

SUPPLEMENT

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Contents

LINE PLANT PRACTICE B, 1975	49
COMPUTERS B, 1975	52
TELECOMMUNICATION PRINCIPLES C, 1975	56
MATHEMATICS C, 1975	59
COMMUNICATION RADIO C, 1975	64
TELEGRAPHY C, 1975	68
TELEPHONY C, 1975	72
LINE TRANSMISSION C, 1975	76

QUESTIONS AND ANSWERS

Answers are occasionally omitted or reference is made to earlier Supplements in which questions of substantially the same form, together with the answers, have been published. Some answers contain more detail than would be expected from candidates under examination conditions.

For economic reasons, alternate issues of the Supplement are published in 32-page and 16-page sizes.

LINE PLANT PRACTICE B, 1975

Students were expected to answer any 6 questions

Q 1 What do you understand by the following terms used in concrete mixing and placing:

- water-to-cement ratio,
- volume batching,
- weight batching,
- curing, and
- vibration?

A 1 (a) The water-to-cement ratio of a concrete mix is the weight of water in the mix divided by the weight of cement. The strength and density of a particular mix are directly governed by the amount of water used: the higher the proportion of water, the weaker the concrete. A small proportion of the water used is needed for the chemical reaction to take place, and the remainder is needed to produce a mix that can be easily compacted. The water-to-cement ratio must be kept as low as possible consistent with the degree of workability required.

(b) Volume batching is a method of measuring out the ingredients of a mix by volume before mixing. There is always the danger of variation between one batch and another and, consequently, extra care must be taken when the quantities are measured. On no account should the "shovelful" be used as a unit of measurement. A special box, known as a *gauge box*, is used to measure the sand and aggregate for the mix, and is designed so that multiples of the volume of a bag of cement (0.034 m³) can be used. Thus, the measuring out of the cement is eliminated except for counting the number of bags used.

(c) Weight batching is the best method of mixing concrete for high-quality work. The weight of sand varies little from one batch to another because of its water content, and a mix specified by weight is more likely to be of consistent quality. Proportioning by weight is usually based on the 50 kg bag of cement, and a mix is usually specified as 50 kg of cement to so many kilograms of sand and aggregate. For example, an average mix may be specified as 50 kg of cement : 120 kg of sand : 190 kg of aggregate.

(d) Concrete hardens as a result of chemical reactions between the cement and the water, the reaction continuing as long as the temperature is favourable and sufficient water is present. The hardening process is not due to drying out and, if drying out is allowed to take place too quickly, the concrete will have a much lower crushing strength than estimated. Thus, in warm or windy weather, concrete should be prevented from drying out.

The hardening of concrete is retarded at low temperature and, at freezing point (0°C), the hardening process almost ceases. Furthermore, if ice forms within the body of unhardened concrete, there is a serious danger of the concrete being broken up and disintegration occurring.

It is essential to protect hardening concrete from sun, wind and frost, and to keep it wet for some days after placing. Portland-cement concrete should be kept moist for 7 d. The practice of preventing the premature drying out of newly placed concrete is called *curing*, and the concrete is referred to as being "green" until it hardens.

(e) Concrete can be placed in position by mechanical vibration. Vibrators, operating at 41.7–200 Hz, are either attached to the formwork or inserted in the concrete.

Vibration enables drier mixes to be used, producing a concrete of greater durability and higher strength for the same cement content. Also, formwork can be removed from vibrated concrete after a much shorter period than for hand-compacted concrete.

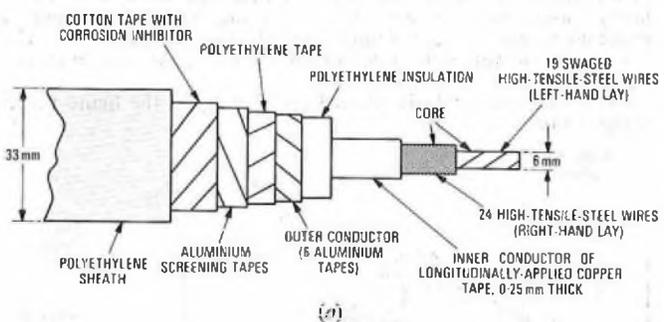
Although vibration has advantages over hand-compaction, the exact manner of its application and duration must be carefully controlled; otherwise, an excess of water in the upper sections, and a deficiency of cement and water in the lower, can result.

Where hand-compaction is used, concrete should be placed in layers of not greater than 300 mm depth at one time. With vibration, the depth can be increased to 600 mm.

Q 2 With the aid of sketches, describe

- the make-up of a lightweight deep-sea telephone cable, and
- the method of joining the core and inner conductor only of a lightweight deep-sea cable.

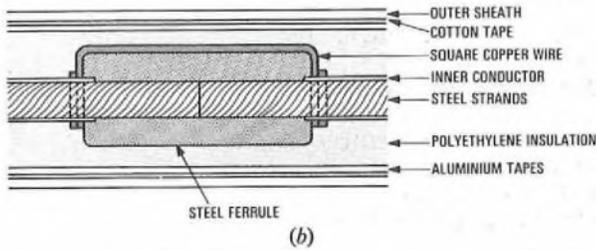
A 2 (a) A section of lightweight deep-sea coaxial cable is shown in sketch (a).



The high-tensile-steel core is torsionally balanced so that it has no tendency to twist under tension. The copper inner conductor is applied tightly over the core and closed with a folded box-seam. Polyethylene insulation is applied to the inner conductor to increase the diameter to 25 mm. Six 0.45 mm thick aluminium tapes are then closely applied to the surface of the polyethylene to form the outer conductor. These are separated from aluminium screening tapes by a layer of polyethylene tape. The screening tapes are held in position by cotton tape, impregnated with barium chromate to prevent the aluminium tapes from corroding. A tightly-extruded sheath of polyethylene over the cotton tape then completes the cable.

(b) A joint in a lightweight deep-sea cable is made with an overlap of approximately 300 mm. The polyethylene sheath is removed for a distance of 200 mm, and the cotton and aluminium tapes are laid back to expose the copper inner conductor. Electrodes are connected about 100 mm apart on the copper tube, and a large current is applied until the tube and its enclosed steel strands are burnt through. The

heat causes the steel strands to become welded together, which ensures that they all take a share of any tensile load applied to the completed joint. When the 2 cable-ends have been thus treated, they are allowed to cool before being cut back and filed to form stubs of circular cross-section, capable of being butted together.



The welded stubs are coated with a layer of silicon-carbide dust to provide extra grip, and are inserted into a 115 mm long steel ferrule. The bore of the ferrule is such that the ends of the copper tubes are just nipped by the ends of the steel ferrule. A press, capable of exerting a force of 2 MN on a 50 mm length of ferrule, is used to swage the ferrule down on to the steel strands; the central section of the ferrule is pressed first, and then the ends. After pressing, the ferrule is approximately 130 mm long.

A groove in the steel ferrule accommodates a 0.315 mm square-section copper wire, which is soldered to each copper tube with high-melting-point solder to maintain the continuity of the inner conductor. The joint is illustrated in sketch (b).

Q 3 (a) Describe 4 major points that have to be considered in the design of a foundation for a supporting leg of a lattice-steel aerial tower.
 (b) With the aid of a sketch, describe a typical foundation block for one leg of an aerial tower.

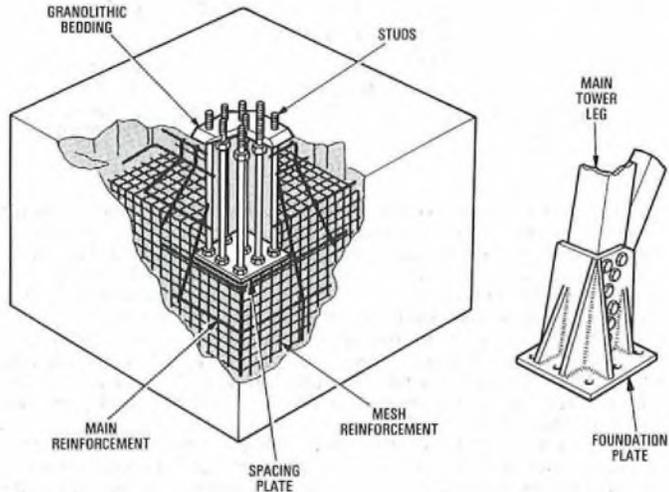
A 3 (a) The foundations of a lattice-steel aerial tower have to be designed so that the forces imposed by the wind and aerial attachments do not fracture the foundation or, in the extreme case, lift the whole foundation out of the ground. The following 4 points have to be considered in the design.

(i) Because the stresses induced in the foundations are both compressive and tensile, the concrete must be reinforced. Where each leg has an individual foundation, the main reinforcement consists of steel bars, and there is a smaller secondary mesh. Where the foundation is a slab, the legs can be joined within the slab by steel joists laid at right angles to each other.

(ii) The foundations are generally designed assuming the worst possible loading conditions.

(iii) Where there is a tendency for the foundation block to lift out of the ground, it can be provided with a toe, but the block should be heavy enough to resist overturning or lifting out of the ground. As undisturbed soil on a deep foundation aids this resistance, every effort is made to excavate only sufficient soil for the foundation to be constructed.

(iv) A factor of safety is allowed on all designs, the figure usually being 1.5 or 2.

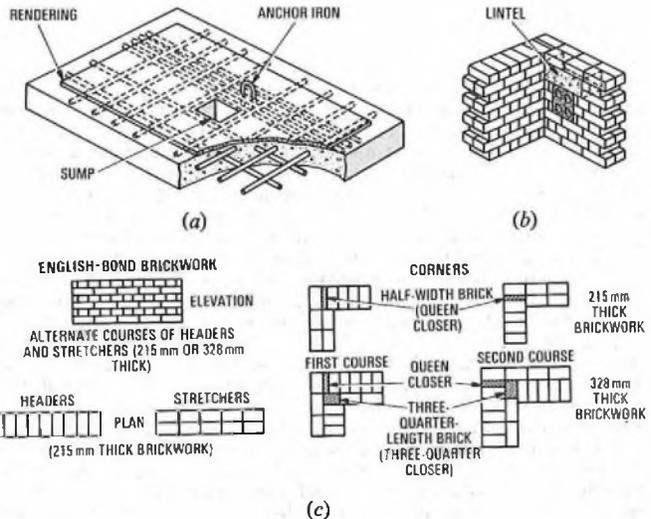


(b) The sketch illustrates a typical foundation block for a tower, showing the main and mesh reinforcement and the method of securing the leg by means of a foundation plate. The foundation plate is bolted

to the foundation block by threaded studding spaced by a steel plate embedded in the concrete. The studding projects through a granolithic bedding; this is a screed of sand, cement and granite chippings which provides an extremely hard base for the foundation plate to rest on. A typical block is 4 m square and 3 m high.

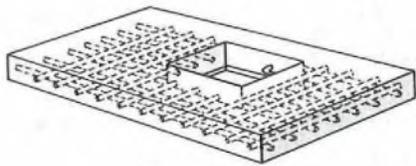
Q 4 A brick manhole is to be built on a duct route in the footway. Describe, with the aid of sketches, the processes of
 (a) constructing the floor,
 (b) building the walls and leading-in the ducts,
 (c) constructing the roof, and
 (d) fitting the anchor irons.

A 4 (a) The floors of brick manholes are constructed in reinforced concrete. With the excavation taken out to the dimensions required and, if necessary, the sides suitably timbered with poling boards, walings and struts, the base is levelled and any soft spots dug out. Hardcore is then placed in the base of the excavation and consolidated to form a solid foundation. A hole, 530 mm square and 300 mm deep, is excavated in the hardcore to form a sump. A layer of concrete of quality A (a 1 : 2 : 4 mix of cement, sand and 19 mm graded aggregate) is placed in the base of the sump to a thickness of 150 mm, and shuttering is erected to give a finished internal measurement of 230 mm square and 300 mm deep. Concrete of quality A is placed evenly over the hardcore foundation to within 35–40 mm of the finished floor level. The floor anchor-iron is placed on the concrete directly beneath the proposed position of the manhole entrance, followed by its two 32 mm diameter mild-steel reinforcing bars and the main floor-reinforcement of 12 mm diameter mild-steel bars. The main floor-reinforcement bars are laid so that their upper surface is 25 mm from the finished floor level. The remainder of the concrete is placed to give a finished floor thickness of 150 mm. Care is taken not to disturb the reinforcement when the concrete is vibrated. On completion of the construction of the manhole, a 20 mm thick cement mortar rendering is placed on the floor, sloped towards the sump. Sketch (a) shows a typical reinforced-concrete floor.



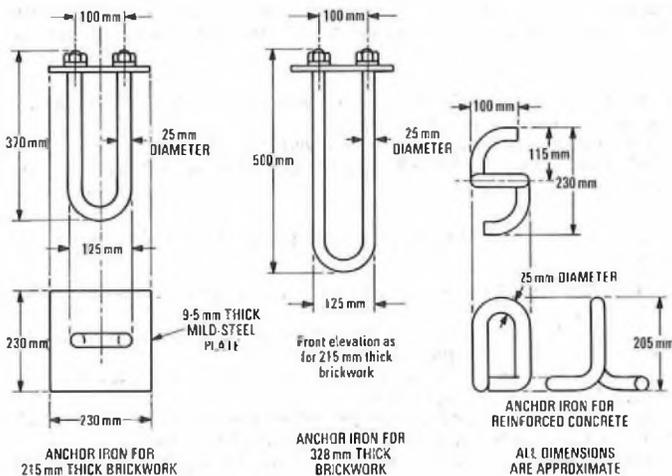
(b) The walls are constructed in either 215 mm or 328 mm thick brickwork, depending upon the size and siting of the manhole. Construction cannot commence until a minimum of 12 h after completion of the floor, when the portion of the floor on which the walls will be placed is cleaned of laitance and dirt. Engineering bricks are used throughout in English-bond brickwork, and are laid in cement mortar (a 1 : 3 mix of cement and sand) and weather-pointed. The brickwork above the point of entry of a duct, or nest of ducts, is supported by a reinforced-concrete lintel. The space between the back of the wall and the face of the excavation is back-filled as the brickwork progresses. Sketch (b) is a view of a section of brickwork, showing a duct-entry and concrete lintel. Sketch (c) shows the details of English-bond brickwork. The lintel is 150 mm high and has the same thickness as the brickwork. For 215 mm thick brickwork, the lintel has two 12 mm diameter reinforcing bars and overlaps the duct-entry by 100 mm each end. For 328 mm thick brickwork, the lintel has three 12 mm diameter reinforcing bars and overlaps the duct-entry by 150 mm.

(c) On completion of the walls, the shuttering for the roof is erected, including that for the shaft entrance. The shuttering is such that it can be dismantled and passed out through the entrance shaft when the roof is completed and adequately set. Prior to placing the roof concrete, a minimum of 12 h after construction of the walls, the upper



(d)

surface of the brickwork is cleaned, and a thin layer of cement mortar placed to make a suitable joint between the walls and roof. A layer of concrete of quality A, approximately 25 mm thick, is placed, and 12 mm diameter mild-steel reinforcing bars are positioned. The remainder of the roof concrete is placed to give a finished thickness of 200 mm. Additional 12 mm diameter mild-steel reinforcing bars are placed around the shaft, 25 mm from the top of the roof. Care is taken not to disturb the reinforcement when the concrete is vibrated. After 12 h, work can commence on the construction of a shaft if necessary, and on the fitting of the frame and cover. Where ordinary Portland cement is used, the shuttering can be removed after 5 d. Back-filling, can be started after 7 d and the carriageway finally opened to traffic. Sketch (d) shows a typical reinforced-concrete roof.



(e)

(d) Four anchor irons are fitted in each end wall, and one in the floor. The floor anchor-iron is sited directly beneath the shaft entrance, and is installed at the time of the construction of the floor, as described in part (a) and illustrated in sketch (a). One wall anchor-iron is fitted in each corner of each end wall, 150 mm from the floor and roof, and 230 mm from the side walls. The wall anchor-irons are fitted during construction of the walls, ensuring that the back-plate is hard up against the back of the wall. Sketch (e) illustrates the 2 types of anchor iron used in brickwork and reinforced concrete.

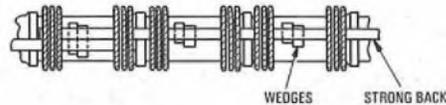
Q 5 (a) Describe in detail, with sketches, a method of slewing and lowering a multivay earthenware duct route.

(b) State 3 disadvantages of slewing and lowering plant.

A 5 (a) Before slewing and lowering plant, it should be checked that any cable left in the duct will not be damaged, and that pulling of the joints in the jointing chambers at each end of the duct route concerned will not occur. Where the route will be lengthened by the operation, sufficient slack in either or both of the jointing chambers is necessary. Particular attention should be paid to coaxial cables that will remain in service during the work.

The exact position of the duct is first determined, and any obstructions that must be removed or negotiated are located. This is achieved by excavating pilot holes at suitable intervals along the route. The duct route is then fully excavated, and the trench is widened and deepened to the new line of the route. The sides of the trench are timbered using poling boards, walings and struts as necessary.

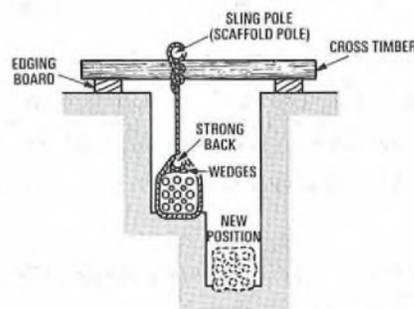
To minimize damage to the duct line during the slewing and lowering operations, a "strong back" of tubular scaffold poles, or lengths of timber, is placed along the top of the ducts. Each length of duct is rigidly secured to this support; the poles or timber forming the strong back must overlap sufficiently to maintain rigidity. Wedges are then driven between the strong back and the duct to ensure that ultimately, when the duct line is supported from the top, there will be no sagging. Sketch (a) shows the plan view of a duct line and strong back.



(a)

Edging boards are placed approximately 450 mm from the edge of the trench on both sides to distribute the load during slewing and lowering of the duct line. Stout baulks of timber are laid across the trench at intervals of just over 1 m, resting on the edging boards. Sling poles are laid on the cross timbers, directly above the duct line, and secured to the strong back with rope. The rope ties at the sling pole should be such that they will hold during the slewing operation, but provide enough friction to enable the duct line to be gently lowered at the appropriate time.

Earth is removed from beneath the ducts until they are freely supported from the sling poles. Any loose earth is cleared from the bottom of the new trench, and the bottom levelled and well punned. In rocky or stony soils, earth that is free from stones is placed in the bottom of the trench and hand punned to a finished thickness of 75 mm. Sketch (b) shows a cross-sectional view of the trench and supported duct line.



(b)

An adequate labour force is necessary on site to ensure that the slewing and lowering operations are carried out smoothly and gently in one operation. The sling poles are gradually slid over the cross timbers until the duct line is directly above its new position. All the rope ties are then simultaneously eased off at the sling poles, and the duct line is lowered into its new position. When the duct line is nearly in position, holes for the duct collars are excavated; this is done at this stage to ensure accurate location and minimize breakages. If the trench is timbered, the struts must be gradually moved towards the bottom of the trench, until a position is reached where they can be replaced above the ducts.

When the duct line is firmly bedded in the new position, the ropes, strong-back and sling poles are removed, and the ducts inspected for damage. Damaged ducts containing no cables are replaced by similar good ducts or split ducts; damaged ducts containing cables are replaced by split ducts. Minor damage can be repaired *in situ* with a cement mortar or cement-compound mixture. Where the duct route has been lengthened, short lengths of ordinary or split duct are inserted in the duct line. On completion of these operations, uncabled ducts are cleaned and tested using a brush and mandrel.

(b) The main disadvantages of any slewing and lowering operation are that

- (i) duct joints are invariably disturbed and become less watertight,
- (ii) despite the extreme care taken, duct breakages do occur and, although the damage can normally be repaired, the quality of the duct line is permanently impaired from the point of view of rodding and cabling, and
- (iii) slight alterations to the length of the duct line can occur, and this puts any cables in the duct line at risk, particularly coaxial cables. Unless precautions are taken, it is possible to pull-fracture joints in the inner conductors of coaxial cables. The alternatives are either to take precautions at the joints or bypass the section concerned with an interruption cable.

Q 6 Describe in detail, with a sketch,

- (a) the make-up of a 2.6/9.5 mm coaxial pair, and
- (b) a method of terminating a 2.6/9.5 mm coaxial pair.

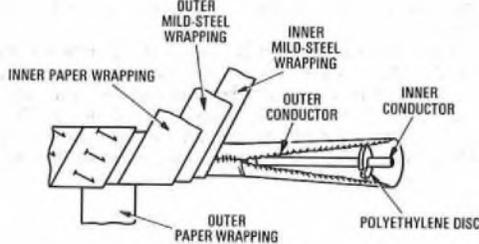
A 6 (a) A 2.6/9.5 mm coaxial pair consists of a solid-copper inner conductor, 2.6 mm diameter, enclosed in an outer cylindrical conductor of 9.5 mm internal diameter. The outer conductor is formed from a single soft-copper tape, 0.25 mm thick, and is supported

LINE PLANT PRACTICE B, 1975 (continued)

concentrically with the inner conductor by polyethylene discs spaced at 33 mm intervals along the inner conductor.

The soft-copper tape has corrugations along its edges which prevent the seam joint from overlapping when the tape is folded into a tube. Asymmetrical corrugations with a left-hand bias are impressed along one edge, and corrugations with a right-hand bias along the other edge. When the corrugations come together at the seam, they cannot interlock or ride over each other as the effective thickness of the seam is increased to 3 or 4 times the thickness of the tape.

To provide adequate mechanical strength and electromagnetic screening, the pair is bound with 2 mild-steel tapes, 0.13 mm thick. To insulate and identify the pair, 2 thicknesses of insulating paper are applied over the steel tapes, the outermost tape having an identifying number printed on it. A complete 2.6/9.5 mm coaxial pair is shown in the sketch.



Normally, this type of coaxial pair is made up into either 2, 4, 6 or 8-pair cables, with conductors filling the interstices to form a core of circular cross-section. Two layers of paper tape are then wound around the complete core and, finally, a lead sheath is applied. Protection against corrosion is obtained by sheathing the lead with polyethylene.

(b) See A5, Line Plant Practice B, 1971, Supplement, Vol. 65, p. 49, Oct. 1972.

Q. 7 A new exchange requires a lead-in for a 24-way PVC duct. Describe, with sketches, the

- preparation of the excavation,
- setting-up of the duct nest,
- method of concreting, and
- method of finally testing the ducts.

A 7 See A7, Line Plant Practice B, 1970, Supplement, Vol. 64, p. 65, Oct. 1971.

Q 8 Alternative methods of providing telephone plant are being considered.

- What is meant by the term annual charges?
- State the 4 factors under which annual charges are assessed.
- Explain, in fuller detail, 2 of the factors stated in part (b).

A 8 (a) The cost of the provision, maintenance, and recovery (at the end of its average life) of an item of plant can be expressed as equal sums of money paid at the end of each year for the average life of the plant. This annual sum is known as the annual charge, and includes interest, depreciation, maintenance and, where applicable, operating costs.

The present value of the annual charge is the amount of money which, if invested at the prevailing long-term interest rate, would enable the annual charge to be paid at the end of each year over a given period of time. This period of time is known as the costing period. For external plant, the annual charge is derived from annual-charge rates, which are expressed as percentages of the capital cost of the plant.

(b) The 4 factors under which annual charges are assessed are

- interest,
- depreciation,
- maintenance costs, and
- operating costs.

(c) For completeness, a brief summary of each factor is given.

Interest The interest rate used is the long-term average interest rate, and the annual interest is the average interest paid on capital raised over a long period.

Depreciation The depreciation rate is the rate at which money is required to be invested annually in an interest-bearing sinking fund, so that, at the end of the average life of the plant, an amount is available to add to the net residual value of the plant either to renew the plant or to pay back the original borrowed capital.

Maintenance The maintenance rate is the average annual cost of maintaining the plant in serviceable condition over its average life.

Operating Costs When a proposed project needs staff to operate the equipment, the resulting operating costs are expressed as an annual charge.

Q 9 Cathodic protection is to be provided on a cable system either by reactive anodes or impressed current.

- State 5 disadvantages of using reactive anodes.
- State 4 factors that influence the magnitude of the current in an impressed-current system.

A 9 (a) Five disadvantages of using reactive anodes are given below.

- Their use is limited to soils of electrical resistivity of less than 300 kΩ m.
- Anodes corrode at different rates, and care has to be taken to ensure their selective replacement.
- For bare or poorly-coated structures, the cost of cathodic protection by reactive anodes is high.
- As no centralized above-ground equipment exists, the installation is liable to be forgotten and so become ineffective.
- Checking a system containing many anodes is expensive and takes a long time.

(b) Four factors that influence the magnitude of current in an impressed-current system are the

- structure-to-soil resistance,
- area over which protection is required,
- economics of one large-current scheme compared with a number of small-current systems, and
- the possibility of interference with other undertakers' plant.

Q 10 An audio cable has been laid and jointed over a 2 km section. Subsequent tapping-out tests have shown that the B-wire and C-wire of one quad are crossed. Describe, with a diagram, a method of finding the position of the fault.

COMPUTERS B, 1975

Students were expected to answer any 6 questions

Q 1 (a) Using Boolean algebra, minimize the following expression:

$$f = A.C + A.\bar{C} + \bar{A}.B + B.\bar{B}$$

(b) Using the expression in part (a), show how a map method produces the same minimized result.

(c) With the aid of examples of each, explain what is meant by the terms minterm and maxterm.

A 1 (a) $f = A.C + A.\bar{C} + \bar{A}.B + B.\bar{B}$

By the distributive law,

$$f = A.(C + \bar{C}) + \bar{A}.B + B.\bar{B}$$

Since $C + \bar{C} = 1$ and $B.\bar{B} = 0$,

$$f = A + \bar{A}.B,$$

$$= \underline{A + B}.$$

	A.B	A.B	A.B	A.B
C	1	1	0	0
\bar{C}	0	0	0	0
	A.C			

(a)

	A.B	A.B	A.B	A.B
C	0	0	0	0
\bar{C}	1	1	0	0
	A. \bar{C}			

(b)

	A.B	A.B	A.B	A.B
C	0	0	1	0
\bar{C}	0	0	1	0
	$\bar{A}.B$			

(c)

	A.B	A.B	A.B	A.B
C	0	0	0	0
\bar{C}	0	0	0	0
	B. \bar{B}			

(d)

(b) The parts of the function can be represented on Karnaugh maps, as shown in sketches (a), (b), (c) and (d). These can then be combined to form the Karnaugh map shown in sketch (e), representing the complete expression. The logic variables can be grouped as shown by the dashed lines, resulting in the expression $f = A + B$.



(e)

(c) The term minterm is applied to Boolean expressions that are written as the sum of products. In this form, there are no brackets. The expression in part (a) is in minterm form, as is the following expression:

$$A + A.B.D + \bar{B} + B.C.$$

The term maxterm is applied to Boolean expressions that are written as the product of sums. In this form, the OR expressions are bracketed and, so, should be evaluated before the AND expressions. For example,

$$(A + B + C).(A + B + C)$$

is in maxterm form. This can be converted to minterm form by multiplying out the brackets.

Q 2 (a) Why is the shift-and-subtract method of division used in digital computers?

(b) Using the shift-and-subtract method, divide binary 100 011 110 by binary 11 010, showing all working.

(c) Using a standard method, convert the answer to part (b) into denary, octal and excess-3-weighted binary-coded-decimal (BCD) form. Show all working.

A 2 (a) Division by the shift-and-subtract method is used in digital computers because the technique uses instructions that are commonly available in computers and can be quickly executed in a simple repetitive manner. This is made possible by the nature of the binary system, which restricts the value of the digits in the quotient to 0 or 1. It is therefore necessary only to check that it is possible to subtract the divisor from the dividend (that is, that the dividend is greater than the divisor) before subtraction is performed and a shift to the right made.

(b) The operations required to divide 100 011 110₂ by 11 010₂ using the shift-and-subtract method are shown below.

```

Quotient:           1 011
Divisor and dividend: 11 010 )100 011 110
Shift:              11 010
Subtract:           1 001 110
Shift:              1 101 0
Shift:              110 10
Subtract:           11 010
Shift:              11 010
Subtract:           00 000
    
```

Therefore, $100\ 011\ 110_2 \div 11\ 010_2 = 1\ 011_2$.

(c) (i) To convert a binary number into denary form, each bit is multiplied by its binary weight and the results added together. Thus,

$$\begin{aligned} 1\ 011_2 &= 1 \times 2^3 + 1 \times 2^1 + 1 \times 2^0, \\ &= 8 + 2 + 1, \\ &= 11_{10}. \end{aligned}$$

(ii) Since $8 = 2^3$, each 3 bit group of the binary number, starting at the binary point, can be converted to octal form by writing down its equivalent denary value.

$$\therefore 001\ 011_2 = 13_8.$$

(iii) In the excess-3-weighted BCD code, each decimal digit is represented by a 4 bit binary group whose denary value (in an 8421 system) is 3 larger than that of the decimal digit. Thus, to convert a binary number to excess-3-weighted BCD form, it is necessary to convert it to denary form, increase the value of each digit by 3, and take the BCD (weighted 8421) equivalent of each digit.

The denary equivalent of $1\ 011_2$ is 11_{10} . Increasing the value of each digit by 3 gives 44_{10} .

Now, $44_{10} = 0100\ 0100_{\text{BCD}8421}$.

$$\therefore 1\ 011_2 = 0100\ 0100_{\text{EXCESS-3 BCD}}$$

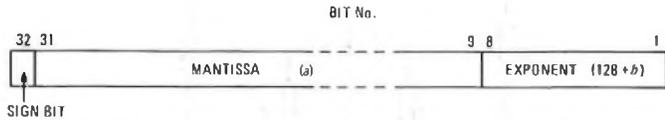
Q 3 (a) Explain what is meant by the term normalization (or standardization).

(b) Show how the following real numbers can be represented in a 16 bit digital computer that uses a 24 bit mantissa and an 8 bit exponent:

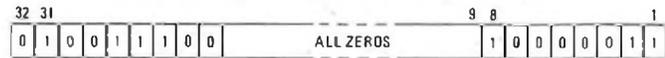
- (i) $+4\frac{7}{8}$,
- (ii) $-\frac{1}{16}$.

A 3 (a) Normalization is used in digital computers to maintain the accuracy of floating-point numbers during arithmetical operations. Following any operation involving floating-point numbers, the binary representation is adjusted to hold the maximum number of significant digits before further operations are made. For each place the binary point is moved to the right, the value of the exponent is reduced by 1.

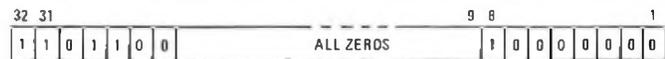
(b) In a 16 bit digital computer that uses a 24 bit mantissa and an 8 bit exponent, numbers can be represented using the format shown in sketch (a).



(a)



(b)



(c)

Floating-point numbers are of the form $A = a \times 2^b$, where a is the mantissa and b is the exponent. Note that $0.5 \leq a < 1$ for positive numbers since, when a binary point is placed in front of the most-significant digit of a binary number, that number cannot be less than 0.5 or greater than 0.9. Similarly, $-1 < a \leq -0.5$ for negative numbers. Also, $-128 \leq b \leq 127$. The exponent is stored as $128 + b$ since it does not accommodate a sign bit.

(i) $+4\frac{7}{8} = +4.875$.

The binary representation of 4.875 is obtained by repeated division of the integral part and repeated multiplication of the fractional part, as shown in the following table.

Integral Part		Fractional Part	
Quotient	Remainder	Result	Product
$4 \div 2$			0.875×2
2	0	1	0.750
1	0	1	0.500
0	1	1	0.000

$$\therefore +4.875_{10} = 100.111_2.$$

In floating-point form, $+4.875_{10} = 0.100\ 111 \times 2^3$, so that the mantissa is 100 111, the exponent is 11, and the sign bit is 0. The bit pattern is shown in sketch (b).

(ii) $-\frac{1}{16} = -0.3125$.

The binary representation of -0.3125 is obtained by repeated multiplication, as shown in the following table.

Result	Product
	0.3125×2
0	0.6250
1	0.2500
0	0.5000
1	0.0000

$$\therefore +0.3125_{10} = +0.0101_2.$$

The one's complement of 0.0101 is 1.1010. The 2's complement of 0.0101 is 1.1011. In floating-point form, $-0.3125 = 1.1011 \times 2^0$, so that the mantissa is 1 011, the exponent is 0, and the sign bit is 1. The bit pattern is shown in sketch (c).

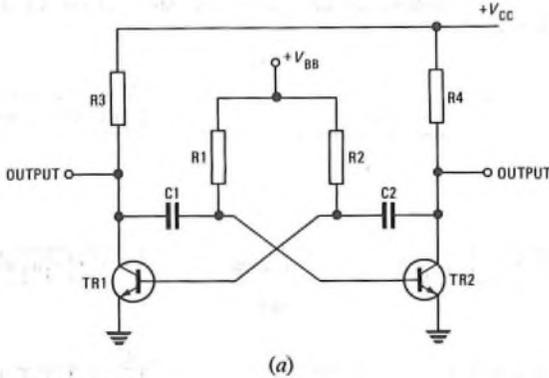
Q 4 (a) Draw a logic diagram of a serial binary subtractor. Registers may be represented by blocks, but the arithmetical logic must be shown in full. EQUIVALENCE and NON-EQUIVALENCE logic symbols will be accepted.

(b) With the aid of a truth table, verify the operation of the circuit by subtracting the binary equivalent of 23₁₀ from 37₁₀.

Q 5 (a) With the aid of a circuit diagram, explain the operation of an astable device suitable for use in digital computers.

(b) Briefly explain 2 different uses for astable devices in digital computers.

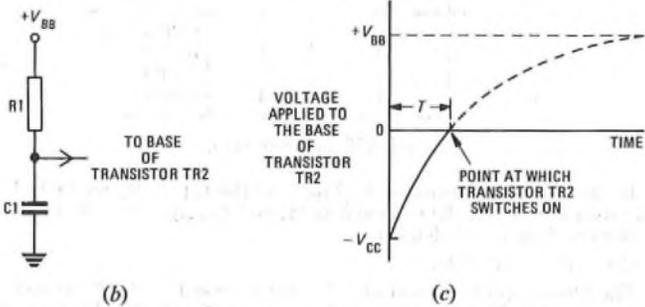
A 5 (a) The circuit diagram of an astable device suitable for use in a digital computer is shown in sketch (a).



Initially, it is assumed that transistor TR1 is in the OFF state, and transistor TR2 is in the ON state. The collector of transistor TR1 is therefore at approximately +V_{CC} volts, and the collector of transistor TR2 is at approximately 0 V. Capacitor C1 is therefore charged to +V_{CC} volts; that is, the potential on its left-hand plate is +V_{CC} volts and the potential on its right-hand plate is approximately zero.

Assuming that a short positive pulse is applied to the base of transistor TR1, the transistor switches on and its collector voltage drops to approximately zero. As capacitor C1 cannot discharge instantaneously, the potential of the right-hand plate falls by V_{CC} volts from zero to -V_{CC} volts, which switches transistor TR2 off. The voltage at the collector of transistor TR2 rises to +V_{CC} volts. Capacitor C2 is therefore charged so that the potential on its left-hand plate is approximately 0 V and that on its right-hand plate is approximately +V_{CC} volts.

Capacitor C1 discharges through the path shown in simplified form in sketch (b), until the voltage at the base of transistor TR2 is sufficient to turn that transistor on, as illustrated in sketch (c).



When transistor TR2 is switched on, its collector voltage falls from +V_{CC} volts to approximately zero. The potential on the left-hand plate of capacitor C2 thus falls to -V_{CC} volts, switching transistor TR1 off. The voltage at the collector of transistor TR1 then rises to +V_{CC} volts, and capacitor C1 is charged as described in the second paragraph. Capacitor C2 discharges until the voltage at the base of transistor TR1 is sufficient to turn on that transistor.

Thus, when transistor TR1 is on, transistor TR2 is off, and vice versa, and the output from each collector is a square-wave oscillation between zero and +V_{CC} volts.

If C₁ = C₂ = C, and R₁ = R₂ = R, the time, T seconds, for which transistor TR1 is on and transistor TR2 is off in each cycle (that is, half the periodic time) is given by

$$T = CR \log_e \left(\frac{V_{BB} + V_{CC}}{V_{BB}} \right) \text{ seconds.}$$

As the circuit is symmetrical, the frequency of oscillation is given by 1/2T hertz.

(b) Each output of a free-running astable device is a train of rectangular pulses. Therefore, the principal use is in timing operations. The pulse train can be synchronized by a low-powered submultiple of the frequency of the master oscillator of a digital computer. These pulses can be used

(i) to synchronize and control the timing of all the operations of the computer,

(ii) as binary-number inputs to the arithmetical circuits, and

(iii) to initiate binary-coded instructions to be carried out by the logic circuits.

Q 6 A digital computer has just obeyed an ADD instruction stored in location 100. With the aid of a block diagram of the internal structure of the computer, explain each microprogramme step from the completion of this instruction to the completion of a DIRECT-JUMP-FORWARD instruction stored in location 101.

Q 7 (a) With the aid of sketches, explain single-address, 2-address and 3-address instructions, and discuss the advantages and disadvantages of each method.

(b) Devise a 3-address instruction set with a comprehensive key, and use it to write a programme to solve the equation $y = 3x^2 + t^2 + 3x$. Variables x and t can be found in locations 10 and 11 respectively, and variable y must be stored in location 12.

Q 8 (a) Explain the difference between active and passive networks.

(b) With the aid of sketches, explain how

(i) an active network, and

(ii) a passive network

can be used to solve the equation $y = (x_1 + x_2 + x_3)/4$.

A 8 (a) A passive network is a group of passive elements, such as resistors, connected so that it can only dissipate any power applied to it, and is incapable of generating any power. A network is said to be active when amplifiers are added to compensate for the losses incurred in the passive components. The main assumption made for resistive networks is that all resistors are linear; that is, the current through each resistor varies directly with the voltage across the resistor.

(b) (i) An active network that will solve the equation

$$y = (x_1 + x_2 + x_3)/4$$

is shown in sketch (a). The variables to be added together are represented by input voltages x_1 , x_2 and x_3 , and the solution, y , is represented by the output of the network. The amplifier is an ideal operational amplifier having infinite gain. Thus, point X is a virtual-earth point.

Assuming the potential at point X to be zero,

$$i_1 = \frac{x_1}{R_1}, i_2 = \frac{x_2}{R_2}, i_3 = \frac{x_3}{R_3} \text{ and } i_f = \frac{y}{R_f}$$

Applying Kirchoff's first law, which states that there is no net current at a junction, gives

$$i_f = i_1 + i_2 + i_3$$

$$\text{Hence, } \frac{y}{R_f} = \frac{x_1}{R_1} + \frac{x_2}{R_2} + \frac{x_3}{R_3}$$

$$\therefore y = \left(\frac{x_1}{R_1} + \frac{x_2}{R_2} + \frac{x_3}{R_3} \right) \times R_f$$

Choosing suitable resistance values, such that $4R_f = R_1 = R_2 = R_3$, gives

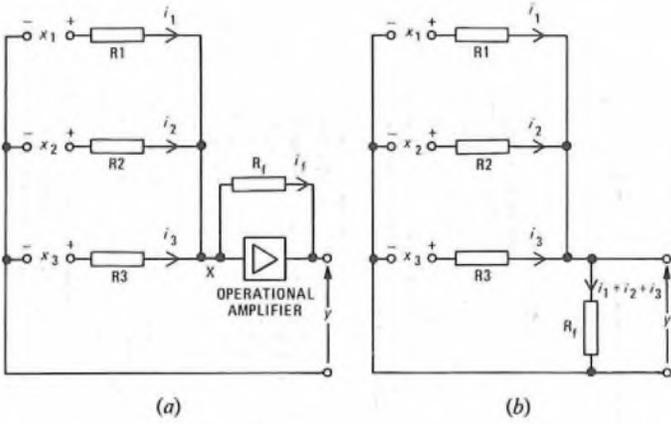
$$y = \left(\frac{x_1}{4R_f} + \frac{x_2}{4R_f} + \frac{x_3}{4R_f} \right) \times R_f \\ = \frac{(x_1 + x_2 + x_3)}{4}$$

Thus, the network can be used to solve the given equation.

(ii) A passive network that will solve the equation

$$y = (x_1 + x_2 + x_3)/4$$

is shown in sketch (b).



The output voltage, y , is given by

$$y = (i_1 + i_2 + i_3) \times R_f$$

$$\therefore \frac{y}{R_f} = \left(\frac{x_1 - y}{R_1} + \frac{x_2 - y}{R_2} + \frac{x_3 - y}{R_3} \right)$$

$$\therefore \frac{y}{R_f} + y \times \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) = \frac{x_1}{R_1} + \frac{x_2}{R_2} + \frac{x_3}{R_3}$$

$$\therefore y = \left(\frac{x_1}{R_1} + \frac{x_2}{R_2} + \frac{x_3}{R_3} \right) \times \frac{1}{\frac{1}{R_f} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Choosing suitable resistance values, such that $R_f = R_1 = R_2 = R_3$, gives

$$y = \frac{1}{R_f} \times (x_1 + x_2 + x_3) \times \frac{R_f}{4}$$

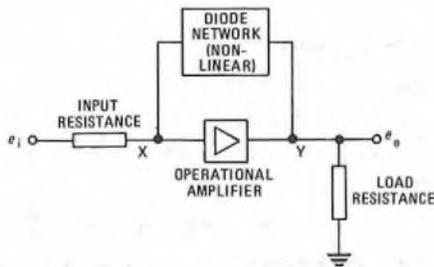
$$= \frac{(x_1 + x_2 + x_3)}{4}$$

Thus, the network can be used to solve the given equation.

Q 9 (a) Why are diode function generators used in analogue computers?

(b) With the aid of a sketch, explain the operation of a diode function generator that will generate a function approximating to the formula $y = x^3 + k$ for both positive and negative values of x , where k is a constant. Sketch a graph of the expected output of the generator and explain the effect of the constant.

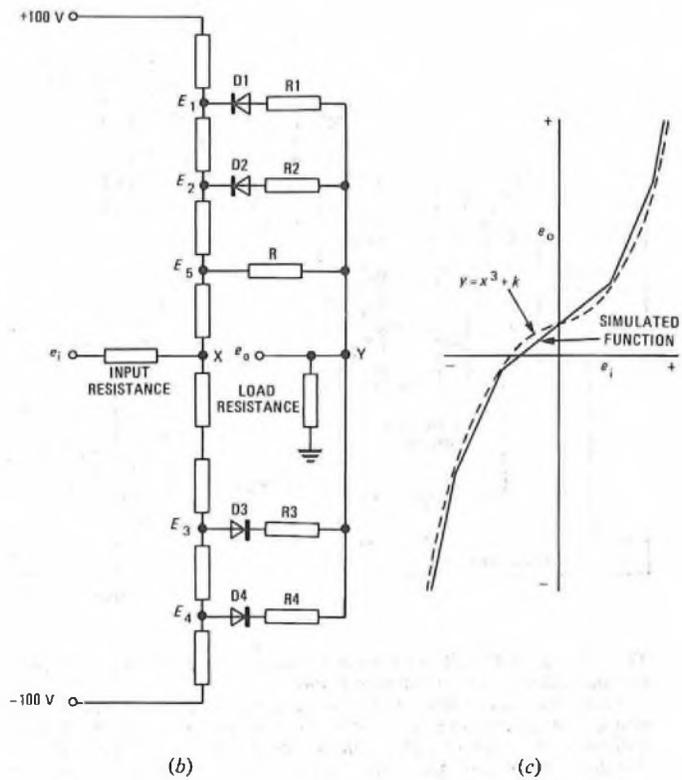
A 9 (a) Diode function generators are used in analogue computers to produce a linear approximation to a required non-linear characteristic.



(a)

This simulation of a non-linear function is achieved by the use of diodes in the feedback network of an operational-amplifier circuit, as shown in sketch (a). The diodes are controlled by biasing potentials and act as voltage-dependent switches, adding or removing feedback resistances at certain values of input voltage, which are known as *break points*. This has the effect of altering the gain of the amplifier in a number of discrete steps when the input voltage is linearly changed, thereby producing a non-linear output voltage. By careful design of the biasing arrangements and feedback resistances, a linear approximation to a given characteristic can be achieved.

(b) The circuit of a diode function generator that will produce an electrical analogy of the formula $y = x^3 + k$ is shown in sketch (b). For simplicity, the operational amplifier has been omitted; the circuit will operate reasonably well without it. The input voltage, e_i , represents the independent variable, x , and the output voltage, e_o , represents



the dependent variable, y . As in all circuits of this type, e_o is in anti-phase with e_i .

When no input voltage is applied, all the diodes are reverse-biased and, hence, are non-conducting. If e_i is sufficiently negative to overcome the positive voltage E_1 (that is, $e_i < -E_1$), diodes D1 and D2 conduct, and current flows through resistors R1 and R2, as well as resistor R.

If e_i becomes more positive, diodes D1 and D2 are cut off when $e_i = -E_1$ and $-E_2$ respectively. At each break point, the number of current paths is reduced by one and, hence, the voltage gain of the amplifier, e_o/e_i , is reduced in discrete steps.

Note that e_o is zero when $e_i = -E_5$, and that e_o is at a positive value (neglecting the phase change) when e_i is zero, due to current flowing through resistor R. It is by this means that the constant value of k is added to the formula $y = x^3$.

If the input voltage is further increased, diodes D3 and D4 conduct when e_i is sufficiently positive to overcome the biasing voltages E_3 and E_4 respectively. At each break point, the number of current paths is increased by one and, hence, the voltage gain of the amplifier is increased in discrete steps.

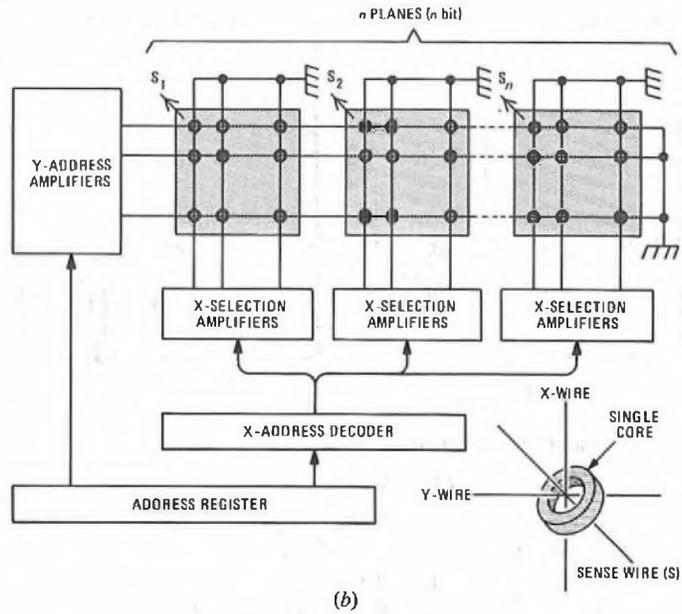
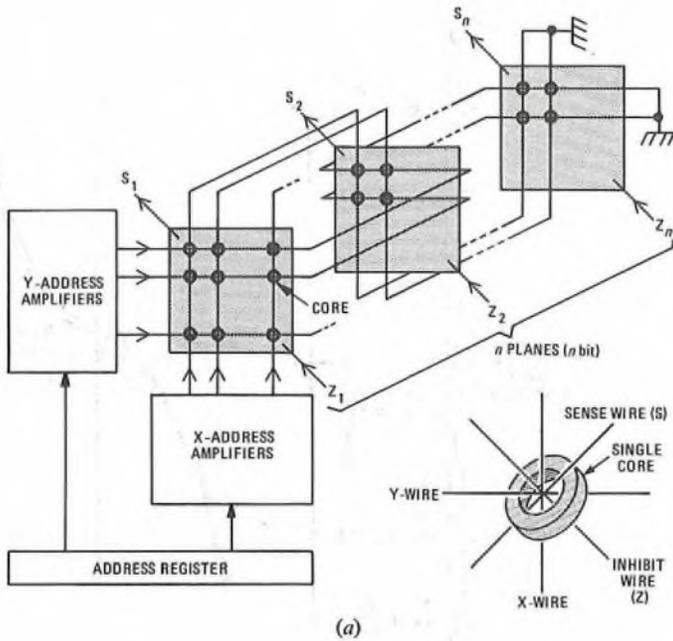
By the careful choice of feedback resistances and biasing voltages, an electrical analogy of the formula $y = x^3 + k$ is achieved. Sketch (c) shows the curve $y = x^3 + k$ (shown dashed) and the simulated function that approximates to it.

Q 10 With the aid of sketches, explain the difference in operation between 3-wire and 4-wire magnetic-core-matrix stores. Give one major advantage and one major disadvantage of each arrangement.

A 10 A magnetic-core store consists of a stack of 2-dimensional planes. In each plane is stored a single bit and, therefore, the number of planes is equivalent to the number of bits in the stored word. Each plane consists of a matrix of X-wires and Y-wires, and there is a magnetic core at each crosspoint.

A 4-wire magnetic-core store is shown in sketch (a). Four wires are threaded through each core. The X-wire and Y-wire are the address lines, and the corresponding address lines on adjacent planes are connected in series such that each word can be selected by addressing a single pair of X- and Y-wires. The SENSE and INHIBIT wires for each plane are threaded through every core on that plane.

To write into the store, currents (known as *half-write* currents) are passed simultaneously along one X-line and one Y-line to select one core in each plane. To maintain a core in the logic 0 state (assuming it was left in that state when last read), it is necessary to cancel the effect of the *half-write* current on the X-wire for the plane concerned. This is achieved by passing an *inhibit* current through the particular plane. To set a core to the logic 1 state, no *inhibit* current is required.



Thus, if the first 3 bits of a word are 101, an *inhibit* current is passed through plane 2 but not planes 1 and 3.

To read from the store, *half-read* currents are passed simultaneously along one X-line and one Y-line (in the opposite direction to the *half-write* current) to switch all the cores of the word to the logic 0 state. For any core previously in the logic 1 state, a current pulse is induced in the SENSE wire threaded through the relevant plane, whereas a core previously in the logic 0 state does not cause a pulse to be induced.

A 3-wire magnetic-core store operates in a similar way, except that the matrices all lie in the same logic plane, as shown in sketch (b).

The corresponding Y-lines in adjacent planes are connected in series, as before, but the X-lines on each matrix are independent. The third wire is the SENSE wire, and reading from the store is carried out in the same way as for the 4-wire system. However, to write a logic 0 into the store, the *half-write* current on the X-wire is omitted. Therefore, there is no need to cancel out its effect, and the INHIBIT wire is not required.

The 3-wire system has the advantage of using smaller cores, giving faster switching times and, hence, reducing the reading-and-writing-cycle time. The disadvantage is that additional electronic circuitry is needed for selecting and driving the X-wires. The 4-wire system has the advantage of being cheaper because the amount of circuitry is kept to a minimum, but it has the disadvantage of being slower.

TELECOMMUNICATION PRINCIPLES C, 1975

Students were expected to answer any 6 questions

Q 1 (a) A capacitor, having a solid dielectric, is connected to a constant-voltage variable-frequency supply. Explain why the power loss usually increases as the source frequency increases.

(b) A 500 pF capacitor has a power factor of 4×10^{-4} , independent of frequency. Determine the equivalent series and parallel loss resistances at frequencies of

- (i) 1 MHz, and
- (ii) 2 MHz.

(c) Hence, or otherwise, calculate the power loss when the capacitor is connected to a 2.5 V source at 2 MHz.

A 1 (a) Capacitors exhibit an effect known as *absorption*. A capacitor, charged through a low-impedance battery, requires a finite time to reach its maximum voltage, and, when discharged through a short circuit, retains some charge after the short circuit is removed.

In an a.c. circuit, this effect means that loss occurs during each cycle of the current. The circuit loss thus increases with frequency, and exhibits itself as *dielectric hysteresis loss*.

(b) Let r be the value of the equivalent series loss resistance (ohms), C the value of the capacitor (farads), I the current in the circuit (amperes), V the voltage across the circuit, V_r the voltage across the loss resistance, and V_C the voltage across the capacitor. The equivalent series circuit is shown in sketch (a), from which the phasor diagram shown in sketch (b) can be drawn.

The power factor is $\cos \phi$ and, for a low-loss capacitor, this is approximately V_r/V_C ; that is,

$$\cos \phi \approx \frac{V_r}{V_C} = \frac{Ir}{I} = 2\pi frC,$$

where f is the frequency (hertz).

$$\therefore r = \frac{\cos \phi}{2\pi fC} \text{ ohms.}$$

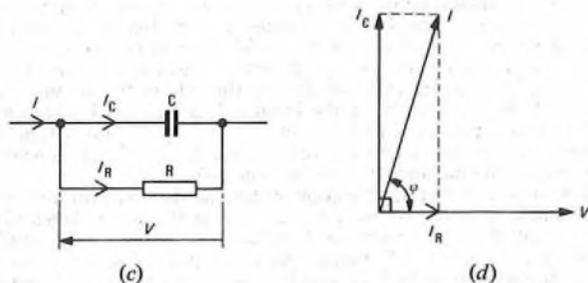
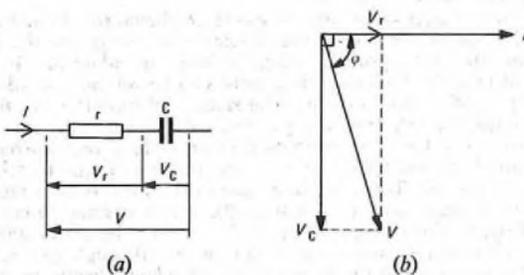
Thus, at 1 MHz,

$$r = \frac{4 \times 10^{-4}}{2\pi \times 10^6 \times 500 \times 10^{-12}} = 0.127 \Omega,$$

and, at 2 MHz,

$$r = \frac{4 \times 10^{-4}}{2\pi \times 2 \times 10^6 \times 500 \times 10^{-12}} = 0.064 \Omega.$$

The equivalent parallel circuit is shown in sketch (c), from which the phasor diagram shown in sketch (d) can be drawn. Let R be the value of the equivalent parallel loss resistance (ohms), I_C the current in the capacitor (amperes), and I_R the current in the loss resistance (amperes).



From sketch (d),

$$\cos \phi \approx \frac{I_R}{I_C} = \frac{\frac{V}{R}}{2V\pi fC} = \frac{1}{2\pi fCR}$$

Thus, at 1 MHz,

$$R = \frac{1}{2\pi \times 10^6 \times 500 \times 10^{-12} \times 4 \times 10^{-4}} \Omega = 796 \text{ k}\Omega,$$

and, at 2 MHz,

$$R = \frac{1}{2\pi \times 2 \times 10^6 \times 500 \times 10^{-12} \times 4 \times 10^{-4}} \Omega = 398 \text{ k}\Omega.$$

(c) The power loss is given by

$$\frac{V^2}{R} = \frac{2 \cdot 5^2}{398 \times 10^3} \text{ W} = 15.7 \mu\text{W}.$$

Q 2 (a) Sketch an equivalent circuit for an iron-cored transformer, and explain the significance of each circuit element.

(b) Such a transformer is connected between a source, of voltage e_s volts and resistance R_s ohms, and a load of resistance R_L ohms, as shown in Fig. 1. Sketch and explain a typical curve showing the variation of output voltage, V_o volts, with frequency.

(c) Comment briefly on the effects of changing

- (i) R_s , and
- (ii) R_L .

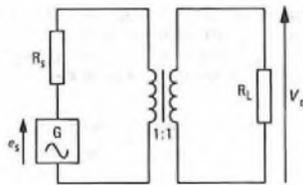


Fig. 1

Q 3 (a) Give a clear explanation of the superposition theorem.

(b) By using this theorem, or otherwise, determine the magnitude of the current, I amperes, in the 120Ω resistor of Fig. 2.

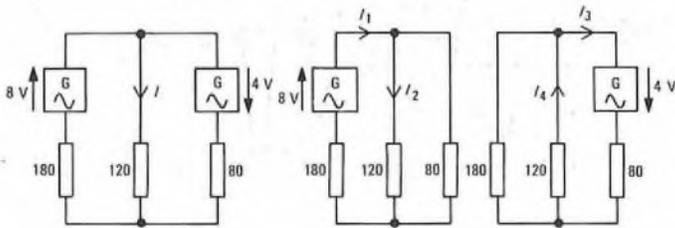


Fig. 2

A 3 (a) In a linear circuit, the total effect of all generators acting together can be determined by obtaining the effect of each generator acting independently and adding the separate effects together. When not under consideration, each voltage generator is replaced by its series resistance, and each current generator by its parallel resistance.

(b) Considering only the 8 V generator, the circuit becomes as shown in sketch (a). The effective series resistance of the circuit

$$= 180 + \frac{120 \times 80}{120 + 80} = 228 \Omega.$$

$$\therefore I_1 = \frac{8}{228} \text{ A} = 35.1 \text{ mA}.$$

$$\therefore I_2 = 35.1 \times \frac{80}{200} = 14 \text{ mA}.$$

Considering only the 4 V generator, the circuit becomes as shown in sketch (b). The series resistance

$$= 80 + \frac{120 \times 180}{120 + 180} = 152 \Omega.$$

$$\therefore I_3 = \frac{4}{152} \text{ A} = 26.3 \text{ mA}.$$

$$\therefore I_4 = 26.3 \times \frac{180}{300} = 15.8 \text{ mA}.$$

Hence, $I = 15.8 - 14 = 1.8 \text{ mA}$,

in the direction of I_4 .

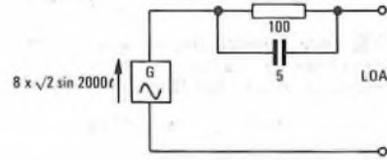
Q 4 An 8 V source, having a frequency of 318 Hz, has an internal impedance equivalent to a 100Ω resistor in parallel with a $5 \mu\text{F}$ capacitor.

(a) Determine the values of the 2 ideal components which will take maximum power from the source when the 2 components are

- (i) in parallel, and
 - (ii) in series.
- (b) Calculate the value of the maximum load power.

A 4 (a) The sketch shows the source described. For maximum power transfer, the load impedance must be the complex conjugate of the source impedance; that is, if the source impedance is $a + jb$ ohms, the load impedance must be $a - jb$ ohms.

The source impedance is a resistance of 100Ω in parallel with a reactance of $-j/(2000 \times 5 \times 10^{-6}) = -j100 \Omega$.



(i) The ideal parallel load is therefore a resistance of 100Ω in parallel with a reactance of $+j100 \Omega$; that is, an inductance of a $100/2000 \text{ H} = 50 \text{ mH}$.

(ii) The equivalent series components for the parallel source are given by

$$\frac{100 \times (-j100)}{100 - j100} = \frac{-j10000 \times (100 + j100)}{100^2 + 100^2},$$

$$= \frac{100 - j100}{2} = 50 - j50 \Omega.$$

The ideal series load is therefore a resistance of 50Ω in series with a reactance of $+j50 \Omega$; that is, an inductance of $50/2000 \text{ H} = 25 \text{ mH}$.

(b) Considering the series circuit, the current taken from the source is $8/(50 - j50 + 50 + j50) \text{ A} = 80 \text{ mA}$. Therefore, the maximum load power

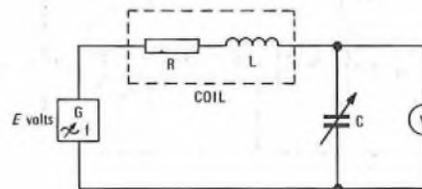
$$= (80 \times 10^{-3})^2 \times 50 \text{ W} = 320 \text{ mW}.$$

Q 5 (a) Describe an experimental method of determining the Q -factor of a coil.

(b) A radio-frequency (RF) coil has an inductance of $50 \mu\text{H}$ and a Q -factor of 200 at 4 MHz. If the coil is connected in series with a loss-free capacitor, determine the capacitance required for resonance at 4 MHz.

(c) Calculate the 2 frequencies at which the impedance is twice that at resonance.

A 5 (a) A circuit suitable for determining the Q -factor of a coil is shown in the sketch.



The impedance of the variable-frequency generator is negligible. Capacitor C is varied until resonance occurs, indicated by a maximum reading on the voltmeter.

At resonance, the circuit impedance is equal to the resistance of the coil, R ohms. The voltage across capacitor C , V_C volts, is therefore $E/\omega_0 CR$ volts, where ω_0 is the angular frequency at resonance (radians/second). The Q -factor is the ratio of one of the reactances to the resistance in the circuit; that is, $Q = 1/\omega_0 CR$.

Thus, $V_C = EQ$ volts,

or $Q = \frac{V_C}{E}$.

Therefore, the Q -factor is given by the ratio of the voltmeter reading to the generator voltage.

(b) At resonance,

$$\omega_0 L = \frac{1}{\omega_0 C},$$

where L is the value of the inductance (henrys), C is the value of the capacitance (farads), and $\omega_0 = 2\pi f_0$, where f_0 is the resonant frequency (hertz).

$$\begin{aligned} \therefore C &= \frac{1}{4\pi^2 f_0^2 L} \text{ farads,} \\ &= \frac{1}{4\pi^2 \times 4^2 \times 10^{12} \times 50 \times 10^{-6}} \text{ F,} \\ &= \underline{31.7 \text{ pF.}} \end{aligned}$$

(c) For frequencies near the resonant frequency, the circuit impedance, Z ohms, is given by

$$Z \approx R \left(1 + j2Q \times \frac{\Delta f}{f_0} \right) \text{ ohms,}$$

where Δf is the difference between the resonance frequency and the frequency under consideration (hertz).

Now, the condition required is that the circuit impedance

$$\begin{aligned} &= R(1 + j \times \sqrt{3}) \text{ ohms.} \\ \therefore \Delta f &= \frac{\sqrt{3} \times f_0}{2 \times Q} \text{ hertz,} \\ &= \frac{\sqrt{3} \times 4 \times 10^6}{2 \times 200} \text{ Hz} = \underline{17.3 \text{ kHz.}} \end{aligned}$$

Hence, the 2 frequencies are $4 \times 10^6 \pm 17.3 \times 10^3 \text{ Hz}$.

Q 6 (a) Give 2 reasons for the use of bias stabilization in common-emitter transistor amplifiers.

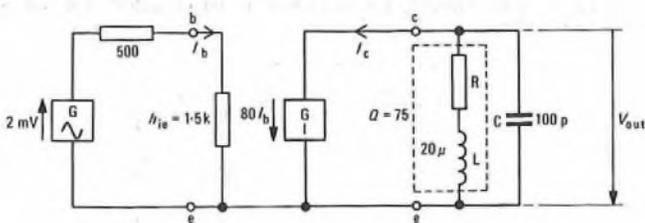
(b) A transistor, having an h_{ie} of $1.5 \text{ k}\Omega$ and an h_{fe} of 80 with both h_{oe} and h_{re} negligible, is connected as a common-emitter tuned amplifier. The collector load comprises a $20 \mu\text{H}$ coil with a Q -factor of 75 in parallel with a 100 pF capacitor of negligible loss. A variable-frequency source, with an open-circuit e.m.f. of 2 mV and a resistive impedance of 500Ω , is connected to the amplifier input terminals. Giving reasons for any assumptions made, determine

- (i) the frequency for maximum output voltage, and
- (ii) the value of the maximum output voltage.

A 6 (a) The small-signal parameters of transistors are dependent on the d.c. conditions under which the transistors are operated, so that the d.c. conditions must be stabilized to give a constant gain. This is particularly important in feedback amplifiers, where instability could result if large variations in gain occur.

Thermal instability can easily occur in power-amplifier stages because of positive thermal-feedback effects; bias stabilization overcomes this problem.

(b) The sketch shows the equivalent circuit for the amplifier.



(i) The maximum value of the output voltage, V_{out} volts, occurs when the impedance of the tuned circuit is a maximum; that is, at resonance. The resonant frequency, f_0 hertz, is given, to a very good approximation, by

$$f_0 = \frac{1}{2\pi \times \sqrt{LC}} \text{ hertz,}$$

where L is the inductance (henrys), and C is the capacitance (farads). The effect of the coil losses can be ignored since the coil has a high Q -factor.

$$\therefore f_0 = \frac{1}{2\pi \times \sqrt{(20 \times 10^{-6} \times 100 \times 10^{-12})}} \text{ Hz} = \underline{3.56 \text{ MHz.}}$$

(ii) The Q -factor of the coil is given by

$$Q = \frac{\omega_0 L}{R},$$

where $\omega_0 = 2\pi f_0$ and R is the resistance of the coil (ohms).

$$\therefore R = \frac{2\pi \times 3.56 \times 10^6 \times 20 \times 10^{-6}}{75} = 5.96 \Omega.$$

The dynamic impedance of the tuned circuit, Z_D ohms, is given by

$$\begin{aligned} Z_D &= \frac{L}{CR} \text{ ohms,} \\ &= \frac{20 \times 10^{-6}}{100 \times 10^{-12} \times 5.96} \Omega = \underline{33.56 \text{ k}\Omega.} \end{aligned}$$

The maximum output voltage is the dynamic impedance multiplied by the collector current, I_c amperes. Now, $I_c = h_{fe} I_b$, where I_b is the base current (amperes), given by the input voltage divided by the source resistance plus h_{ie} .

$$\therefore V_{out} = \frac{33.56 \times 10^3 \times 80 \times 2 \times 10^{-3}}{500 + 1.5 \times 10^3} = \underline{2.68 \text{ V.}}$$

Q 7 (a) Describe an experimental method of measuring the modulation factor of an amplitude-modulated carrier.

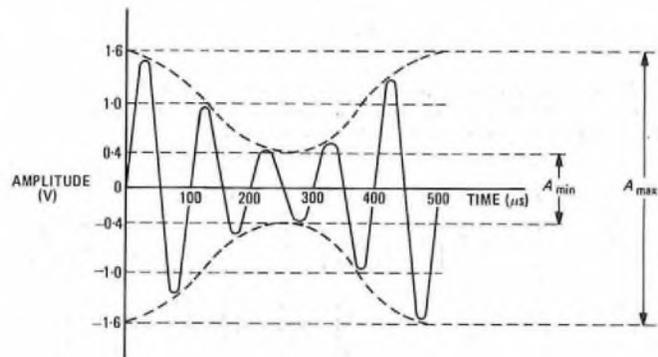
(b) A 10 kHz sinusoidal carrier, of peak amplitude 1 V , is amplitude-modulated by a factor of 0.6 by a 2 kHz sinusoidal modulating signal. Sketch the waveform of the modulated carrier over one cycle of the modulating signal.

(c) Sketch a phasor representation of the modulated wave corresponding to an instantaneous peak value of 1.3 V .

A 7 (a) The modulation factor, m , of an amplitude-modulated wave can be obtained by measuring the maximum and minimum peak-to-peak values (A_{max} and A_{min} respectively) of an oscillogram of the modulated wave, as shown in sketch (a).

Then,

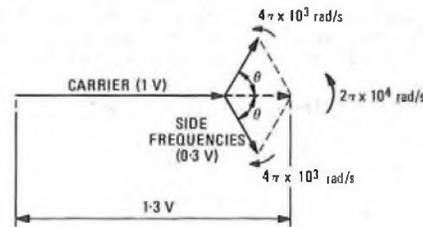
$$m = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$



(a)

(b) The required waveform is shown in sketch (a).

(c) Each side-frequency component has a magnitude of $0.5m$ times the unmodulated peak amplitude of the carrier; that is, $0.5 \times 0.6 \times 1 = 0.3 \text{ V}$ for $m = 0.6$. The phasor diagram is shown in sketch (b).



(b)

From the phasor diagram, it can be seen that

$$1 + (2 \times 0.3 \cos \theta) = 1.3.$$

$$\therefore \cos \theta = 0.5.$$

$$\therefore \theta = 60^\circ.$$

Q 8 (a) Which electrode of a power transistor is usually connected to the transistor case? Explain why this is so, and discuss the complications that this can cause in the circuitry or in the transistor mounting.

(b) Why are heat sinks often used with power transistors?

(c) The transistor in a class-A power amplifier is mounted so that its maximum permitted collector dissipation is 15 W. With the signal applied, the stage has an efficiency of 35%. Estimate the maximum output power if the signal is continuous.

A 8 (a) The collector electrode is usually connected to the transistor case, because most of the internal power dissipation occurs at the collector. It is therefore necessary to reduce the thermal resistance between the collector and the transistor case as much as possible. Consequently, the transistor case must be isolated from the equipment chassis. Alternatively, the circuit must be designed around earthed-collector configurations.

(b) Heat sinks are necessary to provide a large surface area for the dissipation of heat, so that the collector temperature can be kept below the maximum value specified. The heat sink effectively reduces the thermal resistance between the transistor case and the circulating air.

(c) The ratio of the power in the output, P_{out} watts, to the power dissipated as heat is 35 : 65. Thus,

$$P_{out} = 15 \times \frac{35}{65} = 8.08 \text{ W.}$$

Q 9 (a) Discuss the relative advantages of amplitude modulation and frequency modulation.

(b) A sinusoidal carrier of peak value 3 V, having a frequency of 10 MHz, is frequency modulated so that the frequency deviates linearly by 2 kHz for each volt of modulating signal. If the modulating signal is $5 \sin 1000\pi t$ volts, calculate the peak frequency deviation of the modulated carrier and write down an expression for the instantaneous voltage of the modulated wave.

(c) Repeat (b) for modulating signals of

- (i) $10 \sin 1000\pi t$ volts, and
- (ii) $10 \sin 50\pi t$ volts.

A 9 (a) The relative advantages of amplitude modulation and frequency modulation can be briefly summarized by saying that an amplitude-modulated transmission requires only a narrow bandwidth (single-sideband transmissions require only the signal bandwidth), while the noise performance of a frequency-modulated transmission is superior. Secondary relative advantages are that amplitude modulation requires simple modulators and demodulators, and frequency-modulation broadcasts have a greater dynamic range available to them.

(b) The peak frequency deviation, f_d hertz, is given by the frequency deviation per volt multiplied by the peak voltage of the modulating signal.

$$\therefore f_d = 2 \times 10^3 \times 5 \text{ Hz} = 10 \text{ kHz.}$$

The instantaneous voltage, v volts, of a frequency-modulated wave is given by

$$v = V_c \sin(\omega_c t - \eta \cos \omega_m t) \text{ volts,}$$

where V_c is the peak value of the carrier (volts), ω_c is the angular frequency of the carrier (radians/second), t is the time (seconds), η is the modulation index, and ω_m is the angular frequency of the modulating signal (radians/second).

Now, $\eta = f_d/f_m$, where f_m is the frequency of the modulating signal (hertz).

Also, $\omega_c = 2\pi f_c$ radians/second, where f_c is the carrier frequency (hertz), and $\omega_m = 2\pi f_m$ radians/second.

$$\begin{aligned} \therefore v &= 3 \sin\left(2\pi t \times 10^7 - \frac{10^4}{500} \cos 1000\pi t\right) \text{ volts,} \\ &= 3 \sin(2\pi t \times 10^7 - 20 \cos 1000\pi t) \text{ volts.} \end{aligned}$$

(c) (i) For a modulating signal of $10 \sin 1000\pi t$ volts, the peak frequency deviation becomes 20 kHz, since the amplitude is twice that in part (b). Hence,

$$v = 3 \sin(2\pi t \times 10^7 - 40 \cos 1000\pi t) \text{ volts.}$$

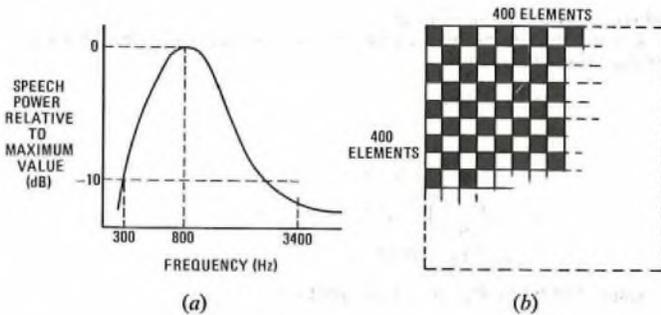
(ii) For a modulating signal of $10 \sin 50\pi t$ volts, the peak frequency deviation is again 20 kHz, but the modulating frequency has been reduced by a factor of 20, so that the modulation index becomes 800. Hence,

$$v = 3 \sin(2\pi t \times 10^7 - 800 \cos 50\pi t) \text{ volts.}$$

Q 10 Give estimates, with reasons, of the required bandwidth (prior to any modulation process) for the following types of signal:

- (a) commercial speech,
- (b) music, and
- (c) a 400-line picture-transmission system that transmits a square picture in 20 s.

A 10 (a) Commercial speech channels usually transmit frequencies in the range 300–3400 Hz. The reason for using only this bandwidth is demonstrated by the typical speech-power/frequency characteristic shown in sketch (a), where it can be seen that most speech power is concentrated in this range. While it is true that a smaller channel bandwidth may be possible, clarity and definition of speech would suffer.



(b) Music channels are usually allocated the somewhat wider bandwidth of 20–12 000 Hz, corresponding to approximately 4 speech channels. This is because of the large range of frequencies that need to be reproduced for different musical instruments, ranging from large bass drums to tiny piccolos. The size of an instrument is a controlling factor of the fundamental frequency it produces.

(c) A square picture of 400 horizontal lines has an equivalent number of vertical lines, assuming equal horizontal and vertical definition. The total number of picture elements is thus 400×400 . The picture requiring the most bandwidth is one in which adjacent elements are different; that is, a chequered pattern, as shown in sketch (b).

The transmitted waveform for this picture is a square wave, alternating between peak-black and peak-white signals. The fundamental frequency is equal to half the number elements divided by the transmission time. Thus, the fundamental frequency

$$= \frac{400 \times 400}{2 \times 20} \text{ Hz} = 4 \text{ kHz.}$$

This is also the minimum bandwidth that could be used to transmit the picture information.

MATHEMATICS C, 1975

Students were expected to answer any 6 questions

Q 1 The resistances R_1 and R_2 of an attenuator are to be chosen so that

$$R_2 + \frac{R_1(R_2 + 800)}{R_1 + R_2 + 800} = 800, \text{ and } R_1 + R_2 = 1200.$$

Calculate, to 3 significant figures, the values of R_1 and R_2 to achieve these requirements.

A 1 $R_1 = 894$, and $R_2 = 306$, to 3 significant figures.

Q 2 (a) Write down and simplify the series for $(1+x)^{-1/2}$ as far as the term in x^4 , and write an expression for the term involving x^r . What restrictions must be placed on the value of x for this series to be true?

(b) The formula $L = \mu N^2 A/l$ relates the inductance, L , of a solenoid to the number of turns, N , its cross-sectional area, A , and its length, l . Use the series derived in part (a) to calculate, to 2 significant figures, the percentage increase in the number of turns required to maintain the same inductance if the cross-sectional area is reduced by 10%.

$$\begin{aligned}
 \text{A 2 (a)} \quad (1+x)^{-1/2} &= 1 + (-\frac{1}{2})x + \frac{-\frac{1}{2}(-\frac{1}{2}-1)x^2}{1 \times 2} \\
 &+ \frac{-\frac{1}{2}(-\frac{1}{2}-1)(-\frac{1}{2}-2)x^3}{1 \times 2 \times 3} \\
 &+ \frac{-\frac{1}{2}(-\frac{1}{2}-1)(-\frac{1}{2}-2)(-\frac{1}{2}-3)x^4}{1 \times 2 \times 3 \times 4} + \dots \\
 &= 1 - \frac{x}{2} + \frac{(\frac{1}{2} \times \frac{3}{2})x^2}{1 \times 2} - \frac{(\frac{1}{2} \times \frac{3}{2} \times \frac{5}{2})x^3}{1 \times 2 \times 3} \\
 &\quad + \frac{(\frac{1}{2} \times \frac{3}{2} \times \frac{5}{2} \times \frac{7}{2})x^4}{1 \times 2 \times 3 \times 4} - \dots \\
 &= 1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \frac{35x^4}{128} - \dots
 \end{aligned}$$

The term involving x^r can be written as

$$(-1)^r \times \frac{1 \times 3 \times 5 \dots \times (2r-1) \times x^r}{2^r \times r!}$$

This series is true provided that the numerical value of x is less than unity.

(b) Now, $N^2 = \frac{IL}{\mu A}$

If the cross-sectional area is reduced by 10%, then

$$A_1 = A - 0.1A,$$

where A_1 is the reduced area.

Let N_1 be the number of turns required to maintain the inductance at the same value. Then,

$$\begin{aligned}
 N_1^2 &= \frac{IL}{\mu A_1} \\
 &= \frac{IL}{\mu A(1-0.1)} \\
 \therefore N_1 &= \left(\frac{IL}{\mu A}\right)^{1/2} \times (1-0.1)^{-1/2}, \\
 &= N(1-0.1)^{-1/2}.
 \end{aligned}$$

Using the binomial series from part (a),

$$\begin{aligned}
 (1-0.1)^{-1/2} &\approx 1 + \frac{0.1}{2} + \frac{3 \times 0.1^2}{8} + \frac{5 \times 0.1^3}{16} + \frac{35 \times 0.1^4}{128} \\
 &\approx 1.05 + 0.00375 + 0.0003125.
 \end{aligned}$$

It is clear that the term in $(-0.1)^4$ can be ignored, since it can affect only the fifth decimal place.

$$\therefore N_1 \approx N \times 1.0540625.$$

Hence, the percentage increase in N , to 2 significant figures, is $0.054 \times 100\%$

$$= 5.4\%$$

Q 3 Fig. 1 shows a cylinder of radius x millimetres just fitting inside a hollow cone of base radius 30 mm and vertical height 80 mm.

Find the value of x for which the cylinder has the greatest volume, and show this volume to be $4/9$ of the cubic capacity of the cone.

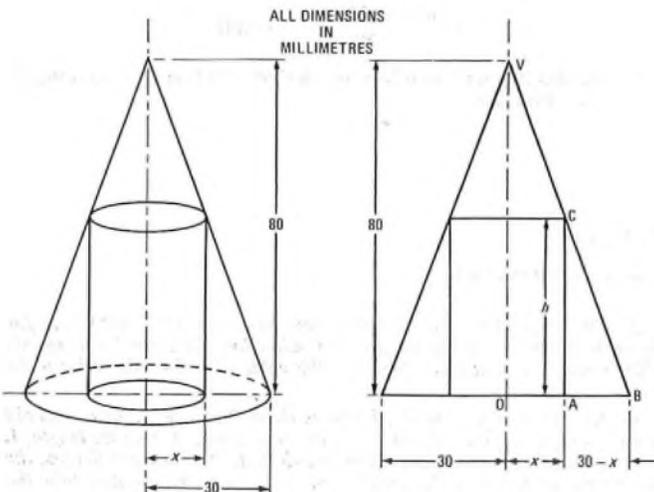


Fig. 1

(a)

A 3 Sketch (a) shows a cross-section through the axis of the cone and cylinder.

If h is the height of the cylinder (millimetres), then the volume of the cylinder, V_C millimetres³, is $\pi x^2 h$.

Since triangles BAC and BOV are similar,

$$\frac{CA}{VO} = \frac{AB}{OB}$$

$$\therefore h = \frac{80 \times (30 - x)}{30} \text{ millimetres,}$$

$$= 80 - \frac{8x}{3} \text{ millimetres.}$$

$$\therefore V_C = \pi x^2 \left(80 - \frac{8x}{3}\right) \text{ millimetres}^3,$$

$$= 80\pi x^2 - \frac{8\pi x^3}{3} \text{ millimetres}^3.$$

$$\therefore \frac{dV_C}{dx} = 160\pi x - 8\pi x^2.$$

For maximum or minimum values of V_C , as x varies,

$$\frac{dV_C}{dx} = 0.$$

$$\therefore 160\pi x = 8\pi x^2.$$

$$\therefore x = 0 \text{ or } 20 \text{ mm.}$$

At $x = 0$, $V_C = 0$; that is, V_C is a minimum. Hence, the maximum volume of the cylinder occurs at $x = 20$ mm.

If V_N is the volume of the cone (millimetres³), then

$$\frac{V_C}{V_N} = \frac{\pi x^2 h}{\frac{\pi}{3} \times 30^2 \times 80}$$

When $x = 20$ mm,

$$\frac{V_C}{V_N} = \frac{400h}{300 \times 80}$$

But

$$h = 80 - \frac{8 \times 20}{3} = \frac{80}{3} \text{ mm.}$$

$$\therefore \frac{V_C}{V_N} = \frac{400 \times 80}{3 \times 300 \times 80} = \frac{4}{9}. \quad \text{QED.}$$

Q 4 (a) By sketching a curve and a straight line (or otherwise), obtain rough estimates of the 3 roots of the cubic equation

$$x^3 - 4x^2 - 4x + 9 = 0.$$

(b) By graphical enlargement, find, to 3 significant figures, the smaller of the 2 positive roots.

A 4 (a) $x^3 - 4x^2 - 4x + 9 = 0, \dots\dots (1)$

or $x^3 - 4x^2 = 4x - 9. \dots\dots (2)$

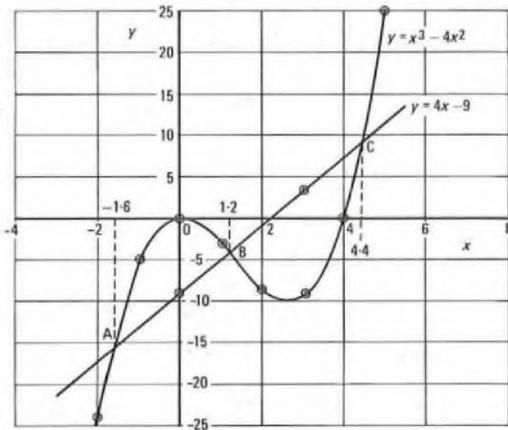
The intersection of the graphs of the functions $y = x^3 - 4x^2$ and $y = 4x - 9$ must satisfy equation (2) and, hence, must satisfy equation (1).

Since $y = 4x - 9$ is a linear function, its graph can be plotted using only 2 points. When $x = 0, y = -9$, and when $x = 3, y = 3$.

The graphs are shown in sketch (a), that of $y = x^3 - 4x^2$ being sketched from the following table of values.

x	-2	-1	0	1	2	3	4	5
x^3	-8	-1	0	1	8	27	64	125
$-4x^2$	-16	-4	0	-4	-16	-36	-64	-100
y	-24	-5	0	-3	-8	-9	0	25

MATHEMATICS C, 1975 (continued)



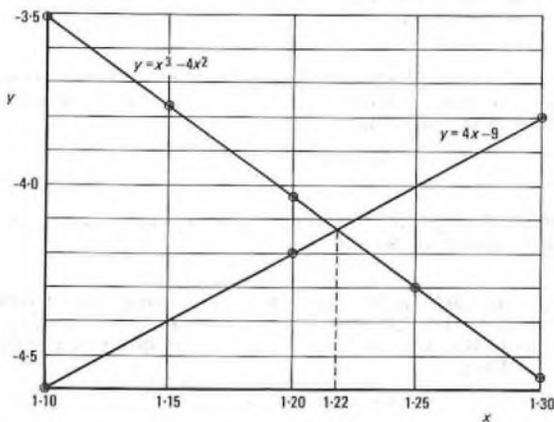
(a)

It can be seen that the line $y = 4x - 9$ intersects the curve $y = x^3 - 4x^2$ at points A, B and C, from which the 3 roots of the cubic equation are estimated to be -1.6 , 1.2 and 4.4 .

(b) From part (a), the smaller of the positive roots is approximately 1.2.

For an enlarged graph, it is necessary to choose a range of x that will ensure accuracy to 3 significant figures and, at the same time, accommodate any inaccuracies in the original sketch. The enlargement is shown in sketch (b), plotted from the values in the following table.

x	1.100	1.150	1.200	1.250	1.300
$y = 4x - 9$	-4.600	-	-4.200	-	-3.800
x^3	1.331	1.521	1.728	1.953	2.197
$-4x^2$	-4.840	-5.290	-5.760	-6.250	-6.760
$y = x^3 - 4x^2$	-3.509	-3.769	-4.032	-4.297	-4.563



(b)

It is sufficient to join the plotted points of the function $y = x^3 - 4x^2$ by straight lines because of the small range used, and because the degree of curvature in the vicinity of point B in sketch (a) can be seen to be very small.

From sketch (b), the smaller positive root of the cubic equation is 1.22 , to 3 significant figures.

Q 5 (a) Find, in the form $a + jb$, the 2 square roots of the complex number $-5 - j8$ and display them in a complex plane diagram.

(b) (i) Express the complex quantity $Z^2 = (R + j\omega L)/j\omega C$ in polar form.

(ii) Hence, or otherwise, find the magnitude of Z and its phase angle (assume the latter to be between -90° and $+90^\circ$) when $\omega = 5000$, $L = 0.3 \times 10^{-3}$, $C = 3.6 \times 10^{-9}$, and $R = 4.4$.

A 5 (a) $-5 - j8 = r(\cos \theta + j \sin \theta)$,

where $r = \sqrt{5^2 + 8^2} = \sqrt{89}$, and $\theta = \tan^{-1}(-8/-5) = 238^\circ$.

$$\therefore (-5 - j8)^{1/2} = 89^{1/4} \left\{ \cos \frac{(238^\circ + 360n^\circ)}{2} + j \sin \frac{(238^\circ + 360n^\circ)}{2} \right\},$$

where $n = 0$ or 1 .

When $n = 0$,

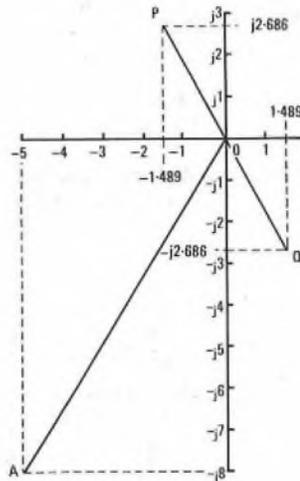
$$\begin{aligned} (-5 - j8)^{1/2} &= 3.071(\cos 119^\circ + j \sin 119^\circ), \\ &= 3.071(-0.4848 + j0.8746), \end{aligned}$$

and, when $n = 1$,

$$(-5 - j8)^{1/2} = 3.071(0.4848 - j0.8746).$$

Hence, $(-5 - j8)^{1/2} = -1.489 + j2.686$ or $1.489 - j2.686$.

The 2 square roots are shown in the sketch, where point A is the complex number $-5 - j8$, and points P and Q are the roots.



(b) (i)
$$\begin{aligned} Z^2 &= \frac{R + j\omega L}{j\omega C}, \\ &= \frac{L}{C} - \frac{jR}{\omega C}, \\ &= \sqrt{\left\{ \left(\frac{L}{C}\right)^2 + \left(\frac{R}{\omega C}\right)^2 \right\}} \angle \tan^{-1}\left(-\frac{R}{\omega L}\right), \\ &= \sqrt{\left(\frac{R^2 + \omega^2 L^2}{\omega^2 C^2}\right)} \angle \tan^{-1}\left(-\frac{R}{\omega L}\right), \\ &= \frac{\sqrt{R^2 + \omega^2 L^2}}{\omega C} \angle \tan^{-1}\left(-\frac{R}{\omega L}\right). \end{aligned}$$

(ii)
$$\begin{aligned} Z^2 &= \frac{\sqrt{(4.4^2 + 5000^2 \times 0.3^2 \times 10^{-6})}}{5000 \times 3.6 \times 10^{-9}} \\ &\quad \angle \tan^{-1}\left(-\frac{4.4}{5000 \times 0.3 \times 10^{-3}}\right), \\ &= \frac{4.649}{18 \times 10^{-6}} \angle \tan^{-1}(-2.933), \\ &= 0.2582 \times 10^6 \angle 288^\circ 50'. \end{aligned}$$

Hence, from part (a),

$$\begin{aligned} Z &= \sqrt{(0.2582 \times 10^6)} \angle 144^\circ 25' \text{ or} \\ &\quad \sqrt{(0.2582 \times 10^6)} \angle 144^\circ 25' + 180^\circ. \end{aligned}$$

Disregarding the first root, which is outside the range under consideration,

$$Z = 508.1 \angle 324^\circ 25' = 508.1 \angle -35^\circ 35'.$$

Thus, the magnitude of Z is approximately 508, and its phase angle is $-35^\circ 35'$.

Q 6 (a) Sketch the curve, whose equation in polar co-ordinates is $3/r = 2 + \cos \theta$, for the range $\theta = -\pi$ to $\theta = +\pi$. Mark on the graph the points A, B, C and D at which $\theta = 0, \pi/3, \pi$ and $-2\pi/3$ respectively.

(b) Derive, in cartesian co-ordinates, the equation of the same curve referred to the pole as the origin and the line $\theta = 0$ as the x-axis of the cartesian system.

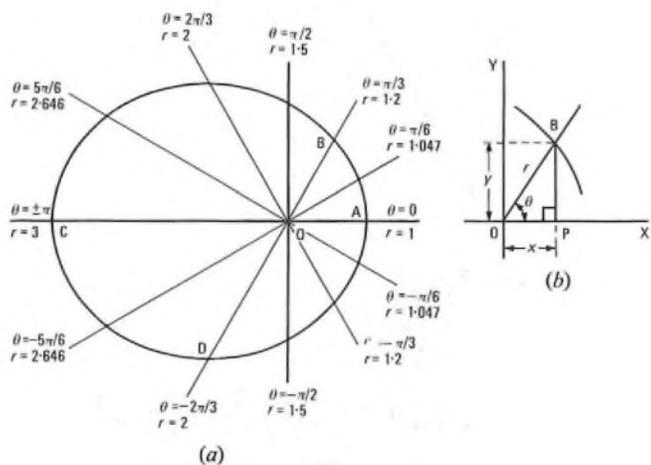
A 6 (a)
$$\frac{3}{r} = 2 + \cos \theta.$$

$$\therefore r = \frac{3}{2 + \cos \theta} \dots \dots (1)$$

Since $\cos \theta = \cos(-\theta)$, the values of r for the 2 ranges, $\theta = 0$ to $+\pi$ and $\theta = 0$ to $-\pi$ radians, can be determined simultaneously, as shown in the following table.

θ	0	$\pm\pi/6$	$\pm\pi/3$	$\pm\pi/2$	$\pm 2\pi/3$	$\pm 5\pi/6$	$\pm\pi$
$\cos \theta$	1	0.866	0.5	0	-0.5	-0.866	-1
$2 + \cos \theta$	3	2.866	2.5	2	1.5	1.134	1
r	1	1.047	1.2	1.5	2	2.646	3

The curve is shown in sketch (a), and the points A, B, C and D, at which $\theta = 0, \pi/3, \pi$ and $-2\pi/3$ respectively, are marked.



(b) In sketch (b), the portion of the same curve, about point B, is drawn on cartesian axes, OX and OY, along the lines $\theta = 0$ and $\theta = \pi/2$ respectively. If BP is the perpendicular from point B to OX, then

$$OP = x = r \cos \theta,$$

$$\text{and } BP = y = r \sin \theta.$$

Also, From equation 1,

$$r = \frac{3}{2 + (x/r)} = \frac{3r}{2r + x}.$$

$$\therefore x + 2r = 3.$$

$$\therefore x + 2 \times \sqrt{x^2 + y^2} = 3.$$

$$\therefore 2 \times \sqrt{x^2 + y^2} = 3 - x.$$

$$\therefore 4(x^2 + y^2) = 9 - 6x + x^2.$$

$$\therefore 3x^2 + 4y^2 + 6x - 9 = 0.$$

This is the equation, in cartesian co-ordinates, of the polar equation $3/r = 2 + \cos \theta$.

Q 7 (a) A star-connected 3-phase system carries a common-return current of $50 \sin \omega t + 64 \sin (\omega t + 2\pi/3) + 48 \sin (\omega t - 2\pi/3)$ amperes, where ω is in radians/second. Express this current in the form $a \cos (\omega t + \alpha)$.

(b) Solve the trigonometrical equation $4 \cos \theta - 3 \sin \theta = 2.4$, giving solutions between -180° and $+180^\circ$ (to the nearest degree).

A 7 (a)
$$50 \sin \omega t + 64 \sin (\omega t + 2\pi/3) + 48 \sin (\omega t - 2\pi/3)$$

$$= 50 \sin \omega t + 64 \left(\sin \omega t \cos \frac{2\pi}{3} + \cos \omega t \sin \frac{2\pi}{3} \right)$$

$$+ 48 \left(\sin \omega t \cos \frac{2\pi}{3} - \cos \omega t \sin \frac{2\pi}{3} \right),$$

$$= 50 \sin \omega t + 64 \left\{ \left(-\frac{1}{2} \right) \sin \omega t + \frac{\sqrt{3}}{2} \cos \omega t \right\}$$

$$+ 48 \left\{ \left(-\frac{1}{2} \right) \sin \omega t - \frac{\sqrt{3}}{2} \cos \omega t \right\}.$$

$$= 50 \sin \omega t - 32 \sin \omega t + 32 \times \sqrt{3} \cos \omega t$$

$$- 24 \sin \omega t - 24 \times \sqrt{3} \cos \omega t,$$

$$= 8 \times \sqrt{3} \cos \omega t - 6 \sin \omega t,$$

$$= \sqrt{\{(8 \times \sqrt{3})^2 + 6^2\}} \left[\frac{8 \times \sqrt{3}}{\sqrt{\{(8 \times \sqrt{3})^2 + 6^2\}}} \cos \omega t \right.$$

$$\left. - \frac{6}{\sqrt{\{(8 \times \sqrt{3})^2 + 6^2\}}} \sin \omega t \right],$$

$$= \sqrt{228} \times \left(\frac{8 \times \sqrt{3}}{\sqrt{228}} \cos \omega t - \frac{6}{\sqrt{228}} \sin \omega t \right),$$

$$= 15.1 \cos (\omega t + \alpha),$$

where $\alpha = \tan^{-1} \{(6/(8 \times \sqrt{3}))\} = \tan^{-1} 0.4330.$

Thus, the right-hand side becomes

$$15.1 \cos (\omega t + 23^\circ 25').$$

(b) $4 \cos \theta - 3 \sin \theta = 2.4.$

$$\therefore 5 \left(\frac{4}{5} \cos \theta - \frac{3}{5} \sin \theta \right) = 2.4.$$

$$\therefore 5 (\cos \theta \cos \alpha - \sin \theta \sin \alpha) = 2.4,$$

where $\alpha = \tan^{-1} (3/4) = 36^\circ 52'.$

$$\therefore \cos (\theta + \alpha) = 0.48.$$

$$\therefore \theta + \alpha = 61^\circ 19' \text{ or } -61^\circ 19'.$$

$$\therefore \theta = 24^\circ 27' \text{ or } -98^\circ 11'.$$

Hence, the solutions of the equation between -180° and $+180^\circ$ are -98° and 24° to the nearest degree.

Q 8 (a) Explain, with aid of a diagram, why the expression $(\sin \theta)/\theta$ (where θ is in radians) tends to 1 as θ tends to zero. Hence, or otherwise, show from first principles that

$$\frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x.$$

(b) Obtain dy/dx for the function $y = x^2 e^{-3x}$, and simplify the result. Sketch the graph of the function.

A 8 (a) In sketch (a), let angle AOB be an acute angle, θ radians, subtended by the arc AB of a circle with centre O and radius r . Let the tangent to the circle at point A meet the radius, OB, (produced) at point C. Then,

area of triangle AOB = $\frac{r^2}{2} \sin \theta,$

area of sector AOB = $\frac{r^2 \theta}{2},$

and area of triangle AOC = $\frac{AO}{2} \times AC = \frac{r^2}{2} \tan \theta.$

Now, the area of triangle AOB is less than the area of sector AOB which, in turn, is less than the area of triangle AOC.

$$\therefore \frac{r^2}{2} \sin \theta < \frac{r^2 \theta}{2} < \frac{r^2}{2} \tan \theta.$$

$$\therefore \sin \theta < \theta < \tan \theta.$$

Dividing by $\sin \theta$ gives

$$1 < \frac{\theta}{\sin \theta} < \frac{1}{\cos \theta}.$$

As θ tends to zero, $\cos \theta$ tends to unity and, hence, $1/\cos \theta$ tends to unity. Thus, $\theta/\sin \theta$ lies between unity and a quantity that tends to unity as θ tends to zero. Hence, $\theta/\sin \theta$ itself tends to unity.

$$\therefore \lim_{\theta \rightarrow 0} \frac{\theta}{\sin \theta} = 1,$$

and

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1.$$

QED.

Let $y = \cot x$, and suppose that, when x increases by a small amount, δx , y changes by a corresponding amount, δy . Then

$$\begin{aligned} y + \delta y &= \cot(x + \delta x), \\ &= \frac{\cos(x + \delta x)}{\sin(x + \delta x)}, \\ \therefore \delta y &= \frac{\cos(x + \delta x)}{\sin(x + \delta x)} - \frac{\cos x}{\sin x}, \end{aligned}$$

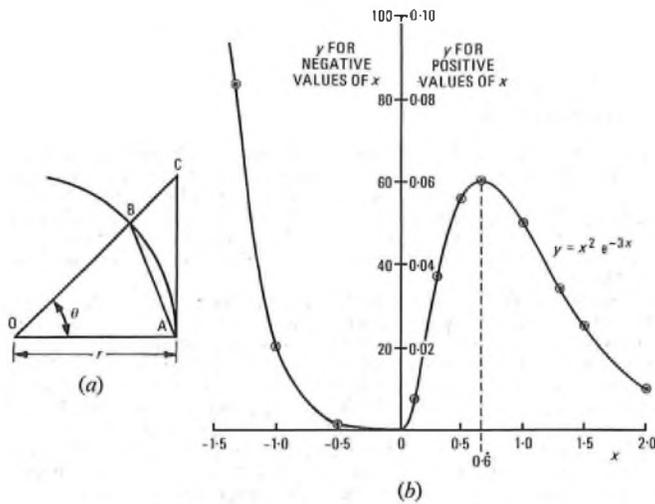
since $y = \cos x/\sin x$.

$$\begin{aligned} \therefore \delta y &= \frac{\sin x \cos(x + \delta x) - \cos x \sin(x + \delta x)}{\sin(x + \delta x) \sin x}, \\ &= -\frac{\sin(x + \delta x - x)}{\sin(x + \delta x) \sin x}, \\ \therefore \frac{\delta y}{\delta x} &= -\frac{\sin \delta x}{\delta x} \times \frac{1}{\sin(x + \delta x) \sin x}, \\ \therefore \frac{dy}{dx} &= \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = -1 \times \frac{1}{\sin x \sin x}, \end{aligned}$$

since, from above, $\lim_{\delta x \rightarrow 0} \frac{\sin \delta x}{\delta x} = 1$.

$$\therefore \frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x.$$

QED.



(b)

$$\begin{aligned} y &= x^2 e^{-3x}, \\ \therefore \frac{dy}{dx} &= x^2 \times \frac{d(e^{-3x})}{dx} + e^{-3x} \times \frac{d(x^2)}{dx}, \\ &= -3x^2 e^{-3x} + 2x e^{-3x}, \\ &= x e^{-3x} (2 - 3x). \end{aligned}$$

When x is negative, x^2 is positive, and e^{-3x} has a positive index; the function $y = x^2 e^{-3x}$ is positive and large (for example, when $x = -2$, $y \approx 1614$).

At $x = 0$, $y = 0$ and, as x becomes positive, e^{-3x} decreases and remains positive, while x^2 increases again.

Maxima or minima occur when $dy/dx = 0$; that is, when $x = 0$ or $2/3$.

Note: When $e^{-3x} = 0$, $x = \infty$, and the curve becomes asymptotic to the x -axis.

It is clear that, at $x = 0$, a minimum value of $y = 0$ occurs, and that a maximum value of $y = (2/3)^2 \times e^{-2} = 0.0601$ occurs at $x = 2/3$.

The curve is shown in sketch (b), plotted from the values derived above and the table below. Because of the large differences in values of the function for negative and positive values of x , the y -axis scales are widely different for the 2 sectors shown.

x	-1.3	-1.0	-0.5	0.1	0.3	0.5
y	83.5	20.1	1.1	0.007	0.037	0.056

x	1	1.3	1.5	2
y	0.05	0.034	0.025	0.01

Q 9 (a) Evaluate

(i) $\int_0^{\pi/4} \cos 3x \, dx$, and

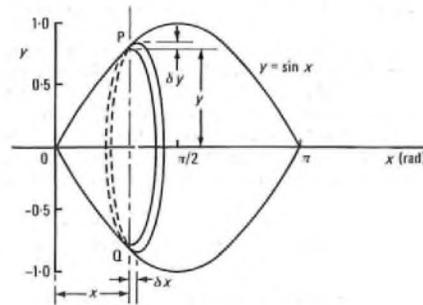
(ii) $\int_1^8 \sqrt{3x+1} \, dx$.

(b) Calculate the volume enclosed when the portion of the curve $y = \sin x$ from $x = 0$ to $x = \pi$ is rotated through 360° about the x -axis.

A 9 (a) (i) $\int_0^{\pi/4} \cos 3x \, dx = \left[\frac{\sin 3x}{3} \right]_0^{\pi/4}$,
 $= \frac{1}{3} (\sin \frac{3\pi}{4} - \sin 0)$,
 $= \frac{0.7071}{3} = 0.2357$.

(ii) $\int_1^8 \sqrt{3x+1} \, dx = \int_1^8 (3x+1)^{1/2} \, dx$,
 $= \left[\frac{1}{3} \times \frac{2}{3} \times (3x+1)^{3/2} \right]_1^8$,
 $= \frac{2}{9} \times (25^{3/2} - 4^{3/2})$,
 $= \frac{2}{9} \times (125 - 8)$,
 $= 26$.

(b) The sketch shows the curve of $y = \sin x$ between the limits $x = 0$ and $x = \pi$ radians, rotated about the x -axis, generating the solid figure shown.



Let $P(x, y)$ be any point on the curve. If the ordinate through P is rotated through 360° , it generates a circle of radius y . If a second circle, with radius $y + \delta y$ and centred at $x + \delta x$, is drawn very close to the first, the volume of the thin solid slice contained between them is approximately $\pi y^2 \delta x$.

MATHEMATICS C, 1975 (continued)

The total volume of the solid between $x = 0$ and $x = \pi$ radians is given by

$$\int_0^\pi \pi y^2 dx = \pi \int_0^\pi \sin^2 x dx,$$

$$= \pi \int_0^\pi \frac{1 - \cos 2x}{2} dx,$$

since $\cos 2x = 1 - 2 \sin^2 x$. Thus, the volume

$$= \frac{\pi}{2} \left[x - \frac{\sin 2x}{2} \right]_0^\pi,$$

$$= \frac{\pi}{2} \times (\pi - 0),$$

$$= \frac{\pi^2}{2}.$$

Q 10 (a) Calculate the mean value of $y = 4e^{-3x}$ from $x = 0$ to $x = 0.6$.

(b) Use Simpson's rule, with 5 ordinates, to evaluate $\int_0^4 \frac{dx}{2x+3}$ to 3 significant figures. Verify the result by exact integration.

A 10 (a) The mean value of y is given by

$$\frac{\int_0^{0.6} y dx}{0.6} = \frac{5}{3} \int_0^{0.6} 4e^{-3x} dx,$$

$$= \frac{20}{3} \left[-\frac{e^{-3x}}{3} \right]_0^{0.6},$$

$$= -\frac{20}{9} \times (e^{-1.8} - e^0),$$

$$= -\frac{20}{9} \times (0.1653 - 1),$$

$$= 1.855.$$

(b) The area under the curve $y = 1/(2x + 3)$ between $x = 0$ and $x = 4$ is given by $\int_0^4 \frac{dx}{2x+3}$. The use of Simpson's rule with 5 ordinates requires the division of the area into 4 strips of width unity, as given in the following table.

x	0	1	2	3	4
y	1/3	1/5	1/7	1/9	1/11

Hence, from Simpson's rule,

$$\int_0^4 \frac{dx}{2x+3} \approx \frac{1}{3} \times (\text{width of a strip})$$

$$\times (\text{sum of the first and last ordinates}$$

$$+ 2 (\text{sum of the other odd ordinates})$$

$$+ 4 (\text{sum of the even ordinates})),$$

$$= \frac{1}{3} \left\{ \left(\frac{1}{3} + \frac{1}{11} \right) + 2 \left(\frac{1}{5} + \frac{1}{9} \right) \right\},$$

$$= \frac{1}{3} \left(\frac{14}{33} + \frac{2}{7} + \frac{4 \times 14}{45} \right),$$

$$= 0.652, \text{ to 3 significant figures.}$$

By exact integration,

$$\int_0^4 \frac{dx}{2x+3} = \left[\frac{\log_e (2x+3)}{2} \right]_0^4,$$

$$= \frac{1}{2} \times (\log_e 11 - \log_e 3),$$

$$= \frac{1}{2} \times (2.3979 - 1.0986),$$

$$= 0.650, \text{ to 3 significant figures}$$

COMMUNICATION RADIO C, 1975

Students were expected to answer any 6 questions

Q 1 (a) Explain the meanings of the following terms:

- (i) critical frequency,
- (ii) selective fading,
- (iii) surface wave, and
- (iv) ground wave.

(b) With what frequency band is each of the above normally associated?

Q 2 (a) A superheterodyne receiver is being designed to receive any 9 kHz channel in the band 500–1500 kHz. State the main disadvantage in using each of the following intermediate frequencies:

- (i) 10 kHz,
- (ii) 100 kHz,
- (iii) 1000 kHz, and
- (iv) 10 000 kHz.

(b) State the approximate frequency limits of the band for which 10 MHz would be an appropriate intermediate frequency.

(c) What would be a typical noise figure for such a receiver?

A 2 (a) The intermediate frequency (IF) of a superheterodyne receiver should be high enough for the image channel to be rejected by the tuned circuits of the signal-frequency amplifier (or input stage), yet low enough for channels adjacent to the wanted channel to be rejected by the IF stages without introducing excessive selectivity requirements. The choice is a compromise between these 2 requirements.

The disadvantages of various IFs are illustrated by the following table.

IF (kHz)	10	100	1000	10 000
Signal Band (kHz)	500–1500	500–1500	500–1500	500–1500
Oscillator Range (kHz)	510–1510	600–1600	1500–2500	10 500–11 500
Image Band (kHz)	520–1520	700–1700	2500–3500	20 500–21 500

(i) With an IF of 10 kHz, the image band is virtually coincident with the signal band, so that the image-channel rejection is poor. Also, signals close to the image frequency produce whistles. For example, with a signal frequency of 1000 kHz, an IF of 10 kHz and oscillator frequency of 1010 kHz, a "near-image" of 1012 kHz is translated to 12 kHz, which then beats with the IF to produce a 2 kHz whistle.

Although an IF bandwidth of 9 kHz appears just adequate for the 9 kHz channel, it is very sensitive to frequency changes in the oscillator. For example, a 0.2% increase in oscillator frequency (from, say, 1010 kHz to 1012 kHz when tuning to a signal frequency of 1000 kHz) results in an IF change of 20%; that is, from 10 kHz to 12 kHz. Furthermore, since 10 kHz is in the audio-frequency band, the detector offers a low impedance and thus produces little output.

(ii) With an IF of 100 kHz, the image band still falls within the signal-frequency band, but, with the tuned circuits in the signal-frequency amplifier (or input stage), the image and intermodulation products are probably less than those for an IF of 10 kHz.

(iii) When the IF is 1000 kHz, the image band falls outside the signal-frequency band. However, the IF is now within the signal-width frequency band, so that the 1000 kHz channel continuously breaks through. In addition, the relatively wide IF band that results when centred at 1000 kHz offers poor selectivity against adjacent channels.

(iv) With an IF of 10 000 kHz, the IF bandwidth is so large that no selectivity is provided.

(b) An IF of 10 MHz would be appropriate for the frequency-modulation broadcast band (80–110 MHz).

(c) A typical noise figure for such a receiver would be between 5–10 dB.

Q 3 (a) What is meant by the term critical coupling?

(b) The coupled circuits shown in Fig. 1 are tuned to the same frequency. For different values of coupling coefficient, sketch curves of the primary and secondary currents against frequency to illustrate the effect of the coupling coefficient on circuit response.

(c) If both circuits are tuned to a frequency of 500 Hkz, and the coupling coefficient is 0.01, what is

- (i) the value of the common Q -factor, and
- (ii) the approximate half-power bandwidth?

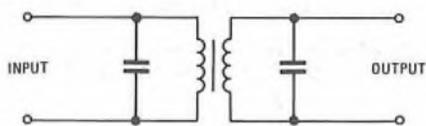
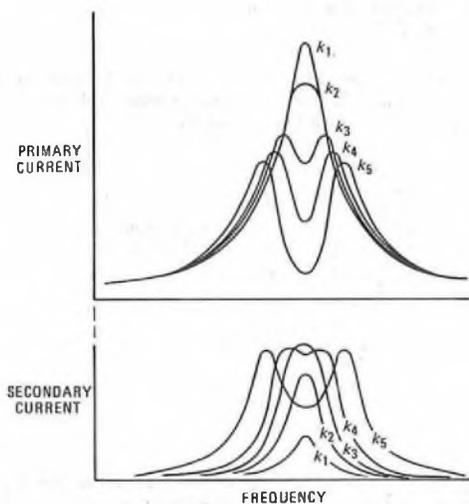


Fig. 1

A 3 (a) Any 2 circuits, arranged so that a current flowing in one circuit (the primary circuit) causes current to flow in the other (the secondary circuit), are said to be coupled. Critical coupling is the condition when the coupling is such that the secondary current is a maximum. For coupled circuits tuned to the same resonant frequency, this maximum transfer of power occurs when the resistance coupled from the secondary circuit to the primary is equal to the primary resistance, and the reactance coupled from the secondary circuit to the primary is equal, and of opposite sign, to the primary reactance.

(b) Curves of primary and secondary current against frequency, for different values of coupling coefficient, k , are shown in sketch (a).



Note: $k_1 < k_2 < k_3 < k_4 < k_5$

(a)

(c) (i) For circuits arranged with mutual-inductance coupling, as shown in Fig. 1, the coupling coefficient is given by the mutual inductance, M henrys, divided by the geometric mean of the primary and secondary inductances, L_1 and L_2 henrys respectively. Thus,

$$k = \frac{M}{\sqrt{L_1 L_2}} \dots \dots (1)$$

The Q -factors of the primary and secondary circuits, Q_1 and Q_2 respectively, are given by

$$Q_1 = \frac{\omega_0 L_1}{R_1} \text{ and } Q_2 = \frac{\omega_0 L_2}{R_2}$$

where ω_0 is the resonant angular frequency (radians/second), and R_1 and R_2 are the respective resistances of the primary and secondary coils (ohms).

Substituting for L_1 and L_2 in equation (1) gives

$$k = \frac{M}{\sqrt{\left(\frac{R_1 Q_1}{\omega_0} \times \frac{R_2 Q_2}{\omega_0}\right)}} = \frac{\omega_0 M}{\sqrt{(R_1 R_2 Q_1 Q_2)}} \dots \dots (2)$$

For critical coupling, the secondary resistance coupled to the primary circuit is equal to the primary resistance. Hence,

$$\frac{\omega_0^2 M^2}{R_2} = R_1 \text{ ohms.}$$

$$\therefore \omega_0^2 M^2 = R_1 R_2 \text{ ohms}^2.$$

Substituting for $R_1 R_2$ in equation (2) gives the critical coupling coefficient as

$$k_{\text{critical}} = \frac{\omega_0 M}{\sqrt{(\omega_0^2 M^2 Q_1 Q_2)}} = \frac{1}{\sqrt{(Q_1 Q_2)}}$$

The question implies that $Q_1 = Q_2 = Q$.

$$\therefore k_{\text{critical}} = \frac{1}{Q}$$

When the critical coupling coefficient is 0.01,

$$Q = \frac{1}{0.01} = 100.$$

(ii) The Q -factor of a tuned circuit is defined as the ratio of the resonant frequency, f_0 hertz, to the half-power bandwidth. Now, the half-power bandwidth, B_{CC} hertz, of a critically-coupled double-tuned circuit is $\sqrt{2}$ times that of the individual circuits of which it is composed.

$$\begin{aligned} \therefore B_{CC} &= \frac{\sqrt{2} \times f_0}{Q} \text{ hertz,} \\ &= \frac{\sqrt{2} \times 500 \times 10^3}{100} \text{ Hz} = 7.07 \text{ kHz.} \end{aligned}$$

Q 4 (a) Explain briefly the term delayed automatic gain control (AGC).

(b) Draw a simplified circuit diagram of the intermediate-frequency (IF) amplifier and detector stages in a superheterodyne broadcast receiver. Indicate clearly the components which comprise the delayed AGC circuit.

(c) The maximum field strength at a communications receiver is 5 mV/m and corresponds to a control voltage of 10 V. The gain control is 2.9 dB/V on each of the 3 IF stages, and 1.3 dB/V on the frequency changer. If the control voltage is proportional to the output voltage, and the maximum allowable change in output is 6 dB, what is

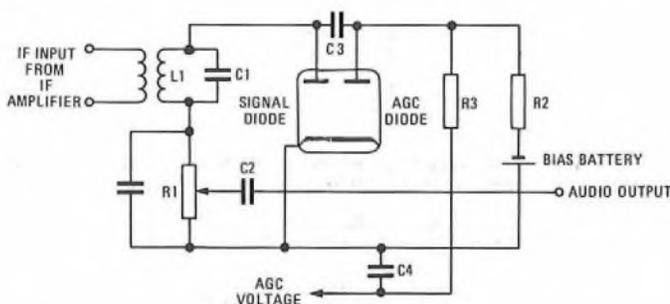
- (i) the corresponding change in control voltage, and
- (ii) the minimum field strength for which the receiver is suitable?

A 4 (a) To overcome the problem of fading in the reception of radio signals, it is common practice to provide AGC. This can be done by developing a control voltage which is applied to the amplifying stages to reduce their gain in proportion to the carrier-signal strength. However, this simple method suffers from 2 disadvantages:

- (i) the weakest incoming signals are attenuated unnecessarily, and
- (ii) the sensitivity is greatest when the receiver is not tuned to a signal.

These problems can be overcome by suppressing the control voltage until a predetermined signal level is reached, and the technique by which this is achieved is called delayed AGC.

(b) A circuit that provides delayed AGC is shown in the sketch. A double-diode tube is used, one section providing signal detection, and the other AGC detection. The radio-frequency signal is developed across the tuned circuit formed by inductor $L1$ and capacitor $C1$, and rectified by the signal diode to develop the audio frequency across load resistor $R1$. This is coupled to the audio stage by capacitor $C2$.



The radio-frequency signal is also applied, via capacitor $C3$, to the AGC detector. Because of the bias battery, the AGC diode does not conduct until the peak signal level exceeds the bias voltage, when the AGC voltage is developed across load resistor $R2$.

Before the AGC voltage is applied to the amplifying stages, the alternating components developed across resistor $R2$ are virtually removed by resistor $R3$ and capacitor $C4$.

(c) Since the control voltage is proportional to the output voltage, a maximum allowable change of 6 dB in the latter gives a similar change in the former. If the maximum control voltage, V_{max} , is 10 V, the minimum control voltage, V_{min} , must be 6 dB less.

$$\therefore 20 \log_{10} \frac{10}{V_{\text{min}}} = 6 \text{ dB.}$$

$$\therefore \frac{10}{V_{\text{min}}} = 2.$$

$$\therefore V_{\text{min}} = 5 \text{ V.}$$

Hence, the change in control voltage

$$= V_{\max} - V_{\min} \text{ volts} = 10 - 5 = \underline{5 \text{ V.}}$$

(ii) The minimum field strength is detected when the control voltage is a minimum. At this condition, the gains of the amplifiers and the mixer are a maximum.

For a control voltage 5 V below its maximum value, one amplifier, with a control sensitivity of 2.9 dB/V, increases its gain above its minimum value by $2.9 \times 5 = 14.5$ dB. Three such amplifiers together raise the gain by $14.5 \times 3 = 43.5$ dB.

The mixer, with a control sensitivity of 1.3 dB/V, increases its gain above its minimum value by $1.3 \times 5 = 6.5$ dB.

The total gain of the receiver is therefore $43.5 + 6.5 = 50$ dB.

However, the output is now 6 dB below the maximum output, and the maximum output corresponds to a field strength of 5 mV/m. The minimum field strength, E_{\min} volts/metre, is therefore 56 dB below 5 mV/m.

$$\therefore 20 \log_{10} \frac{5 \times 10^{-3}}{E_{\min}} = 56 \text{ dB.}$$

$$\therefore \frac{5 \times 10^{-3}}{E_{\min}} = 631.$$

$$\therefore E_{\min} \approx \underline{8 \mu\text{V/m.}}$$

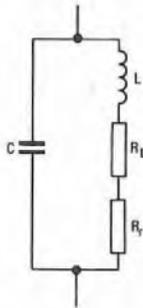
Q 5 A broadcast transmitter has an output circuit whose Q -factor falls from 140 to 14 when the load is coupled. When delivering its maximum output, the tube draws 10 A from an 11 kV supply. The tank-circuit is tuned to 10 MHz when its capacitance is 400 pF. If the anode swing is ± 10 kV, calculate

- the radio-frequency (RF) power delivered to the tank-circuit,
- the RF power delivered to the load, and
- the overall efficiency of the output stage.

A 5 (a) The sketch represents the tank circuit. The loaded Q -factor is given by

$$Q_L = \frac{\omega L}{R} = \frac{1}{\omega CR},$$

where ω is the angular frequency (radians/second), L is the inductance (henrys), C is the capacitance (farads), and $R = R_L + R_r$, where R_L is the effective series resistance of the tank circuit (ohms), and R_r is the resistance reflected into the tank circuit by the load (ohms).



At resonance, the impedance of the tank circuit is known as the dynamic impedance, Z_D ohms, where

$$Z_D = \frac{L}{CR} = \frac{\omega L}{\omega CR} = \frac{Q_L}{\omega C} \text{ ohms,}$$

$$= \frac{14}{2\pi \times 10^7 \times 400 \times 10^{-12}} = 557 \Omega.$$

The power delivered to the tank circuit, P_T watts, is given by E^2/Z_D watts, where E is the r.m.s. anode voltage, equal to $E_{\text{peak}}/\sqrt{2}$ volts.

$$\therefore P_T = \frac{10^8}{2 \times 557} \text{ W} = \underline{89.77 \text{ kW.}}$$

(b) The power dissipated in the load is $I^2 R_r$ watts, where I is the circulating current in the tank circuit (amperes). The power dissipated in the tank circuit is $I^2 R_L$ watts. Thus, the efficiency of the tank circuit, η_T , is given by R_r/R_L .

Now, $Q_L = \omega L/R$ and $Q_U = \omega L/R_L$, where Q_U is the unloaded Q -factor.

$$\therefore \frac{Q_L}{Q_U} = \frac{R_L}{R}.$$

$$\therefore 1 - \frac{Q_L}{Q_U} = 1 - \frac{R_L}{R} = \frac{R - R_L}{R} = \frac{R_r}{R} = \eta_T.$$

$$\therefore \eta_T = 1 - \frac{Q_L}{Q_U} = 1 - \frac{14}{140} = 90\%.$$

Hence, the power transferred to the load, P_L watts,

$$= 0.9 \times P_T \text{ watts} = 0.9 \times 89.77 = \underline{80.79 \text{ kW.}}$$

(c) The overall efficiency of the stage, η_S , is given by the power delivered to the load divided by the power taken from the supply.

$$\therefore \eta_S = \frac{P_L}{I_A V_A},$$

where I_A is the current drawn from the supply (amperes), and V_A is the supply voltage.

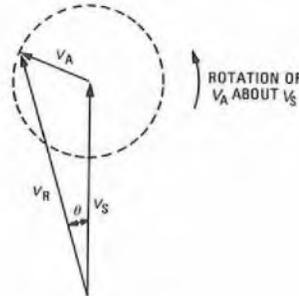
$$\therefore \eta_S = \frac{80.79 \times 10^3}{10 \times 11 \times 10^3} \times 100\% = \underline{73.4\%}.$$

Q 6 (a) Use a phasor diagram to explain how a relatively small adjacent sinusoidal signal gives rise to deviation of the wanted unmodulated carrier in a frequency-modulation (FM) receiver.

(b) The radio-frequency bandwidth required by an FM transmitter is 100 kHz when the modulation index is 4. If the modulation level is increased by 6 dB, what is

- the new modulation index, and
- the new bandwidth?

A 6 (a) If V_S represents the wanted signal-frequency voltage, and V_A represents a small adjacent sinusoidal signal voltage, the effect of V_A upon V_S is shown by the phasor diagram.



Phasor V_A can be considered to rotate about phasor V_S with an angular frequency equal to 2π times the frequency difference between the 2 signals.

The resultant, V_R , varies in amplitude between $V_S \pm V_A$, and in phase between $\pm \theta_{\max} = \tan^{-1}(V_A/V_S)$.

The resultant is therefore amplitude and phase modulated, and since phase changes are associated with frequency, frequency modulated.

(b) (i) The modulation index, η , is defined as $k f_d/f_m$, where f_d is the peak frequency deviation (hertz), f_m is the modulating frequency (hertz), and k is the actual amplitude of the modulating signal divided by the maximum rated amplitude for the system.

An increase of 6 dB in the modulating amplitude is a change from V_1 volts to V_2 volts, so that

$$20 \log_{10} \frac{V_2}{V_1} = 6 \text{ dB.}$$

$$\therefore V_2 = V_1 \text{ antilog}_{10} \frac{6}{20} = 2V_1 \text{ volts.}$$

Thus, the modulating amplitude doubles.

Since the maximum rated amplitude is constant, k increases by a factor of 2. Thus, provided f_m does not change, η increases by a factor of 2. Hence, the new modulation index is 8.

(ii) The bandwidth, B hertz, required for satisfactory reception of a frequency-modulated wave is given by the empirical formula

$$B = 2(k f_d + f_m) \text{ hertz,}$$

$$= 2(\eta f_m + f_m) \text{ hertz.}$$

When $B = 100$ kHz, $\eta = 4$.

$$\therefore 100 \times 10^3 = 2(4 f_m + f_m) \text{ hertz.}$$

$$\therefore f_m = 10 \text{ kHz.}$$

When η becomes 8, the new bandwidth is given by

$$B = 2(8 \times 10 \times 10^3 + 10 \times 10^3) \text{ hertz,}$$

$$= \underline{180 \text{ kHz.}}$$

COMMUNICATION RADIO C, 1975 (continued)

Q 7 (a) List the main differences between a receiver designed for the reception of frequency-modulation (FM) signals and one designed for the reception of amplitude-modulation (AM) signals.

(b) Draw the circuit diagram of the demodulator and first audio stage of an FM receiver, indicating the components which comprise the de-emphasis circuit.

(c) Use phasor diagrams to describe the operation of the demodulator in the circuit diagram you have drawn.

Q 8 (a) Draw the block diagram of a very-high-frequency (VHF) transmitter/receiver suitable for marine use, and briefly describe its operation.

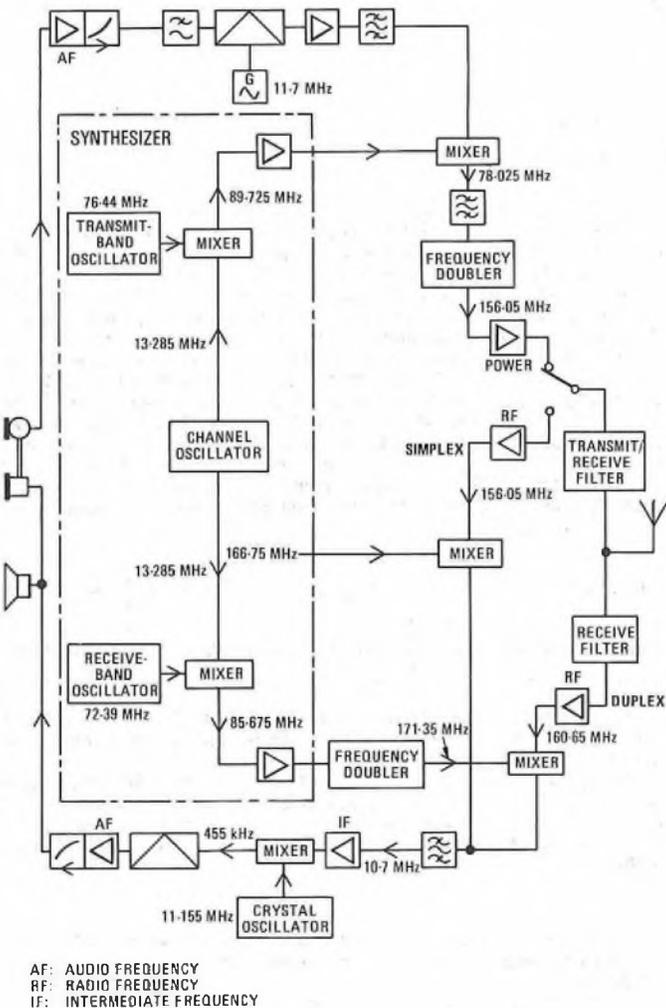
(b) For such a transceiver, give typical values for

- (i) the operating frequency,
- (ii) the output power, and
- (iii) the bandwidth.

(c) What is meant by the following terms:

- (i) muting, and
- (ii) duplex operation?

A 8 (a) The sketch shows a multi-channel frequency-modulated VHF transmitter/receiver suitable for operating in simplex and duplex modes for marine applications.



bandwidth limited in the transmit/receive (simplex) filter before being applied to the aerial.

Depending on whether operation is in the duplex or simplex mode, the received signal is selected by the relevant radio-frequency filter, amplified and translated to a first intermediate frequency of 10.7 MHz. After band limiting and amplification, a further translation is made to produce an intermediate frequency of 455 kHz. The audio-frequency signal is recovered at the discriminator and, after de-emphasis and amplification, is applied to the handset receiver or loudspeaker.

(b) Such a transceiver is likely to have

- (i) an operating frequency range of 156–174 MHz,
- (ii) a radio-frequency output power in the range 5–25 W, and
- (iii) a radio-frequency bandwidth of about 5 kHz.

(c) (i) Muting is the suppression of the receiver output during idle (or very weak signal) conditions

(ii) Duplex operation means that simultaneous transmission and reception is possible.

Q 9 (a) Draw a dimensioned sketch of a Yagi aerial, suitable for operation at 200 MHz, and explain why a folded dipole is normally used.

(b) Draw the horizontal and vertical polar diagrams of

- (i) a vertical dipole, and
- (ii) a vertical dipole with one reflector.

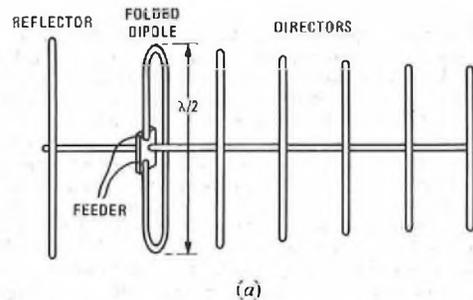
(c) Show how the gain of an aerial relative to a dipole can be obtained from its polar diagram.

A 9 (a) A Yagi aerial (or, more appropriately, a Yagi-Uda aerial) is composed of a radiating half-wave dipole and a number of parasitic elements.

When a parasitic element is slightly less than half a wavelength, it is capacitive and reinforces the field, acting as a director. When it is slightly longer than half a wavelength, it is inductive and opposes the field, acting as a reflector.

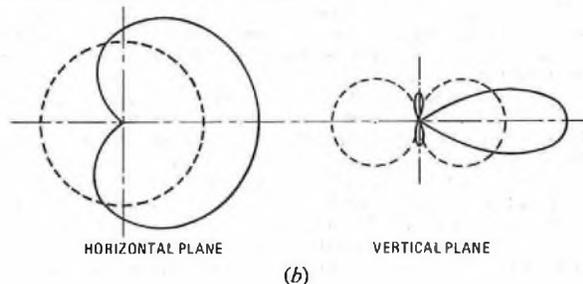
The forward-to-reverse gain ratio for a Yagi aerial depends upon the number of, and spacing between, the parasitic elements. This ratio increases as the element spacing decreases, but the shunt effect of the directors becomes more dominant and reduces the input impedance of the dipole. When there are more than 3 directors, the dipole impedance falls to about 20 Ω.

However, by folding the dipole, the impedance is increased by a factor of 4, so that it is possible to achieve an acceptable match with the impedance of the feeder cable, which is generally 75 Ω.



An example of a typical array, using 6 parasitic elements and a folded dipole, is shown in sketch (a). The dipole is 0.5λ long, where λ is the wavelength (metres), and the reflector is about 6% longer than the dipole and is spaced 0.25λ metres from it. The first director is about 4% shorter than the dipole and is spaced 0.15λ metres from it. Successive directors have the same spacing, but their lengths decrease by about 4% of that of the preceding director each time. An array of the proportions shown has a forward gain of about 10 dB and a bandwidth of about 2% of its operating frequency. For λ = 1.5 m, the operating frequency is 200 MHz.

(b) The horizontal and vertical polar diagrams for a vertical dipole (shown dashed) and a vertical dipole with one reflector (shown by solid lines) are shown in sketch (b).

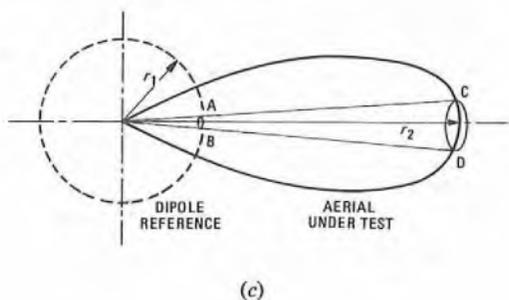


Internationally, channels in the frequency range 156.025–157.425 MHz are used for transmission and simplex reception, and in the range 160.625–162.025 MHz for duplex reception.

The audio-frequency output from the microphone is amplified, with pre-emphasis and amplitude-limiting, and frequency limited by a 3 kHz low-pass filter. It frequency modulates an 11.7 MHz oscillator, using a varactor diode. After amplification and band limiting, the signal is mixed with a synthesizer output appropriate to the channel selected. In the sketch, selection of channel 1 is illustrated. The translated signal frequency is frequency doubled, power amplified, and then

COMMUNICATION RADIO C, 1975 (continued)

(c) Sketch (c) shows the polar diagram of an aerial under test superimposed upon that of a dipole. Although a plan section only is shown, the wavefronts at AB and CD are parts of spherical surfaces of radii r_1 and r_2 metres respectively.



Since the polar diagram defines points of equal power density, the power density at CD for the test aerial, J_T watts/metre², is equal to the power density at AB for the dipole, J_D watts/metre².

The gain, G decibels, of the test aerial relative to the dipole is given by

$$G = 10 \log_{10} \frac{P_T}{P_D} \text{ decibels,}$$

where P_T is the power received from the test aerial (watts), and P_D is the power received from the dipole (watts).

But $P_T = k4\pi r_2^2 J_T$ watts and $P_D = k4\pi r_1^2 J_D$ watts, where k is the fractional solid angle subtended by AB or CD.

$$\begin{aligned} \therefore G &= 10 \log_{10} \frac{k4\pi r_2^2 J_T}{k4\pi r_1^2 J_D} \text{ decibels,} \\ &= 10 \log_{10} \left(\frac{r_2}{r_1} \right)^2 \text{ decibels.} \end{aligned}$$

Q 10 Using diagrams of the expected trace, describe how a cathode-ray oscilloscope can be used to measure:

(a) the modulation depth of a sinusoidally-modulated amplitude-modulated signal by the trapezoidal method for

- (i) 50% modulation,
- (ii) 100% modulation, and
- (iii) overmodulation; and

(b) the phase difference between 2 sinusoidal waveforms equal in amplitude and

- (i) approximately in phase,
- (ii) with a 90° phase difference, and
- (iii) with a 45° phase difference.

A 10 See A9, Communication Radio C, 1970, Supplement, Vol. 64, p. 74, Jan. 1972.

TELEGRAPHY C, 1975

Students were expected to answer any 6 questions

Q 1 (a) What are the advantages and disadvantages of providing telegraph circuits by using multi-channel voice-frequency (MCVF) systems, compared with using physical circuits?

(b) Upon what basis are the carrier frequencies selected for an MCVF system?

(c) For a band-pass filter,

- (i) draw a diagram of the circuit elements, and
- (ii) sketch the principal characteristics.

A 1 (a) Telegraph circuits can be provided by direct physical connexion, using a pair of wires in a cable or a pair of overhead wires. This method has many advantages for short-distance circuits, in that the local telephone network can be used and the equipment required at each end of the circuit is simple and inexpensive.

Disadvantages are that high voltages are used, a signal-shaping network is needed at the receiver, and a filter has to be inserted at the transmitter to prevent interference if the circuit is routed over a telephone cable. Also, the use of d.c. circuits over distances greater than about 40 km has many disadvantages. The telegraph-signal distortion and attenuation due to line constants is excessive, and the circuit requires heavy conductors and repeating relays at regular intervals. The use of 2 pairs of conductors with repeating relays, known as 2-loop working, extends the range of d.c. operation up to about 145 km, but this is expensive in line plant. A further disadvantage of long-distance d.c. working is that few telephone circuits of that length are without amplifiers, and the use of a circuit for telegraph working requires the disconnection of any amplifying equipment.

An MCVF system uses signalling tones having frequencies within the bandwidth of a telephone circuit, so that the system can use the amplified telephone network for long-distance working. A 4-wire circuit is required, with terminal equipment which incurs capital and maintenance costs. In addition, the telephone bearer circuit has to be modified to remove bridging equipment.

For Telex working, voice-frequency systems do not provide the third signalling condition which gives the call-clear signal from the exchange to the subscriber; special arrangements have to be made for this. The voice-frequency system does, however, cater for up to 24 circuits, which is very economical over long distances, although failure of the system means that a large number of circuits are simultaneously out of order.

(b) Carrier frequencies selected for an MCVF system depend upon the available bandwidth of the bearer circuit, the bandwidth required by the separate telegraph channels (related to the modulation rate), and the characteristics of practical channel filters.

Any non-linearity in the transmission equipment that distorts a carrier frequency, f hertz, produces harmonics having frequencies $2f$, $3f$, $4f$, etc. In addition, the simultaneous application of 2 channel frequencies, f_1 and f_2 , could result in a series of sum-and-difference frequencies of f_1 and f_2 and their various harmonics. The

first-order intermodulation products, $2f_1$, $2f_2$, $f_1 + f_2$, and $f_1 - f_2$, are most likely to cause interference. To reduce these products, the odd multiples of a basic frequency that is equal to one half of the width of a channel are chosen as carrier frequencies. These values ensure that the even harmonics and the sum-and-difference products fall midway between the nominal values of carrier frequencies, where the attenuation of the channel filters is at a maximum value.

The fundamental frequency of a 50 baud telegraph signal is 25 Hz. This can be accommodated within a channel bandwidth of 120 Hz, which allows a conservative design for the filter. A telephone circuit has an effective bandwidth of at least 3 kHz, and 24 channels, each of 120 Hz, occupy a bandwidth of 2.88 kHz. The single common frequency chosen is 60 Hz, with the first carrier frequency at $7 \times 60 = 420$ Hz. Subsequent channels are spaced at 120 Hz intervals with channel 24 at 3.18 kHz. The first few channel frequencies are shown in the table.

Channel	1	2	3	4	5	6	7
Carrier Frequency (Hz)	420	540	660	780	900	1020	1140

For channel 1, the second harmonic, $2f_1$, is $2 \times 420 = 840$ Hz. This frequency lies midway between the carrier frequencies for channels 4 and 5.

For channels 1 and 2, $f_1 + f_2 = 420 + 540 = 960$ Hz, which again lies midway between 2 carrier frequencies. In this way, the selection of carrier frequencies ensures minimum interference between channels.

(c) See A2, Telegraphy C, 1969, Supplement, Vol. 63, p. 51, Oct. 1970.

Q 2 A Telex network in a country has about 50 000 subscribers with one international exchange, a few large exchanges and many small exchanges.

(a) Develop and explain a numbering scheme for the system.

(b) What are the merits and limitations of such a numbering scheme?

(c) Draw a trunking diagram to show the connexion of a call from a subscriber on a small exchange to a subscriber on a similar exchange at the other end of the country.

A 2 (a) The choice of a national numbering scheme for a Telex network depends to a certain extent on the type of switching mechanism used, whether registers are employed and whether keyboard characters or dial pulses are used for selection signals. For the UK network, which has about 50 000 subscribers and uses dial-pulse selection, a linked numbering scheme is used in which the first 2 digits

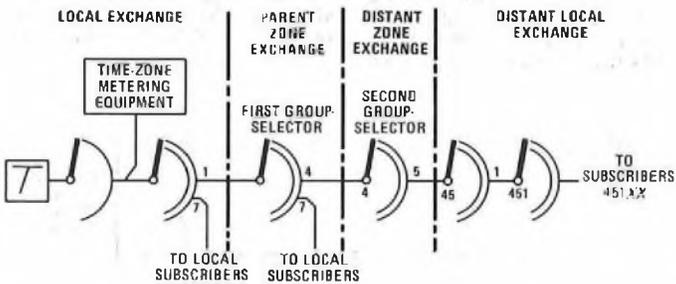
of a subscriber's number route the call to the appropriate charging area. The UK is divided into 50 charging areas. There are 8 main exchanges, known as *zone exchanges*, 3 of which are in London, and each area exchange is dependent on a zone exchange. Normally, one exchange is provided for each charging area, except for London, which has several exchanges, and certain less significant charging areas which are represented by groups of selectors acting as hypothetical exchanges in the nearest zone exchange. The zone exchanges are fully interconnected; that is, they have routes to every other zone exchange. Each area exchange is connected to its parent zone exchange and has a direct route to each London international exchange to cater for the heavy international traffic. There are also some direct area-exchange routes to non-parent zone or other area exchanges, depending on the traffic between them.

The first digit of the subscriber's number is characteristic of the zone, with the second digit indicating the area exchange; the remaining 3 or 4 digits are particular to the subscriber. In this way, a 5-digit or 6-digit numbering scheme gives a linked numbering scheme to cater for 50 000 subscribers.

Local service codes use the prefix digit 0, such as 01 for telegrams and 02 for the test desk. National service codes use the prefix digit 2, such as 29 for the manual switchboard and 2003 for directory enquiries. Access to international circuits is from level 2 in area and zone exchanges, which gives access direct to a London international exchange. Codes 202, 203, 204, 205 and 209 provide connexions to international destinations, such as Hamburg (2032) and Prague (20550). The intercontinental exchange is reached by using the code 207.

(b) An advantage of this numbering scheme is that it provides simple, almost uniform dialling procedures whereby a subscriber on any zone exchange dials the directory number for any other subscriber. A disadvantage is that area-exchange subscribers must dial a prefix digit (1) for any call that is not local, to route the call to the parent zone exchange. As no letters are used, there is no restriction on codes or the format of overseas dials. The geographical basis of the numbering scheme facilitates the use of time-zone metering and makes numbers easy to remember. The numbering scheme is suitable for direct selection and for the provision of direct routes.

Some of the disadvantages of the scheme are that the growth is limited if zones are unbalanced numerically, a list of dialling codes is required for use by the subscriber and, if direct area-area or area-non-parent-zone routes are provided, routing translators must be used.



(c) The sketch shows the required trunking diagram. The numerals denote the levels of the 2-motion selectors.

Q 3 (a) For a switched teleprinter network, what are the typical distortion limits for

- (i) a teleprinter transmitter,
- (ii) a voice-frequency channel,
- (iii) a physical circuit, and
- (iv) the overall network?

(b) Describe, with the aid of a sketch, the principles of operation of equipment used to measure distortion.

(c) At what points in the network is the equipment used?

A 3 (a) Typical distortion limits are

- (i) a teleprinter transmitter: 5%,
- (ii) a voice-frequency channel: 10%,
- (iii) a physical circuit: 10%, and
- (iv) the overall network: 28%.

(b) See A1, *Telegraphy C, 1970, Supplement*, Vol. 64, p. 43, July 1971, and A4, *Telegraphy C, 1974, Supplement*, Vol. 68, p. 68, Oct. 1975.

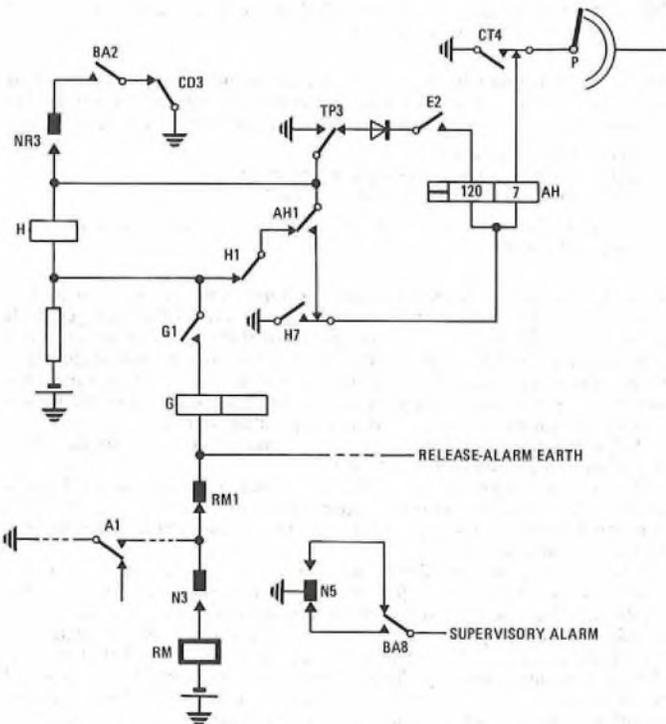
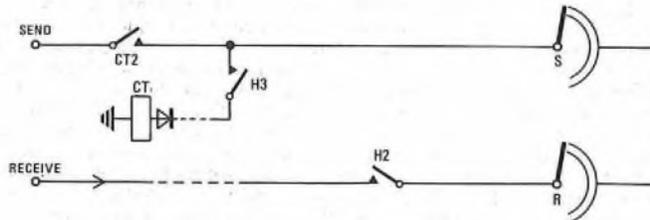
(c) A distortion-measuring set is used at a test-desk and in workshops to measure the start-stop distortion, and at voice-frequency-system terminals to measure the isochronous distortion. The measuring set is used when adjusting teleprinters or lining-up voice-frequency channels.

Q 4 For a Telex final selector,

(a) explain, with the aid of a diagram, the circuit operation when the selector steps to a free line, and

(b) explain how attention is drawn to the selector if it does not release after the call has been cleared.

A 4 (a) The sketch shows the diagram of part of a final-selector circuit. The selector steps in the vertical and rotary directions under the control of the calling-subscriber's dial, which pulses relay A (not shown). After the last pulse has been received, relay CD (not shown) releases and earth potential is connected to the P-wiper through contacts CD3, BA2, NR3, TP3, E2, the 120 Ω and 7 Ω coils of relay AH and contact CT4. If the called-subscriber's line is free, a 550 Ω negative potential is connected to the P-wire, and relay AH operates. Contact AH1 short-circuits the 120 Ω coil of relay AH, and the relay holds over its 7 Ω coil and offers a sufficiently low earth potential to the P-wire to prevent any other call switching to the same subscriber. Contact AH1 also removes the short-circuit from relay H, which operates. Contact H1 disconnects the rotary-magnet circuit. Contact H2 extends a -80 V potential from the calling station to operate relay CM in the called-subscriber's control unit. Contact H3 connects relay CT to the S-wire to detect the -80 V call-connected signal when this is received from the called station. Contact CT2 then connects the S-wire negative potential to the preceding time-zone equipment, which transmits the *who-are-you?* signal to the called subscriber. The answer-back code indicates to the calling subscriber that the distant subscriber is connected and the call can proceed.



(b) When the call is cleared, the selector self-drives to the release-alarm earth. If the rotary magnet (RM) is energized, but the selector fails to restore within 6-12 s, the RELEASE alarm operates to the rotary-magnet battery through the operated off-normal springs, N3. If the selector fails to release within 3-6 min due to a fault in the rotary-magnet circuit, the SUPERVISORY alarm operates from contacts N5 (operated) and BA8 (normal).

Q 5 (a) Explain, with the aid of a diagram, how the various frequencies are generated for a multi-channel voice-frequency system.

(b) How is the accuracy of the frequencies

- (i) controlled, and
- (ii) measured locally?

Q 6 Under what circumstances are the following required for an automatic error-corrected (ARQ) radio system:

- (a) alpha and beta signals,
- (b) a request-to-retransmit (RQ) signal,
- (c) a pulse-controlled automatic transmitter, and
- (d) a buffer store?

A 6 (a) and (b) See A8, Telegraphy C, 1971, Supplement, Vol. 65, p. 37, July 1972.

(c) An ARQ system is used for transmitting telegraph messages over high-frequency radio routes. The system converts each character of the transmitted message to a 7-unit code having a start-to-stop (space-to-mark) signal polarity ratio of 4 : 3. Mutilated characters can thus be detected and retransmitted on receipt of an RQ signal from the distant terminal. Two or three characters may have followed a faulty character before the RQ signal is received from the distant terminal, and the transmitting terminal therefore stores up to 4 characters to take account of the propagation time. As the store can become full when the radio route is incapable of carrying traffic (known as the cycling condition), a pulse-controlled automatic transmitter is used at the sending-subscriber's terminal of a private circuit. The pulses required to release each character from the transmitter are sent from the ARQ equipment over a separate line, and are arranged to take account of the propagation time between the ARQ station and the automatic transmitter.

(d) Similar conditions apply when the ARQ circuit is used for Telex traffic, except that it is not convenient to provide and operate a pulse-controlled automatic transmitter at every Telex station that may wish to use the radio route. In addition to the lack of a pulse-control wire on Telex circuits, many Telex subscribers would wish to use the teleprinter, not an automatic transmitter. For Telex, therefore, a buffer store is provided at or near the ARQ terminal. The store can hold 4000 characters, or more, and may use paper tape, a magnetic drum or a core store. The traffic from the subscriber's teleprinter or tape reader is fed directly into the store and is extracted from it under the control of release pulses from the ARQ equipment when the route is clear. The characters held in the store are transmitted at a rate of 411/min, compared with a maximum input rate of 400/min from a 50 baud Telex circuit. This ensures that the store can clear faster than it can fill after a period of cycling.

Q 7 (a) A manufacturer has offices and factories in various parts of the country. For the transmission of telegraph messages between the various properties, what are the advantages and disadvantages of using

- (i) the Telex network,
- (ii) a manual torn-tape message-relay system, and
- (iii) an automatic message-relay system?

(b) With the aid of a block diagram, explain the operation of a message-relay centre.

A 7 (a) (i) The Telex network offers subscriber-to-subscriber connexion through a common exchange or series of exchanges with selection by dial pulses. The advantages of this type of connexion are that the Telex machine can be used for sending and receiving messages from subscribers other than the parent company and, when a message has been transmitted, answer-back codes can be exchanged, giving assurance that the distant machine is operating correctly.

Telex machines can be used in a "conversational" mode, with questions being answered immediately.

The disadvantages of using Telex are that charges can be high if many calls are made to one destination, and time can be wasted in obtaining a connexion by dialling or by calling a subscriber who is frequently engaged.

(ii) The customer would prefer a message-relay system when this offered a reduction in cost for the volume of traffic, the length of the routes and the facilities required. The private network normally used consists of central relay centres and a teleprinter at each outstation. Messages are sent to the nearest centre, where they are routed to their destination independently. The centre-to-centre circuits are operated in the duplex mode so that messages can be sent in both directions simultaneously. Queueing of messages at relay centres ensures that they reach their destination in order of reception, with no waiting for engaged lines or subscribers, although urgent messages can be given priority. Broadcast messages can also be transmitted. A manual relay system requires operators for the relay centres, and this type of labour may be costly or scarce, depending on the location.

(iii) An automatic message-relay system operates in a similar manner, and has similar advantages to a manual system, with the exception that no operators are employed.

To ensure that each message is processed correctly, a rigid format is required for the preamble. This, in turn, requires that the originator of each message has to pay attention to detail. The relay equipment is expensive to provide, costly to maintain and may require some manual effort to deal with lost, mutilated or incorrectly formulated messages.

(b) For a description of an automatic message-relay centre, see A7, Telegraphy C, 1971, Supplement, Vol. 65, p. 36, July 1972.

For a description of a manual message-relay centre, see A5, Telegraphy C, 1974, Supplement, Vol. 68, p. 69, Oct. 1975.

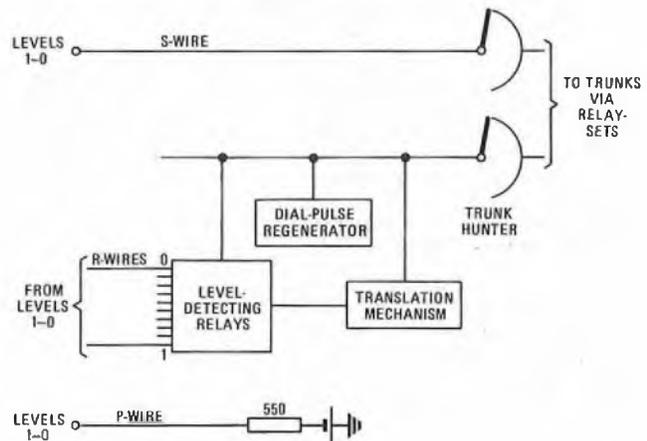
Q 8 (a) Why is a dial-pulse regenerator required as part of a routing translator?

(b) With the aid of a block diagram, explain the operation of a routing translator used in an automatic Telex exchange.

(c) What are the ideal characteristics of Telex dial pulses?

A 8 (a) A routing translator is used to effect economies in trunk circuits by routing calls for lightly-loaded routes over the main trunk routes. The translator can send a translation of up to 3 digits to route a call through a distant zone exchange, but, while the routing digits are being sent, the selection digits continue to be received from the calling subscriber. These are stored in the routing translator and transmitted after the routing digits. For this reason, a dial-pulse regenerator is required as part of a routing translator.

(b) The sketch shows a block diagram of a routing translator. Access to routing translators is obtained from the levels of first group-selectors, because the first digit that the calling subscriber dials indicates the zone centre or London centre to which the call is to be routed. Each translator is served from levels 1-0 of a section of first group-selectors. The S-wires from each outlet are commoned, as are the P-wires, but the R-wires are connected individually to level-detecting relays in the translator to indicate the level from which the call has originated. The first group-selector hunts for and selects the first free translator on the dialled level by detecting the 550 Ω negative potential on the P-wire. The level-detecting relays mark certain contacts on the bank of a uniselector within the routing translator, and the uniselector controls the transmission of the necessary translation digits. A separate trunk hunter, associated with the group of trunks concerned, is energized, a free trunk circuit is seized and the translation digits are transmitted to the distant exchange. The ±80 V dial pulses are generated by a pulse-generating relay, and have a break-to-make pulse ratio within the limits 55-66% : 45-34%, with 9-11 pulses/s. The total number of pulses generated is controlled by the uniselector, which steps once to each pulse until the marked bank contact that indicates the translation is reached.



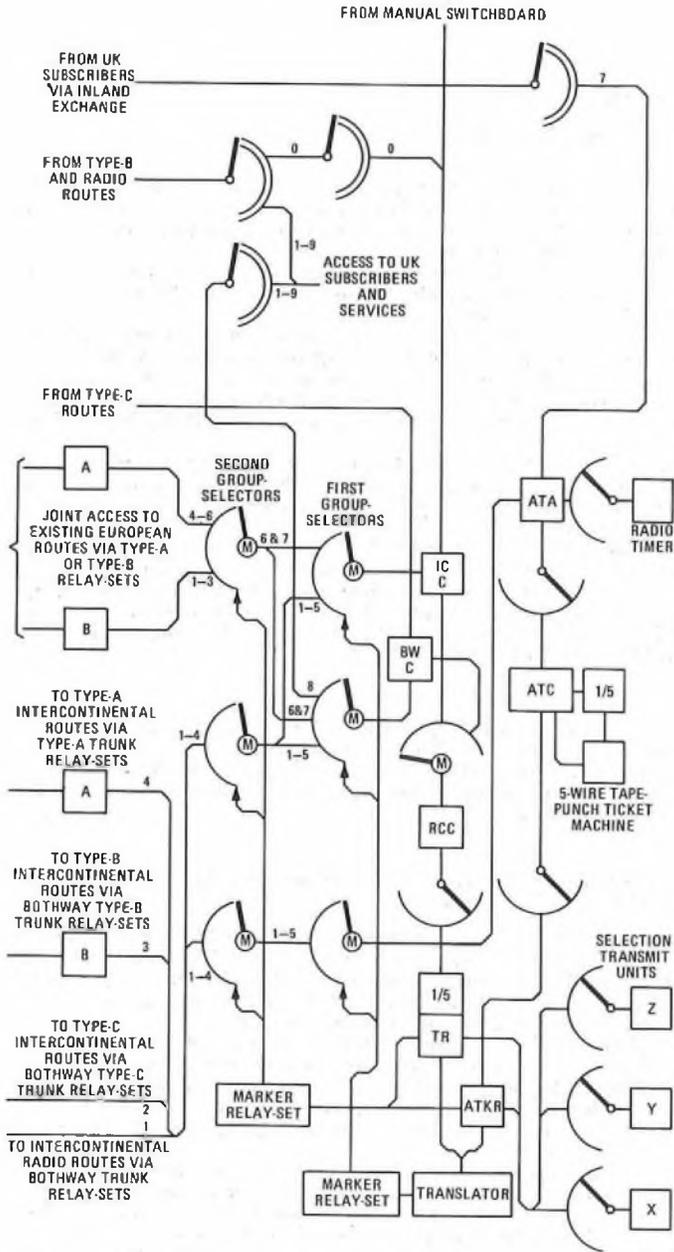
The called-subscriber's selection digits are stored in the pulse regenerator by moving a ratchet wheel once for each pulse received. During the inter-digital pause, the number of teeth through which the wheel has turned is marked by the depression of a pin; the movement of the wheel also winds a spring for the return motion. When the selection digits are to be transmitted, the wheel is allowed to turn by releasing the first pin, causing a dial-type mechanism controlled by a governor to make and break contacts to transmit the ±80 V dial pulses. The number of pulses transmitted is determined by the distance to the next depressed pin, which is then released after an inter-digital pause so that the next pulse train is transmitted.

(c) Ideally, Telex dial pulses have a rate of 10 pulses/s (that is, 100 ms per impulse), with each impulse consisting of the connexion of +80 V to line for 60 ms and -80 V for 40 ms (that is, a 60% : 40% break-to-make ratio). The limits are 9-11 pulses/s, with a break-to-make ratio of 1.2 : 1 to 1.9 : 1 (equivalent to 55% : 45% to 66% : 34%). The inter-digital pause should not be less than 600 ms.

Q 9 (a) Draw a trunking diagram, and explain the operation, of an exchange for switching intercontinental Telex calls.

(b) What arrangements are made for charging a subscriber for an outgoing call through this exchange?

A 9 (a) The sketch shows the trunking diagram of an exchange suitable for switching intercontinental calls. Inland Telex subscribers gain access to the exchange and to the automatic-ticketing equipment by dialling 20 followed by the digit 7; the last digit prevents the time-zone equipment from operating the subscriber's meter. The incoming call seizes an automatic-ticketing-access relay-set, which contains timing equipment and is permanently associated with a motor-uniselector first-group-selector. The access relay-set seizes an automatic-ticketing-control relay-set which, in turn, is connected to a



Note: Numerals denote levels for 2-motion selectors and outlet groups for motor-uniselectors

A and B: Type-A and type-B conversion relay-sets
 ATA: Automatic-ticketing-access relay-set
 ATC: Automatic-ticketing-control relay-set
 ATKR: Automatic-ticketing-keyboard register
 BW C: Bothway type-C trunk relay-set
 IC C: Incoming type-C access relay-set
 1/5: 1-5-wire converter
 RCC: Register-connect circuit
 TR: Transit register

1-5-wire converter associated with a 5-wire tape-punch ticket machine. Signals are exchanged between the equipment and the subscriber to record the subscriber and equipment identities, and the subscriber keys the selection digits into the automatic-ticketing keyboard register in the form of telegraph characters. The selection digits are supplied to the translator, which marks the first group-selector directly and provides the register with information to mark the second stage to give access to the required route. One of the selection transmit units, X, Y or Z, is associated with the outgoing call to give the correct selection conditions for the chosen route.

For transit traffic and terminal incoming traffic, bothway type-C trunk relay-sets are used, while incoming traffic from type-B and radio routes uses incoming type-C access relay-sets. Both types of relay-set gain access to the translator through the register-connect circuit, a 1-5-wire converter and transit register. The first and second group-selector stages are marked as before. As most of the traffic from type-C routes is for inland destinations, access to the inland network is given from the first-group-selector stage.

The manual board has access to the exchange through the incoming type-C access relay-set.

(b) See A4, Telegraphy C, 1972, Supplement, Vol. 66, p. 55, Oct. 1973.

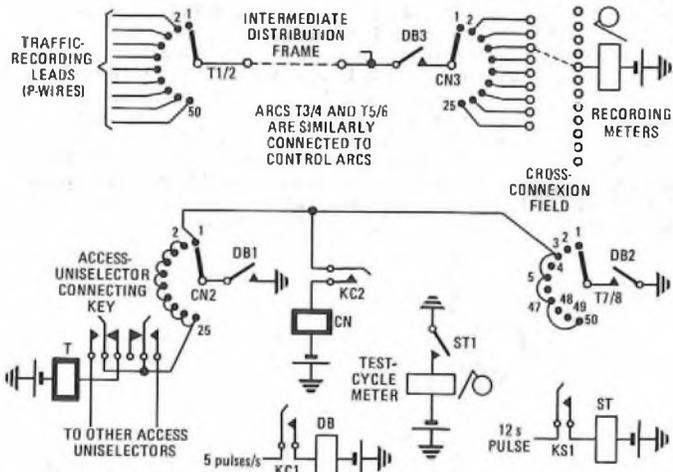
Q 10 (a) Why are the traffic recorders provided in a Telex exchange?

(b) With the aid of a diagram, explain the principles of operation of an automatic traffic-recorder.

(c) List other totalling meters provided in a Telex exchange to record effective and ineffective calls.

A 10 (a) Traffic recorders are provided in a Telex exchange to measure the amount of traffic carried by groups of automatic switching equipment and trunk circuits. By comparing the results with traffic-capacity tables for standard and critical grades of service, the amount of equipment or number of circuits needed to cater for future growth can be determined. Traffic is measured by recording the average number of trunks engaged during any given period. By definition, this is the traffic, in erlangs, carried by that group of circuits. The recorder can be connected to examine all the exchange equipment simultaneously, or used on one section at a time, depending on the size of the exchange. Records are normally taken at 6-monthly intervals.

(b) The sketch shows the principles of a traffic recorder. The recorder consists of access uniselector T, control uniselector CN, common equipment and recording meters. Access uniselectors are located close to the exchange equipment to which they are connected, such as group selectors, final selectors, and level and trunk relay-sets, to economize on cable provision; the rest of the recording equipment is mounted on a traffic-recorder rack. By pairing arcs, the access uniselectors are given 50 outlets. The access uniselector required is connected under key control.



When the traffic recorder is started by operating keys KS and KC, relay ST operates to the next 12 s pulse and relay DB operates and releases to the 5 pulses/s pulse train. At the first pulse, contact DB1 causes the control uniselector to step to contact 2. At the second pulse, contact DB1 energizes the access uniselector, which steps to its second contact.

At the third pulse, contact DB3 connects the traffic-recording leads on the second contacts of arcs T1/2, T3/4 and T5/6 to 3 meters, one on each of 3 arcs of uniselector CN. If any of the leads are connected to an engaged switch, earth potential from the switch operates the meter associated with that lead. At the termination of the third pulse,

relay DB releases, all meters release, and the access unselector steps to the third contact. The next operation of relay DB operates unselectors T and CN, and the testing operation is repeated over the remaining access leads in a similar manner.

At the conclusion of the test cycle, unselector T steps from its fiftieth contact to its first contact, while unselector CN steps from contact 25 to contact 1. As the access unselector step 5 times each second, the whole operation occupies 10 s, so that the next 12 s pulse re-starts the test cycle.

Each recording meter thus records the traffic carried by at least 2 of the items of equipment under test, and a cross-connection field enables connection of the meters to several contacts of the control unselector, depending on the type of record required. The access unselectors are connected to the main equipment through a jumper-field, to give flexibility.

(c) Other totalling meters provided in a Telex exchange are

(i) station-line-group overflow meters, which provide a record of the number of times an incoming call fails to mature because the called-subscriber's line or lines are engaged,

(ii) late-choice meters, which are normally connected to the last contact of a unselector bank to record the number of calls carried by the last trunk; the meter reading gives some indication of when the amount of traffic carried by the grading is approaching a critical value for the grade of service,

(iii) overflow meters, which operate on the eleventh step of a group-selector level or routing-translator-hunter arc for each single or collective demand for the service signal NC (no circuit),

(iv) international-trunk meters, which operate on each trunk circuit; 2 meters are provided, one to indicate the number of calls made over the circuit and the other to indicate the total duration, and the information thus given is used for international accounting, and

(v) total-call meters, which are provided to record the total number of call attempts on the Telex network; the meters are operated by group-selector relay contacts, with up to one rack of 60 selectors sharing one meter.

Meters are also provided to record the number of calls made to ceased or changed-number lines.

TELEPHONY C, 1975

Students were expected to answer any 6 questions

Q 1 (a) With the aid of a circuit diagram, explain how a transistor can be used to operate a relay.

(b) What circuit precautions are necessary to safeguard the transistor?

(c) In what circumstances would this method of operating a relay be used? Give a typical example.

A 1 (a) Sketch (a) shows a simple circuit for operating a relay using a transistor.

With resistor R2 disconnected and the base at earth potential, the relay is normal, because the only current flowing through it is the collector leakage current, and this is much less than 0.1 mA. If resistor R2 is connected, base current I_B flows to turn on the transistor and, hence, the relay operates.

Normally, the transistor is operated in a saturated condition; that is, virtually all the collector supply voltage is developed across the relay. Hence, if the supply voltage is 50 V and the d.c. resistance of the relay is 2.5 k Ω , the collector current, I_C , is given by

$$I_C = 50/2500 \text{ A} = 20 \text{ mA.}$$

If the current gain of the transistor is, for example, 40, the minimum value of I_B required to saturate the transistor is $20 \times 10^{-3}/40 \text{ A} = 0.5 \text{ mA}$. Hence, the value of resistor R2 (ignoring the effect of resistor R1) is $50/(0.5 \times 10^{-3}) \Omega = 100 \text{ k}\Omega$.

(b) Referring again to sketch (a), it can be seen that a diode is connected across the relay coil. This is necessary to protect the transistor when it turns off, since a high voltage is induced across the relay when the current through it collapses. If this voltage were greater than the maximum collector voltage allowed for the transistor, the transistor could be destroyed. However, the induced voltage across the relay tends to make the collector side of the relay negative and, when this voltage exceeds (that is, is more negative than) the -50 V supply voltage, the diode becomes forward-biased and clamps the collector voltage at a value that cannot exceed -50 V. It should be noted that a resistor-capacitor quench circuit could be used in place of the diode. Such an arrangement has the advantage of not increasing the release lag of the relay as much as does a diode.

(c) Sketch (b) shows a simple line-monitoring unit that could be used for the detection of dialling on subscribers' lines. It is designed to detect *make* and *break* conditions without degrading the line by drawing a lot of current from it.

With a dial *break* pulse, or open loop, on the line, point X is at earth potential. Consequently, transistors TR1 and TR2 are switched off and relay P is normal.

With a dial *make* pulse, or loop, on the line, the potential at point X falls to some negative value between approximately 5–25 V, depending on the line resistance and the coil resistance of relay A. Transistor TR1 turns on, turning on transistor TR2 after a short delay depending on the values of resistor R2 and capacitor C1. Relay P therefore operates.

Relay P is thus operated when relay A is operated, and released when relay A is released. A contact of relay P can therefore be used to monitor the number dialled by a subscriber.

Resistor R1 and diode D1 give the monitoring circuit a high input impedance to prevent slugging of relay A, with a consequent increase in its release time. Resistor R2 and capacitor C1 prevent misoperation of the monitor, due to spurious line conditions of short duration, by slowing down the turn-on time of transistor TR2. Resistor R3 is the collector load for transistor TR1, and diode D2 protects transistor TR2 from the back-e.m.f. of relay P.

Q 2 (a) Draw a trunking diagram of a 600-line Strowger-type unit automatic exchange (UAX) with STD facilities.

(b) Describe the sequence of operations on an STD call.

(c) Explain the arrangements made in the trunking to economize in junction provision.

A 2 (a) The sketch shows the trunking diagram of a typical 600-line UAX with STD facilities.

(b) When a subscriber originates a call, the line circuit sends a *start* signal to the allotter, which allocates a line finder and first selector to the call. The line finder finds the calling line and extends the subscriber's loop to the first selector, which returns dial tone.

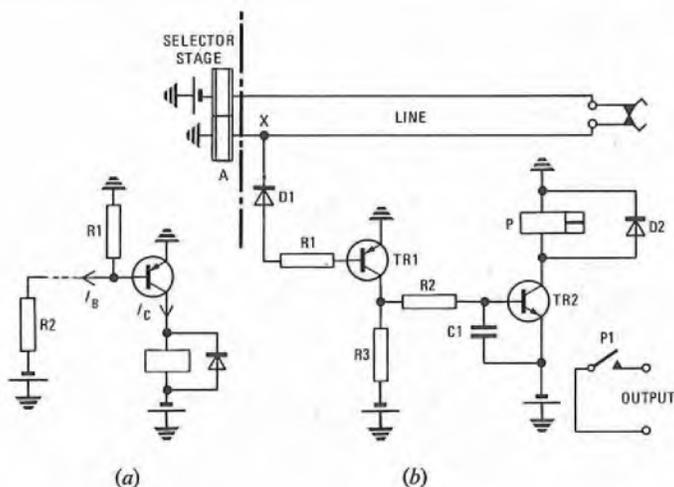
The subscriber dials 0 to step the first selector to level 0, and a level-1-and-0 relay-set is seized. When the relay-set detects the first *break* pulse after the digit 0, the associated junction hunter hunts for a free junction to the group switching centre (GSC). A first selector at the GSC is seized, and the relay-set sends forward the digit 0 to step the first selector, which seizes a controlling register via a register-access relay-set.

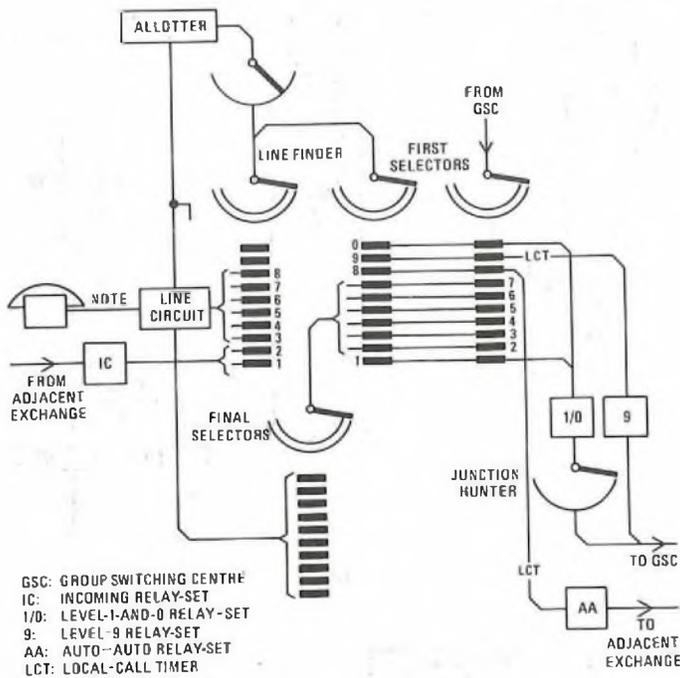
The level-1-and-0 relay-set has received from the line finder a class-of-service discrimination signal which indicates whether the call has originated from an ordinary subscriber or a coin-collecting box. The level-1-and-0 relay-set forwards a class-of-service signal, in the form of a digit in the range 1–6, to the register-access relay-set at the GSC, and this digit indicates

- the charging group whence the call has originated, and
- whether the call is from an ordinary or coin-collecting-box line.

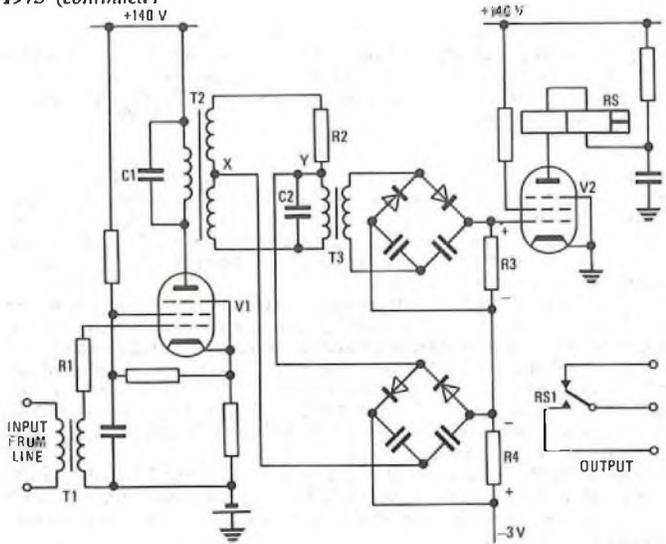
The level-1-and-0 relay-set then repeats forward the rest of the subscriber's dialled digits to enable the controlling register to set up the call through the network.

When the called subscriber answers and conversation is established, periodic metering-over-junction signals are received by the level-1-





Note: Via coin-and-fee-checking circuit for coin-collecting-box lines

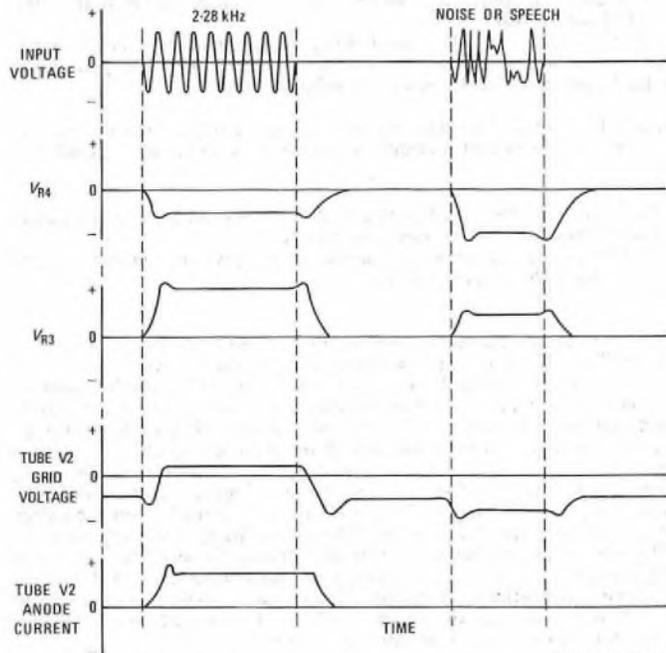


(a)

former T3 and the guard output is taken from the balance points, X and Y, of the bridge. When only a 2.28 kHz signal frequency is received, the bridge is approximately balanced, giving maximum signal output across transformer T3, and minimum guard output across points X and Y. When speech or other signals are received, the bridge is unbalanced for all frequencies apart from any 2.28 kHz element and, under these conditions, the signal output across transformer T3 is a minimum, and the guard output across points X and Y is a maximum.

The signal and guard outputs are both rectified by voltage-doubling circuits, one providing a d.c. signal output across resistor R3, and the other providing a d.c. guard output across resistor R4, the outputs being of opposite polarity. These opposing voltages are superimposed on the -3 V grid-bias voltage of tube V2, so that the voltage at the grid of tube V2 at any instant is $-3 + V_{R3} - V_{R4}$ volts, where V_{R3} and V_{R4} are the voltages across resistors R3 and R4 respectively.

When 2.28 kHz only is received, V_{R3} is sufficient to overcome the -3 V bias and any guard voltage developed across resistor R4, so that the grid of tube V2 is driven positive and relay RS operates. When speech signals are received, V_{R4} is sufficient to overcome any voltage developed across resistor R3; hence, the grid voltage at tube V2 remains at -3 V or below, and relay RS is prevented from false operation.



(b)

The waveforms at various points in the circuit during the receipt of signals are shown in sketch (b).

and-0 relay-set, which converts them into a form suitable for operating the calling-subscriber's meter.

(c) To economize in junction provision, all originating traffic from the UAX to the GSC (that is, from levels 1,9 and 0 of the first selector) is carried on a common group of junctions, rather than as segregated groups. This gives the same grade of service as for segregated groups of circuits, but uses fewer junctions. Arrangements are made within the level-1-and-0 and level-9 relay-sets to bar certain codes so that only calls for which the subscriber has dialled an authorized code are accepted.

It should also be noted that the UAX register-access relay-sets at the GSC can be common to all UAXs served by the GSC, irrespective of the originating charging group.

Q 3 (a) Give typical grades of service adopted for the provision of internal switching stages and external trunk routes, and explain any dissimilarity.

(b) A route accessed from 20-availability switches is re-trunked from a 40-availability switching stage. If the number of circuits in the route remains the same, what is the effect on the traffic-carrying capacity of the route and why?

(c) Explain the relative effect on the grade of service of traffic overload on large and small groups of trunks.

Q 4 (a) Describe 2 of the special features of a relay used for receiving d.c. pulses from a subscriber's dial.

(b) How is the relay performance affected by the characteristics of the subscriber's line?

(c) What features of an electromechanical selector determine the limits of acceptable pulse distortion?

Q 5 Sketch and describe a circuit element suitable for receiving voice-frequency signalling pulses and converting them to d.c. signals. Include the arrangements for securing speech immunity, and fully explain the principle of operation. Illustrate your answer with waveform sketches.

A 5 Sketch (a) shows the basic elements of a single-voice-frequency-signal receiver operating at 2.28 kHz.

Tone (and speech) signals from the line are fed through transformer T1 to the grid circuit of tube V1, which functions as an amplifier-limiter. The operating bias of tube V1, together with the voltage drop across resistor R1, provides balanced limiting of the negative and positive half-cycles of the line signal, so that the output from transformer T2 is approximately constant over a wide range of input signal levels.

The primary winding of transformer T2 in the anode circuit of tube V1 is broadly tuned to 1.4 kHz by capacitor C1 to emphasize the guarding frequencies relative to any 2.28 kHz element in speech. The amplified line signals are applied to a bridge circuit consisting of the 2 halves of the secondary winding of transformer T2, resistor R2, and the primary winding of transformer T3, which is tuned to the 2.28 kHz signal-frequency by capacitor C2.

The signal output is taken from the secondary winding of trans-

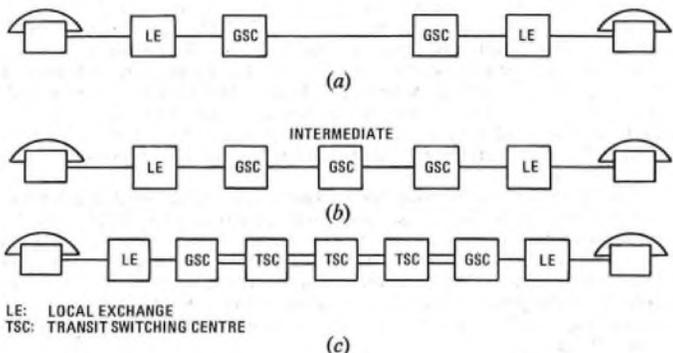
- Q 6 (a)** What is the purpose of a national numbering scheme?
(b) Describe, with the aid of suitable diagrams, 3 ways in which STD calls can be routed between 2 group switching centres (GSCs).
(c) State 3 factors which limit the number of line links that can be connected in tandem.

A 6 (a) The purpose of a national numbering scheme is to provide each telephone subscriber within a country with a unique telephone number, so that any subscriber can gain access to any other subscriber within the national numbering scheme by dialling just the wanted-subscriber's national number, without having to refer to local dialling-code instructions.

(b) Sketch (a) shows a call routed between 2 GSCs over a direct single-link trunk or junction circuit. Such circuits are provided where the level of traffic is sufficient to justify a direct GSC-GSC route.

Sketch (b) shows a call routed between 2 GSCs via an intermediate GSC. This type of routing occurs where the level of traffic is insufficient to justify a direct route, but is sufficient to justify a tandem routing via an intermediate GSC, provided there is sufficient switching capacity at the intermediate GSC.

Sketch (c) shows a call routed between 2 GSCs via 3 transit switching centres and the transit network. This type of routing is provided where the level of traffic is insufficient to justify even a tandem routing, as in sketch (b).



(c) In the UK, the number of links that can be connected in tandem is limited to 2 for the following reasons.

(i) The controlling register-translator equipment may not be capable of sending sufficient routing digits to route the call over more than 2 links.

(ii) Three or more links in tandem can unacceptably reduce the level of transmission.

(iii) An unacceptable post-dialling delay (the delay between the end of dialling and the reception of a supervisory tone) could be introduced, particularly on long routings.

Note: The above comments apply only to Strowger routings; on the 4-wire switched transit network, up to 5 links in tandem are possible.

- Q 7 (a)** With the aid of sketches, describe the principle of operation of the subscriber's private metering system.

- (b)** What is the order of magnitude of the operating current required for the subscriber's private meter?

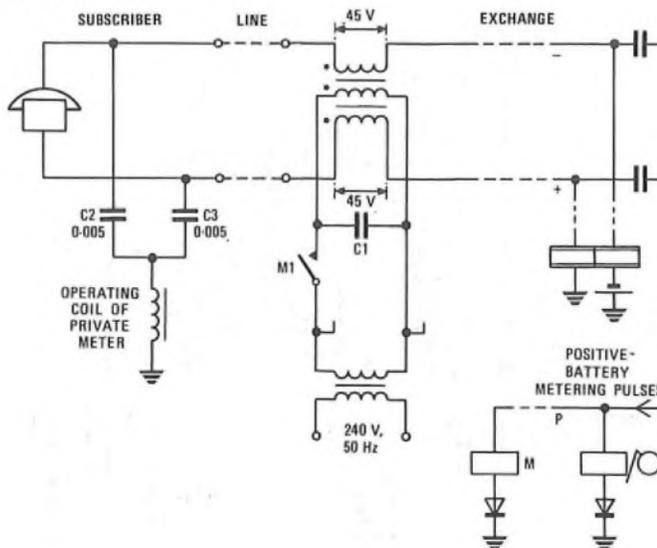
A 7 (a) Sketch (a) shows the circuit elements involved in remotely controlling the operation of a subscriber's private meter.

Relay M is operated in parallel with the subscriber's ordinary exchange meter for a nominal period of 250 ms each time a periodic metering pulse is received. Contact M1 causes a longitudinal voltage of 45 V at 50 Hz to be induced into the phantom circuit of the subscriber's line. The resulting current in the phantom-circuit path, from earth at the exchange transmission bridge to earth at the subscriber's premises, via the line, capacitors C2 and C3, and the meter coil, causes the meter to operate. The meter circuit is sharply tuned to 50 Hz to avoid operation at exchange ringing frequencies, and has a high impedance to ensure minimum shunt transmission loss and dial-pulse distortion. Capacitor C1 and the primary winding of the injection transformer form a parallel tuned circuit at 50 Hz, so that minimum current is drawn from the supply.

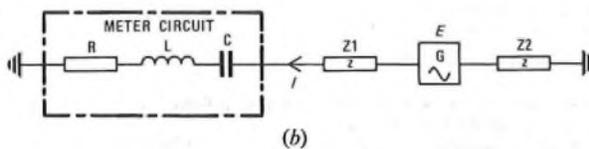
(b) Sketch (b) shows the approximate equivalent circuit for the meter.

Inductor L and capacitor C form a series tuned circuit with a resonant frequency, f_0 , of 50 Hz.

Now, $C = C_2 + C_3 = 0.01 \mu\text{F}$.



(a)



(b)

At the resonant frequency,

$$L = \frac{1}{(2\pi f_0)^2 C} = \frac{1}{(100\pi)^2 \times 0.01 \times 10^{-6}} \text{H} \approx 1 \text{ kH}$$

Resistor R is the d.c. resistance of the meter's operating coil, and is of the order of 40 kΩ. Impedances Z1 and Z2 are the line and exchange impedances respectively, and probably together do not exceed 2 kΩ. The supply voltage, E, is 45 V r.m.s. at 50 Hz.

Thus, ideally, the meter operating current, I amperes, is given by

$$I = \frac{E}{R + Z_1 + Z_2} \text{ amperes,}$$

since L and C together have no impedance at resonance. Therefore, assuming Z1 and Z2 to be resistive,

$$I = \frac{45}{40 \times 10^3 + 2 \times 10^3} \text{ A} \approx 1 \text{ mA}$$

Thus, the operating current for the meter is approximately 1 mA; that is, 0.5 mA in each wire of the subscriber's pair.

- Q 8 (a)** With the aid of a block diagram, describe the principle of operation of a cordless switchboard system.

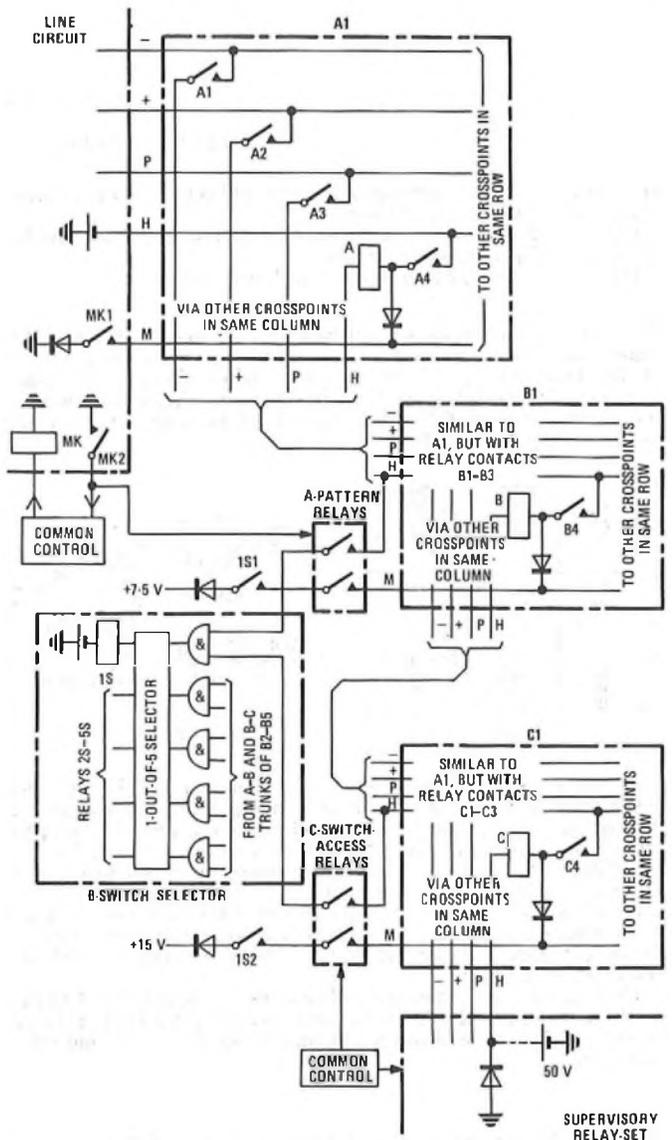
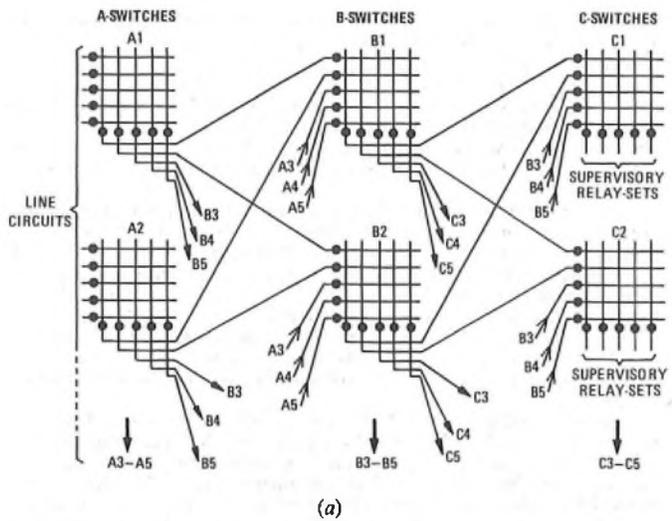
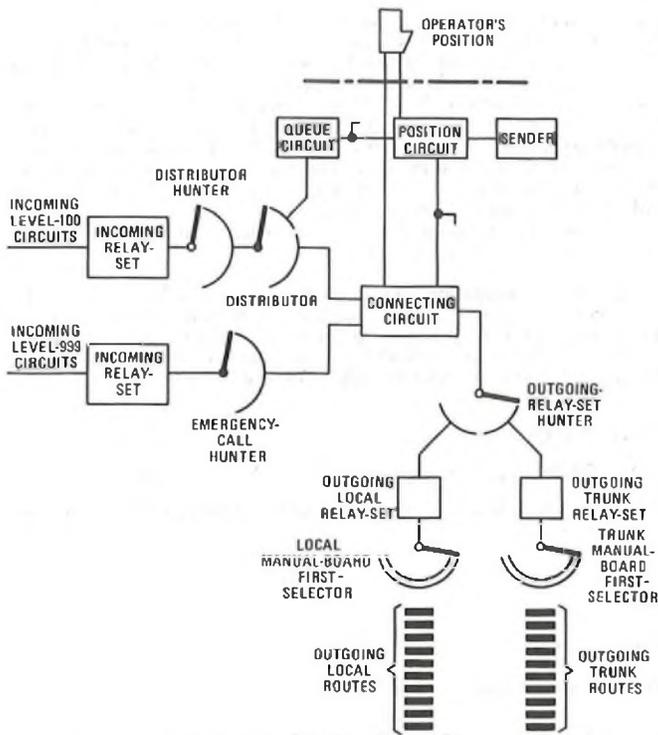
- (b)** What are the main advantages of this system over cord-type switchboards?

A 8 (a) The sketch shows a block diagram of a typical cordless switchboard system.

An incoming assistance call (level 100) seizes an incoming relay-set, and the associated distributor hunter finds and allocates a free distributor. The distributor searches for and seizes the next free place in the queue and the caller receives ringing tone. The queue circuit moves the call progressively along the queue as the preceding calls are answered and, when the call reaches the head of the queue, the distributor hunts round the bank to a connecting-circuit contact marked by the operation of a connecting-circuit SPEAK key and the CONNECT-ANSWER bar by the operator. Ringing tone is disconnected, and the caller is connected to the operator via the connecting circuit.

Emergency calls (level 999) bypass the normal queueing arrangements so that they are not delayed by calls of lower priority. A visual indication and audible alarm is given in the switchroom, and the first operator at one of the nominated emergency positions to operate a free connecting-circuit SPEAK key and depress the CONNECT-ANSWER bar receives the emergency call.

When the caller's requirements are known, the operator depresses one of 4 OUTGOING-ROUTE keys: LOCAL-CODE DIALLING, LOCAL DIRECTOR,



TRUNK-CODE DIALING or TRUNK CONTROLLING REGISTER. This causes coded signals to be sent to the position circuit, which results in seizure of the position sender, seizure of an outgoing local or trunk relay-set via the outgoing relay-set hunter and, in the case of a routing using a director or controlling register, stepping of the local or trunk first selector to the required level by a predetermined prefix digit to gain access to the director or register-translator equipment. After the required digits have been dialed, the FINISH key is depressed, and the sender pulses out the stored digits before releasing from the connecting circuit, which then gives normal supervisory indications to the operator throughout the call.

A call timer is associated with each connecting circuit, and starts automatically, when required, on receipt of called-subscriber-answer conditions.

At the completion of a call, the operator resets the call timer, if necessary, and, by operating the appropriate RELEASE keys, restores the connecting circuit and releases the call from the position.

(b) The main advantages of a cordless switchboard system are that

- (i) call queuing is used, and this ensures that calls are dealt with in chronological order,
- (ii) access to outgoing routes is gained via selectors and not via a switchboard multiple,
- (iii) the speech path is set up via automatic equipment instead of cord circuits, thus reducing the fault liability,
- (iv) the operators' positions can be housed in office-type accommodation, giving operators a much quieter and more pleasant environment in which to work, and
- (v) the operators' positions can be separate from the automatic equipment and, if required, be located in a different building.

Q 9 (a) Distinguish between preventive and corrective maintenance in an electromechanical exchange.

(b) Describe briefly the function and use of the principal classes of testing equipment used to facilitate maintenance in such an exchange.

(c) How do the test limits applied by the testing equipment relate to the operational requirements of the exchange equipment?

Q 10 With the aid of sketches of basic circuit elements, explain the principles of path selection, marking and holding used in setting-up a connexion in a TXE2 exchange between a subscriber's line circuit and the first-choice supervisory relay-set.

A 10 Sketch (a) shows how the A-switches, B-switches and C-switches within a TXE2 switching matrix are interconnected. For simplicity, switches with only 5 inlets and 5 outlets are shown.

Sketch (b) is an expansion of sketch (a), and shows the circuit elements within the switches concerned with marking, path selection and holding. Again, for simplicity, only inlet 1 and outlet 1 of switches A1, B1 and C1 are shown.

When the common-control equipment desires to set up a path between a subscriber and a supervisory relay-set, it operates relay MK in the subscriber's line circuit. Contact MK1 marks the position of the calling subscriber on the A-switch (in this case, switch A1, inlet 1); contact MK2 operates the A-pattern relays in readiness for the choice of a

B-switch and also signals to the common control, which selects and marks a free supervisory relay-set and C-switch, operating the C-switch-access relays.

The 5 A-B trunks of the calling A-switch and 5 B-C trunks of the chosen C-switch (in this case, C1) are examined by the B-switch selector, which attempts to find a free path between the A-switch and the selected C-switch. There are 5 possible paths between the line circuit and the chosen C-switch, and these are distributed one through each B-switch.

The A-pattern and C-switch-access relays extend the marking leads (M-wires) and holding leads (H-wires) to the B-switch selector, which then proceeds to match the pairs of A-B and B-C H-wires for simultaneous free conditions. The AND gates operate only if there are simultaneous free conditions on both H-wires of a matching pair, and the 1-out-of-5 selector chooses one of the free pairs. In this case, one of the inputs to each of the 5 AND gates comes from inlet 1 of switches B1-B5 respectively, and the other input comes from inlets 1-5 respectively of switch C1. This results in the operation of one of relays 1S-5S.

Assuming paths A1-B1 and B1-C1 both to be free, relay 1S is operated. Contacts 1S1 and 1S2 apply marking potentials to the selected B-switch and the chosen C-switch. The +15 V potential applied to the M-wire of the C-switch operates relay C to the +50 V potential on the H-wire from the supervisory relay-set. Contact C4 extends the +15 V potential to the H-wire of the B-C trunk, where it

operates relay B in the B-switch to the +7.5 V potential on the B-switch M-wire. Contact B4 extends the +7.5 V potential to the H-wire of the A-B trunk, where it operates relay A in the A-switch to earth via the A-switch M-wire and contact MK1.

Contact A4 replaces the marking earth on the A-switch M-wire by a -50 V potential on the H-wire, so that the initial end-to-end holding potential is 100 V. However, the +50 V potential in the supervisory relay-set is subsequently removed, leaving the crosspoint relays (A, B and C) holding in series to an earth in the supervisory relay-set and the -50 V potential in the line circuit.

The negative, positive and P-wires are thus extended from the line circuit to the supervisory relay-set.

Note: Under examination conditions, it would be sufficient merely to reproduce the path-selection, marking and holding elements of sketch (b) rather than draw the more complete diagrams shown above; these have been included here to give a fuller appreciation of the principles to those who are not familiar with them.

CORRECTION

TELEPHONY C, 1974 (Supplement, Vol. 68, Oct. 1975)

A 6 The TXE2 is a common-controlled exchange, not register-controlled as stated.

LINE TRANSMISSION C, 1975

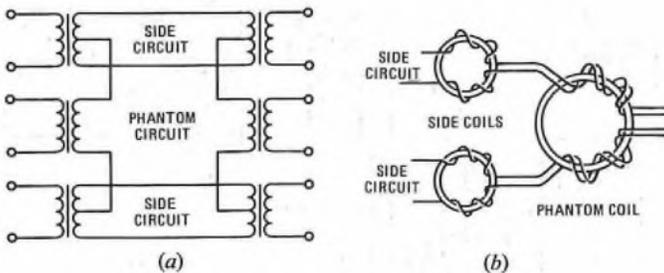
Students were expected to answer any 6 questions

Q 1 (a) Explain how transformers can be used to provide a superposed (phantom) circuit over 2 cable pairs.

(b) State the conditions which must be met to avoid overhearing between the phantom and side circuits.

(c) Explain how a phantom circuit can be coil-loaded.

A 1 (a) A phantom circuit can be provided over 2 cable pairs by using centre-tapped transformers, as shown in sketch (a). Each leg of the phantom circuit consists of the 2 wires of one of the pairs connected in parallel. Signals in the phantom circuit do not pass through the side-circuit transformers, but are cancelled by the action of the centre-tapped windings.



(b) To avoid overhearing, or *crosstalk*, between the phantom and side circuits, it is essential to ensure that the centre-taps on the line transformers are accurately made, and that the 2 wires of each pair are of equal impedance. If the phantom is to be used only for d.c. signalling, it does not need its own line transformers; otherwise, they are necessary for impedance matching.

(c) A specially-wound coil is used to load a phantom circuit. It has bifilar windings, thus presenting inductance to currents in the phantom circuit but appearing non-inductive to the side circuits. Sketch (b) shows the arrangements.

Usually, the phantom-circuit coils are put in the same loading pot as the side-circuit coils. Toroidal cores that have negligible external magnetic fields are used to prevent mutual interference by inductive coupling.

Q 2 A cable pair has a characteristic impedance of 600 Ω and, at a particular frequency, is one quarter of a wavelength long. The far end is short-circuited. Its transmission loss is 6 dB.

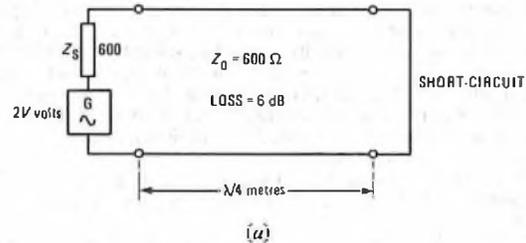
(a) Obtain, by phasor diagrams or otherwise,

- (i) the current, and
- (ii) the voltage

at the input to the line, assuming it to be fed from a matching source.

(b) Calculate the input impedance.

A 2 (a) Sketch (a) shows the circuit arrangement, assuming the source to generate a voltage of 2V volts at the frequency for which the length of the line, l metres, is λ/4 metres, where λ is the wavelength (metres).



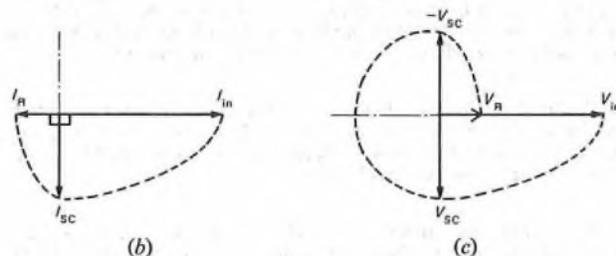
Sketch (b) shows the phasor diagram for the current. The input current, I_{in} amperes, is given by

$$I_{in} = 2V / (Z_S + Z_0) = V / 600 \text{ amperes,}$$

where Z_S is the source impedance (ohms), and Z_0 is the characteristic impedance (ohms).

On reaching the short-circuit, the phase of the current has changed by $-\beta l$ radians, where β is the phase-change coefficient, equal to $2\pi/\lambda$ radians/metre. The phase change is therefore $-\pi/2$ radians. Also, the magnitude of the current has been attenuated by 6 dB; that is, by a factor of 2. Hence, the current at the short-circuit, I_{SC} amperes, has a magnitude of $0.5V/600$ amperes.

No change of phase occurs at the short-circuit, and the current is reflected back to the sending end, undergoing a further phase-change of $-\pi/2$ radians and being again attenuated by a factor of 2 as it traverses the line. The current received at the sending end, I_R amperes, thus has a magnitude of $0.25V/600$ amperes and is in anti-phase with I_{in} .



Sketch (c) shows the phasor diagram for the voltage. Since $Z_S = Z_0$, the input voltage, V_{in} volts, is half the source voltage; that is, $V_{in} = V$ volts.

The voltage reaching the short-circuit, V_{SC} volts, is $0.5V$ volts with

a phase change of $-\pi/2$ radians. A change of phase of π radians occurs at the short-circuit (so that the total voltage is zero), and the voltage reflected back is therefore $-V_{SC}$ volts. Hence, the voltage received at the sending end, V_R volts, has a magnitude of $0.25V$ volts and is in phase with V_{in} .

(i) From sketch (b), the total current at the input, I_T amperes, is given by

$$I_T = I_n - I_R \text{ amperes,}$$

$$= \frac{V}{600} - \frac{0.25V}{600} \text{ amperes} = \underline{1.25V \text{ milliamperes.}}$$

(ii) From sketch (c), the total voltage at the input, V_T volts, is given by

$$V_T = V_{in} + V_R \text{ volts,}$$

$$= V + 0.25V = \underline{1.25V \text{ volts.}}$$

(b) The input impedance, Z_{in} ohms, is given by

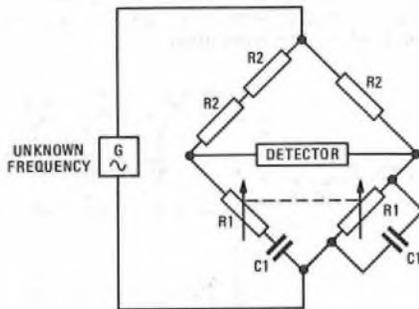
$$Z_{in} = \frac{V_T}{I_T} = \frac{1.25V}{1.25V \times 10^{-3}} \text{ ohms} = \underline{1 \text{ k}\Omega}.$$

Q 3 (a) Sketch a bridge arrangement by which audio frequencies can be measured.

(b) Derive an expression for the measured frequency in terms of the bridge components.

(c) List the precautions necessary to ensure accuracy of measurement.

A 3 (a) The basic circuit of a bridge suitable for the measurement of audio frequencies is shown in the sketch.



The fixed ratio arms have resistances of $2R_2$ ohms and R_2 ohms respectively. One of the variable ratio arms has a resistance of R_1 ohms in series with a capacitance of C_1 farads, and the other has 2 exactly similar components in parallel. Resistors R_1 are ganged variable resistors, which are adjusted for balance. The frequency range of the bridge can be altered by changing the value of capacitors C_1 . The detector is usually a telephone receiver.

(b) At balance,

$$\frac{2R_2}{R_1 + \frac{1}{j\omega C_1}} = \frac{R_2}{\frac{1}{R_1} + j\omega C_1}$$

$$\therefore \frac{2R_2}{\frac{1}{R_1} + j\omega C_1} = R_2 \left(R_1 + \frac{1}{j\omega C_1} \right)$$

$$\therefore \frac{2R_2}{R_2} = \left(R_1 + \frac{1}{j\omega C_1} \right) \left(\frac{1}{R_1} + j\omega C_1 \right)$$

$$\therefore \frac{2}{1} = 1 + \frac{1}{j\omega C_1 R_1} + j\omega C_1 R_1 + 1 \quad \dots \dots (1)$$

Equating imaginary parts gives

$$-\frac{1}{j\omega C_1 R_1} = j\omega C_1 R_1$$

$$\therefore \omega^2 C_1^2 R_1^2 = 1$$

Since $\omega = 2\pi f$ radians/second, where f is the frequency (hertz),

$$f = \frac{1}{2\pi C_1 R_1} \text{ hertz.}$$

The choice of the ratio 2 : 1 for the fixed arms can be explained by substituting X and Y for $2R_1$ and R_1 respectively in equation (1). Thus,

$$\frac{X}{Y} = 1 + \frac{1}{j\omega C_1 R_1} + j\omega C_1 R_1 + 1$$

Equating real parts gives

$$\frac{X}{Y} = 2$$

Since both real and imaginary parts must be equal at balance, X and Y must be in the ratio 2 : 1 for the bridge to be balanced.

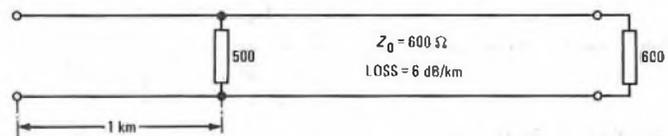
(c) To ensure the accuracy of the measurement, the following precautions must be taken:

- (i) all bridge components must be properly calibrated and of high quality,
- (ii) the fixed resistance arms must be in the correct ratio of 2 : 1,
- (iii) the variable resistance arms must be exactly in step,
- (iv) a layout that avoids stray capacitances must be used,
- (v) a balanced and screened transformer (not shown) must be inserted between the bridge terminals and the source of the frequency to be measured, and
- (vi) there should be a band-pass filter (not shown) after the source to eliminate the effect of harmonics.

Q 4 A cable pair has an attenuation of 6 dB/km at a particular frequency, and the characteristic impedance is 600Ω non-reactive. It is terminated in its characteristic impedance at the distant end. At a distance of 1 km from the near end, a fault has altered the impedance at that point to 500Ω non-reactive. Calculate

- (a) the reflection loss at the fault, and
- (b) the level of the reflected signal at the near end relative to that of the transmitted signal.

A 4 The sketch illustrates the circuit arrangement.



(a) The reflection loss is the ratio of the power dissipated in a load impedance that differs from the source impedance to the power that would be dissipated if the impedances were equal. It is given by

$$\text{reflection loss} = 20 \log_{10} \frac{2 \times \sqrt{(|Z_0| \times |Z_T|)}}{|Z_0 + Z_T|} \text{ decibels,}$$

where Z_0 is the characteristic impedance (ohms), and Z_T is the terminating loss (in this case, the fault impedance) (ohms).

$$\therefore \text{reflection loss} = 20 \log_{10} \frac{2 \times \sqrt{(600 \times 500)}}{600 + 500} \text{ dB,}$$

$$= 20 \log_{10} 0.9959 = 20 \times (-0.0018)$$

$$= \underline{-0.036 \text{ dB.}}$$

(b) The return loss is a measure of the power lost by a signal reflected from a terminating impedance, and is given by

$$\text{return loss} = 20 \log_{10} \left| \frac{Z_0 + Z_T}{Z_0 - Z_T} \right| \text{ decibels,}$$

$$= 20 \log_{10} \frac{600 + 500}{600 - 500} \text{ dB,}$$

$$= 20 \log_{10} 11 = 20 \times 1.0414 = 20.8 \text{ dB.}$$

The total loss of a signal reflected by the fault impedance back to the near end is the sum of the return loss and twice the cable attenuation for the distance between the near end and the fault. Thus, the total loss is

$$20.8 + 2 \times 6 = 32.8 \text{ dB,}$$

so that the reflected signal is $\underline{-32.8 \text{ dB}}$ relative to the transmitted signal.

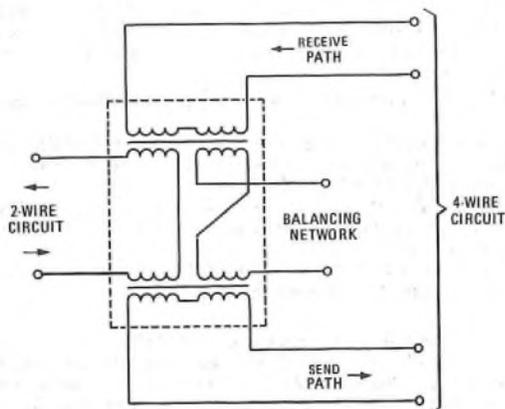
Q 5 (a) Show how the sending and receiving performance of a central-battery telephone set is related to the electrical characteristics of the local line.

(b) Explain the effect of sidetone on a speaking person.

A 5 See A5, Line Transmission C, 1971, Supplement, Vol. 65, p. 89, Jan. 1973.

- Q 6 (a) Sketch the circuit of a 2-wire-4-wire terminating set.
 (b) Explain its function when used in a repeatered 4-wire audio circuit.
 (c) Calculate the balance return-loss for a 2-wire line of impedance $700 \angle -30^\circ \Omega$ and a balancing resistor of 600Ω .

A 6 (a) The elements of a 2-wire-4-wire terminating unit are shown in the sketch. The balancing network is a combination of resistors and capacitors designed to match the impedance of an average 2-wire line. The transformers each have a ratio of 1 : 1.



The theoretical losses for various directions of transmission are given in the following table, assuming ideal transformers and perfect balance. In practice, these conditions cannot be met, and the table also shows the practical values.

Direction of Transmission	Transmission Loss (dB)	
	Theoretical	Practical
2-wire circuit to SEND path	3	4
2-wire circuit to RECEIVE path	3	4
2-wire circuit to balancing network	∞	Very high
RECEIVE path to 2-wire circuit	3	4
RECEIVE path to balancing network	3	4
RECEIVE path to SEND path	∞	Very high

- (b) A terminating unit combines the unidirectional SEND and RECEIVE paths of the amplified 4-wire section of a circuit with the bi-directional 2-wire section, with the minimum of attenuation in each direction of transmission. It also provides a very high-impedance (infinite, when the balance is perfect) path between the RECEIVE and SEND paths; this is necessary because the 4-wire loop would howl if the attenuation were low, unless the loop gain were severely restricted.
 (c) The balance return-loss is given by

$$\text{balance return-loss} = 20 \log_{10} \left| \frac{Z_0 + Z_B}{Z_0 - Z_B} \right| \text{ decibels,}$$

where Z_0 is the line impedance (ohms), and Z_B is the balancing impedance (ohms).

$$\begin{aligned} \text{Now, } Z_0 &= 700 \angle -30^\circ \Omega, \\ &= 700 \cos(-30^\circ) + j700 \sin(-30^\circ) \Omega, \\ &= 606 - j350 \Omega. \end{aligned}$$

$$\begin{aligned} \therefore \text{balance return-loss} &= 20 \log_{10} \left| \frac{606 - j350 + 600}{606 - j350 - 600} \right| \text{ dB,} \\ &= 20 \log_{10} \left| \frac{1206 - j350}{6 - j350} \right| \text{ dB,} \\ &= 20 \log_{10} \left| \frac{(1206 - j350)(6 + j350)}{6^2 + 350^2} \right| \text{ dB,} \end{aligned}$$

$$\begin{aligned} &= 20 \log_{10} \left| \frac{129\,736 + j420\,000}{122\,536} \right| \text{ dB,} \\ &= 20 \log_{10} |1.06 + j3.43| \text{ dB,} \\ &= 20 \log_{10} 3.59 \text{ dB,} \\ &= \underline{11.1 \text{ dB.}} \end{aligned}$$

- Q 7 (a) Draw block diagrams to show the power-supply arrangements for the transmission equipment in a repeater station.
 (b) Explain how interruption due to a short-term mains failure is avoided.
 (c) Outline the steps taken to prevent the power supply from interfering with the transmission circuits.

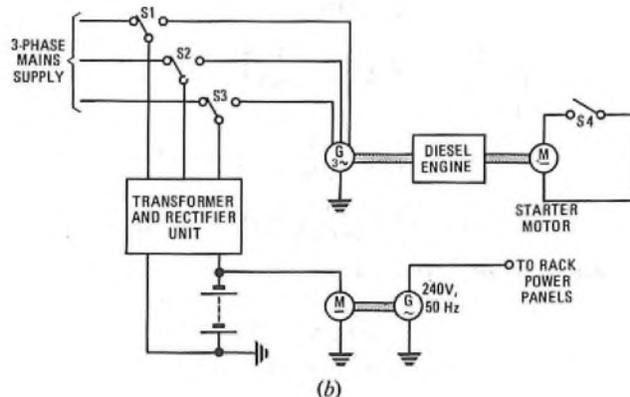
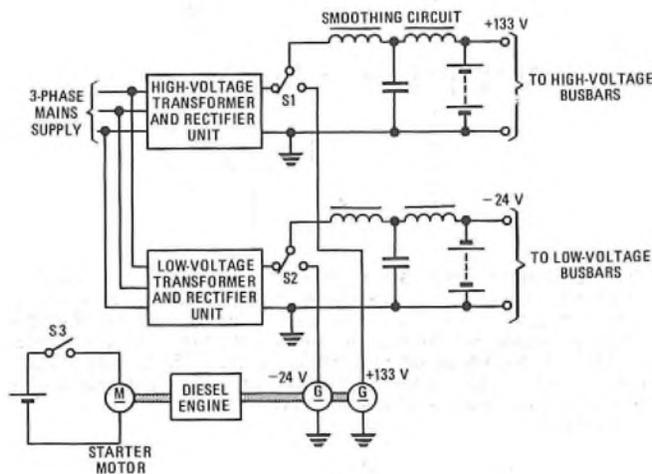
A 7 (a) and (b) The primary source of power is usually the mains supply, and there are 2 main methods of using this for repeater-station equipment:

- (i) by using batteries that are float-charged from the mains, and distributing to individual racks by busbars, and
 (ii) by feeding a mains-type supply to small power packs on individual racks.

The essential requirements are good regulation and absolute reliability. The former is fairly easy to achieve because the load is constant (in contrast to the load in a telephone exchange), but the latter requires some form of alternative supply in case of mains failure. The alternative supply is usually provided by a prime mover, such as an oil engine, coupled to an alternator or d.c. generators.

The examples given in this answer pertain to a repeater station having tube-operated transmission equipment.

A typical arrangement for method (i) is shown in sketch (a). In the event of mains failure, switch S3 operates to start the oil engine. No interruption occurs because the batteries continue to feed the equipment while the engine runs up to speed. Switches S1 and S2 then transfer the loads to the generators.



Sketch (b) shows a typical arrangement for method (ii). The individual power packs provide an a.c. heater supply and a high-voltage d.c. anode supply for the tubes on their racks. An alternator feeds

the individual power packs. This alternator is coupled to a d.c. motor driven by the rectified mains supply, backed up by a floated battery. In the event of mains failure, switch S4 operates to start the oil engine, and when the correct running speed is reached, switches S1, S2 and S3 operate so that the 3-phase alternator takes over the supply. No interruption occurs because the d.c. motor continues to run from the battery while the engine runs up to speed.

(c) Noise can enter the transmission circuits from the power supply if the smoothing arrangements are inadequate. With floated batteries, the smoothing must be sufficient to suppress armature ripple from the generators. With individual power-packs, the smoothing must be sufficient to suppress mains hum from the rectifiers.

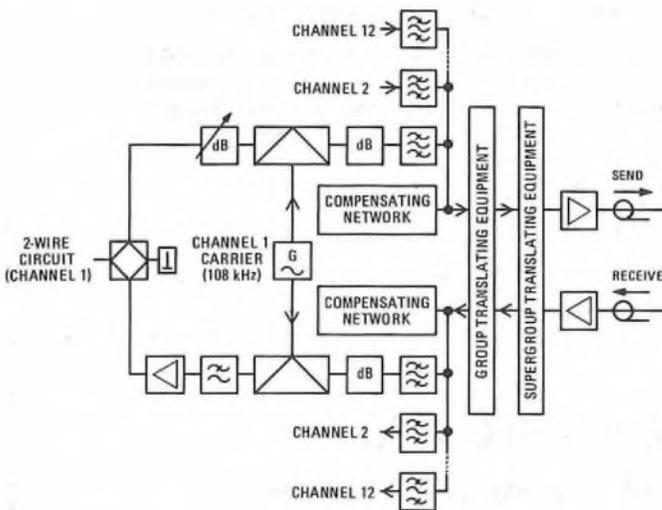
Mutual interference between transmission circuits can occur because of the use of a common supply, and this can be suppressed only by making the internal impedance of the supply very low. This requires a very small impedance at signal frequencies, which is usually achieved by the use of decoupling capacitors.

Q 8 (a) Draw a block diagram to show the essential features of a frequency-division-multiplex transmission system, as used on a coaxial cable.

(b) Explain the function of each block.

(c) Explain how the repeater spacing is determined on such a system.

A 8 (a) The essential features of a frequency-division-multiplex transmission system, over a coaxial cable, are shown in the sketch.



(b) The 2-wire circuit is connected to the 4-wire section by a 2-wire-4-wire terminating unit, which provides the interconnection with the minimum attenuation in each direction, while exhibiting a very high attenuation between the RECEIVE and SEND paths.

In the SEND direction, the signal passes through attenuators, which set the required transmission level, a channel modulator, which amplitude modulates the channel carrier, and a band-pass filter, which selects the lower sideband. The channel is then combined with 11 other channels to form a group. The group is passed through the group translating equipment, where it is modulated, filtered and combined with 4 other groups to form a supergroup.

The supergroup is similarly modulated, filtered and combined with other supergroups by the supergroup translating equipment, and is then passed to the SEND line via an amplifier.

Wideband amplifiers and equalizers compensate for losses in the cable sections along the route.

At the receiving end, the signals are demodulated in the reverse order; that is, the supergroups are selected by band-pass filters and demodulated, the groups are selected and demodulated, and the channels are selected and demodulated. The channel signals are passed to the 2-wire-4-wire termination unit via a low-pass filter and amplifier.

The compensating networks simulate the loading effects of filters for frequencies either side of the first and last channel carriers, so that all the band-pass filters have an equal loading.

(c) The spacing between repeaters depends partly on the cable attenuation and partly on the characteristics of the active devices within the repeater itself. The attenuation of the cable determines the gain of each repeater and, economically, the repeater spacings need to be as high as possible. However, the difference between a repeater's input and output levels must not be so great as to give a small signal-to-noise ratio. The spacing is therefore a compromise between these factors.

Q 9 (a) Show that the attenuation coefficient, α nepers/kilometre, of a high-frequency transmission line is given by the approximate expression

$$\alpha \approx \frac{R}{2} \sqrt{\frac{C}{L}} + \frac{G}{2} \sqrt{\frac{L}{C}} \text{ nepers/kilometre.}$$

(b) Explain how and why the attenuation of a coaxial pair varies with frequency.

A 9 (a) For a uniform transmission line with a loop resistance of R ohms/kilometre, a loop inductance of L henrys/kilometre, a loop capacitance of C farads/kilometre, and a loop leakage of G siemens/kilometre, the propagation coefficient, γ , is given by

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)},$$

where β is the phase-change coefficient (radians/kilometre), and ω is the angular frequency (radians/second).

$$\text{Now, } R + j\omega L = j\omega L \left(\frac{R}{j\omega L} + 1 \right),$$

$$\text{and } G + j\omega C = j\omega C \left(\frac{G}{j\omega C} + 1 \right).$$

$$\therefore (R + j\omega L)(G + j\omega C) = j\omega L j\omega C \left(1 + \frac{R}{j\omega L} + \frac{G}{j\omega C} - \frac{RG}{\omega^2 LC} \right).$$

$$\therefore \gamma = j\omega \sqrt{LC} \left(1 + \frac{R}{j\omega L} + \frac{G}{j\omega C} - \frac{RG}{\omega^2 LC} \right)^{1/2}.$$

At high frequencies, the term containing ω^2 in the denominator can be neglected. Thus, using the binomial expansion and ignoring terms containing ω^2 and higher powers of ω in the denominator,

$$\begin{aligned} \gamma &\approx j\omega \sqrt{LC} \left\{ 1 + \frac{1}{2} \left(\frac{R}{j\omega L} + \frac{G}{j\omega C} \right) \right\}, \\ &= j\omega \sqrt{LC} + \frac{R}{2} \sqrt{\frac{C}{L}} + \frac{G}{2} \sqrt{\frac{L}{C}}. \end{aligned}$$

Now, $\gamma = \alpha + j\beta$. Thus, equating real parts, at high frequencies

$$\alpha = \frac{R}{2} \sqrt{\frac{C}{L}} + \frac{G}{2} \sqrt{\frac{L}{C}} \text{ nepers/kilometre.}$$

(b) As shown in part (a), the attenuation coefficient at high frequencies has 2 terms. The first term represents the series loss, and is proportional to R , and the second represents the shunt loss, proportional to G . At high frequencies, the effective resistance becomes proportional to the square root of the frequency due to the skin effect. In modern high-quality cables, the shunt loss is very small, so that the series loss becomes the main factor of the attenuation coefficient.

Therefore, in a high-quality coaxial cable, the attenuation coefficient at high frequencies is substantially proportional to the square root of the frequency.

Q 10 A cable pair has the following primary coefficients at an angular frequency, ω , of 5000 rad/s:

$$\text{loop resistance, } R, = 25 \Omega/\text{km},$$

$$\text{loop inductance, } L, = 1 \text{ mH}/\text{km},$$

$$\text{loop capacitance, } C, = 0.15 \mu\text{F}/\text{km}, \text{ and}$$

$$\text{loop leakage, } G, = 1 \mu\text{S}/\text{km}.$$

Calculate

(a) the characteristic impedance,

(b) the propagation coefficient, and

(c) the attenuation over a distance of 20 km.

A 10 (a) The characteristic impedance, Z_0 ohms, is given by

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \text{ ohms.}$$

$$\text{Now, } R + j\omega L = 25 + j \times 5000 \times 1 \times 10^{-3} \Omega,$$

$$= 25 + j5 \Omega,$$

$$= \sqrt{25^2 + 5^2} \angle \tan^{-1} \frac{5}{25} \Omega,$$

$$= 25.5 \angle 11.3^\circ \Omega.$$

LINE TRANSMISSION C, 1975 (continued)

$$\begin{aligned} \text{Also, } G + j\omega C &= 1 \times 10^{-6} + j \times 5000 \times 0.15 \times 10^{-6} \text{ S,} \\ &= (1 + j750) \times 10^{-6} \text{ S,} \\ &= 750 \times 10^{-6} \angle 89.9^\circ \text{ S.} \\ \therefore Z_0 &= \sqrt{\left(\frac{25.5}{750 \times 10^{-6}}\right) \angle \frac{11.3^\circ - 89.9^\circ}{2}} \Omega, \\ &= 184.4 \angle -39.3^\circ \Omega. \end{aligned}$$

(b) The propagation coefficient, γ , is given by

$$\begin{aligned} \gamma &= \sqrt{\{(R + j\omega L)(G + j\omega C)\}}, \\ &= \sqrt{(25.5 \times 750 \times 10^{-6}) \angle \frac{11.3^\circ + 89.9^\circ}{2}}, \end{aligned}$$

$$= 0.1383 \angle 50.6^\circ.$$

(c) Since $\gamma = \alpha + j\beta$, where α is the attenuation coefficient (nepers/kilometre), and β is the phase-change coefficient (radians/kilometre),

$$\alpha = 0.1383 \cos 50.6^\circ = 0.0878 \text{ nepers/km.}$$

Hence, the attenuation over a distance of 20 km

$$\begin{aligned} &= 0.0878 \times 20 = 1.756 \text{ nepers,} \\ &= 1.756 \times 8.686 \text{ dB,} \\ &= 15.3 \text{ dB.} \end{aligned}$$

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