Gaussian-Response Filter Design

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FILTERS having a gaussian response shape are useful in pulse systems because of the small under- and overshoots or ringing they produce following rapid signal changes. This paper first gives a resume of the magnitude, phase, impulse response, step response, and effective thermal noise bandwidth of perfect gaussian filters. A physically realizable approximation to the magnitude characteristic is then given. The complex frequency roots of this approximation (up to the 9th order) are supplied. Then considered, in order of synthesis complexity, is the use of these roots to design stagger-tuned systems, lossless and uniformly lossy filters resistively loaded on one end only, and the same filters resistively loaded on both ends; all designs supplying nth-order gaussian magnitude approximations. Then, in order of synthesis complexity, is considered the use of these roots to design stagger-tuned systems, lossless and uniformly lossy filters resistively loaded on one end only, and the same filters resistively loaded on both ends; all designs supplying nth-order gaussian magnitude approximations. Numerical design data is supplied in 7 tables; a low-pass and a band-pass design example are given.

In many communication systems, particularly those involving pulses, it is desirable that the system response-magnitude-versus-bandwidth characteristic be gaussian in shape. This paper does not consider the system aspects of this problem, but it may be mentioned that some of the desirable qualitative results of using gaussian response shapes are very-small over- and undershoots and/or ringing following rapid signal changes, and the symmetrical pulse outputs that can be obtained when impulses are applied.

This paper is written to supply exact data that will enable successful design and construction of both low- and band-pass filters having magnitude-versus-frequency characteristics that are nth-order approximations to the perfect gaussian amplitude response, where n is the number of elements in the over-all filter. (With low-pass filters, the word elements stands for reactances; with band-pass filters, for resonators.) As will be shown in a later section, an infinite number of elements is required to produce a perfect gaussian response and thus it is always necessary to approximate this response when a finite number of elements are used.

1. Perfectly Gaussian Response Characteristics

For reference purposes this section gives resumes of five of the often-used characteristics of the perfect gaussian response.

1.1 Relative Attenuation Magnitude versus Bandwidth

Equation (1) gives the magnitude of the perfectly gaussian relative attenuation shape.

\[
\begin{align*}
|V_\text{p}/V| &= \exp \left( \frac{X}{X_{\text{ad}}} \right)^2 \\
&= \exp \left[ 0.3466 \left( \frac{X}{X_{\text{ad}}} \right)^2 \right],
\end{align*}
\]

where \( V_p \) is the peak output voltage of the filter occurring, in this case, at zero bandwidth, \( V \) is the output at any bandwidth \( X \), and \( X \) is the bandwidth variable (the actual radian frequency \( \omega \) in the low-pass case and the fractional total bandwidth \( BW/f_o = 2(f - f_o)/f_o \) in the symmetrical carrier-centered band-pass case). \( X_\beta \) is the normalizing bandwidth and, as usual, its meaning can be obtained from the equation in which it appears. For instance, a plot of (1) shows that when \( X = X_\beta \), the relative attenuation is 1 neper or 8.68 decibels. From this same relative-attenuation plot, it will be seen that \( X_\text{ad} = 0.588 X_\beta \) and after renormalization to the 3-decibel bandwidth, the second right-hand side of (1) results. Taking 20 log of both sides of (1) to obtain relative attenuation in decibels gives the very-simple and useful (1A).

\[
\text{Decibels} = 3 \left( \frac{X}{X_{\text{ad}}} \right)^2.
\]

At twice the 3-decibel bandwidth, the magnitude of the perfectly gaussian relative attenuation is 12 decibels; at 3 times the 3-decibel bandwidth it is 27 decibels; et cetera.

1.2 Relative Attenuation Phase (Phase Lag) versus Bandwidth

Based on the fact that a series with an infinite number of terms is required to represent the magnitude (1), it can be shown that perfectly