SUPPLEMENT

TO THE POST OFFICE ELECTRICAL ENGINEERS' JOURNAL

Vol. 62 No. 2 July 1969

CITY AND GUILDS OF LONDON INSTITUTE EXAMINATIONS, 1967-68

Contents

QUESTIONS AND ANSWERS

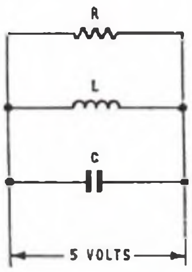
Answers are occasionally omitted or reference is made to earlier Supplements in which questions of substantially the same form, together with the answers, have been published. Some answers contain more detail than would be expected from candidates under examination conditions.

TELECOMMUNICATION PRINCIPLES C, 1968 (*continued*)

Q. 5. A coil of inductance 50 mH and Q-factor 100 is connected in parallel with a 0·002-μF capacitor. A variable frequency source of constant voltage 5 volts r.m.s. is connected across the parallel circuit.

Calculate:
(a) the frequency of resonance,
(b) the current taken from the source at resonance,
(c) the current in the capacitor at resonance,
(d) the magnitude of the current taken from the source at the frequency 0·5 per cent above the resonant frequency.

A. 5. The circuit is shown in the sketch.



(*a*) The frequency of resonance will be,

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}},$$

$$= \frac{1}{2\pi}\frac{1}{\sqrt{50\times10^{-3}\times0\cdot002\times10^{-6}}},$$

$$= \frac{10^5}{2\pi} = \underline{15\cdot9\text{ kHz.}}$$

(*b*) The Q-factor of the parallel-circuit inductance is

$$Q = \frac{R}{\omega_0 L}.$$

$$\therefore\ 100 = \frac{R}{10^5\times50\times10^{-3}},$$

and $R = 0\cdot5$ Mohm.

The current taken by the circuit at resonance is

$$I_s = \frac{5}{0\cdot5\times10^6} = \underline{10\ \mu\text{A}}.$$

(*c*) The current taken by the capacitor at resonance is

$$I_c = 10^5\times0\cdot002\times10^{-6}\times5 = \underline{1\text{ mA.}}$$

(*d*) The admittance of the circuit at a frequency $\omega_0(1+\delta)$ is given by

$$Z = \frac{1}{R} + \frac{1}{j\omega_0L(1+\delta)} + j\omega_0C(1+\delta).$$

By the binomial theorem $(1+\delta)^{-1} \simeq 1-\delta$ if δ is small.

$$\therefore\ Z = \frac{1}{R} + \frac{1}{j\omega_0L} + j\omega_0C - \delta\left(\frac{1}{j\omega_0L} - j\omega_0C\right),$$

$$= \frac{1}{R} + 2j\delta\omega_0C,$$

$$= \sqrt{\frac{1}{R^2} + 4\delta^2\omega_0^2C^2}\ \underline{/\tan^{-1}2\delta\omega_0CR}.$$

$\therefore$ source current at a frequency 0·5 per cent above the resonant frequency is

$$I'_s = 5\sqrt{\frac{1}{R^2} + 4\delta^2\omega_0^2C^2},$$

$$= 5\sqrt{\left(\frac{10^{-6}}{0\cdot5}\right)^2 + 4\left(\frac{0\cdot5}{100}\right)^2(10^5)^2(0\cdot002\times10^{-6})^2}.$$

$$\therefore\ \underline{\text{current} = 10\sqrt{2}\ \mu\text{A.}}$$

Q. 6. An amplitude-modulated current is represented by the expression

$$i = 10[1 + 0\cdot4\sin(3{,}140t)]\sin(6\cdot28\times10^5t)\ \text{amperes.}$$

Deduce:
(a) the modulation depth,
(b) the modulating frequency,
(c) the carrier frequency.

Expand the expression and calculate for each component the r.m.s. current and the frequency.

The modulated current is fed to an aerial whose input resistance is 50 ohms. Calculate the total power fed to the aerial. How much of this power is contributed by the side-frequency components?

A. 6. The expression

$$i = A(1 + m\sin\phi t)\sin\omega t,$$

$$= A\sin\omega t + mA\sin\phi t\sin\omega t,$$

$$= A\sin\omega t - \frac{mA}{2}\cos(\omega+\phi)t + \frac{mA}{2}\cos(\omega-\phi)t,$$

represents an amplitude-modulated sine wave where:

$\frac{\omega}{2\pi}$ is the carrier frequency of amplitude A,

$\frac{\phi}{2\pi}$ is the modulating frequency of amplitude mA,

$\frac{\omega+\phi}{2\pi}$ is the upper side frequency of amplitude $\frac{mA}{2}$,

$\frac{\omega-\phi}{2\pi}$ is the lower side frequency of amplitude $\frac{mA}{2}$,

m is the modulation factor or depth.

Hence, in the given expression,

(*a*) the modulation depth is $\underline{0 \cdot 4.}$

(*b*) the modulating frequency is $\dfrac{3{,}140}{2\pi} = \underline{500 \text{ Hz.}}$

(*c*) the carrier frequency is $\dfrac{6 \cdot 28 \times 10^5}{2\pi} = \underline{100 \text{ kHz.}}$

The r.m.s. carrier current is $\dfrac{10}{\sqrt{2}} = \underline{7 \cdot 07 \text{ amps.}}$

The r.m.s. current at each side frequency is

$$\frac{0 \cdot 4 \times 10}{2\sqrt{2}} = \underline{1 \cdot 41 \text{ amps.}}$$

The total power which would be fed into the aerial is

$$P_T = \left(\frac{10}{\sqrt{2}}\right)^2 50 + \left(\frac{0 \cdot 4 \times 10}{2\sqrt{2}}\right)^2 50 + \left(\frac{0 \cdot 4 \times 10}{2\sqrt{2}}\right)^2 50,$$

$$= 2{,}500 + 100 + 100 = \underline{2{,}700 \text{ watts.}}$$

$$\text{Percentage of power in each sideband} = \frac{100}{2{,}700} \times 100 \text{ per cent,}$$

$$\underline{\simeq 3 \cdot 7 \text{ per cent}}$$

Q. 7. *Explain the difference between amplitude-modulation and frequency-modulation.*

In a frequency modulator, the frequency of the carrier is shifted by 1 kHz (kc/s) per volt of modulating signal. If the carrier and signal are represented by $e_c = 20 \sin(6 \cdot 28 \times 10^6 t)$ volts and $e_s = 5 \sin(3 \cdot 14 \times 10^4 t)$ volts respectively, write down an expression for the instantaneous frequency of the modulator output and calculate its peak frequency deviation and peak phase deviation.

Give an estimate of the transmission bandwidth that would be required in practice for a frequency-modulated carrier when a sinusoidal signal at a frequency of 1 kHz produces a peak frequency deviation of (a) 50 Hz, (b) 50 kHz.

A. 7. Amplitude-modulation is the process by which the amplitude of a carrier wave is modified by the amplitude of the signal. Frequency-modulation is the process by which the frequency of a carrier wave is modified by the amplitude of the signal.

The instantaneous frequency of the modulator output is

$$f = \frac{6 \cdot 28}{2\pi} 10^6 + 5 \times 10^3 \sin(3 \cdot 14 \times 10^4 t),$$

$$= \underline{10^6 + 5 \times 10^3 \sin(3 \cdot 14 \times 10^4)t.}$$

Since the maximum value of $\sin\theta = 1 \cdot 0$,

$\underline{\text{peak frequency deviation} = 5 \text{ kHz.}}$

The instantaneous phase of the modulated wave is given by

$$\theta = \int \omega dt = \int (\omega_c + \Delta\omega \sin \phi t)dt,$$

$$= \omega_c t - \frac{\Delta\omega}{\phi} \cos \phi t.$$

Thus, the phase deviation is $\dfrac{\Delta\omega}{\phi} \cos \phi t$.

When $\cos \phi t = 1 \cdot 0$, the phase deviation has a maximum value of $\dfrac{\Delta\omega}{\phi}$. In the question,

$$\frac{\Delta\omega}{\phi} = \frac{2\pi \times 5 \times 10^3}{\pi \times 10^4} = \underline{1 \text{ radian.}}$$

The expression for the instantaneous value of a frequency-modulated wave is

$$e = A \sin\{(\omega_c + \Delta\omega \sin \phi t)t\}.$$

The expansion of this expression indicates that it contains angular frequencies $\pm(\omega_c + n\phi)$.

The amplitude of the pairs of components is controlled by the ratio $\dfrac{\Delta\omega}{\phi}$ but is not directly proportional to it.

The bandwidth required for the transmission of frequency-modulated signals must include the components which contain a significant amount of energy relative to that of the unmodulated carrier. It will be found that most of these are contained within the range

$$\pm \frac{1}{2\pi}(\Delta\omega_{max} + \phi).$$

$\therefore$ with a peak frequency deviation of 50 Hz,

$$\text{Bandwidth} = 2 \times (50 + 1{,}000) = \underline{2{,}100 \text{ Hz.}}$$

With a peak frequency deviation of 50 kHz,

$$\text{Bandwidth} = 2 \times (50 + 1) \times 10^3 = \underline{102 \text{ kHz.}}$$

Q. 8. *The small-signal voltages and currents for the transistor in Fig. 4 are related by the equations:*

$$v_b = 2{,}000 i_b + 10^{-5} v_c$$
$$i_c = 40 i_b + 10^{-4} v_c$$

for a particular bias condition.

The transistor is connected, so that the same bias conditions apply, to form a common-emitter amplifier with a collector load resistance of 5 kohms. A signal source having an open-circuit e.m.f. 1 mV and a 400-ohm resistive internal impedance is connected between base and emitter.

By drawing the equivalent circuit, or otherwise, calculate the base and collector signal currents and the output signal voltage across the collector load.

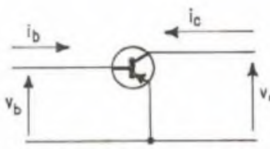
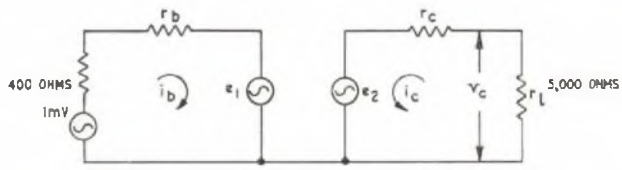


Fig. 4

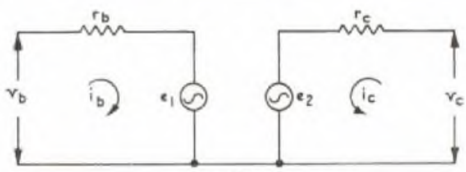
A. 8. The equivalent circuit for the common-emitter transistor amplifier is given in sketch (*a*).



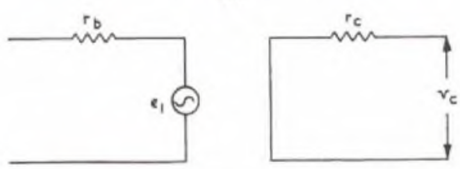
(*a*)

Considering the transistor alone, (see sketch (*b*)), when a voltage v_c is applied across the output terminals and the input is open-circuited, the reverse voltage feedback ratio is

given by $\quad h_{12} = e_1 = 10^{-5} v_c$ as shown in sketch (*c*).



(*b*)



(*c*)

When the output is open-circuited, $e_1 = 0$.

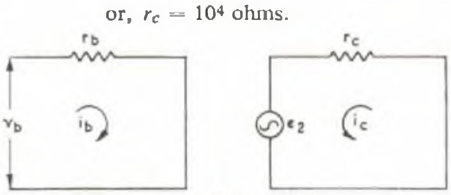
$\therefore$ the input resistance $(h_{11}) = \dfrac{v_b}{i_b} = r_b = 2{,}000$ ohms.

When the input is open-circuited, $i_b = 0$ and $e_2 = 0$.

Therefore, the output admittance of the transistor (h_{22}) is given by

$$h_{22} = \frac{1}{r_c} = 10^{-4},$$

$$\text{or, } r_c = 10^4 \text{ ohms.}$$



(*d*)

When $r_l = 0$, $v_c = 0$ (see sketch (*d*)),

and, $i_c = 40i_b$.

But $i_c = \dfrac{e_2}{r_c}$.

$\therefore\ e_2 = r_c i_c = 40 \times 10^4 i_b$.

Hence, for the amplifier shown in sketch (*a*),

$$v_c = -5{,}000 i_c,$$

i.e. a potential difference not an applied voltage.

$$v_b = 10^{-3} - 400i_b = 2{,}000i_b - 10^{-5} \times 5{,}000i_c,$$

$$\text{or, } 10^{-3} = 2{,}400i_b - 0{\cdot}05i_c, \qquad \ldots\ldots (1)$$

$$\text{and, } i_c = 40i_b - 5{,}000 \times 10^{-4}i_c,$$

$$40i_b = 1{\cdot}5i_c. \qquad \ldots\ldots (2)$$

Substituting the value of i_b from equation (2) into equation (1) gives

$$10^{-3} = 90i_c - 0{\cdot}05i_c.$$

$$\therefore\ i_c \simeq \frac{10^{-3}}{90} = \underline{11{\cdot}1\ \mu\text{A}}.$$

$$\text{Voltage across collector load} = \frac{5{,}000 \times 10^{-3}}{90} = \underline{55{\cdot}5\ \text{mV}}.$$

$$\text{Base current} = i_b = \frac{1{\cdot}5}{40} i_c,$$

$$= \frac{1{\cdot}5}{40} \times \frac{10^{-3}}{90} = \underline{0{\cdot}41\ \mu\text{A}}.$$

Q. **9.** *The characteristics of a pentode valve for currents above the knee are given in the table.*

Anode Voltage V_a	*Anode Current I_a (mA) at grid voltage V_g*					
	0	−2	−4	−6	−8	−10
30	40·0	31·9	23·8	15·7	7·6	2·8
210	40·2	32·1	24·0	15·9	7·8	3·0
390	40·4	32·3	24·2	16·1	8·0	3·2

Plot the anode characteristics and on these plot the curve for a maximum anode dissipation of 5 watts.

The valve is to be used in the output of an audio amplifier with its anode connected in series with the primary of an output transformer to a 200-volt h.t. supply.

Assuming a knee voltage of 30 volts, select a suitable bias point and draw an a.c. load line to give maximum undistorted output power. Calculate for a sinusoidal input signal

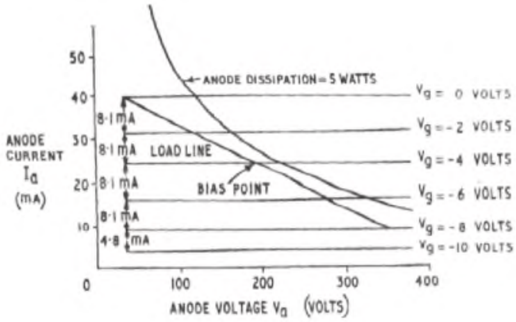
(*a*) *the maximum output power,*
(*b*) *the anode dissipation when the valve is producing this power.*

A. 9. The anode characteristics of the pentode valve are shown in the sketch and on this has been included a curve for a maximum anode dissipation of 5 watts.

Also on the characteristics, a load line has been drawn for the maximum undistorted output power. From this,

$$(a)\ \text{maximum output power} = \frac{1}{8} \times 340 \times \frac{32}{1{,}000},$$

$$= \underline{1{\cdot}36\ \text{watts}}.$$



$$(b)\ \text{anode dissipation for this power} = \frac{200 \times 24}{1{,}000},$$

$$= \underline{4{\cdot}8\ \text{watts}}.$$

Q. **10.** *Describe an experiment to measure the deflexion sensitivities of a cathode-ray tube.*

A cathode-ray tube has equal "X" and "Y" deflexion sensitivities of 50 volts/cm. A waveform displayed on the tube has a peak-to-peak vertical amplitude of 4 cm, and horizontally two full cycles occupy 5 cm. The time-base voltage rises at a rate of 100 volts/ms.

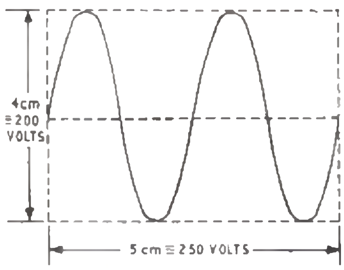
Calculate the r.m.s. voltage and the frequency of the waveform displayed.

A. 10. To measure the deflexional sensitivity of a cathode-ray tube, a known d.c. voltage, V, is applied across each of the pairs of deflector plates (or coils) and the corresponding deflexion, δ, measured in centimetres.

$$\text{The sensitivity} = \frac{\delta}{V}\ \text{cm/volt}.$$

The applied voltage could be supplied from a 50-volt battery across which a voltmeter is connected. The deflexion could be measured on the tube itself or transferred by pencil to a sheet of tracing paper fixed temporarily to the face of the tube.

The trace displayed on the screen of the cathode-ray tube will be as shown in the sketch. From the sketch, amplitude of the waveform is 100 volts.



Assuming the waveform is sinusoidal, its r.m.s. value will be <u>70·7 volts.</u>

Time for one cycle is equivalent to 125 ms, which corresponds to a periodic time of $\dfrac{125}{100} = 1{\cdot}25$ ms.

$$\text{Thus, the frequency is } \frac{1{,}000}{1{\cdot}25} = \underline{800\ \text{Hz}}.$$

MATHEMATICS C, 1968

Students were expected to answer any six questions

Q. **1.** *The energy loss E when water flows at a velocity v through a cylindrical diameter pipe is shown in the table below:*

v	0·76	1·13	1·50	1·85	2·29	2·82	3·51
E	0·085	0·135	0·23	0·35	0·50	0·695	1·25

Show by plotting suitable variables that an assumed law $E = kv^n$ is satisfied over a range of velocities.

From the graph obtain estimates of the constants k and n, and state the range of velocities for which the formula holds.

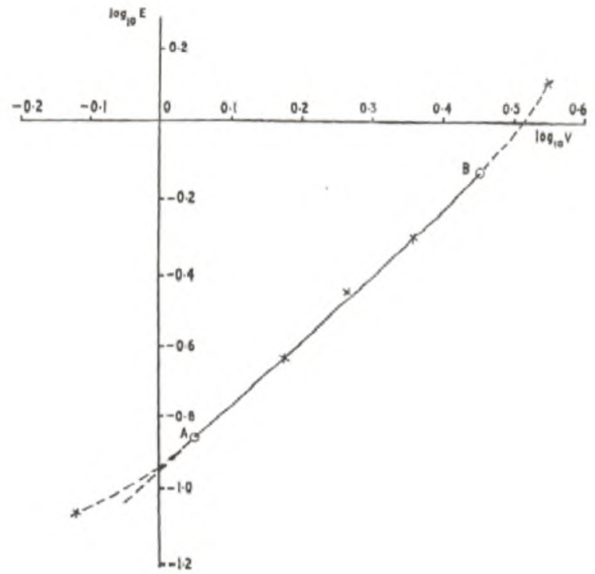
A. 1. If the energy loss E and water velocity v are related by the law $E = kv^n$, then,

$$\log_{10} E = \log_{10} k + n \log_{10} v.$$

As k and n are constants, this logarithmic form of the law is of the linear type $y = ax + b$ and this may be tested by plotting $\log_{10} E$ against $\log_{10} v$ from the following table of values.

v	0·76	1·13	1·50	1·85	2·29	2·82	3·51
E	0·085	0·135	0·23	0·35	0·50	0·695	1·25
$\log_{10} v$	$\bar{1}$·8808 = −0·119	0·053	0·176	0·267	0·360	0·450	0·545
$\log_{10} E$	$\bar{2}$·929 = −1·071	$\bar{1}$·130 = −0·870	$\bar{1}$·362 = −0·638	$\bar{1}$·544 = −0·456	$\bar{1}$·699 = −0·301	$\bar{1}$·842 = −0·158	0·097

The sketch shows that, if the two extreme points are excluded, the points lie approximately on a straight line, as indicated by the dotted extensions.



The gradient of the straight line may be obtained from the co-ordinates of two widely-separated points, e.g. A and B, which actually lie on the line. By measurement from the graph,

$$\text{gradient} \qquad n = \frac{-0\cdot135 - (-0\cdot865)}{0\cdot45 - 0\cdot05},$$
$$= \frac{0\cdot73}{0\cdot4},$$
$$= 1\cdot825.$$

When $\log_{10} v = 0$,
$$\log_{10} E = \log_{10} k.$$

Thus, by a small extension of the straight-line graph from point A to intersect the $\log_{10} E$ axis, the value of $\log_{10} E$ when $\log_{10} v = 0$ may be read from the graph as $-0\cdot955$.

$$\therefore \log_{10} k = -0\cdot955,$$
$$= \bar{1}\cdot045.$$
$$\therefore k \simeq 0\cdot111.$$

Hence, from the graph, estimates of the constants are

$$\underline{k = 0\cdot111} \quad \text{and} \quad \underline{n = 1\cdot825.}$$

The range of velocities for which the formula $E = kv^n$ holds is about from A to B, i.e. from where $\log_{10} v = 0\cdot05$ to where $\log_{10} v = 0\cdot45$, or from $\underline{v \simeq 1\cdot1}$ to $\underline{v \simeq 2\cdot8}$.

Q. 2. *Verify, using sketch-graphs or otherwise, that the equation*

$$x^3 - 10x^2 + 9x + 36 = 0$$

has three real roots.

By graphical enlargement or otherwise obtain the two positive roots to three significant figures, and thence deduce the negative root.

A. 2. $$x^3 - 10x^2 + 9x + 36 = 0.$$

With an equation of this type, i.e. one with reasonably small integral coefficients, it is often advantageous to try an inspection method for its solution.

It is at once apparent that $x = 0$ and $x = \pm1$ cannot be solutions.

Try $x = 2$:

substituting this value in the equation gives

$$8 - 40 + 18 + 36 \neq 0.$$

$\therefore x = 2$ cannot be a root.

Try $x = -2$:

Substituting this value in the equation gives,

$$-8 - 40 - 18 + 36 \neq 0.$$

$\therefore x = -2$ cannot be a root.

Try $x = 3$:

Substituting this value in the equation gives,

$$27 - 90 + 27 + 36 = 0.$$

$\therefore x = 3$ is a root.

$\therefore x - 3$ must be a factor of $x^3 - 10x^2 + 9x + 36$.

Dividing $(x - 3)$ into $(x^3 - 10x^2 + 9x + 36)$ gives,

$$\begin{array}{r|l} & x^2 - 7x - 12 \\ x - 3 & x^3 - 10x^2 + 9x + 36 \\ & x^3 - 3x^2 \\ \hline & -7x^2 + 9x \\ & -7x^2 + 21x \\ \hline & -12x + 36 \\ & -12x + 36 \\ \hline & \cdots \end{array}$$

$$\therefore x^3 - 10x^2 + 9x + 36 = (x - 3)(x^2 - 7x - 12).$$

To test that the remaining roots are real, it is necessary to solve the quadratic equation

$$x^2 - 7x - 12 = 0,$$

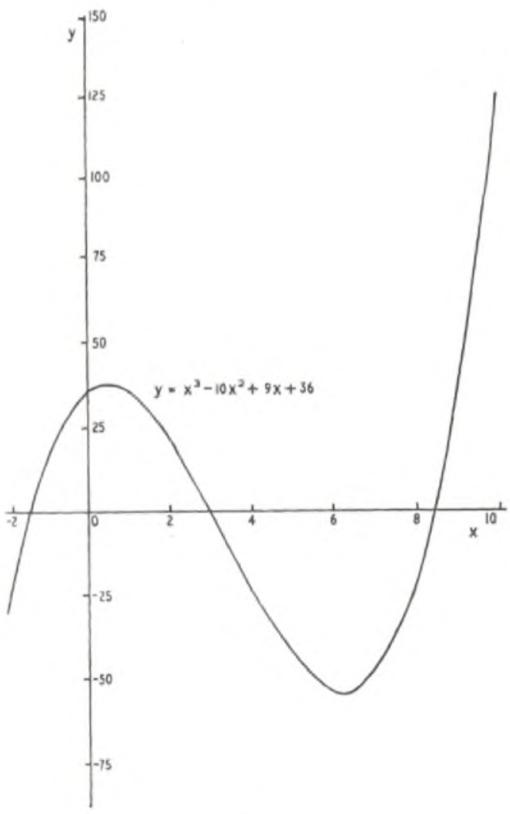
by one of the usual methods.

Using the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ gives,

$$x = \frac{7 \pm \sqrt{49 + 48}}{2},$$
$$= \frac{7 \pm \sqrt{97}}{2},$$
$$= \frac{7 \pm 9\cdot849}{2},$$
$$= 8\cdot4245 \quad \text{or} \quad -1\cdot4245.$$

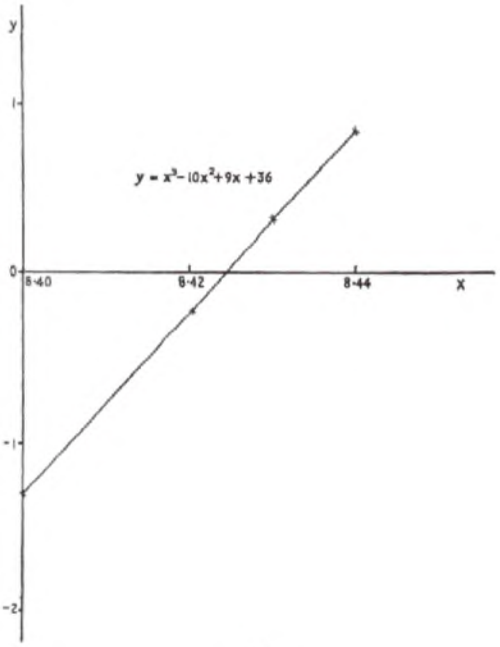
Thus the cubic equation has three real roots. The two positive roots are $\underline{3}$ and $\underline{8\cdot42,}$ the first being an exact root and the second correct to three significant figures.

The negative root is $\underline{-1\cdot42}$ correct to three significant figures.



(*a*)

Note.—Sketch (*a*) shows the graph of $y = x^3 - 10x^2 + 9x + 36$. This cuts the x-axis at $-1 \cdot 42$ approximately, 3 and $8 \cdot 42$ approximately. Sketch (*b*) shows an enlarged graph to determine the root near to $8 \cdot 42$.



(*b*)

Q. **3.** (*a*) *Expand* $(1 - 3x)^{1/3}$ *as a power series as far as the term in* x^4, *and state the coefficient of* x^r *in this series. For what range of* x *values is this expansion valid?*

(*b*) *The maximum velocity* v *of an electron stream in terms of the controlling voltage* V *is given by the formula*

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\left(1 + \frac{2V}{10^6}\right)^2}},$$

where c *is the velocity of light.*

If V *is small compared with* 10^6 *derive an approximate formula of the type*

$$v = cV^{1/2}(a + bV).$$

A. 3. (*a*)
$$(1 - 3x)^{1/3} = 1 + \tfrac{1}{3}(-3x) + \frac{\frac{1}{3}(\frac{1}{3} - 1)}{2!}(-3x)^2$$
$$+ \frac{\frac{1}{3}(\frac{1}{3} - 1)(\frac{1}{3} - 2)}{3!}(-3x)^3$$
$$+ \frac{\frac{1}{3}(\frac{1}{3} - 1)(\frac{1}{3} - 2)(\frac{1}{3} - 3)}{4!}(-3x)^4 + \ldots,$$
$$= 1 - x - \frac{\frac{1}{3} \times \frac{2}{3}}{2.1}9x^2 - \frac{\frac{1}{3} \times \frac{2}{3} \times \frac{5}{3}}{3.2.1}27x^3$$
$$- \frac{\frac{1}{3} \times \frac{2}{3} \times \frac{5}{3} \times \frac{8}{3}}{4.3.2.1}81x^4 - \ldots,$$
$$= 1 - x - x^2 - \frac{5x^3}{3} - \frac{10x^4}{3} - \ldots$$

The coefficient of x^r will be,

$$-\frac{\dfrac{1 \times 2 \times 5 \times \ldots \text{ to } r \text{ terms}}{3^r}}{r!}3^r$$
$$= -\frac{1 \times 2 \times 5 \ldots (3r - 4)}{r!}.$$

The expansion is valid when $|3x| < 1$

or, $|x| < \frac{1}{3}$,

where $|x|$ denotes the magnitude of x.

(*b*)
$$\frac{v}{c} = \sqrt{1 - \frac{1}{\left(1 + \frac{2V}{10^6}\right)^2}},$$
$$= \sqrt{1 - \left(1 + \frac{2V}{10^6}\right)^{-2}}.$$

If V is small compared with 10^6, $2V$ can also be assumed sufficiently small for the binomial expansion to be approximately true after only a few terms.

Therefore,
$$\left(1 + \frac{2V}{10^6}\right)^{-2} \simeq 1 - 2 \times \frac{2V}{10^6} + \frac{(-2) \times (-3)}{1.2}\left(\frac{2V}{10^6}\right)^2,$$
$$= 1 - \frac{4V}{10^6} + \frac{12}{10^{12}}V^2.$$
$$\left(\frac{v}{c}\right)^2 = 1 - \left(1 + \frac{2V}{10^6}\right)^{-2}.$$
$$\therefore \frac{v}{c} \simeq \sqrt{\frac{4V}{10^6} - \frac{12}{10^{12}}V^2},$$
$$\text{or, } v \simeq cV^{1/2}\sqrt{\frac{4}{10^6} - \frac{12}{10^{12}}V},$$
$$= cV^{1/2}\left(\frac{4}{10^6} - \frac{12}{10^{12}}V\right)^{1/2},$$
$$= \frac{2cV^{1/2}}{10^3}\left(1 - \frac{3V}{10^6}\right)^{1/2}.$$

Again, the square root may be expanded by the Binomial Theorem on the assumption that $10^6 \gg 3V$.

$$\therefore v \simeq \frac{2cV^{1/2}}{10^3}\left(1 - \frac{1}{2}\frac{3V}{10^6}\right),$$
$$= \frac{2cV^{1/2}}{10^3}\left(1 - \frac{3V}{2 \times 10^6}\right).$$

This is of the form required by the question except that the coefficient $\frac{2}{10^3}$ has been brought outside the brackets, which would be the usual practice.

Q. **4.** (*a*) *Prove for any angles* θ *and* ϕ

$$\cos^2(\theta - \phi) - \cos^2(\theta + \phi) = \sin 2\theta \sin 2\phi.$$

(*b*) *If* $\tan\theta = t$, *express* $\cos 2\theta$ *and* $\sin 2\theta$ *in terms of* t, *and hence or otherwise find the solutions of*

$$(1 + \tan\theta)(\cos 2\theta - \sin 2\theta) = 1$$

which lie in the range $\theta = 0$ *to* $\theta = 180°$ (*inclusive*).

A. 4. (*a*) $\cos^2(\theta - \phi) - \cos^2(\theta + \phi) = \sin 2\theta \sin 2\phi$.

The left-hand side (L.H.S.) $= \cos^2(\theta - \phi) - \cos^2(\theta + \phi)$,
$$= \{\cos(\theta - \phi) - \cos(\theta + \phi)\} \times \{\cos(\theta - \phi) + \cos(\theta + \phi)\}.$$

$\therefore$ From the sum-to-product formulae,

$$\text{L.H.S.} = 2\sin\theta\sin\phi \times 2\cos\theta\cos(-\phi).$$

Since $\cos(-\phi) = \cos\phi$,

$$\text{L.H.S.} = 2\sin\theta\cos\theta \times 2\sin\phi\cos\phi,$$
$$= \sin 2\theta \sin 2\phi.$$

Q.E.D.

(*b*)
$$\cos 2\theta = \cos(\theta + \theta),$$
$$= \cos^2\theta - \sin^2\theta,$$
$$= \cos^2\theta\left(1 - \frac{\sin^2\theta}{\cos^2\theta}\right),$$
$$= \frac{1 - \dfrac{\sin^2\theta}{\cos^2\theta}}{\sec^2\theta}.$$

But, $\sec^2\theta = 1 + \tan^2\theta$.

$$\therefore \cos 2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta},$$

and $\tan\theta = t$,

$$\therefore \cos 2\theta = \frac{1-t^2}{1+t^2}.$$

$$\sin 2\theta = 2\sin\theta\cos\theta,$$

$$= 2\frac{\sin\theta}{\cos\theta}\cos^2\theta,$$

$$= \frac{2\tan\theta}{\sec^2\theta},$$

$$= \frac{2\tan\theta}{1+\tan^2\theta}.$$

$$\text{or, } \sin 2\theta = \frac{2t}{1+t^2}.$$

$(1 + \tan\theta)(\cos 2\theta - \sin 2\theta) = 1.$

Let $\tan\theta = t$. Then, using the results of the first part of answer (*b*),

$$(1+t)\left(\frac{1-t^2}{1+t^2} - \frac{2t}{1+t^2}\right) = 1,$$

$$\text{or, } (1+t)\left(\frac{1-t^2-2t}{1+t^2}\right) = 1.$$

$$\therefore (1+t)(1-t^2-2t) = 1+t^2.$$

$$1 - t^2 - 2t + t - t^3 - 2t^2 = 1 + t^2,$$

$$\text{or, } -t - 3t^2 - t^3 = t^2.$$

$$\therefore t^3 + 4t^2 + t = 0,$$

$$\text{or, } t^2 + 4t + 1 = 0, \quad \text{unless} \quad t = 0.$$

$$\therefore t = \frac{-4 \pm \sqrt{16-4}}{2}, \text{ or } 0,$$

$$= \frac{-4 \pm \sqrt{12}}{2}, \text{ or } 0,$$

$$= \frac{-4 \pm 2\sqrt{3}}{2}, \text{ or } 0,$$

$$= -2 \pm \sqrt{3}, \text{ or } 0,$$

$$= -2 \pm 1{\cdot}732, \text{ or } 0.$$

$$\therefore \tan\theta = -0{\cdot}268, \text{ or } -3{\cdot}732, \text{ or } 0.$$

$$\therefore \theta = 180^\circ - 15^\circ, \text{ or } 180^\circ - 75^\circ, \text{ or } 0^\circ, \text{ or } 180^\circ.$$

Therefore the solution of the equation for θ between 0° and 180° is

$$\theta = 0^\circ, 105^\circ, 165, \text{ or } 180^\circ.$$

Q. 5. *The cathode emission I amperes, at absolute temperature T, is given by*

$$I = aT^2e^{-b/T}$$

where a, b are constants.

(*i*) *Express b in terms of I, a and T.*

(*ii*) *Derive and simplify an expression for* $\frac{dI}{dT}$.

(*iii*) *For a small rise r per cent in temperature, show that the percentage rise in emission is approximately* $\left(2 + \frac{b}{T}\right)$ *r per cent.*

A. 5. $$I = aT^2e^{-b/T}.$$

(*i*) $$e^{-b/T} = \frac{I}{aT^2},$$

$$\therefore \frac{1}{e^{b/T}} = \frac{I}{aT^2},$$

$$\text{or, } e^{b/T} = \frac{aT^2}{I},$$

Taking logarithms to base e,

$$\frac{b}{T} = \log_e \frac{aT^2}{I},$$

$$\text{or, } b = T\log_e \frac{aT^2}{I}.$$

(*ii*) $$I = aT^2e^{-b/T} = aT^2e^{-bT^{-1}}.$$

$$\frac{dI}{dT} = aT^2e^{-bT^{-1}} \times bT^{-2} + 2aTe^{-bT^{-1}},$$

$$= e^{-b/T}(ab + 2aT),$$

$$= ae^{-b/T}(b + 2T).$$

(*iii*) From the definition of a derivative,

$$\lim_{\delta T \to 0} \frac{\delta I}{\delta T} = \frac{dI}{dT}.$$

Hence, when δT is very small,

$$\frac{\delta I}{\delta T} \simeq \frac{dI}{dT}$$

$$\text{or, } \delta I \simeq \frac{dI}{dT} \times \delta T.$$

For a small rise r per cent in temperature,

$$\delta T = \frac{rT}{100}.$$

$$\therefore \delta I = ae^{-b/T}(b + 2T)\frac{rT}{100}.$$

Expressed as a percentage change in the emission I amperes,

$$\delta I = \frac{ae^{-b/T}(b+2T)\frac{rT}{100}}{aT^2e^{-b/T}} \times 100 \text{ per cent}$$

$$= \frac{(b+2T)r}{T} \text{ per cent},$$

$$= \left(\frac{b}{T} + 2\right)r \text{ per cent.} \qquad \text{Q.E.D.}$$

Q. 6. (*a*) *State and prove a formula for differentiating the quotient of two functions.*

Hence or otherwise find $\frac{dy}{dx}$ *when* $y = \frac{2x-5}{(x-3)^2}$, *giving the result in its lowest terms.*

Sketch the graph of the function, clearly labelling any asymptotes and turning points on the graph.

(*b*) *If* $y = e^{-3x}\cos 2x$ *show that*

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 13y = 0.$$

A. 6. (*a*) Suppose that u and v are two functions of x and that $y = \frac{u}{v}$. Then the formula for differentiating y may be stated as,

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}.$$

To prove this, let x increase by a small amount δx and let δu, δv and δy be the corresponding changes in u, v and y, respectively.

$$\text{Then, } y + \delta y = \frac{u + \delta u}{v + \delta v},$$

$$\text{or, } \delta y = \frac{u+\delta u}{v+\delta v} - y.$$

$$\therefore \delta y = \frac{u+\delta u}{v+\delta v} - \frac{u}{v},$$

$$= \frac{v(u+\delta u) - u(v+\delta v)}{v(v+\delta v)},$$

$$= \frac{v\,\delta u - u\,\delta v}{v(v+\delta v)}.$$

$$\therefore \frac{\delta y}{\delta x} = \frac{v\frac{\delta u}{\delta x} - u\frac{\delta v}{\delta x}}{v(v+\delta v)}.$$

As $\delta x \to 0$, $\frac{\delta u}{\delta x} \to \frac{du}{dx}$, $\frac{\delta v}{\delta x} \to \frac{dv}{dx}$ and $\frac{\delta y}{\delta x} \to \frac{dy}{dx}$.

Hence, in the limit when δx is zero.

$$\frac{dy}{dx} = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}. \qquad \text{Q.E.D.}$$

$$y = \frac{2x-5}{(x-3)^2},$$

Applying the above rule gives,

$$\frac{dy}{dx} = \frac{(x-3)^2 2 - (2x-5)2(x-3)}{(x-3)^4},$$

$$= \frac{(x-3)(2x-6-4x+10)}{(x-3)^4},$$

$$= \frac{-2x+4}{(x-3)^3}.$$

$$y = \frac{2x-5}{(x-3)^2}.$$

To sketch the graph of this function it is necessary to determine its salient features without tabulating a wide range of values and plotting the graph accurately.

It is first clear that, when $x = 2{\cdot}5$, the numerator is zero and hence $y = 0$. Also, when x becomes large in magnitude,

$$y \simeq \frac{2x}{x^2}, \quad \text{or,} \quad \frac{2}{x}.$$

and hence, as $|x| \to \infty$, $\;y \to 0$.

Since $(x-3)^2$ is always positive, y must be negative when x is less than $2{\cdot}5$ and positive when x is greater than $2{\cdot}5$.

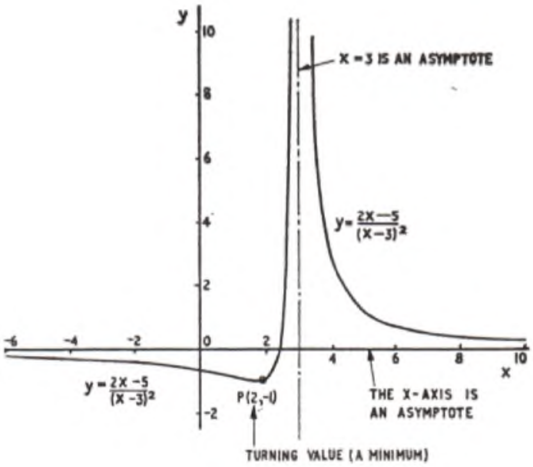
Again, as $x \to 3$, $(x-3) \to 0$ and hence $y \to \infty$. Since y is positive when x is greater than $2{\cdot}5$, $y \to +\infty$ and hence the curve will ascend rapidly towards infinity either side of the line $x = 3$, which is therefore an asymptote.

Turning points occur when $\dfrac{dy}{dx} = 0$,

$$\frac{dy}{dx} = \frac{-2x+4}{(x-3)^3} = 0 \text{ for turning point.}$$

$$\therefore\; x = 2.$$

For $x = 2$, $y = \dfrac{4-5}{(-1)^2} = -1$ and this must be a minimum value because the graph ascends towards $+\infty$ at $x = 3$.



The graph is shown in the sketch with the asymptotes and turning value (a minimum) labelled.

(*b*) $\quad y = e^{-3x}\cos 2x.$

$$\frac{dy}{dx} = e^{-3x}(-\sin 2x)2 + e^{-3x}(-3)\cos 2x,$$

$$= -e^{-3x}(2\sin 2x + 3\cos 2x).$$

$$\frac{d^2y}{dx^2} = -e^{-3x}\{(2\cos 2x)2 - (3\sin 2x)2\}$$
$$\qquad -e^{-3x}(-3)\{2\sin 2x + 3\cos 2x\},$$

$$= \cos 2x(-4e^{-3x} + 9e^{-3x}) + \sin 2x(6e^{-3x} + 6e^{-3x}),$$

$$= 5e^{-3x}\cos 2x + 12e^{-3x}\sin 2x.$$

$$\therefore\; \frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 13y$$

$$= 5e^{-3x}\cos 2x + 12e^{-3x}\sin 2x$$
$$\qquad - 6e^{-3x}(2\sin 2x + 3\cos 2x) + 13e^{-3x}\cos 2x$$

$$= \cos 2x e^{-3x}(5 - 18 + 13) + \sin 2x e^{-3x}(12 - 12),$$

$$= 0. \qquad \text{Q.E.D.}$$

Q. 7. (a) State and prove from first principles an expression for $\dfrac{d}{dx}(\tan x)$, *x measured in radians.*

(b) The power p watts at any instant t seconds in a reactive circuit is given by

$$p = E_m I_m \sin \omega t \sin(\omega t - \phi).$$

Prove that the maximum value of p is $E_m I_m \cos^2 \dfrac{\phi}{2}$, *and state one value of t at which it occurs.*

A. 7. (*a*) $\qquad \dfrac{d}{dx}(\tan x) = \sec^2 x.$

$$\text{Let } y = \tan x, = \frac{\sin x}{\cos x}.$$

Let x increase by a small amount δx and let δy be the corresponding change in y.

$$\text{Then } y + \delta y = \frac{\sin(x+\delta x)}{\cos(x+\delta x)}.$$

$$\text{or, } \delta y = \frac{\sin(x+\delta x)}{\cos(x+\delta x)} - \frac{\sin x}{\cos x},$$

$$\text{since } y = \frac{\sin x}{\cos x}.$$

$$\therefore\; \delta y = \frac{\sin(x+\delta x)\cos x - \cos(x+\delta x)\sin x}{\cos(x+\delta x)\cos x}.$$

Now $\sin(A - B) = \sin A\cos B - \cos A\sin B$.

$$\therefore\; \sin(x+\delta x)\cos x - \cos(x+\delta x)\sin x$$
$$= \sin(x + \delta x - x),$$
$$= \sin \delta x.$$

$$\text{Hence, } \delta y = \frac{\sin \delta x}{\cos(x+\delta x)\cos x},$$

$$\therefore\; \frac{\delta y}{\delta x} = \frac{\dfrac{\sin \delta x}{\delta x}}{\cos(x+\delta x)\cos x}.$$

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x},$$

$$\text{and } \lim_{\delta x \to 0}\left(\frac{\sin \delta x}{\delta x}\right) = 1.$$

$$\therefore\; \frac{dy}{dx} = \frac{1}{\cos x \cos x},$$

$$= \sec^2 x.$$

$$\text{Thus, } \frac{d}{dx}(\tan x) = \sec^2 x.$$

(*b*) $\qquad p = E_m I_m \sin \omega t \sin(\omega t - \phi).$

From the product-to-sum trigonometric formula,

$$2\sin A\sin B = \cos(A - B) - \cos(A + B).$$

$$\therefore\; \sin \omega t \sin(\omega t - \phi) = \tfrac{1}{2}\{\cos(\omega t - \omega t + \phi) - \cos(\omega t + \omega t - \phi)\},$$
$$= \tfrac{1}{2}\{\cos \phi - \cos(2\omega t - \phi)\}.$$

$$\therefore\; p = \frac{E_m I_m}{2}\{\cos\phi - \cos(2\omega t - \phi)\}.$$

$$\therefore\; \frac{dp}{dt} = \frac{E_m I_m}{2} 2\omega \sin(2\omega t - \phi).$$

For maximum or minimum values $\frac{dp}{dt} = 0$.

$$\therefore \sin(2\omega t - \phi) = 0,$$

$$\text{or, } 2\omega t - \phi = n\pi \quad \text{where} \quad n = 0, 1, 2 \ldots$$

$$\therefore t = \frac{\phi + n\pi}{2\omega}.$$

$$\frac{d^2p}{dt^2} = 2\omega^2 E_m I_m \cos(2\omega t - \phi).$$

When n is zero or even,

$$\cos(2\omega t - \phi) = \cos n\pi = 1,$$

and hence $\frac{d^2p}{dt^2}$ is positive, assuming $E_m I_m$ to be positive. Thus for a zero or any even value of n, a minimum value of p occurs.

When n is odd,

$$\cos(2\omega t - \phi) = \cos n\pi = -1,$$

and hence $\frac{d^2p}{dt^2}$ is negative and a maximum value of p occurs.

Taking $n = 1$,

$$2\omega t = \phi + \pi.$$

$$\therefore p_m = \frac{E_m I_m}{2}\{\cos\phi - \cos(\phi + \pi - \phi)\},$$

$$= \frac{E_m I_m}{2}(\cos\phi + 1),$$

$$= \frac{E_m I_m}{2}\left(2\cos^2\frac{\phi}{2} - 1 + 1\right),$$

$$\therefore \underline{p_m = E_m I_m \cos^2\frac{\phi}{2}}. \qquad \text{Q.E.D.}$$

One value of t at which the maximum value occurs is $\underline{\frac{\phi + \pi}{2\omega}}$.

Q. **8.** *(a) Evaluate*

(*i*) $\int_0^{\pi/2} 7 \sin 3x dx$. (*ii*) $\int_0^3 \frac{dx}{(4-x)^2}$.

(b) State without proof Simpson's Rule for evaluating approximately the area under a curve in terms of an odd number of equally-spaced ordinates.

The voltage response to frequency of a tuned circuit is as follows:

Frequency kHz (kc/s)	245	246	247	248	249	250	251	252	253
Volts	35	46	63	83·5	96	100	96	83·5	63

Use Simpson's Rule to estimate the mean voltage response over this range of frequency.

A. 8. (*a*) (*i*) $$\int_0^{\pi/2} 7\sin 3x\, dx = 7\left[-\frac{\cos 3x}{3}\right]_0^{\pi/2},$$

$$= -\frac{7}{3}\left(\cos\frac{3\pi}{2} - \cos 0\right),$$

$$= -\frac{7}{3}(0 - 1),$$

$$= \underline{\frac{7}{3}}.$$

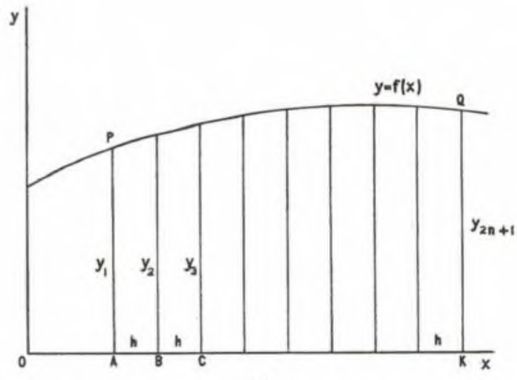
(*ii*) $$\int_0^3 \frac{dx}{(4-x)^2} = \int_0^3 (4-x)^{-2}\, dx,$$

$$= \left[\frac{-(4-x)^{-1}}{-1}\right]_0^3,$$

$$= \frac{1}{4-3} - \frac{1}{4},$$

$$= \underline{\frac{3}{4}}.$$

(*b*) Referring to sketch (*a*), suppose the area under the curve $y = f(x)$ between points P and Q is required. If the ordinates through P and



(*a*)

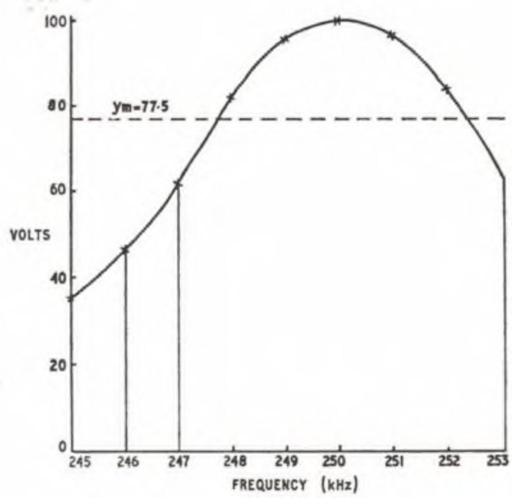
Q intersect the x-axis at A and K as shown, the line AK is divided into an even number, $2n$, of equal parts. Let the ordinates, of which there will be an odd number, be $y_1, y_2 \ldots y_{2n+1}$. Then Simpson's Rule states that the area under the curve from P to Q will be approximately equal to

$$\frac{h}{3}\{y_1 + y_{2n+1} + 2(y_3 + y_5 + \ldots) + 4(y_2 + y_4 + \ldots)\},$$

where $h = AB = BC = \ldots$ is the width of each strip of area. This may be stated in words as:
One third the width of each strip, multiplied by the sum of the first and last ordinates plus twice the sum of the other odd ordinates plus four times the sum of the even ordinates.

The graph of voltage against frequency for the tuned circuit is shown in sketch (*b*).

The values of frequency given total nine, which is an odd number, and hence divide the range into the required even number of strips to apply Simpson's Rule. Although the accuracy of this rule increases with the number of strips there is little point in increasing the number because the gain in accuracy would be offset by reading errors from the graph. The values of the nine ordinates are already given in the table and hence no error will arise from the graph. It is not really necessary to plot the graph but this has been done partly for illustration and partly because it is in any case desirable.



(*b*)

Applying Simpson's Rule, the area under the graph between 245 kHz and 253 kHz is given by,

$$\text{Area} = \frac{1{,}000}{3}\{35 + 63 + 2(63 + 96 + 96) + 4(46 + 83\cdot5 + 100 + 83\cdot5)\}$$

$$= \frac{1{,}000}{3}\{98 + 2 \times 255 + 4 \times 313\},$$

$$= \frac{1{,}000}{3}(98 + 510 + 1{,}252),$$
$$= \frac{1{,}000}{3} \times 1{,}860,$$
$$= 620{,}000.$$

If y_m represents the mean voltage over the same frequency range, then

$$\text{Area under the curve} = 620{,}000 = y_m \times (253 - 245) \times 1{,}000,$$
$$\text{or, } y_m = \frac{620}{8} \text{ volts},$$
$$= \underline{77{\cdot}5} \text{ volts}.$$

This value is shown in sketch (*b*).

Q. 9. *A capacitance C farads, initially charged to Q coulombs, discharges through a resistance R ohms.*

At an instant t seconds the charge q coulombs is given by

$$\frac{dq}{dt} = -\frac{q}{RC}.$$

Integrating $\frac{dt}{dq}$*, obtain t as a function of q, and hence derive q in terms of t.*

Calculate the time required to lose half the initial charge, where $R = 10^7$ *and* $C = 2 \times 10^{-6}$.

A. 9.
$$\frac{dq}{dt} = -\frac{q}{RC},$$
$$\text{or, } dt = -\frac{RC}{q}\,dq.$$
$$\therefore \int dt = \int -\frac{RC}{q}\,dq + c,$$

where c is a constant of integration.

$$\therefore t = -RC\log_e q + c.$$
$$\text{When } t = 0, \quad q = Q.$$
$$\therefore 0 = -RC\log_e Q + c,$$
$$\text{or, } c = RC\log_e Q.$$
$$\therefore t = -RC\log_e q + RC\log_e Q,$$
$$= -RC(\log_e q - \log_e Q).$$
$$\therefore \log_e\frac{q}{Q} = -\frac{t}{RC},$$
$$\text{or, } \frac{q}{Q} = e^{-t/RC}.$$
$$\therefore \underline{q = Q\,e^{-t/RC}}.$$

When $q = \frac{Q}{2}$, half the initial charge has been lost.

$$\therefore e^{-t/RC} = 0{\cdot}5.$$
$$\therefore -\frac{t}{RC} = \log_e 0{\cdot}5,$$
$$= -0{\cdot}6931.$$
$$\therefore t = 0{\cdot}6931\,RC,$$
$$= 0{\cdot}6931 \times 10^7 \times 2 \times 10^{-6},$$
$$= \underline{13{\cdot}862 \text{ seconds.}}$$

Q. 10. (*a*) *The equation of a cam profile in polar coordinates is*

$$r = 5(3 + 2)\cos\theta.$$

Sketch this curve, marking the points A, B, C, D *where*

$$\theta = 0,\ \frac{\pi}{3},\ \pi,\ \frac{3\pi}{2},$$

radians respectively.

(*b*) (*i*) *If* $z = r(\cos\theta + j\sin\theta) = r\angle\theta$ *prove that*

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\angle\theta_1 - \theta_2.$$

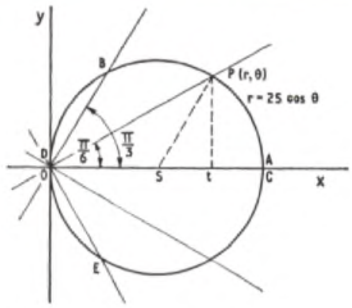
(*ii*) *Express* $z = \frac{1 - j\sqrt{3}}{1 + j}$ *in the polar form* $r\angle\theta$ *and find in the same form the two square roots of z.*

A. 10. (*a*)
$$r = 5(3 + 2)\cos\theta.$$
$$= 25\cos\theta.$$

Note 1.—It seems likely that there was a misprint in this equation, as, in addition to the improbable form in which the equation is set out, it is the equation to a circle, as shown later in this answer. Although a cam, mounted eccentrically, could be circular in shape, it would be an unusual shape for such a purpose. However, in the absence of other information, the question can only be dealt with on the basis of $r = 25\cos\theta$.

The curve may be sketched from the following values:

$\theta°$	0	30	60	90	120	150	180
$\cos\theta$	1	0·866	0·5	0	−0·5	−0·866	−1
$r = 25\cos\theta$	25	21·65	12·5	0	−12·5	−21·65	−25



The points corresponding to $\theta = 0°$, $30°$ and $60°$ are shown as A, P and B in the sketch. When $\theta = 90$, $r = 0$ and the origin is therefore a point on the curve. When $\theta = 120°$, r is negative and hence must be measured off in the negative direction, i.e. backwards along the radius, to give the point E. Similarly the remaining radii for angles up to 180° are measured negatively to give a closed curve at A. The values of θ from 180° to 360° give values of r which repeat those shown in the reverse order and hence trace round the closed curve a second time.

The points A, B, C and D are marked on the sketch, corresponding to $\theta = 0$, $\frac{\pi}{3}$ (60°), π (180°) and $\frac{3\pi}{2}$ (270°) respectively.

The closed curve is actually a circle, whose centre is at $s\left(\frac{25}{2}, 0°\right)$ and this may be proved as follows:

Note 2.—The following proof is not required in answer to the question but has been included for the benefit of the student—see Note 1.

Consider any point P (r, θ) on the curve and drop the perpendicular Pt on to the x axis. Join Ps.

$$\text{Now } Pt = y = r\sin\theta,$$
$$\text{and } ts = Ot - Os,$$
$$= r\cos\theta - \frac{25}{2}.$$

From the right-angled triangle Pst,

$$Ps^2 = st^2 + Pt^2,$$
$$= \left(r\cos\theta - \frac{25}{2}\right)^2 + r^2\sin^2\theta,$$
$$= r^2\cos^2\theta - 25r\cos\theta + \frac{625}{4} + r^2\sin^2\theta,$$
$$= r^2(\cos^2\theta + \sin^2\theta) - 25r\cos\theta + \frac{625}{4},$$
$$= r^2 - 25r\cos\theta + \frac{625}{4},$$
$$= r(r - 25\cos\theta) + \frac{625}{4}.$$

But $r - 25\cos\theta = 0$.

$$\therefore Ps^2 = \frac{625}{4} \text{ which is constant.}$$

Thus P traces out the circumference of a circle with centre s and radius $\frac{25}{2}$. Thus $r = 25\cos\theta$ is the polar equation of a circle.

(*b*) (*i*) $z = r(\cos\theta + j\sin\theta) = r\angle\theta.$

Then $z_1 = r_1(\cos\theta_1 + j\sin\theta_1) = r_1\angle\theta_1$,

and $z_2 = r_2(\cos\theta_2 + j\sin\theta_2) = r_2\angle\theta_2$,

$$\therefore \frac{z_1}{z_2} = \frac{r_1(\cos\theta_1 + j\sin\theta_1)}{r_2(\cos\theta_2 + j\sin\theta_2)},$$

$$= \frac{r_1}{r_2}\frac{(\cos\theta_1 + j\sin\theta_1)(\cos\theta_2 - j\sin\theta_2)}{(\cos\theta_2 + j\sin\theta_2)(\cos\theta_2 - j\sin\theta_2)},$$

$$= \frac{r_1}{r_2}\left(\frac{\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2 + j\sin\theta_1\cos\theta_2 - j\cos\theta_1\sin\theta_2}{\cos^2\theta_2 + \sin^2\theta_2}\right).$$

Therefore from the addition formulae for $\cos(A - B)$ and $\sin(A - B)$.

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left(\frac{\cos(\theta_1 - \theta_2) + j\sin(\theta_1 - \theta_2)}{1}\right).$$

$$= \frac{r_1}{r_2}\angle\theta_1 - \theta_2 \text{ in polar form.} \qquad \text{Q.E.D.}$$

(*ii*)
$$z = \frac{1 - j\sqrt{3}}{1 + j},$$

$$= \frac{(1 - j\sqrt{3})(1 - j)}{(1 + j)(1 - j)},$$

$$= \frac{1 - j\sqrt{3} - j - \sqrt{3}}{1 + 1},$$

$$= \frac{1 - \sqrt{3}}{2} - \frac{j(1 + \sqrt{3})}{2},$$

$$= \sqrt{\tfrac{1}{4}\{(1 - \sqrt{3})^2 + (1 + \sqrt{3})^2\}}\ \angle \tan^{-1}\frac{-\left(\frac{1 + \sqrt{3}}{2}\right)}{\frac{1 - \sqrt{3}}{2}},$$

$$= \tfrac{1}{2}\sqrt{1 - 2\sqrt{3} + 3 + 1 + 2\sqrt{3} + 3}\ \angle \tan^{-1}\frac{\sqrt{3} + 1}{\sqrt{3} - 1},$$

$$= \sqrt{2}\angle \tan^{-1} 3\cdot7321,$$

$$\text{or, } z = \sqrt{2}\angle 255°,$$

since the angle must be in the 3rd quadrant.

Generally, $z = \sqrt{2}\{\cos(255° + n\,360°) + j\sin(255° + n\,360°)\}$,

and $z^{1/2} = 2^{1/4}\left\{\cos\left(\frac{255° + n\,360°}{2}\right) + j\sin\left(\frac{255° + n\,360°}{2}\right)\right\}$.

where $n = 0, 1, 2 \ldots$

The two square roots of z are given by $n = 0$ and $n = 1$. Further values of n merely repeating previously-determined values of $z^{1/2}$.

When $n = 0$,

$$z^{1/2} = 2^{1/4}\{\cos 127\tfrac{1}{2}° + j\sin 127\tfrac{1}{2}°\},$$
$$= 1\cdot19\angle 127\tfrac{1}{2}°.$$

When $n = 1$,

$$z^{1/2} = 2^{1/4}\{\cos 307\tfrac{1}{2}° + j\sin 307\tfrac{1}{2}°\},$$
$$= 1\cdot19\angle 307\tfrac{1}{2}°.$$

Therefore the two square roots of z are $1\cdot19\angle 127\tfrac{1}{2}°$ and $1\cdot19\angle 307\tfrac{1}{2}°$.

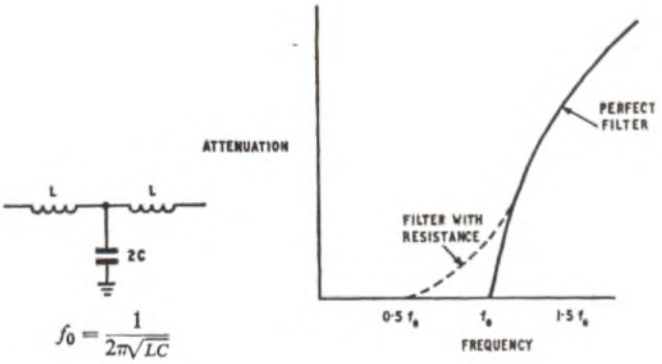
TELEGRAPHY C, 1968

Students were expected to attempt any six questions.

Q. 1. *Draw an approximate attenuation/frequency graph for the low-pass filter (No. 4B) used in teleprinter circuits. What is the purpose of this filter? Why is a 200-ohm resistor connected in series with the input to the filter?*

With the aid of simple diagrams, show the various points at which the low-pass filter would be inserted in a telex exchange system.

A. 1. The attenuation/frequency graph for the low-pass filter used in teleprinter circuits, is shown in sketch (*a*).

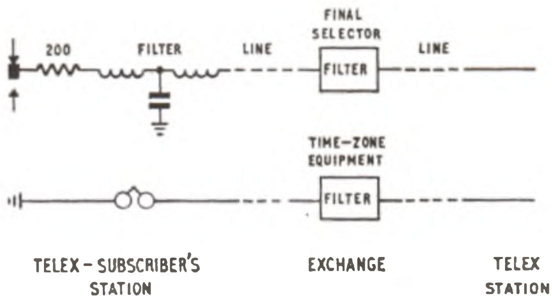


(*a*)

A telegraph signal, consisting of square-topped reversals, may be resolved into a series of sine curves representing the fundamental frequency of the reversals, together with the odd harmonics to infinity, of the fundamental frequency. Those harmonics which lie within the speech band could induce crosstalk and noise into adjacent telephone circuits if the telegraph circuit were routed in a telephone cable. The low-pass filter is normally inserted between the point of modulation and the line, and is provided to attenuate the audio-frequency components of the telegraph signals. For the filter used in the British Post Office network the cut-off frequency is about 140 Hz.

The design impedance of the filter at 50 bauds (25 Hz) is about 1,100 ohms, a compromise value giving the best average match to the impedance of the various types of line plant to which it may be connected. The impedance of the modulating source consists essentially of the 100-ohm bulb resistor in the signalling supply, which does not match the filter impedance. This mismatch induces strong oscillatory effects in the filter, so that the output waveform is severely distorted. With distant terminal equipment having mainly resistive impedance and high sensitivity, the telegraph signals would be mutilated and errors increased. To reduce the tendency of the filter circuit to oscillate, the impedance of the source is increased by the insertion of a 200-ohm resistor between the modulating source and the filter; this value was chosen as the minimum value acceptable without jeopardizing the transmission limits in use on the telex network.

In a telex exchange, the filters are associated with each final selector and with each time-zone equipment, as shown in sketch (*b*). These



(*b*)

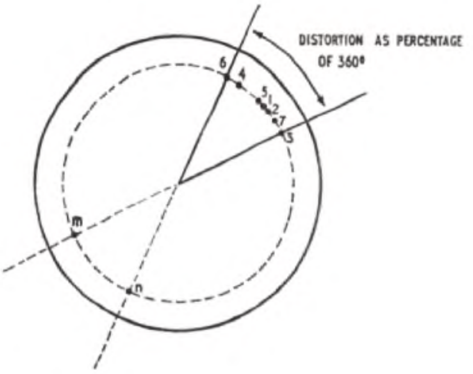
locations were chosen because the majority of telex subscribers are connected to the exchange by a physical circuit, and it is more economical to provide filters for each circuit when it is in use for a call than to provide two filters for each subscriber's circuit.

Filters are also inserted in the send wire at the subscriber's installation and on the send wire at m.c.v.f. terminals if the telegraph signals are transmitted over telephone cables.

Q. 2. *Sketch a typical display observed on a synchronous TDMS when receiving 7-unit signals which are affected by fortuitous distortion. Indicate how the distortion value is read upon the screen. How would the display be affected if the signals originated from a 7½-unit transmitter?*

What degree of fortuitous distortion would you regard as acceptable for a single voice-frequency channel? What other forms of signal distortion have to be taken into account?

A. 2. The sketch shows a typical display observed on a synchronous telegraph distortion-measuring set (TDMS) when receiving 7-unit signals affected by fortuitous distortion. The incoming signals cause a bright spot to appear on a circular trace on a cathode-ray oscilloscope at each transition, the time of revolution of the trace being adjusted to the average time of one element of the received signal. The persistence of the glow of the oscilloscope permits successive trains of signals to be observed, and the amount of distortion is assessed as the sum of the time that the earliest transition is early and the latest



transition is late; this is normally expressed as a percentage of a unit element.

Fortuitous distortion arises from random influence on the apparatus or circuit. It may be caused by mechanical faults in the receiving or transmitting apparatus or by electrical interference from neighbouring circuits in a telephone cable or m.c.v.f. system. This type of distortion is characterized by the random nature of the distortion present at each transition.

If the signals were received from a 7½-unit source, each alternate set of spots on the TDMS would occur 180° out of phase with the preceding set of spots; that is, two displays would be seen on opposite sides of the trace. The reason for this is that the circular trace makes one revolution for each element length; the half unit of the preceding cycle will cause the trace to register the succeeding transitions in the sector m-n on the sketch.

Two other forms of distortion must be taken into account when assessing the performance of a single voice-frequency channel; these are bias distortion and characteristic distortion.

Bias distortion is the distortion which results from the lengthening (or shortening) of the signals of start polarity and the corresponding shortening (or lengthening) of the signals of stop polarity. The bias is said to occur on that signal which is lengthened; that is, if the signals of start polarity are lengthened then the circuit is said to have spacing bias. This type of distortion may be corrected by adjusting the equipment until the received signals are neutral.

Characteristic distortion arises from the inherent characteristics of the circuit, including the sending and receiving terminations; this type of distortion occurs consistently with any given series of signal elements. It may be measured at the output of the receive relay when perfect signals are applied to the circuit, with the circuit free from bias and fortuitous distortion.

The acceptable limit of the degree of isochronous distortion on standardized text for a single voice-frequency channel is 10 per cent; of this, some 3–4 per cent could be regarded as due to fortuitous distortion.

Q. **3.** *For a frequency-modulated voice-frequency channel draw approximate graphs to indicate the following characteristics:*

(*a*) *the relationship between the d.c. input and the output frequency of the oscillator-modulator,*

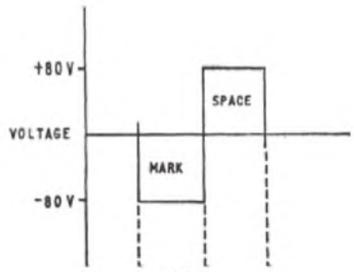
(*b*) *the relationship between the input frequency and the d.c. output of the discriminator.*

The graphs should refer to a mark element followed by a space element. Comment on the shapes of the graphs.

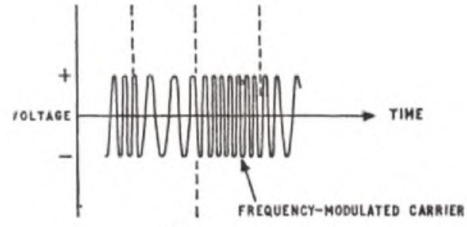
A. 3. The sketches (*a*), (*b*) and (*c*) show the relationship between the d.c. input and the output frequency of the oscillator-modulator for a frequency-modulated voice-frequency channel. The modulating signal varies the frequency of the carrier wave above and below its mean value, according to the polarity of the input signal; the amplitude of the modulated wave remains constant at the amplitude of the carrier wave. The carrier frequency is not modulated directly by the d.c. signal; the frequency of the carrier is increased or decreased by ±30 Hz, depending on the polarity of the input signal.

In order to prevent distortion caused by phase-shift when an abrupt change occurs on the input, the equipment responds with the smooth transition shown in sketch (*c*). This shows the characteristic change in frequency for channel No. 4 of a m.c.v.f. system and gives the response of the oscillator to changes in the current flowing in the modulator circuit.

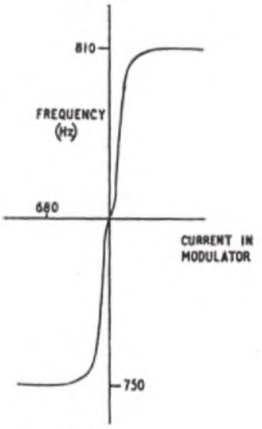
Sketch (*d*) shows the relationship between the input frequency and the d.c. output of the discriminator. The received signals are applied to the receive filter, where the required frequencies are selected; the filter output is connected through three stages of limitation to a transformer. Two tuned circuits are connected across one of the secondary windings of the transformer: one tuned circuit responds to the higher shifted frequency and the other tuned circuit responds to the lower frequency. The tuned circuits select the appropriate frequency and the outputs are amplified and applied to a polarized relay to give the d.c. output signal. The tuned circuits resonate at 50–60 Hz above and below the mid-band frequency to give a linear relationship over the range ±30 Hz for 50-baud signals.



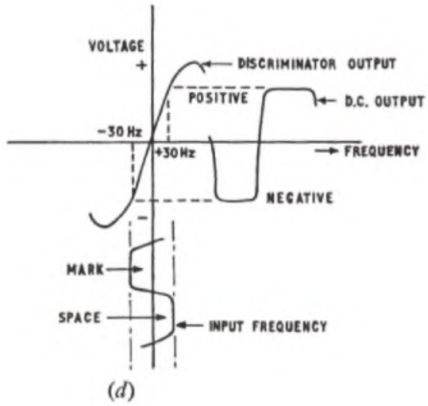
(*a*)



(*b*)



(*c*)

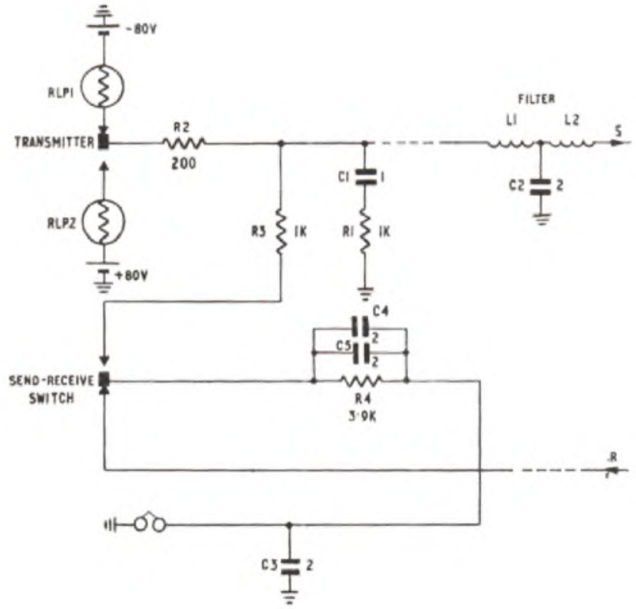


(*d*)

Q. **4.** *The terminal equipment at a telex subscriber's station includes a number of resistors and capacitors. Sketch the basic circuit diagram sufficiently to show the inclusion of five of each of these components and explain the function of each one.*

A. 4. The sketch shows the basic circuit diagram of a telex subscriber's station. The functions of the various resistors and capacitors are described below.

*Resistor R*1 *and capacitor C*1 act as a spark quench for the protection of the transmitter contacts. When the teleprinter is connected to a distant receive electromagnet the inductance of this magnet, together with the inductance of the local electromagnet and the inductance of the line, cause a considerable voltage to be generated when the transmitter tongue moves and disconnects the signalling potential. This induced voltage could form an arc between the transmitter tongue and the contact, causing the contacts to wear and become pitted. The spark-quench capacitor and resistor present a fairly low-impedance



path so that the induced current is dissipated in the form of heat in the resistor.

*Resistor R*2, of value 200 ohms, is connected in the transmit wire to increase the impedance of the source of modulation, so that the tendency of the low-pass filter circuit to oscillate is reduced. If this resistor were not included in the circuit, the filter would tend to oscillate when the line was connected to a distant high-impedance termination, and the teleprinter signals would be mutilated.

*Resistor R*3 connects the transmitter to the local receive magnet, and provides the local-record circuit. When the transmitter operates in response to the depression of one of the keys on the key-board, the send-receive switch is operated mechanically. Resistor R3 connects the transmit line to the receive electromagnet, and in conjunction with the signal-shaping network, provides the correct termination for signal reception.

*Capacitor C*2. To prevent interference from teleprinter signals affecting other circuits in an underground cable, a low-pass filter is provided in the send wire. The filter consists of capacitor C2 and the two inductors, and has a cut-off frequency of about 140 Hz, thus effectively suppressing the higher harmonics of the teleprinter signals.

*Capacitor C*3. When the receive magnet operates in response to incoming signals, the tongue of the magnet moves in the magnetic field of the polarizing magnets and the relay coils. The induced voltage in the line coils of the receive magnet is reflected into the receive line and may cause interference in adjacent circuits in the same underground cable. The capacitor suppresses the induced voltages.

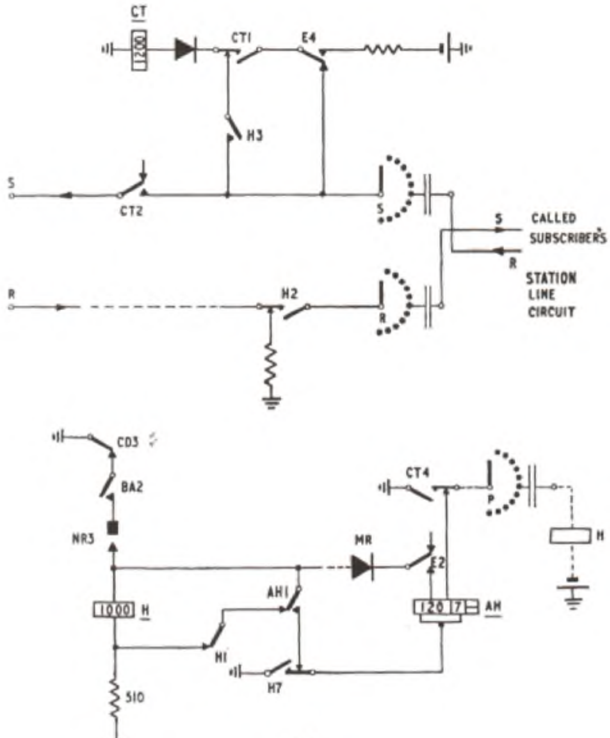
*Capacitors C*4 *and C*5 *and resistor R*4 form a combination known as the signal-shaping network. The resistor is provided to increase the speed of operation of the receive relay. The time required for the current to reach its final value in the electromagnet depends upon the ratio of the inductance of the circuit to the resistance; as the resistance increases, the time for the current to reach its maximum value decreases. This resistance has, therefore, to have as high a value as possible, depending upon the signalling voltage and the current required to operate the magnet. The capacitor also increases the instantaneous flow of current in the electromagnet when the line potential changes from positive to negative. The capacitor interacts with the inductance of the electromagnet to cause the current to build up in excess of the steady current. The amplitude of this oscillation is increased as the capacitance is increased, and the optimum arrangement has been found by experiment to consist of a capacitance of 2 μF. This value, however, is increased to 4 μF to overcome the effect of the capacitor C3.

*Bulb resistors RLP*1 *and* 2 are designed to limit the line current under fault conditions. If the line is faulty and is connected to earth potential, the current flowing in the resistors causes the filament of each bulb to become hot. The metal of which the filament is made has a non-linear temperature coefficient, i.e. as the temperature increases, the resistance increases more rapidly, and the line current is limited to about 70 mA.

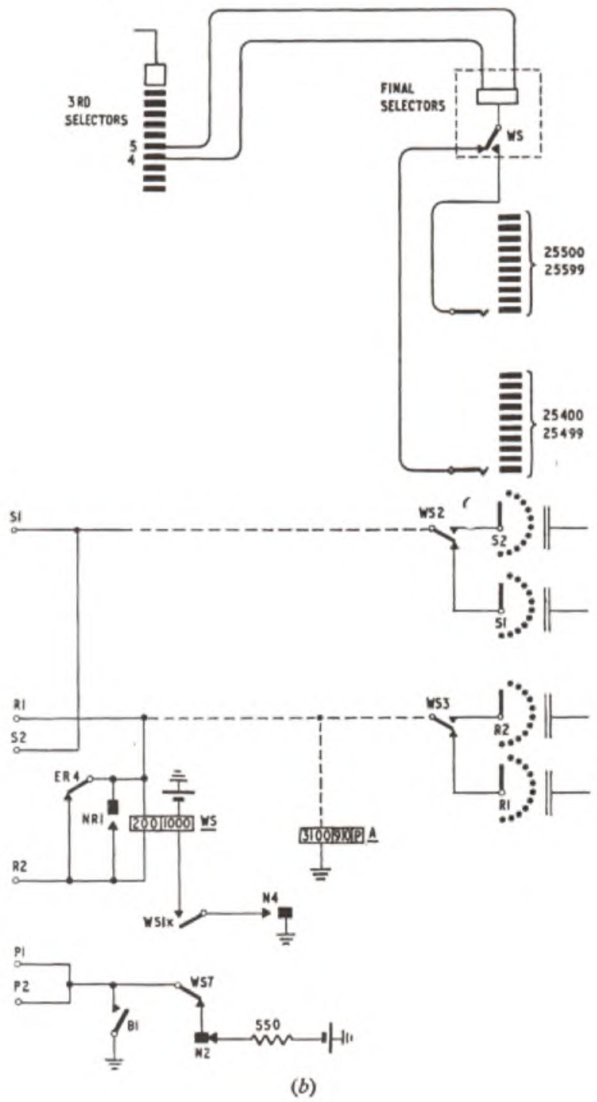
Q. 5. *With the aid of a diagram describe how, in a 200-outlet final selector, the wanted telex subscriber's line is tested on completion of dialling. Explain how the correct set of wipers is selected.*

If the wanted line is found to be free, what signal is used and how is it applied to call the subscriber's station?

A. 5. The sketch (*a*) shows the testing circuit for a 200-outlet final selector. At the conclusion of the rotary pulse-train, i.e. on completion of dialling, relay CD releases slowly and, at contact CD3, connects



(*a*)



(*b*)

earth potential to the P wiper through contacts BA2, NR3, MR, E2, the coils of relay AH in series, and contact CT4. If the called subscriber is disengaged, negative potential is connected to the contact on the P arc, and relay AH operates. Contact AH1 short-circuits the 120-ohm coil of relay AH, but the relay remains operated over the 7-ohm coil; the 7-ohm earth potential on the P-arc contact prevents a second final selector switching to the same subscriber's outlet, as relay AH in the second final selector will not operate in parallel with that of the first. Contact AH1 also disconnects the short-circuit on relay H; relay H operates to earth potential at CD3 and contact H7 holds relay AH. Contact H2 extends the negative-potential calling signal from the incoming R-wire to seize the equipment in the called circuit.

If the called-subscriber's line is engaged, the service signal OCC is connected to the calling equipment.

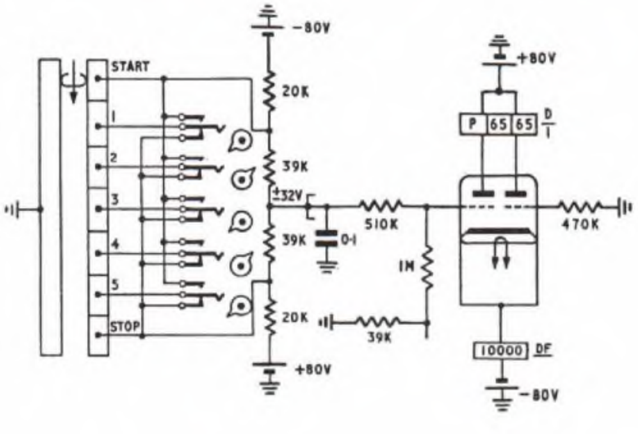
If the called-subscriber's line is not engaged, the calling signal connected at contact H2 causes negative potential to be returned on the R-wire from the subscriber's equipment to the S-wire at the final selector. Relay CT operates to this call-connected signal, and relays the signal to the time zone metering equipment via contact CT2. This causes the service signal WRU to be transmitted to the called-subscriber's teleprinter, which responds by transmitting the answer-back code to the calling subscriber.

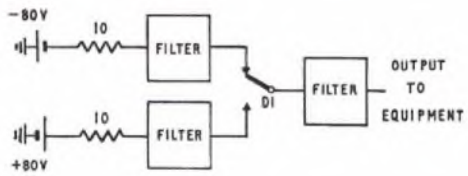
The sketch (*b*) shows the trunking scheme and the final-selector circuit by which the appropriate group of 100 final-selector numbers is selected. If the selector is seized over the S1, R1, P1 input, relay A operates and the call is extended over the S1, R1, P1 wipers to the selected bank contacts. If, however, the selector is seized over the S2, R2, and P2 wires, relay A pulses as before to the vertical pulse-train, but, during the pause before the next selection digits are received, relay ER operates and contact ER4 allows relay WS to operate in series with relay A. Contact WS1 ensures that relay WS remains operated until the off-normal springs, N4, are disconnected when the selector is released. The rotary off-normal contacts, NR1, ensure that the 200-ohm coil of relay WS is effectively short-circuited from the pulsing circuit after the first rotary step. Contacts of relay WS extend the circuit over the wipers S2, R2, P2 and PX2 to the selected bank contacts.

Q. 6. With the aid of a circuit diagram describe how a printed service signal (e.g. OCC) is generated and distributed in an automatic telex exchange. Why is it necessary to phase the signal and how is this achieved?

What is the purpose of the valve included in this equipment?

A. 6. The sketch shows the method of generating and distributing printed service-signals in an automatic telex exchange. The signals





are generated by a motor-driven signal-generator which is accurately speed-controlled. Spring-sets operated by cams, the peripheries of which are notched to provide the correct sequence of operation, apply the appropriate potentials to the distribution circuits. For printed service-signals, the outputs from the cam spring-sets are routed through the segments of a distributor; this gives the accurate timing and correct formation necessary for the teleprinter signals.

The distributor consists of concentric rings, some of which are continuous and some divided into segments. Connexion between the segmented rings and their corresponding solid rings is made by brush arms carrying carbon brushes. To minimize wear on the distributor rings and brushes, the current flowing in each brush circuit is restricted to 4 mA. The signalling current required to operate the teleprinter which is receiving the service signal is about 20 mA, and, as a number of such teleprinters may be receiving the same service signal simultaneously, the service signal output is applied to a number of parallel high-impedance valve circuits. Each valve controls a polarized relay, which distributes the service signal from a low-impedance source to a limited number of lines.

As shown in the sketch, the brushes, moving over the distributor rings, apply earth potential to the potentiometer network as determined by the position of the cams and their associated spring-sets. Earth potential connected to the −80-volt side of the potentiometer network applies a potential of +32 volts to the left-hand grid of the double-triode valve; a potential of −32 volts is applied when earth potential is connected to the +80-volt side of the potentiometer network. The right-hand grid of the triode is maintained at earth potential, and, with the incoming signal at negative potential, the right-hand triode conducts and the left-hand triode is cut-off. Current flowing in the right-hand winding of the polarized relay causes the relay tongue to connect −80 volts (or stop potential) to the line. When the incoming signal is positive, current flows in the left-hand triode only, and the polarized relay connects +80 volts (or start potential) to the line.

The output from the tongue of the polarized relay is from a low-impedance (10-ohm) source of ±80 volts; the resistors limit the surge current under maximum output conditions. The filters are provided to suppress radio-interference signals and form a spark-quench circuit to minimize damage due to arcing across the relay contacts.

The service signals are connected to each line under the control of phasing pulses. The latter are generated from the same machine as the service signals, and are accurately controlled. Phasing pulses are required so that a service signal is connected to the line at the correct instant. The demand for a service signal from any group or final selector will arrive at random, and, if the service signal were connected immediately, the distant teleprinter receive mechanism would probably start in the middle of a character and would print a mutilated signal. To prevent this, the service signal is not connected to the line until the phasing pulse indicates that the service-signal cycle is about to begin.

Q. 7. What are the essential differences between the methods of telex signalling known as type A and type B? Explain, in general terms only, how the problems caused by the co-existence of these two types of signalling are solved for the forward and backward signalling paths on outgoing and incoming international calls.

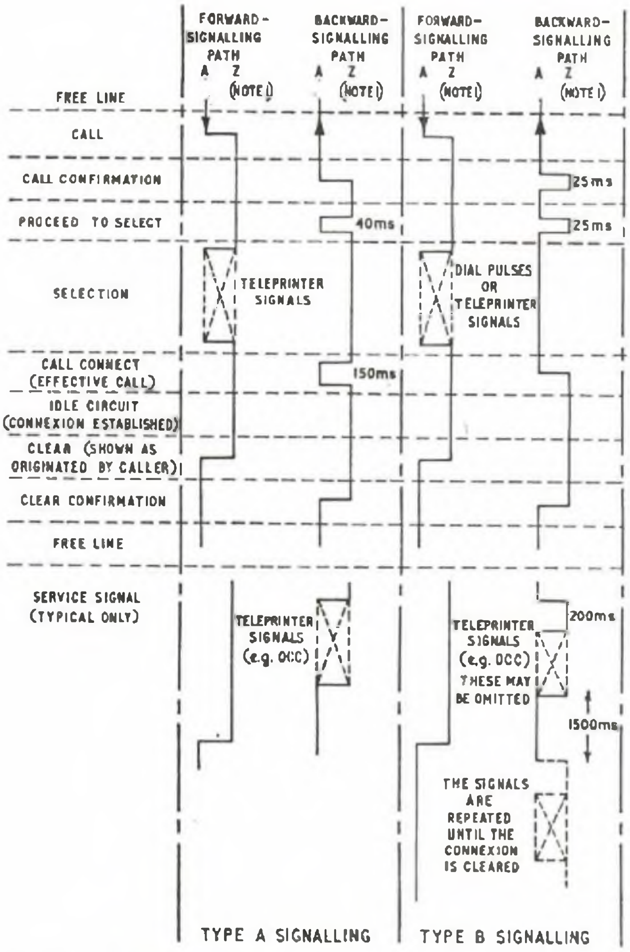
A. 7. The essential difference between the methods of telex signalling type A and type B is the condition on the backward-signalling path whilst the call is being set up; for type A this is stop polarity whilst for type B it is start polarity. The condition for type A thus permits teleprinter characters to be transmitted to the caller to indicate the progress in setting-up the call, e.g. the register code, the character V as the proceed-to-select signal, and the answer-back code to indicate that the call has been connected. For automatic systems using registers, teleprinter-selection signals may be stored as readily as dial-pulse signals, and these systems normally use type A signals. The use of teleprinter signals on both the forward and backward paths facilitates the use of all types of synchronous multiplex systems.

Type B signalling is usually associated with direct-selection systems using dial pulses, although the introduction of register systems has led to the adoption of keyboard selection in this case also. Similarly, the original pulse service-signal has been adapted to include teleprinter-code signals to indicate the reason for the failure of the call. The different calling and call-confirmation signals on the type B signalling system have the advantage on bothway circuits that there is less likelihood of spurious signals causing to lock up, and dual seizure, due to calls originating simultaneously in both directions, is more readily identified.

The sketch shows the conventions recommended for both types of signalling.

The C.C.I.T.T.* recommend that the signalling conditions on an international trunk circuit should conform to the requirements of the country which is receiving the call. This ensures that the signals exchanged are in the form most suited to the equipment in the receiving country, that delay in setting-up a call is minimal, and that the cost of conversion is equitably divided. For calls from subscribers in a country, such as the United Kingdom, where printed service-signals and the called-subscriber's answer-back code are automatically returned to the calling subscriber, it is necessary to convert any non-printed signals received from the distant country into printed signals for presentation to the calling subscriber. Signal conversion is normally effected in the international trunk relay-set, which also incorporates facilities for taking the answer-back of the called subscriber by transmitting the WRU signal after the call-connected signal has been received. Failure to receive signals in response to the WRU signal is interpreted by the trunk relay-set as a faulty line, and the call is forcibly released.

* C.C.I.T.T.—International Telegraph and Telephone Consultative Committee.



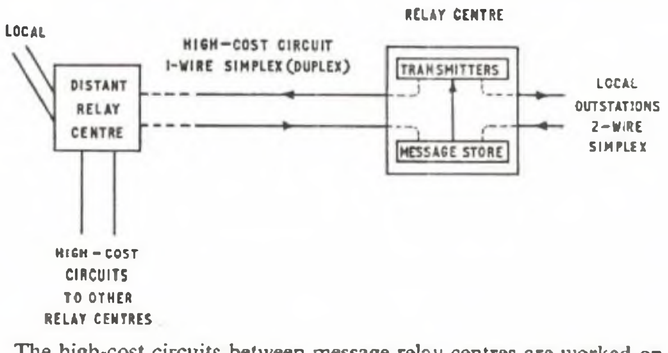
Notes: 1. The conventions used to indicate polarities of signals are: A = start polarity, Z = stop polarity.
2. The durations indicated for the signals are nominal.

Where the distant system employs registers, it is necessary, in a direct-selection system, such as is used in the United Kingdom, to store the subscriber's dial-selection signals until a distant register is available. For this purpose, registers are provided in the United Kingdom system and they are convenient points to convert the dial-pulse signals to teleprinter signals if these are required by the distant system.

Q. 8. *In what circumstances is the use of a message-relay system likely to be preferable to a circuit-switching system? Give details of a typical message format used in a message-relay system and explain why an agreed format is necessary.*

A. 8. Message-relay systems have been developed to give a fast, efficient and economical communication service for use with high-cost point-to-point private circuits. A message-relay system is likely to be preferable to a circuit-switching system if there is a large amount of traffic to be carried between comparatively few terminal stations, if the terminals are a large distance apart and if it is essential that priority messages can be given precedence.

The sketch shows the general arrangement of a message-relay system.



The high-cost circuits between message-relay centres are worked on a 1-wire simplex (duplex) system, ensuring that, with automatic transmission of messages, the circuit is fully occupied in both directions of transmission for the maximum period each day. No circuit time is lost in passing selection digits or service signals, as on a switched network, and both channels of the circuit carry messages simultaneously. With a switched network, as on a telephone network, one channel of the circuit is normally idle and is only used for supervisory purposes.

A further advantage of a message-relay system is that, although an outgoing circuit may be engaged, the message is stored in a queue and is transmitted automatically when the line is free. This ensures that any message will reach its destination in a fairly short, predictable time, eliminating the frustration and delay which may be encountered on a circuit-switched network. In addition, the relay system offers facilities for priority messages to be given predecence, and for broadcast messages to be processed in an economical manner.

Using a message-relay system, each message is normally regenerated and retransmitted at each relay centre. There is no difficulty, therefore, with any message which has to be transmitted over a large number of links; the message will not suffer from cumulative mutilation or distortion.

A message-relay system is most suited to networks, such as those provided for the air-line companies or for the transmission of international telegrams, where speed of working, high traffic density and priority facilities are required.

A typical message format to C.C.I.T.T. recommendations for an automatically-switched system for handling telegrams is shown below.

ZCZC GEB099 WY79
GBLD HL URWA 013
WASHINGTON 13/12 13 1205
3
LT
—MIDBANK LONDON—
3
TEXT
3
SIGNATURE
10
NNNN
10

[The figures between groups indicate the number of line-feed characters transmitted]

Significance of each group of characters in the message is as follows.

(*a*) The first line, known as the numbering line, contains the start-of-message signal, consisting of the characters ZCZC, the channel sequence number, consisting of three letters constituting the indicator of the channel, and three figures constituting the numbers in the series on the channel, together with up to 12 printed characters giving the telegram identification group.

(*b*) The second line is known as the pilot line, and contains the destination indicator, consisting of four characters: the first two indicating the country of destination and the second two characters characterizing the destination office in that country. These are followed by two characters indicating the priority and class of traffic of the message, four characters indicating the country and office of origin, and a 3-figure number giving the number of chargeable words.

(*c*) The third line contains the preamble to the message. This may vary according to the requirements of the originator, but in general the preamble consists of the office of origin in plain language (as opposed to the 4-letter code in line 2, which is used for automatic routing), the time and date of origin, the number of words in the message and the number of chargeable words.

(*d*) The fourth line contains the address; the character—, corresponding to the combination No. 1, upper case, will be inserted at the beginning and end of the address so that this is easily recognized.

(*e*) The text of the message.

(*f*) The signature.

(*g*) The end-of-message signal NNNN

An agreed format is necessary on a manually-operated system in order that the routing and identification information is readily available to the operator in such a form that the possibility of error is minimized. For a fully-automatic system an agreed format is vital, as the automatic equipment is programmed to detect and check destination codes, channel numbering sequences and message-separation signals at fixed points in the message sequence. The codes for start and end-of-message indicators have been chosen to offer the minimum possibility of error and the location of the address and text is designed to present to the recipient the origin and text of the message in the clearest way.

Q. 9. *Explain the purpose of the following in an ARQ multiplex radio-teleprinter system: (a) pulsed 5-unit automatic transmitters, (b) 7-unit telegraph receiver, (c) buffer store, (d) signal alpha, (e) signal beta.*

A. 9. (*a*) An ARQ multiplex radio-teleprinter system is used for transmitting teleprinter messages over radio circuits; as the radio circuits are subject to fading, causing mutilation in the received message, the system is error-correcting. At the ARQ terminal, each character is converted from a 5-unit to a 7-unit code before transmission, so that the distant terminal may easily detect an error; when this happens, the distant terminal transmits an RQ or repeat signal, which causes the originating station to re-transmit the mutilated character.

Owing to the propagation delay over the circuit, it is necessary for the originating station to store characters as they are transmitted, in case the RQ signal is received and the characters have to be re-transmitted; normally a 4-character store is used at the originating station. If a private renter wishes to lease an ARQ circuit, it is necessary to provide a pulse-controlled automatic transmitter at the renter's installation, in order that the characters comprising the message may be transmitted under the control of the ARQ equipment, to prevent the store becoming over-full when the radio circuit is cycling. The ARQ system transmits a pulse to operate the clutch of the automatic transmitter for every character to be transmitted by the ARQ equipment. The sender is therefore aware of any delay on the radio circuit, and a pulsing circuit must be provided in addition to the signalling circuit to the renter's premises.

(*b*) The 7-unit telegraph receiver is used to monitor and test the signals passing on one of the ARQ channels. The signals are checked to verify that a correct translation has been made from the 5-unit input to the 7-unit ARQ signals. The signals are received in parallel form, operating seven magnets on the receiver; a pulse is generated by the ARQ equipment to indicate when the character is to be printed by the receiver.

(*c*) The buffer store is a store capable of holding about 4,000 teleprinter characters, using either perforated-paper tape, or a magnetic drum. The store is provided so that characters received from machines which are not pulse-controlled by the ARQ equipment may be held whilst the radio circuit is cycling and released when conditions are normal. Traffic direct from a renter's teleprinter or automatic transmitter can be stored at the ARQ terminal ready for subsequent transmission to the ARQ equipment. The store is particularly required for telex subscribers, as there is no means in the telex system of pulse-controlling the renter's machine, and messages have to be received at the ARQ terminal from the renter irrespective of the cycling of the radio circuit.

(*d*) and (*e*) The 7-unit code used with the ARQ equipment utilizes 35 combinations. Of these, 32 are used to replace the 5-unit code and the remaining three are used for supervisory purposes, these being generated within the equipment. One of the three signals is the RQ signal, required to initiate a repetition cycle, the remaining two signals are known as the alpha signal and the beta signal.

The alpha signal corresponds to a continuous start polarity, which is the telex clearing signal; this is transmitted continuously whilst a channel is idle between calls.

The beta signal corresponds to a continuous stop polarity; this signal is used as the telex calling signal and is also continuously transmitted during idle periods when a call is in progress.

As the ARQ system is a synchronous time-division system, a signal must always be present in order that the two ends of the system retain the correct time relationship. If either the alpha or beta signal were omitted when teleprinter characters were not being transmitted, this would correspond to a failure of the radio path, and a correction cycle would be initiated.

Q. **10.** *For what purposes are signals at the following frequencies used in phototelegraphy:* 1,020 *Hz* (*c/s*)*;* 1,300 *Hz;* 1,500 *Hz;* 1,900 *Hz;* 2,300 *Hz;* 1,900 *Hz modulated at* 1,020 *Hz?*

A. 10. Phototelegraphy is the transmission of signals to represent black and white photographs and the reproduction of the photographs in the correct tone range. The general principle is for the transmitting apparatus to scan the original picture in a series of lines with a beam of light; the light reflected from the picture is made to act on a photo-electric device, and the derived signals are transmitted to line. At the receiver a recording blank is scanned in a manner similar to that used for the original picture at the transmitter, with the received signals actuating a marking device on the blank. To enable the picture to be scanned, it is mounted on a drum which is caused to rotate and move longitudinally simultaneously, so that the scanning light-beam moves across the picture in the form of a spiral. At the receiver, a sheet of light-sensitive paper is mounted on a similar drum, and the received signals modulate a light-beam so that the picture is built up in a spiral form as at the transmitter. To obtain faithful reproduction, it is necessary that the drums at the transmitter and receiver rotate at the same speed, otherwise the received picture will be distorted; for phototelegraphy a tuning-fork-controlled oscillator is used to govern the speed of the drum motor. The synchronizing frequency of 1,020 Hz has been recommended by the C.C.I.T.T. so that inter-working facilities may be offered to machines at various locations. In certain types of receiver, the fork frequency may be varied by a small amount for checking against the transmitted frequency.

Normal line conditions, and problems of d.c. amplification, prevent the direct transmission of frequencies from the transmitter to the receiver. In all phototelegraphic transmissions a carrier frequency is employed which is amplitude or frequency modulated by the photo-electric-cell signals. For lightly-loaded cable circuits a carrier frequency of 1,300 Hz has been recommended by the C.C.I.T.T.; the system requires a bandwidth of 1,300 $\pm$ 550 Hz, which is readily available in the normal telephone circuit.

For phototelegraph circuits which use carrier or coaxial telephone circuits, a carrier frequency of 1,900 Hz is recommended. This frequency is near the middle of the range 300–3,400 Hz, which is the standard bandwidth of a coaxial or carrier telephone circuit. The maximum photocell frequency may be increased to 1,000 Hz, representing a bandwidth of 1,900 $\pm$ 1,000 Hz, so that the best possible use is made of the telephone circuit. The phase/frequency distortion at the higher frequencies is too great, however, for high-quality work, and the bandwidth for this type of transmission is limited to 1,900 $\pm$ 500 Hz.

For picture transmission over radio circuits, frequency modulation (f.m.) is used, whilst amplitude modulation (a.m.) is more normal over cable circuits. At the transmitting and receiving terminals the standard amplitude-modulation sets are used, and, where a circuit consists of land lines and a radio circuit, a.m.-to-f.m. and f.m.-to-a.m. conversion sets are required at the radio terminals. The conditions recommended by the C.C.I.T.T. for sub-carrier frequency modulation are as follows:

sub-carrier	:	1,900 Hz,
pure white	:	1,500 Hz,
pure black	:	2,300 Hz.

Where there is the possibility that the transmitter and receiver may be connected by a circuit liable to induce frequency changes, e.g. a carrier telephone circuit, the use of simple 1,020 Hz synchronizing tone is unsatisfactory. The preferred method is to transmit the phototelegraph carrier (1,900 Hz or 1,300 Hz) modulated by the 1,020 Hz synchronizing tone.

LINE PLANT PRACTICE B, 1968

Students were expected to answer any six questions.

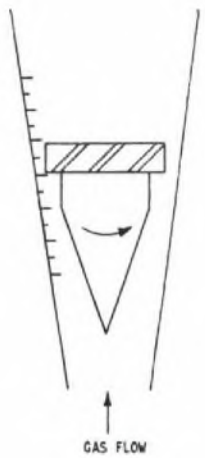
Q. **1.** *Describe briefly, using sketches where necessary, the undermentioned items of equipment used in the pressurization of cables:*

(*a*) *gas-flow indicator,*
(*b*) *contact gauge,*
(*c*) *compression-type contactor.*

A. 1. (*a*) Gas-flow indicators are fitted at exchanges at which continuous-flow gas-pressure schemes are operating. The indicators are used to measure the rate of air flow at cable inputs.

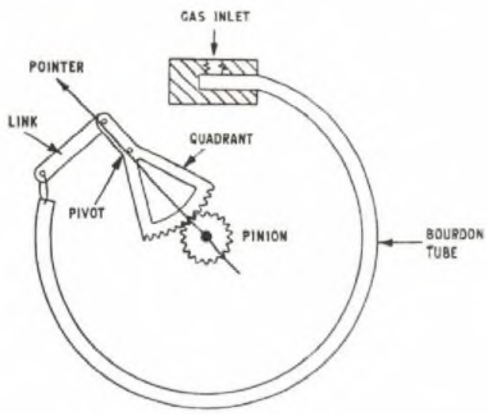
The flow indicator, illustrated in sketch (*a*), consists of a vertical-tapered glass tube containing a cone-shaped float which acts as a metering element. The float is free to move in the tube under the control of the rate of air flow. The position of equilibrium is determined by the annular orifice between the rim of the cap of the float, and the side of the tube. If the rate of air flow is increased, the float will rise up the tube, until the area between the cap and the side of the tube is sufficiently larger for the float to be in equilibrium again. Conversely, if the flow rate is decreased the float will drop to a narrower part of the tube. The glass tube is graduated from 0·1 to 1·0 ft^3/hr, the level of the top of the float indicating the rate of air flow.

Oblique grooves in the cap cause the float to rotate in the air flow, preventing it from sticking to the glass.



(*a*)

(*b*) A contact gauge consists of a Bourdon-tube pressure-gauge, fitted with a pointer-operated contact assembly. The main elements of a Bourdon-tube pressure-gauge are shown in sketch (*b*).

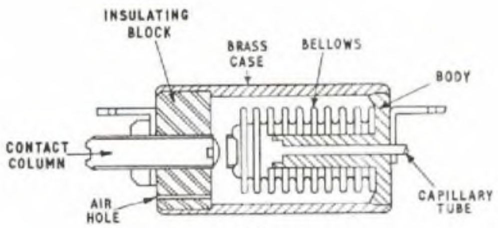


(*b*)

The Bourdon tube is metallic, usually copper alloy, oval in cross-section and bent into circular form. One end of the tube is soldered to a block, forming the gas entry, the other end is sealed. The sealed end is coupled by a link to a pivoted quadrant. The teeth on the quadrant mesh with the teeth of a pinion to which a pointer is secured. An increase in pressure in the tube forces its cross-section into a more circular form, consequently, the tube uncoils slightly, and the pointer is deflected. If the case of the pressure gauge is sealed and evacuated, the gauge will measure absolute pressure. More usually the case is not airtight, and the surroundings of the tube are at atmospheric pressure. The gauge then measures gauge pressure, i.e. the value of the inlet pressure above atmospheric pressure.

The pointer of the gauge carries one contact, the second contact is fitted to a spring-loaded manually-adjusted arm. The pressure gauge is calibrated from 0–10 lb/in^2, and the adjustable arm may be positioned at any value in this range. The contacts are connected to a spare cable pair, which terminates on an alarm circuit. In this manner the contact gauge may be set to operate an alarm circuit when the pressure drops to any predetermined critical value.

(*c*) Contactors are pneumatically-operated switches fitted to cables. They operate an alarm circuit when the gas pressure in the cable falls to a predetermined level. They give no further indication of the state of the gas pressure in the cable.



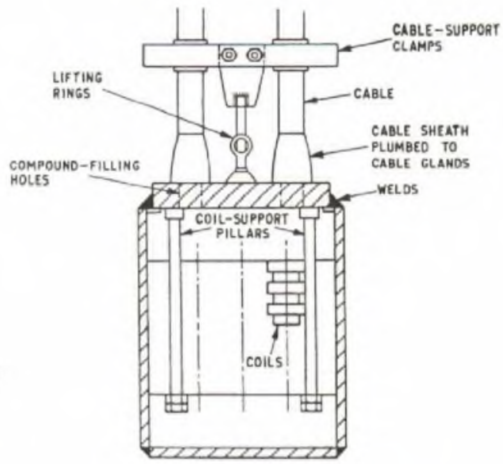
(*c*)

Sketch (*c*) shows the principle of a compression-type contactor. In this type the inside of the bellows is sealed at atmospheric pressure. When the contactor is placed under pressure the bellows compress. If the gas pressure drops below the critical value, the bellows expand to their natural position, and complete the alarm circuit via the contacts on the bellows and the contact column.

A difficulty experienced with this type of contactor is that the operation of the contact occurs gradually, and is not sufficiently positive. The contact can make and break intermittently, over an extended period, operating alarms but making accurate use of the fault-locating equipment difficult.

Q. **2.** *Describe the construction of a loading-coil pot suitable for an aerial-cable route. Draw a sketch to show the method of mounting the pot and jointing it to the aerial cable.*

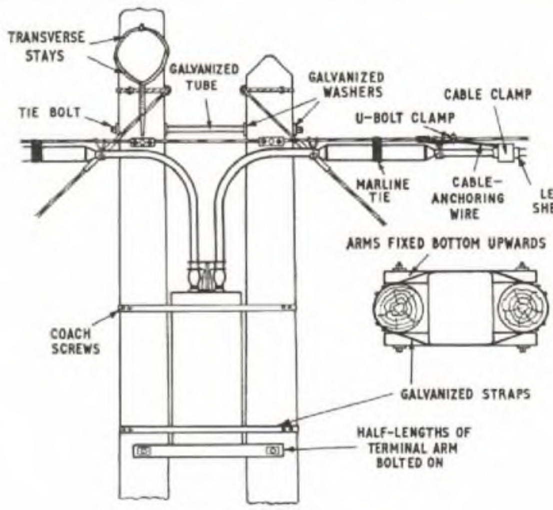
A. 2. Sketch (*a*) shows an aerial-cable loading-coil pot in section. The case of the pot is constructed from sheet steel, arc welded along the edges. The sides and base are of $\frac{1}{16}$ in thick material. Before full assembly, the case with the lid is pressure tested to 20 lb/in^2 using compressed air or fluid. The lid is of thicker material, and has cable-support clamps, cable glands and lifting rings fitted to its upper surface. Support pillars, attached to the underside, carry the individual loading coils firmly clamped in columns or stacks with suitable distance pieces. The lid also has one or two holes to facilitate compound filling and pressure testing during manufacture.



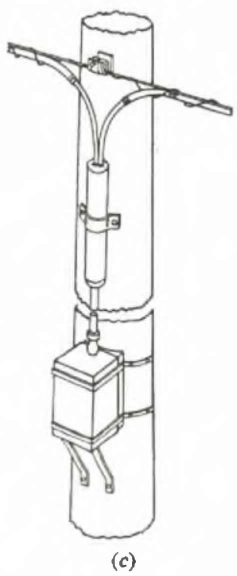
(*a*)

One or two tail cables may be provided depending on the size of the case, i.e. the number of coils. After these have been fitted, terminated and tested, the coil assembly is lowered into the case, and the lid arc welded to the rim of the case. A final air-pressure test at 20 lb/in^2 is then carried out, to ensure that there are no leaks. Using either pressure or vacuum technique, an impregnating compound is introduced into the case to cover the coils and stubs, thus sealing them against ingress of moisture. The compound-filling holes are then sealed.

Corrosion protection of the case is provided as follows. The case is shot blasted to thoroughly clean and roughen the surface, and then a 0·005 in zinc-spray coating is applied. The final finish consists of three coats of black bituminous paint over the case and fittings.



(*b*)



(*c*)

It will be noted that the construction differs slightly from that of the underground-type case. Channel-iron feet are not required, and the impregnated-hessian protection used on the underground case is not suitable for the different corrosion and weathering conditions.

The method of mounting the loading-coil pot on the pole, and jointing to the aerial cable, will depend upon the size of pot involved; sketch (*b*) shows the method for a large pot, and sketch (*c*) for a small pot.

Q. 3. *An audio-frequency paper-core cable has been laid and jointed over a half-mile section. This cable is not loaded, and does not require balancing tests. Enumerate and describe the tests that should be carried out on completion of this half-mile section, and say why each test is necessary.*

A. 3. When a cable has been placed in position, it is necessary to carry out a number of tests on it to ascertain that no damage has occurred during the pulling in, that the characteristics of the cable are still within the specified limits, and that the sheathing and joints are air-tight.

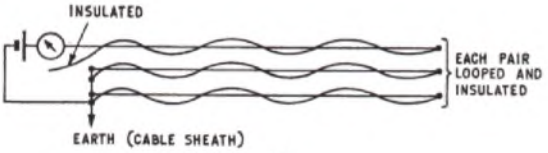
Tests will be made to prove the following:

(*a*) continuity of conductors,
(*b*) absence of wire crosses between pairs,
(*c*) freedom from contacts and earth,
(*d*) correct insulation resistance,
(*e*) absence of leakage under air pressure.

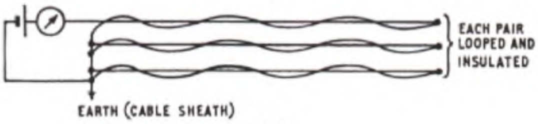
Test for continuity, crosses, contacts and earths

At the down end of a subscriber's cable, or at the end from which the jointing proceeds on a main cable, the two wires of each pair are twisted together and each looped pair is insulated with a paper sleeve. At the testing end of the cable, all the conductors are connected together with tinned-copper wire, the free end of which is earthed by soldering it to the cable sheath.

A pair is taken from the earthed bunch, one wire being connected to a detector and cell, and the other wire insulated as shown in sketch (*a*). If no deflexion is obtained, the pair is clear of contact, cross, and earth.



(*a*)



(*b*)

The insulated wire of the pair is then connected to the earthed bunch as shown in sketch (*b*). A deflexion in the detector proves continuity of the pair. The tested pair is then placed on one side, and the remaining pairs are tested in a similar manner. If, on any pair, a deflexion larger than normal is obtained, this indicates a contact between the wires of the pair under test. Any pair found faulty should be reconnected to the earthed bunch, before testing the remaining pairs.

Failure to detect a split pair will result in severe crosstalk.

Insulation resistance

Measurements are made with a 500-volt megger, in the following manner.

The conductors at the distant end are insulated from each other and from earth. At the testing end the A-wires of every quad are bunched together, similarly the B-wires, C-wires and D-wires. When the four groups have been formed, the insulation resistance is measured between each group, and the three remaining groups joined to the cable sheath (earth). The insulation resistance of the cable in megohms per mile is obtained by multiplying together the reading of the Megger in megohms, the number of wires per group, and the length of the cable in miles.

The insulation resistance of the cable should not be less than 5,000 megohms per mile for local cables, or 10,000 megohms per mile for main cables. The above test proves the absence of contacts between any of the groups, and between any of the wires of the cable and earth. In order to prove the absence of contacts between any of the individual wires of each of the four groups, all the wires of the cable are bunched and earthed. A complete quad is then withdrawn and its insulation resistance is measured to the whole of the remaining quads of the cable. This is repeated until each quad has been tested.

Pressure test

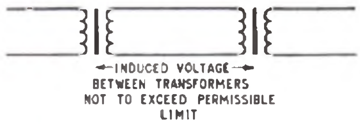
This test is carried out by forcing air, which has been dried by being passed through cylinders containing calcium chloride, into the section of the cable to be tested, the end of which has been sealed. When a pressure of about 20 lb/in^2 has been obtained at both ends of the section, the joints are smeared with soapsuds or leak-detecting fluid, and carefully examined for any sign of bubbles. With the cocks turned off, thus sealing the cable, there should be no observable drop in pressure within 24 hours.

Q. 4. *Describe, with the aid of diagrams, two methods of limiting the effects of the voltage induced in a telephone line running parallel to a high-voltage power-line. Assume that earth fault currents cannot be limited at source, and the separation between telephone line and power line cannot be increased.*

Discuss the relative advantages and disadvantages of the two methods described.

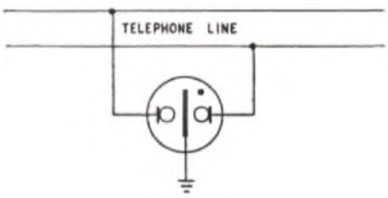
A. 4. Induced voltages may be kept within safe limits, with regard to plant and personnel, by the insertion of isolating transformers, or by fitting gas-discharge tubes.

Isolating transformers may be used on circuits employing a.c. signalling. Each transformer consists of a 1 : 1 line transformer, specially insulated to withstand a high test voltage applied between windings, or between windings and case. The transformers are inserted into portions of the telephone line exposed to high induced voltages. They divide the line into sections, in each of which the maximum permissible induced voltage will not be exceeded, as shown in sketch (*a*).



(*a*)

A gas-discharge tube consists of three electrodes, housed in a glass envelope filled with argon and neon gases. The two outer electrodes, which are evenly spaced from the centre electrode, are connected to the A-wire and B-wire of the telephone line, respectively, as shown in sketch (*b*). The centre electrode is earthed. When the voltage to earth



(*b*)

of the telephone line rises above the operating value of the gas-discharge tube, the tube impedance falls to a very low value, thus clamping the line virtually to earth potential.

Gas-discharge tubes can be fitted to any type of line, but isolating transformers give rise to the following difficulties:

(*a*) the insertion of the transformers may significantly increase the signal attenuation and distortion of the transmission line,
(*b*) d.c. signalling and dialling are not possible,
(*c*) insulation testing and localization of faults on intermediate sections are made more difficu't,
(*d*) any harmonic-distortion products introduced could cause cross-modulation between the channels of a carrier system.

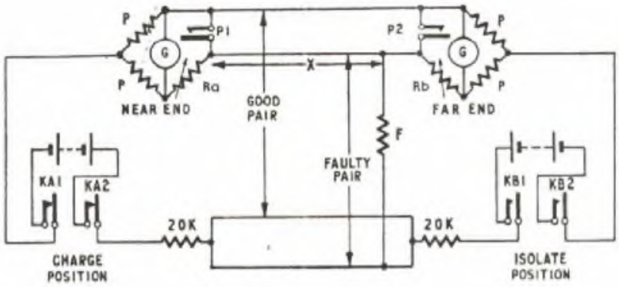
Q. 5. *Describe, with the aid of a diagram, the double-ended Varley method of locating a low-insulation resistance fault on a long length of cable.*

In such a test on a cable pair 6 *miles long the near-end and distant-end resistance readings were* 140 *ohms and* 110 *ohms, respectively. How far was the fault from the near end?*

A. 5. The equipment required to carry out a double-ended Varley test, consists principally of two Wheatstone bridges, one connected at each end of the cable under test. A complete circuit is shown in the sketch. The test is made in the following manner.

(*a*) The line is charged up to the near-end battery, by putting key KA in the charge position. Key KB is left in the isolate position. The line is short-circuited at the far end by inserting plug P2. Plug P1 is left out at the near end.

(*b*) The bridge resistor at the near end (R_a) is adjusted to give zero deflexion on the galvanometer. When the zero reading is constant—i.e. the line is charged up to a steady potential—the value of R_a is recorded.



(*c*) A similar test is then made at the far end by putting key KB in the charge position, and KA at the near end in the isolate position. Plug P2 is taken out and plug P1 inserted. The far-end bridge is balanced and the value of resistor R_b recorded.

These tests are repeated several times to obtain an accurate mean value for R_a and R_b. The distance to the fault is calculated from the following formula:

$$X = \frac{R_b \times L}{R_a + R_a},$$

where X = distance to fault from near end in miles,
R_a = near-end Varley reading,
R_b = far-end Varley reading,
L = length of cable in miles.

Substituting the given values: $R_a = 140$ ohms, $R_b = 110$ ohms, $L = 6$ miles, gives

$$X = \frac{110 \times 6}{140 + 110} = \underline{2.64 \text{ miles}}$$

Q. 6. *A joint in a coaxial cable, which carries power to a repeater station, has to be opened for maintenance purposes. Write an account of the safety precautions that should be taken before the cable joint is opened and after it is closed again. Assume that the joint is located in a large footway joint box.*

Q. 7. *You are required to make a survey for a new aerial-cable route, for 2 miles along a narrow country road. The cable will be of the self-supporting combined type, containing 50 pairs of 10-lb conductors. Give an account of the factors you would take into consideration, and indicate what details you would be expected to settle in advance of the actual construction.*

A. 7. The survey for a new aerial-cable route needs to take account of a number of factors. Initial enquiries will be necessary to ascertain that the route does not pass through or near, places of special interest or beauty, airfields, v.h.f. sites, school playgrounds or power lines. The effects of any proposed road-widening or town and country development schemes must also be considered.

Some initial planning can be done on large-scale ordnance-survey maps, but it will be necessary to walk the route, making a note of any special features which will effect the detailed layout of the route, such as obstructions due to buildings, trees, etc., possibility of joint use of existing electricity poles, and staying facilities. Enquiries must also be made to establish the ownership of land on which poles or stays may need to be sited, in order that the necessary private wayleaves may be applied for. This survey will enable the best pole sitings to be chosen. Road crossings should be avoided if possible.

Sufficient data will now be available to justify formal application for wayleaves and consents from the appropriate local authorities. The route can be designed in detail, and the works estimate prepared for authorization. The latter includes the following information:

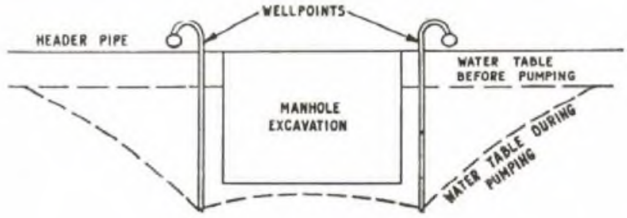
(*a*) length and class of poles, based on the required clearances and calculations of ultimate loading,
(*b*) siting of poles,
(*c*) methods of staying or strutting,
(*d*) treatment at power crossings and other protection details,
(*e*) lengths of cable, jointing and loading points,
(*f*) methods of working, use of special aids,
(*g*) list of stores, and storage sites.

The works estimate, wayleaves, and consents, must all be finalized before the construction work is started.

Q. 8. *A reinforced-concrete manhole has to be constructed in water-logged ground. Describe in detail how ground de-watering would be used to enable the excavation to be made and the concrete placed.*

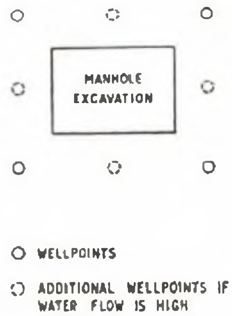
Include in your answer a sketch of a de-watering wellpoint, showing the valve action.

A. 8. In water-logged ground the surface of the water, known as the water-table, is either coincident with the surface of the ground or very close to it. Sketch (*a*) indicates this water-table in relation to a required manhole excavation, before and after de-watering.



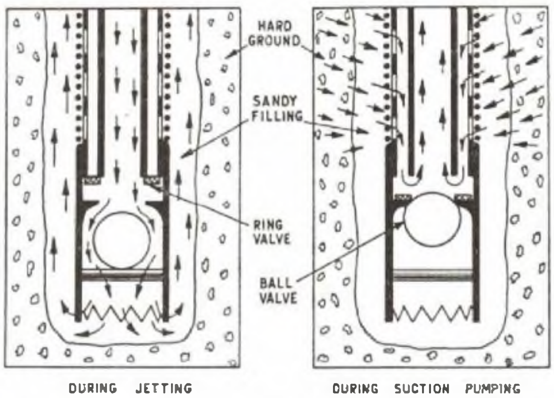
(*a*)

De-watering is carried out by the wellpoint system. This uses a number of riser pipes, or "wellpoints", sunk into the ground around the manhole as shown in sketch (*b*).



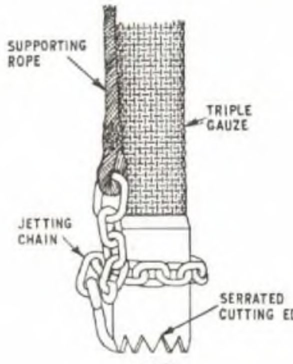
(*b*)

A wellpoint consists of a length of 2 in diameter steel pipe, fitted with a valve system at the lower end. The parts and their function are shown in sketch (*c*).



(*c*)

To set up a de-watering system, holes are first excavated through the paved surface by a pickaxe or bar, and shovel. The top end of the wellpoint is connected to a high-pressure water "jetting-pump" fed from an independent source, and the wellpoint is held vertical in one of the excavated holes. When the pump is turned on, the high-pressure jet of water erodes the soil away, and the wellpoint sinks under its own weight. When it reaches the required depth below the manhole



(*d*)

excavation, the pump is turned off. The top of the triple gauze will then be approximately 1 ft below the deepest part of the excavation. In hard soil, the serrated edge is rotated to assist the sinking.

When the jetting pump is stopped, granular particles of sandy soil normally sink back into the eroded hole and form an additional filter around the triple gauze, but in some soils the wellpoint makes a "slip" fit. To overcome this tight fit a jetting chain is hooked over the cutting edge, as shown in sketch (*d*). The extra gap formed is then filled with sand to form the necessary additional filter. The chain is then unhooked, by lifting the wellpoint slightly, and withdrawn.

The first wellpoint can be used as a source of water to sink the others. When all the points are in position, each one is connected to a common header pipe, usually of 6 in diameter. This header pipe is connected to the input of a powerful suction pump, and after a few hours pumping the water-table will fall to the position shown in sketch (*a*).

A check must be made after 20 minutes pumping that the water is not coloured by fine soil particles held in suspension. This would be caused by inadequate filtering due to a broken gauze, sticking valve or insufficient sand fill. The offending wellpoint must be located and repaired.

Excavation and concreting can be carried out in dry conditions, provided the pump is kept working. Only a minimum of soil-supporting timber is normally necessary.

Q. 9. Discuss the principles upon which cost studies are made, to determine which of the various means of providing telephone line plant to meet future growth, will be the most economic over a given period of years. Illustrate your answer with reference to a practical example.

A. 9. The principle on which these cost studies are made is present valuation of all the charges involved.

Because some costs are capital payments and some are annual payments, it is convenient to reduce all costs to annual charges, and then to calculate the present value of the annual charges (p.v. of a.c.).

The annual charges on capital payments will be made up of interest on the capital borrowed, and an amount to cover depreciation. The latter must accumulate to the cost of replacement of the plant at the end of its estimated life, and would take into account its end-of-life value, which might be negative or positive.

The cost of maintaining plant is usually expressed as an annual charge equal to a fixed percentage of the capital cost of the plant.

In order to compare two alternative schemes, all the annual charges are present valued, or discounted, to the amount of money which must be invested today at the current rate of interest in order to produce annual amounts to pay all the annual charges. The scheme which produces the lowest sum is the most economic.

The discounting factors are available in tables which give the present values for various conditions, e.g. regular annual payments of the same amount, annual payments of amounts increasing regularly each year, and annual payments starting a few years in the future.

Discounting future payments to the present value, enables a true comparison to be made between schemes involving widely varying capital sums and maintenance charges. The method is also useful in determining whether to provide plant to meet a 20-year requirement at the outset, or in instalments.

The principle is illustrated by the following simple example.

Assume that 800 cable-pairs are required over a period of 20 years, each pair to be 4 lb cable, and one mile long in an existing duct.

Scheme 1—provide one mile of 800/4 cable in 1969.

Scheme 2—provide one mile of 400/4 cable in 1969 ,one mile of 200/4 in 1973 and one mile of 200/4 cable in 1978.

Scheme 1

The capital cost of providing the 800/4 cable is £4,500. If the annual charges are assumed to be depreciation 1·3 per cent, maintenance 2·0 per cent, interest 8·0 per cent, total 11·3 per cent, the annual charges will be 11·3 per cent of £4,500 i.e. £510.

The present-value multiplier for a regular annual payment of £510 over 20 years, is 9·82, so that the p.v. of a.c. is £5,000.

Scheme 2

Assuming the same depreciation, maintenance, and interest conditions apply for the smaller cables, the costs for this scheme are:

capital cost of 400/4 cable	£2,500,	
annual charges	£ 285,	
p.v. of a.c.		£2,800.
Capital cost of 200/4 cable in year 5	£1,650,	
annual charges	£ 185,	
p.v. of a.c. (multiplier is 6·5)		£1,200.
Capital cost of 200/4 cable in year 10	£1,650,	
annual charges	£ 185,	
p.v. of a.c. (multiplier is 3·6)		£ 650.
Total p.v. of a.c.		£4,650.

In this example, Scheme 2 is therefore the more economic.

Q. 10. Discuss the factors which have to be taken into account when specifying a concrete mix for a particular application. Give details of four qualities of concrete mix, and state the type of work for which each would be used.

A. 10. The practical design of a concrete mix resolves itself into obtaining the requisite strength, with sufficient plasticity, at the cheapest cost.

The first variable to be determined is the water-cement ratio for which the required compressive strength, and degree of exposure to weathering, must be considered. For example, for load-bearing structures in sheltered conditions, such as buildings and underground works, it is usually the final strength which determines the water-cement ratio. When the concrete is to be exposed to weathering, the water-cement ratio will be dependent on the degree of exposure and the type of structure involved. Table 1 gives approximate values of water-cement ratio which may be used for various conditions of exposure.

Table 1

Condition of Exposure		Water-Cement Ratio (by weight)	
		Thin Sections	Mass Concrete
Regular wetting and drying		0·45	0·55
Normal outside exposure	Severe climate	0·50	0·60
	Temperate climate	0·55	0·65
Continuously under water		0·55	0·65

Another variable is the ratio of aggregate to cement, which must be chosen to give the desired plasticity, i.e. to make the cement sufficiently easily workable. For the same plasticity the required ratio of the mix will vary considerably depending on the shape of the aggregate. For example, with flaky aggregate a greater amount of cement is required than with spherical-shaped aggregate. A slightly leaner mix may be used when the maximum size of the aggregate, or the fineness modulus, are increased.

These factors have been taken into account in the design of suitable mixes for use in various types of work. The proportions of the respective ingredients for four typical concrete mixes are shown in Table 2.

Table 2

Quality	Parts by volume			Maximum size of aggregate	Water gals/ft³ of cement
	Cement	Sand	Aggregate		
A	1	2	4	¾ in	4½ to 5½
B	1	2	7	1½ in	4½ approx.
C	1	3	9	Any suitable size	4 approx.
H	1		15	All-in aggregate ¾ in to 1½ in according to availability	Nil

These different mixes are used in the following ways:

A — Construction of jointing chambers,
B — Reinstatement of trenches when a heavy load is anticipated, and underpinning,
C — Normal trench reinstatement,
H — Stabilized backfill, normally mixed and placed without added water.

The plasticity of concrete can be improved by the addition to the mix of some inert powder which lubricates the aggregate. Surface-active materials such as sulphuretted hydrocarbons, carbohydrate salts or saponin are also effective in increasing the plasticity. The use of these plasticisers enables a lower water-cement ratio to be used, with a consequent increase in strength of the finished concrete.

TELEPHONY B, 1968

Students were expected to answer any six questions

Q. 1. *What developments in electronics have made it possible to energize an amplifier at a subscriber's premises from the line current which flows when the exchange line is in use? In what way does the power available from the line limit the maximum power output of the amplifier?*

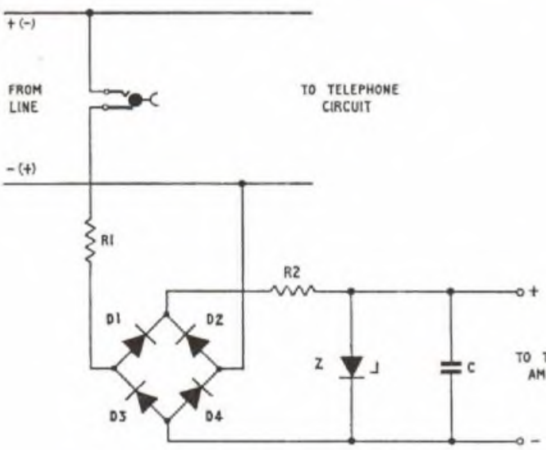
Explain with the aid of a sketch how the power is extracted from the line, and show how correct polarity is maintained during reversal of line polarity at the exchange.

A. 1. The two main developments which have made it feasible to energize an amplifier at a subscriber's premises from the power available from the exchange line, are the availability of transistors for use as the principal amplifying device in the amplifier, and the use of semi-conductor diodes in the circuit used to extract power from the line. Transistors have the following advantages as compared with thermionic valves, which were previously the only available devices:

(*a*) small size and light weight,
(*b*) good reliability,
(*c*) operation under physically more demanding conditions,
(*d*) low operating voltage (of the order of 5–10 volts) and lower power, thus enabling the operating power to be derived from the telephone line and,
(*e*) no heater power is required.

Semi-conductor diodes are also extremely reliable small-size, low-weight devices.

The d.c. power available from the line is given by the product of the voltage at the subscriber's premises and the current being drawn from the line. This governs the audio-frequency power available from the amplifier, because the maximum power available in the load is determined by the maximum current which can be drawn through the transistor plus load, and the maximum voltage which can appear across the load.



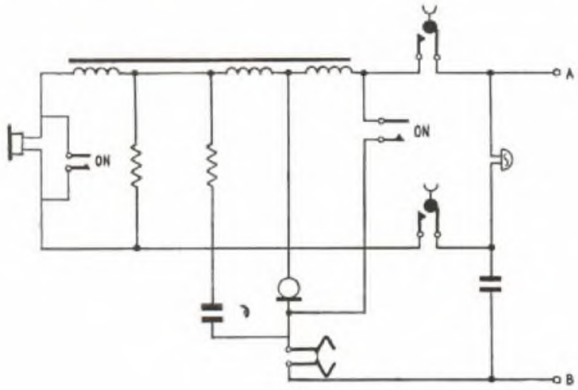
The sketch shows the method used to derive the necessary voltage supply from the line. Diodes D1–4 are connected in bridge formation to the line. Thus, whatever the polarity of the voltage received from the exchange, the output voltage from the bridge has the same polarity. Resistors R1 and R2 act as decoupling and ballast resistors for the zener diode, Z, which is chosen to give the required output voltage, and to stabilize it against differing values of exchange-line resistance. Capacitor C assists in filtering speech currents from the amplifier power supply.

Q. 2. *Sketch the circuit of any telephone instrument suitable for working to an automatic exchange. Explain how two such instruments may be interconnected to provide simple internal extension facilities on an exchange line.*

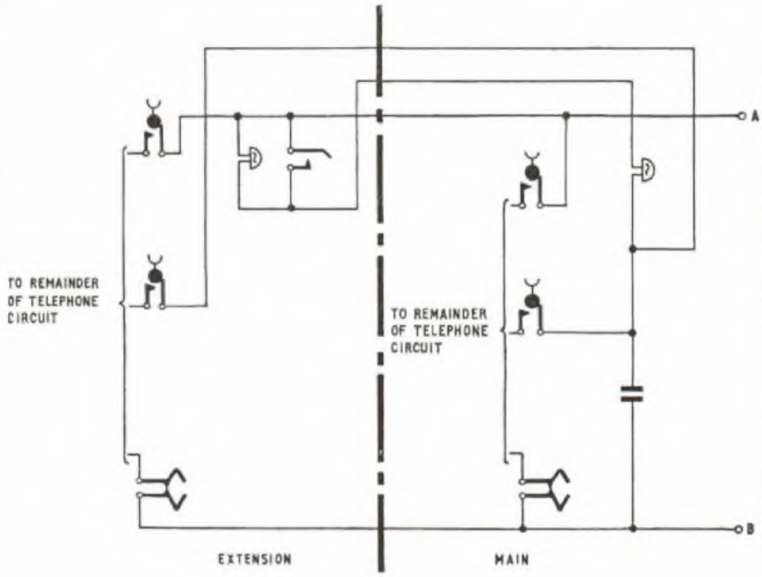
Explain the effect on sending and receiving levels when one of the instruments is used for listening-in to a call already in progress.

A. 2. Sketch (*a*) shows the circuit of a telephone suitable for working to an automatic exchange.

Sketch (*b*) shows the principle of how two such instruments may be inter-connected to provide simple extension facilities. The telephone bells are connected in series, with the capacitor of the extension telephone disconnected. The arrangement allows the capacitor of the main telephone to form part of the spark-quench circuit of the extension-instrument's dial contacts. A switch to short-circuit the bell is provided in the extension telephone. Communication between main and extension instruments is not a normal facility with this arrangement, although this is possible, with some degradation of speech, when an exchange call has been established. If one telephone is taken into use while dialling is taking place from the other, mutilation of the dial pulses will result.



(*a*)



(*b*)

When one telephone is used for listening-in to a call already in progress, speech currents from line will divide between the two telephones and result in a degradation of approximately 3 dB. Speech currents generated by the speaking instrument will divide between the listening instrument and the line, again resulting in a loss of about 3 dB. The feed-current to each transmitter will also be reduced due to the shunting effect of the other transmitter. The effect of this on the generated speech currents will depend partly on whether a regulator is fitted to the telephone, and partly on how much the value of the reduced transmitter current exceeds the minimum design value for satisfactory transmitter operation.

Q. 3. *Exchanges A, B and C form a linked-numbering scheme in a non-director area. Exchange A has 3,000 lines; B and C each have 1,500. Devise a suitable numbering scheme.*

Assuming there are no direct junctions between B and C, name three switching arrangements that are possible for this linked-numbering scheme. Draw a simplified trunking diagram for the linked-numbering scheme using one such arrangement, and state your reasons for preferring it to the others.

A. 3. It is assumed that, whichever scheme is adopted, the division of subscribers' lines between the three exchanges is such that exchange A would be nominated the main exchange, with exchanges B and C acting as satellites.

There are three possible arrangements in a non-director area;

(*a*) the discriminating-selector repeater scheme,
(*b*) the 2,000-type discriminator and,
(*c*) the group-selector satellite.

(There is also a variant of scheme (*c*) known as "trombone" working.)

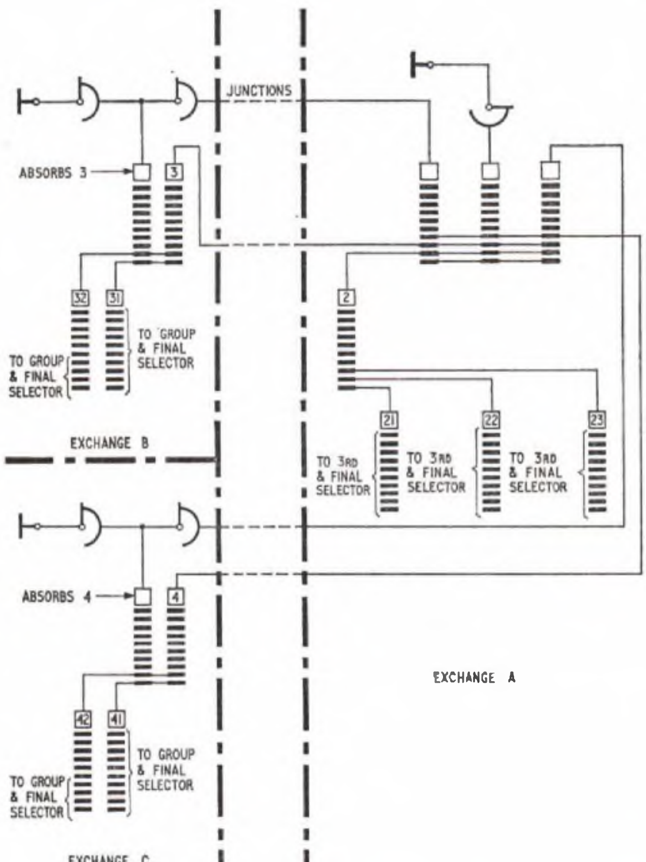
The most suitable scheme, i.e. the least costly in the long term, will depend on the characteristics of the exchange area concerned: namely, the traffic density, the proportion of traffic passing from satellites to main and that confined to the satellites themselves, the topography of the area, and the availability and cost of line plant. If, for example, the aim is to minimize junction costs and at the same time to conserve exchange equipment as far as possible, the discriminating-selector repeater scheme could be used. The 2,000-type discriminator arrangement is also economical in line plant as it requires only one group of junctions between each satellite and the main exchange. The first method involves obsolescent exchange equipment, gives rise to false junction traffic and is not desirable from a maintenance viewpoint, while the second involves a discriminator, which is complex, and which makes call tracing difficult because it operates on a common-control basis.

The group-selector scheme uses comparatively simple, standard switching equipment at both main and satellite exchanges, but requires several junction groups between the exchanges.

Assuming in this case that line plant is at a premium and that too much exchange complexity is to be avoided, the discriminating-selector repeater scheme is chosen.

A suitable numbering scheme would be as follows:

(*a*) exchange A (main) —21000–23999,
(*b*) exchange B (satellite)—31000–32499,
(*c*) exchange C (satellite)—41000–42499.



Q. 4. *A subscriber who complains of general difficulty on outgoing calls is found to have a fast-running dial. Explain in detail why the service should improve if a replacement dial is fitted.*

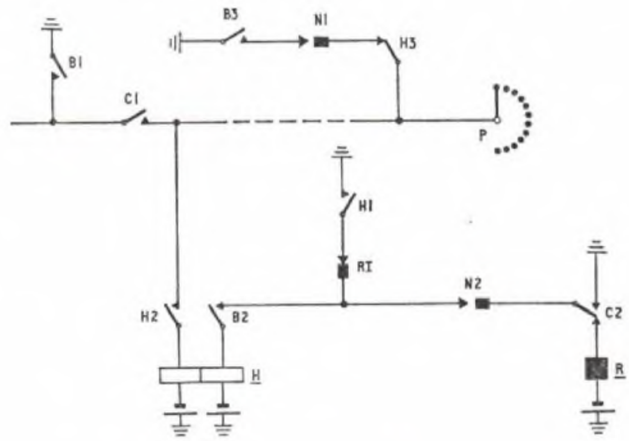
State any principle of measurement that can be used in a practical device for checking the repetition rate of the pulses generated by a subscriber's dial.

A. 4. The nominal speed and ratio of a subscriber's dial are 10 pulses per second (p.p.s.) and 66 per cent break, respectively. This corresponds to a make-pulse of 33⅓ ms duration and a break of 66⅔ ms. When dialling takes place, pulses of current at this periodicity are received by the pulsing relays associated with the selectors in the various switching stages. Provided that a pulsing relay can respond to the dial pulses without introducing appreciable distortion, the pulses repeated to the selector mechanism are of substantially the same duration as those generated by the dial. Assume that the dial is operating at 15 p.p.s., then the make and break pulse durations are $22\frac{2}{9}$ms and $44\frac{4}{9}$ ms, respectively. A typical pulsing relay would have nearly equal operate and release lags of 10–15 ms. As this period is well within the make-pulse time of $22\frac{2}{9}$ ms, the pulsing relay would respond satisfactorily. The operate and release times of the selector operating magnets are likely to be similar to, if not greater than, the pulse times corresponding to the fast dial, with the result that stepping may fail, one or more pulses being absorbed, and giving rise to misrouting. A fast dial reduces the intertrain pause and this could cause a selector to still be hunting for a free outlet to the next stage when the next train of pulses begins. Again, this would mutilate the pulses received by the following stage and result in wrong numbers. During selector stepping under control of dial pulses, a relay is held operated. When the pulse train ends, this relay releases to initiate the hunting action. The relay is held by the receipt of current pulses corresponding to the break period of the dial pulses, in conjunction with a slow-release feature. Should the break-pulse duration be sufficiently short, the relay would release prematurely, and initiate hunting action before all the dial pulses were received. This would give rise to wrong numbers.

The repetition rate of the pulses generated by a subscriber's dial may be measured by charging a capacitor from a source controlled by the dial pulses (of predetermined number), and then comparing the resulting voltage with that existing on a reference capacitor.

Q. 5. *Describe the circuit operation of a 2-motion group selector when searching for and switching to a disengaged outlet. Illustrate your answer with sketches of the circuit elements concerned.*

Identify those features of the circuit operation which determine the stepping speed during the rotary action.



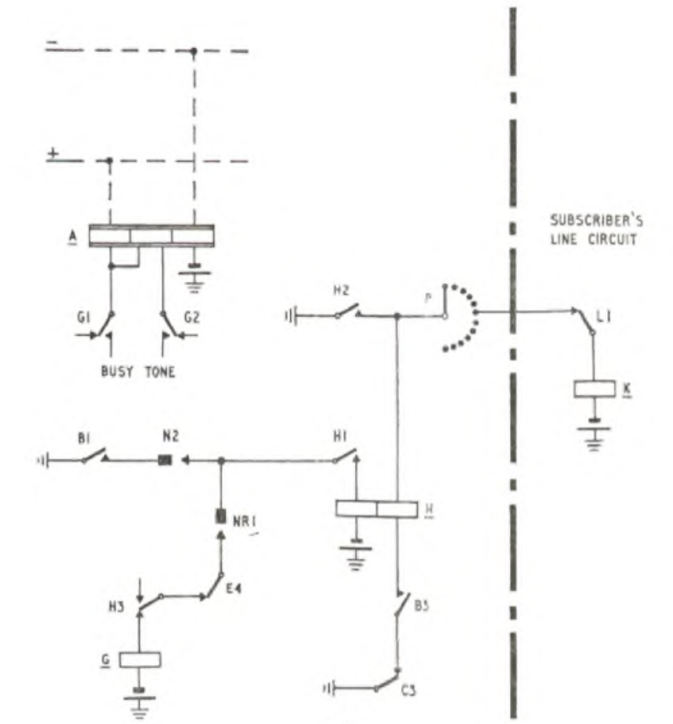
A. 5. The sketch shows the elements of the circuit concerned in the rotary stepping-action of a 2,000-type 2-motion group selector. After completion of vertical stepping under the control of the subscriber's dial, or machine pulses, according to the system, relay C releases. At this stage, relay B is operated, and relay H, previously operated via contact C2, is maintained operated via contact H1 and the rotary interruptor-contacts, RI. The same earth condition now energizes the rotary magnet via contact N2 operated and contact C2 normal. The rotary magnet operates and the wipers step into the bank and onto the first bank contact. Operation of the rotary magnet causes the rotary interruptors to open, and this breaks the hold circuit for relay H, which is now dependent on the condition received from the P-wiper, via contact H2. Should the outlet be busy, the earth on the P-wire will hold relay H operated. The breaking of the rotary interruptors disconnects the magnet circuit and the magnet releases. When the interruptors remake, the operate circuit is again completed since relay H is held, and the wipers are stepped to the second bank contact. The process continues until the wipers step to a free outlet, where a disconnexion is encountered. Relay H then releases, and when the rotary interruptors restore, there is no longer a circuit for the rotary magnet. Contact H3 releasing, applies an earth to the P-wiper to busy the seized outlet, pending the application of a busying and holding earth from the selector in the following stage. The release of relay H causes relay C to re-operate. This causes the positive, negative and P-wires to be switched to the seized outlet and relay H to be re-operated via contacts B2, N2 and C2. Relay H locks, via contact H2, to the guarding earth on the P-wire and, when relay B releases, holds relay C operated. The practice of relay H remaining held during hunting, and releasing when a free outlet is found, is known as the cut-drive method.

The stepping speed during the rotary action is determined mainly by the operate and release times of the rotary magnet, and by the adjustment of the rotary interruptor-contacts. To some extent the hunting speed is also affected by the wiper adjustment, and the state of cleanliness of the bank and bank contacts.

Q. 6. Explain, with the aid of sketches of the circuit elements concerned, how a final selector:

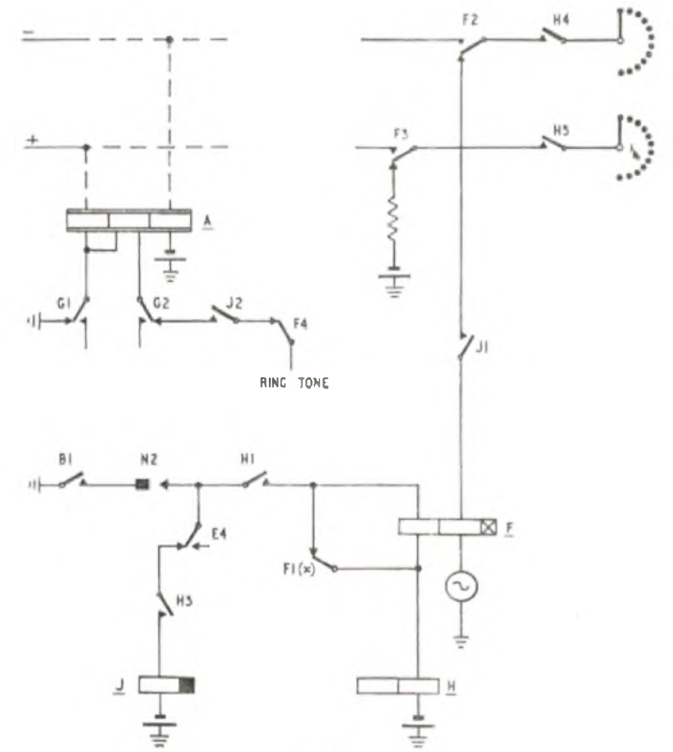
(a) tests the called line to determine whether it is free or engaged,
(b) rings on the called line until it is answered,
(c) initiates metering action when the called subscriber answers.

A. 6. The testing circuit of a 2,000-type final selector is shown in sketch (*a*). When the called subscriber's line is free, relay H operates to the K relay battery on the release of relay C, which has released at the end of the impulse-controlled rotary stepping. Relay H locks via



(*a*)

contacts H1, N2, and B1 which have been operated. Other contacts of relay H, shown in sketch (*b*), extend ringing current to the called subscriber. Contact C3 prevents an earth from being extended to the P-contacts of lines passed over during rotary stepping. Should the called subscriber's line be busy, an earth is present on the P-wire,

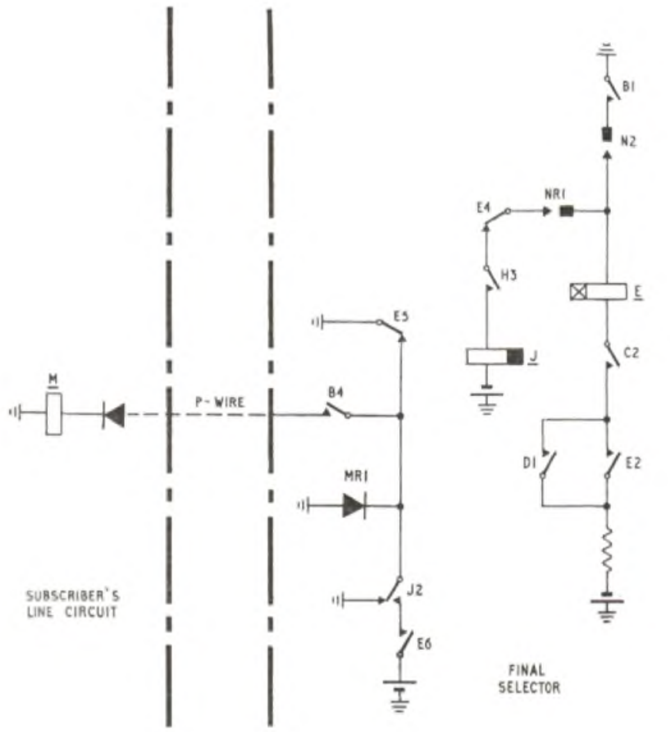


(*b*)

relay H does not operate and, on the release of relay E, a circuit is completed for the operation of relay G via contacts H3, E4, NR1, N2 and B1. Contacts of relay G apply busy tone to the calling subscriber's line via the tone coil of relay A.

Sketch (*b*) shows that part of the final-selector circuit which controls the connexion of ringing current to the called subscriber's line. Ringing current is extended to line via a coil of relay F, and the circuit is completed via the subscriber's bell and capacitor and the ringing return battery at contact F3. Ring tone is connected to the tone winding of relay A via contact J2, and hence to the calling subscriber. A hold circuit is prepared for relay F, via contacts B1, N2, and H1 and one coil of relay H. The slow-to-operate slug on relay F, together with the winding short-circuited by contact F1 normal, prevent the relay operating to the ringing current passing through one coil to line. When the called subscriber answers, the completion of the telephone loop causes direct current to flow around the ringing path, and this operates relay F. Contact F1 quickly removes the short-circuit from the hold coil, and establishes an alternative holding circuit for the relay before the operating circuit is broken at contacts F2 and F3. These contacts extend the called subscriber's loop to the final selector D relay (not shown), which operates and initiates metering.

The elements of the metering circuit which apply current to operate the calling subscriber's meter are shown in sketch (*c*). At the beginning



(*c*)

of the sequence, relays B, H, C, and J are operated, and off-normal contacts N2 and NR1 are also operated. The operation of relay D, when the called subscriber answers, completes a circuit for relay E, which operates. The operation of relay E breaks the circuit of relay J, which is slow to release. During this slow release period, while contact J2 is still operated, positive battery is extended to the P-wire via contacts E6, J2, and B4. Rectifier MR1 does not conduct to the positive battery, but maintains a guarding earth on the final selector P-wire during the transit time of contact J2 as it restores. Having operated, relay E locks independently of relay D so that the metering sequence cannot recur should the called subscriber flash and cause relay D to release and reoperate.

Q. 7. What services are provided by an assistance operator in a network that is almost fully automatic?

Explain how a caller's telephone controls the cord-circuit supervisory lamp on the assistance operator's position at a sleeve-control switchboard. Illustrate your answer with sketches of the circuit elements concerned.

A. 7. The services which are provided by an assistance operator in an almost wholly automatic network may be summarized as follows:

(*a*) to enable automatic subscribers to obtain access to the few exchanges which have not yet been incorporated into the national subscriber trunk dialling network,

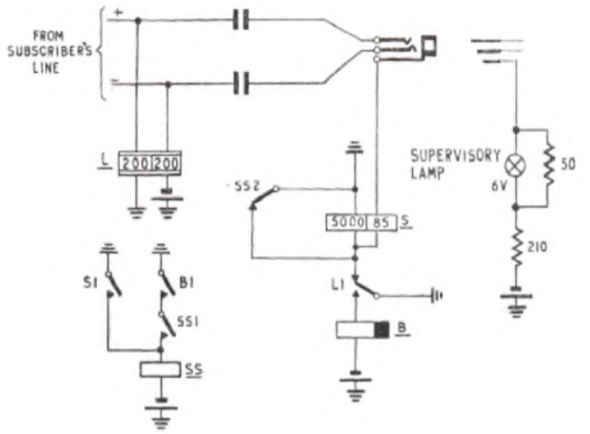
(*b*) to assist subscribers in obtaining numbers in cases of difficulty, e.g. where permanent engaged tone is encountered, or where the connexion, when established, is not satisfactory, (*c*) to complete calls for subscribers who are unable, through physical disability, or are unwilling, to dial the wanted number for themselves,

(*d*) to deal with special services such as reversed-charge calls, service interception, changed-number interception, personal calls, early morning calls, etc.,

(*e*) to receive emergency calls (999 service) and connect the caller to the wanted service,

(*f*) to receive complaints and fault reports,

(*g*) to provide a directory enquiry service.



The sketch shows the circuit elements involved at an assistance operator's position for controlling the cord-circuit supervisory lamp from the subscriber's telephone. Assuming that the subscriber is on the line when the operator plugs her cord into the answering jack, relay L will be operated to the subscriber's telephone loop. Contact L1 operates relay B. The operation of relay L previously caused the calling lamp to glow, but this circuit element is not shown. Battery via the supervisory lamp and associated resistors causes relay S to operate over its 85-ohm coil. Contact S1 completes a circuit for relay SS, which operates and removes the short-circuit from the 5,000-ohm coil of relay S. This relay now holds to the sleeve of the cord circuit over its two coils in series, but the current flowing is insufficient to allow the supervisory lamp to glow. The darkened supervisory lamp informs the operator that the subscriber is on the line. When the subscriber clears, relay L releases and contact L1 short-circuits the 5,000-ohm coil of relay S. Relay S remains held, but the increased current via the 85-ohm coil now causes the supervisory lamp to glow, thus indicating to the operator that the subscriber has replaced his handset. The circuit arrangement shown in connexion with the sleeve conductor applies when the operator's speak key is normal; when her key is operated, marginal relays are introduced in place of the lamp. These relays operate to the conditions received via the coils of relay S, according to the value of current, and cause the supervisory lamp to glow or darken appropriately. Thus the sleeve-control feature is retained.

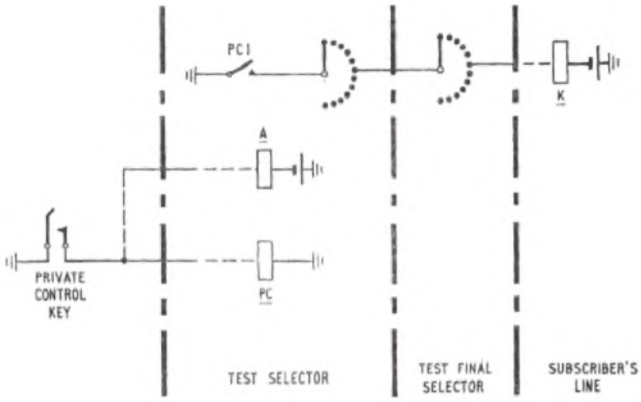
Q. 8. *Describe in general terms the automatic selectors which enable a test clerk to gain access to a line for the purpose of making tests with a voltmeter.*

Explain, with the aid of sketches of the circuit elements concerned, how the test clerk remotely operates and releases the cut-off relay in the subscriber's line circuit.

A. 8. The test clerk gains access to a line for testing purposes by means of test selectors. These consist essentially of 2-motion selectors, which enable any wanted line to be selected. They differ from ordinary final selectors in that they extend a pair of wires from the test cord to the line under test, without interposing a transmission bridge. Control of a test selector is achieved over a separate pair of wires known as the control or operate pair, leaving another pair, the test pair, free for testing purposes.

To set up a test connexion, the test clerk selects the thousands group of the wanted number by plugging his test cord into the appropriate access jack on the test desk. The subscriber's number is then dialled. The first digit steps a test selector to the required level, and the second digit steps the same selector to the outlet in the level to seize the required test final-selector. The last two digits are dialled into the test final-selector to select the required line. Indications are given to the test clerk if the test final-selector is in use by another test clerk, or if the wanted line is busy. Having reached the wanted line, the test clerk operates the HOLD AND TEST key, which in turn operates relay K in the subscriber's line circuit, and the test pair is then extended through to line.

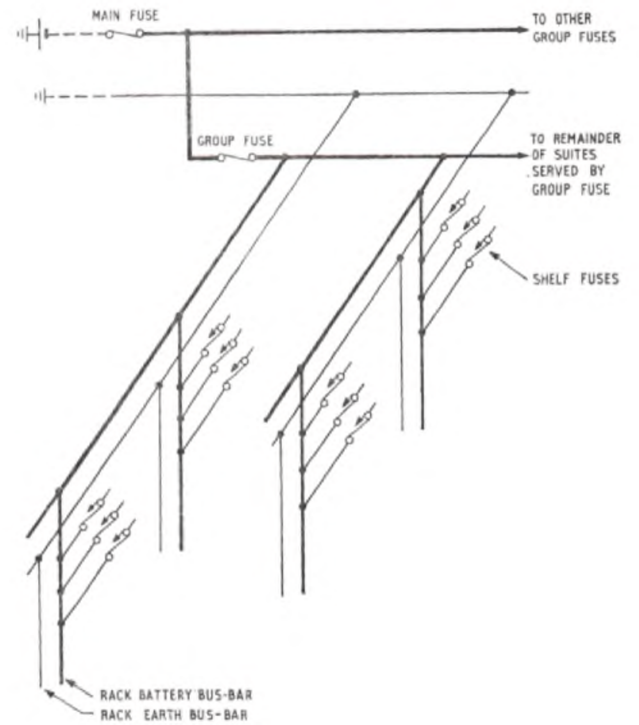
The sketch shows the principle used to control the operation of the subscriber's K relay. The line having been seized for test, relays A and PC in the test selector are held in series, and contact PC1 holds relay K operated. By operating the PRIVATE CONTROL key, the test



clerk causes an earth condition to short-circuit relay PC, which releases and in turn allows relay K to release. When the key is restored, relays PC and K reoperate. Relay A is held throughout the test to hold the switching train. With relay K released, the test clerk can test into the subscriber's line circuit.

Q. 9. *Describe, with the aid of a sketch, a suitable bus-bar and fusing arrangement for distributing power from the negative 50-volt battery to the apparatus racks.*

What maximum voltage-drop should be permitted in the system? What determines the instantaneous voltage-drop which occurs if a short-circuit fault arises in the wiring of a selector on a rack?



A. 9. The sketch shows a typical bus-bar and fusing arrangement for distributing the negative 50-volt power supply to apparatus racks. From the main-battery fuse the negative supply is run along the ends of the apparatus suites by means of copper or aluminium rectangular-section bus-bars. At intervals, a feed is taken to a group-fuse board, which holds a lower-rated fuse, and this serves several suites, according to the loading of the racks comprising the suites. The cross-sectional area of the bus-bars feeding down the length of each suite, is smaller than that of the main bus-bars. Similarly, the cross-sectional area of the bus-bars feeding from the suite bus-bars down the sides of individual racks is less than that of the suite bus-bars. At intervals down the length of the rack bus-bars, fuses are mounted on small panels from which the battery supply is wired to the shelves and individual equipments. A rack fuse can serve a shelf of equipment or more, or only a few items per shelf, according to the loading.

The earth bus-bar runs parallel with the battery bus-bar and they are suitably clamped to supports along their lengths. The battery bus-bar is insulated throughout but the earth bus-bar is not usually insulated along the suites and down the racks.

The main and group fuses are of the cartridge type, with smaller alarm-type fuses in parallel. The rack fuses are of the alarm type.

The voltage drop between the battery terminal and the farthest rack-fuse-panel must not exceed 1·0 volt; this includes both lead and return. The voltage drop across the group fuse should not exceed 100 mV at full-rated current.

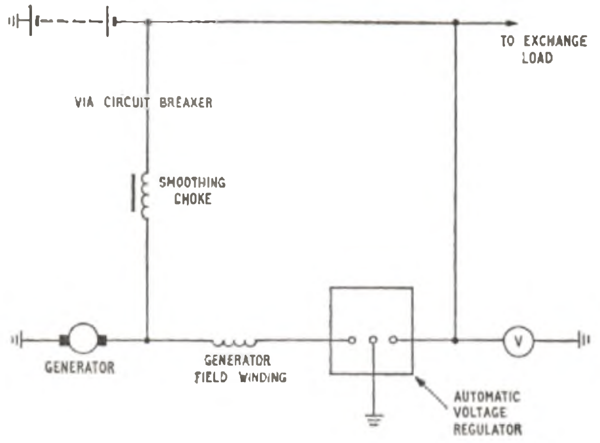
If a short-circuit occurs in the wiring of a selector on a rack, the voltage-drop will depend on the total resistance of the power leads from the battery to the point of short-circuit, together with the internal resistance of the battery. This may be regarded as the instantaneous voltage-drop since, after an interval of the order of milliseconds, a fuse should blow to disconnect the short-circuit condition.

Q. **10.** *Why is it desirable in a negative* 50-*volt floated-battery power plant to include arrangements for controlling the voltage of the floated battery?*

Describe a method of voltage control which permits only a very small variation (less than ± 1 *volt) from the nominal voltage. Sketch a simplified circuit of the method you describe.*

A. 10. Unless suitably controlled, the voltage of the exchange battery, operated on the float principle, will fluctuate over a wide range, because over a period of 24 hours the exchange load varies greatly. Thus, the current passing from the float-charge power plant into the battery can rise to a high value during periods of light traffic, and the battery voltage (and hence the voltage supplied to the exchange) will rise accordingly. This has two undesirable effects; the exchange voltage may exceed the design limits of the apparatus, causing circuit malfunctioning, and the battery life may be shortened by excessive charging, leading to gassing and damage to the plates.

The sketch shows a common method of controlling the battery voltage in a system using a motor-generator as the float-charge source. Typically, the voltage would be maintained between 50·25 and 51·75 volts. The output of the generator is connected in parallel with the battery to supply the exchange load. The voltage at the bus-bars is monitored by an automatic voltage-regulator unit controlling a device which introduces a variable resistance into the field winding of the generator. As the voltage at the bus-bars rises, increasing current in the operating coil of the regulator causes more resistance to be inserted in series with the field winding, thus causing the generator output voltage to fall, and the charging current to be reduced. The converse happens when the exchange voltage starts to fall.



A contact voltmeter is also arranged to monitor the exchange voltage. Should the voltage attain a value outside the limits set, i.e. 50·25 and 51·75 volts, a prompt alarm is activated.

Published by *The Post Office Electrical Engineers Journal*, 2–12 Gresham Street, London, E.C.2, and printed in Great Britain at The Gresham Press by Unwin Brothers Limited, Woking and London