (a \pm b) = a^2 \mp ab + b^2$.

$a^2 - b^2 = (a+b)(a-b)$.

$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$.

$a^4 + a^2b^2 + b^4 = (a^2 + ab + b^2)(a^2 - ab + b^2)$.

$(a^n - b^n) \div (a-b) = a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots\ldots + ab^{n-2} + b^{n-1}$.

$a^2 + b^2 + c^2 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ac)$.

RATIO AND PROPORTION

if $a:b::c:d$ then-, (1) $ad = bc$. (2) $\frac{a}{b} = \frac{c}{d}$. (3) $\frac{b}{a} = \frac{d}{c}$

(4) $\frac{a-b}{b} = \frac{c-d}{d}$ (5) $\frac{a+b}{b} = \frac{c+d}{d}$ (6) $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

QUADRATIC EQUATION

If $x^2 + px = q$ then $x = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} + q}$.

or if $x^2 - px + q = 0$ then $x = \frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$.

or if $ax^2 + bx = c$ then $x = \frac{-b \pm \sqrt{b^2 + 4ac}}{2a}$

and if $x + y = s$ and $xy = p$ then $x = \frac{s + \sqrt{s^2 - 4p}}{2}$

and $y = \frac{s - \sqrt{s^2 - 4p}}{2}$

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CUBIC EQUATION

If $x^3 + ax + b = 0$ then Cardens Solution gives

$$x = \left\{-\frac{b}{2} + \sqrt{\frac{a^3}{27} + \frac{b^2}{4}}\right\}^{\frac{1}{3}} + \left\{-\frac{b}{2} - \sqrt{\frac{a^3}{27} + \frac{b^2}{4}}\right\}^{\frac{1}{3}}$$

BINOMIAL THEOREM

$$(a+x)^n = a^n + na^{n-1}x + \frac{n(n-1)}{1\times 2}a^{n-2}x^2 + \frac{n(n-1)(n-2)}{1\times 2\times 3}a^{n-3}x^3 + \dots\dots x^n.$$

example $(a+x)^5 = a^5 + 5a^4x + \frac{5\times 4}{1\times 2}a^3x^2 + \frac{5\times 4\times 3}{1\times 2\times 3}a^2x^3 + \frac{5\times 4\times 3\times 2}{1\times 2\times 3\times 4}ax^4 + x^5 =$

$$a^5 + 5a^4x + 10a^3x^2 + 10x^3a^2 + 5ax^4 + x^5$$

The $(r+1)^{th}$ term of $(a+x)^n$ is equal to

$$\frac{n(n-1)(n-2)\dots\dots(n-r+1)a^{n-r}x^r}{|\underline{r}}$$

NEWTONS THEOREM OF SUCCESSIVE APPROXIMATIONS

This can be used for solving any equation with only one unknown, and is particularly useful for solving equation which would be difficult or impossible by any other method.

Consider the equation $y = f(x)$ where y is known. Assume x_1 as a first approximation to the value of x, and let this give y_1 as the value of $f(x_1)$, then a second approximation to x much better than x_1 will be $x_2 = x_1 - \frac{y_1 - y}{dy_1 \div dx_1}$. For example solve for

$x^5 + x = 33{\cdot}7$. here $y = 33{\cdot}7$, take $x_1 = 2$ $\therefore$
$y_1 = 2^5 + 2$ or $y_1 = 34$. Now $dy \div dx = 5x^4 + 1$ $\therefore$
$dy_1 \div dx_1 = 5\times 2^4 + 1 = 81$ $\therefore$ $x_2 = 2 - \frac{34 - 33{\cdot}7}{81} = 1{\cdot}9963$

which is a much better approximation than x_1 to the true value of x. By repeating the process with $1{\cdot}9963$ instead of 2 an even better 3^{rd} approximation x_3 can be obtained, viz:-

$$x_3 = 1{\cdot}9963 - \frac{(1{\cdot}9963^5 + 1{\cdot}9963) - 33{\cdot}7}{5(1{\cdot}9963)^4 + 1}$$

This process can be continued indefinitely until such degree of accuracy as is required in the result is obtained.

ALGEBRA

MISCELLANEOUS SERIES

In the following formulae S_n denotes the sum of n terms of the series, and S_∞ the sum to infinity

$S_n = 1+2+3+\dots+n = \frac{n}{2}(n+1)$

$S_n = 1^2+2^2+3^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$

$S_n = 1^3+2^3+3^3+\dots+n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$

$S_n = (1\times 2)+(2\times 3)+(3\times 4)+\dots n(n+1) = \frac{1}{3}n(n+1)(n+2)$

$S_n = (1\times 2\times 3)+(2\times 3\times 4)+(3\times 4\times 5)+\dots+n(n+1)(n+2) = \frac{1}{4}n(n+1)(n+2)(n+3)$

$S_n = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \dots + \frac{1}{n(n+1)} = 1 - \frac{1}{n+1}$

$S_\infty = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \dots = 1$

$S_n = \frac{1}{1\times 2\times 3} + \frac{1}{2\times 3\times 4} + \frac{1}{3\times 4\times 5} + \dots + \frac{1}{n(n+1)(n+2)} = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$

$S_\infty = \frac{1}{1\times 2\times 3} + \frac{1}{2\times 3\times 4} + \frac{1}{3\times 4\times 5} + \dots = \frac{1}{4}$

$S_n = 1+2x+3x^2+4x^3+\dots+nx^{n-1} = \frac{1-x^n}{(1-x)^2} - \frac{nx^n}{1-x}$

EXPONENTIAL AND LOGARITHMIC SERIES

$a^x = 1 + Ax + \frac{A^2x^2}{2\times 1} + \frac{A^3x^3}{3\times 2\times 1} + \frac{A^4x^4}{4\times 3\times 2\times 1} + \dots$ Where $A = \text{Log}_e a$

put $a = e$ and then since $\text{Log}_a e = 1$. $e^x = 1 + x + \frac{x^2}{|\underline{2}} + \frac{x^3}{|\underline{3}} + \dots$

and $e = 1 + 1 + \frac{1}{|\underline{2}} + \frac{1}{|\underline{3}} + \frac{1}{|\underline{4}} + \dots$ and e is the base of the Naperian or Hyperbolic Logarithm system. $e = 2{\cdot}7182818284\dots$

$\therefore \frac{1}{e} = e^{-1} = \frac{1}{|\underline{2}} - \frac{1}{|\underline{3}} + \frac{1}{|\underline{4}} - \frac{1}{|\underline{5}} + \dots$ $\quad \text{Log}_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$

$\text{Log}_e m = 2\left\{\frac{m-1}{m+1} + \frac{1}{3}\left(\frac{m-1}{m+1}\right)^3 + \frac{1}{5}\left(\frac{m-1}{m+1}\right)^5 + \dots\right\}$

$\text{Log}_{10}(n+1) - \text{Log}_{10} n = 2\mu\left\{\frac{1}{2n+1} + \frac{1}{3(2n+1)^3} + \frac{1}{5(2n+1)^5} + \dots\right\}$

where $\mu = \frac{1}{\text{Log}_e 10} = {\cdot}43229448\dots$

$\text{Log}_e(n+1) - \text{Log}_e n = 2\left\{\frac{1}{2n+1} + \frac{1}{3(2n+1)^3} + \frac{1}{5(2n+1)^5} + \dots\right\}$

PERMUTATIONS AND COMBINATIONS

Every arrangement that can be made by taking some or all of a number of things is called a permutation, thus permutations of 4, 5, 6 taken two at a time are 45, 46, 54, 64, 65 and 56. Every group or selection that can be made by taking some or all of a number of things is a combination. Thus combinations of 4, 5, 6 taken two at a time are 45, 46, and 56. 45 and 54 are different permutations, but only one combination of 4 and 5

PERMUTATIONS AND COMBINATIONS

The number of permutations of N things taken S at a time is $N(N-1)(N-2)\ldots\ldots(N-S+1)$

The number of permutations of N different things taken N at a time is $N(N-1)(N-2)\ldots\ldots 1$ or $1 \times 2 \times 3 \times \ldots \times N$

The form $N(N-1)(N-2),\ldots 1$ is shown by $\underline{|N}$ which is called factorial N

The number of combinations of N different things taken S at a time is $\frac{N(N-1)(N-2)\ldots(N-S+1)}{\underline{|S}}$ or $\frac{\underline{|N}}{\underline{|S}\,\underline{|N-S}}$

ARITHMETICAL PROGRESSION

When a number changes by fixed amounts, for exampl[e] 1, 3, 5, 7, etc, with a common difference of 2 in this case

This is called an Arithmetical Progression

Let A = First Term. N = number of terms. L = last term.

D = common difference. S = sum of N terms

Basic Laws $L = A+(N-1)D$. $S = \frac{A+L}{2} \times N$

To find A. given-,

D.L.N. $= L-(N-1)D$. L.N.S. $= \frac{2S}{N} - L$

D.N.S. $= \frac{S}{N} - \left(\frac{N-1}{2} \times D\right)$. D.L.S. $= \frac{D}{2} \pm \frac{1}{2}\sqrt{(2L+D)^2 - 8DS}$

To find D given-,

A.L.N. $= \frac{L-A}{N-1}$. A.L.S. $= \frac{L^2-A^2}{2S-(L+A)}$.

A.N.S. $= \frac{2S-2AN}{N(N-1)}$. L.N.S $= \frac{2NL-2S}{N(N-1)}$.

To find L given-,

A.D.N. $= A+(N-1)D$. D.N.S. $= \frac{S}{N} + \frac{N-1}{2} \times D$.

A.D.S. $= -\frac{D}{2} \pm \sqrt{8DS+(2A-D)^2}$. A.N.S $= \frac{2S}{N} - A$.

To find N. given-,

A.L.S. $= \frac{2S}{A+L}$. A.D.S $= \frac{D-2A}{2D} \pm \frac{1}{2}D\sqrt{8DS+(2A-D)^2}$

D.L.S. $= \frac{2L+D}{2D} \pm \frac{1}{2}D\sqrt{(2L+D)^2-8DS}$. A.D.L. $= 1 + \frac{L-A}{D}$

To find S given-,

A.D.N. $= \frac{N}{2}\left[2A+(N-1)D\right]$. D.L.N. $= \frac{N}{2}\left[2L-(N-1)D\right]$

A.L.N. $= \frac{N}{2}(N+L)$. A.D.L. $= \frac{A+L}{2} + \frac{L^2-A^2}{2D} = \frac{A+L}{2D}(L+D-A)$

GEOMETRICAL PROGRESSION

Is when the term is multiplied by a constant multiplier called the ratio, thus 2, 6, 18, increasing by ratio of 3.

Let A = First term. L = Last or N^{th} term. N = number of terms

R = Ratio. S = sum of N terms.

GEOMETRICAL PROGRESSION

Basic Law $L = AR^{N-1}$. $S = \frac{RL-A}{R-1}$

To find A given-,

L.N.R $= \frac{L}{R^{N-1}}$. L.N.S $= A(S-A)^{N-1} = L(S-L)^{N-1}$

L.R.S $= LR-(R-1)S$. N.R.S. $= \frac{S(R-1)}{R^N-1}$.

To find L given-,

A.N.R. $= AR^{N-1}$. A.N.S. $= L(S-L)^{N-1} = A(S-A)^{N-1}$.

A.R.S $= \frac{1}{R}\left[A+(R-1)S\right]$. N.R.S. $= \frac{S(R-1)R^{N-1}}{R^{N-1}}$

To find N given-,

A.L.R $= \frac{\text{Log } L - \text{Log } A}{\text{Log } R} + 1$. L.R.S. $= \frac{\text{Log } L - \text{Log}\left[LR-(R-1)S\right]}{\text{Log } R} + 1$

A.R.S $= \frac{\text{Log}\left[A+(R-1)S\right] - \text{Log } A}{\text{Log } R}$. A.L.S. $= \frac{\text{Log } L - \text{Log } A}{\text{Log}(S-A) - \text{Log}(S-L)} + 1$.

To find S given-,

A.N.R. $= \frac{A(R^N-1)}{R-1}$. A.L.N. $= \frac{\sqrt[N-1]{L^N} - \sqrt[N-1]{A^N}}{\sqrt[N-1]{L} - \sqrt[N-1]{A}}$.

L.N.R. $= \frac{L(R^N-1)}{(R-1)R^{N-1}}$. A.L.R. $= \frac{LR-A}{R-1}$

To find R given-,

A.L.N. $= \sqrt[N-1]{\frac{L}{A}}$. A.N.S. $R^N = \frac{SR}{A} + \frac{A-S}{A}$

A.L.S. $= \frac{S-A}{S-L}$. L.N.S. $= R^N = \frac{SR^{N-1}}{S-L} - \frac{L}{S-L}$.

HARMONICAL PROGRESSION

Three quantities A, B and C are in Harmonical Progression when $A : C :: A-B : B-C$. Reciprocals of quantities in Harmonical progression are in Arithmetical progression. If A, B, and C are in Harmonical Progression then $B = \frac{2AC}{A+C}$ and B is the Harmonical mean of A and C

A.B and C are in Harmonical Progression when $\frac{A}{C} = \frac{A-B}{B-C}$ and in Harmonical Progression when $A(B-C) = C(A-B)$

$a : x :: x : b$

$x = \sqrt{ab}$ is the Geometric middle term of a and b

$x = \frac{2ab}{a+b}$ " " Harmonic " " " a " b

$x = \frac{a+b}{2}$ " " Arithmetic " " " a " b

ARITHMETIC

SIMPLE INTEREST. P = principal. p = per cent, r = rate of interest expressed decimally. n = the number of years F = interest in £'s. Pn = total of principal and interest after n years. $Pn = P + Prn = P(1+rn)$. $r = F \div Pn$ $N = F \div Pr$ and $P = F \div rn$

COMPOUND INTEREST $Pn = P(1+r)^n$. $P = \frac{Pn}{(1+r)^n}$. $r = \sqrt[n]{\frac{Pn}{P}} - 1$

$N = \frac{\text{Log}\,Pn - \text{Log}\,P}{\text{Log}(1+r)}$. if interest is payable q times per year it will be computed q times per year or qn times in n years ∴ at the end of n years the amount due will be $Pn = P\left(1+\frac{r}{q}\right)^{nq}$.

PRESENT VALUE AND DISCOUNT. the present value of a given amount, due in a given time = V. $V = \frac{Pn}{1+nr}$ at simple interest and $V = \frac{Pn}{(1+r)^n}$ at compound interest. D the true discount is the difference between the amount due at the end of n years and the present value $D = Pn - V = \frac{Pn\,nr}{1+nr}$ at simple interest. $D = Pn - V = Pn\left[1 - \frac{1}{(1+r)^n}\right]$ at compound interest

ANNUITIES. if an annuity is to be paid for n consecutive years, the interest rate being r, then the present value P of the annuity is $P = A\frac{(1+r)^n - 1}{(1+r)^n r}$ interest at compound reckoning the annuity A that a principal P drawing interest at the rate r will give for a period of n years is $A = \frac{Pr(1+r)^n}{(1+r)^n - 1}$. if at the beginning of each year a sum A is set aside at an interest rate r, then the total value of the principal and interest at the end of n years will be $Pn = A\frac{(1+r)[(1+r)^n - 1]}{r}$. if at the end of each year a sum A is set aside at interest rate r, the total value of the principal with interest at the end of n years is $Pn = A\frac{(1+r)^n - 1}{r}$. if a principal P is increased or decreased by a sum A at the end of each year, then the value of the principal after n years will be $P_n = P(1+r)^n \pm A\frac{(1+r)^n - 1}{r}$. if the sum A by which the principal P is decreased each year is greater than the total yearly interest on the principal, then the principal, with accumulated interest, will be entirely used up in n years. $n = \frac{\text{Log}\,A - \text{Log}(A - Pr)}{\text{Log}(1+r)}$.

MENSURATION

CONE FRUSTUM. $V = \frac{1}{12}\pi H(D^2 + Dd + d^2) = \frac{1}{3}\pi H(R^2 + Rr + r^2)$. $A = \pi S(R+r) = 1{\cdot}5708\,S(D+d)$
$S = \sqrt{(R-r)^2 + H^2}$

CYLINDER $V. = \pi R^2 H = {\cdot}785\,D^2 H$. Circular area $= 2\pi RH = \pi DH$. Total area $= 2\pi R(R+H) = \pi D(\frac{1}{2}D + H)$

PROLATE SPHEROID $V = \frac{4}{3}\pi Rr^2 = 4{\cdot}189\,Rr^2 = \frac{1}{6}\pi Dd^2 = {\cdot}5236\,Dd^2$. $A = \frac{4\pi}{\sqrt{2}} r\sqrt{R^2 + r^2}$

PARABOLOID $V = \frac{1}{2}R^2 H = 1{\cdot}5708\,R^2 H = \frac{\pi}{8}D^2 H = {\cdot}3927\,D^2 H$. area $= \frac{1}{2}$ prolate spheroid

PYRAMID $V = \frac{1}{3}$ area of base $\times H = \frac{\text{No of sides} \times SH}{6}\sqrt{R^2 - \frac{S^2}{4}}$
R = radius of inscribed circle

PYRAMID FRUSTRUM $V = \frac{H}{3}(A + a + \sqrt{Aa})$

PORTION OF CYLINDER $V = 1{\cdot}5708\,R^2(H+h) = 3927\,D^2(H+h)$
Cylindrical surface $= \pi R(H+h)$. Where H = major height and h = minor height

HOLLOW CYLINDER $V = \pi H(R^2 - r^2) = {\cdot}7854\,H(D^2 - d^2) = 1{\cdot}5708\,H \times$ Thickness of wall $\times (D+d)$

SPHERICAL ZONE $V = {\cdot}5236\,H\left\{\frac{3C^2}{4} + \frac{3P^2}{4} + H^2\right\}$
area of spherical surface $= 2\pi RH$. Where P = Major chord and C = minor chord.
$R = \sqrt{\frac{P^2}{4} + (P^2 - C^2 - 4H^2)^2}$

CIRCULAR WEDGE $V = \frac{M}{360} \times \frac{4\pi R^3}{3} = {\cdot}0116\,MR^3$
$A = \frac{M}{360} \times 4\pi R^2 = {\cdot}0349\,MR^2$ Where M = angle of wedge

HOLLOW SPHERE $V = \frac{4\pi}{3}(R^3 - r^3) = 4{\cdot}188(R^3 - r^3) = \frac{\pi}{6}(D^3 - d^3) = {\cdot}5236(D^3 - d^3)$

REGULAR POLYGONS Let L = length of each side of a regular Polygon. N = number of sides. R = radius of circumscribing circle. T = radius of inscribed circle
Perimeter $= NL = 2NT\tan\frac{180°}{N} = 2NR\sin\frac{180°}{N} =$
$A = \frac{1}{4}NL^2\cot\frac{180°}{N} = NT^2\tan\frac{180°}{N} = \frac{1}{2}NR^2\sin\frac{360°}{N}$.
$L = 2T\tan\frac{180°}{N} = 2R\sin\frac{180°}{N}$.

MENSURATION

THE TRAPIZOIDAL RULE

To obtain The area of irregular figures divide the base of the figure into a number of equal parts and erect ordinates at point of division. Measure the lengths of these ordinates, then the area is equal to the length of one division X by the sum of ($\frac{1}{2}$ first and last ordinate) + sum of all the remaining ordinates

SIMPSONS RULE for area of irregular figures

Divide base of figure into an even number of equal divisions, then the area of the figure is equal to $\frac{1}{3}$ width of 1 base division $\times$ height of {1st + last ordinate + 4(sum of even ordinates) + 2(sum of odd ordinates), excluding the first and last ordinates}

GENERAL FORMULAS

Radian $= \frac{360°}{2\pi} = 57.3°$ and $\frac{\pi}{180}$ Radians $= 1°$

Now in an angle of 1 radian, the arc is equal to the radius in length $\therefore$ the length of arc of any angle is equal to radius X angle measurement in radians.

In all the succeeding Formulas, the following abbreviations are used -

R = major radius, r = minor radius. D = major diameter d = minor diameter, c = chord. H = height. S = side length V = volume. A = major radius a = minor radius

SPHERE $V = \frac{4\pi R^3}{3} = 4.189 R^3 = \frac{\pi D^3}{6} = .524 D^3$

$A = 4\pi R^2 = \pi D^2 = 12.5664 R^2 = 3.1416 D^2$

$R = \sqrt[3]{\frac{3V}{4\pi}} = .6204 \sqrt[3]{V}$

TORUS or circular shape hollow ring $V = 2\pi^2 R(r)^2$ $= 19.739 Rr^2 = \frac{\pi^2}{4} D(d)^2 = 2.4674 Dd^2$ $A = 4\pi Rr = 39.478 Rr = \pi^2 Dd = 9.8696 Dd$. Where R = main radius of ring, and r = radius of circular form

SPHERICAL SECTOR $V = \frac{2}{3}\pi R^2 H = 2.0944 R^2 H = \frac{2}{3} R^2 (R \pm \sqrt{R^2 - \frac{1}{4}c^2})$. $A = \pi R(2H + \frac{1}{2}c)$. $C = 2\sqrt{H(2R-H)}$

Where H = height of segment above chord

CONE $V = \frac{\pi R^2 H}{3} = 1.047 R^2 H = .2618 D^2 H$

$A = \pi R\sqrt{R^2 + H^2} = \pi RS = 1.5708 DS$. $S = \sqrt{R^2 + H^2} = \sqrt{\frac{D^2}{4} + H^2}$

MENSURATION

ELLIPSE $A = \pi Rr$. Perimeter $= \pi\sqrt{2(R^2+r^2) - \frac{(R-r)^2}{2.2}}$ which is a close approximation

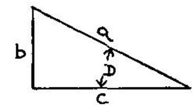
QUADRANT $A = .785 R^2$

FILLET $A = R^2 - \frac{\pi R^2}{4} = .215 R^2$

CIRCULAR RING SECTOR $A = \frac{\text{Angle} \times \pi}{360}(R^2 - r^2) = .000873 \text{ angle}(R^2 - r^2) = \frac{\text{Angle} \times \pi}{4 \times 360}(D^2 - d^2) = .00218 \text{ angle}(D^2 - d^2)$

TRIGONOMETRICAL FORMULAS

Definitions

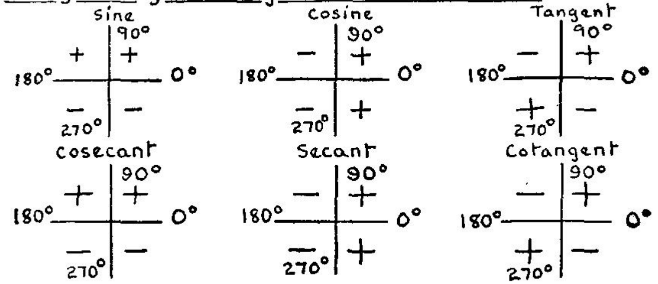


$\frac{b}{a}$ = sine D. $\frac{b}{c}$ = Tangent D

$\frac{c}{a}$ = Cosine D. $\frac{a}{b}$ = Cosecant D

$\frac{c}{b}$ = Cotangent D. $\frac{a}{c}$ = Secant D

Change in sign of Trigonometrical Functions



USEFUL FORMULAS

$\sin^2 A + \cos^2 A = 1$. $\tan A = \frac{\sin A}{\cos A} = \frac{1}{\cot A}$.

$\cot A = \frac{\cos A}{\sin A} = \frac{1}{\tan A}$. $\sec A = \frac{1}{\cos A}$. $\operatorname{cosec} A = \frac{1}{\sin A}$

$\sin A = \sqrt{1 - \cos^2 A} = \frac{\tan A}{\sqrt{1 + \tan^2 A}} = \frac{1}{\sqrt{1 + \cot^2 A}}$.

$\cos A = \sqrt{1 - \sin^2 A} = \frac{\cot A}{\sqrt{1 + \cot^2 A}} = \frac{1}{\sqrt{1 + \tan^2 A}}$

$\sin(A+B) + \sin(A-B) = 2 \sin A \cos B$

$\sin(A+B) - \sin(A-B) = 2 \cos A \sin B$

$\cos(A+B) + \cos(A-B) = 2 \cos A \cos B$

$\cos(A-B) - \cos(A+B) = 2 \sin A \sin B$

TRIGONOMETRY

USEFUL FORMULAS

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$. $\cot(A \pm B) = \dfrac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$

$\tan A \pm \tan B = \dfrac{\sin(A \pm B)}{\cos A \cos B}$. $\cot A \pm \cot B = \dfrac{\sin(B \pm A)}{\sin A \sin B}$

$\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin(A+B)\sin(A-B)$

$\cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A = \cos(A+B)\cos(A-B)$

$\sin A \sin B = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$

$\sin A \cos B = \frac{1}{2}\sin(A+B) + \frac{1}{2}\sin(A-B)$

$\tan A \tan B = \dfrac{\tan A + \tan B}{\cot A + \cot B}$. $\cot A \cot B = \dfrac{\cot A + \cot B}{\tan A + \tan B}$

$\sin A = 2 \sin \frac{1}{2} A \cos \frac{1}{2} A$. $\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A} = \dfrac{2}{\cot A - \tan A}$. $\sin A = \dfrac{2 \tan \frac{1}{2} A}{1 + \tan^2 \frac{1}{2} A}$

$\cot 2A = \dfrac{\cot^2 A - 1}{2 \cot A} = \dfrac{\cot A - \tan B}{2}$. $\cos A = \dfrac{1 - \tan^2 \frac{1}{2} A}{1 + \tan^2 \frac{1}{2} A}$

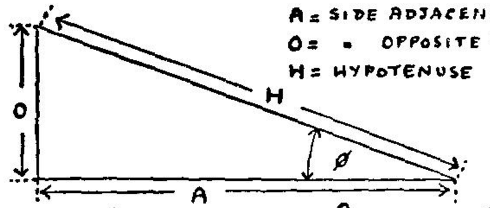
$\sin 3A = 3 \sin A - 4 \sin^3 A$.

$\cos 3A = 4 \cos^3 A - 3 \cos A$. $\tan 3A = \dfrac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$

USEFUL CONSTANTS

π	=	3·14159	g	=	32·16
$3 \div \pi$	=	·95492	$1 \div 2g$	=	·01555
π^2	=	9·8696	$\pi \div \sqrt{g}$	=	·55399
$\sqrt{\pi}$	=	1·77245	$\sqrt[3]{6 \div \pi}$	=	1·2407
$1 \div \sqrt[3]{\pi}$	=	·68278	$\pi \div 3$	=	1·0472
$\pi \div 4$	=	·7854	$1 \div \pi$	=	·31831
$2g$	=	64·32	$1 \div \pi^2$	=	·10132
$1 \div \sqrt{g}$	=	·17634	$\sqrt[3]{\pi}$	=	1·46459
$\pi \div 180$	=	·01745	$\sqrt[3]{3 \div 4\pi}$	=	·62035
2π	=	6·28318	g^2	=	1034·226
$4\pi \div 3$	=	4·18879	$\sqrt{2g}$	=	8·01998
π^3	=	31·00628	e	=	2·71828
$1 \div \sqrt{\pi}$	=	·56419	$180° \div \pi$	=	57·2958°
$\sqrt[3]{\pi^2}$	=	2·14503			

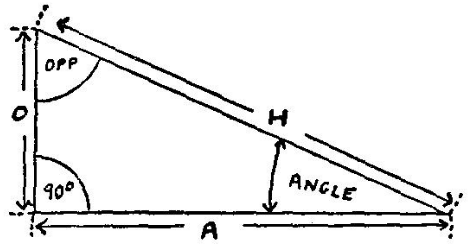
SOLUTION OF RIGHT ANGLE TRIANGLES



A = SIDE ADJACENT TO Ø
O = " OPPOSITE TO Ø
H = HYPOTENUSE

SINE $Ø = \dfrac{O}{H}$ TANGENT $Ø = \dfrac{O}{A}$ SECANT $Ø = \dfrac{H}{A}$

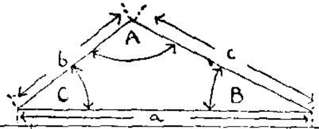
COSINE $Ø = \dfrac{A}{H}$ COTANGENT $Ø = \dfrac{A}{O}$ COSECANT $Ø = \dfrac{H}{O}$



PARTS GIVEN	PARTS TO BE FOUND				
	HYP	ADJ SIDE	OPP SIDE	ANGLE	OPP ANGLE
HYPOTENUSE AND ADJACENT	—	—	$\sqrt{HYP^2 - ADJ^2}$	COSINE $= \dfrac{ADJ}{HYP}$	SINE $= \dfrac{ADJ}{HYP}$
HYPOTENUSE AND OPPOSITE	—	$\sqrt{HYP^2 - OPP^2}$	—	SINE $= \dfrac{OPP}{HYP}$	COSINE $= \dfrac{OPP}{HYP}$
HYPOTENUSE AND ANGLE	—	HYP X COSINE	HYP X SINE	—	90° - ANGLE
ADJACENT AND OPPOSITE	$\sqrt{ADJ^2 + OPP^2}$	—	—	—	COTAN $= \dfrac{OPP}{ADJ}$
ADJACENT AND ANGLE	$\dfrac{ADJ}{COSINE}$	—	ADJ X TANGENT	—	90° - ANGLE
OPPOSITE AND ANGLE	$\dfrac{OPP}{SINE}$	OPP X COTAN	—	—	90° - ANGLE

Solution of Oblique Angled Triangles

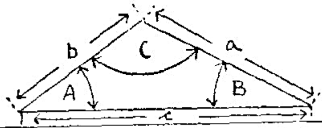
Angles



Parts Given	Angles to be Found		
	Angle A	Angle B	Angle C
a.b.c	$\frac{b^2+c^2-a^2}{2bc} = \operatorname{Cos} A$	$\frac{a^2+c^2-b^2}{2ac} = \operatorname{Cos} B$	$\frac{a^2+b^2-c^2}{2ab} = \operatorname{Cos} C$
b,c, Angle A	—	$\frac{b \operatorname{Sin} A}{c - b \operatorname{Cos} A} = \operatorname{Tan} B$	$\frac{c \operatorname{Sin} A}{b - c \operatorname{Cos} A} = \operatorname{Tan} C$
a.c Angle B	$\frac{a \operatorname{Sin} B}{c - a \operatorname{Cos} B} = \operatorname{Tan} A$	—	$\frac{c \operatorname{Sin} B}{a - c \operatorname{Cos} B} = \operatorname{Tan} C$
a.b Angle C	$\frac{a \operatorname{Sin} C}{b - a \operatorname{Cos} C} = \operatorname{Tan} A$	$\frac{b \operatorname{Sin} C}{b - a \operatorname{Cos} C} = \operatorname{Tan} B$	—
a.b. Angle A	—	$\frac{b \operatorname{Sin} A}{a} = \operatorname{Sin} B$	$180° - (A+B)$
a.b Angle B	$\frac{a \operatorname{Sin} B}{b} = \operatorname{Sin} A$	—	$180° - (A+B)$
a.c. Angle A	—	$180° - (A+C)$	$\frac{c \operatorname{Sin} A}{a} = \operatorname{Sin} C$
a.c. Angle C	$\frac{a \operatorname{Sin} C}{c} = \operatorname{Sin} A$	$180° - (A+C)$	—
b.c Angle B	$180° - (B+C)$	—	$\frac{c \operatorname{Sin} B}{b} = \operatorname{Sin} C$
a Angles A.B.	—	—	$180° - (A+B)$
a Angles A.C.	—	$180° - (A+C)$	—
a Angles B.C	$180° - (B+C)$	—	—
b Angles A.B.	—	—	$180° - (A+C)$
b Angles A.C.	—	$180° - (A+C)$	—
b Angles B.C.	$180° - (B+C)$	—	—
b.c Angle C	$180° - (B+C)$	$\frac{b \operatorname{Sin} C}{c} = \operatorname{Sin} B$	—
c Angles A.B	—	—	$180° - (A+B)$
c Angles A.C	—	$180° - (A+C)$	—

Solution of Oblique Angled Triangles

Sides



Parts Given	Sides to be Found		
	Side a =	Side b =	Side c =
b.c Angle A	$\sqrt{b^2+c^2-2bc \operatorname{Cos} A}$	—	—
a.c Angle B	—	$\sqrt{a^2+c^2-2ac \operatorname{Cos} B}$	—
a.b Angle C	—	—	$\sqrt{a^2+b^2-2ab \operatorname{Cos} C}$
a.b Angle A	—	—	$\frac{a \times \operatorname{Sin} C}{\operatorname{Sin} A}$
a.b Angle B	—	—	$\frac{b \times \operatorname{Sin} C}{\operatorname{Sin} B}$
a.c Angle A	—	$\frac{a \times \operatorname{Sin} B}{\operatorname{Sin} A}$	—
a.c Angle C	—	$\frac{c \times \operatorname{Sin} B}{\operatorname{Sin} C}$	—
b c Angle B	$\frac{b \times \operatorname{Sin} A}{\operatorname{Sin} B}$	—	—
b c Angle C	$\frac{c \times \operatorname{Sin} A}{\operatorname{Sin} C}$	—	—
a Angles A.B	—	$\frac{a \times \operatorname{Sin} B}{\operatorname{Sin} A}$	$\frac{a \times \operatorname{Sin} C}{\operatorname{Sin} A}$
a Angles A.C	—	$\frac{a \times \operatorname{Sin} B}{\operatorname{Sin} A}$	$\frac{a \times \operatorname{Sin} C}{\operatorname{Sin} A}$
a Angles B.C	—	$\frac{a \times \operatorname{Sin} B}{\operatorname{Sin} A}$	$\frac{a \times \operatorname{Sin} C}{\operatorname{Sin} A}$
b Angles A.B	$\frac{b \times \operatorname{Sin} A}{\operatorname{Sin} B}$	—	$\frac{b \times \operatorname{Sin} C}{\operatorname{Sin} B}$
b Angles A.C	$\frac{b \times \operatorname{Sin} A}{\operatorname{Sin} B}$	—	$\frac{b \times \operatorname{Sin} C}{\operatorname{Sin} B}$
b Angles B.C	$\frac{b \times \operatorname{Sin} A}{\operatorname{Sin} B}$	—	$\frac{b \times \operatorname{Sin} C}{\operatorname{Sin} B}$
c Angles A.B	$\frac{c \times \operatorname{Sin} A}{\operatorname{Sin} C}$	$\frac{c \times \operatorname{Sin} B}{\operatorname{Sin} C}$	—
c Angles A.C.	$\frac{c \times \operatorname{Sin} A}{\operatorname{Sin} C}$	$\frac{c \times \operatorname{Sin} B}{\operatorname{Sin} C}$	—
c Angles B.C	$\frac{c \times \operatorname{Sin} A}{\operatorname{Sin} C}$	$\frac{c \times \operatorname{Sin} B}{\operatorname{Sin} C}$	—

ANALYTICAL GEOMETRY

CIRCLE AND TANGENT EQUATIONS

circle radius b, centre at origin, $x^2+y^2=b^2$.
tangent at $x_1 y_1$ on circle. $xx_1+yy_1=b^2$.
circle radius b, centre at $(l\,m)$, $(x-l)^2+(y-m)^2=b^2$.
tangent at $x_1 y_1$ on circle, $(x_1-l)(x-l)+(y_1-m)(y-m)=b^2$.
general equation to circle, $x^2+y^2+2gx+2fy+c=0$.
equation to tangent at $x_1 y_1$ on circle. $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$

ELLIPSE semi axis a along OX, b along OY.
equation to curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
tangent at $x_1 y_1$ is $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$
eccentricity "p" where $p^2=1-\frac{b^2}{a^2}$

PARABOLA. origin at vertex, axis of parabola as axis of x. equation to curve $y^2=4ax$.
tangent at $x_1 y_1$. $yy_1=2a(x+x_1)$
HYPERBOLA. semi axis a along OX, centre at origin. equation to curve $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. tangent at $x_1 y_1$ $\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$. equation to asymptotes $\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$ or $\frac{x}{a}\pm\frac{y}{b}=0$. eccentricity "$p$" where $p = 1+\frac{b^2}{a^2}$.

EQUATIONS TO CURVES. circle $y=\sqrt{ax-x^2}$.
hyperbola. $y=\sqrt{\frac{p}{a}(ax+x^2)}$. cubic parabola. $y=mx^3$
ellipse. $y=\sqrt{\frac{p}{a}(ax-x^2)}$. cissoid $y=\sqrt{\frac{x^3}{a-x}}$.
parabola. $y=\sqrt{px}$. lemniscoid $y=\frac{\sqrt{a^2-x^2}}{a}$
where x = abscissa, y = ordinate, a = the axis, and p = the parameter.

SOLID GEOMETRY

A. B. C. are rectangular coordinates of a point D. E. F. are the polar coordinates of the same point. D = distance. E = co-latitude. F = longitude
To convert rectangular to polar coordinates
$D=\sqrt{A^2+B^2+C^2}$. $E=\cos^{-1}\frac{C}{\sqrt{A^2+B^2+C^2}}$
$F=\tan^{-1}\frac{B}{A}$

SOLID GEOMETRY

To convert polar to rectangular coordinates
$A = D\sin E\cos F$. $B = D\sin E\sin F$. $C = D\cos E$.
d = distance between points $P(A_1 B_1 C_1)$ and $Q(A_2 B_2 C_2)$ = $\sqrt{(A_2-A_1)^2+(B_2-B_1)^2+(C_2-C_1)^2}$

Direction cosines of line PQ.

$$\left.\begin{array}{l} l=\frac{A_2-A_1}{d} = \text{cos of angle between PQ and OX} \\ m=\frac{B_2-B_1}{d} = \text{" " " " PQ " OY} \\ n=\frac{C_2-C_1}{d} = \text{" " " " PQ " OZ} \end{array}\right\} l^2+m^2+n^2=1$$

E = angle between 2 lines whose direction cosines are (l_1, m_1, n_1) and (l_2, m_2, n_2) respectively. $\cos E = l_1 l_2+m_1 m_2+n_1 n_2$.
K. = angle between PQ and OX, $\cos K = l = \frac{A_2-A_1}{d}$
L. = " " PQ " OY, $\cos L = m = \frac{B_2-B_1}{d}$
M. = " " PQ " OZ, $\cos M = n = \frac{C_2-C_1}{d}$
Inclination of line PQ to plane YOZ $=90^\circ-K$
" " " " " " ZOX $=90^\circ-L$
" " " " " " XOY $=90^\circ-M$
Length of projection of PQ on plane YOZ $= d\sin K$
" " " " " " " ZOX $= d\sin L$
" " " " " " " XOY $= d\sin M$
equation to plane, perpendicular from origin to plane $= p$, direction cosines of p being, l, m, n, is $lA+mB+nC=p$
equation to line through point K, L, M, of direction cosines l, m, n is $\frac{A-K}{l}=\frac{B-L}{m}=\frac{C-M}{n}$.

equation to sphere $A^2+B^2+C^2=a^2$. equation to ellipsoid $\frac{A^2}{a^2}+\frac{B^2}{b^2}+\frac{C^2}{c^2}=1$. Prolate spheroid. $\frac{A^2}{a^2}+\frac{B^2}{b^2}+\frac{C^2}{b^2}=1$ $a>b$

oblate spheroid. $\frac{A^2}{a^2}+\frac{B^2}{b^2}+\frac{C^2}{a^2}=1$ $a>b$

MID ORDINATE rule for area of irregular figures. divide base of figure into any number of equal parts, erect ordinates at middle points of these strips. Multiply the average height of the mid-ordinates by the length of the base of the figure, and this is approximately equal to the area of the figure

DIFFERENTIAL AND INTEGRAL CALCULUS

FORMULAS

$y = x^n$, $\frac{dy}{dx} = nx^{n-1}$. $y = \sin\phi$, $\frac{dy}{d\phi} = \cos\phi$.

$y = \cos A$, $\frac{dy}{dA} = -\sin A$. $y = \tan A$, $\frac{dy}{dA} = \sec^2 A$

$y = \cot A$, $\frac{dy}{dA} = -\operatorname{cosec}^2 A$. $y = \sec A$, $\frac{dy}{dA} = \tan A \sec A = \frac{\sin A}{\cos^2 A}$.

$y = \operatorname{cosec} A$, $\frac{dy}{dA} = -\cot A \operatorname{cosec} A = -\frac{\cos A}{\sin^2 A}$. $y = \sin^{-1}\frac{x}{a}$, $\frac{dy}{dx} = \frac{1}{\sqrt{a^2-x^2}}$

$y = \cos^{-1}\frac{x}{a}$, $\frac{dy}{dx} = -\frac{1}{\sqrt{a^2-x^2}}$. $y = \tan^{-1}\frac{x}{a}$, $\frac{dy}{dx} = \frac{a}{a^2+x^2}$

$y = \cot^{-1}\frac{x}{a}$, $\frac{dy}{dx} = -\frac{a}{a^2+x^2}$. $y = \sec^{-1}\frac{x}{a}$, $\frac{dy}{du} = \frac{a}{x\sqrt{x^2-a^2}}$

$y = \operatorname{cosec}^{-1}\frac{x}{a}$, $\frac{dy}{dx} = -\frac{a}{x\sqrt{x^2-a^2}}$. $y = e^x$, $\frac{dy}{dx} = e^x$

$y = e^{ax}$, $\frac{dy}{dx} = ae^{ax}$. $y = a^x$, $\frac{dy}{dx} = a^x \log a$.

$y = \log x$, $\frac{dy}{dx} = \frac{1}{x}$. $y = uv$, $\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

$y = \frac{u}{v}$, $\frac{dy}{dx} = \left(v\frac{du}{dx} - u\frac{dv}{dx}\right) \div v^2$. $\int u\,dv = uv - \int v\,du$.

Maclaurins Theorem. $f(x) = f(0) + xf'(0) + \frac{x^2}{\times 2} f''(0) + \ldots\ldots$

Taylor's Theorem $f(x+h) = f(x) + hf'(x) + \frac{h^2}{1\times 2} f''(x) \ldots$

$\int x^n dx = \frac{x^{n+1}}{n+1}$. $n \neq -1$. $\int \cos A\,dA = \sin A$

$\int \sin A\,dA = -\cos A$. $\int \sec^2 A\,dA = \tan A$. $\int \operatorname{cosec}^2 A\,dA = -\cot A$.

$\int \tan A \sec A\,dA = \sec A$. $\int \cot A \operatorname{cosec} A\,dA = -\operatorname{cosec} A$.

$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}$. $\int \frac{-dx}{\sqrt{a^2-x^2}} = \cos^{-1}\frac{x}{a}$. $\int \frac{a\,dx}{a^2+x^2} = \tan^{-1}\frac{x}{a}$.

$\int \frac{-a\,dx}{a^2+x^2} = \cot^{-1}\frac{x}{a}$. $\int \frac{a\,dx}{x\sqrt{x^2-a^2}} = \sec^{-1}\frac{x}{a}$. $\int \frac{-a\,dx}{x\sqrt{x^2-a^2}} = \operatorname{cosec}^{-1}\frac{x}{a}$.

$\int e^x dx = e^x$. $\int e^{ax} dx = \frac{e^{ax}}{a}$. $\int a^x dx = \frac{a^x}{\log a}$.

$\int \frac{dx}{x} = \log x$. $\int \sinh x\,dx = \cosh x$. $\int \cosh x\,dx = \sinh x$

$\int \operatorname{sech}^2 x\,dx = \tanh x$. $\int \frac{dx}{\sqrt{a^2+x^2}} = \sinh^{-1}\frac{x}{a} = \log\{x + \sqrt{x^2+a^2}\}$

$\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\frac{x}{a} = \log\{x + \sqrt{x^2-a^2}\}$

$\int \frac{dx}{a^2-x^2} = \frac{1}{a}\tanh^{-1}\frac{x}{a} = \frac{1}{2a}\log\frac{a+x}{a-x}$