The secrets of the Smith chart

PART TWO.
by
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Now those outer scales I ignored earlier on. We will consider the scale wavelengths towards and from generator and from and to load. Other outer scales are possible. To show how these scales are used, we are going to calculate the input impedance of a gamma-matched dipole as an example.

This month Alec Jones, GM8HGD, shows us how to design our own antenna systems.

Recently the aerial for the Grampian VHF repeater GB3GN was changed from a folded dipole to a Cushcraft 4D colinear. This uses four gamma-matched half-wave dipoles and a phasing harness to produce the colinear aerial. We shall be considering one element only. I took the physical dimensions from one element of the Cushcraft, and the calculation shows the electrical characteristics at a frequency of 145.5MHz.

On the Smith Chart, the total distance round the outside of the chart is half a wavelength. This is an electrical length, the result of the velocity of phase propagation (or more simply, the wave velocity) in the medium through which the energy passes. With air insulation in coaxial cables and open-wire transmission lines, the velocity of propagation can be taken as 300 000 km/sec. Hence = v/f. For dielectrics other than air, the velocity of propagation is lower, and hence the wavelength for a given frequency is lower. The factor by which the velocity is reduced is given in the tables of information for the cables. It is the square root of the dialectric constant of the medium.

We are going to use the method of calculating the gamma match given in the ARRL Antenna Book, 1974 edition, pages 119 and 120. There are several stages of calculation before the Smith Chart is used.

Firstly, we need to calculate the step-up ratio of the impedance due to the coupling between the dipole and the gamma match. **Fig. 9** shows the details. This impedance step-up is a function of the diameters of the elements and the spacing between them.

$$r = 1 + \left(\frac{\log (2S/d_1)}{\log (2S/d_2)} \right)^2$$

where d_1 is the diameter of the dipole, d_2 is the diameter of the gamma match, and S is the spacing between them. In our example we take the values: $d_1 = 1.905$, $d_2 = 0.95$, and $S = 3.57$ (all centimetres). This gives a value for r of **2.66**.

Next we calculate the characteristic impedance of the "transmission line" formed by the gamma match and the dipole element considered as two parallel conductors. Use the formula:

$$Z_0 = 276 \log \left(\frac{2S}{\sqrt{d_1 . d_2}} \right)$$

where the symbols have the same meaning as previous equation. Hence $Z_0 = 200$

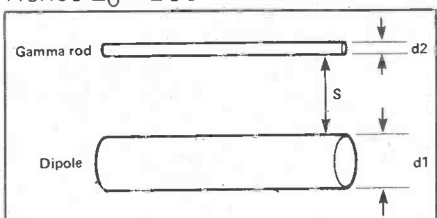

Fig.9 Impedance step-up due to rod diameter and spacing.

The lengths of the gamma rod, 17cm from dipole centre to shorting point, corresponds to a wavelength of 0.008245γ. We can express this in electrical degrees, where 1γ = 360°, and this calculates to 29°41′ (= θ).

With a gamma match, the dipole element is being driven off-centre, and hence has an increased impedance over the centre-fed dipole. The impedance at the centre of the half-wave dipole, with 3% shortening, can be taken for calculation purposes as 25 − j25 ohms (Z_1). Feeding this off-centre gives the new impedance $Z_2 = Z_1/\cos^2\theta$. This calculates to 33.123 − j33.123Ω. Hence the antenna impedance seen at the shortening strip end of the gamma match will be $Z_3 = r \times Z_2$, so Z_3 calculates to 88.114 − j 88.114.

All these calculations to date have been done on my pocket calculator. We now have to move the impedance down the transmission line of the gamma match to reach the feedpoint where the 50 ohm cable is attached. This is where we need the Smith Chart and its peripheral scales. The "transmission line" has a characteristic impedance of 200 ohms, as calculated above, so we normalise our impedances for use on the Smith Chart to 200 ohms. Whence the impedance at the shorting strip becomes 0.44 − j 0.44.

Plot this point on the chart as point A. We now travel down the gamma match rod towards the generator by a distance of 0.08245γ. On a single piece of paper, it is not possible to rotate the inner scales relative to the outer. Use a straight edge to go from the centre 1.0 point to the outer scales, passing through point A (**Fig. 10A**). This cuts the wavelengths towards generator scale at 0.42. We need to travel an extra 0.08245 λ. Draw a line from this point on the outer scale to the centre. Using compasses, draw an arc centred at 1.0 from point A to cut this new line at point B (= 0.35 + j0.005), **Fig. 10B**. We are now at the feedpoint of the gamma match and have the value of the impedance of the aerial due to the aerial alone.

In parallel with this is an induc-