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Technical Papers

IN a recent book ("Technical Literature," George Allen & Unwin Ltd.) G. E. Williams, who is head of the Editorial Department of the Institution of Electrical Engineers, offers some good advice to authors of scientific and technical articles, advice which makes his book worthy of careful study by all such authors.

He has, however, written with the requirements of the journal of an institution in mind, and they are not necessarily quite the same as those of other publications.

We feel, therefore, that it may be useful to say something about what is desirable in a paper submitted to *Wireless Engineer*. In the first place, only one copy of the paper with its illustrations is wanted, and it is not even essential for it to be typewritten, although this is preferred. What is important is that the paper should be legible.

Double-spaced typing with wide margins and on only one side of the paper is ideal, and is not a waste of paper but an economy, for a badly-typed paper may have to be retyped before it is fit for the printer. In typing, ample space should be left for the insertion of mathematical expressions in ink, and great care should be taken to write very plainly and to avoid cramping the symbols. Space is needed for the insertion of instructions to the printer.

Choice of Symbols

It is necessary to take particular care over certain symbols which are easily confused with others. In handwritten copy α , ∞ and ∞ are often almost indistinguishable from one another, as also are h , k and κ , β and ρ , r and s , u and v , w and ω . In typewritten copy, 0 and nought, 1 and l cause endless difficulty.

Even in print some letters and symbols are

to be avoided as being insufficiently differentiated. In *Wireless Engineer* a and α are not likely to be confused, but in some founts they are indistinguishable. However, v and ν are too near each other for it to be wise to use them both in the same paper, as also are x and χ .

One thing which continually causes trouble is the use of what the Americans term a prime ($'$)—more commonly called a foot or minute sign, or simply a tick. Always write it carefully as a slanting line; never type it, or write it as a vertical line, for then it will appear in print as a superscript 1 ($'$).

Some authors use bars, circumflex accents and other signs above letters. These are very awkward. Their use is satisfactory only if a special fount is available in which these signs are integral with the letters. If it is not, they must be hand set above ordinary letters; this is not only expensive, but the pieces of type involved are so small that there is serious risk of their being displaced. Except where they occur perhaps half a dozen times only in an issue, so that the hand-setting is tolerable, such signs cannot be used in *Wireless Engineer*.

It should, perhaps, be made clear that equations generally are hand-set and so the use of bars is not here an insuperable obstacle, although the risk of their displacement remains. However, in many papers most of the symbols are freely used in the body of the text as well as in displayed mathematics; here machine setting is imperative.

There are two remarks made by Williams which we would like to endorse and to emphasize their importance as strongly as possible. The first is in connection with the use of a multiplier ' $\times 10^n$ ' in a column heading of a table (or on the scale of a graph). All too often it is not clear whether it

is inserted to show that the author has multiplied by 10^* or to instruct the reader to multiply by 10^* , and it makes quite a difference! Williams' second remark is that text and drawings should be kept separate and never on any account be mixed up or on the same piece of paper. Few authors do this, but there are some. In the normal routine the MS and the illustrations are separated in the editorial office; the one is sent to the printer and the other to the drawing office and work on the two parts is carried on simultaneously. This cannot be done if they are mixed together and then they must go to the printer and drawing office in sequence with consequent delay and sometimes confusion.

Illustrations

This mention of the drawing office reminds us that it is not always realized that *Wireless Engineer* does not require authors to provide elaborate drawings. Almost invariably they are redrawn or traced in our own drawing office, for only in this way is it possible to secure the requisite uniformity of style throughout the journal. Because of this, only simple sketches are required from contributors; circuits can be freehand pencil sketches, graphs can be in pencil or ink on ordinary graph paper. In the case of mechanical details something a little better will usually be necessary; it would obviously be hard to convey to the draughtsman the detail of, say, the rotating joint of a waveguide system, without providing at least a moderately well-finished machine drawing.

So far we have said nothing about the paper itself. Williams has quite a bit to say about its preparation and arrangement and we do not wholly agree with him. We think, however, that this is very largely a matter personal to each author and that everyone has his own favoured method. Our own methods are not those of Williams and that is all there is to it.

We do not like too great an amount of sectionalizing. So far from being a help to understanding we find it a hindrance. Too many section and sub-section headings distract from the paper itself and interrupt what should be a smooth flow of thought from one aspect of the subject to another. Some headings are useful

and necessary. We feel that there are few articles which need breaking into more than three sections, apart from introductory and acknowledgment paragraphs, and references. We feel also that only rarely do sections need subdividing.

This is our general opinion. There will be variations and no two papers will call for quite the same treatment. There are also cases, the exceptions, however, when many sections and sub-sections will be necessary.

After accuracy, the most important thing of all is clarity of exposition. The author's job is to make matters as easy as he can for his reader. An article on the most important discovery is of no value at all if no one can understand it.

The author must bear in mind always that he is writing for those who know less about the subject than he does. He may be writing an instructional article, and this fact will then be obvious to him. If he is recording some new facts—perhaps some new observations in ionosphere research—it probably will not occur to him. He is, however, still writing for those who know less than he does, for his fellow workers in that field are unacquainted with his new facts until they have read his paper to learn what they are.

The most difficult thing of all for an author to decide is how much prior knowledge he can assume his readers to possess. In general, specialists tend to assume too much prior knowledge, for in *Wireless Engineer* they are not writing solely for the benefit of other specialists in their own field. If they write well, they will be read and understood by specialists in other branches of wireless as well as in their own and also by the non-specialist of good general knowledge. Too much reliance on mathematics should be avoided. This is a fault to which writers on filters and waveguides are particularly prone. Qualitative explanations in electrical and non-mathematical terms are desirable for understanding and often alternative explanations in different forms increase the clarity, for what appeals to one mind does not to another. The mathematics are necessary, of course, but they do not replace the general explanations, they are complementary to them.

LATERALLY-DISPLACED SLOT IN RECTANGULAR WAVEGUIDE

By A. L. Cullen, A.C.G.I., B.Sc. (Eng.)

SUMMARY.—The admittance presented by a resonant slot cut in the top face of a rectangular waveguide is calculated by a simple application of transmission-line theory. The phase of radiation is considered, and the effect of a small deviation from resonance is calculated. Experimental results support the theory remarkably well.

1. Introduction

A RESONANT slot cut in the wall of a waveguide forms an excellent radiating element for the microwave band, and linear arrays of such slots have been used very satisfactorily in radar applications.^{1, 2, 3}

In order to design such an array it is necessary to know the power radiated by each slot. A particularly suitable type of slot is the longitudinal slot displaced laterally from the centre line of the guide by an amount x as shown in Fig. 1. It is found that a slot of this kind is equivalent to a shunt conductance on a transmission line so that if the value of this conductance is known the power radiated can be found. It is also necessary to know the phase relation between the radiated field and the field in the guide. If a band of frequencies is to be covered, the effect on the relevant characteristics of a small departure from the resonant condition must be known.

This paper shows how all these quantities may be calculated in terms of the geometry of the system by a simple application of transmission-line theory.

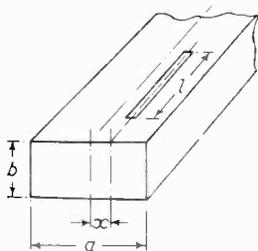


Fig. 1. Laterally-displaced slot.

2. Resonant Slot

In a most important paper⁴ H. G. Booker has shown that a slot in a metal sheet can be used as a radiating element, and that there is a close relationship between the impedance of the slot and that of the complementary dipole. (A strip dipole which would just fit into the slot.) If Z_1 is the impedance of the dipole, and Z_2 is the impedance of the slot, then $Z_1 Z_2 = Z^2/4$, where $Z = 377 \Omega$, the intrinsic impedance of free space. It follows that the admittance of a slot is proportional to the impedance of the complementary dipole, and the variation of both with length and

wavelength can be shown to follow the same curve if the scales are adjusted suitably. The general shape of this curve is shown in Fig. 3.

The curve shows two resonant points, one at $l = \lambda/2$ and one at $l = \lambda$. If the diagram is interpreted as an impedance plot, the resonance in the $\lambda/2$ region will be seen to correspond qualitatively to that of a series-resonant circuit, and the

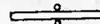


Fig. 2. Slot in metal sheet.

resonance in the λ region to that of a parallel-resonant circuit. On the other hand, if the ordinates are taken to represent admittance, the region near $\lambda/2$ corresponds to parallel resonance, and the region near λ to series resonance. Thus a slot about $\lambda/2$ in length may be represented approximately as a parallel-resonant circuit.

3. Laterally-Displaced Slots

The theory is based on the conception of a waveguide as the limiting case of a transmission line supported by quarter-wave stubs.⁵ The current carried by the transmission line corresponds to the longitudinal current in the actual guide, and the current carried by the stubs corresponds to the lateral current.

Now consider Fig. 1, which shows a rectangular waveguide in which a laterally-displaced slot has been cut. It is clear that lateral current flow in the waveguide will be modified somewhat by the presence of the slot, some of it flowing round the ends of the slot, and some of it terminating in charges near the centre of the slot, and developing a voltage across it. The ratio of this voltage to the current causing it is the impedance of the slot.

MS accepted by the Editor, September 1947.

Returning to the transmission-line and stub model, we may suppose that this impedance is inserted in series with one of the stubs. The input impedance of this stub is no longer infinite but appears as a shunt load on the equivalent transmission line. The value of this shunt load can be calculated by the usual transmission-line methods.

The ratio of this shunt-load impedance to the characteristic impedance of the transmission line will be called the 'normalized' impedance presented by the slot. This quantity can, of course, be found experimentally from measurements of the standing-wave pattern in the waveguide and it will be shown that results obtained in this way are in good agreement with the theory.

With the co-ordinate system in Fig. 4, the field components, in rationalized m.k.s.—coulomb units, for an H_{10} -wave travelling into the paper are:—

$$\left. \begin{aligned} E_y &= E_0 \cos\left(\frac{\pi x}{a}\right) e^{j\phi} \\ H_x &= \frac{I}{Z} \cdot \frac{\lambda}{\lambda_g} \cdot E_0 \cos\left(\frac{\pi x}{a}\right) e^{j\phi} \\ H_z &= \frac{-j}{Z} \cdot \frac{\lambda}{\lambda_c} \cdot E_0 \sin\left(\frac{\pi x}{a}\right) e^{j\phi} \end{aligned} \right\} \dots (I)$$

—in which $\lambda_c (= 2a)$ is the cut-off wavelength, $Z (= \sqrt{\mu/\kappa})$ is the intrinsic impedance of the medium (377Ω for air) and $\phi = 2\pi z/\lambda_g$.

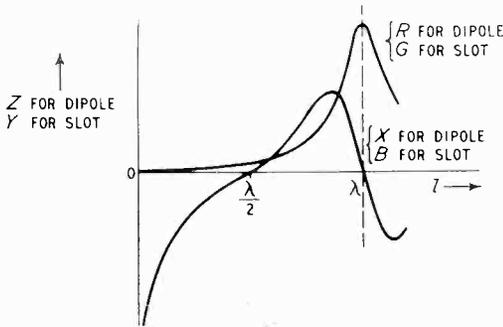


Fig. 3. Resonance curves for dipole or slot.

The maximum voltage between the top and bottom walls of the guide is at $x = 0$, and is—

$$V = \int_0^b E_y dy = E_0 b e^{j\phi} \dots (2)$$

The vector surface current density \mathbf{K} on a metal surface is given by—

$$\mathbf{K} = \mathbf{n} \times \mathbf{H}$$

—where \mathbf{n} is the unit normal, directed out from the metal. On the inside top face of the guide, $\mathbf{n} = -\mathbf{j}$, and therefore—

$$\mathbf{K} = \mathbf{k} H_x - \mathbf{i} H_z$$

$$\begin{aligned} K_x &= H_x \\ K_z &= -H_z \end{aligned} \dots (3)$$

To find the total longitudinal current I_z on the inside top wall of the guide we must integrate K_z across the whole width of the guide. The result is—

$$I_z = \frac{2a}{b} \cdot \frac{\lambda}{\lambda_g} \cdot \frac{E_0}{Z} e^{j\phi}$$

—and this is the current associated with a forward travelling wave of voltage V on the equivalent

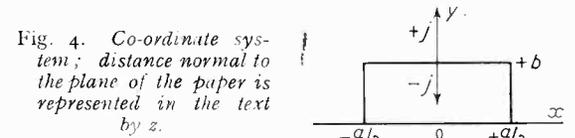


Fig. 4. Co-ordinate system; distance normal to the plane of the paper is represented in the text by z .

transmission line. The characteristic impedance Z_z of the equivalent transmission line is therefore the ratio of the voltage given by (2) to the current I_z , and so we find—

$$Z_z = \frac{V}{I_z} = \frac{\pi}{2} \cdot \frac{b}{a} \cdot \frac{\lambda_g}{\lambda} \cdot Z \dots (4)$$

We now define the effective length of the slot in terms of the total current flowing through the slot and the current density at this position before the slot was cut. Thus the total current I_x flowing through the slot is the integral of the initial lateral current density K_x over the effective length of the slot. If the ratio of effective length to actual length is η , the effective length is ηl . Carrying out the integration we find that—

$$I_x = \frac{-j}{Z} \cdot \frac{\lambda E_0}{\pi} \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi \eta l}{\lambda_g}\right) e^{j\phi} \dots (5)$$

If η is small, this is approximately

$$I_x = \frac{-j}{Z} \cdot \frac{\lambda}{\lambda_g} \cdot \eta l E_0 \sin\left(\frac{\pi x}{a}\right) e^{j\phi} \dots (6)$$

This current must be carried by the equivalent stub. Now the voltage between top and bottom walls at a distance x from the centre of the guide is—

$$V_x = E_0 b \cos\left(\frac{\pi x}{a}\right) e^{j\phi} \dots (7)$$

—by an obvious generalization of (2). Thus—

$$\frac{V_x}{I_x} = jZ \frac{2ab}{\lambda \eta l} \cot\left(\frac{\pi x}{a}\right) \dots (8)$$

This is the formula for the input impedance of a stub of length $\frac{a}{2} - x$, short circuited at $x = \frac{a}{2}$, carrying waves of length $2a$, and having a characteristic impedance—

$$Z \cdot \frac{2ab}{\lambda \eta l}$$

It seems reasonable, therefore, to take this as the characteristic impedance of the quarter-wave stub in series with which the impedance of the slot is considered to be connected.

In the vicinity of half-wave resonance, the slot may be represented by the circuit of Fig. 5,

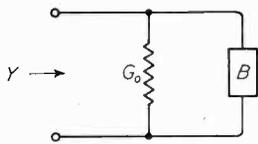


Fig. 5. Equivalent circuit of slot near half-wave resonance.

so that its admittance is given by—

$$Y = G_0 + jB \quad \dots \quad (9)$$

—where G_0 is practically constant, and B depends on l and λ .

The problem now is to calculate the input impedance Z_s of the stub shown in Fig. 6. This can be done very easily by a straightforward application of standard transmission-line formulae and the result is—

$$Z_s = G_0 x^2 Z \operatorname{cosec}^2 \theta - jZ_x (\cot \theta - BZ_x \operatorname{cosec}^2 \theta) \quad \dots \quad (10)$$

—where $\theta = \pi x/a$. This impedance appears in parallel with the equivalent transmission line, so that the whole system is represented by the transmission-line system of Fig. 7.

At resonance, $X_s = 0$, and the corresponding

value of slot susceptance, B_0 say, is given by—
 $\cot \theta = B_0 Z_x \operatorname{cosec}^2 \theta$

$$\text{or } B_0 = \frac{1}{2Z_x} \sin 2\theta \quad \dots \quad (11)$$

At resonance, therefore, the input conductance is—
 $G_{s0} = \frac{1}{G_0 Z_x^2} \sin^2 \left(\frac{\pi x}{a} \right) \quad \dots \quad (12)$

If this is normalized to the characteristic impedance of the equivalent transmission line by writing—

$$g_s = G_s Z_x \quad \dots \quad (13)$$

—we find, on substituting for Z_x and Z_z , that—

$$g_s = \frac{\pi}{8} \left(\frac{\eta^2}{G_0 Z} \right) \frac{\lambda \lambda_g l^2}{a^3 b} \sin^2 \left(\frac{\pi x}{a} \right)$$

—in which η and G_0 are unknown. We can therefore write—

$$g_s = K \frac{\lambda \lambda_g l^2}{a^3 b} \sin^2 \left(\frac{\pi x}{a} \right) \quad \dots \quad (14)$$

—in which K is a constant which can be found by experiment. If various values of a , b , λ , λ_g , l , and x are used, the constancy or otherwise of K will provide a check of the theory.

4. Determination of K

Some experimental results are given in Table I. The results due to Watson are published in Fig. 33(a) and (b) and Fig. 34(a) of his book,³ or in

TABLE I
Comparison of Theory and Experiment

	λ	a	b	x''	l (cm)	g_s	Type of slot	K	K/\bar{K}	$(K/\bar{K} - 1)^2$	
Watson ³ [Fig. 33(b)]	3.20	0.90"	0.40"	0.066"	1.58	0.074	Rect. No cover	0.487	1.072	0.0052	
	3.20	0.90"	"	0.134	1.59	0.280	" "	0.462	1.013	0.0003	
	3.20	0.90"	"	0.267"	1.62	0.805	" "	0.407	0.896	0.0108	
Hirst	8.96	3.00"	1.00"	0.75"	4.00	0.317	Dumb-bell Dialux	0.450	0.990	0.0001	
	8.99	"	"	1"	4.15	0.506	Rect. "	0.440	0.963	0.0010	
	9.02	"	"	0.75"	4.00	0.335	Dumb-bell Mica	0.468	1.030	0.0009	
	9.08	"	"	0.625	4.04	0.249	Rect. Dialux	0.453	0.997	0.0000	
	9.15	"	"	0.875	4.13	0.435	" "	4.436	0.960	0.0016	
	9.15	"	"	0.875	4.10	0.438	Dumb-bell "	0.445	0.980	0.0004	
	9.16	"	"	0.75	4.10	0.355	Dumb-bell "	0.452	0.996	0.0000	
	9.20	"	"	0.50	4.035	0.183	Rect. "	0.477	1.050	0.0025	
	9.35	"	"	0.50	4.00	0.175	Dumb-bell "	0.444	0.977	0.0005	
	9.43	"	"	0.50	4.20	0.200	Rect. "	0.450	0.991	0.0001	
	9.59	"	"	0.50	4.60	0.255	Rect. "	0.459	1.010	0.0001	
	10.20	"	"	0.50	4.60	0.325	Rect. Mica	0.492	1.084	0.0070	
	Watson ³ [Figs. 34(a) & 33(a)]	10.12	2.8"	1.3"	0.78"	4.92	0.735	$\frac{1}{16}$ " Rect. No cover	0.423	0.931	0.0048
10.12		"	"	0.78"	4.90	0.770	$\frac{3}{8}$ " " " "	0.445	0.980	0.0004	
10.21		"	"	0.78"	4.99	0.785	$\frac{1}{4}$ " " " "	0.424	0.934	0.0043	
10.27		"	"	0.78"	5.10	0.805	$\frac{3}{16}$ " " " "	0.409	0.900	0.0100	
10.70		"	"	0.20"	5.08	0.087	$\frac{3}{16}$ " " " "	0.468	1.030	0.0009	
10.70		"	"	0.30"	5.08	0.208	$\frac{3}{16}$ " " " "	0.507	1.118	0.0139	
10.70		"	"	0.40"	5.08	0.351	$\frac{3}{16}$ " " " "	0.495	1.000	0.0081	
								$\Sigma =$		$\Sigma =$	
								9.99		0.0729	
Constant K				Probable error							
$\bar{K} = 0.454$				$p = 1.25\%$							

Fig. 3(a) and (b) and Fig. 4(a) of his recent paper on the subject.² The results due to Hirst are published here for the first time. In the table, the ratio of the normalized conductance to the

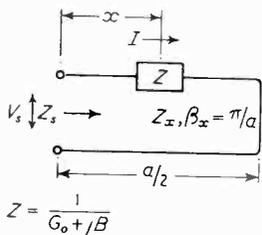


Fig. 6. Equivalent stub to carry lateral current through slot.

function is tabulated, where F is defined by the following equation—

$$F = \frac{\lambda \lambda_0 l^2}{a^3 b} \sin^2 \left(\frac{\pi x}{a} \right) \quad \dots \quad (15)$$

—and this is equal to K for each slot. It will be seen that K is reasonably constant for quite widely differing conditions, and assuming that the errors are purely random, it has the value 0.454 with a probable error in its determination of 1.25%.

The normalized conductance is plotted against F in Fig. 8, and it will be seen that the experimental points lie well on the line of slope 0.454.

5. Resonance Curves and Q

The equivalent circuit of Fig. 7 shows that the laterally-displaced slot behaves like a series-resonant circuit in shunt across the equivalent transmission line. The resonance curves of Fig. 9 to 13 confirm this. The experimental variation of normalized resistance with wavelength is seen to agree fairly well with the theoretical value of normalized resistance given by the reciprocal of equation (14), and so provides additional confirmation of the analysis. One would expect the experimental curve to fall rather faster than the theoretical

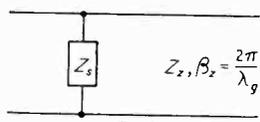


Fig. 7. Equivalent transmission line system $Z_s = R_s + jX_s$ where $R_s = G_0 Z_0^2 \cos^2 \theta$, $X_s = B Z_0^2 \cos^2 \theta - Z_0 \cot \theta$ and $\theta = \pi x/a$.

one, since the theory neglects the slight fall in conductance of the slot itself.

We shall define the Q of a resonant system in terms of the rate of change of reactance with frequency or wavelength in the vicinity of resonance. Thus for a system with resistance R and reactance X , we write—

$$Q = \frac{1}{2} \frac{f_0}{R_0} \left| \frac{dX}{df} \right|_0 = \frac{1}{2} \frac{\lambda_0}{R_0} \left| \frac{dX}{d\lambda} \right|_0 \quad \dots \quad (16a)$$

Alternatively, in terms of conductance and susceptance—

$$Q = \frac{1}{2} \frac{f_0}{G_0} \left| \frac{dB}{df} \right|_0 = \frac{1}{2} \frac{\lambda_0}{G_0} \left| \frac{dB}{d\lambda} \right|_0 \quad \dots \quad (16b)$$

The suffix zero in these formulae indicates that the value at resonance is to be used. It is a simple matter to show that these formulae are equivalent, and that they lead to the usual formula $\omega L/R$ for the Q of a simple series-resonant circuit.

Let us apply this definition to calculate the effective Q of a laterally-displaced slot. Since the characteristic impedance Z_z of the equivalent transmission line [equation (4)] is a function of frequency it might at first sight appear that the normalized value of the equivalent shunt load must be used in the calculation of Q .

But—

$$\frac{d}{d\lambda} \left(\frac{X_s}{Z_z} \right) = \frac{1}{Z_z} \cdot \frac{dX_s}{d\lambda} + X_s \frac{d}{d\lambda} \left(\frac{1}{Z_z} \right)$$

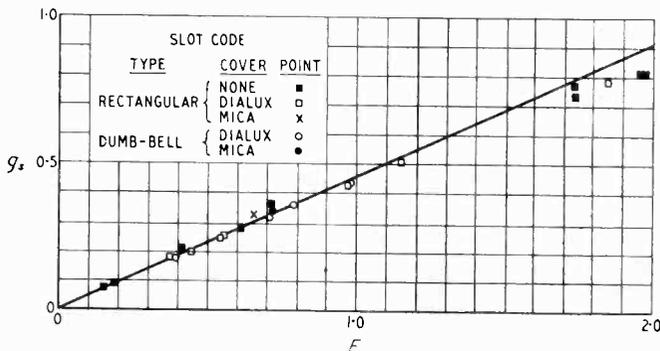


Fig. 8. Experimental determination of Q .

—and so at resonance, when $X_s = 0$, we have—

$$\left| \frac{d}{d\lambda} \left(\frac{X_s}{Z_z} \right) \right|_0 = \frac{1}{Z_z} \left| \frac{dX_s}{d\lambda} \right|_0$$

—and it follows that the effective Q measured in the guide, which we shall denote by Q_g , is given by—

$$Q_g = \frac{1}{2} \frac{\lambda_0}{R_{s0}} \left| \frac{dX_s}{d\lambda} \right|_0 \quad \dots \quad (17)$$

in which the suffix zero indicates resonance, as before.

Differentiating the reactive part of (10), we get—

$$\begin{aligned} \frac{dX_s}{d\lambda} &= \frac{dZ_x}{d\lambda} [\cot \theta - 2Z_x B \operatorname{cosec}^2 \theta] \\ &- \frac{dB}{d\lambda} \cdot Z_x^2 \operatorname{cosec}^2 \theta \end{aligned}$$

Introducing the condition of resonance given by equation (11), and using the fact that—

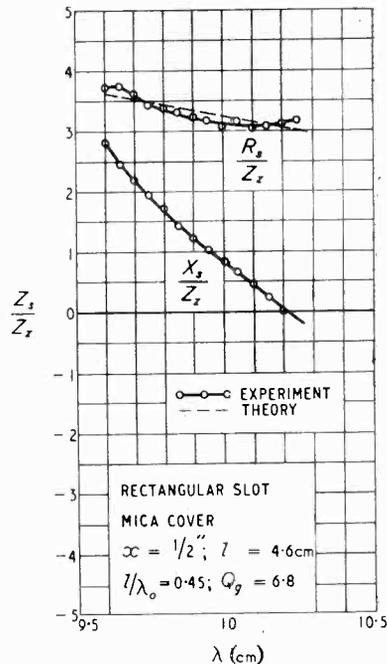
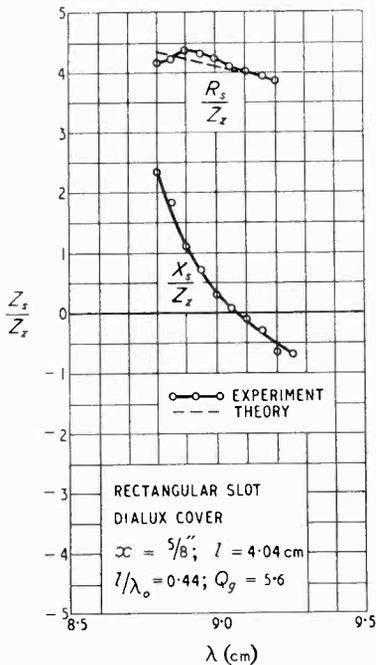
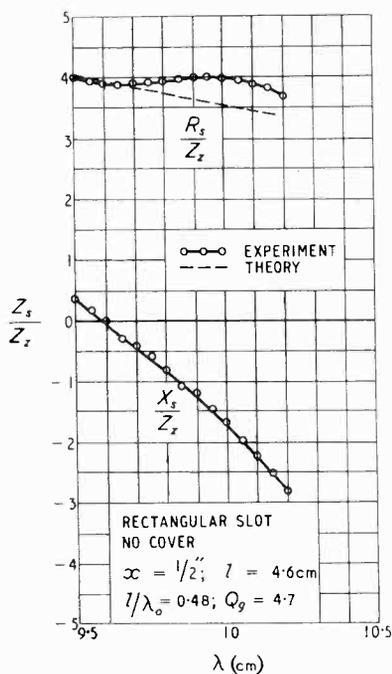
$$\frac{dZ_x}{d\lambda} = -\frac{Z_x}{\lambda}$$

—it is easy to show that

$$Q_g = \frac{1}{2} \cdot \frac{\lambda_0}{G_0} \left[\left| \frac{dB}{d\lambda} \right|_0 - \frac{B_0}{\lambda_0} \right] \dots \dots (18)$$

It follows that Q_g is practically independent of slot offset x . It should be noted, however, that in general Q will depend somewhat on the guide dimensions and wavelength. Referring again to Figs. 9 to 13, it will be seen that the Q_g value is given for each case, and the data are collected in Table II.

It will be seen that the narrower dumb-bell



Figs. 9, 10 and 11. Resonance curves.

We may assume that near resonance, the slot susceptance is proportional to the deviation of wavelength from the wavelength at which the slot itself is resonant; i.e., at which $B = 0$. Thus if $\Delta\lambda$ is the increment required to produce the susceptance B_0 , we have, approximately—

$$B_0 = - \left| \frac{dB}{d\lambda} \right|_0 \cdot \Delta\lambda \dots \dots (19)$$

—in which the negative sign is due to the fact that $dB/d\lambda$ is negative in the vicinity of half-wave resonance.

Thus, we can write—

$$Q_g = Q [1 + \Delta\lambda/\lambda_0] \dots \dots (20)$$

—in which Q is the Q of the slot itself. Now the deviation of the resonant wavelength of the equivalent shunt load from that of the slot itself, is usually only a few per cent, as would be expected on physical grounds; and it is usually accurate enough to write—

$$Q_g \approx Q \dots \dots (21)$$

slot has a somewhat higher Q_g than a rectangular slot with the same cover.

Q_g also depends on the type of cover used, if any. Thus the use of a dialux or mica cover increases Q_g by increasing the capacitance per unit length of slot. This effect is clearly illustrated by the graph of Fig. 14 derived from the curves Figs. 9—13.

Using the author's definition of Q , Watson has given additional experimental results on variation of Q with slot width³.

Summing up, we may say that for a broad-band

TABLE II

Slot	λ_0	l	l/λ_0	Q_g
Rect. Open ..	9.59	4.60	0.48	4.7
Rect. Dialux ..	9.06	4.04	0.45	5.6
Rect. Mica ..	10.21	4.60	0.45	6.8
Dumb. Dialux ..	8.97	4.00	0.45	7.93
Dumb. Mica ..	9.025	4.00	0.44	8.0

system the slots should be as wide as possible, and that if a cover is essential thin material of low dielectric constant should be used.

6: Phase of Radiation at Resonance

a. At Resonance

In designing a waveguide array of slots, it is necessary to know the phase of the radiation from each slot. It will now be shown that this bears a fixed relation to the phase of the field in the guide at the centre of the slot, provided that the condition of resonance is fulfilled, whatever the offset x .

Referring to Fig. 6, the total current through Z is given, by the usual transmission-line formulae as—

$$I = V_s \left[\frac{I}{Z_s} \cos \theta - j \frac{I}{Z_x} \sin \theta \right] \quad \dots (22)$$

—where $\theta = \pi x/a$.

The voltage V_0 appearing across the slot at resonance is—

$$V_0 = \frac{I}{G_0 + jB_0} \quad \dots (23)$$

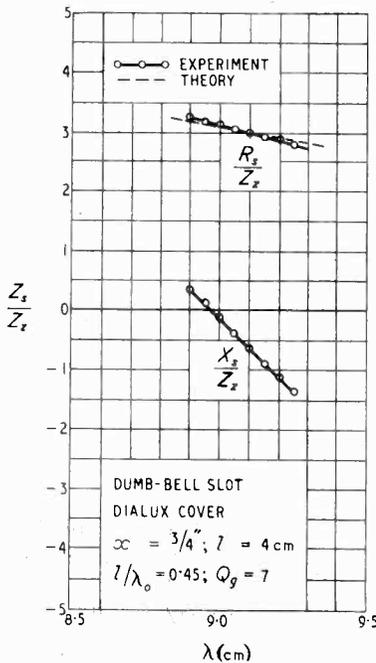


Fig. 12. Resonance curve.

and since—

$$\left. \begin{aligned} \frac{I}{Z_{s0}} &= \frac{I}{G_0 Z_x^2} \sin^2 \theta \quad \dots \dots \dots \\ \text{and } B_0 &= \frac{I}{Z_x} \sin \theta \cos \theta \quad \dots \dots \dots \end{aligned} \right\} (24)$$

—at resonance, we find, on substituting (24) in (23) and (22), that—

$$V_0 = \frac{V_s}{G_0 Z_x} \left[\frac{\sin \theta \cos \theta - j G_0 Z_x}{G_0 Z_x + j \sin \theta \cos \theta} \right] \sin \theta$$

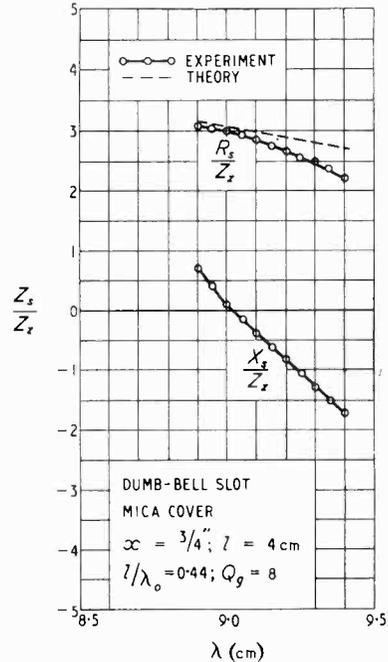


Fig. 13. Resonance curve.

and multiplying top and bottom by j , we get—

$$V_0 = \frac{V_s}{j G_0 Z_x} \sin \theta \quad \dots \dots \dots (25)$$

Thus we find that the voltage across the slot, which of course determines uniquely the phase of radiation from it, is in quadrature with the voltage V_s at the input end of the stub. It follows, therefore, that the voltage across the slot is in quadrature with the electric field in the guide at the centre of the slot. It should be noted that this is independent of θ , and therefore of x . The amplitude of V_0 is of course proportional to $\sin \theta$ for a fixed V_s , in agreement with the previous results on conductance.

b. Slightly off Resonance.

It is a matter of practical importance to know how the phase of radiation is affected by a small departure from resonance.

We have already shown that the effective Q of a laterally-displaced slot as deduced from impedance measurements in the waveguide, is practically equal to the Q of the slot itself. (See Equ. (21) of Section 4.) By definition of Q , we therefore have, to first order—

$$-\frac{\Delta X_s}{R_{s0}} = \frac{\Delta B}{G_0} \approx -\frac{2Q\Delta\lambda}{\lambda_0} = -\alpha, \quad \dots (26)$$

— where $\Delta\lambda$ is a small increase in wavelength above the resonant wavelength. The sign is determined by inspection. (cf. Fig. 3). Equations (23) and (24) may now be replaced by—

$$V = \frac{I}{G_0 + jB} \quad \dots \quad (27)$$

$$\left. \begin{aligned} \text{and } \frac{I}{Z_s} &= \frac{I}{G_0 Z_x^2} \sin^2 \theta \left(\frac{I}{I - j\alpha} \right) \quad \dots \quad (28) \\ B &= \frac{I}{Z_x} \sin \theta \cos \theta - \alpha G_0 \quad \dots \end{aligned} \right\}$$

Substituting (28) in (22), and (27), we get, after a little algebraic manipulation—

$$V = \frac{V_s}{jG_0 Z_x} \sin \theta \left(\frac{I}{I - j\alpha} \right)$$

$$\text{or } V = V_0 \left(\frac{I}{I - j\alpha} \right) \quad \dots \quad (29)$$

For small values of α , therefore, the amplitude of the voltage across the slot is practically the same as at resonance, but it has a phase lead approximately equal to α ; i.e., the phase lead is

$$\alpha = 2Q\Delta\lambda/\lambda_0 \text{ radians} \quad \dots \quad (30)$$

phase lag in relation to the phase at resonance. In this form the equation lends itself readily to experimental verification. The apparatus is shown diagrammatically in Fig. 15. Two slots in the lower guide are separated by $\lambda_g/2$, so that the field in the guide is in the same phase for both. Radiation from the two slots is picked up by another slot cut in a horizontal length of waveguide mounted above them, and the amount of radiation indicated by a crystal detector in this guide. The two slots in the lower guide are cut on the same side of the centre line, so that if both slots are resonant, the position of the upper slot for minimum signal is midway between them. If the length of one of the slots is varied, the position of the upper slot for minimum signal will vary by an amount depending on the phase difference between the slots, and the geometry of the system. The phase difference is clearly given by

$$\alpha = \frac{2\pi}{\lambda} (S_2 - S_1)$$

The procedure is to make one slot resonant, and the other shorter. The short slot is lengthened a little at a time, and S_1 and S_2 found at each stage.

For an open rectangular slot, $\frac{1}{4}$ -in wide. at

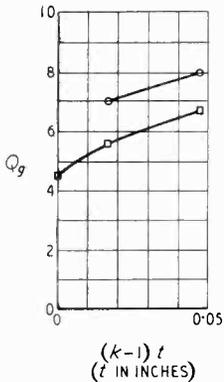


Fig. 14 (above). Variation of Q_g with additional capacitance per unit length of slot.

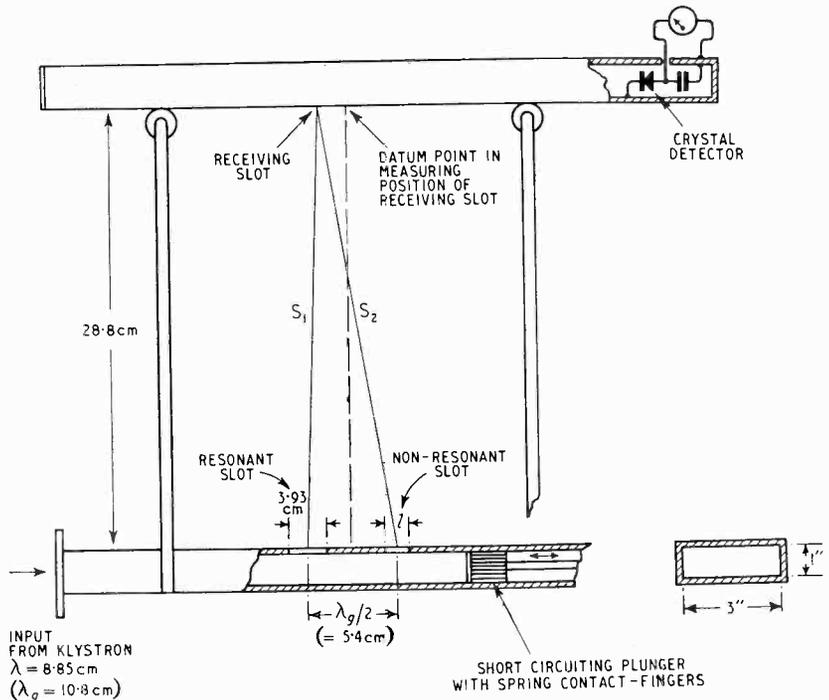


Fig. 15 (right). Apparatus used in comparing phase of radiation from resonant and non-resonant slots.

If the length of the slot is taken as the variable quantity, we can write

$$\alpha = -2Q \Delta l/\lambda_0 \text{ radians} \quad \dots \quad (31)$$

—so that an increase in slot length makes the

$\lambda = 8.85$ cm, resonant length about 4.0 cm, a fairly linear relation between phase and slot length has been found, with a phase change of $13.9^\circ/\text{mm}$, (Fig. 16).

From Table II, we see that $Q_g = 4.7$ for this type of slot, and substituting in Equ. (31), we find a theoretical phase change of $13.8^\circ/\text{mm}$. Thus good agreement of theory and experiment is again found.

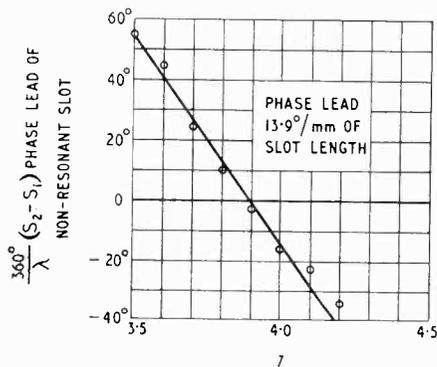


Fig. 16. Variation of phase of radiation with length of slot.

7. Stevenson's Theory

A more advanced treatment of the laterally displaced slot, and other radiating slots, has been given by Stevenson. He considers only resonant conditions, for an infinitely thin slot, and his results give only the normalized conductance at resonance. However, the calculation is complete, and there is no unknown constant to be found experimentally.

The resulting formula is

$$g_s = 2.09 \frac{a}{b} \frac{\cos^2\left(\frac{\pi \lambda}{2 \lambda_g}\right)}{\lambda/\lambda_g}$$

Over the range of values for which experimental results are available, the numerical results obtained from either the formula above or equation (14) are in close agreement, rather surprisingly in view of their different form.

In spite of the superiority of Stevenson's method, which is based on Maxwell's equations and the theory of Green's functions, it is thought that the present simple treatment gives a useful physical picture of the processes involved in radiation from the slot. A discussion of Stevenson's method and a summary of his results is given by Watson³.

8. Acknowledgments

The author wishes to acknowledge the influence of many valuable discussions with Dr. J. L. Michiels, and to thank D. Hirst for permission to use some hitherto unpublished experimental results. Most of the work described was carried out during the war in the Radio Department of

the Royal Aircraft Establishment, and is published with the permission of the Ministry of Supply. The illustrations, with the exception of Figs. 15 and 16, are Crown Copyright and are reproduced with the permission of H.M. Stationery Office.

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RADIO COMPONENT MANUFACTURERS' EXHIBITION

The 1949 R.C.M.F. Private Exhibition of components, valves and test gear will be held in the Great Hall, Grosvenor House, Park Lane, London, W.1, from 1st to 3rd March 1949, from 10 a.m. to 6 p.m. Admission is by invitation only and applications for tickets should be sent to the Secretary, Radio Component Manufacturers' Federation, 22, Surrey St., Strand, London, W.C.2.

ELECTRONICS IN INDUSTRY

A course of six lectures on this subject is being given during February and March 1949 by L. I. Farren, M.B.E., Whit. Schol., A.M.I.E.E., A.C.G.I., D.I.C., at the Polytechnic, Regent St., London, W.1. It includes ultra-sonic, high-frequency heating and control systems.

CITY & GUILDS EXAMINATIONS

The final entry date for the City & Guilds examinations in Telecommunications Engineering (Subject 50), Radio Service Work (Intermediate, Subject 53) and Radio Amateurs (Subject 54), is 1st March 1949. Entries should be made through a local technical college or the office of a Local Education Authority.

In the case of the Radio Servicing Certificate Examination (Final, Subject 53) candidates must apply to the Secretary, Radio Trades Examination Board, 9, Bedford Sq., London, W.C.1, before 1st February, 1949, upon an entry form obtainable therefrom or from the City & Guilds London Institute, Dept. of Technology, 31, Brechin Place, London, S.W.7. Applications must be accompanied by documentary evidence in support of the statements on the entry form and a fee of two guineas. After receiving confirmation of their eligibility, candidates must enter by the normal procedure through a local technical college or the office of a London Education Authority.

TELEGRAPH CONDENSER CO. LTD.

The price and working voltage of the T.C.C. Visconal Condenser, Type CP5800, appeared incorrectly in this firm's advertisement in the December issue. The list price is 15s., not 23s., and the maximum working voltage 7,000, not 6,000.

TRANSMISSION-LINE FILTERS

By E. K. Sandeman, Ph.D., B.Sc., M.I.E.E.

SUMMARY.—The pass and attenuating bands of several types of transmission-line filters are shown in graphical form. The presentation is restricted to filters using transmission lines in which dissipation is negligible and in which the lateral dimensions (e.g., spacing, in the case of open-wire lines or cables, whether balanced or concentric, and internal transverse dimensions in the case of waveguides) are small compared to the wavelength. It will be evident that, in view of the restriction on dimensions, the treatment is not normally applicable to waveguides.

Three general types of filter section are dealt with:

- (i) Filter sections constituted by lengths of transmission line leak-loaded with short-circuited or open-circuited stubs consisting of lengths of the same type of transmission line.
- (ii) Filter sections constituted by lengths of transmission line series-loaded by short-circuited or open-circuited stubs.
- (iii) Filter sections constituted by two lengths of the same type of transmission line in parallel, the electrical lengths of the two transmission lines being unequal. The use of such sections as quarter-wave transformers of infinitely variable impedance is described.

Although the image impedances are different, the pass bands of filter sections constituted by lines leak-loaded with shorted stubs are the same as those of lines series-loaded with open stubs. Similarly, the pass bands of lines leak-loaded with open stubs are the same as those of lines series-loaded with shorted stubs.

The presentation of characteristics is followed by the derivation of formulae.

1. Introduction

1.1. Stubs.—The use of stubs or short sections of transmission line bridged across a feeder circuit, with their far ends open or shorted, is common radio practice when it is required to eliminate an unwanted frequency, or when it is required to select one frequency and reject frequencies each side of it. (The use of such stubs for impedance matching is not being considered here.)

Up to now the use of such stubs has been slightly empirical, and any engineer applying such methods has had to find out by calculation and experiment how far such types of filter were capable of meeting his requirements. Under such conditions it is possible that some of the capabilities of such filters may have escaped notice. For instance, every radio engineer knows that a quarter-wave stub will introduce pass and attenuating bands in infinite series in the range of frequencies above the frequency at which the stub is a quarter-wavelength long, but how many people are aware that when the distance of the stub from neighbouring stubs or the ends of the transmission line is large compared with the length of the stub, a series of pass and attenuating bands is introduced below the frequency at which the stub is a quarter-wavelength long? So far as the writer knows, this property has not been used. It is true that the attenuation so introduced may be very small, but small amounts of attenuation are often adequate and, if more attenuation is required, it is usually a simple matter to introduce more than one stub and a series of short stubs may be preferable to one long stub. (In this connection, it seems quite

practicable to use a folded or coiled stub in order to economize in space.)

As an example of the performance of a stub filter at low frequencies, reference to Fig. 3 shows that a stub-supported line (in which the stubs are evidently shorted), with a spacing between stubs equal to twenty times the length of stub (i.e., $a = 10$) has ten attenuating bands below the frequency at which the stub is quarter of a wavelength long. Considering the band which extends from $u = 0.1$ to $u = 0.17$, the maximum attenuation is about 13 db and the attenuation

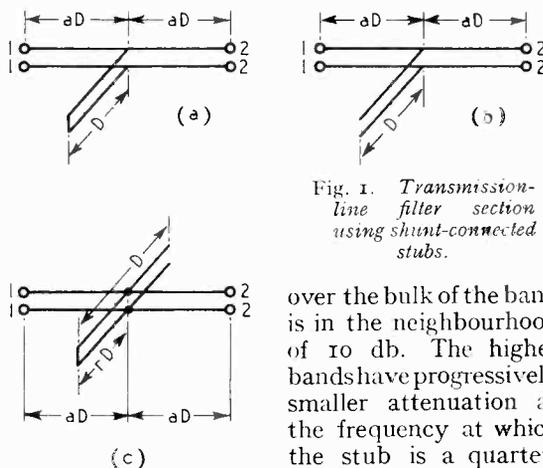


Fig. 1. Transmission-line filter section using shunt-connected stubs.

over the bulk of the band is in the neighbourhood of 10 db. The higher bands have progressively smaller attenuation as the frequency at which the stub is a quarter-wave long is approached.

The attenuation of lines leak-loaded by shorted stubs is easily calculated by means of equation (7) and equation (11) gives the attenuation of lines leak-loaded by open stubs.

Equations (7) and (11) define $\cosh P$. The

MS accepted by the Editor, December 1947

awkwardness of any expression defining $\cosh P$ can be rapidly dissipated by writing for $\cosh P$ the fuller form $\frac{1}{2}(q + 1/q)$ where $q = e^p$. Then $q + 1/q = 2 \cosh P$ and a close approximation to q can be obtained quickly by mental arithmetic once the value of $\cosh P$ is known. The attenuation in db = $20 \log_{10} q$.

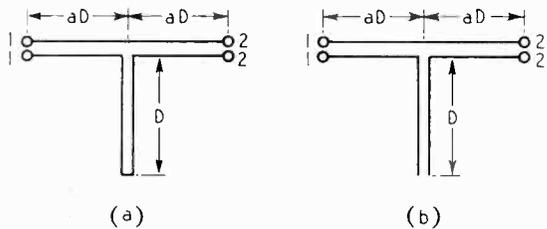


Fig. 2. Transmission-line filter section using series-connected stubs.

For instance, if $\cosh P = 5$, $q + 1/q = 10$, so that $q = 9.9$ very nearly, and the attenuation is 20 db within the accuracy with which attenuations are generally required.

1.2. Two-Path Filter. The two-path filter, of which several varieties are shown in Fig. 8, does not appear to have received the consideration it deserves, since it is extremely simple and cheap and is capable of affording very high values of insertion loss at two frequencies in every attenuating band and a very useful value of attenuation over the greater part of most bands. By inserting two such filters in tandem with their pass bands staggered to any required extent, selection of very narrow bands of frequency can be made.

In addition, as described below, it can be made to provide a quarter-wave transformer of infinitely variable impedance [see Fig. 9(a) and accompanying text], and of course it can be used to provide lengths of inserted section of impedance differing widely from that of any transmission line. In the past the method of impedance matching by means of an inserted section, whether of quarter wavelength or other length, has been severely limited by the non-availability of lengths of trans-

mission line of characteristic impedance varying over a sufficiently wide range.

2. Stub-loaded Lines

Fig. 1 shows a number of leak-loaded filter sections, each consisting of a section of transmission line of length $2aD$ with a leak (shunt) reactance at its centre point constituted by a stub or transmission line of length D . In case (a) the stub is connected to the transmission line at one end and is shorted at its free end; in case (b) the stub is open at its free end, while in case (c) the stub is open at one end and shorted at the other and is connected to the transmission line at a distance rD from the shorted end.

Fig. 2 shows two series-loaded filter sections, each consisting of a section of transmission line of length $2aD$ with a series reactance at its centre point constituted in case (a) by a shorted stub, and in case (b) by an open stub.

It is evident that for each type of filter section shown in Figs. 1 and 2, an infinite range of variations is obtained by varying the parameter 'a' which determines the ratio of half the distance between the input and output terminals to the length of the stub.

2.1—Shorted Shunt-Stub Lines: Open Series-Stub Lines. Fig. 3 shows the pass bands of all varia-

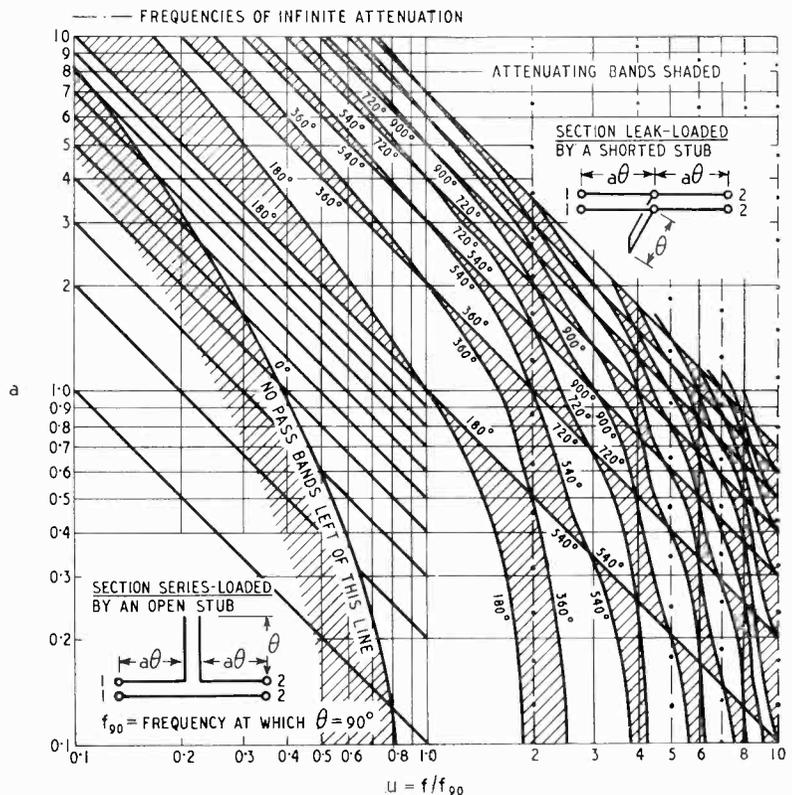


Fig. 3. Pass bands of lossless stub-loaded filter.

tions of section leak-loaded by shorted stubs, or series-loaded by open stubs, for a range of variation of 'a' from 0.1 to 10.0. The scale of 'a' is vertical, while the horizontal scale at the bottom of the diagram is a scale of frequency f normalized in terms of the frequency f_{90} , at which

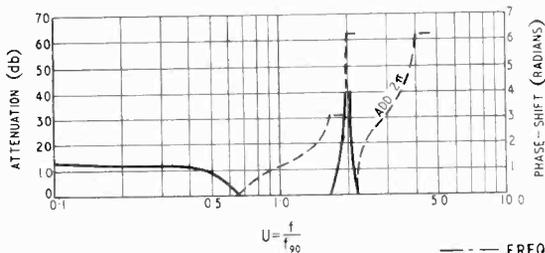


Fig. 4. Attenuation and phase-shift for typical filler section leak-loaded with shorted stubs or series-loaded with open chain dotted lines. for case where $a = \frac{1}{2}$.

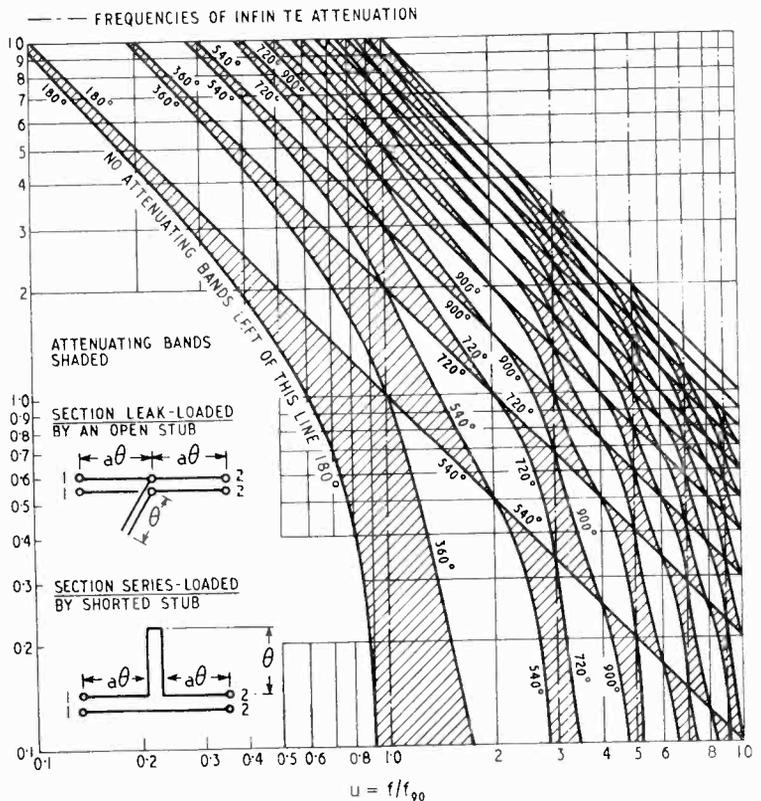
Fig. 5. Pass bands of lossless stub-loaded lines.

the stub is a quarter of a wavelength long; the dimensions of the scale are therefore f/f_{90} , which has been designated by u . The loci of the frequencies of infinite attenuation appear as straight vertical chain dotted lines. The values of phase shift at the cut off frequencies are also indicated. The figures in degrees at the cut-off boundaries indicate the magnitude of the phase-shift constant.

If a ruler is laid horizontally across the diagram at any value of 'a' it selects the pass bands corresponding to that variation of filter having the corresponding value of 'a'. For instance, if it is required to design a stub-supported line (which evidently constitutes an example of a line leak-loaded with shorted stubs) so that the maximum possible pass range is obtained, it can be seen from the diagram, by placing a ruler across Fig. 3 at $a = 1.0$, that two pass bands are confluent at the frequency at which the stub is a quarter of wavelength long; i.e., at f_{90} . There is therefore effectively a pass band extending from just below $0.4u$ to very nearly $1.6u$, a frequency ratio of substantially 4. If 'a' is not exactly equal to unity there is a risk of some attenuation being introduced in the

neighbourhood of $u = 1$ so that the whole of this range is not available for all purposes, because in many sections small attenuations will add up to large amounts. It will be noticed that by locating the stubs at closer intervals, a wide pass band can be obtained, free from the undesirable feature of a possible small bump of attenuation near the middle of the band: if $a = 0.38$ a pass band exists from $u = 0.6$ to $u = 1.6$. A construction with such short spacing is, however, normally of theoretical interest only.

In practice, the widest band can be obtained by making $a = 1$ and using the range from $u = 0.4$ to $u = 1.0$, giving a bandwidth of 0.6.



A solution which is nearly as good, and uses slightly larger spacing is to make $a = 1.5$, and to use the range from $u = 0.75$ to $u = 1.25$.

Suppose that it is required to make a stub-supported line to transmit a frequency which may lie anywhere in the range from 2,700 to 3,300 Mc/s, that is a range of variation of approximately $\pm 10\%$, what is the widest spacing of stubs which can be used, and what is the length of stub?

A series of competitive bands exist with a geometric mid-band frequency at $u = 1$ and

values of a equal to 1.5, 2.5, 3.5, etc. It will be seen that if $a = 3.5$ a pass band extends from $u = 0.9$ to $u = 1.1$, which corresponds to a variation of $\pm 10\%$ on the mid-band frequency.

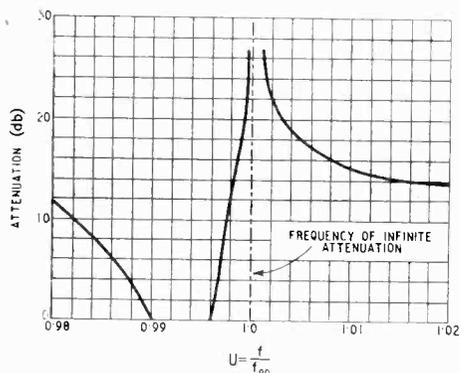


Fig. 6. Attenuation of transmission-line filter section leak-loaded with open stubs, when $a = 1.01$. Image impedance = 114 when $u = 0.994$; image impedance = ∞ when $u = 0.99$.

By going to $a = 2.5$ a larger tolerance still is obtained, while at $a = 4.5$ the tolerance is less. Hence, if the mid-band is located at $u = 1$, and $f_{90} = 3,000$ Mc/s so that the length of stub is a quarter-wavelength at 3,000 Mc/s (i.e., 2.5 cm), the spacing between stubs will be $2 \times 3.5 \times 2.5$ cm = 17.5 cm. It is important to remember that the pass bands in Fig. 3 will only be realized exactly when the diameter of stub is negligible compared with the wavelength, and the diameter of conductor used in conventional types of stub-supported line at centimetre wavelengths cannot be regarded as entirely negligible compared with the wavelength. Considerable caution has therefore to be used in applying the results from Fig. 3.

The use of stubs for securing rejection of unwanted frequencies is well known, and it will be seen that a series of frequencies of 'infinite attenuation' exist at $u = 2, 4, 6, 8$, etc. In practice, of course, the attenuation is never infinite and depends on the amount of dissipation in the stub. With practically realisable values of dissipation, however, very large values of attenuation can be obtained at these frequencies.

Fig. 4 shows the phase shift and attenuation characteristics of one example of the class of filters of which the pass bands are displayed in Fig. 3; i.e., when $a = 1/3$.

2.2. Open Shunt-Stub Lines: Shorted Series-Stub Lines. Fig. 5 shows the pass bands of all variations of section, leak-loaded with open stubs or series-loaded by shorted stubs, for ranges of variation of a and u from 0.1 to 10. As in the case of Fig. 3, the loci of the frequencies of

infinite attenuation appear as vertical straight chain-dotted lines; and the values of phase shift at the cut-off frequencies are also indicated.

By selecting such a value of a that a frequency of infinite attenuation occurs near the edge of an attenuating band, it is theoretically possible to realize a very high degree of discrimination between frequencies which differ by a very small percentage. For instance, examination of Fig. 5 in the region where $a = 1.01$ and u is approximately unity shows that a frequency of infinite attenuation occurs at $u = 1.0$ and a small pass band occurs immediately to the left of this region. Fig. 6 shows the attenuation characteristics in this region. It will be seen that it would, for instance, be theoretically possible to reject with high attenuation a frequency of 1,000 Mc/s while receiving with no attenuation a frequency of 995 Mc/s. In practice, owing to dissipation and the appreciable lateral dimensions of transmission lines compared with the wavelength, the attenuation would not be infinite, the cut-off frequencies would not be so sharply defined, and the attenuation inside the pass band would not be zero. This does not mean that high discriminations of useful size are not obtainable by such means, but only that the exact characteristics of Fig. 6 are not likely to be realized exactly in any practical arrangement.

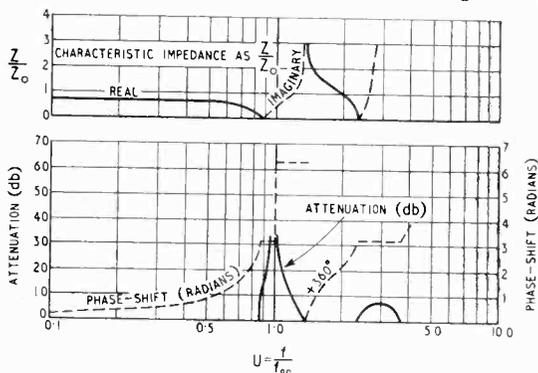
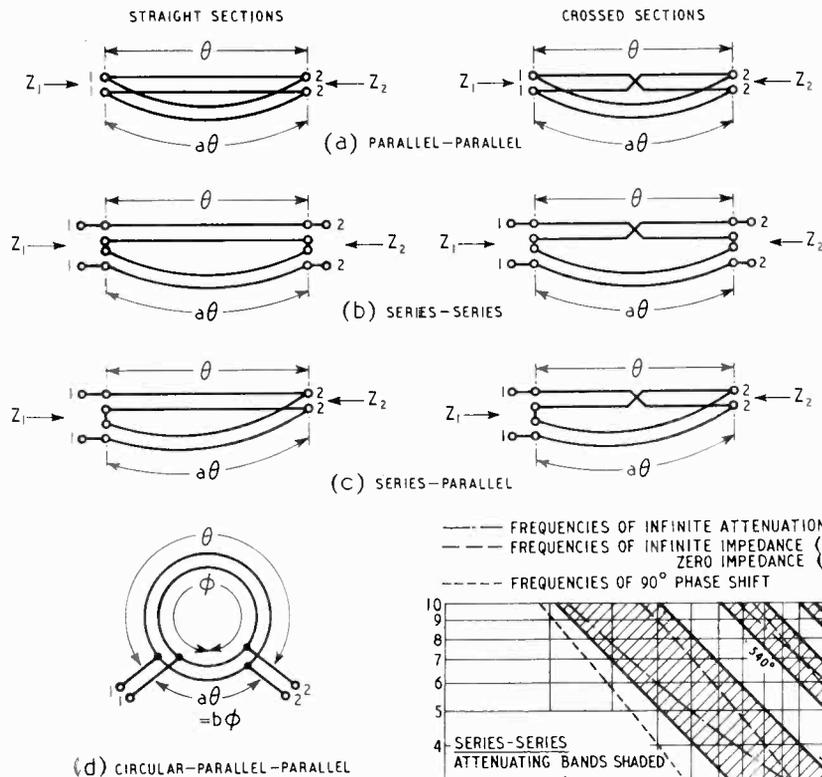


Fig. 7. Characteristics of typical section leak-loaded with open stubs or series-loaded with shorted stubs for case where $a = 1/3$.

Fig. 7 shows the attenuation and phase shift characteristics of a filter section of the type for which the pass bands are mapped in Fig. 5, for the particular case when $a = 1/3$. While the same warnings apply to this case as for the filter of Fig. 6, the frequencies between which high discrimination is obtained are separated by wider frequency intervals, and in general it is to be expected that the general form of the characteristics is more likely to be realized in the case of Fig. 7 than in the case of Fig. 6, for transmission



filter was originally drawn by E. W. Hayes of the B.B.C., who had been matching lines by the method of inserted section and had been modifying the impedance of open-wire lines by hanging on to each leg of the line a second conductor of the same gauge as the main conductor, which was fastened to it at intervals so that it hung from the main conductor in a series of equal loops or festoons. He found that such a circuit gave rise to very

Fig. 8 (above). Types of two-path transmission-line filters.

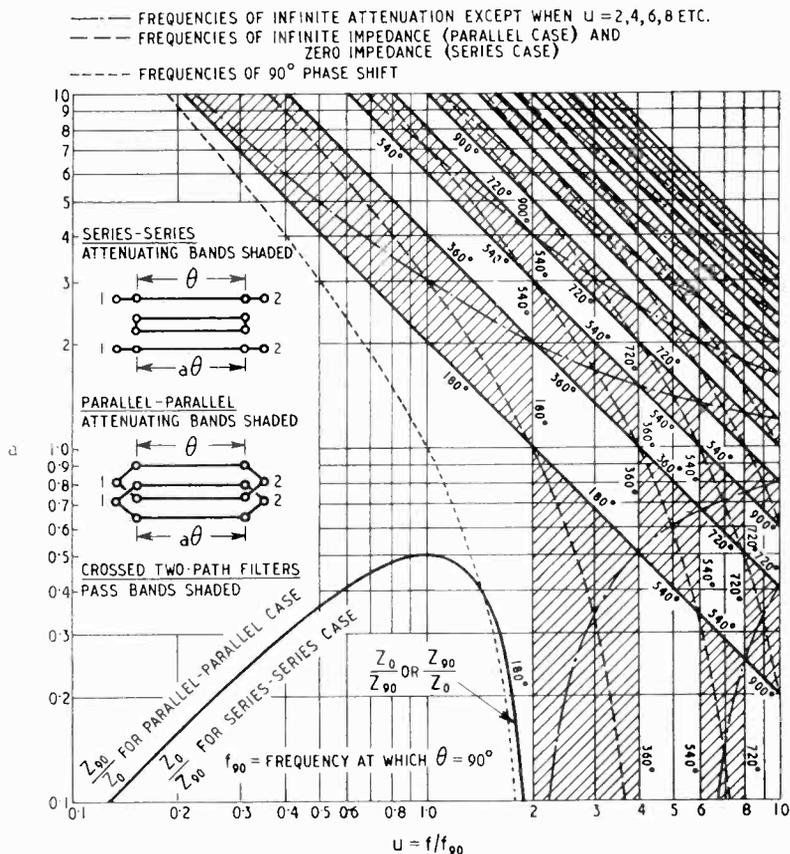
Fig. 9 (right). Pass bands of series-series and parallel-parallel two-path filters. The pass bands of straight filters correspond with the pass bands of crossed filters, and vice versa. Frequencies of infinite attenuation and infinite and zero impedance are not shown for the crossed cases.

lines of equal dissipation and equal ratio of lateral dimensions to wavelength in each case.

The general form of these statements is evidently the conclusion of most probable general utility to be drawn from this analysis: the greater the ratio of the wavelength to the lateral dimensions of the transmission line, the lower the slope of the attenuation/frequency curves, and the lower the transmission-line loss, the closer will any practical filter characteristic approximate to the theoretical.

3. Two-path Transmission-line Filters

The attention of the author to this type of



high attenuation at certain frequencies and that it was possible to regard the arrangement of conductors as a succession of sections each consisting of two paths of different electrical length in parallel.

It is understood that a two-path section has been in use in T.R.E. for impedance-matching purposes.

The possible variations of this type of filter are indicated in Fig. 8. Probably the simplest type of section is the Parallel-Parallel type of section shown at (a) and (d), so called because the inputs and outputs of the component transmission paths are in parallel both at input and output. The section at (a) on the left is called a Straight section because it contains no commutation while that at (a) on the right is called a Crossed section because it contains a commutation in one transmission path only: in more general terms, a crossed section is any section in which the number of commutations in the two component transmission paths is unequal. The method of designating the remaining types of section will be clear by inspection of Fig. 8.

series of loci of frequencies of infinite attenuation. Certain attenuation bands contain both frequencies of infinite attenuation and frequencies of infinite or zero impedance, and in such bands the insertion loss reaches a maximum at two frequencies and the result is a useful value of attenuation over practically the whole of the band. The loci of the frequencies of zero or infinite impedance and of infinite attenuation are indicated on Fig. 9, as well as the values of phase shift at the cut off frequencies.

3.2. *Quarter-Wave Networks.* A two-path transmission-line filter can be used to provide a quarter-wave network with a wide range of possible image impedances. On Fig. 9, in the left-hand pass band, is marked the locus of

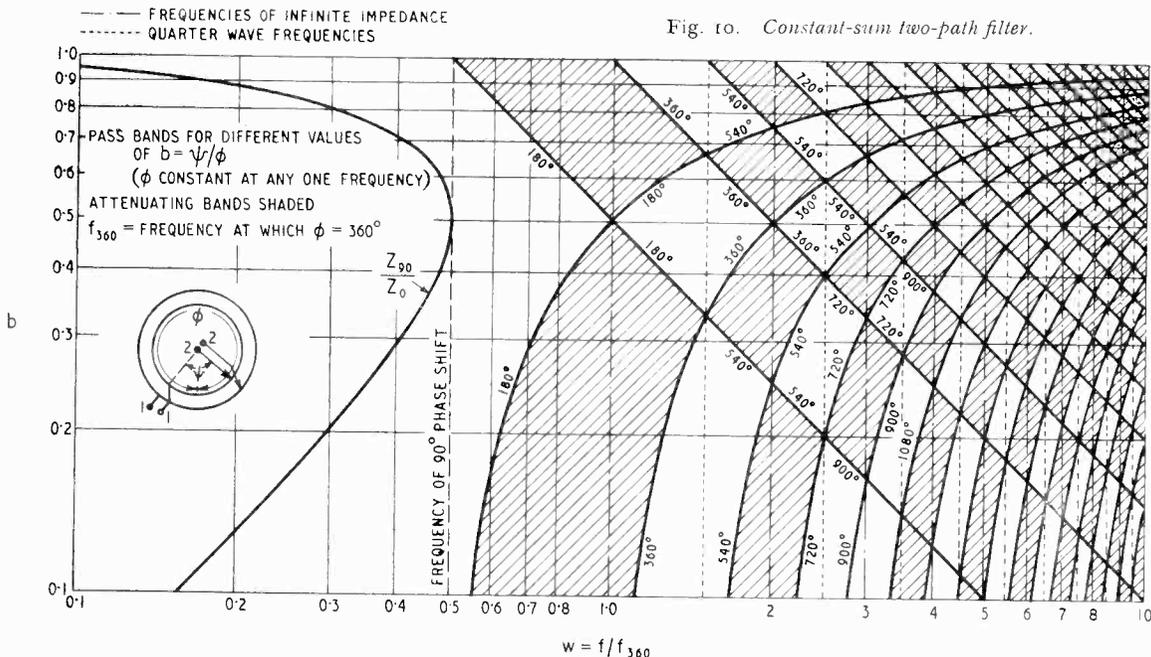


Fig. 10. Constant-sum two-path filter.

3.1. *Series-Series and Parallel-Parallel Two-Path Filters.* Fig. 9 defines the pass and attenuating bands of Series-Series and Parallel-Parallel Filters, both straight and crossed, for the case where the mechanical length of one component path is kept constant and that of the other is varied as defined by the value of a . The shaded areas define attenuating bands for straight filters and pass bands for crossed filters. As might have been expected, the pass and attenuating bands of Parallel and Series Filters are the same. Inserting a commutation in either transmission path interchanges the pass and attenuating bands.

Each attenuating band consists of two adjacent or confluent bands with a frequency of zero or infinite impedance between, and also there is a

frequencies of 90° phase shift. Immediately below this line is a full line marked Z_{90}/Z_0 for parallel-parallel case and Z_0/Z_{90} for series series case. This line defines the image impedance of the section at any frequency, when the value of a is so chosen that at that frequency the phase shift is 90° ; in other words, the section constitutes a quarter-wave network. Hence, in order to design a section as a quarter-wave network with any required image impedance, it is only necessary to proceed as follows:—

1. Decide on the value of the required image impedance: Z_{90} . Calculate the value of Z_{90}/Z_0 or Z_0/Z_{90} , respectively, according to whether the section is parallel or series connected.

- Enter the value of the ratio determined in 1 on the vertical scale (i.e., the scale normally used for a) in Fig. 9; traverse horizontally until the impedance-ratio curve is encountered, then vertically until the curve of frequencies of 90° phase-shift is met, and then horizontally until the vertical scale of a is reached, where the required value of a is read.

Example. Given transmission lines of characteristic impedance 80 ohms, to design a quarter-wave network with an image impedance of 32 ohms. $32/80 = 0.4$, and entering this on the left-hand scale, the impedance ratio curve is encountered at a value of $u = 0.59$; proceeding vertically, the 90° phase-shift curve is encountered at a value of $a = 2.4$. Hence the quarter-wave network should consist of a length of transmission line which is 0.59 of a quarter-wave ($53^\circ 6'$) long, in parallel with a line which is $0.59 \times 2.4 = 1.416$ quarter-waves ($127^\circ 26'$) long at the frequency at which the quarter-wave network is required.

Evidently quarter-wave networks of this kind may be used just like any other quarter-wave networks: for impedance matching purposes, or for impedance inversion, etc. In this connection it may be remarked that a two-path filter may also be used to provide networks of any required impedance and any required electrical length (within the ranges available). Such a network may be useful, for instance, for impedance matching in a transmission line by an inserted section of image impedance differing from that of the main transmission line.

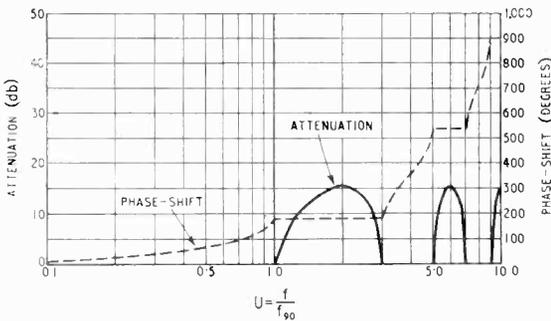


Fig. 11. Attenuation and phase-shift characteristic of straight parallel-parallel or series-series two-path filter when $a = 2$.

Fig. 9 was derived on the assumption that the ratio, between the electrical lengths of the two alternative paths constituting the two-path filter is varied by keeping one path constant and varying the other: the sum of the electrical lengths of the two paths therefore varies. Fig. 10

has been derived on the assumption that the sum of the electrical lengths remains constant while the ratio between them is varied.

Constant-Sum Two-Path Filter. A constant-sum two-path filter is one in which the sum of the electrical lengths of the two component paths remains constant as the ratio between the electrical lengths of the component paths is varied. Such a filter may conveniently be constituted by driving a circular or closed transmission line at any convenient point and deriving an output from any other point which is assumed to slide along the transmission line when it is required to vary the length ratio of the component paths. The characteristics of such a transmission line are shown in Fig. 10

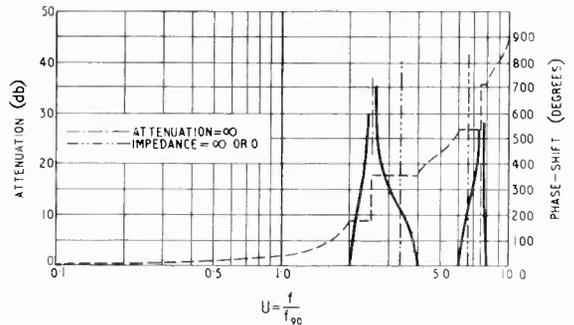


Fig. 12. Attenuation and phase-shift characteristic of parallel-parallel and series-series two-path filter when $a = 0.2$.

in terms of different parameters from those used for Fig. 9. Frequency is normalized in terms of the frequency at which ϕ , the sum of the electrical lengths of the two paths = 360° , while instead of showing the variation of the pass bands in terms of the ratio of the electrical lengths of the two transmission paths, the variation is shown in terms of the ratio of the electrical length of one path to the sum of the electrical lengths of the two paths. In other words, the boundaries between attenuating and pass bands are plotted in a field of w and b where:—

$$w = f/f_{360}$$

$$f_{360} = \text{frequency at which } \phi = 360^\circ$$

$$\psi = \text{electrical length of one path}$$

$$\phi = \text{sum of electrical lengths of the two paths}$$

$$b = \psi/\phi \text{ (evidently } b \text{ must lie between 0 and unity).}$$

It will be evident that Fig. 10 is merely a transform of Fig. 9 and that the same information is conveyed in both figures. If it is required to know how the pass bands of a filter will vary if one transmission path is varied, keeping the other constant, then Fig. 9 should be used. If

it is required to know how they vary when one path is reduced by the same amount as the other is increased, then Fig. 10 should be used.

It will be seen that at the frequency at which $w = 0.5$ (i.e., when $\phi = 180^\circ$), the filter behaves like a quarter-wave network regardless of the value of b . Hence it affords a means of providing at any frequency a quarter-wave transformer having infinitely variable impedance, since as b is varied the image impedance of the structure varies. The relation between b and Z_{90}/Z_0 is plotted on Fig. 10. Referring to this curve it will be seen that over a range of variation of b from 0.18 to 0.82 the variation of image impedance of the structure does not exceed 2:1 so that the variation of insertion loss when matched to any value of its image impedance in this range is only one or two db. In this range of b such a filter may therefore be used to provide an infinitely variable tuning device, although its relative advantages compared with the use of

when $a = 0.2$ respectively. From inspection of Figs. 11 and 12 it will be seen that the magnitude of the attenuation obtainable with this type of filter varies considerably with the value of a . It will also be seen from Fig. 12 that the effective insertion loss of only a single section of such a filter can be appreciable over the greater part of its pass band. The insertion loss of a four-pole of image impedances Z_1 and Z_2 working between impedances Z_a and Z_b (Z_1 connected to Z_a and Z_2 connected to Z_b) and of propagation constant P is given by:—

$$20 \log_{10} \frac{\left(Z_b + \frac{Z_a Z_2}{Z_1}\right) \cosh P + \left(Z_2 + \frac{Z_a Z_b}{Z_1}\right) \sinh P}{(Z_a + Z_b) \sqrt{\frac{Z_2}{Z_1}}}$$

It will be seen that if Z_1 or Z_2 is equal to 0 or infinity a high value of insertion loss is to be expected: in practice, the insertion loss at frequencies where the image impedances are 0 or infinity, is never infinity, on account of dissipation, but the presence of a frequency of zero or infinite impedance may mean that the insertion loss at that frequency approximates to that at the frequency of infinite attenuation.

Fig. 13 defines the pass and attenuating bands of Series-Parallel transmission-line filters, both straight and crossed. The shaded areas define attenuating bands for straight filters and pass bands for crossed filters. The same remarks apply to the series-parallel filters as apply to series-series and parallel-parallel filters except that series-parallel filters provide a means of obtaining higher impedance step down since the parallel-

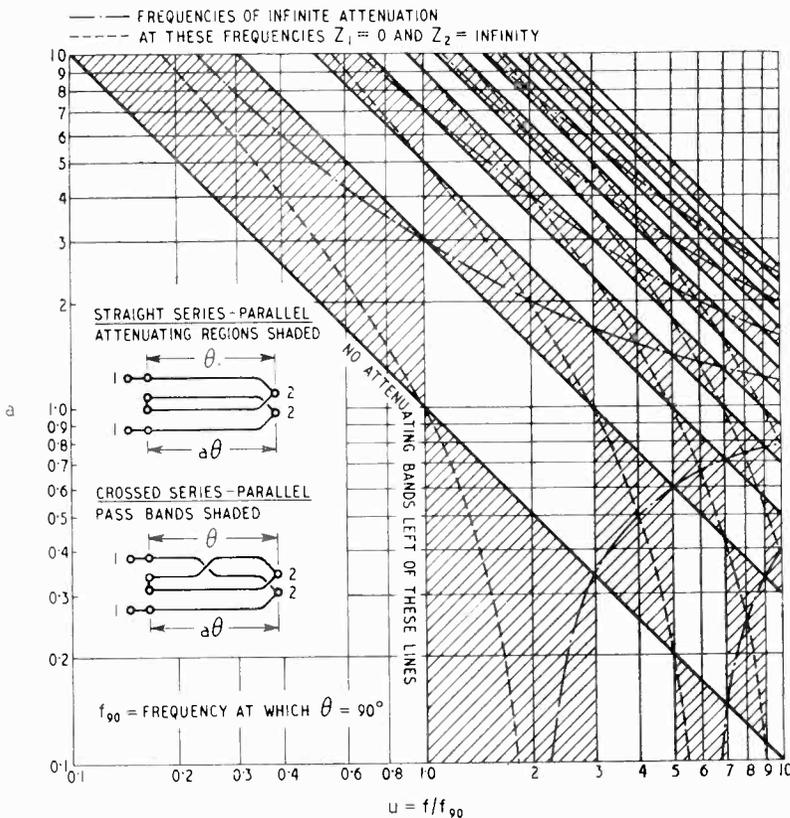


Fig. 13. Pass bands of straight and crossed series-parallel filters. Frequencies of zero and infinite impedance, and frequencies of infinite attenuation are not shown for the crossed case.

the structure portrayed in Fig. 9 will depend on the kind of performance which is required.

Figs. 11 and 12 show typical attenuation and phase-shift characteristics of straight series-series or parallel-parallel sections when $a = 2.0$ and

output impedance is the inverse of the series-input impedance, being inverted about the characteristic impedance of the component transmission lines (which have equal characteristic impedance).

4. Derivation of Formulae

Conventions

- f = frequency in cycles per second
 $\omega = 2\pi f$
 D = length of stub in metres
 a = ratio of half length of filter section to stub length (for stub filters) and also
 a = ratio of lengths of component transmission paths (for two-path filters)
 b = ratio of length of one path to path sum (for constant-sum filters)
 α = phase-shift constant of filter section
 β = attenuation constant of filter section
 P = propagation constant of filter section
 Z, Z_1 and Z_2 = image impedance of filter sections
 Z_{SC} and Z_{OC} = short-circuit and open-circuit impedances
 α_1 = phase shift constant of transmission line per metre = $2\pi/\lambda = \omega/V$
 λ = wavelength in transmission line at frequency f in metres
 V = velocity of propagation in transmission line at frequency f , in metres per second
 $\theta = D\omega/V$ = angular length of stub at frequency f
 f_0 = frequency at which $\theta = 90^\circ$; i.e., at which the stub is a quarter-wave long
 $u = f/f_0$ = normalized frequency
 $\phi = a\theta$ = line angle of length of transmission line of length equal to half length of stub filter section = $aD\omega/V$ and also
 ϕ = sum of electrical lengths of paths in constant sum filter
 Z_0 = characteristic impedance of transmission line
 Z_{90} = image impedance of filter section at frequency at which $\alpha = 90^\circ$
 $//$ = in parallel with
 $A_\phi, D_\phi, Y_\phi, Z_\phi = A, Y, D$ and Z matrices respectively, of line of angle ϕ
 A_S = A matrix of fourpole constituted by a shunt element of impedance S
 A_R = A matrix of fourpole constituted by a series element of impedance R
 $A_F, D_F, Y_F, Z_F = A, D, Y$ and Z matrices respectively, of filter section
 R = value of series reactance element in filter section
 S = value of shunt reactance element in filter section.

The matrices used are described under the designations A, Y, D and W by F. Strecker and R. Feldkeller in *E.N.T.*, Vol. 6, pp. 93-112, 1929, "Grundlagen der Theorie des allgemeinen Vierpols"; see also "Communication Networks," by E. A. Guillemin (pub. John Wiley).

Those who are unfamiliar with the characteristics of filters may find the following helpful in understanding the basis of the calculations which follow.

A filter section is an arrangement of dissipationless reactance elements so disposed as to afford transmission of electrical energy between one pair of terminals 1, 1, and a second pair of

terminals 2, 2. The reactance elements may consist of lumped or distributed reactance, and must be so disposed that attenuation of the transmitted energy occurs at certain frequencies, while at other frequencies transmission occurs free of attenuation. Apart from transmission lines and certain forms of lattice network, almost any form of structure of both positive and negative reactances which provides free transmission in one or more bands of frequency (the pass bands) will provide attenuation in one or more other bands of frequency.

Inside the pass bands the image impedances of the filter section are pure resistances of magnitude varying with frequency, while in the attenuating bands the image impedances are pure reactances varying in magnitude with frequency. The image impedances at the cut-off frequencies, which define the boundaries between the pass and attenuating bands, are either zero or infinity. It will be evident that by examining the image impedances of a structure of reactances it is possible to determine the cut-off frequencies and the location of pass and attenuating bands. This is what is done below: the frequencies at which the image impedances are zero or infinity are first found, and then the image impedances between these frequencies are examined to find out if they are real (resistive) or imaginary (reactive).

It will be evident that if the arrangement of reactances is not symmetrical with regard to the terminals 1,1 and 2,2, the image impedance looking into terminals 1,1 will differ from that looking into 2,2.

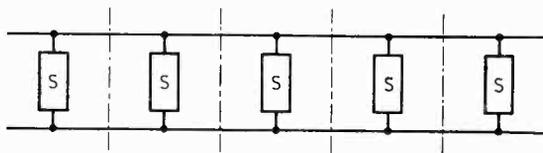


Fig. 14. Leak-loaded transmission line.

5. Leak-loaded Transmission Lines

Fig. 14 shows a transmission line leak-loaded with shunt impedances at regular intervals. This may be regarded as consisting of a number of identical sections as indicated by the vertical lines of section. Each of these sections may be broken down into three parts as indicated in Fig. 15.

The A matrix of the filter section shown in Fig. 15 may be obtained by multiplying together the A matrices of the component fourpoles, and this must evidently be equal to the A matrix

of the whole filter section shown in Fig. 15, formed directly from its image impedance Z and its propagation constant P . Stating this analytically in reverse order:

$$\begin{aligned}
 A_F &= \left\| \begin{array}{cc} \cosh P & Z \sinh P \\ \frac{\sinh P}{Z} & \cosh P \end{array} \right\| = A_\phi \times A_s \times A_\phi \\
 &= \left\| \begin{array}{cc} \cos \phi & jZ_0 \sin \phi \\ \frac{j \sin \phi}{Z_0} & \cos \phi \end{array} \right\| \times \left\| \begin{array}{cc} 1 & 0 \\ \frac{1}{S} & 1 \end{array} \right\| \times \left\| \begin{array}{cc} \cos \phi & jZ_0 \sin \phi \\ \frac{j \sin \phi}{Z_0} & \cos \phi \end{array} \right\| \\
 &= \left\| \begin{array}{cc} \cos \phi & jZ_0 \sin \phi \\ \frac{j \sin \phi}{Z_0} & \cos \phi \end{array} \right\| \times \left\| \begin{array}{cc} \cos \phi & jZ_0 \sin \phi \\ \frac{\cos \phi}{S} + \frac{j \sin \phi}{Z_0} & \frac{jZ_0 \sin \phi}{S} + \cos \phi \end{array} \right\| \\
 &= \left\| \begin{array}{cc} \cos^2 \phi - \sin^2 \phi + j \frac{Z_0}{S} \sin \phi \cos \phi & j2 Z_0 \sin \phi \cos \phi - \frac{Z_0^2}{S} \sin^2 \phi \\ \frac{j2 \sin \phi \cos \phi}{Z_0} + \frac{\cos^2 \phi}{S} & \cos^2 \phi - \sin^2 \phi + j \frac{Z_0}{S} \sin \phi \cos \phi \end{array} \right\|
 \end{aligned}$$

(Note as a check, that the determinant of this matrix is unity).

Equating terms of this matrix with like terms of A_F ,

$$\begin{aligned}
 \cosh P &= \cos^2 \phi - \sin^2 \phi + j \frac{Z_0}{S} \sin \phi \cos \phi \\
 &= \cos^2 \phi + j \frac{Z_0}{2S} \sin^2 \phi \dots \dots (1)
 \end{aligned}$$

$$\text{Also } Z \sinh P = j2Z_0 \sin \phi \cos \phi - \frac{Z_0^2}{S} \sin^2 \phi (2)$$

$$\text{and } \frac{\sinh P}{Z} = j \frac{2 \sin \phi \cos \phi}{Z_0} + \frac{\cos^2 \phi}{S} \dots (3)$$

Dividing (2) by (3)

$$\begin{aligned}
 \therefore Z &= Z_0 \sqrt{\frac{j2S \sin \phi \cos \phi - Z_0 \sin^2 \phi}{j2S \sin \phi \cos \phi + Z_0 \cos^2 \phi}} \dots (4) \\
 &= Z_0 \sqrt{\frac{1 + j \frac{Z_0}{2S} \tan \phi}{1 - j \frac{Z_0}{2S} \cot \phi}}
 \end{aligned}$$

The values of ϕ at which cut off occurs are found by equating Z to zero and to infinity.

$$\begin{aligned}
 \text{Conditions for cut off are therefore:} \\
 S &= -j \frac{Z_0}{2} \tan \phi \dots \dots (5a)
 \end{aligned}$$

$$S = j \frac{Z_0}{2} \cot \phi \dots \dots (5b)$$

These equations are quite general, and apply whatever kind of device is used for providing the reactive elements.

Coil Leak-loaded Line

For instance, by making $S = jL\omega$, and $\phi = aD\omega/V$, equations (1) and (4) respectively define, as functions of frequency, the propagation constant per section and the mid-line image impedance of a transmission line leak-loaded with inductances L spaced $2aD$ metres apart.

The term mid-line image impedance is now introduced to indicate that the section termination occurs in the middle of each length of transmission line half-way between (and distance aD from) each shunt. Equation (5b) defines the cut-off frequency. The structure is evidently a high-pass filter.

Leak-loading by Short-circuited Stubs [Fig. 1(a) and Fig. 3].

Assuming that, as is usually the case, the characteristic impedance of the stub line is the same as that of the main line,

$$S = jZ_0 \tan \theta \dots \dots (6)$$

so that

$$\cosh P = \cos 2\phi + \frac{1}{2} \sin 2\phi \cot \theta \dots (7)$$

$$Z = \sqrt{\frac{1 + \frac{1}{2} \tan \phi \cot \theta}{1 - \frac{1}{2} \cot \phi \cot \theta}} \dots (8)$$

Cut-off occurs when $Z = 0$ or infinity and since $\phi = a\theta$, the conditions for cut-off are

$$\tan a\theta = -2 \tan \theta \text{ or infinity} \dots (9a)$$

$$\cot a\theta = 2 \tan \theta \text{ or infinity} \dots (9b)$$

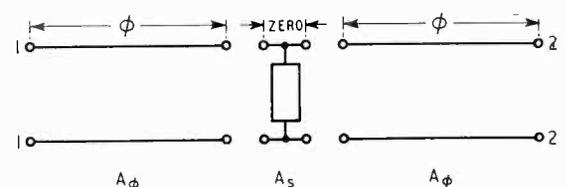


Fig. 15. Division of filter section in two four-poles with matrix conventions for each.

These relations between a and θ are plotted on Fig. 3 and define the boundaries between pass and attenuating regions.

These boundaries are plotted as relations between a and u , derived as follows. From equation (9a),

$$a\theta = \tan^{-1}(-2 \tan \theta)$$

so that, by choosing any value of θ , the corresponding value of a , can be found. The corresponding value of u is then found from the relation $u = \theta/90$. The plot between the values of a and u so derived constitutes one class of boundary between pass and attenuating bands. Another class of boundary is found from the relation $\tan a\theta = \infty$, which means that $a\theta = n \times 90^\circ$ where n is integral and odd, so that, making the substitution $\theta = u \times 90^\circ$, $a = n/u$ which gives rise to a series of lines at 45° drawn through the points $u = 1, 3, 5, 7$, etc. Equation (9b) provides a number of similar lines drawn through the points $u = 1, a = 2, 4, 6, 8$, etc.

Leak-loading by Open-circuited Stubs [Fig. 1(b) and Fig. 5].

In this case $S = -jZ_0 \cot \theta \dots \dots$ (I0)

$\cosh P = \cos 2\phi - \frac{1}{2} \sin 2\phi \tan \theta$ (I1)

$Z = \sqrt{\frac{1 - \frac{1}{2} \tan \phi \tan \theta}{1 + \frac{1}{2} \cot \phi \tan \theta}} \dots$ (I2)

Cut-off occurs when

$\tan a\theta = 2 \cot \theta$ or infinity $\dots \dots$ (I3a)

$\cot a\theta = -2 \cot \theta$ or infinity $\dots \dots$ (I3b)

These relations define the pass bands of Fig. 5.

Leak-loading by Tapped Stubs [Fig. 1(c)].

This might equally well be regarded as loading

$A_F = \left\| \begin{array}{cc} \cosh P & Z \sinh P \\ \frac{\sinh P}{Z} & \cosh P \end{array} \right\| = A_\phi \times A_R \times A_\phi$

$= \left\| \begin{array}{cc} \cos \phi & jZ_0 \sin \phi \\ \frac{j \sin \phi}{Z_0} & \cos \phi \end{array} \right\| \times \left\| \begin{array}{cc} 1 & R \\ 0 & 1 \end{array} \right\| \times \left\| \begin{array}{cc} \cos \phi & jZ_0 \sin \phi \\ \frac{j \sin \phi}{Z_0} & \cos \phi \end{array} \right\|$

$= \left\| \begin{array}{cc} \cos \phi & jZ_0 \sin \phi \\ \frac{j \sin \phi}{Z_0} & \cos \phi \end{array} \right\| \times \left\| \begin{array}{cc} \cos \phi & jZ_0 \sin \phi \\ + \frac{jR \sin \phi}{Z_0} & + R \cos \phi \end{array} \right\|$

$= \left\| \begin{array}{cc} \cos^2 \phi & j2Z_0 \sin \phi \cos \phi \\ + j \frac{R}{Z_0} \sin \phi \cos \phi - \sin^2 \phi & + R \cos^2 \phi \\ \frac{j2 \sin \phi \cos \phi}{Z_0} - \frac{R \sin^2 \phi}{Z_0^2} & -\sin^2 \phi + j \frac{R \cos \phi \sin \phi}{Z_0} + \cos^2 \phi \end{array} \right\| \dots \dots$ (I6)

by a short-circuited stub and an open-circuited stub located at the same point in the transmission line.

$S = jZ_0 \tan r\theta // - jZ_0 \cot (1-r)\theta$
 $= jZ_0 \frac{1 + \tan \theta \tan r\theta}{\cot r\theta + \tan r\theta} \dots \dots$ (I4)

In the case where $r = \frac{1}{2}$,

$S = jZ_0 \frac{\tan \theta/2}{1 - \tan^2 \theta/2} = j\frac{1}{2}Z_0 \tan \theta$

For this case

$Z = Z_0 \sqrt{\frac{1 + \tan \phi \cot \theta}{1 - Z_0 \cot \phi \cot \theta}} \dots \dots$ (I5)

Cut-off frequencies can be obtained as before by equating Z to zero and infinity. These relations have not been plotted.

6. Series-loaded Transmission Lines

Fig. 2 shows two arrangements in which reactances R , constituted by the input impedances of the stubs, are connected so as to be effectively in series with the section of transmission line shown running from left to right. In case (a) a shorted stub is used and in case (b) an open-circuited stub is used.

Evidently these sections may be broken up into three fourpoles, just as was done before, and the matrix of the whole filter section may be obtained by multiplying together the matrices of the component fourpoles, as follows:

(Note as a check that the determinant of this matrix is unity).

$\therefore \cosh P = \cos 2\phi + j \frac{R}{2Z_0} \sin 2\phi \dots \dots$ (I7)

$$Z = Z_0 \sqrt{\frac{1 - j \frac{R}{2Z_0} \cot \phi}{1 + j \frac{R}{2Z_0} \tan \phi}} \quad \dots \quad (18)$$

Conditions for cut-off are therefore :

$$2Z_0 = jR \cot \phi \quad \dots \quad (19a)$$

$$2Z_0 = -jR \tan \phi \quad \dots \quad (19b)$$

These equations are quite general, and apply whatever device is used for providing the series reactance R .

Conventional (Series) Coil-loaded Line

By making $R = jL\omega$ and $\phi = aD\omega/V$, equations (17) and (18) define respectively, as functions of frequency, the propagation constant per section and the mid-line image impedance of a series coil-loaded transmission line with inductances L spaced $2aD$ metres apart.

Equation (19a) defines the cut off frequencies.

Series-loading by Short-circuited Stubs

If the characteristic impedance of the stub is the same as that of the main line :

$$R = jZ_0 \tan \theta \quad \dots \quad (20)$$

so that from (17)

$$\cosh P = \cos 2\phi - \frac{1}{2} \tan \theta \sin 2\phi \quad \dots \quad (21)$$

$$\text{and } Z = Z_0 \sqrt{\frac{1 + \frac{1}{2} \cot \phi \tan \theta}{1 - \frac{1}{2} \tan \phi \tan \theta}} \quad \dots \quad (22)$$

Cut-off occurs when

$$\tan a\phi = 2 \cot \theta \text{ or infinity} \quad \dots \quad (23a)$$

$$\cot a\theta = -2 \cot \theta \text{ or infinity} \quad \dots \quad (23b)$$

In other words the pass and attenuating bands are located as in the case of the line leak-loaded with open-circuited stubs. Fig. 3 therefore represents this case also.

Series-loading by Open-circuited Stubs

If the characteristic impedance of the stub is the same as that of the main line :

$$R = -jZ_0 \cot \theta \quad \dots \quad (24)$$

so that $\cosh P = \cos 2\phi + \frac{1}{2} \cot \theta \sin 2\phi \quad \dots \quad (25)$

$$\text{and } Z = Z_0 \sqrt{\frac{1 - \frac{1}{2} \cot \phi \cot \theta}{1 + \frac{1}{2} \tan \phi \cot \theta}} \quad \dots \quad (26)$$

Cut-off occurs when

$$\tan a\theta = -2 \tan \theta \text{ or infinity} \quad \dots \quad (27)$$

$$\cot a\theta = 2 \tan \theta \text{ or infinity} \quad \dots \quad (28)$$

The pass and attenuating bands are, therefore, the same as those of a line leak-loaded with short-circuited stubs. Fig. 5 therefore shows the pass bands for this case also.

Two-path Transmission-line Filters (Fig. 8)

This class of filter may be solved with equal ease by means of matrices, or by means of opens and shorts using the equations :

$$\tanh P = \sqrt{Z_{sc}/Z_{oc}}$$

$$Z = \sqrt{Z_{sc}/Z_{oc}}$$

where P = the propagation constant of the network being examined

Z = the image impedance of the network

Z_{sc} = the input impedance with the output terminals shorted

Z_{oc} = the input impedance with the output terminals open

The expressions for Z obtained by opens and shorts are more amenable to substitution and rapid estimation than those obtained by matrices, while the expressions for P obtained by matrices are more useful than those obtained by opens and shorts.

Where necessary a brief summary of each method is given below.

Straight Parallel-Parallel Two-path Filter [Figs. 8(a), (d) and Fig. 9]

Solution by Opens and Shorts

By inspection of Figs. 8(a) or (d)

$$Z_{oc} = -j\frac{1}{2}Z_0 \cot \frac{1}{2}(1+a)\theta$$

$$Z_{sc} = \frac{jZ_0 \tan \theta \times jZ_0 \tan a\theta}{jZ_0 \tan \theta + jZ_0 \tan a\theta} = \frac{jZ_0}{\cot \theta + \cot a\theta}$$

Hence :

$$\tanh P = j \sqrt{\frac{2}{(\cot \theta + \cot a\theta) \cot \frac{1}{2}(1+a)\theta}} \quad (29)$$

$$\left(\cosh P = \frac{1}{\sqrt{1 - \tanh^2 P}} \right)$$

$$Z_1 = Z_2 = \frac{Z_0}{\sqrt{2(\cot \theta + \cot a\theta) \tan \frac{1}{2}(1+a)\theta}} \quad (30)$$

When $\frac{1}{2}(1+a)\theta = n\pi/2$ (n integral)

n odd : $\tan \frac{1}{2}(1+a)\theta = \text{infinity}$, $\cot \theta + \cot a\theta = 0$

Hence, Z_1 and Z_2 are finite.

n even : $\tan \frac{1}{2}(1+a)\theta = 0$, $\cot \theta + \cot a\theta = 0$.

Hence $Z_1 = Z_2 = \text{infinity}$.

Cut-off frequencies occur when :

(i) $\frac{1}{2}(1+a)\theta = n\pi/2$, and making the substitution $\theta = u/2$, this condition for cut-off becomes :

$$a = \frac{2n}{u} - 1 \quad (n \text{ integral and even}) \quad \dots \quad (31)$$

This cut-off frequency marks the boundary between contiguous or confluent attenuating bands. At this frequency $Z_1 = Z_2 = \text{infinity}$.

(ii) $\cot \theta$ or $\cot a\theta = \text{infinity}$; θ or $a\theta = n\pi/2$:
that is: $u = n$, or $a = n/u$ (n even).

Solution by Matrices

Evidently, each filter section consists of two fourpoles with their inputs and output respectively in parallel. The Y matrix of the filter section can therefore be obtained by adding together the Y matrices of the component fourpoles:

$$Y_F = \begin{vmatrix} \frac{\coth P}{Z} & -\frac{\operatorname{cosech} P}{Z} \\ \frac{\operatorname{cosech} P}{Z} & -\frac{\coth P}{Z} \end{vmatrix} = Y_\theta + Y_{a\theta} \dots \dots \dots (32)$$

$$= \begin{vmatrix} \frac{-j(\cot \theta + \cot a\theta)}{Z_0} & \frac{j(\operatorname{cosec} \theta + \operatorname{cosec} a\theta)}{Z_0} \\ \frac{-j \operatorname{cosec} \theta + \operatorname{cosec} a\theta}{Z_0} & \frac{j(\cot \theta + \cot a\theta)}{Z_0} \end{vmatrix} \dots \dots \dots (33)$$

Equating like terms in Y_F and $Y_\theta + Y_{a\theta}$:

$$\therefore \cosh P = \frac{\coth P}{\operatorname{cosech} P} = \frac{\cot \theta + \cot a\theta}{\operatorname{cosec} \theta + \operatorname{cosec} a\theta} \quad (29a)$$

Frequencies of infinite attenuation occur when the denominator is zero and the numerator finite; i.e., when

$$a\theta = \theta \pm n\pi$$

$$\text{or } a = 1 \pm \frac{2n}{u} \quad (n \text{ odd}) \dots \dots (29b)$$

$$Z = \frac{Z_0}{\sqrt{\operatorname{cosec} \theta + \operatorname{cosec} a\theta^2 - (\cot \theta + \cot a\theta)^2}} \dots \dots \dots (30a)$$

Straight Parallel-Parallel Constant-sum Two-path Filter (Fig. 10)

In this case the two paths of the filter are respectively of electrical length $b\phi$ and $(1-b)\phi$ and $\phi = \text{sum of electrical lengths of the two paths}$.

Solution by Opens and Shorts

By inspection $Z_{oc} = -j\frac{1}{2}Z_0 \cot \frac{1}{2}\phi$

$$Z_{sc} = \frac{jZ_0 \tan b\phi \times jZ_0 \tan(1-b)\phi}{jZ_0 \tan b\phi + jZ_0(1-b)\phi} = \frac{jZ_0}{\cot b\phi + \cot(1-b)\theta}$$

Hence: $\tanh P$

$$= j \sqrt{\frac{2}{[\cot b\phi + \cot(1-b)\phi] \cot \frac{1}{2}\phi}} \dots (34)$$

$$Z_1 = Z_2 = \frac{Z_0}{\sqrt{2[\cot b\phi + \cot(1-b)\phi] \tan \frac{1}{2}\phi}} \dots \dots (35)$$

When $\frac{1}{2}\theta = \frac{\pi}{n_2}$; i.e. $\phi = n\pi$ (n integral)

n odd: $\cot b\phi + \cot(1-b)\phi = 0$, $\tan \frac{1}{2}\phi = \text{infinity}$:

$$Z_1 = Z_2 = \text{infinity}$$

n even: $\cot b\phi + \cot(1-b)\phi = 0$, $\tan \frac{1}{2}\phi = 0$:

$$Z_1 = Z_2 = \text{infinity}$$

Frequency is normalized in terms of $f_{360} = \text{the}$

frequency at which ϕ is $360^\circ = 2\pi$ radians.

Hence $\phi = 2\pi \times \frac{f}{f_{360}} = 2\pi w$ where $w = f/f_{360}$.

Cut-off frequencies occur when:

(i) $\phi = n\pi$, n integral and even; i.e., $w = n/2$
or

$w = n(n \text{ integral})$. At this frequency, which marks the boundary of confluent attenuating bands,

$$Z_1 = Z_2 = \text{infinity} \dots \dots (36)$$

(ii) $\cot b\phi$ or $\cot(1-b)\phi = \text{infinity}$; i.e., $b\phi$ or $(1-b)\phi = n\pi$

$$\therefore b = \frac{n\pi}{\phi} \text{ or } 1 - \frac{n\pi}{\phi} = \frac{n}{2w} \text{ or } 1 - \frac{n}{2w} \quad (n \text{ integral}) \dots (37)$$

Straight Series-Series Two-path Filter [Figs. 8(b) and 9]

Solution by Opens and Shorts

By inspection of Fig. 8(b)

$$Z_{oc} = -jZ_0(\cot \theta + \cot a\theta)$$

$$Z_{sc} = j2Z_0 \tan \frac{1}{2}(1-a)\theta$$

Hence: $\tanh P$

$$= j \sqrt{\frac{2}{(\cot \theta + \cot a\theta) \cot \frac{1}{2}(1+a)\theta}} \dots (38)$$

$$Z_1 = Z_2 = Z_0 \sqrt{2(\cot \theta \cot a\theta) \tan \frac{1}{2}(1+a)\theta}} \dots \dots (39)$$

$$(\cosh P = \sqrt{1 - \tanh^2 P})$$

The cut-off frequencies, pass bands, and attenuation and phase-shift characteristics of this

filter are the same as those of the straight parallel filter.

The image impedance is equal to $2Z_0^2$ divided by the image impedance of the straight parallel-parallel filter.

Solution by Matrices.

The Z matrix of this section can be obtained by adding together the Z matrices of the component transmission lines :

$$Z_F = \begin{vmatrix} Z \coth P & -Z \operatorname{cosech} P \\ Z \operatorname{cosech} P & -Z \coth P \end{vmatrix} = Z_0 + Z_{a\theta}$$

$$\begin{vmatrix} -jZ_0(\cot \theta + \cot a\theta) & jZ_0(\operatorname{cosec} \theta + \operatorname{cosec} a\theta) \\ -jZ_0(\operatorname{cosec} \theta + \operatorname{cosec} a\theta) & jZ_0(\cot \theta + \cot a\theta) \end{vmatrix}$$

Hence : $\cosh P = \frac{\cot \theta + \cot a\theta}{\operatorname{cosec} \theta + \operatorname{cosec} a\theta} \dots (38a)$

$$D_F = \begin{vmatrix} \sqrt{\frac{Z_1}{Z_2}} \operatorname{sech} P & Z_1 \tanh P \\ -\frac{1}{Z_2} \tanh P & \sqrt{\frac{Z_1}{Z_2}} \operatorname{sech} P \end{vmatrix} = D_\theta + D_{a\theta} \dots \dots \dots (44)$$

$$= \begin{vmatrix} \sec \theta + \sec a\theta & jZ_0(\tan \theta + \tan a\theta) \\ -\frac{j}{Z_0}(\tan \theta + \tan a\theta) & \sec \theta + \sec a\theta \end{vmatrix} \dots \dots \dots (45)$$

and $Z = \frac{Z_0 \sqrt{(\operatorname{cosec} \theta + \operatorname{cosec} a\theta)^2 - (\cot \theta + \cot a\theta)^2}}{\dots \dots \dots} (39a)$

Straight Series-Parallel Two-path Filter [Figs. 8(c) and 13]

Solution by Opens and Shorts

Looking into terminals 1, 1

By inspection of Fig. 8(c)

$Z_{oc} = -j2Z_0 \cot \frac{1}{2}(I + a)\theta$

$Z_{sc} = jZ_0(\tan \theta + \tan a\theta)$

Looking into terminals 2, 2

$Z_{oc} = \frac{-2Z_0}{\tan \theta + \tan a\theta}$

$Z_{sc} = jZ_0 \frac{1}{2} \tan \frac{1}{2}(I + a)\theta$

Hence :

$\tanh P = j\sqrt{\frac{1}{2}(\tan \theta + \tan a\theta) \tan \frac{1}{2}(I + a)\theta} \dots \dots \dots (40)$

$Z_1 = Z_0 \sqrt{2(\tan \theta + \tan a\theta) \cot \frac{1}{2}(I + a)\theta} \dots \dots \dots (41)$

$Z_2 = \frac{Z_0}{\sqrt{2(\tan \theta + \tan a\theta) \cot \frac{1}{2}(I + a)\theta}} \dots \dots \dots (42)$

When $\frac{1}{2}(I + a)\theta = n\pi/2$ (n integral)
 n odd : $\cot \frac{1}{2}(I + a)\theta = 0, \tan \theta + \tan a\theta, Z_1 = 0, Z_2 = \text{infinity}$
 n even : $\cot \frac{1}{2}(I + a)\theta = \text{infinity}, \tan \theta + \tan a\theta = 0, Z_1 = Z_2 = \text{finite}.$

Cut-off frequencies occur when :
 (i) $\frac{1}{2}(I + a)\theta = n\pi/2$; i.e., when $a = \frac{2n}{u} - I$ (n integral and odd) (43)

This cut-off frequency marks the boundary between contiguous or confluent attenuating bands. At this frequency $Z_1 = 0$ and $Z_2 = \text{infinity}$.
 (ii) $\tan \theta$ or $\tan a\theta = \text{infinity}$; i.e., when $u = n$, or $a = n/u$ (n integral and odd).

Solution by Matrices

Hence :

$$\sinh P = j \frac{\tan \theta + \tan a\theta}{\sec \theta + \sec a\theta}$$

$$\cosh^2 P = \frac{(\sec \theta + \sec a\theta)^2 - (\tan \theta + \tan a\theta)^2}{(\sec \theta + \sec a\theta)^2} (40a)$$

$$Z_1 = Z_0 \sqrt{(\sec \theta + \sec a\theta)^2 - (\tan \theta + \tan a\theta)^2} (41a)$$

$$Z_2 = \frac{Z_0}{\sqrt{(\sec \theta + \sec a\theta)^2 - (\tan \theta + \tan a\theta)^2}} (42a)$$

Crossed Parallel-Parallel Two-path Filter [Figs. 8(a) and 9]

Solution by Open and Shorts

By inspection :

$Z_{oc} = j\frac{1}{2}Z_0 \tan \frac{1}{2}(I + a)\theta ; Z_{sc} = \frac{jZ_0}{\cot \theta + \cot a\theta}$

Hence :

$\tanh P = \sqrt{\frac{2}{(\cot \theta + \cot a\theta) \tan \frac{1}{2}(I + a)\theta}} (46)$

$Z_1 = Z_2 = \sqrt{2(\cot \theta + \cot a\theta) \cot \frac{1}{2}(I + a)\theta} (47)$

When $\frac{1}{2}(I + a)\theta = n\pi/2$ (n integral)
 n odd : $\cot \frac{1}{2}(I + a)\theta = 0, \cot \theta + \cot a\theta = 0, Z_1 = Z_2 = \text{infinity}$

n even : $\cot \frac{1}{2}(\mathbf{I} + a)\theta = \text{infinity}$, $\cot \theta + \cot a\theta = 0$.

Cut-off frequencies occur when :

(i) $\frac{1}{2}(\mathbf{I} + a)\theta = n\pi/2$; i.e., when $a = \frac{2n}{u} - \mathbf{I}$ (n odd) (48)

This cut-off frequency marks the boundary of contiguous or confluent attenuating bands. At this frequency $Z_1 = Z_2 = \text{infinity}$.

(ii) $\cot \theta$ or $\cot a\theta = \text{infinity}$:

so that θ or $a\theta = \frac{n\pi}{2}$; i.e., $u = n$ or $a = n/u$ (n even) (49)

Crossed Series-Series Two-path Filter [Figs. 8(b) and 9)].

Solution by Opens and Shorts

By inspection :

$Z_{oc} = -jZ_0(\cot \theta + \cot a\theta)$
 $Z_{sc} = -j2Z_0 \cot \frac{1}{2}(\mathbf{I} + a)\theta$

Hence :

$Z_1 = Z_2 = \frac{-jZ_0 \sqrt{2(\cot \theta + \cot a\theta) \cot \frac{1}{2}(\mathbf{I} + a)\theta}}{2}$ (50)

$\tanh P = \frac{2}{\sqrt{(\cot \theta + \cot a\theta) \tan \frac{1}{2}(\mathbf{I} + a)\theta}}$ (51)

Cut-off frequencies, pass bands, and attenuation and phase-shift characteristics, are the same as in the case of the crossed parallel-parallel filter.

Crossed Series-Parallel Two-path Filter [Figs. 8(c) and 12].

Solution by Opens and Shorts

By inspection of Fig. 8(c)

Looking into Terminals 1,1

$Z_{oc}^{1,1} = j2Z_0 \tan \frac{1}{2}(\mathbf{I} + a)\theta$
 $Z_{sc} = jZ_0(\tan \theta + \tan a\theta)$

Looking into Terminals 2,2

$Z_{oc} = \frac{-jZ_0}{\tan \theta + \tan a\theta}$
 $Z_{sc} = jZ_0 \frac{1}{2} \cot \frac{1}{2}(\mathbf{I} + a)\theta$

Hence :

$\tanh P = \frac{\sqrt{\frac{1}{2}(\tan \theta + \tan a\theta) \cot \frac{1}{2}(\mathbf{I} + a)\theta}}{\sqrt{2(\tan \theta + \tan a\theta) \tan \frac{1}{2}(\mathbf{I} + a)\theta}}$ (52)
 $Z_1 = -jZ_0 \sqrt{2(\tan \theta + \tan a\theta) \tan \frac{1}{2}(\mathbf{I} + a)\theta}$ (53)

$Z_2 = \frac{jZ_0}{\sqrt{2(\tan \theta + \tan a\theta) \tan \frac{1}{2}(\mathbf{I} + a)\theta}}$ (54)

When $\frac{1}{2}(\mathbf{I} + a)\theta = n\pi/2$ (n integral)

n odd : $\tan \frac{1}{2}(\mathbf{I} + a)\theta = \text{infinity}$, $\tan \theta + \tan a\theta = 0$, Z_1 and Z_2 are finite

n even : $\tan \frac{1}{2}(\mathbf{I} + a)\theta = 0$, $\tan \theta + \tan a\theta = 0$, $Z_1 = 0$, $Z_2 = \text{infinity}$.

Cut-off frequencies occur when :

(i) $\frac{1}{2}(\mathbf{I} + a)\theta = n\pi/2$, (n even); i.e., when $a = \frac{2n}{u} - \mathbf{I}$ (55)

At this frequency which marks the boundary between confluent attenuating bands $Z_1 = 0$, $Z_2 = \text{infinity}$.

(ii) $\tan \theta$ or $\tan a\theta = \text{infinity}$: θ or $a\theta = \frac{n\pi}{2}$ (n odd): (56)
 $u = n$, or $a = n/u$.

7. Acknowledgment

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INTERELECTRODE CAPACITANCE OF VALVES

Change with Operating Conditions

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(Communication from the Research Staff of the M.O. Valve Company at the G.E.C. Research Laboratories, Wembley, England)

SUMMARY.—This paper presents measurements, made at a frequency of 1 Mc/s, of the grid-to-cathode and the grid-to-anode capacitances of triode valves. These valves are all of a common basic electrode structure in which the active elements are essentially plane.

The measurements do not quantitatively confirm the theoretically predicted changes and indicate that the geometry of the grid has a marked influence on the change of the grid-to-cathode capacitance.

1. Introduction

THE interelectrode capacitances of valves are an important factor in the design of wide-band radio-frequency amplifiers in which the main tuning capacitances are those of the valves. The capacitances which must be considered are those which obtain under the working conditions of the valve and therefore a knowledge of the change of interelectrode capacitance with the working conditions is important. In the consideration of the frequency stability of valve oscillators, a knowledge of the capacitance change is again an important factor.

The accepted theory^{1, 2, 3} of the internal action of thermionic valves predicts that the grid-to-cathode capacitance of a triode, operating under conditions of space-charge limitation, will be four-thirds of the capacitance existing in the absence of electrons, the increase being due to the electron-transit time.

Results of measurements of the change of interelectrode capacitances of valves with the d.c. operating conditions were given in a paper⁴ by T. I. Jones. His measurements were made on a number of different types of valve and, in general terms, the valves all behaved similarly. However the measurements showed that the change of capacitance was of a magnitude greater than that predicted by theory. Most of the valves used in these measurements had directly-heated cathodes and had electrode structures which were not geometrically simple.

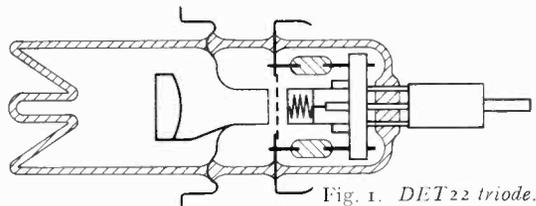
The valves used in this present work were based on either the DET22 or the E1714 valve types, which have similar electrode structures but have different external constructions. The essential features of the valves whose detailed descriptions are given in the following section, are the plane circular active elements and the indirectly-

heated cathode. The geometry of the valve is simple and both the anode and cathode form equipotential surfaces. The valves should therefore be more amenable to a mathematical treatment than other types.

2. Description of Valves

2.1 DET22 (CV273).

The DET22 is a disc-seal triode and is described elsewhere⁵. In this valve the grid is constructed from a number of molybdenum wires brazed to a molybdenum frame. This frame is soldered to a copper disc which is sealed through the glass envelope of the valve. The anode is made integral with a second copper disc, while the cathode is connected to a tube sealed through the glass envelope at the end remote from the anode. A diagram of the valve is given in Fig. 1 and the essentials of the geometry of the electrode structure are shown in Fig. 2. The active elements are all circular and are coaxial.



The diameter of the cathode is 4.2 mm, that of the grid 5.0 mm and that of the anode 4.5 mm. A number of valves were specially constructed for this work and in these valves the anode-to-grid distance is 0.26 mm whilst the grid-to-cathode distance ranges between 0.11 mm and 0.31 mm with the cathode cold. Both distances are measured to a plane through the centres of the grid wires (Fig. 2). The method of measurement of the grid-to-cathode distance has been previously described⁵ and the results are given in the Table.

MS accepted by the Editor, July 1947

The grid wire diameter is 0.03 mm and the spacing between the centres of the wires is 0.127 mm except in one specimen, in which the spacing is 0.106 mm. The cathode coating is a barium—strontium oxide mixture approximately 0.06 mm thick and the working temperature is 780°C.

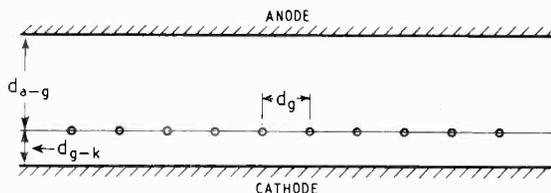


Fig. 2. Electrode system of DET22 triode; d_{a-g} = grid-to-anode distance; d_{g-k} = grid-to-cathode distance; d_g = grid wire spacing.

2.2 E1714.

The E1714 is an experimental triode valve whose electrode structure is essentially that of the DET22. In this valve, however, the structure is supported in a mica frame and is mounted on a seven-pin pressed-glass base. The diameter of both anode and cathode is 4.2 mm and that of the grid 5.0 mm. The anode-to-grid distance is 0.26 mm and that of the grid-to-cathode is approximately 0.09 mm when the cathode is at its correct temperature. The grid structures of these valves were constructed with (a) grid wires of 0.03 mm diameter with grid-wire spacing of 0.106 mm and (b) 0.018 mm diameter with a spacing of 0.075 mm. The cathode itself is the same as that of the DET22 and Fig. 2 again shows the geometry of the electrode structure.

3. Method of Measurement

The measurements of the interelectrode capacitances were made on a radio-frequency bridge⁶ which measures the capacitive and conductive components directly and operates at a frequency of 1 Mc/s. This bridge is very suitable for the measurement of the partial capacitances of valves as the balance of the bridge is independent of the impedance to earth from either of the two bridge terminals. Thus the capacitance between any two electrodes can be measured directly by earthing all the other valve elements. The discrimination of the bridge was 0.01 $\mu\mu\text{F}$, and the radio-frequency signal input, which at balance appears between the two bridge terminals and therefore between the valve electrodes, was a few millivolts.

Minor modifications of the bridge circuit were necessary so that direct voltages could be applied to the valve without affecting the conditions of balance. They were applied through low-resistance chokes so that the impedance to earth should not adversely affect the discrimination of the bridge. The circuit arrangement for supplying the direct voltages to the valve is shown in Fig. 3 (a) and the essentials of the bridge circuit are given in Fig. 3(b). The connections between the valve and the bridge are also indicated and it should be pointed out that the input-signal voltage to the bridge must appear between the cathode and the anode and not between the grid and the anode for the measurement of the grid-to-cathode capacitance. Under the latter condition, the bridge would measure the mutual conductance in parallel with the grid-to-cathode capacitance and conductance

TABLE

Valve	Grid wire		Grid-to-cathode distance		Grid-to-cathode capacitance at cut off ($\mu\mu\text{F}$)	Grid bias negative at		Mutual conductance		Percentage increase of capacitance from cut off to	
	Diam	Spacing	Cold	Working (estimated)		250 V 10 mA	150 V 10 mA	250 V 10 mA	150 V 10 mA	250 V 10 mA	150 V 10 mA
	(mm)	(mm)	(mm)	(mm)		(V)	(V)	(mA/V)	(mA/V)		
A	0.03	0.127	0.11	0.07	1.62	7.5	—	4.2	—	12	—
B	0.03	0.127	0.14	0.08	1.40	4.7	1.5	5.5	6.9	29	46
C	0.03	0.127	0.14	0.09	1.34	3.5	—	5.25	—	41	—
D	0.03	0.127	0.18	0.13	0.86	4.2	—	5.0	—	67	—
E	0.03	0.127	0.25	0.20	0.54	0.9	—	2.4	—	67	—
F	0.03	0.127	0.31	0.24	0.44	1.0*	—	1.9*	—	68*	—
G	0.03	0.106	0.15	0.10	1.26	2.8	—	6.7	—	63	—
H	0.03	0.106	—	—	1.00	3.4	1.5	6.8	7.1	86	116
I	0.018	0.075	—	—	1.20	3.8	1.8	7.8	8.2	63	75

* Measured at 250 V, 7 mA.

and no balance position could be found as the conductance range of the bridge is much less than would then be required. Similarly in the measurement of the anode-to-grid capacitance the input signal must appear between the anode and the cathode and not between the grid and the cathode.

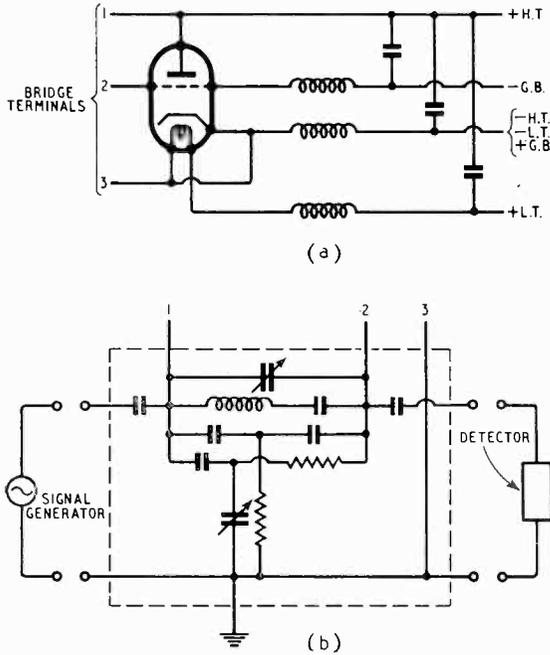


Fig. 3. The method of applying the direct voltages to the valve is shown at (a) and the bridge circuit at (b).

A jig, suited to the type of valve under test, was included as part of the bridge circuit so that the stray capacitance remained constant. The capacitance was measured by balancing the bridge with the jig in position, inserting the valve into the jig and then rebalancing the bridge. A series of measurements of a particular valve was made after a single initial setting of the bridge, as the electrode voltages could be altered without upsetting the initial balance or removing the valve from its jig.

The jig used for the EI714 type of valve was a valveholder mounted rigidly with respect to the bridge chassis and that for the DET22 was similar to the open end of a double coaxial-line circuit. In this latter case the depth of insertion of the valve into the jig affected the stray capacitance between the electrodes, and precautions were taken to ensure that for each particular valve this remained constant.

The true capacitance between the active parts of the grid-cathode structure was determined by a direct measurement of the stray capacitance.

This was done for each of the DET22 types of valve by breaking the glass between the anode and grid seals, removing the grid wires and re-measuring the grid-to-cathode capacitance with the anode again in position. The presence of the earthed anode makes the electric field in the grid-cathode space almost the same as that in the presence of the grid wires, so that the capacitance now measured will give the stray capacitance between grid and cathode. In the case of the EI714 type of valve, a valve was made up without grid wires and the capacitance so obtained was taken as that of the stray. This value was confirmed by the direct measurement of one valve of this type which was measured in a similar manner to that used for the DET22 types.

4. Results of Measurements

4.1 Grid-to-Cathode Capacitance.

The results of measurements of the grid-to-cathode capacitance are shown in graphical form in Figs. 4-9 and the Table gives other relevant data which may assist in the interpretation of the results. Discussion of the results is given in Section 5.

4.2 DET22.

Measurements were made on six valves of this type which had grids with a spacing of 0.127 mm between wires and on one valve with 0.106-mm spacing. The grid-to-cathode distances of these valves with the cathode cold, ranged between 0.11 mm and 0.31 mm and between 0.07 mm and 0.24 mm when hot. The change of the grid-to-cathode capacitance with the anode current for a constant anode voltage of 250 volts is shown in

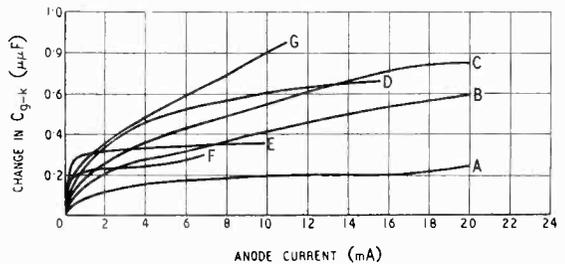


Fig. 4. Valve Type DET22, change of the grid-to-cathode capacitance with anode current ($V_a = 250$ volts).

Fig. 4 and the percentage change with anode current and with the mutual conductance is shown in Figs. 5 and 6. The change with anode current for various anode voltages is shown in Fig. 7 for a particular valve in which the grid-wire spacing was 0.127 mm and the working grid-to-cathode distance was 0.08 mm.

4.3. E1714.

Measurements were made on two valves of this type, one of which had a grid-wire spacing of 0.106 mm and the other a spacing of 0.075 mm.

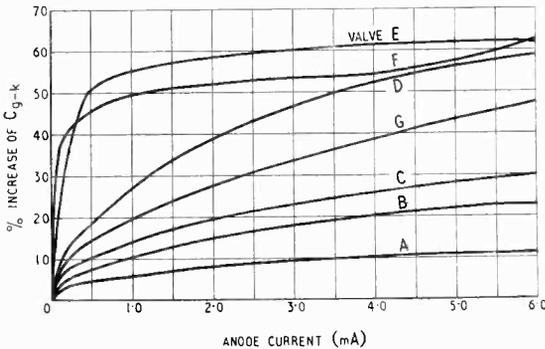


Fig. 5. Valve Type DET22, percentage increase of the grid-to-cathode capacitance with anode current ($V_a = 250$ volts).

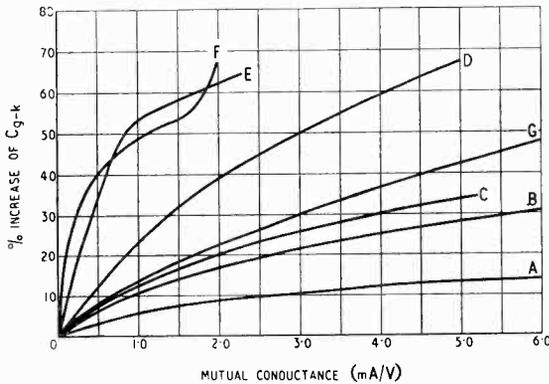


Fig. 6. Valve Type DET22, percentage change of the grid-to-cathode capacitance with mutual conductance ($V_a = 250$ volts).

The grid-wire diameter was 0.03 mm for the former valve and 0.018 mm for the latter. The percentage increase of capacitance with anode current for a constant anode voltage is shown in Fig. 8 for these valves together with two valves of the DET22 type one of which has a grid-wire spacing of 0.127 mm and the other 0.106 mm spacing. The change with anode current for various anode voltages for the two E1714 valves is shown in Fig. 9.

4.4. Anode-to-Grid Capacitance.

Measurements of this capacitance were made on several valves of both types and these showed that there was a small decrease with increase of anode current. However, the magnitude of the change, even at the highest currents, was only of the order of 0.01 $\mu\mu\text{F}$ which is about 3% of

the capacitance between the active parts of the grid-anode structure. The change was independent of the anode voltage within the sensitivity of the measurements.

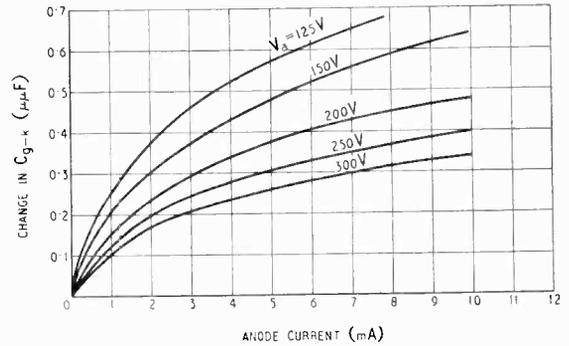


Fig. 7. Valve Type DET22, valve B; change of grid-to-cathode capacitance with anode current for various values of anode voltage.

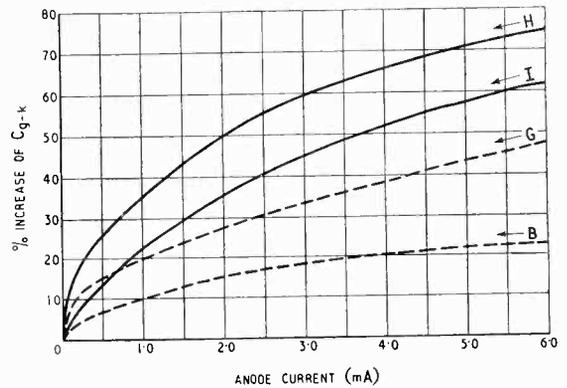


Fig. 8. Valve Type E1714, percentage change of the grid-to-cathode capacitance with anode current ($V_a = 250$ volts).

5. Discussion of Results

An examination of the results of the measurements shows, first, that except for valve A, in which the grid-to-cathode distance has the smallest value of all, the increase of the grid-to-cathode capacitance is much greater than that predicted theoretically. It shows, secondly, that the manner in which this change takes place is greatly influenced by the grid-cathode structure.

The effect of the grid-to-cathode distance is shown in Figs. 4, 5, and 6 and it will be seen that the initial rate of change of the capacitance increases with increase of distance. However, the total percentage change seems to attain an almost constant value in the valves with a grid wire spacing of 0.127 mm for large values of the

grid-to-cathode distance, while for smaller values the total percentage change again increases with increase of distance.

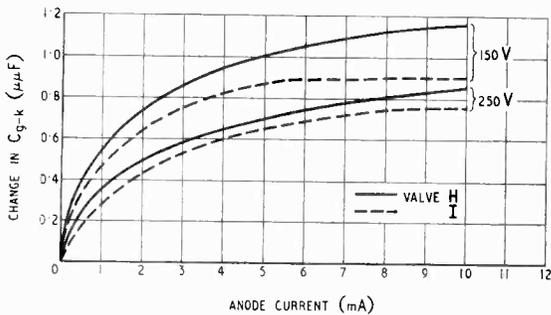


Fig. 9. Valve Type EI714, change of the grid-to-cathode capacitance with anode current for various values of anode voltage.

The reason for the small percentage change of capacitance in valves with small grid-to-cathode distance is probably two-fold. First, the cathode will have a screening effect on the grid, which will increase as the distance is reduced. Secondly, the smaller the grid-cathode distance the less uniform will be the field at the cathode surface due to the penetration of the anode field, so that for any particular anode current and anode voltage, the electrons will be confined into a smaller area between the grid wires. Due to both these causes the electrons, in their transit from cathode to anode, will have a smaller influence on the grid in the valves with small spacing than in those with large spacing.

Due to the same causes one would expect the percentage change in grid-cathode capacitance to increase if the spacing between grid wires is decreased and this is demonstrated by comparing valve G, which has a grid-wire spacing of 0.106 mm with valve C, which has a spacing of 0.127 mm. The change in the capacitance of valve G is about 50% greater than that of valve C.

A similar reasoning may also account for the decrease in capacitance with increase of anode voltage shown in Figs. 7 and 9 and for the smaller value of this decrease which arises in valve I, the grid-wire spacing of which is 0.075 mm compared with 0.106 mm in valve H and 0.127 mm in valve B.

All the measurements indicate that the difference between the 'working' capacitance and the 'cut-off' capacitance of a triode is greatly influenced by the geometry of the electrode structure. It would seem that the assumptions made in the existing transit-time theory of valves are inadequate and that a greater knowledge of the electron paths is required before a theory can

be formulated which will explain the practical results.

It is hoped that the presentation of these results will assist in the development of such a theory.

6. Acknowledgment

In conclusion, the authors desire to tender their acknowledgments to the M.O. Valve Co. Ltd., on whose behalf the work described in this publication was carried out.

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BOOKS RECEIVED

Techniques in Experimental Electronics

By C. H. BACHMAN. Pp. 252 + xvii, with 138 illustrations. Chapman & Hall Ltd., 37, Essex St., London, W.C.2. Price 21s.

Of American origin, this book deals in a rather elementary way with vacuum technique. Pumps, vacuum gauges, joints, glass-blowing and the assembly and processing of electronic devices are among the chapter headings.

B.B.C. Year Book 1949

B.B.C. Publications Dept., The Grammar School, Scarle Rd., Wembley, Middx. Price 3s. 6d.

Graphical Symbols for Telecommunications BS.530: 1948. Pp. 100. British Standards Institution, 24-28, Victoria St., London, S.W.1. Price 10s. 6d.

Radio Measurtests

This booklet is produced by Marconi Instruments Ltd., St. Albans, Herts, from whom it is obtainable. Requests for it must be made on business notepaper. It describes the uses and applications of test equipment manufactured by this firm, with particular reference to broadcast and television receiver maintenance.

Gas Welding and Cutting

By C. G. Bainbridge, M.I.Mech.E., M.Inst.W. Pp. 305 with 186 illustrations. The Louis Cassier Co. Ltd., Dorset House, Stamford St., London, S.E.1. Price 15s.

Radio Valve Practice

Pp. 24. The British Radio Valve Manufacturers' Association, Piccadilly House, 16, Jermyn St., London, S.W.1.

A Method of Determining the Polar Diagrams of Long Wire and Horizontal Rhombic Aerials. Radio Research Special Report No. 16. Pp. 39 + iv, with 32 illustrations. H.M. Stationery Office, London. Price 9d.

RADIATION FROM CAR IGNITION SYSTEMS

Measurements in the 40-650 Mc/s Band

By B. G. Pressey, Ph.D., M.Sc.(Eng.), A.M.I.E.E., and G. E. Ashwell, B.Sc.

(Communication from the National Physical Laboratory)

SUMMARY.—This paper describes measurements which have been made of the field strength of the radiation from a typical motor-car ignition system, a matter of importance in connection with interference to television and other radio services. A wide-band (2.5 Mc/s bandwidth) measuring set with a cathode-ray indicator unit was used and the field strength recorded was that corresponding to the peak of the output signal. The results showed that the general level of the field was maintained throughout the frequency range and that its value was approximately 10 mV/m at a distance of about 30ft (9.14 metres). Measurements of the suppression of the radiation by the insertion of block resistors and resistive leads were also made. The secondary pulses, which are associated with each nominal spark, were also examined under unsuppressed and suppressed conditions.

The second part of the paper describes measurements of the radiation from a basic system consisting of a single plug and loop of wire. The resonances which occur in the lead when block resistors are inserted were also investigated and it was shown that for certain positions of the resistor there was an increase in the observed field strength.

1. Introduction

ONE of the limitations on television is the interference caused by the radiation from the ignition systems of motor vehicles. Measurements of the field strength of the radiation from such systems at frequencies between 20 and 450 Mc/s have been made by several workers^{1, 2, 3}. In these measurements the receivers used had a relatively narrow bandwidth with an output measuring circuit of relatively long time constant. The present measurements were made with a wide-band receiver employing a cathode-ray output indicator and having an effective overall bandwidth of 2.5 Mc/s.

With both the above mentioned types of equipment the amplitude of the output signal is determined by the bandwidth and the detector time constant. For this reason the measured field strength which corresponds to the maximum amplitude of the output voltage, will be referred to as the effective peak field strength. In the case of the wide-band receiver the output voltage is limited only by the bandwidth.

The main purpose of the paper is to give the results of measurements on a typical motor car and to draw attention to the fact that the level of the radiation is maintained up to frequencies of the order of 600 Mc/s and does not fall off rapidly above the h.f. band, as is still sometimes believed in spite of the evidence published by R. W. George¹. Some measurements showing the effect of two standard methods of suppression are also included though no attempt at investigating the efficiency of such methods was made.

MS accepted by the Editor, November 1947

Since the measurements on the car indicated that the radiated field was probably modified by the presence of the engine and the body, it was thought desirable to make measurements on a basic ignition system consisting of a single plug and loop of wire. This system also provided an opportunity of investigating the lead resonances which occur when block suppressors are fitted.

The British system of units has been used in several instances in this paper. The conversion factor required to express the appropriate quantities in metric units is:—

$$1 \text{ ft} = 0.3048 \text{ metres.}$$

2. Measuring Equipment

The measurements were made with the National Physical Laboratory Pulse Field Strength Measuring Set⁴. This equipment was designed for the measurement of the peak field strength of pulse transmissions over the frequency range 20-30 and 40-650 Mc/s.

The equipment consists essentially of a tuned half-wave dipole aerial, a wide-band receiver in which are incorporated calibrated attenuators and a cathode-ray tube indicator unit. The intermediate frequency is 35 Mc/s and the bandwidth is 2.5 Mc/s. The measurement is made by adjusting the attenuators for a standard output amplitude and the field strength is obtained from the attenuator readings and the known calibration constant.

In using the equipment to measure the field strength of the radiation from a car ignition system certain precautions had to be taken on account of the continuous frequency spectrum

of the radiation and the extreme shortness of the pulse. The r.f. tuning circuit gives adequate discrimination against second-channel interference and it was found that there was negligible pick-up of a signal at the intermediate frequency except over the range 330-650 Mc/s where, owing to the use of a different type of circuit in the frequency changer for this range, the rejection of i.f. signals is relatively low. The most convenient way to eliminate this pick-up was to mount the aerial in a waveguide having a critical frequency between 35 and 330 Mc/s. The guide used was of square section with $3\frac{1}{2}$ -ft sides and had a length of 10ft. The aerial was mounted at a quarter of a wavelength from the closed end. This guide, which had a gain of about 4 db with respect to a simple dipole, gave the requisite rejection of i.f. signal and had the additional advantage of being sufficiently directive to prevent rays reflected from nearby objects from causing interference.

The r.f. circuits have a greater bandwidth than the i.f. amplifier and, therefore, since the pulse width was narrower than that for which the equipment was designed (i.e., less than 0.5 microsecond) there was a possibility of the frequency changer being overloaded even when the i.f. amplifier was operating normally. However, check measurements showed that the largest signals measured were 20 db below this overload value.

3. Measurements on Car Systems

3.1 The Test System.

The cars used for the measurements were standard 1940 saloon models of the Vauxhall 12 and Vauxhall 14 with four- and six-cylinder engines respectively.

In both models the distributor unit was mounted on the off-side of the engine block and the plug leads were consequently very short, the longest being about 12 inches long. The coil was mounted on the engine block in one model and on the scuttle in the other. The cars had all-metal welded bodies.

Two sets of suppression equipment were supplied. One consisted of leads with built-in block resistors of 15,000-ohms resistance at the plug end and of 5,000-ohms resistance at the distributor end of the plug lead. There was also a 5,000-ohm resistor at the distributor end of the coil lead. The other equipment consisted of resistive leads which had a core of graphited silk of approximately 5,000 ohms per foot resistance. The plug lead resistances were between 2,700 ohms and 5,000 ohms and the coil lead resistance was 1,600 ohms.

Suppressor capacitors were permanently attached

to the l.t. side of the coil and to the dynamo so that the radiation was confined substantially to that from the plug leads.

Reference to previously published⁵ measurements of the interference from a large number of motor vehicles of various types and makes show that that from a Vauxhall car is of about average value so that there is no reason to believe that the measurements described in this paper are not typical of motor vehicles in general.

3.2 Method of Measurement

The measurements were made at a distance of 30ft from the centre of the bonnet of the car on the off-side; i.e. the same side as the ignition system. An aerial height of 10ft was used for frequencies up to 330 Mc/s and a height of 6ft for the frequencies above. The change in aerial height was made in order to avoid a node in the interference pattern for horizontally-polarized waves which occurs at a height of about 10ft for a frequency of 600 Mc/s.

For the purposes of the measurements the cars were run under no load conditions at an engine speed of about 1,500 r.p.m. Before making the measurements the engines were warmed up for some time to enable them to reach a steady working state, thus eliminating as far as possible variations due to changes of speed and of sparking plug temperature.

3.3 Measurements

Considerable difficulty was encountered in estimating the peak amplitude as the individual peaks varied over the firing cycle by as much as 10 db at some frequencies, with occasional peaks

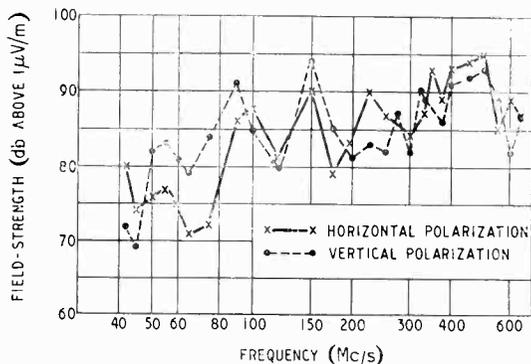


Fig. 1. Field-strength of radiation from 6-cylinder car at distance of 30ft.

up to 5 db above the general maximum level occurring about once a second. A change of frequency of a few megacycles per second was often sufficient to cause a complete change in the relative amplitudes of consecutive peaks. In the measurements it was the maximum of the

regularly occurring peaks which was observed, those peaks which occurred less than twice a second (i.e., at a frequency considerably below that corresponding to the firing cycle) being ignored.

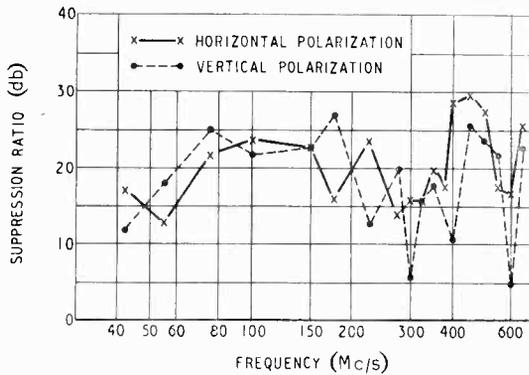


Fig. 2. Suppression ratio for block resistors on 6-cylinder car.

Measurements were made over the frequency range 45 to 650 Mc/s as described above for both horizontal and vertical polarization on both the four- and six-cylinder cars. The results for the six-cylinder car are shown in Fig. 1. Very similar curves were obtained for the four-cylinder car. It will be noticed that although the curves show some variations the general level has a steady rise of 15 db from the lower end of the frequency range to the upper end. Also, the field is approximately the same for horizontal as for vertical polarization.

The field strength was also measured at four selected frequencies in the range 55 to 500 Mc/s at three other positions around the car at the same distance from the bonnet and the results showed random variations up to 10 db with both position and polarization. There was no definite position for maximum field.

The field strength both with block resistor and with resistive-lead suppression was also measured on the six-cylinder car over the frequency range 40 to 650 Mc/s and the suppression ratios are plotted in Figs. 2 and 3. The results show that the suppression ratio is of the order of 20 db in both cases although the suppressors differed both in nature and in total resistance. The curves show some definite maximum and minimum values at particular frequencies but these cannot be regarded as having any real significance for, although they may be due to resonant effects in the plug leads or other parts of the ignition system, they could be accounted for simply by the uncertainty in the determination of the fluctuating field strength.

3.4 Multiple Sparks

All the above measurements were made by observing the maximum peaks, pulses of appreciably lower amplitude being ignored. In a recent paper⁶, Eaglesfield reported that each nominal spark gave rise to the radiation of a train of pulses lasting for as long as half a millisecond. However, as he did not make any measurements of the relative amplitudes of the pulses, it was decided to take the opportunity of examining them on the field-strength measuring equipment.

Measurements were made at frequencies of 50 and 200 Mc/s on a number of unsuppressed cars of different types, and they showed that the radiation consists of a primary pulse followed by a train of secondary pulses occurring up to 1.5 milliseconds later. The train consists of up to 100 or more pulses having a duration of up to 1.5 milliseconds and a mean amplitude of the order of 20 db below that of the primary pulse. Thus, in general, they will be observed only from cars which are sufficiently close to cause excessive interference.

Measurements on the cars when suppressed with a 15,000-ohm block resistor at each plug showed that whereas the amplitude of the primary pulse was reduced by between 10 and 20 db the train of secondary pulses was reduced by between 20 and 40 db and that there was a reduction in the duration of the train to 30 microseconds or less.

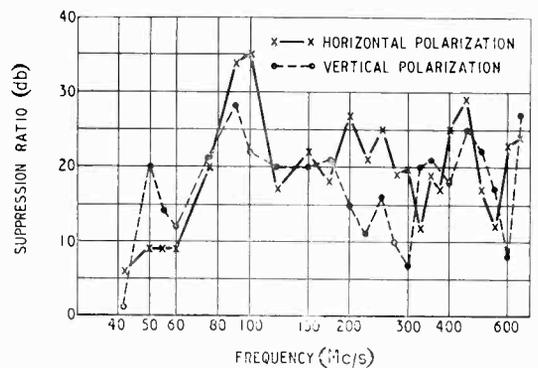


Fig. 3. Suppression ratio given by resistive leads on 6-cylinder car.

In zones of severe interference these trains of secondary pulses show on a television screen as lines of bright dots which are liable to cause greater annoyance than the primary pulses which cause single bright spots. Thus suppressors are likely to give greater reduction of the annoyance factor than is indicated by the measured reduction in amplitude of the primary pulse.

end reduces the damping effect of the magneto upon the remainder of the circuit.

The efficiency of a resistive lead as a suppressor was also measured. A lead 80-cm long and of 10,000-ohms resistance was substituted for the wire loop and the field strength measured. The suppression ratio is given in Fig. 6 and it is to be noted that the ratio is almost constant except for a small decrease at 75 Mc/s.

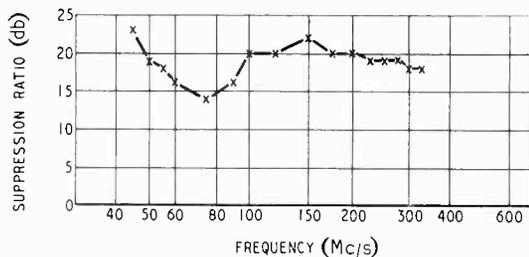


Fig. 6. Suppression ratio given by resistive leads on basic system. Length of loop, 80 cm; resistance of lead 10,000 ohms.

4.4 Current Magnitudes

With a simple radiator like the basic system it is possible to deduce approximately the current in the loop corresponding to the radiated field and so obtain the equivalent current distribution over the frequency spectrum.

The free-space field at a distance d from a single turn loop of area A and carrying a uniform current I is given by the equation

$$E_0 = \frac{120 \pi^2 A I}{\lambda^2 d}$$

Using this relation the loop current equivalent to the free-space field was calculated and its distribution with frequency is shown in Fig. 7.

In the derivation of the equation for E_0 it is assumed that the distribution of current around the loop is uniform. In the basic system this was approximately so for frequencies up to 200 Mc/s but at the higher frequencies it was not, since the periphery of the loop was large compared with the wavelength. Thus the curve of Fig. 7 must be regarded as very approximate in the 200-650 Mc/s region, but it does serve, however, to emphasise the importance of relatively very small currents at the higher frequencies in the production of the radiation.

5. Conclusions

The measurements of the radio-frequency radiation from a motor car and a basic ignition system over the frequency range 40 to 650 Mc/s lead to the following main conclusions:—

(a) The general level of the interference field from typical four- and six-cylinder cars shows no

falling off with increase of frequency even at 650 Mc/s but, on the contrary, has a tendency to rise. The maximum variation from the general level is 9 db. The measurements on the basic system showed that this interference spectrum is not substantially modified by the presence of the engine and body.

(b) At a distance of 30ft from the engine of the car and at a height of 6 to 10ft from the ground the effective peak field strength as measured on a wide-band receiver (2.5 Mc/s bandwidth) is of the order of 10 mV/m for both horizontally- and vertically-polarized components. The effective peak field strength radiated by the basic system is steady to within ± 7 db over the whole frequency range, and at a distance of 60ft has an average free-space value of about 20 mV/m.

(c) The variation in the amplitude of the individual pulses from the cars was normally about 10 db but the variation in the amplitude of those from the single-plug basic system was only 3 db.

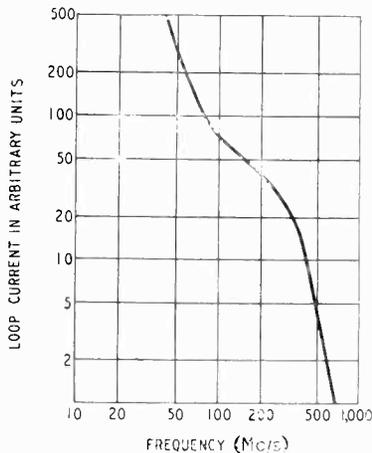


Fig. 7. Equivalent loop current of basic system.

(d) The average suppression ratio obtained with block resistors of about 20,000 ohms and with resistive leads of about 5,000 ohms on the cars is of the order of 20 db in both cases. There are considerable variations of the ratio with frequency, giving values as high as 35 db and as low as 4 db. Measurements on the basic system have shown that abnormally low ratios can be obtained at certain frequencies which are determined by the position and number of the block resistors and the length of the leads, and correspond very closely to the resonant frequencies of the circuit. Although the variations of the ratio for the car may be accounted for by such resonances they may also be due to uncertainties in the determination of the fluctuating field strength.

(e) The basic-system measurements have also shown that it is possible to obtain a negative suppression ratio of as much as 10 db for certain positions of the block resistors in the plug leads.

(f) Since the bandwidth of the measuring equipment is comparable with that of a television receiver, the measurements indicate the order of the interference which will be experienced on a television system. Moreover, the examination of the trains of secondary pulses, which are associated with each nominal spark, has emphasized their importance under conditions of severe interference, but has shown that suppressors are more effective in the reduction of these trains than in the reduction of the primary pulse.

6. Acknowledgments

The work described in this paper was carried out as part of the programme of the Radio Research Board at the suggestion of the Radio Interference Sub-Committee of the British Electrical and Allied Industries Research Association. The paper is published by permission of the Department of Scientific and Industrial Research.

The authors are indebted to Vauxhall Motors Ltd. for supplying the two cars on which the measurements were made and to Mr. W. Nethercot of E.R.A. for his advice and assistance in the organization of these measurements.

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CORRESPONDENCE

"Principles of Radar"

SIR,—In a recent review* of our book "Principles of Radar" (Cambridge University Press), M.G.S. has referred to the fact that the examples in this book are drawn largely from metre-wave systems, and systems which, according to the reviewer, had little or no future even in 1945.

There are good security reasons, as he will be aware, why it is not possible to give details of the latest radar systems, and for this reason we may have tended to draw our examples from older systems. However, we are a little perturbed by the possibility that more may be read into his remarks than was intended. Although the development of microwave radar was the most important single radar development of the war, there are still many uses for longer-wave systems and in several cases they are

definitely the preferred solution. We feel that the large amount of recent literature on microwave technique should not be allowed to blind your readers to this fact.

One of the simplest cases is the omni-directional beacon, whose optimum vertical radiation pattern is fixed by required range and aircraft ceiling. The gain of the ground aerial is therefore fixed, and that of the aircraft aerial has to be low, to allow for banking. Only for wavelengths above, say, 1m does the physical size of the aerial required limit the gain. Now the maximum range is given by†

$$r_m^2 = \frac{\lambda^2}{16\pi^2} \cdot \frac{P_s G_1 G_2}{P_m}$$

so that if transmitter power and receiver sensitivity were constant, the range would be directly proportional to the wavelength. In fact, attainable receiver sensitivities at microwave frequencies are lower than for longer waves, and the advantage of the latter is even greater than the formula suggests. When range only is desired, metre wavelengths are certainly recommended.

For a ground station designed to detect all aircraft within a given range, the optimum vertical polar diagram of the aerial is again fixed, but the horizontal directivity may be fixed by the maximum array width permissible. In this case $G \propto 1/\lambda$. If we assume common aerial working the formula for range is,

$$r_m^4 = \frac{P_s G^2 \lambda^2 A}{64\pi^3 P_m}$$

so that in this case the G and λ factors cancel, and the factor P_s/P_m remains. The receiver sensitivity is so much worse in the microwave band that a very high-power transmitter will be needed to give the same range as is obtainable with longer waves. We believe, therefore, that an optimum frequency would be near the limit of operation of grounded-grid triode amplifiers, say 30-50 cm, although much will depend on the effort put into designing high-power transmitters. The vertical dimensions of an aerial to give the required radiation pattern for $\lambda = 30$ cm is not unreasonable.

This analysis overlooks possibilities of not looking at the whole of space with one aerial. It is perhaps significant that most microwave early warning radars use more than one aerial so as to be able to obtain adequate range and at the same time cover the whole range of elevation. The later 200-Mc/s G.C.I. stations in this country had a switching system which performed the same function. Nevertheless, we would prefer a straightforward solution using a single radar channel for the search radar. There are of course other factors in choice of wavelength, such as vulnerability to jamming, anomalous propagation effects and the reflection coefficient of the earth or sea, but we feel it would be a profound mistake to assume that wavelengths above 10 cm are a dead letter.

An interesting example is the detection of the German V2 rocket. With its streamlined symmetrical form, this object is a highly-directive reflector of microwaves, and is very difficult to detect with them unless its aspect is almost exactly broadside. In fact, in the war it was the 12-m CH stations which gave by far the greatest amount of information about these rockets, simply because with this wavelength they could be detected over a wide range of aspect angles. He would be a bold man who would say that the detection of rocket projectiles would form no part of any future war.

DENIS TAYLOR
C. H. WESTCOTT

* *Wireless Engineer*, August 1948.

Wireless World, September 1948, p. 331.

† "Principles of Radar," whose notation we use throughout.

NEW BOOKS

Principles of Microwave Circuits

Edited by C. G. MONTGOMERY, R. H. DICHE and E. M. PURCELL. Pp. 486 + xvi, with 314 illustrations. (Vol. 8, M.I.T. Radiation Laboratory Series.) Price 36s. (in U.K.) and **Microwave Transmission Circuits**. Edited by GEORGE L. RAGAN. Pp. 725 + xviii, with 624 illustrations. (Vol. 9, M.I.T. Radiation Laboratory Series.) Price 51s. (in U.K.). McGraw-Hill Publishing Co. Ltd., Aldwych House, London, W.C.2.

As we should expect from our knowledge of the earlier volumes of this series on Radar, these books give a most comprehensive treatment of their subject. The two books, together with Vol. 10, "Waveguide Handbook," cover the subjects of waveguides and microwave coaxial-transmission lines and various junction arrangements and switching systems. It is perhaps a criticism of the series that three volumes are required to cover this field, since on a number of occasions material is duplicated, but the presentation is generally good and very thorough.

The first three chapters of Vol. 8 form an excellent introduction to the subject, dealing with the field equations of waveguides in a very general manner. Later chapters deal with waveguide elements (irises, etc.) and cavities, and it is here that the overlap with Vol. 9 begins. A feature of this volume, which is perhaps related to the use of the word 'circuits' in the title, is the large amount of space given to the matrix algebra of the general poly-terminal network and equivalent network theory. Although this approach to high-frequency phenomena may appeal to many trained in low-frequency theory, the reviewer feels that it is much better to start afresh and to use the travelling-wave component approach at these high frequencies. In particular the statement in Table 3.1 (p. 78) that a $\lambda_g/2$ length of line is equivalent to either a T-network with all its elements open-circuits, or to a π -network with all its elements short-circuits, should not have been allowed to pass without comment. Such results serve to illustrate how far wrong one can go with the un-intelligent application of equivalent-circuit theory, and readers might have been warned accordingly.

Vol. 9, on the other hand, is largely devoted to the practical details of construction of waveguide sections, joins, bends, branches and corners. Microwave coaxial-line practice is also described, although readers should realise that this refers exclusively to microwave (i.e., generally to 10-cm band) practice. Thus the capacitance switch developed in this country for 200 and 600 Mc/s, and in America from a British design for 1,000 Mc/s, is not mentioned in the chapter on switching methods; this is understandable because only microwave practice is treated. Otherwise the treatment is comprehensive, and includes chapters on materials, rigid and flexible lines and guides, transition units, rotary couplings, tuners and switches. The last quarter of the book is devoted to microwave filters, including an extensive theoretical treatment. It is, perhaps, doubtful whether the amount of space taken is justified in view of the specialist nature of the subject.

Vol. 9 also opens with two introductory chapters dealing with general principles, and some data are given which clearly overlap the "Waveguide Handbook." Although Vol. 8 is entitled "Principles," the practical man will probably learn all he requires, including the nature of the various modes in the simpler forms of

waveguide, from Vol. 9. Indeed, this volume includes a much better treatment of the important practical matter of the use of the Smith chart, or transmission-line calculator, than does Vol. 8. It describes the use of this chart for broad-band designing in several places, and one's only criticism is that there is no collected treatment of the subject, which is left to appear from time to time as examples arise. It is a general defect of the detailed and erudite theoretical treatment given in Vol. 8 that it is almost entirely devoted to network-equivalents which, in fact, are only equivalents for a spot frequency, and do not touch on the bandwidth problem.

Nevertheless, these volumes form an indispensable part of an invaluable series of publications which should be in every technical library of any size, and to the specialist worker they will be essential. It is understandable that they should generally describe American practice rather than British, although British designs and reports are mentioned occasionally, usually when there is no corresponding American one. We should indeed be grateful to the M.I.T. staff who sat down at the end of the war and wrote up their work for posterity.

C. H. W.

Vacuum Tube Amplifiers

Edited by GEORGE E. VALLEY, JR. and HENRY WALLMAN. (Vol. 18, M.I.T. Radiation Laboratory Series.) Pp. 743 + xvii, with 390 illustrations. McGraw Hill Publishing Co. Ltd., Aldwych House, London, W.C.2. Price 60s. (in U.K.).

This book covers a very large number of dissimilar, yet related subjects, and it is a very welcome addition to the literature on amplifiers. Starting with 'Linear-Circuit Analysis and Transient Response,' the book goes on to discuss in great detail what are usually known as video amplifiers but which are here called pulse amplifiers. Both linear and large-signal amplifiers are dealt with as well as the related subjects of signal mixing and electronic switching.

A jump is made in the next chapters to band-pass r.f. amplifiers for application in i.f. amplification. Resonance-tuned and stagger-tuned single-circuit couplings are treated, as well as coupled pairs of circuits and the use of negative feedback with single-circuit couplings. As usual, the treatment is based on the steady-state response and is very thorough. However, in a subsequent chapter the response of a network to carrier-frequency pulses is examined and a useful feature is the inclusion of curves showing the pulse response of many practical circuits, both singly and in cascade.

Low-frequency amplifiers in which the gain is stabilized to about 0.1% by means of negative feedback are discussed in considerable detail. Much of the discussion naturally centres on the means necessary for the prevention of self-oscillation with the large amount of feedback used. The design of selective i.f. amplifiers is then treated, the methods described being an amplifier with various forms of selective circuit, including the Wien bridge, in the feedback circuit.

A chapter on d.c. amplifiers follows. This is a particularly good one and it includes useful information about those special characteristics of some common valves which are of special importance to d.c. amplification. Circuit arrangements for the avoidance of drift

due to heater-voltage variation are given, and the use of feedback is discussed.

There are two very long chapters on noise. One is highly theoretical and mathematical. The other deals with it from the practical angle and is concerned mainly with what must be done to secure the best noise figure. Another rather short chapter is devoted to methods of measuring noise figure.

The book covers a great deal of ground. It necessarily includes much material that is well known and is to be found in most good books on the subject. It does also include, however, quite a lot which the reviewer has not previously seen published, such as some of the pulse-response curves of band-pass amplifiers and some of the d.c. amplifier material.

On the whole the treatment is good and is essentially practical—nearly every chapter concludes with a number of examples, circuits with values being given.

The first chapter, which deals mainly with the Laplace transform, is the least satisfactory. Its inclusion is justified by the use of this method in circuit analysis in other parts of the book, but it could have been much more helpful than it is. Its great fault lies in being written too much in a technical mathematical style. This results in its being very heavy going for the engineer—who is engineer first and mathematician second. Probably to a mathematician it seems easy, but he would be likely to find the rest of the book as hard as the engineer does this chapter.

This chapter, however, is only one flaw in an otherwise excellent book.

W. T. C.

Vibration and Sound (Second Edition).

By PHILIP M. MORSE. Pp. 468 + xix. McGraw-Hill Publishing Co., Aldwych House, London, W.C.2. Price 33s. (in U.K.).

This book is intended primarily as a text book on the mathematical theory of vibration for students of physics and communications engineering. It is designed to provide on a first reading a systematic course on the fundamental parts of the subject. In addition a considerable amount of more advanced material is included to give the student a grasp of the modern theoretical techniques used in acoustics. The first edition, published in 1936, has gained a high reputation, and in this second edition the work has been extensively revised and enlarged to bring the subject up to date. An important new feature is the comprehensive treatment of transient phenomena by the methods of the operational calculus which the author has developed for acoustical systems.

A sound knowledge of the calculus is assumed. A short mathematical introduction is followed by a chapter on systems of one and two degrees of freedom, in which impedance methods are introduced and the theory is applied to electro-mechanical transducers. The basic ideas of the operational method are presented in a thorough discussion of the transients of the simple and coupled oscillators, and the reader is made familiar with the apparatus of complex integration, the Heaviside step function, the unit impulse or Dirac delta function, and Fourier and Laplace transforms.

In Chapters 3, 4 and 5, on the flexible string, the vibration of bars, and membranes and plates, emphasis is placed on a unified development of the theory of the normal modes of vibration of the systems, and the forms of vibration are illustrated by excellent drawings. The treatment of strings employs impedance concepts and methods analogous to those of electrical-line theory. In conjunction with the theory of normal modes, the operational method is extended to the calculation of the

transients of strings and membranes. Among the subjects covered in these chapters may be mentioned the string with yielding supports, perturbation calculations for the non-uniform string, the forced motion of membranes and plates, and the condenser microphone.

Chapter 6 gives the theory of plane waves of sound, and describes the analogy between acoustical and electrical elements. As in the treatment of strings the methods of electrical-line theory are applied to the propagation of plane waves in tubes, and the important hyperbolic-tangent, or standing-wave, method for the determination of acoustical impedance is described. This chapter contains interesting new material on reed and wind instruments and loudspeaker horns.

Chapter 7 deals with the radiation and scattering of spherical and cylindrical sound waves in more detail than before, in view of developments in ultrasonics, and the theory is applied to the behaviour of loudspeakers and microphones. The author now gives a full discussion of the nature of the acoustical impedance of an absorbing surface and sets out the electrical equivalents of various types of absorbing materials, and he introduces the useful concepts of surfaces of local and extended reaction. Morse's theory of the propagation of sound in absorbent-lined ducts of rectangular and circular section is of special interest in connection with the reduction of noise in ventilating ducts. On this subject reference may also be made to a paper by R. A. Scott (*Proc. Phys. Soc.*, Vol. 58, p. 358) in which an extension of the theory is considered.

Chapter 8 has been largely re-written, and gives a fundamental treatment of the theory of room acoustics in terms of the normal modes of air vibration and the acoustical impedance of the surfaces. A detailed analysis is given for the rectangular room, and the operational method is applied to the transient response. This chapter will be of outstanding importance to students of room acoustics.

The book is written in a clear and vigorous style, and is characterized by a strong sense of the unity of theoretical physics. It must be acknowledged that the mathematical treatment is at times formidable, but care is taken to point out the underlying physics of the reasoning. Particular attention is given to the practical evaluation of the solutions obtained, and for this purpose useful charts of hyperbolic functions and tables are provided in the appendix.

This important book will be indispensable to all students and research workers who wish to keep abreast of modern theoretical techniques and developments in the subject.

E. J. E.

Radio at Ultra-High Frequencies. Vol. II. 1940-1947.

Pp. 485 + x. R.C.A. Review, Radio Corporation of America, R.C.A. Laboratories Division, Princeton, New Jersey, U.S.A. Price \$2.50 (postage abroad 20 cents).

Reprints of 22 papers by R.C.A. authors and summaries of 22 others in the period 1940-1947. The subjects include aerials, propagation, reception, radio relays, microwaves and measurements.

The Measurement of Stress and Strain in Solids

Pp. 114 + xvi, with 41 illustrations. The Institute of Physics, 47, Belgrave Square, London, S.W.1. Price 17s. 6d.

This book consists of a collection of papers read at a conference at the University of Manchester in July 1946. There are four on the characteristics and use of resistance strain gauges, three on photo-elasticity, two on X-rays and one on acoustic methods.

WIRELESS PATENTS

A Summary of Recently Accepted Specifications

The following abstracts are prepared, with the permission of the Controller of H.M. Stationery Office, from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price 2/- each.

DIRECTIONAL AND NAVIGATIONAL SYSTEMS

597 429.—Radiolocation equipment for ships or aircraft in which the effect of rolling or pitching upon the exploring beam is automatically corrected.

F. C. Williams and A. I. Llewelin. Application date 8th August, 1945.

597 662.—Waveguide coupling or switching arrangement, of the stub type, particularly for the aerial system in radiolocation.

The British Thomson-Houston Co. Ltd. Convention date (U.S.A.) 17th July, 1944.

597 698.—Auxiliary prism device for modifying or controlling the polar diagram of a directive aerial, such as a waveguide horn or reflecting mirror.

R. B. Robertson-Shersby-Harvie and R. G. Garfitt. Application date 20th June, 1945.

597 742.—Phase-adjusting device for sense-determination in direction finders, particularly one using an Adcock aerial.

S. B. Smith and J. F. Hatch. Application date 18th September, 1945.

597 769.—Direction-indicating systems for navigating mobile craft, comprising the use of interrogating pulses and response signals having a saw-toothed form.

Hazeltine Corpn., (assignees of B. D. Loughlin). Convention date (U.S.A.) 15th September, 1944.

597 912.—Radiolocation receiver in which, a potentiometer device with a logarithmic characteristic is used to diminish the effect of 'permanent' echoes.

R. H. A. Carter and G. Bradfield. Application date 10th August, 1945.

597 977.—Time-base circuit adapted to generate a strobing voltage with a controlled time-delay, for expanding the echo trace in radiolocation.

F. E. J. Girling. Application date 23rd October, 1945.

597 986.—Cathode-ray indicator for radiolocation in which a translation circuit reduces the visibility of ground or other undesired echo signals relatively to those received from a moving target.

Standard Telephones and Cables Ltd., (assignees of D. D. Greig). Convention date (U.S.A.) 18th March, 1944.

598 752.—Measuring the carrier-wave frequency of radiolocation and like pulsed signals by means of a superheterodyne receiver with a tuned rejector circuit in combination with a cathode-ray indicator.

S. Jefferson. Application date 17th September, 1945.

599 554.—Means for deriving d.c. voltages corresponding to the echo-signals received at the four critical points in a radiolocation receiver utilizing a conical exploring system.

R. S. Webley and J. M. Robson. Application date 2nd May, 1945.

599 572.—Mirror type of aerial, for use in radiolocation, and having a reflecting characteristic suitable for indicating the position of towns or other objects in a given plane.

C. H. Smith, H. G. Booker and P. M. Woodward. Application date 18th September, 1945.

599 600.—Mirror type of aerial which is divided into two phase-differentiated parts in order to produce an asymmetrical polar diagram, say for use in radiolocation.

J. D. Lawson. Application date 11th October, 1945.

599 602.—Phasing circuit and gate-valve device for indicating the phase position of a selected pulse in a series of pulsed signals, say in radiolocation.

Svenska Aeroplan Akt and Akt Bofors. Convention date (Sweden), 29th July, 1944.

RECEIVING CIRCUITS AND APPARATUS

(See also under Television)

597 674.—Receiver in which a single control knob conditions the circuits for any one of the available wavebands, or for press-button selection, or for continuous tuning.

E. K. Cole Ltd. and C. L. Burnard. Application date 24th August, 1945.

597 897.—Method of processing the silicon element of a metal-point contact detector or mixer.

The General Electric Co. Ltd., F. H. Brittain, C. E. Ramsley and J. W. Ryde. Application date 20th April, 1945.

597 904.—Arrangement of a rotary scanning drum and co-operating electrodes for receiving facsimile signals.

W. G. H. Finch. Convention date (U.S.A.) 16th September, 1944.

598 133.—Method of reducing noise in reception by first clipping the carrier to form a train of trapezoidal waves and then converting them into a series of amplitude-modulated pulses.

Standard Telephones and Cables Ltd., (assignees of D. D. Greig). Convention date (U.S.A.) 16th April, 1943.

599 647.—Switch-controlled receiver with auxiliary press buttons arranged to give broad or narrow-band reception on each selected station.

E. K. Cole, Ltd. and H. Hunt. Application date 22nd September, 1945.

599 844.—Coaxial line with concentric shield and cathode-follower valve, for reducing input-capacitance effects when coupling a signal-source to a cathode-ray indicator.

Sperdy Gyroscope Co. Inc. (assignees of D. E. Kenyon). Convention date (U.S.A.), 28th August, 1944.

599 927.—Receiver for frequency-modulated signals in which provision is made for reducing the normal current consumption when the set is automatically muted, during interstation tuning or when on 'stand-by.'

Philips Lamps Ltd. Convention date (U.S.A.) 26th May, 1944.

599 934.—Process for acoustically-damping a radio receiver by lining the interior walls of the cabinet with pulp-board, cork composition, or the like.

W. E. Lord. Application date 25th August, 1945.

599 950.—Resistance-capacitance network, particularly for heterodyne oscillators, whereby a convenient step-by-step tuning-control is provided within a number of different frequency ranges.

The British Broadcasting Corpn. and A. R. A. Rendall. Application date 26th September, 1945.

599 985.—Slot-and-hole method of mounting certain of the control members on a radio panel or chassis, to facilitate removal for servicing.

D. Jackson and Pye, Ltd. Application date 27th September, 1945.

TELEVISION CIRCUITS AND APPARATUS

FOR TRANSMISSION AND RECEPTION

597 395.—Scanning system for generating television signals from a continuously-fed motion-picture film.

The General Electric Co. Ltd., and L. C. Jesty. Application date 20th July, 1944.

597 461.—Distributing multiple television programmes by radiating the picture, sound and synchronizing signals on different selected carrier and pulsing frequencies.

Standard Telephones and Cables Ltd., (assignees of E. Labini). Convention date (U.S.A.) 25th August, 1944.

597 525.—Television receiver of the kind in which the transparency of a sensitive, or deformable, screen is controlled by the scanning stream of a cathode-ray tube.

Scophony Ltd. and G. Wikkenhauser. Application date 23rd May, 1945.

597 580.—Television receiver in which the transparency of a crystalline screen is controlled, in the sense of acting as a diffraction-grating, by the scanning beam of a cathode-ray tube.

Scophony Ltd., G. Wikkenhauser and F. Okolicsanyi. Application date 16th January, 1945.

597 647.—Television system in which the pulsed synchronizing signals are also modulated to carry the sound signals, and are transmitted over a separate channel from the picture signals.

Standard Telephones and Cables Ltd., P. K. Chatterjee and L. W. Houghton. Application date 17th March, 1945.

TRANSMITTING CIRCUITS AND APPARATUS

(See also under Television)

597 736.—Transmission-line coupling and switching arrangement for the aerial of a combined T and R set.

Standard Telephones and Cables Ltd., (assignees of A. G. Kandoian). Convention date (U.S.A.) 16th December, 1942.

597 950.—Frequency-modulating system in which a pair of oppositely-tuned tank circuits are so coupled and controlled as to eliminate amplitude modulation.

Marcou's W. T. Co., Ltd., (assignees of G. L. Usselman). Convention date (U.S.A.) 25th April, 1944.

598 098.—Control circuit for automatically generating a negative feedback voltage in order to stabilize the frequency of an oscillatory circuit, particularly in frequency modulation.

The British Broadcasting Corp., and G. G. Gouriet. Application date 21st September, 1945.

SIGNALLING SYSTEMS OF DISTINCTIVE TYPE

596 078.—Receiving circuit for a pulsed communication system utilizing time or amplitude modulated signals, or both.

Standard Telephones and Cables Ltd. (assignees of D. D. Græg). Convention date (U.S.A.) 18th September, 1942.

596 092.—Multi-channel intercommunication system in which automatically-interlaced time intervals are provided for the incoming and outgoing pulsed signals.

Siemens Bros. & Co. Ltd., M. Reed and S. H. Moss. Application date 18th July, 1945.

596 093.—Pulsed signalling system in which distortion

is minimized by transmitting a pulse of reversed polarity after each signalling pulse.

Siemens Brothers & Co., M. Reed and S. H. Moss. Application date 18th July, 1945.

597 114.—Automatic calling arrangement for a two-way radio communication system working on a single wavelength.

The General Electric Co. Ltd. and A. A. Chubb. Application date 10th August, 1945.

597 260.—Pulse-modulating centimetre waves at audio frequency by varying the frequency of an oscillator coupled to the control grid of a rhumbatron type of generator.

G. R. Shepherd (communicated by Westinghouse Electric International Co.). Application date 2nd August, 1945.

597 323.—Filtering and receiving circuit for eliminating interference from signals transmitted as time-modulated trains of pulses.

Standard Telephones and Cables Ltd., (assignees of E. Labini and D. D. Greig). Convention date (U.S.A.) 24th May, 1943.

597 581.—System of inter-office communication in writing, based on the use of facsimile telegraph apparatus.

W. S. Tandler and D. S. Walker. Application date 12th March, 1945.

597 671.—Signalling system in which built-up characters or codes are transmitted by trains of pulses.

Standard Telephones and Cables Ltd., (assignees of L. A. de Rosa). Convention date (U.S.A.) 18th June, 1943.

598 277.—System of gain-control, particularly for stabilizing a super-regenerative circuit of the kind which responds automatically to an interrogating signal.

Hazeltine Corp., (assignees of J. J. Okrent). Convention date (U.S.A.) 13th October, 1944.

598 278.—Means for monitoring a super-regenerative circuit of the automatic-responder type, so as to prevent it from so-called 'false-firing.'

Hazeltine Corp., (assignees of J. J. Okrent). Convention date (U.S.A.) 13th October, 1944.

SUBSIDIARY APPARATUS AND MATERIALS

597 102.—Construction of screening-can for a valve, and device for fixing it in position.

Cinch Manufacturing Corp., (assignees of S. M. del Camp). Convention date (U.S.A.) 16th August, 1944.

597 262.—Testing and measuring equipment in which the standing waves set up along a waveguide are continuously observed in a cathode-ray indicator.

W. D. Allen and J. E. McF. Johnston. Application date 23rd August, 1945.

597 832.—Oscillation generator with associated reflecting system adapted to generate a concentrated beam of micro-waves for physiological or like purposes.

J. H. Cotton. Application date 23rd July, 1945.

597 948.—Cathode-ray indicator for testing and aligning the circuits of a radio receiver in response to different bands of f.m. waves.

Philco Corp. Convention date (U.S.A.) 31st October, 1944.

598 244.—Automatic gain control for an amplifier designed to measure the different components of a complex input; e.g., an ohmmeter, dynamometer, or the like.

The British Electrical and Allied Industries Research Association and B. Rosenblum. Application date 24th January, 1945.