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The Birthplace of Modern Radio!

IN the August Editorial we discussed the question, "Where was the Telephone Invented?", and we drew attention to the fact that the town of Brantford in Ontario, where the parents of Alexander Graham Bell settled on leaving this country, called itself the Telephone City and regarded itself as the birthplace of the telephone. Something similar has been happening recently in California. How many radio engineers, if asked to name the birthplace of modern radio, would reply Palo Alto?

We have recently received copies of the *Palo Alto Times* of four consecutive days last June carrying the heading "The birthplace of modern radio!" On the first page we read "*The Times* today presents the first of a series of four authoritative articles telling for the first time the full story of Palo Alto's historic place in the development of modern radio and the whole electronics art . . . . and of the Palo Alto laboratories in which two key inventions—the amplifier and the oscillating audion—were discovered in 1912. . . . .What Edison's Menlo Park is to the incandescent lamp, Palo Alto is to radio and the electronics art, ... an historic shrine of science, and the nursemaid of one of the country's giant industries."

Most people who have heard of Palo Alto associate it with the Leland Stanford University, opened there in 1891 by the generosity of a railway constructor of that name, who gave twenty-six million dollars for the purpose in memory of his son, and who died in Palo Alto in 1893. The association with radio is due to Cyril F. Elwell, a familiar name to those whose interest in the subject goes back to the first world war. He was born in Australia of American parents and graduated from Stanford in 1907. He began radio-telephone experiments shortly afterwards, buying a cottage in Palo Alto in 1908 to serve as a home and laboratory. At the cottage he erected two 75-foot masts and suspended a receiving aerial from the tower of the waterworks three-quarters of a mile away. The primary object was to test a system in which several local bankers had invested money; it involved intermittent discharges and damped wave-trains.

The Poulsen Arc

Although speech transmission of a kind was found to be possible, Elwell was dissatisfied and came to the conclusion that for satisfactory telephony continuous waves were essential, so he went to Copenhagen and made arrangements with Poulsen for the use of the arc generator which Poulsen had invented in 1903. In 1909 the Poulsen Wireless Telephone and Telegraph Company was formed with Elwell as president and chief engineer. The first two arc sets were obtained from Denmark, but further sets were made in Palo Alto; Elwell took three men back with him from Denmark to help in the manufacture. In 1910 telephonic communication was established between two towns 50 miles apart. In 1911 the name of the Company was given a more American flavour; it became the Federal Telegraph Co. In 1913 something must have happened, for Elwell severed his connection with the Company, came to Europe and built a 100 kilowatt Poulsen Arc station for the Admiralty at Horsea. He also built several large arc stations on the Continent and installed equipment of the same type on a number of warships. In 1932 the Federal Co., transferred its headquarters from Palo Alto to Newark, N.J., and during the recent war had forty-eight factories manufacturing radio equipment.

To return to our subject, it was in July 1911 that Elwell engaged de Forest, whose mother lived at Palo Alto, and two assistants, C. V. Logwood and H. B. van Etten, to undertake research in the Federal Co. laboratories at Palo Alto. De Forest had invented the three-electrode valve in 1907 by adding the grid to the diode, and it was the development and application of the audion that Elwell wished de Forest to pursue. In August 1912 they constructed a three-stage amplifier which was taken to Washington and New York in September and October and demonstrated to the Naval Authorities and to the Bell telephone engineers. " The amplifier, then, is outstanding invention No. 1 to the credit of Palo Alto." The howls and squeals that occasionally caused them so much annoyance when adjusting the amplifier provided the clue for the second invention. In August 1912 van Etten deliberately set up a feedback circuit to cause sustained oscillations. Luckily he recorded his doings in note-books, which played an important rôle in the subsequent patent litigation that went on until 1934. There were several claimants; Armstrong, of f.m. fame, claimed to have discovered the feedback circuit on 31 January 1913, Meissner on 9 April 1913, and Langmuir on 1 August 1913, but all the courts sustained the decision in favour of de Forest. So the feedback oscillator became Palo Alto's No. 2 invention.

It is interesting to note that in 1911 and 1912 while Elwell was engaged on the installation of arc generator equipment in Chicago, San Francisco, Seattle and Honolulu, the trio who were working in his Palo Alto laboratory were making discoveries which in a few years were to reduce his arc generators to the status of museum specimens. G. W. O. H.

RADIATION RESISTANCE OF RING AERIALS*

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SUMMARY.—Formulae are derived for the radiation resistances of ring aerials. In particular, systems having antifading properties are considered. These consist of vertical aerials, equally spaced around a circle having a radius comparable with the wavelength. The currents in the aerials are all of the same amplitude; the phase changes progressively round the ring and the total phase change is an integral multiple of 2π radians. A special case of in-phase ring currents with a central aerial is also treated.

1. Introduction

Antifading aerials have been in common use for medium-wave broadcasting for many years, aerials or mast radiators of height between 0.5λ and 0.6λ being used for this purpose. Compared with a short aerial radiating the same power, a high aerial leads to a reduction in the field strength at small angles to the vertical and an increase in the field strength along the ground. In other words there is a reduction in the ratio of the ray which may be reflected from the ionosphere to that propagated along the ground, and hence a reduction in fading. The required height of mast radiators to give antifading properties on medium wavelengths is between 400 and 800 feet, depending on the wavelength, and the consequent cost is high. Moreover, for long wavelengths masts of the necessary height would involve engineering difficulties out of all proportion to the performance. There may also be other objections to the use of high masts, such as regulations for the safety of aircraft.

Any alternative system should have a circular horizontal polar diagram and a vertical polar diagram at least as good as that of a high mast. Arrangements of ring aerials have been proposed for this purpose; these consist of a number of short aerials, arranged in the form of a ring, the radius of

* MS accepted by the Editor, February 1947.

which is comparable with the wavelength. The amplitude of the ring aerial currents is the same, but the phase changes progressively round the ring, the total phase change being an integral multiple of 2π radians, say $2\pi n$ radians where n is integral.

In this case there is evidently no discontinuity of phase round the ring, and the horizontal polar diagram will be circular. As far as the vertical polar diagram is concerned it is evident that, except when $n = 0$, the radiation at small angles to the vertical results from the addition of a number of vectors of equal amplitude, equally spaced round a total angle of approximately 2π radians, and will therefore be small. Along the ground, however, by a suitable choice of ring radius, the contributions due to individual aerials may be made to add instead of subtracting. This type of aerial we will call the phased-ring aerial system.

When $n = 0$, the currents in the ring aerials are in phase, and the radiation at small angles to the vertical will be increased in a greater ratio than that along the ground. In other words, the ratio of the reflected ray to ground ray will be increased. If however a central aerial is added, carrying a current of opposite phase to that of the ring, the radiation at small angles of incidence may be cancelled without cancellation of the ground wave, and the system thus given antifading properties. We will call this type of aerial a concentric-ring aerial system.

The phased-ring aerial system was first suggested by Chireix[1] and was used at the French long-wave station at Allouis in 1939. Some years previously a concentric-ring system had been patented by the Telefunken Co.[2] and used in Germany[3] in 1932.

In considering the suitability of any aerial two important characteristics must be examined, the polar diagram and the radiation resistance. The polar diagram tells us whether the distribution of radiation in different directions is satisfactory. In our case, where a circular horizontal polar diagram is required, we have to consider only the vertical polar diagram, and to ascertain whether the reduction in radiation at small angles to the vertical is sufficient. The value of the radiation resistance is necessary in order to calculate the operating currents and voltages and, hence, to design the components in the matching network. Furthermore, it is necessary to have a reasonably high radiation resistance in order that ground and other losses should not unduly reduce the efficiency of the system.

We will next proceed to calculate the vertical polar diagram and radiation resistance of both types of ring aerial systems.

LIST OF SYMBOLS

λ = wavelength

θ = angle to vertical

k = ratio of central-aerial current to ring-aerial current in the concentric-ring aerial system.

n = phase difference between the currents in the two aerials of the phased-ring system subtending 1 radian at the centre of the ring.

$q = \dfrac{2\pi r}{\lambda}$

r = ring radius

R_0 = radiation resistance of vertical aerial of same height as ring.

R'_0 = effective radiation resistance of central aerial in concentric-ring aerial system.

R_r = radiation resistance of ring-aerial system

R'_r = effective radiation resistance of ring aerial in concentric-ring aerial system

2. Vertical Polar Diagram

For the purpose of calculation it is assumed that there is an infinitely large number of aerials in the ring, but in practice a comparatively small number of aerials may be used and a substantially circular horizontal polar diagram still obtained.

Fig. 1. *Ring aerial with centre at O and radius r.*

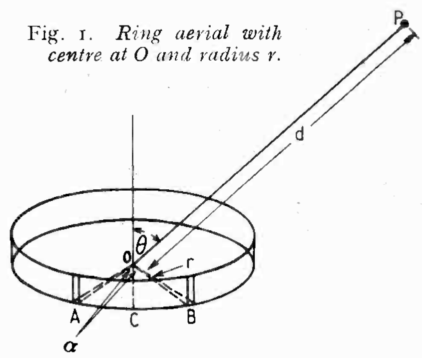


Fig. 1 shows the ring aerial, centre O and radius r. We will calculate the field strength at the point P, at an angle θ to the vertical, where $d = OP \gg r$. OC is the radius perpendicular to the projection of OP in the horizontal plane. A and B are two typical aerial elements, the angles AOC and BOC being α; the angle subtended by two adjacent aerials in the ring is $\delta\alpha$, which is considered infinitesimally small.

We will define the total ring current as the sum of the moduli of the currents in all the elements. A and B therefore each carry a fraction $\delta\alpha/2\pi$ of the total ring current.

Now the total change of phase of aerial current round the ring is $2\pi n$; if the phase lags from A to B, the current in B lags, and in A leads the current in the element at C by an angle $n\alpha$. In addition, the field at P due to A and B will also differ in phase due to the difference in path lengths.

Now $PA \approx d + r \sin\theta \sin\alpha$ if $d \gg r$

and $PB \approx d - r \sin\theta \sin\alpha$.

Hence the path difference is $2r \sin\theta \sin\alpha$, which is equivalent to a phase difference of $2q \sin\theta \sin\alpha$, where $q = \dfrac{2\pi r}{\lambda}$. Compared with an aerial at C, therefore the field due to A lags and due to B leads by an angle $(q \sin\theta \sin\alpha - n\alpha)$. If, therefore, the field strength due to a single vertical aerial, equal in height to the ring, and carrying a current equal to that of the ring, at an angle θ to the vertical is $f(\theta)$, the field due to elements A and B is $f(\theta)\dfrac{\delta\alpha}{2\pi}$ $2\cos(q \sin\theta \sin\alpha - n\alpha)$. The resultant field due to all the ring elements is therefore

$$f(\theta)\frac{1}{\pi}\int_0^{\pi} \cos(q \sin\theta \sin\alpha - n\alpha)d\alpha$$
$$= f(\theta) \,.\, J_n(q \sin\theta)$$

where $J_n(z)$ is the Bessel function of the first kind and order n.

The vertical polar diagram due to a single vertical aerial is, therefore, modified by a factor $J_n(q \sin\theta)$ due to the arrangement of the aerials in a ring, and the ratio

$$\frac{J_n(q \sin\theta)}{J_n(q)}$$

gives the modification at a vertical angle θ compared with that along the ground. The smaller this ratio the smaller is the ratio of indirect ray to ground ray and the more marked the antifading property. The ratio is plotted in Fig. 2 for a ring radius of 0.25λ, and it is seen that the larger the value of n the smaller the ratio, and hence the greater the improvement in the vertical polar diagram.

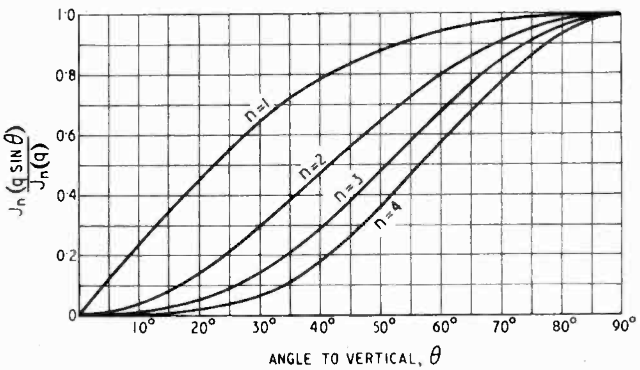


Fig. 2 (above). *Curves showing modification of vertical polar diagram of a single vertical aerial for a ring radius of* 0.25λ. *Total phase change round ring* $= 2\pi n$ *radians.*

In the case of ring aerials short compared with the wavelength $f(\theta) = K \sin\theta$ where K is a constant, and the expression for E, the relative field strength at a vertical angle θ, becomes

$$E = \frac{J_n(q \sin\theta) \sin\theta}{J_n(q)} \quad .. \quad (1)$$

the maximum value of the expression being made unity for comparison purposes.

Typical vertical polar diagrams for this case are shown in Fig. 3.

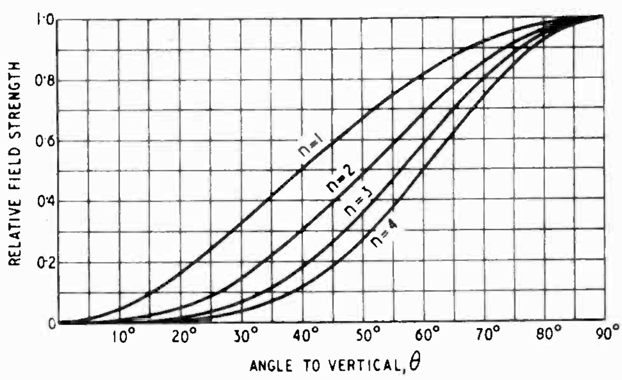


Fig. 3 (right). *Vertical field-strength diagrams for a phased-ring aerial of radius* 0.25λ. *Total phase change round ring* $= 2\pi n$ *radians.*

The corresponding diagrams of typical single antifading vertical aerials are shown in Fig. 4, making the usual assumption of a sinusoidal current distribution. From Figs. 3 and 4 we see that a ring aerial with $n = 2$ gives a polar diagram similar to that of a $\lambda/2$ vertical aerial. Further, n must be greater than 2 in order to obtain a diagram comparable with or better than that of a vertical aerial of optimum height, the diagrams progressively improving from an antifading point of view as n increases. Unfortunately there is a practical difficulty which limits the value of n that can be used. The higher the value of n the greater the number of ring aerials

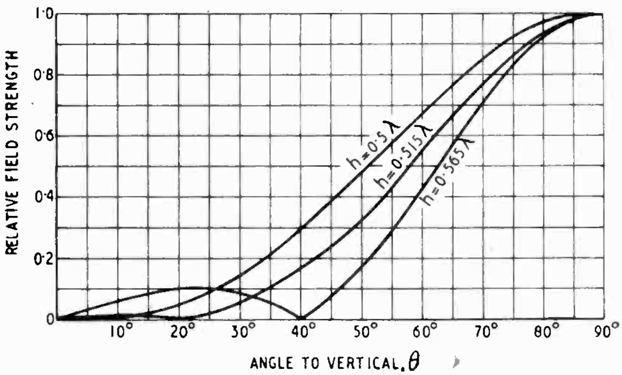
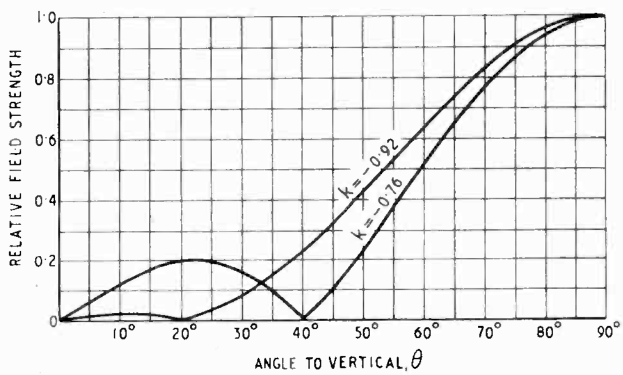


Fig. 4 (above). *Vertical field-strength diagrams for a typical single anti-fading aerial, height h.*

Fig. 5 (right). *Vertical field-strength diagrams for a concentric-ring aerial of radius 0.25λ.*



necessary to obtain a substantially circular horizontal polar diagram, and the larger the ring radius to obtain a sufficiently high radiation resistance for each individual aerial. The permissible value of n in any particular case will therefore usually be limited by considerations of cost.

So far we have not considered the case when $n = 0$; $J_0(z)$ has a maximum value of 1 when $z = 0$, and is smaller for all other values of z. This means that the use of a ring of co-phased aerials will actually increase the radiation at high angles to the vertical compared with a single aerial; as we have seen, the factor modifying the polar diagram is $\dfrac{J_0(q \sin \theta)}{J_0(q)}$ which is maximum, for a fixed value of q, when $\theta = 0$. This difficulty can be overcome by adding an aerial at the centre of the ring carrying a current opposite in phase to that of the ring.

If the ratio of the central-aerial current to ring-aerial current is k, it follows from the previous analysis that the field strength is proportional to $\{k + J_0(q \sin \theta)\}f(\theta)$. Hence by making $k = -J_0(q \sin \theta_0)$ we obtain a zero in the vertical polar diagram at any required angle, θ_0. The negative sign implies that the current is opposite in phase to that in ring, since $J_0(q \sin \theta_0)$ is positive over the range of q in which we are interested. The concentric-ring aerial can therefore be made to give a vertical polar diagram similar to that of a single vertical aerial between 0.5λ and 0.6λ high, consisting of a minor and major lobe.

The expression for the vertical polar diagram of the concentric-ring aerial system, for aerials short compared with λ, becomes

$$E = \left\{\frac{k + J_0(q \sin \theta)}{k + J_0(q)}\right\} \sin \theta \quad (2)$$

the maximum value again being made unity for comparison purposes.

Typical vertical polar diagrams are shown in Fig. 5, and there are two points of interest. First, comparing Figs. 4 and 5, we see that the amplitude of the minor lobe for the ring system is approximately twice that of a high vertical aerial having the same value of θ_0. This is undesirable, as it means that the indirect ray within the service area will be greater than for a simple high vertical aerial. Second, in the case of the ring system the field strength zero at θ_0 can in practice be obtained, as it arises from the difference between two fields of which the amplitude and phase can be separately adjusted. In the case of a high vertical aerial, however, although the zeros in Fig. 4 are often shown in the published literature, in general a minimum, and not a zero, is obtained because the current usually departs appreciably from the postulated sinusoidal distribution. This advantage of the concentric-ring system over a vertical aerial probably outweighs the disadvantage referred to previously.

We will next proceed to calculate the

radiation resistance of ring aerial systems of the type described. These calculations will only apply strictly to aerials for which the field strength at an angle θ to the vertical is proportional to $\sin\theta$, in other words to aerials short compared with the wavelength. The results however will be substantially correct for aerial heights up to $\lambda/4$ as over this range there is no appreciable change in the shape of the vertical polar diagram.

3. Radiation Resistance

We will define the radiation resistance of the ring aerial system, R_r, as that value of resistance which when multiplied by the square of the ring current gives the total power radiated. It follows that, in the case of a ring containing a discrete number of aerials, the radiation resistance of each individual aerial is equal to the ring radiation resistance multiplied by the number of aerials.

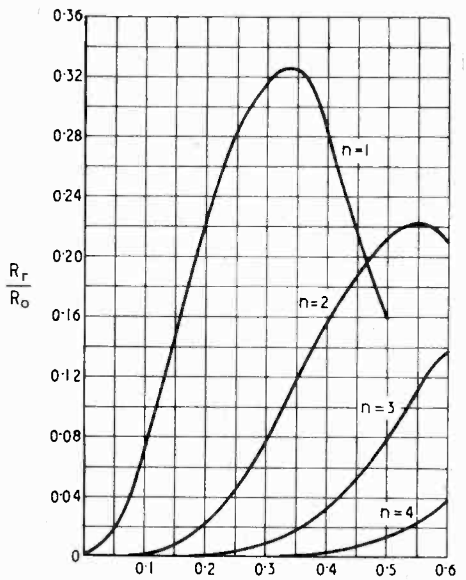


Fig. 6. *Radiation resistance of phased-ring aerial system. R_r = radiation resistance of ring aerial; R_0 = radiation resistance of vertical aerial of same height as ring.*

If we consider the aerial system surrounded by a hemisphere of radius a, by Poynting's vector theorem, the power radiated through an element of the surface included between radii at angles θ and $\theta + \delta\theta$ to the vertical, is proportional to the square of the field strength multiplied by the area of the element of surface.

Comparing the phased-ring aerial, with a single vertical aerial of the same height carrying the same current, we have shown that, at a vertical angle θ, the field strengths at a distant point are in the ratio

$$J_n(q\sin\theta)\sin\theta : \sin\theta$$

The ratio of the powers radiated by the two aerials, through the element of the surface being considered, is therefore :—

$$\{J_n(q\sin\theta)\sin\theta\}^2\, 2\pi a\sin\theta\,\delta\theta : (\sin\theta)^2\, 2\pi a\sin\theta\,\delta\theta.$$

By integrating this ratio between the limits 0 and $\pi/2$ we obtain the ratio of the total power radiated by the two systems.

For equal aerial currents, however, the ratio of the radiated powers is equal to $\frac{R_r}{R_0}$, where R_r is the radiation resistance of the ring and R_0 that of a single vertical aerial of the same height. These radiation resistances may be the values at the base of the aerial, or alternatively referred to the current loop, the ratio being the same in each case.

It follows that

$$\frac{R_r}{R_0} = \frac{\int_0^{\frac{\pi}{2}} \left\{J_n(q\sin\theta)\right\}^2 \sin^3\theta\,d\theta}{\int_0^{\frac{\pi}{2}} \sin^3\theta\,d\theta} = \frac{3}{2}I_1$$

where $I_1 = \int_0^{\frac{\pi}{2}} \left\{J_n(q\sin\theta)\right\}^2 \sin^3\theta\,d\theta$

This integral is evaluated in Appendix I, and gives the result :

$$\frac{R_r}{R_0} = \frac{3}{2}\left[-\frac{(2n-1)}{8q^2}J_{2n}(2q) + \frac{1}{4q}J_{2n+1}(2q) + \left\{\frac{1}{4q} + \frac{(4n^2-1)}{16q^3}\right\}\int_0^{2q} J_{2n}(t)\,dt\right] \quad (3)$$

The integral $\int_0^{2q} J_{2n}(t)\,dt$ is conveniently calculated by expressing it in the form $2\sum_{r=0}^{\infty} J_{2n+2r+1}(2q)$ as this series rapidly converges. R_0, the radiation resistance of a single aerial can be calculated from a knowledge of the aerial height by well-known methods, and the value of R_r then obtained.

The ratio $\frac{R_r}{R_0}$ is plotted in Fig. 6 for a range of values of r and n. We see that, as n increases, the radiation resistance rapidly decreases. This is a disadvantage, as it means that the operating current for a given power is high, and also that losses in the earth will be excessive ; further, the voltage at the base of the aerial is also high, since this is equal to the product of the base current and aerial impedance, and the reactive component of the impedance is usually much greater than the resistive component. The larger the value of n, therefore, the larger the value of r necessary in order to obtain a reasonable ratio of $\frac{R_r}{R_0}$. The site area and the consequent cost of the system will therefore be great, and this places a practical upper limit on the value of n which may be used.

Figs. 3 and 6, showing the vertical polar diagram and radiation resistance of the phased-ring aerial system, summarize the basic information on which a practical design can be based. A related important consideration, the number of ring aerials required to give a substantially circular horizontal polar diagram, is outside the scope of this paper.

Turning next to the case when $n = 0$, the radiation resistance of the ring alone, in the absence of the central aerial, can be deduced from equation (3) by putting $n = 0$. However, as we saw in Section 2, such a ring is usually used together with a central aerial in order to obtain antifading properties, and this case will be examined in more detail.

Let us again call R_r the radiation resistance of the concentric-ring aerial system as a whole, referred to the ring current ; in other words R_r is the power radiated by the complete ring system with unit current in the ring.

The ratio of the field strength at a distant point for the concentric-ring aerial and a vertical aerial of the same height, carrying a current equal to that of the ring, is

$$\{k + J_0(q \sin \theta)\} \sin \theta : \sin \theta$$

Using the same method as for the phased-ring system, the corresponding power ratio, and hence $\frac{R_r}{R_0}$, is given by :

$$\frac{R_r}{R_0} = \frac{\int_0^{\frac{\pi}{2}} \left\{k + J_0(q \sin \theta)\right\}^2 \sin^3\theta \, d\theta}{\int_0^{\frac{\pi}{2}} \sin^3\theta \, d\theta}$$

$$= \frac{3}{2}(I_1 + 2kI_2 + k^2 I_3) \quad .. \quad (4)$$

where $I_1 = \int_0^{\frac{\pi}{2}} \left\{J_0(q \sin \theta)\right\}^2 \sin^3\theta \, d\theta,$

$$I_2 = \int_0^{\frac{\pi}{2}} J_0(q \sin \theta) \sin^3\theta \, d\theta,$$

and $I_3 = \int_0^{\frac{\pi}{2}} \sin^3\theta \, d\theta = \frac{2}{3}.$

I_1 can be derived from the result given in Appendix I, by putting $n = 0$, and I_2 is derived in Appendix II.

Using these results we can determine R_r and thus the operating currents and voltages for a given power. It is also important, however, to know the effective radiation resistances of the central and ring aerials separately. This information is necessary in order to design the aerial matching networks, and also the circuits to obtain the correct power distribution.

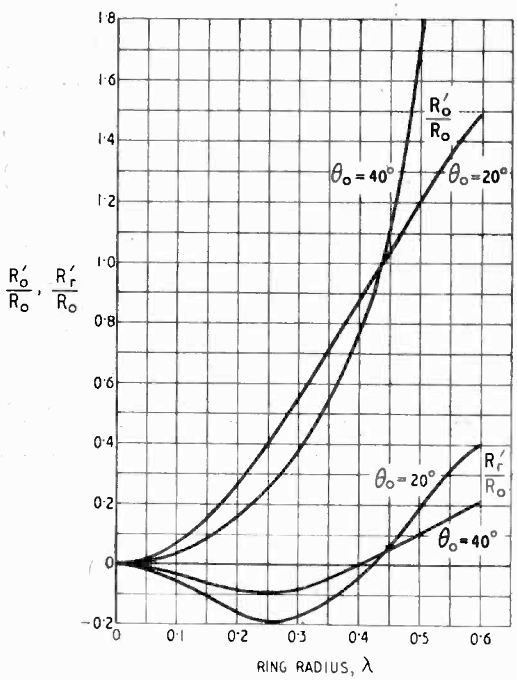


Fig. 7. *Radiation resistance of central and ring aerials in the concentric-ring system. R'_0 = effective radiation resistance of central aerial ; R'_r = effective radiation resistance of ring aerial ; R_0 = radiation resistance of vertical aerial of same height as ring ; θ_0 = angle of zero radiation.*

Let us call R_0 and R_1 the self-radiation resistance of the central and ring aerials respectively, and R_m the mutual resistance due to radiation coupling.

The effective radiation resistance of central and ring aerials, R'_0 and R'_r respectively, will be $R_0 + \dfrac{R_m}{k}$ and $R_1 + R_m k$.

The total power radiated for unit current flowing in the ring aerial and a current k in the central aerial, will be :

$$(R_1 + R_m k) + \left(R_0 + \frac{R_m}{k}\right) k^2$$
$$= R_1 + 2R_m k + k^2 R_0.$$

Since this is also equal to R_r we have

$$\frac{R_r}{R_0} = \frac{R_1}{R_0} + 2k\frac{R_m}{R_0} + k^2$$

Comparing this with equation (4) we find :—

$$\frac{R_1}{R_0} = \frac{3}{2} I_1 ; \quad \frac{R_m}{R_0} = \frac{3}{2} I_2$$

The effective radiation resistance of the component parts of the aerial are therefore as follows :

Effective radiation resistance of ring aerial,

$$R'_r = \frac{3}{2}(I_1 + I_2 k)R_0 \qquad \ldots \quad (5)$$

Effective radiation resistance of central aerial,

$$R'_0 = \frac{3}{2}\left(\frac{2}{3} + \frac{I_2}{k}\right) R_0 \quad \ldots \qquad \ldots \quad (6)$$

The ratios $\dfrac{R'_r}{R_0}$ and $\dfrac{R'_0}{R_0}$ are plotted in Fig. 7 for two typical values of k, namely -0.92 and -0.76. These values result in vertical polar diagrams having zeros at $\theta = 20°$ and $40°$ respectively, the corresponding diagrams being shown in Fig. 5. In the case of the ring aerial it is seen that for a ring radius in the neighbourhood of 0.4λ the effective radiation resistance is zero, in other words the correct current relationship is obtained without any power being fed to the ring, which is operated parasitically. For lower ring radii the radiation resistance is negative, power being fed back into the source by radiation coupling, but at higher values power must be fed to the ring.

Figs. 5 and 7 summarize the basic characteristics of the concentric ring aerial system.

4. Acknowledgments

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[3] H. Harbich and W. Hahnemann : "Wirksame Bekämpfung des Nahschwundes im Rundfunk durch Sende Antennengebilde Bestimmter Form" *Elektrische Nachrichten Technik*, October 1932.

APPENDIX I

Evaluation of $\displaystyle\int_0^{\frac{\pi}{2}} \left\{J_n(q \sin\theta)\right\}^2 \sin^3\theta \, d\theta$

The evaluation of this integral for the particular case when $n = 0$ was first given by W. N. Bailey (*Quarterly Journal of Mathematics*, June 1938, Vol. 9, No. 34, p. 141). The following solution is an extension of Bailey's method to the general case.

$$\text{Let } I_1 = \int_0^{\frac{\pi}{2}} \left\{J_n(q\sin\theta)\right\}^2 \sin^3\theta \, d\theta$$

$$\text{Then}^* \ I_1 = \frac{2}{\pi}\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} J_{2n}(2q\sin\theta\cos\phi)\sin^3\theta \, d\theta \, d\phi$$

$$= \frac{2}{\pi}\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \sin^3\theta \sum_{r=0}^{\infty} \frac{(-1)^r q^{2n+2r}\sin^{2n+2r}\theta\cos^{2n+2r}\phi \, d\theta \, d\phi}{r!\,(2n+r)!}$$

$$= \frac{2}{\pi}\sum_{r=0}^{\infty}\frac{(-1)^r q^{2n+2r}}{r!\,(2n+r)!}\int_0^{\frac{\pi}{2}}\sin^{2n+2r+3}\theta \, d\theta \int_0^{\frac{\pi}{2}}\cos^{2n+2r}\phi \, d\phi$$

$$= \frac{2}{\pi}\sum_{r=0}^{\infty}\frac{(-1)^r q^{2n+2r}}{r!\,(2n+r)!}\left\{\frac{(2n+2r+2)\,(2n+2r)\,\ldots\ldots\,4.2}{(2n+2r+3)(2n+2r+1)\,\ldots\,3.1}\right\}\left\{\frac{(2n+2r-1)(2n+2r-3)\ldots3.1}{(2n+2r)\,(2n+2r-2)\,\ldots\,4.2}\cdot\frac{\pi}{2}\right\}$$

$$= \sum_{r=0}^{\infty}\frac{(-1)^r q^{2n+2r}}{r!\,(2n+r)!}\;\frac{(2n+2r+2)}{(2n+2r+3)\,(2n+2r+1)}$$

$$= \sum_{r=0}^{\infty}\frac{(-1)^r q^{2n+2r}}{r!\,(2n+r)!}\left\{\frac{1}{2(2n+2r+3)} + \frac{1}{2(2n+2r+1)}\right\}$$

* G. N. Watson, "Theory of Bessel Functions." Cambridge 1922, § 5.43.

$$= \frac{1}{2q^3}\sum_{r=0}^{\infty}\frac{(-1)^r}{r!\,(2n+r)!}\int_0^q t^{2n+2r+2}dt$$

$$+\frac{1}{2q}\sum_{r=0}^{\infty}\frac{(-1)^r}{r!\,(2n+r)!}\int_0^q t^{2n+2r}dt$$

$$=\frac{1}{2q^3}\int_0^q \sum_{r=0}^{\infty}\frac{(-1)^r\,t^{2n+2r+2}}{r!\,(2n+r)!}\,dt$$

$$+\frac{1}{2q}\int_0^q \sum_{r=0}^{\infty}\frac{(-1)^r\,t^{2n+2r}}{r!\,(2n+r)!}\,dt$$

$$=\frac{1}{2q^3}\int_0^q t^2 J_{2n}(2t)\,dt+\frac{1}{2q}\int_0^q J_{2n}(2t)\,dt$$

$$=\frac{1}{16q^3}\int_0^{2q} t^2 J_{2n}(t)\,dt+\frac{1}{4q}\int^{2q} J_{2n}(t)\,dt$$

The first term may be integrated by parts :

$$\int_0^{2q} t^2 J_{2n}(t)\,dt$$

$$=\int_0^{2q} t^{-2n+1}\frac{d}{dt}\left\{t^{2n+1}J_{2n+1}(t)\right\}dt$$

$$=\Big[t^2 J_{2n+1}(t)\Big]_0^{2q}+(2n-1)\int_0^{2q} t\,J_{2n+1}(t)\,dt$$

$$=4q^2 J_{2n+1}(2q)$$

$$-(2n-1)\int_0^{2q} t^{2n+1}\frac{d}{dt}\left\{t^{-2n}J_{2n}(t)\right\}dt$$

$$=4q^2 J_{2n+1}(2q)-(2n-1)\Big[t\,J_{2n}(t)\Big]_0^{2q}$$

$$+(2n-1)(2n+1)\int_0^{2q} J_{2n}(t)\,dt$$

$$=4q^2 J_{2n+1}(2q)-(2n-1)2q\,J_{2n}(2q)$$

$$+(4n^2-1)\int_0^{2q} J_{n2}(t)\,dt$$

$$\therefore I_1=\frac{-(2n-1)}{8q^2}J_{2n}(2q)+\frac{1}{4q}J_{2n+1}(2q)$$

$$+\left\{\frac{1}{4q}+\frac{(4n^2-1)}{16q^3}\right\}\int_0^{2q} J_{2n}(t)\,dt.$$

$\int_0^{2q} J_{2n}(t)\,dt$ is conveniently calculated, when $n > -1$, by expressing it in the form

$$2\sum_{r=0}^{\infty} J_{2n+2r+1}(2q)$$

as this series converges rapidly.

APPENDIX II

Evaluation of $\int_0^{\frac{\pi}{2}} J_0(q\sin\theta)\sin^3\theta\,d\theta$

Let $I_2=\int_0^{\frac{\pi}{2}} J_0(q\sin\theta)\sin^3\theta\,d\theta$

To evaluate this we make use of the result : †

$$\int_0^{\frac{\pi}{2}} J_0(q\sin\theta)\sin\theta\cos^{2u+1}\theta\,d\theta=$$

$$\frac{2^u\Gamma(u+1)}{q^{u+1}}J_{u+1}(q)\ \text{ if } u>-1$$

On putting $u=-\frac{1}{2}$ this gives :

$$\int_0^{\frac{\pi}{2}} J_0(q\sin\theta)\sin\theta\,d\theta=\frac{2^{-\frac{1}{2}}\Gamma(\frac{1}{2})}{q^{\frac{1}{2}}}J_{\frac{1}{2}}(q)$$

$$=\sqrt{\frac{1}{2}\frac{\pi}{q}\frac{2}{\pi q}}\sin q$$

$$=\frac{\sin q}{q}$$

On putting $u=+\frac{1}{2}$:

$$\int_0^{\frac{\pi}{2}} J_0(q\sin\theta)\sin\theta\cos^2\theta\,d\theta=\frac{2^{\frac{1}{2}}\Gamma(\frac{3}{2})}{q^{\frac{3}{2}}}J_{\frac{3}{2}}(q)$$

$$=\frac{\sqrt{2}\,\frac{1}{2}\sqrt{\pi}}{q\sqrt{q}}\sqrt{\frac{2}{\pi q}}\left(\frac{\sin q}{q}-\cos q\right)=\frac{\sin q}{q^3}-\frac{\cos q}{q^2}$$

It follows that

$$I_2=\frac{\sin q}{q}-\frac{\sin q}{q^3}+\frac{\cos q}{q^2}$$

† Watson, *loc. cit.* § 12.11.

F.B.I. REGISTER OF BRITISH MANUFACTURERS

A limited further supply of this reference book, referred to in the December 1947 issue, is now available from Kelly's Directories Ltd., 186, Strand, London, W.C.2. Price £2 2s. It provides the only complete guide to members of the Federation of British Industries.

MECHANICAL HANDLING

The first National Mechanical Handling Exhibition, which is being organized by an associated journal *Mechanical Handling*, will be held at Olympia from 12th to 21st July. It features the saving of labour and the increase of speed in production, storage and transport which can be effected by the use of mechanization in handling.

SCIENTIFIC INSTRUMENT MANUFACTURERS' ASSOCIATION

The S.I.M.A. Handbook is a guide to the products of members of the association. It is arranged alphabetically and covers optical, survey, fire control, industrial, laboratory, medical and electronic apparatus. It is obtainable from the Association, 26, Russell Square, London, W.C.1, price 10s. 6d. post free.

DETERMINATION OF AERIAL GAIN FROM ITS POLAR DIAGRAM*

By J. A. Saxton, Ph.D., B.Sc., A.M.I.E.E.

(*Communication from the National Physical Laboratory*)

SUMMARY.—On the assumption that the field-strength distribution in the main forward lobe of a highly directive aerial may be represented by an ellipsoid, and that only a very small fraction of the total energy is radiated in side lobes, it is shown that the power gain of the aerial, compared with a doublet radiator, is $(\frac{8}{3})\phi^2$. Here ϕ is the ratio of the major to the minor axis of the ellipsoid. The validity of this approximation in certain circumstances has been demonstrated by measurements of the polar diagrams and gains of some aerials used for centimetre wavelengths.

1. Introduction

FOR the purpose of defining the gain of an aerial system it is necessary to postulate a standard comparison radiator. In the past the reference source has variously been taken to be a doublet or a half-wavelength dipole, while more recently a hypothetical omni-directional source, radiating equally in all directions, has been used as the standard. In the present investigation the doublet is used principally, though the gain relative to either of the other two comparison radiators may readily be derived, since a doublet has a gain of 1.76 db (i.e., a power gain of 1.5) and a half-wavelength dipole one of 2.15 db (1.65) relative to an omni-directional source. The power gain of an aerial system is thus here defined as the ratio of the power flux through a small solid angle about the direction of maximum radiation of the aerial to the flux through the same solid angle about the direction of maximum radiation of a doublet.

It is well known that there is an intimate relation between the polar diagram of an aerial and its gain, and a graphical method exists for the rigorous determination of the gain, given the polar diagram. This method is first described briefly below for purposes of comparison with a proposed simple solution of the problem for certain types of highly-directive aerials having negligible side-lobe radiation.

2. Graphical Solution

2.1 *Radiation from a Doublet*

Fig. 1 shows the polar diagram of a doublet radiator in the x, y plane, the doublet lying along the y-axis: the three-dimensional pattern is obtained by rotating the figure about the y-axis of symmetry.

The radius vector r is the field strength in volts per centimetre at a given distance D cm from the radiator, and at an angle θ to the x-axis, which is the direction of maximum radiation. D is sufficiently large for the radiation field only of the doublet to be of importance. Then we have :—

$$r = a\cos\theta \quad \ldots \quad (1)$$

where a represents the maximum field strength of the doublet in the equatorial plane ; r and a are peak values of the field strength, which is assumed to be a sinusoidal function of time.

$$\text{Now } a = \frac{60\pi M}{\lambda D} \text{ volts/cm}\dagger \quad \ldots \quad (2)$$

where the moment of the doublet, M, is equal to $i\,\delta l$, i being the peak current in the doublet in amperes, and δl cm being the length. $\delta l \ll \lambda \ll D$, λ being the wavelength of the radiation.

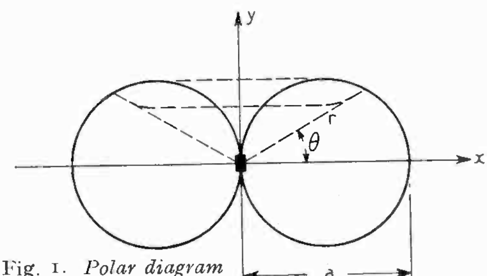


Fig. 1. *Polar diagram of a doublet.*

Since r is in volts/cm the average energy flux at the point where the field strength is r is given by :—

$$\frac{cr^2}{8\pi}\times\left(\frac{1}{300}\right)^2 = \frac{cr^2}{72\pi}\times 10^{-4} \text{ ergs per cm}^2 \text{ per sec} \quad \ldots \quad (3)$$

* MS accepted by the Editor, January 1947.

† See, for example, R. S. Glasgow, " Principles of Radio Engineering " (McGraw-Hill Book Co., 1936) p. 423.

(1 e.s. unit = 300 volts)

i.e., the energy flux $= \dfrac{r^2}{240\pi}$ watts per cm^2 since $c = 3 \times 10^{10}$ cm/sec (the velocity of light). Thus the power flux through a small solid angle about the y-axis contained between θ and $\theta + \delta\theta$ is given by δP where :

$$\delta P = \frac{r^2}{240\pi} \cdot 2\pi\, D^2 \cos\theta\, d\theta \quad \ldots \quad (4)$$

and P, the total power radiated, in watts, is :—

$$P = 2\int_0^{\pi/2} \frac{r^2 D^2 \cos\theta}{120}\, d\theta$$

$$= \frac{D^2}{60}\int_0^{\pi/2} r^2 \cos\theta\, d\theta \quad \ldots \quad (5)$$

[Note that actually P is independent of D, as it must be, since $r \propto 1/D$]

$$\text{Thus} \quad P = \frac{D^2}{60}\int_0^{\pi/2} R^2\, d\theta \quad \ldots \quad (6)$$

where $R = r\sqrt{\cos\theta}$

Now, if a derived polar diagram is plotted, as in Fig. 2, with R as a function of θ from 0 to $\pi/2$, then :—

$$A = \int_0^{\pi/2} \tfrac{1}{2}R^2\, d\theta \quad \ldots \quad (7)$$

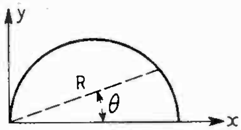


Fig. 2. $R = r\sqrt{\cos\theta}$ *as a function of* θ *for a doublet.*

where A is the area enclosed by the curve in Fig. 2.

Therefore

$$P = D^2A/30 \quad \ldots \quad (8)$$

2.2 *Directive Radiator*

Now consider a radiation pattern of a idealized form, such as that shown in Fig. 3, and consisting of a single forward lobe with ox as an axis of symmetry. It is assumed that the same power (P) is radiated as by the doublet. If M_1 is the moment of the radiator, then the field-strength pattern (again corresponding to a distance D) is :—

$$r = \frac{60\pi M_1}{\lambda D} \cdot f(\theta) \quad \ldots \quad (9)$$

where $f(\theta)$ is the function represented by Fig. 3. At $\theta = 0$, $f(\theta) = 1$ and the maximum forward field is $\dfrac{60\pi M_1}{\lambda D}$. The forward power gain, G (as defined in the Introduction) of the radiator is given by :—

$$G = \left(\frac{M_1}{M}\right)^2 \quad \ldots \quad (10)$$

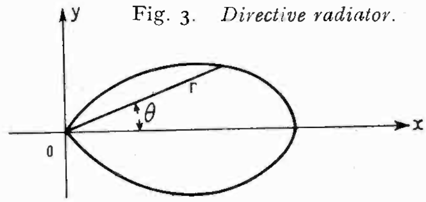


Fig. 3. *Directive radiator.*

$$\therefore \frac{r}{\sqrt{G}} = \frac{60\pi M}{\lambda D} \cdot f(\theta) = a \cdot f(\theta) \ldots \quad (11)$$

$$= r_1, \text{ say.}$$

Thus, if the field-strength polar diagram of the directive radiator is plotted so that the maximum value along the ox axis is represented by the same vector as that considered above for the doublet (that is a), we have for the total power radiated :—

$$P = \int_0^{\pi/2} \frac{r^2D^2 \sin\theta\, d\theta}{120}$$

$$= \frac{GD^2}{120}\int_0^{\pi/2} r_1^2 \sin\theta\, d\theta$$

$$= \frac{GD^2}{120}\int_0^{\pi/2} R_1^2\, d\theta \quad \ldots \quad (12)$$

where $R_1 = r_1\sqrt{\sin\theta}$

Therefore

$$P = \frac{GD^2}{60}A_1 \quad \ldots \quad (13)$$

where A_1 is the area of the polar diagram obtained by plotting R_1 as a function of θ from 0 to $\pi/2$.

$$\text{Then} \quad \frac{GD^2}{60}A_1 = \frac{D^2}{30}A = P$$

$$\text{and} \quad G = \frac{2A}{A_1} \quad \ldots \quad (14)$$

So that by evaluating the areas A and A_1 the power gain may be determined. If appreciable side lobes exist in the original pattern, then their corresponding areas in the derived diagram must also be taken into account.

3. Ellipsoid Theory

It is of interest to see if, in certain special cases, the aerial gain can be determined directly from the (r, θ) polar diagrams without recourse to the construction of the (R, θ) diagrams and measurement of the corresponding areas A and A_1.

It may be observed that with many highly directive aerials, as in the case of those used with centimetre wavelengths, the (r, θ) diagram may frequently be represented by a single lobe which is approximately an ellipsoid of revolution about the axis of maximum radiation. Such aerials are sectorial horns of roughly square aperture, or those using paraboloidal reflectors There are, of course, always small side lobes, but it is often possible to arrange that only a small fraction of the total radiated energy appears outside the main lobe.

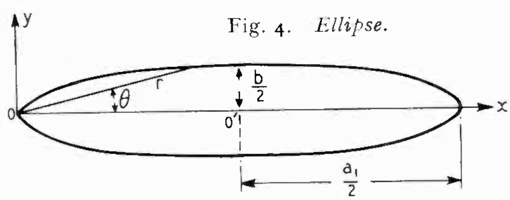
Consider first the total radiation from a doublet. Substituting $r = \dfrac{60\pi M}{\lambda D}\cos\theta$ in equation (5), we have :—

$$P = \frac{D^2}{60}\int_0^{\pi/2}\left(\frac{60\pi M}{\lambda D}\right)^2 \cos^3\theta\, d\theta$$

$$= \frac{40\pi^2 M^2}{\lambda^2} \quad \cdots \quad \cdots \quad \cdots \quad (15)$$

Now let us assume that the polar diagram of a highly directive aerial is in the form of an ellipsoid of revolution about the x-axis, the generating ellipse being as shown in Fig. 4.

Fig. 4. *Ellipse.*



Referred to o′ as origin the equation of the ellipse is :—

$$\frac{x^2}{a_1^2} + \frac{y^2}{b^2} = \frac{1}{4} \quad \cdots \quad \cdots \quad \cdots \quad (16)$$

a_1 being the maximum forward field at a distance D, and

$$a_1 = \frac{60\pi M_1}{\lambda D}\ \text{volts/cm}$$

where M_1 is the moment of the radiator as defined in Section 2.2.

On substituting :—

$$\begin{cases} x = -\left(\dfrac{a_1}{2} - r\cos\theta\right) \\ y = r\sin\theta \end{cases}$$

we find $r = \left(\dfrac{a_1\cos\theta}{\phi^2\sin^2\theta + \cos^2\theta}\right) \quad \cdots \quad (17)$

where $\phi = a_1/b$

r being in volts/cm, the total radiated power, P, in watts is :—

$$P = \frac{D^2}{120}\int_0^{\pi/2} r^2 \sin\theta\, d\theta \quad \cdots \quad \cdots \quad (18)$$

$$= \frac{D^2 a_1^2}{120}\int_0^{\pi/2}\frac{\cos^2\theta\sin\theta\, d\theta}{[\phi^2\sin^2\theta + \cos^2\theta]^2}$$

$$= \frac{D^2 a_1^2}{120}\cdot I \quad \cdots \quad \cdots \quad \cdots \quad (19)$$

where

$$I = \int_0^{\pi/2}\frac{\cos^2\theta\sin\theta\, d\theta}{[\phi^2\sin^2\theta + \cos^2\theta]^2} \quad \cdots \quad (20)$$

I may be obtained by integrating by parts, with the result :—

$$I = \frac{1}{2(\phi^2 - 1)}\left[1 - \frac{1}{2\phi\sqrt{\phi^2 - 1}}\log_e\left(\frac{\phi + \sqrt{\phi^2 - 1}}{\phi - \sqrt{\phi^2 - 1}}\right)\right] \quad \cdots \quad (21)$$

If $\phi \gg 1$ (high gain antenna) equation (16) approximates to :—

$$I \approx \frac{1}{2\phi^2}\left[1 - \frac{\log_e 4\phi^2}{2\phi^2}\right] \quad \cdots \quad \cdots \quad (22)$$

which, for the same conditions, obviously reduces further to :—

$$I \approx 1/2\phi^2 \quad \cdots \quad \cdots \quad \cdots \quad (23)$$

and therefore :—

$$P = \frac{D^2 a_1^2}{240\phi^2} \quad \text{or} \quad P = \frac{15\pi^2 M_1^2}{\lambda^2\phi^2} \quad \cdots \quad (24)$$

Thus, if the directive aerial, with power gain G, radiates the same power, P, as the doublet, we have :—

$$G = \left(\frac{M_1}{M}\right)^2 \text{[from equation (10)]}$$

$$= \frac{8}{3}\phi^2 \quad \cdots \quad \cdots \quad \cdots \quad (25)$$

If $\phi = 15$, $G = 600$, (27.8 db) and the error in using equation (23) instead of (21) is less than 2 per cent. For $\phi = 5$ the corresponding difference is about 10 per cent. The error introduced in using equation (22) instead of (21), however, is still less than

5 per cent for ϕ as low as 2.5, and, within the limitations of the theory, (22) may therefore be used for gains as small as 12 db.

4. Measurements of Aerial Gains and Polar Diagrams

4.1 *Gain Determinations*

The author and Grace[1], have recently described a field-strength meter for centimetre wavelengths. The instrument consists of an electromagnetic-horn receiver incorporating a radio-frequency bolometer, this latter being placed in the throat of the horn and also connected into one arm of a modified form of Wheatstone Bridge, which is calibrated to give directly the power in the bolometer. The horn receiver is shown in Fig. 5; a length of about 25 cm of rectangular-section waveguide (internal dimensions, 7.5 cm × 2.6 cm) is terminated at one end by a horn flared in both directions to make an angle of about 25 degrees with the waveguide section, the final aperture being 20 cm × 14.5 cm. The bolometer, B, is placed in the guide as shown, and the system is tunable by means of the two stubs S_1, S_2 and the piston P closing the rear end of the waveguide.

If we have a transmitter with an aerial power gain G_s relative to a half-wavelength dipole radiating a power of W watts on a wavelength λ, then it is readily shown that the power P, obtained in a receiver, under free-space conditions and at a distance sufficiently great for the field-strength to be inversely proportional to the distance, is :—

$$P = \frac{49\lambda^2 G_S G_R W}{300\pi^2 D^2} \text{ watts} \quad \ldots \quad \ldots \quad (26)$$

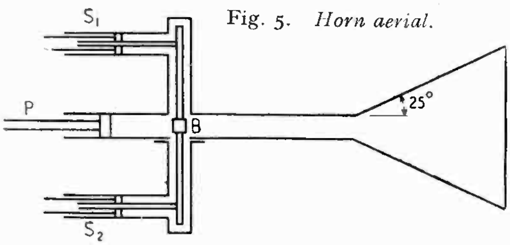


Fig. 5. *Horn aerial.*

where G_R is the gain of the receiver relative to a half-wavelength dipole

D is the distance of transmission

D and λ must, of course, be in the same units.

If two horns, similar to the one described above, are used as transmitter and receiver, and if they have identical gains, G_h, then :—

$$P = \frac{49\lambda^2 G_h^2 W}{300\pi^2 D^2} \quad \ldots \quad \ldots \quad \ldots \quad (27)$$

The conditions under which equation (27) may be applied to determine G_h are discussed in detail in the work referred to above. Using this method it was found that the gain of the horn relative to a half-wavelength dipole was 11.5 db and 13.3 db at the wavelengths of 9.2 cm and 6.2 cm respectively. In terms of the doublet as standard these factors become 11.9 db and 13.7 db.

The second type of aerial system investigated consisted of a half-wavelength dipole at the focus of a paraboloidal mirror having an aperture 122 cm in diameter, the focus being in the aperture plane; a parasitic aerial situated one-eighth of a wavelength in front of the dipole slightly increased the total gain, and also reduced side lobes. The system is shown diagrammatically in Fig. 6. As described in reference 1, the product of the gain of this system and the gain of the horn, investigated previously, was determined using equation (26). Hence knowing G_h, the gain of the paraboloid system was obtained. In this manner it was found that for $\lambda = 9.2$ cm the gain of the system was 27.7 db relative to a doublet. For $\lambda = 6.2$ cm the corresponding value was 31.4 db.

4.2 *Measurement of Polar Diagrams*

The polar diagrams of the aerials were determined by using the paraboloid system and the horn as transmitter and receiver, or vice versa. In all cases it was the receiver which

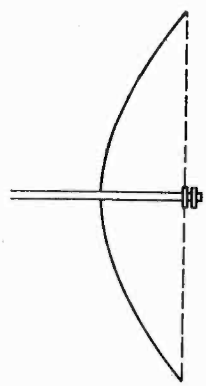


Fig. 6. *Paraboloidal aerial.*

was rotated about a vertical axis, and the received power was observed as a function of azimuth. Measurements were made in the plane of the E-vector by having the dipole in the paraboloid horizontal, and in the plane of the H-vector by having the dipole vertical. The horn was orientated appropriately in the two cases, that is with the short side of the waveguide horizontal or vertical respectively. The observations were made at a spacing between the aerials such that the free-space law of attenuation of field-strength inversely with distance was obtained. The direct-reading form of Wheatstone Bridge mentioned above was used to determine the received power, and it was possible to measure powers in the range 5 microwatts to 1 milliwatt.

With the paraboloidal mirror and with both wavelengths used, (9.2 and 6.2 cm) no side lobes were observed in which the power at the peak of such a lobe was more than 1 per cent of the power at the maximum of the main lobe. It was not possible to delineate these side lobes accurately with the present apparatus, and they are accordingly not plotted in Figs. 7 and 8, but their

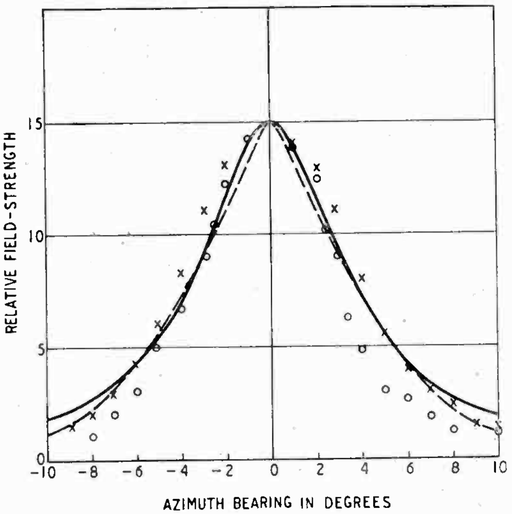


Fig. 7. *Radiation pattern of paraboloid wavelength* = 9.2 cm. *Experimental observations ;* ⊙ *H - vector plane ;* × *E - vector plane. Theoretical curves :— ——ellipse ; — — — Gaussian function.*

absence from these figures should not be taken as indicating that very small side lobes do not exist. The total power radiated in the side lobes is, however, such a very small fraction of the power in the main lobe that it is sufficient for present purposes simply to consider the main lobe. It was also found for the particular size of horn used that, except for the side lobes shown in Figs. 9 and 10, any other lobes were again such that a negligibly small fraction of the total radiated power was contained in them.

5. Comparison of Experimental Results with Theory

It is frequently assumed that a Gaussian function gives a good approximation for the polar diagram of a directive aerial. Fig. 7 shows such a function which has been made to agree with the experimental results at the peak and quarter-peak power points (half maximum field-strength points) for the paraboloid at the wavelength 9.2 cm. The same figure also shows an ellipse, plotted in Cartesian coordinates, and made to agree at the same points. It is quite evident that in the region of maximum radiation the ellipse is in appreciably better agreement with experiment than the Gaussian curve, and this region is, of course, the most important. There is little to choose between the two theoretical curves at angles well away from the direction of maximum radiation.

The two field-strength diagrams, in polar coordinates, given in Figs. 11 and 12 show the immediate resemblance of the main lobe to an ellipse. The rest of the field-strength diagrams are here shown in Cartesian coordinates since this form of presentation perhaps lends itself better to a more critical comparison between theoretical curves and experimental points. In each case the length of the minor axis of the ellipse is the mean of the original experimental widths of the beam in the E- and H-vector planes plotted in polar coordinates.

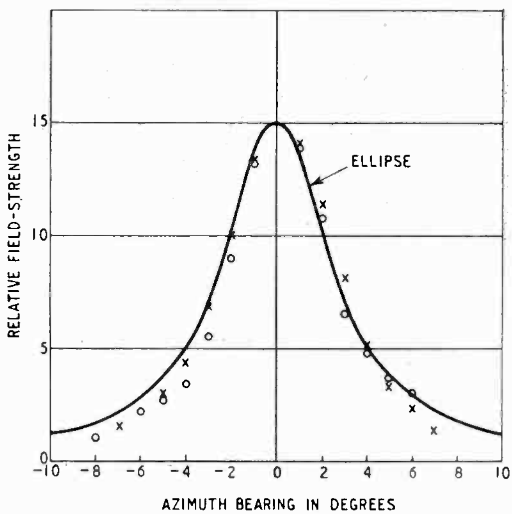


Fig. 8. *Radiation pattern of paraboloid : wavelength* = 6.2 *cm. ;* ⊙ *H - vector plane ;* × *E - vector plane.*

For the cases examined the beam widths in the two planes differed only slightly. Suppose the corresponding values of a_1/b are ϕ and $\phi + \delta\phi$; then probably the appropriate value to insert in equation (25) is $\phi(\phi + \delta\phi)$. To the first order of small quantities this is the same as $(\phi + \delta\phi/2)^2$, so that simply the mean value of ϕ may be used. Table I shows the comparison between the experimentally measured gains of the various aerials, and the gains calculated on the ellipse theory based on the experimental polar diagrams.

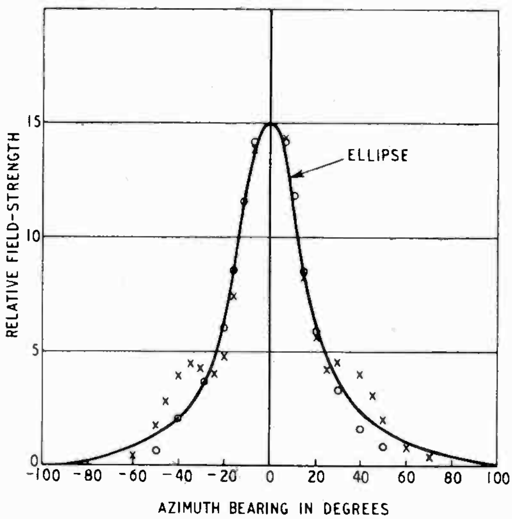


Fig. 9. *Radiation pattern of horn: wave-length = 6.2 cm.;* ⊙ *H-vector plane;* × *E-vector plane.*

The agreement between measured and calculated gains is seen to be quite good. It should be mentioned that, due to errors inherent in the measurement of power by the bolometer method, the accuracy of the directly measured values of the gains is probably not better than 1 db.[1] Also equation (22) instead of (23) should really be used to obtain the gains in the two horn cases on the ellipse theory; this would result in an increase of about 0.5 db in the calculated values.

Robertson and King[2], and Mueller[3] have recently described some high-gain aerials and have given values for the power gains and also the beam widths between half-power points on either side of the maximum. On the ellipse theory, the half-power is obtained by putting $r = \left(\frac{a_1}{\sqrt{2}}\right)$ in equation (17). This gives :—

$$\frac{\cos\theta}{\phi^2 \sin^2\theta + \cos^2\theta} = \frac{1}{\sqrt{2}} \quad \ldots \quad \ldots \quad (28)$$

Now provided $\theta \ngtr 10$ degrees it is a sufficiently good approximation to put $\sin\theta = \theta$ and $\cos\theta = \cos^2\theta = 1$. This will hold for a high-gain aerial. Then equation (28) reduces to

$$\frac{1}{\phi^2\theta^2 + 1} = \frac{1}{\sqrt{2}} \quad \ldots \quad \ldots \quad (29)$$

or $$\phi^2 = \frac{\sqrt{2} - 1)}{\theta^2}$$

and $$G = \frac{8}{3}\phi^2 = \frac{8(\sqrt{2} - 1)}{3\theta^2} \quad \ldots \quad \ldots \quad (30)$$

In (30) θ is in radians; if θ is expressed in degrees we find that :—

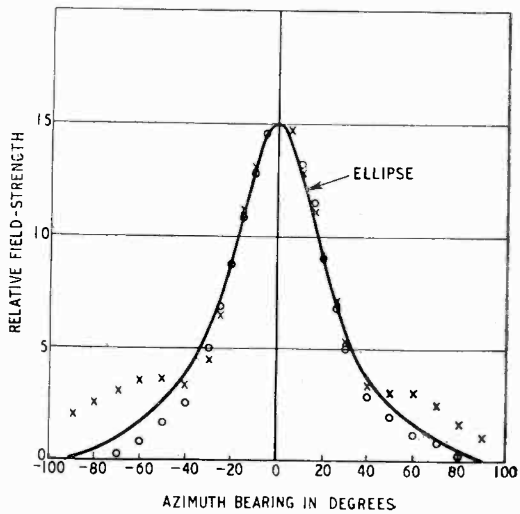


Fig. 10. *Radiation pattern of horn: wave-length = 9.2 cm;* ⊙ *H-vector plane;* × *E-vector plane.*

TABLE I

Aerial System		Gain Relative to Doublet (db) Measured	$(8/3)\,\phi^2$
Paraboloid	Wavelength 9.2 cm	27.7	27.4
	,, 6.2 cm	31.4	31.0
Horn	,, 9.2 cm	11.9	12.2
	,, 6.2 cm	13.7	15.0

$$G = \frac{3600}{\theta^2} \qquad (31)$$

2θ is the total angle between the two half-power points.

The gains quoted by Robertson and King, and by Mueller are relative to an omni-directional source, the values given in Table II are less than those quoted by 1.76 db, so that they are now relative to a doublet and may be compared with the values obtained from equation (31).

In the works referred to the beam widths in one plane only were given, but it is unlikely that the divergence from symmetry would be great. It can be seen from Table II, that the ellipse theory again gives quite good agreement with the quoted aerial gains.

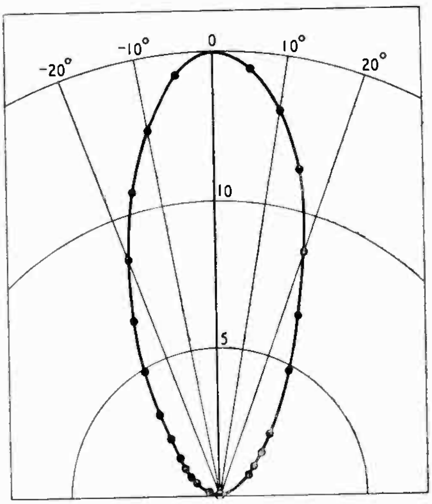


Fig. 11. *Polar diagram of horn : wavelength = 9.2 cm. ; H - vector plane.*

6. Conclusions

It appears that the main lobe of the field-strength pattern of a highly-directive aerial may often be assumed to be ellipsoidal in shape. In such cases, the side-lobe radiation being negligible, a useful approximation to the power gain is given by $\left(\frac{8}{3}\right)\phi^2$, ϕ being the ratio of the major axis to the minor axis of the ellipsoid. This conclusion has been confirmed by comparison with experimental work, both by the author and by others.

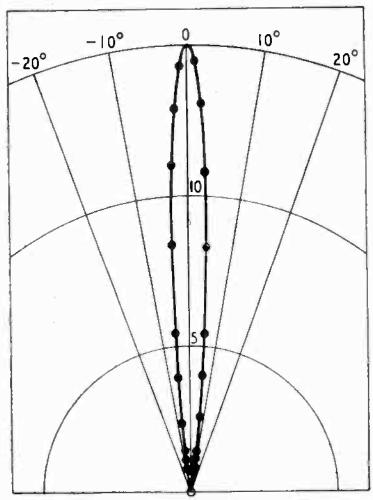


Fig. 12. *Polar diagram of paraboloid : wavelength = 9.2 cm. ; E - vector plane.*

7. Acknowledgments

The work described above was conducted as part of the programme of the Radio Research Board, and is published by permission of the Department of Scientific and Industrial Research. The author desires to acknowledge the work performed by G. W. Luscombe, B.Sc. and J. A. Lane, B.Sc., who assisted in carrying out the observations.

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[2] S. D. Robertson and A. P. King : "The Effect of Rain upon the Propagation of Waves in the 1- and 3-Centimetre Regions." *Proc. Inst. Radio Engrs*, Vol. 34, No. 4, 1946, p. 178.

[3] G. E. Mueller : "Propagation of 6-Millimetre Waves." ibid, p. 181.

TABLE II

Aerial System	2θ (half-power)	Gain Relative to Doublet (db)	
		Quoted	($3600/\theta^2$)
Horn : Wavelength 1.09 cm	3.5°	29.25	30.7
Paraboloid, Aperture 30 in : Wavelength 3.2 cm	3.0°	33.25	32.1
Horn : Wavelength 3.2 cm	10.0°	21.25	21.6
Paraboloid, Aperture 18 in : Wavelength 0.62 cm	0.8°	43.25	43.5

RECTIFIER RESISTANCE LAWS*

Analysis of Non-linear Transmission Circuits

By D. G. Tucker, Ph.D., A.M.I.E.E.

(Post Office Research Station)

SUMMARY.—This paper discusses the applications of an exponential relationship between the resistance of a rectifier and the voltage across it, with particular reference to the analysis of modulator-circuit performance. The measured performance of several types of rectifier—copper-oxide, selenium, silicon, germanium and diodes—is presented as a set of curves relating voltage, current, d.c. resistance, a.c. resistance, temperature and the variation within samples of one type of rectifier. It is shown that in all cases the resistance (d.c. or a.c.) can be fitted to a law of the type :—

$$R = R_0 + \kappa\epsilon^{-qV}$$

where R_0, κ and q are constants for any particular rectifier. The accuracy of representation is best in the case of diodes (although their d.c. resistance is meaningless and is not considered) and is worst in the case of silicon and germanium crystal rectifiers. It is shown that the use of this law in modulator circuit analysis is particularly useful and enables, by its simplicity, some of the more detailed properties of the circuit to be investigated ; these would otherwise be rather formidable. The use of the law in developing new and improved circuits is also illustrated.

1. Introduction

CIRCUITS involving metal rectifiers and diodes generally present considerable difficulty in analysis because of the non-linear characteristics involved. If the non-linearity is associated with capacitance or inductance, the analysis usually becomes virtually impossible, but in many cases only resistance is concerned, and then the analysis may generally be effected by a method falling in one of the following classes :—

(a) The non-linear element is assumed to have a constant low resistance for one direction of current or voltage and an infinite (or constant high) resistance for the other direction. This is the case of "switching."

(b) The non-linear element may have a voltage-current characteristic of the type

$$V = \kappa I^\beta \ (\kappa \text{ and } \beta \text{ constant})$$

over a certain range of current in each direction, the constants κ and β being different for each direction. The transition from one direction to the other is then represented by a switching-function as in (a).

(c) The non-linear characteristic may be represented by a power-series of the type

$$I = \sum_{n=0}^{\infty} a_n V^n$$

which can rarely be dealt with as an infinite series, but is quite manageable if n is restricted to a small range of integers.

(d) The non-linear characteristic may be represented by some continuous function (which can always be expressed as a power series if desired) which can be conveniently manipulated in analysis. Examples are the exponential and the trigonometric functions, the latter being sometimes employed only over a limited angular range.

The above classes may be broadly grouped as "discontinuous" [classes (a) and (b)] and "continuous" [classes (c) and (d)].

Class (a) is frequently used in dealing with rectifier modulator circuits of ring or Cowan types[1,2]. While primarily of value in qualitative assessments of circuit behaviour, the method does in fact give results surprisingly in accord with experience from many points of view, although it is quite inadequate for determining the finer points

* MS accepted by the Editor, December 1946.

of modulator operation, such as carrier leak in a highly-balanced modulator.

Class (b) is intrinsically the most accurate method of analysis of rectifier circuits, since the law given has been shown to apply very closely to contact-type rectifiers over several decades of current magnitude.[3] An approach to circuit analysis using this law has been indicated by Ashworth, Needham and Sillars[4], but this is not adequate for such circuits as modulators, where the switching conception must be used in addition.

The switching action, in so far as it represents a sudden change in the circuit, is analagous to the Heaviside Unit Step function, and this leads to the use of Fourier series in the analysis[5,6], which becomes very involved and difficult. The mathematical complexity of such a method tends to obscure the physical significance of the work.

Class (c) has received considerable attention as it is particularly suitable for dealing with harmonic production and inter-modulation in valve circuits[7,8,9]. The analysis of the synchronized oscillator and its associated problems has also been treated by this

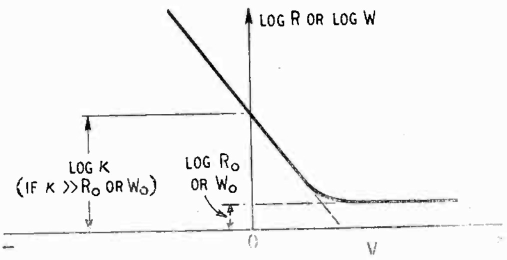

Fig. 1. *The exponential resistance laws,* $R = R_0 + \kappa_1 \epsilon^{-q_1 V}$; $W = W_0 + \kappa_2 \epsilon^{-q_2 V}$.

method[10,11,12], with the restriction of the power series to the first four terms. As far as rectifier circuits are concerned, Caruthers[13] has used the power series for some aspects of the analysis of balanced-modulator circuits.

Class (d) has received little attention, in spite of its great potentialities. In modulator circuits it is, from many points of view, convenient to work in terms of resistance functions, as these can be readily used in transmission equations, and Degawa[14] has used the equation

$$r = r_0 + r_c \cos ct$$

to express the variation of rectifier resistance with carrier frequency and time. This simple cosine function is, of course, a poor representation in general, though by suitable

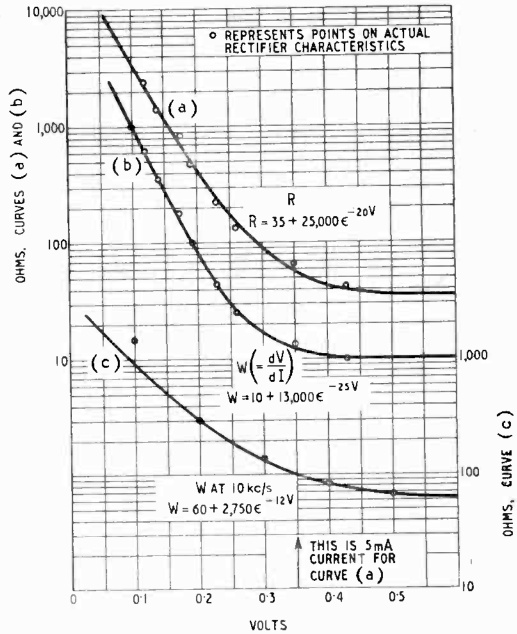

Fig. 2. *Copper-oxide rectifiers. Curve (a) d.c. resistance (forward) of H*1 *disc; curve (b) a.c. resistance (forward) of H*1 *disc; curve (c) a.c. resistance of* $\frac{3}{16}$*-in diam. disc at* 10 *kc/s, according to Caruthers.*

adjustment of the carrier voltage it can be made to apply fairly closely.

Considering the main requirements of a resistance function to represent a rectifier, we can see that a fairly accurate representation is required over the forward voltage range, since the resistance is there low, but all that is really required in the backward range is that the resistance should be high;

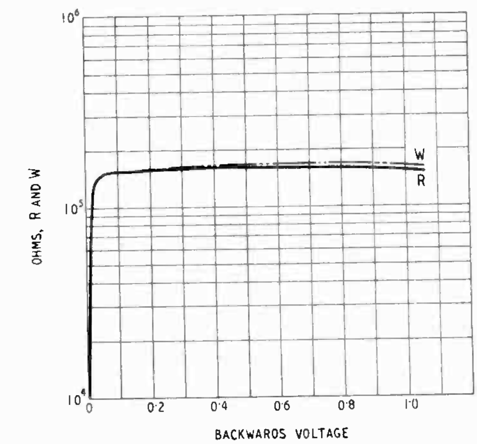

Fig. 3. *Backwards characteristics of copper-oxide rectifier, type H*1.

the exact value is then unimportant, rarely having any significance in the circuit operation. From this consideration it seems that an exponential function would be satisfactory, since a relation such as

$$r = \kappa \epsilon^{-qV}$$

(where κ and q are constants and V is the voltage across the rectifier) gives a high

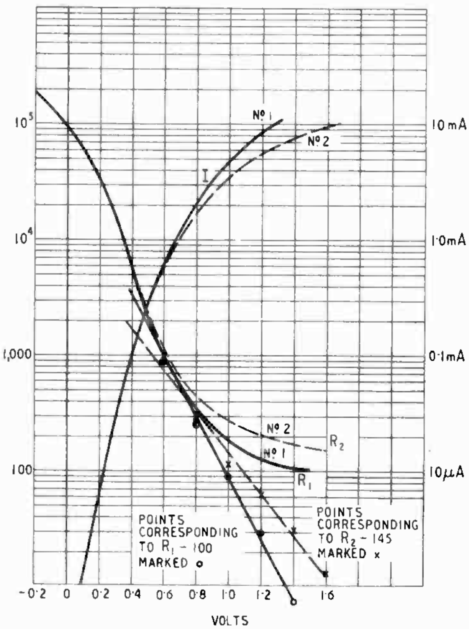

Fig. 4. *D.C. characteristics (forward) after 5 temperature cycles of copper-oxide rectifiers, type G2.*

backward resistance and a low forward resistance falling smoothly as the voltage increases. In the next sections it is shown that such a law can indeed be applied to most types of rectifier (including diodes, in which an exponential relation between current and voltage, and between a.c. resistance and voltage, is well-known) with the modification of the addition of a constant term. The application of the law to circuit analysis is straightforward and accurate, and is discussed in Section 4.

2. Exponential Resistance Law

The resistance of a rectifier is a quantity requiring careful definition. If V is the voltage across the rectifier and I is the current through it, then we can refer to the ratio V/I as the d.c. resistance designated R; and the differential ratio $\frac{dV}{dI}$, being the measure of the resistance effect with a.c. signals, can be called the a.c. resistance, designated W. Now R and W are functions determined explicitly by V, but the effective a.c. resistance depends not only on V, which may be regarded as a bias voltage, but also on the amplitude of the a.c. signal to which it is presenting the resistance. In all the work which follows, this difficulty is avoided by assuming the a.c. signal to be small so that the resistance presented to it is indeed $\frac{dV}{dI}$; i.e., W. Throughout, no consideration will be given to rectifier capacitance; it is almost impossible to allow for it analytically (although Kruse[15] gives a useful discussion of it) and the best way of dealing with the problem is to avoid it by using rectifiers with a sufficiently small capacitance. With the advent of the crystal-valve[16, 17] rectifiers with a capacitance of less than 1 $\mu\mu F$ are available, and meet most requirements.

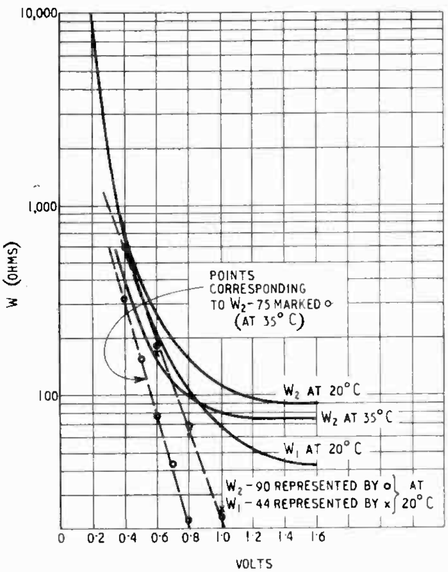

Fig. 5. *A.C. resistance on 5th temperature cycle of copper-oxide rectifiers type G2.*

It is found that both R and W reach constant low values at forward voltages exceeding some particular value, generally about 0.5 volt, and these values may be designated R_0 and W_0. At small voltage values in both directions, the relation of R and W to V is not far from exponential, and the resistance laws suggested for use here are therefore

$$R = R_0 + \kappa_1 \epsilon^{-q_1 V} \quad \quad (1)$$

$$\text{and } W = W_0 + \kappa_2 \epsilon^{-q_2 V} \quad \quad (2)$$

Fig. 1 shows these relations in a general form. It will be seen that κ is determined by the intercept on the resistance axis assuming $\kappa \gg R_0$ or W_0 which is usually the case in practice ; and if resistance is plotted on a logarithmic scale, then the slope of the straight line portion of the graph through the resistance axis gives q. Thus R_0, κ and q are characteristic parameters for any particular rectifier. It is found that κ_1 and κ_2 are somewhat different from one another, as also are q_1 and q_2, even for the same rectifier. The resistance of a real rectifier at large negative voltages ceases to rise, and generally falls somewhat ; this is not indicated by the exponential laws given, but this is of little importance as the resistance is high enough in practical circuits to have a negligible effect on the circuit in any case.

It will be observed, then, that the fit of the exponential law to a measured rectifier characteristic is easily tested by plotting on log paper. If the constant low resistance is subtracted from all measured resistance values, then the resulting values should all lie on a straight line. This method of test is used in the next section.

It should be pointed out that the laws are entirely empirical, and though one is tempted to explain them on physical grounds, there is really no justification for this.

3. Measured Rectifier Characteristics

In this section a large set of rectifier characteristics is presented with the dual purpose of providing useful data and of showing the applicability of the exponential law. For the most part the figures are self-explanatory, but a brief discussion of them follows.

3.1. *Copper-oxide Rectifiers.* (Figs. 2–6).

Figs. 2 and 3 show resistance characteristics of H1-type rectifiers based on earlier measurements made ten years ago. For the forward d.c. and a.c. resistances the measured values have been fitted to an exponential law, and it will be seen that the agreement is remarkably close. Fig. 2, curve (c), shows the a.c. resistance of a 3/16th-in diameter disc at 10 kc/s, the measurements being given by Caruthers.[13] The observed values again fit an exponential law very closely, although it is evidently a different law to that fitting the $\frac{dV}{dI}$ curve, which is the a.c. resistance at very low frequencies, where the capacitance plays no part.

Fig. 4 shows the d.c. resistance of G2 type rectifiers. The test described in Section 2 is applied, and it will be observed that the fit of the exponential law is good from a voltage of about 0.5 (i.e., 0.25 volt per disc) upwards. Below this the fit is poor, but as the resistance is over 10 times the constant value R_0, the accuracy of circuit analysis may well be unaffected. The same remarks apply to the a.c. resistances shown in Fig. 5, where the effect of 15° C rise of temperature is also shown. Fig. 6 shows the effect of temperature on the d.c. characteristics ; the results of the fifth temperature cycle are shown as earlier cycles are not quite cyclic. Conclusions to be drawn from these curves are that, (a), q_1 tends to remain constant with temperature although R_0 and κ_1 change, both decreasing with rising temperature and, (b), W_0 decreases with rising temperature, while q_2 perhaps remains constant.

3.2. *Selenium Rectifiers* (Figs. 7–12).

It will be seen from Fig. 7 that the d.c. resistance of a small selenium rectifier (Sentercell type 7A) varies over a ratio of more than 10,000 to 1 in the forward voltage range. The upper three decades are nearly exponential, and from 1,000 ohms down to the constant resistance the law also fits the exponential relation (1), but with different parameters in the two ranges. The a.c. resistance (Fig. 9) is well fitted by the exponential law from 0.4 volt upwards, and can be approximately fitted from 1,000 ohms downwards.

Figs. 8 and 9 show the variations of characteristic in a sample of 16 rectifiers. The current values at 0.8 volt were shown to follow a normal error-law distribution about the average value, from which the percentages shown in Fig. 8 were deduced. From the R and W curves it appears that q_1 and q_2 are fairly constant throughout the sample, but κ_1 and κ_2 together with R_0 and W_0 vary widely.

Figs. 11 and 12 show the effects of temperature. It will be seen that none of the parameters remains unaffected.

3.3. *Silicon Crystal-Valve Rectifiers* (Figs. 13–15).

It will be seen that the exponential resistance law applies from about 0.2 volt upwards, over a resistance range of about 5 to 1 only. This cannot be considered good agreement, although it may be accurate

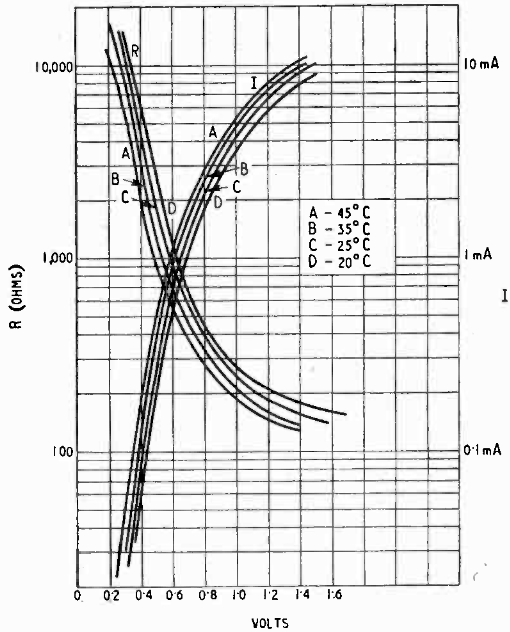

Fig. 6. *D.C. characteristics at various temperatures on the 5th temperature cycle of copper-oxide rectifier type G2.*

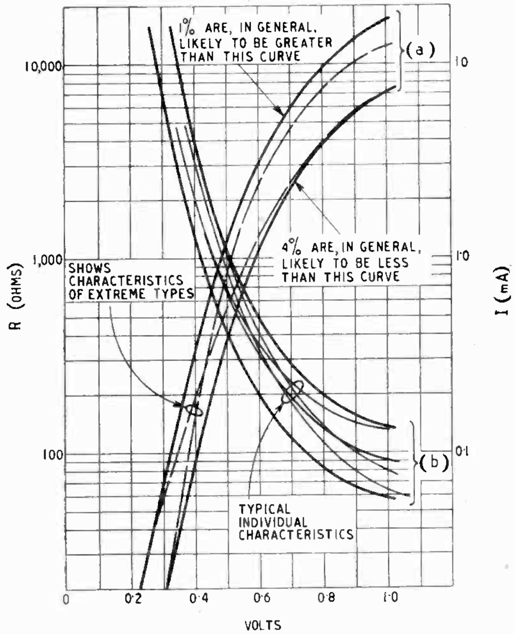

Fig. 8. *Selenium rectifiers, type 7A. Curves (a) show the envelope of the V–I characteristics of 16 sample rectifiers and curves (b) give the envelope of the d.c. resistance characteristics.*

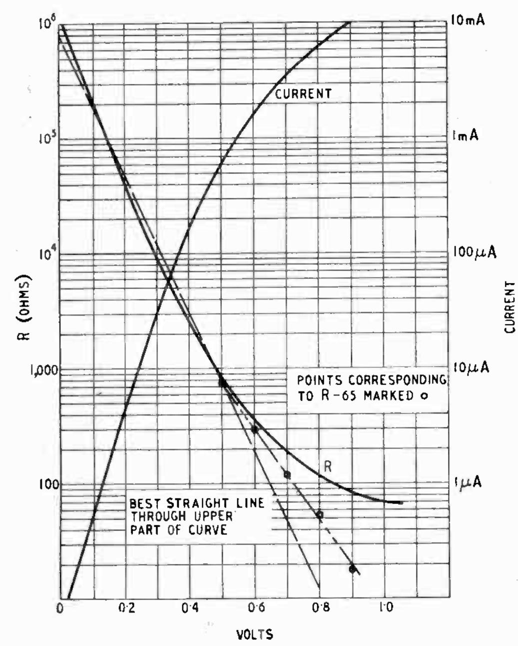

Fig. 7. *D.C. characteristics (forward) of selenium rectifier type 7A.*

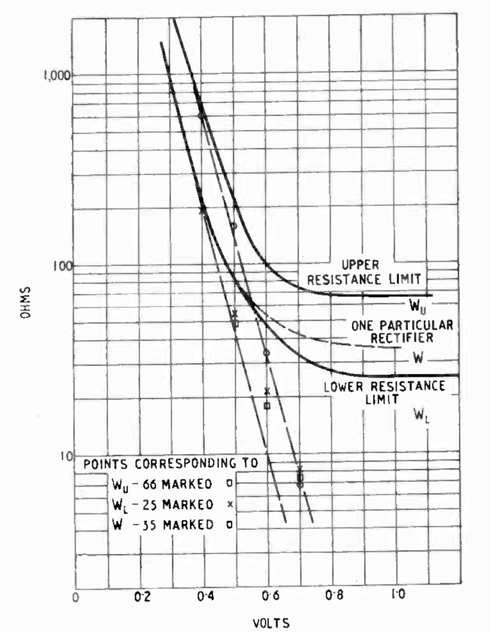

Fig. 9. *Selenium rectifiers type 7A; envelope of a.c. resistance characteristics (forward) of sample of 16 rectifiers.*

enough for the analysis of some circuits.

The effect of temperature on resistance is very nearly the same as for selenium rectifiers.

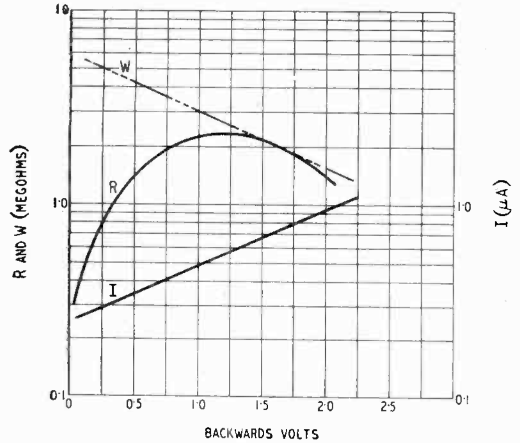

Fig. 10. *Selenium rectifier type 7A ; backward characteristic (N.B. Not the same rectifier as that of Fig. 7.)*

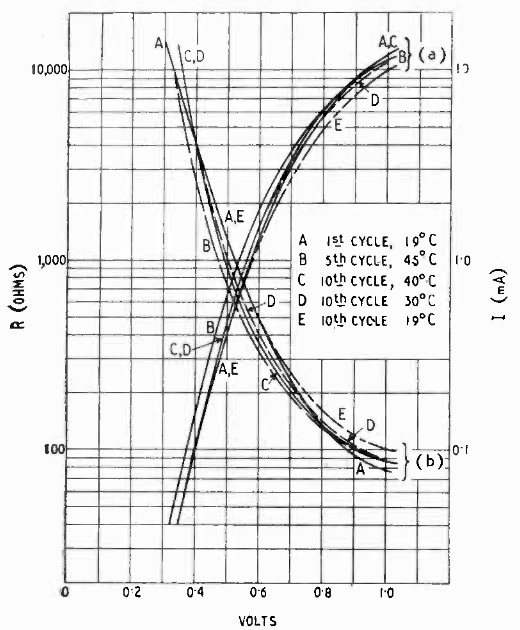

Fig. 11. *Selenium rectifiers type 7A ; forward V–I curves for various temperatures (a) and forward d.c. resistance (b).*

3.4. *Germanium Crystal-Valve Rectifiers* (Figs. 16–17).

Results are shown for two rectifiers of type VX 4047. The exponential law applies very closely over the upper voltage range, but diverges badly at low voltages, indicating resistances for $V = 0$ very seriously in error. Where the exponential law does apply, the value of q_1 and q_2 appears to be the same for both rectifiers, although all other parameters are different.

3.5 *Diodes* (Figs. 18–20).

A diode valve characteristic is different from those previously discussed in that a current exists even when no voltage is applied. This means that the conception of a d.c. resistance is useless. Accordingly only current—voltage and a.c. resistance—d.c. voltage curves are given. The diode is more nearly a perfect, linear rectifier than any of the others discussed, as is shown very clearly by the W–V curves of Fig. 20.

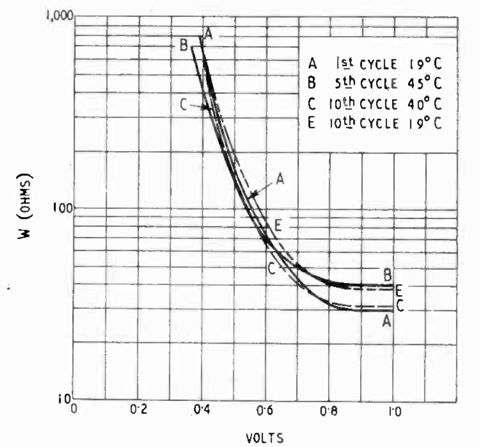

Fig. 12. *Selenium rectifier type 7A ; forward a.c. resistance at various temperatures.*

The backward a.c. resistance rises very rapidly, while the forward resistance is constant. At low currents the V–I relationship is truly exponential ;

i.e., $I \propto \epsilon^{qV}$

and this gives directly $W = \dfrac{dV}{dI} \propto \epsilon^{-qV}$.

Since the extent of the bend in the W curve is small, the law $W = W_0 + \kappa_2 \epsilon^{-q_2 V}$ fits almost perfectly.

Fig. 20 shows that four different diodes chosen at random gave the same value of q_2 although κ_2 and W_0 were different, and also that the same applies to one diode used at different heater currents. Thus q_2 appears to be a constant for a type of diode.

It will be seen from an inspection of Fig. 20 that the operation of the diode as a modulator element or " switch " would often be greatly improved by the addition of a bias so that

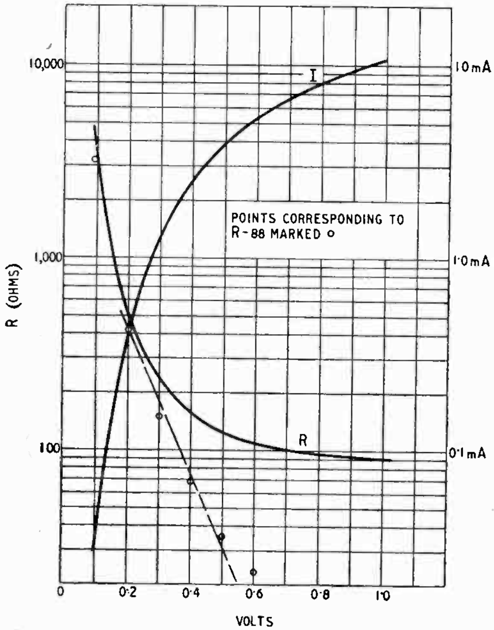

Fig. 13. *Crystal valve CV113 (silicon) ; forward d.c. characteristics.*

the zero applied voltage point occurred at a somewhat higher resistance. A bias of about 0.2 to 0.4 volt appears suitable for the type D1 (Mazda) shown.

3.6 *Conclusions*

It can be concluded from this examination

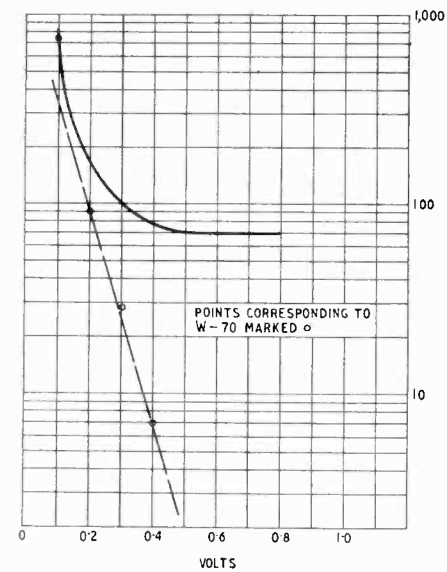

Fig. 14. *Crystal valve CV113 (silicon) ; forward a.c. resistance.*

of rectifier characteristics that the exponential laws of Section 2 apply sufficiently closely for many purposes of circuit analysis. With the exception of diodes, which are fitted almost perfectly, the laws fit well only over the positive voltage range in excess of some value where the resistance is perhaps 10 times the ultimate constant low value. The fit is poorest for the crystal valves, but in all cases is good enough to give good results in the analysis of modulator circuits (see Section 4).

There appears to be a tendency for the index coefficient q to be constant for a particular type of rectifier while the factor κ and the constant term are extremely variable.

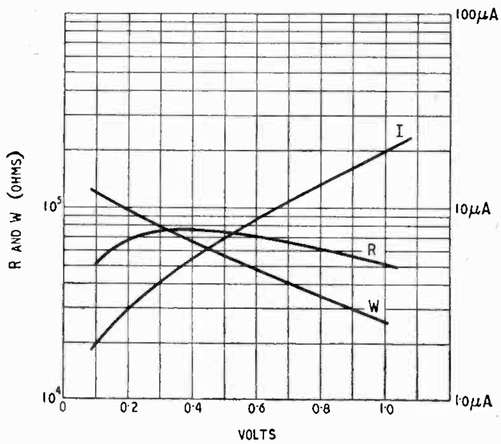

Fig. 15. *Crystal valve CV113 (silicon) ; backward characteristics.*

4. Application to Circuit Analysis

A few examples will now be given to show how greatly the use of the exponential resistance law simplifies the analysis of circuits otherwise rather formidable.

4.1. *Ring Modulator*

A typical ring modulator circuit is shown in Fig. 21. The general qualitative explanation of its operation is given elsewhere.[1] The method of analysis now proposed is to regard the circuit as a lattice network as shown in Fig. 22, where Z_x and Z_y represent the a.c. impedances (which, on the assumption of low capacitances, are actually the a.c. resistances) of the rectifiers, forward and backward respectively, at a particular carrier voltage. Since Z_x and Z_y vary with the carrier voltage, the insertion loss of the lattice between generator and

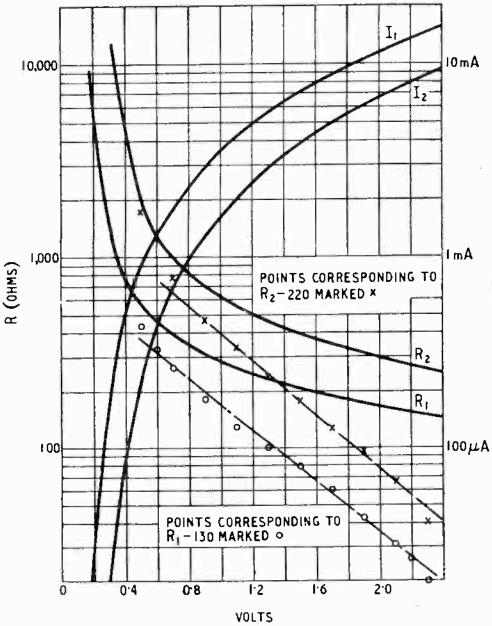

Fig. 16. *Germanium crystal valves type VX4047; d.c. characteristics of two sample crystals.*

load is a function of this voltage and therefore of time. The insertion loss plotted against time may well be regarded as a modulating function, and a Fourier analysis of this function will give the relative magnitudes of the various orders of output frequency components. By considering the mesh equations for the circuit of Fig. 22, it is easy to show that the insertion loss ratio L is given by

$$L = \frac{Z_x + Z_y}{Z_y - Z_x} + \frac{2Z_1Z_2 + 2Z_xZ_y}{(Z_y - Z_x)(Z_1 + Z_2)} \quad (3)$$

The usual case is where $Z_1 = Z_2 = Z$ and then we have

$$L_{[Z_1=Z_2]} = \frac{(Z + Z_x)(Z + Z_y)}{Z(Z_y - Z_x)} \quad \ldots \quad (4)$$

If however the input signal comes from a constant-current or high-impedance generator we may put $Z_1 = \infty$, $Z_xZ_y \ll Z_1Z_2$ and $Z_2 < Z_1$ so that we obtain

$$L_{[Z_1=\infty]} = \frac{Z_x + Z_y + 2Z_2}{Z_y - Z_x} \quad \ldots \quad \ldots \quad (5)$$

Such a condition is involved in a special

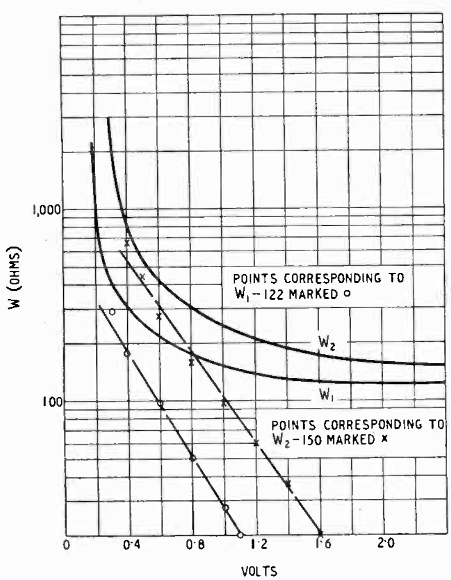

Fig. 17. *Germanium crystal valves type VX4047; a.c. resistance characteristics of two sample crystals.*

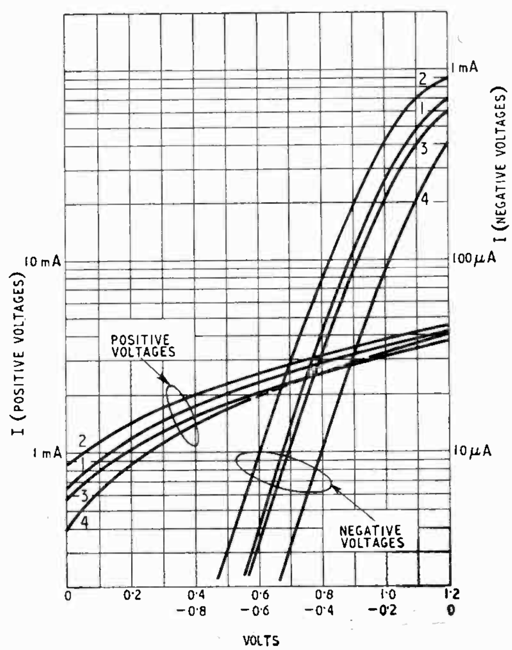

Fig. 18. *Diodes type D1; V–I characteristics of four diodes (4 volts on heaters).*

ring modulator circuit proposed by Cooper[18] to provide very low and stable carrier leak.

Taking the ordinary ring modulator where $Z_1 = Z_2 = Z$ (i.e., equation 4), and assuming a carrier source of instantaneous e.m.f. e and resistance r developing a voltage V across the rectifiers (neglecting the resistance of the transformers), we have to relate V to e in terms of the rectifier resistance, R. Two rectifiers are to be considered as having a forward resistance R_f and two a backward resistance, R_b, such that

$$\left.\begin{aligned} R_f &= R_0 + \kappa_1 \epsilon^{-q_1 V} \\ R_b &= R_0 + \kappa_1 \epsilon^{+q_1 V} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

Thus

$$\frac{V}{e} = \frac{\frac{1}{4}R_f R_b}{\frac{1}{2}(R_f + R_b)} \Big/ \left[r + \frac{\frac{1}{4}R_f R_b}{\frac{1}{2}(R_f + R_b)} \right]$$

Now $R_f R_b = R_0^2 + \kappa_1^2 + 2R_0\kappa_1 \cosh q_1 V$
and $R_f + R_b = 2R_0 + 2\kappa_1 \cosh q_1 V$

So that $\dfrac{V}{e}$

$$= \frac{R_0^2 + \kappa_1^2 + 2R_0\,\kappa_1 \cosh q_1 V}{R_0^2 + \kappa_1^2 + 4rR_0 + 2\kappa_1(R_0 + 2r)\cosh q_1 V} \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

Now e is a sinusoidal e.m.f., say

$$e = E_m \sin \omega_c t$$

Therefore

$$V = \frac{a + b \cosh q_1 V}{c + d \cosh q_1 V} \cdot E_m \sin \omega_c t \quad \ldots \quad (8)$$

where the values of a, b, c and d are easily seen from (7). This equation gives V as a function of time, and is easily solved graphically so that V can be plotted against the angular value of $\omega_c t$ as in Fig. 23(b).

The modulating function, which we can designate as $\phi(t)$, is conveniently defined as a fraction less than unity; i.e., $\phi(t) = 1/L$. Thus from equation (4),

$$\phi(t) = \frac{Z\,(Z_y - Z_x)}{(Z + Z_x)(Z + Z_y)} \quad \ldots \quad \ldots \quad (9)$$

$$\left.\begin{aligned} \text{where } Z_x &= W_0 + \kappa_2 \epsilon^{-q_2 V} \\ \text{and } Z_y &= W_0 + \kappa_2 \epsilon^{+q_2 V} \end{aligned}\right\} \quad \ldots \quad \ldots \quad (10)$$

So that $\phi(t)$

$$= \frac{2Z\kappa_2 \sinh q_2 V}{(Z + W_0)^2 + \kappa_2^2 + 2\kappa_2(Z + W_0)\cosh q_2 V} \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

The values of V obtained from equation (8) can be inserted in this equation, so that $\phi(t)$ can be plotted against the angular values of $\omega_c t$ as in Fig. 23(c).

The modulating function is, of course, the function by which the input signal is modulated; i.e., the output signal is $[e_1 \sin \omega t]\,\phi(t)$, where the input signal is given by $e_1 \sin \omega t$.

It should be noted that the shape of the modulating function can easily be determined in the laboratory for any particular modulator by applying a small d.c. signal to the input to the ring, and observing the output on a cathode-ray oscilloscope; in other words, $\phi(t)$ is the output of the modulator when a zero-frequency input signal is used. If the wave-shape of $\phi(t)$ is analysed into a Fourier series by some graphical or empirical method, then the amplitudes of the various harmonics (all odd) give the relative amplitudes of the second-order, 4th-order, etc, modulation products.

The carrier leak from the modulator can be studied by the use of the exponential law, taking different values of R_0 and κ_1 for each individual rectifier. It is reasonable to assume q_1 constant, as previously pointed out (Section 3.6.). The working is simple, but being rather bulky, is not given here.

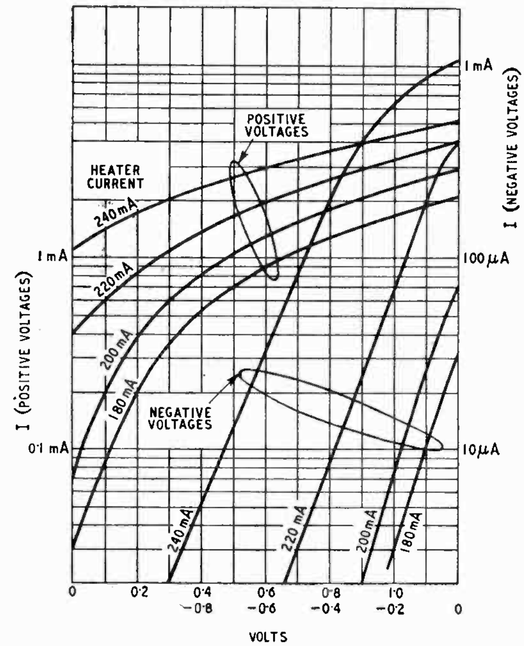

Fig. 19. *Diode type D1; V–I characteristics at various heater currents.*

The general method of dealing with carrier leak in practical equipment is to balance it out as well as possible by means of a potential divider connecting the carrier feed to the two halves of the output transformer. This enables the differences in R_0 to be compensated, but still leaves the errors in κ_1 effective at low carrier voltages. This residual carrier leak is a voltage across the load given very nearly by

$$\tfrac{1}{2}ZV\left[\frac{2R_0 + \kappa_{1A}\,\epsilon^{-q_1 V} + \kappa_{1D}\,\epsilon^{+q_1 V}}{R_0^2 + \kappa_{1A}\,\kappa_{1D} + R_0(\kappa_{1A}\,\epsilon^{-q_1 V} + \kappa_{1D}\,\epsilon^{+q_1 V})} - \frac{2R_0 + \kappa_{1B}\,\epsilon^{-q_1 V} + \kappa_{1C}\,\epsilon^{+q_1 V}}{R_0^2 + \kappa_{1B}\,\kappa_{1C} + R_0\,(\kappa_{1B}\,\epsilon^{-q_1 V} + \kappa_{1C}\,\epsilon^{+q_1 V})}\right] \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

where V is assumed unaffected by the unbalances, and the subscripts A, B, C and D refer to the four rectifiers as shown in Fig. 21. A typical waveshape of this residual leak at small carrier voltages is shown in Fig. 23(d).

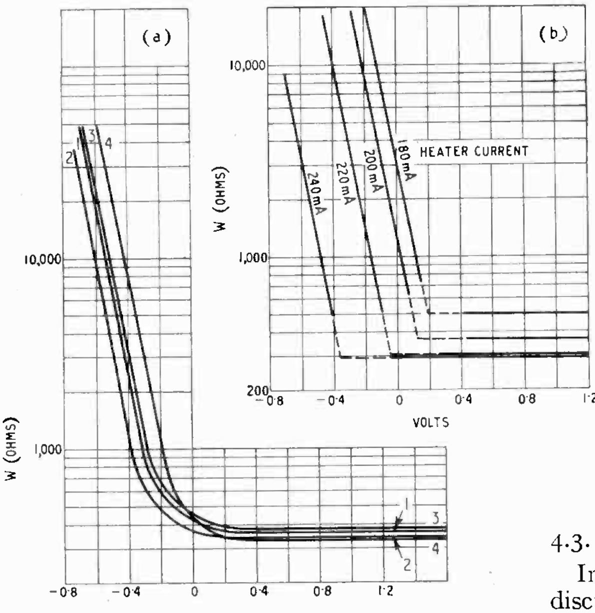

Fig. 20. *Diodes type D1; (a) shows the a.c. resistance of four samples with 4 volts on the heaters and (b) shows the a.c. resistance of one diode for various heater currents.*

4.2 *Cowan Modulator.*

The Cowan modulator is also a 4-rectifier modulator giving carrier suppression, but it has the advantage over the ring modulator that both input and output can be earthed without the use of transformers. It has the disadvantage of being less efficient, the loss from input to one sideband in the output being of the order of 14 db instead of the 7–8 db of the ring modulator. The general arrangement is shown in Fig. 24. On one carrier half-cycle, the rectifiers are all on the forward resistance and partially short-circuit the input; on the other half-cycle, the backward resistance hardly affects the input.

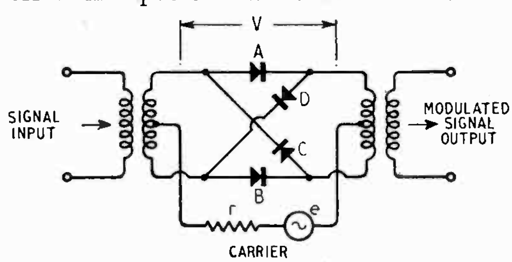

Fig. 21. *Ring modulator.*

Following the general lines of the previous analysis, we have

$$L = \frac{Z_1 Z_2 + W(Z_1 + Z_2)}{W(Z_1 + Z_2)} \quad \ldots \quad (13)$$

$$\frac{V}{e} = \frac{R_0 + \kappa_1 \epsilon_1^{-q_1 V}}{r + R_0 + \kappa_1 \epsilon^{-q_1 V}}$$

i.e.,

$$V = \frac{R_0 + \kappa_1 \epsilon^{-q_1 V}}{r + R_0 + \kappa_1 \epsilon^{-q_1 V}} \cdot E_m . \sin \omega_c t \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

from which the relation between V and $\omega_c t$ is easily found graphically. Then $\phi(t)$

$$= \frac{(Z_1 + Z_2)(W_0 + \kappa_2 \epsilon^{-q_2 V})}{Z_1 Z_2 + (Z_1 + Z_2)(W_0 + \kappa_2 \epsilon^{-q_2 V})} \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

The general nature of this function is shown in Fig. 25.

4.3. *Constant-Impedance Modulator*[19].

In general, both the modulator circuits discussed above have input and output impedances which vary with the carrier voltage. This means that they are not suitable for terminating filters which are calculated to give a prescribed insertion

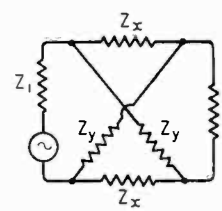

Fig. 22. *Generalized transmission circuit of modulator.*

loss—frequency response. Consequently† it is usual to insert attenuation pads of 3 to 6 db between the modulator and a filter at either end of it.

It is possible, however, to design the

†There is an additional consideration here, in that the output of the modulator is altered if the output terminals are closed by a filter which presents a different impedance to one sideband from that presented to the other; in such a case power is diverted from one sideband into the other. This effect is not readily calculable and confuses the discussion of the filter insertion loss. The present section must be regarded as showing an interesting property of the modulator, which may possibly be useful; at the input it can almost certainly be utilized.

modulator to have a constant impedance. This can be done by taking advantage of the exponential law and using the principle of Zobel's constant-impedance network theory[20].

The ring modulator may be represented as in Fig. 26 (a). By a well-known transformation of lattice networks, the circuit of Fig. 26 (b) may be drawn as identically equivalent. By the principle of constant-impedance networks, it can be seen, regarding

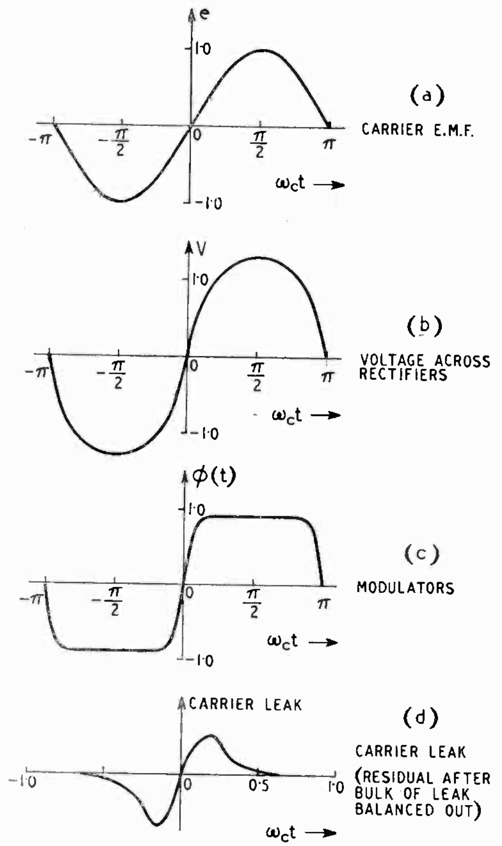

Fig. 23. *The carrier e.m.f. is shown at (a); the voltage across the rectifiers at (b); the modulating function at (c); and the carrier leak voltage at (d). This last is the residual after the bulk of the leak has been balanced out.*

the lattice portion of Fig. 26 (b), that if this lattice is terminated at one side by a resistance $R = \sqrt{\kappa_2\epsilon^{-q_2V} \cdot \kappa_2\epsilon^{+q_2V}} = \kappa_2$, then the resistance at the other side, seen looking into the lattice network, is also $R (= \kappa_2)$.

Returning to the real network of Fig. 26(a), it is now evident that if the terminating impedance at one pair of terminals is made a resistance of value $\kappa_2 - W_0$, then the impedance seen looking into the other pair is $\kappa_2 + W_0$, which is constant and independent of the carrier voltage.

5. Conclusions

A considerable number of rectifier resistance characteristics have been

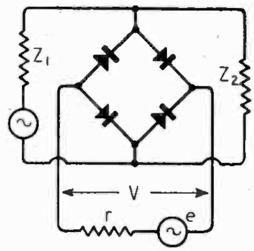

Fig. 24. *The Cowan modulator.*

presented and it has been shown that both the d.c. and the a.c. resistance of rectifiers can be represented as a voltage-dependent function thus :—

$$R = R_0 + \kappa\,\epsilon^{-qV}$$

where R_0 is a constant value which the resistance reaches at large forward voltages, κ is a constant and approximates to the

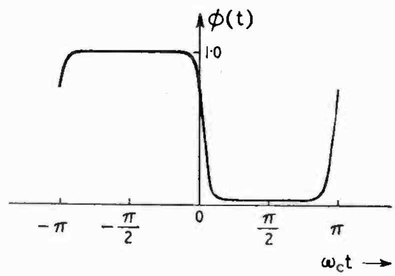

Fig. 25. *Modulating function of a Cowan modulator.*

resistance when the voltage is zero, and q is a constant. V is the d.c. (bias) voltage, and any a.c. voltage is assumed small so that the a.c. resistance is given by $\dfrac{dV}{dI}$. The accuracy of the representation is very high for diode valves, fairly good for copper-oxide and selenium rectifiers, and not very

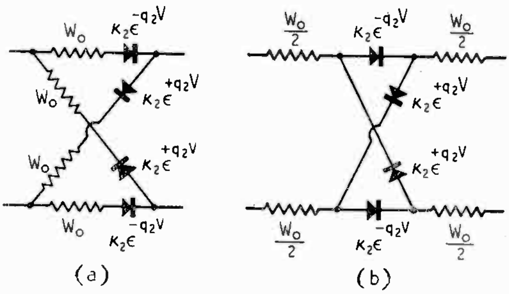

Fig. 26. *Constant-impedance modulator.*

good for crystal valves. In all cases it fulfils one important requirement of any such law; it shows a low forward resistance and a high backward resistance. The index q appears to be fairly constant for any particular type of rectifier, but R_0 and κ vary from one rectifier to another; none of the "constants" is the same for the a.c. as for the d.c. resistance.

It has been shown that this exponential law greatly assists the analysis of non-linear circuits employing rectifiers, particularly such transmission circuits as balanced modulators. It has been shown how, in this way, the finer points of modulator performance may be examined, and improved modulators designed.

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[N.B.—These are not necessarily original references but are those most applicable to the present discussion, or most likely to be accessible.]

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NEW BOOKS

Ultra- and Extreme-Short Wave Reception

By M. J. O. Strutt, D. Tech. Sc. Pp. 387 + xi, with 248 illustrations. Published in U.S.A. by D. Van Nostrand Co., Inc. Macmillan and Co., Ltd., St. Martin's Street, London, W.C.2. Price 37s. 6d.

The list of more than 400 references in this book is evidence of a large (and rapidly growing) pile of literature on the subject. Dr. Strutt has performed a useful service to the practical designer, who lacks the time to read and digest all this, by providing a balanced review, which also incorporates much of his own practical experience in this field.

Although one would hardly infer it from the title, the range of frequencies is 6-30,000 Mc/s, with particular reference to those higher than about 30 Mc/s. The reader is assumed to be well grounded in general principles, and to be in need mainly of quantitative readjustment. So except for shifting the theoretical emphasis where necessary the author devotes most of his space to providing the practical formulæ and numerical data appropriate to the higher frequencies. The theoretical background is provided by verbal explanations rather than formal proofs. Advanced mathematical ability is not required of the reader.

The first chapter, on Waves and Signals, briefly recapitulates the properties of electromagnetic waves and then provides data on propagation, including reflection and disturbances. Under Signals is considered principally the various types of modulation. The key importance of fluctuation noise at v.h.f. and over is recognized by considering it next; first the causes of noise, then a discussion of noise ratio and noise figure. Emphasis is laid on the idea of available power from a source; a conception which is applied in the next chapter, on Antennas, and in fact more or less throughout the book. The theory of transmission lines, waveguides and cavities having been adequately treated elsewhere, it is only summarized in chapter IV, as the basis for practical applications and data. Chapter V, on measuring instruments, their operation, and typical results obtained with them, is welcome as an elementary introduction to this field, since most of the information published hitherto has concerned apparatus for research rather than for design and test work. The design and construction of signal generators is discussed, and instructions given for the measurement of noise figure, gain, impedance and power. Of the two remaining chapters, dealing with receivers, one is devoted to the all-important first stage. The second includes sections on f.m., impulse and single-sideband reception, and over-all design.

The numerous references are listed at the back of the book, and are indicated at the end of each sub-section; that is to say, at intervals of a page or two. With this arrangement it is not always clear which reference should be looked up for further information on any specific point. Nor is the origin or derivation of much of the data evident. Some of the references are as recent as 1947; but unfortunately it does not appear to have been possible to benefit from any of the large amount of information which has been appearing in Part IIIA of the *J. Instn elect. Engrs*, and references to Part III are fewer than might be expected. For example, the waveguide type of attenuator is not mentioned, attenuators being said to be mainly of the capacitive type; and the implications of the capacitive attenuator's being non-dissipative are not clearly brought out.

In general, however, the information is up-to-date and authoritative, and forms a good basis for more advanced study.

M. G. S.

Frequency Modulation

By PAUL GÜTTINGER. Pp. 183 with 99 illustrations. Verlag A.G. Gebr. Leemann & Co., Zürich.

This book is intended for radio engineers and for students of high-frequency technology. It deals with the principles of frequency and phase modulation and their application to transmitters and receivers. Separate chapters are devoted to distortion and to interference. The treatment is necessarily mathematical and there are appendices on Bessel functions and complex integration. The book is divided into 36 sections, each more or less complete in itself, so that it can be used to some extent as a reference book; each section has its own bibliography. The complete bibliography contains 295 references. The printing, diagrams, and general production of the book are all excellent. The section on the construction of f.m. receivers gives, in addition to the diagrams of connections, a complete list of the 114 components.

G. W. O. H.

Philips Manual of Radio Practice for Servicemen

Compiled by E. G. BEARD, M.I.R.E. (Aust). Pp. 495 with 440 illustrations. Philips Electrical Industries of Australia Pty. Ltd., Sidney, Australia. Price 22s. 6d. (Aust).

This book is divided into sections covering Broadcast Reception, Broadcast Receiver Technique, Principles and Components, Service to Radio Receivers, Technical Formulae, Tables and Charts, Mathematical Formulae and Tables, Valve Data, and an Appendix. The treatment is descriptive and non-mathematical, and of an elementary nature. It covers most of the circuits commonly used in broadcast receivers and they are described well, but briefly.

The section on Service is the least satisfactory. It includes descriptions of measuring instruments, discussions of servicing in the home and in the workshop, soldering and masts. The discussion of servicing itself however is very cursory and comprises rather less than $7\frac{1}{2}$ pages.

It is stated in the introduction that "The reader is requested to give words such as frequency, potential, voltage, current, and others the meaning suggested by the context, as they have been used somewhat loosely in order to avoid cumbersome expressions. Lack of precision in the use of these words is now common practice and should not result in ambiguity to the reader even if it irritates the purist."

This is rather an extraordinary attitude to take up for it is just the elementary reader for whom the book is intended who is most likely to be misled by looseness of expression. The purist, who may be irritated by it, is probably the only one who is unlikely to be misled.

In spite of the introductory warning, however, the accuracy of expression and terminology is much better than one might suppose, but there are a few curious statements. On p. 124 it is said that "the term resistance is used only to denote the opposition of a circuit to direct current, although sometimes the term 'Ohmic Resistance' is used to emphasise that only resistance which dissipates energy as heat is referred to." On p. 138 energy seems to be equated with power for it is said "The energy lost can be calculated from the formula: Watts = I^2R".

This is a good example of the kind of looseness which does seriously confuse the thoughtful beginner. If he is once led to believe that energy and power are the same thing he meets with serious difficulty when reading books in which the terms are correctly used.

On the whole, however, the book is well written and contains a very large amount of information of value to more than the serviceman.

W. T. C.

Nomograms of Complex Hyperbolic Functions

By JØRGEN RYBNER. Published by Jul. Gjellerups Forlag, Copenhagen, Denmark. Price Kr. 24.

The text is in both English and Danish with a 9-page introduction explaining the nomograms. There are 10 pages of formulae for circular and hyperbolic functions, Guddermannian angle, integrals and differentials, series, functions of imaginary and complex arguments, inverse complex hyperbolic functions. Formulae for 4-terminal networks and transmission lines are also included.

The nomograms are for cosh $(b + ja) = p + jq$, a range of $b = 0$ to $b = 4$ being covered in 13 pages, sinh $(b + ja) = p + jq$ (13 pages) and tanh $(b + ja) = r/\theta$ (16 pages). A further 8 pages include nomograms covering $x + jy = r/\theta$, $R/\alpha = 1 + r/\phi$, $b_r = \log_e \left|\frac{Z_1 + Z_2}{2\sqrt{Z_1 Z_2}}\right|$, $a_r = \angle\left(\frac{Z_1 + Z_2}{2\sqrt{Z_1 Z_2}}\right)$, $f = 1/2\pi\sqrt{LC}$, $K = \sqrt{L/C}$.

The book includes a transparent straight edge, and it has a ring binding. The 70 leaves are printed on one side only for the nomograms and on both sides for the text.

W. T. C.

The Story of the Telephone

By J. H. ROBERTSON. Pp. 299 + viii. Sir Isaac Pitman & Sons, Ltd., London. Price 10s. 6d.

This book has as sub-title "A History of the Telecommunications Industry of Britain," and this is a good description of the contents. The book does not pretend to deal with the scientific development although, of course, frequent reference is made to it, since the development of the industry depended on the scientific development. There are no illustrations. The book gives a very interesting and readable account of the squabbles and struggles which followed Bell's invention, the Gower-Bell Co. versus the Edison Co., then both combined versus the Post Office. The growth of the manual and automatic systems is discussed very fully and the associated growth of the various concerns manufacturing the apparatus. The Post Office is severely castigated for its lack of initiative and foresight in the early days, but the author emphasizes that this is not so now. The story is brought right down to the present day, and after describing war developments such as radar, the author says "No industry could face the future with less fear. Since August 1945, a good deal of gloomy stuff has been written and spoken about the awful responsibilities of applied science and industry. The early bogy of the 'wicked armaments king' has been to some extent replaced by the bogy of the 'Amoral Scientist'; both are scapegoats for humanity's sense of its own pitiful political ineffectiveness. . . . The telephone industry can claim unhesitatingly that it represents applied science at its best."

The book is well written, although there are some mistakes in the spelling of names, e.g., Branley for Branly, Grinstead for Grinsted and Backhausen and Kurtz for Barkhausen and Kurz. On p. 128 Bell's father-in-law Gardiner G. Hubbard appears to have been split up into two persons; viz., Mr. Gardiner and Mr. S. Hubbard. It is also incorrect to refer to Miss Hubbard as one of Bell's pupils. She had been deaf from the age of four as the result of illness and had been sent to Europe to study lip reading, but she was never a pupil of Bell's in the ordinary sense of the word.

These are minor details, however, and detract little from the value of the book. G. W. O. H.

High-Frequency Measuring Techniques Using Transmission Lines.

By E. N. PHILLIPS, W. G. STERNS and N. J. GAMARA. Pp. 58 ($8\frac{1}{2} \times 11$in). Published by John F. Rider Publisher, Inc., 404, Fourth Avenue, New York 16, U.S.A. Price $1.50.

Describes the procedure for the determination of impedances of, and velocity of propagation and attenuation in, two- and four-terminal networks at frequencies over 100 Mc/s. By using a 7-ft slotted coaxial line the voltage standing-wave ratio and the positions of the voltage nodes are determined. The other characteristics are then found by calculation from these measured quantities.

Radio Questions and Answers. Vol. II.—Radio Receivers.

By E. M. SQUIRE. Pp. 152 + vi with 146 illustrations. Sir Isaac Pitman and Sons Ltd., Kingsway, London, W.C.2. Price 10s. 6d.

Science at War

By J. G. CROWTHER and R. WHIDDINGTON, C.B.E., F.R.S. Pp. 185 with 102 illustrations. H.M. Stationery Office, London. Price 2s. 6d.

A non-technical account of radar, operational research, atomic bombs and science at sea, which is concerned chiefly with history and application.

Electrical Engineering: Scope, Training and Prospects

By F. W. PURSE, M.I.E.E., M.I.Mech.E. Pp. 104. Southern Editorial Syndicate Ltd., 555, Lea Bridge Rd., London, E.10. Price 5s.

Indicates and describes methods of training which are available for the various branches of electrical engineering.

B.B.C. Year Book, 1948

Pp. 152. The British Broadcasting Corporation, B.B.C Publications Dept., The Grammar School, Scarle Rd., Wembley, Middx. Price 2s. 6d. (Postage 4d.)

Elements of Radio Servicing

By WILLIAM MARCUS and ALEX LEVY. Pp. 475+ix with 361 illustrations. McGraw Hill Publishing Co. Ltd., Aldwych House, London, W.C.2. Price $4.50.

General Electrical Engineering

Edited by PHILIP KEMP, M.Sc. (Tech.), M.I.E.E., A.I.Mech.E. Pp. 448 with 522 illustrations. Odhams Press Ltd., Long Acre, London, W.C.2. Price 9s. 6d.

CORRESPONDENCE

Letters to the Editor on technical subjects are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Standard Terms and Abbreviations

SIR,—By at least one who has habitually talked of amps and milliamps with a lurking fear that it was not quite respectable your proposal that "ampere" should be officially shortened is accepted as complete absolution. It should be enough that the suggestion comes from such a source, but if arguments are needed yours seem unanswerable. I would press for full international agreement. The French are not likely to be so touchy in these matters as the Teutons and with the English and Italian syllabic sacrifices before them the loss of national prestige should hardly be worth more than token resistance.

At the same time one might consider bringing 'k' into line with other multiplying abbreviations by elevating it to 'K.' Then with our own house thoroughly in order we should tackle the Americans on their apparently indefensible use of 'm' for both mega- and milli-. Your reference to dimensions of motor coaches in millimetres, by the way, can be matched in our own sphere by the delight taken by Americans in stating the power of their broadcasting stations in watts. Do they really think that their fellow-members of the I.R.E. are dazzled by a display of noughts? I have not actually seen the output of American power stations given in VA but it would be consistent. Their practice of rating valves in micromhos however might perhaps be considered more excusable.

One respect in which American example is, I think, wholly commendable is in using lower-case letters for initial abbreviations of words which would begin with them if written in full; for example i.f., a.g.c. and a.c. This has also been the practice of *J. Instn elect. Engrs* for a long time, and is spreading. But even in the 1945 amendment of B.S. 560: 1934 the only concession to it is a timid note saying that if the abbreviations are used adjectivally lower-case letters *may* be used. The example given is "a.c. motor." But if a.c. is used adjectivally surely the American form a-c, is more correct. Incidentally, can we have an Editorial pronouncement on whether "a.c. current" should still be resisted because it is absurd, or accepted because its absurdity is outweighed by its general use?

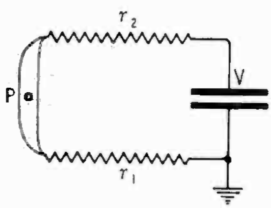
Lastly, may I utter hearty approval of your plea for better technical writing ? The amount of technical matter one has to read these days is such that gratitude to writers who begin by saying exactly what they are going to do and then develop their theme clearly and logically, and irritation with those who do neither, tend towards new intensities.

Bromley, Kent. M. G. SCROGGIE.

Thermal Noise in Resistors

SIR,—The noise voltage generated by a resistor has been derived, by D. A. Bell, on the supposition that a " single event " is the sudden flight of an electron through the mean free path λ. In order to obtain the correct result it is necessary to make the assumption that the proportion of the electron charge which appears on the capacitor plate is $\lambda \div L$, where L is the total length of wire composing the resistor. A short discussion of this result is given by Moullin, " Spontaneous Fluctuations of Voltage," p. 67, who implies that there is no very clear physical interpretation of such an effect, although " if L were the distance between the capacitor plates the result would not be surprising."

It is the purpose of this note to suggest the following explanation ; suppose that unit positive charge is suddenly placed in a small cavity at P, within the metal of the resistor. The walls of the cavity will instantaneously attain a potential V_P, while current will flow away to the capacitor plates. The quantity of charge flowing to the lower capacitor plate, and so to earth, will be



$$\int_0^\infty \frac{V_P}{r_1}\,dt$$

and similarly the quantity flowing to the upper plate will be

$$\int_0^\infty \frac{(V_P - V)}{r_2}\,dt$$

where r_1, r_2 are the respective resistances from the cavity walls to the lower and upper plates, and V is the instantaneous upper-plate potential. Now if V is extremely small throughout the period of flow it may be neglected in comparison with V_P, so that the total quantity of charge which flows away from the vicinity of the cavity

$$= \left(\frac{1}{r_1} + \frac{1}{r_2}\right) \int_0^\infty V_P dt = \text{unity}$$

since it must numerically equal the charge placed at P. Hence the charge which flows to the upper plate is

$$\frac{r_1}{r_1 + r_2}$$

Now if an electron in its free flight moves a distance λ it may be supposed that a charge is suddenly removed from its initial position, leaving there an excess of positive charge, and then suddenly appears at its terminal position, bringing there an excess of negative charge ; if, for example, the movement is along the wire towards the upper plate, it is clear that the net charge appearing on the capacitor plate is

$$\frac{e\,\rho\,\lambda}{\text{TOTAL RESISTANCE}}$$

where ρ = resistance per unit length of wire, at the position of the electron. For a wire of uniform resistance per unit length this result is equivalent to the required result, and it is moreover the correct generalization of the required result for a resistance which is not uniform along its length.

New Barnet, Herts. S. RODDA.

Electro-Encephalograph Amplifier

SIR,—In a personal communication Dr. W. Grey Walter has drawn my attention to a paper which merited inclusion in the bibliography to my article " Electro-encephalograph Amplifier " published in *Wireless Engineer*, August-October 1947.

This paper was by Fritz Buchthal and J. Oskar Nielson, entitled " A Newly Developed Push-Pull D.C. Amplifier for use with the Cathode Ray Oscilloscope for Electro Physiological Phenomena," and appeared in *Skand. Arch. Physiol.* Vol. 74, p. 202, 1936. The circuits described depend entirely upon the use of floating battery supplies, and electronically-stabilized supplies are not used, but the discussion of noise, cathode fluctuations, bandwidth and differential amplification is, even after 12 years, very helpful in understanding the problems of d.c. amplifiers and a.c. amplifiers of long time constant.

My colleagues, W. E. Schneider and R. J. Simpson, have kindly made a translation from the German (reference 24 B), and anyone working on these problems who wishes to have a copy should write to me c/o Research Laboratories of Elliott Brothers (London) Ltd., Elstree Way, Borehamwood, Herts.

Borehamwood, Herts. DENIS L. JOHNSTON.

"Micro-Waves and Waveguides"

SIR,—The review of my book entitled " Micro-Waves and Waveguides " published in your February issue invites me to offer the following comments in reply.

The reviewer has clearly overlooked the fact that this book was written primarily for the engineer approaching the subject for the first time. It is designed to fill a gap in the literature which is all too apparent to so many engineers who, as serious students, have attempted to develop their ideas of ordinary line transmission to include a sound working knowledge of waveguide propagation. The book does not pretend to present the kind of information required to design a complete waveguide system, nor would it achieve its objective if this were done. There are plenty of other books to meet that requirement. The modes of propagation chosen for more detailed consideration are those of particular engineering significance, and in examining them mathematically they are shown quite clearly to represent special cases of the more general forms of H and E waves. The criticism of this book offered by the reviewer is, in my view, based on a misconception of what it sets out to achieve.

University College, London. H. M. BARLOW.

WIRELESS PATENTS

A Summary of Recently Accepted Specifications

The following abstracts are prepared, with the permission of the Controller of H.M. Stationery Office, from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price 1/- each.

AERIAL AND AERIAL SYSTEMS

588 044.—Directional array of folded dipoles arranged to present a substantially-flat impedance curve over a wide range of frequencies.

Standard Telephones and Cables Ltd. and E. O. Willoughby. Application date 31st October, 1944.

DIRECTIONAL AND NAVIGATIONAL SYSTEMS

587 874.—Radiolocation system characterized by the use of exploring pulses, the leading and trailing edges of which are symmetrically sloped.

Standard Telephones and Cables Ltd. and W. A. Beatty. Application date 21st June, 1940.

587 875.—Cathode-ray indicators, suitable for radiolocation, in which a selected portion of a saw-tooth waveform is used to magnify or expand the scan.

Baird Television Ltd. and S. S. West. Application date 1st July, 1940.

587 876.—Radiolocation system in which pulses are radiated at constant or random intervals, and are combined through a variable time-delay circuit with the echo-signals.

Standard Telephones and Cables Ltd. and C. W. Earp. Application date 14th March, 1941.

587 912.—Phase-shifting device for stabilizing the direction of the radiolocation beam from a ship's aerial in spite of pitching and rolling.

F. N. Scaife, F. Hoyle, F. Ehrenstein and C. S. Wright. Application date 17th November, 1944.

588 024.—Equi-signal radio-beacon system in which a centrally-located auxiliary aerial is arranged to discriminate between selected parts of the radiated field-pattern.

W. J. O'Brien. Convention date (U.S.A.) 2nd March, 1942.

588 122.—Flexible waveguide coupling suitable for feeding a rotary aerial, as used for radiolocation.

G. E. Bacon. Application date 30th January, 1945.

588 155.—Preventing the echo-signals from stationary objects from damaging the sensitive screen of a radiolocation indicator-tube.

Standard Telephones and Cables Ltd. (assignees of R. E. Rutherford). Convention date (U.S.A.) 20th December, 1943.

588 159.—Aerial scanning-system comprising a tapered waveguide with multiple apertures, and associated reflectors, for producing a spherical wave-front suitable for radiolocation.

Western Electric Co. Inc. Convention date (U.S.A.) 15th January, 1944.

588 187.—Radiolocation system in which the range of a target is continuously indicated through the medium of an auxiliary train of pulses, the latter being synchronized with the received echo-signals through a variable delay-network.

Western Electric Co. Inc. Convention date (U.S.A.) 22nd June, 1943.

588 242.—Blind-landing system in which differently-modulated carriers are radiated from upper and lower aerials to give a clear-cut gliding path.

Standard Telephones and Cables Ltd. (assignees of C. B. Watts). Convention date (U.S.A.) 29th May, 1942.

588 672.—Time-base circuit for a radiolocation indicator, comprising means for generating and controlling a saw-toothed voltage of variable slope and for terminating it at a predetermined amplitude.

Western Electric Co. Inc. Convention date (U.S.A.) 30th November, 1943.

588 715.—Radiolocation or altimeter device with auxiliary means for giving an automatic indication when the measured range falls within a predetermined distance.

Marconi's W.T. Co. Ltd. (assignees of W. D. Hershberger). Convention date (U.S.A.) 30th January 1943.

588 763.—Radiolocation method of measuring the height of a low-elevation target by comparing the strength of the echo-signals received on two aerials having different vertical polar diagrams.

O. M. Böhm and C. S. Wright. Application date 12th June, 1944.

588 777.—Power-control device for an unmanned unit which is designed to radiate a pulsed identification signal when interrogated by radiolocation equipment.

Standard Telephones and Cables Ltd. (assignees of H. G. Busignies). Convention date (U.S.A.) 26th October, 1943.

588 851.—Increasing the clarity of a desired echo-signal on a plan-position indicator by a device which suppresses all signals outside a predetermined scanning area.

D. S. Watson and C. S. Wright. Application date 11th January, 1945.

589 023.—Waveguide switching device, for the aerial system of a radiolocation set, in which a rhumbatron resonator is periodically damped by a glow-discharge tube.

H. Cooke, H. W. B. Skinner, A. G. Ward and C. S. Wright. Application date 9th March, 1942.

589 136.—Leaky-waveguide "beam" aerial system with rotary-vane switching device, particularly adapted for radiolocation.

Western Electric Co. Inc. Convention date (U.S.A.) not stated. Application in United Kingdom 5th June, 1943.

RECEIVING CIRCUITS AND APPARATUS

(See also under Television)

588 199.—Heterodyne control circuit for continuously sweeping the tuning of a receiving set over a predetermined band of frequencies.

Amalgamated Wireless (Australasia) Ltd. Convention date (Australia) 3rd March, 1944.

588 328.—Arrangement and housing of an elevated

transformer used to reduce the effect of interference picked up by the down-lead from the aerial to the set.

N. M. Best and N. S. Beebe. Application date 9th February, 1945.

588 479.—"Reflexed" receiver for frequency-modulated signals in which the signals are fed back from the output side of the discriminator to the input of the carrier-wave amplifier.

The General Electric Co. Ltd. and L. C. Stenning. Application date 21st February, 1945.

588 517.—Chassis-assembly and screening of the components of a radio set, designed to withstand the effects of heat and moisture in a tropical climate.

The British Thomson-Houston Co. Ltd. and W. S. Melville. Application date 26th February, 1945.

589 153.—Receiver for frequency-modulated signals in which two regenerative circuits having different frequency-response characteristics are combined and periodically quenched to give a super-regenerative effect.

Hazeltine Corpn. (assignees of B. D. Loughlin). Convention date (U.S.A.) 1st February, 1944.

589 229.—Homodyne type of circuit in which provision is made to maintain the amplitude of the received signal constant in spite of variations in the phase angle between the incoming signal and the homodyning oscillation.

A. D. Blumlein. Application date 10th January, 1940.

TELEVISION CIRCUITS AND APPARATUS

For Transmission and Reception

587 498.—A television relaying system in which positive and negative modulation is used alternately at successive repeater-stations in order to maintain the linearity of the signals.

The General Electric Co., Ltd. and D. C. Espley. Application date 6th November, 1944.

587 772.—A comparatively-inexpensive television-receiver which is capable of reproducing a 400-line picture from a signal radiated at a higher rate of line-scanning.

The General Electric Co., Ltd. and D. C. Espley. Application date 6th October, 1943.

588 722.—Television receiver cabinet in which the c.r. tube is pivotally mounted so that it can be swung to project outside the casing and swivelled for convenient viewing from different angles.

Philco Radio and Television Corp. (assignees of E. I. Harman and D. H. L. Jensen). Convention date (U.S.A.) 28th October, 1943.

TRANSMITTING CIRCUITS AND APPARATUS

587 106.—Circuit for generating a rectangular signal-impulse from two component pulses each having an undesirable wave-form.

Hazeltine Corporation (assignees of R. C. Hergenrother). Convention date (U.S.A.) 9th February, 1944.

587 405.—Keying and control arrangements for transmitting pulsed signals, particularly for reproducing facsimile letters, numerals, or coded patterns.

Standard Telephones and Cables Ltd. (assignees of E. M. Deloraine and L. A. de Rosa). Convention date (U.S.A.) 5th June, 1943.

587 419.—Multi-grid oscillator, embodying a non linear voltage regulator, particularly suitable for the transmission of different signal tones for operating selective relays.

B. M. Hadfield. Application date 13th October, 1944.

587 538.—Cam-and-follower device for adjusting the piston-coupling to a resonator-chamber associated with a waveguide.

Standard Telephones and Cables Ltd., and A. S. Wade. Application date 4th May 1944.

587 544.—Automatic control circuit for preventing drift of the mean carrier wave in frequency- or phase-modulating systems.

Marconi's W. T. Co., Ltd. (assignees of N. I. Korman). Convention date (U.S.A.) 30th June, 1943.

587 576.—H. F. attenuator circuit, comprising two partly-coupled single-loop inductances, for dividing an input signal between two transmission lines.

Philco Radio and Television Corp. (assignees of R. G. Clapp). Convention date (U.S.A.) 13th May, 1943.

587 679.—Impedance-matching reactance devices for waveguides, consisting of rotatable elements mounted in one wall of the guide and adapted to protrude into the interior channel.

G. E. F. Fertel, J. A. Barrable and C. S. Wright. Application date 12th June, 1944.

587 714.—Phase-shifting network particularly suitable for determining the frequency of a valve oscillator of the resistance-reactance type, say for frequency modulation.

E. R. Wigan. Application date 9th January, 1945.

587 812.—Partition for separating sections of a waveguide that are operating under different gas-pressures, or at different levels of attenuation.

The British Thomson-Houston Co., Ltd. Convention date (U.S.A.) 22nd January, 1944.

588 360.—Four-terminal network with separate switch controls for charging and discharging, in order to generate voltage pulses at predetermined intervals.

The General Electric Co. Ltd., C. R. Dunham and C. C. Hall. Application date 11th February, 1943.

588 526.—Construction and mounting of a tunable device, comprising a probe wire and rectifying crystal, for detecting and measuring the energy in a waveguide.

D. H. Tomlin and C. S. Wright. Application date 26th February, 1945.

588 638.—Transmitting set in which the components are mounted on a common high-voltage panel, which is bypassed to earth for radio frequencies through a single capacitor.

Marconi's W.T. Co. Ltd. (assignees of J. E. Young). Convention date (U.S.A.) 9th February, 1944.

588 721.—Ultra-high-frequency oscillation-generator of the kind in which the anode and cathode circuits are separately tuned, and are screened from each other by an earthed grid-disc.

Marconi's W.T. Co. Ltd. (assignees of H. C. Lawrence Jr). Convention date (U.S.A.) 28th October, 1943.

588 817.—Stabilizing the frequency of a high-powered oscillation generator of the rhumbatron type against the effect of variations in the voltage applied to the resonator.

Standard Telephones and Cables Ltd. and J. H. Fremlin. Application date 19th November, 1943.

SIGNALLING SYSTEMS OF DISTINCTIVE TYPE

587 447.—Broadcast system in which a low-powered indentification signal is constantly transmitted, but is only rendered audible in the receiver when desired.

H. B. Rantzen. Application date 24th June, 1943.

587 553.—Multi-channel signalling system wherein two sections of the same pentagrid valve are used to amplify or translate two separate carrier waves carrying different signals.

Standard Telephones and Cables Ltd. and J. D. Holland. Application date 1st September, 1944.

587 939.—Delay-network distributor for separating the signals in a multi-channel time-phased pulse communication system.

Standard Telephones and Cables Ltd and M. M. Levy. Application date 26th May, 1944.

587 940.—Multivibrator circuit for producing high peak-powered pulses from waives of low power-rating, particularly for multi-channel signalling systems.

Standard Telephones and Cables Ltd. and M. M. Levy. Application date 26th May, 1944.

587 941.—Distributor device for separating the signals in a multi-channel, time-phased, pulsed communication system.

Standard Telephones and Cables Ltd. and M. M. Levy. Application date 17th April, 1944.

588 043.—System in which a circuit responsive to potential differences, but not to absolute potential changes, is utilized for the immediate transmission of messages in facsimile.

E. H. Cooke-Yarborough. Application date 7th October, 1944.

588 416.—Transitron shaping circuit for generating a variable strobing voltage from a periodic saw-toothed wave, as used say in a multiplex system with interlaced pulsed signals.

Z. Jelonek, E. W. Anderson, J. G. Macmillan and T. J. McDermott. Application date 16th February, 1945.

588 417.—Shaping circuit for generating a number of strobing pulses, from a recurrent saw-toothed wave, for separating the different messages in a multiplex pulsed signalling system.

Z. Jelonek, E. W. Anderson, J. G. Macmillan and T. J. McDermott. Application date 16th February, 1945.

588 937.—Secret signalling system in which a carrier-wave is frequency-modulated between certain limits which are determined by periodic modulating-waves of given form.

Standard Telephones and Cables Ltd. (assignees of M. Silver, C. A. Segerstrom Jr and R. B. Reade). Convention date (U.S.A.) 21st February, 1944.

588 974.—System in which a number of angular-velocity-modulated signals are transmitted over a single channel so that the modulation-components are located in different predetermined bands of frequencies.

Hazeltine Corpn. (assignees of B. D. Loughlin). Convention date (U.S.A.) 20th September, 1943.

589 093.—Two-way system in which the field-intensity of the signals radiated by the responding unit is determined by the strength of the incoming signals.

Hazeltine Corpn. (assignees of L. R. Malling). Convention date (U.S.A.) 7th March, 1944.

CONSTRUCTION OF ELECTRONIC-DISCHARGE DEVICES

587 201.—Stacking and cooling rectifier units of the dry-contact type.

Westinghouse Brake and Signal Co., Ltd. and L. E. Thompson. Application date 30th November, 1944.

587 364.—Thermionic-valve circuit, applicable as a frequency-divider, or as a time-delay device, wherein an applied impulse initiates a definite cycle of voltage-changes followed by a return to normal.

F. C. Williams and N. F. Moody. Application date 22nd August, 1944.

587 392.—Construction and assembly of the electrodes of a thermionic valve of the external grid-disc type, particularly for use with coaxial-line resonators.

Standard Telephones and Cables Ltd. and C. N. Smyth. Application date 15th February, 1943.

587 619.—Velocity-modulation device, of the coaxial-line type, so designed and arranged that the operating frequency is made dependent upon the value of the mean accelerating voltage.

Standard Telephones and Cables Ltd. and S. G. Tomlin. Application date 21st January, 1944.

587 727.—Demountable high-powered transmitting valve with an airtight joint which avoids the necessity for continuous pumping.

" Patelhold " Patentverwertungs etc. A. G. Convention date (Switzerland) 25th November, 1943.

587 730.—Method of constructing and curving the bulb of a cathode-ray tube so as to reduce distortion of the image thrown on the fluorescent screen

L. F. Broadway and V. A. Stanley. Application date 10th January, 1945.

587 741.—Coating the sensitized screens of cathode-ray tubes by centrifugal action.

G. B. F. Goff and Cathodeon Ltd. Application date 13th January, 1945.

587 757.—Construction and spacing of the electrodes of a short-wave triode valve.

M. C. Goodall and C. S. Wright. Application date 15th January, 1945.

587 894.—Construction of an electron-discharge tube designed to secure optimum geometry of the electrodes for ultra-high frequency working.

Standard Telephones and Cables Ltd. (assignees of C. V. Litton). Convention date (U.S.A.) 9th July 1943.

589 361.—Frequency-dividing circuit, comprising a triode valve with a tuned-anode circuit which serves to apply periodic blocking-potentials to the control grid.

Marconi's W.T. Co. Ltd (assignees of S. W. Seeley) Convention date (U.S.A.) 24th December, 1943.