Solving Soma Cube and Polyomino Puzzles Using a Microcomputer

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The genesis of this article was an inexpensive puzzle consisting of twelve plastic pieces which are supposed to be fitted into a rectangular cardboard box. Despite assurances by experts (see bibliography, Martin Gardner) that there are 2339 separate and distinct ways of solving the puzzle, a year’s work by a veritable platoon of people (mainly Yekta) produced only slightly more than 150 solutions.

Introduction

Polyomino puzzles and Soma Cubes are examples of a class of problems which are particularly suited to solution on a small computer. The amount of data needed in each case is relatively small, but the amount of calculation needed to do an exhaustive search for solutions is staggering.

For a set of Pentominoes, for instance, you need only compute the shapes of the twelve pieces and provide an array of sixty spaces into which you try to fit them. For a Soma Cube there are only seven pieces, which fit into an array of twenty-seven spaces. In both cases, all of the necessary data will easily fit into 2 K bytes of memory. However, the number of individual situations that would have to be considered in an unoptimized exhaustive search would be $3.2 \times 10^{11}$ for the Pentomino puzzle and $4.7 \times 10^{19}$ for the Soma Cube.

In this article, we will present a 6502 assembly language program which will solve a wide variety of puzzles of the sort where a given region, either two or three dimensional, must be filled with a given set of pieces. The program has been written in a general manner so that the shape of the region can be easily changed and certain pieces can be specified as fixed, in order to take advantage of symmetry. The number and shape of the pieces themselves can also be easily changed.

Due to a clever search method, the program given here actually considers many fewer cases than the unoptimized search mentioned above. Using a Commodore PET with a clock frequency of 1 MHz, most of the problems for which we have generated a complete set of solutions have taken from a few minutes to a few hours to run. The longest running problem we have considered, that of Pentominoes in a 10 by 6 rectangle, took slightly less than two days to generate all of the 2339 solutions.

If the program is run in BASIC, which we actually tried, this problem takes more than two months. The large difference in running speeds is due to the fact that BASIC on the PET is an interpreted language, each line of which must be decoded every time it is executed. This should serve as a caveat to anyone intending to write a BASIC interpreter version of this program.

The search algorithm used in the program is extremely general, as is illustrated by the fact that there are only three places in the assembly code where a check is made to see if the region under consideration is two or three dimensional. Thus the user should find it easy to modify the program to consider more complicated or exotic problems, such as those involving oddly shaped pieces or more than three dimensions.

The program given here written in the symbolic assembly language of the 6502 microprocessor, but users of other microprocessors should be able to adapt the fundamental algorithm to their own machines without much trouble. The accompanying BASIC routines are written in Commodore's version of BASIC (a Microsoft product), but they should also be easily adaptable to other machines. Since "safe" memory locations vary from machine to machine, users should be aware of the quirks of their own particular computer when they choose the addresses for the variables in the program.

Polyominoes

Polyominoes are planar objects consisting of a number of squares connected at their edges (see figure 1). The simplest such object is a monomino, which is just a single square. Next is the domino, consisting of two squares joined at a side, which has the shape of the familiar game pieces.