EXPERIMENTAL WIRELESS
& The WIRELESS ENGINEER

VOL. VII. DECEMBER, 1930. No. 87.

Editorial.

An Interesting Sideband Problem.

FIG. 1 represents an alternating e.m.f. of amplitude $2E$ and frequency $\omega/2\pi$. Fig. 2 represents an alternating e.m.f. of the same "frequency," the amplitude of which varies between zero and $2E$ according to a sine law.

Is it possible for the e.m.f. shown in Fig. 2 to build up a larger current in a circuit than that shown in Fig. 1? It appears very improbable, and to make it still more improbable we shall assume that in Fig. 2 the phase of the e.m.f. is reversed whenever the amplitude passes through zero, *i.e.*, at the points A and B.

We propose, however, to show that in certain circumstances the e.m.f. of pulsating amplitude and changing phase will build up a much larger current than the e.m.f. of constant amplitude.

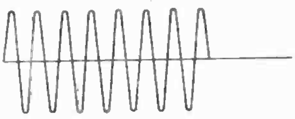

Fig. 1.

Fig. 3 represents the resonance curve of an oscillatory circuit. Electromotive forces represented by the formulæ $E \sin \omega_0 t$, $E \sin \omega t$, and $E \sin \omega_1 t$ would build up currents of I_0, I and I_1 respectively when acting separately. Now let us consider what will happen when two electromotive forces $E \sin \omega_0 t$ and $E \sin \omega_1 t$ act simultaneously on the circuit. This is really the same question as that raised above, for the resultant electromotive

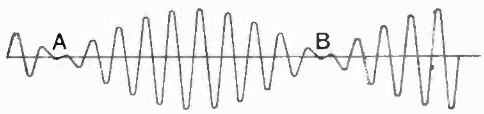

Fig. 2.

force acting on the circuit is now

$$E \sin \omega_0 t + E \sin \omega_1 t = 2E \sin \frac{\omega_0 + \omega_1}{2} t \cos \frac{\omega_0 - \omega_1}{2} t = 2E \cos \frac{\omega_0 - \omega_1}{2} t \sin \omega t$$

where ω is the mean of ω_0 and ω_1.

This is the e.m.f. shown in Fig. 2 with a "frequency" $\omega/2\pi$ and an amplitude varying between $2E$ and $-2E$, or, what is the same thing, varying between $2E$ and 0 with a phase reversal whenever $\frac{\omega_0 - \omega_1}{2}$ goes through 0 or π, since the cosine then changes its sign.

The e.m.f. of constant amplitude $2E$ shown in Fig. 1 would produce a current equal to $2I$, *i.e.*, equal to AB in Fig. 3, because we assume that Fig. 3 has been obtained with an e.m.f. of constant amplitude E and variable frequency. The two constituent electromotive forces $E \sin \omega_0 t$ and $E \sin \omega_1 t$ will produce currents of I_0 and I_1 each of the same frequency as the electromotive forces producing them, and the resultant current will be compounded of these two currents, one of which, *viz.*, I_0, is so much greater than the other that the resultant will approximate closely to a steady alternating current $I_0 \sin \omega_0 t$. The amplitude will merely fluctuate between $I_0 + I_1$ and $I_0 - I_1$, *i.e.*, between CF and CG with a frequency equal to the difference between the two constituent frequencies.

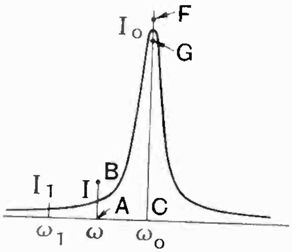

Fig. 3.

With the numerical values assumed in Fig. 3 the e.m.f. of Fig. 2 produces about five times the amplitude of current produced by the e.m.f. of Fig. 1. This surprising result shows up in a striking manner the fallacy of speaking of the frequency in Fig. 2 where there is no regularly recurring phenomenon between the points A and B, except the mere crossing of the base line. It also serves to emphasise the importance of splitting up a modulated e.m.f. into its constituent components of constant amplitude before any attempt is made to predict its action on an oscillatory circuit. The two electromotive forces which we have considered may be the sidebands of a modulated carrier which would have a frequency of $\omega/2\pi$, and it is obvious from Fig. 3 that if an oscillatory circuit is slightly detuned one of the sidebands may produce a much larger current than the carrier and other sideband combined—a fairly conclusive proof of its reality. G.W.O.H.

A Receiver of Outstanding Interest.

The Pye Twintriple Receiver gained the distinction of being voted the outstanding exhibit of the recent Olympia Radio Show by readers of THE WIRELESS WORLD, in a ballot competition which that journal conducted.

A description of the receiver is to be found in the issue of THE WIRELESS WORLD for November 12th.

The illustrations above show the front and back (with cover removed) views of the battery model of this set. It was the all-mains A.C. type which readers of THE WIRELESS WORLD selected.

Loud Speaker Tests and Performance Factors.*

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Introduction.

THE time is rapidly approaching when a detailed set of widely recognised tests for loud speakers will be required similar in character to those already in vogue for other pieces of radio apparatus. Limited agreement is already found in the literature of the subject, but there are many points on which considerable divergences of opinion still exist.

The general effects of eliminating frequency bands, introducing harmonics and making similar variations are now fairly well known but more work will have to be done before a complete correlation can be made between scientific laboratory tests and the subjective impressions as described by selected observers. This is largely due to differences in personal taste which are unavoidable when it has to be decided how a large orchestra should be reproduced in a sitting room. However, the present object is to examine the basis of tests which are or can be applied to reproducers under controlled conditions, even if the test conditions are not those found in practice. Provided accurate comparisons can be made under simple, artificial, but quite definite conditions, some reasonable attempt can be made to correlate observed and calculated effects.

Acoustical Measurements in General.

The general reasons underlying the use of specially constructed sound testing chambers can be readily appreciated by a preliminary consideration of the following example. Consider a perfectly rigid vibrating piston situated in an infinite rigid wall with no free communication between the two sides, and radiating sound waves into free space from each side of the piston. The sound field round the piston can be calculated or explored with an ideal pressure or velocity measuring device. In practice, of course, nothing is ideal; measurements become essential: infinite rigid baffles are replaced by vibratile finite ones, or, if effectively infinite ones are obtained by limiting the medium, an approximation only to free space is secured. The medium is usually air and its boundaries the walls, floor and ceiling of a test chamber into which the piston radiates. If now the diaphragm of a loud speaker is made to form part of one wall and the sound from its back surface is prevented from reaching its front surface, an effectively infinite baffle is realised. This is a convenient test condition to standardise, unless the size of baffle or variations in cabinet design are being investigated. For the present we will consider only diaphragm loud speakers and assume that the sound radiated from the back of the diaphragm is allowed to escape freely without setting up, by reflections, any acoustical reaction on the diaphragm itself. The modifications in procedure, when small horn type reproducers are under test, will be obvious.

Let us turn our attention again to the front of the diaphragm and suppose it working into an ordinary closed room. When the loud speaker is emitting a single note a measurement of the sound pressure at some point in the room would, for the following reason, normally yield a result too complicated for ready interpretation. At the measurement point considered there would be the direct wave from the source and a number of subsidiary waves arriving from various reflecting bounding surfaces, each wave making an unknown contribution to the observed sound pressure. A further element of uncertainty in the test conditions is introduced by the reflected waves altering the acoustical radiation resistance of the source. The difficulties just outlined largely disappear if the room is lined with absorbing material so that in the ideal case all the sound energy reaching the walls, floor and ceiling is completely absorbed. The excess alternating sound pressure or air particle velocity at a given point is then the pressure or velocity in a simplified diverging wave system, and the results are more easily interpreted. The acoustical radiation

* MS. received by Editor, July, 1930.

resistance for each side of the diaphragm is then the same as for an infinite medium. It is not difficult in practice to approximate to a perfectly absorbing room for a wide range of audio frequencies. The lower the frequency, the more difficult it becomes to avoid reflections. Imperfections in absorption usually make themselves apparent below about 300 cycles per second and depend upon the character and quantity of damping material employed. Measurements at low frequencies are therefore better made out-of-doors by fixing the loud speaker with an unobstructed outlook in the side of a large wall or building, and suitably supporting the microphone in front of it. Both microphone and loud speaker should be sufficiently high above the ground level to avoid appreciable errors from reflections from the ground. Weather conditions have to be contended with, and on this account outdoor measurements frequently have to be abandoned continuously for long periods. However, for a given type of reproducer comparable measurements taken indoors and out-of-doors enable a working calibration to be derived for a highly damped acoustic measuring chamber.

To avoid errors due to room reflections it is usual in damped chambers to work with the microphone as near to the source as is permissible. Small standing waves are then set up between the loud speaker and the microphone, especially at the higher frequencies, even when the room is perfectly absorbing. The magnitude of the effect is often not great but is troublesome when the variation of sound pressure with distance is being investigated. Two general methods of overcoming standing wave effects have been put forward. The first is to swing the microphone, but this is open to several objections. As has already been pointed out, the output of the measuring system is not a true mean of the pressure existing in the volume swept out by the microphone due to the variation in the velocity of the microphone[1], and there is also the difficulty of making the amplitude of swing rather more than half a wavelength below certain frequencies.[2] Finally it is important to bear in mind that as the source is approached the true pressure is increasing, and *vice versa*. The result of these simultaneous effects is probably to introduce more uncertainty into the observations than is normally associated with stationary microphone measurements.

The other proposal is to superimpose a cyclical variation in frequency on the applied frequency, to prevent a steady wave system from being formed. This method can be considered partially successful for some purposes, but in the testing of loud speakers it tends to suppress the real narrow frequency-band peaks in the response characteristic, which are valuable in studying the mechanics of the vibrating system. At present, therefore, the writer is inclined to consider that response curves derived from stationary microphone—steady frequency tests, under the best conditions of room damping, are to be preferred, separate investigations being made to determine the errors involved and the general qualifications which must be placed on the measured response characteristics themselves.

The Definition of Acoustical Efficiency and Response.

(*a*) *Efficiency.*

This factor has been considered at some length in a paper by Bostwick.[3] He has pointed out that the function of a loud speaker is to draw maximum power from a supply source and to radiate maximum power to the air or load, and has defined efficiency by the equation.

$$\eta_1 = \frac{P_A}{P_E} \qquad \ldots \quad \ldots \quad \ldots \quad (1)$$

where

P_A = acoustic power radiated.
P_E = maximum electrical power which the supply circuit is capable of delivering under optimum impedance conditions.

P_E is equated to $e^2/4r$ which is the maximum power dissipated in a non-inductive load supplied from an ideal source of power having an internal resistance r and an open circuit voltage e. The load resistance is equal to the internal resistance of the source under these optimum conditions. This definition of efficiency can be fairly criticised from several points of view. In the first place it is an overall efficiency of power stage and loud speaker, and while such a criterion of performance is valuable, it is undesirable to combine a figure for the actual acoustical power output with an estimate of the electrical input which, from an experimental point of view, is artificial and to some extent fictitious in character. In practice the source of power is normally a thermionic power valve, the maximum A.C. output of which depends—besides other factors—upon the watt rating of the valve, the permissible percentage of harmonic distortion and the coupling arrangements used. Moreover, it is well known that for a triode power valve supplied at constant anode voltage the optimum power output into a non-inductive load, under sensibly ideal coupling conditions, is obtained when the load resistance is twice the internal valve resistance. The input impedance of most loud speakers has a reactive term as great as, if not greater than, that due to effective resistance, and in so far that the reactance present limits the maximum power attainable a further variable is introduced. Thus if an overall figure of efficiency is desired on the lines of equation (1) it appears better to re-define the overall efficiency at a given frequency by the relation

$$\eta_2 = \frac{P_a}{P_e} \qquad \ldots \qquad \ldots \quad (2)$$

where

P_a = measured useful acoustic power radiated.

P_e = maximum electrical power the fully loaded power stage under working conditions can deliver with a given percentage of harmonic into a non-inductive load at a specified frequency (say, 1,000 cycles per second).

The overall efficiency as defined by η_2 is therefore placed on a practical and experimental basis and both P_a and P_e are directly measurable. As the term "useful acoustic power" is intended to cover the sound radiation that can be utilised, the wasted sound from the back of a cone working in a large baffle would normally be ignored.

However, in view of the fact that loud speakers are ordinarily intended to work with an innumerable variety of power valves and coupling arrangements there seems to be a strong case for isolating the loud speaker itself and defining its efficiency on orthodox lines. The following definition is suggested

$$[\eta_3]_f = \frac{[P_a]_f}{[P'_e]_f} = \frac{\text{Total useful acoustical output power at a given frequency } f.}{\text{Total input electrical power at the same frequency } f.} \quad \ldots \quad (3)$$

P_a is defined by the same general expression given for P_A by Bostwick,[3] viz. :

$$P_a = \frac{1}{\rho c}\iint p^2 ds \qquad \ldots \qquad \ldots \quad (4)$$

where ρ = density of air, c = velocity of propagation of sound, p is the excess r.m.s. sound pressure and ds is an element of the surface of the sphere on which p is measured. The only difference between P_a and P_A is that for the former the limits of integration may be different. Equation (3) will be further examined later.

P_e' represents the electrical power supplied to the loud speaker at the frequency considered and referred to the same A.C. current at which the polar curves of pressure are taken. Hence if $\hat{Z^\theta}$ be the polar vector input impedance of the loud speaker and i be the current

$$P_e' = i^2 Z \cos\theta \qquad \ldots \qquad \ldots \qquad \ldots \quad (5)$$

or, generally, the efficiency is

$$[\eta_3]_f = \left[\frac{\iint p_i^2 ds}{\rho c i^2 Z \cos\theta}\right]_f \qquad \ldots \quad (6)$$

Bostwick has clearly pointed out the possibility of determining P_A or P_a from polar curves of sound pressure round a source of sound radiating into free space or into a completely absorbing room, but the details have not been elaborated. If p be the r.m.s. excess alternating pressure in dynes per square cm. in an element of a plane wave, and v be the r.m.s. air particle velocity in cm. per second, then we have the well-known relation

$$v = p/\rho c \text{ cm./sec.} \qquad \ldots \qquad \ldots \quad (7)$$

where ρ and c have the same meanings as before. The average acoustical power or rate of doing work per unit area of wave front

$$\left|\frac{dW}{dt}\right|_{av} = pv\cos\psi \text{ . ergs per sq. cm./sec.} \quad (8)$$

where ψ is the time phase angle between p and $v\delta$, and W is work in ergs.

However, in plane waves and in diverging waves at large distances from the source, p and v are in phase and hence $\cos\psi = 1$. Thus in a plane wave

$$\left|\frac{dW}{dt}\right|_{av} = pv = \frac{p^2}{\rho c} \text{ ergs per sq. cm./sec.} \quad (9)$$

Now 10^7 ergs per sec. = 1 watt.

Thus the acoustical power in watts in a plane wave for an element of area ds in which a pressure p is set up is

$$\left|\frac{dW}{dt}\right|_{av} = \frac{1}{\rho c}\, p^2 ds \,.\, 10^{-7} \text{ watts} \quad .. \quad (10)$$

Let us consider simple diverging waves in further detail. The r.m.s. pressure and velocity in a simple diverging wave set up by a point source of sound are given by the following equations.[4]

$$p_r = \frac{A\rho\omega}{4\pi\sqrt{2}\,.\,r} \quad .. \quad .. \quad (11)$$

$$v_r = \frac{A}{4\pi\sqrt{2}}\sqrt{\frac{1}{r^4} + \frac{1}{r^2}\frac{\omega^2}{c^2}} \quad .. \quad (12)$$

Where A is an amplitude factor.

r is the radial distance from the source.

ω is the angular frequency ($2\pi f$).

ρ and c have the same meanings as before.

The power factor angle ψ by which the velocity lags on the pressure is given by the equation

$$\tan\psi = \frac{c}{r\omega} = \frac{\lambda}{2\pi r} \quad .. \quad .. \quad (13)$$

where λ is the wavelength.

The acoustical power factor

$$\cos\psi = \cos(\tan^{-1}\lambda/2\pi r) \quad .. \quad (14)$$

This quantity is shown plotted for distances up to one wavelength in Fig. 1. Thus if p_r and v_r are measured at distances less than one wavelength from a point source, the average power per square cm. of wavefront is given by equation (8) where $\cos\psi$ is calculated from equation (14). It is assumed that the sound field is not disturbed by the measuring apparatus. A small condenser microphone and Rayleigh disc could be used to measure the pressure and air particle velocity respectively.

The ratio of pressure to velocity in a diverging spherical wave is given by

$$\frac{p_r}{v_r} = \rho c \bigg/ \frac{1}{2\pi}\sqrt{\frac{1}{n^2} + 4\pi^2} = \rho c/K \quad .. \quad (15)$$

where $r = n\lambda$.

K is plotted in Fig. 1 and is the ratio increase in velocity as the point source is approached to the velocity which would exist if a simple inverse distance law held. Substituting in equation (8) for the power we have

$$\left|\frac{dW}{dt}\right|_{av} = \frac{p_r^2}{\rho c} K\cos\psi = \frac{p_r^2}{\rho c} \text{ ergs per sq. cm./sec.} \quad .. \quad (16)$$

which is the same as equation (9) because $K\cos\psi$ is equal to unity. The final expression can, of course, be obtained by multiplying the general equations for p_r and v_r together, but the method given here is considered of interest and the curves of $\cos\psi$ and K are useful for other purposes. Equation (10), therefore, holds for a simple diverging wave at all distances, provided the sound can be regarded as being propagated from a point source. In the case of a piston source at low and medium frequencies this is sensibly true if polar measure-

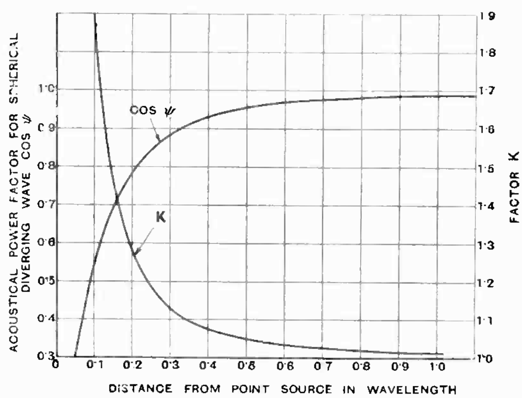

Fig. 1.—*Power and velocity factors for point source of sound.*

ments are made at a radial distance equal to several times the diameter of the source. This condition can usually be satisfied for small domestic loud speakers. For large horn-type speakers out-of-door measurements would appear essential. The evaluation of the quantity $\frac{1}{\rho c}\iint p^2 ds$ over, say, a hemisphere can be carried out graphically or by calculation.

Let XOX' in Fig. 2a represent the plane of the baffle of a loud speaker situated at O and radiating in direction OY'. Let

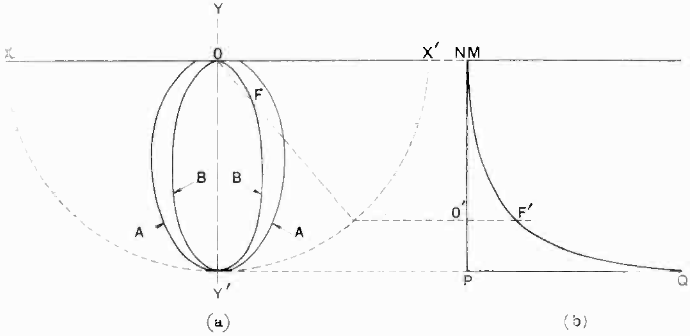

Fig. 2.—*Rousseau construction applied to polar curves of sound pressure.*

curve A be a polar curve of relative sound pressures p_1 for a given current i and frequency f, at a radial distance R. It is usual to let the pressure on the axis be unity. Let the actual axial pressure be q. Curve B is the square of curve A and is the one required. Axial symmetry is assumed. Apply now the well-known Rousseau[10] construction as shown in Fig. 1(b). Then the area $MNPQ$ ($NP = PQ = 1$) is seen to be $\int_0^{\pi/2} p_1^2 \sin\alpha \, . \, d\alpha \; (= S)$ and can be obtained by using a planimeter. Consider Fig. 3. Let p_1 be the relative sound pressure in direction α at radius R. Then $\int p^2 ds$ for this zone is $p_1^2 q^2 2\pi R^2 \sin\alpha \, . \, d\alpha$ or total acoustical power :—

$$\left|\frac{dW}{dt}\right|_{av} = \frac{2\pi R^2 q^2}{\rho c}\int_0^{\pi/2} p_1^2 \sin\alpha \, . \, . \, d\alpha \quad .. \quad (17)$$

$$= \frac{2\pi R^2 q^2}{\rho c} \times (\text{Area } NMQP = S)$$

$$\times 10^{-7} \text{ watts} \quad .. \quad (18)$$

Writing p_θ' for $[p_1^2]_{6^\circ}$ the area $NMQP$ can be derived by calculation by using the formula[5]

$$S = \tfrac{1}{2}[(p'_0 + p'_5)(\cos 0^\circ - \cos 5^\circ) + (p'_5 + p'_{10})(\cos 5^\circ - \cos 10^\circ) \; . \; . \; . \; . \; . \; . \; . \; . \; (p'_{80} + p'_{85})(\cos 80^\circ - \cos 85^\circ) + (p'_{85} + p'_{90})(\cos 85^\circ - \cos 90^\circ)] \quad .. \quad (19)$$

The evaluation of equation (19) can be facilitated by the use of tables of differences of cosines or, provided the polar curve of p_1^2 is not too complicated, the simplest method of determining S is to measure or scale off the values of p_1^2 at the Russell angles.[11]

If now ρc be taken as 42, eq. (18) gives

$$[P_a]_f = 0.150 \, R^2 q^2 S \times 10^{-7} \text{ watts} \quad .. \quad (20)$$

In general $q^2 = \beta^2 i^2$ where q is the sound pressure set up at a point on the axis at a distance of R cm. Hence as an alternative to equation (20) we have

$$[P_a]_f = 0.150 \, (R\beta i)^2 S \times 10^{-7} \text{ watts} \quad .. \quad (21)$$

β is a constant over the working range and is a (dynes per square cm. per ampere) factor. From equation (6), the percentage acoustical efficiency becomes

$$[\eta_3]_f = \frac{0.150 \, R^2 \beta^2 S}{[Z \cos\theta]_f} \times 10^{-5} \quad .. \quad (22)$$

It is, therefore, apparent that to measure an acoustical efficiency at any frequency on the lines indicated, it is necessary to have besides a polar distribution curve a knowledge of the impedance, the value of the radial distance, and any pair of the three variables q, β and i.

(*b*) *Response.*

The term "response" is in general use in electro-acoustical literature to imply the activity of a device when frequency is the

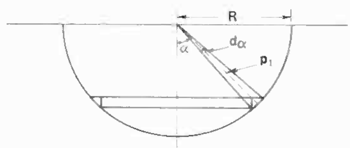

Fig. 3.

independent variable. Curves of amplitude, velocity, or pressure plotted against frequency are all popularly referred to as "response curves." Very few writers seem to have defined the term and those who have already published performance curves of different types of loud speakers have usually been content to leave the results in the form of curves of sound pressure to a logarithmic base of frequency. Such curves make the apparent quality of the reproducer appear much worse than the impression which would be given aurally and the question is raised as to whether it is not possible to depict the results in some way which would enable a graphical impression of quality to be given. This is partially possible and the decibel notation of telephone engineering is valuable in this connection because having a logarithmic basis an approximation is made to the apparent impression experienced by the ear. The decibel system is essentially one of power ratios. Two powers P_1 and P_0 are said to differ by $10 \log_{10} \frac{P_1}{P_0}$ decibels. Here if P_0 is regarded as the reference level and $P_1 > P_0$ the value of decibels, N say, will be positive and P_1 is said to show a gain of N decibels over P_0. If P_1 is taken as the reference level, N will be negative and P_0 is then said to show a loss of N decibels on P_1. When dealing with currents or voltages and reckoning their ratios in decibels it is necessary to associate with them a real or assumed constant resistance. Thus as $P = I^2R = V^2/R$, two currents I_1 and I_0 differ by $10 \log_{10} \frac{I_1^2R}{I_0^2R}$ which equals $20 \log_{10} \frac{I_1}{I_0}$ decibels. Similarly two voltages V_1 and V_0 differ by $10 \log_{10} \frac{V_1^2/R}{V_0^2/R}$ which is equal to $20 \log_{10} \frac{V_1}{V_0}$ decibels. In the same way it is suggested that the response of loud speakers when expressed in decibels should be defined on similar lines. From equations (7) and (9), we have seen that acoustical power is $v^2\rho c$ or $p^2/\rho c$ per square cm. of wavefront in completely absorbing media. v and p have the well-known analogous relations to current and voltage and the constant acoustical resistance term with which they are associated is seen to be ρc. The quantity ρc is now termed the "radiation resistance of the medium per unit area" and was first suggested by Nichols.[6] If, therefore, velocity or pressure and frequency are the dependent and independent variables respectively, two velocities v_1 and v_0 or two pressures p_1 and p_0 can rightly be said to differ by $20 \log \frac{v_1}{v_0}$ decibels and $20 \log \frac{p_1}{p_0}$ decibels respectively when measurements are made in a completely absorbing medium. These definitions differ from the "response" in decibels as defined by Bostwick[3] whose expression involves a quantity having the dimensions of an electrical input power, and is in form an arbitrary and idealised measure of the overall efficiency of power stage and loud speaker. The points raised in connection with the quantity P_E again largely apply and such a definition, although having a certain amount of theoretical justification, has many practical drawbacks. Thus to express a sound pressure-frequency curve in the decibel notation, we have to take the ordinate p_0 at a specified frequency f_0, and if p is the pressure at any other frequency f, the decibel curve can be obtained by plotting $20 \log_{10} (p/p_0)$ to a logarithmic base of f. The response will usually vary above and below zero level, which brings out a further point. A decibel response curve is purely a non-dimensional curve of ratios and thus it is always necessary to state in addition the reference pressure p_0. The recommended specified reference frequency at which to take the reference pressure p_0 is 1,000 cycles per second, which is suitable in practice for most electro-acoustical apparatus. Further necessary information is microphone position and a statement of the value of the constant A.C. input current. In comparing two response curves in decibels of which the reference pressures are p_0 and $n\ p_0$ respectively, if we retain p_0 as the reference level for both curves the second curve must be considered displaced bodily in the appropriate direction by an amount $20 \log_{10}n$. Further, if the ordinates of the output stage current variation, expressed in decibels, be added to the corresponding ordinates of the loud speaker response in decibels, measured at constant current, the general effect on the quality can be seen immediately from the resulting curve.

Caution must be exercised in drawing conclusions from response curves as they are not necessarily completely indicative of the quality of reproduction. It is, nevertheless, widely agreed that a response curve, properly obtained and interpreted, is the best single criterion of performance. Light is thrown on the response to transient excitation by the general uniformity and frequency range of the response curve measured under steady conditions. This follows from the consideration that if a compound system resonates strongly at several frequencies then impulsive or shock excitation tends to set up oscillations at approximately these frequencies, and the effects may be serious from the point of view of good reproduction. Hence if there are no very prominent peaks—indicating strong resonances—and the range of response is good, on general grounds the ability to reproduce transients may be expected to be good, unless serious phase distortion is present.

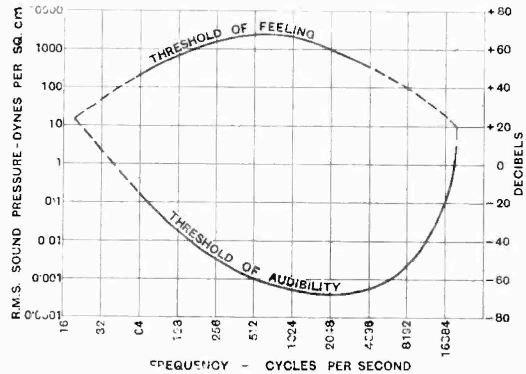

Fig. 4.—*Average threshold characteristics of the ear.*

Approximate limits of the frequency range of reproduction can be derived from complete characteristics, but a really reliable determination of the low frequency limits usually involves special out-of-door measurements. The general aspects of the interpretation of response curves will not be elaborated further except with regard to aural effects which receive a measure of detailed treatment in the next section.

Loudness in Relation to Response Curves.

On the supposition that the response of a loud speaker has been expressed in the decibel notation, the general reasons for which have already been given, the following question quite naturally arises. Are we justified in assuming that equal small departures in decibels from a mean level response will sound equally loud at all frequencies to an average ear? This question is obviously of some importance as we may be better able to tolerate by ear a given measured departure from the mean level at one part of the frequency range than at some other part, or, from another point of view, the tolerance that can be allowed in the response curve due to lack of acuteness of the ear may vary with the frequency. Actually the permissible tolerance does vary to some extent and the magnitude of the effect will now be examined.

Let us consider pure tones to enable some idea to be formed of what is to be expected in the general case. We are popularly familiar with loudness as the magnitude of the sensation produced in our ears by an impressed sound. It is a well-known psychological law that, for a pure tone, equal steps on a logarithmic scale of pressure sound approximately like equal loudness steps. A true scale of loudness is that in which the numbers expressing loudness are proportional to it. It is necessary at this stage to introduce the conception of "sensation level." The sensation level of any sound reaching the air is defined[7] as the number of decibels the sound is above the threshold level for audition, and is just a special way of reckoning the ratio of the pressure in a sound wave to the sound pressure at the same frequency just inaudible to an average ear. For a constant sound pressure the sensation level varies considerably over the frequency range. This is made clear by the well-known and oft-quoted curve Fig. 4 which shows the average threshold characteristics of the ear.[8]

In practice, it is found that different observers having normal hearing can balance two notes of different frequency if the intensity of one note is fixed and the intensity of the other can be varied. The note of fixed intensity is called the reference tone and Kingsbury[9] in his work used a pure note having a frequency of 700 cycles per second. This reference source was made to emit steadily and the adjustable source altered until it appeared equally as loud by ear. This was repeated at different frequencies and with different observers. The sensation level of the reference tone was then changed and the process repeated. Kingsbury's final results are shown in Fig. 5. He found that loudness is directly proportional to sensation level between 700 and 4,000 cycles, and the numbers expressing the loudness were chosen to be the same as the numbers expressing the sensation levels in decibels for this frequency range. The loudnesses of the lower frequency tones were then expressed in the same scale of loudness. The application of these results to our present problem will now be considered.

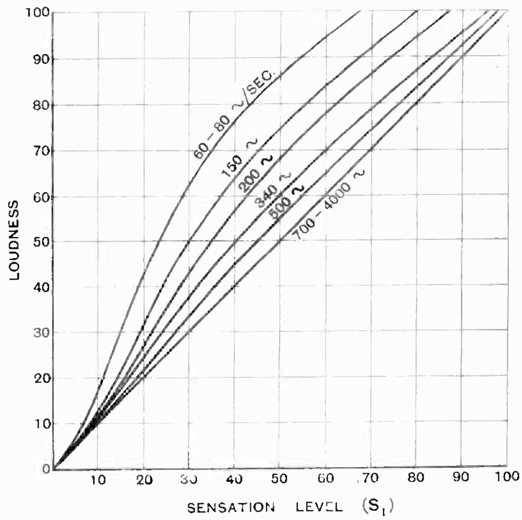

Fig. 5.—*Variation of loudness with sensation level and frequency (Kingsbury).*

The slope of each of the curves in Fig. 5 gives us $(\partial L/\partial S_l)_f$ or the rate of change of loudness with change in sensation level at a given frequency. The curves of Fig. 5 have been differentiated graphically and the derived curves are shown in Fig. 6. These new curves are also curves of relative loudness per decibel at different frequencies if we agree to take the change in loudness per decibel change in sensation level at the higher frequencies as a standard. This follows from the fact that on the loudness scale adopted unit change in sensation level produces unit change in loudness for all sensation levels and all frequencies between 700 and 4,000 cycles per second.

In interpreting loud speaker response curves we require a knowledge of the relative loudness per decibel change about a practically constant level of *sound pressure.* From Fig. 4 we derive the sensation levels at different frequencies for various specified values of constant sound pressure, and by using Fig. 6 we can therefore obtain the relative changes in loudness per unit change in loud speaker response for stated constant sound pressure levels. These are shown in Table I.

From Table I we see that for low and medium sound pressures (1–10 dynes/sq. cm.) the relative loudness is practically constant for all frequencies above about 100 cycles per second. This implies that in a response curve plotted in decibels little error will be made in regarding equal departures from a mean level as equal departures in loudness. If greater precision is desired, then, from the point of view of loudness, more weight must be attached to departures at low frequencies for small volumes. To give an example from Table I, a departure of say 4 decibels at 128 cycles about a sound pressure level of 0.1 dyne/sq. cm. represents an audible step which is just about twice as loud as the same departure at 1,024 cycles. At ten times the pressure level the same step is only 40 per cent. louder under the same conditions. It may be noted that for very loud sounds the relative loudnesses of the lower frequencies

TABLE I

Frequency Cycles Per Second.	Sound Pressure. 0.1 dyne/sq. cm.		Sound Pressure. 1.0 dyne/sq. cm.		Sound Pressure. 10 dynes/sq. cm.		Sound Pressure. 100 dynes/sq. cm.	
	Sensation Level.	Relative Loudness Per Decibel.	Sensation Level.	Relative Loudness Per Decibel.	Sensation Level.	Relative Loudness Per Decibel.	Sensation Level.	Relative Loudness Per Decibel.
4,096	46	1.0	66	1.0	86	1.0	106	1.0
2,048	48	1.0	68	1.0	88	1.0	108	1.0
1,024	45	1.0	65	1.0	85	1.0	105	1.0
512	39	1.1	59	1.0	79	0.9	99	0.9
256	28	1.5	48	1.0_3	68	0.8	88	0.8
128	14	$2._1$	34	1.4	54	0.9	74	0.8
64	—	—	16	2.5	36	1.3	56	0.8_5

tend to fall off rather than to increase. We therefore feel justified in saying that equal departures in decibels from a general response level can, with but little error, be taken to be equal steps in loudness for normal sound volume levels. Nevertheless it is an interesting point that while the same relative values of response at different frequencies may be obtained for a wide range of A.C. currents, the precise loudness values of fluctuations in the response must be assigned with due regard to the actual volume level at which the loud speaker is normally to be worked.

A change in sound output of one decibel is usually considered to be "just detectable" by a trained ear when two pure tones of the same frequency are sounded, the one immediately after the other. If a short interval elapses about twice this amount is required, while approximately three decibels change is necessary in ordinary circumstances. The "three decibel levels" marked on loud speaker response curves have been found useful as indicative of "quite definite" steps in sound output. Variations of ± 1.5 decibels about a general level can be regarded as negligible frequency distortion. Extreme variations in response no greater than about 12 decibels are now found in good moving coil loud speakers within their frequency range, when tested under working supply conditions.

The Principal Variables in Acoustical Response Measurements.

To make the test conditions definite it is necessary at this stage to discuss the variables involved. We will assume that in all cases the reproducer is supplied from the power stage of a thermionic valve amplifier. Broadly speaking, we have :—

(*A*) The loud speaker.

(*E*) The coupling system between loud speaker and power valve.

(*C*) The power valve.

It is possible to combine *A*, *B* and *C* together and specify all the conditions, but in practice this is desirable in particular cases only, because a loud speaker is normally intended to work with a variety of power valves and modes of coupling. Moreover, perfect matching of loud speaker and power valve from an impedance point of view can only be attained over a very limited range of frequencies. Thus it must be possible to test individual loud speakers under controlled conditions which will yield comparable results. Hence if we regard frequency as the independent variable we must fix,

(*a*) The D.C. excitation of the loud speaker, unless fitted with permanent magnets.

(*b*) The input alternating current which is maintained constant at all frequencies.

(*c*) The position of the sound pressure or velocity measuring instrument.

To maintain a constant alternating voltage at all frequencies across the input terminals would be an alternative to (*b*) above, but

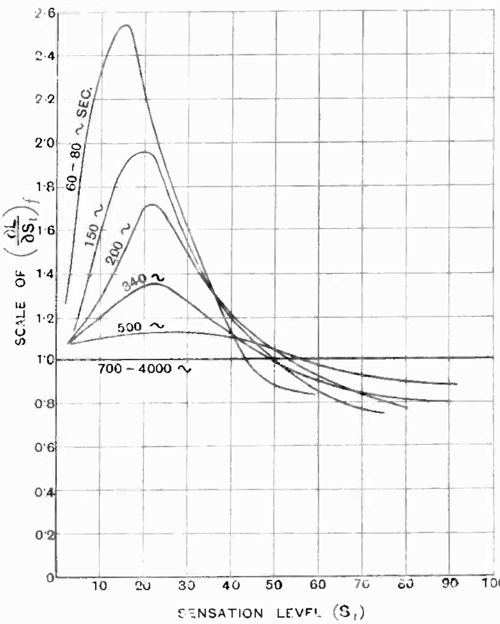

Fig. 6.—*Variation of rate of change of loudness with sensation level at different frequencies.*

this is undesirable in practice as will appear later. For a given frequency, (*a*), (*b*) and (*c*) can each be made the independent variable in turn, the variation of sound output with the variation of D.C. excitation usually being relatively small for the makers' specified range of working voltages. The variation of sound output with A.C. current is of considerable importance. In the general case the sound pressure is proportional to the A.C. input current over the working range. Towards the superior limit of current lack of proportionality gives information on amplitude distortion and is usually accompanied by the production of overtones. For a given frequency and fixed values of (*a*) and (*b*) the variation of sound pressure with distance is not usually a very simple relation and depends on the size and character of the diaphragm and the mode of vibration. There are, however, some guiding principles which have already been pointed out.[3] Considering the ideal piston again, it has been calculated that up to an axial distance of $\frac{D^2f}{4{,}500}$ feet (or $7.3\,D^2f \times 10^{-6}$ cm.), where D is the piston diameter in feet or cm. and f the frequency in cycles per second, there is a series of sound pressure maxima and minima due to interference between sound arriving from different parts of the disc, after which the pressure varies inversely as the distance. Hence in a perfectly absorbing test chamber the response curves when measured beyond this minimum distance in a given polar direction should all have the same shape. This minimum distance is not very great for diaphragm speakers of the ordinary commercial size. Some values are given in Table II.

Curve A.—*Response at constant current of* 5.3 *milliamperes. Zero decibels* ≡ 1.9_5 *dyne/cm.*^{2}.
Curve B.—*Overall response of power stage. Zero decibels* = 7.5 *dyne/cm.*^{2}.

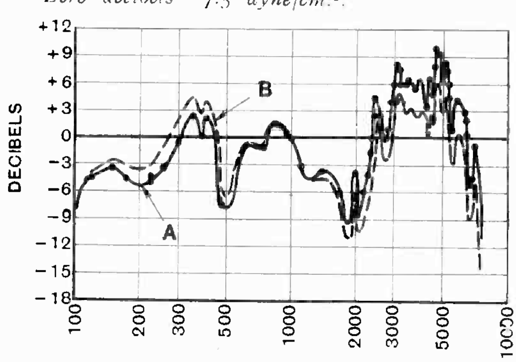

Fig. 7.—*Response curves for moving coil loud speaker.*

Measurements of response taken within the minimum distance have no immediate practical importance for the ordinary sizes of commercial domestic loud speakers and therefore all response measurements should be taken at or beyond this distance whenever possible.

Objections have been raised to response measurements being made at constant A.C.

Power stage, Fig. 9. H.T. voltage 308. *Anode voltage* 270. *Grid bias* −69 *volts. A.C. Input* 48 *volts (r.m.s.).*

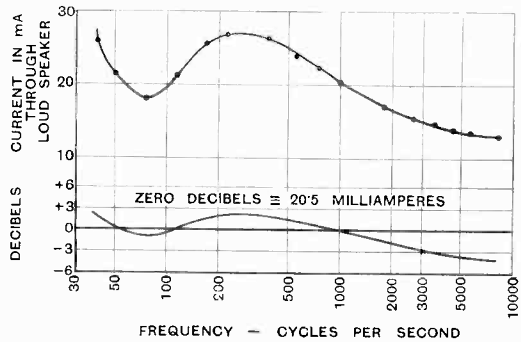

Fig. 8.—*Variation of current with frequency through moving coil loud speaker.*

input current. However, the value of this procedure has proved itself in practice, for to derive the working overall response characteristic with a particular valve and coupling arrangement involves but a measurement of the A.C. current which flows through the loud speaker at different frequencies when connected in the output circuit of the power stage. The grid of the power valve would normally be fully loaded and supplied with an A.C. input voltage, constant at all frequencies. The variation of current with frequency can also be calculated from a knowledge of the speaker impedance at different frequencies and the ordinary power stage particulars. Measurements of speaker impedances for the vibra-

TABLE II

Diameter of Piston. ins.	cm.	Highest Frequency, Cycles per Second.	Minimum Distance Approx. ins.	cm.
6	15	6,000	4	10.5
9	23		9	22.5
12	31		16	37.5
6	15	8,000	5½	14
9	23		12	30
12	31		21½	55

ting condition are of value not only in estimating the effect of different assemblages of valves and coupling arrangements but are necessary for calculating the power consumption and efficiency.

Experimental Results.

Some typical experimental results, with particular reference to acoustical response measurements, will now be given in general support of the recommendations which have been put forward. These tests were carried out in the heavily damped sound-testing chamber at the Research Laboratories of the General Electric Company, but a description of this room and of the measuring equipment which has been installed will not be given here.

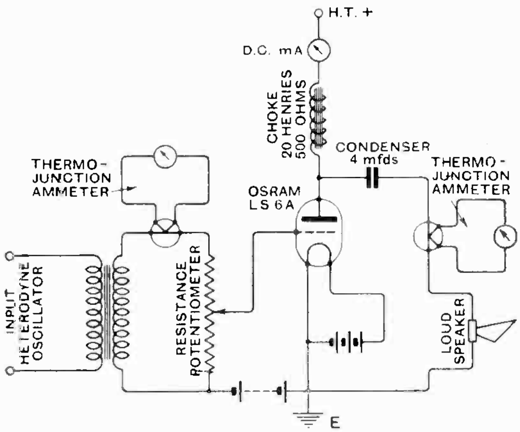

Fig. 9.—*Power stage and loud speaker circuit diagram.*

The response curve of a moving coil loud speaker, taken at constant current, is shown in Curve A, Fig. 7.

The variation of current with frequency, when the loud speaker was connected in a suitable power stage and the A.C. input voltage to the grid was maintained constant at all frequencies, is shown in Fig. 8. The general circuit arrangements are shown in Fig. 9. The overall response of power stage and loud speaker are shown in Curve B, Fig. 7, by superimposing the decibel current variation curve of Fig. 8 on to Curve A, Fig. 7, taken at constant current. The sound output levels have, of course, altered considerably, but the effect on the quality is seen by comparing curves A and B in Fig. 7.

The general effect has been to increase the intensities of the lower frequencies and to diminish the intensities of the higher ones when 1,000 cycles is taken as the reference

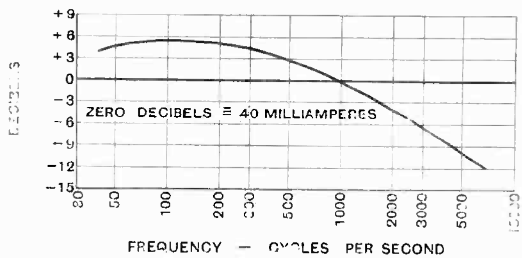

Fig. 10.—*Typical curve of variation of current with frequency for high impedance loud speaker.*

frequency. This type of result is usually found but varies considerably from speaker to speaker. Another curve of current variation only for a higher impedance speaker tested under the same conditions is given in Fig. 10. It is seen that this curve superimposed on a response curve at constant current would profoundly modify the response and would give a valuable indication of the quality under working conditions.

A typical curve of the fall-off in sound pressure with distance for the same moving coil loud speaker is shown in Fig. 11. For

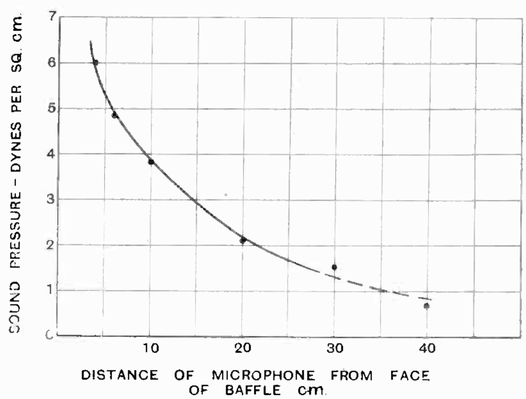

Fig. 11.—*Variation of sound pressure with distance at constant current. Frequency—1,000 cycles per second.*

this loud speaker and for several others of different types the sound pressure set up at a given point was found to be proportional to the current over quite large ranges.

A few curves showing the polar distribution of sound pressure round a cone loud speaker mounted in one end of the test chamber are given in Fig. 12. The axial pressure has been taken as unity in each case. The general shape of the curves is in agreement with the kind of distribution which can be calculated for a piston working in an infinite baffle, but in the case of the cone the beam effect is less marked. The main conclusion at this stage, without going into details, is that a response curve measured at a point on the axis shows up the higher frequencies to the best advantage.

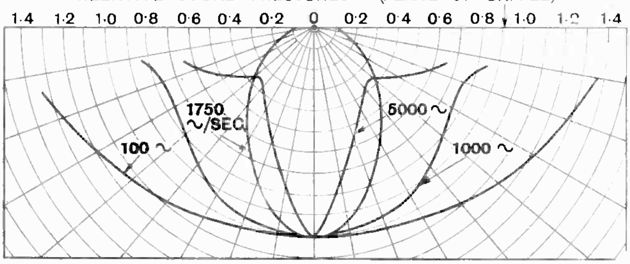

Fig. 12.—*Typical curves of polar distribution of sound pressure around cone loud speaker at different frequencies.*

Summarised Proposals.

It is proposed that in making response measurements on small domestic loud speakers or in expressing the results, the following shall be the conditions and requirements.

(*a*) An effectively infinite baffle.

(*b*) A completely absorbing medium (open air or heavily damped chamber).

(*c*) A stated microphone position at a radius not less than a certain minimum value, already discussed. The normal position to be on the axis.

(*d*) A stated D.C. excitation, if any.

(*e*) A stated constant A.C. input current.

(*f*) The response as already defined to be expressed in decibels, and the reference pressure at 1,000 cycles per second to be stated.

(*g*) In overall response measurements, the current variation with frequency in the output circuit to be expressed in decibels and the reference current at 1,000 cycles per second to be stated. Full power stage and test particuars to be given, with, if possible, the valve characteristics.

Whilst the above list is not complete it should serve, if agreed, as a first step towards a much-needed standardisation of test conditions. Other desirable tests, such as those of transient response, efficiency, handling capacity, harmonic production and impedance, do not lend themselves to easy specification, and much depends on the equipment and on the comprehensiveness of the results desired.

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On the Efficiency-rating of Transmitting Aerials for Broadcasting Distribution.*

By E. T. Glas.

SUMMARY.—A proposal is given for the numerical rating of broadcasting antennas relative to the true half-wave aerial. The result is discussed for theoretical linear radiators.

THE total efficiency of a transmitting station can be calculated if the supplied and emitted power have been measured. The supplied power is measured without any difficulty by well-known methods, but the emitted power—*i.e.*, the total radiation from the antenna, presents difficulties as we are only able in practice to determine the strength of the radiation along the surface of the earth. Unfortunately, the efficiency of the antenna cannot be given beforehand, because the current-distribution along the linear conductors of the antenna—this distribution mainly determining the radiation—is dependent on too many factors (some not very fully investigated) and the influence of masts, stays, and other mechanical details as well as of the ground connection, cannot be expressed by formulæ, at any rate, at present. The antenna efficiency, as is well known, can be expressed as

$$\eta_a = \frac{R_s}{R_s + R_0}$$

where R_s is the radiation resistance relative to the current at the base of the antenna and R_0 its loss resistance, mainly concentrated at the base as ground-lead resistance. The power radiated as space-radiation here enters as useful power, since R_s is the resistance equivalent to the total radiation. When ground radiation is the only thing aimed at, the above formula says nothing about the actual efficiency of the aerial. The distribution of broadcasting within the range of a certain station for reliable reception is effected—as far as we now can see—exclusively by the ground-wave. But the space-wave limits the distribution area available in practice, because fading by interference sets in at a certain more or less definite distance from the transmitter. Thus for a commercial broadcasting distribution the space-wave is only an evil.

Restricting ourselves to wavelengths used for present broadcasting, some other definition must be accepted, if the corresponding efficiency is to indicate the efficiency of the antenna for the purpose that is considered. In practice, instead of the old formula one looks for a value of the power, radiated through a certain surface close to the ground per kilowatt total antenna power, an expression of the general type $k = \frac{(E \cdot d)^2}{P_a}$ where k is a constant and the other quantities will be defined later on. The figures thus obtained may be directly compared for different antennas, but in order to have definite values of the antenna-efficiency it is convenient to take a well-defined, simple aerial as a basis for the comparison.

The most acceptable reference aerial seems to be a no-loss half-wave antenna—*i.e.*, a straight, vertical radiating conductor, the distribution of current along which at resonance forms a simple sine wave with nodes at the base and top (Fig. 3c). Theory tells that this type of single antenna gives the greatest field strength at the ground (the supplied power being the same) if we make it a condition that no additional space lobes are allowed for in the vertical radiation diagram. As for this type the current is zero, or practically very small at the base, we further need not prescribe anything concerning the ground-lead (the predominant loss occurring in the vicinity of the ground-connection for straight vertical aerials). Finally, the efficiency of an antenna that may be constructed in practice will be within 100 per cent. A combination of aerials, particularly half-wave aerials, multiple tuned or vertical phase-reversing types like the Franklin

* MS. first received by the Editor, April, 1929.

radiator, may, however, provide an exception. Theoretically, the half-wave antenna ensures minimum high angle radiation and should thus reduce short distance fading. It should also be observed that a practical half-wave antenna coincides more closely with the theoretical image than does any other type, including the quarter-wave antenna, which is very sensitive to the ground-lead conditions owing to the large current at the base.

In the following considerations we assume that two fundamental measurements on the antenna, which is to be examined, have been carried out, namely:

(1) *A measurement of the total antenna resistance,* relative to the base, the only point in fact which is accessible to common investigation. If an equivalent antenna resistance R_a has been measured at the base of the antenna and the current at the same point is I_a, the total antenna power is $P_a = R_a\ I_a^2$.

(2) *A series of short distance measurements of the field-strength at the ground* in order to get a mean value of the product $E\ d$. The measuring distance is determined by the two facts, that there shall be no appreciable attenuation and that the radiation field shall be fully developed. The conformity with the hyperbolical law governing a true radiation is likely to be good already for a quarter-wavelength distance.

Starting from 100 per cent. efficiency for the no-loss half-wave aerial, it will be shown below how a simple efficiency formula can be deduced according to the following definition.

The efficiency of an antenna for present broadcasting is the percentage ratio of the power, that is radiated between the surface of the earth and a conical surface with the antenna as axis and which forms a sufficiently small angle with the earth's surface to allow for assuming the field-strength as constant and equal to the field-strength at the ground along a spherical zone, that is cut out by the earth's surface and the conical one, to the power, that would be radiated within the same limited space from a no-loss half-wave aerial, which is fed with the same power as the antenna under consideration.

According to Fig. 1 the surface of the spherical zone is $2\pi d^2 \triangle\phi$. Poynting's theorem gives the intensity of the radiation as $\frac{E_0^2}{4\pi c}\sin^2\omega t$. Thus the radiated energy through the spherical zone, mean value per period

$$\triangle P = 2\pi d^2 \triangle\phi \frac{E_0^2}{4\pi c^2}\,\frac{1}{2} = \frac{\triangle\phi}{2c}(E\,.\,d)^2\,;$$

$$[E_0 = E\sqrt{2}]$$

If d is given in km. and E in volt/m. this will become

$$\triangle P = \frac{\triangle\phi}{2\,.\,3\,.\,10^{10}}(E\,.\,10^8.\,10^{-2}.\,d\,10^5)^2\,10^{-7}\,10^{-3}$$

$$= \frac{50}{3}\triangle\phi(Ed)^2\ \text{KW.}$$

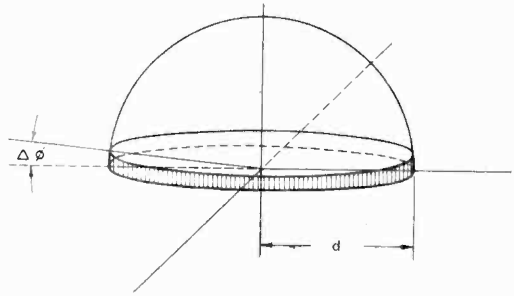


Fig. 1.

For a theoretical half-wave antenna we have

$$h_{\text{eff.}} = \frac{2}{\pi}\,h = \frac{2}{\pi}\,\frac{\lambda}{2} = \frac{\lambda}{\pi}$$

and the field strength at the ground will be

$$E' = 120\pi\frac{h_{\text{eff.}}\,I'_{\text{max.}}}{\lambda d} = 120\frac{I'_{\text{max.}}}{d}\ \text{volt./m.}$$

(*R.M.S.*)

if $I'_{\text{max.}}$ *i.e.*, the current maximum along the antenna is rated in amp. and d here in m. The radiation resistance of the ideal half-wave aerial is $R'_s = 100$ ohm, referred to the current maximum. Thus the total radiated power

$$P' = 100\,I'^2_{\text{max.}}\ 10^{-3} = \frac{1}{10}\,I'^2_{\text{max.}}\ \text{KW.}$$

According to our definition the antenna-power equals

$$P_a = P' = \frac{1}{10}\,I'^2_{\text{max.}}\ \text{KW.}\,; \quad \text{therefore } (E'd)^2 = 120^2\ 10\ P_a$$

$$\because \qquad \triangle P' = 2,4\,\triangle\phi\,P_a\ \text{KW}$$

whence $$\eta = \frac{\triangle P}{\triangle P'} \simeq 7\,.\,\frac{(E\,.\,d)^2}{P_a}$$

where E is in volt/m. d in km., and P_a in KW. Finally, we have the percentage efficiency of the antenna

$$\eta = 700 \frac{(E \text{ volt/m. } d_{km.})^2}{P_{a_{KW}}} \% \quad .. \quad (1)$$

a simple formula, not difficult to remember.

It should be noted that the quantity $E\,d$ by introducing these very units will have a most handy numerical value for practical antennas of reasonable power. By a happy chance the formula for the corresponding meter-amperes is simply

$$MA = \lambda_{m.} \cdot \sqrt{\frac{\eta}{100} \cdot P_{a_{KW}}}$$

with an error less than 1%.

It is interesting to undertake a more intimate study on the values of the efficiency according to formula (1) which will come out for different types of theoretical radiators, *i.e.*, straight, vertical no-loss wires with a certain definite sinusoidal distribution of current. As is well known, practical antennas in many respects differ from these ideal images. Thus the corresponding efficiencies should be looked upon as values wished for, which we can only approach more or less in practice.

For a theoretical radiator over a perfectly conducting ground we have

$$E = 120\pi \frac{h_{\text{eff.}} \cdot I_{\text{max.}}}{\lambda d} = 30\pi \frac{I_{\text{max.}}}{d} \frac{h_{\text{eff.}}}{h} \frac{1}{\frac{\lambda}{4h}};$$

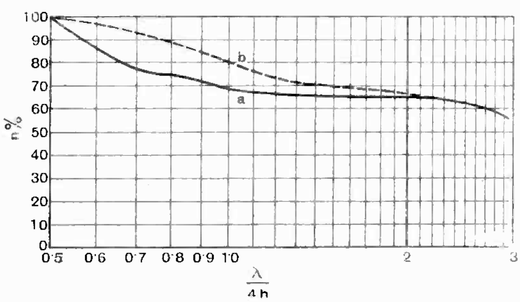


Fig. 2.

The Efficiency of Theoretical Aerials.
a, Antenna-current zero at the top (case a).
b, Antenna-current zero at the base (case b).

therefore

$$(E\,d)^2 = (30\pi)^2 I_{\text{max.}}^2 \left(\frac{h_{\text{eff.}}}{h}\right)^2 \frac{1}{\left(\frac{\lambda}{4h}\right)^2};$$

If $I_{\text{max.}}$ is the current maximum along the no-loss radiator, we further have $P_a = R_s I_{\text{max.}}^2$ where R_s is the radiation resistance at this maximum. Reduction to the chosen units will give the following formula for the theoretical radiator

$$\eta = \frac{630\pi^2}{R_s} \left(\frac{\frac{h_{\text{eff.}}}{h}}{\frac{\lambda}{4h}}\right)^2 = f\left(\frac{\lambda}{4h}, \psi\right) \quad .. \quad (2)$$

Both $\frac{h_{\text{eff.}}}{h} \left(= \frac{\int_0^h I.ds}{I_{\text{max.}}} \right)$ and R_s are functions

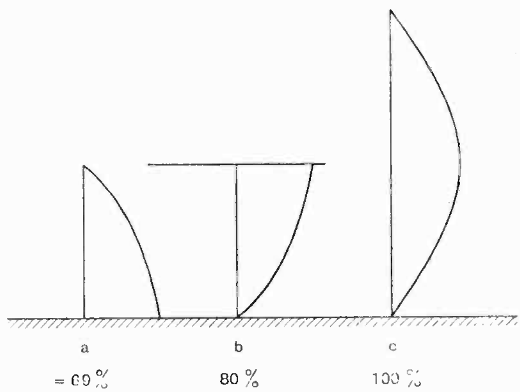


Fig. 3.

only of $\frac{\lambda}{4h}$ and the electric displacement angle ψ of the current distribution curve,* this angle being determined by the load. Zero-current assumed at the top or base of the aerial η will be a function only of $\frac{\lambda}{4h}$. Fig. 2 shows graphically $\eta = f\left(\frac{\lambda}{4h}\right)$ in these special cases. The curve corresponding to zero current at the base of the aerial is calculated without taking into consideration the radiation from the horizontal top of the aerial, which should be zero in a symmetrical case.

Particularly important is the no-loss ¼-wave aerial (Fig. 3a) where $R_s = 36.6$ ohm; $\frac{\lambda}{4h} = 1$; $\frac{h_{\text{eff.}}}{h} = \frac{2}{\pi}$; thus $\eta = 69$ % and the

* For the calculation of the radiation resistance see v. der Pol. Jun., *Proc. Phys. Soc.*, London, 1917, 15/6, p. 279. Jahrbuch der D.T. XIII, 1918, p. 229.

"reversed" $\frac{1}{4}$-wave aerial (Fig. 3b) where $R_s = 31.7$ ohm; $\frac{\lambda}{4h} = 1$; $\frac{h_{eff.}}{h} = \frac{2}{\pi}$; thus $\eta = 80\%$.

The straight, vertical radiator with a maximum radiation resistance at the current maximum, namely, 105.5 ohm, for

$$\frac{\lambda}{4h} = 0.56\,; \quad \frac{h_{eff.}}{h} = 0.695$$

(zero current at the top) gives $\eta = 91\%$.

As soon as $\frac{\lambda}{4h} < 0.5$ space-radiation lobes will be developed. Consequently, such aerials fall out of the scope of this article, which is restricted to the antenna-efficiency for antennas, suitable to a commercial broadcasting distribution.

Although the short-distance measurements of the field-strength on which the efficiency-rating is based in practice, do not simply state the quantitative distribution—for this we need systematical long-distance measurements all over the distribution area—they nevertheless make it possible quickly to gain an idea of the efficiency of the antenna itself, which is of a very considerable importance when the point is to estimate successively the value of different antenna arrangements, for instance, while the building of a station is in progress. I think personally that the aerial problems should be solved more quickly if the broadcasting governments made it a duty to publish figures of their aerial-efficiencies according to some system of which this article may give a possibility.

The Estimation of the Sensitivity of the Grid Rectifier for Large Inputs.*

By the late C. D. Hall, B.Eng.

THE author undertook some measurements on the sensitivity of the grid rectifier using low impedance valves of the "L" type with high anode voltages followed by resistance coupling. As a check on the resulting curve, the output was calculated by the method suggested by W. A. Barclay† using measured valve characteristics. When the practical results were compared with the theoretical curve, the former showed a much smaller output than the latter. This was thought at first to be due to a change in the values of the components due to the higher frequency. But, as the result of experiment, it was found to be due to the small time constant of the grid condenser and leak circuit.

In order that the grid potential may be able to follow the modulation, this time constant must be small compared with the time of an audio-frequency oscillation, but if the carrier frequency is not very great, compared with the highest modulation frequency, the time constant of the grid circuit becomes comparable with the periodic time of the carrier input wave. When this is so, the grid condenser will discharge between each positive half wave and thus reduce the effective input.

In considering this discharge of the grid condenser, there are two paths open for the charge to leak away, one through the grid leak, which is constant, the other through the grid-filament conductance, which varies from zero up to a maximum, when the input E.M.F. is at its maximum positive value.

For the purpose of approximate calculation, the conductance has been assumed to be the mean, taken over the whole wave. The results obtained by making this assumption seem to justify it. The grid-filament conductance will vary with the input and must be computed for each value of input,

* We regret to learn that the author, who was a research student at Liverpool University, has died since submitting this article for publication on 26th June, 1930.

† "Grid Signal Characteristics and other aids to the Numerical Solution of Grid Rectification Problems." W. A. Barclay. *E.W. & W.E.*, 1927, pp. 450-466 and 552-558.

and the appropriate value taken in making the correction.

The capacity in circuit consists of the grid condenser in parallel with the grid filament capacity of the valve. This latter is of the order of $15\mu\mu$F. in the valves tested.

If the grid-filament resistance is R_1 and the grid leak resistance is R, then the effective resistance

$$R_0 = \frac{R_1 R}{R_1 + R}$$

The voltage across the condenser will fall according to the equation

$$e = E_0 e^{-t/T_0} \text{ (see Fig. 1).}$$

Where $T_0 = R_0 C$, and E_0 is the initial value of e which would be maintained if there were no leakage.

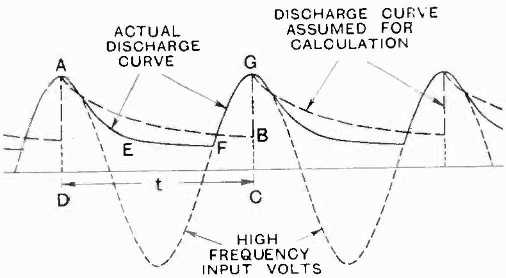


Fig. 1.—*In this figure the manner in which the grid voltage falls during the negative half-cycle is shown by the curve AEF. The mean grid potential will be represented by the mean height of the curve AEFG, i.e., the area AEFGCD ÷ DC. If we assume the grid-filament conductance to be constant at its mean value, the grid voltage will fall along the curve AB whose equation is* $E = E_0 e^{-t/T_0}$ *where* $AD = E_0$. *The approximation will be true if the area of ABCD is the same as that of AEFGCD.*

Now the mean value E of e over a complete cycle is

$$1/t \int_0^t (E_0 e^{-t/T_0}) dt$$

where $t = 1/f$.

$$E = E_0 \frac{T_0}{t} \left[- e^{-t/T_0} \right]_0^t$$

$$E = E_0 \frac{T_0}{t} (1 - e^{-t/T_0})$$

$$E = E_0 R_0 C f (1 - e^{-1/R_0 C f})$$

and the correction factor is :—

$$F = E/E_0$$
$$= R_0 C f (1 - e^{-1/R_0 C f})$$

to be applied to the *ACTUAL* input voltage to give the *EFFECTIVE* grid voltage.

Values of F have been computed for different values of $R_0 Cf$ and these are plotted in Fig. 2.

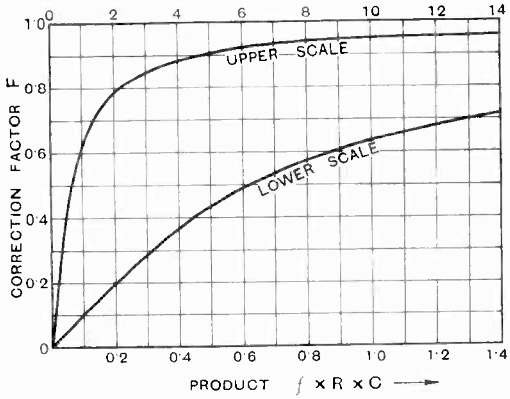


Fig. 2.

The method used by W. A. Barclay to calculate the sensitivity is supposed to give the potential to which the grid becomes biased for a given input voltage. If with the working anode load, the characteristic is taken of grid voltage against anode current, this can be used to find the actual current which will flow for each value of input voltage. From these values, the *CHANGE* in anode current produced by each value of input can be determined, and when this is multiplied by the load resistance, the change in anode voltage is obtained.

If the input is v volts (peak), and the anode current i_a is changed by an amount δi_a, then the change in anode voltage will be $\delta i_a \times R_a = v_a$ which will be the peak value of the audio-frequency output when the radio frequency input v is 100 per cent. modulated (assuming linear rectification). Thus the most satisfactory way of representing the results is by plotting $\delta i_a \times R_a$ against v peak.

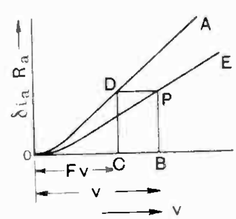


Fig. 3.

The method of calculation is as follows :—

The theoretical curve is obtained using the method suggested by W. A. Barclay.* Then using the values of effective grid capacity, effective leak resistance and frequency, the correction factor F is calculated.

In Fig. 3 suppose OA is the theoretical curve neglecting the effect of short grid circuit time constant and it is required to find the practical curve corresponding to a value of correction factor equal to F. Taking an input voltage $OB = v$ along the input axis, multiply this by F obtaining $F \cdot v$ for the effective input. Measure OC equal to $F \cdot v$, draw CD vertically to cut the curve OA in D, then CD is the output corresponding to an input OB; if now BP is drawn equal and parallel to CD, then P is a point on the new curve OE and similarly for the different values of the input.

With some valves, this method will not apply because the value of R_0 varies with the input. In this case, for a given input v a value of R_0 must be chosen arbitrarily, the value of F_1 calculated and the effective input $F_1 v$ determined. The new $R_0{}^1$ corresponding to an input $F_1 v$ is then found and

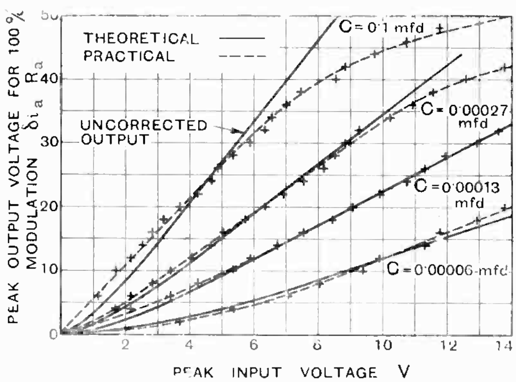


Fig. 4.—f = 110,000 ∼/*sec.*
Rg = 100,000 *ohms.*

F_2 is calculated and so on by successive approximation till the correct value is found. Alternatively the process can be reversed, taking an effective input v the value of R_0 is determined and F calculated, then the actual input v/F is obtained.

The method has only been applied to low impedance valves whose grid input re-

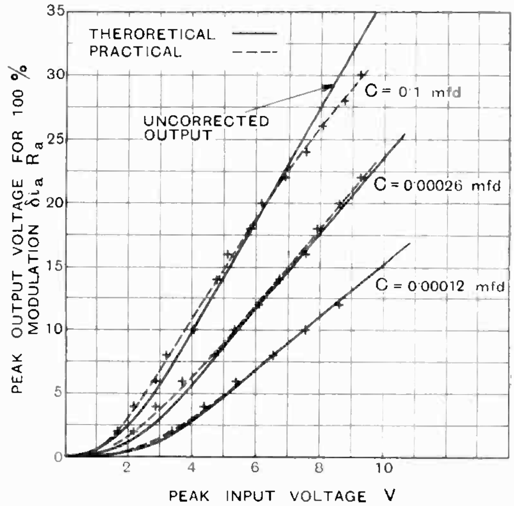


Fig. 5.—f = 110,000 ∼/*sec.*
Rg = 100,000 *ohms.*

sistance does not change greatly from zero input up to the full input. This will be found approximately true when the grid current curve starts near the point of zero grid voltage, and when the working point is at or near to the point of maximum curvature of this characteristic.

In the curves shown, Figs. 4 and 5, which refer to different valves, it will be noticed that above a certain value of effective input, the output tends to fall away from that indicated by theory even after the correction has been applied, this is due to the fact that the anode characteristic is not linear in the neighbourhood of zero anode current. Thus when the input is large enough, the grid potential traverses this curved portion and the mean anode current will be greater than that indicated by the mean grid potential, that is anode rectification is taking place which causes a reduction in sensitivity.

* *E.W. & W.E.*, 1927, pp. 450-466 and 552-558.

Method of Alignment Applied to Anti-logarithmic Triode Characteristics.

By W. A. Barclay, M.A.

1. Introduction.

IN a former article, " The Algebraic Representation of Triode Valve Characteristics," *E.W. & W.E.*, April, 1929), it was shown that the ordinary lumped characteristics of a triode can be fairly accurately represented by the antilogarithmic formula

$$i_a = I_s\{1 - 10^{a(1-10^{bv_l})}\} \quad .. \quad (1)$$

where a and b are positive constants and I_s is the value of saturation current for the particular filament temperature considered. It was established that, for any characteristic having the form of (1) above, there exists the relation

$$s \,.\, ab \,.\, 10^{bv_l} = 0.1886 \quad .. \quad (2)$$

where the quantity s denotes the length (measured in volts) of the subtangent MT for any point P on the characteristic, taken

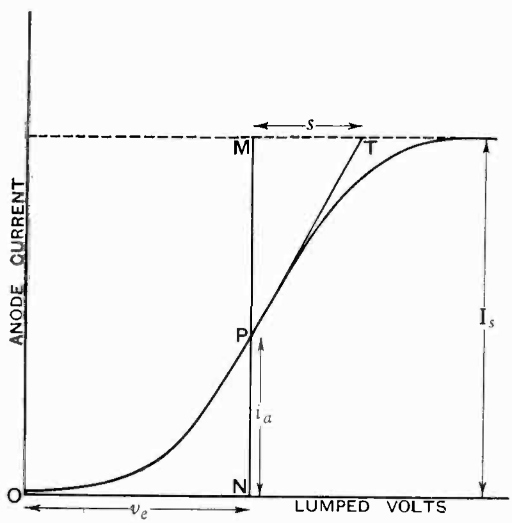


Fig. 1.—*Derivation of s from lumped characteristic.*

on the line of saturation current, I_s (see Fig. 1). From equation (2) the values of a and b may be readily derived for any particular characteristic, and an example of the process was given. When the values of a and b are known, the actual characteristics may be plotted with the help of a table of

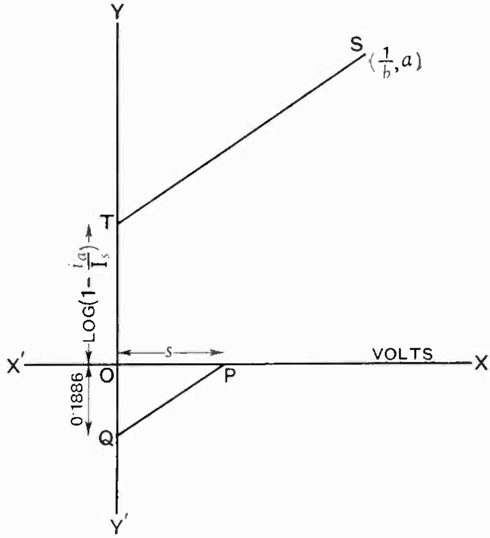


Fig. 2.

antilogarithms by means of the equations

$$i_a = I_s\{1 - \text{antilog.}\, ap\} \quad .. \quad (3)$$

$$p = 1 - \text{antilog.}\, bv_l \quad .. \quad .. \quad (4)$$

which are the equivalent of (1) above, p being a parameter.

It was observed (*loc. cit.*) that both in the matter of determining the constants a and b from a given characteristic, and in the inverse process of deriving the characteristic from equations (3) and (4), the Principle of Alignment could provide material assistance. It is now proposed to show briefly how this may be done.

2. The Alignment Derivation of a and b.

Since from (1)

$$(10^{a(1-10^{bv_l})}) = 1 - \frac{i_a}{I_s}$$

i.e., $$a\,(1 - 10^{bv_l}) = \log\left(1 - \frac{i_a}{I_s}\right),$$

C

we may re-write (2)

$$sb \,.\, \left\{a - \log\left(1 - \frac{i_a}{I_s}\right)\right\} = 0.1886 \qquad (5)$$

If, now, on Cartesian Axes (Fig. 2) P and Q denote the points (s, o) and $(o, -0.1886)$, while S and T denote the points $\left(\frac{1}{b}, a\right)$ and $\left(o, \log\left(1 - \frac{i_a}{I_s}\right)\right)$, we shall find that ST is parallel to PQ in virtue of equation (5), since this may be written in the form

$$\frac{o - s}{-0.1886 - o} = \frac{\frac{1}{b} - o}{a - \log\left(1 - \frac{i_a}{I_s}\right)}$$

If, therefore, on Fig. 2 we mark off P at a distance s along OX, and Q at a distance 0.1886 in the negative direction along OY' and join PQ, and if, further, we find the point T on the Y-axis of ordinate $\log\left(1 - \frac{i_a}{I_s}\right)$ for the value of i_a corresponding to that used in finding s, and draw through T a line parallel to PQ, *this line will pass through the point whose co-ordinates are* $\left(\frac{1}{b}, a\right)$.

If this procedure be repeated for several corresponding values of s and i_a, we shall obtain several lines each of which, theoretically, ought to pass through this point. In practice, however, it will be found that, owing to a variety of causes, these lines do not intersect accurately in one point, but will meet each other in various points whose mean position may be taken as that of the required point $\left(\frac{1}{b}, a\right)$. The polygon of error which thus encloses the ideal point of intersection will be an index of the accuracy or otherwise of our observational data, and also of the correctness of the type of formula assumed in equation (1). This process of selecting an ideal point from a field of several approximately coincident points has its counterpart in the graphical process previously used (*loc. cit.*) of drawing the best straight line among a set of approximately collinear points. The latter process is always more or less of a hazard, as the eye has to estimate the optimum position of a line over its whole length—often a considerable distance. With the alignment method, on the other hand, the points are all close together, and the optimum position is considerably easier to assess. More

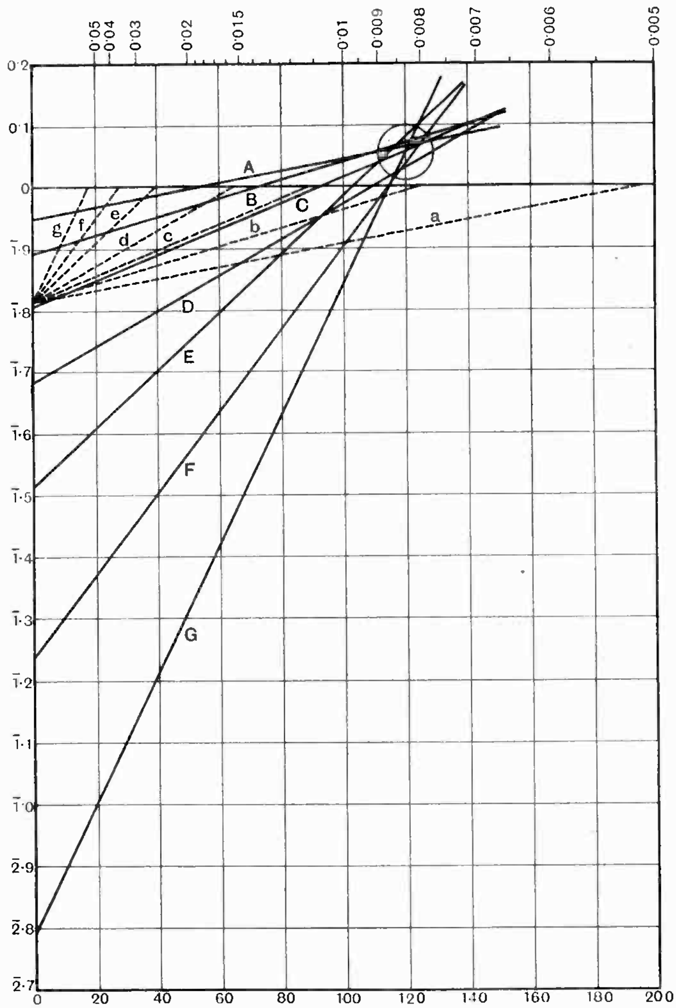


Fig. 3.—*The alignment derivation of a and b.*

important still, the limits of possible error lie patent to the view, and it becomes possible to judge not only the values of the desired quantities, *but also of the accuracy of our approximation.* Here the superiority of the new method is undoubted.

These remarks may be illustrated by applying the method to a characteristic the constants of which were determined in the previous article. The saturation current was 4.2 milliamps., and the corresponding values of v_l, i_a and s were ascertained to be as under :

TABLE.

v_l	i_a	s	$\frac{i_a}{I_s}$	$1 - \frac{i_a}{I_s}$	$\log\left(1 - \frac{i_a}{I_s}\right)$	Key Letter.
40	0.47	196	0.11	0.89	$\bar{1}.95$	A
60	0.92	125	0.22	0.78	$\bar{1}.89$	B
80	1.51	89	0.36	0.64	$\bar{1}.81$	C
100	2.17	65	0.52	0.48	$\bar{1}.68$	D
120	2.83	40	0.67	0.33	$\bar{1}.52$	E
140	3.48	28	0.83	0.17	$\bar{1}.23$	F
160	3.94	18	0.94	0.06	$\bar{2}.79$	G

The values of $\log\left(1 - \frac{i_a}{I_s}\right)$ having been tabulated as shown, the diagram of Fig. 3 was prepared to receive the data as explained above, as large a scale as possible being used for the work. The lines for the various values of s are drawn through the fixed point $(0, -0.1886)$, and are shown dotted, being lettered with the small key letters a, b, c, d, e, f, g. The solid lines A, B, C, D, E, F, G were drawn parallel to the former through the appropriate values of $\log\left(1 - \frac{i_a}{I_s}\right)$ taken on the Y-axis. The field of intersection of the latter lines is delimited on the diagram, and optimum values selected as follows : $a = 0.06$; $\frac{1}{b} = 120$ or $b = 0.008$. For convenience a scale of reciprocals has been inserted at the top of the diagram to enable values of b to be estimated at sight. The possible error incurred by adopting the above values of a and b is not very large, in the case of b being almost certainly less than 0.001.

It will be remembered that the values previously derived for the same constants were, $a = 0.048$; $b = 0.0088$. In the light afforded by the diagram of Fig. 3, it will be realised that the accuracy implied by the last significant figure in these values cannot properly be justified on our data. The values in question were derived by drawing an arbitrary line through certain datum points, and this line was evidently by no means the best that might have been chosen. The alignment method now used has the merit of giving the required values *directly* and without further calculation, while indicating at the same time the amount of confidence we can place in our result. All that we are entitled to say from our observational data is that a lies between 0.04 and 0.08, while b lies between 0.008 and 0.009. This is clearly shown by Fig. 3.

Into the general question of how far the concurrence of the intersecting lines is due to errors of observation and how far to error in the assumed type of formula, the writer does not propose to enter in this article. The subject is one of much interest to experimenters generally, and it may be said here that the differentiation of systematic from casual errors and their relation to various types of assumed formulæ constitute a field which is peculiarly suited for the application of alignment methods. These methods, as has been already remarked in *E.W. & W.E.*, while of quite recent date, deserve a prominent place in the armoury of modern mathematics on account of their generality, adaptability and simplicity. The writer is happy to think that through the medium of *E.W. & W.E.* he may have been in some measure instrumental in their introduction to the great field of electrical research.

3. The Alignment Derivation of the Characteristic.

Turning now to the inverse problem of setting out the characteristic from known values of a and b, we find that the alignment principle very greatly simplifies the work. The two equations (3) and (4) which enable us to compute first the parameter p and thence the values of i_a are capable of simple representation by alignment, as in Figs. 4a and 4b. All that need be done, therefore, is to seek on Fig. 4a values of v_l and b on their respective scales, when the value of $-p$ will be found in alignment with them. (N.B.—p is always a negative quantity.)

Then, seeking the same value of $-p$ on the left-hand scale of Fig. 4b, we join it to the matic, and its accuracy is limited only by the scale of the drawing.

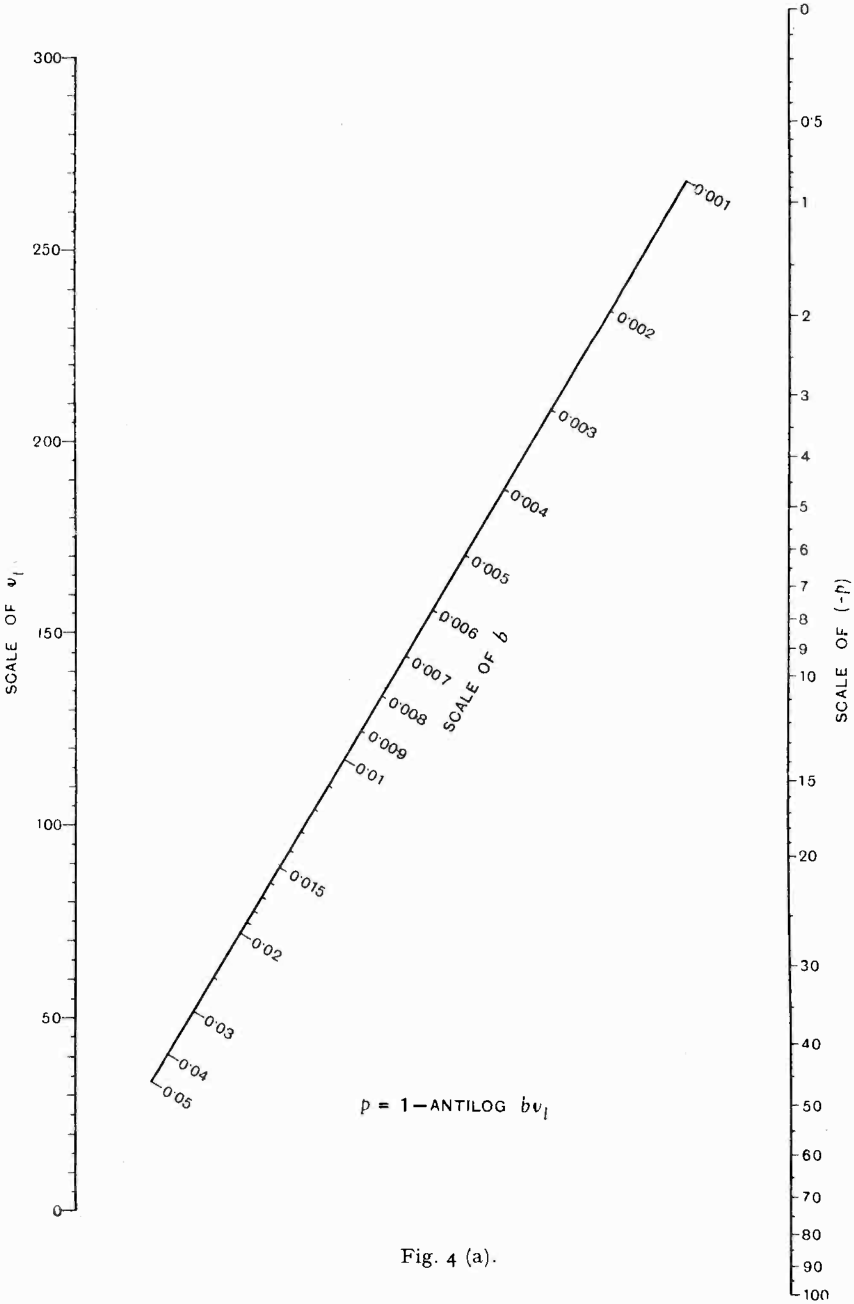


Fig. 4 (a).

value of a to obtain that of $\frac{i_a}{I_s}$. From this the value of i_a is obtained by slide-rule or simple multiplication. The whole process is automatic, and its accuracy is limited only by the scale of the drawing.

The method of constructing these charts would take too long to describe in the present article, which is concerned primarily with their use, but it may be said that they are both very simple examples of the alignment of three variables. Once drawn out, the charts are permanent, *i.e.*, they can be used as often as required to plot the characteristic for any values of a and b. It is not too much to say that by this means what was formerly the work of minutes becomes that of seconds only. Arithmetic is reduced to a minimum, and the characteristics obtained in this way have all the accuracy which is required in practical work.

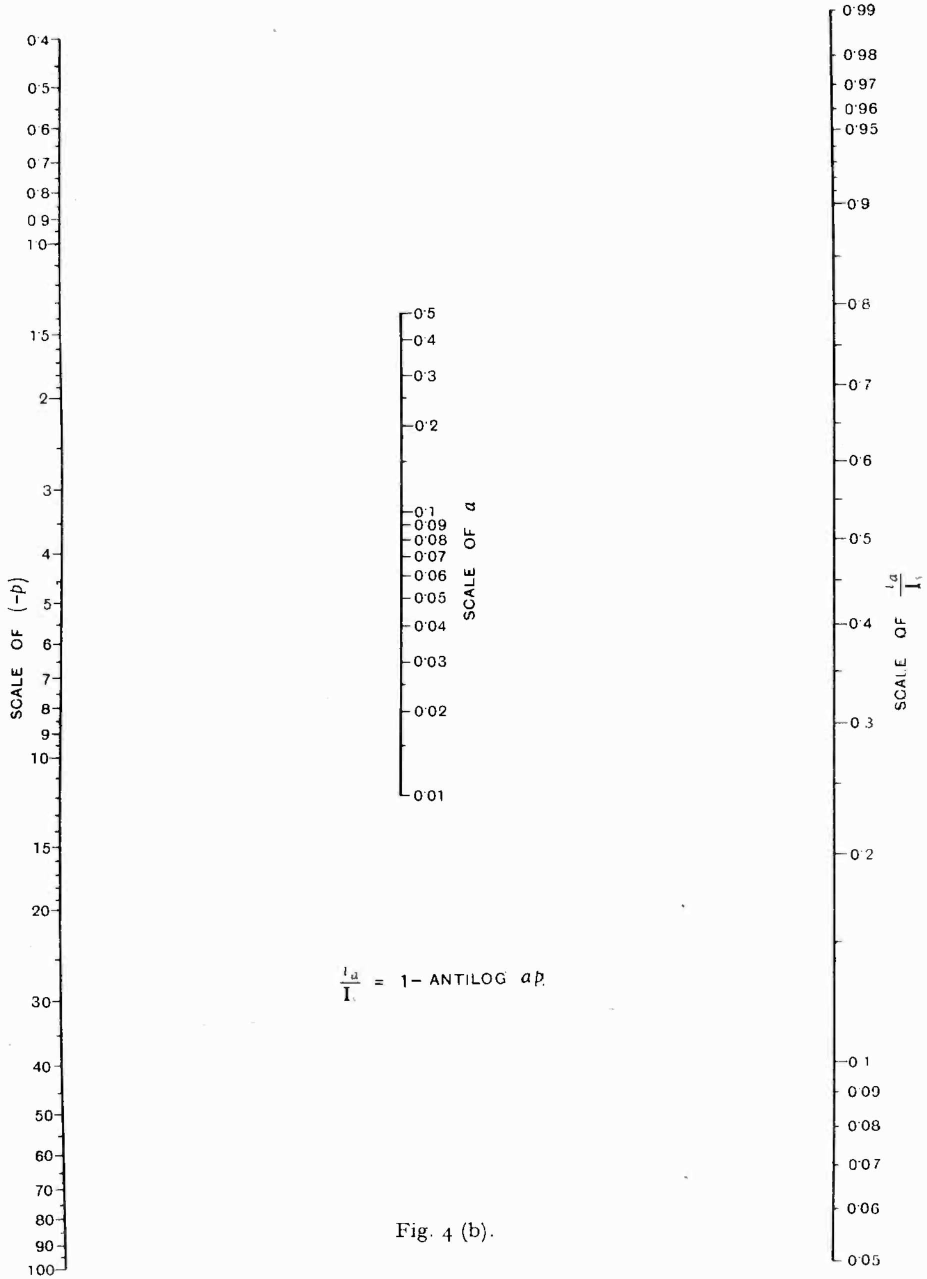


Fig. 4 (b).

A Review of Wireless Progress.

I.E.E. Wireless Section Chairman's Address.

THE opening meeting of the I.E.E. Wireless Section was held on Wednesday, 5th November, when the new Section-Chairman, Mr. C. E. Rickard, O.B.E., delivered his inaugural address.

The address took the form of a very comprehensive review of wireless progress, especially in its commercial aspects. The lecturer opened by referring to the development of wireless which had occurred due to the increase of broadcasting, and to the technique which that development had made available in other applications. Passing to Marine Communication, he said that this was the first application of wireless on a commercial basis, and was still the most important application. He then reviewed progress in marine work. An increasing number of ships that were not compelled by law to carry wireless were nevertheless doing so because of its value to navigation, etc. One thousand two hundred ships were fitted with automatic call-devices and 450 ships' lifeboats were now equipped with wireless. Short waves were in increasing use for ship-to-shore communication and 150 vessels were equipped with short-wave transmitting apparatus. Direction-finding gear was compulsory on vessels of over 5,000 tons and most countries had coastal beacon transmitters, automatically operated, for the benefit of ships using d.f. The rotating beacon (e.g., as at Orfordness) provided a solution for ships not fitted with d.f. apparatus, and shore d.f. stations were also available for the communication of bearings to ships. Hitherto the aerials used at the latter stations had been of the Bellini type, but the development of the Adcock type of aerial had now led to a direction-finder with much greater freedom from night errors. Errors of only 2 or 3 degrees were obtained while a Bellini Tosi aerial gave errors up to 90 degrees, and two-thirds of the observations had errors greater than 5 degrees. The Adcock aerial, however, seemed unlikely to be applicable to installations on shipboard.

During the past year the British Post Office and the American Telegraph and Telephone Company had commenced a public service of telephony between ship and shore. The working of the system had produced difficulties due to the lack of beam facilities on the vessel and its varying distance from shore, while its varying direction also precluded the advantage of directional aerials at the shore receiver. Five wavelengths appeared to be advisable for use on this service, but experiments on the subject were not yet quite definite.

Amongst small transmitters the chief development had been of circuits for short waves, for military, air and other purposes, where the small weight and long range (compared with the medium wave-band) was a great advantage. Transmitters for this purpose range from sets using hand generators up to gear of 3 kw. Design difficulties were chiefly due to the master oscillator. Screened-grid transmitter valves had opened up new possibilities in stabilising the amplifier by dispensing with neutralisation of inter-electrode capacity. The efficiency, however, was lower and voltage had to be supplied for the screening electrode. For military and air purposes progress had been made in the field of picture transmission for the direct reception of printed messages. Up to 150 miles had been found possible from ground to air at a speed of 7 sq. in. per minute.

In long-distance communications the past year had shown a falling-off of the success of short-wave work, which had been more seriously affected by magnetic storms. A slide was reproduced showing the interruptions to the Australian service due to this cause in the months of April and May. The interference was worse on channels whose great-circle path passed over the Poles, and a suggested method of mitigation was to use relay stations so that the great-circle bearings should not pass through Polar regions. Long waves were relatively unaffected and still remained a reliable means of long-distance communication.

Turning to facsimile transmissions over long distances, he said that high speed by wireless was not yet solved. The high modulation-frequencies necessary for these speeds were impossible due to multiple reflections in the course of propagation, examples being quoted in illustration. This limited the speed at which reception was possible to, sometimes, less than 1/30th of that of which the instrument was capable. The method was, however, of considerable importance in yielding information on effects in connection with the propagation of short waves.

Discussing transmitter design, a notable power tendency was the use of 3-phase supply with static transformation and rectification, while, in another connection, reference was also made to the use of mercury rectifiers for purposes of power supply, slides of a recent Russian installation being shown.

The need for frequency stabilisation at transmitters was most important, and the lecturer reviewed the available methods, i.e., tuning fork, quartz crystal and valve master-oscillator. While the behaviour of the master-oscillator might be quite satisfactory in the laboratory, its performance might be very different when spread over periods of a week or a month. Slides were shown illustrating the drift of a master-oscillator during the initial period after switching on. A new type of double-valve oscillator had been devised by Mr. C. J. Franklin and a slide of its performance over wide temperature ranges was shown.

The speaker lastly referred to the increase of high-power broadcast stations in Europe and to the development of a new large water-cooled valve, in which the filament leads were also water-cooled. The shortage of available wavelengths was a serious matter and was leading in certain quarters to the development of local receiving stations to pick up and relay the broadcast on a short wave.

The meeting concluded with a vote of thanks to the new Chairman, moved by the Institution President, Mr. C. C. Paterson, O.B.E.

Correspondence.

Letters of interest to experimenters are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Leaky Grid—Anode Bend Switching.

To the Editor, E.W. & W.E.

SIR,—In your November editorial a switch for changing from leaky grid to anode bend rectification is cited as a special feature of recent German practice. The Megavox Three, designed by the writer, published in the *Wireless World*, 12th September, 1928, and exhibited at the National Radio Exhibition, was the first receiver to be fitted with a change-over switch for this purpose. In this receiver the quality with leaky grid rectification was definitely superior to anode bend.

Coupled with the experiences of others—in power grid detection—it would appear necessary in the department of applied science, that to meet with approval or recognition in this country, methods must bear the hall mark of German or American publication, and usage.

N. W. MCLACHLAN.

[So far as we are aware, only one German firm fit such a switch, and the reason given for the added complication is that better quality is obtained with anode-bend detection, but when quality has to be sacrificed for added sensitiveness, leaky grid detection can be employed. We should not be surprised to learn that many experimenters had employed such a switch, but we were somewhat surprised to find it incorporated in a commercial set just at a time when the superior quality obtained with anode bend detection was becoming a matter of considerable doubt. If the German firm finds anode bend superior in quality their action is quite logical, but we do not see the point of fitting such a switch to a set which is so designed that leaky grid detection gives superior results. We doubt whether such methods have the hallmark of German or American publication and usage. Although somewhat irrelevant, it is interesting to consider what constitutes power grid detection; if one tries to improve the quality of leaky grid detection by a proper design of the components and an increase in the anode voltage applied, is there some definite boundary, upon crossing which we enter the realms of power grid detection? It would be interesting to learn from those who use the term exactly what constitutes "power" grid rectification as distinct from the c.g.s., that is the common or garden system of grid rectification.—G.W.O.H.]

A Rule for the Impedance of Parallel Circuits.

To the Editor, E.W. & W.E.

SIR,—The impedance of parallel circuits is not always easily obtained, due to lengthy calculations. In wireless we are always coming across parallel circuits and an easy way of obtaining their impedance will be of considerable utility.

The symbolic theory of A.C. does not offer a very simple solution, although it considerably reduces the steps involved in arriving at the solution.

I therefore suggest the following rule, which is quite useful and general in its application.

Without going into the details of the symbolic theory of A.C., let us denote by z the impedance operator or $R + jX$ of a circuit, where R is the resistance and X the reactance of the circuit; further, if z is of the form

$$\frac{R_0 - jX_0}{(R_0')^2 + (X_0')^2}$$

and $\frac{1}{z}$ of the form

$$\frac{R_0'' + jX_0''}{(R_0''')^2 + (X_0''')^2}$$

where the different R's represent the resistances and the various X's the reactances of the circuit under consideration then the impedance Z of the circuit can be obtained by

$$\frac{\sqrt{(R_0''')^2 + (X_0''')^2}}{\sqrt{(R_0')^2 + (x_0')^2}}$$

or, in words, by dividing the root of the denominator of the admittance operator by the root of the denominator of the impedance operator.

This rule is quite general, providing the denominators are real and of the second degree throughout. This condition is obtained more easily than solving the complex expression for the impedance of a parallel circuit.

Below are given a few examples illustrating the method, from which the number of steps saved is easily appreciated.

Consider first of all a simple circuit of a condenser shunted by a resistance.

In this case

$$\frac{1}{z} = \frac{1}{R} + j\omega c = \frac{1 + jR\omega c}{R}$$

and

$$z = \frac{R}{1 + jR\omega c} = \frac{R(1 - jR\omega c)}{1 + R^2\omega^2c^2}$$

By applying the rule given above we obtain the impedance as

$$\frac{\sqrt{R^2}}{\sqrt{1 + R^2\omega^2c^2}} = \frac{R}{\sqrt{1 + R^2\omega^2c^2}}$$

As another example, consider the case of an inductive resistance R_1L_1 in parallel with a non-inductive resistance R_2.

In this case

$$\frac{1}{z} = \frac{1}{R_2} + \frac{1}{R_1 + j\omega L_1} = \frac{R_1^2 + (\omega L_1)^2 + R_2(R - j\omega L_1)}{R_2(R_1^2 + \omega^2 L_1^2)}$$

and

$$z = \frac{R_2(R_1 + j\omega L_1)}{R_2 + R_1 + j\omega L_1} = \frac{R_2(R_1 + j\omega L_1)\,(R_2 + R_1 - j\omega L_1)}{(R_2 + R_1)^2 + \omega^2 L_1^2}$$

By the application of the rule we obtain the impedance as

$$Z = \frac{\sqrt{R_2^2(R_1^2 + \omega^2 L_1^2)}}{\sqrt{(R_2 + R_1)^2 + \omega^2 L_1^2}} = \frac{R_2\sqrt{(R_1^2 + \omega^2 L_1^2)}}{\sqrt{(R_2 + R_1)^2 + \omega^2 L_1^2}}$$

Lastly, let us take the example of a condenser shunted by an inductive resistance, a circuit which occurs most frequently in wireless.

In this case

$$\frac{1}{z} = \frac{1}{R + j\omega L} + j\omega c$$
$$= \frac{(R - j\omega L) + (R - j\omega L)(R - \omega^2 Lc)}{R^2 + \omega^2 L^2}$$
$$z = \frac{R + j\omega L}{(1 + \omega^2 Lc) + jR\omega c}$$
$$= \frac{R + j\omega L\{(1 + \omega^2 Lc) - jR\omega c\}}{(1 + \omega^2 Lc)^2 + \omega^2 c^2 R^2}$$

By applying the rule we obtain the impedance as

$$Z = \frac{\sqrt{R^2 + \omega^2 L^2}}{\sqrt{(1 + \omega^2 Lc)^2 + \omega^2 c^2 R^2}}$$

From these examples the reader can appreciate the usefulness of the rule.

K. Fleming.

[In publishing the above letter we feel it our duty to draw attention to the following points: (1) An impedance cannot be represented by a formula of the type suggested, since the numerator and denominator are of the same dimensions and their quotient would be a mere ratio and not an impedance; (2) z and $\frac{1}{z}$ cannot both be represented by formulæ of the same type; (3) correct results have been obtained in the examples only by ignoring the so-called rule which they are alleged to illustrate.

The problem can be solved very simply as follows: The admittance $\left(= \frac{1}{\text{impedance}}\right)$ of each branch is expressed thus:

$Y_1 = \frac{1}{Z_1} = G_1 + jB_1$ and $Y_2 = \frac{1}{Z_2} = G_2 + jB_2$; then the joint admittance $Y = Y_1 + Y_2 = G + jB$, where $G = G_1 + G_2$ and $B = B_1 + B_2$; then $Z = \frac{1}{Y} = \frac{1}{G + jB}$, and the numerical value of Z is $\frac{1}{\sqrt{G^2 + B^2}}$. This is the usual routine procedure.

G.W.O.H.]

Book Review.

Photocells and their Application. By Zworykin and Wilson. xi + 209 pp., with 98 Figs. Chapman & Hall. 12s. 6d.

This book by two research engineers of the American Westinghouse Company, aims at introducing the modern "electric eye" with its characteristics and special idiosyncrasies to the general public. The authors "have attempted to assemble reliable information from the thousands of articles scattered throughout the world's literature, sift the significant from the inconsequential, and present the residue in the light of their own training and long experience with the subject." The book is, in our opinion, admirably adapted to give a reader a good working knowledge of the subject provided he has already a general scientific training; we do not believe that an educated but non-scientific general reader would persevere beyond the first few pages. This is not due to any fault in the book but to the nature of the subject. The book deals both with theory and practice, and traces the development of the subject and other closely allied subjects, from the early discoveries down to their modern applications to talkies and television. A careful study of the book will give the reader not only an insight into the construction, characteristics, and operation of photo-electric cells, but will also give him sufficient information about the quantum theory and the modern ideas concerning the dual nature of the electron to enable him to take an intelligent interest in these branches of modern physics. There are several points, however, that we would criticise. The application of "nu-spelling" methods to the name of Hertz—it is spelt throughout Herz—is quite unwarranted. The use of the word billion is very annoying; when one is told that "a frequency of approximately 750 thousand billion vibrations per second produces the sensation of violet colour," one has to know the answer and work backwards to discover whether a billion is a million million or only a thousand million; the authors use the latter; it certainly sounds more impressive, especially on page 69 where we read of "The amazingly small value of one one-hundred-thousand-million-billion-trillionth of a pound." In plain English this is 10^{-32} lb. The statement on p. 16 that the eye just fails to detect radiations beyond 3.80×10^{-6} cm. is wrong; it should be 3.80×10^{-5} cm.; also from 1.36×10^{-6} to 3.80×10^{-6} is not "about 3 octaves." We note that on page 63, electrostatic field strength is denoted by H—a very unfortunate choice, as the authors discover on page 65 where they also have magnetic fields to deal with.

These are minor details, however, and the book can be thoroughly recommended. A large number of references to original works are given at the end of each chapter.

G.W.O.H.

Books Received.

Handbook of Technical Instruction for Wireless Telegraphists. By H. M. Dowsett. (Fourth Edition.)

Revised and greatly enlarged to meet the more exacting requirements of the sea-going operator's duties of to-day and providing a complete theoretical course for the P.M.G. certificate. Pp. 487 + XIX, with 462 diagrams and illustrations. Published by Iliffe & Sons, Ltd., London. Price 25s. net.

Factory Electrification for Works Directors and Managers.

A pamphlet by the well-known Consulting Engineer, Mr. W. J. Crampton, M.I.E.E., setting down briefly the points to be considered in the negotiations between the Consumer and the Supply Authority when installing Electrical Power in Factories, with a discussion on the comparative merits of the three tariff systems in general use.

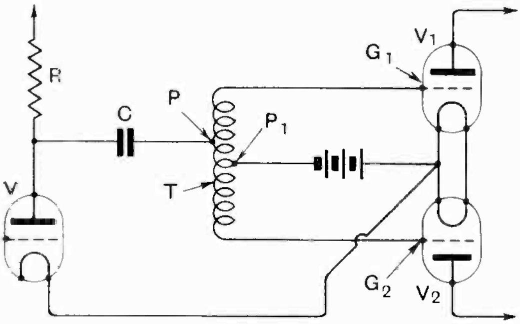
Some Recent Patents.

The following abstracts are prepared, with the permission of the Controller of H.M. Stationery Office, from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price 1/- each.

PUSH-PULL AMPLIFIERS.

Application date, 15th April, 1929. *No.* 329118.

The voltage gain from an amplifier V is applied to a subsequent push-pull stage V_1, V_2 through an auto-transformer T at a point P which is so selected as to give the required potential step-up across the grids G_1, G_2 and the mid-point tapping P_1 to the



No. 329118.

filaments. A resistance or Choke R in the plate circuit of the valve V diverts speech frequencies through the coupling-condenser C. The latter prevents the flow of any direct-current component through the windings of the auto-transformer T.

Patent issued to Kolster-Brandes, Ltd. and F. V. A. Green.

PORTABLE SETS.

Application date, 15th February, 1929. *No.* 329036.

In order to prevent a lowering of pitch due to reflection effects when one side of the loud-speaker diaphragm is permanently enclosed within the lid or casing of the set, the diaphragm is mounted independently so that it is protected as usual when the lid is closed, but can be moved out and away from the lid when the latter is open.

Patent issued to W. M. Holbeach.

POWER AMPLIFIERS.

Application date, 8th December, 1928. *No.* 328960.

When valves with oxide-coated filaments are used for low-frequency power amplification (*e.g.*, public address systems) parasitic oscillations are sometimes generated owing to secondary emission from the grids—should these happen to acquire an instantaneous positive potential, although the system is normally stable when no input is being applied.

To obviate this effect an auxiliary diode is formed inside the valve consisting of a cathode in parallel with the main filament and an electrode connected with the grid. The combination serves to increase the forward or normal grid current during those periods when the main grid is positive, thus swamping-out the effect of secondary emission and preventing any reverse grid-current.

Patent issued to E. Y. Robinson and Metropolitan Vickers Electrical Co., Ltd.

SUPERHETERODYNE RECEIVERS.

Application date, 9th January, 1929. *No.* 327710.

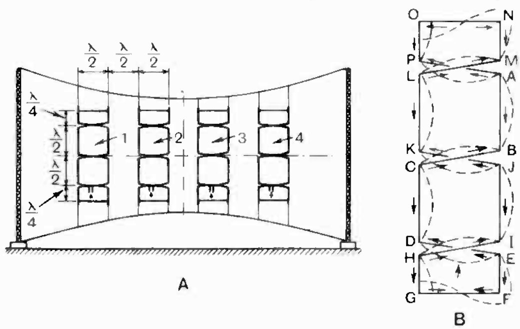
In a supersonic receiver in which two or more straight-line-frequency tuning-condensers are ganged together, a constant beat-difference between the incoming and locally generated oscillations is ensured for different settings of the tuning-condensers, by the provision in the frame-aerial or input circuit of an auxiliary variometer adjusted so as to compensate or equalise the inductance in these two circuits.

Patent issued to Gramophone Co., Ltd., and C. E. G. Bailey.

DIRECTIVE AERIAL SYSTEMS.

Application date, 13th February, 1929. *No.* 329321.

An aerial array, producing a sharply-concentrated beam of energy, consists of a number of sections of the kind shown in Fig. A, each comprising a series of single wire antennæ 1, 2, 3, 4, Fig. A. Each antenna or radiating unit is of the shape shown in Fig. B, and consists of a single conductor $A \ldots\ldots P$ bent into the form of square sections. The side of each square is a half wave-length, except the top and bottom sections, which are bisected. The result of bending the wire into the form shown is to suppress radiation from each alternate half wavelength. Each vertical side



No. 329321.

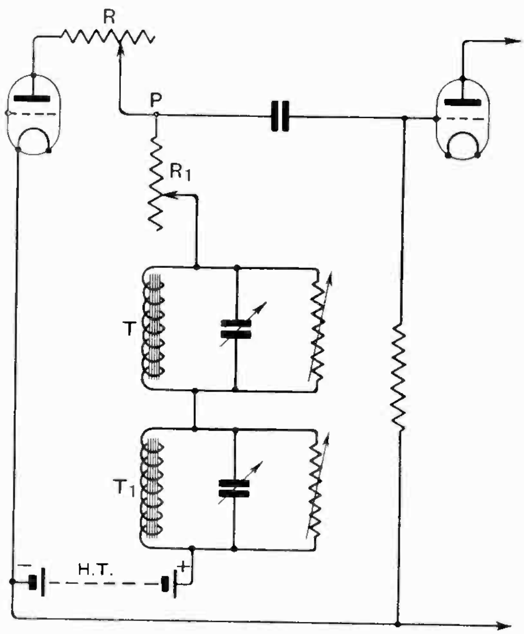
carries in-phase current, whilst the folded-over wires carry currents in phase-opposition, as shown by the dotted-line curve. The result is an intensified directional effect, such that a deviation of 5° on either side of the axis of the beam produces a falling-off of 50 per cent. in field strength.

Patent issued to Standard Telephones and Cables, Ltd.

L.F. AMPLIFICATION.

Application dates, 28th January and 21st February, 1929. No. 328331.

Electrical reproduction from a gramophone tends to be deficient on the lower notes owing to the restricted amplitude of the cutter on the record; it also lacks "brilliance" on the higher notes owing to inherent defects in the pick-up and stylus.



No. 328331.

According to the invention "shaping" or correcting circuits are applied to compensate for both these defects. The shaping circuit may be inserted directly across the terminals of the pick-up, or, as shown in the Figure, across the output of a valve amplifier. The output shown comprises two variable resistances R, R_1, a tuned circuit T having a resonant frequency lying between 20 and 350 cycles, and a second tuned circuit T_1 having an effective resonance in the neighbourhood of 4,000 cycles. The input to the second valve is tapped off at a point P between the resistances R and R_1.

Patent issued to N. W. McLachlan.

ELIMINATING HUM.

Application date, 20th April, 1929. No. 328117.

In order to minimise "hum" when an AC mains-fed receiver is sharply tuned, two condensers are connected in series across the secondary of the supply transformer, and a connection is taken from a point between the two condensers either to earth or to the negative terminal of the rectified current. With a full-wave rectifier, the two condensers are connected across the centre-tapped secondary which feeds the two anodes of the rectifying-valve.

Patent issued to H. V. Carlisle and Ferranti, Ltd.

TESTING VALVES.

Application date, 25th January, 1929. No. 328049.

The valve to be tested is plugged into terminals which supply filament and plate current through the secondary of a transformer energised directly from an AC mains supply. The grid of the valve can be connected by a reversible key to one side or other of the secondary windings. In its normal position the key applies a grid bias 180° out of phase with the plate voltage, thus reducing the plate current to a minimum. When the key is reversed, the grid bias is in phase with the plate voltage, giving a maximum reading in a milliammeter in the plate circuit. The difference between the two readings gives a direct measure of the mutual conductance of the valve.

Patent issued to Sir G. C. Marks.

PREVENTING INTERFERENCE.

Convention date (Germany), 21st March, 1928. *No.* 308312.

Relates to the method of balancing-out atmospherics by using two aerial circuits, one of which is tuned to the signal frequency and the other slightly de-tuned. According to the invention a local oscillator having the frequency of the incoming carrier is coupled to the de-tuned circuit in order more exactly to equalise the grid voltage applied to the two detector valves. This ensures equal rectifier efficiency, and a more complete balance of the outputs when these are combined in opposition.

Patent issued to Loewe-Radio G.M.B.H.

AUTOMATIC DF SYSTEMS.

Application date, 28th December, 1928. No. 328315.
(Addition to No. 300697.)

The bearing of a ship or aeroplane relative to a beacon station is indicated automatically by utilising the maximum and minimum effect of a rotating directional aerial to actuate relays, which measure the intervals corresponding to angles moved through by the rotating aerial system. The various moving parts are driven by a spring or electric motor under the control of tuned-reed vibrating contacts, which are intermittently closed by the action of special modulation notes imposed on the carrier wave from the beacon station.

Patent issued to C. H. P. Wheatley.

PORTABLE SETS.

Application date, 10th January, 1929. No. 327467.

In order to prevent the pick-up of a frame aerial from being reduced by the metallic screens incorporated in the set, and also to avoid the setting-up of eddy-currents and undesirable reaction effects, the aerial is wound on a former adapted to be withdrawn bodily from the set during reception, and to be replaced when the receiver is not in use. The withdrawal of the frame aerial is arranged automatically to light-up the valves, switch on the high-tension, and connect the aerial in circuit.

Patent issued to H. J. Round.

TELEVISION SYSTEMS.

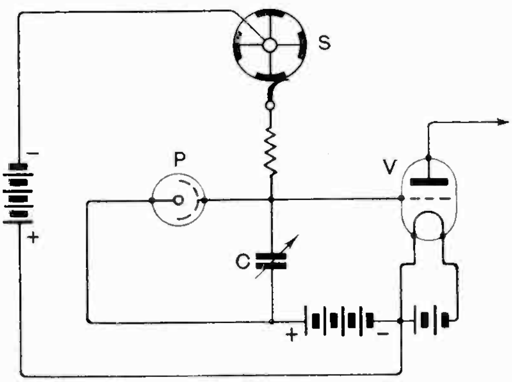
Convention date (U.S.A.), 29th February, 1928. No. 306962.

The picture or scene is subjected as usual to a point-by-point analysis at the transmitter, but in reception the corresponding signal impulses are first stored up in sequence and are then released simultaneously, so that the scene is reproduced as a whole. The receiver comprises a number of strip electrodes each aligned with a series of smaller electrodes. The strip electrodes are connected through time-lag networks to a bank of condensers which form the storing means. A commutator switch, rotating synchronously with the analysing-device at the transmitter, transfers the stored-up impulses so as to produce luminous discharge effects simultaneously between the strip and smaller electrodes.

Patent issued to Postal Telegraph-Cable Co.

Convention date (Austria), 22nd August, 1928. No. 317778.

To secure half-tone effects either in still-picture or television systems, the amplitude fluctuations produced by an exploring device are translated into corresponding impulses of varying time duration. The conversion is effected by applying a negative charge to the grid of the valve *V* and to one electrode of a photo-sensitive cell *P*, at regular intervals, by means of a rotating switch *S*. A variable condenser *C* allows the effective charge to be controlled.



No. 317778.

The cell *P*, when subjected to the exploring ray of light, acts as a discharge path for the imposed grid potential, so that the intermittent anode current of the valve is proportional in duration to the brightness of the light incident on the cell *P*.

Patent issued to O. Fulton.

Application dates, 25th October, 1928, and 4th February, 1929. No. 328286.

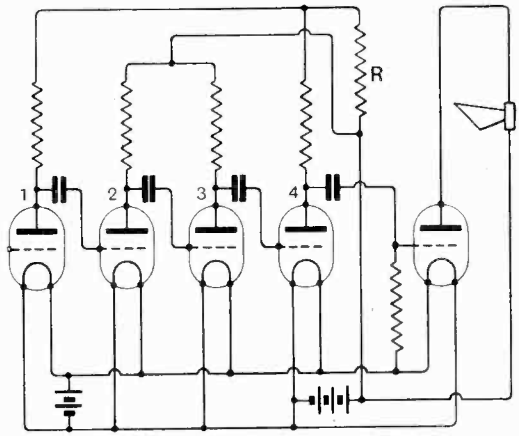
The picture is considered as being made up of a series of parallel lines and means are provided for displacing these laterally and then connecting each line in succession to the preceding one so as to resolve the complete picture into one continuous "line image." The displacement is effected by reflecting the image from a "stepped" or echelon mirror, the process being reversed at the receiving end. The method is stated to render the use of synchronising-means unnecessary in still-life picture transmission.

Patent issued to G. W. Walton.

LOW FREQUENCY AMPLIFIERS.

Application date, 18th March, 1929. No. 329435.

In an amplifier consisting of four or more resistance-coupled stages, the first and fourth stages



No. 329435.

each comprise a common anode resistance *R* which is not included in the plate circuits of the second and third stages. If the number of stages is even, and at least four, the additional resistance *R*, or an equivalent impedance, may be common to all the anode circuits. The arrangement is stated to give uniform amplification over a broad range of frequencies.

Patent issued to S. G. S. Dicker.

SOUND DIAPHRAGMS.

Application date, 13th February, 1929. No. 329294.

A conical diaphragm is tapered in thickness from the apex outwards, the cross-section of the material being such that the weights of concentric rings of equal width are substantially the same at all distances from the apex, except in the immediate vicinity of the latter.

Patent issued to M. E. Elliott.

Application date, 18th February, 1929. No. 329376.

A small rigid central disc or diaphragm is surrounded by several separate concentric rigid rings to form a composite sound-reproducing surface. The central disc is attached to the first ring by a flexible suspension of sheet-rubber, oiled silk, etc., whilst successive rings are similarly attached to each other. The drive is applied only to the central disc; which responds mainly to the higher frequencies, the lower notes being distributed over the whole system.

Patent issued to R. H. Parkinson.

TRANSMITTING AERIALS.

Convention date (Germany), 11th August, 1928. No. 317297.

In energising a short-wave aerial from a source of high-frequency power, a double feed-line is employed, one of the wires being connected directly to the aerial and the other to earth through a condenser equal to the aerial impedance at the working frequency. This simple arrangement gives a close approximation to the working requirement that the wave resistance of the feeder system must equal the radiation resistance of the aerial.

Patent issued to Telefunken Gesell. für Drahtlose Telegraphie M.B.H.

REMOTE VOLUME CONTROL.

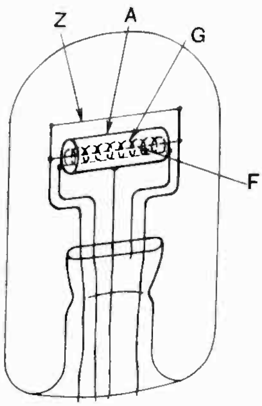
Application date, 24th January, 1929. No. 329689.

A fixed biasing-resistance is inserted in the lead connecting the cathode to the control grid of a screened-grid amplifier, and a separate variable resistance is located at a distance, in parallel with the first resistance, in order to adjust the potential drop across the latter. This in turn varies the effective grid-bias in the HF stage and provides an effective volume control.

Patent issued to A. Hall and Ferranti, Ltd.

THERMIONIC VALVES.

Convention date (Holland), 15th December, 1928. No. 329854.



No. 329854.

In addition to the filament *F*, grid *G*, and anode *A*, an extra wire *Z* of zirconium is inserted, as shown, in parallel with the filament, in order to absorb any residual gas that may be liberated whilst the valve is in operation. The wire *Z* is of high specific resistance, so that it is raised by a comparatively small current to a very high temperature.

Patent issued to N. V. Philips Gloeilampen-Fabricken.

TUNING CONTROL.

Application date, 29th January, 1929. No. 328575.

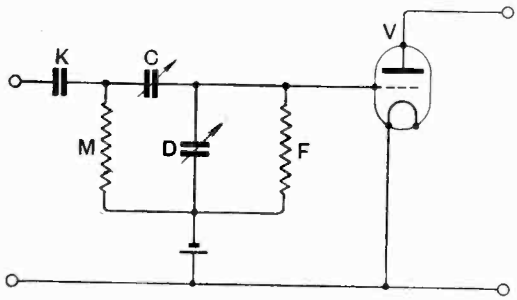
When using single-knob control, in order to overcome the difficulty of accurately "matching" successive tuned stages, the circuits are given substantially different degrees of damping, but are identical in all other respects. The degrees of damping, though widely different, are so chosen that the overall resonance curve corresponds to that which would be obtained if all the stages were uniformly damped.

Patent issued to H. J. Round.

VOLUME CONTROL.

Application date, 20th March, 1929. No. 329747.

Two condensers *C*, *D* are arranged in series across the input of a valve amplifier *V*. They are coupled together in such a way that when one is at maximum the other is at minimum value. The condenser *C*



No. 329747.

is preferably of the "square law" type whilst *D* is an ordinary straight-line condenser. As the capacity *C* is reduced, the impedance of the input path is increased, and the amplification drops. Similarly when *D* is increased more of the applied impulses are by-passed through it, and fail to reach the grid. An additional fixed condenser *K* prevents any *DC* voltage from reaching the control condensers. *F* is the ordinary grid leak, whilst *M* is an additional resistance to allow any charge on the condenser *C* to leak away.

Patent issued to P. G. A. H. Voigt.

SELECTIVE TUNING.

Convention date (Germany), 21st June, 1928. No. 314089.

Two or more piezo-electric crystals are combined with variable inductances or capacities in the grid circuit of a transmitting or receiving valve, and are arranged so that they can be switched into circuit either in series or parallel, or in series-parallel. This enables a comparatively wide frequency range to be covered. For instance with four crystals there are 100 different combinations possible, each giving a distinct tuning adjustment.

Patent issued to Telefunken Gesell. für Drahtlose Telegraphie M.B.H.

PIEZO-ELECTRIC SUBSTITUTES.

Convention date (Germany), 1st August, 1928. No. 316628.

By allowing certain substances, such as waxes, resins, asphalt, etc. (preferably with some content of powdered quartz), to cool under the action of high-tension direct current, they acquire piezo-electric properties and can be used as microphones. A microphone of this type is inserted under the control of a stabilising voltage in the input circuit of a valve amplifier.

Patent issued to Telefunken Gesell. für Drahtlose Telegraphie M.B.H.